JOURNAL OF
The British Institution of Radio Engineers

(FOUNDED IN 1925—INCORPORATED IN 1932)

"To promote the advancement of radio, electronics and kindred subjects by the exchange of information in these branches of engineering."

Vol. XII (New Series) No. 7 JULY 1952

NOTICE OF THE TWENTY-SEVENTH ANNUAL GENERAL MEETING

NOTICE IS HEREBY GIVEN that the TWENTY-SEVENTH ANNUAL GENERAL MEETING (the nineteenth since Incorporation) of the Institution will be held on WEDNESDAY, OCTOBER 8th, 1952, at 6.30 p.m., at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.

AGENDA

1. **To confirm the Minutes of the 26th Annual General Meeting held on October 11th, 1951.** (Reported on pages 413-416 of Volume XI (New Series) of the *Journal* dated October, 1951.)

2. **To receive the Annual Report of the General Council.** (To be published in the September, 1952 *Journal.*)

3. **To elect the President.**
   The Council is unanimous in recommending the election of William E. Miller, M.A. (Cantab.) as President of the Institution for the year 1952/53.

4. **To elect the Vice-Presidents of the Institution.**
   The Council unanimously recommends the re-election of Leslie H. Paddle and the election of Rear Admiral (L) Charles P. Clarke, C.B., D.S.O., and Professor E. E. Zepler, Ph.D.

5. **To elect the General Council.**
   The retiring members of the Council are:—
   E. A. H. Bowsher (Member).
   H. E. Drew (Member).
   Professor E. E. Zepler, Ph.D. (Member).
   Lt.-Cdr. J. R. Christophers (Associate Member).
   H. A. Brookes (Associate Member).
   In addition, Sir Louis Sterling, D.Lit. (Hon. Member) has to retire because of ill-health.
   Consequently, under Article 29, vacancies arise for six ordinary members of Council as follows:— a maximum of three Members, a maximum of two Associate Members and one Honorary Member.
   In accordance with Article 32, the Council has nominated:—
   (*a*) Members for re-election: E. A. Bowsher and H. E. Drew.
   (*b*) Member for election: H. J. Leak.
   (*c*) Associate Members for election: Commander (L) H. W. Young, R.N., and Major S. R. Rickman.
   Any member who wishes to nominate a member or members for election to the Council must deliver such nomination in writing to the Secretary, together with the written consent of such person or persons, to accept office, if elected, not later than September 15th, 1952. Such nomination must be supported by not less than 10 corporate members.

6. **To elect the Honorary Treasurer.**
   The Council unanimously recommends the re-election of Mr. S. R. Chapman, M.Sc. (Member).

7. **To receive the Auditors' Report, Accounts and Balance Sheets for the year ended March 31st, 1952.**
   The Accounts for the General and other Funds of the Institution will be published in the September *Journal.*

8. **To appoint Auditors.**
   Council recommends the re-appointment of Gladstone, Jenkins & Co., 42 Bedford Avenue, London, W.C.1.

9. **To appoint Solicitors.**
   Council recommends the re-appointment of Braund & Hill, 6 Grays Inn Square, London, W.C.1.

10. **Awards to Premium and Prize Winners.**

11. **Any other business.** (*Notice of any other business must reach the Secretary* 40 *days before the meeting.*)

(*Members unable to attend the Annual General Meeting are urged to appoint a proxy.*)

ANGLO-FRENCH TELEVISION LINK

The first Anglo-French television exchange resulted in some 17 programmes being televised from France to Britain between July 8th and July 14th. Including the two TV transmitters in Paris, one in Lille and four in Britain, the programmes were transmitted simultaneously from seven stations—Europe's largest TV link to date.

During the week preceding the programme transmission, the President-elect (Mr. W. E. Miller), the Chairman of the Technical Committee (Mr. E. A. Bowsher), and the General Secretary were in a large British party which visited France for a preview of the technical arrangements.

Technical Arrangements

The television signals were sent from the Eiffel Tower to Senate House, University of London, in seven links, as follows:—

Paris-Lille (136 miles)

The 819-line vision signals were carried by the RTF experimental radio link with intermediate stations at Villers-Cotterets (44 miles north-east of Paris) and Sailly-Saillissel, near Peronne (50 miles from Villers-Cotterets and 42 miles from Lille). This radio link works on an approximate frequency of 900 Mc/s and is used by Radiodiffusion et Télévision Françaises to supply programmes to the Lille television transmitter until the permanent radio link now being installed by the French Post Office is ready.

Lille

The programmes were broadcast from the RTF transmitter at Lille for the benefit of viewers in that area.

Cassel

The Lille transmissions on 180 Mc/s* were picked up at Cassel by a special receiver, and RTF also installed a temporary radio link working on 9,000 Mc/s.* The signals thus received were fed to the converter developed by the B.B.C. Research Department, for changing pictures from French standards to British standards (819-405 lines). A demonstration of this method of conversion was given in April, 1952, when pictures originating in Paris were taken from Lille, the nearest point on the French television network, converted from the French standard to the British at Cassel, and transmitted by a series of radio links to London.

Cassel-Alembon (18 miles)

B.B.C. radio link on 7,000 Mc/s.*

The three remaining links, **Alembon-Swingate (Dover)** (40 miles), **Swingate-Wrotham** (49 miles), **Wrotham-London** (23 miles) operated on 4,500 Mc/s.*

On Tuesday, July 1st, delegates were given an official reception at the Eiffel Tower by Radiodiffusion et Télévision Françaises. Le Général Leschi (Directeur des Services Techniques) revealed that plans were being made for an exchange of television programmes between France and Italy, and France and Germany. The B.B.C. have also announced that next year they will collaborate in a relay to European countries of the Coronation television programme.

Conversion

There are at present four different television standards in use in Europe, each employing a different number of scanning lines. Thus, one of the major problems which may arise in relaying television programmes from one country to another is that of converting the signals from the standard of the first to that of the second.

The French are at present using two television systems: one operates on 819 lines, the other on 441 lines, although it is anticipated that only one system will be used as from 1955. For the present, two different programmes are transmitted by the two systems; but during the Anglo-French television week, RTF used a similar type of converter to that developed by the B.B.C. for converting the 819-line picture to the 441-line standard.

The basic idea of the present conversion process is very simple; it consists of displaying the picture with the original number of lines on the screen of a cathode ray tube which is re-scanned by a television camera operating on the number of lines normally used at the receiving end of the circuit. In developing the converter, a considerable amount of experimental work was carried out, and this showed that it was necessary to display the picture to be converted on a tube having a phosphor with a decay time comparable with that necessary to scan a single frame of the television image. By this means the camera views an image that is virtually continuous during one frame. The use of a phosphor with long persistence has the further advantage that the exposure time is increased in a greater signal output from the camera.

*Approximately.

NEW AMPLIFIER TECHNIQUES*

by

V. J. Cooper, B.Sc.(Eng.), A.C.G.I.†

A Paper presented at the Fifth Session of the 1951 *Radio Convention on August 22nd in the Cavendish Laboratory, Cambridge*

SUMMARY

Three new techniques in television amplifier design are discussed. These are:—

(a) The Cathode Repeater.

(b) Shunt-regulated Amplifiers.

(c) Feedback Amplifiers with desired amplitude frequency characteristics.

(a) The Cathode Repeater represents the first of three attempts to improve upon the performance of simple valve amplifiers at video frequencies and it is shown that by its use, the designer has choice of input and output impedance.

(b) Shunt-regulated Amplifiers are shown to have improved amplitude linearity, higher gain, improved frequency response and greater conversion efficiency than conventional amplifiers and are particularly attractive for television modulator applications.

Some types have the properties of positive, zero or negative output impedance, positive, infinite or negative input impedance, and are stabilized inherently against supply fluctuations.

(c) Feedback Amplifiers with desired amplitude frequency characteristics.

It is shown that by the use of complex feedback, amplifiers may have amplitude frequency response characteristics of: Aperiodic flatness; Maximal flatness; Optimal flatness.

It is shown that by the application of optimal feedback, stage gains comparable with those possible with 4-terminal coupling networks are easily achieved.

Practical circuits and experimental results are given for all the techniques described.

SYMBOLS

A transfer factor or "gain."

i_{a1} current fluctuation in valve 1.

i_{a2} current fluctuation in valve 2.

i_L current fluctuation in load.

k attenuation or amplification factor.

r_{a1} anode a.c. resistance of valve 1.

r_{a2} anode a.c. resistance of valve 2.

V_{g1} grid-cathode voltage fluctuation in valve 1.

V_{g2} grid-cathode voltage fluctuation in valve 2.

V_{a1} anode-cathode voltage fluctuation in valve 1.

V_{a2} anode-cathode voltage fluctuation in valve 2.

V_i input voltage fluctuation.

V_o output voltage fluctuation.

V_r supply voltage fluctuation.

Z_1, Z_2 etc. impedances.

Z_L load impedance.

Z_i input impedance; i.e., the ratio of fluctuation of input voltage to the resulting fluctuation of input current.

Z'_i input impedance at terminals other than the normal input terminals of the amplifier.

Z_o output impedance; i.e., the ratio of a fluctuation of output voltage, consequent upon a fluctuation of output current, to the fluctuation of output current.

* Manuscript received July 20th, 1951.
† Marconi's Wireless Telegraph Co., Ltd.
U.D.C. No. 621.397.645.37

Z_{o1} output impedance of valve 1.

Z_{o2} output impedance of valve 2.

Z_s source impedance.

R resistance.

C capacitance.

β fraction of output voltage returned to input to produce negative feedback. (The feedback fraction)

A_x external amplification.

A_o the value of A in the region where it does not vary significantly with frequency.

A_{xo} the value of A_x in the region, etc.

N feedback factor defined as $1 + A_o\beta$ or $1 + A_o\delta$.

a magnitude of the reciprocal of frequency response, i.e.

$$a = \left| \frac{A_o}{A_x} \right|$$

x ωCR

N' frequency conscious feedback defined as $A_o\gamma$

γ the fractional feedback due to the frequency conscious part of the feedback network.

$\beta' = \delta + j\gamma$.

δ the real part of β'

k, K constants.

X_c capacitive reactance $= \dfrac{1}{\omega C}$

THE CATHODE REPEATER*

Introduction

The use of a triode with the cathode as the signal input terminal has certain advantages for wide band amplifiers. It can be shown that under certain circumstances its input impedance can be purely resistive or can have a positive or negative susceptance.

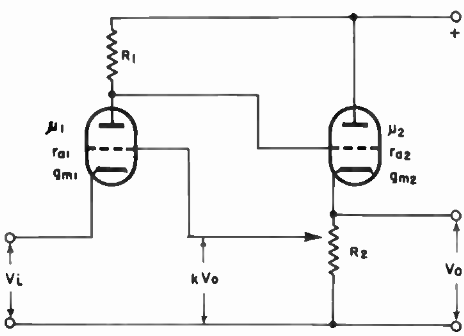


Fig. 1.—The basic cathode repeater circuit.

In combination with a cathode follower, a circuit has been devised which can amplify from a low impedance input into a low impedance output with considerable control of the input and output impedance.

The new circuit has been given the name of "cathode repeater."

In general form, the "cathode repeater" is illustrated in Fig. 1, from which it will be seen that it is the direct combination of a cathode input stage with a cathode output stage, i.e., an anode follower and a cathode follower with interstage feedback.

The voltages at input and output are in phase while the signal currents in the valves are in antiphase. It is, therefore, possible to arrange for the cathode repeater to present a constant load to the supply at all signal amplitudes.

From the analysis the output impedance is shown to be adjustable from approx. $\dfrac{1}{g_m}$ to $\dfrac{1}{Ag_m}$ where g_m is the mutual conductance of the output valve and A the gain of the input valve. The input impedance is adjustable from $\dfrac{r_a + R_1}{\mu + 1}$ without feedback to something approaching $r_a + R_1$ with full feedback, R_1 being the anode resistor in the input valve circuit.

Analysis

The basic circuit is shown in Fig. 1.

It may be shown that for this circuit

$$\text{Gain } \frac{V_o}{V_i} = \frac{(\mu_1 + 1)(\mu_2 R_2 R_1)}{k\mu_1\mu_2 R_1 R_2 + (r_{a2} + R_2\overline{\mu_2 + 1})(r_{a1} + R_1)}$$

* The greater part of this section is abstracted from Ref. 1.

Output impedance

$$Z_o = \frac{1}{g_{m2}\left(kA_1 + \frac{\mu_2 + 1}{\mu_2}\right)}$$

where A_1 is the gain of the 1st stage as an ordinary grid input amplifier.

i.e. $A_1 = \frac{\mu_1 R_1}{R_1 + r_{a1}}$

Input impedance

$$= \frac{R_1 + r_{a1}}{\mu_1 + 1 - \mu_1 k A_2}$$

where A_2 = overall gain $= \frac{V_o}{V_i}$

In practice these simple expressions are complicated by the source impedance of driving stage.

When the source resistance is R_s the properties are

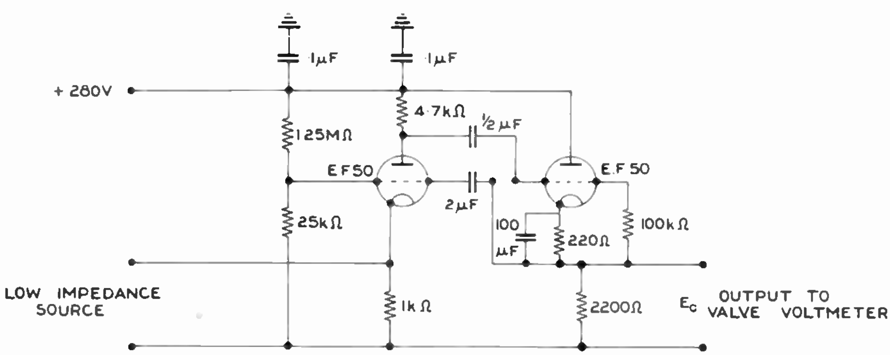
Gain

$$\frac{V_o}{V_i} = \frac{(\mu_1 + 1)(\mu_2 R_1 R_2)}{k\mu_1\mu_2 R_1 R_2 + \left(r_{a2} + R_2\overline{\mu_2 + 1}\right)\left(r_{a1} + \overline{\mu_1 + 1}\, R_s + R_1\right)}$$

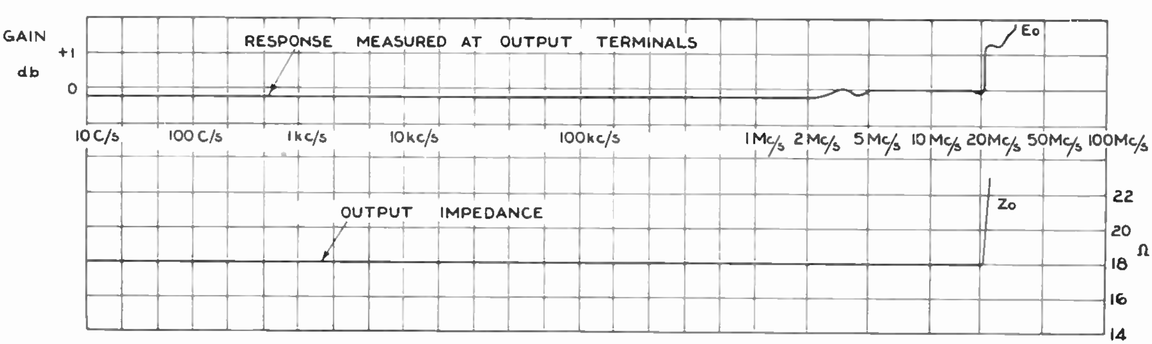
Output impedance

$$Z_o = \frac{1}{g_{m2}\left(kA_3 + \frac{\mu_2 + 1}{\mu_2}\right)}$$ where A_3 is the gain of the first valve reduced by the negative feedback resistance R_s. Using the circuit shown in Fig. 2*a*, the amplitude frequency characteristic and the output impedance frequency characteristic are shown in Fig. 2*b*. The control of output impedance is shown by the measured value using the amplifier of Fig. 3 (*a*) as plotted in Fig. 3 (*b*).

Further design details are disclosed in Ref. (1).



(*a*)



(*b*)

Fig. 2.—Response curve and output impedance of "cathode repeater" using EF50 valves.

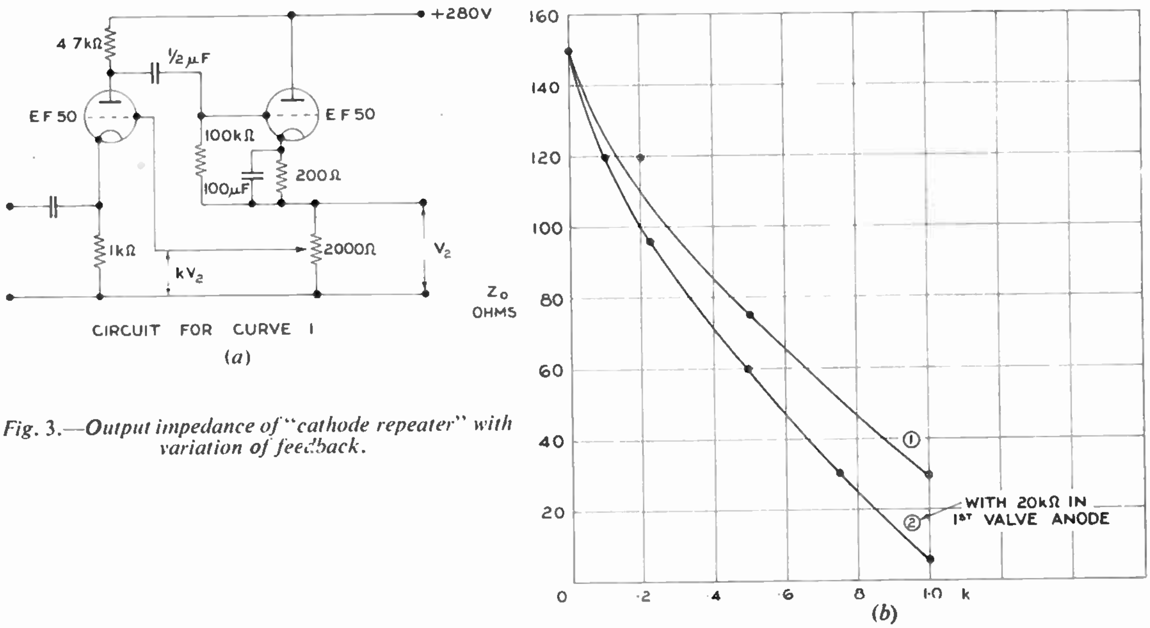


Fig. 3.—Output impedance of "cathode repeater" with variation of feedback.

SHUNT-REGULATED AMPLIFIERS*

Introduction

The development of shunt-regulated amplifiers arose because of the difficult conditions encountered when modulating television transmitters at high level.

In general terms a high-level video-frequency modulator must have a wide frequency response, and must be capable of maintaining this response across a load consisting of a large shunt capacitance and a shunt resistance which, due to the non-linear flow of grid current will vary from a very high value to a very low value with the amplitude of the signal.

In the particular case of a 50-kW transmitter, grid modulated in its final amplifier, the modulator must produce with good linearity and good frequency response a voltage output of 1,100 V peak-to-peak across the load presented by the grid-modulated amplifier. This consists of a capacitance of 500 pF shunted by a resistance, due to the grid-current load, falling from a resistance of infinity at zero volts to a resistance that may be as low as 400 ohms.

Also, whereas the reactive loading is a direct function of frequency, the grid-current loading is a function of amplitude and can be a maximum at any frequency at which full amplitude components are present.

General Approach

We may conceive the general problem as consisting of two parts:—

1. Providing the required voltage excursion into a substantially constant load.
2. Providing a regulating system connected in association with the output terminals of 1 to ensure that the voltage excursions across the load are a faithful reproduction of the excursions across the output terminals of 1 despite the load variations of current, either resistive or reactive.

Two ways of fulfilling these requirements suggest themselves and are illustrated in Fig. 4. Various series arrangements (*a*) are possible, but none appears particularly attractive for the problem in hand, but there are some useful shunt forms (*b*).

The performance of any such system is completely specified by the input and output impedances Z_i and Z_o and the transfer factor; i.e., the "gain."

The output impedance Z_o is defined as the ratio of a change of output voltage to the initiating change of output current, while the

* The greater part of this section is abstracted from Ref. 2.

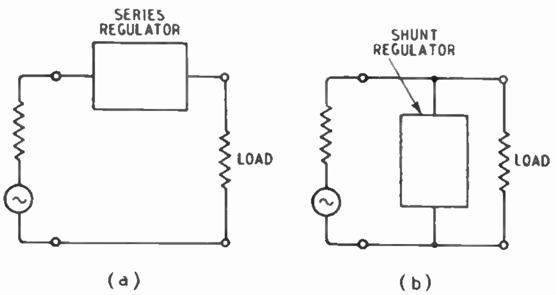


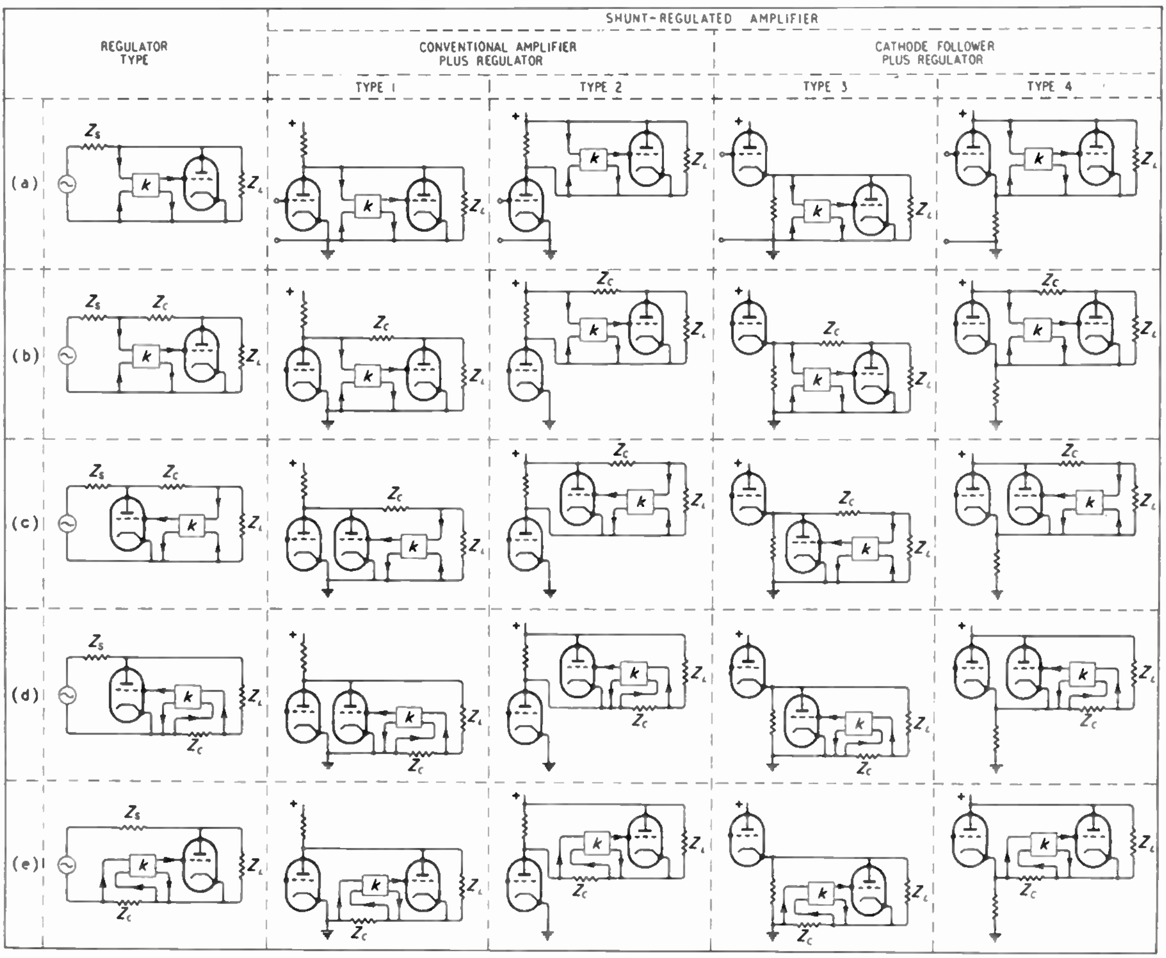
Fig. 4.—Basic regulator circuit: (a) series, and (b) shunt.

input impedance Z_i is the ratio of an input voltage fluctuation to the consequent input current fluctuation.

