A COURSE IN

ELECTRICAL ENGINEERING

VOLUME I

DIRECT CURRENTS

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ELECTRICAL ENGINEERING TEXTS

A COURSE IN
ELECTRICAL ENGINEERING

VOLUME I
DIRECT CURRENTS

BY

CHESTER L. DAWES, S.B., A.M.
Associate Professor of Electrical Engineering, Graduate School of Engineering, Harvard University; Fellow American Institute of Electrical Engineers, Etc.

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PREFACE TO THE THIRD EDITION

Since the revision of this textbook in 1927, the standards and requirements in courses in Electrical Engineering have been materially raised and many teachers have requested the author to add material of more advanced character, both in the text and in the problems. In the first two editions the use of calculus was avoided. However, practically all students now using the book have studied elementary calculus at least. Since the use of the calculus greatly facilitates the development of many important relations among electrical and magnetic quantities, elementary calculus is now used when necessary. Furthermore, its use has permitted the development of relations, under transient conditions, of current, quantity, and voltage in circuits having resistance, and inductance, and also resistance and capacitance in series. The calculus has been included in such a manner that those who do not know calculus will have no difficulty in using the text.

At the suggestion of several users of the book, the chapter on Magnetism and Magnets, and that on Electromagnetism, have been transferred from the beginning of the book to positions immediately preceding Chapter VIII, The Magnetic Circuit, thus giving continuity to the entire subject of magnetism. Because of added magnetic material, self- and mutual inductance have been removed from Chapter VIII, The Magnetic Circuit, and made into a new Chapter IX, Self- and Mutual Inductance.

During the last few years the manufacture of electrical machinery has been made to conform to modern methods of fabrication made possible by welding. Accordingly, the descriptions and illustrations of generator construction are those showing the latest designs made in accordance with modern-fabrication methods. Also, such new developments as the diverter-pole generator and arc-welding generator are included in this revision.

Notwithstanding the more advanced material which has been added, the original fundamental and elementary concepts and developments of principles have not been removed. Also, it seemed that the interest of the student would be stimulated if brief footnote biographies of the men who have made substantial contributions to the development of electrical engineering were added. These biographies appear where the names first occur in the text.

Much of the improvement in the book is due to many suggestions received from teachers and students using it. Among those who have been particularly helpful are Lieut. Col. C. L. Fenton, Professor and Head of the Department of Chemistry and Electricity at the United States Military Academy, West Point; David G. Howard, Professor of Electrical Engineering at the United States Naval Academy, Annapolis; Professor A. A. Nims of the Newark College of Engineering, Newark, New Jersey; Walter Criley, Instructor in Electrical Engineering, Rochester Athenaeum and Mechanics Institute, Rochester, New York; Raymond T. Gibbs, Instructor in Electrical Engineering, The Graduate School of Engineering, Harvard University.

The author is also indebted to H. W. Beedle, Engineer and Manager of the Boston Office, Electric Storage Battery Company, and E. W. Allen, Sales Engineer of the Edison Storage Battery Division of Thomas A. Edison, Inc., for their assistance in preparation of Chapter IV on Batteries, and to Arthur L. Russell, Instructor in Electrical Engineering, of the Franklin Union, Boston, for his assistance in the preparation of the book, particularly in the matter of the problems.

The author cannot express too greatly his appreciation of the assistance and interest of Harry E. Clifford, Dean Emeritus of the Graduate School of Engineering, Harvard University, to whom, in a large measure, the success of the book is due.

CHESTER L. DAWES.

CAMBRIDGE, MASS.,
May, 1937.

PREFACE TO THE FIRST EDITION

For some time past the editors of the McGraw-Hill Electrical Engineering Texts have experienced a demand for a comprehensive text covering in a simple manner the general field of Electrical Engineering. Accordingly, these two volumes were written at their request, after the scope and general character of the two volumes had been carefully considered.

As the title implies, the books begin with the most elementary conceptions of magnetism and current-flow and gradually advance to a more or less thorough discussion of the many types of direct and alternating current machinery, transmission devices, etc., which are met in practice. These two books are intended for Electrical Engineering students as a stepping stone to the more advanced Electrical Engineering Texts which are already a part of the series.

These two volumes should be useful also to students not planning to specialize in the electrical engineering field, who are taking courses in Electrical Engineering as a part of their general training. Such men often find difficulty in obtaining detailed and straightforward discussions of the subject in any one text and the brevity of their course does not give them time to assimilate fragmentary information obtainable only by consulting a number of references. Men taking foremen's and industrial courses in Electrical Engineering, which as a rule are carried on only in the evening, require text books sufficiently comprehensive, but at the same time not involving much mathematical analysis. Ordinarily, this type of student does not have ready access to reference libraries and is usually out of contact with his instructors except during the short time available for class-room work. In preparing this work the needs of the foregoing types of students have been carefully kept in mind and as a result, a liberal use of figures and illustrative problems has been made. Also frequent discussions of the methods of making measurements and laboratory tests are included.

In any course in Electrical Engineering, even though it be intended for non-electrical engineers, the author feels that the student gains little from a hurried and superficial treatment of the subject, as such treatment tends only to develop the memorizing of certain formulæ which are soon forgotten. Accordingly the attempt has been made in this text to develop and explain each phenomenon from a few fundamental and well-understood laws rather than to give mere statements of facts. Such treatment will develop the student's reasoning powers and give him training that will be useful in the solution of the more involved engineering problems that may arise later in his career.

Throughout the text, especially in the treatment of the more abstract portions, attempt has been made to show the ultimate bearing upon general engineering practice. The student takes more interest in the theory when he sees that it can be applied to the solving of practical problems. Because this work is not intended for advanced students in Electrical Engineering, little or no calculus is used and the mathematics is limited to simple equations.

The author is indebted to several of the manufacturing companies who have coöperated in the matter of supplying photographs, cuts and material for the text; and particularly to Professor H. E. Clifford of The Harvard Engineering School, for his many suggestions and for the care and pains which he has taken in the matter of editing the manuscripts.

C. L. D.

HARVARD UNIVERSITY, CAMBRIDGE, MASS.,
January, 1920.

CONTENTS

CHAPTER III

CHAPTER IV

CHAPTER VII

CHAPTER VIII

CHAPTER IX

CHAPTER X

CHAPTER XIII

A COURSE IN

ELECTRICAL ENGINEERING

VOLUME I

DIRECT CURRENTS

CHAPTER I

RESISTANCE

Introduction.—The importance which electricity has in the life of both the community and the individual is so evident that it hardly needs be emphasized. No city, even of moderate size, could exist today without electrical energy for light, transportation, water supply, elevators, communication, power, etc. Even automobiles become inoperative without electrical energy for ignition, starting, and lights. There are several factors which make electricity so useful and so important. Electrical energy can be readily and efficiently converted into any form of energy, such as heat, light, mechanical, and chemical energy. It may be generated at the most favorable locations, such as at a water-power site where hydraulic energy is available, near a coal mine where fuel is readily obtainable, or at the shore of navigable water to which fuel can be economically shipped and where ample cooling water is available.

Electrical energy may be economically transmitted for great distances to regions where it can be used effectively, as at centers of population, for electric railways, in mills, and at industrial centers.

Electrical energy is convenient in that it can be easily applied for numerous and varied purposes such, for example, as to operate elevators, to drive electric drills, to operate electric furnaces, and for lighting. It is readily concentrated to give extremely high temperatures, as in spark plugs and in arc lights, in welding and in electric furnaces. It is easily and quickly controlled.

Moreover, when used in connection with magnets, electrical energy can be used for operating relays, telephones, bells, and tractive magnets (p. 239).

Electrical energy is free from the products of combustion such as smoke, ashes, and fumes.

The numerous uses and applications of electricity can be accomplished only through a thorough understanding of the laws which govern its flow in circuits, its relations with magnetism, as well as its generation and its electromechanical effects.

1. Nature of the Flow of Electricity.—According to modern theory, which has been substantiated by the experimental results

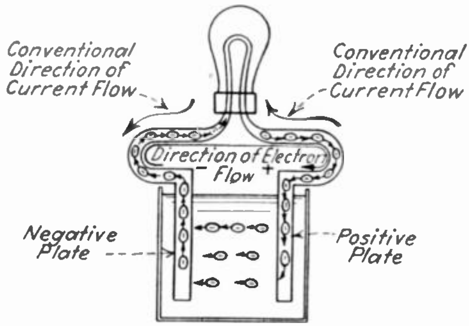


FIG. 1.—Electron movement and conventional current flow.

of many investigators, the atoms of all matter consist of a positively charged nucleus, around which infinitesimal negative charges rotate with high angular velocity. The individual negative charges, which are called *electrons*, are found to be identical for all matter. In conductors, some of these electrons are free to pass from atom to atom when a difference of potential is impressed across the ends of the conductor. The movement of these electrons constitutes the electric current. Hence, the electric current may be considered as electricity in motion, and is called *dynamic electricity.*

Since electrons are *negative* charges, the direction of their motion is opposite to the conventional direction of current flow. This is illustrated in Fig. 1, which shows the electrons moving from the negative plate of a battery, through an incandescent-lamp filament and into the positive plate of the battery. The conventional direction of current flow in the circuit external to the battery is from the positive to the negative plate as indicated. The electrons, being negative charges, are *repelled* by the negative plate and are *attracted* by the positive plate.

In non-conductors of electricity, or insulators,[1] the electrons are very closely bound to the nucleus and it is difficult to remove an electron from the atom. Hence, as compared with conductors, a relatively high potential difference is required to remove only a few electrons from the atom, and the corresponding current flow is extremely small.

2. Electrical Resistance.—The current flowing through an electric circuit depends not only on the electromotive force impressed on the circuit but on the circuit properties as well. For example, if a copper wire be connected across the terminals of a battery, a current will flow through this wire. If a poor contact be made at one of the battery terminals or at some other point in the circuit, the current will decrease, even with the e.m.f. remaining constant. Also, heat will be dissipated at the point of poor contact. Likewise, if the copper wire be cut and a small incandescent lamp be inserted in the circuit, the lamp filament will be heated and may become incandescent. At the same time the current in the circuit will decrease in magnitude. In both cases heat is noticed particularly at the points in the circuit where the poorer conducting medium is inserted. Also in each case a decrease in current accompanies the insertion of the poorer conducting medium, even with a constant e.m.f.

This property of an electric circuit tending to prevent the flow of current and at the same time causing electric energy to be converted into heat energy is called *resistance*.

Resistance may be accounted for by the electron theory of current flow discussed in Par. 1. The electrons in moving through the conductor must pass through the molecules or the atoms. In doing so they collide with other electrons and with the nucleus itself. The collision results in the evolution of heat and accounts

[1] No substance is a perfect insulator, but the current which the usual insulator will conduct with a given potential difference is extremely small as compared with that for conductors of electricity under similar conditions.

for the heat which accompanies current flow through resistance. Also, because of the collisions, the velocity of the electrons is reduced and a higher resulting potential is necessary in order to maintain a given current.

Resistance in the electric circuit may be likened in its effect to friction in mechanics. For example, if a street car is running at a uniform speed on a straight, level track, friction tends to prevent the moving of the car. The power which is used in moving the car is converted by friction into heat. Friction tends to impede the flow of water in a pipe or in a flume, some of the energy of the water being expended in overcoming this friction. The loss of energy is represented by a loss of head. This energy loss is largely absorbed by the water, and careful measurements would show a slight increase in its temperature.

As will be shown in the next chapter, the energy loss which occurs when an electric current flows through a resistance is directly proportional to the amount of resistance and to the square of the current. Also the current is equal to the applied e.m.f. divided by the resistance (Ohm's law). That is, the current $I = E/R$ (see p. 35).

3. Conductors and Insulators.—It was stated in Par. 1 that, with some substances, electrons are able to pass readily from atom to atom, and such substances are conductors. On the other hand, with other substances, electrons can be removed from the atom only with difficulty, and such substances are insulators. However, all substances[1] offer some resistance to current flow and are therefore not perfect conductors; moreover, all insulating substances are conducting to some extent. For the most part there is a marked distinction between conductors and insulators. The metals and carbon are good conductors; most organic and vitreous substances are insulators, such, for example, as rubber, oils, glass, and quartz. (At very high temperatures, vitreous

[1] Professor Kamerlingh-Onnes of Leyden, in 1914, was able to produce a circuit in which an electric current showed no diminution in strength 5 hr. after the e.m.f. had been removed. The current was induced magnetically in a short-circuited coil of lead wire at −270°C. produced by liquid helium, and the inducing source was then removed. Liquid helium has the lowest temperature known, being in the neighborhood of absolute zero (−273°C.). This experiment indicates that the resistance of the lead was practically zero at this extremely low temperature.

substances become conductors in an *electrolytic* sense. The Nernst lamp was based on the principle that porcelain becomes conducting at the temperature of incandescence.) Electrolytes have varying degrees of conductivity, but in the electrolytic sense only.

Of the usual metals, silver is the best conductor and copper is second best (see Appendix G, p. 595). The other metals and their alloys have varying degrees of conductivity. Oils, glass, silk, paper, cotton, ebonite, fiber, paraffin, rubber, etc., may be considered as non-conductors or good insulators. Wood, either dry or impregnated with oil, is a good insulator, but wood containing moisture is a partial conductor.

The marked differences between conductors and insulators are illustrated as follows:

The resistance of a centimeter cube of copper at 20°C. is 1.7241×10^{-6} ohm; the resistance of a centimeter cube of hard rubber is approximately 10^{16} ohms, giving for the ratio of the resistivity of hard rubber to that of copper the value 6×10^{21} approximately. Likewise, the resistance of a centimeter cube of glass is of the order of 10^{14} ohms, making its ratio to that of copper also very high.

4. Unit of Resistance.—The ohm[1] is the practical unit of resistance and is defined as that resistance which will allow 1 amp. to flow if 1 volt is impressed across its terminals (also see p. 30).

An ohm has such a value that 1 amp. flowing through it for 1 sec. produces as heat 1 joule of energy.

The resistance of insulating substances is ordinarily of the magnitude of millions of ohms, so that it is awkward to express this resistance in terms of a unit as small as the ohm. The *megohm*, equal to 1,000,000 (10^6) ohms, is the unit ordinarily used under these conditions. (The prefix "mega" means million.)

On the other hand, the resistance of bus-bars and short pieces of metals may be so low that the ohm is too large a unit for con-

[1] The unit is named for Georg Simon Ohm (1787–1854), of Germany, the mathematician who about 1827 evolved the principle now known as *Ohm's law*. He also had a prominent part in the development of other basic laws of electricity.

veniently expressing it. Under these conditions the *microhm* is used as the unit, and is equal to 1/1,000,000 of an ohm (10^{-6}). (The prefix "micro" means one-millionth.)

5. Resistance and Geometry of Conductors.—The resistance of a body of a given material depends both on its geometry and on the direction of flow of current. For example, consider the rectangular prism shown in Fig. 2, composed of two equal cubes 1 cm. on edge and of the same conducting material. If the current I_1 flows from side A to side B, which is the side opposite A, it must flow successively through the two cubes. Assume the resistance between opposite faces of each cube to be 2 microhms. The current I_1 in going from side A to side B must therefore encounter the resistance of two cubes in series, or a total of 4 microhms.

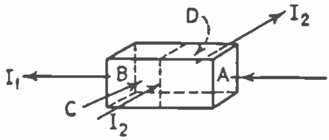


FIG. 2.—Resistance and direction of current flow.

Hence, it may be stated that with *constant cross-section*, resistance varies *directly as the length* of the conductor.

Next consider the current I_2 equal to I_1 but flowing in a direction perpendicular to I_1 from C to D (Fig. 2). The cross-section over which I_2 distributes itself is now twice that of a single cube, but the length of current path is that of a single cube. Since only one-half the current flows in each cube, the voltage drop across each cube is one-half that produced by I_1 in a single cube. Hence the resistance in each cube encountered by the current I_2 between surfaces C and D is 1 microhm, or one-half that of a single cube. Thus with a current path of *constant length*, the resistance *varies inversely* as the cross-section of conductor path.

Also, the resistance between surfaces C and D is but *one-fourth* that between surfaces A and B. This illustrates the fact that the resistance of a conducting body depends not only on its geometry but also on the *direction* of current flow. In the usual electric circuit the length of the path is so great as compared with its cross-section that the direction of current flow is obvious. However, there are instances, such for example as the flow of current in insulation (p. 10), in which it is easily possible to choose the incorrect direction of current flow.

6. Specific Resistance or Resistivity.—In Par. 5 it was shown that, with constant cross-section, the resistance of a conducting body varies as the length, and with constant length the resistance varies inversely as the cross-section. Hence it follows that *the resistance of a homogeneous body of uniform cross-section varies directly as its length and inversely as its cross-section,* the length being taken in the direction of current flow and the cross-section perpendicular to the direction of current flow.

That is,

$$R = \rho \frac{L}{A} \tag{1}$$

where R is the resistance in ohms, L is the length in the direction of the current flow, A is the area at right angles to the current flow, and ρ is a constant of the material known as its *resistivity* or *specific resistance.*

If L is 1 cm. and A is 1 cm. square, the substance in question must have the form of a cube, 1 cm. on an edge, and

$$R = \rho \frac{1}{1 \times 1}$$

or

$$R = \rho.$$

ρ is called the *specific resistance* or the *resistivity* of the substance, in this case per centimeter cube. ρ may be expressed in terms of an inch cube or in other units, as will be shown later. The resistivity of copper is 1.7241 microhms, or 1/580,000 ohm, per centimeter cube at 20°C. It is evident that the cube is a perfectly definite unit of resistivity since the resistance between any two opposite faces is the same. The resistivities of various substances are given in Appendix G (p. 595). Knowing the specific resistance in terms of the centimeter cube, the resistance of a wire, bar, etc., may be readily computed from Eq. (1).

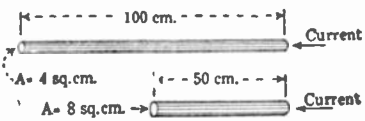


FIG. 3.—Brass rods having equal volumes but unequal resistances.

Example.—Determine the resistance of the two brass rods A and B (Fig. 3), the resistivity of the brass being 11.4 microhms per centimeter cube. Rod A is 100 cm. long and has a circular cross-section of 4 sq. cm.; rod B is 50 cm. long and has a circular cross-section of 8 sq. cm.

$$\text{Rod } A. \quad R = 11.4\ \frac{100}{4} = 285 \text{ microhms.} \quad \textit{Ans.}$$

$$\text{Rod } B. \quad R = 11.4\ \frac{50}{8} = 71.25 \text{ microhms.} \quad \textit{Ans.}$$

Although both rods have the same volume, rod A has four times the resistance of rod B, because its length in the direction of current flow is twice that of B and its cross-section, perpendicular to the direction of current flow, is one-half that of B.

Example.—Determine the resistance of 3,000 ft. of annealed 0000 copper wire having a diameter of 0.460 in., the specific resistance of copper being taken as 1.724 microhms (= 0.000001724 ohm) per centimeter cube (20°C.) (see Par. 15).

$$3{,}000 \text{ ft.} = 3{,}000 \times 12 \times 2.54 = 91{,}400 \text{ cm.}$$

$$\text{Cross-section} = \frac{\pi}{4}(0.460 \times 2.54)^2 = 1.07 \text{ sq. cm.}$$

$$R = \rho\frac{L}{A} = (0.000001724) \times \left(\frac{91{,}400}{1.07}\right) = 0.1472 \text{ ohm.} \quad \textit{Ans.}$$

7. Volume Resistivity.—Since the volume of a body is

$$V = LA$$

where L is its length and A its uniform cross-section, Eq. (1) may be written

$$R = \rho\frac{L}{A} = \rho\frac{L^2}{V} = \rho\frac{V}{A^2}. \qquad (2)$$

That is:

The resistance of a conductor varies directly as the square of its length when the volume is fixed.

The resistance of a conductor varies inversely as the square of its cross-section when the volume is fixed.

Example.—A kilometer of wire having a diameter of 11.7 mm. and a resistance of 0.031 ohm is drawn down so that its diameter is 5.0 mm. What does its resistance become?

The original cross-section of the wire

$$A_1 = \frac{\pi}{4}11.7^2 = 107.5 \text{ sq. mm.}$$

The final cross-section

$$A_2 = \frac{\pi}{4}5.0^2 = 19.64 \text{ sq. mm.}$$

Applying Eq. (2),

$$R_1 = \rho\frac{V}{(107.5)^2} = 0.031 \text{ ohm,}$$

$$R_2 = \rho\frac{V}{(19.64)^2}.$$

Since the volume of the wire does not change during the drawing process and the resistivity constant ρ remains the same,

$$\frac{R_2}{R_1} = \frac{R_2}{0.031} = \frac{\rho\frac{V}{(19.64)^2}}{\rho\frac{V}{(107.5)^2}}.$$

$$R_2 = 0.031\frac{(107.5)^2}{(19.64)^2} = 0.031\frac{11{,}560}{386} = 0.93 \text{ ohm.} \quad \textit{Ans.}$$

Also see the example, Par. 6, p. 7 (Fig. 3). The volume and resistivity of the two brass rods are the same. Their resistances are proportional to the square of their lengths. Likewise their resistances are inversely proportional to the square of their cross-sections.

8. Conductance.—Conductance is the reciprocal of resistance and may be defined as being that property of a circuit or of a material which tends to permit the flow of an electric current. The unit of conductance is the reciprocal ohm or *mho*. Conductance is usually expressed by g or G.

$$g = \frac{1}{R}, \tag{3}$$

also

$$g = \gamma\frac{A}{L} \tag{4}$$

where γ is the *specific conductance* or the *conductivity* of a substance, A the uniform cross-section, and L the length.

The conductivity of copper at 20°C. is 580,000 mhos per centimeter cube.

Example.—Determine the conductance at 20°C. of an aluminum bus-bar 0.5 in. thick, 4 in. wide, and 20 ft. long.

The conductivity of aluminum is 61 per cent that of copper and copper has a conductivity of 580,000 mhos per centimeter cube at 20°C.

The conductivity of aluminum is

$$\gamma = 0.61 \times 580{,}000 = 354{,}000 \text{ mhos per centimeter cube.}$$

The cross-section of the bus-bar

$$A = 0.5 \times 4 \times 2.54 \times 2.54 = 12.9 \text{ sq. cm.}$$

The length

$$L = 20 \times 12 \times 2.54 = 610 \text{ cm.}$$

The conductance

$$g = 354{,}000 \times \frac{12.9}{610} = 7{,}490 \text{ mhos.} \quad \textit{Ans.}$$

9. Resistance Paths of Variable Cross-section.—Occasionally the cross-section of a resistance path is not uniform but changes with its length. An excellent example is the path of the leakage current in the insulation between the conductor and the outer wall of a cylindrical cable. For example, in Fig. 4 is shown the cross-section of a cylindrical cable in which R_1 is the radius of the conductor, R_2 is the radius to the outer surface of the insulation, ρ is the resistivity of the insulating material, and l is the length of the cable in centimeters. The actual resistance to the leakage current from the conductor to the outer wall is found readily by integration. (Electrical contact with the outer wall is usually made by means of a lead sheath or by immersion of the cable in water.) An annulus at a distance r cm. from the center of the cable and having an infinitesimal thickness of dr cm. is first considered.

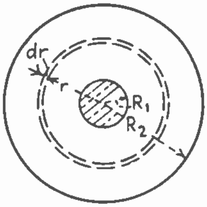


FIG. 4.—Resistance path in cable insulation.

The length of this annulus in the direction of current flow is dr cm., and its cross-section perpendicular to the direction of current flow is $2\pi rl$ cm. Hence, from Eq. (1), the resistance of the annulus is

$$dR = \rho\frac{dr}{2\pi rl} \text{ ohms.}$$

The total resistance is

$$R = \frac{\rho}{2\pi l}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{\rho}{2\pi l}\Big[\log_e r\Big]_{R_1}^{R_2}$$

$$= \frac{\rho}{2\pi l}\log_e\frac{R_2}{R_1} \text{ ohms.} \tag{5}$$

Since R_2 and R_1 appear in Eq. (5) as a ratio, the unit in which they are expressed is immaterial so long as both are expressed in the same unit. It will be noted that in Eq. (5) l, the length of the cable, occurs in the denominator. This is due to the fact that the direction of current flow in the insulation is perpendicular to the length of the cable.

Example.—In a cylindrical rubber-insulated cable, the conductor is No. 6 solid A.W.G., the diameter of which is 0.162 in., and the thickness of the wall of insulation is ¼ in. The resistivity of the rubber is 10^{14} ohms per centimeter cube. Determine: (*a*) the insulation resistance of a 1,000-ft. length of the cable in megohms; (*b*) similarly, the insulation resistance of a 2,100-ft. length. (1 megohm = 1,000,000 ohms (see p. 5).)

(*a*) The length of the cable is $1{,}000 \times 12 = 12{,}000$ in.

$$R_1 = \frac{0.162}{2} \text{ or } 0.081 \text{ in.},$$

$$R_2 = 0.081 + 0.250 = 0.331 \text{ in.}$$

$$\rho \text{ (per inch cube)} = 10^{14}\frac{2.54}{(2.54)^2} = 3.94 \times 10^{13} \text{ ohms.}$$

Hence, from Eq. (5),

$$\begin{aligned} R &= \frac{3.94 \times 10^{13}}{2\pi \times 12{,}000} 2.303 \log_{10} \frac{0.331}{0.081} \\ &= 1.203 \times 10^9 \log_{10} 4.09 \\ &= 1.203 \times 10^9 \times 0.6117 = 7.36 \times 10^8 \text{ ohms} \\ &= 7.30 \times 10^2, \text{ or } 736 \text{ megohms.} \quad \textit{Ans.} \end{aligned}$$

(*b*) Since the direction of current flow in the insulation is at right angles to the length of the cable, the area of the current path must be proportional to the length of the cable. Hence the resistance of the insulation must be *inversely* as the length of the cable. Therefore the resistance of a 2,100-ft. length

$$R = 736\frac{1{,}000}{2{,}100} = 350 \text{ megohms.} \quad \textit{Ans.}$$

10. Resistances in Series.—A series circuit is one in which the resistances or other electrical devices are connected end to end as shown in Fig. 5. In such a system, the same current flows in each part of the circuit, but the total line voltage divides among the different elements of the circuit. If a number of resistances R_1, R_2, R_3, etc. (Fig. 5), are connected in series, that is, end to end, the total resistance of the combination is

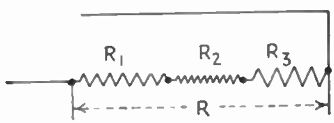


FIG. 5.—Resistances in series.

$$R = R_1 + R_2 + R_3 + \cdots \tag{6}$$

That is:

In a series circuit the total resistance is the sum of the individual resistances.

Example.—Four resistances R_1, R_2, R_3, R_4 are connected in series. The values of the resistances are $R_1 = 24.2$ ohms; $R_2 = 36.4$ ohms; $R_3 = 18.5$ ohms; $R_4 = 42.9$ ohms. Determine the value of a single resistance R, which would be equivalent to these four in series. From Eq. (6),

$$R = 24.2 + 36.4 + 18.5 + 42.9 = 122.0 \text{ ohms.} \quad \textit{Ans.}$$

11. Resistances in Parallel.—A parallel circuit is one in which one terminal of each element is connected to a common point to form one terminal of the system, and the other terminal of each element is connected to a second common point to form the other terminal of the system. Under these conditions, each element of the parallel system is across the same voltage, but the total current divides among the elements of the circuit. A parallel circuit of three resistances is shown in Fig. 6(*a*).

With resistances in parallel, the total equivalent resistance must always be less than that of any one of the single resistances, since the addition of a resistance in parallel with another resistance increases the available current path and hence decreases the resistance to current flow.

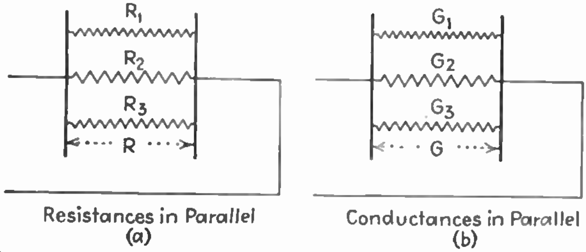


FIG. 6.—(*a*) Resistances in parallel. (*b*) Conductances in parallel.

The equivalent of resistances in parallel may be determined directly if they are first considered as conductances. Thus, in Fig. 6(*a*), are shown three resistances R_1, R_2, R_3 in parallel. Their conductances are G_1, G_2, G_3, where $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$ (Fig. 6(*b*)). The total equivalent conductance is

$$G = G_1 + G_2 + G_3. \tag{7}$$

If R is the total equivalent resistance, $G = 1/R$ and Eq. (7) may be written

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{8}$$

That is:

In a parallel circuit, the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the individual resistances.

For a circuit with two resistances R_1 and R_2 in parallel, the equivalent resistance R is found as follows:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1R_2}. \tag{I}$$

Taking the reciprocal of (I) gives

$$R = \frac{R_1R_2}{R_1 + R_2}. \tag{9}$$

With three resistances R_1, R_2, R_3 in parallel, the equivalent resistance R is found as follows:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2R_3 + R_3R_1 + R_1R_2}{R_1R_2R_3}. \tag{II}$$

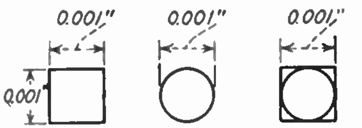
Taking the reciprocal of Eq. (II),

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}. \tag{10}$$

Example.—Determine the equivalent resistance of a circuit having four branches, the individual resistances of which are 3, 4, 6, 8 ohms.

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = 0.333 + 0.250 + 0.167 + 0.125$$
$$= 0.875 \text{ mho.}$$
$$R = \frac{1}{0.875} = 1.142 \text{ ohms.} \quad \textit{Ans.}$$

12. Circular Mil.—In the English and American wire tables the circular mil is the standard unit of wire cross-section.



(*a*) The square mil (*b*) The circular mil (*c*) Comparison of the square mil and the circular mil

FIG. 7.—The square mil and circular mil.

The term *milli* means one-thousandth; for example, a millivolt = 1/1,000 volt. A *mil* is *one-thousandth of an inch*. A square mil is a square, each side of which is 1 mil (0.001 in.), as shown in Fig. 7 (*a*). The area of a square mil is 0.001 × 0.001 = 0.000001 sq. in.

A *circular mil* is the area of a circle whose *diameter* is 1 *mil* (0.001 in.) (Fig. 7(*b*)) and is usually written C.M. or cir. mil. As will be seen from Fig. 7(*c*), a circular mil is a smaller area than a square mil. The area in square inches of a circular mil = $(\pi/4)(0.001)^2$ = 0.0000007854 sq. in.

The circular mil is the unit in which the cross-section of wires and cables is measured, just as the square foot is the unit in which larger areas such as floors, land, etc., are measured. The advantage of the circular mil as a unit is that circular areas measured in terms of this unit bear a very simple relation to the diameters. Also, with the circular mil as the unit, the factor π does not enter computations of cross-sections.

In Fig. 8, A represents the cross-section of a wire having a diameter of 1 in. Required: to determine its area in circular mils.

$$\text{The area } A = \frac{\pi}{4}(1)^2 \text{ sq. in.}$$

$$\text{The area } a \text{ of a circular mil} = \frac{\pi}{4}(0.001)^2 \text{ sq. in.}$$

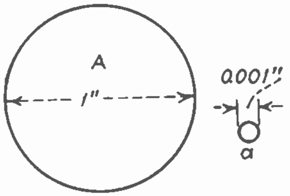


FIG. 8.—Cross-section expressed in circular mils.

The ratio A/a obviously gives the number of circular mils in A.

Therefore,

$$\frac{A}{a} = \frac{\frac{\pi}{4}(1)^2}{\frac{\pi}{4}0.000001} = 1{,}000{,}000 \text{ cir. mils.}$$

The general relation may be written

$$\text{Circular mils} = \frac{D_1^2}{(0.001)^2} = 1{,}000{,}000(D_1)^2 = D^2 \qquad (11)$$

where D_1 is the diameter of the wire in *inches*.

D is the diameter of the wire in *mils*.

The matter may be summed up in two rules:

To obtain the number of circular mils in a solid wire of given diameter, express the diameter in mils and then square it.

To obtain the diameter of a solid wire having a given number of circular mils, take the square root of the circular mils and the result will be the diameter of the wire in mils.

Example.—Number 00 wire (A.W.G.) has a diameter of 0.3648 in. What is its circular milage?

$$0.3648 \text{ in.} = 364.8 \text{ mils}$$
$$(364.8)^2 = 133{,}100 \text{ cir. mils.} \quad \textit{Ans.}$$

Example.—A certain wire has a cross-section of 52,640 cir. mils. What is its diameter?

$$\sqrt{52{,}640} = 229.4 \text{ mils} = 0.2294 \text{ in.} \quad \textit{Ans.}$$

13. Circular-mil-foot.—Another convenient unit of resistivity, especially in the English system, is the resistance of a circular-mil-foot. This unit is the resistance of a wire having a cross-section of 1 cir. mil and a length of 1 ft., as shown in Fig. 9. The resistance of a circular-mil-foot of copper at 20°C. is 10.37 ohms. (In practical work this resistance may frequently be taken as 10 ohms.) Knowing this resistivity, the resistance of any length and size of wire may be determined by Eq. (1).

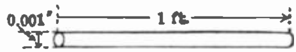


FIG. 9.—The circular-mil-foot.

Example.—What is the resistance of a 750,000-cir.-mil. copper cable, 2,500 ft. long?

If the cable had a cross-section of 1 cir. mil it would have a resistance of 2,500 × 10.37 = 25,900 ohms. However, the cross-section is actually 750,000 cir. mils, therefore,

$$R = \frac{25{,}900}{750{,}000} = 0.0346 \text{ ohm.} \quad \textit{Ans.}$$

or Eq. (1), p. 7, may be used directly:

$$R = 10.37\frac{2{,}500}{750{,}000} = 0.0346 \text{ ohm.}$$

When applying Eq. (1), L must be expressed in *feet* and A in *circular mils.*

14. Circular-mil-inch.—The circular-mil-inch is also a very convenient unit of resistivity, particularly in connection with coils and windings. Such windings usually operate at a temperature which gives a resistivity of approximately 12 ohms per circular-mil-foot or 1 ohm per circular-mil-inch. (A temperature of 60°C. gives this resistivity.) It follows that the resistance of a copper conductor

$$R = 1\frac{l}{\text{cir. mils}}$$

where l is in inches.

That is, *the resistance is equal to the length in inches divided by the cross-section in circular mils.*

Example.—A certain circular magnet coil having an inner diameter of 6 in., an outer diameter of 9 in., and a length of 4 in. is wound with 1,000 turns of No. 14 d.c.c. magnet wire, which has a cross-section of 4,110 cir. mils. Assuming that the resistance of 1 cir.-mil-in. of the wire is 1 ohm, find the resistance of the coil.

The mean length of turn is $\pi\frac{9+6}{2} = \pi 7.5 = 23.6$ in.

The total length of copper $1{,}000 \times 23.6 = 23{,}600$ in.

The resistance $R = \frac{23{,}600}{4{,}110} = 5.74$ ohms. *Ans.*

15. Resistivities of Copper.—Formerly, the standard for the conductivity of copper was based on results obtained in 1862 by Matthiessen, who made careful measurements of the resistance of supposedly pure copper. He found the resistivity to be 1.594 microhms per centimeter-cube at 0°C. In view of the uncertainty of the quality of his copper, the Bureau of Standards[1] made a large number of measurements upon commercial copper. Its recommendation that the standard of resistivity be 0.15328 ohm per meter-gram at 20°C. was accepted by the International Electrotechnical Commission at a meeting in Berlin, Sept. 1–6, 1913. This became known as the *International Annealed Copper Standard* and is the internationally accepted resistivity for annealed copper. The meter-gram standard is a *mass* resistivity and is equal to the resistance of a uniform annealed copper wire 1 m. long and weighing 1 g. The international standard for the density of copper is 8.89 g. per centimeter-cube at 20°C. and corresponds to 0.3212 lb. per cubic inch. The resistivity at 20°C. of the International Annealed Copper Standard expressed in several units is as follows:

0.15328	ohm (meter-gram)
875.20	ohms (mile-pound)
1.7241	microhms (centimeter-cube)
0.67879	microhm (inch-cube)
10.371	ohms (mil-foot)
0.017241	ohm (meter-square millimeter)

The conductivity of commercial annealed copper is about 98 per cent that of the International Standard and the conductivity of hard-drawn copper is from 96 to 97 per cent that of the International Standard.

16. Resistivities of Aluminum.—Aluminum is next to copper in importance as a commercial conductor (see Par. 25, p. 25). Its volume conductivity is 61 per cent that of copper, but its mass conductivity is 200.7 per cent that of copper. That is,

[1] See "Copper Wire Tables," *Circ.* 31, Bureau of Standards, 1914.

for equal weights and lengths the conductance of aluminum is practically twice that of copper. Below is given the resistivity at 20°C. of aluminum expressed in several units.[1]

0.0764	ohm (meter-gram)
436.	ohms (mile-pound)
2.828	microhms (centimeter-cube)
1.113	microhms (inch-cube)
17.01	ohms (mil-foot)

The density of aluminum is 2.70 g. per cubic centimeter, or 0.0975 lb. per cubic inch.

17. Resistivity Specifications.—Formerly, the conductivity of commercial copper was specified and expressed as a percentage of the conductivity of the International Annealed Copper Standard given in Par. 15 (p. 16). The conductivity of commercial copper was about 98 per cent.

In the Standards of the American Institute of Electrical Engineers,[2] it is now recommended that resistivity shall not exceed the value of 891.58 for a pound-mile-ohm or ohms (mile-pound) at 20°C. (68°F.).

The equivalents of this resistivity in other units are as follows:

891.58 pound-mile-ohm is equal to

0.15614	ohm (meter-gram)
1.75614	microhms (centimeter-cube)
0.69150	microhm (inch-cube)
10.565	ohms (mil-foot)

Example.—The resistance of a 000 A.W.G. copper rod, the diameter of which is 10.40 mm., is measured between points 106.8 cm. apart and is found to be 0.0002193 ohm. Determine: (*a*) the resistivity of this copper per centimeter-cube; (*b*) whether or not it meets the A.I.E.E. Standard; (*c*) the weight of a mile-ohm of this copper.

(*a*) From Eq. (1), p. 7,

$$\rho = R\frac{A}{l}$$

$$= 0.0002193\frac{(\pi/4)(1.040)^2}{106.8} = 1.746 \times 10^{-6} \text{ ohm (centimeter-cube).}$$

[1] See "Copper Wire Tables," *Circ.* 31, Bureau of Standards, 1914.

[2] Standards A.I.E.E., No. 61 "Specifications for Soft or Annealed Copper Wire," adopted Dec. 16, 1927.

(*b*) This value of resistivity is less than the A.I.E.E. specification.

(*c*) From Eq. (2), p. 8, $V = \rho\frac{L^2}{R}$, hence for a centimeter-cube the volume:

$$V_1 = \rho\frac{1^2}{1.746 \times 10^{-6}} = 1 \text{ c.c.}$$

A mile equals 2.54 × 12 × 5,280 = 161,000 cm. Hence the volume of a mile-ohm is

$$V_2 = \rho\frac{(161,000)^2}{1} \text{ c.c.}$$

$$\frac{V_2}{V_1} = \frac{\rho\frac{(161,000)^2}{1}}{\rho\frac{1^2}{1.746 \times 10^{-6}}}$$

and

$$V_2 = 1\frac{(161,000)^2/1}{1/(1.746 \times 10^{-6})} = (161)^2(1.746) = 45,200 \text{ c.c.}$$

Since a cubic centimeter of copper weighs 8.89 g. and 1 kg. equals 2.20 lb., the weight of a mile-ohm equals 45,200 × 8.89 × 10^{-3} × 2.20 = 884 lb *Ans.*

This value is slightly less than the A.I.E.E. Standard because of the lesser resistivity of this sample.

18. Resistor Materials and Alloys.—Resistor materials are used where it is desired to introduce resistance into a circuit. Frequently, the resistor materials must operate at high temperature, as in heating devices, such as electric furnaces, ranges, or toasters. The material must have high resistivity, high melting point, and must be able to resist oxidation at high temperatures. With field rheostats and controllers, long life and overload capacity are highly desirable. Hence they are not normally operated at the high temperatures of many of the heating devices and thus the effects of corrosion and disintegration are reduced. Resistor materials are also used in connection with electrical measuring instruments, as for ammeter shunts or series resistances for voltmeters, and although the temperature may not be high when the materials are used for these purposes, it is highly desirable that the temperature coefficient of resistance be small so that the instrument indications will not change appreciably with temperature.

One of the earliest resistor alloys is *german silver* or nickel silver, which is composed of copper, nickel, and zinc, in varying proportions. The alloy is usually listed on the percentage of nickel, 18 per cent alloy containing 18 per cent nickel. The 18 per cent alloy has a resistivity about 18 times that of copper, and the 30 per cent about 28 times. German silver has a high temperature coefficient of resistance. Manganin is a copper-manganese alloy of about 65 per cent copper, 30 per cent ferro-manganin, 5 per cent nickel. It has a very low temperature coefficient and hence is used extensively with instruments for multipliers, shunts, etc. Iron-nickel alloys have high resistivity but are far inferior to chromium-nickel alloys in resistance to corrosion. Copper-nickel alloys have resistivities of from 10 to 30 times the resistivity of copper and are used commonly for resistor materials. The chromium-nickel alloys, such as "Nichrome," have resistivities from 60 to 70 times that of copper and are used for resistor materials where high resistivity is essential; these alloys also have a high melting temperature and resist oxidation at high temperatures. Hence they are used in wire form for electrical heating units, as in household appliances or electric furnaces. Iron wire and cast iron are also used as resistors, cast-iron grids being commonly used for starting and controller resistances. The table in Appendix G (p. 595) gives the electrical properties of the more common metals and resistor materials.

19. Temperature Coefficient of Resistance.—The resistance of copper and other non-alloyed metals increases appreciably with temperature. Since the temperature at which electrical conductors operate varies with the current and depends on the surrounding or ambient temperature as well, it is important to know the relation between temperature and resistance. For example, the temperature of electrical machinery when in operation must necessarily be higher than that of the surrounding medium, and, as will be shown later, the temperature of the machinery may be found by resistance measurements. Also, the temperature of various electrical devices such as wires and cables, the filaments of incandescent lamps, and the wire in resistor units increases during operation.

The relation between resistance and temperature may be expressed as follows:

$$R_t = R_0(1 + \alpha t) \tag{12}$$

where R_t is the resistance at the temperature t, R_0 the resistance at 0°C., and α is the *temperature coefficient of resistance* at 0°. For copper, α is 0.00427 and for most of the unalloyed metals is sensibly of this value.[1] This means that with copper the resistance increases 0.427 of 1 per cent for each degree centigrade increase of temperature above 0°. For example, assume that a coil has a resistance of 100 ohms at 0°C. For every degree increase of temperature the coil resistance will increase

$$100 \times 0.00427 \text{ ohm or } 0.427 \text{ ohm.}$$

At 40°C. the increase of resistance will be $40 \times 0.427 = 17.08$ ohms, and the resistance at 40° will be $100 + 17.08 = 117.08$ ohms.

If the resistance at some definite temperature other than 0°C. is known, ordinarily the resistance at 0°C. must first be found before the resistance at other temperatures can be calculated. For this purpose Eq. (12) may be put in the form

$$R_0 = \frac{R_t}{1 + \alpha t}. \tag{13}$$

Example.—The resistance of an electromagnet winding of copper wire at 20°C. is 30 ohms. What is its resistance at 80°C.?

The resistance at 0°C.

$$R_0 = \frac{30}{1 + (0.00427 \times 20)} = \frac{30}{1.085} = 27.65 \text{ ohms}$$
$$R_{80} = 27.65[1 + (0.00427 \times 80)] = 37.11 \text{ ohms.} \quad \textit{Ans.}$$

This process of working back to 0° is a little inconvenient, but it is fundamental and easy to remember. It is possible, however, to determine the temperature coefficient for *any* initial temperature.

Let R_1 and R_2 be the resistances at temperatures t_1 and t_2 respectively. Then

$$R_1 = R_0(1 + \alpha t_1) \tag{I}$$
$$R_2 = R_0(1 + \alpha t_2) \tag{II}$$

Dividing Eq. (II) by Eq. (I) and solving for R_2,

$$R_2 = R_1\frac{(1 + \alpha t_2)}{(1 + \alpha t_1)} = R_1[1 + \alpha_1(t_2 - t_1)] \tag{III}$$

where α_1 is the temperature coefficient at the initial temperature t_1. Solving Eq. (III) for α_1,

[1] For example, the value of α for aluminum is 0.0039.

$$\frac{1 + \alpha t_2}{1 + \alpha t_1} - 1 = \alpha_1(t_2 - t_1),$$

$$\alpha_1 = \frac{1 + \alpha t_2 - 1 - \alpha t_1}{(1 + \alpha t_1)(t_2 - t_1)} = \frac{\alpha(t_2 - t_1)}{(1 + \alpha t_1)(t_2 - t_1)} = \frac{\alpha}{1 + \alpha t_1}. \quad \text{(IV)}$$

Dividing the right-hand member of Eq. (IV) by α gives

$$\alpha_1 = \frac{1}{\frac{1}{\alpha} + t_1} \quad (14)$$

If α (for copper) equals 0.00427, $1/\alpha = 234.5$.

Then $\alpha_1 = \dfrac{1}{234.5 + t_1}$, which is easy to remember.

For example, $\alpha_{40} = 1/274.5 = 0.00364$ (see Par. 20).

Paragraph 20 gives the temperature coefficients of copper for various initial temperatures. By using this table, temperature-coefficient problems are simplified, as shown by solving the example of Par. 19 (p. 20).

Example.—The temperature coefficient of copper at 20°C. initial temperature from Par. 20 is 0.00393. The rise in temperature = 80° − 20° = 60°. Then the resistance at 80°C.

$$R_{80} = 30(1 + 0.00393 \times 60) = 37.07 \text{ ohms.} \quad \textit{Ans.}$$

20. Temperature Coefficients of Copper at Various Initial Temperatures. (From formula $1/(234.5 + t)$, see Eq. (14)

Initial Temperature	Increase in Resistance per 1°C.
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364
45	0.00358
50	0.00352

21. Inferred Zero Resistance.—If the resistance of copper at ordinary temperatures be plotted as ordinates and temperature as abscissas, the graph is practically a straight line (Fig. 10). If this line be extended, it will intersect the zero-resistance line at −234.5°C. (an easy number to remember), as shown in Fig. 10.

This is equivalent to saying that between ordinary limits of temperature, copper behaves as if it had zero resistance at $-234.5°C$. (Actually the curve departs from a straight line at extremely low temperatures, as shown by the dotted line in Fig. 10.) This gives a convenient method for determining temperature-resistance relations.

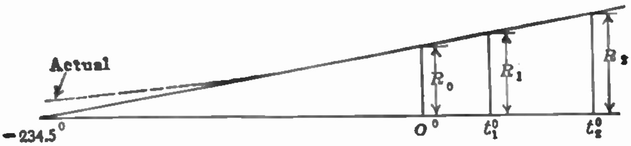


FIG. 10.—Variation of resistance with temperature.

By the law of similar triangles (Fig. 10),

$$\frac{R_0}{234.5°} = \frac{R_1}{234.5° + t_1}, \tag{15}$$

$$\frac{R_1}{234.5° + t_1} = \frac{R_2}{234.5° + t_2}. \tag{16}$$

Applying Eq. (16) to the example of Par. 19 (p. 20),

$$\frac{30}{234.5° + 20°} = \frac{R_{80}}{234.5° + 80°}.$$

$$R_{80} = 30\,\frac{234.5° + 80°}{234.5° + 20°} = 30\,\frac{314.5°}{254.5°} = 37.07 \text{ ohms.} \quad \textit{Ans.}$$

22. American Wire Gage (A.W.G.).—The A.W.G. (formerly Brown & Sharpe Gage) is based on a constant ratio between diameters of successive gage numbers; that is, the diameters taken in order form a geometrical progression. The diameter of No. 0000 is defined as 0.4600 in. (1.168 cm.) and the diameter of No. 36 as 0.0050 in. (0.0127 cm.). Between No. 0000 and No. 36 are 38 gage numbers. Hence the ratio of any diameter to the diameter of the next greater gage number must be

$$\sqrt[39]{\frac{0.4600}{0.0050}} = \sqrt[39]{92} = 1.123.$$

The square $(1.123)^2$ of this ratio is 1.2610. Since the cross-section varies as the square of the diameter, the ratio of any cross-section to the cross-section of the next greater gage number is 1.261. The sixth power of 1.123 is 2.0050. Hence the ratio of any diameter to the diameter of a gage number which is 6 greater is 2, practically. It follows that the cross-section either doubles or halves for every three gage numbers. Since $(1.261)^2$ is 1.590, the ratio of any cross-section to the cross-section of a gage number greater by 2 is 1.60, practically. The ratio of any cross-section to one of a gage number differing by 10 is $(92^{19/39})^2 = 10$ (practically), or the reciprocal, $1/10$, practically.

A working table for annealed solid copper wire, A.W.G., is given in Appendix D (p. 592) and a similar table for stranded cable is given in Appendix E (p. 593).

23. Approximations with the A.W.G.—The diameter of No. 10 wire is 0.102 in. (0.259 cm.) and the wire has a cross-section of 10,400 cir. mils. As an approximation, the diameter may be considered as being 0.1 in. (0.254 cm.) and the cross-section as 10,000 cir. mils.

From the foregoing and the fact that the weight of 1,000 ft. of No. 10 wire is 31.4 (10π) lb., the following approximate rules may be given:

(1) No. 10 wire has a diameter of 0.1 in. and a resistance of 1 ohm per 1,000 ft. (2) The resistance of the wire doubles with every increase of three gage numbers. (3) Therefore the resistance increases $\sqrt[3]{2} = 1.26$ ($1\frac{1}{4}$) times for each successive gage number and $(1.26)^2 = 1.6$ times for every two numbers. (4) The resistance is multiplied or divided by 10 for every difference of 10 gage numbers. (5) The weight of 1,000 ft. of No. 10 wire is 31.4 lb. and the weight of 1,000 ft. of No. 2 wire is 200 lb. These rules make it a comparatively simple matter to determine without reference to the table the weight or resistance for any gage number.

Example.—What are the resistance and weight of 1,000 ft. of No. 0000 wire?

The resistances will decrease as follows:

Gage No........	10	7	4	1	000
Resistance.......	1	0.5	0.25	0.125	0.0625 (rules 1 and 2)

Resistance of 0000 = 0.0625/1.25 = 0.050 ohm (rule 3). *Ans.*

Weight of 1,000 ft. No. 2 = 200 lb.

Weight of 1,000 ft. 00 = 400 lb.

Weight of 1,000 ft. 0000 = 400 × 1.6 = 640 lb. (rules 5, 2,* and 3).* *Ans.*

The example might have worked more quickly by rule 4.

Resistance of 1,000 ft. of No. 10 = 1 ohm.

Resistance of 1,000 ft. of 0 = 0.1 ohm (rule 4).
Resistance of 1,000 ft. of 0000 = 0.050 ohm (rule 2).

Example.—What are the resistance and weight of 1,800 ft. of No. 34 wire?

Resistance of 1,000 ft. of No. 10 = 1 ohm (rule 1).
Resistance of 1,000 ft. of 20 = 10 ohms (rule 4).
Resistance of 1,000 ft. of 30 = 100 ohms (rule 4).
Resistance of 1,000 ft. of 33 = 200 ohms (rule 2).
Resistance of 1,000 ft. of 34 = 250 ohms (rule 3).

$1.8 \times 250 = 450$ ohms. *Ans.*

Weight of 1,000 ft. of No. 10 = 31.4 lb. (rule 5).
Weight of 1,000 ft. of 20 = 3.14 lb. (rule 4).*
Weight of 1,000 ft. of 30 = 0.314 lb. (rule 4).*
Weight of 1,000 ft. of 33 = 0.157 lb. (rule 2).*
Weight of 1,000 ft. of 34 = 0.157/1.25, or 0.126 lb. (rule 3).*

$1.8 \times 0.126 = 0.227$ lb. *Ans.*

* The weight is inversely as the resistance.

24. Stranded Cables.—Solid copper wire of greater cross-section than 0000 gage is practically never used since, because of its rigidity, it cannot be readily bent. Wires of greater cross-section therefore are stranded to give flexibility. Also, for certain uses, wires of comparatively small cross-section are frequently stranded to obtain greater flexibility. (Lamp cord and cords for portable devices are excellent examples.)

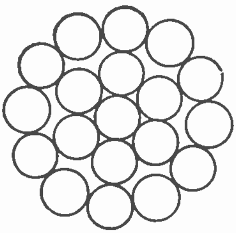


FIG. 11.—Nineteen-strand cable.

Stranding is accomplished geometrically as follows:

Six wires will just fit about a center wire. In each successive layer six additional wires will just fill up that layer. Thus, the first layer contains six wires; the second, 12 wires; the third, 18 wires, and so on, as is shown in Fig. 11. Hence the number of strands which standard cable will contain are 1–7–19–37–61–91–127– . . . Sometimes in order to obtain even greater flexibility, the unit strands themselves are also stranded.

Wires greater than 0000 gage are designated by their circular mils rather than by a gage number. The standard sizes differ successively by 50,000 cir. mils, beginning with 250,000 cir. mils. Appendix E, p. 593, gives the properties of stranded annealed-copper cables. Obviously the diameter over the outside strands is greater than the diameter of a solid cylindrical conductor of equal cross-section.

25. Conductors.—Although silver is a better conductor than copper, its use as a conductor is very limited because of its cost. In a few instances it is used where a delicate and highly conducting material is necessary, such as in the brushes and occasionally in the commutator of watthour meters. Copper, because of its high conductivity and moderate cost, is used more extensively as a conductor than any other material. It has many good qualities, such as ductility, high tensile strength, is not easily abraided, is not corroded by the atmosphere, and is readily soldered.

Aluminum has only 61 per cent of the conductivity of copper, but for the same length and weight it has about twice the conductance of copper. It is softer than copper, its tensile strength is much less, and it cannot be easily soldered. It is not affected by exposure to the atmosphere. The large diameter for a given conductance prohibits its use where an insulating covering is required. Aluminum is used extensively as a conductor for high-voltage transmission lines, where its lightness and large diameter (because of corona) are an advantage (see p. 345 and Vol. II, p. 470). It is used to some extent for low-voltage busbars as it offers much greater radiating surface than copper of the same conductance.

Iron and steel have about nine times the resistance of copper for the same cross-section and length. The large cross-section for a given conductance prohibits their use where an insulating covering is necessary and the increased weight prevents their use in most cases where the conductors must be placed on poles. These materials are most commonly used as resistors in connection with rheostats and for third rails of electric railways. Iron and steel ordinarily must be protected from oxidation by galvanizing or other protective covering. Copperclad steel (Copperweld) consists of a steel wire coated or covered with a layer of copper, fused or welded to the steel. The advantages claimed for it are that it possesses the high tensile strength of steel, combined with the high conductivity of copper. Further, the copper protects the steel from corrosion. Its field is the transmission-line conductor, where long spans make high tensile strength necessary. It is also used as an overhead ground wire on transmission lines.

CHAPTER II

OHM'S LAW AND THE ELECTRIC CIRCUIT

It is stated in Chap. I that electricity consists of infinitesimal charges called *electrons*[1] and, if these electrons are forced to travel in the same direction, an electric current results. The flow of electricity through a circuit resembles in many ways the flow of water through pipes, for electricity acts as an incompressible fluid would act, undergoes pressure drop, etc., as will be shown later.

26. Absolute Systems of Electrical Units.—There are two basic or fundamental systems of electrical units, the *electrostatic* system and the *electromagnetic* system.

The units in the *electrostatic system* are based on Coulomb's experimental law (see p. 318) which states that the force acting between two electrostatic point charges Q_1 and Q_2, placed r cm. apart in vacuum, is $\frac{Q_1Q_2}{r^2}$ dynes. Other electrical units, such as those of current and potential, may be derived from this simple relation. The electrostatic system is used to calculate the capacitance of condensers from their geometry (see Eq. 179, p. 336) and is also used to a large extent in the calculation of electron phenomena, since electrons themselves are minute electric charges. The units of the electrostatic system ordinarily have the prefix *stat*, for example, statvolt, statfarad, etc.

The units in the c.g.s. (cm.-gm.-sec.) *electromagnetic* or *absolute* system are based fundamentally on the law which gives the force between magnetic poles (Eq. 92, p. 206). The unit of current is derived by means of the Biot-Savart law, which gives the relation existing between the current in a conductor and the force which the current exerts on a unit magnetic pole. This law is discussed in Chap. VII, Electromagnetism (pp. 232 and 234). It follows

[1] See Vol. II, Chap. XIV. (All references to Vol. II are to the third edition (revised).)

from Eq. (103) (p. 235) that if a conductor carrying a current of I' absolute amperes be bent into an arc of a circle 1 cm. in length and of 1 cm. radius (Fig. 12) and the unit pole be placed at the center of the *circle*, the force acting on the unit pole is

$$f = I' \text{ dynes.} \tag{17}$$

(Note that the leading-in connections to the arc (Fig. 12) are radial so that the current in them exerts no force on the unit pole.) If the current is adjusted until the force on the unit pole is 1 *dyne*, the value of the current, from Eq. (17), is 1 c.g.s. or 1 *absolute ampere* (*absampere*). All other units in the system may be derived from the current as just defined. For example, the quantity Q is equal to the product of current and time, etc. The units of this c.g.s. electromagnetic system are commonly used in magnetic calculations (see Chap. VIII). They also serve as the basis for the units of the practical system (Par. 27). Absolute units ordinarily have the prefix *ab* or *abs*, for example, abvolt, absampere, etc.

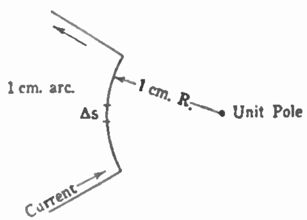


FIG. 12.—Absolute determination of current.

27. Practical System of Electrical Units.[1]—Most of the units of the absolute electromagnetic system are many times smaller than the quantities used in practice. For example, the absolute volt or abvolt has 1/100,000,000 or (10^{-8}) the magnitude of the practical volt. In the practical system, which is derived from the absolute system, the units are of the order of magnitude of the quantities which are ordinarily met in practice. These units of the practical system differ from the units of the absolute system

[1] At its plenary meeting in June, 1935, at Scheveningen-Brussels, the International Electrotechnical Commission unanimously adopted the MKS (meter-kilogram-second) system of electrical units, which is substantially the present practical system. The quantities are so defined that the system can be derived on a fundamental basis from the meter, kilogram, and second, rather than from the centimeter, gram, and second, as the present c. g. s. system is derived. In the MKS system the weber (see p. 251, footnote 2) was adopted as the practical unit of magnetic flux. The weber is equal to 10^8 maxwells. (See A. E. KENNELLY, "I.E.C. Adopts MKS System of Units," *Elec. Eng.*, Dec., 1935, p. 1373.)

only by definite numerical ratios. For example, the practical ampere has one-tenth (10^{-1}) the magnitude of the absampere (Par. 26), the practical ohm has one billion (10^9) times the magnitude of the absohm, the henry has one billion (10^9) times the magnitude of the abhenry, etc.

Current.—The practical unit of current is the *ampere*[1] and its magnitude is one-tenth that of the absolute ampere as defined in Par. 26. However, since it is difficult to determine experimentally the value of current by measuring the force which it exerts on a unit pole (Fig. 12), it is necessary to use methods which are more practicable but which give values corresponding to the fundamental definition. Hence the ampere is defined by an act of Congress, 1894, as follows:

> The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units and is the practical equivalent of the unvarying current, which when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second.

It should be kept in mind that the ampere is the *rate* of flow of electricity. It corresponds in hydraulics to the rate of flow of water, which is expressed as cubic feet per second, gallons per minute, etc.

Quantity.—The unit of quantity is the *coulomb.*[2] This is equal to the quantity of electricity conveyed by 1 amp. in 1 sec. The

[1] Ampère, André Marie (1775–1836). A French physicist who from his early days showed unusual aptitude for mathematics and the sciences. In 1801 he was appointed Professor of Physics and Chemistry at Bourg, in 1804 was appointed Professor of Physics and Chemistry at the Lycée of Lyons, in 1809 was appointed Professor of Mathematics at the École Polytechnique at Paris, and later at the Collège de France. His important contributions were the establishment of the relations existing between electricity and magnetism, in which he gave a far more complete exposition than Oersted. He developed mathematical theory which not only explained the relations which had been observed between current and magnetic field but predicted many new ones.

[2] Coulomb, Charles A. (1736–1806). A French philosopher who began his career as a military engineer but later became interested in science. He is distinguished for his investigations in both mechanics and electricity. His noteworthy contributions were the construction of a refined torsion

coulomb is analogous to the unit quantity of water in hydraulics, such as the cubic foot, the gallon, etc.

From this definition it is evident that an electric current may be expressed in *coulombs per second* rather than in *amperes.*

Difference of Potential and Electromotive Force (e.m.f.).—The practical unit of potential difference is the *volt.*[1] The difference of potential in volts between two points is defined fundamentally as measured by the work in joules expended in moving a unit quantity of electricity, the coulomb, from one point to the other. Difference of potential and e.m.f. tend to cause a flow of electricity. The unit of potential difference or of e.m.f. is the volt and is that potential difference which when impressed across the terminals of a resistance of 1 ohm will cause a current of 1 amp. to flow. The *international volt* is now more specifically defined as 1/1.01830 of the voltage of a normal Weston cell (see Par. 73, p. 100).

The mechanical analogue of potential difference is pressure. The difference in hydraulic pressure between the ends of a pipe causes or tends to cause the flow of water. The pressure of water behind the dam tends to cause water to flow through the penstock or through any leaks. The pressure in a boiler tends to cause steam to flow through the pipes, valves, etc. Likewise electric pressure or difference of potential tends to cause current to flow.

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balance and the determination experimentally as well as mathematically of the laws governing the attraction and repulsion of magnetic poles and of electric charges.

[1] Volta, Alessandro (1745–1827). An Italian physicist and a pioneer in electrical science. In 1774 he was appointed Professor of Physics in the Gymnasium at Como, and in 1779 he was appointed to a similar chair at Pavia.

His first notable contribution to the science of electricity was the invention of the electrophorus (1775) with which electrical energy is evolved by the mechanical separation of a conducting disc from a layer of insulating material. By the application of a condenser to the terminal of the electroscope (1782) he increased the sensitivity many times. His greatest contribution, however, was the galvanic pile (1799) which showed that an electrical potential exists between two dissimilar metals which are in joint contact with an electrolyte. This is the basis of all electric batteries. He also was the first to discriminate between metallic and electrolytic conduction.

He received numerous medals and other honors for his work.

Resistance.—The ohm, the unit of resistance, has already been defined in Chap. I as that resistance which will allow 1 amp. to flow if 1 volt is impressed across its terminals. The *international ohm* is specifically defined as the resistance of a column of mercury at the temperature of melting ice (0°C.), 14.4521 g. in mass, of a constant cross-sectional area and of a length of 106.300 cm.

28. Absolute Potential.—The absolute potential of a body in c.g.s. units is defined as measured by the work in ergs necessary to bring a c.g.s. unit of charge up to the body from infinity. It is practically impossible to determine the absolute potential of a body. For example, the absolute potential of the earth is not even approximately known, but for convenience the earth is assumed to be at zero potential and the potentials of bodies are usually given with reference to earth potential.

It is only occasionally that absolute potential is of interest. Usually it is desired to know *difference of potential.* The difference of potential (in c.g.s. units) between any two points is defined fundamentally as measured by the work in ergs necessary to move a c.g.s. unit of charge from one point to the other. In the practical system the difference in potential would be given by the work in joules (10^7 ergs) necessary to move 1 coulomb from one point to the other (p. 29). Practically, difference of potential is measured by means of some measuring device such as a voltmeter (see Par. 109, p. 154).

29. Nature of the Flow of Electricity.—The flow of electricity through a circuit resembles in many ways the flow of water through a closed system of pipes. For example, in Fig. 13 water enters the mechanically driven centrifugal pump P at a pressure h_1 (represented by the length of a column of mercury) above the point of zero pressure shown by the line h_0. In virtue of the action of the pump blades, the pressure of the water through the pump is increased from h_1 to h_2, representing a net increase of pressure H_1. The water then flows out along pipe F_1 to the hydraulic motor W. Because of the friction loss in the pipe F_1, the pressure h_3 at the motor terminals is slightly less than h_2. In other words, a pressure of $h_2 - h_3$ is required to overcome the frictional resistance of the pipe F_1. The line *ab* gives the pressure at each point along the pipe F_1. The pressure decreases uniformly in F_1, as shown by line *ab*.

In Fig. 14 the mechanically driven electrical generator G raises the potential of the current entering its negative terminal from v_1 to v_2 where v_1 and v_2 are measured from the earth whose potential is ordinarily assumed as zero. (The various voltages

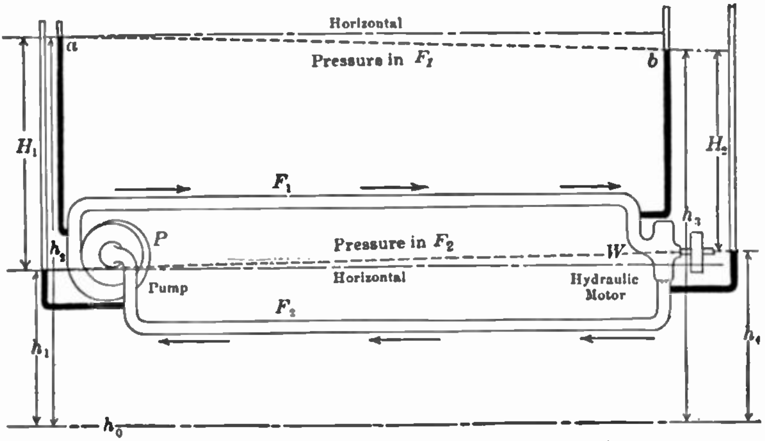


FIG. 13.—Flow of water through a hydraulic motor and pipe system.

are measured with voltmeters v_1', v_2', etc.) The generator, in raising the potential of this portion of the circuit from v_1 to v_2, produces a net increase in pressure $v_2 - v_1 = V_1$. The current now flows out through the wire L_1 to the + terminal of the motor

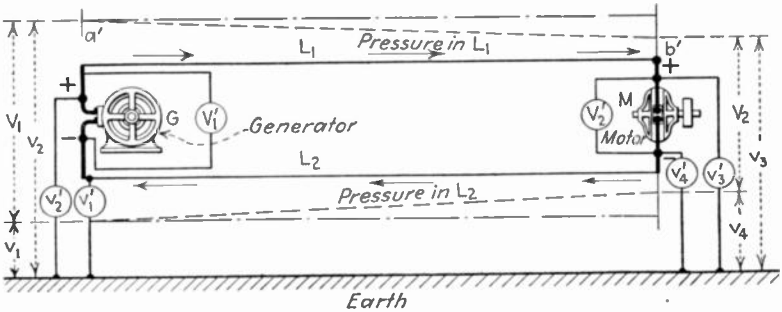


FIG. 14.—Flow of an electric current through an electric motor and the connecting feeder system.

M. Because of the line resistance, the potential drops from v_2 at the generator to v_3 at the motor in practically the same manner that the water pressure drops in pipe F_1 (Fig. 13). A voltage $v_2 - v_3$ is necessary to overcome the resistance of the wire L_1.

The line $a'b'$ shows the actual voltage at each point along the wire, the distance of $a'b'$ from the ground line being proportional to the voltage at each point. The voltage drop is uniform.

Referring to Fig. 13, the water enters the hydraulic motor W and in overcoming the back pressure of the revolving blades the pressure of the water drops from h_3 to h_4, representing a net drop in pressure H_2. Pressure h_4 must necessarily be greater than h_1 in order that the water may flow back through the pipe F_2. The pressure $h_4 - h_1$ is necessary to overcome the friction loss in the pipe F_2. It is to be noted that H_2, the net pressure at the motor terminals, is less than the pressure H_1 at the pump, by the sum of the pressures necessary to overcome the friction in the two pipes F_1 and F_2.

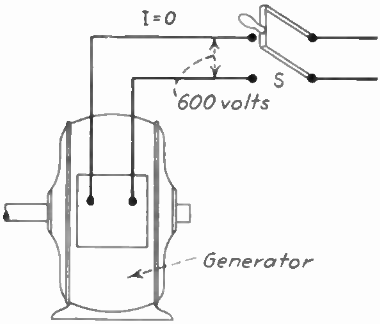


FIG. 15.—Illustrating the existence of potential difference without current.

In a similar manner, the pressure of the electric current in passing through the motor M (Fig. 14), drops from v_3 to v_4, representing a net drop in pressure V_2. A large percentage of this voltage V_2 is necessary to overcome the back e.m.f. of the motor. v_4 is necessarily greater than v_1, or the current could not flow along L_2 back to the negative terminal of the generator. It is to be noted that, as in Fig. 13, the net potential difference V_2 at the motor M is less than the potential difference V_1 at the generator, by the drop in potential due to the resistance of both the *outgoing* and *return* wires.

Difference of potential is therefore the equivalent of pressure and *tends* to send current through a circuit; current is quantity of electricity per second. Potential difference may exist with no resulting current flow, in the same manner that a boiler may have a very high steam pressure with no steam flow, due to all the valves being closed. Likewise a generator (Fig. 15) may have a very high potential difference at its terminals, yet, because the switch S is open, no current flows.

30. Difference of Potential.—In order that current may flow between two points, there must be a *difference of potential* between the two points, as shown in Fig. 14. This is further illustrated in Fig. 16. A large reservoir and a small tank are connected by a pipe P. The water level in the tank and in the reservoir is the same. There is pressure in each, but there is no *difference in*

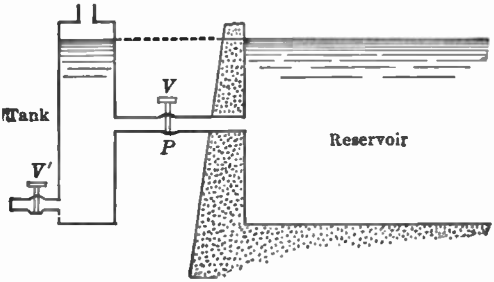


FIG. 16.—Tank and reservoir at the same pressure.

pressure between them. Under these conditions when the valve V is opened, no water flows from the reservoir to the tank. However, if the valve V' is opened, allowing the water level in the tank to fall, a difference of pressure results and water flows from the reservoir to the tank.

Figure 17 shows two batteries A_1 and A_2 each having an e.m.f. of 2 volts. The positive terminal a of A_1 has a potential of +2 volts above its negative terminal; likewise the positive terminal b of A_2 has a potential of +2 volts above its negative terminal. The negative terminals of both batteries are at the same potential because they are connected by a copper wire through which no current flows, and consequently there can be no potential difference between the ends of the copper wire. Therefore points a and b must each be at the same potential of +2 volts. If now the switch S be closed, *no* current will flow between a and b, because there is no *difference of potential* between a and b. This is analogous to the conditions in Fig. 16 when the water level in the tank is the same as that in the reservoir.

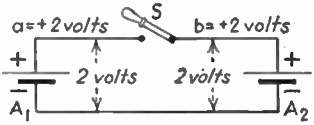


FIG. 17.—Two batteries having equal e.m.fs.

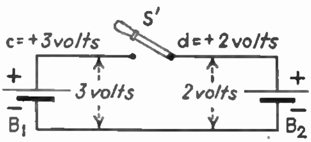


FIG. 18.—Two batteries having unequal e.m.fs.

In Fig. 18 the e.m.f. of battery B_1 is 3 volts and therefore the potential of its positive terminal *c* is 3 volts above that of its negative terminal. The e.m.f. of battery B_2 is 2 volts and therefore the potential of its positive terminal *d* is 2 volts above that of its negative terminal. The negative terminals are at the same potential being connected and, if this potential be assumed as zero, the point *c* is at a potential of +3 volts and the potential of *d* is +2 volts. Therefore, the point *c* is at a potential of $3 - 2$, or 1 volt higher than *d*. When switch S' is closed, a current will flow from *c* to *d*, in virtue of *c* being at a higher potential than *d*. This is analogous to the conditions in Fig. 16 when the water level in the tank is lower than that in the reservoir.

31. Measurement of Voltage and Current.—Voltage or potential difference is ordinarily measured with a voltmeter. It is

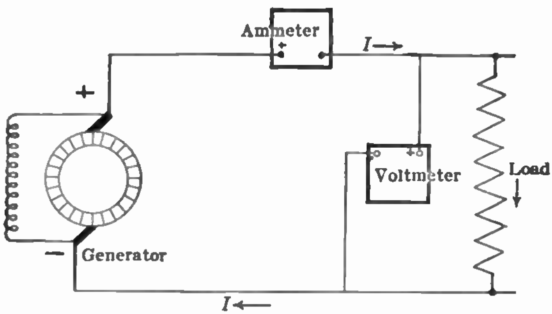


FIG. 19.—Proper method of connecting a voltmeter and an ammeter.

rarely that *absolute potential* is of interest. Ordinarily *difference of potential* is desired. The voltmeter, therefore, should be connected *across* or *between* the wires whose difference of potential is to be measured. This is illustrated in Fig. 19 which shows a shunt generator delivering power to a resistance load over wires which have negligible resistance. The voltmeter measures the potential difference at the load and also measures the potential difference at the generator terminals if the very small drop in the ammeter be neglected.

Current is ordinarily measured with an ammeter. As current is the *quantity of electricity* per second passing in the wire, the ammeter must be connected so that only the current to be measured passes through it. This is accomplished by opening one of the wires of the circuit and inserting the ammeter, just as a water meter is inserted in a pipe when it is desired to measure the flow of water in the pipe. When the ammeter is so connected, the current passing through the wire is measured by the ammeter. This is illustrated in Fig. 19 in which the ammeter in series with the generator measures the current which the generator delivers. The ammeter also measures the combined current to the load and voltmeter, but the voltmeter current is usually so small that the ammeter measures essentially the current to the load.

An ammeter should never be connected across the line.

The resistance of an ammeter is very low. Even if connected across a low voltage, it will ordinarily take a current several times its rating, which results in a bent pointer and may result in the instrument's burning out. For example, the resistance of a 10-amp. instrument is approximately 0.005 ohm and, if connected across a voltage as low as 10 volts, it will take 10/0.005, or 2,000, amps. (Ohm's law, Par. 32). Such a large current would ruin the ammeter.

32. Ohm's Law.—Ohm's law states that for a steady current, the current in a circuit is *directly* proportional to the *total* e.m.f. acting in the circuit and is *inversely* proportional to the total resistance of the circuit.

The law may be expressed by the following equation if the current I is in *amperes,* the e.m.f. E is in *volts,* and the resistance R is in *ohms:*

$$I = \frac{E}{R}. \tag{18}$$

That is, the current in amperes in a circuit is equal to the total e.m.f. of the circuit in volts divided by the total resistance of the circuit in ohms. Potential difference may be represented by either the letter "V" or "E," V usually signifying terminal voltage and E e.m.f. or induced voltage.

Example. - The resistance of the field winding of a shunt motor is 30 ohms. What current will flow through the winding when it is connected across 115-volt mains?

$$I = \frac{E}{R} = \frac{115}{30} = 3.83 \text{ amp.} \quad \textit{Ans.}$$

Example.—Figure 20 shows a simple series circuit in which are two batteries whose e.m.fs. are 6 volts and 10 volts, and two resistances of

2 ohms and 6 ohms. Neglecting the resistance of the batteries, find the current in the circuit.

It is obvious from a study of Fig. 20 that the 6-volt battery tends to send current in a clockwise direction, and the 10-volt battery tends to send current in a counterclockwise direction. Hence they are in *opposition* and the net e.m.f. is 10 − 6, or 4, volts acting in the counterclockwise direction. The total resistance is 2 + 6, or 8, ohms. Hence, by Ohm's law,

$I = \frac{10 - 6}{2 + 6} = \frac{4}{8} = 0.5$ amp. flowing in the counterclockwise direction. *Ans.*

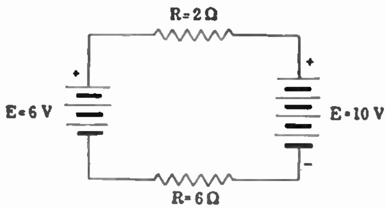


FIG. 20.—Current with two e.m.fs. and two resistances.[1]

By transformation, Eq. (18) becomes

$$E = IR. \tag{19}$$

That is, the voltage across any part of a circuit is equal to the product of the current in amperes and the resistance in ohms, *provided the current is steady and there are no sources of e.m.f. within this part of the circuit.*

Example.—The resistance of the field winding of a shunt generator is 48 ohms and the resistance of its rheostat is 22 ohms (see Fig. 21). If the field current is 3.2 amp., what is the voltage across the field winding terminals, the voltage across the rheostat, and the voltage across the generator terminals?

$E_1 = IR_1 = 3.2 \times 22 = 70.4$ volts across rheostat.

$E_2 = IR_2 = 3.2 \times 48 = 153.6$ volts across field winding.

Total $\overline{224.0}$ volts at generator terminals.

Also,

$E = I(R_1 + R_2) = 3.2(22 + 48) = 224.0$ volts. *Ans.* (*check*).

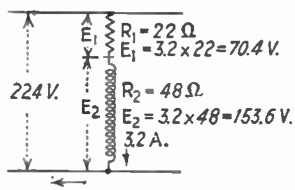


FIG. 21.—Voltage drops across a generator field and its rheostat.

Again, if Eq. (18) be solved for the resistance, the result is

$$R = \frac{E}{I}. \tag{20}$$

That is, the resistance of a circuit, or any part of a circuit, is equal to the voltage divided by the current, provided the current

[1] The Greek letter Ω (capital omega) is used in electrical engineering as the symbol for ohms (see Appendix J, p. 598).

is steady and there are no sources of e.m.f. within the part of the circuit considered. This formula is very useful in making resistance measurements (see Par. 113, p. 158).

Example.—The voltage across the terminals of a generator field is 220 volts and the field current is 4 amp. What is the resistance of the field circuit?

$$R = \frac{E}{I} = \frac{220}{4} = 55 \text{ ohms.} \quad \textit{Ans.}$$

33. Series Circuit.—As is stated in Par. 10 (p. 11), if several resistances are connected in series, the total resistance is the sum of the individual resistances. That is,

$$R = R_1 + R_2 + R_3 + \ . \ . \ . \qquad (21)$$

and the current

$$I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3 + \ . \ . \ .} \qquad (22)$$

Example.—A 50-ohm relay is connected in series with a resistance tube of 30 ohms and with a small pilot lamp having a resistance of 5 ohms. The operating voltage is 115 volts. What current flows in this relay circuit?

$$I = \frac{115}{50 + 30 + 5} = \frac{115}{85} = 1.35 \text{ amp.} \quad \textit{Ans.}$$

34. Parallel Circuit.—In Par. 11 (p. 11), the relation of total resistance to the component resistances in a parallel circuit is proved by transforming conductances into resistances. This relation may be proved by Ohm's law as follows: Consider the circuit of Fig. 22, consisting of resistances R_1, R_2, and R_3 in parallel across the voltage E. Let I_1 = the current in resistance R_1, I_2 = the current in R_2, and I_3 = the current in R_3.

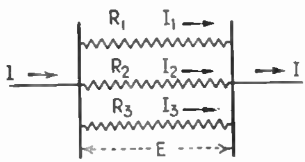


FIG. 22.—A parallel circuit.

Then

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \\ I_3 &= \frac{E}{R_3} \end{aligned}\right\} \quad \text{(Eq. 18)}$$

Adding,

$$I_1 + I_2 + I_3 = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

The total current is $I = I_1 + I_2 + I_3$.
Let the equivalent resistance be R, so that

$$I = \frac{E}{R}.$$

Substituting I for $I_1 + I_2 + I_3$,

$$I = \frac{E}{R} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right),$$

or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{23}$$

That is, *the reciprocal of the equivalent resistance of a parallel circuit is the sum of the reciprocals of the individual resistances.*

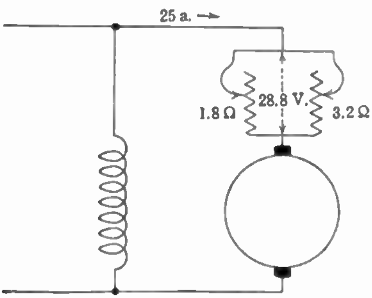


FIG. 23.—Parallel-connected rheostats in series with a motor armature.

If but two resistances are involved,

$$R = \frac{R_1 R_2}{R_1 + R_2}. \tag{24}$$

(See p. 13.)

Example.—Two rheostats, connected in parallel, are in series with the armature of an electric motor (Fig. 23). One rheostat is adjusted until its resistance is 3.2 ohms and the other until its resistance is 1.8 ohms. (*a*) What resistance is connected in series with the armature? (*b*) If the armature current is 25 amp., what is the voltage drop in the two resistances?

(*a*) Using Eq. (23),

$$\frac{1}{R} = \frac{1}{3.2} + \frac{1}{1.8} = 0.312 + 0.556 = 0.868 \text{ mho}$$

$$R = \frac{1}{0.868} = 1.152 \text{ ohms.} \quad \textit{Ans.}$$

Using Eq. (24),

$$R = \frac{3.2 \times 1.8}{3.2 + 1.8} = 1.152 \text{ ohms } (\textit{check})$$

(*b*) $$E = 25 \times 1.152 = 28.8 \text{ volts.} \quad \textit{Ans.}$$

(The voltage drop must be the same in each rheostat, since they are in parallel.)

Example.—Determine the total current in a circuit consisting of four resistances of 4, 6, 8, and 10 ohms, connected in parallel across a 10-volt source.

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = 0.25 + 0.167 + 0.125 + 0.10 = 0.642 \text{ mho.}$$

$$R = \frac{1}{0.642} = 1.56 \text{ ohms.}$$

$$I = \frac{10}{1.56} = 6.42 \text{ amp.} \quad \textit{Ans.}$$

The example may also be solved as follows: (see Eq. (10), p. 13).

$$R = \frac{4 \times 6 \times 8 \times 10}{4 \times 6 \times 8 + 6 \times 8 \times 10 + 8 \times 10 \times 4 + 10 \times 4 \times 6} = 1.56 \text{ ohms.}$$

(check)

35. Division of Current in Parallel Circuit.—In Fig. 24, two resistances R_1 and R_2 are connected in parallel across the voltage E. Then,

$$I_1 = \frac{E}{R_1},$$

$$I_2 = \frac{E}{R_2},$$

$$\frac{I_1}{I_2} = \frac{E/R_1}{E/R_2} = \frac{R_2}{R_1}. \qquad (25)$$

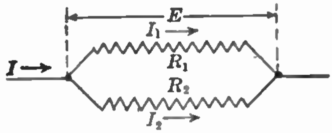


FIG. 24.—Division of current in a two-branch parallel circuit.

That is, *in a parallel circuit of two branches, the currents are inversely as the resistances.* (This relation does not hold when there is a source of e.m.f. in either branch. For example, *it does not give the division of current through the field and armature of a shunt motor when the motor is running, since the motor armature generates an e.m.f.*)

Example.—A current of 12 amp. divides between two branches in parallel, one branch having a resistance of 8 ohms, the other branch having a resistance of 12 ohms. How much current passes through each branch?

If I_1 be the current in the 8-ohm branch and I_2 the current in the 12-ohm branch,

$$\frac{I_1}{I_2} = \frac{12}{8} \text{ (Eq. 25).} \qquad \text{(I)}$$

Also

$$I_1 + I_2 = 12.$$

$$I_1 = I_2\frac{12}{8} = I_2\frac{3}{2} \text{ (from Eq. (I)).} \qquad \text{(II)}$$

Substituting in Eq. (II),

$$I_2\frac{3}{2} + I_2 = 12.$$

$$\frac{5I_2}{2} = 12; \quad I_2 = 4.8 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 = 4.8\frac{3}{2} = 7.2 \text{ amp.} \quad \textit{Ans.}$$

The voltage drops,

$$7.2 \times 8 = 57.6 \text{ volts.}$$
$$4.8 \times 12 = 57.6 \text{ volts } (\textit{check}).$$

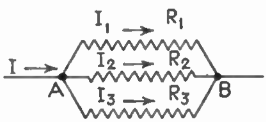


FIG. 25.—Division of current in a three-branch parallel circuit.

If the circuit consists of three branches in parallel (Fig. 25), the resistances of which are R_1, R_2, R_3, the currents may be found as follows:

Let the total current $I = I_1 + I_2 + I_3$; let R be the equivalent resistance of the circuit, that is, the resistance measured between points A and B, and let E be the voltage drop from A to B.

Then

$$E = IR;$$

and also

$$E = I_1R_1.$$

Hence,

$$IR = I_1R_1$$

and

$$\frac{I}{I_1} = \frac{R_1}{R} \tag{I}$$

Similarly,

$$\frac{I}{I_2} = \frac{R_2}{R} \tag{II}$$

and

$$\frac{I}{I_3} = \frac{R_3}{R}. \tag{III}$$

Combining Eq. (I) with Eq. (10), p. 13, gives Eq. (26). Likewise combining Eqs. (II) and (III) with Eq. (10) gives Eqs. (27) and (28).

$$I_1 = I\left(\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{26}$$

$$I_2 = I\left(\frac{R_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{27}$$

$$I_3 = I\left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{28}$$

(Note the cyclic order of the subscripts.)

Example.—A current of 25 amp. flows in a circuit consisting of three resistances 2.5, 4.0, and 6.0 ohms in parallel. Determine the division of current among the resistances.

Let R be the equivalent resistance of the combination. From Eq. (8), p. 12,

$$\frac{1}{R} = \frac{1}{2.5} + \frac{1}{4.0} + \frac{1}{6.0} = 0.817 \text{ mho.}$$
$$R = 1.225 \text{ ohms.}$$

From Eq. (I),

$$\frac{I}{I_1} = \frac{R_1}{R},$$
$$\frac{25}{I_1} = \frac{2.5}{1.225}; \quad I_1 = 12.25 \text{ amp.} \quad \textit{Ans.}$$

Similarly,

$$I_2 = \frac{25 \times 1.225}{4.0} = 7.65 \text{ amp.} \quad \textit{Ans.}$$

and

$$I_3 = \frac{25 \times 1.225}{6.0} = \underline{5.10} \text{ amp.} \quad \textit{Ans.}$$

The total current = 25.00 amp. (*check*).

The example may also be solved by means of Eqs. (26), (27), and (28):

$$I_1 = 25 \frac{4.0 \times 6.0}{(2.5 \times 4.0) + (4.0 \times 6.0) + (6.0 \times 2.5)}$$
$$= 25 \frac{24}{10 + 24 + 15} = 12.25 \text{ amp.}$$
$$I_2 = 25 \frac{6.0 \times 2.5}{49} = 7.65 \text{ amp.}$$
$$I_3 = 25 \frac{2.5 \times 4.0}{49} = \underline{5.10} \text{ amp.}$$

Total 25.00 amp. (*check*).

36. Series-parallel Circuit.—A circuit may consist of groups of parallel resistances in series with other resistances as shown in Fig. 26. When such is the case, each group of parallel resistances is first replaced by its equivalent single resistance and the entire circuit is then treated as a series circuit.

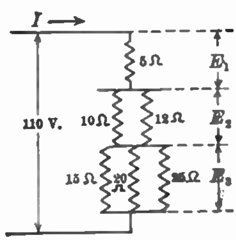


FIG. 26.—Series-parallel circuit.

Example.—Determine the total current in the circuit shown in Fig. 26; determine the voltage across each portion of the circuit; determine the current in each resistance.

Replace the 10- and 12-ohm resistances by a resistance R_1 where

$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{12} = 0.10 + 0.0833 = 0.1833 \text{ mho.}$$
$$R_1 = 5.45 \text{ ohms.}$$

Replace the group of three resistances by R_2 where

$$\frac{1}{R_2} = \frac{1}{15} + \frac{1}{20} + \frac{1}{25} = 0.0667 + 0.050 + 0.040 = 0.1567 \text{ mho.}$$

$$R_2 = \frac{1}{0.1567} = 6.38 \text{ ohms.}$$

$$I = \frac{110}{5 + 5.45 + 6.38} = \frac{110}{16.83} = 6.54 \text{ amp.}$$

$$\begin{aligned} E_1 &= 6.54 \times 5.0 &= 32.7 \text{ volts.} \\ E_2 &= 6.54 \times 5.45 &= 35.6 \text{ volts.} \\ E_3 &= 6.54 \times 6.39 &= \underline{41.7} \text{ volts.} \end{aligned}$$

Total 110.0 volts (*check*).

$$\text{Current in 10 ohms} = \frac{35.6}{10} = 3.56 \text{ amp.}$$

$$\text{Current in 12 ohms} = \frac{35.6}{12} = \underline{2.97} \text{ amp.}$$

Total 6.53 amp. (*check*).

$$\text{Current in 15 ohms} = \frac{41.7}{15} = 2.78 \text{ amp.}$$

$$\text{Current in 20 ohms} = \frac{41.7}{20} = 2.09 \text{ amp.}$$

$$\text{Current in 25 ohms} = \frac{41.7}{25} = \underline{1.67} \text{ amp.}$$

Total 6.54 amp. (*check*).

37. Electrical Power.—The unit of electrical power is the *watt*[1] which is defined fundamentally as 10^7 ergs, or 1 joule, per second (see Par. 38). One joule of work is done when 1 coulomb is carried through a potential difference of 1 volt (see Par. 27, p. 29). If 1 coulomb *per second* is carried through a potential difference of 1 volt, the *power* must be equal to 1 joule per second or 1 *watt*.

[1] Watt, James (1736–1819). A Scottish engineer and the inventor of the modern condensing steam engine. His father was a small unsuccessful merchant, so that at an early age James was thrown on his own resources. He first became a mathematical-instrument maker, finally obtaining employment at Glasgow College. This gave him an opportunity to repair Newcomen's engine in which the steam was condensed in the working cylinder. Seeing the advantages of a condenser external to the cylinder, in 1765 he invented and made the first engine operating on this principle. He also invented many improvements which are still used, such as the condenser air pump, the steam-jacketed cylinder, the double-acting cylinder, the early cut-off of steam to the cylinder, and the throttle valve and centrifugal governor. He also invented the engine indicator which has been of extreme importance in the evolution of the engine. His inventions were very successful commercially.

One coulomb per second gives a current of 1 *amp.* Hence the watt may be defined as the power developed by 1 amp. in flowing through a potential difference of 1 volt. The watts are therefore equal to the product of the volts and the amperes. Thus the power

$$P = EI \text{ watts.} \tag{29}$$

Since $E = IR$ in a circuit containing resistance only (Eq. (19), p. 36), Eq. (29) may be written

$$P = (IR)I = I^2R. \tag{30}$$

Substituting for I its value $\left(I = \frac{E}{R}\right)$ in Eq. (29),

$$P = \frac{E^2}{R} \tag{31}$$

Equation (29) is useful when the volts and the amperes are known; Eq. (30) is useful when the current and the resistance are known; and Eq. (31) is useful when the voltage and the resistance are known.

Equation (31) can be used only for parts of a circuit in which there is *resistance alone.* It cannot be used for parts of circuits which contain sources of e.m.f., such as batteries, or armatures which generate an e.m.f.

Example.—The resistance of a 150-scale voltmeter is 12,000 ohms. What power is consumed by this voltmeter when it is connected across a 125-volt circuit?

Since the voltage and the resistance are known, Eq. (31) is most convenient:

$$P = \frac{(125)^2}{12,000} = 1.3 \text{ watts. } \quad \textit{Ans.}$$

This may be checked by Eq. (29).

$$I = \frac{125}{12,000} = 0.0104 \text{ amp.}$$
$$P = 125 \times 0.0104 = 1.3 \text{ watts } (\textit{check}).$$

Example.—A field rheostat is adjusted until the field current is 4.7 amp. The resistance of the rheostat is found to be 12.8 ohms. Determine the power lost as heat in the rheostat.

Since the current and the resistance are given, Eq. (30) is most convenient.

$$P = (4.7)^2 \times 12.8 = 283 \text{ watts. } \quad \textit{Ans.}$$

The watt is often too small a unit for commercial use and the *kilowatt* (equal to 1,000 watts) is used when large amounts of power are being considered. It is often necessary to transform from mechanical horsepower to electrical power and conversely, and a knowledge of the relation of the two is therefore useful:

$$746 \text{ watts} = 1 \text{ hp.} \tag{32}$$
$$0.746 \text{ kw.} = 1 \text{ hp.} \tag{33}$$

and

$$1 \text{ hp.} = \tfrac{3}{4} \text{ kw. very nearly.} \tag{34}$$
$$1 \text{ kw.} = \tfrac{4}{3} \text{ hp. very nearly.} \tag{35}$$

Example.—An electric motor takes 28 amp. at 550 volts and has an efficiency of 89 per cent. What horsepower does it deliver?

$$\text{Input} = 28 \times 550 = 15{,}400 \text{ watts.}$$
$$\text{Output} = 15{,}400 \times 0.89 = 13{,}700 \text{ watts.}$$
$$\frac{13{,}700}{746} = 18.37 \text{ hp. at the pulley.} \quad \textit{Ans.}$$

38. Electrical Energy.—Power is the *rate of doing work*, or is the *rate of expenditure of energy*. Therefore electrical energy is equal to the product of electrical power and time (see Par. 37).

The fundamental c.g.s. unit of work or energy is the *erg* or *dyne-centimeter* which is the work done when a force of 1 dyne is exerted through a distance of 1 cm. in the direction of the force. This unit, however, is too small for practical purposes, hence the *watt-second* or *joule*,[1] equal to 10,000,000 (10^7) ergs, is the unit of energy in the practical electrical system.

It follows that electrical energy

$$W = EIt \text{ watt-sec. or joules,}$$

where t is in seconds, E is in volts, and I is in amperes.

[1] Joule, James Prescott (1818–1889). An English physicist, who is best known for his pioneer researches on the mechanical equivalent of heat. He was the first to evaluate an electric current quantitatively, the value being determined by the weight of water which a given current decomposed in an hour. He enunciated and proved experimentally the law that whenever mechanical energy is converted into heat energy, an exact equivalent of heat energy is always obtained. It was he who first determined the absolute value of the B.t.u. and also its value in terms of electrical energy.

Even the watt-second is ordinarily too small a unit for commercial purposes, so the larger unit, the *kilowatt-hour* (kw.-hr.) is commonly used. 1 kw.-hr. = $1{,}000 \times 60 \times 60 = 3{,}600{,}000$ joules or watt-sec.

The distinction between power and energy (or work) should be kept clearly in mind. Power is *rate* of doing work, just as velocity is rate of motion. On the other hand, energy is the total work done and is equal to the power multiplied by the time during which the power acts just as distance covered is the velocity or rate of motion multiplied by the time. To speak of a train traveling at a rate of 40 miles per hour gives no information as to the total distance which the train travels. Likewise, to speak of 50 kw. does not state the amount of energy that is involved. The statement "electricity is sold for so many cents per kilowatt" is incorrect. The correct expression is "electrical *energy* is sold for so many cents per kilowatt-*hour*." To illustrate:

Example.—If energy is sold for 10 cts. per kilowatt-hour (kw.-hr.), how many kilowatts may be purchased for 20 cts.? This question as it stands cannot be answered, since the *time* is not given. If, however, it is assumed that the power is to be used for 1 hr.,

$$\frac{20 \text{ cts.}}{10 \text{ cts.}} = 2 \text{ kw.-hr. available.}$$

$$\frac{2 \text{ kw.-hr.}}{1 \text{ hr.}} = 2 \text{ kw. are available.} \quad \textit{Ans.}$$

If used in 0.5 hr.,

$$\frac{2 \text{ kw.-hr.}}{0.5 \text{ hr.}} = 4 \text{ kw.;} \quad \textit{Ans.}$$

if used in 0.001 hr.,

$$\frac{2 \text{ kw.-hr.}}{0.001 \text{ hr.}} = 2{,}000 \text{ kw.,} \quad \textit{Ans.}$$

so that the 20 cts. could purchase *any number of kilowatts*, depending on the time during which the power is supplied.

In a similar way, horsepower is *rate of doing work* and is equivalent to 33,000 ft.-lb. *per minute* and *not* to 33,000 ft.-lb. A motor developing ⅛ hp. could do 33,000 ft.-lb. of work if allowed 8 min. in which to do it. When speaking of *work* in connection with horsepower, the *horsepower-hour* is the unit ordinarily used.

Example.—How many watt-seconds are supplied by a motor developing 2 hp. for 5 hr.?

$$2 \times 5 = 10 \text{ hp.-hr.}$$
$$10 \text{ hp.-hr.} \times 746 = 7{,}460 \text{ watt-hr.}$$
$$7{,}460 \times 3{,}600 = 2.69 \times 10^7 \text{ watt-sec.} \quad \textit{Ans.}$$

39. Heat and Energy.—It is well known that heat energy may be converted into mechanical and into electrical energy, and, conversely, that electrical and mechanical energy may be converted into heat energy. The complete cycle of energy transformation is well illustrated by a steam-power plant. The energy is brought to the plant in the coal as *chemical energy.* The constituents of the coal combine with the oxygen of the air, thus converting the chemical energy into *heat energy.* A certain percentage of this heat energy is transferred to the boiler and produces steam. The expansion of the steam in the engine cylinders, or through the buckets and blades of the turbine, converts the heat energy of the steam into *mechanical energy.* This mechanical energy drives the electrical generator which converts a large proportion of the mechanical energy into *electrical energy.* A portion of this electrical energy is transformed into heat in the wires, bus-bars, transformers, and the transmission system. Finally, the remainder is used to operate motors, supply lamps, propel electric cars, and some may be used for chemical processes. Ultimately all the energy appears again as heat energy or is converted into other forms of energy.

The following table shows approximately what becomes of each 100 heat units existing initially in the coal in the most effi-

EFFICIENCY OF ENERGY CONVERSION

	Form of energy	Efficiency, per cent	Heat units converted
Coal	Chemical	....	100.0
Boiler	Heat	85	85.0
Turbine	Mechanical	30	25.5
Generator	Electrical	96	24.5
Transmission system (to point of utilization)	Electrical	80	19.6
Substation transformers	Electrical	98	19.2
Large motors (average)	Mechanical	85	16.3
Small motors (average)	Mechanical	65	12.5
Lamps	Light	3.5	0.67

cient modern steam-power plants, using superheaters, condensers, and large units, and operating under the best conditions.

Figure 27[1] illustrates the flow of energy (expressed in terms of power) in a typical modern electrical system, from the chemical energy in the coal as it is fed to the stokers beneath the boilers to the point of utilization which in this case is represented by motors.

Energy is delivered to the stokers in the form of coal at a rate of 94,860 B.t.u. per second, corresponding to 100,000 kw. The power loss and the power transferred are indicated by the flow lines at the top of the diagram.

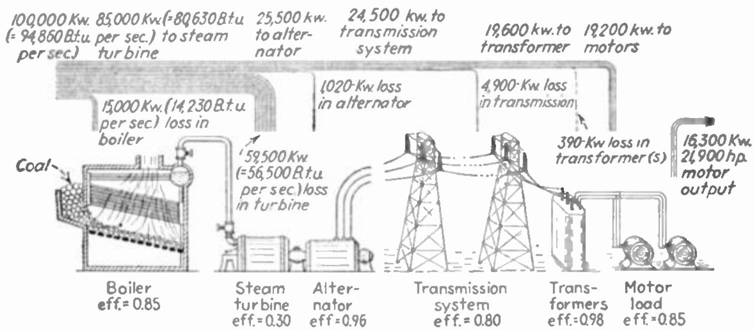


FIG. 27.—Energy flow—chemical, thermal, mechanical, and electrical—in typical power system.

This system is a typical high-efficiency system, and the efficiencies given are those obtained under very favorable operating conditions. For simplicity many details in the system, such as switches, step-up transformers, bus-bars, have been omitted (also see Fig. 413, p. 559, and Vol. II, Fig. 359, p. 451).

It is to be noted that in the most modern steam plants operating under the most favorable conditions, the steam turbine is very wasteful, converting only 30 per cent of the received thermal energy into mechanical energy. The over-all efficiency of such modern systems is not high, being from coal to motors only 19.2/100, or 19.2 per cent. The efficiencies of the motors, lights, and other devices reduce the total over-all efficiency to a still lower value.

[1] This diagram is based on one developed by R. A. Philip; see *Trans. A.I.E.E.*, Vol. 34, p. 779, 1915.

40. Thermal Units.—The unit of heat energy in the English system is the B.t.u. (British thermal unit) and is equal to the amount of heat required to raise the temperature of 1 lb. of water 1°F. It is equal to 778 ft.-lb. (called the *mechanical equivalent of heat*).

In the c.g.s. system, the heat unit is the gram-calorie and is equal to the amount of heat required to raise the temperature of 1 g. of water 1°C.[1]

A gram-calorie is equal to 4.2 watt-sec. or joules.

By Joule's law the heat developed in a circuit is

$$W = \frac{1}{4.2} I^2Rt = 0.24\ I^2Rt \text{ cal.} \tag{36}$$

where t is in seconds, I in amperes, and R in ohms.

Example.—Ten horsepower is delivered by a pump in circulating 400 gal. of water per minute through a certain cooling system. How many degrees Fahrenheit is the temperature of the water raised by the action of the pump?

$$10 \text{ hp.} = 10 \times 33{,}000 = 330{,}000 \text{ ft.-lb. per minute.}$$
$$\frac{330{,}000}{778} = 424 \text{ B.t.u. per minute.}$$
$$400 \text{ gal.} = 400 \times 8.34 = 3{,}336 \text{ lb.}$$
$$\frac{424}{3{,}336} = 0.13°\text{F.} \quad \textit{Ans.}$$

Example.—An incandescent lamp taking 0.5 amp. from 110-volt mains is immersed in a small tank, containing 2,000 c.c. of water. Neglecting radiation, by how many degrees per minute is the temperature of the water raised?

$$W = 0.24 \times 0.5 \times 110 \times 60 = 792 \text{ cal. per minute.}$$
$$\frac{792}{2{,}000} = 0.396°\text{C.} \quad \textit{Ans.}$$

41. Potential Drop in a Feeder Supplying One Concentrated Load.—Figure 28 shows a feeder (consisting of a positive and a negative wire) supplying a motor load. The feeder is connected to bus-bars having a constant potential difference of 230 volts. The feeder is 1,000 ft. long and consists of two 250,000-cir.-mil conductors. The maximum load on the feeder is 250 amp. It is required to determine the voltage at the motor terminals and the efficiency of transmission.

[1] See Appendix A, p. 589.

As is stated in Par. 30 (p. 32), the voltage at the motor must be less than that at the bus-bars because of the voltage lost in supplying the resistance drop in the feeder.

From Appendix E (p. 593), the resistance of 1,000 ft. of 250,000-cir.-mil cable is 0.0431 ohm. As is shown in Par. 30, the net voltage at the receiving end of the line is less than the voltage at the sending end by the voltage loss in both the *outgoing* and the *return* wire. Therefore the drop in 2,000 ft. of cable must be taken, the total resistance being 0.0862 ohm. The current is 250 amp.

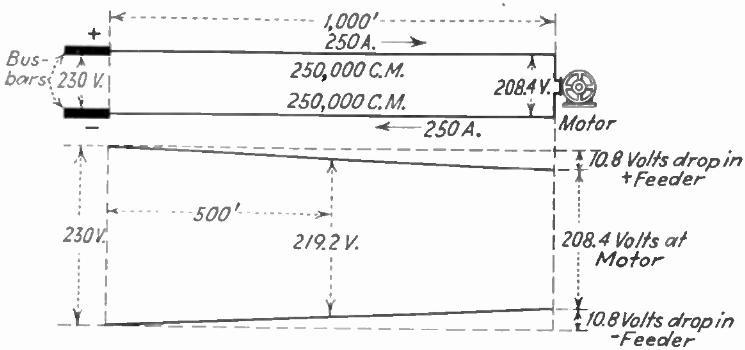


Fig. 28.—Voltage drop in a feeder due to a single load.

By Eq. (19), p. 36, the voltage drop in the line

$$E' = 250 \times 0.0862 = 21.55 \text{ volts.}$$

Therefore, the voltage at the motor terminals is

$$230 - 21.6 = 208.4 \text{ volts.} \quad \textit{Ans.}$$

In Fig. 28 the voltage along the line is shown graphically. The voltage at the sending end of the line is 230 volts, and there is a uniform drop in each wire, this drop increasing uniformly to 10.8 volts, making a total voltage loss of 21.6 volts. The potential difference between the two wires 500 ft. from the sending end will be 230 − 10.8 = 219.2 volts as shown.

$$\text{The power delivered to the motor} = 208.4 \times 250 \quad \text{watts.}$$

$$\text{The power delivered to the line} = 230 \times 250 \quad \text{watts.}$$

$$\text{The efficiency of the line} = \frac{\text{output}}{\text{input}} = \frac{208.4 \times 250}{230 \times 250}$$

$$= \frac{208.4}{230}, \text{ or } 90.6 \text{ per cent.}$$

With one concentrated load the efficiency of transmission is given by the voltage at the load divided by the voltage at the sending end of the line.

42. Potential Drop in a Feeder Supplying Two Concentrated Loads at Different Points.—In Fig. 29 a 300,000-cir.-mil feeder supplies 200 amp. to a load 800 ft. from the bus-bars, and 150 amp. to a load 400 ft. farther on. If the bus-bar voltage is maintained constant at 240 volts, determine the voltage at each load, the total line loss, and the efficiency of transmission.

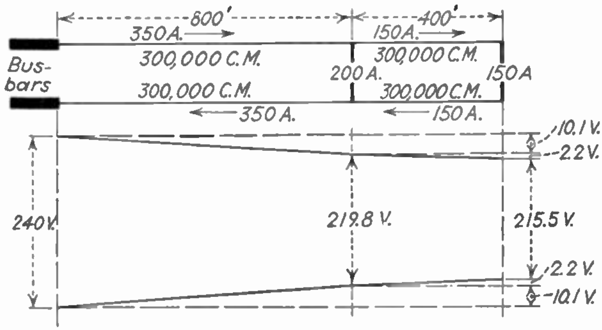


FIG. 29.—Voltage drops in a feeder supplying two loads.

From Appendix E (p. 593), the resistance of 1,000 ft. of 300,000-cir.-mil cable is 0.0360 ohm. The resistance of 800 ft. = $800/1{,}000 \times 0.0360 = 0.0288$ ohm.

Voltage drop to the 200-amp. load

$$E' = 350(2 \times 0.0288) = 20.16 \text{ volts.}$$

Voltage at 200-amp. load

$$E_1 = 240 - 20.2 = 219.8 \text{ volts.} \quad \textit{Ans.}$$

Resistance of one conductor from the 200-amp. load to the 150-amp. load = $400/1{,}000 \times 0.0360 = 0.0144$ ohm.

Voltage drop from 200-amp. load to 150-amp. load

$$E'' = 150(2 \times 0.0144) = 4.32 \text{ volts.}$$

Voltage at 150-amp. load

$$E_2 = 219.8 - 4.3 = 215.5 \text{ volts.} \quad \textit{Ans.}$$

The voltage distribution along the line is shown graphically in Fig. 29.

To determine the efficiency:
Line loss to 200-amp. load

$$P_1 = (350)^2(2 \times 0.0288) = 7{,}060 \text{ watts (Eq. (30), p. 43).}$$

Line loss from 200-amp. load to 150-amp. load

$$P_2 = (150)^2(2 \times 0.0144) = 649 \text{ watts (Eq. (30)).}$$

Total line loss

$$P_1 + P_2 = 7{,}060 + 649 = 7{,}709 \text{ watts, or } 7.709 \text{ kw.}$$

Efficiency =

$$\frac{\text{input} - \text{losses}}{\text{input}} = \frac{(240 \times 350) - 7{,}709}{240 \times 350} = \frac{76{,}290}{84{,}000} \text{ or } 90.8 \text{ per cent.}$$

43. Estimation of Feeders.—It is stated in Par. 13 (p. 15) that a circular-mil-foot of copper has a resistance of 10.37 ohms. In many cases it is sufficiently exact to assume this value as 10 ohms. Assume the current density in a feeder to be one ampere per 1,000 cir. mils, or 0.001 amp. per circular mil. Call this the *normal* current density. (Bus-bars and large feeders operate at a density very nearly equal to this.)

The voltage drop through a circular-mil-foot carrying 0.001 amp. is

$$E = IR = 0.001 \times 10 = 0.01 \text{ volt.}$$

Another circular-mil-foot, carrying 0.001 amp., will also have a drop of 0.01 volt between its ends. If these be placed side by side, the drop across the two will still be 0.01 volt. With any number of wires, each having 1-cir.-mil cross-section, a length of 1 ft., and a current of 0.001 amp., the drop between the ends of each wire will be 0.01 volt. The wires may be separated, or they may be made into a cable.

In Fig. 30(a) are shown four separate conductors, each of 1 cir.-mil-ft. and each carrying 0.001 amp. The voltage across each must be 0.01 volt. In Fig. 30(b) these same four conductors are grouped together and, as each carries 0.001 amp., the total current must be 0.004 amp. The voltage drop across the group is still 0.01 volt. If any number of circular-mil-foot conductors each carrying 0.001 amp. are added in parallel to the group of Fig. 30(b), the drop remains 0.01 volt.

From the foregoing the following rule may be deduced:

The voltage drop per foot of copper conductor whose resistivity is 10 *ohms-cir.-mil-ft. is* 0.01 *volt if the current density is* 0.001 *amp. per circular mil. Further, if the current density is other than* 0.001 *amp. per circular mil, the voltage drop will be in direct proportion to the current density.* This last follows from Eq. (19), p. 36.

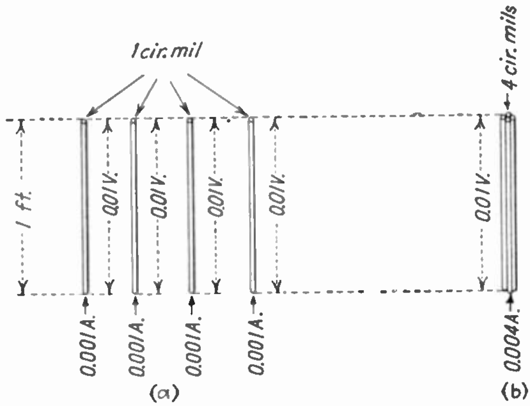


FIG. 30.—Voltage drop in a cir.-mil-foot.

Example.—A motor 800 ft. from the power house is to take 500 amp. from 230-volt bus-bars. What size cable is necessary in order that the voltage drop shall not exceed 20 volts?

A cable to operate at the *normal* density must have

$$500 \times 1{,}000 = 500{,}000 \text{ cir. mils.}$$

The total voltage drop then becomes

$$0.01 \times 800 \times 2 = 16 \text{ volts.}$$

The allowable drop is 20 volts, so a smaller cable may be used.

$$500{,}000 \times \frac{16}{20} = 400{,}000 \text{ cir. mils.} \quad \textit{Ans.}$$

This makes the actual current density

$$\frac{500}{400} = 1.25 \text{ amp. per } 1{,}000 \text{ cir. mils.}$$

The foregoing relation may also be treated from the following point of view.

The voltage drop per unit length of conductor is known as the *voltage gradient* in the conductor and is the slope of the voltage graph in Fig. 28

(p. 49). The relation between voltage gradient and current density is shown by combining Eq. (19), p. 36, with Eq. (1), p. 7. That is, $E = IR = I\rho L/A$, from which

$$\frac{E}{L} = \rho\left(\frac{I}{A}\right). \qquad \text{(I)}$$

Hence, the voltage gradient is equal to the product of the current density and the resistivity. As has just been shown for copper conductor, the resistivity of which is 10 ohms-cir.-mil-ft., the voltage drop per foot is 10 times the current density in amperes per circular mil.

The example may also be solved from the voltage-gradient point of view. The voltage gradient in the cable is $20/(2 \times 800)$ volts per foot.

From Eq. (I),

$$\frac{20}{2 \times 800} = 10\frac{500}{A}$$

where $\rho = 10$ and $I = 500$.

Hence,

$$A = 400{,}000 \text{ cir. mils}$$

giving an actual current density of 500/400,000, or 0.00125, amp. per circular mil.

This density is somewhat in excess of the normal density but not unduly so.

44. Power Loss in a Feeder.—The method of Par. 43 may be used to determine the power loss in a copper conductor. At the normal density,

$$P' = I^2R = (0.001)^2 10 = 0.00001 \text{ (or } 10^{-5}\text{) watt per circular-mil-foot.} \qquad \text{(I)}$$

The total power loss at the *normal density* is

$$P_0 = 0.00001 \times \text{C.M.} \times l \qquad \text{(II)}$$

where C.M. is the conductor cross-section in circular mils, and l the length in feet.

The *actual* power loss is proportional to the *square* of the ratio of the actual to the normal current density.

That is,

$$P = P_0 D^2 \qquad \text{(III)}$$

where P is the actual power loss, P_0 the power loss at the normal current density, and D is the actual current density in amperes per 1,000 cir. mils.

Example.—Determine the power loss in the example of Par. 43.
$P_0 = 0.00001 \times 400{,}000 \times 800 \times 2 = 6{,}400$ watts at the normal density.
The actual power loss

$$P = 6{,}400 \times (1.25)^2 = 10{,}000 \text{ watts} = 10 \text{ kw.} \quad \textit{Ans.}$$

Also if the voltage drop and the current are known, the power loss is readily determined. For example, in the foregoing problem the voltage drop is 20 volts and the current is 500 amp. Hence the power loss

$$P = 20 \times 500 = 10{,}000 \text{ watts} = 10 \text{ kw. } (\textit{check}).$$

The foregoing gives an easy and rapid method of solution for many problems. It is sufficiently exact under most practical conditions.

The foregoing relations may be developed also in the following manner. From Eq. (30), p. 43, and Eq. (2), p. 8, the power loss in a conductor,

$$P = I^2R = I^2\rho\frac{V}{A^2} = \left(\frac{I}{A}\right)^2\rho V \qquad \text{(IV)}$$

where V is the volume.

Also from Eq. (31), p. 43, and Eq. (2), p. 8,

$$P = \frac{E^2}{R} = \frac{E^2}{\rho L^2/V} = \left(\frac{E}{L}\right)^2\frac{V}{\rho}. \qquad \text{(V)}$$

From (IV) and (V), the power loss per unit volume,

$$\frac{P}{V} = \rho\left(\frac{I}{A}\right)^2 = \frac{1}{\rho}\left(\frac{E}{L}\right)^2 \qquad \text{(VI)}$$

If with the circular-mil-foot of copper, $\rho = 10$ and $I/A = 0.001$, (the *normal* current density), $P/V = 10 \times (0.001)^2 = 10^{-5}$ watt per circular-mil-foot at the normal current density, which agrees with (I).

Likewise at the normal current density the voltage drop per foot is $E/L = 0.01$ volt, $P/V = 10^{-5}$ watt (Eq. (VI)).

The *total* power loss is given by Eqs. (IV) and (V).

Example.—Repeat the foregoing example, using Eq. (IV):

$$P = \left(\frac{500}{400{,}000}\right)^2 10(1{,}600 \times 400{,}000) = 10 \text{ kw.} \quad \textit{Ans.}$$

In this example the actual power loss per unit volume (Eq. VI),

$$\frac{P}{V} = 10\left(\frac{500}{400{,}000}\right)^2 = \frac{1}{10}\left(\frac{20}{1{,}600}\right)^2 = 1.56 \times 10^{-5} \text{ watt per circular-mil-foot}$$

(which is not excessive).

CHAPTER III

BATTERY ELECTROMOTIVE FORCES—KIRCHHOFF'S LAWS

45. Battery Electromotive Force and Resistance.—If a voltmeter be connected across the terminals of a battery (Fig. 31), the switch S being open, the instrument will indicate a certain voltage E. If the switch S be closed, allowing the current I

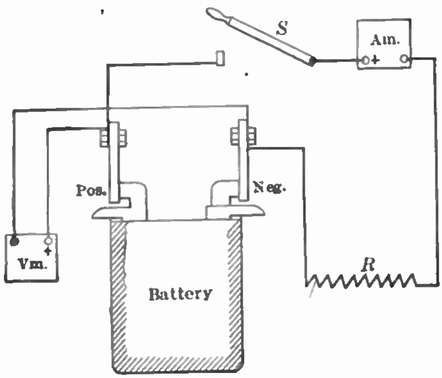


FIG. 31.—Connections for measuring battery resistance.

to flow, the instrument will indicate a voltage V which is less than E.

The voltage E, measured when the battery delivers no current, is the *internal voltage* or the *e.m.f.* of the battery; the voltage V, measured when a current I flows, is known as the *terminal voltage* of the battery for that particular current value.

The difference between the open-circuit voltage E and the voltage V, measured when current is being taken from the battery, is the *voltage drop* in the battery due to the passage of current through the battery resistance. Every cell has resistance, lying for the most part in the electrolyte, but partly in the plates and terminals. When the external circuit is closed so that current flows, voltage is required to send this current through the battery resistance, just as voltage is required to send current through an external resistance.

If the voltage E, measured at the battery terminals when the circuit is open, drops to V when the circuit is closed, the voltage $e = (E - V)$ is the voltage drop through the cell due to the passage of the current I. Let the cell resistance be r. Then, by Ohm's law,

$$E - V = e = Ir \text{ (by Eq. (19), p. 36)}$$

or

$$r = \frac{e}{I} = \frac{E - V}{I} \text{ (by Eq. (20), p. 36),} \tag{37}$$

$$E = V + Ir. \tag{38}$$

That is, the internal resistance of the battery is equal to the difference of the open-circuit and closed-circuit terminal voltages divided by the current.

Also the e.m.f. of the battery is equal to the closed-circuit terminal voltage *plus* the resistance drop in the battery.

Example.—The open-circuit voltage of a storage cell is 2.20 volts. The terminal voltage, measured when a current of 12 amp. flows, is found to be 1.98 volts. What is the internal resistance of the cell?

The voltage drop through the cell

$$E - V = 2.20 - 1.98 = 0.22 \text{ volt.}$$

Then

$$r = \frac{0.22}{12} = 0.0183 \text{ ohm.} \quad \textit{Ans.}$$

In making a measurement of this character, it must be remembered that even under open-circuit conditions the ordinary voltmeter takes some current. If the cell capacity is small (as in the case of a Weston cell) the voltmeter current alone may reduce the terminal voltage to a value one-half, or even less, of the open-circuit voltage. Under these conditions the ordinary voltmeter cannot be used to measure the e.m.f. of the cell.

Moreover, it is impossible to measure directly the internal voltage of the battery when the battery delivers current, for the voltage drop occurs *within* the cell itself. Figure 32 represents these conditions so far as their effect on the external circuit is concerned. A battery cell B is enclosed in a sealed box. Its resistance r is considered as removed from the cell itself and connected external to the cell, but within the sealed box. The cell then may be considered as having no resistance, its resistance having been replaced by r. The connections are brought through bushings in the box to terminals a and b. When no current is being delivered by the cell, if a voltmeter which takes negligible current be connected across the two terminals a and b, the instrument, will measure the e.m.f. E of the cell. If, however, a current I flows, the terminal voltage will drop from E to V, due to the voltage drop in the resistance r. Under these conditions it is impossible to measure E while the current is flowing, since the voltmeter can be connected only across the terminals a and b and therefore must include the voltage drop in the resistance.

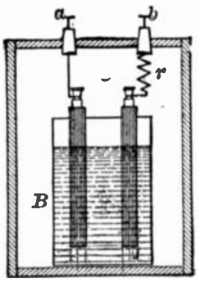


FIG. 32.—The internal resistance of a cell.

The voltage E and the resistance r are seldom constants but are more or less dependent on the current. They are also affected by temperature, change in specific gravity of the electrolyte, polarization, etc. (see p. 93).

46. Battery Resistance and Current.—As is shown in Par. 45, the resistance within the battery tends to reduce the current. If, in Fig. 31, the switch S be closed, the cell e.m.f. E will be acting on a circuit consisting of the internal resistance of the cell r and the resistance of the external circuit R in series, and the total resistance in the circuit is their sum. Then by Ohm's law (p. 35), the current is

$$I = \frac{E}{r + R}. \tag{39}$$

The power lost in the battery is

$$P = I^2r. \tag{40}$$

If the cell is short-circuited, R becomes zero and $I = E/r$. Under these conditions all the electrical energy developed by the cell is converted into heat within the cell itself.

Example.—A battery cell having an e.m.f. of 2.2 volts and an internal resistance of 0.03 ohm is connected to an external resistance of 0.10 ohm.

Determine the current and the efficiency of the battery under this condition of operation.

$$I = \frac{2.2}{0.03 + 0.10} = \frac{2.2}{0.13} = 16.9 \text{ amp.} \quad \textit{Ans.}$$

Power lost in the battery

$$P = (16.9)^2 \times 0.03 = 8.57 \text{ watts.}$$

The useful power

$$P' = (16.9)^2 \times 0.10 = 28.6 \text{ watts.}$$

P' is equal to the total power P_0 developed by the battery minus the battery loss.

$$P_0 = 2.2 \times 16.9 = 37.2 \text{ watts.}$$
$$P' = 37.2 - 8.6 = 28.6 \text{ watts.}$$
$$\text{Efficiency} = \frac{28.6}{28.6 + 8.6} \text{ or } 76.9 \text{ per cent.} \quad \textit{Ans.}$$

The foregoing is a further illustration of Ohm's law, namely: *The current in a circuit is equal to the total e.m.f. acting in the circuit divided by the total resistance of the circuit* (see Par. 32, p. 35).

47. Maximum Power Delivered by a Battery.—Assume that the e.m.f. E and the internal resistance r of the battery (Fig. 31) are constant and that the external resistance R may be varied. If the battery delivers a current i, the power p, delivered to the external circuit, is $vi = i^2R$ where v is the terminal voltage of the battery. Since the terminal voltage $v = E - ir$, v will decrease as the current i increases, as was shown in Par. 46. Hence the power p, delivered to the external circuit, is the product of two variables, one of which, v, decreases as the other, i, increases. The external power will be a maximum when the product vi is a maximum. The conditions under which this occurs may be determined as follows:

$$p = vi = (E - ir)i = Ei - i^2r. \tag{I}$$

Differentiating Eq. (I) with respect to the variable i and equating to zero,

$$\frac{dp}{di} = E - 2ir = 0; \quad i = \frac{E}{2r} = \frac{E}{r + r}.$$

From Eq. (39) it follows that

$$R = r.$$

That is: *Maximum power is obtained when the external resistance is made equal to the internal resistance of the battery.*

Since

$$E = v + ir \quad \text{and} \quad v = iR = ir, \tag{41}$$

$$E = 2v; \quad v = \frac{E}{2}. \tag{42}$$

That is: *When a battery is delivering the maximum power, the terminal voltage is equal to one-half the internal e.m.f. of the battery.*

The *total* power which the battery is developing equals the product of its internal e.m.f. E and the current i. That is, this total power

$$P_0 = Ei. \tag{43}$$

The power lost within the battery is i^2r and the power given to the external circuit is i^2R. Since $r = R$, it follows that $i^2r = i^2R$.

That is, *when a battery is delivering its maximum power, half the total power is lost within the battery itself.* Therefore the efficiency of the battery under these conditions is 50 *per cent.*

As has been shown, the *maximum current* occurs when the external resistance R is zero. Under these conditions, the terminal voltage v is zero and the power delivered to the external circuit is zero. The total power Ei, developed by the battery, is lost in heating the battery itself.

Usually, it is not economical to operate a battery so that it delivers maximum power, because of the low efficiency and the fact that the battery heats unduly. Occasionally, however, batteries are so operated for short periods. For example, automobile-starting batteries during the cranking period frequently operate at such a current that the terminal voltage approaches one-half the e.m.f., and therefore the power that the battery delivers is not far from maximum.

Example.—A battery cell has an e.m.f. of 1.2 volts and an internal resistance of 0.8 ohm. (*a*) For what value of current will the power delivered by the cell be a maximum? (*b*) What is the power under these conditions? (*c*) What will be the value of the external resistance under these conditions? (*d*) Determine the current and the power when the external resistance is 1.0 ohm. (*e*) When it is 0.6 ohm. (*f*) What is the maximum current which this cell can deliver? (*g*) What is the external power under these conditions?

(*a*) The power is a maximum when the terminal voltage $v = \frac{E}{2} = 0.60$ volt. The current $I = \frac{1.2 - 0.60}{0.8} = \frac{0.6}{0.8} = 0.75$ amp. *Ans.*

(*b*) $p = 0.60 \times 0.75 = 0.45$ watt. *Ans.*

(*c*) The external resistance $R = \frac{v}{i} = \frac{0.60}{0.75} = 0.8$ ohm. (= internal resistance). *Ans.*

(*d*) $i = \frac{1.2}{0.8 + 1.0} = \frac{1.2}{1.8} = 0.667$ amp. *Ans.*

$v = 1.2 - 0.667 \times 0.8 = 1.2 - 0.533 = 0.667$ volt.

$p = vi = 0.667 \times 0.667 = 0.444$ watt. *Ans.*

(*e*) $i = \frac{1.2}{0.8 + 0.6} = \frac{1.2}{1.4} = 0.857$ amp. *Ans.*

$v = 1.2 - 0.857 \times 0.8 = 1.2 - 0.686 = 0.514$ volt.

$p = vi = 0.514 \times 0.857 = 0.440$ watt. *Ans.*

(*f*) $i = \frac{1.2}{0.8} = 1.5$ amp. *Ans.*

(*g*) Since $v = 0$,

$p = 0 \times 1.5 = 0.$ *Ans.*

It will be noted that the product vi is a maximum when $v = E/2$. In (*d*), v is greater than in (*a*), but i is so much less that the product vi is less. In (*e*) i is greater than in (*a*) but v is so much less that the product vi is less. (*d*) and (*e*) may be solved more readily by using $p = i^2R$, but this equation does not so clearly illustrate the fact of the product vi being a maximum.

48. Batteries Receiving Energy.—If a resistance load be connected across a battery, current will flow from the positive terminal of the battery through the external circuit and will return to the battery through the negative terminal. As has been pointed out, the battery-terminal voltage will be less than its open-circuit value, due to current flowing through the internal resistance of the battery. Under these conditions the battery is a source of energy and is acting as a generator, or it *delivers* energy.

If current is forced to *enter* at the positive terminal of the battery, the battery will no longer be supplying energy but will be receiving energy. This energy must be supplied from some external source, as from another battery, or, as is more common, from a generator. The cell shown in Fig. 33 has an e.m.f. of 2 volts, and a voltmeter V, connected across its terminals, indicates 2 volts when no current flows, that is, when switch S is open. Another source of electrical energy, such as a direct-current generator, is adjusted so that its terminal voltage is exactly 2 volts. The negative(−) terminal of the generator is connected to the negative(−) terminal of the battery. The positive(+) terminal of the generator is connected to the switch *S*, which at first is open. However, when the switch *S* is closed, connecting the positive(+) terminal of the battery to the positive(+) terminal of the generator as shown in the figure, no

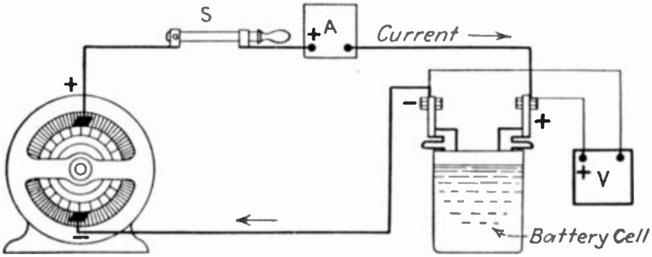


FIG. 33.—Generator charging a battery.

current flows. The voltmeter *V* still reads 2 volts, and the ammeter *A* still reads zero. That is, the battery neither delivers nor receives energy and no effects are noted other than those observed when the battery is open-circuited. Under these conditions the battery is said to be "floating."

If, however, the voltage of the generator be raised slightly, the ammeter *A* will indicate a current flowing from the positive terminal of the generator *into* the positive terminal of the battery,

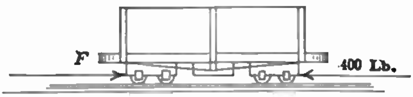


FIG. 34.—Force necessary to start a car.

a direction just opposite to that which the current has when the battery *supplies* energy. The voltmeter will no longer read 2 volts but will indicate a potential difference somewhat in excess of 2 volts.

The relation in the generator-battery combination may be illustrated by a mechanical analogy. Figure 34 shows a car standing on the track. A force of 400 lb. is necessary to overcome the standing friction of the car on the track. At one end of the car a force *F* is applied. Before the force *F* can move the car, its value must equal at least 400 lb. When F is exactly 400 lb., the car will not move, just as no current flows into the battery when the generator voltage is exactly equal to that of the battery. When the force F exceeds 400 lb., however, the car will move, the force effective in producing this motion being the amount by which F exceeds 400 lb. Thus, if $F =$ 450 lb., 400 lb. of this is utilized in overcoming the 400-lb. opposing force due to friction, and the 50 lb. is effective in moving the car. (The standing or static friction ordinarily exceeds the dynamic friction.)

In the case of the battery no current will flow until voltage in excess of 2 volts is produced by the generator. Thus, if the generator voltage be raised to 2.4 volts, 2.0 volts of this is utilized to "buck" the 2.0 volts of the cell, and 0.4 volt is effective in sending current into the cell. Thus, if the cell resistance be 0.1 ohm, the current will be

$$I = \frac{0.4}{0.1} = 4.0 \text{ amp.}$$

This assumes that the resistance of the leads is negligible.

Therefore, if E is the e.m.f. of a battery, r its resistance, and V the terminal voltage when current flows *in* at its positive terminal,

$$I = \frac{V - E}{r}, \tag{44}$$

and

$$E = V - Ir. \tag{45}$$

That is, the e.m.f. of the cell is *less than the terminal voltage* by the amount of the resistance drop in the cell itself.

These equations should be compared with Eqs. (37) and (38), p. 56.

Under the foregoing conditions, the cell is *receiving* electric energy, as when a storage battery is being charged.

Example.—The e.m.f. of a three-cell storage battery is 5.80 volts, and the internal resistance is 0.072 ohm. The battery is being charged, the current being 12 amp. Determine: (*a*) the terminal voltage of the battery; (*b*) the power loss in internal heating of the battery; (*c*) the energy stored per second.

(a) The terminal voltage must at least equal the counter e.m.f. before current can flow and in addition must supply the internal-resistance drop. Hence,

$$V = 5.80 + 12 \times 0.072 = 5.80 + 0.864 = 6.664 \text{ volts.} \quad \textit{Ans.}$$

(b) $p = (12)^2 \times 0.072 = 10.38$ watts. *Ans.*

(c) The storage of energy must be represented by the flow of current *against* the counter e.m.f. of the battery. Hence, the energy stored per second

$$w = 5.80 \times 12 = 69.6 \text{ watt-sec. or joules.} \quad \textit{Ans.}$$

49. Battery Cells in Series.—Strictly speaking, a battery consists of more than one unit or cell. However, the term battery has come also to mean a single cell, when this cell is not acting in conjunction with others.

When cells are connected in series, their e.m.fs. are added together to obtain the total e.m.f. of the battery, and their resistances are added together to obtain the total resistance of the battery.

Thus, if several cells, having e.m.fs. E_1, E_2, E_3, E_4, etc., and resistances r_1, r_2, r_3, r_4, etc., are connected in series, the total e.m.f. of the combination is

$$E = E_1 + E_2 + E_3 + E_3 + \cdots, \tag{46}$$

and the total resistance is

$$r = r_1 + r_2 + r_3 + r_4 + \cdots. \tag{47}$$

Equation (46) assumes that the cells are all connected with positive to negative terminals so that their e.m.fs. are additive. If any cell be connected so that its e.m.f. opposes the others, its voltage in Eq. (46) must be preceded by a minus sign.

If an external resistance R is connected across these cells in series, by Ohm's law (p. 35), and also from Eq. (39), the current is

$$I = \frac{E}{r + R} = \frac{E_1 + E_2 + E_3 + E_4 + \cdots}{r_1 + r_2 + r_3 + r_4 + \cdots + R} \tag{48}$$

Example.—Four dry cells having e.m.fs. of 1.30, 1.30, 1.35, 1.40 volts and resistances of 0.3, 0.4, 0.2, 0.1 ohm, are connected in series to operate a relay having a resistance of 10 ohms. What current flows in the relay?

$$I = \frac{1.30 + 1.30 + 1.35 + 1.40}{0.3 + 0.4 + 0.2 + 0.1 + 10} = \frac{5.35}{11.0} = 0.486 \text{ amp} \quad \textit{Ans.}$$

A battery consisting of n equal cells in series has an e.m.f. n times that of one cell, but has the current capacity of one cell only.

50. Equal Batteries in Parallel.—To operate satisfactorily in parallel, all the batteries should have the same e.m.f. The behavior of batteries having unequal e.m.fs. can be treated as a special problem (see Par. 53).

Figure 35 shows a battery of six cells, each having an e.m.f. of 2.0 volts and a resistance of 0.2 ohm. It is clear that the e.m.f. of the entire battery is no greater than the e.m.f. of any one cell. The current, however, has six paths through which to flow. Therefore, for a fixed external current, the voltage drop in each cell is one-sixth that occurring if all the current passed

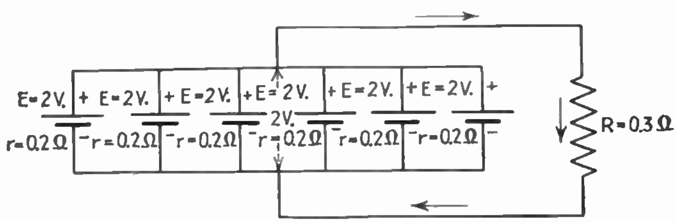


FIG. 35.—Parallel connection of equal cells.

through one cell. If the internal resistance of one cell is 0.2 ohm, the resistance of the battery as a whole must be $0.2/6 = 0.033$ ohm.

Example.—If the external resistance connected across the terminals of the battery in Fig. 35 is 0.3 ohm, what current flows?

Resistance of battery $= 0.2/6 = 0.033$ ohm.

$$I = \frac{2.0}{0.033 + 0.3} = \frac{2.0}{0.333} = 6 \text{ amp.} \quad \text{(Eq. 39).} \quad \textit{Ans.}$$

If the e.m.fs. are equal but the resistances of the cells are not equal, being r_1, r_2, r_3, r_4, etc., the battery resistance r is found by considering these resistances as in parallel (Eq. (8), Chap. I, p. 12).

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \cdots. \tag{49}$$

Example.—A battery consists of four cells connected in parallel, each having an e.m.f. of 2.0 volts and resistances of 0.30, 0.25, 0.22, 0.20 ohm. If a resistance of 0.50 ohm is connected across the terminals of the battery, what current flows and how much current does each cell supply? What is the voltage across the battery terminals?

$$\frac{1}{r} = \frac{1}{0.30} + \frac{1}{0.25} + \frac{1}{0.22} + \frac{1}{0.20} = 16.87 \text{ mhos.}$$

$$r = \frac{1}{16.87} = 0.0593 \text{ ohm.}$$

$$I = \frac{2.0}{0.0593 + 0.50} = \frac{2.0}{0.5593} = 3.576 \text{ amp.} \quad \textit{Ans.}$$

The terminal voltage

$$\begin{aligned} V = IR &= 3.576 \times 0.5 = 1.788 \text{ volts} \\ &= 2.0 - 3.576 \times 0.0593 \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

The current in each cell may be found by means of Eq. (37), p. 56.

$$\frac{2.0 - 1.788}{I_1} = 0.30.$$

Solving

$$I_1 = \frac{2.0 - 1.788}{0.30} = \frac{0.212}{0.30} = 0.707 \text{ amp.}$$

Likewise,

$$I_2 = \frac{2.0 - 1.788}{0.25} = \frac{0.212}{0.25} = 0.848 \text{ amp.}$$

$$I_3 = \frac{2.0 - 1.788}{0.22} = \frac{0.212}{0.22} = 0.964 \text{ amp.}$$

$$I_4 = \frac{2.0 - 1.788}{0.20} = \frac{0.212}{0.20} = 1.060 \text{ amp.}$$

Total current 3.579 amp. *Ans.* (*check*)

That is, *the current in any cell is equal to the voltage drop in the cell divided by the resistance of the cell*, or

$$I = \frac{E - V}{r}. \tag{50}$$

Since the terminal voltages and the e.m.fs. of all the cells are equal, it follows that the product of current and resistance is the same for each cell. That is,

$$0.707 \times 0.30 = 0.848 \times 0.25 = 0.964 \times 0.22 = 1.060 \times 0.20 = 0.212.$$

Cells connected in parallel *must have the same terminal voltage* since all the positive terminals are connected together and all the negative terminals are connected together. If the e.m.fs. of the cells are all equal, the total battery e.m.f. is equal to the e.m.f. of but one cell. The total battery resistance may be found by the equation for resistances in parallel. *If the e.m.fs. are all equal*, the current in each cell is inversely proportional to the resistance of the cell. The current capacity of the battery is the sum of the current capacities of the individual cells.

51. Series-parallel Grouping of Cells.—Rows of series-connected cells may be connected so that the rows themselves are grouped in parallel. Figure 36 shows a row of four cells in series, and five of these rows in parallel.

If there are m equal cells in series in each row, the e.m.f. of each row must be

$$E = mE' \text{ (by Eq. (46))} \quad (51)$$

where E' is the e.m.f. of one cell.

The resistance of each row must be

$$r_1 = mr' \text{ (by Eq. (47))} \quad (52)$$

where r' is the resistance of one cell.

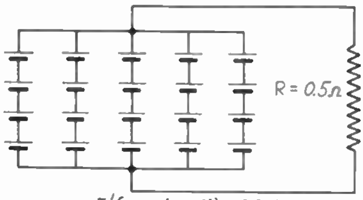


Fig. 36.—Series-parallel grouping of cells.

Since there are n rows in parallel, the resistance of the whole combination must be

$$r = \frac{r_1}{n} = \frac{m}{n}r'. \quad (53)$$

If an external resistance R is connected to the battery, the current is

$$I = \frac{mE'}{\frac{m}{n}r' + R} \quad (54)$$

Example.—Let each of the cells of Fig. 36 have an e.m.f. of 0.9 volt and an internal resistance of 0.08 ohm. If the external resistance R is 0.5 ohm, what current flows?

$$I = \frac{4 \times 0.9}{\frac{4}{5}0.08 + 0.5} = \frac{3.6}{0.564} = 6.4 \text{ amp.} \quad \textit{Ans.}$$

52. Grouping of Cells.—A primary consideration in the grouping of cells is the required voltage. The cells must be so arranged as to give this voltage, regardless of other considerations. However, if no definite terminal voltage is required, the following considerations may govern the manner of grouping.

(*a*) *To obtain the best economy,* group the cells so that the battery resistance is as low as possible. This usually means a large number of parallel connections. Under these conditions the life of the battery will be prolonged and the initial cost is excessive.

(b) *To obtain the maximum current with fixed external resistance* make the internal resistance $\left(\frac{m}{n}r'\right)$ of the battery equal to the external resistance. This is not economical, since only one half of the energy developed by the battery is available in the external circuit; the other half is lost in the cells themselves. Under these conditions the battery delivers maximum power (see Par. 47, p. 58).

(c) *To obtain quick action* for intermittent operation, as of relays or bells, group the cells in series if possible.

Example.—In the example of Par. 51, how should the cells be arranged to obtain the maximum current?

The total battery resistance $\frac{m}{n}0.08$ must be equal to the external resistance. That is,

$$\frac{m}{n}0.08 = 0.5.$$

Also

$$m \times n = 20; \quad n = \frac{20}{m}.$$

Solving

$$\frac{m}{(20/m)}0.08 = 0.5.$$

$$m^2 = \left(\frac{20}{0.08}\right)0.5 = 125.$$

$$m = 11+. \quad \textit{Ans.}$$

The best arrangement is 10 cells in series, and two rows in parallel. (Eleven cells in series would not operate satisfactorily if connected in parallel with the remaining nine cells in series.)

53. Batteries with Unequal Electromotive Forces and Resistances.—Two batteries in parallel having unequal e.m.fs. and unequal resistances may be replaced by a single or equivalent battery having an e.m.f. equal to the common terminal voltage of the two batteries in parallel and with no external load. The internal resistance of the equivalent battery is the joint resistance of the two batteries in parallel. These relations may be proved as follows:

First consider Fig. 37(*a*), which shows two batteries A and B in parallel with no external load and having e.m.fs. of E_1 and E_2 volts and internal resistances of r_1 and r_2 ohms. Assume that E_1 is greater than E_2.

The circulating current through the batteries is

$$I_0 = \frac{E_1 - E_2}{r_1 + r_2} \text{ amp.}$$

The common terminal voltage is

$$V = E_1 - I_0 r_1 = E_1 - \frac{E_1 - E_2}{r_1 + r_2} r_1 \text{ volts}$$

$$= E_1 \frac{r_2}{r_1 + r_2} + E_2 \frac{r_1}{r_1 + r_2} \text{ volts.} \qquad \text{(I)}$$

Next consider Fig. 37(b) which shows the batteries of Fig. 37(a) connected in parallel and with an external resistance R across

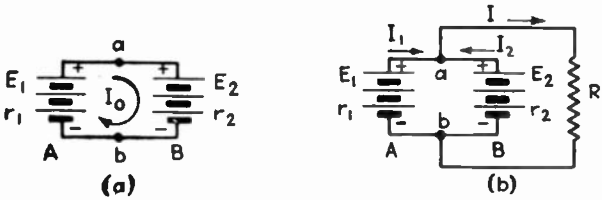


FIG. 37.—Equivalent of unequal batteries in parallel.

their terminals. Let the current in battery A be I_1, that in battery B be I_2; and let the current in the resistance R be I.

The common terminal voltage

$$V_{ab} = E_1 - I_1 r_1 = E_2 - I_2 r_2 \text{ volts}$$

from which

$$I_1 = \frac{E_1 - E_2 + I_2 r_2}{r_1} \text{ amp.} \qquad \text{(II)}$$

The load current

$$I = \frac{V_{ab}}{R} = \frac{E_1 - I_1 r_1}{R} \text{ amp.} \qquad \text{(III)}$$

Also,

$$I = I_1 + I_2 \qquad \text{(IV)}$$

Solving Eqs. (II), (III), and (IV) simultaneously gives

$$I = \frac{E_1 \dfrac{r_2}{r_1 + r_2} + E_2 \dfrac{r_1}{r_1 + r_2}}{R + \dfrac{r_1 r_2}{r_1 + r_2}} \text{ amp.} \qquad (55)$$

The numerator of Eq. (55) is Eq. (I) which gives the common terminal voltage of the two batteries when they are in parallel without external load. The denominator of Eq. (55) is the sum of the external resistance R and the equivalent resistance of the batteries in parallel (see Eq. (49), p. 64).

Example.—In Fig. 37(*b*), the e.m.fs. of batteries A and B are 12 and 10 volts, and their internal resistances are 0.5 and 0.3 ohm. The external resistance R is 1.156 ohms. Determine the current in the external resistance.

The circulating current

$$I_0 = \frac{12 - 10}{0.5 + 0.3} = 2.5 \text{ amp.}$$

The corresponding terminal voltage, the numerator of Eq. (55),

$$V_{ab} = 12 - (2.5 \times 0.5) = 10.75 \text{ volts} = 10 + (2.5 \times 0.3) \text{ volts.}$$

The equivalent parallel resistance of the two batteries

$$r = \frac{0.5 \times 0.3}{0.5 + 0.3} = 0.1875 \text{ ohm.}$$

The external current

$$I = \frac{10.75}{1.156 + 0.1875} = 8.00 \text{ amp.} \quad \textit{Ans.}$$

The same result is obtained if Eq. (55) is used directly.

54. Division of Current among Unequal Batteries in Parallel.—In the method for determining the current delivered by two batteries having unequal e.m.fs. and unequal resistances, Par. 53, the resistance of the external load is given. Below are given two methods by which the division of current between two such batteries may be determined when the current to the load is given rather than the resistance of the load. The division of current among such batteries is based on two principles.

For simplicity consider two batteries only (Figs. 38 and 39). First, an initial current is allowed to flow either in one battery or in both batteries, with the object of bringing the terminal voltages of the two batteries to equality. This current has no direct relation to the total load current. After the terminal voltages have been brought to equality, the two batteries behave like any two batteries having equal e.m.fs. but different internal resistances. Accordingly (Par. 50, p. 64) they will divide all *addi-*

tional current inversely as their internal resistances. The total current delivered by each battery is then readily found by adding to the initial current of each battery the additional current, the division of which between the batteries is determined by their internal resistances.

There are two common methods. In the first method the battery with the greater e.m.f. delivers an initial current which

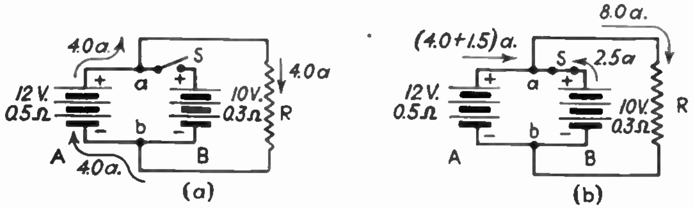


FIG. 38.—Unequal batteries in parallel—floating-battery method.

is just sufficient to make its terminal voltage equal to the e.m.f. of the second battery, so that the second battery neither takes nor delivers current but "floats" (see p. 61). In the second method the load itself is first considered as being open-circuited and the resulting circulatory current through the two batteries (Fig. 39(*a*)) constitutes the initial current which brings the two batteries to equality of terminal voltage. (These two methods

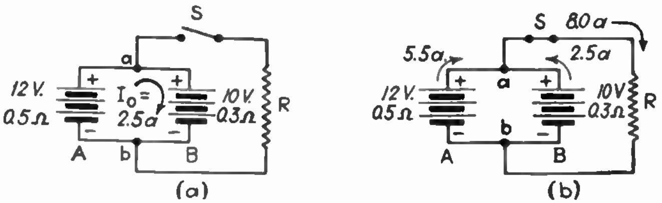


FIG. 39.—Unequal batteries in parallel—circulatory-current method.

are very useful in the analysis and solution of more complex problems, such as determining the division of load among direct-current generators, alternators, and transformers, in parallel.)

The application of the two methods to two batteries in parallel is illustrated by the following example.

Example.—Two batteries A and B (Fig. 38), having e.m.fs. of 12 and 10 volts and internal resistances of 0.5 and 0.3 ohm are connected in parallel. An external resistance R connected across the batteries takes 8 amp. (This battery system is the same as that shown in Fig. 37.) Determine: (*a*) the current delivered by each battery; (*b*) the voltage V_{ab} across their terminals; (*c*) the value of the external resistance; (*d*) the total power developed by each battery; (*e*) the power delivered to the load by each battery.

I. *Floating-battery Method.*—(*a*) In this method the battery with the lower e.m.f. is first considered as being disconnected, as by the opening of switch S in series with the battery B (Fig. 38(*a*)). In the battery A having the higher e.m.f., a current is assumed which will just make the terminal voltage of battery A equal to the e.m.f. of B. Thus in Fig. 38(*a*) the value of this current is $(12 - 10)/0.5 = 4.0$ amp. Since the terminal voltage of battery A is now 10 volts, or is equal to the e.m.f. of battery B, switch S may be closed (Fig. 38(*b*)) without causing any change in the currents of the system. For any additional current to the load, resistance R, the parallel combination behaves like two batteries of equal e.m.fs. but unequal resistances (Par. 50, p. 64). The additional current necessary to produce a total current of 8 amp. in resistance R is 4.0 amp. This current will divide inversely as the battery resistances, or the portions of the additional current taken by batteries A and B are

$$I_A' = \frac{0.3}{0.8}4 = 1.5 \text{ amp.}; \quad I_B' = \frac{0.5}{0.8}4 = 2.5 \text{ amp.}$$

Hence the total currents of batteries A and B are

$$I_A = 4.0 + 1.5 = 5.5 \text{ amp.}; \quad I_B = 0 + 2.5 = 2.5 \text{ amp.} \quad \textit{Ans.}$$

(See Fig. 37(*b*).)

(*b*) $V_{ab} = 12 - 5.5 \times 0.5 = 10 - 2.5 \times 0.3 = 9.25$ volts. *Ans*

(*c*) $R = \dfrac{9.25}{8} = 1.156$ ohms. *Ans.* (compare with example in Par. 53, p. 69).

(*d*) $P_A' = 12 \times 5.5 = 66$ watts.

$P_B' = 10 \times 2.5 = 25$ watts. *Ans.*

(*e*) $P_A = 9.25 \times 5.5 = 50.88$ watts.

$P_B = 9.25 \times 2.5 = 23.13$ watts. *Ans.*

II. *Circulatory-current Method.*—In this method the external load is first considered as being disconnected, as by the opening of the switch S (Fig. 39(*a*)). Under these conditions a circulatory current I_0 will flow equal to the difference of the battery e.m.fs. divided by the sum of their internal resistances. In Fig. 39(*a*) the current

$$I_0 = \frac{12 - 10}{0.5 + 0.3} = 2.5 \text{ amp.}$$

Since the e.m.f. of battery A is greater than that of battery B, the direction of the circulatory current will be clockwise, as shown. It is positive with respect to battery A and negative with respect to battery B. This current brings the terminal voltages of the two batteries to equality. Thus, $12 - 2.5 \times 0.5 = 10 + 2.5 \times 0.3 = 10.75$ volts.

After such equality of terminal voltages has been established, any additional current will divide inversely as the resistances of the two batteries as in Method I and in Par. 50 (p. 64). Since until now the load current has been considered as zero, the total load current must be divided between the batteries. Accordingly, switch S is closed (Fig. 39(b)) and

$$I_A' = \frac{0.3}{(0.8)}8 = 3.0 \text{ amp.};\quad I_B' = \frac{0.5}{(0.8)}8 = 5.0 \text{ amp.}$$

Since battery A already delivers +2.5 amp., its total current

$$I_A = 3.0 + 2.5 = 5.5 \text{ amp.} \quad \textit{Ans.}$$

The battery B already delivers −2.5 amp., so that its total current is

$$5.0 + (-2.5) = 2.5 \text{ amp.} \quad \textit{Ans.}$$

These values of current are the same as those obtained in Method I(a).

The solutions of (b), (c), (d), and (e) are identical with those in Method I.

55. Kirchhoff's[1] Laws.—By means of Kirchhoff's laws it is possible to solve many circuit networks that would otherwise be difficult of solution. In fact the solution of practically all complex circuits requires the application of the principles of Kirchhoff's laws.

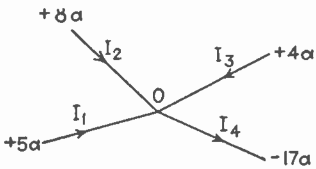


FIG. 40.—Illustrating Kirchhoff's first law.

1. *In any electrical network, the algebraic sum of the currents that meet at a point is zero.*

2. *In any complete electrical circuit the sum of all the e.m.fs. and all the resistance drops, taken with their proper signs, is zero.*

The first law is obvious. It states that the total current *leaving* a junction is equal to the total current *entering* the junction.

The law is illustrated by Fig. 40. Four currents, I_1, I_2, I_3, I_4, meet at the junction O. The first three currents flow *toward*

[1] Kirchhoff, Gustav Robert (1824–1887). A German physicist, born and educated at Königsberg; became Extraordinary Professor of Physics at Breslau in 1850, and in 1854 was appointed to a similar position at Heidelberg. He transferred to Berlin in 1875. His contributions to electrical science were numerous. Besides originating the laws which bear his name, he worked out the solution of many difficult problems, such as electrical conduction in thin plates and in curved sheets, the distribution of electricity in two influencing spheres, and laws of induced currents and of magnetic distribution. He also made notable contributions in radiation and in spectrum analysis.

the junction so have plus signs as they *add* to the quantity of electricity at the point O. The current I_4 flows away from the junction, so has a minus sign as it subtracts from the quantity at the point O. Then,

$$I_1 + I_2 + I_3 - I_4 = O. \tag{56}$$

Assume that $I_1 = 5$ amp., $I_2 = 8$ amp., and $I_4 = 17$ amp. Then,

$$5 + 8 + I_3 - 17 = O$$

and $I_3 = +4$ amp., the plus sign indicating that the current flows toward the junction.

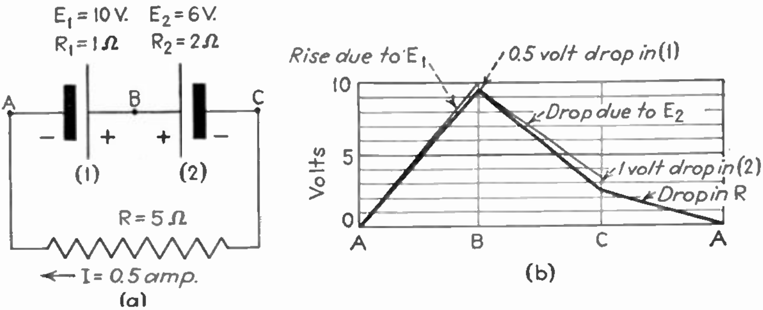


FIG. 41.—Variation of potential in circuit—Kirchhoff's second law.

The second law is merely another application of Ohm's law (Eq. (19), p. 36). The basis of the law is obvious; if one starts at a certain point in a circuit, and follows continuously around the paths of the circuit until the starting point is again reached, he must be at the same potential with which he started. Therefore the sources of e.m.f. encountered in this passage must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign.

This second law is illustrated by the following example:

Two batteries (Fig. 41), having e.m.fs. of 10 and 6 volts and internal resistances of 1 and 2 ohms, are connected in series opposing (their positive terminals connected together) and in series with an external resistance of 5 ohms. Determine the current and the voltage at each part of the circuit.

Since the two batteries act in opposition, the net e.m.f. of the two batteries is $10 - 6 = 4$ volts.

The current is

$$I = \frac{10 - 6}{1 + 2 + 5} = \frac{4}{8} = 0.5 \text{ amp.}$$

Consider the point A as being at zero or reference potential. In passing from A to B there is a 10-volt *rise* in potential due to the e.m.f. of battery 1, but around the circuit in the direction of current flow there occurs a simultaneous 0.5-volt drop of potential due to the current flowing through the 1-ohm resistance of battery 1. Therefore the net potential at B is but 9.5 volts greater than that at A, as is shown in Fig. 41(b). In passing from B to C there is a drop of 6 volts due to passing from the positive to the negative terminal of battery 2, and there is also a further drop of 1 volt due to the current of 0.5 amp. flowing through the 2-ohm resistance of battery 2. This makes the net potential at $C = 9.5 - 6 - 1 = +2.5$ volts. In passing from C to A there is a drop in potential of 2.5 volts due to the current of 0.5 amp. flowing through the 5-ohm resistance. When point A is reached the potential has dropped to zero.

Therefore the sum of all the e.m.fs. in the circuit, taken with their proper signs, is equal to the sum of the Ir drops. This is illustrated as follows:

Electromotive Forces	Ir Drops
Battery 1 = +10 volts.	Battery 1 = −0.5 × 1 = −0.5 volt
Battery 2 = − 6 volts.	Battery 2 = −0.5 × 2 = −1.0 volt
Total + 4 volts.	5-ohm res. = −0.5 × 5 = −2.5 volts
	Total −4.0 volts

$$+4 + (-4) = 0.$$

The current also may be found by applying Kirchhoff's second law directly. Starting at point A which is assumed to be at zero or reference potential,

$$+10 - I(1) - 6 - I(2) - I(5) = 0.$$
$$8I = +4; \quad I = +0.5 \text{ amp.}$$

56. Applications of Kirchhoff's Laws.—In the application of Kirchhoff's second law to specific problems, the question of algebraic signs may be troublesome and is a frequent source of error. If, however, the following rules are kept in mind, no difficulties should occur.

A rise in potential should be preceded by a + sign.

A drop in potential should be preceded by a − sign.

For example, in passing *through a battery* from the − terminal to the + terminal, the potential *rises*, so this voltage should be preceded by a + sign. On the other hand, when passing from the + terminal to the − terminal, the potential *drops*, so a − sign should precede this voltage. These voltages are all due to sources of e.m.f. *Hence, the sign preceding them is independent of the direction of current flow.*

When going through a resistance in the *same* direction as the current flow, the voltage drops in the same manner that the level of a stream of water decreases when one goes in the direction of stream flow. Hence, a voltage taken through a resistance in the direction of current flow, whether a battery resistance or an external resistance, should be preceded by a − sign. When going through a resistance in the direction *opposite* to the current flow, the voltage rises in the same manner that the level of a stream of water rises when one goes in the direction opposite to the stream flow. Hence, a voltage taken through a resistance in a direction opposed to that of the current flow, whether a battery resistance or an external resistance, should be preceded by a + sign.

It should be noted that the algebraic signs preceding the voltages across resistances depend *only on the direction of current flow and are independent of the polarity of any sources of e.m.f. in the circuit.*

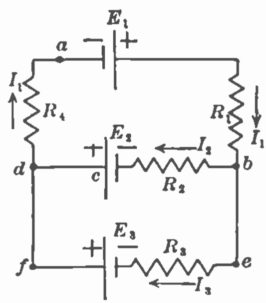


Fig. 42.—Application of Kirchhoff's laws.

This is further illustrated by the electric circuit shown in Fig. 42. Three batteries having e.m.fs. E_1, E_2, E_3 are connected as shown in different parts of the network of resistances R_1, R_2, R_3, R_4. The *assumed* directions for the various currents are indicated by the arrows. The battery resistances are assumed negligible as compared with the other circuit resistances.

Starting at the point *a*, and applying Kirchhoff's second law to the path *abcda*, an equation may be written

$$+E_1 - I_1R_1 - I_2R_2 + E_2 - I_1R_4 = 0 \tag{1}$$

Starting at f and passing along the path *febcdf*,

$$-E_3 + I_3R_3 - I_2R_2 + E_2 = 0 \tag{II}$$

This gives but two equations for the determination of three unknown currents. Three equations are necessary. A third equation may be obtained by applying Kirchhoff's second law to a third path *febadf*. This equation, however, when combined with either Eq. (I) or Eq. (II) would give either Eq. (II) or Eq. (I), showing that another condition must be placed on the network so that a solution may be found. This third condition must be obtained by applying Kirchhoff's first law to some junction as b.

$$+I_1 - I_2 - I_3 = 0,$$

since I_1 is assumed to flow *toward* the junction and I_2 and I_3 *away* from the junction.

With these three equations it is possible to determine the three currents.

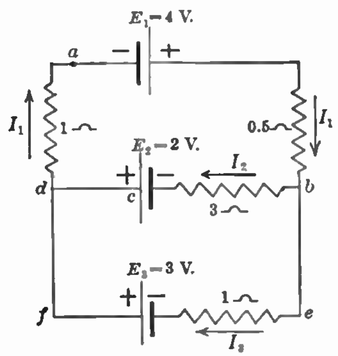


FIG. 43.—Application of Kirchhoff's laws.

In applying Kirchhoff's laws to any network, the following conditions are necessary for the solution:

Kirchhoff's first law must be applied to a sufficient number of junctions to include every unknown current at least once.

Kirchhoff's second law must be applied a sufficient number of times to include every branch in the network at least once.

Example.—Figure 43 shows a network identical with that shown in Fig. 42, except that numerical values are used. The battery resistances are assumed to be negligible compared with the circuit resistances.

Considering path *abcda*,

$$+4 - (I_1 0.5) - (I_2 3) + 2 - (I_1 1) = 0$$

or

$$1.5I_1 + 3I_2 = 6. \tag{I}$$

Similarly, for path *febcdf*, starting at f,

$$-3 + (I_3 1) - 3I_2 + 2 = 0$$

or

$$3I_2 - I_3 = -1. \qquad \text{(II)}$$

At the junction b,

$$+I_1 - I_2 - I_3 = 0$$

or

$$I_1 = I_2 + I_3. \qquad \text{(III)}$$

Substituting I_1, (Eq. (III)), in Eq. (I),

$$1.5(I_2 + I_3) + 3I_2 = 6,$$
$$4.5I_2 + 1.5I_3 = 6,$$

and combining with Eq. (II),

$$\begin{aligned} 9I_2 - 3I_3 &= -3. \quad \text{(II)} \\ 9I_2 + 3I_3 &= 12. \\ -6I_3 &= -15. \\ I_3 &= 2.5 \text{ amp.} \quad \textit{Ans.} \end{aligned}$$

Substituting this value in Eq. (II),

$$\begin{aligned} 3I_2 - 2.5 &= -1. \\ 3I_2 &= 1.5. \\ I_2 &= 0.5 \text{ amp.} \\ I_1 = I_2 + I_3 &= 3.0 \text{ amp.} \quad \text{(III)} \quad \textit{Ans.} \end{aligned}$$

As a check on the correctness of these solutions, Kirchhoff's second law may be applied to circuit *abefda*, which was not utilized in forming the network equations:

$$+4 - (3.0 \times 0.5) - (1 \times 2.5) + 3.0 - (1 \times 3.0) = 0 \ (\textit{check})$$

57. Procedure in Network Solutions.—The solution of networks by means of Kirchhoff's laws is greatly facilitated, and likelihood of error is minimized if a systematic procedure, as suggested below, is followed.

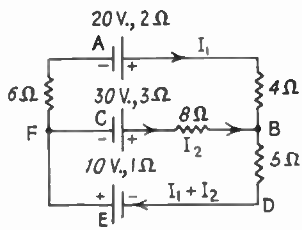


FIG. 44.—Procedure in network solution.

1. Make a clear diagram of the network. Letter the diagram, and mark on it every given quantity in the network.

2. Indicate the assumed direction of flow of current in each branch of the network. Then designate the currents in the different branches by I_1, I_2, I_3, etc. (The number of unknown currents may be reduced by applying Kirchhoff's first law to certain junctions, as is explained in Par. 59.)

3. The number of unknowns may be found from 2 and the same number of *independent* equations is necessary in order to determine these unknowns. Write the equations in accordance with Kirchhoff's first and second laws.

Each equation must contain some element which has not been considered in any previous equation. Verify the sign of each quantity.

4. Solve the equations algebraically for the unknowns, and verify the solutions by substituting in the equation of a circuit not already utilized.

Example.—In the circuit shown in Fig. 44, determine the direction and value of the currents in each of the batteries A, C, and E. The assumed directions of currents are indicated by the arrows. It will be noted that the current in battery E is the sum of I_1 and I_2 and is represented as such (see Par. 59). Since there are two unknowns, two independent equations are necessary.

In circuit $ABCFA$, starting at A,

$$+20 - 2I_1 - 4I_1 + 8I_2 - 30 + 3I_2 - 6I_1 = 0. \qquad \text{(I)}$$

In circuit $FCBDEF$, starting at F,

$$30 - 3I_2 - 8I_2 - 5(I_1 + I_2) + 10 - 1(I_1 + I_2) = 0. \qquad \text{(II)}$$

Simplifying,

$$-10 = 12I_1 - 11I_2. \qquad \text{(I)}$$
$$40 = 6I_1 + 17I_2. \qquad \text{(II)}$$

Solving,

$$I_1 = 1 \text{ amp.} = \text{current through } A. \quad \textit{Ans.}$$
$$I_2 = 2 \text{ amp.} = \text{current through } C. \quad \textit{Ans.}$$
$$I_1 + I_2 = 3 \text{ amp.} = \text{current through } E. \quad \textit{Ans.}$$

The assumed directions of current flow are correct.

In circuit $ABDEFA$, starting at A,

$$20 - 2I_1 - 4I_1 - 5(I_1 + I_2) + 10 - 1(I_1 + I_2) - 6I_1 = 0.$$
$$30 = 18I_1 + 6I_2. \qquad \text{(III)}$$

The substitution of the numerical values of I_1 and I_2 in Eq. (III) gives 30 and verifies the solutions of Eqs. (I) and (II).

58. Assumed Direction of Current.—In the application of Kirchhoff's laws to the solution of electric networks, the question of assuming the proper direction of current often arises. The current may be *assumed* to flow in either direction. If the assumed direction of the current is not the actual direction, this current will be found to have a minus sign when the equations are solved. However, when a direction has been assumed for a current, that direction must remain unchanged throughout the solution of the problem.

Example.—This is illustrated by assuming that the three currents of Par. 56 have such a direction that they all flow toward point d as shown in Fig. 45. This condition is of course impossible.

Considering circuit *abcda*, starting at *a*,

$$+4 + 0.5I_1 - 3I_2 + 2 + I_1 = 0.$$
$$1.5I_1 - 3I_2 + 6 = 0.$$

Similarly with circuit *febcdf*, starting at *f*,

$$-3 + I_3 - 3I_2 + 2 = 0.$$
$$I_3 - 3I_2 - 1 = 0.$$

The three currents I_1, I_2, I_3 all flow toward junction *d*, therefore,

$$I_1 + I_2 + I_3 = 0.$$

Substituting and solving,

$I_1 = -3$ amp. *Ans.*
$I_2 = 0.5$ amp. *Ans.*
$I_3 = 2.5$ amp. *Ans.*

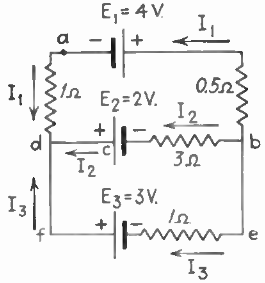


FIG. 45.—Assumed direction of current.

The minus sign occurring in the value of I_1 signifies that this current flows in the opposite direction to that assumed and is indicated by the arrow (Fig. 45). The positive values of I_2 and I_3 indicate that the assumed directions for these two currents are the actual directions of flow.

59. Further Applications of Kirchhoff's Laws.—It was stated in Par. 56 (p. 76) that Kirchhoff's first law must be applied to a sufficient number of junctions to include every current at least once. Also, Kirchhoff's second law must be applied to every branch of the network at least once. By applying these two rules, the number of independent equations can be made equal to the number of unknowns. These facts are illustrated in the examples of Par. 56 and 58.

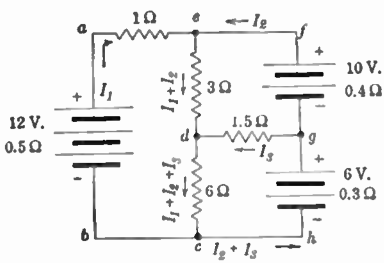


FIG. 46.—Electric network.

It is possible, however, to reduce the number of unknown currents, and hence the number of equations, by combining the currents directly on the diagram, in accordance with Kirchhoff's first law. This is in part illustrated by the example in Par. 57 but is more completely illustrated by the following example, in which the resistance of the batteries is no longer negligible.

Example.—Determine all the currents in the network of Fig. 46 and also the voltages between points *ed* and *dc*. There are six unknown currents, but by combining at the junctions *e* and *d*, as shown in the diagram, the number of unknown currents is reduced to three. Applying Kirchhoff's second law to path *abcdèa*,

$$-12 + 0.5I_1 + 6(I_1 + I_2 + I_3) + 3(I_1 + I_2) + (1)I_1 = 0$$

or

$$10.5I_1 + 9I_2 + 6I_3 = 12. \tag{I}$$

Path *efgde*,

$$-10 + 0.4I_2 - 1.5I_3 + 3(I_1 + I_2) = 0$$

or

$$3I_1 + 3.4I_2 - 1.5I_3 = 10. \tag{II}$$

Path *cdghc*,

$$+6(I_1 + I_2 + I_3) + 1.5I_3 - 6 + 0.3(I_2 + I_3) = 0$$
$$6I_1 + 6.3I_2 + 7.8I_3 = 6. \tag{III}$$

(I), (II), and (III) give three independent equations, and since there are but three unknowns, I_1, I_2, I_3, these can be determined by the simultaneous solutions of Eqs. (I), (II), and (III).

For example, if I_3 is eliminated between Eqs. (I) and (II) and then between Eqs. (II) and (III), there result

$$22.5I_1 + 22.6I_2 = 52$$

and

$$32.4I_1 + 35.97I_2 = 87,$$

from which

$$I_1 = -1.244 \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = +3.540 \text{ amp.} \quad \textit{Ans.}$$

By substituting these values in Eq. (II),

$$I_3 = -1.126 \text{ amp.} \quad \textit{Ans.}$$

Hence, I_1 and I_3 actually flow in directions opposite to those assumed.

The current from *e* to *d* = $I_1 + I_2$ = +2.296 amp. *Ans.*

The current from *d* to *c* = $I_1 + I_2 + I_3$ = +1.17 amp. *Ans.*

The current from *c* to *h* to *g* = $I_2 + I_3$ = +2.414 amp. *Ans.*

The voltage across *ed* = $+2.296 \times 3$ = 6.89 volts. *Ans.*

The voltage across *dc* = $+1.17 \times 6$ = 7.02 volts. *Ans.*

The voltage across *ce* = $12 - (-1.244)(1.5)$ = 13.9 volts (*check*).

As a further check on the correctness of the solutions (Par. 57), Kirchhoff's second law may be applied to circuit *abchgfea*, which was not utilized in deriving Eqs. (I), (II), and (III).

$$-12 + (-1.244)0.5 + 6 - (2.414)0.3 + 10 - (3.540)0.4 + (-1.244)1 = 0.$$
$$+16 - 16 = 0 \ (\textit{check}).$$

60. Applications of Kirchhoff's Laws to Railway Systems.—Kirchhoff's laws may be applied to problems involving electric railways, distribution systems, etc., where power is fed to the loads through different feeders and from different substations. In practice, however, it is not always possible to apply Kirchhoff's laws directly to electric railway systems, since the widely fluctuating loads which are constantly shifting their location make it difficult to formulate a definite problem. These laws may be applied in the computation of power and lighting systems. However, the size and arrangement of feeders in such systems are

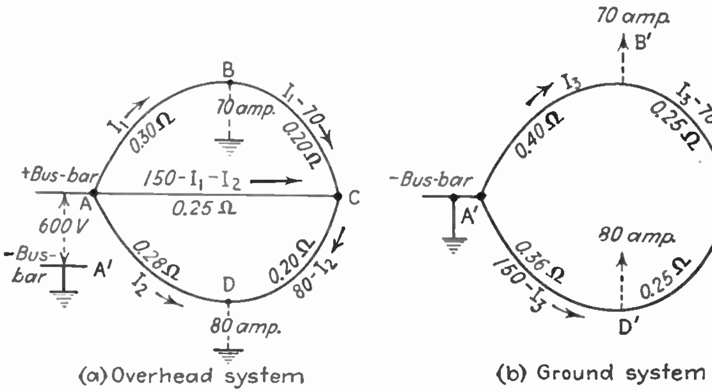


FIG. 47.—Railway system.

in considerable measure determined by operating considerations, such as the required voltage at different parts of the system. But Kirchhoff's laws underlie the voltage and current relations in such systems, irrespective of the method of computation which is used. Hence, the applications of these laws to representative systems give a better understanding of the factors which influence operation.

The following example illustrates the application of these laws to a simple railway system.

Example.—In Fig. 47 a simple railway system, with a ring-connected trolley and a single feeder, is shown at (*a*). For simplicity, the rail and ground-return circuit is shown separately at (*b*), *A′* corresponding to *A*, etc. (see Fig. 213*A*, p. 629). The station bus-bars at *A* are maintained at 600 volts above ground potential. The resistance of the overhead trolley is as follows: *A* to *B*, 0.30 ohm; *B* to *C*, 0.20 ohm; *C* to *D*, 0.20 ohm; *D* to *A*, 0.28

ohm. A feeder is connected from A to C, and its resistance is 0.25 ohm The resistance of rail and ground return is as follows: A' to B', 0.40 ohm; B' to C', 0.25 ohm; C' to D', 0.25 ohm; D' to A', 0.36 ohm. A trolley car at BB' takes 70 amp., and a trolley car at DD' takes 80 amp. Determine: (a) the current in each section of the trolley and in the feeder; (b) the voltage at each car and at the feeding point CC'.

The current flowing from A to B is designated as I_1 and that from A to D as I_2. Since the total current is 150 amp., the current in the feeder AC is equal to $150 - I_1 - I_2$. By applying the principle of Par. 59, the currents between points B and C, and C and D, may be expressed in terms of I_1 and I_2, as shown.

Applying Kirchhoff's second law to path $ABCA$,

$$(a)\quad -0.30I_1 - 0.20(I_1 - 70) + 0.25(150 - I_1 - I_2) = 0.$$
$$-0.75I_1 - 0.25I_2 + 51.5 = 0. \qquad \text{(I)}$$

Applying the law to path $ACDA$,

$$-0.25(150 - I_1 - I_2) - 0.20(80 - I_2) + 0.28I_2 = 0.$$
$$0.25I_1 + 0.73I_2 - 53.5 = 0. \qquad \text{(II)}$$

Multiplying Eq. (II) by 3 and adding it to Eq. (I),

$$0.75I_1 + 2.19I_2 - 160.5 = 0.$$
$$-0.75I_1 - 0.25I_2 + 51.5 = 0.$$
$$1.94I_2 = 109.$$
$$I_2 = 56.2 \text{ amp.} \quad \textit{Ans.}$$

Substituting the value of I_2 in Eq. (II),

$$0.25I_1 + 41.0 - 53.5 = 0.$$
$$I_1 = 50.0 \text{ amp.} \quad \textit{Ans.}$$

The current from B to C is

$$50.0 - 70.0 = -20.0, \text{ amp.,} \quad \textit{Ans.}$$

and actually flows from C to B.

The current from C to D is $80 - I_2 = 23.8$ amp. *Ans.*

The current in the feeder AC is $150 - 50.0 - 56.2 = 43.8$ amp. *Ans.* (These solutions may be verified by application of Kirchhoff's second law to path $ABCDA$.)

(b) The voltage at B (above that of the station negative bus-bar) is

$$600 - 50.0 \times 0.30 = 585 \text{ volts.}$$

Likewise, the voltage at C is

$$600 - 43.8 \times 0.25 = 588.5 \text{ volts,}$$

and that at D is

$$600 - 56.2 \times 0.28 = 584.2 \text{ volts.}$$

Before the voltage at the two cars and from point C to ground can be found, the voltage drops in the rail and ground must be determined. This may be done by applying Kirchhoff's second law to the ground circuit shown in (b), the path being $A'B'C'D'A'$,

$$-0.40I_3 - 0.50(I_3 - 70) + 0.36(150 - I_3) = 0.$$
$$1.26I_3 = 89; \quad I_3 = 70.7 \text{ amp.}$$

The current from A' to D' is

$$150 - 70.7 = 79.3 \text{ amp.}$$

The voltage drop from A' to B' is

$$70.7 \times 0.40 = 28.3 \text{ volts.}$$

The voltage drop from A' to D' is

$$79.3 \times 0.36 = 28.6 \text{ volts.}$$

The voltage drop from B' to C' and from D' to C' is about 0.2 volt and may be neglected.

Hence the voltage at car BB' is

$$585 - 28.3 = 556.7 \text{ volts.} \quad \textit{Ans.}$$

and the voltage at car DD' is

$$584.2 - 28.6 = 555.6 \text{ volts.} \quad \textit{Ans.}$$

The voltage *from C to ground* is practically 588.5 volts. *Ans.*

It will be noted that this type of problem is different from the preceding ones in that currents rather than resistances are given. Also it is to be noted that the division of currents in the overhead system depends only on the resistances of the overhead system and is independent of the ground system. Likewise, the division of currents in the ground system depends only on the resistances in the ground system.

61. Application of Kirchhoff's Laws to Power Systems.—The following example illustrates the application of Kirchhoff's laws to a 240-volt power system. This system is in general similar to the railway system of Par. 60, except that the outgoing and return systems are now symmetrical. This simplifies the solution since corresponding currents in the return and outgoing system are the same. For illustration, two methods of solution are given, the second being the simpler.

Example.—In Fig. 48, a 240-volt substation at A supplies two distributing centers B and C, by a ring system of feeders. Between A and B, a distance of 800 ft., two 1,000,000-cir.-mil. feeders are paralleled; between A and C, a distance of 1,200 ft., three 1,000,000-cir.-mil. feeders are paralleled; between

B and C, a distance of 600 ft., a 1,000,000-cir.-mil. tie line is connected. Determine the current in each feeder and the voltage at each distributing center, when the load at B is 2,000 amp. and that at C is 3,500 amp.

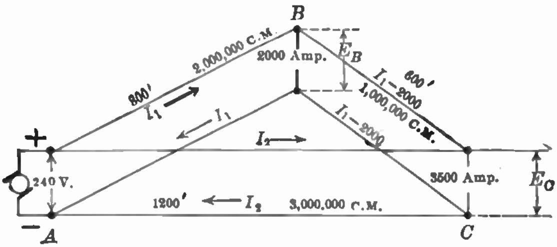


FIG. 48.—Ring-feeder system.

Assuming 10 ohms per circular-mil-foot:

$$\text{Resistance per wire } A \text{ to } B = \frac{800 \times 10}{2,000,000} = 0.004 \text{ ohm.}$$

$$\text{Resistance per wire } B \text{ to } C = \frac{600 \times 10}{1,000,000} = 0.006 \text{ ohm.}$$

$$\text{Resistance per wire } A \text{ to } C = \frac{1,200 \times 10}{3,000,000} = 0.004 \text{ ohm.}$$

Going from A to B to C, out on the positive and back on the negative conductor,

$$240 - I_1(0.004) - (I_1 - 2,000)0.006 - E_C - (I_1 - 2,000)0.006 - I_1(0.004) = 0.$$

$$240 - I_1(0.02) + 24 = E_C. \qquad \text{(I)}$$

Likewise, going directly from A to C,

$$240 - I_2(0.004) - E_C - I_2(0.004) = 0.$$

$$240 - I_2(0.008) = E_C. \qquad \text{(II)}$$

Equating Eqs. (I) and (II),

$$240 - I_1(0.02) + 24 = 240 - I_2(0.008).$$

$$0.02I_1 - 0.008I_2 = 24. \qquad \text{(III)}$$

At the junction C,

$$I_1 - 2,000 + I_2 = 3,500.$$

$$I_1 + I_2 = 5,500. \qquad \text{(IV)}$$

Substituting in Eq. (III) for $I_1 = 5,500 - I_2$,

$$0.02(5,500 - I_2) - 0.008I_2 = 24.$$

$$110 - 0.02I_2 - 0.008I_2 = 24.$$

$$0.028I_2 = 86.$$

$$I_2 = 3,070 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 = 2,430 \text{ amp.} \quad \textit{Ans.}$$

From Eq. (II),

$$E_C = 240 - 3{,}070(0.008) = 215.44 \text{ volts.} \quad Ans.$$
$$E_B = 240 - 2{,}430(0.008) = 220.56 \text{ volts.} \quad Ans.$$

The problem may be solved, however, without involving the voltages at A, B, or C. Taking a path from the + bus at A and going along the positive wires of the two feeders to B and C and then returning to A along the positive wire of the 3,000,000-cir.-mil feeder, the following equation is obtained:

$$-0.004I_1 - 0.006(I_1 - 2{,}000) + 0.004I_2 = 0$$

or

$$0.01I_1 - 0.004I_2 = 12,$$

which is equal to Eq. (III) divided by 2; and at C,

$$I_1 - 2{,}000 + I_2 = 3{,}500$$

or

$$I_1 + I_2 = 5{,}500 \text{ amp.},$$

which is the same as Eq. (IV).

In this example it is to be noted that, as in the railway type of problem, currents as well as e.m.fs. and resistances are included in the given data.

62. Equivalent Delta and Star (or Y) Systems.—The solution, by means of Kirchhoff's laws, of electric networks having a

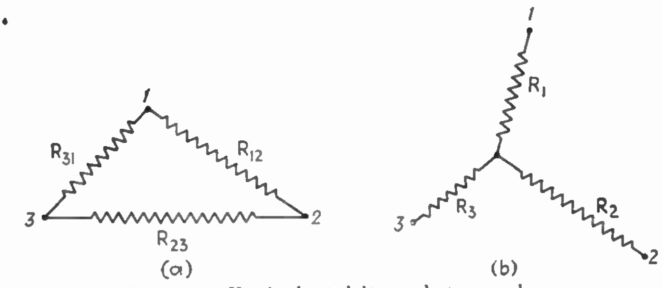


FIG. 49.—Equivalent delta and star meshes.

considerable number of branches may become much involved owing to the large number of simultaneous equations which must be solved. In many cases such networks may be reduced to very simple circuits by successively replacing delta meshes with star systems and *vice versa*. So far as the respective three terminals are concerned, any delta system of passive[1] resistances may be replaced by an equivalent three-terminal star or Y sys-

[1] A passive network or system is one in which there is no source of energy.

tem. In a similar manner, the three-terminal star system may be replaced by an equivalent delta system.

I. *Delta System Replaced by Star System.*—In Fig. 49(a) is shown a delta system consisting of three resistances, R_{12}, R_{23}, R_{31}, the three terminals being 1, 2, 3. In (b) is shown a star system consisting of three resistance members, R_1, R_2, R_3, which connect respectively to terminals 1, 2, 3. Let it be required to find the values of R_1, R_2, R_3 which will make the star system in (b) equivalent to the delta system in (a) so far as the terminals 1, 2, 3 are concerned.

In (b) the resistance between terminals 1 and 2 is $R_1 + R_2$. In (a) the resistance between terminals 1 and 2 consists of R_{12} in parallel with R_{31} and R_{23} in series. Hence,

$$R_1 + R_2 = \frac{R_{12}(R_{31} + R_{23})}{R_{12} + R_{23} + R_{31}}. \qquad \text{(See Eq. (9), p. 13).} \qquad \text{(I)}$$

In a similar manner, for terminals 2 and 3, and terminals 3 and 1,

$$R_2 + R_3 = \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad \text{(II)}$$

and

$$R_3 + R_1 = \frac{R_{31}(R_{23} + R_{12})}{R_{12} + R_{23} + R_{31}}. \qquad \text{(III)}$$

There are now three unknowns and three equations, (I), (II), and (III). Solving these equations simultaneously gives

$$R_1 = \frac{R_{12}R_{31}}{\Sigma R_n}, \qquad (57)$$

$$R_2 = \frac{R_{23}R_{12}}{\Sigma R_n}, \qquad (58)$$

$$R_3 = \frac{R_{31}R_{23}}{\Sigma R_n}, \qquad (59)$$

where

$$\Sigma R_n = R_{12} + R_{23} + R_{31}.$$

II. *Star System Replaced by Delta System.*—The conversion of the star system to the delta system is quite readily made by the use of Eqs. (57), (58), and (59). First let

$$\Sigma R_0 = R_1R_2 + R_2R_3 + R_3R_1. \qquad \text{(IV)}$$

Multiplying Eq. (57) by (58), Eq. (58) by (59), and Eq. (59) by (57), and then adding, gives

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2R_{23}R_{31} + R_{12}R_{23}^2R_{31} + R_{12}R_{23}R_{31}^2}{(R_{12} + R_{23} + R_{31})^2}. \tag{V}$$

From Eqs. (IV) and (V),

$$\Sigma R_0 = \frac{R_{12}R_{23}R_{31}(R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}.$$

$$R_{12} = \frac{\Sigma R_0(R_{12} + R_{23} + R_{31})}{R_{23}R_{31}}.$$

From Eq. (59),

$$R_{23}R_{31} = R_3(R_{12} + R_{23} + R_{31}).$$

Hence,

$$R_{12} = \frac{\Sigma R_0(R_{12} + R_{23} + R_{31})}{R_3(R_{12} + R_{23} + R_{31})} = \frac{\Sigma R_0}{R_3}. \tag{60}$$

In a similar manner,

$$R_{23} = \frac{\Sigma R_0}{R_1} \tag{61}$$

and

$$R_{31} = \frac{\Sigma R_0}{R_2}. \tag{62}$$

Example.—In Fig. 50(*a*) is shown a resistance network *AB*. The number near each element gives its resistance in ohms. Determine the equivalent resistance of the network between points *A* and *B*.

One method of solving the problem is to convert the delta meshes *AOC* and *aOB* into equivalent stars. These equivalent stars are shown in (*b*) and (*c*), Eqs. (57), (58), (59) being used for the conversion.

For example in (*b*),

$$AP = \frac{8 \times 9}{22} = 3.27; \quad OP = \frac{8 \times 5}{22} = 1.82; \quad CP = \frac{5 \times 9}{22} = 2.04.$$

In (*c*),

$$Oq = \frac{4 \times 6}{20} = 1.20; \quad aq = \frac{4 \times 10}{20} = 2.00; \quad Bq = \frac{10 \times 6}{20} = 3.00.$$

Substituting the stars in (*b*) and (*c*) for the corresponding deltas gives the network structure shown at (*d*). This again may be simplified by converting the delta *AqP* into a star as shown in (*e*),

$$AS = \frac{3.27 \times 14}{20.29} = 2.26; \quad qS = \frac{3.02 \times 14}{20.29} = 2.08; \quad PS = \frac{3.27 \times 3.02}{20.29} = 0.487.$$

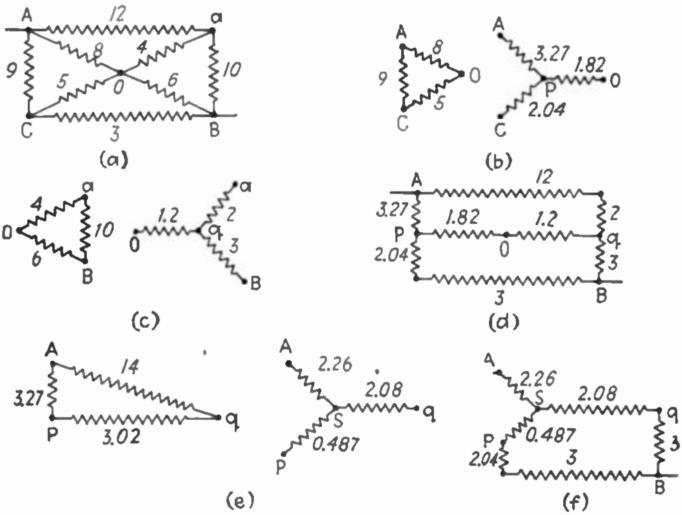


FIG. 50.—Simplification of complex network.

Substituting this star for its equivalent delta in (*d*) gives the structure shown in (*f*). This consists merely of a series-parallel circuit, the resistance of which is

$$R = 2.26 + \frac{(3.0 + 2.08)(0.487 + 2.04 + 3.0)}{3.0 + 2.08 + 0.487 + 2.04 + 3.0} = 2.26 + 2.65$$
$$= 4.91 \text{ ohms.} \quad \textit{Ans.}$$

CHAPTER IV

PRIMARY AND SECONDARY BATTERIES

63. Principle of the Electric Battery.—If two copper strips or plates be immersed in a dilute sulphuric acid solution (Fig. 51(*a*)) and be connected to the terminals of a voltmeter, no appreciable deflection of the voltmeter will be observed. This

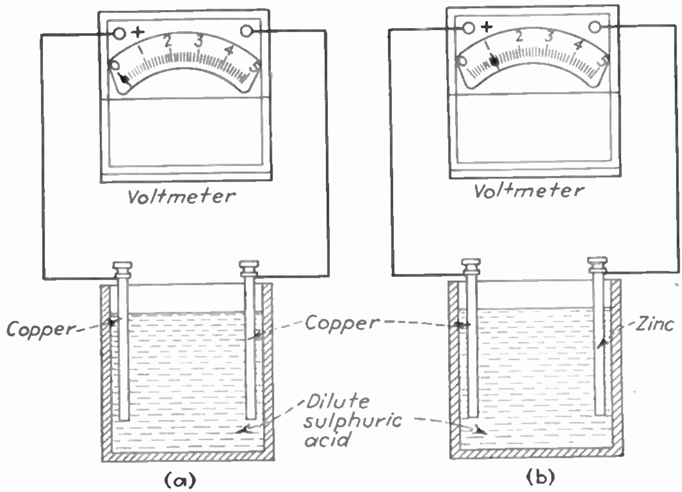


FIG. 51.—Simple primary cell.

shows that no appreciable difference of potential exists between the copper strips. If, however, one of the copper strips (Fig. 51(*b*)) be replaced by a zinc strip, the voltmeter needle will deflect and will indicate approximately 1 volt, showing that a potential difference between strips now exists. It will be necessary to connect the copper to the positive terminal and the zinc to the negative terminal of the voltmeter for the voltmeter to read up scale. This shows that so far as the *external* circuit is concerned, the copper is positive to the zinc.

The experiment may be repeated with various metals. For example, carbon or lead may be substituted for the copper, and a potential difference will be found to exist between each of these and the zinc, although it will not be of the same value as for the copper-zinc combination. Various metals may be substituted for the zinc, and potential differences will be found to exist between them and the copper.

Furthermore, it is not necessary that sulphuric acid be used for the solution. Other acids, such as hydrochloric acid, and chromic acid, may be substituted for the sulphuric acid; or even salt solutions as of common salt (sodium chloride), as of sal ammoniac (ammonium chloride), as of copper sulphate, or as of zinc sulphate may be used.

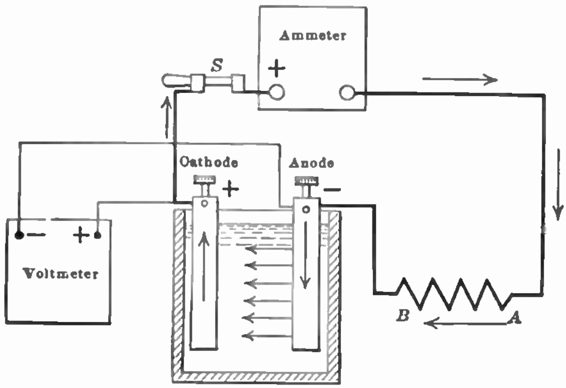


Fig. 52.—Current flow in a single cell.

In order to obtain a difference of potential between the two metal plates, only two conditions are necessary:

1. The plates must be of different metals.

2. They must be immersed in some electrolytic solution, such as an acid, an alkali, or a salt.

Again, if current be taken from the cell shown in Fig. 51(*b*) by connecting a resistance AB across its terminals (Fig. 52), current will flow out of the cell from the copper through the resistance AB and then into the cell at the zinc. Inside the cell, however, the current will flow *from the zinc* through the solution *to the copper* as shown in Fig. 52. Since current flows *from zinc to copper within the cell,* zinc is said to be electrochemically positive to copper. Therefore, when considering such an electrochemical cell, the copper is positive to the zinc when the external circuit is considered, but the zinc is electropositive to the copper when the plates and the solution alone are considered.

64. Definitions.—The metal strips or plates of a cell are called *electrodes.* The electrode at which current enters the solution (the zinc, Fig. 52) is the *anode,* and the electrode at which current leaves the solution (the copper, Fig. 52) is the *cathode.*

The solution used in a cell is called the *electrolyte.*

If current be taken from the cell the zinc plate will diminish in weight. This is true not only for this particular type of cell, but in practically all cells the flow of current is accompanied by a loss in weight of at least one of the plates. Energy is stored in the cell as *chemical energy,* and the electrical energy is delivered at the expense of the plate which goes into solution. That is, one plate is oxidized or else is converted into another chemical compound, this change being accompanied by a decrease in the available chemical energy of the system. Therefore *chemical energy* is converted into *electrical energy,* when the cell delivers current. Hence,

An electric cell or battery is a device for transforming chemical energy into electrical energy.

Such cells or batteries are divided into two classes: *primary cells* and *secondary cells.*

In a *primary cell* it is necessary from time to time to renew both the electrolyte and the electrode which goes into solution.

In a *secondary cell* the electrolyte and the electrodes, which undergo change during the process of supplying current, are restored by the electrochemical action of a current sent through the cell in the reverse direction.

65. Primary Cells.—Although it is stated in Par. 63 that there are many combinations of metals and solutions capable of generating an e.m.f. and so forming a cell, only a limited number of such combinations are commercially practicable. The general requirements of a good cell are as follows:

a. There must be no local action, that is, little or no wastage of the materials when the cell is not delivering current.

b. The e.m.f. must be of such magnitude as to enable the cell to deliver a reasonable amount of energy with a moderate current.

c. Frequent replacement of materials must not be necessary and such materials must not be expensive.

d. The internal resistance and the polarization effects must not be large, otherwise the battery cannot supply even moderate values of current for any substantial time.

As an illustration, the cell shown in Fig. 51(*b*) would not be practicable, because both copper and zinc would waste away, even were the battery delivering no current. Polarization (see Par. 67) would be substantial, and the battery would be capable of delivering only a comparatively small current.

66. Internal Resistance.—As is pointed out in Chap. III, every cell or battery has an internal resistance, which tends to reduce the magnitude of the current and also of the terminal voltage, when current is taken from the cell. Such resistance lies in the electrodes, in the contact surface between the electrodes and the electrolyte, and in the electrolyte itself. This resistance may be reduced by changing the dimensions of the cell in the same way as would be done for any electric conductor. The cross section of the path through which the current flows inside the cell should be made as large as is practicable. This means large area of electrodes in contact with the electrolyte. Also the transverse cross-section of the plates must be large enough to carry the current to the cell terminals without excessive drop in voltage. Little difficulty is experienced in making the voltage drop negligible in the plates themselves. It will be appreciated that larger electrodes mean a larger cell, with a greater current capacity. In addition to increasing the area of the electrodes in contact with the electrolyte, the resistance of the cell may also be diminished by decreasing the distance between the electrodes. This reduces the length of the path through which the current flows within the cell and correspondingly reduces the cell resistance.

Increasing the size of the cell does not increase its e.m.f. This e.m.f. depends solely on the material of the two electrodes and of the electrolyte. Thus, Fig. 53 shows two gravity cells, made up of the same materials but differing substantially in size. The cells are in opposition, that is, their positive terminals are joined and their negative terminals are joined. A galvanometer G connected in one of the leads reads zero, indicating that no current flows from the larger to the smaller cell. Hence their e.m.fs. must be equal.

67. Polarization.—If a test be made to determine the decrease of terminal voltage when current is taken from a cell, as, for example, a dry cell, by connecting voltmeter, ammeter, and external resistance as in Fig. 52, the results will be somewhat as follows:

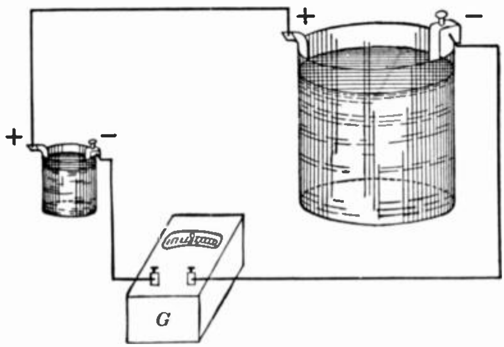


Fig. 53.—Equality of electromotive forces in cells of unequal sizes.

When the cell is on open circuit, the voltmeter will indicate the cell e.m.f. E (Fig. 54[1]). When the switch S is closed, current will flow and the terminal voltage will drop immediately by the amount AB. The distance AB represents approximately the voltage drop due to the internal resistance of the cell and this

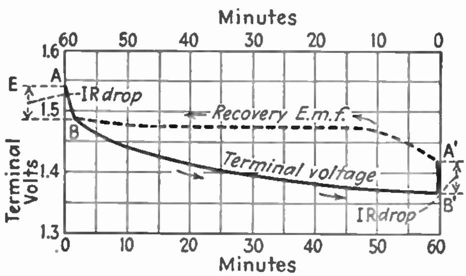


Fig. 54.—Discharge and recovery characteristics of a dry cell.

has just been considered. As time elapses, the terminal voltage will continue to drop, as shown by the portion BB' of the graph (Fig. 54), even though the external resistance be maintained

[1] "Standard Handbook for Electrical Engineers," 6th ed., Sec. 24, p. 2379, McGraw-Hill Book Company, Inc.

constant. This further drop of voltage is due almost entirely to *polarization*.

If at the time B' the external resistance is open-circuited, the terminal voltage almost immediately rises by the amount $B'A'$, this distance representing the internal-resistance drop in the cell. With further lapse of time, now represented from right to left, as shown by the top scale (Fig. 54), the e.m.f. of the cell gradually recovers, due to the escape of hydrogen bubbles at the cathode.

Polarization is due to the fact that when the cell delivers current, small bubbles of hydrogen form upon the positive plate or cathode, practically covering it and these bubbles have two effects.

1. They cause a substantial increase in the resistance at the contact surface between the cathode and the electrolyte.
2. Hydrogen acting in conjunction with the cathode or positive plate sets up an e.m.f. which opposes that of the cell.

These two effects explain the reduction in the current capacity of many types of cells after they have delivered current for some time.

Remedies for Polarization.—These hydrogen bubbles may be removed *mechanically* by brushing them off or by agitating the electrolyte. This is impracticable under commercial conditions. If the plate be roughened, the bubbles form at the projections and come to the surface more readily.

The more practicable method is to remove the hydrogen bubbles *chemically* by bringing oxidizing agents, such as chromic acid or manganese dioxide, into intimate contact with the cathode. The hydrogen readily combines with the oxygen of these compounds to form water (H_2O). This method is used in the bichromate cell, in the Le Clanché cell, and in dry cells.

68. Daniell Cell.—This cell (Fig. 55) is a two-fluid cell having copper and zinc as electrodes. It consists of a glass jar, inside of which is a porous cup containing zinc sulphate solution or a solution of zinc sulphate and sulphuric acid. The anode or negative electrode is immersed in this electrolyte. The porous cup is placed in a solution of copper sulphate with copper sulphate crystals in the bottom of the jar. The copper plate, or cathode, surrounds the porous cup. The porous cup keeps the two solutions separated. As the copper is in a copper sulphate solution, there is no polarization. This cell is designed for use in a circuit which is continuously closed. If left on open circuit, the electrodes waste away. When the cell is taken out of service for some time, the electrodes should be removed and the porous cup should be thoroughly washed. The e.m.f. of this cell is about 1.1 volts.

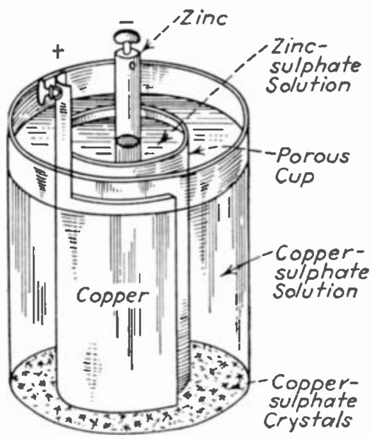


Fig. 55.—Daniell cell.

69. Gravity Cell.—The gravity cell is similar to the Daniell cell, except that gravity, rather than a porous cup, is depended upon to keep the two electrolytes separated. This cell is shown in Fig. 56. The cathode, which is of copper, is made of thin strips riveted together and placed in the bottom of the cell together with copper sulphate crystals. A solution of copper sulphate is then poured to within a few inches of the top of the jar. The connection to the copper is usually an insulated copper wire fastened to the copper and carried out through the solution to the top of the jar. There should always be copper sulphate crystals at the bottom of the cell.

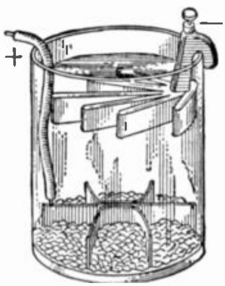


Fig. 56.—Gravity cell.

The anode is zinc, is usually rather massive, and is cast in the form of a crow's foot and hung on the top of the jar. This is surrounded by a zinc sulphate solution. The solutions are kept separated by gravity. The copper sulphate is the heavier of the two solutions and therefore tends to remain at the bottom. The solutions should be poured in carefully for, if the copper sulphate solution comes in contact with the zinc, copper will be deposited on the zinc. This copper should be removed, if by chance it becomes deposited in any way. In the operation of the cell the zinc goes into solution as zinc sulphate, and metallic copper comes out of the copper sulphate solution and is deposited upon the copper electrode. The cathode will therefore gain in weight, whereas the anode will lose in weight. This is the reason for having the zinc electrode massive and the copper electrode of very thin sheet copper when the cell is set up initially.

Due to capillary action, the zinc sulphate tends to creep up over the top of the jar forming a crystalline deposit. To prevent this creeping, the top of the jar should be paraffined. To prevent evaporation, the upper surface of the electrolyte may be covered with oil. When the cell is replenished, metallic zinc and copper sulphate crystals are supplied and metallic copper and zinc sulphate are removed.

The gravity cell is a *closed-circuit* battery, and the circuit should be kept closed for the best results. Otherwise the copper sulphate will gradually mix with the zinc sulphate. The cell has been found very useful in connection with railway signals, fire-alarm systems, and telephone exchanges, all closed-circuit work, although the storage battery has replaced it in many instances. The e.m.f. of the cell is practically that of the Daniell cell, being about 1.09 volts, and varies slightly with the concentration of the solutions.

70. Edison-Lalande Cell.[1]—The Edison-Lalande cell (Fig. 57) is still used to some extent. The cathode is of copper oxide and is suspended between two zinc plates which form the anode. All the plates are fastened to a porcelain cover by means of bolts

[1] Thomas A. Edison, 1847–1931. An outstanding American inventor. In his youth he sold newspapers on trains, and became acquainted with the telegraph through using it to send advance notice of important news. On one occasion he was ejected from the train because his improvised chemical laboratory set fire to a railroad car. He studied and repeated Faraday's experiments, became interested in telegraphy, and invented the stock ticker and in 1869 duplex telegraphy. He started a shop and made many improvements in telegraph instruments and stock tickers, and invented the carbon telephone transmitter. In 1877, he invented the phonograph. In 1879 he invented the incandescent lamp, his most important invention. He improved dynamos, and invented methods for the distribution of electrical energy, including the three-wire system which bears his name. In 1889 he invented motion pictures. He also invented the primary battery and the storage battery which bear his name. He was a tireless worker and extremely resourceful. He took out more than a thousand patents. His laboratories and factories were located at Menlo Park, West Orange, New Jersey.

which serve as binding posts as well as supports for the plates. The electrolyte is caustic soda (NaOH), 1 part by weight of soda to 3 of water. To prevent the soda being acted upon by the air, the electrolyte is covered with a layer of mineral oil. The copper oxide of the cathode gives up its oxygen readily to the hydrogen which forms on it, thus preventing any substantial polarization. These cells are capable of delivering a heavy current. The e.m.f. is about 0.95 volt, and when delivering normal current the terminal voltage drops to 0.75 volt. There is little or no local action in this cell and therefore it can be used to advantage on both open-circuit and closed-circuit work. Its chief disadvantage is its low e.m.f. Because of the convenience and the superiority of dry cells and of storage batteries, the Edison-Lalande cell is almost obsolete.

