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VOL. XI

mathematics
- calculations

by FRANK D. GRAHAM, B.S., M.S., M.E., E.E.

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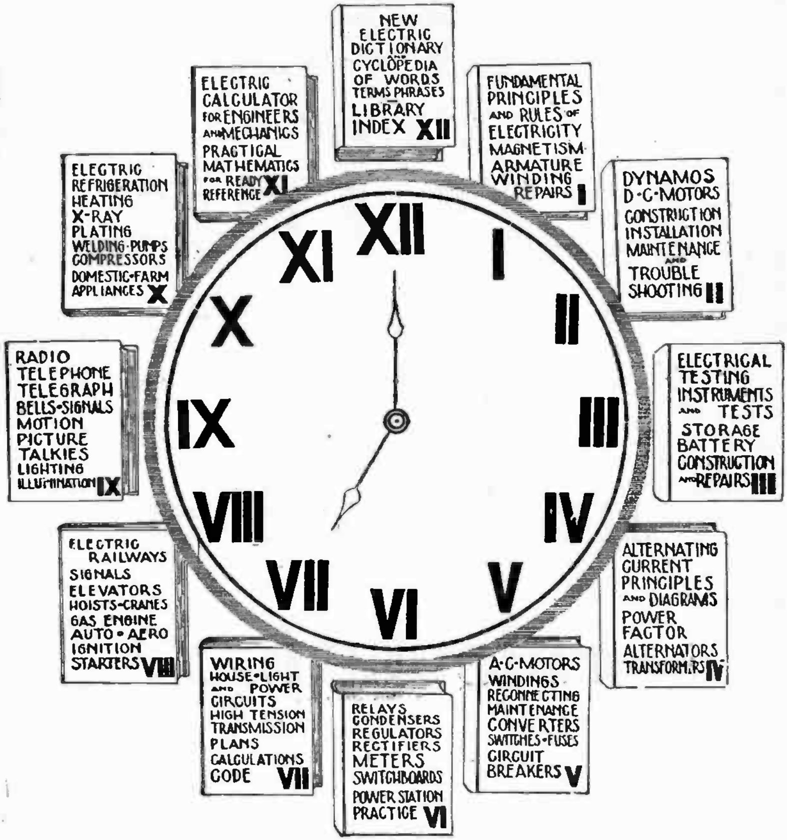

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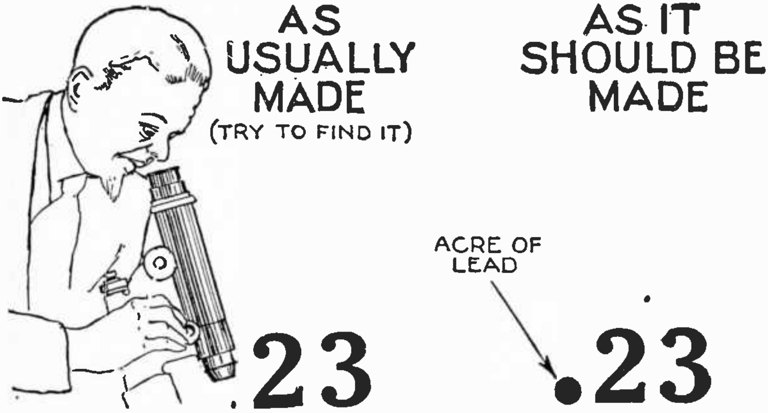

Usual method of making a decimal point and ***correct method.*** The author here tries to impress upon the student the importance of making a decimal point that can be seen.

AUDELS MATHEMATICS AND CALCULATIONS *for* Mechanics

A READY REFERENCE

by

FRANK D. GRAHAM
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Foreword

The chief aim of the author in the preparation of this book on Mathematics, is to present the subject in all its branches in a way so different from that usually employed, as to render a difficult subject not only ***easy*** but ***interesting.***

It is suggested that the student when using this book have at hand a ***pencil*** and ***blank*** sheets of ***paper.*** In this way the book can be picked up for 15 or 20 minutes at a time and one subject after another mastered.

To master a subject, *the student's interest must be aroused, and to this end the author has exhausted every means of unusual and forceful presentation that he could devise.*

To impress the fundamentals upon the mind thoroughly, the explanations are further brought out by illustrations of an exceptional and totally unexpected character.

The subjects are presented concisely and progressively to higher mathematics with calculations covering a great variety of important subjects.

The general arrangement of the book comprises: principal divisions of practical mathematics with sub-classifications and illustrations presented in a step by step easily understood manner such as to arouse the interest of the student.

This book with its entirely different manner of presentation, it is hoped will inspire a wider interest in mathematics, because it puts at the student's fingers ends a greater knowledge of applied mathematics, simplified for home study and ready reference.

The slide rule will be found very useful, especially to those who have to make many calculations. It is explained in this book in such a simple manner that it can be mastered in a very short time.

The author desires to express his appreciation:

To M. E. Guissinger for capable assistance in checking calculations, etc.

To E. K. Watson for efficient help on make up, checking of pages, etc.

Also to J. J. O'Riordan for efficient type setting, collating and make up.

—*The Author.*

How to Use This Index

Get the habit of using this Index. It will quickly reveal a vast mine of valuable information.

In using this index, it should be noted that the book is divided into four sections indicated by the letters A. B, C, D, *that is*:

Section A. Mathematics
" B. Electrical Calculations
" C. Mechanical Calculations
" D. Slide Rule

Accordingly the references in the index are given with both the section letter and page number thus:

Magnet calculations B27

which means that the item will be found in the second or B section on page 27.

The index will be found a valuable guide for quickly finding any item relating to mathematics, electrical and mechanical calculations and the slide rule.

"An hour with a book would have brought to your mind,
The secret that took the whole year to find;
The facts that you learned at enormous expense,
Were all on a library shelf to commence."

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SECTION A

ARITHMETIC

AND

PRACTICAL MATHEMATICS

INCLUDING

- Arithmetic
- Plane Geometry
- Geometrical Propositions
- Geometrical Problems
- Mensuration
- Solid Geometry
- Spherical Geometry
- Descriptive Geometry
- Analytical Geometry
- Trigonometry
- Algebra
- Calculus

CHAPTER 1

Arithmetic

Arithmetic is *the science of numbers and the art of reaching results by their use.*

Signs and Abbreviations.—The various processes in mathematics to be performed are usually indicated by signs, both for convenience and brevity; for instance, 2×4 means that 2 is to be multiplied by 4.

The table on page 2 gives the numerous signs and abbreviations commonly used.

Definitions

Abstract Number.—One that is not applied to any object; as four, six.

Addition.—The process of finding the sum of two or more numbers.

Aliquot Part.—Contained in another number an integral number of times. Thus 6¼, 10, 12½ are aliquot parts of 100.

Alligation.—The method of finding the proportion and relation of prices of the various ingredients of a mixture.

Amount.—The sum total of numbers or quantities.

Analysis.—The process of resolving a problem into its first elements.

Antecedent.—The first term, or dividend, of a ratio.

Area.—The number of square units contained in a surface.

Signs and Symbols

$+$	plus (addition)
$+$	positive
$-$	minus (subtraction)
$-$	negative
$\pm$	plus or minus
$\mp$	minus or plus
$=$	equals
$\times$	multiplied by
$\cdot$	.. ..
$\div$	divided by
$/$	.. ..
$\sqrt{\ }$	square root
$\sqrt[3]{\ }$	cube root
$:$	is to $::$ as $:$ is to
$>$	greater than
$<$	less than

$\square$	square
$\bigcirc$	circle
$^\circ$	degrees
$'$	minutes or feet
$''$	seconds or inches
$\{\}()$	parentheses
$[\ ]$	brackets
$\overline{\quad}$	vinculum
2^2	two squared
2^3	.. cubed
π	**3.1416**
$\frac{\pi}{4}$	**.7854**
$\%$	per cent
$\log_e$	hyperbolic logarithm
∞	infinity

Average.—The calculated mean of several amounts; the number which can be put for each of them without changing their sum.

Base.—The number, in calculating percentage, of which the per cent is taken.

Board Measure.—A unit for measuring lumber being a volume of a board 12 ins. wide, 1 ft. long and 1 in. thick.

Cancellation.—The striking out of a common factor from the numerator and denominator.

Circle.—A plane figure bounded by a curved line, called the circumference, every point of which is equally distant from a point within called the center.

Circulating Decimal.—A decimal in which a figure or set of figures is constantly repeated in the same order; a recurring decimal.

Common Denominator.—A denominator common to two or more fractions.

Common Divisor.—A factor common to two or more numbers.

Common Multiple.—A number exactly divisible by two or more numbers.

Complex Fraction.—One whose numerator or denominator is a fraction.

Composite Number.—A number which can be exactly divided by other integers besides itself and one.

Compound Fraction.—A fraction of a fraction.

Compound Numbers.– Units of two or more denominations of the same kind.

Compound Quantity.—A quantity expressed in two or more denominations, as 3 qts., 1 pt.

Concrete Number.—A number used to designate objects or quantities.

Consequent.—The second term, or divisor, of a ratio.

Couplet.—A ratio, or each member of a proportion.

Cube.—The third power of a quantity.

Cube Root.—One of the three equal factors of a number.

Cubic Measure.—A measure of volume involving three dimensions: length, breadth and thickness.

Currency.—The current medium of exchange in commerce and trade; coin, bank notes, etc.

Decimal Scale.—One in which the order of progression is uniformly ten.

Decimal Fraction.—One or more of the decimal divisions of a unit; also called decimal.

Denomination.—Name of the units of a concrete number.

Diameter of Circle.—A straight line passing through the center and terminating at the circumference.

Difference.—The number found by subtraction.

Division.—The process of determining how many times one number is contained in another of the same kind.

Duodecimals.—Method of computing in divisions of 12; as fractions of a foot formed by dividing by 12 successively; as $^1/_{12}$, $^1/_{144}$, etc.

Equation.—A statement of equality between two expressions or numbers.

Equivalent.—Equal in value.

Even Number.—One that can be exactly divided by two.

Evolution.—The process of finding the root of a number.

Exact Divisor.—A whole number that will divide a number without a remainder.

Exponent.—A small figure placed at the upper right of a number indicating how many times the number is to be taken as a factor.

Extremes.—The first and fourth terms of a proportion.

Factor.—One of two or more quantities which, when multiplied together, produce a given quantity.

Figure.—Any of the nine digits or cipher.

Finite Decimal.—A decimal that has no recurring figures; one that terminates with the written figures.

Fraction.—A number which expresses equal parts of a unit or quantity

Fractional Unit.—One of the equal parts into which any integral unit may be divided.

Greatest Common Divisor.—The greatest number that will exactly divide two or more numbers.

Improper Fraction.—A fraction in which the numerator is equal to or exceeds the denominator.

Incommensurable Quantities.—Quantities that are not measured by the same standard.

Index.—A small figure written at the left and above the radical sign indicating the root to be extracted of the number under the radical sign.

Integers.—Numbers which represent whole things. Numbers are either integral, fractional or mixed.

Involution.—The multiplication of a quantity by itself any number of times; raising a number to a given power.

Least Common Denominator.—The least common multiple to which the denominators of two or more fractions can be reduced.

Least Common Multiple.—Least number that is exactly divisible by two or more numbers.

Like Numbers.—Numbers which represent the same kind of quantity.

Long Division.—The method of dividing a number by another number of two or more figures and indicating the process in full.

Mean Proportional.—A number that is both the second and third terms of a proportion.

Means.—The second and third terms of a proportion.

Measure.—That by which the extent, quantity, capacity, volume or dimensions in general are ascertained by some fixed standard.

Mensuration.—The process of measuring.

Metric System.—A decimal system employing as a unit of measure the meter which is equal to 39.37079 inches. The unit of capacity is the liter and of weight, the gram.

Minuend.—The larger number, in subtraction, from which the subtrahend is taken.

Mixed Number.—A whole number and fraction combined.

Multiple of a Number.—Any number exactly divisible by that number.

Multiplicand.—The number to be multiplied.

Multiplication.—The process of taking a number a given number of times.

Multiplier.—The number denoting how many times the multiplicand is to be taken.

Notation.—A system in which numbers are expressed by symbols.

Number.—A group of digits indicating how many times a unit is taken.

Numeration.—The system of reading numbers.

Odd Numbers.—Numbers not exactly divisible by 2.

Percentage.—One or more hundredths of a number.

Perimeter.—The length of the boundary line of a plane figure.

Period.—One of the groups into which a number is divided, as when a root is to be extracted.

Power of a Number.—The product obtained by using the number a given number of times as a factor.

Prime Number.—A number that cannot be exactly divided by any number except itself and one. Numbers are prime to each other when they have no common factor.

Product.—The result of multiplying.

Proportion.—An equation expressing equality of ratios.

Quantity.—That which can be increased, diminished or measured.

Quotient.—The result of division.

Ratio.—The relation between two numbers of the same kind.

Reciprocal of a Number.—One divided by that number.

Reduction.—Changing terms of a problem into other terms of equivalent value to make it easier to solve.

Remainder.—The number found by subtracting.

Repetend.—A figure or set of figures continually repeated.

Root.—One of the equal factors of a number.

Rule.—The statement of a method.

Scale.—Order of progression on which any system of notation is founded.

Short Division.—Process of dividing by numbers less than 12.

Significant Figure.—Any figure except a cipher.

Solution.—The process of obtaining the answer.

Square Root.—One of the two equal factors of a number.

Subtraction.—The process of taking one number from another.

Subtrahend.—The number to be taken from the minuend.

Sum.—The result of addition.

Surd.—An indicated root that cannot be extracted. A quantity that cannot be expressed in figures.

Surface.—A magnitude having length and breadth.

Unit.—The standard by which separate things are counted or measured. A single thing or a definite quantity.

Unity.—Unit of an abstract number.

Uniform Scale.—One in which the order of progression is the same throughout the entire succession of units.

Unlike Numbers.—Numbers used to express different kinds of quantities.

Varying Scale.—One in which the order of progression is not the same throughout the entire succession of units.

Volume.—The contents, or amount of space included within the boundary surfaces of a solid.

Notation.—By definition, *notation* in arithmetic is the writing down of a figure or figures to express a number. There are two systems of notation.

1. Roman; 2. Arabic.

Roman System.—In this method numbers are expressed by means of letters. It is so called because it was used by the ancient Romans. It is now occasionally used, as in the Bible, for chapter headings, corner stones, etc.

ROMAN TABLE

I denotes One	XII denotes Twelve	L denotes Fifty
II denotes Two	XIII denotes Thirteen	LX denotes Sixty
III denotes Three	XIV denotes Fourteen	LXX denotes Seventy
IV denotes Four	XV denotes Fifteen	LXXX denotes Eighty
V denotes Five	XVI denotes Sixteen	XC denotes Ninety
VI denotes Six	XVII denotes Seventeen	C denotes One hundred
VII denotes Seven	XVIII denotes Eighteen	D denotes Five hundred
VIII denotes Eight	XIX denotes Nineteen	M denotes One thousand
IX denotes Nine	XX denotes Twenty	$\bar{X}$ denotes Ten thousand
X denotes Ten	XXX denotes Thirty	$\bar{M}$ denotes One million
XI denotes Eleven	XL denotes Forty	

In the Roman notation, when any character is placed at the right hand of a larger numeral, its value is added to that of such numeral; as, VI, that is, V + I; XV, that is, X + V; MD, that is, M + D; and the like. I, X, and rarely C, are also placed at the left hand of other and larger numerals, and when so situated their value is subtracted from that of such numerals as, IV, that is, V — 1; XC, that is, C — X; and the like. Formerly the smaller figure was sometimes repeated in such a position twice, its value being in such cases subtracted from the larger; as, IIX, that is, X — II XXC, that is, C — XX; and the like. Sometimes after the sign IↃ for D, the character Ↄ was repeated one or more times, each repetition having the effect of multiplying IↃ by ten: as IↃↃ, 5,000; IↃↃↃ, 50,000; and the like. To represent numbers twice as great as these, C was repeated as many times before the stroke I, as the Ↄ was after it; as, CCIↃↃ, 10,000; CCCIↃↃↃ, 100,000; and the like. ***The ridiculous custom*** of using the Roman notation for chapter numbers, year of copyright, sections, etc., should be discontinued.

Arabic System.—In this system ten figures are used to express numbers. The figures are:

0 1 2 3 4 5 6 7 8 9

From left to right these figures are called: cipher, one, two, three, four, five, six, seven, eight, nine.

With exception of the cipher, these figures are called *significant* figures because each has a value of its own. They are also called *digits*.

Figures have two values:

1. Simple;
2. Local.

The simple value of a figure is its value taken alone, thus:

1. 2. 3.

The local value of a figure is its value when used with another figure or figures, thus:

12. 23. 475.

When two or more figures are used together, the position of each figure relative to the other figure or figures is called its *place* or *order*, thus:

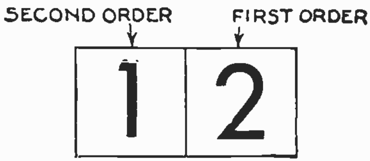


FIG. 1.—Analysis of the number 12.

When one of the figures stands by itself, it is called a *unit;* but if two of them stand together, the right hand one is still called a unit, but the left hand one is called *tens*, thus:

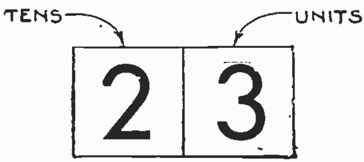


FIG. 2.—Analysis of the number 23.

Again, if three figures stand together, the left hand one is called *hundreds*.

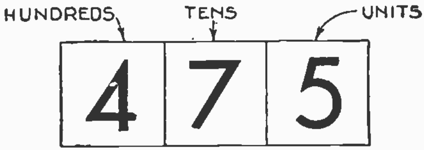


FIG. 3.—Analysis of the number 475.

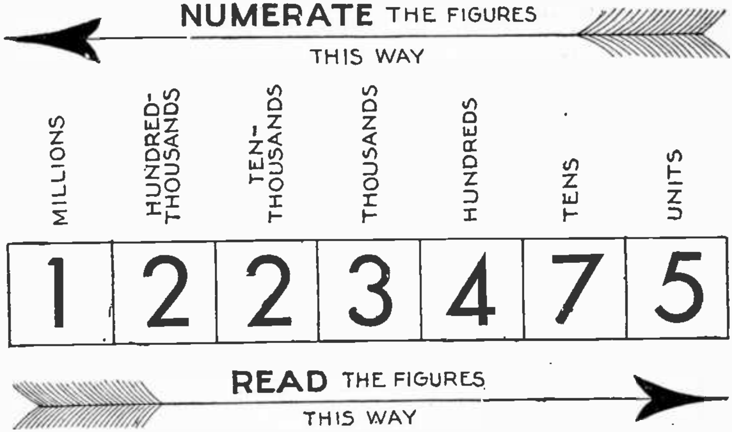


FIG. 4.—Large number illustrating how to numerate.

The number 12 (twelve) is a collection of 2 units and one set of ten units, thus:

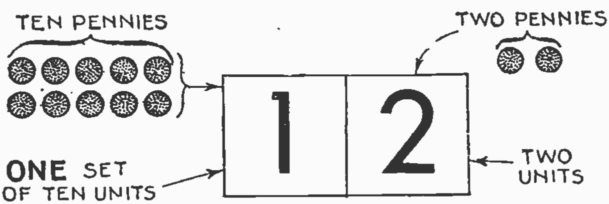


FIG. 5.—Analysis of the number 12.

The number 23 (twenty-three) is a collection of 3 units and two sets of ten units, thus:

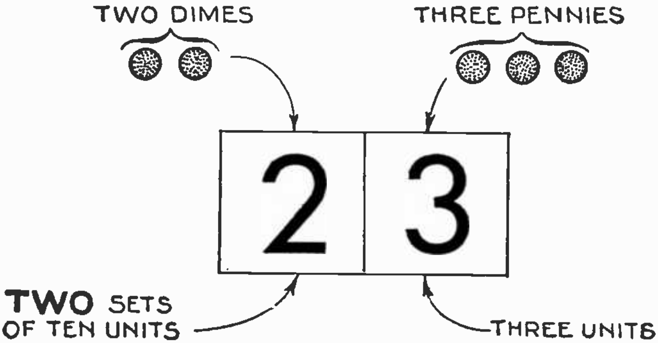


FIG. 6.—Further analysis of the number 23.

In this way various numbers are expressed, thus:

Eleven,	one ten and one,	is expressed by 11.
Twelve,	one ten and two,	is expressed by 12.
Thirteen,	one ten and three,	is expressed by 13.
Fourteen,	one ten and four,	is expressed by 14.
Fifteen,	one ten and five,	is expressed by 15.
Sixteen,	one ten and six,	is expressed by 16.
Seventeen,	one ten and seven,	is expressed by 17.
Eighteen,	one ten and eight,	is expressed by 18.
Nineteen,	one ten and nine,	is expressed by 19.
Twenty-one,	two tens and one,	is expressed by 21.
Twenty-two,	two tens and two,	is expressed by 22.
Forty-three,	four tens and three,	is expressed by 43.
Fifty-four,	five tens and four,	is expressed by 54.
Sixty-five,	six tens and five,	is expressed by 65.

In order to easily read large numbers made up of many figures they are divided by commas into *periods*. A period is a group of figures containing the *hundreds*, *tens* or *units* of any denomination. Periods are separated by commas, thus:

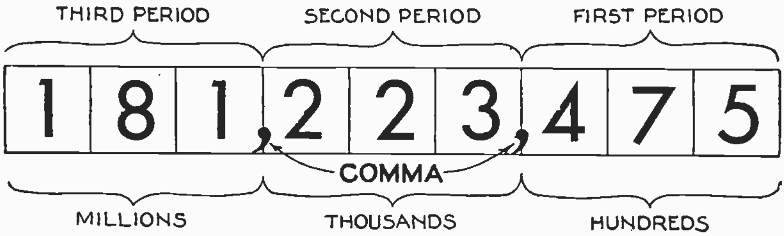


FIG. 7.—Method of placing commas in large numbers.

the number being read thus: one hundred and eighty-one million, two hundred and twenty-three thousand, four hundred and seventy-five.

Scheme of Arithmetic.—An interesting notion of what arithmetic consists of as given in Funk & Wagnall's Standard Dictionary is shown on page 13.

The following 10 formulæ include the elementary operations of arithmetic.

1. *The sum = all the parts added.*
2. *The difference = the minuend − the subtrahend.*
3. *The minuend = the subtrahend + the difference.*
4. *The subtrahend = the minuend − the difference.*
5. *The product = the multiplicand × the multiplier.*
6. *The multiplicand = the product ÷ the multiplier.*
7. *The multiplier = the product ÷ the multiplicand.*
8. *The quotient = the dividend ÷ the divisor.*
9. *The dividend = the quotient × the divisor.*
10. *The divisor = the dividend ÷ the quotient.*

Scheme of Arithmetic

Notation

Basis — 1 — unit

Arithmetic Alphabet

0 1 2 3 4 5 6 7 8 9

Increased			***Diminished***			
By tens			*By tens*			
1, 10, 100, 1,000, etc.			1, .1, .01, .001, etc.			
By varying scales			By varying scales			
1 oz.	1 lb.	1 cwt.	$\frac{1}{4}$	$^6/_7$	$\frac{1}{3}$ oz.,	$^{11}/_{23}$ etc.
1 pt.	1 qt.	1 gal.	$\frac{1}{6}$ lb.	$^9/_8$ oz.		$\frac{3}{8}$ cwt.
1 in.	1 ft.	1 yd.		etc.		

According to the Four Ground Rules

Addition Subtraction	Multiplication Division
By involution (powers)	By evolution (roots)

Relations Expressed by

Ratios 2 : 3 5 : 6 8 : 9 etc.

Proportion (equality of ratios) 2 : 3 : : 4 : 6 etc.

Practical Applications

Percentage, interest, profit and loss, reduction of weights and measures, mensuration, etc.

Addition

Symbol ☞ +

Addition.—Uniting two or more numbers or groups of objects of the same kind into one is called *addition*, and the number obtained by adding is called the *sum*.

The sign of addition is + and is read "plus," thus 7 + 3 is read "seven plus three."

Rule A.—*Write the numbers to be added so that like orders of units stand in the same column.*

B.—*Commencing with the lowest order, or at the right hand, add each column separately, and if the sum can be expressed by one figure, write it under the column added.*

C.—*If the sum of any column contain more than one figure, write the unit figure under the column added, and add the remaining figure or figures to the next column.*

EXAMPLES FOR PRACTICE

7,060	248,124	13,579,802
9,420	4,321	93
1,743	889,866	478,652
4,004	457,902	87,547,289
22,227 Ans.		