The transfer factor is the ratio of a voltage fluctuation at the output to the initiating voltage fluctuation at the input. For the purpose in hand we need to find regulators with transfer factors approaching unity and with output impedances significantly lower than the source impedances.

In Table 1 are shown the basic types of shunt regulator in association with amplifier and cathode-follower sources. This table is not

TABLE 1



In these diagrams rectangle k represents an amplifier or attenuator of output/input voltage ratio k, with or without a phase reversal according to the requirements of the particular circuit.

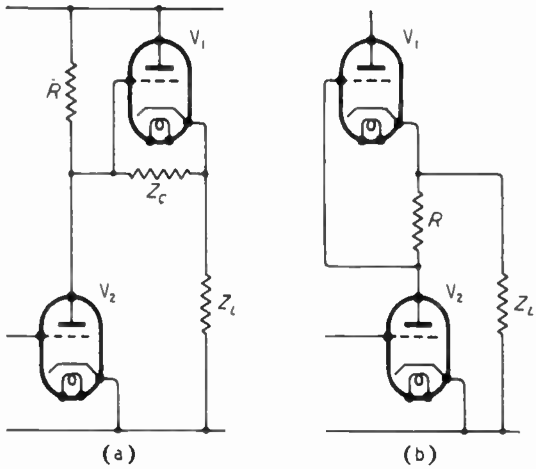


Fig. 5.—Circuit of shunt-regulator Type 2(e): (a) general form; (b) simplified form for similar valves.

intended to be exhaustive, but to typify the approach, and an examination of these combinations reveals interesting possibilities. An examination of these arrangements reveals that those using valves in series are more attractive than those using valves in parallel. This was foreseen by Rudolf Urtel[3] in 1935, and later in a different form by N. H. Clough[4] in 1940.

Shunt-regulated Amplifier Type 2 (e)

This type shown in Table 1 as Type 2 (*e*) shows very considerable advantages over normal technique. Since A can approach unity with k unity and Z_c small, we can simplify the arrangement to the form of Fig. 5 (*a*). Now if Z_c approaches infinity the regulator valve V1 is a conventional cathode-follower and supplies substantially all the load demand of current. On the other hand, if $Z_c = 0$ the regulator valve acts merely as a resistance and makes no individual contribution of current to the load, all the load now being transferred to the first valve. The intermediate conditions are interesting.

Since the excitation of valve V1 and therefore its contribution of current to the load are directly proportional to the current in its grid-to-cathode impedance Z_c, the two valves V1 and V2 can be made to share the non-linear resistive load currents and we may conceive the valve V1 as a current "booster."

From Fig. 5 (*a*) we can say that V1 supplies current in accordance with the demands of the load and it does this because of the current-monitoring impedance Z_c. Also, as the signal current output of the valve V2 increases, the current in the valve V1 tends to decrease and a partial current-balancing effect occurs.

A further simplification is possible where the valves are similar, for then their mean currents can be made equal; the circuit can then be reduced to that of Fig. 5 (*b*). The output impedance of this arrangement lies between the a.c. resistance r_a of the valve V2 and $1/g_m$ of the valve V1 depending upon the value of the resistor R. The operation of this system can be explained quantitatively by reference to the characteristics of the valves which, for simplicity, are taken to be similar.

Graphical Analysis

Typical characteristics for type ACM3 valves are shown in Fig. 6. Assuming a given instantaneous current flowing through the system and a given H.T. voltage, we can derive the grid-cathode excitation of the valve V1 by taking the product of the current and the resistance R.

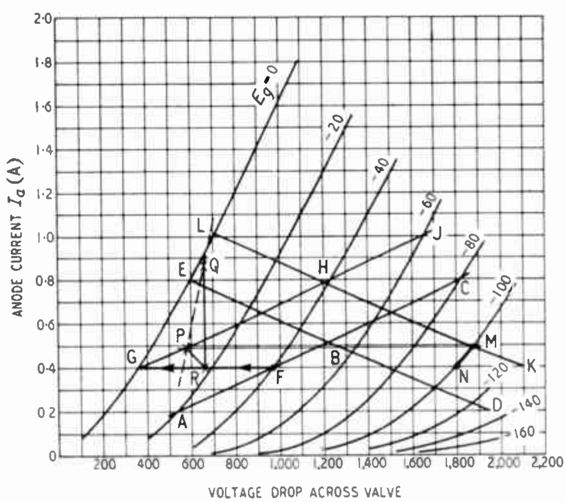


Fig. 6.—Characteristics of ACM3 valve with load lines drawn for a shunt-regulated amplifier Type 2(e).

This settles the voltage drop V_{a1} across the valve V1 because both current and grid voltage are known and, therefore, the voltage V_{a2} across the valve V2. Current and voltage relations in the valve V2 are then known and the voltage excursions for the two valves can be drawn.

Assuming an H.T. supply of 2,500 V and R = 100Ω then $V_{a2} = 2{,}500 - iR - V_{a1}$.

TABLE 2

i	iR	V_{a1}	V_{a2}
0·2	20	530	1,950
0·4	40	980	1,480
0·6	60	1,420	1,020
0·8	80	1,820	600

The anode excursion of valve V1 is shown in Fig. 6 by the plot ABC and that of valve V2 by plot DBE. It is clear from these plots that the valve V2 works into a normal load condition determined by the slope of the line DBE and the effective anode load is $r_a + (\mu + 1) R$.

If now the lower limit of current excursion is decided, say, point F on curve ABC, this value of current can be made to coincide with the $E_g = 0$ characteristic (for class A operation) for the valve V1 by adding bias. This moves point F to point G and transposes both load lines to the positions shown by the plots GHJ and KHL. It is approximately equivalent to increasing the H.T. voltage by μ times the bias applied to effect the transposition. In this condition the a.c. resistance of the valve V1 with its feedback resistor remains constant at the previous value of $r_a + (\mu + 1) R$ while the d.c. resistance is appreciably reduced. The linearity of the voltage excursion is also considerably improved by this transposition, as can readily be seen in Fig. 6.

When working into a reactive load as in Fig. 7, the capacitor C is charged by a rising voltage at the output terminal Y. This corresponds to a falling current in valve V2.

With the valve V1 replaced by the conventional resistor the total charging current must flow in the circuit of valve V2, and the maximum charging rate is determined by the magnitude of the current change in V2. With V2 operative, however, the voltage at the point Y tends to rise more rapidly since additional charging current can be supplied by the valve V1, which acts partially as a cathode follower.

The output impedance and the sharing of the load-current demands by the two valves V1 and V2 can be found quantitatively from the usual form of circuit analysis. The results only are quoted here. The output impedance is easily shown to be

$$Z_o = \frac{r_{a1}(r_{a2} + R)}{r_{a2} + R(\mu_1 + 1) + r_{a1}} \quad \ldots\ldots(6)$$

and is equivalent to two current generators in parallel of output impedances

$r_{a2} + R$ (Valve V2) and $\dfrac{r_{a1}(r_{a2} + R)}{r_{a2} + (\mu_1 + 1) R}$ (Valve V1)

and the non-linear resistive and reactive load currents are supplied from the two equivalent generators in the inverse ratio of the generator impedances.

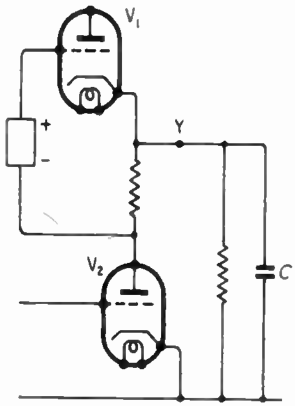


Fig. 7.—Shunt-regulated amplifier Type 2(e) with capacitive load circuit.

For the ACM3 valves taken to illustrate the system, the ratio becomes approximately 4 to 1 for a value of R of 100Ω; i.e. 4/5 of the load current is supplied by the "booster" valve. The output impedance, using the same numerical values, is about 150 ohms and this reduced output impedance may be demonstrated graphically with the aid of Fig. 6.

Assume that a load has been applied at the output terminal Y and a voltage drop occurs, causing the current in the resistance R and the valve V2 to fall. Let this fall of current be 0·095 A from point M to point N. The drop of voltage at the anode of valve V2 is then 75V, and so the drop of voltage at the cathode of valve V1 is 75 + 100 × 0·095 = 84·5V. The 9·5V drop across the resistor R changes the grid-cathode voltage of valve V1 from −10 to −0·5V.

The new current in valve V1 is thus approximately 0·88A, as shown by the construction

PRQ, the point Q being 84·5V higher than point P and at a grid-cathode voltage 9·5V more positive. The current flowing into the load is thus

$$QR = 0{\cdot}88 - 0{\cdot}4 = 0{\cdot}48\text{A}$$

and the output impedance is

$$\frac{84{\cdot}5}{0{\cdot}48} = 176\Omega$$

Therefore, the load division ratio between valves is

$$= \frac{0{\cdot}48 - 0{\cdot}095}{0{\cdot}095} = \frac{0{\cdot}375}{0{\cdot}095} = 4 : 1$$

This checks roughly with the theoretical figures obtained from the foregoing equation by substituting values of r_a and μ obtained from the valve characteristic at the appropriate points; i.e. $r_{a_1} = 600\Omega$, $r_{a_2} = 800\Omega$, $\mu = 16{\cdot}5$, $Z_o \simeq 171\Omega$, load division $\sim 4{\cdot}1$. The agreement is within the tolerance of estimation from the curves.

Regarded as a single equivalent valve, the characteristics for $E_g = -100$V pass through the points PQ. The usual limitations associated with current cut-off with reactive loading are reduced since the load lines are raised well away from the zero-current region. It is interesting to note that the voltage obtainable at the anode of the valve V2 is higher than at the cathode of valve V1 by the iR drop in the resistor R. In practice, therefore, one or other output terminal can be used depending upon the relative importance of output impedance and output voltage.

The gain of the shunt-regulated amplifier of this type is given by

$$\frac{V\,out}{V\,in} = \frac{\mu_2\,(r_{a_1} + \mu_1 R)}{r_{a_1} + r_{a_2} + (\mu_1 + 1)R} \qquad (7)$$

which, for the practical values, already taken is approximately $0{\cdot}75\mu$.

A direct comparison between the properties of two ACM3 valves arranged as above, and arranged conventionally as two valves in parallel to give the same gain, is given in Table 3.

TABLE 3

	Conventional	Shunt Regulated
ACM3 taken as $\mu = 15$, $r_a = 600\Omega$.		
Valves	2 ACM3 in parallel	2 ACM3 in series
Gain	11	11
Output voltage (p-p) ..	1,000	1,000
Resistor	825 Ω	100 Ω
Watts dissipated in resistor	2·1 kW	64 W
Input capacitance (approx.)	400 pF	200 pF
Departure from amplitude linearity in response to a sawtooth waveform	6% approx.	1% approx.
H.T. required	2,900 V	2,500 V
Mean current	1·6 A	0·8 A
H.T. power input ..	4,640 W	2,000 W
Output impedance ..	228 Ω	175 Ω
Current fluctuation in supply for 1,000-V swing	1·2 A	0·6 A

Shunt-regulated Amplifier Type 3 (*e*).

The same form of regulator Type (*e*) but associated with a cathode-follower source, as in Table 1, arrangement 3 (*e*), is also of particular interest.

Simplified as for Type 2 (*e*) we get the arrangement of Fig. 8, the current-monitoring resistor being transferred for convenience to the anode side of the cathode-follower source valve to get a fractional value of k and a phase reversal. The output impedance is

$$Z_o = \frac{(r_{a1} + R)\,r_{a2}}{(\mu_1 + 1)\,r_{a2} + r_{a1} + \left[(\mu_1 + 1)\mu_2 + 1\right] R} \qquad \ldots\ldots\ldots\ldots\ldots(8)$$

and is composed of $\frac{r_{a1} + R}{\mu_1 + 1}$ due to the cathode-follower valve in parallel with

$$\frac{r_{a2}\,(R + r_{a1})}{r_{a1} + \left((\mu_1 + 1)\mu_2 + 1\right) R};$$

and in the limit when $R \gg r_{a1}$

$$Z_o \text{ becomes } \frac{1}{(\mu_1 + 1)\,g_{m2}}$$

In practice this order of improvement is not possible, but very worth-while reductions of output impedance are easily achieved.

Taking a pair of ACM3 valves, as before, with 100 Ω as the control resistor the output impedance becomes approximately 10 Ω as compared with $1/g_m$ for this valve of approximately 36 Ω. As for the amplifier arrangement, the

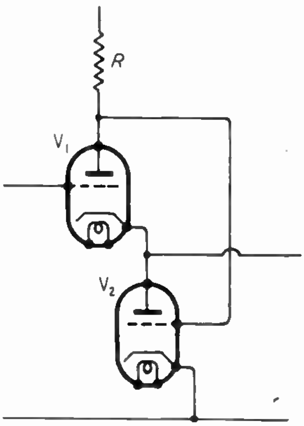


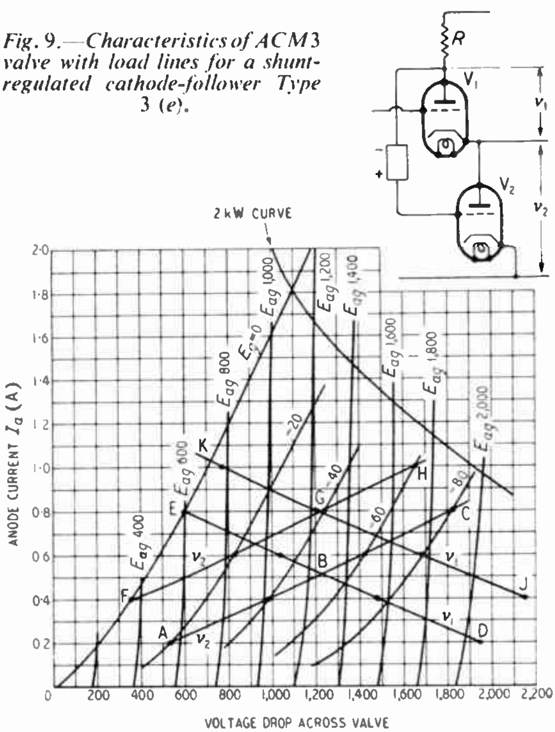
Fig. 8. — Shunt-regulated cathode-follower Type 3(e).

operation can be studied graphically. This is done in Fig. 9 for two ACM3 valves.

As before, assuming a given instantaneous current flowing through the system with no load connected to the output terminals, and a given H.T. supply voltage, we can derive the grid-cathode excitation of the valve by taking the product of the current and the resistance *R*. This settles the voltage drop across the valve V2 and, therefore, the voltage across the valve V1. Current and voltage excursions for the two valves can then be drawn.

This is demonstrated in Fig. 9 by the excursions ABC and DBE. The addition of positive bias to the valve V2 will transpose the excursions to FGH and JGK. It should be noted that the positive bias is only relative to the signal circuit and that, in fact, the potential of the grid of the valve V2 will be very much less positive than the anode of the valve V1 when all the mean d.c. components are added.

Fig. 9.—Characteristics of ACM3 valve with load lines for a shunt-regulated cathode-follower Type 3 (e).



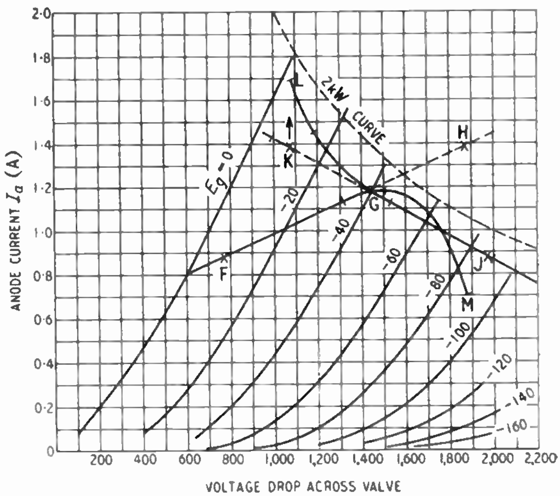


Fig. 10.—Characteristics of ACM3 valve with curved load lines due to grid current in the following stage; shunt-regulated cathode-follower Type 3(e).

The load lines drawn in Fig. 9 show the excursions when there is no current demand by the load. If such an arrangement is used to grid modulate a transmitter, the non-linear grid current loading changes the shape of the "load lines" as in Fig. 10. The ordinate amplitude HM represents the share of grid current delivered by the "regulator" valve; the ordinate KL represents the share of grid current delivered by the cathode-follower valve, the total grid current being represented by the vertical distance between L and M.

It will be seen that the curvature of the load line FGM has been used to raise the mean current of the valves to permit the maximum possible margin for reactive current.

A comparison in Table 4 is made between the shunt-regulated cathode-follower described and a conventional cathode-follower designed to give the same output voltage swing with the same reactive-current handling capacity.

TABLE 4

	Conventional	Shunt Regulated
ACM3 taken as $\mu = 14$, $r_a = 600\Omega$.		
Valves	2 ACM3 in parallel	2 ACM3 in series
Output voltage (p-p) ..	1,000	1,000
Gain	0·91	0·91
Resistor	875Ω	100Ω
Watts dissipated in resistor	5 kW	140 W
Output impedance ..	18Ω	10Ω
Mean current	2·4 A	1·2 A
H.T. required	3,480 V	3,000 V
H.T. power input ..	8·3 kW	3·6 kW

Two further types are of special interest and are shown in Figs. 11 and 12, and are Types 2(*e*/*e*) and 3(*e*/*e*) respectively.

These types can be analysed graphically in a manner similar to that already adopted for the simpler types and all the properties of Types 2(*e*) and 3 (*e*) are present.

Additional properties of considerable value are also found.

It can be shown that the input impedance of these types is of the form

$$Z_i = \frac{Z_v}{Z_w + \dfrac{Z_x - Z_y}{Z_L}}$$

where Z_v, Z_w, Z_x and Z_y are expressions involving the valve and circuit parameters and Z_L is the load impedance.

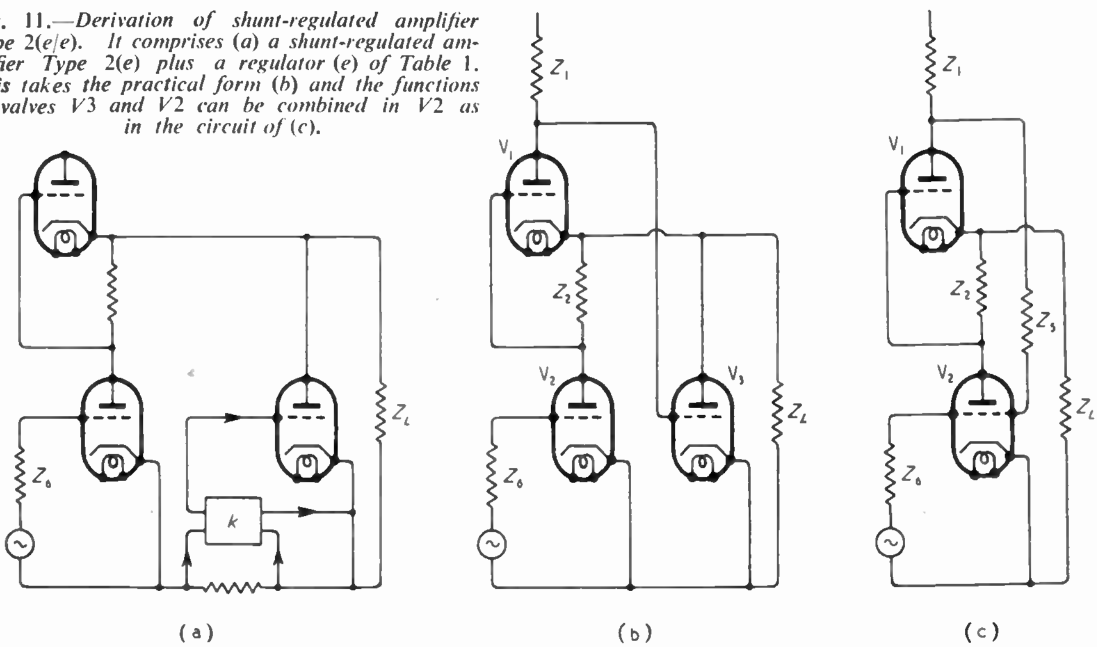
It is thus possible to reflect an impedance from the load into the input circuit with change of sign if desired or to make the input impedance infinite.

The condition for infinite input impedance is usually where $Z_L < r_a$.

If the capacitive component of the load fulfils the condition for negative input capacitance while the resistive component of the load maintains a satisfactory input resistance then the overall effect on the input circuit is to improve its frequency response, the demand for reactive current by the input circuit now being supplied from the output.

Curves showing the properties of these two types are shown in Figs. 13 and 14.

Fig. 11.—Derivation of shunt-regulated amplifier Type 2(e/e). It comprises (a) a shunt-regulated amplifier Type 2(e) plus a regulator (e) of Table 1. This takes the practical form (b) and the functions of valves V3 and V2 can be combined in V2 as in the circuit of (c).



