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DISCUSSION

of

Professor L. A. Hazeltine's Paper on

"Losses and Capacity of Multilayer Coils"

Presented before the Radio Club of America

February 16, 1917

(This paper was published in the April, 1917 issue of "QST.")

Edwin H. Armstrong: As a matter of history, I believe that multilayer coils were used by Dr. Stone as early as 1906 and also by Dr. deForest at about the same time This type of coil was also used by the Poulsen company on the Pacific coast in 1911.

I suppose that we should all feel fully qualified to construct coils from the formulas given in this paper, but I believe that dimensions, suggested by Professor Hazeltine, would be more useful.

L. A. Hazeltine: There is no one ideal proportion for multilayer coils. In Figure 1 (taken from my written discussion of Mr. Eastham's paper previously referred to) are shown three representative proportions of coils, all having the same value of

$$\frac{£}{r}$$

and each having the minimum capacity for a given copper volume. The approximate relative capacities and copper volumes are denoted by C and V respectively; and the ratio of insulation thickness between layers to pitch of layer is denoted by x. It is believed that the best form of coil for any particular case will lie between those of (a) and (c), the former being used when the capacity of the coil is of relatively little importance (as when the external tuning capacity is large), the latter when the capacity is to be kept as small as practicable. These proportions are based on the assumption that eddy-current loss is of negligible importance. When eddy-current losses are of controlling influence in the design, they may be reduced: (a) by using smaller wire or strands; (b) by using a larger radius and therefore fewer turns; and (c) by increasing the perimeter of the winding section (i. e., by making the depth much greater than the width or vice versa). I cannot give any general directions for this case, other than

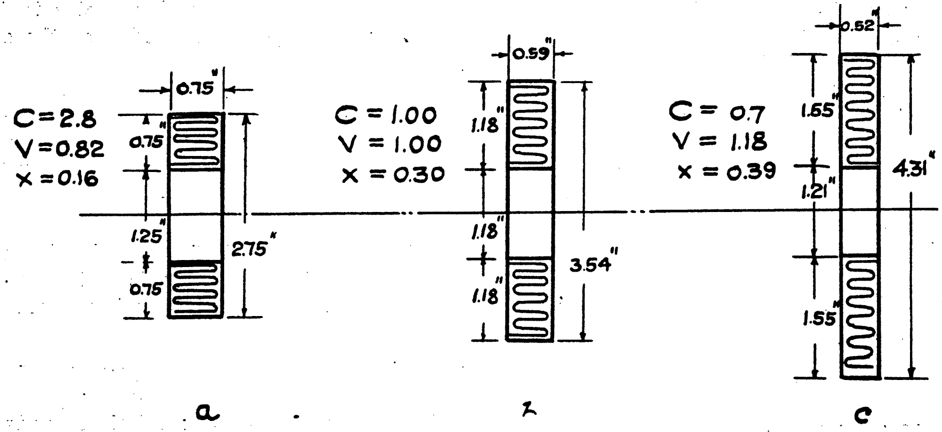


Figure 1

to apply the formulas of my paper to various assumed proportions and to choose that giving the best characteristics. In making such calculation the wave length and the capacity will be ordinarily known or assumed and the required self-inductance is then given by formula (10). (The equation numbers and notation are those of the paper under discussion, as published in the April, 1917 issue of "QST." This is then compared with the self-inductance for the assumed coil proportions, which may be calculated from the following approximate formula (also given in my discussion of Mr. Eastham's paper):

$$£ = \frac{0.0008\ a^2\ N^2}{6a+9b+10c} \text{ millihenries}$$

When the eddy-current resistance for any reasonable proportions of a multilayer coil is too high, the single-layer type should be employed. The various formulas may be used in this case, though with less accuracy than for multilayer coils, by putting ($c = 0$), except that in formula (1) the first term is zero.

One of the coils here exhibited was intended for use in the reception of signals at wave lengths up to about 15,000 meters. It is wound in a wooden form with 1212 turns of 10—No. 38 Litzendraht in thirty layers, with 6.5 mil paper between layers and has been boiled in paraffin. The dimensions are; mean radius, a, =1.125in.; length, b, =0.76 in.; depth, c, =0.75 in. This was wound over a year ago and the proportions are susceptible of some improvement in the light of more recent studies. I believe the constants of this coil compare as follows with those of a coil constructed by Professor Morecroft for a similar purpose.