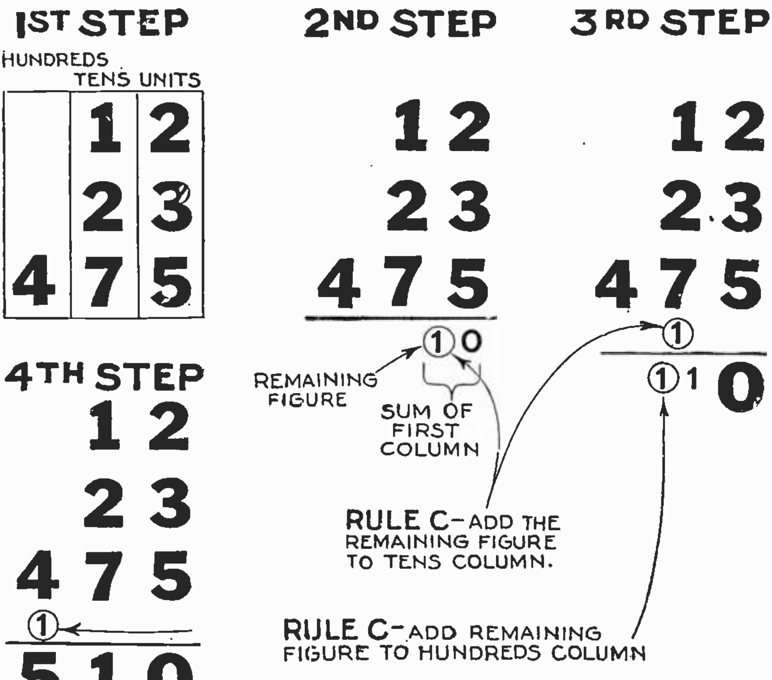
Addition Table

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 3	2 " 4	2 " 5	2 " 6	2 " 7
3 " 4	3 " 5	3 " 6	3 " 7	3 " 8
4 " 5	4 " 6	4 " 7	4 " 8	4 " 9
5 " 6	5 " 7	5 " 8	5 " 9	5 " 10
6 " 7	6 " 8	6 " 9	6 " 10	6 " 11
7 " 8	7 " 9	7 " 10	7 " 11	7 " 12
8 " 9	8 " 10	8 " 11	8 " 12	8 " 13
9 " 10	9 " 11	9 " 12	9 " 13	9 " 14
10 " 11	10 " 12	10 " 13	10 " 14	10 " 15

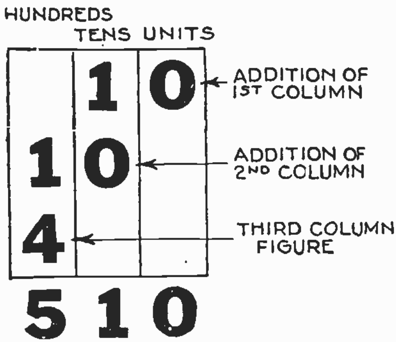
6 and	7 and	8 and	9 and	10 and
1 are 7	1 " 8	1 are 9	1 are 10	1 are 11
2 " 8	2 " 9	2 " 10	2 " 11	2 " 12
3 " 9	3 " 10	3 " 11	3 " 12	3 " 13
4 " 10	4 " 11	4 " 12	4 " 13	4 " 14
5 " 11	5 " 12	5 " 13	5 " 14	5 " 15
6 " 12	6 " 13	6 " 14	6 " 15	6 " 16
7 " 13	7 " 14	7 " 15	7 " 16	7 " 17
8 " 14	8 " 15	8 " 16	8 " 17	8 " 18
9 " 15	9 " 16	9 " 17	9 " 18	9 " 19
10 " 16	10 " 17	10 " 18	10 " 19	10 " 20

Example.—Find the *sum* of 12, 23 and 475.

The rules are applied as in figs. 8 to 11:



FIGS. 8 to 11.—Example in addition.



Use great care in placing the numbers in vertical lines, as irregularity in writing them is one cause of mistakes.

To check the addition, add each column; arrange the several sums according to place, and add the several sums so arranged, as shown in fig. 12.

FIG. 12.—Method of checking example given in figs. 8 to 11.

Subtraction

Symbol ☛ —

By definition, subtraction is *the process of taking one number called the* ***subtrahend*** *from another number called the* ***minuend.*** The result thus obtained, or "difference" between the two numbers, is called the remainder, thus:

$$\begin{array}{rl} 10 & \text{minuend} \\ \underline{7} & \text{subtrahend} \\ 3 & \text{remainder.} \end{array}$$

that is, 7 subtracted from 10 leaves a remainder of 3. It may also be expressed as:

$$10-7=3$$

that is, 10 minus 7 equals 3.

To subtract one number from another proceed as follows:

Rule—A. *Write down the sum so that the units stand under the units, the tens under the tens, etc., etc.*

B. *Begin with the units, and take the under from the upper figure, and put the remainder beneath the line.*

C. *But if the lower figure be the larger, add ten to the upper figure, and then subtract and put the remainder down—this borrowed ten must be deducted from the next column of figures where it is represented by* 1.

Subtraction Table

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 “ 1	3 “ 1	4 “ 1	5 “ 1	6 “ 1
3 “ 2	4 “ 2	5 “ 2	6 “ 2	7 “ 2
4 “ 3	5 “ 3	6 “ 3	7 “ 3	8 “ 3
5 “ 4	6 “ 4	7 “ 4	8 “ 4	9 “ 4
6 “ 5	7 “ 5	8 “ 5	9 “ 5	10 “ 5
7 “ 6	8 “ 6	9 “ 6	10 “ 6	11 “ 6
8 “ 7	9 “ 7	10 “ 7	11 “ 7	12 “ 7
9 “ 8	10 “ 8	11 “ 8	12 “ 8	13 “ 8
10 “ 9	11 “ 9	12 “ 9	13 “ 9	14 “ 9
11 “ 10	12 “ 10	13 “ 10	14 “ 10	15 “ 10

6 from	7 from	8 from	9 from	10 from
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 leaves 0
7 “ 1	8 “ 1	9 “ 1	10 “ 1	11 “ 1
8 “ 2	9 “ 2	10 “ 2	11 “ 2	12 “ 2
9 “ 3	10 “ 3	11 “ 3	12 “ 3	13 “ 3
10 “ 4	11 “ 4	12 “ 4	13 “ 4	14 “ 4
11 “ 5	12 “ 5	13 “ 5	14 “ 5	15 “ 5
12 “ 6	13 “ 6	14 “ 6	15 “ 6	16 “ 6
13 “ 7	14 “ 7	15 “ 7	16 “ 7	17 “ 7
14 “ 8	15 “ 8	16 “ 8	17 “ 8	18 “ 8
15 “ 9	16 “ 9	17 “ 9	18 “ 9	19 “ 9
16 “ 10	17 “ 10	18 “ 10	19 “ 10	20 “ 10

Example.—Subtract 12 from 23:

$$\begin{array}{r} 23 \\ \underline{12} \\ 11 \end{array}$$

Here, two units from three units leaves one unit, and one ten from two tens leaves one ten.

Example.—Subtract 275 from 928:

$$\begin{array}{r} 928 \\ \underline{275} \\ 653 \end{array}$$

First subtract 5 units from 8 units which leaves 3 units for the first column remainder. As 7 tens cannot be taken from 2 tens, borrow one of the 9 hundreds and change it to tens which equals 10 tens. Add the two tens which makes 12 tens. Take 7 tens from 12 tens which leaves 5 tens for the second column remainder. Since 1 hundred was borrowed from the hundred column, there are 8 hundreds left. 2 from 8 leaves 6 for the third column remainder; the total remainder being 653.

Another method is as follows:

First subtract 5 units from 8 units, and obtain 3 units for a partial remainder. As 7 tens cannot be taken from 2 tens, add 10 tens to the 2 tens, making 12 tens; then 7 tens from 12 tens leave 5 tens, the second partial remainder. Now since 10 tens, or 1 hundred, has been added to the minuend, add 1 hundred to the subtrahend, and the true remainder will not be changed; thus, 1 hundred added to 2 hundreds makes 3 hundreds, and this sum subtracted from 9 hundreds leaves 6 hundreds; the total remainder being 653.

NOTE.—The process of taking ten from one column of the minuend and adding it to the next column is sometimes called *borrowing*, and that of adding 1 to the next figure of the subtrahend is called *carrying* 1.

To check the subtraction, *add the remainder to the subtrahend and if the remainder be correct the sum will be equal to the minuend.*

Examples for Practice

892	2,572	9,999
46	1,586	8,971
846 remainder.		

Multiplication

Symbol ☛ X

By definition, multiplication consists in taking one of two given numbers as many times as there are units in the other.

The number to be multiplied or increased is called the *multiplicand;* the number by which the multiplicand is multiplied is called the *multiplier;* and the result thus obtained, the *product.*

The multiplier and multiplicand which produce the product are called its *factors.* This is a word frequently used in mathematical works and its meaning should be remembered.

The sign of multiplication is × and is read "times" or multiplied by; thus 6×8 is read, 6 times 8 is 48, or, 6 multiplied by 8 is 48.

The principle of multiplication is the same as addition, thus $3 \times 8 = 24$ is the same as $8 + 8 + 8 = 24$.

Rule.—*Place the unit figure of the multiplier under the unit figure of the multiplicand. Thus:*

Multiplication Table

Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
11 " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

7 times	8 times	9 times	10 times	11 times	12 times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 49	7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 56	8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 63	9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 70	10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 77	11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

```
846        487,692
  8            143
```

Example.—Multiply 346 by 8:

```
 346
   8
----
2768
```

Taking the units of each order 8 times is equivalent to taking the entire number 8 times. Therefore, commencing at the right hand, 8 times 6 units are 48 units, or 4 tens and 8 units. Write the 8 units in the product in units' place and reserve the 4 tens to add to the next product; 8 times 4 tens are 32 tens, and the 4 tens reserved in the last product added, are 36 tens, or 3 hundreds and 6 tens; write the 6 tens in the product in tens' place, and reserve the 3 hundreds to add to the next product; 8 times 3 hundreds are 24 hundreds, and the 3 hundreds reserved in the last product added, are 27 hundreds, which being written in the product each figure in the place of its order, gives for the entire product, 2768.

Example.—Multiply 758 by 346:

```
    758
    346
-------
   4548
  3032
 2274
-------
262,268
```

758 multiplied by 6 units is 4,548 units; 758 multiplied by 4 tens is 3,032 tens, which is written with its lowest order in tens' place, or under the figure used as a multiplier; 758 multiplied by 3 hundreds is 2,274 hundreds, which is written with its lowest order in hundreds' place. Since the sum of these products must be the entire product of the given numbers, add the results which will give 262,268, the total product.

When there are ciphers between the significant figures of the multiplier, *put the first figure of each partial product directly under the figure multiplied by.*

NOTE.—When the multiplier contains two or more figures, the results obtained by multiplying by each figure are called *partial products.*

Example.—Multiply 312 by 203:

```
  312
  203
-----
  936
 624
-----
63336
```

Rule—*When there are ciphers after the significant figures of the multiplier, place the ciphers at the right of the product of the significant figures.*

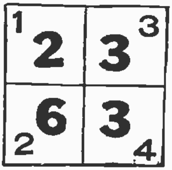
Example.—Multiply 12 by 300:

```
 12
 300
----
3600
```

Place the multiplier under the multiplicand so that the 3 comes under the 2 as above. Multiply 12 by 3 and add the two ciphers.

```
2347          234700
  12000          900000
------        --------
```

Italian Short Proof Method.—This method of checking multiplication is used to quickly check the result of multiplying two large numbers by each other.



The method is as follows:

Example.—Multiply 172,856 by 375.

This is expressed as:

A. . . . 172856 Add digits giving 29. Divide 29 by 9 giving 3 and remainder of 2.
B. . . . 375 Add digits giving 15. Divide 15 by 9 giving 1 and a remainder of 6.

```
  864280
1209992
518568
--------
```

C. . 64821000 Add digits giving 21. Divide 21 by 9 giving 2 and a remainder of 3.

In line A, add together all the digits, divide the sum by 9 and place the remainder in the square designated as 1, (page 23).

In line B, add together all the digits, divide the sum by 9, and place the remainder in the square designated as 2.

Multiply the number in square 1 by the number in square 2, divide the product by 9 and place the remainder in square designated as 3. In this case 2×6=12 and 12 divided by 9=1; with a remainder of 3.

In line C, add together all the digits, divide the sum by 9, and if the multiplication has been properly performed, the remainder will be the same as the number placed in the square designated as 3.

Place the remainder in square designated as 4. It happened in this particular case that the remainder number in square 3, was 3.

American Short Proof Method.—This is another method of proving the product of multiplying large numbers.

Example.—Multiply 24532 by 13685.

A........24532 Adding digits gives 16, adding again; 1+6=7

B........13685 Adding digits gives 23; adding again; 2+3=5.

```
      122660        Multiplying 7 by 5=35
     196256              Adding 3 and 5= 8
    147192
    73596
   24532            D——————————E
C....335720420 Adding digits gives 26; adding again gives 8.
```

Add the digits in the multiplicand; designated by A. In this case giving the number 16. Again add digits giving 7. Add the digits in the multiplier B, giving in this case 23. Again add digits, giving 5. Multiply 7 by 5 giving 35 and adding digits 3+5 giving 8.

Add all the digits in the product C, giving in this case 26. Again add digits 2+6 giving 8.

Always add digits until a number containing only one figure remains.

If the result of the operations indicated above the line D E be the same as the result of the operations indicated below the line, the multiplication has been performed correctly.

Involution or Powers of Numbers

Symbol Exponent

By definition, involution is *the continued multiplication of a number by itself a given number of times.*

The number is called the root or first power, and the product is called the power.

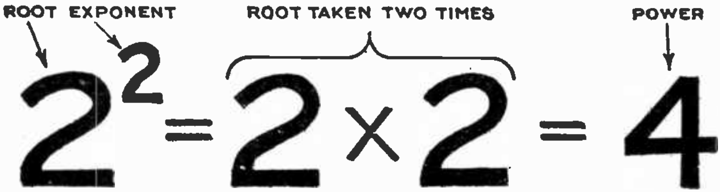
The second power is called the *square;* the third power the *cube.* The higher powers are called the *fourth power*, *fifth power*, etc.

Examples.—

square of $2 = 2 \times 2 = 4$

cube of $2 = 2 \times 2 \times 2 = 8$

The power to which a number is raised is indicated by a small "superior" figure called an "exponent." Thus:



from which it is seen that the exponent indicates the number of times the number or "root" is to be taken.

The square of a number contains *either twice as many figures as the number or twice as many, less one, thus:*

$1^2 = 1$

$9^2 = 81$

$99^2 = 9{,}801$

$100^2 = 10{,}000$

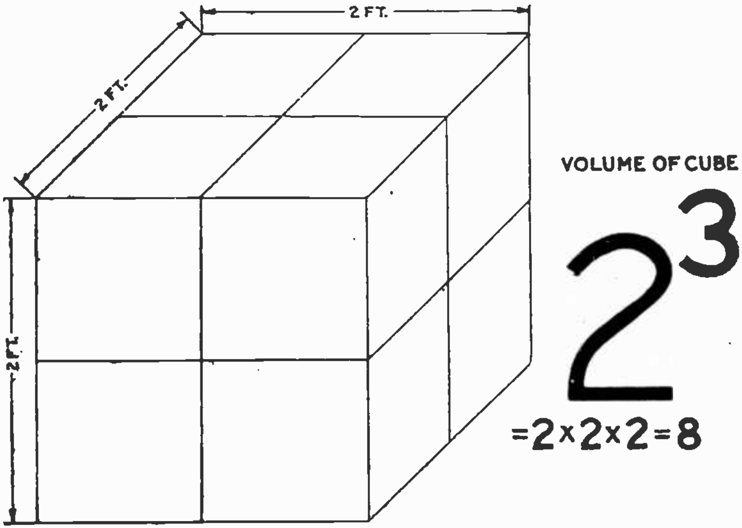


FIG. 13.—Two foot cube illustrating cube of a number.

Examples for Practice

$12^2 = ?$	$138^3 = ?$
$23^2 = ?$	$256^2 = ?$
$84^2 = ?$	$475^3 = ?$
	$1223^2 = ?$

Division

Symbol ☞ ÷

Division consists in finding the value of one of a given number of equal parts into which a quantity is to be divided.

When one number is divided by another number, the first one is called the ***dividend***, and the second one, the ***divisor***, the result thus obtained is called the *quotient*.

The sign of division is ÷ and is read "divided by," thus 8 ÷ 2 is read "eight divided by two."

There are two methods of division known as:

1. Short.
2. Long.

In the method of short division the continued subtraction is effected mentally, the quotient alone being set down without any working. Evidently this method is suitable only for small divisors, as 8 ÷ 2, 1,272 ÷ 12.

In the method of long division the operations are written down in full, the method being applied with large divisors of two or more figures as 13,765 ÷ 126. To apply short division with such a large divisor would involve too great a mental process.

1. *Short division.*

To divide by any number up to 12.

Rule.—*Put the dividend down with the divisor to the left of it, with a small curved line separating it, as in the following:*

Example.—Divide 7,865,432 by 6.

$$\begin{array}{r} 6\overline{)7,865,432} \\ \overline{1,310,905} \text{ and 2 over} \end{array}$$

At the last it is evident that 6 is contained in 32, 5 times and 2 over. When there is a number left over as shown, make it a fraction, with the divisor, 6 (denominator) below the line and the number left over, 2 (numerator) above the line. Place the fraction close to the quotient thus 1,310,905 $^2/_6$.

To divide any number with a cipher or ciphers after the last significant figure in both divisor and dividend.

Rule.—*Place the numbers down as in the last example, then mark off from the right of the dividend as many figures as there are ciphers in the divisor; also mark off the ciphers in the divisor; then divide the remaining figures by the number remaining in the divisor, thus:*

Example.—Divide 6,000,000 by 12,000.

Cancel all ciphers to the right of the last significant figure of the divisor and cancel as many ciphers to the right of the last significant figure of the dividend as were cancelled from the divisor.

Thus: $12,000\overline{)6,000,000}$

Cancel ciphers

$$\begin{array}{r} 12\overline{)6,000} \\ \overline{500} \end{array}$$

2. *Long division.*

To divide any number by a large divisor of two or more figures.

Example.—Divide 18,149 by 56.

$$\begin{array}{r} 56)18149(\underline{324\,^5/_{56}} \\ \underline{168}\phantom{49(324\,^5/_{56}} \\ 134\phantom{9(324\,^5/_{56}} \\ \underline{112}\phantom{9(324\,^5/_{56}} \\ 229\phantom{(324\,^5/_{56}} \\ \underline{224}\phantom{(324\,^5/_{56}} \\ 5\phantom{(324\,^5/_{56}} \end{array}$$

Division Table

1 in	2 in	3 in	4 in	5 in
1, 1 time	2, 1 time	3, 1 time	4. 1 time	5, 1 time
2, 2 times	4, 2 times	6, 2 times	8, 2 times	10, 2 times
3, 3 "	6, 3 "	9, 3 "	12, 3 "	15, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "	20, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "	25, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "	30, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "	35, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "	40, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "	45, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "	50, 10 "
6 in	**7 in**	**8 in**	**9 in**	**10 in**
6, 1 time	7, 1 time	8, 1 time	9, 1 time	10, 1 time
12, 2 times	14, 2 times	16, 2 times	18, 2 times	20, 2 times
18, 3 "	21, 3 "	24, 3 "	27, 3 "	30, 3 "
24, 4 "	28, 4 "	32, 4 "	36, 4 "	40, 4 "
30, 5 "	35, 5 "	40, 5 "	45, 5 "	50, 5 "
36, 6 "	42, 6 "	48, 6 "	54, 6 "	60, 6 "
42, 7 "	49, 7 "	56, 7 "	63, 7 "	70, 7 "
48, 8 "	56, 8 "	64, 8 "	72, 8 "	80, 8 "
54, 9 "	63, 9 "	72, 9 "	81, 9 "	90. 9 "
60, 10 "	70, 10 "	80. 10 "	90, 10 "	100, 10 "

In the above operation the process is as follows: As neither 1 nor 18 will contain the divisor, take three figures 181, for the first *partial* dividend. 56 is contained in 181 three times, and a remainder. Write the 3, as the first figure in the quotient, and then multiply the divisor by this quotient figure thus: 3 times 56 is 168, which when subtracted from 181 leaves 13. To this remainder annex or "bring down" 4 the next figure in the dividend thus forming 134, which is the next partial dividend. 56 is contained in 134 two times and a remainder. Thus 2 times 56 is 112, which subtracted from 134 leaves 22. To the remainder bring down 9 the last figure in the dividend, forming 229, the last partial dividend. 56 is contained in 229 four times and a remainder. Thus: 4 × 56 = 224, which, subtracted from 229, gives 5, the final remainder which write in the quotient with the divisor, below it, thus completing the operation of long division.

Exact Divisor.—*A number is said to be divisible by another when there is no remainder after dividing.*

Any number is *divisible:*

1. By 2, if it be an *even* number.

2. By 3, if the sum of its digits be divisible by 3.

3. By 4, if its two right hand figures be ciphers or express a number divisible by 4.

4. By 5, if it end with a cipher or 5.

5. By 6, if it be an *even* number and divisible by 3.

6. By 8, if its three right hand figures be ciphers, or express a number divisible by 8.

7. By 9, if the sum of its digits be divisible by 9.

8. By 10, if it end with one or more ciphers.

9. By 7, 11 and 13 if it consist of but *four* places, the first and fourth being occupied by the same significant figures, and the second and third by ciphers.

10. An *odd* number is not divisible by an *even* number.

11. If an *even* number be divisible by an *odd* number, the quotient will be an *even* number.

12. If an even number be divisible by an odd number, it is also divisible by *twice* that number.

13. Every odd number except 1, increased or else diminished by 1, is divisible by 4.

14. Every prime number except 2 and 3, increased or else diminished by 1, is divisible by 6.

Prime Numbers

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

Factors.—By definition a factor is *one of two or more quantities which, when multiplied together produce a given quantity.*

Thus, 4 and 5 are factors of 20 because 4 multiplied by 5 equals 20.

A prime factor of a number is *one which cannot be separated into factors.*

Thus 4 is a factor of 20 but is not a prime factor because it is made up of two factors, two and two; that is, $2 \times 2 = 4$.

Apply the following rule to obtain the prime factors of a given number.

Rule.—*Divide the given number by any prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.*

Example.—What are the prime factors of 798?

$$\begin{array}{r|l} 2 & 798 \\ 3 & 399 \\ 7 & 133 \\ 19 & 19 \\ & 1 \end{array}$$

Since the given number is even, divide by 2, and obtain the odd number 399 for a quotient. Now divide by the prime numbers 3, 7, and 19 as above, the last quotient being 1. The divisors 2, 3, 7, and 19 then are the prime factors of 798.

Another method is to find the ***composite factors*** of a number and then find the prime factors of the composite factors.

Thus,

$$180 = \frac{20}{2\times2\times5} \times \frac{9}{3\times3}$$

Greatest Common Divisor.—By definition, the greatest common divisor of two or more numbers is *the greatest number that will exactly divide each of them.*

To find the greatest common divisor.

Rule.—1. *Write the numbers in a line, with a vertical line at the left, and divide by any factor common to all the numbers.* 2. *Divide the quotient in like manner, and continue the division till a set of quotients is obtained that are prime to each other.* 3. *Multiply all the divisors together and the product will be the greatest common divisor sought.*

Example.—What is the greatest common divisor of 72, 120 and 440?

$$\begin{array}{r|rrr} 4 & 72 & 120 & 440 \\ \hline 2 & 18 & 30 & 110 \\ \hline & 9 & 15 & 55 \end{array}$$

4 will exactly divide each of the given numbers, and 2, each of the quotients obtained by dividing by 4. The last quotients 9, 15 and 55 are prime to each other, hence greatest common divisor is $4 \times 2 = 8$.

Least Common Multiple.—By definition the least common multiple of two or more numbers is *the least number exactly divisible by those numbers.*

To find the least common multiple.

