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CONTROLLED AND UNCONTROLLED MULTIVIBRATORS**

By Eugene R. Shenk*

SUMMARY

IN this paper*** the analysis of the multivibrator is developed on the basis of simple capacitor-resistor time constants. An equation relating the natural frequency of the multivibrator to the characteristics of the tubes and circuit components is derived and discussed. The waveform of the synchronizing voltage is considered, as well as the conditions that determine the phase in which this voltage must be supplied to each tube. Equations are given which relate the variations permissible in the time constants of the circuit to the order of division of the stage. Three conditions which must be satisfied in the design of a synchronized multivibrator to allow for the above variations are stated and illustrated. A method of designing the multivibrator so as to fulfill these requirements is developed, together with a practical method for adjusting the amplitude of the synchronizing voltage to the calculated optimum value. The application of either positive or negative pulses to only one tube and to both tubes is discussed. Percentage variations in the frequency of the synchronizing pulses over which a given order of division can be maintained are presented. Equations and curves are included which facilitate the design of both synchronized and unsynchronized multivibrators.

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** This paper covers the material included in a lecture delivered before the Radio Club of America on November 9, 1944 in Havemeyer Hall, Columbia University. A large part of the material was published originally in the January, February and March, 1944 issue of "ELECTRONICS". The author and The Radio Club of America appreciate the ready approval granted by the Editors of "ELECTRONICS" for the lecture and this reprint.

*** List of symbols is at end of paper.

General Multivibrator Considerations

To familiarize the reader with the author's concept of the operation of the multivibrator, a preliminary explanation is given. The multivibrator circuit described by Abraham and Bloch consists of two resistance-capacitance coupled amplifier stages with the output of each stage connected to the input of the other. This is illustrated in Fig. 1.0.

If V_2 is thought of as a switch which closes a circuit and applies a negative step of voltage to terminal 2, it is a simple matter to write the equation that describes the voltage across R_{d1}. When the switch closes, the voltage across R_{d1} will decrease instantaneously from zero to the value of the negative step and will thereafter increase exponentially toward zero. Finally this voltage will become less negative than that required to prevent the flow of current in V_1.

Due to the regenerative action of the circuit, an infinitesimal current through V_1 will be rapidly amplified and will result, according to the above logic, in closing switch V_1, thereby applying a negative step of voltage to terminal 3. When this switch closes, it simultaneously opens switch V_2, i.e., the plate current of V_2 is stopped. Now the previous cycle of operations is repeated with the functions of the two sections of the circuit interchanged. The sum of the nonconducting times of V_1 and V_2 is one period of the multivibrator frequency.

It is apparent that the natural frequency of the MV (multivibrator) is determined by the characteristics of the tubes, the values of the circuit time constants and the magnitude of the change of voltage applied to these time constant circuits when the tubes change from the nonconducting to the conducting state.

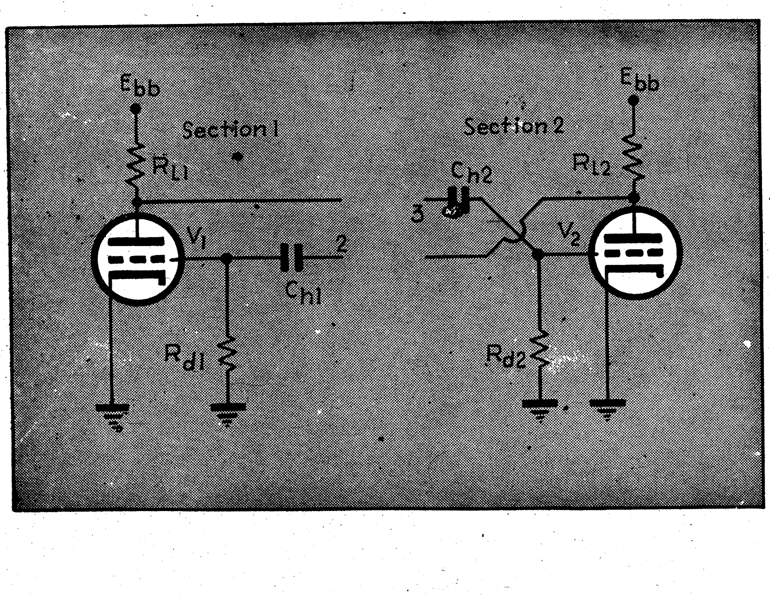
The Unsynchronized Multivibrator

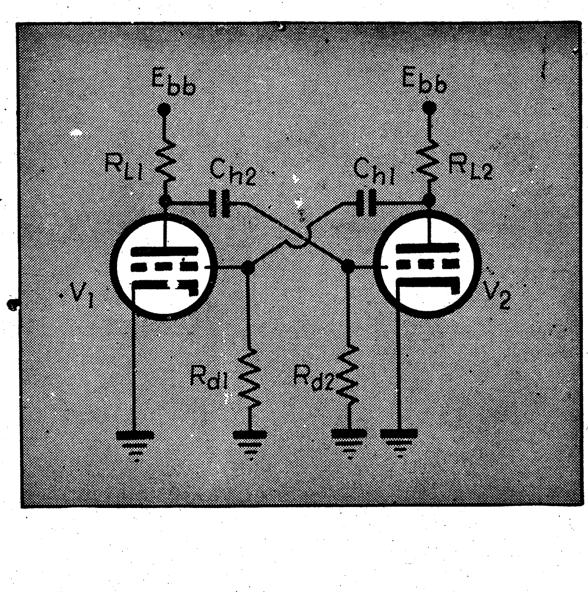
During one portion of the MV cycle, section 1 generates a step function of voltage which is applied to the time constant circuit of section 2. The duration of this portion of the cycle depends only upon the magnitude of the voltage step, the value of the time constant $C_{h2}R_{d2}$ and the critical grid voltage of V_2. In practice, the internal resistance of the generator is not zero. Consequently, the resistive component of the above time constant is larger than R_{d2}.

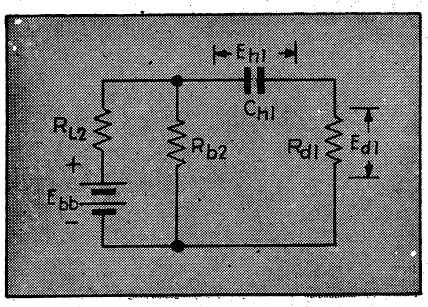
It should be noted that section 1 would continue to supply the step of voltage indefinitely, but that this portion of the cycle is terminated when the grid voltage of V_2 reaches the critical (cut-off) value $-E'_{co2}$. For the other portion of the cycle, section 2 becomes a generator of a voltage step which is applied to V

Thus, it is seen that both tubes are in a static condition except for the extremely short time during which they are in the process of interchang-

FIG. 1.0—This Abraham and Bloch multivibrator circuit consists of two resistance-capacitance coupled amplifier stages. The output of each stage is connected to the input of the other stage. FIG. 1.1—(below at right)—Basic two-triode multivibrator circuit







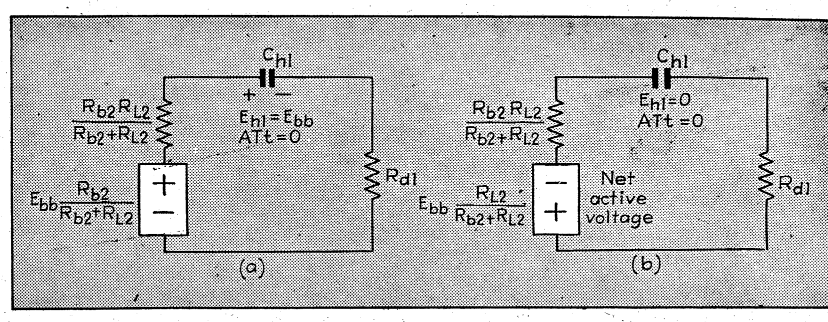


FIG. 1.2—Equivalent discharge circuit of C_{h1} in the multivibrator of Fig. 1.1. V_2 is conducting, and V_1 is nonconducting. $E_{h1} = E_{bb}$ at $t = 0$. Time is measured from the instant V_2 becomes conducting. FIG. 1.3 (above at right)—Simplification of Fig. 1.2 by the use of Thevenin's Theorem

ing their conduction states. The present paper assumes that the time involved in these changeovers is negligible in comparison with the MV period. For this reason, static values of plate resistance and amplification factor are used in developing equations for the period of the MV.

Referring to the MV circuit of Fig 1.1 the equivalent discharge (decreasing potential difference across the capacitor terminals) circuit of C_{h1} is as indicated in Fig. 1.2[1]. By the use of Thevenin's theorem, Fig. 1.2[1] can be simplified to Fig. 1.3a. For the solution of the transient voltage across R_{d1}, Fig. 1.3b is equivalent to Fig. 1.3a. The difference between these figures is that in Fig. 1.3b the net active voltage has been indicated as the only voltage in the circuit. This net voltage is readily obtained from Fig. 1.3a by taking the difference between the voltage across the capacitor and the voltage applied to the circuit

It is apparent from Fig. 1.1 also that the magnitude of the active voltage is equal to the drop across R_{L2}, since it is by this amount that the charging voltage for C_{h1} has been reduced. The same current will flow in the circuit of Fig. 1.3b as in the circuit of Fig. 1.3a. The actual voltage across the capacitor and the voltage applied to the circuit are as indicated in Fig. 1.3a.

From Fig. 1.3b

$$E_{d1} = -\left[E_{bb}\frac{R_{L2}}{R_{b2}+R_{L2}}\right]\left[\frac{R_{d1}}{R_1}\right]\exp\left[-\frac{t}{C_{h1}R_1}\right] \quad (1.1)$$

$$= -k_2 E_{bb}\exp[-\alpha_1 t] \quad (1.2)$$

$$\text{Where } k_2 = \left[\frac{1}{1+\frac{R_{b2}}{R_{L2}}}\right]\left[\frac{1}{1+\frac{1}{R_{d1}}\left(\frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}}\right)}\right] \quad (1.3)$$

$$\text{and } \alpha_1 = \frac{1}{C_{h1}R_1} \quad (1.4)$$

$$\text{and } R_1 = \left[R_{d1} + \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}}\right]$$

Equation (1.2) is plotted in general terms in Fig. 1.4. From this curve it is noted that when V_1 first becomes nonconducting ($t = 0$) its grid voltage is equal to $-k_2E_{bb}$. As time increases, this grid voltage increases exponentially toward zero. When E_{d1} reaches the cut-off value $-E_{co1}$, the tube will become conducting. Suppose $-E_{co1}$ is equal to $-0.1k_2E_{bb}$. Then from Fig. 1.4 the tube will become conducting when $\alpha_1 t$ is equal to 2.3.

Eq. (1.2) can be written

$$\frac{E_{d1}}{-k_2E_{bb}} = \exp[-\alpha_1 t]. \quad (1.2)$$

Substituting the specific values $-E_{co1}$ for E_{d1} and T_1 for t, (1.2) becomes

$$\frac{E_{co1}}{k_2E_{bb}} = \frac{1}{k_2\mu_{co1}} = \exp[-\alpha_1 T_1] \quad (1.5)$$

$$\alpha_1 T_1 = -\log_e\left[\frac{1}{k_2\mu_{co1}}\right] = \log_e[k_2\mu_{co1}] = 2.30\log_{10}[k_2\mu_{co1}]. \quad (1.5a)$$

(A modification of this equation for use with pentodes is given later in this paper.) Usually μ_{co1} is $\frac{1}{2}$ to $\frac{3}{4}$ of the rated amplification factor of the tube and equals E_{bb}/E_{co1}. T_1 is directly proportional to $C_{h1}R_1$ and to the logarithm of $k_2\mu_{co1}$. Therefore, a 2 to 1 change in the value of $C_{h1}R_1$

[1] R_{b2} of Fig. 1.2 is defined as the ratio of plate voltage to plate current at zero grid voltage. A resistance somewhat smaller than this would be more accurate as an average value, due to the fact that for a portion of the cycle the grid voltage is positive rather than zero.

It will be noted that E_{h1} of Fig. 1.2 is assumed to be E_{bb} at time equal to zero. This assumption can be validated by proper selection of the resistance associated with C_{h1} during its charging time. This charge time constant is considered later.