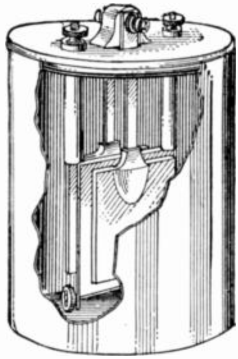


FIG. 57.—Edison-Lalande cell or Edison primary battery.

71. Le Clanché Cell.—The Le Clanché cell is perhaps the most familiar type of primary battery, because it forms the basis of the dry cell which has such a wide application. In fact the dry cell is merely a form of the Le Clanché cell in which the electrolyte is retained by an absorbent medium. The cathode is molded carbon and the anode is amalgamated zinc. The electrolyte is sal ammoniac (ammonium chloride). Because of the rapidity with which it polarizes, this type of cell is suited for open-circuit work only. The e.m.f. is from 1.4 to 1.5 volts, but because of the drop due to internal resistance and that due to polarization the *terminal voltage* may be considered as being 1 volt when the cell is in service. The most common method of reducing polarization is to bring manganese dioxide into intimate

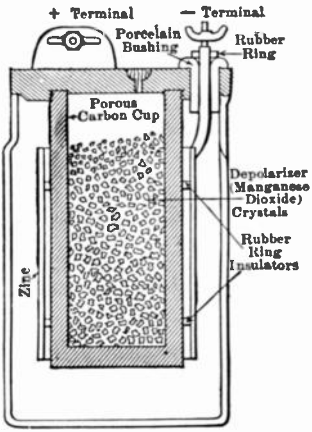


FIG. 58.—Porous-cup Le Clanché cell.

contact with the carbon. The manganese dioxide gives up oxygen readily, which unites with the hydrogen bubbles to form water.

In one type of Le Clanché cell, a pencil zinc is suspended in the center of a hollow cylinder of carbon and manganese dioxide. An improved type, the porous-cup cell, is shown in Fig. 58. In this form a carbon cup is filled with manganese dioxide, and the zinc, bent into cylindrical form, surrounds the carbon cup, being separated from it by rubber rings.

The solution should consist of 3 oz. of sal ammoniac to 1 pt. of water. A more concentrated solution produces zinc-chloride crystals on the zinc and carbon. To prevent the solution "creeping," the top of the cell is dipped in paraffin and the top of the carbon is covered with wax.

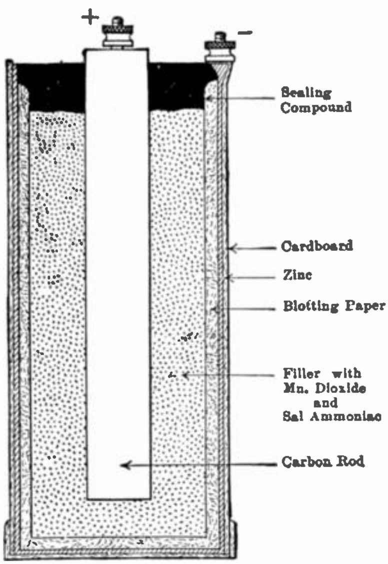


Fig. 59.—Sectional view—dry cell.

Because of its simplicity and the fact that it contains no injurious acids or alkalies, this type of cell was once widely used for intermittent work such as for door bells, telephone work, and open-circuit telegraph work. Because of its more convenient form, the dry cell has to a large extent replaced this type of cell.

72. Dry Cell.—The dry cell is in reality a Le Clanché cell with the electrolyte held by an absorbent material, and the entire cell is sealed to prevent evaporation. Since this type of cell is relatively very light, is portable, and convenient, it has practically replaced other types of primary cells. The name "dry cell" is really a misnomer, for no cell that is dry will deliver any appreciable current. In fact the chief cause of dry cells becoming exhausted is their actually becoming dry.

A cross-section of a typical dry cell is shown in Fig. 59. The anode is sheet zinc, in the form of a cylindrical cup, and acts as the container of the cell. The binding post is soldered to the top of the zinc. The zinc is lined with some non-conducting material such as blotting paper or plaster of paris. The cathode consists of a carbon rod, with the mixture of coke, carbon, etc., which surrounds this rod. The rod itself varies in shape among manufacturers. It is located axially in the zinc container, and the binding post is secured to the top of the rod. The depolarizing agent, powdered manganese dioxide, is mixed with finely crushed coke and pressed solidly into the container between the carbon rod and the non-conducting material which lines the zinc. It fills the cell to within about an inch of the top. Sal ammoniac, with a little zinc sulphate, is added and the cell is then sealed with wax or some tar compound. The outside of the zinc is frequently lacquered, and the cells are always set in close-fitting cardboard containers.

The e.m.f. of a dry cell is from 1.5 to 1.6 volts when new, but this drops to about 1.4 volts with time, even though the cell remains idle. An e.m.f. of much less than 1.5 volts usually indicates that deterioration has begun. A cell is practically useless after 12 to 18 months, even if not used meanwhile. The internal resistance of the cell is about 0.1 ohm when new and increases to several times this value with time. The polarization effect is large as compared with the internal resistance, so that a low value of internal resistance is not important except as an indication of the condition of the cell. A method for testing the condition of a cell is to short-circuit it through an ammeter, when it should deliver an instantaneous value of 1.5/0.1, or 15 amp., if in good condition. When new, the current under these conditions may reach even 25 amp. When delivering appreciable current, the terminal voltage is very nearly 1 volt.

One of the chief causes of a cell's becoming useless is the using up of the zinc as a result of electrochemical reactions within the cell. This allows the solution to leak out and to dry up, and the cell then becomes worthless. The life of a cell may be prolonged temporarily by introducing fresh solution, but the results are usually far from satisfactory.

As is well known, dry cells have many applications. Their field is limited to supplying moderate currents intermittently, but they are capable of supplying continuously very small currents of the magnitude of 0.1 amp. They are used extensively for door bells, electric bells, buzzers, telephones, telegraph instruments, gas-engine ignition, flash lamps, and for many other purposes. They are also used as *A*, *B*, and *C* batteries in direct-current radio receiving sets.

73. Weston Standard Cell.—It is essential in practical work to be able to reproduce accurately standards of current, voltage, and resistance. Obviously if two of these quantities are known, the third is readily obtainable by Ohm's law. It is a matter of no great difficulty to make and to reproduce resistance standards, as such standards are nothing more than metals in strips and in other forms, carefully mounted and calibrated. Such standards are practically permanent, so that their resistance remains constant indefinitely.

A standard of current or of voltage is much more difficult to reproduce and to maintain than a standard of resistance. It has been found more practicable to produce and maintain a voltage standard rather than a current standard. This voltage standard is the e.m.f. of a *standard cell.* The e.m.f. of a cell depends on its materials and their impurities, the concentration of the electrolyte, the temperature, the polarization effects, etc. It is difficult, therefore, to select such materials for a cell as will enable it to be reproduced at different times and at various places with a high degree of accuracy. The Clark cell was the first of the standard cells to prove commercially successful. The cathode was mercury, the anode zinc, and the electrolytes were mercurous sulphate and zinc sulphate. The objections to this cell were that the e.m.f. changed appreciably with the temperature and that this change lagged the change in temperature.

In the Weston cell, cadmium is substituted for the zinc of the Clark cell. An unmounted Weston cell is shown in Fig. 60. The cathode is mercury, located at the bottom of one leg of an H-tube. Above this is mercurous-sulphate paste. These materials are held in position by means of a porcelain tube, expanded at the bottom and packed with cotton. This tube extends to the top of the cell and acts as a vent for any gases that are formed. In the bottom of the other leg of the H-tube is the anode, of cadmium amalgam. This is held in place by another porcelain tube packed with cotton. The electrolyte is cadmium sulphate. The leads from the cathode and the anode are sealed into the tubes at the bottom. The top of the cell is sealed with cork, paraffin, and wax. The entire cell is mounted in a metal case with binding posts at the top.

The cell is made in two forms, the *normal cell* and the unsaturated or *secondary cell.* In the normal cell, cadmium sulphate crystals are left in the solution so that it is always saturated. Its e.m.f. is affected slightly by temperature, but corrections can be accurately made. It is possible to reproduce such cells with e.m.fs. differing among themselves by only a few parts in 100,000.

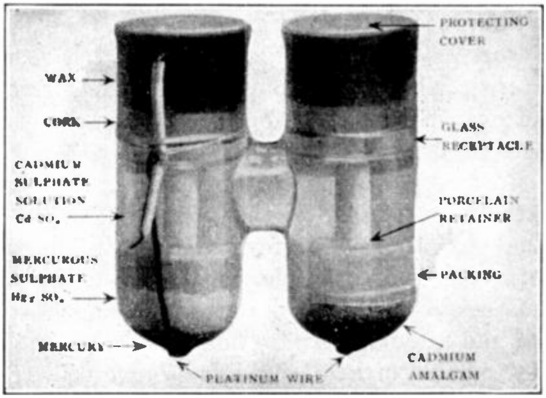


FIG. 60.—Unmounted Weston unsaturated cell.

In the unsaturated cell, the solution is saturated at 4°C. and as no crystals are left in the solution, its concentration is substantially constant at other temperatures. Such cells have practically no temperature coefficient. They are not so accurately reproducible as is the normal cell. A certificate should accompany each one giving its e.m.f., which usually is about 1.0186 volts. The unsaturated type of cell rather than the normal cell is used almost entirely in practical work.

The terminal voltage of any cell differs from its e.m.f. by the *IR* drop due to the cell resistance. As the resistance of a Weston cell is about 200 ohms, it is evident that, if any appreciable current be taken from the cell, its terminal voltage will be quite different from its e.m.f. The cell must be used, therefore, in such a manner that it delivers no appreciable current. By means of the so-called *Poggendorff method*, described in Par. 125 (p. 181), the cell is used without delivering current. Not more than 0.0001 amp. should be taken from the cell at any time. If appreciable current is taken, the e.m.f. drops, but when the circuit is again opened the e.m.f. slowly recovers its initial value.

STORAGE BATTERIES

74. Storage Batteries.—A storage or secondary cell (sometimes called an *accumulator*) involves the same principles as a primary cell, but the two differ from each other in the manner in which they are renewed. The materials of a primary cell which are used up in the process of delivering current are replaced by new materials, whereas, in the storage cell, the cell materials are restored to their initial condition by sending a current through the cell in a reverse direction. For this reason the electrochemical products resulting from discharge must remain within the cell. Therefore, if a cell in its operation gives off material, usually in the form of gas, so that it cannot be brought back to its original condition with a reverse current, it is not suitable for a storage cell. For example, the Le Clanché cell gives off free ammonia gas and therefore cannot be used as a storage cell. The Daniell and gravity cells are both reversible and hence are theoretically capable of being used as storage cells; but, as the active materials go into solution and do not all return during the reverse cycle, the life of such a cell would be short. There are but two types of storage cells in common use, the *lead-lead-acid* type and the *nickel-iron-alkali* type. In both of these cells the active materials do not leave the electrodes.

75. Lead-lead-acid Cell.—The principle underlying the lead cell may be illustrated by the following simple experiment. Two plain lead strips (Fig. 61) are immersed in a glass of dilute sulphuric acid (sp. gr. = 1.200 approximately). These are connected in series with an incandescent lamp or equivalent resistance and are supplied from 115-volt direct-current mains, or from a battery. When current flows through this cell, bubbles of gas will be given off from each plate, but it will be found that a much greater number come from one plate than from the other. After a short time, one plate will be observed to have changed to a dark chocolate color, and the other apparently will not have changed its appearance. A careful examination, however, will show that the metallic lead at the surface of the latter plate has started to change from solid metallic lead to spongy lead.

When the current is flowing into the cell, as shown in Fig. 61, the cell is being charged and the voltmeter connected across the cell indicates about 2.5 volts. If the current be interrupted, as

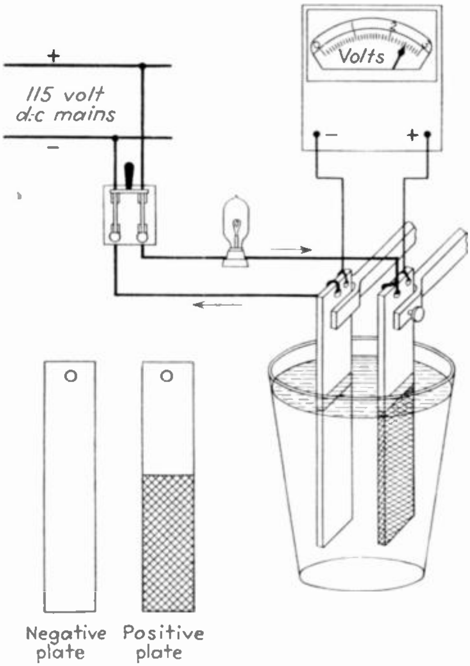


FIG. 61.—Forming the plates of an elementary lead storage cell.

by pulling the switch, the voltmeter reading will fall to about 2.05 volts, and the cell will now be found capable of delivering a small current. This current is of sufficient magnitude to operate a small buzzer for a very short period, but the amount of energy that such a cell can deliver is very limited; even the small current taken by the voltmeter is sufficient to exhaust the cell in a very short time. As the cell discharges, the voltage drops off slowly to about 1.75 volts, after which it drops more rapidly until it becomes zero and the cell is apparently exhausted. The color of the dark-brown plate will now have become lighter and will more nearly resemble its initial lead color. After a short rest the cell will recover slightly and will again deliver current for a very brief time.

The plate which is a dark chocolate color in the above experiment is the positive plate or cathode, and the one which is partially converted to spongy lead is the negative plate or anode. The bubbles which are noted come mostly from the negative plate and are free hydrogen gas. Some bubbles, however, come from the positive plate and these are oxygen. When the current is made to flow into such a cell, the metallic lead of the positive plate is converted into lead peroxide, whereas the negative plate is not changed chemically but is converted from solid lead into the spongy form of lead, which is softer and more porous than ordinary metallic lead. When the cell is discharged, the lead peroxide of the positive plate is changed to lead sulphate, and the spongy lead of the negative plate also is changed to lead sulphate so that both plates tend to become electrochemically equivalent.

The electrolytic principle of the cell is the same as that of a primary cell. When the two lead plates are the same electrochemically, that is when both are lead sulphate, no potential difference exists between them. When the positive plate is converted to the peroxide and the negative plate to spongy lead by the action of the current during charge, the two plates become dissimilar and an e.m.f. now exists between them. This e.m.f. is about 2.05 volts, the difference of 0.45 volt between this and the 2.50 volts at the terminals during charge being necessary to overcome the internal resistance and polarization effects. This simple experiment illustrates the principle underlying the operation of lead storage cells.

The chemical reactions which take place in a lead storage cell are as follows:

Battery Discharged					Battery Charged				
(+ plate)		(− plate)			(+ plate)		(− plate)		
$PbSO_4$	+	$PbSO_4$	$+ 2H_2O$	$\leftrightarrows$	PbO_2	+	Pb	+	$2H_2SO_4$
Lead sulphate	+	lead sulphate	+ water	is changed to	Lead peroxide	+	lead	+	sulphuric acid

When read from left to right, the equation shows the reactions which occur in the battery on charge. When read from right to left, it shows the reactions which occur on discharge.

It will be noted that when the battery is being charged the only change that takes place in the electrolyte is that water is converted into sulphuric acid. This accounts for the rise of specific gravity on charge. On discharge, the sulphuric acid is dissociated and reacts with the lead peroxide to form water. Therefore, on discharge, the specific gravity of the electrolyte decreases. When charging, free hydrogen is given off at the negative plate and oxygen at the positive plate. Because of the explosive nature of hydrogen, *no flame should be allowed to come in proximity to a storage battery while the battery is charging.*

76. Planté Plates.—It would not be practicable to construct storage cells of plain lead sheets as used in the foregoing experiment. The current capacity of the cell would be so small that the cell could not deliver current of commercial value for any length of time unless the cell were made prohibitively large in order to secure the necessary plate area.

If the charging of the elementary cell (Fig. 61) were carried further, the dark lead peroxide of the positive plate would fall off in flakes and drop to the bottom of the glass. Therefore, in a commercial cell, provision must be made to minimize this flaking of the active material.

Early in the art, it was recognized that, in order to make the storage cell commercial, a large plate area must be exposed to the action of the acid and a large amount of the lead must be converted into the peroxide and so become active material. There are two methods of obtaining the result, the Planté process and the Faure process. In the Planté process the active material on the plates is formed from the metallic lead by causing a current to flow through the cell, first in one direction and then in the reverse direction, which procedure works the lead on the surface of the plates into active material. This process is slow but may be accelerated by adding certain acids to the sulphuric acid during the forming process. The Gould plate shown in Fig. 62 is made by this process. The plate is first passed under revolving steel wheels which convert its surface into ridges and furrows, increasing the surface area of the plate. As this process weakens the plate mechanically, certain portions of it are not acted upon by the wheels. These portions act as ribs which give support and mechanical strength to the plate and tend to prevent buckling.

The active material is then formed electrolytically by the Planté process. The negative plate is made from the positive plate by reducing the peroxide to spongy lead by an electric current, the plate being made the cathode.

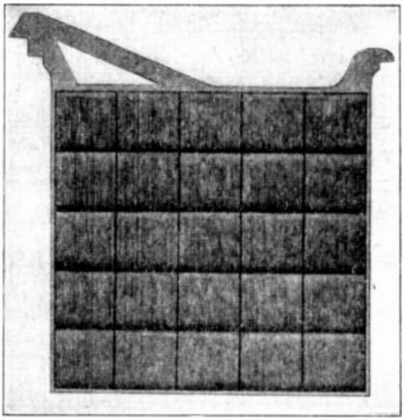


FIG. 62.—Gould ploughed plate—Planté process.

Another type of Planté plate, the Exide-Manchester type, is shown in Fig. 63. A grid made of lead and antimony is perforated. The active material consists of a corrugated lead ribbon, which is coiled into spirals and pressed into the perforations of the grid. The peroxide has a greater volume than the lead from which it is derived. Therefore, when the cell is charged, these spirals expand and become more firmly embedded in the

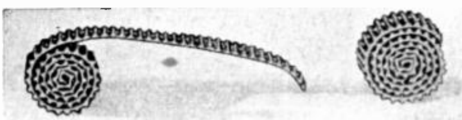


FIG. 63.—Planté (Manchester) button and positive group.

plate. The grid itself is not acted upon to any great extent but serves as a mechanical support. The advantage of this type of plate is its rigidity and mechanical strength. Since the grid acts merely as the support for the lead buttons, the plate can be used until all the lead is converted into active material, without having the plate as a whole disintegrate.

Planté positive plates should be designed to give from 1,800 to 2,400 complete cycles of charge and discharge. With Planté plates, the charging and discharging during the use of the battery convert more and more of the plate into active material. Hence, the positive plates must gradually shed some of the active material to make room for this new active material. Therefore, sufficient space must be allowed between the bottom of the jars and the bottom of the plates to prevent the accumulation of lead peroxide from short-circuiting the plates.

Planté negative plates should give between 2,500 and 3,000 complete cycles of charge and discharge before their capacity falls to 80 per cent of its initial value. They fail from a loss of capacity due to the lead losing its spongy form rather than from mechanical disintegration of the plate. Only one or two companies use Planté negative plates, the Faure or pasted plate being almost always used, even with Planté positives.

77. Faure or Pasted Plate.—This type of plate consists of a lead-antimony lattice work or grid into which lead oxide is applied in the form of a paste. The battery is then charged. The paste on the positive grid is converted into peroxide and on the negative grid into spongy lead. The grid of a pasted-type positive plate is shown in Fig. 64(*a*). In Fig. 64(*b*) is shown a part-microscopic view of a grid partially filled with active material.

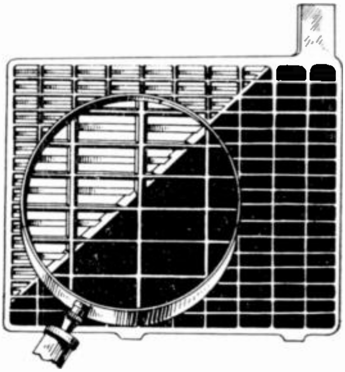
The chief advantage of the pasted plate is its high energy capacity, especially for short periods, together with its lesser size, cost, and weight for a given discharge rate. It is therefore very useful where lightness and compactness are necessary, such as in electrical-vehicle batteries, and in ignition and starting batteries for automobiles.

Flat-pasted positive plates also lose active material by shedding or by the erosive action of the bubbles. A pasted plate should give from 300 to 400 complete cycles of charge and discharge before its capacity falls to 80 per cent of its initial value. Pasted positive plates gain in capacity during the early part of their life, increasing in capacity to about 120 per cent of the initial value. This extra capacity is available during the greater part of their life, since new active material is available as rapidly as it is shed. However, as soon as all the reserve active material has been utilized, further erosion results in a considerable loss in capacity. The erosion and the rate at which the plates lose capacity then become so rapid that the useful life of the plate is practically ended.

At high rates of discharge, pasted plates have a much greater ampere capacity per unit area than Planté plates. On the other hand, the Planté plate is more rugged and has a much longer life.

In all batteries there is one more negative than positive plate. This allows all the positives to be worked on *both* sides. Were any of the positives to be worked on one side only, the expansion of the active material,

(*a*) Positive grid.



(b) Grid partially filled with active material—positive or negative.

Fig. 64.—Faure or pasted plates. *Electric Storage Battery Company.*

which occurs when it is converted to the peroxide on charge, would be unequal on the two sides of the plate and buckling would result.

78. Exide-Ironclad Cell.—For the propulsion of vehicles of various kinds and for many purposes where it is desirable to combine the characteristics of the pasted plate with much of the ruggedness and long life of the Planté plate, the Exide-Ironclad cell is the type of lead storage cell most generally used. Its positive plate consists of a lead-antimony frame which supports a number of slotted hard-rubber tubes. An irregular-shaped lead-antimony core passes through the center of each tube and serves as a collecting device for the current. The peroxide or active material is pressed into the tubes, filling the space between the irregular core and the inner wall of the tube. The perforations are so small that the peroxide does not drop out readily and erosion of the positive plate is practically eliminated. An ordinary pasted plate, somewhat thickened, is used for the negative plate of this cell. Although slightly more expensive, this type of cell has a life of from two to three times that of the usual flat-pasted-plate cell. Also this type of battery can stand considerable rough usage. A view of an Exide-Ironclad cell, cut away to show the assembly, is given in Fig. 65.

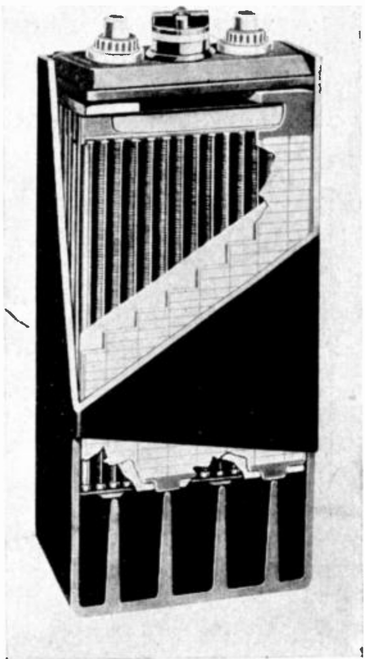


FIG. 65.—Cutaway of an Exide-Ironclad cell. (*Electric Storage Battery Company.*)

Storage batteries are divided into two general classes, *stationary batteries* and *portable batteries.*

79. Stationary Batteries.—The plates of this type of battery may be either of the Planté type or of the pasted type, depending on the nature of the service. For continuous "cycling" or regulating duty at a moderate though continual rate of charging and discharging, the Planté plate is preferable. Where a battery is installed for emergency service, to carry an extreme overload for a very short period during a temporary shut-down of the generating apparatus, the Faure or pasted plate is preferable. For a given floor area, a pasted-plate battery can discharge at the 1-hr. rate twice the current that a Planté plate can, and at higher ampere rates this ratio becomes even greater. This is a very important factor in thickly settled city districts where such batteries are usually located and where floor area is very valuable.

80. Tanks.—The containing tanks for stationary batteries are usually of two types: glass, and lead-lined wooden tanks. Glass jars are used for cells up to 600 amp.-hr. capacity with Planté-type plates and 1,000 amp.-hr. capacity with Faure-type plates. In the largest sizes they would be expensive and would not have the requisite mechanical strength.

When glass jars are used, the cells may be of either the open or sealed type (see Figs. 72 and 73, pp. 116 and 117). In the open type, the plates are suspended by projecting lugs which rest on the edges of the jar (Fig. 72). In the sealed type, the plates are usually hung by the connecting strap and post from the cover, which is sealed with compound to the top edge of the jar (Fig. 73).

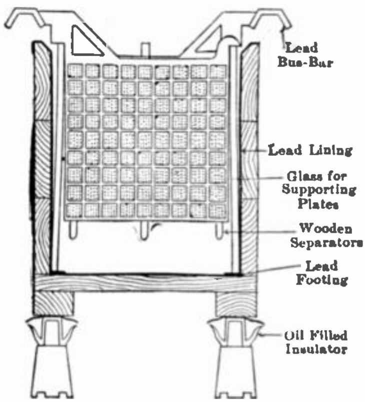


FIG. 66.—Lead-lined wooden tank storage cell.

The wooden tanks must be strong and well made (Fig. 66). They are lined with sheet lead. The seams of the lead lining must be sealed by "lead burning" with a non-oxidizing flame. Solder should never be used. The wood should be painted with an acid-resisting paint, such as asphaltum. An occasional application of linseed oil will prevent decomposition of the wood due to the acid.

In the lead-lined tanks, the plates are also suspended by projecting lugs resting on two glass slabs, 3/8 in. thick, which in turn rest on the bottom of the tank (see Fig. 66).

The plates of like polarity are burned to a heavy lead strip or bus-bar to which the current-carrying conductor is either burned or bolted. There should always be a liberal space between the plates and the bottom of the tank to allow the material shed from the plates to accumulate without short-circuiting the plates. All open types of stationary cells should have a glass cover to reduce evaporation and to intercept the fine acid spray which occurs during the charging periods.

81. Separators.—Except for some special assemblies where wide spacing between plates is provided, all battery cells, whether sealed or open, are assembled with some form of mechanical separation held in place between the plate surfaces. To prevent contact of plates of opposite polarity, the separation must be of insulating material and is preferably a continuous sheet or diaphragm.

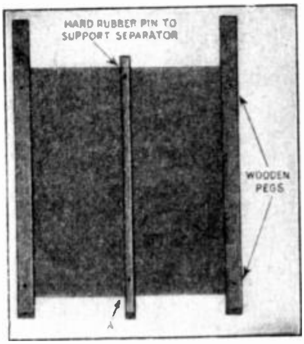


Fig. 67.—Assembly of a wooden separator.

For over 30 years, the standard separation for batteries has been wood sheets with vertical channels provided either through grooving the wood or by using thin sheets with wood or hard-rubber dowels for support and to give the desired distance between adjacent plates (Fig. 67).

The wood for this purpose must be treated to remove ingredients that would be detrimental to the electrolyte and if the wood is not allowed to dry out, it will be serviceable for many

Fig. 68.—Exide-Mipor separator. Because of the porosity of the Exide-Mipor separator, the application of a liquid upon one surface is quickly apparent upon the opposite surface. By means of a mirror this characteristic of the new separator is clearly shown.

years in battery electrolyte of standard gravities and at normal temperatures.

Hard-rubber dowels are preferred because of their greater mechanical strength and because they are unaffected by contact with the oxygen liberated by the positive plate on charge. Perforated hard-rubber sheets are also used in some assemblies between a grooved-wood separator and the positive plate in order to protect the wood and to ensure maximum service life. Such sheets serve an additional purpose of preventing in some degree the erosion of the positive active material.

Depending on the degree of perforation, the hard-rubber sheets interpose resistance to the flow of current between the plates of the cell, and some of the effectiveness of the battery at high rates of discharge is sacrificed for greater life.

A recent development by the Electric Storage Battery Company, a rubber separator having microscopic pores—hence the trade name "Mipor" (Fig. 68)— is made as sheets or is grooved in the same manner as the wood separators. This separator has all the advantages of the wood separator and does not add any undue restriction to the cell action. It is not affected either by heat or by the electrolyte and is of ample mechanical strength.

82. Electrolyte.—The sulphuric acid of the electrolyte should be chemically pure. When fully charged, the specific gravity should be 1.210 for Planté plates and not higher than 1.300 for pasted plates. The solution may be made from concentrated acid (oil of vitriol, sp. gr. 1.84) by *pouring the acid into water* in the following ratios:

PARTS WATER TO 1 PART ACID

Specific gravity	Volume	Weight
1.200	4.3	2.4
1.210	4.0	2.2
1.240	3.4	1.9
1.280	2.75	1.5

Considerable heat is evolved when acid and water are mixed resulting in a large amount of steam being generated if the water is added to the acid. This should be avoided as it may scatter the acid, break the container, and cause personal injury.

The specific gravity of a solution may be determined directly by the use of a hydrometer, consisting of a weighted bulb and a graduated tube which floats in the liquid as shown in Fig. 69. The bulb floats in the liquid whose specific gravity is to be measured, and the specific gravity is read at the point where the surface of the liquid intercepts the tube. In stationary batteries of the open type such a tube may be left floating permanently in a representative cell called a *pilot cell* (Fig. 69).

The small amount of liquid and the design of vehicle and starting batteries as well as stationary batteries of the sealed type

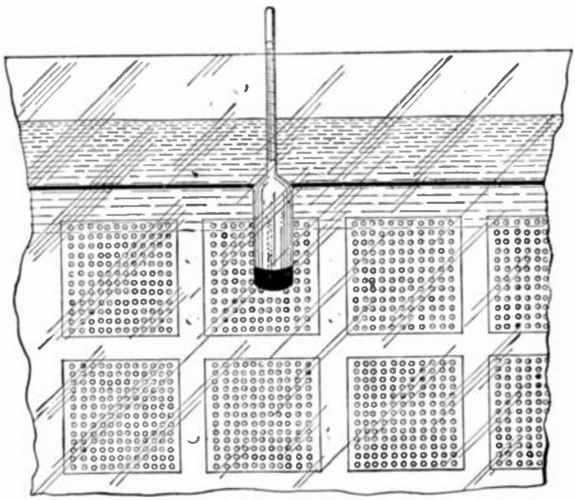


FIG. 69.—Measurement of specific gravity in a stationary battery.

make the use of such a hydrometer impossible in such batteries. To determine the specific gravity with such batteries, the syringe hydrometer is used (Fig. 70). The syringe contains a small hydrometer, and, when sufficient liquid is drawn into the syringe tube, the small hydrometer floats and may be read directly. The upper portion of the hydrometer for readings less than 1.225 is shown dark and is marked "unsafe." A gravity reading in this portion shows that a pasted-plate battery, particularly of the starting type, is more than half discharged and indicates approaching failure either through lack of sufficient charging or because of deterioration in the battery.

Figure 71 shows the change in specific gravity during charge and discharge at constant normal-current rate. This relation is very important, as the specific gravity of the electrolyte is an accurate indication of the condition of charge of the battery.

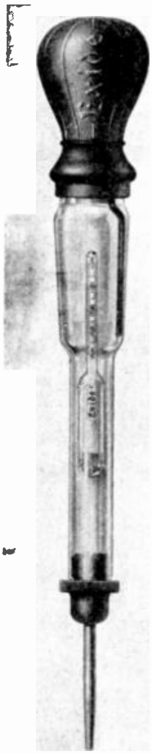


FIG. 70.—Syringe hydrometer.

83. Specific Gravity.—When the battery is charged, oxygen (O_2) is given to the positive plate to convert it into the peroxide, and sulphate ions (SO_4) are given off at the negative plate, leaving spongy lead. The oxygen given to the positive plate is obtained from the oxygen ion of the water, leaving the hydrogen ion of the water to combine with the SO_4 ion given up by the negative plate, to form H_2SO_4, or sulphuric acid (see chemical equation, p. 104). Hence the electrolyte gives up water and sulphuric acid is formed, which means that during charge the solution becomes more concentrated. For example, the specific gravity will rise from the complete discharge value of 1.160 to a value of 1.210 when the battery becomes fully charged, as shown in Fig. 71. Point *a* is called the gassing point because it is the point at which all the hydrogen or oxygen or both are not absorbed in the chemical reactions and are given off rapidly as gas. At this point the specific gravity drops off slightly due to the presence of the gas bubbles in the electrolyte. After the charging has ceased, the specific gravity continues to rise for some time. This is due to the very concentrated acid in the pores of the active material diffusing out into the solution and also to the fact that the gas bubbles are escaping from the solution. The discharge curve shown in Fig. 71 is similar to the charge curve. The specific gravity continues to decrease even after the battery has ceased to deliver current. This is due to the dilute acid in the pores of the active material diffusing into the solution. The specific gravity is such a good indicator of the state of charge of the battery that the hydrometer reading is generally used to determine how nearly charged or discharged the battery is.

As the hydrogen and oxygen which escape from the battery during the charging and discharging periods are merely dissociated water, the battery loses nothing but the equivalent of water. Ordinarily, therefore, only water need be added to replace the electrolyte. A small amount of the acid is carried away as a spray by the gas bubbles, but this loss is hardly appreciable. Acid need be added only when an actual loss of electrolyte takes place, such as occurs with a leaky tank.

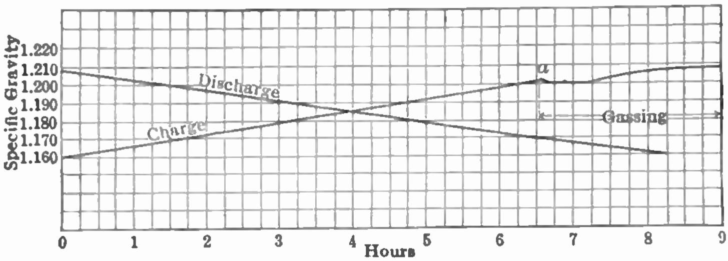


FIG. 71.—Variation of specific gravity in a stationary battery.

Distilled water is used, as a rule, to replace the evaporation of the electrolyte. If any doubt exists as to the suitability of local water, the battery companies, upon receipt of a sample, will analyze the water and report on the matter.

84. Temperature.—Below is given the relation between the freezing point of the electrolyte and its specific gravity. It will be noted that the freezing point is considerably lowered with increasing specific gravity, so that, if a battery is well charged, there is no danger of its freezing in the temperate zone.

Specific Gravity	Freezing Temperature, Fahrenheit
1.180	− 6°
1.200	−16°
1.240	−51°
1.280	−90°

At the higher temperatures, the rate of diffusion of the acid throughout the pores of the active material is increased so that the rating of a battery increases markedly with increasing temperature. Above 70°F. this increase is of the order of from 0.5 to 1.0 per cent per degree Fahrenheit.

85. Installing and Removing from Service.—The sealed-type batteries are shipped complete with electrolyte in the cells and are ready for service. They should be given a freshening charge after assembling and before using. The plates, tanks, electrolyte, and containers of stationary batteries of the open type are packed separately when shipped. When received, the separators should be placed immediately where they may be kept wet. The jars should be set in sand trays, as shown in Fig. 72. The plates should be handled carefully, particularly when being placed in the jars. The separators should be carefully slid into position (Fig. 73). As the active material on the plates is more or less converted into lead salts during exposure to the atmosphere, these salts must be reduced electrically before the battery is ready for service. Therefore the battery should be given an initial charge at the rate and for the time recommended by the manufacturer.

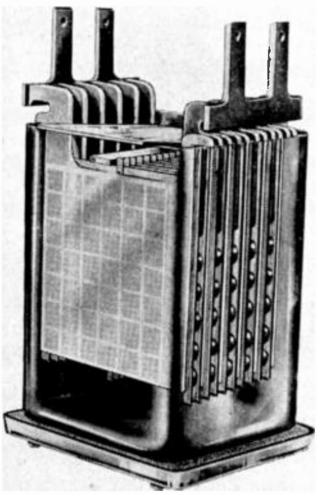


FIG. 72.—Open-type cell on glass sand tray.

If the battery stands without being used over a long period of time, the active material becomes more or less converted into inactive lead sulphate, which is a non-conductor and so is difficult to reduce electrically. Therefore a battery if idle should be charged occasionally. If the battery is to remain idle for a long time, and it is impracticable to charge it periodically, the following procedure is necessary to prevent sulphation. Give the battery a full charge, then siphon off the electrolyte, which may be saved and again used. Fill the cells with water and allow them to stand for 12 to 15 hr. Siphon off the water and the cells will stand indefinitely without injury to the plates. To put back in service, replace the separators if warped or cracked, fill the battery with the electrolyte having a specific gravity of 1.210, and charge in the same manner as with the initial charge.

86. Portable Batteries.—In the design of batteries for propelling vehicles and for automobile starting and lighting, it is necessary to obtain a high discharge rate with minimum weight and size. For portable batteries, and even for some stationary installations, these last two factors are important. Therefore, pasted plates are used for both positives and negatives. These are

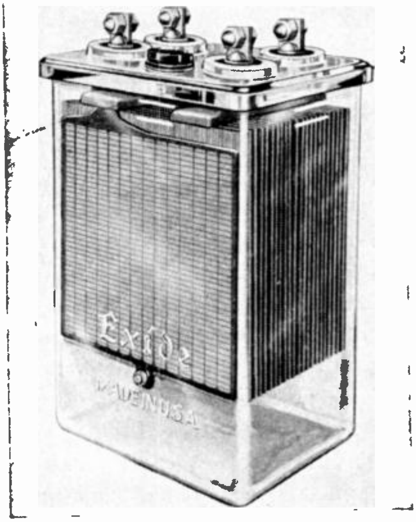


FIG. 73.—Storage cell in sealed glass.

made relatively thin and are insulated from one another by thin separators. They are then packed tightly into a hard-rubber jar or container which is practically the same as that of the Ironclad cell (Fig. 65, p. 109). The cover is sealed with an asphaltum compound to prevent the splashing of the liquid. There is a hole in the top of the jar which is closed with a cap. This permits the replenishing of water of the electrolyte. A vent in the cap allows the gases to escape. The discharge rates of this type of battery may be high as, for example, when doing starting duty. Furthermore, the ampere-hour capacity of the battery for its weight and size must be high. Hence, the volume of the electrolyte is small and the specific gravity must vary between wide limits. When the battery is fully charged, the specific gravity is as high as 1.280 and 1.300, and when it is completely discharged, the specific gravity is as low as 1.100.

The individual cells are mounted snugly beside one another in boxes or crates and are connected together on top by lead connectors which may be "lead-burned" or held by lead nuts. The number of cells in such a unit depends on the voltage which is desired.

Portable batteries are usually shipped assembled, charged and complete with the electrolyte so that they are ready for use when received. However, a freshening charge is advisable.

Because of its ruggedness, the "Exide-Ironclad" (Par. 78, p. 108) is used to a large extent in electric vehicles.

As the space for the electrolyte is very limited in vehicle batteries, and as considerable gassing occurs, the level of the electrolyte falls quite rapidly, so that frequent additions of water are necessary, usually once for each week of active service.

87. Rating and Batteries.—Practically all batteries have a nominal rating based on the 8-hr. rate of discharge. Thus, if a Planté battery can deliver a current of 40 amp. continuously for 8 hr., the battery will have a rating of $40 \times 8 = 320$ amp.-hr. The normal charging rate of such a battery would be 40 amp. Assuming the battery to be capable of delivering just 40 amp. for 8 hr., it would not be able to deliver 64 amp. for 5 hr. (320 amp.-hr.) but only 88 per cent of this, or 56.4 amp. for 5 hr. Fifty-six and four-tenths amperes is called *the 5-hr. rate*.

Below is given a table showing the percentage capacity with various discharge rates.

	Hours				Minutes	
Discharge rate	8	5	3	1	20	6
Percentage of capacity at 8-hr. rate:						
Planté type	100	88	75	55.8	37	19.5
Pasted type	100	93	83	63	41	25.5

This falling off in capacity with higher rates of discharge is due to the inability of the free solution to penetrate rapidly the pores of the active material. Consequently it is not possible to reduce all the active material during the short periods of discharge. After such a battery has stood for a short time, it will be found to have recovered to some extent and is therefore capable of delivering more current, even after apparently having become exhausted. This is due to the final penetration of the pores of the active material by the free solution.

Batteries are able to discharge at extreme rates for very short intervals. For example, a starting battery having an 8-hr. rating of 10 amp. is often called upon to supply 450 amp. when doing starting duty.

88. Charging.—The following rule may be observed in charging a lead battery: The charging rate in amperes may always be made equal to the number of ampere-hours that have been discharged by the battery. For example, if 200 amp.-hr. are out of the battery, a charging rate of 200 amp. may be used. As the battery charges, the ampere-hours out of the battery decrease and the charging rate must correspondingly decrease. The rate should never be such that violent gassing occurs.

Gassing represents a waste of energy because a considerable portion of the charging energy is used in merely breaking up the water into hydrogen and oxygen. In addition, gassing causes the battery to become heated, the acid is carried out in a fine spray by the bubbles, and active material may be carried from the plates by the erosive action of the bubbles.

When a battery is fully charged, any rate will produce gassing, but the rate can be reduced to such a low value that the gassing is not excessive and is practically harmless. This rate is called the *finishing rate*.

A battery may be charged in 5 hr. by beginning at a rate several times the finishing rate and tapering off to the finishing rate as the charge progresses (constant-voltage method). On the other hand, a constant current of moderate value may be used over a much longer period, even as long as 16 hr. (constant-current method).

A common example of the constant-current method is the charging of low-voltage batteries from 110-volt mains. This is illustrated by Fig. 74, which shows the charging of a 6-volt starting battery. The e.m.f. of the battery is so small in comparison with the 110 volts of the mains that the current is determined almost entirely by the resistance of the lamp bank or of any similar series resistance. Since such resistance is practically constant, the current remains substantially constant, irrespective of the small changes in battery e.m.f. Before connecting the battery, it should be definitely determined that the mains supply *direct current*, and it is also necessary to know which main is positive. If doubt exists as to the polarity and a voltmeter is not available, dip the two ends of the wires which connect the mains to the battery into a glass of slightly acidulated water or into salt water. Bubbles form about the *negative* wire. When using the constant-current method of charging, the charging rate must be reduced as the battery approaches the fully charged condition.

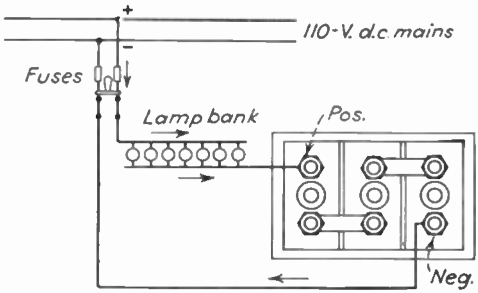


Fig. 74.—Charging a starting battery from 110-volt mains.

The constant-potential method of charging is frequently to be preferred, as the charging current automatically tapers off due to the rise in the cell e.m.f. as the cell approaches the completely charged condition, and little or no attention is required. The applied voltage should be about 2.3 volts for each cell of the battery when there is no series resistance in the circuit. The diverter-pole generator is well adapted to this method of charge (see Par. 289, p. 469).

With 2.3 volts per cell and no series resistance, the current at the beginning of charge is usually too great, so that it is advisable to use a small series resistance. If a series resistance is used, a voltage source of 2.5 or 2.6 volts per cell is desirable, as otherwise adjustments must be made during the charging period.

Many installations, such, for example, as oil-switch control batteries in power stations, are operated as "floating" batteries connected continuously to the bus. Usually these installations consist of 60 cells, and the bus is held at an average voltage of

129 volts, or 2.15 volts per cell, at which voltage the cells are kept in a charged condition.

If a recharge of the battery is required after an extended discharge or to give a periodic equalizing charge, the bus voltage is raised somewhat, the limit being determined by the character of connected load.

When it is not feasible to raise the bus voltage for this purpose, a series booster may be used for raising the charging potential to a value sufficiently high to send current into the battery. The booster ordinarily consists of a low-voltage, separately excited

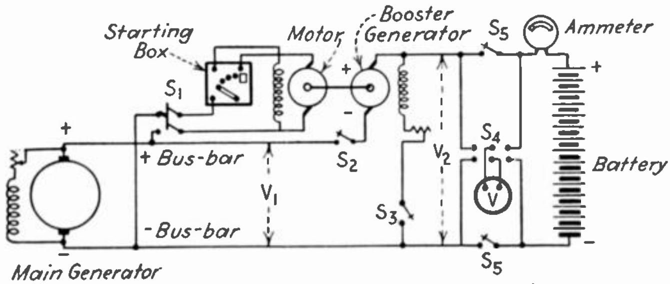


FIG. 75.—Booster method of charging a storage battery.

shunt generator, driven by a shunt motor. Figure 75 shows the connections of a booster system. The booster generator is connected with its negative terminal to the + bus-bar and raises the bus-bar voltage V_1 to a value V_2, which can be adjusted to give the desired charging current. To put the set in operation, switch S_1 is first closed and the motor started. Switches S_2 and S_3 are then closed, and the field current of the booster is adjusted until the voltage V_2 is just equal to the battery e.m.f. This condition is determined by the voltmeter V using the D.-P., D-T. (double-pole, double-throw) switch S_4. The switches S_5, usually of the D.-P., S.-T. (double-pole, single-throw) type are then closed, and the field of the booster is readjusted to give the desired charging current.

As an example, consider a 129-volt installation with a floating battery. As the average cell voltage is about 2.15 volts, 60 cells are necessary. Assume that the battery has a 320 amp.-hr. rating. The charging current will be 320/8, or 40, amp. (the normal 8-hr. rating). The voltage of each cell should be boosted to approximately 2.5 volts, on charge. Therefore the total voltage necessary will be $2.5 \times 60 = 150$ volts. Of this 150 volts, the bus-bars can supply 129 volts. The booster supplies the remaining 21 volts, and its rating will be

$$\frac{21 \times 40}{1,000} = 0.84 \text{ kw.}$$

The *total* power utilized in charging the battery is, however,

$$\frac{150 \times 40}{1,000} = 6.0 \text{ kw.}$$

(Also see pp. 62 and 585.)

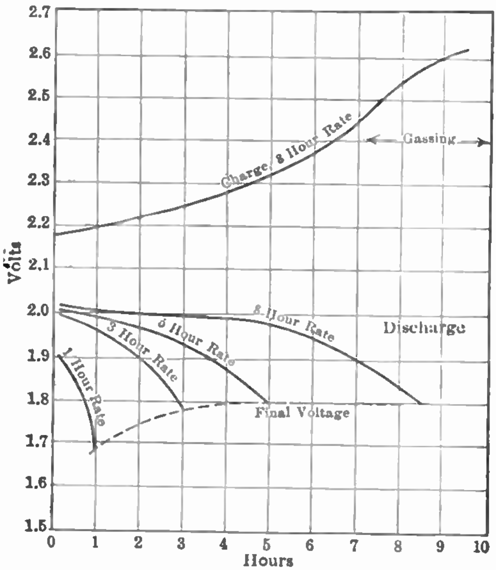


FIG. 76.—Voltage curves on charge and discharge for lead cell.

The terminal voltage of a cell rises on being charged, as is shown in Fig. 76. The terminal voltage is about 2.18 volts at the beginning of charge at the normal, or 8-hr., rate and rises slowly to about 2.4 volts, after which it rises very rapidly to 2.6 volts. This last rise occurs in the gassing period. This final rise of voltage also indicates that the cell is nearing the completion of charge. It is this rise of voltage which automatically cuts down the charging rate when the constant-potential method is used. The voltage does not rise so rapidly when the charging rate is reduced toward the end of charge, because of the lesser *IR* drop in the cell itself.

In Fig. 76 the charge characteristic holds for a temperature of approximately 70°F. (21°C.). The final voltage ranges from 2.75 to 2.45 volts with variations of temperature ranging from 30°F. (−1°C.) to 110°F. (43°C.). Hence the final cell voltage cannot be used as a criterion of charge unless the temperature of the electrolyte is taken into consideration.[1]

The *drop* of voltage at various rates of discharge is also shown in Fig. 76. It will be noted that the battery voltage at the 8-hr. discharge rate is practically constant up to 5 hr. and at even greater values of time the change in voltage is not great. This is a very distinct advantage if the battery is used to supply incandescent lamps.

89. Battery Installations.—Batteries should be installed in dry, well-ventilated rooms. Sealed glass-jar cells may be mounted directly on wooden racks painted with asphaltum paint. A separate battery room is not required for batteries of this type of assembly. Open glass jars are set in glass trays containing sand (Fig. 72, p. 116), the trays being set on wooden racks painted with asphaltum paint. The larger battery tanks should be set on insulating pedestals 6 in. or so above the floor. For tank batteries the floor should preferably be of acid-resisting material, such as mastic asphalt. A plain cement floor is preferable to tile or vitrified brick as the acid seeps into the cracks which inevitably occur in the grouting. All wooden surfaces should be covered with asphaltum paint. The room should be well ventilated, to remove the gases and the spray which is carried out of the tanks on charge. As hydrogen gas is given off, no flame should be allowed in the room and no switches should be installed in the room. In addition to the danger of explosion caused by arcing at the switch contacts, the acid in the air will corrode the copper. Lead-covered insulated copper wire and lead lugs should be used for making connections, or copper bus-bar painted for protection.

[1] For an excellent discussion of lead-storage-battery characteristics, see J. Lester Woodbridge, "Storage Battery Charging," *Trans. A.I.E.E.*, Vol. 54 (1935), p. 516.

90. Capacities and Weights of Lead Cells.—Below is given the relation of weight to kilowatt capacity for the various types of cells just described.

KILOWATT CAPACITY (AS RELATED TO WEIGHT) OF REPRESENTATIVE CELLS
Manufactured by The Electric Storage Battery Company

Size of plates	H	G	MV
Number of plates per cell	81	41	13
Type of plates	Exide	Chloride Accumulator	Exide-Ironclad
Weight of plates in cell, pounds	2,102	886	28.5
Weight of cell, pounds*	3,660	1,864	42.75
Kilowatts—per cell:			
For 1 hr	10.80	2.80	0.235
For 4 hr	4.14	1.23	0.0895
For 8 hr	2.41	0.772	0.0541
Kilowatts—per pound of plates:			
For 1 hr	0.00513	0.00315	0.00825
For 4 hr	0.00197	0.00138	0.00314
For 8 hr	0.00115	0.000871	0.00190
Kilowatts—per pound of cell:			
For 1 hr	0.0029	0.00150	0.00549
For 4 hr	0.001096	0.000659	0.00209
For 8 hr	0.000638	0.000414	0.00127

* The necessary insulating supports for the H- and G-cells and tray for the MV-cell are not included—these add approximately 2 and 10 per cent, respectively, to the cell weights.

Type H Exide batteries, used in central-station stand-by service. Largest battery 150 cells, 169 plates per cell, capacity 3,460 kw. for 1 hr.; two other batteries 3,420 kw. each.

Type G Chloride Accumulator, used in power plants for peak, regulating and exciter-bus service; also in telephone exchanges, large isolated plants, etc. Largest battery 288 cells, 85 plates per cell, capacity 1,700 kw. for 1 hr.; two other batteries same size and capacity.

Type MV Exide-Ironclad, used in electric vehicles, locomotives, industrial trucks and tractors, and for yacht lighting, etc.

EDISON NICKEL-IRON-ALKALINE CELL

History.—This type of cell was invented by Edison in 1901 when the use of battery-propelled trucks, tractors, and locomotives, as well as electrically operated lighting equipment for railway passenger cars, created the need for a light, durable type of storage battery. As the Edison cell is the only nickel-iron-alkaline type in commercial use in the United States it will be the only one discussed. The light weight and durability of this cell are due mainly to the employment of steel in the construction of plates and container.

91. Plate Construction.—The positive plate consists of a nickeled-steel grid, holding nickeled-steel tubes which contain the positive active material. When inserted in the tubes the active material is in the form of nickel hydrate, but this changes to an oxide of nickel after the formation treatment. In order to give the electrolyte free access to the active material, the tubes are perforated. To obtain improved electrical conductance, the active material is alternated with layers of pure metallic nickel flake at the time it is tamped into the tubes. The tubes are either 3/16 or 1/4 in. (0.476 or 0.635 cm.) inside diameter and about 4 in. (10.16 cm.) long and are reinforced by eight encircling seamless steel rings equidistantly spaced over the tube length.

The negative plate is generally similar in construction to the positive plate except that a finely divided oxide of iron is used as active material and is contained in rectangular perforated nickeled-steel pockets instead of tubes. Positive and negative plates are shown in Fig. 77.

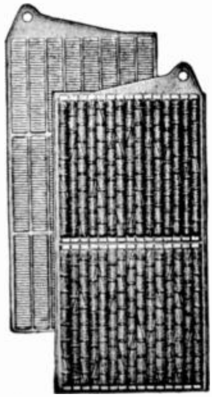
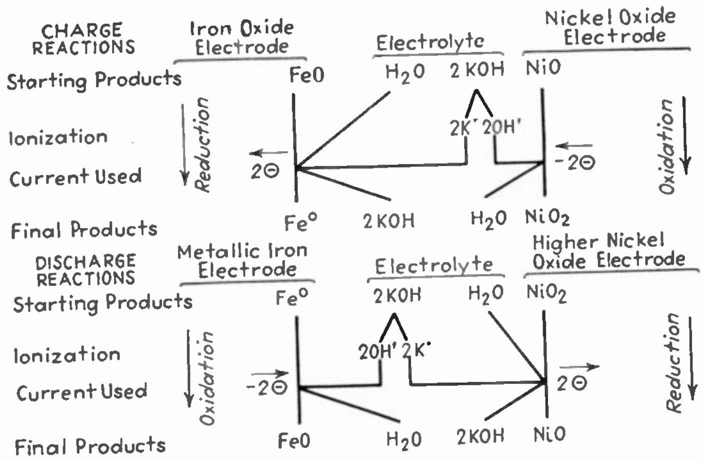


FIG. 77.—Positive and negative plates of an Edison storage cell.

92. Electrochemical Principle.—Instead of using an acid, the Edison cell employs an alkaline electrolyte consisting of a 21 per



cent solution of potassium hydroxide to which has been added a small amount of lithium hydroxide. The electrolyte, instead of attacking the steel tubes, pockets, grids, container, etc., actually preserves them. The chemical reactions which take place within the cell are complex, but their nature is indicated by the equations on p. 125.

In any condition of the electrolyte it does not cause disintegration or solution of the active materials, and, since only a relatively small quantity of the potassium and lithium hydroxide is absorbed by the higher nickel-oxide electrode, the composition of the electrolyte does not vary appreciably throughout the cycle of charge and discharge. The conductivity and specific gravity of the electrolyte are therefore practically constant.

This is also shown by the reaction diagram. On charge, the active material of the negative plate, iron oxide (FeO), is reduced to iron (Fe). The active material of the positive plate, nickel oxide (NiO), is oxidized to nickel dioxide (NiO_2). On discharge, the reverse process occurs, the negative plate being oxidized to iron oxide and the positive plate being reduced to nickel oxide. It will be noted that throughout charge and discharge, the solution consisting of $2KOH + 2H_2O$ remains unchanged, both chemically and in concentration. Hence, unless evaporation is allowed to take place, the specific gravity of the electrolyte does not change with charge and discharge as it does in the lead-acid cell.

FIG. 78.—Assembly, Edison battery plates removed from container.

93. Assembly.—By passing a steel connecting rod through holes at the top of the grids, the positive and negative plates are assembled into positive and negative groups. Steel spacing washers between adjacent plates on the connecting rod ensure proper plate spacing. A lock washer and nut are drawn tight at each end of the connecting rod, binding the plate group firmly together. Positive and negative plate groups are intermeshed to form complete elements, separation between alternate positive and negative plates being accomplished by vertical hard-rubber grids and pins.

The assembled elements, shown in Fig. 78, are placed in a corrugated nickeled-steel container after which the steel cover is welded in position. The poles are insulated from the cover by a series of hard-rubber and soft-rubber washers which also provide a gas-tight and liquid-tight packing. Projecting from the cell cover is the filling aperture, on which is mounted a hinged filler cap. The filler cap is held either positively open or positively closed by a steel clip spring. Suspended from the filler cap is a hard-rubber valve which seats by gravity when the cap is closed, thus excluding external air and reducing evaporation, yet permitting the escape of gas.

The tops of the pole pieces are threaded. Immediately below they are tapered to fit the lugs of the intercell and intertray connectors. The connector lugs are steel forgings bored to fit the taper of the pole pieces and are swedged upon heavy-copper connecting links. All lugs, links, and nuts are nickel-plated. Individual cells are mounted in wooden trays to form a battery as in Fig. 79.

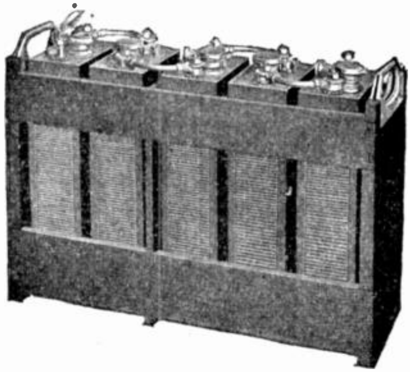


Fig. 79.—Five Edison storage cells mounted in a tray.

94. Discharge and Charge Characteristics.—The rated capacity of the Edison cell is based on a normal 5-hr. discharge until the voltage becomes 1 volt per cell for *A*, *B*, *C*, and *N* types and on a normal 3⅓-hr. discharge to 1 volt per cell for *G* and *L* types. Figure 80 shows normal charge and discharge curves for the Edison *A*-type cell. It will be noted that the average discharge voltage at these rates is about 1.2 volts per cell. At discharge rates other than normal, voltage values will vary above or below this average figure, as shown on Fig. 81. These discharge characteristics are based on a preceding normal-rate charge for a period of 7 hr. for *A*-, *B*-, *C*-, and *N*-type cells and for 4¾ hr. for *G*- and

L-type cells. The specific gravity of the electrolyte changes but slightly between charge and discharge and cannot be used to determine the state of charge. The completion of full charge is indicated when the voltage ceases to rise over a period of ½ hr

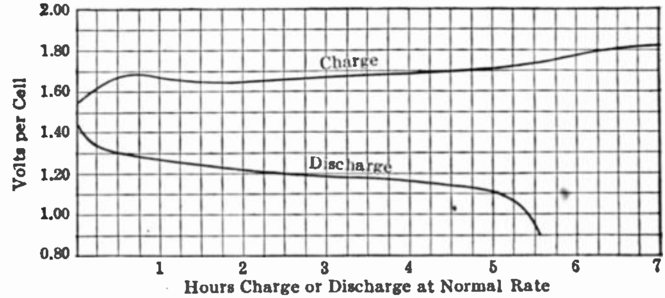


FIG. 80.—Voltage changes during the charge and discharge of an Edison cell.

during charge with constant current flowing. The state of charge can be approximated at any time by use of an ampere-hour meter or by a charge test fork which indicates from a pilot cell the voltage delivered at a given rate of discharge.

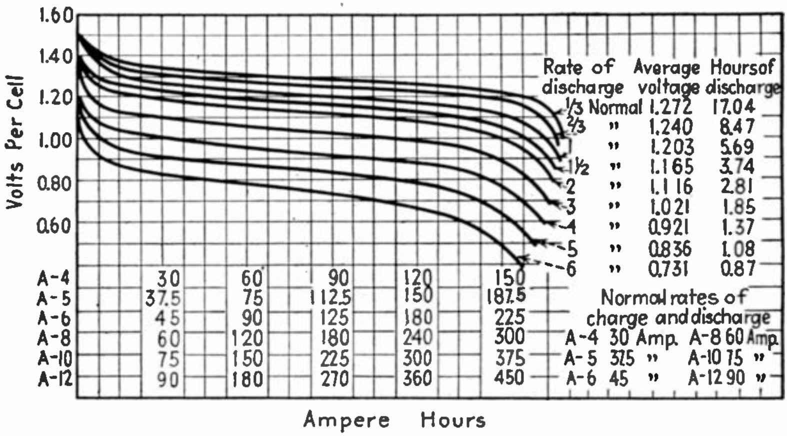


FIG. 81.—Discharge characteristics of type "A" Edison cells at various rates subsequent to normal charges.*

95. Electrolyte.—As with other cells, the solution level gradually drops during service due to the electrolysis of water into hydrogen and oxygen gas on charge. Bubbles of gas carry with them small particles of the solution, most of which break against

* See also Curve 1.

the special valve in the filler cap and drain back into the cell. A small quantity, however, escapes into the air, and, because only distilled water is used to replace this loss, the strength of the solution is gradually reduced. After an extended period of time, depending on the usage to which the cell has been put, the specific gravity of the solution reaches 1.160 at 60°F., at which point it should be replaced with new electrolyte of proper specific gravity. The normal specific gravity of the electrolyte in new cells is between 1.195 and 1.215 with the solution at the proper level above plate tops and when correction for temperature is made.

96. Advantages.—The nickel-iron-alkaline cell possesses important advantages which make it well suited to the various services to which it is applied. These advantages result from the nature of the materials used, the method of construction, and the fundamental electrochemical principle involved. The extensive use of steel permits precision manufacturing and results in a cell capable of withstanding the vibration and shock incidental to commercial service. Moreover, the use of steel for tubes, pockets, grids, etc., makes possible a plate construction which securely retains the active materials and which eliminates buckling and warping.

Aside from its rugged construction, additional advantages of the Edison cell are its light weight per watthour of capacity and its ability to withstand electrical abuse. It may be overcharged, overdischarged, accidentally short-circuited, charged in the reverse direction, or left standing idle in a discharged condition indefinitely without injury. It is free from corrosive acid fumes and is not subject to ordinary storage-battery diseases such as sulphation, sedimentation, or terminal corrosion. Its tray assembly and cell connections are simple, it requires no replacement of separators throughout its life, and it is not damaged by freezing. Operation is dependable over a long period of time.

97. Application.—Edison cells find their greatest usefulness in storage-battery-propelled street trucks, industrial trucks, tractors, mine and industrial locomotives; for railway passenger-car lighting and air conditioning; multiple-unit car control; all types of railway signaling, such as for track circuits, target lighting, highway crossing signals, and interlocking plants; for marine power and lighting, miners' electric safety-cap lamps, emergency lighting, isolated electric light plants; for time clocks, fire-alarm systems, and other services. The various types manufactured at present are not adapted to low-voltage automobile-starting service and are not sold for this purpose. Below are tabulated the various types and sizes of Edison cells together with their capacities and weights.

98. Data on Edison Cells.

Cell type	Rating†		Weight,‡ pounds per cell	
	Ampere-hour capacity	Normal rate, amperes	Standard	High type
N2	11¼	2¼	1.9	
B1, B1H*	18¾	3¾	5.3	6.7
B2, B2H*	37½	7½	6.0	7.2
B4, B4H*	75	15	9.5	10.9
B6, B6H*	112½	22½	13.0	15.2
A4, A4H*	150	30	16.5	19.3
A5, A5H*	188	37½	19.6	22.5
A6, A6H*	225	45	22.4	25.5
A7, A7H*	263	52½	25.8	28.6
A8, A8H*	300	60	31.2	35.9
A10, A10H*	375	75	38.1	43.8
A12, A12H*	450	90	47.3	53.5
A14, A14H*	525	105	56.6	60.8
A16, A16H*	600	120	62.9	68.0
C4	225	45	24.3	
C5	281	56¼	29.3	
C6	338	67½	34.4	
C7	394	78¾	39.8	
C8	450	90	45.5	
C10	563	112½	61.3	
C12	675	135	71.0	
L20	12½	3¾	1.9	
L30	18¾	5⅝	2.6	
L40	25	7½	3.2	
G4, G4H	100	30	12.5	14.6
G6, G6H	150	45	16.1	18.5
G7, G7H	175	52½	20.8	23.0
G9, G9H	225	67½	22.6	25.7
G11, G11H	275	82½	29.0	33.4
G14, G14H	350	105	36.9	42.4
G18, G18H	450	135	49.5	56.0

* The letter H indicates high-type cells; these cells have the same characteristics as the standard-type cells but are built higher so as to hold more electrolyte and are used in installations where frequent flushing is not convenient.

† Ratings are on basis of 5-hr. rate for A-, B-, C-, and N-type cells and 3⅓-hr. rate for G- and L-type, with average of 1.2 volts per cell and final of 1.0 volt per cell.

‡ Weights are for completely assembled cells, including trays, connectors, etc.

99. Efficiency of Storage Batteries.—The efficiency of a storage battery is the ratio of the watthour output to the watthour input.

Example.—A fully charged cell is discharged at a uniform rate of 38 amp. for 6 hr. at an average voltage of 1.95 volts. The cell is now charged at a uniform rate of 40 amp. for 6 hr., the average voltage being 2.3 volts. The cell is then in its original condition of charge. What is the efficiency of the cell?

$$\text{Watthours output} = 38 \times 1.95 \times 6 = 445.$$
$$\text{Watthours input} = 40 \times 2.3 \times 6 = 552.$$
$$\text{Efficiency} = 445/552, \text{ or } 80.7 \text{ per cent.}$$

One often hears of the ampere-hour efficiency of a storage battery. As ampere-hours do not represent energy, the ampere-hour efficiency is not a measure of a battery's ability to store energy. In the example the ampere-hour efficiency may be found as follows:

$$\text{Ampere-hours output} = 38 \times 6 = 228.$$
$$\text{Ampere-hours input} = 40 \times 6 = 240.$$
$$\text{Ampere-hour efficiency} = 228/240, \text{ or } 95 \text{ per cent}$$

The much lower watthour efficiency is due to the difference between the voltage of charge and that of discharge, as shown in Figs. 76 and 80. pp. 122 and 128.

The efficiency of a storage battery varies with the rate of charge and discharge, and with the temperature. As high charge and discharge rates produce relatively high I^2R and polarization losses, the efficiency is lowered under these conditions. Further, a cell may be charged at the 8-hr. rate and discharged at the 3-hr. rate and have an apparent efficiency of 60 per cent. This does not represent the true efficiency as the cell actually will not be completely discharged, even if it appears to be. Owing to the inability of the free acid to permeate the active material, much of the active material has not been reduced, and after a short time the cell will be found to have recuperated to a considerable extent and will be able to deliver more energy.

The ampere-hour efficiency of a storage battery is of the order of magnitude of 95 per cent. For a complete cycle the watthour efficiency of a stationary battery of moderate size is about 75 per cent at the 8-hr. charge and discharge rates. The watthour efficiency of a large stationary battery is about 85 per cent under the same conditions. Where a battery merely "floats" and the cycle of charge and discharge is a matter of minutes or perhaps of seconds even, the watthour efficiency may be as high as 95 or 96 per cent.

In selecting a battery, the efficiency is only one of the factors to be considered. The initial cost and the maintenance of batteries form a substantial proportion of the total cost and hence are important considerations. Also, such factors as weight, ruggedness, voltage regulation and overload capacity must be considered for each individual installation.

The uses of storage batteries in the generation and distribution of power are considered in Chap. XV.

100. Electrolysis.—Pure water is a very poor conductor of electricity. In fact, it may be considered as practically an insulator. If, however, even a very small amount of acid, alkali, or salt be added to water, the solution becomes a good conductor. Moreover, if an electric current be caused to flow through such a solution, it dissociates the molecules of the substance in solution, or the molecules of the water itself, into simpler substances which appear at the anode and at the cathode. The phenomenon of the electric current in causing such dissociation under these conditions is called *electrolysis*. The solution which becomes conducting under these conditions is called an *electrolyte*.

If current be caused to flow in slightly acidulated water, using an inert material such as platinum for the electrodes, hydrogen is released at the cathode and oxygen is released at the anode. Moreover, the volume of hydrogen released is twice that of oxygen. Hence, the water molecule H_2O is broken up or dissociated into its two constituent elements.

The theory of electrolysis is based on electrolytic dissociation. In solution, the molecules of acids, alkalies, and salts dissociate into positive and negative ions. Thus hydrochloric acid (HCl) dissociates into a positive, (+), H ion and a negative, (−), Cl ion. Likewise, the copper sulphate molecule ($CuSO_4$) becomes dissociated into the +Cu ion and the $-SO_4$ ion. According to the electron theory, the negative ion has an excess of electrons, and the positive ion has a deficiency of electrons (see p. 315 and also Vol. II, Chap. XIV). When no potential difference is applied between electrodes, the ions drift about in the solution. However, when a potential difference is applied to the electrodes in the solution, the positive ions migrate to the negative electrode or cathode and accordingly are called *cations*. The negative ions go to the anode or positive electrode and accordingly are called *anions*.

The positive ions give up their charge to the cathode and the negative ions give up their charge to the anode, thus constituting the current. Hence, the conduction of current through an electrolyte is a convection effect, the charges being carried to the electrodes by the ions. Thus, electrolytic conduction differs from the ordinary metallic conduction in that it involves a transfer of matter and is accompanied by chemical change.

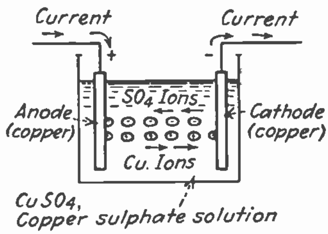


FIG. 82.—Electrolysis of copper in copper-sulphate solution.

In Fig. 82 are indicated the effects which occur when two copper plates are immersed in copper sulphate solution, and current is caused to flow from one plate to the other. The positive copper (Cu) ions move to the cathode and deposit metallic copper. The negative sulphate (SO_4) ions move to the anode and combine with the hydrogen ion of the water to form sulphuric acid, an oxygen molecule being given off at the anode. The reaction is as follows:

$$SO_4 + H_2O = H_2SO_4 + O.$$

This reaction occurs in the electrolytic refining of copper. The copper ingot to be refined is used as anode, and a thin plate of copper is used as cathode. Copper goes into solution from the anode and deposits as pure copper on the cathode. Impurities are precipitated as *anode mud* in the bottom of the cell. However, considerable value in metals, such as silver and gold, may be recovered from the sludge.

Electrolysis is not confined to substances in water solution, but fused salts may also be subjected to electrolysis. For example, the electrolysis of fused table salt (NaCl) produces sodium (Na) at the cathode and chlorine (Cl) at the anode. Aluminum is produced by the electrolysis of the fused salt, aluminum oxide (Al_2O_3), at a temperature of from 900° to 1000°C.

101. Faraday's Laws of Electrolysis.—As a result of his experiments, Faraday discovered two fundamental laws of electrolysis.

The first law states that *the weight of the products of electrolysis is proportional to the quantity of electricity which has passed through the electrolyte.* That is, the weight

$$w = \epsilon q = \epsilon i t \tag{63}$$

where ϵ is a constant called the *electrochemical equivalent* of the substance, q is the quantity of electricity in coulombs, i is the current in amperes, and t is the time in seconds.

The second law states that *for a given quantity of electricity the weight of the products of electrolysis is proportionate to their electrochemical equivalents.* For example, the atomic weight of hydrogen is 1.008 and 1 coulomb will cause 0.0104 mg. to be released. The value 0.0104 is the electrochemical equivalent of hydrogen. Below are given the electrochemical equivalents of a few of the elements.

ELECTROCHEMICAL EQUIVALENTS, MILLIGRAMS PER COULOMB

Aluminum	0.09316	Nickel	0.3041
Chlorine	0.3674	Oxygen	0.08291
Chromium	^{3}0.1797 ^{6}0.0898	Potassium	0.40516
Cobalt	0.3054	Silver	1.118
Copper	^{1}0.6588 ^{2}0.3294	Sodium	0.2382
Gold	^{1}2.0436 ^{3}0.6812	Tin	0.3075
Lead	1.0737	Tungsten	0.3180

The small superior numbers give the valency.

Example.—A current of 25 amp. flows for 8 hr. through an electrolytic cell with copper electrodes and copper sulphate solution similar to that shown in Fig. 82. Determine the kilograms of copper deposited on the cathode. With copper sulphate, the valence of copper is 2. Eight hours equals $8 \times 3{,}600 = 28{,}800$ sec. Hence, using Eq. (63) and the electrochemical equivalent of copper in the table, the weight of copper

$$w = 0.3294 \times 10^{-6} \times 25 \times 28{,}800 = 0.237 \text{ kg.} \quad \textit{Ans.}$$

102. Electroplating.[1]—Electroplating is merely an application of the principles of electrolysis given in Par. 100 and 101, by which thin coatings of metal are deposited on other metal. Because of the desirability of plating the baser metals with protective

[1] See "Standard Handbook for Electrical Engineers," 6th ed., Sec. 23, Par. 136 to 180, for a more complete discussion.

coverings, such as copper, nickel, chromium, and silver, electroplating is an important industry.

As a simple illustration, assume that it is desired to copper-plate a carbon brush. The portions of the brush to be plated are immersed in a solution of copper sulphate as shown in Fig. 83. A copper strip is also immersed in the solution and is connected to the positive terminal of a dynamo or some other source of direct-current supply. The brush is connected to the negative terminal of this supply. Under these conditions the current will carry copper from the solution and deposit it on the carbon brush.

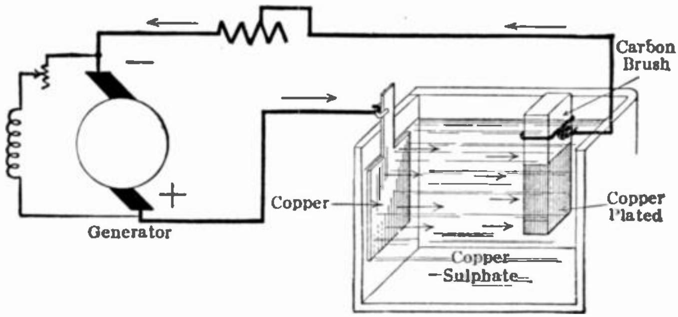


FIG. 83.—Copper-plating bath.

This copper which leaves the solution is replaced by copper which is carried from the copper strip (the anode) into the solution so that there is no change in the solution itself. The current should be such that the density is about 0.02 amp. per square inch of the surface to be plated.

It is not necessary that the anode be of the metal which it is desired to deposit. Other metals may be used. Under these conditions, however, the solution in time becomes contaminated by the going into solution of the anode. If an inert substance such as carbon is used as anode, acid is formed in the solution.

The *only opposing* e.m.f. in the bath just described is the *IR* drop in the solution. This may be reduced by bringing the electrodes close together, but if the electrodes are too close together the deposit will not be uniform. The amount of metal deposited per second is proportional to the current. Because of the nature of electroplating baths, they are naturally low-voltage devices. When practicable, several are connected in series. A low-voltage and high-current generator is generally used for plating purposes. In practice there are many refinements to be observed.

Acid is added to the solution to prevent impurities from depositing. A cyanide solution of copper is found to give better results than the sulphate Nickel, tin, zinc, silver, gold, etc., may be deposited by the use of suitable baths and electrodes.

A gravity cell is an example of electroplating in which the source of current is derived from the cell itself. The current flows from the zinc to the copper within the solution, zinc is carried into the solution as sulphate, and copper is deposited or plated from its sulphate on the positive electrode.

Electrotyping is another common example of electroplating. An impression is made in wax with the type or object to be reproduced. The surface of the wax is made conducting by applying a thin coating of graphite. Copper is then plated on this surface. It is later backed by type metal to give it the necessary mechanical strength.

CHAPTER V

ELECTRICAL INSTRUMENTS AND ELECTRICAL MEASUREMENTS

103. Principle of Direct-current Instruments.—Most direct-current instruments operate on the principle that a coil may be so placed in a magnetic field that the coil will tend to turn when a current flows in it. This turning action is the underlying principle of the electric motor and is discussed more in detail in Chap. XIII. It may be explained briefly as follows:

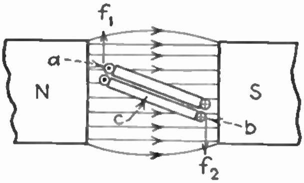


FIG. 84.—Turning forces of a coil carrying current.

It is a fundamental law that a conductor carrying a current and lying in a magnetic field tends to move at right angles to both the direction of the field and the direction of the current (p. 476). For example, in Fig. 84 is shown the top view of a coil C placed in a magnetic field between two pole pieces N and S. The direction of the current in the conductors a at the left-hand side of the coil is outward, and in the conductors b at the right-hand side is inward. As a result, conductors a develop a force f_1 and tend to move upward, and conductors b develop an equal force f_2 and tend to move downward, thus producing a couple tending to turn the coil in a clockwise direction.

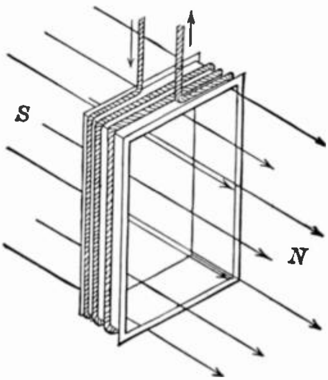


FIG. 85.—Magnetic field produced by an instrument coil.

This turning moment may be explained also on the basis of attraction and repulsion between magnetic poles (Chaps. VI and VII). The current in the turns of the coil produces a magnetic field through the coil with N- and S-poles at opposite faces of the coil (Fig. 85).

If the coil be placed in a magnetic field, the coil will tend to turn in such a direction that the ampere-turns of the coil will increase the strength of the magnetic field and the total flux of the system will be a maximum (see Par. 150, p. 219).

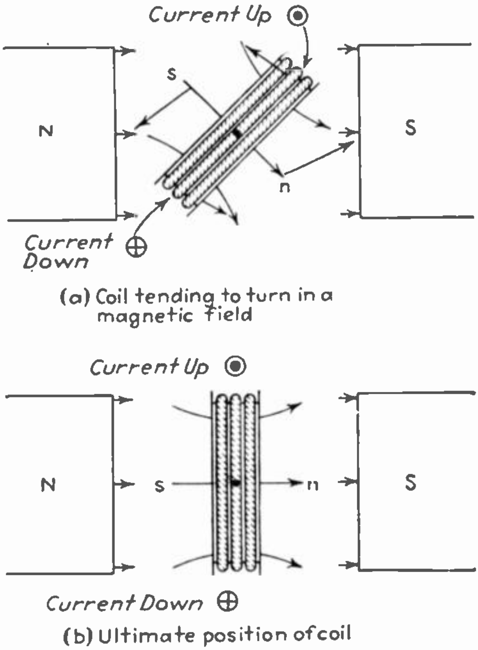


FIG. 86.—Turning moment of an instrument coil.

The N-pole of the coil will be attracted toward the south pole of the magnetic field and the S-pole of the coil will be attracted toward the north pole of the magnetic field.

This tendency of the coil to turn is shown in Fig. 86(a) where the coil tends to turn in the direction indicated by the arrows. If the coil is pivoted and free to turn, it will reach the position shown in Fig. 86(b). Under these conditions the coil has placed itself in such a position that its flux is acting in the same direction as that of the main field in which the coil is placed. Also, the unlike poles are as near each other as possible and the like poles are as far away from each other as possible.

This behavior of a coil carrying a current and placed in a magnetic field should be thoroughly understood, for it is the underlying principle of most current-measuring instruments and is in addition the principle on which all electric motors operate.

104. The D'Arsonval Galvanometer.—A galvanometer is a sensitive instrument used for detecting and measuring small electric currents. The D'Arsonval galvanometer, which is based on the principle of a coil turning in a magnetic field, is the most common type of galvanometer. Due to its simplicity, it has superseded practically all other types. In addition it is comparatively rugged and is not appreciably affected by stray magnetic fields. Figure 87 shows the principle of its construction. A coil of very fine wire is suspended between the poles of a permanent magnet by means of a filament, usually a flat strip of phosphor bronze. The coil may be wound with or without a bobbin. The bobbin is usually either of fiber or of aluminum. The advantage of an

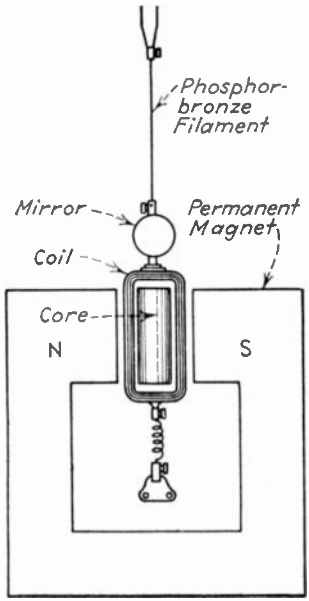


Fig. 87.—Principle of the D'Arsonval galvanometer.

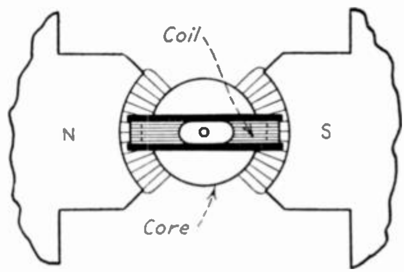


Fig. 88.—Radial field produced by cylindrical core and pole faces.

aluminum bobbin will be considered later. The poles of the magnet are usually cylindrical in shape and a cylindrical soft-iron core is placed between the poles and coaxial with them (Fig. 88).

The cylindrical form of pole faces and core is advantageous for two reasons. The length of the air path is reduced so that the amount of flux linking the coil is increased, thus making the galvanometer more sensitive; the flux between the pole faces and the core is practically radial. In such a radial field the deflections of the coil are directly proportional to the current in the coil, so that a uniform scale is obtained.

In Fig. 87 the coil is shown suspended by the phosphor-bronze filament. Any turning of the coil produces torsion in the filament which opposes the turning of the coil and is called the *restoring force.* When the moment of the restoring force and the

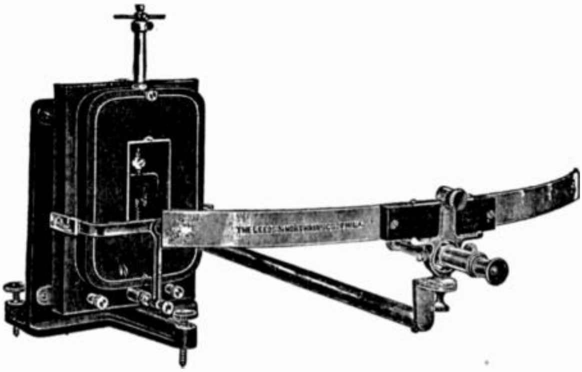


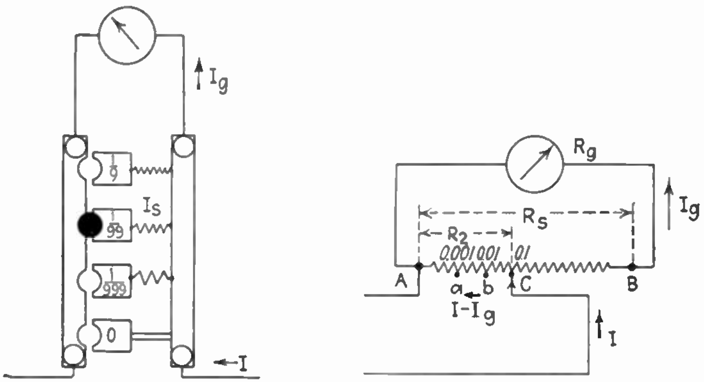
FIG. 89.—Telescope and scale method of reading a galvanometer.

turning moment due to the current are equal, the galvanometer assumes a steady deflection. For all practical purposes the galvanometer deflection is proportional to the current. The phosphor-bronze filament usually serves as one of the leading-in wires which carry current to the coil. The other leading-in wire consists of a very flexible spiral filament fastened to the bottom of the coil, as shown in Fig. 87.

There are two common methods of reading the deflection of a galvanometer. A plane mirror is mounted on the coil system, and a scale and telescope are mounted about 0.5 m. from the galvanometer. The reflection of the scale in the mirror can be seen with the telescope (Fig. 89). When the mirror turns, the reflection of the scale in the mirror deflects. The value of this deflection is determined by means of a crosshair in the telescope.

Another method is to use a concave mirror on the galvanometer moving system. A lamp filament is placed some distance from the mirror and its image is focused on a ground glass to which a scale graduated in centimeters is fastened. As the mirror deflects, the beam of light travels across the scale.

Damping.—If a galvanometer coil, which is hung freely, starts to swing, it will continue swinging for some time unless it is in some way retarded or damped. Electrodynamic damping is the simplest and most effective method of damping instruments of the D'Arsonval type. If the coil be wound on an aluminum bobbin, the motion of the bobbin through the magnetic field will



(a) Galvanometer shunt (b) Ayrton shunt

FIG. 90.—Types of galvanometer shunt.

induce currents in the bobbin, and these will be in such a direction as to put an electric load on the moving coil as in an electric generator. This opposes the motion of the coil. The same result may be obtained by binding short-circuited copper coils on the main coil, by shunting the galvanometer externally with a resistance (see Ayrton shunt, Par. 105), or even by short-circuiting.

105. Galvanometer Shunts.—When galvanometers are used to detect small currents, as in null methods (see Wheatstone bridge, p. 165), the conditions may be such that a comparatively large current flows through the galvanometer. This causes a violent deflection of the coil and may result in injury to the galvanometer. In certain other measurements, the current that it is desired to measure with the galvanometer may be so large that the deflection is considerably beyond the scale.

In either case the sensitivity of the galvanometer may be reduced by the use of a shunt, that is, a resistance which by-passes a certain known proportion of the current from the galvanometer. There are two common types of shunt. One type is shown in Fig. 90(a) and consists of three or four separate resistances which are plugged across the galvanometer one at a time. These are so adjusted in value that with a given external current to be measured, each two successive galvanometer currents are in the ratio of 10 to 1. For example, if the galvanometer is to measure one-tenth the external current, the top resistance (Fig. 90(a)) is of such a value that when it is plugged across the galvanometer, it shunts nine-tenths of the current away from the galvanometer.

The values of these resistances are determined as follows:
Let

R_g = galvanometer resistance.
I_g = galvanometer current for full-scale deflection.
I = circuit current.
I_s = shunt current.
R_s = shunt resistance.

To reduce the galvanometer current to one-tenth the value which it would have if all the current I flowed through the galvanometer, I_g must be one-tenth I. That is,

$$\frac{I_g}{I} = \frac{1}{10}. \tag{I}$$

The shunt current

$$I_s = I - I_g. \tag{II}$$

But the shunt current and the galvanometer current are inversely as the respective resistances. Hence,

$$\frac{R_g}{R_s} = \frac{I_s}{I_g} = \frac{I - I_g}{I_g}. \tag{III}$$

If

$$I_g = \frac{I}{10} \text{ (Eq. (I))},$$

$$\frac{R_g}{R_s} = \frac{I - I/10}{I/10} = 9,$$

$$R_s = \frac{1}{9} R_g \text{ ohms.} \tag{64}$$

For a reduction of 100 to 1,

$$\frac{R_g}{R_s} = \frac{I - I/100}{I/100} = 99$$

and

$$R_s = \frac{1}{99} R_g. \tag{65}$$

Example.—The resistance of a galvanometer is 600 ohms. What resistances should be used to shunt it in order that its deflections may be reduced in the ratio of 10 to 1 and 100 to 1?

$$R_1 = \frac{600}{9} = 66.7 \text{ ohms.} \quad \textit{Ans.}$$

$$R_2 = \frac{600}{99} = 6.06 \text{ ohms.} \quad \textit{Ans.}$$

Ayrton Shunt.—The *Ayrton shunt* is shown in Fig. 90(*b*). A permanent resistance AB is connected across the galvanometer terminals. One line terminal A is permanently connected to one end of this resistance, and the other line terminal C is movable and can be connected to various points along AB. With a fixed line current, the maximum deflection is obtained when C is at B. If point C be moved to a, where resistance Aa is 1/1,000 of the total resistance AB, the galvanometer deflection will be 1/1,000 its maximum value. If C be moved to b, where Ab is 1/100 the resistance AB, the galvanometer deflection will be 1/100 its maximum value, etc. This may be proved as follows:

Let I be the line current, I_g the galvanometer current, R_g the resistance of the galvanometer, and R_s the total resistance of the shunt. R_2 is the resistance from the point A to the contactor C which may make contact at various points such as a and b.

From Fig. 90(*b*),

$$\frac{I_g}{I - I_g} = \frac{\frac{AC}{AB}R_s}{\frac{CB}{AB}R_s + R_g} = \frac{\overline{AC}R_s}{\overline{CB}R_s + \overline{AB}R_g}$$

and

$$I_g(\overline{CB}R_s + \overline{AC}R_s + \overline{AB}R_g) = I\overline{AC}R_s.$$

Hence,

$$I_g = I\left(\frac{AC}{AB}\right)\frac{R_s}{R_s + R_g} \tag{66}$$

Equation (66) shows that for any fixed line current I, the galvanometer current I_g, and hence the galvanometer deflection, will be proportional to the fractional setting AC/AB of the Ayrton shunt, since, with any particular galvanometer-shunt combination, the quantity $R_s/(R_s + R_g) = k$, a constant.

If the shunt is set with contact C at B, so that $AC/AB = 1$,

$$I_{1g} = I_1\frac{R_s}{R_s + R_g} = kI_1, \tag{67}$$

which gives the maximum value of galvanometer current with a fixed line current I_1. Were the shunt removed, the entire line current would flow through the galvanometer. Hence the presence of the shunt reduces both the maximum galvanometer current and the galvanometer sensitivity by the factor $R_s/(R_s + R_g) = k$.

In order not to reduce the galvanometer sensitivity by too great an amount, R_s should be large compared with R_g. The ratio of R_s to R_g is usually from 8 to 10 and may be even greater (see example, p. 146).

Let I_g' be the galvanometer current for any setting of the shunt, as at C. Then, with a fixed line current I, from Eqs. (67) and (66),

$$\frac{I_{1g}}{I_g'} = \frac{I\dfrac{R_s}{R_s + R_g}}{I\left(\dfrac{AC}{AB}\right)\dfrac{R_s}{R_s + R_g}} = \frac{AB}{AC} = \frac{R_s}{R_2}$$

or

$$I_{1g} = I_g'\frac{R_s}{R_2}. \tag{68}$$

$R_s/R_2 = M$, where M is the *multiplying* power of the shunt. This means that the galvanometer current, with C set at 1, is equal to the galvanometer current with C at any other setting multiplied by R_s/R_2. For example, with C at 0.1, $R_2 = R_s/10$ and the multiplying power is 10. With C at $b = 0.01$, $R_2 = R_s/100$ and the multiplying power is 100. Equation (68) shows the universal nature of the Ayrton shunt, as the multiplying power does not involve the resistance of the galvanometer, and the shunt can therefore be used with any galvanometer practically. Actually, the shunt gives best results when the total resistance of the shunt is about 10 times the resistance of the galvanometer.

Ordinarily the line current is not fixed but varies over a wide range. Under these conditions, the Ayrton shunt is very useful in finding the ratio of the line currents in terms of the multiplying powers of the shunt and the deflections, or current, of the galvanometer.

Equation (66) may be expressed as follows:

$$I = I_g\left(\frac{AB}{AC}\right)\frac{R_s + R_g}{R_s} = I_g\left(\frac{AB}{AC}\right)K = I_g\frac{R_s}{R_2}K = I_gMK \tag{69}$$

where $K = 1/k$.

$(R_s + R_g)/R_s = K$ is a constant for any particular galvanometer-shunt combination. With the contact C at B, which gives maximum galvanometer current for any fixed line current, the line current $I_1 = KI_{1g}$.

From Eq. (69), with any fixed value of galvanometer current, I_g, the ratio of any two line currents I_1 and I_2, is given by

$$\frac{I_1}{I_2} = \frac{I_g M_1 K}{I_g M_2 K} = \frac{M_1}{M_2} \tag{70}$$

where M_1 and M_2 are the multiplying powers of the shunt. The shunt markings are usually the ratios AC/AB, such as 0.001, 0.01, etc., so that the multiplying powers are the reciprocals of these markings, as 1,000, 100, etc.

The values of two different line currents I_1 and I_2, in terms of galvanometer current, are determined as follows:

The galvanometer deflection D is proportional to the galvanometer current. Thus, the deflections D_1 and D_2 corresponding to galvanometer currents I_{1g} and I_{2g} are

$$I_{1g} = sD_1 \quad \text{and} \quad I_{2g} = sD_2 \qquad \text{(I) and (II)}$$

where s is a constant.

The corresponding line currents from Eq. (69) are

$$I_1 = KM_1 I_{1g} \quad \text{and} \quad I_2 = KM_2 I_{2g}. \qquad \text{(III) and (IV)}$$

Hence, from Eqs. (I), (II), (III), (IV),

$$\frac{I_1}{I_2} = \frac{M_1 I_{1g}}{M_2 I_{2g}} = \frac{M_1 D_1}{M_2 D_2}. \tag{71}$$

For example, let I' be the line current and I_g' the galvanometer current with the shunt set at 0.1. Let I'' be the line current and I_g'' the galvanometer current with the shunt set at 0.01. Then, from Eq. (69),

$$I' = I_g' \frac{R_s}{0.1R_s} K = 10KI_g' \tag{V}$$

and

$$I'' = I_g'' \frac{R_s}{0.01R_s} K = 100KI_g''. \tag{VI}$$

Dividing Eq. (V) by Eq. (VI) gives

$$\frac{I'}{I''} = \frac{10I_g'}{100I_g''} = \frac{10D'}{100D''}. \tag{72}$$

The deflections D' and D'' of the galvanometer being proportional to the currents producing them, 10 and 100 are the multiplying powers of the shunt for settings at 0.1 and 0.01.

It will be noted that Eq. (72) is independent of the resistances of the galvanometer and shunt and involves only the galvanometer deflections and the fractional settings of the shunt. Application of this relation will be found on p. 146.

The advantages of the Ayrton shunt are:

Within the limits of the required sensitivity, the shunt is applicable to any galvanometer, regardless of the galvanometer resistance.

A fixed resistance is shunted across the galvanometer, which gives a constant value of damping in open-circuit ballistic measurements (see Par. 230, p. 347).

Example.—The resistance of a galvanometer is 800 ohms, and a 10,000-ohm Ayrton shunt is employed in conjunction with it. The shunt is marked in decimal proportions from 0.0001 to 1. The sensitivity of the galvanometer is such that a current of 8×10^{-8} amp. produces full-scale deflection of 50 cm. The deflections are proportional to the current.

(*a*) When the line current is 2.4×10^{-5} amp., determine the necessary setting of the shunt and the corresponding deflection. (*b*) Determine the line current which will produce 0.5-cm. deflection with the shunt set at maximum sensitivity. (*c*) Determine the reduction in the ultimate sensitivity of the galvanometer due to the presence of the shunt.

With the shunt set at 0.0001 the galvanometer deflection is 18.5 cm. due to the current through a certain resistance. A second resistance is substituted and the deflection is 5.2 cm. with the shunt set at 0.1. Determine: (*d*) the ratio of the first to the second current; (*e*) the absolute value of each line current.

(*a*) From Eq. (I) or (II),

$$8 \times 10^{-8} = s \times 50$$

and

$$s = \frac{8}{50}10^{-8} = 1.6 \times 10^{-9}.$$

As a trial, assume that 1/1,000 the line current flows through the galvanometer. This gives a galvanometer current of 2.4×10^{-8} amp., which is less than 8×10^{-8} amp., the current corresponding to full-scale deflection of the galvanometer. Hence, the shunt may be set at 0.001 ($= AC/AB$) *Ans.*

Then, from Eq. (66),

$$I_g = 2.4 \times 10^{-5}(0.001)\left(\frac{10,000}{10,800}\right) = 2.22 \times 10^{-8} \text{ amp.}$$

From Eq. (I) or (II),

$$2.22 \times 10^{-8} = 1.6 \times 10^{-9}D.$$

$$D = \frac{2.22 \times 10^{-8}}{1.6 \times 10^{-9}} = 13.9 \text{ cm.} \quad \textit{Ans.}$$

(*b*) 0.5 cm. corresponds to a galvanometer current of

$$\frac{0.5}{50}8 \times 10^{-8} = 8 \times 10^{-10} \text{ amp.}$$

For maximum sensitivity, $M = 1$. Hence, from Eq. (69),

$$I = 8 \times 10^{-10}\left(\frac{10,800}{10,000}\right) = 8.64 \times 10^{-10} \text{ amp.} \quad \textit{Ans.}$$

(*c*) From Eq. (67).

$$k = \frac{10{,}000}{10{,}800} = 0.926. \quad \textit{Ans.}$$

(*d*) $M_1 = 10{,}000$ and $M_2 = 10$. Hence, from Eq. (71),

$$\frac{I_1}{I_2} = \frac{10{,}000 \times 18.5}{10 \times 5.2} = 3{,}560. \quad \textit{Ans.}$$

(*e*) From Eqs. (I) and (II), p. 145.

$$I_{1g} = 1.6 \times 10^{-9} \times 18.5 = 29.6 \times 10^{-9} \text{ amp.}$$
$$I_{2g} = 1.6 \times 10^{-9} \times 5.2 = 8.32 \times 10^{-9} \text{ amp.}$$

From Eqs. (III) and (IV),

$$I_1 = 1.08 \times 10{,}000 \times 29.6 \times 10^{-9} = 3.20 \times 10^{-4} \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = 1.08 \times 10 \times 8.32 \times 10^{-9} = 8.99 \times 10^{-8} \text{ amp.} \quad \textit{Ans.}$$
$$\frac{I_1}{I_2} = \frac{3.20 \times 10^{-4}}{8.99 \times 10^{-8}} = 3{,}560 \ (\textit{check}).$$

106. Early Indicating Instruments.—In general the measurement of current and voltage requires instruments that are rugged as well as portable, and at the same time their precision should be high. The galvanometer itself obviously is not adapted to such measurements. In the early days of electrical engineering, the portable and switchboard types of instruments depended for their operation on the pull exerted by a solenoid on some type of iron plunger. The amount of pull depends on the value of the current in the solenoid. Hence, by restraining the movement of the plunger by some means, as by gravity or a spring, the deflection of a pointer attached to the plunger might be made to indicate amperes. Figure 91 shows a typical instrument of this class. Inherently such an instrument is inaccurate, because it is crude and clumsy. Also, there is error due to magnetic hysteresis or magnetic lag in the iron core, and for a given current there results a higher reading for decreasing values of current than for increasing values. The weight of the plunger makes it difficult

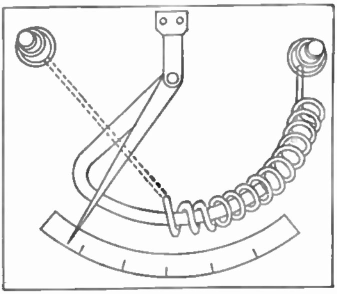


FIG. 91.—Early type of plunger ammeter.

if not impossible to mount the moving system so that the friction error is negligible, and as the instrument is not damped, it fluctuates violently on slightly fluctuating current. Stray magnetic fields acting on the plunger also cause errors. These difficulties are overcome in the Weston type of instrument.

107. Weston-type Instrument.—For the measurement of direct current and voltage, the Weston-type instrument is in universal use. The instrument is based on the principle of the D'Arsonval

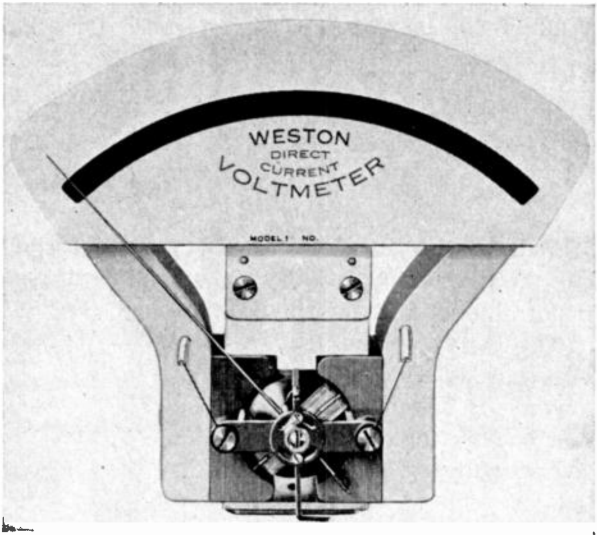


Fig. 92.—Movement of Weston instrument.

galvanometer but is so constructed that it is portable, and is provided with a pointer and scale for indicating the deflections of the moving coil. The same instrument movement may be used as either ammeter or voltmeter. For use as ammeter, except for extremely small currents, it is provided with a shunt through which the greater portion of the current flows; for use as voltmeter, a resistance is connected in series with the moving coil so that the instrument can be connected across the line without taking excessive current.

The essential parts of the instrument are shown in Fig. 92. As in the D'Arsonval galvanometer, a permanent magnet is necessary, being made in horseshoe form. Two soft-iron pole pieces are fitted to the magnet poles, and a cylindrical core is held between these pole pieces by a strip of brass. This gives uniform air-gaps and a radial field. The length of the air-gap is very much shorter than is usual with D'Arsonval galvanometers. The moving coil consists of very fine silk-covered copper wire wound on an aluminum bobbin The aluminum bobbin, besides supporting the coil mechanically, makes the instrument highly damped. This damping is due to the currents induced in the aluminum because of its cutting the magnetic field.

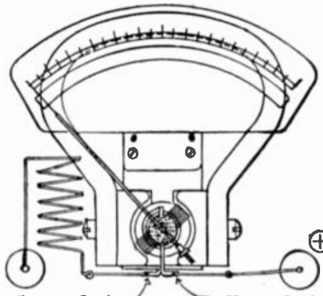


FIG. 93.—A typical Weston direct-current millivoltmeter.

Instead of suspending the coil by a filament, it is supported at the top and bottom by hardened-steel pivots turning in cup-shaped jewels, usually sapphire. This method of supporting the moving coil is almost frictionless and makes the instrument portable, whereas the D'Arsonval galvanometer is not portable. The current is led in and out of the coil by two flat spiral springs, one at the top of the coil and the other at the bottom. These springs also serve as the controlling device for the coil, any tendency of the coil to turn being opposed by these two springs. The top and bottom springs are coiled in opposite directions so that the effect of change of temperature, which causes a spiral spring to coil or uncoil, will not cause the needle to change its zero position. A very light and delicate aluminum pointer is attached to the moving element to indicate the deflection of the coil. The pointer is carefully balanced by very small counterweights so that the whole moving element holds its zero position very closely, even if the instrument is not level. The pointer moves over a graduated scale, which may be marked in volts or in amperes as the case may be. Because of the radial field, the deflection of the moving coil in this type of instrument is practically proportional to the current in the moving coil, so that the scale of the instrument has substantially uniform graduations, which is desirable. The internal connections of a Weston instrument are shown in Fig. 93.

Instruments of this construction having very weak springs are often used for portable galvanometers. Although lacking the extreme sensitivity of the suspended type, they can be made sufficiently sensitive for certain classes of work, and their ruggedness and portability make them very useful. Such a galvanometer is shown in Fig. 94.

FIG. 94.—Weston portable galvanometer and microammeter.

108. Ammeters.—The moving coils of Weston portable instruments deflect to the full-scale value with from 0.01 to 0.05 amp. in the coil, depending on the condition of use. When the instrument movement is designed for use in an ammeter, the coil should be wound for low resistance so that the voltage drop across the instrument is low. The ammeter then will not introduce appreciable resistance when connected in the circuit. Hence, the cross-section of the wire in the coil should be as large as is practicable, and the number of turns will be correspondingly small. When so wound, currents of from 0.02 to 0.05 amp. produce full-scale deflection. In order to measure currents greater than this, a portion of the current must be diverted from the moving coil by a *shunt*. The shunt is merely a low resistance, usually made of manganin strip M brazed to comparatively

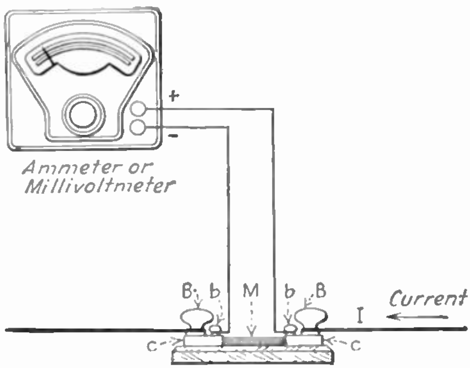


FIG. 95.—Ammeter with an external shunt.

heavy copper blocks *cc* as shown in Fig. 95. Two sets of binding nuts are fastened to the copper blocks. The heavy wing nuts *BB* are for carrying the *main* current through the shunt. The small posts *bb* are used to connect the ammeter leads. The copper blocks serve two purposes. They are an excellent conductor of heat and so carry the heat away from the manganin strip,

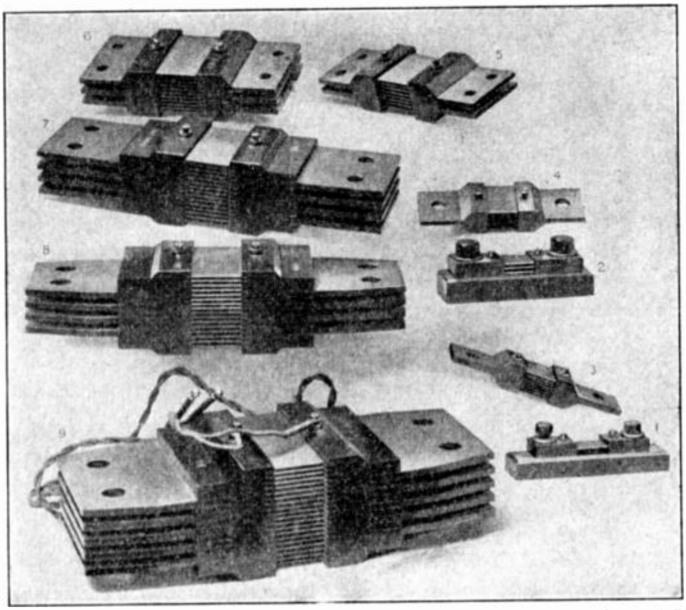


FIG. 96.—Ammeter shunts. No. 1 from 25 to 200 amp. No. 9 from 4,500 to 6,000 amp.

and their low resistance keeps all parts of each copper block at very nearly the same potential. The ammeter is in reality a voltmeter reading the voltage drop across a resistance. A complete set of shunts, with their current ratings, is shown in Fig. 96. The heavy copper terminals for connection to bus-bars should be noted.

The voltage drop across the shunt is

$$V_{sh} = I_{sh}R_{sh}$$

where I_{sh} and R_{sh} are the shunt current and the shunt resistance. If R_{sh} is constant, the voltage drop across the shunt is proportional to the current in the shunt, so that the instrument readings are proportional to the current in the shunt. For this reason, the ammeter itself (Fig. 93) is often marked "millivoltmeter." For full-scale deflection, the drop across a shunt is about 50 millivolts. The current taken by the instrument itself is usually from 0.02 to 0.05 amp., so that it is almost always negligible compared with the main current. Therefore, in most cases the line current practically equals the shunt current.

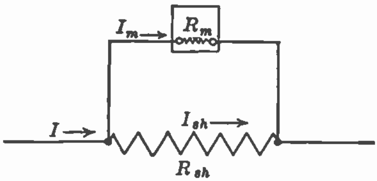


FIG. 97.—Division of current between an ammeter and its shunt.

An ammeter and its shunt may be considered also as a divided circuit. In Fig. 97, let R_{sh} and I_{sh} be the shunt resistance and shunt current, and let R_m and I_m be the instrument resistance and instrument current. By the law of divided circuits,

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}},$$

that is, the current divides between the instrument and the shunt inversely as their resistances.

Example.—Assume that an instrument has a resistance of 4 ohms, the shunt a resistance of 0.0005 ohm, and that the line current is 90 amp. What is the value of the instrument current?

As the current in the line differs from the shunt current by a very small amount, the two may be assumed equal. Then,

$$\frac{90}{I_m} = \frac{4}{0.0005},$$

$$I_m = 0.01125 \text{ amp.} \quad \textit{Ans.}$$

For accuracy, the current must always divide between the instrument and the shunt in a fixed ratio. This means either that the resistance of the shunt and the resistance of the instrument must not change at all or that both must change in the same ratio. As the shunt operates at a higher temperature than the instrument, it should be made of a metal such as manganin (see p. 19) whose resistance does not change appreciably with temperature. The resistance of the instrument circuit should also remain constant. The resistance of the leads connecting the shunt to the instrument should remain constant, and the leads with which the instrument is calibrated should always be used to connect the shunt to the instrument. The lugs and binding-post contacts should be kept clean from oxide and dirt. A low adjustable resistance (the spiral, Fig. 93) is connected inside the instrument. By varying this resistance, the instrument is adjusted to its shunt.

An ammeter with an external shunt may be made to have a large number of scales or ranges. Assume that an instrument gives full-scale deflection when the instrument current is 0.02 amp. and that its resistance is 2.5 ohms. The resistance of the shunt is 0.0005 ohm. The voltage across the instrument terminals is $0.02 \times 2.5 = 0.050$ volt, or 50 millivolts. Dividing this voltage by the shunt resistance, the shunt current is

$$I = \frac{0.050}{0.0005} = 100 \text{ amp.}$$

The instrument then deflects full scale with 100 amp. in the line.

If the foregoing shunt be replaced by one having a resistance of 0.005 ohm, the 50 millivolts drop across the shunt may be obtained with 10 amp. ($10 \times 0.005 = 0.050$). Therefore, a 10-scale ammeter results. By the choice of suitable shunts, the same instrument may be made to give full-scale deflection with 1 amp. and with 5,000 amp. For example, all the shunts shown in Fig. 96 could be used with the same instrument and as many different scales obtained thereby.

In the smaller sizes of instruments up to 50 amp. and where only one scale is desired, the shunt is usually placed within the instrument. For ranges between 50 and 100 amp., the use of an internal or an external shunt is optional. Above 100 amp., it is usual to have the shunt external to the instrument on account of its size and its heating loss.

An ammeter can usually be distinguished from a voltmeter by the fact that the ammeter's binding posts are heavy and are of bare metal, except in the case of an instrument having an external shunt. The binding posts of millivoltmeters and voltmeters are of much lighter construction and the metal posts are covered with hard rubber, mostly for insulation purposes. Since the binding posts of millivoltmeters and voltmeters are usually identical, *care should be taken not to mistake a millivoltmeter for a voltmeter and use it as a voltmeter.* Even a comparatively low voltage across a millivoltmeter will injure it seriously.

An instrument when connected in a circuit should disturb the circuit conditions as little as possible. An ammeter shunt, as it goes in series with the line, should have as low a resistance as is practicable, so that when it is connected, very little additional resistance is introduced into the circuit. To protect ammeters from heavy currents, provision may be made for *short-circuiting* them when readings are not being taken.

109. Voltmeters.—The construction of a voltmeter does not differ materially from that of an ammeter in so far as the move-

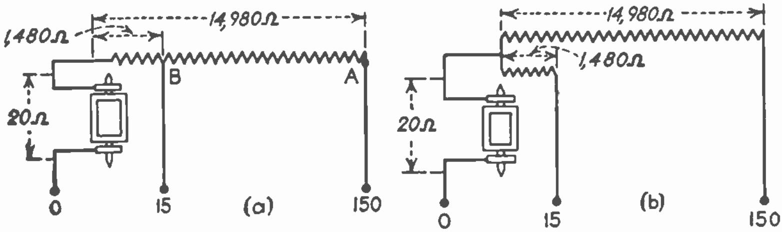


Fig. 98.—Methods of connecting a voltmeter resistance.

ment is concerned (see Fig. 92). The moving coil of the voltmeter is usually wound with more turns and of finer wire than that of the ammeter and so has a higher resistance. The principal difference, however, lies in the manner of connecting the instrument to the circuit. As a voltmeter is connected directly across the line to measure the voltage, it is desirable that the voltmeter take as little current as is practicable. Because of its comparatively low resistance, the moving coil of the voltmeter cannot be connected directly across the line, as it would ordinarily take an excessive current and be burnt out. Therefore it is necessary to connect a high resistance in series with the moving coil, as shown in Fig. 98. By Ohm's law the current through the instrument is proportional to the voltage at its terminals, so that the instrument scale can be graduated in volts. The resistance required is easily determined. Assume that an instrument gives full-scale deflection with 0.01 amp. in the moving coil, and that the coil resistance is 20 ohms. If it is desired that the instrument indicate 150 volts, full scale, the total resistance of the instrument circuit must be

$$R = \frac{V}{I} = \frac{150}{0.01} = 15{,}000 \text{ ohms.}$$

As the instrument has a resistance of 20 ohms, 14,980 ohms additional are necessary (Fig. 98(*a*)). If it be desired that this same instrument also have full-scale deflection with 15 volts, the resistance of 14,980 ohms may be tapped so that the resistance OB (Fig. 98(*a*)) $= 15/0.01 = 1{,}500$ ohms, and this tap can be brought to a binding post. Another method of securing the same result is shown in Fig. 98(*b*). A separate resistance equal to $1{,}500 - 20 = 1{,}480$ ohms is connected from the 15-volt binding post to the junction of the resistance and the moving coil. This last method is advantageous in that it permits independent adjustment of each resistance; also injury or repair in one resistance does not affect the other.

110. Multipliers or Extension Coils.—The range of a voltmeter, having its resistance incorporated within the instrument, may be increased by the use of external resistance connected in series with the instrument.

Example.—A 150-scale voltmeter has a resistance of 17,000 ohms. What external resistance should be connected in series with it in order that its range may be (*a*) 300 volts? (*b*) 600 volts?

(*a*) In order to obtain full-scale deflection at 300 volts, the voltmeter current must be equal to the current which produces full-scale deflection at 150 volts. Accordingly, the total resistance of the instrument for 300 volts must be twice the 150-volt value. Therefore, a total resistance of $17{,}000 \times 2 = 34{,}000$ ohms is necessary. As the instrument already has 17,000 ohms, the added resistance will be

$$34{,}000 - 17{,}000 = 17{,}000 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) The total resistance must now be

$$\frac{600}{150} \times 17{,}000 = 68{,}000 \text{ ohms.}$$

As 17,000 ohms is already within the instrument, $68{,}000 - 17{,}000 = 51{,}000$ ohms must be added external to the instrument. *Ans.*

External resistances used in this manner are called *multipliers*, or sometimes *extension coils*. They are usually placed within a perforated box and the terminals brought out to binding posts. The multiplying power of the multiplier is marked near a terminal.

The equation giving the relation of the resistance of the multiplier R_x, the resistance of the instrument R_m, and the multiplying power M is as follows:

$$M = \frac{R_x + R_m}{R_m}. \tag{73}$$

Example.—In the above example (*b*), the multiplying power of the multiplier is as follows:

$$M = \frac{51{,}000 + 17{,}000}{17{,}000} = 4.$$

111. Hot-wire Instruments.—In the instruments thus far considered, the action of the instrument depends on the electrodynamic action of the current. There is another type of instrument which depends for its indications on the *heating* action of the current. A diagram of this instrument is shown in Fig. 99. AB is a fine wire of platinum-silver through which the current flows. At C, a wire CF is attached to AB. At E, on CF, a silk fiber EH is attached. This passes around the pulley W and is held in tension by the spring H. When current flows through AB, the heat expands the wire AB, reducing the tension in the wire CF and allowing the spring H to pull the silk fiber to the left. This fiber, acting on the pulley W, moves the pointer P over the scale.

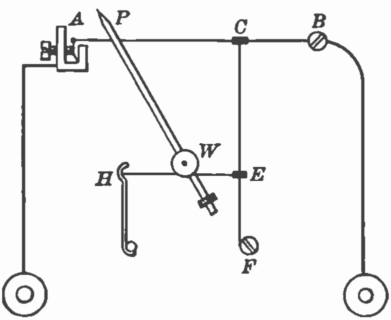


FIG. 99.—Principle of Hartmann and Braun hot-wire instruments.

When used as an ammeter, a shunt is necessary unless the current is very small. When used as a voltmeter, a high resistance is connected in series with the wire AB.

This type of instrument is "dead beat," that is, it is very sluggish in its behavior and it comes slowly to its ultimate deflection. This is an advantage in the measurement of fluctuating currents, as the needle follows the fluctuations very slowly and can be accurately read. Another advantage of the hot-wire type of instrument is that it can be used for alternating as well as for direct currents. It is often used as a transfer instrument to measure alternating currents in terms of direct current. This type of instrument is particularly useful for the measurement of high-frequency alternating currents, as its indications are independent of the frequency, provided a shunt is not used. For this reason this type is useful for measuring currents in radio circuits. Such instruments are affected by temperature and do not hold their calibration for long periods. Therefore, for accurate work they should be calibrated at the time of use.

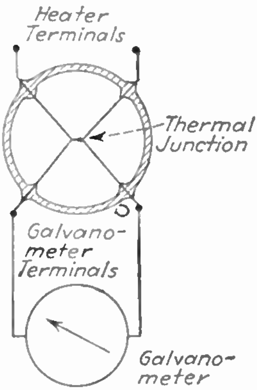


FIG. 100.—Vacuum thermocouple and galvanometer.

112. Vacuum Thermocouple.—The vacuum thermocouple in conjunction with an associated galvanometer or sensitive deflecting instrument is a current-measuring device also based on the heating effect of the current. A very small thermal junction is in thermal contact with a small resistor or heater through which the current to be measured flows (Fig. 100). The junction and the small connecting wires are sealed within a small evacuated glass bulb, thus reducing heat dissipation from the junction. The thermal-junction terminals are connected to a galvanometer or to some other sensitive type of D'Arsonval instrument. The current in the heater circuit raises the temperature of the thermal junction and as a result of the thermal e.m.f. the indicating instrument deflects. The deflections are practically proportional to the square of the current. Since the capacitive and inductive effects in the heater circuit are practically nil, such instruments are well adapted to radiofrequency measurements. The heater circuit is delicate, and its rating is limited to very small currents. With large currents a shunt is necessary. This type of thermal instrument is far more precise and sensitive than the hot-wire type of instrument described in Par. 111.

(The wattmeter and the watthour meter are described on pp 193 and 194.)

ELECTRICAL MEASUREMENTS

Measurement of Resistance

113. Voltmeter-ammeter Method.—The resistance of any portion of an electric circuit which does not contain a source of e.m.f. is, by Ohm's law,

$$R = \frac{V}{I}$$

where V is the voltage across that portion of the circuit and I is the steady current flowing in that portion of the circuit.

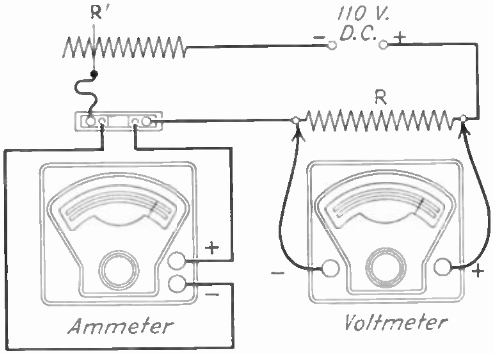


Fig. 101.—Voltmeter-ammeter method of measuring resistance.

Obviously, the voltage V may be measured with a voltmeter, the current I measured with an ammeter, and the resistance R computed.

Let it be required to determine the resistance R in the circuit shown in Fig. 101. The source of power is the 110-volt supply. The resistance R is comparatively small and if connected directly across 110 volts would take an excessive current. Therefore, it is necessary to insert a resistance R' in series with R to limit the current. The voltmeter, however, must be connected directly across R as it is desired to know the resistance of this portion of the circuit only.

Example.—The voltmeter (Fig. 101) reads 19 volts when the ammeter reads 24 amp. What is the value of the resistance R?

The resistance is

$$R = \frac{19}{24} = 0.792 \text{ ohm.} \quad \textit{Ans.}$$

As a matter of interest let it be required to determine the resistance of R'. The voltmeter terminals are transferred from across R to across R'. Under these conditions the voltmeter reads 91 volts and the ammeter still reads 24 amp. Therefore,

$$R' = \frac{91}{24} = 3.79 \text{ ohms.}$$

Elimination of Contact Resistance.—It is sometimes desired to measure resistances of such low value that, if a voltmeter were connected directly across their terminals, the contact resistance, which may be comparatively large, would introduce considerable error and might even exceed in magnitude the resistance which

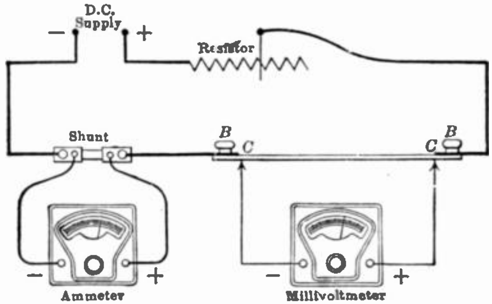


FIG. 102.—Measuring the resistance of a metal rod.

it is desired to measure. To eliminate this error due to contact resistance, the voltmeter terminals are connected well inside the terminals BB (Fig. 102) through which the current is led to the specimen. Since the voltmeter takes but a very small current, small sharp-pointed contacts CC may be used. As the resistance of the voltmeter is comparatively high, it is only necessary that the contact resistances at CC be negligible compared with the resistance of the instrument. This condition is easily met. As these contacts are small and sharp, the points of contact on the specimen can be determined very accurately.

Example.—When the ammeter (Fig. 102) reads 50 amp., the millivoltmeter indicates 40 millivolts. The contacts CC are 23 in. apart. What is the resistance per inch length of the rod?

The resistance for 23 in. is

$$R = \frac{0.040}{50} = 0.00080 \text{ ohm.}$$

The resistance per inch is

$$R = \frac{0.00080}{23} = 0.0000348 \text{ ohm.} \quad \textit{Ans.}$$

114. Voltmeter Method.—It is possible to measure a resistance by means of a voltmeter alone provided the resistance to be measured is comparable with that of the voltmeter. In Fig. 103(a), let it be required to measure the resistance R. The voltmeter is first connected across the source of supply and a reading V_1 taken. It is then transferred so that the resistance R is in series with it across the source of supply and the voltmeter reading is again taken. Let this reading be V_2.

As V_1 is the total circuit voltage and V_2 is the voltage across the instrument, the voltage across the unknown resistance R is obviously $V_1 - V_2$. When the voltmeter is in series with R, the same current i must flow through each so that the voltages are as follows:

$$V_2 = iR_v \tag{I}$$

$$V_1 - V_2 = iR \tag{II}$$

where R_v is the resistance of the voltmeter.

Dividing Eq. (II) by Eq. (I) and solving for R,

$$R = R_v\frac{V_1 - V_2}{V_2}. \tag{74}$$

If desired, the voltage V_1 may be measured with a second voltmeter. This makes it possible to increase the voltage V_1 above the range of the voltmeter in series with R and thus increases the sensitivity of the measurement.

This method of measuring resistance is particularly useful in determining insulation resistance, such as for dynamo windings or cables. As such resistances are very high, they are usually expressed in megohms (1 megohm = 1,000,000 ohms). It will be seen from Eq. (74) that the greater the value of R_v the greater the resistance that can be measured by this method. For this reason special 150-scale voltmeters, having resistances of 100,000 ohms (0.1 megohm) are available. These give a sensitivity about six times as great as can be obtained with the ordinary 150-scale voltmeter.

Figure 103(*b*) shows the application of this method to the measurement of the insulation resistance of a cable. The ground connection to the sheath represents the condition usually met in practice where the lead sheath is in actual contact with the ground and is frequently bonded to pipes and other grounded systems. Accordingly, if one side of the supply system is grounded, as is frequently the case, *that side* should be connected to the sheath. (Theoretically, under these conditions no connection from the grounded conductor to the sheath is necessary,

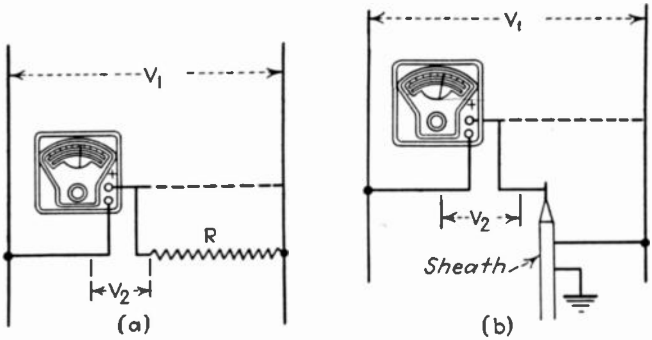


FIG. 103.—Measurement of resistance by the voltmeter method.

but nevertheless such connection should be made to avoid any uncertainty.)

If any one of the conductors of the supply system is grounded, the connecting of the grounded sheath to any conductor other than the grounded one will result in a *short circuit.* If the cable is isolated, that is, if it is coiled on a reel and the sheath is insulated from ground, the ground connection shown in Fig. 103(*b*) may be removed.

Example.—When a 100,000-ohm voltmeter is connected across a direct-current line it reads 120 volts. One terminal of the voltmeter is then connected to the core of a lead-covered cable and the sheath of the cable is connected to the other side of the line as in Fig. 103(*b*). The voltmeter now reads 10 volts. What is the insulation resistance of the cable?

$$X = 0.1\frac{120 - 10}{10} = 1.1 \text{ megohms.} \quad \textit{Ans.}$$

115. Megger.—The megger is an instrument of the ohmmeter type by means of which the measured value of a resistance is directly indicated on a scale. In the ohmmeter type of instrument the indication must be independent of the voltage. The megger consists of two primary elements, a direct-current generator of the magneto type, usually hand driven, which supplies the current for making the measurement; and the instrument movement by means of which the value of the resistance under measurement is indicated.

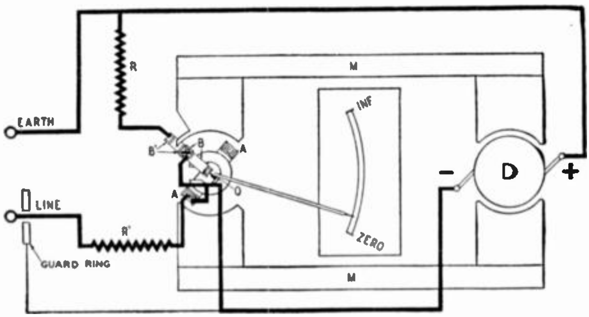
A diagram of the instrument is shown in Fig. 104(*a*). *MM* are two parallel permanent bar magnets. The generator armature *D*, together with its iron pole pieces, is bridged across one pair of poles of the two magnets and the iron pole pieces and the core of the instrument movement are bridged across the other pair of poles. The armature of the generator is usually hand driven, its speed being stepped up through gears. For ordinary insulation testing, 500 volts is most common, but in order to apply a simultaneous high-test potential, ratings as high as 2,500 volts are available.

The instrument system consists primarily of two coils *A* and *B*, mounted on the same moving system (see Fig. 104(*b*)). Coil *A*, called the *current coil*, is identical with the moving coil of the Weston movement (Fig. 92, p. 148); one terminal is connected to the negative brush of the generator and the coil is in series with the resistance *R′* to the LINE terminal. When a resistance is connected from the line to the EARTH terminal, the coil *A*, the resistance *R′*, and the resistance under measurement are all in series across the armature brushes.

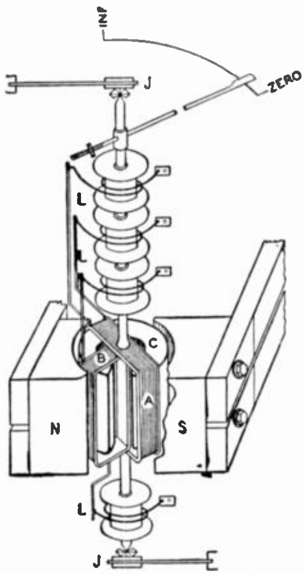
Coil *B*, called the *potential coil*, is connected across the armature brushes in series with a suitable resistance *R*. Coil *B* is narrower than coil *A* and moves so that in some positions it encircles a part of the C-shaped iron core *C* in the manner shown in Fig. 104(*b*).

The moving system (Fig. 104(*b*)) is mounted in spring-supported jewel bearings *J* and is free to rotate on its axis, there being no restraining force such as the spring used in the Weston ammeter and voltmeter. Current is led to the coils by flexible conducting ligaments *LLL* having negligible tension. Hence, when the generator is not being operated, the pointer "floats" over the scale and under these conditions may remain in any position whatsoever.

If the line and earth terminals are open-circuited, or if a resistance substantially infinite in value is connected across the



(a) Magnetic circuit and electrical connections.



(b) Moving element.
FIG. 104.—Megger construction.
(*Courtesy of James G. Biddle Company, Philadelphia.*)

terminals, no current flows in coil *A*, so that the potential coil *B* alone controls the movement of the moving system. Coil *B* will take a position opposite the gap in the core *C* and the pointer will indicate INF (infinity). When, however, resistance is connected between the terminals, current flows in coil A and the corresponding torque will draw the system away from the *infinity* position into a field of gradually increasing strength, until equilibrium between the torques of coils A and B is reached. Hence the scale may be calibrated in terms of resistance, and the instrument becomes direct reading. Since changes in voltage affect both the coils A and B in the same proportion, the position of the moving system is independent of the voltage. Should

Fig. 105.—Standard high-range megger testing set. *(Courtesy of James G. Biddle Company, Philadelphia.)*

the instrument become short-circuited, the ballast resistance R' is sufficient to protect the current coil.

In order to obtain better scale proportions and to make the instrument astatic, that is, not affected by stray fields, compensating coil B' (Fig. 104(a)) is added to the moving system. This coil reacts with one of the pole tips. It is wound in an astatic relation with coil B and is connected in series with it.

The guard wire (Fig. 104(a)) should be noted. It performs the same function as the guard wires of Fig. 116 (p. 180). Any leakage current over the terminals or within the megger itself is shunted to the negative terminal of the generator without going through coil A and so does not affect the indications of the instrument. Usually a terminal is provided by means of which this guard wire may be connected to a guard wire on the insulation under measurement (see Fig. 105).

If the megger is used to measure the resistance of devices having relatively large capacitance, fluctuations of voltage affect the current in resistance R', in series with the capacitance, to a greater extent than fluctuations of voltage affect the current in the pure resistance R. For this reason, some types of megger are provided with a clutch which slips when a certain maximum speed of the armature is reached. Therefore, so long as the driving speed exceeds this value, the speed of the armature and hence the e.m.f. remain constant, and the difficulty due to capacitance is eliminated.

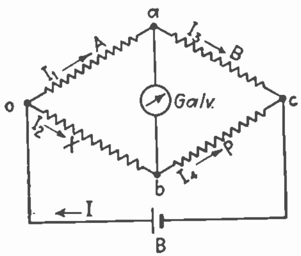


FIG. 106.—Elementary Wheatstone bridge.

The resistance range of meggers is very great. For insulation measurements, their range is in thousands of megohms. They are also designed to measure resistance of only a few ohms, such as the resistance to ground of transmission-tower footings or ground wires. The megger is widely used for measuring insulation resistance, as the resistance between the windings and the frame of electric machinery, the insulation resistance of cables, of insulators, or of bushings.

Figure 105 shows an exterior view of a megger.

116. Wheatstone Bridge.—In distinction to the foregoing methods of measuring resistance, the Wheatstone-bridge method is one in which the *unknown* resistance is balanced against other *known* resistances. The bridge, in its simplest form, is shown in Fig. 106. Three known resistances A, B, P and the unknown resistance X are connected to form a diamond. The arms A and B usually consist only of decimal values of resistance such as 1, 10, 100, 1,000 ohms. The arm P is adjustable, usually so that integral values of resistance from 1 ohm to as high as 11,000 ohms and more may be obtained. A battery B is connected to the two opposite corners o and c of the diamond. Across the other two corners a and b a galvanometer is connected.

To make a measurement, each of the two arms A and B is set at some fixed value of resistance, usually at some decimal value, such as 1, 10, 100, 1,000 ohms. The arm P is then adjusted until the galvanometer does not deflect. If the galvanometer does not deflect, no current flows through it and therefore the two points a and b must be at the *same potential*. Since no current flows through the galvanometer, $I_1 = I_3$ and $I_2 = I_4$.

If the points a and b are at the same potential, the voltage drop $oa = ob$ and

$$I_1A = I_2X. \tag{I}$$

Also the voltage drop $ac = bc$ and

$$I_3B = I_4P.$$

And since $I_1 = I_3$ and $I_2 = I_4$,

$$I_1B = I_2P. \tag{II}$$

Dividing Eq. (I) by Eq. (II),

$$\frac{I_1A}{I_1B} = \frac{I_2X}{I_2P} \quad \text{or} \quad \frac{A}{B} = \frac{X}{P};$$

$$X = \frac{A}{B}P, \tag{75}$$

which is the equation of the Wheatstone bridge. A and B are called the *ratio arms* and P the *balance* or *rheostat arm*. Obviously the battery and the galvanometer may be interchanged without affecting the relation given in Eq. (75).

The many types of Wheatstone bridge found in practice do not differ in principle from that shown in Fig. 106. The differences lie in the geometrical positions of the arms A, B, P on the bridge, as well as in the manner in which the coils in these arms are cut in and out of circuit.

Most of the early bridges were of the plug type, the function of the plugs being to short-circuit the resistance units when not in use. Oxide and dirt readily accumulated on the plugs and they required frequent cleaning in order that the contact resistance should not be excessive. Moreover, the use of plugs in obtaining a balance was inconvenient and tedious.

Plug-type bridges have been superseded by the dial type employing the decade arrangement of resistances.

117. Decade and Dial Bridges.—The decade arrangement of the resistance units of the rheostat arm P is shown in Fig. 107. The resistances are arranged in groups of equal resistances, one group consisting of ten 1-ohm coils, the next of ten 10-ohm coils, the next of ten 100-ohm coils, etc.

Each group is called a *decade*. Only one plug per decade is necessary. This arrangement has the advantage that the plugs are always in service and so are not so likely to be mislaid or to become dirty; there is less probability of error in reading; it is a simple matter to see that the few plugs used are fitting tightly, and a balance can be quickly obtained. It is obvious that nine coils per decade are sufficient for obtaining any desired resistance, although 10 coils per decade are often used.

The decade principle has been extended to an even more convenient type of bridge, the dial bridge. Instead of using plugs, a dial arm similar to the type used in rheostats is employed to select the required resistances.

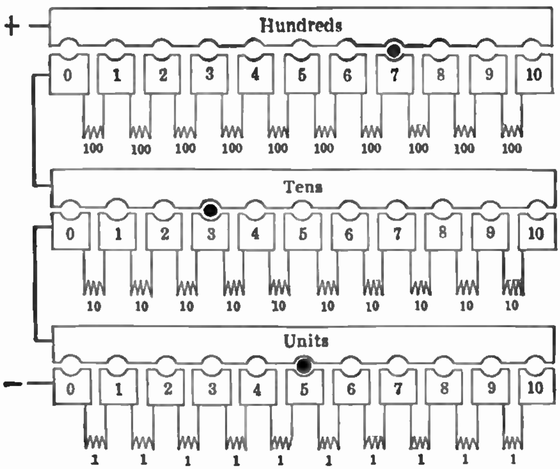
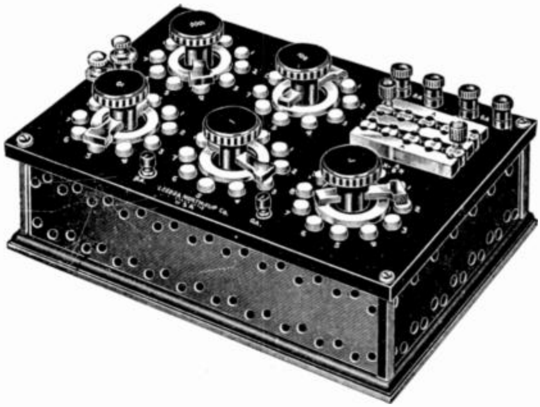


FIG. 107.—Arrangement of rheostat arm resistances in a decade bridge.

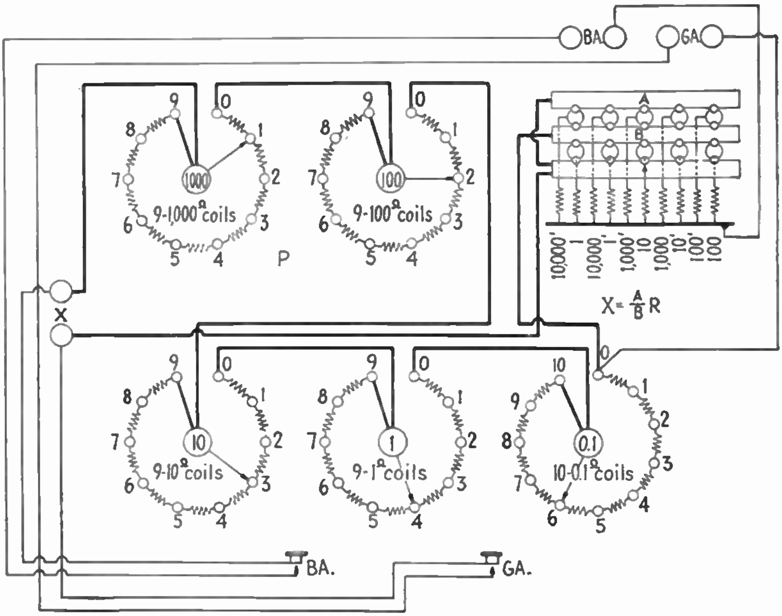
Because of its ease of manipulation, this type has come into extensive use. Care should be taken to keep the dials and contacts free from dirt and oxides. Figure 108(*a*) shows a five-dial Wheatstone bridge of the Leeds and Northrup type and Fig. 108(*b*) shows the connections.

118. Method of Balancing a Bridge.—In using the bridge, much time may be saved if a systematic procedure is followed in obtaining a balance. Assume that it is desired to measure a certain unknown resistance. The battery and galvanometer are connected to their respective terminals *BA* and *GA* (Fig. 108), and the unknown resistance is connected at *X*. A shunt should be used with the galvanometer (Par. 105, p. 141) to protect it from deflecting violently when the bridge is considerably out of balance. The ratio arms *A* and *B* should at first be given a one-to-one ratio, preferably 1,000 ohms each.

With the galvanometer well shunted and all the dials in *P* set to 0 ($P = 0$), depress first the battery key and then the galvanometer key. The galvanometer is observed to deflect to the left. Now start turning the contactor



(a) Five-dial type of bridge.



(b) Connections of five-dial type of bridge.

FIG. 108.—Leeds and Northrup dial Wheatstone bridge.

in the 1,000 dial, depressing the battery key and then the galvanometer key at each contact. At contact 1 the galvanometer deflects to the right.

From these observations, three facts are determined. The unknown resistance is less than 1,000 ohms; when the galvanometer deflects to the left, the value of resistance in P is too small; when it deflects to the right, the value of P is too large. The contactor in the 1,000 dial is returned to zero and the contactor in the 100 dial is then moved to contact 1 and the galvanometer still deflects to the right, indicating that 100 ohms in P is too large. This procedure is repeated with 10 ohms, and then with the 1 ohm where it is found that the galvanometer deflection now reverses, that

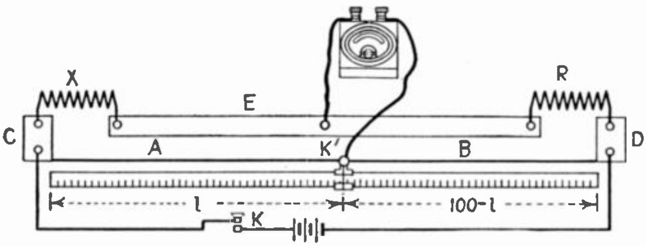


FIG. 109.—Slide-wire bridge.

is, it deflects to the left. It does not reverse and deflect to the right again until 3 ohms are selected by the contactor. Hence, the unknown resistance is narrowed down to between 2 and 3 ohms. To obtain a more precise measurement, the ratio arms must be changed. A is now made 1 ohm, B remaining at 1,000 ohms, and the dials in P are set at 2,000 ohms. By successive trials, all the time reducing the effect of the galvanometer shunt, a balance is obtained at 2,761 ohms in P. Then,

$$X = \frac{A}{B}P = \frac{1}{1,000}2,761 = 2.761 \text{ ohms.}$$

In obtaining a balance, the battery key should always be depressed before the galvanometer key, so that the current in the bridge has time to reach a constant value. Otherwise any e.m.f. of self-induction may introduce an error.

119. Slide-wire Bridge.—The slide-wire bridge is a simplified Wheatstone bridge, in which the balance is obtained by means of a slider which moves over a German-silver or manganin resistance wire. A typical slide-wire bridge is shown in Fig. 109. The resistance wire AB, 100 cm. long, is stretched tightly between two heavy copper blocks CD, 100 cm. apart. A meter scale is placed along this wire. A contact key K' is movable along the scale, and, when the key K' is depressed, a knife edge makes contact with the wire. The rest of the bridge consists of a heavy copper bar E, a known resistance R, and the unknown resistance X. R is connected between D and E, and X between C and E. The positions of R and X are interchangeable.

The galvanometer is connected between the key K' and E, and the battery terminals are connected to C and D. A balance is obtained by moving K' along the wire until the galvanometer shows no deflection.

Let l be the distance in centimeters from the left-hand end of the scale to K' when a balance is obtained. Then $100 - l$ is the distance from K' to the right-hand end of the scale. Let r be the resistance per unit length of the slide wire. Then the resistance of l is lr, and that of the remainder of the wire is $(100 - l)r$.

By the law of the Wheatstone bridge,

$$\frac{X}{lr} = \frac{R}{(100 - l)r}. \tag{76}$$

r cancels, and Eq. (76) becomes

$$X = l\frac{R}{(100 - l)}. \tag{77}$$

Equation (76) may also be written

$$\frac{X}{R} = \frac{l}{100 - l}. \tag{78}$$

This is equivalent to stating that when a balance is obtained, the slide wire is divided into two parts which are to each other as X is to R.

The slide wire is not so accurate as the coil bridge, because the slide wire may not be uniform; the solder at the points of contact at C and D makes the length of the wire uncertain, although it is possible to correct for this error; the slide wire cannot be read so accurately as the resistance units of a bridge can be adjusted.

Example.—Assume that R (Fig. 109) equals 10 ohms and that a balance is obtained at 74.6 cm. from the left-hand end of the scale. Find the unknown resistance X.

From Eq. (77),

$$X = 74.6\frac{10}{100 - 74.6} = 74.6\frac{10}{25.4} = 29.38 \text{ ohms.} \quad \textit{Ans.}$$

120. Kelvin Bridge.—In the measurement of very low resistances with the Wheatstone bridge, the contact resistances between the sample and the bridge terminals may be so large, compared with the resistance of the sample itself, that values of resistance obtained for the sample are practically worthless (also see p. 159). The effect of these contact resistances is eliminated in the Kelvin bridge (Fig. 110). A and B are two ratio arms, similar to those of the usual type of bridge. The values of A and B may be made so high that the contact resistances at p' h, d are negligible in comparison.

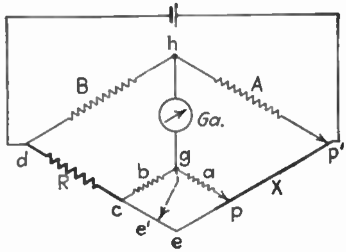


FIG. 110.—Wiring diagram of the Kelvin double bridge.

Let X be the unknown resistance, which may be the resistance included between two knife-edge contacts $p'p$ on a copper rod as indicated in Fig. 110. The arm R or the resistance between points c and d is the rheostat arm. The resistance of this arm

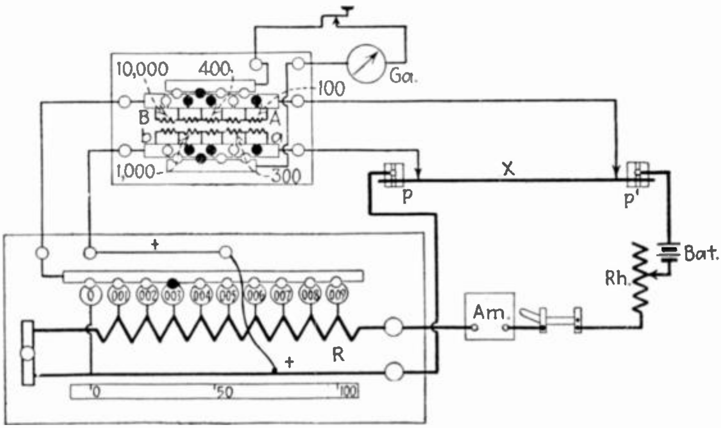


FIG. 111.—Actual connection of Kelvin bridge. (Leeds and Northrup type.)

must be low and it is made adjustable (see Fig. 111). The contact resistance between X and R is the resistance of the connection pec, and this resistance may be a large portion of the resistance of arms X and R and may even exceed the resistance of the arm R or of the sample X. Hence, unless this contact resistance is eliminated, it introduces so large an error that the measured value of the unknown resistance is worthless. However, this contact resistance *pec* may be eliminated by connecting the galvanometer to the junction of two auxiliary resistances a and b, rather than to either p or c as in the usual Wheatstone bridge. The other end of arm a is connected to contact p, and the other end of arm b is connected to point c (Fig. 110). If a and b are adjusted so that $a/b = A/B$, the contact resistance is eliminated, and $X/R = A/B$ as in the ordinary bridge.

In order to obtain sufficient sensitivity, it is necessary that the current to the bridge be from 15 to 150 amp. depending on the range of resistance for which the bridge is designed.

The relation $X/R = A/B$, when $a/b = A/B$, may be proved by writing Kirchhoff's laws for the network and solving the equations when $A/B = a/b$. The following demonstration, however, is a simpler proof.

Assume that the bridge is in balance. There must be some point e' between p and c which is at the same potential as the point g at the junction of a, b and the galvanometer. If, therefore, g and e' are connected as shown by the dotted line, no effect is produced in the bridge.

Since points g and e' are connected, two parallel circuits are formed, one consisting of a and pe' in parallel and the other consisting of b and ce' in parallel. Hence the resistance of the bridge arm from p' to e' is $X + \dfrac{a(pe')}{a + pe'}$ and the resistance of the bridge arm from d to e' is $R + \dfrac{b(ce')}{b + ce'}$.

Then, by the ordinary Wheatstone bridge relation,

$$\frac{A}{B} = \frac{X + \dfrac{a(pe')}{a + pe'}}{R + \dfrac{b(ce')}{b + ce'}}. \tag{I}$$

Also,

$$\frac{A}{B} = \frac{a}{b}. \tag{II}$$

The four resistances a, b, pe', and ce' form a miniature Wheatstone bridge in balance, since g and e' are at the same potential. Hence

$$\frac{a}{pe'} = \frac{b}{ce'}. \tag{III}$$

Treating Eq. (III) by composition gives

$$\frac{a}{a + pe'} = \frac{b}{b + ce'}. \tag{IV}$$

Substituting Eq. (IV) and then Eq. (III) in Eq. (I) gives

$$\frac{A}{B} = \frac{X + \dfrac{b(pe')}{b + ce'}}{R + \dfrac{b(ce')}{b + ce'}} = \frac{X + \dfrac{a(ce')}{b + ce'}}{R + \dfrac{b(ce')}{b + ce'}}. \tag{V}$$

Simplifying Eq. (V) and substituting for A/B in Eq. (II) gives

$$\frac{X(b + ce') + a(ce')}{a} = \frac{R(b + ce') + b(ce')}{b}. \tag{VI}$$

Dividing each side of Eq. (VI) by ce', and treating it by division,

$$\frac{X(b + ce') + a\,(ce') - a(ce')}{a(ce')} = \frac{R(b + ce') + b\,(ce') - b(ce')}{b(ce')}$$

or

$$\frac{X(b + ce')}{a(ce')} = \frac{R(b + ce')}{b(ce')}. \tag{VII}$$

Hence,

$$\frac{X}{R} = \frac{a}{b} = \frac{A}{B}. \quad Q.E.D. \tag{79}$$

The actual arrangement of the bridge, as manufactured by Leeds and Northrup Company, is shown in Fig. 111. The lettering corresponds to the diagram of Fig. 110.

In Fig. 111, $A = a = 300$ ohms, and $B = b = 10{,}000$ ohms. $R = 0.003$ ohm plus the slide-wire reading.

CABLE TESTING

121. Murray Loop.—The slide-wire bridge offers a convenient method of locating grounds in cables and wires. Figure 112 shows a cable AB which has become grounded at the point O, owing to a defect in the insulation. CB, another cable, is the return conductor for AB and is similar to AB except that it has no ground or "fault." The two cables are looped together at B, the far end of the two cables, which may be at some power station, telephone exchange, etc.

The slide wire is then connected to the home ends of the cable, as shown. It will be noted that the battery and the galvanometer are not in the positions shown in Fig. 109, but have been interchanged. This is done in order that earth currents shall not disturb the galvanometer readings. Also, if the resistance of the fault to ground is high, the e.m.f. of the battery Ba may be increased until sufficient current to operate the bridge is sent through this resistance. The resistance of the fault to ground does not produce any error in the measurement so long as the conductor is not broken. If the conductor in the cable is broken, with both ends lying on the ground, the resistance of the conductor is increased and a false location of the fault may result.

In Fig. 112, the distance X to the fault may be found as follows:

$$\frac{X}{l'} = \frac{L + (L - X)}{l} \tag{80}$$

where L is the length of one cable.

Equation (80) shows that the slide wire is divided into two sections which are to each other as the total length of cable on

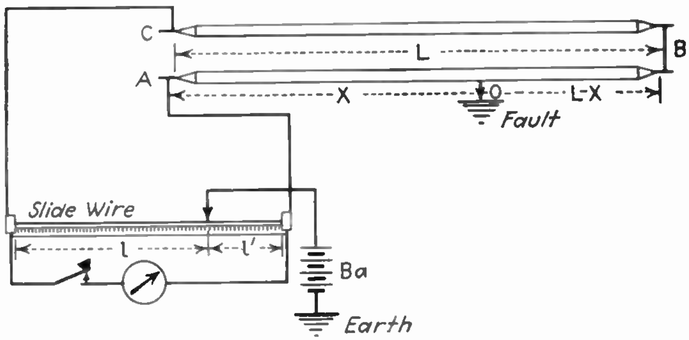


FIG. 112.—Murray-loop test.

one side of the fault is to the total length of cable on the other side of the fault.

Solving Eq. (80) for X,

$$X = \frac{2Ll'}{l + l'}. \tag{81}$$

This assumes that the resistance per foot of both conductors is the same and is uniform. The jumper tying the cable ends together at B should make good connection, as contact resistance at this point may introduce an appreciable error. A ratio arm and rheostat arm of a bridge box may be used instead of the slide wire.

Example.—A cable 2,000 ft. long consists of two conductors. One conductor is grounded at some point between stations. A Murray-loop test, with a 100-cm. slide-wire bridge, is connected as in Fig. 112 to locate the fault. A balance is obtained at 85 cm. How far from the station is the fault or ground? Using Eq. (81):

$$L = 2{,}000; \quad l' = 15; \quad l = 85.$$

$X = (4{,}000 \times 15)/100 = 600$ ft. from the station at which the measurement is made. *Ans.*

122. Varley Loop.—The Varley loop is also used to locate cable faults. It is similar in principle to the Murray loop, a bridge box being necessary, however. The connections are shown in Fig. 113. A and B are the two ratio arms of a bridge, and P is the rheostat arm. It is necessary that the battery and

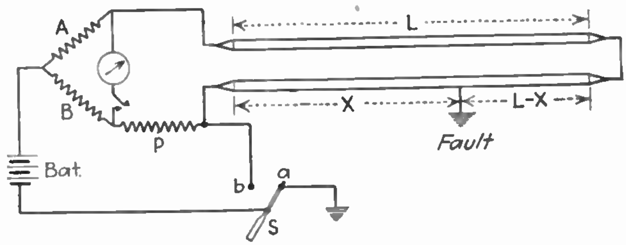


FIG. 113.—Varley-loop test.

the galvanometer occupy the positions shown, in order to avoid disturbances in the galvanometer due to earth currents. A balance is first obtained by means of P, with the switch S at a. Let r be the resistance per foot length per conductor, assumed uniform. (It will be noted that P and X together form one arm of the bridge.)

$$\frac{A}{B} = \frac{r(L + L - X)}{P + rX}. \tag{82}$$

Before X can be found, it is necessary to know r. To obtain r, the switch S is thrown to position b. This connects both lengths of cable in series and makes them the fourth arm of a bridge. A simple bridge measurement of the total loop resistance is then made. Let this resistance be R. Then the resistance per foot of cable

$$r = \frac{R}{2L}.$$

(This measurement is not necessary if the resistance per foot or the total resistance of the cable is already known.)

Substituting this value of r in Eq. (82)

$$\frac{A}{B} = \frac{(R/2L)(2L - X)}{P + RX/2L}.$$

Solving for X,

$$X = \frac{2L}{R}\left(\frac{BR - AP}{A + B}\right), \tag{83}$$

This equation gives the distance in feet to the fault. The equation is frequently given as follows:

$$R_x = \frac{BR - AP}{A + B}. \tag{84}$$

In this case, R_x is not the *distance* to the fault but is the *resistance* along the grounded conductor to the fault.

If $A = B$ in Eq. (83),

$$X = \frac{L}{R}(R - P), \tag{85}$$

which is simpler in form than Eq. (83).

Example.—In locating a fault by the Varley-loop test, the connections shown in Fig. 113 are used. Each conductor is 2,800 ft. long. With the switch at a and $A = 10$, $B = 1{,}000$, P is found to be 137 when a balance is obtained. Switch S is then thrown over to b. Under these conditions, a balance is obtained when $A = 10$, $B = 1{,}000$, $P = 221$, making $R = 2.21$.

By Eq. (83), the distance in feet to the fault

$$X = \frac{2 \times 2{,}800}{2.21}\left[\frac{(1{,}000 \times 2.21) - (10 \times 137)}{1{,}010}\right] = 2{,}100 \text{ ft.} \quad \textit{Ans.}$$

123. Insulation Testing.—In practice it is necessary to measure the resistance of the insulation of cables, both at the factory and after the cable is installed. A low value of insulation resistance may indicate that the insulation is of an inferior grade. A low insulation resistance after installation may indicate improper handling or faulty installation. The voltmeter method described in Par. 114 (p. 160) is applicable in many cases, but where the insulation resistance is high, even a high-resistance voltmeter is not sufficiently sensitive.

To make the measurement, a sensitive galvanometer is utilized. A source of considerable potential, from 100 to 500 volts, is usually necessary. Such potential may be secured from direct-current mains, although dry cells, silver chloride cells, and test-tube batteries connected in series are more satisfactory. A simple diagram of connections is shown in Fig. 114.

The method is one of substitution. A known resistance, usually 0.1 megohm (100,000 ohms), is first connected in the circuit and the galvanometer deflection noted. The unknown resistance X is then substituted and the galvanometer reading again noted. As the currents in the two cases are inversely proportional to the circuit resistances, the unknown resistance can be determined,

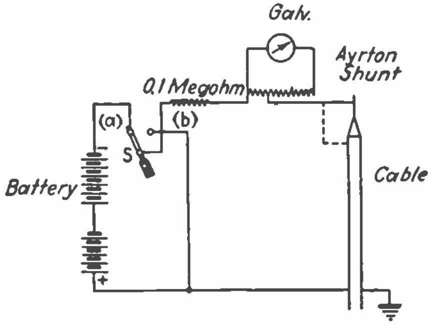


FIG. 114.—Measurement of the insulation resistance of a cable.

the galvanometer deflections being used rather than actual values of current. Let D_1 be the deflection with the 0.1 megohm and D_2 be the deflection with the unknown resistance.

$$\frac{X}{0.1} = \frac{D_1}{D_2}.$$

$$X = 0.1\frac{D_1}{D_2}. \tag{86}$$

Under ordinary circumstances it is not possible to obtain accurate results by the direct comparison of the galvanometer deflections (Eq. (86)), because the unknown resistance may be in the hundreds of megohms and the known resistance is only 0.1 megohm. This makes the deflection D_2 so many times smaller than D_1 that D_2 would not be readable.

This difficulty is overcome by the use of the Ayrton shunt described in Par. 105 (p. 143). When only the 0.1 megohm is in circuit, the galvanometer sensitivity is such ordinarily that the deflection would be off the scale unless the galvanometer were shunted.

Therefore the shunt is adjusted to some low value as 0.0001. Let this reading of the shunt be S_1 and the galvanometer deflection be D_1. The multiplying power of the shunt $M_1 = 1/S_1$. The cable is now introduced into the circuit and the shunt adjusted until a reasonable deflection is obtained. Let this deflection be D_2 and the value of the shunt be S_2. Its multiplying power is now $M_2 = 1/S_2$.

The ratio of the currents in the circuit in the two cases

$$\frac{I_1}{I_2} = \frac{M_1D_1}{M_2D_2}, \tag{87}$$

and the unknown resistance, from Eq. (86), is

$$X = 0.1\frac{I_1}{I_2} = 0.1\frac{M_1D_1}{M_2D_2}. \tag{88}$$

In practice, instead of substituting the cable for the 0.1 megohm, the cable is first short-circuited by the wire shown dotted (Fig. 114) and the constant determined. This wire is then removed, placing the cable in circuit. The 0.1 megohm is left permanently in circuit to protect the galvanometer in case of accidental short circuit of the cable. The 0.1 megohm is usually not appreciable compared with the insulation resistance of the cable, so that ordinarily no correction is necessary for it.

A switch or key S is usually provided. When in position (a) (Fig. 114) the circuit is closed through the cable. When thrown over to (b), the cable, which is charged electrostatically, discharges through the galvanometer.

When the switch S is first closed at (a), there is a rush of current which charges the cable electrostatically (see Par. 216, Chap. X). Because of the "absorption" characteristic of the insulation, it takes time to charge the cable, and for some time this charging current flows, decreasing continuously. This is shown in Fig. 115, giving the relation of galvanometer deflection to time. As it is often inconvenient to wait for the galvanometer to reach a steady deflection, it has been agreed to take the deflection at the end of 1 min. as the arbitrary value to be used in determining insulation resistance.

When the switch S is thrown to (*b*), the electrostatic charge in the cable rushes out through the galvanometer in the reverse direction. Again, due to absorption, it requires considerable time for the cable to become totally discharged. This is also shown in Fig. 115.

In making insulation-resistance measurements, precautions must be taken to insulate thoroughly the apparatus itself. Hard-

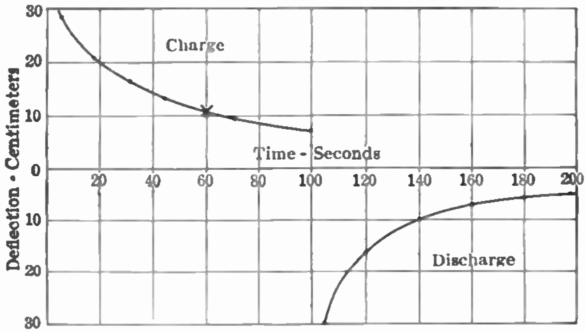


FIG. 115.—Charge and discharge curves of a cable.

rubber posts should be used for supports and, wherever possible, the leads should be carried through the air rather than be allowed to rest on the ground. As the insulation resistance varies widely with temperature, the temperature at which the measurements are made should be carefully determined and stated.

Example.—The cable whose insulation characteristics are shown in Fig. 115 was tested for insulation resistance. The deflection with 0.1 megohm only in circuit was 20 cm. and the shunt read 0.0001. When the curve shown in Fig. 115 was obtained, the shunt read 0.1. The cable is 2,200 ft. long.

(*a*) What is its insulation resistance?

(*b*) What is its insulation resistance per mile?

$$M_1 = 1/0.0001 = 10{,}000.$$
$$M_2 = 1/0.1 = 10.$$
$$D_2 \text{ (from curve)} = 11 \text{ cm.}$$
$$(a)\ X = 0.1\left(\frac{10{,}000 \times 20}{10 \times 11}\right) = 182 \text{ megohms.} \quad \textit{Ans.}$$

(*b*) The insulation resistance per mile will be *less* than that of the 2,200-ft. length because the amount of leakage current is directly proportional to the length of the cable. Therefore the resistance of this leakage path is inversely proportional to the length of cable. The cross-sectional area of the leakage path for the mile length is greater than it is for the 2,200-ft. length. Therefore the insulation resistance per mile

$$R = \left(\frac{2{,}200}{5{,}280}\right)182 = 75.8 \text{ megohms.} \quad \textit{Ans.}$$

124. Guard Wire.—In determining the insulation resistance of cables and of other insulation, precautions against end leakage must frequently be taken. Moisture, combined sometimes with dust, provides comparatively low-resistance paths over the surface of the insulation between the core and sheath. These conducting paths are in shunt with the insulation, so that this end leakage may give values of insulation resistance which are entirely too low. To prevent such leakage, the ends of the cable are frequently dried with a torch, care being taken not to carbonize the insulation, and hot paraffin is then poured over the ends.

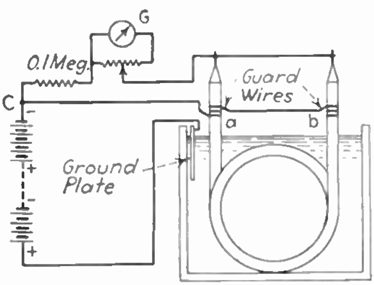


FIG. 116.—Use of guard wires to by-pass surface-leakage current of cable immersed in water.

Where both ends of the cable are accessible, the end leakage may be shunted around the galvanometer by the use of *guard wires*, and its effect, so far as the measurement is concerned, is thus eliminated. The method is shown in Fig. 116. A few turns of bare wire are wrapped tightly around the insulation at *a* and *b* between the sheath or ground and the core of the cable. These turns of wire are connected directly with the terminal *C* of the battery, which also connects with the galvanometer (Fig. 116). A study of Fig. 116 shows that any leakage current is conducted back to the negative terminal of the battery without passing through the galvanometer. That is, when leakage current reaches *a* and *b*, it has the choice of two paths to the negative terminal of the battery, one through the low-resistance wire *aC* and the other over the remainder of the insulation through the galvanometer to the battery. Practically all the leakage current will pass through the wire *aC* and will not affect the galvanometer deflection.

POTENTIOMETERS

125. Potentiometer.—The potentiometer is an instrument for making accurate measurements of voltage. Its standardization depends primarily on the Weston standard cell (see Par. 73, Chap. IV). The principle is as follows:

Assume in Fig. 117(*a*) that a standard cell S has an e.m.f. of exactly 1 volt. Let a storage cell Ba. supply current to a wire AB through a rheostat R. Let the wire AB be divided into 15 divisions, each of 1 ohm resistance, making the total resistance of AB 15 ohms. The standard cell is connected with its negative terminal to the negative terminal A of the storage cell, and its positive terminal is connected to the tenth 1-ohm coil C through a key and galvanometer. If 0.1 amp. flows through the wire AB, the voltage drop through each division of AB will be 0.1 volt, and the voltage drop across AC will be 1.0 volt. If the key be depressed, no current will flow through the galvanometer, as the standard-cell e.m.f. of 1 volt is in exact opposition to this 1-volt drop. If, however, the current in AB is not exactly 0.1 amp., current will flow through the standard-cell circuit due to the voltage drop from A to C being either greater or less than 1 volt. If the current is less than 0.1 amp., the galvanometer deflects in one direction, and if it is greater than 0.1 amp., the galvanometer deflects in the reverse direction. Obviously it is possible to adjust the current in AB to such a value that the galvanometer deflection is zero. Under these conditions the current in AB is exactly 0.1 amp., and the potential drop across each division in AB is 0.1 volt. Therefore AB may be marked in volts, as shown.

Let it be required to measure some unknown e.m.f. E whose value is known to be less than 1.5 volts. The negative terminal of E is connected to the end A of the wire AB (Fig. 117(*b*)). The positive terminal of E is connected through the galvanometer and key to a movable contact b. It is assumed that the current in AB has been adjusted to exactly 0.1 amp. Contact b is moved along AB until the galvanometer deflection is zero. This means that the e.m.f. E is just balanced against an equal drop in the wire AB. As AB is calibrated in volts, the value of E may be read directly on AB. This method of measuring voltage is the *Poggendorff Method* and is the fundamental principle of the potentiometer.

The two diagrams (*a*) and (*b*) (Fig. 117) may be combined into one by the use of the single-pole, double-throw (S.-P., D.-T.) switch *Sw* (Fig. 117(*c*)). When the switch is in its left-hand position the standard cell is in circuit for calibration as in (*a*).

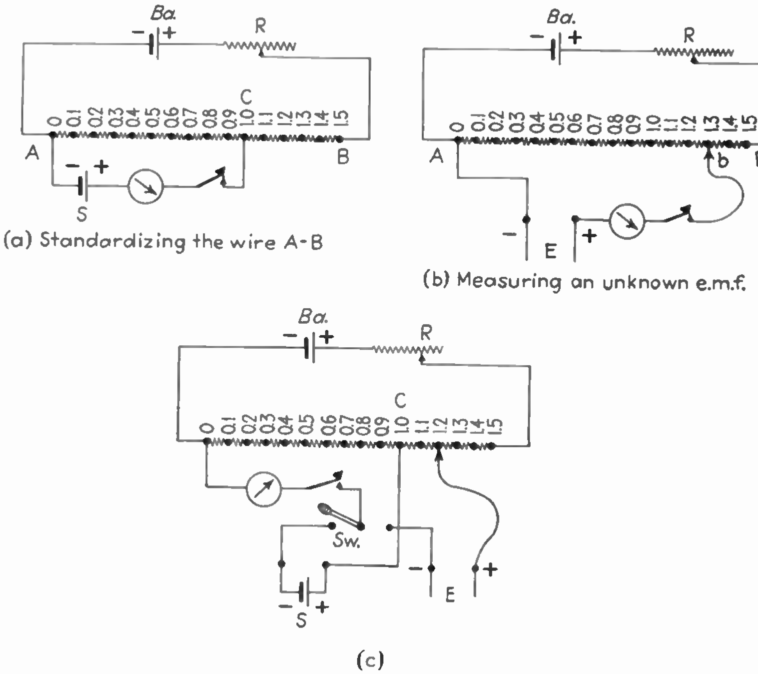


FIG. 117.—Simple potentiometer.

When the switch is in its right-hand position, the unknown e.m.f. is in contact with the wire *AB* so that its value may be determined.

126. Leeds and Northrup Low-resistance Potentiometer.—Figure 118 shows the Leeds and Northrup low-resistance potentiometer. In most respects it is similar to the simple potentiometer shown in Fig. 117.

A 2-volt battery is connected to the terminals +BA and −BA. The potentiometer working current is controlled by two adjustable rheostats (shown as MEDIUM and FINE) and by three fixed resistances (COARSE), each of which may be short-circuited by plugs at 1, 2, and 4. The path of the current is then through the shunted dial B, to which one of the standard-cell connections is made, through the dial D divided into fifteen 5-ohm, 0.1-volt divisions and the spiral slide wire H and its shunting resistance. Since the resistance corresponding to 0.1 volt is 5 ohms, the working current is $0.1/5 = \frac{1}{50}$, or 0.02 amp.

One standard-cell connection is made at the 0.5-volt contact. The resistance from the 0.5-volt contact to the 1.5-volt contact

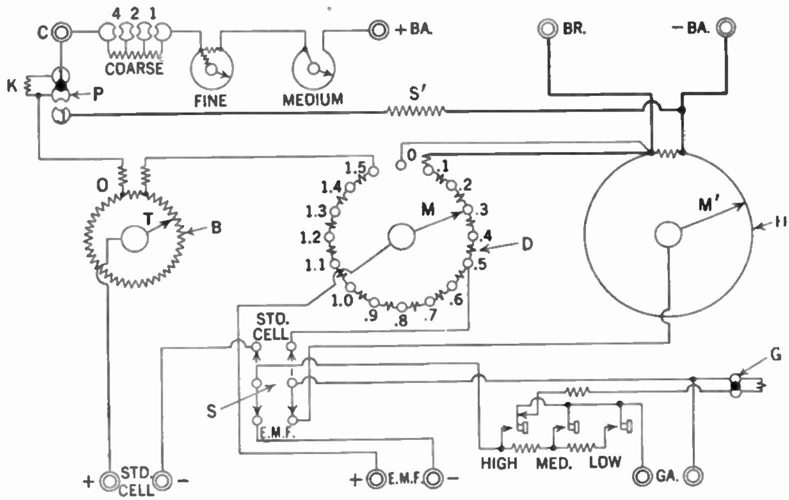


FIG. 118.—Connections of Leeds and Northrup low-resistance potentiometer.

corresponds to 1 volt. The e.m.f. of an unsaturated standard cell is slightly over 1 volt but may vary for different cells, the e.m.f. being about 1.0186 volts. In order to take care of the excess e.m.f. over 1 volt, the dial B with its series resistance and shunt is designed to give added e.m.f. The dial is graduated in volts, and the contact T is set to correspond with the actual value of the e.m.f. of the standard cell.

The resistance of the slide-wire dial H with its shunting resistance is 5.5 ohms, so that when in adjustment the voltage drop across it is 0.11 volt. The slide wire consists of 11 turns of resistance wire mounted on a marble cylinder. Each turn represents 0.01 volt and the entire wire is divided into 1,100 divisions.

M and M′ are movable contacts, which are adjusted to balance the unknown e.m.f. M moves over the 15 contacts, each corresponding to 0.1 volt, and M′ moves over the slide wire. A D.-P., D.-T. switch S (corresponding to *Sw*, Fig. 117(*c*)) changes the connection of the galvanometer from the standard cell to the unknown e.m.f. There are three galvanometer keys, HIGH, MED., LOW. "LOW" should first be depressed as it inserts a high resistance in series with the galvanometer and prevents a violent deflection if there is considerable unbalance. "MED." inserts less resistance, and there is no resistance in series with "HIGH," which is depressed when the final balance is obtained.

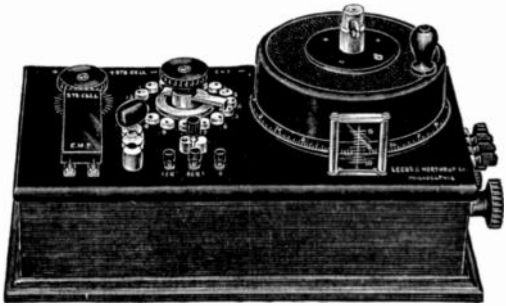


FIG. 119.—Leeds and Northrup potentiometer without accessories.

By means of a plug G, the resistance in series with the galvanometer can be changed to correspond more nearly to the resistance for critical damping of the galvanometer. The binding post BR is used only when it is desired to measure the resistance of the potentiometer wire in order to check the instrument.

A resistance S′ shunts nine-tenths of the current from the potentiometer system, when the plug at P is changed. The resistance K is automatically put in circuit, keeping the total potentiometer resistance, and therefore the load on the battery, constant. By this arrangement, the readings on the potentiometer are all one-tenth their previous values.

An external view of this potentiometer is shown in Fig. 119.

127. Other Potentiometer Methods.—In the simple potentiometer (Fig. 117) and in the Leeds and Northrup potentiometer (Fig. 118), there are no contacts in series with the working current within the calibrated portion of the potentiometer circuit.

This type of potentiometer is advantageous in that there can be no errors due to contact resistance. When a balance is obtained, no current flows in either the e.m.f. or standard-cell contacts, so that resistance in these contacts causes no error. However, with this type of potentiometer it is impossible to have more than two dial or slide-wire readings. For example, in the Leeds and Northrup type, the tenths of a volt are obtained on one dial with contact M (Fig. 118) and all the smaller divisions, such as hundredths, thousandths, and ten-thousandths of a volt, must be obtained on the single slide wire with the contact M′. To

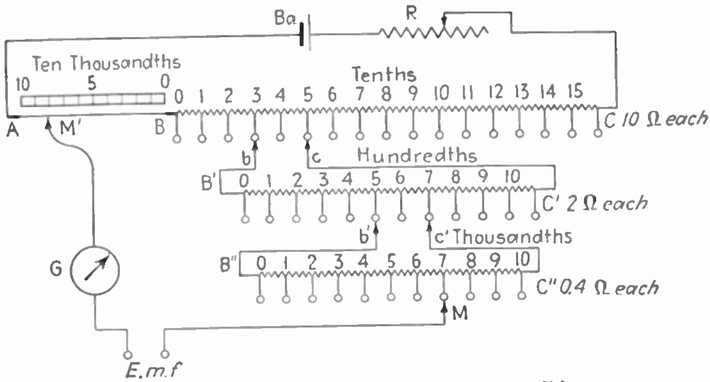


Fig. 120.—Principle of Thomson-Varley slide.

obtain more than two significant figures, it is necessary that the hundredths, thousandths, etc., of a volt be obtained on the slide wire. It is possible, however, to arrange a potentiometer so that the tenths, hundredths, thousandths, etc., of a volt are each read on a separate dial. In any of such types of potentiometer, it is essential to maintain a fixed resistance in the potentiometer itself, for the current must remain constant irrespective of the positions of the dials. There are two common methods of obtaining more than two significant figures by separate dials and at the same time maintaining constant potentiometer resistance, the Thomson-Varley method and the Wolff method.

The Thomson-Varley method is illustrated in Fig. 120. It is desired that tenths, hundredths, thousandths, and ten-thousandths of a volt be obtained on a separate resistance unit or dial divided into 10 or more equal divisions. In the potentiometer shown in Fig. 120, the tenths, hundredths, and thousandths of a volt are obtained on a separate dial, and the ten-thousandths of a volt are obtained on a slide wire. Fundamentally, *it is necessary to maintain constant resistance in the entire potentiometer circuit AC irrespective of the position* of the e.m.f. dials. Assume that the resistance corresponding to one-tenth volt is 10 ohms. The resistance BC consists of sixteen 10-ohm resistances. Two contacts b and c, fastened rigidly together, always span two of the 10-ohm coils in BC. Between b and c there are connected eleven 2-ohm coils, $B'C'$. Two contacts b' and c', fastened rigidly together, always span two of the 2-ohm coils in $B'C'$. Between b' and c' there are connected ten 0.4-ohm coils $B''C''$. One contact arm M operates over the contacts in the dial $B''C''$. These resistances are usually arranged in circular dials. The total resistance of $B''C''$ is 4 ohms, and it is connected across two 2-ohm resistances in $B'C'$. Hence, the equivalent resistance between the contactors $b'c'$, and therefore between any two of the contact points which contactors $b'c'$ may bridge, is 2 ohms. This is the value of the resistance of each of the resistance units in the resistance bank $B'C'$. Hence the effect of bridging resistance bank $B''C''$ across any two resistance units in resistance bank $B'C'$ is to reduce the resistance between B' and C' by 2 ohms, or by one resistance unit. However, since there are 11 resistance units of 2 ohms each in bank $B'C'$, and since the contacts $b'c'$ are always across two such units, the resistance between points $B'C'$ always remains constant at 20 ohms.

In a similar manner the resistance bank $B'C'$, the total resistance of which is 20 ohms, is always bridged across two resistance units in resistance bank BC by contactors bc, making the resistance between contactors bc equal to 10 ohms. Hence the bridging effect of contacts bc is to reduce the resistance of BC by 10 ohms, the resistance of a single unit. There are, however, 16 such units in BC so that the resistance of BC is reduced to 150 ohms, corresponding to 15 units. This resistance remains constant irrespective of the position of contactors bc. With the normal current of 0.01 amp., the total voltage across BC is 1.5 volts; the total voltage across $B'C'$ is 0.1 volt; and the total voltage across $B''C''$ is 0.01 volt. Hence, the tenths of volts are read at b on dial BC; the hundredths of volts are read at b' on dial $B'C'$; and the thousandths of volts are read at M on scale $B''C''$. The ten-thousandths of volts may be read at contact M' on a slide wire AB, having a total resistance of 0.1 ohm. The reading with the dials set as in Fig. 120 is 0.3578 volt. It is obvious that the resistance between points A and C remains constant at 150.1 ohms irrespective of the position of any of the contact arms.

Since contact resistance, except at M and M', introduces error, it is desirable that potentiometers of this type have a high inher-

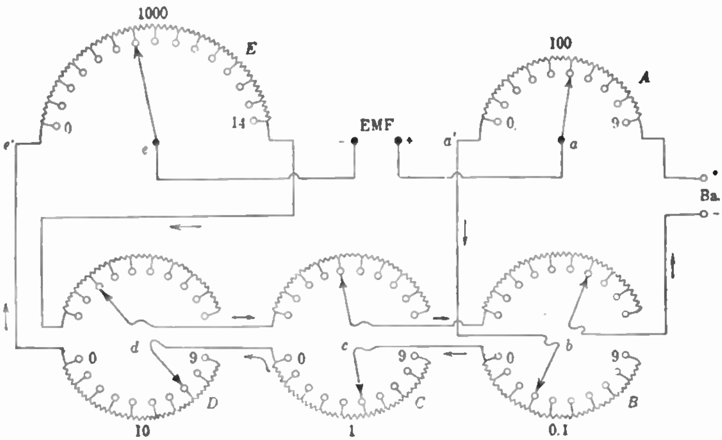


Fig. 121.—Principle of the Wolff potentiometer.

ent resistance, so that the contact resistance may be neglected. This reduces the sensitivity. Another disadvantage of this type is the fact that the contacts must be kept clean from oxide and dust.

Wolff Method.—The principle of the Wolff potentiometer is illustrated in Fig. 121. The potentiometer has a total resistance of 15,000 (actually 14,999.9) ohms and the working current is 0.0001 amp., so that it is a high-resistance potentiometer. The dial E consists of fourteen 1,000-ohm coils, and the dial A consists of nine 100-ohm coils. The dials B, C, D are double, and have double dial switches, the purpose of which is to maintain constant resistance in the battery circuit. There are nine 10-ohm coils in each half of dial D, nine 1-ohm coils in each half of dial C, and nine 0.1-ohm coils in each half of dial B. The path of the current, shown by arrows, is from the battery positive terminal through the entire dial A, through those portions of the lower halves of dials, B, C, D which are not cut out by the lower parts of the dial switches b, c, d, thence through the entire dial E and so through those portions of the upper halves of dials D, C, B which are not cut out by the upper parts of the dial switches d, c, b, and thus to the battery negative terminal.

A study of Fig. 121 shows that the resistance of the battery circuit is constant irrespective of the positions of the dial switches, for each double dial switch always cuts out as much resistance in the upper half of the dial as it inserts in the lower

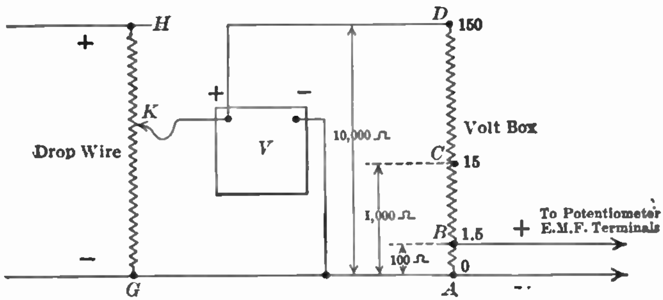


FIG. 122.—Volt-box and drop-wire connections.

half. On the other hand, the moving of any of the double dial switches changes the resistance between points a' and e' and hence causes a change in the potential difference between points a and e.

With the setting of the dials shown in Fig. 121, the potentiometer reads 0.65753 volt.

128. Voltage Measurements with Potentiometer.—Potentiometers themselves are designed to measure potentials up to only 1.6 volts. For the measurement of potentials in excess of this value, a *volt box* is necessary. A volt box is merely a very high resistance from which suitable taps are brought. This is illustrated by the resistance AD (Fig. 122). Assume AD to have a resistance of 10,000 ohms and AB a resistance of 100 ohms. If no current leaves the wire at B, the voltage drop across AB will be $100 \div 10{,}000$ or $1/100$ that across AD. If leads be carried from AB to the potentiometer, the potentiometer will measure one-hundredth the voltage across AD, since the potentiometer principle is an opposition method so that no current is taken from B. Therefore, if a voltmeter V is being calibrated, it should be connected in parallel with AD. If the voltmeter reads 119.0 volts and the potentiometer reads 1.184 volts, the true voltage across the voltmeter will be $1.184 \times 100 = 118.4$ volts. Therefore the correction to the voltmeter is -0.6 volt.

In a similar manner, voltages from 1.5 to 15 volts are connected across AC, the multiplying factor in this case being 10.

Drop Wire.—GH is a resistance wire connected directly across the line. One voltmeter terminal and one terminal of the volt box

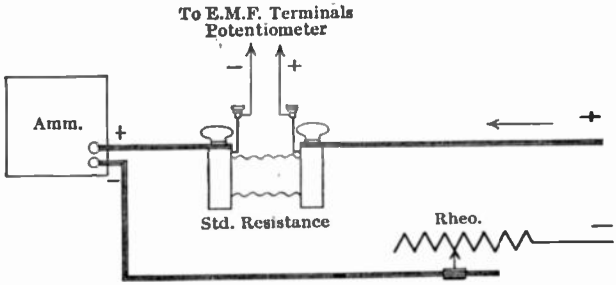


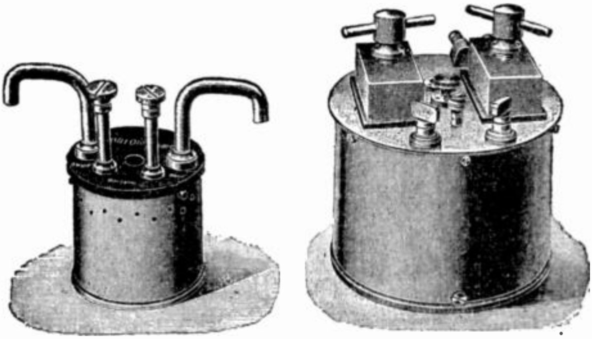
FIG. 123.—Calibration of an ammeter with a potentiometer.

are connected to the end G of this wire. The other terminal of the voltmeter and the remaining terminal of the volt box are connected to a movable contact K. By sliding K along GH any desired voltage can be obtained. When used in this manner, GH is called a *drop wire.* It is not necessary to the operation of the volt box but is merely a convenient means for adjusting the voltage.

129. Measurement of Current with Potentiometer.—As has just been pointed out, a potentiometer is designed primarily to measure *voltage.* By merely applying Ohm's law, the *current* in a circuit may also be determined with the potentiometer. Let an unknown current I flow through a known resistance R. If E, the voltage drop across R, be measured, the current I is immediately determined, since for this part of the circuit both the voltage and the resistance are known. Therefore,

$$I = \frac{E}{R}.$$

The method of making the measurement is shown in Fig. 123. It is desired to know the exact current to the ammeter, in order to determine its errors, if any exist. The ammeter is connected in series with the standard resistance and also with a rheostat to control the current. Standard resistances are provided with four terminals as a rule, two heavy ones for current and two smaller binding posts for potential (see Fig. 124). The two potential binding posts are connected to the potentiometer, the proper polarity being observed. The voltage across the standard resistance is then measured by means of the potentiometer.



(*a*) 0.01 ohm. (*b*) Self-contained 0.001 ohm.

FIG. 124.—Standard resistances.

Standard resistances are usually adjusted to even decimal values such as 10, 1, 0.1, 0.01 ohms. They are ordinarily rated to carry a current that will give 1.0 volt drop. Thus the 1 ohm can carry 1 amp., the 0.001 ohm, 1,000 amp., etc. To keep the resistances cool they are often immersed in oil. The type shown in Fig. 124(*a*) is set in a water-jacketed oil bath provided with a motor-driven stirrer. The type shown in Fig. 124(*b*) is rated for larger currents, 1,000 amp. and more. The water jacket, the stirrer, etc., are included within the unit itself.

Knowing that the potentiometer is limited to 1.5 volts, it is easy to select the proper standard resistance. An instrument having a range of 100 amp., would require $1.5 \div 100 = 0.015$ ohm, and 0.01 ohm would be used. Likewise a 15-scale instrument would require $1.5 \div 15 = 0.1$ ohm.

When instruments are calibrated, they should be checked at 10 or 15 points on the scale, and the corresponding corrections at each point are plotted as ordinates, the instrument readings being plotted as abscissas. As an instrument scale is subject to scale errors, it is customary to connect successive points of the correction curve by straight lines, as shown in Fig. 125. For

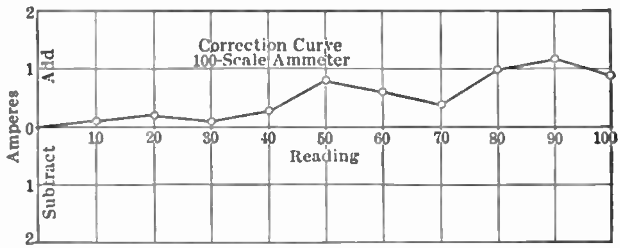


FIG. 125.—Ammeter correction curve.

example (Fig. 125), the correct current when the instrument reads 50 amp. is $50 + 0.8 = 50.8$ amp.

130. Measurement of Power.—Direct-current power is usually measured by means of a voltmeter and an ammeter. Since

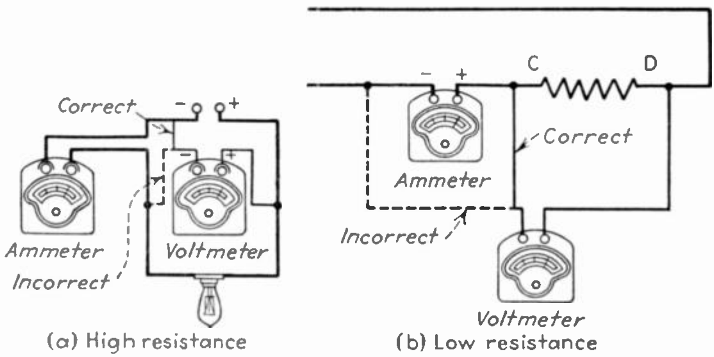


FIG. 126.—Correct and incorrect methods of connecting voltmeters and ammeters in power measurements.

the power is the product of the volts and the amperes ($P = EI$), it is merely necessary to multiply the volts by the amperes to obtain the power in watts. However, certain precautions, may be necessary in measuring the power.

Assume that it is desired to measure the power taken by an incandescent lamp. If the voltmeter is connected as shown by the dotted line in Fig. 126(a), the current taken by the voltmeter is being measured by the ammeter. In other words, the voltmeter is a load connected in parallel with the lamp. As the current taken by the lamp is small, this voltmeter current, although of itself small, may introduce an appreciable error into the measurement; that is, the power taken by the voltmeter will be included in the measurement. There are three methods of eliminating this error. The voltmeter power may be calculated, knowing the voltmeter resistance, and proper correction made. The voltmeter may be open-circuited when the ammeter is being read, if it is certain that this will not alter the voltage across the lamp. The voltmeter lead may be connected as shown by the solid line (Fig. 126(*a*)), so that the voltmeter current does not flow through the ammeter. Under this last condition, the voltmeter is not reading the true voltage across the lamp, but its reading is too high by the drop through the ammeter. As the resistance of the lamp is high and that of the ammeter low, this last error is usually negligible.

However, if a low resistance *CD* is being measured (Fig. 126(*b*)), the drop across the resistance is necessarily low, and if the voltmeter is now connected outside the ammeter, an appreciable error may be introduced, as the voltmeter reading includes the voltage drop in the ammeter. The voltmeter should now be connected *inside* the ammeter. This will not introduce an appreciable error, for presumably a large current is required for the measurement of the low resistance, and the very small voltmeter current is negligible in comparison with the ammeter current.

These precautions should be observed also in making resistance measurements.

Example.—It is desired to measure the power taken by a 40-watt tungsten lamp. A 0.5-scale ammeter having a resistance of 0.15 ohm and a 150-scale voltmeter having a resistance of 16,000 ohms are used for the measurement. When the voltmeter is connected inside the ammeter it reads 120 volts, and the ammeter reads 0.35 amp. What is the true power taken by the lamp, and what is the apparent power if the voltmeter loss is neglected?

Apparent power $= 120 \times 0.35 = 42$ watts. *Ans.*

$$\text{Power taken by voltmeter} = \frac{(120)^2}{16{,}000} = 0.9 \text{ watt.}$$

True power to lamp $= 41.1$ watts. *Ans.*

The voltmeter introduces a 2 per cent error in this case.

If the voltmeter is connected outside the ammeter, the ammeter will now read

$$0.35 - \frac{120}{16{,}000} = 0.3425 \text{ amp.}$$

The voltmeter will now read

$$120 + (0.15 \times 0.3425) = 120.05 \text{ volts,}$$

and the apparent power is $120.05 \times 0.3425 = 41.12$ watts, an error of 0.05 per cent, which is negligible.

131. Wattmeter.—The wattmeter measures power directly. It consists of fixed coils *FF* and a pivoted coil *M*, free to turn within the magnetic field produced by coils *FF* (Fig. 127). The coils *FF* are wound with comparatively few turns of wire and are capable of carrying the entire current of the circuit. The moving

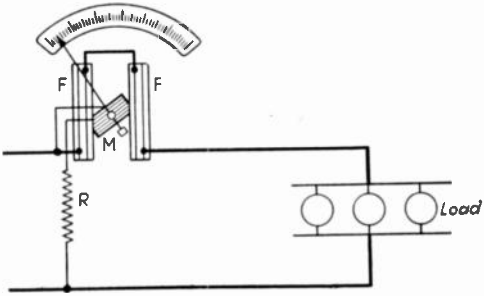


FIG. 127.—Indicating wattmeter.

coil *M* is wound with very fine wire, and the current is led into it through two control springs in the same manner that current is led into the movable coil of a Weston instrument. The fixed coil is connected in series with the load in the same manner as an ammeter is connected. The moving coil is connected across the line in series with a high resistance *R* in the same manner as a voltmeter coil is connected ordinarily.

The field of the coils *FF* is proportional to the current and the current in the coil *M* is proportional to the voltage. Therefore, the turning moment is proportional to both the current and the voltage and hence to the power of the circuit. It also depends on the angular position of *M* with respect to *FF*, which is taken into consideration when the scale is marked.

Owing to the high degree of accuracy obtainable by the use of voltmeter and ammeter, the wattmeter is seldom used for direct-current measurements of power. As it is subject to stray fields, reversed readings should be taken, that is, both the current and voltage should be reversed and the average of the two readings used. The wattmeter is used almost exclusively for the measurement of power with alternating currents. A more complete description, together with its uses, is found in Chap. IV, Vol. II.

132. Watthour Meter.—The watthour meter is a device for measuring electrical *energy* (see Par. 38, p. 44). As energy is the

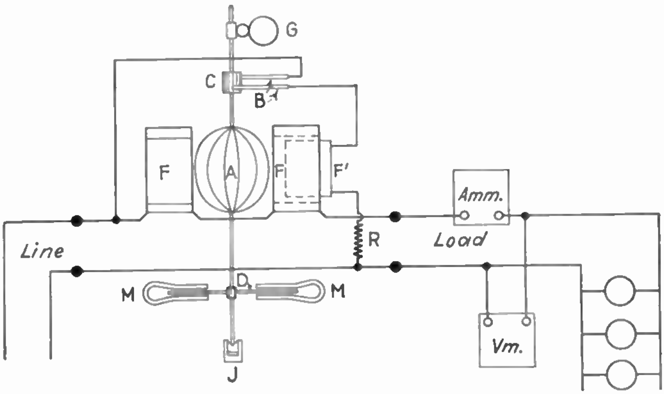


Fig. 128.—Connections of watthour meter.

product of power and time, the watthour meter must take into consideration both of these factors. As power is usually sold on an energy basis, a saving of many dollars may depend on the accuracy of such a meter. Therefore a proper understanding of its mechanism and the method of adjustment is essential.

In principle the watthour meter is a small motor whose instantaneous speed is proportional to the power passing through it, and whose total revolutions in a given time are proportional to the total energy or watthours during that time.

Referring to Fig. 128, the line is connected to two terminals on the left-hand side of the meter. The upper terminal is connected to two coils *FF* in series, wound with wire sufficiently heavy to carry the maximum current taken by the load, which should not greatly exceed the rated current of the meter. The connection through coils *FF* terminates at the upper binding post on the right-hand side of the meter. The coils *FF* are wound so that they aid each other, and they supply the field in which the armature rotates. The other line wire runs straight through the meter to the load. A shunt circuit is tapped to the upper line on the left-hand side. It runs first to the armature, through the silver brushes *B*, which rest on the small commutator *C*. From the brushes, connection is made through coil *F'* and through a resistance *R* to the lower line wire. This resistance *R* is omitted in certain types of meters.

As the load current flows through *FF*, and there is no iron in the circuit, the magnetic field produced by these coils is proportional to the *load current*. As the armature, in series with resistance, is connected directly across the line, the current in the meter armature is proportional to the *line voltage*, the counter e.m.f. of the armature being negligible in comparison with the line voltage. Neglecting the small voltage drop in *FF*, the torque acting on the armature will be proportional to the product of the load current and the load voltage or, in other words, proportional to the power passing through the meter to the load.

It can be proved[1] that if the meter is to register correctly, there must be a retarding torque acting on the moving element proportional to its angular velocity. To meet this condition an aluminum disc *D* is pressed on the motor shaft. This disc rotates between the poles of two permanent magnets *MM*. In cutting the field produced by these magnets, eddy currents are induced in the disc, retarding its motion. As the value of these currents is proportional to the angular velocity of the disc, and they are acting in conjunction with a magnetic field of constant strength, their retarding effect is proportional to the angular velocity, so that the condition for correct registration is fulfilled.

Friction cannot be entirely eliminated in the rotating element, even with the most careful construction. Near the rated load of the meter the effect of friction is practically negligible, but since the friction torque is nearly constant, it has a much greater proportionate effect at light loads. As the ordinary meter may

[1] See LAWS, F. A., "Electrical Measurements," McGraw-Hill Book Company, Inc.

operate at light loads during a considerable portion of the time, it is desirable that the error due to friction be eliminated. This is accomplished by means of coil F' connected in series with the armature. F' is so connected that its field acts in the same direction as that due to coils FF. Therefore it assists the armature A to rotate. Being connected in the shunt circuit and across practically constant voltage, it is acting continuously and pro-

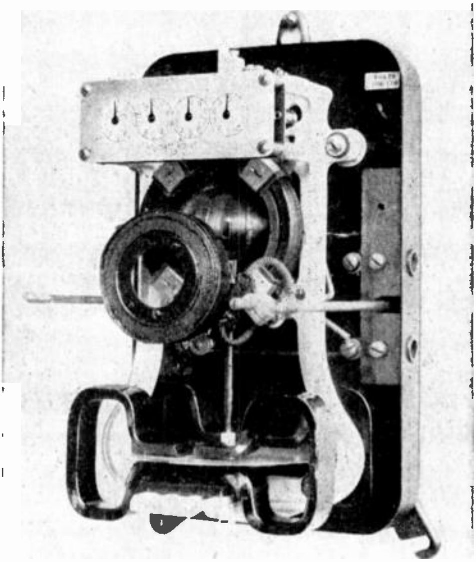


FIG. 129.—Thomson watthour meter. (*General Electric Company.*)

duces nearly constant torque. The coil F' is movable and its position can be adjusted so that the friction error is compensated.

To reduce friction and wear, the rotating element of the meter is made as light as possible. The element is supported by a hardened steel pivot turning in a jewel bearing J, which is a sapphire in the smaller sizes and a diamond in the heavier types. The jewel is supported on a spring. In time the pivot becomes dulled and the jewel roughened, which increases friction and causes the meter to run more slowly unless F' is readjusted. The moving element turns the clockwork of the meter dials through a worm and the gears G.

Figure 129 shows the interior view of a Thomson watthour meter. The friction-compensating coil is plainly shown at the front of the meter.

133. Adjustment of the Watthour Meter.—Even if the initial adjustment be accurate, the registration of a watthour meter may, in time, become incorrect. This is due to many causes, such as pitting of the commutator, roughening of the jewel, wear on the pivot, or change in the strength of the retarding magnets. As the cost of energy to consumers is largely based on the registration of such meters, it is important that they be kept in adjustment, as a small error, particularly in the larger sizes, may ultimately mean a difference of many dollars to either the consumer or the power company.

To adjust the meter it may be loaded as shown in Fig. 128. The power taken by the load is measured by a calibrated voltmeter and ammeter. The revolutions of the disc D are counted over a period of time which is measured with a stop watch. In most meters, the relation between watthours and the revolutions of the *disc* is

$$W \times H = K \times N \tag{89}$$

where W is in watts.

H is in hours.

K is the meter constant or watthour constant, usually marked on the disc.

N is the revolutions of the disc.

This equation means that the meter constant multiplied by the revolutions of the disc gives the watthours registered by the meter. The gear ratios and clockwork take care of the dial registration.

When checking a meter, the time is usually measured in seconds.

Equation (89) then becomes

$$\frac{W \times t}{3{,}600} = K \times N \tag{90}$$

where t is the time in seconds.

When the meter is tested, the voltmeter and ammeter are read at intervals, while the revolutions of the disc are being counted. A run of about 1 min. gives good results.

Let the average watts determined from the corrected voltmeter and ammeter readings be W_1.

The average watts as indicated by the meter during the same period are, from Eq. (90),

$$W = \frac{K \times N \times 3{,}600}{t}. \tag{91}$$

The per cent accuracy of the meter is

$$\frac{100W}{W_1}$$

Example.—In the test of a 10-amp. watthour meter having a constant of 0.4, the disc makes 40 revolutions in 53.6 sec. The average volts and amperes during this period are 116 volts and 9.4 amp. What is the per cent accuracy of the meter at this load?

Average standard watts $W_1 = 116 \times 9.4 = 1{,}090$.

Average meter watts, from Eq. (91),

$$W = \frac{0.4 \times 40 \times 3{,}600}{53.6} = 1{,}074.$$

$$\text{Per cent accuracy} = \frac{100 \times 1{,}074}{1{,}090} = 98.5 \text{ per cent.} \quad \textit{Ans.}$$

This means that the meter is 1.5 per cent slow and should be speeded up slightly. With calibrated indicating instruments and careful adjustment, a meter may easily be brought within 0.5 per cent of accurate registration.

There are two adjustments to be made. Near full load, the magnets are moved. If the meter is running slow, the magnets are moved nearer the center of the disc where the effect of the retarding currents is reduced, and if the meter is running fast the magnets are moved farther from the center. If the meter has been correctly adjusted near full load, and is found to be in error near light load, the error is obviously due to friction. The light-load adjustment, made at from 5 to 10 per cent rated load, is effected by moving the friction-compensating coil F'. If the meter is slow, the coil F' is moved in nearer the armature, and if the meter is fast, it is pulled out farther from the armature. This adjustment of F' may affect the full-load adjustment slightly, so that the meter should be rechecked at full load and then again at light load.

134. Other Types of Watthour Meter.—The three-wire meter is designed to register energy in a three-wire system. It does not differ materially from the meter shown in Fig. 128 except that the two coils *FF* are connected in opposite sides of the line as in Fig. 130. Hence, the field in which the armature *A* rotates is produced by the joint effect of the current in the positive conductor to neutral and the current in the negative conductor to neutral. Therefore, the torque is proportional to the power to

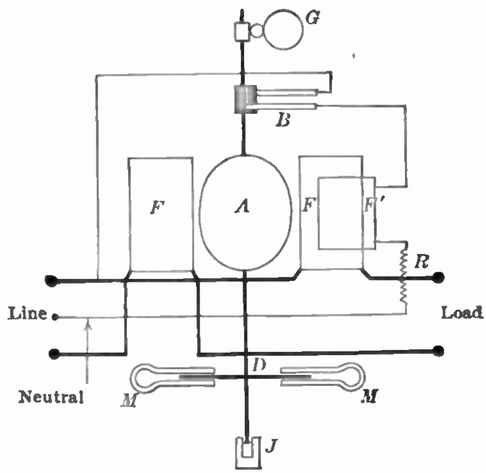


FIG. 130.—Diagram of a 3-wire watthour meter.

the three-wire system irrespective of any unbalancing of the current. The armature circuit may be connected to the neutral as shown, or it may be connected across the outer wires. If this latter connection is used the neutral connection to the meter is omitted. The meter does not register accurately unless the voltages between the two outer lines and neutral are equal. This error is usually small.

The meters already described should not be installed near bus-bars carrying heavy currents because the strength of the meter field and of the retarding magnets may be affected by the stray fields. To eliminate the effect of stray fields, an astatic type of meter is available. There are two armatures on the spindle, each of which rotates in a magnetic field produced by a few turns or a single turn in series with the load. These two magnetic fields act in opposite directions so that any stray field presumably will strengthen the field in which one armature rotates as much as it will weaken the field in which the other armature rotates, and the resulting effect will be nil. There are two sets of retarding magnets. These magnets are placed so that if a stray field increases the strength of one set, it simultaneously reduces the strength of the other. For further protection these magnets are surrounded by an iron box.

CHAPTER VI

MAGNETISM AND PERMANENT MAGNETS

Magnets and magnetism are involved in the operation of practically all electrical apparatus. Therefore an understanding of their underlying principles is essential to a clear conception of the operation of such apparatus.

Magnets may be divided into two general classes: *permanent* magnets, which have the property of retaining their magnetism indefinitely and which require no exciting ampere-turns; *electromagnets*, the magnetism of which depends on the magnetic action of electric currents (Chap. VII).

Permanent magnets are made of hardened steel and its alloys, and of alloys of elements such as cobalt and aluminum, which, alone, are weakly magnetic. Electromagnets are made of soft iron and soft steel, which are highly responsive to changes in the magnetizing effect of electric currents. Electromagnets are discussed in Chaps. VII and VIII.

135. Magnetic Materials.—Iron (or steel) is far superior magnetically to any other single substance. Cobalt and nickel individually possess magnetic properties, which are far inferior, however, to those of iron. Liquid oxygen is attracted to the poles of magnets. Recently developed are alloys of iron, cobalt, nickel and other metals, such as aluminum, for example, having excellent magnetic properties. Also, certain iron-nickel alloys have unusual magnetic properties. *Permalloy* (Par. 186, p. 272), an alloy composed of 80 per cent nickel and 20 per cent iron, has extremely high permeability at very low flux densities, but at the higher values of flux density its magnetic properties are inferior to those of good iron. *Hipernick* (Par. 187, p. 274) has low hysteresis loss and *Coupernick* has almost constant permeability. *Perminvar* (Par. 186, p. 273) is an iron-nickel-cobalt alloy which has low hysteresis loss and constant permeability. Also, *Alnico* (Par. 152, p. 222), an iron-aluminum-nickel-cobalt alloy, has excellent permanent magnetic properties.