Rule.—1. *Resolve the given numbers into their prime factors.* 2. *Multiply together all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number. Their product will be the least common multiple.* 3. *When a prime factor is repeated in any of the given numbers it must be taken as many times in the multiple, as the greatest number of times it appears in any of the given numbers.*

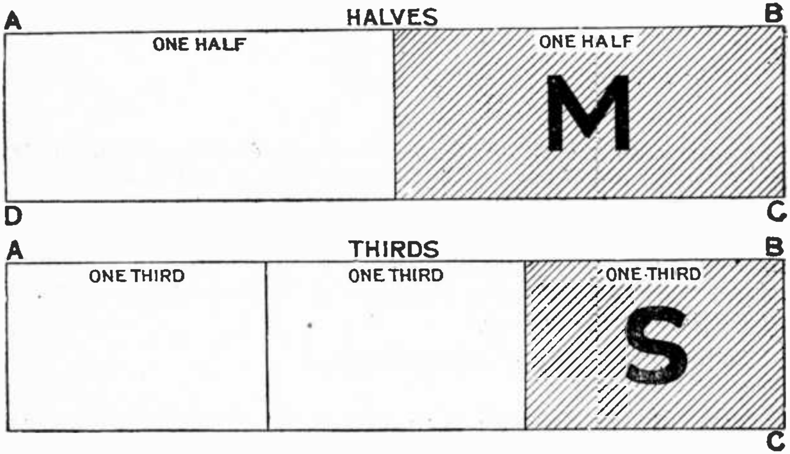
Example.—Find the least common multiple of 60, 84 and 132.

$$60 = 2 \times 2 \times 3 \times 5$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$132 = 2 \times 2 \times 3 \times 11$$
$$(2 \times 2 \times 3 \times 11) \times 5 \times 7 = 4{,}620$$

The factor 2 appears twice in each number, hence, applying 3, of the rule it is written down in the least common multiple twice. By inspection the factors not found in the largest number are 5 and 7, these are written down as above together with the factors of 132, giving 4,620 the least common multiple.

Fractions

A fraction is *a quantity less than a unit or whole number.*



FIGS. 14 and 15.—Graphic representation of fractional parts.

Fractions take their *name* and *value* from the *number* of parts into which the unit is divided. Thus, if the unit be divided into 2 equal parts, one of these parts is called *one-half*, as **M**, in fig. 14; if divided into 3 equal parts, one of these parts is called *one-third*, as **S**, in fig. 15.

Evidently from the figures, one-half or **M** is larger than one-third or **S**.

To express a fraction by figures two numbers are required: one to express the number of parts into which the unit is divided and the other to express the number of these parts taken.

The number expressing the number of parts taken called the *numerator* is written *above* a diagonal or horizontal line and that expressing the number of parts into which the unit is divided called the *denominator* is written below the line, thus:

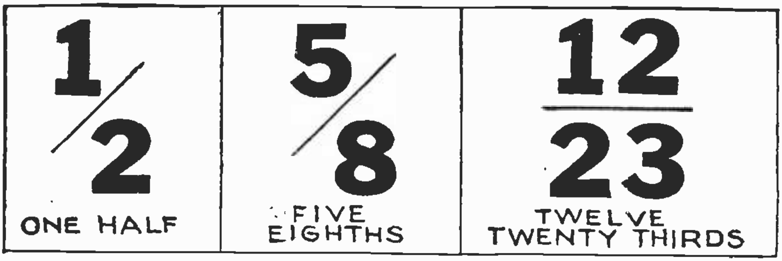


FIG. 16.—Method of writing fractions.

It is seen that the line may be either diagonal or horizontal. Usually a diagonal line is used where numerator and denominator consist of one figure each, and a horizontal line for two or more figures.

Definitions

Complex Fraction.—One whose numerator or denominator is a fraction.

Compound Fraction.—A fraction of a fraction.

Improper Fraction.—One whose numerator equals or exceeds its denominator.

Mixed Number.—An integer and a fraction united.

Partial Fractions.—Fractions whose sum may be reduced to the original fraction.

Proper Fraction.—One whose numerator is less than its denominator.

Simple Fraction.—One whose numerator and denominator are whole numbers.

Vulgar Fraction.—One expressed by a numerator and denominator as distinguished from a decimal fraction.

The following general principles should be noted:

1. *A change in the numerator produces a like change in the value of the fraction.*

2. *A change in the denominator produces an opposite change in the value of the fraction.*

3. *Multiplying the denominator divides the fraction and dividing the denominator multiplies the fraction.*

4. *Multiplying or dividing both terms of the fraction by the same number does not alter the value of the fraction.*

To reduce a fraction to its lowest terms.

Rule.—*Divide both numerator and denominator by their greatest common divisor.*

Example.—Reduce $^{20}/_{40}$ to its lowest terms.

A Find greatest common divisor:

$$\begin{array}{r|rr} 2 & 20 & 40 \\ \hline 2 & 10 & 20 \\ \hline 5 & 5 & 10 \\ \hline & 1 & 2 \end{array}$$

B Divide both terms by greatest common divisor:

$$\frac{20|20}{20|40} = \frac{1}{2}$$

$2 \times 2 \times 5 = 20$ greatest common divisor

Find the greatest common divisor, 20 as at **A**, divide both terms by 20 as at **B**, obtaining ½ which is the lowest terms of $^{20}/_{40}$.

To change an improper fraction to a mixed number.

Rule.—*Divide the numerator by the denominator.*

Example.—Change $^{49}/_5$ to a mixed number.

$$^{49}/_5 = 49 \div 5 = 9\,^4/_5$$

To change a mixed number to an improper fraction.

Rule.—*Multiply the whole number by the denominator of the fraction; to the product add the numerator and place the sum over the denominator.*

Example.—Change 12⅝ to an improper fraction.

multiply whole number by denominator $12 \times 8 = 96$
add the numerator 5
sum 101

place sum over denominator $\frac{101}{8}$

Before fractions can be added or subtracted their denominators must be the same. The least common multiple is the least common denominator.

To reduce fractions to a common denominator.

Find the least common multiple or use the method according to the following rule:

Rule.—*Multiply each numerator by all of the denominators except its own for the new numerator and all the denominators together for the common denominator.*

Example.—Reduce ½, ⅓ and ⅗ to a common denominator.

New numerator of first fraction $= 1 \times 3 \times 5 = 15$
New numerator of second fraction $= 1 \times 2 \times 5 = 10$
New numerator of third fraction $= 3 \times 3 \times 2 = 18$
Common denominator $= 2 \times 3 \times 5 = 30$

from which the fractions become

$$\frac{15}{30} \quad \frac{10}{30} \quad \frac{18}{30}$$

To add fractions.

Rule.—*Reduce them to a common denominator, add the numerators and place their sum over the common denominator.*

Example.—Add ½, ⅓ and ⅗

A Reduce to common denominator:

$1 \times 3 \times 5 = 15$
$1 \times 2 \times 5 = 10$
$3 \times 3 \times 2 = 18$
$2 \times 3 \times 5 = 30$

B Add numerators:

15
10
18
43 (sum)

C Place sum over common denominator:

$\frac{43}{30}$

The sum of the fractions ($^{43}/_{30}$) is an improper fraction, hence, reduce to mixed number.

$$^{43}/_{30} = 43 \div 30 = 1\,^{13}/_{30}$$

To subtract fractions.

Rule.—*Reduce them to a common denominator, subtract the numerators and place the difference over the common denominator.*

Example.—Subtract $^3/_7$ from $^4/_5$.

A Reduce to common denominator

$3 \times 5 = 15$
$4 \times 7 = 28$
$7 \times 5 = 35$

B Subtract the numerators

28
15
13 (difference)

C Place difference over the common denominator

$$\frac{13}{35}$$

To multiply fractions.

Rule.—(*Case I. Multiplying by a whole number.) Multiply the numerator or divide the denominator by the whole number.*

Example.—Multiply $^7/_{12}$ by 4.

A Multiplying numerator

$^7/_{12} \times 4 = {}^{28}/_{12} = 2\,^4/_{12} = 2\tfrac{1}{3}$

B Dividing denominator

$^7/_{12} \times 4 = {}^7/_3 = 2\tfrac{1}{3}$

Rule.—(*Case II. Multiplying by a fraction.) Multiply the numerators for a new numerator and the denominators for a new denominator.*

Example.—Multiply ¾ by $^5/_7$.

$$\tfrac{3}{4} \times {}^5/_7 = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

Division of fractions.

Rule.—(*Case I. Dividing by a whole number.) Divide the numerator, or multiply the denominator by the whole number.*

Example.—Divide $^{10}/_{13}$ by 5.

A Dividing numerator
$^{10}/_{13} \div 5 = {}^{2}/_{13}$

B Multiplying denominator
$^{10}/_{13} \div 5 = {}^{10}/_{65} = {}^{2}/_{13}$

Rule.—(*Case II. Dividing by a fraction.*) *Invert the divisor and proceed as in multiplication.*

Example.—Divide $^{3}/_{4}$ by $^{5}/_{7}$.

A Invert divisor-
$^{5}/_{7}$ inverted is $^{7}/_{5}$

B Multiply by inverted divisor
$^{3}/_{4} \times {}^{7}/_{5} = {}^{21}/_{20} = 1\,{}^{1}/_{20}$

The two operations are expressed thus:

$$^{3}/_{4} \div {}^{5}/_{7} = {}^{3}/_{4} \times {}^{7}/_{5} = {}^{21}/_{20} = 1\,{}^{1}/_{20}$$

Decimals

Symbol ● (decimal point)

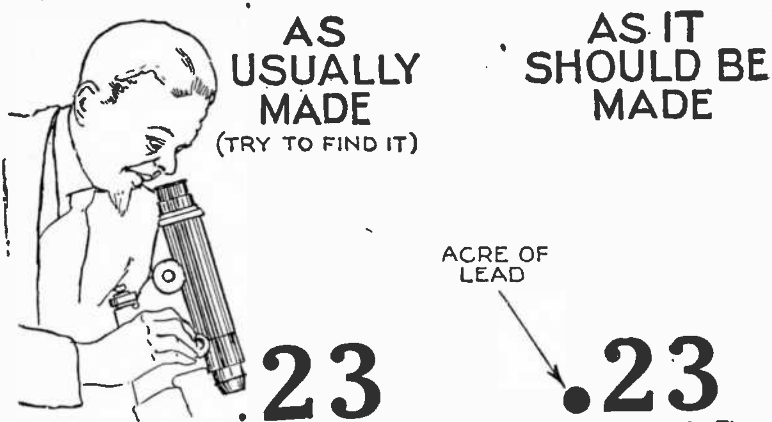
The word decimal signifies ten, being derived from the Latin word *decem*.

By definition a decimal fraction is *a fraction whose denominator is* 10 *or a power of* 10. It is usually written without the denominator, the number of ciphers in the denominator being indicated by the number of places occupied by the numerator preceded if necessary by ciphers, and placed after a point or period called the "decimal point."

It should be understood absolutely once and for all that the decimal point is an item of extreme importance.

Most errors are due to writing the decimal point carelessly so as to require a microscope to see it—use an "acre of lead" if necessary to make the decimal point plainly visible.

In the formation of a fraction a single unit is divided into 10 parts as in fig. 19.



FIGS. 17 and 18.—*Usual* method of making a decimal point and *correct* method. The author here tries to impress upon the student the importance of making a decimal point that can be seen.

Here the big rectangle or unit is divided into ten parts, then any one of these parts as LARF, is $^1/_{10}$ of the unit. In the decimal system however it is not necessary to write the denominator because the same law of local value governs the decimals as the integral numbers. The "*decimal point*" (.) is always placed before the decimal figures to distinguish them from integers.

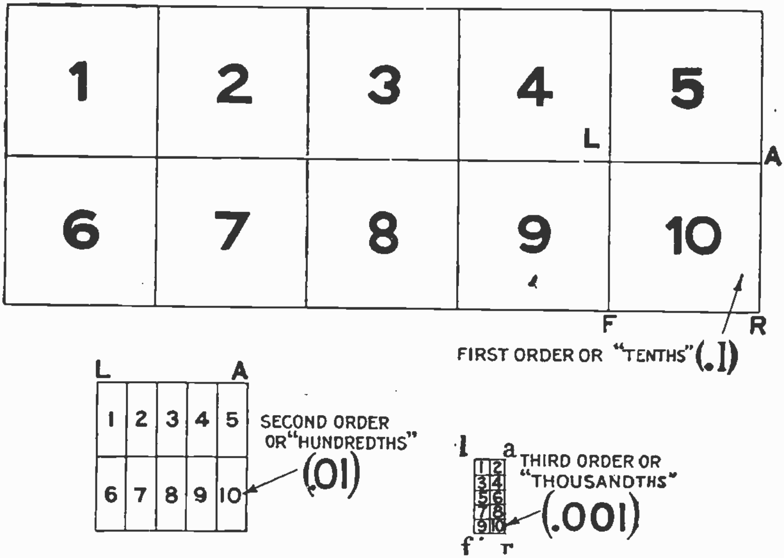
The law of local value for decimals assigns:

The first place at the right of the decimal point to 10ths—(1st order) the second place at the right of the decimal point to 100ths—(2nd order)

The third place at the right of the decimal point to 1,000ths—(3rd order)

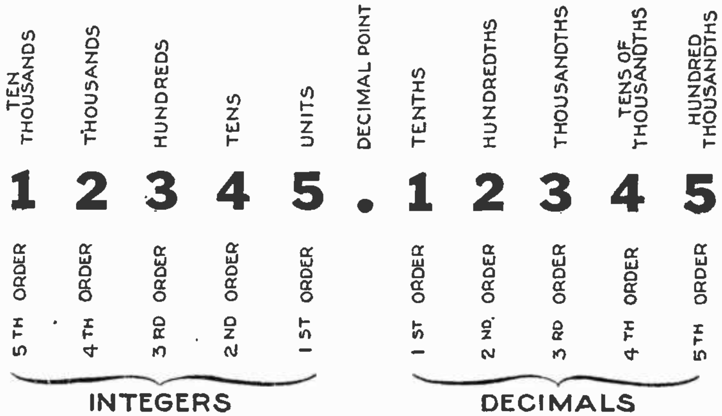
Thus in fig. 26 LARF, or $^1/_{10}$ of the unit is written .1. Fig. 27 shows section LARF of fig. 26 divided into 10 parts. Evidently one of these parts is equal to one hundredth of the unit and is expressed as .01.

Similarly one of the ten parts of *l'a'r'f'* (fig. 28) is equal to one thousandth of the unit and is expressed as .001. Evidently if several of the parts were taken they would be expressed for instance, as .2, .03, .009. The decimal may include parts of the several orders as .23, .145, etc.



FIGS. 19 to 21.—Graphic representation of decimal fractions or "decimals."

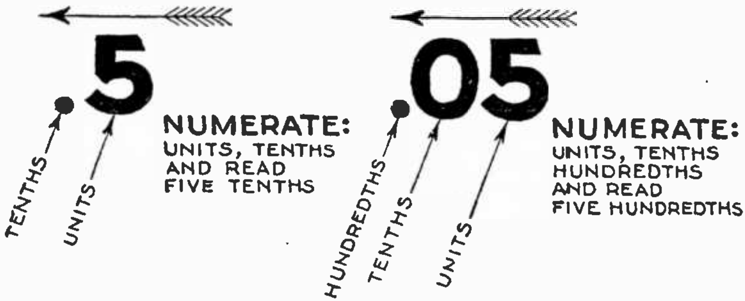
Numeration of Decimals



Any decimal or combination of a decimal and integer may be read by applying the table on page 41.

A number may be made up of one or more integers and a decimal as 23.35; this is read twenty-three and thirty-five hundredths.

In the case of a column of figures as in addition, care should be taken to have all the decimal points exactly under each other.

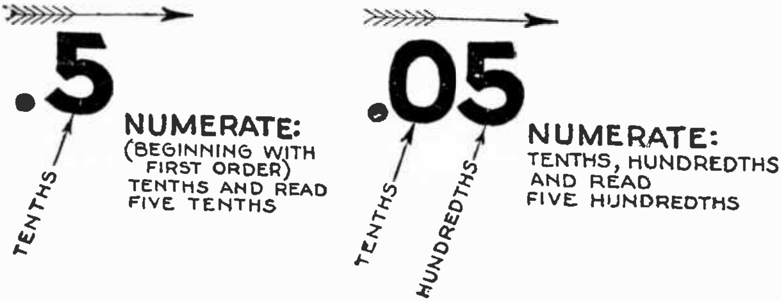


FIGS. 22 and 23.—How to read decimals (*first method*). ***Rule.**—Numerate toward the decimal point (units, tenths, hundredths, etc.), numerating each order and the decimal point.*

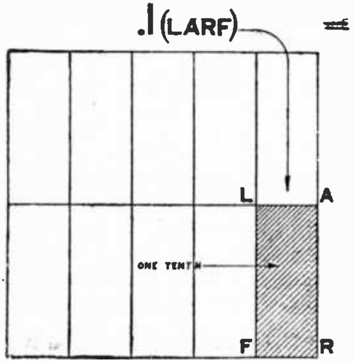
The practice of putting a cipher to the left of the decimal when there is no integer, as, for instance, 0.5, is *unnecessary*.

There are several methods of numerating or reading decimals; one method is to *numerate* **toward** *the decimal point numerating each figure and the decimal point* as shown in figs. 22 and 23.

A second method, numerating *from* the decimal point (beginning with the first order), is shown in figs. 24 and 25. It is immaterial which method be used, the result is the same although some theoretical highbrows might object to the first method.



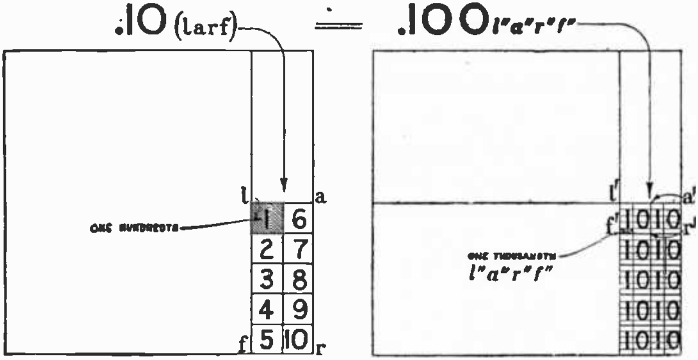
FIGS. 24 and 25.—How to read decimals (*second method*). *Rule.—Numerate from the decimal point, beginning with the first order "tenths."*



Annexing ciphers after a decimal does not change its value.

In fig. 26, LARF, is equal to one-tenth (.1) of the large rectangle, and

FIGS. 26 to 28.—Diagrams showing that *annexing ciphers after a decimal does not change its value.*



is equal to *larf* in fig. 27. If *larf*, be divided into 10 parts, each of these parts =one-hundredth (.01) of the large rectangle. Similarly if each of these parts, as *l'a'r'f'*, fig. 28, be again divided into 10 parts, each of the parts thus obtained = one-thousandth (.001) of the large rectangle. Hence .1 = .10 = .100.

The practice of annexing ciphers after decimals ***is a useless waste of time.***

To reduce decimals to a common denominator:

Rule.—*Annex ciphers after each decimal so that each will have the same number of figures or places thus:*

.5	annexing ciphers	.500
.27	for same number	.270
.325	of places become	.325

that is to say

.5 = .500	which is read	five tenths = five hundred thousandths
.27 = .270		twenty-seven hundredths = two hundred and seventy thousandths.
.325 = .325		three hundred and twenty-five thousandths. = three hundred and twenty-five thousandths.

in other words *adding ciphers after a decimal does not change its value.* This is apparent from figs. 26 to 28. The practice of adding ciphers after decimals is quite unnecessary except perhaps in a very large column of figures to be added.

To reduce common fractions to decimals.

Rule.—*Divide the numerator by the denominator and carry out the division to as many decimal places as desired.*

Examples.

Change $^4/_5$ to decimal

$$\frac{4}{5}: \quad 5)4.0 = .8$$

Change $^5/_8$ to decimal

$$\frac{5}{8}: \quad 8)5.000 = .625$$

To add decimals.

Rule.—*Place the numbers in a column with the decimal points under each other and add as in whole numbers.*

Example.—Add .5 .25 1.75.

$$\begin{array}{r} .5 \\ .25 \\ 1.75 \\ \hline 2.50 \end{array}$$

To subtract decimals.

Rule.—*Place the numbers so that the decimal points are under each other and proceed as in simple subtraction.*

Example.—Subtract .72 from 1.25.

$$\begin{array}{r} 1.25 \\ .72 \\ \hline .53 \end{array}$$

To multiply decimals,

Rule.—*Proceed as in simple multiplication and point off as many places as there are in multiplier and multiplicand*

Thus

$$.1 \times .0025 = .00025$$

Here there is one place in multiplicand and four in multiplier, or five altogether.

To multiply a decimal by 10 or any power of 10.

Rule.—*Move the decimal point to the right in the multiplicand as many places as there are ciphers in the multiplier.*

Example.—Multiply .023 by 100

$$.023 \times 100 = 2.3$$

To divide decimals.

Rule.—*Proceed as in simple division, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

Example.—Divide 3.642 by .3

$$3.642 \div .3 = 12.14$$

To divide a decimal by 10 or any power of 10.

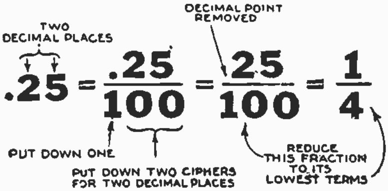
Rule.—*Move the decimal point as many places to the left as there are ciphers in the divisor, and divide by the number on the left of the ciphers in the divisor.*

Example.—Divide 47.5 by 1,000

$$47.5 \div 1{,}000 = .0475$$

To convert decimals to common fractions.

Rule.—*Set down the decimal as a numerator, and place as the denominator* 1 *with as many ciphers annexed as there are decimal places in the numerator; erase the decimal point in the numerator, and reduce the fraction thus formed to its lowest terms, thus:*



Fractions and Decimal Equivalents

8ths	
$\frac{1}{8}$	= .125
$\frac{1}{4}$	= .250
$\frac{3}{8}$	= .375
$\frac{1}{2}$	= .500
$\frac{5}{8}$	= .625
$\frac{3}{4}$	= .750
$\frac{7}{8}$	= .875

16ths	
$\frac{1}{16}$	= .0625
$\frac{3}{16}$	= .1875
$\frac{5}{16}$	= .3125
$\frac{7}{16}$	= .4375
$\frac{9}{16}$	= .5625
$\frac{11}{16}$	= .6875
$\frac{13}{16}$	= .8125
$\frac{15}{16}$	= .9375

32ds	
$\frac{1}{32}$	= .03125
$\frac{3}{32}$	= .09375
$\frac{5}{32}$	= .15625
$\frac{7}{32}$	= .21875
$\frac{9}{32}$	= .28125
$\frac{11}{32}$	= .34375
$\frac{13}{32}$	= .40625
$\frac{15}{32}$	= .46875
$\frac{17}{32}$	= .53125
$\frac{19}{32}$	= .59375
$\frac{21}{32}$	= .65625
$\frac{23}{32}$	= .71875
$\frac{25}{32}$	= .78125
$\frac{27}{32}$	= .84375
$\frac{29}{32}$	= .90625
$\frac{31}{32}$	= .96875

64ths	
$\frac{1}{64}$	= .015625
$\frac{3}{64}$	= .046875
$\frac{5}{64}$	= .078125
$\frac{7}{64}$	= .109375
$\frac{9}{64}$	= .140625
$\frac{11}{64}$	= .171875
$\frac{13}{64}$	= .203125
$\frac{15}{64}$	= .234375
$\frac{17}{64}$	= .265625
$\frac{19}{64}$	= .296875
$\frac{21}{64}$	= .328125
$\frac{23}{64}$	= .359375
$\frac{25}{64}$	= .390625
$\frac{27}{64}$	= .421875
$\frac{29}{64}$	= .453125
$\frac{31}{64}$	= .484375
$\frac{33}{64}$	= .515625
$\frac{35}{64}$	= .546875
$\frac{37}{64}$	= .578125
$\frac{39}{64}$	= .609375
$\frac{41}{64}$	= .640625
$\frac{43}{64}$	= .671875
$\frac{45}{64}$	= .703125
$\frac{47}{64}$	= .734375
$\frac{49}{64}$	= .765625
$\frac{51}{64}$	= .796875
$\frac{53}{64}$	= .828125
$\frac{55}{64}$	= .859375
$\frac{57}{64}$	= .890625
$\frac{59}{64}$	= .921875
$\frac{61}{64}$	= .953125
$\frac{63}{64}$	= .984375

To reduce a recurring decimal to a common fraction.

