JOURNAL OF

The British Institution of Radio Engineers

(FOUNDED IN 1925 - INCORPORATED IN 1932)

"To promote the general advancement of and to facilitate the exchange of information and ideas on Radio Science."

Vol. X (New Series) No. 7 JULY 1950

NOTICE OF THE TWENTY-FIFTH ANNUAL GENERAL MEETING

NOTICE IS HEREBY GIVEN that the TWENTY-FIFTH ANNUAL GENERAL MEETING (the Seventeenth since Incorporation) of the Institution will be held on WEDNESDAY, SEPTEMBER 27th, 1950, at 6.30 p.m., at the London School of Hygiene and Tropical Medicine, Keppel Street (Gower Street), London, W.C.1.

AGENDA

1. **To confirm the Minutes of the 24th Annual General Meeting held on September 22nd, 1949.** (Reported on pages 353-356 of Volume 9 (New Series) of the Journal dated October, 1949.)

2. **To receive the Annual Report of the General Council.** (To be published in the August 1950 Journal.)

3. **To elect the President**

   The Council is unanimous in recommending the election of Mr. Paul Adorian as President of the Institution for the year 1950/51.

4. **To elect the Vice-Presidents of the Institution.**

   The Council unanimously recommends the re-election of Mr. William E. Miller, M.A.(Cantab), and the election of Mr. Leslie Paddle and Mr. J. W. Ridgeway, O.B.E.

5. **To elect the General Council.**

   The retiring members of the Council are :—

   H. A. Brooks (Associate Member).
   J. W. Ridgeway, O.B.E. (Member).
   W. J. Thomas, Ph.D., B.Sc.(Hons.) (Associate Member).
   Professor H. E. M. Barlow, Ph.D., B.Sc.(Hons.), (Member).
   L. Paddle (Member).

   Commander A. J. B. Naish, M.A. (Associate Member) has already retired from the Council.

   In accordance with Article 32, the Council has nominated :—

   E. E. Zepler, Ph.D. (Member).
   R. G. Kitchenn, B.Sc.(Eng.) (Associate Member).
   H. E. Drew (Member)
   E. A. H. Bowsher (Member).
   Commander H. F. Short, M.B.E., R.N. (Associate Member).

   Hon. Treasurer : S. R. Chapman, M.Sc. (Member).

   Any member who wishes to nominate a member or members for election to the Council must deliver such nomination in writing to the Secretary, together with the written consent of such person or persons, to accept office, if elected, not later than September 15th, 1950. Such nomination must be supported by not less than 10 corporate members.

6. **To receive the Auditors' Report, Accounts and Balance Sheets for the year ended March 31st, 1950.**

   The Accounts for the General and other Funds of the Institution will be published in the August Journal.

7. **To appoint Auditors.**

   Council recommends the re-appointment of Messrs. Gladstone, Titley & Co., 74 Victoria Street, S.W.1.

8. **To appoint Solicitors.**

   Messrs. Braund & Hill, 6 Grays Inn Square, London, W.C.1, are recommended for re-election as Solicitors.

9. **Awards to Prize Winners.**

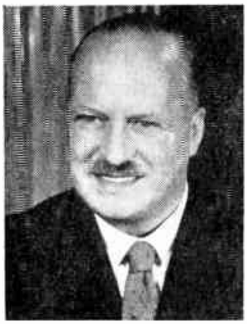
10. **Any other business.** (*Notice of any other business must reach the Secretary* 40 *days before the meeting.*)

(*Members unable to attend the Annual General Meeting are urged to appoint a proxy.*)

World Radio History

NOMINATED FOR ELECTION TO COUNCIL

Edward Albert Henry Bowsher, who has been nominated for election to the General Council, was born in London in May, 1900. He received his technical education at Leyton Technical Institute whilst training as a telegraph plant engineer with Western Union Cable Company. Mr. Bowsher served as a pilot in the R.F.C. and R.A.F. during the 1914-18 War, and in 1921 joined the Standard Telephones and Cables Ltd. (then known as Western Electric Ltd.), supervising the installation of numerous telephone exchanges, etc.



For four years prior to the 1939-45 War he was in charge of the Circuit Laboratory of the Telephone Division and was responsible for a number of patents, which included a method of signalling by D.C. over power supply networks. Between 1939 and 1945 Mr. Bowsher was for some time in charge of production and planning for radar equipment and later became Chief Inspector for the Company. Since 1945 he has been in charge of the Development Department of Central Rediffusion Services Ltd.

Soon after his election to Membership of the Institution Mr. Bowsher was invited to join the Technical Committee on which he has served for over a year in addition to assisting one of the Professional Purposes Sub-Committees.

Herbert Frank Short was born at Portsmouth in May, 1907, and started his Naval career in 1922. After qualifying for promotion through the ranks to Warrant Officer he was commissioned in 1938.

He served as W/T officer in H.M.S. *Resolution* and subsequently as a radar instructional officer at H.M. Signal School. After a period of executive duty, he was transferred in 1942 to Combined Operations Headquarters where, as Radio Technical Adviser, he was responsible for the efficient installation and operation of W/T and radar equipment in landing ships and craft; he subsequently held a similar appointment under the Director of Craft and Amphibious Material at the Admiralty. At present Commander Short is with the Naval Electrical Department at the Admiralty.

He was appointed a M.B.E. in 1944.

Elected an Associate Member of the Institution in 1947, Commander Short was co-opted to the Council in February of this year in the place of Commander Naish, who was unable to complete his period of elected service.

Ronald Graley Kitchenn was born at Letchworth, Hertfordshire, in March, 1920. He was educated at Letchworth Grammar School and joined the G.P.O. Engineering Department as a Youth-in-Training in 1936. Between 1937 and 1940 he passed various City and Guilds examinations in all branches of telecommunications, and he was elected A.M.I.E.E. in 1946. Subsequent studies at Northampton Polytechnic and North Staffordshire Technical College led to his obtain- a B.Sc.(Eng.) degree in 1948.

A period of defence telecommunications duties preceded his transfer, in 1944, to the Telegraph Branch of the Engineer-in-Chief's Office, G.P.O., where he was engaged initially in the provision of telegraph installations for the Services, and subsequently on circuit design. In 1946, Mr. Kitchenn became a Lecturer in transmission subjects at the G.P.O. Engineering Department's Central Training School at Stone, Staffordshire. Two years later, he joined the laboratories of the Local Lines and Wire Broadcasting Branch of the Engineer-in-Chief's Office, since when he has been responsible for the design, development and prototype work on transmitting equipment for carrier wire broadcasting.

Mr. Kitchenn was elected an Associate of the Institution in July, 1942, and an Associate Member in September, 1947. He has served on the Programme and Papers Committee since January, 1950.

World Radio History

ON THE SPACE-CHARGE SMOOTHING OF SHOT FLUCTUATIONS IN TRIODE SYSTEMS RESPONDING TO VERY HIGH FREQUENCIES*

by

I. A. Harris (*Associate Member*)

SUMMARY

After a brief survey of existing theoretical results, a detailed description of the simplified theory of space-charge smoothing of shot fluctuations in a diode, based on the Benham-Llewellyn theory, is presented. This is readily applied to the case in which the response is such that electron transit times are appreciable (i.e. in the V.H.F. range of response). Unlike the one reached formerly, the conclusion is that the "low-frequency" smoothing factor remains appreciably unchanged up to transit angles of approximately one radian.

Application to the noise of grounded-grid triode circuits gives results which are in better agreement with experimental results in the V.H.F. range than are the results based on earlier theory.

In conclusion, the effect on noise of a non-uniform field at the cathode of a triode is briefly discussed.

1.0 Survey of Existing Results

1.1. The normal shot effect arises on account of the completely random distribution in time of the thermionic emission of electrons. Questions of importance to the engineer concern the effect of this random distribution on an instrument (e.g. a network, such as a resonant circuit) which alone can be made the subject of analysis and can be measured quantitatively[1].

If the general characteristic of such an "instrument" be described as a function of frequency f, the response to fluctuations can then be determined from the response of an elemental bandwidth df, which is expressed as a mean square fluctuation :—

$$d\overline{i^2} = 2eI_s \,.\, df \quad \ldots\ldots (1)$$

where e is the positive magnitude of the charge on an electron and I_s is the temperature limited emission current[2].

1.2. The meaning of formula (1) is illustrated in Fig. 1, where a source of random electrical events (r) is connected to the input of a noise-free four-terminal network. The output terminals are short-circuited, and the "current gain" M is defined as the ratio of the output current in the short-circuit to the input current. M is a function of frequency. Then the mean square current in the output circuit is given by:—

$$\overline{i^2} = 2eI_s \int_0^{\infty} |M|^2 \,.\, df \quad \ldots\ldots (2)$$

The four-terminal network is the "instrument" on which the sequence of random events (r) acts, and, in the case of the thermionic diode, this sequence of events is the emission of electrons as a function of time.

1.3. The normal shot effect of a temperature limited current I_s in a planar diode system responding to very high frequencies, such that the mean transit angle β ($= 2\pi f \tau$ where τ is the electron transit time in the diode) is appreciable, has been calculated by various authors.[3,4,5] The result is :—

$$d\overline{i^2} = 2eI_s \left\{ \frac{4}{\beta^4} (2 + \beta^2 - 2\cos\beta - 2\beta\sin\beta) \right\} df \quad \ldots\ldots (3)$$

$$\simeq 2eI_s \left(1 - \frac{1}{18}\beta^2 \right) df \text{ (for small transit angles).}$$

When this is expressed in the form of formula (2), the part which is a function of β (and therefore of f) must be placed in the integrand. Therefore, in principle, the effect of the transit time is to modify the four-terminal network of Fig. 1.

Both formulæ (1) and (3) require that the capacitance and any residual conductance of the diode be included in the network as part of

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World Radio History

the "instrument." They constitute a basis on which circuit problems, involving the normal (or full) shot-effect of temperature limited emission current, can be solved.

1.4. The theory of the space-charge smoothed shot-effect in planar diode systems responding to moderately high radio frequencies was, after much inconclusive work, finally established, and gained general acceptance during the years 1938 to 1942.[6, 7, 8]

The result was expressed by Schottky and North in the form :—

$$d\overline{i^2} = 2eI \, . \, \Gamma^2 \, . \, df \quad \ldots\ldots (4)$$

where the *smoothing factor* Γ^2 is expressed by :—

$$2k\theta g/(eI) \, . \times 0{\cdot}644,$$

and by Rack in the form :—

$$d\overline{i^2} = 4k\,(0{\cdot}644\,\theta)\,g\,.\,df \quad \ldots\ldots (5)$$

where the effect is likened to a thermal fluctuation in a resistor of conductance g at a temperature $0{\cdot}644\theta$. In these formulæ, θ is the cathode temperature (°K) and g is the differential conductance of the diode.

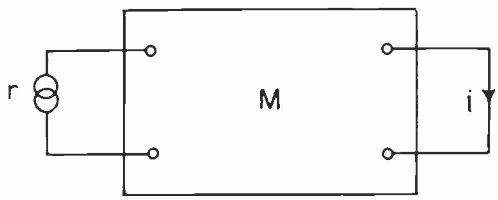

Fig. 1.—Source of random electrical events (r) feeding into a noise-free four-terminal network with a current gain M. The output current i is the effect of r on the instrument typified by the network.

The range of validity of (4) and (5) has been stated to be $I < 0{\cdot}8I_s$ with the accelerating voltage greater than about 2, where I is the space-charge limited current and I_s is the total emission current.

These formulae have been applied extensively to triodes and other electrode configurations with success.

1.5. The mean square fluctuation current in a system responding to frequencies, such that the transit angle β in the diode is appreciable, has been calculated, under various assumptions[4] with the result :—

$$d\overline{i^2} = 2eI\Gamma^2 \left\{ \frac{4}{\beta^4}(2 + \beta^2 - 2\cos\beta - 2\beta\sin\beta) \right\} df \quad \ldots\ldots (6)$$

The factor in β is identical with that in (3) for the temperature limited current.

1.6. These relations for fluctuations in space-charge limited currents have been deduced on the assumption that the electrodes are planar. In this case, only the component of initial emission velocity (which takes part in the smoothing of fluctuations) normal to the planar cathode is considered. With cylindrical structures, however, it has been suggested that[9], in the extreme case where the anode diameter is many times the cathode diameter, *two* components of the initial emission velocity take part. This may have the effect of substituting $\lambda\theta$ for the cathode temperature θ in the above formulæ for planar electrodes, where λ is a number between 1 and 2, according to the ratio of electrode diameters.

1.7. Transit time effects on fluctuation currents, which are of most interest in application, are not so much those connected directly with the diode, as those effects connected with induced electrode currents in triodes and multi-electrode valves. Thus, in a negative grid triode there is the *induced grid noise*, calculated in terms of the shot noise as follows :—

$$d\overline{i_g^2} = (\beta^2/9)\,.\,d\overline{i^2} \quad \ldots\ldots (7)$$

to the order β^2 in powers of β.[10] Formulæ derived from this have been verified experimentally. On the other hand, experimental and theoretical investigations into the noise of a grounded-grid triode[11] have shown that existing theory is at variance with the results of experiment. Experiment shows the existence of an induced noise current which, according to the theory leading to formulæ (6) and (7), does not exist. This discrepancy brings to light the assumptions made in the derivation of (6), and the aim of the present work is to examine these assumptions and suggest an alternative approach which leads to a more satisfactory comparison between theory and experiment.

2.0. A Simplified Theory of the Space-charge Smoothing of Fluctuations in a Diode

2.1. The rigorous derivation of equation (4), as, for example, carried out by North,[8] involves a detailed consideration of the change in the total space-charge limited current brought about by a small excess or deficiency in the normal emission current associated with the initial velocity range u_c to $u_c + du_c$. The ratio of the change in the total space-charge limited current to the change in the stated part of the emission

World Radio History

is denoted by γ, and it is a function of the associated initial velcity u_c. This means that each fluctuation in the part of the emission, with initial velocity between u_c and $u_c + du_c$, results in a fluctuation in the total space-charge limited current, which is reduced linearly by the factor $\gamma(u_c)$.

2.2. As long as the electron transit times are small compared with the main oscillation period of the response of the external circuit, there is no difficulty in applying the rigorous method of North (or of Rack) in calculating the value of γ. When transit times are appreciable, however, a straightforward method of solving problems involving space-charge limited currents in planar structures is provided by the Benham-Llewellyn theory.[12] This theory unfortunately precludes the actual case in which the initial emission velocity is distributed over a range of values, owing to the serious inherent limitation that in the theory all electrons leaving the cathode at a given instant must have the same velocity and acceleration. For this powerful method to be used, therefore, it is necessary to simplify the mathematical picture of the actual physical problem ; but before this is discussed, it is helpful to have a clear physical picture in terms of which the mechanism of space-charge limitation of space-current can readily be understood.