The substances just mentioned, possessing magnetic properties, are *paramagnetic substances.* On the other hand, most other substances tend to be less magnetic than a vacuum, although the magnetic effect is usually so slight that it is difficult to detect it. Such substances are *diamagnetic.* Bismuth is the most highly diamagnetic substance known. Its permeability is 0.9998.

136. Natural Magnets.—Magnetic phenomena were noted by the ancients. Certain stones, notably at Magnesia, Asia Minor, were found to have the property of attracting bits of iron, and the name "magnets" was given to these magic stones. The fact that such stones had the property of pointing north and south, if suspended freely, was not discovered until the tenth or twelfth century. The practical use of such a stone in navigation gave it the name of "lodestone," or leading stone. Natural magnets are composed of an iron ore known in metallurgy as *magnetite*, an iron oxide having the chemical composition Fe_3O_4. Iron filings, when brought in contact with lodestone, concentrate at two or more regions, showing that the lodestone possesses two or more localized magnetic regions or poles.

137. Permanent (Artificial) Magnets.—If a piece of hardened steel be magnetized, either by coming in contact with another magnet, or by means of an electric current, the steel will be found to have acquired a considerable amount of magnetism, which it will retain indefinitely. Such a steel magnet is called a *permanent magnet.* If a piece of soft steel or soft iron be similarly treated, it retains only a very small portion of the magnetism initially imparted to it.

These properties make it desirable to use hardened steel (and its alloys) when a permanent magnet is desired, and to use soft iron or soft steel when it is essential that the magnetism respond closely to changes in magnetizing force.

138. Magnetic Poles and Magnetic Field.—When a piece of iron or other magnetic body becomes magnetized, magnetism appears to emerge from the body in some regions and appears to enter the body in other regions. These regions where the magnetism appears to emerge or to enter are called the *poles* of the magnet. For example, in the bar magnet shown in Fig. 131, magnetism appears to emerge from the left-hand end of the magnet and to enter at the right-hand end. The region at which magnetism appears to emerge is called the *north* or *N-pole* of the magnet; the region at which magnetism appears to enter is called the *south* or *S-pole* of the magnet. Magnetic lines are not necessarily confined to the ends of a magnet, but they may emerge or enter at any portion of a surface (Par. 140, p. 205). Hence, a magnetic pole may be defined *as any surface from which magnetic lines are emerging or into which magnetic lines are entering.* The two poles are distinguished by the position which they seek if

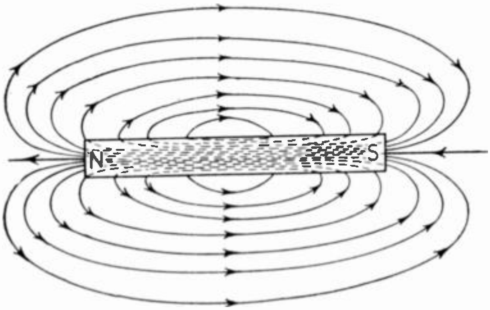


FIG. 131.— Magnetic field about a bar magnet.

the magnet is suspended freely. The pole which points north is called the *north-seeking pole* or *north pole* for short, and the other pole is the *south-seeking pole*, or *south pole*.

Magnetism manifests itself as if it existed in lines,[1] as indicated in Fig. 131. The force exerted on a small piece of iron is in the direction of these lines and the magnitude of the force is proportional to the density of the lines taken normal to their direction. Moreover, the mapping of a magnetic field by such lines conveys to the mind a picture of the magnetic effects. For example, the lines (Fig. 131) indicate the magnetic field about a bar magnet.

If the magnetic lines be determined experimentally, it is found that they seem to emerge from the N-pole of the magnet and to enter the S-pole of the magnet as shown in Fig. 131. These lines may be assumed to continue through the magnet from the S- to the N-pole, each line forming a closed loop. Such lines are called *lines of induction* (see Par. 143, p. 208).

[1] The line is the c.g.s. unit of magnetic flux and is called the *maxwell* (see p. 250).

In a magnet of the general shape shown in Fig. 131, the plane midway between the poles is the *neutral zone*, or *equator*, of the magnet. The entire path through which the lines of induction pass is called the *magnetic circuit*. Since each line of induction is continuous (Fig. 131), it follows that in a single magnet neither a N-pole nor a S-pole can exist alone. Also, all the N-poles taken together must be equal magnetically to all the S-poles taken together.

That the N-poles and S-poles of a magnet are equal in strength is substantiated by the fact that when a magnet is free to move in a uniform magnetic field, like that of the earth, no tendency toward translation has ever been observed. The magnet merely tends to align itself in the direction of the field.

The influence of a magnet extends to a region well outside the magnet itself, as is shown by the fact that the magnet exerts forces on iron or on electric currents which are located in this region. The region in which magnetic influence exists constitutes the *magnetic field*. The lines also represent the magnetic field as is shown in Fig. 131. The strength of the magnetic field at any point is represented by the density of these lines at that point (Par. 144, p. 211).

139. Effect of Breaking a Bar Magnet.—Even if a magnet be broken, N-poles and S-poles appear on each fragment, and for each fragment the N-poles all taken together are equal in strength to the S-poles all taken together. Consider Fig. 132(b), which shows the ordinary bar magnet of (a) broken near its center. A S-pole appears at the broken end of the fragment containing the original N-pole, and a N-pole appears at the broken end of the fragment containing the original S-pole.

In Fig. 132(c) the bar magnet is broken in two places, and again equal and opposite N- and S-poles are created on each fragment, as shown. The phenomenon is easily explained by noting that the lines of induction still continue to pass from one fragment to the next adjacent one, and in so doing constitute N- and S-poles as shown in Fig. 132(b) and (c).

However, it does not necessarily follow that the same number of lines of induction pass through each of the fragments, so that the N- and S-poles on one fragment are not necessarily equal to those on some other fragment.

Experimentally this phenomenon is easily illustrated by magnetizing a highly tempered steel knitting needle and breaking it at various points. The resulting magnetic field is readily observed by the sprinkling of iron filings (Par. 148, p. 216).

140. Consequent Poles.—Consequent poles are poles which form on the sides of a magnet rather than at the ends. Such poles for a bar magnet are shown in Fig. 133. Consequent poles

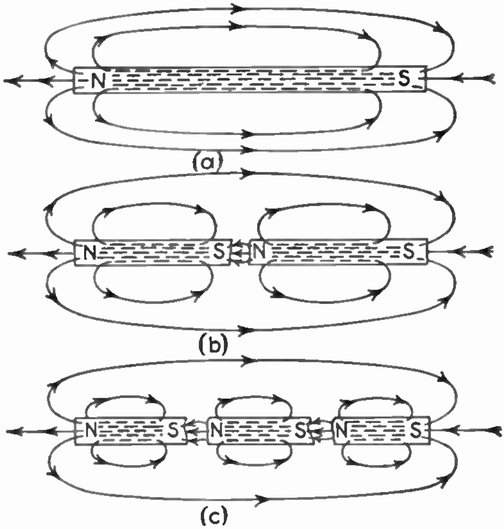


FIG. 132.—Effect of breaking a bar magnet.

are occasionally found in bar magnets where different portions have been rubbed by a N-pole, or a S-pole, or when exciting coils, acting in opposition, have been placed over the bar. Consequent poles are in reality due to the fact that the bar consists of two or more magnets arranged so that two N- or two S-poles exist in the same portion of the magnet. Consequent poles are illustrated in Fig. 133. The magnetic field shown in Fig. 143 (p. 217) is in a way illustrative of the field resulting from consequent poles. In this case, however, two bar magnets are used and a small air-gap exists between the adjacent N-poles. Consequent poles are frequently produced on iron and steel bars due to their having come in contact with lifting magnets (see p. 241).

141. Mechanical Forces between Magnetic Poles.—When a freely-suspended N-pole is brought in the vicinity of another

N-pole, it is repelled, whereas, if a S-pole is brought into the presence of a N-pole, it is attracted toward the N-pole. S-poles are also found to repel one another. These attractions and repulsions are mutual. From this it may be stated that *like poles repel one another and unlike poles attract one another.*

Coulomb's Law of Magnetic Repulsion and Attraction.—Coulomb,[1] by means of a torsion balance, proved experimentally that

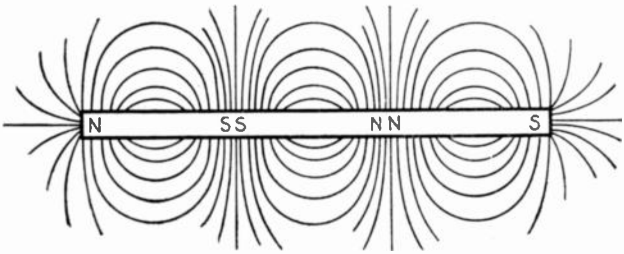


FIG. 133.—Consequent poles.

the force of attraction (or repulsion) between two given poles is inversely as the square of the distance between the poles, provided the dimensions of the poles are small compared with the distance between them. This law holds strictly for point-poles only. It may be proved by analytical methods.

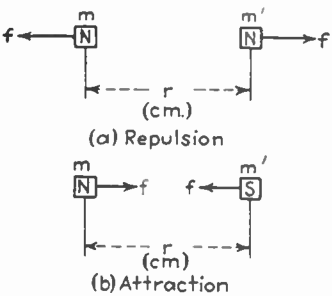


FIG. 134.—Repulsion and attraction between magnetic poles.

The unit pole is defined as follows:

A unit magnetic pole is one of such strength that if placed at a distance of 1 *cm. in free space from a like pole of equal strength will repel it with a force of* 1 *dyne.*

Pole strength is measured by the number of unit poles which are equivalent to the pole in question.

The force f, existing between poles in air, may be formulated as follows:

$$f = \frac{mm'}{r^2} \text{ dynes} \tag{92}$$

[1] See footnote, p. 28.

dynes 981

where m and m' are the pole strengths (in terms of unit pole) of two magnetic poles, placed a distance apart of r cm. in vacuum or air, as shown in Fig. 134. This force may be attraction or repulsion, as the poles are unlike or like.

Example.—Two N-poles, one having a strength of 500 units and the other a strength of 150 units, are placed a distance apart of 4 in. in air. What is the force in grams acting between these poles, and in what direction does the force act?

$$4 \text{ in.} = 4 \times 2.54 = 10.16 \text{ cm.}$$

$$f = \frac{500 \times 150}{(10.16)^2} = \frac{75{,}000}{103.2} = 726 \text{ dynes.}$$

$$\frac{726}{981} = 0.740 \text{ g.} \quad \text{Poles repel each other.} \quad \textit{Ans.}$$

in grams

When the dimensions of the poles are not small in comparison with the distance between them, Eq. (92) cannot in general be applied. However, the pole may be divided into very small areas and the effect of each area determined, the distance to the center of each area being used.

142. Weber and Ewing Theory.—In spite of the rapid advance made during the past few years in the knowledge of atomic and molecular phenomena, little if anything has been developed as yet to account for the factors which are responsible for the magnetic properties of magnetic substances. For example, there seems to be little that is unique in the structure of the iron molecule to give iron its magnetic qualities, which are far superior to those of other substances. Moreover, the fact that the magnetic properties of certain alloys of iron and nickel greatly exceed under certain conditions those of either metal makes the ultimate causes of magnetism even more mystifying.[1]

The molecular theory first advanced by Weber[2] and later expanded by Ewing explains many magnetic phenomena, such as the appearance of N- and S-poles on the breaking of a magnet, the character of the saturation curve, hysteresis, and certain other phenomena. In fact, modern physics has been unable to improve to any substantial degree on these early theories.

[1] An excellent discussion of this subject is given by KARL K. DARROW, "Contemporary Advances in Physics XIII; Ferromagnetism," *Bell System Tech. Jour.*, April, 1927, p. 295.

[2] See footnote, p. 251.

In the Weber-Ewing theory each of the molecules which constitute the magnet is assumed to be a very small magnet with its own N- and S-pole, as shown in Fig. 135(a). The molecules themselves are not subject to hysteresis, saturation, etc. When no magnetizing force is being applied, these small magnets are arranged in a haphazard manner, as in Fig. 135(a), so that the various molecular N- and S-poles all neutralize one another, and no external magnetic effect is produced. On the application of a magnetizing force, however, the small magnets tend to arrange themselves so that their axes are parallel and their N-poles are all pointing in the same general direction as the magnetizing force. This is shown in Fig. 135(b). The theory further explains the formation of N- and S-poles on the cutting of a magnet. For example, if a magnet be cut along the line XX (Fig. 136) a new N-pole and a new S-pole will result, which, before the fracture took place, neutralized each other. This explains the formation of the N-pole and S-pole at the fracture when a magnet is broken (Fig. 132).

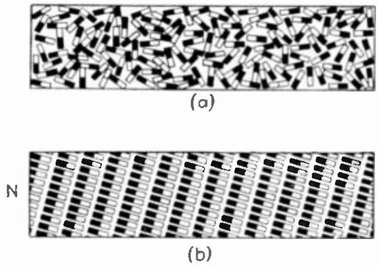


FIG. 135.—Weber-Ewing theory of magnets.

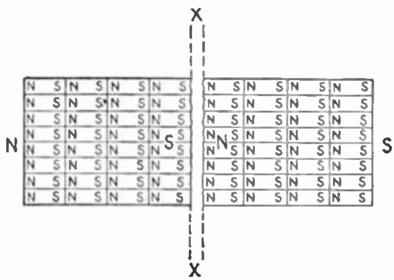


FIG. 136.—Weber-Ewing theory and cutting of magnet.

This theory is further supported by the fact that if a permanent magnet is ground into very small particles, each of the particles possesses the properties of a bar magnet, each particle having its own N- and S-pole.

The explanation of the saturation curve and of hysteresis is considered later (Par. 174 and 183, pp. 253 and 266).

143. Lines of Magnetization, Force, and Induction.—The magnetic relations in a permanent bar magnet at first appear relatively simple since from Fig. 131 the lines of induction merely pass through the iron as closed loops. However, even with the

Weber-Ewing theory, it is not possible to explain some of the more simple phenomena, such as the fact that some of the magnetic lines leave the magnet at the ends and others leave at the sides. In an attempt to explain magnetic phenomena, Ewing evolved

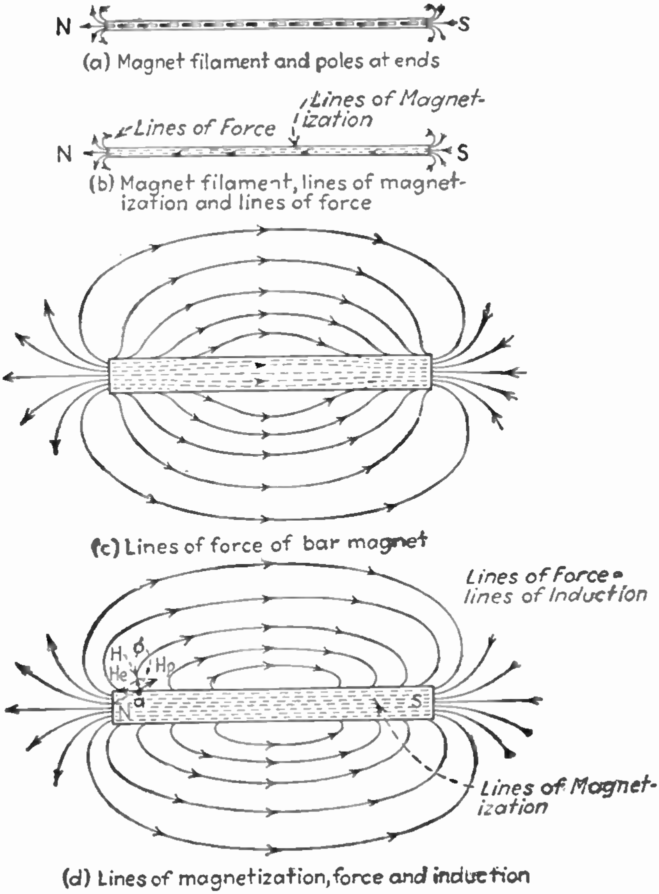


FIG. 137.—Permanent magnetism in bar magnet.

the following theories, and, although there has been some disagreement with him, a more rational explanation has not been advanced as yet.

The magnet may be considered as consisting of a number of very thin filaments (Fig. 137(*a*) and (*b*)), and a certain magnetizing force exists in each filament due to the alignment of the molecular magnets as shown in Fig. 137(a). Accordingly a N-pole is produced at one end and a S-pole at the other end of each filament. Experiments, as for example with a steel knitting needle, show that in this type of magnet the poles exist almost entirely at the ends. Owing to this inherent magnetizing force within the filament, *lines of magnetization* pass within the magnet from south to north, as in Fig. 137(b). The lines of magnetization remain entirely within the magnet and were there no free poles at the ends, that is, were each filament closed on itself, these lines of magnetization would constitute the lines of induction also.

However, because of the poles formed at the ends, lines of force leave the poles (Figs. 137(a) and (b)). The cross-section of the filaments is so small, however, that practically none of these lines passes back through the filament itself.

Lines of force *originate* on a N-pole and *terminate* on a S-pole and *are not closed lines.* This is indicated in Fig. 137(b) in which the solid lines are lines of force, and in Fig. 137(c) which shows roughly the lines of force in and about a bar magnet. The *lines of induction within the magnet* are defined *as the vector difference within the magnet of the lines of magnetization and the lines of force.* Since the lines of magnetization do not leave the magnet, it follows that in the region outside the magnet the lines of induction are equal to the lines of force. However, with very small filaments (Fig. 137(b)), the number of lines of force passing back through the filament is negligible, so that within the filamentary magnet the lines of induction are essentially equal to the lines of magnetization; outside the magnet the lines of induction are equal to the lines of force.

As a bar magnet of substantial cross-section is built up by combining these filaments, as in Fig. 137(c), a considerable proportion of the lines of force leaving the single N-pole formed at the end now pass back through the magnet, opposing the lines of magnetization. This reduces the lines of induction passing through the magnet.

Hence, the poles formed at the ends of a magnet are the source of a countermagnetizing force which tends to demagnetize the magnet. With this theory it is possible to account for the fact that a considerable proportion of the lines of induction leave the sides of the bar magnet as in Fig. 137(*d*). The force H at any point a is due to the combined effect of the N-pole at the end of the magnet, which exerts a component of force H_p, and the internal magnetizing force H_e, which produces the lines of magnetization. (The S-pole also exerts force at a, but at points near the N-pole the effect of the S-pole is negligible by comparison.) The resultant of H_p and H_e produces the line of induction ϕ at the point a and determines its direction. The total lines of induction are as indicated in Figs. 131 and 132(*a*). The lines of force are not closed but originate at a N-pole and terminate at a S-pole. Within the magnet they are opposed to the lines of magnetization, since the lines of force go from north to south. With a long bar magnet, one half the lines of induction existing at the middle cross-section, or neutral plane, leaves through the end surfaces and one half leaves through the sides.

Experience shows that, as the area of the poles becomes greater in proportion to the length of the magnet, permanent magnetization becomes more difficult. It becomes extremely difficult to produce permanent magnetism in short specimens of even the best permanent magnetic material.

It follows that the lines of magnetization, which are inherent within the magnet, originate at the S-pole and pass through the magnet to the N-pole; the lines of force originate at the N-pole, pass through the air and the magnet, and terminate at the S-pole; the lines of induction are continuous closed lines, passing outside the magnet from N-pole to S-pole and passing within the magnet from S-pole to N-pole.

The distinction of lines of magnetization, lines of force, and lines of induction should be kept clearly in mind.[1]

144. Lines of Force.—From Par. 143, the lines of force and the lines of induction are identical in air. If a small N-pole be placed in a magnetic field, two effects will be observed.

1. The N-pole will be urged along a line of force and if free to move will go to the S-pole of the magnet.

2. The force urging the N-pole will be greatest where the lines of force are most dense, and, moreover, the force will be proportional

[1] For a more detailed discussion, see PENDER, "Principles of Electrical Engineering," Par. 43, p. 56, McGraw-Hill Book Company, Inc.

to the number of lines per unit area taken normal to the direction of the lines.

It follows that the *density of the lines of force* represents the force at the various points in the magnetic field. The direction of the force is the direction of the line of force at the point, and the magnitude of the force is given by the number of lines per square centimeter taken normal to the direction of the lines at the point.

145. Field Intensity.—It has been stated that the force at a point acting on a magnetic pole in a magnetic field is proportional to the density of the lines of force at that point. *Unit field intensity is defined as the field strength which will act on a unit pole with a force of 1 dyne.* One *line of force* perpendicular to and passing through a square centimeter represents *unit field intensity.* Field intensity is given in *dynes per unit pole* and is usually represented by the symbol H.

Recently, by international agreement, the unit of field intensity was named the *oersted* in honor of Hans Christian Oersted[1] of Copenhagen, who in 1819 showed that a magnet tends to place itself at right angles to an electric current (Par. 159, p. 229; also see p. 257).

If a pole of m units be placed in a field of intensity H, the force acting on this pole is

$$f = m \times H \text{ dynes.} \tag{93}$$

A pole placed in such a field must be of such small strength that it will have no appreciable disturbing effect on the magnetic field.

Example.—When a small pole having a strength of 25 unit poles is placed in a magnetic field, it is acted on with a force of 200 dynes. What is the field intensity at this point?

The force per unit pole

$$H = 200 \div 25 = 8 \text{ dynes per unit pole, or 8 oersteds.} \quad \textit{Ans.}$$

[1] Oersted, Hans Christian (1777–1851). A Danish physicist, who at first studied medicine but in 1806 was appointed Professor of Physics at the University of Copenhagen. In 1829 he became Director of the Polytechnic of the same city and in 1850 Privy Councilor. His outstanding works were the establishment of the intimate interrelations of electricity, "galvanic" currents, and magnetism.

It also follows that the density of the lines of force, taken for an area perpendicular to the direction of these lines, is 8 lines per square centimeter, or 8 gausses.

Example.—A total flux of 200,000 lines exists in air between two parallel pole faces, each 8 centimeters square. The field is uniformly distributed. With what force (grams) will a pole, having a strength of 100 units, be acted upon if placed in this field?

Flux density $= \frac{200,000}{8 \times 8} = 3,125$ lines per square centimeter, or 3,125 gausses. Being in air this value of flux density also equals the field intensity H.

$$f = m \times H = 100 \times 3,125 = 312,500 \text{ dynes.}$$
$$312,500 \div 981 = 319 \text{ g.} \quad \textit{Ans.}$$

146. Flux Density.—Flux density is the number of lines of induction per unit area taken perpendicular to the induction. In free space, flux density and field intensity are the same numerically, but within magnetic material the two are quite different. The two should not be confused. The c.g.s. unit of flux density (one line per square centimeter) is the *gauss*.[1] The expressions "lines per square centimeter" and "lines per square inch" are often used in practical work when speaking of flux density.

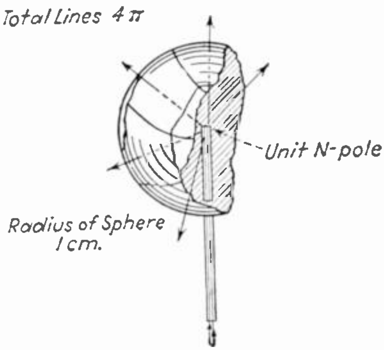


FIG. 138.—Lines of force leaving a unit N-pole.

By definition, the force exerted by a unit pole upon another unit pole at centimeter distance in air is 1 dyne. The field intensity on a spherical surface of 1 cm. radius and with a unit pole

[1] Gauss, Karl Friedrich (1777–1855). A German mathematician and astronomer, who was Director of the Göttingen Observatory from 1807 until his death. He became much interested in the study of the earth's magnetism and in conjunction with Weber invented new apparatus for its measurement, apparatus of the same type being still in use. In 1833, with Weber's assistance, he erected a magnetic observatory at Göttingen almost entirely free from iron, where he made many valuable magnetic observations of the earth's magnetism. He wrote a large number of papers on mathematical and magnetic subjects. His work resulted in a number of important contributions to the knowledge of magnetism.

at its center must then be unity and can be represented by one line per square centimeter over the entire spherical surface, as in Fig. 138.

Since there are 4π sq. cm. on the surface of a sphere of 1-cm. radius, each unit pole must have 4π or 12.57 lines of force leaving it.[1] Figure 138 represents a portion of a spherical surface of 1-cm. radius and shows the passage of one line of force through each square centimeter of surface, each line originating in the unit N-pole. This also explains the appearance of the 4π term encountered so often in magnetic formulas; $4\pi m$ lines of force leave a N-pole having a strength of m units.

Example.—A pole having a strength of 400 units is placed at the center of a sphere having a radius of 3 cm. What is the flux density at the surface of the sphere, and what force will be exerted on a pole of 10 units placed at the surface of the sphere?

Total lines leaving pole at center $= 400 \times 4\pi = 5{,}027$ lines.

Area of surface of sphere $= 4\pi r^2 = 4\pi 9 = 113.1$ sq. cm.

Flux density $= \dfrac{5{,}027}{113.1} = 44.4$ gausses.

Force upon pole of 10 units $= 44.4 \times 10 = 444$ dynes. *Ans.*

As a check, the force may be determined by the law of inverse squares (see Par. 141, p. 205).

$$f = \frac{mm'}{r^2} = \frac{400 \times 10}{3 \times 3} = 444 \text{ dynes.}$$

147. Compass Needle.—The compass consists of a hardened-steel needle or small bar, permanently magnetized and accurately balanced upon a sharp pivot. The north-seeking end or N-pole points north, and the south-seeking end or S-pole points south. The N-pole of the needle is frequently colored blue, or is given some distinguishing mark. With the exception of a few magnet needles used for lecture purposes, the needle is enclosed in an

[1] Obviously the magnetic flux does not leave in individual lines but occupies the entire field, filling it uniformly if the field is uniform. The actual condition is represented by *tubes of force* (an expression originated by Faraday) the axis of each tube coinciding with the direction of the field. With the unit pole, there would be 4π, or 12.57, *tubes* of force. Each tube is represented by a single line at its axis as in Fig. 138. The conception of tubes of force explains the possibility of the fractional 0.57 line. At the surface of a sphere of unit radius, the area of each of the 12 tubes of force is 1 sq. cm., and the area of the remaining tube of force is 0.57 sq. cm. (4π = 12.57). In air or vacuum, tubes of induction would coincide with the tubes of force.

air-tight case for mechanical protection. Mariners' compasses are mounted carefully upon gimbals, so that they always remain level. Upon steel ships, heavy iron balls are placed near the compass to neutralize the magnetic effect of the ship itself.

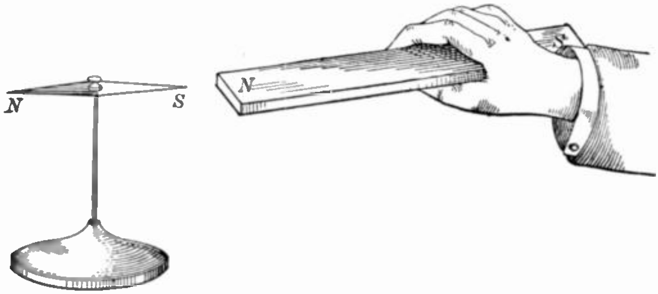


FIG. 139.—Compass needle and bar magnet.

By means of the compass needle, the polarity of a magnet is readily determined. The *S*-pole of the compass needle points to the *N*-pole of the magnet, as in Fig. 139. Likewise, the *N*-pole of the compass needle points to the *S*-pole of the magnet. This

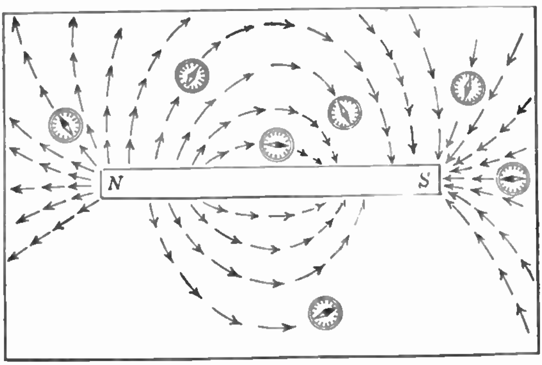


FIG. 140.—Exploring the field about a bar magnet with a compass.

action of the compass needle follows immediately from the law that like poles repel and unlike poles attract each other. This is very useful in practical work for it enables one to determine the polarity of the poles of motors and generators and to determine if the exciting coils are correctly connected.

Further, the compass needle always tends to set itself in the direction of the magnetic field in which it finds itself, the north end of the needle pointing in the direction of the lines of force or magnetic lines. This is illustrated in Fig. 140. By placing a small compass needle at the various points in the region of a magnet, and drawing an arrow at each point, the arrow pointing in the same direction as the needle, the field around the magnet may be mapped, as in Fig. 140. In mapping a field in this way it must be remembered that the earth's field also may exert con-

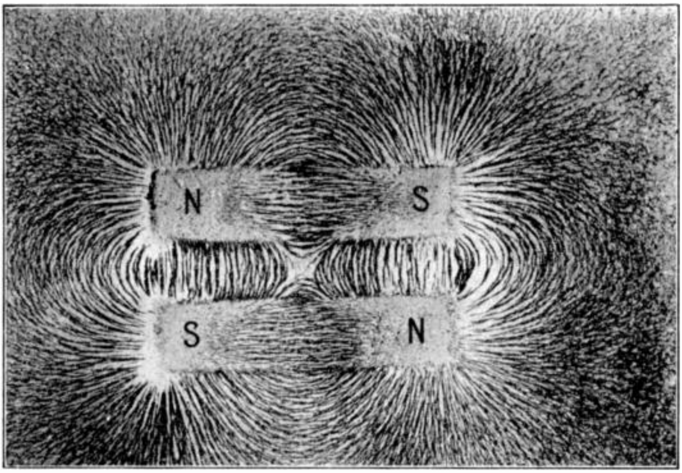


FIG. 141.—Magnetic figure, unlike poles adjacent.

siderable influence on the compass needle in addition to the effect of the field being studied.

148. Magnetic Figures.—If a card be placed over a magnet and iron filings be sprinkled over the card, a magnetic figure is obtained. At each point the filings set themselves in the direction of the line of force at that point, and the resultant figure shows in detail the character of the magnetic field. Figure 141 shows the magnetic field due to two bar magnets placed side by side, with unlike poles adjacent. On the other hand, Fig. 142 shows the field due to these same bar magnets when like poles are adjacent. It will be noted in Fig. 141 that the lines of force seem like elastic bands stretched from one pole to the other, acting to pull the unlike poles together. In Fig. 142 the lines of force from the two like poles appear to repel one another, indicating a state of repulsion between the poles. Figure 143 shows the field obtained by placing the bar magnets end to end, having the two N-poles adjacent.

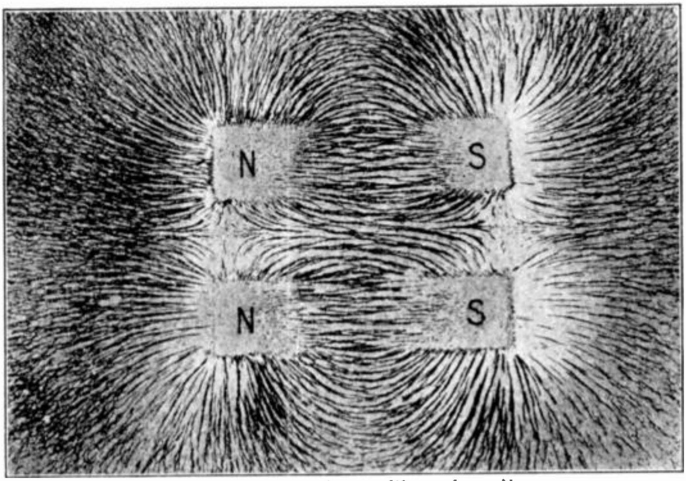


FIG. 142.—Magnetic figure, like poles adjacent.

149. Magnetic Induction.—If a magnet is brought near a piece of soft, non-magnetized iron, the piece of iron becomes *magnetized*

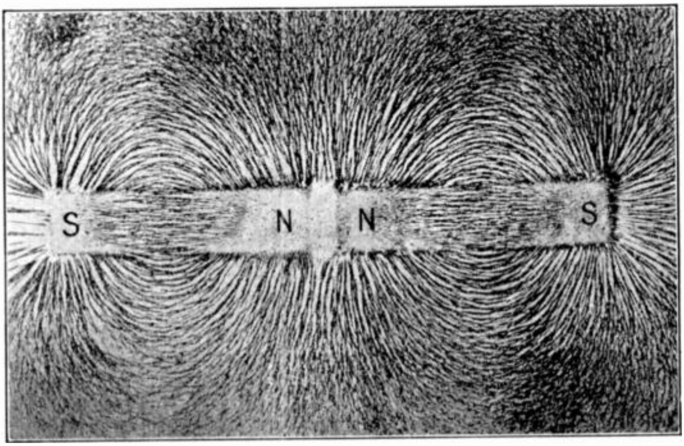


FIG. 143.—Magnetic figure, like N-poles adjacent.

by induction. If the N-pole of the magnet is brought near the soft iron, a S-pole is induced in that part of the iron nearest the inducing magnet, and if the S-pole of the magnet is brought near the iron a N-pole is similarly induced. This is illustrated in Fig. 144(a).

The reason that a N-pole induces a S-pole and a S-pole induces a N-pole is illustrated in Fig. 144(b). The lines of induction, which leave the N-pole of the bar magnet, will concentrate in the soft iron, because iron permits the passage of magnetic lines far better than air does. Since the magnetic lines which leave the N-pole of the bar magnet must *enter* the soft iron at the end which is adjacent to the N-pole of the bar magnet, a S-pole is formed

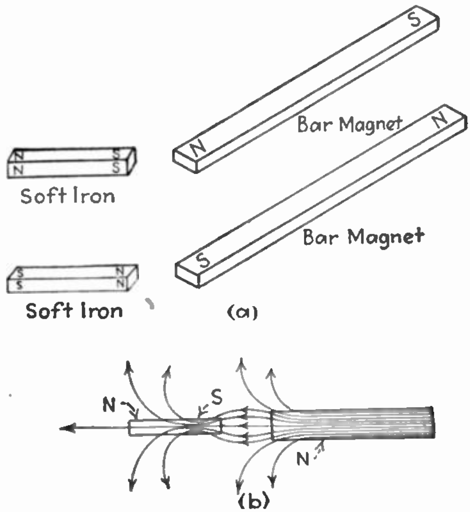


Fig. 144.—Inducd magnetic poles.

at that end of the soft-iron bar which is adjacent to the N-pole of the bar magnet. As lines of magnetic induction are continuous, they must also *leave* the soft-iron bar, and they do so at the end which is more remote from the magnetizing N-pole. Hence a N-pole is formed at the more remote end of the soft-iron bar.

The inducing N-pole attracts the induced S-pole and repels the induced N-pole on the soft iron. As the induced S-pole is nearer to the inducing pole, attraction predominates.

It is sometimes noticed that if a comparatively weak N-pole be brought into the vicinity of a strong N-pole, attraction between the two results, rather than the repulsion which might be expected. This is no violation of the laws governing the attraction and repulsion of magnetic poles but comes from the fact that the strong *N*-pole induces a *S*-pole which overpowers the existing weak *N*-pole and hence attraction results. In this way it is easy to reverse the polarity of a compass needle by holding one end close to a strong magnetic pole of the same polarity.

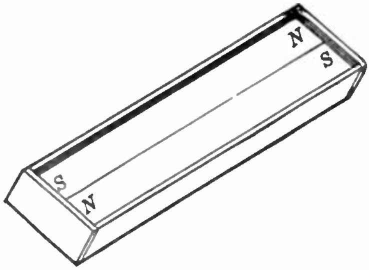


Fig. 145.—Proper method of "keeping" bar magnets.

For a similar reason, when two bar magnets are put away in a box, the adjacent ends should be of opposite polarity, as in Fig. 145. They will retain their magnetism better under these conditions. When a horseshoe magnet is not in use, a "keeper" of soft iron should be placed across the poles.

150. Law of the Magnetic Field.—*The magnetic field always tends to conform itself so that the maximum amount of flux is attained.* This offers further explanation of the attraction of iron to poles of magnets. The iron is drawn towards the magnet so

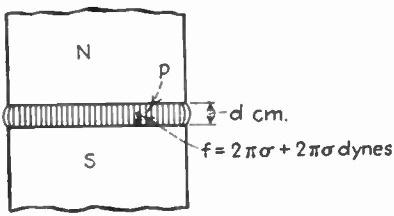


Fig. 146.—Force of attraction between parallel magnetized surfaces.

that the magnetic lines may utilize the iron as a part of their path, since iron conducts these lines much better than air. This is illustrated in the horseshoe magnet of Fig. 149. The armature is drawn toward the poles of the magnet, and the return path through the air is materially shortened, so that the number of magnetic lines is materially increased. The maximum flux exists when the armature is in contact with the poles.

151. Force between Magnetized Parallel Surfaces.—Consider two magnetic poles of opposite polarity having parallel surfaces of equal area separated by a short air-gap, *d* cm. in length

(Fig. 146). The area of each pole face is A sq. cm., and the magnetization is distributed uniformly over the two pole faces. Let the number of unit poles per square centimeter of each surface be σ. If the length of air-gap d is small compared with the length and breadth of the poles, the lines of force between the surfaces may be considered as straight, parallel, and uniformly distributed, except for a small number of lines at the edges of the poles. Since 4π lines leave each unit pole, the flux density in the air-gap will be $4\pi\sigma$ lines per square centimeter, or $4\pi\sigma$ gausses (see Par. 146, p. 213). $4\pi\sigma$ per cm^2

If a unit N-pole P be placed in the air-gap, a force of H dynes will act on it, where H is the field intensity (Par. 145, p. 212). Since the flux density is $4\pi\sigma$ lines per square centimeter, H must be equal to $4\pi\sigma$ oersteds or dynes per unit pole. This force is independent of the position of the unit pole, since the field, everywhere within the gap (except at the edges), is uniform. Half of the force, or $2\pi\sigma$ dynes, is due to the repulsion of the N-pole and the remaining $2\pi\sigma$ dynes is due to the attraction of the S-pole.

Now consider the unit pole P as placed on the surface of the N-pole. The S-pole will attract it with a force of $2\pi\sigma$ dynes. However, there are σ unit poles in each square centimeter of the N-pole. Hence, the S-pole will exert a force of $2\pi\sigma^2$ dynes on each square centimeter of the N-pole. Therefore, the total force exerted by the S-pole on the N-pole must be

$$f = 2\pi\sigma^2 A \text{ dynes.} \tag{94}$$

Since the flux density $B = H = 4\pi\sigma$ (numerically),

$$f = 2\pi\sigma^2 A = \frac{B^2 A}{8\pi} \text{ dynes} \tag{95}$$

33

where B = maxwells or lines per square centimeter. Also, since 981 dynes = 1 g. force, the force

$$f = \frac{B^2 A}{24.64} \text{ kg.,} \tag{96}$$

if B is expressed in kilolines per square centimeter. If B is expressed in lines per square inch and A in square inches,

$$f = \frac{B^2 A}{72{,}130{,}000} \text{ lb.} \tag{97}$$

Example.—The core of a solenoid is 2 in. in diameter and a uniform flux of 200,000 lines passes from the end of the core into an iron armature of equal area. What is the force of attraction in pounds between the core and the armature?

$$A = \frac{\pi}{4}(2)^2 = 3.14 \text{ sq. in.}$$

$$B = \frac{200{,}000}{3.14} = 63{,}800 \text{ lines per square inch.}$$

$$f = \frac{63{,}800^2 \times 3.14}{72{,}130{,}000} = 177 \text{ lb.} \quad \textit{Ans.}$$

The fact that the flux density is $4\pi\sigma$ in the gap at the center of the bar magnet also follows from the fact that $4\pi m$ lines leave each of the poles at the ends of the magnet. Assume that the cross-section of the bar magnet is 1 sq. cm. Hence, $m = \sigma$. The total lines leaving each of the poles at the end of the magnet is $4\pi m = 4\pi\sigma$. *All* these lines must go through the center section of the magnet. Hence, the flux density in a short transverse gap at the center of the magnet must be $4\pi\sigma = B$ lines per square centimeter.

152. Materials for Permanent Magnets.—The magnetic properties of materials to be used for permanent magnets are better understood if the relation of permanent magnetism to the hysteresis loop is known (see Par. 183, p. 266). The permanent magnet must not only retain its magnetism when left to itself, but it must also maintain it when used in connection with air-gaps, soft-iron pole pieces and other parts of magnetic circuits with which it is associated. After a piece of iron has been magnetized to a value of flux density well above saturation, and the magnetizing force is then reduced to zero, the remaining residual magnetism is called *remanence*. This is represented for cobalt-chromium steel by the ordinate $0a$ (Fig. 147). To bring the magnetism to zero, a reversed m.m.f. called the *coercive* force is necessary. This is represented by the abscissa $0b$ (Fig. 147). The coercive force is frequently called the *retentivity*. With permanent magnets, both high remanence and high retentivity are essential.

With the magnetic circuit composed entirely of magnetic material, that is, with no air-gaps or other non-magnetic substances in the closed magnetic circuit, the remanence would represent the flux density existing after the exciting ampere-turns have been removed. However, in order to utilize the magnetic flux produced by the magnet, an air-gap is usually necessary. Since a magnetizing force is required to send a flux across an air-gap, the effect of such an air-gap is to produce the equivalent of a demagnetizing force on the magnet. Such a demagnetizing force for tungsten steel is shown at Pp (Fig. 147), so that with the given air-gap the tungsten steel is operating at point P. It will be noted that with this material a very small increase in the air-gap, and hence in the demagnetizing force, causes a rapid decrease in the flux density. With too long an air-gap the magnet may

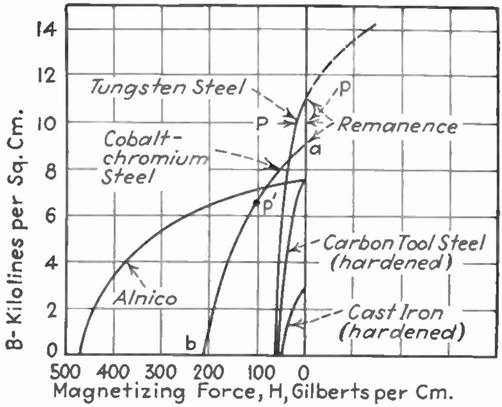


FIG. 147.—Characteristics of permanent-magnet materials.

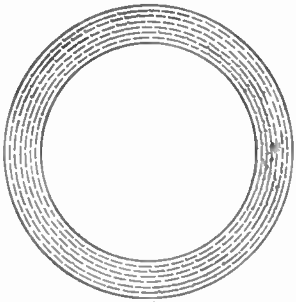
become very nearly completely demagnetized. Hence, tungsten steel is best adapted to magnets operating with very short air-gaps and no considerable amount of soft iron should be introduced into the magnetic circuit when tungsten-steel magnets are used.

The characteristic of cobalt-chromium steel shows that although the remanence is less than for tungsten steel, the retentivity is much greater. Hence with normal operation, as at point p', the magnet is highly stable. Moreover, this alloy is well adapted for magnetic circuits with long air-gaps, since an increase in the demagnetizing force causes only moderate decrease in the flux density.

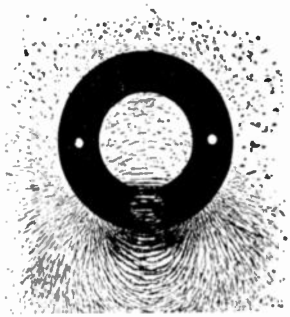
Recently a permanent-magnet alloy *Alnico*,[1] having nearly $2\frac{1}{2}$ times the retentivity of the cobalt-chromium steel, has been

[1] EDGAR, R. F., "Permanent Magnets," *Gen. Elec. Rev.*, October, 1935, p. 466.

developed by the General Electric Company. The characteristic of Alnico is shown in Fig. 147. This alloy consists of aluminum,



(a) Closed ring.



(b) Field of circular instrument magnet.

FIG. 148.—Ring magnets.

iron, nickel, and cobalt. Another magnetic alloy, *Alnic*, consists of aluminum and nickel. The high retentivity of Alnico opens up a much wider field of usefulness for permanent magnets.

The area $0ab0$ (Fig. 147) (shown for the cobalt-chromium steel) is proportional to the *energy* per unit volume of the magnetic material (see Par. 184, p. 268), and such areas are considered as the criteria for permanent-magnet materials.

In Fig. 147, characteristics are also shown for hardened cast iron and for carbon tool steel, both of which are inferior to the tungsten and cobalt-chromium alloys.[1]

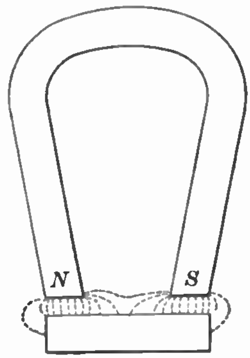


FIG. 149.—Horseshoe magnet attracting a soft-iron armature.

153. Forms of Permanent Magnets.—The simple bar magnet usually is not suitable for commercial work. For the same amount of material, other forms are more powerful and more compact. Figure 148(a) shows a closed ring magnet for which the mag-

[1] For further discussion, see J. FERDINAND KAYSER, "Cobalt Steels for Permanent Magnets," *Engineer*, Jan. 19, 1923, and Jan. 26, 1923; JOHN WALTER ESTERLINE, "Permanent Magnets," publ. by the Esterline Company, Indianapolis, Ind.

netic flux is contained within the ring, and as no external effect is noted, such a ring is not very useful. However, if an air-gap be introduced in the ring (Fig. 148(*b*)), the magnetic flux within the ring becomes available for use. This type of ring magnet is used for electrical instruments (p. 148), usually of the miniature type. The intense field near the poles should be noted.

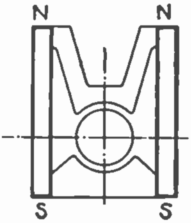


FIG. 150.—Application of straight-bar cobalt-steel magnets to measuring instrument.

The horseshoe magnet (Fig. 149) is very useful for two reasons. As the two poles are near each other, a comparatively strong field exists between them. The proximity of the two poles, combined with the considerable length of magnetic circuit, reduces the demagnetizing effect of the poles on the magnet (Par. 143, p. 208). If the function of the magnet is to exert a pull on an armature, both poles are equally effective. Figure 92 (p. 148) shows a horseshoe magnet as used in Weston direct-current instruments. This form of magnet is also well adapted to magnetos (Fig. 153).


FIG. 151.—Typical watthour meter brake magnets.

Figure 150 shows the application of straight-bar cobalt-steel permanent magnets to measuring instruments (see Par. 152). It is possible to use bar magnets in this manner only when the retentivity of the material is very high (Fig. 147, p. 222).

FIG. 152.—Compound or laminated bar magnet.

Figure 151 shows magnets of the horseshoe type with very short air-gap, as used to damp the armatures of watthour meters (Par. 132, p. 194). Small horseshoe magnets are also used in telephone receivers.

154. Laminated Magnets.—Thin steel magnets are stronger in proportion to their weight than thick ones. This is due to the more uniform hardening which can be obtained with the thinner metal during the quenching which follows heat treatment.

Hence, for a given amount of material, a magnet made up of laminations or leaves (Fig. 152) is more powerful and retains its magnetism better than one made of a single piece of metal of the same total mass. Figure 153 shows the form of horseshoe magnet, in laminated form, used for telephone and ignition magnetos.

155. Magnetizing.—A magnet may be magnetized by merely rubbing it with another magnet. The resulting polarity at any point is opposite to that of the last pole which came in contact with this point. Therefore, it is well to rub one end with the N-pole of the inducing magnet and the other end with the S-pole. This

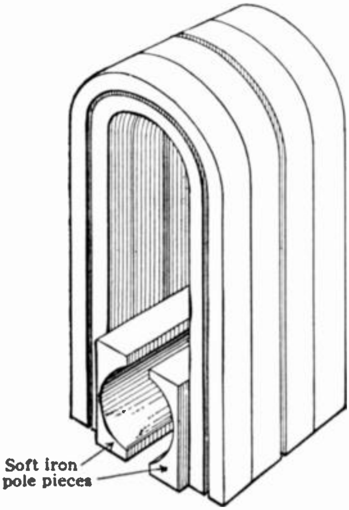


Fig. 153.—Compound horseshoe magnet used in magnetos.

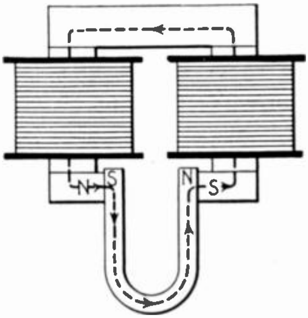


Fig. 154.—Magnetizing horseshoe magnet with electromagnet.

method, however, is not efficient and is not used commercially. One commercial method is to place the material to be magnetized between the poles of a powerful electromagnet (Fig. 154). An armature or "keeper" should be placed across the poles of a horseshoe magnet before removing it from the field of the electromagnet.

A common method of magnetizing watthour magnets is shown in Fig. 155. A large number of the magnets are placed over a single cylindrical copper conductor capable of carrying a large current. Only one such magnet is shown in Fig. 155. A large direct current is then made to flow in the conductor which magnetizes the magnets, the direction of the magnetizing force for the particular direction of current being shown by the dotted line in Fig. 155 (also see Fig. 157, p. 229). This method is convenient and permits a large number of magnets to be magnetized simultaneously. Magnetization may also be secured by inserting the magnet in a suitable exciting coil and allowing a heavy current to flow in the coil. A few turns of low-resistance wire may be wound around the magnet and connected in series with a storage battery. On closing the switch, a large current flows for a short time, but the switch may be opened before damage to the battery occurs. The large value of temporary current is usually sufficient to leave the magnet in a strongly magnetized condition.

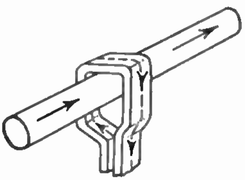


Fig. 155.—Magnetizing with single current.

156. Artificial Aging.—The rapid quenching of permanent magnets during heat treatment produces sudden changes in their physical structure, and it may be several weeks before the magnet becomes adjusted to a stable condition. A change in magnetic properties accompanies these physical changes. When permanent magnets are used for instruments and meters, it is most important that their strength shall not change with time. Artificial aging is therefore used to bring magnets to the stable condition within a short period of time.

The methods of aging vary. One method is to immerse the magnets in oil at 120°C. for 1 hr.; another method is to subject them to a temperature of 100°C. for from 24 to 40 hr. During this process the magnets are sometimes subjected to mechanical vibration. Some manufacturers apply a slight demagnetizing force, then successively smaller magnetizing and demagnetizing forces. This process reduces the residual magnetism by possibly 20 per cent, but it tends to bring the magnet to a permanent magnetic state.

157. Magnetic Shielding.—There is no known insulator for magnetic flux. No appreciable change in the flux or in the pull of a magnet is noticed if glass, paper, wood, copper, or other such non-magnetic material be placed in the magnetic field. However, it is often desirable to shield galvanometers and electrical measuring instruments from the earth's field and from stray fields due to generators, conductors carrying currents, etc. This is done by surrounding the instrument with an iron shell as in

Fig. 156. This shell by-passes *practically* the entire flux and thus prevents it from affecting the sensitive portions of the instrument. The smaller the openings in the shell, the more effective the screening. Three or four shells, with air spaces between, are found to be more effective than one shell of the same total thickness. Such shells, however, are used only in screening the most sensitive galvanometers.

It should be emphasized that perfect screening by this method cannot be attained since there is always an air path in parallel

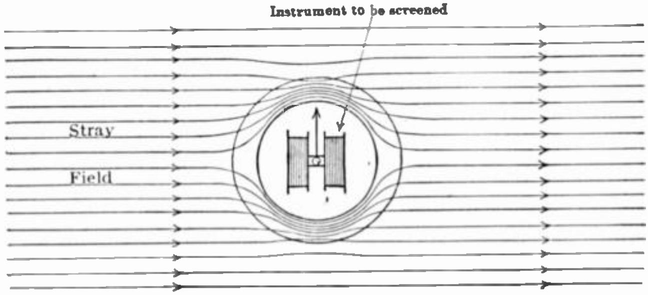


FIG. 156.—Magnetic screen.

with the iron path of the shell. However, it is usually possible to reduce the stray field within the shell to a negligible value.

158. Earth's Magnetism.—The earth itself produces a magnetic field, acting as if a huge bar magnet were contained within it, the direction of the axis being approximately north and south. The center of the northern pole (corresponding in polarity to the *S*-pole of a magnet) is situated near Boothia Felix about 1,000 miles from the geographical pole. In this region the magnetic needle, if suspended freely, will assume a vertical position. The exact location of the southern magnetic pole has not been determined, but experiment points to the existence of two *S*-poles. Due to the non-coincidence of the geographical and magnetic poles and to the presence of magnetic materials in the earth, the compass points to the true north in only a few places on the earth's surface. The deviation from the true north is called the *magnetic declination,* and magnetic maps are provided showing the declination at various parts of the earth. At New York it is about 9° west. The declination undergoes a gradual variation from year to year, called the *variation* change. A careful record is kept of this secular variation, and scientific measurements, such as are used in astronomy, surveying, and navigation, must be corrected correspondingly. The needle undergoes a very small daily variation and also an annual variation.

A freely suspended and balanced needle does not take up a position parallel to the earth's surface, when under the influence of the earth's magnetism alone, but assumes a position making some angle with the horizontal. This angle is called the *dip* of the needle. At New York it is about 70° north. The dip undergoes changes similar to those in the variation. The field intensity (total, not horizontal) of the earth's field at New York is about 0.61 c.g.s. unit, although this value changes slightly from time to time.

CHAPTER VII

ELECTROMAGNETISM

159. Magnetic Field Surrounding a Conductor.—It had long been suspected that some relation exists between electricity and magnetism, but it remained for Oersted, in 1819, to show that this relation not only exists but that it is a definite one.

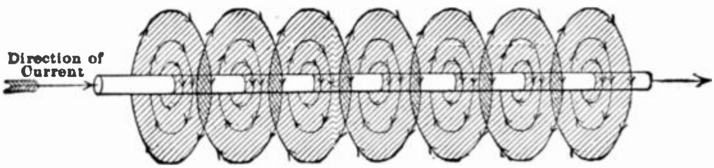


FIG. 157.—Magnetic field about a straight conductor.

If a compass be brought into the neighborhood of a single conductor carrying an electric current, the needle deflects, indicating the presence of a magnetic field. It is further observed that the needle always tends to set itself at right angles to the current.

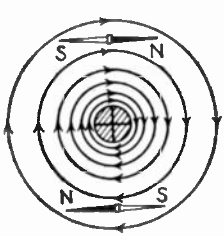


FIG. 158.—Lines of flux surrounding a cylindrical conductor—current inwards.

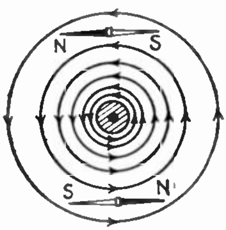


FIG. 159.—Lines of flux surrounding a cylindrical conductor—current outwards.

When it is held above the conductor, the needle points in a direction opposite to that which it assumes when held beneath the conductor. Further investigation shows that the magnetic flux exists in circles about the conductor (if there is no other magnetic field in the vicinity), as in Figs. 157, 158, and 159. These circles have their centers at the axis of the conductor and their planes are perpendicular to the conductor. If the current in the conductor be reversed, the direction in which the compass needle is deflected will also reverse, showing that the direction of this magnetic field depends on the direction of the current. The relation between the two is shown in Fig. 157. The fact that the magnetic flux exists in circles perpendicular to the conductor explains the reversal of the direction of the compass needle when moved from a point above the conductor to a point beneath it, since the direction of the field above the conductor must be opposite to that beneath the conductor. This is illustrated in Figs. 158 and 159.[1]

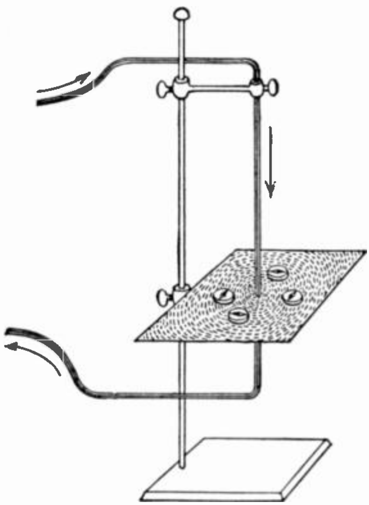


FIG. 160.—Investigation of the magnetic field surrounding a conductor.

The experiment shown in Fig. 160 illustrates this concentric relation of flux to current. A conductor carrying a current is brought vertically down through a horizontal sheet of cardboard. Iron filings sprinkled on the cardboard form concentric circles. (A current of about 100 amp. is necessary to obtain distinct figures.) If four or more compasses are arranged as in Fig. 160, they will indicate by the direction in which their needles point that the magnetic lines are circles having the axis of the wire as a center.

160. Relation between Magnetic Field and Current.—A definite relation exists between the direction of the current in a conductor and the direction of the magnetic field surrounding the conductor. There are two simple rules by which this relation may be remembered.

[1] A circle having a cross inside (⊕) represents the feathered end of an arrow and indicates that the current is flowing into the paper, away from the observer. A circle having a dot at the center (⊙) represents the approaching tip of an arrow and indicates that the current is flowing out of the paper, toward the observer.

Hand Rule.—Grasp the conductor in the *right hand* with the thumb pointing in the direction of the current. The fingers will then point in the direction of the lines of flux (Fig. 161).

Corkscrew Rule.—The direction of the current and that of the resulting magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated.

This last rule is probably the most common and the most easily remembered. However, it must not be inferred from this rule

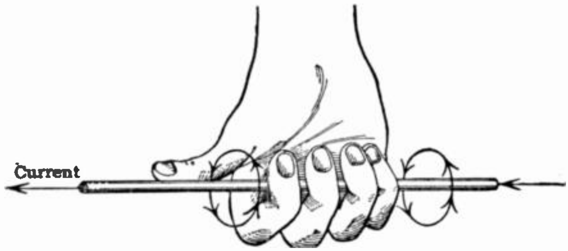


FIG. 161.—Hand rule.

that the magnetic field exists in spirals about the conductor. It exists actually in planes perpendicular to the conductor.

161. Magnetic Field of Two Parallel Conductors.—When each of two parallel conductors carries an electric current, flowing in

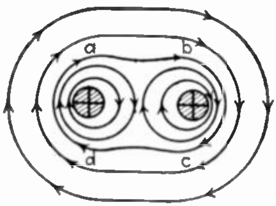


FIG. 162.—Magnetic field about two parallel conductors—current in same direction.

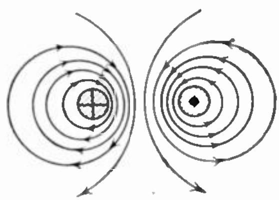


FIG. 163.—Magnetic field about two parallel conductors—current in opposite directions.

the same direction, there is a tendency for the two conductors to be drawn together. The reason for this is obvious. In Fig. 162 the lines of force encircle each conductor in the same direction (corkscrew rule) and the resultant field is an envelope of lines acting like elastic bands tending to pull the conductors together. Further reason for this attraction is given by the rule of Par. 150 (p. 219) stating that the magnetic field tends to conform itself so that the number of magnetic lines is a maximum. The pulling together of the conductors reduces the length of path *abcd* through which the magnetic lines must pass. The field due to each conductor separately is still circular in form, but the resultant magnetic field is no longer circular, as is shown in Fig. 162.

In Fig. 163 is shown the field which exists when two parallel conductors carry current in opposite directions. The magnetic lines are circles, but these circles are not concentric either with one another or with the conductor. The lines are crowded between the conductors and therefore exert a repelling action tending to push the conductors farther apart. Again, when the conductors separate, the area through which the flux passes is increased, so that the magnetic circuit in this case also tends to conform itself so that the magnetic flux is a maximum.

From the foregoing, the following rules may be formulated:

Conductors carrying current in the same direction tend to be drawn together; conductors carrying current in opposite directions tend to be repelled from one another.

All electric circuits tend to take such a position as will make their currents parallel and flowing in the same direction.

This effect is especially pronounced in modern large-capacity power systems. Bus-bars have been wrenched from their clamps; transformer coils have been pulled out of place and transformers wrecked by the forces produced by the excessive currents arising under short-circuit conditions.

162. Biot-Savart[1] Law, and Field Intensity Due to Current.—The Biot-Savart law gives a quantitative relation between an electric current and the field intensity produced by it. The law thus permits the calculation of the forces which exist between electric currents and magnetic fields. In Fig. 164 let I' be a current expressed in absolute amperes (1 absolute ampere (absamp.) = 10 practical amperes), flowing in a wire ab. The current in a very short element Δx of the wire produces a field intensity at P,

$$\Delta H = \frac{I'\Delta x}{z^2} \sin \theta \text{ oersteds} \tag{98}$$

where z is the distance in centimeters from point P to Δx, and θ is the angle which the element Δx makes with the line connecting

[1] Biot, Jean Baptiste (1774–1862); A French physicist and collaborator of La Place. Savart, Felix (1791–1841); a French physician and physicist.

P and Δx. The field intensity at P is the force in dynes on a unit pole placed at P.

From relation (98) it may be shown that the field intensity H at a point P located at a perpendicular distance h cm. from an infinitely long straight wire carrying a current of I' absamp. is

$$H = \frac{2I'}{h} \text{ oersteds.} \tag{99}$$

This may be proved as follows: In Fig. 165 let the current in the wire be I' absamp. Let the origin be taken at the point where the perpendicular h from P intersects the conductor. The field intensity at P due to the current I' in an element dx situated at a positive distance x from the origin (from Eq. (98)) is

$$df_p = \frac{I'\,dx}{z^2} \sin\theta. \tag{I}$$

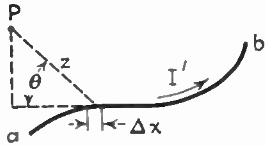


FIG. 164.—Biot-Savart law.

In order to determine the force due to the entire conductor integrate with respect to x, expressing as functions of x both $\sin\theta$ and z.

$$z^2 = x^2 + h^2 \quad \text{and} \quad \sin\theta = \frac{h}{\sqrt{x^2 + h^2}}.$$

An integration difficulty is avoided if the integration is made between 0

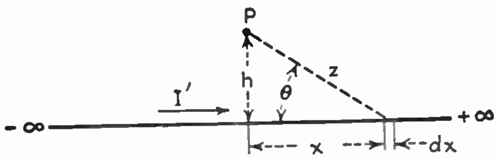


FIG. 165.—Field intensity due to current in single conductor.

and $+\infty$, rather than from $-\infty$ to $+\infty$, and the integral then multiplied by 2. Hence,

$$f = 2\,I'h\int_0^\infty \frac{dx}{(x^2 + h^2)^{3/2}} = 2I'h\left[\frac{x}{h^2\sqrt{x^2 + h^2}}\right]_0^\infty. \tag{II}$$

The difficulty of evaluating Eq. (II) (∞ in both numerator and denominator) may be avoided by dividing both numerator and denominator by x.

$$= \frac{2I'}{h}\left[\frac{1}{\sqrt{1 + \dfrac{h^2}{x^2}}}\right]_0^\infty \tag{III}$$

$$= \frac{2I'}{h} \text{ oersteds.} \quad Q.E.D. \tag{100}$$

The difference between the effect of an infinitely long wire and one whose length is 30 or 40 times the distance h is so small as to be negligible. The direction of the field intensity at P is *perpendicular* to the plane of the paper.

It is to be noted that if a unit pole at P be carried at constant radius h, about the conductor, the work done in making one complete circuit is

$$w = \left(\frac{2I'}{h}\right)2\pi h = 4\pi I' \text{ ergs.} \tag{101}$$

Hence the work is independent of the radius. By definition, w of Eq. (101) is the m.m.f. of a single long conductor carrying a current I' absamp. (p. 249).

Example.—A current of 60 amp. flows in a long straight wire. Determine (*a*) the field intensity at a point P, 10 cm. perpendicularly from the axis of the wire; (*b*) the work done in carrying a unit pole once around the wire; (*c*) the force exerted on a pole of 60 unit-pole strength at point P.

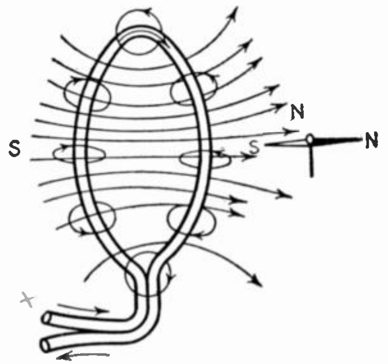


FIG. 166.—Magnetic field produced by a single turn.

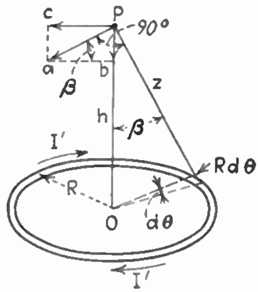


FIG. 167.—Field intensity due to a single turn.

$$60 \text{ amp.} = 6.0 \text{ absamp.}$$

(*a*) Using Eq. (100),

$$H = \frac{2 \times 6.0}{10} = 1.2 \text{ oersteds.} \quad \textit{Ans.}$$

(*b*) $w = 1.2 \times 2\pi \times 10 = 75.4$ ergs. *Ans.*
(*c*) $f = 1.2 \times 60 = 72$ dynes. *Ans.*

163. Magnetic Field of a Single Turn.—If a wire carrying a current be bent into a loop, a field results similar to that shown in Fig. 166. This magnetic field has a N-pole and a S-pole which possess all the properties of similar poles of a short bar magnet. A compass needle placed in this field assumes the direction shown, the N-pole pointing in the direction of the magnetic lines.

The field intensity at any point on a perpendicular to the plane of a circular turn at its center is readily computed by the Biot-Savart law (Par. 162). In Fig. 167 is shown a single turn, of radius R cm., in which a current I' absamp. flows. The length of a perpendicular OP at the center O of the coil is h cm. Consider the very small length of arc intercepted by the angle $d\theta$. The peripheral length of this arc is $Rd\theta$ cm. A line Z drawn from the arc to P makes angle β with OP. The field intensity at P, due to the small length of arc $Rd\theta$ is,

$$df' = \frac{RI'd\theta}{Z^2}. \tag{1}$$

The direction of the force at P is at right angles to the line Z and is represented by the vector Pa. Vector Pa can be resolved into a horizontal component Pc and a vertical downward component Pb. When the effect of the turn throughout the complete 2π radians is considered, only the vertical components remain, the horizontal components canceling. Therefore, Eq. (1) must be multiplied by $\sin \beta$ or R/Z, which is the same for all elements. Hence, the total force at P,

$$f = \int_0^{2\pi} \frac{RI'R}{Z^3}\, d\theta = \frac{2\pi R^2 I'}{(R^2 + h^2)^{3/2}} \text{ oersteds.} \tag{102}$$

$F = \frac{2\pi N R^2 I}{10(R^2+h^2)^{3/2}}$ 368

To determine the field intensity at O, the center of the turn, $h = 0$, and

$$f_0 = \frac{2\pi I'}{R} \text{ oersteds.} \tag{103}$$

$F_0 = \frac{2\pi NI}{10R}$

If a coil consists of n closely grouped turns, so that the thickness of the coil is small as compared with its radius,

$$f_0 = \frac{2\pi n I'}{R} \text{ oersteds} \tag{104}$$

SHORT COIL 378

where I' is the current in absamperes.

If, in Eq. (103), $R = 1$ and the field intensity at the center of the coil is 2π dynes, the current I' is 1 *absamp*. This should be compared with the definition of the ampere (p. 26).

Example.—A current of 20 amp. flows in a circular coil of 25 turns, the radius of the coil being 12 cm. and its thickness being small as compared with its radius. Determine: (*a*) the force on a unit pole on a perpendicular to the plane of the coil at its center, and 16 cm. from the plane of the coil; (*b*) the force on a unit pole placed at the center of the coil.

(*a*) 20 amp. = 2.0 absamp. From Eq. (102) (with n added),

$$f = \frac{2\pi \times \overline{12}^2 \times 2.0 \times 25}{(\overline{12}^2 + \overline{16}^2)^{3/2}} = 5.66 \text{ dynes.} \quad \textit{Ans.}$$

(*b*) Using Eq. (104),

$$f_0 = \frac{2\pi \times 25 \times 2.0}{12} = 26.2 \text{ dynes.} \quad \textit{Ans.}$$

164. Solenoids.—An electric conductor wound spirally, in the form of a helix, about an axis is called a *solenoid*. The operation of all electromagnets is based fundamentally on the properties of the solenoid, so that an understanding of these properties is important. A simple solenoid and the magnetic field produced within it when current flows through the conductor are shown in Fig. 168. The solenoid may be considered as consisting of a

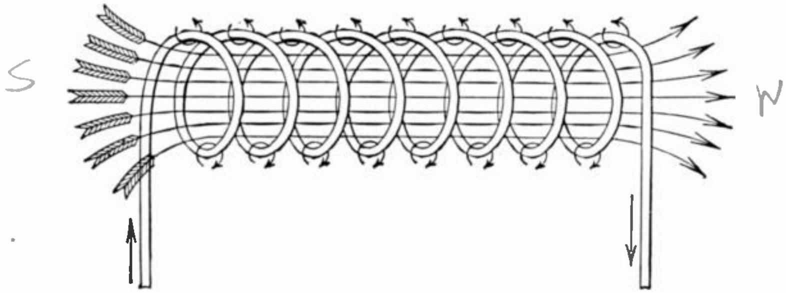


Fig. 168.—Magnetic field produced by a helix or solenoid.

large number of the turns, shown in Fig. 166 (p. 234), placed together. The solenoid winding may also consist of several layers as shown in the plunger type of Fig. 171 (p. 239).

The relation of the direction of the flux within the solenoid to the direction in which the current flows in the helix may be

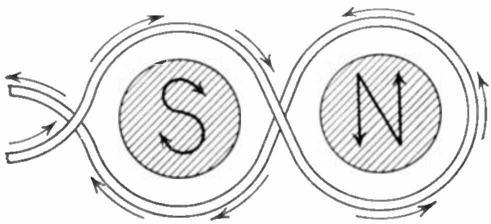


Fig. 169.—Relation of magnetic poles to direction of exciting current.

determined by the hand rule, or by the corkscrew rule of Par. 160 (p. 230). Another simple method is shown in Fig. 169, where the arrows at the ends of the *N* and the *S* show the direction of current in the coil. For example, when looking down on a *N*-pole the current direction in the coil will be counterclockwise as shown by the *N*; when looking down on a *S*-pole the direction of the current will be clockwise as shown by the *S*.

165. Field Intensity within a Long Air Solenoid.—Long air solenoids of uniform cross-section are used as standards for magnetic flux, since the total flux across the center section may be accurately computed when the dimensions of the solenoid and the value of the current are known (see p. 276). First, the equation for the field intensity at the center of the solenoid will be derived for a solenoid assumed to be infinite in length. It will then be shown that the field intensity at the center of a solenoid which is finite in length and in which the ratio of length to radius is of the order of 40 to 1 is essentially the same as that of the infinite solenoid.

Consider the long cylindrical solenoid in Fig. 170, made of helically wound turns of wire. There are n turns per centimeter length and the mean radius of the solenoid is R cm. The solenoid is assumed to be infinite in length. The current is I' absamp. The origin is taken at P, the center of the solenoid.

The effect of the current in a very thin transverse section of the solenoid, dx in thickness and at a distance x from the origin, is first considered.

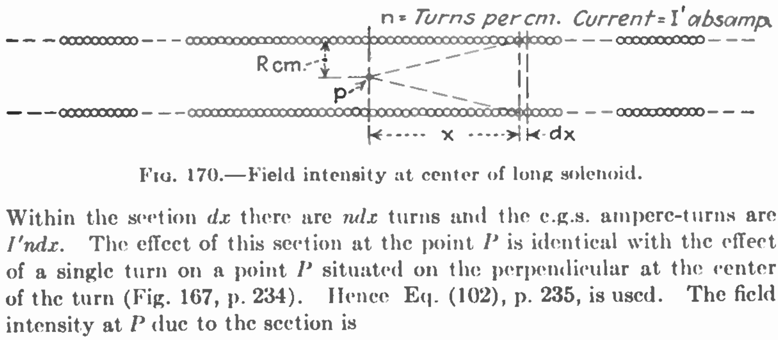


FIG. 170.—Field intensity at center of long solenoid.

Within the section dx there are ndx turns and the c.g.s. ampere-turns are $I'ndx$. The effect of this section at the point P is identical with the effect of a single turn on a point P situated on the perpendicular at the center of the turn (Fig. 167, p. 234). Hence Eq. (102), p. 235, is used. The field intensity at P due to the section is

$$dH_p = \frac{2\pi R^2 I'n\,dx}{(R^2 + x^2)^{3/2}} \text{ oersteds.} \tag{I}$$

The total field intensity at P is found by integrating Eq. (I) between 0 and $+\infty$, and then multiplying by 2. These limits are taken in order to avoid the troublesome evaluation of the integral when the limits are $-\infty$ and $+\infty$ (see p. 233).

$$H_p = 4\pi R^2 I'n \int_0^\infty \frac{dx}{(R^2 + x^2)^{3/2}} = 4\pi R^2 I'n \left(\frac{x}{R^2\sqrt{R^2 + x^2}} \right) \Bigg|_0^\infty .$$

As in Par. 162 (p. 233) the numerator and denominator of this expression are each divided by x before inserting the limits of integration. Hence,

$$H_p = 4\pi n I' \left(\frac{1}{\sqrt{\left(\frac{R}{x}\right)^2 + 1}} \right) \Bigg|_0^\infty = 4\pi n I' \text{ oersteds.} \tag{105}$$

If the current is expressed in amperes,

$$H_p = 0.4\pi n I \text{ oersteds.} \tag{106}$$

Consider a solenoid which is not infinite in length, but in which the ratio of the length l to the radius R is 40 to 1. The ratio R/x in Eq. (105) will then be 1 to 20. Hence,

$$H_p = 4\pi n I' \left(\frac{1}{\sqrt{1/400 + 1}} \right) = 4\pi n I' \ (0.9987).$$

Hence, the field intensity at the center of this finite solenoid differs from that of the infinite solenoid by a little over 0.1 per cent. (The use of the foregoing type of solenoid in making magnetic measurements is given in Par. 189 (p. 276).

166. Commercial Solenoids or Electromagnets.—When turns of a conductor carrying current are wound about an iron core, an electromagnet results. The electromagnetic action of the current in producing magnetic flux in the iron core is the same as that which produces the magnetic flux in the air solenoid (Fig. 168, p. 236). However, due to the much greater permeability of iron, a given number of ampere-turns produces a very much greater flux in the iron core than is produced in air.

Electromagnets are used in practice for tripping circuit-breakers (Par. 333, p. 556); for operating contactors in automatic motor starters (Par. 306, p. 508); for operating voltage-regulating devices (Par. 285, 286, pp. 463, 464); for arc-lamp feeds, for operating valves, and for many other purposes. In practically all cases, either a soft-iron (or soft-steel) plunger or an armature is necessary for obtaining the required tractive force.

A very useful type of electromagnet is the solenoid and plunger type, the operation of which is indicated in Fig. 171. The flux due to the solenoid produces magnetic poles on the plunger. The pole nearer the solenoid will be of such sign that it will be urged along the lines of force (see Par. 144, p. 211), and in such a direction as to be drawn within the solenoid.

A position of equilibrium is reached when the center of the plunger is at the center of the solenoid (Fig. 171). Figure 172 shows an "ironclad" solenoid commonly used for tractive work.

The ironclad feature increases the range of uniform pull and produces a marked increase of pull as the plunger approaches the end of the stroke. When a stop a is used (Fig. 172), the solenoid

right hand rule

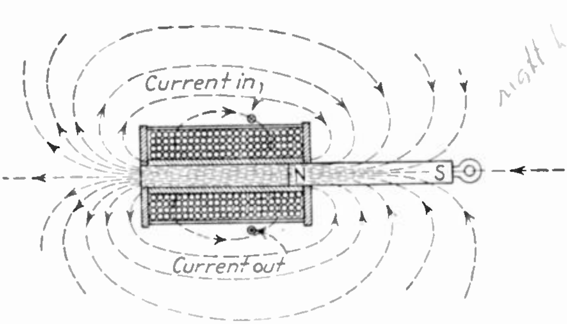


FIG. 171.—Simple solenoid and plunger.

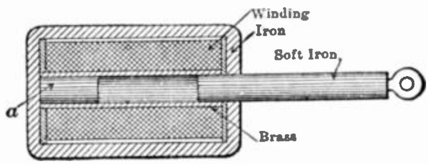


FIG. 172.—"Ironclad" solenoid and plunger with stop.

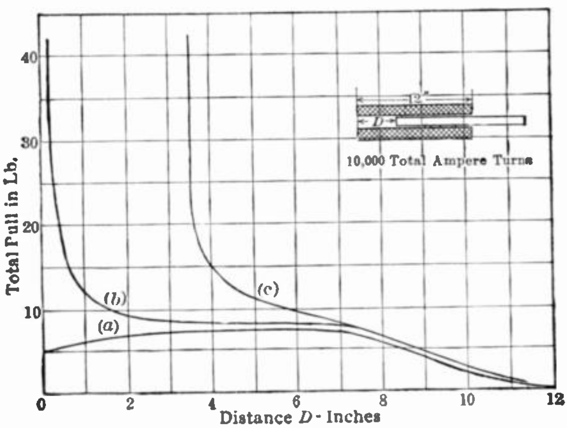


FIG. 173.—Pull of solenoid on plunger.

becomes a *plunger electromagnet*. This changes the characteristics of the solenoid in that the maximum pull now occurs when the end of the plunger is near the stop. Figure 173 shows the results of solenoid tests made by C. R. Underhill.[1] Curve (a) shows the pull on the plunger of a simple solenoid like that of Fig. 171; curve (b) shows the pull when this solenoid is ironclad as in Fig. 172 but without a stop; curve (c) shows the effect of the "stop" on the pull. It will be noted that the ironclad feature as well as the stop causes the pull to attain very high values as the plunger approaches the end of its stroke. By making the end of the plunger cone-shaped and cutting a hollow cone into the end of the stop (Fig. 174), a higher value of pull over a greater proportion of the stroke may be obtained. It will be noted (Fig. 173) that the usual ironclad feature and the usual stop have little effect on the pull except near the end of the stroke.

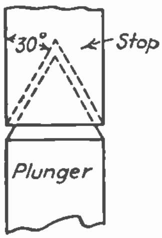


FIG. 174.—Curved plunger and stop.

An important practical application of the solenoid occurs in the braking of elevators and cranes. When the power is removed

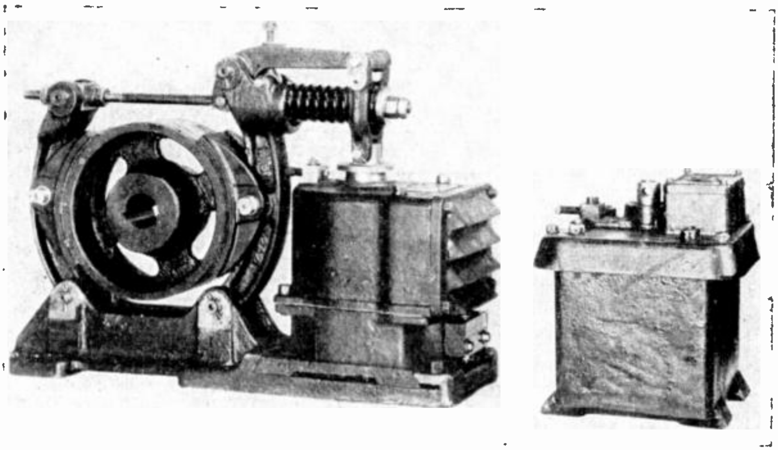


FIG. 175.—Magnetic brake. (*Westinghouse Electric & Manufacturing Company.*)

from the lifting motor or when the power is interrupted due to a broken wire or other accident, the brake must be applied immediately. One method of accomplishing this is shown in the Westinghouse type of brake (Fig. 175). The brake is applied by the

[1] "Standard Handbook for Electrical Engineers," Sec. 5, McGraw-Hill Book Company, Inc.

spring which acts through the rod to cause both brake shoes to be pressed against the brake drum. The brake is released by energizing a vertical solenoid located in the metallic housing at the right. The plunger pulls the lever downward, and the resulting lever action on the rod and on the right-hand brake shoe pushes the shoes away from the drum and at the same time compresses the spring. With any failure of the power, the brake is applied immediately. A plunger electromagnet is best adapted to this purpose since the stroke is short and the pull must be positive in its action. In this particular brake the cone-shaped plunger (Fig. 174) is used.

The principle of operation of the solenoid and plunger is illustrated by the following example.

Example.—Assume in Fig. 171 that the field intensity at the center of the solenoid is 2,000 oersteds and that one end of the plunger is at the center. The cross-section of the plunger is 6 sq. cm., and the pole induced on the end of the plunger is 1,920 unit poles. The other end of the plunger is well outside the field.

Determine the pull on the plunger in kilograms.

Since the definition of field intensity is the force in dynes exerted on a unit pole, the field will exert a force of 2,000 dynes on each unit pole on the end of the plunger. Hence the total force

$$f = \frac{2{,}000 \times 1{,}920}{981 \times 1{,}000} = 3.92 \text{ kg.} \quad \textit{Ans.}$$

167. Lifting Magnets.—Lifting magnets are used commercially to handle iron and steel having various forms. A great saving of time and labor is effected by their use, because chains and slings for holding the load are not necessary. They are very useful for handling steel billets in rolling mills, although the billets cannot be picked up when red hot as they lose their magnetic properties at this temperature. Lifting magnets are especially useful in loading and unloading steel rails, for an entire layer may be picked up and laid down again without being disarranged. Lifting magnets effect a great saving of labor when small pieces of iron, such as scrap iron, are handled, for they will pick up large quantities at every lift. Without such a magnet it would be necessary to move each individual piece by hand. Figure 176 shows in cross-section a typical Cutler-Hammer lifting magnet.

Figure 177 shows a lifting magnet in actual operation.

Formulas for the holding force of electromagnets are given in Par. 151 (p. 220).

It should be understood that the magnet itself does little or no work in the lifting but merely serves as a holding device. The

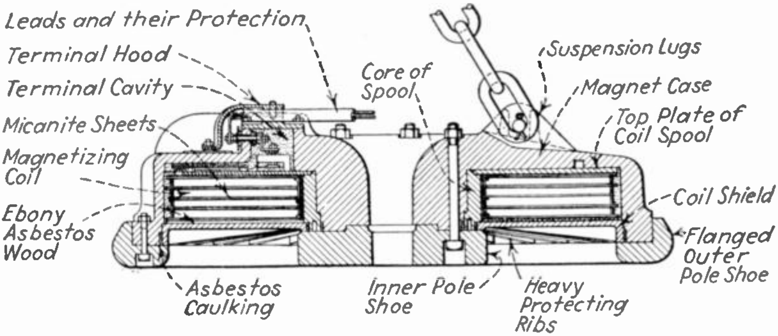


FIG. 176.—Cross-section of Cutler-Hammer lifting magnet.

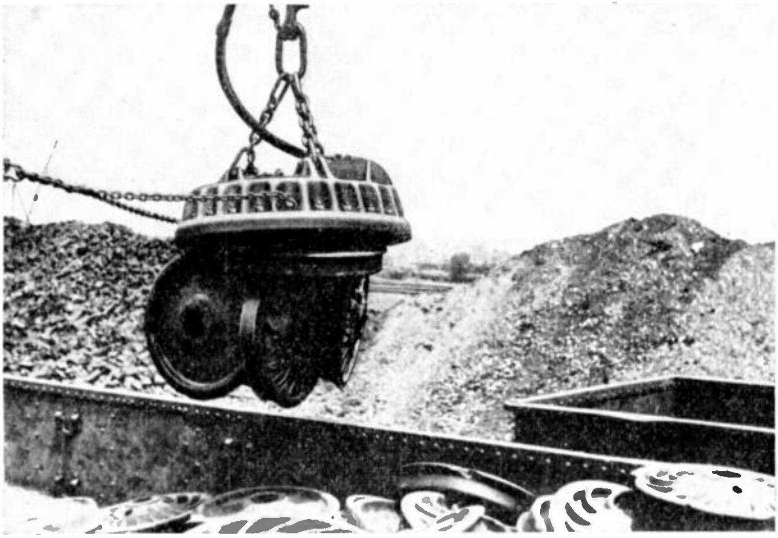


FIG. 177.—Cutler-Hammer lifting magnet handling car wheels.

actual work is performed by the engine or motor which operates the ropes or chains attached to the magnet.

168. Magnetic Separator.—Another important application of magnetic principles is found in the magnetic separator of Fig. 178.

It is especially designed to remove steel and iron from coal, rock, ore, etc., but it can be used for separating steel shot from molding sand, iron chips from machine-shop turnings, etc. The material is fed on an endless belt running at a speed of about 100 ft. per minute. The belt passes over a magnetized pulley. The non-magnetic material immediately drops off into a hopper, but the magnetic material is held by the pulley until the belt leaves the pulley when the material drops into a second hopper. The

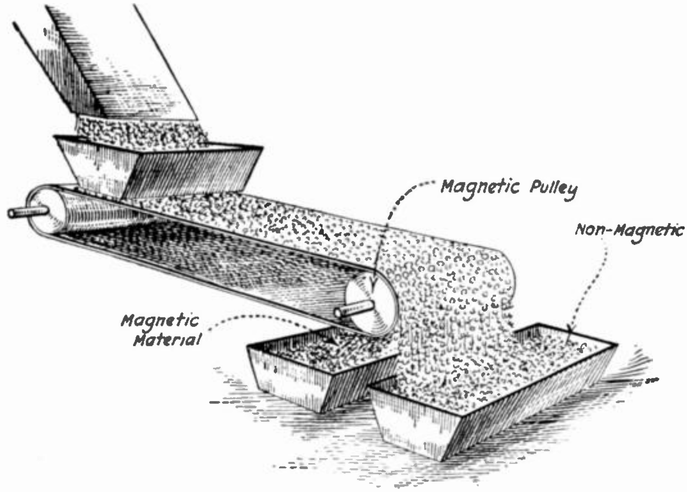


FIG. 178.—Magnetic separator.

pulley is magnetized by coaxial exciting coils, to which current is carried by means of slip-rings.

169. Magnetic Track Brakes.—From early days the magnetic track brake has been employed to some extent on electric railway cars, particularly in regions where grades are severe. The brake as made by the General Electric Company is shown in Fig. 179. It is a simple electromagnet and usually consists of two parallel side plates *A* (Fig. 179), with an iron core *B* between them, all bolted together. The brake is magnetized by a long longitudinal coil *C* which surrounds the core *B*. The brake shoes *D* are of opposite polarity and are made of a hard wear-resisting steel. They are readily removed from the side plates *A* and can be renewed without difficulty. The path of the magnetic flux, when the shoes are on the rail, is shown in the figure.

In the simplest method of operation, the brake is suspended on springs so that the shoes normally clear the rail by from 1/4 to 3/8 in. The brake moves in the guides *G* which also serve to transmit the braking force to the car. When the coil *C* is energized, the brake is magnetically attracted to the rail and the magnetism then provides the necessary adhesion for braking. The braking action is readily controlled by a rheostat in series with the magnet coil. When the exciting current is interrupted, the springs pull the brake up to its normal position clear of the rails.

The magnetic brake provides a large braking surface and thus acts more effectively than the wheel brake, particularly with a

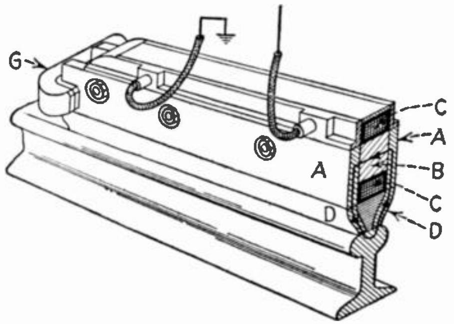


Fig. 179.—Magnetic track brake.

slippery rail. Since the force between the brake and the rail is due to magnetic attraction, there is no reduction in the weight on the wheels during braking such as occurs when track brakes are applied mechanically. Another distinct advantage of the magnetic brake is that it does not produce flat wheels.

170. Magnetic Circuits of Dynamos.—One of the most important uses of electromagnets is in the magnetic fields of motors and generators. An early and simple type of magnetic circuit is illustrated by the Edison bipolar generator in Fig. 180. The type of magnetic circuit shown is very inefficient, because of its great length in comparison with its sectional area. There results a considerable magnetic leakage which reduces, therefore, the amount of flux passing through the armature. Moreover, the flux, in taking the shortest path, tends to crowd through the upper half of the armature. This may produce unsatisfactory commutation.

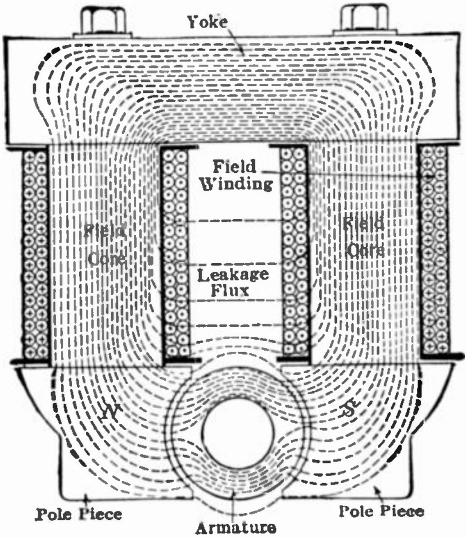


FIG. 180.—Magnetic circuit and field windings of an Edison bipolar generator.

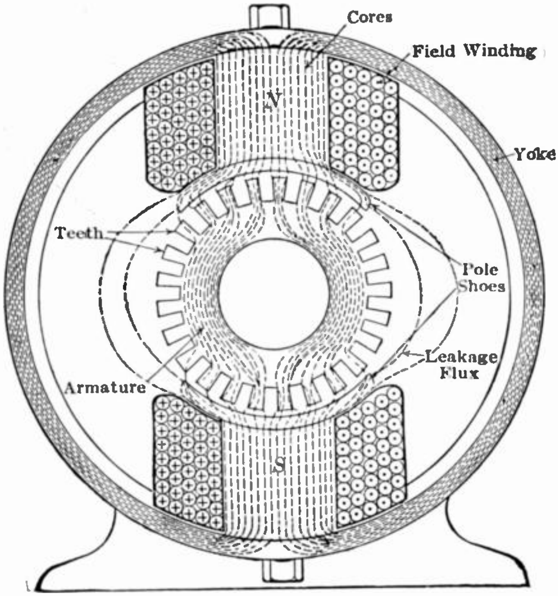


FIG. 181.—Magnetic circuit and field windings of a modern bipolar generator.

The magnetic circuit of a bipolar generator of modern design is shown in Fig. 181. Because of the symmetry of the magnetic circuit, the flux divides evenly through the two sides of the armature. The long air path existing between the pole shoes reduces the magnetic leakage to a minimum. It is to be noted

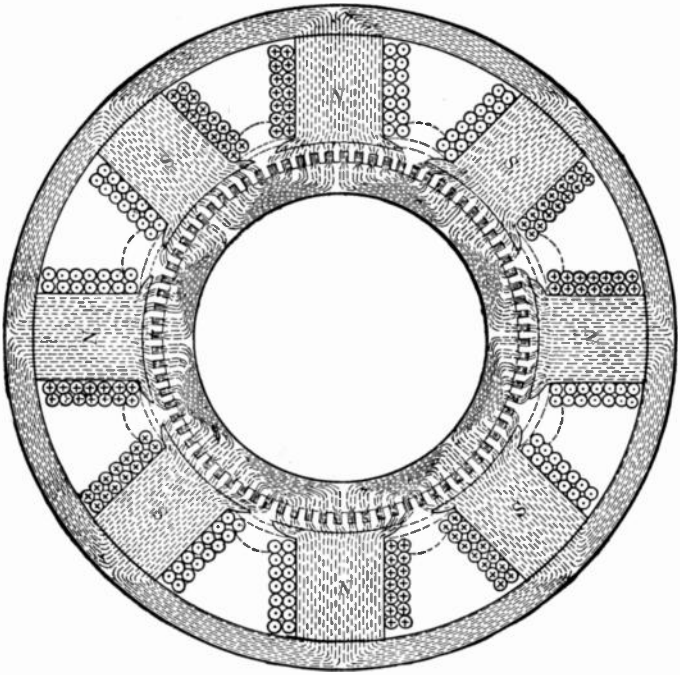


FIG. 182.—Magnetic circuits of a multipolar generator.

that the flux in the cores divides as it passes into the yoke. Ordinarily the yoke need be only one-half the cross-section of the field cores. Direct-current machines of the bipolar type are made usually in small units.

Figure 182 shows the more complex magnetic circuits of a multipolar generator with eight poles. It is to be noted that the poles are alternately north and south. The flux passing through the field cores divides, both on reaching the yoke and on reaching the armature path, and the cross-section of the yoke need be only one-half that of the cores. In both Fig. 181 and Fig. 182 the magnetic leakage is materially reduced by placing the exciting ampere-turns as near the armature as possible. This result is not secured in the Edison bipolar generator of Fig. 180. To illustrate, Fig. 183 shows the same generator as that in Fig. 182 but with the exciting coils placed upon the yoke. They still act upon the magnetic circuits, but because of their remoteness from the armature, a large magnetic leakage exists around the outside of the yoke and through the interpolar space, resulting in a smaller percentage of the total flux passing through the armature.

Magnetic leakage does not lower the power efficiency of a dynamo, but it does result in increased weight and cost. A

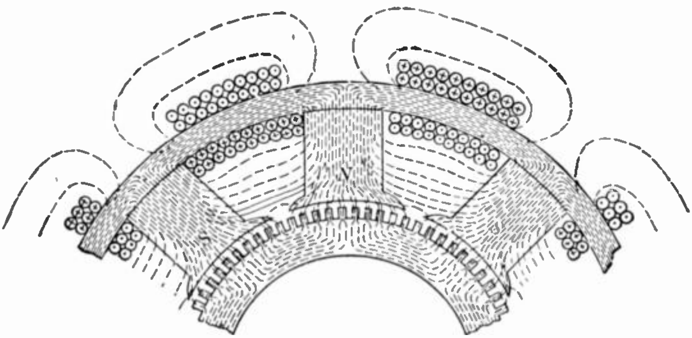


Fig. 183.—Magnetic leakage produced by incorrect position of exciting coils.

steady magnetic flux cannot cause any power loss, since it represents merely stored energy. No power is required to maintain a steady magnetic field. After the flux has reached a steady value, the power to the exciting coils is all accounted for by the copper loss in the coils themselves (see Par. 201, p. 301). Hence, since the leakage flux is steady, it cannot cause any power loss and therefore cannot affect the power efficiency.

However, the leakage flux must pass through the field cores and yoke (Fig. 180, p. 245). Therefore, these parts of the machine must be of greater cross-section than would be necessary for the useful flux alone. Moreover, the larger cross-section of the field cores results in a greater amount of copper in the exciting coils if the field loss is to remain constant. Hence, the effect of leakage flux is not to lower the power efficiency of dynamos but to make them heavier and more expensive to manufacture.

CHAPTER VIII

THE MAGNETIC CIRCUIT

171. Magnetic Circuit.—Although the general nature and characteristics of magnetism are discussed in both Chaps. VI and VII, the quantitative relation of the magnetic flux to both the ampere-turns and the properties of the magnetic circuit is not considered. If the magnetic resistance or the reluctance of a circuit and the ampere-turns linked with this circuit be known, the magnetic flux can be calculated in the same manner that the current in the electric circuit can be calculated if the resistance and voltage be known. In this respect the two circuits are similar. The magnetic circuit differs from the electric circuit in three respects, making it difficult to attain the same degree of accuracy in magnetic calculations as is obtained in electrical calculations.

The electric current has been considered as confined to a definite path, for example, a wire. The surrounding air and the insulating supports for the wire have a very high resistance, so that any leakage current which escapes from the wire is almost always negligible compared with the current in the wire itself. There is no known insulator for magnetic flux. In fact, air itself is a fairly good magnetic conductor. Therefore, it is impossible to restrict magnetic lines to definite paths in the same way that electric currents are restricted. This is illustrated by the fact that even in the best designed dynamos, from 15 to 20 per cent of the total flux produced leaks across air paths where it cannot be utilized. The presence of this leakage flux may be detected with a compass, and its intensity is often sufficient to magnetize watches even when they are several feet distant from the dynamo.

Magnetic paths are usually short and have a large cross-section in proportion to their length. They are often so complicated in their geometry that only approximations to their magnetic resistance can be obtained. This may cause errors of considerable magnitude in magnetic calculations. This is well illustrated by the air-gap of a dynamo. The geometry of the magnetic path between the armature teeth and slots and the pole face is very complicated, and at best only approximations to the actual distribution of the magnetic flux can be made (see Fig. 182, p. 246).

Under ordinary conditions of use, the resistance of electric conductors is substantially constant, although temperature changes may cause variations of several per cent. Correction for the effect of temperature changes can be accurately made. The magnetic resistance of materials, however, is not constant but varies over wide ranges. This resistance depends to a large extent on the magnetic history of the material. The magnetic resistance of iron may increase 50 times when the magnetic flux density alters from a low to a high value.

Although the foregoing factors prevent magnetic calculations being made with the same degree of accuracy as can be obtained with electric circuits, yet if one understands magnetic relations he can, by the use of certain approximations, make computations that are sufficiently close for most purposes.

172. Magnetic Units.—The magnetic units with their definitions are given as follows:

Ampere-turns (IN).—The ampere-turns acting on a magnetic circuit are given by the product of the turns linked with the circuit and the amperes flowing through these turns. For example, 10 amp. flowing through 150 turns give 1,500 ampere-turns. The same result is produced by 15 amp. flowing through 100 turns. If any ampere-turns act in opposition, they must be subtracted.

In magnetic calculations, the c.g.s.[1] (cm.-gm.-sec.) or absolute system is almost always used. The following units are c.g.s. units.

Magnetomotive Force (m.m.f., also F).—Magnetomotive force tends to drive the flux through the circuit and corresponds to e.m.f. in the electric circuit. It is directly proportional to the ampere-turns of the circuit and only differs from the numerical value of the ampere-turns by the constant factor $0.4\pi = 1.257$. That is, $F = 0.4\pi IN = 1.257IN$.

In the c.g.s. magnetic system, the m.m.f. of a circuit is the work in ergs in carrying a unit N-pole once through the entire magnetic circuit.

[1] Par. 26, p. 26.

The unit of m.m.f. is the *gilbert*.[1] The gilberts acting on a circuit are obtained by multiplying the ampere-turns by 0.4π or 1.257.

Reluctance ($\mathcal{R}$).—Reluctance is resistance to the passage of magnetic flux and corresponds to resistance in the electric circuit. The unit of reluctance is that of a centimeter cube of air. As yet no name has been given to the unit of reluctance.

Permeance ($\mathcal{P}$).—The permeance of a circuit is the reciprocal of the reluctance ($\mathcal{P} = 1/\mathcal{R}$) and may be defined as that property of the circuit which permits the passage of the magnetic flux or of the lines of induction. It corresponds to conductance in the electric circuit.

Permeability (μ).—The permeability of a material is the ratio of the flux or the number of lines of induction existing in the material to the flux or number of lines of induction which would exist in the same space if the material were replaced by vacuum, the m.m.f. acting on the space remaining unchanged. The permeability of vacuum is taken as unity and with the exception of iron, steel, nickel, liquid oxygen, and certain iron oxides, most other materials including air may be considered as having a permeability of unity. The permeability of commercial iron and steel ranges from 50 and even lower to about 2,000. In special investigations, vacuum-treated iron has attained a permeability of 5,000 and even greater (also see Permalloy, p. 272).

Example.—In a ring solenoid wound on an iron core similar to that of Fig. 148(*a*), p. 223, the magnetic flux is 4,000 lines or maxwells. When the iron core is removed, the flux in air is only 20 lines. What is the permeability of the iron?

Removing the iron core does not change the ampere-turns or the geometry of the magnetic circuit. Therefore,

$$\mu = \frac{4,000}{20} = 200. \quad \textit{Ans.}$$

Flux (ϕ).—The magnetic flux is equal to the total number of lines of induction existing in the magnetic circuit and corresponds

[1] Gilbert, William (1544–1603). The most distinguished man of science in England during the reign of Queen Elizabeth. He was probably the first to demonstrate that the earth was in reality a large permanent magnet. His principal work was an excellent treatise on magnetism which embodied several years of systematic experimentation and research. This was the first really great contribution to the science of electricity and magnetism.

to current in the electric circuit. The c.g.s. unit of flux is the *maxwell*,[1] but "line of induction" or simply "line" is often used.

At the meeting of the International Electrotechnical Commission at Scheveningen-Brussels in June, 1935, at which the MKS system of electrical units was adopted (see p. 27), the *weber*[2] was adopted as the practical or MKS unit of magnetic flux. The weber is equal to 10^8 maxwells.

Magnetic Induction or Flux Density (B).—The flux density is the number of maxwells or lines of induction per unit area, the area being taken at right angles to the direction of the flux. The unit of flux density in the c.g.s. system is one line per square centimeter and is called the *gauss*.[3] Flux density is also expressed in "lines per square centimeter" or "lines per square inch."

$$B = \frac{\phi}{A} \tag{107}$$

where A is the area and ϕ the flux through and normal to this area.

173. Reluctance of the Magnetic Circuit.—The unit reluctance is defined as that of a centimeter-cube of vacuum or of air. If the portion of a magnetic circuit between pole faces a and b

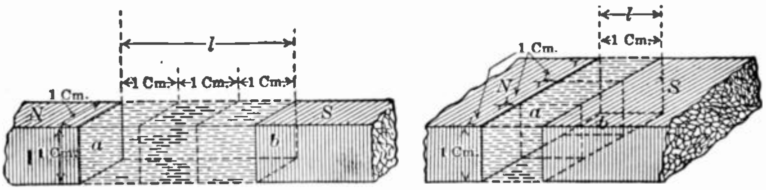
[1] Maxwell, James Clerk (1831–1879). An eminent British physicist who for more than half of his brief life held a prominent position in the very front rank of natural philosophers. Although he made admirable contributions to the subjects of mechanics and heat, his great contributions were in the field of electricity and magnetism. His work "Electricity and Magnetism," which appeared in 1873, is still considered to be one of the most outstanding scientific contributions of all time. In it he reduced all electric and magnetic phenomena to stresses in a material medium, expressing these facts by general mathematical equations. In addition he also deduced the electromagnetic theory of light and actually predicted the existence of electromagnetic (radio) waves years before their discovery by Hertz in 1888.

[2] Weber, Wilhelm Eduard (1804–1891). A German physicist, who for most of his life occupied the chair of Professor of Physics at Göttingen. He showed, as did his colleague Gauss for magnetic quantities, that electrical quantities could be defined absolutely in terms of length, mass, and time. He also conducted researches in magnetism and invented a system of electromagnetic telegraphy.

[3] See footnote, p. 213.

(Fig. 184(a))[1] consists of a path in air having a length of 3 cm. and a cross-section of 1 sq. cm., as in the figure, this path is equivalent to three centimeter cubes placed in series. As the total flux must pass in succession through each cube, it is evident that the total reluctance is 3 units. *The reluctance is proportional to the length of the flux path.*

On the other hand, if the path has a length of 1 cm. and a cross-section of 3 sq. cm., as in Fig. 184(b), the reluctance of the



(a) Path whose reluctance is 3 units. (b) Path whose reluctance is ⅓ unit.
FIG. 184.—Reluctance of simple magnetic paths.

path through which the flux passes is one-third that of one centimeter cube alone, or is ⅓ unit. *The reluctance is inversely proportional to the cross-section of the path.*

Moreover, if these paths were in iron, having a permeability μ, the flux would be μ times its value in air, provided the same m.m.f. were maintained between the two pole faces. This results in lower reluctance.

It follows that *the reluctance of any portion of a magnetic circuit is proportional to its length, inversely proportional to its cross-section, and inversely proportional to the permeability of the material.* The constant of proportionality is unity, since the reluctance of a path in air 1 cm. long and 1 sq. cm. cross-section is 1 unit. Hence,

$$\mathcal{R}_1 = \frac{l_1}{A_1\mu_1} \tag{108}$$

where l_1 is the length in centimeters of that part of the circuit under consideration, A_1 is the uniform cross-section in square centimeters of that part of the circuit, and μ_1 is the permeability of that part of the circuit.

[1] The actual flux path between pole faces would not exist as shown in Fig. 184(a), but the flux would "fringe" as shown in Figs. 148(b) and 149, p. 223.

If a magnetic circuit consists of four parts in series as in Fig. 185, the total reluctance is

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 = \frac{l_1}{A_1\mu_1} + \frac{l_2}{A_2\mu_2} + \frac{l_3}{A_3\mu_3} + \frac{l_4}{A_4\mu_4}. \quad (109)$$

Permeances in parallel are added together to find the total permeance just as conductances in parallel are added together to find the total conductance.

The total permeance

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4.$$

Reluctances in parallel combine just as resistances in parallel,

$$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \frac{1}{\mathcal{R}_4}. \quad (110)$$

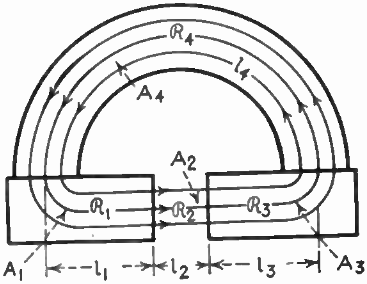


Fig. 185.—Reluctances in series.

174. Magnetization Curve.—It has been stated that the reluctivity of magnetic materials such as iron is not constant but varies with the flux density and also with the previous magnetic history.

Except over a limited range at the lower flux densities, the flux is not proportional to the ampere-turns or m.m.f. acting on the magnetic circuit because of the changing reluctivity. With increasing m.m.f., the rate of increase of flux diminishes as the iron approaches saturation.

The relation of the flux in iron or steel to the m.m.f. can not be expressed simply, and it is necessary to show this relation by a curve called the *magnetization curve.*

Magnetization curves almost always give the magnetic properties of a unit cube, flux density being plotted as ordinates and m.m.f. per unit length as abscissas. The more rational method is to use c.g.s. units, lines per square centimeter, or gausses, being the ordinates and gilberts per centimeter the abscissas. However, the common use of the British system for the dimensions of electric machinery in the United States has made it common also to use lines per square inch as ordinates and ampere-turns per inch as abscissas.

A magnetization curve for one grade of cast steel is shown in Fig. 186. Abscissas are m.m.f. in gilberts per centimeter (H)[1] and ordinates are the corresponding flux densities (B).

From 0 to A the curve concaves upward slightly. According to Weber's theory (see p. 207), this is due to the fact that before the application of magnetizing force, the molecular magnets, of which the iron is formed, are arranged in a haphazard manner, and a considerable magnetizing force is required before these molecular magnets begin to take directions corresponding to the applied m.m.f.

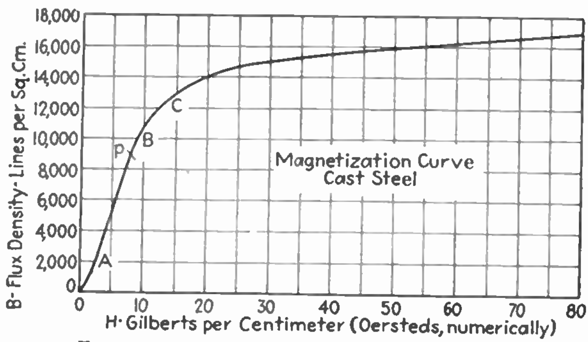


FIG. 186.—Magnetization curve for cast steel.

From A to B, the curve is practically a straight line. Beyond B, the flux density increases much less rapidly for a given increase in m.m.f., and the iron approaches saturation. The point C, where the bend in the curve is pronounced, is the *knee of the curve*. Beyond C, the flux can be increased but slightly even with a great increase in the m.m.f. This, according to Weber's theory, is due to the fact that the molecular magnets are now nearly parallel to the direction of the magnetizing force (Fig. 135, p. 208) and a large increase in magnetizing force has little further effect on their direction. The steel is now said to be *saturated*. The type of curve shown in Fig. 186, which starts with zero induction and is taken with increasing values of magnetizing force, is called the *normal* saturation or induction curve. Figure 190 (p. 261) shows normal induction curves for commercial grades of iron.

[1] H is the symbol for field intensity, but H is *numerically* equal to the m.m.f. per centimeter (see Par. 177, p. 257).

175. Permeability.—Permeability is defined as the ratio of the flux in the magnetic substance to the flux in air, the m.m.f. and the geometry of the magnetic circuit being the same in both cases (Par. 172, p. 249). The magnetization curve (Fig. 186) gives the relation of the flux in a centimeter cube of steel to the m.m.f. acting across two opposite faces of the cube. If the steel were removed, the same m.m.f. is acting across a centimeter cube of air. Also, since the flux now is equal numerically to the m.m.f. acting between the two opposite faces of the cube, the flux is also

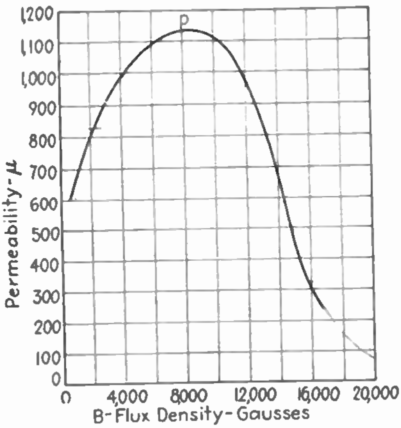


Fig. 187.—Permeability characteristic of cast steel.

equal to H (Par. 145, p. 212). Hence, from definition, the permeability

$$\mu = \frac{B}{H}. \tag{111}$$

Thus the permeability is found by dividing each ordinate B of the magnetization curve (Fig. 186) by the corresponding value of H.

Figure 187 shows the permeability curve of cast steel (Figs. 186 and 190), determined by this method, plotted as a function of the flux density B.

It will be noted that the permeability varies over a wide range. It begins at a comparatively low value, corresponding to the portion $0A$ of the magnetization curve (Fig. 186), increases to a maximum at the point p, and then decreases with increasing saturation to about one-fifth its maximum value.

The permeability of iron or steel depends on the quality of the material, the flux density, and the previous magnetic history. Also, in general, the permeability of iron increases as the proportion of impurities, such as sulphur, carbon, and phosphorus, diminishes.

176. Law of the Magnetic Circuit.—The relation among flux, m.m.f., and reluctance, for the magnetic circuit, is identical with

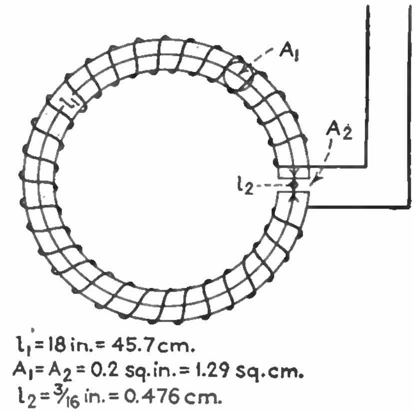


Fig. 188.—Ring-type electromagnet.

the relation among current, e.m.f., and resistance for the electric circuit:

$$\phi = \frac{F}{\mathcal{R}}. \tag{112}$$

The flux is directly proportional to the m.m.f. and inversely proportional to the reluctance of the circuit.

Since $F = 0.4\pi NI$ (Par. 172, p. 249), and $\mathcal{R} = l/A\mu$ (Par. 173, p. 251),

$$\phi = \frac{0.4\pi NI}{l/A\mu}. \tag{113}$$

If the magnetic circuit consists of several distinct parts in series having reluctances $\mathcal{R}_1$, $\mathcal{R}_2$., etc., and m.m.fs. F_1, F_2, etc., from Eq. (113),

$$\phi = \frac{F_1 + F_2 + F_3 + \cdots}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} = \frac{0.4\pi(I_1N_1 + I_2N_2 + I_3N_3 + \cdots)}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} \quad (114)$$

Example.—A ring magnet (Fig. 188) is wound with 250 turns of wire, through which a current of 1.5 amp. flows. Assume the permeability of the iron to be 800. Neglecting fringing, determine the flux in the ring and also the flux density.

$F = 0.4\pi \times 1.5 \times 250 = 471$ gilberts.

$l_1 = 18$ in. $= 18 \times 2.54 = 45.7$ cm.

$l_2 = \frac{3}{16}$ in. $= \frac{3}{16} \times 2.54 = 0.476$ cm.

$A_1 = A_2 = 0.2$ sq. in. $= 0.2 \times 2.54 \times 2.54 = 1.29$ sq. cm.

From Eq. (114),

$$\phi = \frac{471}{\dfrac{45.7}{1.29 \times 800} + \dfrac{0.476}{1.29 \times 1.0}} = \frac{471}{0.0443 + 0.369}$$

$= 1{,}140$ lines (maxwells). *Ans.*

The flux density

$$B = \frac{1{,}140}{1.29} = 884 \text{ lines per square centimeter (gausses)}$$

$= 5{,}700$ lines per square inch. *Ans.*

177. Magnetomotive Force per Centimeter and Field Intensity H.—In air the m.m.f. in gilberts per centimeter is equal *numerically* to the field intensity in dynes per unit pole, or oersteds. This may be shown as follows.

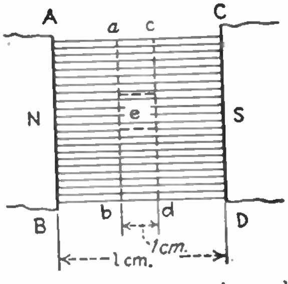


FIG. 189.—Field intensity and m.m.f.

In Fig. 189 is shown a portion of a uniform magnetic field in air between two parallel planes AB and CD situated l cm. apart. This field might be produced between the poles of an electromagnet or in the center of a long solenoid (pp. 252 and 237). The cross-section of the field perpendicular to the paper is A sq. cm. The m.m.f. between the planes AB and CD is F gilberts. The magnetic flux between planes AB and CD is, by Eq. (113),

$$\phi = \frac{F}{(l/A)}. \quad \text{(I)}$$

First consider two other planes *ab* and *cd* perpendicular to the field and 1 cm. apart. The m.m.f. between these planes is F/l and the reluctance is $\left(\frac{1}{A}\right)$. Hence the flux $\phi = \frac{F/l}{1/A}$ and is the same as that given by Eq. (I).

Next consider a centimeter cube *e* between the planes *ab* and *cd*. The m.m.f. across the two opposite faces of this cube, which lie in the planes *ab* and *cd*, is F/l. The reluctance of the cube is unity. Hence the flux between the opposite faces of the cube *e* is equal to $(F/l)/1 = B$ maxwells or lines per square centimeter.

Since the m.m.f. between any two points is defined as the work in ergs performed in carrying a unit N-pole between these points, the work done in carrying a unit pole from surface *ab* to surface *cd* is F/l ergs. The magnetic field is uniform, and the force on a unit pole is therefore everywhere the same. Since work is equal to the product of force and distance, the force is equal to the m.m.f. divided by the distance which is unity between the opposite faces of the cube. Hence, the force on a unit pole or the field intensity everywhere between the surfaces *ab* and *cd* is F/l dynes. But F/l is the m.m.f. *per centimeter length* or *m.m.f. gradient* of this uniform magnetic field.

Thus, in air, the field intensity H, which is equal to the force in dynes per unit pole, is *numerically* equal to the m.m.f. per centimeter length. However, field intensity and m.m.f. per centimeter in air are physically different quantities, field intensity H being *force* per unit pole, and m.m.f. per centimeter being the *work* performed in carrying a unit N-pole 1 cm. in the direction of the magnetic field.

It is to be noted that since the field between planes AB and CD is uniform, the field intensity everywhere in this region is equal to F/l oersteds.

Example.—The distance between two parallel pole faces, similar to those in Fig. 189, is 10 cm., and the cross-section in which the field is uniform is 8 by 5 cm., perpendicular to the paper. The m.m.f. between surfaces AB and CD is 4,800 gilberts. Determine: (*a*) the work done in carrying a unit N-pole between surfaces CD and AB; (*b*) the reluctance of the flux path; (*c*) the total flux; (*d*) the flux density in gausses; (*e*) the m.m.f. per centimeter length of the flux path; (*f*) the flux in the cross-section of a centimeter cube perpendicular to the field, the surfaces of the cube being either parallel with or perpendicular to the field; (*g*) the field intensity in oersteds or dynes per unit pole.

(a) Since m.m.f. between two points is defined as work performed in carrying a unit pole between those points, the answer to (a) must be 4,800 ergs. *Ans.*

(b) $\mathcal{R} = 10/(8 \times 5) = 0.250$ c.g.s. reluctance units. *Ans.*

(c) $\phi = 4{,}800/0.250 = 19{,}200$ maxwells. *Ans.*

(d) $B = 19{,}200/(8 \times 5) = 480$ gausses. *Ans.*

(e) M.m.f. per centimeter is $4{,}800/10 = 480$ gilberts. *Ans.*

(f) Since the reluctance of a centimeter cube is unity, the flux in the cross-section of the cube must be

$$\phi = \frac{480}{1} = 480 \text{ maxwells, which is also the flux density } B. \quad \textit{Ans.}$$

(g) Since the m.m.f. per centimeter length is 480 gilberts, 480 ergs are involved in carrying a unit *N*-pole 1 cm. distance in the direction of the field. Since the field is uniform, the force is constant and must be $480/1 = 480$ dynes (force = work/distance). Field intensity is defined as force in dynes per unit pole, hence $H = 480$ oersteds. *Ans.*

It is to be noted that the gausses (d), the m.m.f. per centimeter in gilberts (e), and the field intensity in oersteds (g) are all equal *numerically*.

178. Method of Trial and Error.—Magnetic problems cannot always be solved readily by the method used in Par. 176 (p. 256). This is due to the fact that the permeability (which is a variable but is given in the example as a constant value of 800) is not ordinarily known until the flux density and therefore the answer are known. The answer of course depends in turn on the permeability. If the permeability could be expressed as a function of the flux density, the problem could be solved directly. As a rule, however, the relation of permeability to flux density is not simple, so that a direct solution of the problem is difficult. However, such problems can usually be solved by trial and error as illustrated by the following example.

Example.—The iron ring of Fig. 188 and Par. 176 (p. 256) is made of cast steel whose permeability curve is given in Fig. 187. The air-gap is reduced to $\frac{1}{16}$ in. by inserting additional iron. Determine the flux and the flux density.

Assume the permeability as 900.

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 900} = 0.0397.$$

$$\mathcal{R}_2 = \frac{\frac{1}{16} \times 2.54}{1.29} = 0.123.$$

$$\phi = \frac{471}{0.0397 + 0.123} = 2{,}890 \text{ maxwells.}$$

$$B = \frac{2{,}890}{1.29} = 2{,}240 \text{ gausses.}$$

From Fig. 187, the permeability at this flux density is 830. Therefore $\mathcal{R}_1$ must be recalculated using the new value of permeability:

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 830} = 0.0431.$$

$$\phi = \frac{471}{0.0431 + 0.123} = 2{,}840 \text{ maxwells.}$$

The new value of $B = 2{,}200$ gausses.

As the value of μ corresponding to this flux density is 820, or sufficiently close to the value 830 just used, the last two values of flux and flux density are substantially correct.

With the magnetic problems usually met in practice, the problem is simplified since either the flux or the flux density is known, and it is required to determine the corresponding ampere-turns.

179. Determination of Ampere-turns.—It is shown in Par. 43 (p. 51) that the voltage drop per unit length of a conductor depends only on the *current density* and the resistivity of the conductor. In a similar manner the m.m.f. per unit length depends only on the *flux density* and the reluctivity of the material. This is proved as follows:

From Eq. (113), p. 256,

$$\phi = \frac{F}{l/A\mu} = BA.$$

Since $\phi = BA$, where B is the uniform flux density and A is the cross-section of the path,

$$\frac{F}{l} = \frac{B}{\mu}. \tag{I}$$

Multiplying both sides of Eq. (I) by l,

$$F = \frac{Bl}{\mu}. \tag{115}$$

Equation (115) shows that the m.m.f. is equal to the product of the *flux density* and the length of the magnetic path, divided by the permeability of the material. To determine the m.m.f. for a unit length of a circuit, it is necessary to know only the flux density and the permeability. Instead of plotting the permeability as a function of the flux density, the magnetization curve is usually plotted with ampere-turns per unit length as abscissa and the corresponding flux density as ordinate. This is more convenient and avoids using both 0.4π and the permeability. Such curves are shown in Fig. 190 for various commercial irons and steels used in the manufacture of electrical machinery.

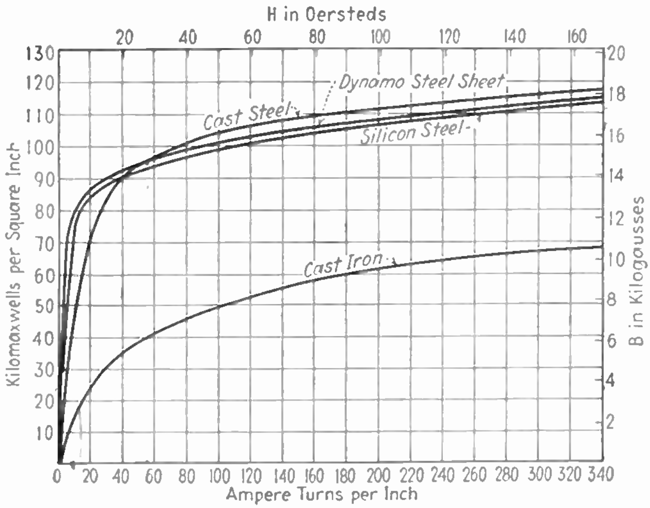


Fig. 190.—Typical magnetization curves for iron and steel.

In problems where the flux and the cross-section of the magnetic paths are known, and it is desired to find the ampere-turns necessary to produce this known flux, the curves just referred to enable the solution to be obtained readily.

180. Ampere-turns for a Simple Air-gap.—Let it be required to determine the ampere-turns IN necessary to produce a uniform flux ϕ in a simple air-gap between two parallel planes having areas of A sq. cm. and spaced a distance of l cm.

The flux $\phi = BA = \dfrac{0.4\pi IN}{(l/A)}$ (from Eq. (113), p. 256; $\mu = 1.0$)

where B is the flux density. Hence,

$$IN = \frac{1}{0.4\pi}Bl = 0.796Bl$$

$$= 0.8Bl \text{ (nearly) since } \mu = 1.0. \qquad (116)$$

Hence, the ampere-turns for an air-gap depend only on the *flux density* and the *effective length* of gap. (Equation (116) should be compared with Eq. (115).)

Since the dimensions of machines are frequently given in inch units and the operating flux densities are usually given in lines per square inch, it is convenient to express Eq. (116) in inch units. Let B' be the flux density in lines per square inch and l' the length of the gap in inches.

$$B' = (2.54)^2B; \quad l' = \frac{l}{2.54}.$$

Hence,

$$IN = 0.796\frac{B'}{(2.54)^2}\cdot 2.54l' = 0.313B'l'. \tag{117}$$

Example.—The effective length of the air-gap of a dynamo, after correcting for the effect of the teeth and slots,[1] is 0.25 cm. and the pole faces have an area of 400 sq. cm. If the total flux is 2,400,000 maxwells, determine the ampere-turns necessary to produce this flux in the gap.

$$B = \frac{2,400,000}{400} = 6,000 \text{ gausses.}$$

$$IN = 0.796 \times 6,000 \times 0.25 = 1,194 \text{ (from Eq. (116)).} \quad \textit{Ans.}$$

If inch units are used, the area

$$A' = \frac{400}{6.45} = 62.0 \text{ sq. in.}$$

$$B' = \frac{2,400,000}{62.0} = 38,700 \text{ lines per sq. in.}$$

The length

$$l' = \frac{0.25}{2.54} = 0.0984 \text{ in.}$$

Using Eq. (117),

$$IN = 0.313 \times 38,700 \times 0.0984 = 1,192 \ (\textit{check}).$$

181. Use of the Magnetization Curves.—In Fig. 190 are plotted the magnetization curves for common types of magnet iron and steel. Both inch units and c.g.s. units are used for abscissas and ordinates. To illustrate the use of the magnetization curves the following example is given.

[1] Methods of making this correction are given in LANGSDORF, "Principles of Direct-current Machines," Chap. IV, McGraw-Hill Book Company, Inc.

Example.—Determine the ampere-turns necessary to produce an air-gap flux of 750,000 lines in the electromagnet of Fig. 191. The cores are cast iron, and the yoke and pole pieces are cast steel. Neglect fringing and leakage.

The flux density in the lower yoke is

$$B_1 = \frac{750{,}000}{3 \times 4} = 62{,}500 \text{ lines per sq. in.}$$

The ampere-turns per inch for a density of 62,500 from Fig. 190 (cast steel) is 18.

The mean length of flux path is (approximately) 16 in.

$$I_1N_1 = 16 \times 18 = 288 \text{ amp.-turns,}$$

or 288 ampere-turns is required to produce a flux of 750,000 lines in the lower yoke.

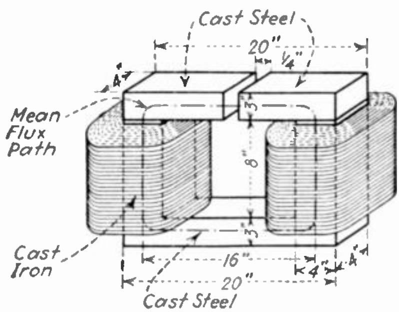


FIG. 191.—Typical electromagnet.

The density in the cores is

$$B_2 = \frac{750{,}000}{4 \times 4} = 46{,}900 \text{ lines per sq. in.}$$

From the curve (cast iron) the ampere-turns per inch = 86.

As there are two cores, the total length is 16 in., neglecting the quarter-turn at each corner of the magnetic circuit.

$$I_2N_2 = 16 \times 86 = 1{,}376 \text{ amp.-turns.}$$

The pole pieces are in every way identical with the yoke, except that the path is 0.25 in. shorter. This small difference if neglected will not introduce an appreciable error, so for the two pole pieces

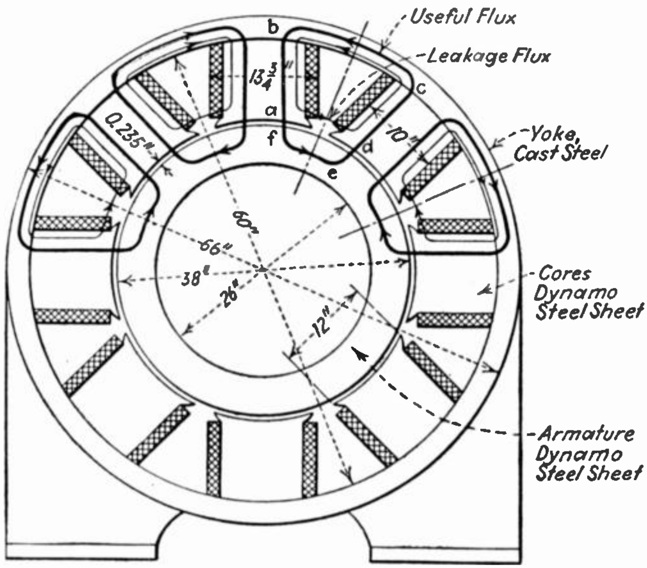
$$I_3N_3 = 288 \text{ amp.-turns.}$$

For the air-gap (from Eq. (117))

$$I_4N_4 = 0.313 \times 62{,}500 \times 0.25 = 4{,}900 \text{ amp.-turns.}$$

As all the various parts are in series,
The total ampere-turns = 288 + 1,376 + 288 + 4,900 = 6,852. *Ans.*

182. Magnetic Calculations in Dynamos.—The general character of the magnetic circuits of dynamos is discussed in Chap. VII. The calculation of the exciting ampere-turns is somewhat complicated by the irregular nature of the air-gap, due to the armature teeth, air ducts, fringing, etc. The amount of leakage



Axial length of armature stampings and pole-faces = 16 in.
FIG. 192.—Eight-pole, 400 r.p.m., 250-volt d.c. generator.

flux between poles introduces another factor which must be considered.

However, by making some approximations as to leakage and the geometry of the magnetic-flux paths, the relation of flux to ampere-turns may be computed with reasonable accuracy. Below is given an example illustrating the general method of procedure in computing the ampere-turns per pole in a generator. This example is not intended, however, to include the detailed computations such as are found in texts which specialize in dynamo design.

Example.—In the dynamo shown in Fig. 192, it is desired to compute the number of ampere-turns per pole necessary to produce an air-gap flux of 7,500,000 lines from each pole into the armature. The air-gap has an effective length of 0.235 in., after correction has been made for armature teeth, fringing, etc.[1] The leakage coefficient (ratio of core flux to armature flux) is assumed to be 1.15.

The paths of the fluxes from the various poles, including the leakage flux, are shown in the figure. The lengths of paths are easily determined. Consider the flux path *abcdef*.

The length $ab = \frac{60 - 38}{2} - 0.235 = 10.8$ in. (approximately); *bc* (approximately one-eighth the mean circumference of the yoke, less 5 in.) $= \frac{\pi 63''}{8} - 5 = 24.7 - 5 = 19.7$ in.

$$fe = \frac{\pi 32}{8} = 12.6 \text{ in. (approximately).}$$

The flux densities are as follows:

Flux in cores $= 7{,}500{,}000 \times 1.15 = 8{,}630{,}000$, as the flux in the core is equal to the armature flux plus the leakage flux.

Flux density in cores $= \frac{8{,}630{,}000}{16 \times 10} = 54{,}000$ lines per sq. in.

This should be increased by about 10 per cent to allow for the thickness of the oxide on the surface of the laminations:

$$54{,}000 \times 1.10 = 59{,}400.$$

Flux density in yoke $= \frac{8{,}630{,}000}{2(16 \times 3)} = 90{,}000$ lines per sq. in., as the pole flux divides, one-half going each way in the yoke.

Flux density in armature $= \frac{7{,}500{,}000}{2(6 \times 16)} = 39{,}000$ lines per sq. in.

This must be increased by about 25 per cent to allow for the air-duct space and the spaces between laminations.

This makes the density in the armature

$$39{,}000 \times 1.25 = 48{,}800 \text{ lines per sq. in.}$$

The air-gap density $= \frac{7{,}500{,}000}{16 \times 12} = 39{,}000$ lines per sq. in.

Knowing the above factors, and utilizing the magnetization curves of Fig. 190, it is a comparatively simple matter to determine the total ampere-turns per pole.

For example, with dynamo steel sheet 4 ampere-turns per inch are necessary with 59,400 lines per square inch. Hence, for the core *ab*,

$$I_1N_1 = 4 \times 10.8 = 43 \text{ ampere-turns.}$$

The ampere-turns are then found as follows:

[1] See footnote, p. 262.

Part	Material	Flux, lines	Area, square inches	Flux density, lines per square inch	Ampere-turns per inch	Length, inches	Ampere-turns
Core *ab*	Dynamo steel sheet	8,630,000	160	59,400	4.0	10.8	43
Yoke *bc*	Cast steel	4,315,000	48	90,000	40.0	19.7	788
Core *cd*	Dynamo steel sheet	8,630,000	160	59,400	4.0	10.8	43
Gap *de*	Air	7,500,000	192	39,000	12,200	0.235	2,870
Armature *ef*	Dynamo steel sheet	3,750,000	76.8	48,800	3.0	12.6	38
Gap *fa*	Air	7,500,000	192	39,000	12,200	0.235	2,870
			Total ampere-turns for two poles				6,652
			Total ampere-turns per pole				3,326

As the machine is symmetrical, each complete magnetic circuit requires the same number of ampere-turns per pole. The design of the exciting coils themselves is not difficult.

183. Hysteresis.—If the m.m.f. per centimeter or magnetizing force acting on an iron sample begins at zero and is increased, the relation between magnetizing force and flux density (or induction) will be similar to that shown by curve $0a$ (Fig. 186, p. 254). This curve is called the *normal* saturation or magnetization curve and already has been discussed.

If the magnetizing force is now decreased (Fig. 193), the induction will *not* decrease along the line aO, but will decrease less rapidly, along ab. When point b is reached, the magnetizing force is zero but the magnetic induction has not reached zero. The flux density Ob is called the *remanence*. Before the flux density can be reduced to zero, the magnetizing force must be reversed in direction. That is, it requires a negative magnetizing force Oc to reduce the flux density to zero. The magnetizing force Oc is called the *coercive force*. (These magnetic properties are discussed in connection with permanent magnets, Par. 152, p. 221.)

If the magnetizing force now be increased in the negative direction to d', where $Od' = Oa'$, the flux density will be carried to a negative maximum $d'd$. The negative maximum flux density $d'd$ is equal to $a'a$. If the magnetizing force now is increased toward zero, the curve will pass through point e when the magnetizing force is again zero and the negative remanence $Oe = Ob$. A positive coercive force $Of = Oc$ is necessary to bring the flux density again to zero. When the magnetizing force again becomes Oa', the flux density will return to its original value at a, closing the loop.

It is seen that the induction always lags the magnetizing force. For example, at b the magnetizing force has reached zero; the

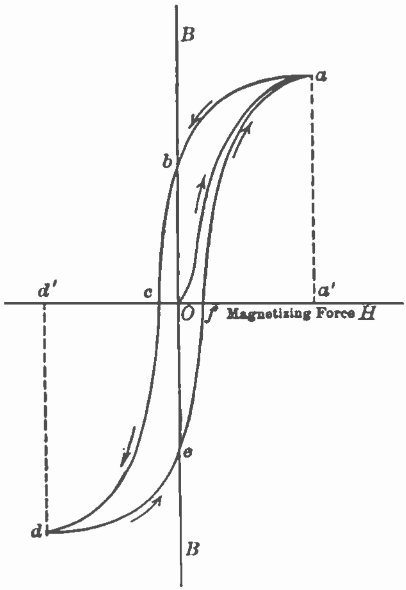


FIG. 193.—Hysteresis loop.

induction, however, does not reach zero until the iron has been carried farther along the cycle of magnetization to c. *This lag of the induction with respect to the magnetizing force has been given the name "hysteresis." The cycle of magnetization abcdefa* (Fig. 193) *is called a "hysteresis loop."*

The iron does not return to a previous condition of magnetization without the application of a magnetizing force. For example, it requires a magnetizing force Oc to bring the iron to zero induction at c. (The state of magnetization at c is also different from that at O.) An expenditure of energy is required to carry the iron through the cycle of magnetization. According to the

Weber theory of molecular magnetism (Par. 142, p. 207), this expenditure of energy is due to the friction of the molecular magnets in changing their directions with change of magnetizing force.

If several loops are taken, having different maximum flux densities, they will have the general appearance of the three loops

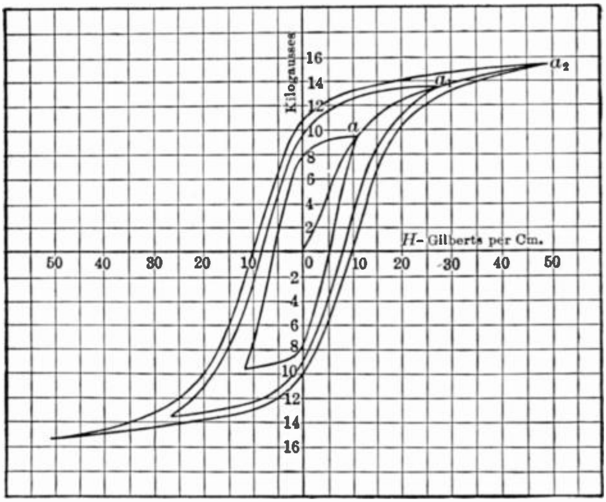


FIG. 194.—Hysteresis loops for three maximum flux densities.

in Fig. 194. The maximum points a, a_1, a_2 all lie along the normal saturation curve Oa_2.

184. Hysteresis Loss.—The hysteresis loss is proportional to the area of the hysteresis loop (Figs. 193 and 194). In fact the hysteresis loss may be obtained by finding the area of the loop to scale, and dividing by 4π. This gives the loss in ergs per cycle.

The relation may be proved as follows: A coil wound on an iron core of 1 sq. cm. cross-section is connected across the voltage V. The current i which results is determined not only by the resistance of the coil but by the counter e.m.f. $Nd\phi/dt = NdB/dt$, where ϕ is the flux in the core in maxwells, which is equal to B, the flux density, as the cross-section is 1 sq. cm. (see Par. 194, p. 287). V and i are given in absolute c.g.s. units.

By Kirchhoff's second law, the voltage V must be equal to the resistance drop iR plus the counter e.m.f. NdB/dt. That is,

$$V = iR + N\frac{dB}{dt} \quad \text{abvolts.} \tag{I}$$

The power

$$Vi = i^2R + Ni\frac{dB}{dt} \quad \text{ergs per second.} \tag{II}$$

Vi is the total power input and i^2R is the heating loss in the winding. The term $NidB/dt$ must be the power p_m which goes to store energy in the magnetic field. The total stored energy, equal to the integrated product of power and time, is

$$W = \int p_m dt = \int Ni\frac{dB}{dt}dt \quad \text{ergs.} \tag{III}$$

Let n be the turns per centimeter length of the coil. The m.m.f. per centimeter length, $H = 4\pi ni$.

Rewriting Eq. (III) for a centimeter length of the core,

$$W_h = \int ni\frac{dB}{dt}\, dt = \frac{1}{4\pi}\int HdB \quad \text{ergs per centimeter cube.} \tag{118}$$

The $\int HdB$ is the expression for the area of the hysteresis loop.

For example, let the area of the smallest loop (Fig. 194) be A sq. in. The scale is such that 1 in. on the abscissa scale represents 10 gilberts per centimeter, and 1 in. on the ordinate scale represent 4 kilogausses. The ergs loss per cycle per cubic centimeter is

$$W_h = \frac{A \times 10 \times 4{,}000}{4\pi} \quad \text{ergs.} \tag{119}$$

To convert this energy loss into joules or watt-seconds, divide by 10^7.

The hysteresis loss per cubic centimeter per cycle depends on two factors, the magnetic material and the maximum flux density. The loss within certain limits may be expressed by the Steinmetz law as follows:

$$W_h = \eta B^{1.6} \tag{120}$$

W_h is the hysteresis loss per cubic centimeter in ergs per cycle, η is a constant depending on the material, and B is the maximum flux density in gausses.

Below are given a few typical values of η.

Hard cast steel	0.025	Sheet iron	0.004
Forged steel	0.020	Silicon sheet steel	0.0010
Cast iron	0.013	Silicon steel	0.0009

Example.—What will be the ergs loss per cycle in a core of sheet iron having a net volume of 40 c.c., in which the maximum flux density is 8,000 gausses?

$W_h = 0.004 \times 8{,}000^{1.6}$.

$$\log 8{,}000 = 3.9031.$$
$$1.6 \times 3.9031 = 6.2449.$$
$$\log 1{,}757{,}000 = 6.2449.$$

$W_h = 0.004 \times 1{,}757{,}000 = 7{,}028$ ergs per cubic centimeter per cycle.
Total loss $W = 7{,}028 \times 40 = 281{,}000$ ergs per cycle,

$$\text{or } 281{,}000 \times 10^{-7} = 0.0281 \text{ joule per cycle.} \quad \textit{Ans.}$$

Since the area of the hysteresis loop, divided by 4π, gives the hysteresis loss in ergs per cycle, the hysteresis loss in ergs per second must be equal to the ergs per cycle multiplied by the cycles per second, or frequency. It follows that hysteresis loss is directly proportional to the frequency.

185. Core Losses.—In cores subjected to a varying or an alternating magnetic flux, there exists not only hysteresis loss but eddy-current loss as well. Eddy-current loss is reduced to a very low value by laminating, the loss varying as the square of the thickness of the laminations. It also varies as the *square* of both frequency and flux density (see Vol. II, Par. 138, p. 240). In transformer cores, low core loss is distinctly essential, for the rating of a transformer is based to considerable extent on the core loss (see Vol. II, Chap. VII). Silicon steel, which has low core loss, is used therefore almost entirely in transformers. About 4 per cent of silicon alloyed with steel reduces the hysteresis loss materially, and, since silicon also increases the electrical resistivity, the eddy-current loss is reduced. These effects of silicon may be seen by examining the following table.

In order to make a general comparison of the core losses in different grades of iron or steel, the standard has been adopted of watts loss per pound at maximum flux density of 10,000 gausses at 60 cycles. The following Table I[1] shows in a concise manner the various important properties of some standard types of magnetic core materials, as well as of the more perfect laboratory products which are valuable for purposes of comparison.

[1] From "Magnetic Core Materials Practice," Allegheny Steel Company.

Table I.—Properties of Commercial Grades of Magnetic Core Materials at 10,000 Gausses
(By courtesy of the Allegheny Steel Company)

Magnetic Core Materials	Silicon content, per cent (1)	B_r Residual flux from 10,000 gausses, maximum (2)	H_c Coercive force from 10,000 gausses, maximum (3)	$H_c \times B_r$ (4)	Hysteresis loss, ergs/cm.-cube/cycle, for 10,000 gausses (5)	H/pound (watt/pound) 60-cycle hysteresis loss for 10,000 gausses (6)	Electrical resistivity, microhms/cm. cube (7)	P_e/pound (watts/pound) 60-cycle eddy-current loss for No. 29 U.S.S. gauge, for 10,000 gausses (8)	Maximum total core loss, 60 cycles—10,000 gausses, Epstein test, No. 29 U.S.S. gauge (9*)
Full, hard, cold-rolled steel sheets, or strip	....	8,500	5.5	46,700	21,400	7.56	11	0.4	7.96
Hot-rolled, annealed soft-steel sheets	....	8,400	0.86	7,220	3,540	1.25	11	0.4	1.65
Allegheny "Armature"-grade sheet steel	0.5	.....	....	.....	2,750	0.958	18	0.356	1.30*
Allegheny "Dynamo"-grade sheet steel	2.5	8,100	0.68	5,510	2,530	0.913	40	0.132	1.01*
Allegheny "Dynamo-Special"-grade sheet steel	3.25	7,300	0.65	4,740	2,030	0.733	48	0.128	0.82*
Allegheny "Radio-Transformer"-grade sheet steel	3.8	.....	....	.....	1,890	0.683	56	0.128	0.76*
Allegheny "Transformer 'C' "-grade sheet steel	4.25	8,100	0.4	3,240	1,815	0.658	60	0.114	0.72*
Allegheny "Audio-Transformer 'B' "-grade sheet steel	3.60	8,100	0.4	3,240	1,475	0.532	51	0.128	0.66*
Allegheny "Audio-Transformer 'A' "-grade sheet steel	3.60	8,100	0.4	3,240	1,305	0.472	51	0.128	0.60*
Products from the Research Laboratories									
Pure iron (Yensen—1925)	....	8,600	0.2	1,720	600	0.207	7.64	0.60	0.807
Vacuum-melted 4 per cent silicon iron alloy (Yensen—1925)	.	5,200	0.15	780	500	0.179	55	0.12	0.299
Hipernik (Yensen—1925)		7,300	0.05	365	200	0.072	45	0.13	0.202
Permalloy (78 per cent nickel) (Yensen—1925)		5,500	0.05	275	200	0.063	25	0.28	0.343

* The values in column (9) which are starred are commercial guarantees which only approximate the total of values in columns (6) and (8). Number 29 U.S.S. gauge is 0.014 in. (14 mils) or 0.0356 cm. thick.

FERROMAGNETIC ALLOYS

Certain alloys of iron and nickel, when properly prepared and heat-treated, have magnetic properties far superior to those of iron itself, particularly at very low flux densities. The most common of these alloys are Permalloy, Perminvar, Hipernik, and Coupernik (also see Par. 135, p. 201).

186. Permalloy.—Permalloy is a nickel-iron alloy, which has abnormally high magnetic properties at very low values of magnetizing force. Its composition is approximately 78.2 per cent nickel and 21.4 per cent iron, the remainder being impurities.

The maximum permeability of ordinary Permalloy is approximately 87,000, although values much higher than this have been found in specially treated samples. This is very much higher than any values of permeability which have been found heretofore for iron or for any of the other iron or steel alloys. This maximum permeability in Permalloy occurs at a flux density of 5,000 gausses, corresponding to a magnetizing force of 0.058 gilbert per centimeter, a value very much lower than those at which ordinary iron and steel obtain their maximum permeabilities. In fact, Permalloy becomes highly saturated in the earth's field alone ($H = 0.45$) so that extreme care must be exercised in testing it.

Figure 195 gives a B-H curve for Permalloy, and, for comparison, the curve of Armco iron, which is almost pure iron. It will be noticed that only at low values of H is Permalloy superior to Armco iron in its magnetic properties.

The hysteresis loop of Permalloy, when carried to a maximum induction of 5,000 gausses, has only one-sixteenth the area of that of iron, and accordingly the hysteresis loss is in the same proportion.

The magnetic properties of Permalloy are very sensitive to heat treatment, so that it must be carefully prepared and in very thin strips. Also, it is very unstable and may lose much of its high permeability if subjected to magnetic and mechanical shocks. Its principal use at present is in communication work, particularly in the "loading" of submarine cables.[1]

[1] For further details see ARNOLD, H. D. and ELMEN, G. W., "Permalloy, an Alloy of Remarkable Magnetic Properties." *Jour. Franklin Inst.*

Permalloy powder or "dust" is mixed with a resinous binder and then compressed under very high pressure to form cores for transformers used at high frequencies in communication work.

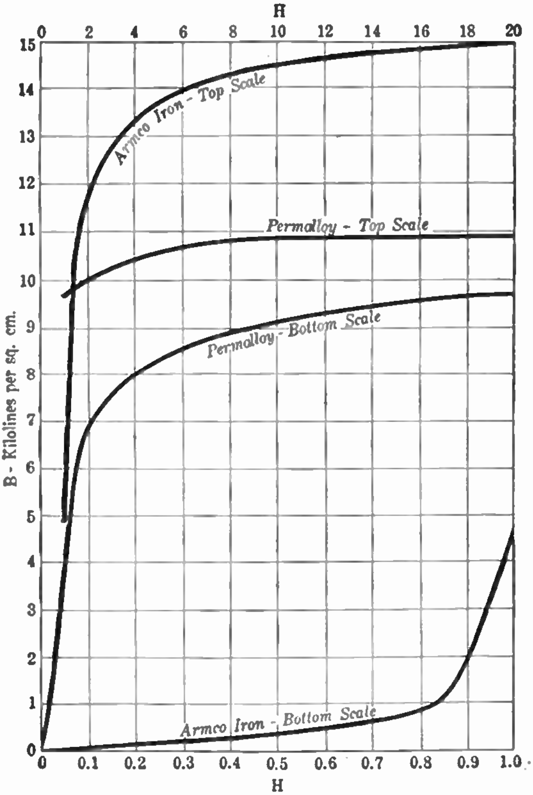


FIG. 195.—*B-H* curves of permalloy and Armco iron.

The fact that the particles are all insulated from one another makes the eddy-current loss practically zero.

Perminvar is an iron-nickel-cobalt alloy which has constant permeability over a wide range of *B*. Since the area of the hysteresis loop accordingly must be very small, the hysteresis

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May, 1923, p. 621. Reprinted July, 1923, in *Bell System Tech. Jour.* KENNELLY, A. E., "The Reluctivity of the Recently Discovered Magnetic Metal Permalloy," *Jour. Franklin Inst.*, May, 1924 loss is correspondingly small. The permeability is only moderate, being less than half that of Armco iron.

187. Hipernik.—Hipernik is a highly refined 40 to 60 per cent iron-nickel alloy. It is found that, in this type of alloy, impurities of a few hundredths of 1 per cent affect the permeability by several hundred per cent so that in the process of manufacture every trace of impurity must be removed. Ordinarily the permeability reaches values as high as 57,000, which may be determined from Fig. 196, which gives both the *B-H* curve and two

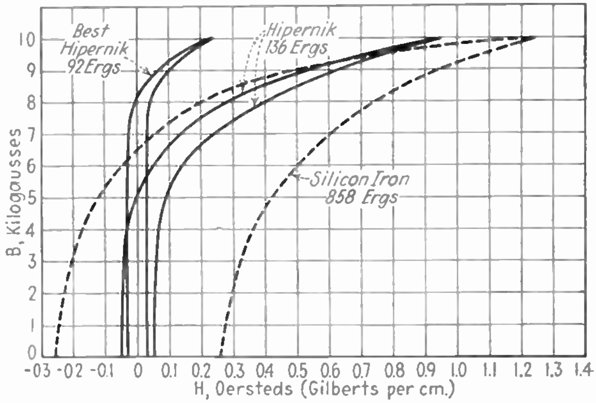


FIG. 196.—Hysteresis loops of Hipernik and silicon iron.

hysteresis loops of Hipernik. Under special conditions, the permeability may reach higher values.[1] For comparison, a hysteresis loop for silicon iron also is given. It will be noted that the area of the Hipernik loops is materially less than for silicon iron. The ergs loss per cycle is indicated on each loop.

The principal use of Hipernik is for the coils of current transformers (Vol. II, Chap. VIII). For this purpose a long ribbon of Hipernik is coiled like a clock spring to form the core, so that waste due to punching is practically eliminated.[1]

Coupernik is an alloy of the same composition as Hipernik, but it is given a different heat treatment which produces a constant permeability over a wide range of flux density.

[1] YENSEN, T. D., "Permeability of Hipernik Reaches 167,000," *Elec. Jour.*, June, 1931, p. 386.

MAGNETIC-FLUX MEASUREMENTS

Magnetic flux cannot be measured by the mere insertion of instruments in a circuit as can electric current, voltage, and power. Magnetic flux is ordinarily measured in two ways: *indirectly* by reading the deflection of a ballistic galvanometer or by the indication of a voltmeter; and *directly* by permeameters of the Koepsel type. The first method depends on the fact that an e.m.f. is induced in a coil when the flux linking the coil is changed; the second method depends on the fact that the deflections of Weston-type direct-current instruments are proportional to the flux if the current in the moving coil is maintained constant. In the first method, the voltmeter is used if the flux is alternating, the flux being determined by measuring the e.m.f. which it induces in a coil of a known number of turns, the voltmeter being of the alternating-current type (see Vol. II, Chap. VIII).

188. Ballistic Galvanometer.—It is shown in Par. 230 (p. 347) that the ballistic throw of a galvanometer is proportional to the quantity Q of electricity discharged through it, provided the discharge all takes place before the galvanometer coil has begun to move.

If a galvanometer be connected in series with a coil (Fig. 197) and the flux linking this coil be changed in any manner, a current will flow, and hence a quantity of electricity will be discharged through the galvanometer. It can be shown that if the flux is changed by an amount $\Delta\phi$, the quantity of electricity discharged through the galvanometer,

$$Q = N\frac{\Delta\phi}{R} \tag{121}$$

where N is the number of turns in the coil, and R is the combined resistance of the coil and galvanometer. Since the galvanometer deflection is proportional to Q, it must also be proportional to the *change of flux*, provided the resistance R of the entire circuit is kept constant. Therefore, a ballistic galvanometer may be used to measure flux. The change of flux

$$\phi_1 - \phi_2 = KD \tag{122}$$

where ϕ_1 is the initial flux, ϕ_2 the final flux, K the galvanometer constant, and D the ballistic deflection of the galvanometer.

189. Standard Solenoid.—In order to determine the constant K in Eq. (122), a flux of known value is necessary. This flux, which links a secondary coil of known number of turns, is caused to change by a known amount, either by decreasing it to zero or by building it up from zero. Simultaneously, the ballistic deflection of the galvanometer in series with the coil is read. This gives sufficient data for obtaining the galvanometer constant.

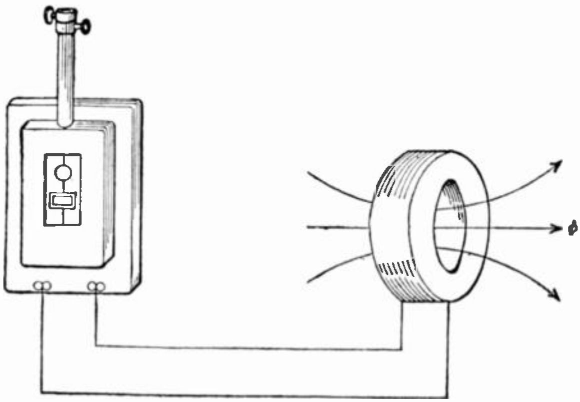


FIG. 197.—Ballistic galvanometer used to measure flux.

The long straight solenoid discussed in Par. 165 (p. 237) is used to obtain the flux of known value. It is shown that the field intensity at the center of such a solenoid is equal practically to $0.4\pi nI$ oersteds, if the ratio of length to radius of the solenoid is large. (n is the number of turns per centimeter length and I the current in amperes.) Since in air, field intensity is *numerically* equal to flux density, the total flux across the center cross-section is $0.4\pi nIA$ maxwells, where A is the cross-section in square centimeters. Hence, such a solenoid provides a flux the value of which can be calculated.

The properties of the straight solenoid may also be developed by a consideration of the ring solenoid (Fig. 198), the cross-section of which is circular and the core is of non-conducting, non-magnetic material such as wood. Let the cross-sectional area be A sq. cm. and the mean circumferential length be l cm. (Fig. 198).

The solenoid is wound uniformly with n turns per mean circumferential centimeter and a current of I amp. flows in the winding. The total m.m.f. F is obviously $0.4\pi nlI$ and, if the cross-sectional diameter of the solenoid is small compared with the internal diameter of the ring, the reluctance is l/A.
Hence the flux

$$\phi = \frac{F}{\mathcal{R}} = \frac{0.4\pi nlI}{l/A} = 0.4\pi nIA \text{ maxwells} \tag{123}$$

and is independent of the length of winding.

The mean flux density

$$B = \frac{\phi}{A} = 0.4\pi nI \text{ gausses.} \tag{124}$$

That is, the flux density *is equal to the m.m.f. per centimeter length* and is also independent of the length of winding.

It will be noted that Eq. (124) is identical numerically with Eq. (106), p. 238, which gives the field intensity at the center of a long, straight solenoid. Equation (123) also gives the flux across the center cross-section of a long, straight solenoid.

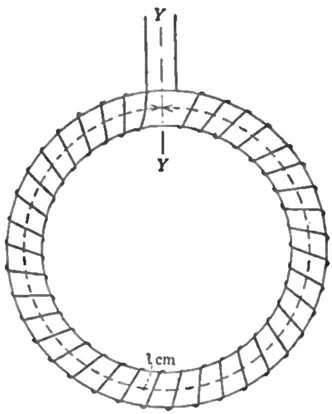


FIG. 198.—Ring solenoid.

Referring to Eq. (123),

$$\phi = \frac{0.4\pi nI}{(1)/(A)}.$$

$1/A$ is the reluctance *per centimeter length* of the solenoid and $0.4\pi nI$ is the m.m.f. *per centimeter length* of the solenoid. Hence, it may be said that in such a solenoid the m.m.f. in each unit length is utilized entirely in sending the flux through that unit length.

If the ring solenoid (Fig. 198) be cut along the plane YY and straightened out, a long solenoid is obtained as in Fig. 170 (p. 237) or Fig. 199. The long solenoid (Fig. 199) also has a length of l cm. and n turns per centimeter length. The flux, which is shown as acting from right to left within the solenoid, leaves the left-hand end of the solenoid and spreads, theoretically, over an infinite area and returns to enter the right-hand end. If there were no reluctance exterior to the solenoid, all the m.m.f. within the solenoid would be utilized in overcoming the reluctance of the solenoid, as in the ring solenoid (Fig. 198). Hence, the flux in this solenoid also can be obtained from Eq. (123). Since, in the exterior return path, the flux spreads theoretically over an infinite area, the reluctance is small except for the parts $\mathcal{R}$ near the ends of the solenoid, where the area is finite.

If, however, the reluctance $l \div A$ of the solenoid itself, is large compared with $2\mathcal{R}$, the end reluctances, the total m.m.f. of the solenoid may be considered as being utilized in overcoming its own reluctance. Hence, the flux near the center of the solenoid

$$\phi = \frac{0.4\pi nlI}{l/A} = 0.4\pi nIA, \tag{125}$$

the same as Eq. (123). Therefore, the flux ϕ in such a solenoid is equal numerically to the *m.m.f. per centimeter* multiplied by the cross-section of the solenoid in square centimeters. It is a simple matter to make such a solenoid, having a known diameter and a known number of turns per centimeter. If the length of the solenoid is 10 times the diameter, Eq. (125) is correct within 0.5 per cent, and if the length is 20 times the diameter, the error is only 0.1 per cent.

The error due to the two end reluctances $2\mathcal{R}$ is identical with the error resulting from assuming that a finite solenoid is infinite in length (Par. 165, p. 237).

190. Calibration of Galvanometer.—To calibrate the galvanometer, connections are made as in Fig. 199, in which a ring test specimen is included as an example. The current in the primary turns of the standard solenoid is obtained from some steady direct-current source. Its value is controlled by the rheostat R' and read with the ammeter. A secondary of a known number of turns, n_2, is wound about the solenoid near its center. This secondary is connected in series with the secondary of the test specimen and with the galvanometer. Both secondaries are always in circuit in order to keep the value of R constant in Eq. (121), p. 275. A reversing switch Sw is connected in the galvanometer circuit. To calibrate, open switch Sw and close switch S, which energizes the solenoid without causing the galvanometer to deflect. Close switch Sw and read the current I with the ammeter. Then open S, noting the ballistic throw D_1 of the galvanometer. Several check readings should be taken.

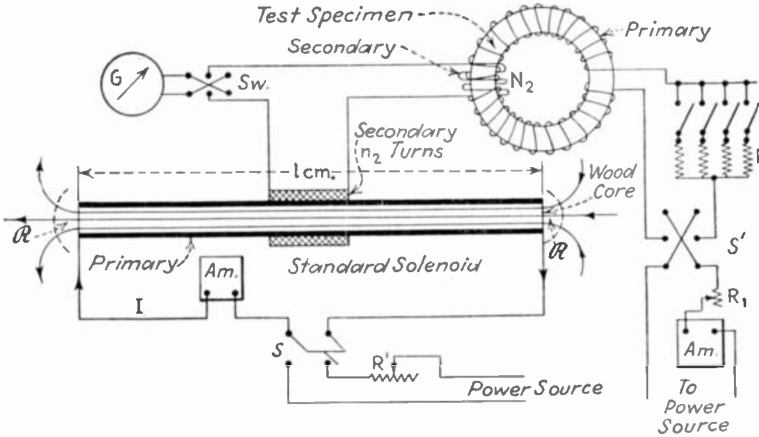


FIG. 199.—Standard solenoid with ring sample and galvanometer.

When the switch S is opened, the total change of flux $\Delta\phi$ is $0.4\pi nIA$, and the quantity Q_1 is discharged through the galvanometer (Eq. (121)).

$$Q_1 = \frac{n_2 \Delta\phi}{R} = n_2 \frac{0.4\pi nIA}{R}. \tag{126}$$

When the flux ϕ_2 in the specimen is changed by an amount $\Delta\phi_2$, or from ϕ_2 to ϕ_1, from Eq. (121), the quantity

$$Q_2 = N_2 \frac{\Delta\phi_2}{R} = N_2 \frac{\phi_2 - \phi_1}{R} \tag{127}$$

where N_2 is the number of secondary turns on the test specimen.

Dividing Eq. (127) by Eq. (126),

$$\frac{Q_2}{Q_1} = \frac{\dfrac{N_2(\phi_2 - \phi_1)}{R}}{n_2 \dfrac{0.4\pi nIA}{R}} = \frac{N_2(\phi_2 - \phi_1)}{0.4\pi n n_2 IA}.$$

$$\phi_2 - \phi_1 = \frac{Q_2}{Q_1}\left(\frac{0.4\pi n n_2 IA}{N_2}\right).$$

Since the galvanometer deflections are proportional to the quantities Q_2 and Q_1 (Eq. (122))

$$\Delta\phi_2 = \phi_2 - \phi_1 = \frac{D_2}{D_1}\left(\frac{0.4\pi n n_2 I A}{N_2}\right)$$
$$= \left(\frac{0.4\pi n n_2 A I}{D_1 N_2}\right) D_2 = K D_2. \tag{128}$$

The quantity in parentheses is the galvanometer constant K in Eq. (122), p. 275. That is,

$$K = \frac{0.4\pi n n_2 A I}{D_1 N_2} \tag{129}$$

where n is the turns per centimeter in the *primary* of the standard solenoid; n_2 is the total turns in the *secondary* of the standard solenoid; N_2 is the total turns in the secondary of the test specimen; A is the cross-section of the standard-solenoid core in square centimeters; I is the current in the primary of the standard solenoid; D_1 is the galvanometer deflection when the primary of the standard solenoid is opened.

Example.—An air solenoid 48 in. long has an inside diameter of 1.75 in., a primary winding of 960 turns, and a secondary winding of 2,000 turns. There are 40 turns in the secondary of the test specimen. The galvanometer is connected in series with the secondaries of both the standard solenoid and the test specimen (see Fig. 199). When the current in the primary of the standard solenoid has become steady at 3.82 amp., the circuit is opened and the resulting ballistic deflection of the galvanometer is 12.4 cm. Determine: (*a*) the flux density (gausses) in the standard solenoid; (*b*) the total flux in the standard solenoid; (*c*) the galvanometer constant; (*d*) the flux change in the test specimen when the ballistic throw of the galvanometer is 8.9 cm.

The turns per centimeter length in the standard solenoid

$$n = \frac{960}{48 \times 2.54} = 7.88.$$

The cross-section of the standard solenoid $A = (\pi/4)(1.75)^2 \times (2.54)^2 = 15.5$ sq. cm.

(*a*) From Eq. (124), $B = 0.4\pi \times 7.88 \times 3.82 = 37.8$ gausses. *Ans.*

(*b*) $\phi = BA = 37.8 \times 15.5 = 586$ maxwells. *Ans.*

(*c*) Using Eq. (129),

$$K = \frac{0.4\pi \times 7.88 \times 2{,}000 \times 15.5 \times 3.82}{12.4 \times 40} = 2{,}360. \quad \textit{Ans.}$$

(*d*) $\phi_2 - \phi_1 = \Delta\phi_2 = KD_2 = 2{,}360 \times 8.9 = 21{,}000$ maxwells. *Ans.*

191. Yoke Method.—In the yoke or divided-bar method, which is due to Hopkinson, the test specimen is a cylindrical rod of from ¼ to ½ in. (0.64 to 1.27 cm.) diameter and about 20 in. (50.8 cm.) long. The rod is in two sections A and B (Fig. 200(*a*)), the two ends butting at e. These two ends should have accurately ground flat surfaces so that they make good magnetic contact. The rod fits tightly at both ends into a massive iron yoke Y whose reluctance is negligible in comparison with that of the rod. The effective length of the rod is taken as l cm., the distance between the inside surfaces of the yoke. The exciting coil PP is in two sections and has a total of N turns. The flux is determined by means of a small secondary coil S, through which the rod A passes. By means of a spring, the coil S springs out of the yoke when the part A of the rod is withdrawn.

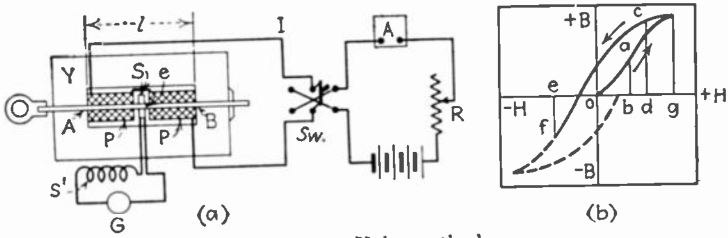


FIG. 200.—Yoke method.

The galvanometer is connected in series with the coil S and the secondary S' of the standard solenoid (Fig. 200(a)). The current to the exciting coil is supplied by a battery, or by any source of steady current, through a rheostat R, an ammeter A, and a reversing switch Sw. A drop wire often gives better adjustment than a series resistance. The m.m.f. in gilberts per centimeter

$$H = \frac{0.4\pi NI}{l}.$$

To determine some point a on the normal induction curve, the current is increased to a value which gives a m.m.f. ob (Fig. 200(b)). The end A of the rod is withdrawn, allowing S to spring out, and the ballistic throw of the galvanometer is read. Since the flux linking the coil S becomes nearly zero when S springs out, the galvanometer deflection must be proportional to the ordinate ba. To determine point c on the hysteresis loop, the excitation is increased to og and then decreased to od. The end A of the rod is again withdrawn and the galvanometer deflection, which is proportional to cd, is read. To determine the point f on the hysteresis loop, the excitation is increased to og, is reduced to zero at o, the reversing switch is thrown, and the excitation is increased negatively to e. The end A of the rod is again withdrawn, and the resulting galvanometer deflection, which is proportional to the ordinate ef, is read.

The galvanometer is calibrated by the method given in Par..190. The flux density B is readily determined, knowing the flux ϕ and the cross-section A of the rod.

This method is subject to slight errors, due to leakage from the rod to the yoke, imperfect magnetic contacts, and the hysteresis effects in the yoke.

192. Ring Method.—In the ring method of magnetic testing, which was devised by Rowland, the form of the sample is such as to give a closed magnetic circuit. It usually consists of a ring (Figs. 199 and 201), although if the sample cannot be made conveniently into ring form, it may be in the form of a square. For example, a square is readily made from long, rectangular laminations which are made alternately to butt and lap at the joints. If properly made, the joints in this latter case introduce slight error. The connections in the ring method (Figs. 199 and 201(a)) are essentially the same as those for the yoke method (Fig. 200(a)). The principal difference lies in the procedure in carrying the sample through the magnetic cycle.

The primary is wound uniformly on the ring (Fig. 201(a)) or along the four sides, if the sample is in the form of a square. The value of H is given approximately by dividing the total m.m.f. by the mean length of the sample. The secondary is wound about the specimen in the manner shown in Fig. 199 and Fig. 201(a). The success of the experiment depends on the current being quickly changed by definite amounts. Hence, the parallel arrangement of resistances R (Fig. 201(a)), each of which is provided with a switch, is preferable to a sliding-contact resistance. The auxiliary rheostat R_1 is also useful, in that it may be set to correspond to different values of maximum current. A single series resistance, having several switches which short-circuit definite amounts, may also be used to change the current.

Since the sample is a closed iron circuit, there may be initially considerable residual magnetism or remanence. It is necessary to reduce this residual magnetism to a very small value. This is done by reversing the exciting current more or less rapidly by means of switch $S'w$ and at the same time slowly reducing the value of the current to zero with resistances R_1 or R. This carries the iron through hysteresis loops of diminishing amplitude, and, if properly done, the residual magnetism or remanence may be reduced to a very small value.

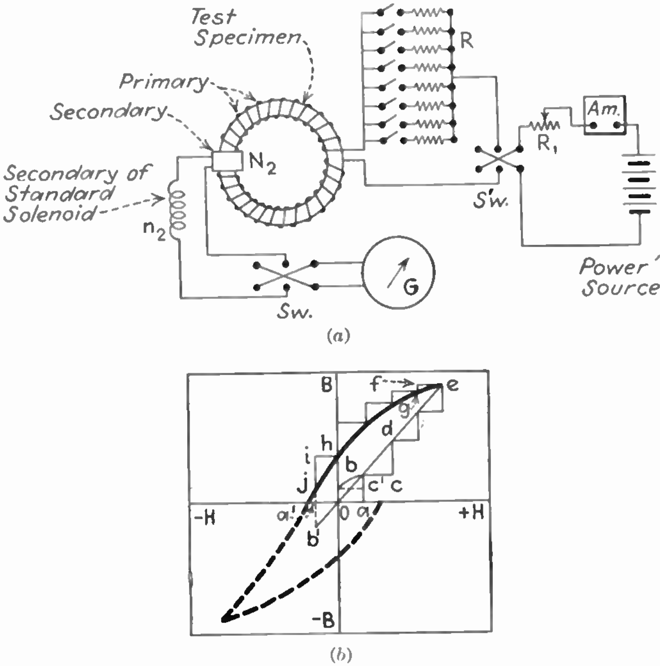


FIG. 201.—Ring method.

The normal curve may be obtained by reversing the magnetizing force each time that a measurement is made. For example, in Fig. 201(*b*), let it be desired to obtain the point *b* on the normal curve, corresponding to a flux ϕ_1. If the magnetizing force is carried to *a* and the circuit then opened, the change of flux will not be $ab = \phi_1$ but bc', due to the remanence. However, if the switch $S'w$ is reversed, the total change of flux will be $ba + a'b' = 2\phi_1$ where $oa' = oa$. Thus each point on the curve may be determined by obtaining the desired magnetizing force, reversing it, and observing the galvanometer deflection. The galvanometer deflection corresponds to *twice* the value of the flux.

The normal curve may also be obtained by an increment method. The hysteresis loop must be obtained in this manner.

When the magnetizing force is increased from *o* to *a* (Fig. 201(*b*)), by throwing in a single switch (Fig. 201(*a*)), it causes a change of flux *ab*, the value of which is determined by the ballistic deflection of the galvanometer. A second switch is then closed, increasing H by the amount *bc*, and the flux by the amount *cd*, the value of which is determined by the ballistic deflection of the galvanometer. The flux is thus increased in *increments* to *e*, the total flux at *e* being $ba + cd$, etc. The galvanometer measures the *increment* (or *decrement*) of flux between successive magnetizing forces. To obtain points on the hysteresis loop, the magnetizing force is decreased by an amount *ef* by opening a single switch (Fig. 201(*a*)), and the decrement of flux *fg* is read with the galvanometer. When the remanence *oh* is reached, the switch $S'w$ is reversed and a single switch (Fig. 201 (*a*)) is closed. This gives a decrement of magnetizing force *hi* and a corresponding flux decrement *ij*. The sample is thus carried around the hysteresis loop.

Since the flux at any point is the sum of successive flux increments, a single error vitiates the entire data. It is, therefore, advisable to practice carrying the flux once or twice through the cycle represented by the loop before actually taking data.

Although the ring method gives more precise results than the yoke method, it is more difficult to manipulate and the samples are not of such convenient form. Moreover, the entire data may be rendered worthless by a single error, such as failing to read the galvanometer correctly, inadvertently closing a switch, etc.

193. Koepsel Permeameter.—In the preceding two methods of magnetic testing, the measurements are indirect, quite laborious, and involve considerable computation. The Koepsel permeameter is an indicating instrument on whose scale the flux density B in a specimen is read directly, just as current is read on an ammeter scale. The magnetizing force H is also read directly by means of an ammeter. The instrument operates on the principle of the Weston ammeter (Par. 107, p. 148), that is, if a coil carries current in a magnetic field, it tends to assume a position perpendicular to the field. In the ordinary Weston instrument, the flux is constant and the deflections are practically proportional to the current in the moving coil; in the Koepsel permeameter, the current in the moving coil is constant and the deflections are practically proportional to the flux in the gap.

The instrument (Fig. 202) consists of two massive soft-iron yokes JJ, a moving coil M which turns in a very short air-gap, two compensating coils CC, and the exciting coil S. The specimen to be tested consists of an iron rod P held into the yokes

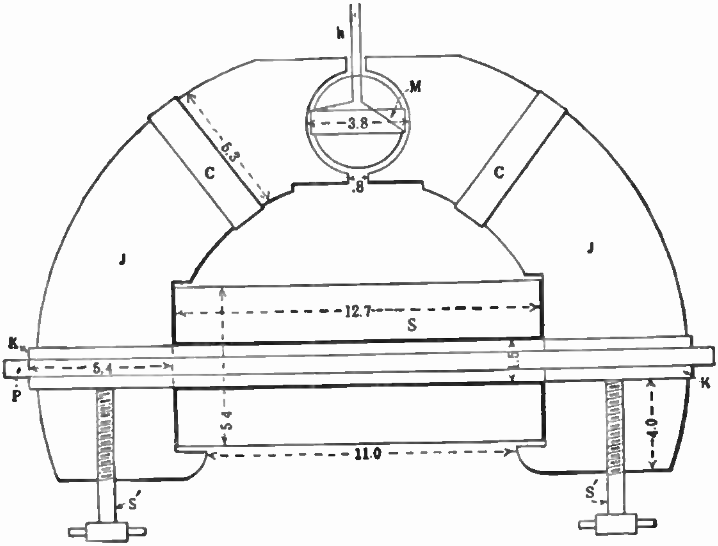


FIG. 202.—Diagram of the Koepsel permeameter (dimensions in centimeters).

by iron clamps KK and screws $S'S'$. The test specimen may be square in cross-section or it may be round, the clamps ordinarily being adapted to hold specimens 6 mm. (0.236 in.) square or 6 mm. diameter.

When the current in the moving coil M is equal to $0.005/A$ amp., where A is the cross-section of the test specimen in square centimeters, the instrument reads directly the flux density B in the specimen. The current for the moving coil is supplied by a 4-volt source, such as three or four dry cells in series, and is controlled by a suitable rheostat. This current, when once adjusted, must remain constant. The exciting coil S is wound with 79.6 $(= 100/0.4\pi)$ turns per centimeter, and the m.m.f. per centimeter $H = 0.4\pi(100/0.4\pi)I = 100I$ gilberts where I is the exciting current in amperes. Therefore, the m.m.f. per centimeter may be readily obtained by multiplying the ammeter reading by 100. Ordinarily, the same ammeter is used to measure the moving-coil current and the exciting current, by connecting different shunts in circuit by means of plugs.

With current in the exciting coil S, the instrument would normally deflect even if no specimen were in place, since the m.m.f. of the exciting coil is sufficient to produce considerable flux in the yokes. To prevent such deflection, two compensating coils CC, placed on the yoke near the gap, are connected in series with and opposing S. They are so adjusted that there is no deflection when current flows in S. To minimize deflections of the coil caused by the earth's field, the instrument should be so placed that the axis of the coil is in the plane of the magnetic meridian. A view of the instrument is given in Fig. 203. Although this instrument is simple in its operation and repeats itself very closely, it is subject to some error, particularly at high flux densities. It is assumed that a fixed percentage of the flux in the rod reaches the gap. This fixed percentage may vary as much as 2 per cent, depending on the flux density. Also, because of the reluctance of the air-gap, leakage, etc., the error in H under extreme conditions may be as large as 10 per cent. Correction curves usually accompany the instrument so that the effect of such errors may be reduced. The fact that ordinarily the errors are not large makes the instrument particularly useful in comparing magnetic materials and in detecting non-uniformity.

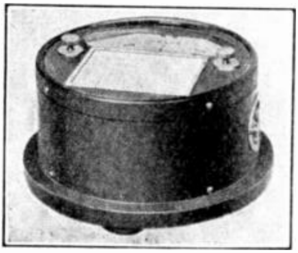


FIG. 203.—Koepsel permeameter.

CHAPTER IX

SELF- AND MUTUAL INDUCTANCE

In earlier chapters the current in a circuit has been considered as being determined by sources of e.m.f., such as generators or batteries acting in the circuit, and by resistance. When the current is in the *steady state*, these factors alone do determine the value of the current. If, however, a change in current occurs, it causes a change in the magnetic field associated with the circuit. The change in magnetic field induces an e.m.f. in the circuit itself and this e.m.f. is also effective in determining the value of current. Such an e.m.f. exists only during the time when the current is *changing*. It does not exist when the current is steady.

A circuit in which a change of current causes an e.m.f. to be induced within the circuit itself is said to have *self-inductance*, or frequently, just *inductance*. Methods for determining the effect of inductance in electric circuits are given in the paragraphs which follow.

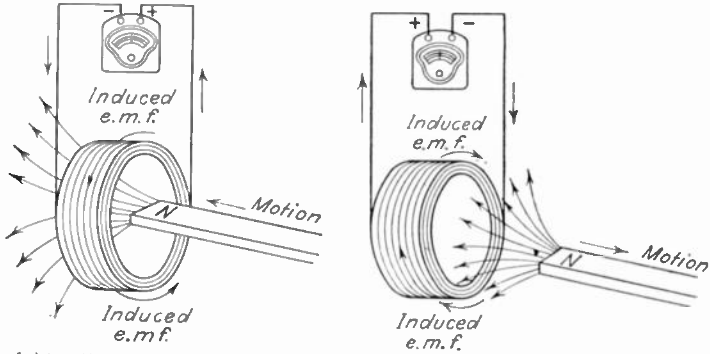
If an electric circuit is associated with a second circuit, so that a change of current in the second circuit causes a change in the flux linking any of the turns of the first circuit, the two circuits are said to have *mutual inductance*. Mutual inductance is also discussed in the paragraphs which follow.

The effects of inductance are in reality the effects of *induced e.m.f.* As the operation of electrical apparatus, such as generators, motors, and transformers, depends fundamentally on induced e.m.f., an understanding of the principles of induced e.m.f. is important.

194. Induced Electromotive Force.—If the terminals of an insulated coil (Fig. 204(a)) be connected to a galvanometer, and a magnetic field be set up through this coil, either by thrusting a bar magnet into the coil or by some other means, the galvanometer will deflect momentarily and then return to rest. This shows that an e.m.f. has been induced temporarily in the coil.

When the flux through the coil has ceased to change, this e.m.f. ceases. If investigation be made, it will be found that the direction of this induced e.m.f. is that shown in the figure and that this direction is such that if the e.m.f. be allowed to produce a current, this current will tend to push the bar magnet *out* of the coil, or, what is the same thing, will oppose its entering the coil.

If the magnet be withdrawn from the coil (Fig. 204(*b*)), the galvanometer will deflect again, momentarily as before, but the



(a)-North pole inserted in coil. (b)-North pole withdrawn.

FIG. 204.—Induced electromotive force.

deflection is opposite to its direction in the first case. The direction of the induced e.m.f. is now such that, if the e.m.f. produces a current, this current will tend to prevent the magnet from being withdrawn from the coil. The e.m.f. in each case is transient and ceases when the *change* of flux through the coil ceases.

If careful measurements be made, the value of this induced e.m.f. will be found to depend upon (1) the number of turns in the coil; (2) the rate at which the flux linked with the coil changes.

The *average* induced e.m.f. is given by

$$e = -\frac{N\phi 10^{-8}}{t} \text{ volts} \tag{130}$$

where N is the number of turns in the coil, ϕ is the total change of flux in maxwells linked with the coil, and t is the time in seconds required to produce the change in flux. 10^{-8} reduces the flux ϕ to practical units so that e is in volts. The minus sign indicates that the induced e.m.f. is in opposition to the effect which produces it.

ϕ/t is the average rate of change of flux, so that the induced e.m.f. may be said to be proportional to the *number of turns* and the *rate of change of flux*.

Example.—A flux of 1,500,000 lines links a coil having 350 turns. This flux through the coil is decreased to zero at a uniform rate in 0.2 sec. What is the induced e.m.f. during this time?

$$e = 350\frac{1,500,000}{0.2}10^{-8}$$
$$= 26.25 \text{ volts.} \quad \textit{Ans.}$$

When the flux does not change at a uniform rate, but the rate varies from instant to instant, the induced e.m.f. can still be calculated if the *instantaneous* rate of change of flux $d\phi/dt$ is used. This condition occurs, for example, in transformers in which the flux usually varies sinusoidally with time. When the flux is going through zero its rate of change is a maximum, and when the flux is a maximum its rate of change is zero (see Vol. II, Fig. 195, p. 227). With changing flux,

$$e = -N\frac{d\phi}{dt} 10^{-8} \text{ volts.} \qquad (131)$$

In the foregoing example, the rate of change of flux $d\phi/dt$ is 1,500,000/0.2 or 7.5×10^6 maxwells per second.

The fact that the currents produced by induction oppose the effect producing them should be carefully noted, for this principle is manifest in practically all types of electric machinery. This principle was first formulated by Lenz, in a form known as *Lenz's law* which in effect says:

In all cases of electromagnetic induction, the induced voltages have a direction such that the currents which they produce oppose the effect which produces them.

This law is also based on the law of the conservation of energy. That is, the induced currents are produced at the expense of the mechanical energy required to push the magnet into the coil against their opposition, or the energy required to withdraw the magnet against the opposition of the induced currents, which try to prevent this withdrawal. When an e.m.f. is induced in a circuit by a change of current in that circuit, the e.m.f. is due to changes in the flux-current linkages of the circuit.

195. Flux-current Linkages and Self-inductance.—If a current flows in a conductor, a magnetic flux is set up about the conductor. This magnetic flux completely encircles the conductor, and the current in the conductor completely encircles the flux. Some familiar examples of this are given in Fig. 205, in which the currents and related fluxes are shown.

In Fig. 205(a), current in a single turn is shown. The flux which it produces is indicated by the rings. The rings encircle

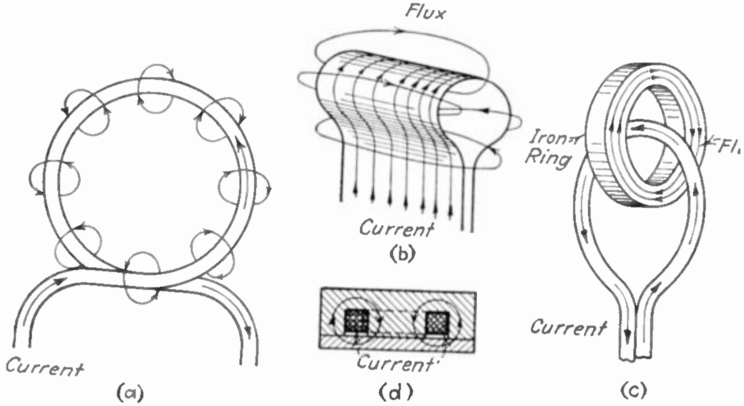


Fig. 205.—Illustrations of flux-current linkages.

the current, and the current encircles the rings. In (b), the current in a thin sheet forming a loop is shown. The flux lines encircle the current, and the current encircles the flux lines which it produces. A still more striking example is shown in (c), which shows the current in a loop of wire interlinking a circular magnetic core. Another excellent example of current-flux linkages is given in (d), which shows the flux in the magnetic circuit of a lifting magnet linking the current in the exciting coil.

These figures illustrate the fact that a current and the flux which it produces always completely encircle each other. Hence, they are said to *link* each other. This mutual linking action is particularly well illustrated in Fig. 205(c).

The product of the turns of conductor and the number of lines of flux linking these turns is called the *flux linkages* of the circuit. These linkages will be given in c.g.s. units if the flux is given in maxwells.

Example.—A certain solenoid has 800 turns. A current of 5 amp. flowing in the winding produces a flux of 2,500,000 lines. What are the c.g.s. linkages?

$$800 \times 2{,}500{,}000 = 20 \times 10^8 \text{ flux linkages (c.g.s. units).} \quad \textit{Ans.}$$

When the reluctance of the magnetic circuit is constant (see Par. 198, p. 295), the number of these *linkages per unit current* is called the *self-inductance* or just the *inductance* of the circuit and is represented by the symbol L, implying linkages. The unit of inductance is the henry.[1]

From definition, the inductance

$$L = \frac{N\phi}{I \times 10^8} \text{ henrys} \tag{132}$$

where ϕ is the flux in maxwells and I the current in amperes.

It is necessary to divide by 10^8 because 10^8 c.g.s. magnetic lines, or maxwells, are equal to *one* line in the practical system of volts, amperes, etc. (see p. 250).

Example.—In the foregoing circuit, determine: (*a*) the inductance; (*b*) the inductance if the number of turns is increased to 1,200, other factors remaining unchanged.

(*a*) $L_1 = \dfrac{20 \times 10^8}{5 \times 10^8} = 4.0$ henrys. *Ans.*

(*b*) With 1,200 turns and the same current of 5 amp. the flux will now be

$$\frac{1{,}200}{800}2{,}500{,}000 = 3{,}750{,}000 \text{ maxwells.}$$

The new flux linkages are

$$3{,}750{,}000 \times 1{,}200 = 4.5 \times 10^9.$$

[1] Henry, Joseph (1797–1878). An eminent American physicist, who was educated at Albany Academy and prepared himself for the medical profession. However, he became interested in science and in 1825 was elected to the chair of Mathematics and Natural Philosophy at the Albany Academy. With the crudest of apparatus and very little spare time, he made many improvements in electromagnetic apparatus and presented important papers on the subject of electromagnetism.

In 1832 he was elected to the chair of Natural Philosophy at Princeton University and in 1846 he became Secretary and Director of the Smithsonian Institution in Washington. His most important contribution to the science was the establishment of the relations which exist among magnetism, current, and induced e.m.fs. and currents.

The inductance

$$L_2 = \frac{4.5 \times 10^9}{5 \times 10^8} = 9.0 \text{ henrys.} \quad \textit{Ans.}$$

The definition of self-inductance given in the Smithsonian Physical Tables follows from the foregoing relations.

Self-inductance is for any circuit the e.m.f. produced in it by a unit rate of change of current (also see Eq. (137), p. 294).

From Eq. (132) it would appear that the inductance of a circuit is primarily a function of the flux. Also, it might be inferred that inductance is directly proportional to the turns. These two relations are not true, however, under the conditions of constant reluctance because the flux ϕ is proportional to both the turns and the current. That is, $\phi = 0.4\pi NI/\mathcal{R}$ where $\mathcal{R}$ is the reluctance of the magnetic circuit. Hence,

$$L = \frac{0.4\pi N^2}{\mathcal{R}10^8} \text{ henrys.} \tag{133}$$

Since $\mathcal{R} = l/\mu A$ (Par. 173, p. 251),

$$L = \frac{0.4\pi N^2 \mu A}{l10^8} \text{ henrys.} \tag{134}$$

Thus, inductance is primarily a property of the geometry of the circuit, the number of turns, and the permeability.

From Eqs. (133) and (134) it should be noted that under the conditions of constant reluctance, inductance varies as the *square* of the turns, even though the term N appears in Eq. (132) to the first power only. This is illustrated by the foregoing example.

$$L_2 = 4.0\left(\frac{1{,}200}{800}\right)^2 = 9.0 \text{ henrys.} \quad \textit{Ans. (check).}$$

196. Electromotive Force of Self-induction.—If a coil be connected to a battery and a switch S closed (Fig. 206), current will begin to flow in the coil. This current produces a flux linking the coil. As this flux increases it must induce an e.m.f. in the coil, the magnitude of which depends on the number of turns in the coil and the rate at which the flux increases. By Lenz's law, and also from a consideration of Fig. 204(a), the e.m.f. thus induced must have such a direction as to oppose the increase in the flux linking the coil and hence must *oppose* any increase of current. Thus, in Fig. 206, as viewed from the top, the direction of the current in the coil is clockwise, the direction of the flux is downward, and the direction of the induced e.m.f. is counter-clockwise or opposite to that of the current. The current cannot, therefore, reach its maximum value at once but is retarded by

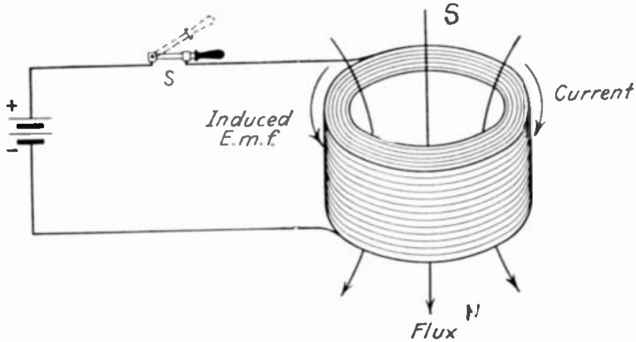


FIG. 206.—Relation of e.m.f. of self-induction to current.

an e.m.f. which the current itself produces. Such an e.m.f., which acts in the same circuit as the current which produces it, is an *e.m.f. of self-induction.*

In contrast, the rise of current in a circuit containing resistance only is shown in Fig. 207, the impressed voltage being 10 volts and the resistance 20 ohms. When the switch S is closed, the

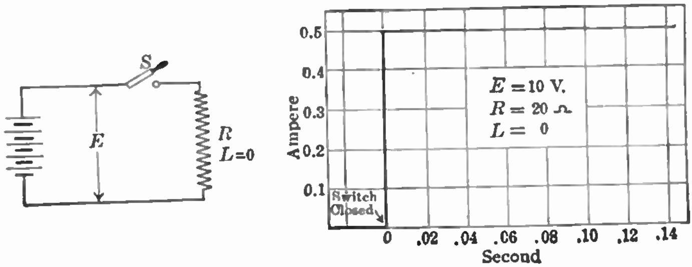


FIG. 207.—Rise of current in a non-inductive circuit.

current reaches its maximum or Ohm's-law value of 0.5 amp. at once.

With the inductive circuit, the current gradually approaches its Ohm's-law value as shown in Fig. 209(*b*), (p. 296). To be exact, it takes an infinite time for the current to reach its Ohm's-law value, although in a comparatively short time it reaches substantially this value. The relation of current to time in an inductive circuit is derived in Par. 199 (p. 296).

197. Calculation of the Electromotive Force of Self-induction.—From Eq. (130), p. 288, the e.m.f. induced in a coil due to a change in the flux linking the coil is

$$e = -N\frac{\phi}{t}10^{-8} \text{ volts} \tag{135}$$

where N is the number of turns and ϕ/t the rate at which the flux changes.

Remembering that

$$L = \frac{N\phi}{I}10^{-8} \quad \text{or} \quad N\phi 10^{-8} = LI \text{ (Eq. (132), p. 291)},$$

and also that the e.m.f. of self-induction opposes the change in current, its value may be written

$$e = -\frac{N\phi 10^{-8}}{t} = -L\frac{I}{t}. \tag{136}$$

If the rate of change of current with respect to time is not constant, the derivative di/dt of current with respect to time must be used. From Eq. (131), $e = -N\frac{d\phi}{dt}10^{-8}$ volts, and from Eq. (132), $N\phi 10^{-8} = Li$. Multiplying by -1 and differentiating

$$-N\frac{d\phi}{dt}10^{-8} = -L\frac{di}{dt} = e \text{ volts},$$

and

$$e = -L\frac{di}{dt} \text{ volts} \tag{137}$$

where L is in henrys.

Hence, the e.m.f. of self-induction is equal to the product of the inductance and the rate of change of current with respect to time. The minus sign indicates that this e.m.f. *opposes* the change of current.

If the inductance varies as well as the flux, Eq. (137) may be written

$$e = -\frac{d}{dt}(Li) = -\left(L\frac{di}{dt} + i\frac{dL}{dt}\right), \tag{138}$$

the additional term accounting for the induced e.m.f. due to change in the inductance.

Example.—The field circuit of a generator has an inductance of 6 henrys. If the field current of 12 amp. is interrupted in 0.05 sec., what is the average induced e.m.f. in the field winding?

$$e = -6\frac{12}{0.05} = -1{,}440 \text{ volts.} \quad \textit{Ans.}$$

198. Self-inductance with Varying Reluctance.—When the reluctance of the magnetic circuit is *not constant*, the inductance is equal to the product of the number of turns and the *rate of*

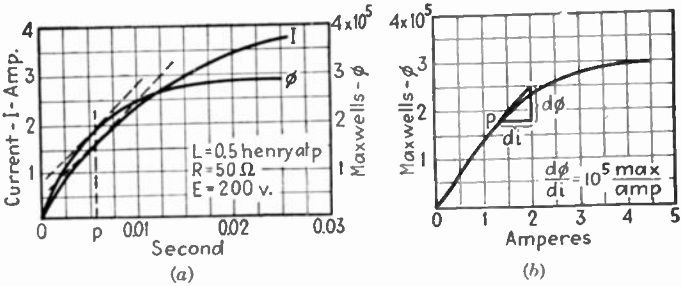


FIG. 208.—Inductance with variable flux-current relation.

change of flux divided by the *rate of change of current* and by 10^8. This relation is derived as follows: Let L now be a *variable* inductance. At any instant the induced e.m.f. is

$$-L\frac{di}{dt} = -N\frac{d\phi}{dt}10^{-8} \text{ volts.} \tag{139}$$

Solving for L gives

$$L = N\frac{d\phi}{di}10^{-8} \text{ henrys.} \tag{140}$$

The relation is illustrated in Fig. 208. An electric circuit of 500 turns is linked with a magnetic circuit. When the electric circuit is switched across 200 volts, the current increases from zero in the manner shown by curve I in Fig. 208(a). However, with increase of current the iron becomes saturated, as may be seen from the saturation curve of the magnetic circuit in (b). Therefore, as the current becomes greater in value in (a), the rate of increase of flux diminishes with respect to the rate of increase of current. At the instant p (Fig. 208(a)), corresponding to 0.0055 sec., the flux is increasing at a rate of 2×10^7 maxwells per second and at this same instant the current is increasing at a rate of 200 amp. per second. These rates of increase may be determined from the tangents drawn to the curves at this instant of time.

Example.—Determine the value of self-inductance at the instant p. (From Eq. (139)),

$$L = \frac{\left(N\frac{d\phi}{dt}\right)}{di/dt}10^{-8} \text{ henrys.}$$

Hence,

$$L = 500\frac{2 \times 10^7}{200 \times 10^8} = 0.5 \quad \text{henry.} \quad \textit{Ans.}$$

In (*b*), which shows the saturation curve for this circuit, the point p corresponds to point p in (*a*). At point p in (*b*) the rate of change of flux

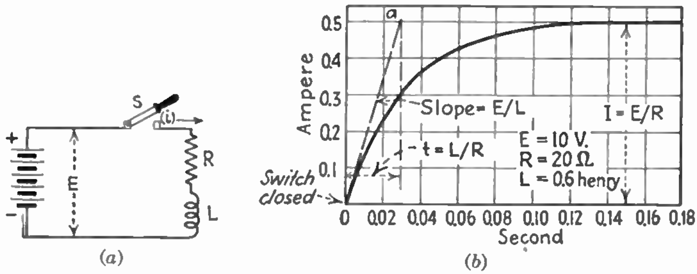


FIG. 209.—Rise of current in an inductive circuit.

with respect to current ($d\phi/di$) is 10^5 maxwells per ampere. Hence, using Eq. (140),

$$L = 500 \times 10^5 \times 10^{-8} = 0.5 \text{ henry,} \quad \textit{Ans. (check).}$$

which value may be determined by the tangent drawn at the point p in (*b*).

199. Rise of Current in Inductive Circuit.—In Fig. 209(*a*) is shown a resistance R in series with a self-inductance of L henrys. The inductance itself is assumed to be resistanceless, its actual small resistance being included in R. When the switch S is closed, the circuit is connected suddenly across a potential difference of E volts. Let the instant of closing S be taken as zero time. The current i in amperes may then be found as a function of the time t expressed in seconds.

The applied voltage E must not only supply the resistance drop in the circuit, but it must also *overcome* the e.m.f. of self-induction, $-L(di/dt)$. Thus,

$$E = Ri + L\frac{di}{dt} \tag{141}$$

where dt is the differential time. This is a linear differential equation of the first order and is readily integrated.

First, multiplying Eq. (141) by dt and transposing,

$$Edt = Ridt + Ldi,$$
$$(E - Ri)dt = Ldi,$$
$$\frac{di}{E - Ri} = \frac{1}{L}dt.$$

Integrating,

$$-\frac{1}{R}\log_\epsilon (E - Ri) = \frac{1}{L}t + K$$

where ϵ is the Napierian logarithmic base = 2.718, and K is a constant of integration.

$$\log_\epsilon (E - Ri) = -\frac{Rt}{L} - KR.$$

When the time $t = 0$, $i = 0$. This must be true, for, if the current rose to any finite value whatsoever in zero time, its rate of change would be infinite and the corresponding e.m.f. of self-induction accordingly would be infinite. Hence,

$$K = -\frac{1}{R}\log_\epsilon E,$$
$$\log_\epsilon (E - Ri) - \log_\epsilon E = \log_\epsilon \frac{(E - Ri)}{E} = -R\frac{t}{L}.$$

Taking the exponential,

$$\frac{E - iR}{E} = \epsilon^{\frac{-Rt}{L}},$$

from which

$$i = \frac{E}{R}\left(1 - \epsilon^{\frac{-Rt}{L}}\right). \tag{142}$$

This is an exponential equation, the graph of which is shown in Fig. 209(b) for a circuit in which E = 10 volts, R = 20 ohms, and L = 0.6 henry.

The rate of current increase is large at first and then diminishes until at $t = \infty$ it becomes zero. Theoretically, the current does not reach its Ohm's-law value until infinite time. Practically, it reaches this value in a relatively short time.

The rate of change of current at any time t,

$$\frac{di}{dt} = \frac{E}{R}\frac{d}{dt}\left(1 - \epsilon^{\frac{-Rt}{L}}\right) = \frac{E}{L}\epsilon^{\frac{-Rt}{L}} \text{ amp. per second.} \quad (143)$$

When $t = 0$,

$$\frac{di}{dt} = \frac{E}{L} \text{ amp. per second.} \quad (144)$$

That is, at the instant of closing the switch, the current begins to increase at a rate of E/L amp. per second.

A line drawn tangent to the curve at the origin o must have a slope E/L, as shown by line oa (Fig. 209(b)). Also, the rate of increase of current at the instant when the switch is closed is found directly from Eq. (141), for, when $t = 0$, $i = 0$ and $di/dt = E/L$.

Since the current approaches its Ohm's-law value gradually and theoretically never reaches this value, the time to reach or even to approach the Ohm's-law value cannot be a criterion of the rate of increase of the current. For this reason the time L/R has been chosen as the criterion. This ratio L/R of the inductance in henrys to the resistance in ohms is called the *time constant* of the circuit and is the time in seconds required for the current to reach 63.2 per cent of its final Ohm's-law value. Substituting this value L/R for t in Eq. (142) gives

$$i = \frac{E}{R}\left(1 - \epsilon^{\frac{-R}{L}\left(\frac{L}{R}\right)}\right) \text{ amp.,}$$

$$i = \frac{E}{R}\left(1 - \frac{1}{2.718}\right) = 0.632\frac{E}{R} \text{ amp.} \quad (145)$$

That is, in the time L/R sec. the current reaches 63.2 per cent of its final or Ohm's-law value. This relation is shown in Fig. 209(b), in which the time constant is $0.6/20 = 0.03$ sec.

It is to be noted that if the current continues at its initial rate of increase E/L, it will reach its Ohm's-law value in L/R sec. That is,

$$i = \frac{E}{L}t = \frac{E}{L}\cdot\frac{L}{R} = \frac{E}{R} \text{ amp.}$$

This is shown by the tangent *oa* (Fig. 209(*b*)), which intersects the 0.03-sec. ordinate at 0.5 amp.

Example.—The resistance of a relay is 400 ohms and the inductance is 0.4 henry. The relay is connected suddenly across 120 volts. Determine: (*a*) the equation of the current; (*b*) the rate at which the current begins to increase; (*c*) the value of current at 0.0005 sec.; (*d*) the time constant of the circuit; (*e*) the value of current corresponding to the time in (*d*).

(*a*) Substituting in Eq. (142),

$$i = \frac{120}{400}\left(1 - \epsilon^{\frac{-400t}{0.4}}\right) = 0.3(1 - \epsilon^{-1,000t}). \quad \textit{Ans.}$$

(*b*) When $t = 0$,

$$\frac{di}{dt} = \frac{E}{L} = \frac{120}{0.4} = 300 \text{ amp. per second.} \quad \textit{Ans.}$$

(*c*) $i = 0.3(1 - \epsilon^{-1,000 \times 0.0005})$

$$= 0.3(1 - \epsilon^{-0.5}) = 0.3\left(1 - \frac{1}{\sqrt{\epsilon}}\right) = 0.1179 \text{ amp.} \quad \textit{Ans.}$$

(*d*) The time constant $T = \dfrac{L}{R} = \dfrac{0.4}{400} = 0.001$ sec. *Ans.*

(*e*) $i = 0.3\left(1 - \dfrac{1}{\epsilon}\right) = 0.3(1 - 0.368) = 0.1896$ amp. *Ans.*

This delayed rise of current in a circuit, due to self-inductance, should be carefully kept in mind, since it accounts for some of the time lag observed in relays, trip coils, etc. When a short circuit takes place, there may be considerable delay between the time at which the short circuit occurs and the time of opening of the breaker or switch controlled by the relay. The effect of inductance is also one of the controlling factors in the initial current rush on short circuit.

200. Decay of Current in Inductive Circuit.—If an inductive circuit carrying current be short-circuited, the current does not cease immediately, as it does in a non-inductive circuit under the same conditions, but continues to flow and does not become zero until an appreciable time after the instant of short circuit. Theoretically, the current becomes zero only after infinite time. This flow of current, after the applied voltage has been discontinued, is due to the e.m.f. of self-induction. The flux linking the coil is due to the current, and when the current decreases, this flux also decreases. The decreasing flux induces an e.m.f. in the coil. In the same way that the current due to the induced e.m.f. tends to prevent the flux being withdrawn in Fig. 204(*b*), p. 288, so the e.m.f. of self-induction tends to prevent the decrease of the current and the flux. This e.m.f. diminishes as the rate of decrease of current and flux diminishes.

A circuit in which inductance and resistance may be short-circuited safely is shown in Fig. 210(*a*). The switch S is arranged to

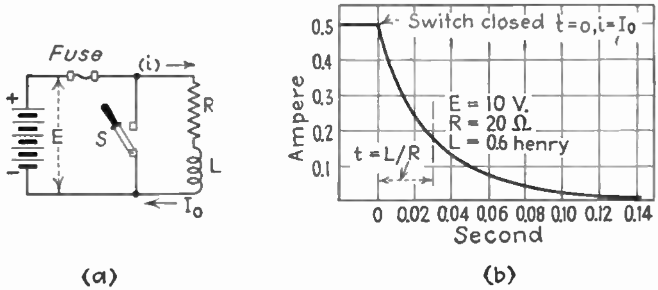


FIG. 210.—Decay of current in inductive circuit.

short-circuit the resistance R and the inductance L in series. At the instant of short circuit, a current I_0 amp. is flowing in R and L. A fuse is inserted in series with the battery to protect it from the short circuit. It is possible to solve for the current in the resistance and inductance as a function of time by making $E = 0$ in Eq. (141), p. 296. However, the circuit is fundamentally a simple one in which an e.m.f., $-L\,di/dt$, is impressed across a resistance R. Hence, by Ohm's law,

i'R/L.

$$iR = -L\frac{di}{dt}, \tag{146}$$

the minus sign showing that the e.m.f. of self-induction is in opposition to the effect producing it (Lenz's law). This equation is readily integrated:

$$\frac{di}{i} = -\frac{R}{L}\,dt,$$

$$\log_\epsilon i = -\frac{Rt}{L} + K$$

where K is a constant of integration.