FIG. 1.4—Plot of Eq. (1.2). Starting from the value $-k_2E_{bb}$, the grid voltage of V_1 increases exponentially toward zero

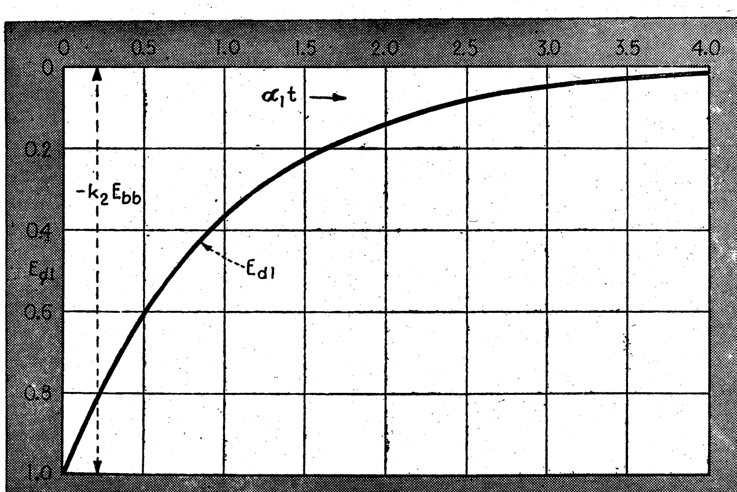
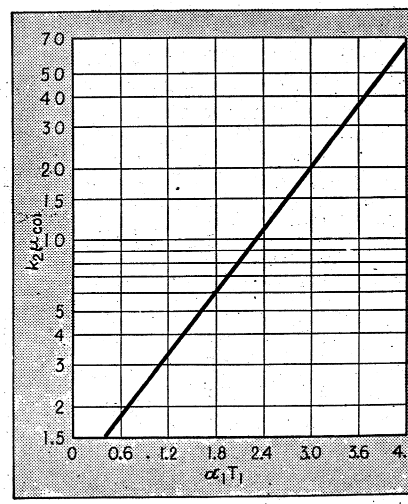


FIG. 1.5—Plot of Eq. (1.5a). By means of this curve, the multivibrator can be designed to operate at a given natural frequency



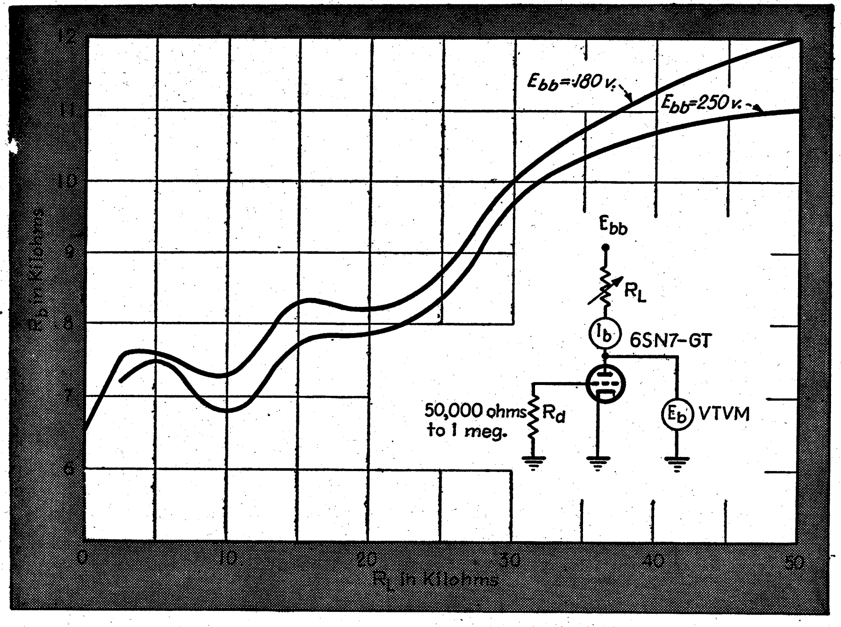


FIG. 1.6—Measured average d-c plate resistance (R_b) of type 6SN7-GT at zero bias as a function of the plate load resistor. R_b is practically independent of R_d for values of R_d in the range of 50,000 ohms to 1 megohm. $R_b = E_b/I_b$

Since R_b is a function of the plate current through the tube, it will depend upon the values of R_L and E_{bb}.

Measurements

The measured values given in Fig. 1.6 for type 6SN7-GT twin triode are somewhat lower than those obtained from static characteristics given in tube data books. While making these plate resistance measurements, R_d was varied from 50,000 ohms to 1 megohm with essentially no change in plate resistance. For lower values of R_d the plate resistance is less than that given in Fig 1.6. The decrease in R_b for R_d equal to zero is as much as 10 to 40 percent of the values on Fig. 1.6 the greater percentage decreases taking place at the higher values of R_L.

From the information on Fig. 1.6, the value of k as a function of R_L can

will produce a 2 to 1 change in T_1. A 2 to 1 increase or decrease in $k_2\mu_{co1}$ will add or subtract $\log_e 2 = 0.693$ to the value of the logarithm term. The percentage change that this represents depends upon the original value of $k_2\mu_{co1}$. If $k_2\mu_{co1} = \epsilon = 2.718$, the log term of Eq. (1.5a) becomes unity. If, in addition, $R_{d1} >> R_{b2}R_{L2}/(R_{b2} + R_{L2})$, the equation reduces to the frequently mentioned "order of magnitude" form, i.e., $T_{mv} = C_{h1}R_{d1} + C_{h2}R_{d2}$.

Figure 1.5 is a plot of Eq. (1.5a). For a given k_2 and μ_{co1}, this curve gives the value of $\alpha_1 T_1$ at which the tube will become conducting. Knowing $\alpha_1 T_1$, the value of α_1 required to keep V_1 nonconducting for a time T_1 can be calculated.

The portion T_2 of the MV period is calculated by using Eq. (1.6).

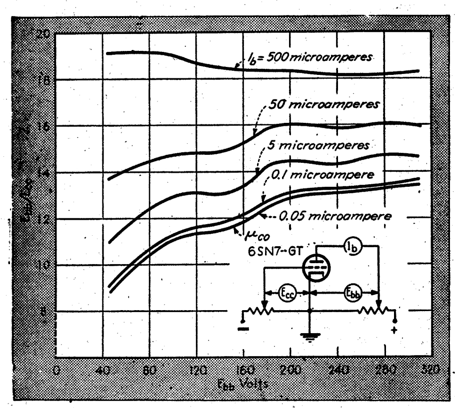
$$T_2 = \frac{1}{\alpha_2} \log_e [k_1 \mu_{co2}] \qquad (1.6)$$

The period of the MV is then T_{MV} where $T_{MV} = T_1 + T_2$ (1.7)

Note that as long as $\mu_{co} = E_{bb}/E_{cc}$ remains constant, the period of the MV is independent of the plate supply voltage.

The quantity k is a function of the d-c plate resistance and associated load resistor of one tube and the resistor in the grid circuit of the other tube. If $R_{d1} >> \frac{R_{b2} R_{L2}}{R_{b2} + R_{L2}}$ then k_2 is very nearly equal to $1/[1 + (R_{b2}/R_{L2})]$.

FIG. 1.8—Measured average values of the ratio E_{bb}/E_{cc} required to provide various values of plate current as a function of plate voltage for the type 6SN7-GT tube. Note that μ_{co} is considerably less than the rated dynamic mu which is 20



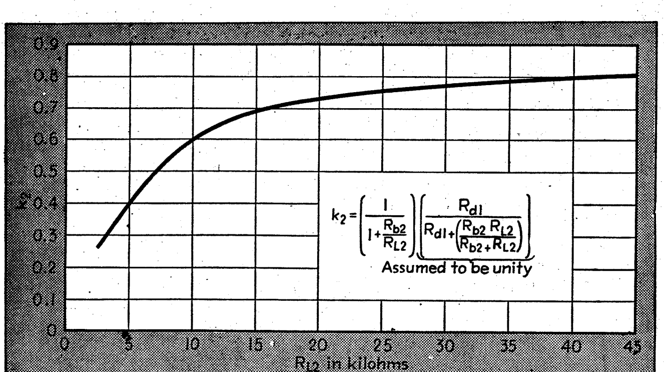


FIG. 1.7—If the second bracket in the expression for k_2 is assumed equal to 1; then, by reading from Fig. 1.6, average values of k_2 can be plotted as a function of R_{L2}. This curve is for type 6SN7-GT in the E_{bb} range of 150 to 250 volts

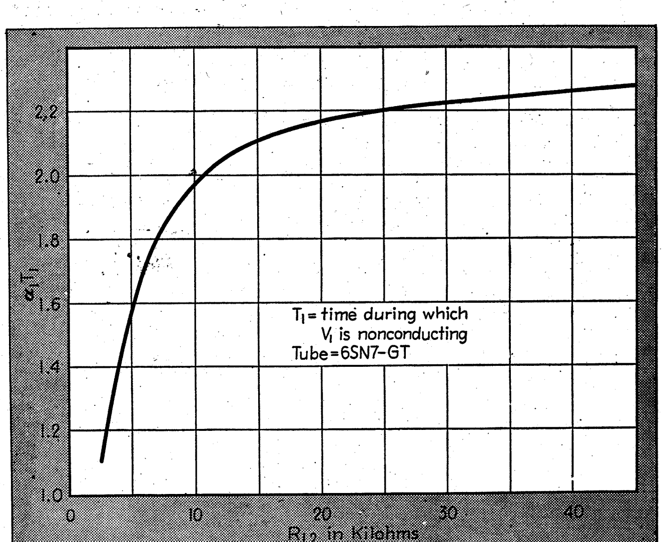
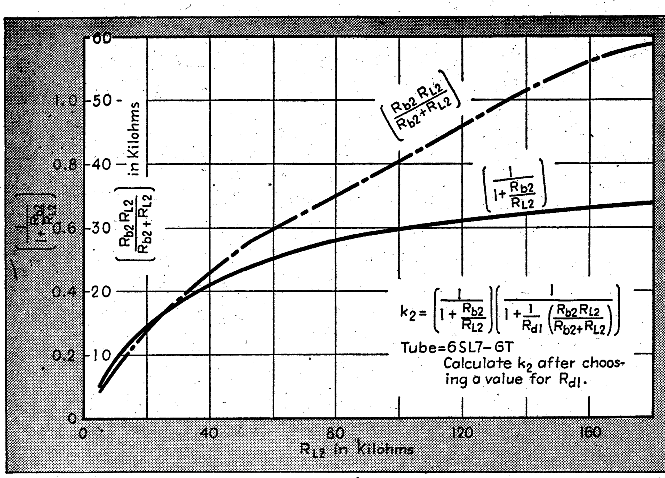


FIG. 1.9 (at right)—For a given tube type, $\alpha_1 T_1$ can be plotted as a function of R_{L2}. This type of curve simplifies multivibrator design



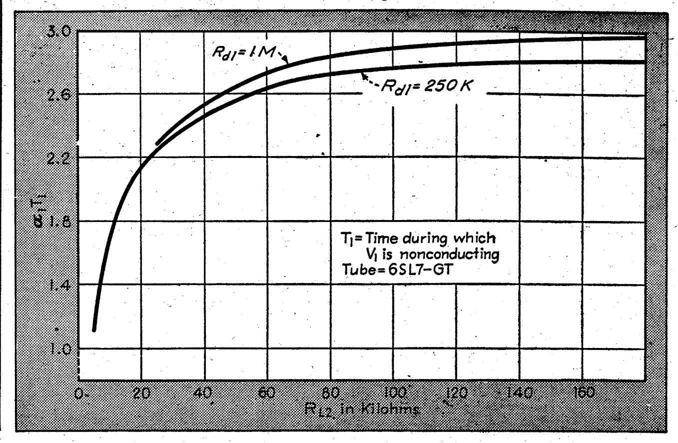


FIG. 1.10—A method of plotting the information necessary to compute the value of k as a function of R_L in cases where the value of the second bracket in the expression for k cannot be considered equal to unity. Measured average values for type 6SL7-GT in the E_{bb} range of 150 to 250 volts. FIG. 1.12 (above at right)—Similar to Fig. 1.9, but applicable to type 6SL7-GT tubes

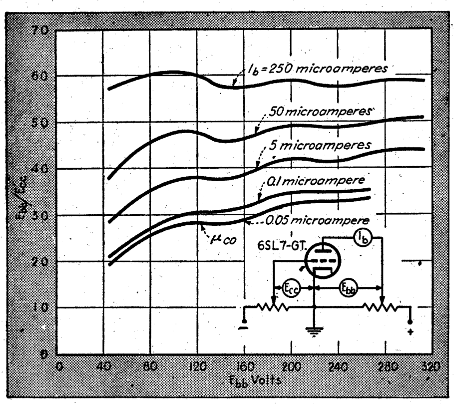


FIG. 1.11—Same as Fig. 1.8, but applicable to type 6SL7-GT tubes. The curve designated μ_{co} is useful in multivibrator design

be obtained. Figure 1.7 is such a plot for the type 6SN7-GT tube. Because k vs. R_L varies most rapidly for values of R_L less than about 20,000 ohms, the natural frequency of the MV will change more for a given percentage variation of R_L in this range than it will with higher values of R_L.

Figure 1.9 is a plot of $\alpha_1 T_1$ vs. R_{L2} (or $\alpha_2 T_2$ vs. R_{L1}) for type 6SN7-GT. This curve is for average values of $k_2\mu_{co1}$ for various tubes and E_{bb} values in the range of 150 to 250 volts.