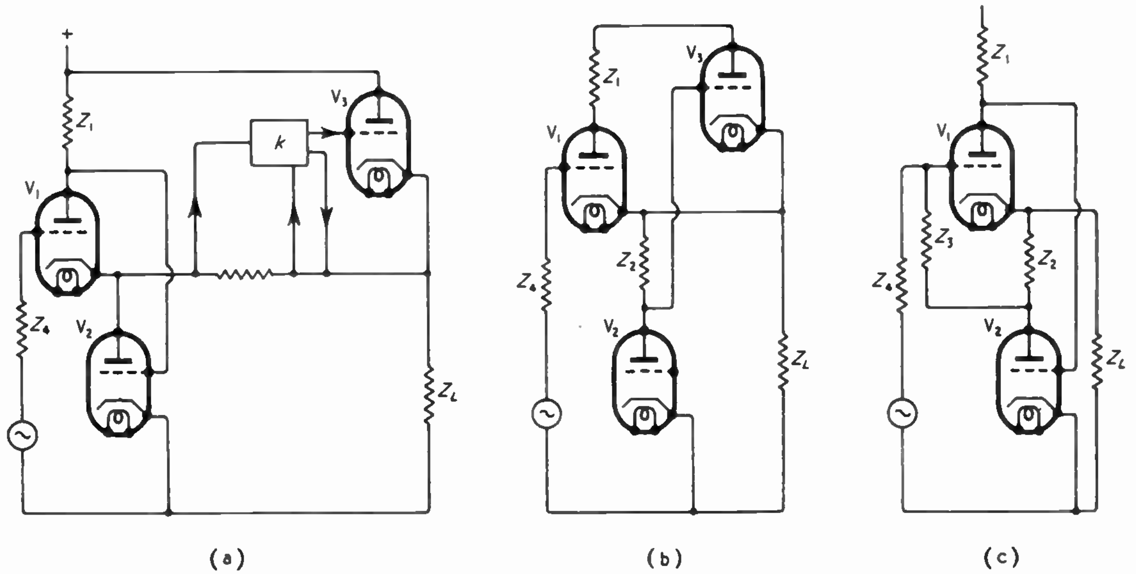


Fig. 12.—*Derivation of shunt-regulated cathode-follower Type 3(e/e). It comprises a shunt-regulated cathode-follower 3(e) plus a regulator (e) of Table 1. The functions of V1 and V3 can be combined in V1 as in the circuit (c).*

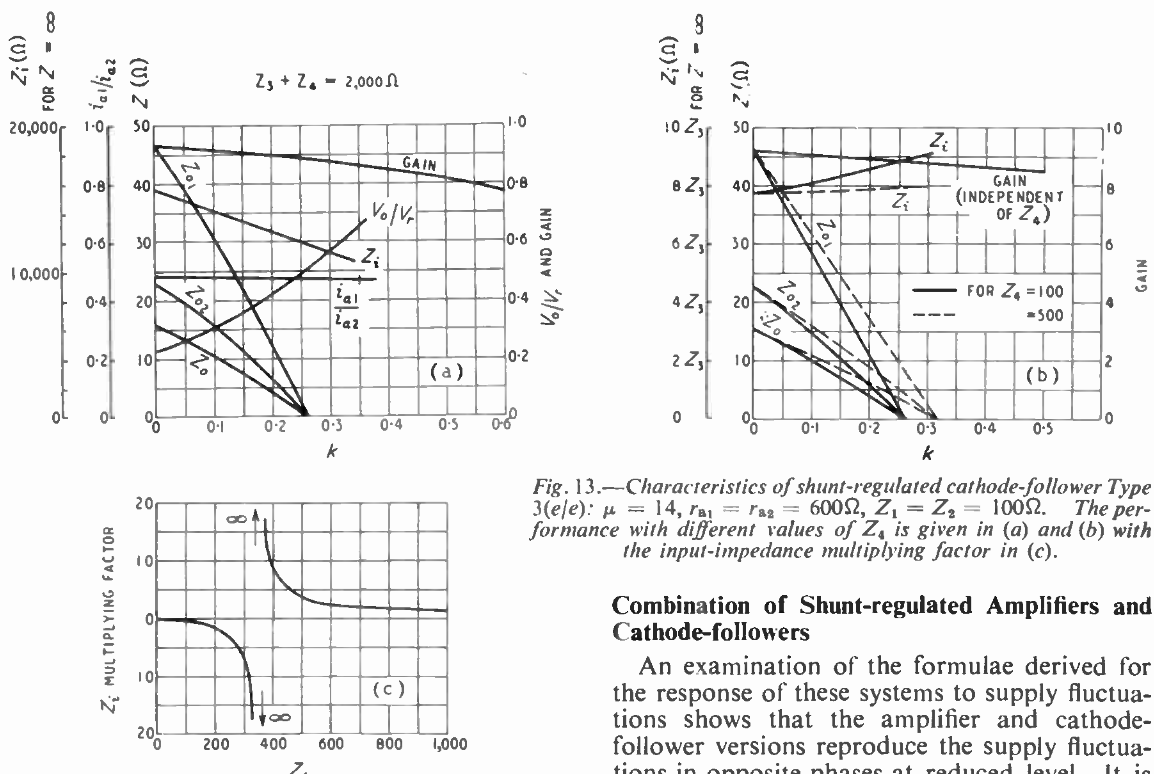


Fig. 13.—*Characteristics of shunt-regulated cathode-follower Type 3(e/e):* $\mu = 14$, $r_{a_1} = r_{a_2} = 600\Omega$, $Z_1 = Z_2 = 100\Omega$. *The performance with different values of* Z_4 *is given in (a) and (b) with the input-impedance multiplying factor in (c).*

Combination of Shunt-regulated Amplifiers and Cathode-followers

An examination of the formulae derived for the response of these systems to supply fluctuations shows that the amplifier and cathode-follower versions reproduce the supply fluctuations in opposite phases at reduced level. It is thus possible, in practice, to proportion the operating conditions so that a shunt-regulated amplifier, in combination with a shunt-reguated cathode-follower, will attenuate supply fluctuations to zero at the output terminal. Even if this precise adjustment is not made, the reduction of the effect of supply fluctuations is sufficient to reduce the degree of supply voltage stabilization required.

Applying the numerical values r_{a1}, μ_1, Z_1, etc., used to illustrate the properties, we find the following responses to supply fluctuations

Type	*Response*
2(*e*)	0·26
3(*e*)	−0·25

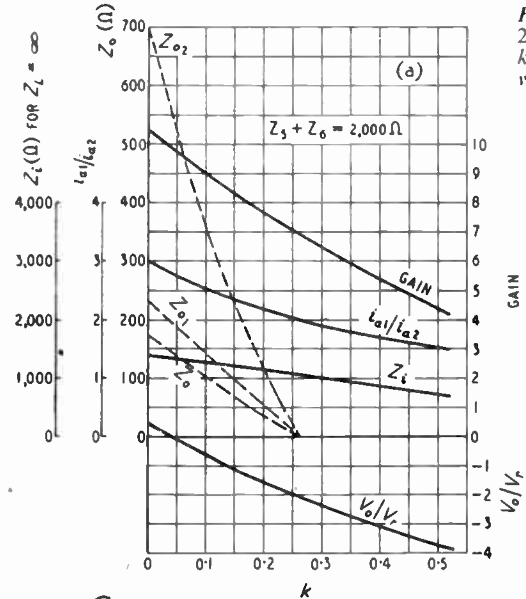
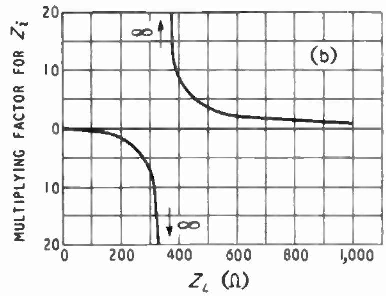
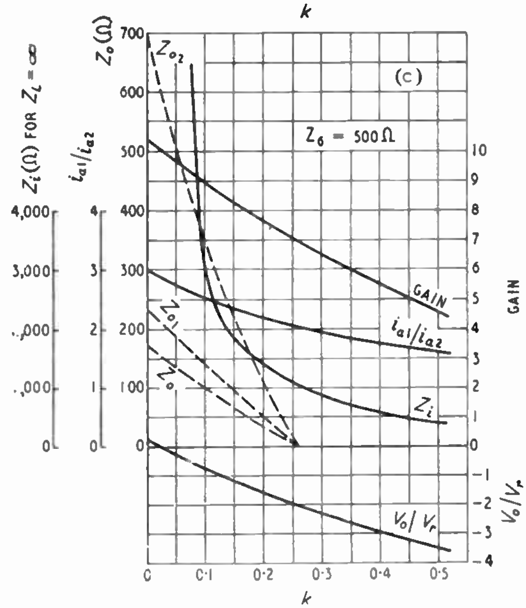
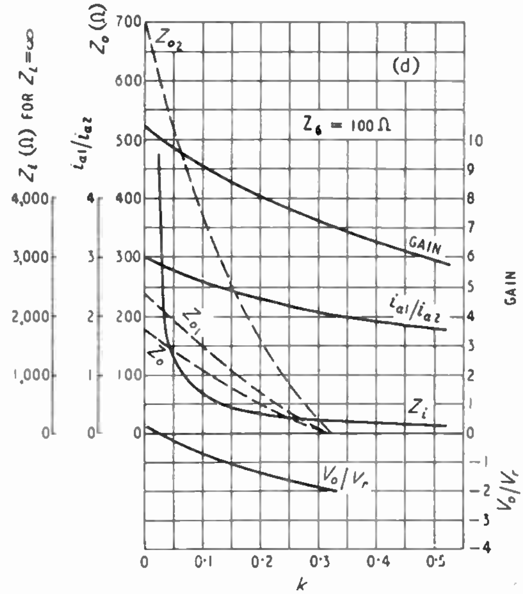


Fig. 14.—Characteristics of shunt-regulated amplifier Type 2(e/e); $\mu = 14$, $r_{a1} = r_{a2} = 600\Omega$, $Z_1 = Z_2 = 100\Omega$, $k = Z_6/(Z_5 + Z_6)$. The performance for three different values of Z_6 is given in (a), (c) and (d) while (b) gives the multiplying factor for a finite load impedance.







Therefore, a combination of a shunt-regulated amplifier and a shunt-regulated cathode-follower will, with practical values of regulating impedance, give almost complete isolation from supply fluctuations.

In the case of Types 2(*e*/*e*) and 3(*e*/*e*) shunt-regulated amplifiers in combination, the balancing out of supply-voltage fluctuations is also possible and the necessary conditions can be read from the curves of Figs. 13 and 14.

Practical Applications

As a demonstration of the impedance reversing properties of the shunt-regulated amplifier, Fig. 15 shows the response of the arrangement of Fig. 16 for different values of capacitance added across the output terminals.

The circuit shown is exactly that of the experimental model and does not represent a good finished design. It will be noted that the feedback from the lower output valve has been taken via the amplifier-coupling circuit for practical convenience and that no attempt has been made to deal with low-frequency response, the sole purpose of the experiment being to verify the impedance reversing property at the higher frequencies.

It should also be noted that no attempt has been made in the circuit to achieve either good intrinsic frequency response or good response in the feedback loop. It was considered satisfactory, however, that this test under these conditions adequately verified the theory.

A combination of the techniques described in this article has been incorporated at low and high levels in a recently completed modulator for a 50-kW television transmitter and a comparison of its performance with the estimated performance of a conventional modulator is made in Table 5.

Frequency Compensation in Shunt-regulated Amplifiers

As an indication of what can be done with inductance compensating the feedback resistor (Fig. 17), the change of response is shown in Fig. 18 for compensation in this one circuit for maximal flatness.

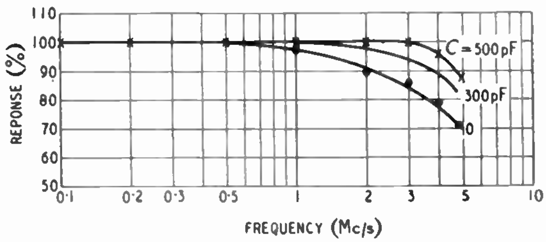


Fig. 15.—Effect of load capacitance on the frequency response of the shunt-regulated amplifier of Fig. 16.

The natural frequency response of the Type 2 (*e*) shunt-regulated amplifier is typified by curve A, Fig. 18, and is not a natural RC law.

An inductance included with the coupling resistor corrects the law and produces, for all practical purposes, a natural RC law.

This is shown in curve B. This first degree of compensation removes substantially all the load capacitance from the input valve, the RC law being determined by the inherent capacitance

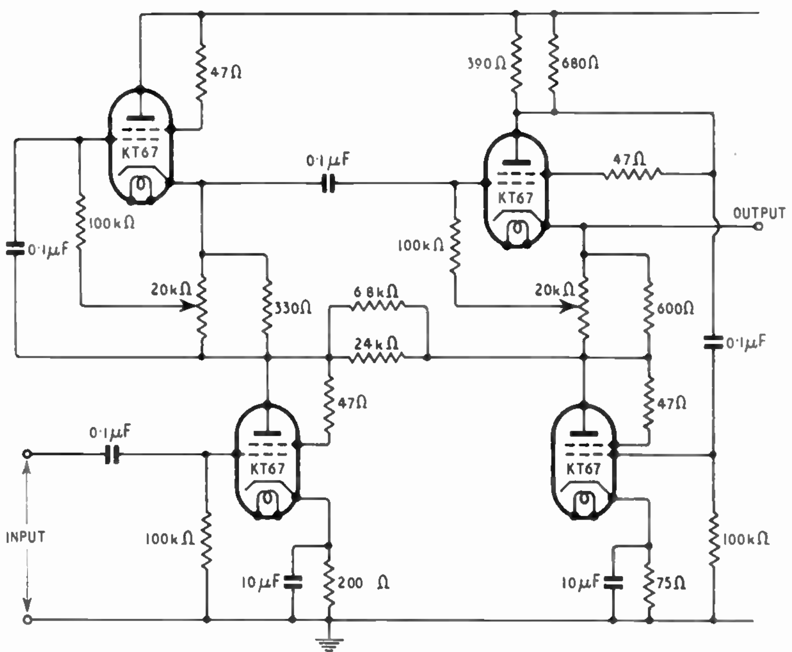


Fig. 16.—Experimental shunt-regulated amplifier.

TABLE 5

	Conventional (estimated)	Shunt Regulated (measured)
Total number of ACM3 valves (including stabilization)	16	11
Frequency response ..	±5% to 4 Mc/s	±5% to 7 Mc/s
Total power input including H.T. and fils and air cooling ..	90 kW	36 kW
Rise time of output step with 0·07 μsec rise input step	0·14 μsec 5% overshoot	0·09 μsec* no overshoot
Max. voltage swing ..	1,250	1,250
Non-linearity for transmitter grid current of 3 A	7-10%	<1%

* On the assumption that the input step is exponential this gives an indicial response of <0·05 μsec.

in the circuit. Further compensating networks on the same two-terminal system produces the curve C. (Fig. 17).

With inductance compensation taken to the limit, the 50-kW modulator quoted has in its laboratory form a response flat ±5 per cent to 11 Mc/s.

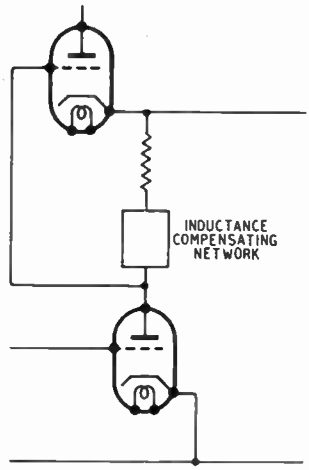


Fig. 17.—Method of inductance-compensating the feedback resistor.

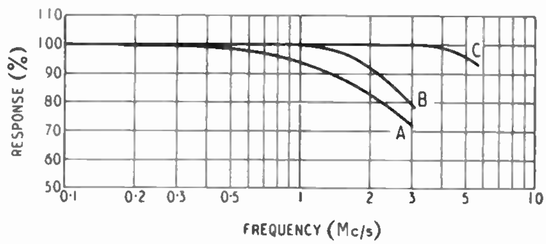


Fig. 18.—Frequency response of shunt-regulated amplifier with one stage of inductance compensation. Curve A is the natural response of the Type 2(e) circuit while curve B shows the effect of adding one stage of inductance compensation. The effect of further compensation is shown by curve C.

NEGATIVE FEEDBACK AMPLIFIERS OF DESIRED AMPLITUDE FREQUENCY CHARACTERISTICS*

Introduction

As is well known, when negative feedback is applied to an amplifier of two or more stages, the degree of feedback is severely limited unless peaks in the gain/frequency response curve are tolerable.

Ways of overcoming this feature have been suggested by Brockelsby[6] and Mayr.[7]

A more recent paper by Flood[8] has analysed the conditions for what herein is defined as critical flatness and uses a complex feedback network to achieve an indicial response without overshoot.

There are, however, still gaps in the design criteria and an attempt is made to fill the gaps with a simple presentation.

As far as the author is aware, apart from Flood's recent article, no simple means have been presented for designing feedback amplifiers with desired response curves using frequency concious feedback, although it would appear that considerable advantages are obtained by so doing.

As is well known, the amplification of an amplifier with feedback is given by the relation

$$A_x = \frac{A}{1 + A\beta}$$

Brockelsby deals with the design of maximally flat amplifiers assuming β is aperiodic.[6]

* The greater part of this section is abstracted from Ref. 5.

With the feedback complex several interesting conditions are possible and are described below.

High-frequency Response-Single-stage Amplifier

(*a*) *Voltage Feedback* (*Fig.* 19)

For a conventional pentode or tetrode stage we may write

$$A = \frac{g_m R}{1 + jx}$$

where

$$x = \omega CR$$

R being the anode load resistance and

C the total effective shunt capacitance.

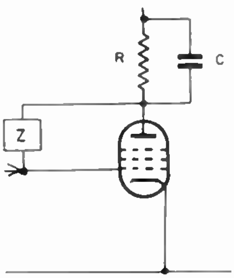


Fig. 19.—*Single valve amplifier with voltage feedback.*

The feedback will be $A\beta'$

Thus, the gain formula is

$$A_x = \frac{\dfrac{g_m R}{1 + jx}}{1 + \dfrac{g_m R}{1 + jx}\beta'} = \frac{g_m R}{1 + jx + g_m R\beta'}$$

$$= \frac{A_0}{1 + jx + A_0\beta'}$$

(*b*) *Aperiodic Feedback*

For aperiodic feedback leaving the response curve unaltered in shape and extent so that only the gain is changed by the feedback, $A\beta'$ is made aperiodic,

i.e. β' must be a constant product or fraction of $(1 + jx)$

Writing $\beta' = \delta + j\gamma$

Then for any given value of δ, $\gamma = \delta x$

and the feedback fraction β' for aperiodic feedback is $\delta + j\delta x$.

(*c*) *Critical Feedback.*

For the same amplifier to achieve critical flatness the response must be changed from

$$A = \frac{A_0}{1 + jx} \text{ to } A = \frac{A_0}{N\left(1 + j\dfrac{x}{N}\right)}$$

so that the shape of the response curve is retained but the frequency response is extended by the feedback.

Then

$$\frac{\dfrac{A_0}{1 + jx}}{1 + \dfrac{A_0}{1 + jx}(\delta + j\gamma)} = \frac{A_0}{N\left(1 + j\dfrac{x}{N}\right)}$$

$$\text{so } 1 + jx + A_0\delta + A_0 j\gamma = N + jx$$

$$jx + A_0 j\gamma = jx$$

$$\gamma = 0$$

i.e., the condition is satisfied with constant feedback and the feedback factor is $1 + A_0\delta$, with δ of any value.

(*d*) *Maximal feedback*

For maximal flatness the shape of the response curve will have no maxima and minima and is of the form

$$\frac{1}{\sqrt{K + x^{2n}}}$$

For the amplifier already assumed

$$A_x = \frac{A_0}{1 + jx + A_0\beta'}$$

$$\frac{A_0}{A_x} = 1 + jx + A_0\delta + A_0 j\gamma$$

$$= jx + N + jN'$$

$$\left|\frac{A_0}{A_x}\right|^2 = N^2 + (N' + x)^2$$

$$= N^2 + N'^2 + 2N'x + x^2$$

For maximal flatness

$$N'^2 + 2N'x = 0$$

$N' = 0$ (corresponding to critical flatness)

or $N' = -2x$

Substituting $N' = -2x$ we find this solution corresponds to a rising frequency characteristic with $-$C or $-$R replacing the original C or R in the amplifier, thus retaining the shape of response curve in the opposite amplitude sense.

We therefore conclude that, for a single valve amplifier with voltage feedback, maximal flatness is also critical flatness.

(e) Optimal Flatness

From the equation

$$\left|\frac{A_o}{A_x}\right|^2 = N^2 + N'^2 + 2\,N'\,x + x^2$$

we found that for maximal flatness the terms $N'^2 + 2N'$ were made zero, leaving the required response curve shape

$$\left|\frac{A_o}{A_x}\right|^2 = N^2 + x^2$$

We can, however, go a step further and compensate for the frequency variable x^2, i.e. we can make

$$N'^2 + 2N'\,x + x^2 = 0$$

giving a frequency response shape

$$\left|\frac{A_o}{A_x}\right|^2 = N^2 = \text{constant}$$

The solution to this condition is $N' = -x$ which can be seen equally well by a consideration of the original formula

$$A_x = \frac{A_o}{1 + jx + A_o\beta'} = \frac{A_o}{1 + jx + A_o\delta + A_o j\gamma}$$

from which we see that all the frequency conscious terms disappear if

$$A_o\gamma = -x$$

i.e. $N' = -x$ as before.

The feedback fraction β' then becomes

$$\delta - \frac{jx}{A_o}$$

the real and complex parts being quite independent.

We can then, if we wish, make $\delta = 0$ in which case the gain remains constant at its original value of A_o to infinite frequency, at least theoretically.

We may also apply a feedback somewhat less than that required for optimal flatness, in which case we get a response shape similar to that of a single circuit but with reduced C, i.e., we can improve the gain bandwidth product without changing the response shape.

Summarizing the relations derived from the single valve amplifier with voltage feedback we thus have

Aperiodic feedback fraction			$\delta + j\delta x$
Critical	,,	,,	δ
Maximal	,,	,,	δ
Optimal	,,	,,	$\delta - \dfrac{jx}{A_o}$

Applying the same method to two successive stages of amplification we get the following relations.

Aperiodic feedback fraction

$$\beta' = (\delta_1 + \delta j\gamma_1)(\delta_2 + j\gamma_2)$$
$$\beta' = (1 + jx_1)(1 + jx_2)$$

Critical feedback fraction

$$\beta' = \delta + j\gamma$$
$$= \delta + j\,\frac{[2\sqrt{x_1 x_2 N} - (x_1 + x_2)]}{A_o}$$

Maximal feedback fraction

$$\beta' = \delta + j\,\frac{[\sqrt{2x_1 x_2 N} - (x_1 + x_2)]}{A_o}$$

Optimal feedback (identical stages)

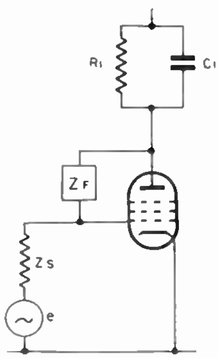
$$\beta' = -\frac{(1 + jx)^2 - (1 + A_0\delta)}{A_o}$$

Optimal feedback (dissimilar stages)

$$\beta' = -\frac{(1 + jx_1)(1 + jx_2) - (1 + A_0\delta)}{A_o}$$

Reviewing the forms of feedback fraction required we find that of the complex factors the form $\delta + jkx$ is the most common, i.e., in a number of cases the feedback voltage is of the form $(\delta + jkx)\,V_o$.

Simple feedback networks capable of producing a voltage of this form at the input terminals are only approximate.



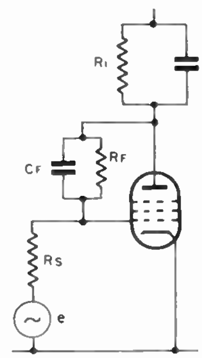


Fig. 20 (left).—Single valve voltage feedback amplifier with driving source impedance Z_s.
Fig. 21 (right).—Single valve amplifier with complex voltage feedback.