When $t = 0$, $i = I_0$. Hence,

$$\log_\epsilon I_0 = K,$$

$$\log_\epsilon \frac{i}{I_0} = -\frac{Rt}{L}.$$

Taking the exponential and solving for i,

$$i = I_0 \epsilon^{-\frac{Rt}{L}}. \tag{147}$$

Equation (147) is a decaying exponential function. Theoretically, the current becomes zero only after infinite time. It becomes practically zero in a relatively short time. Equation (147) is represented graphically in Fig. 210(b) for a circuit in which, prior to short circuit, $E = 10$ volts. R is 20 ohms and L is 0.6 henry, the same constants as for the circuit of Fig. 209(b).

The time constant $T = L/R$ is now a criterion of the rate of decay of the current. For example, when $t = T = L/R$,

$$i = \frac{I_0}{\epsilon} = 0.368 I_0. \tag{148}$$

Example.—In the circuit, the constants of which are given in Fig. 210(b), the current is $I_0 = 0.5$ amp. when the switch S is closed. Determine: (a) the current when $t = 0.01$ sec.; (b) the time constant of the circuit; (c) the current corresponding to the time in (b).

(a) Using Eq. (148),

$$i = 0.5\epsilon^{-\frac{20\times 0.01}{0.6}}$$
$$= \frac{0.5}{2.718^{1/3}} = \frac{0.5}{1.396} = 0.358 \text{ amp.} \quad \textit{Ans.}$$

(b) $T = \frac{L}{R} = \frac{0.6}{20} = 0.03$ sec. *Ans.*

(c) $i = I_0\epsilon^{-1} = I_0/\epsilon = 0.368 I_0 = 0.184$ amp. *Ans*

201. Energy of the Magnetic Field.—To *establish* a magnetic field, energy must be expended. To maintain a constant field does not require an expenditure of energy so far as the *field* is concerned. The energy supplied to the exciting coils of electromagnets is accounted for as heat in the copper and is not concerned with the energy of the magnetic field itself. The energy of the magnetic field is stored as potential energy and is similar to the energy of a raised weight (Fig. 211). Work is performed in raising the weight to its position, but *no expenditure of energy is required to maintain the weight in this position.* The energy of the weight due to its position is Wh ft.-lb., where W is the weight in pounds and h is the height in feet through which the weight has been raised above the floor. This energy is available and can be utilized in many ways.

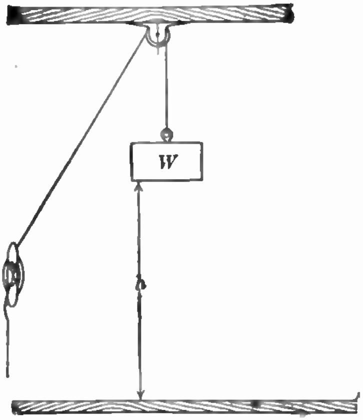


Fig. 211.—Energy of a suspended weight.

In the same way the energy stored in the magnetic field is available and may make itself manifest in many ways, as, for example, in the arc at the switch contacts. In an alternating-current circuit all this energy may be returned to the circuit.

The energy of the field in joules or watt-seconds is

$$W = \tfrac{1}{2}LI^2 \qquad (149)^*$$

where L is the circuit inductance in henrys and I is the circuit current.

Equation (149) is readily derived from the self-inductance relation. Consider the current i flowing in the inductive circuit (Fig. 209(a)). Of the energy input, $\int_0^t Eidt$ joules, up to time t, some is dissipated as heat in the resistance $\left(= \int_0^t i^2Rdt\right)$, and the remainder is stored as energy in the magnetic field. This energy may be determined as follows. When a current flows against an e.m.f., either mechanical power is developed (motor, p. 489) or energy is being stored (charging a battery, Par. 48, p. 60). In the inductive circuit the current flows against the counter e.m.f. $e = -L\ (di/dt)$, and energy must be stored.

* The resemblance of this expression to that for stored energy in a moving mass should be noted. For example, when a mass M moves with a velocity V, the kinetic energy is $\tfrac{1}{2}MV^2$. With a rotating body having a moment of inertia I and angular velocity ω, the kinetic energy is $\tfrac{1}{2}I\omega^2$.

The instantaneous power is $ie = i(L di/dt)$ and the energy

$$W = \int_0^T ie\,dt = L\int_0^I i\,di = \tfrac{1}{2}LI^2. \tag{149}$$

(When $t = T$, $i = I$.)

Equation (149) shows that the energy of the magnetic field is proportional to the *square* of the current. Therefore, if the current can be reduced by a suitable resistance to one-half its initial value before opening a highly inductive circuit, the energy of the arc at the switch contacts can be reduced to one-fourth the value which it would have had without the additional resistance. This fact should be remembered when opening the field circuit of a dynamo.

Example.—In a circuit having an inductance of 4 henrys, the current is 10 amp. What is the energy of the magnetic field? If this circuit is interrupted in 0.2 sec., what is the average value of the power expended by the magnetic field during this time?

$$W = \tfrac{1}{2} \times 4 \times 10^2 = 200 \text{ watt-sec. or joules.} \quad \textit{Ans.}$$

$$P = \frac{200}{0.2} = 1{,}000 \text{ watts} = 1 \text{ kw.} \quad \textit{Ans.}$$

202. Magnetic and Heat Energy.—If, after having established the current in the circuit of Fig. 209(*a*), the switch S be opened, a noticeable arc will appear at the switch contacts. This arc will be much greater in magnitude than that formed at the switch contacts in the circuit of Fig. 207, which has only resistance in the circuit, although the current and circuit voltage are the same in both cases. This arcing at the switch contacts may be attributed to two effects of self-inductance.

When the switch is being opened, the current is caused to decrease rapidly. This decrease of current causes an e.m.f. of self-induction, $-L\,di/dt$, (Par. 196, p. 294), which attempts to prevent the decrease of current. The induced e.m.f. becomes sufficiently high to start and to maintain an arc of substantial length across the switch contacts. The arc burns the switch contacts and usually causes their rapid deterioration.

Another and probably a better method of considering the phenomenon is from the viewpoint of energy. The energy stored in the magnetic field is $\tfrac{1}{2}LI^2$ joules. When the current becomes zero, the energy stored in the field must become zero and by the law of the conservation of energy it must appear elsewhere. In this case it is converted into the heat energy of the arc at the switch contacts.

The induced e.m.f. and the arc resulting from the opening of an inductive circuit may become dangerous, both from the point of view of personal injury and of damage to apparatus, as, for example, with the fields of alternators which are usually separately excited and it is necessary to open the circuit when shutting down the alternator. The resulting e.m.f. of self-induction has been

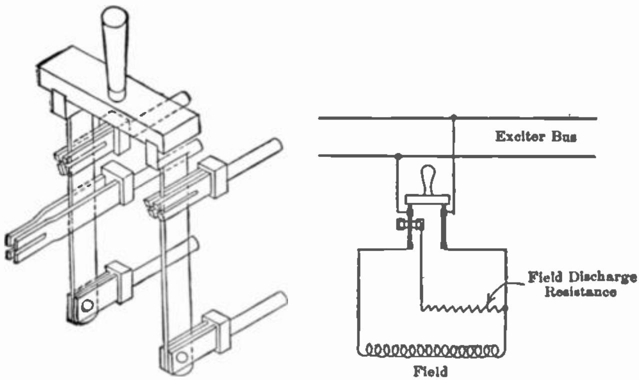


FIG. 212.—Field-discharge switch with connections.

known to reach such values as to puncture the insulation when the field circuit is opened. To protect the field from puncture, a field-discharge switch (Fig. 212) is often used. At the instant of opening the switch, the field (and the line, temporarily) is paralleled by the field-discharge resistance. The energy of the field is dissipated partly in this resistance rather than in the arc at the switch contacts.

The fields of shunt generators and motors are usually connected across the armatures, so that when the machine is shut down the field discharges gradually through the armature and a field-discharge switch is unnecessary.

Contact with switches opening inductive circuits, even in the case of very low voltages, should be carefully avoided. Not only is there danger of being burned by the arc, but also of being injured from the high induced e.m.fs.

In measuring the resistance of an inductive circuit by the voltmeter-ammeter method, the voltmeter, shown dotted (Fig. 213) should never be left connected across the *load side* of the switch. When the switch is opened, the high e.m.f. of self-induction is almost certain to injure the voltmeter. It may be left connected to the *line side* of the switch since this voltage will be

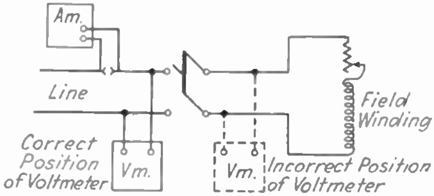


FIG. 213.—Position of voltmeter with inductive circuit.

maintained at the nominal value by the generating apparatus of the system.

203. Utilization of Magnetic-field Energy.—A common example of the utilization of energy stored in the magnetic field occurs in the ignition systems of automobiles. A diagram of a typical system is shown in Fig. 214. The primary of an ignition coil is connected across the battery in series with the interrupter contacts. The ignition coil consists of a primary winding of relatively few turns and a secondary winding with a comparatively

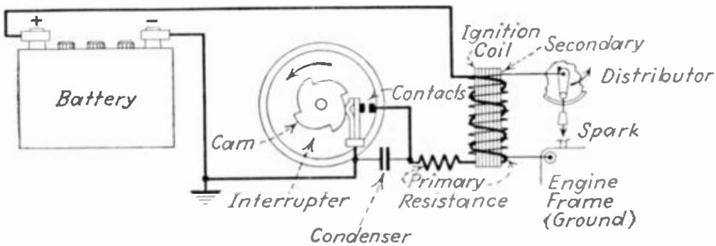


FIG. 214.—Automobile ignition system.

large number of turns, both wound over a laminated iron core. The contacts in the interrupter are closed and opened by a rotating cam, the number of lobes of the cam corresponding to the number of cylinders. The contacts are closed for a time which is sufficient to permit the flux in the core of the ignition coil to build up to a substantial value. The contacts then open suddenly. This induces a high e.m.f. in the secondary ($e_2 = -N_2\, d\phi/dt$), which causes the ignition spark to jump across the points of the spark plug. The energy which is stored relatively slowly in the magnetic field is thus suddenly released and appears as heat energy between the points of the spark plug. The condenser shunted across the contacts serves to suppress the arc when the contacts open. Otherwise some of the energy stored in the magnetic field of the core would burn the contacts. This energy is now stored in the condenser, from which it is released when the contacts close again. The resistance in series with the primary has a high temperature coefficient. Hence, when the ignition switch is inadvertently left closed and the contacts happen also to be closed, the primary will not overheat.

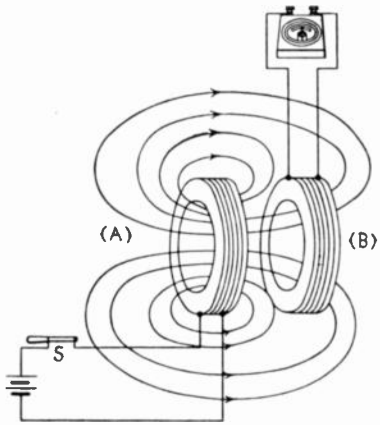


Fig. 215.—Mutual inductance between two coils.

This principle of storing energy slowly in a magnetic field and then releasing it suddenly is applied to "make-and-break" ignition used with internal-combustion engines. Two contacts within the cylinder head are in series with a low-voltage source, usually six dry cells, and an "ignition coil." The ignition coil consists of a laminated iron core wound with several turns of wire. It is similar to the core and primary of the ignition coil (Fig. 214). The contacts are made to open and close by means of a cam mechanism. When the contacts are closed, a magnetic field is built up in the ignition coil, thus storing magnetic energy. When the contacts are made to open, this energy appears as a hot spark at the contacts, and this spark ignites the explosive mixture in the cylinder.

204. Mutual Inductance.—In Fig. 215 are shown two coils A and B. Coil A is connected to a battery through a switch S. Coil B is connected to a galvanometer but not to a source of voltage. Coil B is placed so that its axis is nearly coincident with that of A and the two coils are close together. When the switch S is closed, current flows in coil A, building up a field which links A. The position of B with respect to A results in a considerable part of the magnetic flux produced by A linking B also. Therefore, if the current in A be interrupted by opening the switch S, or if it be altered in magnitude, a *change of flux simultaneously* occurs in B inducing an e.m.f. in B. This e.m.f. is detected by the galvanometer connected across the terminals of B. Upon closing the switch S, the galvanometer will deflect momentarily and then come to zero, showing that a transitory e.m.f. has been induced in coil B. On opening the switch S the galvanometer deflection will reverse, showing that the induced e.m.f. on opening the circuit is opposite in direction to the e.m.f. induced on closing the circuit. Because coil B is in such a relation to A that an e.m.f. is induced in B due to the change of flux in A, these two coils are said to possess *mutual inductance*. The induced e.m.f. is an e.m.f. of *mutual* induction, and its average value, Eq. (135), p. 294, is

$$e_2 = N_2\frac{\phi_2}{t}10^{-8} \text{ volts}$$

where N_2 is the number of turns in coil B, ϕ_2 the change in magnetic flux from coil A which links coil B, and t the time in seconds required to change the flux by ϕ_2 lines.

Even though coils A and B be brought close together, all the flux ϕ_1, produced by coil A, does not link coil B. Only a certain proportion, K, of ϕ_1 links B, K being less than unity. That is,

$$e_2 = N_2\frac{K\phi_1}{t}10^{-8} \text{ volts.} \tag{150}$$

K is called the *coefficient of coupling* of the circuits A and B. As N_2 and K are constants for any given geometry of the circuits, and ϕ_1 may be assumed proportional to the current I_1, in coil A, Eq. (150) may be written

$$e_2 = M\frac{I_1}{t} \text{ volts*} \tag{151}$$

* If instantaneous values are considered, the induced e.m.f. becomes $e_2 = M\dfrac{di_1}{dt}$.

where M is the *mutual inductance*, or coefficient of mutual induction, in henrys between coil A and coil B.

$$M = \frac{KN_2\phi_1}{I_1}10^{-8} \tag{152}$$

Thus two circuits have mutual inductance when a change of current in one circuit causes an e.m.f. to be induced in the other.

In the Smithsonian Physical Tables, mutual inductance is defined quantitatively as follows:

Mutual inductance of two circuits is the e.m.f. produced in one per unit rate of variation of the current in the other.

Example.—Coil A (Fig. 215) has 400 turns and coil B has 600 turns. When 5 amp. flow in coil A, a flux of 500,000 lines links with A, and 200,000 of these lines link coil B also. What is the self-inductance of coil A with B open-circuited, and what is the mutual inductance of the two coils?

$$L_1 = \frac{N_1\phi_1}{I} = \frac{400 \times 500{,}000}{5}10^{-8} = 0.4 \text{ henry.} \quad \textit{Ans.}$$

The induced e.m.f. in B due to the current in A rising to 5 amp. in 1 sec. will be

$$e_2 = N_2\frac{\phi_2}{t} = 600 \times 200{,}000 \times 10^{-8} = 1.2 \text{ volts,}$$

as a change of 5 amp. in coil A changes the flux in coil B by 200,000 lines. Therefore,

$$e_2 = M\frac{I_1}{t}.$$

$$1.2 = M \times \frac{5}{1}.$$

$$M = 0.24 \text{ henry.} \quad \textit{Ans.}$$

Or, using Eq. (152),

$$M = \frac{0.4 \times 600 \times 500{,}000}{5}10^{-8} = 0.24 \text{ henry.} \quad \textit{Ans.}$$

It can be shown that if M is the mutual inductance of coil B with respect to A, then M is also the mutual inductance of coil A with respect to B. That is, if the rate of change of current in coil B is I_2/t amp. per second, an e.m.f. is induced in A,

$$e_1 = M\frac{I_2}{t} \text{ volts.} \tag{153}$$

The mutual inductance of two circuits may be substantially increased by linking the circuits with an iron core. Thus, if two coils, similar to those shown in Fig. 215, be placed upon an iron core (Fig. 216), the coefficient of coupling K may be made

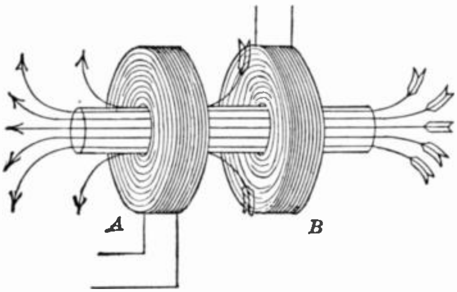


FIG. 216.—Effect of iron core on mutual inductance.

very nearly unity. That is, practically all the flux linking coil A also links coil B.

205. Mutual and Self-inductance.—The mutual inductance of two circuits having inductances L_1 and L_2 is

$$M = K\sqrt{L_1L_2} \tag{154}$$

where K is the coefficient of coupling. This may be shown as follows.

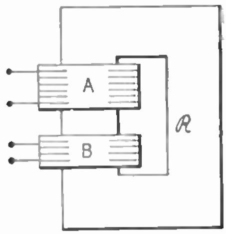


FIG. 217.—Self- and mutual-inductance coils on iron core.

In Fig. 217 are shown two coils A and B adjacent to each other on the same magnetic circuit. There are N_1 turns in coil A and N_2 turns in coil B. The reluctance of the magnetic circuit is $\mathcal{R}$ c.g.s. units and is assumed to be constant. Let a current i_1 flow in A alone.

The flux in A,

$$\phi_1 = \frac{0.4\pi N_1 i_1}{\mathcal{R}} \text{ maxwells.} \tag{I}$$

The flux in B,

$$\phi_2 = K\phi_1 = K\frac{0.4\pi N_1 i_1}{\mathcal{R}} \text{ maxwells.} \tag{II}$$

The e.m.f. induced in B, due to a rate of change of current di_1/dt in A,

$$e_2 = KN_2\frac{d\phi_1}{dt}10^{-8} = K\frac{0.4\pi N_1}{\mathcal{R}}N_2 10^{-8}\frac{di_1}{dt} \text{ volts.} \tag{III}$$

Hence, from Eq. (151),

$$M = K\frac{0.4\pi N_1 N_2}{\mathcal{R}}10^{-8} \text{ henrys.} \tag{155}$$

From Eqs. (132), and (133), pp. 291 and 292,

$$L_1 = \frac{N_1\phi_1}{i_1}10^{-8} = \frac{0.4\pi N_1^2 i_1}{\mathcal{R}i_1}10^{-8} = \frac{0.4\pi N_1^2}{\mathcal{R}}10^{-8} \text{ henrys.} \tag{IV}$$

Likewise, if a current i_2 flows in coil B alone,

$$L_2 = \frac{N_2\phi_2}{i_2}10^{-8} = \frac{0.4\pi N_2^2 i_2}{\mathcal{R}i_2}10^{-8} = \frac{0.4\pi N_2^2}{\mathcal{R}}10^{-8} \text{ henrys.} \tag{V}$$

Multiplying Eqs. (IV) and (V) together and taking the square root gives

$$\sqrt{L_1L_2} = \frac{0.4\pi N_1 N_2}{\mathcal{R}}10^{-8} \text{ henrys.} \tag{VI}$$

From Eq. (155) it follows that

$$M = K\sqrt{L_1L_2} \text{ henrys.} \tag{156}$$

If the coils A and B are connected in series aiding, the total inductance is

$$L = L_1 + L_2 + 2M \tag{157}$$

and, if connected opposing,

$$L = L_1 + L_2 - 2M. \tag{158}$$

Equation (157) may be readily proved. Let the current in the two coils in series be i. The total e.m.f. induced in the circuit, due to a change of current with respect to time di/dt, is the sum of the e.m.fs. due to the self-inductance of the individual coils and the e.m.f. induced in each coil due to its mutual inductance with the other. That is,

$$e = \left(L_1\frac{di}{dt} + L_2\frac{di}{dt}\right) + \left(M\frac{di}{dt} + M\frac{di}{dt}\right)$$
$$= (L_1 + L_2 + 2M)\frac{di}{dt} = L\frac{di}{dt}.$$

Equation (158) may be proved in a similar manner.

Example.—Two coils A and B having 800 and 1,200 turns are linked with a magnetic circuit the reluctance of which is 0.0015 c.g.s. unit. The coefficient of coupling is 0.8, and a current of 0.5 amp. flows in the two coils in series. Determine: (*a*) the self-inductance of each coil; (*b*) the mutual inductance of the two coils; (*c*) the total inductance with the two coils in series aiding; (*d*) the total inductance with the two coils in series opposing.

$$(a)\quad \phi_1 = \frac{0.4\pi 800 \times 0.5}{0.0015} = 335{,}000 \text{ maxwells.}$$

$$L_1 = \frac{N_1\phi_1}{i_1}10^{-8} = \frac{800 \times 335{,}000}{0.5}10^{-8} = 5.36 \text{ henrys.} \quad \textit{Ans.}$$

$$\phi_2 = \frac{0.4\pi 1{,}200 \times 0.5}{0.0015} = 502{,}000 \text{ maxwells.}$$

$$L_2 = \frac{N_2\phi_2 10^{-8}}{i_2} = \frac{1{,}200 \times 502{,}000}{0.5}10^{-8} = 12.05 \text{ henrys.} \quad \textit{Ans.}$$

(*b*) Using Eq. (152), p. 308,

$$M = 0.8\frac{1{,}200 \times 335{,}000}{0.5}10^{-8} = 6.44 \text{ henrys.} \quad \textit{Ans.}$$

Also, using Eq. (154),

$$M = 0.8\sqrt{5.36 \times 12.05} = 6.44 \text{ henrys } (\textit{check}).$$

(*c*) Using Eq. (157),

$$L = 5.36 + 12.05 + (2 \times 6.44) = 30.29 \text{ henrys.} \quad \textit{Ans.}$$

(*d*) Using Eq. (158),

$$L = 5.36 + 12.05 - (2 \times 6.44) = 4.53 \text{ henrys.} \quad \textit{Ans.}$$

206. Stored Energy.—If a current i_1 flows in coil A and a current i_2 flows in coil B, the total stored energy[1] is

$$W = \tfrac{1}{2}L_1i_1^2 + \tfrac{1}{2}L_2i_2^2 \pm Mi_1i_2 \text{ joules.} \tag{159}$$

207. Measurement of Self- and Mutual Inductance.—Self-inductance is measured most readily with an impedance bridge (see Vol. II, Par. 74, p. 108). Mutual inductance also is measured most conveniently on such a bridge. There are several bridge methods of measuring mutual inductance.[2]

The simplest methods, however, are as follows: Let inductances L_1 and L_2 (Fig. 218) have mutual inductance M. Using an impedance bridge, measure L_1 alone, L_2 alone, and then

[1] See LAWRENCE, R. R., "Principles of Alternating Currents," 2d ed., p. 187, McGraw-Hill Book Company, Inc.

[2] See LAWS, F. A., "Electrical Measurements," McGraw-Hill Book Company, Inc.

measure the inductance L with the two connected in series. From Eqs. (157) and (158),

$$L = L_1 + L_2 \pm 2M \text{ henrys.} \tag{I}$$

If the inductances are aiding magnetically,

$$M = \frac{L - (L_1 + L_2)}{2} \text{ henrys.} \tag{160}$$

If opposing,

$$M = \frac{L_1 + L_2 - L}{2} \text{ henrys.} \tag{161}$$

Also, the total inductance L' may be measured with the inductances aiding and then the inductance L'' measured with them opposing. Then,

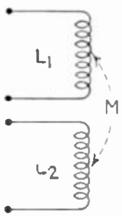


FIG. 218.—Measurement of mutual inductance.

$$L' = L_1 + L_2 + 2M \text{ henrys.} \tag{II}$$
$$L'' = L_1 + L_2 - 2M \text{ henrys.} \tag{III}$$

Subtracting Eq. (III) from Eq. (II) gives

$$M = \frac{L' - L''}{4}. \tag{162}$$

208. Inductance and Inertia.—*Inductance* in the electric circuit is analogous to *inertia* in mechanics. Inertia opposes change of velocity in a mass. When a mass is at rest, inertia opposes the imparting of velocity to it. When a mass is in motion, or has velocity, inertia opposes the bringing of the mass to rest. A flywheel is an excellent example of mechanical inertia. It resists acceleration and its inertia opposes change of speed. The flywheel thus tends to stabilize the angular velocity of the device with which it is connected, such as a reciprocating steam engine or an internal-combustion engine. Moreover, the energy stored in a moving body is $\frac{1}{2}Mv^2$, where M is the mass and v the velocity. With a rotating mass the stored energy is $\frac{1}{2}\omega^2 I$ where ω is the angular velocity in radians per second and I the moment of inertia (see p. 598). Inertia opposes any attempt to change the stored energy.

Likewise, inductance in the electric circuit has no effect so long as the current is steady. It does, however, oppose any *change* in the current. It opposes the increase of current in a circuit. Likewise, it opposes the reduction of current in a circuit. Hence, inductance frequently can be introduced in order to stabilize electric circuits and electrical devices, such as rectifiers (see Vol. II, Chap. XV). The energy stored in the magnetic field is $\frac{1}{2}Li^2$ joules. Inductance opposes any attempt to change this energy.

209. Induction Coil.—A very common example of mutual inductance occurs in the induction coil (Fig. 219). A primary winding P, of comparatively few turns of coarse wire, is wound

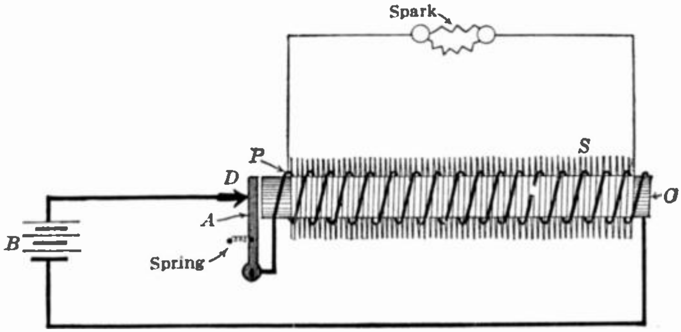


FIG. 219.—Induction coil.

on a laminated iron core C. This winding is connected to a battery B. The primary current is interrupted by the contact D, against which the iron armature A is held by a spring. When the core C is magnetized by the primary current, the armature A is drawn toward the core and away from D, opening the circuit and causing the flux in the core to drop to zero practically. The spring then pulls the armature A against the contact D, and the cycle is repeated. By this process the flux in the core C is continually being established and then destroyed.

On the same core is placed a secondary winding S, consisting of many turns of fine wire. This winding is thoroughly insulated from the primary winding, but, as it is wound on the same core as P, the two coils have a high value of mutual inductance. Because of the change of flux in the core, due to the interruptions of the primary current, a high alternating e.m.f. is induced in the secondary. This induced e.m.f. may be considered as due to the mutual inductance existing between the primary and secondary coils. The induction coil has many practical applications. A very common use at present is in obtaining the 90 volts or so for the *B* battery in automobile radio sets. By means of an induction coil, the 6 volts supplied by the storage battery is converted into an alternating-current voltage across the secondary. This voltage is rectified by a suitable electronic tube to give the necessary direct current. A filter, too, may be used to eliminate objectionable frequencies.

CHAPTER X

ELECTROSTATICS: CAPACITANCE

Thus far, only the electric current or electricity in motion has been considered. However, electricity may also be static, or at rest.

Electricity, though at rest, has important effects on the media surrounding it. For example, the design and the applications of insulation, particularly for high voltages, depend on a knowledge of the laws governing static electric charges. Also, the design and operation of transmission lines, the causes of lightning and protection against it involve these same laws. It is the purpose of this chapter to consider some of the laws which govern the behavior of electricity at rest, and to determine its effects on the operation of electrical systems.

210. Dynamic and Static Electricity.[1]—In Chap. I, Par. 1 (pp. 2 and 3), dynamic electricity, or electricity in motion, is discussed with relation to the movement of electrons from atom to atom of the conducting medium. With static electricity, or electricity at rest, the electrons behave as if they were at rest, although there may have been an initial movement of electrons with respect to the atoms. Consider first the conditions in the atom itself. When the total charge (negative) of the electrons in an atom is just equal to the net positive charge of the nucleus or proton, the atom is said to be in the *uncharged* or *neutral* state. If, however, one or more electrons be removed, the atom is said to be *positively charged.* If the removed electrons associate themselves with other neutral matter, this matter is said to be *negatively charged.*

In the dynamic circuit, the applied potential difference causes the electrons in any given group of atoms to pass on to the next group of atoms thus giving an electric current. Their places are immediately taken, however, by electrons coming from the preceding adjacent group of atoms which are being acted on by a

[1] See Vol. II, 3d Ed., Chap. XIII.

similar potential difference. Hence, the same number of electrons is always associated with any single atom. If, however, the conducting circuit is not closed, electrons are withdrawn from the positive terminus of the circuit and transferred to the negative terminus. Since the conducting circuit is open, there is no opportunity for the electrons displaced at the positive terminus to be replaced by electrons from adjacent atoms.

If this transfer of electrons is accomplished by a steady source of potential, the positive and negative charges appear to be at rest. For example, if the insulated parallel conducting plates A and B (Fig. 220) are connected to the positive and negative

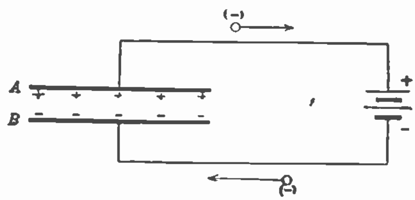


FIG. 220.—Transfer of electrons in condenser plates.

terminals of a battery or of an influence machine, the action of the applied potential will withdraw some of the free electrons from A and transfer them to B. Hence, plate A has become positively charged, since negative charges have been withdrawn from it, and plate B has become negatively charged. Under these conditions, the charge on the plates is called *static electricity*.

From the preceding brief discussion, it appears that dynamic and static electricity are identical in their ultimate nature. With dynamic electricity, there is a movement of electrons between adjacent atoms of the conductor. With static electricity, free electrons have been displaced from the positive to the negative plate and are maintained in this condition by the electric field. This displacement has the effect of producing a single negative charge on the negative plate and a single positive charge on the positive plate.

The displacement of charges at the positive and negative plates (Fig. 220) is frequently associated with high voltage, particularly if a source, such as an influence machine, is used. The high voltage and small quantity sometimes convey the impression that static and dynamic electricity are different in nature.

211. Electrostatic Charges.—If the terminals of an electrostatic-induction machine be connected to two insulated elongated conducting bodies (Fig. 221), the body connected to the positive terminal will be charged with positive electricity and that connected to the negative terminal will be charged with an equal amount of negative electricity. The charge will distribute itself over the entire surface of each body, but the density of the charges will be greatest on the adjacent ends of the two bodies. This is due to the fact that positive and negative charges attract each other.

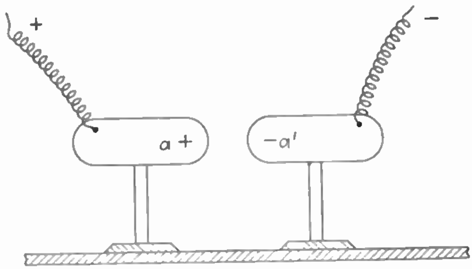


FIG. 221.—Electrostatic charges on insulated conducting bodies.

If the two wires from the electrostatic machine be disconnected, the two charges will not be sensibly affected at first. In time they will leak away through the insulating supports.

If the two bodies are free to move, they will come together. If they are connected by a wire, a spark will be observed at the instant that contact is made, showing that, for an instant, current flows from one body to the other. Both of these effects are due to the fact that the positive and negative charges attract each other.

212. Electrostatic Induction.—If a positively charged conducting body A (Fig. 222(a)) be brought near a perfectly insulated conducting body B, which initially has no charge, a negative charge b will be found on the end of B nearest A. As B did not have any initial charge, and is assumed to be perfectly insulated, no electricity can have gone from B and none can have reached it from external sources, so that the total charge on B must still be zero. Therefore, a positive charge b' must also appear on B at the end farthest from A, and this charge must be equal to b. As the two charges are equal and of opposite sign, the total charge on B is zero. It will be noted that the negative charge b is as near as possible to the positive inducing charge a, whereas the positive charge b' is as far as possible from the positive inducing charge a. This is due to the fact that unlike charges attract and like charges repel.

Charges a and b are *bound charges*, and charge b' is a *free charge.* This may be proved by connecting B to ground (Fig. 222(b)). The charge b' will escape to ground and will seek a position as far as possible from a, whereas the two charges a and b will remain.

If a were a negative charge, b would be positive and b' would be negative.

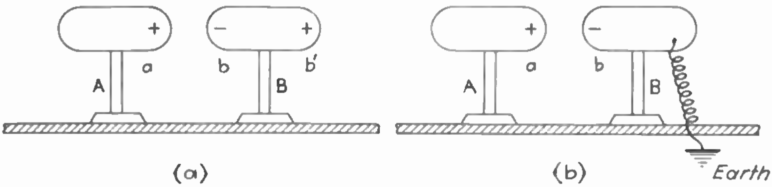


Fig. 222.—Electrostatic induction.

These experiments illustrate the following laws of electrostatics:

Charges of unlike sign attract each other and charges of like sign repel each other.

A positive charge will induce a negative charge on a body near it.

A negative charge will induce a positive charge on a body near it.

This is similar to magnetic induction, where a N-pole induces a S-pole, etc. (see Par. 149, p. 217).

213. Unit Charge and Coulomb's Law.—In the fundamental relations of electrostatics, the electrostatic system of units (see p. 26) is employed. Conversion of these units into those of the practical system is readily made (see Appendix B, p. 590).

A unit electrostatic charge (e.s.u.) is defined as that charge which when placed 1 *cm. in a vacuum from an equal charge, repels it with a force of* 1 *dyne.* The unit charge is called the *statcoulomb* as well as the e.s.u.

Coulomb's Law.[1]—By conducting experiments with charged spheres, Coulomb, between the years 1785 and 1789, proved the following law.

[1] See p. 28, footnote.

The force acting between two charged bodies in air is proportional to the product of the charges and inversely proportional to the square of the distance between them.

With q and q' (see Fig. 223) expressed in electrostatic units and r in centimeters,

$$f = \frac{qq'}{r^2} \text{ dynes.} \tag{163}$$

If the charges are in a medium whose dielectric constant is κ, Eq. (163) becomes

$$f = \frac{qq'}{\kappa r^2} \text{ dynes.} \tag{164}$$

It is assumed that the charges are *concentrated at points.*

FIG. 223.—Force between electrostatic charges.

Example.—Two small spheres in air, spaced 14 cm. between centers, are charged with 2 positive and 5 negative e.s.u. What is the force in dynes acting between the spheres?

From Eq. (163),

$$f = \frac{2 \times 5}{14^2} = \frac{10}{196} = 0.0510 \text{ dyne.} \quad \textit{Ans.}$$

214. Dielectric or Electrostatic Field.—It is shown in Chap. VI that the medium in the neighborhood of a magnet appears to be in a stressed condition, and that a force acts on a N-pole or a S-pole placed in such a magnetic field. Likewise, a condition of stress appears to exist in the medium in the neighborhood of an electric charge, and force acts on a positive or a negative charge placed in the medium. This condition of stress makes itself evident if the charges are sufficiently large, and the stress may become so great as to cause mechanical rupture of the medium, followed by an arc discharge (see Par. 217, p. 326).

The region in which the condition of stress exists is called the *dielectric or electrostatic field.* As with the magnetic field, the condition of stress may be represented by lines. A field in which there exists one line per square centimeter, taken normal to the direction of the field, exerts a force of 1 dyne on a unit charge.

It follows that 4π lines must leave each unit positive charge (see Par. 146, p. 213). Hence, $4\pi q$ lines must leave a positive charge of q units.

These lines of force in the dielectric field have properties similar to those of magnetic *lines of force* (*not* magnetic lines of induction).

1. *Every line originates on a positive charge and terminates on a negative charge.*

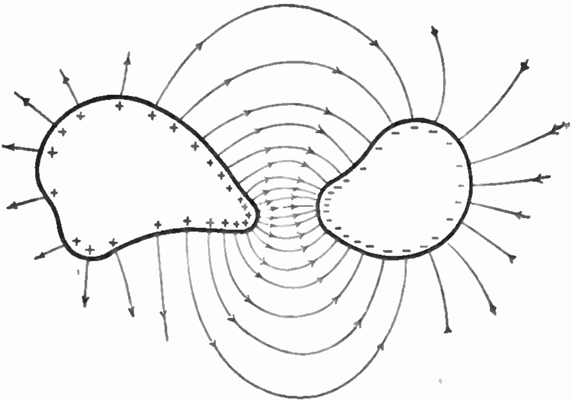


FIG. 224.—Electrostatic field between charged electrodes.

2. *A unit positive charge, if placed at the surface of the positive electrode, will be urged along the lines of force to the negative electrode, with a force in dynes at each point equal to the number of lines per square centimeter at that point, the area being taken normal to the direction of the lines.*

3. *The dielectric field tends to conform itself so that the number of lines is a maximum.*

The lines behave like stretched rubber bands and act as if they repelled one another.

A dielectric field between two irregular electrodes is illustrated in Fig. 224. It will be noted that each line originates at a positive charge and terminates at a negative charge. That is, the lines are not closed. Any positive charge is urged along a line of force from the positive to the negative electrode. The lines have the appearance of elastic bands tending to contract, and therefore tending to reduce the dielectric reluctance of the field to a minimum. They distribute themselves exactly as the flow or stream lines of an electric current do, or as the lines of force in the magnetic circuit distribute themselves.

A dielectric line of force must always be normal to a conducting surface where it leaves or enters that surface. If this were not true, there would be a component of electric force tangential to the conductor at its surface and a resulting potential difference which would cause a flow of current. Current cannot flow, since the *static* condition is assumed. Hence, there cannot be a component of force tangential to the surface of the conductor, and every line must therefore be normal to the surface at the point where it leaves or enters the conductor.

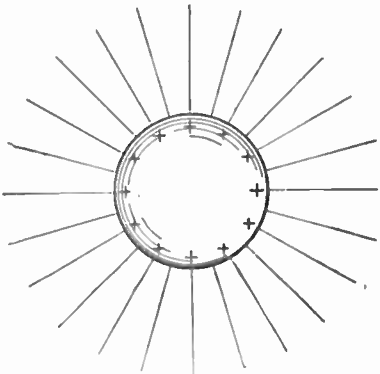


FIG. 225.—Isolated charged sphere.

It will also be noted (Fig. 224) that the dielectric-flux lines concentrate in those places where the radius of curvature is least. In these regions, therefore, the electric stress is greatest. In the design of insulation, sharp points and small radii of curvature are avoided so far as possible, so that the electric stress may not become too highly concentrated.

Dielectric flux is denoted by the symbol ψ.

215. Charged Spheres.—Figure 225 shows a positively charged[1] isolated sphere in air. The charge must lie wholly on the surface of the sphere. As the individual positive charges repel one another, they will separate as far as possible from one another, and therefore must all take positions on the surface of the sphere.

If the corresponding negative charge is sufficiently far removed, or if it exists on an outer concentric spherical shell, the positive charge must be uniformly distributed on the surface of the

[1] For every positive charge, there must be an equal negative charge. The corresponding negative charge may exist on the walls of the room, on the surface of the earth, or at some remote place.

sphere from symmetry. Again, from symmetry, the dielectric lines must all leave radially and uniformly from the surface of the sphere. If these lines were continued inward, they would meet at the center of the sphere. Hence, so far as points in space outside the sphere are concerned, the charge has the same effect as if it were concentrated at the center of the sphere.

If there are q unit charges on the sphere, then, from Par. 214, $4\pi q$ lines leave the sphere. If the sphere has a radius of r cm., the area of its surface is $4\pi r^2$ sq. cm. Therefore, the density of the lines of force at its surface must be $4\pi q/4\pi r^2$ or q/r^2 lines per square centimeter. Hence, the force in air at the surface of a

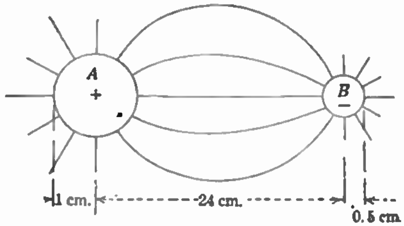


FIG. 226.—Charged spheres in air.

sphere of radius r, charged with q e.s.u., is q/r^2 which, from Eq. (163), p. 319, is the same as if the entire charge were concentrated at the center of the sphere.

Example.—Two spheres A and B (Fig. 226) with radii of 1 cm. and 0.5 cm. are spaced 24 cm. between centers in air. If the first sphere is charged positively and the second negatively with 20 e.s.u. determine: (*a*) the number of dielectric lines leaving each sphere; (*b*) the dielectric-flux density at the surface of each sphere, due to its own charge; (*c*) the maximum force at the surface of each sphere; (*d*) the intensity of the dielectric field at a distance of 10 cm. from the center of sphere A and on the line joining the centers of the two spheres; (*e*) the force of attraction between the spheres.

Assume the charges to be concentrated at the centers of the spheres.

(*a*) $\psi = 4\pi 20 = 251$ lines from or to each sphere. *Ans.*

(*b*) $D_A = \dfrac{4\pi 20}{4\pi(1)^2} = 20$ lines per square centimeter. *Ans.*

$D_B = \dfrac{4\pi 20}{4\pi(0.5)^2} = 80$ lines per square centimeter. *Ans.*

(*c*) If each sphere were isolated, the force at the surface of each would be given by (*b*), but if the spheres are not isolated, the charge on each sphere exerts a force at the surface of the other. This latter force will be a maximum at that point on either sphere which is nearest the other, since the force varies inversely as the square of the distance.

This force at the surface of sphere B due to sphere A, and on the line that joins the centers of the spheres, acts toward the center of B, since the positive charge on A repels a unit-positive charge at B. The force on this same positive-unit charge, due to the charge on B, acts toward the center of B since the negative charge on B attracts this charge. Hence both forces act in the same direction at this point. Likewise, the charges on both spheres A and B act in the same direction at the point nearest B on the surface of sphere A. Therefore, the force at the surface of each sphere is a maximum at the points where a line joining their centers intersects their surfaces. Hence,

$$f_A = 20 + \frac{20}{(23)^2} = 20 + 0.038 = 20.04 \text{ dynes. } \quad \textit{Ans.}$$

$$f_B = 80 + \frac{20}{(23.5)^2} = 80 + 0.036 = 80.04 \text{ dynes. } \quad \textit{Ans.}$$

(*d*) As the intensity of the field at any point is equal to the force exerted on a unit-positive charge placed at that point,

$$f = \frac{20}{(10)^2} + \frac{20}{(14)^2} = 0.20 + 0.102 = 0.302 \text{ dyne. } \quad \textit{Ans.}$$

(*e*) From Eq. (163),

$$f = \frac{20 \times 20}{(24)^2} = 0.695 \text{ dyne. } \quad \textit{Ans.}$$

It follows that, with an isolated sphere, the density of the dielectric lines of force varies *inversely as the square of the distance* from the center of the sphere.

When the distance between two spheres, such as those in Fig. 226, is small compared with their radii, the centers of the charges are no longer at the centers of the spheres. Due to proximity effect, that is the mutual attraction of the charges the centers of charge are displaced, each toward the other sphere.

216. Capacitance.—Two conductors separated by a dielectric constitute a condenser. When a potential difference is applied between the plates of a condenser, electricity is stored in the condenser, a positive charge being on one plate or set of plates, and an equal negative charge being on the other plate or set of plates. This property of a condenser to store electricity is called *capacitance*. The mechanism by which these charges are stored is discussed at the beginning of the chapter and is illustrated in Fig. 220 (p. 316).

The performance of a condenser when connected in an electric circuit is illustrated in Fig. 227 which shows two conducting plates connected to a battery, the plates being separated by a dielectric. There is also a single-pole, double-throw (S.-P., D.-T.) switch S and a galvanometer G in the circuit. If the switch S be closed to the left, the galvanometer will deflect

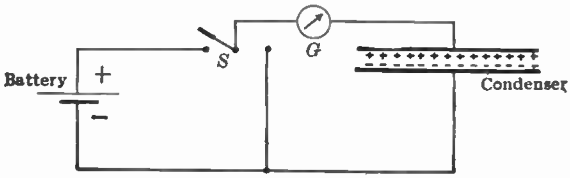


FIG. 227.—Charging and discharging a condenser.

momentarily and then come back to zero. This indicates that, when the switch is closed, a quantity of electricity passes through the galvanometer, but that the current ceases to flow almost immediately. The current flows for a time sufficient to charge the condenser. After the condenser has become fully charged, the current ceases because the e.m.f. of the condenser is equal and opposite to that of the battery. As this condenser e.m.f. opposes the current entering the condenser, it may be considered as a back e.m.f. Any current which may flow after the condenser has become fully charged is a leakage current flowing through the insulation. If the switch S be opened for a short time, and

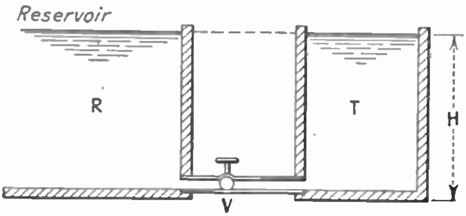


FIG. 228.—Reservoir and connected tank.

then closed again, no deflection of the galvanometer will be noted unless there has been leakage through the insulation.

The charging of a condenser from a battery is not unlike the filling of a tank T from a reservoir R (Fig. 228). When the valve V is first opened, water will rush through the pipe connecting R and T and will continue to flow at a diminishing rate until the level H of the water in the tank T is equal to the level of the water in the reservoir. If the tank does not leak, no water flows through the pipe after the water levels have become equal. In the same manner, the condenser (Fig. 227) takes current until its potential difference is equal to that of the battery, after which current ceases to flow. Again, if tank T does not leak, no further flow of water occurs when valve V is closed and then opened.

To prove that electricity has actually been stored in the condenser (Fig. 227), the switch S may be closed to the right. This short-circuits the condenser through the galvanometer. The galvanometer now deflects momentarily in a direction opposite to that on charge, showing that the current now flows *out* of the positive plate. The condenser now becomes completely discharged, as is shown by there being no longer any deflection of the galvanometer. Also, if the condenser is not leaky, the ballistic deflection on discharge is the same as that on charge, the quantity of electricity being the same in both cases.

If the voltage of the battery (Fig. 227) be increased, the galvanometer deflection on charge and on discharge will increase also. This is due to the fact that the charge given to the condenser is proportional to the voltage across its terminals, just as the amount of water in the tank will be proportional to the height H (Fig. 228). The relation between the voltage and the charge in a condenser is expressed by the equation

$$Q = CE. \tag{165}$$

That is, the quantity of electricity in a condenser is equal to the voltage multiplied by a constant C. This constant C is the *capacitance* of the condenser. The practical unit of capacitance is the *farad.*[1] If C is in farads and E in volts, Q is in coulombs or ampere-seconds.

[1] Faraday, Michael (1791–1867). An English chemist and physicist who was son of a blacksmith and until 1813 was an apprenticed bookbinder. At the instance of Sir Humphry Davy, he was appointed assistant in the laboratory of the Royal Institution of Great Britain. He was made Director in 1825 and in 1833 was appointed Fullerian Professor of Chemistry for life. Although he made important discoveries in chemistry, his outstanding work was in the field of electricity. He was the first to produce continuous rotation about each other (motor action) of wires conducting current, and magnets; the first to cause a current in a circuit to be induced by

The farad is too large a unit for practical purposes. A condenser having a capacitance of 1 farad would be prohibitively large. The capacitance of the earth as an isolated sphere is less than one-thousandth of a farad. The *microfarad* ($\mu f.$), equal to one-millionth of a farad, is the unit of capacitance ordinarily used.

In radio-telegraph and radio-telephone work, where the capacitances are very small, the microfarad is too large a unit, and the micromicrofarad ($\mu\mu f. = 10^{-12}$ farad) is used. However, in Eq. (165), and Eqs. (166) and (167) which follow, if Q is expressed in *coulombs* and E in *volts*, C *must* be expressed in *farads*.

By transposition, Eq. (165) may be written as follows:

$$C = \frac{Q}{E}. \tag{166}$$

$$E = \frac{Q}{C}. \tag{167}$$

As an illustration of the use of the above relations, consider the following example:

Example.—A condenser has a capacitance of 200 $\mu f.$ and is connected across 600-volt mains. If the current is maintained constant at 0.1 amp., how long must it flow before the condenser is fully charged?

The quantity in the condenser, when fully charged, is $Q = 0.000200 \times 600 = 0.12$ coulomb or ampere-second.

$$0.12 = 0.1t,$$
$$t = 1.2 \text{ sec.} \quad \textit{Ans.}$$

217. Dielectrics.—If electrostatic or dielectric phenomena are being considered, the medium between two conductors is called a *dielectric*. The *dielectric* properties of a medium are determined by the relation between dielectric lines and potential. On the other hand, the *insulation* properties of this same medium concern the relation between current and potential. For example, air is not a particularly good dielectric so far as flashover is concerned, its dielectric strength being only about 75,000 volts to the inch, but it is one of the best insulators.

magnetism or by a current in another circuit (generator action and mutual induction). He discovered the two fundamental laws of electrolysis (Par. 101, p. 133), the effect of magnetism on polarized light, diamagnetism, and made notable contributions to the understanding of electrostatic phenomena; the law of equal and opposite charges and the Faraday tubes of force being typical examples.

Thin samples of rubber will withstand voltages as high as 450,000 volts to the inch, but they will permit a much greater leakage current than air will permit. That is, rubber is a better dielectric but a poorer insulator than air.

It is stated in Par. 214 (p. 319) that the dielectric field resembles both the electric circuit and the magnetic field, in that dielectric lines distribute themselves as lines of current flow and magnetic lines do. No matter how much current flows in a conductor, the conductor is not injured mechanically, provided it can be kept cool. Neither is a magnetic conductor injured, no matter how many magnetic lines exist in it. Dielectric media, however, have the property of permitting only a limited number of dielectric lines per unit of area without rupture occurring. When the dielectric-flux density exceeds this limiting value, rupture occurs and may be followed by a dynamic arc, which burns and chars the dielectric. At the present time, the mechanism of the breakdown of solid and liquid dielectrics is not well understood although several rational theories have been advanced.

The ability of a substance to resist dielectric breakdown is called its *dielectric strength*. This is expressed in volts per unit thickness when the substance is placed between flat electrodes having rounded edges. For example, the dielectric strength of air is approximately 3,000 volts per millimeter. Rubber and varnished cambric have a much greater dielectric strength than air, an average value for rubber being 16,000 volts per millimeter, or 400,000 volts per inch. Varnished cambric has about twice the dielectric strength of rubber.

The volts per unit thickness impressed across a dielectric is the *voltage gradient*. For example, if 24,000 volts is impressed across 30 mils of insulation, the gradient is 24,000 ÷ 30 or 800 volts per mil. With insulating substances, both the insulation properties and the dielectric properties must be considered.

218. Specific Inductive Capacity or Dielectric Constant.[1]—A parallel-plate condenser (Fig. 229(*a*)), with air as a dielectric, has a measured capacitance C_1. If a slab of glass or of hard rubber be inserted between the plates so as to fill the intervening space completely (Fig. 229(*b*)), and the capacitance of the condenser

[1] This is also called *relative permittivity*. Permittivity is dielectric conductivity. The term *relative* refers to air or vacuum.

again be measured, it will be found to be greater than its previous value. Let this new value be C_2. The increase in capacitance must be due to the presence of the glass or the rubber.

The ratio $C_2/C_1 = \kappa$ is called the *specific inductive capacity*, or *dielectric constant*, or *relative permittivity* of the material between

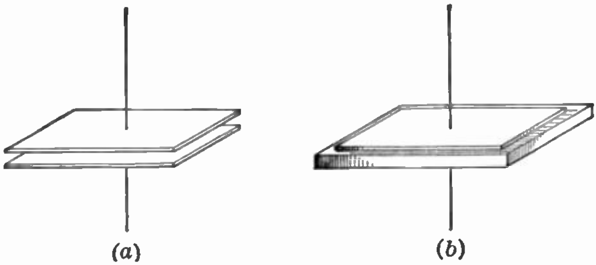


FIG. 229.—Plate condenser having air and then glass as a dielectric.

the condenser plates. The specific inductive capacity of air is assumed to be unity, just as the magnetic permeability of air is assumed to be unity.

In the table are given the dielectric constants of some of the more common dielectrics.

DIELECTRIC CONSTANTS

Bakelite*	4.5 to 5.5	Paper	2.0 to 2.6
Ebonite	2.8	Paraffin	2.1 to 2.5
Fiber	2.5 to 5	Porcelain	5.7 to 6.8
Glass	5.4 to 9.9	Rubber	2.0 to 3.5
Mica	2.5 to 6.6	Water	81
Oil	2.2 to 4.7	Wood	2.5 to 7.7

* For more complete data see "Standard Handbook," 6th ed., Sec. 4, Par. 450 *et seq.*

219. Equivalent Capacitance of Condensers in Parallel.—Let it be required to determine the capacitance C of a number of condensers in parallel, the condensers having capacitances C_1, C_2, C_3. This arrangement of condensers is shown in Fig. 230. Let the common voltage across the condensers be E and the total resulting charge Q. Obviously,

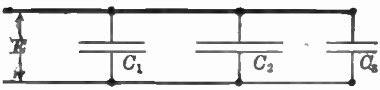


FIG. 230.—Capacitances in parallel.

$$Q = CE$$

and

$$Q_1 = C_1E, \quad Q_2 = C_2E, \quad Q_3 = C_3E.$$

The total charge

$$Q = Q_1 + Q_2 + Q_3 = CE.$$
$$CE = C_1E + C_2E + C_3E.$$
$$CE = (C_1 + C_2 + C_3)E.$$

Therefore,

$$C = C_1 + C_2 + C_3. \tag{168}$$

That is, if condensers are connected in parallel, the resulting capacitance is the sum of the individual capacitances.

This is analogous to the grouping of conductances in parallel in the electric circuit.

Example.—Three condensers, having capacitances of 5, 10, 12 $\mu f.$, are connected in parallel across 600-volt mains. (*a*) What single condenser would replace the combination? (*b*) What is the charge on each condenser?

(*a*) $C = 5 + 10 + 12 = 27\ \mu f.$ *Ans.*

(*b*) $Q_1 = 5 \times 600 =$ 3,000 microcoulombs.

$Q_2 = 10 \times 600 =$ 6,000 microcoulombs.

$Q_3 = 12 \times 600 =$ 7,200 microcoulombs. *Ans.*

Total charge = 16,200 microcoulombs = 27×600 microcoulombs (*check*).

220. Equivalent Capacitance of Condensers in Series.—In Fig. 231, three condensers, having capacitances C_1, C_2, C_3, are connected in series across the voltage E. It is desired to determine the capacitance of an equivalent single condenser. Let E_1, E_2, E_3 be the potential differences across the condensers C_1, C_2, C_3. After the voltage E is applied to the system, there will be $+Q$ units of charge on the positive plate of C_1, and, by the law of electrostatic induction, $-Q$ units must be on the negative plate of C_1.

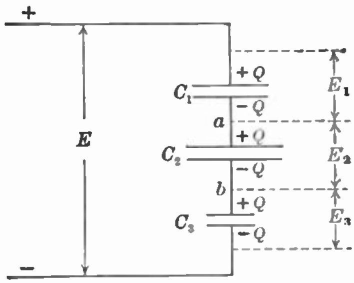


FIG. 231.—Capacitances in series.

Now consider the region *a* which consists of the negative plate of C_1, the positive plate of C_2, and the lead connecting them. This system is insulated from all external potentials, since it is assumed that the condensers have perfect insulation. Before the voltage was applied to the system of condensers, no charge existed in the region a. After the application of the voltage, the net charge in this region must still be zero, as no charge can flow through the insulation (see Par. 212, p. 317). Therefore, $+Q$ units must come into existence in order that the net charge in the region a may remain zero. $((+Q) + (-Q) = 0)$. This charge of $+Q$ units will go to the plate of C_2 since it is repelled by the $+$ charge on C_1 just as the charge b' (Fig. 222(a), p. 318) took a position on the end of the conducting body as far as possible from the positive inducing charge a. The same reasoning holds for the region b, between C_2 and C_3. Therefore, each of the three condensers in series has the same charge Q. (This is analogous to resistances in series, each of which must carry the same current if no leakage exists.)

Consider the voltages E_1, E_2, E_3.

$$E_1 = \frac{Q}{C_1}, \quad E_2 = \frac{Q}{C_2}, \quad E_3 = \frac{Q}{C_3} \text{ (from Eq. (167), p. 326).}$$

The sum of the three condenser voltages must equal the line voltage.

$$E_1 + E_2 + E_3 = E.$$

$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Also, $E = Q/C$, as by definition the equivalent condenser C must have a charge Q.

Substituting this value for E,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \tag{169}$$

That is, *the reciprocal of the equivalent capacitance of a number of condensers in series is equal to the sum of the reciprocals of the capacitances of the individual condensers.*

In assuming for condensers connected in series that with direct current the potential across each condenser is inversely proportional to its capacitance, the factor of leakage is neglected. If the condensers are even slightly leaky, however, a current flows through the series and eventually the potential distributes itself according to Ohm's law.

$$E_1 = IR_1, \quad E_2 = IR_2, \quad E_3 = IR_3,$$

where I is the leakage current, and R_1, R_2, R_3 are the ohmic resistances of the three condensers.

Example of Condensers Connected in Series.—Consider that the three condensers of Par. 219, having capacitances of 5, 10, 12 $\mu f.$, are connected in series across 600-volt mains. Determine: (*a*) the equivalent capacitance of the combination; (*b*) the charge on each condenser; (*c*) the potential across each condenser, assuming no leakage.

$$(a) \qquad \frac{1}{C} = \frac{1}{5} + \frac{1}{10} + \frac{1}{12} = 0.383.$$

$$C = \frac{1}{0.383} = 2.61\ \mu f. \quad \textit{Ans.}$$

(*b*) $Q = 2.61 \times 600 = 1{,}566$ microcoulombs, on each condenser. *Ans.*

$$(c)\ E_1 = \frac{1{,}566 \times 10^{-6}}{5 \times 10^{-6}} = 313 \text{ volts.}$$

$$E_2 = \frac{1{,}566 \times 10^{-6}}{10 \times 10^{-6}} = 157 \text{ volts.}$$

$$E_3 = \frac{1{,}566 \times 10^{-6}}{12 \times 10^{-6}} = 130 \text{ volts.} \quad \textit{Ans.}$$

$$E_1 + E_2 + E_3 = \overline{600} \text{ volts } (\textit{check}).$$

221. Field Intensity between Parallel Plates.—Some of the electrostatic phenomena, such as the energy stored in a condenser, for example, depend on the intensity of the field between charged parallel plates.

Consider two parallel conducting plates each having an area A sq. cm., with a distance of separation which is small compared with their area, so that edge effects may be neglected (Fig. 233, p. 335). Let the dielectric be air. Place a charge $+Q$ e.s.u. on one plate and $-Q$ e.s.u. on the other. These charges will reside entirely on the adjacent surfaces. The charge per unit area $\sigma = Q/A$. Since 4π lines leave each positive-unit charge and terminate on each negative-unit charge, there will be $4\pi\sigma$ lines per square centimeter between the plates. Hence, from definition, the field intensity $D = 4\pi\sigma$ dynes per unit charge.

The relation may also be developed in the same manner as is used to determine the force adjacent to a magnetized surface (Par. 151, p. 219). Since the law relating to the force between unit charges is the same as that relating to unit poles, the force just outside a plate charged with σ unit charges per square centimeter is $2\pi\sigma$ dynes. The field adjacent to the charged plate is normal to it (Par. 214, p. 321) and is uniform. It follows that the force must be $2\pi\sigma$ dynes at any distance from the plate so long as the field remains uniform. The field intensity between the two parallel, oppositely charged plates is uniform throughout. Each plate must therefore exert a force of $2\pi\sigma$ dynes on a unit-positive charge in the region between the plates, so that the total force is $4\pi\sigma$ dynes and the field intensity is

$$D = 4\pi\sigma \text{ dynes per unit charge.} \tag{170}$$

The force of attraction between the plates is readily computed. Consider a unit charge on the negative plate. The positive plate acts on this charge with a force of $2\pi\sigma$ dynes. The force on each square centimeter of the negatively charged plate is $2\pi\sigma^2$ dynes, and the force on the entire negative plate is

$$f = 2\pi\sigma^2 A \text{ dynes.} \tag{171}$$

(Compare with Eq. (94), p. 220.)

222. Energy Stored in Condensers.—As a certain quantity of electricity is stored in a condenser and a difference of potential exists between the positive and negative plates, energy must be stored in the condenser. The existence of this energy is shown by the spark resulting from short-circuiting the condenser.

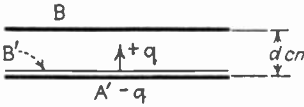


FIG. 232.—Charging a plate condenser.

The stored energy may be computed by finding the mechanical work done in charging the condenser. In Fig. 232, the equal, parallel flat conducting plates A' and B' are separated from each other by an infinitesimal distance. A charge $+q$ e.s.u. is placed on B' and a charge $-q$ e.s.u. is placed on A'. Since these charges are separated by an infinitesimal distance, no finite work has been done in separating them. The force between the two plates is $2\pi\sigma^2 A$, where σ is the density of charge on each plate in electrostatic units per square centimeter, and A is the area of each plate in square centimeters (see Par. 221). This force is constant so long as the charge is constant and the field between the plates is uniform. The plate B' is now moved away from A' in a direction perpendicular to its plane to a position B, d cm. from A' and parallel to A'. The work done in separating the charges $+q$ and $-q$ is the product of force and distance. That is,

$$w = 2\pi\sigma^2 Ad \text{ ergs.} \tag{I}$$

From Par. 224 (p. 335), the capacitance of such a condenser,

$$C = \frac{A}{4\pi d}. \tag{II}$$

Substituting d from Eq. (II) in Eq. (I), remembering that $q = \sigma A$,

$$w = \frac{q^2}{2C} \text{ ergs.} \tag{172}$$

If practical units are used, the work is given in joules.

Let the condenser be further charged by moving another plate B' from A' to B, bringing an additional charge $+q_1$ to plate B. There will be two forces to overcome, that due to the separation of charges $+q_1$ and $-q_1$ which is equal to $2\pi\sigma_1{}^2A$ or $2\pi(q_1/A)^2A$ and that due to the intensity of the field produced by charges $+q$ and $-q$. This last force is equal to $4\pi\sigma q_1$ or $4\pi(q/A)q_1$. The *total work* is now

$$W = \frac{q^2}{2C} + \frac{4\pi q q_1 d}{A} + \frac{2\pi q_1{}^2 d}{A}.$$

Substituting,

$$C = \frac{A}{4\pi d},$$

$$W = \frac{q^2}{2C} + \frac{qq_1}{C} + \frac{q_1{}^2}{2C} = \frac{1}{2C}(q + q_1)^2 = \tfrac{1}{2}\frac{Q^2}{C} \text{ ergs,} \tag{173}$$

where $Q = q + q_1$. The same procedure can be followed with additional charge q_2, q_3, etc.

Also the work may be readily computed using calculus. In Fig. 232, the work done in moving q units through a difference in potential dv (Par. 27, p. 29) is

$$dw = qdv,$$

and the total work

$$W = \int_0^V qdv.$$

But, $q = Cv$. Hence,

$$W = C\int_0^V vdv = \tfrac{1}{2}CV^2 \text{ ergs.} \tag{174}$$

Since $Q = CV$, Eqs. (173) and (174) may also be expressed as

$$W = \tfrac{1}{2}QV \text{ ergs.} \tag{175}$$

If Q, C, and E are expressed in coulombs, farads, and volts,

$$W = \tfrac{1}{2}\frac{Q^2}{C} \text{ joules.} \tag{176}$$

$$W = \tfrac{1}{2}CV^2 \text{ joules.} \tag{177}$$

$$W = \tfrac{1}{2}QV \text{ joules.} \tag{178}$$

The similarity in form of Eq. (177) and the equation for the energy stored in the magnetic field should be noted (see Eq. (149), p. 303, Par. 201). The energy stored in the dielectric field is proportional to the square of the *voltage*, whereas the energy stored in the electromagnetic field is proportional to the square of the *current*.

Example.—Determine the stored energy in each of the condensers in series of Par. 220 (p. 331) and the total stored energy.

$$W_1 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{5 \times 10^{-6}} = 0.2453 \text{ joule.} \quad \textit{Ans.}$$

$$W_2 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{10 \times 10^{-6}} = 0.1225 \text{ joule.} \quad \textit{Ans.}$$

$$W_3 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{12 \times 10^{-6}} = 0.1020 \text{ joule.} \quad \textit{Ans.}$$

The total energy $W = \tfrac{1}{2}(1{,}566 \times 10^{-6} \times 600) = 0.4698$ joule. *Ans.*

Using Eq. (177),

$$W_1 = \tfrac{1}{2} \times 5 \times 10^{-6} \times (313)^2 = 0.245 \text{ joule } (\textit{check}).$$

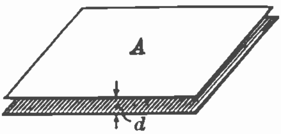
Using Eq. (178),

$$W_1 = \tfrac{1}{2} \times 1{,}566 \times 10^{-6} \times 313 = 0.245 \text{ joule } (\textit{check}).$$

223. Calculation of Capacitance.—If the geometry is not too complicated, it is possible to calculate the capacitance of a condenser by either analytical or graphical methods. In such calculations the electrostatic system of units is used, since the capacitance may be computed in terms of the dimensions of the condenser. It is a simple matter to convert to farads by dividing by 9×10^{11}, the numerical ratio of the units of capacitance of the two systems. The method of procedure is to determine the work done in carrying a unit charge through the field from one electrode to the other. By definition this gives the potential difference between the plates. From Eq. (166), p. 326, the ratio of charge to potential difference gives the capacitance.

The computation of the capacitance of concentric spherical and cylindrical condensers is in part dependent on the fact that *there can be no force in a region totally enclosed by a conductor when there is no charge within the region.* If a force does exist within such a region, there must be lines of force. These lines cannot exist for two reasons. They must begin at a positive charge and end at a negative charge. This is impossible since the space is entirely surrounded by a charge of one sign. Also, there must be a difference of potential between the beginning and the end of a line of force. This is impossible under these conditions since the entire surrounding conductor is at one potential. If it were not, current would flow, which is contrary to the assumed static conditions.

224. Capacitance of Parallel-plate Condensers.—The parallel-plate condenser is the simplest type of condenser and is widely used in various forms. Because of its simple geometry it is easy to calculate the capacitance if the spacing of the plates is small compared with their area making the effect of edge fringing negligible.



$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5}$$

FIG. 233.—Capacitance of a plate condenser.

In the simple two-plate condenser (Fig. 233) the area of each plate is A sq. cm. and the distance between plates is d cm. The dielectric constant of the insulating medium is κ. A positive charge $+Q$ e.s.u. is placed on the upper plate and an equal negative charge $-Q$ e.s.u. is placed on the lower plate. The density of charge σ on each plate is Q/A e.s.u. per square centimeter.

In Par. 221 (p. 331) it is shown that with air as a dielectric, the field intensity between such charged plates is $4\pi\sigma$ dynes per unit charge and the field is uniform. With the dielectric constant κ, the field intensity is

$$D = f = \frac{4\pi\sigma}{\kappa} \text{ dynes per unit charge.}$$

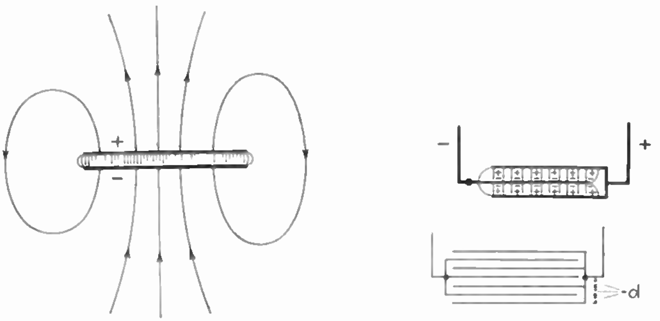
Since the field is uniform, the force throughout the region between the plates is constant. The work done in carrying a unit-positive charge from the negative to the positive plate is

$$W = fd = \frac{4\pi\sigma d}{\kappa} = V \qquad (1)$$

where V is the potential difference between the plates. Substituting, $\sigma = Q/A$, and since $C = Q/V$,

$$C = \frac{\kappa A}{4\pi d} \text{ statfarads.} \qquad (179)$$

Since the *microfarad* is 9×10^5 times as great as the statfarad, it is necessary to divide Eq. (179) by 9×10^5 in order to obtain



(*a*) Electrostatic leakage lines of a plate condenser.

(*b*) Multiplate condensers.

FIG. 234.

the capacitance in microfarads. Hence,

$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5} \mu f. \qquad (180)$$

The total capacitance of a simple plate condenser of this type cannot be accurately calculated for the following reason: All the dielectric lines do not lie in the region between the plates, as certain lines pass from the back of the positive plate to the back of the negative plate, as in Fig. 234(*a*). This results in the actual capacitance being greater than the value just calculated.

This error may be avoided by using one more plate in one group than in the other (Fig. 234(*b*)). In this case the area A (Eq. (179)) includes both sides of all the plates with the exception of the two outside ones. As the charge on both outer plates is of the same sign and the plates have the same potential, no dielectric lines can pass between them. An error may occur due to the bulging or "fringing" of the lines near the edges of the plates unless the plate area is large compared with the distance between plates.

Example of Condenser Design.—It is desired to construct a plate condenser having a total capacitance of 8 μf. The plates are of tin-foil 6 × 8 in. and 1 mil thick. The dielectric is of paper 7 × 9 in. and 2 mils thick, having a dielectric constant 3. How many sheets of paper and of tin-foil are necessary? What will be the dimensions of the condenser?

The area of each plate is

$$6 \times 8 \times (2.54)^2 = 309.6 \text{ sq. cm.}$$

The distance between plates

$$d = 0.002 \times 2.54 = 0.00508 \text{ cm.}$$

The capacitance between two plates (from Eq. (179))

$$C = \frac{3 \times 309.6}{4\pi \times 0.00508 \times 9 \times 10^5} = 0.01616 \ \mu f.$$

Therefore,

$$\frac{8}{0.01616} = 495 \text{ sections are needed.}$$

These sections are indicated at d (Fig. 234(*b*)). This means that 496 plates and 495 sheets of paper are necessary.

Thickness:

$$\begin{aligned} \text{Tin-foil} &= 496 \times 0.001 = 0.496 \text{ in.} \\ \text{Paper} &= 495 \times 0.002 = \underline{0.990} \text{ in.} \\ & \qquad\qquad\qquad\quad\ 1.486 \text{ in.} \end{aligned}$$

Volume of condenser proper = 7 × 9 × 1.49 in. *Ans.*

In addition, outside insulation and a protective covering are necessary.

225. Capacitance of Concentric Cylindrical Condensers.—Concentric cylindrical condensers are in common use. Their widest use is exemplified by the single-conductor cable, particularly the underground power cable with a lead sheath. It is not difficult to compute the capacitance of such a cable. However, before making this computation, the force exerted by an infinitely long charged filament is first determined.

In Fig. 235, a thin straight filament A in air, extending from $-$ infinity to $+$ infinity, is charged with q e.s.u. per centimeter length. Let it be required to determine the force exerted by this linearly distributed charge at a point P, h cm. from the filament. Let the origin O be the intersection of the perpendicular h and the filament. Consider a differential length of filament

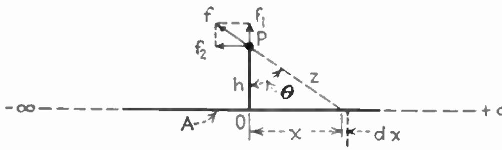


FIG. 235.—Force due charged filament of infinite length.

dx, at x cm. from O. The charge included within the distance dx is $q\,dx$ e.s.u. By Coulomb's law (p. 319), it exerts a force $f = qdx/z^2$ dynes at P. Each component of force parallel to the filament due to a charge on one side of the origin is balanced by an equal component due to a charge similarly situated on the other side of the origin, and the resultant force due to the entire charge on the filament will be along the perpendicular h. Hence, the component f_1, in the direction of h, of the force f needs be alone considered. Force f_1 is equal to $f \cos \theta = f\dfrac{h}{\sqrt{x^2 + h^2}}$. Also, $z^2 = x^2 + h^2$. Let the resultant force be f_0. Difficulty in evaluating the integral is avoided if the integration limits are made 0 and ∞ and the integral is multiplied by 2. Hence,

$$f_0 = 2qh\int_0^\infty \frac{dx}{(x^2 + h^2)^{3/2}} = 2qh\frac{x}{h^2\sqrt{x^2 + h^2}}\Bigg|_0^\infty.$$

Dividing numerator and denominator by x and inserting the limits,

$$f_0 = \frac{2q}{h}\frac{1}{\sqrt{1 + (h^2/x^2)}}\Bigg|_0^\infty = \frac{2q}{h} \text{ dynes.} \tag{181}$$

If the medium is a dielectric of constant κ, the force

$$f_0 = \frac{2q}{\kappa h} \text{ dynes.} \tag{182}$$

Equation (181) should be compared with Eq. (99), p. 233, which gives the force due to a current in a long straight, conducting filament.

From the symmetry of the charges, the dielectric lines between concentric cylinders (Fig. 236) must be radial, and, if continued inward, will intersect the axis. Hence it follows that for points external to the inner cylinder, the charge on the cylinder acts as if concentrated along the axis and Eq. (182) may be applied.

Consider the two concentric cylinders (Fig. 236) in which the radius of the outer cylinder is R_2 cm. and the radius of the inner cylinder is R_1 cm. The dielectric constant of the insulating medium is κ. The charge on the inner cylinder is $+q$ e.s.u. per centimeter length and that on the outer cylinder is $-q$ e.s.u. per centimeter length. The cylinders are considered as being infinite in length, although when the ratio of length to radius is only moderately large, the effect is practically that of infinite cylinders. A unit charge is placed at point P in the dielectric, a distance r cm. from the axis. From Eq. (182) the force due to the inner cylinder is $2q/\kappa r$ dynes. There can be no force at P due to the charge on the outer cylinder, since P is in a region entirely enclosed by a conductor, the open ends at infinity having no effect. The work done in carrying a unit charge from the surface of the inner cylinder to the outer cylinder, and hence the potential difference,

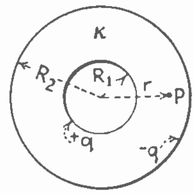


FIG. 236.—Concentric cylindrical condenser.

$$W = V = \int_{R_1}^{R_2} \frac{2q}{\kappa r} dr = \frac{2q}{\kappa} \log_\epsilon r \Big|_{R_1}^{R_2} \text{ ergs.}$$

Inserting the limits and remembering that $\log_\epsilon R_2 - \log_\epsilon R_1 = \log_\epsilon (R_2/R_1)$,

$$V = \frac{2q}{\kappa} \log_\epsilon \frac{R_2}{R_1} \text{ statvolts.}$$

Since C_s, the capacitance per centimeter, is equal to q/V where q is the e.s.u. per centimeter,

$$C_s = \frac{q}{V} = \frac{\kappa}{2 \log_\epsilon (R_2/R_1)} \text{ statfarads} \tag{183}$$

$$= \frac{\kappa}{2 \times 2.303 \log_{10} \dfrac{R_2}{R_1}} \text{ statfarads.} \tag{184}$$

By substituting the microfarad for 9×10^5 statfarads, and changing centimeters to miles, Eq. (184) becomes

$$C = \frac{0.0388\kappa}{\log_{10} (R_2/R_1)} \ \mu f. \text{ per mile.} \tag{185}$$

The equation in this form is convenient for use with underground cables.

Example.—A 1,200-ft. length of No. 4 A.W.G., single-conductor, rubber-insulated underground cable has a $\frac{5}{32}$-in. wall of insulation. The diameter of the conductor is 204.3 mils. The capacitance of the cable is measured and found to be 0.105 μf. What is the dielectric constant κ of the rubber?

$$R_1 = \frac{0.2043}{2} = 0.1022 \text{ in.}$$

$$\tfrac{5}{32} \text{ in.} = 0.1563 \text{ in.}$$

$$R_2 = \frac{0.1563 + 0.1563 + 0.2043}{2} = 0.2585 \text{ in.}$$

The capacitance of a mile length

$$C = \left(\frac{5{,}280}{1{,}200}\right)0.105 = 0.462\mu f.$$

Using Eq. (185),

$$\kappa = \frac{0.462 \log_{10} \dfrac{0.2585}{0.1022}}{0.0388} = \frac{0.462 \times 0.4031}{0.0388} = 4.8. \quad \textit{Ans.}$$

226. Capacitance of Concentric Spherical Condenser.—In Fig. 237 is shown a concentric spherical condenser. The radius of the inner sphere A is R_1 cm. and the radius of the outer sphere B is R_2 cm. The dielectric constant of the insulating medium is κ. A charge of $+Q$ e.s.u. is placed on the inner sphere and a charge of $-Q$ e.s.u. on the outer sphere. The potential difference between the spheres is to be determined by carrying a unit charge from sphere A to sphere B. The force at any point P in the dielectric, r cm. from the center, due to the charge on the inner sphere, is $Q/\kappa r^2$ dynes (see Fig. 225, p. 321). The force at P due to the charge on the outer sphere is zero since point P is within the sphere (Par. 223, p. 335). Therefore, the potential difference between spheres A and B is

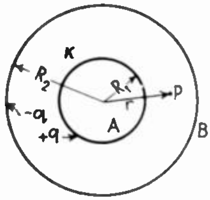


FIG. 237.—Concentric spherical condenser.

$$W = V = \int_{R_1}^{R_2} \frac{Q\,dr}{\kappa r^2} = -\frac{Q}{\kappa r}\Big|_{R_1}^{R_2} = -\frac{Q}{\kappa}\left(\frac{1}{R_2} - \frac{1}{R_1}\right).$$

The capacitance

$$C = \frac{Q}{V} = \frac{\kappa}{\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)} = \kappa \frac{R_1 R_2}{R_2 - R_1} \text{ statfarads.} \tag{186}$$

It is interesting to note that, if R_2 becomes infinite, the capacitance becomes

$$C = \kappa R_1. \tag{187}$$

That is, the capacitance of an isolated sphere in statfarads is equal to its radius in centimeters, if the sphere is in air ($\kappa = 1$). The capacitance of the earth as an isolated sphere is approximately 720 μf.

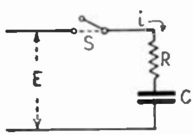


FIG. 238.—Capacitance charged through resistance.

227. Current to Resistance and Capacitance in Series.—If a capacitance C and a resistance R in series be connected suddenly across a voltage E (Fig. 238), current will flow through the resistance to charge the capacitance. The voltage across the capacitance will come to line voltage only after a time has elapsed. Theoretically, the time required is infinite, but practically it is usually relatively short. The charging current may be determined as a function of time.

The voltage across the condenser at any instant is q/C volts. After the switch S is closed (Fig. 238), the voltage E must not only supply the resistance drop but also must overcome the counter e.m.f. q/C of the condenser. That is,

$$E = iR + \frac{q}{C} \text{ volts.} \tag{I}$$

The quantity $q = \int idt$. Substituting in Eq. (I),

$$E = iR + \frac{1}{C}\int idt. \tag{II}$$

Differentiating with respect to t,

$$0 = R\frac{di}{dt} + \frac{i}{C}. \tag{III}$$

Rearranging terms, and integrating,

$$\frac{di}{i} = -\frac{dt}{CR}. \tag{IV}$$

$$\log_\epsilon i = -\frac{t}{CR} + K \tag{V}$$

where K is a constant of integration.

When the time t is zero, the quantity q in the condenser must be zero, since finite energy cannot be stored in zero time. Hence when $t = 0$, $q = 0$ and the voltage q/C across the condenser is zero. Therefore, the line voltage E is equal to I_0R where I_0 is the initial current. Hence at $t = 0$, $i = I_0 = E/R$. Substituting in Eq. (V),

$$K = \log_\epsilon \frac{E}{R},$$

$$\log_\epsilon i - \log_\epsilon \frac{E}{R} = \log_\epsilon \frac{i}{(E/R)} = -\frac{t}{CR}. \tag{VI}$$

Expressing in exponential form,

$$\frac{i}{(E/R)} = \epsilon^{-\frac{t}{CR}} \tag{VII}$$

and

$$i = \frac{E}{R}\epsilon^{-\frac{t}{CR}} = I_0\epsilon^{-\frac{t}{CR}} \text{ amp.} \qquad (188)$$

Equation (188) is an exponential decaying function having an initial value $E/R = I_0$, when $t = 0$. Theoretically the current becomes zero only when $t = \infty$, but practically it reaches this value in a relatively short time. The function is plotted in Fig. 239 for $E = 200$ volts, $R = 2{,}000$ ohms, $C = 0.00004$ farad $= 40$ μf. (see example).

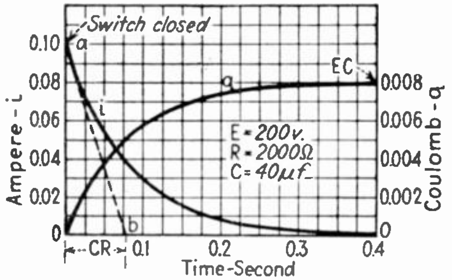


FIG. 239.—Charge characteristics of capacitance and resistance in series.

The time constant of the circuit is

$$T = CR \text{ sec.} \qquad (189)$$

Substituting T for t in Eq. (188),

$$i = \frac{E}{R}\epsilon^{-\frac{CR}{CR}} = \frac{E}{R}\frac{1}{2.718} = 0.368\frac{E}{R} \text{ amp.} \qquad (190)$$

That is, at time $t = CR$, the current has dropped to 36.8 per cent its initial value.

After closing the switch, the rate of change of the current

$$\frac{di}{dt} = \frac{E}{R}\left(-\frac{1}{CR}\right)\epsilon^{-\frac{t}{CR}} = -\frac{E}{CR^2}\epsilon^{-\frac{t}{CR}} \text{ amp. per second.} \qquad (191)$$

When $t = 0$, $di/dt = -E/CR^2$. If the current continued to decrease at this rate it would reach zero in a time equal to the time constant CR. For example, if the uniform rate of change of current $-E/CR^2$ is multiplied by the time CR, the total change of current during this time is $(-E/CR^2)(CR) = -E/R$.

This is equal numerically to the initial current, so that the resulting current is zero (see line ab, Fig. 239).

The quantity on the condenser is readily determined by integrating Eq. (188). That is,

$$q = \int i\,dt = \frac{E}{R}\int \epsilon^{-\frac{t}{CR}} = \frac{E}{R}\left(-CR\epsilon^{-\frac{t}{CR}} + K'\right) \qquad (1)$$

where K' is a constant of integration. When $t = 0$, $q = 0$ and $K' = (E/R)(CR) = EC$.

Substituting in Eq. (1),

$$q = EC\left(1 - \epsilon^{-\frac{t}{CR}}\right). \qquad (192)$$

Equation (192) is similar in character to Eq. (142), p. 297, giving the rise of current in an inductive circuit. It is plotted in Fig. 239.

Since in an uncharged condenser the counter e.m.f. is zero, the current must be infinite at the instant of connection across a constant voltage without series resistance. The duration of the current under this condition is zero. However, the inevitable resistance in the circuit and condenser prevent the current becoming infinite, though it does reach high values. It is also clear that with any current which is not infinite at the instant of switching, the voltage across any uncharged condenser is zero.

Example.—A capacitance of 40 $\mu f.$ in series with 2,000 ohms is suddenly connected across a 200-volt source. Determine: (*a*) the initial current; (*b*) the equation of current as a function of time; (*c*) the equation of quantity as a function of time; (*d*) the time constant; (*e*) the value of current when the time is equal to the time constant; (*f*) the charge on the condenser when the time is 0.04 sec.; (*g*) the energy stored in the condenser in (*f*). (*h*) Plot the functions (*b*) and (*c*).

(*a*) Since, at the instant of switching, the voltage across the capacitance is zero, the impressed voltage is utilized entirely in the resistance drop. Hence,

$$I_0 = \frac{200}{2{,}000} = 0.10 \text{ amp.} \quad \textit{Ans.}$$

(*b*) Substituting in Eq. (188),

$$i = 0.1\epsilon^{-\frac{t}{40\times 10^{-6}\times 2{,}000}} = 0.1\epsilon^{-\frac{t}{0.08}} \text{ amp.} \quad \textit{Ans.}$$

(c) Substituting in Eq. (192),

$$q = 200 \times 40 \times 10^{-6}\left(1 - \epsilon^{-\frac{t}{0.08}}\right)$$
$$= 0.008\left(1 - \epsilon^{-\frac{t}{0.08}}\right) \text{ coulombs.} \quad \textit{Ans.}$$

(d) $T = CR = 40 \times 10^{-6} \times 2{,}000 = 0.08$ sec. *Ans.*
(e) $i = 0.1\epsilon^{-1} = 0.0368$ amp. *Ans.*
(f) From (c),

$$q = 0.008\left(1 - \epsilon^{-\frac{0.04}{0.08}}\right) = 0.008(1 - \epsilon^{-\frac{1}{2}})$$
$$= 0.008\left(1 - \frac{1}{\sqrt{2.718}}\right) = 0.00315 \text{ coulomb.} \quad \textit{Ans.}$$

(g) From Eq. (176), p. 334,

$$w = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(0.00315)^2}{(40 \times 10^{-6})} = \frac{9.92 \times 10^{-6}}{80 \times 10^{-6}}$$
$$= 0.124 \text{ joule.} \quad \textit{Ans.}$$

(h) See Fig. 239.

228. Discharge of Capacitance.—In Fig. 240 a capacitance C has been charged through resistance R by closing the switch S upward. The switch S is then thrown downward disconnecting the line and short-circuiting the resistance and capacitance in series. The voltage across the capacitance at the instant of closing the switch downward is E_0 volts. E_0 may differ from E, owing, for example, to leakage or to the fact that the switch S was closed downward before the capacitance had become completely charged. The capacitance now becomes a diminishing source of e.m.f. q/C and is supplying current to the resistance. The direction of the current is the reverse of that on charge. By Ohm's law,

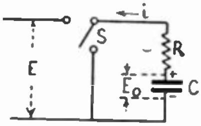


FIG. 240.—Capacitance discharged through resistance.

$$-iR = \frac{q}{C} \text{ volts.} \tag{I}$$

Since $q = \int idt$,

$$-iR = \frac{1}{C}\int idt \text{ volts.} \tag{II}$$

Differentiating and rearranging,

$$-R\frac{di}{dt} = \frac{i}{C}; \quad \frac{di}{i} = -\frac{dt}{CR}. \tag{III}$$

Integrating,

$$\log_\epsilon i = -\frac{t}{CR} + K_0 \tag{IV}$$

where K_0 is a constant of integration.

When $t = 0$, $i = -I_0 = -E_0/R$, so that $K_0 = \log_\epsilon (-I_0)$. Hence,

$$\log_\epsilon \frac{i}{-I_0} = -\frac{t}{CR}. \tag{V}$$

Expressing in exponential form, and solving for i,

$$i = -I_0\epsilon^{-\frac{t}{CR}} = -\frac{E_0}{R}\epsilon^{-\frac{t}{CR}} \text{ amp.} \tag{193}$$

Equation (193) is identical in form with Eq. (188) except that the current is negative. When $E_0 = E$, Eq. (193) becomes the negative of Eq. (188). The discharge characteristic of the capacitance and resistance of the example (Par. 227) is shown in Fig. 241.

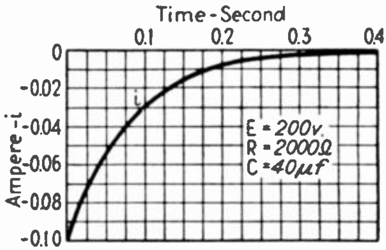


FIG. 241.—Discharge characteristics of capacitance and resistance in series.

229. Ionization of Air; Corona.—When solid dielectrics are subjected to sufficiently high dielectric stress, they rupture and the dynamic arc which follows chars and disintegrates the dielectric (Par. 217, p. 326). Although the exact mechanism of the rupture is not understood, undoubtedly there first occurs a destruction of the molecular structure of the medium in some one spot, due to a separation of the electrons from their atomic nuclei, as a result of the high potential gradient.

With gaseous dielectrics such as air, the mechanism of dielectric rupture is much better understood. This is due in part to the fact that the molecular structure of gases is much simpler than that of most other substances. Also, the gas particles are not destroyed by the rupture, and since they are mobile, they can be collected and analyzed even after they have been subjected to rupturing stresses. A gas atom consists of a positive nucleus with one or more minute electric charges or electrons moving in orbits about the nucleus. A hydrogen atom has but one electron, a helium atom has two electrons.

In a dielectric field the electrons of a gas, being negative charges, are attracted to the positive electrode; the positive nuclei or ions are likewise attracted to the negative electrode. Hence, the dielectric field tends to separate the electrons from their nuclei. The dielectric forces holding the electrons to their nuclei are so great, however, that a prohibitively high voltage gradient would be necessary to separate them by this effect alone.

There are always some free ions and electrons in any gaseous medium. Under the action of the electric field the ions and, more particularly, the electrons will accelerate. While moving toward the electrodes they will collide with the neutral atoms that happen to be in their paths. These collisions knock other electrons from their nuclei, thus producing more free ions and electrons. Hence, the process tends to be a cumulative one. When the potential gradient becomes sufficiently great (approximately[1] 30,000 volts per centimeter for air at 760 mm. pressure and at 25°C.), the velocities acquired by the ions become sufficiently high to produce ions by collision more rapidly than the ions are withdrawn from the field. The gas is then said to be *ionized*. Ionized gas conducts current but has, notwithstanding, a *very high* resistance. This conduction of current through an ionized gas is a convection phenomenon not unlike the conduction of electricity through an electrolyte (Par. 100, p. 132). Thus the dielectric strength of the ionized gas has become practically nil. If a source of potential having considerable power behind it is used, rupture of the gas occurs and a dynamic arc follows.

A partial ionization of a gas without complete rupture may be accomplished, if an arrangement somewhat like that shown in Fig. 242 is employed. One electrode, the upper in Fig. 242, is sharp and may be conveniently a needle, the lower one being a flat plate. With the application of potential, dielectric lines arise

[1] This corresponds to 100 electrostatic lines per square centimeter.

between the needle point and the plate. Owing to the sharpness of the needle point, the flux density near it is very great, whereas the density near the plate is much smaller because of its larger area. Hence, the air in the vicinity of the needle point first becomes ionized, the ionized region extending at first to line *aa*. Beyond *aa*, the dielectric field is not sufficiently intense to permit further ionization and the remainder of the air therefore prevents any dynamic current flowing. With further increase in potential, the ionized zone may extend to *bb* without complete rupture. Ultimately, however, the potential may reach a value which is sufficient to rupture the non-ionized region, and a dynamic arc follows.

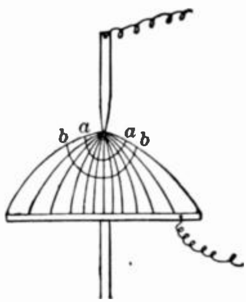


FIG. 242.—Dielectric stress lines between a needle point and a plate.

Ionized air at the surfaces of conductors appears as small tufts and streamers, giving a bluish-reddish light easily visible in the dark. Such ionized air is called *corona* because of its resemblance to solar corona. Corona is also accompanied by a hissing sound, and the odor of ozone is noticed. In the presence of moisture nitrous acid is formed, and this phenomenon is the basis of one of the methods of manufacturing by electrical means, nitrates from the nitrogen of the atmosphere. Corona forms on high-voltage apparatus and on transmission lines (see Vol. II, Chap. XIII).

MEASUREMENT OF CAPACITANCE

230. Ballistic Method.—There are two common methods of measuring capacitance, the direct-current or ballistic method and the alternating-current or bridge method.

The direct-current method employs a galvanometer which is used ballistically. It can be shown that if the moving coil of the ordinary galvanometer has considerable inertia and is properly damped, its maximum throw, due to the impulse produced by the sudden passage of a current through the coil, is proportional to the total quantity of electricity passing through the galvanometer. This assumes that the entire charge passes through the coil before the coil begins to move (see Par. 188,

p. 275). Let D be the maximum galvanometer throw in centimeters. Then,

$$Q = KD \tag{194}$$

where Q is the quantity and K is the galvanometer constant.

To make the measurement, the apparatus is connected as shown in Fig. 243. A battery B supplies the current for the apparatus. The measurement may be made on either charge or discharge of the condenser, or check measurements may be made using both charge and discharge. If the condenser is at all leaky, the discharge method is preferable.

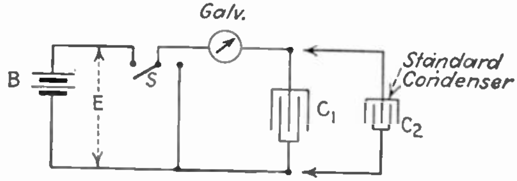


FIG. 243.—Ballistic method of measuring capacitance.

When the switch S is closed to the left, the condenser C_1 is charged through the galvanometer and the maximum throw of the galvanometer is read. Several check readings should be taken. The galvanometer should return immediately to zero. If it shows a steady deflection, a leaky condenser is indicated. In a corresponding manner the ballistic throw of the galvanometer may be read on discharge by closing switch S to the right after charging. Let D_1 be the deflection of the galvanometer when C_1 is connected, Q_1 the quantity going into the condenser and E the voltage across the condenser. Then, by Eq. (194),

$$Q_1 = KD_1,$$

also

$$Q_1 = C_1E,$$

where C_1 is the unknown capacitance. Therefore,

$$C_1E = KD_1. \tag{I}$$

If now the standard capacitance C_2 be substituted for the unknown capacitance and another set of readings taken,

$$Q_2 = KD_2,$$
$$C_2E = KD_2. \tag{II}$$

Dividing Eq. (I) by Eq. (II),

$$\frac{C_1E}{C_2E} = \frac{KD_1}{KD_2}.$$

$$C_1 = C_2\frac{D_1}{D_2}. \tag{195}$$

$E\frac{C_2}{D_2}$ is the galvanometer constant.

It is often desirable to use an Ayrton shunt in such measurements, as it gives the apparatus greater range. When such a

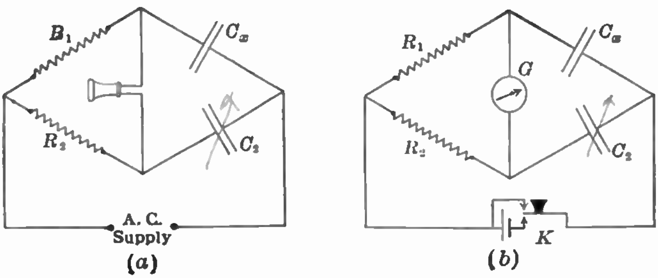


FIG. 244.—Bridge methods of measuring capacitance.

shunt is used, proper correction must be made for its multiplying power. Also, it is convenient to install either a single-pole or a double-pole, double-throw switch so that C_2 may be substituted easily for C_1, and *vice versa*.

231. Bridge Method.—In the bridge method, two capacitances form adjacent arms of a Wheatstone bridge and two resistances form the other two arms (Fig. 244(a)). An alternating-current supply is preferable. The secondary of an induction coil may be used as the source of power or a battery with a key may be made to charge and discharge the system as in Fig. 244(b). A telephone is used as a detector except in (b). Let C_x be the unknown capacitance and C_2 a standard which may or may not be adjustable. R_1 and R_2 are two known resistances, one of which should be adjustable unless C_2 is so.

Either C_2, or one of the resistances, is adjusted, until there is no sound in the telephone, showing that the bridge is in balance. Under these conditions,

$$\frac{C_x}{C_2} = \frac{R_2}{R_1},$$

$$C_x = C_2\frac{R_2}{R_1}. \tag{196}$$

When a battery is used, a double-contact key K is necessary (Fig. 244(b)). K is pressed and released, and, until the bridge is balanced, the galvanometer will deflect both upon charge of the system when the key is pressed, and upon discharge when the key is released. The bridge is balanced when the galvanometer does not deflect on either charge or discharge. Equation (196) is then applicable.

In the above measurements, it is assumed that there is little if any leakage through the condensers.

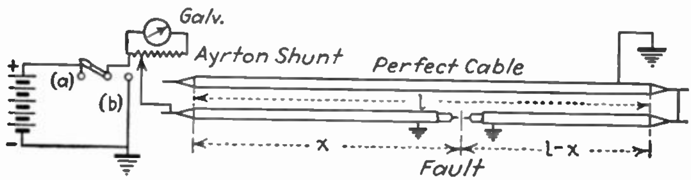


FIG. 245.—Locating an open in a cable.

232. Location of a Total Disconnection in Cable.—In Chap. V, it is shown that a grounded fault in a cable can be located by suitable resistance measurements, such as the Murray and Varley loop tests. If a cable be totally disconnected, and its broken ends remain insulated, these loop tests are impossible. The distance to the fault may now be determined by capacitance measurements. The connections are shown in Fig. 245. The capacitance C_1 of the length x to the fault is first measured by the ballistic method. If a *similar* perfect cable, of length l, parallels the faulty cable, the two are looped at the far end and a measurement is made of the combined capacitance. This capacitance C_2 is the sum of the capacitance of the length l of the perfect cable and the capacitance of the length $l - x$ of the faulty cable. Hence C_2 is the capacitance of the length of cable $l + l - x = 2l - x$.

Let c be the capacitance per foot, assumed to be the same for each cable.

$$C_1 = xc = KD_1 \tag{I}$$

where K is the galvanometer constant and D_1 is the deflection corresponding to C_1.

Likewise,

$$C_2 = (2l - x)c = KD_2. \tag{II}$$

Dividing Eq. (I) by Eq. (II),

$$\frac{x}{2l - x} = \frac{D_1}{D_2},$$

$$x = l\frac{2D_1}{D_1 + D_2}. \tag{197}$$

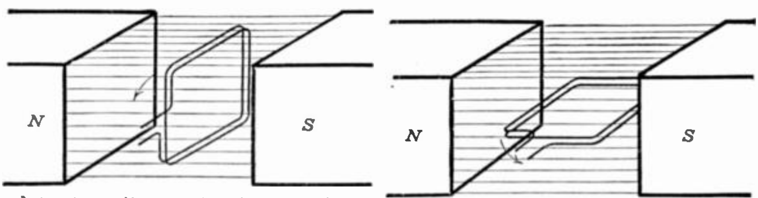
The capacitance per unit length and the total capacitance do not enter into the final equation, so that it is not necessary to use a standard condenser for the calibration of the galvanometer. The capacitances of the various lengths are proportional to the galvanometer deflections when corrected for the setting of the Ayrton shunt.

.637 = $\frac{2}{\pi}$

CHAPTER XI

THE GENERATOR

Definition.—A generator is a machine which converts mechanical energy into electrical energy. This is accomplished by means of an armature carrying conductors on its surface and acting in conjunction with a magnetic field. Electromotive force is generated by the relative motion of the armature conductors and the magnetic field, and when current is delivered to an external circuit, electrical energy is given out by the armature. In the direct-current generator the field is usually stationary and the armature rotates. In most types of alternating-current generator, the armature is stationary and the field rotates. Either the armature or the field may be driven by mechanical power applied to its shaft.



(*a*) Maximum lines passing through coil (*b*) No lines passing through coil

Fig. 246.—Simple coil rotating in a magnetic field.

233. Generated Electromotive Force.—It is shown in Chap. IX that if the flux linking a coil is varied, an e.m.f. is *induced* in the coil. The action of the generator is based on this principle. The flux linking the armature coils is varied by the relative motion of armature and field.

In Fig. 246 a coil rotates in a uniform magnetic field produced by a N-pole and a S-pole. In Fig. 246(a) the coil is perpendicular to the magnetic field and the maximum possible flux links the coil. Let this flux be ϕ.

If the coil be rotated counterclockwise a quarter of a rotation, it will lie in the position shown in (b) and as the coil is parallel to the flux, no lines now link the coil. Therefore, in a quarter-rotation the flux which links the coil has been decreased by ϕ lines. The average e.m.f. induced in the coil during this period is, therefore,

$$e = N\frac{\phi}{t}10^{-8} \text{ volts (Chap. IX, Eq. (130), p. 288)}$$

where N is the number of turns in the coil, and t is the time required for a quarter-rotation. But $t = \frac{1}{4s}$ where s = *revolutions per second.* Therefore, the average voltage during a quarter-revolution is

$$e = 4Ns\phi 10^{-8} \text{ volts.} \qquad (198)$$

Hence, in a generator, the induced e.m.f. is proportional to the armature turns, the speed in revolutions per second, and the flux which links the armature turns.

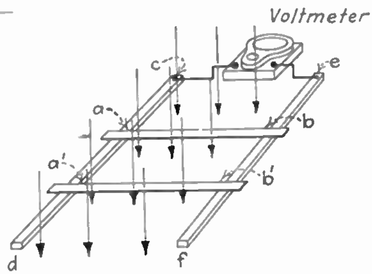


FIG. 247.—Conductor cutting uniform magnetic field.

The generation of e.m.f. in a moving coil of this type, which is similar to those used in dynamos, may be analyzed also by considering the total e.m.f. as due to the sum of the e.m.fs. induced in each side of the coil. The e.m.f. of one turn is the sum of the e.m.fs. in each conductor forming the sides of the turn, since these conductors are connected in series by the end connections of the turn. The e.m.fs. are thus considered as being induced in the *conductor* rather than induced in the coil. This in no way conflicts with the fact that the induced e.m.f. is also due to the change of flux *linked* with the coil. The same total e.m.f. is obtained under either assumption, the flux linkages of the coil are changed, or the coil sides cut the magnetic flux.

Consider the conductor *ab* (Fig. 247), free to slide along two parallel metal rails *cd* and *ef* spaced *ab* cm. apart. The rails are connected at one end by a voltmeter. A uniform magnetic field, having a density of B lines per square centimeter, passes perpendicularly through the plane of the rails and conductor.

Let the conductor ab move parallel to itself at a uniform velocity v to the position $a'b'$. While this movement is taking place, the voltmeter will indicate a certain voltage. This voltage may be attributed to either of two causes:

1. As conductor ab moves to position $a'b'$, the flux linking the conducting loop formed by ce, the rails, and ab, is increased, because of the increasing area of this loop.
2. An e.m.f. is generated in the conductor ab since it cuts the magnetic field.

Similarly, the e.m.f. developed by the coil in Fig. 246 may be attributed to the e.m.fs. generated in the conductors on opposite sides of the coil by their *cutting* of magnetic lines. These conductors are connected in series by the end conductors, or connectors, which in themselves generate no appreciable e.m.f. The direction of the e.m.fs. developed in the coil sides are such that these e.m.fs. are additive.

The e.m.f. induced in a single conductor which cuts a magnetic field is

$$e = Blv10^{-8} \text{ volts} \tag{199}$$

where B, l, and v are mutually perpendicular.

B is the flux density of the field in gausses, l is the length of conductor in *centimeters*, and v is the velocity of the conductor in centimeters per second.

That the e.m.f. induced by a change of flux linked with a coil is the same as that obtained by considering the e.m.fs. generated by the cutting of magnetic lines by the conductors which make up the coil may be illustrated by a concrete example. Let the density of the flux be 100 lines per square centimeter (Fig. 247). The distance ab is 30 cm. and $aa' = bb' = 20$ cm. The conductor ab moves at a uniform velocity v to position $a'b'$ in 0.1 sec. What is the e.m.f. across ce?

The change of flux linking the coil is

$$\phi = 30 \times 20 \times 100 = 60{,}000 \text{ lines.}$$

This change occurs in 0.1 sec.

By Eq. (130), p. 288,

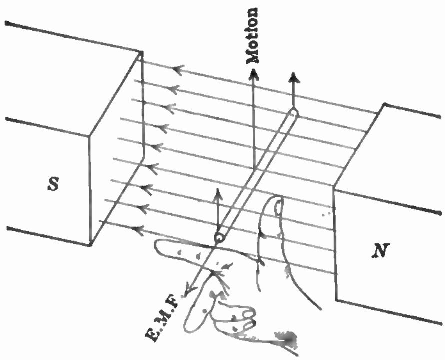
$$e = 1\,\frac{60{,}000}{0.1}\,10^{-8} = 0.006 \text{ volt.}$$

Applying Eq. (199),

$$v = \frac{20}{0.1} = 200 \text{ cm. per second.}$$

$$e = 100 \times 30 \times 200 \times 10^{-8} = 0.006 \text{ volt.}$$

The same result is obtained whether the e.m.f. is considered as being induced by the conductor itself *cutting* the field or as being induced by the change in flux *linking* the coil.



Forefinger along lines of force. Thumb in direction of motion. Middle finger gives direction of induced e.m.f.

FIG. 248.—Fleming's right-hand rule.

234. Direction of Induced Electromotive Force. Fleming's Right-hand Rule.—A definite relation exists among the direction of flux, the direction of motion of the conductor, and the direction of e.m.f. induced in the conductor.

A convenient rule for determining this relation is the *Fleming right-hand rule.* In this rule the fingers of the *right* hand are utilized as follows:

Set the forefinger, the thumb, and the middle finger of the right hand at right angles to one another (Fig. 248). If the *forefinger points along the lines of flux and the thumb in the direc-*

tion of motion of the conductor, the middle finger will point in the direction of the induced e.m.f.

This rule is illustrated by Fig. 248.

235. Electromotive Force Generated by Rotation of a Coil.—A coil of a single turn is shown in Fig. 249(*a*). The coil rotates in a counterclockwise direction at a uniform speed in a uniform magnetic field. As the coil assumes successive positions, the e.m.f. induced in it changes. When it is in position 1, the e.m.f. generated is zero, for in this position neither active conductor is *cutting* magnetic lines but is moving parallel to these lines. When the coil reaches position 2 (shown dotted), its conductors are

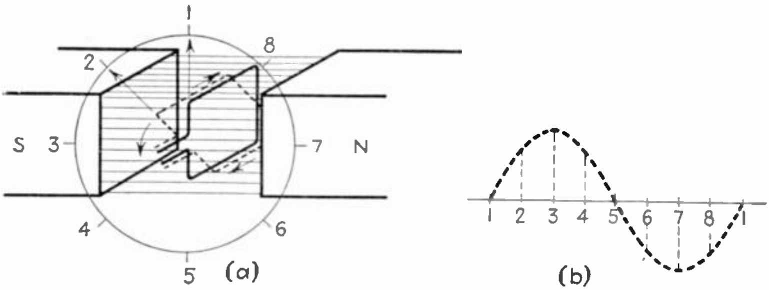


FIG. 249.—E.m.f. induced in coil rotating at constant speed in uniform magnetic field.

cutting across the lines *obliquely* and the e.m.f. has a value indicated at 2 in Fig. 249(*b*). When the coil reaches position 3, the conductors are cutting the lines *perpendicularly* and are cutting therefore at the maximum possible rate. Hence, the e.m.f. is a maximum when the coil is in this position. At position 4, the e.m.f. is less, due to a lesser rate of cutting. At position 5, no lines are being cut and, as in 1, there is no e.m.f. In position 6, the direction of the e.m.f. in the conductors will have reversed, as each conductor is under a pole of opposite sign to that for positions 1 to 5. The e.m.f. increases to a negative maximum at 7 and then decreases until the coil again reaches position 1. After this the coil merely repeats the cycle.

This induced e.m.f. is alternating in direction and an e.m.f. varying in the manner shown is a *sine wave* of e.m.f. This alternating e.m.f. may be impressed on an external circuit by means of two *slip-rings* (Fig. 250). Each ring is continuous and is insulated from the other ring and from the shaft. A metal or a carbon brush rests on each ring and conducts the current from the coil to the external circuit (see Vol. II, Chap. I).

If a *direct current* is desired, that is, one whose direction is always the same, such rings cannot be used. A direct current must always flow into the external circuit in the *same direction.*

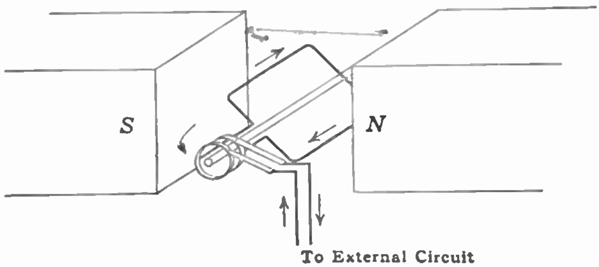


FIG. 250.—Current taken from rotating coil by means of slip-rings.

As the coil current must necessarily be alternating, since the e.m.f. which produces it is alternating, as just shown, this current must be rectified before it is allowed to enter the external circuit. This rectification can be accomplished by using a split ring as shown in Fig. 251. Instead of using two rings, as in Fig. 250, only one ring is used. This is split by saw cuts at two points diametrically opposite, and the ends of the coil are connected to the sections or segments so produced.

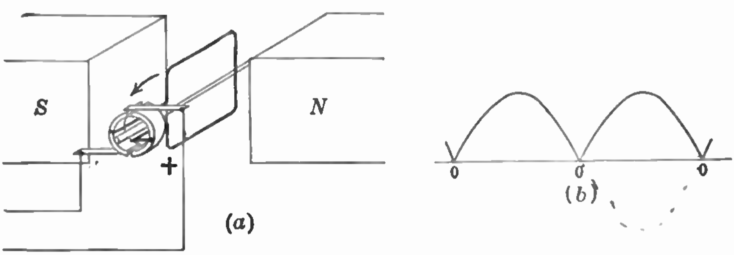


FIG. 251.—Rectifying effect of a split ring or commutator.

A consideration of Fig. 251 will show that as the direction of the current in the coil reverses, the connections to the external circuit are reversed. Therefore, the direction of flow of current in the external circuit is not changed. The brushes pass over the cuts in the ring when the coil is perpendicular to the magnetic field, or in the so-called *neutral plane,* and no e.m.f. is being generated, as at points 1 and 5 of Fig. 249. These neutral points are marked 0-0-0 in Fig. 251(*b*).

By comparing Fig. 249(*b*) with Fig. 251(*b*), it will be seen that the negative half of the wave has been reversed and made positive.

A voltage with zero value twice in each cycle (Fig. 251) could not be used commercially for direct-current service. Also, a single-coil machine would have a small output for its size and

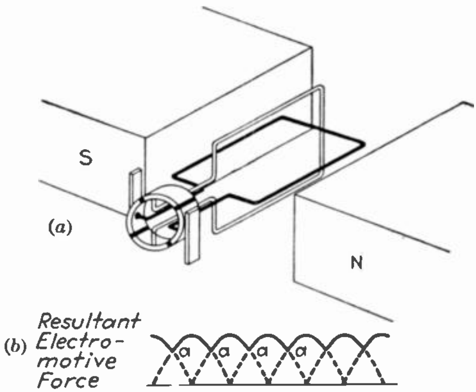


Fig. 252.—Effect of two coils and four commutator segments on the electromotive force wave.

weight. The e.m.f. wave of Fig. 251 may be made less pulsating by the use of two coils and four commutator segments (Fig. 252(*a*)). This gives an *open-circuit* type of winding, since it is impossible to start at any one commutator segment and return to this segment by following through the entire winding. In this particular winding the full e.m.f. generated in each coil is not utilized, as one coil passes out of contact with the brushes at points *a, a, a* (Fig. 252(*b*)), and the voltage shown by the dotted lines is not used.

236. Armature.—It has just been shown that in a generator there must be conductors which cut a magnetic field, and a commutator must be provided for reversing the induced currents at the proper time and for conducting them to the external circuit. These elements are supplied by the armature. Also, the armature must be designed so that the power output of the generator is sufficiently large in proportion to the weight and cost to justify the generator economically.

The commercial armature must have a core of iron which not only carries the copper conductors through the magnetic field but also forms part of the magnetic circuit. Without the iron core the magnetic field would be so weak that little power output would result. The armature consists of a cylindrical core of iron laminations (p. 392) mounted on a shaft so that it may rotate in the magnetic field, which is produced by the field poles. The faces of the poles are practically concentric with the armature, so that the lengths of the air-gaps in the magnetic circuit are relatively small, thus reducing the magnetic reluctance.

The conductors are placed in slots on the surface of the armature (p. 394). This construction is advantageous in that the conductors are held firmly in position, and the teeth permit a short air-gap to be used. Every effort is made to place as much copper as possible in the slots, since the generator output is proportional to the active copper. Also, the conductors must be placed and connected so that their e.m.fs. are additive and the full generated e.m.f. is delivered to the brushes. Except in the smallest machines, air ducts are provided for ventilating the armature.

In order to reverse the alternating current induced in the conductors, and to conduct the resulting rectified current to the external circuit as direct current, a commutator with appropriate brushes is necessary. A relatively large number of commutator segments is essential in order that the e.m.f. ripples shall not be too pronounced (Fig. 254), and that the voltage between segments may not be so high that flash-overs occur.

It is necessary to study armature construction, and the methods of winding the armature, if one is to select the type of armature which will give the best results under given conditions. Also, an understanding of the operation of the generator itself depends on a thorough knowledge of the armature assembly.

237. Gramme-ring Armature.—The gramme-ring type of armature was used with the early generators. It is simple in its electrical connections and in its general construction. The armature core consists of an iron ring or more often of a hollow cylinder with insulated wire wound spirally about the ring (Fig.

253) with taps taken from the wire at regular intervals and connected to commutator segments. This winding also has the advantage that the same winding is adapted to any number of poles, if the voltage limitations do not prevent On the other

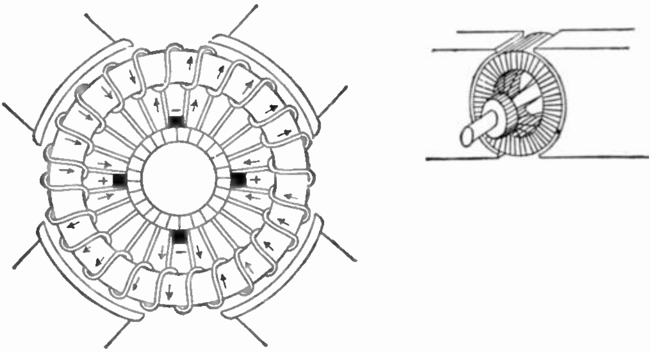


FIG. 253.—Gramme-ring winding.

hand, it has the disadvantage that the portions of the conductors which lie inside the ring cut no flux and act merely as connectors for the active portions of the conductors. Because of the small proportion of active conductors, a relatively large amount of copper is required in such a winding. In small machines, there is not sufficient room to carry these inactive conductors back through the armature core. In a gramme-ring winding, formed coils cannot be used, and this makes the winding expensive.

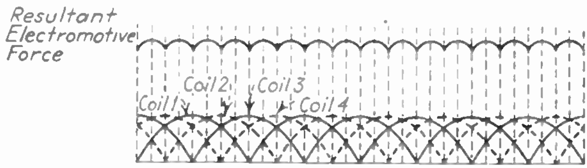


FIG. 254.—Resultant electromotive force due to four series-connected coils between brushes.

It will be noted that in a gramme-ring winding the e.m.f. between brushes is the sum of the e.m.fs. of all the coils that lie between brushes. When one coil connection passes a brush, an adjacent coil connection moves forward to take its place. Figure

254 shows the e.m.f. between brushes due to four coils it being assumed that the e.m.f. curve for each coil is a sine wave. The e.m.f. of each coil is plotted separately. Owing to the positions of the individual coils, these e.m.fs. do not all have their zero value at the same time, nor do they reach their maximum value at the same time. The resultant e.m.f. at any instant is the sum of these individual e.m.fs. at this instant. The resultant e.m.f. should be compared with the resultant e.m.f. obtained with the open-coil winding shown in Fig. 252, in which the resultant e.m.f. does not equal the sum of the individual e.m.fs. but is made up of the tops of the individual waves. That is, in the gramme-ring type of winding, every coil is contributing e.m.f. at all times except during those brief intervals when it is undergoing commutation.

It will be noted that a fairly smooth resultant e.m.f. is obtained with four coils, the "ripples" being noticeable but small. A gramme-ring winding is a *closed-coil* winding, since it is possible to start at any one point in the winding and return to the same point by passing continuously through the winding.

Although this type of winding is not used in the modern type of dynamo, it is frequently advantageous to employ it diagrammatically in considering armatures in actual operation. The turns do not cross one another, and consequently their electric and magnetic effects are readily visualized. Qualitatively, these effects are identical with those of the drum-wound armature type used in modern dynamos.

238. Drum Winding.—The objections to the ring winding are overcome by the use of the drum winding. The conductors of this winding all lie upon the surface of the armature and are connected to one another by front and back connections or coil ends (*ad* and *bc*, Fig. 255, are coil ends). With the exception of these end connections, all the armature copper is "active," that is, it cuts flux and so is active in generating e.m.f.

The sides of each coil should be separated by about one pole pitch (the distance between centers of adjacent poles). If one coil side is under a *N*-pole, the other is under a *S*-pole, and as both move in the same direction but under different polarities, the e.m.fs. of these conductors, in space, will be in opposite directions (Fig. 255). Owing to the manner in which these conductors are connected at their ends, the e.m.fs. in the two coil sides of the individual coils are additive.

In most gramme-ring windings, and in the earlier drum-wound armatures, the surface of the armature core was smooth. The conductors were held in position partly by projecting pins, and were prevented by binding wires from flying out under the action of centrifugal force. The smooth core construction has been superseded by the "ironclad" type where the conductors are embedded in slots (Fig. 258). The slots are lined with insulation

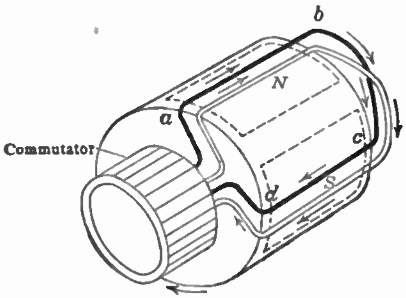


FIG. 255.—Two coils in place on a four-pole, drum-wound armature.

and the conductors are held firmly by fiber or other non-conducting wedges in the larger machines (Fig. 285, p. 394), and by binding wires in the smaller machines (Fig. 275, p. 387). These constructions are much better mechanically than the smooth-core type and also permit a much shorter air-gap. On the other hand, as the coils are embedded in iron, they have relatively high self-inductance. This makes satisfactory commutation more difficult, and the flux pulsations due to the armature teeth give pole-face and tooth losses.

239. Lap Winding.—Direct-current armatures are usually wound with form-made coils (Fig. 256). These coils are wound on machines and the turns are then bound together with cotton or mica tape. The coils are bent into proper shape by another machine. The ends are left bare so that they may be soldered later to the commutator bars. The span of the coil, or the *coil pitch*, should be equal or nearly equal to the pole pitch, so that when one side of a coil is under a N-pole the other side is under a S-pole.

A saving in copper in the end connections and slightly better commutation result if the coil span is made less than the pole pitch. Such a winding is called a *fractional-pitch winding*. With direct-current machines, the pitch may be as low as nine-tenths full pitch. If the pitch is too small, too great a reduction in the induced e.m.f. results.

Usually two coil sides which may be made up of multiple coils (p. 367), occupy one slot, one coil side lying at the top and

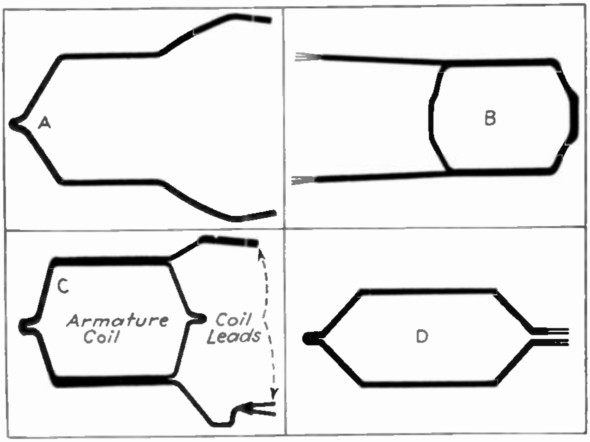


FIG. 256.—Formed armature coils.

the other at the bottom of the slot. That is, if the side of one coil is in the bottom of a slot, its opposite side lies in the top of some other slot. This allows the end connections to be easily made as the coil ends can be bent around one another in a systematic manner, passing from the bottom to the top layer by means of the peculiar twist in the ends of the coils.

In the simple-lap type of drum winding, the coil connection is made from one segment, through the two sides of a coil and thence *back* to the next adjacent segment, from which similar connection is made to the next coil, etc.

In the study and design of windings, the group of wires which constitutes the side of a single coil, and which is almost always wrapped with tape as a unit is considered as being a *winding element*. Such winding elements are shown at *ab* in the two coils in Fig. 257. Even when there are several conductors in a coil side, they will be shown as a single conductor in the wiring diagram, as in Fig. 257. Obviously there will be twice as many of these elements as there are coils. The number of elements that the coil advances on the back of the armature is the *back pitch* of the winding and will be denoted by y_b. This back pitch is determined by the span of the connection bc (Fig. 255). The number of elements spanned on the commutator end of the armature is called the *front pitch* and will be designated by y_f. This may be *greater or less* than the back pitch *but not equal* to it.

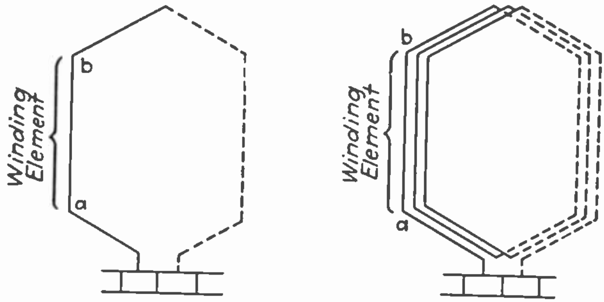


FIG. 257.—Single coil representing a three-turn coil of an armature winding.

If it be greater, the winding is retrogressive; that is, it advances in a counterclockwise direction when viewed from the commutator end. If the front pitch be less than the back pitch, the winding is progressive; that is, it advances in a clockwise direction when viewed from the commutator end. This is illustrated in Figs. 258 and 259. Element 1 is connected on the back of the armature to element 10. Therefore, the back pitch $y_b = 9$. Element 10 is then connected back to 3 on the front of the armature, the connection being made at the commutator segment. Therefore, the front pitch $y_f = 7$. This winding is therefore *progressive.*

As most windings are now made in two layers, only two-layer windings will be considered. The elements lying in the tops of the slots will be given odd numbers and those in the bottoms of the slots even numbers (Fig. 258). If one side of a coil lies in the bottom of a slot, the other side must lie in the top of some other slot. Hence y_b and y_f must both be odd. If y_b

and y_f were both even, all the elements would lie in *either* the tops of the slots or in the bottoms of the slots and it would

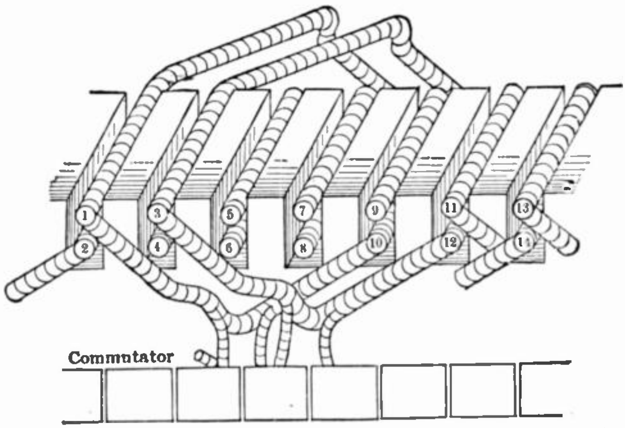


Fig. 258.—Simplex lap winding having back pitch of 9 and front pitch of 7.

be impossible for one side of a coil to lie in the bottom of a slot and the other side of the same coil to lie in the top of some

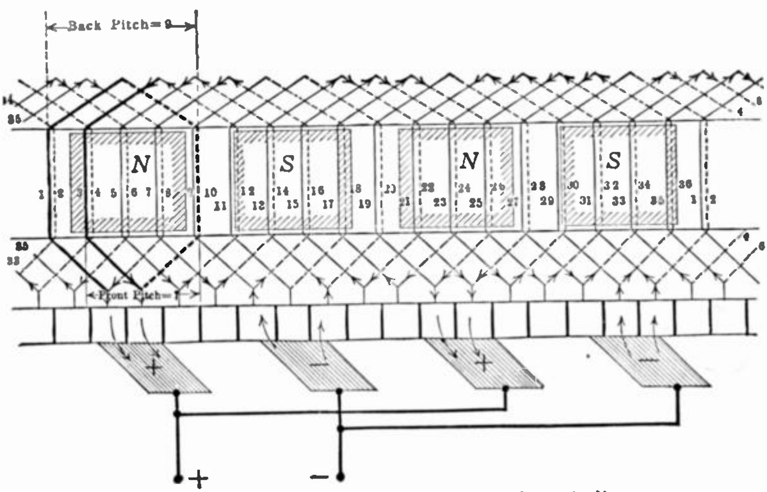


Fig. 259.—Development of a four-pole lap winding.

other slot. Hence y_b and y_f must both be odd if the winding is to be properly placed on the armature.

It follows that the front and back pitches must differ from each other by 2. That is,

$$y_b = y_f \pm 2 \tag{200}$$

The average pitch is

$$y = \frac{y_b + y_f}{2}. \tag{201}$$

The plus sign in Eq. (200) indicates that the winding is progressive, that is, progresses in a clockwise direction when viewed from the commutator end. The minus sign indicates a retrogressive winding whose advance is in a counterclockwise direction when viewed from the commutator end.

240. Commutator Pitch.—It will be seen that for every coil one commutator segment is necessary. Therefore, the number of commutator segments

$$N_c = N = \frac{Z'}{2}$$

where Z' is the total number of winding elements on the surface of the armature and N is the number of coils.

From Figs. 258 and 259 it will be seen that the winding advances *one commutator segment for each complete turn*. Hence, in such a winding, the commutator pitch $y_c = 1$.

In designing a winding it is necessary that the opposite sides of each coil lie under different poles so that the two e.m.fs. generated in the coil sides may be additive. Hence, the average pitch should be nearly equal to the number of elements per pole.

The three fundamental conditions to be fulfilled by a *lap winding* are:

1. *The pitch must be such that the opposite sides of a coil lie under unlike poles.*
2. *The winding must include each element once and only once.*
3. *The winding must be reentrant or must close on itself.*

Example.—Assume that the armature of a four-pole dynamo has 18 slots. Design a two-layer lap winding having two elements per slot.

There are 36 elements. The average pitch should be nearly equal to $36 \div 4 = 9$. The back pitch can be made equal to 9.

$$y_b = 9; \quad y_f = 7$$

Starting at 1, the winding will progress as follows:

1–10–3–12–5–14–7–16–9–18–11–20–13–22–15–24–17–26–19
28–21–30–23–32–25–34–27–36–29–2–31–4–33–6–35–8–1.

This is a *winding table.* It is useful in checking the winding. By inspection it may be determined whether or not each conductor is included once, and only once, and whether the winding closes at the same conductor, 1 in this case, at which it began. The winding is shown in Fig. 259 as if it were split axially and laid out flat. It will be noted that the brushes rest on segments to which are connected elements as 1 and 19, for example, which lie midway between the poles. That is, the brushes are connected to conductors in which practically no e.m.f. is being induced.

241. Multiple Coils.—In machines of larger ratings it is often necessary to place several coil sides or elements in one slot, usually four, six, or eight. More than eight coil sides per slot are rarely used. The reason for placing several coil sides in a slot is as follows:

The commutator segments must not be too few in number. The average voltage between adjacent segments cannot safely exceed 15 volts or thereabouts, because of danger of flash-over. Therefore, the number of segments is determined by the voltage between brushes, and there is a lower limit to the number. Moreover, with too few segments the e.m.f. ripple may become objectionable (Fig. 252(*b*), p. 358). Hence, in many machines with the necessary number of commutator segments, if two elements per slot were used, one in the top layer and one in the bottom layer, a large number of slots would be necessary. This would reduce the size of the slots and make the space factor (ratio of the copper cross-section to the slot cross-section) low. Also, the tooth root would be so narrow that the teeth would be mechanically weak. By placing more than two elements in each slot the number of slots is reduced and larger slots result. This also reduces the cost of winding.

Two, three, or four individual coils are taped together to form a single coil as shown at *B* in Fig. 256. Such a coil is called a *multiple coil,* and it may be more specifically designated as a double coil, a triple coil, etc. Such a single coil may be placed as a unit in the appropriate slots. An examination of the armature of Fig. 275(*a*) (p. 387) shows four wires running from the top of each slot to the commutator, indicating a quadruple coil.

The numbering and connections of the elements are in no way different from those already described in the case of only two elements per slot.

The selection of the pitch, where several elements per slot are used, is more restricted than with two elements per slot.

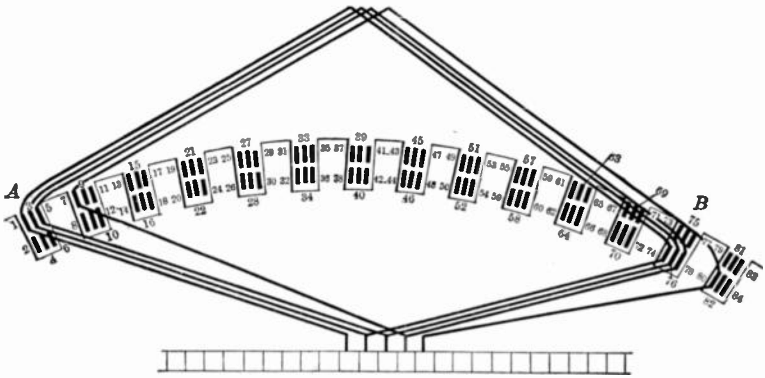


FIG. 260.—Method of connecting a triple coil.

Assume that a six-pole machine has 72 slots and six elements per slot. The total number of elements on the armature surface

$$Z = 72 \times 6 = 432.$$

The pitch should be approximately

$$y = \frac{432}{6} = 72.$$

Let

$$y_b = 71,$$
$$y_f = 69.$$

If this back pitch is used, a coil must reach from element 1 to element 72 (Fig. 260). Then the next element 3, which is taped to element 1, will obviously reach to element 74. These two coils, therefore, span different distances on the armature, since elements 72 and 74 lie in different slots. Hence, although the left-hand sides of these two coils and element 5 are taped together as a unit, the right-hand elements cannot be taped together, since they lie in different slots. This can be seen from a study of Fig. 260. In practice, it is desirable that if any number of coil sides are to be placed together in the top of any slot, their opposite sides should all be placed together in the bottom of some other slot. This makes it possible to tape all these coils together and place them as a unit in both slots.

Therefore, if in the above case $y_b = 73$ and $y_f = 71$, the coil containing element 1 will connect from the upper *left-hand* side of slot A to the lower *left-hand* side of slot B, that is, from element 1 to element 74. Element 3 will connect from the center and top

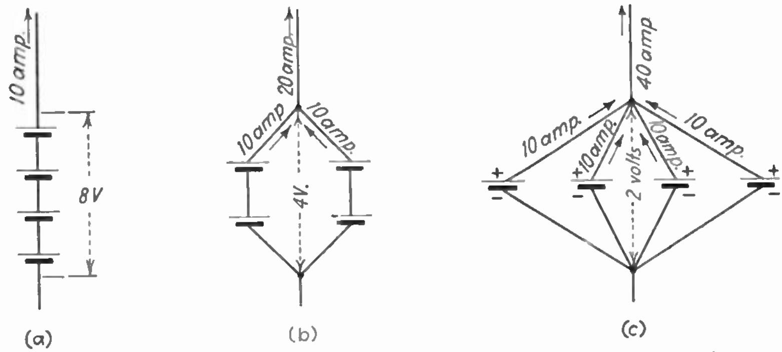


FIG. 261.—Series, series-parallel, and parallel arrangement of batteries.

of slot A to the center and bottom of slot B, and element 5 will connect from the upper right-hand side of slot A to the lower right-hand side of slot B. As all three coils now span the same distance on the armature, they will be equal in size, form, etc. Moreover, the three single coils can be taped together to form a *triple coil* and placed in the two slots as a unit. Therefore, if the sides of three adjacent coils are in the top of a slot, their other sides should lie together in the bottom of some other slot. This condition is obtained by making the back pitch one greater than a multiple of the number of coil sides or elements per slot. For example, in the illustration just given, y_b is equal to 73, that is, one greater than 72, and 72 is a multiple of 6.

Coils taped together and placed in the slots in the foregoing manner are called *multiple coils*.

242. Paths through Armature.—In Fig. 261(a) are shown four battery cells in series. Each has an e.m.f. rating of 2 volts, a current rating of 10 amp., and a power rating of 20 watts. When the four cells are connected in series aiding, the e.m.f. rating of the battery becomes 8 volts, the current rating 10 amp., and the power rating 80 watts. With this series connection there is but one current path through the battery. If the four cells be arranged in two groups of two in series as in (*b*), the result is two paths for the current through the battery, the e.m.f. rating is 4 volts, the current rating 20 amp., and the power rating is $4 \times 20 = 80$ watts.

If the four cells be arranged in parallel as in (*c*), the result is four paths for the current, the e.m.f. rating is 2 volts, the current rating is 40 amp., and the power rating remains 80 watts.

Let Z be the number of equal cells in a battery, e the e.m.f. rating per cell, i the current rating per cell, and P' the paths through the battery.

Then, the e.m.f. rating of the battery

$$E = \frac{Ze}{P'} \text{ volts.} \tag{202}$$

The current rating

$$I = P'i \text{ amp.} \tag{203}$$

The power rating

$$P = EI = Zei \text{ watts.} \tag{204}$$

It is to be noted that the power rating is independent of the manner of connecting the batteries, so long as all act in conjunction to supply current to the external circuit.

These relations among the cells of a battery also apply to the conductors or inductors in an armature. Each inductor is a source of e.m.f., and in the normal machine these e.m.fs. are all equal. These inductors may be connected in different combinations so that there are two, four, or more current paths through the armature. In the usual type of winding there must be at least two paths through the armature. The e.m.f. and current ratings are determined by Eqs. (202) and (203), where e and i are the e.m.f. and current ratings of each inductor. Moreover, as with the battery, the power rating of the armature is independent of the manner of connecting the inductors, so long as all act in conjunction to supply current to the external circuit.

To determine the number of parallel paths through an armature, start at one of the machine terminals, as, for example, the negative, and see how many different paths through the armature it is possible to follow in order to reach the positive terminal.

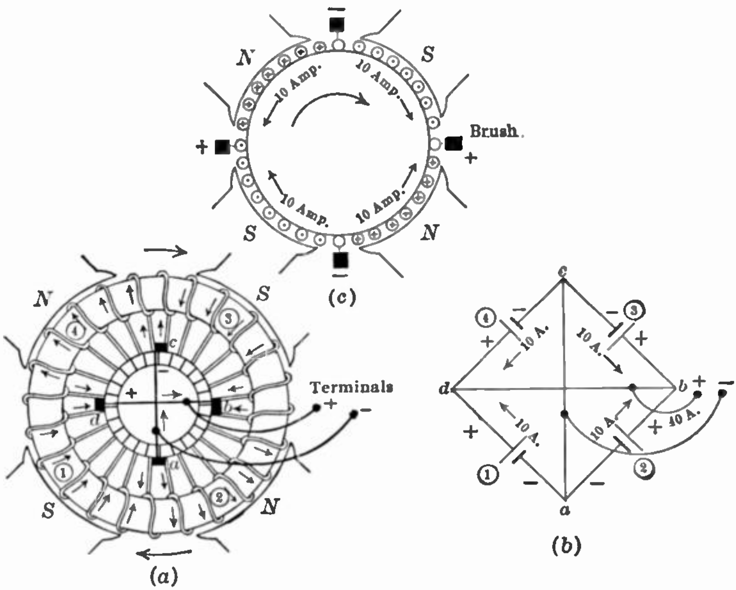


Fig. 262.—Four paths in parallel through an armature.

The simplest arrangement of conductors occurs in the gramme-ring winding. Figure 262(*a*) shows such a winding for a four-pole machine.

Starting at the − terminal, one path may be followed by going to brush *a*, through the winding at 1 to brush *d* and then to the + terminal.

A second path is obtained by going to brush *a*, through path 2 to brush *b* and then to the + terminal.

A third path is obtained by going to brush *c*, through path 3, then through brush *b* to the + terminal.

A fourth path is obtained by going to brush *c*, through path 4 to brush *d*, then to the + terminal.

This makes four separate paths between the − and + terminals, these paths being in parallel.

Assume that there are 10 amp. per path and 20 volts between brushes. The armature may be considered as equivalent to four batteries connected as in Fig. 262(*b*), each battery delivering 10 amp. at 20 volts. Battery 1 corresponds to path 1, battery 2 to path 2, etc.

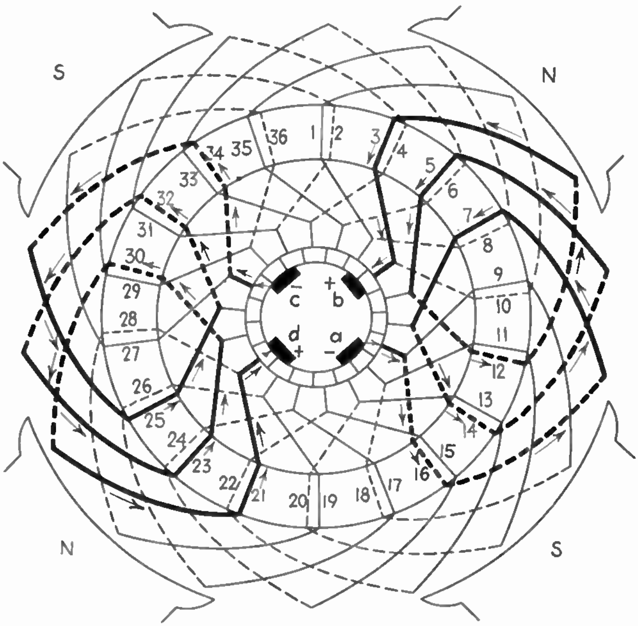


FIG. 263.—Heavy lines show two of the four parallel paths of lap winding.

It will be seen that the four batteries are connected in parallel because their four positive terminals are connected together, and their four negative terminals are connected together. The total current delivered will be 40 amp. at 20 volts. In a similar manner each path in the gramme-ring winding will deliver 10 amp., making 20 amp. per brush or 40 amp. per terminal. The potential difference between brushes will be 20 volts.

The paths through a drum winding are not so easy to follow as those through a ring winding since the end connections cross one another as the winding spirals along the armature surface.

Figure 263 shows developed in circular form the 18-slot drum winding of Fig. 259 (p. 365). For the sake of simplicity two paths are shown in heavy lines, one from brush *a* to brush *b*, the other from brush *c* to brush *d*. These constitute two paths. By tracing through the lighter lines, two more paths may be found, one

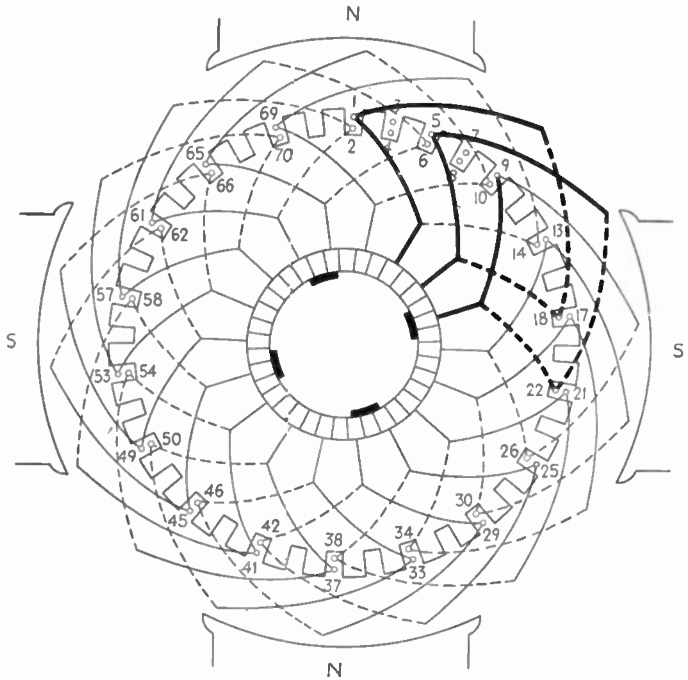
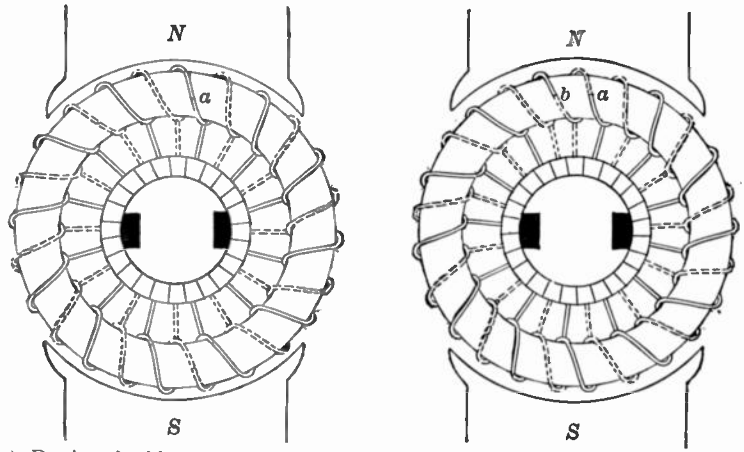


Fig. 264.—Duplex doubly reentrant lap winding—one winding only being shown.

between brushes *c* and *b*, the other between brushes *a* and *d*, making four paths in all.

In all simplex lap windings there are as many paths through the armature as there are poles.

243. Multiplex Windings.—Figure 264 shows a 36-slot, four-pole winding, in which every alternate slot is filled. There are two coil sides per slot. The back pitch y_b is 17, and coil side 1 connects to coil side 18 on the back of the armature. Coil side 18 then connects to 5 on the front of the armature, making the front pitch $y_f = 13$. Instead of returning to the coil side differing by 2 from the initial coil side, the return is made to a coil side differing by 4 from the initial coil side. Likewise, from coil side 5, connection is made at the back of the armature to coil side 22 and thence back to 9. Coil sides 3, 4 and 7, 8 are not connected to this winding. Furthermore, only alternate commutator segments are utilized. It will be seen that this winding closes on itself after going once around the armature; that is, this winding is reentrant and is in itself complete in the same manner as any simplex 18-slot winding (see Fig. 263).



(*a*) Duplex doubly reentrant gramme-ring winding.

(*b*) Duplex singly reentrant gramme-ring winding.

Fig. 265.

As this winding uses only alternate slots and alternate commutator segments, another winding, the duplicate of the first, can be placed in the vacant slots, this second winding having the same front and back pitch as the first, and being connected to the commutator segments not utilized by the first. This second winding will also close on itself and is reentrant.

These two windings are separate, and are insulated from each other on the armature, but are connected together electrically by the span of the carbon brushes on the commutator. This condition is more clearly shown in the simple gramme-ring winding of Fig. 265(*a*), where one winding is in solid lines and the other in dotted lines. The two windings on the armature of Fig. 264 and on the armature of Fig. 265(*a*) are in parallel, so that the number of paths is now twice that of a simplex lap winding. As each of the two windings closes on itself, the winding is *doubly reentrant*. With this type of winding it is necessary that the brush span at least two commutator segments.

When there are two such windings in parallel, the winding is *duplex*. Therefore, this is a *doubly reentrant duplex winding*. Three or more such windings can be placed on an armature, making the winding triplex, quadruplex, etc., the number of such windings being the multiplicity of the winding.

Let m = the multiplicity of the winding. The number of paths p' in a lap winding is

$$p' = mp \tag{205}$$

where p is the number of poles.

The relation between the back and the front pitch is

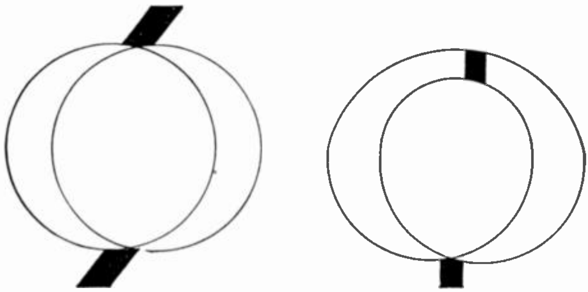
$$y_b = y_f \pm 2m. \tag{206}$$

This should be compared with Eq. (200), p. 366, where $m = 1$.

If the number of slots and hence of coils (Fig. 264) be odd, that is, if there are 35 or 37 coils and commutator segments, the winding will not close after having gone once around the armature, but will return one slot, or two coil sides, to the right or to the left of the coil side at which it started. If there are more than four coil sides per slot, the winding may return to the same slot at which it started, but removed by two coil sides from the coil side at which it started. Therefore, this winding does not close, or become reentrant, after having passed once around the armature, but must pass around once again before closing. This is illustrated in Fig. 265(b). The initial winding shown with the solid line starts at a. After passing once around the ring armature it does not close at a, as does the winding in Fig. 265(a), but terminates at b, one coil side removed from a. The second winding, shown dotted, starts at b and, after passing once around the armature, closes at a. Although the winding passes around the armature twice, it closes only once and so is *singly* reentrant. Therefore, this is a *singly reentrant duplex* winding. The singly reentrant and the doubly reentrant windings are the same electrically. Their difference is best illustrated by the two simple diagrams of Fig. 266.

From the foregoing it is seen that, with duplex lap windings, the commutator pitch $y_c = 2$; with triplex windings $y_c = 3$; etc.

244. Equalizer Connections in Lap Windings.—Lap windings may consist of several paths in parallel, the parallel connections being made through the brushes. If several batteries are connected in parallel and their e.m.fs. are not equal, currents circulate among the batteries, even when no external load is being supplied. This means a constant loss of energy and heats the batteries.



(a) Duplex doubly reentrant winding. (b) Duplex singly reentrant winding.
Fig. 266.—Duplex windings in diagrammatic form.

The same condition exists in generator armatures. Because of very slight inequalities in the air-gap, due to the wearing of the bearings, lack of mechanical alignment, etc., there may be slight differences of e.m.f. in the different paths through the armature. These differences of e.m.f. will cause currents to flow between different points in the armature, and these currents must flow through the brushes even when no current is being delivered by the generator. To relieve the brushes of this extra current, several points in the armature which should be simultaneously at the same potential are connected together by heavy copper bars or equalizers. This allows these circulating currents to flow from one point in the armature to another without passing through the brushes.

The action of the equalizer connections, however, differs from that of the brushes. The equalizing currents through the brushes are direct currents, which bring the different paths to equality of voltage by the IR drops which these currents produce in the different armature paths. The currents through the equalizer connections are alternating currents. These alternating currents produce m.m.fs. which react on the poles of the machine in such a manner that they tend to bring the *induced* e.m.fs. in every path to equality. They accomplish the equalization of voltage without the heating which the brush currents would produce.[1]

To make these equalizer connections, the number of coils should be a multiple of the number of poles, and the coils per pole should be divisible by some small number as 2 or 3. As an

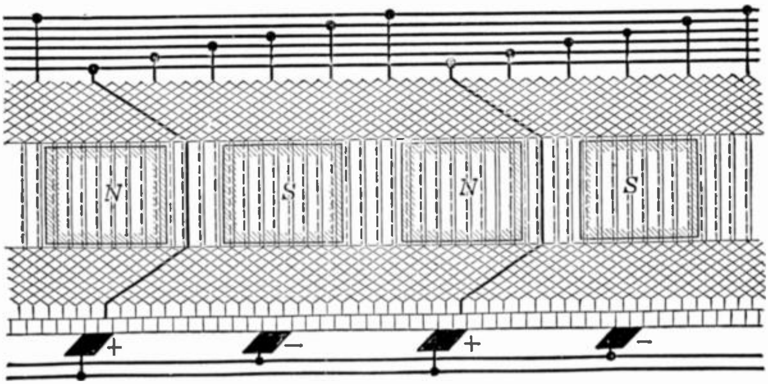


FIG. 267.—Simplex lap winding with equalizer connections.

example, assume an eight-pole generator having 12 slots per pole and two coil sides per slot. There will be 96 slots and 192 coil sides. The number of coil sides per pole will be 24. Let $y_b = 25$, and $y_f = 23$. A portion of the winding is shown in Fig. 267. It will be noted that every fourth coil is connected to an equalizer. The coils that are connected to the same equalizer occupy the same positions relative to the poles. (See the two half-coils drawn with heavy lines.) This is necessary as such coils at any instant should be generating the *same* e.m.f. It will be noted in Fig. 267 that the two segments under the two positive brushes are connected together by an equalizing connection.

Theoretically, every coil should be connected to an equalizer, but, as this would require an undue number of equalizers, it is sufficient, practically, to connect every third or fourth coil. This is the reason that the number of coils per pole should be divisible

[1] See MOORE, A. D., "Theory of the Action of Equalizer Connections in Lap Windings," *Elec. Jour.*, December, 1926, p. 624.

by a small number, as 2, 3, or 4. Figure 268 shows a large direct-current armature with the equalizer connections at the back of the armature.

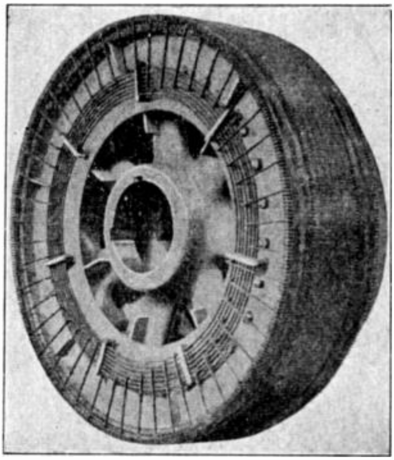
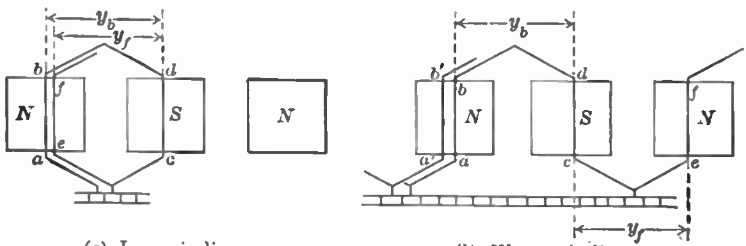


FIG. 268.—General Electric Company direct-current armature with equalizer connections.

245. Wave Winding.—It has been shown that with the *lap* winding a coil side under one pole is connected directly to a coil side which occupies a nearly corresponding position under the next pole. This second coil side is then connected *back* again to a coil side under the *original* pole, but removed two or more coil sides from the initial coil side. This is shown in Fig. 269(*a*), where coil side *ab* under a *N*-pole is connected to coil side *cd* having a corresponding position under the adjacent *S*-pole. Coil side *cd* is then connected back to *ef* which is adjacent to *ab* under the original *N*-pole. Obviously it would make no difference so far as the direction and magnitude of the induced e.m.f. in the winding are concerned if the connection, instead of returning to the same *N*-pole, advanced *forward* to the next *N*-pole, as in Fig. 269(*b*). The winding then passes successively



(*a*) Lap winding (*b*) Wave winding

FIG. 269.—Lap and wave windings.

every *N*-pole and *S*-pole before it returns to the original pole, as shown at *a′b′* in Fig. 269(*b*). The winding after passing once around the armature reaches conductor *a′b′* lying under the same pole as the initial conductor *ab*. When a winding advances from pole to pole in this manner, it is a *wave winding*. The number of winding elements spanned by the end connections on the back of the armature is called the *back* pitch and is denoted by y_b in Fig. 269(*b*). This is similar to the corresponding term in the lap winding shown in Fig. 269(*a*). The number of elements which the end connections span on the commutator end of the armature is the *front* pitch and is denoted by y_f. This should also be compared with Fig. 269(*a*). As in the lap winding, y_f and y_b must both be odd in order that one side of a coil may lie in the top of a slot and the other side in the bottom of a slot. Unlike the lap winding, y_f may equal y_b in the wave winding. The above is illustrated as follows:

A certain wave winding has a back pitch of 23 and a front pitch of 19. The average pitch

$$y = \frac{23 + 19}{2} = 21.$$

Both the front and back pitch may be 21 making the average pitch 21. In any event, the average pitch

$$y = \frac{y_b + y_f}{2} \tag{207}$$

may be either even or odd.

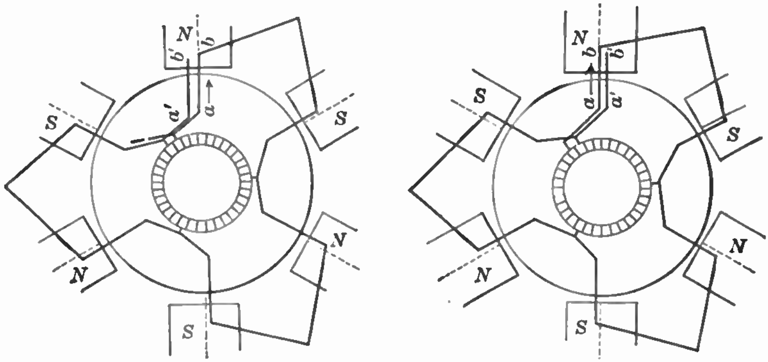
When the winding viewed from the commutator end falls in a slot to the left of its starting point as *a'b'* (Figs. 269(*b*) and 270(*a*)) after passing once around the armature, the winding is *retrogressive*. If, on the other hand, it falls to the right of its starting point, as shown in Fig. 270(*b*), it is *progressive*.

The wave winding is much more restricted in its relation to the number of slots and coils than the lap winding, for the following reason. In a simplex wave winding, after having passed once around the armature, the winding must fall *two* elements either to the right or left of the element at which it started. Thus in Fig. 270(*a*), if there are two elements per slot, and element *ab* lies in the bottom of one slot, element *a'b'* must lie in the bottom of the slot next to *ab*. As there are two elements in each slot, this means that elements *ab* and *a'b'* will differ from each other by 2.

Let y be the average pitch. Assume that the winding closes after passing once around the armature, which, of course, it should not do as this would constitute a short circuit. Then,

$$py = Z$$

where p is the number of poles and Z the number of coil sides or elements. But the winding must *not* close after passing once



(*a*) Retrogressive wave winding—34 commutator segments.

(*b*) Progressive wave winding—32 commutator segments.

FIG. 270.

around; in fact, it must not close until every slot is filled. Therefore, after having passed once around the armature, the product py cannot equal Z but must be $Z \pm 2$. That is

$$py = Z \pm 2,$$

or

$$y = \frac{Z \pm 2}{p}. \tag{208}$$

The plus sign indicates a *progressive* winding and the minus sign a *retrogressive* winding.

As an illustration, assume that a four-pole armature has 63 slots and four coil sides per slot, making 252 winding elements. Let the average pitch be 63, the front and back pitch being 63. As in the lap-winding diagrams, a single-turn coil will be used to represent a coil having several turns, as in Fig. 271. Starting at element 1, the winding will advance as follows:

1–64–127–190–(253 or 1).

That is, the winding will close on itself after going once around the armature, which condition constitutes a short circuit and makes the winding impossible (The method by which a winding may be placed in these slots will be shown later, p. 382.) Therefore, a simplex wave winding is impossible in a four-pole machine if 252 winding elements are to be included.

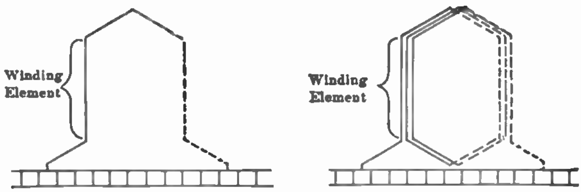


Fig. 271.—Single-turn coil representing a three-turn coil for winding diagram.

Let N_c be the number of commutator segments, which is also the number of coils.

$$N_c = \frac{Z}{2}; \quad Z = 2N_c.$$

Let

$$p_1 = \text{pairs of poles} = \frac{p}{2}; \quad p = 2p_1.$$

Substituting in Eq. (208)

$$y = \frac{2N_c \pm 2}{2p_1};$$

$$N_c = p_1 y \pm 1. \tag{209}$$

If p_1 and y are odd, the product p_1y is odd, as the product of two odd numbers is always odd. Adding or subtracting unity makes N_c even.

Therefore, with a wave winding whose average pitch is *odd* and having 6, 10, 14 poles, or 3, 5, 7 pairs of poles, the number of commutator segments and coils must each be *even*. If the average pitch is *even*, the number of commutator segments and coils must each be *odd*.

On the other hand, if p_1 is *even*, corresponding to 4, 8, or 12 poles, the product p_1y is always even, so that N_c must be *odd*. The application of Eq. (209) is illustrated in Fig. 270. There are 6 poles and the average pitch y is 11. Applying Eq. (209),

$N_c = 3 \times 11 + 1 = 34$ and $N_c = 3 \times 11 - 1 = 32$. The 34 segments are shown in Fig. 270(a), which gives a retrogressive winding, and the 32 are shown in (b), which gives a progressive winding. N_c is even in either case.

246. Commutator Pitch.—It can be seen from a study of Fig. 270 that the commutator pitch in a wave winding is quite different from that in a lap winding. In both (a) and (b) the commutator pitch $y_c = 11$ segments. The commutator pitch *cannot* be equal to the total number of segments divided by the pairs of poles, for if this were the case the winding would close on itself after one passage around the armature. For example, if the total number of segments in Fig. 270(a) and (b) be divided by the pairs of poles, there results $11\frac{1}{3}$ and $10\frac{2}{3}$. The difference between 11, the actual commutator pitch, and these quantities represents the creepage of the wave winding. This creepage is necessary in order that the winding shall not close on itself after one passage around the armature.

This gives another limitation of the wave winding and shows why the 252 element (126-coil) winding just considered is impossible. The number of coils must be *odd* in a four-pole winding. However, if one coil were omitted, making 250 elements, the winding would progress as follows:

1–64–127–190–(253 or 3)
–66–129–192–5, etc.

That is, the winding would advance by two elements after each passage around the armature, which condition makes the winding possible. The omission of a coil reduces the number of commutator segments and coils from 126 to 125, an odd number. If the armature stampings were standard, having 126 slots, the winding would be possible by omitting one coil. This coil would be inserted in the slots just the same as the other coils, except that its ends would not be connected to the commutator segments but would be taped and thus insulated from the main winding. The coil would serve only as a filler and is a "dummy coil." In this case there would be a slight "creeping" of the winding with respect to the commutator, as shown in Fig. 272. This is a *forced* winding.

If the coils in a wave winding consist of more than one turn, they will have the ends brought out and connected in the manner shown in Fig. 256, (p. 363) and in Fig. 271. When coils for a wave winding are formed, the ends must be spread as in *A* and *C* (Fig. 256). With the lap winding the coil ends are taped in close to each other so that they may be connected to adjacent commutator segments as in Fig. 258 (p. 365). (With a duplex winding the ends connect to alternate segments, etc.)

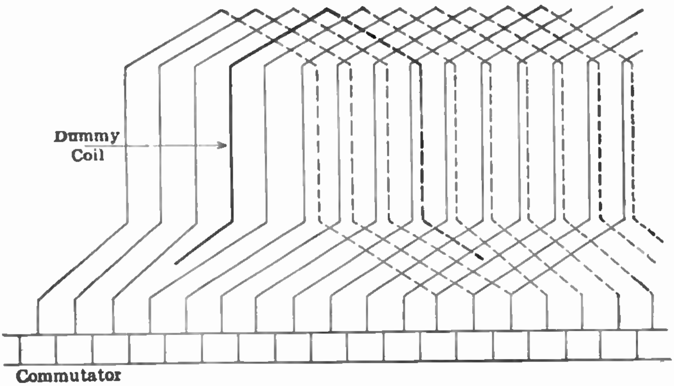


FIG. 272.—Dummy coil and "creeping" in forced wave winding.

247. Number of Brushes.—Figure 273 shows a wave winding after a single passage around the armature. It starts at positive brush *a* and returns to this brush after advancing once around the armature. This is a six-pole winding in a machine having 44 commutator segments. The pitch is found from Eq. (209):

$$44 = 3y \pm 1.$$
$$y = \frac{44 \pm 1}{3} = 15,$$

using the plus sign. This is also equal to the number of commutator segments by which the winding advances per *pair* of poles, or each time that it is connected to the commutator. Therefore, the segment connections, starting with 1, are 1–16–31–2–17, etc., as in Fig. 273. The winding ends one segment beyond the starting point for each complete passage around the armature, showing that the correct pitch has been chosen.

There are three positive brush sets *a, b, c,* and also three negative brush sets, the same number that would be used with a lap winding. It should be noted that the three positive brushes, *a, b, c,* are connected together *directly* by the winding. Moreover, the conductors which connect these three brushes all lie between the poles in the neutral plane, where they are not cutting magnetic lines and are for the instant, therefore, dead conductors. Hence, if brushes *b* and *c* were removed, the current could readily flow

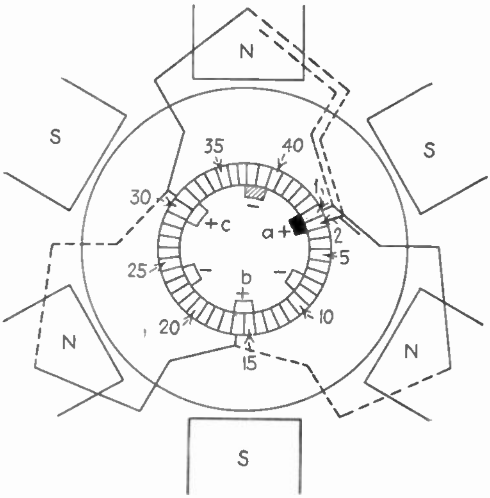


Fig. 273.—Wave winding—three positive brushes connected by the winding itself.

through these dead conductors to brush *a* and thence to the external circuit. In like manner, two of the negative brushes could be removed without serious disturbance. When possible, it is desirable to utilize all six brush sets, since with only two sets the brush area per set must be three times the value when all six brush sets are used. This would mean that the commutator must be nearly three times as long as before.

In a wave winding only two brushes are necessary, regardless of the number of poles, although it is usually desirable to use the same number of brushes as poles.

There are cases, however, where it is desirable to use only two brushes. The best example is in railway motors where it would be difficult to obtain access to four or six brushes. By means of a small hand hole in the motor casing, it is a comparatively simple matter to reach two brushes located on the top of the commutator.

248. Paths in Wave Winding.—In a simplex wave winding there are always *two* parallel paths, regardless of the number of poles. Figure 274 shows a four-pole, 17-slot, simplex wave winding, having two coil sides per slot. One of the paths is

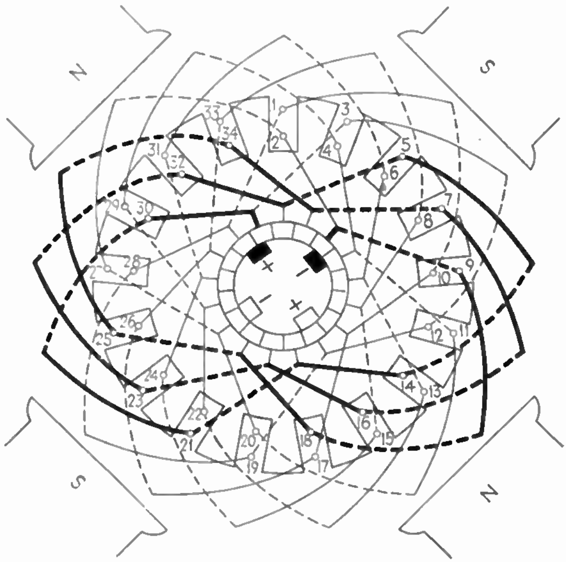


Fig. 274.—Seventeen-slot, four-pole, simplex wave winding; back pitch = 9, front pitch = 7; one of two parallel paths shown heavy.

shown by heavy lines. Approximately half the winding is heavy, the other half constituting the other path. (The coils short-circuited by the brushes are not included.) A wave winding may be duplex, triplex, or have any degree of multiplicity, just as the lap winding may be.

The number of paths through the armature depends only on the degree of multiplicity and not on the number of poles. A simplex wave winding always has two paths, a duplex winding four paths, etc.

It is interesting to compare the current and voltage of an armature for the various ways of connection. Consider a six-pole machine. When connected as a simplex lap winding let its e.m.f. be 300 volts and the armature current per terminal be 120 amp. The following table gives the values of current and e.m.f. obtainable when the winding is changed, the total number of armature conductors remaining fixed.

For 6 Pole machine.

	Paths	Volts	Amperes	Kilowatts
Simplex lap	6	300	120	36
Duplex lap	12	150	240	36
Triplex lap	18	100	360	36
Simplex wave	2	900	40	36
Duplex wave	4	450	80	36
Triplex wave	6	300	120	36

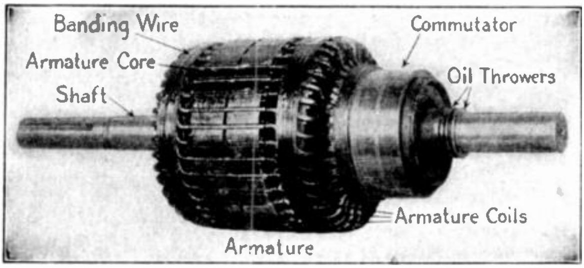
It will be noted that in this particular machine the triplex wave winding gives the same result as the simplex lap winding. The kilowatt capacity is not affected by the connection used. These relations should be kept in mind when it is desired to change a machine from one current and voltage rating to another. This may often be done merely by changing the commutator connections.

249. Uses of the Two Types of Winding.—A wave winding has the advantage that it gives a higher e.m.f. with a given number of poles and armature conductors. It is used, therefore, in small machines, especially those designed for 600-volt circuits. In this case a lap winding would result in a very large number of small conductors. This, in turn, means a higher winding cost and less efficient utilization of the space in the slots.

The wave winding has an additional advantage that the e.m.f. in each path is induced in series-connected conductors, which pass under *all* the N- and S-poles successively. Any magnetic unbalancing, therefore, due to such causes as air-gap variation and difference in pole strength, does not produce cross currents, because the corresponding conductors of every armature path move by the same poles and the effect of such unbalancing is the same in each path. Hence no equalizer connections are necessary.

The use of only two brush sets in railway motors has already been mentioned

When large currents are required, the lap winding is more satisfactory, since it gives a larger number of paths. As 200 amp. per path is practically the limit, a large number of paths must be used where heavy current output is desired. This is particularly true for large engine-driven multipolar generators.



(*a*) Twenty-five-hp. wave-wound generator armature. (*Westinghouse.*)

(*b*) End view of an armature showing spider and commutator. (*Westinghouse.*)
FIG. 275.

In Fig. 275(*a*) is shown the side view of a completed 25-hp. armature. The fact that the direction of the end connections at one end of the armature is opposite to that at the other end of the armature indicates that a wave winding is used. In (*b*) is shown the end view of a wound armature with the shaft removed. The armature spider and the method of connecting the winding to the commutator are clearly shown.

Figure 276 shows an armature in process of being wound. The use of treated-duck strips between the two layers in the end connections, the use of fish paper in the slots, and the method of

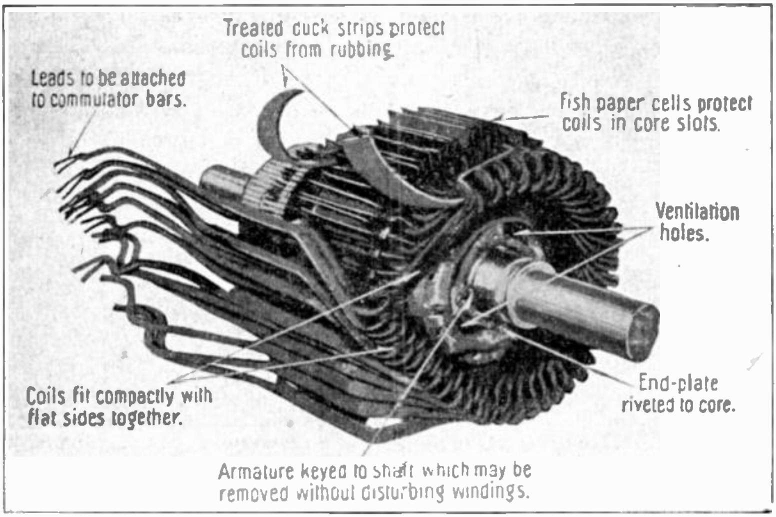


FIG. 276.—Partly wound armature showing method of assembling coils. (*Westinghouse.*)

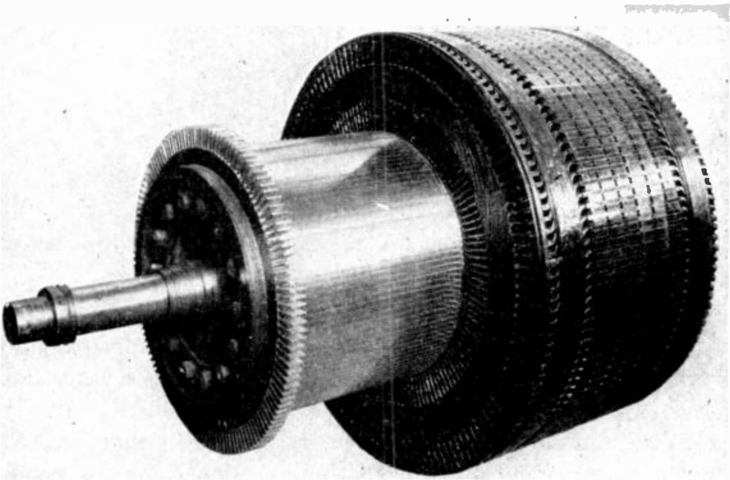


FIG. 277.—Wound armature for 1,000-kw., 250-volt, 720-r.p.m., d.-c. generator. (*General Electric Company.*)

overlapping the coils should be noted (see Par. 252, p. 392). In Fig. 277 is shown the completed armature of a 1,000-kw., 720-r.p.m. generator. The vanes on the near end of the commutator conduct heat from the commutator bars and reduce the commutator temperature by 5° to 10°C.

DYNAMO CONSTRUCTION

250. Frame and Cores.—The frame or yoke of a dynamo has two functions. It is a portion of the magnetic circuit (see

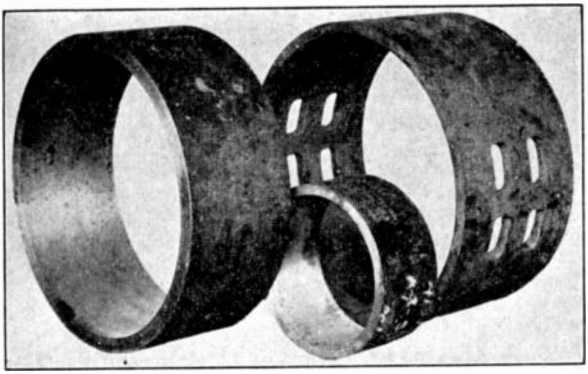


Fig. 278.—Frame rings—Westinghouse type SK motor.

Figs. 180, 181, and 182, pp. 245 and 246) and it acts as a mechanical support for the machine as a whole. In small machines, where weight is of little importance, the yoke is often made of cast iron. The feet almost always form a part of the casting. Except in dynamos of very small size, however, the modern method of forming the yoke is to roll a steel slab around a cylindrical mandrel and then to weld the slab at the bottom (Figs. 278 and 280). With large dynamos the frame is formed by means of a "bender." (See footnote, p. 390.) This type of construction makes a rigid frame with uniform magnetic characteristics. Also the permeability of steel is high (see Fig. 190, p. 261).

In the Westinghouse type SK dynamos, an outer end ring, to hold the front bracket, is supported from the frame by welded-steel spacing bars (Fig. 279). The frame feet consist of steel slabs welded to the frame and outer ring (Fig. 279). In large dynamos, the field frame, the feet, and the bed plate are fabricated from steel as in Figs. 280[1] and 281. In such dynamos the field frame is fabricated in two pieces which are bolted

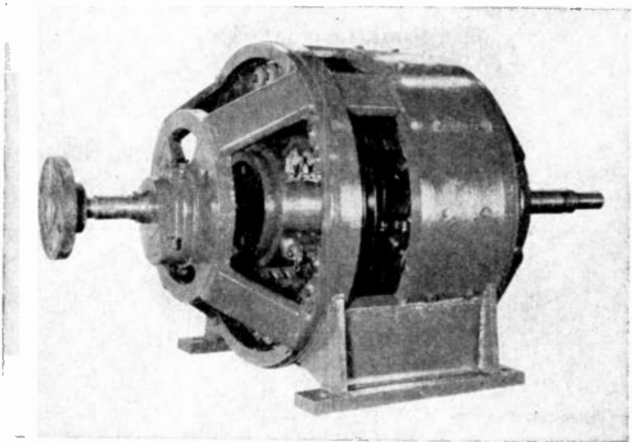


FIG. 279.—Type SK d.-c. motor with fabricated yoke and feet. (*Westinghouse.*)

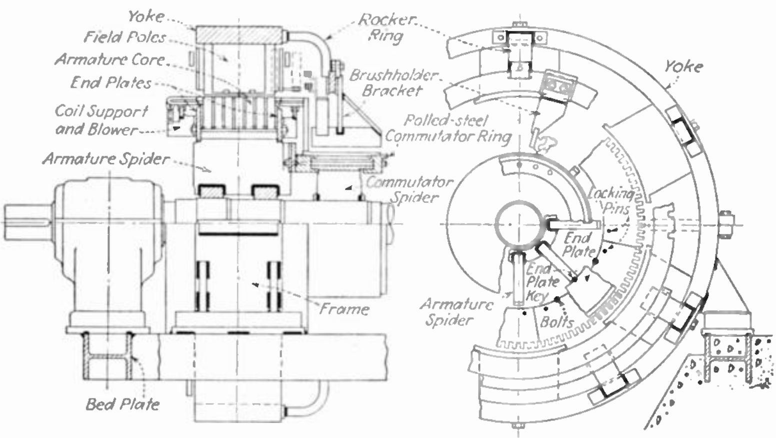


FIG. 280.—A typical, fabricated d.-c. machine assembly. (*Westinghouse.*)

together (Figs. 280 and 281). This facilitates shipment and also permits the armature to be removed by means of an overhead crane.

[1] See MARTHENS, R. S., BRINTON, C. C., HAGUE, F. T., "Standard Line of Direct-current Machines Fabricated by Arc Welding," *Elec. Jour.*, December, 1928, p. 575.

251. Field Cores and Shoes.—Although in the past it has been the practice to use cast steel and forged steel for the field cores, in many modern designs the field cores are built up of annealed sheet-steel laminations riveted together under hydraulic pressure. The complete field core with the windings is bolted to the frame, the

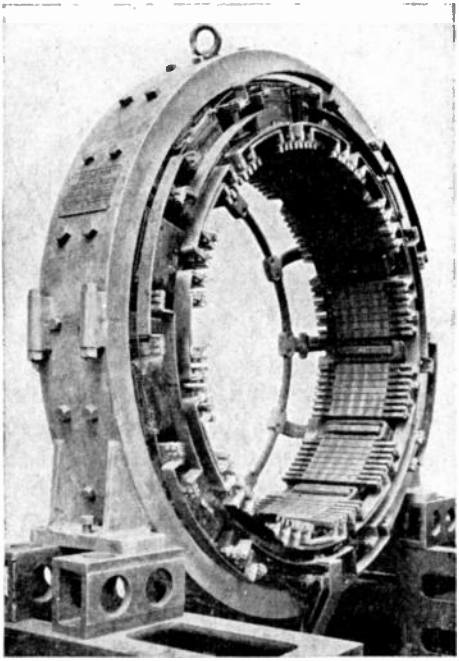


FIG. 281.—Stator frame with brush-holder yoke for 12-pole, 2,300 kw., 600 volt. 500-r.p.m., d.-c. generator. *(General Electric Company.)*

heads of which extend through the frame and tap into the pole cores (see Figs. 279, 280, and 281). In order to minimize the distortion of the flux due to armature reaction (Par. 265 and 266) each lamination is sometimes made with only a single pole tip (Fig. 282). These laminations are then stacked so that a pole tip comes alternately on one side and the other. This results in there being but half the normal amount of iron in a pole-tip cross-section, giving a saturated pole tip, which assists commutation.

An advantage of pole cores built up of laminations is that eddy-current losses in the pole faces are minimized (p. 530).

When solid pole cores are used, it is usually necessary to have laminated pole shoes.

252. Armature.—The armature is made of sheet-steel discs (14 to 25 mils thick) punched out by a die. The slots may be cut by the die, or they may be punched with a slotting

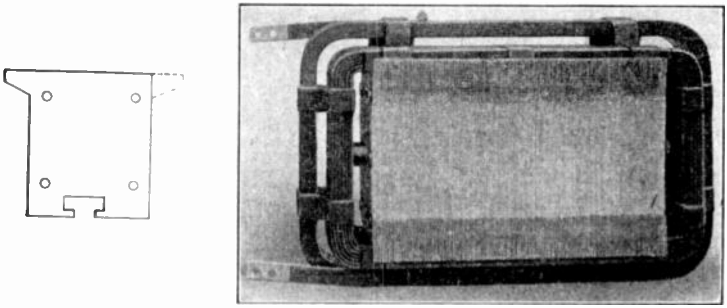


FIG. 282.—Field core lamination, and pole piece assembled. (*Westinghouse.*)

machine. In small motors these stampings are keyed directly to the shaft. After every 2 or 3 in. of laminations a suitable spacer is inserted to form a ventilating duct (Fig. 275(*a*)). The laminations are clamped together by end plates (Fig. 283), which are in turn held by nuts on the shaft or by bolts passing through the laminations. The laminations are perforated to allow air to pass through the armature axially, and out radially

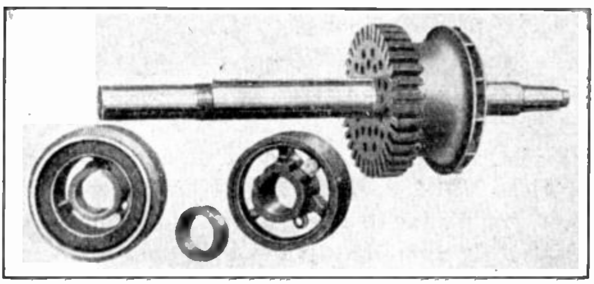


FIG. 283.—Armature construction of a small motor.

through the ducts. Frequently a blower is attached to the end plate (Fig. 283) to increase ventilation.

In machines of medium size, the stampings are assembled and keyed to an armature spider, which is in turn keyed to the shaft (Figs. 275(*b*) and 284). This reduces the amount of sheet steel necessary and at the same time permits a free passage of air through the center of the armature. This air is thrown out through the ventilating ducts by centrifugal action. The stampings are usually clamped together by end plates held by through bolts. These end plates serve sometimes as supports for the overhang of the armature coils.

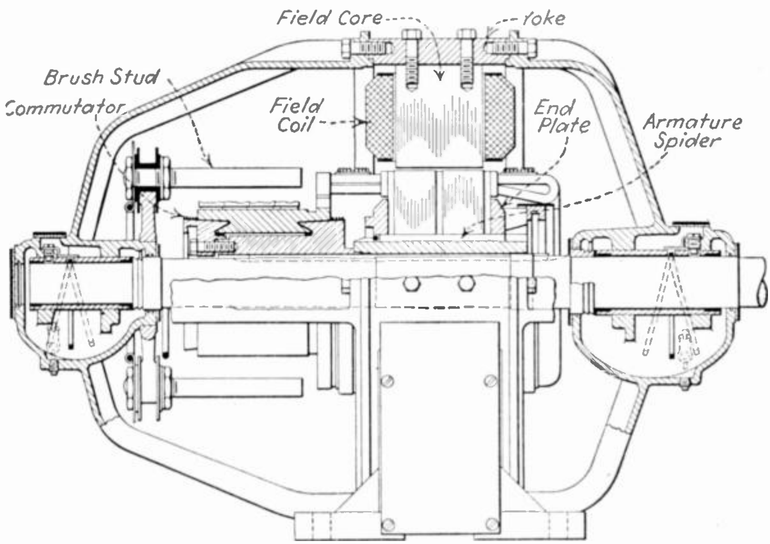
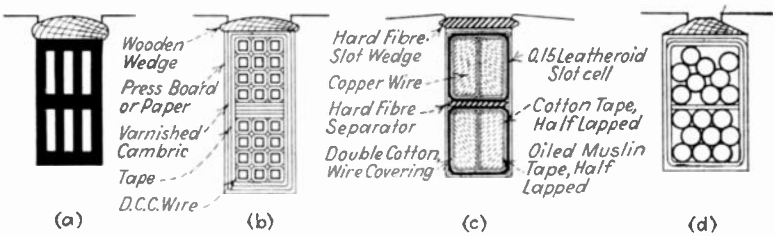


FIG. 284.—Direct-current generator with shield bearings. Longitudinal section. (*General Electric Company.*)

When the armature becomes greater than 30 in., or so, in diameter, it is not economical to punch out a complete ring. Such armatures are made up of segments similar to those in Fig. 280, each segment lapping the joint in the next layer. The segments are either dovetailed to the armature spider or are clamped between the two end plates by through bolts, and the end plates are in turn keyed to the armature spider (Fig. 280). In the fabricated construction of Fig. 280 the spider arms are welded to the hub rings, the welds being shown in heavy black.

The slots may be straight sided (Fig. 181, p. 245) in which case the conductors are held in the slots by binding wires. In the larger machines the conductors are held in the slots by hard-fibre or wooden wedges (Fig. 285). The slots must be well insulated, as grounds are troublesome and expensive to repair. A layer of a hard substance, such as fish paper, fiber, or press board, should be placed next to the laminations (see Fig. 276, p. 388). This layer, in turn, should be lined with varnished cambric or muslin. In Fig. 285(*a*) is shown the arrangement of triple coils in a slot, a wooden wedge being used to hold them in the slot. In 285(*b*) are shown two coil sides of 12 turns each held by a wooden wedge. In 285(*c*) is shown the type of slot used by the Reliance Electric and Engineering Company, in which a hard-fibre wedge and a hard-fibre separator are used.



(*a*) Open slot containing triple coil.
(*b*) Open slot containing two coil sides, 12 turns per coil.
(*c*) Slot used by Reliance Electric and Engineering Co.
(*d*) Semi-closed slot and "mush" winding.
FIG. 285.—Types of slot.

The conductors themselves are usually covered with cotton insulation, either single cotton-covered (s.c.c.), double cotton-covered (d.c.c.), or enamel and cotton. The conductors of a coil are bound together with cotton tape to form the coil. Two or more coils may be taped together to form multiple coils (see Fig. 256, p. 363, and Par. 241, p. 367).

Some manufacturers now use mica for coil insulation. After the coil is formed, the individual conductors are insulated with mica tape; a specially prepared mica sheet is wrapped around the portions of the coil which lie in the slots and this is wrapped with cotton tape. The coil is then dipped in an impregnating compound and baked. This may be repeated three or four times before the coil is ready to be placed in the slot.

To reduce the flux irregularities in the air-gap, due to the teeth, a semiclosed slot (Fig. 285(*d*)) may be used. Because of the restricted slot opening, the individual conductors must be placed in the slot one by one, so the coil ends must be taped after the coils are placed in the slots. Such a winding is called a *mush* winding. The expense of winding prevents the general use of this type of slot in direct-current machines.

253. Commutator.—The commutator is made of wedge-shaped segments of hard-drawn or drop-forged copper, insulated from one another by thin layers of built-up mica cut from segment plate. The segments are held together by clamping flanges (Fig. 286), which pull the segments inward when the flanges are

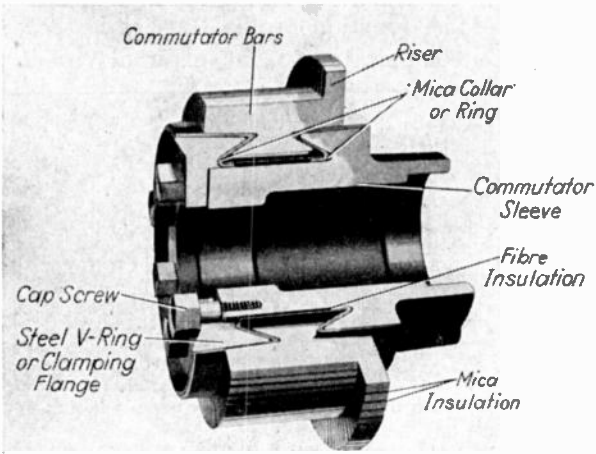


FIG. 286.—Commutator construction. (*Reliance Electric and Engineering Company.*)

drawn together by either through-bolts or cap screws. These flanges are prevented from short-circuiting the segments by two collars or rings of built-up mica (Fig. 286). This type of construction is illustrated by the commutators of the machines in Figs. 280 and 284.

The leads from the armature coils may be soldered into small longitudinal slits in the ends of the segments or the segments may have risers (Figs. 280, 284, 286) to which these leads are soldered (see also Fig. 275(*b*)).

254. Field Coils.—The field coils are usually wound with d.c.c. wire. The coils are dried in a vacuum and then impregnated with an insulating compound. The outer cotton insulation is often protected by tape or cord. In the larger machines, an air space is often left between layers for ventilating purposes. Some manufacturers place two U-shaped pieces of Fuller board over the pole core (Par. 251, p. 391) and the shunt winding is then wound directly over this under considerable tension. Another method is to wind the shunt coil on a metal spool (Fig. 287). Such a spool is readily removed from the pole core. The series winding is usually of bare copper strap, wound edgewise and spaced away from the shunt winding to assist ventilation (Fig. 287).

255. Brushes.—The function of the brushes is to carry the current from the commutator to the external circuit. They

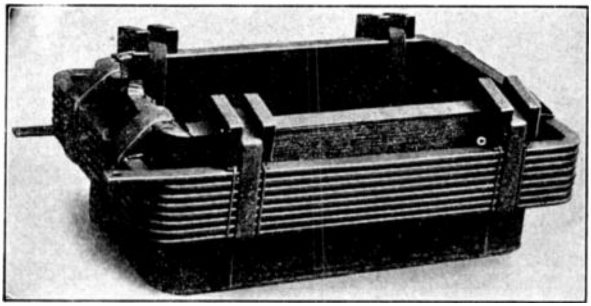


FIG. 287.—Shunt-field coil and edgewise series winding.

are usually made of carbon, although in very low-voltage machines they may be made of copper gauze or patented metal-carbon compounds. The brush holder is fastened to the brush stud (Fig. 284) and holds the brush in its proper position on the commutator. The brush stud is insulated from the rocker arm by means of an insulating bushing and washer (Fig. 284). The brush should be free to slide in its holder in order that it may follow any irregularities in the commutator. The brush is made to bear down on the commutator by a spring. The pressure should be from 1 to 2 lb. per square inch. To decrease the electrical resistance, the upper portion of the brush is copper-plated (Fig. 83, p. 135), and this plating is connected to the brush holder by a pig-tail made of copper ribbon. A brush-holder yoke attached to the stator frame is shown in Fig. 281.

CHAPTER XII

GENERATOR CHARACTERISTICS

256. Electromotive Forces in an Armature.—The method of determining the average e.m.f. in simple single-coil and two-coil generators is discussed at the beginning of Chap. XI. Also it is shown that the e.m.f. induced in a single conductor cutting the magnetic field at right angles is $e = Blv\ 10^{-8}$ volts (Eq. 199, p. 354). The armature forms part of the magnetic circuit and carries the conductors mechanically through the magnetic field in such a manner that e.m.fs. are induced in them. The windings on the armature are arranged so that certain groups of conductors are in series between brushes, care being taken in making the connections that no induced e.m.fs. are in opposition. The induced e.m.f. between brushes can be determined in terms of flux per pole, speed, number of armature conductors, parallel paths, etc.

In the elementary generators (Figs. 246 and 249, pp. 352 and 356) the e.m.f. varied sinusoidally with time because simple coils rotated in a uniform magnetic field. In commercial generators these conditions usually do not exist. Instead of being uniform the field has a maximum density directly under the poles. The density diminishes irregularly between poles, becoming zero at some place in the region between poles (Fig. 289). Commercial generators also have closed-coil windings and a number of conductors in series are distributed over the armature surface.

Consider Fig. 288, which shows a portion of an armature under a N-pole and moving from left to right. The flux entering the armature from this north pole is also shown. Below, the curve B shows the flux density along the armature. That is, the ordinate at each point gives the *flux density* at that point, expressed preferably in lines per square centimeter. The total flux is given by the area under the curve multiplied by the effective length of the armature. Midway between poles this flux density is zero and under the pole, where the air-gap is practically uniform, the flux density is practically uniform.

In Fig. 288 are also shown 12 conductors (1, 2, . . . , 12) on the surface of the armature. At the instant shown these conductors lie between the two brushes *bb*. A study of armature windings shows that these 12 conductors are all connected in series between brushes, as, for example, in Fig. 263 (p. 372). Therefore, the total e.m.f. between brushes must be the sum of the e.m.fs.

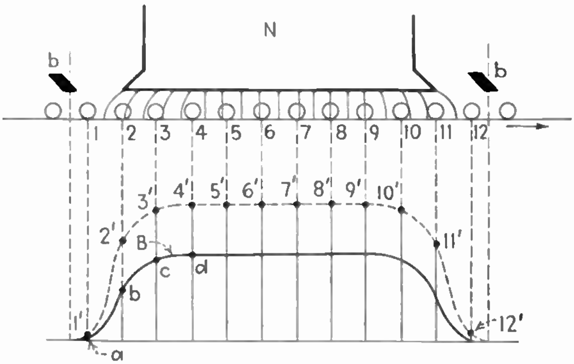


FIG. 288.—E.m.f. induced in conductors between brushes.

induced in these 12 conductors. The e.m.f. per single conductor at any instant is given by Eq. (199), p. 354:

$$e = Blv10^{-8} \text{ volts.}$$

If the speed is constant, the induced e.m.f. in any single conductor is proportional to the flux density B in which that conductor finds itself. For example, conductor 2 finds itself in a field whose density is given by ordinate b. The e.m.f. induced in this conductor will be given by the ordinate 2′, which is equal to ordinate b multiplied by a constant of proportionality. Likewise, the e.m.f. induced in conductor 3 is given by ordinate 3′, which is equal to ordinate c multiplied by the same constant of proportionality. Conductor 1 lies in the interpolar space, where the flux density, given by ordinate a, is low and the e.m.f. is given by ordinate 1′. From similar reasoning it is seen that the total e.m.f. between brushes *bb* is given by the sum of all the ordinates 1′ to 12′ inclusive. It is also seen that if a smooth curve be drawn through points 1′ to 12′, it is identical in shape with the curve B, being equal to B multiplied by the constant of proportionality. The sum of all the ordinates 1′ to 12′ inclusive is equal to the total induced e.m.f. between brushes *bb*. This total e.m.f. is also equal to the average value of these ordinates multiplied by the number of ordinates, 12 in this case.

The e.m.f. ordinates (Fig. 288) give the e.m.fs. in the various conductors which at the instant shown happen to be lying between the two brushes *bb*. Since the conductors are all moving, these ordinates likewise give the e.m.f. induced in any single conductor as it takes successive positions 1, 2, etc., between brushes *bb*.

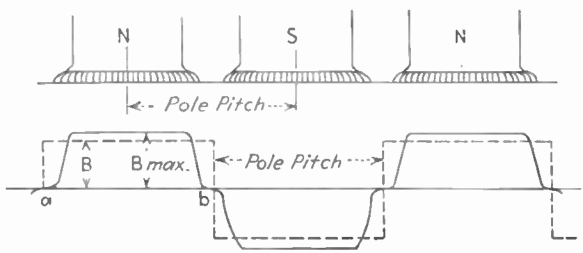


Fig. 289.—Flux distribution at no load of d.-c. generator.

Hence it follows that the total e.m.f. between brushes is given by the average e.m.f. *per conductor* multiplied by the number of conductors. Further, it follows that the curve giving the variation with time of the e.m.f. in a single conductor, as it cuts a flux at constant speed, is of the same shape as the flux density curve.

In Fig. 289 are shown two N-poles and one S-pole of a generator. For simplicity, the armature surface is shown as if it were rolled out flat. The flux density curve is given below. The positive ordinates of the distribution curve are N-pole flux entering the armature and the negative ordinates are flux leaving the armature and entering a S-pole. The total flux leaving a N-pole is given by the area under one of the positive parts of the distribution curve multiplied by the axial length of the pole. Similarly, the total flux leaving the armature and entering a S-pole is given by the area of one of the negative parts of the distribution curve multiplied by the axial length of the pole. The maximum flux density is given by the ordinate B_{max}.

Each positive part and each negative part of the curve may be replaced by a rectangle having the same area, as shown by the dotted line in Fig. 289. The height of this rectangle will be B maxwells per square centimeter, which is equal to the *average* value of the flux density over an entire pole pitch. Let it be required to determine the average e.m.f. induced in a single conductor as it passes through the flux of successive poles.

Let the total flux leaving a N-pole or entering a S-pole be ϕ maxwells. Let l be the active length of the conductor in centimeters, s the speed of the armature in *revolutions per second*, and P the number of poles.

When the conductor passes through the distance ab, or one pole pitch, the average induced e.m.f., by Eq. (199), p. 354, and from p. 399 is

$$e = Blv10^{-8} \text{ volts}$$

where B is the *average* flux density per pole pitch, l the active length of the conductor in centimeters, and v the velocity of the conductor in centimeters per second.

$$v = \frac{ab}{t}$$

where t is the time required for the conductor to travel the distance ab, equal to the pole pitch.

$$e = \frac{Bl(ab)}{t}10^{-8} = \frac{\phi}{t}10^{-8} \text{ volts,}$$

since $Bl(ab)$ gives the total flux cut by the conductor between the points a and b and is therefore equal to ϕ, the flux per pole in maxwells.

The time

$$t = \frac{1}{sP}.$$

Therefore, the average e.m.f. per conductor is

$$e = \frac{\phi}{1/sP}10^{-8} = \phi sP10^{-8} \text{ volts.}$$

If there are Z such conductors and P' paths through the armature, there must be Z/P' such conductors in series (see Eq. (202), p. 370).

Hence the total e.m.f. generated between brushes is

$$E = \frac{\phi s P Z}{P' 10^8} \text{ volts.} \tag{210}$$

Example.—A 900-r.p.m., six-pole generator has a simplex lap winding. There are 300 conductors on the armature.

The poles are 10 in. square, and the average flux density is 50,000 lines per square inch. What is the e.m.f. induced between brushes?

$$\phi = 10 \times 10 \times 50{,}000 = 5{,}000{,}000 \text{ lines.}$$
$$s = {}^{900}\!/_{60} = 15 \text{ r.p.s.}$$
$$P = 6.$$
$$P' = 6 \text{ (see Par. 242, p. 369).}$$
$$E = \frac{5{,}000{,}000 \times 15 \times 6 \times 300}{6 \times 10^8} = 225 \text{ volts.} \quad \textit{Ans.}$$

257. Saturation Curve.—Equation (210) may be written as follows:

$$E = \left(\frac{PZ}{60P'10^8}\right)\phi S \tag{211}$$

where S is revolutions per minute.

The quantity within the parentheses is constant for a given machine and may be denoted by K.

Therefore,

$$E = K\phi S. \tag{212}$$

The induced e.m.f. in a dynamo is directly proportional to the flux and to the speed. If the speed be kept constant, the induced e.m.f. is directly proportional to the flux ϕ.

The flux is produced by the field ampere-turns, and, as the turns on the field remain constant, the flux is a function of the field current. It is not directly proportional to the field current because of the varying permeability of the magnetic circuit.

Figure 290 shows the relation existing between the field ampere-turns and the flux per pole. The flux does not start at zero, ordinarily, but at some value slightly greater, owing to the residual magnetism in the magnetic circuit. At first the line is practically straight, since most of the reluctance of the magnetic circuit is in the air-gap. At the point q the iron begins to be saturated and the curve falls away from the straight line.

The number of field ampere-turns for the air-gap and for the iron can be approximately determined for any point on the curve.

Let it be required to determine the ampere-turns for the gap and for the iron at the point c. From the origin, draw ob tangent to the saturation curve and also draw the horizontal line ac. The

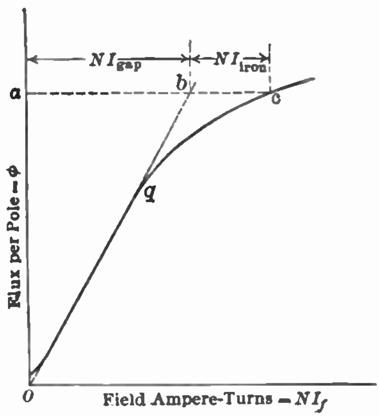


FIG. 290.—Saturation curve.

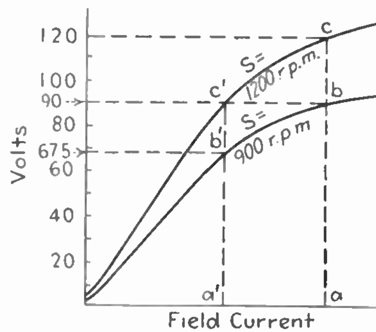


FIG. 291.—Saturation curves for two different speeds.

line ob is the magnetization curve of the air-gap, if the reluctance of the iron at low saturation be neglected. Therefore, the ampere-turns required by the gap are equal to ab and required by the iron are equal to bc.

From Eq. (212) the induced e.m.f. is proportional to the flux, if the speed is maintained constant. Therefore, if the induced e.m.f. be plotted with field current as abscissas, a curve similar to that of Fig. 290 is obtained. This is shown in Fig. 291 and the ordinates differ from those of Fig. 290 only by a constant quantity (KS). Two curves are shown in Fig. 291, one plotted for 1,200 r.p.m. and the other for 900 r.p.m. The curves are similar, any ordinate of the lower curve being 900/1,200 of the value of the corresponding ordinate of the upper curve. Thus, at ordinate ac,

$$\frac{ab}{ac} = \frac{900}{1{,}200}.$$

Also at ordinate $a'c'$,

$$\frac{a'b'}{a'c'} = \frac{900}{1{,}200}.$$

If the saturation curve of a generator for one speed has been determined, saturation curves for other speeds may be readily found by the proportionality method just indicated.

258. Hysteresis.—The saturation curve *oab* (Fig. 292(*a*)) is determined for *increasing* values of the field current. If when point *b* is reached the field current be decreased, the curve will not retrace its path along the curve *bao*. For any given field

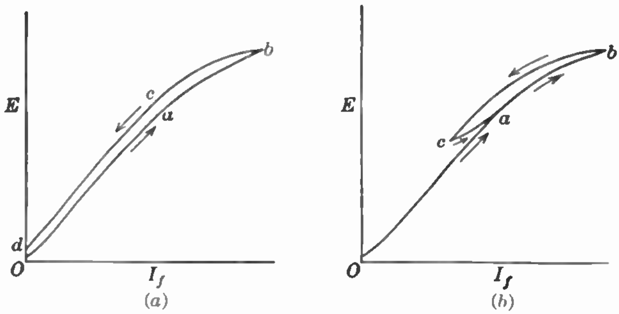


Fig. 292.—Hysteresis loops.

current, the corresponding induced e.m.f. will now be greater than it was for *increasing* field currents. This is shown by the curve *bcd*. This effect is due to hysteresis in the iron (see Par. 183, p. 266).

Figure 292(*b*) shows the effects obtained when the curve is carried up along the path *oab*, back to *c*, and at *c* the field current is again increased, the curve ultimately coming back to *oab* at the point *a*.

It is evident that for any given value of field current there may be more than one value of flux. The value of flux for any given field current depends on whether the field current was *increased* until it reached the value in question or whether it was *decreased*. This characteristic of the magnetic circuit should be borne in mind, for the operating characteristics of both generators and motors are affected to a considerable degree by hysteresis in the magnetic circuit.

259. Determination of Saturation Curve.—The saturation curve of a dynamo has important effects on the operating characteristics for both generator and motor. For example, Figs. 290, 291, 292 show that saturation definitely limits the voltage at which a dynamo can operate. Hence, a knowledge of the saturation curve is necessary for the understanding of the performance of dynamos.

To determine the saturation curve experimentally, connect the field, in series with an ammeter and rheostat, across a direct-current source of power (Fig. 293). A voltmeter should be

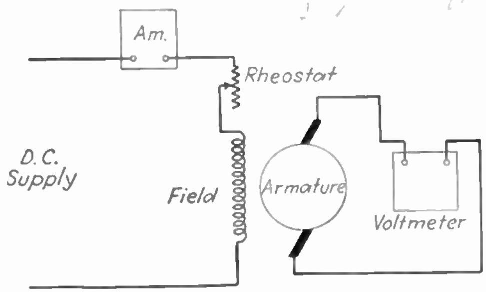


FIG. 293.—Connections for obtaining saturation curve.

connected across the armature terminals. The ammeter measures the field current, plotted as abscissa; the voltmeter measures induced armature e.m.f. plotted as ordinate. The connections are shown in Fig. 293. As the voltage drop within the armature due to the voltmeter current is negligible, the terminal volts and the induced volts are identical under these conditions. During the experiment the speed should be determined each time that the other readings are taken (see p. 524). If the speed cannot be maintained constant, corrections can be made for any variation, using the method of Par. 257 (p. 401).

When the saturation curve of a shunt generator is determined, it may be difficult to obtain a sufficiently high resistance to reduce the field current to its lower values. A drop-wire or potentiometer connection (Fig. 294) allows field currents as low as zero to be obtained without using excessive resistance. Such a connection is easily made with the well-known three-point type of field rheostat (Fig. 294).

In determining the saturation curve experimentally, the field current should be varied continuously in *one direction*, either up or down, as in Fig. 292(*a*). Otherwise minor hysteresis loops will be introduced (Fig. 292(*b*)).