	Prof. Hazeltine's Coil	Prof. Morecroft's Coil
£	72.4 millihenries	72. millihenries
C	0.0205 millimicrofarads	0.032 millimicrofarads
Natural Wave Length	2300 meters	2850 meters
r_{dc}	48 ohms	80 ohms
r at 12,000 m.	63 ohms	140 ohms

A second coil shown here has about 16.000 turns of No. 34 single-silk-covered wire in sixty layers with 3.5 mil paper between layers. It has no mechanical support other than the paraffin with which it is impregnated. The dimensions are: mean radius, a, =1.11 in.; length, b, =0.26 in.; depth, c, =0.72 in. Its self-inductance is about 160 millihenries with the very low capacity of about 0.007 millimicrofarads.

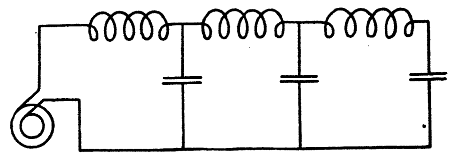


Figure 2

This coil is particularly useful in the wing circuit of an audion (as in Figure 3 of this paper) to produce an oscillation in circuits of relatively large loss or with a stubborn audion.

Edwin H. Armstrong: It is interesting to note that while these coils are here considered for use in making an audion oscillate, an engineer of the Western Electric Company is rumored to have obtained a patent on an idea for stopping such oscillation.

L. A. Hazeltine: This is accomplished merely by the inversion of the coil in question.

John H. Morecroft: In considering the matter of inductance and capacity in this manner a thorough knowledge of the subject is very essential. The development of complicated formulas is quite useless if such a knowledge is not had regarding these subjects to which the formulas apply.

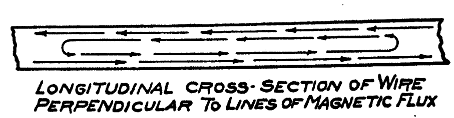


Figure 3

I would like to outline an elementary view of inductance and capacity, and desire to refer to Figure 2. Assumed that the inductance, capacity and resistance of this current are measured at various frequencies. At certain frequencies only one of these characteristics seem to exist. Considering such results it is difficult to believe that the application of a formula is justified in the case of a coil.

I wish to call attention to the fact that there have been differing opinions regarding what the elements of predominant capacity in a coil were; also to the fact that the Bureau of Standards has found that the resistance of an antenna may be varied by placing a stick of wood in the electrostatic field thereof. In general it seems advisable to keep dielectrics other than air out of strong electrostatic fields.

Speaking as one working in the general field of alternating currents, it appears most remarkable and in fact unbelieveable that it is possible to obtain zero skin effect and yet such large eddy currents. May I ask where these eddy currents will flow? And what is the area of the magnetic circuit?

L. A. Hazeltine: The eddy currents flow longitudinally through one half of the wire and return through the other half, as illustrated in Figure 3. The circuit is completed by current flowing across the wire in weak parts of the magnetic field, or even at the leads. The area of the magnetic circuit for the flux which induces the eddy current at the outermost filaments is the longitudinal cross-section of the wire represented in Figure 3.

John H. Morecroft: But in this case the length of eddy current path is twice the length of the wire in the coil. Only one current can exist in the wire. I am confident that you will find that the "eddy current" losses are merely due to redistribution of actual current, **THE** current in the wire. May I inquire as to how Professor Hazeltine's measurements were made? Mine were made with a Wheatstone bridge.

L. A. Hazeltine: My measurements were made as described in the paper. The Wheatstone bridge is not generally considered reliable at the high frequencies to which my tests extended (about 1,000,000 cycles per sec., or 300 meters wave length) on account of stray capacity effects. Also, the effective resistance of a coil, as connected to a Wheatstone bridge, would usually exceed **one megohm** at its natural frequency, and would change by an extraordinary amount with a slight change in frequency; so measurements by the Wheatstone bridge at the natural frequency would be practically impossible. With my method measurements at the natural frequency were quite as convenient as at lower frequencies.

John H. Morecroft: It seems to me that this idea of Prof. Hazeltine's blaming the increased resistance of the coil upon eddy currents in the wire, there being at the same time negligible skin effect in the wire, is more likely to confuse one than to clarify the phenomena. It indeed seems to me to be erroneous.

It will be noted that these so-called eddy currents flow the whole length of the conductor constituting the coil, being in one direction on one side of the conductor and in the reverse direction on the other side of the conductor. And that these eddy currents are at 90 degrees phase difference from the so-called working current. (Altho I do not like the term working current I use it to make my remarks directly applicable to Prof. Hazeltine's explanation.) Now evidently if the eddy current flows the whole length of the wire and there is at the same time the working current flowing uniformly thruout the cross-section of the conductor, the combination will be a non-uniform current distribution in the wire; there will be a greater current density on the side of the wire closer to the center of the coil. But such a redistribution of the current is nothing but the skin effect; it is well known that the skin effect in conductors wound into coils is not an effect symmetrical with respect to the axis of the wire, but makes the current crowd into one side of the wire. This increase in skin effect, due to mutual induction between the various parts of the wire making up the coil, has been approximately figured for single layer coils but I recollect no solution for multiple layer coils. I much doubt if anyone has solved the problem.