Two such approximations are possible:—

1. If in Fig. 20 Z_s is small and substantially resistive (R_s) then Z_F may take the form of a parallel RC network of component values R_F and C_F as in Fig. 21. ($R_s \ll R_F$)

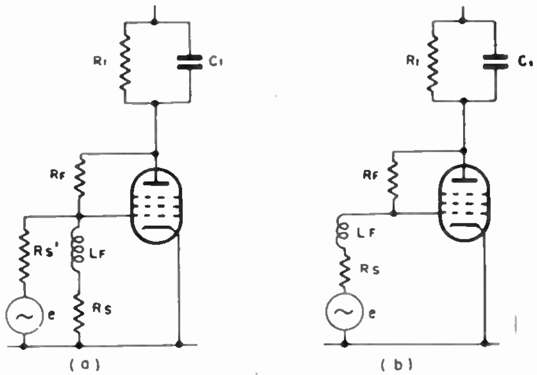


Fig. 22.—Single valve amplifier with alternative form of complex feedback $R_s' \gg R_s$).

2. Alternatively, we may use the reciprocal equivalent network to the above, i.e., an inductance and resistance in series as in Fig. 22 where

$$\frac{L_F}{R_S} = R_F C_F$$

With these approximate networks the feedback voltage when $R_S \ll R_F$ and when $X_C > R_F$ or $X_L < R_S$ is of the form

$$\frac{R_S}{R_F}(1 + j\omega R_F C_F) \text{ for (1)}$$

$$\frac{R_S}{R_F}\left(1 + j\omega\frac{L_F}{R_S}\right) \text{ for (2)}$$

and may thus be used in many of the cases derived and is in fact directly applicable to the case for

(*a*) Aperiodic feedback over one stage

(*b*) Critical and maximal feedback over two stages.

For optimal feedback the real and complex components are of opposite sign and must be dealt with separately.

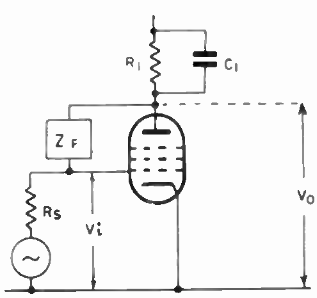


Fig. 23.—Single valve amplifier with complex voltage feedback and resistive driving impedance.

For a single stage the optimal feedback fraction is

$$\delta - j\frac{x}{A_O}$$

i.e., with the stage shown as in Fig. 23.

$$V_i = \left[\delta - \frac{jx}{A_O}\right] V_O.$$

This is theoretically possible if R_s is small and if Z_F is composed of a resistor shunted by a negative capacitance.

Suppose we consider a capacitor with a voltage V across it, the current will be $V\omega C$

Suppose now we insert with the capacitor a generator of $2V$ volts in opposition so that a current of $V\omega C$ now flows in the opposite direction. The capacitor plus its inserted generator will now appear as a negative capacitance of numerical value equal to the original positive capactance. We may thus simulate the negative capacitance with suitable circuitry and one possible arrangement is shown in Fig. 24.

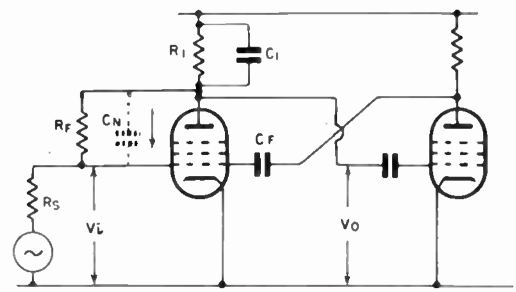


Fig. 24.—Single valve amplifier with optimal feedback.

The negative capacitance required is shown dotted, the current flowing in this capacitor will be in the direction of the arrow and of magnitude $(V_O + V_1)\,\omega C_N$ since V_O and V are of opposite signs.

Suppose the gain of the second valve is $-A_2$ and the gain of the first valve $\frac{-V_O}{V_1} = -A_1$.

Then the current flowing into the grid circuit of the first valve through C_F is

$$(A_2 V_O - V_1)\,\omega C_F$$

Thus to simulate the negative capacitance C_N

$$(A_2 V_O - V_1)\,\omega C_F = (V_O + V_1)\,\omega C_N$$

and if $|C_F| = |C_N|$

$$A_2 = \frac{V_o + 2V_1}{V_o}$$

$$\text{or } A_2 = 1 + \frac{2}{A_1}$$

Thus the gain of the feedback phase reversing valve is small and it will not play a very significant part in the frequency response due to its own limitations.

The approximate feedback network chosen is such that its law will only approximate to the required law up to the frequency at which X_c is still larger than R_s and thus little profit can be found.

Suppose however, that we postulate that C_F must be $\frac{1}{4}C_N$. Then we extend its usefulness at the expense of providing gain in the extra valve.

Then for $C_F = \frac{1}{4}C_N$

$$A_2V_o - V_1 = 4V_o + 4V_1$$

$$A_2 = \frac{4V_o + 5V_1}{V_o}$$

$$\text{or } A_2 = 4 + \frac{5}{A_1}$$

If the frequency response of this amplifier is still adequate, the system will be improved by this modification. An example of this is given in conjunction with a multi-stage amplifier later.

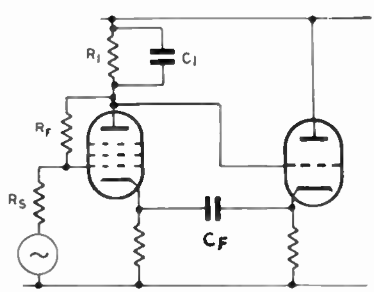


Fig. 25.—Single valve amplifier with alternative form of optimal feedback.

An alternative form is shown in Fig. 25. Shown diagrammatically this is a better practical form than the original for frequency response but still suffers from the response of the approximate feedback network.

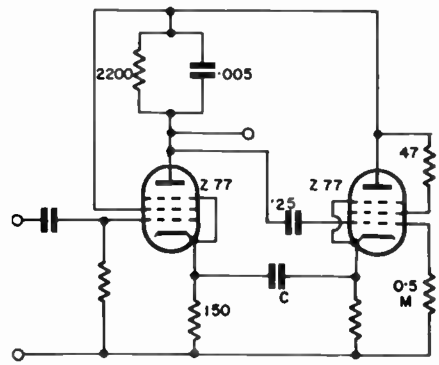


Fig. 26.—Experimental amplifier to verify optimal feedback.

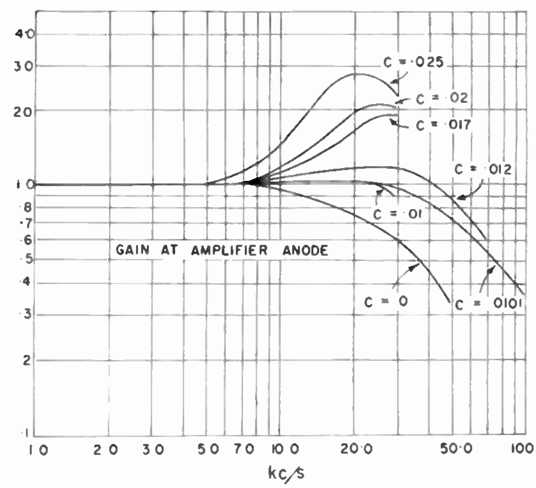


Fig. 27.—Practical results using amplifier of Fig. 26.

A system as in Fig. 26 gives the results plotted in Fig. 27 and shows the profit that can be obtained by optimal feedback in an amplifier that is followed by a cathode follower.

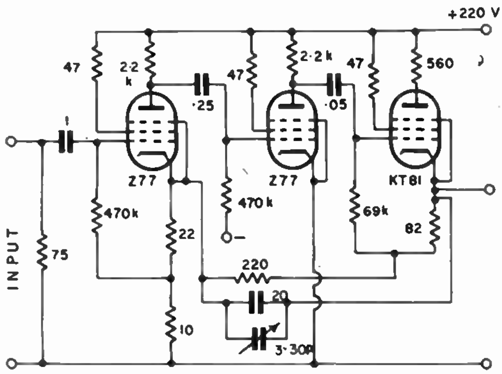
A check on the feedback voltage appearing at the amplifier cathode proves the inadequacy of the feedback system to give more than the 2½ to 1 improvement in frequency response (without loss of gain) over the simple RC circuit, the comparison being made between the frequencies in the two cases at which the loss of gain is 5 per cent.

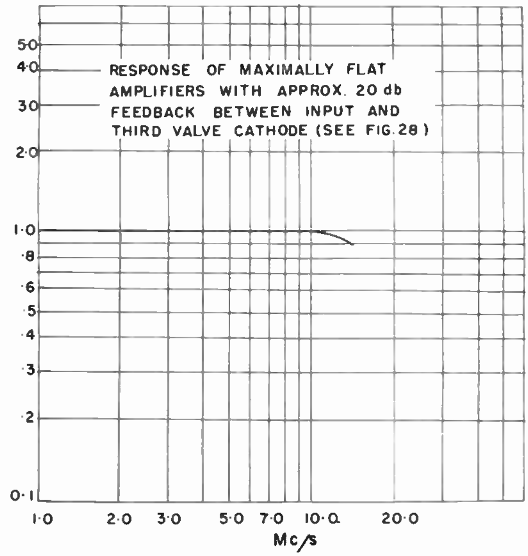
Care in design could probably improve on this result as is amply shown in later examples.

Practical Applications: Two-stage Amplifier

A two-stage amplifier designed for the input stages of a high-power television modulator to give maximally flat response is as shown in Fig. 28.

The gain of each stage and its natural response without feedback were checked carefully and the feedback network calculated by the aid of the foregoing formulae.





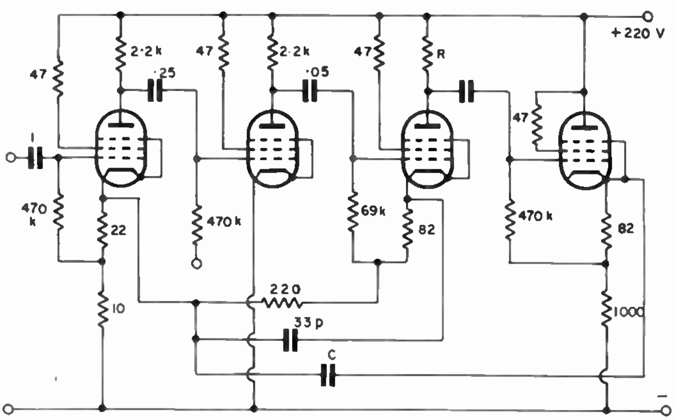
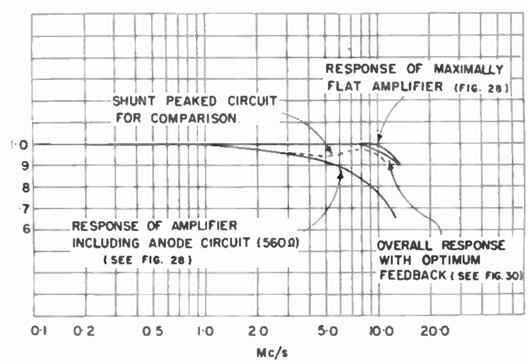


Fig. 28 (above left).—Practical two stage amplifier and cathode-follower output stage with maximal feedback.

Fig. 29 (above right).—Response of amplifier shown in Fig. 28.

Fig. 30 (left).—Three stage amplifier and cathode-follower output stage with maximal and optimal feed-back.

Fig. 31 (below).—Response of amplifier of Fig. 30.



The feedback capacitor C for measured maximal flatness was within 5 per cent. of the calculated value—the response is shown in Fig. 29.

This amplifier was then modified by including a resistor in the anode of the third valve and following it with a cathode follower as Fig. 30. optimal feedback being applied to this added stage through the maximally flat amplifier by the capacitor C.

Results obtained are shown in Fig. 31 and compared with a similar amplifier without optimal feedback but with shunt peaking.

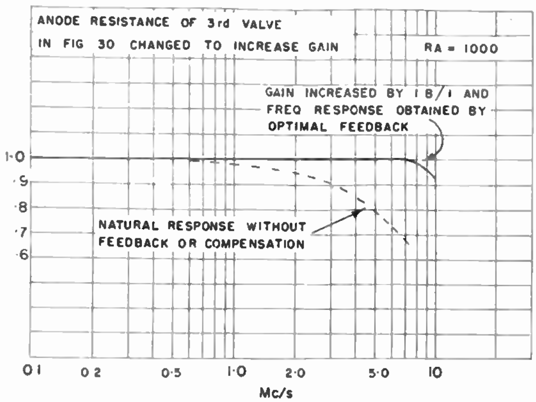


Fig. 32(*a*).—*Response of amplifier of Fig.* 30 *with gain increased* 80 *per cent.*

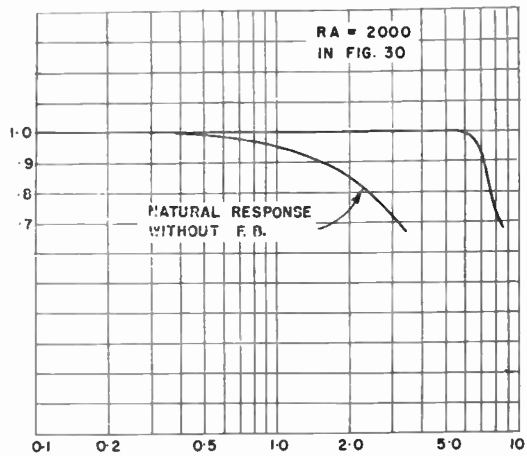


Fig. 32(*b*).—*Response of amplifier of Fig.* 30 *with gain increased by a factor of* 3·6.

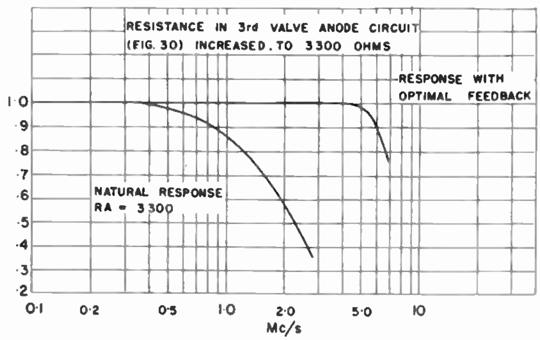


Fig. 32(*c*).—*Response of amplifier of Fig.* 30 *with gain increased by a factor of* 5·9.

The application of overall optimal feedback showed that up to the limit of response of the complete feedback loop, complete compen-

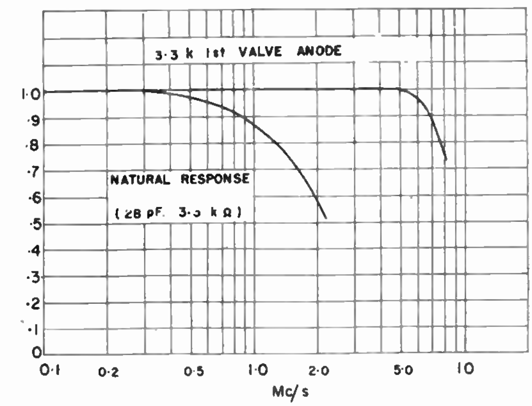


Fig. 33.—*Response of amplifier of Fig.* 35 *with* $R = 3{\cdot}3k\Omega$.

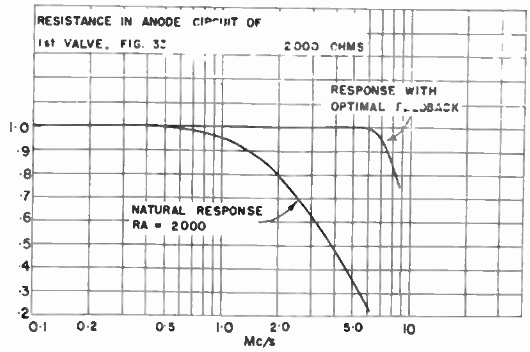


Fig. 34.—*Response of amplifier of Fig.* 35 *with* $R = 2k\Omega$.

sation was possible and the simple expedient of adjusting C at 8 Mc/s to give the low-frequency gain was sufficient to achieve the result shown.

The resistor R was then increased to 1,000 Ω and the result showed that the feedback loop was still adequate, the response curve now being as shown in Fig. 32 (*a*). Further results are shown in Figs. 32 (*b*) and 32 (*c*).

A further check of optimal feedback was made by preceding the maximally flat amplifier with a low response amplifier and correcting the poor response with optimal feedback.

The circuit was as Fig. 35 and response curves are shown in Figs. 33 and 34 for various values of R.

A further observation on the practical feedback networks possible is of interest. If the gain of the feedback loop is unity then the approximate reciprocal network of a capacitance and a

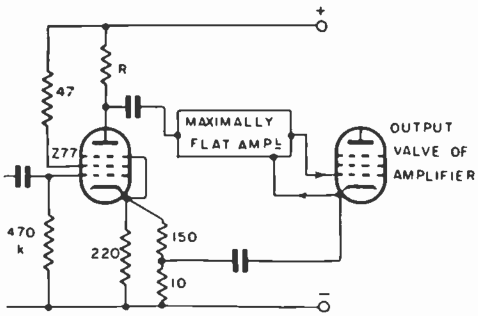


Fig. 35.—*Circuit of amplifier added to system of Fig.* 30 *and compensated by overall optimal feedback.*

resistance in series only holds good up to a frequency less than the normal frequency for 3 db response and little profit is possible, i.e., the optimal feedback conditions producing the flat response will fall away beyond this frequency.

With a gain of A on the feedback loop, the frequency at which the optimal flatness can be realized will extend to A times the frequency without gain.

We may therefore conclude that given a good amplifier of flat response to frequency f_1 and of gain A_1 associated with a poor amplifier of frequency response flat to f_2, we may, by optimum feedback improve the response of the two systems in cascade to give a response approximately flat to frequency Af_2.

It may be shown that optimal flatness also corresponds to a linear phase characteristic and the rate of fall away from the condition of flat frequency response and linear phase response need only be governed by the failure of the simple R-C feedback network to maintain the correct law for the feedback voltage and is therefore relatively slow.

Conclusion

Considerable profit results from the use of maximal and optimal feedback and there seems no reason why the relations should not apply equally well to band-pass systems using band-pass low pass equivalents.

Acknowledgment

The author is indebted to Messrs. Marconi's Wireless Telegraph Company, Ltd., for permission to present this paper.

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4. N. H. Clough. British Patent 526,418.
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A search of the literature shows that amplifier arrangements have been used with valves in series d.c. connection. The following references are given:—

M. Artzt, "Survey of D.C. Amplifiers," *Electronics*, **18**, August 1945, p. 112.

G. E. Valley and H. Wallman. "Vacuum Tube Amplifiers." Radiation Laboratory Series, Vol. 18. (McCraw-Hill, 1949).

CONTRIBUTION

ANALYSIS OF THE OSCILLATOR IN FIG. 5(*a*) OF "R.C. TUNED OSCILLATORS"*

by

H. Stibbé (*Associate Member*)

Although the relative directions of i_1 and i_2 are not shown in Fig. 5 (*a*), the direction of i_2 is shown in Fig. 5 (*b*) to be from cathode to anode of V2. The direction of i_1 is then given from the expression in Mr. Kundu's analysis

$$e_{gk} = (i_1 - i_2) R_k = e_{gc2}$$

i.e., i_1 flows from anode to cathode of V1. Fig. 5 (*a*) is redrawn to include these currents.

Mr. Kundu's analysis is correct until the statement "The conditions for oscillations will be satisfied when

$$e_o = e_{gc1} + i_1 R_k\text{"}$$

By Kirchoff, $e_o = e_{gc1} + e_{gk} = e_{gc1} + R_k(i_1 - i_2)$. Thus for $e_o = e_{gc1} + i_1 R_k$ to be true, i_2 must be zero. When i_2 is zero, the voltage across the anode load of V2 is zero, and consequently e_o, i_1, e_{gc1} and e_{gk} are also zero. Clearly under these conditions the circuit does not oscillate.

The condition for oscillation may be expressed as follows: The "gain" (which will be fractional) from V2 anode to V1 grid via C must be equal to the reciprocal of the gain from V1 grid to V2 anode via the cathode coupling resistance R_k. From Figs. 5 (*b*) and 5 (*c*) page 236, the gain

from V2 anode to V1 grid via C $= \dfrac{Z_2}{Z_2 + Z_3}$

The gain from V1 grid to V2 anode via R_k

$$= \frac{i_2 Z}{e_o} = \frac{i_2 Z}{e_{gk}} \cdot \frac{e_{gk}}{e_o} = g_{m2} Z \frac{e_{gk}}{e_o} \quad ..(1)$$

Mr. Kundu has shown that

$$e_{gk} = e_{gc2} = \frac{g_{m1} e_{gc1} R_k}{1 + g_{m2} R_k} \quad ....(2)$$

Also $e_o = e_{gc1} + e_{gk}$ and substituting from (2)

$$e_o = e_{gc1} + \frac{g_{m1} e_{gc1} R_k}{1 + g_{m2} R_k} \quad ....(3)$$

Substituting (2) and (3) for e_{gk} and e_o respectively in (1)

* P. Kundu, "R.C. Tuned Oscillators," *J.Brit.I.R.E.* **11**, No. 6, June 1951, pp. 233-241.

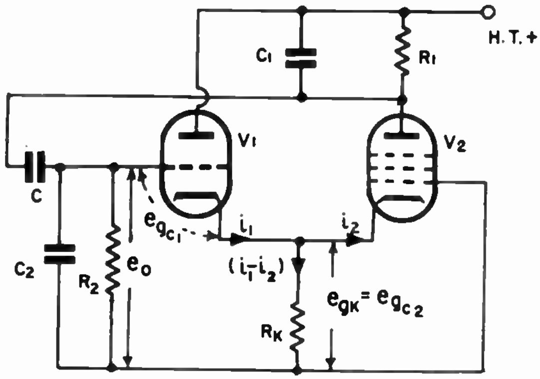


Fig. 5(a), redrawn.

$$\frac{i_2 Z}{e_o} = g_{m2} Z \left[\frac{\left(\dfrac{g_{m1} e_{gc1} R_k}{1 + g_{m2} R_k}\right)}{e_{gc1} + \dfrac{g_{m1} e_{gc1} R_k}{1 + g_{m2} R_k}} \right]$$

$$= g_{m2} Z \left[\frac{1}{1 + \dfrac{1 + g_{m2} R_k}{g_{m1} R_k}} \right]$$

$$= g_{m2} Z \left[\frac{g_{m1} R_k}{1 + g_{m1} R_k + g_{m2} R_k} \right]$$

$$= \frac{g_{m1} g_{m2} R_k}{1 + g_{m1} R_k + g_{m2} R_k} \cdot \frac{Z_1 (Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}$$

Thus for oscillation

$$\frac{g_{m1} g_{m2} R_k}{1 + g_{m1} R_k + g_{m2} R_k} \cdot \frac{Z_1 (Z_2 + Z_3)}{Z_1 + Z_2 + Z_3} = \frac{Z_2 + Z_3}{Z_2}$$

i.e. $$\frac{g_{m1} g_{m2} R_k}{1 + g_{m1} R_k + g_{m2} R_k} \cdot \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} = 1$$

Equations (i), (iii), (v) and (vi) of Mr. Kundu's appendix are revised as follows:—

Equation (i) becomes

$$\frac{g_{m1} g_{m2} R_k}{1 + R_k(g_{m1} + g_{m2})} \cdot \frac{R_1 R_2 C}{\sum CR + \dfrac{1}{j\omega}\left[1 - \omega^2 R_1 R_2 \sum CC\right]}$$

Equation (iii) becomes

$$\frac{g_{m1}\,g_{m2}\,R_k\,R_1}{1+R_k\,(g_{m1}+g_{m2})}\cdot\frac{1}{1+\dfrac{R_1}{R_2}+\dfrac{C_2}{C}+\dfrac{C_1R_1}{CR_2}}=1$$

Equation (v) becomes

$$\frac{g_{m1}\,g_{m2}\,R_k\,R_1}{1+R_k(g_{m1}+g_{m2})}=2$$

Equation (vi) becomes $g_m\,R_1=6$

Equations (ii) and (iv) remain unchanged.