2.3. The mechanism of the space-charge limitation of space-current is illustrated in Figs. 2*a* and 2*b*. In the customary manner, the space between the cathode and the anode is divided into the "α" and the "β" spaces, the division being marked by a potential minimum or barrier, which forms in front of the cathode whenever the positive anode voltage is insufficient to draw off all the emission. The field distribution is illustrated in Fig. 2*a*. Mathematically, the α- and β- spaces are considered separately, and the barrier is characterized by the condition $dV/dx = 0$.

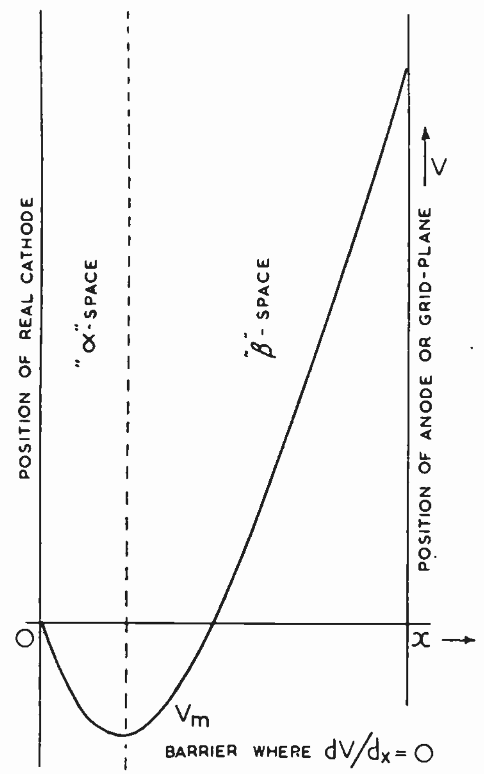

Fig. 2a.—Potential distribution in the presence of a space-charge limited current. The β-space current is limited by the anode voltage V, the height and location of the barrier adjusting themselves so that just sufficient electrons have the initial velocity to pass the barrier and form the space-charge limited current.

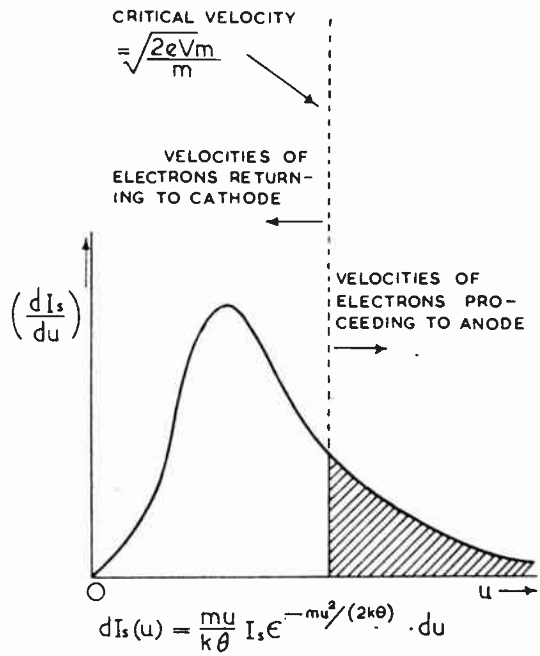

Fig. 2b.—Distribution of the emission current over values of the component of initial velocity normal to the cathode surface. Only those electrons with initial velocities in excess of a critical value pass the barrier (Fig. 2a) and enter the β-space at a rate sufficient to form the space-charge limited current. Electrons with smaller initial velocities return to the cathode, never leaving the α-space.

The Maxwellian distribution of the component of initial emission velocity normal to the planar cathode surface is shown in Fig. 2*b*, in which it is illustrated how only those electrons with initial velocity components normal to the cathode in excess of a critical value enter the β-space to form the space-charge limited current.

In the steady state, equilibrium is attained between the anode voltage V_d, the cathode temperature θ, the emission current I_s, and the space-charge limited current I. The rigorous theory has been given in the classical papers of Langmuir and others.

2.4. The resulting relation between the applied anode voltage V_d and the space-charge limited current, which is very nearly correct provided that V_d is greater than about 2 volts when the oxide-coated cathode is used, is expressed as follows :—

$$I = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{\frac{1}{2}} \cdot \frac{(V_d - V_m)^{\frac{3}{2}}}{b^2}$$
$$\left[1 + \frac{3\pi^{\frac{1}{2}}}{2}\left(\frac{k\theta}{e(V_d - V_m)}\right)^{\frac{1}{2}}\right]$$

where b is the potential-minimum to anode distance, and V_m is the barrier potential relative to the cathode. This relation may also be obtained by applying the Benham-Llewellyn theory to the β-space, provided that the electrons passing the barrier into the β-space are treated as if they all had the same barrier velocity, i.e. the actual *average* velocity $[\pi k\theta/(2m)]^{\frac{1}{2}}$. The result follows from equation (A15) in Appendix I, in which δu_o is replaced by this average velocity.

This averaging of the initial velocities is the necessary simplification referred to earlier.

2.5. The Benham-Llewellyn theory may also be applied to the β-space in calculating the value of γ, as set out in 2.1, provided the average velocity $\bar{u}_o$ at the barrier is applied to all electrons, even though in fact they have a whole range of velocities.[4, 10, 14] Thence, equation (4) or (5) may be derived, and the justification for the simplification is that it leads to the correct solution which has also been obtained by more rigorous, if more cumbersome, methods.[7, 8]

2.6. In fluctuation problems, it is also convenient to regard all fluctuations as being expressible as a sum of sinusoidal components in the manner of a Fourier series or integral, only one general component then entering into the detailed discussion.

Thus the excess or deficiency of emission current with initial velocity in the range u_c to $u_c + du_c$ is accounted for by superimposing a sinusoidal component $\delta(dI_s)\varepsilon^{j\omega t}$ on the (mean) value dI_s. Resulting fluctuations in other quantities are also represented by sinusoidal components. For the sake of briefness, the time-factor $\varepsilon^{j\omega t}$ will be omitted in much of the following analysis. The justification for such a Fourier representation is that it is used in the sense of Fig. 1, where the circuit or system *response* is the subject of analysis, rather than the statistical analysis of the random events at their source.

2.7. The components of the fluctuations in the β-space current are strictly governed by the space-current limitation in this space.

The actual smoothing of the normal shot fluctuations is brought about by correlated fluctuations in the magnitude of the potential barrier, and, to a lesser extent, in its position. In most examples of practical interest, fluctuations in the barrier location can be ignored.

The space-charge limited current in the β-space can be increased either by an increase in the average velocity at the barrier by $\delta\bar{u}_o$, or by an increase in the P.D. between the barrier and the anode which is achieved by changing the barrier magnitude by δV, or by both changes. Therefore, if the fluctuation in the emission at the cathode is to produce a fluctuation δI in the β-space current, it must change either or both $\bar{u}_o$ and V.

In systems responding to low or moderately high radio frequencies, a simple application of the theory to the β-space, as shown in Appendix I, leads to the result :—

$$\delta I = \left(\frac{3m}{4\pi^2 e I \tau^4}\right) . \delta V + \left(\frac{3m}{2\pi e \tau^2}\right) . \delta\bar{u}_o$$
$$= g . \delta V + \left(\frac{3m}{2\pi e \tau^2}\right) . \delta\bar{u}_o \qquad (8)$$

where I is the β-space current, τ is the mean transit time, and g is the diode differential conductance $\partial I/\partial V$.

2.8. In deriving (8), the average initial velocity $\bar{u}_o$ has been neglected in so far as it affects the current change δI, only the change $\delta\bar{u}_o$ in $\bar{u}_o$ having been included. This simplification, made throughout the present work, makes no significant difference to the results, and it is in accordance with the customary neglect of initial velocities when discussing space-charge limited currents in connection with theoretical valve characteristics.

2.9. The smoothing of the normal shot fluctuations (component δI_s) by correlated fluctuations in the barrier (component δV_m) giving rise to smaller fluctuations in the β-space (component δI), is expressed by the relation :—

$$\delta I = \delta I_s - \left(\frac{eI}{k\theta}\right) . \delta V_m \quad \ldots\ldots\ldots\ldots (9)$$

which follows from the equation expressing the Maxwellian velocity distribution. The fluctuations are only those associated with the velocity range u_o to $u_o + du_o$ at the barrier.

From (9), together with the equation for the initial velocity distribution, follows the approximate formula[10, 13]

$$\delta \bar{u}_o = (u_o - \bar{u}_o) \frac{\delta I_s}{I} \quad \ldots\ldots\ldots\ldots (10)$$

This expresses the change $\delta \bar{u}_o$ in the average velocity at the barrier in terms of the change δI_s in the emission at the cathode, associated with the velocity range u_o to $u_o + du_o$ at the barrier (or u_c to $u_c + du_c$ at the cathode). In so far as δV_m is small, $\delta \bar{u}_o$ does not depend explicitly upon it, as is seen from inspection of (10).

On combining equation (10) with equation (8), in which the term in δV is neglected by comparison with the term in $\delta \bar{u}_o$, the following is obtained :—

$$\delta I = \frac{3m(u_o - \bar{u}_o)}{2\pi e I \tau^2} \delta I_s \quad \ldots\ldots\ldots\ldots (11)$$

This has been obtained elsewhere.[10, 13] From this relation, the mean square smoothing factor for those fluctuations associated with the stated initial velocity group, namely $\gamma^2 = |\delta I/\delta I_s|^2$, can be formed. If this factor is weighted according to the Maxwellian distribution law, and is then integrated over all those velocity groups which pass the barrier, the correct value of Γ^2 is obtained.

This procedure depends on the fact that the shot fluctuations associated with one small range of initial velocity are completely uncorrelated with those of any other range, so that the resultant is obtained by summing the mean square values.

2.10. The following points concerning the component δI of the smoothed fluctuations follow from the foregoing discussion :—

(*a*) While, in the first place, fluctuations of the barrier correlated to the normal shot fluctuations are the physical cause of the "reduction" or "smoothing," the nature of the initial velocity distribution and of the dependence of space current on initial velocity makes δI effectively dependent only on $\delta \bar{u}_o$.

(*b*) There exists the relation :—

$$\delta I = \left(\frac{3m}{2\pi e \tau^2}\right) . \delta \bar{u}_o \quad \ldots\ldots\ldots\ldots (12)$$

and the fluctuations in the β-space are wholly accounted for by fluctuations in the average initial velocity at the barrier.

3.0. Space-charge Smoothing in Diode Systems Responding to V.H.F.

3.1. If the Benham-Llewellyn theory is applied to the β-space when the fluctuations are resolvable into components with time factors $\varepsilon^{j\omega t}$, and it is assumed that, as before, the constant component of the initial velocity u_o is zero, then from Appendix II :—

$$\delta V = \frac{8\pi^2 e I \tau^4}{m\alpha^4}\left\{2\delta i\left[2-(\alpha+2)\varepsilon^{-\alpha} - \alpha + \frac{\alpha^3}{6}\right] - \frac{m.\delta\bar{u}_o}{2\pi e \tau^2}\alpha^2\left[1-(1+\alpha)\varepsilon^{-\alpha}\right]\right\}\varepsilon^{j\omega t} \quad ..(13)$$

where δi is an alternating component of the *total* current, and $\alpha = j\omega\tau$, τ being the mean transit time in the β-space. In the V.H.F. range of 30 to 300 Mc/s, the transit angles in the α-space are assumed to be negligible compared with those in the β-space.

Now, it can readily be shown, by the method of Appendix I, that the low-frequency conductance g is expressible in the form $3m/(4\pi^2 e I \tau^4)$, so that (13) may be rewritten in the form :—

$$\delta i = g . \delta V \frac{\alpha^4}{12\left[2-(\alpha+2)\varepsilon^{-\alpha} - \alpha + \frac{\alpha^3}{6}\right]} + \frac{3m\delta\bar{u}_o}{2\pi e \tau^2} . \frac{\alpha^2[1-(1+\alpha)\varepsilon^{-\alpha}]}{6\left[2-(\alpha+2)\varepsilon^{-\alpha} - \alpha + \frac{\alpha^3}{6}\right]} \quad \ldots\ldots\ldots\ldots (14)$$

It will be noted that the time factor has been omitted. This equation corresponds to equation (8), the transit angle $\omega\tau$ having been brought into account through $\alpha (= j\omega\tau)$.

World Radio History

The factor of δV represents the small-signal admittance of the diode at the angular frequency ω, while the term in $\delta\bar{u}_0$ represents a component of fluctuating current arising from the component of fluctuation $\delta\bar{u}_0$ in $\bar{u}_0$.

3.2. According to earlier work[4], $\delta V = 0$ and, by taking the low-frequency value of the conductance, g, instead of the V.H.F. value, the expression :—

$$\delta i = \frac{3m\delta\bar{u}_0}{2\pi e\tau^2} \cdot \frac{2[1-(1+\alpha)\varepsilon^{-\alpha}]}{\alpha^2} \quad \ldots\ldots(15)$$

is obtained. This was originally interpreted as arising from an equivalent voltage generator of value :—

$$\delta v = \frac{3m\delta\bar{u}_0}{2\pi e\tau^2 g} \cdot \frac{2[1-(1+\alpha)\varepsilon^{-\alpha}]}{\alpha^2}$$

This acts through the conductance g of the diode. If the usual analysis from equation (15) is continued, formula (6) is obtained, the square of the modulus of the second main factor in (15) being equal to the factor in β in (6). By taking the full V.H.F. expression for the diode admittance and putting $\delta V = 0$ as before, the value of δi is given by the second term in equation (14). When $\omega\tau$ is small (say < 1), this may be written :—

$$\delta i \simeq \frac{3m\delta\bar{u}_0}{2\pi e\tau^2}\left(1 - \frac{11}{30}\alpha\right) \quad \ldots\ldots\ldots\ldots(16)$$

3.3. In either relation (15) or (16), δi and δu_0 are alternating components with a phase relation such that δi lags on $\delta\bar{u}_0$. Now, δi is a component of the fluctuation that is constituted by an excess (or deficiency) of electrons entering the β-space, and which, by its presence, produces the change $\delta\bar{u}_0$ in the corresponding component of the average velocity at the barrier.

Also, it is a well-known result[12] that when $dV/dx = 0$, as at the barrier, then the total current at and very near the barrier is wholly conductive, i.e., it is constituted solely by moving charges.

Again, the total current, comprising the conduction current and the displacement current, is uniform throughout the space at any instant. Therefore it is not a function of x, the distance measured from the barrier, subject to the limitation that all electron velocities present are small compared with the velocity of light.

It is concluded from these arguments that the total current δi in the β-space is vectorially equal to the conduction current leaving the barrier which constitutes the increase $\delta\bar{u}_0$ in $\bar{u}_0$, and, therefore, that δi must be in phase with $\delta\bar{u}_0$. This result is at variance with relations (15) and (16), and points the way to another approach to the problem of fluctuations at V.H.F.