Rule.—*Subtract decimal figures that do not recur from the whole* *decimal including one set of recurring figures; set down the remainder as the numerator of the fraction, and as many nines as there are recurring figures, followed by as many ciphers as there are non-recurring figures, in the denominator.*

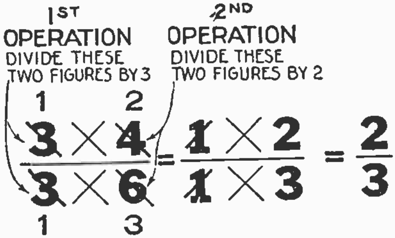
Example.—In the decimal .79054054, the recurring figures are 054.

$$\begin{array}{l} \text{Subtract} \quad \underline{79} \\ \qquad\qquad\quad \dfrac{78975}{99900} = \text{(reduced to its lowest terms) } \dfrac{117}{148} \end{array}$$

Cancellation

Cancellation is *the process of shortening calculations by rejecting equal factors from numerator and denominator*, that is from dividend and divisor.

Example.—Divide 3×4 by 3×6.



Rule.—*Reject from the dividend and divisor all factors common to both, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

When all the factors of both dividend and divisor are cancelled, the quotient is 1, for the dividend will then exactly contain the divisor once.

Example.—Divide 72×66×49 by 63×40×21.

$$\frac{\overset{\cancel{8}}{\cancel{72}} \times \overset{22}{\cancel{66}} \times \overset{\cancel{7}}{\cancel{49}}}{\underset{\cancel{7}}{\cancel{63}} \times \underset{5}{\cancel{40}} \times \underset{\cancel{3}}{\cancel{21}}} = \frac{22}{5} = 5\overline{)22.0}$$

$$4.4$$

THAT IS $4\frac{2}{5}$

Since 9 is a factor of both 72 and 63 it may be rejected from both, leaving 8 instead of 72 in the dividend and 7 instead of 63 in the divisor. Next cancel 8 from 8 and 40, leaving 5 instead of 40 in the divisor. Now cancel 7 from 7 and 49, leaving 7 instead of 49 in the dividend; and cancel 7 again from 7 and 21, leaving 3 instead of 21. Rejecting the factor 3 from both 66 and 3, there is left for a dividend 22, and for a divisor 5, which gives a quotient of $4^2/_5$.

Evolution or Roots of Numbers

Symbol $\sqrt{}$

The word *evolution* means *the operation of extracting a root.* The root here is a factor repeated to produce a power. Thus in the equation $2\times2\times2=8$, 2 is the root from which the power (8) is produced. Evolution is indicated by the symbol $\sqrt{}$ called the radical sign, which placed over a number means that the root of the number is to be extracted. Thus:

$\sqrt{4}$ means that the *square* root of 4 is to be extracted

The *index* of the root is a small figure placed over the radical sign which denotes what root is to be taken. Thus $\sqrt[3]{9}$ indicates the cube root of 9; $\sqrt[4]{16}$, the fourth root of 16. ***When there is no index the radical sign alone always means the square root.***

Sometimes the number under the radical sign is to be raised to a power before extracting the root, thus:

$$\sqrt[3]{4^3} = \sqrt[3]{4 \times 4 \times 4} = \sqrt[3]{64} = 4$$

The power and the root are often combined and expressed as a fractional exponent, thus $8^{2/3}$ which is read the cube root of 8 squared, that is:

$$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

Square Root.—By definition the square root of a number or quantity is that number which, multiplied by itself, produces the given number or quantity; thus 8 is the square root of 64, because 8 multiplied by 8 equals 64.

When there is ***no index*** *the radical sign alone always means the* ***square root,*** *as before stated.*

Rule.—1. *Point off the given number into periods of two places each, beginning with units.* **2.** *If there be decimals, point these off likewise, beginning at the decimal point and supplying as many ciphers as may be needed.* **3.** *Find the greatest number whose square is contained in the left hand period, and place it as the first figure in the quotient.* **4.** *Subtract its square from the left hand period, and to the remainder annex the two figures of the second period for a dividend.* **5.** *Double the first figure of the quotient for a partial divisor; find how many times the latter is contained in the dividend exclusive of the right hand figure in the quotient, and annex it to the right of the partial divisor, forming the complete divisor.* **6.** *Multiply this divisor by the second figure in the quotient, and subtract the product from the dividend.* **7.** *To the remainder bring down the next period and proceed as before, in each case doubling the figures* *in the root already found to obtain the trial divisor.* 8. *Should the product of the second figure in the root by the completed divisor be greater than the dividend, erase the second figure both from the quotient and from the divisor, and substitute the next smaller figure, or one small enough to make the product of the second figure by the divisor less than or equal to the dividend.*

Example.—Extract the square root of 186,624.

```
      18'66'24)432
      16
 83)   266
       249
862)    1724
        1724
```

From right to left point off the given number into periods of two places each. Begin with the last period pointed off (18). Largest square in 18 is 4; put this down in the quotient and the square (16) under the 18. Write down remainder (2) and bring down next period (66). Multiply 4 (in quotient) by 2 for first number of next divisor and say 8 goes into 26, three times.

Place 3 after 4 in quotient and also after 8 in the divisor. Multiply the 83 by 3, placing product 249 under 266 and subtract,obtaining remainder 17. Bring down last period 24 and proceed as before, obtaining 432 as the square root of 186,624.

Cube Root.—Extracting the cube root is a more complicated though a similar process, as indicated by the rule following.

Rule.—*(Cube root).* 1. *Separate the number into groups of three figures each, beginning at the units.* 2. *Find the greatest cube in the left hand group and write its root for the first figure of the required root.* 3. *Cube this root, subtract the result from the left hand group, and to the remainder annex the next group for a dividend.* 4. *For a partial divisor, take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the second figure of the root.* 5. *To this partial divisor add three times the product of the first figure of the root considered as tens by the second figure, and also the square of the second figure. This sum will be the complete divisor.* 6. *Multiply the complete divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.* 7. *Proceed in this manner until all the groups have been annexed. The result will be the cube root required.*

Example.—Extract the cube root of 42,875.

		42,875 | 35
$3^3 =$	27	27
$3\times30^2 =$	2,700	15,875
$3\times(30\times5) =$	450	
$5^2 =$	25	
	3,175	15,875

Since 42,875 consists of two groups or periods, the cube root will consist of two figures.

The first group, 42, contains the cube of the tens' figure of the root. The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence, 3 is the tens' figure of the root.

The remainder, 15,875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units. Each of these three parts contains the units' figure as a factor. Hence, the 15,875 consists of two factors, one of which is the units' figure of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The largest part of this second factor is three times the square of the tens. If the 158 hundreds of the remainder be divided by the $3\times30^2=27$ hundreds, the quotient will be the units' figure of the root.

The second factor can now be completed by adding to the 2700, $3\times(30\times5)=450$ and $5^2=25$.

Example.—Extract the cube root of 1,881,365,963,625.

		1,881,365,963,625 | 12345
		1
300×1^2	$=300$	881
$30\times1\quad\times2=$	60	
$2^2=$	4	
	364	728
		153365
300×12^2	$=43200$	
$30\times12\quad\times3=$	1080	
$3^2=$	9	
	44289	132867
		20498963
300×123^2	$=4538700$	
$30\times123\quad\times4=$	14760	
$4^2=$	16	
	4553476	18213904
		2285059625
300×1234^2	$=456826800$	
$30\times1234\times5=$	185100	
$5^2=$	25	
	457011925	2285059625

Simple Rule for Cube Root.—*Separate the number into its prime factors* when this method is possible.

Example.—Find the cube root of 74,088.

2	74,088
2	37,044
2	18,522
3	9,261
3	3,087
3	1,029
7	343
7	49
7	7

Proof.

$$2^3\times3^3\times7^3=74{,}088$$

$$\sqrt[3]{74{,}088}=2\times3\times7=42$$

As stated this is an easy way to find the cube root but it is not always practicable; accordingly a general method is necessary as given above.

The easiest, quickest and best way to obtain a cube root, especially for those having many calculations to make, is to look it up in a table of powers and roots. Moreover there is less chance of error.

Mental Cube Root.—To extract the cube root of a cube of two periods, have a small table firmly fixed in the mind.

It consists of the cubes of the numbers from 1 to 9. The cube of 1 is 1, of 2 is 8, of 3 is 27, of 4 is 64, of 5 is 125, of 6 is 216, of 7 is 343, of 8 is 512, of 9 is 729. Examine these cubes and it will be seen that the cube of 8 ends in 2, and the cube of 2 ends in 8.

Then again, the cube of 3 ends in 7 and the cube of 7 ends in 3. All others end with the same figure which has been used as a factor.

When a cube is given to extract the root, first note the "thousands." In the number 32,768, for example, the thousands consist of two figures, 32. Now 32 is between 27 and 64, the cubes of 3 and 4, hence the root is between 30 and 40, or the first figure must be 3.

Disregard the other figures until the last one is reached. As the figure is 8, it is evident the second figure of the root must be 2, because the cube of 2 always ends in 8. Thus:

Cube root of the fraction $\frac{12{,}167}{15{,}625}$ is $\frac{23}{25}$

This also applies to the first and last figure of a cube root of any number of figures.

Compound Quantities

A compound quantity expresses *units of two or more denominations of the same kind*, as five yards, one foot and four inches.

A denominate fraction is a concrete fraction whose integral unit is a denominate number. Thus $^3/_7$ of a day is a denominate fraction, the integral unit being one day; so are $^5/_8$ of a bushel; $^2/_3$ of a mile, etc., denominate fractions.

To reduce a compound number to a lower denomination—**Reduction descending.**

Rule.—*Multiply the highest denomination of the given number by the scale number which will reduce it to the next lower denomination and add to the product the given number, if any, of that lower denomination, continuing until the number is reduced to the denomination required.*

Examples.—Reduce 1 yd., 8 ft. and 7 ins. to ins.

1 yd. 8 ft. 7 ins.
3 (scale factor 3 ft. = 1 yd.)
3 ft.
8 ft. to be added
11 ft.
12 (scale factor 12 ins. = 1 ft.)
132 ins.
7 ins. to be added
139 ins. total

Reduction descending

To reduce a denominate number to a compound number of higher denominations—**Reduction ascending.**

Rule.—*Divide the denominate number by that number of the ascending scale which will reduce it to the next higher denomination; the quotient is in the higher denomination and the remainder if any, in the lower denomination. Continue the division until the number is reduced to the highest denomination required.*

Example.—Reduce 139 ins. to a compound number of higher denominations.

139 ÷ 12* = 11 ft., 7 ins.

*NOTE.—12 is scale number to reduce ins. to ft.

In any of the measures which follow in the next section the compound numbers may be reduced by following the rules.

Weights and Measures

By definition a measure is *that by which the extent, quantity, capacity, volume or dimensions in general are ascertained by some fixed standard.* There are several kinds of measure as:

1. Linear (length).
2. Square (area).
3. Cubic (volume).
4. Weight.
5. Time.
6. Angular.

etc.

Linear Measure.—There are several kinds of linear measure known as: 1, long; 2, surveyors' or old land; 3, nautical.

TABLE

Long Measure

12 inches (ins. or ″)	make 1 foot (ft. or ′)
3 feet	make 1 yard (yd.)
5½ yards or 16½ feet	make 1 rod (rd.)
40 rods	make 1 furlong (fur.)
8 furlongs or 320 rods	make 1 statute mile (mi.)

Unit equivalents

mi.		fur.		rd.		yd.		ft.		ins.
								1	=	12
						1	=	3	=	36
				1	=	5½	=	16½	=	198
		1	=	40	=	220	=	660	=	7,920
1	=	8	=	320	=	1,760	=	5,280	=	63,360

Scale—ascending, 12. 3. 5½, 40, 8; descending, 8, 40, 5½, 3, 12.

TABLE

Surveyors' or Old Land Measure

7.92 ins.	make 1 link (l.)
25 links	make 1 rod (rd.)
4 rods or 66 ft.	make 1 chain (ch.)
80 chains	make 1 mile (mi.)

Unit equivalents

mi.		ch.		rd.		l.		ins.
						1	=	7.92
				1	=	25	=	198
		1	=	4	=	100	=	792
1	=	80	=	320	=	8,000	=	63,360

Scale—ascending, 7.92, 25, 4, 80; descending, 80, 4, 25, 7.92.

NOTE.—The denomination *rods* is seldom used in chain measure, distances being taken in chains and links.

Table

Measures Occasionally Used

1000 mils	=1 in.
1 hand	=4 ins.
1 span	=9 ins.
1 military pace	=2½ ft.
1 fathom	=6 ft.
1 cable length	=120 fathoms.

TABLE

Nautical Measure

6,080.26 ft. or 1.15156 statute miles	= 1 nautical mile or knot*
3 nautical miles	= 1 league
60 nautical miles or 69.168 statute miles	= 1 degree (at the equator)
360 degrees	= circumference of earth at equator

*NOTE.—The British Admiralty takes the round figure 6,080 ft. for length of the "measured mile" used in trials of vessels. The length between knots on the log line is 1/120 of a nautical mile, or 50.7 ft. when a half minute glass is used; so that a speed of 10 knots is equal to 10 nautical miles per hour.

Square Measure.—This kind of measure is used to measure the area of a surface; it involves two dimensions, length and breadth, that is:

$$\text{area} = \text{length} \times \text{breadth}$$

The dimensions length and breadth may be taken in any denomination as inches, feet, yards, etc., but both must be

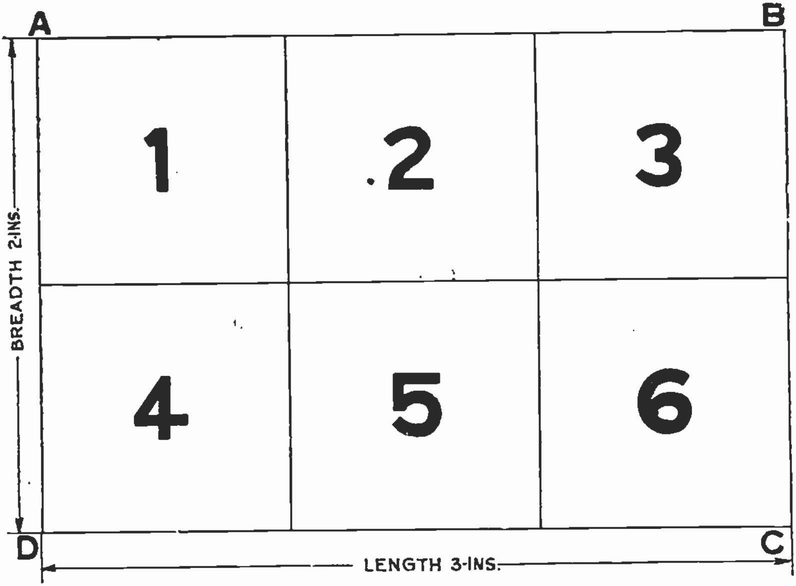


FIG. 29.—Diagram illustrating square measure. If the rectangle ABCD, measure 2 ins. on one side and 3 ins. on the other, and lines be drawn at each inch division, then each of the small squares will have an area of 1 sq. in. and the area of the rectangle will be area ABCD = breadth × length = 2 × 3 = 6 sq. ins.

taken in the same denomination. The word "square" is used to denote the product of the two dimensions, thus:

inches (length) × inches (breadth) = square *inches*

Square measure is shown graphically in fig. 29.

TABLE

Square Measure

144 square inches (sq. ins.) make 1 square foot (sq. ft.)
9 sq. ft. make 1 square yard (sq. yd.)
30¼ sq. yds. make 1 square rod or perch (sq. rd. or P.)
40 sq. rods make 1 rood (R)*
4 roods make 1 acre (A)
640 acres make 1 square mile (sq. mi.)

Unit equivalents

sq. mi.	A	R	sq. rd.	sq. yd.	sq. ft.	sq. ins.
					1	144
				1	9	1,296
			1	30¼	272¼	39,204
		1	40	1,210	10,890	1,568,160
	1	4	160	4,840	43,560	6,272,640
1	640	2,560	102,400	3,097,600	27,878,400	4,014,489,600

Scale—ascending, 144, 9, 30¼, 40, 4, 640; descending, 640, 4, 40, 30¼, 9, 144.

TABLE

Surveyors' Square Measure

625 square links (sq. l.) make 1 pole (P)
16 poles make 1 square chain (sq. ch.)
10 square chains make 1 acre (A)
640 acres make 1 square mile (sq. mi.)
36 square miles (6 miles square) make 1 township (Tp.)

Unit equivalents

Tp.	sq. mi.	A	sq. ch.	P.	sq. l.
				1	625
			1	16	10,000
		1	10	160	100,000
	1	640	6,400	102,400	64,000,000
1	36	23,040	230,400	3,686,400	2,304,000,000

Scale—ascending, 625, 16, 10, 640, 36; descending, 36, 640, 10, 16, 625.

Cubic Measure.—This measure is used to find the volume or

*NOTE.—The denomination *rood* is practically obsolete. An acre equals a square whose side is 208.71 feet.

amount of space within the boundary surfaces of a body. It involves three dimensions, that is:

$$\text{volume} = \text{length} \times \text{breadth} \times \text{thickness}$$

As in square measure these dimensions may be taken in any denomination but all must be of the same denomination.

The word "cubic" is used to denote the product of the three dimensions, thus:

inches (length) × inches (breadth) × inches (thickness) = ***cubic inches***

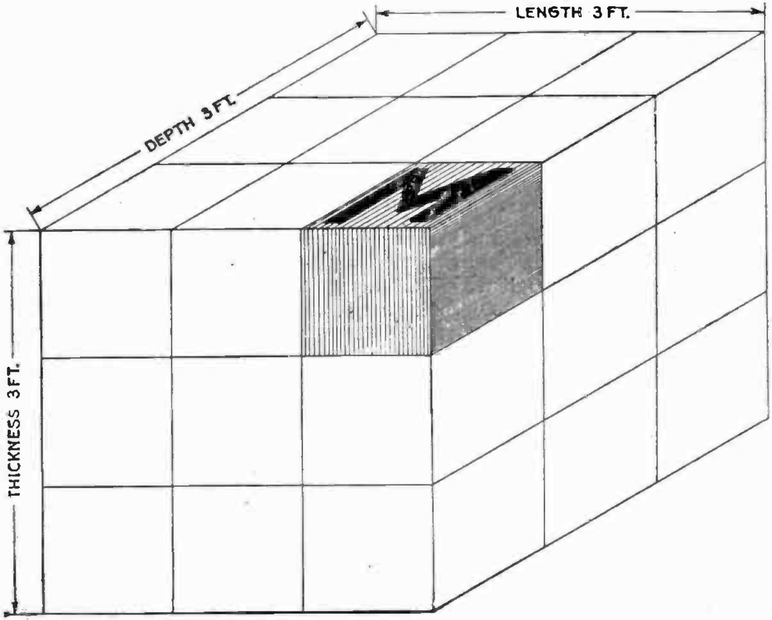


FIG. 30.—Diagram illustrating cubic measure. If each side of the cube measure 3 ft. and it be cut as indicated by the lines, each little cube as **M**, will have each of its sides 1 ft. long and will contain $1 \times 1 \times 1 = 1$ cu, ft. Accordingly the large cube will contain $3 \times 3 \times 3 =$ 27 cu. ft. or 1 cu. yd.

Cubic measure is shown graphically in fig. 30.

TABLE

Cubic Measure

1,728 cubic inches (cu. in.)	make 1 cubic foot (cu. ft.)
27 cubic feet	make 1 cubic yard (cu. yd.)
40 cubic feet of round timber or 50 cubic feet of hewn timber	make 1 ton or load (T)
16 cubic feet	make 1 cord foot (cd. ft.)
8 cord feet or 128 cubic feet	make 1 cord of wood (Cd.)
24¾ cubic feet	make 1 perch of stone or masonry (Pch.)

Scale—Most of the unit equivalents are fractional except 1,728 and 27, and are therefore omitted.

There are other kinds of cubic measure known collectively as measures of capacity. These are divided into two classes:

1. Liquid.
2. Dry.

Liquid measure also known as wine measure is used in measuring various liquids as water, molasses, liquors, etc.

TABLE

Liquid measure

4 gills (gi.)	make 1 pint (pt.)
2 pints	make 1 quart (qt.)
4 quarts	make 1 gallon (gal.)*
31½ gallons	make 1 barrel (bbl.)
2 barrels or 63 gallons	make 1 hogshead (hhd.)

Unit equivalents

hhd.		bbl.		gal.		qt.		pt.		gi.
								1	=	4
						1	=	2	=	8
				1	=	4	=	8	=	32
		1	=	31½	=	126	=	252	=	1,008
1	=	2	=	63	=	252	=	504	=	2,016

Scale—ascending, 4, 2, 4, 31½, 2; descending, 2, 31½, 4, 2, 4.

*NOTE.—There are two kinds of gallons; the U. S. gallon = 231 cu. ins.; the British Imperial gallon = 277.274 cu. ins.

Dry measure is used for measuring such articles as grain, salt, fruit, ashes, etc.

TABLE

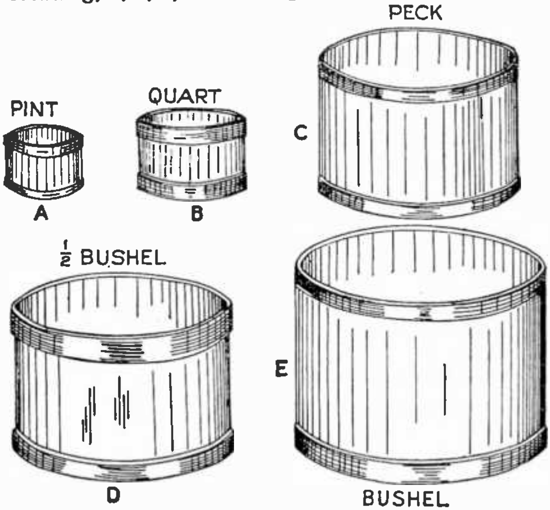
Dry measure

2 pints (pt.)	make 1 quart (qt.)
8 quarts	make 1 peck (pk.)
4 pecks	make 1 bushel (bu.)*

Unit equivalents

bu.		pk.		qt.		pt.
				1	=	2
		1	=	8	=	16
1	=	4	=	32	=	64

Scale—ascending, 2, 8, 4; descending, 4, 8, 2.



FIGS. 31 to 35.—Various dry measure containers. **A**, pint; **B**, quart; **C**, peck; **D**, ½ bushel· **E**, bushel basket.

*NOTE.—The standard U. S. bushel is the Winchester bushel, which is, in cylinder form, 18½ ins. in diameter and 8 ins. deep; it contains 2150.42 cu. ins. A struck bushel contains 2150.42 cu. in. or 1.2445 cu. ft. A heaped bushel is a cylinder 18½ ins. in diameter and 8 ins. deep, with a heaped cone not less than 6 ins. high. The British Imperial bushel = 8 imperial gallons = 2218.192 cu. ins. or 1.2837 cu. ft.

TABLE

Board Measure

1 board 1 in. thick × 1 ft. wide × 1 ft. long = 1 ft. board measure (B. M.)
1 board 2 in. thick × 1 ft. wide × 1 ft. long = 2 ft. board measure
1 board ½ in. thick × 1 ft. wide × 1 ft. long = 1 ft. board measure
etc.

from which follows

Board Measure Rule.—*Multiply length in ft. by width in ft. of the board and multiply this product by 1 for board an inch or less than an inch in thickness, and by the thickness in inches and fractions of an inch for board over 1 in. in thickness.*

Example.—How many feet board measure (B. M.) in a board 12 ft. long by 18 ins. wide by ½ in. thick?; by 1¾ in. thick?

$$18 \text{ ins.} = 18 \div 12 = 1\tfrac{1}{2} \text{ ft.}$$

$$\text{board } \tfrac{1}{2} \text{ in. thick} = 12 \times 1\tfrac{1}{2} \times 1 = 18 \text{ ft. B. M.}$$

$$\text{board } 1\tfrac{3}{4} \text{ in. thick} = 12 \times 1\tfrac{1}{2} \times 1\tfrac{3}{4}$$

$$= 12 \times 1.5 \times 1.75 = 31.5 \text{ ft. B. M.}$$

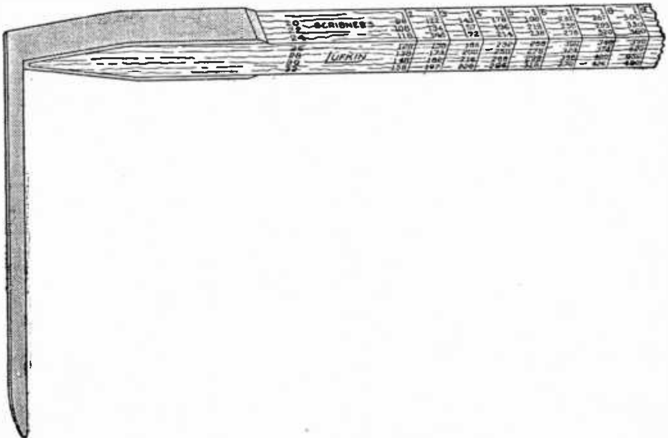


FIG. 36.—Lufkin Pacific coast log rule for large timber especially adapted to Pacific coast requirements, ⅞ in. square, and has 12 ins. forged steel hook (Seattle pattern). Marking is Scribner's scale for even length logs 20 to 48 ft. inclusive.