Prof. Hazeltine notes the phase of these eddy currents with respect to the working current as lagging 90 degrees. It is to be remembered that all skin effects have strong's paper presented before the Institute of Radio Engineers, March 3, 1915. Immediately after the presentation of this paper, I worked out mathematically that this should occur in certain cases, and verified the theory by experiments at audio frequencies, where there could be no doubt of the change in frequency or the existence of beats. Even at radio frequencies the click and sudden drop in wing current due to change in frequency are readily distinguishable from those due to the start of an oscillation.

In regard to the calculation of r_2, it is common knowledge that energies of oscillation of two tuned coupled circuits are equal, provided the coupling is not too loose. There is thus a definite resistance in one circuit which will be equivalent to any given resistance in the other—contrary to a statement of Professor Morecroft. In the circuit used (Figure 3 of this paper), this is exactly true as long as the coupling exceeds the power factor (resistance divided by reactance) of the test circuit, which means less than one percent. Tests could be made at any coupling closer than this critical value, were it not for the tendency of the audion to "hold on" to the original frequency after the tuning point is passed. Practically, then, tests were made at the critical coupling, which is not difficult to determine. With still looser coupling, of course the computed resistance would be too low. The correctness of the method was checked by making measurements with and without a known resistance in series in the test circuit. I expect to give the mathematics of this circuit in a paper before the Institute of Radio Engineers in the near future.

I do not claim that this method of measurement is the best possible. It is simply a method available to practically any experimenter as it employs common apparatus only. It is my hope that others will try it out and develop the technique. My results with crude apparatus were probably accurate within 10%, but the method should be capable of giving much greater precision—say within 2% under good conditions.

President Armstrong's remarks on the early use of multilayer coils in radio work add evidence to the contention in the introduction of my paper—that they have not the novelty supposed by Professor Morecroft.

Mr. Austen Curtis's experience with coils of different self-inductance indicates that his coils must have had considerable eddy-current loss. I must refer to the conclusion of my paper, that efficient radio coils must be made with **stranded wire.** When this is used, the multilayer type will be preferable to the single-layer type for all wave lengths down to about 1,000 meters. Below this limit, with the commercially available Litzendraht, the single-layer type will usually be preferable.

ERRATA in "LOSSES AND CAPACITY OF MULTILAYER COILS" (February 16, 1917) by Professor L. A. Hazeltine, Proceedings of the Radio Club of America, April, 1917.

In the notation table the following corrections should be noted. The symbol for permittivity (line 9) should be the Greek letter "epsilon" ,as in equation (1). The symbol "I" should precede "current" (line 14) and the four symbols now following "I" should each be moved up one line. On the last line the symbol "x" represents "insulation thickness between layers" divided by "pitch of layer". In equation (18) the expression in brackets should be squared. In Figure 3 a stopping condenser should be inserted in series with the grid.

DISCUSSION

of

Professor J. H. Morecroft's Paper on

"Inductance and Capacity Phenomena"

Presented before the Radio Club of America
January 19, 1917

(This paper was published in the March, 1917, issue of "QST.")

Hawley O. Taylor: Professor Morecroft's observation that the potential wave on an antenna in which a condenser is placed has no node (that is, the potential line seems to jump across the axis of zero potential without touching it) may seem

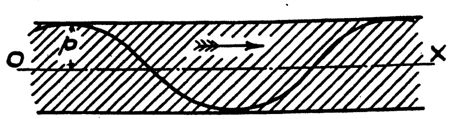


Figure 1

curious but it is borne out by work in other lines of physics. An instance that may be cited is in connection with an acoustical investigation described in an article entitled, "A Direct Method of Finding the Value of Materials as Sound Absorbers," (Taylor, Phys. Rev., Oct., 1915, pp. 270-298).

It must be remembered that a stationary wave cannot be represented by a single curve, since the elements in vibration are displaced equally on both sides of their equilibrium positions, but of two lines placed symmetrically with respect to this axis and constituting the envelope of the maximum displacements of the vibrating elements of the wave.

In the article referred to it is shown that when a train of progressive sound waves of amplitude, p, Figure 1, moves in the positive direction, say, then all points in the medium in the path of the waves move through a distance 2p (shown by the shaded portion). If a second train of progressive waves of the same wave length but of amplitude, r, traveling in the opposite direction meets and combines with this first train then there will be some points in the medium which will move through a greater, and others through a less, distance than before, as shown in Figure 2, shaded portion.