3.4. Given that δi remains in phase with $\delta\bar{u}_0$ as the responding frequency is increased, it is reasonable to suppose that the relation between them is similar to equation (12), which gives correct results when applied to the problem of low-frequency response. Therefore, we write :—

$$\delta i \,.\, \varepsilon^{j\omega t} = \frac{3m\delta\bar{u}_0}{(2\pi e\tau^2)} \,.\, \varepsilon^{j\omega t} \quad \ldots\ldots\ldots\ldots\ldots(17)$$

This leads to a result similar to that for the low-frequency case, Γ^2 being the same as for low frequencies. This result has its limitations. If equation (17) is used to eliminate $\delta\bar{u}_0$ in equation (14), there results :—

$$\delta V = \frac{\delta i}{g}\left\{\frac{12}{\alpha^4}\left[2 - (\alpha+2)\varepsilon^{-\alpha} - \alpha + \frac{\alpha^3}{6}\right] - \frac{2}{\alpha^2}\left[1-(1+\alpha)\varepsilon^{-\alpha}\right]\right\}\varepsilon^{j\omega t}$$

or approximately (for $\alpha < j1$) :—

$$\delta V = \frac{11}{30}\alpha \,.\, \frac{\delta i}{g} \,.\, \varepsilon^{j\omega t} = \frac{11}{30}\omega\tau \,.\, \frac{\delta i}{g}\,\varepsilon^{j(\omega t + \pi/2)} \quad \ldots\ldots\ldots\ldots\ldots(18)$$

This means that, with V.H.F. components, there are additional fluctuation components of the barrier correlated to the current fluctuations, resulting in displacement currents in the β-space which maintain dynamic space-charge equilibrium while allowing $\delta\bar{u}_0$ to be in phase with δi. According to (18), as the frequency increases so δV increases and will, when α becomes large, probably invalidate equation (10). Therefore, the low-frequency value of Γ^2 probably holds for a frequency response up to about $\omega\tau = 1$ radian, and the present approximate theory will give little information on fluctuations at greater transit angles.

3.5. Experimental results[15,16] on diodes operating at transit angles around 7 radians at a frequency of 3,000 Mc/s show considerable reduction in the normal shot effect, approximately 10 : 1 due to transit time, and a further

World Radio History

10 : 1 due to space-charge smoothing. This appears to give qualitative support to equation (6). However, it is difficult to reconcile these results on diodes with the observed considerable increase with frequency in the noise experienced in triodes, both at V.H.F. and U.H.F. ; unless, as will be shown in the next section, the law of smoothing as a function of transit angle differs from the laws derived from (15) or (16).

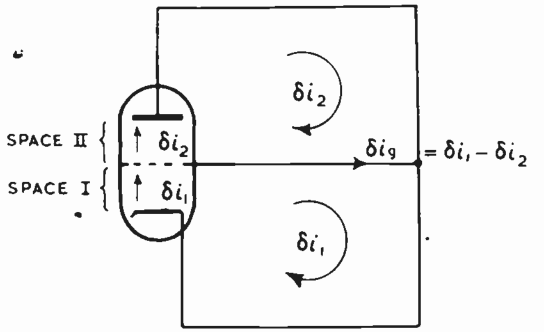

Fig. 3.—Basic triode system. The currents δi_1 and δi_2 are mesh currents associated with spaces I and II respectively. Not shown in this figure are the positive anode and negative grid direct voltages.

4.0. Fluctuations in Triode Systems Responding to V.H.F.

4.1. The basic triode system is shown diagrammatically in Fig. 3. Following the normal custom, the triode is divided into two spaces, the cathode-grid space and the grid-anode space. Direct induction through the grid is neglected, i.e., the cathode to anode direct capacitance through the active part of the electrode system is negligible compared with the other direct capacitances. Let the transit time of the mean state in space I be τ_1 and the transit time in space II be τ_2, and let $\alpha_1 = j\omega\tau_1$ and $\alpha_2 = j\omega\tau_2$. Direct voltages not shown in Fig. 3, are applied to the circuit, which cause a mean space current to flow to the anode with the grid at a negative potential relative to the cathode. There is an effective short-circuit to alternating currents across each space, as indicated in the figure.

The current component δi_1 originates in space I in the manner explained in the previous section, but now the effective voltage V_d in the grid plane replaces the diode-anode direct voltage, and g_m/σ replaces the diode g in the usual manner. Here, g_m is the low-frequency triode mutual conductance, and σ is a factor which is slightly less than unity. The component of current δi_2 in space II results from density modulation of the electron stream by the fluctuations in space I, the electrons passing between the grid wires and entering space II. It remains to investigate the magnitude and phase of the component of current δi_2 relative to δi_1.

4.2. We first examine the relation between δi_1, a component of the total fluctuation current in space I, and the purely conduction current which passes through the grid plane. The current in space II is not space-charge limited. Since δi_1 is the total current, the electron conduction current at the grid plane is, therefore :—

$$\delta i_{o2} = \delta i_1 - \frac{1}{(4\pi)} \cdot \left(\frac{\partial E}{\partial t}\right) \quad \ldots\ldots\ldots(19)$$

i.e., the total current, less the displacement current at the grid plane. The coefficient $(\partial E/\partial t)$ in (19) is to be evaluated at the position of the grid plane.

Now the method of Appendix II shows that :—

$$\frac{1}{4\pi}\cdot\frac{\partial E}{\partial t} = \left\{1 - \frac{2}{\alpha_1^2}\left[1-(1+\alpha_1)\varepsilon^{-\alpha_1}\right]\right\}\delta i_1 - \frac{\alpha_1\varepsilon^{-\alpha_1} m\,\delta u_o}{2\pi e\tau_1^2} \quad \ldots\ldots\ldots\ldots(20)$$

and if $\delta \bar{u}_o$ is eliminated by equation (17),

$$\frac{1}{4\pi}\cdot\frac{\partial E}{\partial t} = \left\{1 - \frac{2}{\alpha_1^2}\left[1-(1+\alpha_1)\varepsilon^{-\alpha_1}\right] - \frac{1}{3}\alpha_1\varepsilon^{-\alpha_1}\right\}\delta i_1 \quad \ldots\ldots\ldots\ldots(21)$$

$$\doteqdot \left(\frac{1}{3}\alpha_1 + \frac{1}{12}\alpha_1^2\right)\delta i_1 \text{ (when } \alpha_1 < 1).$$

If, at first, we assume that the transit time in space II, τ_2, is negligible, then the current δi_{o2} entering space II may approximately be identified with δi_2 and we may write :

$$\delta i_2 \doteqdot \delta i_{o2} = \left(1 - \frac{1}{3}\alpha_1 - \frac{1}{12}\alpha_1^2\right)\delta i_1 \quad (22)$$

4.3. More accurately, the total electron current in space II is expressed in terms of the conduction current δi_{o2} entering the space by :

$$\delta i_2 = \left(1 - \frac{2}{3}\alpha_2 + \frac{1}{4}\alpha_2^2\right)\delta i_{o2} \quad \ldots\ldots(23)$$

provided the mean potential at the grid plane is small compared with the mean bias on the anode.[17,18] In formula (23) δi_2 is the induced

World Radio History

electron current, but since the anode and the mean grid-plane potentials are constant, there is no displacement current component.

Formulæ (22) and (23) together give the result :—

$$\delta i_2 = \delta i_1 \left(1 - \frac{1}{3}\alpha_1 - \frac{1}{12}\alpha_1^2\right)$$

$$\left(1 - \frac{2}{3}\alpha_2 + \frac{1}{4}\alpha_2^2\right) \quad \ldots\ldots(24)$$

To the first power in α_1 and α_2, this is in agreement with existing results[10, 18, 19] but, differs in the coefficient of the second power in α_1.

The induced grid current component is (from Fig. 3) $\delta i_g = \delta i_1 - \delta i_2 \simeq 1/3\ \alpha_1$, to the first power in α_1 and neglecting α_2, which results in equation (7). Therefore, the present theory gives the same result as the accepted and proven theory.

4.4. The work of van der Ziel and Versnel on the noise factor of the grounded grid triode[11], where the component δi_g in the grid lead has no practical effect, has resulted in the introduction of a new induced noise effect, expressible as an equivalent resistance R_o where :—

$$\frac{1}{R_o} = \text{real part of } \left(y_1 - y_2 \frac{\delta i_1}{\delta i_2}\right) \quad \ldots(25)$$

In this, y_1 is the admittance of space I, not including the cold capacitance, and y_2 is the transadmittance of the current in space II, relative to the signal voltage across space I.

With low frequencies, $y_1 = y_2 = g_m$, but it can be shown by application of the Benham-Llewellyn theory[12] that at V.H.F.,

$$y_1 = g_m \left(1 - \frac{1}{5}\alpha_1 + \frac{7}{300}\alpha_1^2\right) \quad \ldots(26)$$

$$\text{and } y_2 = g_m \left(1 - \frac{11}{30}\alpha_1 + \frac{11}{150}\alpha_1^2\right)$$

$$\cdot\left(1 - \frac{2}{3}\alpha_2 + \frac{1}{4}\alpha_2^2\right) \quad \ldots\ldots(27)$$

Using (24) with (26) and (27) in (25), there results :—

$$\left(y_1 - y_2 \frac{\delta i_1}{\delta i_2}\right)$$

$$= g_m \left(1 - \frac{1}{5}\alpha_1 + \frac{7}{300}\alpha_1^2\right)$$

$$- g_m \frac{\left(1 - \frac{11}{30}\alpha_1 + \frac{11}{150}\alpha_1^2\right)}{\left(1 - \frac{1}{3}\alpha_1 - \frac{1}{12}\alpha_1^2\right)} \quad \ldots(28)$$

The real part of this is $- g_m \cdot \frac{11}{90}\alpha_1^2$ to the second power in α_1.

So :

$$\frac{1}{R_o} = \frac{11}{90} \cdot g_m \omega^2 \tau_1^2 \ldots\ldots\ldots\ldots\ldots\ldots(29)$$

According to the theory which follows from the adoption of equation (16), the term $- 1/12\alpha_1^2$ in equation (24) would become $+ 7/180\,\alpha_1^2$, which, in equation (28), would lead to the result $1/R_o = 0$.

4.5. In the grounded grid triode measurements[11] discussed by van der Ziel and Versnel, $g_m = 12 \times 10^{-3}$ (mhos) and $\omega = 2{\cdot}7 \times 10^8$ (radians/sec), and it is probable that $\tau_1 \simeq 5 \times 10^{-10}$ (sec). Putting these values in formula (29) gives $R_o \simeq 40{,}000$ (ohms). The value derived from the measurements[11] was of the order of 3,000 ohms, with a large probable error.

5.0. The Effect of Non-uniformity of the Cathode-Grid Field of Triodes on Fluctuations at V.H.F.

5.1. In the foregoing discussion of the triode, it has been assumed that the field near the cathode is uniform for all parts of the cathode surface, as in a diode. With most modern triodes operating with a negative grid, however, the field near the cathode is by no means uniformly distributed. Fig. 4(*a*) illustrates the field in a typical negative grid condition. Immediately behind each grid wire, the field between the cathode and grid is retarding ; while between each pair of grid wires, the field near the cathode is an accelerating field.

It is known that in a retarding field in which the transit time is appreciable, the electron excursions induce fluctuations (total emission noise) in the circuit containing the space in question.[20] It follows, therefore, that in a triode in which the grid is sufficiently negative and of suitable geometry to produce a field such as

World Radio History

that of Fig. 4 (*a*), there will be additional noise on account of parts of the cathode being associated with a retarding field.

If, on the other hand, the grid is less negative or is of different geometry so as to eliminate island formation when in the operating state, then with a field distribution such as that shown in Fig. 4 (*b*) there will be no appreciable number of electrons in a retarding field, and the total emission noise will be negligible.

These considerations apply whenever the transit time in any retarding field portions of the cathode-grid space is appreciable.

5.2. When the times of transit in the α-spaces associated with those parts of the cathode opposite an accelerating field are also appreciable, one would expect total-emission noise, associated with these spaces, to be manifest. However, the tentative theory is now advanced

(a)

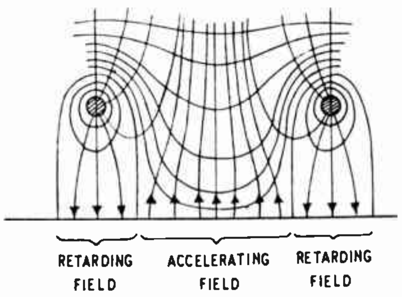

Fig. 4(a).—Field plot of the cathode-grid space of a triode exhibiting island formation.

(b)

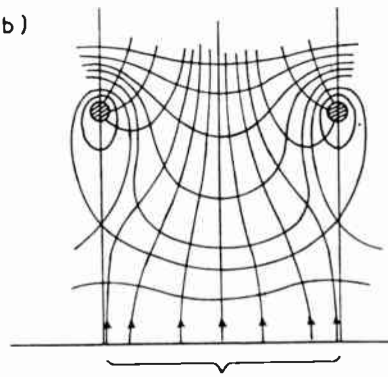

Fig. 4(b).—Field plot of the cathode-grid space of a triode proportioned and operated so as to eliminate island formation at the cathode surface.

that the potential-minimum, when situated relatively close to the cathode, as is the case with high anode current density, not only acts as a barrier to low-velocity electrons, but acts also as an effective barrier to electric induction. Total-emission fluctuations in the α-space would then be manifest in an outer circuit only through intermediate fluctuations in the barrier magnitude or position. Judging by the relative contribution of the α-space fluctuations to Γ^2 in the low frequency theory,[8] it is expected that, when close to the cathode, the barrier will exercise considerable smoothing on the effect of the α-space fluctuations. When the barrier is nearer the anode (or grid), due to very close electrode spacing without an appropriate increase in current density, it will produce little smoothing of the α-space fluctuations.

Such a theory appears to be in accordance with experimental observations[15, 16] on the noise of a space-charge limited diode, with not too close an electrode spacing, responding to 3,000 Mc/s. The observed noise was accountable in terms of β-space fluctuations, in qualitative agreement with equation (6).

The theory is also consistent with conclusions drawn from recent measurements of total emission damping.[21]

6.0. Acknowledgement

Acknowledgement is made to the Ministry of Supply for permission to publish this paper, but the opinions expressed do not necessarily represent the views of the department.

LIST OF PRINCIPAL SYMBOLS

- E Electric field intensity.
- I Space current (amp.).
- I_s Current leaving cathode.
- R Resistance (ohms).
- V Potential difference.
- a Acceleration (cm/sec^2.).
- e Magnitude of electronic charge (e.s.u.).
- f Frequency (c/s).
- g Conductance (mho).
- g_m Triode mutual conductance.
- i Alternating component of current.