Measures of Weight.—By definition, weight is *the measure of the force with which bodies tend toward the earth's center; the downward pressure due to gravity minus the centrifugal force due to the earth's rotation.* Weight differs from gravity in being the effect of gravity or the downward pressure of a body under the influence of gravity. Weight is *the measure of the quantity of matter a body contains*. Three scales of weight are used in the U.S.:

1. Troy (for weighing gold silver, etc.).
2. Apothecaries (used by druggists in compounding medicines).
3. Avoirdupois (for all ordinary purposes).

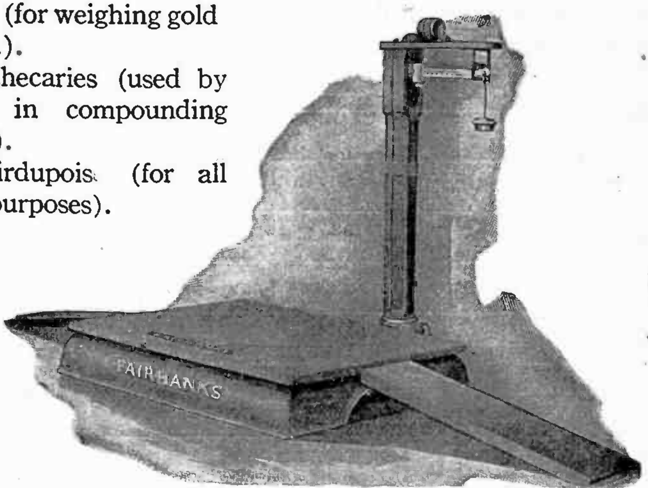


FIG. 37.—Fairbanks platform scale with incline brackets especially adapted to weighing wheelbarrow loads or for general use. The brackets cast on the ends of the frame form rests for incline planks so that the wheelbarrow loads may be easily run on and off the scale.

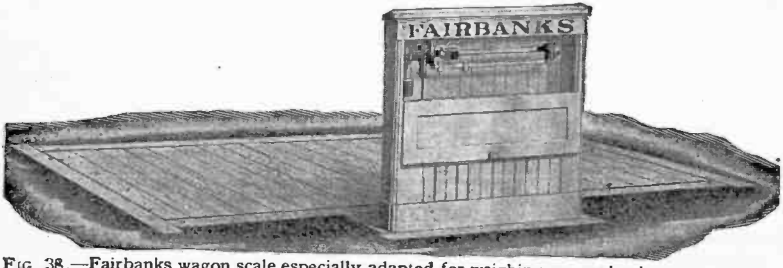


FIG. 38.—Fairbanks wagon scale especially adapted for weighing wagon loads.

***Troy Weight.**—This measure is used in weighing gold, silver and jewels; in philosophical experiments, and generally where great accuracy is required. The unit is the pound, and of this all the other denominations in the table are divisors.

Table

Troy Measure

24 grains (gr.)	make	1 pennyweight	pwt. or dwt.
20 pennyweights	"	1 ounce	oz.
12 ounces	"	1 pound	lb.

Unit equivalents

		pwt.	gr.
	oz.	1 =	24
lb.	1 =	20 =	480
1 =	12 =	240 =	5,760

Scale.—Ascending, 24,20,12; descending, 12,20,24.

Apothecaries' Weight.—This measure is used by apothecaries and physicians in compounding medicines.

Table

Apothecaries' Measure

20 grains (gr.)	make	1 scruple	sc. or ℈
3 scruples	"	1 dram	dr. or ʒ
8 drams	"	1 ounce	oz. or ℥
12 ounces	"	1 pound troy	lb. or ℔

Unit equivalents

			sc.	gr.
		dr.	1 =	20
	oz.	1 =	3 =	60
lb.	1 =	8 =	24 =	480
1 =	12 =	96 =	288 =	5,760

Scale.—Ascending, 20,3 8,12; descending, 12,8,3,20.

*NOTE.—Troy weight is sometimes called goldsmiths' weight.

Apothecaries' Fluid Measure.—The measures for fluids, as adopted by apothecaries and physicians in the United States, and used in compounding medicines, and putting them up for market, are as follows:

Table

Apothecaries' Fluid Measure

60 minims (♏)	make	1 fluidrachm (dram)	f ʒ
8 fluidrachms	"	1 fluidounce	f ℥
16 fluidounces	"	1 pint	O
8 pints	"	1 gallon	Cong.

Unit equivalents

						f ʒ		♏
				f ℥		1	=	60
		O		1	=	8	=	480
Cong.		1	=	16	=	128	=	7,680
1	=	8	=	128	=	2,048	=	61,440

Scale.—Ascending, 60,8,16,8; descending, 8,16,8,60.

Table

Avoirdupois Weight

16 drachms (dr.) or 437.5 grains (gr.)	make 1 ounce (oz.)
16 ounces	make 1 pound (lb.)
100 pounds	make 1 hundred weight (cwt.)
2,000 pounds	make 1 short ton
2,240 pounds	make 1 long ton

Unit equivalents

						oz.		dr.
				lb.		1	=	16
		cwt.		1	=	16	=	256
T.		1	=	100	=	1,600	=	25,600
1	=	20	=	2,000	=	32,000	=	512,000

Scale—ascending, 16, 16, 100, 20; descending, 20, 100, 16, 16.

Long ton Table

28 lbs.	make 1 quarter (qr.)
4 quarters	make 1 long hundred weight (cwt.)
20 hundred weight	make 1 long ton (T.)

				qr.		lbs.
		cwt.		1	=	28
l.t.	=	1	=	4	=	112
1	=	20	=	80	=	2,240

Scale—ascending, 28, 4, 20; descending, 20, 4, 28.

Also (in Great Britain):

14 lbs.	= 1 stone
2 stone = 28 lb.	= 1 quarter
4 quarters = 112 lb.	= 1 long hundredweight (cwt.)
20 hundredweight	= 1 long ton

Shipping Measure

1 Register ton	= 100 cu. ft.
1 U. S. shipping ton	= 40 cu. ft.
	= 32.14 U. S. bu.
	= 31.14 imp. bu.
1 British shipping ton	= 42 cu. ft.
	= 32.70 imp. bu.
	= 33.75 U. S. bu.

Circular measure Table

60 seconds (″)	make 1 minute (′)
60 minutes	make 1 degree (°)
30 degrees	make 1 sign (S)
360 degrees	make 1 circle (C)

Unit equivalents

				°		′		″
						1	=	60
		S		1	=	60	=	3,600
C		1		30	=	1,800	=	108,000
1	=	12	=	360	=	21,600	=	1,296,000

Scale—ascending, 60, 60, 30, 12; descending, 12, 30, 60, 60.

TABLE

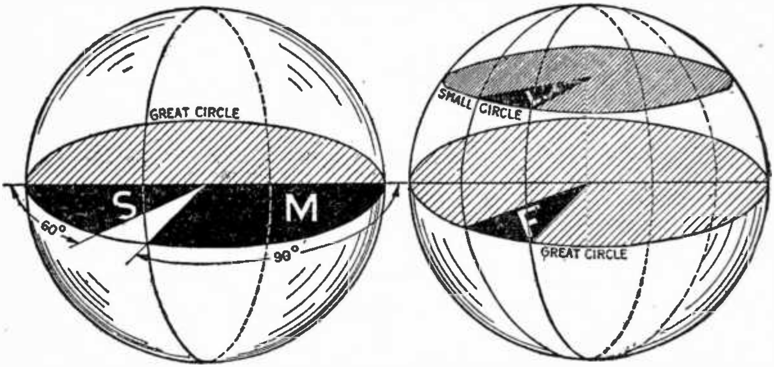
Time

60 seconds (sec.″)	make 1 minute (min.′)
60 minutes	make 1 hour (hr.)
24 hours	make 1 day (da.)
365 days	make 1 common year (yr.)
366 days	make 1 leap year
12 calendar months	make 1 year
100 years	make 1 century (C.)

Unit equivalents

yr.	mo.	wk.	da.	hr.	min.	sec.
					1	= 60
				1	= 60	= 3,600
			1	= 24	= 1,440	= 86,400
		1	= 7	= 168	= 10,080	= 604,800
1	= 12 =		{ 365	= 8,760	= 525,600	= 31,536,000
			{ 366	8,784	= 527,040	= 31,622,400

Scale—ascending, 60, 60, 24, 7· descending, 7, 24, 60, 60.



FIGS. 39 and 40.—Globes illustrating circular measure. Fig, 39 shows a great circle, or circle passing through the center of the globe. In contrast with this is the small circle shown in fig. 40. In circular measure each of these circles is divided into 360°. Fig. 39 shows a quadrant M, made up of two radii and a 90° arc, also a sextant S, measured with a 60° arc. Note in fig. 40, the angles L, and F, are the same and accordingly have the same number of degrees, but the length of each degree of L, is less than that of F.

The Metric System.—This system was adopted in France in 1795 and its use was authorized in Great Britain in 1864, and in the United States in 1866.

The important feature of the metric is that it is based upon the ***decimal scale,*** hence, the student should first acquire a knowledge of decimals before taking up the metric system.

The *meter* is the base or unit of the system and is defined the one ten-millionth part of the distances on the earth's surface from the equator to either pole. Its value in inches should be remembered.

1 meter = 39.37079 ins.

The theory of the system is that the meter is a $\frac{1}{10000000}$ part of a quadrant of the earth through Paris; the liter or unit of volume is a cube of $\frac{1}{10}$ meter side; the gramme or unit of weight is (nominally) $\frac{1}{1000}$ of the weight of a liter of water at 4° C. The idea of adopting scientific measurements had been suggested as early as the 17th century, particularly by the astronomer Jean Picard (1620-1682) who proposed to take as a unit the length of a pendulum beating one second at sea-level, at a latitude of 45°.

NOTE.—The meter, as adopted by England, France, Belgium, Prussia and Russia, is that determined by Capt. A. R. Clarke, R.E., F.R.S., 1866, which at 32° in terms of Imperial standard at 62° Fahr. is 39.370432 ins. or 1.009362311 yards, its legal equivalent by Metric Act of 1864 being 39.3708 ins., the same as adopted in France.

NOTE.—Captain Kater's comparison, and the one formerly adopted by the U. S. Ordinance Corps, was = 39.3707971 inches, or 3.28089976 feet, and the one adopted by the U. S. Coast Survey was as above noted, 39.37043235 ins.

It must be apparent that the meter is used like tho yard in measuring cloths and short distances.

Units of other denominations are named by prefixing to the word *meter* the Latin numerals for the lower denominations and Greek numerals for the higher denominations.

Lower denomination		*Higher denomination*	
Deci	= 1/10	Deka	= 10
Centi	= 1/100	Hecto	= 100
Milli	= 1/1000	Kilo	= 1,000
		Myria	= 10,000

Thus one decimeter = 1/10 of a meter; 1 millimeter = one-thousandth of of a meter and one kilometer = one-thousand meters. From this the linear table which follows is easily understood.

Metric Table of Linear Measure

Metric Denomination			Meter		U. S. value
	1 millimeter	=	.001	=	.03937 in.
10 millimeters	= 1 centimeter	=	.01	=	.3937 in.
10 centimeters	= 1 decimeter	=	.1	=	3.937 in.
10 decimeters	= 1 *meter*	=	1.	=	39.3707 ins.
10 meters	= 1 dekameter	=	10.	=	32.809 ft.
10 dekameters	= 1 hectometer	=	100.	=	328.09 ft.
10 hectometers	= 1 kilometer	=	1,000.	=	.62137 mi.
10 kilometers	= 1 myriameter	=	10,000.	=	6.2137 mi.

The kilometer is commonly used for measuring long distances.

The square meter is the unit for measuring ordinary surfaces; as flooring, ceilings, etc.

Metric Table of Square Measure

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter	= .155+ sq. in.
100 sq. centimeters (sq. cm.)	= 1 sq. decimeter	= 15.5 + sq. in.
100 sq. decimeters (sq. dm.)	= 1 sq. meter (sq. m.)	= 1.196+ sq. yd.

1 sq. meter = 10.7639 sq. ft.

Metric Table of Land Measure

1 centiare (ca.)	= (1 sq. meter)	=	1.196034 sq. yd.
100 centiares (ca.)	= 1 are	=	119.6034 sq. yd.
100 ares (A.)	= 1 hectare	=	2.47114 acres
100 hectares (ha.).	= 1 sq. kilometer	=	.3861 sq. mi.

The cubic meter is the unit for measuring ordinary solids: as excavations, embankments, etc.

Metric Table of Cubic Measure

1,000 cu. millimeters (cu. mm.)	= 1 cu. centimeter	= .061+ cu. in.
1,000 cu. centimeters (cu. cm.)	= 1 cu. decimeter	= 61.023+ cu. in.
1,000 cu. decimeters (cu. dm.)	= 1 cu. meter	= 35.314 + cu. ft.

The stere is the unit of wood or solid measure, and is equal to a cubic meter or .2759 cord.

Metric Table of Wood Measure

	1 decistere	= 3.531+ cu. ft.
10 decisteres (dst.)	= 1 stere	= 35.316+ cu. ft.
10 steres (st.)	= 1 dekastere (dst.)	= 13.079+ cu. yd.

The liter is the unit of capacity, both of liquid and of dry measures, and is equal in volume to a cube which has an edge of one-tenth of a meter, equal to 1.05673 qt. liquid measure, and .9081 qt. dry measure.

Metric Table of Capacity

10 milliliters (ml.)	= 1 centiliter
10 centiliters (cl.)	= 1 deciliter
10 deciliters (dl.)	= 1 liter
10 liters (l.)	= 1 dekaliter
10 dekaliters (dl.)	= 1 hectoliter
10 hectoliters (hl.)	= 1 kiloliter, or stere
10 kiloliters (kl.)	= 1 myrialiter (ml.)

The hectoliter is the unit in measuring liquids, grain, fruit, and roots in large quantities.

		Dry	Liquid
1 myrialiter	= 10 cubic meters	= 283.72+ bu.	= 2641.7+ gal.
1 kiloliter	= 1 cubic meter	= 28.372+ bu.	= 264.17 gal.
1 hectoliter	= 1/10 cubic meter	= 2.8372+ bu.	= 26 417 gal.
1 dekaliter	= 10 cu. dm.	= 9.08 quarts	= 2.6417 gal.
1 liter	= 1 cu. dm.	= .908 quart	= 1.0567 qt.
1 deciliter	= 1/10 cu. dm.	= 6.1022 cu. in.	= .845 gil.
1 centiliter	= 10 cu. cm.	= .6102 cu. in.	= .338 fluid oz.
1 milliliter	= 1 cu. cm.	= .061 cu. in.	= .27 fluid dr.

The gram is the unit of weight, and equal to the weight of a cube of distilled water, the edge of which is one hundredth of a meter, equal to 15.432 Troy grains.

Metric Table of Weight Measure

10 milligrams (mg.)	= 1 centigram	= .15432+ gr. troy
10 centigrams (cg.)	= 1 decigram	= 1.54324+ gr. troy
10 decigrams (dg.)	= 1 gram	= 15.43248+ gr. troy
10 grams (g.)	= 1 dekagram	= .35273+ oz. avoir.
10 dekagrams (Dg.)	= 1 hectogram	= 3.5274 + oz. avoir.
10 hectograms (hg.)	= 1 kilogram or kilo	= 2.20462+ lb. avoir.
10 kilograms (kg.)	= 1 myriagram	= 22.04621+ lb. avoir.
10 myriagrams (Mg.) or 100 kilograms	= 1 quintal	= 220.46212+ lb. avoir.
10 quintals or 1,000 kilos	= 1 tonneau, or 1 ton	= 2204.62125+ lb. avoir.

Symbol ☞ :

Ratio

Ratio is *the relation of one number to another of the same kind*. Thus the ratio of 12 to 23 is expressed as 12:23 or in the form of a fraction as $\frac{12}{23}$.

When it is required to determine *what the relation of one number to another is*, it is evident that the *first* is the dividend, and the *second* the divisor.

When it is required to determine *the relation between two numbers*, either may be regarded as dividend or divisor.

The first number is commonly regarded as the dividend.

Note the following definitions:

Terms of a ratio.—The numbers compared.

Antecedent.—The first term.

Consequent.—The second term.

Couplet.—The antecedent and consequent together.

Observe from the definitions:

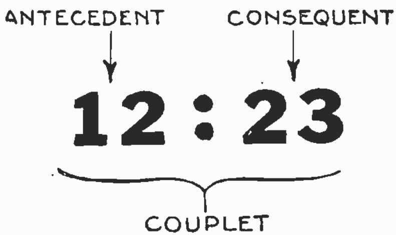


FIG. 41.—Example of a couplet.

Ratio problems should be solved according to the following rules:

Rule 1.—*The terms of a ratio must be like numbers.*

Rule 2.—*The ratio is an abstract number.*

Rule 3.—*Multiplying or dividing both terms of a ratio by the same number does not change the ratio of the terms.*

Rule 4.—*Fractions should be reduced to fractions having the same denominator; when thus reduced they will have the ratio of their numerators.*

Example.—When steam is cut off at ¼ stroke what is the ratio of expansion? This ratio is expressed as:

$$\text{stroke} \div \text{cut off}$$

$$1 \div \tfrac{1}{4} = 1 \times \tfrac{4}{1} = 4$$

Proportion

Symbols ☞ : :: or =

Proportion is *an equality of ratios*, that is, when two ratios are equal the four terms form a proportion. A proportion is expressed by putting the sign = or : : between the ratios, thus:

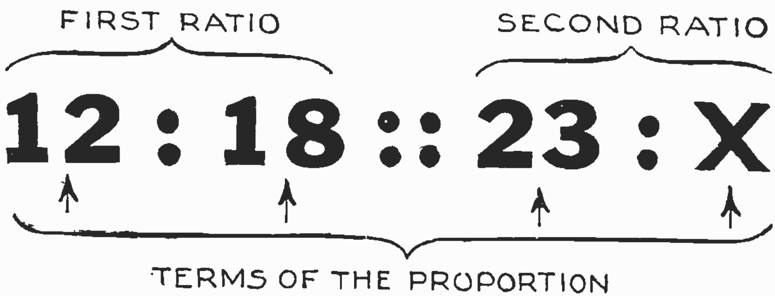


FIG. 42.—Method of writing out a proportion.

The proportion may also be written:

$$\frac{12}{18} = \frac{23}{X}$$

Note the following definitions:

Antecedents.—The antecedents of the proportion are *the first and third terms.*

Consequents.—The consequents of the proportion are *the second and fourth terms.*

***Extremes.**—The first and fourth terms.*

***Means.**—The second and third terms.*

Note extremes and means in the following proportion:

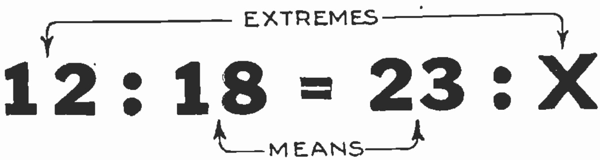


FIG. 43.—Example illustrating extremes and means in a proportion.

Problems in proportion should be solved according to the following rules:

Rule 1.—*The product of the extremes is equal to the product of the means.*

Rule 2.—*The product of the extremes divided by either mean gives the other mean.*

Rule 3.—*The product of the means divided by either extreme gives the other extreme.*

There are several kinds of proportion defined as follows:

Simple proportion.—An equality between two simple ratios.

Compound proportion.—One in which either ratio is compound, that is, either ratio is the product of two or more simple ratios.

Direct proportion.—One in which each term *increases* or *diminishes*, as the one on which it depends *increases* or *diminishes*.

Inverse proportion.—One in which each term *increases* as the term upon which it depends *diminishes*, or *diminishes* as it *increases*.

Example in direct proportion.—If an engine make 475 revolutions in one minute, how many will it make in two minutes?

small number of rev.		large number of rev.		short time		long time
475	:	X	=	1	:	2

Note that each ratio must contain quantities of the same kind. The unknown quantity is represented by the letter X. Since from the example it is evident that the consequent of the first ratio will be greater than the antecedent, write down the second ratio so its consequent is greater than its antecedent, thus:

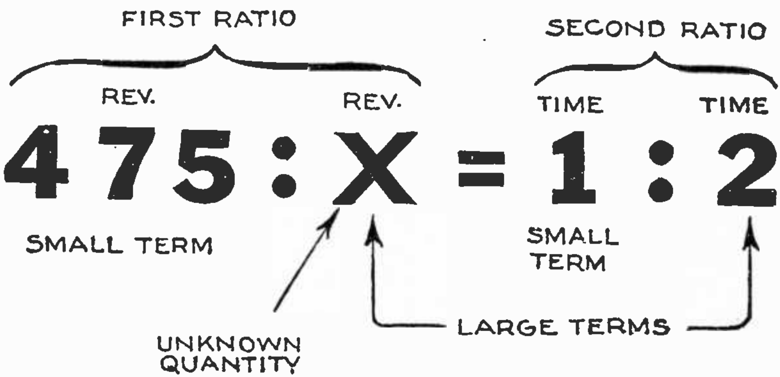


FIG. 44.—Example illustrating how to arrange a proportion.

Multiplying extremes and means according to Rule 1.

$$X \times 1 = 475 \times 2$$

from which

$$X = \frac{475 \times 2}{1} = 950 \text{ rev.}$$

Example in inverse proportion.—If 12 men can dig a ditch in 18 hours, how long would it take 23 men to dig the same ditch?

Evidently 23 men will dig the ditch in *less* time than 12 men, and the proportion will be arranged thus:

$$\left\{\begin{matrix}\textit{small number}\\ \textit{of men}\end{matrix}\right\} : \left\{\begin{matrix}\textit{large number}\\ \textit{of men}\end{matrix}\right\} = \left\{\begin{matrix}\textit{short}\\ \textit{time}\end{matrix}\right\} : \left\{\begin{matrix}\textit{long}\\ \textit{time}\end{matrix}\right\}$$

Substituting

$$\begin{matrix}\text{men} & & \text{men} & & \text{time} & & \text{time}\\ 12 & : & 23 & = & X & : & 18\end{matrix}$$

$$23X = 216$$

$$X = 9.3 \text{ hrs., or } 9 \text{ hrs., } 18 \text{ min.}$$

Short Form for Simple Proportion.—A simple proportion as for instance:

$$12 : 18 = 23 : X$$

may be expressed in practical form, as:

$$\frac{12}{18} = \frac{23}{X}$$

which by putting X in the numerator becomes:

$$\frac{X}{18} = \frac{23}{12} \text{ or } X = 18 \times \frac{23}{12} = 34\tfrac{1}{2}$$

Hence in a simple proportion the value of X, the unknown term, is the product of one of the known terms multiplied by the ratio of the other two terms. *From the nature of the problem it is easily determined whether the ratio will be greater than or less than* 1.

The problem may then be expressed by one equation according to the following rules.

Rule 1.—*Find the ratio terms and put down in fractional form making the fraction greater than or less than one as determined by the nature of the problem.*

Rule 2.—*Multiply the ratio by the third known term.*

Example.—If a pump deliver 23 gals. of water in 5 minutes, how long will it take to pump 475 gals.?

The third term will be of the same kind as the unknown term; in the example, 5 minutes. The two other known terms, 23 and 475, form the ratio. Evidently the answer will be greater than 23 the third term, and the ratio greater than 1. Knowing this the equation is written

unknown term		third term	ratio
X	$=$	5	$\times \frac{475}{23} = 103.3$ minutes

Compound Proportion.—This is an expression of equality between a compound and a simple ratio, or between two compound ratios. The principle of compound proportion is that *the product of two or more proportions is a proportion.*

In stating problems in compound proportion the quantity that corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required.

Example.—If 4 men cut 15 trees in 5 days of 14 hrs., in how many days of 13 hrs. can 7 men cut 19½ trees?

As the answer is to be in days, make 5 days the third term. It will require *less days* for 7 men to cut 15 trees than for 4 men. Therefore, make 7 the first term, and 4 the second.

It will require *more days* for the same number of men to cut 19½ trees than to cut 15 trees. Therefore, make 15 the first term and 19½ the second

It will require *more days* of 13 hrs. than of 14 hrs. for the same number of men to cut the same number of trees. Therefore, make 13 the first term and 14 the second.