The maximum amplitude (where crest meets crest and trough meets trough) will be $p+r$, and the minimum amplitude (where crest meets trough) will be $p-r$. Such a condition as this is brought about when a train of waves is reflected, those reflected meeting and combining with the on-coming waves forming a stationary wave and may be observed in any branch of physics where waves or vibration may occur. It is thus seen that in general the potential wave on an antenna is nowhere zero except in the special case when waves traveling in opposite direction are of the same amplitude (which cannot occur on reflection.)

The coefficient of absorption of potential on reflection of the waves at the end of the antenna may be computed from the values of the maximum and minimum amplitudes of the stationary wave on the antenna.

If P is the coefficient of reflection, then

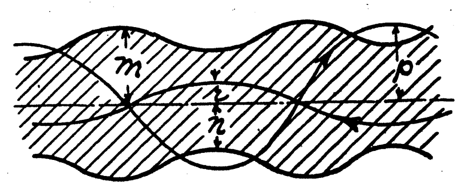


Figure 2

Let m = maximum amplitude of the stationary wave and n = minimum amplitude of the stationary wave

Then $m = p + r$

and $n = p - r$

from which

$$p = \frac{m+n}{2}$$

and

$$r = \frac{m-n}{2}$$

Therefore

$$P = \frac{m-n}{m+n}$$

The coefficient of absorption, oc, of the electric potential on the antenna, would be given by

$$\alpha = 1 - p = 1 - \frac{m-n}{m+n} = \frac{2n}{m+n}$$

L. A. Hazeltine. Professor Morecroft is a firm believer in the idea that phenomena at high frequencies may be studied and described by experiments made at low frequencies. Though this may be legitimate when purely qualitative results are desired, it is likely to mislead when quantitative results are presented, as in his paper. Thus the tests of Professor Morecroft(Figure 1 and 2 of his paper) showed the reactance of the test coil to be of the order of 15 times its resistance; while in any well designed radio coil the reactance will be over 150 times the resistance. Moreover, the ratio of reactance to resistance in some cases enters as a square; for example, his Figure 2 shows a coil at 38% coupling to have an equivalent resistance at the resonant frequency of about 20 times the actual resistance; while in a radio circuit at the same coupling the equivalent resistance will exceed 2000 times the actual resistance. It is, therefore, evident that the curves of Figures 1 and 2 are in their proportions entirely different from those which would be encountered in radio apparatus. This is also evident from the fact that the equivalent resistance, as shown in these curves, remains at a large fraction of its maximum value for a range in frequency of 10%, which indicates so broad a tuning as to be out of question in efficient radio work.

Professor Morecroft proposes definitions of inherent capacity and self-inductance of coils in terms of dielectric and magnetic energies, respectively. These definitions give the result that both capacity and self-inductance will vary with the frequency—usually in a manner that is difficult to determine, either by calculation or by test. These definitions also make inapplicable the usual formula for calculating wave length (or frequency) from capacity and self-inductance. The writer believes that in general the customary method of assuming the self-inductance to be constant and then calculating the effective capacity from the self-inductance and the measured wave length is preferable from a stand point of simplicity. The capacity, alone, will then vary with the frequency, in a manner which can be determined by experiment. Professor Morecroft suggests no way in which his definitions of capacity and self-inductance can be applied to actual measurements.

The following statement appears in Professor Morecroft's paper: "A long single layer solenoid with **no resistance** (italics his) has indeterminate capacity from the standpoint of energy because there is no difference of potential between any of its parts and hence no electrostatic field is set up." This is quite in error, for the difference in potential in a coil is due almost entirely to its alternating magnetic flux, which would remain even if the resistance were zero. In fact the effect of resistance on the value of the potential difference is ordinarily so small as to be negligible. Possibly Professor Morecroft had in mind the conditions with **a direct current** in the coil, but this is not apparent from the context as in the preceding sentence (and, in fact, throughout the entire preceding portion of the paper) he refers to **varying frequency.**

The above discussion omits reference to Professor Morecroft's statements in regard to the supposed novelty of multilayer coils and of his method for calculating their capacity, as this matter is covered in the writer's recent paper before The Radio Club of America, as published in the April, 1917 issue of "QST."

John H. Morecroft: Professor Hazeltine's surmisal that conditions with a direct current in the long single layer solenoid were had in mind is quite correct.

Motional Impedance Circle of the Telephone Receiver*

By Hawley O. Taylor, Ph. D.
(Research Engineer, National Electric Signalling Company)

IN order to find motional impedance, means are required for determining the total impedance of telephone receivers for various frequencies. The ordinary impedance bridge, as shown in Figure 1, is suitable for this purpose.