World Radio History

j $+\sqrt{-1}$.
k Boltzmann's constant.
m Mass of electron (gm).
t Time (sec.)
u Initial velocity (cm/sec.).
v Alternating e.m.f.
x Distance from barrier.
y Admittance.
α $j\beta$ (or space division label).
β $\omega\tau$ (or space division label).
θ Temperature (°K).
ρ Space-charge density (e.s.u.).
σ Geometrical constant for triodes.
τ Transit time (sec.).
ω Angular frequency (radians/sec.).
Γ Root mean square space-charge smoothing factor.

7.0. References

1. N. R. Campbell and V. J. Francis. *J.I.E.E.*, Vol. 93, Part III, 1946, pp. 45-52.
2. W. Schottky. *Ann. d. Phys.*, Vol. 57, 1918, pp. 541-567.
3. E. Spenke. *Wiss. Veröff aus den Siemens Werken*, Vol. 16, 1937, pp. 127-136.
4. A. J. Rack. *Bell System Tech. Jour.* Vol. 17, 1938, pp. 612-619.
5. D. B. Fraser. *Wireless Engineer*, Vol. 26, 1949, pp. 129-132.
6. W. Schottky and E. Spenke. *Wiss. Veröff aus den Siemens Werken*, Vol. 16, 1937, pp. 1-41.
7. A. J. Rack. *Bell System Tech. Jour.*, Vol. 17, 1938, pp. 592-612.
8. D. O. North. *R.C.A. Review*, Vol. 4, 1940, pp. 441-473.
9. D. A. Bell. *Proc. Phys. Soc.*, Vol. 62B, 1949, pp. 334-335.
10. C. J. Bakker. *Physica*, Vol. 8, 1941, pp. 23-43.
11. A. van der Ziel and A. Versnel. *Philips Research Reports*, Vol. 3, 1948, pp. 255-270.
12. F. B. Llewellyn. "Electron Inertia Effects," Cambr. Univ. Press, 1941 ; W. E. Benham *Proc.I.R.E.*, Vol. 26, 1938, pp. 1093-1170.
13. E. J. Schremp. "Vacuum Tube Amplifiers," M.I.T., McGraw-Hill, 1949, pp. 563-568.
14. J. R. Pierce. *Bell System Tech. Jour.*, Vol. 27, 1948, pp. 167-172.
15. R. Kompfner, J. Hatton, E. E. Schneider and L. A. G. Dresel. *J.I.E.E.*, Vol. 93, Part IIIA, No. 9, 1946, p. 1441.
16. N. T. Lavoo. *Proc.I.R.E.*, Vol. 37, 1949, pp. 383-386.
17. D. O. North. *Proc.I.R.E.*, Vol. 24, 1936, pp. 108-136.
18. M. J. O. Strutt and A. van der Ziel. *Physica*, Vol. 8, 1941, pp. 81-108.
19. M. J. O. Strutt. "Ultra and Extreme Short Wave Reception" (van Nostrand), 1947, p. 62.
20. A. van der Ziel and A. Versnel. *Nature*, Vol. 159, 1947, pp. 640-641 ; *Philips Research Reports*, Vol. 3, 1948, pp. 13-23.
21. G. Diemer and K. S. Knol. *Physica*, Vol. 15, 1949, pp. 459-462.

8.0. Appendix I

Quasi-steady State of Space-charge Limited Currents

The current between two parallel plane electrodes is considered. The total current I is given by :—

$$I = \rho u + \frac{1}{(4\pi)} \cdot \frac{\partial E}{\partial t} \quad \ldots\ldots\ldots\ldots (A1)$$

where ρ is the space-charge density and E is the electric field intensity at a distance x from the first plane at a time t.

The equation of motion of an electron is :—

$$eE = ma \quad \ldots\ldots\ldots\ldots (A2)$$

where e is the magnitude of the electronic charge, m is the electronic mass, and a is the acceleration imparted to the electron by the field. Starting from the relation (A1), it has been shown[12] that :

$$I = \frac{1}{(4\pi)} \cdot \frac{dE}{dt} = \frac{m}{(4\pi e)} \cdot \frac{da}{dt} \ldots\ldots (A3)$$

For an electron which leaves the first planar electrode (cathode or barrier) at $t = O$, we have an integrating (A3) with respect to t successively, between the limits $t = O$ to $t = \tau$:—

$$\frac{m}{(4\pi e)} \cdot a = I\tau \qquad \text{(A4)}. \ (a=0 \text{ at } t=0)$$

$$\frac{m}{(4\pi e)} \cdot u = \tfrac{1}{2}I\tau^2 \qquad \text{(A5)}. \ (u=o \text{ at } t=0)$$

$$\frac{m}{(4\pi e)} \cdot x = \tfrac{1}{6}I\tau^3 \qquad \text{(A6)}. \ (x=o \text{ at } t=0)$$

if $x=0$ at the first plane and if τ is the time of transit to a point distant x from this plane. Also, if $-V$ is the potential at any point in the space, it can also be shown that[12] :—

$$V = 2\pi I \int_0^{\tau} a\tau^2 d\tau \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(A7)}$$

Let the initial velocity be increased from zero to δu_o, and let the potential difference between the electrodes be increased from V to V + δV. As a result, let the current increase to I + δI and let the velocity at any point in the space become $u + \delta u$. Since x does not increase, let τ be defined by equation (A6). We stipulate that, in the change, the acceleration a_o at $x = 0$ remains at zero. Then, corresponding to (A4), (A5) and (A6) we have :—

$$\frac{m}{(4\pi e)} \cdot (a+\delta a) = (I + \delta I) \cdot (\tau + \delta\tau)$$

$$= I\tau + I \cdot \delta\tau + \tau \cdot \delta I \quad \ldots\text{(A8)}$$

$$\frac{m}{(4\pi e)} \cdot (u+\delta u) = \frac{1}{2}(I + \delta I) \cdot (\tau + \delta\tau)^2 + \frac{m\delta u_o}{(4\pi e)}$$

$$= \frac{1}{2} I\tau^2 + \frac{1}{2}\tau \cdot \delta I + I\tau \cdot \delta\tau + \frac{m\delta u_o}{(4\pi e)} \ldots\ldots\ldots\ldots\text{(A9)}$$

$$\frac{m}{(4\pi e)} \cdot x = \frac{1}{6}(I + \delta I) \cdot (\tau + \delta\tau)^3 + \frac{m\delta u_o\tau}{(4\pi e)}$$

$$= \frac{1}{6} I\tau^3 + \frac{1}{2} I\tau^2 \cdot \delta\tau + \frac{1}{6}\tau^3 \cdot \delta I + \frac{m\delta u_o\tau}{(4\pi e)} \ldots\ldots\ldots\text{(A10)}$$

From each of these equations we subtract the corresponding equation of the set (A4) to (A6), giving from (A10) :—

$$I \cdot \delta\tau = -\frac{1}{3}\tau \cdot \delta I - \frac{m\delta u_o}{(2\pi e\tau)} \quad \ldots\ldots\text{(A11)}$$

and from (A8) :—

$$\frac{m}{(4\pi e)} \cdot \delta a = I \cdot \delta\tau + \tau\delta I,$$

or with (A11) eliminating $\delta\tau$:—

$$\frac{m}{(4\pi e)} \cdot \delta a = \frac{2}{3}\tau \cdot \delta I - \frac{m\delta u_o}{(2\pi e\tau)} \quad \ldots\ldots\ldots\text{(A12)}$$

The equation corresponding to (A7) is :—

$$V + \delta V = 2\pi \int_0^{\tau} (I + \delta I) \cdot (a + \delta a) \cdot \tau^2 \cdot d\tau \quad \ldots\ldots\ldots\ldots\text{(A13)}$$

(The τ^2 and $d\tau$ in this integral are unchanged)[12], and from this and equation (A12) we obtain :—

$$\delta V = \frac{8\pi^2 eI}{m} \int_0^{\tau} \left[\frac{2}{3}\tau^3 \cdot \delta I - \frac{m\tau \cdot \delta u_o}{(2\pi e)}\right] d\tau,$$

and :—

$$\delta V = \frac{8\pi^2 eI}{m}\left[\frac{1}{6}\tau^4 \cdot \delta I - \frac{m\tau^2\delta u_o}{(4\pi e)}\right] \quad \text{(A14)}$$

Therefore,

$$\delta I = \frac{3m \cdot \delta V}{4\pi^2 e I\tau^4} + \frac{3m \cdot \delta u_o}{2\pi e\tau^2} \quad \ldots\ldots\text{(A15)}$$

is the increase in the total space current δI, due to a slow increase δV in the potential difference across the diode, and an (independent) increase in the initial velocity δu_o.

9.0. Appendix II

Fluctuations of Space-charge Limited Currents Resulting from Fluctuations in Potential Difference and Initial Velocity

In equation (A3) of Appendix I, we place :—

$$\frac{m}{(4\pi e)} \cdot \frac{da}{dt} = I_o + \delta I \cdot \varepsilon^{j\omega t} = I \quad \ldots\ldots\ldots\text{(A16)}$$

where I_o is a steady current with a small sinusoidal component δI superimposed to form the total current I.

In the same manner as in Appendix I, this is integrated from $t = t_o$ to t, putting $t = 0$ at $x = 0$ and $a = 0$ at $x = 0$. Then :—

$$\frac{m}{(4\pi e)} \cdot a = I_o(t - t_o) + \frac{\delta I}{(j\omega)} \cdot (\varepsilon^{j\omega t} - \varepsilon^{j\omega t_o}) \ldots\text{(A17)}$$

World Radio History

Integrating again, with $u = \delta u_o \varepsilon^{j\omega t_o}$ at $x = 0$,

$$\frac{m}{(4\pi e)} \cdot u = \frac{1}{2} I_o (t - t_o)^2 - \frac{\delta I}{\omega^2} \cdot (\varepsilon^{j\omega t} - \varepsilon^{j\omega t_o})$$

$$- \frac{\delta I}{(j\omega)} \cdot (t - t_o) \varepsilon^{j\omega t_o} + \frac{m\delta u}{(4\pi e)} \cdot \varepsilon^{j\omega t_o} \quad \ldots\ldots\ldots\ldots (A18)$$

and again, where $x=0$ at $t=t_o$,

$$\frac{m}{(4\pi e)} \cdot x = \frac{1}{6} I_o (t - t_o)^3 - \frac{\delta I}{(j\omega^3)} \cdot (\varepsilon^{j\omega t} - \varepsilon^{j\omega t_o})$$

$$+ \frac{\delta I}{\omega^2} \cdot (t - t_o) \varepsilon^{j\omega t_o} - \frac{\delta I}{(2j\omega)} \cdot (t - t_o)^2 \varepsilon^{j\omega t_o}$$

$$+ \frac{m\delta u_o}{(4\pi e)} \cdot (t - t_o) \varepsilon^{j\omega t_o} \quad \ldots\ldots\ldots (A19)$$

When $\delta I = 0$, let $\tau = t - t_o$, and when δI is introduced, let $t - t_o$ become $\tau + \delta\tau$, following Llewellyn[12]. Then $m/(4\pi e) . x = 1/6 . I_o \tau^3$ defines τ. In (A19), we then have, if we write $\alpha = j\omega\tau$ and note that $\varepsilon^{j\omega t_o} = \varepsilon^{-\alpha} . \varepsilon^{j\omega t}$,

$$\frac{1}{2} I_o \delta\tau = \frac{\delta I}{j\omega} \left\{ \frac{\varepsilon^{-\alpha} - 1}{\alpha^2} + \frac{\varepsilon^{-\alpha}}{\alpha} + \frac{\varepsilon^{-\alpha}}{2} \right\} \varepsilon^{j\omega t}$$

$$- \frac{\alpha}{j\omega\tau^2} \cdot \frac{m\delta u_o}{4\pi e} . \varepsilon^{-\alpha} . \varepsilon^{j\omega t} \quad \ldots . (A20)$$

This is neglecting small quantities of the second order. The potential difference is given by (A7), in which τ is defined by (A6).

Now

$$\frac{ma}{(4\pi e)} = I_o \tau + I_o \delta\tau + \frac{\delta I}{(j\omega)} . \varepsilon^{j\omega t} (1 - \varepsilon^{-\alpha})$$

and therefore

$$\frac{m\delta a}{(4\pi e)} = \frac{\varepsilon^{j\omega t}}{j\omega} \left\{ \delta I \left[\frac{2(\varepsilon^{-\alpha} - 1)}{\alpha^2} + \frac{2\varepsilon^{-\alpha}}{\alpha} \right.\right.$$

$$\left.\left. + \varepsilon^{-\alpha} + (1 - \varepsilon^{-\alpha}) \right] - \frac{m\delta u_o}{2\pi e \tau^2} \alpha \varepsilon^{-\alpha} \right\}.$$

It then follows that :—

$$\delta V = \frac{8\pi^2 e I}{m\omega^4} \int_0^{\alpha} \left\{ \delta I \left[2(\varepsilon^{-\alpha} - 1) + 2\alpha\varepsilon^{-\alpha} + \alpha^2 \right] \right.$$

$$\left. + \frac{m\delta u_o}{2\pi e} . \omega^2 \alpha \varepsilon^{-\alpha} \right\} \varepsilon^{j\omega t} . d\alpha$$

$$= \frac{8\pi^2 e I \varepsilon^{j\omega t}}{m\omega^4} \left\{ 2\delta I \left[2 - (\alpha + 2) \varepsilon^{-\alpha} \right.\right.$$

$$\left. - \alpha + \frac{\alpha^3}{6} \right] + \frac{m\delta u_o}{(2\pi e)} . \omega^2 [1 -$$

$$\left. (1 + \alpha) \varepsilon^{-\alpha}] \right\} \ldots\ldots\ldots\ldots\ldots (A21)$$

World Radio History

NOTICES

Increased Power of Midland Home Service Transmitters

From Sunday, June 25th, the power of the Midland Home Service, transmission on 276 metres (1,088 kc/s) from Droitwich was increased from 60 kilowatts to 150 kilowatts, the maximum permitted by the Copenhagen Wavelength Plan. The power of the Norwich transmitter, which broadcasts this programme on the same wavelength, was also increased.

The increased power from Droitwich has been achieved by modifying a high-power long-wave transmitter to work on medium waves. This transmitter, which was built in 1934 to replace the original Daventry " 5XX " transmitter, broadcast the Light Programme on 1,500 metres until the Copenhagen Plan came into force on March 15th this year.

The increase in power will give clearer reception of the Midland Home Service, especially for those listeners who live at some distance from the transmitting stations.

Radio and Television Electricity Load

Sir Vincent de Ferranti, M.C., in his Presidential address to the second British Electrical Power Convention at Harrogate on Tuesday, June 20th, pointed out that with an electric system all sorts of unexpected and considerable loads appeared.

" As an example of this," continued Sir Vincent, " is radio and television, which have not only made a supply of electricity an absolute necessity in every home, as indicated by the rapid increase in the number of consumers connected during the boom years of that industry, but have brought a connected load of 1,200,000 kW to the supply system."