The ratio E_{bb}/E_{cc} for several values of plate current is plotted against E_{bb} for the type 6SN7-GT tube in Fig. 1.8. Variations of this ratio for different tubes, particularly between different brands, were as great as 20 percent for each current value plotted on this figure. The average for several brands of tubes is shown. The curve for 0.05 microampere of current has been designated μ_{co}.

For a given tube and plate supply voltage there is a corresponding value of μ_{co}. Also if $R_{d1} >> \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}}$, a given R_{L2} will provide a definite value of k_2. Therefore, when a tube and plate load resistor are selected, the value of $k_2\mu_{co1}$ is fixed. This is sufficient information to solve for $\alpha_1 T_1$ by Eq. (1.5a) or Fig. 1.5.

Figures 1.10, 1.11 and 1.12 are curves for the type 6SL7-GT tube, similar to Figs. 1.7, 1.8 and 1.9 for the type 6SN7-GT.

The only assumption that has been made in the development of Eqs. (1.1) through (1.6) is that C_{h1} charges to E_{bb} during the time T_2 (the discharge time of C_{h2}) and that C_{h2} charges to E_{bb} during T_1. Figure 1.13 is the

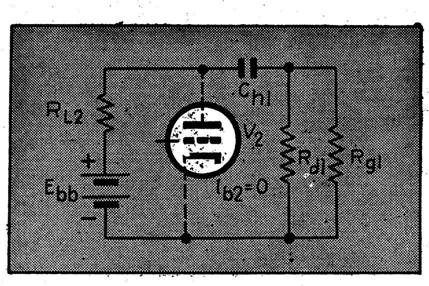


FIG. 1.13—Equivalent charge circuit of C_{h1} in the multivibrator of Fig. 1.1. V_2 is nonconducting, and V_1 is conducting

equivalent charge circuit for C_{h1}. The time constant of this circuit is

$$C_{h1}\left(R_{L2}+\frac{R_{d1}R_{g1}}{R_{d1}+R_{g1}}\right) \quad (1.8)$$

A steady-state analysis shows that if

$$C_{h1}\left(R_{L2}+\frac{R_{d1}R_{g1}}{R_{d1}+R_{g1}}\right) \lesssim \frac{T_2}{5} \quad (1.9)$$

and if

$$C_{h2}\left(R_{L1}+\frac{R_{d2}R_{g2}}{R_{d2}+R_{g2}}\right) \lesssim \frac{T_1}{5} \quad (1.9)$$

then the voltage across the capacitors will be equal to or greater than 0.993 E_{bb} at the ends of the charging intervals. If the 5 in (1.9) is decreased to 2.5, the capacitor voltage will be equal to or greater than $0.91E_{bb}$.

As long as (1.9) is satisfied, Eqs. (1.1) through (1.6) are accurate for frequencies at which the shunt capacities of the circuit can be neglected. The upper frequency limit is influenced by the same factors that determine the high-frequency response of an amplifier stage. (The effects of shunt capacities are discussed later in this paper.)

Note that in (1.9) R_d is shunted by the relatively small R_g. A reasonable average value for R_g in (1.9) is 1500 ohms. Therefore R_d can be increased and C_h decreased to maintain the necessary product C_hR_d of Eq. (1.5a) and at the same time satisfy (1.9). See Appendix I for a discussion of the relative importance of fully satisfying (1.9).

Example I

A symmetrical MV is to operate at a natural frequency of 1000 cps[2]. The tube is a type 6SN7-GT. E_{bb} is 180 volts. Design the MV.

Solution:

(a) Since the MV is symmetrical, $T_1 = T_2 = T_{MV}/2$, i.e., each tube contributes half the total period.

(b) Choose $R_{L1} = R_{L2} = 20{,}000$ ohms.

[2] By a symmetrical MV is meant one in which all the components of one section are identical to the corresponding components of the other section.

(c) To satisfy (1.9)

$$C_{h1\,max} = \frac{T_2}{5\left[R_{L2} + \frac{R_{d1}R_{g1}}{R_{d1}+R_{g1}}\right]} = \frac{1}{2\times10^3\times5\,[20\times10^3 + 1.5\times10^3]} = 0.00465\ \mu f$$

(d) From Fig. 1.9 read $\alpha_1T_1 = \alpha_2T_2 = 2.16$.

$$\alpha_1T_1 = \frac{T_1}{C_{h1}\left[R_{d1} + \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}}\right]} = 2.16$$

(e) If C_{h1} is chosen as 0.0005 μf,

$$R_{d1} + \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}} = \frac{T_1}{2.16\,C_{h1}} = \frac{1}{2\times10^3\times2.16\times5\times10^{-10}} = 4.63\times10^5\ \text{ohms.}$$

(f) The value of R_{b2} for R_{L2} = 20,000 ohms is read from Fig. 1.6 as 8.4×10^3 ohms. Therefore,

$R_{d1} = 463\times10^3 - 6\times10^3 = 457{,}000$ ohms.

Figure 1.14 is a schematic diagram of the MV. If it is desired to adjust the frequency accurately to 1000 cps, a 250,000 ohm resistor in series with a 500,000 ohm rheostat can be used for R_{d1} and R_{d2}. This will permit adjusting the MV for symmetrical wave shape and correct frequency for different tubes.

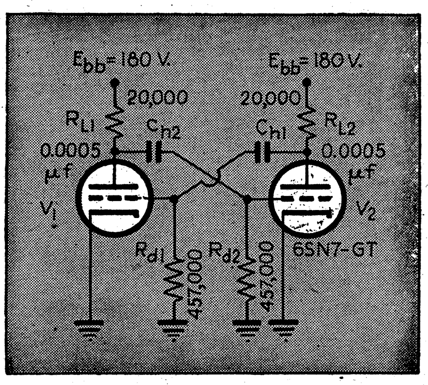


FIG. 1.14—Multivibrator designed in Example I

A MV was built using resistors and capacitors having values within 1 percent of those shown on Fig. 1.14. The natural frequency of the circuit for nine different tubes—three each of two different standard brands and one inferior brand—and three values of plate supply voltage is tabulated in Table I. The design, which was based upon 1000 cps at E_{bb} = 180 volts, is in error by a maximum of 5.8 percent for the first seven tubes. The maximum error between tubes (No. 1 and No. 6) is 6 percent. While the center frequency obtained with tubes No. 8 and No. 9 differs appreciably from the average of the first seven tubes, the percentage change in frequency with change in E_{bb} is about the same for all tubes.

If the total change of MV period as E_{bb} is decreased (from 180 volts) to 150 volts and increased to 210 volts is attributed to variation of μ_{co} (reference to Fig. 1.6 shows this to be reasonable; since, for R_L = 20,000 ohms, R_b does not change sufficiently to affect k), the result of reading Figs. 1.7 and 1.8 and substituting into Eq. (1.52) of Appendix I is (for a decrease of E_{bb} from 180 volts to 150 volts)

TABLE I

Tube	Brand	Natural Frequency For E_{bb} of		
		150v.	180v.	210v.
1	Standard 1	1022	998	985
2	"	1061	1034	1016
3	"	1070	1044	1020
4	Standard 2	1041	1017	997
5	"	1064	1034	1020
6	"	1085	1058	1038
7	Inferior	1085	1050	1031
8	"	1186	1151	1133
9	"	1211	1180	1163

Measured natural frequency for various tubes and plate supply voltages used with the multi-vibrator of Fig. 1.14.

$$\frac{dT_1}{T_1} = -\frac{0.72\,(12.4 - 11.6)}{0.72\times12.4}\left[\frac{1}{\log_e(0.72\times12.4)}\right] = -0.0295 = -2.95\%$$

The minus sign enters because the change in μ_{co} is in a negative direction. (Although the variations involved are too large to be considered differential changes, the accuracy obtained from the equation is still quite good.) Hence, the period at E_{bb} = 150 volts will be 97 percent of that at 180 volts or the frequency should be 1031 cps, an increase of 31 cps. A similar calculation shows that at 210 volts the frequency should be 980 cps.

Example II

A MV is to have a natural frequency of 50 cps. The tube is a type 6A6. E_{bb} is 200 volts. The value of E_{cc} required for plate current cutoff is −10 volts. Design the MV to supply a 20/80 output waveshape.

Solution:

(a) $T_1 = 1/50 \times 2/10 = 0.004$ sec.

(b) $T_2 = 1/50 \times 8/10 = 0.016$ sec.

(c) Let $R_{L1} = R_{L2}$ = 50,000 ohms. Then from the I_b vs. E_b family of curves, R_b = 25,000 ohms.

(d) If $R_{d1} >> \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}}$, then $k_2 = 1/[1 + (R_{b2}/R_{L2})] = 0.67$.

(e) $k_2\mu_{co1} = 0.67 \times 200/10 = 13.4$.

(f) Reading from Fig. 1.5, for $k_2\mu_{co1} = 13.4$, $\alpha_1T_1 = 2.60$.

Since $k_1\mu_{co2} = k_2\mu_{co1}$, $\alpha_2T_2 = \alpha_1T_1 = 2.60$. Then $\alpha_1 = 2.60/T_1 = 650$. Similarly, $\alpha_2 = 162$.

(g) Substituting in (1.9) to find maximum values of C_{h1} and C_{h2} gives

$$C_{h1\,max} = \frac{0.016}{5}\times\frac{1}{(50\times10^3 + 1.5\times10^3)} = 0.062\ \mu f.$$

$$C_{h2\,max} = C_{h1\,max}\times\frac{0.004}{0.016} = 0.0154\ \mu f.$$

If $C_{h1} = 0.002\ \mu$f,

$$R_{d1} = \frac{1}{C_{h1}\alpha_1} - \frac{R_{b2}R_{L2}}{R_{b2}+R_{L2}} = \frac{1}{2\times10^{-9}\times6.5\times10^2} - \frac{25\times50\times10^6}{75\times10^3} = 752{,}000\ \text{ohms}$$

and, choosing $C_{h2} = 0.008\ \mu$f, R_{d2} is also equal to 752,000 ohms.

Figure 1.15 is a schematic diagram of the MV. In order that the wave shape might be adjusted to 20/80, it would be well to use a 500,000-ohm rheostat and a 500,000-ohm resistor in series as R_{d1} and R_{d2}.

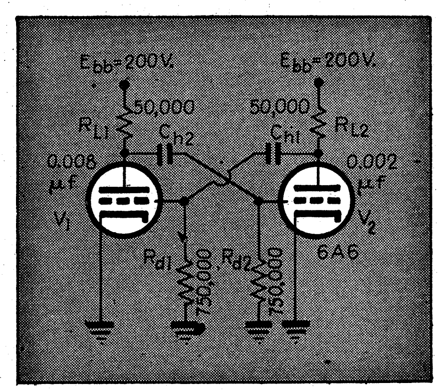


FIG. 1.15—Multivibrator designed in Example II

Selecting components to an accuracy of 1 percent, this circuit was built and the natural frequency resulting with four different tubes of a standard brand was 48.9, 50.1, 50.3 and 52 cps respectively. Figure 1.16 is a plot of natural frequency as a function of E_{bb} for two of these tubes. All values for the other two tubes fell between these curves. This fig-

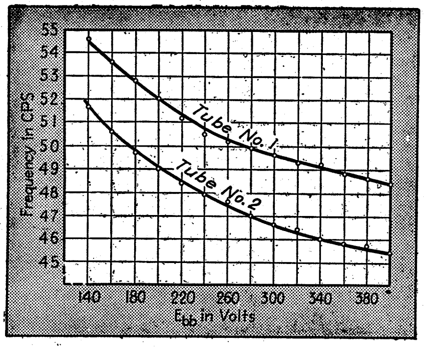


FIG. 1.16—Plots of natural frequency vs. E_{bb} for two type 6A6 tubes in the multivibrator circuit of Fig. 1.15.

ure shows that changes in E_{bb} affect the frequency in a similar manner as changes in C_h. Therefore, the effect of a given change in E_{bb} can be stated in terms of an equivalent change in C_h. From Fig. 1.16, as E_{bb} increases from 180 to 360 volts—a 100 percent increase—the natural frequency (No. 1 tube) decreases from 52.8 to 48.8 cps, an increase of 8.2 percent in the period of the MV. In this case, a 100 percent increase in E_{bb} is equivalent, in its effect upon the natural period, to an 8.2 percent increase in the capacity of both C_{h1} and C_{h2}. Decreases of 10 percent in E_{bb} from 200 volts and 400 volts are equivalent to decreases of 1.5 percent and 0.8 percent in the values of *both* time constants. In general, rate of change of frequency vs E_{bb} decreases as E_{bb} increases.