The field current in this experiment should be obtained from a supply other than the generator itself, for two reasons. If the generator excites its own field, the induced e.m.f. and field current are interdependent. Hence, any adjustment of field current causes a change in induced e.m.f. which in turn changes

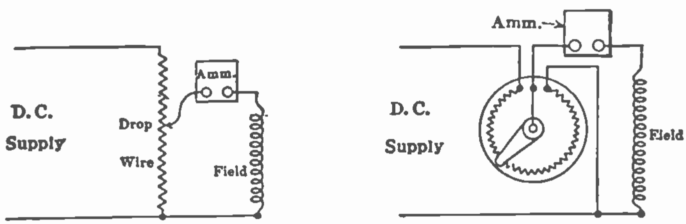


FIG. 294.—Drop-wire connections for obtaining field current.

the field current. This makes it difficult to adjust the field current to a definite value. Also, a voltage drop exists in the armature due to field current. The voltmeter does not indicate, therefore, the true induced e.m.f., although the error is slight.

260. Field-resistance Line.—By Ohm's law, the current in a simple circuit is proportional to the voltage for constant resistance. If the volts be plotted against current (Fig. 295), a straight line passing through the origin results. For example, if the resistance of a field circuit be 50 ohms, the current will be 2 amp. when the voltage is 100 volts; 1.5 amp. when the voltage is 75 volts; and 1 amp. when the voltage is 50 volts. This relation is shown in curve II (Fig. 295). Curve I shows the resistance line for 80 ohms field resistance. It will be noted that at 80 volts the current is 1.0 amp., at 40 volts it is 0.5 amp., etc. Curve III shows the same relation for a field resistance of 40 ohms.

It will be noted that the higher the resistance the greater the slope of the resistance line. In fact the slope of the line is equal to the field resistance in ohms, since the tangent of the angle which the line makes with the axis of abscissas is E/I.

261. Types of Generators.—There are three general types of direct-current generator, the shunt, the compound, and the series. In the shunt type the field circuit is connected across the armature terminals, usually in series with a rheostat (Fig. 296). The shunt field, therefore, must have a comparatively high resistance in order that it may not take too great a proportion of the

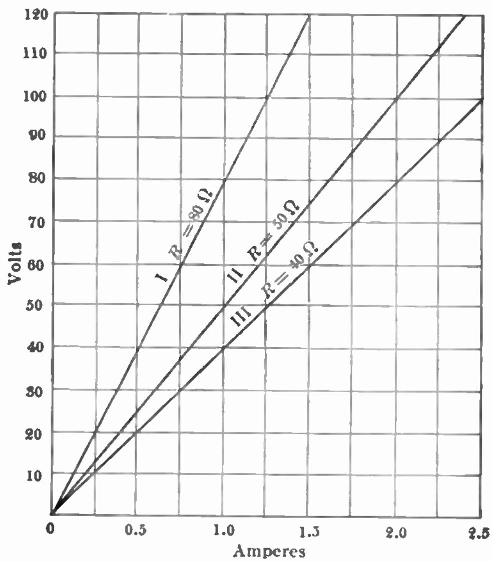


FIG. 295.—Field-resistance lines.

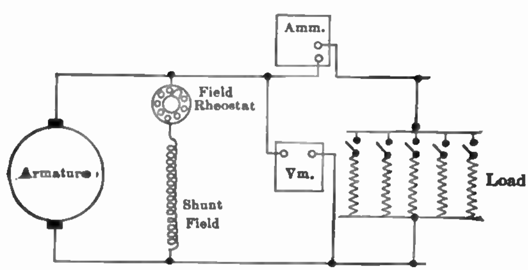


FIG. 296.—Shunt-generator connections.

generator current. The compound generator is similar to the shunt but has an additional field winding connected in series with the armature or load (see Fig. 334, p. 448). The series generator is excited entirely by a winding of comparatively few turns connected in series with the armature and load (see p. 459).

262. Shunt Generator.—Figure 297 shows the saturation curve of a shunt generator and its shunt-field resistance line, both drawn on the same plot. The shunt field being connected across the armature terminals, the ordinates of the field-resistance line must represent the terminal voltage of the generator. The resistance of the field, represented by the field-resistance line

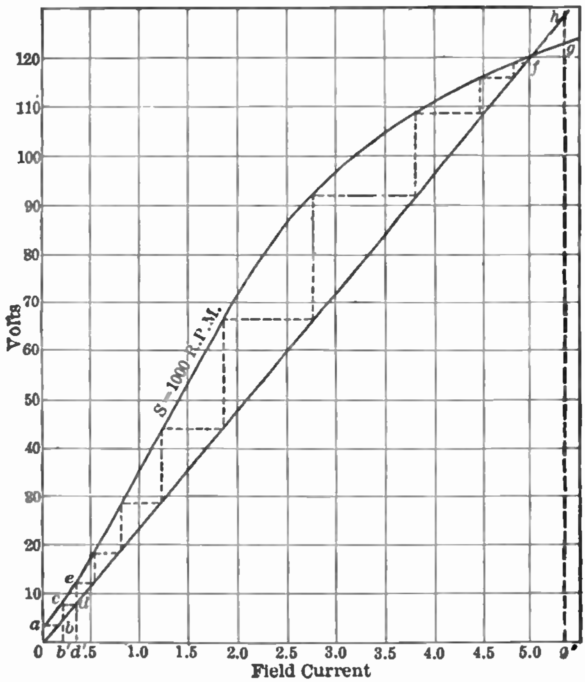


FIG. 297.—Method of shunt generator building up.

in Fig. 297, is 24 ohms, so that at 120 volts the field takes 5 amp., at 60 volts 2.5 amp., etc.

When starting a generator from rest, the induced e.m.f. is zero. The generator may come up to voltage in the following manner: As the generator comes up to speed, there is a small e.m.f. *oa*, in Fig. 297 about 4 volts, induced in the armature due to residual magnetism in the magnetic circuit. These 4 volts also exist across the field, because the field is connected across the armature terminals. The value of field current which flows in virtue of these 4 volts can be obtained by drawing a horizontal line from *a* until it meets the field resistance line at *b*. The field current in this particular case is *ob'* or about 0.2 amp. By consulting the saturation curve, it is seen that for this field current the induced e.m.f., *b'c*, is about 8 volts. The 8 volts produces about 0.33 amp. in the field, as is seen by projecting across to the field resistance line at *d*. This field current *od'* produces a voltage *d'e*, which in turn produces a higher value of field current. Thus it is seen that each value of field current produces an e.m.f. in excess of the previous value and this increased e.m.f. in turn increases the field current, or the action is cumulative. The machine continues to build up until point *f* is reached, where the field-resistance line crosses the saturation curve. The machine cannot build up beyond this point for the following reasons.

Consider a point *h* above *f* on the field-resistance line. This point represents a field current *og'* of about 5.3 amp. To produce this field current requires a voltage *g'h* of about 128 volts. But this field current of 5.3 amp. produces an induced e.m.f. *g'g* of only 122 volts. If 128 volts are required to produce the field current of 5.3 amp., and the machine can only produce 122 volts at this field current, it is obvious that the machine cannot build up to the point *h*.

It is evident that a machine would build up indefinitely if its iron did not become saturated.

263. Critical Field Resistance.—If the resistance of the field (Fig. 297) be increased to 60 ohms, the field-resistance line will be represented by *oa* (Fig. 298). This line crosses the saturation curve at point *a'*, corresponding to about 6 volts. Therefore, with this value of field resistance, the generator will not build up beyond *a'*. If the field resistance be slowly decreased until the field-resistance line reaches *ob*, tangent to the saturation curve, the generator will begin to build up. It will stop building up at the point *b'*. The value of the field resistance, corresponding to *ob*, which makes the field-resistance line tangent to the saturation curve is called the *critical field resistance*. In this particular case, the critical field resistance is 120 ÷ 3.25, or 36.9 ohms. If the field resistance exceeds the critical value, the generator cannot build up

264. Failure to Build Up.—If a generator fails to build up, the cause may be one or more of the four following.

1. The shunt field may be connected in such a way that the initial current through it on starting is in such a direction as to "buck" or reduce the residual magnetism, instead of increasing it. Under these conditions, the generator cannot build up. To test for this, open the field circuit. If the voltage rises when the field is opened, the field current is bucking the residual magnetism and the field should be reversed. If opening and closing the field produce no effect on the voltmeter it may be assumed that the field circuit is open.

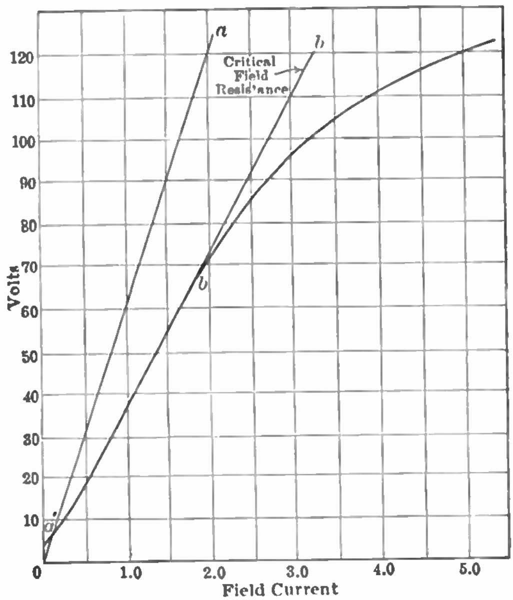


FIG. 298.—Critical field resistance.

2. The field resistance may be greater than the critical field resistance. In this case, the procedure is to reduce the field resistance until the machine builds up.

3. Imperfect brush contact may cause high resistance from commutator to brush. Since the field is the only load across the armature, poor brush contact is equivalent to high resistance in the field circuit. The effect is the same as that shown by

the resistance line *oa* (Fig. 298), and the generator cannot build up to any substantial voltage. Low current density causes high brush-contact resistance (Par. 271, p. 429). High contact resistance may be determined by exerting slight pressure on the individual brushes.

4. There may be no residual magnetism in the machine, due to jarring or to too long a period of idleness. If the armature circuit is not open and the voltmeter is known to be all right, the absence of residual magnetism will be indicated by the voltmeter not reading. To remedy the difficulty, it may be necessary to connect the field terminals temporarily across a separate supply circuit in order to build up the residual magnetism. This is called *flashing* the field. If the generator has a series field, a convenient method is to connect a low-voltage source, such as a storage battery or even a dry cell, across the series field. This may produce enough magnetism to cause the machine to begin to build up. One or two trials may be necessary in order to secure the proper polarity.

265. Armature Reaction.—Figure 299(*a*) shows the flux passing from the field poles through the armature of a bipolar generator when there is no current in the armature conductors. This flux is produced entirely by the ampere-turns of the field. Moreover, it is distributed symmetrically with respect to the polar axis, that is, the center line of the *N*- and the *S*-poles. The neutral plane, which is a plane perpendicular to the direction of the flux, coincides with the geometrical neutral of the system. At the right in Fig. 299(*a*) is shown a vector *F* which represents in magnitude and direction the m.m.f. producing this flux. At right angles to this vector *F* is the neutral plane.

In Fig. 299(*b*) there is no current in the field coils, but the armature conductors are shown as carrying current. This current is in the same direction in the armature conductors as it would be were the generator under load. The current flows in the same direction in all the conductors that lie under one pole. The current is shown as flowing into the paper on the left-hand side of the armature. (This current direction may be checked by Fleming's right-hand rule, Par. 234, p. 355.) These conductors combine their m.m.fs. to send a flux *downward* through the armature, as shown in the diagram, this direction being determined by the corkscrew rule. The conductors on the right-hand side of the armature are shown as carrying current

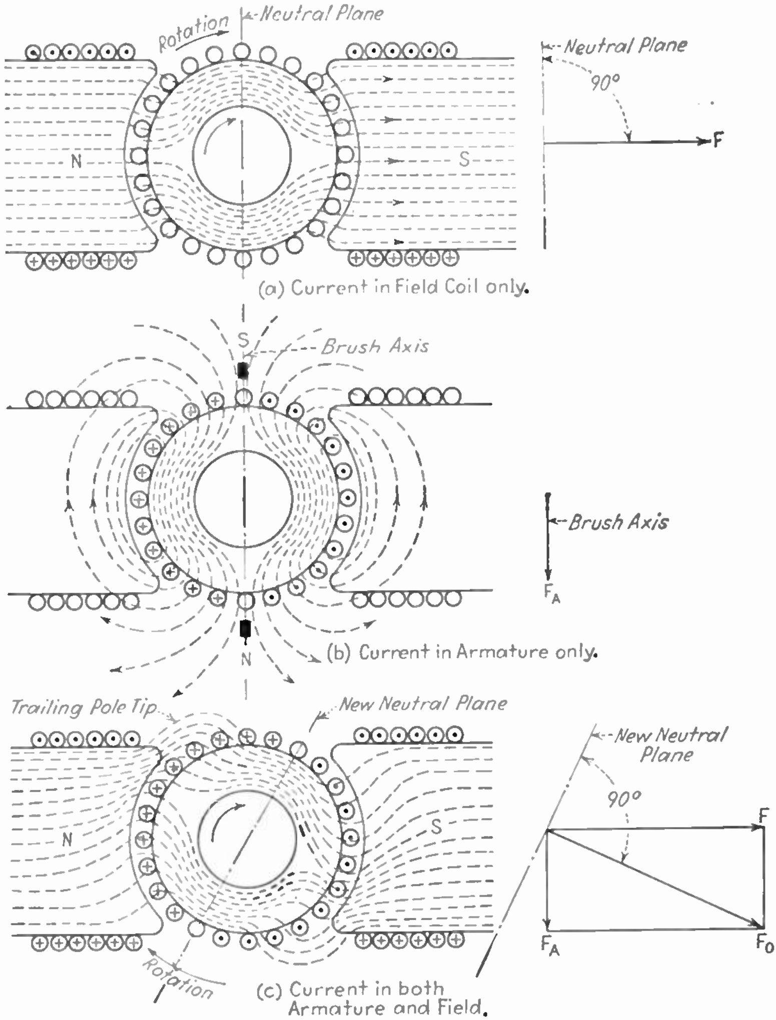


FIG. 299.—Effect of armature reaction on field of generator.

coming out of the paper. They also combine their m.m.fs. to send a flux *downward* through the armature. That is, the conductors on both sides of the armature combine their m.m.fs. in such a manner as to send flux downward through the armature. The direction of this flux is perpendicular to the polar axis. To the right of the figure, the armature m.m.f. is represented in magnitude and direction by the vector F_A.

Figure 299(*c*) shows the result obtained when the field current and the armature current are acting simultaneously, which occurs when the generator is under load. The armature m.m.f. crowds the symmetrical field flux shown in (*a*) into the upper pole tip in the *N*-pole and into the lower pole tip in the *S*-pole. As the generator armature is shown rotating in a clockwise direction, it will be noted that the flux is crowded into the *trailing* pole tip in each case. On the other hand, the flux is weakened in the two *leading* pole tips.

The effect of the armature current is to displace the field in the direction of rotation of the generator. *It should be kept in mind that the flux is not pulled around by the mechanical rotation of the armature.*

To the right of Fig. 299(*c*) the effect of armature reaction is shown by vectors. The field m.m.f. vector F and the armature m.m.f. vector F_A combine at right angles to form the resultant-field m.m.f. vector F_0. The direction of F_0 is downward and to the right, which corresponds to the general direction of the resultant flux in the drawing. The neutral plane must be at right angles to F_0, provided the direction of the resultant flux is the same as that of the resultant m.m.f.

As the neutral plane is perpendicular to the resultant field, it will be observed that it too has been advanced. It is shown in Chap. XI that the brushes should be set so that they short-circuit the coil undergoing commutation as it passes through the neutral plane. When the generator delivers current, the brushes should be set a little ahead of this neutral plane, as will be shown later. If the brushes are advanced to correspond to the advance of the neutral plane, all the conductors to the left of, but between, the two brushes must still carry current into the paper and those to the right must carry current out of the paper. The result is shown in Fig. 300. *The direction of the armature field moves with the brushes. Its axis always lies along the brush axis.* Therefore F_A, instead of pointing vertically downward, now points downward and to the left, as is shown by the vector.

F_A may be resolved into two components, F_D parallel to the polar axis and F_C perpendicular to this axis. Since the geometrical distributions about the armature of these three m.m.fs. are quite different from one another (see Fig. 307), a coefficient must be applied to each in order to make F_A equal to the vector sum of F_D and F_C.

It will be noted that F_D acts in direct opposition to F, the main field (Fig. 302). Therefore, it tends to reduce the total flux and so is called the *demagnetizing component* of armature reaction.

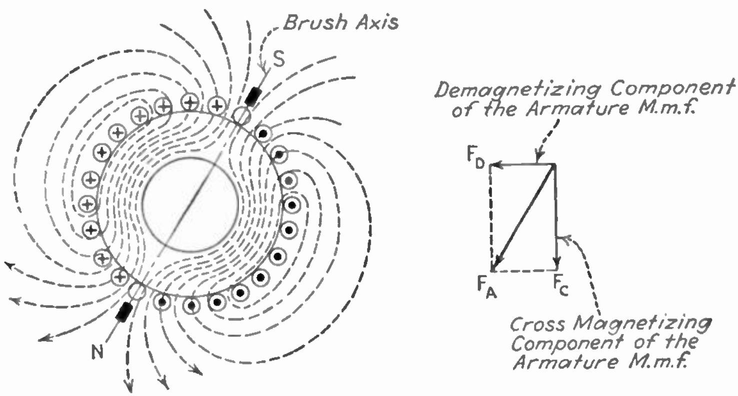


Fig. 300.—Relation of armature field to brush axis.

F_C acts at right angles to F, producing distortion, and is called the *cross-magnetizing component* of armature reaction.

The exact conductors which produce these two effects are shown in Fig. 301. In (*a*) the brushes are shown as advanced by an angle β to correspond to the advance of the neutral plane. All the conductors within the angle 2β, at both the top and bottom of the armature, carry current in such a direction as to send a flux through the armature from right to left. This may be checked by the corkscrew rule. These conductors thus act in direct opposition to the main field and are called the *demagnetizing armature conductors*. Their m.m.f. is represented by the component F_D (Fig. 300).

Figure 301(*b*) shows the flux produced by the conductors not included within twice the angle of brush advance. The direction of this flux is downward and perpendicular to the polar axis.

These conductors cross-magnetize the field. The m.m.f. producing this flux is represented by the component F_C (Fig. 300).

It should be remembered that the number of demagnetizing and cross-magnetizing *ampere-turns* is equal to one-half the number of the corresponding *ampere-conductors.*

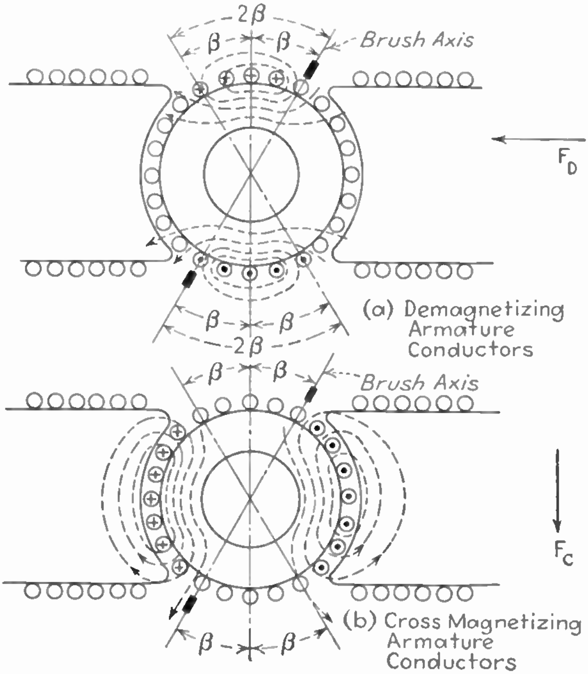


FIG. 301.—Demagnetizing and cross-magnetizing *components* of armature reaction.

Example.—A four-pole dynamo has 288 surface conductors. The armature is lap-wound and the armature current is 120 amp. The brushes are advanced 15 space-degrees. How many demagnetizing and how many cross-magnetizing armature ampere-turns are there?

Twice the angle of brush lead is 30°. There are four brushes, so that the total number of degrees covered by the demagnetizing conductors is 120°. Therefore one-third the conductors on the armature, or 96 conductors, are demagnetizing conductors.

As the armature is lap-wound, there are four paths through it. The current per path = 120 ÷ 4 = 30 amp.

Demagnetizing *ampere-conductors* = 30 × 96 = 2,880.

Demagnetizing *ampere-turns* = 2,880 ÷ 2 = 1,440. *Ans.*

The number of cross-magnetizing conductors must be two-thirds of the conductors on the armature. Therefore, the number of cross-magnetizing *ampere-turns* is

$$\frac{192 \times 30}{2} = 2{,}880. \quad \textit{Ans.}$$

Figure 302 gives the m.m.f. diagram. F is the field m.m.f., and F_A is the armature m.m.f. acting along the brush axis after the brushes have been advanced. F_0 is the resultant of the two, being less than F due to the demagnetizing component of F_A. F_A can be resolved into two components at right angles to each other: F_D, the demagnetizing component of the armature m.m.f.; and F_C, the cross-magnetizing component of the armature m.m.f. Due to the fact that F_C and F_D are produced by distributed windings, and F is produced by the field coils, the space distributions of all three m.m.fs. are quite different (Figs. 303 and 307). Hence, quantitatively, F_A is only equal to the vector sum of F_D and F_C when each m.m.f. has been multiplied by a coefficient which corrects for space distribution.

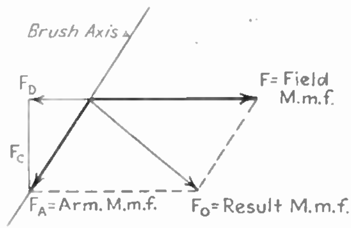


FIG. 302.—Resultant of armature and field m.m.fs.

266. Armature Reaction in Multipolar Machines.—Reactions occur in multipolar machines in the same manner as in bipolar machines as just described. To the eye, the effect may be somewhat different. In Fig. 303 the armature and the field poles of a multipolar generator are shown, the armature being shown as a flat surface, for convenience.

In (*a*) are shown the alternate *N*- and *S*-poles, together with the magnetic flux entering and leaving the armature. There is no current flowing in the armature conductors. In (*b*) the flux distribution is shown. It will be observed that it is symmetrical about the polar axis. It is substantially constant under the pole shoe and drops off gradually at the edges, due to fringing. It falls to zero and reverses in the interpolar spaces. The neutral plane is the region where the flux is zero and under no-load conditions is midway between the poles.

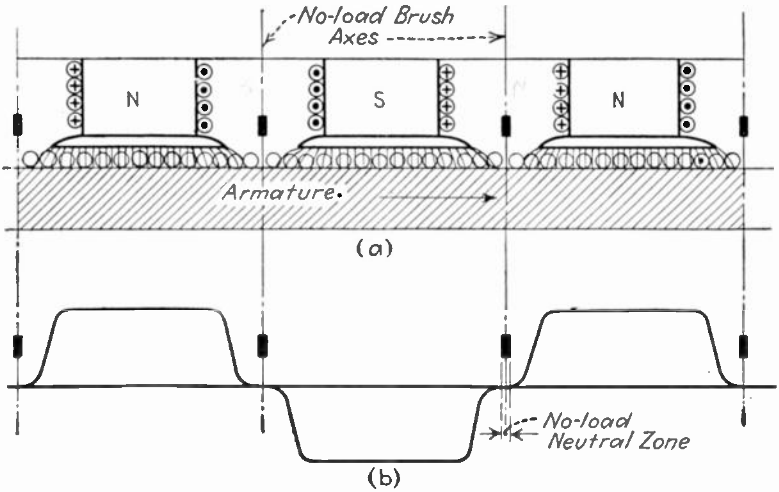


FIG. 303.—Field flux density in a multipolar generator.

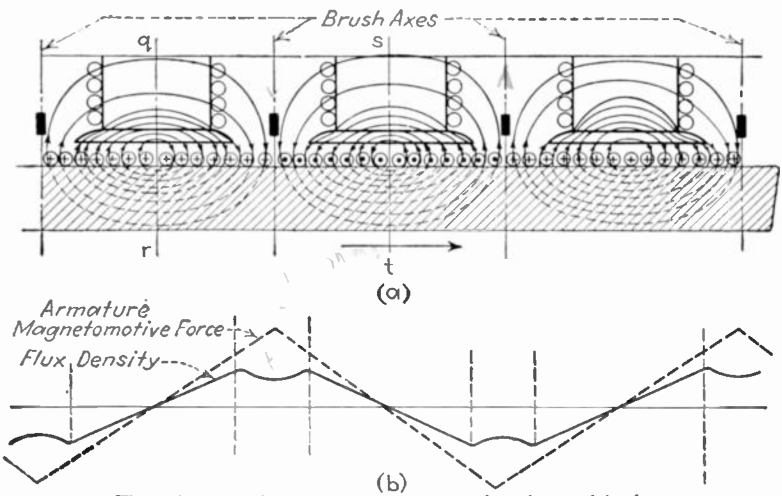


FIG. 304.—Flux density due to armature reaction in multipolar generator.

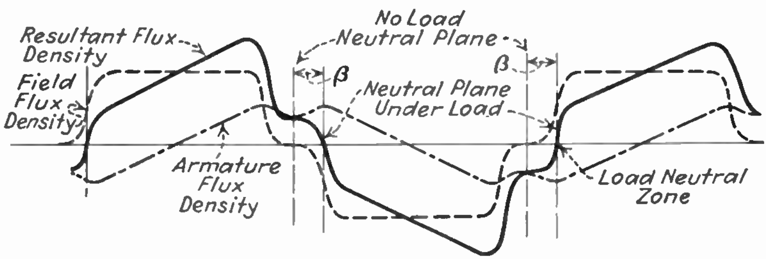


FIG. 305.—Resultant flux density found by combining field flux density (Fig. 303) and armature flux density (Fig. 304).

Figure 304(a) shows the armature conductors carrying current, the field current being zero. These armature conductors produce a flux in a manner similar to that in Fig. 299(b). The m.m.f. of the armature along the armature surface is not uniform, but *varies uniformly* from zero at the pole axis to a maximum in the center of the interpolar space. The armature conductors between the lines qr and st may be considered as constituting a pancake coil, the current flowing into the paper in the conductors on the left and out of the paper in the conductors on the right. Midway between qr and st, the m.m.f. will be a maximum, as the m.m.fs. of all the conductors on both sides are acting in conjunction at this point. The m.m.f. directly under the pole centers is zero, since for every ampere-conductor on one side of the pole axis there is a symmetrically spaced ampere-conductor on the other side carrying an equal current in the *same* direction. The net m.m.f. at the pole center, due to all such ampere-conductors, is zero. The m.m.f. distribution along the air-gap is shown by the dotted line (Fig. 304(b)). Magnetomotive force tending to send flux into the armature is shown as positive; m.m.f. tending to send flux out of the armature is shown as negative. If the reluctance of the air-gap were constant, the flux-density curve would be identical in shape with the m.m.f. curve. Owing to the high reluctance of the interpolar space, the *flux-density* curve has not the same shape as the m.m.f. curve, but droops in the interpolar space, as in Fig. 304(b).

The resultant flux density at each point is found by adding the two flux-density curves of Figs. 303 and 304, as is done in Fig. 305. *This assumes constant permeability in the iron.* It will be noted that the flux peaks on the trailing pole tip (Fig. 305), just as in the bipolar generator. The neutral plane has advanced by an angle β in the direction of rotation. In order to keep the brushes in the neutral plane, they must be advanced as the neutral plane advances. It should also be noted that the width of the load neutral zone is practically zero. This makes satisfactory commutation more difficult to obtain (Par. 269, p. 422). Figure 306 shows the crowding of flux in the trailing pole tips in a four-pole generator.

267. Cross and Demagnetizing Ampere-turns in Multipolar Generators.—From Fig. 305 it is seen that under load the neutral plane advances by the angle β. As in the bipolar generator, the brushes must be advanced by an amount at least equal to this angle if good commutation is to be obtained. In order to compensate for the e.m.f. of self-induction, the brushes are advanced actually beyond this angle (see Par. 270, p. 426). As in the bipolar generator, the advance of the brushes converts some of the cross-magnetizing ampere-turns into demagnetizing ampere-turns.

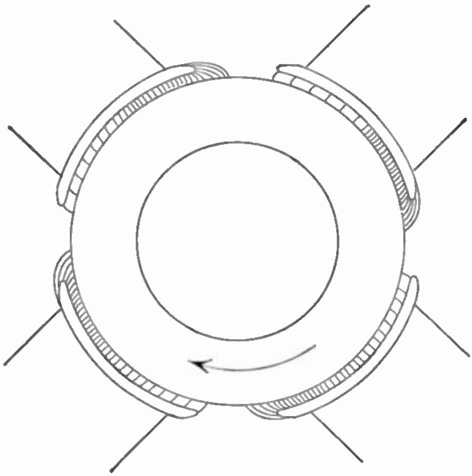


FIG. 306.—Field distortion in a four-pole generator.

Consider Fig. 307 which shows the armature of Figs. 303 and 304 with the brushes advanced by an angle β beyond the no-load neutral plane. The conductors *abcd*, lying in the interpolar space, are included within an angle 2β. Conductors $b'a'$ to the left and $d'c'f'e'$ to the right are also located in interpolar space. Since the current in conductors *ab* is inward and the current in conductors $a'b'$ is outward, they may be shown, for purposes of analysis, as being connected to form turns shown by the solid lines. Likewise, conductors *cd* and $c'd'$, and *ef* and $e'f'$, may be connected to form turns as indicated. The turns aa' and bb' link the magnetic circuit of the *N*-pole, and their m.m.fs. F_D act upward. Thus the ampere-turns of these two coils are in direct opposition to the ampere-turns of the *N*-pole. The turns cc' and dd' link the magnetic circuit of the *S*-pole and their m.m.fs. F_D' act downward. Thus the ampere-turns of these two coils are in direct opposition to the ampere-turns of the S-pole. Similarly, the m.m.fs. of turns ee' and ff' act in opposition to a second N-pole. Thus, as in the bipolar generator (Fig. 301), the conductors included within an angle 2β, twice the angle of brush advance, are demagnetizing ampere-conductors.

Beneath, in Fig. 307, the value of the demagnetizing armature m.m.f. is plotted as a function of distance along the armature and is shown by the solid line.

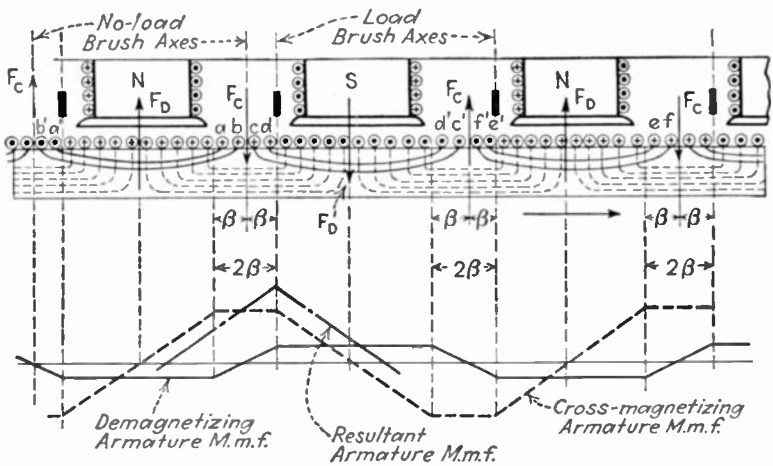


FIG. 307.—Cross and demagnetizing ampere-turns in multipolar generator.

In Fig. 307, the armature conductors not included within twice the brush angle β are connected with dotted lines to form turns. Except for the omission of the demagnetizing ampere-conductors included within the angles 2β, the positions and currents of these conductors are identical with those on the armature of Fig. 304. Their m.m.f. graph, shown beneath in Fig. 307 by the dashed line, is similar to that of Fig. 304, with the effect of the demagnetizing ampere conductors omitted. The m.m.f. is zero at the centers of the pole faces and is a maximum across the interpolar spaces. Accordingly, the m.m.f. acts transversely to the direction of the flux in the armature core, and the ampere-turns must be cross-magnetizing as in Fig. 304, where the brushes are in the no-load neutral plane. At each interpolar space, the cross-magnetizing m.m.f. is denoted by F_C.

Thus the rules used in bipolar generators for separating demagnetizing and cross-magnetizing ampere-conductors apply to multipolar generators.

If the cross-magnetizing and demagnetizing m.m.f. graphs (Fig. 307) are combined, the resulting armature m.m.f. graph is identical with that in Fig. 304, except that the graph as a whole has moved to the right by the angle β, with the movement of the brushes. The peaks of the graph still coincide with the brush positions. For simplicity, only a small portion of the resultant armature m.m.f. is shown in Fig. 307.

268. Compensating Armature Reaction.—As the cross-magnetizing effect of the armature usually necessitates the shifting of the brushes with load, it is desirable to minimize armature reaction if this can be done without too much complication and expense. The magnetic circuit for the flux produced by the armature m.m.f. is not the same as that for the flux produced by the field m.m.f., although parts of the circuit, such as the air-gap, are common to both. Hence, one possible method of reducing armature reaction is to interpose reluctance in the magnetic circuit of the armature flux, without at the same time affecting to any considerable extent the reluctance to the main flux.

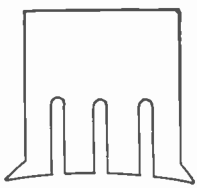


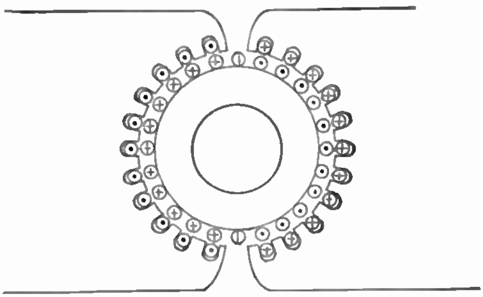
Fig. 308.—Longitudinal slots in pole face for reducing armature reaction.

One practical method, when laminated pole cores are used, is to use a stamping having but one pole tip, as in Fig. 282, p. 392. These are alternately reversed when the core is built up. This leaves spaces between the pole-tip laminations, and the pole tips have but one-half the cross-section of iron along their lengths. Therefore, the pole tip becomes highly saturated and its permeability greatly reduced. This tends to prevent the flux from crowding into the trailing tip. Actually, the magnetic circuit for the armature flux becomes highly saturated, whereas that for the main flux is scarcely affected.

An interesting method, though rarely used, is the Lundell pole (Fig. 308), having longitudinal slots in the pole cores and pole faces. These slots offer high reluctance to the transverse armature flux but have little effect on the main flux. The effect of these slots is partly neutralized by the passage of the armature flux through the iron back of the slots.

Since the m.m.fs. which produce the armature flux are in a different location from those which produce the main flux, it is possible without affecting the main field to neutralize the armature m.m.fs. by opposing m.m.fs. This is the principle of the Thompson-Ryan method.

In order to be effective, these compensating m.m.fs. should be equal in magnitude and opposite in direction to those of the armature at every point. This principle is illustrated by Fig. 309, which shows conductors embedded in the pole faces close to the armature. Each conductor carries a current opposite in direction



(a)

FIG. 309.—Compensation of armature reaction with pole-face conductors.

to that of its corresponding armature conductor. This winding is connected in series with the armature so that the m.m.fs. are opposite and equal at all loads. These windings allow the use of a very short air-gap, with the accompanying reduction in field copper and field loss. This particular method is used in the alternating-current series motor in which armature reaction is particularly objectionable (see Vol. II, Chap. X).

The Thompson-Ryan method is also applied to rolling-mill generators and motors which operate under unusually severe conditions, such as rapid reversals of direction of rotation and extremely high loads for short periods of time (see Fig. 281, p. 391).

The method is also used in the Ridgway dynamo (Fig. 310). Each compensating winding consists of three coils embedded in the pole-face slots and connected in series with the armature. Small T-shaped poles are placed between the main poles, and the coil wound around each of these T-poles usually has more turns than the other coils in the winding. The crossbar at the top of the small pole becomes highly saturated for any leakage flux between the main poles but does not become highly saturated for the compensating flux.

Armature reaction is also reduced by increasing the length of the air-gap, thus offering higher reluctance to the armature flux. This also increases the reluctance to the main flux and, in order that the voltage of the generator be maintained, the field ampere-turns must be increased. Thus the main flux is brought back to normal, but the armature-reaction flux is reduced. It is to be noted that the ratio of field to armature ampere-turns has been increased. *The ratio of field to armature ampere-turns is a criterion*

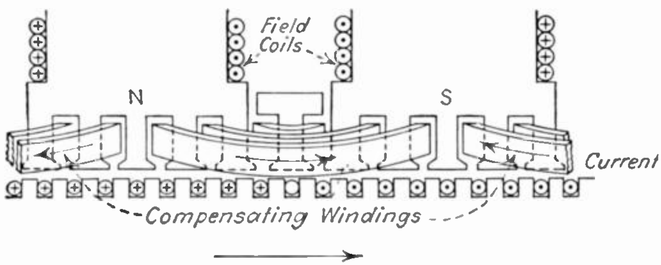


FIG. 310.—Turns in pole faces to compensate armature reaction.

of armature reaction. Dynamos with a long air-gap, and thus a high ratio of field to armature ampere-turns, are said to have a "stiff field." The increased field ampere-turns increase the field copper and the field loss. By the use of commutating poles, however, modern dynamos operate satisfactorily with short air-gaps (Par. 272, p. 431).

269. Commutation.—It has been shown that the e.m.f. induced in any single coil of a direct-current generator is alternating. In order that the current may flow always in the same direction to the external circuit, a commutator is necessary. Figure 311 shows the change of current in an armature coil as it approaches and recedes from a brush. It is assumed that ideal commutation is being realized. That is, the current, as it leaves the commutator, is distributed uniformly over the brush. Accordingly, in each coil, the current change or reversal from maximum positive to maximum negative value is uniform. This is called *straight-line* commutation. The load is such that 20 amp. flow in each path of the armature, and 40 amp. leave the armature by this one brush.

When in positions 1, 2, 3, each coil, and therefore any one particular coil in its successive positions, carries 20 amp. As the brush covers four commutator segments, and the current distribution is uniform, 10 amp. must flow into the brush from each segment. Therefore, in moving from 3 to 4, the coil must lose the 10 amp. which flow from segment 3 into the brush. Hence, in position 4 the coil carries only 10 amp.

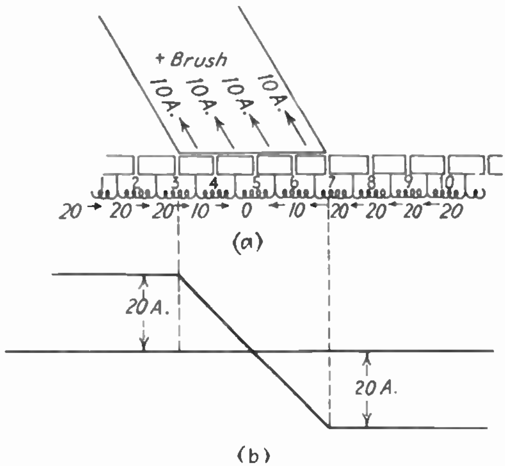


FIG. 311.—Current in coil undergoing commutation—ideal conditions.

Before reaching position 5, the coil gives up another 10 amp. so that the current is zero when the coil reaches position 5. When the coil reaches position 6, the current flows through the coil in the reverse direction, due to the fact that the current enters the brush from the other armature path. The current reaches 20 amp. in position 7 and remains 20 amp. in positions 8, 9, 10.

Therefore, commutation consists of two parts:

1. Reversing the current in any coil from its full positive value to an equal negative value. This reversal must take place in the very short time interval required for a segment to pass under the brush.

2. The current supplied by the paths meeting at the brush must be conducted to the external circuit.

Part 1 is illustrated by Fig. 311(*b*). The current in the coil is +20 amp. until the brush is reached, when it reverses at a uniform rate to a value of −20 amp. This is ideal commutation.

The foregoing ideal commutation is only approximated in practice. There are two causes preventing its complete realization.

It will be noted that when the coil is in positions 4, 5, 6, it is short-circuited by the brush. If any e.m.f. is induced in the coil in these positions, a large current will flow, since the resistance of the short-circuited path is very low. This resistance consists merely of the resistance of the coil plus the contact resistance of the brush, the contact resistance constituting the greater portion of the total resistance. Figure 312 shows assumed currents of 15 and 5 amp. flowing in coils 4 and 5, due to e.m.fs. induced in 4 and 5 while they are short-circuited by the brush.

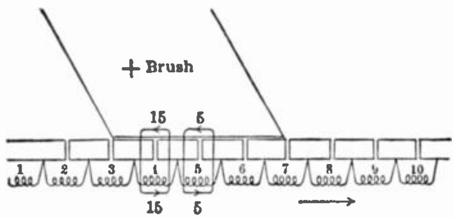


FIG. 312.—Short-circuit currents through brush.

If the local short-circuit currents of Fig. 312 be superposed on those of Fig. 311(*a*), the current distributes itself over the brush in the manner shown in Fig. 313(*a*). There are 45 amp. entering the brush and 5 amp. leaving it. Therefore, the brush must carry 50 amp. instead of 40 amp., and in one segment there are 20 amp., or twice the current flowing under the ideal conditions of Fig. 311. This will tend to produce undue heating and undue sparking under the heel of the brush.

Figure 313(*b*) shows the manner in which the current in the coil varies under these new conditions. Instead of dropping uniformly from 20 amp., it *rises* to 25 amp. before starting to reverse. It will be noted that the time for reversing from +20 amp. to −20 amp. has been reduced from time t to time t_1, making commutation more difficult. The curve of Fig. 313 occurs when the brush is too far back of the neutral plane and electromotive forces are induced in the coils as they undergo commutation.

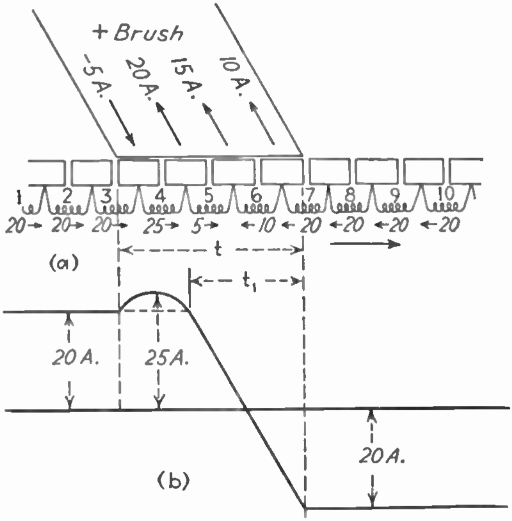


FIG. 313.—Change of current in coil when brushes are too far back of neutral plane.

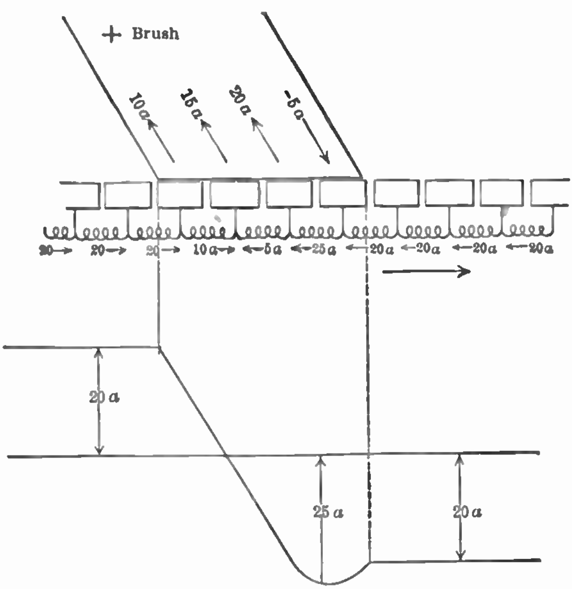


FIG. 314.—Commutation with brushes too far ahead.

The curves of Fig. 311(*b*) and 313(*b*) are called *commutation curves.*

If the brushes are placed too far ahead of the neutral plane, short-circuit currents flow under the toe of the brush, resulting in the current distribution and commutation curve of Fig. 314. This condition produces undue sparking under the toe of the brush.

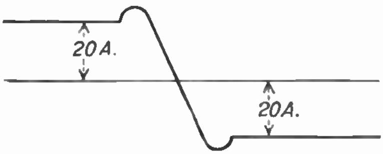
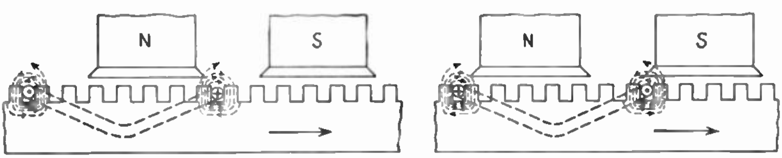


FIG. 315.—Commutation with too wide a brush.

If the brushes are too wide, both the heel and the toe of the brush will short-circuit coils in which e.m.fs. are induced, resulting in the commutation curve of Fig. 315. Moving the brushes either backward or forward does not improve matters in this case. The only remedy is a narrower brush.

270. Electromotive Force of Self-induction.—Figure 316(*a*) shows an armature coil just as it is entering the commutation zone. The slot conductors are embedded in iron giving a good magnetic circuit and therefore considerable flux links the coil, due to the current in the coil. Most of this flux encircles or links the coil sides in the slots, passing through the armature iron, the teeth, and across the slot opening. Since most of the reluctance of the path is in air, the flux is practically proportional to the current in the coil. When the coil is just entering the commutating zone, the general direction of the flux is upward through the coil



(*a*) Before commutation (*b*) After commutation

FIG. 316.—Change of flux through coil undergoing commutation.

(Fig. 316(*a*)). Since the flux is practically proportional to the current in the coil, the values of flux, as the coil passes through the commutating zone, follow the law of the current (Figs. 311, 313, 314, 315).

Let ϕ_1 be the magnitude of the total flux linking the coil. In Fig. 316(*b*) the coil is shown just after it has left the commutating zone. The current in the coil is the same as in 316(*a*) but now flows in the reverse direction. The flux linking the coil has still the magnitude ϕ_1 but is reversed in direction. Therefore, in the time t sec. required for a segment to pass the brush or through the commutating zone, the flux has changed by $2\phi_1$ lines. This is shown in Fig. 317, where ideal commutation is assumed. This change of flux will induce an e.m.f.

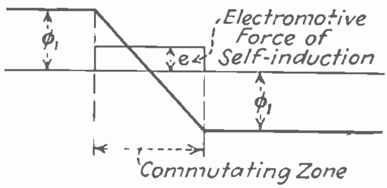


FIG. 317.—Electromotive force of self-induction in coil undergoing commutation.

$$e = -N\frac{d\phi_1}{dt}10^{-8} \text{ volts (Eq. (131), p. 289).}$$

When ideal commutation takes place (Fig. 317),

$$e = -N\frac{2\phi_1}{t}10^{-8} \text{ volts (Eq. (130), p. 288).}$$

N being the number of turns in the coil.

This e.m.f. with its proper direction is shown in Fig. 317. It is an e.m.f. of self-induction. That is, flux produced by the current in the coil itself links the coil and the coil has self-inductance. During the time that an e.m.f. is induced in the coil, the coil is short-circuited by the brush. The e.m.f. is acting in a circuit of very low resistance, and, unless this e.m.f. is in large measure neutralized, it will produce a large short-circuit current through the commutator and brush resulting in severe sparking. If the brushes are set so that the coils undergoing short circuit are in the geometrical neutral, where they are cutting no flux and hence have no induced e.m.f. due to rotation, there will still be the e.m.f. of self-induction and severe sparking will result.

In order to neutralize this e.m.f. of self-induction in a generator without interpoles, it is necessary to set the brushes *ahead* of the neutral plane so that the sides of the short-circuited coils find themselves in a field of the same polarity as that which they enter on leaving the commutating zone (see Fig. 305). Under these conditions the e.m.f. induced by rotation is in opposition to the e.m.f. of self-induction. By choosing a correct position for the brushes, the e.m.f. induced by the cutting of the field can be made theoretically to neutralize the e.m.f. of self-induction, making the net e.m.f. in the coil zero and thus eliminating sparking. Actually, due to the slope of the flux-density curve in the fringe of the pole (see B_1, Fig. 321, p. 432), it is impossible to find a brush position for which perfect neutralization of the e.m.f. of self-induction is obtained. Some sparking always takes place.

The effect of moving the brushes may be considered also from another point of view. The self-inductance of the coil undergoing commutation acts as electrical inertia, tending to prevent the reversal of the current. The direction of the e.m.f. induced by the cutting of the pole flux in advance of the neutral plane is the same as the direction in which the reversed current is about to flow. Hence, by moving the brushes ahead, the e.m.f. induced in the coil by the cutting of the pole flux helps the current to reverse.

Thus in a *generator* it is necessary that the brushes be kept *in advance* of the neutral load plane in order to obtain satisfactory commutation under load conditions.

Practically all modern generators and motors have commutating poles, and this movement of the brushes with change of load is therefore unnecessary. However, there is still in operation a large number of generators and motors with no commutating poles. The function of the commutating pole is best understood when the effects of armature reaction and brush position on commutation are known.

271. Sparking at the Commutator.—The e.m.fs. induced in a coil due to the shifting of the neutral plane and to its own self-inductance are comparatively low in value, being of the order of magnitude from a few tenths of a volt to 4 or 5 volts possibly. But they are acting in a circuit having very low resistance. The resistance of a single coil is low so that most of the circuit resistance is due to the brush contact. If the brush-contact resistance is too low, the short-circuit current may reach such values as to produce severe sparking at the brushes. On the other hand, a brush with low contact resistance is desirable from the standpoint of conducting the current to the external circuit with minimum contact loss.

Copper brushes have a very low contact resistance, but the short-circuit currents are excessive when they are used. Therefore, their application is limited to low-voltage, high-current dynamos. Copper gauze, rolled and pressed into a rectangular cross-section, is often used. Another disadvantage of using copper brushes is that they "cut" the commutator mechanically. These disadvantages have been overcome in considerable measure by brushes of a graphite-copper composition which have low resistance and do not abrade the commutator.

Carbon brushes have a much higher contact resistance than copper and thus limit the short-circuit currents, giving much more satisfactory commutation. In addition, they are more or less graphitic in composition and lubricate the commutator to a degree. Unusually hard carbon brushes may cut the commutator. Different grades of carbon are required for different operating conditions.

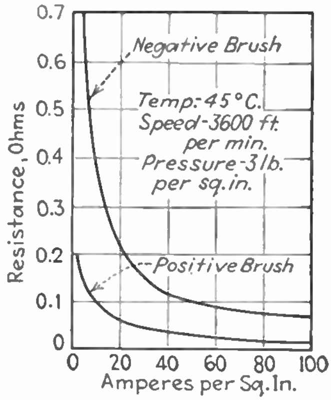


FIG. 318.—Contact-resistance characteristics of carbon brushes.

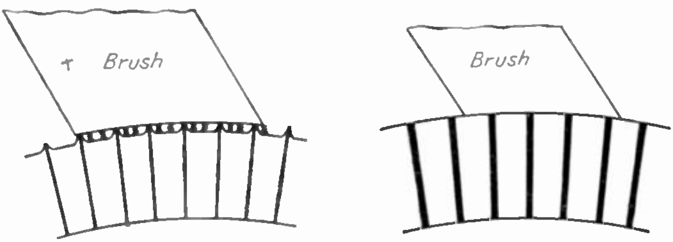
The passage of the current from the commutator to the brush is more of an arc phenomenon than one of pure conduction. A careful examination will show myriads of minute arcs existing between the brush surface and the commutator. The voltage drop between the commutator and the brush, instead of being proportional to the current (as it would be with linear conduction), is substantially constant and is equal to about 1 volt per brush. Bits of copper may be found in the *positive* brush due to the arcing. The voltage drop across the *negative* brush is *greater* than that across the *positive* brush, due to the commutator copper being positive in one case and negative in the other. The resistance of the armature, *including brushes*, diminishes with increasing current.

These relations are illustrated in Fig. 318,[1] which shows the variation of contact resistance for positive and negative brushes

[1] From HESSLER, VICTOR P., "The Effect of Various Operating Conditions upon Electrical Brush Wear and Contact Drop," *Iowa Eng. Exp. Sta.*

when operating on copper rings. All these facts substantiate the arcing theory.

Another proof of the theory is "high mica." After a machine has been in operation for a considerable time, it often happens that the mica insulation between the commutator segments protrudes above the surface of the commutator, resulting in "high mica" (Fig. 319(*a*)). It was supposed at one time that this was due to mica being harder than copper, which resulted in the wearing away of the copper more readily than the mica. The fallacy



(*a*) Commutator with high mica (*b*) Undercut mica commutator

FIG. 319.—Brush on commutator.

of this supposition is evident. Even though mica is harder than copper, the two must always wear down evenly, for the brush cannot grind the copper until it comes in contact with it. Hence, the brush must grind down the mica before it can touch the copper, if high mica is due to mechanical abrasion alone.

The rational explanation of high mica is given in detail by B. G. Lamme in a paper presented before the American Institute of Electrical Engineers.[1] The copper is not worn away, as was formerly supposed, but is carried away by the minute arcs that exist between the brush and the commutator, as in Fig. 319(*a*). This may be proved by running two similar machines for the same period of time, one of the machines delivering current and the other having no current in brushes and commutator. High mica will ultimately appear on the commutator which

Bull. 122, Iowa State College. The negative-brush characteristic was taken from Fig. 60 and the positive-brush characteristic was plotted from the data of Fig. 59 in the paper.

[1] LAMME, B. G., "Physical Limitations in D. C. Commutating Machinery," *A.I.E.E. Trans.*, Vol. 34, Part II, p. 1739, 1915.

carries current, whereas it will be found impossible to produce high mica on the machine which carries no current.

High mica may be reduced by the use of fairly hard brushes which grind the mica down. Amber mica appears to be softer than white mica and is often preferred for commutator-segment insulation. In modern practice the mica is undercut by many manufacturers, that is, the top of the mica is below the commutator surface, as in Fig. 319(*b*). There is some disadvantage in this construction, in that small bits of copper, carbon, and dirt collect in the grooves and may ultimately short-circuit the segments. However, these grooves can be cleaned out easily.

The result of any arcing under the brush is to pit the commutator. As irregularities and depressions in the commutator tend to prevent intimate contact between the brush and commutator, arcs of increasing magnitude will be formed. The deeper the depressions, or the higher the mica, the larger and more vigorous these arcs become. Hence, any condition which produces sparking and so roughens the commutator increases the sparking and roughening, these being cumulative actions. If a commutator is sparking badly and the cause of the sparking is not corrected, the commutator will deteriorate very rapidly and soon will become inoperative.

The brushes should be fitted very carefully to the commutator surface by grinding with sand paper or "sanding in" in the manner shown in Fig. 320. Carbon on the surface of the commutator should be removed with an oily cloth. *Do not use waste.* A slightly roughened commutator may be partially smoothed with fine sandpaper. *Do not use emery,* as the particles of emery are conducting and may short-circuit the commutator bars. If the commutator is grooved by the brushes or is otherwise in poor condition, it should be turned down in a lathe.

Other difficulties, such as loose mica and loose segments, are more serious in character. It is often possible to rectify these last difficulties by tightening up the commutator clamp-bolts.

272. Commutating Poles (Interpoles).—Figure 321 shows the geometrical neutral or no-load neutral plane and the neutral plane of a generator when under load. It is to be noted that this figure is taken from Fig. 305 (p. 416). If the brushes remain in the no-load neutral plane, there will be severe sparking under load conditions, because of the flux density B_2, due to armature reaction and existing in the neutral zone. The brushes will not

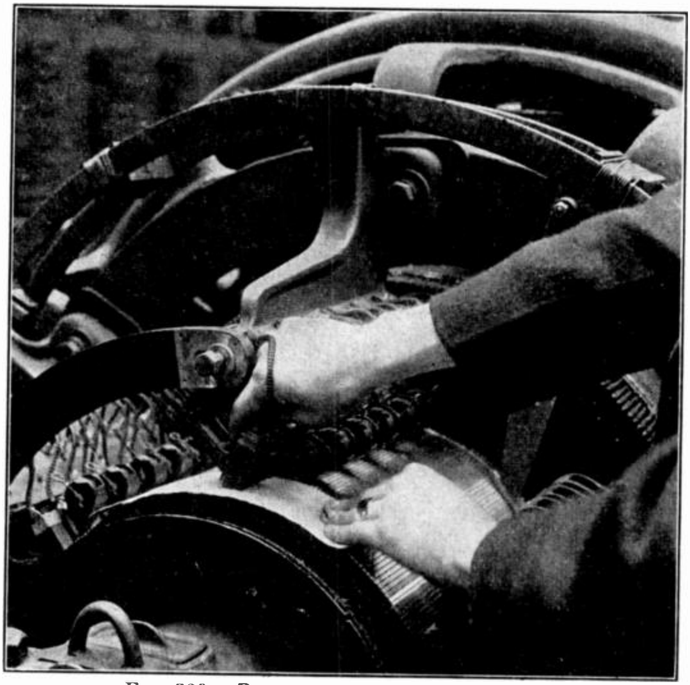


FIG. 320.—Proper method of fitting brushes.

commutate properly even if advanced to the load neutral plane. This is due to the fact that the e.m.f. of self-induction exists in the coils undergoing short circuit, even if the e.m.f. due to the cutting of the pole flux is zero. The brushes must be advanced so that

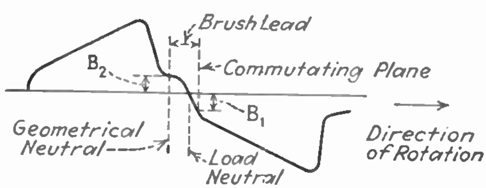


FIG. 321.—Brush advance to proper commutating plane.

the short-circuited coils are cutting the flux density B_1 of the next pole, as in Fig. 321, in order that an e.m.f. may be generated which will balance the e.m.f. of self-induction. It will be noted that this position is in the fringe of the next pole-flux. As a very slight movement of the brushes in either direction brings a very marked change in the flux density, it is difficult to obtain good commutation under these conditions. In fact it may be impossible to obtain satisfactory commutation because of the steepness of the flux-distribution curve. When the best position of the brushes is obtained, the trailing tip of each brush may be in too strong a field and the leading tip in too weak a field.

If a flux density having the same value as B_2, but opposite to it in direction, can be produced in the geometrical neutral, it is clear that the flux density in the neutral plane can be brought to zero notwithstanding armature reaction. If a flux density of a value $B_2 + B_1$ is produced, satisfactory commutation is obtained without moving the brushes. It is the function of commutating poles to produce the desired flux density in the neutral zone.

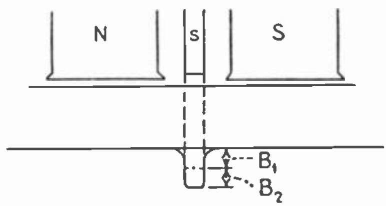


FIG. 322.—Flux density produced by commutating pole alone.

Commutating poles are narrow poles located between the main poles. They send a flux into the armature of the proper magnitude to produce satisfactory commutation. For example,

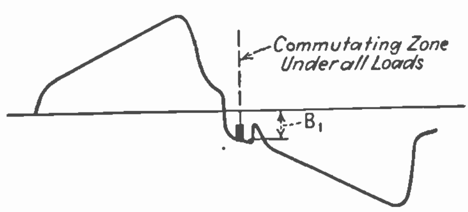


FIG. 323.—Resultant of main flux and commutating-pole flux—machine loaded.

in Fig. 321 the commutating pole must first produce a flux density equal to B_2 so as to neutralize in the neutral zone the increase of flux density due to armature reaction. It must also produce an additional flux density B_1 in order to balance the e.m.f. of self-induction in the coil undergoing commutation. This commutating-pole flux density is shown in Fig. 322. The pole producing it at this point must be a S-pole. Figure 323 shows the resultant flux obtained by combining Figs. 321 and 322.

As the armature reaction and the e.m.f. of self-induction in the coils undergoing commutation are both proportional to the armature current, the flux density necessary to compensate for them, produced by the commutating poles, must also be proportional to the armature current. Therefore, the commutating poles are wound with a few turns of comparatively heavy wire and are connected in series with the armature, as in Fig. 324. The

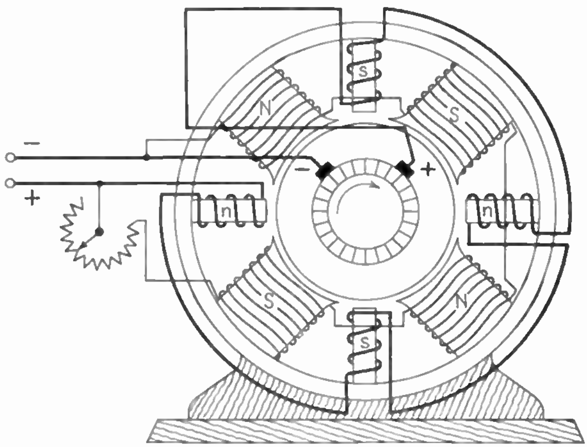


FIG. 324.—Connections of shunt field and commutating poles.

air-gap between the commutating poles and the armature is large, so that at all loads the commutating-pole flux is nearly proportional to the armature current.

It should be noted that the sequence of poles in the direction of rotation in a generator is *NsSn*, where the capitals refer to the main poles and the small letters refer to the commutating poles. Figure 325 shows an interpole with its winding separate from the dynamo.

In Fig. 326 the frame and poles of a dynamo are shown. The arrangement and connections of both main and commutating poles are clearly indicated (also see Fig. 281, p. 391).

It is possible to use only two interpoles in a four-pole dynamo. This comes from the fact that, with the usual windings, when one side of a coil is under a *N*-pole, the other side is under a *S*-pole. Hence by using a single interpole for each pair of poles, this single interpole having twice the magnetic strength that would be necessary for each of two interpoles, the correct e.m.f.

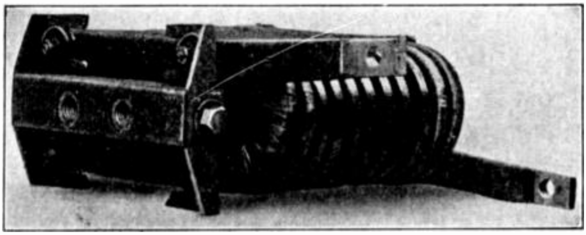


FIG. 325.—Commutating pole and winding.

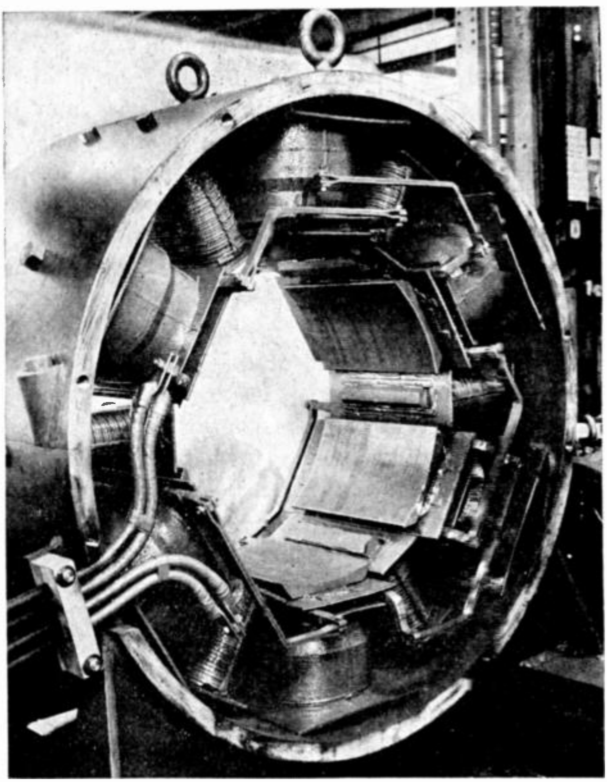


FIG. 326.—Frame and poles of dynamo showing arrangement of commutating poles. (*Reliance Electric and Engineering Company.*)

for commutation is induced in the coil. With a wave winding, a single interpole would suffice, irrespective of the number of poles, if it could be made of sufficient magnetic strength (see Fig. 273, p. 384). In practice, however, the number of interpoles is usually made equal to the number of main poles.

Commutating poles are so designed that they produce a flux density of greater magnitude than is necessary. Formerly, the method of obtaining the correct density was to shunt the entire commutating-pole circuit by a low resistance, the resistance being adjusted until the best condition of commutation was obtained. Modern practice is to dispense with such shunt resistance and to adjust the effect of the commutating poles by the use of shims between the commutating poles and the yoke.

Commutating poles increase the leakage flux between the main poles. For this reason the main-pole arc, normally 0.70 the pole pitch, is reduced to 0.65 the pole pitch or less (Figs. 324 and 326). Also, the commutating poles must be made very narrow, and at moderate values of overload they usually become saturated. It is for this reason that the air-gaps are made relatively large so that the commutating-pole flux will tend to remain proportional to the armature current.

The distinction between the action of the commutating poles and of the Thompson-Ryan compensating windings (p. 421) should be kept in mind. Both are connected in series with the armature and with both the m.m.fs. act along an axis midway between the main poles. The commutating poles, however, act only *locally* in the commutating zone, giving the flux in that zone the correct value for good commutation. On the other hand, the Thompson-Ryan poles practically neutralize the entire armature reaction and are larger and more expensive than commutating poles. However, in the commutating zone, the Thompson-Ryan poles must do more than merely to neutralize the armature m.m.f. if the e.m.f. of self-induction is to be neutralized (see B_2, Fig. 322). In some generators and motors operating under unusually severe conditions, both types of poles are used in the same dynamo (Fig. 281, p. 391).

273. Shunt-generator Characteristics.—If a shunt generator, after building up to voltage, be loaded, the terminal voltage will drop. This drop in voltage will increase with increase in load. Such a drop in terminal voltage is undesirable, especially when it occurs in generators which supply power to incandescent lamps.

It is important to know the voltage at the terminals of a generator for each value of current that it delivers, because the ability to maintain its voltage under load conditions determines in large measure the suitability of a generator for a specified service.

The relation between the terminal volts and the current which a generator delivers is called its *characteristic* and is usually shown graphically. When the relation is between the *terminal* volts and the current to the load, it is called the *external* characteristic. The *internal* or *total* characteristic is usually the relation between the induced e.m.f. in the armature and the total current from the armature (see Par. 275, p. 441). Unless otherwise specified, the term *shunt characteristic* usually refers to the external characteristic of the shunt generator.

To test a generator to determine the relation of terminal volts to current, it is connected as in Fig. 296 (p. 406). The machine is self-excited and a voltmeter is connected across its terminals to indicate the terminal volts. An ammeter is connected in the line to measure the load current. In performing this test, it is often desirable to connect an ammeter in the field circuit so that the change in field current as load is applied may be determined.

In starting the test, rated load should first be applied and the field current adjusted until rated voltage is obtained. It is desirable to run the machine under these conditions for 20 min. or longer in order to give the field opportunity to warm up (see Fig. 404, p. 548). The load should then be removed and the no-load voltage read on the voltmeter. The field rheostat should remain unchanged. The load should then be gradually applied, reading the volts and the current for each load. The speed of the generator should be maintained constant throughout. If the readings be plotted as in Fig. 327, the shunt characteristic is obtained. If, in a small generator, the load be carried far enough, the terminal voltage will begin to decrease rapidly as shown at point *c* and beyond, Fig. 327. This is called the *break-down* point and the generator is said to "unbuild." Further application of load results in a very rapid decrease of voltage, and beyond a certain point any attempt at increase of load results in a *decrease* of current rather than an increase, as at point *d* in Fig. 327. The load may even be carried to short-circuit as at e and yet the current will actually decrease as this short-circuit condition is approached. At short circuit the field is short-circuited and any current flowing is due merely to the residual magnetism of the machine.

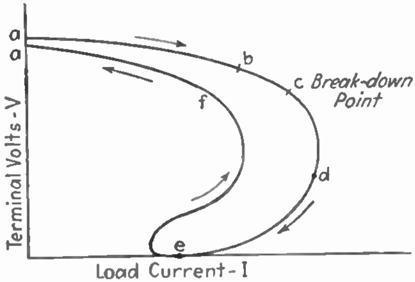


FIG. 327.—Shunt-generator characteristic.

If the external resistance now be increased, the voltage will rise slowly and will ultimately reach a value not far below that at which it started. The fact that the voltage follows a different curve when the short circuit is removed is due primarily to hysteresis. When the load is being applied, the voltage is dropping and the iron is on the part of the cycle represented by c (Fig. 292(a),

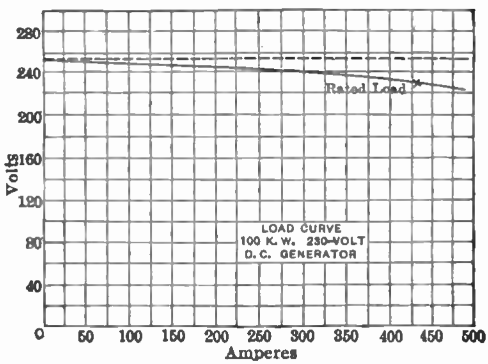


FIG. 328.—Typical shunt characteristic.

p. 403). When the voltage starts to increase, the return curve is along the path a (Fig. 292(a)). On path a there is less flux for a given field current than on path c and consequently less e.m.f. is induced. This fact, together with a lesser field current resulting from the lower terminal voltage, accounts for the return curve *efa'* lying below *abde*.

In practice, generators are operated only on the portion *ab* of the characteristic (Fig. 327). Figure 328 shows this portion of the curve for a 100-kw., 230-volt generator. The rated current is 100,000 ÷ 230 = 435 amp. The generator field rheostat is set so that the terminal voltage is 230 volts when the generator is delivering 435 amp.

There are three reasons for the drop in voltage of a shunt generator under load.

1. The terminal voltage is less than the induced e.m.f. by the resistance drop in the armature. That is, the terminal voltage

$$V = E - I_a R_a \tag{213}$$

where E is the induced e.m.f., I_a the armature current, and R_a the armature resistance.

Example.—The e.m.f. induced in the armature of a shunt generator is 600 volts. The armature resistance is 0.1 ohm. What is the terminal voltage when the armature delivers 200 amp.?

Applying Eq. (213),

$$V = 600 - (200 \times 0.1) = 600 - 20 = 580 \text{ volts.} \quad \textit{Ans.}$$

2. Armature reaction weakens the field and so reduces the induced e.m.f.

3. The drop in terminal voltage due to 1 and 2 results in a decreased field current. This in turn results in a decreased induced e.m.f.

The effect of each of these three factors is shown in Fig. 329.

It might appear that the voltage of the generator would drop to zero, practically, of its own accord when the load is first applied, because the foregoing cycle is cumulative. That is, a lesser terminal voltage results in a weaker field, a weaker field results in a lesser induced e.m.f., and therefore a lower terminal voltage, which still further reduces the field, etc. The cycle *would* result in the terminal volts reaching zero, if the iron were not saturated in some degree. If a 10 per cent drop in terminal voltage resulted in a 10 per cent drop in flux, the generator would be unable to supply any substantial load. However, a 10 per cent drop in terminal voltage and thus in field current results probably in a 1 or 2 per cent drop in flux, due to saturation and hysteresis, as in Fig. 292(a) (p. 403). Therefore, a generator when operating at high saturation maintains its voltage better than when operating at low saturation.

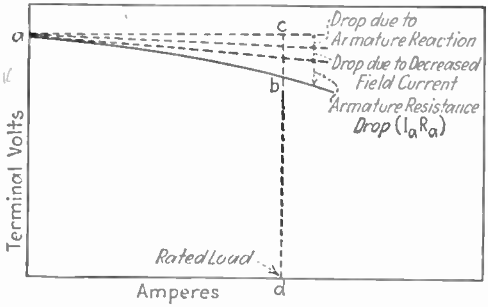


Fig. 329.—Voltage drops in shunt generator.

This is illustrated in Fig. 330, which shows two saturation curves for a 230-volt generator, one at 900 r.p.m. and the other at 1,200 r.p.m. If the no-load voltage of the generator in each case is 230 volts, the generator will be operating at point (a) on the 1,200-r.p.m. curve and at point (b) on the 900-r.p.m. curve.

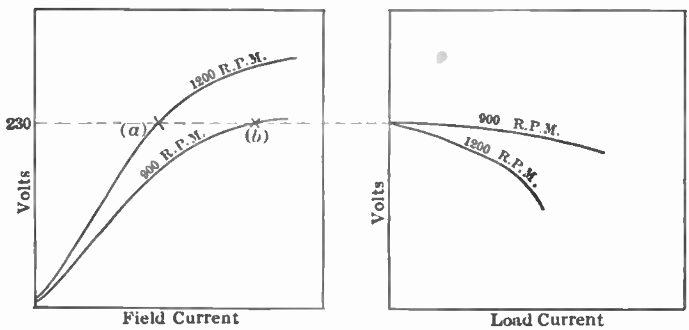


Fig. 330.—Relation of shunt characteristics to speed.

As point (b) corresponds to a much higher saturation of the armature and field iron than (a), the generator will maintain its voltage better at 900 r.p.m. than at 1,200 r.p.m., as shown by the characteristics in Fig. 330. (The effect of saturation is further discussed in Par. 276, p. 443.)

274. Generator Regulation.—The ability of a generator to maintain its voltage under load is a measure of its suitability for constant-potential service. The *regulation* shows quantitatively the change in terminal voltage from rated load to no load.

Regulation is defined in the A.I.E.E. Standards as follows:[1]

> The regulation of a d.-c. generator is usually stated by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads. The regulation of d.-c. generators refers to changes in voltage corresponding to gradual changes in load, and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

Other conditions of test are that the generator shall be tested at rated speed, allowing the specified drop in speed inherent in the prime mover; that unless specified, the load shall be increased to rated load and the regulation obtained with decreasing load; with self-excited machines the resistance in the field circuit shall remain constant; the machine shall have reached constant temperature.

As an example, the regulation of the generator whose characteristic is shown in Fig. 328 is given by 250 volts at no load and 230 volts at rated load.

275. Total Characteristic.—Reference is often made to the total characteristic of a shunt generator. The *shunt* characteristic, to which reference has been made, is the relation between *load* current and *terminal* volts. The *total* characteristic is the relation between *armature* current and *induced* volts.

The armature current differs from the load current by the current flowing in the field.

The armature current

$$I_a = I + I_f \tag{214}$$

where I is the load current and I_f the shunt-field current.

The induced volts

$$E = V + I_a R_a \tag{215}$$

[1] A.I.E.E. Standards, "Direct-current Rotating Machines, Generators, and Motors," No. 5, July, 1925.

where V is the terminal voltage and R_a the armature resistance, including brush and brush-contact resistance. The total characteristic is the curve showing the relation of I_a and E. It may be found graphically from the shunt characteristic as follows:

Let qr (Fig. 331) be the shunt characteristic. Draw the field resistance line oa, as is done in Figs. 297 and 298 (pp. 407 and 409). The line will have the *appearance* of being nearly vertical, owing to the fact that the abscissas are plotted to armature-current scale. The horizontal distances from the oY-axis to line oa give the value of field current for each value of terminal voltage. By adding these distances horizontally to the shunt characteristic, the total current is given by the resulting characteristic qe. For example, at point c on the shunt characteristic, the distance $c'd'$ is added horizontally at cd, giving point d on the characteristic qe.

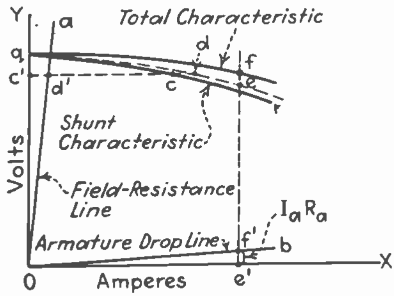


FIG. 331.—Total characteristic of shunt generator.

The armature-resistance-drop line ob is then plotted, assuming that the brush-contact resistance is constant. The voltage drop in the armature is then proportional to the current. It is necessary merely to determine the drop $e'f'$ at some value of current oe'. That is, the voltage drop

$$e'f' = (oe')R_a.$$

Draw the line $of'b$. The vertical distances from the oX-axis, oe' to of', give the armature drop for each value of current. Adding these drops to the characteristic qe, as $ef = e'f'$ is added at the point e, the total characteristic qf is obtained.

It should be borne in mind that the total *induced* e.m.f. multiplied by the total current gives the total power developed by the armature. All of this power is not available, however, for two reasons.

1. Some of this power is lost in the armature itself, appearing as $I_a^2R_a$ loss in the armature copper.

2. Some of the armature output is consumed in heating the shunt field.

Example.—A 20-kw., 220-volt, shunt generator has an armature resistance of 0.07 ohm and a shunt-field resistance of 200 ohms. What power is developed in the armature when it delivers its rated output?

Rated current

$$I = \frac{20{,}000}{220} = 90.9 \text{ amp.}$$

Field current

$$I_f = \frac{220}{200} = 1.1 \text{ amp.}$$

Armature current

$$I_a = 90.9 + 1.1 = 92.0 \text{ amp.}$$

Induced volts

$$E = 220 + (92.0 \times 0.07) = 226.4 \text{ volts.}$$

Power developed in armature

$$P = 226.4 \times 92.0 = 20{,}830 \text{ watts} = 20.83 \text{ kw.} \quad \textit{Ans.}$$

The same result may be obtained by adding power losses as follows:

Field loss

$$P_f = \frac{(220)^2}{200} = 242 \text{ watts.}$$

Armature loss

$$P_a = (92.0)^2 0.07 = 592 \text{ watts.}$$

Power developed in armature

$$P = 20{,}000 + 242 + 592 = 20{,}834 \text{ watts} = 20.83 \text{ kw. } (\textit{check}).$$

When current flows in the armature of a generator it develops a motor torque or countertorque (p. 481) which tends to drive the armature in the *reverse* direction. The prime mover, such as a motor, steam turbine, or water wheel, must overcome this countertorque. Thus this countertorque is the reaction in the generator which causes the prime mover to develop the energy necessary to drive the generator (also see Par. 297, p. 480).

276. Relation of Shunt Characteristic to Saturation Curve.—It is pointed out in Par. 273 (p. 439) that the shunt characteristic depends on the degree of saturation of the magnetic circuit and hence on that part of the saturation curve at which the generator is operating. A method of determining the shunt characteristic directly from the saturation curve and the field-resistance line is as follows.

a. Armature Reaction Negligible.—Assume first that the armature reaction is so small as to be negligible. This condition often occurs with interpole machines in which the brushes remain in the geometrical neutral. Figure 332 gives the saturation curve of a shunt generator taken at some definite speed. The dotted line is taken with increasing values and the solid line with decreasing values of field current. The field-resistance line is *oa* and the generator will build up along the dotted line to point *a*, neglecting

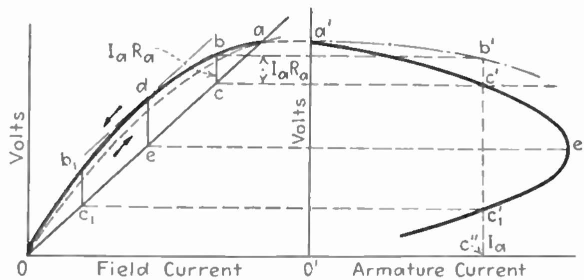


FIG. 332.—Relation of shunt characteristic to saturation curve—no armature reaction.

the small drop in the armature due to the field current. Hence, (neglecting field current) a' on the right-hand plot will be the generator terminal voltage when the armature current is zero. With the application of load, two effects are the first to cause the terminal voltage to drop, the resistance drop (I_aR_a) in the armature and the decreased field current due to the lessened terminal voltage. The armature reaction is assumed to be negligible.

When the generator has reached a condition of stability, two conditions must be fulfilled. The induced e.m.f. must lie on the descending portion of the saturation curve *abo*, as at some point *b*; the corresponding terminal voltage must lie on the field-resistance line at point *c*, since the field-circuit terminals are connected directly to the armature terminals. Moreover, points *b* and *c* must have the same abscissa, for the field current which gives the induced e.m.f. *b* is also caused to flow by the terminal voltage corresponding to *c*. Furthermore, the distance *bc* must be equal to I_aR_a, since the difference between the induced e.m.f. and the terminal voltage is equal to the armature-resistance drop. That is, the induced e.m.f. E at b is equal to the terminal voltage V at c plus the I_aR_a drop (see Eq. (215)). Hence, c gives one value of terminal voltage. The corresponding armature current I_a is found by dividing bc by R_a; that is, $I_a = bc \div R_a$. This value of armature current is laid off at $o'c''$ along the axis of abscissas in the right-hand graph. The ordinate c is projected horizontally to meet at c' the ordinate erected at c''. The point c' is a point on the characteristic. Since b gives the induced e.m.f. for this value of armature current, b' is a point on the total characteristic. If the usual shunt characteristic is desired, the load current may be found by subtracting the field current from I_a, either graphically by the method given in Fig. 331 or by subtracting numerically the value of field current corresponding to point c on the field-resistance line.

A study of Fig. 332 shows that ordinarily there is a second place, such as at b_1c_1, where bc may be fitted vertically between the saturation curve and the field-resistance line. Hence, there must be two values of terminal voltage corresponding to the same value of armature current. The load current will be slightly different in the two cases, since the values of field current are not the same. Point c_1', on the lower portion of the characteristic, gives the second value of terminal voltage corresponding to the given value of armature current.

The maximum value of armature current may be found by drawing a tangent to the saturation curve at d parallel to the field-resistance line. The corresponding value of armature current is found by dividing the vertical distance de by the armature resistance. This gives point e' on the characteristic.

With constant armature resistance, the distances bc, de, etc., are proportional to the armature current. Hence, the abscissas for the characteristic may be found also by drawing a number of vertical lines between the saturation curve and the field-resistance line, determining the armature current for each of these lines by scaling.

As is pointed out in Par. 271 (p. 429), the armature resistance is not constant but is a function of the armature current. A considerable error results in assuming the armature resistance to be constant. Also, the ordinates of the saturation curve depend so much on the previous magnetic history of the iron that the curve does not ordinarily repeat itself accurately. A small percentage change in either the saturation curve or the field-resistance line makes a much larger percentage change in their difference. Therefore, as would be expected, characteristics obtained by this method may only approximate the characteristics obtained by test. However, the method is extremely valuable in a qualitative sense. For example, if in Fig. 332 the speed be increased and the no-load voltage be maintained constant by increasing the field resistance, the distances *bc*, *de* will be decreased. Hence, for any given terminal voltage, the armature current

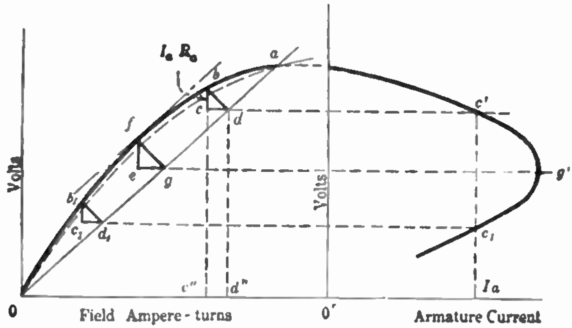


FIG. 333.—Relation of shunt characteristic to saturation curve including armature reaction.

will be decreased, resulting in a more drooping characteristic (see Fig. 330, p. 440).

b. Armature Reaction Not Negligible.—In Fig. 333, let *abfo* be the descending saturation curve with field *ampere-turns* as abscissa, and *oa* the field-resistance line in terms of field ampere-turns rather than field current. Consider a terminal voltage *dd''*. Point *d* must be on the field-resistance line since the field is connected across the armature terminals. If there were no demagnetizing action of the armature, the induced e.m.f. corresponding to the field ampere-turns *od''* would lie on the saturation curve vertically over *d*, as *b* lies over *c* in Fig. 332. Under load, the armature reduces the total ampere-turns of the magnetic circuit by an amount *d''c''*, where *d''c''* is equal to the armature demagnetizing ampere-turns (see Par. 265, p. 410). Hence, the *net* ampere-turns acting on the field are given by *oc''*. The corresponding induced e.m.f. must be *c''b*.

The condition that the terminal voltage must be equal to the induced e.m.f. minus the I_aR_a drop must be fulfilled. Hence, bc must be equal to the I_aR_a drop in the armature, where cd is equal and parallel to $c''d''$. Therefore, triangle bcd must have such a position that point b lies on the saturation curve, point d lies on the field-resistance line, and cd is parallel to the axis of abscissas. To determine the two sides of triangle bcd, side bc, or the I_aR_a drop, may be calculated as in a (p. 444), using the desired value of armature current. The armature demagnetizing ampere-turns per pole cd may also be calculated (see Par. 265). Knowing bc and cd, the right triangle bcd is determined. It is then merely necessary to fit this triangle between the saturation curve and the field-resistance line, so that cd is parallel to the axis of abscissas, point b lies on the saturation curve, and point d on the field-resistance line. The ordinate $d''d$ is the value of terminal voltage corresponding to the value of the chosen armature current. If the load current is desired, it may be found readily by the method outlined in a. The point c' on the characteristic is then found by projecting horizontally to meet the ordinate I_a, as before.

If the saturation curve is plotted with field current as abscissa, rather than with field ampere-turns, it is merely necessary to divide the armature demagnetizing ampere-turns by N_f, where N_f is the number of shunt-field turns per pole, in order to use the given scale of abscissas.

Other values of armature current may be found as follows: If the armature resistance be assumed constant, both the sides bc and cd of triangle bcd are proportional to the armature current. Hence, the triangles for all values of armature current are similar, and when fitted properly between the saturation curve and the field-resistance line, their corresponding sides are parallel. Therefore, the hypotenuses are parallel to one another. That is, fg is parallel to bd, etc. To determine the armature current for the terminal voltage corresponding to g, draw fg parallel to bd, intersecting the saturation curve at f. The armature current will be equal to fg to the same scale as bd. Triangle feg is drawn to correspond to the maximum value of armature current, the tangent drawn to the saturation curve at f being parallel to the field-resistance line. Point g' on the characteristic is found by projecting horizontally to meet the ordinate corresponding to the value of armature current determined from *fg*. This method is open to the same errors as those of *a*, that is, the armature resistance is not constant and the saturation curve varies because of variations in the magnetic history of the iron. In addition, it is not possible (on account of pole-tip saturation, etc.) to determine with high precision the effect of the demagnetizing ampere-turns.

277. Compound Generator.—The drop in terminal voltage with load, which is characteristic of the shunt generator, makes this

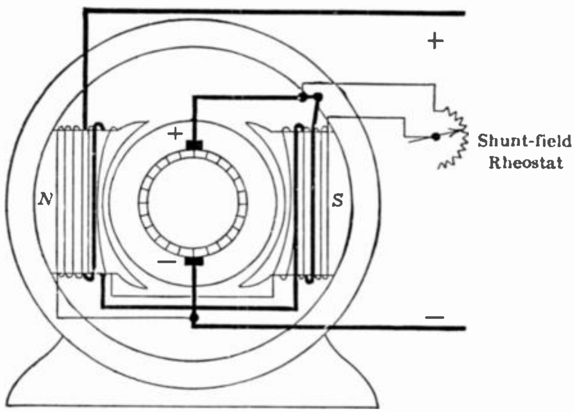


Fig. 334.—Connections of compound generator (short shunt.)

type of generator undesirable where constancy of voltage is essential. This applies particularly to lighting circuits, where a very slight change of voltage makes a material change in the candle-power of incandescent lamps. A generator may be made to produce a substantially constant voltage, or even a rise in voltage as the load increases, by placing on the field core a few turns which are connected *in series* with either the load or the armature. These turns are connected so as to *aid* the shunt turns when the generator delivers current (Fig. 334). As the load increases, the current through the series turns also increases and, therefore, the flux through the armature increases. The effect of this increased flux is to increase the induced e.m.f. By adjustment of the series ampere-turns, this increase in armature voltage may be made to balance the combined drop in voltage due to armature reaction and to armature resistance. If the terminal voltage is maintained substantially constant, the field current will not drop as the load increases. Therefore, the three causes of voltage drop, namely, armature reaction, I_aR_a drop, and drop in field current (Fig. 329), are neutralized more or less completely by the effect of the series ampere-turns.

The shunt field may be connected directly across the armature terminals (Fig. 335(*a*)), in which case the machine is *short-shunt*. If the shunt field be connected across the machine terminals outside the series field (Fig. 335(*b*)), the machine is *long-shunt*. The operating characteristic is about the same in both cases.

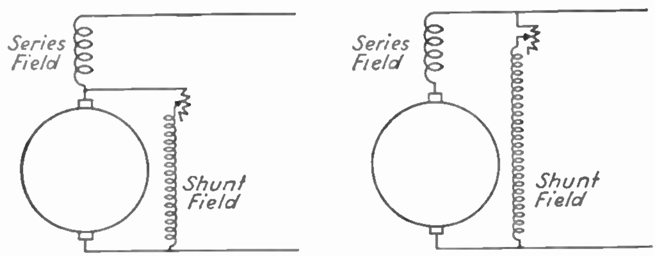


FIG. 335.—Compound-generator connections.

If the effect of the series turns is to produce the same voltage at rated load as at no load, the machine is *flat-compounded* (see Fig. 336). It is seldom possible to maintain a constant voltage for all values of current from no load to rated load. The tendency is first for the voltage to rise and then to drop, reaching the same voltage at rated load as at no load. The shape of the characteristic is due to the saturation of the iron, so that the series ampere-turns do not increase the flux at full load proportionately as much as they do at light load. When the rated-load voltage is greater than the no-load voltage, the machine is *overcompounded*. When the rated-load voltage is less than the no-load voltage, the machine is *undercompounded*. Generators are seldom undercompounded.

Flat-compounded generators are used principally in isolated plants, such as hotels and office buildings. The size of the conductors in the distribution system of such plants is determined almost entirely by underwriters' requirements as to safe current-carrying capacity. Wires conforming to these requirements are usually of such size that only a very small voltage drop takes place between the generator and the various loads.

Overcompounded generators are used where the load is located at a considerable distance from the generator. As the load

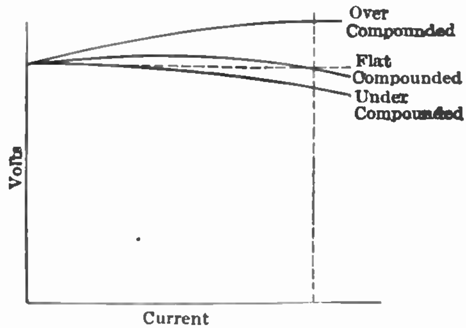


FIG. 336.—Compound-generator characteristics.

increases, the voltage at the load tends to decrease, due to the voltage drop in the feeder. If, however, the generator voltage rises just enough to balance this feeder drop, the voltage at the load remains constant.

Example.—Consider the conditions shown in Fig. 337(*a*). A certain load is 4,000 ft. distant from the generator. The load is supplied over a

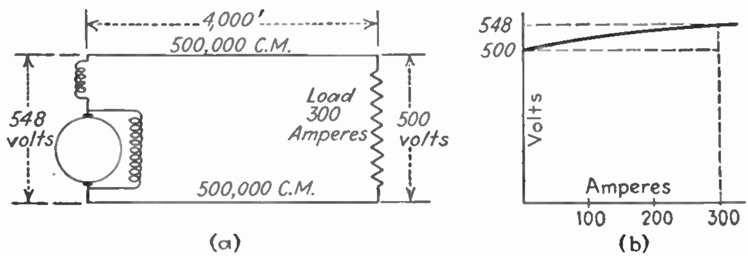


FIG. 337.—Overcompounded generator maintaining constant voltage at the end of feeder.

500,000-cir.-mil feeder. The no-load voltage of the generator is 500 volts. It is desired to maintain the load voltage at a substantially constant value of 500 volts from no load to the maximum demand of 300 amp. What must be the characteristic of the generator?

If the feeder operated at "normal" density, the current would be 500 amp., or 0.001 amp. per circular mil (Par. 43, p. 51), and the drop would be 0.01 volt per foot, making a total drop of 80 volts.

The actual drop is

$$(300 \div 500) \times 80 = 48 \text{ volts}.$$

The generator terminal voltage should rise from a no-load value of 500 volts to 548 volts when 300 amp. is being delivered to the load (Fig. 337(*b*)).

Compound generators are usually wound so as to be somewhat overcompounded. The degree of compounding can then be regulated by shunting more or less current from the series field. To do this a low-resistance shunt, called a *diverter*, is used (Fig. 338).

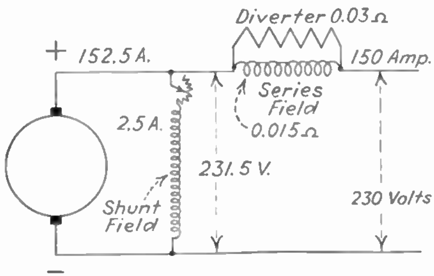


FIG. 338.—Series-field diverter.

Compound generators which supply three-wire distribution systems usually have two series-field windings, one connected to each side of the armature. There are two separate series windings on each pole, one winding being connected to the positive terminal and the other to the negative terminal of the generator (see Fig. 411, p. 556).

In a compound generator, the induced e.m.f. in the armature is

$$E = V + I_sR_s + I_aR_a \qquad (216)$$

where V is the terminal voltage, I_s the series-field current, I_a the armature current, and R_s and R_a the resistance of the series field and armature. If a diverter is used, R_s is the equivalent parallel resistance of the series field and diverter, and I_s is the combined current in the series field and diverter. In a long-shunt generator, $I_s = I_a$.

Example.—In a compound generator, connected short-shunt, the terminal voltage is 230 volts when the generator delivers 150 amp. (Fig. 338). The shunt-field current is 2.5 amp., armature resistance 0.032 ohm, series-field resistance 0.015 ohm, and diverter resistance 0.030 ohm. Determine the induced e.m.f. in the armature, the total power generated in the armature, and the distribution of this power.

The combined series-field and diverter current is 150 amp.

Series-field current

$$I_s = 150\frac{0.030}{0.015 + 0.030} = 100 \text{ amp.}$$

Diverter current

$$I_d = 150\frac{0.015}{0.015 + 0.030} = 50 \text{ amp.}$$

Combined equivalent resistance of series field and diverter

$$\frac{1}{R'} = \frac{1}{0.015} + \frac{1}{0.030}; \quad R' = 0.010 \text{ ohm.}$$

Voltage drop in series field and diverter

$$E' = 150 \times 0.010 = 1.50 \text{ volts.}$$

Armature current

$$I_a = 152.5 \text{ amp.}$$

Induced e.m.f.

$$E = 230 + 1.5 + 152.5 \times 0.032 = 236.4 \text{ volts.}$$

Total power generated

$$P_a = 236.4 \times 152.5 = 36{,}050 \text{ watts} = 36.05 \text{ kw.}$$

Armature loss

$$P_a' = 152.5^2 \times 0.032 = \quad 744 \text{ watts.}$$

Series-field loss

$$P_s = 100^2 \times 0.015 = \quad 150 \text{ watts.}$$

Diverter loss

$$P_d = 50^2 \times 0.030 = \quad 75 \text{ watts.}$$

Shunt-field loss

$$P_{sh} = (230 + 1.5)2.5 = \quad 579 \text{ watts.}$$

Power delivered

$$P = 230 \times 150 = \underline{34{,}500} \text{ watts.}$$

Total 36,048 watts (*check*).

278. Effect of Speed.—Figure 339 shows the saturation curve of a 230-volt, compound generator, taken at 900 r.p.m. The shunt-field rheostat is adjusted so that the generator builds up to a no-load voltage of 230 volts, a certain number of shunt-field ampere-turns being necessary, as indicated by the distance *oa*. When load is applied to the generator, a certain number of series ampere-turns is added. Let the number of series ampere-turns be represented by the distance *ab*. Neglecting armature reaction, the induced e.m.f. will be increased by a value *cd*.

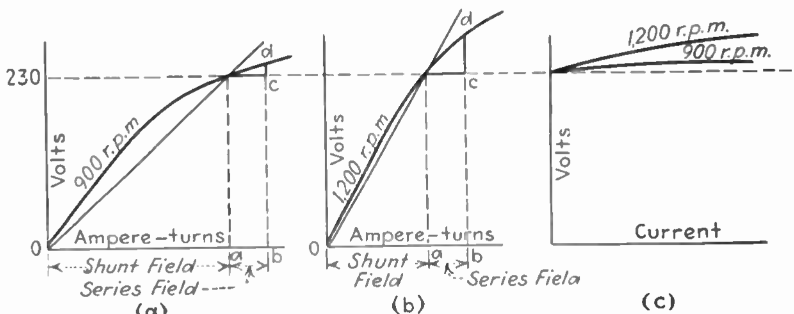


FIG. 339.—Effect of speed on compound characteristic.

Let the same generator be speeded up to 1,200 r.p.m. (Fig. 339(*b*)) and let the no-load terminal voltage still be 230 volts. The distance *oa* will be less than it was in Fig. 339(*a*), owing to the increased speed. But the distance *ab* will be the same in each case, as the increase of series turns depends solely on the current. The *increase of voltage cd* is much greater in (*b*) than in (*a*), owing to the lesser saturation of the iron. Therefore, the higher speed generator will have the more rising characteristic, as in Fig. 339(*c*). It will be noted that the effect of speed on the compound characteristic is just opposite to the effect of speed on the shunt characteristic (see Fig. 330, p. 440). This is due to the fact that in each case saturation opposes change of flux.

279. Relation of Compound Characteristic to Saturation Curve.—The characteristic of the compound generator may be determined from the saturation curve and field-resistance line just as with the shunt generator. In Fig. 340 is shown the saturation curve of a generator, plotted with shunt-field ampere-turns per pole as abscissa, and field-resistance line *oa*. Assume that the generator is long-shunt, so that the armature and series-field currents are the same. At no load, the voltage drop in the armature and series field, due to the shunt-field current, may be neglected. Moreover, this small drop is at no load offset by the series ampere-turns due to the shunt-field current in the series field. Thus at no load the terminal voltage will be equal to $o'a'$. Consider some value of terminal voltage bb''. Since bb'' represents *terminal* voltage, point b must lie on the field-resistance line. The corresponding shunt-field ampere-turns are given by

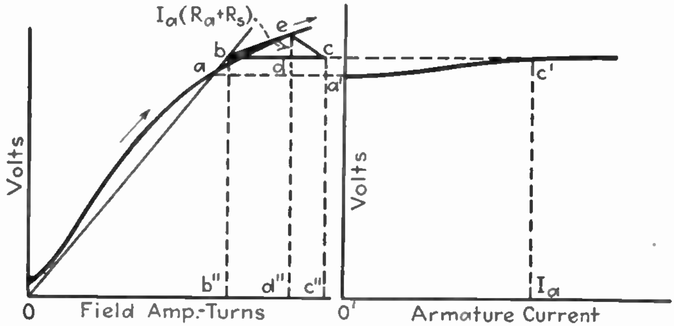


FIG. 340.—Relation of compound characteristic to saturation curve.

ob''. Let bc equal the series-field ampere-turns per pole, I_aN_s, where I_a is the armature current and N_s the number of series turns per pole. The *total* ampere-turns per pole must be the sum of the shunt ampere-turns ob'' and the series ampere-turns $b''c''$, giving total ampere-turns oc''. Were it not for the demagnetizing ampere-turns, the induced e.m.f. would be found on the saturation curve at the point of intersection with ordinate $c''c$. The armature demagnetizing ampere-turns are given by cd or $c''d''$. These must be subtracted from the total ampere-turns oc'' in order to obtain the *net* field ampere-turns per pole od''. Hence, the induced e.m.f. is equal to $d''e$.

As before, the terminal voltage bb'' must be equal to the induced e.m.f. minus the $I_a(R_a + R_s)$ drop, where R_s is the resistance of the series field. Hence, ed is equal to $I_a(R_a + R_s)$.

To plot the characteristic, the value of I_a is assumed. The triangle bec is then determined; $bc = I_aN_s$; $cd = I_aN_D \div P'$; $ed = I_a(R_a + R_s)$. (N_D = armature demagnetizing turns per pole;

P' = parallel paths in armature.) The triangle *bec* is so placed that *bc* is parallel to the axis of abscissas, point *e* lies on the saturation curve, and point *b* lies on the field-resistance line. The line *bc* is extended to the right to meet the ordinate erected at I_a, giving c' as a point on the characteristic. If it is desired to plot the load current as abscissa, the field current may be subtracted, either graphically or numerically, its value ($= ob'' \div N$) being determined from the saturation curve (see p. 442).

The method of determining some other point c_1' on the characteristic is indicated in Fig. 341, which shows the upper portion of the saturation curve of Fig. 340 to larger scale. Assume that

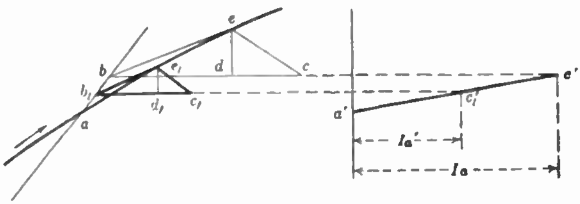


FIG. 341.—Compound characteristic and saturation curve.

point c_1' corresponds to an armature current $I_a' = \frac{1}{2} I_a$. The triangle $b_1e_1c_1$, similar to *bec*, is drawn, each side being one-half the length of the corresponding side of triangle *bec*. Triangle $b_1c_1e_1$ is placed so that side $b_1d_1c_1$ is parallel to the axis of abscissas, e_1 lies on the saturation curve, and b_1 lies on the field-resistance line. The point c_1' on the characteristic is found as before by extending the line $b_1d_1c_1$ to meet the ordinate at I_a'.

If the load current is being increased, ae_1e must be the saturation curve taken with *increasing* values of field current; if the load current is being decreased, ee_1a must be the saturation curve taken with *decreasing* values of field current.

If the saturation curve at ae_1e has curvature, the characteristic $a'c_1'c'$ will not be a straight line (see Figs. 336 and 337 (*b*), p. 450). If I_a is the rated armature current of the generator and the generator is flat-compounded, points *a* and *b* coincide (Figs. 340 and 341).

280. Determination of Series Turns; Armature Characteristic.—It is often desired to determine experimentally the number of series turns necessary to be placed on the poles of a shunt generator in order to make it either flat-compounded or to give it any desired degree of compounding.

To make the determination, adjust the no-load voltage to the desired value, and let this value of shunt-field current be I_1. Load the generator to its rated load and by means of the field rheostat bring the terminal volts to the desired value. Let the corresponding value of field current be I_2. The necessary increase of field ampere-turns is

$$(I_2 - I_1)N_{sh}$$

where N_{sh} = shunt-field turns. Either turns per pole or total turns may be used.

Let I be the rated-load current of the machine and N_s the necessary series turns for the desired degree of compounding. Then

$$N_sI = (I_2 - I_1)N_{sh} \text{ ampere-turns.}$$

$$N_s = \frac{(I_2 - I_1)}{I}N_{sh} \text{ turns.} \qquad (217)$$

Since, in Eq. (217), I is the *rated-load* current, the equation applies to the *short-shunt* connection, for with this connection the series field is directly in series with the load. If it were desired to use Eq. (217) with the *long-shunt* connection, the experiment should be conducted with the ammeter in the armature circuit. The armature current I_a should then be substituted for the load current I in Eq. (217). Little difference would be noticed in the two cases since the shunt-field current rarely exceeds 3 per cent of the armature current and the differences due to hysteresis in the magnetic circuit of the generator would probably exceed any effect due to the shunt-field connection.

The number of series turns for flat-compounding may be obtained also by means of the armature characteristic. The load is applied to the armature in the usual way. It is preferable to excite the field separately, as shown in Fig. 342. Load is applied, and the terminal voltage is maintained constant by means of the shunt-field rheostat. Corresponding values of field current and armature current are noted. When the two are plotted (Fig. 343), the resulting curve is the *armature characteristic.*

Owing to saturation the field current increases more rapidly than the armature current.

The shunt-field ampere-turns necessary to maintain constant voltage across the armature terminals are given by $N_{sh}(bc)$. If

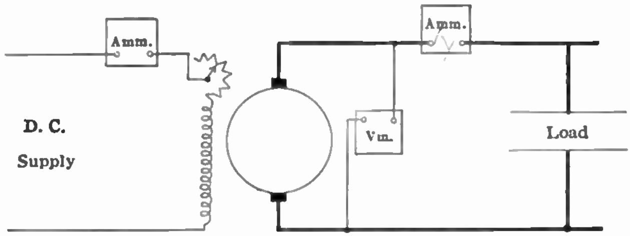


FIG. 342.—Connections for obtaining armature characteristic.

oa is the rated-load current and the generator is connected short-shunt, the number of series turns for flat-compounding, with reference to the voltage at the *armature terminals*, is

$$N_s = N_{sh}\frac{bc}{oa} \text{ turns.} \tag{218}$$

Because of the drop in the series field the voltage at the *generator terminals* is slightly less than the value given by Eq. (218) for flat-compounding. If the series-field resistance be known, the field current *ac* (Fig. 343) may be changed to a value *ac′* (not shown in Fig. 343) which at rated load gives a value of voltage equal to the no-load voltage plus the series-field drop. The series turns for flat-compounding then take the value

$$N_s = N_{sh}\frac{bc'}{oa} \text{ turns.} \tag{219}$$

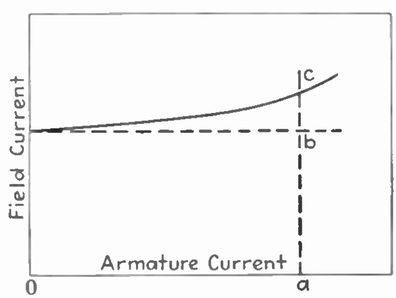


FIG. 343.—Armature characteristic.

In both Eqs. (218) and (219), the very small drop in the armature, due to the shunt-field current, is neglected.

If the generator is connected long-shunt, the value *oa* represents the value of *armature* current at rated load. Equations (218) and (219) may then be used.

281. Compounding Effect of Interpoles.—Flux enters and leaves the armature through the interpoles (Figs. 323 and 324, pp. 433 and 434), and the magnitude of this flux is nearly proportional to the armature current. If the flux from each interpole divides equally on the two sides of the conductor connected to the commutator segment directly under the center of the brush, there is negligible compounding effect of the interpoles. Prac-

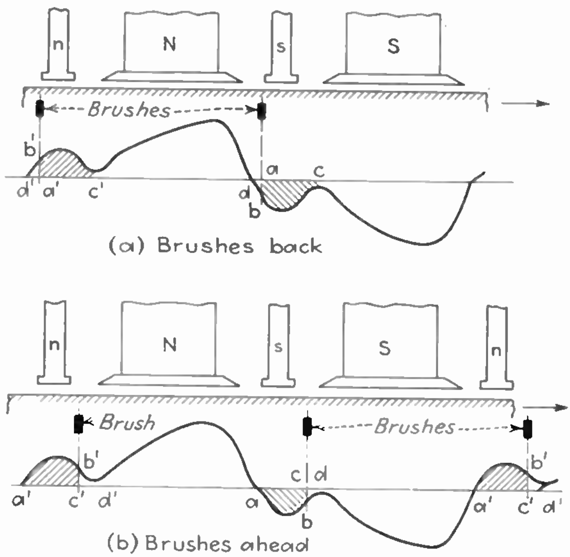


FIG. 344.—Compounding effect of interpoles.

tically, it is not always possible to place the brushes in this ideal position.

It also happens frequently that when the brushes are adjusted to give what appears to be the best commutation conditions, their positions may be such that a coil side to which the brush is connected at the instant, is at one side of the interpole flux, as in Fig. 344(*a*) and (*b*), rather than at the center of this flux. For example, in (*a*) the brushes are back of the center of the interpole flux. Consequently the portion *abc* of this flux adds to the *S*-pole flux between brushes and the portion *a'b'c'* adds to the *N*-pole flux. The small area *abd* subtracts from a *N*-pole flux and the small area *a'b'd'* subtracts from a *S*-pole flux. The net effect, however, is to increase the flux between brushes, and the interpoles tend to overcompound the generator.

If, on the other hand, the brushes are moved ahead of the center of the interpole flux, as in (*b*), the flux corresponding to the area *abc* will subtract from a *N*-pole flux and the flux corresponding to the area *a'b'c'* will subtract from a *S*-pole flux. The small flux areas *bcd* and *b'c'd'* add to *S*-pole and *N*-pole flux respectively. Thus, the net effect of the interpoles in this case is to compound the generator differentially. This is usually undesirable and some care should be taken, therefore, when determining the brush position. Thus the interpoles may have the effect of a series winding on the operating characteristics of a generator. These compounding effects may become distinctly detrimental in the operation of a motor.

282. Series Generator.—In the series generator the field winding is connected in series with the armature and the external circuit. It consists necessarily of comparatively few turns of wire having sufficiently large cross-section to carry the rated current of the generator.

The series generator in most instances is used for constant-current work, in distinction to the shunt generator which maintains constant potential. Figure 345 shows the saturation curve of a series generator and also its characteristic. The saturation curve differs in no way from that of the shunt generator. Also, at low saturation the external characteristic is similar in shape to the saturation curve. The voltage at each point on the characteristic is less than that shown by the saturation curve, by the amount due to the drop through the armature and field $I_a(R_a + R_s)$ and the drop due to armature reaction. The curve reaches a maximum beyond which armature reaction becomes so great as to cause the curve to droop sharply, and the terminal voltage drops rapidly to zero. Series generators are designed to have a very high value of armature reaction.

The generator builds up as follows: If the series field is connected in such a manner that the current due to residual magnetism aids this magnetism, the generator will build up, provided the external resistance equals or is less than that indicated by the external-resistance line *oa*. The line *oa* is called the *critical external-resistance line*. As the external resistance decreases, the external-resistance line swings down to the right, as is discussed for tne shunt generator in Par. 263, p. 408. The line *ob* is such a line. It would be practically impossible to operate with an external resistance corresponding to the line *oa*, or to any line cutting the curve to the left of *d*, as a small increase in external resistance would swing the resistance line away from the curve, resulting in the dropping of the load. The generator is designed

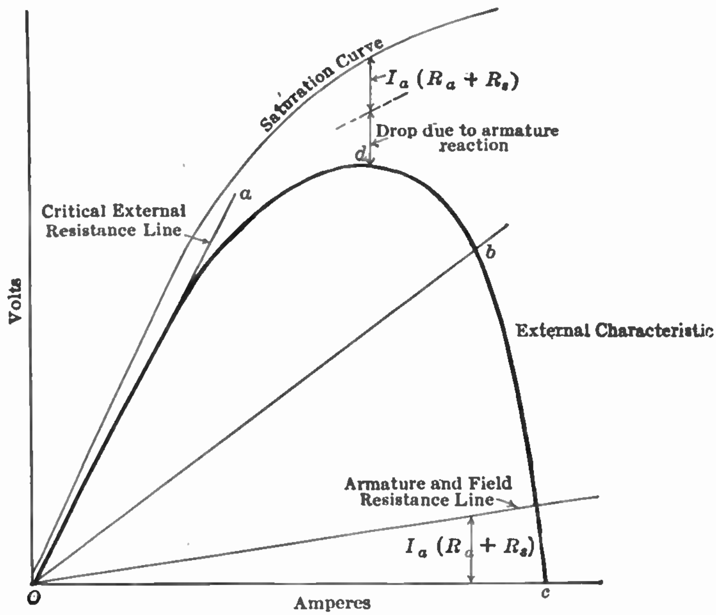


Fig. 345.—Series-generator characteristic.

to operate along the portion *bc* of the characteristic, which corresponds to substantially constant current. The current is not affected by a considerable change in external resistance. This may be seen by swinging the external-resistance line *ob* over a considerable arc. To obtain close regulation, the series field is shunted by a rheostat, whose resistance is controlled by a solenoid connected in series with the line. In this way the current delivered by the generator may be held very nearly constant.

The series generator has been much used in series arc lighting. The Brush arc generator and the Thomson-Houston generator are common examples of such machines. Both of these have open-circuit armatures (see Par. 235, p. 358). As the voltage on the commutator is 2,000 to 3,000 volts, the commutators have wide gaps between segments. In the Brush arc generator there are two or three separate commutators connected in series to reduce the voltage per commutator and to smooth out the ripples in the voltage wave (see Fig. 252, p. 358). There are but four segments per commutator.[1]

In Europe, power is transmitted by direct current at potentials as high as 50,000 volts, by means of the Thury system.[2] This voltage is obtained by connecting several series generators in series and transmitting at constant current. The voltage increases with the load. Each generator has two commutators, one at each end of the armature. The potential may run as high as 5,000 volts per commutator. Regulation is obtained by shunting the fields. The power is utilized by series motors connected in series with the line at the desired points.

283. Series Boosters.—Series generators are often used as boosters on direct-current feeders. When a drop on a particular feeder becomes excessive, it may be cheaper to install a booster and utilize it at peak load than to invest in more copper. The booster is a series generator operating on the straight portion of the saturation curve, the terminal voltage being proportional to the current flowing through the machine. Likewise, the voltage drop in the feeder is proportional to the current in the feeder. If the generator be connected in series with the feeder (Fig. 346(*a*)) and adjusted properly, its terminal volts may be made always equal to the drop in the feeder (Fig. 346(*b*)), and the voltage at the load may be maintained constant. The booster is direct-connected to a shunt motor taking its power from the bus-bars. If the driving power should in any way be removed, the series generator will reverse and operate as a motor. The speed of a series motor without load is practically unlimited, and it will run away and tear itself to pieces. Therefore, such a booster should never be belt-driven and should have some device to prevent its running away.

[1] For a more complete description see S. P. Thompson, "Dynamo Electric Machinery," Vol. I.

[2] See "Standard Handbook for Electrical Engineers," 6th ed., Sect. 14. McGraw-Hill Book Company, Inc.

284. Effect of Variable Speed on Characteristics.—When a generator is being tested to determine its characteristic or its regulation, it is assumed that the generator speed is maintained at a constant value, the rated speed of the generator. Any drop in voltage resulting from a drop in speed of the prime mover or driving motor is not chargeable to the generator.

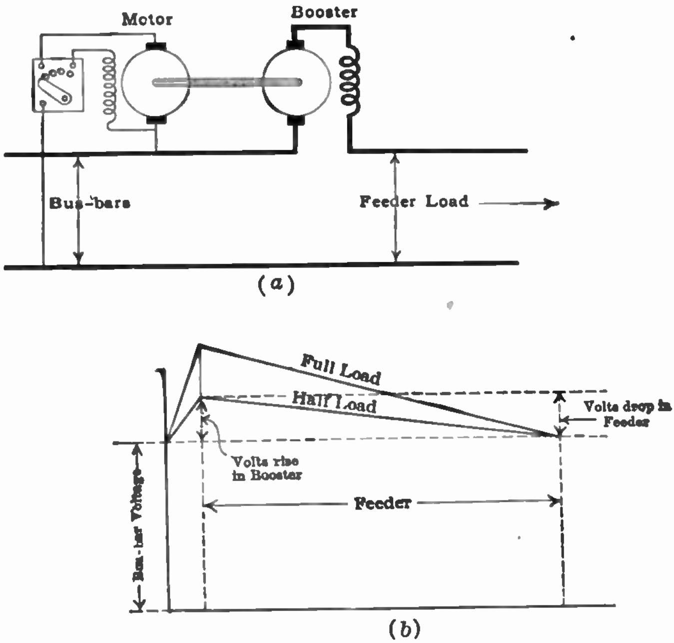


Fig. 346.—Series booster.

In practice, a drop in speed with load in the case of the prime mover is often unavoidable, and the regulation of the generator is made to include the voltage drop due to this decreased speed. When making out specifications, the regulation of the generator when *driven by its prime mover should be specified.* Speed correction applied to characteristics of generators is somewhat involved, because of the many factors which enter the computation[1] (also see Par. 274, p. 441).

[1] For a more complete discussion see W. B. Kouwenhoven, "A Solution for an Acceptance-Test Problem," *Elec. World*, Vol. 71, p. 138, 1918.

AUTOMATIC VOLTAGE REGULATORS

It has been pointed out that the voltage of a generator varies with the load, speed, etc. By means of an automatic regulator, the voltage of a generator can be maintained constant even under rapid fluctuations of load. In addition, compensation may be made for line drop. These automatic regulators usually operate through the field of the generator or the field of an exciter to vary the field current with changes in line voltage.

285. Tirrill Regulator.—In the Tirrill regulator, the voltage is controlled by small relay contacts, which short-circuit the shunt-field rheostat, the duration of the short circuit depending on the amount of regulation required. The field rheostat is usually set so that the generator voltage is 35 per cent below normal when the regulator is disconnected.

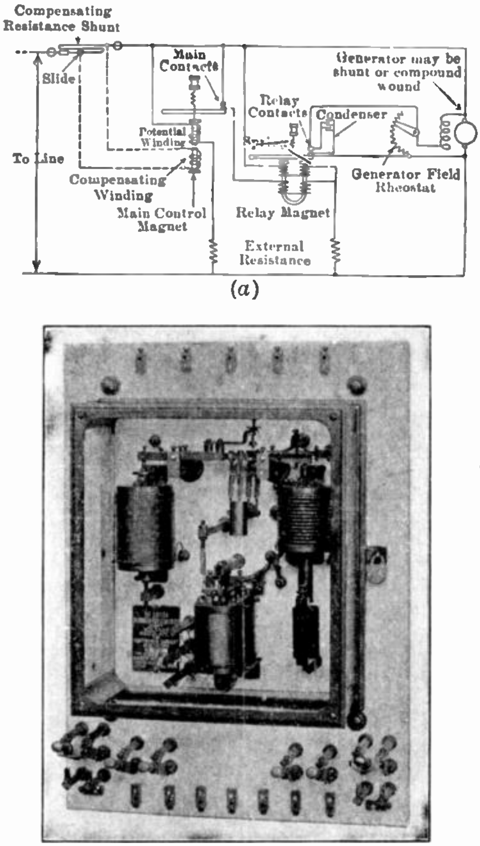
The diagram of the apparatus is shown in Fig. 347. The relay magnet is U-shaped and has two solenoids, differentially wound, upon its core. One winding is directly across the line; the other is connected across the line through the main contacts. The relay contacts intermittently short-circuit the generator-field rheostat.

The main-control magnet can open the main contacts or allow them to close. These contacts are normally held closed by a spring. Assume that the voltage rises. The potential winding of the main-control magnet strengthens this magnet and opens the main contacts. This opens one of the windings on the relay magnet and so nullifies the differential action. The relay contacts are then pulled open and the short circuit removed from the generator-field rheostat. This reduces the generator voltage. The reverse action takes place when the voltage drops. Actually both relays are constantly vibrating so that the changes in the generator voltage are very small.

The relay contacts are shunted by a condenser to reduce sparking. Owing to the fact that these contacts can carry only a very small current, it is usually necessary to have the regulator act on an exciter field, and so to maintain the bus-bar voltage constant through the exciter.

A compensating winding on the main-control magnet may be connected across the compensating resistance to give the system a rising voltage characteristic and so compensate for line drop.

286. Direct-acting Regulator.—The type GDD regulator, manufactured by the General Electric Company, is a direct-acting rheostatic type with the regulating rheostat as part of the



(a)

(b)

FIG. 347.—The Tirrill regulator.

regulator itself. The rheostat consists of stacks of graphite plates, each plate being pivoted at the center by a metallic member. At the back the plates are separated from one another by an insulating spacer, and at the front by a silver button. When the plates are all tilted forward, the silver buttons form a continuous short circuit, and the resistance of the regulating rheostat is a minimum. When the plates are all tilted backward, the current path is through all the plates and pivots, and the regulating resistance of the rheostat is a maximum. The value of the regulating resistance thus depends on the number of plates that are tilted either backward or forward.

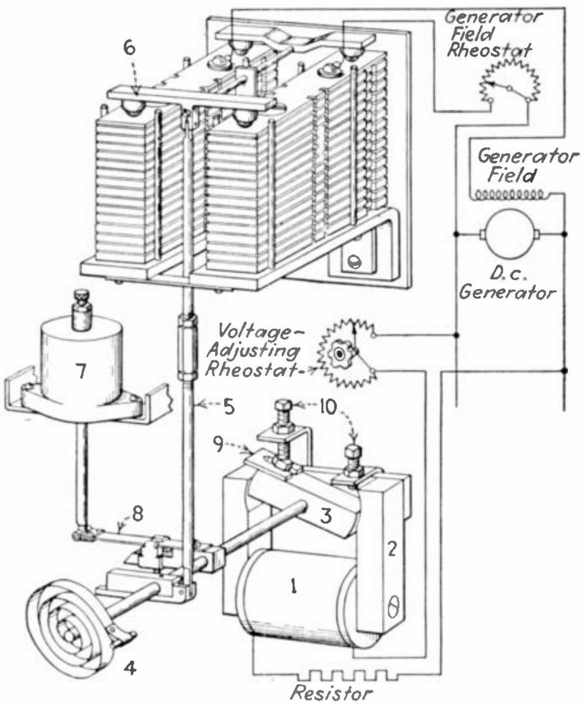


FIG. 348.—Schematic drawing and connection diagram of type GDD-2 generator-voltage regulator. (*General Electric Company.*)

A diagram of the regulator is shown in Fig. 348. A U-shaped iron core 2 is magnetized by a coil 1, connected across the generator terminals in series with a resistor and voltage-adjusting rheostat. A torque armature of iron 3 tends to align itself between the poles of the iron core 2, but is restrained by the spring 4. The torque system actuates an adjustable link 5, which in turn operates the rheostatic element through the equalizing bar 6. In order to prevent oscillations, a dashpot 7 is connected to the torque-system shaft through a leaf spring 8, acting as a lever. The spring 8 permits a rapid instantaneous movement of the mechanism at the instant a change in voltage occurs. The plate 9 and screws 10 are for purposes of adjustment.

The regulator is normally at rest. If a drop in generator voltage occurs, the current in coil 1 decreases and the flux is weakened. The spring 4 overcomes the torque of the armature 3 and the shaft rotates in a clockwise direction, causing a downward pull on the link 5. This causes some of the plates to tilt forward, thus cutting out some of the resistance in the generator-field circuit. The voltage rises and equilibrium between the spring and armature is restored. The reverse process occurs with an increase in generator voltage.

SPECIAL TYPES OF GENERATOR

287. Unipolar or Homopolar Generator.[1]—In the ordinary direct-current generator, an alternating e.m.f. is generated and

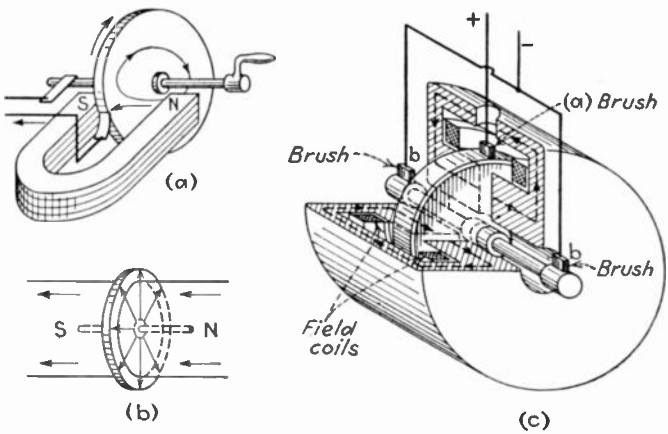


Fig. 349.—Unipolar generator.

the resulting current must be rectified or commutated. In the unipolar generator, however, a unidirectional e.m.f. is generated and no commutator is necessary.

The principle of the unipolar generator is that of Faraday's disc dynamo (Fig. 349(a)). If a disc be rotated between the poles of

[1] For a more complete discussion see "Standard Handbook for Electrical Engineers," 6th ed., Sec. 8, Par. 213, McGraw-Hill Book Company, Inc.

a magnet, an e.m.f. is generated between the center and rim of the disc. A current can be taken from the disc by placing a brush at the center and another at the rim. The disc shown in Fig. 349(*a*) would not be practicable because the e.m.f. is generated at one portion of the disc only, and current can flow back through the disc even when the external circuit is open. If an annular pole be used (Fig. 349(*b*)), an equal e.m.f. is generated along each radius of the disc and the current has no return path in the disc itself.

Figure 349(*c*) shows in perspective a commercial type of unipolar generator with one-quarter cut away The brushes *bb* are of one polarity, shown negative, and the brush *a* is of the opposite polarity, shown positive.

A hole in the casing allows access to brush *a*. Such generators are sometimes made with a rotating cylinder and are said to be of the axial type.

The chief disadvantage of the unipolar type of generator is the very low e.m.f. generated, even at high speeds. It is necessary to connect several discs in series in order to obtain commercial values of voltage. The generator in Fig. 349(*c*), having an armature diameter of about 20 in. and running at 3,000 r.p.m., would give only about 40 volts. Another disadvantage is the difficulty of conducting the current from the disc at the high speeds at which these generators are necessarily run. Their field of application is that of a high-speed, turbo-driven generator, designed for high currents at low voltages. Because of the superiority of coil-wound generators, the use of the unipolar generator has been very limited.

288. Third-brush Generator.—In some types of small generator, particularly those which operate at variable speed, such as automobile generators, advantage is taken of armature reaction to regulate the current. Were an ordinary shunt or compound generator used in an automobile to charge the battery, the generator would be lightly loaded at low speeds and too heavily loaded at the higher speeds.

A common method of regulating automobile generators is the third-brush method in which the shunt field is connected between one of the main brushes and a small third brush located between the two main brushes, as in Fig. 350. In the figure, *A* is the positive main brush, and B is the negative main brush which is usually grounded to the frame of the car or engine.

The auxiliary or third brush C is placed at an angle of about 60° from brush B in the direction opposite to the direction of rotation. The shunt field is connected between the brush C and the positive brush. In Fig. 350(*a*) the flux through the armature under light-load conditions is shown diagrammatically. There is negligible armature reaction (see Fig. 299(*a*), p. 411). The voltage across the field will then be substantially the e.m.f. induced

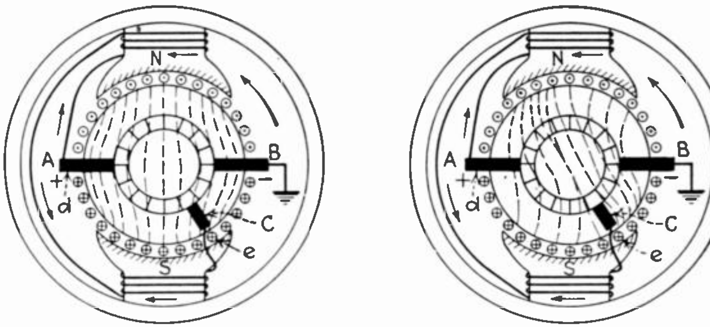


Fig. 350.—Third-brush generator.

in the conductors connected in series between d and e, which cut the flux included between brushes A and C.

If the speed of the generator increases, other factors remaining constant, the e.m.f. will increase and the generator will deliver a greater current. This current, however, distorts the magnetic field in the direction of rotation as in Fig. 350(*b*) (also see Fig. 299(*c*), p. 411). The effect is to transfer the flux from between conductors d and e to the region between brushes C and B. Since the total e.m.f. between conductors d and e, and hence the voltage across the field, is proportional to the flux between conductors d and e, the effect of the increased speed will be compensated in part at least by the lesser flux, and the current will increase only slightly if at all. A study of Fig. 350(*b*) shows that the current may be increased by moving the third brush C in the direction of rotation, and the current may be decreased by moving it against the direction of rotation.

When such generators are used to charge batteries, as in automobiles, there should be a cutout relay which connects the battery to the generator only after the generator has built up to a voltage slightly higher than that of the battery. If the generator slows down, this relay opens when the current reverses and begins to flow from battery to generator.[1]

289. Diverter-pole Generator.[2]—The diverter-pole generator is a compound generator in which the voltage is controlled by

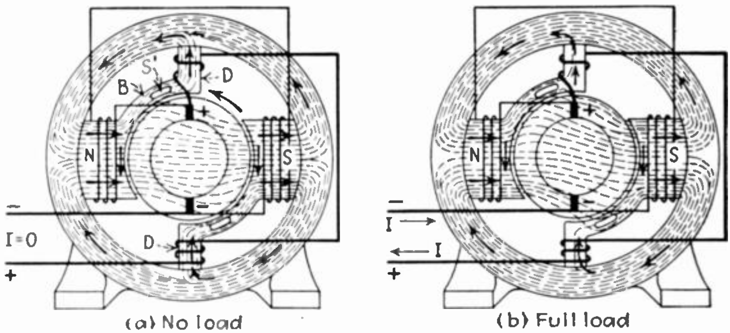


FIG. 351.—Magnetic flux distribution in diverter-pole generator.

shunting more or less of the main flux away from the armature by means of a magnetic shunt. The characteristics thus obtained have certain advantages over those of the usual compound generator, particularly for battery charging. Diagrams of the generator at no load and at full load are shown in Fig. 351. Small diverter poles D are placed midway between the main poles, and each diverter pole is connected to a main pole by means of a magnetic bridge B. In each magnetic bridge there is a longitudinal slot S', the object of which is to restrict the cross-section of the magnetic bridge at that point and thus to cause the iron at the slot S' to be easily saturated. Series turns are wound on the diverter pole, the winding being in such a direction as to produce a polarity which is the same as that of the main pole to which the diverter pole is connected.

[1] See DAWES, C. L., "Industrial Electricity," Part I, p. 305, McGraw-Hill Book Company, Inc.

[2] SMITH, E. D., "The Diverter Pole Generator," *Trans. A.I.E.E.*, Vol. 47, p. 1412, October, 1928.

The no-load conditions are shown in Fig. 351(*a*). The excitation of the diverter poles is zero, and a considerable portion of the main flux is shunted to the yoke by the diverter pole, the shunting effect being limited by the saturation in the bridge at slot S'. The effect of the slot may be considered also from the point of view of magnetic potential. The considerable drop in magnetic potential through the saturated portion S' of the bridge

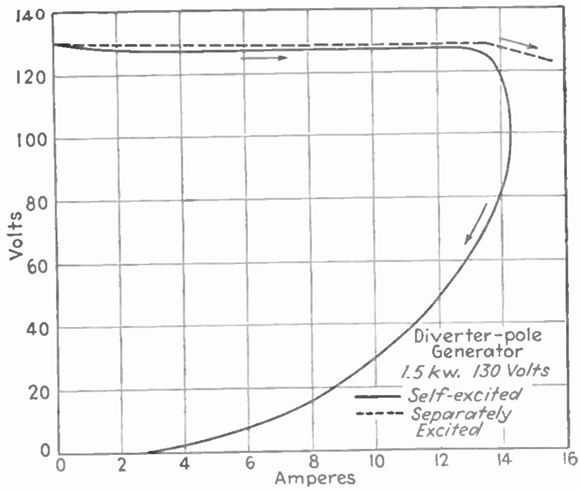


FIG. 352.—Load characteristic of diverter-pole generator.

raises the magnetic potential of the main pole face with respect to the armature, and thus causes a substantial portion of the total flux to enter the armature from the bridge itself.

In Fig. 351(*b*) the magnetic conditions of the generator under load are shown. The series turns on the upper diverter pole act to drive the flux downward and therefore to force the leakage flux, which went through the diverter pole, into the bridge and down into the armature. This action may also be considered from the point of view of magnetic potential. The series ampere-turns act to raise the magnetic potential of the bridge, and as a result more flux will enter the armature.

Typical characteristics of this type of generator, operating shunt or self-excited and separately excited, are shown in Fig. 352. The very flat nature of the characteristics should be noted.

The principal use of this generator is in the charging of storage batteries. It is not open to the objection of the shunt generator in which the voltage drops with load, nor of the compound generator, the characteristics of which are convex, due to saturation (Fig. 336, p. 450). The linear characteristic of the diverter-pole generator is due to the fact that, with increase of load, additional flux is not added to the main pole to saturate it, but leakage flux is merely diverted into the armature. Unlike the straight-compound type, this generator will not run away when it accidentally "motors." The series turns cannot cause any reversal of flux in the main field because the restricted sections of the bridge limit the flux which can go through the bridge. Moreover, flux from the diverter poles into the armature is limited by the saturation of these poles themselves. As these poles act along the brush axis, they have little effect on the e.m.f. in the armature. The diverter poles also have the advantage that they provide along the brush axis a flux of the correct polarity for assisting commutation and hence act like commutating poles.

290. Electric-welding Generators.—Electric welding is now widely used in the fabrication of iron and steel. Because of the steady stream of ions between anode and cathode, a direct-current arc is more easily maintained than an alternating-current arc and is preferred for many types of welding. The characteristics of the arc-welding generator must be adapted to meet the special conditions determined by the arc.

The generator must be able to operate intermittently from open circuit, before the arc is struck, to short circuit, which occurs when the arc is struck or when globules of molten metal short-circuit the arc. The volt-ampere characteristics of the generator must be such that the arc is stable. The length of arc, and hence the current and voltage, vary rapidly with time so that the generator operates continuously under transient conditions, changes taking place with considerable rapidity.

In Fig. 353 is shown a conventional shunt characteristic *A*. This type of characteristic is not suited to arc welding since, with welding, the current would vary widely with small changes of voltage, and the arc demands that the voltage shall go at times to short circuit. When the shunt characteristic goes to short circuit, conditions become unstable (Fig. 327, p. 438). However, the characteristics B and C are well adapted to arc welding. In each of these characteristics the voltage at no load is 90 volts, which is well above the voltage, about 45 volts, necessary to maintain an arc. The currents at short circuit are nominal and definite. The current varies moderately with voltage, so that the operator can control the welding current. Thus, with such characteristics, the arc is stable and the current can be controlled by the operator.

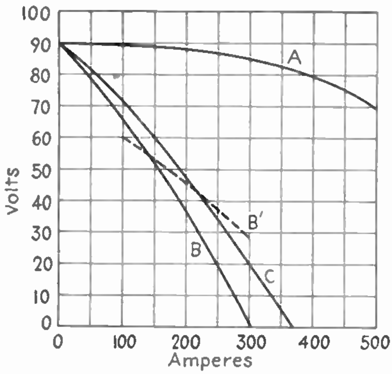


FIG. 353.—Operating characteristics of shunt and arc-welding generators. *A*. Conventional shunt generator. *B*. Steady-state characteristic of welding generator. *B′*. Dynamic or transient characteristic of welding generator. *C*. Steady-state characteristic of welding generator with larger values of current.

It is to be noted that B and C are steady-state characteristics. That is, the value of the voltage for each value of current is determined after conditions have become steady.

In practice, the generator would not ordinarily operate according to characteristic B or C, because of the transient conditions of the arc. Consider the usual shunt generator. The load characteristic is determined by the armature-resistance drop, the variation in field current due to change in terminal voltage, and the armature reaction (p. 440). As time is required to change a magnetic field (p. 312), the change in flux due to armature reaction cannot occur immediately with a sudden change of load. Therefore, at the instant that a new value of current is reached, the terminal voltage will not be that corresponding to the steady-state value. These same magnetic relations exist in the welding generator. For example, with rapidly increasing load a portion of the *transient characteristic* corresponding to the steady-state characteristic B may be B'. The characteristic B' is not well adapted to welding.

Moreover, a change of flux in the magnetic circuit induces currents in the iron and in the field circuit. By Lenz's law (p. 289), these induced currents oppose any change in the flux producing them. Hence, the induced currents cause an additional retardation in the change of flux with change in load, and also cause a further increase in the departure of the dynamic from the steady-state characteristic.

In welding generators, these difficulties are overcome in part by laminating the magnetic circuit so as to reduce the induced currents in the iron. An inductance or reactor is frequently connected in series with the load. The inductance opposes change in current (p. 312). Actually, with change of current, the e.m.f. of self-induction acts in such a direction as to bring the transient characteristic B' in closer proximity to the steady-state characteristic B.

To overcome the effect of the transient change in field current produced by the change in flux, the inductance in series with the load is sometimes provided with a secondary connected to a winding on the magnetic circuit. The direction of this winding is such that its induced current is in opposition to that induced in the field winding and hence practically counteracts the effect of change in current in the field winding.

There are many special types of welding generators designed to give the required characteristics under transient as well as under steady-state conditions. The General Electric Type WD generator[1] is such a type, and the design is such that an inductance or a reactor for stabilizing the arc is not required.

A diagram of the generator with connections is shown in Fig. 354. Four main poles are shown, but the two N-poles and the two S-poles are adjacent to each other and thus this is in reality a two-pole generator, the two main poles being split. The brushes A, B lie along the geometrical neutral, and commutating poles act along the same axis. Let ϕ_M be the flux in the poles along the vertical axis and ϕ_C be the flux in the poles along the horizontal axis. The armature m.m.f. R lies along the brush axis (p. 413). R may be resolved into two components, R_C in opposition to ϕ_C, and R_M in conjunction with ϕ_M. Hence, armature reaction reduces the flux ϕ_C and tends to increase the flux ϕ_M. However, the poles along the vertical axis are operated

[1] Hornby, F. B., "Control of Transients in Welding Generators," *Trans. A.I.E.E.*, Vol. 53, p. 1598, 1934. Also see *Elec. Eng.*, December, 1934, p. 1598; and April, 1935, p. 441.

at high saturation, and the added m.m.f. R_M has little effect on ϕ_M.

The shunt coils on all four poles are in series, and the circuit is connected between brush A and a third brush E midway between the two N-poles. It will be recognized that this constitutes a third-brush generator (Par. 288), and the excitation voltage is that induced in the series-connected conductors lying under the upper N-pole. The flux ϕ_M in this pole is substantially constant at all loads, and hence the shunt-field voltage is likewise substantially constant at all loads.

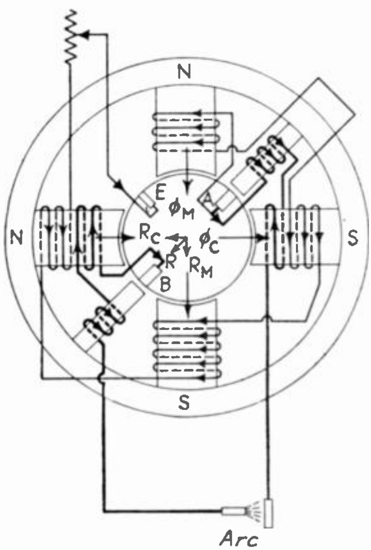


FIG. 354.—Arc-welding generator. (*General Electric Company.*)

The horizontal poles operate at low saturation. Hence, with increase of load, the m.m.f. R_C readily reduces ϕ_C and may even reverse it. The voltage at the terminals of the generator is that between brushes A and B. This voltage is due to the e.m.f. induced in *all* the series-connected conductors between A and B, and hence in the conductors under both vertical and horizontal poles. Therefore, the terminal voltage is proportional to the sum of the fluxes ϕ_M and ϕ_C. The flux ϕ_M is substantially constant and the flux ϕ_C decreases as the load increases. Hence, the terminal voltage will consist of a steady component and a component which decreases rapidly with increase of load. For example, if at no load the e.m.f. due to ϕ_M and ϕ_C is 45 volts each, the terminal voltage is 90 volts. If, at nearly half load, the e.m.f. due to ϕ_C is 5 volts, the terminal voltage is 50 volts. Hence, under such conditions, the generator characteristic becomes similar to B and C (Fig. 353).

Inductance in the shunt-field circuit reduces the transient induced current which causes flux lag. In this type of generator the flux ϕ_M does not change. The turns about the vertical poles, however, do give inductance to the entire shunt circuit and hence reduce induced currents. Obviously this inductance is due to the leakage flux in the coils about the vertical poles. Inductance is also produced by the armature flux-linkages. The horizontal poles offer a path of low reluctance for the armature flux and thus increase the armature inductance. These inductances, due to field and armature, are sufficient to stabilize the arc without the use of an external reactor.

Different characteristics such as *B* and *C* (Fig. 353) adapted to different types of welding are obtained by means of the series winding shown, together with the taps which cut turns in and out as is found necessary. The series winding opposes the shunt coils. The generator also has commutating poles which are excited by a series winding.

CHAPTER XIII

THE MOTOR

It is stated in Chap. XII that a generator is a machine for converting mechanical energy into electrical energy.

In a similar way the motor is a machine for converting *electrical* energy into *mechanical* energy. The same machine, however, may be used either as motor or as generator.

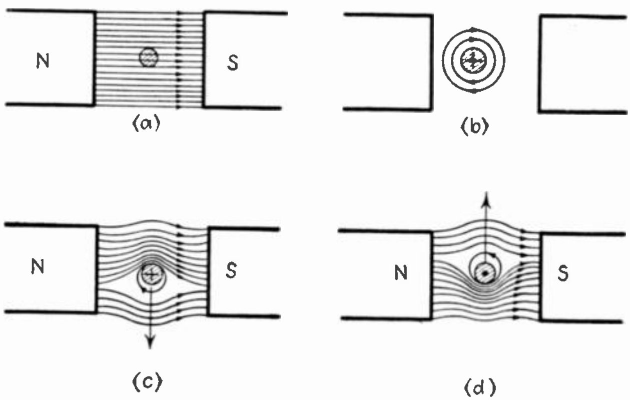


FIG. 355.—Force acting on a conductor carrying current in a magnetic field.

291. Principle of the Motor.—Figure 355(*a*) shows a magnetic field of uniform strength or intensity in which is placed a conductor that carries no current. In (*b*) the conductor is shown as carrying a current into the paper, but the field due to the *N*- and *S*-poles has been removed. A cylindrical magnetic field now exists about the conductor due to the current in it. The direction of this field, which may be determined by the corkscrew rule, is clockwise.

Figure 355(*c*) shows the resultant field obtained by combining the main field and that due to the current. The field due to the current in the conductor acts in conjunction with the main field above the conductor, but it opposes the main field below the conductor. The result is to crowd the flux into the region directly *above* the conductor and to reduce the flux density in the region directly *below* the conductor.

It will be found that a force acts on the conductor, trying to push the conductor *down*, as shown by the arrow.

It is convenient to think of this phenomenon as due to the crowding of lines on one side of the conductor. Magnetic lines of force may be considered as acting like elastic bands under tension. These lines always are endeavoring to contract to minimum length. The tension in these lines on the upper side of the conductor tends to pull it down as shown in the figure.

If the current in the conductor be reversed, the crowding of lines will occur *below* the conductor, which will tend to move it *upward*, as in Fig. 355(*d*).

The operation of the electric motor depends on the principle illustrated by Fig. 355. A conductor carrying current in a magnetic field tends to move at right angles to the field.

292. Force Developed on Conductor Carrying Current.—The force acting on a conductor carrying a current in a magnetic field is directly proportional to three quantities: the strength of the field, the magnitude of the current, and the length of the conductor lying in the field. The force in *dynes* is given by

$$F = Bl\frac{I}{10} \text{ dynes} \tag{220}$$

where B is the flux density in lines per square centimeter or gausses, l the active length of the conductor in centimeters, and I the current in amperes. In Eq. (220) the current must be perpendicular to the direction of the field. If it is not, the right-hand side of the equation must be multiplied by the sine of the angle between the two.

The relation is developed as follows:

In Par. 163 (p. 235), Eq. (103), it is shown that the force on a unit pole at the center of a circular turn of radius R cm. carrying current I' absamp. is

$$f_0 = \frac{2\pi I'}{R} \text{ dynes.} \tag{I}$$

The force per centimeter length of the turn

$$f_1 = \frac{2\pi I'}{R(2\pi R)} = \frac{I'}{R^2} \text{ dynes.} \tag{II}$$

The turn itself must be acted on by an opposite and equal force. The flux from the unit pole is 4π maxwells (p. 214). The flux density at the surface of a sphere of R cm. radius is

$$B = \frac{4\pi}{4\pi R^2} = \frac{1}{R^2} \text{ gausses.} \tag{III}$$

This must be the flux density at the circular turn. Substituting in Eq. (II) the value of R^2 from Eq. (III),

$$f_1 = BI' \text{ dynes.}$$

The force on a length of l cm. is

$$f = BlI' \text{ dynes,} \tag{221}$$

$$F = Bl\frac{I}{10} \text{ dynes (Eq. (220))} \quad Q.E.D.$$

where I is in amperes. Flux density B, length of conductor l, and current I are all mutually perpendicular to one another.

Example.—A rectangular flat coil of 20 turns lies with its plane parallel to a magnetic field (see Fig. 360), the flux density in the field being 3,000 lines per square centimeter. The axial length of the coil is 8 in. The current is 30 amp. Determine the force in pounds which acts on each side of the coil (see arrows in Fig. 360(a), p. 481).

$$B = 3{,}000 \text{ gausses.}$$
$$l = 8 \times 2.54 = 20.32 \text{ cm.}$$
$$I = 30 \text{ amp.}$$
$$F = 3{,}000 \times 20.32 \times {}^{30}\!/_{10} = 182{,}900 \text{ dynes.}$$

As there are 20 turns,

$$F_1 = F_2 = 20 \times 182{,}900 = 3{,}658{,}000 \text{ dynes.}$$
$$\frac{3{,}658{,}000}{981} = 3{,}730 \text{ g.}$$
$$= 3.73 \text{ kg.}$$
$$3.73 \times 2.204 = 8.22 \text{ lb.} \quad \textit{Ans.}$$

293. Fleming's Left-hand Rule.—The relation among the direction of a magnetic field, the direction of motion of a conductor in that field, and the direction of the induced e.m.f. in the conductor is given by Fleming's right-hand rule (see Par. 234, p. 355).

In a similar manner, the relation among the direction of a magnetic field, the direction of a current in that field, and the direction of the resulting *motion* of the conductor carrying the current can be determined by using Fleming's left-hand rule.

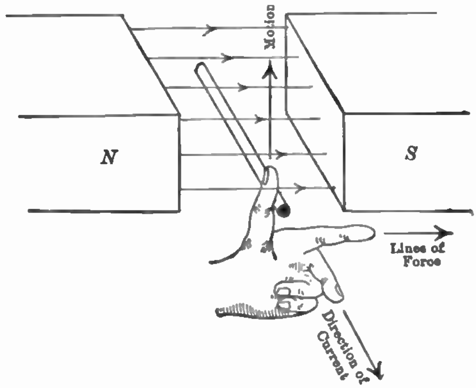
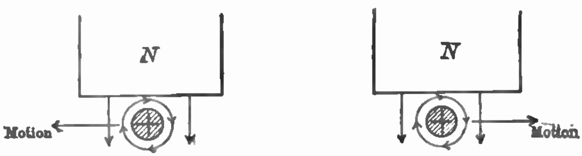


FIG. 356.—Fleming's left-hand rule.

Fleming's *left-hand* rule:

Point the forefinger in the direction of the field or flux, the middle finger in the direction of the current in the conductor, and the thumb will point in the direction in which the conductor tends to move. This is illustrated by Fig. 356.

Another convenient method for determining this relation is to make use of the fact that the crowding of the magnetic lines behind the conductor tends to push it along. It is necessary merely to sketch the main field and the lines about the conductor, as in Fig. 357(*a*). It is evident that the lines will be crowded



(*a*) Motor (*b*) Generator

FIG. 357.—Motor and generator action.

at the right of the conductor so that the direction of motion is to the left.

In Fig. 357(*b*) is shown a similar condition for a generator. In this case the conductor, as a generator, moves to the right.

Hence, in a generator the conductor must move *against* a force tending to oppose its motion, and so the conductor requires a driving force to keep it in motion. This driving force is supplied by the prime mover to which the generator is connected.

294. Torque.—When an armature, a flywheel, or any other device is rotating about its center, a tangential force is necessary to produce and maintain rotation. This force may be developed within the machine itself, as in a motor or steam engine, or it may be applied to a driven device such as a pulley, a shaft, a generator, or the driving gears on the wheels of a street car

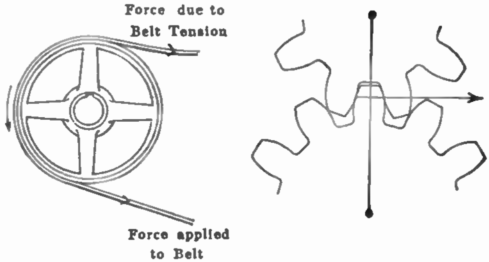


Fig. 358.—Torque developed by a belt and by gears.

(Fig. 358). The total effect of the force is determined not only by its *magnitude* but also by its *arm*, or radial distance from the center of the pulley or gear to the line of action of the force.

The product of this force and its perpendicular distance from the axis is called *torque*.

Torque may also be considered as a mechanical couple tending to produce rotation. Its value is expressed in units of force and distance.

In the English system, torque is usually expressed in pounds-feet. (This distinguishes it from foot-pounds which represent *work*.)

In the c.g.s system the unit of torque is the dyne-centimeter (a very small unit), and in the metric system the unit is the kilogram-meter.

Example.—A belt is driving a 36-in. pulley, as in Fig. 359. The tension in the tight side of the belt is 90 lb. and that in the loose side is 30 lb. Determine the torque applied to the pulley.

The two sides of the belt are acting in opposition, so that the net pull on the rim of the pulley is

$$90 - 30 = 60 \text{ lb.}$$

This force is acting 18 in. or 1.5 ft. from the center of the pulley Therefore the torque

$$T = 60 \times 1.5 = 90 \text{ lb.-ft.} \quad \textit{Ans.}$$

295. Torque Developed by a Motor.—Figure 360(*a*) shows a rectangular coil of a single turn, whose plane lies parallel to a magnetic field. Current flows into the paper in the left-hand side of the coil and out of the paper in the right-hand side of the coil. Therefore, the left-hand conductor tends to move downward with a force F_1 and the right-hand conductor tends to move upward with an equal force F_2. As the current in each of the conductors is the same and they lie in magnetic fields of the same strength, force $F_1 = F_2$. Both forces act to develop a torque which tends to turn the coil in a counterclockwise direction. In Fig. 360(*a*) the coil is in the position of maximum torque because the perpendicular distance from the coil axis to the forces acting is a maximum.

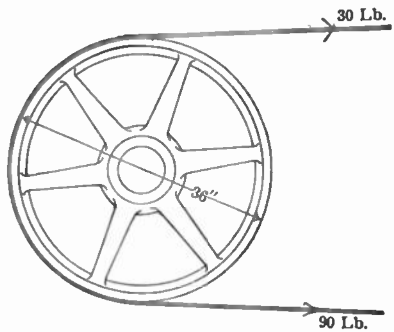


FIG. 359.—Example of torque produced upon a pulley by a belt.

When the coil reaches the position (*b*) neither conductor can move any farther without the coil itself spreading. This is a

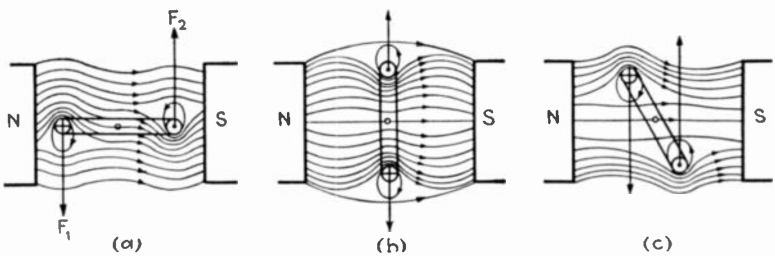


FIG. 360.—Torque developed at different positions of a coil.

position of zero torque because the perpendicular distance from the coil axis to the line of action of the forces is zero.

If, however, the current in the coil be reversed when the coil reaches position (*b*) and the coil be carried beyond the dead center, as in (c), a torque is developed which tends to continue to turn the coil in the counterclockwise direction.

To develop a continuous torque in one direction, the current in each coil on the armature must be reversed as it is passing

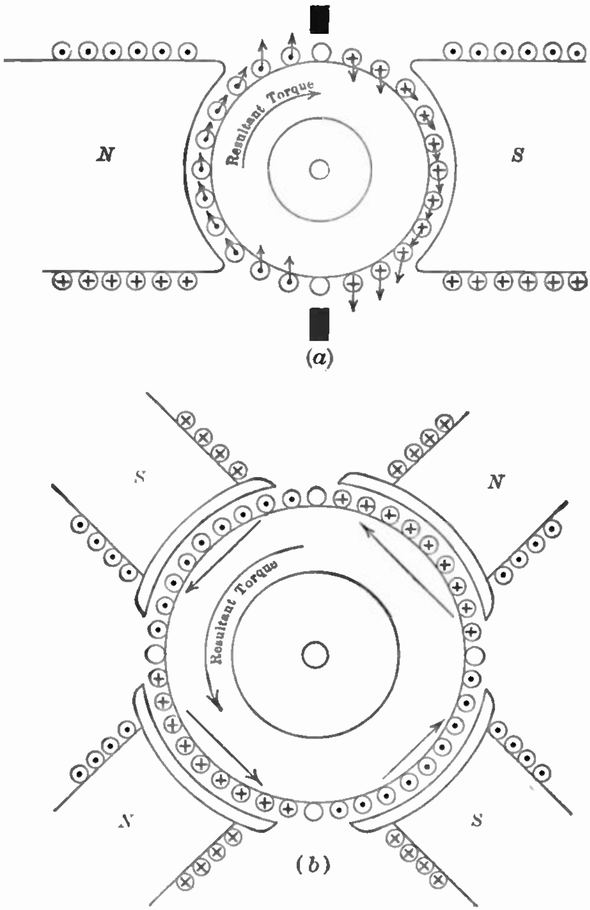


FIG. 361.—Torque developed by belt conductors in motor armatures.

through the neutral plane or plane of zero torque. A commutator is therefore necessary. This is analogous to using a commutator in connection with a generator in order that the current delivered to the external circuit may be unidirectional.

A single-coil motor, like that shown in Fig. 360, would be impracticable as it has dead centers and the torque which it develops is pulsating. A two-coil armature would eliminate the dead centers, but the torque developed would still be more or less pulsating in character.

The best results are obtained when a large number of coils is used, just as in the armature of a generator. In fact there is no difference in the construction of a motor armature and a generator armature. In Fig. 361(a) an armature and a field are shown for a two-pole motor and the torque developed by each individual conductor is indicated. Figure 361(b) shows armature and field for a four-pole machine. The direction of the torque developed by each belt of conductors is indicated by the arrow at that belt.

In armatures of this type a very small portion of the total number of coils is undergoing commutation at any one instant. Therefore, the variation in the number of active conductors is so slight that the torque developed is substantially constant for constant values of armature current and of main flux.

From Eq. (220), the torque developed by an armature is

$$T = K_t'ZI_a\phi \tag{222}$$

where K_t' = a constant, involving the number of poles, the parallel paths through the armature, the choice of units, etc.

Z = number of conductors on the surface of the armature.

I_a = current to the armature in amperes.

ϕ = flux from one N-pole entering the armature.

Equation (222) is derived as follows.

From Eq. (220) the force on each conductor $f = Bl\frac{I}{10}$ dynes where I is the amperes per path. The sum of the forces on Z conductors is $ZB_{av}l\frac{I}{10}$ dynes where B_{av} is the average flux density over any one pole pitch (see B, Fig. 289, p. 399). The corresponding torque is

$$T = \left(\frac{d}{2}\right)\left(ZB_{av}l\frac{I}{10}\right) \text{ dyne-cm.} \tag{I}$$

where d is the diameter of the armature in centimeters.

The flux entering the armature from one N-pole

$$\phi = B_{av}l\left(\frac{\pi d}{P}\right) \tag{II}$$

where P is the number of poles.

Substituting into Eq. (I) the value of B_{av} from Eq. (II), and $I_a = IP'$, where P' is the number of parallel paths,

$$\begin{aligned} T &= \frac{d}{2}Z\left(\frac{\phi P}{l\pi d}\right)l\frac{I_a}{10P'} \text{ dyne-cm.} \\ &= \frac{P}{20\pi P'}ZI_a\phi \text{ dyne-cm.} \\ &= K_t'ZI_a\phi \text{ dyne-cm.} \quad Q.E.D \end{aligned}$$

where K_t' is a constant of proportionality and is equal to $P/(20\pi P')$.

The torque in pounds-feet is found by dividing by $445{,}000 \times 2.54 \times 12$. Hence,

$$T = 0.117\frac{P}{P'}ZI_a\phi \times 10^{-8} \text{ lb.-ft.} \tag{223}$$

For any particular machine Z is fixed, so that the torque

$$T = K_tI_a\phi \tag{224}$$

where K_t is a new constant.

That is, in a given motor, *the torque is proportional to the armature current and to the strength of the magnetic field.*

This is a very important relation to keep in mind, for by its use the variation of torque with load in the various types of motors can be readily determined.

Example.—When a certain motor armature is taking 50 amp. from the line, it develops 60 lb.-ft. torque. If the field strength is reduced to 75 per cent of its original value and the current increases to 80 amp., what is the new value of the torque developed?

If the current remained constant, the new value of torque, due to weakening the field, would be

$$0.75 \times 60 = 45 \text{ lb.-ft.}$$

Due to the increase in the value of the current, however, the final value of torque will be

$$(^{80}\!/\!_{50})\ 45 = 72 \text{ lb.-ft.} \quad \textit{Ans.}$$

It must be remembered that the torque expressed by the above equations is the *entire* torque or the *internal* torque developed by the armature. The torque available at the pulley will be slightly less than this, due to the torque lost in overcoming friction and windage and in supplying the iron losses of the armature.

296. Counter Electromotive Force.—The resistance of the armature of the ordinary 10-hp., 110-volt motor is about 0.05 ohm. If this armature were connected directly across 110-volt mains, the current, by Ohm's law, would be

$$I = \frac{110}{0.05} = 2{,}200 \text{ amp.}$$

This value of current is not only excessive but unreasonable, especially when one considers that the rated current of such a motor is in the neighborhood of 90 amp. When a motor is in operation, the *current through the armature is evidently not determined by its ohmic resistance alone.*

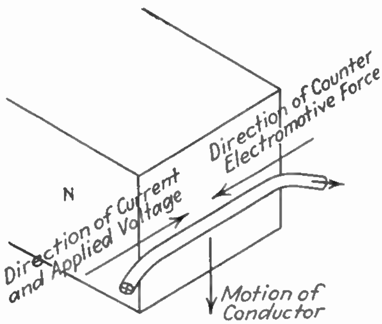


FIG. 362.—Relation of the direction of currents and voltages in a motor conductor.

The armature of a motor is in every way similar to that of a generator. The conductors on its surface, in addition to carrying current and so developing torque, are cutting flux. Therefore, they *must* be generating an e.m.f.

Consider Fig. 362 which shows a conductor on the armature of a motor, moving downward in front of a N-pole. In order to move downward, the current in this conductor must be inward or from left to right (Fig. 360(c)).

If the right-hand rule be applied to determine the direction of the e.m.f. *induced* in this conductor, due to its downward motion (see Fig. 362), it will be found to be acting from right to left or in opposition to the current.

If the direction of the induced e.m.f. in any conductor on a motor armature be similarly determined, it will be found to act always in *opposition* to the current. That is, it *opposes* the current entering the armature. This induced e.m.f. is called the *counter electromotive force* or *back electromotive force*. As the counter e.m.f. opposes the current it must also oppose the line voltage. Therefore, the net e.m.f. acting in the armature circuit is the difference of the line voltage and the back e.m.f. Let V equal the line voltage and E the back e.m.f. The net voltage acting in the armature circuit is

$$V - E.$$

The armature current follows Ohm's law and is

$$I_a = \frac{V - E}{R_a} \tag{225}$$

where R_a is the armature resistance.

This equation may be transposed and written

$$E = V - I_aR_a. \tag{226}$$

This should be compared with Eq. (215), p. 441, which is the similar equation for a generator.

In a generator the induced e.m.f. is equal to the terminal voltage *plus* the armature-resistance drop. In a motor the induced e.m.f. is equal to the terminal voltage *minus* the armature-resistance drop. The counter e.m.f. must always be less than the terminal or impressed voltage if current is to flow *into* the armature at the positive terminal.

Example.—Determine the back e.m.f. of a 10-hp. motor when the terminal voltage is 110 volts and its armature is taking 90 amp. The armature resistance is 0.05 ohm.

$$E = 110 - (90 \times 0.05) = 110 - 4.5 = 105.5 \text{ volts.} \quad \textit{Ans.}$$

An interesting experiment for demonstrating the existence of counter e.m.f. is shown in Fig. 363. A lamp bank is connected in series with the armature of a shunt motor. First close switch S_2 which closes the field circuit. Then close S_1. At the instant of closing S_1 the lamps will burn brightly, being practically up to candlepower. As the armature speeds up, these lamps will become dimmer and dimmer, showing that the armature is generating a *counter* e.m.f. which opposes the line voltage and so results in less voltage for the lamps. When the armature is up to speed, the lamps will be very dim. If, however, the field switch S_2 now be opened, the flux and, therefore, the counter e.m.f. will be reduced to zero, practically, which will be shown by the lamps again coming up to full candlepower. (In practice, when a motor is in operation, *the field circuit should not be opened under any conditions whatsoever.*)

Equation (210), p. 401, for the induced e.m.f. in a generator will obviously apply to a motor. That is, the counter e.m.f.

$$E = \frac{\phi s P Z}{P' 10^8} \text{ volts}$$

where ϕ is the total flux entering the armature from one N-pole, s is the speed of the armature in revolutions per second, P is

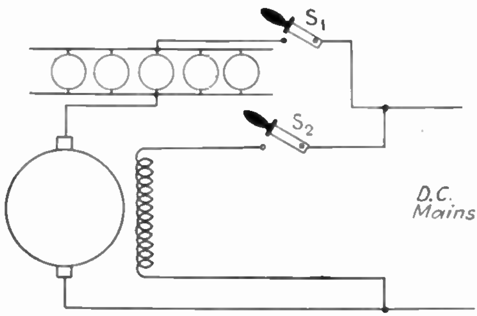


FIG. 363.—Demonstration of counter electromotive force.

the number of poles, Z is the number of conductors on the surface of the armature, and P' is the number of parallel paths through the armature.

As Z, P, P', and 10^{-8} are all constant for any given motor, the counter e.m.f. is

$$E = K_1 \phi S,$$

which is identical with Eq. (212), p. 401, S being given in r.p.m.

Solving for speed

$$S = K\frac{E}{\phi} \tag{227}$$

where $K = 1/K_1$.

The speed of a motor is directly proportional to the counter e.m.f. and inversely proportional to the field.

Substituting for E in Eq. (227), its value given in Eq. (226), the speed becomes

$$S = K\frac{V - I_a R_a}{\phi}. \tag{228}$$

This is a very important relation, for it shows the law of speed variation of a motor with changes of load.

Example.—In a certain motor the armature resistance is 0.1 ohm. When connected across 110-volt mains the armature takes 20 amp., and its speed is 1,200 r.p.m. What is its speed when the armature takes 50 amp. from these same mains, with the field increased 10 per cent?

Applying Eq. (228),

$$\frac{S_2}{S_1} = \frac{K\dfrac{110 - 50 \times 0.1}{\phi_2}}{K\dfrac{110 - 20 \times 0.1}{\phi_1}} = \frac{\dfrac{105}{\phi_2}}{\dfrac{108}{\phi_1}} = \frac{105}{\phi_2} \cdot \frac{\phi_1}{108}$$

$$S_1 = 1{,}200.$$

Therefore,

$$S_2 = 1{,}200\frac{105}{108} \cdot \frac{\phi_1}{\phi_2}.$$

But

$$\phi_2 = 1.10\phi_1.$$

Therefore,

$$S_2 = 1{,}200\frac{105}{108}\left(\frac{\phi_1}{1.10\phi_1}\right) = 1{,}060 \text{ r.p.m.} \quad \textit{Ans.}$$

Although, from Eq. (227), the speed decreases when the field is strengthened, it might appear that the stronger field would produce a greater torque (Eq. (224)) which would accelerate the armature to a higher speed. Momentarily, however, the speed of the armature can not change appreciably, due to its inertia. The strengthened field increases the counter e.m.f. E. Since the difference between the terminal voltage V and the counter e.m.f. E is small, a small proportionate increase in E causes a large proportionate decrease in the current (Eq. (225), p. 486). Hence the current I_a in Eq. (224) decreases more than ϕ increases and the torque therefore *diminishes*. The speed of the armature will drop, therefore, and ultimately reach such a value that the electromagnetic torque will be in equilibrium with the torque required of the armature under the changed conditions. The new value of speed will be that given by Eq. (228).

For example, assume that I_aR_a drop is $0.1V$ where V is the terminal voltage.

From Eq. (225), the current

$$I_a = \frac{V(1 - 0.9)}{R_a} = \frac{0.1V}{R_a}.$$

The flux ϕ is suddenly increased by 5 per cent, and the speed is assumed to remain constant for the moment. The induced e.m.f. E' now equals $1.05E$, and the new value of current

$$I_a' = \frac{V(1 - 0.9 \times 1.05)}{R_a} = \frac{0.055V}{R_a} = 0.55I_a.$$

The flux ϕ has been increased by 5 per cent, and the current has decreased by 45 per cent. The momentary torque is

$$T' = 1.05 \times 0.55T = 0.58T$$

where T is the original torque. Hence the speed of the armature will decrease until the current has such a value that the reaction with the new value of flux brings the electromagnetic torque into equilibrium with the load and friction torques.

297. Counter Electromotive Force and Mechanical Power.— Let the input to a motor armature be VI_a watts. A part P_a of this power is lost in heating the armature.

$$P_a = I_a^2R_a$$

where R_a is the armature resistance. The remainder of the power VI_a must appear as mechanical power P_m. This follows from the law of the conservation of energy. That is,

$$\begin{aligned} P_m &= VI_a - I_a^2R_a \\ &= (V - I_aR_a)I_a. \end{aligned}$$

But $V - I_aR_a$ is the counter e.m.f. of the motor (Eq. (226)).

Hence, *the mechanical power developed within the armature of a motor is equal to the product of the counter e.m.f. and the armature current.* The power available at the pulley is slightly less, since some of this internal power is accounted for by friction, windage, and iron losses; that is, it is lost as stray power (see p. 531).

Example.—Determine the mechanical power developed in the armature of the motor given in the example on p. 486. The counter e.m.f. is 105.5

volts and the armature current is 90 amp. Hence, the mechanical power developed within the armature or the internal power

$$P_m = 105.5 \times 90 = 9{,}495 \text{ watts.} \quad \textit{Ans.}$$
$$= 12.73 \text{ hp.} \quad \textit{Ans.}$$

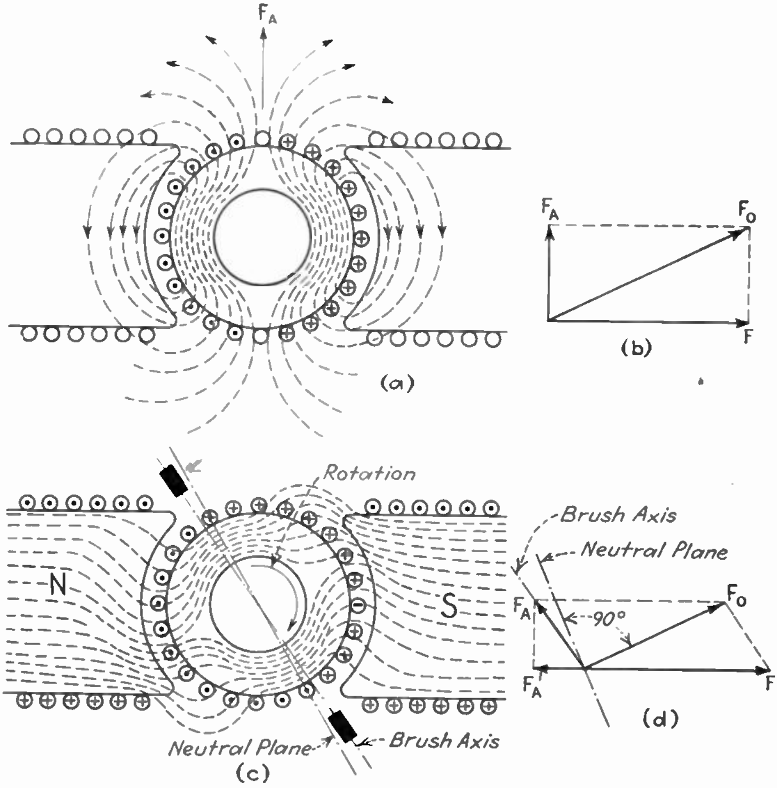


Fig. 364.—Armature reaction in a motor.

298. Armature Reaction and Brush Position in a Motor.—Figure 364(a) shows a motor armature carrying current, the directions of the currents in the armature conductors corresponding to clockwise rotation and to a N-pole at the left. Due to the armature ampere-turns, a m.m.f. F_A is produced in the armature, and the direction of flux produced by this m.m.f. is upwards and at right angles to the polar axis. Figure 364(b) shows the vectors representing the magnitudes and directions of the armature m.m.f. F_A and the field m.m.f. F. By adding these two m.m.fs. vectorially, the resultant m.m.f. F_0 is obtained. The total flux produced by F_0 is distorted as shown in Fig. 364(*c*). It will be noted that (1) the flux has been crowded into the *leading* pole tips, and (2) the neutral plane perpendicular to the resultant field has moved *backward*. Therefore, in a motor it is necessary to move the brushes *backward* with increase of load, whereas in a generator they are moved *forward*. Were it not for the e.m.f. of self-induction (see Par. 270, p. 426), the brush axis would coincide with the neutral plane. Due, however, to the necessity of counteracting this e.m.f. of self-inductance, the brushes are set behind this load neutral-plane, as in Fig. 364(*c*). That is, in both motor and generator it is necessary to set the

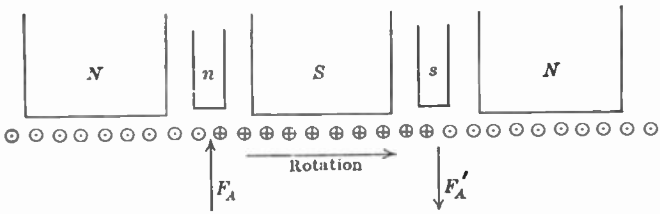


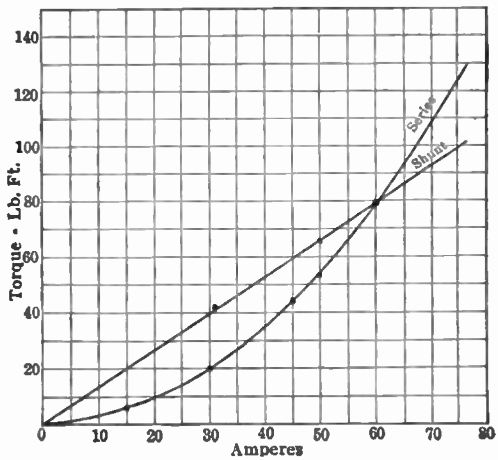
Fig. 365.—Relation of commutating poles to main poles in a motor.

brushes beyond the load neutral-plane in order to counteract the e.m.f. of self-induction.

This backward movement of the brushes is accompanied by a demagnetizing action of the armature on the field, as indicated in Fig. 364(*d*), where F_A' is the demagnetizing component of F_A. Therefore, as the load is increased on a motor, the armature reaction tends to increase the motor speed. In fact instances have been known where motors with short air-gaps (producing high armature reaction) have run away when the load was applied.

Figure 365 shows the armature conductors of a motor carrying current and moving under successive N- and S-poles. The directions of the currents correspond to a direction of rotation from left to right. These directions are such that the armature reaction F_A in the first interpolar space is *upward* (see Fig. 304, p. 416). Therefore, if a commutating pole is to be used, it must be a N-pole, in order to oppose this m.m.f. of the armature by tending to send a flux downward into the armature. $F_A{}'$ must likewise be opposed by a S-pole. Therefore, in a motor, the relation of main poles and commutating poles, *in the direction of rotation*, is $NnSs$, or opposite to the corresponding relation for a generator (see Fig. 324, p. [redacted]). 434

If a motor happens to be sparking badly from some unknown cause, the polarity of the interpoles should be carefully investigated with a compass as the sparking may be due to their being incorrectly connected.



$= T = K I_a^2$

$= T = K I_a \Phi$

Fig. 366.—Shunt and series motors; torque-current characteristics. (Electromagnetic or developed torque.)

299. Shunt Motor.—The shunt motor is connected in the same manner as a shunt generator. That is, its field is connected directly across the line in parallel with the armature. A field rheostat is usually connected in series with the field.

If the *applied* torque to any rotating power-transforming device is increased, the resulting reactions must be such as to cause an increase in the *developed* torque. Otherwise the device will not operate.

If load is applied to a motor, the motor immediately tends to slow down. With the shunt motor, the flux remains substantially constant and the decrease of speed decreases the back e.m.f.

If the back e.m.f. is decreased, more current flows into the armature (see Eq. (225), p. 486). This continues until the increased armature current produces sufficient torque to meet the demands of the increased load. Therefore, the shunt motor is always in a condition of stable equilibrium, since the reactions caused by a change of load always adapt the power input to the changed load conditions.

The suitability of a motor for any particular duty is determined almost entirely by two factors, the variation of its *torque* with load, and the variation of its *speed* with load.

In the shunt motor the flux is substantially constant. Therefore, from Eq. (224), p. 484, the electromagnetic torque will vary almost directly proportional to the armature current. For example, in Fig. 366, when the armature current is 30 amp., the motor develops 40 lb.-ft. torque; and when the current is 60 amp., the motor develops 80 lb.-ft. torque. That is, when the current doubles, the torque doubles.

The speed of a motor varies according to Eq. (228), p. 487, where

$$S = K\frac{V - I_aR_a}{\phi}.$$

In the shunt motor, K, V, R_a, and ϕ are all substantially constant. Therefore, the only variable is I_a. As the load on the motor increases, I_a increases and the numerator of the fraction decreases. As a rule the denominator changes only by a small amount. The speed of the motor will drop with increase of load, as in Fig. 367. As I_aR_a is ordinarily from 2 to 6 per cent of V, the percentage drop in speed of the motor is of the same order of magnitude. For this reason the shunt motor is considered a constant-speed motor, even though its speed does drop slightly with increase of load.

Owing to armature reaction, ϕ ordinarily decreases slightly with increase of load and this tends to maintain the speed constant. Occasionally the armature reaction is sufficiently great to give a rising speed characteristic with increase of load.

Speed ~~*regulation* is defined by the A.I.E.E. Standards~~ Rules as follows:

In constant-speed direct-current motors the regulation is the ratio of the difference between rated-load and no-load speeds to the rated-load speed at the final temperature attained at operation under rated load for the time specified in the rating.

That is, in Fig. 367, the speed regulation is

$$\frac{ca - ba}{ba} = \frac{cb}{ba}.$$

Example.—The speed of a shunt motor falls from 1,100 r.p.m. at no load to 1,050 r.p.m. at rated load. What is its speed regulation?

$$\text{Regulation} = \frac{1{,}100 - 1{,}050}{1{,}050} = 0.0476 \text{ or } 4.76 \text{ per cent.} \quad \textit{Ans.}$$

The speed regulation is a measure of the ability of a motor to maintain its speed when load is applied.

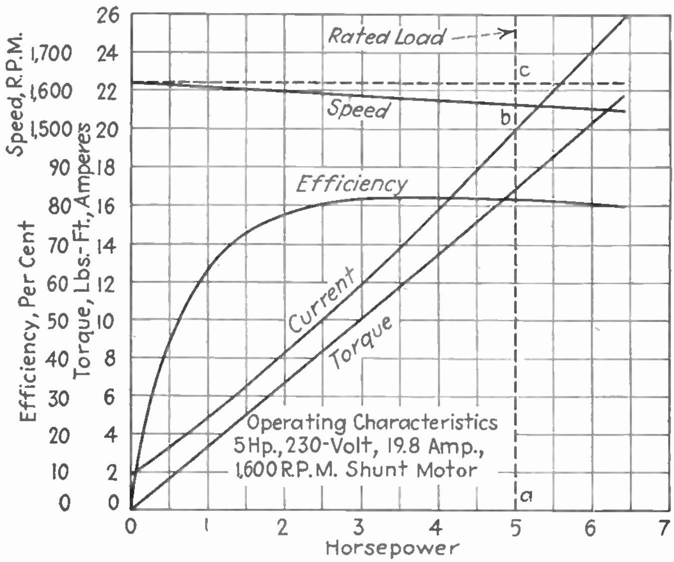


FIG. 367.—Typical shunt-motor characteristics.

Figure 367 shows the four essential characteristics of a shunt motor, namely torque, speed, current, and efficiency, each plotted as a function of horsepower output. The effect of machine losses on efficiency will be discussed in the next chapter. It will be noted that the shunt motor has a definite no-load speed. Therefore it does not run away when the load is removed, provided the field circuit remains closed.

Shunt motors are used where a substantially constant speed is required, as in machine-shop drives, spinning frames, or blowers. No type of motor is better adapted than the shunt motor to speed control and to speed adjustment. Hence it is also used where the speed must be varied or must be adjustable. With *adjustable* speed, the speed is set at the desired value and then remains substantially constant as the load varies. An engine lathe is an excellent example of adjustable-speed operation. Different sizes of cuts require different speeds. However, when the speed for a definite cut has been fixed, the speed should remain substantially constant (also see p. 511).

There is an erroneous impression that shunt motors have low starting torque. This is undoubtedly due to the fact that the ordinary starting box is designed for starting under light load only. Although such boxes will safely allow 125 per cent full-load current on the first notch, they quickly overheat if this value of current is sustained sufficiently long to bring the motor with its load up to speed.

Torque is proportional to the product of current and flux (Eq. (224), p. 484). The motor is started with full-field excitation (Par. 302, p. 501). Hence, the motor on starting can develop full-load torque, or even 150 per cent and over of full-load torque provided the starting rheostat can carry the requisite current.

When motors are started under load, a controller (Par. 305, p. 507) rather than a starting box is provided. In a controller, the resistors are designed to carry continuously full-load and overload currents.

In modern shunt motors, the shunt-field ampere-turns and hence the shunt-field loss are made very small by making the air-gap short. The short air-gap causes high armature reaction (see Fig. 384, p. 514), but good commutation is secured by the commutating poles. However, due to high armature reaction, there is a danger that with increasing load the armature m.m.f. may overcome the field m.m.f., causing the motor to run away. To prevent this, a few series turns aiding the shunt turns are wound on each pole. Although such a machine might be considered a compound motor, the series turns are so few that the characteristics are actually those of a shunt motor, and such machines are considered as shunt motors.

300. Series Motor.—In the series motor, the field is connected in series with the armature, as in Fig. 368. The field has comparatively few turns of wire and this wire must be of sufficient cross-section to carry the rated armature current of the motor.

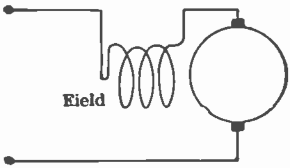


FIG. 368.—Connections of a series motor.

In the series motor the flux ϕ depends entirely on the armature current. If the iron of the motor is operated at moderate saturation, the flux will be almost directly proportional to the armature current. Therefore, in the expression for torque (Eq. (224), p. 484),

$$T = K_t I \phi.$$

If ϕ is assumed to be proportional to I, the torque

$$T = K_t' I^2 \tag{229}$$

where K_t' is a constant.

The torque is proportional to the *square* of the armature current, as in Fig. 366. When the current is 30 amp., the torque is 20 lb.-ft.; at 60 amp., the torque is 80 lb.-ft. That is, doubling the armature current results in quadrupling the torque. It will be noted that, as the current increases above 60 amp., the torque rises very rapidly. This characteristic of the series motor makes its use desirable where a large increase in torque is desired with a moderate increase in current. In practice, saturation and armature reaction both tend to prevent the torque increasing as rapidly as the square of the current.

When Eq. (228) is applied to the series motor, the speed

$$S = K\frac{V - I_a(R_a + R_s)}{\phi} \tag{230}$$

where K is a constant, V the terminal voltage, I_a the motor current, R_a the armature resistance including brushes, R_s the series-field resistance, and ϕ the flux entering the armature from a

N-pole. R_s, the resistance of the series field, is now added to the armature resistance in order to obtain the total motor resistance. Both I_a and ϕ vary with the load.

As the load increases, the voltage drop in the field and armature resistance increases, this voltage drop being proportional to the current. Therefore, the back e.m.f. and the numerator of Eq. (230) become less, which tends to decrease the speed, although, as with the shunt motor, the decrease due to this factor is only a few per cent. The flux ϕ in the denominator, however, increases almost *directly proportional to* the current. Hence an increase in current decreases the numerator and increases the denominator of Eq. (230), and the speed of the motor must decrease with increase of load. The resistance drop ordinarily is from 3 to 8 per cent of the terminal voltage V so that its effect in decreasing the speed is of this magnitude. The speed is inversely proportional to the flux ϕ and a given percentage change in ϕ produces the same percentage change in the speed.

When the load torque is increased, the reactions are as follows: There must be a drop in speed, at least momentarily, since load torque exceeds electromagnetic torque, and, for the instant, current and flux have not changed. This reduces the counter e.m.f., and the difference between terminal voltage and counter e.m.f. increases. Hence the current increases and causes field flux and electromagnetic torque to increase. The speed and current will then adjust themselves until electromagnetic torque is equal to the sum of load torque and loss torque, and equilibrium is reached. Since electromagnetic torque increases nearly as the square of the current, the increase of current with the same increase in torque is less than with a shunt motor.

When the load torque is decreased, the armature accelerates, at least momentarily, increasing the back e.m.f., since, for the moment, current and flux have not changed. Accordingly, current and field flux will decrease, and electromagnetic torque will also decrease. Speed and current will adjust themselves until equilibrium is reached.

If the load be removed altogether, ϕ becomes extremely small, resulting in a very high speed. It is dangerous to remove the load from series motors, as their armatures are almost certain to reach speeds where centrifugal action will wreck them.

Figure 369 shows the characteristic curves of a 5-hp. series motor plotted with horsepower as abscissa. The torque curve concaves upward for the reasons just stated. The speed varies practically inversely as the current. At large values of current the speed is low and at small values of current the speed is high. The characteristics cannot be determined for small values of current because the speed becomes dangerously high.

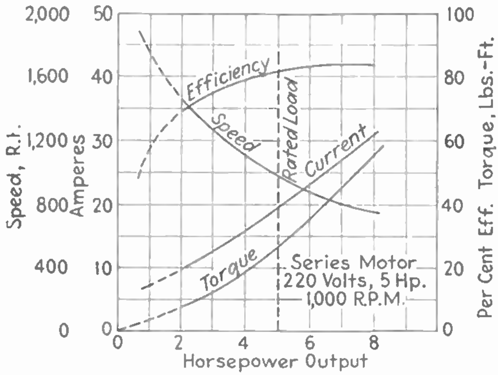


FIG. 369.—Typical series-motor characteristics.

The efficiency increases rapidly at first, reaches a maximum at about half load, and then decreases. This is due to the fact that at light loads the friction and iron losses are large compared with the load. The effect of these losses becomes less as the load increases. The field and armature loss varies as the *square* of the current (I^2R), so that these losses increase rapidly with the load. The maximum efficiency occurs when the friction and iron losses are practically equal to the copper losses. These characteristic curves should be carefully compared with the corresponding curves for the shunt motor (Fig. 367).

Series motors are used for work which demands large starting torque, such as street cars, locomotives, or cranes. In addition to the large starting torque, there is another characteristic of series motors which makes them especially desirable for traction purposes. Assume that a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed of the car at approximately the same value that it has when the car is running on level ground. The motor therefore tends to take an excessive current. A series motor, on the other hand, automatically slows down on such a grade, because of the increased current demand. It therefore develops more torque at reduced speed. The drop in speed allows the motor to develop a large torque with but a moderate increase of power. Hence, the rating of a series motor

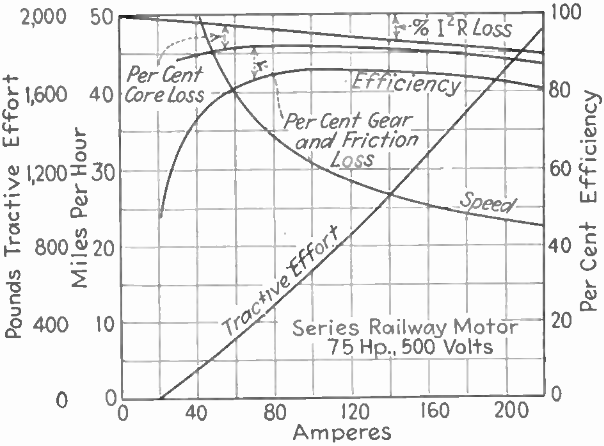


FIG. 370.—Typical characteristics of a 75-hp., series railway motor.

would be less than that of a shunt motor under the same conditions.

When the characteristics of railway motors are plotted, the curves refer to the output at the track and not at the motor shaft. Figure 370 gives such characteristics for a 500-volt, 75-hp., General Electric railway motor. It will be noted that tractive effort is plotted rather than torque. The speed of the car in miles per hour is given rather than the r.p.m. of the motor armature. These curves differ from those for torque and r.p.m. obtained at the pulley by a constant of proportionality, determined by the gear ratio and by the diameter of the driving wheels. There is a slight loss of torque in the gears. The efficiency curve is the efficiency at the rails. These characteristic curves resemble closely the curves of Fig. 369. Figure 371 shows a General Electric railway motor as viewed from the pinion and suspension side. The hand hole through which access to the brushes is obtained is shown at the far end.

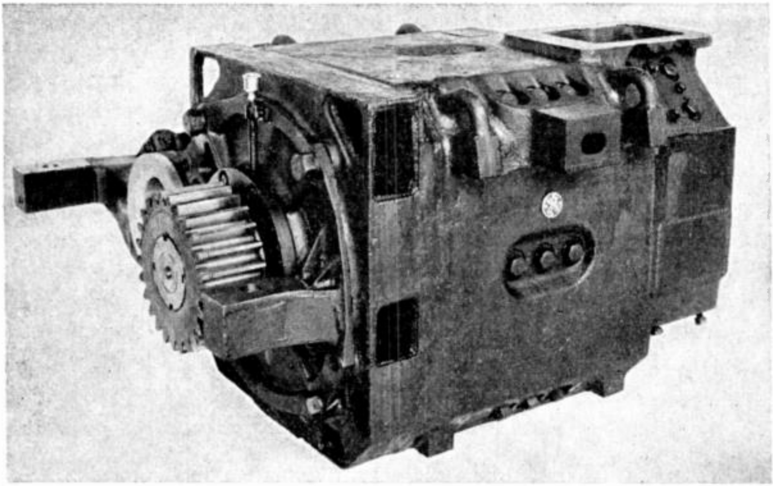


FIG. 371.—Railway motor, three-quarter view commutator end, axle side. (*General Electric Company.*)

301. Compound Motor.—A shunt motor may have an additional series-field winding in the same manner as a shunt generator. This winding may be connected so that it aids the shunt winding, in which case the motor is said to be *cumulative compound;* or the series winding may oppose the shunt winding, in which case the motor is said to be *differential compound.*

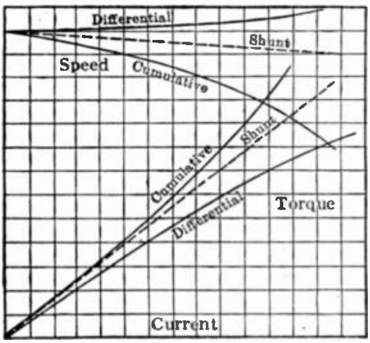


FIG. 372.—Electromagnetic torque (developed) and speed characteristics of shunt and compound motors.

The characteristics of the cumulative-compound motor are a combination of shunt and series characteristics. As the load is applied, the series turns increase the flux, causing the torque for any given current to be greater than it would be for the shunt motor. On the other hand, this increase of flux causes the speed to decrease more rapidly than it does in the shunt motor. These characteristics are shown in Fig. 372. Values are given for electromagnetic torque or torque developed within the armature. The cumulative-compound motor develops a high torque with sudden increase of load. It also has a definite no-load speed, so does not run away when the load is removed.

The field of application of the cumulative-compound motor lies principally in driving machines which are subject to sudden applications of heavy load, such as occur in rolling mills, shears, or punches. This type of motor is used also where a large starting torque is desired, but where a straight series motor cannot be used conveniently. Cranes and elevators are representative of such loads. In elevators, the series turns are usually short-circuited when the motor reaches running speed.

In the *differential*-compound motor, the series field opposes the shunt field so that flux is decreased as load is applied. This results in the speed remaining substantially constant or even increasing with increase of load. This speed characteristic is obtained with a corresponding decrease in the rate at which the torque increases with load. Such motors are used where a very constant speed is desired. Because of the substantially constant speed of the shunt motor, there is little occasion to use the differential-compound motor. Moreover, since the field is weakened with increase of load, there is a tendency to speed instability and the motor running away. In starting a differential-compound motor, the series field should be short-circuited, as the large starting current in the series field may be sufficient to overbalance the shunt-field ampere-turns and to cause the motor to start in the wrong direction. Typical torque and speed curves of the differential-compound motor are shown in Fig. 372. The values of torque are for electromagnetic torque or gross torque developed by the armature.

To reverse the direction of rotation in any motor, either the armature alone or the field alone must be reversed. If both are reversed the direction of rotation remains unchanged. Therefore, in so far as the direction of rotation of the motor is concerned, it is immaterial which line is positive.

MOTOR STARTERS

302. Three-point Box.—It is shown in Par. 296 (p. 485) that if a 10-hp., 110-volt motor is connected directly across 110-volt mains, the resulting current is 110 ÷ 0.05, or 2,200 amp. Such a current would not be permissible under commercial conditions. Hence, resistance must be connected in series with the motor *armature* when starting. This resistance may be gradually cut out as the armature comes up to speed and develops a back e.m.f.

Figure 373 shows the use of a simple resistance R for starting a shunt motor. It will be noted that this resistance is in the *armature* circuit and that the field is connected directly across the line and outside the resistance R. If the field were connected across the armature terminals, putting the resistance R in series with the whole motor, there would be little or no voltage across the field at starting. There would be little torque developed and difficulty in starting would be experienced.

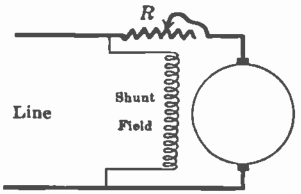


FIG. 373.—Resistance used for starting purposes.

Figure 374 shows a three-point starter. This does not differ fundamentally from the connections shown in Fig. 373. One line connects directly to an armature terminal and a field terminal tied together. It makes no connection whatever with the starting box. The other line goes to the line terminal of the starting box, which is connected directly to the starting arm. The starting arm moves over contacts set in the slate front of the starting box. These contacts connect with taps distributed along the starting resistance. The armature terminal of the starting box, which is the right-hand end of the starting resistance, is connected to the other armature terminal of the motor. The field connection in the starting box is connected from the first starting contact, through the hold-up magnet, to the field terminal of the box. This field terminal is connected directly to the other terminal of the shunt field.

When the starting arm makes connection with the first contact, the field is connected directly across the line and at the same time all the starting resistance is put in series with the armature. As the starting arm is moved, the starting resistance is gradually cut out. When the arm reaches the running position, the starting resistance is all cut out, and, to insure good contact, the line and armature conductors frequently are connected directly by a laminated copper brush, as in Fig. 374. The field current now flows back through the starting resistance. This resistance is so low, compared with the resistance of the field itself, that it has no material effect on the value of the field current. A spring tends to pull the starting arm back to the starting position. When

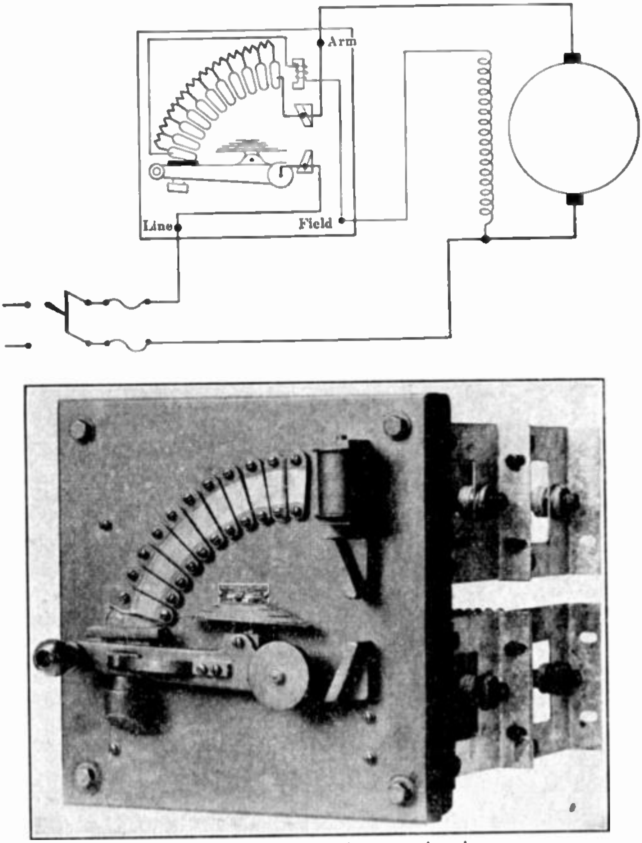


Fig. 374.—Three-point starting box.

the arm reaches the running position, it is held against the action of this spring by a soft-iron magnet or hold-up magnet, connected in series with the shunt field. A soft-iron armature is often attached to the starting arm as shown in the figure. If, for any reason, the line is without voltage, the starting arm will spring back to the starting position. Otherwise, if the voltage returned to the line after a temporary shut-down, the stationary motor armature would be connected directly across the line and a short circuit would result.

The advantage of connecting the hold-up coil in series with the shunt field is that, should the field circuit become opened, the arm

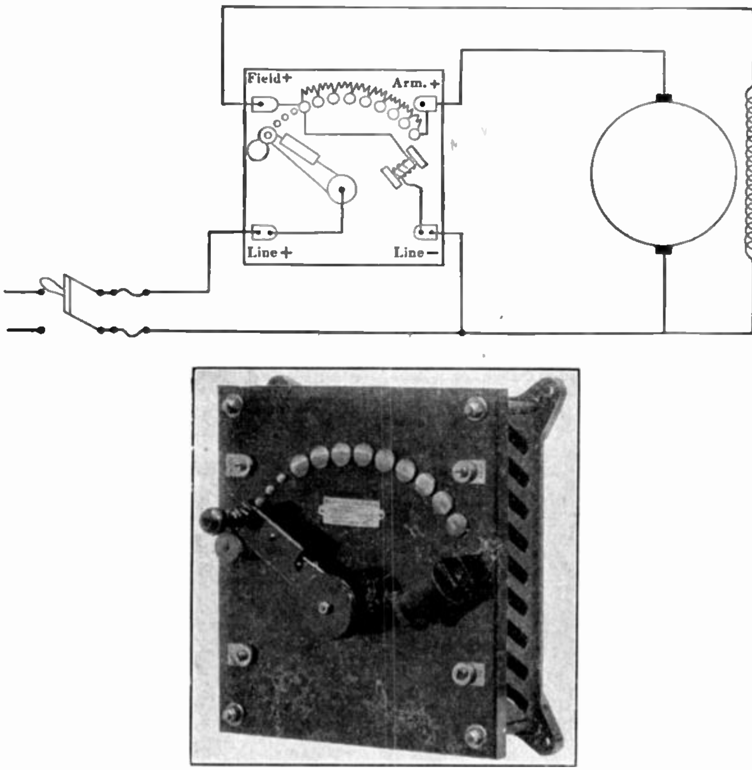
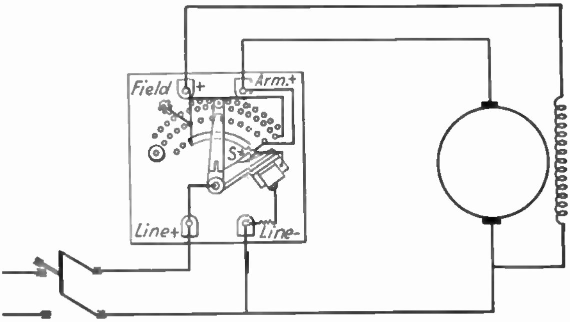


Fig. 375.—Connections for a four-point starting box.

springs back to the starting position and so prevents the motor running away.

303. Four-point Box.—The three-point starting box cannot be used to advantage on variable-speed motors having field control. Such motors frequently have a speed variation of five to one. This results in the field current having approximately the same range. The hold-up magnet may be too strong, therefore, at the higher values of field current and too weak at the lower values. To obviate this difficulty a four-point box is used (Fig. 375).

It is similar to the box shown in Fig. 374, except that the hold-up coil is of high resistance and is connected *directly across the line.* The only difference in the connection is that the "line −" terminal must be connected to the side of the line which runs directly to the common armature and field terminals. When the voltage



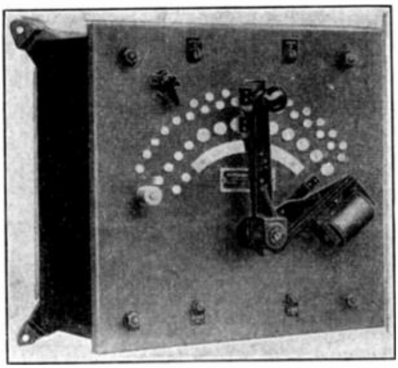


FIG. 376.—Westinghouse starting and speed-adjusting rheostat.

leaves the line, the hold-up coil becomes dead and allows the arm to spring back to the starting position.

Sometimes the field resistance is contained within the starting box. For example, in Fig. 376, it is connected between the contacts in the two upper rows. The box then has two arms. The shorter arm connects to the lower row of contacts and cuts out the armature starting resistance in the ordinary manner. This arm is pushed forward by the longer arm. During the starting period the field rheostat is short-circuited by the finger S moving over the metallic sector (Fig. 376). When the starting resistance is all cut out, the shorter arm is held by the magnet and the short circuit of the field resistance is removed by this arm pushing S to the right of the sector. The longer arm, which has no spring, inserts resistance in the field circuit when moved backward. If the voltage goes off, the magnet becomes de-energized and the shorter arm springs back carrying the longer one with it.

In stopping a motor, the *line switch* should always be opened rather than throwing back the starting arm. With shunt motors,

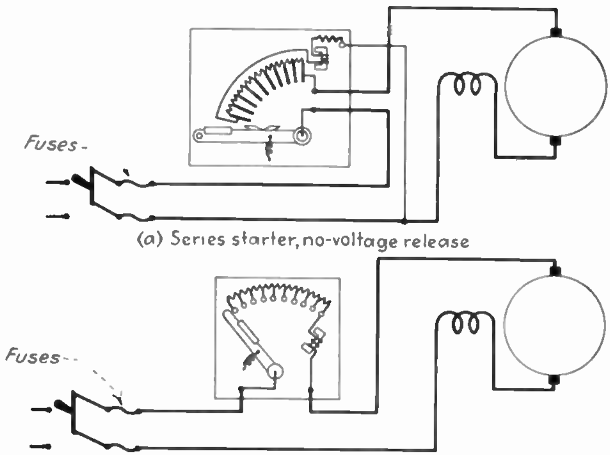


FIG. 377.—Series-motor starters.

the line switch can be opened with no appreciable arc, since the motor develops a back e.m.f. nearly equal to the line voltage and the net voltage across the switch contacts is small. The electromagnetic energy stored in the field does not appear at the switch but is discharged gradually through the armature. On the other hand, if the starting arm is thrown back, the field circuit is broken at the last contact button. Owing to the inductive nature of the field, this results in a hot arc which burns the contact. To prevent the contact from being burned, a small finger breaks the arc (Fig. 376).

304. Series-motor Starters.—The series-motor starter needs no shunt-field connection. There are two principal types, one having a no-voltage release (Fig. 377(a)), and one having a no-load release

(Fig. 377(*b*)). In the no-voltage-release type, the hold-up coil is connected directly across the line and releases the arm when the voltage goes off the line. In the no-load-release type, the hold-up coil consists of a few turns in series with the motor. When the motor current falls below a determined value, the starting arm is released. This last type is particularly adapted to series

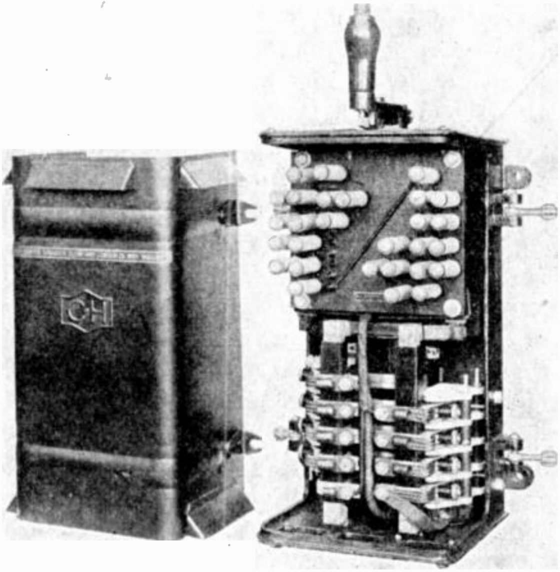


Fig. 378.—Compound revolving-drum controller with finger-type field control. (*The Cutler-Hammer Manufacturing Company.*)

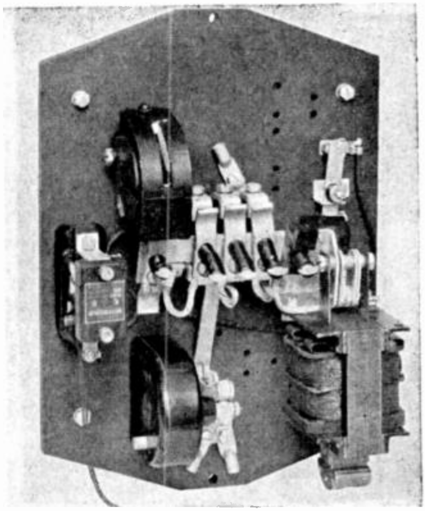
motors where there is a possibility of the load dropping to such a low value that the motor speed may become dangerous.

305. Controllers.—Controllers are used where the operation of the motor is continually under the direct control of an operator, as in street-car, crane, and elevator motors. The controller must be more rugged than the starting box, since the controller is used for continual starting, stopping, and reversing the motor. A typical controller is shown in Fig. 378. Such controllers usually have an external resistance which is cut in and out by fingers in the controller. A shunt-motor field rheostat may also be incorporated within the controller. Controllers are usually fitted with a "reverse," so that the motor may be run in either direction. The controller in Fig. 378 contains both the shunt-field resistance and the reverse.