Once again, so far as the manufacturers were concerned, the supply industry both at home and abroad, was not their only customer, as very many concerns included the radio and telecommunications industries. Of the total number employed in electrical manufacturing industry of 500,000, about 200,000 were working in the electrical machinery and wires and cables group, and probably 50,000 of those were employed in plant and equipment for the B.E.A. and Area Boards.

Television Aerials

The Radio and Electronic Component Manufacturers' Federation and the British Radio Equipment Manufacturers' Association have recently published a booklet which poses the question " Outdoor or Indoor."

It is intended primarily for the guidance of municipal authorities and other property owners in the selection of the type of television aerial required in a particular location. The booklet is written in a non-technical manner and should certainly facilitate a clearer understanding of the subject by a layman.

Copies may be obtained from the Secretary, B.R.E.M.A., 58 Russell Square, London, W.C.1, price 1s.

United Nations

The formative work of U.N.E.S.C.O. in regard to its relationship with engineering and technical associations on a world basis is now in progress. A Conference is being convened by U.N.E.S.C.O. in October, 1950, when it is proposed to form a " Union of International Specialist Engineering Organizations " which will be sponsored by U.N.E.S.C.O.

The Secretariat of the Economic and Social Council U.N. have also stated that it is to international organizations that they will look for assistance in finding the technical personnel required for the development of their scheme for giving aid to the " Under-developed " countries.

New Television Equipment

The B.B.C. has ordered four sets of television camera equipment, two from Marconi's Wireless Telegraph Co., Ltd., and two from Pye, Ltd.

Two of these equipments, each comprising a mobile control room with three operational cameras and associated equipment, will be used initially at the Festival of Britain next year.

The other two equipments, each providing three operational cameras with their control equipment, will be used for the new television studios at Lime Grove, Shepherds Bush, and for outside broadcasts.

World Radio History

CONTRIBUTIONS

SOME NOTES ON METHODS OF PULSE MODULATION*

by

E. G. Beard

Lately much development has taken place in methods of communication based upon the use of pulses instead of continuous waves, and many articles have been published dealing with various types of pulse modulation. The principal features and defects of the more common methods of pulse modulation are briefly stated in the following notes.

In all pulse transmission systems samples of the signal to be transmitted are taken at specified intervals of time, and these samples are transmitted. The rate at which the samples are taken must be higher than the highest frequency signal.

At the receiving end these pulses can be integrated into a smooth signal with the aid of a low-pass filter or an integrating circuit according to the system used.

Pulse Amplitude Modulation

Pulse amplitude modulation is probably the simplest method of transmitting information by pulses. The overall envelope of the pulses is modulated just as the envelope of a continuous wave is modulated in the usual amplitude modulation.

The most common application of pulse amplitude modulation is in connection with time division telephony.

Probably pulse amplitude modulation is the least satisfactory from the viewpoint of noise, as limiters and pulse regenerators cannot be used. The receiver makes use of a linear peak detector as with normal amplitude modulation and a low-pass filter.

Pulse Frequency Modulation

Pulse frequency modulation resembles the frequency modulation of continuous wave. The modulation varies the recurrent rate of the pulses.

Maximum and minimum amplitude limiters may be used with pulse frequency modulation, and so it has advantages over pulse amplitude modulation so far as noise is concerned.

Pulse regeneration does not eliminate noise because the exact time of arrival of a pulse is not known beforehand, so noise can occur because it may displace the pulses in time.

The system is not suitable for time division telephony for the same reason.

Demodulation involves the use of a frequency discriminator as in normal f.m. practice. The system is not suitable for time division telephony.

Pulse Phase Modulation

In pulse phase modulation the intelligence is conveyed by the time displacement of the pulses from specified instants. The recurrent frequency does not alter, but the spacing of the pulses does.

Maximum and minimum amplitude limits can be used, but pulse regeneration cannot be used to eliminate noise because of the unknown time of arrival of a pulse.

The noise advantages are probably not so marked as with pulse frequency modulation, the difference being similar to that between frequency modulated and phase modulated continuous waves. Reception is possible with the aid of an integrating circuit.

Pulse Width Modulation

Pulse width modulation may be based on amplitude, phase or frequency modulation, the variation in the duration of the pulses representing the factor concerned.

Pulse width modulation involves the times of commencement and termination of the pulse and, therefore, it is probable that although maximum limiters can be used, distortion may result from the use of a minimum limiter.

Noise will also cause time displacement of the start and finish of the pulse and so have some effect which cannot be eliminated by pulse regeneration.

Reception is more difficult than in systems not dependent upon pulse width.

*Reprinted from the *Proc.I.R.E. (Aust)*, January, 1950.

World Radio History

Pulse Code Modulation

Pulse code systems can be regarded as the application of telegraph practice to telephony. A code group of pulses is used to represent the different instantaneous levels of the signal to be transmitted at regularly spaced sampling times.

In pulse code signalling the information necessary to actuate the receiver correctly is confined to the simple information as to whether or not a pulse is transmitted at certain predetermined times. As the times of arrival of the pulses are specified, the pulses can be regenerated and noise cannot affect the result by causing changes in either amplitude or times of starting or stopping a pulse. Hence pulse code modulation has a marked advantage over the preceding systems so far as noise encountered during transmission is concerned. Unfortunately, the system introduces another kind of noise called quantitization noise. This noise is introduced because of the limited number of levels which can be transmitted by the code used. A five-unit code enables 32 levels to be transmitted, and a seven-unit code 128 levels. The complication of coding and decoding the levels rapidly increases as the number of units in the code is increased and so quantitization noise represents a very serious limitation.

In some systems the coding is carried out by a cathode-ray tube which has an arrangement of collector plates designed to suit the code used. The collectors are scanned horizontally by the beam at a rate synchronized with the pulse repetition frequency, while the height at which the scanning takes place is controlled by the level to be transmitted. Thus, the shape of collectors at different heights performs the coding.

Decoding at the receiver is much simplified if the code is suitably designed. For instance, the pulses may be caused to put electrical charges into a leaky capacitor and the voltage of this capacitor tapped off after a specified time interval. The voltage will, therefore, depend upon the number of pulses received and the times at which they charge the capacitor.

Pulse code modulation is suitable for the time division telephony because the pulses occur at specified intervals of time.

Delta Pulse Modulation

Whereas in practically all other pulse modulation systems the information transmitted is the level of the signal at sampling times above some reference data line, in delta pulse transmission advantage is taken of the fact that the level of the signal at the preceding sample time has already been transmitted to the receiver. The information transmitted is as to whether the instantaneous level has increased (or decreased) by a specified quantity since the last sampling time. In other words, the transmitted signals convey the slope or the rate of change of the level of the signal in between the sampling times. Hence the name delta pulse modulation.

Signals are received by integrating the pulses in a resistance capacity circuit.

At the transmitter the signals are integrated in a local receiver. At the next sampling time the difference between the level in this local receiver and the signal determines whether or not a pulse is to be transmitted. Transmission of a pulse applies an additional unit charge to the integration condensers. Absence of a pulse leaves the charge to decay. Hence the information to be transmitted can be conveyed by the presence or absence of a pulse at specified intervals of time.

As the timing of the pulses is known, the pulses can be regenerated and the system is as free from noise introduced in transmission as is the pulse code system. The equivalent to quantitization noise can be controlled by increasing the frequency of the pulses and so there is no serious practical limitation.

As the information transmitted refers to slope and not amplitude, there is no theoretical limit to the amplitude of the signals which can be transmitted.

The receiving equipment is simple, being in the main an integration circuit.

Delta Pulse Code Modulation

Delta pulse code modulation is, as its name suggests, a combination of the delta system with the code system. A code is adopted for transmitting the changes in level instead of simple pulses. This, of course, introduces the problems of coding and decoding, but avoids many of the problems of quantitization noise as the range of levels which has to be transmitted by the code is restricted to that necessary to define the slope.

Delta pulse code modulation probably represents, at the moment, the most complete solution of the noise problem in communications.

World Radio History

MULTI-STATION V.H.F. COMMUNICATION SYSTEMS USING FREQUENCY MODULATION*

by

W. P. Cole, B.Sc.,† and E. G. Hamer, B.Sc.(Eng.)† (*Associate*)

A paper read before the London Section on May 25th, 1950

SUMMARY

The reasons for the use of multi-station V.H.F. systems of communication are discussed, and an historical survey of previous technical experiments on F.M. systems is given.

The standard equipment normally used for a single-station scheme is described and its technical performance stated. From the basis of this standard equipment the required performance of multi-station equipment is derived. It is shown that one of the most important factors for such schemes is that the main carrier frequencies should be nearly identical and the permissible difference of frequency determined.

Linking frequencies available for a control link are discussed and, in conjunction with the permissible frequency difference, the basic design factors of the equipment are evaluated.

Details of a particular type of equipment are described, this equipment being that used for the initial experiments and for a completed installation.

The factors affecting distortion caused by multipath transmission are discussed and a quantitative analysis made of the distortion occurring under the worst conditions. From this analysis the maximum permissible path difference for reasonable speech quality is determined, this being ± 30 miles and this is shown to agree with practical tests.

The practical tests carried out in the London area and in Scotland are described. Possible future developments of such systems are indicated.

TABLE OF CONTENTS

1.0. Introduction

Recent operational requirements in the V.H.F. mobile communication field require expansion of the service area normally covered by a single fixed station using frequency modulated transmission. It is anticipated that requirements will also arise for a restriction of the service area.

A single high-power station using frequency modulation enables large areas of difficult territory to be covered, provided that a good

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† Communication from the Staff of the Research Laboratories of The General Electric Co., Ltd., Wembley, England.
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World Radio History

site is available. In many cases the use of such a station gives unwanted coverage in areas a considerable distance away, causing mutual interaction with other V.H.F. systems.

Due to the large number of services requiring frequency allocations in the V.H.F. bands, many services must be allocated the same frequencies in the hope that the geographical separation of stations will be sufficient to prevent mutual interaction. For these requirements, low powered multi-stations on relatively poor sites are to be preferred, as the service area can be more accurately controlled. In general the cost of a multi-station scheme will compare favourably with that of a single-station scheme as site development, including such items as extension of access roads and power lines, forms a large percentage of the total cost. Sites for schemes to restrict the service area and hence reduce the possibility of mutual interaction would normally be located nearer to existing facilities than a single station sited on a hill.

2.0. Historical Survey

Tests on area coverage systems using frequency modulated transmissions have been carried out as follows :—

(*a*) North London—July 1945.

(*b*) South Middlesex and Berkshire—July 1946.

(*c*) Fifeshire and West Lothian—October 1946.

(*d*) S.E. London—January 1947.

These tests showed that technically such systems were practical. In areas of mutual interaction, between stations approximately synchronized in frequency, reception of signals was entirely satisfactory.

In view of later operational requirements for such systems further tests were carried out in the London area to resolve the engineering problems associated with more recently developed types of equipment.

3.0. Technical Description of Standard Equipment

Standard radio equipment as used for single station V.H.F. frequency-modulated systems is employed as far as possible. The mobile station radio equipment is identical for multi-station V.H.F. area coverage systems and single-station V.H.F. systems.

A brief technical summary of the standard radio equipment used in the recent tests is as follows :—

3.1. *Receiver*

The receiver is of the " fixed-tuned " double-superheterodyne type, the first intermediate frequency being 8-9 Mc/s and the second intermediate frequency approximately 455 kc/s. A crystal-controlled local oscillator is used to heterodyne the first intermediate frequency, and a multiple of the oscillator frequency to heterodyne the radio frequency input signal. Any frequency errors in the initial grinding of the transmitter and receiver crystals may be eliminated by adjustment of the exact frequencies of the first and second intermediate frequencies.

Frequency	30-180 Mc/s
Bandwidth	±15 kc/s for 6 db. loss in sensitivity.
Sensitivity	1 μV depresses noise output by 20 db. at 100 Mc/s 2·5 μV depresses noise output by 20 db. at 180 Mc/s
Spurious Responses	Better than −50 db. apart from second channel −35 db.
Audio Frequency Output Power.	1·0 watt for ±10 kc/s deviation

Muting operates at below a 3 db. quieting signal.

3.2. *Transmitter (for mobile use and as driver for higher-powered fixed station)*

The output of the transmitter is directly derived from a quartz crystal mounted in a thermostatically controlled oven. The audio frequency input voltage is integrated and then applied to a radio frequency phase modulator, providing frequency modulation of the crystal frequency.

Frequency Range :—	30-180 Mc/s.
Nominal Peak Deviation :—	±12·5 kc/s.

Mean Deviation :—	±5 kc/s.
Power Output :—	12 watts at 100 Mc/s, 9 watts at 180 Mc/s.
Spurious Outputs :—	better than 60 db. down on fundamental.

The minimum receiver bandwidth was determined from conditions of temperature drift of components, mainly crystals, and it leaves adjacent channels available for other users.

The transmitter peak deviation is nearly equal to the receiver bandwidth and thus efficient use is made of the bandwidth required from receiver design considerations. The transmitter mean deviation is kept high, and if a peak of speech does cause an instantaneous deviation greater than the nominal peak deviation, the only deleterious effect is to cause an instantaneous peak overload of the receiver audio output stage.

4.0. Description of Multi-station System

In order to meet the technical requirements of the narrow bandwidth standard mobile receivers, the frequency of the main outgoing transmitter at the main and satellite stations must either be the same, or differ by an amount less than 2 kc/s. If the transmitted frequencies differ by more than 2 kc/s from the centre frequency to which the receiver is tuned, distortion of the speech intelligence will occur, and there will be a somewhat reduced signal-to-noise ratio.

The return circuit from the mobile stations may be demodulated at the satellite and main stations. The speech intelligence may then be conveyed to the central control position by means of separate land lines or radio links. At this central control position the various speech signals will require to be combined to give a common output.

Fig. 1 shows a block schematic diagram of a system fulfilling the above requirements. The frequency of all the main outgoing transmitters is principally controlled by a single quartz crystal oscillator. A radio link to the satellite stations is required for frequency controi purposes and for the conveyance of the intelligence to be transmitted by this station.

Each station is equipped with a receiver tuned to the frequency of the mobile transmitters. The outputs of these receivers are transmitted to the audio mixing unit at the control point by means of landlines or radio links.