Reply by P. Kundu, M.Sc. (*Associate Member*)

The analysis as worked out by Mr. Stibbé, and his statement for the condition of oscillation, which is generally expressed as $A=1/\beta$ is quite sound.

It may be interesting to point out my own approach and the reason for preferring the one in the appendix.

With reference to Fig. 5(*a*)

$$e_o=e_{gc1}+e_{gk}$$

$$\text{and}\quad e_{gk}=(i_1-i_2)\,R_k=e_{gc_2}$$

e_{gc2} has been shown to be $=\dfrac{g_{m_1}\,e_{gc1}\,R_k}{1+g_{m2}\,R_k}$

∴ Voltage across the load of V_2

$$=i_2\,\frac{Z_1\,(Z_2+Z_3)}{Z_1+Z_2+Z_3}$$

Then the voltage at the input of V_1 fed back through the potential divider from the anode of V_2

$$=i_2\,\frac{Z_1Z_2}{Z_1+Z_2+Z_3}$$

The circuit will be oscillatory when

$$i_2\,\frac{Z_1Z_2}{Z_1+Z_2+Z_3}=e_o=e_{gc1}+(i_1-i_2)\,R_k \quad\ldots\ldots\ldots\ldots\ldots(1)$$

or,

$$i_2\left(\frac{Z_1Z_2}{Z_1+Z_2+Z_3}+R_k\right)=e_{gc1}\,(1+g_{m1}\,R_k)\ldots(2)$$

where i_2 was found to be $=g_{m2}\,\dfrac{g_{m1}\,e_{gc1}\,R_k}{1+g_{m2}\,R_k}$

A comment should be made at this point that i_2 was not intended to be zero, as is argued by Mr. Stibbé, but as $Z_1Z_2 \gg R_k$, R_k was neglected in order to present the note in the appendix in a simpler manner.

On substitution of the value of i_2 in equation (2), we have (when R_k is not neglected)

$$\frac{g_{m1}\,g_{m2}\,R_k}{(1+g_{m1}\,R_k)\,(1+g_{m2}\,R_k)}\cdot\left[\frac{Z_1Z_2}{Z_1+Z_2+Z_3}+R_k\right]=1\quad\ldots(3)$$

or,

$$\frac{g_{m1}\,g_{m2}\,R_k}{1+g_{m1}\,R_k+g_{m2}\,R_k}\cdot\frac{R_1\,R_2\,C}{\sum CR+\dfrac{1}{j\omega}\left(1-\omega^2R_1\,R_2\sum CC\right)}=1\ldots(4)$$

At the frequency of oscillation and when $R_1 \ll R_2$ and $C=C_2 \gg C_1$

$$\frac{g_{m1}g_{m2}\,R_k\,R_1}{1+g_{m1}\,R_k+g_{m2}\,R_k}=2\ldots\ldots\ldots\ldots(5)$$

This equation is the same as Mr. Stibbé's modification.

When $\quad g_{m1}=g_{m2}=g_m$

$$g_m\,R_1=4+\frac{2}{g_m\,R_k}\ldots(\text{I})$$

But when R_k is neglected as it was in the appendix, the relation becomes:

$$\frac{g_{m1}g_{m2}\,R_1\,R_k}{(1+g_{m1}\,R_k)\,(1+g_{m2}\,R_k)}=2$$

i.e.

$$g_m\,R_1=4+\frac{2}{g_m\,R_k}\,(1+g_m^{\,2}\,R_k^{\,2})\ldots(\text{II})$$

$$\text{when } g_{m1}=g_{m2}=g_m$$

The equation (II) remains very nearly equal to equation (I) so long as the value of g_mR_k is less than unity, which is generally the case.

However, the equation (5) as above, without any further simplification, is a more accurate statement for the condition of oscillation.

My thanks are due to Mr. Stibbé for his kind criticism.

THE WORK OF THE RADIO RESEARCH STATION

A representative of the Institution was recently invited by the Director to visit the Radio Research Station at Slough. The Station is closely associated, under the Department of Scientific and Industrial Research, with the National Physical Laboratory, and, in fact, the Director is Superintendent of the Radio Division of the Laboratory.

The major activity of the Station is investigation into the propagation of radio waves, and most of the work which was seen dealt with measurements for radio waves of all frequencies. At the low-frequency end of the radio waveband, an interesting investigation is proceeding in the direct correlation of phase errors with ground conductivity in the Decca Navigator system. Observations were taken along a path at right-angles to the base line joining two transmitters and compared with soil conductivities along the path which, in one case under investigation, lay across the London Basin. The variation of the error observed closely followed the changes in conductivity, and work is continuing in investigating the nature of the phase error change at the point where the conductivity changes, for example, at a boundary between chalk and clay.

One of the best-known activities at Slough is the prediction of ionospheric conditions and observations taken on the automatic recorder developed by the Station were demonstrated. The Station sent observers to points on the path of the 1952 Solar eclipse—Khartoum (Sudan) and Ibadan (Nigeria); the University Colleges in both these centres maintain ionospheric observatories in co-operation with the Research Station. The recorder traces showed the instantaneous M.U.F., and the variation of this during the period of the eclipse illustrated its effect on the F-layer. Unusual deviations in the trace can be correlated with the presence of coronae around the sun's circumference. It is interesting to recall that the present Director of the Research Station, Dr. R. L. Smith-Rose, C.B.E., read a paper to the Institution in 1946 describing the ionospheric effects during previous eclipses,* and these latest observations will help considerably in the interpretation of the mechanism of ionospheric propagation.

Observations were also made during the eclipse at Slough on the variation in the angle of elevation of ionospheric rays received from Delhi, that is, across the path of the eclipse. Such investigations, as well as measurements of the absorption in the ionosphere, are an important part of the Station's regular research programme.

The effect of variations in the tilt of the lower surface of the E-layer on the accuracy of high-frequency direction finding was demonstrated, using an Adcock system receiving alternately pulsed and C.W. transmissions over a path of about 700 km. The reception of multiple pulses, and the continually differing phases of the C.W. signals, were shown on a cathode ray direction finder. These variations give rise to errors of up to 20 deg. or more.

A new programme of research has recently been started into long-range back-scatter. A modified high-power radar transmitter, operating between 10 and 20 Mc/s, transmits pulses of low repetition rate (25 per sec) by a directional rhombic aerial beamed in an east-west direction. These pulses undergo a number of oblique reflections at the ionosphere and the earth, and it is found that back-scattering takes place at the surface of the earth, giving rise to measurable echoes on an A-display; echoes are also to be observed from Sporadic-E clouds in the ionosphere. The time scale of the display naturally gives a measure of the distance from the transmitter of all these reflecting surfaces. The method will, it is expected, prove a useful tool in assessment of ionospheric conditions at remote points along propagation paths, and it has the advantage of not requiring co-operation from a distant station. Rotatable Yagi-type arrays will eventually extend the value of the experiments.

The Research Station has been carrying out a series of tests for the B.B.C. in the siting of V.H.F. stations, in particular for the television network, with the principal object of assessing the interference between two geographically separated transmitters operating on the same frequencies. Continuous recordings of the existing television transmitters are made at Slough and these are analysed in conjunction with meteorological data.

One noteworthy item of research conducted at Slough which is not connected with propagation is an investigation into semi-conductors. At the moment this is mainly confined to the germanium diode, and measurements have been made on the effect of light on the electrical properties. Noise measurements are also in progress.

* R. L. Smith-Rose, "The Influence of an Eclipse of the Sun on the Ionosphere." *J.Brit.I.R.E.*, 6, No. 3, June 1946, pp. 82-97.

POSITIVE FEEDBACK OPERATOR NETWORKS*

by

A. W. Keen (*Member*)†

SUMMARY

A special case of the general feedback system in which a passive bilateral shaping network is used in the forward (μ) section and a unidirectional unit gain transformer in the return (β) path, thereby forming a positive feedback loop, is shown to be capable of acting as an operator network, i.e., of performing a prescribed mathematical operation on an input quantity. The set of possible circuit developments derived from the basic network includes certain previously published, but hitherto un-correlated, circuits. Application of localized negative feedback in the β section is used to hold the positive loop gain at unity thereby inhibiting instability. The method is compared with a more conventional arrangement using negative feedback.

NOTATION

a_1, a_2 = constant coefficients in operational expressions.

α = factor of μ of feedback system.

β, β' = feedback ratio of feedback system.

C = capacitance.

$e_i(t)$ = input (signal) voltage.

$e_o(t)$ = output (signal) voltage.

$f(s)$ = voltage transfer function of portion of μ section of feedback system.

$F(s)$ = $e_o(t)/e_i(t)$ = voltage transfer function of entire feedback system.

g = mutual conductance of V1, V2 in special case $g_1 = g_2$.

g_1, g_2 = mutual conductances of V1, V2 in general case $g_1 \neq g_2$.

$i_i(t)$ = input (signal) current.

k = numerical constant (impedance ratio).

L = inductance.

m = numerical constant (impedance ratio).

n = numerical constant (<1) (voltage ratio).

r = differential anode resistance of V1, V2 in special case $r_1 = r_2$.

r_1, r_2 = differential anode resistances of V1, V2 in general case $r_1 \neq r_2$.

R = resistance.

$s = \sigma + j\omega$ = complex frequency variable.

t = time variable.

τ = time constant (CR or L/R).

u = voltage magnification factor of V1, V2 in special case $u_1 = u_2$.

u_1, u_2 = voltage magnification factors of V1, V2 in general case $u_1 \neq u_2$.

μ, μ' = voltage gain of forward section of feedback system.

$Y_1(s) \equiv Z^{-1}(s)$ = an operational admittance.

$Z_1(S), Z_2(S)$ = operational impedances.

* Manuscript received June 11th, 1951.

† E.M.I. Research Laboratories, Ltd. (Now at Coventry Technical College.)

U.D.C. No. 621.392.5.

1.0. Introduction

In certain applications, notably computing and automatic control systems, it is required to synthesize a network having a prescribed voltage transfer characteristic in order to perform a particular mathematical operation[1] or control the response of a servo.[2] Very often a network having the desired transfer function will be known from experience or obtainable from the literature[1,3] but where an unusual characteristic is required it may be necessary to proceed by trial and error or to employ some general method of synthesis. The problem will usually be more difficult if the network is assumed to be a quadripole of unspecified configuration having a voltage transfer ratio $e_o(t)/e_i(t)$ of the desired operational expression than if the network is sought as a two-terminal network whose driving point admittance $i_i(t)/e_i(t)$ has the required form, relying upon subsequent $i \rightarrow e$ conversion where voltage output is essential.

Ideally the network should consist only of passive elements (viz., C, L, R) to ensure stability; the possibilities of synthesis are limited by the use of such a restricted set of elements so that it may not be possible, or conveniently practicable, to obtain certain functions exactly, or even to a sufficiently close approximation. The most important, and possibly the most difficult, particular cases are those of exact integration and differentiation. Simple networks capable of achieving approximate integration and differentiation are well known,[4] and, using them as prototypes, it is possible to obtain greater accuracy by correcting processes which develop the basic network into more complex forms.[5] A more powerful approach involves the use of high-gain amplifiers subject to negative feedback, which, being a limiting process, can be applied in such a manner that the voltage transfer over the whole amplifier is determined almost entirely by a restricted group of passive elements, and can have forms not readily obtainable from networks involving the latter elements alone.[6]

Successful application of negative feedback may have diverted attention from the possibility of employing positive feedback to achieve similar results. The present note describes a method of using such feedback in conjunction with a simple function control network. A theory of the method is derived from the fundamental feedback equation, and a basic network obtained which may be developed into a set of practical circuits. These include as special cases a number of previously published, but hitherto uncorrelated, isolated circuit arrangements, notably Beale and Stansfield's[7] circuit for integration or differentiation, Schmitt and Tolles'[8] feedback differentiator and Newsam's "bootstrap" integrator.[9, 10] Thus it is demonstrated that these and related circuits are, considered in their entirety, examples of positive feedback, although each may, with advantage, incorporate localized negative feedbacks in order to define and stabilize the positive feedback loop gain.

2.0. Derivation from Feedback Theory

2.1. The voltage transfer function of the basic feedback system (Fig. 1 (a)) is conventionally expressed

$$e_o(t)/e_i(t) = \mu/(1 - \mu\beta) \quad \ldots\ldots (1)$$

and in succeeding sections will be denoted $F(s)$,

i.e. $F(s) \equiv e_o(t)/e_i(t)$.

μ and β are the transfer functions of the two sections considered as separate entities and either one (or both) must be to some extent unidirectional, as indicated in Fig. 1 by the use of the triangular block symbol.

2.2. Taking the case $\mu = f(s)$, where $f(s)$ is an arbitrary function of the complex frequency variable ($s = \sigma + j\omega$) (as determined by a passive transmission-type network), with β unidirectional, positive and unity (see Fig. 1(b)) gives, by (1),

$$F(s) = f(s)/\{1 - f(s)\} \quad \ldots\ldots (2)$$

from which

$$f(s) = F(s)/\{1 + F(s)\} \quad \ldots\ldots (3)$$

Thus the result of applying 100 per cent. positive feedback over the passive quadripole denoted $f(s)$ is to transform its voltage transfer function in the manner expressed in equation (2). Conversely, the passive network needed in the μ section to realize a required $F(s)$ is given by (3).

2.3. In most cases a required $F(s)$ will be rational, i.e. expressible as the quotient of two polynomials in s, say $Z_2(s)/Z_1(s)$, so that (3) may be rewritten

$$f(s) = Z_2(s)/\{Z_1(s) + Z_2(s)\}, \quad \ldots\ldots (4)$$

and $f(s)$ may be realized in a simple L-type network having $Z_2(s)$ in the shunt and $Z_1(s)$ in the series arm. In this case the positive feedback ($\beta = +1$) reduces the voltage transfer function of the μ network to a simpler form, thus

$$Z_2(s)/\{Z_1(s) + Z_2(s)\} \rightarrow Z_2(s)/Z_1(s) \quad \ldots (5)$$

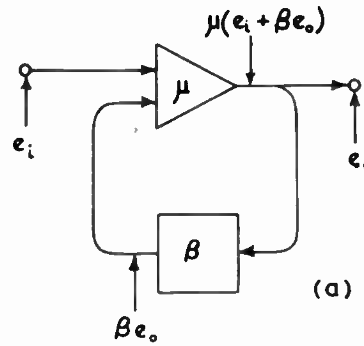
2.4. More generally β may have any positive value, provided the μ section includes an amplitude-transforming section α such that $|\alpha\beta| = 1$ (See Fig. 1(c)). Then (2) becomes

$$F(s) = \alpha . f(s)/\{1 - f(s)\}, \quad \ldots\ldots (2A)$$

giving

$$f(s) = \alpha . F(s)/\{1 + \alpha . F(s)\}, \quad \ldots (3A)$$

which is the principal design equation for the class of networks under discussion. $F(s)$ must now be expressed as $\alpha . Z_2(s)/Z_1(s)$ for equation (3) to remain valid.



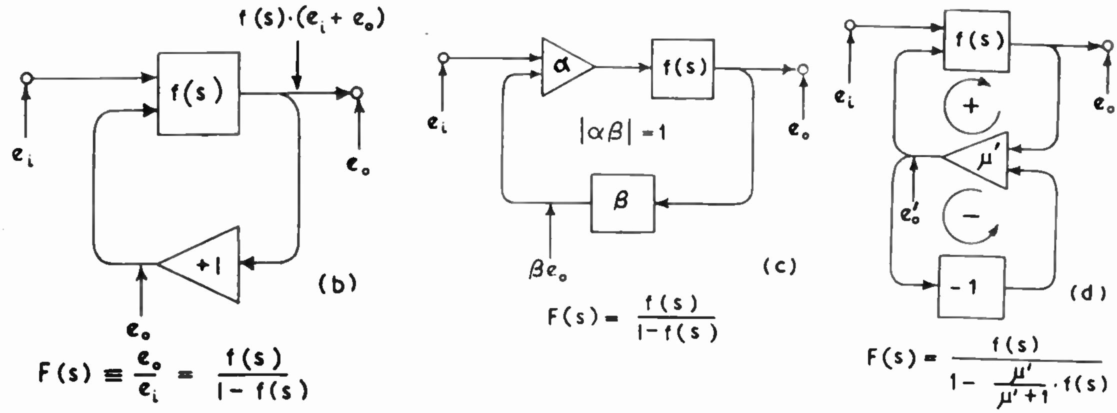


Fig. 1.—Functional diagrams of feedback systems:
(a) The basic feedback system
(b) Unit β positive feedback over shaping network $f(s)$;
(c) Unit $\alpha\beta$ positive feedback over cascade consisting of multiplier α and shaping network $f(s)$ i.e., over $\mu = \alpha.\ f(s)$
(d) As (b) with auxiliary feedback loop ($\mu = \mu'$, $\beta' = -1$) to set upper limit of unity to β.

2.5. The transformation expressed at (5) is particularly useful where very simple functions $F(s)$ are required as will now be shown for the important cases of integration and differentiation.

Exact integration involves obtaining $f(s)$ such that $F(s) = (\tau s)^{-1}$ where τ is a constant of dimension time; by (3)

$$f(s) = (\tau s)^{-1}/\{1 + (\tau s)^{-1}\} = 1/(1 + \tau s)\,..(6)$$

and can be realized exactly with

$$Z_1(s) = R,\ Z_2(s) = (Cs)^{-1}$$

or with

$$Z_1(s) = Ls,\ Z_2(s) = R.$$

Exact differentiation may be expressed

$F(s) = \tau s$; it needs (equation (3))

$$f(s) = \tau s/(1 + \tau s)\ \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

which is characteristic of an L-type network having

$$Z_1(s) = (Cs)^{-1},\ Z_2(s) = R,$$

or $Z_1(s) = R,\ Z_2(s) = Ls.$

The error incurred by these $f(s)$ networks used alone (i.e., in the absence of feedback) can approach zero only in the limiting case $\tau \rightarrow \infty$, with which the magnitude of the output $\rightarrow 0$. In effect, application of feedback overcomes this difficulty by cancelling the unit additive term in the denominator of the transfer function (equation (6), (7)).

The effect of applying feedback is most clearly illustrated by the behaviour of the pole-zero pattern representation of the network under varying β. Thus, with $Z_1(s) = R, Z_2(s) = 1/Cs$ (integrator) the pattern consists, in the absence of feedback, of a single pole at $s = -1/CR$ (See Fig. 2(*a*)). With increasing positive feedback, this pole moves along the abscissa and reaches the origin at $\beta = 1$. The case of the differentiator ($Z_1(s) = 1/Cs, Z_2(s) = R$) is similar except that a fixed zero occurs at the origin (see Fig. 2(*b*)). In the special case $Z_2(s) = R = 1$ the pole-zero pattern transfer function is the inverse of that of $Z_2(S)$ so that, with unit $\alpha\beta$, poles go to zeros and zeros to poles; this result is similar to that obtained with

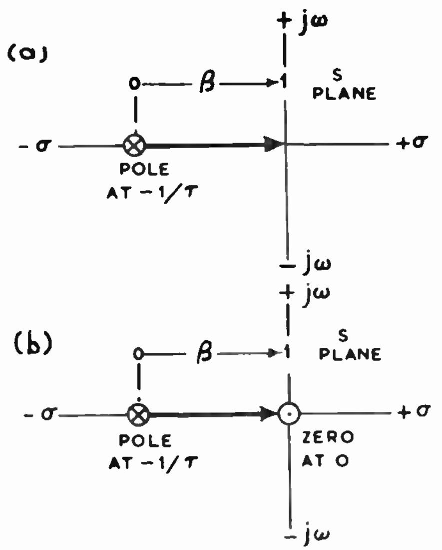


Fig. 2.—Pole-zero patterns of simple unit-β positive feedback operators: (a) Integrator; (b) Differentiator.

a negative feedback system having infinite μ, at which the system pattern is inverse to that of the β section.

2.6. Suitable L-type networks for achieving other simple operations than pure integration and differentiation are included in Table 1. In each particular case the possibility of instability should, of course, be investigated prior to embarking upon detailed circuit design by applying Nyquist's criterion to determine whether the equation

$$1 - f(s) = 0 \qquad \text{(8)}$$

has any roots in the positive real half of the complex frequency plane. Thus Beale and Stansfield (*loc. cit.*), having used two capacitors for anode-grid d.c. blocking in series with the feedback loop of an integrator, found it necessary to insert a third to achieve stability. The effect of any anticipated additional reactances should be included in $f(s)$ when considering equation (8).

2.7. In view of the stability problem it would appear to be preferable to use for β a voltage amplifier (μ') subject to 100 per cent. negative feedback ($\beta' = -1$) in order to make $\beta \to 1$ (more generally $1/\alpha$) as $\mu' \to \alpha$ rather than provide an available β of greater than 1 (or $1/\alpha$) together with an attenuator to allow β to be set exactly to 1 (or $1/\alpha$) as shown in Fig. 1(*d*). Apart from improved stability the required positive loop gain would be set up automatically thereby avoiding an inconvenient, and possibly ill-defined, manual adjustment. β would be held on the stable side of unity and the problem of stability would be largely transferred to the auxiliary negative feedback loop. Using equation (1), with μ' for μ and $\beta' = -1$, in place of $\beta = +1$, equation (3) then becomes:—

$$F(s) = f(s)/[1 - \mu'/(1+\mu')] \cdot f(s)$$
$$\to f(s)/[1 - f(s)] \qquad \text{(9)}$$

as $\mu' \to \infty$.

3.0. Basic Network

3.1. An alternative approach which brings out the principle of the method more clearly and leads more directly to practical circuits is illustrated by Fig. 3.

TABLE 1

Forms of Z_1, Z_2 for Given Operators

No.	Operator	Z_1*	Z_2*	α_1	α_2
1.	α_1/p	R	C	$1/CR$	—
		L	R	R/L	—
2.	$\alpha_1 p$	C	R	CR	—
		R	L	L/R	—
3.	α_1/p^2	L	C	$1/LC$	—
4.	$\alpha_1 p^2$	C	L	LC	—
5.	$\alpha_1/(\alpha_2+p)$	R	$C \times kR$	$1/CR$	$1/kCR$
		$L+kR$	R	R/L	kR/L
6.	$\alpha_1(\alpha_2+p)$	$C \times kR$	R	CR	$1/kCR$
		R	$L+kR$	L/R	kR/L
7.	$\alpha_1/p(\alpha_2+p)$	$L+R$	C	$1/LC$	R/L
8.	$\alpha_1 p(\alpha_2+p)$	C	$L+R$	LC	R/L
9.	$\alpha_1 p/(\alpha_2+p)$	$C+kR$	R	$1/k$	$1/kCR$
		R	$L \times kR$	k	kR/L
10.	$\alpha_1(\alpha_2+p)/p$	R	$C+kR$	k	$1/kCR$
		$L \times kR$	R	$1/k$	kR/L
11.	$\alpha_1(\alpha_2+p)/p^2$	L	$C+R$	R/L	$1/CR$
12.	$\alpha_1 p^2/(\alpha_2+p)$	$C+R$	L	L/R	$1/CR$

* The + sign, as in $L+kR$, denotes series connection. The × sign, as in $C \times kR$, denotes parallel connection.