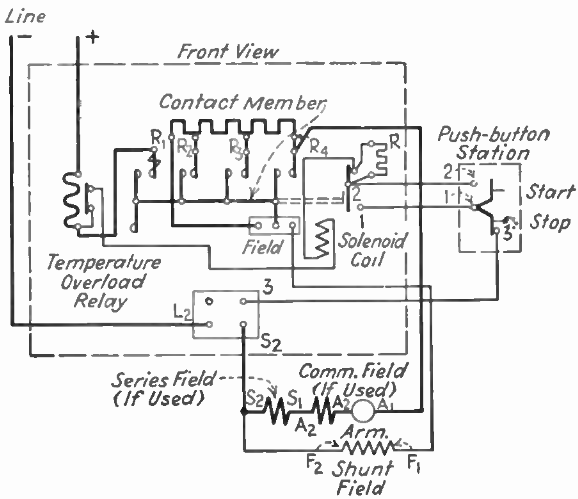
306. Automatic Starters.—Automatic starters are often used in practice. They have many advantages over the hand-operated starter. They cut out the starting resistance at a definite rate, so that the blowing of fuses and the opening of circuit-breakers, due to too rapid acceleration, are avoided. In many installations where a motor is used intermittently, it may be started and stopped by merely pushing buttons. Employees will be more likely to shut the motor down when the power is not being used, because of the ease with which starting and stopping are effected. In the larger sizes of motors, especially when extremely rapid operation is necessary, as in rolling mills, only automatic starters can give satisfactory results.

A typical automatic starter is shown in Fig. 379. The mechanism mounted on a molded-insulation base and with the casing removed is shown in (*a*). The wiring diagram is given in (*b*). In (*a*) the solenoid at the lower right, when energized, tends to cause the contactors to close. The time between the closing of successive contacts, however, is regulated by the timing mechanism shown at the lower left. This mechanism consists of gears, the speed of which is regulated by an escapement. The solenoid can cause each contactor to close only in accordance with the time determined by the escapement. At the left is shown a temperature-overload relay which protects the motor from overloads. The device at the top is an arc shute which restricts and de-ionizes the direct-current arc.

Referring to the wiring diagram in (*b*), when the "start" button is actuated, the solenoid coil becomes energized, pulling the contact member to the right (upward in (*a*)). The extreme left-hand contactor R_1 closes immediately, connecting the shunt field across the line, and the armature, the series and commutating fields across the line in series with the starting resistances R_2, R_3, R_4. These resistances are short-circuited successively by the contactors in accordance with the action of the timing mechanism. After the contactors are all closed, a switch, which short-circuits resistance R in series with the solenoid coil, is opened, thus reducing the heating in the solenoid coil. However, the reduced current is sufficiently large to hold the contacts closed.



(a)



(b)

FIG. 379.—Definite-time, automatic-magnetic starter. (*General Electric Company.*)

When the "stop" push button is actuated, the solenoid circuit is opened, the coil is de-energized, and the contacts all open, disconnecting the motor from the line. By using three- and four-way switches, this type of controller may be operated from widely separated units. Instead of a simple snap switch, the motor may be controlled by a float switch, a pressure switch, or any other automatically-operated switch.

307. Thermostat Protection.—Automatic starters and controllers are frequently equipped with overload devices such as circuit-breakers, fuses, and thermal-overload relays (Fig. 379(*b*)). These devices protect the motor from (1) failure to start; (2) stalled condition; (3) continuous overload. In the Thermoguard motors of the Westinghouse Electric and Manufacturing Company, a small thermostat is built into the windings of the motor. The control circuit of the starter is in series with the contacts of the thermostat. Normally the thermostat contacts are closed, but when the temperature reaches a value which may overheat the motor, the contacts open, thus opening the control circuit, and the motor is disconnected.

Should the shutting down of the motor interfere seriously with production, a lamp may be connected in series with the thermostat. The lamp when lighted gives advance warning of overheating. Similarly, an electric bell, short-circuited by the thermostat, can be used to give warning.

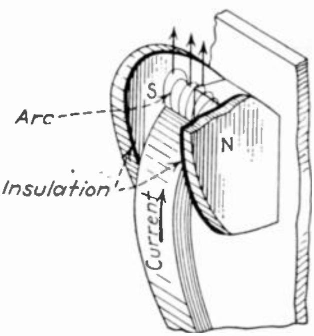


Fig. 380.—Magnetic blowout.

308. Magnetic Blowouts.—Controllers and circuit-breakers are often equipped with magnetic blowouts whose function is to elongate the arc resulting from opening the circuit, thus causing the arc to be extinguished quickly. The arc is also made to move out rapidly from the space directly between the contacts so that burning of the current-carrying surfaces, due to the persistence of the arc, is materially reduced. The principle of blowouts is as follows: The contacts between which the arc is to be broken are placed between the poles of a magnet. This is illustrated in Fig. 380, which shows the pole of a circuit-breaker just as the brush contact is moving away from the copper block to open the circuit. The contact brush is mounted between the

N- and S-poles of an electromagnet, so that a magnetic field acts across the gap. When the contacts open, the current tends to persist in the form of an arc. This arc finds itself in a magnetic field, and motor action results. The arc moves across the field in the direction determined by Fleming's left-hand rule. In Fig. 380 the motion is upward. The arc moves rapidly out of the gap to the upper edges of the contacts and is drawn out to such an extent that it ruptures easily.

The inner surfaces of the magnets or pole faces are lined with insulation (Fig. 380) so that the arc cannot jump to them. In some breakers of high interrupting capacity, the arc is blown into a restricted channel called an *arc shute*. There it comes into intimate contact with the walls of the arc shute where it is cooled and rapidly de-ionized.

SPEED CONTROL

In the equation for motor speed, $S = KE/\phi$, there are but two factors that can be changed to secure speed control without

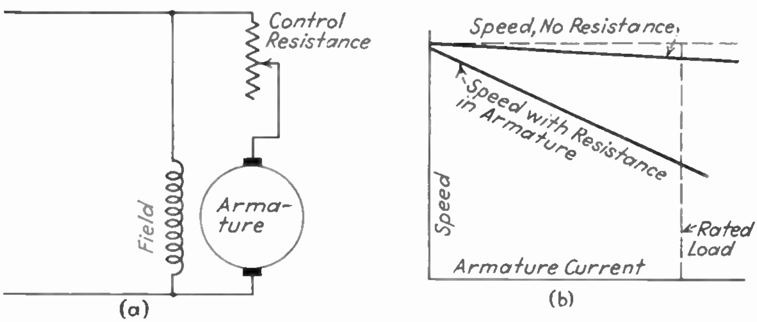


FIG. 381.—Speed control and regulation—armature-resistance method.

making changes in the motor construction. These factors are the back e.m.f. E and the flux ϕ.

309. Armature-resistance Method.—In this method, the speed control is obtained by connecting a resistance directly in series with the motor *armature*, keeping the field across full-line potential, as in Fig. 381(a). Because of the voltage drop in this resistance, the back e.m.f. E is changed. A wide range of speed can be obtained by this method and at the same time the motor will develop any desired torque over its working range, for the *torque* depends only on the *flux* and armature *current*.

The principal objections to this method of speed control are that an excessive proportion of power is lost in the armature series resistance and the speed *regulation* is very poor. In Fig. 381(*b*) are the speed-load curves of a shunt motor with and without resistance in series with the armature. The speed-load curve with armature series resistance shows that half-speed is obtained at rated load. It will be observed that the speed at no load rises to a value which is practically equal to the speed of the motor when there is no armature series resistance. In this case the speed regulation with resistance is about 100 per cent. About 50 per cent of the power supplied to the armature circuit is lost in the series resistance. Without series resistance the speed regulation is the usual 3 or 4 per cent.

Another objection to this method is that the control resistance must have sufficient carrying capacity for rated and overload current of the motor, and provision must be made for dissipating the large amount of heat developed in the resistance.

Example.—A 220-volt, 7-hp. motor has an armature resistance of 0.25 ohm. When running without load at 1,200 r.p.m., the armature takes 6 amp. (*a*) What resistance should be connected in series with the armature to reduce the speed of the motor to 600 r.p.m. with 30 amp. to the armature? (*b*) How much power is lost in the resistance? (*c*) What percentage of the power delivered to the armature circuit appears at the armature terminals? (*d*) What is the speed regulation of the armature? Neglect armature reaction.

(*a*) $$E_1 \text{ (at no load)} = 220 - (6 \times 0.25) = 218.5 \text{ volts.}$$

$$E_2 \text{ (at 600 r.p.m.)} = \frac{600}{1{,}200}218.5 = 109.3 \text{ volts.}$$

$$\text{Total } (R + R_a) = \frac{220 - 109.3}{30} = \frac{110.7}{30} = 3.69 \text{ ohms.}$$

Subtracting the armature resistance,

$$R = 3.69 - 0.25 = 3.44 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) Power lost in the series resistance

$$P_1 = (30)^2 \times 3.44 = 3{,}096 \text{ watts.} \quad \textit{Ans.}$$

(*c*) Power delivered to armature circuit

$$P_2 = 220 \times 30 = 6{,}600 \text{ watts.}$$

Power delivered to armature

$$P_3 = 6{,}600 - 3{,}096 = 3{,}504 \text{ watts.}$$

Percentage power delivered to armature

$$= \frac{3{,}504}{6{,}600} = 0.531 \text{ or } 53.1 \text{ per cent.} \quad \textit{Ans.}$$

(*d*) Speed regulation

$$\frac{1{,}200 - 600}{600} = 100 \text{ per cent.} \quad \textit{Ans.}$$

310. Multivoltage System.—In this system several different voltages are available at the armature terminals of the motor.

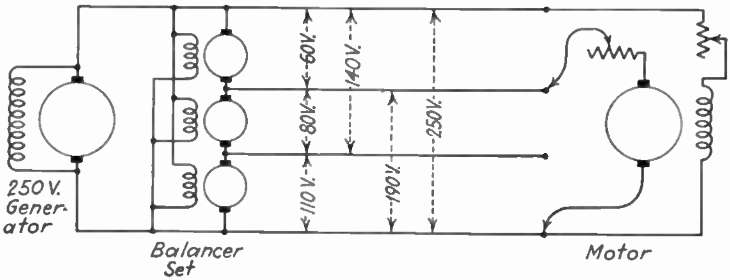


FIG. 382.—Multivoltage speed control.

These voltages are often supplied by a balancer set (Fig. 382). The shunt field of the motor is connected permanently across a fixed voltage and, with the four-wire system shown, six voltages are available for the armature. Intermediate speed adjustments can be made with a limited field control. Owing to the necessity of having a balancer set, or its equivalent; and due to the large number of wires necessary, this system is little used in this country for ordinary power drive. It is used extensively, however, to give the starting voltages for direct-current elevators, where there is a large group of elevators. For this type of service the use of the multivoltage system is particularly advantageous in that there is a large saving of energy in the starting resistors, an important factor where starting occurs during so large a proportion of the time.

311. Ward Leonard System.—In this system (Fig. 383) variable motor voltage is obtained by means of a separate generator G driven by a motor M_1. By varying the field of the generator, the desired voltage across the motor terminals M_2 is obtained. The motor field is connected across the supply mains in parallel with the fields of the other two machines. In Fig. 383, M_1 is a motor driving generator G. G, in turn, supplies variable voltage to the armature of motor M_2 whose speed

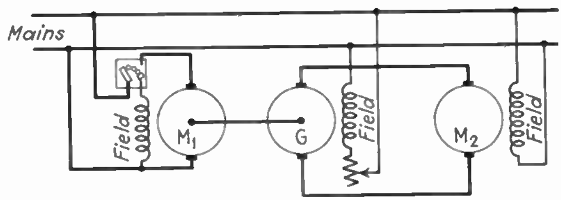


Fig. 383.—Ward Leonard system of speed control.

is to be varied. This system is very flexible and gives close adjustment of speed as well as good speed regulation. The chief disadvantages are the low over-all efficiency of the system, especially at light loads and the necessity of having two extra machines. This system has been used extensively for turning the turrets of battleships but is now superseded for this purpose. At the present time the system is used in reversing mills and in mine-hoist duty.

312. Field Control.—In the foregoing methods of speed control, the armature volts have been varied. A change of speed also may be obtained by varying the flux ϕ by means of a field rheostat. This method is very efficient so far as power is concerned,

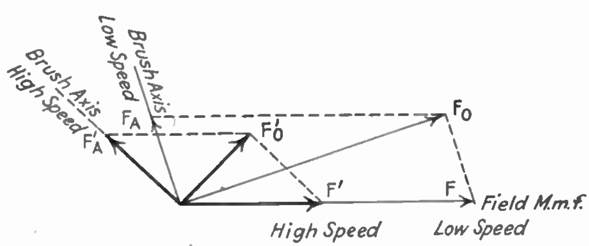


Fig. 384.—Effect of weak field on brush position.

and for any particular speed adjustment the speed regulation from no load to full load is excellent. The range of speed obtainable by this method with the ordinary motor is limited by commutation difficulties. Referring to Fig. 384, F is the field m.m.f. at low speed and F_A is the corresponding armature m.m.f. The resultant m.m.f. is F_0. If it be attempted to double the speed of the motor by weakening its field, neglecting saturation, the new field m.m.f. will be F'. It will now be necessary to move the brushes farther backward so that the armature m.m.f. will be at F_A'. The resultant m.m.f. is F_0'.

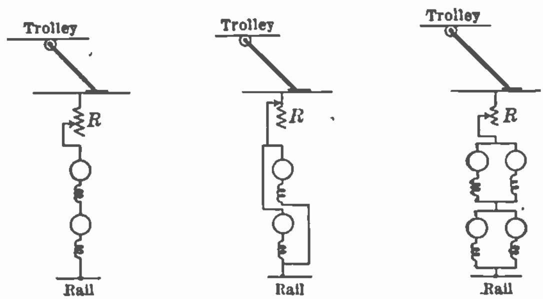
It is evident that the neutral plane has been moved backward to a considerable extent, the demagnetizing component of the armature m.m.f. has increased, and the armature m.m.f. is about equal to the field m.m.f. In addition to severe sparking at the commutator, the strong armature m.m.f. may so weaken the main field that the motor tends to run away. In order to eliminate the demagnetizing action due to moving the brushes, commutating-pole motors should be used where the speed range is large. A range of five to one in speed variation is obtainable with properly designed machines having commutating poles.

313. Lincoln Motor.—In the Lincoln motor, made by the Reliance Electric and Engineering Company, the flux entering the armature is varied by moving the rotating armature in and out of the field structure. As the armature is moved out of the field, the length of armature conductor cutting flux is reduced. Therefore the armature must rotate faster in order to develop the requisite back e.m.f. This gives smooth speed control over wide ranges, ratios as high as 10 to 1 being obtained. These motors are provided with commutating poles.

314. Series-parallel Railway-motor Control.—In a two-motor trolley car, two different speeds can be efficiently obtained. The motors are first connected in series through a starting resistance R as in Fig. 385(a). This resistance is gradually cut out by the controller as the car comes up to speed, and then each motor receives one-half the line voltage. This is the first running position. For any given value of armature current each motor will run at half its rated speed. As there is no external resistance in the circuit, the motors are operating at an efficiency very nearly equal to that obtainable with full-line voltage across the terminals of each motor. Since torque is a function of motor current only, each motor can develop any value of torque up to the values which it develops when connected across full voltage.

When it is desired to increase the speed of the car, the two motors are connected in parallel with each other and in series with a portion of the resistance R. This resistance is gradually cut out and when the running position is reached, each motor receives full-line voltage, as in Fig. 385(b).

In a four-motor car, the motors are usually divided into two groups, each group consisting of two motors which are always in parallel with each other. In starting, these two groups are connected in series, each group taking the place of the single motor of a two-motor car. This starting condition is shown in Fig. 385(c). When the full-speed running position is reached, both groups are



(a) Starting (b) Running with Motors in Parallel (c) Starting, 4 Motors

FIG. 385.—Series-parallel control of series motors.

connected in parallel across the line. Each motor then receives full-line voltage.

315. Multiple-unit Control.—In the heavier electric cars and locomotives, the currents become so large that direct platform control is out of the question from the standpoint of size of controller, safety, and expense. Moreover, when cars are operated in trains, it is necessary that the motors on all the cars shall be under a single control and that each control operation shall take place simultaneously on all cars.

In the multiple-unit system, all the heavy-current switching is done by solenoid-operated contactors located beneath the car. These contactors in turn are operated by an auxiliary circuit called the *train line*, which runs the entire length of the train (Fig. 386). The train line is made continuous through plug and socket connectors located in the car couplers. The wires of this train line receive their power through the master controller operated by the motorman. As this train-line current is of the magnitude of 2.5 amp., a small platform controller can be used. Another distinct advantage of this system is that the rate of cutting out the starting resistance during the acceleration period is outside the control of the motorman, being accomplished by automatically-operated contactors which close in sequence at the proper times. This insures uniform acceleration and eliminates the opening of the car circuit-breakers and the shocks to the

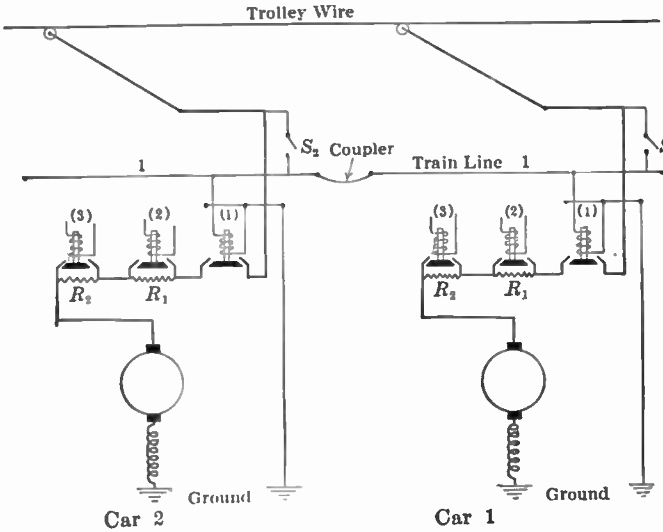


FIG. 386.—Principle of multiple-unit control.

equipment caused by the too great changes in acceleration which may occur when manual operation is used.

Figure 386 shows the underlying principle of the system, no attempt being made to give the many details which must necessarily accompany such a system. Each car has its own trolley or third-rail shoe for collecting the current. A train line of small wires runs the entire length of the train, the connections being made by the use of couplers between cars. This line usually consists of six wires. Solenoids, which operate contactors, are connected across the train lines. Some of the contactors are operated directly by the controller in the hands of the motorman, and others operate automatically after the manual controller has been turned to the desired position. For example, in Fig. 386

are shown two motors, one in each car. One line of the train line is shown running between cars and connected by the coupler. It is assumed that the train is to be operated from car 1. If the switch S_1 in the controller of car 1 be closed, train line 1-1 becomes alive. This energizes relay (1) in each car and both relays simultaneously close the motor circuits, the starting resistances R_1, R_2 being in series with the motors. As the motors "pick up," the current drops, and relay (2) in each car becomes automatically energized and some of the starting resistance R_1, R_1 is cut out in each car. Then the next set of relays becomes energized in a similar manner, until all the starting resistance is cut out and the motors are across the line.

In the complete system there are six train lines, some of which cause reverse, change from series to parallel, etc. The great advantage of this system is that every motor on the train can be operated from either of the two controllers on any one car, that all the motors are controlled simultaneously, the acceleration cannot exceed a certain value regardless of the motorman, and, as there are driving wheels on every car, high accelerations can be obtained. This system is also used extensively on single cars.

316. Dynamic and Regenerative Braking.—It is often desirable to brake a motor when it is being driven by its load, as in the case of descending elevators and crane loads. This is often done by using a controller which connects the field across the line and at the same time puts a resistance load across the armature terminals. This produces generator action and therefore retards the armature. If series motors are used, their fields must be connected across the line in series with a resistance. Such braking is not effective for completely stopping the motor armature, as the braking action diminishes proportionally to the speed of the armature.

Dynamic braking for a series motor is illustrated in Fig. 387. In (*a*) the motor is shown connected for raising the load. When the load reaches the desired position, the armature circuit is broken by means of the controller, as in (*b*). This de-energizes the brake solenoid, resulting in the brake being set (see Fig. 175, p. 240) and so holding the load in position. To lower the load, the operator, by means of the controller, makes the connections in (*c*). A series resistance in circuit allows just enough current I_L to flow to energize and disengage the brake solenoid. This current also produces flux in the series field of the motor and then flows to the negative side of the line without having passed through the armature and thus without having produced any motor action. Since the load is being lowered, the armature is rotating in a direction opposite to that which it had when the load was being raised. The torque which now produces the

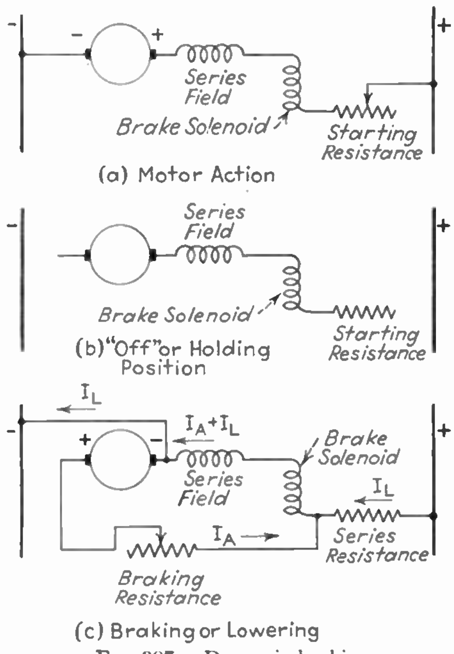


FIG. 387.—Dynamic braking.

rotation is supplied by the load. This rotation of the armature in the magnetic field, produced by the series winding, causes an e.m.f. to be generated in the armature and, as a closed external path is provided, a current I_A flows. Application of both right- and left-hand rules shows that, since the direction of the field is the same as it was with motor action and the direction of rotation is opposite, the direction of the current in the armature as generator must be the same as that corresponding to motor action. This current reinforces the small current I_L from the line to the series field and hence determines the magnitude of the e.m.f.

generated. The magnitude of the load is regulated by varying the braking resistance. Greater load requires greater torque and hence results in greater braking action.

Regenerative braking is based on this same principle, except that the power is returned to the line rather than being wasted in resistance.

Series and overcompounded generators when operated in parallel with each other or with a constant-potential system are unstable (see Par. 332, p. 553). Hence, when series motors are used for regenerative braking, as in railroad operation, the series fields are separately excited in parallel with each other from a

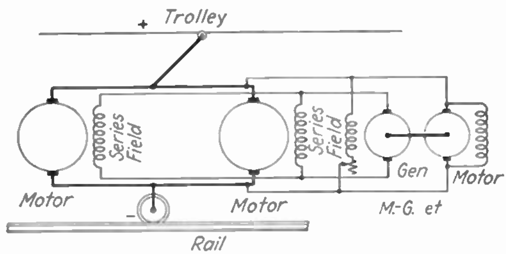


FIG. 388.—Series railroad motors connected for regenerative braking.

low-voltage motor-generator set (Fig. 388). Such a system is used on the electric locomotives of the Chicago, Milwaukee, St. Paul and Pacific Railroad.

317. Motor Testing—Prony Brake.—It is often necessary to determine the efficiency of a motor at certain definite loads and frequently over its entire range of operation. A knowledge of the efficiency may be necessary, as in the case of an acceptance test; further, the motor may be used as a power-measuring device for determining the power taken by some machine, such as a generator, pump, or blower. Knowing the motor input, which can be measured with ammeter and voltmeter, and also knowing the motor efficiency, the output for any given input can be computed. This output will be the power delivered to the generator, the pump, etc.

The most common method of making direct measurements of efficiency in motors up to about 50 hp. is to use a prony brake. Such brakes are made in various forms. One typical form is shown in Fig. 389. It consists of a wooden arm of the proper length, a canvas brake band and a hand wheel for applying tension to the brake band. By means of this hand wheel the motor load can be controlled. An oil dashpot is advisable, to prevent vibration of the brake arm.

The balance measures the pull on the arm due to the rotation of the drum, plus the dead weight of the arm. By multiplying the net balance reading by the distance L, the torque of the motor can be determined.

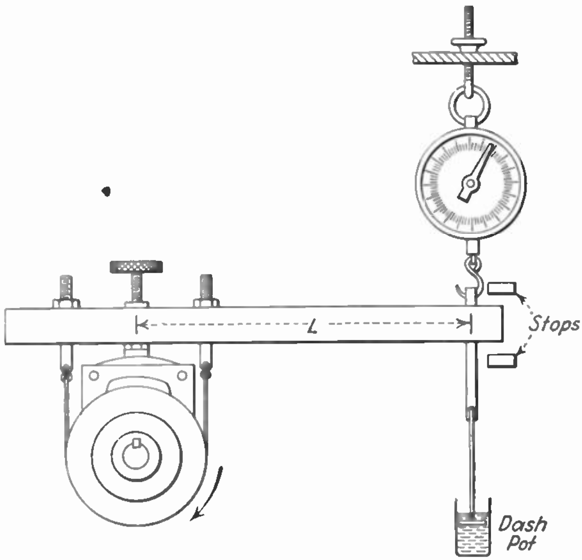


Fig. 389.—Typical prony brake.

There are two simple methods for determining the dead weight or tare of the brake arm. The brake band is loosened and some sort of knife edge, such as a pencil, is placed between the top of the drum and the brake carriage. This acts as a substantially frictionless fulcrum, so that the balance registers the dead weight of the arm alone. Another and easier way is to turn the drum toward the balance by hand, stop, and read the balance. In this case the friction of the brake causes the balance to read too high. If this operation be repeated, rotating the drum in the opposite direction, the balance reading will be too low, due to the same friction. The average of these two balance readings will give very closely the correct value for the dead weight of the arm.

Brakes of this type are cooled ordinarily by pouring water into the hollow brake drum. The water prevents the drum from becoming excessively hot. As the maximum temperature which water can reach in the open air is 100°C., the drum temperature cannot much exceed this. The heat developed in the drum is utilized in converting the water into steam. As a considerable number of heat units is required to convert a small amount of water into steam, a moderate amount of water will keep the drum comparatively cool.

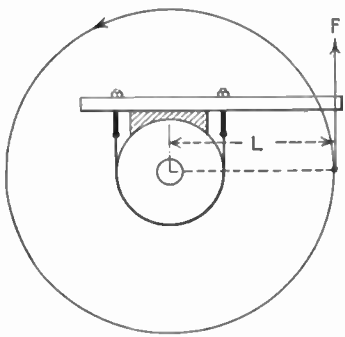


FIG. 390.—Work developed by a prony brake.

To determine the equation for the horsepower absorbed by such a brake, consider Fig. 390. Let F be the net force in pounds acting at a perpendicular distance L ft. from the center of the drum. First assume that the drum is stationary and that the arm is pulled around the drum by means of the force F. The distance per revolution through which the force F acts is $2\pi L$ ft. The work done in one revolution of this arm around the drum is the product of force and distance $= F(2\pi L)$ ft.-lb.

The work done in S revolutions $= F(2\pi L)S$ ft.-lb.

If S is the revolutions per minute, the horsepower

$$\text{Hp.} = \frac{2\pi(FL)S}{33{,}000}.$$

But FL is the torque T; therefore,

$$\text{Hp.} = \frac{2\pi TS}{33{,}000}. \tag{231}$$

$$\frac{2\pi}{33{,}000} = 0.00019.$$

Therefore,

$$\text{Hp.} = 0.00019TS. \tag{232}$$

Obviously, the same amount of work is done on the brake surface whether the drum is stationary and the arm rotates, or the arm is stationary and the drum rotates. Therefore, Eq. (231) applies to brakes of the type shown in Figs. 389 and 390. It will be noted that in this particular type of brake the horsepower is independent of the diameter of the drum.

Example.—In a brake test of a shunt motor, ammeter and voltmeter measuring the input read 34 amp. and 220 volts. The speed of the motor is found to be 910 r.p.m. and the balance on a 2-ft. brake arm reads 26.2 lb. The dead weight of the arm is found to be +2.4 lb. (*a*) What is the output of the motor? (*b*) What is its efficiency at this particular load?

(*a*) Net reading of balance = 26.2 − 2.4 = 23.8 lb.

Torque $T = 23.8 \times 2 = 47.6$ lb.-ft.

Hp. output $= 0.00019 \times 47.6 \times 910 =$ 8.23 hp. *Ans.*

(*b*) Output $= 8.23 \times 746 = 6{,}140$ watts.

Input $= 220 \times 34 = 7{,}480$ watts.

$$\text{Efficiency } \eta = \frac{6{,}140}{7{,}480} 100 = 82.1 \text{ per cent.}$$

Ans.

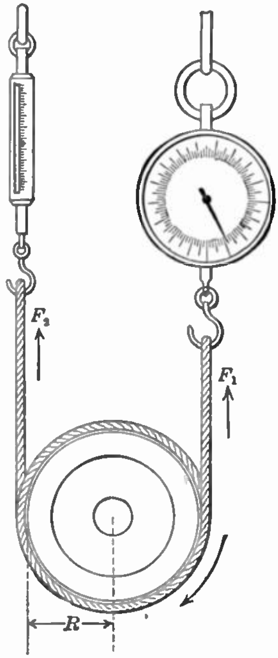


Fig. 391.—Rope brake.

In brakes of this type, the brake arm should be kept approximately horizontal.

Another simple type of brake is the rope brake shown in Fig. 391. A rope is given a turn and a half around a drum and the two free ends are each held by a spring balance. The larger balance is on the end of the rope which is being pulled downward by the rotation of the drum. Let F_1 be the reading in pounds of the larger balance and F_2 that of the smaller balance. As F_1 and F_2 pull in opposite directions with respect to the rotation of the drum, the net pull at the drum periphery is $(F_1 - F_2)$ lb.

The torque in pound-feet is

$$T = (F_1 - F_2)R$$

where R is the radius of the pulley in *feet.*

Example.—In a rope brake of the type shown in Fig. 391, $F_1 = 32.4$ lb. and $F_2 = 8.2$ lb. The drum is 10 in. in diameter. If the motor speed is 1,400 r.p.m., what horsepower does the motor develop?

The torque

$$T = (32.4 - 8.2)\tfrac{5}{12} = 24.2 \times \tfrac{5}{12} = 10.08 \text{ lb.-ft.}$$

The horsepower

$$\text{Hp.} = 0.00019 \times 10.08 \times 1{,}400 = 2.68 \text{ hp.} \quad \textit{Ans.}$$

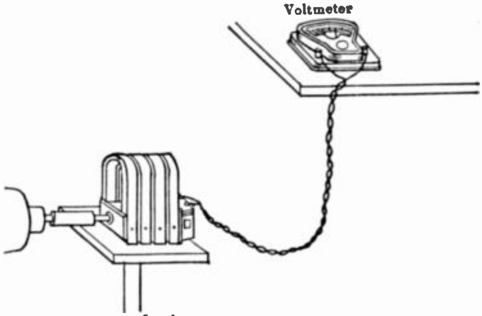
373. Measurement of Speed.—The measurement of the speed of machines is much simpler as a rule than the measurement of torque. The most common method is to use a simple revolution counter having a conical rubber tip which fits into the countersink of the shaft. The Veeder type is a convenient form of revolution counter. The revolutions are recorded directly on the counter. As this counter cannot be set to zero, the actual speed must be found by subtracting the counter reading before from that after the measurement.

FIG. 392.—Jagabi tachoscope.

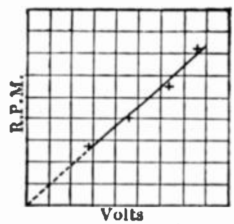
The Jagabi tachoscope (Fig. 392) is a combination of speed counter and stop watch. The spindle may be inserted in the countersink of the shaft without recording. A little pressure, however, causes the counter and stop watch to start simultaneously. They also stop simultaneously when the pressure on the tachoscope is removed. Measurements made with this type of instrument are free from personal error.

Tachometers indicate the instantaneous speed. There are mechanical tachometers, where the indicator is actuated by centrifugal action. This type should be carefully checked at each occasion of use, as it is especially subject to error after having been in service for some time.

A simple and convenient type of tachometer is the combination of a direct-current magneto and a voltmeter, as in Fig. 393(*a*). In the magneto, the flux is produced by permanent magnets and so is constant. Therefore, the e.m.f. induced in the magneto armature is directly proportional to the speed. If this e.m.f. be measured with a voltmeter, the voltmeter reading multiplied by a constant gives the speed directly. The relation of speed to volts may be plotted as in Fig. 393(*b*) and the speed read directly from the plot. This plot is ordinarily a straight line through the origin, one point of which is thus accurately determined. It is possible, however, to procure a magneto-voltmeter combination in which the voltmeter reads directly in r.p.m. It is convenient to attach the magneto by a piece of rubber tubing to the shaft of the machine whose speed is being measured. It is usually necessary to thread a small stud into the end of the shaft whose speed is to be measured, as in Fig. 393(*a*).



(*a*)



(*b*) Speed-voltage curve of magneto.

FIG. 393.—Speed measurement with magneto and voltmeter.

319. Dynamotor.—The A.I.E.E. Standards give the following definition of a dynamotor:

A dynamotor is a transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

The two windings of the dynamotor armature, when the machine has but a single armature, may or may not lie in separate slots. The machine is ordinarily used to convert direct-current power from one voltage to another. One winding develops motor action and causes rotation; the other winding develops generator action and delivers electrical energy. Since the motor and generator currents are in opposite directions, the net armature ampere-turns will be small, being just sufficient to overcome the torque due to rotational losses. Hence, there is but slight armature reaction.

As both windings cut the same magnetic field at the same speed, their induced e.m.fs. must be directly proportional to their turns. The e.m.f. ratio cannot be changed by changing the field excitation. If, for example, the field is strengthened, the motor conductors as well as the generator conductors are cutting more magnetic lines. This, however, will slow down the armature (motor principle), and the induced e.m.fs. will remain unchanged. If the field is weakened, the speed will increase and again the induced e.m.fs. do not change. The ratio between volts at the two commutators is affected to some extent by the armature-resistance drops.

The dynamotor is used as both starter and generator in one type of automobile electric system.

CHAPTER XIV

LOSSES; EFFICIENCY; OPERATION

320. Dynamo Losses.—A certain part of the energy given to any motor or generator is lost within the machine itself, being converted into heat and wasted. Not only is this energy lost, but there is the further objection that it heats the machine and so limits its output. If the energy loss in the machine becomes excessive, the resulting temperature rise may injure the insulation.

As motor and generator are similar, they have the same types of losses throughout. Therefore, the following applies to either motor or generator.

COPPER LOSSES

Armature.—The armature windings have resistance, and when current flows through them, power must be lost. In addition to the loss in the armature copper, there is an electrical loss in the brushes and in the commutator. Let this total power loss be P_a. Then,

$$P_a = I_a^2 R_a \tag{233}$$

where I_a is the armature current and R_a is the armature resistance measured between the terminals of the machine and including the commutator, the brushes and their contact resistance. This contact resistance is not constant, but under the usual conditions it is assumed constant. Where conditions require it, correction for change in current density may be made (see Par. 271, p. 428, and Fig. 318, p. 429). The armature-resistance measurement is often made with the connections shown in Fig. 394. The resistance R is inserted to limit the current flowing through the stationary armature (see Par. 113, p. 158). The measurement should be made with the armature in three or four different positions in order to obtain an average resistance. As the low-reading scale of the voltmeter is used ordinarily in making this measurement, the instrument may be injured on opening the circuit, due to the e.m.f. of self-induction of the armature. Therefore, the voltmeter should be disconnected when the circuit is opened or closed and also when the armature is turned.

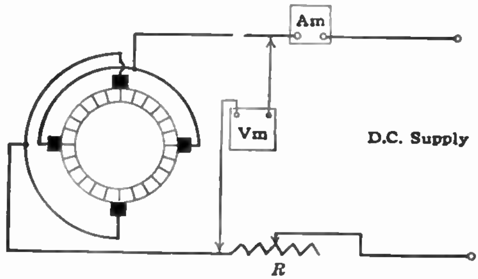


FIG. 394.—Measurement of armature resistance.

Shunt Field.—The field takes a current I_f at the terminal voltage V of the generator or motor. Therefore, the power lost in the field is

$$P_f = VI_f. \tag{234}$$

This includes the power lost in the field rheostat which is chargeable to the field circuit.

Series Field.—The series-field loss is

$$P_s = I_s^2R_s \tag{235}$$

where I_s is the series-field current, which may or may not be equal to the armature current, depending on whether the machine is long- or short-shunt.

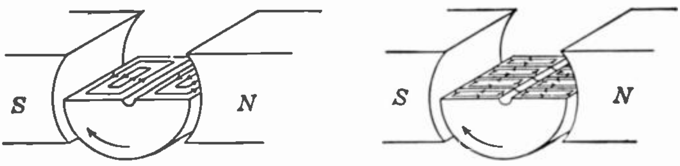
R_s is the series-field resistance. If a series-field shunt or diverter is used, R_s is the equivalent parallel resistance of diverter and series field and I_s is the current of the series field plus that of the diverter.

The losses in the commutating-pole circuit are determined in the same way as are those of the series field.

The foregoing losses are all copper losses and can be either measured directly or calculated with a high degree of precision from instrument readings.

IRON LOSSES

Eddy Currents.—As the armature iron rotates in the same magnetic field as the copper conductors, e.m.fs. are also induced in this iron. As the iron is a good conductor of electricity and the current paths are short and of large cross-section, large currents would be set up in the armature iron were it a solid mass, as in Fig. 395(*a*). These currents represent a power loss which would not be tolerated in a commercial machine. By laminating the armature iron in the manner indicated in Fig. 395(*b*), the paths of these currents are broken up and the currents are reduced. Laminating does not entirely eliminate these eddy-current losses, but it does reduce them to a small value. It will be noted that although the laminations break up the eddy-current paths, they do not interpose reluctance in the magnetic



(*a*) (*b*)

FIG. 395.—Eddy currents in armature iron without and with laminations.

circuit, since they are parallel to the direction of the magnetic flux.

These eddy currents are proportional to both speed and flux. As the loss varies as the square of the current (I^2R), *eddy-current loss varies as the square of both speed and flux.*

Example.—The eddy-current loss in a certain dynamo is 600 watts when the total flux is 2,000,000 lines per pole and the speed is 800 r.p.m. What is the loss when the flux is increased to 2,500,000 lines and the speed is increased to 1,200 r.p.m.?

$$P_e = 600 \times \left(\frac{2,500,000}{2,000,000}\right)^2 \times \left(\frac{1,200}{800}\right)^2 = 2,100 \text{ watts.} \quad \textit{Ans.}$$

Hysteresis.—It is shown in Chap. VIII that when iron is carried through a cycle of magnetization (Pars. 183 and 184, pp. 266 and 268), an energy loss results proportional to the area of the hysteresis loop. The iron in an armature undergoes a cyclic change of magnetization when the armature rotates. Consider the small section of the armature iron at (*a*) (Fig. 396) when it happens to be under a N-pole. This small section has a N-pole and a S-pole at its ends. When the section reaches position (*b*), its poles have become reversed, as shown. Practically all the armature iron is continually going through similar cycles of magnetic reversals. Therefore, there results a hysteresis loss in the armature iron as the armature rotates. This loss is directly proportional to the speed and is proportional to the 1.6 power of the maximum flux density, by the Steinmetz formula (Eq. (120), p. 269). *Laminating the iron does not affect the hysteresis loss.*

Pole-face Loss.—The flux enters and leaves the armature in tufts through the teeth as has already been shown (Fig. 181, p. 245). As these tufts of flux pass across the pole face, they

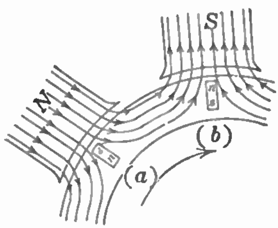


FIG. 396.—Reversal of magnetic flux in armature iron.

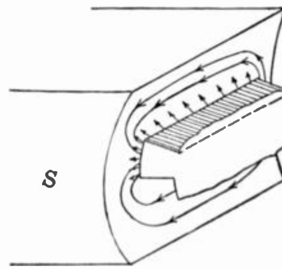


FIG. 397.—Pole-face loss due to tufts of flux from teeth.

produce flux pulsations in the pole face. These pulsations set up eddy currents in the pole face, in the manner shown in Fig. 397. This results in a power loss. A hysteresis loss also accompanies these flux pulsations. These combined losses are some function of flux and speed. Being in part due to eddy currents, they are reduced by laminating the pole faces (see Fig. 282, p. 392).

FRICTION LOSSES

These losses consist of bearing friction, brush friction, and windage, and all are functions of the speed.

INDETERMINABLE OR STRAY-LOAD LOSSES

The indeterminable losses[1] are caused for the most part by flux distortion produced by the load current. They include

[1] See "Direct-current Rotating Machines, Generators, and Motors," Table III, A.I.E.E. Standards, No. 5, July, 1925. Also see Par. 5–350 to 5–368.

iron losses, eddy-current losses in conductors due to transverse fluxes, and tooth-frequency losses. Another indeterminable loss is that due to the short-circuit currents during commutation. In the A.I.E.E. Standards it is recommended that these losses be arbitrarily assumed as 1 per cent of the output, except for motors of 200 hp., 575 r.p.m. and smaller, and, since no value has been assigned to such motors, these losses will be omitted for the present.

SUMMARY

The foregoing losses may be summarized as follows.

Copper losses:

Armature $I_a^2R_a$.
Shunt field VI_f.
Series field $I_s^2R_s$.

Stray power:
- Iron losses (armature and pole face):
  - Eddy current—function of flux and speed.
  - Hysteresis—function of flux and speed.
- Friction losses (bearings, brushes; windage)—function of speed.

Stray-load losses:
- Iron and conductor losses—function of load-flux distortion.
- Commutation short-circuit losses—function of load current.

(The stray-load losses will be neglected in the following discussion.)

The copper losses can be directly measured or can be calculated. The iron and friction losses can neither be directly measured nor so readily calculated. Moreover, as they are all some function of flux, or speed, or both, these losses are combined and are called *stray losses,* the power that they represent being called *stray power.*

As stray power is a function of speed and flux only, it is constant in a given machine if speed and flux are constant. Therefore, no matter what the load is, the stray power does not change unless either flux or speed changes.

In distinction to the copper losses, the stray power is all supplied *mechanically.* In a motor, for example, a mechanical torque is required to supply the stray power, making the torque available at the pulley less than that developed by the armature. In a generator the stray power is supplied by the prime mover and not by the generator itself. On the other hand, the electrical losses are supplied by the generator itself.

321. Efficiency.—The efficiency of a machine is the ratio of output to input. Thus,

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

This may be written also in either of the following ways.

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}}. \tag{236}$$

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}. \tag{237}$$

Therefore, if the losses in a machine be known, the efficiency may be found for any given input or output.

Example.—A shunt motor takes 40 amp. at 220 volts. The total motor losses are 1,800 watts. What is the motor efficiency?

Using Eq. (237),

$$\text{Efficiency} = \frac{(220 \times 40) - 1{,}800}{220 \times 40} = 0.796 \text{ or } 79.6 \text{ per cent.} \quad \textit{Ans.}$$

As electrical rather than mechanical quantities are ordinarily used in efficiency determinations, Eq. (236) is used for generators (output is electrical) and Eq. (237) for motors (input is electrical.)

322. Efficiencies of Motors and Generators.—As a rule the efficiency of electrical apparatus is high.

The table shown on page 533 gives the approximate weights and efficiencies of a few typical motors. The efficiencies of generators, having corresponding ratings, are practically the same as for motors.

The efficiency of a motor may be determined from simultaneous measurements of input and output, as is shown in Par. 317 (p. 520), where a prony brake is used.

Theoretically, the efficiency of a generator may be determined in a similar manner, by measurements of input and output. The output is readily measured with ammeter and voltmeter. The input, however, is difficult to measure. The difficulty lies in the measurement of the torque transmitted to the generator.

Horsepower	Speed, revolutions per minute	Weight of motor alone, pounds	Efficiencies, per cent		
			Load		
			½	¾	Full
½	1,140	54	64	71	74
¾	1,725	54	66	71	73
¾	1,140	82	70	73	76
1	1,725	80	68	74	76
2	1,725	170	71	77	79
3	1,725	180	73	78	80
5	1,750	260	74	80	83
10	1,150	500	82	85	86
25	1,150	890	84	86	87
50	850	1,640	85	88	89
100	850	2,890	87	89	90
200	850	4,410	89	91	92

Torsion dynamometers have been devised, but as a rule are unsatisfactory. The generator may be suspended in a *cradle*, as in Fig. 398. The ends of the generator shaft are supported in bearings, so that the frame is free to turn. The torque is determined by measuring the torque necessary to prevent the frame turning. Such a cradle is expensive, is not easily adapted to all generators, and necessitates that the generator shaft shall protrude beyond both generator bearings.

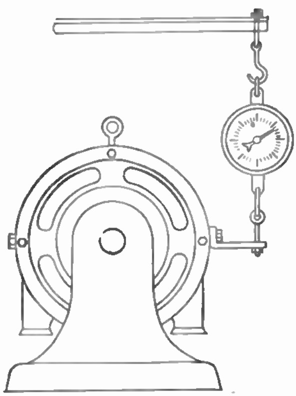


Fig. 398.—Cradle dynamometer.

In any direct measurement of efficiency, any percentage error in the measurement of either output or input introduces the same percentage error in the efficiency.

In the direct measurement of efficiency, the power necessary for the test must be equal to the rating of the machine. In addition to *supplying* this power there must be means for *absorbing* it. This is not a serious matter with small machines, but when large machines are tested, supplying and absorbing the necessary power may be difficult or even impossible.

Because of the foregoing reasons, it is often desirable and may even be necessary to obtain the efficiency by determining the losses.

Example.—A 250-kw., 230-volt, compound generator is delivering 800 amp. at 230 volts. The shunt-field current is 12 amp. The armature resistance is 0.007 ohm and the series-field resistance is 0.002 ohm. The stray power at this load is 5,500 watts. The generator is connected long-shunt. What is the generator efficiency at this load?

Output = 230 × 800 = 184,000 watts.

Shunt-field loss	$= 230 \times 12$	=	2,760 watts.
Armature loss	$= 812^2 \times 0.007$	=	4,620 watts.
Series-field loss	$= 812^2 \times 0.002$	=	1,320 watts.
Stray power		=	5,500 watts.
	Total loss	=	14,200 watts.

$$\text{Efficiency} = \frac{184{,}000}{184{,}000 + 14{,}200} = \frac{184{,}000}{198{,}200} = 0.928 \text{ or } 92.8 \text{ per cent.} \quad \textit{Ans.}$$

323. Measurement of Stray Power.—In order to duplicate the stray-power loss, it is necessary merely to duplicate flux and speed in motor or generator. As the speed (Eq. (227), p. 487) is $S = KE/\phi$, it is necessary merely to duplicate the speed S and the e.m.f. E in order to obtain the proper value of ϕ.

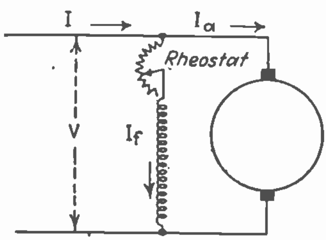


Fig. 399.—Determination of stray power in a dynamo.

To measure stray power, the machine, whether it be a motor or a generator, is run light (without load) as a motor, as in Fig. 399. The field, in series with a rheostat, is connected across the line.

The total power input to the machine is

$$VI = V(I_a + I_f) = VI_a + VI_f$$

where V is the terminal voltage, I the line current, I_a the armature current, and I_f the field current.

The total power input VI is distributed as follows: Part goes to supply the field loss, part supplies the armature $I_a^2R_a$ loss, and the remainder is the stray power (S.P.), the output being zero. Therefore,

$$\left.\begin{aligned} VI_a + VI_f &= VI_f + I_a^2R_a + \text{S.P} \\ \text{S.P.} &= VI_a - I_a^2R_a. \end{aligned}\right\} \qquad (238)$$

The *stray power* is equal to the total input to the *armature* minus the armature-resistance loss.

Example.—A shunt generator when running light as a motor at 1,000 r.p.m. takes 12 amp. from 115-volt mains. The field current is 7 amp. and the armature resistance is 0.03 ohm. What is the stray-power loss of the machine at this particular value of flux and speed?

The armature current $I_a = 12 - 7 = 5$ amp.

The stray power S.P. $= 115 \times 5 - (5)^2 0.03 = 575 - 0.75 = 574$ watts. *Ans.*

It will be observed that the armature $I_a^2R_a$ loss is negligible in this case.

Assume that the above generator is delivering 100 amp. at 110 volts and 1,000 r.p.m. The field current is 7 amp. It is desired to determine the value of the stray power under these conditions.

If the full-load e.m.f. E and speed S be duplicated when the generator is running light, the stray power will be the same as under full-load conditions. When the machine is running light as a motor, the stray power is readily measured as follows: When carrying load as generator, the induced e.m.f.

$$E = 110 + (107 \times 0.03) = 113.2 \text{ volts.}$$
$$S = 1{,}000 \text{ r.p.m.}$$

To make the adjustments of E and S, the generator is run as a motor, connected as in Fig. 400. A rheostat R and an ammeter are connected directly in the armature circuit, and a voltmeter is connected directly across the armature terminals. The rheostat R is first adjusted so that $V_1 = 113.2$ volts, the small armature drop under these load conditions being negligible. The field rheostat is then adjusted to give a speed of 1,000 r.p.m. The machine is now operating at the same value of speed and flux as it did under the given load. Therefore, the stray power is the same in the two cases and is equal to $V_1I_a - I_a^2R_a$.

Example.—Assume that with the connections of Fig. 400 and with the foregoing adjustments, the current I_a is 4.8 amp. and $V_1 = 113.2$ volts. (This neglects the small drop in the armature, 4.8×0.03.) The stray power

$$\text{S.P.} = 113.2 \times 4.8 - (4.8)^2\, 0.03 = 542 \text{ watts.} \quad \textit{Ans.}$$

The efficiency of the generator can now be determined.

$$\begin{aligned}
\text{Output under load} &= 110 \times 100 = 11{,}000 \text{ watts.}\\
I_a^2R_a &= (100 + 7)^2\, 0.03 = 344 \text{ watts.}\\
VI_f &= 110 \times 7 = 770 \text{ watts.}\\
\text{S.P.} &= 542 \text{ watts.}\\
\hline
\text{Total loss} &= 1{,}656 \text{ watts.}
\end{aligned}$$

$$\text{Efficiency} = \frac{11{,}000}{11{,}000 + 1{,}656} = \frac{11{,}000}{12{,}660} = 0.869 \text{ or } 86.9 \text{ per cent.} \quad \textit{Ans.}$$

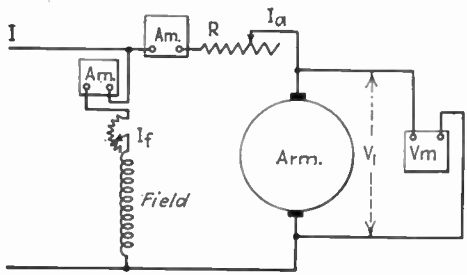


FIG. 400.—Connections for stray-power measurement.

324. Stray-power Curves at Different Field Currents.—It is desired sometimes to determine the stray power of a dynamo over a considerable range of output, in order to have sufficient data to obtain the stray power under various operating conditions. Stray power is a function of two variables, flux and speed, and a single curve cannot express the relation under all conditions. To plot the relation, one quantity, either flux or speed, is held constant and the other is varied. As being more convenient, the flux is usually held constant and the speed is varied, the connections being shown in Fig. 400. The flux is held constant by means of the field rheostat and the speed is varied by means of the rheostat R in the armature circuit.

The flux is a function of field current. Hence, stray power may be considered as a function of field current and speed. This introduces small errors. The flux at no load, in the stray-power test, may be greater than its value when the machine is under load and with the same excitation, because armature reaction tends to reduce the flux.

Therefore, if field current instead of flux is used to determine stray power, the stray power may be too large with the

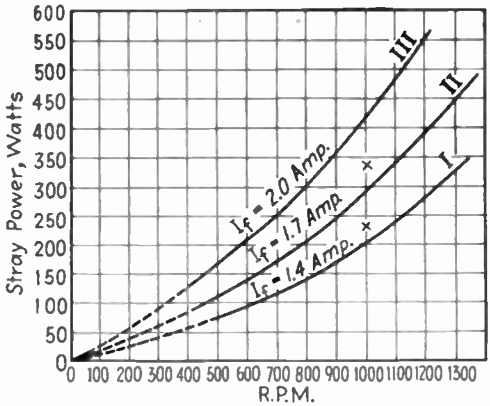


FIG. 401.—Typical stray-power curves.

machine running light, owing to armature reaction. This is in part compensated by the fact that the flux "peaks" under load (see p. 530) so that the loss for any value of total flux is increased, even though the average flux be the same (see Fig. 305, p. 416). Also, due to hysteresis, the flux for a given value of field current may vary, which may cause errors.

In a stray-power run, the field current may be held at a definite value and the speed varied over the probable working range of the machine. The field current may then be adjusted to another value and the run repeated. At least three values of field current should be used, the maximum and the minimum values under which the machine is likely to operate and an intermediate value. In this manner curves similar to those in Fig. 401 are obtained.

Example.—The curves of Fig. 401 were obtained with a 10-kw., 230-volt generator by the method just described, the generator being run light as a motor. The rated current of this generator is 43.5 amp. and the armature resistance is 0.14 ohm.

Determine the efficiency as a generator at half-load and at rated load, the voltage being 230 volts in each case, the values of field current being 1.5 and 1.8 amp. The speed is constant at 1,000 r.p.m.

At half-load,

$$\begin{aligned} I &= 43.5/2 = 21.8 \text{ amp.} \\ I_a &= 21.8 + 1.5 = 23.3 \text{ amp.} \\ I_a^2R_a &= (23.3)^2\, 0.14 = 76 \text{ watts.} \\ VI_f &= 230 \times 1.5 = 345 \text{ watts.} \end{aligned}$$

In Fig. 401, on the 1,000-r.p.m. ordinate, take one-third the distance from curve I to curve II (= 1.5 amp.). The corresponding stray power is 230 watts.

The efficiency at this load is

$$\text{Efficiency} = \frac{230 \times 21.8}{(230 \times 21.8) + 76 + 345 + 230} = \frac{5,000}{5,655} \text{ or } 88.5 \text{ per cent.} \quad Ans.$$

At rated load,

$$\begin{aligned} I &= 43.5 \\ I_a &= 43.5 + 1.8 = 45.3 \text{ amp.} \\ I_a^2R_a &= (45.3)^2\, 0.14 = 287 \text{ watts.} \\ VI_f &= 230 \times 1.8 = 414 \text{ watts.} \end{aligned}$$

In Fig. 401, on the 1,000-r.p.m. ordinate, take one-third the distance from curve II to curve III, (= 1.8 amp.) The corresponding stray power is 330 watts.

$$\text{Efficiency} = \frac{230 \times 43.5}{(230 \times 43.5) + 287 + 414 + 330} = \frac{10,000}{11,030} \text{ or } 90.7 \text{ per cent.} \quad Ans.$$

Assume that it is desired to determine the efficiency of this dynamo when running as a motor at 900 r.p.m. and taking 45 amp. at 230 volts from the line. Under these conditions the field current is found to be 1.6 amp.

$$\begin{aligned} I_a &= 45 - 1.6 = 43.4 \text{ amp.} \\ I_a^2R_a &= (43.4)^2\, 0.14 = 264 \text{ watts.} \\ VI_f &= 230 \times 1.6 = 368 \text{ watts.} \end{aligned}$$

On the 900-r.p.m. ordinate (Fig. 401), take two-thirds the distance from curve I to curve II (= 1.6 amp.). The corresponding stray power is 225 watts.

$$\text{Efficiency} = \frac{(230 \times 45) - 264 - 368 - 225}{230 \times 45} = \frac{9,490}{10,350} \text{ or } 91.6 \text{ per cent.} \quad Ans.$$

325. Stray Power as a Function of Flux and Speed.—As is pointed out in Par. 324, stray power is not strictly speaking a function of the field current. The net flux depends not only on the field current but on armature reaction and hysteresis as well. It is possible, however, to determine stray power as a function of flux only and not of field current.

It is shown that stray power is a function of flux and speed (p. 534). That is,

$$\text{S.P.} = f(\phi, S).$$

It is not a simple matter to measure the flux with ordinary instruments. But from Eq. (212), p. 401, the induced e.m.f. is given by

$$E = K\phi S$$

where K is a constant depending on the winding, number of poles, etc., ϕ is the total flux entering the armature from one N-pole, and S is the speed. Solving for the flux,

$$\phi = \frac{E}{KS}. \tag{239}$$

For any given machine, K is constant. Hence, the stray power is a function of E/S and S; that is,

$$\text{S.P.} = f'\left(\frac{E}{S}, S\right). \tag{240}$$

If the ratio E/S and S are the same for two different operating conditions, the stray power must be the same, neglecting the changes in the shape of the flux-distribution curve (indeterminable losses). The following example illustrates the application of Eq. (240).

Example:—A 20-kw., 220-volt shunt generator, having an armature resistance of 0.096 ohm, is delivering 85 amp. at 222 volts. Its speed is 920 r.p.m. and the field current is 2.24 amp. (*a*) Under what no-load conditions must it be operated in order that its no-load stray power may be equal to that which exists under the given conditions of operation? (*b*) If the armature input at no load is 4.02 amp. at 230.4 volts, what is the value of the stray power? (*c*) Determine the efficiency of the generator under the given operating conditions.

(*a*) $$E = 222 + (85 + 2.24)\,0.096 = 230.4 \text{ volts.}$$

$$\frac{E}{S} = \frac{230.4}{920} = 0.251.$$

Although the dynamo is a generator, it is operated as a motor to determine its stray power. The connections are as in Fig. 400. The *field* rheostat is adjusted until the terminal voltage divided by the speed is equal to 0.251. The armature-resistance drop is negligible at no load. The speed is then adjusted by means of the *armature* rheostat until it is 920 r.p.m. Changing the armature rheostat does not change the flux and hence does not change the ratio E/S.

(*b*) From Eq. (238), the stray power is

$$\text{S.P.} = 230.4 \times 4.02 - (4.02)^2\, 0.096 = 926 \text{ watts.} \quad \textit{Ans.}$$

(Also see point *a*, Fig. 402)

(*c*) Output = 222 × 85 = 18,870 watts.

$$\begin{aligned} I_a{}^2R_a &= (85 + 2.24)^2\, 0.096 &= 730 \text{ watts.}\\ VI_f &= 222 \times 2.24 &= 497 \text{ watts.}\\ \text{S.P.} & &= 926 \text{ watts.}\\ \hline & \text{Total losses} &= 2{,}153 \text{ watts.} \end{aligned}$$

$$\text{Efficiency, } \eta = \frac{18{,}870}{18{,}870 + 2153} \text{ or } 89.8 \text{ per cent.} \quad \textit{Ans.}$$

Owing to the peaking of the flux curve caused by armature reaction (see Fig. 305, p. 416) the stray-power loss will be slightly greater than it is with a flat-top flux curve, even though the total areas under the two curves are the same. This is due to the fact that the stray-power losses are not directly proportional to the flux density but vary to a power higher than the first power.

326. Stray-power Curves with Constant Flux.—Let it be required to plot a series of stray-power curves, similar to those of Fig. 401, covering the maximum probable range of operation of a dynamo as both motor and generator. The stray power is to be plotted as a function of speed for constant values of flux rather than of field current. Three curves are usually sufficient.

Consider the generator of Par. 325. Let it be required to determine three stray-power curves that shall cover the probable operating range of the dynamo as both motor and generator. This range of operation will depend on each particular dynamo and the conditions under which it is to be used. The values given below would apply to an ordinary constant-speed dynamo.

The maximum value of flux occurs when E is a maximum and S is a minimum. E is a maximum for generator operation at overload. Assume 25 per cent overload. The rated current is 91 amp. Assume 3 amp. for the field current.

$$E_1 = 220 + (114 + 3)(0.096) = 231.2 \text{ volts.}$$

The dynamo will probably not operate at less than 900 r.p.m. Hence, the maximum value of flux is determined by

$$\frac{E}{S} = \frac{231.2}{900} = 0.26 \text{ (nearly).}$$

The minimum value of flux occurs when E is a minimum and S is a maximum. E is a minimum under motor operation at overload. Assume 25 per cent overload.

$$E = 220 - 114(0.096) = 209.0 \text{ volts.}$$

Assume that the motor speed will not exceed 1,050 r.p.m. Hence, the minimum value of flux is determined by

$$\frac{E}{S} = \frac{209.0}{1{,}050} = 0.2 \text{ (nearly).}$$

An intermediate value of flux would be determined by $E/S = 0.23$.

To obtain the three curves, the connections of Fig. 400 are used, the machine being operated as a motor in all the stray-power tests. It may be necessary to excite the field from a source greater than 220 volts in order to obtain the maximum value of flux.

The field rheostat is adjusted until $E/S = 0.26$ and a run is made with varying speed. The speed is varied by means of the armature rheostat. Changing this rheostat does not change the flux and hence does not change E/S. The other two curves are obtained in a similar manner, the field rheostat being adjusted to give values of $E/S = 0.23$ and $E/S = 0.20$. A set of curves obtained in this manner is shown in Fig. 402.

In order to determine the efficiency, input, etc., these curves are used in precisely the same manner as those in Fig. 401, as illustrated in Par. 324 (p. 537).

It is also possible to determine the stray power of a dynamo by driving it without load by means of a motor whose efficiency is known. In using this method it is possible to separate the friction and windage losses from the core loss by measuring the power delivered to the dynamo when the field circuit is closed and also when it is open.

Since stray power is a small proportion of the total input of a dynamo, large errors in its determination produce only small errors in the efficiency.

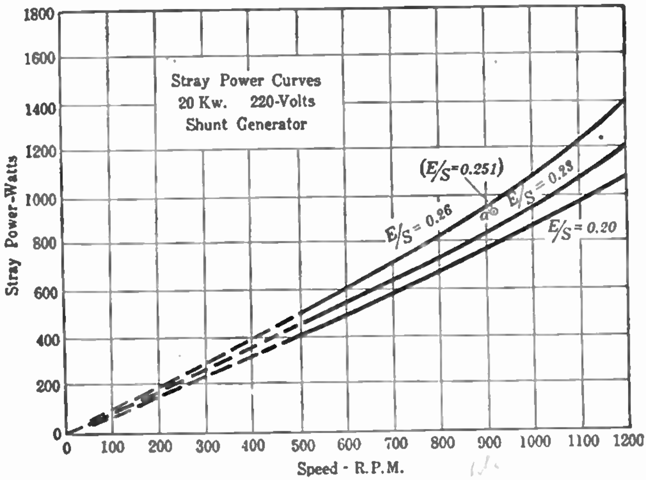


FIG. 402.—Stray power as a function of flux and speed.

327. Opposition Test—Kapp Method.—The objection to the foregoing stray-power methods of measuring losses is that the dynamo is not under load when the losses are being measured, so that their values may be in error. If two similar dynamos are available, their losses may be determined when both dynamos are loaded, and yet the line supplies only the *losses* of the two dynamos. The connections for making such a test are shown in Fig. 403.

The two similar dynamos are coupled together mechanically and are then connected to the line, as shown. The motor should have a starting box. Five ammeters are used, one in each field, one in each armature circuit, and one in the line supplying the two armatures. The fields are connected directly to the line so that their currents are not indicated by the ammeter A_1.

The operation of the set is as follows: The motor supplies mechanical power to the generator. This in turn supplies electrical power to the motor The power delivered by the generator is less than that required by the motor, owing to the losses in the two machines. Therefore, this deficit must be made up by the line which supplies the current I.

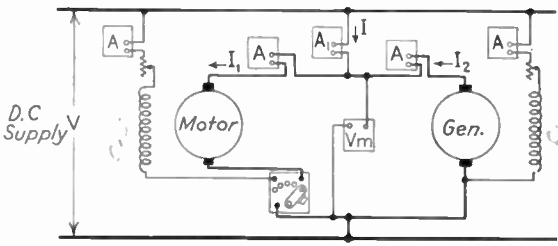


FIG. 403.—Kapp opposition method for determining losses.

The total input to the two armatures is VI.

This power is distributed as follows:

Motor-armature loss $= I_1{}^2R_1$.
Generator-armature loss $= I_2{}^2R_2$.
Motor stray power.
Generator stray power.

R_1 and R_2 are the resistances of motor and generator armature.

As the generator field is necessarily stronger than that of the motor because it requires the higher internal voltage, its stray power will be greater than that of the motor, as stray power increases with increase of flux. As a close approximation, the total stray-power loss may be divided between the two machines in proportion to their induced e.m.fs.

Let E_1 equal the motor induced e.m.f. and E_2 the generator induced e.m.f.

$$E_1 = V - I_1R_1.$$
$$E_2 = V + I_2R_2.$$

Let P_1 and P_2 be the values of stray power in the two machines. Then,

$$\frac{P_1}{P_2} = \frac{E_1}{E_2}. \tag{241}$$

The total input to the two machines goes to supply their armature and stray-power losses, because the output of the system is zero and the field power is supplied separately. By subtracting the armature losses from the input, the total stray power $(P_1 + P_2)$ remains.

That is,

$$P_1 + P_2 = VI - I_1^2R_1 - I_2^2R_2.$$

The field currents are measured directly by the ammeter in each field circuit.

The advantages of this method are that each dynamo is operating under load conditions; the regulation of each dynamo may be determined; the line needs supply only the losses.

The principal disadvantage is that the method requires two similar dynamos. The assumptions made in regard to the stray-power distribution may be slightly in error.

The dynamos are brought into operation by first starting the motor, using the starting box. Then the generator voltage is made equal to the motor terminal voltage and the generator terminals are connected directly across the motor terminals, exactly as generators are connected in parallel. Care should be taken that the polarity is correct. The generator field is strengthened and the motor field weakened until the desired conditions of load and speed are obtained.

Example.—Two similar 120-volt, 7.5-hp. motors are connected in the manner shown in Fig. 403. The armature resistance of each is 0.12 ohm. The fields are adjusted so that the motor current I_1 is 57 amp., and the generator current I_2 is 45 amp. Under these conditions the line is supplying a current I of 12 amp. at 120 volts. Find the stray power of each machine under these conditions.

The power supplied by the line

$$P = 120 \times 12 = 1{,}440 \text{ watts.}$$
$$I_1^2R_1 = 57^2 \times 0.12 = 390 \text{ watts.}$$
$$I_2^2R_2 = 45^2 \times 0.12 = \underline{243} \text{ watts.}$$
$$\text{Total} = 633 \text{ watts.}$$

Total stray power = 1,440 − 633 = 807 watts.

$$E_1 = 120 - (57 \times 0.12) = 113.2 \text{ volts.}$$
$$E_2 = 120 + (45 \times 0.12) = 125.4 \text{ volts.}$$

The motor stray power

$$P_1 = \frac{113.2}{113.2 + 125.4}807 = 383 \text{ watts.} \quad \textit{Ans.}$$

$$P_1 = \frac{E_1}{E_1 + E_2} = W,$$

The generator stray power

$$P_2 = \frac{125.4}{113.2 + 125.4}807 = 424 \text{ watts.} \quad \textit{Ans.}$$

$P_2 = \frac{E_2}{E_1 + E_2} = W_2$

Knowing the stray power and the armature and field losses, the efficiency is readily calculated.

328. Ratings and Heating.—Practically all power apparatus, whether it be steam engines, gas engines, or dynamos, has a definite power rating. The rating is determined by the manufacturer and is supposed to give the power which the apparatus can safely or efficiently deliver under specified conditions. It is interesting to consider what, in general, determines the rating of various power devices.

Both a steam engine and a steam turbine are usually rated at the load for which their *efficiency* is a maximum. These two types of prime mover can carry a large overload without difficulty. Ordinarily, they can carry easily 100 per cent overload but at reduced efficiency.

Owing to the excessive weights and costs, large gas engines are usually rated as high as possible, which is near the point at which they cease to operate. Their thermal efficiency is ordinarily so much greater than that of the steam engine or turbine that the question of weight is more important than the question of efficiency.

Electrical apparatus is usually rated at the load which it can carry without *overheating*. *Commutation* may at times limit the output of direct-current machines.

If the temperature of electrical apparatus becomes too high, the cotton insulation on armature and field conductors, and the insulating varnishes, become carbonized and brittle. This may result ultimately in grounds and short circuits within the machine. The A.I.E.E. Standardization Rules[1] specify insulations with reference to their safe temperature limits when embodied in machines, as follows:

Class O Insulation.—Cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in oil.

[1] "Direct-current Rotating Machines, Generators, and Motors," A.I.E.E. Standards, No. 5, July, 1925.

Class A Insulation.—Cotton, silk, paper, and similar organic materials when impregnated or immersed in oil; also enamel as applied to conductors.

Class B Insulation.—Inorganic materials such as mica and asbestos in built-up form combined with binding substances.

The A.I.E.E. has drawn up the following specifications as to the upper limits of operating temperatures of the important parts of direct-current machines.

Limiting Temperature Rise for Machines Having Continuous and Short-time Ratings.—The temperature rise of each of the various parts, (of d.-c. machines) above the temperature of the cooling medium when tested in accordance with the rating, shall not exceed the values given in the following table. All temperatures shall be determined by the *thermometer* method.

TABLE II

Item		Limiting-temperature rise, degrees centigrade	
		Class *A* insulation	Class *B* insulation
1	Armature windings, wire field windings and all windings other than 2	55	75
2	Single-layer field windings with exposed uninsulated surfaces and bare copper windings	65	85
3	Cores and mechanical parts in contact with or adjacent to insulation	55	75
4	Commutators and collector rings	65	85
5	Miscellaneous parts (such as brush holders, brushes, pole tips, etc.) other than those whose temperatures affect the temperature of the insulating material may attain such temperatures as will not be injurious. (Agreement on the limiting-temperature rise of class *O* insulation has not been reached.)		

The method of measuring the temperatures is given in Rule 5-156 as follows:

This method (thermometer) consists in the determination of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermo-couples, any of these instruments being applied to the hottest part of the machine accessible to mercury or alcohol thermometers.

Although this method gives the temperature at the surface and not at the "hottest spot," the values of limiting temperature given in Table II presumably take this factor into consideration.

329. Temperature Measurement by Resistance Method.—Although the A.I.E.E. Standards Rules no longer specify that the temperature rise in the windings of direct-current motors and generators be determined by the change in resistance of the windings, yet the method is valuable in that it is convenient and gives temperature data not obtainable by the thermometer methods.

It has already been shown that the resistance of copper conductors changes with the temperature. By utilizing this principle, the average temperature within a winding may be very closely determined. For copper, the increase of resistance per degree centigrade rise of temperature may be obtained from the formula $1/(234.5 + t)$,[1] where t is the surrounding or ambient temperature. For example, at an ambient or room temperature of 30°C., the increase of resistance per degree rise is $1/264.5 = 0.00378$.

Example.—With an ambient temperature of 30°C., the resistance of the field of a shunt generator increases from 104 to 112 ohms. What is its temperature rise?

$$\text{The fractional change in resistance} = \frac{112 - 104}{104} = 0.077.$$

$$\text{Temperature } \textit{rise} = 0.077/0.00378 = 20.4°\text{C.} \quad \textit{Ans.}$$

Owing to the time required to reach a constant temperature, motors and generators should be run from 6 to 18 hr. in order that an accurate test of their temperature may be made. As such a long time is usually prohibitive, the heating is often accelerated by running overload for an hour or so and then

[1] See Par. 19, p. 19.

dropping back to rated load. By this procedure a very good idea of the ultimate temperature may often be obtained in a run of 2 or 3 hr.

In order to obtain an idea as to how close a machine is to its ultimate temperature, it is often desirable to plot a curve of temperature rise during the test. A typical curve for a shunt field is shown in Fig. 404.[1] At the beginning of the test, there is but a slight difference of temperature between the field coils and the room. Therefore, but a small amount of heat is given out by the coils and as a result the temperature rises rapidly. As the difference between the coil temperature and the room temperature increases, more and more heat is given out by the coils, and the temperature rises less rapidly. Therefore, the rate of temperature increase becomes less as the time increases. This is illustrated by the curve of Fig. 404. When the curve becomes practically horizontal, the total heat developed in the coils is equal to the heat dissipated by the coils and the coils have reached a constant temperature. Similar curves would hold for other parts of the machine.

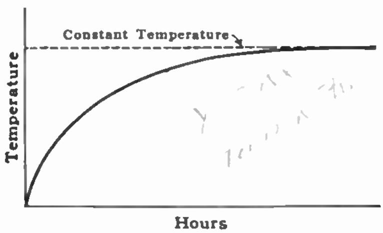


FIG. 404.—Curve of temperature rise with time, for a dynamo.

Care must be taken in measuring the armature resistance when determining temperature rise. The object of this measurement is not to determine the resistance for use in the calculation of loss, but to determine the *change of resistance in the armature copper, due to change of temperature.* Therefore, it is essential that the resistance of the *copper alone* be measured and that the current path through the copper be the same in every measurement. To exclude all resistance except that of the copper, the brush and contact resistances must not be included in the measurement. Therefore, the voltmeter leads must be held on the commutator segments inside the brushes, as in Fig. 405(*a*). Moreover, these segments should be marked, and in every subsequent measurement they should be directly under the same

[1] This curve is identical in character with the curve giving the rise of current with time in an inductive circuit (see Fig. 209, p. 296).

brushes. This insures the same conducting path for each measurement.

When a multipolar armature is thus measured, the *division* of current in the various paths is determined in part by the brush contact resistance. Thus in Fig. 405(*b*), the current from brush *a* to brush *b* is I_1 and that from brush *a* to brush *c* is I_2. The

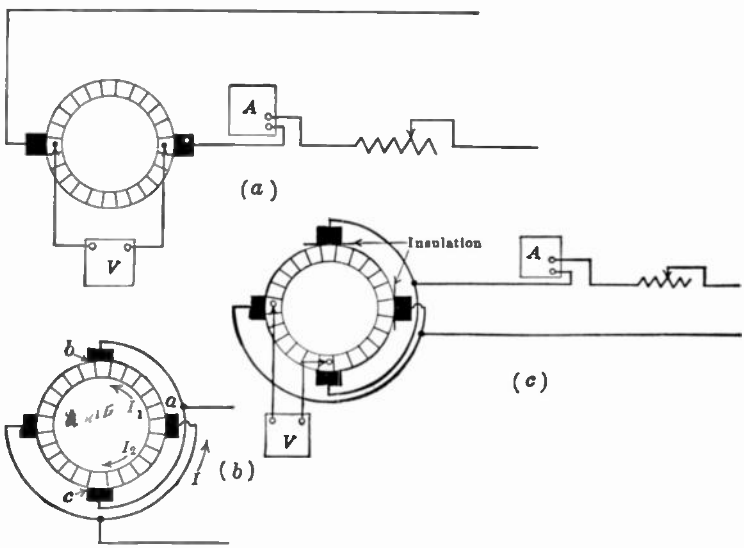


FIG. 405.—Measurement of armature resistance for temperature test.

total current entering the brush *a* is their sum, *I* amp. The division of the current *I* between brushes *b* and *c* is in part determined by the contact resistance at these two brushes. As contact resistance is a variable quantity, the current *division* in the armature may change considerably with different measurements. To keep the current in definite paths, two brushes may be insulated as in Fig. 405(*c*). In this case the current paths are not symmetrical, but the division of current is determined by the copper resistance itself and not by brush contact resistance.

In measuring the shunt-field resistance, the voltmeter should be connected directly across the winding so as to exclude the drop in the field rheostat.

This resistance method gives an average value of the temperature of the windings. To find the "hottest spot" temperature, something like 10°C. should be added.

In addition to measuring the temperature of the windings, the rise of temperature of bearings and of commutator should be measured with a thermometer.

In many modern dynamos, thermocouples are inserted in the windings and are connected to millivoltmeters on the switchboard, so that the operator can determine the "hot-spot" temperatures at any time. (Also see Par. 307, p. 510.)

330. Parallel Operation of Shunt Generators.—In most power plants it is necessary or at least desirable that the power be supplied by several small units rather than by a single large unit.

(*a*) Several small units are more reliable than a single large unit, for if one of the small units is disabled the entire power supply is not cut off. (*b*) The units may be connected in service and taken out of service to correspond with the load on the station. This keeps the units loaded up to their rated capacity and increases the efficiency of operation. (*c*) A unit can be repaired more readily if there are several in the station. (*d*) Additional units may be installed to correspond with the growth of station load. (*e*) The station load may exceed the capacity of any single available unit.

Shunt generators, because of their drooping characteristic, are particularly well suited for parallel operation. In Fig. 406 are shown the characteristics of two shunt generators, designated as No. 1 and No. 2. It will be noted that generator 1 has the more drooping characteristic. If the two generators are connected in parallel (Fig. 407), their terminal voltages must be the same, neglecting any very small voltage drop in the connecting leads. Therefore, for a common terminal voltage V_1 (Fig. 406), generator 1 delivers I_1 amp. and generator 2 delivers I_2 amp. That is, the generator with the more drooping characteristic carries the smaller load.

Assume that some condition arises which temporarily causes generator 1 to take more than its share of the load. This condition might arise from a temporary increase in the speed of its prime mover, or it might be occasioned by a momentary change of load on the system. If the increased current persisted after normal conditions were restored, generator 1 would tend to operate at some point *a* on its characteristic. This results in a drop in its terminal voltage, which tends to make it take *less* load. Moreover, at the time that generator 1 takes more load, generator 2 must take less load, the total load remaining constant, and accordingly generator 2 will tend to operate at some point *b* on its characteristic. This will raise its terminal voltage and cause it to take more load. Therefore, any tendency of one generator to take more than its share of the load results in changes

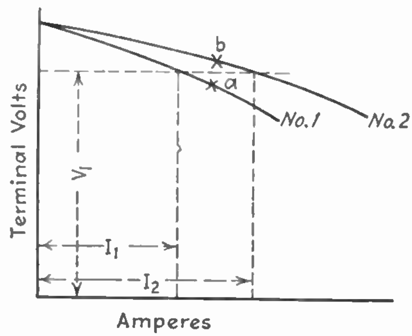


FIG. 406.—Characteristics of shunt generators in parallel.

of voltage in the system which oppose this tendency. Hence, shunt generators in parallel may be said to be in *stable* equilibrium. The reactions of the system are such as to hold the generators in parallel. Moreover, if any *change* of load on the system occurs, each generator must supply some of the increase or decrease of load.

331. Connections of Shunt Generators in Parallel.—The connections for operating shunt generators in parallel are shown in Fig. 407. Each generator should have its own ammeter. A common voltmeter is sufficient for all the generators. The individual generators can be connected to the voltmeter or potential bus through suitable plug connectors or selective switches. Assume that generator 2 is out of service and that generator 1 is supplying the load. It is desired to put No. 2 in service. The prime mover of No. 2 is started and the generator is brought up to speed. Its field is then adjusted so that its voltage is just equal to that of the bus-bars, which condition may be determined by the voltmeter. The breaker and switch are now closed and No. 2 is connected to the system. Under these conditions, however, it is not taking any load, as its *induced* e.m.f. is just equal to the bus-bar voltage, and no current flows between points at the same potential. That is, the generator is "floating" on the bus-bars. Its induced e.m.f. must be greater than the voltage

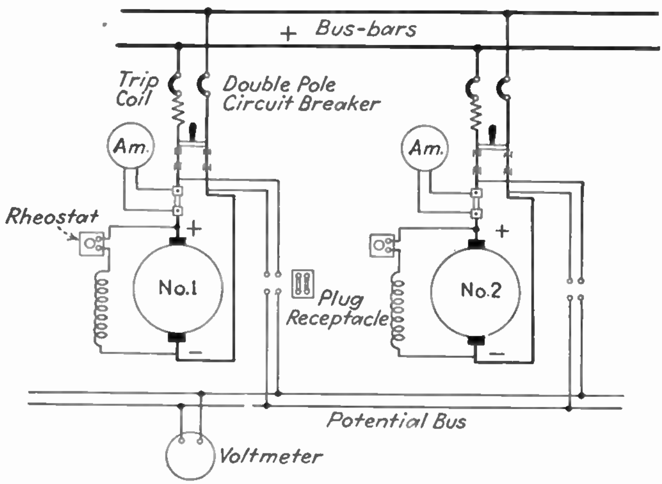


FIG. 407.—Connections for parallel operation of shunt generators.

of the bus-bars in order that it may deliver current. Therefore, the field of No. 2 is strengthened until the generator takes its share of the load. It may be necessary to weaken the field of generator 1 at the same time in order to maintain constant the bus-bar voltage.

To take a generator out of service, its field is weakened and that of the other generator is strengthened until the load of the first generator is zero. The breaker and then the switch are opened, clearing the generator. Connecting in and removing a generator from service in this manner prevents any shocks or disturbance to the prime mover or to the system.

If the field of one generator be weakened too much, power will be delivered to this generator, which will then run as a motor and tend to drive its prime mover.

It is evident that if shunt generators are to divide the load properly at all points, *their characteristics must be similar in form and each must have the same voltage drop from no load to full load.*

332. Parallel Operation of Compound Generators.—Figure 408 shows two overcompounded generators connected to bus-bars, positive and negative terminals being properly connected as regards polarity. Each generator is taking its proper share of the load.

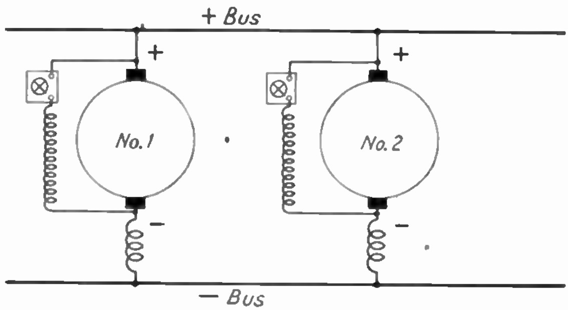


FIG. 408.—Compound generators in parallel.

Assume that for some reason generator 1 takes a slightly increased load. The current in its series winding must increase, which strengthens its field and raises its e.m.f., thus causing it to take still more load. On the other hand, as the system load is assumed to be fixed, generator 2 will at the same time drop some of its load, resulting in a weakening of its series field and a consequent further dropping of its load. In a very short time No. 1 will be driving No. 2 as a motor, and almost immediately the breaker of at least one of the generators will open.

This condition is also illustrated by Fig. 409, which shows the individual characteristics of the two generators. Assume that the generators are operating at voltage V_1, which corresponds to currents I_1 and I_2. Assume that generator 1 takes a slightly increased load. Its voltage will then tend to rise to some point *a*. This increased voltage means that the generator takes still more current and the effect will continue until ultimately a breaker opens.

These compound generators may be considered as in *unstable* equilibrium. That is, any action tending to throw the generators out of equilibrium is accentuated by the resulting reactions.

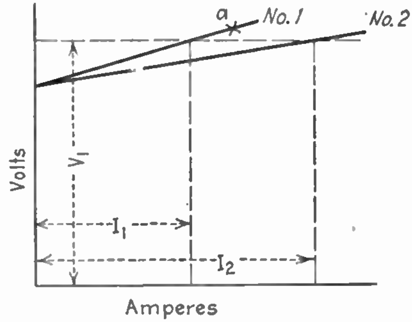


FIG. 409.—Characteristics of compound generators in parallel.

The generators may be made stable by connecting the series fields in parallel (Fig. 410). This connection, which in Fig. 410 ties the two negative brushes together, is a conductor of low

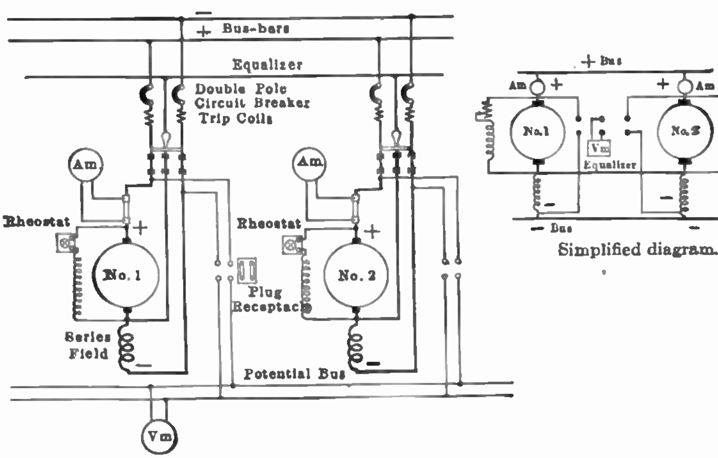


FIG. 410.—Typical connections for two compound generators operating in parallel.

resistance and is called the *equalizer*. Its operation is as follows: Assume that generator 1 starts to take more than its proper share of the load. The increased current will flow in part through the series field of generator 1 and also, by means of the equalizer, will flow in part through the series field of generator 2. Therefore, both generators are affected in a similar manner and No. 1 cannot take the entire load.

To maintain the proper division of load from no load to full load, the following conditions must be satisfied:

a. The regulation of each armature must be the same.

b. The series-field resistances must be inversely proportional to the generator ratings.

It is not always possible to adjust compound-generator characteristics by means of series-field diverters so that the generators divide the load properly. Assume (Fig. 410) that the series field of generator 1 is shunted by a diverter. If the equalizer and bus-bar have negligible resistance, this diverter shunts the series field of generator 2 as well as that of generator 1. Therefore, the diverter merely drops the characteristic of the entire system but does not affect the *division* of load. The proper load adjustments may be made by means of a very low resistance *in series* with one of the series fields.

It should be noted that the desired division of load among either shunt or compound generators for any given external load may be obtained by adjusting their shunt-field rheostats. However, it is usually desirable that this division remain constant at all loads, especially if an operator is not in continuous attendance. Therefore, it is desirable that shunt and compound generators operating in parallel shall have similar characteristics.

A compound generator with a single series field usually has a three-pole switch, one blade of which connects the equalizer, as in Fig. 410. If a three-wire generator (see p. 573) having two series fields is to be connected, a four-pole switch is necessary, as there are two equalizers (see Fig. 411). The load ammeter in a compound generator should always be connected between the *armature* terminal and the bus-bars (Fig. 411). If it is connected in the series-field circuit, the ammeter may not indicate the generator current, due to the fact that some of the generator current may be flowing through the equalizer.

Compound generators are put in service and taken out of service in the same manner as shunt generators, that is, the load is adjusted and shifted by means of the shunt-field rheostat.

333. Circuit-breakers.—Circuit-breakers are in many ways superior to fuses in protecting electrical circuits and apparatus from overloads and short circuits. The circuit-breaker is far more flexible, particularly as its setting may be easily adjusted. A time element and an undervoltage trip may be added. The delay incident to replacing blown fuses does not occur with the circuit-breaker. Moreover, the circuit-breaker may be opened and closed by remote control.

Two new features have been added to modern circuit-breakers, namely, reclosing action and tripping in accordance with the rate of current rise.

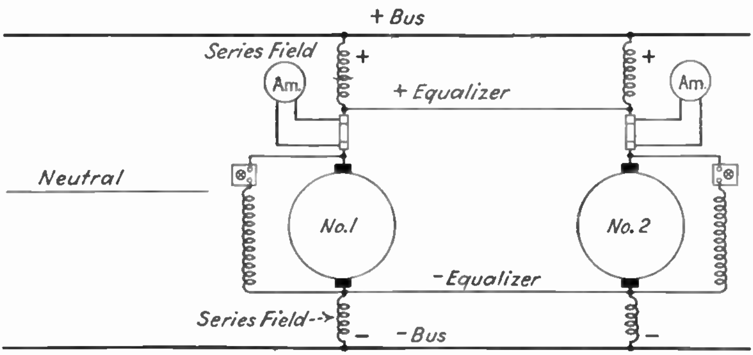


Fig. 411.—Compound generators requiring two equalizers (neutral connection to generators omitted).

Many times circuit-breakers open on a short circuit or overload which is cleared a moment or so later, frequently by the opening of a local breaker or fuse. In order that service shall not continue to be interrupted unnecessarily, automatically reclosing circuit-breakers have been developed. After tripping, a mechanism operates to reclose the breaker. If the short circuit still exists, the breaker cannot reclose. If the short circuit has been cleared, the breaker recloses and restores service. The breaker may attempt to reclose two or three times and if the fault is not then cleared, the breaker becomes permanently locked out and can be closed only by hand.

In the *rate-of-current-rise* feature, the breaker tripping mechanism anticipates in a way the short-circuit current. The more nearly a fault approaches a dead short circuit, the more rapidly the circuit-breaker should open to clear the system. The method used by the I.T.E. Circuit Breaker Company is shown in Fig. 412. The bottom current lead of each pole is divided into two parallel branches. The current in the lower or non-inductive branch links with and actuates the trip-magnet armature. The upper branch is made highly inductive by surrounding it with iron laminations (see Section A-A, Fig. 412). With a slowly

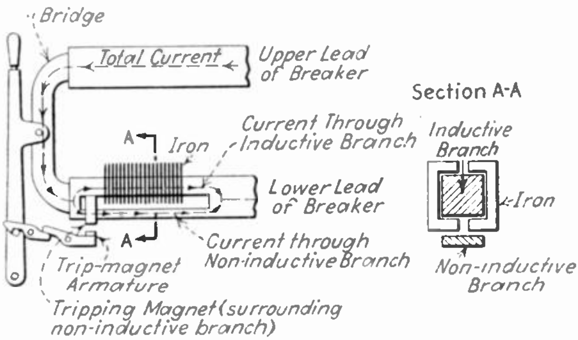


FIG. 412.—Rate of current-rise circuit breaker. (*I.T.E. Circuit Breaker Company.*)

rising current, the currents in the two branches divide almost inversely as the resistances, and the breaker trips according to the normal calibration. However, if a fault is of such a character that the current rises with great rapidity, current into the inductive branch is choked, and a much larger proportion of the total current is then forced into the lower or tripping branch, causing the breaker to trip for a lower value of total current. It follows that the greater the rate at which the current rises, the lower the value of the total current at which the breaker trips. Hence, the breaker may trip at the beginning of trouble rather than when the trouble becomes fully developed.

CHAPTER XV

TRANSMISSION AND DISTRIBUTION OF POWER

334. Power-distribution Systems.—Under modern conditions, most central stations generate power on a large scale as alternating current and transmit this power as alternating current. The reason for using alternating current in transmitting the power is that the voltage may be efficiently raised and lowered by means of transformers.[1] Much less copper is required to transmit power at high voltages. The Thury system does transmit power as direct current at high voltage (see Par. 282, p. 461), but this system is not used in the United States.

Power is utilized ordinarily at comparatively low voltages (115, 230, 575 volts), but it cannot be economically transmitted to any considerable distance at these voltages. In fact, direct current for commercial use can be economically transmitted and distributed only in the congested districts of large cities. Some of its advantages for city distribution are that a storage-battery reserve can be maintained readily, which is a very important consideration; direct-current motors are admirably suited to elevators and printing presses, which form an important part of the power load in a city. The absence of inductive and capacitive effects, which are present with alternating current, and the absence of eddy-current losses in cables are further considerations. Figure 413 shows the general method of power distribution. Power is generated at the power station, is transmitted as alternating current at high voltage to the substation (66,000 volts is shown; the transmission voltage is seldom less than 6,600 volts). At the substation it is either transformed to 2,300 volts or 4,000 volts alternating current by transformers, or to 575 volts or 230 volts direct current by motor-generator sets, synchronous converters, or mercury-arc rectifiers. Figure 413 shows the substation supplying a trolley with 600 volts direct current; a

[1] Also see Vol. II, Par. 1, p. 1.

2,300-volt alternating-current circuit supplies power for lighting, the voltage being transformed on or near the consumer's premises into a 230/115-volt three-wire system; a three-phase, 2,300-volt, alternating-current power line supplies a factory, the voltage being transformed to 550 volts, three-phase, by transformers. These systems are discussed more fully in Chap. XIII, Vol. II. The substation receives the power in large amounts and distributes it to the various consumers in smaller amounts. It bears the same general relation to the power system as the middleman and retailer do to an industrial system.

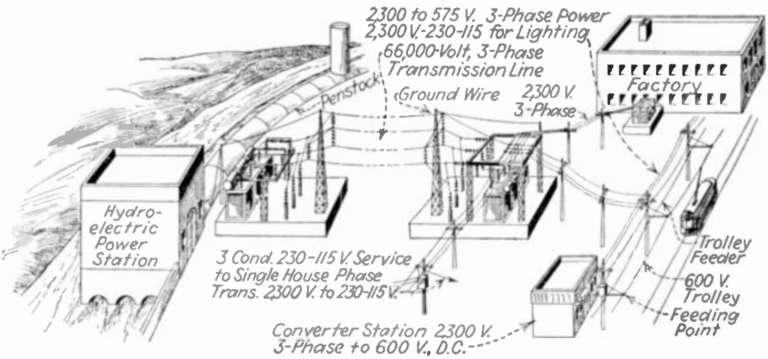


FIG. 413.—Typical power system.

335. Voltage and Weight of Conductor.—*The weight of conductor varies inversely as the square of the voltage*, when the power transmitted, the distance, and the loss are fixed.

Let it be required to transmit power P at voltage V_1 and current I_1 over conductors having resistance R_1.

The current

$$I_1 = \frac{P}{V_1}.$$

The power loss

$$P_1 = I_1^2 R_1.$$

Assume that the voltage is raised to V_2, power, distance, and loss remaining fixed.

The current

$$I_2 = \frac{P}{V_2}.$$

The power loss

$$P_2 = I_2^2R_2 = P_1.$$

Therefore,

$$I_1^2R_1 = I_2^2R_2.$$

$$\frac{R_1}{R_2} = \left(\frac{I_2}{I_1}\right)^2 = \frac{(P/V_2)^2}{(P/V_1)^2} = \frac{V_1^2}{V_2^2}. \tag{242}$$

That is, the conductor resistance varies *directly* as the square of the voltage. But the volume or the weight of a conductor of given length varies *inversely* as the resistance.

Let the weight of copper in the two cases be W_1 and W_2.

$$\frac{W_1}{W_2} = \frac{V_2^2}{V_1^2}. \tag{243}$$

Therefore, the conductor weight varies inversely as the square of the voltage, when power, distance, and loss are fixed.

If the voltage of a system is *doubled*, the weight of the copper is *quartered*, other conditions being the same.

Example.—50 kw. are delivered at a distance of 500 ft. at 110 volts over a 400,000-cir.-mil feeder. (*a*) What is the power loss? (*b*) Repeat for 220 volts.

(*a*) The current

$$I_1 = \frac{50{,}000}{110} = 454 \text{ amp.}$$

If the cable had 454,000 cir. mils (see Par. 44, p. 53), the loss would be 454,000 × 1,000 × 10^{-5} watts = 4,540 watts. Actually the loss is

$$\left(\frac{454}{400}\right)^2 \times 1{,}000 \times 10^{-5} \times 400{,}000 = 5{,}150 \text{ watts.} \quad \textit{Ans.}$$

(*b*) $I_2 = \frac{50{,}000}{220} = 227$ amp.

The loss is

$$\left(\frac{227}{400}\right)^2 \times 1{,}000 \times 10^{-5} \times 400{,}000 = 1{,}290 \text{ watts.} \quad \textit{Ans.}$$

The loss in (*b*) is one-fourth that in (*a*). Therefore, a 100,000-cir.-mil feeder, having just one-fourth the weight of the feeder in (*a*), would transmit the same power the same distance with the *same loss*.

336. Size of Conductors.—In transmitting or distributing power by direct current, four factors must be considered in determining the size of conductor.

1. The wires must be able to carry the required current without overheating.

This is particularly important with inside wiring where fire risk exists. Tables of the permissible current-carrying capacity of wires are given in the Appendices F and H (pp. 594 and 596).

2. The voltage drop to the load must be kept within reasonable limits. This is particularly important when incandescent lamps constitute the load.

3. The wires must be of sufficient mechanical strength. This is important when the wires are strung on poles. It is not advisable to use wires smaller than No. 8 A.W.G. for pole lines.

4. The economics of the problem must be considered. Increasing the size of conductor means higher investment costs but less energy loss in transmission. That size of conductor should be chosen which makes the cost of the energy loss plus the interest on the investment a minimum. This may be modified in view of the considerations stated in 1, 2, and 3.

CONSTANT-POTENTIAL DISTRIBUTION

337. Distribution Voltages.—About 110 volts has been found to be the most convenient voltage for incandescent lighting. It is not so high as to be dangerous to persons. Incandescent-lamp filaments for voltages much in excess of 110 volts become so long and of so small a cross-section that they are fragile. An even lower voltage than this would be desirable from the standpoint of the filament, but a lower voltage would be accompanied by an increase in the required weight of copper. Therefore, 110 to 115 volts has been standardized for lighting and for domestic use as being the most desirable when all factors are taken into consideration. Six hundred volts is commonly used for trolley distribution, because it is not so high as to give operating difficulties and saves considerable copper as compared with systems of lower voltage. At the present time, 1,200, 2,400, and even 3,000 volts direct current are used at the trolley in railway electrification, these higher voltages being for trunk-line electrification, not for municipal traction.

338. Distributed Loads.—The load on a feeder or main may be concentrated at one or two points, as is generally the case with feeders, or it may be distributed uniformly or non-uniformly along the conductors, as when lamp loads are located at various points along mains (see Fig. 414).

The conductors may be of uniform cross-section throughout their entire length (Fig. 414(*a*)). This system is used where the mains are short and the voltage drop is small.

The minimum amount of copper for a given voltage drop is obtained when the mains are uniformly tapered (Fig. 414(*b*)).

As it is impracticable to have a uniformly tapering conductor, a conductor of constant cross-section is run for a part of the

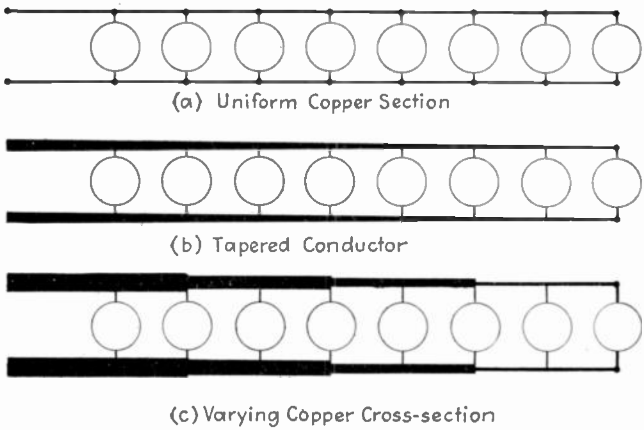


FIG. 414.—Copper cross-section of distributing system or of mains.

distance, followed by another uniform conductor of lesser cross-section, and so on, as shown in Fig. 414(*c*). A good rule to remember is that the current *density* in each section should be the same. For example, the first section may consist of a 250,000-cir.-mil conductor, carrying 200 amp.; assume the second section carries 150 amp.; it should be a $^{150}/_{200} \times 250{,}000 = 190{,}000$-cir.-mil conductor. Ordinarily 4/0 wire would be used for this second section.

339. Systems of Feeding.—In order to keep a number of lamps at the same voltage without excessive copper, the *return-loop* or *anti-parallel* system (Fig. 415(*a*)) is often used. The two feeding wires are connected to opposite ends of the load. This system allows all the lamps to operate at nearly the same voltage, and yet the voltage drop in the feeding wires may be large.

The objection to the return-loop system is the extra length of wire required. This objection is often overcome by arranging the loads in the manner shown in Fig. 415(*b*), the *open-spiral* system. Where large groups of lamps are switched off and on

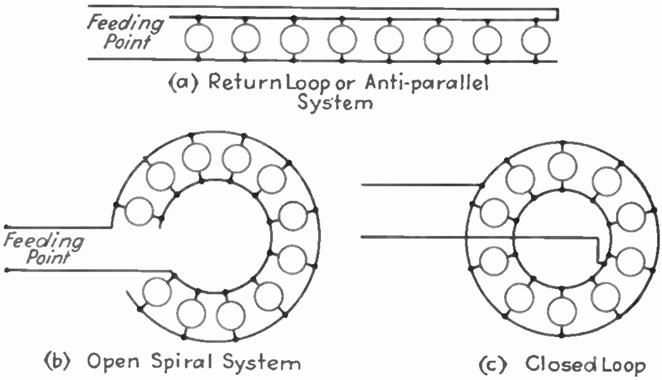


Fig. 415.—Systems of feeding.

at the same time, as in theaters and auditoriums, it is often possible to arrange the lamps in this way.

The open spiral may be closed at its ends, resulting in the *closed-loop* system of Fig. 415(*c*).

340. Series-parallel System.—Doubling the voltage of a system results in reducing the weight of required copper to one-fourth its initial value. If 110-volt lamps be arranged so that two are always in series, as in Fig. 416, the system may be operated at 220 volts. The copper section will then be one-fourth that required for straight 110-volt distribution. The

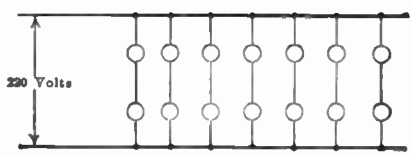


Fig. 416.—Series-parallel system.

obvious disadvantages of the series-parallel system are that lamps can be switched only in groups of two, and if one lamp burns out, the lamp to which it is connected ceases to operate. Also, both of the lamps in series must be of the same voltage and current rating. It is obvious that this system is wholly unsuited to the usual 110-volt appliances.

THE EDISON THREE-WIRE SYSTEM

341. Advantages of Edison Three-wire System.—The objections to the series-parallel system may be eliminated by running a third wire, a *neutral,* between the two outer wires, a method invented by Edison. This neutral maintains all the lamps at approximately 110 volts. The advantage of a higher voltage in reducing the weight of copper is secured by the use of this system. If there were no neutral wire, the 220-volt system would require one-fourth the copper of an equivalent 110-volt system. If it be assumed that the neutral of the Edison system is of the same cross-section as the two outer wires, the total copper for the Edison system is $\frac{3}{8}$, or $37\frac{1}{2}$ per cent of that for a 110-volt system of the same kilowatt capacity. Therefore the saving in copper is $62\frac{1}{2}$ per cent. In practice, the neutral can be made smaller than the two outer wires so that the saving in copper is even greater than $62\frac{1}{2}$ per cent.

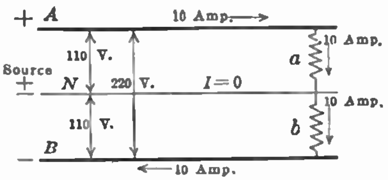


FIG. 417.—Edison three-wire system—balanced loads.

The general plan of the system under balanced conditions is shown in Fig. 417. Two outer wires A and B have 220 volts maintained between them, A being the positive and B the negative. A third wire N is maintained at a difference of potential of 110 volts from each of the two outer wires. Hence, N must be negative with respect to A and positive with respect to B. That is, current tends to flow from A to N, and from N to B.

Another advantage of the system is that two values of voltage are available. The 110 volts is better adapted to lamps and appliances, whereas 220 volts is better adapted to motors.

342. Balanced and Unbalanced Loads.—Figure 417 shows the conditions which exist when the load on each side of the system is the same. Each of the loads a and b takes 10 amp. The 10 amp. taken by load a passes through to load b and then back through wire B to the source. This is equivalent to a series-parallel system as both loads are equal and are in series. Under these conditions the loads are balanced and the current in the neutral wire is zero.

Figure 418(*a*) shows the conditions existing when the load *a* on the positive side of the system is 10 amp., and the load *b* on the negative side is 5 amp. Under these conditions the extra 5 amp. taken by load *a* must *flow back* through the neutral to the generator or source. Therefore there are 5 amp. returning to the generator in the neutral.

In Fig. 418(*b*) the load *b* is 10 amp., and load *a* is 5 amp. Under these conditions the extra 5 amp. must *flow out* to the load through the neutral. It will be observed that the current in the

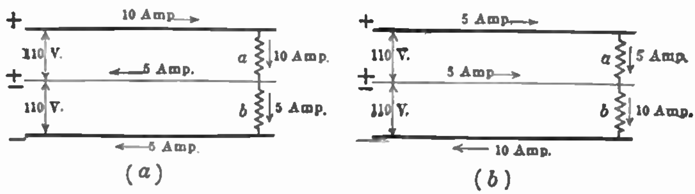


FIG. 418.—Unbalanced three-wire systems.

neutral may flow in either direction, depending on which load is the greater. Therefore, if an ammeter is used in a neutral, it should be of the zero-center type. Moreover, it will be observed that the neutral carries the *difference* of the currents taken by the two loads. In practice the loads are usually so disposed that they are nearly balanced. Ten per cent unbalancing, that is, a neutral current which is 10 per cent that in the outer wires, is usually allowed in the larger systems, while in a system of small capacity the unbalance may at times reach 25 per cent. The current in the neutral is ordinarily much smaller than that in the outer wires and a much smaller conductor can be used safely. In practice the neutral is usually grounded.

343. Effect of Opening Neutral.—In practice, it is very desirable to keep the neutral of the three-wire system closed under *all* conditions. The reason for this is illustrated by the following example.

Figure 419 shows two lamp loads on a three-wire system. The load on the positive side consists of six lamps each taking 2 amp., making a total of 12 amp. The load on the negative side consists of four lamps each taking 2 amp., making this load 8 amp. The voltage across each load is 110 volts so that the resistance R_1 of the positive load is

$$R_1 = \frac{110}{12} = 9.17 \text{ ohms.}$$

The resistance of the negative load is

$$R_2 = \frac{110}{8} = 13.75 \text{ ohms.}$$

If the neutral be opened at the point S, the two loads R_1 and R_2 are in series and must take the same current. The total resistance $R = R_1 + R_2 = 22.92$ ohms.

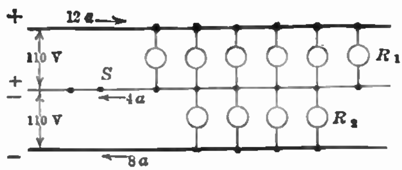


FIG. 419.—Effect on the balance of three-wire system of opening the neutral.

There is now 220 volts across these two loads in series, so that the current

$$I = \frac{220}{22.92} = 9.60 \text{ amp.}$$

The voltage across load R_1 is

$$V_1 = 9.60 \times 9.17 = 88.0 \text{ volts.}$$

The voltage across load R_2 is

$$V_2 = 9.60 \times 13.75 = 132 \text{ volts.}$$

This assumes that the resistance of the lamp filaments does not change. It will be observed, however, that the larger bank of lamps is operating at a much reduced voltage, resulting in a substantial decrease of candlepower, and that the smaller bank is operating considerably above rated voltage, which would result in the burning out of the lamps in a short time.

For this reason the neutral of the three-wire system is almost always grounded, and when circuit-breakers are used, there is rarely a pole in the neutral wire. Also, in modern building and house wiring, no fuses are used in the neutral wire.

344. Voltage Unbalance.—The voltage on the two sides of a three-wire system may be considerably unbalanced if the loads on the two sides of the system are unequal.

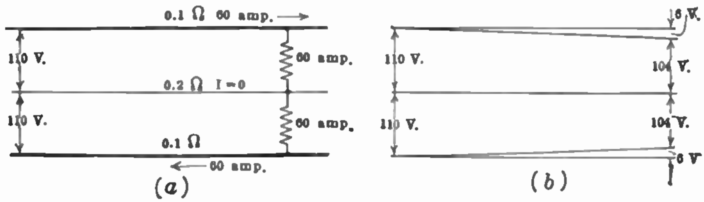


FIG. 420.—Voltage drop in three-wire system having balanced loads.

In Fig. 420(a) a load of 60 amp. exists on each side of the system. The resistance of each outer wire is 0.1 ohm and of the neutral is 0.2 ohm. The applied voltage is 220 volts across the two outer wires.

As the two loads are equal, there is no current in the neutral wire. Therefore, the voltage drop per wire for the outers is

$$e = 60 \times 0.1 = 6.0 \text{ volts.}$$

The voltage across each load is 104 volts. There is no voltage drop along the neutral, as it carries no current. Figure 420(b) shows a plot of the voltage distribution.

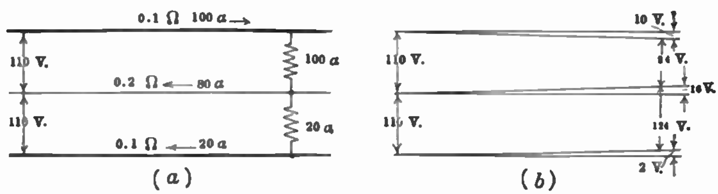


FIG. 421.—Voltage unbalance in three-wire system having unbalanced loads.

Assume that the loads are 100 amp. on the positive side of the system and 20 amp. on the negative side (Fig. 421). This represents the same total amperes as in Fig. 420.

The drop in the positive wire

$$e_1 = 100 \times 0.1 = 10 \text{ volts.}$$

The drop in the neutral

$$e_2 = 80 \times 0.2 = 16 \text{ volts.}$$

Voltage across positive load

$$V_1 = 110 - 10 - 16 = 84 \text{ volts.} \quad \textit{Ans.}$$

The drop in the negative wire

$$e_2 = 20 \times 0.1 = 2 \text{ volts.}$$

Voltage across negative load

$$V_2 = 110 - 2 + 16 = 124 \text{ volts.} \quad \textit{Ans.}$$

There is now 40 volts difference between the voltages on the two sides of the system.

Under these conditions, the voltage across the load on the negative side is *greater* than the voltage on the negative side of the system at the sending end or power source. This rise in voltage from power source to load is due to the direction of the drop in the neutral. Figure 421(*b*) shows these conditions graphically.

When motor loads are to be connected to a three-wire system, they are usually connected between the two outer wires rather than between an outer wire and neutral in order that they shall not produce any voltage unbalancing. In fact some power companies will not permit motor loads exceeding 1 hp. to be connected to neutral.

METHODS OF OBTAINING THREE-WIRE SYSTEM

There are several methods of obtaining a three-wire system.

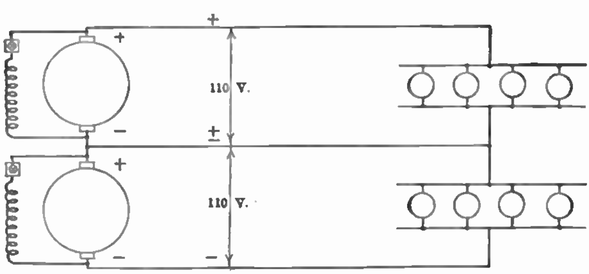


FIG. 422.—Two generators supplying a three-wire system.

345. Two-generator Method.—Two shunt generators may be connected in series, as in Fig. 422. The positive terminal of one should be connected to the negative terminal of the other; that is, the generators are in series between the outers. Both generators may be driven by the same prime mover. When connected in this manner, each generator supplies the load on its own side of the line only. The obvious objection to this method is that two separate generators are required.

346. Storage Battery.—A *storage battery* may be "floated" across the line, as in Fig. 423. The neutral wire is connected to the mid-point of the battery. When the load is unbalanced, that half of the battery on the more heavily loaded side will discharge and the other half will be charged. Figure 423 shows

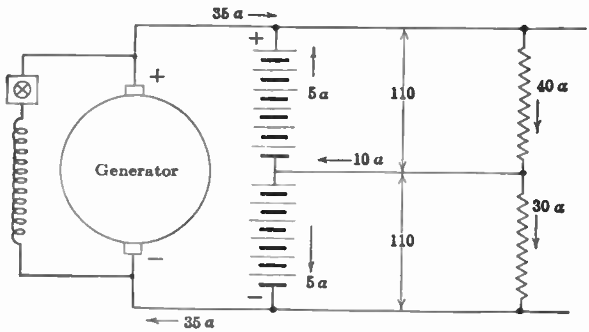


FIG. 423.—Storage battery giving neutral in a three-wire system.

an unbalancing of 10 amp. In this particular case, 5 amp. of the neutral current is shown as returning to the positive wire through the upper half of the battery, which is discharging to supply 5 amp. of the unbalanced current. The other 5 amp. of the neutral current flows to the negative wire through the lower half of the battery and charges this half. In order that the upper half of the battery may discharge, it is necessary that its terminal voltage drop (Eq. (38), p. 56) and in order that the lower halt may charge, its terminal voltage must rise (Eq. (45), p. 62). Hence the system voltages must become unbalanced. The objections to this method of obtaining a neutral are the high maintenance cost of a storage battery, the difficulty of maintaining both halves of the battery at the same condition of charge, and the voltage unbalance at the battery.

347. Balancer Set.—A balancer set is a common method of obtaining the neutral. This set consists of two dynamos mechanically coupled together. They are connected in series across the outer wires and the neutral is brought to their common terminal, as in Fig. 424. With little or no unbalance in current, both dynamos operate as motors. With any appreciable

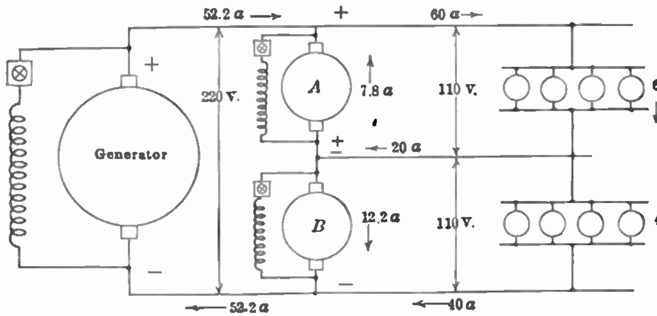


FIG. 424.—Balancer set giving neutral in a three-wire system.

unbalance, one dynamo operates as a motor to drive the other as a generator to supply a part of the unbalanced current.

The action of this set may best be illustrated by the hydraulic analogy shown in Fig. 425. Water is supplied by the canal *A*. This water falls over a weir into canal *B* and may be made to do

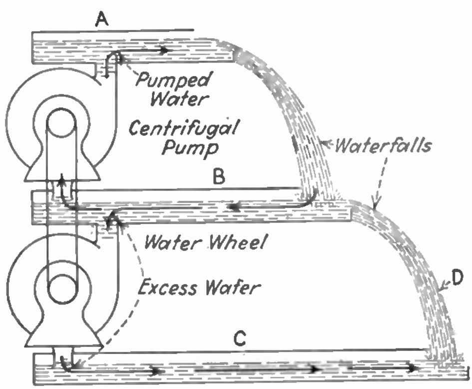


FIG. 425.—Water-wheel analogue of balancer set.

useful work. All this water is not needed at the other point of utilization *D* between the canal *B* and the tail race *C*. Some of the water which is not needed at *D* passes to *C* through the water wheel shown in the figure. This water wheel is belted to a centrifugal pump operating between B and A. In virtue of the water passing through the water wheel some of the water in the canal B is pumped back to A by the pump, where it may be utilized again. The water wheel corresponds to the motor or dynamo B (Fig. 424) and the centrifugal pump to the generator or dynamo A.

If in Fig. 425 more water is required between canals B and C than can be supplied by the weir at A, the centrifugal pump may act as a water wheel and the water wheel as a pump. Some of the extra water required in B will be supplied through the upper machine operating as a water wheel and discharging into B. In so doing the upper machine drives the lower machine as a pump. The lower machine then pumps water from C back to B. This condition corresponds to an excess of load on the negative side of the system of Fig. 424.

If in Fig. 424 there is an excess of load on the positive side of the system, as represented by 20 amp. in the neutral, 12.2 amp. of this 20 amp. flows through the motor and in dropping through 110 volts gives up its energy. The motor then causes the generator to pump 7.8 amp. back to the positive side of the line. This current distribution is determined in the following manner.

Each of the dynamos A and B is assumed to have 80 per cent efficiency. Let I_1 be the generator current in dynamo A, and I_2 be the motor current in dynamo B. The generator output will be $0.8 \times 0.8 = 0.64$ times the motor input. Assuming that the voltages are equal, actually they will be slightly unbalanced,

$$110I_2 \times 0.64 = 70.4I_2 = 110I_1.$$
$$I_1 + I_2 = 20.$$

Solving,

$$I_1 = 7.8 \text{ amp.}$$
$$I_2 = 12.2 \text{ amp.}$$

The machines will respond more readily to unbalanced loads if their fields are crossed, that is, if the motor field is across the generator side of the system and the generator field is across the motor side of the system. In order that a generator may supply additional current, either its terminal voltage must drop, or its induced e.m.f. must rise. In order that a motor may take additional load, either its terminal voltage must rise or its induced e.m.f. must drop. The excess load on the positive side of the system (Fig. 424) tends to reduce the field of dynamo A and to increase that of dynamo B. These effects are the reverse of what is desired. If the generator field is across the motor side of the line, the increased voltage is across the generator field and will raise the generator induced e.m.f. Therefore, its terminal voltage need not drop so much to take care of unbalanced

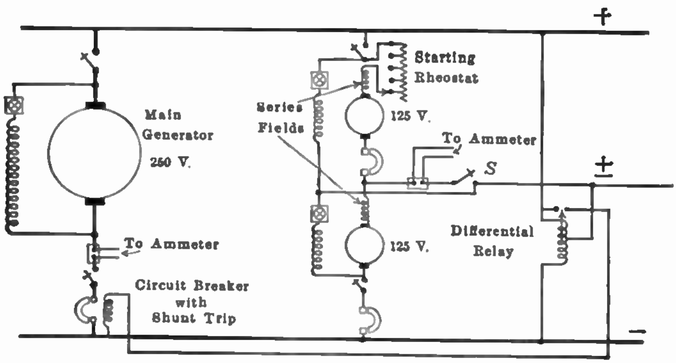


Fig. 426.—Connections of a three-wire system using a balancer set.

currents. The same result may be obtained by compounding the two dynamos. The series fields should be so connected that the dynamo acting as generator is cumulatively compounded, and that acting as motor is differentially compounded. If the compounding is properly adjusted, the set may take care of the neutral current with practically no change in the voltages across the terminals of the two machines.

Figure 426 shows standard connections for a balancer set with series fields. The machines are started in series, with the neutral switch S open and the shunt fields in series across the line. When the machines are up to speed the neutral switch S is closed. If the voltages on the two sides of the system become widely different, the currents in the two halves of the differential relay become unbalanced. This relay then closes the tripping-coil circuit of the main generator breaker, resulting in the main generator circuit opening, even though *its* load is not excessive. This prevents injury to apparatus connected between the outer wires and neutral, due to its being subjected to improper voltage.

348. Three-wire Generator.—The three-wire-generator or Dobrowolsky method is a very efficient means of obtaining a neutral. The details of the method can be understood better after alternating currents and the synchronous converter have been studied.[1] The principle of the method is as follows: Alternating current is generated in a direct-current armature as is shown in Chap. XI, p. 356. If slip-rings be employed, alternating current is obtained from the generator. A coil wound on

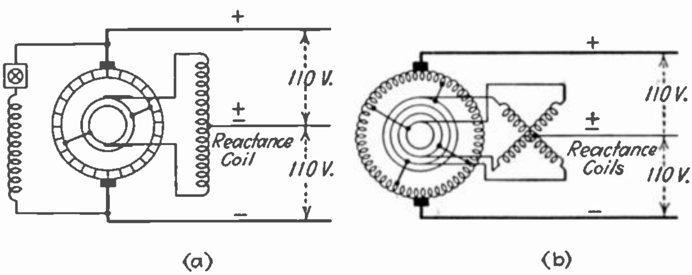


FIG. 427.—Three-wire generator connections (Dobrowolsky method).

an iron core, therefore having high inductance and offering high impedance to alternating current, is connected across the slip-rings. The center of this inductance or reactance coil is at the center of gravity of the voltages generated in the armature. Further, the inductance coil offers very little resistance to the flow of direct current. Therefore, if the neutral be connected to the center of this coil, the voltage to either brush from the neutral will be the same. Moreover, any current flowing back through the neutral can flow back readily into the armature through the reactance coil. The connections of such a generator are shown in Fig. 427(a). Sometimes, to obtain better balancing, two and even three reactance coils are employed. All have their neutrals connected together, as in Fig. 427(b). Occasionally, the reactance coils are placed within the armature. This arrangement requires only one slip-ring, but increases the weight of the armature.

[1] See Vol. II (revised), Chap. XII.

The Edison three-wire system may be extended to four-, five- six-, seven-wire systems (see Fig. 382, p. 513). The complications and number of wires prevent these multiwire systems being extensively used.

349. Feeders and Mains.—In thickly settled districts, the mains form an underground network. This network is supplied at various points, called *centers*, by feeders connected to the direct-current bus-bars at the power station. It requires a careful study of the various loads, amount of copper, etc., in order to determine the most advantageous feeding points or centers. Two or more substations may simultaneously feed the same

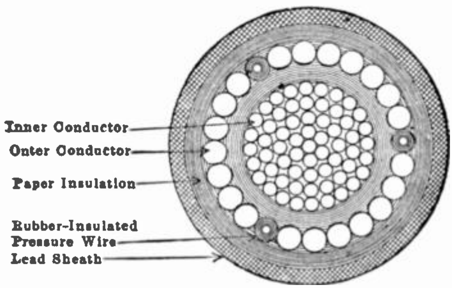


FIG. 428.—Cross-section of a 220-volt, 1,000,000-cir.-mil concentric cable.

centers. In order that the voltages at these centers may be determined and so maintained at the desired values, pilot or pressure wires run back to the station voltmeter. By means of a dial switch, the operator is able to read the voltages at the various centers. Figure 428 shows the cross-section of a concentric 1,000,000-cir.-mil cable. The outer and inner conductors are the outer wires of the Edison three-wire system. The neutral is usually a separate wire of much smaller cross-section, or there may be one large neutral common to several feeders and mains. The three pilot or pressure wires are connected one to each outer wire and one to the neutral at the feeding point. If the operator finds that the voltage is too low at the feeding point, he connects a feeder to a bus-bar of higher voltage. A large voltage drop can exist in such feeders, as no loads are taken off at intermediate points.

In practice, the following are the percentage drops usually allowed: In feeders, 5 to 10 per cent; in the distribution mains, 3 per cent.

The services are usually taken directly from the distribution mains. Occasionally, large loads may be taken directly from junction boxes. Junction boxes are circular iron castings containing a set of insulated bus-bars, to which either the distribution mains or the feeders are connected. Distribution

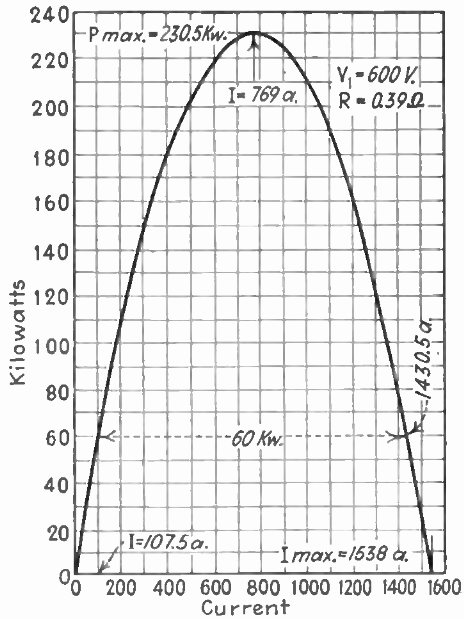


FIG. 429.—Power and current over a feeder.

mains are connected, through fuses, to suitable terminals already installed in the junction boxes. A junction box thus provides a convenient method of connecting the single feeding wires to the several distribution wires. The mains are generally fused, but for the feeders only disconnecting links are used, it being deemed advisable to allow the feeders to burn themselves clear of any short circuits.

350. Maximum Power over a Feeder.—In Par. 47 (p. 58), it is shown that the maximum power which a battery can deliver occurs when the external resistance is equal to the internal resistance of the battery. Under these conditions the voltage drop within the battery is equal to its terminal voltage. This same rule applies to any condition where a constant-voltage source is supplying a concentrated load through a fixed resistance. Consider a direct-current feeder or transmission line which is fed from constant-voltage bus-bars. The line resistance is constant. As load is applied at the end of the line, the current I increases and the voltage V at the load decreases, as is shown in Par. 41 (p. 48). The power delivered is VI. This power will increase so long as I increases faster than V decreases. When the load voltage V is equal to one-half the sending-end voltage, VI is a maximum. Further decrease in the load resistance results in V decreasing faster than I increases, and the power therefore decreases with further increase in I. This relation is shown in Fig. 429. The following example is given to illustrate these conditions.

Example.—A power station supplies 60 kw. to a load over 2,500 ft. of 000, two-conductor copper feeder the resistance of which is 0.078 ohm per 1,000 ft. The bus-bar voltage is maintained constant at 600 volts. Determine: (*a*) the current; (*b*) the voltage at the load; (*c*) the efficiency of transmission; (*d*) the maximum power which can be transmitted; (*e*) the maximum current which can be supplied.

(*a*) Total resistance $R = 5 \times 0.078 = 0.39$ ohm.

Let I be the current and V the voltage at the load:

$$60{,}000 = VI. \tag{I}$$
$$V = 600 - 0.39I. \tag{II}$$

Substituting V from Eq. (II) in Eq. (I),

$$60{,}000 = (600 - 0.39I)I.$$
$$0.39I^2 - 600I = -60{,}000. \tag{III}$$

Dividing Eq. (III) by 0.39,

$$I^2 - 1{,}538I = -153{,}800. \tag{IV}$$

This is a quadratic equation and must have two roots. Hence, two values of current will satisfy the given conditions. Completing the square and solving,

$$I^2 - 1{,}538I + (769)^2 = -153{,}800 + (769)^2.$$
$$(I - 769)^2 = 437{,}600.$$
$$I - 769 = \pm\sqrt{437{,}600}.$$
$$I = 769 \pm 661.5$$
$$= 1{,}430.5, \text{ or } 107.5 \text{ amp.}$$

Both values of current satisfy Eqs. (I) and (II), but if the larger value of current were used, the efficiency would be too low. Hence, the current

$$I = 107.5 \text{ amp.} \quad \textit{Ans.}$$

(b) $V = 600 - (0.39 \times 107.5) = 600 - 42 = 558$ volts. *Ans.*
$P = 558 \times 107.5 = 60{,}000$ watts or 60 kw. (*check*).

(c) $\eta = \dfrac{558}{600} = 0.93$, or 93 per cent. *Ans.*

(d) Repeating Eqs. (I), (II), (III), (IV) with power represented by P,

$$P = VI = (600 - 0.39I)I. \qquad \text{(V)}$$
$$0.39I^2 - 600I = -P.$$

Dividing by 0.39 and completing the square ($591{,}400 = \overline{769^2}$),

$$I^2 - 1{,}538I + 591{,}400 = 591{,}400 - 2.564P.$$
$$I - 769 = \pm\sqrt{2.564}\sqrt{230{,}500 - P}.$$
$$I = 769 \pm 1.6\sqrt{230{,}500 - P}.$$

If the power P exceeds 230,500 watts, the quantity under the radical is negative, the radical is an imaginary quantity and the current is the sum of a real and an imaginary quantity, showing that this condition is impossible. Hence, the maximum value which the power P can have is 230,500 watts, or 230.5 kw.

Since, under these conditions, the radical is zero, the two roots of the equation are equal. Hence, the current I has but a single value and is equal to 769 amp. (see Fig. 429).

The voltage at the load

$$V = 600 - (769 \times 0.39) = 600 - 300, \text{ or } 300 \text{ volts.}$$

Under these conditions, one-half the power is lost in the line, and the efficiency is 50 per cent. This corresponds to the similar condition for batteries (p. 58).

The value of maximum power is more readily determined by the use of calculus. Let V_1 be the bus-bar voltage, V the load voltage, I the current, r the total line resistance, and P the power to the load. Then,

$$V = V_1 - Ir.$$
$$P = VI = V_1I - I^2r.$$

Taking the first derivative of the power with respect to the current, and equating to zero to obtain the condition for maximum power,

$$\frac{dP}{dI} = V_1 - 2Ir = 0,$$
$$Ir = \frac{V_1}{2},$$

and

$$V = V_1 - Ir = V_1 - \frac{V_1}{2} = \frac{V_1}{2}. \qquad (244)$$

(e) The maximum current occurs when the load is short-circuited. That is,

$$I = \frac{600}{0.39} = 1{,}538 \text{ amp.} \quad \textit{Ans.}$$

The power supplied to the load under these conditions must be zero since the voltage at the load is zero.

To illustrate further these relations between power and voltage, the curve (Fig. 429) is given. Current is plotted as abscissa and power as ordinate. Different values of current are assumed, substituted in Eq. (V), and the equation solved for the power P. The power must be zero when the current is zero (voltage V a maximum) and when the voltage V is zero (current a maximum). It is clear that for any given value of power, except the maximum, there are two different values of current. The maximum power is 230.5 kw. and occurs when the current is 769 amp. At this point both roots of the quadratic equation are equal. In practice the power transmitted is usually a small proportion of the maximum power, as can be seen by the position of the 60-kw. point on the curve (Fig. 429).

Also the other solution of Eq. (IV), I = 1,430.6 amp., is found at the second intersection of the characteristic with the 60-kw. ordinate.

351. Electric-railway Distribution.—Electric-railway generators are generally compounded, the series field being on the negative side. The negative terminal is usually connected directly to ground or to the rail through a switch. The positive terminal feeds the trolley through an ammeter, a switch, and a circuit-breaker.

On short lines, with light traffic, the trolley alone may suffice to carry the current to the car, as in Fig. 430(a). Except in small installations, the trolley is not of sufficient cross-section to supply the required power and at the same time to keep the voltage drop within the necessary limits. As the size of the trolley wire is limited by the trolley wheel, it cannot be conveniently increased. The same result as increasing the size of the trolley may be obtained by running a feeder in parallel with the trolley and connecting the feeder to the trolley at short intervals, as in Fig. 430(b). This is called the *ladder system* of feeding. The trolley and feeder together may be considered as forming a single conductor.

Where the density of traffic requires several feeders, better results are obtained by connecting the feeders in the manner of Fig. 430(c). Each feeder is protected by a circuit-breaker.

The objections to the preceding methods of feeding are that trouble, due to a ground, for example, at any point on the trolley, involves the entire system. In cities where traffic is particularly dense, it is not permissible to take chances of having the entire system shut down because of a ground at one point only. Therefore, the trolley is sectionalized, as in Fig. 430 (*d*). In this method the trolley is divided into insulated sections, each of which is supplied by a separate feeder. Trouble in one section is not

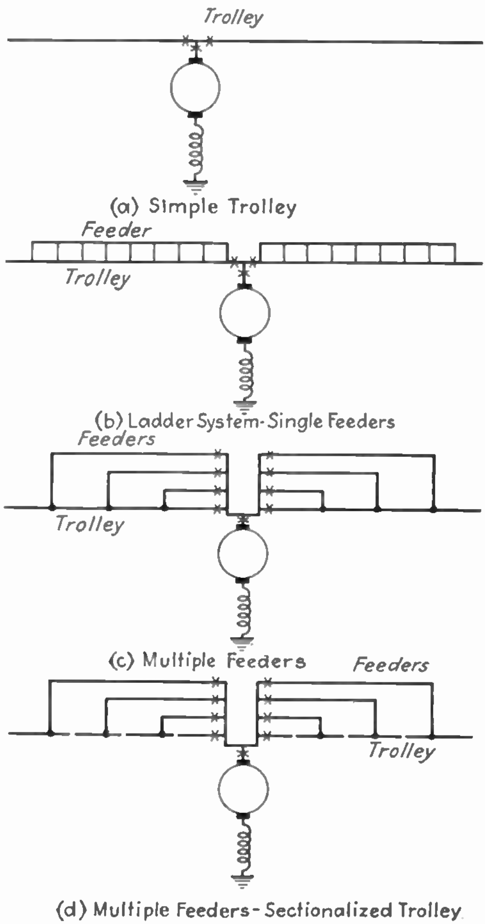


Fig. 430.—Methods of feeding a trolley system.

readily communicated to the other sections. This increased reliability is obtained at the expense of efficiency in the use of the copper, as the feeders are unable to assist one another. In the preceding systems this mutual assistance exists.

In (*d*), when the trolley wheel passes over the insulation between sections, the motorman is required to move the controller to the "off" position, after which he may move it rapidly back to the "full-speed" position. Were the controller left in the full-speed position, even a small voltage difference between sections would cause a large momentary current to the motors. This is due to the fact that the difference between the terminal voltage and the induced e.m.f. is small, and a small change in either makes a large proportionate change in their difference. Unless resistance is introduced in the armature circuit, the change in current will be large (see Eq. (225), p. 486).

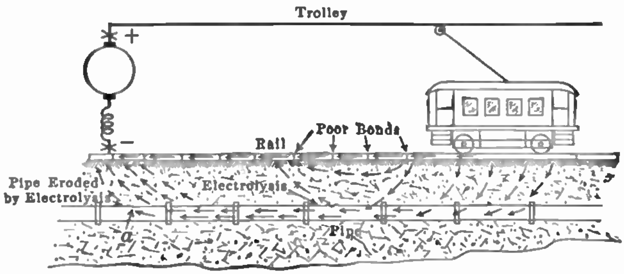


FIG. 431.—Electrolysis by earth currents.

352. Electrolysis by Earth Currents.—Most trolley systems use the track as the return conductor for the current taken by the car. The return currents not only flow through the tracks themselves, but, by spreading through the earth, seek the paths of least resistance by which they may return to the negative terminal of the station generator. Such currents in spreading through the earth follow such low-resistance conductors as water pipes, gas pipes, and cable sheaths, as in Fig. 431. The fact that current *enters* and flows in these conductors does no harm. However, such currents must ultimately leave these pipes, etc., as at *a* (Fig. 431). In leaving, they tend to carry the metal of the pipe into electrolytic solution, which ultimately results in the pipe being eaten away. To decrease the effects of electrolysis, several expedients have been used. Two most successful are the following. (*a*) Provide as good a return path as is possible. This is done by good bonding and by using insulated negative feeders, that is, heavy copper feeders that are run back to the negative bus from various points along the track. Figure 431 indicates how poor rail bonds may cause the current to leave the track and enter the pipe. In some cities the total permissible drop in the ground-return circuit must not exceed 10 to 15 volts. (*b*) Discourage the entering of the current into the pipes by inserting occasional insulating joints in the pipes.

In testing for electrolysis, the usual method is to measure the voltage existing between the track and the water pipes (as at a hydrant). The magnitude of this voltage indicates roughly the magnitude of the current which must be flowing from one to the other. The polarity shows which way the current is flowing. For example, if the track is positive to the pipe, current must be flowing from the track to the pipe.

STORAGE-BATTERY SYSTEMS

353. Central-station Batteries.—Figure 432 shows a typical load curve of a central station. Between 11:00 p.m. and 5:00

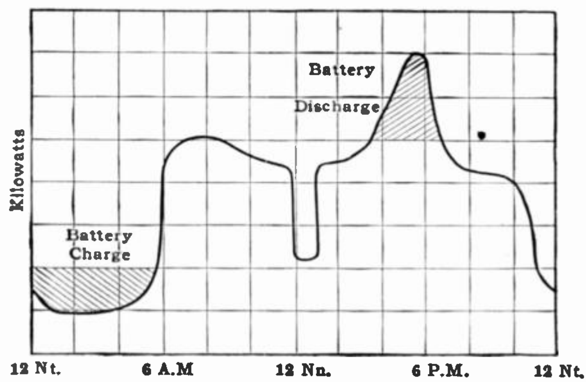


FIG. 432.—Battery smoothing out central-station load curve.

a.m. the load is comparatively small, consisting of street lights and a few all-night commercial loads. This portion of the load curve is called a *valley*.

The load increases rapidly from 5:00 to 7:00 a.m. due to commercial power loads, lights, and perhaps to the beginning of street-car service. The morning peak occurs about 8:00 a.m. The load drops off gradually until noon.

The valley between 12:00 and 1:00 p.m. is due to the shutting down of the commercial loads because of the noon hour. The evening peak, which is usually the highest, occurs generally between 5:00 and 6:00 p.m. This peak may continue for an hour or more, after which it drops to the evening load, which consists mostly of lighting. This load gradually diminishes to the all-night value.

Obviously the power company must have sufficient station and distributing capacity to carry the peak. Even though this apparatus is in use for only a limited period each day, the investment charges are in effect 24 hr. a day.

The ratio of the average load to the maximum load of a station is called the *load factor.*

Example.—A station delivers 192,000 kw.-hr. in a day and its peak load is 20,000 kw. What is the daily load factor?

$$\text{The average load} = \frac{192{,}000}{24} = 8{,}000 \text{ kw.}$$

$$\text{The load factor} = \frac{8{,}000}{20{,}000} = 0.40, \text{ or } 40 \text{ per cent.} \quad \textit{Ans.}$$

Obviously a *high* load factor is most desirable. In fact, power companies welcome loads that will fill in the valleys of the load curve and are usually prepared to offer attractive rates for such loads in order to improve their load factors and thus to utilize apparatus at times when it would otherwise be idle.

The load curve of a direct-current station may be smoothed out by the use of a storage battery. The battery may be charged at night and early morning and so fill in the valley of the load curve and then be discharged during the peak of the load curve, as in Fig. 432. This equalizes the load on the station and increases its load factor.

As a rule, batteries are not installed for the purpose of smoothing out the load curve. A storage battery operating under the best conditions is good for only a limited number of complete charges and discharges. Therefore, the battery maintenance is usually found to more than offset the economies effected by taking some of the load off the peak. Such batteries may be useful in isolated plants, because it is often possible to shut down the entire lighting plant and run on the batteries at night, thus eliminating considerable labor expense.

However, batteries are installed as *reserve* in large central-station systems. They are placed near the center of the load. In the event of a shut-down in the generating system or in the transmission system, the battery can help maintain service. For this reason pasted-plate batteries are more often used, because of their high overload capacity (see Par. 79, p. 109).

Storage batteries are also useful in taking care of unexpected loads. For example, a thunderstorm may give rise to a sudden demand which could not be foreseen and so cannot be met immediately by the generating station, as it takes time to get up steam and put a generator on the line. A battery may be put on the line immediately and so carry a substantial part of the load increase until boilers and turbines can be brought into service. If the battery is already "floating" across the line, it takes the load increase automatically.

354. Resistance Control.—In controlling the load taken by a generator connected to the bus-bars, it is necessary to change its induced e.m.f. by changing the field current. It is not possible to adjust the e.m.f. of a storage battery in this manner. One method of controlling the output of the battery is to have the battery voltage several volts higher than the bus-bar voltage and to insert resistance in series with the battery, as in Fig. 433. By adjusting this resistance, the load delivered by the battery may be controlled. The disadvantage of this method is the loss of power, and the voltage drop in the resistance which varies with the load. Even with constant load the resistance must be adjusted occasionally to compensate for the drop of battery e.m.f. during discharge.

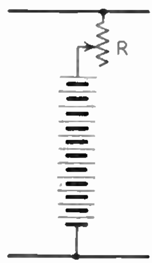


FIG. 433.—Resistance control of battery discharge.

Example.—It is desired to discharge a storage battery, consisting of 115 cells each having an e.m.f. of 2.1 volts and an internal resistance of 0.001 ohm, into 220-volt bus-bars so that the battery delivers 100 amp. To what value must the series resistance be adjusted?

The total battery e.m.f.

$$E = 115 \times 2.1 = 242 \text{ volts.}$$

The bus-bar voltage

$$V = 220 \text{ volts.}$$

The battery resistance

$$r = 115 \times 0.001 = 0.115 \text{ ohm.}$$

Let R equal the added external resistance.

$$100 = \frac{242 - 220}{0.115 + R}.$$

$$100R = 22 - 11.5 = 10.5.$$

$$R = 0.105 \text{ ohm.} \quad \textit{Ans.}$$

355. Counter-electromotive-force Cells.—It is characteristic of storage cells, of either the lead or alkaline type, that when being charged, their terminal voltage does not change greatly over a wide range in the value of the charging current. This principle is utilized in controlling the current delivered by a battery.

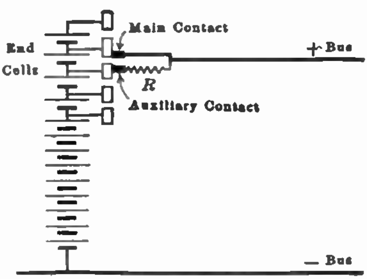


FIG. 434.—End-cell control of storage battery.

A few cells are connected at an end of the battery so that their e.m.fs. oppose the flow of current. The rate of discharge is regulated by the number of cells in circuit, and some means, such as the end-cell switch in Fig. 434, is provided for switching the cells in and out of circuit. Since such cells are not required to store energy, they are made up of plain lead or nickel plates and the appropriate electrolyte. The advantage of this method over resistance control is that the opposing or control e.m.f. is largely independent of changes in load.

356. End-cell Control.—A battery consists usually of a sufficient number of cells to give an e.m.f. exceeding by an ample margin that of the bus-bars. The e.m.f. of the battery, and hence its load, may be controlled by cutting in or out the cells at the end of the battery.

It is essential to do this without opening the circuit. For this purpose a switch is used similar to that shown in Fig. 434. The main contact is connected to the auxiliary contact by a resistance R. When sliding from one battery contact to the next, the auxiliary contact maintains the circuit connections through the resistance R. Were there zero resistance between the main contact and its auxiliary contact, the individual cells would be dead-short-circuited during the transition period. The resistance R is usually so chosen as to allow the normal battery current to flow during the transition period. In Fig. 434 the position of the switch is that corresponding to the transition period. The end-cell switches become rather massive in large battery installations and are often operated by a motor-driven worm. This also permits remote control.

The end cells, not being in continuous service, are discharged to a lesser degree than the main battery cells. Therefore the end cells require individual attention on charging.

357. Regulating Battery.—A battery is sometimes used to equalize sudden fluctuations of load. The battery e.m.f. should be such that with average load it is just equal to the bus-bar voltage. The battery is then delivering no current and is merely "floating."

When a sudden load comes on the station, the bus-bar voltage drops, the battery discharges, and assists the generators. On the other hand, if the load drops to a low value, the bus-bar voltage rises and the battery charges.

As a rule the bus-bar voltage does not change enough to cause the battery to respond sufficiently to the load changes. In fact with overcompounded generators, such as are used for railway service, charging of the battery with increase in load might well occur. There are several methods of causing the battery to charge and discharge at the proper time. The generator may be under-compounded if the resultant voltage regulation of the bus-bars is still satisfactory. However, if close regulation is desired, it can be obtained by the method shown in Fig. 435, the effect of which is to produce overcompounding of the battery. The battery is connected to the bus in series with a motor-driven booster. Two carbon rheostats R_1 and R_2 are connected in series across the battery. The booster field is connected from their common point to the middle of the battery. If $R_1 = R_2$, the booster field is connected across two points of equal potential, the field current is zero, and no e.m.f. is induced in the armature of the booster. An increase of load, however, causes solenoid P to pull down on the lever. This compresses R_1 and releases the pressure on R_2. The resistances R_1 and R_2 now differ considerably so that the booster field is no longer across points of equal potential. A current now flows through the booster field causing the booster to generate an e.m.f. of such polarity as to assist the battery to discharge. S is a spring.

In order to reduce the current flowing through R_1 and R_2 and the battery, the change of booster excitation is often accomplished through an intermediary exciter, whose field is connected in the same manner as that of the booster of Fig. 435.

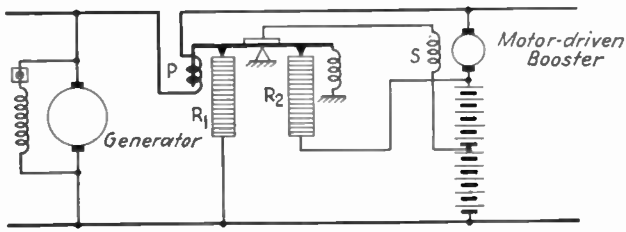


Fig. 435.—Regulating storage-battery discharge with booster set.

358. Series Distribution.—In the parallel system of distribution, the loads are all independent of one another. That is, a load applied to any one point does not affect any of the other loads, provided the voltage does not change. In the series system the loads are all in series with one another so that the same current passes through each. Therefore, if the circuit of any one load be opened, the current to all the other loads will be interrupted. As this is not permissible in practice, a load must be *short-circuited* when it is desired to remove it from service.

Power is usually supplied to a series or constant-current system by one of two methods: the series generator, of which the Brush-arc and Thomson-Houston machines are examples; and the constant-current transformer operating in conjunction with the mercury-arc rectifier (see p. 276, Chap. VIII, Vol. II). Both of these methods tend to maintain constant current under all conditions of load. Therefore, if the circuit be opened and a very high resistance thus introduced, a constant current is maintained across a high resistance and a very high voltage results. For this reason, the lamps in constant-current systems are protected by a thin disc of paper between the lamp terminals (film cutout). If the lamp burns out, the high voltage across this paper punctures it and so prevents the opening of the circuit.

The advantage of the series system is the small amount of copper required. This is due to the fact that the copper carries only the current of a single load. As the loads are in series, the resulting voltage is high. Therefore, this system is applicable only to outside work, such as street lighting, because it would ordinarily be dangerous to have such high voltages in buildings.

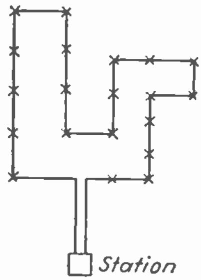


FIG. 436.—Open-loop series circuit.

Another important advantage of the series lighting system is that incandescent lamps on a constant-current system maintain a high lumen efficiency (about 97 per cent) throughout their life, while the lumen efficiency of the constant-voltage incandescent lamp drops to a value of about 79 per cent at the end of its rated life. The lumen efficiency will continue to decline to much lower values if the lamps are not replaced. The power to an incandescent lamp is I^2R, and, with the continual evaporation of the filament which occurs in service, R increases. Since, in the series lamp, the current I is maintained constant, the power to the lamp and hence the filament temperature increase, which nearly compensates for the blackening of the inside of the lamp. For the multiple lamp the power is E^2/R where E is the voltage across the lamp. As R increases with the reduction of filament cross-section, the power decreases. Hence, the reduction in wattage and blackening of the glass act in conjunction to reduce the lumens.

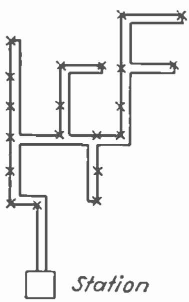


FIG. 437.—Parallel-loop series circuit.

There are two general methods of connecting such series loads. In the open-loop system (Fig. 436), the circuit is connected to the loads without reference to the separation of the two conductors. This system is economical of copper. In the parallel-loop system the outgoing and return conductors are always kept near each other, as in Fig. 437. This system requires more copper than the open-loop system but facilitates testing for faults and reduces the inductive effect on neighboring circuits.

APPENDIX A

Relations of Units

Length

1 inch	=	2.54 centimeters
1 foot	=	30.48 centimeters
1 mile	=	1.609 kilometers

Area

1 circular mil	=	0.7854 square mil
1 circular mil	=	0.000507 square millimeter
1 square inch	=	6.452 square centimeters
1 square meter	=	10.76 square feet

Volume

1 cubic inch	=	16.39 cubic centimeters
1 liter	=	1,000 cubic centimeters
	=	0.2642 U. S. gallon
1 gallon	=	231 cubic inches
	=	3.785 liters
	=	8.345 lb.

Weight

1 gram	=	981 dynes
1 ounce (avoirdupois)	=	28.35 grams
1 kilogram	=	2.205 pounds
1 ton	=	2,000 pounds
1 long ton	=	2,240 pounds
1 metric ton	=	1,000 kilograms
	=	2,205 pounds

Work

1 joule (watt-second)	=	10,000,000 = 10^7 ergs
1 gram-degree centigrade (gram-calorie)	=	4.183 joules
1 pound-degree Fahrenheit (British thermal unit)	=	252.1 gram-degree centigrade (gram-calorie)
	=	777.5 foot-pounds

APPENDIX B

Relations among Electrical Units

Quantity	Practical system	C.g.s. magnetic system	C.g.s. electrostatic system
Electromotive force	1 volt	10^8 abvolts	$\frac{1}{300}$ statvolt
Current	1 ampere	$\frac{1}{10}$ absampere	3×10^9 statamperes
Quantity	1 coulomb	$\frac{1}{10}$ abcoulomb	3×10^9 statcoulombs
Resistance	1 ohm	10^9 absohms	$1/(9 \times 10^{11})$ statohm
Capacitance	1 farad	$\frac{1}{10^9}$ abfarad	9×10^{11} statfarads
Capacitance	1 microfarad	$\frac{1}{10^{15}}$ abfarad	9×10^5 statfarads
Inductance	1 henry	10^9 abhenrys	$1/(9 \times 10^{11})$ stathenry
Energy	1 joule	10^7 abjoules or ergs	10^7 statjoules or ergs
Power	1 watt	10^7 abwatts or ergs per second	10^7 statwatts or ergs per second

APPENDIX C

Specific Gravities

Aluminum	2.67	Mercury	13.60
Copper	8.89	Nickel	7.83
Gold	19.26	Platinum	20.30
Iron, bar	7.48	Silver	10.55
Iron, wrought	7.79	Tin	7.29
Steel	7.85	Zinc	6.86
Lead	11.45		

1 cu. ft. of water weighs 62.5 lb.

APPENDIX D

Standard Annealed Copper Wire, Solid*

American Wire Gage (B.&S.). English Units

Gage number	Diameter, mils	Cross-section		Ohms per 1,000 ft.		Ohms per mile	Pounds per 1,000 ft.
		Circular mils	Square inches	25°C. (=77°F.)	65°C. (=149°F.)	25°C. (=77°F.)	
0000	460.0	212,000.0	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000.0	0.132	0.0630	0.0727	0.333	508.0
00	365.0	133,000.0	0.105	0.0795	0.0917	0.420	403.0
0	325.0	106,000.0	0.0829	0.100	0.116	0.528	319.0
1	289.0	83,700.0	0.0657	0.126	0.146	0.665	253.0
2	258.0	66,400.0	0.0521	0.159	0.184	0.839	201.0
3	229.0	52,600.0	0.0413	0.201	0.232	1.061	159.0
4	204.0	41,700.0	0.0328	0.253	0.292	1.335	126.0
5	182.0	33,100.0	0.0260	0.319	0.369	1.685	100.0
6	162.0	26,300.0	0.0206	0.403	0.465	2.13	79.5
7	144.0	20,800.0	0.0164	0.508	0.586	2.68	63.0
8	128.0	16,500.0	0.0130	0.641	0.739	3.38	50.0
9	114.0	13,100.0	0.0103	0.808	0.932	4.27	39.6
10	102.0	10,400.0	0.00815	1.02	1.18	5.38	31.4
11	91.0	8,230.0	0.00647	1.28	1.48	6.75	24.9
12	81.0	6,530.0	0.00513	1.62	1.87	8.55	19.8
13	72.0	5,180.0	0.00407	2.04	2.36	10.77	15.7
14	64.0	4,110.0	0.00323	2.58	2.97	13.62	12.4
15	57.0	3,260.0	0.00256	3.25	3.75	17.16	9.86
16	51.0	2,580.0	0.00203	4.09	4.73	21.6	7.82
17	45.0	2,050.0	0.00161	5.16	5.96	27.2	6.20
18	40.0	1,620.0	0.00128	6.51	7.51	34.4	4.92
19	36.0	1,290.0	0.00101	8.21	9.48	43.3	3.90
20	32.0	1,020.0	0.000802	10.4	11.9	54.9	3.09
21	28.5	810.0	0.000636	13.1	15.1	69.1	2.45
22	25.3	642.0	0.000505	16.5	19.0	87.1	1.94
23	22.6	509.0	0.000400	20.8	24.0	109.8	1.54
24	20.1	404.0	0.000317	26.2	30.2	138.3	1.22
25	17.9	320.0	0.000252	33.0	38.1	174.1	0.970
26	15.9	254.0	0.000200	41.6	48.0	220.0	0.769
27	14.2	202.0	0.000158	52.5	60.6	277.0	0.610
28	12.6	160.0	0.000126	66.2	76.4	350.0	0.484
29	11.3	127.0	0.0000995	83.4	96.3	440.0	0.384
30	10.0	101.0	0.0000789	105.0	121.0	554.0	0.304
31	8.9	79.7	0.0000626	133.0	153.0	702.0	0.241
32	8.0	63.2	0.0000496	167.0	193.0	882.0	0.191
33	7.1	50.1	0.0000394	211.0	243.0	1,114.0	0.152
34	6.3	39.8	0.0000312	266.0	307.0	1,404.0	0.120
35	5.6	31.5	0.0000248	335.0	387.0	1,769.0	0.0954
36	5.0	25.0	0.0000196	423.0	488.0	2,230.0	0.0757
37	4.5	19.8	0.0000156	533.0	616.0	2,810.0	0.0600
38	4.0	15.7	0.0000123	673.0	776.0	3,550.0	0.0476
39	3.5	12.5	0.0000098	848.0	979.0	4,480.0	0.0377
40	3.1	9.9	0.0000078	1,070.0	1,230.0	5,650.0	0.0299

NOTE 1.—The *fundamental resistivity* used in calculating the tables is the International Annealed Copper Standard, *viz.*, 0.15328 ohm (meter, gram) at 20°C. The *temperature coefficient* for this particular resistivity is $\alpha_{20} = 0.00393$, or $\alpha_0 = 0.00427$. The *density* is 8.89 g. per cubic centimeter.

NOTE 2.—The values given in the table are only for annealed copper of the standard resistivity. The user of the table must apply the proper correction for copper of any other resistivity. Hard-drawn copper may be taken as about 2.7 per cent higher resistivity than annealed copper.

NOTE 3.—Pounds per mile may be obtained by multiplying the respective values above by 5.28.

* From *Circ.* 31, U.S. Bureau of Standards.

APPENDIX E

Bare Concentric Lay Cables of Standard Annealed Copper

English Units

A.W.G. number	Circular mils	Ohms per 1,000 ft.		Pounds per 1,000 ft.	Standard concentric stranding		
		25°C. (=77°F.)	65°C. (=149°F.)		Number of wires	Diameter of wires, mils	Outside diameter, mils
	2,000,000	0.00539	0.00622	6,180	127	125.5	1,631
	1,700,000	0.00634	0.00732	5,250	127	115.7	1,504
	1,500,000	0.00719	0.00830	4,630	91	128.4	1,412
	1,200,000	0.00899	0.0104	3,710	91	114.8	1,263
	1,000,000	0.0108	0.0124	3,090	61	128.0	1,152
	900,000	0.0120	0.0138	2,780	61	121.5	1,093
	850,000	0.0127	0.0146	2,620	61	118.0	1,062
	750,000	0.0144	0.0166	2,320	61	110.9	998
	650,000	0.0166	0.0192	2,010	61	103.2	929
	600,000	0.0180	0.0207	1,850	61	99.2	893
	550,000	0.0196	0.0226	1,700	61	95.0	855
	500,000	0.0216	0.0249	1,540	37	116.2	814
	450,000	0.0240	0.0277	1,390	37	110.3	772
	400,000	0.0270	0.0311	1,240	37	104.0	728
	350,000	0.0308	0.0356	1,080	37	97.3	681
	300,000	0.0360	0.0415	926	37	90.0	630
	250,000	0.0431	0.0498	772	37	82.2	575
0000	212,000	0.0509	0.0587	653	19	105.5	528
000	168,000	0.0642	0.0741	518	19	94.0	470
00	133,000	0.0811	0.0936	411	19	83.7	418
0	106,000	0.102	0.117	326	19	74.5	373
1	83,700	0.129	0.149	258	19	66.4	332
2	66,400	0.162	0.187	205	7	97.4	292
3	52,600	0.205	0.237	163	7	86.7	260
4	41,700	0.259	0.299	129	7	77.2	232

APPENDIX F

Allowable Carrying Capacities of Wires

National Electrical Code

A.W.G. number	Diameter of solid wires, mils	Area, circular mils	Table A, rubber insulation, amperes	Table B, varnished-cloth insulation, amperes	Table C, other insulation, amperes
18	40.3	1,624	3	.....	6
16	50.8	2,583	6	.....	10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	30
10	101.9	10,380	25	30	35
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325.0	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460.0	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		750,000	525	630	800
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1,000
		1,100,000	690	830	1,080
		1,200,000	730	880	1,150
		1,300,000	770	920	1,220
		1,400,000	810	970	1,290
		1,500,000	850	1,020	1,360
		1,600,000	890	1,070	1,430
		1,700,000	930	1,120	1,490
		1,800,000	970	1,160	1,550
		1,900,000	1,010	1,210	1,610
		2,000,000	1,050	1,260	1,670

APPENDIX G

Electrical Properties of the Metals and Alloys

Metals	Resistivity at 20°C.		Temperature coefficient of resistance at 20°C.	Approximate maximum working temperature, degrees centigrade	Melting point, degrees centigrade
	Centimeter cube, microhms	Circular-mil-foot, ohms			
Aluminum	2.828	17.02	0.0039	....	659
Antimony	41.7	251.0	0.0036	....	630
Bismuth	110.0	663.0	0.004	....	271
Carbon:					
Amorphous	3.800 to 4.100	....	(−)	....	>3500
Retort (graphite)	†720 to 812	....	(−)		
Copper (drawn)	1.724	10.37	0.00393	260	1083
Gold	2.44	14.7	0.0034	....	1063
Iron:					
Electrolytic	9.96	59.9	....	....	1530
Cast	74.4 to 97.8	448 to 588			
Lead	*20.4	*123	0.00387	....	327
Mercury	*94.07	*566	0.00072	....	−38.9
Nickel	*6.93	*41.7	0.0062	600	1452
Platinum	*10.96	*66.0	0.003	1500	1754
Silver	1.629	9.8	0.0038	....	961
Steel:					
Soft	15.9	95.8	0.0016	....	1430
Glass hard	45.7	275			
Silicon 4 per cent	51.15	308			
Transformer	11.09	66.8			
Tungsten	5.51	33.2	0.005	....	3400
Zinc	*5.75	*34.6	0.0037	....	419
Alloys					
Brass	7.0	42.1	0.002	....	940
Constantan (Cu 60, Ni 40)	49.0	295.0	0.000005	....	1290
German silver (Ni 18)	33.0	198.5	0.0004		
Iala	49.0	283.0	0.000005	370	1230
Ideal	50.0	296.0	0.000005	350	1090
Manganin (Cu 84, Mn 12, Ni 4)	44.0	265.0	0.000006	100	
Monel metal (Ni-Cu)	42.0	256.0	0.0019	450	1360
Nichrome (FeNiCr)	109.0	660.0	0.00019	950	
Phosphor-bronze (Cu, Sn 2 to 6, Ph 0.005 to 0.13)	3.95	23.7			

* 0°C.

† Furnace electrodes, 3000°C.

APPENDIX H

Carrying Capacity in Amperes of Lead-covered Cables
Simplex Wire and Cable Company

Size	Rubber			Cambric; Paper		
	Temperature rise—30°C. = 54°F. Maximum temperature, 125°F.			Temperature rise—37.5°C. = 67.5°F. Maximum temperature, 150°F.		
	Single conductor	Two conductors	Three conductors	Single conductor	Two conductors	Three conductors
10 A.W.G.	30	26	23	34	29	26
8 A.W.G.	45	39	34	50	44	38
6 A.W.G.	60	52	45	67	58	50
4 A.W.G.	78	68	59	87	76	65
2 A.W.G.	122	106	92	136	118	102
1 A.W.G.	146	127	110	163	142	122
0 A.W.G.	169	147	127	189	165	142
00 A.W.G.	192	167	144	215	187	161
000 A.W.G.	245	213	184	274	238	206
0000 A.W.G.	285	248	214	319	278	239
250,000 C.M.	320	278	240	358	311	269
300,000 C.M.	370	322	278	414	360	311
350,000 C.M.	415	361	311	464	404	348
400,000 C.M.	460	400	345	514	447	386
500,000 C.M.	550	479	412	615	535	461
600,000 C.M.	630	548	473	705	613	529
700,000 C.M.	710	618	532	794	691	596
750,000 C.M.	750	653	563	838	730	630
800,000 C.M.	790	688	593	883	770	665
900,000 C.M.	840	732	627	940	818	700
1,000,000 C.M.	900	783	675	1,005	875	755
1,250,000 C.M.	1,055	916	790	1,180	1,028	885
1,500,000 C.M.	1,200	1,045	900	1,343	1,168	1,007
1,750,000 C.M.	1,340	1,165	1,005	1,500	1,305	1,126
2,000,000 C.M.	1,400	1,220	1,050	1,565	1,362	1,175

This table should be used only under ordinary conditions and does not apply in special cases.

APPENDIX I

Table of Turns per Square Inch; Solid Layer Winding*

The Acme Wire Company

Size, A.W.G. number	Single-cotton covered	Enamel and cotton	Single-silk covered	Enamel and silk	Enamel
10	87.5	84.5	......	......	92.5
11	109	105	......	......	117
12	136	130	......	......	147
13	169	161	......	......	184
14	210	199	......	......	231
15	260	248	......	......	292
16	321	304	......	......	366
17	396	374	......	......	458
18	488	456	......	......	572
19	598	556	......	......	715
20	772	722	865	807	907
21	947	890	1,075	1,010	1,150
22	1,155	1,075	1,330	1,230	1,425
23	1,410	1,303	1,650	1,510	1,780
24	1,720	1,575	2,045	1,860	2,220
25	2,080	1,910	2,520	2,290	2,800
26	2,500	2,310	3,090	2,830	3,540
27	3,020	2,770	3,810	3,460	4,440
28	3,630	3,300	4,690	4,220	5,570
29	4,270	3,910	5,650	5,100	6,950
30	5,100	4,630	6,950	6,200	8,730
31	5,920	5,330	8,410	7,300	10,650
32	6,950	6,300	10,000	8,900	13,500
33	8,120	7,300	12,080	10,650	16,900
34	9,430	8,410	14,500	12,600	21,000
35	10,850	9,610	17,300	14,900	26,000
36	12,350	10,850	20,400	17,300	31,900
37	......	......	23,700	20,400	40,000
38	......	......	27,800	23,700	49,300

* "Standard Handbook for Electrical Engineers," 5th ed., Sec. 5, Par. 102, McGraw-Hill Book Company, Inc.

APPENDIX J

Greek Alphabet

Name	Large	Small	Quantity commonly used to designate
Alpha	A	α	angles; coefficients
Beta	B	β	flux density; angles; coefficients
Gamma	Γ	γ	(small) conductivity
Delta	Δ	δ	variation; density
Epsilon	E	ϵ	logarithmic base
Zeta	Z	ζ	coefficients
Eta	H	η	(small) hysteresis coefficient; efficiency;
Theta	Θ	θ	temperature; angle of phase displacement
Iota	I	ι	
Kappa	K	κ	dielectric constant
Lambda	Λ	λ	(small) wave length
Mu	M	μ	(small) permeability; micro; amplification factor
Nu	N	ν	reluctivity
Xi	Ξ	ξ	output coefficient
Omicron	O	o	
Pi	Π	π	circumference ÷ diameter = 3.1416
Rho	P	ρ	(small) resistivity
Sigma	Σ	σ, ς	(cap.) summation; (small) surface density; mutual conductance
Tau	T	τ	time constant; time phase
Upsilon	Υ	υ	displacement
Phi	Φ	ϕ, φ	magnetic flux
Chi	X	χ	
Psi	Ψ	ψ	angular velocity in time; dielectric flux;
Omega	Ω	ω	(cap.) ohms; (small) angular velocity

QUESTIONS AND PROBLEMS

QUESTIONS ON CHAPTER I

Resistance

1. Name the principal factors which make electricity so useful and so important, with particular reference to flexibility, generation, transmission, and uses.

2. Describe the mechanism of current flow according to modern theory. In what important respect does the atomic structure of insulators differ from that of conductors?

3. Discuss the principal ways by which resistance manifests itself in an electric circuit in which current flows. According to modern theory, how may the power loss in resistance be accounted for by the action of electrons? What is the mechanical analogue of resistance?

4. What substances are the best conductors? What substances are insulators? Discuss any boundary which may exist between conductors and insulators. Compare the relative resistivities of a good conductor and a good insulator.

5. Name and define the unit of resistance.

6. With constant cross-section, how does the resistance of a homogeneous substance vary with its length? With constant length, how does the resistance of a homogeneous substance vary with its cross-section? What factors other than the geometrical shape of a body of given resistivity determine its resistance?

7. Define volume resistivity. Upon what factor does it depend? If the volume of a given substance is fixed, how does the resistance vary with the length? with the cross-section?

8. What is conductance and how does it vary with length and with cross-section? Distinguish between conductance and conductivity.

9. How is the resistance determined for a current path if the cross-section varies with the length? Cite a simple example of such a path.

10. What is the relation of the total resistance of a circuit to the resistances of its individual parts when these are connected in series?

11. What is the relation of the total conductance of a circuit to the conductances of its individual parts when these are connected in parallel? From this relation show how resistances connected in parallel may be combined into an equivalent resistance.

12. What is the meaning of the term *mil?* What is a square mil? A circular mil? What relation does one bear to the other? When is the circular mil usually chosen as the unit of cross-section? What are its advantages over such units as the square mil and the square inch? What is the relation between the number of circular mils in a circular cross-section and its diameter?

13. What is a circular-mil-foot? What is its approximate resistance for copper? How may the resistance of a copper wire be determined if its length in feet and its cross-section in circular mils be known?

14. Define a circular-mil-inch. What is its resistance at 60°C.? For what types of resistance calculations is this unit of resistivity very convenient?

15. For what purposes are resistor materials used? Enumerate some of the more common alloys, with their advantages and uses.

16. How is the resistance of most of the unalloyed metals affected by temperature? What is *temperature coefficient of resistance?* How is it used? How is the temperature coefficient of resistance at any initial temperature determined?

17. At what temperature would the resistance of copper be zero if the resistance decreased at the same rate that it decreases within ordinary ranges of temperature? How may this principle be used to solve problems involving resistance and temperature?

18. What is the basis of the American Wire Gage (A.W.G.)? What relations do the diameters of successive gage numbers bear to one another?

19. What relation do the cross-sections of the wires in the A.W.G. bear to one another? How does this relation enable one to determine readily the resistance and weight of any given size of wire? What is the resistance of 1,000 ft. of No. 10 wire? What is the weight of 1,000 ft. of No. 10 wire? Of No. 2 wire?

20. Why is stranding with the larger sizes of wire necessary? What is the gage number of the largest size of solid copper wire? State the relation of the number of wires in successive layers of a stranded cable and the number of wires in standard cables.

21. What are the best conductors among the metals? Which is most commonly used and why? Compared with copper, what are the advantages and the disadvantages of aluminum as a conductor? When are iron and steel used as conductors? Explain.

PROBLEMS ON CHAPTER I

Resistance

1. Two conductors A and B, of the same material, have the same length, but the cross-section of A is twice that of B. If the resistance of A is 12 ohms, what is that of B?

2. Two cylindrical conductors C and D, of the same material, have the same length, but the diameter of C is twice that of D. If the resistance of C is 12 ohms, what is that of D?

3. The diameter and length of a cylindrical conductor A are twice the diameter and length of a cylindrical conductor B. The resistance of conductor B is 0.8 ohm. What is the resistance of conductor A?

4. If the resistance of copper is 1.724 microhms per centimeter cube at 20°C., what is the resistance of an inch cube at the same temperature?

OCT 12 '41

Determine the resistance of a circular-mil-foot (diameter 0.001 in. and length 1 ft.).

5. The resistivity of a copper-cadmium alloy wire is 2.03 microhms per centimeter cube at 20°C. Determine at the same temperature the resistance of a 200-ft. length of this alloy, the diameter of which is 0.128 in.

6. The resistance of a 300-m. length of an aluminum-bronze alloy wire is 1.635 ohms per centimeter cube at 20°C. and the diameter is 5.18 mm. Determine at the same temperature the resistivity of this alloy in microhms per centimeter cube and ohms per inch cube.

7. Nichrome (nickel-chromium alloy) has a resistivity of 109 microhms per centimeter cube and its temperature coefficient is so small as to be negligible over ordinary ranges of temperature. Find the resistance in microhms between the three pairs of opposite faces of a rectangular sheet 8 mils thick, 6 in. on one side and 14 in. on the other.

8. It is desired to make nichrome-ribbon resistor elements having a resistance of 0.8 ohm each (see Prob. 7). The width of the ribbon must be ⅛ in. and the thickness 10 mils, in order that it may carry the current. What length of ribbon will be required for each element?

9. A certain commercial copper has a volume resistivity of 1.765 microhm-cm. at 20°C. Find the resistance at 20°C. of a cylindrical copper rod 24 ft. long and ½ in. diameter.

10. Determine the resistance at 20°C. of a commercial copper bus-bar 30 ft. long and having a cross-section 3 in. by ½ in. The resistivity of the International Copper Standard is 1.724 microhms per centimeter cube at 20°C., and commercial copper has a conductivity of only 97.8 per cent of this standard. Copper weighs 0.32 lb. per cubic inch. What is the weight of the bus-bar in pounds? in kilograms?

11. Repeat Prob. 10 for an aluminum bus-bar of the same dimensions. Aluminum has a volume resistivity of 2.828 microhm-cm. at 20°C. The density of copper is 8.89 and that of aluminum 2.67 (see Appendix C).

12. The resistivity of a certain commercial copper is 1.765 microhms per centimeter cube at 20°C. and the resistance of a 400-m. length of a cylindrical wire is 3.39 ohms. Determine the cross-section of the wire in square millimeters and the diameter in millimeters.

13. Determine the cross-section of an aluminum bus-bar which will have the same length and the same total resistance as the copper bus-bar (Prob. 10). What is the ratio of the weight of aluminum to the weight of copper in the two cases? Copper has a density of 8.89 and aluminum has a density of 2.70, nearly. The resistivity of aluminum is given in Prob. 11.

14. The cross-section of No. 8 A.W.G. copper wire is 8.385 sq. mm. and the resistivity of the copper at 20°C. is 1.724×10^{-6} ohm per centimeter cube. What should be the length in meters of such a wire to give a resistance of 0.08 ohm at 20°C.?

15. A third rail has a cross-section of 7.9 sq. in. and the steel has a resistivity of 22.2 microhm-cm. Neglecting the effect of joints, determine the resistance of 1 mile of such rail.

16. A 100-lb. (per yard) steel rail has a cross-section of 9.84 sq. in. and its volume resistivity is 12.5 times that of copper at 20°C. (see Par. 15, p. 16). Determine the resistance of 5 miles of such rail.

17. Nichrome, a resistor alloy of nickel and chromium, has a resistivity of 109.7 microhm-cm. Determine the resistance of a 90-ft. strip ¼ in. wide and 0.08 in. thick.

18. Ideal wire, a copper-nickel alloy, has a resistivity of 50 microhm-cm. Determine the resistance of a 100-ft. length of such wire having a diameter of 64 mils.

19. A 1,000-ft. length of No. 10 A.W.G. copper wire has a resistance of 1.02 ohms at 25°C. What would be the resistance of this same mass of copper if it were drawn down to one-half the cross-section?

20. A cylindrical copper rod having a diameter of 0.25 in. has a resistance of 0.01 ohm. Taking the resistivity of copper as 1.724 microhm-cm. at 20°C., determine: (*a*) the length of the rod in feet; (*b*) the resistance of the rod when it is drawn down to one-third the initial diameter, the volume remaining unchanged. Assume that the resistivity of the copper is not affected by the drawing process.

21. The density of copper is 8.89 g. per centimeter cube. Assuming a resistivity of 1.724 microhms per centimeter cube, determine the length in meters and the resistance of 5 kg. of copper, the cross-section of which is 1.31 sq. mm.

22. A copper rod 10 m. long has a resistance of 17.0 microhms at 20°C. This rod is drawn into a wire having a diameter of 1.015 mm. What is the resistance of the resulting wire? Assume constant temperature and also assume that owing to successive annealings, the resistivity does not change during the drawing process. The resistivity of this copper is 1.744 microhm-cm. at 20°C.

23. The resistivity of a certain quality of copper is 0.686 microhm per inch cube at 20°C. Determine the area and the resistance at 20°C. of 100 lb. of this copper when the length is 1,260 ft. The weight per cubic inch of copper is 0.32 lb.

24. Aluminum has a conductivity of 353,600 mhos per centimeter cube at 20°C. Find the conductance and the resistance at 20°C. of an aluminum bus-bar 6 in. by 0.5 in., and 22 ft. long. What is its weight in pounds? in kilograms? The density of aluminum is 2.70 g. per cubic centimeter, nearly.

25. The resistivity of a certain copper conductor is found to be 1.740 microhm-cm. What is its conductivity in mhos per centimeter cube? per inch cube?

26. Determine the conductance of a copper bus-bar of the dimensions given in Prob. 24, the copper being of a resistivity as given in Prob. 25. The specific gravity of copper is 8.89. Determine the weight of the bus-bar in kilograms; in pounds.

27. What should be the cross-section of the bus-bar of Prob. 26, in order that it shall have the same resistance as the bus-bar of Prob. 24? Under these conditions what is the ratio of the weight of the copper to the weight of the aluminum bus-bar?

28. The cross-section of No. 8 A.W.G. wire is 0.0130 sq. in. and the resistivity at 25°C. of the copper is 1.770 microhms-cm. Determine the conductance of 1,250 ft. of such wire.

29. The resistance at 20°C. of a rod of high-tensile-strength bronze is 5.76×10^{-4} ohm. The diameter of the rod is 0.930 cm. and the length for which the resistance was measured is 1.2 m. Determine the conductivity of the bronze in mhos per centimeter cube. Determine the ratio of this conductivity to that of the International Copper Standard (resistivity = 1.724 microhm-cm).

30. The diameter over the copper of a 0, stranded, A.W.G., rubber-insulated cable is 0.373 in. and the wall of insulation is 3⁄8 in. The resistivity of the rubber is 10^{16} ohms per centimeter cube. Determine the insulation resistance of 1 mile length of this cable.

31. Current flows into surface A and around the semicircular path to surface B (Fig. 31A). The inner radius of the path is 2 cm., the outer radius is 3 cm., and the axial thickness of the conductor is 1.5 cm. The resistivity is ρ ohms per centimeter cube. Determine the resistance of the path between surfaces A and B by integration. NOTE.—In order to perform the integration it is necessary first to find the *conductance* of the path.

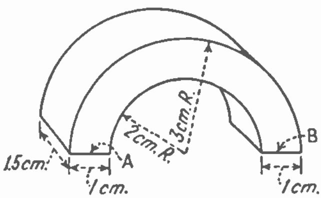


FIG. 31A.

32. The insulation resistance of a 0000, stranded, A.W.G., paper-insulated cable is 100 megohms per mile at 25°C. The diameter of the conductor is 0.528 in. and the thickness of the wall of insulation is 0.25 in. Determine the resistivity of the paper insulation in ohms per inch cube and ohms per centimeter cube.

33. The resistance of a 00, solid copper rod is measured between contact points 38 in. apart and found to be 0.000251 ohm. The temperature is 20°C. and the diameter of the rod is 0.365 in. Determine: (a) the pounds per mile-ohm; (b) the resistance per centimeter cube. State whether or not the rod comes within the resistivity specifications of the A.I.E.E. (Par. 17, p. 17). (The density of copper is 8.89 g. per cubic centimeter.)

34. The resistance of a bus-bar 20 ft. long, the cross-section of which is 6 in. by ½ in., is to be measured between contact points 18 ft. apart. The temperature is 20°C. In order to conform to the A.I.E.E. resistivity specifications, what must be the maximum value of the measured resistance? See p. 17.

35. A resistance of 6.8 ohms is connected in series with resistances of 11.2, 18, and 35 ohms, all in series. What is the resistance of the combination?

36. Three resistances, each of 6.5 ohms, are in series and are in turn connected in series with a series grouping of two resistances of 4.8 ohms each. What is the total resistance of the combination?

37. A conductor consists of a straight bar 1 cm. by 1.5 cm. cross-section bent at its center in semicircular form similar to that shown in Fig. 31A. The parallel portions of the rod are each 12 cm. long (Fig. 37A). Using the data of Prob. 31 determine the resistance between the two end surfaces C and D.

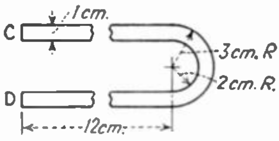


FIG. 37A.

38. A resistance of 18.2 ohms is connected in parallel with a resistance of 12.4 ohms. What is the resistance of the combination?

39. Three conductances of 0.1, 0.2, and 0.4 mho are connected in parallel. (*a*) What is the total conductance of the combination? (*b*) the total resistance? (*c*) Determine the resistance resulting from connecting the three conductances in series. (*d*) Determine the resulting conductance in (*c*).

40. Determine the resistance between the terminals *A* and *B* of the combination in Fig. 40*A*.

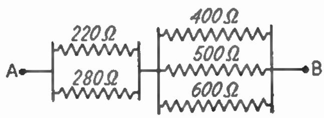


FIG. 40A.

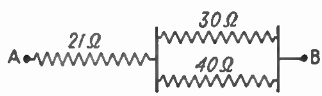


FIG. 41A.

41. Determine the resistance between the terminals *A* and *B* of the combination in Fig. 41*A*.

42. In Fig. 42*A* determine: (*a*) the resistance of branch *ACB*; (*b*) of branch *ADB*; (*c*) the total resistance between terminals *A* and *B*.

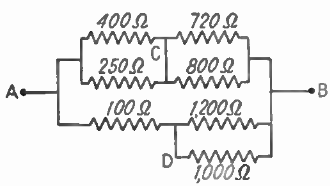


FIG. 42A.

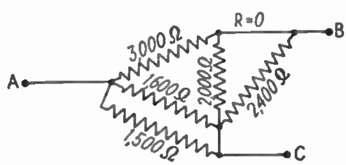


FIG. 44A.

43. A resistance of 12 ohms, a parallel combination of two resistances of 20 and 25 ohms, and a parallel combination of three resistances of 40, 50, and 80 ohms, are all connected in series. Determine: (*a*) the resistance of the entire combination; (*b*) the conductance of each of the three portions of the circuit; (*c*) the conductance of the entire combination.

44. In Fig. 44*A* determine the resistance of the electric mesh between the terminals *A* and *B*.

45. In Fig. 44*A* determine the resistance of the electric mesh between the terminals *B* and *C*.

46. In Fig. 44*A* determine the resistance of the electric mesh between the terminals *A* and *C*.

47. In Fig. 44*A* determine the resistance of the electric mesh between the terminals *A* and *C* connected together and terminal *B*.

48. A galvanometer having a resistance of 890 ohms is shunted by 90 ohms (see p. 141). What is the combined resistance of galvanometer and shunt?

42, 54, 41, 54, 62, 65, 66, 68, 70
Due Oct. 18

49. A 0000 trolley wire (hard-drawn) having a resistance of 0.0523 ohm per 1,000 ft. extends 5 miles from the power station. It is paralleled for 3 miles by a 400,000-cir.-mil cable, having a resistance of 0.0270 ohm per 1,000 ft., the feeder and the trolley being connected every half mile by taps (Fig. 49*A*). The resistance of *each* rail is 0.096 ohm per 1,000 ft. Determine: (*a*) the total resistance of the overhead circuit from the power house to the end of the trolley line; (*b*) the total resistance from the power house

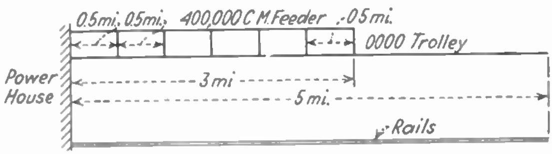


Fig. 49*A*.

to the end of the line and the track return to the power house, assuming that the return current is confined to the two rails.

50. In Prob. 49, assume that a negative feeder consisting of a 250,000-cir.-mil cable having a resistance of 0.0431 ohm per 1,000 ft. is connected from the negative bus-bar at the power house to the rail, 3 miles out. Repeat (*b*), Prob. 49.

51. A cylindrical copper rod has a diameter of 0.25 in. What is its cross-section in square inches? in square mils? in circular mils? Repeat for 0.50 in., 0.75 in., and 1.0 in.

52. Number 15 A.W.G. wire has a diameter of 0.057 in. What is its circular milage? Number 0 A.W.G. wire has a cross-section of 106,000 cir. mils. What is the diameter in inches of solid No. 0 wire?

53. What are the diameters in mils of the solid, cylindrical conductors having the following cross-sections: 168,000, 20,800, 509, and 25.0 cir. mils?

54. The diameter of each strand of a seven-strand, No. 2 A.W.G. conductor is 97.4 mils. Determine the circular mils and compare with those of a solid No. 2 wire the diameter of which is 258.0 mils.

55. The diameter of each strand of a 19-strand, 000 A.W.G. conductor is 94.0 mils. Determine the circular mils and compare with the circular mils of a solid, 000 A.W.G. wire the diameter of which is 410 mils.

56. Determine the total resistance of a two-conductor, 1,600-ft.-length, 800,000-cir.-mil feeder, assuming the resistance of a circular-mil-foot to be 11.0 ohms at the operating temperature of the feeder. (Include the resistance of the outgoing and return conductor.)

57. A telephone cable is made up of 240 pairs of No. 19 A.W.G., copper conductor. What is the resistance of a 40-mile loop (consisting of a pair), using 10.37 as the circular-mil-foot resistivity of the copper? The diameter of No. 19 wire is 36.0 mils.

58. Hard-drawn copper wire is used for trolleys because of its toughness. The resistance of the circular-mil-foot standard of annealed copper is 10.37 ohms at 20°C. The conductivity of hard-drawn copper is approximately 97.3 per cent that of the annealed standard at the same temperature.

Determine the resistance of a 4-mile length of No. 0000, hard-drawn, trolley wire at a temperature of 20°C. The diameter is 460 mils.

59. Copperweld is copperclad steel wire in which an outer jacket of copper forms a weld with the inner core of steel. Such wire combines moderate conductivity with high tensile strength and protection of the steel against corrosion. The diameter of No. 0000 A.W.G., Copperweld wire is 0.460 in. and the cross-section is 211,600 cir. mils (both being the same as with solid copper). The resistivity of the copper at 20°C. is 10.5 ohms per circular-mil-foot, and that of the steel is 7.5 times this amount. The cross-section of the copper is 65,000 cir. mils, and that of the steel is 146,600 cir. mils. Determine the resistance of 1,000 ft. of such wire and the ratio of the conductance of this wire to that of an all-copper wire of the same outside diameter.

60. The outside diameter of No. 00 A.W.G., Copperweld wire is 0.365 in. and the resistance of 1,000 ft. at 20°C. is 0.197 ohm. The resistivity of the copper and the steel are the values given in Prob. 59. Determine the cross-section in circular mils of the copper and of the steel.

61. Steel-reinforced, hard-drawn aluminum cable is used for high-voltage transmission-line conductors. A 0000 (0 copper equivalent) cable consists of six strands of aluminum and a solid steel core. The diameter of the individual aluminum strands and of the steel core is 0.1672 in. At 20°C. aluminum has a resistivity of 17.02 ohms per circular-mil-foot and the steel a resistivity of 78 ohms per circular-mil-foot. What is the resistance of a single, 100-mile length of such conductor: (*a*) neglecting the conductance of the steel core; (*b*) including the conductance of the steel core?

62. The temperature coefficient of resistance of copper at 0°C. is 0.00427 (see p. 20). The resistance of a coil of copper wire is 15.7 ohms at 28°C. Determine its resistance: (*a*) at 0°C.; (*b*) at 42°C.

63. The resistance of a 5-mile length of hard-drawn, 0000 trolley wire is 1.34 ohms at 20°C. What is the resistance at a minimum winter temperature of −10°C.? At a maximum summer temperature of 45°C.?

64. The resistance at 25°C. of 1,000 ft. of 250,000 cir.-mil (copper equivalent), hard-drawn aluminum cable is 0.0452 ohm. The temperature coefficient of resistance at 20°C. for aluminum is 0.0039 (see p. 20). Determine the resistance at 0°C. and at 40°C.

65. The resistance of the field coils of a shunt generator measured after the machine has been standing for some time in a room the temperature of which is 21°C. is found to be 184 ohms. After the machine has been in operation for 3 hr., the resistance is again measured and found to be 205 ohms. (*a*) What is the average temperature of the field coils after the 3 hr.? (*b*) What is the temperature rise?

66. The resistance of an armature, excluding brush and contact resistance, is 0.0668 ohm at a room temperature of 22°C. Determine its resistance at 0°C. and at its allowable operating temperature, which is 40°C. above room temperature.

67. The resistance of the copper circuit of a direct-current armature is measured at a room temperature of 24°C. and found to be 0.024 ohm. After

the machine has been in operation 4 hr., the resistance of the same circuit is again measured and found to be 0.0263 ohm. Determine the rise in temperature at the end of the 4 hr.

68. Tungsten wire has a temperature coefficient of resistance of 0.0050 at 0°C. The hot resistance of a 100-watt lamp is 120 ohms at its operating temperature of 2200°C. Determine its cold resistance at a room temperature of 25°C.

69. Determine the temperature coefficient of resistance for copper at reference temperatures of 15°C.; of 32°C.

70. The temperature coefficient of resistance at 20°C. for aluminum is 0.0039 (Prob. 64). Determine: (*a*) the "inferred zero resistance" (p. 21); (*b*) the temperature coefficient at 0°C.; (*c*) at 28°C.

The following problems 71 to 76, should be solved without consulting the wire tables.

71. Determine the approximate resistance and cross-section in circular mils of a 1,000-ft. length of No. 13 A.W.G. copper wire; of No. 16; of No. 7.

72. Determine the approximate diameter of each of the wires (Prob. 71) and also the weight in pounds of a 1,000-ft. length of each.

73. Determine approximately the cross-section, the diameter, the resistance, and the weight of 1,800 ft. of No. 00 A.W.G., annealed-copper wire.

74. Determine the approximate resistance, weight, cross-section, and diameter of a 1,200-ft. length of No. 37 A.W.G. copper wire.

75. Repeat Prob. 74 for No. 27 A.W.G. copper wire.

76. Determine the resistance, weight, circular mils, and diameter of a 5-mile length of No. 0000 A.W.G. trolley wire.

QUESTIONS ON CHAPTER II

Ohm's Law and the Electric Circuit

1. Upon what fundamental relation is the electrostatic system of units based? Where is this system used?

2. Upon what fundamental relation is the electromagnetic c.g.s., or absolute, system of electrical units based? Where is this system generally used? How is the absolute unit of current defined?

3. What is the basis of the *practical* electrical system? How is the ampere, the unit of current, defined so that it may be determined experimentally? How is the unit quantity related to the unit current?

4. Give the fundamental definition of potential difference. How is unit-potential difference related to current and resistance? Name some mechanical analogues of potential difference.

5. How is the international ohm defined?

6. State briefly the meaning of *absolute potential.* Why is it usually not important to know absolute potential?

7. What is the nature of voltage drop in a line? Compare it with pressure drop in a pipe. Show that it is impossible to supply power over a simple direct-current power line with the voltage at the load equal to the voltage at the sending end of the line. Show that there must be a drop in potential in the return wire to the generator as well as in the outgoing wire. Can potential exist without a current flow? Illustrate.

8. What is meant by *difference of potential?* Show that it is possible for two or more e.m.fs. to exist within a circuit without there being any difference of potential between the circuit terminals.

9. By means of a diagram show the correct method of connecting a voltmeter in a typical electric circuit. On the same diagram show the connections of an ammeter to measure the current to a load. Why should an ammeter never be connected across the power wires?

10. What fundamental relation does Ohm's law express? Give the three forms in which the law may be expressed. Name the conditions under which it is most convenient to use each of these expressions.

11. How are series-connected resistances combined to give a single equivalent resistance? Derive the expression by means of which a number of parallel-connected resistances may be replaced by a single equivalent resistance. In a two-branch parallel circuit, derive the relation between each current and the total current. In a parallel circuit of three branches, derive the relation between each current and the total current.

12. Name and define the unit of power. Express power in terms of volts, amperes, and ohms, taken two at a time. Differentiate carefully between power and energy. What is the practical unit of electrical energy and what relation does it bear to the unit of power? What is the unit of mechanical horsepower? What relation does it bear to the watt and to the kilowatt?

13. How is energy related to power? Name and define the fundamental c.g.s. unit of energy. Name the practical c.g.s. unit of energy, and state its relation to the fundamental unit. Discuss the various forms in which energy is stored or in which energy may appear. Describe the energy cycle involved in a steam-driven electrical power plant. In what forms does the energy appear ultimately? Approximately what is the over-all efficiency of a modern power system?

14. Define the British thermal unit; the gram-calorie. What is the relation between the gram-calorie and the watt-second?

15. What simple relation exists between the voltages at the sending and receiving ends of a power feeder having a single concentrated load at its far end, and the efficiency of transmission?

16. Under what conditions is the voltage drop in each foot of wire independent of the total current? How is this principle utilized in solving electrical problems? Show how this method may be expanded to obtain the power loss.

PROBLEMS ON CHAPTER II

Ohm's Law and the Electric Circuit

77. A wire is in the form of a circle, the diameter of which is 2 cm., and connection to a source of current is made by carrying the ends of the wire away radially and adjacent to each other. The diameter of the wire is small compared with the radius of the circle. The current flowing in the wire exerts a force of 12 dynes on a small magnetic pole placed at the center of the circle and having a strength of four unit poles. Determine the current in the wire both in c.g.s. absolute amperes and in practical amperes.

78. A resistance of 4.0 ohms is connected across a storage battery the terminal voltage of which is 6.08 volts. Determine the current.

79. The hot resistance of a 50-watt incandescent lamp is 260 ohms. Determine the current which it takes from 115-volt, direct-current mains.

80. The cold resistance of a carbon-filament, incandescent lamp is 340 ohms and the hot resistance is 250 ohms. Determine the current which it takes at the instant that it is connected across 110-volt mains and the current that it takes when the constant operating temperature is reached.

81. The cold resistance of a 60-watt, gas-filled tungsten lamp is 18.4 ohms and the hot resistance at its operating voltage of 110 volts is 202 ohms. Determine the current at the instant that the lamp is connected across the voltage, and the current after it has reached the steady operating condition.

82. The resistance of a relay is 286 ohms and it requires 0.024 amp. to operate it. Determine the necessary operating voltage.

83. The maximum resistance of a rheostat is 2.4 ohms and the minimum resistance is 0.84 ohm. Determine the voltage across the rheostat for each condition when the current in the rheostat is 28.4 amp.

84. The resistance of the field winding of a shunt generator is 320 ohms (Fig. 84*A*) and the resistance of the field rheostat is 46 ohms. The field current is 1.64 amp. Determine: (*a*) the terminal voltage of the generator; (*b*) the voltage across the field winding; (*c*) the voltage across the rheostat.

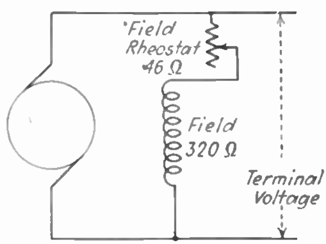


FIG. 84*A*.

85. The hot resistance of an incandescent lamp used with a projector is 10 ohms and the rated voltage is 50 volts. Determine: (*a*) the required series resistance for operation from a 118-volt power source; (*b*) the voltage across the rheostat.

86. An electric toaster takes 4.8 amp. from the 120-volt service wires. Determine its resistance.

87. A telegraph relay which has a resistance of 260 ohms and operates with 48 milliamp. (1 milliamp. = 0.001 amp.) is connected at the far end of a telegraph line. The loop or circuit resistance of the line is 160 ohms.

Determine the voltage which it is necessary to impress at the sending end in order to operate the relay.

88. An ammeter in the field circuit of a shunt generator (see Fig. 84*A*) indicates 2.4 amp., and the voltages across the field winding and the rheostat are 196 volts and 44 volts. Determine: (*a*) the resistance of the field winding itself; (*b*) the resistance of the rheostat at this particular setting; (*c*) the total resistance of the field circuit.

89. Three wires *A*, *B*, and *C* (Fig. 89*A*) of a relay system are short-circuited at their far ends *O* by jumpers. When a voltage of 6 volts is impressed across the two home-end terminals *a* and *b*, the current is 3.0 amp.; when 7.2 volts is impressed across the two home-end terminals *b* and *c*, the current is 3.13 amp.; when 6.75 volts is impressed across the home-end terminals *c* and *a*, the current is 2.5 amp. Determine the resistance of the wires *A*, *B*, and *C*.

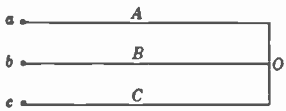


Fig. 89*A*.

90. The resistance of the shunt-field winding of a 230-volt generator is 174 ohms (see Fig. 84*A*). (*a*) With the terminal voltage of the generator adjusted for 230 volts, to what value must the rheostat be adjusted in order that the current in the field circuit may be 1.2 amp.? (*b*) Determine the voltage across the winding and across the rheostat under the conditions of (*a*).

91. Four resistances of 9.4, 8.6, 7.5, and 10.3 ohms are connected in series across a 115-volt, direct-current source. Determine: (*a*) the equivalent resistance of the system; (*b*) the current; (*c*) the voltage across each resistance.

92. A current of 26 amp. is necessary to operate a large electromagnet. There are four series-connected spools on the magnet the resistances of which are 2.4, 2.4, 1.9, and 1.9 ohms. A rheostat, the resistance of which is 0.40 ohm, is in series with the magnet, and the resistance of the connecting wires is 0.16 ohm. Determine: (*a*) the voltage which must be applied to the system; (*b*) the voltage across each coil; (*c*) the voltage across the rheostat; (*d*) the voltage drop in the connecting wires.

93. A series lighting system consists of 92 incandescent lamps, each having a resistance of 7.2 ohms and requiring 6.6 amp. The total series line resistance is 92 ohms. Determine the voltage of the generator supplying this system.

94. An electric flatiron takes 4.8 amp. at 115 volts. Determine its resistance.

95. The current in the shunt-field circuit of a shunt generator is 3.4 amp. and the terminal voltage of the generator is 120 volts. (*a*) Determine the resistance of the shunt field circuit. The voltage drop across the rheostat (see Fig. 84*A*) is 24.4 volts. (*b*) Determine the resistance of the rheostat and of the field winding itself.

96. In order to measure the resistance of an armature with 115 volts as the source of supply, a rheostat in series with the armature is necessary to limit the current (see p. 528). When the current is 38.6 amp., the voltage across the armature terminals is 6.4 volts. Determine: (*a*) the resistance of the armature; (*b*) the resistance of the series-connected rheostat.

97. The voltage between two points 16 ft. apart on a steel rail is 30.5 millivolts (1 millivolt = 0.001 volt) when the current is 200 amp. Determine the resistance of the rail in ohms per mile assuming that the rail joints have no effect on the uniformity of the rail.

98. The current in an aluminum bus-bar, 6 by ½ in. cross-section, is 500 amp., and the voltage drop between points 8 ft. apart is 17 millivolts. Determine: (*a*) the resistance per foot of the bus-bar; (*b*) the voltage drop in 40 ft. when three such bus-bars are connected in parallel and the total current is 3,000 amp.

99. The resistance of the series field, alone, of a compound generator is measured by measuring the voltage drop when the current is adjusted to 90 amp. The voltage drop is found to be 0.60 volt. The series field is then shunted by its diverter (Fig. 99*A*) (also see p. 451). When 90 amp. flows through the two in parallel, the voltage drop is 0.4 volt. Determine the resistance of the series field and of the diverter.

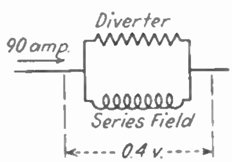


FIG. 99*A*.

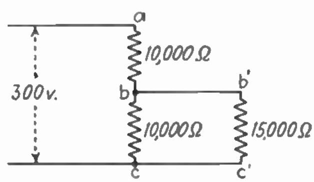


FIG. 100*A*.

100. The resistance of a voltage divider *ac* (Fig. 100*A*) is 20,000 ohms, the center tap *b* being at the mid-point. A resistance *b′c′* of 15,000 ohms is connected across *bc*. The voltage divider is connected across a 300-volt source. Determine: (*a*) the equivalent resistance between points *b* and *c*; (*b*) the resistance between points *a* and *c*; (*c*) the currents in *ab*, *bc*, and *b′c′*; (*d*) the voltage across *bc*.

101. Repeat Prob. 100 with the resistance *b′c′* changed to 20,000 ohms.

102. When the current to a lamp load is 80 amp., the voltage at the lamps is 116 volts. The total resistance of the connecting leads from the load to the bus-bars is 0.06 ohm. Determine the voltage at the bus-bars. (Figure 104*A* shows the connections of this system.)

103. A direct-current, multiple-arc lamp takes 6.4 amp. at 115 volts and the voltage drop across the arc is 68 volts. Determine the resistance of the "ballast" or stabilizing resistance in series with the arc.

104. Figure 104*A* shows a lamp bank, having a total resistance of 6.46 ohms, being supplied from a 120-volt generator over connecting wires having a resistance of 0.12 ohm per wire. Determine: (*a*) the current to the lamp bank; (*b*) the voltage across the lamp bank.

105. The resistor units of an enamel baking oven take 30 amp. at 120 volts. It is desired to reduce the current to 26 amp. Determine: (*a*) the resistance which must be connected in series; (b) the voltage across this resistance and that across the units.

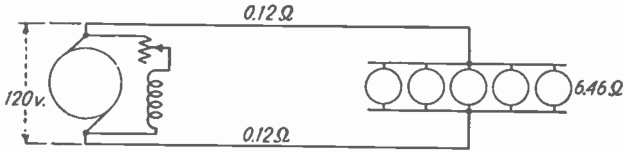


FIG. 104A.

106. It is desired to operate a projection incandescent lamp which is rated at 32 volts and 4 amp. from 120-volt, direct-current mains. Determine the necessary series resistance.

107. Three resistances, 60, 80, and 92.4 ohms, are connected in parallel. (a) Determine the equivalent resistance of the combination. The current in the 92.4-ohm resistance is 2.6 amp. Determine: (b) the voltage across the system; (c) the current in each resistance.

108. Four resistances A, B, C, and D are connected in parallel (Fig. 108A). The resistances A, B, and D are 36, 30, and 16 ohms. A current of 2.5 amp. flows in resistance D and the current to the system is 7.0 amp. Determine: (a) the resistance C; (b) the equivalent resistance of the combination; (c) the current in resistances A, B, and C.

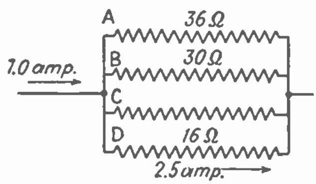


FIG. 108A.

109. In a system similar to that shown in Fig. 108A the resistances A, B, and C are 24, 30, and 48 ohms. The total current to the system is 9.0 amp., of which 2.58 amp. flows in resistance B. Determine: (a) the currents in resistances A, C, and D; (b) the unknown resistance D.

110. The resistance of the series-field winding of a compound generator is 0.0084 ohm, and it is shunted by a diverter the resistance of which is 0.025 ohm (see p. 451). A current of 400 amp. is delivered by the generator, part going through the series field and the remainder through the diverter. Determine: (a) the current in the series field; (b) the current in the diverter; (c) the voltage across the two.

111. Three resistances, 24, 32, and 40 ohms, are connected in parallel, and a current of 42 amp. flows to the combination. Determine: (a) the current in each resistance; (b) the voltage across the combination.

112. A relay, the resistance of which is 20 ohms, operates with a minimum current of 0.05 amp., and it is desired that it operate when the current in a system reaches a value of 0.75 amp. Determine the resistance which should be used to shunt the relay.

113. Three resistances A, B, and C are connected in parallel (Fig. 113A). The resistance of A is 4 ohms and of B 6 ohms. C is adjustable. A current of 8 amp. flows to the circuit. To what value should resistance C be adjusted in order that it may take 2 amp.?

114. A resistance of 12 ohms is connected in series with a combination of two resistances of 16 and 24 ohms in parallel. This entire combination is connected across 100-volt mains. Determine: (*a*) the equivalent resistance of the entire circuit; (*b*) the total current; (*c*) the voltage across the 12-ohm resistance; (*d*) the voltage across the parallel combination; (*e*) the current in the 12-, 16-, and 24-ohm resistances.

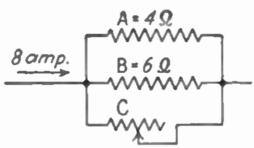


FIG. 113*A*.

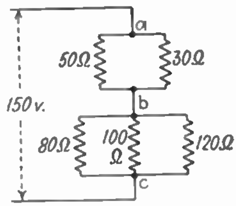


FIG. 115*A*.

115. A series-parallel circuit shown in Fig. 115*A* is connected across a 150-volt supply. Find: (*a*) the equivalent resistance of the entire combination; (*b*) the total current; (*c*) the voltage between points *ab* and *bc*; (*d*) the current in each resistance.

116. A lamp load having a total hot resistance of 6 ohms (Fig. 116*A*) is supplied from a 120-volt service by conductors consisting of a 1,000-ft. length of two-conductor, No. 0 A.W.G. copper wire having a cross-section of 106,000 cir. mils, and an 800-ft. length of two-conductor, No. 1 A.W.G. copper wire having a cross-section of 83,700 cir. mils. Using 10.8 ohms as the resistance of a circular-mil-foot of copper, find the current taken by the system.

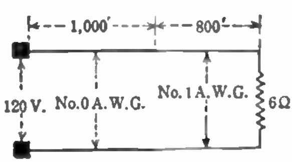


FIG. 116*A*.

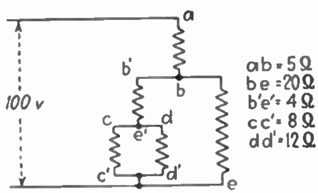


FIG. 117*A*.

117. In Fig. 117*A* is shown a series-parallel combination of resistances connected across a 100-volt source. Determine: (*a*) the equivalent resistance of the entire system; (*b*) the voltage across the resistances *ab*, *b'e'*, *cc'*, *dd'*, and *be*; (*c*) the current in each of the resistances in (*b*).

118. Repeat Prob. 117 with the resistance *be* changed to 10 ohms.

119. The rating of a compound generator is 400 kw. at 250 volts. (*a*) Determine the current rating. (*b*) The efficiency of the generator at its rated load is 0.91. Determine the kilowatt and the horsepower input.

120. The efficiency of a 25-hp., 230-volt motor at its rated load is 0.87. Determine: (*a*) the horsepower input; (*b*) the kilowatt input; (*c*) the rated current of the motor.