Hence,

$$\left.\begin{array}{r}7\\15\\13\end{array}\right\}:\left.\begin{array}{l}4\\19.5\\14\end{array}\right\}=5\text{ days}:X$$

Therefore, the fourth term, or the time required is

$$\frac{4\times19.5\times14\times5\text{ days}}{7\times15\times13\times X}=4\text{ days}$$

Short Form for Compound Proportion.—In every compound proportion all the terms appear in ratios except one which is of the same kind as the answer and called the *odd term*. Find the unknown term according to the following rules.

Rule 1.—*Multiply the odd term by the product of all the ratios.*

Rule 2.—*Make each ratio greater than, or less than one according as the unknown term (depending upon each ratio separately) should be greater than or less than the odd terms.*

Example.—If it cost \$4,320 to supply 32 men with provisions for 18 days, when the rations are 15 ozs. per day, what will it cost to supply 24 men for 34 days, when the rations are 12 ozs. per day?

men days ounces

$$\$4{,}320\times\frac{24}{32}\times\frac{34}{18}\times\frac{12}{15}=\$4{,}896$$

Percentage

Symbol %

Percent is briefly, *the rate per hundred;* from the Latin *per centum* meaning by the hundred, that is a certain part of every hundred. Thus 23 per cent means 23 out of every hundred.

To illustrate, 23 per cent of one dollar, or 100 cents $= \frac{23}{100}$ of $100 = 23$ cents.

The sign for per cent is %. Thus 12% is read twelve per cent.

Since per cent is a number of hundredths, it is usually expressed as a decimal. 12% or $\frac{12}{100}$ is written .12.

Note carefully how to express percentage as follows:

In a statement write twelve per cent 12%.

In a calculation write twelve per cent .12.

Note carefully how to express less than one per cent.

Example.—Express ¼ of 1% as a decimal.

$$\frac{1}{4} \text{ of } 1\% = \frac{1}{4} \text{ of } \frac{1}{100} = \frac{1}{400} = .0025$$

The following terms should be understood:

Rate.—The number of hundredths taken.

Base.—The number on which the percentage is computed.

Percentage.—The number which is a certain number of hundredths of the base.

Amount.—The sum of the base and percentage.

Difference.—The base less the percentage.

Percentage involves the following kinds of problems:

Case 1.—To find the percentage when the base and rate are given.

Case 2.—To find the base when the percentage and rate are given.

Case 3.—To find the rate when the base and percentage are given.

Case 4.—To find the base when the amount and rate are given.

Case 5.—To find the base when the difference and rate are given.

Case 1.—*To find the percentage when the base and rate are given:*

Rule.—*Multiply the base by the rate.*

Formula: $P = B \times R$ ....(1)

Example.—What is 23% of 475?

Here 475 is the base and 23% the rate. Substituting in formula (1)

$$P = 475 \times .23 = 109.25$$

Case 2.—*To find the base when the percentage and rate are given.*

Rule.—*Divide the percentage by the rate.*

Formula: $B = P \div R$ ....(2)

Example.—A farmer lost 24 sheep which was 12% of his flock. How many sheep did he have?

Here 24 is the percentage and 12 % the rate. Substituting in formula (2)

$$B = 24 \div .12 = 200 \text{ sheep}$$

Case 3.—*To find the rate when the base and percentage are given.*

Rule.—*Divide the percentage by the base.*

Formula: $R = P \div B$ ....(3)

Example.—If Gilbert deposit \$26 with Stevens and draw \$10 what part of it is withdrawn?

Here \$26 is the base and \$10 the percentage. Substituting in formula (3)

$$R = 10 \div 26 = .384 \text{ or } 38\tfrac{1}{2}\% \text{ withdrawn by Gilbert}$$

Case 4.—*To find the base when the amount and rate are given.*

Rule.—*Divide the amount by 1 + the rate.*

Formula: $B = A \div (1 + R)$ . . . . (4)

Example.—What number increased by 25% of itself equals 475?

Here 475 is the amount and 1.25 is one plus the rate. Substituting in formula (4)

$$475 \div 1.25 = 380$$

Case 5.—*To find the base when the difference and rate are given.*

Rule.—*Divide the difference by 1 − the rate.*

Formula: $B = D \div (1 - R)$ . . . . (5)

Example.—What number diminished by 27% of itself equals 401.5?

Here 401.5 is the difference and 1 − .27, or .73, is 1 minus the rate. Substituting in formula (5)

$$401.5 \div 73 = 550$$

TEST QUESTIONS

1. *What is arithmetic?*
2. *Give list of signs and abbreviations used.*

3. *What is the difference between Roman and Arabic systems?*
4. *Numerate the number* 1223475.
5. *Give ten formulae which include the elementary operations of arithmetic.*
6. *Give rules for addition.*
7. *What is the difference between the subtrahend and the minuend?*
8. *Multiply* 4175 *by* 1223.
9. *Give the Italian short proof method for multiplication.*
10. *What is the difference between a root and an exponent?*
11. *What kind of numbers are divisible by:* 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, *and* 13, *odd number, even number, etc.?*
12. *What is the greatest common divisor?*
13. *Find the least common multiple of* 60, 84 *and* 102.
14. *What is a vulgar fraction?*
15. *What is a decimal?*
16. *Express* .23 *in the form of a fraction.*
17. *Numerate the decimal* .0000138.
18. *How are calculations shortened by cancellation?*
19. *Find the square root of* 12143785.

20. *Explain the method of mental cube root.*

21. *Give a simple rule for cube root.*

22. *How is lumber measured by the board measure rule?*

23. *What is the difference between direct and inverse proportion?*

CHAPTER 2

Plane Geometry

By definition, geometry is *that branch of pure mathematics that treats of space and its relations.*

In other words, it is the science of the mutual relations of points, lines, angles, surfaces and solids, considered as having no properties except those arising from extension and difference of situation.

Proposition.—This is a statement of *something to be done.* Thus,—

The sum of the angles of a triangle is equal to two right angles, is a proposition to be proved. A proposition is either a theorem or a problem.

The student should clearly understand the difference between the following:

Axiom.—A self-evident truth; a proposition or principle that needs no demonstration.

Theorem.—A proposition not self-evident that is clearly demonstrably true or acknowledged as such; 2. a proposition setting forth something to be proved, as opposed to problem.

Corollary.—A proposition following so obviously from another that it requires little or no demonstration.

Postulate.—A self-evident statement claimed as basis of argument especially regarding a geometrical construction.

Problem.—A proposition in which some operation or construction is required, also the demonstration showing how the task is to be accomplished.

Axioms.—The self-evident truths which follow are important axioms used as bases of proofs in goemetry.

Axioms

1. *Things equal to the same thing are equal to each other.*
2. *If equals be added to equals the sums will be equal.*
3. *If equals be subtracted from equals, the remainders will be equal.*
4. *If equals be multiplied by equals the products will be equal.*
5. *If equals be divided by equals, the quotients will be equal.*
6. *If equals be added to unequals, the sums will be unequal.*
7. *If equals be subtracted from unequals, the remainders will be unequal.*
8. *The whole is greater than any of its parts and is equal to the sum of all its parts.*
9. *Things which are halves of the same thing are equal to each other.*
10. *Things which are doubles of the same thing are equal to each other.*
11. *A quantity may be substituted for its equal in an equation or in an inequality.*

Postulates.—The self-evident propositions or postulates which follow are also used as bases of proofs:

1. *One straight line and only one can be drawn through two given points.*
2. *A straight line may be produced to any required length.*
3. *A straight line is the shortest distance between two points.*
4. *A circle may be described with any given point as a center and any given line as a radius.*
5. *Any figure may be moved from one place to another without altering its size or shape.*
6. *The two adjacent angles which one straight line makes with another are together equal to a straight angle.*
7. *If the sum of two adjacent angles be a straight angle, their exterior sides are in the same straight line.*
8. *A right angle is half a straight angle.*

9. *All straight angles are equal.*

10. *Two straight lines whose extremities coincide must meet throughout their whole extent.*

11. *All radii of the same circle, or of equal circles, are equal and all circles of equal radii are equal.*

Corollaries.—The corollaries here given should be carefully noted:

1. *Two points determine a straight line.*
2. *Two straight lines can intersect in only one point.*
3. *All right angles are equal.*
4. *From a given point in a given line only one perpendicular can be drawn to the line.*
5. *Equal angles have equal complements, equal supplements, and equal conjugates.*
6. *The greater of two angles has the less complement, the less supplement and the less conjugate.*
7. *In a triangle there can be but one right angle or one obtuse angle.*
8. *Corresponding parts of equal figures are equal.*
9. *Two points each equidistant from the extremities of a line determine the perpendicular bisector of the line.*

Notation.—There are a few symbols used so frequently that they should be understood before taking up theorems. They are as follows:

$=$ equality sign	$\parallel$ parallel	▱ parallelogram
$<$ is less than	$\perp$ perpendicular	⊙ circle
$>$ is greater than	$\angle$ angle	$\triangle$ triangle

∴ therefore — **Theo.** theorem

□ square — **Cor.** corollary

▭ rectangle — **Post.** postulate

Q. E. D. which was to be proved (Latin: *quod erat demonstrandum*).

Q. E. F. which was to be done (Latin: *quod erat faciendum*).

Definitions

Acute Angle.—An angle less than a right angle.

Adjacent Angles.—Angles lying next to each other. Angles having a common side and vertices at the same point.

Altitude.—The elevation of an object above its base, or the perpendicular distance between the top and bottom of a figure.

Angle.—The difference in direction of two lines which meet or tend to meet. The lines are called the sides and the point of meeting, the vertex of the angle. Angles are distinguished in respect to magnitude by the terms right, acute and obtuse angles.

Apex.—The summit or highest point of an object.

Arc.—Part of the circumference of a circle.

Axis of a Figure.—A straight line passing through the center of a figure, and dividing it into two equal parts.

Base.—The lowest part.

Bisect.—To divide into two equal parts.

Bisector.—A line which bisects.

Circle—A plane figure bounded by one uniformly curved line, all of the points in which are at the same distance from a certain point within, called the center. The circle is the space contained within the circumference. Of all plane figures the circle has the greatest area within the same perimeter.

Circumscribe.—To draw the line of a figure about or outside, such as a circle drawn around a square touching its corners.

Complement of an Angle.—The angle added to it to form a right angle.

Concave.—Curving inwardly.

Conic Section.—A figure formed by the intersection of a plane with a right circular cone; a triangle, ellipse, parabola or hyperbola according to the position of the cutting plane.

Conjugate of an Angle.—The angle added to it to make a perigon.

Constant.—Remaining unchanged or invariable.

Contour.—The outline of an object.

Convergence.—Lines extending toward a common point.

Converse Propositions.—Propositions so related that what is given in each is what is to be proved in the other.

Convex.—Rising or swelling into a round form—the opposite to concave.

Curve.—A line of which no part is straight.

Degree.—The 360th part of a circle.

Describe.—To make or draw a curved line; to draw a plan.

Develop.—To unroll or lay out.

Diagonal.—A right line drawn from angle to angle of a quadrilateral or many angled figure and dividing it into two parts.

Diameter.—A right line passing through the center of a circle or other round figure terminated by the curve and dividing the figure symmetrically into two equal parts.

Edge.—The intersection of any two surfaces.

Elevation.—The term elevation, vertical projection and front view—applied to drawings—all have the same meaning.

Ellipse.—The locus of a point which moves so that the sum of its distances from two fixed points called the foci, is a constant.

Exterior Angle.—Angle formed by the side of a polygon and an adjacent side produced.

Foci.—Two points the sum or difference of whose distances to a conic section is a constant. In an ellipse the *sum* of the distances is a constant.

Foreshortening.—Apparent decrease in length, owing to objects being viewed obliquely; thus a wheel, when seen obliquely, instead of appearing round, presents the appearance of an ellipse.

Generate.—To form a geometric magnitude by moving a point, line or surface. A line is generated by moving a point; a surface by moving a line.

Hemisphere.—Half a sphere obtained by bisecting a sphere by a plane.

Horizontal.—Parallel with the surface of smooth water. In drawing, a line drawn parallel with the top and bottom of the sheet is called horizontal

Hypothesis.—A supposition on which a demonstration may be founded.

Hypotenuse.—The side of a right angled triangle opposite the right angle.

Inscribe.—See circumscribe—its opposite.

Intercepted Arc.—The part of the circumference between the intersection of two lines with the circumference.

Interior Angle.—One of the four inside angles made by a line cutting two parallel lines.

Locus.—A straight line, surface or curve regarded as traced by one or more points or a line moving under specified conditions. The locus of the tip of a clock hand is a circle.

Longitudinal.—In the direction of the length of an object.

Normal.—A perpendicular to a line at the point of tangency to a curve.

Oblique.—Neither horizontal nor vertical.

Oblong.—A rectangle with unequal sides.

Obtuse Angle.—Greater than a right angle.

Opposite Angles.—Angles which do not lie on the same side of a line.

Oval.—A plane figure resembling the longitudinal section of an egg; or elliptical in shape.

Overall.—The entire length.

Parallel.—Having the same direction and everywhere equally distant.

Perigon.—The entire space around a point. The sum of four right angles.

Perimeter.—The boundary of a closed plane figure.

Periphery.—Circumference.

Perpendicular.—At an angle of 90°.

Perspective.—View; drawing objects as they appear to the eye from any given distance and situation, real or imaginary.

Plan.—Plan, horizontal projection and top view have the same meaning.

Plane Figure.—A part of a plane surface bounded by straight or curve lines, or by both combined

Polygon.—A plane figure bounded by straight lines called the sides of the polygon. The least number of sides that can bound a polygon is three. Polygons bounded by a greater number of sides than four are designated only by the number of sides.

Projection.—The view of an object obtained upon a plane by projecting lines perpendicular to the plane.

Quadrant.—The fourth part; a quarter; the quarter of a circle.

Quadrilateral.—A polygon having four sides.

Quadrisect.—To divide into four equal parts.

Radius.—A straight line from the center of a circle to the circumference; half the diameter.

Rectangle.—A rectangle is a parallelogram having its angles right angles.

Rectilinear.—Right lined; straight.

Reflex Angle.—An angle greater than a straight angle.

Right Line.—Straight line.

Scholium.—A remark pertaining to one or more preceding propositions.

Section.—A projection upon a plane parallel with a cutting plane which intersects any object. The section generally represents the part behind the cutting plane, and represents the cut surfaces by diagonal lines.

Sector.—The part of a circle included between two radii and the intercepted arc.

Segment.—The portion of a circle included between a chord and the arc which it subtends.

Straight Angle.—One in which the sides of the angle extend in opposite directions and form a straight line. A straight angle is equal to two right angles.

Subtended Arc.—Portion of the circumference between the intersections of a chord.

Supplement of an Angle.—The angle added to it to make a straight angle.

Surface.—Space having only two dimensions—length and breadth.

Symmetry.—A proper adjustment or adaptation of parts to one another and to the whole.

Tangent.—A line which touches the circumference in only one point.

Triangle.—A polygon having three sides and three angles.

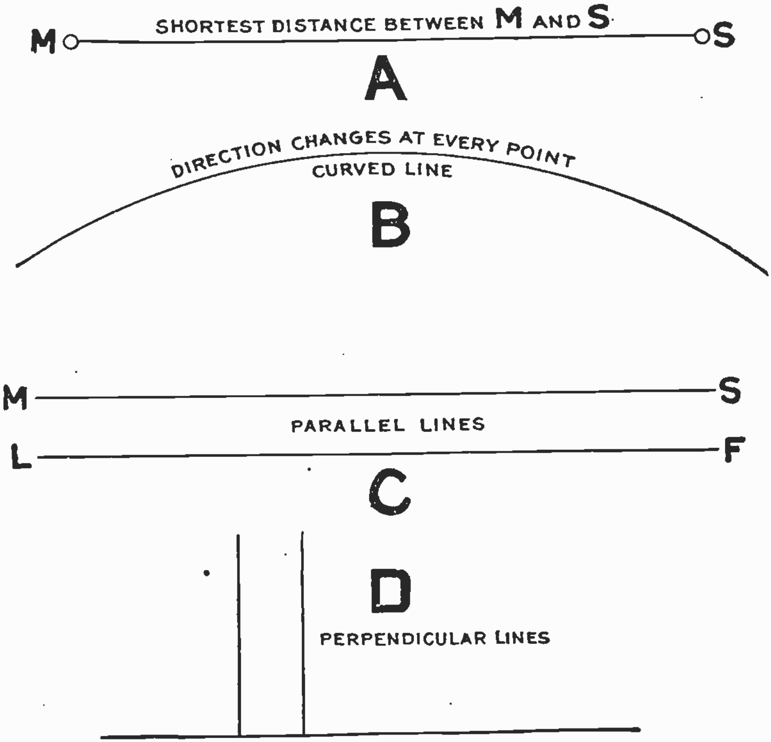
Trisect.—To divide into three equal parts.

Variable.—A quantity which, by the conditions of a problem, is susceptible of continuous change of value.

Vertex.—See angle, quadrilateral, triangle. The vertex of a cone is the point in which its axis intersects the lateral surface.

Vertical.—Upright or perpendicular to a horizontal line or plane. Vertical and perpendicular are not synonymous terms.

Lines.—There are two kinds of lines: straight and curved. A straight line is *the shortest distance between two points*. A curved line is *one which changes its direction at every point*. Two lines are said to be parallel *when they have the same direction*. A horizontal line is *one parallel with the horizon or surface of the water*. A line is perpendicular to another line *when it inclines no more to one side than the other*.

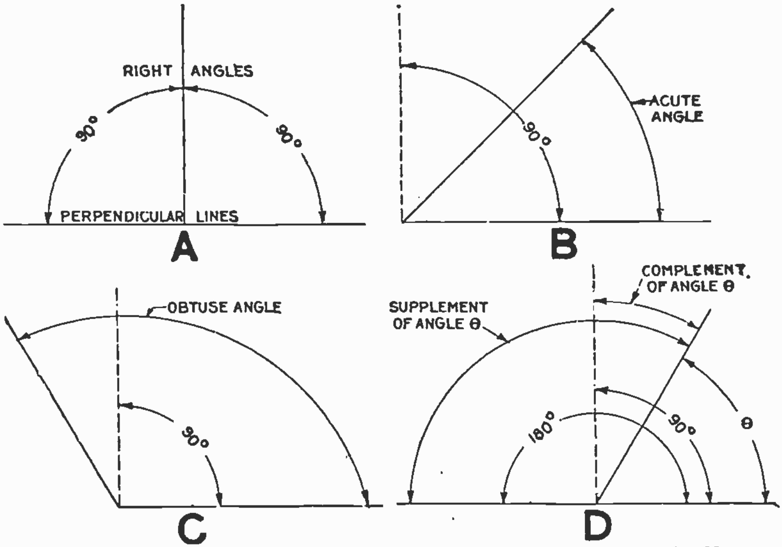


FIGS. 45 to 48.—Various lines. Fig. 45, straight; fig. 46, curved; fig. 47, parallel; fig. 48, perpendicular.

Angles.—An angle is *the difference in direction of two lines proceeding from the same point called the vertex.*

Angles are said to be, *right,* when formed by two perpendicular lines.

Plane Figures.—The term plane figure denotes *a* **plane** *surface bounded by straight or curved lines.* A proper conception



FIGS. 49 to 52.—Various angles. Fig. 49, right; fig. 50, acute; fig. 51, obtuse; fig. 52, complement and supplement of an angle.

of the term "plane" is essential. A plane or plane surface is *one such that any straight line joining any two points lies wholly in the surface.*

Fig. 53 defines a plane surface, and figs. 54 and 55 the ordinary and erroneous idea of such surface. There is a great variety of plane figures known as *polygons* when their sides are straight lines. The sum of the length of the sides is called the *perimeter*. A regular polygon has all its sides and angles equal. Plane figures of three sides are known as triangles; of four sides, *quadrilaterals*, etc. Various plane figures are also formed of curved sides as *circles*, *ellipses*, etc.

Lines and Triangles

Theorem 1.—*If two straight lines intersect, the opposite angles are equal.*

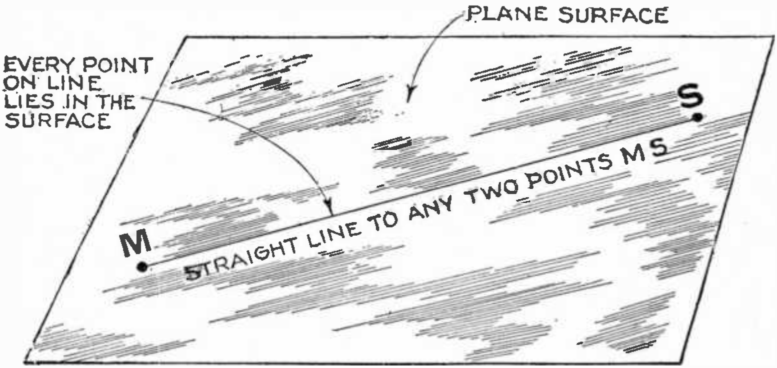
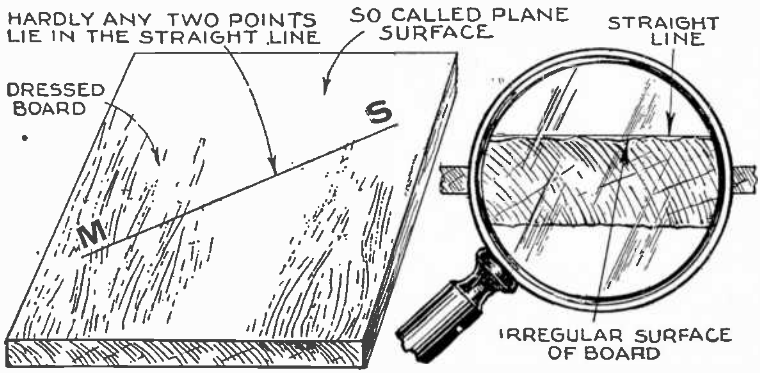


Fig. 53.—Proper conception of a *plane* or *plane surface*.



Figs. 54 and 55.—Popular and erroneous conception of a plane or plane surface.

In fig. 56,

Given: Lines A C and B D intersecting at O.

To Prove: $\angle AOB = \angle COD$.

Proof

$\angle AOB + \angle BOC =$ a st. $\angle$ or $180°$ **Post. 6**

Likewise $\angle BOC + \angle COD =$ a st. $\angle$ or $180°$.

$\therefore \angle AOB + \angle BOC = \angle BOC + \angle COD$. **Post. 9**

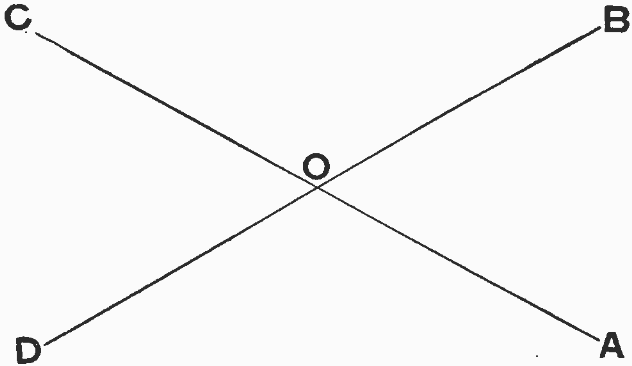


Fig. 56.—*Theorem 1.*

Subtract $\angle BOC$ from both sides of the equation **Ax. 3**

$\therefore \angle AOB = \angle COD$. **Q.E.D.**

Theorem 2.—*If two angles and the included side of one triangle be equal respectively to two angles, and the included side of another triangle, the two triangles are equal.*

In figs. 57 and 58,

Given: $\triangle$ ABC and $\triangle$ XYZ with angle A equal to angle X; angle B equal to angle Y, and with AB equal to XY.

To Prove: $\triangle ABC = \triangle XYZ$.

Proof.