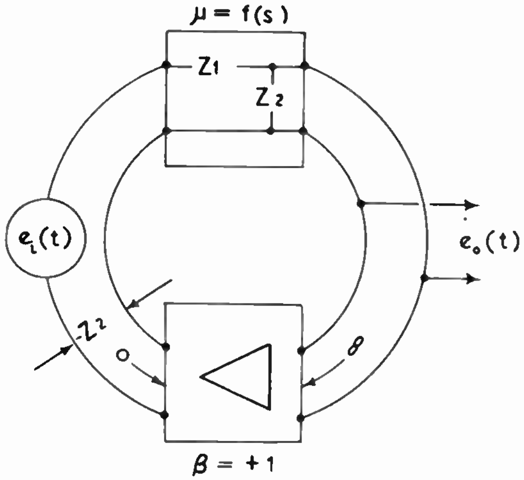


Fig. 3.—Practical scheme for the basic positive feedback operator network.

A two-terminal network is chosen which has a driving point admittance $Y_1(s) = Z_1^{-1}(s)$ of the same form as the voltage transfer function $F(s)$ required of the whole network, considered as a quadripole. Thus for integration $Y_1(s)$ must be proportional to s^{-1} and an inductor $(Ls)^{-1}$ would be suitable. With an applied input voltage e_i the inductor by itself would pass a current $i_i = Y_1(s) \, . \, e_i = (Ls)^{-1} \, . \, e_i$. In order to develop a proportional output voltage a resistor $Z_2 = R$ is inserted in series with L thereby giving $e_o = R \, . \, i_i$, and to prevent the back e.m.f. developed across R from introducing an integration error this e.m.f. is fed back in such a manner as to aid e_i.

The effect of such positive feedback is to introduce an element $-R$ into the mesh. Ideally the feedback network (β) should be a unidirectional unit-ratio transformer having infinite input impedance and zero output impedance, as can be realized in a feedback amplifier (notably of the cathode-follower type) subject to 100 per cent. negative feedback.

3.2. Alternatively $Y^{-1}(s)$ and R may be transposed with the result that the inverse transfer function is obtained. This arrangement is preferable where the source impedance, which will usually be substantially resistive, is not negligibly small compared with the series branch of the function control network.

3.3. More generally both branches of the control network may be functions of s, e.g. $Z_1(s)$, $Z_2(s)$, giving for the entire network a transfer function $F(s) = Z_2(s)/Z_1(s)$. The second derivative and second integral operators may then be obtained, as shown (Ex. 3, 4) in the table.

4.0. Typical Circuit Developments

4.1. With regard to the synthesis of practical circuit arrangements, it will be realized that amplifier-type valves may be used to advantage for buffering the function control network $f(s)$ from the source of input and the load circuit, and for obtaining the characteristics required of the network (Section 3.1), including β gain stabilization (Section 2.7) where desired, as well as facilitating combination of input and feedback voltages. In general, therefore, the network $f(s)$ may be assumed to operate in association with at least two valves, an input or driver valve (V1) and a feedback valve (V2), as in Fig. 4; in addition, further valves may be needed for impedance transformation or buffering.

4.2. In the light of the feedback aspect, as stated generally in Section 2.4, the design problem may be interpreted as of setting up a positive feedback amplifier containing a suitable operator network (as obtained from the Table) in a forward coupling and having unit gain from the output of the operator network back to its input. Avoiding the use of screens and suppressors as signal electrodes where V1 and V2 have more than one grid, the required polarity of feedback allows three possible feedback paths, as follows:—

(*a*) Anode of V2 to grid of V1.

(*b*) Cathode of V2 to cathode of V1.

(*c*) Cathode of V2 to anode of V1.

All three possibilities have been exploited as separate isolated circuit developments for particular purposes such as integration or differentiation, but the close relationship existing between them does not appear to have been recognized and no attempt to have been made to relate the basic principle in any particular case to general feedback theory.

4.3. The basic circuits are given in Fig. 4. Fig. 4(*a*) is a modification of Beale and Stansfield's circuit, the principal change being rearrangement of the grid circuit of V1 to allow a single-ended input. Figs. 4(*b*), 4(*c*) are substantially as used by Schmitt and Tolles for differentiation and by Newsam for integration, respectively. Arrangements (*a*) and (*b*) differ from (*c*) in that V1 is included in the feedback path so that its gain must be neutralized by insertion of an attenuator. Circuit (*c*) represents the most direct realization of the basic network of Fig. 3 and has the advantage over (*a*) and (*b*) of incorporating the negative feedback stabilization of β, since β is composed entirely of the cathode follower formed by V2. It has the disadvantages that β will be less than unity and (in certain cases) of needing a *CR* feedback coupling because of the wide disparity between V2 cathode and V1 anode circuit potentials.

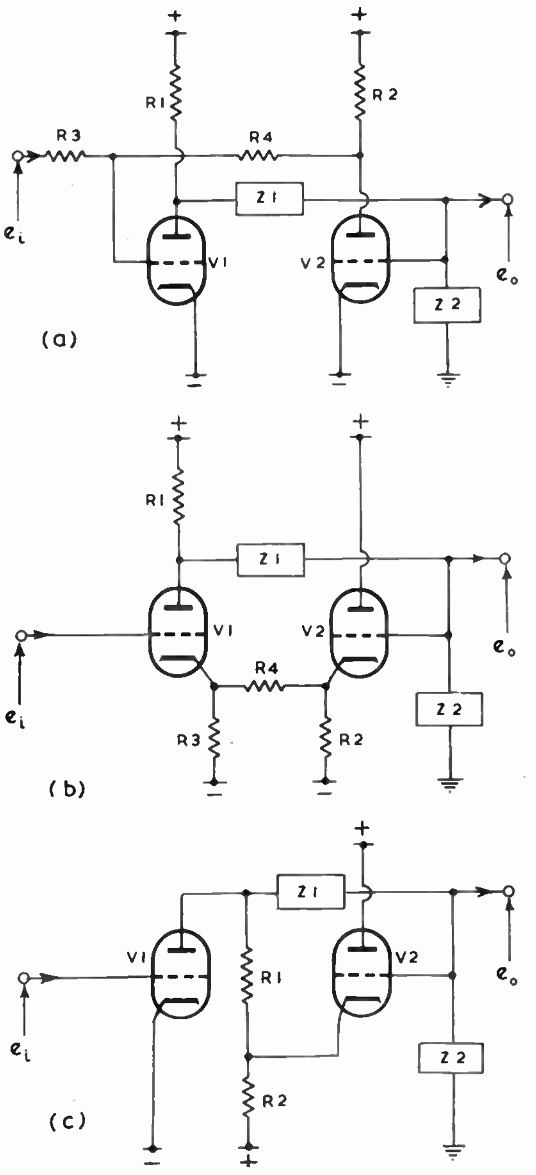


Fig. 4.—Skeleton circuits of three practical developments of Fig. 3: (a) After Beale and Stansfield; (b) After Schmitt and Tolles; (c) After Newsam and Williams.

In all three circuit developments the anode load of V1 should be negligible compared with Z_1 and the output impedance of V1; alternatively it may be preferable to insert a cathode follower between V1 and Z_1 and between Z_1 and the load; use of V2 as an output cathode follower generally incurs a waveform error as a result of forward transmission over the feedback path.

In the case of Fig. 4(*b*) V1 may be arranged as an anode follower in order to obtain the desired low output impedance.

4.4. With regard to the possibility of applying automatic gain definition and stabilization to arrangements (*a*) and (*b*), this is readily achieved in (*a*), but not in (*b*) without resorting to an additional (anode follower) stage. One method of application in case (*a*) is shown in Fig. 5 where the result is achieved by operating V2 as a phase splitter, i.e. with high and equal anode cathode loads to achieve a grid-anode gain of (very nearly) -1, and by arranging V1, V2 as a self-balancing paraphase pair to obtain an anode (V2) to anode (V1) gain of -1 (anode follower). Three feedbacks may be distinguished in the entire circuit, viz.:—

(i) Positive feedback over the path grid 2, anode 2, grid 1, anode 1, grid 2.

(ii) Negative current feedback from cathode to grid of V2, which improves the linearity of V2.

(iii) Negative voltage feedback from anode to grid of V1, which reduces the output impedance of V1 (anode follower).

5.0. Comparison with Negative Feedback Operators

5.1. It is, of course, equally possible to obtain $F(s)$ as the ratio of two branch impedances in networks, notably of the Miller type, which are conventionally regarded as cases of negative

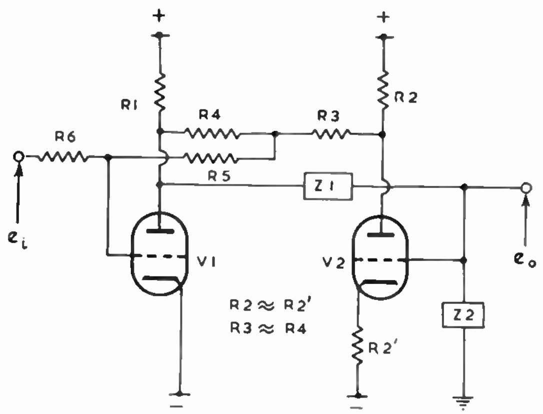


Fig. 5.—Application of negative feedback gain definition to circuit of type Fig. 4(a), cf. Fig. 1(d).

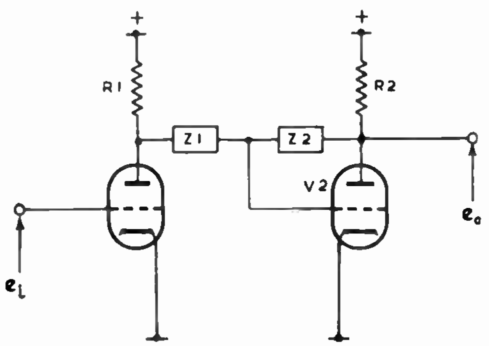


Fig. 6.—A negative feedback equivalent circuit of Fig. 4(c).

feedback. For the sake of completeness and to facilitate comparison with Fig. 4, an equivalent negative feedback circuit is shown in Fig. 6. Anode (V1)-to-anode (V2) gain tends to $-Z_2/Z_1$ as μ of V2 $\rightarrow \infty$ so that Table 1 is applicable in choosing Z_1, Z_2 for a desired $F(s)$. This correlation, and other correspondences such as the pattern inversion mentioned in paragraph 2.5, suggest a general equivalence between a negative feedback amplifier with infinite μ and a positive feedback amplifier having unit $\alpha\beta$, which will be discussed in greater detail in a subsequent paper.

6.0. Acknowledgments

This paper is published by permission of E.M.I. Research Laboratories, Ltd. and is a modified version of Report No. RM/40 of the same title, dated February, 1951.

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8.0. Appendix

Analysis of Basic Circuit Types

Analysis of the circuits given is straightforward and yields the following results:—

Type (a) (cf. Fig. 4(a))

With $R_2 \ll (R_3 + R_4)$ (arbitrarily) and $R_1 \ll (Z_1 + Z_2)$ (desirably):—

$$\mu = \frac{-u_1 R_1 R_4 Z_2}{r_1 (R_3 + R_4)(X_1 + Z_2)} = \alpha \cdot \frac{Z_2}{Z_1 + Z_2}$$

$$\beta = \frac{-u_2 R_2 R_3}{r_2 (R_3 + R_4)}$$

The condition for $F = \mu/(1 - \mu)$ to reduce to $\alpha \cdot Z_2/Z_1$ i.e., $\alpha\beta = 1$, gives

$$\frac{R_3 R_4}{(R_3 + R_4)^2} = \frac{u_1 u_2 R_1 R_2}{r_1 r_2}$$

Putting $r_1 = r_2 = r$, $u_1 = u_2 = u$, $R_3/R_4 = m$ gives

$$\frac{m}{m^2 + 2m + 1} = \frac{r^2}{u^2 R_1 R_2}$$

$$\text{or } m^2 + \left(2 - \frac{u^2 R_1 R_2}{r^2}\right) m + 1 = 0$$

which gives a solution for m in terms of

r, u, R_1, R_2

Type (*b*) (*cf. Fig.* 4(*b*)).

In checking the capability of this type of circuit of providing a system gain of the form $\alpha . Z_2/Z_1$ it is convenient to use $R = R_2 R_3/(R_2 + R_3)$ with $R_4 = 0$ and introduce attenuation by tapping $1/n$th of e_o from Z_2.

Then, with $u_1 = u_2 = u$, $r_1 = r_2 = r$ and $R_1 \ll (Z_1 + Z_2)$:—

$$\mu = \frac{ur' R_1}{r'(r + R_1) + (u + 1) rR} \cdot \frac{Z_2}{Z_1 + Z_2},$$

where $r' = r + (u + 1) R$,

$$\mu = \alpha . \frac{Z_2}{Z_1 + Z_2}$$

$$\beta = \frac{(u + 1)}{nr'}$$

The condition for $F = \mu/(1 - \mu\beta)$ to reduce to $\alpha . Z_2/Z_1$, i.e., $\alpha \beta = 1$, gives

$$n = \frac{u(u + 1) RR_1}{r'(r + R_1) - (u + 1) rR}$$

Type (*c*) (*cf. Fig.* 4 (*c*)).

Assuming:—

(1) $R_1 \ll r_1$ (i.e., V1 a constant current source),

(2) $u_2 \gg 1$ (i.e., V2 a perfect cathode follower —unit gain, zero output impedance),

(3) $R_1 \ll (Z_1 + Z_2)$, as before,

then the circuit equations reduce to

$$e_o(t) = \frac{Z_2}{Z_1 + Z_2} \{g_1 R_1 . e_i(t) + e_o(t)\}$$

whence

$$\frac{e_o(t)}{e_i(t)} = g_1 R_1 . \frac{Z_2}{Z_1}$$

which is of the form $(\alpha . Z_2/Z_1)$ required.

In this case $g_1 R_1$ is not a factor of μ of the feedback loop, so that $\alpha = 1$.

Negative Feedback Circuit (*cf. Fig.* 6).

The gain of this comparative arrangement may be obtained from the appendix of Ref. 6 as:—

$$F = \frac{u_1 R_1}{r_1} . \frac{Z_2}{Z_1}, \quad R_1 \ll Z_1, \quad R_1 \ll r_1$$

Since V1 is not included in the feedback loop, $\alpha = 1$.

RAPID COIL CALCULATIONS FOR MAGNETIC DEVICES*

by

A. E. Maine (*Associate*)†

SUMMARY

The relationships existing between the various physical and electrical quantities encountered in the design of coils for electromagnetic devices are derived by setting up simple equations. These are evaluated and the results displayed in nomographical form, thus providing a means whereby the values of all the important parameters may be settled quickly and with sufficient accuracy for most engineering purposes. The question of the power required, and the current density involved in producing a specified value of ampere turns in a winding, is given special emphasis. Although the charts are primarily intended for the solution of coil problems associated with solenoid-operated devices, this by no means defines their scope. In general it is true to say that the nomograms may be applied usefully to the design of practically any coil winding.

SYMBOLS

A = Window area.
A_1 = Effective winding section area.
a_1 = Conductor section area.
a_p = Interleaving section area/sq. in.
d_1 = Conductor diameter—bare.
d_2 = Conductor diameter—insulated.
E = Voltage.
F = Correction factor to "ρ"
h = Coil length.
I = Current.
I_D = Current density.
K = Ohms per cubic in.
l = Conductor length.
n = Conductors per sq. in.
R = Resistance.
r_1 = Inside radius of coil.
r_2 = Outside radius of coil.
ρ = Specific resistance of conductor material.
S = Additive space factor.
T = Turns.
t_m = Mean turn length.
t_h = Interleaving material thickness.
U = Utilization factor = d_2^2/d_1^2
V = Volume of coil.
W = Watts.
$Wt.$ = Weight.
W_f = Weight factor.

* Manuscript received June 4th, 1951.
† de Havilland Propellers, Ltd.
U.D.C. No. 518.3 : 621.318.3.

1. Introduction

A coil design may proceed from any one of several directions according to which of the many quantities involved assumes special importance. It is rarely possible to treat the coil more or less as a separate entity, instead, it is usual for several quantities to be fixed at the outset by the requirements of the associated magnetic circuit. Available space, permissible weight, and power economy are prominent amongst the many factors which will ultimately decide the form that the coil will take. In view of this, it is, of course, impossible to lay down any rigid design processes general to all coils. Nevertheless, by means of setting up simple equations in terms of the ratios between certain selected quantities, some measure of generality may be achieved. These equations, in themselves, tend to lead to a simplification of the work involved merely on account of their approaching the final result more directly. To further reduce arithmetic tedium, these expressions and certain others are plotted out in the form of nomographs and these, together with correction charts, enable most designs to be worked out very quickly.

2. Theoretical Basis

Before proceeding to the use of the design charts, it is instructive to review their algebraic basis and at the same time consider other expressions relative to coil design. The mean turn length

$$t_m = \pi(r_1 + r_2) \quad \text{.....................(1)}$$

Neglecting space factors for the moment, the total turns

$$T = \frac{h(r_2 - r_1)}{d_2^2} \quad \text{.................... (2)}$$

$$R = \frac{T t_m \rho}{a_1} \quad \text{.................... (3)}$$

Writing E/I in place of R, and re-arranging for a_1, we have an expression for the conductor section area:—

$$a_1 = \frac{IT}{E} \cdot \rho t_m \quad \text{.................... (4)}$$

This equation is valuable in the initial stages of many designs as it readily determines the winding gauge number. A second design equation is deduced as follows:—

Substituting (1) and (2) into (3):—

$$R = \frac{h(r_2^2 - r_1^2)\pi\rho}{d_2^2 a_1} \quad \text{.................... (5)}$$

Multiplying (2) by I, to give ampere turns and (5) by I^2 to give power, dividing these out we get:—

$$\frac{IT}{W} = \frac{a_1}{\pi(r_1 + r_2)\rho I}$$

Writing a_1/I as $1/I_D$ we have

$$\frac{IT}{W} = \frac{1}{\pi(r_1 + r_2)\rho I_D} \quad \text{.................... (6)}$$

The equation is made general for coils of any shape in the form:—

$$\frac{IT}{W} = \frac{T}{E} = \frac{1}{t_m \rho I_D} \quad \text{.................... (7)}$$

It is of interest to note that alternatively, this equation may be derived from equation (4) by re-arranging and multiplying through by $1/I$.

The ampere turn per watt ratio may be regarded in a sense as a figure of merit for a particular design and application; furthermore, in many cases it introduces the current density factor at an early stage in design. This is valuable when overall rating is being considered before the main body of the design has proceeded.

The value of ampere turns required is frequently known at the beginning, and knowledge of this, together with the value of the current density selected, enables design to proceed from this standpoint. These two quantities determine winding cross-sectional area on the basis of square section conductor. It is necessary to convert this to the more usual case of round section wire by multiplying by $4/\pi$, hence:—

$$A_1 = \frac{4IT}{\pi I_D} \quad \text{.................... (8)}$$

This expression describes the wholly impractical case of close-wound bare wire. The winding section area is obtained by multiplying equation (8) by a factor U. The window area follows by adding a factor S.

The factor U is the ratio between the squares of the insulated and bare conductor diameters and varies with the covering material and also the gauge of the wire. The factor S makes allowance for the space occupied by the coil former and additional material inserted into the coil, such as paper interleaving, binding, outgoing leads, etc. Hence:—

$$A = \frac{4ITU}{\pi I_D} + S \quad \text{.................... (9)}$$

The various components making up the factor S must be calculated from their respective dimensions. As regards interleaving, however, a simple algebraic expression may be used, which is deduced as follows:—

Let n = the number of insulated wires per sq. in., and assume these to be arranged in two layers and interleaved. The interleaving section area is therefore:—

$$a_p = \frac{n d_2 t_h}{2}$$

n, however, is equal to $1/d_2^2$, hence paper section area per sq. in. of conductors is given by:—

$$a_p = \frac{t_h}{2d_2} \quad \text{.................... (10)}$$

The total paper area for a coil of fixed dimensions is, then:—

$$A_1\left(\frac{a_p}{1 + a_p}\right) \quad \text{.................... (11)}$$

The amount of paper calculated on this basis is too small in a practical case as it is usual to provide an overlap of up to 50 per cent. The paper, however, tends to be compressed into the spaces around the wires, hence a multiplying factor of 1·2 to 1·3 should be used.

The total resistance of a winding is calculated in the usual way using an equation such as (3). With regard to the weight of the winding, it is convenient in the construction of the design charts to calculate this in two parts: firstly the weight of the conductor, and secondly the weight of the insulant. The latter part, which may be comparatively large for a coil wound with fine wire, is calculated from:—

$$\text{Wt.(Insulant)} = la_1(u - 1) \times \text{Sp. Wt. (Insulant)} \quad \ldots\ldots(12)$$

Often the property of a coil which assumes particular importance for a given application is its electrical resistance. In this case a concept which is useful to adopt, is that of resistance per unit volume—more usually ohms per in³. This unit is actually the product of t.p.s.i. (for a given conductor size) and ohms per linear inch. Assuming once again the case of close wound bare wire,

$$K = \Omega/\text{in.}^3 \frac{4\rho}{\pi d_1^4} \quad \ldots\ldots(13)$$

For a given gauge, but insulated wire, n becomes smaller hence the value of K drops. Also if the expression is worked out for some value of ρ, a correction must be made for other values. In the case of the design charts, equation (13) becomes general for all conductor materials and insulant thicknesses by multiplying by F/U, where F is the ratio between the specific resistance being considered and that of a material arbitrarily selected (actually, ρ for copper given in B.S.128) hence,

$$K = \frac{4\rho F}{\pi d^4 U} \quad \ldots\ldots(14)$$

It follows then, that for a coil of fixed dimensions, $R = V/K$ (allowance in V being made for S).

3. Descriptions of the Charts

The foregoing outlines the principles upon which the charts are constructed and indicates some of the main processes involved. It now remains to consider one or two practical examples, after first having considered the construction of each chart in some slight detail.

3.1. *Chart* 1

This chart evaluates equations (4), (7) and (8). The value of ρ is taken as $6{\cdot}67 \times 10^{-7}$ Ω/in^3 (B.S.128). For evaluating equation (4), lay a rule between the value of t_m on scale D and T/Ω on scale J. The correct value for a_1 appears where the rule intersects the F scale. Projecting horizontally from this point through scales G and H gives the corresponding S.W.G. number and the optimum paper interleaving thickness if this material is to be used. For solving equation (7), a rule is placed between the appropriate points on scales A and D. The value of IT/W appears at the intersection of the rule with scale B. Equation (8) is evaluated by placing a rule between I_D on scale A and IT on scale E. The answer is read off from scale C.