With 50,000 ohm plate load resistors, as shown in Fig. 1.15, the natural frequency decreased smoothly by 4.4 and 4.3 cps for tubes No. 1 and No. 2 respectively in the E_{bb} range from 180 to 400 volts. When these 50,000 ohm resistors were decreased to 10,000 ohms and the condensers increased in capacity to bring the frequency back to approximately 50 cps, the frequency varied 7.8 and 6.2 cps for the same tubes in the same E_{bb} range. This confirms the desirability of using values of R_L that are large in comparison with R_b in order to blot out the effect upon k of variations in R_b. The resulting large value of k is also desirable, as found in Appendix I, because it helps to minimize the effect upon T_{mv} of variations in μ_{co} and in k. To sum up—experiment shows that in a group of four type 6A6 tubes the variations of natural frequency from tube to tube as well as those variations resulting from changes in E_{bb} with a given tube, are approximately half as great for $R_L = 50{,}000$ ohms as they are for $R_L = 10{,}000$ ohms.

With some types of tubes, k varies appreciably (from tube to tube or with changes in E_{bb}) and in the opposite direction from the changes in μ_{co}. As a result, the percentage variations in the product $k\mu_{co}$ are likely to be less than those of either k or μ_{co}. If, in these cases, R_L is made small in order to allow R_b greater control over k, the normal value of the product $k\mu_{co}$ will be decreased proportionately. In Appendix I it is shown that the effect upon T_{mv} of a given percentage change in $k\mu_{co}$ increases as $k\mu_{co}$ decreases. Hence, if small values of k are employed to use changes in k to compensate changes in μ_{co}, the compensating effect must be rather pronounced if a net improvement is to be obtained over the minimizing action of the larger k. Experiment indicates that, for given circuit conditions, the degree of compensation is subject to considerable variation with replacement of the tubes. Generally, the larger values of R_L (and, therefore, of k) are preferable.

The Synchronizing Voltage

Equations relating the natural period of a MV to the characteristics of the tubes and associated circuit parameters have been developed and discussed. If the MV is to be uncontrolled, the information already provided is sufficient to complete the design. If the MV is to be synchronized with a controlling frequency, additional design formulas are needed. Before considering the synchronizing problem, some attention will be given to the wave form of the synchronizing voltage and the point of its application to the MV stage.

Of the three common wave shapes, the sine wave, the square wave and the impulse, the impulse is preferable as the synchronizing voltage for several reasons.

Considering first the case of the sine wave, any variation from the nominal values of the time constants will result in a variation of the phase of the output voltage from the MV. The reason for this is that the sine wave requires a finite time to change amplitude. Mathematically,

$$-\Delta E \exp\left[-\frac{t}{CR}\right] + E_s \sin \omega_s t = -E_{co} \tag{2.1}$$

The MV will synchronize at a value of time just greater than that which satisfies Eq. (2.1). From this equation it is apparent that any variation of ΔE, CR or E_s, or a tube with a slightly different value of E_{co} will cause a change in the time at which the MV synchronizes.

For the case of rectangular-wave synchronizing voltage of sufficient amplitude, phase variations cannot occur in the output voltage. However, due to the time duration of the square wave, the percentage decrease from the nominal value of CR that can be tolerated without the MV dividing by a smaller number is less than for an impulse synchronizing voltage. The sine wave suffers from this disadvantage to a considerable extent also.

With an impulse synchronizing voltage the MV is given only a momentary opportunity to trip. If it does not trip on a given impulse, it will wait for the next one. (If the MV is not properly designed, it is possible that its natural period will lie between impulses.) Thus there is no possibility of phase variations in the output voltage.

Applying the Synchronizing Voltage to the MV

The synchronizing voltage can be applied in the grid, plate or cathode circuit of either tube or both tubes or in any combination of these places.[3] If both polarities of synchronizing voltage are present at any of these points, one polarity will be amplified by the tube that is passing current and will be coupled into the grid circuit of the other tube.

Figure 2.1 illustrates one method of applying the synchronizing voltage to a MV. The synchronizing voltage is supplied, in proper phase, to the grids of limiter tubes V_3 and V_4. The relation between ϕ_1 and ϕ_2 is determined by whether each tube of the MV is to divide by an integer or an integer plus a fraction. (This is explained later in conjunction with Fig. 2.2.) The time constants of which C_1 and C_2 are parts should be small as compared with the period of the synchronizing wave. This will result in pulses of synchronizing voltage being applied to the MV.

In the circuit of Fig. 2.1 both positive and negative synchronizing pulses are applied to the MV. Consider the portion of the MV cycle

[3] Wherever the synchronizing voltage is applied to the circuit, the effect of the impedance of the synchronizing voltage generator upon the operation of the MV should be considered.

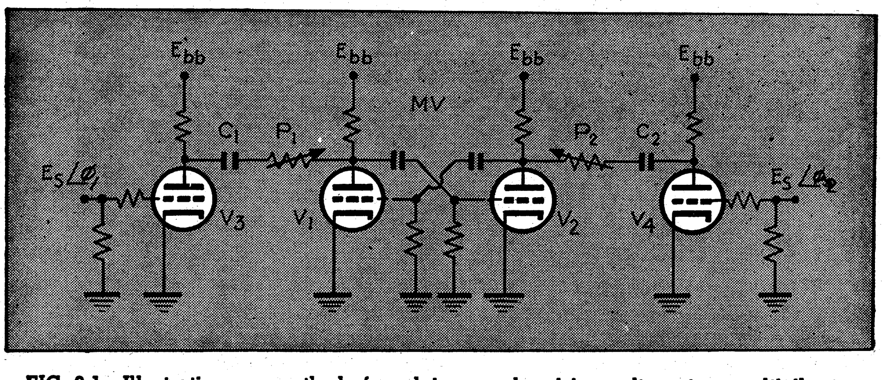


FIG. 2.1—Illustrating one method of applying synchronizing voltage to a multivibrator

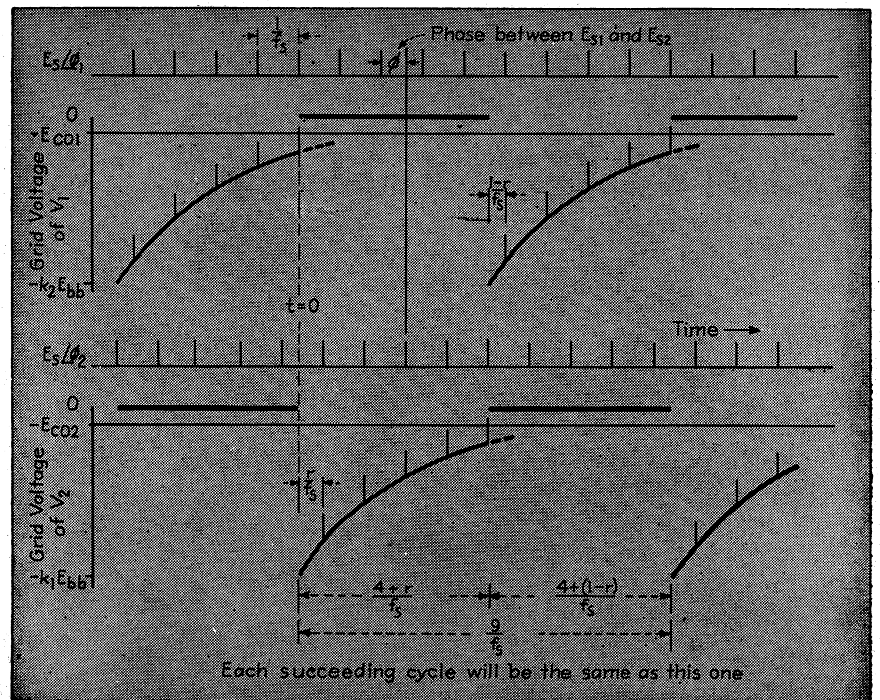


FIG. 2.2—A phase difference between the synchronizing pulses applied to the two tubes of a multivibrator adds a fraction to the order of division of each tube. The two fractions are always complementary

when V_1 is "on" and V_2 is "off". V_3 is supplying positive and negative pulses of $E_s\angle\phi_1$ to the grid of V_2. At the same time V_4 supplies both positive and negative pulses of $E_s\angle\phi_2$ to the grid of V_1. The negative pulses of $E_s\angle\phi_2$ will be amplified and inverted in V_1 and coupled into the grid circuit of V_2. Therefore, two voltages are acting to synchronize V_2—the positive pulses of $E_s\angle\phi_1$ and the amplified, inverted negative pulse of $E_s\angle\phi_2$. Under these conditions, $E_s\angle\phi_2$ is almost certain to take control of V_2. Similarly, $E_s\angle\phi_1$ will control V_1.

The Order of Division

Suppose one section of a MV is to divide the synchronizing frequency by a fraction, r, or an integral number plus a fraction. Then the order of division of the other section of the MV must include a fraction $(1 - r)$. If both sections of the MV are to be synchronized in such a case, the synchronizing voltage must be supplied to the two tubes in different phases. The phase difference between the two voltages must be ϕ where

$$\phi = 360r \text{ deg.} \qquad (2.2)$$

This is illustrated in Fig. 2.2. At time equal to zero a synchronizing pulse causes V_1 to conduct. Therefore, the grid voltage of V_2 decreases to $-k_1E_{bb}$. Due to the phase difference between the two synchronizing voltages, the first pulse occurs at the grid of V_2 at time equal r/f_s. The additional nonconducting time of V_2 must be an integral number of periods of the synchronizing wave. On Fig. 2.2 four periods are included. The fifth pulse trips V_2.

The first pulse at the grid of V_1 occurs $(1 - r)/f_s$ seconds after V_1 becomes non-conducting. As a result, the "off" time of V_1 includes, in addition to an integral number, a fraction $(1 - r)$ of the period of the synchronizing voltage. Thus, the period of the MV includes one period of the synchronizing wave which is contributed partly by V_1 and partly by V_2.

If synchronizing voltage is supplied to both tubes in the same phase, r is zero and each tube divides by an integer only.

It should be noted that it is not necessary to use push-pull synchronization with every MV that is to divide by an odd number. The only cases for which two phases of synchronizing voltage are required are those for which r has a value other than zero. Consider as an example a MV that is to divide by seven. If a 50/50 output waveshape is needed, then each tube must divide by $7/2 = 3.5$. In this case $r = 0.50$, so push-pull synchronization *is required.* However, if it were not necessary to provide a 50/50 output waveshape, then one tube could divide by four and the other by three. Now $r = 0$ and the synchronizing voltage *must* be supplied to both tubes in the same phase. Different orders of division can be obtained in the two tubes either by using a different value of time constant with each tube or by supplying a different amplitude of synchronizing voltage to each tube. Both of these methods are entirely feasible. How either can be done and still maintain optimum synchronizing conditions is considered later in this paper.[4]

Suppose a MV is to divide by an even number, say 6, and a 2 to 3 output waveshape is wanted. Then one tube must divide by 2.4 and the other by 3.6. Now $r = 0.4$ and the phase difference between the two synchronizing voltages must be $360 \times 0.4 = 144$ deg.

When the synchronizing voltage is supplied to the two tubes in the same phase, it acts to make them both conducting at the same time. Assume V_1 is conducting. When V_2 begins to conduct, it must operate to stop the plate current of V_1. Therefore, V_1 is acted upon by two independent voltages of opposite polarity. For reliable synchronization, V_2 must easily overcome the action of the syn-

[4] If sinusoidal synchronizing voltage is used, the statements in this and the succeeding paragraph are modified.

chronizing voltage and stop the flow of plate current through V_1. This can be insured by using a value of $k_2E_{bb} >> E_{s1}$, where E_{s1} is the value of the synchronizing voltage as referred to the grid of V_1. This condition is ordinarily satisfied without giving it any special consideration, particularly for values of N equal to or greater than three.

Correlation of Synchronizing Voltage Amplitude with MV Natural Frequency

Three conditions must be satisfied in the design of a synchronized MV, if the greatest possible variations in the amplitude of the synchronizing signal and from the nominal values of the capacitor-resistor time constants are to be allowed for. This is desirable to stabilize the order of division of the circuit against changes of temperature and power supply voltage, replacement of tubes, etc. If it is assumed that once the amplitude of the sychronizing voltage is adjusted it remains constant, then the nominal value of CR (the discharge time constant) and the amplitude of the synchronizing voltage must be so selected that:

Condition 1 The desired percent decrease of CR can be tolerated without the MV dividing by a smaller number.

Condition 2 The synchronizing pulse is of greater amplitude than the change of grid voltage along its exponential decay curve between the (N-1)th and the Nth pulses[5]. This is necessary to insure that the natural period of the MV cannot lie between these pulses.

Condition 3 The desired percent increase of CR can be tolerated without the MV slipping synchronism or dividing by a larger number.

It can be shown that if the MV design satisfies Conditions 1 and 3, then Condition 2 is automatically satisfied.