Place the △ ABC upon the △ XYZ so that AB shall coincide with its equal XY. **Post. 5**

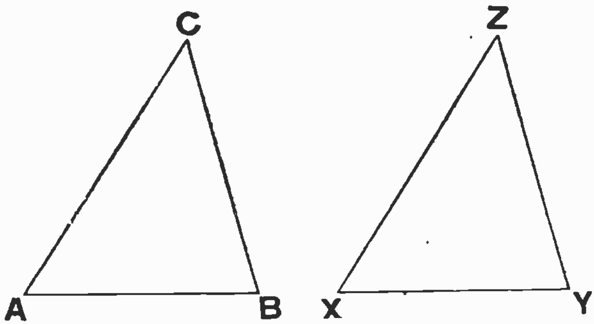
Then AC will fall along XZ and BC along YZ, because

$$\angle A = \angle X \text{ and } \angle B = \angle Y.$$

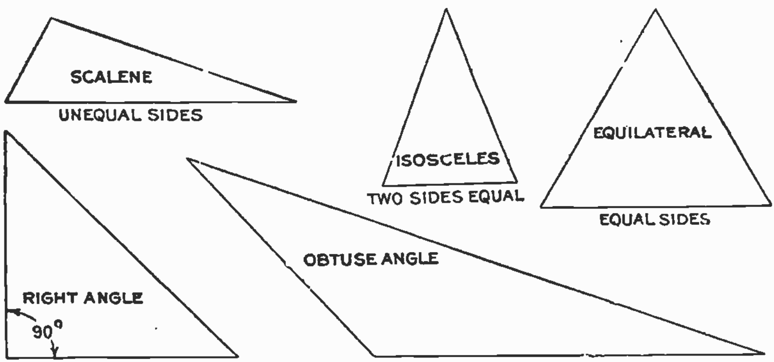
∴ C will fall on Z **Cor. 2**

∴ the two ⨺ are equal

(If two figures can be made to coincide in all their parts, they are equal.)



Figs. 57 and 58.—*Theorem 2.*



Figs. 59 to 63.—Various triangles.

Theorem 3.—*The angles opposite the equal sides of an isosceles triangle are equal.*

In fig. 64,

Given: △ ABC having AC = BC.

To Prove: ∠A = ∠B.

Proof.

Assume CD drawn so as to bisect ∠ACB.

Then in the △s ACD and BCD

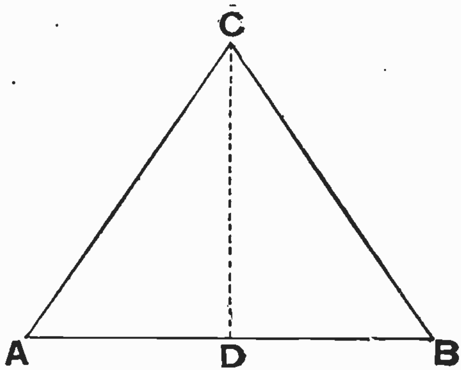


FIG. 64.—*Theorem 3.*

AC = BC,

CD = CD,

(*That is,* CD *is common to the two triangles.*)

and

∠ACD = ∠BCD

(*For* CD *bisects* ∠ ACB.)

∴ △ACD = △BCD **Theo. 16**

and accordingly

∠A = ∠B. **Cor. 8. Q.E.D.**

Theorem 4.—*The sum of the three angles of a triangle is equal to two right angles.*

In fig. 65,

Given: $\triangle ABC$.

To Prove: $\angle A + \angle B + \angle C = 2$ rt. $\angle$s or $180°$

Proof.

Assume line BY drawn ∥ with AC, and produce AB to X.

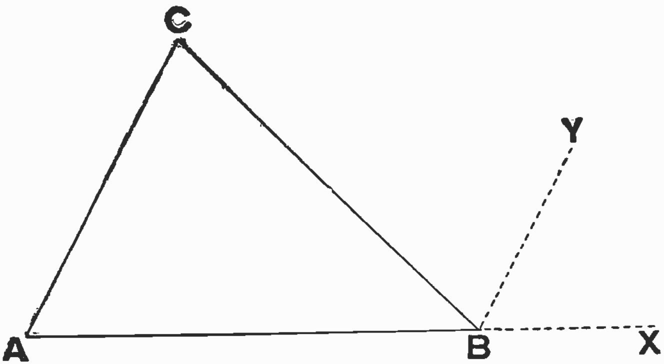


FIG. 65.—*Theorem 4.*

Then $\angle XBY + \angle YBC + \angle CBA = 2$ rt. $\angle$s		**Post. 8**
But	$\angle A = \angle XBY$	**Theo. 12**
and	$\angle C = \angle YBC$	**Theo. 13**

(*Because* BY *is parallel with* AC.)

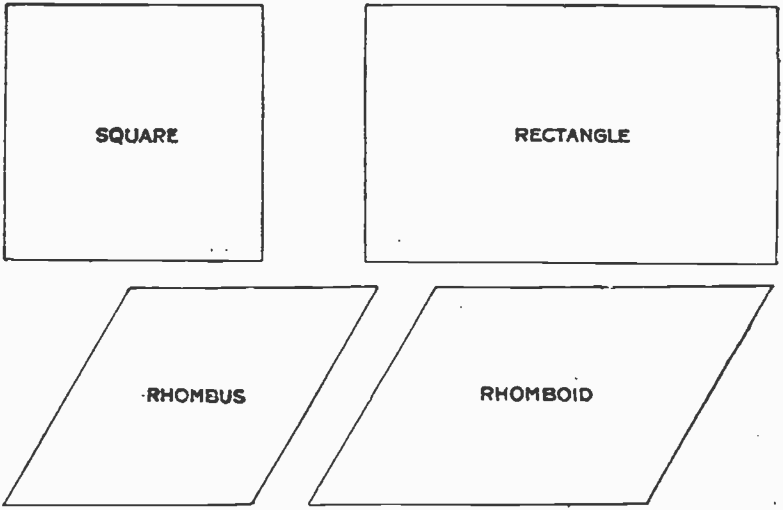
$\therefore \angle A + \angle B + \angle C = 2$ rt. $\angle$s **Ax. 11. Q.E.D.**

The $\angle$s A *and* C *are the opposite interior* $\angle$s *of the exterior* $\angle$ CBX.

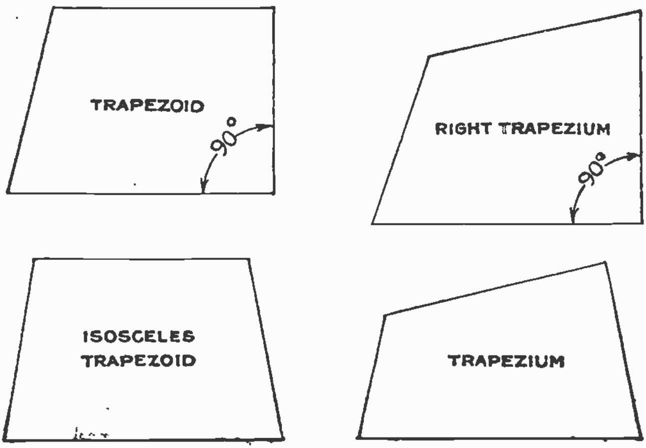
Quadrilaterals

Theorem 5.—*The opposite sides of a parallelogram are equal.*

In fig. 74,



FIGS. 66 to 69.—Various quadrilaterals. 1. *Opposite sides parallel.*



FIGS. 70 and 72.—Various quadrilaterals. 2. Two sides parallel.
FIGS. 71 and 73.—Various quadrilaterals- 3. Opposite sides not parallel.

Given: ▱ ABCD.

To Prove: BC = AD and AB = DC.

Proof.

Draw diagonal AC.

In the △ ABC and ADC

$$AC = AC,$$
$$\angle BAC = \angle DCA,$$
$$\angle ACB = \angle CAD. \qquad \textbf{Theo. 13}$$

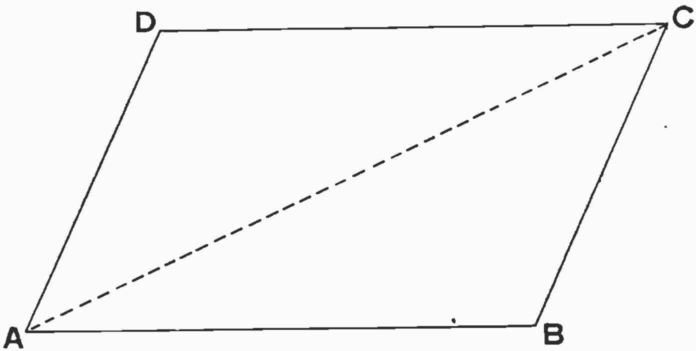


FIG. 74.—*Theorem 5.*

$$\therefore \triangle ABC = \triangle ADC \qquad \textbf{Theo. 2}$$
$$\therefore BC = AD \text{ and } AB = DC \qquad \textbf{Cor. 8. Q.E.D.}$$

Theorem 6.—*The diagonals of a parallelogram bisect each other.*

Given: ▱ ABCD with diagonals AC and BD which intersect at O.

In fig. 75,

To Prove: AO = OC and BO = OD.

Proof.

In the ⊿ ABO and CDO

$AB = CD$ **Theo. 5**

also

$\angle BAO = \angle DCO,$

$\angle OBA = \angle ODC.$ **Theo. 13**

hence $\triangle ABO = \triangle CDO$ **Theo. 2**

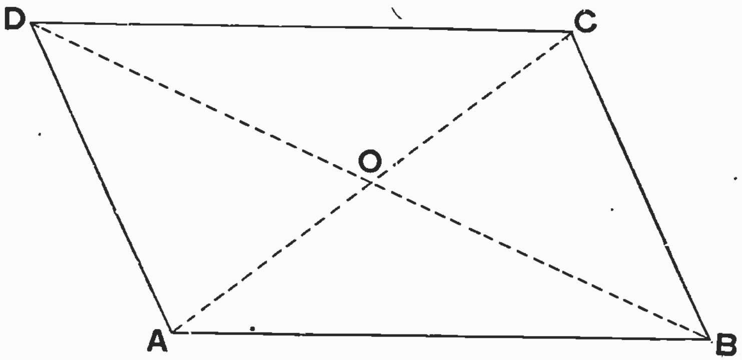


FIG. 75.—*Theorem 6.*

$\therefore AO = OC$

$\therefore BO = OD$ **Cor. 8. Q.E.D.**

Theorem 7.—*The sum of the interior angles of a polygon is equal to two right angles, taken as many times, less two, as the figure has sides.*

In fig. 76,

Given: Polygon ABCDEF having *n* sides.

To Prove: Sum of the interior ∠s $= (n-2) \times 2$ rt ∠s

Proof.

Draw diagonals from A. This divides the polygon into a number of triangles.

The sum of the ∠s of the △s is equal to the sum of the ∠s of the polygon.

The sum of the ∠s of each △ = 2 rt. ∠s and there are $(n-2)$ △s **Theo. 4**

∴ the sum of the ∠s of the $(n-2)$ △s, that is,

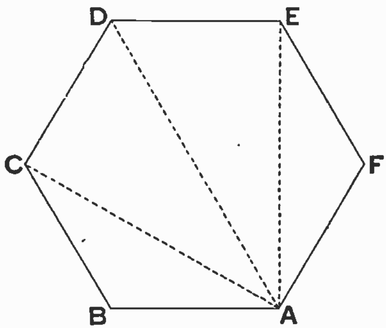


FIG. 76.—*Theorem 7.*

the sum of the ∠s of the polygon, is equal to $(n-2) \times$ 2 rt. ∠s **Ax. 4. Q.E.D.**

Circles.

Theorem 8.—*If any two points be taken in the circumference of a circle, the straight line which joins them will fall within the circle.*

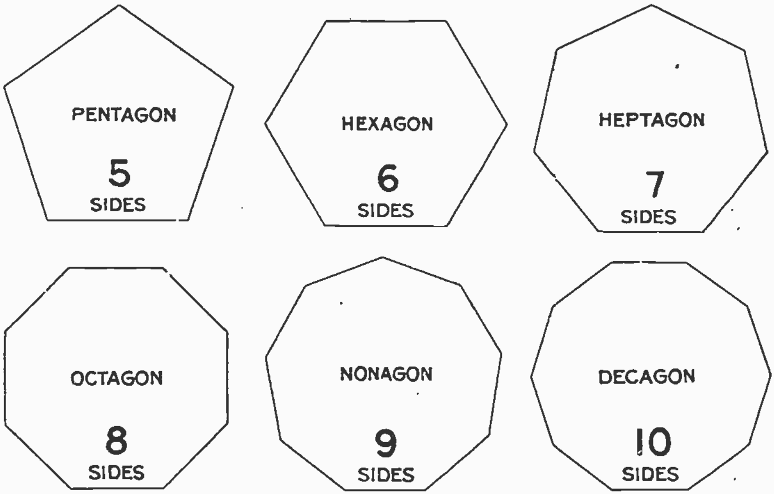
Given: ⊙ ABC and points A and B in the circumference.

To Prove: The straight line drawn from A to B will fall within the circle.

Proof.

In fig. 83,

Take E, any point in AB, join DE, and let it meet the circle in F.



FIGS. 77 to 82.—Various polygons having from five to ten sides. Of these the most important are the *hexagon* (six sides) and the *octagon* (eight sides).

	$DA = DB$	**Post. 11**
	$\therefore \angle DAB = \angle DBA$	**Theo. 3**
	$\angle DEB > \angle DAE$	**Theo. 17**
hence	$\angle DEB > \angle DBE$	
	$\therefore DB > DE$	**Theo. 18**
hence		
	$DF > DE$	

and E is within the circle. **Q.E.D.**

The same may be proved of any point of AB.

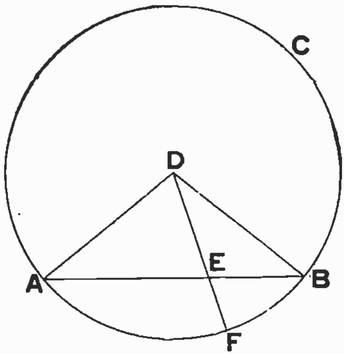


FIG. 83.—*Theorem 8.*

Theorem 9.—*Through three points not in a straight line one circle, and only one, can be drawn.*

In fig. 84,

Given: The three points A,B,C.

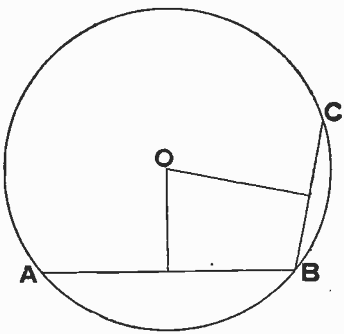


FIG. 84.—*Theorem 9.*

To Prove: Only one circle can be drawn through A, B and C.

Proof.

Draw AB and BC.

Bisect AB and BC and erect ⊥s, intersecting at some point O.

Any point in a ⊥ bisector is equidistant from the ends of the line bisected. **Theo. 15**

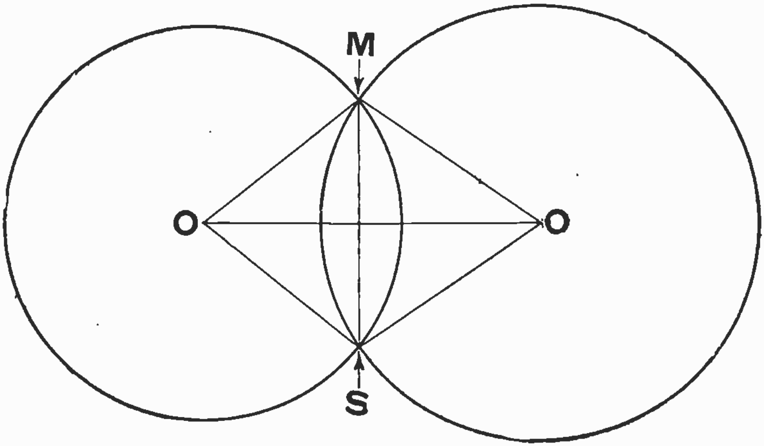


FIG. 85.—*Theorem 10.*

∴ common point O is equidistant from points A, B and C.

∴ A circle, described with center O and radius equal to the distance between O and any one of the points, will pass through the three points.

(The locus of a point in a plane at a given distance from a fixed point is a circle.)

Moreover, only one circle can be described because there is only one point O lying in both ⊥s. **Cor. 2. Q.E.D.**

Theorem 10.—*The line of centers of two intersecting circles is the perpendicular bisector of their common chord.*

In fig. 85,

Given: Two circles intersecting at M and S and the line OO′ connecting the centers OO′.

To Prove: OO′ bisects MS, the common chord.

Proof.

Join the points O and O′ to M and S.

OM = OS and O′M = O′S. **Post. 11**

∴ the two points O and O′ are each equidistant from M and S.

∴ OO′ is the ⊥ bisector of MS. **Cor. 9. Q.E.D.**

Additional Theorems

11.—*If a triangle and a parallelogram have equal bases and equal altitudes, the triangle will be half the parallelogram.*

12.—*If two parallel lines be cut by another line the exterior interior angles are equal.*

13.—*If two parallel lines be cut by another line the alternate interior angles are equal.*

14.—*A right angle is half a straight angle.*

15.—*The locus of a point equidistant from the extremities of a given line is the perpendicular bisector of that line.*

16.—*Two triangles are equal if two sides and the included angle of one be equal respectively to two sides and the included angle of the other.*

17.—*If one side of a triangle be produced, the exterior angle is greater than either of the opposite interior angles.*

18.—*The greater angle of every triangle has the greater side opposite to it.*

NOTE.—The presentation of the subject-matter of geometry as a connected and logical series of propositions, prefaced by "*Opoi*" or foundations, had been attempted by many; but it is to Euclid that we owe a complete exposition. There is probably little in the Elements that is original except the arrangement; but in this Euclid surpassed such predecessors as Hippocrates, Leon, pupil of Neocleides, and Theudius of Magnesia, devising an apt logical model, although when scrutinized in the light of modern mathematical conceptions the proofs are riddled with fallacies. According to the commentator Proclus, the Elements were written with a twofold object, first, to introduce the novice to geometry, and secondly, to lead him to the regular solids; conic sections found no place therein.

NOTE.—What Euclid did for the line and circle, Appolonius did for the conic sections, but there we have a discoverer as well as editor. Their works contain the greatest contributions to ancient geometry. Between Euclid and Apollonius there flourished the illustrious Archimedes, whose geometrical discoveries are mainly concerned with the mensuration of the circle and conic sections, and of the sphere, cone and cylinder, and whose greatest contribution to geometrical method is the elevation of the method of exhaustion to the dignity of an instrument of research.

NOTE.—The extraordinary mathematical talent which came into being in the 16th and 17th centuries reacted on geometry and gave rise to all those characters which distinguish modern from ancient geometry. The first innovation of moment was the formulation of the principle of geometrical continuity by Kepler. The notion of infinity which it involved permitted generalizations and systematizations hitherto unthought of, and the method of indefinite division applied to rectification, and quadrature and cubature problems avoided the cumbrous method of exhaustion and provided more accurate results.

NOTE.—The Romans, essentially practical and having no inclination to study science *qua* science, had a geometry which only sufficed for surveying, and even here there were abundant inaccuracies, the empirical rules employed being akin to those of the Egyptians and Heron. The Hindus, likewise, gave more attention to computation, and their geometry was either of Greek origin or in the form presented in trigonometry, more particularly connected with arithmetic.

TEST QUESTIONS

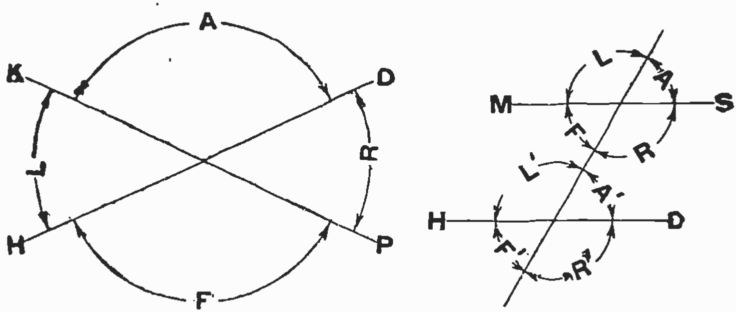
1. *What is a proposition?*
2. *Define: axiom, theorem, corollary, postulate, problem.*
3. *What do the abbreviations* **Q.E.D.** *and* **Q.E.F.** *stand for?*
4. *How is a plane determined?*
5. *If two straight lines intersect, how do the opposite angles compare?*
6. *Name the various triangles.*
7. *What is the difference between a rhombus and a rhomboid?*
8. *Prove that the diagonals of a parallelogram bisect each other.*
9. *What is the sum of the interior angles of a polygon equal to?*
10. *Show that if two points be taken in the circumference of a circle the straight line which joins them will fall within the circle.*
11. *Prove that through three points not in a straight line one circle and only one can be drawn.*
12. *Show that the line of centers of two intersecting circles is the perpendicular bisector of their common chord.*
13. *If two parallel lines be cut by a straight line how do the exterior interior angles compare?*
14. *What is the difference between a right angle and half a straight angle?*
15. *What relation does the locus of a point equidistant from the extremities of a given line bear to the given line?*

16. *Under what conditions are two triangles equal?*
17. *How do the angles of a triangle and the sides opposite compare?*
18. *What is the magnitude of the sum of three angles of a triangle?*

CHAPTER 3

Geometrical Propositions

In this Chapter, a number of propositions relating to lines, triangles, quadrilaterals, etc., are given without proofs for convenient reference. Numerous theorems with proofs are given in the chapter preceding.



FIGS. 86 and 87.—*Propositions 1, 2 and 3, relating to lines.*

Propositions Relating to Lines

1. *If two lines intersect, then the opposite angles formed by the intersecting lines are equal.*

In fig. 86

Angle L = angle R
Angle A = angle F

2. *When two lines intersect, any two adjacent angles are equal to two right angles.*

In fig. 86

$$L + A = 180°$$
$$R + F = 180°$$

3. *If a line intersect two parallel lines, the corresponding angles formed by the intersecting line with the parallel lines are equal.*

In fig. 87

$$L = L';\ A = A';\ R = R';\ F = F'$$

Propositions Relating to Triangles

1. *The sum of the three angles in a triangle always equals* 180 *degrees. Hence, if two angles be known, the third angle can always be found.*

In fig. 88

$$A + B + C = 180°$$
$$A = 180° - (B + C).$$
$$B = 180° - (A + C).$$
$$C = 180° - (A + B).$$

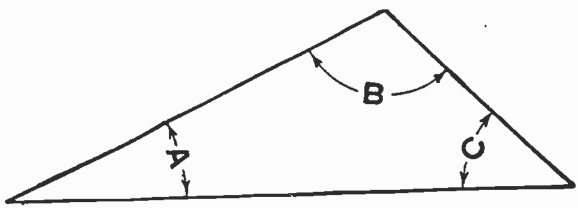
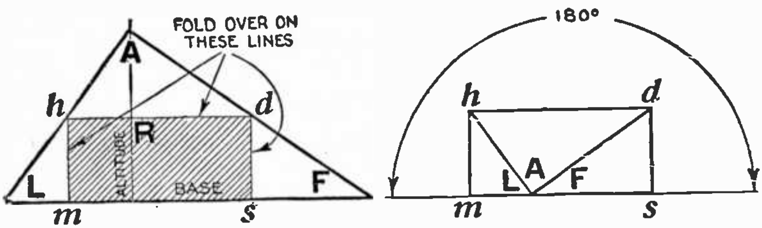


FIG. 88.—*Proposition 1, relating to triangles.*

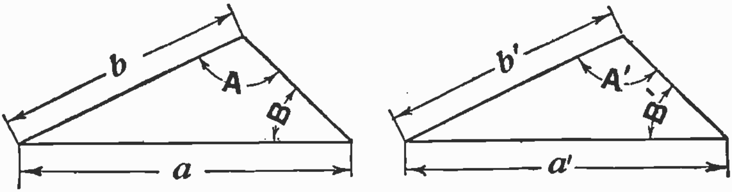


FIGS. 89 and 90.—Visual proof that *the sum of the three angles of a triangle is equal to two right angles, or* 180°. Draw altitude and bisect it at R Through R, draw *hd*, parallel with base and complete the rectangle *hd,ms* Cut out the triangle and fold over the ends on the lines *mh*, *hd* and *ds*, as in fig. 90. Evidently the sum of the three angles L, A, and F, thus folded equals two right angles or 180°

2. *If one side and two angles in one triangle be equal to one side and similarly located angles in another triangle, then the remaining two sides and angle are also equal.*

In figs. 91 and 92

If $a = a'$, A = A′ and B = B′, then the two other sides and the remaining angle are also equal.

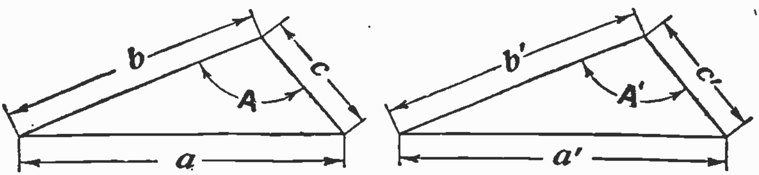


FIGS. 91 and 92.—*Proposition 2, relating to triangles.*

3. *If two sides and one angle in one triangle be equal to two sides and a similarly located angle in another triangle, then the remaining side and angles are also equal; provided, however, that the triangles be either both acute angled triangles, both obtuse angled triangles, or both right angled triangles.*

In figs. 93 and 94.