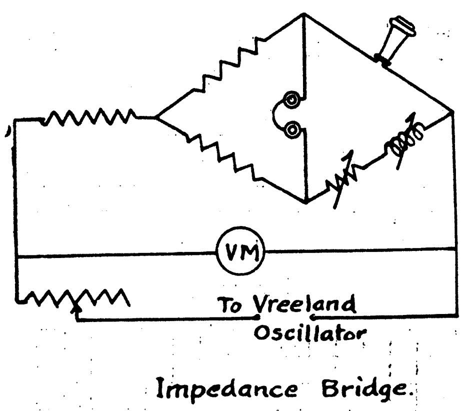


Figure 1

(Bibliography 3, p. 446) The telephone receiver is placed in one arm of the bridge and the resistance and reactance components of its impedance are balanced by adjustment of the resistance and inductance in another arm, as shown, until silence is given by the indicating telephone which spans the bridge. In the observations made, the source of oscillating electromotive force was the Vreeland Oscillator, and the potential across the bridge was maintained constant at about 15 volts.

Readings were made for the damped impedance of the receiver. This is the impedance when the diaphram is not allowed to vibrate. The damping of the diaphram's vibration was accomplished by connecting the diaphram to a large brass cylinder by means of a brass rod; the receiver was supported in a horizontal position and the connection was so made that the diaphram was not strained or deflected from its normal rest position. Impedance observations were made for a range of frequency which included the

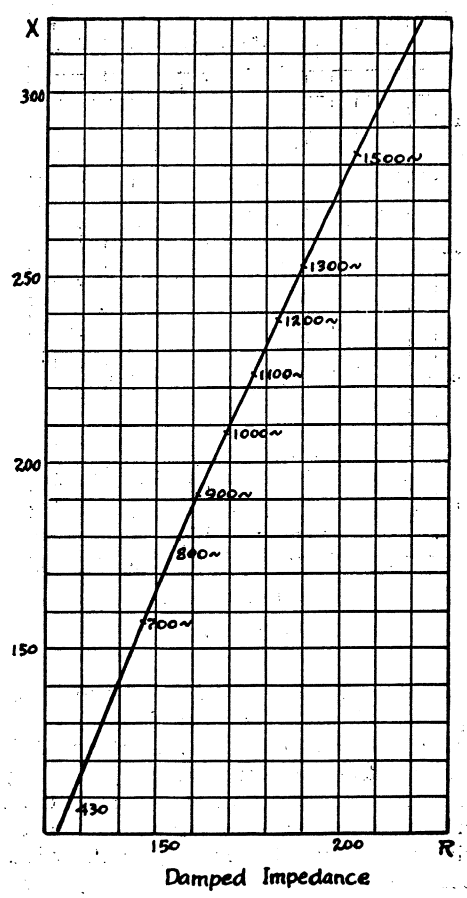


Figure 2

natural frequency of the diaphram, and when the resistance for each frequency was plotted as abscissa and the corresponding reactance as ordinate, a curve was obtained like that shown in Figure 2.

*Presented before THE RADIO CLUB OF AMERICA, March 23, 1917.

The impedance of the telephone receiver at any given frequency is representd by th line joining the origin with the point on the curve corresponding to this frequency.

Following the set of readings for damped impedance, a set was taken for the free impedance of the receiver, that is, when the diaphram was free to vibrate. When the resistance and reactance components of the free impedance were plotted in the same way as those for the damped impedance, a curve was obtained like that shown in Figure 3.

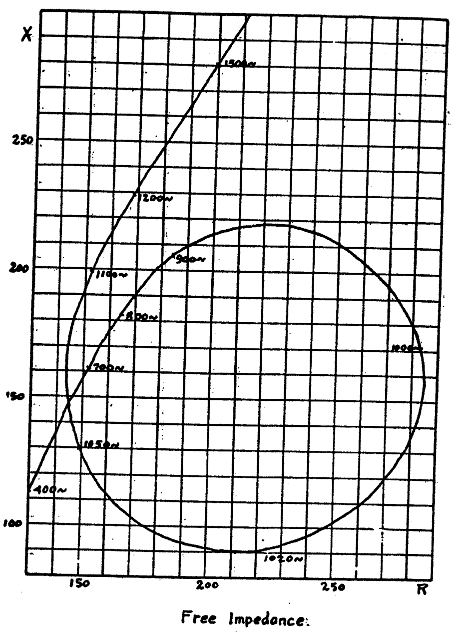


Figure 3

If the curve for free impendance is superimposed upon that for damped impedance and the points in each corresponding to the same frequency joined by straight lines, as shown in Figure 4, these lines will represent the impedance for the given frequency due entirely to the vibrational motion of the diaphram. By drawing these lines radially from a single point, one end being at the point and the line extended in a direction parallel to that in Figure 4, the far ends of the line will lie upon a circle, as shown in Figure 5. This circle is the motional-impedance circle of the telephone receiver, the first account of which was published by Kennelly and Pierce in September, 1912.