Condition 1 sets a maximum value on the amplitude of the synchronizing pulse and Conditions 2 and 3 limit its minimum amplitude. If the maximum value permitted by 1 is at least as large as the larger of the two values required by 2 and 3, all three conditions can be satisfied simultaneously.

These conditions are illustrated graphically in Figs. 2.3, 2.4 and 2.5. A method of designing the MV that satisfies all three conditions will be outlined later. It is shown in Electronics Magazine for February 1944 that if the synchronizing voltage is an impulse and if (1.9) are satisfied, then

$$\frac{1+\delta_i}{1-\delta_d} = \frac{N}{N-1} \qquad (2.8)$$

$$\delta_d = {}_i = \frac{1}{2N-1} \qquad (2.8a)$$

$$\delta_d = \frac{1}{N} \text{ if } \delta_i = 0 \qquad (2.8b)$$

$$\delta_i = \frac{1}{N-1} \text{ if } \delta_d = 0 \qquad (2.8c)$$

Relations (2.8) a, b and c are plotted in Fig. 2.6. Note that when listed according to decreasing range of permissible variation, the order is δ_i, $\delta_{d=i}$, δ_d. As an example, if a symmetrical MV is to divide by 8 then N is 4. From (2.8c), by proper design, it is possible to allow for a 33⅓ percent increase over the nominal value of $C_{h1}R_1$. From (2.8a), ±14.3 percent or a total range of 28.6 percent could be permitted, while by (2.8b) only 25 percent decrease could be tolerated in the value of $C_{h1}R_1$. Or, solving (2.8) for δ_d = 5 percent, the value of δ_i is

[5] Although N is not limited to integral values, the pulse preceding the one that normally trips the MV will be referred to as the (N-1)th pulse.

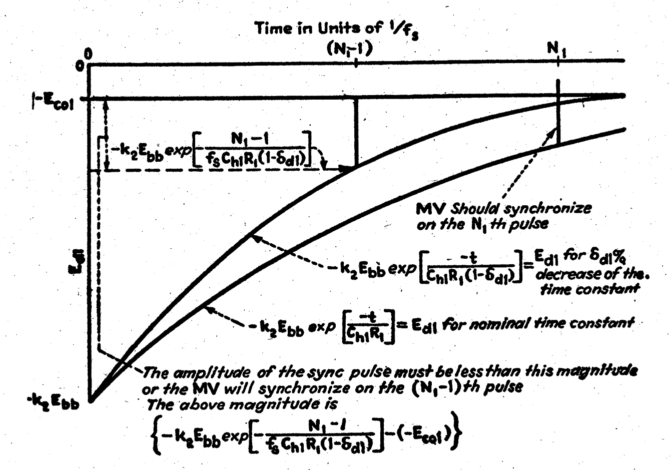
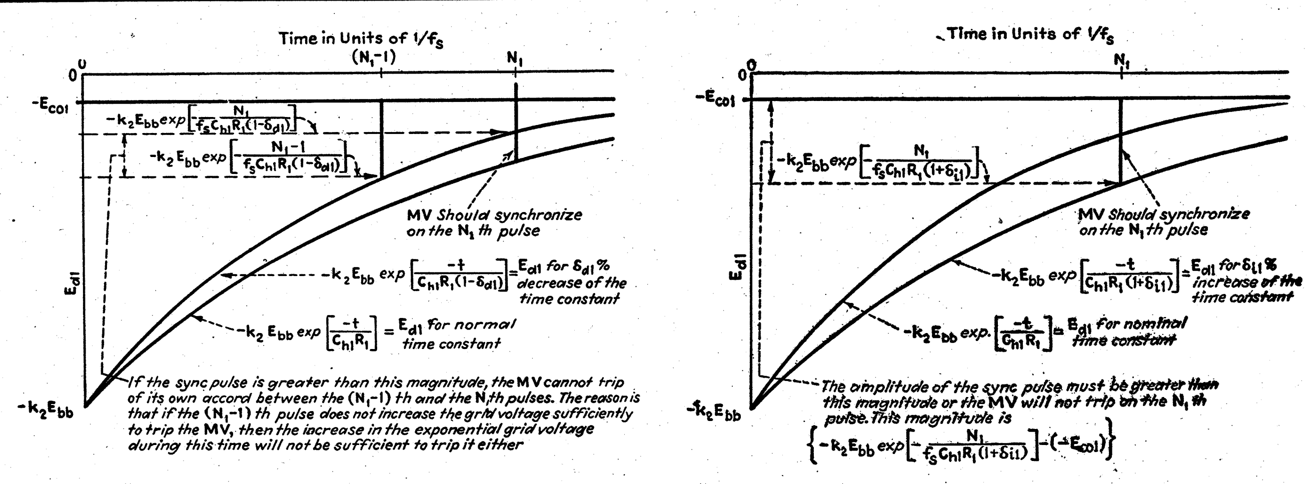


FIG. 2.3, 2.4 and 2.5—These illustrate conditions 1, 2 and 3 which must be satisfied in the optimum design of a synchronized MV



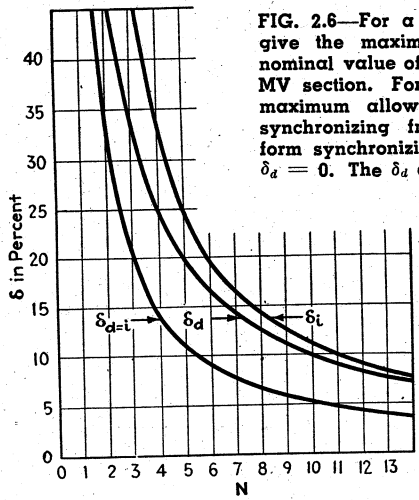


FIG. 2.6—For a constant synchronizing frequency, these curves give the maximum allowable percentage variations from the nominal value of C_hR as a function of the order of division of the MV section. For a constant value of C_hR, the curves give the maximum allowable percentage variations from the nominal synchronizing frequency. Constant amplitude, impulse waveform synchronizing voltage is assumed. The δ_i curve assumes $\delta_d = 0$. The δ_d curve assumes $\delta_i = 0$. The $\delta_d =_i$ curve allows for equal values of δ_d and δ_i

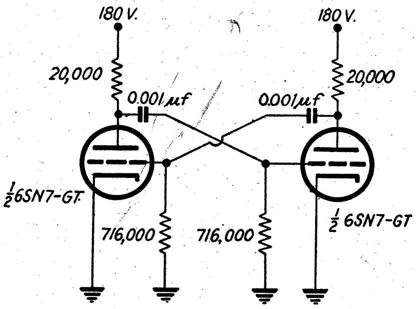


FIG. 2.9—Multivibrator designed in Example III. No specific synchronizing circuit is shown. In general, plate circuit synchronizing is desirable

26.7 percent and the range is 31.7 percent. See Appendix III for experimental verification of (2.8b) and (2.8c).

Note that for values of N_1 less than one, time $(N_1 - 1)/f_s$ occurred before V_1 became non-conducting. Therefore, (2.8) are useful only for values of N_1 greater than one. If $N_1 = 1$, the only limit on C_hR_1 is its minimum value.[7] This minimum value is the one for which the natural period of the MV is equal to the desired controlled period. The product $C_{h1}R_1$ can be made as large as desired, if the synchronizing pulse is of sufficient amplitude. The maximum value of synchronizing voltage required for V_1 cannot exceed k_2E_{bb}. Therefore, if $E_{s1} \geqq k_2E_{bb}$, any variations can be tolerated in the value of $C_{h1}R_1$ as long as it does not decrease below the above-mentioned minimum value.

[7] All multivibrators dividing by one as well as those dividing by two, in which each tube divides by one, fall in this category.

Designing a Synchronized Multivibrator

The problem now becomes that of designing the MV so that the maximum percentage variations of $C_{h1}R_1$ (and $C_{h2}R_2$) can be tolerated. First, δ_d and δ_i should be determined for the given order of division of each tube. These values are calculated by means of (2.8) or read from Fig. 2.6.

The characteristics of the tube and the size of the plate load resistor fix a maximum allowable value of $N_1/f_sC_{h1}R_1$. This maximum value is that for which the natural period is equal to the controlled period and is given by Eq. (1.5a).

$$\alpha_1T_1 = \log_e (k_2\mu_{col}) \qquad (1.5a)$$

If $N_1/f_sC_{h1}R_1$ is written for α_1T_1 Eq. (1.5a) becomes

$$\frac{N_1}{f_sC_{h1}R_1} = \log_e (k_2\mu_{col}). \qquad (2.9)$$

Eq. (2.9) is plotted in Fig. 2.7. The value of $N_1/f_sC_{h1}R_1$ as read from this figure must be multiplied by $(1 - \delta_{d1})$. This is necessary to prevent the natural period from becoming shorter than the controlled period for δ_{d1} percent decrease of $C_{h1}R_1$.

It is well known that when a MV is to be synchronized, its natural period should be longer than its controlled period.[8] If such is the case, some variation is allowable in the values of the circuit components and the characteristics of the tubes for which the natural period of the MV will not become shorter than

[8] It is possible to employ a natural period which is shorter than the desired controlled period. In this case a synchronizing voltage of suitable polarity (negative as referred to the grid of the tube to be synchronized) and of sufficient time duration, must be supplied to the circuit to prevent the MV from tripping at the time determined by its natural frequency. This method of synchronizing is illustrated later.

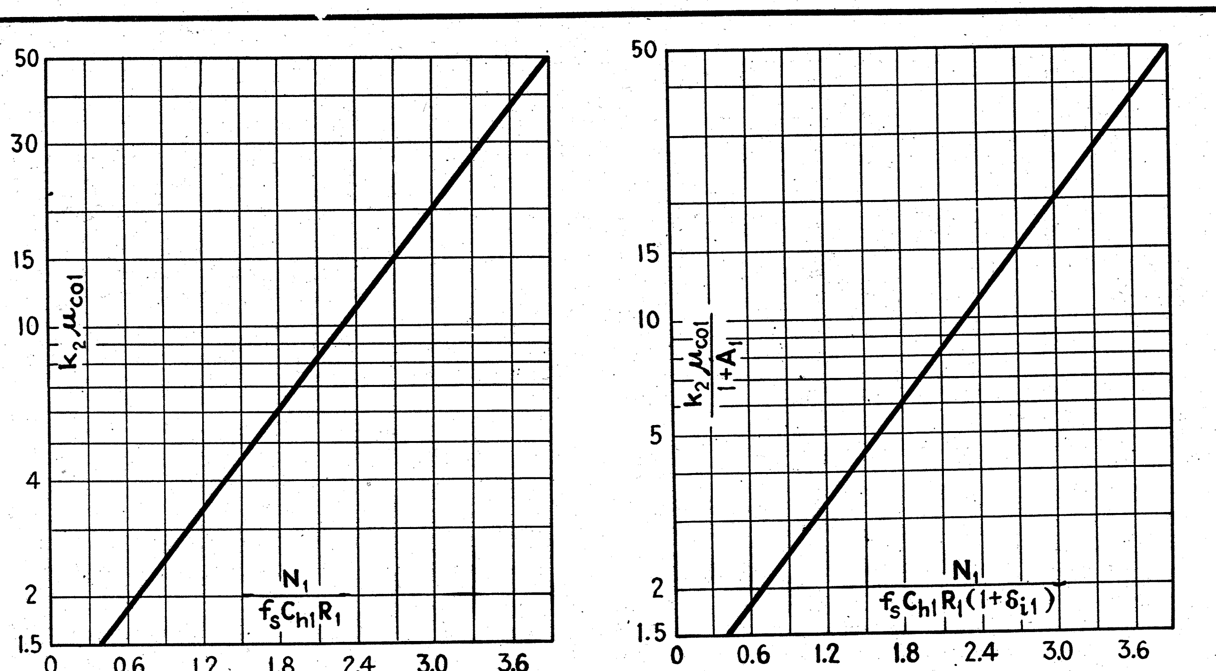


FIG. 2.7 and 2.8—Plots of Eq. (2.9) and (2.10), useful in MV design

the controlled period. Therefore, the value of $N_1/f_sC_{h1}R_1$ as obtained so far should be decreased by some arbitrary amount. Twenty percent is usually sufficient, if components of commercial tolerance and non-selected tubes are to be used. Thus a maximum has been established for $N_1/f_sC_{h1}R_1$ equal to $0.8(1 - \delta_{d1})$ times the value read from Fig. 2.7.