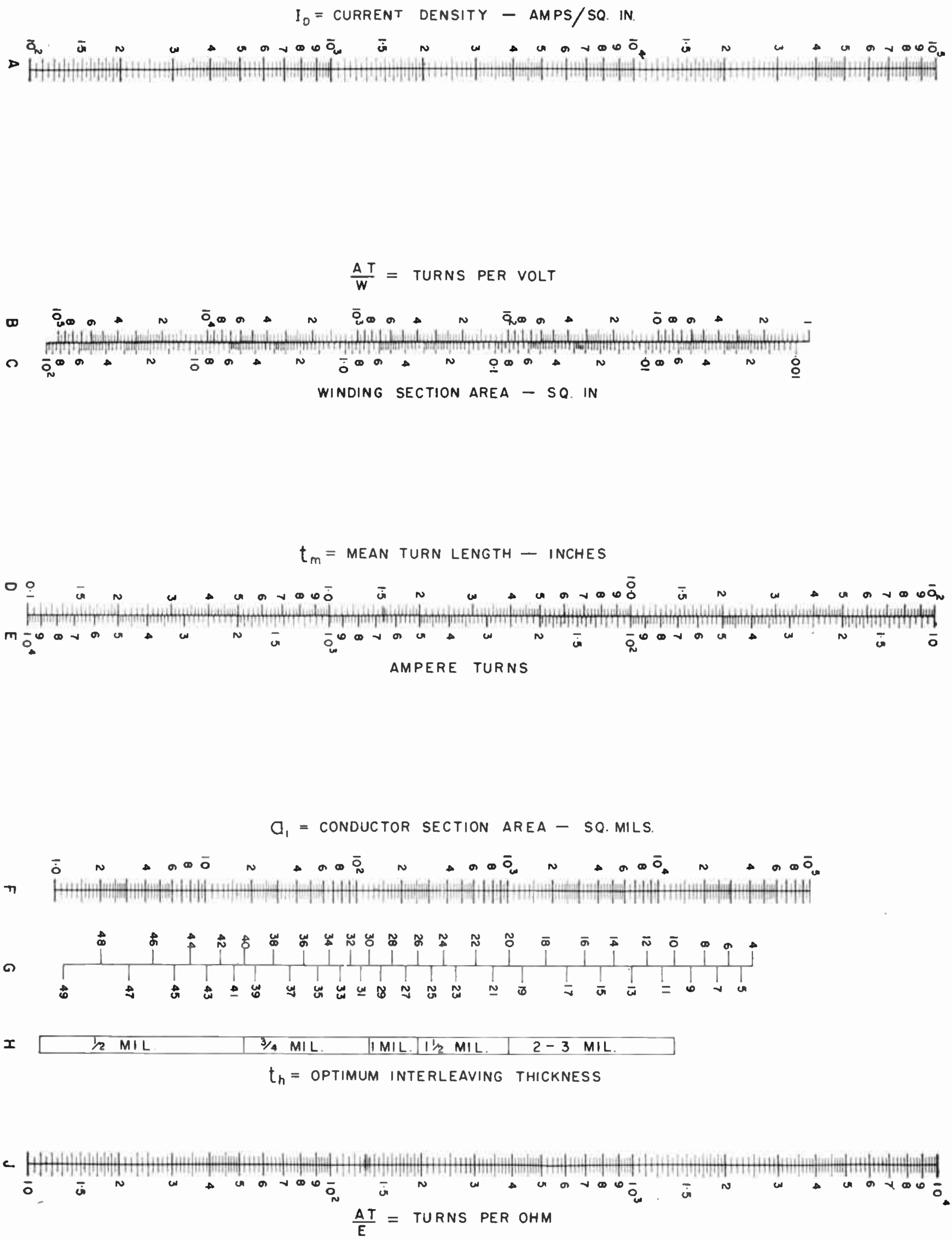
3.2. *Chart* 2

This chart evaluates equation (14) which gives the value of ohms per in³. for any gauge of wire from 5 to 47 S.W.G. Before the chart can be used, reference must first be made to Chart 4 in order to find the value of U for a particular gauge number and type of covering. Reference must also be made to the "F" factor scale on this chart if a value of specific resistance other than that specified in B.S.128 is used. To use Chart 2, lay a rule between the gauge number and the value of U on the short mid-way scale. The opposite main scale is used merely as a reference line from which to project back through the mid-way scale, this time using the F factor, and the answer appears on the first used main scale. For example, project from A through C (reading the value of U) to D, project back through C (reading F) read answer at B. The sequence of dealing with U first and F afterwards must always be adhered to.

3.3. *Chart* 3

This chart evaluates equation (3) and also determines conductor and insulant weight. To find the value of resistance lay a straight-edge between t_m on scale B and T on scale D. The corresponding value of l appears at the intersection with scale H. From this point project back through the appropriate conductor size at scale E. The value of resistance is read off scale A. To find conductor weight, proceed as before to find l on scale H, from here project a line to wire size on scale K. Between the intersection of this line and reference line—scale J, project through the weight factor scale (G) and read off the answer on scale C. For insulant weight, repeat the foregoing, but use a new point on the weight factor scale appropriate to the insulant. Read off the weight on scale C *and multiply by* $(U - 1)$. It is pointed out here that the calculation of insulant weight involves assumptions that cannot be fully met in practice, i.e., constant cross-section and density. Nevertheless, values obtained from the chart agree fairly well with published data and measured samples, differences above 10 per cent. being rare.

CHART 1


I_D = CURRENT DENSITY — AMPS/SQ. IN.
A
$\frac{AT}{W}$ = TURNS PER VOLT
B
C
WINDING SECTION AREA — SQ. IN
t_m = MEAN TURN LENGTH — INCHES
D
E
AMPERE TURNS
a_1 = CONDUCTOR SECTION AREA — SQ. MILS.
F
G
H
½ MIL.
¾ MIL.
1 MIL.
1½ MIL.
2 – 3 MIL.
t_h = OPTIMUM INTERLEAVING THICKNESS
J
$\frac{AT}{E}$ = TURNS PER OHM

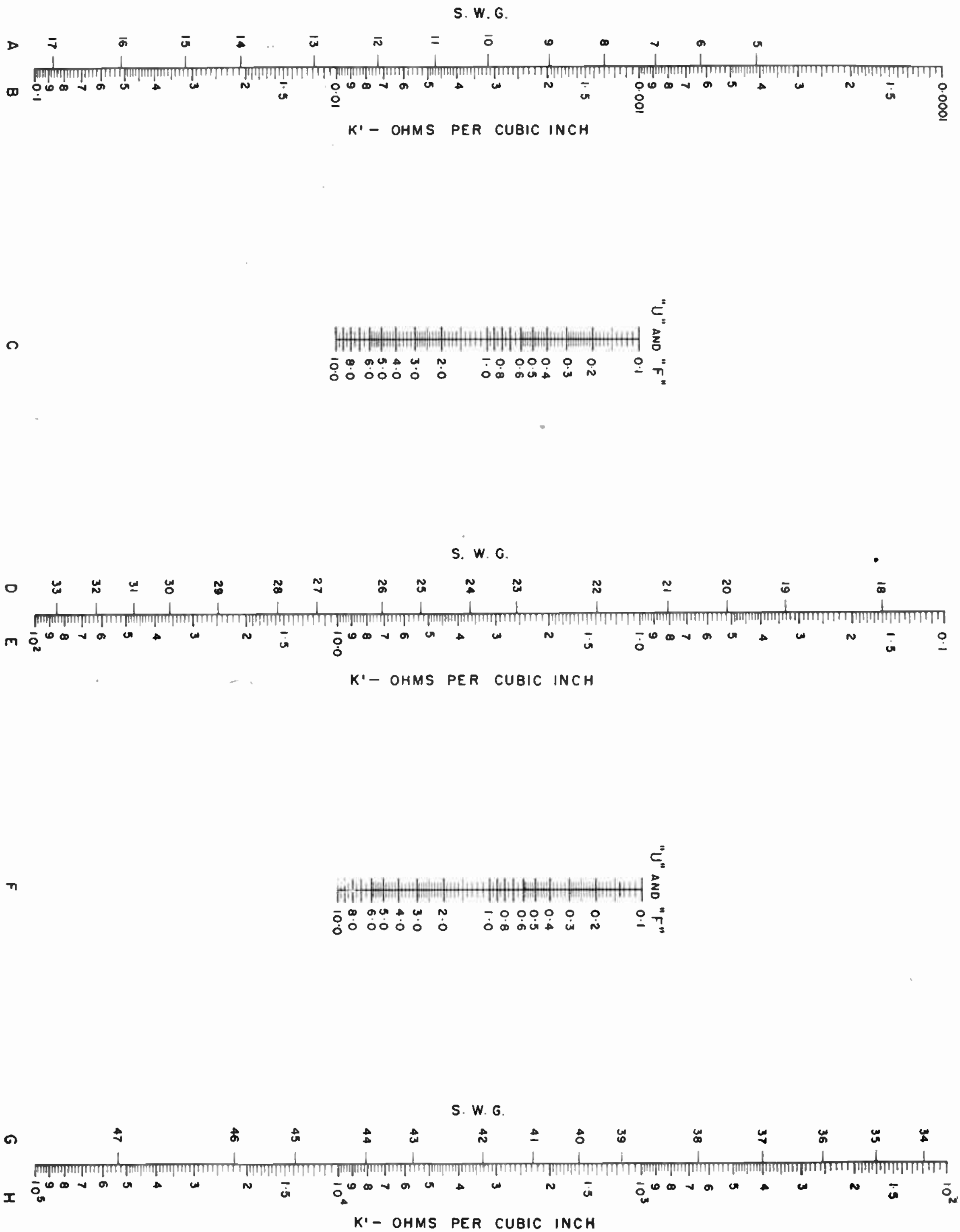

S. W. G.
A
B
K' – OHMS PER CUBIC INCH
C
"U" AND "F"
S. W. G.
D
E
K' – OHMS PER CUBIC INCH
F
"U" AND "F"
S. W. G.
G
H
K' – OHMS PER CUBIC INCH

CHART 2

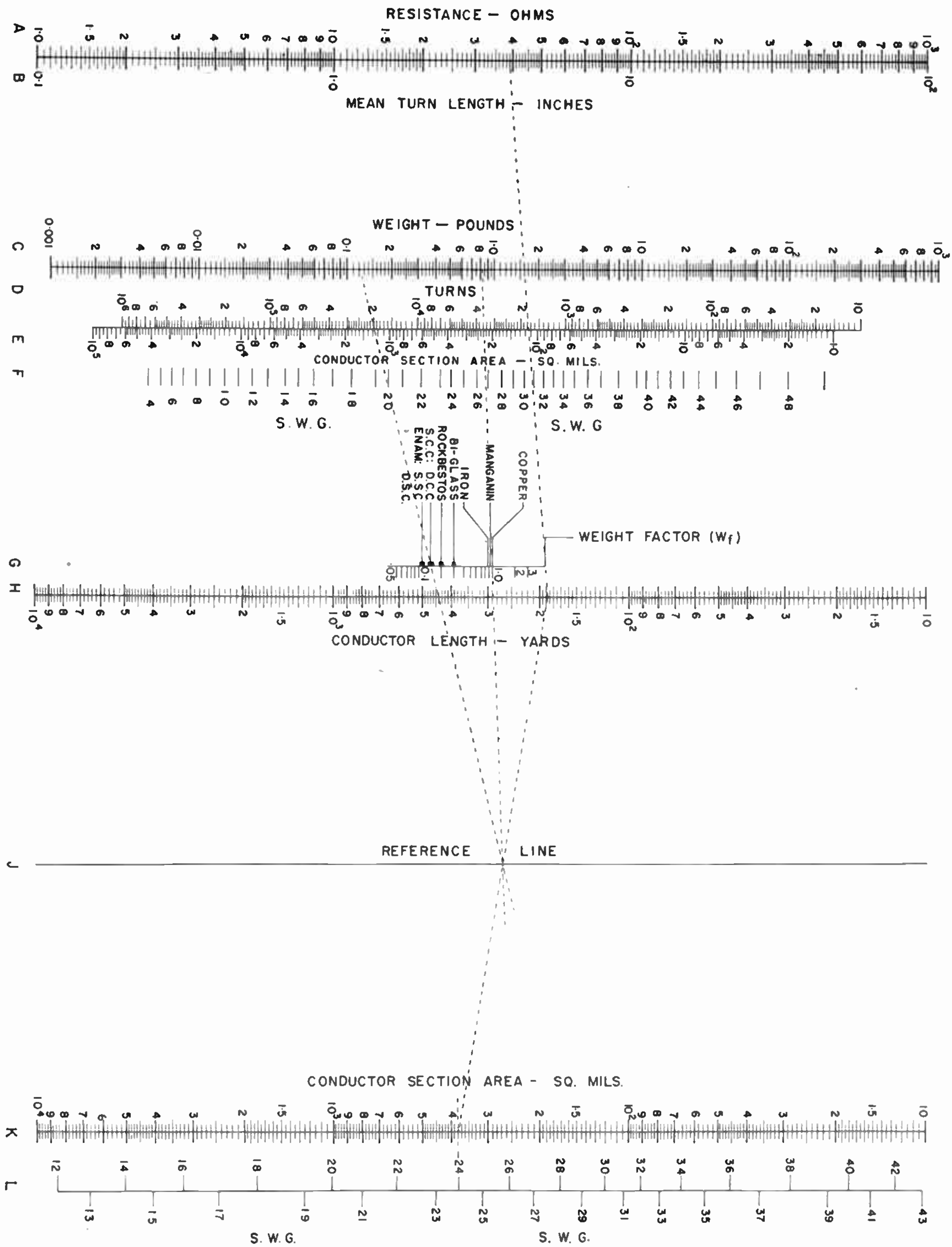
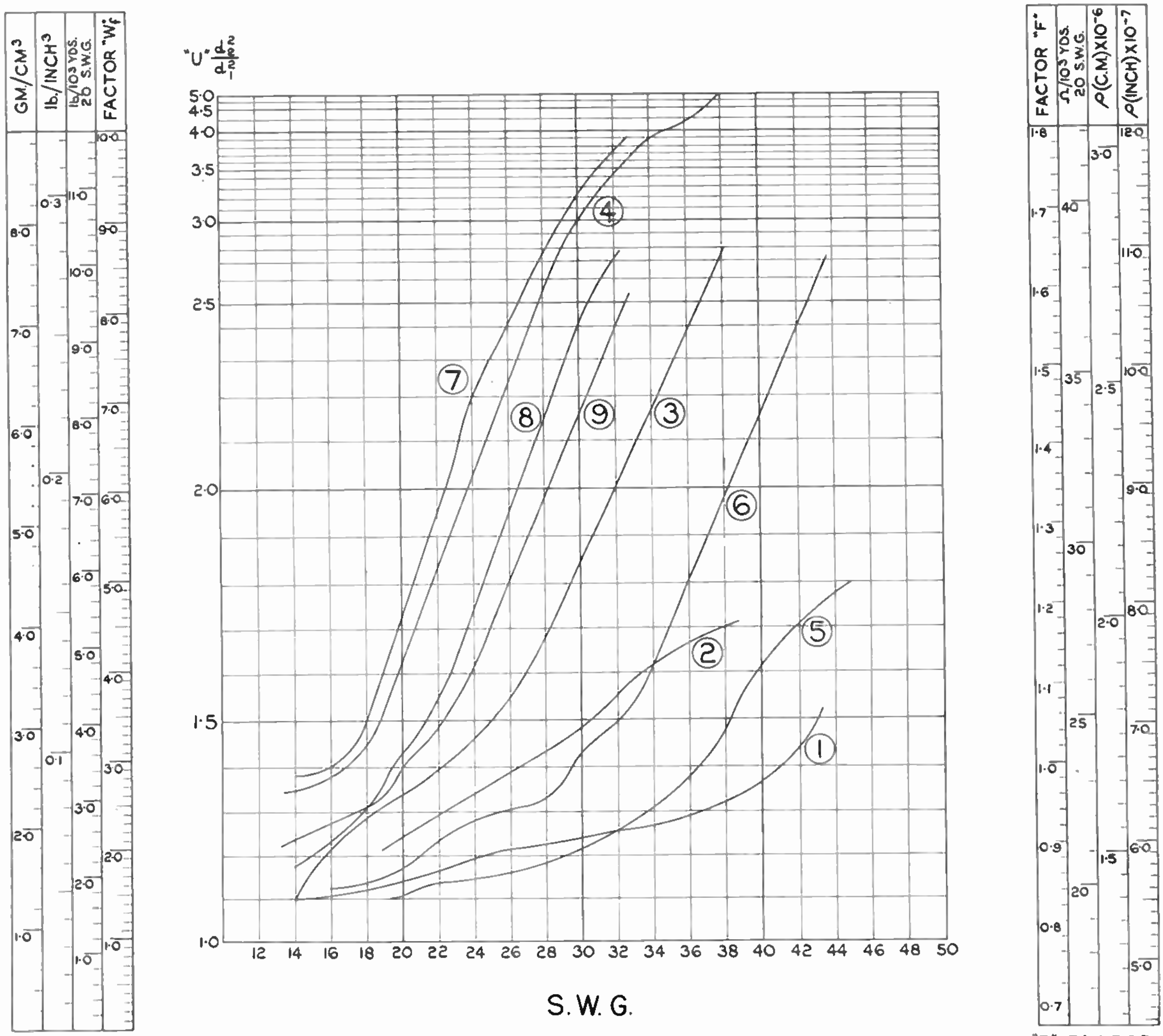

RESISTANCE — OHMS
MEAN TURN LENGTH — INCHES
WEIGHT — POUNDS
TURNS
CONDUCTOR SECTION AREA — SQ. MILS.
S. W. G.
S. W. G
IRON
BI-GLASS
ROCKBESTOS
S.C.C. D.C.C
ENAM. S.S.C
D.S.C
MANGANIN
COPPER
WEIGHT FACTOR (Wf)
CONDUCTOR LENGTH — YARDS
REFERENCE LINE
CONDUCTOR SECTION AREA - SQ. MILS.
S. W. G.
S. W. G.

CHART 3

3.4. *Chart* 4

This chart comprises two conversion scales for finding the F and W_f factors for use with the various equations outlined. These scales are laid out in terms of inches and metric units and also in terms of ohms and pounds weight per 1,000 yd. of 20 S.W.G. bare wire. The latter-mentioned units are useful when adapting information obtained from wire tables, etc., for use with the design charts. Chart 4 also displays the value of U on a square-law basis for a range of wire gauges and for nine different types of insulant covering. This chart is based on information kindly supplied by Messrs. British Insulated Callenders Cables, Ltd. The types of covering and graph line identification are listed below.

1. Enamel Oil Base.
2. Enamel Bicolon "T".
3. Single Cotton Covered.
4. Double Cotton Covered.
5. Single Silk Covered.
6. Double Silk Covered.
7. "Rockbestos."
8. Bi-glass Ordinary.
9. Bi-glass Fine.



"Wf" FACTOR SCALE.

"U" FACTOR CHART.

"F" FACTOR SCALE.

CHART 4

4. Applications

Example 1

A solenoid with an effective winding area of 1·8 in.^{2} and mean turn length of 3·9 in. is required to produce 1,500 A turns at 10 V. Find complete specification for a suitable winding using D.C.C. wire.

Using Chart 1, scales D, J and F, find a_1. The value for a_1 turns out to be 390 sq. mils. Projecting to the right, the nearest gauge, in this case 24, is given on scale G. If paper interleaving were required, scale H shows the optimum thickness to be 1½ mils. Having selected 24 S.W.G. it is necessary to work back and find the slightly modified value of T. The chart shows this to be 146. In other words, with this size of coil and selected voltage only 1,460 A turns are possible. From Chart 4, "U" for 24 S.W.G.—D.C.C. is shown to be 2·03. The conductor winding area is given by dividing the effective area by "U", in this case the result is 0·887. Returning to Chart 1 and using scales E and C read I_D on scale A; in this case the current density is 2,070 A/sq. in. Using this value and t_m on scale D, read off $IT/W = 185$ on scale B. From this last result W is calculated and found to be 7·89 W. The current and the resistance are easily found, using Ohm's Law and are ·789 and 12·68 respectively. From the IT/W ratio, the turns work out at 1,850. Transferring to Chart 3 the conductor weight is given as ·83 lb. and the insulant weight is ·129 lb. The total wire weight is thus ·96 lb. Summarizing, the specification is as follows:—

IT = 1,460 A turns.
I_D = 2,070 A in.^{2}
I = 0·789 A.
W = 7·89 W.
R = 12·68 Ω.
T = 1,850 turns.
S.W.G. = 24 D.C.C.
Copper Wt. = 0·83 lb.
Total Wt. = 0·96 lb.

The above figures are correct to within the reading accuracy of the charts, namely 2 per cent. maximum error. In practice, only small modifications, if any, need be made to these results.

Example 2

What effective winding sectional area is required for a coil with a mean turn length of 3 in. in order to produce a resistance of 180 Ω using 32 S.W.G. "Rockbestos" covered wire having a resistance of 40 Ω per 1,000 yd. for 20 S.W.G. wire ?

From Chart 4, F is given as 1·74 and U = 3·72. Using these values in association with Chart 2, K = 29 Ω/in.^{3} V = 180/29 = 6·2 in.^{3} Dividing this by t_m yields the effective winding area, namely, 2·07 in.^{2}

5. Conclusions

It is hoped that the two examples chosen may have given an idea of the method of using the charts when applied to a practical case; it is pointed out, however, that the examples considered are particular ones and are intended to show the use of the three parameters, turns per Ω, turns per V and Ω/in.^{3} In general, different techniques are required for designs proceeding from other standpoints, but the user of the charts should find little difficulty in adapting and interpreting the charts to suit his own needs.

6. Acknowledgments

In conclusion the author wishes to express his thanks to his Company, de Havilland Propellers, Ltd., who gave permission to publish the charts and to British Insulated Callenders Cables Company, Ltd., who provided helpful technical data and advice.

THEORETICAL PERFORMANCE OF SIMPLE MULTI-CHANNEL SYSTEMS USING FREQUENCY MODULATION*

by

E. G. Hamer, B.Sc.Eng.(Hons.) (*Associate Member*)†

SUMMARY

Formulae normally used for multiplex systems using large numbers of channels are extended to cover systems using small numbers of channels, and the results expressed graphically. The improvements due to non-simultaneous loading in the F.D.M. case or companding in the P.A.M. case are discussed, and typical values suggested.

1. Introduction

The theoretical performance with respect to fluctuation or random noise of various types of multichannel systems when used in conjunction with radio links has been dealt with extensively in previous papers. In nearly all cases the comparisons have been made between systems using large numbers of channels, and wide radio frequency bandwidths. Usually the performance is expressed in graphical form, these graphs not being readily interpolated for systems with small numbers of channels. It is important that when comparing systems the same basic assumptions are made, and the ones used in the cases to be discussed as those made by Feldman and Bennet.[1]

These are:—

The radio frequency signal bandwidth is that portion of the signal spectrum which must be preserved in order to make the received signal sufficiently undistorted. In the case of a system using frequency modulation, this is assumed to be the peak-to-peak frequency swing plus twice the highest modulating frequency.

Interference is assumed to be due only to fluctuation noise, and the effects of man-made noise, echoes, etc., are not treated. When expressing the audio-to-noise ratio in a channel, the total noise power in the channel is compared with the level of a test tone which would fully load the channel. No account is taken of the psophometric weighting curve which is very often used in connection with practical tests on multichannel systems.

The two multiplex systems in most general use for small numbers of channels are:—

(*a*) Frequency Division Multiplex (referred to as F.D.M.). In this case the available spectrum is shared on a frequency basis, each channel, being heterodyned with a suitable oscillator and one sideband suppressed, is fitted in its appropriate part of the spectrum. Such systems often use filters to combine or select the various channels.

(*b*) Pulse Amplitude Multiplex (referred to as P.A.M.). The whole of the available spectrum in this case is used for every channel, and the sharing is made on a basis of time. Each channel is allocated a given amount of the total time, and the samples from each channel are sent consecutively. Such systems often use gating circuits to combine or select the various channels, and their performances when used in conjunction with frequency modulated radio links are evaluated.

Each of these systems has certain advantages; in the F.D.M. case advantage can be taken of the "non-simultaneous load advantage," and in the P.A.M. case the advantage of instantaneous companding may be used. These factors will be discussed later in connection with the systems to which they are applicable.