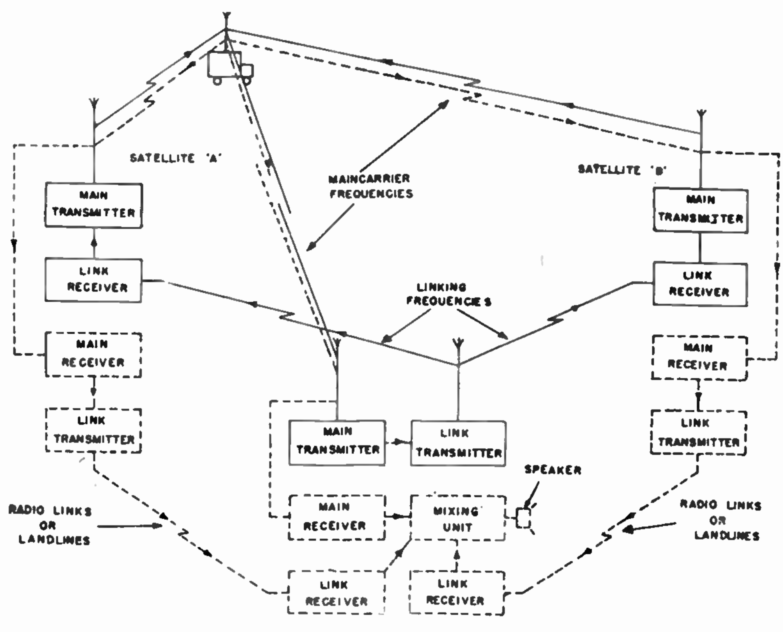

Fig 1.—Simplified block diagram of complete system.

5.0. Control of Frequencies of Stations

Earlier experiments had shown that, to prevent the mutual interaction of the sidebands of several frequency modulated transmissions, which carry the same intelligence, thus causing distortion, the carrier frequencies must be:—

(*a*) maintained within 30 c/s of one another,

or (*b*) separated by a frequency greater than the highest modulating frequency.

The temperature coefficient for a standard 6-10 Mc/s crystal is 2 parts/10^6/deg. C and, even when the thermostatically-controlled oven with limits of ± 1 deg.C is used, the maximum

World Radio History

TABLE 1

BANDWIDTHS OF RECEIVERS AT 6DB. POINTS

	SIMPLEX SINGLE FREQUENCY		SIMPLEX DOUBLE FREQUENCY	
	Any No. of satellites. Essential Radio Links Only	2 Satellites. All Radio Links.	Any No. of satellites. Radio Links Only.	2 Satellites. All Radio Links.
Main Outgoing Circuit at approx. 100 Mc/s	±15 kc/s	±15 kc/s	±15 kc/s	±15 kc/s
Main Incoming Circuit at approx. 100 Mc/s			±15 kc/s	±15 kc/s
Linking Frequency In 150 Mc/s Band	±40 kc/s	±40 kc/s	±40 kc/s	±40 kc/s
Talk-Back Circuit No. 1 Satellite		±15 kc/s		±15 kc/s
Talk-Back Circuit No. 2 Satellite		±15 kc/s		±15 kc/s
Total Frequency Spectrum	110 kc/s	170 kc/s	140 kc/s	200 kc/s

error between two independent transmitters operating at 100 Mc/s is 400 c/s.

In order to use separate crystals a more complex type of oven and smaller temperature coefficient quartz crystals would be required.

In the interests of simplicity a common crystal may be used to control the frequencies of all the stations.

6.0. Available Frequencies for Fixed Radio Links

A linking frequency is required for transmitting a master control frequency and the intelligence to the satellite stations. In this country the available frequency allocations for these types of service are in the region of 150 Mc/s, 470 Mc/s and still higher frequencies. These frequency allocations also apply to any radio links which may be required for talk-back circuits if land lines are not used.

In general these linking frequencies will be in higher frequency bands than main frequency. The master control frequency, therefore, may be multiplied, transmitted to the satellite station and then divided again to give the master control frequency at the satellite station.

By the use of suitable multiplication and division ratios this linking frequency may be placed approximately in any of the frequency bands to be used. The linking frequency is, however, "tied" to the master control frequency, so that any one master control frequency has a series of linking frequencies associated with it.

To enable any linking frequency to be used, the originally derived linking frequency may be altered slightly by means of an auxiliary crystal and transmitted to a satellite station, where a similar auxiliary crystal is used to regain approximately the same master control frequency. As separate auxiliary crystals are used at master and satellite stations, a frequency error will exist between main radiated frequencies, but this can be made less than 30 c/s. (See Appendix I.)

7.0. Total Frequency Spectrum used by Such Systems

In the case of the outgoing circuit, the mobile receiver bandwidth is independent of the number of satellite stations. The frequency spectrum used, as calculated on the basis of the receiver bandwidths and using linking frequencies in the 150 Mc/s band, is shown in Table I.

World Radio History

8.0. Description of Equipment Used

Fig. 1 shows a block schematic diagram of a typical installation comprising a master and two satellite stations.

8.1. *Outgoing Circuit from Main Station to Mobile Station*

Fig. 2 shows the outgoing circuits, comprising the equipments at main and satellite stations.

The main station transmitting equipment comprises the master control frequency crystal oscillator (frequency f). The output voltage from this oscillator is fed into two chains. In chain (a), this voltage is phase modulated, fed to a series of frequency multiplier and amplifier stages to provide a power output of 100 watts at the main carrier frequency, and radiated to the mobile receivers. In chain (b), the master control frequency f is shifted by an amount Δf by the use of an auxiliary crystal oscillator and a frequency changer unit. The voltage at frequency $f \pm \Delta f$ is then phase modulated and fed to a series of frequency multiplier and amplifier stages to provide a power output of 25 watts at the linking carrier frequency and radiated to the satellite stations.

The phase modulators at the master station are fed from a common audio frequency source, an audio frequency time delay network being interposed between the audio frequency source and the phase modulator of the main transmitter. The reason for this is explained in a later section.

The satellite receiving equipment comprises a receiver which accepts a voltage at the linking frequency and gives an output voltage whose frequency is within 30 c/s of the main carrier frequency. This voltage is then amplified to provide a power output of 100 watts which is radiated to the mobile receivers.

Fig. 3a shows a more detailed block schematic diagram of the "link receiver." The input voltage at the linking frequency F is amplified and then heterodyned with a voltage whose frequency is derived from a quartz crystal oscillator of frequency fx. The voltage at the intermediate frequency IF_1 thus produced is then amplified and heterodyned with a voltage whose frequency is derived from the auxiliary crystal of frequency fy. The voltage at the intermediate frequency IF_2 then produced is amplified and fed to a wide bandwidth regenerative divider network producing a voltage at frequency IF_3. The voltage at frequency IF_3 is heterodyned with a voltage derived from the quartz crystal oscillator, frequency fx. The output voltage of this frequency changer is fed to a frequency multiplier and amplifier stages to produce an output voltage whose frequency is within 30 c/s of the main carrier frequency.

It will be seen that the frequency of the output voltage is independent of any frequency errors due to the local oscillator frequency fx, the only frequency error being due to differences of frequency caused by the separate auxiliary crystals in the main and satellite stations.

It will be noted that no demodulation of the intelligence conveyed on the outgoing circuit takes place at the satellite stations. This simplifies the accurate control of modulation levels of the main carrier frequencies.

If the linking frequency allocated is such that auxiliary crystals are not required there will be no frequency error between the main carriers at the main and satellite stations.

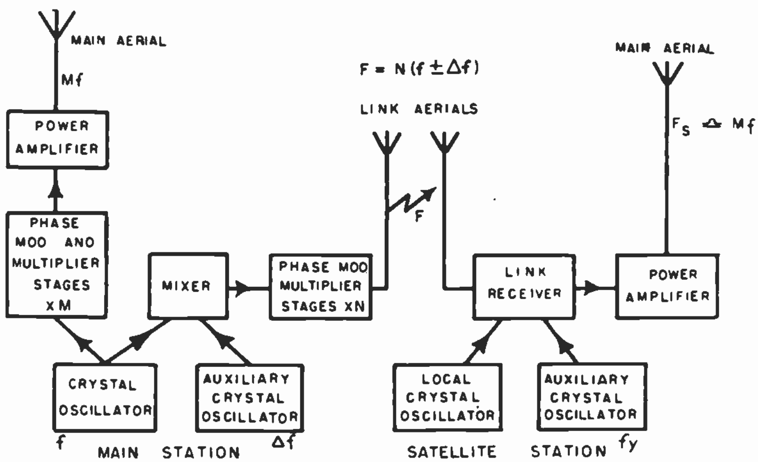

Fig. 2.—Block schematic diagram of outgoing circuit.

World Radio History

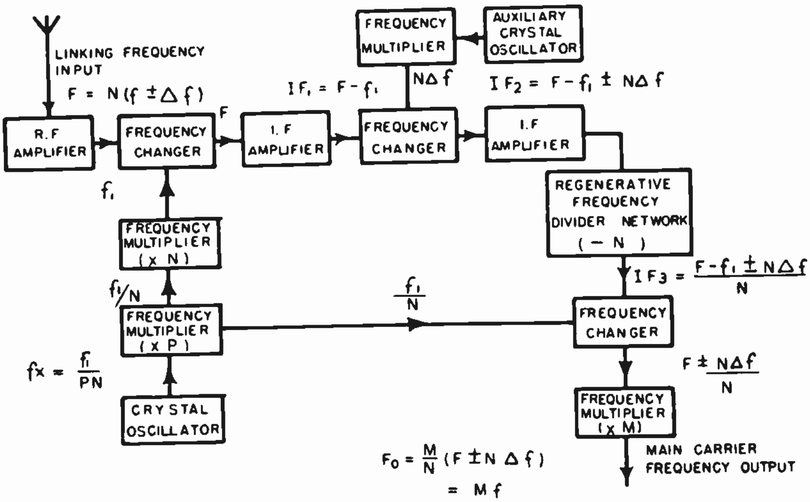

Fig. 3a.—Block schematic diagram of link receiver.

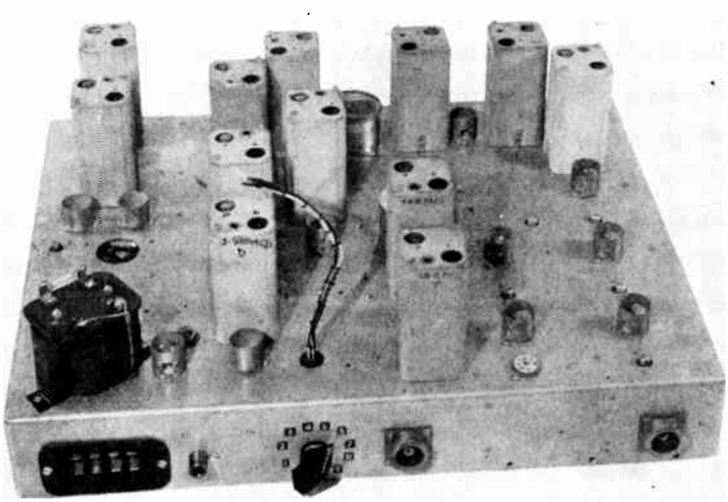

Fig. 3b.—Photograph of link receiver.

8.2. *Incoming Circuit from Mobile Station to Main Station*

Fig. 4 shows the incoming circuits comprising the equipments at main and satellite stations.

The main station receiving equipment consists of :—

(*a*) a receiver tuned to the same frequency as the mobile station transmitter.

(*b*) receivers tuned to the satellite "talk-back" frequency if radio linkages are used, or alternatively the terminating units of the land lines from the satellite stations.

The audio frequency output voltages of the receivers and land lines if used are combined in an audio frequency mixing unit. The audio frequency signals are combined in proportions determined by their individual signal-to-noise ratios.

The satellite station equipment comprises a receiver tuned to the mobile station transmitter, the audio frequency output of this receiver being fed to the main station using a land line or radio link.

8.3. *Aerials*

The main outgoing and return transmissions are vertically polarized, as omnidirectional radiation is required. A ground plane whip aerial is used on the vehicle fitted with the mobile radio equipment, and a vertical half-wave aerial at the main station.

The linking frequency is transmitted with vertical polarization, since by this means it is possible to link it to several satellite stations using one aerial. In the event of a long distance link to a remote satellite station, an extra directive aerial may be fed from the link transmitter.

Horizontally polarized aerials are employed on the return circuit radio link from the satellite to the main station (if radio links are used). This has been done to minimize mutual interaction between local transmitters and

World Radio History

receivers, and also for ease of mounting the aerials used at the top of a 78-ft. mast.

9.0. Distortion due to Multi-path Transmission

The distortion due to multi-path propagation of frequency-modulated signals has been treated in papers,[1, 2, 3, 4] both from the theoretical and experimental view points. The resultant distortion is influenced by several factors and the complete analysis is exceedingly complex.

In the interests of simplicity the following assumptions are made :—

(*a*) A single sinusoidal modulating frequency is used.

(*b*) The receiver limiter characteristic is ideal.

(*c*) The effects of noise are neglected.

When a frequency-modulated signal arrives at a receiver via two different paths there will, in general, be amplitude and phase differences between the carrier waves. The receiver detector output, assuming distortionless discriminator and detector circuits, will consist of:—

(*a*) A direct current.

(*b*) The fundamental component of the modulating frequency.

(*c*) The distortion caused by the multi-path propagation.

The distortion produced is a complex quantity, but it can be shown[1, 2, 3, 4] that the distortion is zero when the relative phase difference of the modulating frequency in the two paths is small.

When a phase difference exists between the modulating frequency in the two paths, the amplitude distortion increases as the transmitter modulating input is increased to a certain critical value ; beyond this point the distortion decreases, finally fluctuating by a small amount about zero as the input is steadily increased. Beyond this critical region all the distortion characteristics for different phase shifts and carrier amplitude ratios converge in an undulating fashion as the transmitter modulating input is increased, and distortion is reduced by maintaining the useful range of input above this critical region.

Distortion increases rapidly as the ratio of the carrier amplitudes approaches unity, the distortion being at a maximum when the phase difference of the carriers is zero or a multiple of π. Variation of deviation ratio, modulating frequency and the relative phase difference of modulating frequency in the two paths, also affect the distortion.

It is interesting to note that, if the transmitter is operated below its critical region of maximum distortion due to multi-path transmission, an increase of the modulating frequency will decrease the distortion. If, however, the transmitter is operated above this critical level an increase of modulating frequency will have the effect of increasing the distortion.