For this value of $N_1/f_sC_{h1}R_1$ calculate $N_1/f_sC_{h1}R_1(1 + \delta_{i1})$. This will be the same as the value of $(N_1 - 1)/f_sC_{h1}R_1(1 - \delta_{d1})$, since values of δ_i and δ_d given by (2.8) through (2.8c) make these quantities identical. To satisfy (2.7), $\log_e[k_2\mu_{co1}/(1 + A_1)]$ must lie between $(N_1 - 1)/f_sC_{h1}R_1(1 - \delta_{d1})$ and $N_1/f_sC_{h1}R_1(1 + \delta_{i1})$. Since the last two quantities are equal, set $\log_e[k_2\mu_{co}/(1 + A_1)]$ equal to them also. Then

$$\log_e\left(\frac{k_2\mu_{co1}}{1 + A_1}\right) = \frac{N_1}{f_sC_{h1}R_1(1 + \delta_{i1})}. \qquad (2.10)$$

The right-hand side of this equation is known. Refer to Fig. 2.8 and read the value of $k_2\mu_{co1}/(1 + A_1)$. From this the values of A_1 and $E_{s1} = A_1E_{co1}$ can be obtained. Any other magnitude of E_{s1} will increase the permissible range of either δ_{d1} or δ_{i1} at the expense of the other. It should be noted that any value of $N_1/f_sC_{h1}R_1$ *less* than that obtained above can be used. The natural period will increase; and A_1 and therefore E_{s1}, will increase as $N_1/f_sC_{h1}R_1$ is decreased. No change will take place in the permissible range of δ_{d1} or δ_{i1}. However, the percentage variations in the amplitude of E_{s1} that will maintain the correct order of division always decreases.

The MV as designed so far satisfies Conditions 1 and 3. It is desirable to show in the general case, if possible, that values of $N_1/f_sC_{h1}R_1$ which satisfy these conditions also always satisfy Condition 2. This proof is given in Appendix IV. Use of the curves of Fig. 2.6, 2.7, and 2.8 is best illustrated by means of examples.

Example III

A symmetrical MV is to divide 6000 cps by 14. The tube is a type 6SN7-GT. E_{bb} is 180 v. Design the MV to permit the maximum allowable variations equally plus and minus from the nominal values of the time constants. What is the magnitude of these permissible variations? What value of E_s should be used?

Solution:

a) Each tube must divide by 7, because the MV is symmetrical. From Fig. 2.6 the maximum value of $\delta_{d=i}$ that can be provided for is 7.7 percent.

b) Choose $R_{L1} = R_{L2} = 20{,}000$ ohms. Then from Fig. 1.7, $k_1 = k_2 = 0.72$. Reading from Fig. 1.8, $\mu_{co1} = \mu_{co2} = 12.4$. For $k_2\mu_{co1} = 9$, Fig. 2.7 gives $N_1/f_sC_{h1}R_1 = 2.2$.

c) To provide a longer natural than controlled period and allow δ_{d1} percent decrease in $C_{h1}R_1$ without the natural period becoming shorter than the controlled period, multiply 2.2 by $0.8(1 - \delta_{d1})$. This gives $N_1/f_sC_{h1}R_1 = 2.2 \times 0.8(1 - 0.077) = 1.62$.

d) By Eq. 2.10, $\log_e[9/(1 + A_1)] = 1.62/(1 + 0.077) = 1.51$.

Reading Fig. 2.8,

$$9/(1 + A_1) = 4.53$$
$$A_1 = 0.99$$
$$E_{s1} = A_1E_{co1} = 0.99 \times 180/\mu_{co1} = 14.4 \text{ volts.}$$

Regardless of where the synchronizing voltage is injected into the circuit, its effective value as referred to the grid of V_1 must be 14.4 peak volts.

e) Before selecting values for C_{h1} and R_1, (1.9) should be checked for the maximum value of C_{h1}.

$$C_{h1max} = \frac{T_2}{5} \times \frac{1}{\left(R_{L2} + \dfrac{R_{d1}\,R_{g1}}{R_{d1} + R_{g1}}\right)}$$
$$= \frac{7}{5 \times 6 \times 10^3} \times \frac{1}{20 \times 10^3 + 1.5 \times 10^3} = 0.0108\ \mu\text{f.}$$

If C_{h1} is chosen as 0.001 μf,

$R_1 = N_1/1.62f_sC_{h1} = 7/1.62 \times 6 \times 10^3 \times 0.001 \times 10^{-6} = 720{,}000$ ohms.

This R_1 is the total effective resistance in the discharge circuit of C_{h1}. Therefore

$$R_1 = R_{d1} + R_{L2}R_{b2}/(R_{L2} + R_{b2}) = R_{d1} + 4.35 \times 10^3 \text{ ohms.}$$

Solving for R_{d1} gives 716,000 ohms.

f) Since the MV is to be symmetrical, $C_{h2} = 0.001\ \mu$f, $R_{d2} = 716{,}000$ ohms and $E_{s2} = 14.4$ volts.

Fig. 2.9 is a schematic diagram of the multivibrator.

Figure 2.10 is a plot of the exponential grid voltage plus synchronizing voltage for V_1 of this example. Since the MV is symmetrical, the plots for V_1 and V_2 are identical. The solid curve is a plot of $-k_2E_{bb}\ \exp(-n/f_sC_{h1}R_1)$ and n takes on values from 0 to 7. Synchronizing pulses shown as light, solid lines are associated with this curve. The 7th pulse synchronizes the MV. A plot of $-k_2E_{bb}\ \exp[-n/f_sC_{h1}R_1(1 - \delta_{d1})]$, which represents the case of δ_{d1} percent decrease of $C_{h1}R_1$ is shown as the curve made up of short dash lines. The dashed extensions of the solid synchronizing pulses indicate the heights reached by the pulses when they are added to the dashed curve. Note that any further decrease of $C_{h1}R_1$ would permit the 6th pulse to trip the circuit. There is no possibility of the natural period occur-

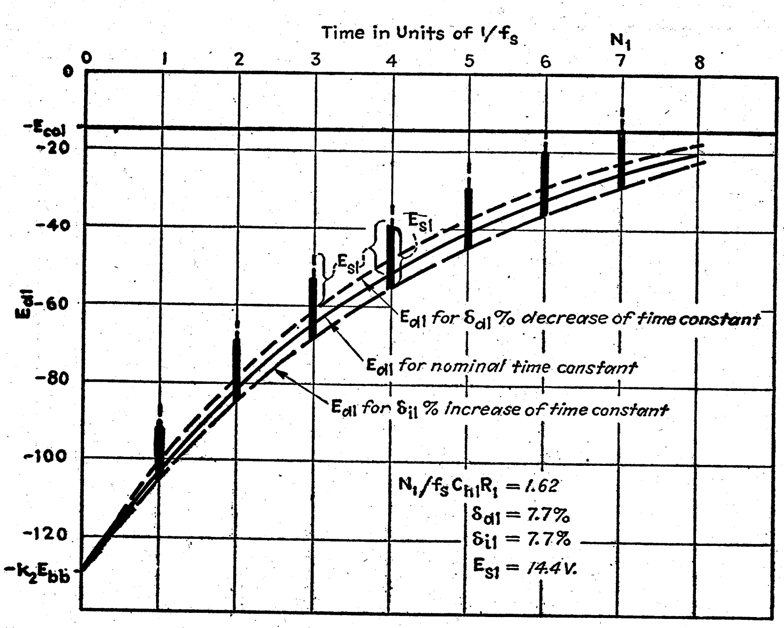


FIG. 2.10—Plot of the exponential grid voltage plus synchronizing pulses for V_1 in the multivibrator designed in Example III. Since the MV is symmetrical, this plot is the same for V_2

ring between the 6th and 7th synchronizing pulses, because the 6th pulse is of greater amplitude than the increase of E_{d1} between the 6th and 7th pulses. Therefore, with a δ_{d1} percent decrease of $C_{h1}R_1$, the 7th pulse is still the one that synchronizes the MV.

The long dash curve is a plot of $-k_2E_{bb}\ \exp\ [-n/f_sC_{h1}R_1(1 + \delta_{i1})]$. This curve represents the condition for δ_{i1} percent increase of $C_{h1}R_1$. The heavy, solid pulse representing the same synchronizing voltage as before added to this curve, is still sufficient to trip the MV at $n = 7$. However, any further increase of $C_{h1}R_1$ would cause the circuit to divide by 8 instead of 7.

The value $N_1/f_sC_{h1}R_1 = 1.62$ as obtained in the design, was used in plotting Fig. 2.10. As has been explained, this represents the maximum value of this factor that should be used. Figure 2.11 is a plot of this example for $N_1/f_sC_{h1}R_1 = 0.81$, i.e., one-half the value used on Fig. 2.10. Note that while the synchronizing voltage had to be increased from 14.4 volts to 46.8 volts, its permissible variation in volts has remained approximately the same. Therefore, the percentage tolerance permissible in the magnitude of the synchronizing voltage has been decreased considerably. The larger magnitude of E_s is of value in certain cases of noise in the synchronizing circuit.

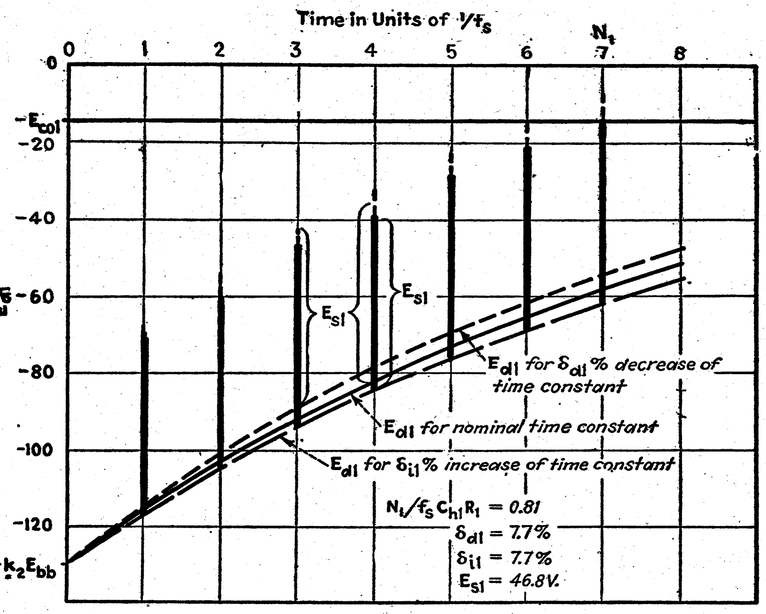


FIG. 2.11—Identical to Fig. 2.10, except for the smaller value of $N_1/f_sC_{h1}R_1$ and the larger required synchronizing voltage

Example IV

A type 6N7 tube is to be used in an MV to divide 300 cps by 8. A 40/60 plate voltage waveshape is desired. It is expected that the greatest changes in the time constants will be in an increasing direction. Provision should be made to allow for a 15 percent increase in each time constant, if this will still permit a reasonable margin for decrease. E_{bb} is 200 v, $E_{co1} = E_{co2} = 10$ v, $R_{L1} = R_{L2} = 60{,}000$ ohms, $R_{b1} = R_{b2} = 28{,}000$ ohms (from the I_b vs. E_b curves).

What percentage decrease in the time constants can be allowed? Give some information on the method of synchronizing this MV.

Solution:

(a) To provide a 40/60 output voltage waveshape, V_1 must divide by $8 \times 4/10 = 3.2$ and V_2 must divide by 4.8.

(b) Reference to Fig. 2.6 indicates that 26 percent is the maximum increase in the value of the time constant that can be tolerated in the section of the MV that divides by 4.8. Because the percentage variation permissible in the time constant increases as the order of division decreases, and because the other section of the MV divides by only 3.2, it is possible to provide for 15 percent increase as well as some decrease from the nominal value of each time constant. Substituting $\delta_i = 0.15$ in (2.8) and solving for the value of δ_d that results in an equality gives $\delta_{d1} = 20.7$ percent for $N_1 = 3.2$ and $\delta_{d2} = 8.75$ percent for $N_2 = 4.8$.

(c) $k_1 = k_2 = 1/(1 + R_b/R_L) = 0.68$.

(d) $\mu_{co1} = \mu_{co2} = 200/10 = 20$.

(e) From Fig. 2.7 for $k_2\mu_{co1} = 13.6$, $N_1/f_sC_hR_1 = 2.61$.

(f) $(N_1/f_sC_{h1}R_1)\,[0.8(1 - \delta_{d1})] = 2.61 \times 0.8 \times 0.79 = 1.65$.

(g) $(N_2/f_sC_{h2}R_2)\,[0.8(1 - \delta_{d2})] = 2.61 \times 0.8 \times 0.91 = 1.90$.

(h) Use Fig. 2.8 to solve for A_1 and A_2 from which the values of synchronizing voltage required for V_1 and V_2 can be found. For $N_1/f_sC_{h1}R_1(1 + \delta_{i1}) = 1.65/1.15 = 1.44$, Fig. 2.8 gives $k_2\mu_{co1}/(1 + A_1) = 4.22$. Therefore, $A_1 = 2.22$ and $E_{s1} = A_1E_{co1} = 22.2$ volts. Similarly $A_2 = 1.61$ and $E_{s2} = 16.1$ volts.