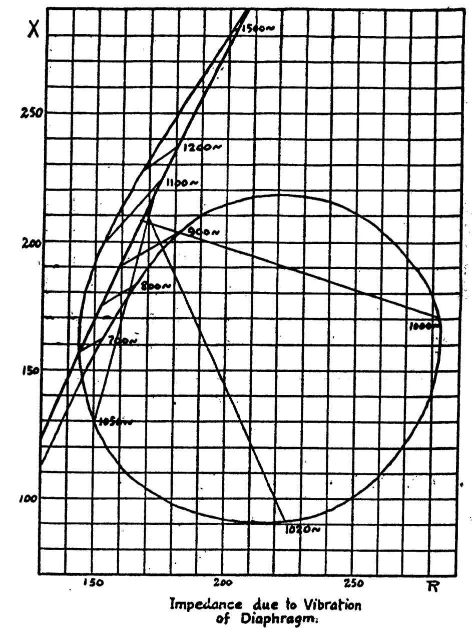


Figure 4

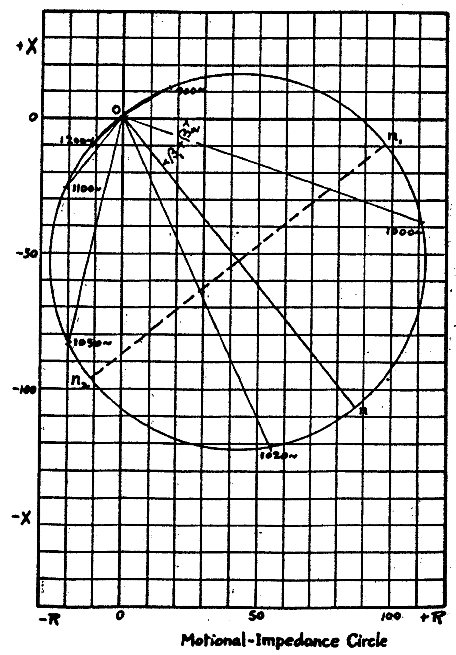


Figure 5

(Bibliography 1.) If the differences between the resistance components of the free and damped impedance are taken for each frequency and plotted as abscissas against the corresponding difference of the reactance components of the motional-impedance as ordinates, the same circle will be obtained, and this is the usual way of handling the data for plotting the motional-impedance circle.

The frequency corresponding to the diameter of this circle is the natural frequency of the diaphram and the angle $(\beta_1+\beta_2)$ which this diameter makes with the resistance axis, represents the lag in phase between the velocity of the diaphram and the current in the telephone coils, this lag being due to hysteresis and eddy current losses and to vibrational variation in the air gap between the diaphram and the pole pieces.

The equation of motion of the diaphram is

$$m\ddot{x} + r\dot{x} + sx = f \qquad (1)$$

where m = equivalent mass of the diaphram (grams) r = mechanical resistance constant (dynes per cm. per sec.) s = elastic constant (dynes per cm.) and f = instantaneous pull on the diaphram (dynes) $= Fe^{jwt}$ where F = the maximum pull at any frequency.

The solution of this equation for any velocity is

$$\dot{x} = \frac{f}{\sqrt{r^2 + (m\omega - \frac{s}{\omega})^2}}$$

+ (a vanishing factor) (2)

This solution corresponds to

$$i = \frac{e}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} \qquad (3)$$

of the well-known electrical equation

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = e \qquad (4)$$

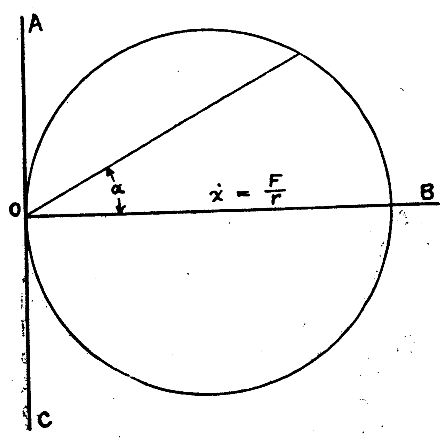
The phase angle, e, between e and i is given by

$$\tan^{-1} e = \frac{L\omega - \frac{1}{C\omega}}{R} \qquad (5)$$

and similiarly the phase angle, α, between f and x in the case of mechanical vibration is given by

$$\tan^{-1} \alpha = \frac{m\omega - \frac{s}{\omega}}{r} \qquad (6)$$

If the phase angle, α, is computed by means of equation (6) for a number of frequencies distributed throughout the motional-impedance circle and the corresponding diaphram velocities are computed by means of equation (2), the curve plotted to polar coordinates between α and x will be a circle, as shown in Figure 6.