121. The field circuit of the motor (Prob. 120) takes 2.5 amp. and the resistance of the field coils, exclusive of the rheostat, is 68 ohms. Determine: (*a*) the power taken by the entire field circuit; (*b*) the percentage of input to the motor taken by the field circuit in (*a*); (*c*) the power taken by the field coils themselves; (*d*) the power dissipated in the rheostat.

122. Determine the power taken by each of the resistances (Fig. 115*A*). Compare the sum of the power taken by the individual resistances with that computed from the product of the total voltage and the total current.

123. In Fig. 123*A* is shown a series-parallel circuit in which a resistance *bc* of 2 ohms is in series with two parallel combinations of resistances, one of which consists of *de* and *fg*, of 20 and 8 ohms. The voltage across this parallel combination is 80 volts. The second parallel combination consists of three resistances *eh*, *jk*, and *gl*, these resistances being 25, 40, and 50 ohms. Determine: (*a*) the voltage across the resistance *bc*; (*b*) the voltage across *am*, the entire circuit; (*c*) the power dissipated in each of the six resistances; (*d*) the total power to the circuit.

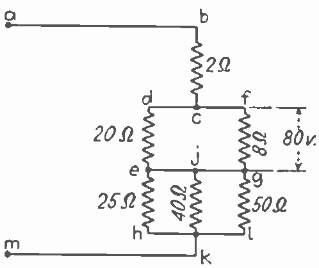


FIG. 123*A*.

124. In Prob. 99 determine the power lost in the series field and in the diverter when the total current is (*a*) 90 amp.; (*b*) 180 amp.

125. The resistance of a 2,000-amp. shunt (see p. 150) is 0.0000225 ohm. Determine the loss in the shunt when the current is 1,200 amp.; when it is 2,000 amp.

126. Determine the power in each of the six resistances (Fig. 123*A*) when the current to the system is 12 amp.

127. The hot resistance of two Mazda C incandescent lamps *A* and *B* when connected across 115-volt mains is 220 and 66.2 ohms. Determine the watt rating of each lamp.

128. The resistance of a 150-volt voltmeter (see p. 154) is 17,500 ohms. Determine the power lost in the voltmeter when it is connected across (*a*) 85 volts; (*b*) 120 volts; (*c*) 150 volts.

129. The resistance of the heater element of an electric toaster is 32 ohms. Determine: (*a*) the watt rating when connected across 120 volts; (*b*) the resistance of a new heater element which would make the rating equal to 500 watts at 120 volts.

130. (*a*) Determine the current rating and the hot resistance of a 115-volt, 60-watt, Mazda C incandescent lamp; (*b*) of a 115-volt, 100-watt lamp.

131. The energy to an electric oven which is operating under steady load is registered by a watthour meter which shows 3,070 kw.-hr. over a period of 96 hr. Determine: (*a*) the average power to the oven; (*b*) the average current if the voltage is 220 volts; (*c*) the total cost of energy if the first 80 kw.-hr. are sold at 6 cts. per kilowatt-hour and the remainder at 3½ cts. per kilowatt-hour.

132. The output of a 230-volt, 10-hp., blower motor is equal to 9.6 hp. at an efficiency of 83.2 per cent. At 3.2 cts. per kilowatt-hour, determine the monthly cost of energy if the motor operates steadily 24 hr. per day for 30 days.

133. At 6½ cts. per kilowatt-hour, determine the cost of raising the temperature of 1½ qt. of water in an electric kettle from 20 to 100°C. Neglect losses. One quart = 0.946 l. or liter. Determine the cost for a month of 30 days, if the kettle is operated twice each day.

134. Energy costs 5½ cts. per kilowatt-hour. Determine the cost of heating 2 qt. of water at room temperature of 24°C. to the boiling point (100°C.). Assume that the efficiency of the heater is 80 per cent (water weighs 8.345 lb. per gallon).

135. A water-barrel rheostat contains 42 gal. of water. How long must 50 amp. at 230 volts flow through the rheostat before the temperature of the water is raised to 200°F. from a room temperature of 70°F.? (One gallon of water weighs 8.345 lb.) Neglect losses.

136. A current of 20 amp. flows in a water-immersed resistance of 14 ohms within an electric boiler. If there are 40 l. of water in the boiler and the thermal losses are neglected, what temperature will the water reach at the end of 30 min.? The room temperature is 20°C.

137. Figure 137*A* shows a drop wire, used for regulating the field current of a generator from zero to a maximum value (see p. 405). The total resistance of the drop wire *ab* is 16 ohms and that of the field is 40 ohms. The line voltage is 120 volts. Determine the current to the generator field when the contact *x* is one-fourth the distance from *a* to *b*; one-half the distance; three-fourths the distance. In each case determine the percentage of the total power received by the field.

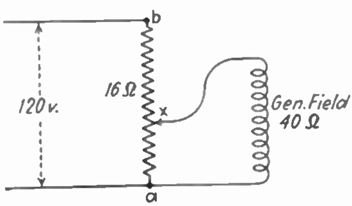


Fig. 137*A*.

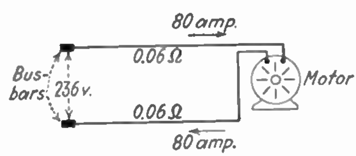


Fig. 139*A*.

138. Repeat Prob. 137 except that a field rheostat of 10 ohms is directly in series with the generator field.

139. In Fig. 139*A* is shown a motor which takes 80 amp. over a feeder, each conductor of which has a resistance of 0.06 ohm. The bus-bar voltage is 236 volts. Determine: (*a*) the voltage at the motor; (*b*) the efficiency of power transmission.

140. In a system similar to that shown in Fig. 139*A* the motor delivers 15 hp. at an efficiency of 0.86, and the voltage at the terminals of the motor is 230 volts. Determine: (*a*) the bus-bar voltage; (*b*) the efficiency of transmission; (*c*) the over-all efficiency of the system.

141. Power is delivered over a feeder to a combined power-and-lighting load with 240 volts at the sending-end bus-bars. The resistance of each of the two wires of the feeder to the load is 0.12 ohm. Determine the voltage at the load and the efficiency of transmission: (*a*) when the current to the load is 80 amp.; (*b*) 60 amp.

142. An electric railway is fed by a 5-mile trolley wire of No. 0000 A.W.G. hard-drawn copper. A 300,000-cir.-mil feeder parallels the trolley wire for 3 miles, being tapped in every half mile (Fig. 142*A*). The resistance of the trolley is 0.269 ohm per mile and that of the feeder 0.190 ohm per mile. The resistance of the ground return may be considered as 0.04 ohm per mile. The station voltage is 600 volts. When the car is 3½ miles

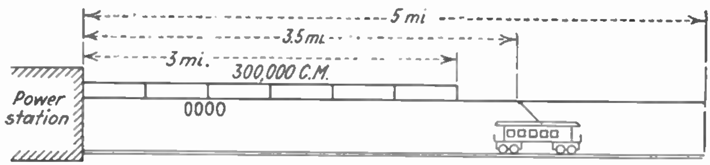


Fig. 142*A*.

from the station and is taking 110 amp., determine: (*a*) the voltage at the car; (*b*) the voltage at the end of the line; (*c*) the efficiency of transmission.

143. In Prob. 142 the car is at the end of the line and is just starting, the starting current being 130 amp. Determine: (*a*) the voltage at the car; (*b*) the efficiency of transmission.

144. It is desired to supply a load of 8 kw. at a distance of 600 ft. from the station bus-bars and a second load of 4 kw. at a farther distance of 400 ft. The voltage at the bus-bars is 245 volts; that at the 8-kw. load must be 220 volts and that at the 4-kw. load must be 218 volts. Determine: (*a*) the resistance of each wire of the feeder from the 8-kw. to the 4-kw. load; (*b*) the resistance of each wire of the feeder from the bus-bars to the 8-kw. load; (*c*) the efficiency of transmission.

145. A 100-kw. load is situated 1,800 ft. from 240-volt bus-bars. It is desired that the voltage at the load be not less than 220 volts when the load is 100 kw. Determine: (*a*) the resistance of each conductor of the feeder; (*b*) the size, in circular mils, of the feeder taking 10.8 ohms as the resistance of a circular-mil-foot of copper; (*c*) the efficiency of transmission; (*d*) the weight in pounds of the copper.

146. In Prob. 145 determine the voltage along the feeder at distances of 400, 950, and 1,500 ft. from the 240-volt bus-bars.

147. Repeat Prob. 145 with the bus-bar and load voltages each halved. The load, the distance, and the efficiency of transmission remain unchanged.

148. Two motor loads are to be supplied from 600-volt bus-bars, the load *A* being located 1,000 ft. away and load *B* being located 600 ft. farther. Load *A* requires a maximum current of 60 amp., and load *B* a maximum current of 40 amp. With these loads the voltage at load *A* should be 562 volts and that at load *B* not less than 531 volts Determine: (*a*) the resistance per conductor of the feeder between loads B and A; (b) the resistance per conductor between load A and the bus-bars; (c) the kilowatts of loads A and B and the kilowatts supplied from the bus-bars; (d) the copper loss in each feeder; (e) the efficiency of transmission; (f) the size wire, A.W.G., which should be used for the conductor in (a); the size wire, A.W.G., which should be used for the conductor in (b).

149. In a system similar to that of Prob. 148, No. 7 A.W.G. conductor, the resistance of which is 0.508 ohm per 1,000 ft., is used between loads B and A, and a No. 4 A.W.G. conductor, the resistance of which is 0.253 ohm per 1,000 ft., is used between load A and the bus-bars. The voltage at the bus-bars is 600 volts, and the currents remain unchanged. Determine: (a) the voltage at load A; (b) the voltage at load B; (c) the kilowatts of loads A and B, and the kilowatts supplied by the bus-bars; (d) the copper loss in each feeder; (e) the efficiency of transmission.

150. A feeder from a direct-current substation supplies two concentrated loads as in Fig. 150A. A load of 700 amp. is located 800 ft., in terms of length of feeder, from the bus-bars, and the feeder to this load consists of a two-conductor, 1,500,000-cir.-mil cable. (The outgoing and the return conductor each has a cross-section of 1,500,000 cir. mils.) At a distance 500 ft. beyond, is a 600-amp. load which is fed by a two-conductor, 1,000,000-cir.-mil cable. The voltage at the 600-amp. load is 230 volts. The resistivity at the operating temperature may be taken as 11 ohms per circular-mil-foot. Determine: (a) the voltage at the 700-amp. load; (b) the voltage at the bus-bars; (c) the total power transmitted; (d) the total power loss; (e) the efficiency of transmission; (f) the voltage at distances of 400 and 1,000 ft. from the bus-bars. (The circular milage is for each conductor.)

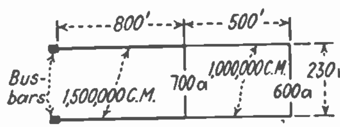


Fig. 150A.

151. In the car system, in Fig. 142A, determine the voltage at each of two cars, one of which is 2 miles from the station and taking 60 amp. and the other 4 miles from the station and taking 75 amp. Also determine efficiency of transmission to the cars.

In the following problems, 152 to 157, assume that the resistance of a circular-mil-foot of copper is 10 ohms. The *normal current density* is 0.001 amp. per circular mil.

152. A 1,200-ft. feeder consisting of two (one positive and one negative) 300,000-cir.-mil copper conductors is supplying a single load at its far end and the sending-end voltage is 250 volts. Determine: (a) the total voltage drop in the feeder if it is operating at the *normal* density of 0.001 amp. per circular mil; (b) the efficiency of transmission under these conditions.

153. If the voltage drop (Prob. 152) is limited to 18 volts, determine the size feeder which is necessary. Also determine the circular mils per ampere and the efficiency of transmission.

154. A load of 400 amp. is located 1,800 ft. from 240-volt bus-bars and it is desired that the voltage at this load be not less than 228 volts. With the feeder operating at the *normal* density, determine: (*a*) the circular milage of the feeder; (*b*) the total voltage drop; (*c*) the loss in this feeder; (*d*) the efficiency of transmission; (*e*) the size feeder necessary to give 228 volts at the load; (*f*) the loss in this feeder; (*g*) the efficiency of transmission.

155. A 50-amp. load is located 1.5 miles from 600-volt bus-bars. Determine: (*a*) the size wire A.W.G. which should be used if the feeder is to operate at the *normal* density; (*b*) the voltage at the load; (*c*) the line loss; (*d*) the efficiency of transmission; (*e*) the size wire, A.W.G., which must be used if the voltage at the load is to be not less than 550 volts; (*f*) the circular mils of this wire; (*g*) the loss in this feeder; (*h*) the efficiency of transmission. (Note.—Number 10 wire may be assumed as having a cross-section of 10,000 cir. mils; No. 0, 100,000 cir. mils; etc.)

156. A No. 0000 A.W.G. hard-drawn trolley wire is paralleled for 6 miles by a 400,000-cir.-mil feeder. The circular milage of the trolley wire is 211,000. When this parallel system is operated at the *normal* current density, determine: (*a*) total voltage drop in feeder and trolley; (*b*) the power loss in feeder and trolley. If it is desired to keep the voltage drop in the overhead system within 50 volts, determine: (*c*) the maximum current density at which the system may operate; (*d*) the total power loss in the overhead system.

157. It is desired to supply power to an 80-hp. motor, 3,000 ft. from 600-volt bus-bars, the minimum permissible voltage at the motor being 550 volts. The motor efficiency at rated load is 0.80. Determine: (*a*) the size of copper conductor if the copper operates at the *normal* current density; (*b*) the size of copper conductor necessary to give 550 volts at the load; (*c*) the total power loss in (*b*); (*d*) the efficiency of transmission.

QUESTIONS ON CHAPTER III

Battery Electromotive Forces—Kirchhoff's Laws

1. What is the effect on the terminal voltage of a battery of applying a load to the terminals? Explain. Why does the e.m.f. of a cell differ from the terminal voltage? Under what conditions are they the same?

2. Discuss the possibility of making a *direct* measurement of the internal voltage of a cell when it is delivering current. How may this internal voltage be calculated if the battery resistance be known?

3. To what factors is the internal resistance of a battery due? Is this resistance a constant quantity?

4. Show the method of calculating the current delivered to an external resistance if the e.m.f. and the resistance of a battery be known. If the battery becomes short-circuited what value of current does it deliver? Account for the energy that the cell develops under these conditions.

5. When a battery has constant e.m.f. and constant internal resistance, for what value of external resistance is the power delivered a maximum?

What is the efficiency under these conditions? Discuss the advisability of operating a battery so that it delivers maximum power. Explain.

6. Under what conditions may a battery be made to receive electrical energy? What relation does the direction of current flow bear to its direction when the battery delivers energy? The voltage of a generator is equal to that of a battery. Discuss any effects which may be noted when the generator is connected to the battery, terminals of like polarity being connected together. What effect is noted when the generator voltage is raised above this value? Explain what is meant by the battery "floating."

7. Before current can be made to flow into the positive terminal of a battery, what voltage must be applied? Explain why any voltage in excess of the battery e.m.f. is effective in causing the flow of current. Give a very common illustration of a battery's receiving energy.

8. If several cells are connected in series, what is the resultant e.m.f. of the combination? What is the resultant resistance of the combination? Give the method of computing the current if the external resistance be known.

9. Under what conditions do batteries operate most satisfactorily in parallel? What is the e.m.f. of the combination under these conditions? What is the relation between the external current and the current in the individual cells? What is the relation between the total battery resistance and the resistances of the individual cells?

10. With batteries in parallel having equal e.m.fs. but unequal internal resistances, explain how the resistance of the entire battery may be found. What relation does the current delivered by each cell bear to the resistance of that cell? What relation exists among the terminal voltages of individual cells when connected in parallel?

11. Show a series-parallel grouping of cells. How is the voltage of the entire battery determined? How may the resistance of the battery be found if the resistance of the individual cells be known? Derive the equation which gives the current in an external circuit if the external resistance, the e.m.fs., and resistances of the individual cells and their arrangement be known.

12. In general, how should cells be grouped to obtain the best economy? How should cells be grouped to obtain the maximum power output?

13. Derive the equation which gives the e.m.f. and the internal resistance of a battery, equivalent to a number of batteries in parallel having different e.m.fs. and resistances.

14. Discuss the principle on which are based the "*floating*"-*battery* method and the *circulatory-current* method for determining the division of current among unequal batteries in parallel.

15. Analyze the "*floating*"-*battery* method of analysis, showing the method of determining the total current in each battery

16. Repeat Question 15 for the *circulatory-current* method.

17. State the two fundamental principles which are enunciated in Kirchhoff's two laws. If several currents meet at a junction, how should their directions of flow be taken into account?

18. How should a rise in potential be represented? A drop in potential? When passing from a negative to a positive terminal of a battery, what should be the sign of the potential change and why? When passing from positive to negative? When passing through a resistance in the direction of the current does a rise or a drop in potential occur? What, then, would be the proper sign to use? When passing through the resistance in opposition to the current, what sign should be used? Explain.

19. If the assumed direction of a current in a network is in error, how is this fact indicated in the result?

20. In applying Kirchhoff's first law to a network, what rule regarding every current must be followed? In applying Kirchhoff's second law to a network, what rule regarding each path through the network must be followed?

21. Outline the procedure by which Kirchhoff's two laws may be systematically applied to a network.

22. Discuss the method by which the number of equations for any network may be reduced.

23. Discuss the limitations in the application of Kirchhoff's laws to railway and other power systems. In what manner do these circuits differ from the usual battery circuits in which the e.m.fs. and resistances only are given?

24. Show how an electric network may be simplified, and its solution rendered comparatively easy by (*a*) substituting star meshes for delta meshes; (*b*) substituting delta meshes for star meshes.

PROBLEMS ON CHAPTER III

Battery Electromotive Forces—Kirchhoff's Laws

158. The open-circuit e.m.f. of a dry cell is 1.35 volts. When the cell delivers a current of 1.2 amp., its terminal voltage drops to 1.29 volts. What is its apparent internal resistance?

159. The terminal voltage of a gravity cell is 0.96 volt when it delivers a current of 2.4 amp. If its internal e.m.f. is 1.07 volts, what is its apparent internal resistance?

160. A starting battery consists of three lead cells connected in series. On open circuit, the e.m.f. of the battery is 6.30 volts. When it delivers a current of 90 amp., its terminal voltage drops to 5.20 volts. Determine: (*a*) its internal resistance; (*b*) the terminal voltage when the battery delivers 50 amp.

161. A battery of six dry cells in series has an open-circuit e.m.f. of 6.5 volts and a total internal resistance of 0.44 ohm. When an external load is applied, the terminal voltage drops to 6.1 volts. Determine the current which the battery is delivering.

162. The open-circuit e.m.f. of a 50-cell storage battery is 102 volts, and the total internal resistance is 0.06 ohm. (a) Determine the maximum current which the battery can deliver without having the terminal voltage drop below 96 volts. (*b*) What current does it deliver when the terminal voltage is 99 volts?

163. When the external resistance in Prob. 158 is 0.24 ohm, determine: (*a*) the current; (*b*) the total power developed by the battery; (*c*) the power lost in battery heating; (*d*) the power to the external circuit.

164. In Prob. 160 determine: (*a*) the external resistance; (*b*) the total power developed by the battery; (*c*) the power lost in internal heating; (*d*) the power to the external circuit.

165. The e.m.f. of a dry cell is 1.20 volts and the internal resistance is 0.06 ohm. (*a*) For what value of external resistance will the power delivered by the cell be a maximum? Determine: (*b*) the power delivered; (*c*) the power lost within the cell; (*d*) the power to the external circuit.

166. What is the maximum power which the battery in Prob. 160 can deliver to an external resistance? Determine: (*a*) the value of the resistance; (*b*) the power lost within the battery.

167. A starting battery has a total e.m.f. of 6.0 volts and each of the three series-connected cells has an internal resistance of 0.006 ohm. Determine: (*a*) the maximum power which the battery can deliver to the starting motor, including the resistance of the leads; (*b*) the value of current under these conditions; (*c*) the terminal voltage of the battery.

168. Determine: (*a*) the terminal voltage when the battery of Prob. 160 is being charged at the 18-amp rate; (*b*) the power delivered to that battery; (*c*) the power lost in heating the battery; (*d*) the chemical energy being stored each second.

169. Determine: (*a*) the terminal voltage necessary to charge the storage battery of Prob. 162, at the 100-amp. rate; (*b*) the total power delivered to the battery; (*c*) the power loss in the battery; (*d*) the chemical energy being stored each second.

170. A storage battery consists of 118 cells, connected in series. Each cell has an e.m.f. of 2.10 volts and an internal resistance of 0.0012 ohm. Determine: (*a*) the voltage necessary to charge the battery at a 60-amp. rate; (*b*) the total power delivered to the battery and the power loss in the battery; (*c*) the terminal voltage when disconnected from the charging source and a resistance of 3 ohms is connected across the terminals.

171. A storage battery consists of 60 cells, each of which has an e.m.f. of 2.10 volts and an internal resistance of 0.0008 ohm. This battery is charged at the 80-amp. rate from 240-volt bus-bars. Determine: (*a*) the terminal voltage of the battery under these conditions; (*b*) the resistance which must be connected in series with the battery; (*c*) the charging current if the bus-bar voltage should fall to 230 volts.

172. Three dry cells with e.m.fs. of 1.35, 1.30, and 1.28 volts are connected in series aiding, and in series with an external resistance of 25 ohms. The resistances of the cells are 0.08, 0.09, and 0.13 ohm. Determine: (*a*) the current; (*b*) the terminal voltage across each cell; (*c*) the internal power developed by each cell; (*d*) the internal power loss in each cell; (*e*) the power delivered to the 25-ohm resistance.

173. In Prob. 172 the 25-ohm resistance is short-circuited. Repeat (*a*) to (*d*) inclusive.

174. The e.m.fs. of four storage cells, connected in series, are 2.00, 2.05, 2.10, and 2.08 volts. The resistances are 0.001, 0.0008, 0.0012, and 0.001 ohm. When the battery delivers a current of 25 amp., determine: (*a*) the terminal voltage of the battery; (*b*) the terminal voltage of each cell; (*c*) the internal power developed by each cell; (*d*) the power loss in each cell; (*e*) the external resistance; (*f*) the power to the external resistance.

175. A station battery consists of 120 storage cells in series, each of which has an e.m.f. of 2.15 volts and an internal resistance of 0.0004 ohm. In order to test the battery, a resistance of 0.812 ohm is connected across its terminals. Determine: (*a*) the current; (*b*) the power delivered by the battery; (*c*) the terminal voltage; (*d*) the power loss in internal heating.

176. In Fig. 176*A* are shown three cells connected in series and in series with a 2.75-ohm resistance. Determine: (*a*) the current I; (*b*) the power p_1, p_2, p_3 developed in each cell; (*c*) the power P_1, P_2, P_3 delivered by each cell; (*d*) the power lost in each cell; (*e*) the voltage v_1, v_2, v_3 across each cell; and (*f*) the voltage V across the resistance. (If a cell is absorbing energy, the power developed is negative.)

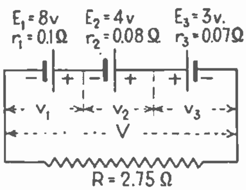


FIG. 176*A*.

177. Five dry cells have the following e.m.fs. and internal resistances: 1.40, 1.27, 1.38, 1.32, and 1.23 volts; 0.09, 0.14, 0.12, 0.13, and 0.12 ohm. All are connected in series aiding and in series with an external resistance of 21.4 ohms. Determine: (*a*) the current; (*b*) the power loss in each cell; (*c*) the terminal voltage of each cell; (*d*) the voltage across the external resistance.

178. Repeat Prob. 177 reversing the third cell, that is, the cell whose e.m.f. is 1.38 volts and internal resistance is 0.12 ohm.

179. Four dry cells, each of which has an e.m.f. of 1.20 volts and an internal resistance of 0.08 ohm, are connected in parallel with terminals of like polarity together. Determine: (*a*) the current to an external resistance of 1.98 ohms; (*b*) the terminal voltage of the battery; (*c*) the internal loss in each cell; (*d*) the power to the external circuit.

180. For Prob. 179 determine: (*a*) the maximum power which the battery can deliver; (*b*) the terminal voltage of the battery under these conditions; (*c*) the value of the external resistance.

181. Six batteries, each of which has an e.m.f. of 12 volts and an internal resistance of 0.6 ohm, are connected in parallel with terminals of like polarity together. An external resistance of 23.9 ohms is connected across the terminals. Determine: (*a*) the current to the external circuit; (*b*) the terminal voltage of the battery; (*c*) the power loss in each battery.

182. Each of four dry cells has an e.m.f. of 1.30 volts, but their internal resistances are 0.08, 0.10, 0.09, and 0.09 ohm. These cells are connected in parallel with terminals of like polarity together. Determine: (*a*) the e.m.f. and internal resistance of a single cell that would replace the battery; (*b*) the current which the battery delivers to an external resistance of 0.60 ohm; (*c*) the current delivered by each cell.

183. In Prob. 182 determine: (*a*) the maximum power which the battery will deliver; (*b*) the corresponding external resistance.

184. Each of two batteries has an e.m.f. of 18 volts, but the internal resistance of one is 0.8 ohm and that of the other is 1.0 ohm. These batteries are connected in parallel with terminals of like polarity together. When the external current is 4.0 amp., determine: (*a*) the current delivered by each battery; (*b*) the terminal voltage of the battery; (*c*) the value of the external resistance; (*d*) the e.m.f. and resistance of a single battery which would replace these two in parallel.

185. A battery consists of four storage cells connected in parallel, each having an e.m.f. of 2.10 volts. The internal resistances of these cells are 0.005, 0.006, 0.004, and 0.0025 ohm. Determine: (*a*) the e.m.f. and resistance of a single equivalent cell; (*b*) the current when the terminal voltage is 1.95 volts; (*c*) the current in each cell.

186. Twenty-four dry cells are arranged in rows of six in series, and the four rows in parallel. The e.m.f. of each cell is 1.25 volts and the resistance of each is 0.1 ohm. Determine: (*a*) the total battery e.m.f.; (*b*) the total resistance; (*c*) the current to an external resistance of 2 ohms; (*d*) the terminal voltage.

187. Repeat Prob. 186 with the cells arranged in rows of four in series, and six rows in parallel.

188. Arrange the cells of Prob. 186 so that the maximum power may be supplied to a load resistance of 0.6 ohm. Under these conditions, determine the power to the resistance and that lost in the battery.

189. A certain load is such that the potential difference at its terminals must not be less than 6.0 volts. Twelve storage cells, each having an e.m.f. of 2.1 volts and a resistance of 0.002 ohm, are available. (*a*) How should these be connected so that the maximum efficiency is obtained? When the load requires 100 amp., determine: (*b*) the battery terminal voltage; (*c*) the load resistance; (*d*) the battery efficiency.

190. Arrange the cells in Prob. 189 so that the maximum current will be delivered to the load resistance. What is the efficiency of the battery under these conditions?

191. The total loop resistance of a telegraph circuit is 20 ohms, and the resistance of a series-connected relay is 80 ohms. Forty gravity cells are available. Each has an e.m.f. of 1.1 volts and an internal resistance of 0.25 ohm. Determine: (*a*) the method of connecting these cells in order that they may deliver the maximum power to the circuit; (*b*) the battery efficiency; (*c*) the efficiency of the telegraph line; (*d*) the method of connection in order that the maximum current may be delivered.

192. The e.m.f. and internal resistance of one of two starting batteries are 6.30 volts and 0.007 ohm, and the e.m.f. and resistance of the other are 6.10 volts and 0.004 ohm. These batteries are connected in parallel with terminals of like polarity together. Determine: (*a*) the e.m.f. and internal resistance of a single equivalent battery; (*b*) the current to an external-load resistance of 0.20 ohm; (*c*) the current in each battery; (*d*) the terminal voltage.

193. The e.m.f. and internal resistance of two batteries *A* and *B* are 25 volts and 0.4 ohm, and 24 volts and 0.6 ohm, respectively. These batteries are connected in parallel with terminals of like polarity together (see Fig. 195*A*). Determine: (*a*) the e.m.f. and internal resistance of a single equivalent battery: (*b*) the current delivered by each battery when the external current is 5 amp.; (*c*) the terminal voltage; (*d*) the external resistance.

194. The e.m.fs. and internal resistances of two storage batteries in parallel, with like terminals connected together, are 120 volts and 0.04 ohm, and 122 volts and 0.06 ohm, respectively. Determine: (*a*) the e.m.f. and internal resistance of a single equivalent battery; (*b*) the current when the load resistance is 1.0 ohm; (*c*) the terminal voltage; (*d*) the current in each battery; (*e*) the power developed by each battery; (*f*) the power delivered by each battery.

195. In Fig. 195*A* are shown the two batteries of Prob. 193 with a switch *S* in series with *B*. The division of the load current of 5 amp. to the external circuit *R* is to be determined by means of the "*floating*"-*battery* method (p. 71). (*a*) With the switch *S* open (and *S'* closed), determine the current which battery *A* must deliver in order that battery *B* may just "float" when switch *S* is closed. (*b*) Determine the division of the remaining load current between the two batteries; (*c*) determine the total current delivered by each battery. (Compare results with those obtained in Prob. 193.)

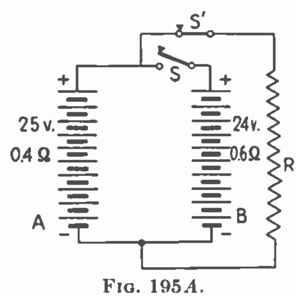


FIG. 195*A*.

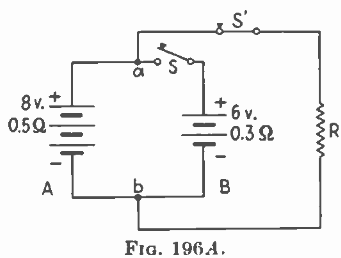


FIG. 196*A*.

196. In Fig. 196*A* are shown two batteries *A* and *B* in parallel, with terminals of like polarity connected together, and having e.m.fs. and internal resistances of 8 volts and 0.5 ohm, and 6 volts and 0.3 ohm, respectively. A switch *S* is in series with battery *B*. The external-load resistance *R* takes 10 amp. It is desired to determine the division of this current between the two batteries by means of the floating-battery method (see p. 71). (*a*) With switch *S* open (and *S'* closed), determine the current which must be delivered by A in order that B may float when switch S is closed. Determine: (*b*) the division of the remainder of the load current between the two batteries; (*c*) the total current delivered by each battery; (*d*) the load resistance R.

197. In Prob. 194, determine by means of the floating-battery method, when the external current is 100 amp: (*a*) the current in each battery: (*b*) the terminal voltage; (*c*) the load resistance.

198. Two storage cells, having e.m.fs. of 2.10 and 2.20 volts and internal resistances of 0.025 and 0.035 ohm, are in parallel with terminals of like polarity connected together. Determine: (*a*) the total current delivered by each cell when a load of 10 amp. is connected across the terminals; (*b*) the power lost in each cell; (*c*) the power delivered by each cell; (*d*) the load resistance.

199. Two batteries A and B (Fig. 199A), having e.m.fs. of 51 and 49 volts, and internal resistances of 0.14 and 0.06 ohm, are connected in parallel with terminals of like polarity together. The current taken by R when the switch S is closed is 280 amp. The division of current between the batteries is to be determined by the circulatory-current method (p. 71). Determine: (*a*) the circulatory current with switch S open (see p. 71); (*b*) the currents, with proper sign, delivered by A and B; (*c*) the additional current taken by A and B when switch S is closed; (*d*) the total current delivered by A; (*e*) the total current delivered by B; (*f*) the terminal voltage across ab; (*g*) the resistance R.

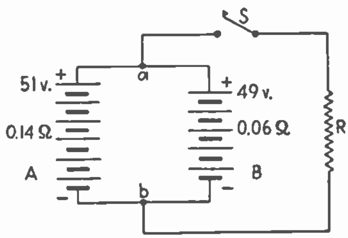


Fig. 199A.

200. Problem 195 is to be solved by the circulatory-current method (p. 71). With switch S closed and S' open (Fig. 195A) determine: (*a*) the circulatory current with its correct sign in batteries A and B. With switch S' closed so that the load resistance R takes 5 amp., determine: (*b*) the division of this current between batteries A and B; (*c*) the total current delivered by A and B; (*d*) the terminal voltage; (*e*) the resistance of R. (Compare results with those obtained in Prob. 195.)

201. Solve the circuit shown in Fig. 196A by the circulatory-current method, that is, with switch S closed and switch S' open at first. The load current is now 12 amp. Determine: (*a*) the current delivered by each battery; (*b*) the terminal voltage; (*c*) the power developed by each battery; (*d*) the power loss in each battery.

202. Solve Prob. 199 by the circulatory-current method with the e.m.f. of battery A equal to 48 volts, and that of battery B equal to 50 volts. The external-load current is still 280 amp. and the resistance of each battery remains unchanged. Determine: (*a*) the current delivered by each battery; (*b*) the terminal voltage; (*c*) the heating in each battery; (*d*) the resistance of R.

Kirchhoff's Laws

203. Two batteries A and B (Fig. 203A), having e.m.fs. of 5.0 and 4.0 volts and resistances of 1.2 and 1.0 ohm, are connected in parallel, positive terminal to positive terminal. A resistance R of 2.5 ohms is connected across the common terminals. Determine: (a) the current in resistance R; (b) the current in each battery; (c) the terminal voltage of the combination.

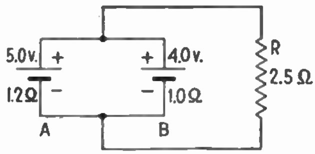


FIG. 203A.

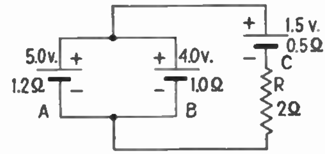


FIG. 204A

204. In Fig. 204A is shown the circuit of Fig. 203A with the addition of a third battery C, e.m.f. 1.5 volts and resistance 0.5 ohm, in series with the 2-ohm resistance R. Determine: (a) the currents in batteries A, B, C; (b) the common terminal voltage of batteries A and B; (c) the terminal voltage of battery C.

205. In Fig. 205A is shown an electric network consisting of three batteries A, B, C and three resistances R_1, R_2 R_3. Find: (a) the currents I_1, I_2, I_3 with their proper signs; (b) the terminal voltages of batteries A, B, C.

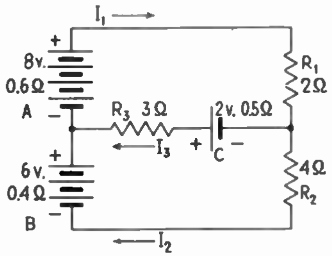


FIG. 205A.

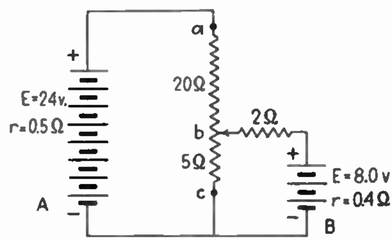


FIG. 207A.

206. Solve Prob. 205 with battery C reversed.

207. In the battery system in Fig. 207A, the e.m.f. of battery A is 24 volts and its internal resistance r is 0.5 ohm. A resistance abc of 25 ohms is connected across the battery terminals, the end a being connected to the positive terminal. The e.m.f. of battery B is 8 volts and its internal resistance r is 0.4 ohm. Its negative terminal is connected to the negative terminal of battery A and to the end c of the resistance. Its positive terminal is connected through a 2-ohm resistance to the point b in the resistance abc, where resistance bc = 5 ohms. Determine: (a) the currents in batteries A and B; (b) the point to which b should be moved (in terms of ohms from b to c) in order that the current in battery B may be zero. The total resistance ac remains unchanged.

208. In Fig. 208*A* is shown a network in which the three currents I_1, I_2, I_3 all flow toward the common junction at *a*. Determine I_1, I_2, I_3.

209. Repeat Prob. 208 with the polarity of battery *D* reversed.

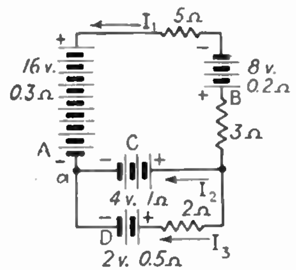


FIG. 208*A*.

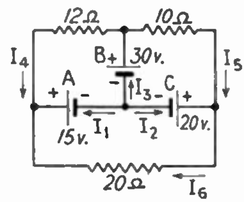


FIG. 210*A*

210. In Fig. 210*A* is shown an electric network in which there are three batteries *A*, *B*, and *C* having e.m.fs. of 15, 30, and 20 volts and negligible internal resistances. Determine the six network currents, I_1 to I_6 inclusive, and show their values and actual directions on the diagram.

211. In Fig. 211*A* is shown a railway system with two cars *A* and *B* taking 80 and 50 amp. For a portion of its length there is a feeder in parallel with the trolley. The voltage at the station bus-bars is 600 volts. The resistances of each part of the overhead and track systems are given. Determine the voltage at each car.

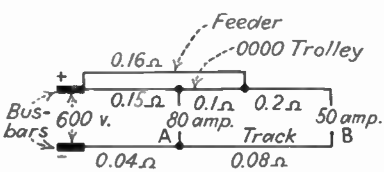


FIG. 211*A*.

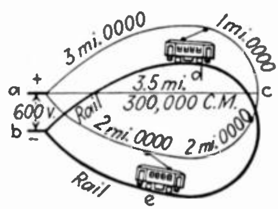


FIG. 213*A*.

212. Repeat Prob. 211 with car *A* at the junction of the feeder and the trolley. The track resistance from station to junction is 0.06 ohm, and from junction to end of line is also 0.06 ohm.

213. In Fig. 213*A* is shown the wiring diagram of a loop trolley system with a single 300,000-cir.-mil feeder. A potential difference of 600 volts is maintained between trolley and rail at the bus-bars *ab*. The No. 0000 trolley wire has a resistance of 0.265 ohm per mile and the 300,000-cir.-mil feeder has a resistance of 0.187 ohm per mile and is connected from *a* to *c*, a distance of 3.5 miles. A car at *d*, 3 miles from the station, takes 80 amp.; and a car at *e*, 2 miles from the station on the other side of the loop, takes 60 amp. Neglecting the resistance of the rail, determine: (*a*) the voltage at the car *d*; (*b*) the voltage at the car *e*.

214. Repeat Prob. 213 when the car at *e* has reached *c* and the car at *d* has moved 1 mile toward *a*. The currents to the cars remain unchanged.

215. Solve Prob. 213, assuming the resistance of the rail to be 0.08 ohm per mile.

216. Solve Prob. 214, assuming the resistance of the rail to be 0.08 ohm per mile.

217. Determine the voltage at each car when car *d* takes 70 amp. and car *e* takes 80 amp., both being in the positions shown in Fig. 213*A*. The resistance of the rail (0.08 ohm per mile) is to be taken into consideration.

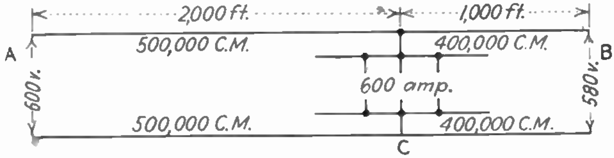


FIG. 218*A*.

218. Two substations *A* and *B* feed into the same distributing center *C* (Fig. 218*A*). The voltage at the bus-bars of station *A* is maintained constant at 600 volts and that at station *B* is maintained at 580 volts. Station *A* feeds a distance of 2,000 ft. through 500,000-cir.-mil cable and station *B* a distance of 1,000 ft. through 400,000-cir.-mil cable to a distributing center *C*. The load at the distributing center is 600 amp. Determine: (*a*) the current supplied by each station; (*b*) the power supplied by each station; (*c*) the power delivered at the distributing center *C*. The resistance of a circular-mil-foot may be taken as 10 ohms.

219. Figure 219*A* shows a 240-volt power-distribution system. The voltage at the substation *AA′* is maintained constant at 240 volts. The

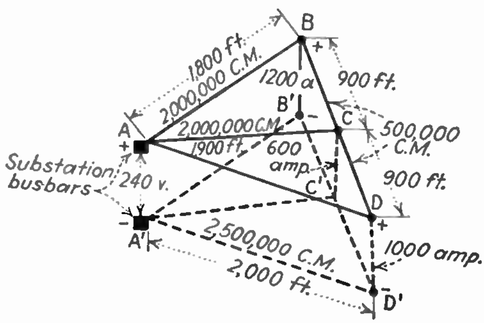


FIG. 219*A*.

positive conductor of a radial feeder extends from *A* to each of the distributing centers *B*, *C*, *D*. The feeder to *B* is 1,800 ft. long and 2,000,000 cir. mils equivalent; that to *C* is 1,900 ft. long and 2,000,000 cir. mils equivalent; that to *D* is 2,000 ft. long and 2,500,000 cir. mils equivalent (per wire in every case). A tie line, 900 ft. long and of 500,000-cir.-mil cross-section connects *B* and *C*, and another similar line connects *C* and *D*. The negative side of the system, which is in every way identical with the positive side, is shown by dashes in the figure. Across BB' there is connected a load of 1,200 amp., across CC' a load of 600 amp., and across DD' a load of 1,000 amp. Find the voltage at each of the distributing centers BB', CC', DD'. (The resistance of a circular-mil-foot may be taken as 10 ohms.)

Equivalent Delta and Star Systems

220. In Fig. 220*A* is shown a Y-combination of resistances in which $AO = 12$ ohms, $BO = 18$ ohms, and $CO = 20$ ohms. Determine and construct the equivalent delta having terminals *ABC*. Verify the correctness of the delta by computing the resistances between different terminals and comparing them with the corresponding resistances in the Y-combination.

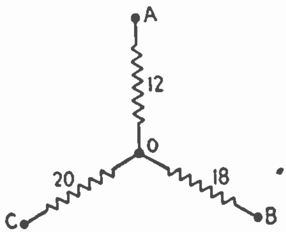


FIG. 220*A*.

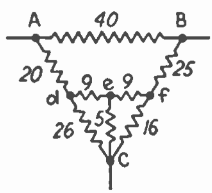


FIG. 221*A*.

221. In Fig. 221*A* is shown an electric network *ABC*. The numbers in the figure designate the ohms resistance of each member. By the use of the delta-star relation, determine the resistances between terminals *AB*, *BC*, *CA*. With 10 volts impressed across the terminals *AC*, determine the current in member *de*.

222. In Fig. 222*A* is shown an electric network between the terminals *AB*. The numbers in the figure designate the ohms resistance of each member. Simplify the network by converting the Y-sections *Ab*, *Ob*, *Bb* and *Aa*, *Oa*, *Ba* into equivalent deltas. (*a*) Determine the resistance between the terminals *AB*. (*b*) With 100 volts impressed across *AB*, determine the current in the members *AO* and *OB*.

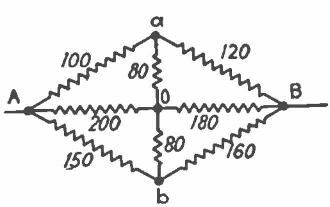


FIG. 222*A*.

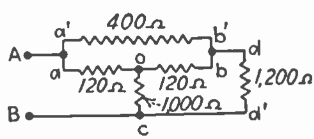


FIG. 223*A*.

223. In the network in Fig. 223*A* determine the resistance between the terminals *A* and *B* and the current in each member when the voltage across AB is 20 volts. (A simple solution is to convert the Y-system ao, bo, co into an equivalent delta.)

224. In Fig. 50 (p. 88) determine the current in the 10-ohm member when the voltage across the terminals AB is 10 volts.

225. In Fig. 225A determine the current in the 800-ohm resistance when 10 volts is impressed across terminals AB.

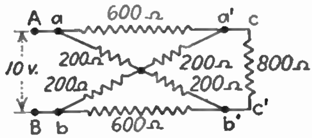


FIG. 225A.

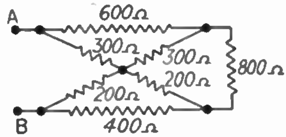


FIG. 226A.

226. In Fig. 226A determine the resistance between terminals AB and find the current in the 800-ohm member when 10 volts is impressed across terminals AB.

QUESTIONS ON CHAPTER IV

Primary and Secondary Batteries

1. State the reactions which occur when two copper strips are immersed in a dilute sulphuric acid solution and a voltmeter is connected between them. Repeat when the two copper strips are replaced by two zinc strips; by two lead strips. In general, what conditions are necessary in order that there may be an e.m.f. between the strips?

2. With dissimilar metals as electrodes, discuss the possibilities of obtaining an e.m.f. if the sulphuric acid be replaced by some other solution. Name three solutions which would cause an e.m.f. under these conditions.

3. What is meant by one metal being electrochemically positive to another? If metal A is electrochemically positive to metal B, what will be the direction of the current flow between them within the cell? What will be the direction of the current flow between them through the external circuit?

4. Define electrode, cathode, anode.

5. In what form is the energy stored within the cell? What changes take place in the electrodes when the cell delivers current? Distinguish between a primary cell and a secondary cell.

6. Name four requirements for a satisfactory primary cell.

7. Discuss the nature of the internal resistance of a cell. In what manner may this resistance be reduced? In what way does increasing the size of the elements of a cell affect its current capacity? Its e.m.f.?

8. What voltage does a voltmeter indicate when it is connected to the terminals of a cell which is open-circuited? If the circuit is suddenly closed, to what is the initial voltage drop due? To what is the excess drop over this initial drop due? Explain the part that hydrogen plays in polarization. Describe two general methods of reducing polarization.

9. Discuss the change with time of the terminal voltage of a cell under constant-resistance load. Discuss the recovery of the cell when the load is removed.

10. Describe the construction of the Daniell cell, stating the materials of the electrodes and the electrolytes. For what type of work is it best adapted? What is the e.m.f. of this cell?

11. In what way does the gravity cell differ from the Daniell cell? Which electrode requires replacing? What changes occur in the other electrode? What is the cell e.m.f., and for what type of work is the gravity cell best adapted?

12. Describe the Edison-Lalande cell, including the electrolyte and the electrodes. In what way does its electrolyte differ from those in the cells in Questions 10 and 11? What is the chief advantage of this type of cell? What is its e.m.f., and what is its terminal voltage when delivering a current?

13. What materials are used for positive and negative electrodes in the Le Clanché cell, and what is used for the electrolyte? What is the approximate value of the e.m.f.? When planning to use the cell commercially, about what voltage per cell should be allowed? What materials are introduced in the cell to reduce polarization? How is the cell renewed? For what type of work is this cell best adapted?

14. In what way does a dry cell resemble a common type of wet cell? Is a "dry" cell really dry? Of what material is the positive electrode composed? the negative? What is the electrolyte, and how is it placed in the cell? What materials are placed between the carbon and the zinc, and what are their functions?

15. What is the e.m.f. of a dry cell when new? After it has stood idle for some time? What is the magnitude of the internal resistance when new? Discuss the effect of time on the internal resistance. How does the polarization effect compare with the internal-resistance effect? About what current should a good cell deliver on short circuit? What is the approximate value of the terminal voltage when the cell delivers a moderate current?

16. What is the principal cause of the cell becoming exhausted? By what means can the cell be temporarily revived? Name some of the commercial applications of dry cells.

17. What is the function of a Weston standard cell in distinction to the uses made of other types of cells? In practice, what two common electrical quantities are most easily reproduced and maintained as standards? What must be the characteristics of a standard cell? Describe the construction of the Weston cell and its elements. How is its permanency insured? In what way does the saturated cell differ from the normal cell? Why cannot the e.m.f. of the Weston cell be measured with an ordinary voltmeter?

18. In what way is a storage cell renewed when it becomes discharged? What condition concerning the materials of the cell is necessary for proper functioning of the cell? What two general types of storage cells are in commercial use?

19. Describe an elementary experiment which illustrates the underlying principle of the lead cell. State the change that occurs in each of the lead strips. What voltage is observed to exist at different times in the experiment? What gases are evolved, and at which plate is each gas given off?

20. Even though both of its plates are of lead, show that the existence of an e.m.f. in a lead storage cell does not in any way violate the principle governing the e.m.f. of electric cells in general. When the cell is approaching complete discharge, what changes in the materials would account for the approach of the voltage to zero? Account for the 2.5 volts per cell utilized in the process of charging.

21. What change in the electrolyte during charge and discharge of a cell is shown by the chemical equation? Why is a cell composed of plain-lead plates not useful in practice? Give two reasons. Describe briefly the Planté process and describe two commercial plates that are formed by this process.

22. Describe the Faure or pasted process for making battery plates. What are the advantages and disadvantages of pasted plates over Planté plates? What commercial conditions demand a pasted plate and why? How does the life of a pasted plate compare with that of a Planté plate?

23. Describe briefly the construction of the Exide-Ironclad cell and its principal use in practice.

24. What are the two general classes into which lead storage batteries may be divided? What types of plate are best suited for regulating duty and for emergency duty in stationary batteries? Why?

25. What two types of containing tanks are used for stationary batteries? Under what conditions is each used and why? In what manner are the joints and seams in lead-lined tanks made non-leakable? How are the plates suspended in the lead tank? What factors must be considered in designing and installing a lead-lined wooden tank?

26. What types of separators are in general use? Name the advantages and the disadvantages of the wooden type, of perforated hard rubber, of Mipor. For what type of battery is each kind commonly used? What precaution must be taken in handling wood separators? Why? Describe a method of reinforcing the wood separators, used in the better types of starting batteries.

27. What should be the specific gravity of a fully charged battery having Planté plates? pasted plates? What precaution should be taken in diluting sulphuric acid for storage-battery use? What simple device is used for determining specific gravity? How is this device adapted for use with vehicle and portable batteries?

28. What change takes place in the electrolyte during the charging period? What is the effect of gassing on the specific gravity? What change takes place in the specific gravity after the charging has ceased? Explain. How does the specific gravity of the electrolyte change during discharge? What practical use is made of these changes of specific gravity?

29. When a stationary battery of the open type is received, what special attention should be given to the wood separators? In what manner should the jars be installed? How should the plates be placed in position? Why is an initial charge necessary?

30. What happens to the active material in a cell if the cell is allowed to stand idle over long periods? In what way may injury to the battery from this cause be avoided? If it is desired to withdraw a battery from service for an indefinite period, what procedure should be followed?

31. What are the requirements of a portable battery that make its design different from that of a stationary battery? What changes are made in the plates? separators? specific gravity of the electrolyte? How is a battery made up? In what way does a portable battery differ from a stationary battery in the manner of shipment? What special attention should be paid to the electrolyte?

32. In what way is the rating of a storage battery expressed? What is meant by the 8-hr. rate? Can as many ampere-hours be extracted from a cell at the 3-hr. rate as at the 8-hr. rate? To what is any difference due? If a cell is apparently exhausted after discharging at the 3-hr. rate, would it be possible later to extract any further current from it? What can be said of the overload capacity of a storage battery?

33. What general principle governs the charging rate of a battery? When does it become necessary to reduce this current? What are the objections to pronounced gassing in a cell? What is the *finishing rate?*

34. Name a very common example of constant-current method of charging. What care should be taken in the connecting up of the battery? Describe a simple test by which the correct terminal polarity may be ascertained.

35. What is the one great advantage of the constant-potential method of charging? About what voltage per cell is necessary in this method? Why is the use of some resistance desirable?

36. When a battery is just floating on a bus-bar and it is desired to charge it, in what manner may the necessary excess potential for charging be obtained? What proportion of the entire energy necessary for charging is supplied by the booster?

37. What change occurs in the e.m.f. of a cell during the charging period? What corresponding changes occur in the terminal voltage? To what is the difference between the cell e.m.f. and the terminal voltage due? Can it be said that the voltage characteristic of a storage battery is such that its use upon lighting circuits is practicable?

38. What is lost by a lead storage battery during its period of service? With what should this loss be replaced except in rare instances? What circumstances justify the addition of acid to a cell? What care should be taken in the selection of water for use with storage batteries?

39. In what manner can the freezing of the electrolyte in a storage battery be prevented? How does a rise of temperature affect the rating of a storage battery?

40. Compare roughly the kilowatts per pound of plate for a given lead cell at different discharge rates. Repeat for kilowatts per pound of cell.

Compare the above factors for three different types of cell, stating the type of service for which each type is best adapted.

41. In the Edison storage cell, what is the general type of construction of the positive electrode? the negative electrode? What solution is used for the electrolyte? What means are taken to give the electrolyte free access to the active material of the electrodes and to increase the conductivity of the active material?

42. In the chemical reaction that takes place on both charge and discharge, what part does the electrolyte play?

43. On complete charge, what is the material of the positive electrode? of the negative electrode? To what do these materials change on discharge? Discuss the changes, if any, which occur in the electrolyte on charge and on discharge, and compare with the changes which occur in the electrolyte of the lead cell.

44. Describe briefly the mechanical construction and assembly of the Edison cell, stating the method of holding the plates and connecting them with the binding posts. What kind of tank is used for this cell? What is the advantage of this type of construction? For what purpose is the valve necessary and what care does the valve require? How is the battery mounted?

45. In what way does the normal rating of an Edison cell differ from that of a lead cell? What is the voltage per cell? Is it possible to tell accurately the condition of charge by readings of either voltage or specific gravity? How can the condition of complete charge be determined?

46. Why does the level of the electrolyte gradually decrease with service? Discuss the replacement of the electrolyte.

47. State the advantages of the Edison battery over other types of storage batteries. What are some of the commercial applications of this battery and what factors limit its applications?

48. In what terms may the efficiency of a storage battery be expressed? Discuss the ampere-hour efficiency as a true criterion of efficiency.

49. State the reason why the ratio of the kilowatt-hours of discharge at the 3-hr. rate to those of charge at the 8-hr. rate in a lead battery does not give the true efficiency. Give some of the factors which determine the efficiency of a battery.

50. What is the order of magnitude of the kilowatt-hour efficiency of a lead storage battery? the ampere-hour efficiency? Why do the two differ? In what manner does the cycle of operation of a storage battery affect the efficiency?

51. State some of the factors which govern the selection of a storage battery for a particular purpose.

52. In what manner may water be made a good conductor? Define *electrolysis* and *electrolyte.* What are the products of the electrolysis of water and where is each released?

53. Define *electrolytic dissociation, ion, positive ions* or *cations, negative ions* or *anions.* Relate the movement of the ions to the electric current.

54. Analyze the electrolysis of copper sulphate, showing that sulphuric acid is formed. Give examples of industrial applications of electrolysis.

55. State briefly Faraday's two laws of electrolysis. In simple language, explain their meaning.

56. State a simple method of producing copper-plating upon a carbon brush such as is used with generators. Which electrode is connected to the positive terminal of the supply and which is connected to the negative terminal? When copper is used in connection with a copper sulphate solution, is there any marked change in the electrolyte? Explain.

57. Can copper be plated from a solution in which neither electrode is copper? What voltages in the plating bath must the supply voltage overcome? How are these voltages reduced to a minimum? Is electroplating a high-voltage or a low-voltage process? In what way are plating baths connected, when possible?

58. Show how the gravity cell is an electroplating bath which supplies its own electroplating current.

59. Describe briefly the process of electrotyping.

PROBLEMS ON CHAPTER IV

Nov 19

Primary and Secondary Batteries

227. A Daniell cell has an e.m.f. of 1.08 volts and an internal resistance of 0.25 ohm. (*a*) What is the maximum current which it can deliver? The size of the cell is increased in such a manner that the plate area is doubled. (*b*) What is the new e.m.f.? (*c*) What is the approximate maximum current which the cell can now deliver? (*d*) Determine the maximum power which this last cell can deliver.

228. Two gravity cells have electrodes of the same materials, and solutions of the same kind, concentration, etc., but one cell has each linear dimension twice that of the other, making its volume eight times greater. The two cells are connected with terminals of like polarity together. The e.m.f. of the smaller cell is 1.04 volts and its internal resistance 0.3 ohm; the internal resistance of the larger cell is 0.1 ohm. (*a*) How much current flows between the two cells? Give reasons for the answer. (*b*) What is the short-circuit current of the larger cell? (*c*) What is the maximum power that the combination can deliver to a resistance connected across the cells in parallel?

229. A Le Clanché cell has an e.m.f. of 1.46 volts on open circuit. A load of 2 amp. is applied and the terminal voltage drops to 1.28 volts almost instantly. After a lapse of some time it drops to 1.06 volts, the current remaining at 2 amp. What is the actual internal resistance of the cell and what is the *e.m.f. of polarization?* What is the total apparent cell resistance?

230. It is desired to use Le Clanché cells, similar to those of Prob. 229, to operate a watchman's signal system. A current of approximately 0.25 amp. is required and the total circuit resistance is 54.4 ohms. What is the minimum number of cells that can be used to supply this current and how should they be connected? From data given in Prob. 229 compute the apparent internal resistance of the battery, taking polarization into consideration and then determine the actual current in the signal system.

231. Assume that gravity cells are to be used in operating the signal system (Prob. 230). Each cell has an e.m.f. of 1.08 volts and an internal resistance of 0.12 ohm. (*a*) How many cells should be used and how should they be connected? (*b*) What should be the value of the external resistance in order that the maximum power may be drawn from the battery?

232. There are four series relays in a telegraph system, each of which requires 50 milliamp. for its operation. Each relay has a resistance of 120 ohms and the total line resistance is 378 ohms. (*a*) How many gravity cells, similar to those of Prob. 231, are necessary? (*b*) How should they be connected?

233. The ignition system of a gasoline engine requires a voltage somewhat in excess of 6 volts for its operation. (*a*) How many dry cells, connected in series, should be used? (*b*) If each cell has an e.m.f. of 1.25 volts and an internal resistance of 0.1 ohm, how much current can the battery deliver without its terminal voltage falling below 6 volts? (*c*) What should be the resistance of the ignition coil if it is to operate at this value of current and at 6 volts?

234. The open-circuit e.m.f. of a dry cell is 1.05 volts and the short-circuit current is found to be 6.0 amp. (*a*) What is its internal resistance? (*b*) What does this test show as regards the condition of this cell?

235. Each of five dry cells of a six-cell ignition battery has an e.m.f. of 1.5 volts and an internal resistance of 0.08 ohm. The cells are connected in series aiding. One cell, which has become exhausted, has an e.m.f. of 0.9 volt and an internal resistance of 0.9 ohm. (*a*) When the battery delivers 100 milliamp., what is the voltage across the defective cell? (*b*) When the battery is short-circuited, what current does the battery deliver? (*c*) What is the voltage, with its proper sign, across the defective cell under the conditions of (*b*)?

236. A certain flashlight has a 3-cp. lamp and is operated by three small dry cells, each of which has an e.m.f. of 1.3 volts and an internal resistance of 0.2 ohm. Assuming that the efficiency of the lamp is 1.2 watts per candlepower, what current does it take, and what is the terminal voltage of the battery when the lamp is delivering its rated candlepower? (NOTE.—Two values of current can be found to satisfy these conditions, but the more rational value should be used.)

237. A Weston cell having an e.m.f. of 1.0183 volts is connected to a potentiometer wire as shown in Fig. 237*A*, in order to calibrate the wire *AC*. Between *A* and *B* is a resistance of 0.915 ohm, and 10 coils having a resistance of 5 ohms each. The resistance of the cell is 180 ohms and the resistance of the galvanometer is 100 ohms. (*a*) When the current in *AB* is 0.019 amp., determine the current in the galvanometer and show its direction. (*b*) Repeat (*a*) with the current equal to 0.0205 amp. (*c*) For what value of current in the wire *AC* will the current through the cell and galvanometer be zero? (*d*) Under these conditions, what will be the voltage across each of the 5-ohm coils? (This problem illustrates the principles of the potentiometer (see Par. 125, p. 181, and Fig. 117, p. 182).)

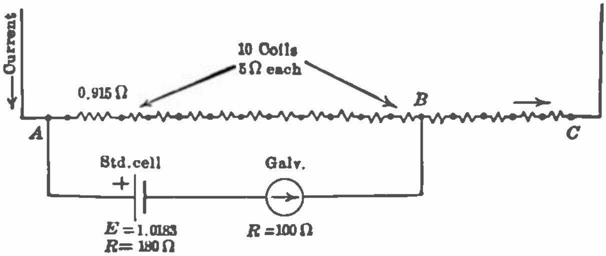


FIG. 237A.

238. In Prob. 237 determine (*a*) the current in the galvanometer when the current in the wire *AB* is 0.0195 amp. In the process of adjustment this current is increased 2 per cent. (*b*) What is the corresponding percentage change in the galvanometer current? (Note that this arrangement gives a very sensitive method of adjustment.)

239. A voltmeter, the resistance of which is 1,000 ohms, is used in an attempt to measure the e.m.f. of the Weston cell in Prob. 237. What will the voltmeter read? Is this a practicable method of using the Weston cell as a standard?

240. It is desired to dilute a liter of concentrated sulphuric acid (sp. gr. = 1.84) to make acid having a specific gravity of 1.20. Determine: (*a*) the liters of water which it is necessary to add; (*b*) the total volume of acid when the solution is mixed; (*c*) the total weight of the final solution in kilograms. (The table (p. 112) must be used. Solving the problem algebraically does not give the correct result, owing to the interdiffusion of the molecules of water and acid.)

241. A certain carboy contains 10 l. of concentrated sulphuric acid (sp. gr. = 1.84). (*a*) How many kilograms does the acid weigh? (*b*) How many liters of dilute sulphuric acid (sp. gr. = 1.200) can be made from it? (*c*) To how many gallons is this equivalent? (See Appendix A.) (*d*) What is the weight in kilograms of the final solution?

242. Immediately after a certain battery has been completely charged, the specific gravity is found to be 1.210. Each battery jar then contains 2.4 l. of electrolyte. Due to evaporation, the specific gravity in time becomes 1.230. (*a*) How much water by volume (liters) must now be added to each cell to bring the specific gravity back to 1.210? (*b*) Determine the weight in kilograms of the electrolyte before and after adding water.

Due Nov 26

243. The specific gravity of the electrolyte in the pilot cell of a 640-amp.-hr. storage battery is found to be 1.165 when the battery is in the discharged (not totally) condition. The specific gravity varies in accordance with the charge characteristics (Fig. 71, p. 115). The battery is then charged at the normal 8-hr.-rate of 80 amp. (*a*) When the specific gravity reaches 1.190 (after the charge is stopped sufficiently long to permit the specific gravity to become constant) how many ampere-hours have been put into the battery? (*b*) How many more ampere-hours are necessary to charge the battery completely to its rating?

244. After the battery (Prob. 243) has been completely charged, it is discharged until its specific gravity becomes 1.180. Theoretically, how many available ampere-hours remain in the battery? (See discharge characteristic, Fig. 71, p. 115.)

245. A certain battery with Planté plates, after being completely charged, (that is, the terminal voltage reaches 2.55 volts) is able to deliver 50 amp. for 8 hr. before being completely discharged. How many ampere-hours would it deliver under similar conditions, except that the rate is adjusted so that the battery is apparently discharged in 5 hr.? What is the discharge rate under these conditions? Repeat for complete discharge in 3 hr. (see p. 118).

246. Repeat Prob. 245 for pasted plates. A 4,480-amp.-hr. Planté-type battery is "floated" across the 240-volt bus-bars for emergency duty. (*a*) If fully charged how many amperes could it deliver for a period of 1 hr.? (*b*) for 20 min.? (See p. 118.)

247. A 6-volt lead storage battery, consisting of three cells in series, is charged from 120-volt mains, the connections being similar to those shown in Fig. 74 (p. 120). The terminal voltage of each cell is 2.4 volts. The charging rate is 16 amp. (*a*) Of the total power taken from the circuit, what percentage is delivered to the battery? (*b*) How many ohms series resistance are necessary? (*c*) If the resistance per cell is 0.008 ohm, how much power is lost as heat in the battery?

248. Repeat Prob. 247 with two 6-volt batteries of this same type in series.

249. A storage battery, consisting of 16 cells in series, is charged at the 40-amp. rate from 115-volt mains. The e.m.f. of each cell is 2.2 volts and the internal resistance of each cell is 0.007 ohm. (*a*) Of the total energy taken from the mains, what percentage is effective in producing chemical reactions within the cell? (*b*) What percentage of the energy taken from the mains is delivered to the battery? (*c*) What value of series resistance is necessary?

250. If the battery (Prob. 249) were accidentally connected with the polarity reversed, the same resistance being retained, what current would it take?

251. Repeat Prob. 249 for a 30-amp. rate.

252. The average charging voltage per cell of an 80-amp. (8-hr. rating) storage battery consisting of 120 cells in series is 2.3 volts. It is desired to charge the battery from a constant-voltage bus-bar. In order to stabilize the system, a 0.2-ohm resistance is connected between the bus-bar and the battery. (*a*) What is the voltage of the bus-bars? (*b*) At 3.5 cts. per kilowatt-hour, what is the energy cost of charging the battery for 8 hr. at this normal rate?

253. A booster set (see Fig. 75, p. 121) is used to charge a 60-cell, 110-volt storage battery from 110-volt bus-bars. The battery requires a maximum charging current of 60 amp., and, when charging at this rate, the terminal voltage per cell is 2.45 volts. (*a*) What is the voltage and current rating of the booster? (*b*) If the generator of the booster set has an efficiency of 0.78 and the motor an efficiency of 0.76, how much power does the booster set take from the bus-bars? (*c*) What percentage of the power to the battery is supplied by the booster?

254. A booster set is used to charge a 120-cell lead storage battery from 220-volt bus-bars. When the battery is being charged at the normal 60-amp. rate, the terminal voltage per cell is 2.5 volts. (*a*) What should be the rating of the booster generator? (*b*) If the generator has an efficiency of 0.80 and the motor an efficiency of 0.82, what power does the booster set take from the bus-bars? (*c*) What percentage of the power to the battery is supplied by the booster? (*d*) If energy costs 3 cts. per kilowatt-hour, how much is the energy cost of charging the battery at the 60-amp. rate for a period of 8 hr.? Assume that the battery terminal voltage has an average value of 2.5 volts per cell during the charging period.

255. A 480-amp.-hr. Planté-plate storage battery, consisting of 60 cells in series, after being practically discharged, is being charged at the 8-hr. rate. (*a*) At the end of 5 hr., what is the voltage across its terminals and how much power is it taking? (See Fig. 76, p. 122.) (*b*) Repeat (*a*) for 7 hr.

256. If the battery (Prob. 255) is completely charged and then discharged at the 5-hr. rate, how much power does it deliver at the end of 2 hr.? At the end of 5 hr.? (See Fig. 76, p. 122.) At the end of 6 hr.?

257. A Planté-plate storage cell has an 8-hr. rating of 40 amp. This rate is maintained constant for the 8 hr. of charge. During this period the voltage rises according to the curve shown in Fig. 76 (p. 122). How many ampere-hours are delivered to the cell? how many watthours? (Note.—Mark several equally spaced points on the voltage curve and take their average.)

258. If the cell (Prob. 257) discharges at the 8-hr. rate and its voltage follows the 8-hr. discharge curve of Fig. 76 (p. 122), how many watthours are discharged? (See note, Prob. 257.)

259. Repeat Prob. 258 for the 3-hr. rate of discharge (see note, Prob. 257).

260. The total resistance of a storage battery consisting of 50 cells in series is 0.4 ohm and it is charged from 120-volt, direct-current bus-bars. At the beginning of charge its e.m.f. is 1.8 volts per cell. (*a*) What current does it take? After charging 3 hr., the e.m.f. rises to 2.0 volts per cell. (*b*) What current does it now take? (*c*) What will be the e.m.f. per cell when the battery ceases taking current? What method of charging is this and is the method a desirable one?

261. It is desired to install a 120-cell stationary lead battery having a total capacity of 497 kw. at the 4-hr. rate. (*a*) What will be the approximate weight of the plates of this battery? (*b*) of the total battery? (See Par. 90, p. 124.)

262. An electric truck operates with 48 Exide-Ironclad MV, 13 cells in series. Determine: (*a*) the kilowatt rating of the battery; (*b*) the kilowatt-hour rating of the battery; (*c*) the total weight of the plates; (*d*) the weight of the entire battery. (*e*) If the vehicle goes 50 miles per charge, determine the kilowatt-hours per mile. Use the data of Par. 90 (p. 124).

263. A storage cell, in a fully charged condition, discharges 25 amp. for 10.5 hr. at an average terminal voltage of 2.02 volts. The battery is then put on charge. It requires 31 amp. for 9 hr. at an average terminal voltage of 2.40 volts to bring it back to its original condition. Determine its ampere-hour and its watthour efficiencies over this cycle.

264. A lead storage battery of the pasted-plate type, consisting of 60 cells in series in a fully charged condition, is discharged at a 40-amp. rate for 8 hr., when the battery is practically exhausted. The terminal voltage per cell varies according to the 8-hr. discharge curve (Fig. 76, p. 122). The battery is then put on charge for 8 hr. at the 42.5-amp. rate, when it is restored to its original condition. The terminal voltage per cell varies according to the upper curve (Fig. 76). Determine the ampere-hour and the watthour efficiencies of the battery over this cycle.

265. Repeat Prob. 264, except that the battery is discharged at the 3-hr. rate. Use the 3-hr. curve (Fig. 76).

266. An Edison battery of 15 cells in series, fully charged, is discharged for 5 hr. at the 30-amp. rate with an average terminal voltage of 18.0 volts. It is then charged for 6 hr. at a 26.5-amp. rate with an average terminal voltage of 25.5 volts, when it is restored to its original condition. Determine its ampere-hour and its watthour efficiencies over this cycle.

267. A 60-cell Edison battery, after being charged for 6 hours at a 45-ampere rate, is discharged for 5 hr. at the 50-amp. rate, the voltage per cell varying according to the discharge curve (Fig. 80, p. 128). (*a*) How many watthours are given to the battery? (*b*) At 3.5 cts. per kilowatt-hour, what is the cost of the energy delivered to the battery? (*c*) How many watthours are delivered by the battery? (*d*) What is the ampere-hour efficiency of the battery over this cycle? (*e*) What is the watthour efficiency of the battery over this cycle?

268. Nickel is being plated in a nickel sulphate bath with a current of 12 amp. What is the weight of nickel in grams which is plated on the cathode each hour? (See Par. 101, p. 133.)

269. Repeat Prob. 268 except that copper is plated and the electrolyte is copper sulphate.

270. From the Table of Electrochemical Equivalents (p. 134) determine: (*a*) the grams of copper which will be plated from a copper sulphate bath for each ampere-hour. (*b*) In an electrolytic refining process how many kilograms of copper will be refined by 200 amp. flowing for 8 hr.? (*c*) If the voltage across the bath is 12 volts, how many kilowatt-hours are consumed?

QUESTIONS ON CHAPTER V

Electrical Instruments and Electrical Measurements

1. If a coil carrying a current be placed in a magnetic field, what effect is noticed? Explain this effect in two ways, showing that it is based on fundamental laws of the magnetic field. Of what importance is this principle?

2. How is the principle of the moving coil adapted to measuring small currents in the D'Arsonval galvanometer? How is the coil suspended? How is the current led in and out of the coil? Why is a soft-iron core usually placed between the poles?

3. What two common methods are used to read the galvanometer deflection? What is meant by the *damping* of a galvanometer? How may this damping be accomplished?

4. How may a galvanometer be protected from excessive currents? Sketch the connections of two types of shunt. What are the advantages of the Ayrton shunt? Define the multiplying power of the Ayrton shunt.

5. What was the underlying principle of the early types of electrical instruments? What factors caused these instruments to be inaccurate?

6. Show that the movement of a Weston direct-current instrument is an evolution of the D'Arsonval galvanometer. How is the moving coil pivoted? How is the current led to the coil? What means are used to oppose the motion of the coil? Discuss the damping of the coil. What is meant by a *radial field* and what effect does it have on the calibration of the instrument scale? Why are the top and the bottom springs coiled in opposite directions? Discuss the application of the movement of a Weston instrument to a galvanometer.

7. Of what order of magnitude is the current that will give full-scale deflection in a Weston instrument? Describe the method by which the instrument may be used for measuring current in excess of this value.

8. Describe briefly the construction of a shunt. Why are four posts or terminals necessary? Show that when a Weston instrument is used in connection with a shunt, it also acts as a voltmeter.

9. What law does the current follow in dividing between the shunt and the instrument? Why should the resistance of the shunt and the resistance of the instrument remain constant? What errors may be caused by the heating of the shunt or of the instrument?

10. In what way may an ammeter be made to have several scales? In general, when is an internal shunt used? an external shunt?

11. Compare the movement of the voltmeter with that of the ammeter. In what important respect does the voltmeter differ from the ammeter? How is the current in the coil of a voltmeter limited when the voltmeter is connected across the line?

12. Show that it is possible for a voltmeter to have more than one scale. Explain. What is meant by a multiplier or extension coil?

13. In what manner may the heating effect of an electric current be utilized to measure the value of the current? State some of the advantages and disadvantages of hot-wire instruments.

14. Describe the construction and the method of operation of the vacuum thermocouple. To what type of measurements is it well adapted?

15. Show the connections that are used in measuring resistance with a voltmeter and an ammeter. What precaution should be taken in connecting the voltmeter? What special type of voltmeter contact should be used in measuring very low resistances?

16. Show the connections whereby resistance may be measured by means of a voltmeter alone. What is the order of magnitude of resistances that can be measured by this method? What special type of voltmeter is often desirable for this work and why? To what type of resistances is this method especially applicable?

17. Give the uses of the *megger* and describe briefly the construction of the magnetic circuit. Compare the instrument movement with that of the Weston instrument, stating the functions of the two principal coils and showing their connections. Describe the operation of the instrument showing how it gives correct indications without any spring control.

18. What is the function of the compensating coil? Show the connection of the guard wire and state its purpose. Why is a slipping clutch frequently used?

19. Sketch an arrangement of four resistances, three of which are known, a galvanometer, and a battery, whereby the fourth resistance may be measured. Prove the law of proportionality that exists when the condition of balance has been reached.

20. Describe briefly a systematic procedure which should be followed in obtaining a balance with a Wheatstone bridge.

21. Compare the plug bridge with the dial bridge from the standpoint of ease of manipulation; plug-contact resistance; convenience.

22. In what way does the slide-wire bridge resemble the Wheatstone bridge? Compare it with the Wheatstone bridge from the standpoint of simplicity and accuracy.

23. Make a sketch showing the connections of the Kelvin bridge. Indicate the auxiliary arms and show the relation which they must have with the other bridge arms in order that the Wheatstone-bridge equation may be applied to determine the resistance of the unknown. For what types of measurement is the bridge used and why?

24. Give the connections whereby the slide-wire bridge may be put to practical use in locating an earth fault in a cable. What is the name of this method? Explain why the galvanometer and battery do not occupy the same positions in the slide-wire bridge of Fig. 112 (p. 174) as they do in Fig. 109 (p. 169).

25. Sketch the connections used in the Varley loop. With what arm is the balance obtained? What additional factor must be known before the position of the fault can be determined? Was it necessary to know this factor in the Murray-loop test? Which is the simpler method? What possible sources of error exist?

26. Why is it desirable in practice to know the insulation resistance of cables? Why is the voltmeter method not always practicable? What is the general principle of the method described in Par. 123 (p. 176)?

27. What method is used to obtain readable deflections of the galvanometer under all conditions of circuit resistance? Why is it desirable to keep the 0.1 megohm in circuit continually? Discuss any error which it may introduce.

28. What factor other than the resistance of the insulation affects the value of the current flowing in the circuit? What time of electrification has been adopted as standard in commercial measurements of insulation resistance? What precautions should be observed in the installation of cable-testing apparatus?

29. Why is a guard wire sometimes necessary in insulation measurements? Explain the principle of the guard wire, showing the path of the end-leakage current.

30. Upon what standard of electrical quantity are potentiometer measurements primarily based? Make a simple diagram of connections and show how the e.m.f. of a standard cell may be utilized without its delivering any appreciable current. Why is a null method necessary in such measurement? What care as regards polarity must be observed if a balance is to be obtained?

31. Show how a potentiometer resistance may be calibrated and marked in volts, after the standard-cell balance has been obtained. Describe and illustrate the method by which unknown e.m.fs. may be measured by means of the standardized resistance.

32. Compare the Leeds and Northrup potentiometer, the connections of which are shown in Fig. 118 (p. 183), with the simple potentiometer diagram shown in Fig. 117 (p. 182). Indicate any minor differences, giving the reasons. Where are the 0.1-volt divisions located, and how are they utilized when obtaining a balance? How are the smaller decimal divisions obtained? What is the resistance of the 0.1-volt units? What is the working current of this potentiometer?

33. Describe the device in the Leeds and Northrup potentiometer by means of which compensation for variations in the e.m.fs. of standard cells may be secured. In what manner is protection provided for the galvanometer?

34. What is the disadvantage of a potentiometer in which there are no sliding contacts in the circuit of the working current? Make a diagram showing how the number of dials may be increased by the Thomson-Varley method. Indicate how the total resistance of the potentiometer remains unchanged, irrespective of the positions of the dial contacts.

35. Make a diagram of connections of the Wolff potentiometer showing how the several decimal values of e.m.f. are obtained without the resistance of the potentiometer changing with the different positions of the dial contacts.

36. What is the maximum voltage ordinarily measurable with a potentiometer alone? Make a wiring diagram of the device by means of which the voltage range of the potentiometer may be increased indefinitely. Also make a diagram of a *drop wire* and explain how it may be used to vary the voltage when the supply is at constant voltage.

37. Show that the potentiometer, which is fundamentally a voltage-measuring device, is adapted to measuring currents. On what fundamental law is the measurement of current based? What is meant by a standard resistance and why are four terminals employed? In what units of resistance are standard resistances generally manufactured? Why is it desirable that their temperature remain normal and what means are adopted to accomplish this?

38. What instruments are generally used in measuring the power in a direct-current circuit? Show that these instruments themselves take power. What should be the relative positions of the voltmeter and the ammeter when the power delivered to a high resistance is being measured? When that delivered to a low resistance is being measured? Show the methods by means of which correction for power taken by the instruments may be made.

39. Describe the construction of a wattmeter and show the principle of its operation. In what way do the fixed and moving coils differ in construction? in their manner of connection to the circuit? Why are the instrument deflections a function of the power? What care is necessary when using this type of instrument with direct currents?

40. What quantity does a watthour meter measure? Upon what familiar electrical device is it based? Show the connections of the field coils and state the factor which determines the current in them. What source supplies the armature current and to what is the armature current proportional? To what quantity is the torque developed by the armature proportional?

41. Why is a retarding device necessary and what must be the law of retardation? Upon what principle does this device operate?

42. At what values of meter load does friction produce the greatest error? Explain. How is this friction error practically eliminated?

43. What methods are used to reduce friction in a watthour meter? What are some of the causes of a meter's running slow? How is the recording dial of a meter actuated?

44. Why is it usually very important that a watthour meter register accurately? What load and measuring devices are necessary in testing a meter?

45. Give the fundamental relation which exists in some meters between the revolutions of the disc and the energy. What measurements are made in checking the meter?

46. What two adjustments are made to change the meter speed? What is the effect of moving the magnets nearer the center of the disc? nearer the periphery? At what loads is this adjustment made?

47. What adjustment is made to correct the meter registration at light loads? Why is this adjustment made at light rather than at heavy loads?

48. Show the connections of a three-wire watthour meter and compare them with those of the two-wire watthour meter. Explain the reason why the connection of the three-wire meter is different from that of the two-wire meter.

49. Describe in a general way the construction of a watthour meter which makes the meter practically astatic and therefore enables it to be used near bus-bars carrying heavy currents. What two elements in a meter are most likely to be affected by stray fields? How are these elements safeguarded from these effects?

PROBLEMS ON CHAPTER V

Electrical Measurements and Electrical Instruments

271. The resistance of a galvanometer is 640 ohms. Compute the three resistances of a shunt for use with this galvanometer so that one-tenth, one-hundredth, and one-thousandth the total current of the line will go to the galvanometer (see Fig. 90(*a*), p. 141).

272. Repeat Prob. 271 for a galvanometer, the resistance of which is 1,200 ohms. With a galvanometer current of 1.2×10^{-9} amp., determine by Ohm's law the line current for each value of the shunt resistance. By means of these line currents and the specified ratios, verify the computed values of the shunt resistance.

273. In Prob. 271 determine the resistance introduced into the system by the galvanometer alone and by the galvanometer and shunt together for each value of the shunt resistance.

274. The resistance from A to B of an Ayrton shunt (Fig. 274A) is 10,000 ohms. The shunt is used in connection with a galvanometer, the resistance of which is 1,200 ohms. (*a*) When the shunt is set at the 0.001 point (C) (the resistance $AC = 10$ ohms), determine the current through the galvanometer when 1.6×10^{-5} amp. flows in the line. (*b*) Repeat (*a*) when the shunt is set at the 0.01 point (D), the resistance AD being 100 ohms. (*c*) Compare the currents in (*a*) and (*b*), together with their theoretical values as determined by Eq. (66), p. 143.

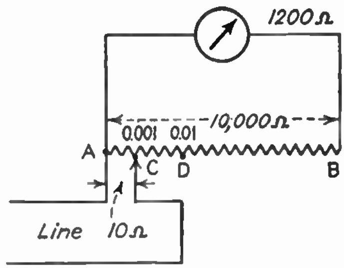


FIG. 274A.

275. In Prob. 274 a galvanometer current 2.4×10^{-7} amp. produces full-scale deflection. (*a*) With the galvanometer current equal to this value, determine the line current when the shunt is set at 0.001, 0.01, and 0.1. (*b*) By what ratio is the maximum sensitivity of the galvanometer reduced by the presence of the shunt?

276. The total resistance of an Ayrton shunt is 30,000 ohms. (*a*) What is the largest resistance which a galvanometer used in conjunction with the shunt may have without the maximum sensitivity of the galvanometer being reduced by more than 10 per cent? (*b*) The galvanometer resistance is 3,000 ohms and a galvanometer current of 1.4×10^{-10} amp. will produce a deflection of 1 cm. Determine the line current with this galvanometer current when the shunt is set at 0.0001. (*c*) With the shunt in circuit, determine the minimum line current which will produce 1-cm. deflection in the galvanometer. (*d*) With the shunt set at 0.01, determine the ratio of line to galvanometer current with the galvanometer in (*b*).

277. The resistance of a 0-to-45-scale millivoltmeter is 10 ohms. (*a*) Determine the resistance of a shunt which, when used in conjunction with the instrument, will produce full-scale deflection with 75 amp. (*b*) Repeat for a 150-amp. shunt. (*c*) In (*a*) and (*b*) compute the ratio of the millivoltmeter current to the shunt current.

278. The resistance of a 50-scale millivoltmeter is 5.2 ohms. It is desired that it measure a current of 100 amp. with full-scale deflection. (*a*) What should be the resistance of the shunt under these conditions? (*b*) Determine the current in the instrument and whether or not it may be neglected as compared with the current in the shunt.

279. Find the resistances of shunts necessary for measuring currents of 150 and 600 amp., full-scale deflection, with the instrument of Prob. 278. In each case compute the power loss in the shunt.

280. The resistance of an instrument of the Weston type is 3.0 ohms. It is used to measure a current of 30 amp. The resistance of the shunt is 0.00090 ohm. (*a*) Determine the current in the instrument; (*b*) in the shunt. (*c*) When the current through the shunt is 75 amp., the instrument needle makes full-scale deflection. Determine the rating of the instrument in millivolts.

281. It is desired to measure a current which does not exceed 60 amp. An internal-shunt, 5-scale ammeter, the resistance of which is 0.01 ohm, is available. What should be the resistance of an additional shunt to be used with this instrument?

282. The resistance of a 0-30-scale millivoltmeter is 1.45 ohms. In order to measure current, this instrument is to be adapted to a 0.0667-ohm shunt, the rating of which is 1.5 amp. (*a*) Determine the additional resistance which must be connected in series with the 30-millivolt scale. Determine the resistances of shunts which are to be rated at (*b*) 30 amp.; (*c*) 75 amp.

283. The resistance of a 25-scale ammeter with its shunt is 0.00020 ohm. In order to increase its range, another shunt, the resistance of which is 0.00010 ohm, is connected in parallel with the complete instrument. Determine the current in the external circuit when the instrument reads (*a*) 18 amp.; (*b*) 24 amp.

284. The resistance of the shunt of a 50-scale ammeter is 0.0012 ohm. A manganin strip, the resistance of which is 0.00024 ohm, is brazed between the copper terminal blocks of this shunt. Determine the range of the instrument under these conditions.

285. A 50-scale, external-shunt ammeter consists of the shunt and a 50-scale millivoltmeter adjusted to be used with it. A current of 0.020 amp. in the millivoltmeter gives full-scale deflection. In attempting to measure an unknown current, the pointer of the millivoltmeter goes off scale by a few divisions and no larger scale instrument is available. A resistance of 1.5 ohms is connected in series with the leads connecting the millivoltmeter to the shunt and the millivoltmeter now indicates 35.0 on the 50-millivolt scale. Determine the correct value of the unknown current.

286. The resistance of the moving coil of a direct-current voltmeter is 62.1 ohms and it gives full-scale deflection when 0.54 volt is impressed across the coil alone. (*a*) What resistance in series with the moving coil is necessary if the instrument is to have a 5-volt range? (*b*) 150-volt range? (*c*) What are the ohms per volt in this instrument?

287. The resistance of a 5-scale, direct-current voltmeter is 540 ohms. (*a*) What additional resistance in series will give this instrument a range of 150 volts? (*b*) 300 volts?

288. It is desired to determine the resistance of the coil system of a 0-15-150-scale, Weston-type voltmeter which is connected as shown in Fig. 288*A* (also see Fig. 98(*b*), p. 154). The following values of resistance are measured by means of a Wheatstone bridge: 0 to A = 16,000 ohms; 0 to B = 1,600 ohms; A to B = 17,500 ohms. (*a*) Determine the resistance of the coil system. (*b*) Show that with the foregoing values of resistance the voltmeter must be connected as shown in Fig. 288*A* and not in the manner shown in Fig. 98(*a*) (p. 154). (*c*) What measured value of resistance between terminals A and B would have shown that the voltmeter is connected in the manner shown in Fig. 98(*a*)?

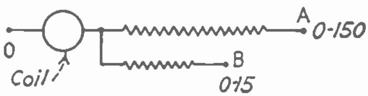


Fig. 288*A*.

289. In measuring an unknown voltage the voltmeter of Prob. 288 deflects slightly beyond the 148-volt range. The 15-volt terminal B is then connected to the 0 terminal and the voltmeter indicates 145 volts. Determine the unknown voltage.

290. It is desired to measure the voltage between trolley and rail of a 1,200-volt railway system. A 300-scale, direct-current voltmeter, the resistance of which is 34,200 ohms, is available. (*a*) What additional resistance in series with this voltmeter is necessary in order that the scale may have a multiplying factor of five? (*b*) Should this resistance be connected between the instrument and trolley or between the instrument and ground? Explain.

291. The resistance of a 150-scale voltmeter is 17,500 ohms. The instrument is slightly off scale when connected across a potential somewhat in excess of 150 volts. When a 2,000-ohm resistance is connected in series with it, the instrument reads 145 volts. What is the unknown voltage?

292. A 300-scale voltmeter, the resistance of which is 40,000 ohms, in series with its five-to-one multiplier is used to measure the direct-current voltage of an electron-tube, plate-circuit generator. It is found that the voltmeter with its multiplier has insufficient range, so the voltmeter is shunted with a resistance of 60,000 ohms. The voltmeter now reads 225 volts. Determine the voltage of the generator.

293. A resistance R (Fig. 293*A*) is connected across a known voltage of 250 volts. A 150-scale voltmeter V, the resistance of which is 15,000 ohms, is connected between the center of the resistance and one side of the line and indicates 115.5 volts. Determine the value of the resistance R.

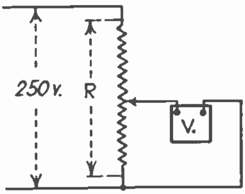


FIG. 293A.

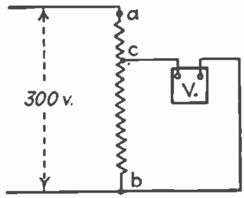


FIG. 294A.

294. It is desired to measure the potential between point c on a resistance ab (Fig. 294A), connected across a 300-volt power source, and point b, when nothing is connected at c. A 300-scale voltmeter, the resistance of which is 32,000 ohms, is first connected across ab and indicates 300 volts. It is then connected between b and c and indicates 197.5 volts. If the resistance ac = 6,000 ohms and the resistance bc = 18,000 ohms, determine the error of measurement in volts and in per cent of the true volts.

295. A 150-scale voltmeter, the resistance of which is 18,000 ohms, and a 300-scale voltmeter, the resistance of which is 32,000 ohms, are connected in series across a potential difference of 410 volts. What does each voltmeter read?

296. A 100-scale ammeter reads 78.2 amp.; a millivoltmeter connected across its terminals reads 32.5 millivolts. (*a*) Determine the resistance of the ammeter, neglecting the current taken by the millivoltmeter. The instrument coil of the ammeter deflects full scale when the current in it is 0.015 amp. (*b*) Determine the resistance of the instrument itself, exclusive of the shunt.

297. It is desired to measure a low resistance, and the maximum current which it can carry without overheating is 25 amp. The only source of supply is a 110-volt, direct-current circuit. An adjustable resistance is connected in series with the low resistance and is adjusted until the current is 25 amp. A voltmeter connected across the low resistance indicates 8.4 volts. (*a*) Determine this low resistance. (*b*) What is the value of the series resistance? (*c*) Determine the voltage across the series resistance when it is adjusted so that the current is 20 amp.

298. When the armature of a 250-volt, 30-hp. motor is stationary, a current of 52.4 amp. gives a voltage drop across its terminals of 4.2 volts. (*a*) Determine the resistance of the armature. (*b*) The current is obtained from direct-current mains, the potential difference of which is 115 volts. Determine the value of the resistance in series with the armature.

299. An ammeter is connected to measure the current to a series-parallel circuit (Fig. 299A) consisting of a fixed resistance R in series with resistances R_1 and R_2 in parallel. R_1 is fixed at 25 ohms, and R_2 is adjustable. The supply voltage is 120 volts. With the resistance R_2 open-circuited, the voltmeter across R indicates 54 volts. (*a*) To what value must R_2 be adjusted in order that the voltmeter may indicate 66 volts? (*b*) What is the reading of the ammeter under these conditions?

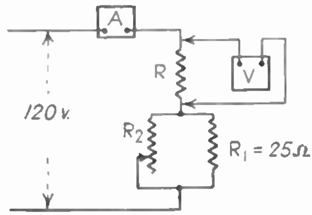


FIG. 299*A*.

300. The resistance of a sample of copper bus-bars is measured at 20°C. by the method shown in Fig. 102 (p. 159). When the ammeter reads 150 amp., the millivoltmeter reads 4.38 millivolts. The bus-bar is 0.5 by 2 in. in cross-section and the distance between voltmeter contacts is 3 ft., 6 in. (*a*) Determine the resistance of the sample; (*b*) the resistance per centimeter cube. (*c*) Compare the resistivity with the commercial standard (see Par. 17, p. 17). Resistance of standard copper is 1.756 microhm-cm. at 20°C.

301. The series field of a compound generator is shunted by a diverter (see p. 451). It is desired to know the resistance of the series field and of the diverter. With the diverter connected across the series field, a voltmeter across the two reads 4.5 volts when the total current is 124 amp. The diverter is then disconnected and the voltmeter reads 5.8 volts, the total current remaining unchanged. What is the resistance of the diverter and of the series field?

302. In order to measure the resistance of a 100-lb. rail, it is connected in series with a standard 0.0001-ohm resistance (Fig. 302*A*). Contact points

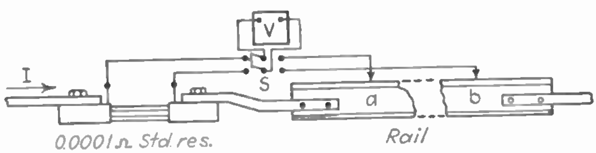


FIG. 302*A*.

a and *b* are then set in the rail 11.6 ft. apart. A millivoltmeter is first connected across the standard resistance and indicates 42.5 millivolts; the millivoltmeter connection is then transferred to the contact points *a* and *b* on the rail by means of the D.-P., D.-T. switch *S* and reads 52.4 millivolts. Determine the resistance per foot of the rail.

303. A 300-scale voltmeter, the resistance of which is 34,000 ohms, is connected across direct-current mains and indicates 250 volts. It is then connected in series with an unknown resistance across these same mains. It now indicates 26 volts. Determine the value of the unknown resistance.

304. The resistance of a special 150-scale voltmeter is 100,000 ohms. When connected across direct-current mains, it indicates 124 volts. The iron frame of a generator is connected to one wire of these mains, and the copper of the field coil is connected to the other wire through the voltmeter, as in Fig. 304*A*. Under these conditions the voltmeter reads 4.5 volts. (*a*) What is the resistance in megohms of the insulation of the field circuit to the frame of the machine? (*b*) When the voltmeter is similarly connected in series with the commutator it indicates 6.4 volts. What is the insulation resistance in megohms between the armature and the frame?

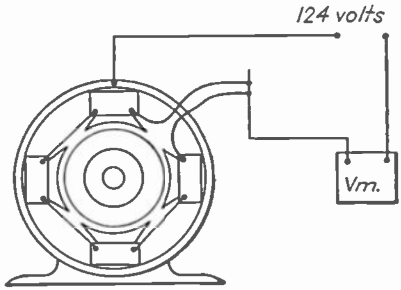


FIG. 304A.

305. In order to obtain greater precision in the measurement of the resistance of some high-voltage insulation, the insulation is connected in series with the special 100,000-ohm voltmeter of Prob. 304 across a voltage of approximately 600 volts and the voltmeter indicates 4.6 volts. A 600-scale voltmeter is then connected across the voltage and indicates 582 volts. Determine the resistance of the insulation.

306. In a Wheatstone-bridge measurement, the unknown resistance is connected at X between one end of the arm A and the arm P (see Fig. 106, p. 165). When a balance is obtained, $A = 1{,}000$ ohms; $B = 10$ ohms; $P = 1{,}462$ ohms. What is the value of the unknown resistance?

307. It is desired to measure a resistance which is known to lie between 4 and 5 ohms. (*a*) What should be the ratio of the A to the B arm in order that the unknown resistance may be measured to four significant figures? (See Fig. 106, p. 165.) (*b*) Repeat (*a*) for a resistance lying between 32 and 40 ohms; (*c*) 640 and 700 ohms; (*d*) 10,000 and 11,000 ohms.

308. Choose the proper values of A and B (Fig. 106, p. 165) to give four significant figures in the value of X, when $X = 14{,}240$ ohms; when $X = 2.562$ ohms; when $X = 45.71$ ohms; when $X = 2{,}479$ ohms. Give the value of P for each balance.

309. A Wheatstone bridge, similar to that shown in Figs. 106 and 108 (pp. 165 and 168) is used to measure an unknown resistance X. With $A = 100$, $B = 1{,}000$, an approximate balance is obtained when $P = 34$ ohms, the galvanometer deflecting decidedly to the right when $P = 33$ ohms and deflecting to the right to a considerably less degree when $P = 34$ ohms. (*a*) What is the apparent value of the unknown resistance? (*b*) Have the best values of the ratio arms A and B been chosen? Explain. Give the values of A and B that should be chosen in order that the value of the resistance of X may be measured to four significant figures.

310. In Fig. 310A is shown a Wheatstone bridge in balance. (*a*) Determine the resistance between points o and c. (*b*) Determine the current in arms A, B, P, and X. The e.m.f. of the battery is 2.2 volts, and its internal resistance is negligible.

311. (*a*) Determine the resistance between points *o* and *c* in Fig. 310*A* with the bridge out of balance, the arm *P* being equal to 2,000 ohms. (*b*) Under these conditions, determine the current in arms *A*, *B*, *P*, *X* and in the galvanometer. The internal resistance of the battery is negligible (see Par. 62, p. 85).

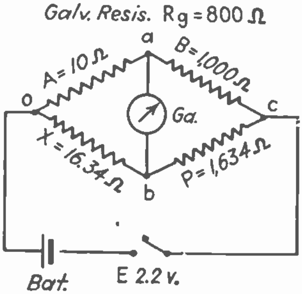


FIG. 310*A*.

312. In Fig. 310*A* with arm *P* 2,000 ohms, determine: (*a*) the resistance of the bridge between points *a* and *b*, the galvanometer and the battery being interchanged; (*b*) the current in the four bridge arms *A*, *B*, *P*, *X*. The internal resistance of the battery is negligible.

313. In Fig. 310*A*, with the galvanometer and the battery interchanged and the bridge out of balance (arm $P = 2{,}500$ ohms) determine: (*a*) the resistance between points *a* and *b*; (*b*) the current in the arms *A*, *B*, *P*, *X* and in the galvanometer. The internal resistance of the battery is negligible.

314. An unknown resistance *X* is measured by means of a slide-wire bridge, similar to that shown in Fig. 109 (p. 169). (*a*) With $R = 100$ ohms, a balance is obtained when the slider reads 28.4 cm. Determine the value of *X*. (*b*) At what reading of the slider will the bridge balance if a 10-ohm resistance is used at *R*?

315. An unknown resistance is measured by means of a 100-cm. slide-wire bridge. A known resistance of 100 ohms is inserted in the position of *X* (Fig. 109, p. 169). A balance is obtained when the slider reads 18.5 cm. (*a*) What is the value of the unknown resistance? (*b*) If a 200-ohm resistance were inserted at *X*, at what reading on the slide wire would a balance be obtained?

316. A Kelvin bridge, similar to that in Fig. 111 (p. 171), is used to measure the resistance of a sample of No. 000 A.W.G., solid copper wire. The distance between the contact points *p* and *p′* is 28 in. The temperature is 20°C. With $A = a = 100$ ohms; $B = b = 1{,}000$ ohms, a balance is obtained when $R = 0.001452$ ohm. Determine: (*a*) the resistance of the copper sample; (*b*) the resistance per centimeter cube. The diameter of the wire is 0.410 in.

317. A No. 0 stranded wire is measured for its resistance in the Kelvin bridge (Fig. 111, p. 171), as was done in Prob. 316. The distance between contacts *p* and *p′* is 28.2 in. The bridge balances when $R = 0.009423$ ohm, with the ratio arms $A = a = 300$ ohms and $B = b = 10{,}000$ ohms. Determine: (*a*) the resistance per centimeter cube of the copper sample; (*b*) the resistivity in ohms per circular-mil-foot. The wire is made up of 19 strands, the diameter of each of which is 0.0745 in.

318. A single-conductor cable 1,600 ft. long, wound on a reel and with its two ends accessible, is known to have a fault in its insulation. It is immersed in a tank of water and the Murray-loop test is used to locate the fault. The slide-wire bridge, 100 cm. long, reads 32.4 cm. when the balance is obtained (see Fig. 112, p. 174). What is the distance from one end of the cable to the fault? Make a wiring diagram showing the location of the fault with respect to the two ends of the slide wire.

319. A ground is known to exist in conductor B of a lead-covered No. 10 A.W.G. signal pair. The entire cable is 3,600 ft. long. To locate the fault,

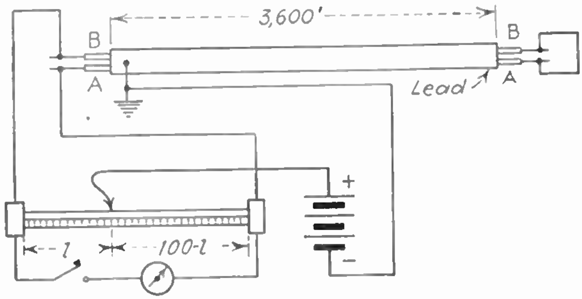


FIG. 319A.

the far end of the pair is looped (Fig. 319A), and the Murray-loop test is made by means of a slide-wire bridge. A balance is obtained when l = 34.6 cm. How far from the near end is the ground in B?

320. An installed two-conductor cable of No. 000 A.W.G. copper wire is 2,800 ft. long. Due to a burn-out, both conductors are short-circuited and grounded at the same point. To locate the fault, a single No. 00 A.W.G. conductor of another cable, which parallels the faulty one, is looped to one conductor of the faulty cable at the far end, as in Fig. 320A. The perfect conductor is connected to the 0-end of the slide wire and one of the faulty conductors to the 100-cm. end. A balance is obtained at 79.5 cm. The

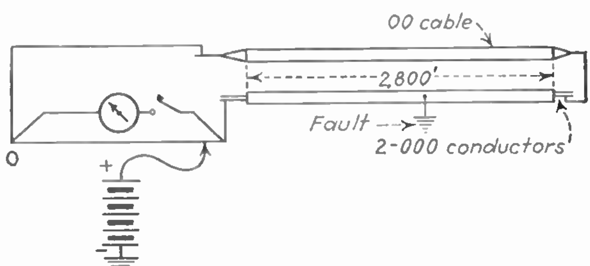


FIG. 320A.

resistance of the 000 conductor is 0.0630 ohm per 1,000 ft. and the resistance of the 00 conductor is 0.0795 ohm per 1,000 ft. How far out on the faulty conductor is the burn-out situated?

321. A 2,400-ft. length of twin-conductor, No. 14 A.W.G., underground, lead-covered cable develops a ground fault in one conductor. The far ends of the two conductors are looped and tested for the fault by means of the

Varley-loop test, the connections of which are shown in Fig. 113 (p. 175). With the switch S at b, so that the loop resistance of the cable is measured, the bridge balances under the following conditions: $A = 10$, $B = 1{,}000$, $P = 1{,}248$ ohms. With the switch S at a, in order to locate the fault, the bridge balances under the following conditions: $A = 10$, $B = 1{,}000$, $P = 802$ ohms. Determine: (a) the resistance to the fault; (b) the distance from the home end to the fault.

322. The underground cable of a series, open-loop, arc-lighting system (see Fig. 436, p. 587) is grounded at one point. The cable is solid No. 6

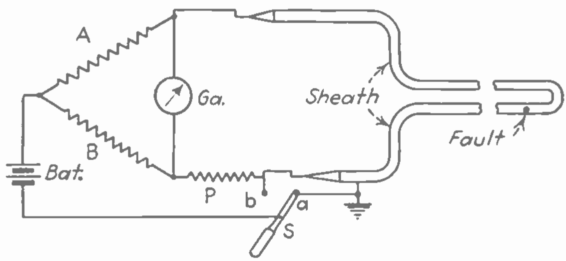


FIG. 322*A*.

A.W.G. copper and the loop circuit is 10,500 ft. in length. To locate the fault a Varley-loop test is made, the connections being shown in Fig. 322*A*. With the switch S at b, so that the bridge is connected to measure the linear resistance of the cable, the bridge balances when, with $A = 10$ and $B = 1{,}000$, the arm P is made equal to 423 ohms. The switch S is then moved to position a in order to determine the location of the fault. The bridge balances under the following conditions: $A = 10$, $B = 1{,}000$, $P = 130$ ohms. Determine the distance from point b to the fault.

323. One conductor in a cable containing two No. 12 wires a and b, each 6,000 ft. long, is known to be grounded. The two are looped at the far end and the Varley-loop test is made. P is connected in series with conductor a. The two arms A and B (Fig. 113, p. 175) are each set at 100 ohms. A balance cannot be obtained with P, as the galvanometer is found to deflect the same way with $P = 0$ and $P = \infty$. P is then shifted over in series with b, the other conductor, and a balance obtained when $P = 5.8$ ohms. The resistance of the conductors is known to be 1.62 ohms per 1,000 ft. In which conductor is the fault, and how far is it from the home end of the cable?

324. In an insulation test of a single-conductor cable 1,800 ft. long, the connections are made as in Fig. 114 (p. 177) When the cable is short-circuited and the Ayrton shunt is set at 0.0001, the galvanometer deflection is 24.2 cm. The short circuit is then removed, putting the cable in circuit, and the galvanometer deflection is 19.8 cm., with the shunt set at 1.0, after the cable has been charged for 1 min. Determine: (a) the insulation resistance of the cable; (b) the resistance of the cable in megohms per 1,000 ft.; (c) the megohms per mile.

325. In Prob. 324, the resistance of the Ayrton shunt is 12,000 ohms and the resistance of the galvanometer is 1,000 ohms. The e.m.f. of the battery is 320 volts and its internal resistance negligible. Determine: (*a*) the battery current when the cable is short-circuited; (*b*) the galvanometer current when the cable is short-circuited; (*c*) the galvanometer current with the 0.1 megohm and the cable in circuit; (*d*) the battery current under the conditions of (*c*); (*e*) By means of (*a*), (*b*), (*c*), and (*d*), and Eq. (87), p. 178, verify the insulation resistance of the cable.

326. The insulation resistance of an 8,000-ft. length of No. 6 solid, single-conductor, underground cable is measured between conductor and lead sheath, the connections in Fig. 114 (p. 177) being used. The diameter of the conductor is 0.162 in. and the wall of rubber insulation is ⅛ in. With the cable short-circuited, and the Ayrton shunt set at 0.0001, the deflection of the galvanometer is 8.2 cm. With the cable in circuit and the shunt set at 0.1, the deflection of the galvanometer becomes 22.6 cm. after 1-min. electrification. Determine: (*a*) the insulation resistance of the cable in megohms; (*b*) the insulation resistance in megohms per 1,000 ft.; (*c*) the resistivity of the rubber in megohms per inch cube; (*d*) the resistivity in megohms per centimeter cube.

327. In order to measure the insulation resistance of a cable in which the insulation resistance is known to be low, connections similar to those shown in Fig. 327*A* are made, the galvanometer, 0.1 megohm, the cable insulation, and the battery, all being in series. The e.m.f. of the battery is 300 volts and its resistance negligible. The resistance of the galvanometer is 560 ohms. With the cable short-circuited, and the galvanometer shunt in the short-circuit position, the switch *Sw* is closed. The resistance of the galvanometer shunt is then gradually increased until it is equal to 12 ohms. The galvanometer deflection is then 24.0 cm.

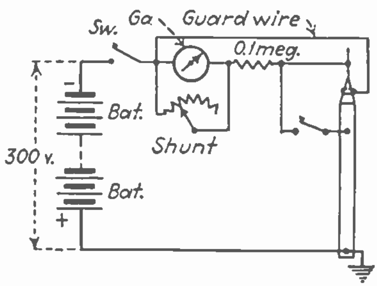


FIG. 327*A*.

The short circuit of the cable is then removed and the galvanometer deflection becomes very small. The resistance of the galvanometer shunt is now gradually increased until it becomes infinite. The deflection of the galvanometer then becomes 0.8 cm. after 1-min. electrification. Determine: (*a*) the galvanometer current with the cable short-circuited; (*b*) the line current in (*a*); (*c*) the galvanometer current with the cable in circuit; (*d*) the insulation resistance of the cable. The internal resistance of the battery is negligible.

328. Figure 328*A* shows a 100-cm. slide wire *ab*, the total resistance of which is 2.5 ohms, connected in series with the adjustable resistance *R* to the terminals of the storage cell *B*, the end *a* being connected directly to the negative terminal of *B*. The e.m.f. of the storage cell is 2.10 volts and the internal resistance is 0.04 ohm. A is a dry cell, the e.m.f. of which is 1.25 volts. Its negative terminal is also connected to a, and its positive terminal is connected through a galvanometer G to a slider c on ab. With the resistance R adjusted to 0.96 ohm, determine the position of c, in centimeters from a, which will make the galvanometer deflection zero.

Explain the effect on the current delivered by the battery B of connecting the battery A to the system under the conditions that make the galvanometer current equal to zero.

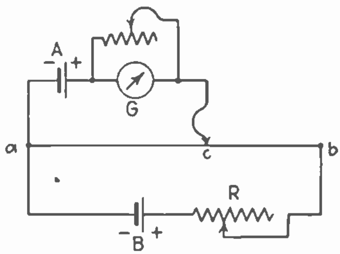


FIG. 328A.

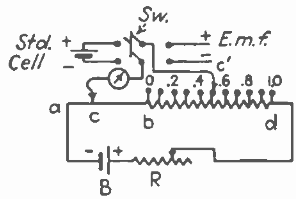


FIG. 329A.

329. A resistance bd, equal to 25 ohms and divided into 10 equal resistances, is connected in series with the slide wire ab (Prob. 328) (see Fig. 329A). The battery B still supplies the current to the system through the resistance R. It is desired that this system operate as a potentiometer, the resistance ab corresponding to 0.1 volt and each division, 0 to .1, .1 to .2, etc., of bd corresponding to 0.1 volt. (*a*) To what value must the current in the circuit be adjusted? (*b*) To what value of resistance must R be adjusted? A Weston cell, the e.m.f. of which is 1.0184 volts, is connected to the two contacts cc' through the galvanometer by throwing switch Sw to the left. (*c*) Determine the positions of contacts c and c', the position of contact c being given in centimeters from b, which will make the galvanometer current equal to zero when the value of the current is equal to that determined in (*a*). (*d*) An unknown e.m.f. is connected to the contacts cc' through the galvanometer by throwing switch Sw to the right. The galvanometer deflection is zero when the contact c' is at 0.5 and the contact c is 64 cm. from b. Determine the value of the unknown e.m.f.

330. It is desired to measure the terminal voltage of a storage battery by means of a standard cell. The ratio arms A and B and the rheostat arm P of a Wheatstone bridge (Fig. 330A) are connected in series across the terminals of the storage battery, and a standard cell having an e.m.f. of 1.0186 volts is connected across a 1,000-ohm coil in arms A and B. The galvanometer in the standard-cell circuit stands at zero when 1,020 additional ohms are unplugged in P. What is the terminal voltage of the storage battery?

331. The storage battery of Prob. 330 is of such large capacity that its e.m.f. and terminal voltage are sensibly the same when delivering the small current required by the resistances of the magnitude of 2,020 ohms. To measure the e.m.f. of another cell, which is not capable of delivering any appreciable current, its negative terminal is connected to the point a and its positive terminal to the point b through a key and galvanometer (see Fig. 330A). P is then adjusted until this galvanometer reads zero. P is now read and found to be 842 ohms. What is the e.m.f. of this cell?

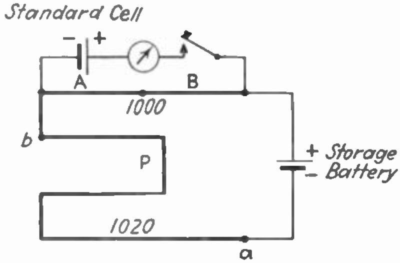


FIG. 330A.

332. Figure 332A shows a portion of a high-resistance potentiometer arranged on the Thomson-Varley-slide principle. Between a and b are sixteen 50-ohm resistances, each corresponding to 0.1 volt. $a'b'$ consists of eleven 10-ohm resistances. $a''b''$ is a 20-ohm resistance divided into ten 2-ohm resistances. (*a*) What is the working current of the potentiometer? With the contacts in the positions shown, determine: (*b*) the potential drop from a to c; (*c*) the current in the resistance $a'b'$; (*d*) the potential drop from a' to c'; (*e*) the current in $a''b''$; (*f*) the potential drop between a'' and c''; (*g*) the total potential drop from a to c'', or the e.m.f. across ef. (Compare this potential drop with the readings of the potentiometer.)

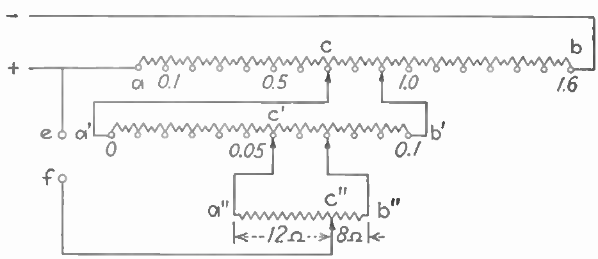


FIG. 332A.

333. In the Thomson-Varley potentiometer, the diagram of which is shown in Fig. 120 (p. 185), the current to the potentiometer is adjusted to the value which gives the correct values of voltage. Determine the current in: (*a*) resistance BC; (*b*) resistance $B'C'$; (*c*) resistance $B''C''$. The e.m.f. of the battery Ba, which supplies the current, is 2.08 volts and its internal resistance together with the resistance of the connecting wires is negligible in comparison with that of the potentiometer. (*d*) Determine the value to which the resistance R must be adjusted in order that the correct value of current be obtained.

334. In the Wolff potentiometer, the diagram of which is given in Fig. 121 (p. 187), the working current is 0.0001 amp. This current is supplied by two dry cells in series, the e.m.f. of each being 1.2 volts. (*a*) Determine the resistance which must be connected between these dry cells and the battery terminals *Ba* of the potentiometer. (*b*) With the dials in the positions shown in the figure, determine the e.m.f. between the two moving contactors in dial *b*.

335. It is desired to calibrate a 15-scale voltmeter near the upper end of its scale. The voltmeter is connected across a battery as shown in Fig. 335*A*. A resistance *abc* is connected in parallel with the voltmeter and battery. Between point *b* and the negative terminal of the battery a standard cell *S*, whose e.m.f. is 1.0186 volts, is connected in series with a key *K* and galvanometer, the negative terminal of the standard cell being connected to the negative side of the battery at *c*. The resistance *bc* is made equal to 100 ohms. When the resistance of *ab* is adjusted to 1,235 ohms, the galvanometer does not deflect when the key *K* is depressed. At this instant the voltmeter reads 13.65 volts. What is the correction to the voltmeter scale for this reading?

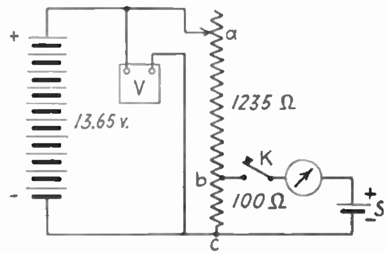


FIG. 335*A*.

336. In Prob. 335, determine the value to which the resistance *ab* must be adjusted in order that the corrected reading of the voltmeter shall be 15 volts if it is possible to vary the battery voltage.

337. It is desired to calibrate a voltmeter at the 120-volt point. No potentiometer is available. The voltmeter is connected in parallel with the arms of a bridge box (Fig. 337*A*), and the supply voltage is adjusted until the voltmeter reads just 120 volts. A standard cell *S*, which is known to have an e.m.f. of 1.0183 volts, is connected across the two ratio arms *A* and *B* in series with a key and galvanometer, the proper polarity being observed, and a total of 100 ohms are unplugged in these two arms. The galvanometer reads zero with the key depressed when 11,940 ohms are unplugged in *P*. What correction should be applied to the voltmeter at this point?

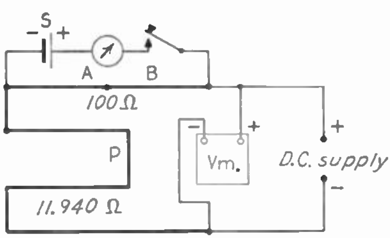


FIG. 337*A*.

338. The power to a 25-watt tungsten lamp is being measured with a voltmeter and an ammeter. The voltmeter, the resistance of which is 14,160 ohms, is connected directly across the lamp terminals. When the ammeter reads 0.206 amp., the voltmeter reads 119 volts. What is the true power taken by the lamp? What percentage error is introduced if the instrument power be neglected?

339. The resistance of the ammeter of Prob. 338 is 0.06 ohm. The voltmeter is now connected directly across the line. With the line voltage remaining unchanged, determine: (*a*) the voltmeter reading; (*b*) the true power now taken by the lamp; (*c*) the power given by the product of the voltmeter and ammeter readings; (*d*) the percentage error if the product in (*c*) is used.

340. In measuring the power taken by a low-resistance rheostat, an ammeter, the resistance of which is 0.0006 ohm, and a voltmeter, the resistance of which is 300 ohms, are used. When the ammeter reads 80 amp., the voltmeter, which is connected directly across the resistance, reads 2.48 volts. Determine: (*a*) the true value of the resistance; (*b*) the true value of the power; (*c*) the percentage error introduced in (*b*) by the voltmeter current; (*d*) the error introduced by connecting the voltmeter outside the ammeter.

341. In a test of a direct-current, 25-amp. watthour meter the corrected average voltmeter reading is 220 volts and the corrected average ammeter reading is 24.5 amp. During the test interval 40 revolutions are counted and the time is found to be 39.5 sec. (*a*) If the meter constant is 1.5, what is the percentage accuracy of the meter at this load? (*b*) What adjustment should be made to bring it nearer the correct registration?

After the meter has been brought to within 0.5 per cent accuracy by the movement of the damping magnets, the load is dropped to 1.5 amp., but the voltage remains at 220 volts. It takes 59.7 sec. for the disc to make 4 revolutions. (*c*) What is the percentage accuracy of the meter at this point? (*d*) What adjustment should be made in order to bring it nearer the correct registration?

342. A 10-amp., 120-volt watthour meter is tested for its accuracy near rated load and at light load. Near rated load the corrected average voltmeter reading is 118.4 volts, and the corrected average ammeter reading is 9.85 amp. The time required for the disc to make 30 revolutions is measured by means of a stop watch and is found to be 47.6 sec. The watthour meter constant is 0.5. (*a*) Determine the percentage accuracy at this load and state the adjustment which should be made. (*b*) The load is reduced and the corrected average voltage is 118.8 volts, and the corrected average current is 0.82 amp. The time required for the disc to make 3 revolutions is measured with the stop watch and found to be 57.7 sec. Determine the percentage accuracy. The meter is not adjusted between (*a*) and (*b*). From an analysis of the percentage accuracy in the two cases, discuss any adjustments of the light-load coil which should be made.

343. In order to make a laboratory test of a 2,000-amp., 220-volt, astatic watthour meter (Fig. 343*A*), its current coils are supplied with current from a 4-volt storage battery, and its armature circuit, which has a resistance of 2,200 ohms, is connected across 235-volt mains. A calibrated voltmeter is connected in parallel with the armature circuit, and an external-shunt ammeter is connected in series with the current terminals of the meter. The meter constant is 150. The corrected voltmeter reading is 235 volts, and the corrected ammeter reading is 2,060 amp. The meter makes 40 revolutions in 45.7 sec. (a) What is the percentage accuracy of the meter

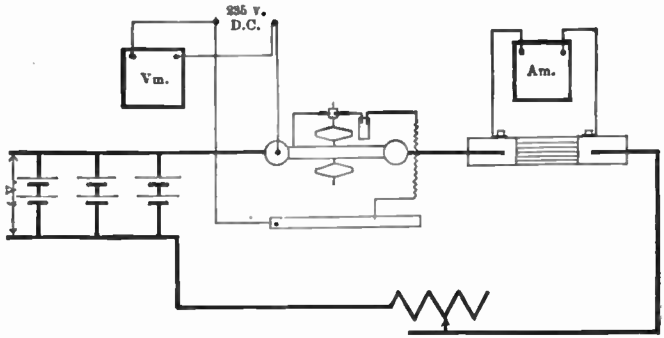


FIG. 343A.

at this load? (b) How much power is required for this test? (c) How much power would be required if the meter current were supplied at 235 volts?

QUESTIONS ON CHAPTER VI

Magnetism and Permanent Magnets

1. Into what two general classes may magnets be divided? Discuss the principal differences between permanent magnets and electromagnets.

2. Name the principal magnetic material and compare it with other materials which have magnetic properties. What substances may be alloyed to produce superior magnetic characteristics?

3. Discuss natural magnets. How are artificial magnets produced? Compare soft iron with hardened steel for permanent magnets.

4. What is meant by a magnetic field? Discuss the significance of magnetic lines. Do such lines actually exist? What experimental effects are related to direction and density of magnetic lines? Name an important property of a line of induction. Distinguish between a north-seeking pole and a north pole.

5. What is the effect of breaking a bar magnet near the neutral zone? Explain how the newly created poles come into existence. Discuss the effects resulting from breaking a long bar magnet into several pieces.

6. What is meant by consequent poles? Distinguish between consequent poles, and poles obtained by breaking a bar magnet.

7. When a freely suspended S-pole is brought into the presence of a N-pole, what effect is noted? What effect is noted if the freely suspended S-pole is brought into the presence of a S-pole? State the general law governing attraction and repulsion between like poles, and between unlike poles.

8. Define a unit pole. How is pole strength determined? State Coulomb's law governing the force of attraction and the force of repulsion between magnetic poles.

9. Discuss the Weber-Ewing molecular theory of magnetism. How does it explain the phenomenon that occurs when a bar magnet is broken?

10. Assuming that a bar magnet consists of a large number of filaments of very small cross-section, define a line of magnetization. Show the field created by a N-pole which exists at one end of an elementary filament.

11. What is the effect of the lines of force, emanating from the poles at the ends of a magnet of considerable cross-section, on the magnetic lines going through the magnet? In a long bar magnet, what is the ratio of the lines of induction at the center cross-section to the lines emanating from the end cross-section? Why is it difficult, if not impossible, to maintain the magnetism in a magnet in which the ratio of cross-section to length is large?

12. What is the direction of the lines of *induction* within and outside the magnet? The lines of *force*? Under what conditions are the lines of force and the lines of induction identical?

13. Define unit field intensity. State the relation between the lines of force and field intensity. What relation exists in the magnetic field between field intensity and "lines of force per square centimeter"?

14. How many lines of force emanate from each unit pole? From a pole of strength m units? If B is the flux density near the center of a long steel rod of 1 sq. cm. cross-section, what is the pole strength at the end of the rod?

15. Of what does a compass needle consist? How is it used in practice to determine the correct polarity of motors and of generators? How is it possible to obtain accurate indications from a compass when it is used upon steel ships? How may a compass needle be used to map out an electrical field in the vicinity of a magnet?

16. Describe the method of determining the flux distribution in a certain region by means of iron filings; by means of small compass needles. Explain the relation of pole attraction and repulsion to the distribution of the magnetic lines existing near the poles.

17. Describe the production of magnetic poles by magnetic induction. What is the relation between the inducing and the induced pole? How does magnetic induction explain the attraction of soft iron to magnetic poles? How may a compass needle become reversed? Explain the use of a "keeper" with a horseshoe magnet.

18. State a fundamental law which governs the configuration of a magnetic field. Show that this law explains the attraction of an iron armature to the poles of a magnet.

19. On each of two parallel, oppositely magnetized surfaces, separated by a narrow, uniform air-gap, there are σ unit poles per square centimeter. What is the flux density in the gap in lines per square centimeter? What is the force in dynes acting on a unit pole placed within the air-gap?

20. In Question 19, what is the force in dynes exerted by the *S*-pole on each unit *N*-pole in the surface of the *N*-pole? on each square centimeter of the *N*-pole? From these factors, derive the equation which gives the force in dynes between the two magnetized surfaces.

21. Sketch the portion of the hysteresis loop over which permanent magnets operate. Discuss, by means of a sketch, the properties of high remanence and of high coercive force or retentivity. For what type of magnetic circuit is high remanence desirable? high retentivity? Explain.

22. What type of essentially pure steel is best adapted to permanent magnets? For what type of magnets is soft steel best adapted and why.

23. Describe the properties of tungsten steel which adapt it to permanent magnets for certain types of service. What is the property of cobalt-chromium steel and similar alloys which makes them highly desirable for permanent magnets? What determines the energy per unit volume of a permanent-magnet material?

24. What is the objection to the use of the bar magnet in practical work? What advantages have the ring and the horseshoe magnets over the bar magnet? When is the ring magnet used?

25. Why is it possible to employ cobalt-chromium magnetic alloys in bar magnets to supply magnetization to certain types of apparatus?

26. What is the principle underlying the compound or laminated magnet? Where are laminated magnets used in practical work?

27. Indicate the method of magnetizing a steel bar by means of a bar magnet; by means of two bar magnets. Show how magnets may be magnetized by means of electromagnets.

28. Describe a method of magnetizing by means of a coil and a current; by means of a single conductor.

29. Why does the magnetization of a permanent magnet change with time unless it is in some manner aged artificially? Describe a method of artificial aging.

30. Describe the principle of shielding sensitive instruments from stray magnetic fields. Show that complete shielding is impossible.

31. State why the compass needle does not point to the true north. What information is necessary in order to determine the true north from the indication of the compass needle? What is meant by declination or variation? What is meant by the dip of the needle?

PROBLEMS ON CHAPTER VI

Magnetism and Permanent Magnets

344. In a long bar magnet *abc* (Fig. 344*A*) the end *a* is a *N*-pole, and the end *c* is a *S*-pole. The magnet is cut transversely at *b*, its mid-zone, and the half *bc* is placed at right angles to its original position as indicated by the dotted lines and by *b′c′*; *b′* and *c′* corresponding to *b* and *c*. Sketch the resulting magnetic field, indicating the N-poles and the S-poles as well as the direction of the lines in the magnetic fields.

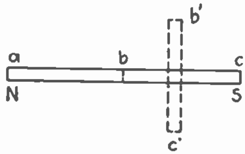


FIG. 344A.

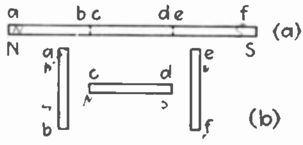


FIG. 345A.

345. In a long bar magnet af (Fig. 345A (a)) the end a is a N-pole and the end f is a S-pole. The magnet is cut into three equal parts ab, cd, ef; and these three parts are arranged in the manner shown in Fig. 345A (b). Sketch the magnetic field produced by this system, showing the direction of the magnetic lines and indicating the several S-poles and N-poles.

346. Two N-poles of strength $m = 1{,}200$ unit poles and $m' = 1{,}500$ unit poles are spaced 15 cm. apart in air. (a) Determine the force in dynes acting between the poles. (b) Indicate the direction in which this force acts (c) Determine the force in grams.

347. If each of the poles of a horseshoe magnet has a strength of $m = 2{,}500$ unit poles, and the centers of the poles are a distance apart of 1.5 in., find the force in pounds tending to pull the two poles together. Assume the poles to be concentrated at their centers.

348. A N-pole m of 800 unit poles and a S pole m' of 1,000 unit poles are spaced 5 cm. apart in air. Determine: (a) the force in dynes acting between the poles; (b) the force in dynes exerted by the N-pole on a unit pole midway between the two poles; (c) the force in dynes exerted by the S-pole on a unit pole midway between the two poles; (d) the total force with its direction acting on the unit pole of (b) and (c); (e) the field intensity at the point midway between the two poles.

349. One N-pole of 800-unit-pole strength and two S-poles each of 400-unit-pole strength are arranged so that the N-pole is at the apex of the right angle of an isosceles right triangle (Fig. 349A). The other two poles are located at the vertices of the two 45° angles, each being 16 cm. from the 800 unit pole. Determine the magnitude and direction of the force acting on each pole, using a diagram.

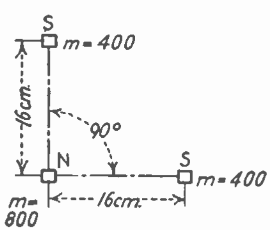


FIG. 349A.

350. A bar magnet (Fig. 350A) is 20 cm. long and the poles at the ends of the magnet are each of strength $m = 600$ unit poles. (a) Find the magnitude and direction of the force which the N-pole exerts on a unit N-pole at point P, where the angle aPb is 90° and point P is equidistant from both the N- and the S-pole. (b) Find the magnitude and direction of the force which the S-pole exerts on the unit N-pole at point P. (c) Find the magnitude and direction of the resultant force acting on the unit pole at point P. (d) Find the intensity and direction of the magnetic field at point P.

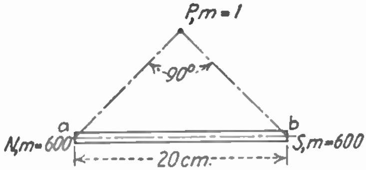


FIG. 350A.

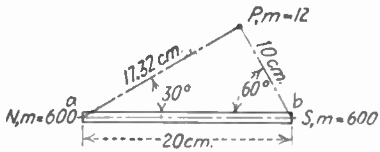


FIG. 351A.

351. In Fig. 351A is shown a bar magnet similar to that of Prob. 350. The point P is located a distance of 17.32 cm. from the N-pole and 10 cm. from the S-pole, the angle aPb being 90°. A N-pole of 12-unit-pole strength is located at P. Determine: (a) the direction and magnitude of the force exerted by the N-pole at a on the pole at P; (b) the direction and magnitude of the force exerted by the S-pole at b on the pole at P; (c) the magnitude and direction of the resultant force at P; (d) the field intensity at P.

352. Two similar bar magnets each 24 cm. long are pivoted together at their centers (Fig. 352A). The pole strength of each of the N- and S-poles is 500 unit poles. One of the bars is held rigidly. Determine the turning moment in centimeter-dynes acting on the other bar.

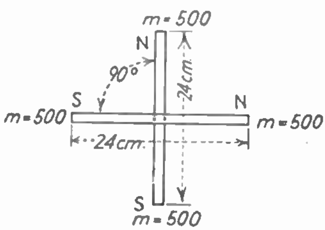


FIG. 352A.

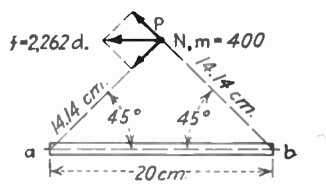


FIG. 353A.

353. A N-pole of 400-unit-pole strength is located at P (Fig. 353A) 14.14 cm. from each of the two ends a and b of a bar magnet 20 cm. long. The triangle aPb is a right-isosceles triangle. There is a N-pole at one end of the magnet and an equal S-pole at the other end. The resultant force f exerted on the N-pole at P is 2,262 dynes acting to the left and parallel to the direction of the magnet as indicated. Determine the magnitude of the N- and S-poles. Indicate the north end and the south end of the magnet.

354. A uniform field is produced between two parallel polar surfaces of a magnet, the surfaces being 20 cm. apart. The field intensity is 400 oersteds or dynes per unit pole. A bar magnet 12 cm. long is inserted in this field along the lines of force. The two poles at the ends of the bar magnet are each 50 unit poles. The N-pole of the bar magnet points to the N-pole of the magnet, and its S-pole points to the S-pole of the magnet. Determine: (a) the magnitude and direction of the force acting on the N-pole of the bar magnet; (*b*) on the *S*-pole of the bar magnet; (*c*) the resultant force acting on the bar magnet. (*d*) Sketch the resulting field. (*e*) What change might ultimately occur in the magnetism of the bar magnet?

355. The bar magnet Prob. 354 is turned so that its direction is at right angles to the uniform magnetic field (Fig. 355*A*). Determine: (*a*) the magnitude and direction of the force acting on each of the poles of the bar magnet; (*b*) the turning moment acting on the magnet in centimeter-dynes. Illustrate with a sketch.

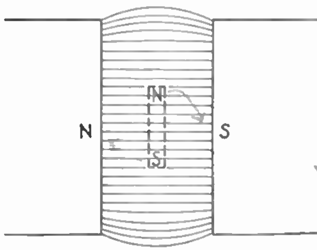


FIG. 355*A*.

356. A bar magnet 16 cm. long is placed in a uniform magnetic field and perpendicular to the direction of the field. The *N* and *S*-poles of the magnet are each 100-unit-pole strength. The turning moment of the bar magnet is 384,000 cm.-dynes. (*a*) Determine the field intensity. (*b*) Make a sketch indicating the general appearance of the resulting magnetic field. (*c*) Determine the turning moment acting on the bar magnet when its direction is 45° to the direction of the uniform field.

357. A bar magnet 20 cm. long, with pole strength $m = 200$ unit poles, is placed in a uniform magnetic field and at 45° to its direction. The turning couple acting on the bar magnet is 1,600 cm.-g. Determine field intensity.

358. The pole strength of a *N*-pole is 260 unit poles. Determine: (*a*) the number of lines of force leaving this pole in air; (*b*) the force exerted on a unit pole 8 cm. distant; (*c*) the density of the lines of force 2 cm. distant; (*d*) 8 cm. distant. (*e*) Compare (*d*) with (*b*).

359. A point *N*-pole having a strength $m = 400$ unit poles and a point *S*-pole having a strength $m' = 600$ unit poles are 18 cm. apart in air. (*a*) How many lines of force leave each? (*b*) What is the density of the lines of force leaving the *N*-pole at a point 8 cm. distant? (*c*) What is the density of the lines of force leaving the *S*-pole at a point 10 cm. distant? (*d*) What resultant force do the two poles exert on a unit *N*-pole on their common axis and 8 cm. from the *N*-pole?

2ND

360. Repeat Prob. 359, substituting a *N*-pole for the *S*-pole.

361. In Prob. 359 determine: (*a*) the density of the lines of force 10 cm. from the *N*-pole; (*b*) the density of the lines of force 8 cm. from the *S*-pole; (*c*) the field intensity at a point on their common axis 10 cm. from the *N*-pole.

362. Repeat Prob. 361, substituting a *S*-pole for the *N*-pole.

363. A long cylindrical magnetized steel rod has a diameter of 2 cm. The flux density in the cross-section taken at the center of the rod is 2,500 maxwells per square centimeter. Determine: (*a*) the strength of pole at each end of the rod; (*b*) the number of maxwells leaving or entering the pole at each end of the rod.

364. The poles at the ends of a long cylindrical bar magnet each consists of 6,000 unit *N*- and *S*-poles. The cross-section of the magnet is 1.6 sq. cm. Determine: (*a*) the number of magnetic lines leaving each of the poles; (*b*) the number of magnetic lines crossing the center zone of the magnet; (*c*) the flux density in lines per square centimeter crossing the center zone of the magnet.

365. In Prob. 364 a transverse saw cut is made at the center of the bar magnet. Determine: (*a*) the unit poles σ per square centimeter on each of the pole surfaces created by the saw cut; (*b*) the field intensity within the short air-gap; (*c*) the force exerted by one of the pole faces on a unit pole at the surface of the other; (*d*) the total pull in dynes and in kilograms between the two pole faces.

366. The diameter of a long cylindrical magnet is 4 cm. and the magnetic pull between two surfaces formed by a transverse saw cut at its center is 2,040 g. Determine: (*a*) the magnetic lines per square centimeter on each of the pole-face surfaces at the saw cut; (*b*) the pole strength at the ends of the magnet.

367. It is desired to make a magnet of the shape shown in Fig. 367*A*, capable of holding an armature load of 1,000 lb. The cores of the magnet are circular in cross-section. The pole faces are flat and make contact with the armature over their entire area. The flux density at the area of contact of the poles and the armature is to be 20,000 magnetic lines per square inch. Determine the diameter *D* of the pole cores.

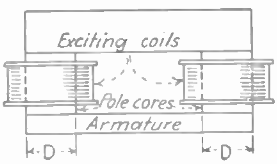


Fig. 367*A*.

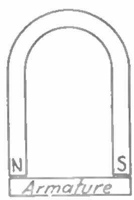


Fig. 368*A*.

368. The cross-section of the poles of the horseshoe magnet in Fig. 368*A* is 1 by 4 cm. These poles are each in contact with a soft-iron armature. There are 10,000 magnetic lines from each pole to the armature. Determine the force, in grams, necessary to remove the armature from the pole faces, when the force is applied to the center of the armature.

369. A small soft-iron bar *ab* (Fig. 369*A*) 4 cm. long is placed with one of its ends directly in line with a long bar magnet, one end being 5 cm. distant from the *N*-pole of the bar magnet. The pole strength of the *N*-pole of the bar magnet is 400 unit poles and the induced *S*-pole on the small iron bar is 150 unit poles. Assume that the effective center of the *N*-pole on the soft-iron bar is at its end. Determine the approximate force of attraction, in grams, between the two magnets. Assume that the effective centers of all the magnetic poles are at the centers of the pole surfaces.

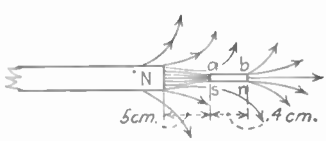


Fig. 369*A*.

QUESTIONS ON CHAPTER VII

Electromagnetism

1. What are the nature and general shape of the magnetic field about a cylindrical conductor carrying an electric current? What relation exists between the direction of the current and the direction of the field produced about the conductor?

2. How may the above relations be shown experimentally? What two simple rules enable one to remember the relation which exists between the current direction and the direction of the magnetic field?

3. The current in a conductor flows from left to right. In what direction will the north end of a compass needle point if held over the wire? If held beneath the wire?

4. By means of a sketch illustrate the Biot-Savart law, which gives a quantitative relation between the magnitude of a current and the field intensity which it produces.

5. Derive the equation from which the field intensity due to a current in an infinitely long, straight conductor can be calculated.

6. From the result in Question 5 derive an expression which gives the work done in carrying a small magnetic pole once around a conductor in which a current flows. Of what quantity is the work independent?

7. If two parallel conductors carry current in the same direction, do these conductors tend to separate or to come together? Give two reasons for the answer. Repeat for two conductors carrying current in opposite directions.

8. A single loop of wire lying in the plane of the paper carries a current in a clockwise direction. What effect will be noticed if a compass is placed within this loop? Compare the magnetic properties of this loop with those of a bar magnet.

9. Derive the equation which gives quantitatively the field intensity at any point on a perpendicular at the center of the plane of a circular loop of wire in which there is current.

10. From Question 9 derive the field intensity at the center of the loop itself. Compare this relation with the fundamental definition of an electric current.

11. Show how several loops similar to the one mentioned in Question 8 may be combined to form a long solenoid.

12. State three methods whereby the polarity of the poles at the ends of a solenoid may be determined, provided the direction of the current through the solenoid turns be known.

13. Derive the equation which gives the field intensity at the center of an infinitely long, straight solenoid. Compare the result obtained when the ratio of the length to the radius of the solenoid is moderately large, with that obtained with the solenoid of infinite length.

14. Make a sketch showing the construction of: (*a*) a simple solenoid with plunger; (*b*) an ironclad solenoid and plunger; (*c*) of (*b*) with a stop. By means of graphs show the relation between the pull of the plunger and its position within the solenoid. Point out the effect of the ironclad feature and of the stop on the tractive pull of the plunger as it is drawn into the solenoid.

15. Explain by the fundamental laws of magnetism why the plunger is drawn into a solenoid when current flows in the solenoid winding. State seven uses of commercial solenoids.

16. Show the principle whereby a U-shaped solenoid attracts an armature. Explain the principle of operation of the telegraph relay; the ordinary electric door bell.

17. Sketch a lifting magnet, showing its general construction. Where are such magnets used commercially, and in what way are they more economical than the older methods of handling material? Does the magnet itself do appreciable work when it is being used to handle iron and steel?

18. What is the disadvantage of the early types of magnetic circuits of dynamos, as represented by the Edison bipolar type? How has the design of the magnetic circuits of the more modern generators overcome some of the disadvantages of the earlier ones? What should be the approximate ratio of the cross-section of the field cores of a multipolar generator to the cross-section of the yoke? What general rule should be followed in placing the exciting ampere-turns on a magnetic circuit? Does magnetic leakage between the poles of a generator represent a direct loss of power?

PROBLEMS ON CHAPTER VII

Electromagnetism

370. In Fig. 370*A* are shown four conductors carrying direct current and entering a terminal box. The *N*-pole of a compass needle when placed over conductor *A* deflects away from the observer, as shown. When placed beside conductor *B*, it deflects to the right as indicated. Determine the direction in which the current in conductor *A* is flowing; conductor *B*.

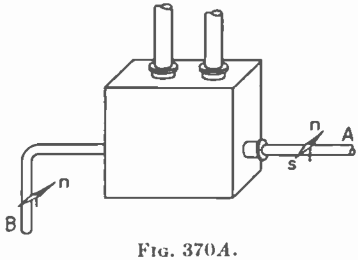


FIG. 370*A*.

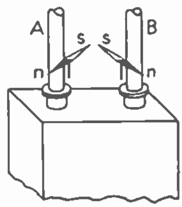


FIG. 371*A*.

371. In Fig. 371*A* two vertical conductors *A* and *B*, next to a wall and carrying direct current, enter two conduits, as shown. A compass needle is held near each conductor and it deflects in the directions shown. State the direction in which the current in each conductor flows.

372. A current of 150 amp. flows from left to right in an indefinitely long, straight wire *ab* (Fig. 372*A*). Determine: (*a*) the magnitude in dynes and the direction of the force which is exerted on a unit pole *n* at a distance of 10 cm. from the center of the conductor; (*b*) the work done in ergs, with the proper sign, in carrying the unit pole once around the conductor in a counterclockwise direction when viewed in the direction *a* to *b*.

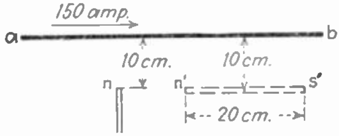


Fig. 372*A*.

A narrow bar magnet *n's'*, 20 cm. long, the pole strength of which is 40 unit poles, is laid parallel to, and 10 cm. from, the center of conductor *ab*, as is indicated by the dotted lines (Fig. 372*A*). (*c*) Determine the turning couple acting on this bar magnet, indicating the direction in which it acts. (*d*) If the position of the bar magnet is maintained parallel to the wire, state whether or not it tends to move around the wire.

373. A current of 250 amp. flows around the 90° arc *ab* (Fig. 373*A*) being led in and out by the two wires *a'a* and *b'b*, the directions of which are radial. A small *S*-pole *s* is located at the center of the arc *S*. Determine the direction and the magnitude in grams of the force acting on the pole.

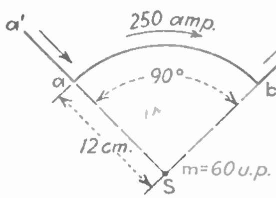


Fig. 373*A*.

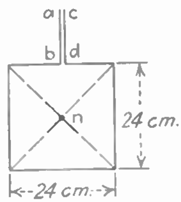


Fig. 374*A*.

374. A wire (Fig. 374*A*) is bent into the form of a square 24 cm. on a side. A current of 160 amp. flows in the wire, being conducted to the square by the two leads *ab* and *cd* which are placed very close together. Determine the magnitude in grams and the direction of the force acting on a *N*-pole *n* at the center of the square, *n* having a pole strength of 25 unit poles.

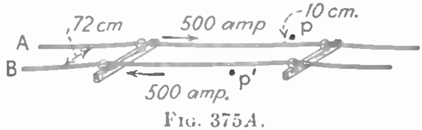


Fig. 375*A*.

375. In Fig. 375*A* are shown two parallel wires *A* and *B* on crossarms, the wires being spaced 72 cm. between centers, and each wire carrying a direct current of 500 amp. in opposite directions, as indicated by the arrows.

(a) Determine the direction of the magnetic field in the region between wires A and B. (b) Determine the direction of the magnetic field at point p just outside conductor A; at p' just outside conductor B. Points p and p' lie in the same plane as conductors A and B.

376. In Prob. 375 determine the field intensity in oersteds on a line parallel to conductors A and B midway between their centers.

377. In Prob. 375 point p is 10 cm. outside of conductor A and in the plane of the two conductors. Determine the field intensity at this point due to: (a) the current in conductor A; (b) the current in conductor B; (c) the currents in conductors A and B.

378. A flat coil of wire 24 cm. in diameter consists of 10 turns of small wire bound closely together. A current of 20 amp. flows in the coil, the leads carrying the current to and from the coil being very close to each other. Determine: (a) the field intensity at a point on a perpendicular to the plane of the coil at the center and 30 cm. above the plane of the coil; (b) the force acting on a small pole of 18-unit-pole strength, placed in the plane of the coil at its center.

379. Two plane coils A and B (Fig. 379A) each consisting of 12 turns are placed parallel to each other, and the center of each coil lies in a perpendicular oo' to their planes. The diameter of coil A is 20 cm. and the diameter of coil B is 30 cm., and the distance between the planes of the coils is 60 cm. A current of 12 amp. flows in each turn in both of the coils, and the coils are so connected ~~that~~ their magnetic fields both act downward. Determine: (a) the field intensity at a point P on the perpendicular oo' and 20 cm. from the plane of coil A; (b) the field intensity at the center of coil A; (c) the field intensity at the center of coil B.

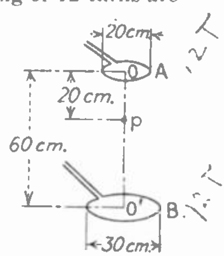


FIG. 379A.

380. In Prob. 379 coil A is placed in the plane of coil B and concentric with it. The magnitudes of the currents remain unchanged and they flow in the same direction. Determine the field intensity at the common center of the two coils.

381. In Prob. 380 the current in coil B remains at 12 amp. To what value must the current (reversed) in coil A be adjusted in order that the field intensity at their common center be zero?

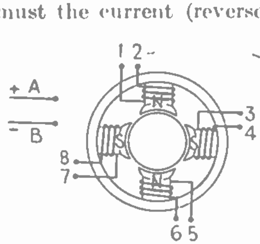


FIG. 382A.

382. In Fig. 382A is shown an end view of a four-pole generator. The four poles are to be so connected that they will be alternately north and south as indicated. The direct-current power is supplied by terminals A and B, terminal A being positive. The terminals of the four coils are designated 1, 2, . . . 8. Indicate the method of connection of the eight coil terminals with the supply terminals A and B which will produce the polarities shown.

383. In Fig. 383*A* is shown a magnetic circuit with a winding, linking each of the poles *A*, *B*, *C*, the polarity of terminals T_1 and T_2 of the winding being plus and minus. Indicate the polarity of the poles *A*, *B*, *C*.

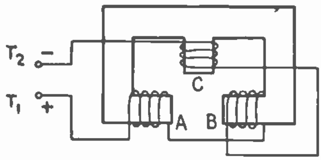


FIG. 383*A*.

384. The mean diameter of a cylindrical solenoid is 5 cm. and the length is 100 cm. There are 12 turns of No. 21 s.c.c. wire per centimeter length. A current of 2 amp. flows in the winding. Determine: (*a*) the field intensity at the center of the solenoid, assuming the solenoid to be infinite in length; (*b*) the flux density in lines per square centimeter at the center of the solenoid. The flux density over the center section of the solenoid is uniform. (*c*) Determine the flux across this center section.

385. (*a*) In Prob. 384 determine the exact field intensity at the center of the solenoid, the actual length of the solenoid being taken, rather than assuming it to be infinite. (*b*) Determine the field intensity at the extreme end of the axis of the solenoid.

386. In Prob. 384 determine the field intensity at a point on the axis within the solenoid: (*a*) 2 cm. from either end; (*b*) 10 cm. from either end.

387. In a simple solenoid and plunger, similar to that shown in Fig. 171 (p. 239), the field intensity is 3,000 oersteds just at the end of the plunger, and the approximate pole strength at the end of the plunger is 480 unit poles. The plunger is cylindrical and the diameter of the plunger is 3 cm. Determine the force on the plunger in grams, the other end of the plunger being well outside the magnetic field.

388. Repeat Prob. 387 with the exciting current in the solenoid reduced 20 per cent, field intensity and pole strength being proportionately reduced.

389. A solenoid similar to that of Prob. 387 is ironclad and a stop of the same diameter as the plunger is added. When the end surface of the plunger is nearly in contact with the surface of the stop, the flux density between them is 4,000 gausses. Determine the pull in kilograms between plunger and stop.

390. Repeat Prob. 389 with the exciting current in the solenoid reduced 20 per cent. The iron may be assumed as not saturated so that the flux density decreases proportionately.

If the magnetic members of a solenoid are not saturated, how does the pull on the plunger vary with the exciting current?

391. The flat pole pieces of an electromagnet are in contact with each other, and a total flux of 500,000 lines passes from one to the other. If the face of each pole is circular and 5 in. in diameter, what force in pounds is necessary to pull these pole pieces apart? Assume that the flux is distributed uniformly over the pole faces.

392. In Fig. 392*A* is shown the cross-section of a simple, circular, lifting magnet having an outside diameter of 24 in. The radial thickness of the annular pole piece is 2.2 in., and the diameter of the center pole is 9.5 in.

With a maximum flux density of 30,000 lines per square inch at the surface of contact, when a flat iron load is being lifted, determine the holding power of the magnet in pounds.

$\frac{\pi}{4} = .785$

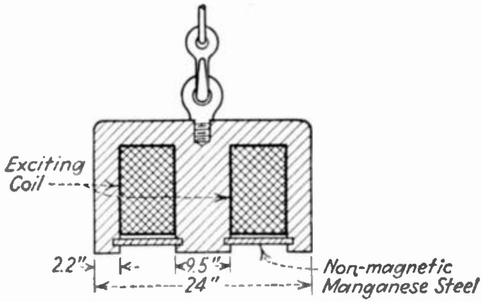


FIG. 392A.

393. In a lifting magnet constructed similar to that shown in Fig. 392*A*, the outside diameter of the annular pole shoe is 36 in., and the inner diameter is 33.75 in. The diameter of the center shoe is 12.5 in. Determine the flux density in lines per square inch at the surfaces of the pole shoes when the magnet is holding a flat iron piece with a force of 2,120 lb. Assume uniform flux distribution over the pole surfaces.

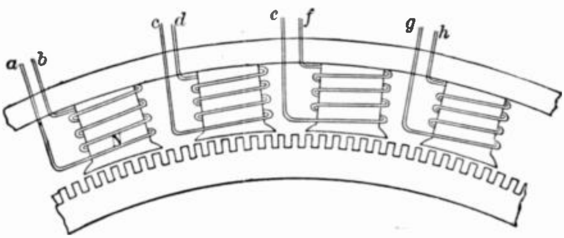


FIG. 394A.

394. Connect the coils *ab*, *cd*, *ef*, *gh* in the multipolar machine in Fig. 394*A*, so that the proper sequence of poles is obtained. Make the left-hand pole a *N*-pole as shown. Sketch the paths of the magnetic lines.

QUESTIONS ON CHAPTER VIII

The Magnetic Circuit

1. In what way does the magnetic circuit resemble the electric circuit? In what three respects do they differ from each other? Why cannot magnetic flux be confined to definite paths? In a general way, how does the accuracy obtainable in magnetic calculations compare with that obtainable in electrical calculations?

2. Define ampere-turns. What is the numerical relation between m.m.f. in gilberts and ampere-turns? Which is the larger unit, the gilbert or the ampere-turn? To what quantity in the electric circuit does m.m.f. correspond?

3. Define reluctance. To what quantity in the electric circuit does it correspond? What is the fundamental unit of reluctance? How is permeance related to reluctance and to conductance? How should permeances in parallel be added?

4. Define permeability. To what quantity in the electric circuit does magnetic flux correspond? Name the c.g.s. unit of flux; the practical unit. Define induction or flux density. Name the c.g.s. unit of flux density.

5. How is the reluctance of a magnetic path related to its length? to its cross-section? its permeability? Write the equation for reluctance. How are reluctances in series combined? in parallel? How are permeances in parallel combined?

6. In iron and steel, why is it usually necessary to represent the relation between magnetizing force and flux by a curve? What is the general shape of the lower part of such curves? the upper part? What is meant by saturation? How may a permeability curve be obtained from a *B-H* curve? How does the variation of permeability compare with the usual variation of electrical resistance in conductors, due to heating?

7. Show that in air the m.m.f. per centimeter is equal *numerically* to the field intensity *H* in oersteds.

8. State the simple law governing the relation which exists among flux, m.m.f., and reluctance. Repeat, when there are a number of different reluctances in series. To what law in the electric circuit does this law correspond?

9. Why is a method of trial and error sometimes necessary in solving magnetic problems? Show that, when the flux or flux density is given and it is required to determine the ampere-turns, the method of trial and error is no longer necessary.

10. Upon what three factors is the m.m.f. acting on a magnetic circuit dependent? Give the numerical equation for ampere-turns in centimeter units; in inch units.

11. How are magnetization curves plotted to reduce computations to the simplest basis? From the magnetization curves, compare the magnetic properties of cast iron, silicon steel, dynamo sheet steel, and cast steel.

12. Indicate the method of using the magnetization curves for determining the ampere-turns for different parts of a magnetic circuit. Indicate the procedure for determining the ampere-turns per pole in a multipolar dynamo.

13. The m.m.f. acting on a sample of iron is increased from zero to some definite value and then decreased again to zero. Plot the relation between the m.m.f. and the flux density. Compare the part of the curve for increasing values of m.m.f. with that for decreasing values.

14. In Question 13 what value does the magnetic flux reach when the excitation is decreased to zero? How may the magnetic flux be brought back to zero? Define a cycle; a hysteresis loop; remanence; coercive force.

15. Account for hysteresis loss by the Weber theory of molecular magnetism. What relation exists between hysteresis loss and the area of the hysteresis loop? How does hysteresis loss per cycle vary with the maximum flux density? What is the relation of hysteresis loss to the frequency?

16. What method is used to reduce eddy-current loss in iron? How does eddy-current loss vary with thickness of laminations; with frequency; with flux density? On what general basis are core losses compared?

17. What type of ferromagnetic alloy shows unusual magnetic properties particularly at the lower flux densities? What is the general composition of permalloy? What remarkable magnetic characteristics has it? Compare its magnetic properties with those of a pure iron, at both low and high flux densities. To what uses is it limited?

18. Describe Perminvar, and state its particular magnetic property.

19. Give the composition of Hipernik. What precautions are necessary in its manufacture? What particularly valuable properties has it, and what is its principal use? State the nature of Coupernik.

20. Describe the method by which flux may be measured by means of a ballistic galvanometer. Upon what three factors does the ballistic throw depend? Describe the means by which known values of flux are obtained.

21. Derive the equation which gives the flux in a ring-type solenoid having a cross-sectional area A sq. cm., a mean circumferential length l cm., and wound with n turns per centimeter (circumferential). Under what conditions does this equation give the flux within a long, straight solenoid near its center?

22. Describe the method of calibrating the ballistic galvanometer by means of a standard solenoid. Derive Eq. (126), p. 279, which gives the quantity Q_1 discharged through the galvanometer when the flux in the standard solenoid is caused to change. In a similar manner derive Eq. (127), p. 279, which gives the quantity Q_2 discharged through the galvanometer when the flux in the test specimen is changed from ϕ_1 to ϕ_2. By means of these two equations derive the galvanometer constant (Eq. (129), p. 280).

23. Describe the yoke method of magnetic testing. What are its advantages? Name two important sources of error.

24. Describe the ring method of magnetic testing, comparing the procedure with that of the yoke method. State its advantages and disadvantages.

25. Upon what well-known principle does the Koepsel permeameter operate? What is the form of the test specimen, and where is it placed? Describe the exciting coil and give the function of the compensating coils. What is the advantage of this instrument as a method of testing? What errors may exist in the instrument?

PROBLEMS ON CHAPTER VIII

The Magnetic Circuit

395. In Fig. 395*A* is shown a closed-ring type of solenoid, the core being of non-magnetic material. The average diameter of the ring is 3.6 cm., and the cross-sectional diameter of the ring is 0.6 cm. The winding on the ring is uniform and there is a total of 288 turns. A current of 0.2 amp. flows in the winding in the direction shown. Determine: (*a*) the direction of the flux in the ring; (*b*) the ampere-turns per centimeter of average circumference; (*c*) the gilberts per centimeter in (*b*). (*d*) From (*b*) and (*c*) determine the ratio of the ampere-turn to the gilbert, stating which unit is larger. The flux at the average diameter is 1.81 maxwells. If the non-magnetic core is replaced by iron, the flux becomes 1,632 maxwells. (*e*) Determine the permeability of the iron.

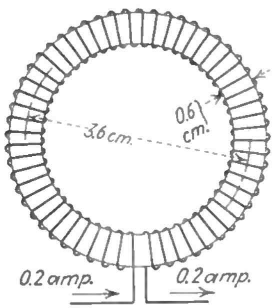


FIG. 395*A*.

396. In Fig. 396*A* is shown an electromagnet on which there are four equal exciting coils *A*, *B*, *C*, *D*, with 2,400 turns each, the resistance of each coil being 12 ohms. At first, coils *A* and *B* are connected in series across 120-volt mains, the connection being such that the coils are acting in conjunction on the magnetic circuit. (*a*) Determine the total ampere-turns acting on the magnetic circuit. (*b*) The coils *C* and *D* are connected in series electrically with *A* and *B* in such a way that their ampere-turns are acting in conjunction with those of *A* and *B*. The four coils in series are then connected across the 120-volt mains. Determine the total ampere-turns now acting on the magnetic circuit. (*c*) Determine the total power lost in the exciting coils in (*a*) and (*b*) and find the ratio of the power loss in (*b*) to that in (*a*). This illustrates the principle that by increasing the amount of copper and likewise the amount of iron, the excitation loss may be reduced to almost any desired value without changing the ampere-turns. (*d*) Determine the total m.m.f. in gilberts. (*e*) Determine the ampere-turns and the excitation loss when the coils *AB* in series are connected in parallel with coils *CD* in series, across 120 volts.

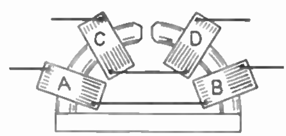


FIG. 396*A*.

397. (*a*) In Prob. 396(*b*) determine the total ampere-turns when the current in both pairs of coils, *AB* and *CD*, in series, is adjusted until the heating in each coil is the same as in (*a*). (*b*) Determine the total ampere-turns when the current is adjusted until the total heating in Prob. 396(*b*) is the same as the total heating in Prob. 396(*c*).

398. An exciting coil, with 14,000 turns of No. 37 A.W.G., enamel wire. is designed for a 120-volt circuit, the resistance being 3,370 ohms. (*a*) Determine the number of turns and the resistance of another exciting coil of the same size and having the same winding space factor (ratio of net copper section to total coil section), which is wound with No. 34 wire having twice the cross-section of No. 37. (*b*) With the same voltage, what is the ratio of the ampere-turns in the two cases? (*c*) Compare the heating in the two cases. (*d*) What voltage should be impressed across the second coil in order that the ampere-turns in the two cases may be the same? (c) in order that the heating may be the same?

399. Two coils *A* and *B* have 4,200 and 3,600 turns and 30- and 25-ohm resistance. Both coils are designed to operate at 120 volts without overheating. Both are placed on the same magnetic circuit. Determine the ampere-turns acting on the circuit: (a) with coil *A* alone; (b) with coil *B* alone; (c) with coils *A* and *B* in series; (d) with coils *A* and *B* in parallel. The voltage is 120 volts in each case, and in (c) and (d) the coils act in conjunction on the magnetic circuit.

400. (a) In Prob. 395 determine the flux density in gausses with the ring of non-magnetic material; (b) with the iron ring. (c) Convert the flux density in (a) to lines per square inch; (d) convert the flux density in (b) to lines per square inch.

401. It is desired to operate a magnetic core of circular cross-section at a flux density of 12,000 gausses, the total flux to be 150,000 maxwells. Determine the diameter of the core cross-section.

402. The cross-section of the core of an electromagnet is to be square. It is desired to operate the core at a flux density of 80,000 maxwells per square inch, the total flux to be 1,200,000 maxwells. (a) Determine the dimensions of the core. (b) Repeat (a) with a rectangular cross-section, one dimension of which is 1.5 times that of the other.

403. In Fig. 403*A* is shown the steel core of a magnetic circuit with the two pole pieces *P*. The cross-sections of the core and of the pole pieces are square, the core being 5 cm. on a side and the pole pieces being 10 cm. on a side. The mean length of the core is 21 cm., and the length of the air-gap is 0.5 cm. The permeability of the steel in the core is 800. Determine: (a) the reluctance of the air-gap; (b) the reluctance of the steel core; (c) the total reluctance. Assume that the reluctance of the pole pieces is negligible and that fringing may be neglected.

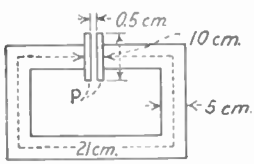


Fig. 403*A*.

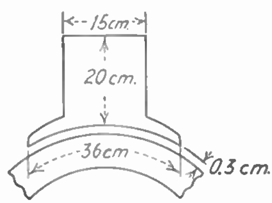


Fig. 404*A*.

404. Figure 404*A* shows the pole core and a portion of an armature of a four-pole dynamo. The cross-section of the pole core is square, 15 by 15 cm., and its effective length is 20 cm. The permeability at the operating flux density is 600. The length of pole arc is 36 cm., and the pole face is square. The length of air-gap is 0.3 cm., and the armature surface may be considered as smooth. Determine the combined reluctance and permeance of the pole core and air-gap in series.

405. The core of a ring magnet of the dimensions given in Fig. 395*A* is made of steel, the permeability of which is 1,200. A saw cut of uniform length of 1 mm. is cut transversely to the ring. Determine: (*a*) the reluctance of the iron ring using the mean length; (*b*) the reluctance of the air-gap; (*c*) the reluctance of the entire magnetic circuit; (*d*) the permeance of the entire magnetic circuit.

✓ **406.** Exciting coils having a total of 2,000 turns are placed on the magnetic core shown in Fig. 406*A*. The cross-section of the core is 3 in. square. When the current is 1.2 amp., the resulting flux density makes the permeability 1,100. Determine: (*a*) the flux; (*b*) the flux density in the steel; (*c*) the flux density in the air-gap. Neglect leakage and fringing.

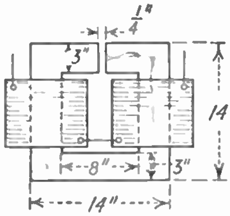


FIG. 406*A*.

407. In the ring magnet of Prob. 405 determine: (*a*) the m.m.f. in gilberts necessary to produce a flux of 2,250 lines in the magnetic circuit; (*b*) the ampere-turns.

408. In a ring solenoid of the type shown in Fig. 395*A*, the average diameter across the ring is 6 cm. and the cross-sectional diameter of the ring is 1.2 cm. A transverse saw cut, 1.5 mm. in length, is made in the iron ring. The permeability of the steel may be taken as 1,000. Determine: (*a*) the reluctance of the steel; (*b*) the reluctance of the air-gap; (*c*) the total reluctance; (*d*) the flux when a m.m.f. of 1,600 ampere-turns is placed on the magnetic core; (*e*) the flux density in the steel and in the air-gap. Neglect leakage and fringing.

409. In the magnetic circuit in Fig. 409*A*, the cross-section of the iron core is square, 3 cm. on a side, and the length of each air-gap is 1.5 mm.

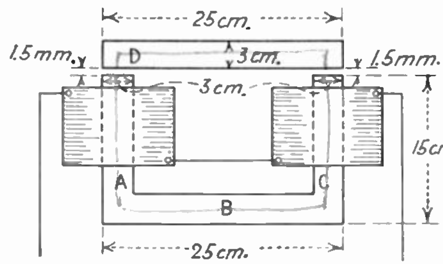


FIG. 409*A*.

The permeability of the parts *A*, *B*, *C* is to be taken as 1,200 and that of the armature *D* as 900. Each of the two exciting coils has 1,600 turns, and the exciting current is 0.9 amp. Determine: (*a*) the combined reluctance of the members *A*, *B*, *C*; (*b*) the reluctance of the armature *D*; (*c*) the reluctance of the two air-gaps; (*d*) the total m.m.f. in gilberts acting on the circuit; (*e*) the total flux; (*f*) the flux density in gausses. Neglect fringing and leakage.

410. Figure 410*A* shows the core of an electromagnet, composed of a semicircular portion *A* and two cores, and pole pieces *B* and *C* in the form of rectangular prisms. The dimensions and permeability of each portion are given. (*a*) Compute the reluctance of the magnetic circuit; (*b*) an exciting winding of 10,000 turns is wound over the core. (*c*) Determine the flux in the core when the exciting current is 1.0 amp. Neglect fringing and leakage.

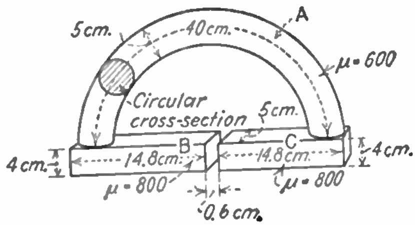


FIG. 410*A*.

411. Figure 411*A* shows a magnetic circuit composed of two branches, which are similar and in parallel. The permeability of the iron is 600 throughout. The mean length of path in the iron core may be taken as the distance *l* shown on the left-hand branch of the circuit. (*a*) Compute the reluctance of each branch and the total reluctance of the circuit. A coil of 3,000 turns is placed on the center core. When the exciting current is 2.6 amp., (*b*) compute the flux in each air-gap. (*c*) Compute the flux in the center core. Neglect fringing and leakage.

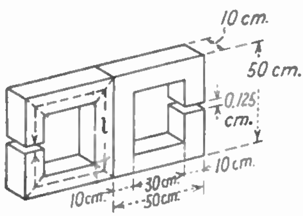


FIG. 411*A*.

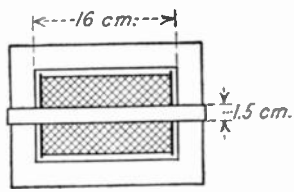


FIG. 412*A*.

412. In an ironclad solenoid (Fig. 412*A*), the reluctance of the yoke is negligible compared with that of the cylindrical cast-steel plunger. The diameter of the plunger is 1.5 cm. and the net length inside the yoke is 16 cm. The magnetic characteristics of the plunger are given in Figs. 186 and 187 (pp. 254 and 255). In the exciting coil are 400 turns, and the exciting current is 1.67 amp. By the method of trial and error (Par. 178, p. 259), determine the flux and the flux density in the plunger.

413. In a ring solenoid of the type of Fig. 395*A*, the core is of cast steel, the magnetization curve of which is given in Fig. 186 (p. 254) and the permeability curve in Fig. 187 (p. 255). The average diameter across the ring is 8 cm. and the cross-sectional diameter of the ring is 1.5 cm. A transverse saw cut, 0.6 mm. long, is made in the ring. There are 1,200 turns in the exciting coil and the current is 1.09 amp. By the method of trial and error (Par. 178, p. 259), using the characteristics of the steel given in Figs. 186 and 187, determine the flux and the flux density in the core.

414. Repeat Prob. 413 with the exciting current decreased to 0.766 amp.

415. Assume the core of the electromagnet shown in Fig. 411*A* to be made of cast steel, the magnetic characteristic of which is given in Figs. 186 and 187 (pp. 254 and 255). The exciting current is adjusted to 2 amp. Using the method of trial and error (Par. 178, p. 259), determine the flux and the flux density in the core. Neglect fringing and leakage.

416. Repeat Prob. 415 with the exciting current adjusted to 2.5 amp.

417. Using the magnetization curve of Fig. 186 (p. 254), determine, for the plunger of Fig. 412*A*, the gilberts necessary to produce a flux density of (*a*) 5,000 gausses; (*b*) 8,000 gausses; (*c*) 14,000 gausses; (*d*) 16,000 gausses. (*e*) Convert (*a*), (*b*), (*c*), and (*d*) into ampere-turns.

418. In Fig. 418*A* is shown a cast-iron rod, 1.25 in. in diameter, held in an iron yoke similar to that shown in Fig. 412*A*, the reluctance of which is negligible. The net length of rod between the inner faces of the yoke is 15 in. Using the magnetization curve for cast iron (Fig. 190, p. 261), determine the number of ampere-turns necessary to produce, in the rod, flux densities in maxwells per square inch of (*a*) 30,000; (*b*) 45,000; (*c*) 60,000. (*d*) Express (*a*), (*b*), and (*c*) in gilberts.

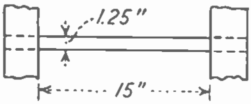


Fig. 418*A*.

419. Assume that the rod (Fig. 418*A*) is of cast steel. Using the characteristic for cast steel (Fig. 190, p. 261), determine the ampere-turns necessary to produce a *total* flux in maxwells in the rod of (*a*) 86,000; (*b*) 107,000; (*c*) 130,000; (*d*) 135,000. (*e*) Convert (*a*), (*b*), (*c*), to gilberts.

420. In Fig. 403*A*, which shows a 0.5-cm. air-gap between two flat steel surfaces 10-cm. square, compute the ampere-turns necessary to produce in the air-gap the following flux densities in gausses: (*a*) 4,000; (*b*) 6,000; (*c*) 10,000. (*d*) In (*a*), (*b*), and (*c*), compute the m.m.f. in gilberts between the flat surfaces. Neglect fringing.

421. In the dynamo air-gap (Fig. 404*A*), determine the ampere-turns necessary to produce the following flux densities in gausses: (*a*) 4,500; (*b*) 6,400; (*c*) 8,000. (*d*) In (*a*), (*b*), and (*c*), compute the m.m.f. in gilberts between the armature surface and the pole face. Neglect fringing.

422. In an air-gap similar to that shown in Fig. 404*A*, the length of air-gap is 0.2 in. and the pole faces are 12 in. square. Determine the ampere-turns necessary to produce the following flux densities in lines per square inch: (*a*) 20,000; (*b*) 35,000; (*c*) 45,000. (*d*) In (*a*), (*b*), and (*c*), compute the total flux per pole entering the armature. Neglect fringing.

423. Repeat Prob. 422 with an air-gap of 0.22 in., and pole faces with axial lengths of 12 in. and peripheral lengths of 14 in.

424. The diameter of the non-slotted armature of a four-pole generator is 12 in. and its axial length is 6.75 in. The poles are square and cover seven-tenths of the armature circumference. The air-gap is 0.22 in. Determine the ampere-turns necessary to send a flux of 2,000,000 maxwells per pole across the gap.

425. In a ring magnet similar to that in Fig. 395*A*, the core consists of a cast-steel ring 1 cm. in diameter, and the average diameter across the ring is 5 cm. The length of the transverse air-gap is 1.5 mm. Using the magnetization curve of Fig. 190 (p. 261), determine: (*a*) the ampere-turns necessary to produce a flux density of 9,000 gausses in the steel; (*b*) the ampere-turns necessary to produce this flux density in the air-gap; (*c*) the total ampere-turns; (*d*) the total flux in maxwells. From (*c*) and (*d*) determine the total reluctance of the magnetic circuit.

426. A ring, similar to that of Fig. 395*A*, is made up of dynamo-sheet-steel laminations, each with a radial thickness of 0.5 in., an average diameter of 4 in., and a transverse air-gap of 0.075 in. The laminations are stacked to a thickness of 0.5 in. making a gross cross-section of 0.5 by 0.5 in. Owing to the oxide on the surfaces of the laminations, the net steel is only nine-tenths the gross cross-section. Using the appropriate characteristic in Fig. 190 (p. 261), determine: (*a*) the ampere-turns necessary to produce in the core a total flux of 21,500 maxwells; (*b*) the ampere-turns for the gap; (*c*) the total ampere-turns. Neglect fringing.

427. In Fig. 406*A* is shown an electromagnet, the core of which is cast iron and the magnetic characteristic of which is given in Fig. 190 (p. 261). The core is 3 in. square and there is a transverse air-gap ¼ in. long. There are 800 turns in each of the exciting coils. Using the curve (Fig. 190), determine the exciting current necessary to produce a flux of 540,000 maxwells in the air-gap. Neglect fringing and leakage.

428. Repeat Prob. 427 with the air-gap flux equal to 450,000 maxwells.

429. Repeat Prob. 427, assuming the core to be of cast steel and the flux in the air-gap to be 810,000 maxwells. Use the magnetization curve of Fig. 190 (p. 261).

430. Assume that the magnetic cores (Fig. 411*A*) are made of cast steel and that it is desired to produce a flux of 1,400,000 maxwells in each air-gap. Using the characteristic curve for cast steel in Fig. 190 (p. 261), determine: (*a*) the ampere-turns necessary to send this flux through each iron core; (*b*) the ampere-turns necessary for each air-gap; (*c*) the total ampere-turns necessary on the central core; (*d*) the flux density in the iron in the central core; (*e*) the total flux linking the exciting coil. (*f*) Compare the ampere-turns in (*c*) with those necessary to produce a flux of 700,000 maxwells in the air-gap of one branch alone. Neglect fringing and leakage.

431. Repeat Prob. 430 with a flux of 1,100,000 maxwells in each gap. Omit (*f*).

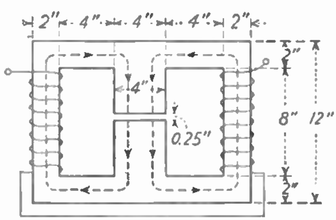


Fig. 432*A*.

432. In Fig. 432*A* is shown a magnetic circuit, the core of which is of cast steel. The dimensions are given on the figure, the thickness perpendicular to the paper being 4 in. The length of the air-gap is 0.25 in. There are two equal exciting coils connected in series. The air-gap flux is 1,600,000 maxwells. Determine: (*a*) the ampere-turns necessary to send this flux through the iron of each half of the core; (*b*) the ampere-turns for the air-gap; (*c*) the ampere-turns in each coil.

433. Repeat Prob. 432 with a total flux of 1,440,000 maxwells in the air-gap.

434. In Prob. 432, consider the magnetic circuit (Fig. 432*A*) split in halves vertically along its center. With a total flux of 800,000 maxwells, repeat (*a*), (*b*), and (*c*). Compare the flux densities with those in Prob. 432.

435. In Fig. 435*A* is shown the magnetic circuit of a two-pole dynamo. The field cores are of cast steel and are 4.5 by 4.5 in. The armature is of dynamo sheet steel and has a net axial length of 5.2 in. over the iron; the yoke is of cast steel, the cross-section being 1.75 by 6 in. The other necessary directions are given on the figure. Using the magnetization curves of Fig. 190 (p. 261), determine the field ampere-turns necessary to produce an average flux density of 40,000 lines per square inch in the air-gap.

436. Repeat Prob. 435 with an average air-gap density of 45,000 lines per square inch.

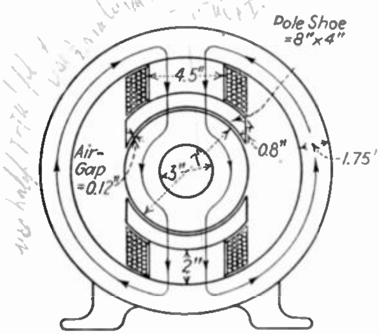


FIG. 435*A*.

437. Repeat Prob. 435 with an air-gap length of 0.1 in.

438. Repeat Prob. 435 assuming that the leakage factor is 1.25; that is, only 80 per cent of the flux in the field cores and yoke enters the armature. (Hence leakage factor = 100 ÷ 80 = 1.25.)

439. In Fig. 439*A* is shown a portion of an eight-pole, 250-volt, direct-current generator. Two complete magnetic circuits are shown as well as portions of two others. The outside diameter of the armature is 36 in., the radial thickness of the armature is 4 in., and the axial length of the

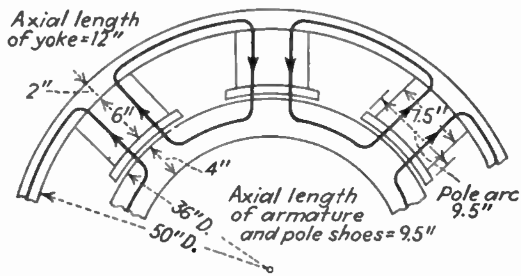


FIG. 439*A*.

armature is 9.5 in. The pole cores are 7.5 in. square and 6 in. long, and are of dynamo sheet steel, nine-tenths of the total volume being net iron. The pole faces are 9.5 in. square and are practically 1 in. thick (less the length of air-gap). The inside diameter of the yoke is 50 in., and the cross-section of the yoke is 12 by 2 in. The armature is of dynamo sheet steel, nine-tenths of the gross volume being net iron. The yoke is of rolled steel, the magnetic characteristics of which may be assumed the same as those for cast steel (Fig. 190, p. 261). Of the flux in the pole cores and yoke, five-sixths enter the armature. (Leakage factor = 6 ÷ 5 = 1.20.) The air-gap has an equivalent length of 0.2 in. Using the magnetization curves of Fig. 190, determine the ampere-turns per pole necessary to produce a flux density of 45,000 maxwells per sq. in. in the air-gap.

440. Repeat Prob. 439 with an air-gap flux density of 50,000 maxwells per sq. in.

441. Repeat Prob. 439 with an air-gap of 0.24 in.

442. Repeat Prob. 439 with an air-gap flux density of 50,000 maxwells per sq. in. and an air-gap of 0.24 in.

443. The total volume of a core built up of silicon-sheet-steel laminations is 6,000 c.c. and the gross cross-sectional area is 144 sq. cm. The core operates with an alternating flux which varies between positive and negative values of 1,280,000 maxwells at a frequency of 50 cycles per second. Owing to the oxide on the surfaces of the laminations, the ratio of the net steel to the gross cross-section is 9 to 10. Using the data of Par. 184 (p. 268), determine: (*a*) the ergs loss per cubic centimeter per cycle; (*b*) the total loss in ergs per cycle; (*c*) the ergs loss per second; (*d*) the power loss in watts owing to hysteresis (1 watt = 10^7 ergs per second).

444. The volume of a transformer core made of silicon-sheet-steel laminations is 800 cu. in., and the gross cross-sectional area is 16 sq. in. The ratio of net iron to total core cross-section is 9 to 10. (*a*) Determine the hysteresis loss in ergs per cubic inch per cycle if the maximum flux density is 50,000 lines per square inch. (*b*) With a frequency of 60 cycles per second, determine the hysteresis loss in watts. (See Par. 184, p. 268.)

445. In a transformer core of laminated silicon steel the hysteresis loss is 110 watts when the total flux (maximum value) is 128,000 maxwells, and the frequency is 50 cycles per second. Determine the loss: (*a*) when the frequency is 60 cycles per second, the flux remaining unchanged; (*b*) when the flux is 100,000 maxwells and the frequency is 50 cycles per second; (*c*) when the flux is 100,000 maxwells and the frequency is 60 cycles per second.

446. In Prob. 445 the eddy-current loss is 20 watts at 128,000 maxwells and 50 cycles. Determine the eddy-current loss under the conditions of (*a*), (*b*), and (*c*).

447. In Table II (p. 271), the hysteresis and eddy-current loss in watts per pound for "Allegheny Transformer C Grade Sheet Steel" are given as 0.658 and 0.114 watt per pound at 60 cycles and 10,000 gausses. Determine these two losses at (*a*) 60 cycles, 12,000 gausses; (*b*) 50 cycles, 10,000 gausses; (*c*) 50 cycles, 12,000 gausses.

448. Repeat Prob. 447 for "Allegheny Dynamo-special"-grade sheet steel, in which the hysteresis and eddy-current losses are 0.733 and 0.128 watt per pound.

449. A straight air-core solenoid, 120 cm. long, consists of a single-layer primary winding of d.c.c. wire, with six turns to the centimeter, and a secondary of 1,000 turns is wound over this primary winding near the center. The inside diameter of the solenoid is 3.5 cm. When the primary current is 0.8 amp., determine: (*a*) the flux density in the solenoid; (*b*) the total flux; (*c*) the c.g.s. flux linkages with the secondary.

450. A secondary coil having 12 turns, and used with the yoke method and a ballistic galvanometer, is connected in series with the secondary of the solenoid of Prob. 449. When the current of 0.8 amp. in the primary of the standard solenoid is suddenly reduced to zero by opening the switch, the ballistic deflection of the galvanometer is 16 cm. (*a*) Determine the galvanometer constant (see Eq. (129), p. 280). (*b*) The current in the primary of the yoke specimen is increased from zero to 0.4 amp. and one section of the test specimen is withdrawn, thus reducing the flux suddenly to zero. The corresponding galvanometer deflection is 20.5 cm. What is the value of the flux in the specimen? (*c*) If the diameter of the yoke specimen is 1 cm., what is the flux density?

451. In Prob. 450, the current in the yoke specimen is increased to 1.5 amp., decreased to zero, and then increased to 0.9 amp. in the reverse direction. At this point, half of the specimen is again withdrawn and the corresponding galvanometer deflection is 22.4 cm. (reversed). What is the flux density corresponding to this value of current? Make a sketch of the hysteresis loop indicating this point and the similar one given in Prob. 450.

452. The normal saturation curve of a ring specimen is being determined by the ring method. The cross-section of the ring is 1 cm. square and the average diameter of the ring is 10 cm. The galvanometer of Prob. 450 is used in the measurement, the calibration remaining unchanged. There are 10 turns on the secondary of the ring. With a complete *reversal* of the current in the primary of the ring, the deflection of the galvanometer is 28 cm. Determine the flux in the ring for this value of current. K14 – 0

QUESTIONS ON CHAPTER IX

Self- and Mutual Inductance

1. Describe the method of inducing an e.m.f. in a circuit by means of a bar magnet.

2. If an induced current is allowed to flow in a coil, what reaction will exist between this current and the inducing agent? If the inducing agent as, for example, a bar magnet be withdrawn from a coil, compare the direction of the induced e.m.f. with its direction when the bar magnet is inserted into the coil. In each case what reaction is produced between the induced current and the bar magnet?

3. Upon what three factors does induced e.m.f. depend when expressed in terms of flux? when expressed in terms of self-inductance? How is the value of the e.m.f. determined in volts? State and explain Lenz's law.

4. How is the geometrical position of the lines of induction related to the current in a circuit? Compare this relation with the term *linkages.* How may these linkages be calculated? With constant permeability, what relation does inductance bear to the total linkages? Define inductance under these conditions, giving the unit.

5. How does inductance vary with the turns, the permeability remaining constant?

6. If the flux linking a coil be made to change by altering the value of the current in the coil itself, show that an e.m.f. must be induced. What is the relation between the direction of this e.m.f. and the direction of the current in the coil? How does the induced e.m.f. affect the rapidity with which the current builds up to its Ohm's-law value?

7. From the equation which gives induced e.m.f. in terms of rate of change of flux—turn linkages, derive the equation which gives induced e.m.f. as function of inductance and rate of change of current.

8. Define inductance under the conditions of variable permeability.

9. Derive the equation which gives the rise of current as a function of time, in a circuit having both resistance and inductance in series. Sketch the graph which shows this relation.

10. What is meant by the *time constant* of a circuit, and by what two quantities is it expressed? In a general way, what does it indicate as regards the circuit? What is the practical importance of the time lag of current due to self-inductance?

11. Explain why current continues to flow after an inductive circuit is short-circuited. Derive the equation which gives the relation of current to time. Indicate the value of current at the value of time given by the time constant of the circuit.

12. Comment on the following two statements. It requires power over an interval of time to store energy in a magnetic field. It requires no power to maintain a steady magnetic field.

13. Compare the energy stored in the magnetic field with a mechanical analogue. Discuss the relation of the energy stored in the magnetic field with any expenditure of energy that may be necessary to maintain it. Account for the energy supplied to the exciting coils of electromagnets after the current reaches a steady value. How does stored magnetic energy vary with the current?

14. In what way does the energy of the magnetic field manifest itself when the current maintaining the field is interrupted? How can it be shown that this condition is not produced by the mere interruption of the current itself? Analyze the arcing at switch contacts which occurs when an inductive circuit is opened. Under what conditions in practice may such arcing become a menace? How may this menace be partially or wholly eliminated?

What personal dangers may result from opening inductive circuits? Make a wiring diagram showing the correct position of the voltmeter connected across an inductive circuit.

15. Show, with a wiring diagram, the method of utilizing stored magnetic energy for ignition in automobile engines.

16. Show that it is possible for a change of current in one coil to induce an e.m.f. in a second coil which is completely insulated from the first coil. Show that this is analogous to the production of e.m.f. by the insertion of a bar magnet into the second coil. What is the relation between the direction of the induced e.m.f. in the secondary when the primary circuit is closed, and its direction when the primary circuit is opened? Upon what three factors does the induced e.m.f. depend?

17. Discuss the possibility of all the flux produced by one coil, linking all the turns of a second coil. Define *coefficient of coupling*.

18. With unit current in one coil, to what three factors is mutual inductance proportional? Define mutual inductance in terms of rate of change of current in one of the circuits.

19. Derive the equation by means of which the e.m.f. induced due to mutual inductance may be computed.

20. Discuss methods of increasing mutual inductance between circuits. Show that if two electric circuits are linked with the same magnetic circuit, the mutual inductance $M = K\sqrt{L_1L_2}$, where K is the coefficient of coupling, and L_1 and L_2 are the self-inductances of the two circuits.

21. If the two coils in Question 20 are connected in series, show that the total inductance $L = L_1 + L_2 \pm 2M$.

22. Describe methods of measuring mutual inductance by means of an alternating-current bridge.

23. State the general effect of self-inductance on changes in the current. To what mechanical property is it analogous? Give mechanical analogues of inductance.

24. Describe the operation of the induction coil showing that it depends on mutual inductance. By what method is the primary current interrupted, and why is it necessary that this current be interrupted?

PROBLEMS ON CHAPTER IX

Self- and Mutual Inductance

453. A coil (Fig. 453A) is wound with 600 turns and, when a current I of 5 amp. flows in the coil a flux ϕ = 7,500 maxwells, links these turns. Compute: (*a*) the flux linkages in maxwell-turns; (*b*) the maxwell-turn flux linkages per ampere divided by 10^8; (*c*) the self-inductance of the coil in henrys.

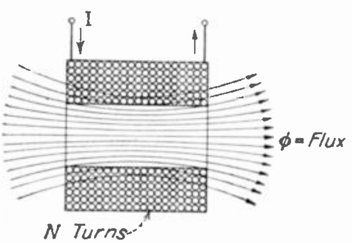


Fig. 453A.

454. A coil of the same dimensions as that in Fig. 453A is wound with twice the number of turns but the wire is one-half the cross-section. Determine (*a*), (*b*), and (*c*) of Prob. 453. (*d*) Compare these results with those of Prob. 453 and state the relation of inductance to turns which they illustrate if the magnetic reluctance is constant.

455. In a ring solenoid of the type shown in Fig. 455A, the reluctance of the non-magnetic core is 4.0 e.g.s. units and the ring is wound with 400 turns

When the current is 0.25 amp., determine: (a) the flux in maxwells; (b) the maxwell-turn linkages; (c) the maxwell-turn linkages per ampere; (d) the inductance of the ring in henrys and mil-henrys (1 mil-henry = 0.001 henry).

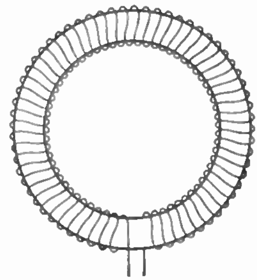


FIG. 455A.

456. A steel ring of the same dimensions as that in Prob. 455, but with a transverse air-gap of 0.20 mm., is wound with 400 turns. The cross-sectional area of the ring is 0.283 sq. cm. The permeability of the steel may be assumed to be constant at 800. The current is 0.25 amp. Repeat (a), (b), (c), and (d).

457. Solve Prob. 456 with the core wound with 200 turns.

458. A plunger-type tractive solenoid (p. 239) is wound with 3,000 turns of No. 12 d.c.c. wire. When the current is 3.6 amp., the net flux linking the turns is 50,000 maxwells. Assuming constant permeability, determine: (a) the c.g.s. linkages; (b) the c.g.s. linkages per ampere divided by 10^8; (c) the inductance of the solenoid in henrys. (d) Would the inductance of such a solenoid, when in operation, be constant even if the permeability of the iron remained constant?

459. The solenoid of Prob. 458 is rewound with 6,000 turns of No. 15 d.c.c. wire having half the cross-section of No. 12 wire. It now operates at double voltage so that the current is 1.8 amp. Determine: (a) the c.g.s. linkages; (b) the inductance in henrys; (c) the relation of inductance to number of turns.

460. Assuming that the permeability of the iron does not change, determine the inductance of the solenoid of Prob. 459 when the current is doubled. In general, how would the inductance of a coil having a core of ordinary iron vary with increase of current?

461. The two exciting coils of a bipolar dynamo are wound with 1,800 turns each. When the exciting current is 2.8 amp., the flux crossing an air-gap is 2,500,000 maxwells. The coefficient of leakage is 1.25 (see Prob. 439, p. 682). Assuming constant permeability for the iron, determine the inductance of this field circuit.

462. In Prob. 461, when the exciting current is 3.2 amp., the flux in the air-gap is 2,750,000 maxwells. The coefficient of leakage remains unchanged. Determine: (a) the self-inductance of the field circuit, neglecting any change in reluctance; (b) the average rate of change of flux in maxwells per second, if the field circuit is opened in 0.1 sec.; (c) the average induced e.m.f. across the terminals under the conditions of (b); (d) the average induced e.m.f. calculated from the equation $e = -L di/dt$.

463. In a four-pole generator are 2,400 turns per field pole, and when the exciting current is 3 amp. the air-gap flux is 3,600,000 maxwells. The coefficient of leakage is 1.25 (Prob. 461). Determine: (a) the maxwells linking each field-turn; (b) the maxwell-turn linkages per pole; (c) the total maxwell-turn linkages; (d) the self-inductance of the field circuit, neglecting any change in reluctance.

464. In Prob. 463, the field circuit is interrupted in 0.15 sec. Determine: (*a*) the average rate of change of flux in maxwells per second; (*b*) the average rate of change of flux-turn linkages per second divided by 10^8; (*c*) the average induced e.m.f. (*d*) Calculate (*c*) on the basis of $e = -Ldi/dt$. (*e*) Determine the energy stored in the field in joules.

465. The exciting current to an electromagnet causes 2,400,000 lines of induction to link the turns of the exciting coils. The exciting coils have a total of 5,400 turns. (*a*) If the exciting current with the resulting arc is interrupted in 0.2 sec., determine the average induced e.m.f. across the ends of the exciting winding. (*b*) If the exciting current is 1.2 amp., determine the inductance of the exciting winding. (*c*) How much energy is stored in the magnetic field? (*d*) Determine the average power delivered by the field during the current interruption. (*e*) Recompute (*a*) and (*d*), assuming that the circuit is interrupted in 0.05 sec.

466. The inductance of a generator field circuit is 5 henrys and the exciting current is 150 amp. The induced e.m.f. across the field terminals must not exceed 1,000 volts. (*a*) What is the minimum time which can be allowed for opening the field? (*b*) How much energy is liberated in opening this field? (*c*) What is the average power during the period of opening?

467. Repeat Prob. 466 with the total field resistance doubled by means of the field rheostat, the voltage remaining unchange.

468. A solenoid is wound with 400 turns. The relation of the flux linking these turns to the current is given by Fig. 208(*b*) (p. 295). Determine the self-inductance of the solenoid when the exciting current is (*a*) 1 amp.; (*b*) 3 amp. (see Par. 198, p. 295). (*c*) Determine the induced e.m.f. in (*a*), if the current is increasing at the rate of 120 amp. per second; (*d*) in (*b*) if the current is increasing at the rate of 30 amp. per second. (The value of $d\phi/di$ at 1 amp. is 1.3×10^5 and at 3 amp is 3×10^4 maxwells per ampere.)

469. The following data give the relation of flux to exciting current in a solenoid circuit in which there are 800 turns in the exciting winding:

Amperes	0.2	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Kilomaxwells	14	42	54	62	67	71	74	76.5	78.5

Determine the inductance when the current: (*a*) varies from 0 to 0.6 amp. (straight part of curve); (*b*) equals 1.0 amp.; (*c*) equals 1.6 amp. When the current is increasing at the rate of 100 amp. per second, determine the induced e.m.f.: (*d*) in (*a*); (*e*) in (*b*); (*f*) in (*c*). (*g*) In (*a*) at what rate is the flux changing with respect to current?

470. A circuit consisting of a resistance of 12 ohms in series with 0.48-henry inductance is connected suddenly across a constant 120-volt source. Determine: (*a*) the equation giving the current in this circuit as a function of time, and plot: (*b*) the time constant of the circuit; (*c*) the current at the time in (*b*); (*d*) the current when the time is 0.08 sec.

471. In Prob. 470, determine the energy stored in the magnetic field when the current is (*a*) 4 amp.; (*b*) 8 amp.; (*c*) when the current reaches its Ohm's-law value. (*d*) Determine the rate of increase of current at the instant of closing the circuit.

472. The inductance of the field of an alternator is 5 henrys and the total field resistance is 25 ohms. Determine: (*a*) the equation of the current (and plot) when the field is connected across a constant 120-volt source; (*b*) the energy stored in the magnetic field at the value of time corresponding to the time constant of the circuit; (*c*) the power loss in the resistance in (*b*); (*d*) the rate of increase of current at the instant of closing the circuit.

473. The armature of a relay on a 240-volt circuit is not actuated until the current reaches 0.2528 amp. It is desired that the relay close 0.0035 sec. after the relay circuit is closed, this time corresponding to the time constant of the circuit. (*a*) What must be the resistance and the inductance of the relay? (*b*) At what rate does the current begin to increase at the instant the switch is closed?

474. The circuit of Prob. 470 is suddenly short-circuited after the current has attained its Ohm's-law value. Determine: (*a*) the equation of the current as a function of time; (*b*) the rate at which the current begins to decrease; (*c*) the current at the instant corresponding to the time constant of the circuit; (*d*) the energy stored in the field in (*c*).

475. With the field current equal to the Ohm's-law value the field of the alternator (Prob. 472) is to be disconnected. It is first temporarily connected in series with a field-discharge resistance of 25 ohms (see Fig. 212, p. 304). Determine: (*a*) the initial value of the current; (*b*) the initial value of voltage across the field winding and rheostat; (*c*) the equation of the current; (*d*) the rate at which the current is diminishing at 0.1 sec.; (*e*) the energy stored in the magnetic field at the time in (*d*).

476. Repeat Prob. 475 with the field-discharge resistance equal to 15 ohms.

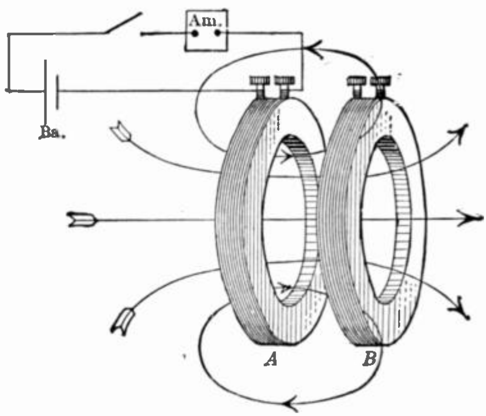


FIG. 478*A*.

477. A series circuit, in which the resistance is 20 ohms and the inductance is 1 henry, is connected momentarily across a 120-volt source. At the end of 0.05 sec., the circuit becomes short-circuited. Determine: (*a*) the value of current at the instant of short circuit; (*b*) the energy stored in the magnetic field at this instant. (*c*) Plot the current as a function of time until the current has almost reached its zero value. At what rate does the current begin (*d*) to increase, (*e*) to decrease?

X **478.** Two coils *A* and *B* (Fig. 478*A*) are insulated electrically but are so placed that 70 per cent of the flux produced by one of the coils links the other. In coil *A* there are 200 turns and in coil *B* there are 150 turns. When 1.5 amp. flows in coil *A*, 120,000 magnetic lines link the coil. (*a*) How many lines link *B*? (*b*) What is the coefficient of coupling of the two circuits? (*c*) If the current in *A* is interrupted in 0.05 sec. what induced e.m.f. results in *B*? (*d*) in *A*?

479. The same flux that was produced in *A* by 1.5 amp. is produced in *B* by 2.0 amp. (*a*) Determine the e.m.f. induced in *A* on interrupting the 2.0 amp. of *B* in 0.1 sec. (*b*) What e.m.f. is induced in *B*?

480. (*a*) Determine the mutual inductance in henrys of coils *A* and *B*, in Probs. 478 and 479, based on the definition: *e.m.f. produced in one circuit per unit rate of variation of the current in the other* (see p. 308). Determine: (*b*) the self-inductance of *A*; (*c*) of *B*. (*d*) Verify (*a*) by use of Eq. (152), p. 308, and Eq. (154), p. 309.

481. Repeat Prob. 480 with the coils *A* and *B* moved farther apart, so that the coefficient of coupling is now 0.60.

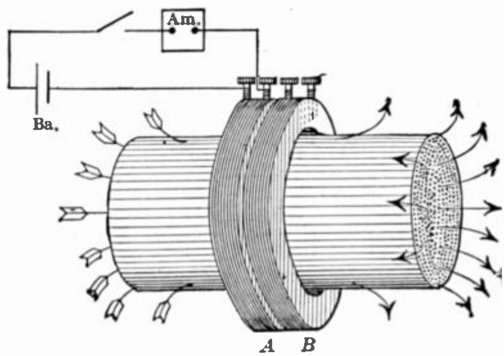


Fig. 482*A*.

482. Coils *A* and *B* of Prob. 478 are now linked magnetically by an iron core, as in Fig. 482*A*, so that practically all the flux of one links with the other. One-tenth ampere in *A* now produces 120,000 lines in the joint magnetic circuit. (*a*) How many amperes in *B* will produce this same flux? (*b*) What is the self-inductance of *A*? (*c*) of *B*?

483. (*a*) If the 0.1 amp. in *A* of Prob. 482 is interrupted in 0.05 sec., what e.m.f. is induced in *A*? (*b*) in *B*? (*c*) What is the mutual inductance of the circuits? (*d*) the self-inductance of *A*? (*e*) of *B*?

484. In Prob. 483, (*a*) at what rate must the current in *B* be interrupted to induce 10 volts in *A*? (*b*) If the current in *B* is 0.05 amp., in what time should the circuit be opened? (c) If the current in coil A and that in coil B is 2 amp., and the two coils act in conjunction magnetically, what is the total stored energy of the magnetic circuit?

485. In Fig. 482A the coils A and B are connected in series aiding. Determine: (a) the self-inductance of the entire circuit; (b) the stored energy in the magnetic field using both Eq. (149), p. 303, and Eq. (159), p. 311.

486. Repeat Prob. 485 with the two coils connected in opposition.

487. Two coils A and B are both on the same magnetic circuit. There are 400 turns in coil A and 600 turns in coil B. A current of 1 amp. in coil A produces 80,000 maxwells in coil A. The coefficient of coupling of the two coils is 0.75. Determine: (a) the flux produced by 1 amp. in coil B; (b) the self-inductance of A and of B; (c) the mutual inductance of A and B; (d) the self-inductance with the two coils connected in series aiding. The reluctance of the magnetic circuit remains constant.

488. In Prob. 487, (a) compute the self-inductance of the two coils in series opposing; (b) the stored energy in (a) with the current equal to 2 amp.; (c) the stored energy with the coils in series aiding; current equal to 2 amp.

489. Two coils A and B are on the same magnetic circuit. The inductance of coil A alone is measured and is found to be 0.4 henry; the inductance of coil B is likewise found to be 0.225 henry. The inductance of the two coils in series aiding is then measured and is found to be 0.985 henry. Determine: (a) the mutual inductance of the coils; (b) the coefficient of coupling.

490. Two coils C and D are on the same magnetic circuit. The inductance of the two in series aiding is first measured and found to be 1.25 henrys; the inductance of the two in series opposing is likewise found to be 0.75 henry. Determine: (a) the mutual inductance; (b) the self-inductance of coil C if that of D is 0.4 henry; (c) the coefficient of coupling.

QUESTIONS ON CHAPTER X

Electrostatics: Capacitance

1. According to modern electron theory, under what conditions is an atom electrically neutral? When does the atom become positively charged? When does matter become negatively charged?

2. Compare the dynamic electric circuit and associated electron phenomena with the static condition of electricity. Also compare dynamic electricity with static electricity, stating why the two sometimes appear to be different in their nature.

3. If two insulated conducting bodies near each other are connected to the terminals of an influence machine, upon what portions of the bodies will the density of charge be greatest? Discuss any changes which would be observed in these charges if the wires to the machine were disconnected. How can it be shown that charges are "bound"?

4. What is the character of the force existing between the conducting bodies in Question 3? the charges? State the laws relating to the forces between charges.

5. If a positive charge is brought into the neighborhood of an insulated and uncharged conducting body or a sphere, what phenomenon occurs? What is the relation of the induced charge to the inducing charge? State the laws relating to induced charges. Distinguish between free and bound charges. How may it be shown experimentally that free and bound charges behave differently?

6. Define a unit charge. State Coulomb's law.

7. What effect does an electric charge produce in the surrounding medium? Compare this condition with that surrounding a magnet. How many lines of force must leave each unit charge?

8. State three laws which govern the configuration of a dielectric field of force. Compare the dielectric field with current flow; with magnetic lines of force; with magnetic lines of induction. Show that dielectric lines must be normal to a conducting surface where they enter or leave such a surface.

9. Why must a positive charge on an isolated sphere reside on the surface of the sphere and be uniformly distributed over it? Prove by two methods that a charge distributed uniformly over the surface of a sphere behaves, so far as its external effects are concerned, as if the charge were concentrated at the center of the sphere.

10. Define a condenser. What is the effect of applying a voltage to a condenser? Define capacitance. What is the order of magnitude of the time required to charge a condenser directly from a direct-current source? Why does the current cease to flow?

11. How can it be shown that electricity is actually stored in a condenser? How does the quantity which can be stored in a condenser vary with the voltage? What simple relation does this give among charge, capacitance, and voltage? State the equation, written in three different ways, which gives the relation among charge, capacitance, and voltage.

12. What is the practical unit of capacitance? Why is the microfarad commonly used as a unit? Under what conditions is the micromicrofarad also used as a unit?

13. Distinguish between the dielectric properties and the insulating properties of a medium. In what one important respect do dielectric lines differ from current and magnetic lines? Give examples of excellent insulators and of excellent dielectrics. In what terms is dielectric strength measured? Discuss the probable mechanism of dielectric rupture according to modern electron theory.

14. What is the usual effect of inserting a dielectric medium other than air between condenser plates? What is *specific inductive capacity* and to what magnetic property is it analogous? What is the approximate dielectric constant of glass? of mica? of rubber?

15. Determine the equivalent capacitance of condensers connected in parallel. To what electric-circuit condition is this analogous?

16. Determine the equivalent capacitance of condensers connected in series. What is the relation among the electric charges on a number of condensers connected in series? To what equation in the electric circuit is the equation relating to the equivalent capacitance of condensers in series similar?

17. Derive the relation among the voltages across a number of condensers in series, if the individual capacitances are known. Are these voltage relations dependent on the insulating properties of the dielectrics? In the case of leaky condensers, on what does the ultimate voltage distribution depend?

18. Show that the field intensity between charged parallel plates is $4\pi\sigma$, provided the distance between the plates is small compared with their areas. σ is the density of charge on the plates in e.s.u. per square centimeter. How many dielectric lines leave from each square centimeter of each plate?

19. Show that the force of attraction between charged parallel plates, close together, is $2\pi\sigma^2 A$ dynes.

20. How may it be shown that energy is stored in a condenser? Outline the procedure by which it may be proved that the stored energy in a parallel-plate condenser is $q^2/2C$ ergs where q is the charge in statcoulombs or e.s.u., and C is the capacitance in statfarads. Prove that the same equation is obtained even when an additional charge is given to the condenser.

21. How does stored energy vary with the voltage of a condenser? Express the stored energy in terms of voltage and charge.