(i) Solve (1.9) to find the maximum values of C_{h1} and C_{h2}.

$$C_{h1\max} = \left[\frac{T_2}{5}\right]\left[1/\left(R_{L2} + \frac{R_{d1}\,R_{g1}}{R_{d1} + R_{g1}}\right)\right]$$
$$= [4.8/300 \times 5]\,[1/(60 \times 10^3 + 1.5 \times 10^3)]$$
$$= 0.052\ uf.$$

It is apparent that $C_{h2\max} = (3.2/4.8)C_{h1\max}$. Hence $C_{h2\max} = 0.0347$ μf.

(j) Choose $C_{h1} = 0.01$ μf. Then since $N_1/f_sC_{h1}R_1 = 1.65$, $R_1 = N_1/1.65f_sC_{h1} = 3.2/1.65 \times 300 \times 0.01 \times 10^{-6} = 647{,}000$ ohms.

$$R_{d1} = R_1 - \frac{R_{b2}\,R_{L2}}{R_{b2} + R_{L2}} = 647 \times 10^3 - 19 \times 10^3 = 628{,}000 \text{ ohms.}$$

Similarly if C_{h2} is selected as 0.01 μf, then $R_{d2} = 823{,}000$ ohms.

Figure 2.12 is a schematic diagram of the MV. Because the order of division of each tube includes a fraction of a period of the synchronizing frequency, it is necessary to supply the synchronizing pulses to V_1 and V_2 in different phases. By Eq. (2.2) this phase difference must be $360 \times 0.2 = 72$ deg (or $360 \times 0.8 = 288$ deg which is the same result).

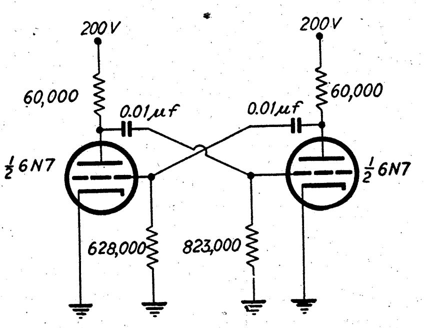


FIG. 2.12—Multivibrator designed in Example IV

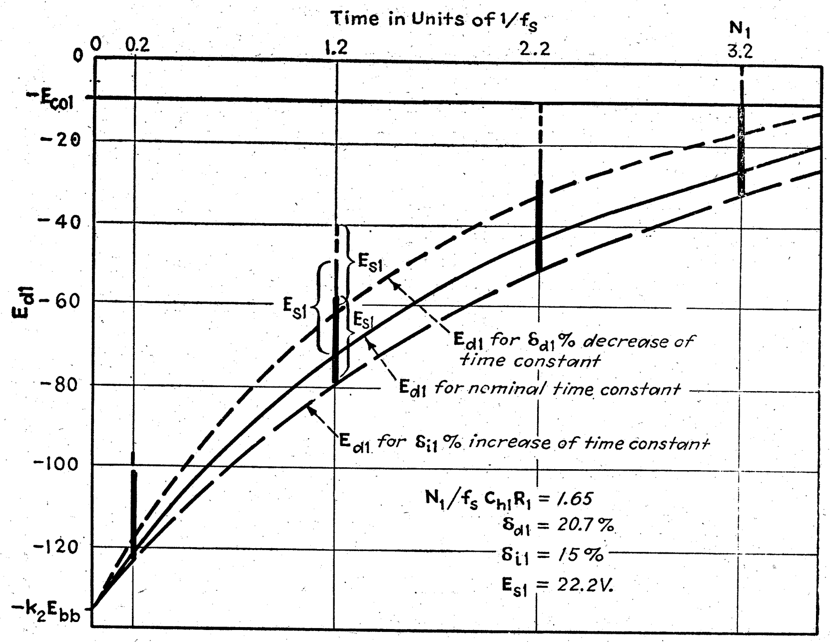


FIG. 2.13—Plot of the exponential grid voltage plus synchronizing pulses for V_1 of the MV designed in Example IV. Note that the first pulse occurs 0.2 T_s second after V_1 becomes nonconducting. This condition is obtained by properly phasing these pulses with respect to those supplied to V_2

the proper order of division. In such a case, the same amplitude of synchronizing voltage is provided for both tubes. Therefore, the value used will usually be optimum for only one of the tubes. Independent adjustment of the synchronizing voltage supplied to each tube becomes more desirable with increasing value of N.

Since the exact value of $C_{h1}R_1$ has been shown to be unimportant as long as it is larger than a certain minimum, C_{h1} and R_{d1} can be selected as the nearest standard sizes. In rounding off condenser and resistor values, it is preferable to choose the standard value on the high side of the calculated one. The only modification that this rounding off process necessitates in the MV design is in the amplitude of the synchronizing voltage. The new magnitude of E_{s1} could be easily calculated, but this is not necessary if the synchronizing pulse amplitude is adjusted according to the procedure outlined. Whatever value of C_{h1} is used, it should be shunted with a capacity $\delta_i C_{h1}$ while the synchronizing voltage is being adjusted.

Further, if the amplitude of the synchronizing voltage can be adjusted independently to each tube, then it is not necessary to select equal or exact values for C_{h1} and C_{h2} or R_{d1} and R_{d2}. All that needs to be known is δ_{i1} percent of $C_{h1}R_1$, and this is always δ_{i1} percent of C_{h1}.

Figures 2.13 and 2.14 are plots of the grid voltages of the two tubes of this MV similar to Fig. 2.10 for the symmetrical MV of Example III.

Adjusting the Amplitude of the Synchronizing Voltage

When the design of the MV is completed, the problem becomes that of supplying the calculated optimum value of synchronizing voltage to the circuit. For the reason that with most synchronizing methods some synchronizing voltage finds its way into both the grid and plate circuits of the tube, it is not easy to supply, by direct measurement, the required value of voltage as referred to the grid circuit. A simple and straightforward method of adjusting the synchronizing voltage to its optimum value, and a method which automatically takes into account any discrepancy between the value of μ_{co} for the tube actually in the circuit and the average μ_{co} as used in the calculations is as follows:

By means of a bridge select C_h and R_d. Make a coarse adjustment of the amplitude of the synchronizing voltage such that each section of the MV is dividing by the proper number. Then, if synchronizing voltage is supplied to each tube independently, shunt C_{h1} with a capacity $\delta_{i1}C_{h1}$. Adjust the amplitude of the synchronizing voltage to the minimum value for which the MV continues to operate properly. Then remove $\delta_{i1}C_{h1}$. Repeat the above procedure for C_{h2}, using a shunting capacity $\delta_{i2}C_{h2}$.

In cases where the synchronizing voltage is supplied to both tubes in common, either one or both of the condensers can be paralleled with the capacity $\delta_i C_h$ and the synchronizing voltage adjusted to the minimum value required to maintain

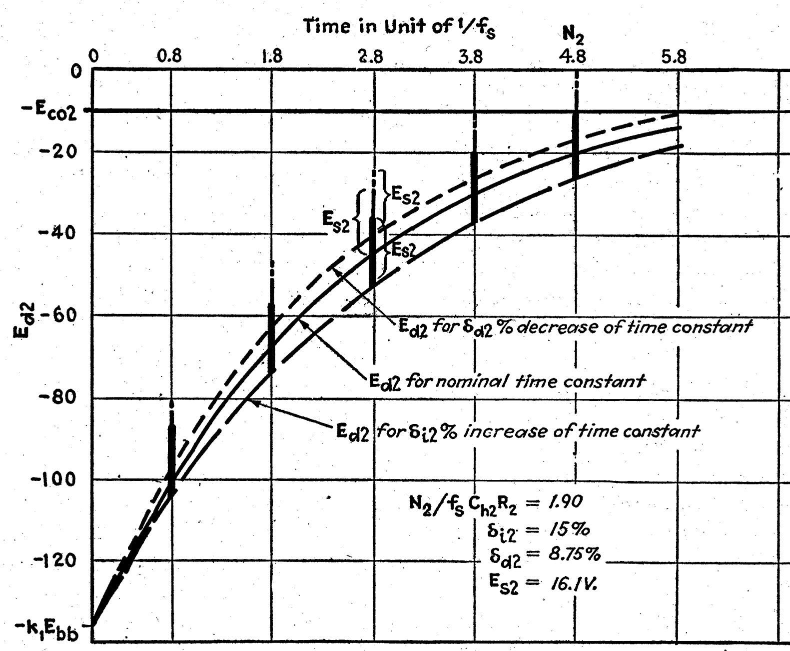


FIG. 2.14—Similar to Fig. 2.13, but applying to the grid voltage of V_2

The experimental adjustment of the synchronizing voltage could be made by decreasing R_1 to the value $R_1(1 - \delta_{d1})$ instead of increasing C_{k1} to $C_{k1}(1 + \delta_{i1})$. However, with some synchronizing methods the effective amplitude of the synchronizing voltage is a function of R_{d1}. Then too, since δ_{d1} percent of R_1 is not equal to δ_{d1} percent of R_{d1}, this method is not as convenient as paralleling C_{k1}. However, for practical purposes R_{d1} can, in some cases, be considered equal to R_1. To decrease R_{d1} by δ_{d1} percent, parallel it with a resistance of $R_{d1}(1 - \delta_{d1})/\delta_{d1}$ ohms. Increase the amplitude of the synchronizing voltage to the value at which any further increase causes the MV to become unstable or to divide by a smaller number. Then remove the shunt resistor.

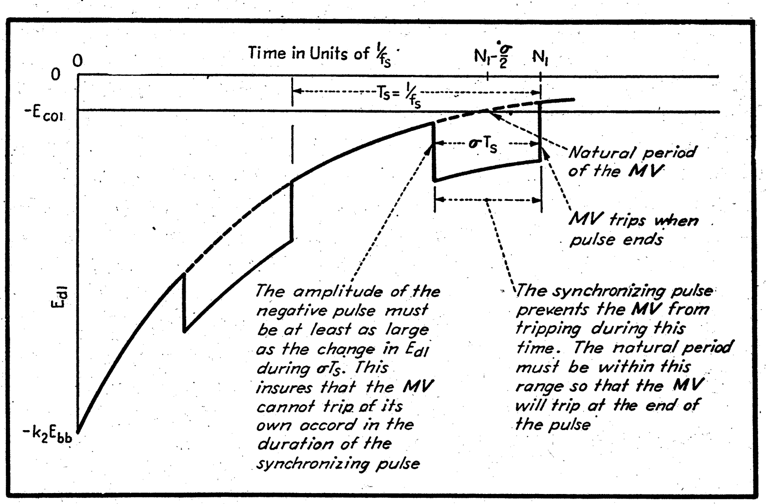
If a common amplitude of synchronizing voltage is to be supplied to the two tubes of an unsymmetrical MV, it is possible to make this one value optimum for both tubes. Remember first that, other things remaining the same for both sections, the amplitude of synchronizing voltage required increases with decreasing N. Consequently, the section of the MV which divides by the smaller number will require the greater amplitude of synchronizing voltage. Hence, the section of the MV having the smaller value of N should be designed first to obtain the magnitude of E_s required for it. Using this value of E_s, work backward to obtain the value of N/f_sC_kR which will make this same E_s optimum for the other section. For example, if $k_1\mu_{co2} = k_2\mu_{co1}$ and if $A_1 = A_2$ which will be true if the same type of tube is used in each section,[10] it is merely necessary to equate $N_2/f_sC_{k2}R_2(1 + \delta_{i2})$ to $N_1/f_sC_{k1}R_1(1 + \delta_{i1})$. This is apparent from Eq. (2.10).

Note that in every case the factors $(1 - \delta_d)$ and $(1 + \delta_i)$ operate on the product f_sC_kR. Therefore, if C_kR remains constant, the MV will divide by the same number over a synchronizing frequency range of minus δ_d percent and plus δ_i percent. In some cases, when adjusting the amplitude of the synchronizing voltage to its optimum value, it is convenient to change the synchronizing frequency to $f_s(1 - \delta_d)$ or $f_s(1 + \delta_i)$ rather than to alter the time constant of the circuit. Then, depending upon whether $f_s(1 - \delta_d)$ or $f_s(1 + \delta_i)$ is used, the amplitude of the synchronizing voltage is set to the maximum or minimum value for which the desired order of division is obtained. If this method of adjusting the amplitude of the synchronizing voltage is accepted, it must be assumed that, in the circuit used and in the frequency range involved, the amplitude of the synchronizing pulse is not a function of its frequency. It is important to remember in connection with discussions of the maximum allowable variations of C_kR or f_s, that the synchronizing voltage is assumed to be a constant amplitude impulse.

Once the design of an MV has been completed for given values of N, f_s and C_kR, the same order of division will be maintained for different synchronizing frequencies provided f_sC_kR is a constant. The optimum amplitude of E_s remains constant for a given order of division as long as f_sC_kR is unchanged.

[10] Assuming $\mu_{co1} = \mu_{co2}$.