2. Frequency Modulated Receiver

The performance of the receiver may be evaluated for the signal which just safely exceeds the threshold below which fluctuation noise would cause failure of the system. This minimum received signal power is termed the "Marginal Power," and has been evaluated for various receiver bandwidths. In determining the marginal powers for the various bandwidths the following assumptions have been made:—

(*a*) The peak noise voltage never exceeds the r.m.s. noise voltage by more than 12 db.

* Manuscript first received July 9th, 1951, and in final form October 12th, 1951.

† Research Laboratories of The General Electric Co., Ltd., Wembley, England.

U.D.C. No. 621.395.43:621.396.619.13.

(*b*) The r.m.s. thermal noise power has been assumed to be −204 db below 1 W per cycle of bandwidth.

(*c*) The noise bandwidth (assumed rectangular in shape) is equal to the radio frequency signal bandwidth.

(*d*) A receiver noise factor of 10 db has been assumed, and a safety factor of 3 db has been added.

The lower curves of Figs. 1 and 3 show the marginal power plotted against bandwidth for a frequency modulated receiver, whilst the upper curves show the audio/noise ratio in the worst channel for various types of systems, using the formulae given in Appendices 1 and 2.

3. The F.D.M.-F.M. System

Any multichannel system using frequency division multiplex has a "non-simultaneous load advantage"; this means that the peaks of modulation in the various channels very rarely occur simultaneously. Under these conditions the equipment may be designed so that its peak power handling capacity is less than the sums of the individual peak powers in the various channels. Overloading will occur, but it will only be for a very small percentage of the total time. The actual ratio of system peak power to individual channel power requires a careful statistical analysis of the percentage of time when overload occurs, with each channel in normal use. For small numbers of channels the peak system power remains nearly constant, whereas for larger numbers of channels the system peak power is approximately the product of the square root of the number of channels multiplied by the average individual channel power. The following figures have been extracted from reference 1:—

No. of channels	1	10	100	500	1,000
Required Relative Power Capacity (db)	0	+6	+9	+13	+16
Loading Advantage (db)	0	14	31	41	44

It is again emphasized that overloading can and does occur, but it will only be for a very small percentage of the total time, and may be neglected.

From these figures the fractional deviation per channel may be derived and the audio signal-to-noise ratio for the highest frequency channel determined. The audio-to-noise ratio at or above marginal signal level is improved by:—

(*a*) The reduced post-detector bandwidth,

(*b*) The normal F.M. improvement due to the triangular noise spectrum,

(*c*) The fact that any channel only occupies part of the total post-detector bandwidth.

These improvements are derived in Appendix 1 and are added to the original carrier/noise ratio at marginal signal level (i.e. 12 db) to obtain the audio/noise ratio in the channel concerned.

The audio/noise ratio for marginal signals has been evaluated for various bandwidths, and numbers of channels and the results are presented in graphical form in Fig. 1, using the expressions derived in Appendix 1 for the F.D.M.-F.M. system. If the received carrier power is greater than the marginal power, the audio/noise ratio will be improved by a similar amount. In all cases the total signal bandwidth has been assumed to be the peak-to-peak deviation plus twice the highest modulating frequency. With this assumption, almost all of the sideband power will fall inside the receiver radio frequency signal bandwidth, and the remainder may be neglected. When the radio frequency bandwidth is small the allowable deviation falls rapidly, and the minimum bandwidth which can be used in normal designs of equipment is determined by the highest modulating frequency.

4. The P.A.M.-F.M. System

A time division multiplex system has the advantage that instantaneous compression and expansion may be employed. Companding may be applied to any system, and is very often done manually in the case of broadcast transmissions. In this case the programme engineer increases the level of the small audio signals, and, to avoid overloading or over-modulation, decreases the level of the large audio signals. To some extent this can be done automatically by using automatic gain controls, but severe distortion may take place with rapidly varying audio signals. Where pulse multiplex systems are used, instantaneous compression and expansion may be used and will cause no additional distortion. This does not require any increase of bandwidth since the audio-to-noise ratio of small amplitude audio signals is improved at the expense of the larger amplitude audio signals. No improvement to the system as a whole has resulted, but the audio-to-noise ratio has been made less dependent on

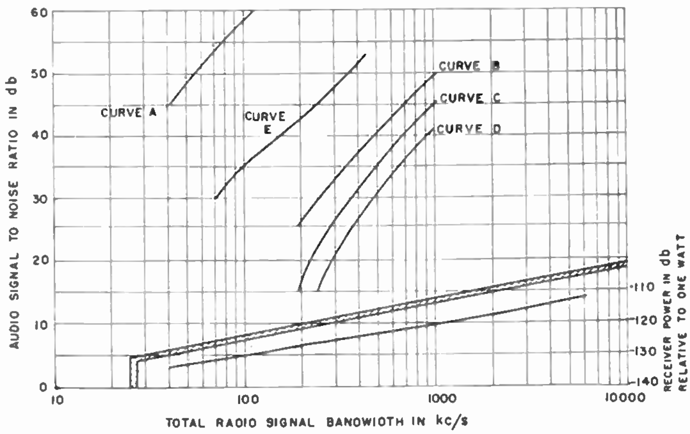


Fig. 1.—Diagram showing graph of performance with respect to fluctuation noise of F.D.M.-F.M.

Audio signal/noise ratios for 4 and 6 channels.
Audio channels 4 kc/s wide and double-spaced (except for curve E.)
Peak-to-peak deviation = signal bandwidth—2 × highest modulating frequency.
Values calculated for worst channel.
All graphs for marginal power.

Curve A — Single channel.
Curve B — 4 channel (8-36 kc/s)
Curve C — 4 channel (32-60 kc/s)
Curve D — 6 channel (32-72 kc/s)
Curve E — 3 channel (0-12 kc/s)

Lower curves show marginal power for F.M. receiver 10-db noise factor.

the amplitude of the audio signal. It is normally found that subjectively no impairment of speech signals by noise occurs if the audio-to-noise ratio is greater than 22 db. In the absence of companding large audio-to-noise ratios of, say, 70 db are required when a channel is fully modulated to ensure a reasonable audio-to-noise ratio for small amplitude speech signals. This method is uneconomic and this is shown in Fig. 2. Curve A is the normal variation of audio-to-noise ratio with varying speech levels for a maximum audio-to-noise ratio of 71 db. Subjectively, curve B would be just as effective since the low-level speech audio-to-noise ratio is unaffected, and, although the audio-to-noise ratio has been reduced on average and loud speech signals by some 15 to 30 db, the resultant audio-to-noise ratio is still greater than 22 db. Curve B can be considered as representing a received radio frequency signal power which has been reduced 26 db, but with a 26-db compander added to give the same subjective performance. Curve C is the audio/noise ratio to be expected from this lower powered circuit if the compander circuits were removed. All these curves have been plotted with the average speech power referred to a tone which would fully load the channel. Considering curve B, the weak speech audio-to-noise ratio has been improved by 26 db without subjectively degrading the stronger signals, and hence can be considered as a 26-db improvement.

For companders with a greater gain, yielding more low-level improvement, the high speech level audio-to-noise ratio is further degraded, and the limit to the amount of companding permissible from the viewpoint of the high speech level performance is determined by the original uncompanded audio-to-noise ratio. The permissible low-level improvement increases several decibels for every db improvement of the original uncompanded audio-to-noise ratio. Thus, the amount of compander gain is determined by the original uncompanded audio-to-noise ratio, the permissible compander gain

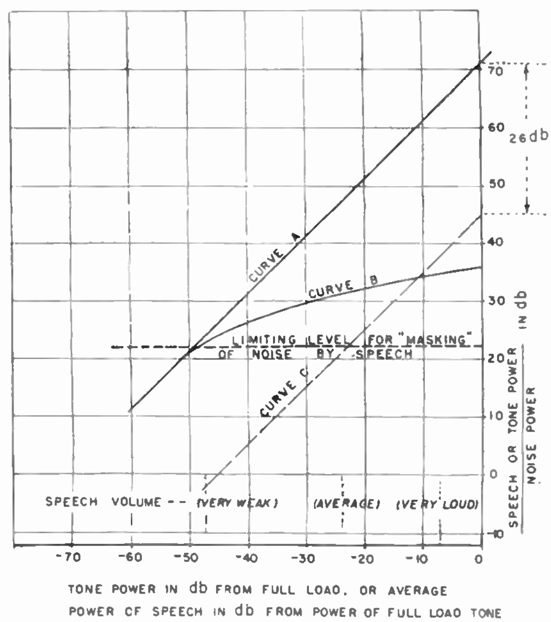


Fig. 2.—Typical compander performance.

Curve A — required audio/noise performance.
Curve B — "equivalent" audio noise performance.
Curve C — this performance with 26-db compander produces curve B.

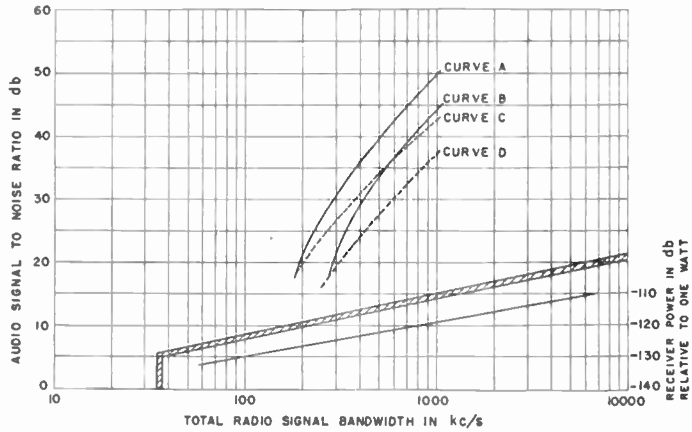


Fig. 3.—Diagram showing graph of performance with respect to fluctuation noise of P.A.M.-F.M.

Audio signal noise ratios for 4 and 6 channels.
Audio channels 4 kc/s wide—sampling rates 8 kc/s.
Peak-to-peak deviation = signal bandwidth—2 × highest modulating frequency.
All graphs for marginal power.
Curve A — 4-channel rectangular gating.
Curve B — 6-channel rectangular gating.
Curve C — 4-channel instantaneous sampling.
Curve D — 6-channel instantaneous sampling.
Curves A and B—sinusoidal non-overlapping pulses.
Curves C and D — sinusoidal overlapping pulses.
Lower curves show marginal power for F.M. receiver 10-db noise factor.

increasing nearly as quickly as the final "equivalent" audio-to-noise ratio. On circuits with a good audio-to-noise ratio a large compander gain may be added, but on circuits with a low audio-to-noise ratio at full modulation little, if any, compander gain is permissible. Also it is not permissible to reduce the radio frequency signal input to below marginal level, and then try to improve the result by companding. A companded system is more vulnerable to the effects of fading and a larger allowance must be made in the original system's design to cater for these effects.

In the system to be considered no allowance has been made for companding, and if any compander gain is permissible the figures for audio-to-noise ratio of Fig. 3 should be increased by the compander gain at weak speech levels. The pulses are assumed to be approximately sinusoidal, and the receiver channel gate is assumed rectangular when the pulses are non-overlapping. If overlapping pulses are used, then instantaneous sampling of the signal must take place. Owing to the instantaneous sampling the audio-to-noise ratio will be reduced by 8 db, and at the higher bandwidths this more than counterbalances the improvement due to the increased permissible frequency deviation of the radio frequency signal.

In deriving the system performance, harmonics of the gating function have been assumed to act as a carrier to the noise spectrum centred about them, and contribute audio/noise components to the channel within the limits of the channel filter (Ref. 1).

The audio-to-noise ratio of a channel

$$= \frac{9{\cdot}64 \times 10^{-2} \times B\beta^2}{N^2\,(N + \text{correction})}$$

(See Appendix 2)

The curves of Fig. 3 are based on this result and show the audio-to-noise ratio at marginal power for various types of pulses and numbers of channels.

5. Conclusions

For average circuits using radio links with an audio/noise ratio in the worst channel of 40 db, the two systems have a very similar performance. This is due to the fact that companding cannot be used. Where a better circuit is required, say 70 db audio-to-noise ratio, and the radio frequency input signal to the receiver is much greater than the marginal level, then the P.A.M.-F.M. system is superior, when companding is used to give the same "equivalent" performance. Suggested permissible amounts of companding are:—

Initial audio/noise ratio in db	30	35	40	45
Maximum permissible companding in db	—	3	10	30
Final "equivalent" audio/noise ratio in db	30	38	50	75

It will be seen that the effect of doubling the radio frequency signal bandwidth is to increase the audio-to-noise ratio in all channels by approximately 9 db. At the same time the marginal power level has increased by 3 db, so that a 3-db increase in power has given a 9-db increase in audio-to-noise ratio. Hence, for a given transmitter radio frequency power, the maximum audio-to-noise ratio will occur when the maximum bandwidth is used which is consistent with marginal operation of the receiver.

Provided that sufficient receiver input power is available to ensure marginal operation, improved overall performance will result from wide bandwidth operation; and thus a system using U.H.F. radio links where wide bandwidths are required for other reasons will have a much improved performance. This last statement is only true if some of the increased bandwidth is available for the purpose of transmitting speech intelligence, and if the receiver noise factor has not deteriorated too greatly.

6. References

1. C. B. Feldman and W. R. Bennet, "Band Width and Transmission Performance." *Bell System Technical Journal*, **28**, July 1949, No. 3, pp. 490-595.
2. L. A. Meacham and E. Petersen, "An Experimental Multichannel Pulse Code Modulation System," *Bell System Technical Journal*, **27**, January 1948, No. 1, pp. 1-43.

7. Symbols

To avoid confusion, the symbols in most cases are the same as those used by Feldman and Bennet, although they may not be those normally preferred.

- B = total radio frequency bandwidth in kc/s
- β = peak-to-peak deviation in kc/s
- F_B = receiver post-detector bandwidth in kc/s
- f_R = sampling rate in kc/s
- f_m = modulating frequency in kc/s
- n = fractional deviation per channel
- P_n = mean power of fluctuation noise per kc/s
- W_c = carrier power at receiver input terminals
- W_n = total noise power
- N = number of channels
- K = arbitrary constant
- k = ratio of peak noise power to mean noise power (= 16)

8. Appendix 1

Improvements in F.D.M.-F.M. System

(a) Due to reduced Post-detector Bandwidth

Assuming the fluctuation noise power is equal over the whole of the radio frequency signal bandwidth.

$$\text{Improvement} = \frac{B}{2F_B} \text{ (power ratio)}$$

(b) F.M. Improvement

$$\text{Improvement} = \sqrt{3}\,\frac{n\beta}{2f_m} \text{ (voltage ratio)}$$

(c) Reduced width of Highest Frequency Channel compared to Audio Bandwidth

If true frequency modulation is used, the post-detector r.m.s. noise voltage plotted against the audio frequency will take the form of a triangular spectrum and the highest frequency channel will have the greatest noise power. If phase modulation is used the noise spectrum will be rectangular and the r.m.s. noise power per channel will not vary.

Assuming frequency modulation, and audio channels 4 kc/s wide:

$$\text{Total noise power in band} = \int_0^{F_B} KP_n f^2 df.$$

$$\text{Noise power in top channel} = \int_{F_B-4}^{F_B} KP_n f^2 df.$$

$$\text{Improvement} = \frac{F_B^3}{F_B^3 - (F_B - 4)^3} \text{ (power ratio)}$$

9. Appendix 2

Improvements in P.A.M.-F.M. System

The speech output power of the channel $= \dfrac{\beta^2}{32N^2}$

Noise output power of the channel is

$$W_n = \frac{P_n f_R^3}{\pi^2 W_c} \sum_{m=1}^{m=2N} \left(1 + \frac{1}{12m^2}\right) \sin^2 \frac{m\pi}{N}$$

and if N is large

$$W_n = \frac{NP_n f_R^3}{\pi^2 W_c}$$

For small values of N, expanding the summation gives

$$N = 4 \qquad W_n = \frac{P_n f_R^3}{\pi W_c}\left[N + 0{\cdot}42\right]$$

$$N = 6 \qquad W_n = \frac{P_n f_R^3}{\pi W_c}\left[N + 0{\cdot}22\right]$$

and at marginal signal $W_c = kP_nB$

when k = ratio of peak noise power to mean noise power (= 16).

$$\frac{\text{Audio}}{\text{Noise}} = \frac{\beta^2}{32N^2} \frac{\pi k P_n B}{P_n f_R^3 (N + \text{correction})}$$

$$= \frac{9{\cdot}64 \times 10^{-2} \times B\beta^2}{N^2 (N + \text{correction})} \text{ (power ratio)}$$

If a sinusoidal gate is used instead of a rectangular gate the audio-to-noise ratio will be decreased by a small amount.

For non-overlapping pulses $F_B = 2Nf_R$

For overlapping pulses $F_B = Nf_R$

APPLICANTS FOR MEMBERSHIP

New proposals were considered by the Membership Committee at a meeting held on June 18th, 1952, as follows: 15 proposals for direct election to Graduateship or higher grade of membership and 13 proposals for transfer to Graduateship or higher grade of membership. In addition 64 applications for Studentship registration were considered. This list also contains the names of 2 applicants who have subsequently agreed to accept a lower grade than that for which they originally applied.

The following are the names of those who have been properly proposed and appear qualified. In accordance with a resolution of Council and in the absence of any objections being lodged, these elections will be confirmed 14 days from the circulation of this list. Any objections received will be submitted to the next meeting of the Council, with whom the final decision rests.

Direct Election to Full Member

BHAR, Jatindra Nath, D.Sc. *Chandernagore, India.*
MITRE, Sisir Kumar, D.Sc. *Calcutta.*

Direct Election to Associate Member

BUCKMAN, Vernon, Squadron Leader. *Malvern, Worcestershire.*
EUGENE, Antony Felix, Lieutenant Colonel, B.E. *Trinchinopoly, India.*
FREELAND, Maurice Montague, B.Sc. *London, S.W.17.*
JONES, John Hugh, Flight Lieutenant. *Victoria, Australia.*
SMITH, Norman Arthur, Wing Commander. *Harrow, Middlesex*
SRINIVASAN, Satagopan, B.Sc. *Madras.*
STEELE, Henry Albert, Flight Lieutenant. *Carshalton, Surrey.*

Transfer from Associate to Associate Member

McGOWAN, Kenneth Miller, Lieutenant Commander (L), R.N. *Portsmouth.*
MOSS, Lawrence Edward, Flight Lieutenant. *Ruislip, Middlesex.*
OSBOURNE, Basil Whitworth, M.Sc. *Datchet, Bucks.*
WARDEN, Charles Alexander. *Hounslow, Middlesex.*

Transfer from Graduate to Associate Member

ANSTEY, Harold George. *Bromley, Kent.*

Transfer from Student to Associate Member

CAKEBREAD, John Robert Gordon, Flight Lieutenant, D.F.C. *London, W.13.*

Direct Election to Associate

FREWIN, Ernest Edward. *Accra, Gold Coast.*
McLELLAN, William Robert McRobert. *Bangor, Co. Down.*
THIRUMULPAD, Cheathempadi Mookunni, Warrant Officer, *Malabar, India.*
WHITLEY, Richard Harold. *Peterborough.*

Transfer from Student to Associate

ARORA, Mahendra Nath. *New Delhi.*
KANGALINGAM, Suppiah. *Urumpirai, Ceylon.*

Transfer from Student to Graduate

ANIKHINDI, Ramesh Gamesh, M.Sc. *Indore ,India.*
KAIWAR, Badri Nath, B.Sc.(Hons.). *Adyar, Madras.*
PALMER, Donald Ridgeway. *Bexley, Kent.*

Studentship Registrations

AGARWAL, Ravendra Kumar. *Moradabad, U.P., India.*
BAHRUDDIN. *Banaras, India.*
BENNELL, Donald Alfred. *Portsmouth.*
BHAGWAT, Prabhakar Sadashiv. *Bombay.*
BHANDARI, Rameshwar Chand. *Ambala Cantt, India.*
BHARGAVA, Uma Shankar. *Ferozepur Cantt, India.*
BHAT, R. Venkateshwara, B.Sc. *New Delhi, India.*
BOUGHEN, Walter Henry. *Wisbech, Cambridgeshire.*
CHADHA, Amar Nath, B.A. *Bangalore, India.*
CHANDER, Krishnan. *Delhi.*
CHORADIA, Bansilal Deepchand. *Poona City, India.*
CHUI, Toni Yim. *Macao.*
DAVIES, Mervyn William. *Perak, Malaya.*
DESAI, Ramesh Chandra. *Delhi.*
DE SELLAS, Peter Augustus. *New Delhi.*
DIN, Mohammad, B.A.(Hons.). *Sialkot, Pakistan.*
D'SILVA, Cecil. *Bombay State.*
GOLDBLATT, Daniel. *Randfontein, South Africa.*
GOLF, Madhusoodan Shankar, B.Sc. *Poona, India.*
HOLMES, Edward Ernest. *Ferryhill, Co. Durham.*
INDULKAR, Vasant Vishwasrao. *Igatpuri, Bombay State.*
ISLAM, Saiyid Mohammad Azizul. *Budaun, India.*
JANAKIRAMAN, Nagavedu B. *Madras.*
JEYNES, Graham Frank. *London, S.W.16.*
JOSHIRAO, Bhalachandra G. *Poona, India.*
KACHARE, Madhukar Ramchandra. *Bombay.*
KAMAL, Nayar. *Banaras, India.*
KANDASAMY, Manicavasagam. *Palam Cottah, Madras State.*
KANWAR, Abhai Singh, M.A. *Amritsar, India.*
KHAN, Ahtram Ali, M.Sc. *Lahore, Pakistan.*
KHER, Mukund Jagannath, B.Sc. *Ahmedabad, India.*
KRISHNAMURTI, C., B.Sc. *Madras.*
KRISHNA SWAMY, T. L. *Bangalore, India.*
LAKSHMINA, Rayanan A. *Madras.*
MADHAVA, RAO, V., B.Sc. *Bangalore, India.*
MENGHANEY, Mengho, B.Sc. *Bombay.*
MENON, Koopat Krishnankutty. *Cochin, India.*
MITRA, Satish Chandra, B.Sc. *Midnapore, West Bengal.*
MUDHER, Parkash Singh. *Bangalore, India.*
MURTHY, R. Aswatha Narayana, B.Sc. *Bangalore, India.*
NAIK, Yeshwant Baburao, B.Sc. *Poona, India.*
PABRAI, Om Parkash. *Bombay.*
PIKE, Derek Warren. *Letchworth, Hertfordshire.*
RAO, Krishna M., B.Sc. *London, W.2.*
ROY, Pronab Kumar, B.A.(Hons.). *Lucknow, India.*
SATHE, Gajanan Vishwanath. *Poona City, India.*
SHARAN, Mata. *Banaras, India.*
SINGH, Inder-Jit. *London, N.W.1.*
SINGH, Kirpal. *Amritsar, India.*
SRINIVASAN, R., B.Sc. *New Delhi.*
SRINIVASAN, T. R., M.A. *Madras.*
SRIVASTAVA, Ram Prakash, B.Sc. *Banda, India.*
STEVENSON, Harold. *Plymouth, Devon.*
THOO, Kim Lan. *Kuala Lumpur, Malaya.*
VENKATESWARAN, K. *Madras State.*
VISWANATHAN, Cadambangudi Ramachandran. *Madras.*
WALLACE, Reginald Antony. *Bangalore, India.*
WILLIAMS, Peter Owen. *Keighley, Yorkshire.*