22. Outline the procedure which is used in calculating the capacitance of a condenser. Show that an electrostatic field cannot exist within a region totally enclosed by a conducting body provided there is no source of charge within the region.

23. Derive the equation which gives the capacitance of a parallel-plate condenser. Upon what factors does the capacitance depend? What is the effect on the capacitance of changing the area of the plates? of decreasing the distance between them? of substituting hard rubber or glass for air? Show that the equation for capacitance is in error when two simple plates alone are used.

24. Prove that the force exerted by a straight, infinitely long charged filament on a unit charge, h cm. from the filament, is $2q/h$ dynes where q is the e.s.u. per centimeter length.

25. From the relation in Question 24, derive the equation which gives the capacitance of concentric cylinders. How does the capacitance vary with the length of cylinder? the radius of the outer cylinder? the radius of the inner cylinder? State a common use of this type of condenser.

26. Derive the equation for the capacitance of concentric spherical condensers. Show that the capacitance of an isolated sphere in e.s.u. is equal to its radius in centimeters. What is the approximate capacitance in microfarads of the earth as an isolated sphere?

27. A capacitance C is suddenly connected across a line of constant voltage E. What is the theoretical instantaneous value of current? Explain. If a resistance R is connected in series with the capacitance, what is the current I_0 at the instant of switching? Explain.

28. Derive the equation which gives the current i to a capacitance C in series with a resistance R as a function of time t. Sketch and discuss this current function.

29. In Question 28 derive the equation for the quantity in the condenser. Sketch and discuss the function.

30. In Questions 28 and 29 determine the time constant. Discuss its significance with respect to current and to quantity during the time of charge.

31. A charged capacitance C at e.m.f. E_0 is discharged through a resistance R. Derive the equation of current as a function of time. Sketch and discuss the function.

32. Describe the constitution of a simple type of gaseous atom according to modern electron theory. Describe the mechanism by which ions are produced by collision. What is meant by ionization? State the properties of ionized air. What is meant by corona? Sketch an arrangement of electrodes which will permit a high degree of ionization at one electrode in the surrounding air without complete rupture following.

33. What two methods are commonly employed in the measurement of capacitance? On what fact does the ballistic galvanometer method depend? What relation exists between the quantity passing through the galvanometer and its maximum ballistic throw?

Should the measurement be made on *charge* or on *discharge?* Explain. How is the galvanometer calibrated?

34. Describe the bridge method of capacitance measurement. Compare it with the Wheatstone-bridge method of resistance measurement. How does the bridge formula for capacitance differ from the formula employed when resistance is measured? What is the source of power and what simple detector is used in the capacitance bridge?

35. How may a disconnection in a cable be located? On what principle does this method of measurement depend? Is this method applicable if the fault is grounded?

PROBLEMS ON CHAPTER X

7th week 5-17 rc

Electrostatics: Capacitance

491. Each of two small spheres, spaced 16 cm. between centers in air has a charge of 20 e.s.u. (statcoulombs), one charge being positive and the other negative. Determine: (*a*) the force in dynes acting between the two spheres; (*b*) the force acting between the spheres when they are immersed in oil, having a dielectric constant of 2.5; (*c*) the direction of the force in (*a*) and (*b*). The distance between spheres and the charges remain unaltered.

The charges may be assumed to act as if concentrated at the centers of the spheres in this problem as well as in those which follow.

492. The charges on two small spheres, spaced 15 cm. between centers in air, are 30 and 40 positive e.s.u. Determine: (*a*) the force in dynes between the spheres; (*b*) the force in dynes on these spheres when immersed in oil, the dielectric constant of which is 3.0; (*c*) the direction of the force in (*a*) and (*b*); (*d*) the total dielectric flux leaving from each sphere when in air and when in oil.

493. An isolated sphere in air, 10 cm. in diameter, is charged with 30 statcoulombs of positive electricity. Determine: (*a*) the total dielectric flux leaving this sphere; (*b*) the density of the dielectric flux in lines per square centimeter at the surface of the sphere; (*c*) the force on a unit charge at the surface of the sphere, determined by two methods.

494. Two spheres, each 0.5 cm. in diameter, are spaced 30 cm. between centers in air. One is given a positive charge of 36 statcoulombs and the other a negative charge of 30 statcoulombs. Determine: (*a*) the force in dynes between spheres; (*b*) the dielectric flux leaving or entering each; (*c*) the force in dynes on a unit charge at the surface of each neglecting the effect of the other; (*d*) the force at a point midway between the centers of the spheres; (*e*) the force at a point on the line joining the centers of the spheres, and 4 cm. from the center of the second sphere.

495. Two small spheres in air are spaced 12 cm. between centers. There is a charge of 60 e.s.u. on one sphere, and the force of attraction between the spheres is 12.5 dynes. Determine: (*a*) the magnitude and sign of the charge on the second sphere; (*b*) the force in dynes on a unit charge placed on a line joining their centers and midway between them.

496. Two small spheres *A* and *B* in air, each of radius of 0.5 cm., are charged with positive 20 and minus 24 e.s.u. The force between them is 30 dynes. Determine: (*a*) the distance in centimeters between the centers of the spheres; (*b*) the number of lines leaving sphere *A*; (*c*) entering sphere *B*; (*d*) the field intensity at the surface of sphere *A*, on the line joining the centers of the two spheres; (*e*) of sphere *B*; (*f*) the force on a unit-positive charge at the mid-point of the line connecting the centers of the spheres.

497. Repeat Prob. 496 with the spheres immersed in oil, the dielectric constant of which is 2.5. The charges remain unchanged.

498. In a spherical condenser similar to that shown in Fig. 237 (p. 340) the radius of the inner sphere is 4 cm. and the inside radius of the outer spherical shell is 10 cm. There are 8,045 dielectric lines leaving the inner sphere *A* to the outer sphere *B*. The dielectric is air. Determine: (*a*) the charge in e.s.u. on the inner sphere; (*b*) the number of lines per square centimeter at the surface of the inner sphere *A*; (*c*) the force exerted on a unit-positive charge at the surface of the inner sphere; (*d*) the number of lines per square centimeter at the inner surface of the outer spherical shell *B*. (*e*) State the law of variation of the density of the dielectric lines with respect to the distance from the center of the inner sphere.

499. A condenser the capacitance of which is 40 μf. is connected across a 200-volt source. (*a*) What charge in coulombs does it take? (*b*) If 320 volts are impressed across the condenser, what charge does it take? (*c*) The condenser in each case is charged for 0.16 sec. with a current whose value is maintained constant. What is the value of the current in each case?

500. The capacitance of a condenser is 24 μf. (*a*) What is its counter e.m.f. when its charge is 3,200 microcoulombs? (*b*) To what value must the charge be increased to make the counter e.m.f. equal to 220 volts?

501. A condenser of 60-μf. capacitance is charged at a uniform rate by a current of 0.001 amp. For how long a time must this current flow in order to raise the e.m.f. of the condenser to 250 volts?

502. A condenser is charged by a steady current of 0.0015 amp. for 20 sec. at which time the e.m.f. of the condenser is 600 volts. Determine the capacitance of the condenser.

503. The capacitance of a Leyden-jar type of condenser is 0.003 μf. How many coulombs are necessary to raise its potential to 12,000 volts?

504. The condensers used with surge or "lightning" generators are 0.5 μf. each, and each condenser is charged to 20,000 volts before discharging to give the lightning stroke. In one surge generator the charging current to each condenser is essentially constant at 0.002 amp. How many seconds are required to charge the condenser?

505. An air condenser consists of three equidistant parallel plates, the two outer plates being connected together to form one electrode, and the center plate forming the other electrode (see Fig. 234(b), p. 336). This condenser has a capacitance of 0.00018 μf. When it is immersed in Transil oil the capacitance is found to be 0.000468 μf. What is the dielectric constant of the oil?

506. A condenser consisting of three parallel plates, with air as dielectric, has a capacitance of 0.00016 μf. A slab of glass is placed between each outer plate and the center plate occupying the entire space between the plates. The capacitance is now found to be 0.00072 μf. What is the specific inductive capacity of the glass?

507. The condenser of Prob. 506, with air as a dielectric, is charged to a potential of 400 volts between plates and the supply then disconnected. The glass of Prob. 506 is then inserted between the plates, completely filling the space. This insertion of the glass in no way changes the value of the electric charge on the plates. (a) What is the condenser voltage after the insertion of the glass? (b) What does the charge become if the 400-volt supply is again connected?

508. A plate condenser, with air as dielectric, has a capacitance of 0.0024 μf. and 360 volts is impressed across its terminals. The condenser is then immersed in a bath of transformer oil having a dielectric constant of 2.4, the voltage supply remaining connected. (a) What is the charge on this condenser before and after immersion in the oil? The voltage supply is disconnected while the condenser is immersed in the oil, and the condenser is then removed from the oil, so that the dielectric is now air. (b) What does the voltage across the condenser become? Neglect any leakage.

509. The capacitance of an air plate condenser is 0.00072 μf. A potential difference of 250 volts is impressed across it. This voltage supply is then disconnected and the condenser is immersed in oil without loss of charge. An electrostatic voltmeter, which takes no current shows the voltage to be 100 volts. (a) Determine the dielectric constant of the oil. The 250-volt source is now reconnected momentarily to the condenser while it is in the oil. (b) Determine the charge in the condenser. The condenser is now removed to the air without loss of charge. (c) Determine its voltage.

510. The charges taken by four condensers are 1,000, 1,250, 1,500, and 2,250 microcoulombs each when they are connected in parallel across 250 volts. What is the equivalent capacitance of the combination?

511. A condenser of 40-μf. capacitance is connected across 500 volts. The voltage supply is removed and the condenser is connected across three uncharged condensers *A*, *B*, *C*, in parallel. The charges are 3,000. 5,000, and 4,000 microcoulombs. Determine: (*a*) the capacitance of *A*, *B*, *C*; (*b*) the voltage across the combination.

512. Three condensers of capacitances 20, 24, and 32 μf. are charged to 200, 250, and 400 volts. They are then connected in parallel with terminals of like polarity together. (*a*) Determine the voltage across the combination. The three condensers are now separated and recharged to the initial potentials of 200, 250, and 400 volts. They are then connected again in parallel with terminals of like polarity together but with a fourth uncharged condenser connected in parallel with the system. The voltage of the system is now 228 volts. (*b*) Determine the capacitance of the fourth condenser.

513. Three condensers of capacitance 4, 8, and 10 μf. are connected in series across a 60-volt battery. (*a*) What single capacitance will replace the three? (*b*) Assuming negligible leakage, what is the charge on each? (*c*) What is the voltage across each?

514. The three condensers of Prob. 513, after being charged in series, are disconnected and then connected in parallel, with plates of like polarity together. (*a*) What is the total charge of the combination? (*b*) What is the charge on each condenser? (*c*) What is the voltage across the parallel combination of condensers?

515. Repeat Prob. 514 with the polarity of the 4-μf. condenser reversed.

516. Determine the equivalent capacitance of four condensers in series having capacitances of 16, 20, 25, and 40 μf. If 250 volts are impressed across the four in series, determine the charge on each and the voltage across each, assuming negligible leakage.

517. The four condensers of Prob. 516, after being charged at 250 volts in series, are disconnected and connected in parallel. The last three are connected with terminals of like polarity together and the 16-μf. condenser is connected in parallel with them, but with its positive terminal to their negative terminal. Determine: (*a*) the net charge on the entire combination; (*b*) the charge on each condenser; (*c*) the voltage of the combination.

518. When four condensers are connected in series across a 200-volt source, their voltages are 30, 70, 40, and 60 volts, and the charge on each is 8,400 microcoulombs. (*a*) What is the capacitance of each? (*b*) What is the capacitance of the series combination? The four condensers are disconnected in the charged condition and are then connected in parallel, the last three with terminals of like polarity together and the first condenser (30 v.) with reversed polarity. Determine: (*c*) the voltage of the combination; (*d*) the charge on each condenser.

519. A parallel-plate condenser consists of two flat circular plates 60 cm. in diameter and spaced 0.4 cm. apart in air. The charge on each plate is 800 e.s.u. Determine: (*a*) the total dielectric lines leaving each plate; (*b*) the density in lines per square centimeter leaving each plate; (*c*) the field intensity between the plates; (*d*) the work in ergs done in carrying a unit charge from one plate to the other; (*e*) the mutual force in dynes acting between the plates. Neglect the effect of leakage flux and fringing.

520. In a condenser identical with that in Prob. 519, except that the plates are spaced 0.60 cm., the mutual force of attraction between the plates is 2.0 dynes. Determine: (*a*) the density of charge on each plate; (*b*) the total charge on each plate; (*c*) the field intensity between the plates; (*d*) the work done if the two plates are allowed to move 0.20 cm. nearer each other. The plates are perfectly insulated.

521. Two condensers in parallel have charges of 1,600 and 2,400 microcoulombs when connected across a 200-volt source. Determine the energy on each and the total energy.

522. Four condensers having capacitances of 20, 25, 36, and 50 $\mu f.$ are connected in series across a 600-volt source. (*a*) What is the total energy of the combination? (*b*) How much energy is stored in each of the condensers? (*c*) The condensers in the charged condition are disconnected from the 600-volt source and are then connected in parallel, terminals of like polarity being connected together. Determine the energy of the system. Account for the energy lost.

523. Three 0.5-$\mu f.$ condensers in a surge generator are connected in series across 48,000 volts. (*a*) Determine the energy of the system. (*b*) Repeat (*a*) with the voltage raised to 60,000 volts.

524. A 12-$\mu f.$ condenser is connected in series with a 4-$\mu f.$ condenser and an 8-$\mu f.$ condenser, which are in parallel. The combination is then connected across a 250-volt source. Determine: (*a*) the total energy; (*b*) the energy in the 12-$\mu f.$ condenser; (*c*) the energy in the 4-$\mu f.$ condenser; (*d*) the energy in the 8-$\mu f.$ condenser.

525. Repeat Prob. 524 with the voltage increased to 500 volts. How does the stored energy in a condenser vary with the voltage?

526. A condenser is charged with 8,000 microcoulombs and the energy stored is 0.8 joule. Determine: (*a*) the voltage; (*b*) the capacitance in microfarads.

527. The capacitance of a parallel-plate condenser is 3,000 statfarads and the charge is 4500 e.s.u. Determine the stored energy: (*a*) in ergs; (*b*) in joules.

528. Three condensers *A*, *B*, *C* (Fig. 528*A*), without initial charge, are connected in series-parallel across voltage *E*. The capacitance of condenser *A* is 40 $\mu f.$ and that of condenser *B* is 20 $\mu f.$ The charge on condenser *A* is 3,200 microcoulombs and that on condenser *B* is 2,000 microcoulombs. Determine: (*a*) the charge on condenser *C*; (*b*) the line voltage *E*; (*c*) the energy stored in condensers *A*, *B*, *C*; (*d*) the capacitance of condenser *C*.

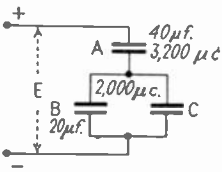


Fig. 528*A*.

529. An air condenser consists of three plates. The two outer ones are connected together as one terminal, and the other terminal is connected to the intermediate plate between the two outers. The dimensions of each plate are 40 by 40 cm. and the plates are spaced

0.12 cm. apart. Determine: (*a*) the capacitance of the condenser; (*b*) the stored energy if the voltage is 300 volts. (*c*) If the space between the plates of the condenser is filled with paraffin, having a dielectric constant 2.2, what does the capacitance become? (*d*) the stored energy at 300 volts?

530. A high-voltage condenser is to be made of alternate layers of glass and tin-foil, the glass having a dielectric constant 7.2. The glass is $\frac{3}{64}$ in. thick and the tin-foil is 2 mils thick and its dimensions are 4 by 6 in. (*a*) How many plates of glass and sheets of tin-foil are necessary to make a condenser having a capacitance 0.0653 $\mu f.$? (*b*) If the glass plates are 6 by 8 in., what is the size of the completed condenser?

531. It is desired to construct a 2-$\mu f.$ condenser each plate being a long sheet of tin-foil, 4 in. wide and 1 mil thick, by rolling it between two long sheets of thin paraffined paper, 1.5 mils thick and having a dielectric constant 2.0. What is the length of the foil and the paraffined paper before they are rolled up?

532. (*a*) Determine the capacitance in micromicrofarads of the plate condenser of Prob. 519. (*b*) Determine the stored energy when it is connected across a 1,000-volt source. With the charged plates disconnected from the power source, they are moved to 0.6 cm. apart. Determine: (*c*) the voltage; (*d*) the stored energy. From what source is the increased energy obtained?

533. A long, straight wire of small diameter is charged with 12 e.s.u. per centimeter length. Determine: (*a*) the force in dynes acting on a positive-unit charge located 8 cm. distant perpendicularly from the center of the wire; (*b*) the work done in moving the charge 10 cm. farther away from the wire.

534. An air condenser consists of two concentric cylinders, 3 m. long. The outer diameter of the inner cylinder is 4 cm. and the inner diameter of the outer cylinder is 8 cm. (*a*) Determine the capacitance of this condenser in micromicrofarads. (*b*) What is the the charge when 2,000 volts are impressed between cylinders. Neglect end effect.

535. A single-conductor cable consists of a solid, No. 4 A.W.G., copper conductor, the diameter of which is 204 mils, and a wall of rubber with an outside diameter of 0.454 in. and a dielectric constant 4.5. The rubber is surrounded by a $\frac{1}{16}$-in. lead sheath. Determine the capacitance of 1-mile length of this cable.

536. A varnished-cambric single-conductor cable consists of No. 0 A.W.G. stranded copper conductor, the outside diameter of which is 0.373 in. There is a $\frac{5}{64}$-in. wall of insulation and an outside lead sheath $\frac{3}{32}$ in. thick. The dielectric constant of the cambric is 4.0. Determine: (*a*) the capacitance of 1-mile length of the cable; (*b*) the charge and the stored energy when 4,000 volts is impressed between conductor and sheath.

537. In a 500,000-cir.-mil, single-conductor, impregnated-paper cable, the diameter over the conductor is 0.84 in. The conductor is wrapped with a $\frac{9}{32}$-in. wall of impregnated paper having a dielectric constant 2.8. The paper is surrounded by a $\frac{1}{8}$-in. lead sheath. Determine: (*a*) the capacitance in microfarads of a 5-mile length of this cable. The cable is tested with high-voltage direct current obtained from a Kenotron set. When 20,000 volts is impressed on the cable determine: (*b*) the charge in coulombs; (*c*) the stored energy in joules.

538. The inside diameter of the outer shell of a concentric spherical condenser is 16 cm. and the outside diameter of the inside sphere is 6 cm. The dielectric is air. Determine: (*a*) the capacitance of the condenser in statfarads; (*b*) the charge when the potential difference between the spheres is 2 statvolts, the outer shell being positive (1 statvolt = 300 volts); (*c*) the force on a unit-positive charge at the surface of the inside sphere; (*d*) the work done in carrying a unit-positive charge radially from the outer surface of the inside sphere to the inner surface of the outside shell.

539. In the spherical condenser of Prob. 538, but with the potential difference between spheres equal to 2.5 statvolts, determine the work in ergs in carrying a unit charge (e.s.u.) from the surface of the inner sphere to the inner surface of the outer shell.

540. A circuit consisting of a 100-μf. capacitance in series with a resistance of 1,200 ohms is connected suddenly across a 120-volt source. Determine: (*a*) the initial value of the current; (*b*) the equation of current as a function of time; (*c*) the time constant of the circuit; (*d*) the value of current at the instant in (*c*); (*e*) the rate at which the current begins to decrease. (*f*) Plot the function in (*b*).

541. A 60-μf. condenser is charged to 150 volts and is then discharged through a resistance of 10,000 ohms. Determine: (*a*) the initial value of current; (*b*) the equation of current as a function of time; (*c*) the rate at which the current begins to decrease.

542. (*a*) In Prob. 540, derive the equation which gives the charge q in the condenser. (*b*) What is the charge is coulombs when the time is equal to the time constant? (*c*) Plot the function in (*a*).

543. A capacitance of 120 μf., in series with a 150-scale voltmeter of 20,000 ohms resistance, is connected suddenly across a 120-volt source. Determine: (*a*) the equation of the current function; (*b*) the reading of the voltmeter when the time $t = 1$ sec.; (*c*) the voltage across the capacitance in (*b*); (*d*) the value of the current when the voltmeter reads 20 volts; (*e*) the time corresponding to (*d*). (*f*) Plot the function in (*a*).

544. (*a*) Derive the equation for the quantity q in Prob. 543. Determine: (*b*) the rate at which the quantity begins to increase; (*c*) the quantity when the time $t = 0.4$ sec.; (*d*) the time when the charge in the capacitance is 99 per cent the ultimate value. (*e*) Plot the function in (*a*).

545. In Prob. 543 the capacitance is charged to 150 volts and the circuit is then disconnected from the power source and closed on itself. (*a*) What is the charge in the capacitance when the voltmeter reads 75 volts? (*b*) At what rate is the current decreasing in (*a*)? (*c*) What is the charge in the capacitance when the current has reached 10 per cent its initial value? (*d*) What is the energy stored in the capacitance when the time t is equal to the time constant of the circuit?

546. In a ballistic measurement of capacitance (see Fig. 243, p. 348) the ballistic deflection of the galvanometer is 24.3 cm. when the unknown condenser is connected, the Ayrton shunt used in connection with the galvanometer being set at 0.1. The galvanometer deflection becomes 10.5 cm. with the Ayrton shunt set at 1.0 when the galvanometer is calibrated with a 1-μf. standard condenser. The same battery is used in each case. (*a*) What is the galvanometer constant? (*b*) What is the value of the unknown capacitance?

547. In a bridge measurement of condenser capacitance, the bridge is connected as in Fig. 244(*a*) (p. 349). When a balance is obtained, $R_1 = 100$, $R_2 = 796$, $C_2 = 0.8\ \mu f$. What is the value of C_x, the unknown capacitance?

548. In a test for a cable fault, the apparatus is connected as in Fig. 245 (p. 350). In the capacitance measurement of the part x, the ballistic deflection of the galvanometer is 8.4 cm. In the measurement of the capacitance of the perfect cable plus the looped end of the faulty cable, the deflection is 22.6 cm. If the length of each conductor is 2,400 ft., how far from the point of test is the cable broken?

QUESTIONS ON CHAPTER XI

The Generator

1. In what way is the flux linking any coil of a generator armature made to vary? Describe the manner in which this variation of flux causes an e.m.f. to be induced. How does this induced e.m.f. vary with the speed? the flux? the number of turns in the coil?

2. If instead of regarding this e.m.f. as being due to the change of flux linking a coil, it is considered as being due to the individual conductors cutting flux, how is the ultimate result affected? If the e.m.f. is considered as being due to the cutting of lines by individual conductors how does the e.m.f. vary with length of conductor? flux density? velocity of conductor?

3. What definite relation exists between direction of induced e.m.f., the direction in which conductor moves, and direction of flux? What simple rule enables one to determine the relation?

4. What are the relative values of the e.m.f. induced in a rotating coil: (*a*) when the coil is in the plane perpendicular to the flux? (*b*) When its plane lies parallel to the flux? Discuss any reversal of direction in the induced e.m.f. during the rotation of the coil.

5. Show how the alternating e.m.f. induced in the coil may be converted into direct-current e.m.f. for the external circuit. What is the effect on the e.m.f. wave form of adding coils to the rotating member? To what are "ripples" in an e.m.f. wave due?

6. What operations does the commercial armature perform? Why is the core made of iron? Why are the faces of the poles practically concentric with the armature? Name two advantages of slotted construction. What factors must be considered when the winding is designed?

7. In what way is the open-coil type of armature different from the closed-coil type? Which type is the gramme-ring armature (Fig. 253, p. 360)? Show that in the open-coil type of winding the contribution of the individual inductor to the total e.m.f. is different from the closed-coil winding.

8. What characteristics of the gramme-ring winding made it so widely used in early generators? Name two serious objections to the ring winding. How are these objections overcome in the drum winding? What two methods are used to fasten conductors on armatures? Which is the better method and why?

9. What factor determines the spread of the coil on the armature surface?

10. What is meant by *coil pitch* and what is its relation to pole pitch? What relative positions in the slots do the two sides of any coil occupy? Why? What is meant by a *winding element?* May it consist of more than one conductor? Explain.

11. What are *front pitch, back pitch average pitch?* What is the relation between the number of winding elements and the number of coils and the number of commutator segments?

12. In a simplex lap winding, how many commutator segments does the winding advance each time that a coil is added? *Define commutator pitch.* What three fundamental conditions must be fulfilled by a winding? What is a winding table and what is its practical value?

13. Why is it sometimes desirable to place more than two winding elements in a slot? In what type of generator is this necessary? Are the winding relations and the conductor numbering in any way affected? What one condition should be imposed on multiple-coil type of winding and why?

14. What is meant by "paths through the armature"? How is the current output of a machine affected by increasing the number of paths? How is the voltage affected? the power output? How many paths are there in all simplex lap windings?

15. What is meant by a duplex winding? Show that such a winding may be composed of two simplex windings each lying in alternate slots. How many closures may such a winding have, and what is its degree of reentrancy in each case?

16. If a duplex winding does not close after one passage around the armature, is the number of segments even or odd? When does such a winding close? How many times does it close, and what is its degree of reentrancy?

17. In a lap winding the multiplicity of which is m, how many winding elements does the winding advance each time that the winding is lapped back? How many armature paths are there in a six-pole machine having a simplex lap winding? a duplex lap winding? a triplex lap winding?

18. To what causes are unequal e.m.fs. in different paths of an armature winding due? Do equalizing connections do away with these inequalities? What is the purpose of equalizing connections? What care should be taken regarding the number of slots per pole when equalizing connections are used? Why?

19. What is the fundamental difference between a lap winding and a wave winding? Does the direction of induced e.m.f. in opposite sides of a coil differ in the two types? Explain.

20. After a wave winding has passed under every pole in going around an armature, what relation should it bear to its starting position if the winding is simplex? What would a closure after one passage around the armature mean?

21. Show that the definitions of *front pitch* and *back pitch* in a wave winding do not differ from the definitions in a lap winding. Can the front pitch be even? odd? Can the back pitch be even? odd? Can the two be equal? Can the average pitch be even? odd? When is a winding progressive? retrogressive? Explain.

22. Is it always possible to fit a wave winding to an armature having a fixed number of slots if all the slots are to be utilized? Explain. What make-shift may be used to accomplish the desired result?

23. If the number of pairs of poles is even, is the number of commutator segments even or odd? Answer if the number of pairs of poles is odd?

24. Explain the meaning of *forced winding.* Under what conditions is it necessary to use such a winding? Compare the formed coil used for the wave winding with that used for the lap winding.

25. What is the minimum number of brush sets that can be used in a wave winding? What is the maximum number that it is possible to use? When would only two sets be used and why? Why is the maximum possible number usually desirable?

26. How many paths are there in a simplex wave winding? In what way is the number of such paths affected by the number of poles? How many paths in a duplex wave winding? a triplex wave winding?

27. When is it desirable to use a wave winding and why? a lap winding? Give specific reasons.

28. In addition to forming a part of the magnetic circuit, what other function does the yoke of a generator perform? Of what two materials is it made, and why? Describe a process whereby the yoke is made without casting.

29. Of what materials are the field cores made? the pole shoes? What are the two general shapes of the core sections? Where is each used?

30. Is the armature a solid casting? If not, how is it built up? By what two methods are the stampings produced? How are they held in position when placed upon the armature? What is the purpose of the ventilating ducts?

31. Sketch two general types of slot. Where is each used? What two methods are used to prevent the conductors from being affected by centrifugal force?

32. Of what is the commutator made? What insulation is used between segments? How are the segments clamped together? How are the coil connections made?

33. What is the purpose of the brushes? Of what material are brushes made usually? What pressure is used to hold the brush on the commutator? What is the purpose of the plating on the brush? What is the purpose of the pig-tail?

PROBLEMS ON CHAPTER XI

The Generator

549. A coil 20 cm. square, having 60 turns, lies perpendicular to a uniform magnetic field the density of which is 600 gausses. This coil is made to revolve a quarter-turn about its axis in 0.1 sec., so that it then lies parallel to the direction of the field. Determine: (*a*) the maximum flux passing through the coil; (*b*) the average induced e.m.f. during the period that the coil is making the quarter-turn. (*c*) If the coil revolves 2.5 r.p.s., what is the average induced e.m.f.? (*d*) What is the induced e.m.f. at the instant that the coil is perpendicular to the field? (*e*) when it is parallel to the field?

550. A coil 25 cm. square, having 40 turns, rotates at a speed of 600 r.p.m. in a uniform magnetic field, the density of which is 240 lines per square centimeter. Determine: (*a*) the maximum flux passing through the coil; (*b*) the time in seconds for the coil to make a quarter-turn; (*c*) the average e.m.f. induced in the coil. (*d*) If flux and speed are both doubled, what is the average induced e.m.f.?

551. (*a*) In Prob. 550, determine the e.m.f. per conductor when the coil is parallel to the direction of the field. (*b*) What is the induced e.m.f. per coil side at this instant? (*c*) What is the total e.m.f. induced in the coil at this instant? (*d*) Show that the ratio of the maximum induced e.m.f. to the average induced e.m.f. under these conditions is $\pi/2$.

552. In Prob. 550 the coil is stationary and its plane is perpendicular to the field. The field circuit is interrupted so that the flux becomes zero in 0.05 sec. (*a*) Determine the average induced e.m.f. during this period. The coil is turned so that its plane makes an angle of 45° with the direction of the field, and the coil remains stationary in this position. After the flux is brought to the density of 240 lines per square centimeter it is reduced to zero in 0.05 sec. (*b*) Determine the average induced e.m.f. during this interval.

553. Figure 553*A* shows a conductor 30 cm. long on the surface of an armature the diameter of which is 28 cm. The armature rotates at a speed of 20 r.p.s. Determine the induced e.m.f. in the conductor when it is in the position shown, directly under the pole, where the flux density is uniform at 6,400 gausses.

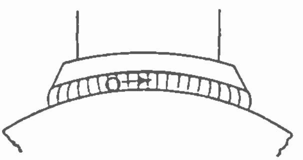


Fig. 553*A*.

554. A uniform magnetic field, the density of which is 1,200 gausses, is just sufficient in cross-section to pass perpendicularly through a coil 24 by 18 in. The coil has 120 turns. If the coil slides parallel to itself out of this field in 0.001 sec. and at a uniform speed, what e.m.f. is induced in the coil due to the change in flux linking the coil? What e.m.f. is generated by the cutting of the flux by the individual conductors? (Work with the coil sliding in two directions, one parallel to the 18-in. side and one parallel to the 24-in. side.)

555. At what speed in r.p.m. must the armature of Prob. 553 rotate in order that the induced e.m.f. in the conductor shall be 8.45 volts?

556. There are 47 slots in an armature of a four-pole generator. (*a*) How many winding elements are there? (*b*) Design a two-layer, simplex lap winding, in which the back pitch is 23 and the front pitch is 21. (*c*) Make a winding table.

557. Repeat Prob. 556 making the front pitch 23 and the back pitch 21. Which winding is progressive and which is retrogressive?

558. Repeat Prob. 556 making the back pitch 21 and the front pitch 19.

559. There are 65 slots in the armature of a six-pole generator and the winding has 65 coils. (*a*) How many winding elements are there? (*b*) Select appropriate values of back pitch and front pitch for a simplex lap progressive winding. (*c*) Sketch a portion of the winding. (*d*) Make a winding table.

560. In an armature for an eight-pole generator there are 112 slots and the winding is to be a two-layer, simplex lap winding, with two coil sides per slot. (*a*) How many winding elements are there? (*b*) Select suitable values of back pitch and front pitch to give a progressive winding. (*c*) Sketch a portion of the winding. (*d*) Make a winding table.

561. In a six-pole generator there are 74 armature slots. It is desired to wind the armature with 222 coils, necessitating the employment of triple coils. The winding is to consist of a two-layer simplex, progressive, lap winding. Determine: (*a*) the number of winding elements; (*b*) a suitable back pitch; (*c*) a suitable front pitch; (*d*) the number of commutator segments. (*e*) Make a sketch of a portion of the winding. (*f*) Make a portion of the winding table.

562. For the back pitch (Prob. 561) select a value which is nearest to but less than the average pitch and yet will permit the proper placing of the triple coils in the slots.

563. Repeat Prob. 561 for a quadruple coil winding, the number of slots remaining unchanged.

564. On the armature of an eight-pole generator there are 112 slots and six winding elements per slot. (*a*) Determine a correct value of back and front pitch for a simplex lap winding. Choose the value of back pitch which is nearest the value of the average pitch and yet fulfills the conditions which such a winding necessitates. (*b*) Sketch a few slots with their winding elements and connections. (*c*) How many commutator segments are necessary?

565. A six-pole, simplex, lap-wound armature delivers a total current of 420 amp. at 240 volts. (*a*) How many amperes per path through the armature are there? (*b*) How many volts per path? (*c*) What is the kilowatt rating of the machine?

566. An armature similar to that in Prob. 565 is wound with identical coils and with the same number of coils. The winding, however, is connected as a duplex lap winding. Determine: (*a*) the current per path;

(*b*) the voltage per path; (*c*) the voltage, current, and kilowatt rating of the machine.

567. Determine the current per parallel path in a six-pole, 500-kw., 120-volt, electroplating generator, which has a duplex lap winding. What is the current per brush set?

568. Sketch a portion of a duplex, doubly reentrant, progressive, lap winding for a four-pole, 68-slot armature, making the back pitch 33. There are two winding elements per slot. Make a complete winding table.

569. Repeat Prob. 568 with 67 slots, changing the degree of reentrancy if necessary.

570. Design a duplex, doubly reentrant, progressive, lap winding for a four-pole, 72-slot armature in which there are four winding elements per slot. Make a winding table.

571. Repeat Prob. 570 for an armature having 73 slots, changing the degree of reentrancy if necessary.

572. The armature of a four-pole generator has 39 slots. Sketch a simplex wave winding with two elements per slot, making $y_b = 21$ and $y_f = 19$. Check with Eq. (208), p. 380, and Eq. (209), p. 381. Is this winding progressive or retrogressive?

573. Repeat Prob. 572, making $y_b = 19$ and $y_f = 19$. Check with Eq. (208), p. 380, and Eq. (209), p. 381. Is this winding progressive or retrogressive?

574. In a six-pole generator are 85 slots on the armature with two elements per slot. Sketch a portion of a wave winding making $y_b = 29$, $y_f = 27$. Make a winding table. Is the winding progressive or retrogressive?

575. It is desired to wind a six-pole armature with a progressive wave winding, two elements per slot, and the front and back pitch are both to be 29. How many slots must there be on the armature?

576. Repeat Prob. 575 for a retrogressive winding.

577. In a four-pole generator there are 72 armature slots and it is desired that the armature be simplex wave-wound. (*a*) Attempt to design a winding which will fulfill the necessary conditions. (For example, try $y_b = y_f = 35$.) (*b*) By the use of a dummy coil again try (*a*). Make a winding table.

578. In a six-pole armature are 83 slots and there are to be two winding elements per slot. Attempt to place a simplex wave winding on this armature making $y_b = y_f = 27$. Omit a single coil (dummy coil) and investigate as to whether or not the winding is possible. Use Eq. (208), p. 380.

579. (*a*) In Prob. 577 design a duplex wave winding without the use of a dummy coil. (*b*) What is the degree of reentrancy of this winding? (*c*) What is the approximate ratio of the voltage rating of this armature to that in Prob. 577? (*d*) the current rating?

580. In a six-pole 250-kw., 250-volt generator the armature winding is a simplex lap winding. (*a*) Give the voltage, current and kilowatt rating if the armature is reconnected as a simplex wave winding. (*b*) Repeat (*a*) for a duplex wave winding.

581. The armature of an eight-pole, 500-kw., 250-volt generator is wound with a duplex wave winding. There are eight brush sets. (*a*) What is the current per armature path? (*b*) What is the current per brush holder? (*c*) What type of winding will give this generator a rating of 500 kw., 500 volts? (*d*) Repeat (*a*) and (*b*) under the conditions of (*c*).

QUESTIONS ON CHAPTER XII

Generator Characteristics

1. What fundamental relation gives the e.m.f. in a single armature conductor, while it is cutting flux under the pole of a generator?

2. Show that in a simplex lap winding, the e.m.f. between brushes at any instant is equal to the sum of the e.m.fs. in all the conductors between any two commutating planes. Show from this that the e.m.f. between brushes is equal to the average e.m.f. induced in any single conductor during the time it is passing between any two commutating planes, multiplied by the number of conductors between commutating planes. Why is the curve of e.m.f. and time for any one conductor identical in shape with the curve of flux density along the air-gap?

3. Derive the equation giving the e.m.f. in a generator in terms of flux per pole, speed, number of armature conductors, number of poles, and number of paths through the armature.

4. A certain armature has a fixed number of conductors on its surface. What are the separate effects on the induced e.m.f. of: (*a*) doubling the speed of the armature; (*b*) doubling the flux entering the armature; (*c*) reconnecting the armature so that the number of paths through the armature is doubled?

5. In a given generator, on what two factors does the induced e.m.f. depend? If the speed of the generator be maintained constant, on what one factor does the induced e.m.f. depend?

6. Show that a similarity should exist between two curves plotted as follows: (*a*) The field ampere-turns of a generator as abscissa and the flux leaving one of its N-poles as ordinate. (*b*) The field current of the same generator as abscissa and the induced armature e.m.f. at constant speed as ordinate.

7. In the curve relating ampere-turns of the field and flux of one N-pole, why does not the flux start at zero value? Why is the first part of the curve practically a straight line? At the higher values of field current why does the induced e.m.f. increase less and less rapidly for any given increase in field current?

8. Discuss any difference that may exist between the saturation curve obtained with *increasing* values of field current and that obtained with *decreasing* values.

9. Sketch the connections used in determining a saturation curve: (*a*) using a simple field rheostat; (*b*) using a drop wire with the field. Give two reasons why the generator should be separately excited.

10. Show that Ohm's law can be expressed graphically. What two quantities are plotted when expressing Ohm's law in this manner?

11. Sketch the connections of a shunt generator. Is the field of comparatively low resistance or of high resistance? Explain.

12. Explain in detail how a shunt generator "builds up." What limits the voltage to which a machine can build up?

13. What is meant by critical field resistance? Give four causes, each of which may prevent the building up of a generator. What tests and remedies should be used for each cause?

14. What is the general direction of the flux produced by the current in the armature conductors? What effect does this have on the resultant flux in a machine? How does it affect the position of the neutral plane? What effect does the change in position of the neutral plane have on the brush position?

15. What is the relation of the direction of the armature field to the brush axis? When the brushes are moved forward in a generator what is the resulting direction of the armature field? Into what two components can this field be resolved? What is the effect of each component on the resultant flux?

16. Which conductors on an armature produce a demagnetizing effect? Which produce a cross-magnetizing effect?

17. Sketch the conductors on the armature, together with the poles, for a loaded multipolar generator, indicating the current directions in the various conductors. Sketch a curve showing the values of armature m.m.f. along the armature surface. Show the flux produced by this m.m.f. when acting alone.

18. Show the effect of the flux due to the armature m.m.f. on the distribution of the total flux along the armature surface. How is the neutral zone affected? What change must be made in the brush position? Designate the cross-magnetizing and the demagnetizing ampere-turns on a multipolar armature.

19. Name four methods by which armature reaction is either practically eliminated or reduced. State the principle of each method.

20. Sketch an *ideal* commutation curve assuming uniform current distribution over the brush.

21. What is the effect of having e.m.fs. induced in a coil during the time it is being short-circuited by the brush? What limits the current in such a coil? How does this current affect the distribution of current over the brush?

22. Sketch commutation curves for the following conditions: (*a*) brush too far forward; (*b*) brush too far back; (*c*) brush too wide.

23. Why does an armature coil have self-inductance? What is the effect of this self-inductance during the commutation period? What effect does the e.m.f. of self-induction have on the relation of the brush position to the neutral zone?

24. What is the order of magnitude of the e.m.fs. induced in a coil undergoing commutation? If such e.m.fs. are small what makes them objectionable?

25. What is the advantage of copper over carbon brushes? Why are carbon brushes used almost universally?

26. What evidence points to the fact that the taking of current from the commutator by the brushes is not pure conduction? To what is "high mica" due? How may it be reduced or even eliminated? Name two methods.

27. In general, what is the effect of arcing on the commutator? Why should any appearance of arcing be a reason for eliminating the cause of the arcing as soon as possible? Why is it not desirable to use emery paper or emery cloth in grinding brushes or smoothing the commutator?

28. What changes occur in the flux at the geometrical neutral of a generator as load is applied? What is the effect of these changes on the brush position? Why is it necessary to move the brushes ahead of the load neutral plane?

29. Show that instead of moving the brushes forward in order to obtain the proper commutating flux, the same result may be obtained by the use of commutating poles.

30. Why are the commutating poles connected in series with the armature? Why have they an unusually long air-gap?

31. What is the relation of the polarities of main poles and commutating poles to the direction of rotation, in a generator? In practice, how are the commutating poles adjusted to the proper strength?

32. Sketch the connections used in obtaining the shunt characteristic. Sketch the characteristic. Why does the generator finally "break down"? Why does the return curve from short circuit not follow the curve obtained with decreasing values of load resistance?

33. Give three reasons why the voltage of a shunt generator drops as load is applied. Why are the three reactions cumulative? What prevents a generator from "unbuilding" as load is applied?

34. What effect has running a shunt generator at higher than rated speed on its characteristic, provided the field current is adjusted so that the no-load volts remain unchanged?

35. What is meant by generator regulation? Does a large value of the regulation indicate that a generator is desirable for supplying lamp loads? Explain.

36. What is meant by the *total characteristic* of a generator? What is its relation to the shunt characteristic? How may the total power developed within an armature be determined?

37. Determine graphically the shunt characteristic of a generator, given the saturation curve, field resistance, armature resistance, and armature reaction: (*a*) neglecting armature reaction; (*b*) taking the demagnetizing effect of armature reaction into consideration. Show the effects of speed and degree of saturation on the shunt characteristic.

38. How may the objectionable drooping characteristic of the shunt generator be improved? How are the additional turns connected, and in what way do they differ from the shunt-field turns?

39. Show the difference between *long-shunt* and *short-shunt* connection. What is the effect of the connection on the characteristic? Sketch the characteristics of an overcompounded, a flat-compounded and an under-compounded generator. Where is each used and why?

40. How is the degree of compounding in a generator adjusted? When do generators have two separate series fields?

41. What is the effect of speed on the degree of compounding, if the no-load voltage is the same in each case? Compare this with the effect of speed on the shunt characteristic and explain.

42. Determine graphically the compound characteristic of a generator, given the saturation curve, shunt, series-field and armature resistances, armature reaction and shunt- and series-field turns.

43. Show how the number of series turns for a desired degree of compounding may be determined experimentally. What is the armature characteristic and how may it be utilized?

44. Show how the interpoles may have the effect on the characteristic of: (*a*) cumulative compounding; (*b*) differential compounding.

45. In what way does the series generator differ fundamentally from the shunt generator in construction? in the type of load that it supplies?

46. Describe the external characteristic of the series generator and show its relation to the saturation curve.

47. In what way does the series generator "build up"? What is meant by the critical external resistance? Why is it desirable to operate on the right-hand side of the external characteristic?

48. Name a very common use of the series generator. Name two common types of machines. Why are special commutators necessary?

49. What is the *Thury system* of power transmission? Where is it used?

50. How may series generators be used to control the voltage at the end of a feeder? On what portion of the characteristic does such a generator operate? Sketch the connections. What precautions must be taken in the installation and operation of a series booster?

51. How may the speed of a prime mover affect the generator characteristic? Is a drop in speed chargeable to the generator? How may it be taken into consideration?

52. State one essential difference between a unipolar generator and the ordinary type of generator. What design is necessary to prevent the armature being short-circuited on itself? What is the advantage of this type of machine over the ordinary type and for what type of work is it best adapted? What are its disadvantages?

53. What is the basic principle of the Tirrill regulator? What is the function of the main control magnet? of the relay magnet? Why cannot this regulator be applied directly to the fields of generators of large capacity? How may it be applied to such machines?

54. Describe the construction and operation of the type GDD direct-acting voltage regulator of the General Electric Company. What are the advantages of this type of regulator?

55. Make a diagram showing the connections of the third-brush generator Analyze the reactions which cause the generator to deliver a substantially constant current. What is the effect of moving the third brush forward? backward? For what type of work is this generator used?

56. Make a sketch of the diverter-pole generator, showing the flux relations at no load. Analyze the changes which occur when load is applied. To what type of work are the characteristics of this generator particularly well adapted?

57. What three characteristics are necessary in a welding generator? Compare the transient with the steady-state characteristic. Why is the comparison particularly necessary with arc-welding generators?

58. Describe the construction of the General Electric arc-welding generator. What means are used to obtain the desired volt-ampere characteristics? Why does this generator do away with the use of an external inductance?

PROBLEMS ON CHAPTER XII

Generator Characteristics

582. Fig. 582*A* shows the flux leaving a *N*-pole and entering the armature of a four-pole generator. Directly under the pole the flux density is 7,000 gausses, as shown by the graph. The diameter of the armature is 30 cm. and the speed is 1,800 r.p.m. The axial length of active conductor is 18 cm. Determine: (*a*) the peripheral velocity in centimeters per second of a conductor *a* on the surface of the armature; (*b*) the e.m.f. induced in this conductor when it is directly under the pole where the flux density is constant at 7,000 gausses.

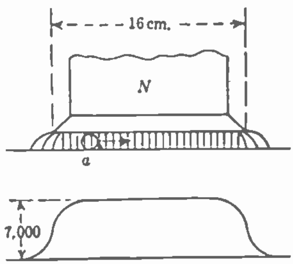


Fig. 582*A*.

583. In Prob. 582 the pole arc is 16 cm. The *average* value of the flux density directly under the pole and included in a distance equal to the pole arc is now 7,200 gausses. The average flux density over the entire *pole pitch* is 5,600 gausses. (*a*) What is the average value of the e.m.f. induced in a conductor during the time it is directly under the pole? (*b*) What is the average value of the e.m.f. induced in a conductor during the time it is passing between two brushes? (*c*) If there are 98 such conductors in one path between any pair of brushes, what is the induced e.m.f. between brushes?

584. The axial length of the pole faces of a four-pole, 250-volt, 12-kw. generator is 4.5 in. and the pole arc is 6 in. The diameter of the armature is 11 in. and the speed is 1,200 r.p.m. The flux density directly under the pole faces is 45,000 lines per square inch and the flux-density curve may be considered as a rectangle, the peripheral length of which is equal to the pole arc. Determine: (*a*) the e.m.f. induced in a single conductor on the surface of the armature when it is directly under a pole face; (*b*) the average e.m.f. induced in the conductor during the time it is passing between any two brushes; (*c*) the induced e.m.f. between brushes. There are 55 slots on the armature and 12 conductors per slot. The armature is lap-wound, giving the same number of parallel paths as there are poles.

585. The pole faces of a four-pole shunt generator are 8 in. square, and the average flux density *under the poles* is 42,000 lines per square inch. The flux-density curve may be considered as a rectangle with a peripheral length equal to that of the pole arc. There are 336 surface conductors on the armature. The armature is wave-wound, giving two parallel paths through the winding. What is the induced e.m.f. when the armature rotates at 800 r.p.m.; 1,000 r.p.m.?

586. If the current per path in Prob. 585 is 25 amp., what is the rating of the machine in kilowatts for each speed?

587. Repeat Probs. 585 and 586 for a simplex lap winding; number of conductors, speeds, flux, etc., remain the same.

588. In a six-pole, 550-volt generator, the diameter of the armature is 36 in., the pole faces are 12 in. square. The average flux density directly under the pole faces at no load is 48,300 lines per square inch and the flux-density curve may be considered as rectangular with a peripheral length equal to the pole arc. There are 16 slots per pole. The speed of the generator is 900 r.p.m. The armature is wave-wound. Determine: (*a*) the maximum induced e.m.f. per conductor; (*b*) the average induced e.m.f. per conductor; (*c*) the conductors per slot necessary to give 600 volts at no load.

589. In an eight-pole, 250-volt generator are 18 slots per pole, there is a double coil in each slot, and there are four turns per coil. The armature is simplex lap-wound. The total flux per pole into the armature is 7,240,000 maxwells. Determine: (*a*) the speed in r.p.m. necessary to give 250 volts; (*b*) the current per armature path which will make the rating 200 kw.

590. The following data are given for the saturation curve of a 60-kw., 240-volt, 800-r.p.m., direct-current shunt generator, the data being taken at 800 r.p.m. and for increasing values of field current:

Field current......	0	0.4	0.8	1.2	1.6	2.0	2.5	3.2	4.0	4.8
Electromotive force	11	44.0	88	126	154	176	200	224	246	264

(*a*) Plot the saturation curve for speeds of 800 and 720 r.p.m. Determine the critical field resistance at (*b*) 800 r.p.m.; (*c*) 720 r.p.m.

591. In the generator of Prob. 590 are four poles and 1,280 shunt-field turns per pole. The armature is simplex lap-wound and there are 72 slots and eight conductors per slot. Plot a curve with the field ampere-turns as abscissa and flux per pole as ordinate. What is the flux per pole when the generator operates at 800 r.p.m. and generates 240 volts?

592. (*a*) In Prob. 590, what value of field resistance will give 240 volts at 800 r.p.m. at no load? (*b*) 220 volts at 720 r.p.m.? (*c*) When the speed is 800 r.p.m. to what voltage will the generator build up when the field resistance is 60 ohms? Draw the field-resistance line in each case. The voltage drop in the armature due to field current may be neglected.

593. When the generator of Prob. 590 has been brought up to the speed of 800 r.p.m. and the adjustment of the field resistance is such that the machine builds up to 240 volts, what initial current flows through the field due to the residual magnetism? What induced e.m.f. results from this field current? What field current results from this last e.m.f.? Trace the successive increments of field current and e.m.f. which follow each other until the machine reaches the condition of stability.

594. A shunt generator when operating at 600 r.p.m. builds up to 125 volts. The speed and the field resistance are both doubled. To what voltage does it now build up?

595. A generator when running at rated speed fails to build up. With the shunt field connected across the armature, a voltmeter across the armature reads 5 volts. When the field circuit is opened the voltmeter reads 7 volts. What is the probable reason that the machine does not build up and what remedy is suggested?

596. The no-load flux per pole of a bipolar generator is 3,000,000 lines and there are 3,000 ampere-turns per field pole. There are 8,400 ampere-conductors uniformly distributed, on the surface of the armature. The brushes are advanced 20° ahead of the no-load neutral plane. Determine: (*a*) the cross-magnetizing armature ampere-turns; (*b*) the demagnetizing armature ampere-turns; (*c*) the net ampere-turns acting on the magnetic circuit if the effect of (*a*) is neglected; (*d*) the net flux entering the armature from the *N*-pole neglecting (*a*) and saturation.

597. There are 280 conductors on the surface of the armature of a bipolar generator. The generator delivers 80 amp., giving 40 amp. in each conductor. If the brushes are advanced 20° how many demagnetizing and cross-magnetizing ampere-conductors are there? How many demagnetizing and cross-magnetizing ampere-turns are there?

598. The brushes of a four-pole generator are advanced 12 space-degrees. The armature is simplex lap wound and there are 432 surface conductors. The generator delivers 96 amp., the shunt-field current is 2.4 amp., and there are 600 shunt-turns per pole. Determine: (*a*) the cross-magnetizing ampere-turns; (*b*) the demagnetizing ampere-turns; (*c*) the net ampere-turns per pole, taking into consideration the field ampere-turns and (*b*).

599. In an eight-pole, 400-kw., 240-volt generator are 160 slots and four conductors per slot. The winding is simplex lap and there are 320 commutator segments. The field current is 13 amp., and there are 800 shunt-turns per pole. The brushes are moved ahead of the geometrical neutral by a distance of five commutator bars. At rated load determine: (*a*) the cross-magnetizing ampere-turns per pole; (*b*) the demagnetizing ampere-turns per pole; (*c*) the net field ampere-turns per pole, considering the effect of (*b*) only.

600. On the armature of a 12-pole, 500-kw., 250-volt shunt generator are 192 slots, 576 commutator segments, 1,152 surface conductors, and the armature is simplex lap wound. The diameter of the armature is 94.5 in. and the pole faces cover 70 per cent of the armature surface. The brushes are moved ahead two commutator segments. Determine: (*a*) the rated current of the generator; (*b*) the current per armature path; (*c*) the demagnetizing ampere-turns per pole; (*d*) the cross-magnetizing ampere-turns per pole. (*e*) Make a sketch of the poles and armature surface over a distance equal to at least twice the pole pitch (see Fig. 307, p. 419). Show the demagnetizing and cross-magnetizing ampere-turns. (*f*) Plot separately the m.m.f. of the demagnetizing and cross-magnetizing ampere-turns.

601. In Prob. 600 the field excitation at rated load is such that if acting alone, 3,600 ampere-turns per pole would be consumed in the air-gap and 1,200 ampere-turns in the teeth. Draw the approximate resultant m.m.f. distribution along the armature surface and the resulting flux distribution.

602. In a four-pole, 25-kw., 125-volt, 1,200-r.p.m. generator the diameter of the armature is 30 cm., there are 64 slots on the armature, and four conductors per slot. The diameter of the commutator is 22 cm., there are 64 commutator segments, and the peripheral brush width is 1 cm. The armature is simplex lap wound with two coil sides per slot. When the generator delivers rated current, the equivalent of 8,000 maxwells links each coil side. The coil consists of four conductors per slot (see Fig. 316, p. 426). Determine: (*a*) the peripheral velocity of the armature in centimeters per second; (*b*) the time of commutation; (*c*) the total change in flux, linking each coil each time the coil goes through the commutating zone; (*d*) the average rate of change of flux in maxwells per second while the coil is going through the commutating zone; (*e*) the average induced e.m.f. per coil while it is going through the commutating zone.

603. Repeat Prob. 602 with 6,800 maxwells linking the four conductors of each slot and the speed reduced to 1,000 r.p.m.

604. The armature of a 20-kw., 240-volt generator is simplex lap wound and there are eight conductors per coil, making 16 conductors per slot. The commutator is 8 in. in diameter and the peripheral width of each brush is $\frac{5}{8}$ in. The speed is 1,600 r.p.m. When the generator is carrying its rated load, 10,800 magnetic lines link each coil side (see Fig. 316, p. 426). Determine: (*a*) the peripheral speed of the commutator in inches per second; (*b*) the time taken for the current in any one coil to reverse from its full-positive to its full-negative value; (*c*) the total flux, linking each coil before commutation begins; (*d*) the total change of this flux during the commutation period; (*e*) the e.m.f. induced in a single coil during the commutation period, assuming straight-line commutation.

605. Repeat Prob. 604 with the speed reduced to 1,200 r.p.m. and the current reduced to 0.80 of its value in that problem.

606. The terminal voltage of a 150-kw. shunt generator is 600 volts when it delivers rated-load current. The resistance of the shunt-field circuit is 100 ohms, and the armature resistance is 0.08 ohm. Determine the induced e.m.f. in the armature.

607. Repeat Prob. 606 when the generator delivers one-half the rated-load current corresponding to 150 kw. at 600 volts. The terminal voltage is now 620 volts. The field resistance remains unchanged.

608. In a 100-kw., 250-volt shunt generator, 258 volts is induced in the armature when the generator delivers rated load at 250 volts. At the same time 6.0 amp. are taken by the shunt field. (*a*) What is the armature resistance? (*b*) The no-load voltage of the generator is 264 volts; what is the regulation? (*c*) Why is the induced e.m.f. at rated load not equal to the no-load volts?

609. The e.m.f. induced in the armature of a 250-kw., 600-volt shunt generator is 628 volts when the load current is at rated-full-load value, and the field current is 6.0 amp. The armature resistance is 0.06 ohm. Determine: (*a*) terminal voltage of the generator when operating under the foregoing conditions; (*b*) power generated; (*c*) power output; (*d*) electrical efficiency of the generator.

610. The following data for the saturation curve of a four-pole, 60-kw., 240-volt compound generator are taken at 800 r.p.m. with decreasing values of field current, the generator operating as a simple shunt machine (see Prob. 590).

Field current...	4.35	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0
Electromotive force.	254	249	240	225	207	185	158	120	66	12

The armature resistance is 0.035 ohm. The field rheostat is adjusted to give 254 volts at no load. (*a*) Neglecting armature reaction, determine and plot a characteristic with armature current as abscissa and terminal volts as ordinate to 200 per cent rated current (see Par. 276, p. 443). (*b*) From (*a*) plot also load current as abscissa and armature e.m.f. as ordinate. (*c*) What is the terminal voltage when the load current is equal to the rated value?

611. From Prob. 610 determine: (*a*) the maximum current which the armature can deliver; (*b*) the two values of field current corresponding to 200 per cent *rated-load* current. (*c*) Determine and plot the external or load, characteristic beginning with 200 per cent rated-load current and decreasing the load current to zero.

612. Repeat Prob. 610 for a speed of 850 r.p.m., discussing any difference in the characteristics obtained.

613. Repeat Prob. 611 for a speed of 850 r.p.m.

614. In the generator of Prob. 610 there are 1,280 shunt-turns per pole. There are 72 slots on the armature, eight conductors per slot, and the armature is simplex lap wound (see Prob. 591). The brushes are moved ahead 4 space-degrees. (*a*) Determine the demagnetizing ampere-turns per pole when the generator delivers rated current. (*b*) Using the data of (*a*), determine the characteristic to 200 per cent rated load with armature current as abscissa and terminal voltage as ordinate. (*c*) Plot a curve giving load current as abscissa and terminal voltage as ordinate.

615. In Prob. 614 determine: (*a*) the maximum load current which the armature can deliver. (*b*) Continue the characteristic of (*b*), Prob. 614, back to no load. (See Prob. 590.)

616. The following are the constants of a 15-kw., 230-volt, 1,200-r.p.m. compound generator: armature resistance 0.16 ohm; series-field resistance 0.04 ohm; series-field-diverter resistance 0.60 ohm; shunt-field resistance 180 ohms. The generator is connected short-shunt. With the generator delivering rated current at rated voltage, determine the power loss in the following: (*a*) armature; (*b*) series field; (*c*) diverter; (*d*) shunt field. Determine: (*e*) induced e.m.f.; (*f*) total power generated within the armature; (*g*) electrical efficiency (output divided by (*f*)).

617. The following are the constants of a 400-kw., 240-volt, 400-r.p.m. compound generator: armature resistance 0.0048 ohm; series-field resistance 0.0008 ohm; commutating-pole resistance 0.00006 ohm; shunt-field resistance 12.8 ohms. The generator is connected long-shunt. When it is delivering rated load at rated terminal voltage, determine; (*a*) power loss in shunt field; (*b*) power loss in series field; (*c*) power loss in commutating-pole circuit; (*d*) power loss in armature; (*e*) total power loss; (*f*) total power generated; (*g*) electrical efficiency of generator (output divided by (*f*)).

618. Repeat Prob. 617 with the load current at one-half rated value and the terminal voltage 245 volts.

619. Repeat Prob. 616 with the generator delivering one-half rated current at 235 volts.

620. A 250-kw., 550- to 650-volt compound railway generator is compounded so that it maintains the voltage constant at 550 volts at all loads for a point 3 miles away. The rated-load current is 455 amp. The overhead system consists of a No. 4/0 hard-drawn trolley wire, having a resistance of 0.28 ohm per mile, paralleled by three 500,000-cir.-mil feeders in parallel, each feeder having a resistance of 0.114 ohm per mile. The resistance of the track return is 0.025 ohm per mile. (*a*) What should be the rated-load voltage of this generator? (*b*) What is the efficiency of transmission, when the generator is delivering rated load?

621. The no-load voltage of a compound generator is 250 volts. It supplies a 500-kw. load, situated 800 ft. distant, over a 1,000,000-cir.-mil cable. It is desired to maintain the voltage at the load constant at 250 volts from no load to full load of 500 kw. What must be the no-load and the full-load voltage rating of the generator? Assume that a circular-mil-foot of copper has a resistance of 11 ohms at the operating temperature of the cable.

622. Repeat Prob. 621 for the condition that the voltage at the load shall rise from 250 to 265 volts from no load to full load.

623. The generator of Prob. 621 is connected long-shunt. Armature resistance is 0.010 ohm, shunt-field resistance is 35 ohms, series-field resistance is 0.0025 ohm, diverter resistance is 0.009 ohm, and interpole-circuit resistance is 0.0015 ohm. (*a*) What is the voltage induced in the armature with the 500-kw. load of Prob. 621. (*b*) How much power is lost in the armature, in the shunt field, in the series field, in the diverter, in the interpole circuit, and in the cable? (*c*) Of the total power generated, what percentage reaches the load?

624. Repeat Prob. 623 for the 500-kw. load of Prob. 622. It is now necessary to remove the series-field diverter.

625. In the generator of Probs. 610 and 614, there are 6½ series turns per pole. The entire resistance of the series field is 0.010 ohm. The generator is connected long-shunt. (*a*) From the saturation curve taken at 800 r.p.m. and with increasing values of field current (see Prob. 590), determine the current in the series field which is necessary to make the generator flat-compounded. (*b*) What should be the resistance of the series-field diverter? (See Par. 279, p. 453.) Neglect armature reaction.

626. Using the method described in Par. 279 (p. 453), determine the terminal voltage of the generator in Prob. 625 when the armature current is equal to the rated-load current and the diverter is removed. The no-load voltage of the generator is 240 volts.

627. In the four-pole, 60-kw., 240-volt generator of Probs. 610 and 625, with 6½ series turns per pole, at 800 r.p.m., determine the compound characteristic to 150 per cent rated load from the saturation curve of Prob. 590, taking into consideration armature reaction. The brushes are moved ahead four segments. Begin with a no-load voltage of 240 volts. The generator is connected long-shunt. The diverter is removed from the series field.

628. It is desired to add series turns to a shunt generator, so that its rated-load terminal voltage is the same as the no-load terminal voltage. There are 652 shunt turns per pole. When load is applied, it is found necessary to increase the shunt-field current from 4.0 to 5.70 amp. in order to keep the rated-load terminal voltage equal to the no-load terminal voltage. The generator is connected long-shunt. The rated current is 250 amp. (*a*) How many series turns per pole should be added? (*b*) If 7½ turns are added, what should be the ratio of diverter to series-field resistance? The voltage drop in the series field may be neglected; that is, the voltage across the brushes at rated load and at no load are the same. The armature current at rated load may be taken as 250 amp.

629. It is desired that the voltage of a 600-kw., 600-volt compound generator, connected long-shunt, shall increase from 550 volts at no load to 600 volts at rated load. With the series field out of circuit and the shunt field excited from an external source, it is found that the desired increase of voltage, in addition to the combined series-field and diverter voltage drop may be obtained by increasing the shunt-field current from its no-load value of 10.5 to 17.5 amp. There are 400 shunt turns per pole and 6½ series turns per pole. The series-field resistance is 0.008 ohm. What must be the resistance of a shunt or diverter to be connected across the series field?

630. A series generator supplies power to forty-two 6.6-amp., 500-watt, series magnetite-arc lamps. The armature resistance is 20 ohms and the field resistance 18 ohms. The line resistance is 12 ohms. Determine: (*a*) terminal voltage of the generator; (*b*) e.m.f. induced in the armature.

631. A 160-kw. load is situated 2,000 ft. distant from the 240-volt bus-bars of a station. The load is supplied over a 750,000-cir.-mil feeder. It is desired that when the load is 160 kw., the load voltage shall be not less than 230 volts. What must be the current and voltage rating of a series booster designed to maintain this voltage at this value? Assume that a circular-mil-foot has a resistance of 11 ohms at the operating temperature of the feeder.

632. The efficiency of the booster of Prob. 631 is 80 per cent. It is driven by a shunt motor connected across the bus-bars, the motor efficiency being 80 per cent. What is the over-all efficiency of the feeder?

QUESTIONS ON CHAPTER XIII

The Motor

1. In what way does a motor differ from a generator in the work which it performs? in general construction?

2. What effect is noted when a conductor carrying a current is placed in a magnetic field? How can this action be explained by two elementary laws of magnetism? What is the effect of reversing the current in the conductor?

3. To what three factors is this force proportional? If the flux density is doubled how is the force affected? If the current is doubled?

4. From the relation giving the field intensity at the center of a coil carrying current derive the equation which gives the force on a current flowing in a magnetic field.

5. State a convenient rule by which the relation among the direction of the current, the direction of the field, and the direction of the force can be determined. What other simple method enables one to determine this relation?

6. Define torque. In what units is it expressed in the British system? in the metric system?

7. Show that a coil carrying current when placed in a magnetic field may develop a torque. In what position of the coil is the torque a maximum? When is it zero? If continuous rotation is desired what change in the connection to the coil should be made when the torque reaches its zero value?

8. Why is a large number of conductors on the armature desirable? To what three factors is the torque of an armature proportional? In any one machine, to what two factors is the torque proportional?

9. How can it be shown that resistance alone does not determine the amount of current taken by a motor armature? Why must a motor of necessity be generating an e.m.f. when it is rotating? What is the relation of this e.m.f. to the direction of the current; to the direction of the applied voltage?

10. Is the counter e.m.f. greater or less than the applied voltage? Why? By what quantity do the two voltages differ from each other?

11. Show that the mechanical power developed by a motor armature is equal to the product of the current and the counter e.m.f.

12. Fundamentally, upon what two quantities does the speed of a motor depend?

13. In what direction is the flux of a motor distorted by armature reaction? In what direction should the brushes be moved as the load is applied to a motor? What general effect on the field flux does this movement of the brushes have? What is the effect on the speed?

14. What is the relation among the main poles, the interpoles, and the direction of rotation of a motor? How does this relation compare with the similar one for a generator?

15. When load is applied to a motor what is its first reaction? With the shunt motor, how does this reaction affect the back e.m.f.? the current flowing into the armature?

16. What two characteristics are very important in considering the adaptability of a motor for commercial work?

17. How does the torque of the shunt motor vary with the load? Why? How does the speed vary with the load? Demonstrate. Ordinarily is the change of speed with load excessive? What effect does armature reaction have on speed? What is meant by *speed regulation?* Does the percentage speed regulation have any significance as regards a motor's performance?

18. To what general type of work is a shunt motor adapted and why? Compare adjustable-speed operation with variable-speed operation of a motor. Discuss the possibilities of starting shunt motors under load.

19. How does the flux in a series motor vary with the load? How does this affect the variation of torque with load? To what extent is the speed of a series motor affected by the application of load? By the removal of load? What precautions should be taken when the series motor is being installed for industrial purposes?

20. To what general types of load is a series motor adapted and why? For what reasons is it especially adapted to railway work?

21. What quantities are plotted as the characteristics of a railway motor? Why?

22. In what way do the windings of a compound motor differ from those of a shunt motor? a series motor? In what two ways, with respect to the shunt winding, may the series winding be connected?

23. Discuss the speed characteristic of the cumulative-compound motor. the torque characteristic. What advantage has it over the series motor? For what general type of work is it best adapted?

24. What is the nature of the speed and torque characteristics of the differential-compound motor? Is this type of motor in general use? Explain. What precaution is necessary in starting this type of motor?

25. How may a motor be reversed in direction of rotation? What is the effect of reversing the line terminals?

26. Why is a starting rheostat necessary for direct-current motors? With the shunt motor, in what circuit is the starting resistance connected? Why should it not be connected in the line?

27. What two additions to the starting resistance of Fig. 373 (p. 502) are incorporated in a three-point starting box? Why? Sketch the connections of a three-point box. Show that the starting resistance which is in series with the shunt field when the arm is in the running position has little effect on the field current.

28. Under what conditions of motor operation is a three-point box undesirable? Why? Show that this objection is overcome by the use of a four-point box. Sketch the connections of a four-point box. What is the principal advantage of having the hold-up magnet in series with the shunt field?

29. Sketch the connections of a starting box containing the field resistance. Why is it necessary to short-circuit this resistance on starting? How is this accomplished?

30. How should a shunt motor be stopped? Give reasons. What is the effect of stopping the motor by throwing back the starting arm?

31. Sketch the connections of series-motor starters. What is the advantage of the no-load release over the no-voltage release?

32. When are controllers used and why? What two functions may a controller perform outside actual starting duty?

33. What are two advantages of automatic starters in medium sizes of motors? In the larger sizes of motors? Describe the method of operation of the General Electric automatic starter shown in Fig. 379 (p. 509). Compare the operation of the temperature overload relay with that of a fuse.

34. What are the advantages of the Thermoguard system of protection used by the Westinghouse Electric and Manufacturing Company, over devices which depend on line current for the initial operation?

35. What is the principle of the magnetic blow-out? When is it used?

36. What two factors only can be varied in obtaining speed control of a motor? In the armature-resistance-control method, which of these factors is varied? What are the advantages of this method of control? Name two serious disadvantages.

37. What is the principle of the multivoltage system? How are coarse adjustments of speed obtained? fine adjustments? What is the objection to this system? Where is it used to advantage?

38. What factor in the speed equation is varied in the Ward Leonard system of speed control? How many machines are necessary in this system? What is its chief advantage and where has it been used extensively? Name two disadvantages.

39. What factor in the speed equation is varied in the field-control method? Name two distinct advantages of this method. What limits the range of speed obtainable? What type of motor is especially adapted to this type of speed control?

40. Upon what principle does the Lincoln motor operate? What are its advantages?

41. What is meant by *series-parallel* control of railway motors? Why is such control desirable? Sketch the half-speed and the maximum-speed connections in a two-motor car; in a four-motor car.

42. Give three reasons why it is objectionable to place the main controller on the platform in the larger sizes of electric-car equipment. How are these

objections overcome? Give two other reasons why automatic control is desirable.

43. What is the general principle underlying the multiple-unit control? What is the train line?

44. Name briefly the sequence of closing of the contactors in starting a train.

45. What is meant by *dynamic braking?* Where is it used? Can a motor armature be brought to a standstill by this method of braking? Explain. Show the sequence of operations by which a series motor is brought from operation as a motor to full dynamic braking.

46. What is regenerative braking and where is it used? What special auxiliary equipment is necessary when regenerative braking is used with series motors?

47. Under what circumstances is it desirable to know the efficiency of a motor? What type of brake is often used for loading motors? Does this type lend itself readily to calculation of torque and power output of the motor? Explain. What is meant by the dead weight of the brake arm and how can it be determined and correction be made?

48. Describe a simple type of rope brake. How many balances are necessary in this type? What is a common method of cooling prony brakes?

49. In what way does a speed counter differ from a tachometer? Upon what principle is the magneto-voltmeter method of measuring speed based?

50. Define a dynamotor. What factors determine the voltage ratio between the two commutators? Discuss the effect on the voltage ratio of varying the field excitation.

PROBLEMS ON CHAPTER XIII

The Motor

633. The flux density directly under a *N*-pole of a motor is 6,000 gausses. The active length of the armature conductors is 20 cm.; each conductor carries a current of 32 amp. (*a*) What is the force in dynes acting on each conductor when under the poles? (*b*) in grams? (*c*) in kilograms? (*d*) If the current in the conductors is inwards in what direction is the force acting?

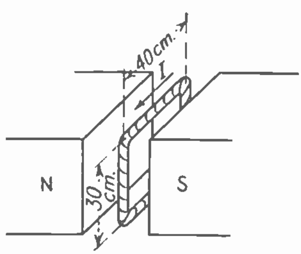


FIG. 634*A*.

634. A coil of 25 turns (Fig. 634*A*), horizontal length 40 cm. and vertical length 30 cm., lies in a uniform magnetic field, the density of which is 1,600 gausses. The plane of the coil is perpendicular to the direction of the field; the direction of the flux is from left to right; and the current in the upper coil side flows toward the observer. The current in each turn of the coil is 18 amp. Determine: (*a*) the force developed by each of the horizontal coil sides; (*b*) the force developed by each of the vertical coil sides; (*c*) the vertical stress in the vertical coil sides; (*d*) the horizontal stress in the horizontal coil sides.

635. A flat coil 20 cm. square and having 24 turns is so placed in a uniform magnetic field that the plane of the coil is parallel to the field and two opposite sides of the coil have the same direction as the field (Fig. 635A). The field intensity is 800 gausses and is uniform. The current is 12.0 amp. Determine: (*a*) the force in kilograms acting on each side of the coil; (*b*) the turning couple in kilogram-meters acting on the coil when it is in this position; (*c*) the pound-feet. (*d*) If the direction of the current in the right-hand side of the coil is away from the observer, in what direction does the coil tend to turn?

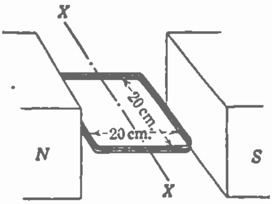


Fig. 635A.

636. Determine the turning couple in pound-feet for the coil of Prob. 635: (*a*) when it is turned an angle of 30° about its axis *XX*; (*b*) 45°.

637. A pulley having a diameter of 18 in. drives a 48-in. pulley with a 6-in. belt. The tension in the tight and loose sides of the belt are 1,400 and 350 lb. What net torque in pound-feet is developed by each pulley?

638. A gear having 120 teeth drives another gear having 48 teeth. The distance from the center of the first gear to the point of contact of the teeth is 8 in., the pitch circle having a diameter of 16 in. The pressure between the teeth at the point of contact is 400 lb. What is the torque in pound-feet developed by each of the gears?

639. The motor of Prob. 633 has four poles, and the diameter of the armature is 32 cm. There are 36 slots on the armature and 12 conductors per slot. There are 72 of these conductors under each pole at any one instant. The flux density directly under each pole is 6,000 gausses and may be considered as uniform (the armature reaction is compensated), and fringing may be neglected. Determine: (*a*) the force in kilograms developed under each pole; (*b*) the total force developed by the armature conductors; (*c*) the torque in kilogram-meters developed by the armature; (*d*) the torque in pound-feet.

640. Repeat Prob. 639 with 24 amp. per conductor and the flux density reduced to 5,000 gausses.

641. A coil consisting of 24 turns of wire lies parallel to a magnetic field having a density of 36,000 lines per square inch (see Fig. 360(*a*), p. 481). The distance across this coil parallel to the field is 12 in., and 18 in. of active conductor lie within the magnetic field and perpendicular to it. What torque in kilogram-meters is developed by the coil when the current per conductor is 15 amp.? Sketch the coil and the resultant magnetic field, indicating the directions of the forces acting.

642. Repeat Prob. 641 with the plane of the coil making an angle of 60° with the direction of the field (see Fig. 360(*c*), p. 481).

643. Repeat Prob. 641 with the coil turned 90° in its own plane so that the 12-in. side is now perpendicular to the field and the 18-in. side is parallel to the field.

644. The pole faces of a four-pole shunt motor are 8 in. square and due to armature reaction the flux density under the trailing pole tip is 30,000 lines per square inch and that under the leading pole tip is 40,000. The flux density may be considered as varying uniformly from one pole tip to the other, and fringing is neglected. The diameter of the armature is 14.5 in. and there are 336 surface conductors on the surface of the armature, and four parallel paths through the armature. Determine the torque in pound-feet of the motor when the current *to the armature* is 120 amp.

645. In Prob. 644, with the armature speed 1,000 r.p.m., determine: (*a*) the counter e.m.f.; (*b*) the mechanical power in watts developed within the armature; (*c*) the internal horsepower; (*d*) the internal torque in pound-feet.

646. Repeat Prob. 644 with the current to the armature 75 amp. and the armature simplex wave wound. The flux distribution at the two pole tips is now 28,000 and 40,000 lines per square inch.

647. In Prob. 646, with the armature speed equal to 1,000 r.p.m., determine: (*a*) the counter e.m.f.; (*b*) the mechanical power in watts developed by the armature; (*c*) the internal horsepower; (*d*) the internal torque in pound-feet.

648. When the flux density in the air-gap of a shunt motor is 42,000 lines per square inch and the armature current is 75 amp., the motor develops 90 lb.-ft. internal torque. (*a*) What is the internal torque developed when the armature takes 40 amp., the flux remaining constant? (*b*) 50 amp.?

649. When the load is entirely removed from the armature of Prob. 648, the motor armature requires 8 amp. to keep it running. What torque is required to overcome the motor losses? What is the torque available at the pulley in each case of Prob. 648, assuming that the no-load torque remains constant?

650. Determine the torque developed in the motor (Prob. 648) when the flux density is 50,000 lines per square inch and the current is 80 amp.

651. With an armature current of 84 amp. the flux per pole of a shunt motor is 2,800,000 lines and the electromagnetic torque is 120 lb.-ft. It is desired that the torque be 200 lb.-ft. when the armature current is 100 amp. and the motor is connected cumulative compound. By how many lines must the flux per pole be increased? Neglect the effect of armature reaction.

652. The armature resistance of a shunt motor is 0.048 ohm. When this motor is connected across 125-volt mains, it develops a counter e.m.f. of 118 volts. (*a*) What current does the armature take? (*b*) What current would the armature take if it were connected across the same mains while stationary? (*c*) What is the counter e.m.f. when the armature current is 160 amp.?

653. What counter e.m.f. does the motor armature of Prob. 652 develop when it is taking 175 amp. from the mains? If this same machine were running as a generator, what would be its internal e.m.f. when the armature is delivering 175 amp. at 125 volts?

654. A shunt dynamo having an armature resistance of 0.15 ohm is connected across 240-volt bus-bars. Determine the induced e.m.f. when the armature current is 75 amp., operating as a motor and as a generator.

655. On the armature of a four-pole shunt motor are 480 surface conductors, and the armature is simplex wave wound. The flux is 3,070,000 lines per pole. What back e.m.f. does the armature develop when rotating at 1,500 r.p.m.? The armature resistance is 0.2 ohm. What is the terminal voltage when armature current is 60 amp. if speed and flux remain constant?

656. Determine the current in the armature of the motor in Prob. 655 when the speed drops to 1,460 r.p.m., the terminal voltage and flux remaining constant.

657. The armature of a four-pole, 15-hp., 240-volt motor has 45 slots, 16 conductors per slot; the winding is simplex wave and the armature resistance, including brushes, is 0.15 ohm. The poles have an axial length of 4.25 in. and the pole arc is 6 in. When the speed is 1,200 r.p.m. and the average flux density under the pole is 31,500 lines per square inch, determine: (*a*) the counter e.m.f. of the armature; (*b*) the current when the terminal voltage is 240 volts.

658. In Prob. 657 determine the armature current when the average flux density becomes 32,800 lines and the speed drops to 1,100 r.p.m.

659. The resistance of the armature of a 20-hp., 250-volt shunt motor is 0.16 ohm. When running without load, the motor takes 6.4 amp. at rated voltage, the field current is 2.2. amp., and the speed is 1,280 r.p.m. Neglecting armature reaction, determine: (*a*) the speed when the motor takes 75 amp. from the line, the field current remaining unchanged; (*b*) the speed regulation. The rated current of the motor is 68 amp.

660. (*a*) Determine the speed of the motor of Prob. 657 when the armature takes 50 amp., and, due to armature reaction, the average flux density under a pole is 30,000 lines per square inch. (*b*) What is the speed regulation of the motor under these conditions, neglecting the no-load current? (*c*) Find the ratio of the torque which the motor develops to that developed under the conditions of Prob. 657.

661. A compound winding on the motor of Prob. 659 is connected long-shunt and aids the shunt field. Its resistance is 0.06 ohm. When the motor is taking its rated current of 68 amp. at 250 volts, this compound winding increases the flux per pole 20 per cent. Assume that the increase in flux is proportional to the armature current and neglect armature reaction. (*a*) Find the speed when the motor takes 6.4 amp. and when it takes 72 amp., the shunt-field current remaining unchanged. (*b*) Compare the torques at 72 amp. with and without the series field.

662. At no load the flux density in the motor of Prob. 659 is 31,500 maxwells per square inch. It is desired that the motor (Probs. 659 and 661) have the same speed at no load and when the armature current is 75 amp. Determine the flux density under the poles when the current is 75 amp.

663. Determine: (*a*) the mechanical power in kilowatts and horsepower developed in the armature of Prob. 659 when the motor takes 72.2 amp.; (*b*) the internal torque under these conditions.

664. A 60-hp., 250-volt, 850-r.p.m. shunt motor takes 168 amp. at 250 volts. The field current is 2.8 amp. and the armature resistance is 0.072 ohm. The motor speed is 850 r.p.m. when the line current is 12 amp. Determine: (*a*) internal power which is being developed in kilowatts and horsepower; (*b*) internal torque when the line current is 168 amp. Neglect armature reaction.

665. In a 100-hp., 600-volt, 1,200-r.p.m. shunt motor the field resistance is 400 ohms and the armature resistance is 0.22 ohm. The efficiency of the motor at its rated load is 90 per cent. At rated load determine: (*a*) rated line current; (*b*) field current; (*c*) counter e.m.f.; (*d*) internal power developed; (*e*) internal torque; (*f*) torque at the pulley.

666. In Prob. 665 the armature is blocked so that it remains stationary, and an external resistance is connected in series with one of the *line* wires and is adjusted so that the rated current flows. Determine: (*a*) field current; (*b*) armature current. (*c*) Compare this division of field and armature current with that of Prob. 665 and explain.

667. The resistance of the armature of a 20-hp., 250-volt series motor is 0.18 ohm, and the resistance of the series field is 0.10 ohm. When the motor takes 70 amp. from the line, the speed is 600 r.p.m. Determine: (*a*) the speed when the current is 80 amp.; (*b*) 40 amp. Assume that the saturation curve is a straight line and neglect armature reaction.

668. In Prob. 667, determine the speed in (*a*) and (*b*) with the series field shunted with 0.5-ohm resistance.

669. In a 60-hp., 550-volt series motor, the series-field resistance is 0.10 ohm and the armature resistance is 0.25 ohm. When taking rated current of 92 amp. from the line, the speed is 420 r.p.m. Determine the speed when the current is: (*a*) 100 amp.; (*b*) 40 amp. (*c*) At what value of current will the speed reach 1,200 r.p.m.? Assume that the saturation curve is a straight line and neglect armature reaction.

670. A shunt motor takes 90 amp. at 250 volts and develops 156 lb.-ft. internal torque at a speed of 960 r.p.m. The field resistance is 220 ohms. Determine: (*a*) armature resistance; (*b*) internal torque and speed developed when the motor current is 45 amp. Neglect armature reaction.

671. The rated current of a 10-hp., 230-volt, 1,350-r.p.m. shunt motor is 37.5 amp. The field current is 0.75 amp. At rated current and speed, the internal torque is 41.5 lb.-ft. Determine: (*a*) the internal torque when the line current is 18 amp.; (*b*) when the line current is 30 amp. (*c*) Find the speed in (*a*) and (*b*). Neglect armature reaction. (*d*) What is the speed regulation of this motor at 37.5 amp. if the no-load current is 2.8 amp. (*e*) Determine the torque at the pulley at rated load.

672. The flux of the motor (Prob. 671) is reduced to 0.88 of the value in Prob. 671. The field current is now 0.6 amp. Find: (*a*) internal torque and speed at rated current with this new value of field current; (*b*) internal torque and speed when the motor current is 20 amp., and the field current is 0.6 amp. Neglect armature reaction.

673. A 5-hp., 240-volt series motor develops 40 lb.-ft. internal torque when the current is 15 amp. Determine the internal torque when the current is:

(*a*) 8 amp.; (*b*) 12 amp. Neglect armature reaction and assume a straight-line saturation curve.

674. The armature resistance of the motor (Prob. 673) is 0.25 ohm and the field resistance is 0.12 ohm. (*a*) What is the speed when the current is 11 amp. at 240 volts? (*b*) What internal horsepower does the motor develop when the current is 20 amp.? (*c*) when the current is 11 amp.?

675. The resistance of the armature of a 50-hp., 600-volt series railway motor is 0.4 ohm, and the resistance of the series field is 0.25 ohm. At rated voltage and when the current is 70 amp., the speed is 600 r.p.m. Determine: (*a*) the speed and internal torque when the current is 90 amp.; (*b*) when the current is 40 amp. Assume that the saturation curve is a straight line and neglect armature reaction.

676. In Prob. 675 repeat (*a*) and (*b*) when the series field is shunted with a resistance of 1.0 ohm.

677. When a street car, operating with two series motors in parallel, takes 120 amp., it develops a total tractive effort of 1,800 lb. (*a*) What approximate tractive effort will be developed when the car is starting with the two motors in series and the car is taking 100 amp.? (*b*) When motors are in parallel and the total current to the car is 140 amp. Assume that the saturation curve is a straight line and neglect armature reaction.

678. The speed of a 25-hp., 250-volt, cumulative-compound motor, when running light, is 1,200 r.p.m. and the line current is 5.40 amp. The shunt-field resistance is 220 ohms, the series-field resistance is 0.04 ohm, and the armature resistance is 0.15 ohm. The motor is connected long-shunt. The line current at rated load is 85 amp. and the corresponding speed is 820 r.p.m. Assume that the increase in flux is proportional to the armature current. Determine: (*a*) counter e.m.f. at rated-load speed; (*b*) ratio of rated- to no-load flux; (*c*) internal torque at rated load.

679. Determine (*a*) the counter e.m.f.; (*b*) the ratio of flux to no-load flux; (*c*) the speed; (*d*) the internal torque of the motor, Prob. 678 when the line current is 42 amp.

680. The rated current of a 20-hp., 220-volt shunt motor is 79 amp. The no-load speed of the motor is 1,000 r.p.m., the armature resistance is 0.14 ohm, and the field resistance is 208 ohms. It is desired that the starting torque of the motor be equal to the rated-load torque. (*a*) What should be the total initial resistance of the starting box? (*b*) Determine the current taken by the armature when the speed becomes 25 per cent of the no-load value, with the entire starting resistance still in circuit. Neglect armature-resistance drop at no load and armature reaction.

681. When the motor of Prob. 680 has reached 25 per cent no-load speed, as in (*b*), it is desired that the armature current be brought again to its rated value by moving the starting arm to the next contact. How much resistance remains in the starting box? Neglect armature-resistance drop at no load and armature reaction.

682. The motor of Prob. 680 reaches one-half the no-load speed with the resistance determined in Prob. 681 still in circuit. (*a*) What current is the armature now taking? (*b*) If it is desired to bring the armature current back to its rated value by moving the starting arm to the next contact, how much resistance must be left in the starting box? Neglect armature-resistance drop at no load and armature reaction.

683. The resistance of the armature of a 60-hp., 550-volt shunt motor is 0.27 ohm and the field resistance is 520 ohms. The rated current is 90 amp. It is desired that the motor develop 150 per cent rated torque on starting. What should be the total resistance of the first step of the controller?

684. The resistances of the armature and field of a 75-hp., 600-volt series motor are 0.25 and 0.12 ohm. The rated current is 100 amp. Determine the resistance of the first position of the starting controller in order that the motor may develop on starting: (*a*) rated-load torque; (*b*) 200 per cent rated-load torque. Assume that the saturation curve is a straight line and neglect armature reaction.

685. The resistance of the armature of a 10-hp., 230-volt shunt motor is 0.35 ohm. When the armature current is 1.6 amp., the speed is 1,040 r.p.m. It is desired that the speed be reduced to 600 r.p.m. at 40 amp. by inserting external resistance in the armature circuit (see Fig. 381, p. 511). (*a*) What value of external resistance is necessary? (*b*) With this external resistance in circuit, at what speed will the motor run when the armature current is 22 amp.? (*c*) The rated motor current is 38.5 amp. and the field current is 0.8 amp. In (*a*) and (*b*) determine the per cent rated-load torque. (*d*) In (*a*) and (*b*), what per cent of power delivered by the line to the armature circuit goes to the armature, and what per cent is converted into mechanical power?

686. Repeat Prob. 685 for 300 r.p.m.

687. A motor when connected across 110-volt mains of Fig. 382 (p. 513) runs at 400 r.p.m. What speeds can be obtained by the use of the armature-voltage method of speed control if the shunt field is kept constant? Neglect the I_aR_a drop in the motor armature.

688. In a Ward Leonard system of speed control the efficiencies of the machines are as follows: M_1 (Fig. 383, p. 514), 86 per cent; G, 85 per cent; M_2, 84 per cent. The voltage across the mains is 250 volts. When M_2 delivers 15 hp., how much current is being supplied by the line? What is the over-all efficiency of the system?

689. In Prob. 688, the load on the motor is 7.5 hp. and the corresponding efficiencies are M_1, 82.5 per cent; G, 81 per cent; M_2, 80 per cent. Determine the over-all efficiency of the system.

690. In a brake similar to that shown in Fig. 389 (p. 521), the length L is 2 ft. The balance reading is 52.6 lb., the dead weight of the arm is 2.8 lb., and the speed of the armature is 1,240 r.p.m. Determine: (*a*) the horsepower developed by the motor; (*b*) the efficiency if the motor input is 83.0 amp. at 240 volts.

691. Repeat Prob. 690 for a balance reading of 36 lb. and a speed of 1,210 r.p.m. The motor input is now 55.3 amp. at 240 volts. The dead weight of the arm remains unchanged.

692. It is desired to conduct a brake test on a 10-hp., 250-volt, 1,500-r.p.m. shunt motor, and a brake with a 2-ft. arm similar to that in Fig. 389 (p. 521) is available. The test is to be carried to 25 per cent overload. Determine the rating in pounds of the balance allowing 5 lb. for tare.

693. In a brake similar to that shown in Fig. 391 (p. 523), the diameter of the drum is 10 in. The speed is 1,400 r.p.m. One balance reads 12.5 lb. and the other reads 3.7 lb. Determine: (*a*) torque developed by the motor at this load; (*b*) horsepower output; (*c*) efficiency if the input is 8.92 amp. at 115 volts.

694. Compute the horsepower output developed by the rope brake shown in Fig. 694*A*. The speed of the drum is 1,180 r.p.m.

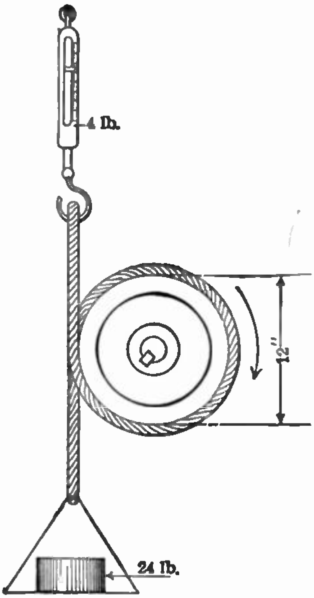


FIG. 694*A*.

QUESTIONS ON CHAPTER XIV

1-13

Losses; Efficiency; Operation

1. Discuss the disposition of the energy which is lost within electrical apparatus. Explain in detail its effect on the apparatus.

2. Into what three groups can the losses in either motor or generator be classified? Name the losses under the first group, indicating how they are determined. Are they readily determinable?

3. What constitutes the losses of the second group? How are these losses supplied, electrically or mechanically? On what do they depend? How is the eddy-current loss made small? What is meant by pole-face loss and to what is it due? How is it reduced?

4. To what factor are indeterminable or stray-load losses due? Enumerate some of these losses.

5. Omitting stray-load losses, why are all the losses except the copper loss grouped as one? On what factors do losses other than copper depend? If it is desired to duplicate stray-power losses under two different conditions of load, what two factors must also be duplicated?

6. Show that, if the losses in a dynamo are known at any particular load, its efficiency at that load can be calculated. Why is the formula for generator efficiency different from that for motor efficiency?

7. How may the efficiency of a generator be measured directly? What practical conditions make such measurements difficult? What effect do errors in the measurements have on the precision of the results? What other objections are there to direct measurements of efficiency?

8. How is a machine ordinarily operated in order to measure its stray power? What measurements are made? To what is the stray power then equal?

9. In stray-power measurements, how is the flux adjusted to the proper value? How is the speed adjusted? Does the flux adjustment have any effect on the speed and if so how are any readjustments made?

10. For what purpose is a set of stray-power curves desirable? Why cannot the stray power over the entire operating range of the dynamo be shown with a single curve? What errors are introduced by using the field current as a measure of the flux and how is one of these errors partially neutralized?

11. For determining losses, what is the advantage of the opposition method over the stray-power method? On what principle does the opposition method depend?

12. What assumption is it necessary to make in the opposition method? Does this assumption introduce appreciable error? In this method how are the two machines started and adjusted? What instruments are used and what measurements are necessary? State the disadvantages of this method.

13. In general, what factor determines the rating of a steam engine? a steam turbine? a gas engine? an electric machine? Give reasons in each case.

14. State the effects of excessive temperature on the insulation of electric machinery. What insulating materials can withstand the highest temperatures?

15. What method of measuring the temperatures of the different parts of a dynamo is recommended by the A.I.E.E.? What is meant by *ambient* temperature?

16. Describe the method of determining temperature rise in a dynamo by the change-of-resistance method. Compare this method with the thermometer method.

17. For what length of time should a temperature test be run? How may the temperature rise be accelerated? In what way may a machine's approach to constant temperature be determined? Why does the temperature of a machine rise more rapidly at the beginning of a test than at the end? What relation exists between the heat supplied and the heat dissipated when a constant temperature is reached?

18. Why must care be taken not to include the brush and contact resistance when measuring the armature resistance for temperature determination? Where must the voltmeter leads be held?

19. What difficulties arise when the resistance of a multipolar armature is measured for temperature determination? How may these difficulties be eliminated? What precautions should be taken when the field temperature is being determined by resistance measurements? Discuss the relation between the temperature determined by the resistance method and the maximum or "hottest spot" temperature.

20. Give five reasons why it is either necessary or desirable to operate shunt generators in parallel. What, in their characteristic, makes them especially adapted to parallel operation?

21. Analyze the reactions which follow when one generator begins to take more than its share of the load. What is meant by *stable equilibrium?*

22. State in detail the steps necessary to connect a generator in service. If the generator is connected in service with its voltage equal to that of the bus, why does it not take load? What must be done in order that it may take load?

23. Describe the steps necessary to remove a generator from service. Why is it undesirable to open the generator switch when the machine is delivering load? What is necessary as regards the generator characteristics in order that the machines may properly divide the load over their entire range of operation?

24. Show that overcompounded generators in parallel are in unstable equilibrium. What simple connection makes their operation stable?

25. What two conditions are necessary for two compound generators to divide the load properly over their entire range of operation?

26. Why does not a diverter change the division of load between two compound generators in parallel? What adjustment can be made to change the load division?

27. How many equalizers may be necessary in certain types of compound generators? How many poles must the switch have for such generators?

28. Compare circuit-breakers and fuses, stating the advantages and disadvantages of each.

29. Describe the operation of the reclosing type of circuit-breaker and state its advantages to electric service.

30. What is the advantage of the *rate-of-rise-of-current* circuit-breaker? Describe a method used on one type of this breaker to give the circuit-breaker this characteristic.

PROBLEMS ON CHAPTER XIV

Losses; Efficiency; Operation

695. The eddy-current loss in a shunt generator is 420 watts when it is running at 1,000 r.p.m. and with a flux of 1,200,000 lines per pole. Determine: (*a*) loss when the speed is 1,200 r.p.m. the flux remaining unchanged; (*b*) loss at 1,000 r.p.m. with a flux of 1,000,000 lines per pole; (*c*) loss with the same flux as in (*b*) but with a speed of 1,200 r.p.m.

696. The hysteresis loss in the generator of Prob. 695 is 640 watts when the speed is 1,000 r.p.m. Find the hysteresis loss under the conditions of (*a*), (*b*), and (*c*).

697. When the induced e.m.f. in a 50-kw., 250-volt shunt generator is 265 volts and the speed is 1,200 r.p.m., the hysteresis loss in the armature iron is 800 watts. Determine the hysteresis loss under the following conditions:

(*a*) induced e.m.f. 280 volts, speed 1,200 r.p.m.; (*b*) induced e.m.f. 230 volts, speed 1,020 r.p.m.; (*c*) induced e.m.f. 240 volts, speed 1,020 r.p.m.

698. In Prob. 697 the eddy-current loss is 420 watts. Determine the eddy-current loss, under the conditions of (*a*), (*b*), and (*c*).

699. A 100-kw., 250-volt, 900-r.p.m. shunt generator delivers 350 amp. at 245 volts and rated speed. The total losses are 9,580 watts. Determine: (*a*) power input in horsepower; (*b*) torque required to drive the generator; (*c*) efficiency.

700. When the generator of Prob. 699 delivers its rated current at rated voltage and speed, the total losses are 11,090 watts. Repeat (*a*), (*b*), and (*c*).

701. In a 50-hp., 250-volt, 1,200-r.p.m. shunt motor the losses, when the motor delivers its rated output at rated speed and voltage, are 4,740 watts. Determine: (*a*) motor torque; (*b*) watts input; (*c*) current; (*d*) efficiency.

702. The resistance of the armature of a 15-hp., 250-volt, 1,500-r.p.m. shunt motor is 0.168 ohm and the field resistance is 210 ohms. The stray power, when it delivers rated load at rated voltage, is 1,400 watts. Determine: (*a*) input at rated load; (*b*) efficiency; (*c*) percentage error in the efficiency caused by a 10 per cent error in determining the stray power.

(Hint:—A simple quadratic equation involving the armature current I_a may be used to find I_a. A trial-and-error method is an alternative.)

703. When the motor (Prob. 702) takes 60 amp. from the line, the speed remaining at 1,500 r.p.m., the stray power is 1,420 watts. Determine: (*a*) input; (*b*) output in horsepower, (*c*) efficiency; (*d*) torque at the pulley.

704. The armature resistance of a 400-kw., 600-volt, 750-r.p.m. shunt generator is 0.038 ohm; the shunt-field resistance is 58 ohms; the stray power at rated load and speed is 18.0 kw. Determine at rated load and speed: (*a*) total losses; (*b*) efficiency; (*c*) torque necessary to drive the generator.

705. A 10-kw., 220-volt, 1,400-r.p.m. shunt generator is running light at rated speed as a motor. The armature takes 2.95 amp. from 230.8-volt mains. The armature resistance is 0.26 ohm. What is the stray-power loss of the machine under these conditions?

706. In Prob. 705, when the generator is delivering rated-load current at 1,400 r.p.m., its induced e.m.f. is 230 volts and the field resistance is 212 ohms. Determine the rated-load efficiency.

707. The generator of Prob. 705 is operating at 1,400 r.p.m. as a shunt motor. The line current is 40 amp., the induced e.m.f. is 230 volts, and the field current is 2.1 amp. Determine: (*a*) total losses; (*b*) output in horsepower; (*c*) efficiency; (*d*) torque at the pulley.

708. When a 25-hp., 250-volt, 1,500-r.p.m. shunt motor is running light, the line voltage is 250 volts and the current is 6.8 amp. The shunt-field resistance is 200 ohms and the armature resistance is 0.118 ohm. Determine the stray-power loss in the motor under these conditions.

709. It is desired to determine the stray power of the shunt generator in Prob. 699 under the conditions given in that problem. The generator is run light as a motor, the connections being those in Fig. 400 (p. 536). The armature resistance is 0.025 ohm. (*a*) To what value should the voltage V_1 across the armature be adjusted, and how is this adjustment made? Neglect armature-resistance drop when the generator is run light as a motor. (*b*) What adjustment of the field is necessary? (*c*) If the stray-power loss is 4,500 watts, what is the reading of the ammeter in the armature circuit?

710. The stray-power curves in Fig. 402 (p. 542) were taken for a 20-kw., 220-volt, 960-r.p.m. shunt generator, the armature resistance of which is 0.12 ohm. When the machine delivers its rated output at rated voltage and speed, the field current is 2.44 amp. Determine: (*a*) total losses; (*b*) generator efficiency; (*c*) losses in per cent of input; (*d*) driving torque which is applied to the shaft.

711. The generator of Prob. 710 runs as a motor with load, and takes 80 amp. at 220 volts from the line. The speed is 1,050 r.p.m. and the field current is 1.95 amp. Determine: (*a*) value of E/S; (*b*) stray power; (*c*) total losses; (*d*) output of the motor in horsepower; (*e*) efficiency; (*f*) external torque; (*g*) total losses in per cent of input.

712. Repeat Prob. 711 for a current of 50 amp. from the line. The speed is now 1,080 r.p.m.

713. Two similar 10-kw., 230-volt generators are connected for the Kapp opposition test (Fig. 403, p. 543) for the purpose of having their losses measured. When the machine operating as a generator is delivering its rated armature current of 45.0 amp., the line current I is found to be 10.4 amp. The generator field current is 1.8 amp. and the motor field current is 1.2 amp. Each machine has an armature resistance of 0.26 ohm. Determine the efficiency of each machine at these conditions of load.

714. When the current in the generator armature of Prob. 713 is 25 amp., the line current I is 6.61 amp. The generator field current is now 1.6 amp. and the motor field current is 1.0 amp. Determine the stray power and the efficiency of each machine at this load. Why are the field currents different in the two machines?

715. The armature and field resistances of a 550-volt shunt generator are measured after the machine has been standing idle for some time in a dynamo room, the temperature of which is 30°C. The voltage across the field winding, exclusive of the rheostat, is found to be 430 volts and the field current 5.2 amp. The armature resistance between two marked commutator segments is found to be 0.18 ohm. After the machine has been running under load for 4 hr., these same measurements are repeated. The field voltage is now 435 volts and the field current 4.92 amp. The armature resistance is now 0.197 ohm. What is the measured temperature rise of the armature winding and of the field winding? Do these maximum temperatures appear to be safe for untreated cotton insulation?

716. A 10-hp., 230-volt, 1,340-r.p.m. shunt motor has been standing idle for some time in a room the temperature of which is 23°C. The armature resistance between marked segments is measured and found to be 0.252 ohm; the resistance of the field coils themselves is measured and found to be 340 ohms. After the machine runs at rated load for 5 hr., these measurements are repeated. The armature resistance between the same two segments is again measured and found to be 0.281 ohm, the resistance of the field copper is found to be 371 ohms. Determine the measured temperature rise in: (*a*) the armature; (*b*) the field.

717. Two 50-kw., 250-volt shunt generators are operating in parallel. They are both adjusted to 250-volts at no load and are then switched in parallel. When operating alone, the terminal voltage of generator 1 drops uniformly from 250 volts at no load to 242 volts at rated load; the terminal voltage of generator 2 drops uniformly from 250 volts at no load to 238 volts at rated load. When the load on the system is 360 amp., what current does each generator deliver? What kilowatt load does each deliver?

718. Repeat Prob. 717 for a system load of 400 amp.

719. The field rheostat of generator 2 of Prob. 717 is adjusted so that at rated-load current and 242 volts, both generators deliver equal currents. The adjustment raises the characteristic 4 volts at every point from rated load to no load. Determine the current delivered by each generator when the system load is: (*a*) 0 amp.; (*b*) 200 amp.; (*c*) 300 amp.

720. It is desired to operate a 100-kw., 240-volt shunt generator and a 60-kw., 240-volt shunt generator in parallel. The terminal voltage of the first generator drops uniformly by 10 volts from a no-load voltage of 250 when its rated load is applied. What should be the drop in terminal voltage of the second generator from no load to rated load in order that each generator may take its proper share of the load at all times? Assume that the voltage-load characteristic is a straight line and the no-load voltage is 250 volts in each case. How much current does each deliver when the system demand is: (*a*) 300 amp.; (*b*) 700 amp.? (*c*) What is the kilowatt output of each generator under the conditions of (*a*) and (*b*)?

721. Two 250-volt compound generators are operating in parallel. One has a rating of 150 kw. and the other a rating of 100 kw. The resistance of the series field of the first is 0.005 ohm. (*a*) What should be the resistance of the series field of the second machine for proper division of the load? The two generators are overcompounded so that the voltage of each rises in a straight line from a no-load voltage of 240 volts to 250 volts at rated load. Determine: (*b*) the load of each generator when the system load is 400 amp.; (*c*) 800 amp.

QUESTIONS ON CHAPTER XV

Transmission and Distribution of Power

1. Why cannot the ordinary direct-current voltages be used for transmitting considerable amounts of power over long distances? Under what conditions is direct current utilized for general light and power distribution? What are its advantages under these conditions?

2. Describe the general scheme for transmitting large amounts of power from a remotely situated power station to the consumers' premises. What are the ranges of transmission voltages? of distribution voltages? What part does the substation play in the system?

3. How does the weight of conductor vary with the transmission voltage? Demonstrate. If the transmission voltage were doubled, how would the weight of copper be affected, the other factors remaining unchanged?

4. Enumerate the five conditions which in general determine the size of conductor to be used. For what conditions does the question of heating particularly apply? In what way may the economics of the problem determine the size of conductor? State the disadvantage of having too large a conductor; too small a conductor.

5. Why is 110 volts most convenient for incandescent lighting? Why is a substantially higher voltage undesirable? What are the advantages and disadvantages of a lower voltage for this purpose?

6. What are the common trolley voltages? Why are these voltages so chosen?

7. What is meant by *distributed loads?* Where do such loads occur? Under what conditions are conductors of uniform cross-section throughout most commonly used?

8. Theoretically, what type of conductor is most economical for uniformly distributed loads? What is the practical condition that most nearly approaches this theoretical condition?

9. Why is the *return-loop* system of distribution used? What is its disadvantage?

10. What system overcomes the disadvantage of the return-loop system? Make a sketch and show how this system may be further modified to form a still more efficient system.

11. What advantage is gained by connecting 110-volt loads in series groups of two, and utilizing 220-volt supply? What are the disadvantages of so grouping the loads?

12. How are the objections to the series-parallel system overcome? What are the relations existing among the voltages of the Edison three-wire system?

13. If the neutral wire be of the same size as the two outers, what are the relative weights of copper in the three-wire system with 220 volts across outers and the simple 110-volt system, other conditions being the same?

14. What is meant by *balanced loads?* Under this condition how much current flows through the neutral?

15. In what direction does the neutral current flow if the positive load is the greater? the negative load? What relation does the neutral current bear to the current in the outer wires? What type of ammeter should be used in the neutral? What are the commercial limits of unbalancing?

16. State briefly the effect of opening the neutral with (*a*) balanced loads and (*b*) unbalanced loads. Why is the neutral usually grounded? Why are circuit-breakers and fuses usually omitted in the neutral?

17. What in general is the effect of putting too heavy a load on one side of a three-wire system on the voltage of that side of the system? on the voltage of the other side of the system?

18. Sketch a method of obtaining a neutral by the use of two shunt generators. What is the principal disadvantage of this method?

19. How may a storage battery be used for obtaining a neutral? In general, how does the current in the neutral wire divide when it reaches the center of the battery? State the objection to this method.

20. On what principle does the balancer set operate? What factor determines which machine shall operate as motor? as generator? What two methods are used to accentuate the motor and generator actions?

21. On what principle does the three-wire generator operate? Where does the alternating current flow? the returning direct current from the neutral? How is the direct current able to pass so readily back into the armature?

22. How in general is power supplied to direct-current loads in the more congested districts? What is the function of the feeders? the mains? the junction boxes? Where are the house services connected? How are the voltages at feeding points generally determined?

23. A load is supplied from constant-voltage bus-bars over a line of constant resistance. Under what conditions is the power taken by the load a maximum? What is the efficiency of transmission under these conditions? When is the current a maximum? What is the efficiency of transmission when the current is a maximum?

24. What type of generator is most commonly used to supply power for railways? How are such generators connected to the system?

25. Under what conditions does a single trolley suffice for transmitting the power to the car? If a single trolley of the ordinary size is not of sufficient cross-section, what means can be taken to assist it in supplying the required power? Why is the size of trolley not increased? Describe the ladder system.

26. Under what conditions are multiple feeders employed? What is the disadvantage of their use? How may this disadvantage be overcome?

27. Why does the return current from a trolley car leave the track? What determines the paths which it follows? What damage, if any, occurs at the point where the current enters a pipe? where it leaves the pipe?

28. Name two methods by which electrolysis may be reduced. What measurements give a good idea of the magnitude of stray currents between pipes and track?

29. Sketch a typical central-station load curve. Show how the habits of a community determine the general shape of such a curve. Why is such a load curve far more undesirable than a uniform load curve having the same total kilowatt-hours? What is meant by load factor? Is a high or a low load factor desirable? Why?

30. How may a storage battery smooth out a station-load curve? When should the battery be charged? discharged? Why are storage batteries not more generally used for this purpose?

31. Under what conditions can storage batteries be used efficiently to carry the load in off-peak times? For what purposes are they now commonly used by central stations? Where should they be located? Under what conditions is a battery very useful to a central station?

32. What difficulty is met when attempt is made to operate storage batteries in conjunction with a power plant? What simple method may be used to control the battery load? What is the objection to this method?

33. Upon what simple principle do counter-e.m.f. cells operate? What is the chief advantage of this method of control over the resistance method?

34. What is meant by end-cell control? In what manner is the connection changed from one cell to the next without opening the circuit or dead-short-circuiting the battery?

35. What is meant by a "regulating" battery? What is the purpose of such a battery? Why is it often necessary to install auxiliary means for increasing the battery discharge with change of bus-bar load? Sketch the connections of one simple method for accomplishing this purpose.

36. What is the essential difference between the series system and the parallel system of distribution? In the series system what is the effect of attempting to remove a load by opening the circuit? How is a load cut out in a series system?

37. By what devices is a series system supplied? What are the advantages of the series system? Where does its field of application lie? Sketch the layout of two different systems of series-lighting distribution. State the advantages of each.

PROBLEMS ON CHAPTER XV

Transmission and Distribution of Power

722. Power is being transmitted over a feeder to a distance of 600 ft. from 250-volt bus-bars. The current at the load is 300 amp. and the voltage at the load is 232 volts. Determine: (*a*) resistance of the feeder; (*b*) power loss; (*c*) efficiency of transmission; (*d*) power which could be transmitted over this feeder from 125-volt bus-bars, with the efficiency remaining unchanged.

723. In Prob. 722 a feeder of the same length but of four times the cross-section is used. With 125 volts at the bus-bars determine: (*a*) power which can be transmitted from 125-volt bus-bars with the load voltage equal to 116 volts; (*b*) power loss; (*c*) efficiency of transmission. Compare (*b*) and (*c*) with (*b*) and (*c*) of Prob. 722.

724. A motor takes 200 amp. at 224 volts over a 1,200-ft. length of 300,000-cir.-mil feeder (two wires, each 1,200 ft.), having a loop resistance of 0.0360 ohm per 1,000 ft. Determine: (*a*) kilowatts transmitted; (*b*) voltage at the sending end of the line; (*c*) power loss; (*d*) efficiency of transmission; (*e*) weight of copper. (A 1,000-ft. length of No. 10 A.W.G. wire, having a cross-section of 10,400 cir. mils, weighs 31.4 lb.)

725. In Prob. 724, assume that the power has been transmitted to the motor at 112 volts, and the power loss and efficiency have remained the same. Determine: (*a*) current; (*b*) total resistance of the feeder; (*c*) resistance per 1,000 ft.; (*d*) circular milage of the feeder; (*e*) weight of the copper;

(*f*) sending-end voltage. (*g*) Compare the weight of copper with that in Prob. 724.

726. A feeder of the same weight as that of Prob. 723 but of twice the length is used to transmit direct-current power. With the same sending-end and receiving-end voltages as in Prob. 723, and hence the same efficiency of transmission, determine: (*a*) the resistance of the feeder; (*b*) the power at the receiving end. (*c*) Repeat (*b*) with the sending-end voltage raised to 250 volts, the efficiency remaining unchanged.

727. A 10-hp. motor is fed from a switchboard, the bus-bars of which are maintained at 125 volts. The motor is located at a distance of 800 ft. from the switchboard, and it is desired to have a potential difference of 115 volts at the motor terminals when the motor is carrying its full load of 10 hp. (*a*) What must be the diameter in mils of the copper wire used to connect the motor to the switchboard? The resistance of a circular-mil-foot at the operating temperature is 11.0 ohms. 1 hp. = 746 watts. The efficiency of the motor at full load is 86 per cent. (*b*) If copper weighs 0.32 lb. per cubic inch, what will be the weight of the wire in (*a*)? (*c*) Repeat (*a*) and (*b*) for a switchboard voltage of 250 volts and the same per cent drop to the motor. (*d*) Repeat (*a*) and (*b*) for a switchboard voltage of 625 volts and the same per cent drop to the motor.

728. It is desired to transmit 120 kw. a distance of 1,500 ft. from 240-volt bus-bars, and the voltage at the load must not be less than 230 volts. The resistance of a circular-mil-foot of copper may be assumed to be 10 ohms. Determine: (*a*) the circular mils in the feeder; (*b*) its weight (see Prob. 724).

729. Repeat Prob. 728 when the distance is 3,000 ft. How does the weight of copper vary with the distance if power and efficiency remain fixed?

730. A certain street is 2,000 ft. long. It is illuminated by eleven 250-watt multiple-connected lamps, placed 200 ft. apart. Number 3 A.W.G. conductors are used to supply this system (resistance = 0.200 ohm per 1,000 ft. of wire). The voltage at the feeding end of the street is 125 volts. Determine: (*a*) the voltage drops between adjacent lamps; (*b*) the voltage at the last lamp. Assume that each lamp takes 2.0 amp.

731. If the lamps of Prob. 730 are fed by the anti-parallel system (see Fig. 415(*a*), p. 563), No. 3 A.W.G. wire still being used, determine the voltage at the lamps at the ends of the street. Compare their absolute voltage and their difference of voltage with the results of Prob. 730.

732. The lamps of Prob. 730 are now connected with No. 5 A.W.G. wire (resistance = 0.320 ohm per 1,000 ft. of wire). They are fed, however, at the center of the system by two No. 4 A.W.G. wires (resistance = 0.250 ohm per 1000 ft. of wire) from a junction box 800 ft. away, the bus-bars of which are maintained at 125 volts. Determine: (*a*) the voltage at each lamp; (*b*) the transmission efficiency of the system.

733. Power is to be transmitted from 250-volt bus-bars to a motor 800 ft. distant delivering 50 hp., and to a motor 600 ft. farther away delivering 20 hp. The efficiency at rated load of the 50-hp. motor is 89 per cent and that of the 20-hp. motor 86 per cent. With each motor operating at rated load the voltage at the 20-hp. motor shall be not less than 225 volts and that at the 50-hp. motor not less than 235 volts. Determine: (*a*) the resistance of the necessary feeders; (*b*) the nearest size, A.W.G. or circular mils; (*c*) the weight of each feeder; (*d*) the efficiency of transmission.

734. Determine: (*a*) the size of uniform feeder which will have the same weight as the two feeders in Prob. 733; (*b*) the voltage at each load with this feeder; (*c*) the efficiency of transmission. Which system uses the copper more effectively? (Use the values of resistance in Prob. 733 and compute the size wire accordingly, even if it does not equal a standard-gage size.) Assume the same load currents as in Prob. 733.

735. It is desired to operate a group of sixty 115-volt lamps, located 500 ft. from a 240-volt source of supply. Two No. 6 A.W.G. wires, having a resistance of 0.4 ohm per 1,000 ft., are used to transmit the power. Assume that each lamp takes 1.0 amp. and that the lamps are connected in series-parallel with two in series. Determine: (*a*) the voltage at the lamps; (*b*) the power loss in the line; (*c*) the efficiency of transmission. (*d*) If rubber-insulated wire is used, verify its carrying capacity (see Appendix F, p. 594).

736. Determine the size of wire required to supply the lamps in Prob. 735 with the lamps all in parallel and taking their power from a 120-volt source. The distance and power loss remain unchanged. What is the ratio of the weight of copper to that in Prob. 735? Verify the carrying capacity by Appendix F (p. 594).

737. Find the weight of copper in Prob. 735, using: (*a*) a neutral with one-fourth the cross-section of that of the outer wires; (*b*) one-half the cross-section.

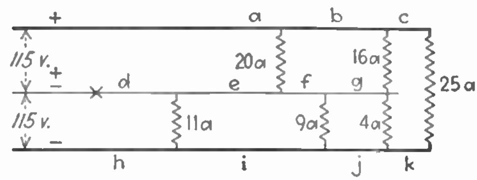


FIG. 738*A*.

738. In Fig. 738*A* is shown an Edison three-wire system with various loads. Indicate the current and its direction at each of the points *a* to *k* inclusive.

739. If the neutral is cut at point *X* (Fig. 738*A*), determine the voltages across the two sides of the system, assuming that the load resistances do not change. Neglect the drop in the mains themselves.

740. Determine the voltages across loads *A* and *B* (Fig. 740*A*), if loads *A* and *B* are each 60 amp.

741. Determine the voltages across loads *A* and *B* (Fig. 740*A*), if load *A* is 50 amp. and load *B* is 70 amp.

742. Determine the voltages across loads *A* and *B* (Fig. 740*A*), if load *A* is 60 amp. and load *B* is 30 amp.

743. In Prob. 742 determine the voltage across each load, if the neutral wire is opened.

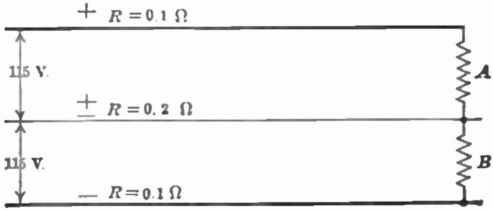


FIG. 740*A*.

744. In Fig. 744*A* is shown the diagram of a typical Edison 240-120-volt, three-wire system with lamp loads *AB* and *CB* connected to neutral and a motor across the outers. With the motor taking 100 amp., load *AB* = 200 amp., and load *CB* = 150 amp., determine the voltages across *AB*, *CB*, and at the motor. The resistance of a circular-mil-foot may be taken as 11 ohms

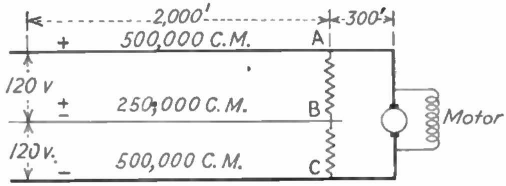


FIG. 744*A*.

745. Repeat Prob. 744 with the motor current 150 amp., the load *AB* = 100 amp., and the load *CB* = 180 amp.

746. Repeat Prob. 744 with the motor connected between the neutral and the negative conductor. Owing to the fact that the voltage is halved the mōtor must now take 200 amp. to develop its former power.

747. In Fig. 747*A* is shown a balancer set consisting of two dynamos and *B* connected between the positive wire and neutral, and the negative wire and neutral of a three-wire system. The efficiency of each machine may be taken as 0.85. With the current I_{ab} = 150 amp., and the current I_{fe} = 100 amp., determine the current in each machine.

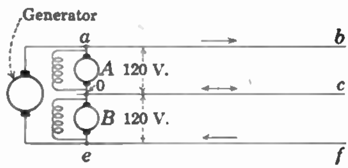


FIG. 747*A*.

748. Repeat Prob. 747 with I_{ab} = 100 amp. and I_{fe} = 200 amp.

749. Repeat Prob. 747 with a load of 160 amp. across the negative side of the system and with no load across the positive side.

750. A direct-current load is located 750 ft. from direct-current bus-bars and is supplied by a two-conductor feeder. When the load current is 400 amp., the voltage at the load is 231.8 volts; when the load current is 250 amp., the voltage at the load is 237.9 volts. Determine: (*a*) the voltage at the bus-bars; (*b*) the resistance of the feeder; (*c*) the circular milage of the feeder. The resistance of a circular-mil-foot may be taken as 10.8 ohms.

751. At the end of an 800,000-cir.-mil, 1,600-ft. feeder there is connected a load of 150 kw., and the bus-bar voltage at the sending end of the feeder is 240 volts. The resistance of a circular-mil-foot may be taken as 10.8 ohms. Determine: (*a*) the resistance of the feeder; (*b*) the current; (*c*) the voltage at the load; (*d*) the efficiency of transmission; (*e*) the theoretical maximum power which the feeder could deliver to the load; (*f*) the theoretical maximum current. Discuss the alternate solution of (*b*).

752. In Prob. 751, but with the load equal to 200 kw.: (*a*) repeat (*b*); (*b*) repeat (*c*); (*c*) repeat (*d*).

753. In Prob. 751, determine (*e*) and (*f*) with 120 volts at the sending end.

754. An electric car on a suburban line, in climbing a grade 8 miles from the power house, requires 60 kw. The bus-bars at the power house are maintained at 600 volts. The No. 000 hard-drawn trolley wire (= 168,000 cir. mils) is paralleled by a 400,000-cir-mil, single-conductor feeder. The total resistance of the rail and ground return is 0.5 ohm. The resistance of a circular-mil-foot, for both trolley and feeder, may be taken as 11 ohms. Determine: (*a*) the current; (*b*) the voltage at the car; (*c*) the power delivered to the line at the station; (*d*) the power lost in overhead system and ground return; (*e*) the efficiency of transmission.

755. Repeat Prob. 754, attempting to obtain 80 kw. at the car.

756. What is the maximum power which the car, Probs. 754 and 755, can take? If the combined resistance of the motors is 0.6 ohm, what is the maximum current which they can take? Plot a curve with power at the car as ordinate and current as abscissa (see Fig. 429, p. 575).

757. Repeat Prob. 754 with 750 volts at the bus-bars.

758. Repeat Prob. 755 with 750 volts at the bus-bars.

759. Repeat Prob. 756 with 750 volts at the bus-bars.

760. A 0000 hard-drawn copper trolley wire runs from 600-volt bus-bars to a station 5 miles out. For 4 miles it is paralleled by a 300,000-cir.-mil feeder which is tapped at every quarter-mile (see Fig. 430(*b*), p. 579). The resistance of the 0000 wire is 0.265 ohm per mile and the resistance of the 300,000 cir.-mil feeder is 0.190 ohm per mile. The resistance of the track and ground return is 0.05 ohm per mile. Determine: (*a*) the voltage at a car 4 miles out and taking 70 amp.; (*b*) the voltage at the end of the line.

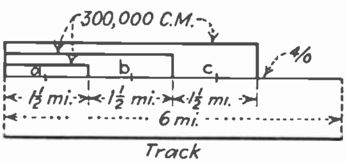


FIG. 762*A*.

761. (*a*) Determine the voltage at the car in Prob. 760 when the car is 4.5 miles from the power station and taking the same current; (*b*) when the car is 2.5 miles from the station and taking 60 amp.

762. Figure 762*A* shows a 6-mile length of hard-drawn 0000 copper trolley wire (211,000 cir. mils). This is fed by three 300,000-cir.-mil multiple feeders, feeding at points 1½ miles apart. Determine the equivalent resistance of the trolley and feeders to the end of the line. The resistance of a circular-mil-foot may be taken as 11 ohms.

763. Determine the voltage at a car (Fig. 762*A*) when it is at the end of the line and taking 110 amp. The station voltage is 600 volts and the ground and track resistance is 0.05 ohm per mile.

764. Determine the voltage at the car of Prob. 762 when the car is 2.5 miles from the station and taking 100 amp.

765. Repeat Probs. 763 and 764 for a sectionalized trolley, the insulated sections being at *a*, *b*, *c* (Fig. 762*A*). Which system of feeding is the more economical of copper?

766. The peak load of a central station is 8,200 kw. The station output is 68,700 kw.-hr. over a 24-hr. period. What is its daily load factor?

767. A storage battery helps the station of Prob. 766 by taking 1,400 kw. off the peak, it being necessary to use this battery for 1½ hr. If the battery efficiency is 80 per cent and it is charged off peak, what is the new load factor of the station?

768. In Probs. 766 and 767, determine the daily load factor with and without the battery, when the peak load is 8,800 kw. and the station output is 82,400 kw-hr. over a 24-hr. period. The battery delivers energy and is charged according to Prob. 767.

769. A storage battery consists of 72 cells conneeted in series, the e.m.f. and resistance of each cell being 2.0 volts and 0.0025 ohm. It is desired that this battery discharge at the 40-amp. rate into 126-volt bus-bars. Determine: (*a*) the necessary series resistance; (*b*) the power developed within the battery; (*c*) the power lost in the battery resistance; (*d*) the power lost in the series resistance.

770. Determine the discharge current of the battery of Prob. 769 when the e.m.f. per cell of the battery drops to 1.98 volts, the other factors remaining unchanged. To what value must the series resistance be adjusted to bring the discharge rate back to 40 amp.?

771. If counter-e.m.f. cells, each having an e.m.f. of 2.16 volts, are used in Prob. 769, how many would be necessary? Neglect the internal resistance of the counter-e.m.f. cells.

772. If end-cell control were used in Prob. 769, how many cells would be cut off the end of the battery?

773. A series-arc generator supplies 54 series-connected, 510-watt, 6.6-amp. magnetite arcs over a No. 6 A.W.G. cable, the resistance of which is 0.403 ohm per 1,000 ft. The length of the arc circuit is 9.6 miles. Determine the terminal voltage of the generator and the efficiency of transmission.

774. Repeat Prob. 773 for a 72-lamp circuit of which the length is 12 miles.

INDEX

C

H

I

J

K

L

M

N

ANSWERS TO PROBLEMS
A COURSE IN ELECTRICAL ENGINEERING
BY
CHESTER L. DAWES
VOLUME I. DIRECT CURRENTS
THIRD EDITION

CHAPTER I
Resistance

1. 24 ohms
2. 48 ohms
3. 0.4 ohm
4. 0.679 ohm
10.37 ohms
5. 0.1491 ohm
6. 11.49 microhms (cm.-cube)
4.52×10^{-6} ohms (in.-cube)
7. 2,300 microhms
12,510 microhms
0.00409 microhm
8. 23.3 in.
9. 0.001019 ohm
10. 166.6 microhms
172.8 lb.
78.4 kg.
11. 267 microhms
51.9 lb.
23.5 kg.
12. 2.08 sq. mm.
1.63 mm.
13. 2.41 sq. in.
0.488 to 1
14. 38.9 m.
15. 0.0701 ohm
16. 0.273 ohm
17. 2.33 ohms
18. 7.34 ohms
19. 4.08 ohms
20. (*a*) 60.3 ft.
(*b*) 0.81 ohm
21. 429 m.
5.65 ohms
22. 2,730 ohms
23. 0.0206 sq. in.
0.504 ohm
24. 10,200 mhos
98.1 microhms
77.3 lb.
35.1 kg.
25. 575,000 mhos-cm.-cube
1,460,000 mhos-in.-cube
26. 16,590 mhos
115.4 kg.
254 lb.
27. 1.843 sq. in.
2.02 to 1
28. 1.243 mhos
29. 307,000 mhos-cm.-cube
0.529 to 1
30. 10,900 megohms
31. 5.16ρ ohms
32. 0.597×10^{14} ohms-in.-cube
1.517×10^{14} ohms-cm.-cube
33. (*a*) 891 lb.-mile-ohm
b) 1.756 microhms-cm.-cube
Yes
34. 49.79 microhms
35. 71.0 ohms
36. 29.1 ohms
37. 21.16ρ ohms
38. 7.38 ohms
39. (*a*) 0.70 mho
(*b*) 1.429 ohms
(*c*) 17.5 ohms
(*d*) 0.0571 mho
40. 285 ohms
41. 38.2 ohms
42. (*a*) 533 ohms
(*b*) 645 ohms
(*c*) 292 ohms
43. (*a*) 40.5 ohms
(*b*) 0.0833 mho
0.0900 mho
0.0575 mho
(*c*) 0.0247 mho
44. 1,149 ohms
45. 845 ohms
46. 651 ohms
47. 800 ohms
48. 81.7 ohms

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49. (*a*) 0.834 ohm
(*b*) 2.10 ohms
50. 1.70 ohms
51. {0.0491 sq. in.
49,100 sq. mils
62,500 cir. mils}
{0.1964 sq. in.
196,400 sq. mils
250,000 cir. mils}
{0.442 sq. in.
442,000 sq. mils
562,500 cir. mils}
{0.785 sq. in.
785,000 sq. mils
1,000,000 cir. mils}
52. 3,260 cir. mils
0.325 in.
53. 410 mils
144 mils
22.6 mils
5.0 mils
54. 66,400 cir. mils
55. 168,000 cir. mils
56. 0.044 ohm
(Change 8,000,000 cir. mils to 800,000 cir. mils in statement of Prob. 56)
57. 3,380 ohms
58. 1.062 ohms
59. 0.1244 ohm
0.399 to 1
60. 41,100 cir. mils
91,900 cir. mils
61. (*a*) 53.6 ohms
(*b*) 51.7 ohms
62. (*a*) 14.02 ohms
(*b*) 16.53 ohms
63. 1.182 ohms
1.471 ohms
64. 0.0409 ohm
0.0478 ohm
65. (*a*) 50.3°C.
(*b*) 29.3°C.
66. 0.0611 ohm
0.0772 ohm
67. 24.8°C.
68. 11.25 ohms
69. 0.00401
0.00375
70. (*a*) −236.4°C.
(*b*) 0.00423
(*c*) 0.00378
71. {2 ohms
5,000 cir. mils}
{4 ohms
2,500 cir. mils}
{0.5 ohm
20,000 cir. mils}
72. {71 mils
15.8 lb.}
{50 mils
7.9 lb.}
{141 mils
63 lb.}
73. 126,000 cir. mils
355 mils
0.144 ohm
720 lb.
74. 606 ohms
0.076 lb.
20 cir. mils
4.5 mils
75. 60.6 ohms
0.75 lb.
198 cir. mils
14.1 mils
76. 1.32 ohms
16,900 lb.
200,000 cir. mils
447 mils

CHAPTER II

Ohm's Law and the Electric Circuit

77. 0.478 absamp.
4.78 amp.
78. 1.52 amp.
79. 0.442 amp.
80. 0.324 amp.
0.440 amp.
81. 5.98 amp.
0.545 amp.
82. 6.86 volts
83. 68.2 volts
23.9 volts
84. (*a*) 600 volts
(*b*) 525 volts
(*c*) 75 volts
85. (*a*) 13.6 ohms
(*b*) 68 volts
86. 25.0 ohms
87. 20.2 volts
88. (*a*) 81.7 ohms
(*b*) 18.3 ohms
(*c*) 100.0 ohms
89. 1.20 ohms (*A*)
0.80 ohm (*B*)
1.50 ohms (*C*)
90. (*a*) 17.7 ohms
(*b*) 209 volts
21 volts
91. (*a*) 35.8 ohms
(*b*) 3.21 amp.
(*c*) 30.2 volts
27.6 volts
24.1 volts
33.1 volts
92. (*a*) 238 volts
(*b*) 62.3 volts
62.3 volts
49.4 volts
49.4 volts
(*c*) 10.4 volts
(*d*) 4.2 volts
93. 4,980 volts
94. 24.0 ohms
95. (*a*) 35.3 ohms
(*b*) 7.2 ohms
28.1 ohms
96. (*a*) 0.1658 ohm
(*b*) 2.81 ohms
97. 0.0503 ohm
98. (*a*) 4.25 microhms
(*b*) 170 millivolts
99. 0.00667 ohm
0.01333 ohm
100. (*a*) 6000 ohms
(*b*) 16,000 ohms

(*c*) 18.75 milliamp.
11.25 milliamp.
7.50 milliamp.
(*d*) 112.5 volts
101. (*a*) 6,670 ohms
(*b*) 16,670 ohms
(*c*) 18.0 milliamp.
12.0 milliamp.
6.0 milliamp.
(*d*) 120 volts
102. 120.8 volts
103. 7.34 ohms
104. (*a*) 17.91 amp.
(*b*) 115.7 volts
105. (*a*) 0.616 ohm
(*b*) 16 volts
104 volts
106. 22 ohms
107. (*a*) 25 ohms
(*b*) 240 volts
(*c*) 4.0 amp.
3.0 amp.
108. (*a*) 19.49 ohms
(*b*) 5.72 ohms
(*c*) 1.111 amp.
1.333 amp.
2.053 amp.
109. (*a*) 3.22 amp.
1.61 amp.
1.59 amp.
(*b*) 48.8 ohms
110. (*a*) 300 amp.
(*b*) 100 amp.
(*c*) 2.51 volts
111. (*a*) 17.87 amp.
13.40 amp.
10.73 amp.
(*b*) 429 volts
112. 1.429 ohms
113. 7.20 ohms
114. (*a*) 21.6 ohms
(*b*) 4.63 amp.
(*c*) 55.6 volts
(*d*) 44.4 volts
(*e*) 4.63 amp.
2.78 amp.
1.85 amp.
115. (*a*) 51.2 ohms
(*b*) 2.93 amp.
(*c*) 55.0 volts
95.0 volts
(*d*) 1.10 amp.
1.83 amp.
1.19 amp.
0.95 amp.
0.79 amp.
116. 18.72 amp.
117. (*a*) 11.11 ohms
(*b*) 45.0 volts
25.0 volts
30.0 volts
30.0 volts
55.0 volts
(*c*) 9.00 amp.
6.25 amp.
3.75 amp.
2.50 amp.
2.75 amp.
118. (*a*) 9.68 ohms
(*b*) 51.6 volts
22.0 volts
26.4 volts
26.4 volts
48.4 volts
(*c*) 10.34 amp.
5.50 amp.
3.30 amp.
2.20 amp.
4.84 amp.
119. (*a*) 1,600 amp
(*b*) 440 kw.
590 hp.
120. (*a*) 28.7 hp.
(*b*) 21.4 kw.
(*c*) 93.2 amp.
121. (*a*) 575 watts
(*b*) 2.68 per cent
(*c*) 425 watts
(*d*) 150 watts
122. 60.5 watts
100.6 watts
113.0 watts
90.2 watts
75.0 watts
439.3 watts
123. (*a*) 28.0 volts
(*b*) 272.6 volts
(*c*) 392 watts
320 watts
800 watts
1,084 watts
678 watts
542 watts
(*d*) 3,816 watts
124. (*a*) 24 watts
12 watts
(*b*) 96 watts
48 watts
125. 32.4 watts
90.0 watts
126. 288 watts
235 watts
588 watts
797 watts
498 watts
398 watts
127. 60 watts
200 watts
128. (*a*) 0.413 watt
(*b*) 0.823 watt
(*c*) 1.286 watts
129. (*a*) 450 watts
(*b*) 28.8 ohms
130. (*a*) 0.522 amp.
220.4 ohms
(*b*) 0.870 amp.
132.2 ohms
131. (*a*) 32.0 kw.
(*b*) 145.3 amp.
(*c*) $109.45
132. $198.35
133. 51.5 cts.
134. 1.149 cts.
135. 1.15 hr.
136. 80.2°C.
137. $\left\{\begin{array}{l}\text{0.698 amp.}\\ \text{2.12 per cent}\end{array}\right.$
$\left\{\begin{array}{l}\text{1.363 amp.}\\ \text{7.57 per cent}\end{array}\right.$

{2.093 amp.
{16.1 per cent
138. {0.566 amp.
{1.397 per cent
{1.111 amp.
{5.11 per cent
{1.698 amp.
{10.96 per cent
139. (*a*) 226.4 volts
(*b*) 95.9 per cent
140. (*a*) 236.8 volts
(*b*) 97.1 per cent
(*c*) 83.5 per cent
141. (*a*) 220.8 volts
92.0 per cent
(*b*) 225.6 volts
94.0 per cent
142. (*a*) 533.0 volts
(*b*) 533.0 volts
(*c*) 88.8 per cent
143. (*a*) 460.6 volts
(*b*) 76.8 per cent
144. (*a*) 0.0543 ohm
(*b*) 0.229 ohm
(*c*) 89.5 per cent
145. (*a*) 0.022 ohm
(*b*) 884,000 cir. mils
(*c*) 91.7 per cent
(*d*) 9,710 lb.
146. 235.6 volts
229.5 volts
223.3 volts
147. (*a*) 0.0055 ohm
(*b*) 3,540,000 cir. mils
(*c*) 91.7 per cent
(*d*) 38,840 lb.
148. (*a*) 0.388 ohm
(*b*) 0.190 ohm
(*c*) 33.72 kw.
21.24 kw.
60.0 kw.
(*d*) 3.80 kw.
1.24 kw.
(*e*) 91.6 per cent
(*f*) No. 8 A.W.G.
No. 2 A.W.G.
149. (*a*) 549.4 volts
(*b*) 525.0 volts
(*c*) 32.96 kw.
21.0 kw.
60.0 kw.
(*d*) 5.06 kw.
0.976 kw.
(*e*) 89.9 per cent
150. (*a*) 236.6 volts
(*b*) 251.9 volts
(*c*) 303.6 kw.
(*d*) 23.9 kw.
(*e*) 92.7 per cent
(*f*) 244.3 volts
234.0 volts
151. 559.1 volts
524.5 volts
90.0 per cent
152. (*a*) 24 volts
(*b*) 90.4 per cent
153. 400,000 cir. mils
1,333 cir. mils per ampere
92.8 per cent
154. (*a*) 400,000 cir. mils
(*b*) 36 volts
(*c*) 14.4 kw.
(*d*) 85 per cent
(*e*) 1,200,000 cir. mils
(*f*) 4.8 kw.
(*g*) 95 per cent
155. (*a*) No. 3 A.W.G.
(*b*) 441.6 volts
(*c*) 7.92 kw.
(*d*) 73.6 per cent
(*e*) No. 000 A.W.G.
(*f*) 160,000 cir. mils
(*g*) 2.48 kw.
(*h*) 91.8 per cent
156. (*a*) 316.8 volts
(*b*) 193.6 kw.
(*c*) 6.340 cir. mils per ampere
(*d*) 4.82 kw.
157. (*a*) No. 000 A.W.G.
(*b*) No. 0000 A.W.G.
(*c*) 5.55 kw.
(*d*) 93.2 per cent

CHAPTER III

Battery Electromotive Forces—Kirchhoff's Laws

158. 0.05 ohm
159. 0.0458 ohm
160. (*a*) 0.0122 ohm
(*b*) 5.69 volts
161. 0.909 amp.
162. (*a*) 100 amp.
(*b*) 50 amp.
163. (*a*) 4.65 amp.
(*b*) 6.28 watts
(*c*) 1.08 watts
(*d*) 5.20 watts
164. (*a*) 0.0578 ohm
(*b*) 567 watts
(*c*) 99 watts
(*d*) 468 watts
165. (*a*) 0.06 ohm
(*b*) 12 watts
(*c*) 6 watts
(*d*) 6 watts
166. 813 watts
(*a*) 0.0122 ohm
(*b*) 813 watts
167. (*a*) 500 watts
(*b*) 167 amp.
(*c*) 3.00 volts
168. (*a*) 6.52 volts
(*b*) 117.4 watts
(*c*) 3.96 watts
(*d*) 113.4 watts
169. (*a*) 108 volts
(*b*) 10.8 kw.
(*c*) 0.6 kw.
(*d*) 10,200 Joules each sec.
170. (*a*) 256.3 volts

(b) 15.38 kw.
0.51 kw.
(c) 236.6 volts
171. (a) 129.8 volts
(b) 1.38 ohms
(c) 72.9 amp.
172. (a) 0.1553 amp.
(b) 1.338 volts
1.286 volts
1.260 volts
(c) 0.209 watt
0.202 watt
0.199 watt
(d) 0.00193 watt
0.00217 watt
0.00314 watt
(e) 0.603 watt
173. (a) 13.1 amp.
(b) 0.302 volt
0.121 volt
0.423 volt
(c) 17.69 watts
17.03 watts
16.77 watts
(d) 13.73 watts
15.45 watts
22.31 watts
174. (a) 8.13 volts
(b) 1.975 volts
2.03 volts
2.07 volts
2.055 volts
(c) 50.0 watts
51.25 watts
52.5 watts
52.0 watts
(d) 0.625 watt
0.500 watt
0.750 watt
0.625 watt
(e) 0.325 ohm
(f) 203.3 watts
175. (a) 300 amp.
(b) 73.1 kw.
(c) 243.6 volts
(d) 4.32 kw.
176. (a) 3.00 amp.
(b) 24.0 watts
12.0 watts
−9.0 watts
(c) 23.1 watts
11.28 watts
−9.63 watts
(d) 0.90 watt
0.72 watt
0.63 watt
(e) 7.70 volts
3.76 volts
3.21 volts
(f) 8.25 volts
177. (a) 0.300 amp.
(b) 0.0081 watt
0.0126 watt
0.0108 watt
0.0117 watt
0.0108 watt
(c) 1.373 volts
1.228 volts
1.344 volts
1.281 volts
1.194 volts
(d) 6.42 volts
178. (a) 0.1745 amp.
(b) 0.00274 watt
0.00426 watt
0.00366 watt
0.00396 watt
0.00366 watt
(c) 1.384 volts
1.246 volts
1.401 volts
1.297 volts
1.209 volts
(d) 3.735 volts
179. (a) 0.600 amp.
(b) 1.188 volts
(c) 0.0018 watt
(d) 0.7128 watt
180. (a) 18 watts
(b) 0.6 volt
(c) 0.02 ohm
181. (a) 0.500 amp.
(b) 11.95 volts
(c) 0.00417 watt
182. (a) 1.30 volts
0.0224 ohm
(b) 2.089 amp.
(c) 0.584 amp.
0.467 amp.
0.519 amp.
0.519 amp.
183. (a) 18.89 watts
(b) 0.0224 ohm
184. (a) 2.22 amp.
1.78 amp.
(b) 16.22 volts
(c) 4.06 ohms
(d) 18.0 volts
0.444 ohm
185. (a) 2.10 volts
0.000984 ohm
(b) 152.5 amp.
(c) 30.0 amp.
25.0 amp.
37.5 amp.
60.0 amp.
186. (a) 7.50 volts
(b) 0.15 ohm
(c) 3.49 amp.
(d) 6.98 volts
187. (a) 5.00 volts
(b) 0.0667 ohm
(c) 2.42 amp.
(d) 4.84 volts
188. 12 in series
2 in parallel
93.75 watts
93.75 watts
189. (a) 3 in series
4 in parallel
(b) 6.15 volts
(c) 0.0615 ohm
(d) 97.6 per cent
190. 12 in series
295 amp.
71.9 per cent
191. (a) 40 in series
(b) 90.9 per cent
(c) 80.0 per cent
(d) 40 in series
192. (a) 6.173 volts

0.00255 ohm
(*b*) 30.5 amp.
(*c*) 29.27 amp.
1.23 amp.
(*d*) 6.095 volts
193. (*a*) 24.6 volts
0.24 ohm
(*b*) 4.00 amp.
1.00 amp.
(*c*) 23.4 volts
(*d*) 4.68 ohms
194. (*a*) 120.8 volts
0.024 ohm
(*b*) 118.0 amp.
(*c*) 118.0 volts
(*d*) 50.8 amp.
67.2 amp.
(*e*) 6.096 kw.
8.198 kw.
(*f*) 5.994 kw.
7.930 kw.
195. (*a*) 2.50 amp.
(*b*) 1.50 amp.
1.00 amp.
(*c*) 4.00 amp.
1.00 amp.
196. (*a*) 4.00 amp.
(*b*) 2.25 amp.
3.75 amp.
(*c*) 6.25 amp.
3.75 amp.
(*d*) 0.4875 ohm
197. (*a*) 40.0 amp.
60.0 amp.
(*b*) 118.4 volts
(*c*) 1.184 ohms
198. (*a*) 4.16 amp.
5.84 amp.
(*b*) 0.433 watt
1.192 watts
(*c*) 8.30 watts
11.66 watts
(*d*) 0.1996 ohm
199. (*a*) 10.0 amp.
(*b*) +10.0 amp.
−10.0 amp.
(*c*) 84.0 amp.
196.0 amp.
(*d*) 94.0 amp.
(*e*) 186.0 amp.
(*f*) 37.84 volts
(*g*) 0.1351 ohm
200. (*a*) +1.00 amp.
−1.00 amp.
(*b*) 3.00 amp.
2.00 amp.
(*c*) 4.00 amp.
1.00 amp.
(*d*) 23.4 volts
(*e*) 4.68 ohms
201. (*a*) 7.0 amp.
5.0 amp.
(*b*) 4.5 volts
(*c*) 56 watts
30 watts
(*d*) 24.5 watts
7.5 watts
202. (*a*) 74 amp.
206 amp.
(*b*) 37.64 volts
(*c*) 766 watts
2,540 watts
(*d*) 0.1344 ohm
203. (*a*) 1.463 amp.
(*b*) 1.119 amp.
0.344 amp.
(*c*) 3.657 volts
204. (*a*) 0.896 amp.
0.075 amp.
0.971 amp.
(*b*) 3.926 volts
(*c*) 1.985 volts
205. (*a*) 2.588 amp.
1.653 amp.
0.935 amp.
(*b*) 6.447 volts
5.339 volts
1.533 volts
206. (*a*) 2.098 amp.
1.942 amp.
0.156 amp.
(*b*) 6.741 volts
5.224 volts
2.078 volts
207. (*a*) 0.840 amp.
0.513 amp.
(*b*) 8.50 ohms
208. −2.232 amp.
1.022 amp.
1.210 amp.
209. −2.357 amp.
−0.033 amp.
2.390 amp.
210. −1.50 amp.
−0.75 amp.
2.25 amp.
1.25 amp.
1.00 amp.
0.250 amp.
211. 584.3 volts
571.2 volts
212. 579.5 volts
566.5 volts
213. (*a*) 564.4 volts
(*b*) 570.9 volts
214. (*a*) 564.1 volts
(*b*) 570.7 volts
215. (*a*) 548.8 volts
(*b*) 558.9 volts
216. (*a*) 549.7 volts
(*b*) 554.7 volts
217. (*a*) 550.5 volts
(*b*) 551.4 volts
218. (*a*) 385 amp.
215 amp.
(*b*) 231 kw.
124.7 kw.
(*c*) 341.6 kw.
219. 220.8 volts
225.5 volts
224.5 volts
220. 40.8 ohms (*A-B*)
68.0 ohms (*B-C*)
45.3 ohms (*C-A*)
221. 23.6 ohms
22.8 ohms
21.4 ohms
0.213 amp.
222. (*a*) 95.9 ohms
(*b*) 0.247 amp.
0.281 amp.

223. 642 ohms
8.44 milliamp. ($I_{a'b'}$)
22.71 milliamp. (I_{ao})
5.43 milliamp. (I_{cb})
17.28 milliamp. (I_{oc})
13.87 milliamp. ($I_{dd'}$)
224. 0.416 amp.
225. 2.27 milliamp.
226. 361.7 ohms
2.94 milliamp.

CHAPTER IV

Primary and Secondary Batteries

227. (*a*) 4.32 amp.
(*b*) 1.08 volts
(*c*) 8.64 amp.
(*d*) 2.33 watts
228. (*a*) None
Same e.m.f.
(*b*) 10.4 amp.
(*c*) 3.61 watts
229. 0.09 ohm
0.22 volt
0.20 ohm
230. 12 cells
In series
11.6 ohms
0.265 amp.
231. (*a*) 13 cells
Series
(*b*) 1.56 ohms
232. (*a*) 40 cells
(*b*) Series
233. (*a*) 6 cells
(*b*) 2.5 amp.
(*c*) 2.4 ohms
234. (*a*) 0.175 ohm
(*b*) Poor condition
235. (*a*) 0.81 volt
(*b*) 6.46 amp.
(*c*) −4.91 volts
236. 1.115 amp.
3.23 volts
237. (*a*) 0.182 milliamp.
To left
(*b*) 0.0911 milliamp.
To right
(*c*) 0.02000 amp.
(*d*) 0.1000 volt
238. (*a*) 0.0911 milliamp.
(*b*) 78 per cent
239. 0.863 volt
240. (*a*) 4.3 l.
(*b*) 5.12 l.
(*c*) 6.14 kg.
241. (*a*) 18.4 kg.
(*b*) 52.2 l.
(*c*) 13.79 gal.
(*d*) 62.6 kg.
242. (*a*) 0.192 l.
(*b*) 2.712 kg.
2.904 kg.
243. (*a*) 320 amp.-hr.
(*b*) 320 amp.-hr.
244. 260 amp.-hr.
245. 352 amp.-hr.
70 amp.
300 amp.-hr.
100 amp.
246. 372 amp.-hr.
72 amp.
332 amp.-hr.
111 amp.
(*a*) 2,500 amp.
(*b*) 4,970 amp.
247. (*a*) 6 per cent
(*b*) 7.05 ohms
(*c*) 6.14 watts
248. (*a*) 12 per cent
(*b*) 6.60 ohms
(*c*) 12.28 watts
249. (*a*) 30.6 per cent
(*b*) 34.5 per cent
(*c*) 1.88 ohms
250. 75.5 amp.
251. (*a*) 30.6 per cent
(*b*) 33.6 per cent
(*c*) 2.55 ohms
252. (*a*) 292 volts
(*b*) $6.54
253. (*a*) 37 volts
60 amp.
(*b*) 3.75 kw.
(*c*) 25.2 per cent
254. (*a*) 80 volts
60 amp.
(*b*) 7.32 kw.
(*c*) 26.7 per cent
(*d*) $4.92
255. (*a*) 139 volts
8.35 kw.
(*b*) 147 volts
8.81 kw.
256. 9.98 kw.
9.12 kw.
Discharged
257. 320 amp.-hr.
739 watt-hr.
258. 630 watt-hr.
259. 458 watt-hr.
260. (*a*) 75 amp.
(*b*) 50 amp.
(*c*) 2.4 volts
Constant voltage
Yes
261. (*a*) 126 tons
(*b*) 220 tons
262. (*a*) 2.6 kw.
(*b*) 20.8 kw.-hr.
(*c*) 1,370 lb.
(*d*) 2,050 lb.
(*e*) 0.416 kw.-hr. per mile
263. 94.1 per cent
79.2 per cent
264. 94.1 per cent
80.3 per cent
265. 78.1 per cent
64.5 per cent
266. 94.3 per cent
66.6 per cent

267. (Add to problem statement: Battery charged for 6 hr. at 45-amp. rate.)
(*a*) 27,000 watt-hr.
(*b*) $0.95
(*c*) 18,300 watt-hr.
(*d*) 92.5 per cent
(*e*) 67.7 per cent
268. 13.14 g.
269. 14.23 g.
270. (*a*) 1.186 g.
(*b*) 1.897 kg.
(*c*) 19.2 kw.-hr.

CHAPTER V

Electrical Measurements and Electrical Instruments

271. 71.2 ohms
6.47 ohms
0.641 ohm
272. 133.3 ohms
12.12 ohms
1.202 ohms
1.2×10^{-8} amp.
1.2×10^{-7} amp.
1.2×10^{-6} amp.
273. 640 ohms
64 ohms
6.4 ohms
0.64 ohm
274. (*a*) 1.428×10^{-8} amp.
(*b*) 1.428×10^{-7} amp.
(*c*) 1 to 10
275. (*a*) 2.69×10^{-4} amp.
(*b*) 2.69×10^{-5} amp.
(*c*) 2.69×10^{-6} amp.
(*d*) 0.893
276. (*a*) 3,333 ohms
(*b*) 1.54×10^{-6} amp.
(*c*) 1.54×10^{-10} amp.
(*d*) 110 to 1
277. (*a*) 0.000600 ohm
(*b*) 0.000300 ohm
(*c*) 1 to 16,670
1 to 33,300
278. (*a*) 0.000500 ohm
(*b*) 0.00961 amp.
Negligible
279. 0.000333 ohm
0.0000833 ohm
7.5 watts
30 watts
280. (*a*) 0.0090 amp.
(*b*) 29.991 amp.
(*c*) 67.5 millivolt
281. 0.000909 ohm
282. (*a*) 3.38 ohms
(*b*) 0.00100 ohm
(*c*) 0.000400 ohm
283. (*a*) 54 amp.
(*b*) 72 amp.
284. 300 amp.
285. 56 amp.
286. (*a*) 514 ohms
(*b*) 17,200 ohms
(*c*) 115 ohms per volt
287. (*a*) 15,660 ohms
(*b*) 31,860 ohms
288. (*a*) 50 ohms
(*b*) R_{AB} greater than R_{AO}
(*c*) 14,400 ohms
289. 149.7 volts
290. (*a*) 136,800 ohms
(*b*) Instrument and trolley
291. 161.6 volts
292. 1,725 volts
293. 4,940 ohms
294. 27.7 volts
12.3 per cent
295. 147.6 volts
262.4 volts
296. (*a*) 0.000416 ohm
(*b*) 2.77 ohms
297. (*a*) 0.336 ohm
(*b*) 4.064 ohms
(*c*) 103.28 volts
298. (*a*) 0.0802 ohm
(*b*) 2.114 ohms
299. (*a*) 50.7 ohms
(*b*) 3.23 amp.
300. (*a*) 29.2 microhms
(*b*) 1.766 microhms per cm. cube
(*c*) 100.6 per cent
301. 0.1618 ohm
0.0468 ohm
302. 10.63 microhms
303. 293,000 ohms
304. 2.66 megohms
1.838 megohms
305. 12.55 megohms
306. 146,200 ohms
307. (*a*) 1 to 1,000
(*b*) 10 to 1,000
(*c*) 100 to 1,000
(*d*) 1,000 to 100
308. {1,000 to 100; 1,424 ohms}
{1 to 1,000; 2,562 ohms}
{10 to 1,000; 4,571 ohms}
{1,000 to 1,000; 2,479 ohms}
309. (*a*) 3.4 ohms
(*b*) 1 to 1,000
310. (*a*) 626.5 ohms
(*b*) 2.18 milliamp.
1.333 milliamp.
311. (*a*) 673 ohms
(*b*) 2.17 milliamp.
(*A*)

2.175 milliamp. (*B*)
1.095 milliamp. (*P*)
1.10 milliamp. (*X*)
0.0046 milliamp. (Galv.)

312. (*a*) 26.11 ohms
(*b*) 83.5 milliamp. (*A*)
0.780 milliamp. (*B*)
0.710 milliamp. (*P*)
83.6 milliamp. (*X*)
0.067 milliamp. (Galv.)

313. (*a*) 26.14 ohms
(*b*) 83.4 milliamp. (*A*)
0.725 milliamp. (*B*)
0.590 milliamp. (*P*)
83.5 milliamp. (*X*)
0.135 milliamp. (Galv.)

314. (*a*) 39.7 ohms
(*b*) 79.9 cm.

315. (*a*) 441 ohms
(*b*) 31.2 cm.

316. (*a*) 0.0001452 ohm
(*b*) 1.739 microhms per cm. cube

317. (*a*) 2.12 microhms per cm. cube
(*b*) 12.69 ohms cir.-mil-ft.

318. 1,082 ft.

319. 2,490 ft.

320. 1,299 ft.

321. (*a*) 4.416 ohms
(*b*) 1,698 ft.

322. 7,200 ft.

323. Fault in *b* 4,210 ft.

324. (*a*) 1,222 megohms
(*b*) 2,200 megohms per 1,000 ft.
(*c*) 417 megohms per mile

325. (*a*) 3,200 microamp.
(*b*) 2.95 microamp.
(*c*) 2.42 microamp.
(*d*) 2.62 microamp.
(*e*) 1,222 megohms

326. (*a*) 36.2 megohms
(*b*) 290 megohms per 1,000 ft.
(*c*) 2.35×10^7 megohms per in. cube
(*d*) 5.96×10^7 megohms per cm. cube

327. (*a*) 62.9 microamp.
(*b*) 3,000 microamp.
(*c*) 2.10 microamp.
(*d*) 143 megohms

328. 83.3 cm.

329. (*a*) 0.04 amp.
(*b*) 25.0 ohms
(*c*) 18.4 cm. 1.0 contact
(*d*) 0.564 volt

330. 2.058 volts

331. 0.941 volt

332. (*a*) 0.002 amp.
(*b*) 0.700 volt
(*c*) 0.001 amp.
(*d*) 0.060 volt
(*e*) 0.0005 amp
(*f*) 0.006 volt
(*g*) 0.766 volt

333. (*a*) 0.01 amp.
(*b*) 0.005 amp.
(*c*) 0.0025 amp.
(*d*) 58.0 ohms

334. (*a*) 9,000 ohms
(*b*) 1.40999 volts

335. −0.04 volt

336. 1.372 ohms

337. +2.6 volts

338. 23.51 watts 4.25 per cent

339. (*a*) 119.01 volt
(*b*) 23.51 watts
(*c*) 23.52 watts
(*d*) 0.042 per cent

340. (*a*) 0.0310 ohm
(*b*) 0.01 per cent
(*c*) 1.9 per cent

341. (*a*) 101.5 per cent
(*b*) Move magnets
(*c*) 109.7 per cent
(*d*) Move light-load coil

342. (*a*) 97.3 per cent
(*b*) 96.1 per cent

343. (*a*) 97.6 per cent
(*b*) 8.27 kw.
(*c*) 484 kw.

CHAPTER VI

Magnetism and Permanent Magnets

344. Sketch

345. Sketch

346. (*a*) 8,000 dynes
(*b*) Repulsion
(*c*) 8.15 g.

347. 0.967 lb.

348. (*a*) 32,000 dynes
(*b*) 128 dynes
(*c*) 160 dynes

(*d*) 288 dynes
(*e*) 288 oersteds
349. 1,768 dynes on *N*
1,052 dynes on *S*
1,052 dynes on *S*
350. (*a*) 3 dynes
(*b*) 3 dynes
(*c*) 4.24 dynes
(*d*) 4.24 oersteds
351. (*a*) 24 dynes
(*b*) 72 dynes
(*c*) 75.9 dynes
(*d*) 6.32 oersteds
352. 29,400 cm.-dynes
353. 800 units
S at *a*
354. (*a*) 20,000 dynes
(*b*) 20,000 dynes
(*c*) 0 dynes
(*d*) Sketch
(*e*) Reversal
355. (*a*) 20,000 dynes
(*b*) 240,000 cm.-dynes
356. (*a*) 240 oersteds
(*b*) Sketch
(*c*) 272,000 cm.-dynes
357. 555 oersteds
358. (*a*) 3,270 lines
(*b*) 4.06 dynes
(*c*) 65.0 gausses
(*d*) 4.06 gausses
359. (*a*) 5,020 lines
7,530 lines
(*b*) 6.25 gausses
(*c*) 6.00 gausses
(*d*) 12.25 dynes
360. (*a*) 5,020 lines
7,530 lines
(*b*) 6.25 gausses
(*c*) 6.00 gausses
(*d*) 0.25 dyne
361. (*a*) 4.00 gausses
(*b*) 9.38 gausses
(*c*) 13.38 gausses
362. (*a*) 4.00 gausses
(*b*) 9.38 gausses
(*c*) 5.38 gausses
363. (*a*) 625 unit poles
(*b*) 7,850 lines
364. (*a*) 75,400 lines
(*b*) 75,400 lines
(*c*) 47,100 gausses
365. (*a*) 3,750 unit poles
(*b*) 47,100 gausses
(*c*) 23,600 dynes
(*d*) 141,500,000 dynes
144.3 kg.
366. (*a*) 2,000 gausses
(*b*) 1,000 unit poles
367. 10.7 in.
368. 2,030 g.
369. 1.69 g.

CHAPTER VII

Electromagnetism

370. In *A*
In *B*
371. Down *A*
Up *B*
372. (*a*) 3.0 dynes
(*b*) +188.4 ergs
(*c*) 2,400 dyne-cm.
(*d*) No
373. 0.20 g.
374. 0.192 g.
375. (*a*) Down
(*b*) Up
Up
376. 5.55 oersteds
377. (*a*) 10 oersteds
(*b*) 1.22 oersteds
(*c*) 8.78 oersteds
378. (*a*) 0.537 oersted
(*b*) 188.4 dynes
379. (*a*) 1.07 oersteds
(*b*) 9.14 oersteds
(*c*) 6.07 oersteds
380. 150.8 oersteds
381. 8.0 amp.
382. *A*—1
2—4
3—5
6—8
7—*B*
383. *A*—South
B—South
C—North
384. (*a*) 30.16 oersteds
(*b*) 30.16 lines per square centimeter
(*c*) 592 lines
385. (*a*) 30.12 oersteds
(*b*) 15.06 oersteds
386. (*a*) 24.46 oersteds
(*b*) 29.65 oersteds
387. 1,468 g.
388. 939 g.
389. 4.59 kg.
390. 2.94 kg.
391. 176.5 lb.
392. 1,300 lb.
393. 24,900 lines per square inch
394. $b - d$
$c - e$
$f - h$

CHAPTER VIII

The Magnetic Circuit

395. (*a*) Clockwise
(*b*) 5.09 ampere-turns per centimeter
(*c*) 6.38 gilberts per centimeter
(*d*) 1.257
Ampere-turn larger
(*e*) 902
(See correction, Prob. 400)
396. (*a*) 24,000 ampere-turns

(b) 24,000 ampere-turns
(c) 600 watts
300 watts
1 to 2
(d) 30,200 gilberts
(e) 48,000 ampere-turns
1,200 watts

397. (a) 48,000 ampere-turns
(b) 48,000 ampere-turns

398. (a) 7,000 turns
842.5 ohms
(b) 2 to 1
(c) 4 to 1
(d) 60 volts
(e) 60 volts

399. (a) 16,800 ampere-turns
(b) 17,280 ampere-turns
(c) 17,020 ampere-turns
(d) 34,080 ampere-turns

400. (a) 6.40 gausses
(b) 5,760 gausses
(c) 41.3 lines per square inch
(d) 37,060 lines per square inch
(In Prob. 395 change 0.568 to 1.81 and 512 to 1,632)

401. 3.99 cm.

402. (a) 3.87 by 3.87 in.
(b) 4.75 by 3.16 in.

403. (a) 0.005 c.g.s. reluctance unit
(b) 0.00105 c.g.s. reluctance unit
(c) 0.00605 c.g.s. reluctance unit

404. 0.000378 c.g.s. reluctance unit
2,650 c.g.s. permeability units

405. (a) 0.033 c.g.s. reluctance unit
(b) 0.353 c.g.s. reluctance unit
(c) 0.386 c.g.s. reluctance unit
(d) 2.591 c.g.s. permeability units

406. (a) 239,000 maxwells
(b) 26,600 maxwells per square inch
(c) 26,600 maxwells per square inch

407. (a) 869 gilberts
(b) 689 ampere-turns

408. (a) 0.01653 c.g.s. reluctance unit
(b) 0.1326 c.g.s. reluctance unit
(c) 0.1491 c.g.s. reluctance unit
(d) 13,490 maxwells
(e) 11,920 gausses

409. (a) 0.00454 c.g.s. reluctance unit
(b) 0.00309 c.g.s. reluctance unit
(c) 0.0333 c.g.s. reluctance unit
(d) 3,620 gilberts
(e) 88,500 maxwells
(f) 9,830 gausses

410. (a) 0.0353 c.g.s. reluctance unit
(b) 356,000 maxwells

411. (a) 0.00391 c.g.s. reluctance unit
0.001955 c.g.s. reluctance unit
(b) 2.51×10^6 maxwells
(c) 5.02×10^6 maxwells

412. 28,300 maxwells
16,000 gausses

413. 26,500 maxwells
15,000 gausses

414. 23,000 maxwells
13,000 gausses

415. 3.06×10^6 maxwells
15,300 gausses

416. 3.15×10^6 maxwells
15,750 gausses

417. (a) 80 gilberts
(b) 112 gilberts
(c) 320 gilberts
(d) 840 gilberts
(e) 63.6 ampere-turns
89.1 ampere-turns
255 ampere-turns
669 ampere-turns

418. (a) 435 ampere-turns
(b) 1,170 ampere-turns
(c) 2,700 ampere-turns
(d) 546 gilberts
1,469 gilberts
3,390 gilberts

419. (a) 300 ampere-turns
(b) 525 ampere-turns
(c) 1,800 ampere-turns
(d) 2,550 ampere-turns
(e) 377 gilberts
660 gilberts

2,260 gilberts

420. (*a*) 1,592 ampere-turns
(*b*) 2,390 ampere-turns
(*c*) 3,980 ampere-turns
(*d*) 2,000 gilberts
3,000 gilberts
5,000 gilberts

421. (*a*) 1,075 ampere-turns
(*b*) 1,530 ampere-turns
(*c*) 1,910 ampere-turns
(*d*) 1,350 gilberts
1,920 gilberts
2,400 gilberts

422. (*a*) 1,252 ampere-turns
(*b*) 2,190 ampere-turns
(*c*) 2,820 ampere-turns
(*d*) 2.88×10^6 maxwells
5.04×10^6 maxwells
6.47×10^6 maxwells

423. (*a*) 1,378 ampere-turns
(*b*) 2,410 ampere-turns
(*c*) 3,100 ampere-turns
(*d*) 3.36×10^6 maxwells
5.88×10^6 maxwells
7.56×10^6 maxwells

424. 3,170 ampere-turns

425. (*a*) 86 ampere-turns
(*b*) 1,072 ampere-turns
(*c*) 1,158 ampere-turns
(*d*) 7,160 maxwells
(*e*) 0.203 c.g.s. reluctance unit

426. (*a*) 724 ampere-turns
(*b*) 2,020 ampere-turns
(*c*) 2,744 ampere-turns

427. 7.85 amp.

428. 5.30 amp.

429. 5.50 amp.

430. (*a*) 2,580 ampere-turns
(*b*) 1,390 ampere-turns
(*c*) 3,970 ampere-turns
(*d*) 14,000 gausses
(*e*) 2.8×10^6 maxwells
(*f*) 3,970 ampere-turns

431. (*a*) 1,260 ampere-turns
(*b*) 1,091 ampere-turns
(*c*) 2,351 ampere-turns
(*d*) 11,000 gausses
(*e*) 2.2×10^6 maxwells

432. (*a*) 2,410 ampere-turns
(*b*) 7,820 ampere-turns
(*c*) 10,230 ampere-turns

433. (*a*) 1,270 ampere-turns
(*b*) 7,040 ampere-turns
(*c*) 8,310 ampere-turns

434. (*a*) 2,410 ampere-turns
(*b*) 7,820 ampere-turns
(*c*) 10,230 ampere-turns
100,000 maxwells per square inch

435. 1,700 ampere-turns per pole

436. 1,935 ampere-turns per pole

437. 1,455 ampere-turns per pole

438. 1,790 ampere-turns per pole

439. 4,100 ampere-turns per pole

440. 6,490 ampere-turns per pole

441. 4,660 ampere-turns per pole

442. 7,120 ampere-turns per pole

443. (*a*) 2,460 ergs
(*b*) 13.3×10^6 ergs
(*c*) 66.5×10^7 ergs
(*d*) 66.5 watts

444. (*a*) 27,400 ergs
(*b*) 118.3 watts

445. (*a*) 132 watts
(*b*) 74.1 watts
(*c*) 89.0 watts

446. (*a*) 28.8 watts
(*b*) 12.21 watts
(*c*) 17.58 watts

447. (*a*) 0.881 watt per pound
0.164 watt per pound
(*b*) 0.548 watt per pound
0.079 watt per pound

(c) 0.734 watt per pound
0.114 watt per pound

448. (a) 0.981 watt per pound
0.184 watt per pound
(b) 0.611 watt per pound
0.089 watt per pound
(c) 0.818 watt per pound
0.128 watt per pound

449. (a) 6.03 gausses
(b) 58.0 maxwells
(c) 58,000 maxwell-turns

450. (a) 302
(b) 6,190 maxwells
(c) 7,880 gausses

451. 8,620 gausses
Sketch

452. 5,080 maxwells

CHAPTER IX

Self- and Mutual Inductance

453. (a) 4.5×10^6 maxwell-turns
(b) 0.009 maxwell-turns per ampere $\times 10^{-8}$
(c) 0.009 henry

454. (a) 18.0×10^6 maxwell-turns
(b) 0.036 maxwell-turns per ampere $\times 10^{-8}$
(c) 0.036 henry
(d) Proportional to turns squared

455. (a) 31.4 maxwells
(b) 12,560 maxwell-turns
(c) 50,240 maxwell-turns per ampere
(d) 0.000502 henry
0.502 mil-henry

456. (a) 1,660 maxwells
(b) 664,000 maxwell-turns
(c) 2.66×10^6 maxwell-turns per ampere
(d) 0.0266 henry
26.6 mil-henrys

457. (a) 830 maxwells
(b) 166,000 maxwell-turns
(c) 664,000 maxwell-turns per ampere
(d) 0.00664 henry
6.64 mil-henrys

458. (a) 1.5×10^8 maxwell-turns
(b) 0.417 maxwell-turn per ampere $\times 10^{-8}$
(c) 0.417 henry
(d) No

459. (a) 3.0×10^8 maxwell-turns
(b) 1.668 henrys
(c) Proportional to turns squared

460. 1.668 henrys
No variation

461. 40.2 henrys

462. (a) 38.7 henrys
(b) 34.4×10^6 maxwells per second
(c) 1,239 volts
(d) 1,239 volts

463. (a) 4.5×10^6 maxwells
(b) 108×10^8 maxwell-turns
(c) 432×10^8 maxwell-turns
(d) 144 henrys

464. (a) 30×10^6 maxwells per second
(b) 2,880 maxwells per second $\times 10^{-8}$
(c) 2,880 volts
(d) 2,880 volts
(e) 648 joules

465. (a) 648 volts
(b) 108 henrys
(c) 77.8 joules
(d) 388 watts
(e) 2,592 volts
1,552 watts

466. (a) 0.75 sec.
(b) 56,250 joules
(c) 75.0 kw.

467. (a) 0.375 sec.
(b) 14,060 joules
(c) 37.5 kw.

468. (a) 0.52 henry
(b) 0.12 henry
(c) 62.4 volts
(d) 3.6 volts

469. (*a*) 0.56 henry
(*b*) 0.27 henry
(*c*) 0.107 henry
(*d*) 56 volts
(*e*) 27 volts
(*f*) 10.7 volts
(*g*) 7×10^6 maxwells per second

470. (*a*) $10(1 - \epsilon^{-25t})$ amp.

t (sec.)	i (amp.)
0.01	2.21
0.02	3.94
0.03	5.28
0.04	6.32
0.05	7.14
0.06	7.74
0.07	8.26
0.08	8.65

(*b*) 0.04 sec.
(*c*) 6.32 amp.
(*d*) 8.65 amp.

471. (*a*) 3.84 joules
(*b*) 15.36 joules
(*c*) 24.0 joules
(*d*) 250 amp. per second

472. (*a*) $4.8(1 - \epsilon^{-5t})$ amp.
(*b*) 23.0 joules
(*c*) 230 watts
(*d*) 24 amp. per second

473. (*a*) 600 ohms
2.10 henrys
(*b*) 114.3 amp. per second

474. (*a*) $10\epsilon^{-25t}$ amp.
(*b*) 250 amp. per second
(*c*) 3.68 amp.
(*d*) 3.25 joules

475. (*a*) 4.8 amp.
(*b*) 120 volts
(*c*) $4.8\epsilon^{-10t}$ amp.
(*d*) 17.67 amp. per second
(*e*) 7.79 joules

476. (*a*) 4.8 amp.
(*b*) 72 volts
(*c*) $4.8\epsilon^{-8t}$ amp.
(*d*) 17.22 amp. per second
(*e*) 11.62 joules

477. (*a*) 3.792 amp.
(*b*) 7.18 joules
(*c*)

t (sec.)	i (amp.)
0.01	1.09
0.02	1.98
0.03	2.71
0.04	3.31
0.05	3.79
	(Shorted)
0.06	3.10
0.07	2.54
0.08	2.08
0.09	1.70
0.10	1.39
0.11	1.14
0.12	0.94

(*d*) 120 amp. per second
(*e*) 75.8 amp. per second

478. (*a*) 84,000 maxwells
(*b*) 0.70
(*c*) 2.52 volts
(*d*) 4.80 volts

479. (*a*) 1.68 volts
(*b*) 1.80 volts

480. (*a*) 0.084 henry
(*b*) 0.160 henry
(*c*) 0.090 henry
(*d*) 0.084 henry

481. (*a*) 0.072 henry
(*b*) 0.160 henry
(*c*) 0.090 henry
(*d*) 0.072 henry

482. (*a*) 0.1333 amp.
(*b*) 2.40 henrys
(*c*) 1.35 henrys

483. (*a*) 4.80 volts
(*b*) 3.60 volts
(*c*) 1.80 henrys
(*d*) 2.40 henrys
(*e*) 1.35 henrys

484. (*a*) 5.56 amp. per second
(*b*) 0.009 sec.
(*c*) 14.7 joules

485. (*a*) 7.35 henrys
(*b*) 14.7 joules

486. (*a*) 0.15 henry
(*b*) 0.30 joule

487. (*a*) 120,000 maxwells
(*b*) 0.320 henry
0.720 henry
(*c*) 0.360 henry
(*d*) 1.76 henrys

488. (*a*) 0.32 henry
(*b*) 0.64 joule
(*c*) 3.52 joules

489. (*a*) 0.180 henry
(*b*) 0.60

490. (*a*) 0.125 henry
(*b*) 0.60 henry
(*c*) 0.255

CHAPTER X

Electrostatics: Capacitance

491. (*a*) 1.563 dynes
(*b*) 0.625 dyne
(*c*) Attraction

492. (*a*) 5.33 dynes
(*b*) 1.78 dynes
(*c*) Repulsion
(*d*) $\begin{cases} 377 \text{ lines} \\ 502 \text{ lines} \end{cases}$

{126 lines
{167 lines

493. (*a*) 377 lines
(*b*) 1.2 lines per square centimeter
(*c*) 1.2 dynes

494. (*a*) 1.2 dynes
(*b*) 452 lines
377 lines
(*c*) 575 dynes
480 dynes
(*d*) 0.293 dyne
(*e*) 1.928 dynes

495. (*a*) 30 e.s.u.
Opposite
(*b*) 2.5 dynes

496. (*a*) 4 cm.
(*b*) 251.4 lines
(*c*) 302 lines
(*d*) 82.0 dynes per e.s.u.
(*e*) 97.6 dynes per e.s.u.
(*f*) 11.0 dynes per e.s.u.

497. (*a*) 2.53 cm.
(*b*) 251.4 lines
(*c*) 302 lines
(*d*) 34.3 dynes per e.s.u.
(*e*) 40.3 dynes per e.s.u.
(*f*) 11.0 dynes

498. (*a*) 640 e.s.u.
(*b*) 40 lines per square centimeter
(*c*) 40 dynes
(*d*) 6.4 lines per square centimeter
(*e*) Density varies as $\frac{1}{\text{distance}^2}$

499. (*a*) 0.0080 coulomb
(*b*) 0.0128 coulomb
(*c*) 0.05 amp.
0.08 amp.

500. (*a*) 133.3 volts
(*b*) 5,280 microcoulombs

501. 15 sec.

502. 50 μf.

503. 0.000036 coulomb

504. 5 sec.

505. 2.6

506. 4.5

507. (*a*) 88.9 volts
(*b*) 0.288 microcoulomb

508. (*a*) 0.864 microcoulomb
2.074 microcoulombs
(*b*) 864 volts

509. (*a*) 2.5
(*b*) 0.45 microcoulomb
(*c*) 625 volts

510. 24 μf.

511. (*a*) 15 μf.
25 μf.
20 μf.
(*b*) 200 volts

512. (*a*) 300 volts
(*b*) 24 μf.

513. (*a*) 2.105 μf.
(*b*) 126.3 microcoulombs
(*c*) 31.6 volts
15.8 volts
12.6 volts

514. (*a*) 379 microcoulombs
(*b*) 68.9 microcoulombs
137.8 microcoulombs
172.2 microcoulombs
(*c*) 17.22 volts

515. (*a*) 126.3 microcoulombs
(*b*) 23.0 microcoulombs
45.9 microcoulombs
57.4 microcoulombs
(*c*) 5.74 volts

516. 5.63 μf.
1,407 microcoulombs
87.9 volts
70.5 volts
56.3 volts
35.3 volts

517. (*a*) 2,814 microcoulombs
(*b*) 446 microcoulombs
557 microcoulombs
696 microcoulombs
1,115 microcoulombs
(*c*) 27.86 volts

518. (*a*) 280 μf.
120 μf.
210 μf.
140 μf.
(*b*) 42 μf.
(*c*) 22.4 volts
(*d*) 6,270 microcoulombs
2,690 microcoulombs
4,700 microcoulombs
3,140 microcoulombs

519. (*a*) 10,050 lines

(*b*) 3.56 lines per square centimeter
(*c*) 3.56 dynes per unit charge
(*d*) 1.422 ergs
(*e*) 1,422 dynes

520. (*a*) 0.01061 e.s.u. per square centimeter
(*b*) 300 e.s.u.
(*c*) 0.1335 dyne per unit charge
(*d*) 0.4 erg

521. 0.16 joule
0.24 joule
0.40 joule

522. (*a*) 1.307 joules
(*b*) 0.474 joule
0.379 joule
0.264 joule
0.190 joule
(*c*) 1.16 joules
Lost in spark

523. (*a*) 192 joules
(*b*) 300 joules

524. (*a*) 0.1875 joule
(*b*) 0.0938 joule
(*c*) 0.0313 joule
(*d*) 0.0625 joule

525. (*a*) 0.750 joule
(*b*) 0.375 joule
(*c*) 0.125 joule
(*d*) 0.250 joule
Voltage^2

526. (*a*) 200 volts
(*b*) 40 μf.

527. (*a*) 3,375 ergs
(*b*) 0.0003375 joule

528. (*a*) 1,200 microcoulombs
(*b*) 180 volts
(*c*) 0.128 joule
0.100 joule
0.060 joule
(*d*) 12 μf.

529. (*a*) 0.00236 μf.
(*b*) 0.0001061 joule
(*c*) 0.00519 μf.
(*d*) 0.000233 joule

530. (*a*) 79 glass
80 tin-foil
(*b*) 6 by 8 by 3.86 in.

531. 139 ft.

532. (*a*) 625 $\mu\mu f$.
(*b*) 312.5 microjoules
(*c*) 1,500 volts
(*d*) 468 microjoules
Source supplying motion

533. (*a*) 3 dynes
(*b*) 19.46 ergs

534. (*a*) 240.3 $\mu\mu f$.
(*b*) 0.481 microcoulomb

535. 0.503 μf.

536. (*a*) 1.022 μf.
(*b*) 4,088 microcoulombs
8.18 joules

537. (*a*) 2.44 μf.
(*b*) 0.0488 coulomb
(*c*) 488 joules

538. (*a*) 4.8 statfarads
(*b*) 9.6 statcoulombs
(*c*) 1.067 dynes
(*d*) 2 ergs

539. 2.5 ergs

540. (*a*) 0.1 amp.
(*b*) $0.1\epsilon^{-8.33t}$ amp.
(*c*) 0.12 sec.
(*d*) 0.0368 amp.
(*e*) 0.833 amp. per second
(*f*) Plot

541. (*a*) 0.015 amp.
(*b*) $-0.015\epsilon^{-1.667t}$ amp.
(*c*) 0.025 amp. per second

542. (*a*) $0.012\ (1 - \epsilon^{-8.33t})$ coulomb
(*b*) 0.00757 coulomb
(*c*) Plot

543. (*a*) $0.006\epsilon^{-0.417t}$ amp.
(*b*) 79.0 volts
(*c*) 41.0 volts
(*d*) 0.001 amp.
(*e*) 4.30 sec.
(*f*) Plot

544. (*a*) $0.0144\ (1 - \epsilon^{-0.417t})$ coulomb
(*b*) 0.006 coulomb per second
(*c*) 0.0022 coulomb
(*d*) 11.04 sec.
(*e*) Plot

545. (*a*) 0.009 coulomb
(*b*) 0.001564 amp. per second
(*c*) 0.0018 coulomb
(*d*) 0.1828 joule

546. (*a*) 0.0952 μf. per centimeter
(*b*) 23.1 μf.

547. 6.37 μf.

548. 1,300 ft.

CHAPTER XI

The Generator

549. (*a*) 240,000 maxwells
(*b*) 1.44 volts
(*c*) 1.44 volts
(*d*) 0 volts
(*e*) 2.26 volts

550. (*a*) 150,000 maxwells
(*b*) 0.025 sec.
(*c*) 2.40 volts
(*d*) 9.60 volts
551. (*a*) 0.0471 volt
(*b*) 1.885 volts
(*c*) 3.77 volts
(*d*) Proof
552. (*a*) 1.20 volts
(*b*) 0.848 volt
553. 3.38 volts
554. 4,010 volts
33.4 volts
555. 3,000 r.p.m.
556. (*a*) 94 elements
(*b*) Winding
(*c*) 1–24–3–26–5–
557. (*a*) 94 elements
(*b*) Winding
(*c*) 1–22–93–20–91–
(556) Progressive
(557) Retrogressive
558. (*a*) 94 elements
(*b*) Winding
(*c*) 1–22–3–24–5–
(Progressive)
559. (*a*) 130 elements
(*b*) 21 (y_b)
19 (y_f)
(*c*) Winding
(*d*) 1–22–3–24–5–
560. (*a*) 224 elements
(*b*) 29 (y_b)
27 (y_f)
(*c*) Winding
(*d*) 1–30–3–32–5–
561. (*a*) 444 elements
(*b*) 73
(*c*) 71
(*d*) 222 commutator segments
(*e*) Winding
(*f*) 1–74–3–76–5–78–

562. 67 (y_b)
563. (*a*) 592 elements
(*b*) 97 (y_b)
(*c*) 95 (y_f)
(*d*) 296 commutator segments
(*e*) 1–98–3–100–5–
564. (*a*) 85 (y_b)
83 (y_f)
(*b*) Winding
(*c*) 336 commutator segments
565. (*a*) 70 amp.
(*b*) 240 volts
(*c*) 100 kw.
566. (*a*) 70 amp.
(*b*) 120 volts
(*c*) 120 volts
840 amp.
100 kw.
567. 347 amp.
1,390 amp.
568. 33 (y_b)
29 (y_f)
1–34–5–38–9–
3–36–7–40–11–
569. 33 (y_b)
29 (y_f)
1–34–5–38–9–
–28–133–32–3–36–7–
–26–131–30–1
570. 73 (y_b)
69 (y_f)
1–74–5–78–9–82–
3–76–7–80–11–84–
571. 73 (y_b)
69 (y_f)
1–74–5–78–9–
3–76–7–80–11–
Doubly reentrant
572. 1–22–41–62–3–24–
Progressive
573. 1–20–39–58–77–18–37–
Retrogressive

574. 1–30–57–86–113–142–169–28–55–
Retrogressive
575. 86 slots
576. 88 slots
577. Impossible
35 (y_f) (y_b)
One dummy coil
578. Impossible
27 (y_f) (y_b)
One dummy coil
579. (*a*) 35 (y_f) (y_b)
(*b*) Singly reentrant
(*c*) 1 to 2
(*d*) 2 to 1
580. (*a*) 750 volts
333 amp.
250 kw.
(*b*) 375 volts
667 amp.
250 kw.
581. (*a*) 500 amp.
(*b*) 500 amp.
(*c*) Simplex wave
(*d*) 500 amp.
250 amp.

CHAPTER XII

Generator Characteristics

582. (*a*) 2,830 cm. per second
(*b*) 3.56 volts
583. (*a*) 3.66 volts
(*b*) 2.85 volts
(*c*) 279 volts
584. (*a*) 1.40 volts
(*b*) 0.972 volt
(*c*) 160.0 volts
585. 241 volts
301 volts
586. 12 kw.
15 kw.
587. 121 volts

150 volts
12 kw.
15 kw.

588. (*a*) 9.83 volts
(*b*) 6.25 volts
(*c*) 2

589. (*a*) 180 r.p.m.
(*b*) 100 amp.

590. (*a*) Plot
(*b*) 110 ohms
(*c*) 99 ohms

591. Plot
3.125×10^6 maxwells

592. (*a*) 63.5 ohms
(*b*) 55.7 ohms
(*c*) 249 volts

593. 0.17 amp.
20 volts

Amperes	Volts
0.32	35
0.55	60
0.95	101
1.60	154
2.43	197
3.12	221
3.48	232
3.65	236
3.72	238
3.78	240

594. 250 volts

595. Reversed field connections

596. (*a*) 3,270 ampere-turns
(*b*) 935 ampere-turns
(*c*) 5,065 ampere-turns
(*d*) 2,533,000 maxwells

597. 2,490 ampere-conductors
8,700 ampere-conductors
1,245 ampere-turns
4,350 ampere-turns

598. (*a*) 3,890 ampere-turns
(*b*) 1,418 ampere-turns
(*c*) 1,085 ampere-turns

599. (*a*) 6,260 ampere-turns
(*b*) 2,080 ampere-turns
(*c*) 8,320 ampere-turns

600. (*a*) 2,000 amp.
(*b*) 166.7 amp.
(*c*) 667 ampere-turns
(*d*) 7,330 ampere-turns
(*e*) Sketch
(*f*) Sketch

601. Sketch

602. (*a*) 1,884 cm. per second
(*b*) 0.000724 sec.
(*c*) 32,000 maxwells
(*d*) 4.42×10^7 maxwells per second
(*e*) 0.884 volt

603. (*a*) 1,570 cm. per second
(*b*) 0.000868 sec.
(*c*) 27,200 maxwells
(*d*) 3.13×10^7 maxwells per second
(*e*) 0.626 volt

604. (*a*) 670 in. per second
(*b*) 0.000937 sec.
(*c*) 21,600 maxwells
(*d*) 43,200 maxwells
(*e*) 3.69 volts

605. (*a*) 502 in. per second
(*b*) 0.00125 sec.
(*c*) 17,280 maxwells
(*d*) 34,560 maxwells
(*e*) 2.21 volts

606. 620.5 volts

607. 630.5 volts

608. (*a*) 0.0197 ohm
(*b*) 250 volts full load
264 volts no load
(*c*) Increased field current
Armature reaction

609. (*a*) 602.6 volts
(*b*) 266 kw.
(*c*) 251 kw.
(*d*) 94.5 per cent

610. (*a*)

Armature current, amp.	Terminal voltage, volts
4.4	254
100	249.5
200	245.5
250	243
300	240.5
400	236
500	231

(b)

Line current, amp.	Armature e.m.f., volts
0	254
95.7	253
195.8	252.5
245.8	251.8
295.9	251
395.9	250
496	248.5

(c) 243 volts

611. (a) 2,000 amp.
(b) 3.97 amp.
0.33 amp.

(c)

Line current, amp.	Terminal voltage, volts
500	224
400	231
300	236
250	239
200	242
100	248
0	254

612. (a)

Armature current, amp.	Terminal voltage, volts
3.5	254
100	248
200	243
250	239
300	236
400	231
500	224

(b)

Line current, amp.	Armature e.m.f., volts
0	254
96.6	252.5
196.7	250
246.7	247.8
296.7	246.5
396.8	245
496.9	241.5

(c) 239 volts

613. (a) 1,700 amp.
(b) 3.08 amp.
0.25 amp.

(c)

Line current, amp.	Terminal voltage, volts
500	208
400	215
300	221
250	225
200	229
100	235
0	242

614. (a) 405 ampere-turns per pole

(b)

Armature current, amp.	Terminal voltage, volts
4.4	254
100	247
200	240
250	235
300	231
400	219
500	203

(c)

Line current, amp.	Terminal voltage, volts
0	254
95.8	247
195.9	240
246	235
296	231
396.2	219
496.5	203

615. (a) 655 amp.

(b)

Armature current, amp.	Terminal voltage, volts
500	183
400	202
300	218
250	224
200	231
100	243
4.4	254

616. (a) 707 watts
(b) 149 watts
(c) 10 watts
(d) 300 watts
(e) 243.0 volts
(f) 16.17 kw.
(g) 92.8 per cent

617. (a) 4.50 kw.
(b) 2.27 kw.
(c) 0.17 kw.
(d) 13.65 kw.
(e) 20.59 kw.
(f) 420.6 kw.
(g) 95.1 per cent

618. (a) 4.69 kw.
(b) 0.581 kw.
(c) 0.044 kw.
(d) 3.48 kw.
(e) 8.79 kw.

(*f*) 212.9 kw.
(*g*) 95.9 per cent

619. (*a*) 183.9 watts
(*b*) 37.3 watts
(*c*) 2.4 watts
(*d*) 310 watts
(*e*) 241.4 volts
(*f*) 8,181 watts
(*g*) 93.5 per cent

620. (*a*) 629.8 volts
(*b*) 87.3 per cent

621. 250 volts
285.2 volts

622. 250 volts
298.2 volts

623. (*a*) 312.2 volts
(*b*) 40.3 kw.
2.32 kw.
6.16 kw.
1.71 kw.
6.05 kw.
70.4 kw.
(*c*) 79.8 per cent

624. (*a*) 324.8 volts
(*b*) 36.0 kw.
2.54 kw.
9.00 kw.
0 kw.
5.40 kw.
62.6 kw.
(*c*) 81.2 per cent

625. (*a*) 89 amp.
(*b*) 0.0154 ohm

626. 260 volts

627.

Line current, amp.	Terminal voltage, volts
375	146
313	178
250	186
187	207
125	218
63	229
0	240

628. (*a*) 4.43 turns
(*b*) 1.5 to 1

629. 0.00466 ohm

630. (*a*) 3,261 volts
(*b*) 3,512 volts

631. 696 amp.
30.8 volts

632. 79.8 per cent

CHAPTER XIII

The Motor

633. (*a*) 384,000 dynes
(*b*) 391 g.
(*c*) 0.391 kg.
(*d*) Left

634. (*a*) 2.936 kg.
(*b*) 2.202 kg.
(*c*) 2.936 kg.
(*d*) 2.202 kg.

635. (*a*) 0.470 kg.
(*b*) 0.094 kg.-m.
(*c*) 0.680 lb.-ft.
(*d*) Clockwise

636. (*a*) 0.589 lb.-ft.
(*b*) 0.481 lb.-ft.

637. 788 lb.-ft.
2,100 lb.-ft.

638. 267 lb.-ft.
106.7 lb.-ft.

639. (*a*) 28.18 kg.
(*b*) 112.7 kg.
(*c*) 18.03 kg.-m.
(*d*) 130.4 lb.-ft.

640. (*a*) 17.61 kg.
(*b*) 70.4 kg.
(*c*) 11.27 kg.-m.
(*d*) 81.5 lb.-ft.

641. 2.854 kg.-m.

642. 1.427 kg.-m.

643. 2.854 kg.-m.

644. 106.1 lb.-ft.

645. (*a*) 125.5 volts
(*b*) 15,060 watts
(*c*) 20.18 hp.
(*d*) 106.1 lb.-ft.

646. 128.9 lb.-ft.

647. (*a*) 243.7 volts
(*b*) 18,280 watts
(*c*) 24.5 hp.
(*d*) 128.9 lb.-ft.

648. (*a*) 48 lb.-ft.
(*b*) 60 lb.-ft.

649. 9.6 lb.-ft.
38.4 lb.-ft.
50.4 lb.-ft.

650. 114 lb.-ft.

651. 1,120,000 maxwells

652. (*a*) 145.8 amp.
(*b*) 2,604 amp.
(*c*) 117.3 volts

653. 116.6 volts
133.4 volts

654. 228.8 volts
251.3 volts

655. 736.7 volts
748.7 volts

656. 157.5 amp.

657. (*a*) 231.4 volts
(*b*) 57.3 amp.

658. 127.3 amp.

659. (*a*) 1,224 r.p.m.
(*b*) 4.06 per cent

660. (*a*) 1,266 r.p.m.
(*b*) 3.24 per cent
(*c*) 0.831 to 1

661. (*a*) 1,263 r.p.m.
994 r.p.m.
(*b*) 1.212 to 1

662. 30,120 lines per square inch

663. (*a*) 16.72 kw.
22.4 hp.
(*b*) 96.0 lb.-ft.

664. (*a*) 39.34 kw.
52.7 hp.
(*b*) 341 lb.-ft.

665. (*a*) 138.2 amp.
(*b*) 1.5 amp.
(*c*) 570 volts
(*d*) 77.9 kw.
(*e*) 457 lb.-ft.
(*f*) 438 lb.-ft.

666. (*a*) 0.076 amp.
(*b*) 138.12 amp.
(*c*) No armature e.m.f.
667. (*a*) 519 r.p.m.
(*b*) 1,089 r.p.m.
668. (*a*) 627 r.p.m.
(*b*) 1,311 r.p.m.
669. (*a*) 384 r.p.m.
(*b*) 1,000 r.p.m.
(*c*) 33.5 amp.
670. (*a*) 0.119 ohm
(*b*) 77.0 lb.-ft.
982 r.p.m.
671. (*a*) 19.48 lb.-ft.
(*b*) 33.02 lb.-ft.
(*c*) 1,394 r.p.m.
1,367 r.p.m.
(*d*) 5.93 per cent
(*e*) 38.9 lb.-ft.
672. (*a*) 36.66 lb.-ft.
1,534 r.p.m.
(*b*) 19.27 lb.-ft.
1,579 r.p.m.
673. (*a*) 11.38 lb.-ft.
(*b*) 25.6 lb.-ft.
674. (*a*) 849 r.p.m.
(*b*) 6.24 hp.
(*c*) 3.48 hp.
675. (*a*) 456 r.p.m.
753 lb.-ft.
(*b*) 1,087 r.p.m.
148.7 lb.-ft.
676. (*a*) 575 r.p.m.
602 lb.-ft.
(*b*) 1,364 r.p.m.
119 lb.-ft.
677. (*a*) 5,000 lb.
(*b*) 2,450 lb.
678. (*a*) 234.1 volts
(*b*) 1.375 to 1
(*c*) 168.7 lb.-ft.
679. (*a*) 242.2 volts
(*b*) 1.183 to 1
(*c*) 986 r.p.m
(*d*) 70.7 lb.-ft.
680. (*a*) 2.68 ohms
(*b*) 58.5 amp.
681. 1.98 ohms
682. (*a*) 51.9 amp.
(*b*) 1.27 ohms
683. 3.85 ohms
684. (*a*) 5.63 ohms
(*b*) 3.87 ohms
685. (*a*) 2.09 ohms
(*b*) 799 r.p.m.
(*c*) 106.1 per cent
58.4 per cent
(*d*) 63.7 per cent
80.0 per cent
57.6 per cent
76.7 per cent
686. (*a*) 3.75 ohms
(*b*) 633 r.p.m.
(*c*) 106.1 per cent
58.4 per cent
(*d*) 34.9 per cent
64.1 per cent
28.8 per cent
60.8 per cent
687. 218 r.p.m. to zero
290 r.p.m. to zero
400 r.p.m. to zero
510 r.p.m. to zero
690 r.p.m. to zero
910 r.p.m. to zero
688. 72.9 amp.
61.4 per cent
689. 53.5 per cent
690. (*a*) 23.5 hp.
(*b*) 88.0 per cent
691. (*a*) 15.29 hp.
(*b*) 85.9 per cent
692. 26.9 lb.
693. (*a*) 3.67 lb.-ft.
(*b*) 0.978 hp.
(*c*) 71.1 per cent
694. 2.25 hp.

CHAPTER XIV

Losses; Efficiency; Operation

695. (*a*) 605 watts
(*b*) 292 watts
(*c*) 420 watts
696. (*a*) 768 watts
(*b*) 478 watts
(*c*) 574 watts
697. (*a*) 874 watts
(*b*) 703 watts
(*c*) 753 watts
698. (*a*) 469 watts
(*b*) 316 watts
(*c*) 344 watts
699. (*a*) 127.8 hp.
(*b*) 746 lb.-ft.
(*c*) 89.9 per cent
700. (*a*) 148.9 hp.
(*b*) 869 lb.-ft.
(*c*) 90.0 per cent
701. (*a*) 219 lb.-ft.
(*b*) 42,040 watts
(*c*) 168.2 amp.
(*d*) 88.7 per cent
702. (*a*) 13.35 kw.
(*b*) 83.9 per cent
(*c*) 1.0 per cent
703. (*a*) 15,000 watts
(*b*) 17.03 hp.
(*c*) 84.7 per cent
(*d*) 59.6 lb.-ft.
704. (*a*) 41.6 kw.
(*b*) 90. 6 per cent
(*c*) 4,150 lb.-ft.
705. 679 watts
706. 87.1 per cent
707. (*a*) 1,556 watts
(*b*) 10.78 hp.
(*c*) 83.8 per cent
(*d*) 40.4 lb.-ft.
708. 1,384 watts
709. (*a*) 253.8 volts
Vary resistance in armature circuit
(*b*) To make speed 900 r.p.m.
(*c*) 17.8 amp.
710. (*a*) 2,530 watts

(*b*) 88.8 per cent
(*c*) 11.23 per cent
(*d*) 165.3 lb.-ft.
711. (*a*) 0.201
(*b*) 920 watts
(*c*) 2,080 watts
(*d*) 20.8 hp.
(*e*) 88.2 per cent
(*f*) 104.1 lb.-ft.
(*g*) 11.8 per cent
712. (*a*) 0.199
(*b*) 950 watts
(*c*) 1,657 watts
(*d*) 12.5 hp.
(*e*) 84.9 per cent
(*f*) 60.9 lb.-ft.
(*g*) 15.06 per cent
713. 87.9 per cent (motor)
86.9 per cent (generator)
714. 531 watts
86.4 per cent (motor)
566 watts
83.1 per cent (generator)
Unequal armature e.m.fs.
715. 24.9°C.
18.2°C.
Yes
716. (*a*) 29.7°C.
(*b*) 23.5°C.
717. 216 amp.
144 amp.
52.2 kw.
34.7 kw.
718. 240 amp.
160 amp.
57.7 kw.
38.5 kw.
719. (*a*) −40 amp.
40 amp.
(*b*) 80 amp.
120 amp.
(*c*) 140 amp.
160 amp.
720. 10 volts
(*a*) 187.5 amp.
112.5 amp.
(*b*) 437.5 amp.
262.5 amp.
(*c*) 46.0 kw.
27.6 kw.
(*d*) 104.8 kw.
62.9 kw.
721. (*a*) 0.0075 ohm
(*b*) 240 amp.
160 amp.
(*c*) 480 amp.
320 amp.

CHAPTER XV

Transmission and Distribution of Power

722. (*a*) 0.0600 ohm
(*b*) 5.40 kw.
(*c*) 92.8 per cent
(*d*) 17.4 kw.
723. (*a*) 69.6 kw.
(*b*) 5.40 kw.
(*c*) 92.8 per cent
724. (*a*) 44.8 kw.
(*b*) 241.3 volts
(*c*) 3.46 kw.
(*d*) 92.8 per cent
(*e*) 2,170 lb.
725. (*a*) 400 amp.
(*b*) 0.0216 ohm
(*c*) 0.0090 ohm
(*d*) 1,200,000 cir. mils
(*e*) 8,680 lb.
(*f*) 120.7 volts
(*g*) 4 to 1
726. (*a*) 0.060 ohm
(*b*) 17.4 kw.
(*c*) 69.6 kw.
727. (*a*) 365 mils
(*b*) 645 lb.
(*c*) 183 mils
161 lb.
(*d*) 73 mils
25.8 lb.
728. (*a*) 1,560,000 cir. mils
(*b*) 14,170 lb.
729. (*a*) 3,130,000 cir. mils
(*b*) 56,700 lb.
(*c*) As square
730. (*a*) 1.60 volts
1.44 volts
1.28 volts
1.12 volts
0.96 volt
0.80 volt
0.64 volt
0.48 volt
0.32 volt
0.16 volt
(*b*) 116.2 volts
731. 111.8 volts
732. (*a*) 112.4 volts
112.6 volts
113.1 volts
113.9 volts
114.9 volts
116.2 volts
114.9 volts
113.9 volts
113.1 volts
112.6 volts
112.4 volts
(*b*) 90.9 per cent
733. (*a*) 0.0587 ohm
0.1297 ohm
(*b*) 300,000 cir. mils
No. 0 A.W.G.
(*c*) 1,482 lb.
391 lb.
(*d*) 92.9 per cent
734. (*a*) 217,000 cir. mils
(*b*) 229.6 volts
225.0 volts
(*c*) 91.5 per cent
Prob. 733 better

735. (*a*) 228.0 volts
(*b*) 360 watts
(*c*) 95.0 per cent
(*d*) Large enough
736. No. 0 A.W.G.
4 to 1
Large enough
737. (*a*) 89.4 lb.
(*b*) 99.3 lb.
738. 61 amp. at *a*
41 amp. at *b*
25 amp. at *c*
12 amp. at *d*
23 amp. at *e*
3 amp. at *f*
12 amp. at *g*
49 amp. at *h*
38 amp. at *i*
29 amp. at *j*
25 amp. at *k*
739. 92 volts
138 volts
740. 109 volts
109 volts
741. 114 volts
104 volts
742. 103 volts
118 volts
743. 67.4 volts
154.8 volts
744. 102.4 volts
113.4 volts
214.5 volts
745. 116.0 volts
98.4 volts
212.5 volts
746. 124.4 volts
91.4 volts
88.8 volts
747. 29.0 amp.
21.0 amp.
748. 58.1 amp.
41.9 amp.

749. 92.9 amp.
67.1 amp.
750. (*a*) 248.1 volts
(*b*) 0.0406 ohm
(*c*) 400,000 cir. mils
751. (*a*) 0.0432 ohm
(*b*) 718 amp.
(*c*) 209.0 volts
(*d*) 87.1 per cent
(*e*) 333.4 kw.
(*f*) 5,560 amp.
752. (*a*) 1,021 amp.
(*b*) 195.9 volts
(*c*) 81.6 per cent
753. 83.3 kw.
2,780 amp.
754. (*a*) 148.4 amp.
(*b*) 404.5 volts
(*c*) 89.0 kw.
(*d*) 29.0 kw.
(*e*) 67.4 per cent
755. Impossible
756. 68.3 kw.
313 amp.

Current amp.	Power kw.
50	26.7
100	46.8
150	60.4
200	67.3
228	68.3
250	67.6
300	61.4

757. (*a*) 96.3 amp.
(*b*) 623.1 volts
(*c*) 72.2 kw.
(*d*) 12.2 kw.
(*e*) 83.1 per cent
758. (*a*) 142.2 amp.
(*b*) 562.6 volts
(*c*) 106.7 kw.
(*d*) 26.7 kw.
(*e*) 75.0 per cent
759. 106.7 kw.
391 amp.

Current, amp.	Power, kw.
50	34.2
100	61.8
150	82.9
200	97.3
250	105.1
285	106.7
300	106.4
350	101.0

760. (*a*) 555.0 volts
(*b*) 555.0 volts
761. (*a*) 544.0 volts
(*b*) 575.9 volts
762. 0.802 ohm
763. 478.8 volts
764. 563.1 volts
765. 425.8 volts
515.6 volts
766. 0.349
767. 0.425
768. 0.390
0.467
769. (*a*) 0.27 ohm
(*b*) 5,760 watts
(*c*) 288 watts
(*d*) 432 watts
770. 36.8 amp.
0.234 ohm
771. 5 cells
772. 5 cells
773. 4,309 volts
96.87 per cent
774. 5,734 volts
97.06 per cent

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