Velocity Circle of Telephone Diaphragm.

Figure 6

The diameter corresponds to the velocity when

$$m\omega = \frac{s}{\omega},$$

that is, when

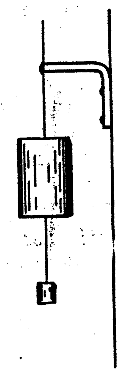
$$\omega = \sqrt{\frac{s}{m}} \qquad (7)$$

The value of this velocity is

$$\dot{x} = \frac{F}{r} \qquad (8)$$

and the frequency corresponding to it is the natural frequency of the diaphram. All other velocities make an angle, α, with the diameter.

The meaning of the velocity circle may be obtained by considering the angular vibration of a system composed of a torsion pendulum, to which is suspended another very small torsion pendulum, as shown in Figure 7. (Bibliography 4, p.



Torsion Pendulum Model for Telephone Receiver Theory

Figure 7

439.) The angular displacement of the large pendulum twists the top of the wire suspension of the small pendulum in the same way that the current in the coils of the telephone receiver produces a pull on the diaphram. The length of the suspension of the large pendulum is first adjusted so that its frequency of vibration is less than the natural frequency of the small pendulum. The vibrations of the two pendulums are then in phase and the amplitude of the small pendulum is very small. Remembering that vibrational velocity leads vibrational amplitude by an angle of $\frac{\pi}{2}$, it is evident that the velocity of the small pendulum is leading the amplitude of the larger pendulum by an angle of $\frac{\pi}{2}$ This is shown by the angle BOA of Figure 6 where OB is the axis to which phase is referred, that is, it represents the vector displacement of the large pendulum which corresponds to the current in the telephone receiver; and OA represents the vector velocity of the small pendulum which corresponds to the vibrational velocity of the telephone diaphram (barring the lag angle $\beta_1 + \beta_2$) Suppose now that the suspension of the large pendulum is shortened so that its frequency is the same as the natural frequency of the small pendulum. Its amplitude of vibration is seen to be very large and to lag behind the displacement of the large pendulum, the vibration of the two pendulums being in quadrature. The velocity of the small pendulum is therefore in phase with the displacement of the large pendulum and the motion of both pendulums is given by the vector OB of the velocity circle, Figure 6. This corresponds in the telephone receiver to the case of resonance when the frequency of the current is adjusted to the natural frequency of the diaphram; the relation between the velocity of the diaphram and the current pulses is indicated by that between the two pendulums. A further shortening of the large pendulum causes its frequency to be greater than the natural frequency of the small pendulum and the amplitude of vibration of the small pendulum is again very small and in phase opposition to the large pendulum. The vector velocity of the small pendulum being at an angle $\frac{\pi}{2}$ ahead of phase opposition is represented in Figure 6, by the line OC. The vibrational velocity of the small pendulum thus leads the amplitude of the large pendulum by an angle of $\frac{\pi}{2}$, in the same way that the vibrational velocity of the telephone diaphram leads the current pulses by an angle of $\frac{\pi}{2}$ when the frequency of the current is greater than the natural frequency of the diaphram. The torsion pendulum model thus reproduces in a leisurely way the vibrational phenomenon of the telephone receiver which is transpiring so rapidly that the eye can not follow it, and all the elements of this phenomenon are represented quantitively by the velocity circle of Figure 6 (except for the lag angle, $\beta_1 + \beta_2$).

The intimate relation between the velocity of the diaphram and the motional-impedance circle are thus clearly shown; in fact the velocity of the diaphram is the cause of the motional impedance circle and for this reason the properties of the motional-impedance circle may be used to enable facts to be discovered concerning the motional constants of the diaphram.

If the amplitude of vibration is the same at all points of its surface, that is, if the diaphram has the motion of a piston, then the equivalent mass, m, is equal to the active mass, M, of the diaphram. The active mass of the diaphram is the total mass of the diaphram within the clamping ring. In the ideal case when the amplitudes along a diameter are given by the Bessel's equation for a diaphram with a fixed boundary, (Bibliography 2, p. 131) the equivalent mass is given by

$$m = .183 \text{ M} \qquad (9)$$

Due to dissymetries in construction and pull, the ideal curve of amplitude is distorted so that the equivalent mass is usually increased. For the diaphram in the usual commercial type of telephone receiver, an average of many motional-impedance circles gives an approximate value of the equivalent mass,

$$m = .3 \text{ M} \qquad (10)$$

The elastic constant, s, is given by

$$s = k \frac{t^3 Y}{D^2} \qquad (11)$$

where t = thickness of diaphram (cms.)
Y = Young's modulus (dynes per cm.)
D = diameter of clamping ring, (cm.)