In general, when the multi-path propagation occurs due to the use of synchronized stations, the phase relationship of the modulation at one or more equi-signal positions may be varied by the use of suitable time delay

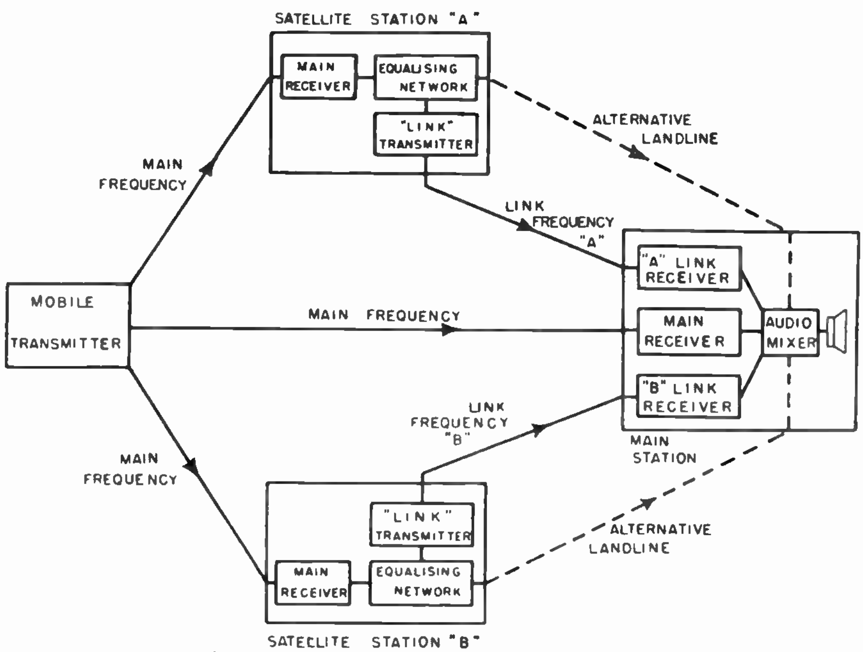

Fig. 4.—Block schematic diagram of incoming circuit.

World Radio History

equipment at one or more of the stations. This modulating frequency phase difference at any one equi-signal position may be made zero and, if the phase difference at other equi-signal positions is not excessive, usable signals will be received throughout the whole area.

Assuming all other variable quantities at equi-signal positions are such as to cause maximum distortion, the maximum modulating frequency phase difference may be evaluated for a given total distortion.

9.1. *Distortion due to Multi-path Transmission under Worst Conditions*

A quantitative analysis of the distortion which would occur under extreme conditions has been made in order to assist in appreciating how distortion arises.

The subjective distortion actually encountered and its importance is dealt with later since it proved impossible in practice to find during trials conditions as bad as those given in this analysis. The distortion takes the form of a chip in a sinusoidal wave.

Assuming that the distortion is caused by simple two-path propagation only, the distortion encountered in the equipment described under the following conditions has been evaluated :—

Ratio of carrier frequency amplitudes 0·8:1

Phase difference between carrier frequencies π

Peak frequency deviation of the transmitter $\pm 12{\cdot}5$ kc/s.

A graphical analysis of the resultant distortion may be made by considering the resultant of the individual carriers during the modulating cycle. This resultant differs in phase from the phase of the individual carriers and it is this phase difference (or rather the time derivative of it) which gives directly the addition to the signal conveyed by the first carrier because of the presence of the second carrier. (See Appendix II.)

Normally the harmonic distortion of a modulating frequency of 3,000 c/s will be reduced by the high frequency attenuation in the audio frequency circuits of the receiver. The 800 c/s modulating frequency will produce harmonics in the speech range but a path difference of 30 miles is permissible for good intelligence.

In general, if the modulating frequency phase error is corrected at a suitable equi-signal position, this correction will be adequate for all other normally encountered equi-signal positions, and the distortion will be undetectable by direct listening tests.

In the case of a partially synchronized system the distortion may be calculated at any instant by considering the phase of the carrier frequencies. The previous calculations have been based on the worst conditions (a carrier phase difference of π at an equi-signal area). The distortion will vary at a rate equal to the difference between the carrier frequencies, and from the maximum distortion already calculated to some smaller value.

10.0. Tests in London Area

Recent tests were carried out with an installation in the London area using a main and two satellite stations. Fig. 1 shows a block schematic diagram of the equipment used. Due to the frequencies allocated it was possible to use a synchronized system, the linking frequency being 3/2 times the main carrier frequency. Under such conditions the standing wave pattern produced is stationary, and it is possible to detect this pattern and place mobile stations at nulls and other equi-signal positions.

In the case of an approximately synchronized system, the space pattern of standing waves will be continuously moving at a rate equal to the difference frequency between the transmitters, and it is impossible to investigate the moving equi-signal position. The effect is similar to that produced by a mobile station passing through equi-signal positions caused by a synchronous system of transmitters.

If, in an approximately synchronized scheme, the mobile station were stationary, and so placed that a null position passed the mobile aerial, a small burst of noise having a rate equal to the frequency difference of the transmitters would be heard.

Investigation of standing wave patterns for a synchronized system have shown that the no-signal areas due to the complete cancellation of the carriers are very few and a movement of the mobile receiver aerial by a distance of

World Radio History

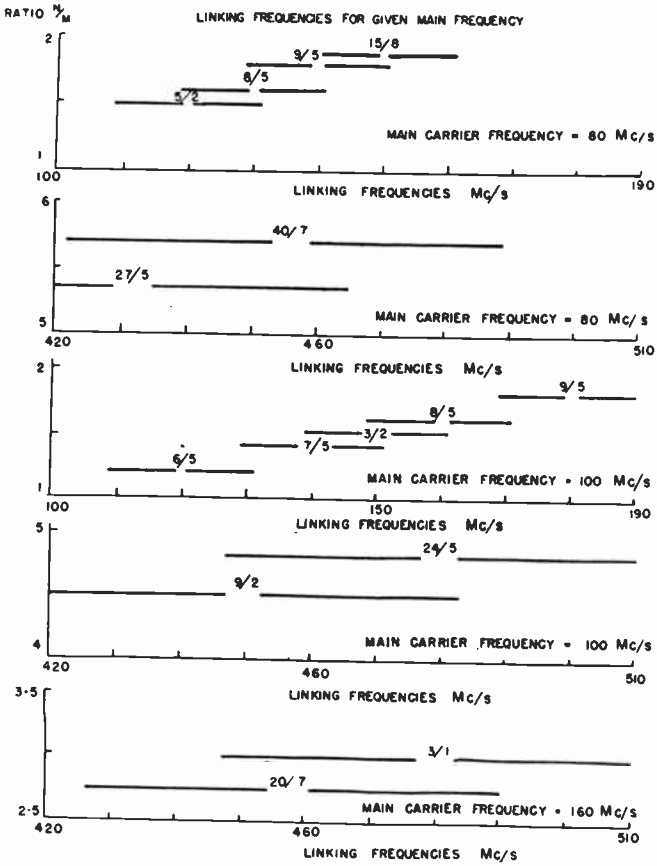

Fig. 5.—Linking frequencies for given main frequency.

6 in. from the null is sufficient to increase the signal to a usable level.

The above investigations were made in areas where individual signals were of the order 1-3μV at the receiver.

Equi-signal areas, where the signal was 10-50μV input to the receiver were also investigated, but it was impossible to detect any difference between single-station and multi-station working. The method employed in these strong equi-signal areas was to apply continuous common speech or tone modulation to all the outgoing circuits, the mobile station being moved through the area in question. Observers were unable to detect any transition, although it was shown by sequential switching of stations, observations of signal-to-noise ratio, and field strength indicators that a transition from the sphere of influence of one station to that of another was taking place.

Fig. 6 is a map of the London area showing the location of sites and equi-signal areas. Thick black lines show routes traversed, in the limited time available, by mobile stations where reliable two-way communication was obtained. It will be seen that the satellite stations are located fairly near to the master station and approximately in line. This was due to a desire to reduce the area of coverage and to the difficulty of obtaining suitable temporary sites. To obtain effective coverage of such an area, a triangular system of stations would normally be used.

It is interesting to note that the service area of this system is approximately the same as that of a single 500 watt frequency-modulated station operating on a high site in South London.

Fig. 7 shows the distortion which takes place in an equi-signal area owing to an 800 c/s modulating tone, as the audio frequency delay to the main transmitter is varied. A series of photographs were taken, the delay being varied to simulate path differences in steps of 2½ miles. The distortion due to the use of incorrect audio frequency delay is mainly harmonic, and, owing to the attenuation of the higher audio frequencies in the receiver audio circuits, is difficult to hear. Perfect intelligibility is, however, maintained even under an artificially introduced path difference of 35 miles. The photographs were taken with the oscilloscope directly connected to the receiver discriminator circuit in order to eliminate the effects of high audio frequency attenuation in the receiver circuits.

The experimental results are seen to agree closely with those predicted theoretically.

11.0. Tests in Scotland

To illustrate the application of such systems in localities over which it would be impossible to maintain reliable communication with a single station, some recent tests made in Ayrshire will be briefly described.

Fig. 8 is a map of Ayrshire showing the

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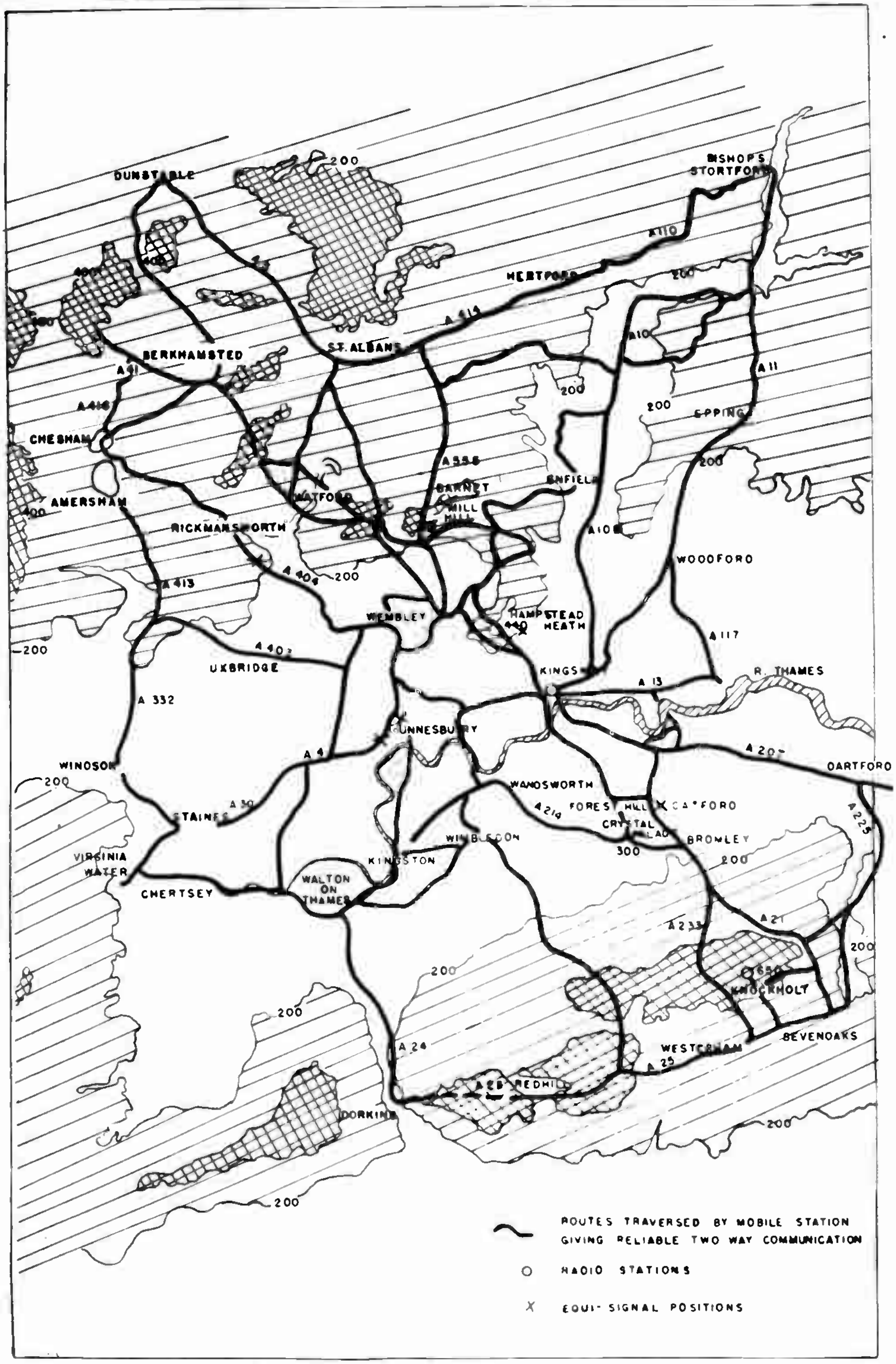

Fig. 6.—Map of the London area, showing location of sites and radio coverage.

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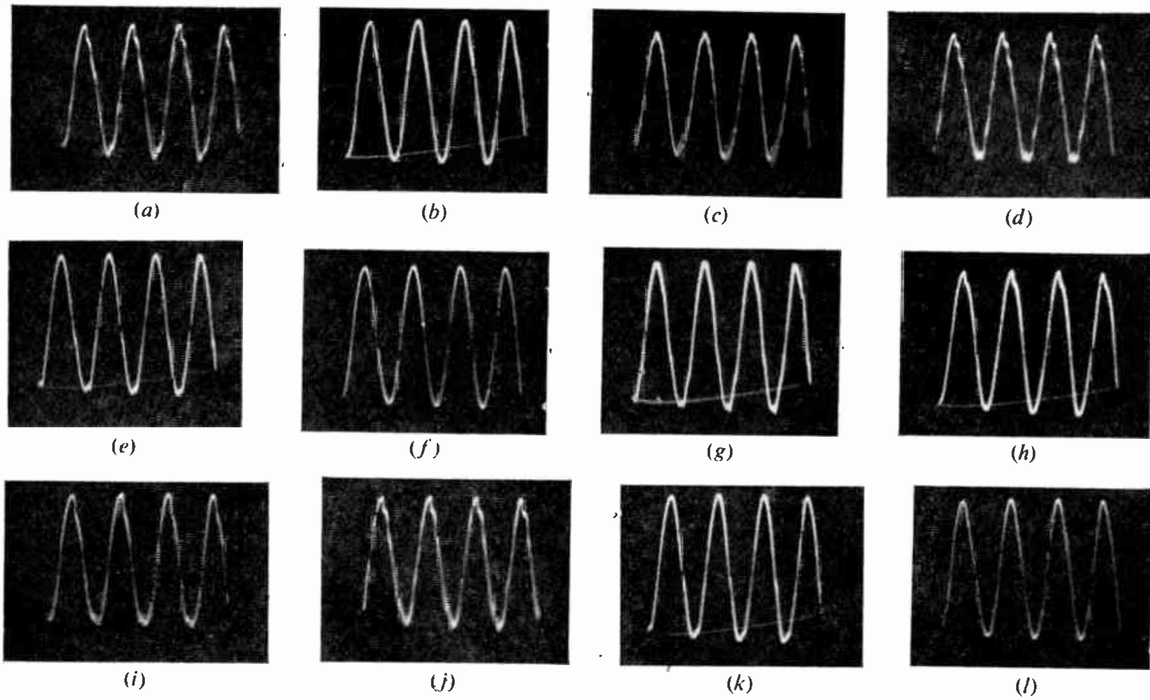

Fig. 7.—Photographs of 800 c/s modulating tone showing distortion due to multi-path transmission with variation of path difference of carriers.