Fig. 3.1—The MV can be synchronized by a negative pulse in the grid circuit of the tubes



Negative Synchronizing Pulses

When synchronizing pulses of negative polarity are used, the natural frequency of the MV *must* be greater than the frequency at which it is to be controlled. As indicated in Fig. 3.1, the synchronizing pulse then prevents the tube from conducting at the time determined by the natural period of the circuit. The MV is not permitted to trip until the end of the synchronizing pulse.[12]

It can be seen from Fig. 3.1 that the percent variations permissible in the time constant are proportional to the duration (σT_s) of the synchronizing pulse. For the case illustrated, the natural period occurs at the center of the synchronizing pulse. Therefore, equal positive and negative variations from the nominal $C_{k1}R_1$ product are permissible. The magnitude of these allowable variations, expressed as a fraction of the original $C_{k1}R_1$ is

$$\frac{\sigma T_s/2}{N_1 T_s - \sigma T_s/2} = \frac{\sigma}{2N_1 - \sigma}$$

This result corresponds with Eq. (2.8a) for $\sigma = 1$.

Synchronizing Only One Tube

If the synchronizing voltage is applied to only one tube of the MV, say V_1, then the value of N_1 is a function of V_2, $C_{k2}R_2$, and R_{L2}. The reason for this is that since no synchronizing voltage is applied to V_2, the time T_2 depends entirely upon the values of the components in that section of the circuit. Therefore, T_2 is subject to variation with changes in temperature and humidity as well as with replacement of V_2. If the period of the MV is to be maintained constant under these conditions, the portion T_1 of the MV period must increase or decrease by whatever amounts T_2 decreases or increases.

The net result of synchronizing only one tube is that the magnitudes of the variations that can be tolerated in the individual time constant circuits are approximately one-half those allowable if both tubes are synchronized.

[12] This is in direct contrast to the case of positive synchronizing pulses where the natural frequency *must* be less than the controlled frequency and the MV trips on the front edge of the synchronizing pulse.

Effect of Shunt Capacitances

Shunt capacitances were not considered in the development of the design equations of this analysis; since, in the light of experimental evidence, it was decided they would add unnecessary complication. For example, using 2500 ohm plate load resistors with type 6SN7-GT tubes, multivibrators were designed for frequencies as high as 100 kc. In most cases, experiment checked these designs within ±5 percent.

The frequency of a symmetrical MV operating at 100 kc using a type 6SN7-GT tube with $R_L = 2500$ ohms, $R_d = 80{,}000$ ohms, $C_h = 50$ $\mu\mu$f and $E_{bb} = 200$ volts, decreased 4.5 kc when 10 $\mu\mu$f capacitors were connected from each grid to ground. Connecting 10 $\mu\mu$f from each plate to ground decreased the frequency only 2.5 kc; 100 $\mu\mu$f capacitors similarly connected decreased the frequency 10 kc, i.e., the natural frequency of the MV became 90 kc. When 100 $\mu\mu$f capacitors were connected from each grid to ground, the frequency *increased* to 107 kc. By increasing the value of C_h to 200 $\mu\mu$f and decreasing R_d to 20,000 ohms, the natural frequency of the MV was made to decrease when the additional 100 $\mu\mu$f capacitors were connected from the grids to ground.

The natural frequency of the MV changed 15 kc as E_{bb} (obtained from a regulated power supply) was varied from 150 volts to 300 volts. Frequency increased as E_{bb} was decreased. This is to be expected, since μ_{co} decreases as E_{bb} decreases. (See Fig. 1.8.) The percentage variation of frequency is greater in this case than it would be at lower frequencies where higher values of R_L and therefore of k could be used.

Comparison of Characteristics of Triode and Pentode Tubes in MV Circuits

1. The a-c plate resistance of the average pentode is many times that of the average triode.

2. The d-c plate resistance, $R_b = E_b/I_b$, is of the same order for both triodes and pentodes.

3. For a given value of grid bias, the d-c plate resistance of triodes decreases as the plate voltage is increased.

4. For given values of control grid bias and screen grid voltage, the d-c plate resistance of pentodes increases almost linearly with plate voltage, except for plate voltages near zero. Therefore, I_b versus E_{bb} and I_b versus R_L approximate horizontal lines.

5. In the case of triodes, the d-c plate current is a function of the plate load resistor. The factor k increases slowly with R_L.

6. Because the d-c plate current of a pentode is practically independent of the plate load resistor, the factor k increases directly with R_L. Therefore, the MV can be designed to provide a logarithmic variation of *period* with linear variation of R_L.

The equations developed for triode tubes in multivibrators hold equally well for pentodes, if the proper interpretation is placed upon the quantities involved. For pentode tubes, Eq. (1.5a) becomes

$$\alpha_1 T_1 = \log_e \left(k_2\, \mu_{co1} \right) \left(\frac{I_{b2} R_{L2}}{E_{sg2}} \right) \quad (1.51a)$$

where E_{sg} = Screen grid voltage. (Assumed constant)

$$k_2 = \left[\frac{1}{1 + \frac{1}{R_{d1}} \left(\frac{R_{b2} R_{L2}}{R_{b2} + R_{L2}} \right)} \right].$$

$\mu_{co1} = E_{sg1}/E_{co1}$.

I_{b2} = d-c plate current through V_2.

When interpreting Eq. (1.51a), one should remember that I_{b2} is a function of E_{sg2}.

Conclusions

The most important conclusions may be summarized as follows:

1. The percentage variation in the natural period of an MV through which a given order of division can be maintained is not a function of the tube type or of the ratio of the controlled period to the natural period; rather, it depends only upon the order of division.

2. The required amplitude of synchronizing pulse increases rapidly with decreasing ratio of controlled to natural period.

3. The number of volts variation in the amplitude of the synchronizing pulse for which the order of division remains constant varies slowly with the ratio of the controlled to the natural period. Consequently, the allowable *percentage* variation in the amplitude of the synchronizing pulse decreases rapidly with decreasing ratio of controlled to natural period.

4. If only one tube of an MV is synchronized, the percentage variation that can be tolerated in the time-constant circuits is approximately half that permissible if both tubes are synchronized.

5. The preceding conclusions are based upon the use of a positive impulse of synchronizing voltage. Use of rectangular synchronizing pulses of finite duration is equivalent, in its effect upon the variations that can be tolerated in the time constant circuits and in the amplitude of the synchronizing pulse, to an increase in the order of division.

Acknowledgment

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Appendix I

A MV with a natural frequency of 50 cps was built using a 6SN7-GT tube. Type 884 tubes were used in blocking oscillator circuits to supply short pulses of negative polarity in series with the plate load resistor of each MV tube. The single polarity pulses prevented interlocking of the synchronizing voltage controls. The 884's were driven by pulses obtained after limiting the sinusoidal tone supplied by a variable frequency oscillator. Provision was made to provide a zero or 180 deg. phase relation between the trains of pulses supplied to the MV tubes, V_1 and V_2, by the 884 tubes. Thus, the order of division of V_1 and V_2 could be integral, or it could include the fraction 0.5. See Eq. (2.2). The amplitude of the synchronizing pulses was variable from zero to greater than k_1E_{bb} or k_2E_{bb}. If a-c ripple or feedback voltages are superimposed upon E_{bb}, the values of δ_{d1} and δ_{11} will be reduced. This is especially true in cases where such voltages are not synchronous with the MV frequency. Hence, the plate supply voltage was obtained from a *well regulated and filtered* pack. The amplitude and duration of the synchronizing pulses supplied to the MV was, for practical purposes, independent of frequency in the range employed.

Values of δ_{d1} and δ_{11} calculated by Eqs. (2.8b) and (2.8c) are tabulated against measured values in Table II. To check δ_{11}, the synchronizing voltage was adjusted to the maximum value for which V_1 continued to divide by N_1. Then f_s was increased to the maximum value for which the selected order of division (N_1) was maintained.

The synchronizing pulses supplied to V_2 were adjusted such that N_2 equaled N_1-1 for the small values of N_1, and N_2-2 for the larger values of N_1. The maximum allowable variation in f_s for a MV is determined by the section with the greater order of division. Thus, the adjustment of E_{s2} did not have to be made with the same care as that of E_{s1}. Further, a better check on the theory was obtained, because only one section of the MV limited the value of δ being measured. To measure δ_{d1}, the synchronizing pulse was adjusted to the minimum amplitude for which V_1 divided by N_1, and f_s was then decreased to the minimum frequency for which the order of division remained at N_1.

It is apparent that having measured either δ_{d1} or δ_{11}, all the information needed to calculate the other is at hand. For example, in checking either δ_{d1} or δ_{11}, two frequencies, f_{max} and f_{min}, are obtained. Then $\delta_{d1} = (f_{max} - f_{min})/f_{max}$ and $\delta_{11} = (f_{max} - f_{min})/f_{min}$. However, each value in Table II was calculated from an individual measurement, as previously outlined.

TABLE II

N_1	$\delta_{11} = 1/(N_1-1)$ Calculated	Measured	$\delta_{d1} = 1/N_1$ Calculated	Measured
1.5	2.00	2.11	0.667	0.680
2.0	1.00	0.935	0.500	0.487
2.5	0.667	0.660	0.400	0.392
3.0	0.500	0.487	0.333	0.325
3.5	0.400	0.397	0.286	0.280
4.0	0.333	0.331	0.250	0.238
5.0	0.250	0.243	0.200	0.198
6.0	0.200	0.196	0.167	0.162
7.0	0.167	0.166	0.143	0.140
8.0	0.143	0.140	0.125	0.125
9.0	0.125	0.122	0.111	0.108
10.0	0.111	0.110	0.100	0.099
12.0	0.091	0.092	0.083	0.084
14.0	0.077	0.074	0.071	0.070
16.0	0.067	0.065	0.063	0.060
20.0	0.053	0.050	0.050	0.049

Measured vs. calculated values of δ_{11} and δ_{d1}. The natural frequency of the symmetrical multivibractor was 50 cps and f_s covered the range of 200 to 3000 cps.

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Symbols Used In This Paper

The numerals 1 or 2 appended to a subscript indicate the section of the circuit in which the component or voltage is located. See Fig. 1.0.

$A = E_s/E_{co}$.

C_k = Plate to grid coupling capacitor.

E_{bb} = Plate supply voltage.

E_{cc} = Grid supply voltage.

E_{co} = *Magnitude* of d-c grid voltage required for plate current cutoff.

E_s = Peak amplitude of the synchronizing voltage. Except where it is specifically stated to the contrary, E_s is considered positive.

f_s = Frequency of the synchronizing voltage.

$$k_1 = \left[\frac{1}{1 + \frac{R_{b1}}{R_{L1}}}\right]\left[\frac{R_{d2}}{R_{d2} + \frac{R_{b1} R_{L1}}{R_{b1} + R_{L1}}}\right].$$

$$k_2 = \left[\frac{1}{1 + \frac{R_{b2}}{R_{L2}}}\right]\left[\frac{R_{d1}}{R_{d1} + \frac{R_{b2} R_{L2}}{R_{b2} + R_{L2}}}\right].$$

MV = Multivibrator.

N = Order of division of *one synchronized section* of multivibrator. See Fig. 1.0.

$N_1 = T_1/T_s$ = Order of division of V_1 and its associated components.

r = Fractional part of N.

r_1 = Fractional part of N_1.

$$R_1 = \left[R_{d1} + \frac{R_{b2} R_{L2}}{R_{b2} + R_{L2}}\right]$$

$$R_2 = \left[R_{d2} + \frac{R_{b1} R_{L1}}{R_{b1} + R_{L1}}\right]$$

R_b = d-c plate resistance of the tube.

R_d = Grid resistor.

R_g = Grid-cathode resistance of the tube.

R_L = Plate resistor.

t = time.

T_1 = Non-conducting time of V_1.

T_2 = Non-conducting time of V_2.

$T_{MV} = T_1 + T_2$ = Period of the multivibrator.

T_s = Period of the synchronizing voltage.

$$\alpha_1 = \frac{1}{C_{k1}\left(R_{d1} + \frac{R_{b2} R_{L2}}{R_{b2} + R_{L2}}\right)}.$$

$$\alpha_2 = \frac{1}{C_{k2}\left(R_{d2} + \frac{R_{b1} R_{L1}}{R_{b1} + R_{L1}}\right)}.$$

$100\delta_d$ = Percent decrease from nominal value of the product $f_s\, C_{k1}\, R_1$

$100\delta_i$ = Percent increase from nominal value of the product $f_s\, C_{k1}\, R_1$.

$\mu_{co} = E_{bb}/E_{co}$. Usually $\frac{1}{2}$ to $\frac{3}{4}$ of the rated amplification factor of the tube.

σ = (Width of rectangular synchronizing pulse)$/T_S$.

Many of the equations are written only for section 1 of the MV. The corresponding equation for section 2 can be obtained in every case by replacing the sub-numeral 1 with 2 and the sub-numeral 2 with 1.