If therefore, s, t, and D are known for one telephone diaphram and t and D are known for a second telephone diaphram, s for this diaphram may be computed approximately. The following constants were ascertained for a certain telephone diaphram:

D = 5 cm.
t = .03 cms.
S = 40,000,000 dynes per cm.
M = 3.87 grams
m = .3 M = 1.16 grams

The elastic constant for another telephone diaphram is, therefore, given by

$$s = \frac{40,000,000 \times 5^2 \times t^3}{(.03)^3 \times D^2} \qquad (12)$$

Having found m and s approximately, the natural frequency of the diaphram may be computed by means of equation (7).

$$n = \frac{1}{2\pi}\sqrt{\frac{S}{m}}$$

where n = natural frequency

For the particular case given

$$n = \frac{1}{2\pi}\sqrt{\frac{40,000,000}{1.16}} = 935 \text{ cycles}$$

If, in Figure 5, a diameter is constructed perpendicular to that corresponding to the resonance impedance of the receiver, the frequencies corresponding to the quadrantal points so located make it possible to compute the mechanical resistance, r, of the diaphram. (Bibliography 1, p. 146). Let n_1 ,be the lower, and n_2 the higher frequency, then

$$\Delta = \pi(n_2 - n_1) \qquad (13)$$

where

$$\Delta = \left(= \frac{r}{2m}\right) \quad \text{(hyp. rad. sec.)}$$

the damping factor of the diaphram

Therefore

$$\frac{r}{m} = 2^{\pi} (n_2 - n_1) \qquad (14)$$

where $(n_1\text{-}n_2)$ represents the range of the resonance.

Thus, when m is constant, the range of resonance increases as the resistance constant increases. Also, for constant r, the range of resonance increases as the equivalent mass decreases. Hence the sharpness of tuning of the diaphram is regulated by the mechanical resistance and equivalent mass of the diaphram.

In radio work, it is important to know how to construct a diaphram for a given purpose, as for highly selective or broad tuning; and the expressions deduced from the analysis of the motional-impedance circle give the necessary information. As an example, the reasonance curve may be plotted using as coordinates, x and n. For a diaphram of mass m, thickness, t, and diameter, D, let the resonance be represented by the ordinate, n, in Figure 8.

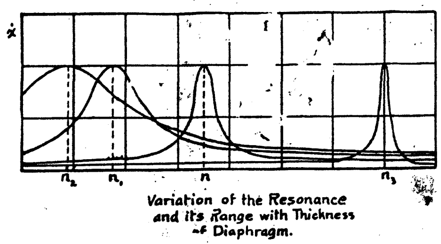


Figure 8

If this diaphram be split in half so that

its thickness is $t_1 = \frac{t}{2}$, its mass will

then be $m_1 = \frac{m}{2}$

and its elastic constant will be

$$s_1 = K\left(\frac{t}{2}\right)^3 = K\frac{t^3}{8} = \frac{s}{8}$$

The new natural frequency will be

$$n_1 = \frac{1}{2\pi}\sqrt{\frac{2s}{8m}} = \frac{1}{2} \times \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

corresponding to a tone one octave below that of the original diaphram; and the new range of resonance will be

$$\frac{r}{m/2} = \pi(n_2 - n_1) = \frac{2r}{m}$$

which is twice as large as the range of resonance of the original diaphram if r is assumed to be constant, r is largely due to air friction but not entirely, therefore a slight decrease in this quantity is probable as the diaphram is made thinner. An interesting research would be to ascertain the proportion of air friction and internal friction making up the mechanical friction, r. Some idea of this might be gained by noting the resonant velocity of the diaphram as it is made thinner and thinner. If r is due entirely to air, this velocity will remain the same in every case when the vibrating pull on the diaphram has a constant maximum, as seen from equation (8). The resonance curves of Figure 8 are plotted for constant r and for the thickness of the diaphram reduced one-half twice and doubled once. It is readily seen that for a fairly good response over a wide range of frequencies, that is, for broad tuning, a very thin diaphram is required, but if highly selective tuning is required, the diaphram should be made thicker. The location of the natural period may then be adjusted by means of other variable properties of the diaphram.

If the diaphram is not held uniformly around the boundary, parasitical vibrations (Bibliography 4, pp. 434-438.) may occur which are superimposed upon the vibration of the diaphram and derive their energy from it. These parasitical vibrations reduce the amplitude in the frequency region where they occur, and if this frequency should be very near the natural frequency of the diaphram, the single maximum amplitude would be replaced by two maxima of lesser values. The possibility of this occuring should be borne in mind when computing the motional constants of the diaphram without having found the motional-impedance circle of the telephone receiver.

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Note: The discussion of this paper will be available for the following issue of the Proceedings.