Ratio of Carrier X to Carrier Y with no modulation = 1·2 : 1. *With zero correction X was in advance of Y by* 18 *miles.*

(*a*) *X* 18 *miles in advance of Y*
(*b*) *X* 15 ,, ,, ,, ,, *Y*
(*c*) *X* 12 ,, ,, ,, ,, *Y*
(*d*) *X* 9 ,, ,, ,, ,, *Y*
(*e*) *X* 6 *miles in advance of Y*
(*f*) *X* 3 ,, ,, ,, ,, *Y*
(*g*) *X* 0 ,, ,, ,, ,, *Y*
(*h*) *Y* 3 ,, ,, ,, ,, *X*
(*i*) *Y* 6 *miles in advance of X*
(*j*) *Y* 9 ,, ,, ,, ,, *X*
(*k*) *Y* 12 ,, ,, ,, ,, *X*
(*l*) *Y* 42 ,, ,, ,, ,, *X*

location of stations, and it gives an indication of the topography of the county. Thick black lines show the routes traversed by mobile stations where reliable two-way communication was obtained, during a recent survey.

The main problems encountered in Ayrshire are not the same as those experienced in built-up areas. Rather, the difficulties lie in obtaining the reliable coverage of rugged mountain country traversed by a relatively small number of roads, mainly in the valleys, but with an important road following the coast line at sea level with steeply rising hills from the coast line. The hills are mainly solid rock covered by a thin layer of soil and many points where reliable two-way communication occurs are not "line of sight" from any of the transmitting stations. This particularly applies to certain sections of the coast road and some of the roads in the valleys, and it is evident that considerable refraction and reflection is taking place.

Theoretical considerations and practical tests have shown that it is impractical to obtain reliable two-way communication over the complete highway system by the use of a single station.

12.0. Possible Trends of Future Development

Due to the large number of services requiring frequency allocations in the V.H.F. band, it is desirable that the total frequency spectrum required, determined from considerations of receiver bandwidth, should be kept to a minimum in these bands. The use of centimetric wave-lengths for the linking frequencies would reduce the total ether space required in the V.H.F. band to 30 kc/s for single frequency simplex working.

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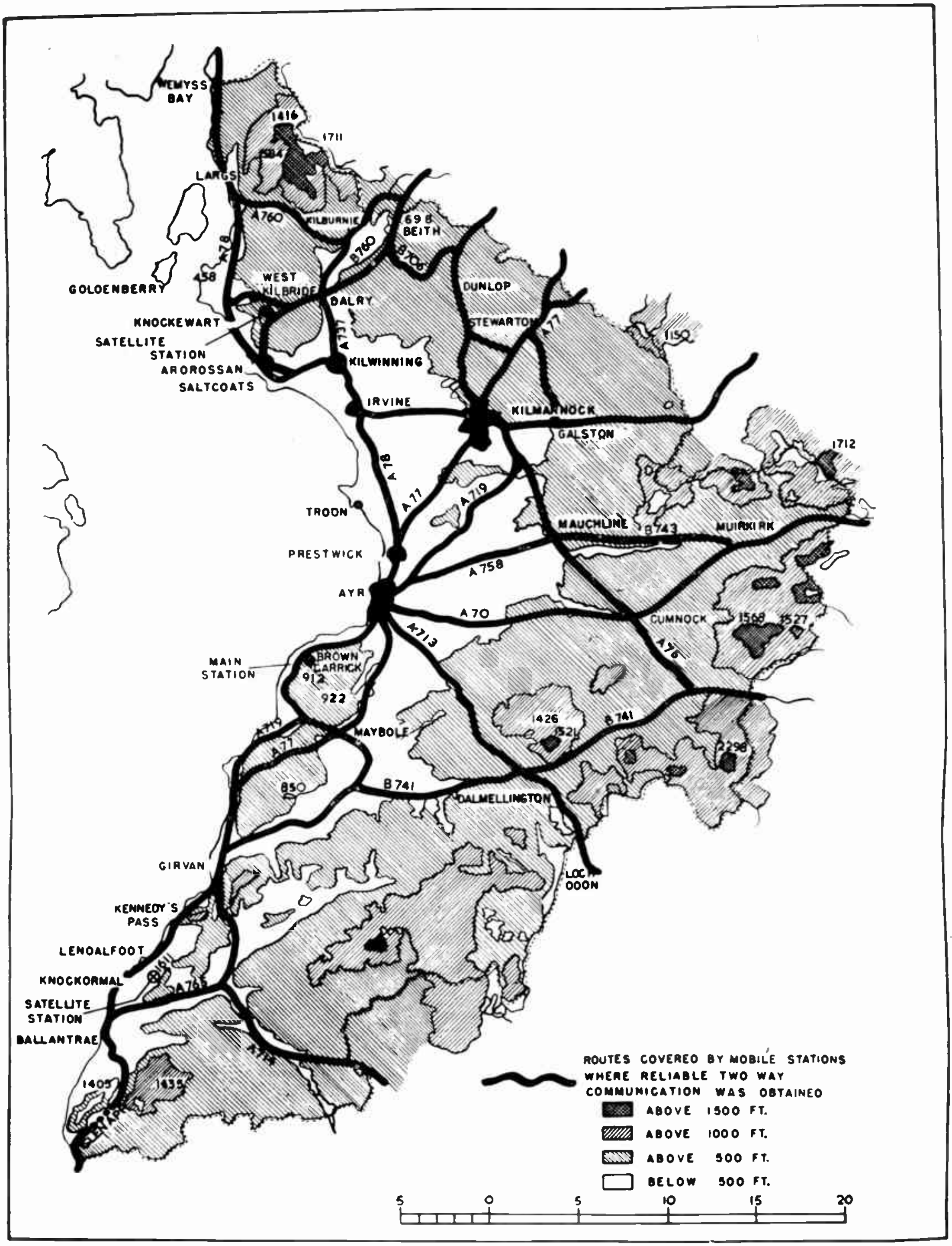

Fig. 8.—Map of Ayrshire showing location of sites and radio coverage.

The linking frequency in the centimetric band could be used either to directly synchronize the satellites or to carry the master control frequency in the form of modulation.

Fig. 9 shows a tentative system of main and satellite stations for use with a simplex double frequency system. The main transmitted carrier at the satellite station could be accurately controlled in frequency from the receiver input carrier by modification of the system already described or alternatively by the use of a stable auxiliary crystal.

In the event of a linking frequency not being available, a system such as that used to synchronize television stations could be used. This consists of a central receiving station monitoring the main and satellite carrier frequencies and sending correcting signals by land line to the satellite station.

13.0. Conclusions

The various methods of synchronizing or partially synchronizing frequency modulated V.H.F. transmitters have been described and discussed. The results obtained by the methods at present in use have fully justified these systems, and show that there are no inherent difficulties, such as the effects of multi-path propagation, to prevent satisfactory service in F.M. area coverage systems.

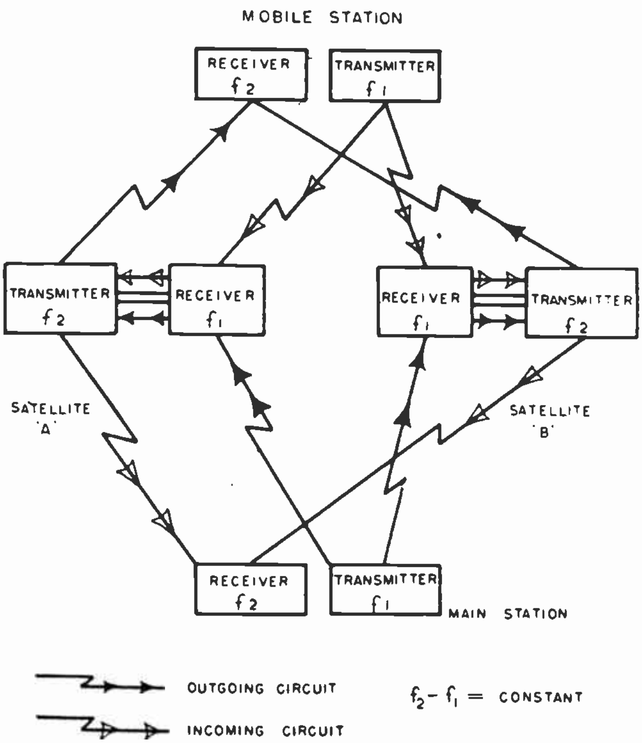

Fig. 9.—Simple double frequency multi-station scheme.

14.0. References

1. M. G. Crosby. "Frequency-modulation propagation characteristics." *Proc.I.R.E.*, June, 1936. Vol. 24. pp. 898-913.
2. M. G. Crosby. "Observations of frequency-modulation propagation on 26 megacycles." *Proc.I.R.E.*, July, 1941. Vol. 29. pp. 398-403.
3. M. S. Corrington. "Frequency modulation distortion caused by multi-path transmission." *Proc.I.R.E.*, December, 1945. Vol. 33. pp. 878-891.
4. S. T. Meyers. "Non-linearity in frequency modulation radio systems due to multi-path propagation." *Proc.I.R.E.*, May, 1946. Vol. 34. pp. 256-265.

15.0 Appendix I

Effects of Inaccuracy in Auxiliary Crystal Frequencies

It is assumed in the following that the initial crystal grinding errors have been corrected by the use of suitable circuits.

The thermostatic ovens hold the crystal temperature within $\pm$ 1 deg. C, and the temperature coefficient of the crystals is, as before, 2 parts/10^6/deg. C.

Let F be the linking frequency obtained by direct multiplication of the main control frequency, and the ratio of F to the main frequency be N : M.

Let ΔF be the frequency by which it is required to move frequency F to the allocated linking frequency.

As already stated, the maximum permissible difference between the main carriers is 30 c/s. The maximum frequency error at linking frequency F due to the auxiliary crystals is :—

$$\frac{2 \times 2 \times \Delta F}{10^6} \text{ c/s.}$$

$\therefore$ The maximum error at the main carrier frequency is :—

World Radio History

TABLE 2

HARMONIC DISTORTION DUE TO MULTI-PATH PROPAGATION UNDER WORST CONDITIONS

Modulating Frequency	Delay		Distortion as Percentage of Fundamental										
	Degrees of Audio Cycle	Time in μ secs.	1	3	5	7	9	11	13	15	17	19	21
800 c/s	5°	17·3	7	7	5	3	2	1·3	Less than 0·5				
800 c/s	45°	155	2·5	6·5	10·5	0·6	22	26	10	6·6	2·4	10·9	8·1
3,000 c/s	5°	4·7	23	10	5	2	2	1·4	Less than 0·5				
3,000 c/s	45°	44	10	40	33	29	25	20	18	14	16	6	1·8

$$\frac{4 \times \Delta F \times M}{10^6 N} \text{ c/s}$$

$$\text{Hence } 30 = \frac{4 \times \Delta F \times M}{10^6 N}$$

$$\Delta F = \frac{N \times 7{\cdot}5}{M} \times 10^6 \text{ c/s}$$

Hence for a main carrier frequency of 100 Mc/s and a ratio of

$$\frac{N}{M} = \frac{3}{2}$$

$$\text{then } \Delta F = \frac{7{\cdot}5 \times 3}{2} \times 10^6$$

$$= 11{\cdot}25 \text{ Mc/s}$$

∴ The linking frequency may be between 138·75 and 161·25 Mc/s.

Technical considerations, on the grounds of simplicity of equipment, make it desirable not to use the band of frequencies within ±1 Mc/s of the direct multiple. Fig. 5 shows how the linking frequencies for several main carrier frequencies may be accommodated in the various frequency bands without causing an error of more than 30 c/s between the main carriers at the master and satellite stations.

16.0. Appendix II

Calculation of Distortion due to Multi-path-Propagation

The distortion caused by multi-path propagation may be evaluated mathematically[4] though the resulting analysis is complex.

The present paper uses a graphical method of approach requiring only simple mathematics and giving at the same time a clearer concept of the physical phenomena that take place.

In this method the resultant of the two carrier components is plotted at a number of equi-distant intervals during the modulation cycle. The time derivative of the varying phase difference of this resultant with respect to that of the principal carrier gives the components added to the signal conveyed by the first carrier, due to the presence of the second carrier.

Numerical values for the components are obtained by subjecting the time derivative of the varying phase difference to a Fourier Analysis.

If we consider modulation by a single sinusoidal signal it will be found that the added signal consists mainly of harmonics of the modulating signal and contributes little to the fundamental. (Table 2.)

Graphical Method

For a given modulating frequency and peak carrier deviation, the peak phase excursion of the carrier vector is :—

$$\varphi \max = \frac{\text{Peak Deviation}}{\text{Modulating Frequency}}$$

World Radio History

The phase excursion of the first carrier vector is plotted as an ordinate against the modulating frequency cycle as abscissa (Fig. 10). The second carrier vector carries out the same phase excursion as the first but delayed by a constant time, hence its phase excursion can be plotted on Fig. 10 by moving the first curve to one side by the appropriate amount.

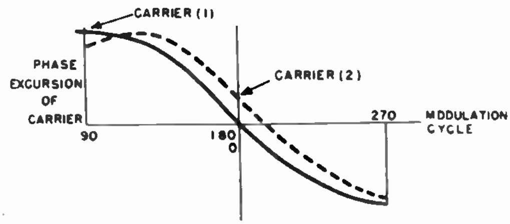

Fig. 10.

The difference between the ordinates of the two curves at a given position indicates the phase shift that exists between the two carriers, as far as it is due to the application of the modulating signal. This variable phase difference is additional to any fixed phase difference between the two unmodulated carriers due to propagation path differences.

In practice this graph need not be drawn as the values may be obtained directly from trigonometrical tables.

For the purposes of the next stage of the calculation it is assumed that the ratio of the two carrier amplitudes is 0·8:1, and the fixed phase difference is π.

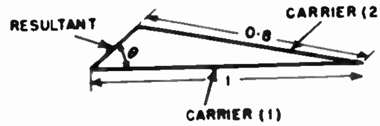

Fig. 11.

The angle θ, the resultant vector makes with respect to the first carrier vector, is obtained by drawing the vector diagrams for selected points during the modulating cycle (Fig. 11) and Fig. 12 is a graph of θ during the modulating cycle.

By differentiating this curve of θ the added signal arising from the presence of the second carrier is obtained, the value of the fundamental and harmonic components may be evaluated by the application of Fourier Analysis to this derivative.

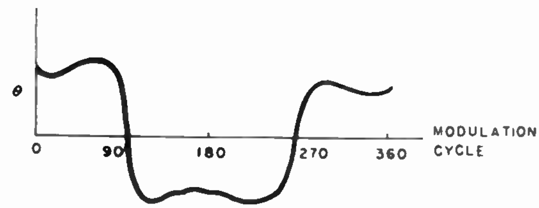

Fig. 12.

A more convenient and accurate method is to apply Fourier Analysis directly to Fig. 12 and to differentiate the components thus obtained. In the interests of accuracy it is advisable that the points chosen for the initial analysis should be those points suitable for the application of Fourier Analysis to Fig. 12.