If $a = a'$, $b = b'$ and A = A′, then the remaining side and angles are also equal, the triangles in this case being both acute angled.



FIGS. 93 and 94.—*Propositions 3 and 4, relating to triangles.*

4. *If the three sides of one triangle be equal to the three sides of another triangle, then the angles in the two triangles are also equal.*

In figs. 93 and 94.

If $a = a'$, $b = b'$, and $c = c'$, then the angles between the respective sides are also equal.

5. *If the three sides of one triangle be proportional to corresponding sides in another triangle, then the triangles are called similar, and the angles in the one are equal to the angles in the other.*

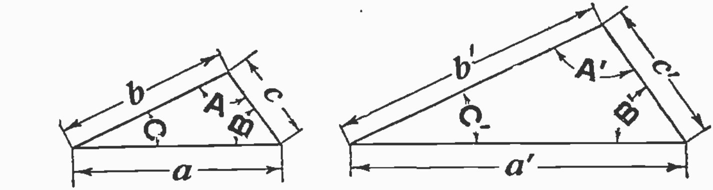
In figs. 95 and 96

If $a:b:c = a':b':c'$ then $A = A'$, $B = B'$ and $C = C'$.

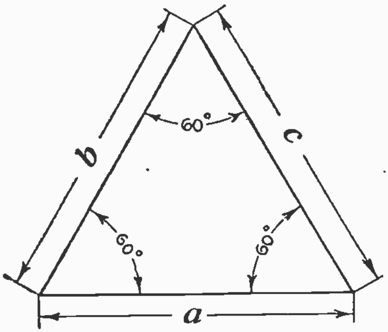
6. *If the angles in one triangle be equal to the angles of another triangle, then the triangles are similar and their corresponding sides are proportional.*

In figs. 95 and 96

If $A = A'$, $B = B'$, and $C = C'$ then $a:b:c = a':b':c'$.



FIGS. 95 and 96.—*Propositions 5 and 6, relating to triangles.*



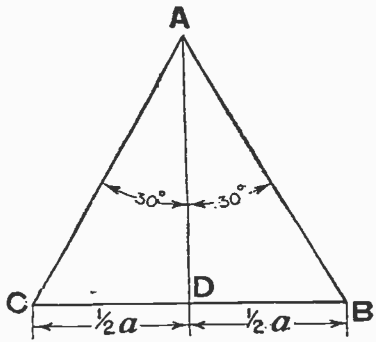


FIG. 97.—*Proposition 7, relating to triangles.*

FIG. 98.—*Proposition 8, relating to triangles.*

7. *If the three sides of a triangle be equal, that is, if the triangle be equilateral, then the three angles are also equal as in fig. 97.*

Each of the three equal angles in the equilateral triangle, fig. 97, is 60 degrees. If the three angles in a triangle be equal, then the three sides are also equal.

8. *A line which in an equilateral triangle bisects or divides any of the angles into two equal parts, will bisect, also the side opposite the angle and be at right angles to it as shown in fig.* 98

If line AD, divides angle CAB, into two equal parts, it also divides line CB, into two equal parts and is at right angles to it.

9. *If two sides in a triangle be equal—that is, if the triangle be an isosceles triangle, then the angles opposite these sides are also equal.*

In fig. 99, if side *b*, equal side *c*, then angle B, equals angle C.

10. *If two angles in a triangle be equal, then the sides opposite these angles are also equal.*

In fig. 99, if angles B and C, be equal then side *b* equals side *c*.

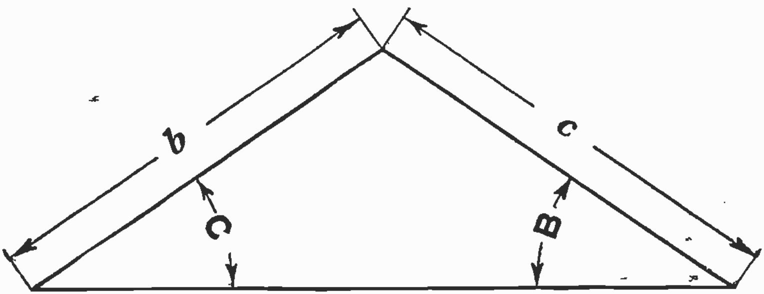


Fig. 99.—*Propositions 9 and 10, relating to triangles.*

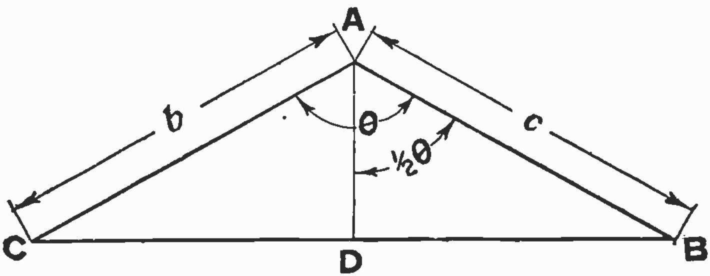


Fig. 100.—*Proposition 11, relating to triangles*

11. *In an isosceles triangle, if a straight line be drawn from the point where the two equal sides meet, so that it bisects the third side or base of the triangle, then it also bisects the angle between the equal sides and is perpendicular to the base.*

In fig. 100

If $b = c$ and AD bisects CB, then angle CAD = DAB.

12. *In every triangle, that angle is greater which is opposite a longer side. In every triangle, that side is greater which is opposite a greater angle.*

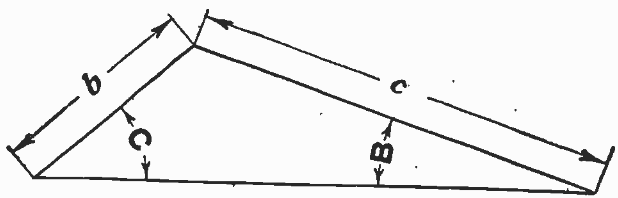


FIG 101.—*Proposition 12, relating to triangles.*

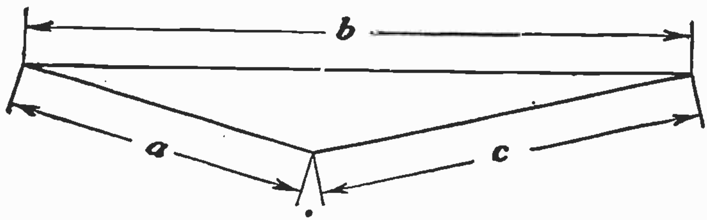


FIG. 102.—*Proposition 13, relating to triangles.*

In fig. 101

If side c, be greater than side b, then angle C, is greater than angle B; also, if angle C, be greater than angle B, then side c, is greater than side b.

13. *In every triangle, the sum of the lengths of two sides is always greater than the length of the third.*

In fig. 102

Side a + side c is greater than side b.

14. *If one side of a triangle be produced, then the exterior angle is equal to the sum of the two opposite interior angles.*

In fig. 103

Angle D = angle A + angle C. In the case of an equilateral triangle, angle D = 60° + 60° = 120°.

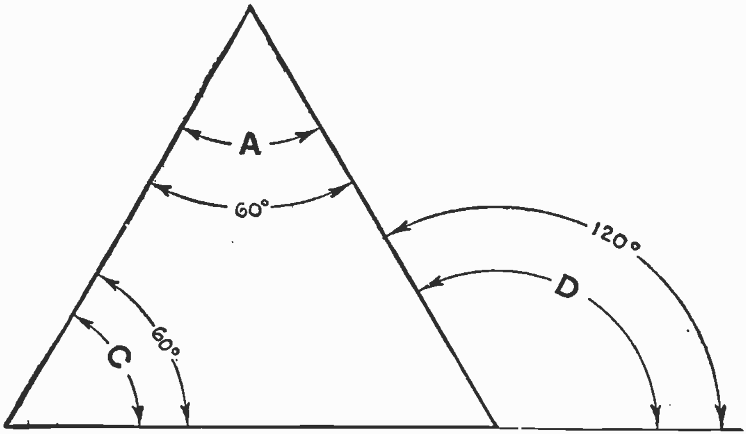


FIG. 103.—*Proposition 14, relating to triangles.*

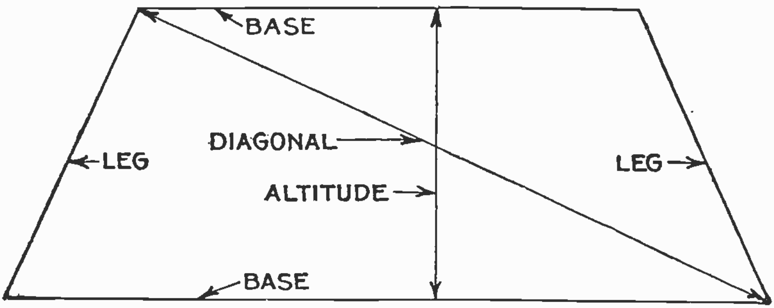


FIG. 104.—Quadrilateral illustrating legs, bases, etc. The parallel sides are the *bases;* the distance between the bases, the *altitude;* a line joining two opposite vertices, a *diagonal.*

Propositions Relating to Quadrilaterals

1. *In any figure having four sides, the sum of the interior angles equals* 360 degrees.

In fig. 105

$$L + A + R + F = 360°$$

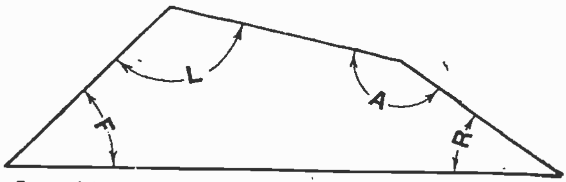
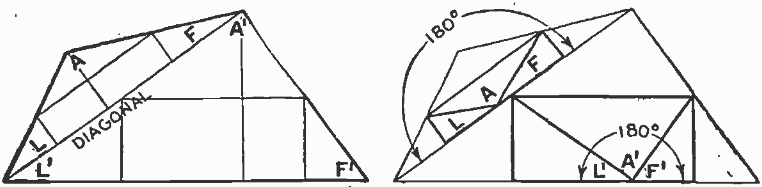


Fig. 105.—*Proposition 1, relating to quadrilaterals.*



Figs. 106 and 107—Visual proof that the sum of the angles of a quadrilateral is equal to four right angles or 360° Draw a diagonal which will divide the figure into two triangles giving the angles L,A,F and L',A',F'. Fold over the angles of each triangle as directed in figs. 89 and 90, thus obtaining angles L,A,F, = 180° and angles L',A',F', = 180°, a total of four right angles or 360°, as in fig. 107.

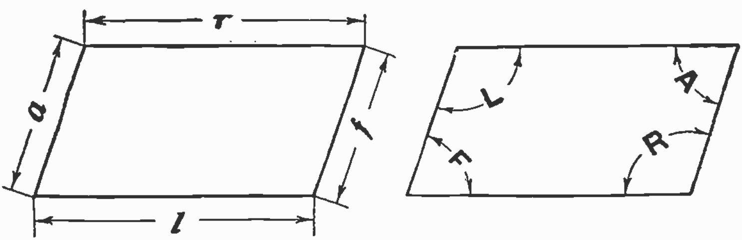


Fig. 108.—*Proposition 2, relating to quadrilaterals*

Fig. 109.—*Proposition 3, relating to quadrilaterals.*

2. *The sides which are opposite each other in a parallelogram are equal.*
In fig. 108

$$a = f \text{ and } r = l.$$

3. *The angles which are opposite each other in a parallelogram are equal.*
In fig. 109

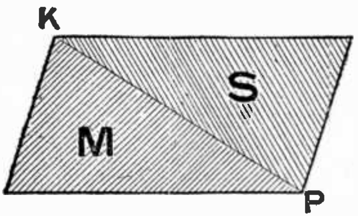
$$A = F \text{ and } L = R.$$

4. *The diagonal divides a parallelogram into two equal parts.*
In fig. 110

triangle M = triangle S

5. *If two diagonals be drawn in a parallelogram, they bisect each other.*
In fig. 111

$$HO = OD \text{ and } KO = OP$$



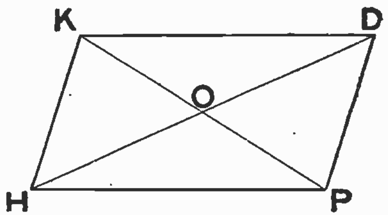


FIG. 110.—*Proposition 4, relating to quadrilaterals.*

FIG. 111.—*Proposition 5, relating to quadrilaterals.*

Propositions Relating to Polygons

1. *By drawing all the diagonals possible from one vertex of a polygon, the number of triangles thus obtained is always two less than the number of sides.*

In fig. 112, there are

$6 - 2 = 4$ triangles, L,A,R and F.

2. *The sum of the angles of a polygon is equal to* 180° *multiplied by the number of sides minus two.*

In fig. 112

Sum of angles $= 180° \times (6 - 2) = 720°$ or 8 right angles

Propositions Relating to Regular Polygons

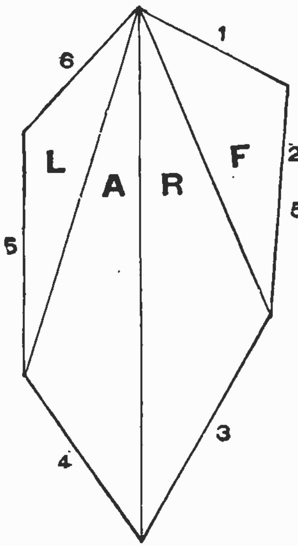
1. *The angles at the center of a regular polygon are equal.*

In fig. 113

$$L = A = R = F = G.$$

2. *Straight lines from the center to all the vertices of a regular polygon divide the polygon into as many equal isosceles triangles as there are sides.*

In fig. 113, the five lines, 01, 02, 03, 04 and 05 divide the polygon into 5 triangles.



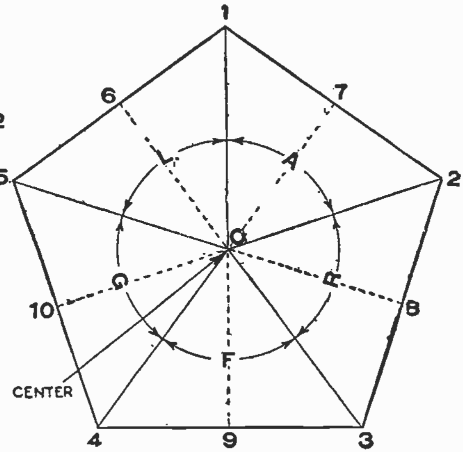


FIG. 112.—*Propositions 1 and 2, relating to polygons.*

FIG. 113.—*Propositions 1, 2 and 3, relating to regular polygons*

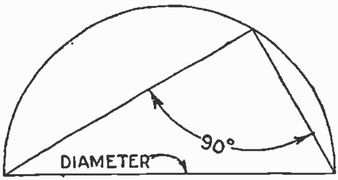
3. *Straight lines from the center to all the vertices of a regular polygon and perpendicular to the sides divide the polygon into twice as many right triangles as there are sides of the polygon.*

In fig. 113, the lines 01, 02, 03, 04 and 05 and perpendiculars 06, 07, 08, 09 and 010 divide the polygon into $5 \times 2 = 10$ right triangles.

Propositions Relating to the Circle

1. *An angle inscribed in a semi-circle is a right angle.* As in fig. 114.

2. *The perpendicular to a radius at its extremity is a tangent to the circle.* As in fig. 115.



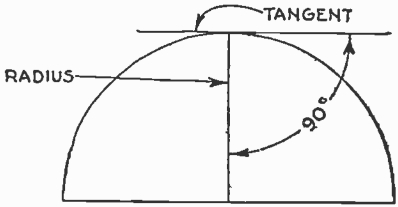
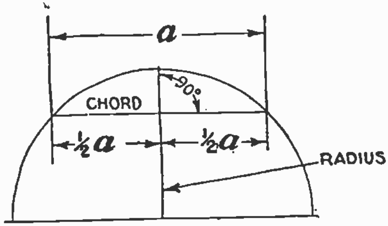


Fig. 114.—*Proposition 1, relating to circles.*
Fig. 115.—*Proposition 2, relating to circles.*

3. *A radius perpendicular to a chord bisects the chord.* As in fig. 116.

4. *Two tangents from an exterior point make equal angles with the straight line which joins the exterior point to the center of the circle.* As in fig. 117.



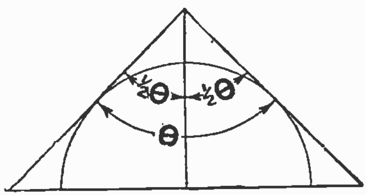
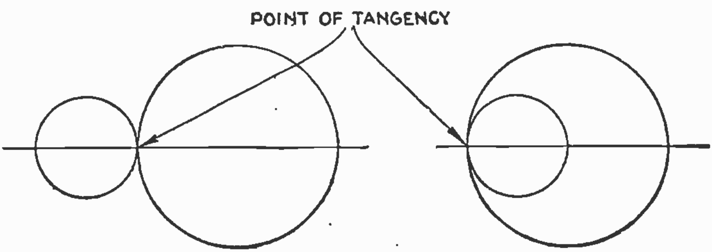


Fig. 116.—*Proposition 3, relating to circles.*
Fig. 117.—*Proposition 4, relating to circles.*



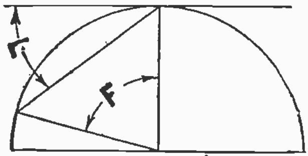
Figs. 118 and 119.—*Proposition 5, relating to circles.*

5. *If two circles be tangent to each other then the straight line which passes through the centers of the two circles must also pass through the point of tangency.* As in figs. 118 and 119.

6. *The angle between a tangent and a chord drawn from the point of tangency equals one half the angle at the center subtended by the chord.*

In fig. 120

Angle L = ½ angle F.



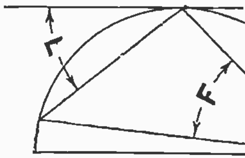


FIG. 120.—*Proposition 6, relating to circles.*
FIG. 121.—*Proposition 7, relating to circles.*

7. *The angle between a tangent and a chord drawn from the point of tangency equals the angle at the periphery subtended by the chord.*

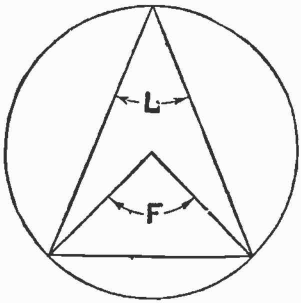
In fig. 121

Angle L = angle F.

8. *If an angle at the circumference of a circle, between two chords, be subtended by the same arc as the angle at the center between two radii, then the angle at the circumference is equal to one half of the angle at the center.*

In fig. 122

L = ½ F.



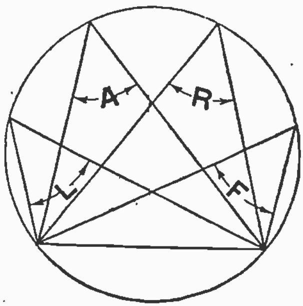


FIG. 122.—*Proposition 8, relating to circles.*
FIG. 123.—*Proposition 9, relating to circles.*

9. *All angles having their vertices at the periphery of a circle and subtended by the same chord are equal.*

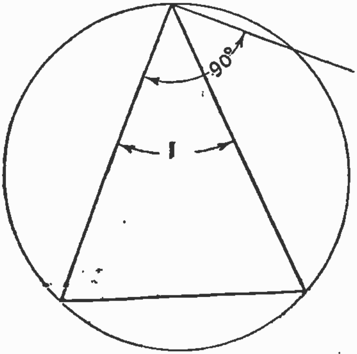
In fig. 123

$$L = A = R = F.$$

10. *An angle subtended by a chord in a circular arc larger than one half the circle is an acute angle—an angle less than 90 degrees.*

In fig. 124

Angle L is *less* than 90°.



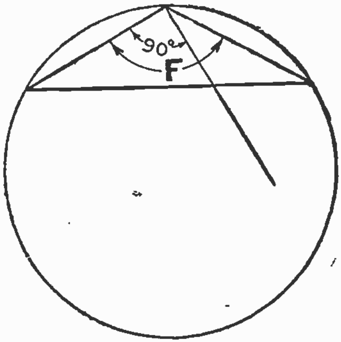
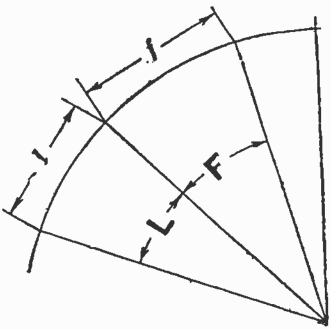


FIG. 124.—*Proposition 10, relating to circles.*
FIG. 125.—*Proposition 11, relating to circles..*



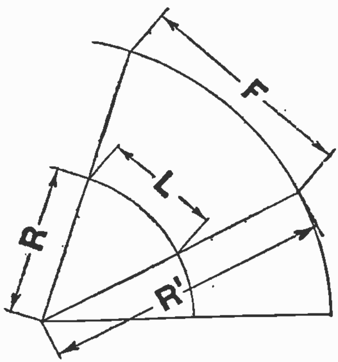


FIG. 126.—*Proposition 12, relating to circles.*
FIG. 127.—*Proposition 13, relating to circles.*

11. *An angle subtended by a chord in a circular segment less than one half the circle is an obtuse angle, that is, an angle greater than 90 degrees.*

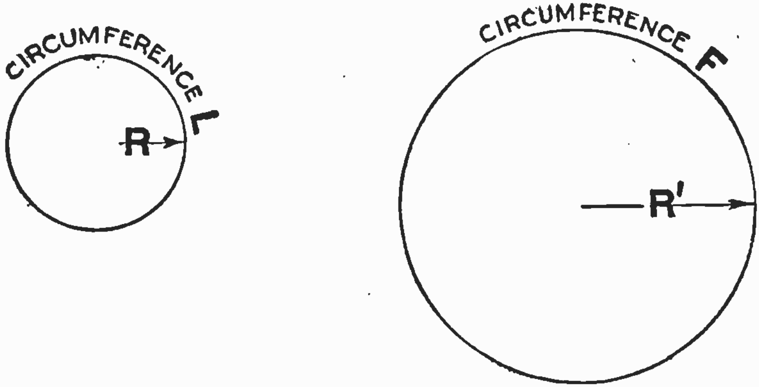
In fig. 125

Angle F is *greater* than 90°.

12. *The length of circular arcs of the same circle are proportional to the corresponding angles at the center.*

In fig. 126

$$L : F = l : f.$$



Figs. 128 and 129.—*Proposition 14, relating to circles.*

13. *The length of circular arcs having the same center angle are proportional to the length of the radii.*

In fig. 127

$$L : F = R : R'$$

14. *The circumferences of two circles are proportional to their radii.*

In figs. 128 and 129

$$L : F = R : R'.$$

Propositions Relating to Ellipses

1. *The foci are at equal distances from the extremities of the major axis, and are also equidistant from the center of the ellipse.*

In fig. 130

$$ML = FS \text{ and } OL = OF.$$

2. *The positions of the foci depend upon the ratio of the axes.*

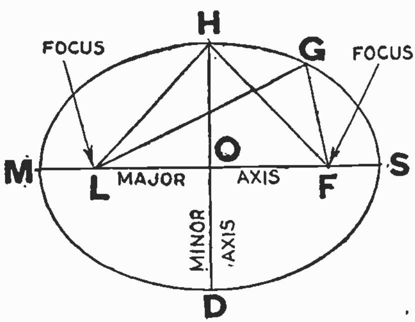
In fig. 130

$$ML : LO = HO : MO.$$

3. *The major and minor axes of an ellipse divide the figure into four similar parts.*

In fig. 130

$$MOH = HOS = MOD = DOS$$



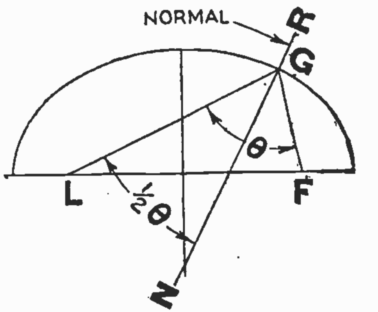


FIG. 130.—**Propositions 1 to 5,** *relating to ellipses.*

FIG. 131.—**Proposition 6,** *relating to ellipses.*

4. *The sum of the lengths of lines from the foci to any point on the curve is a constant.*

In fig. 130

$$LG + FG = \text{constant}.$$

5. *The sum of the lengths of the lines joining the foci to any point on the curve equal length of major axis.*

In fig. 130

$$LG + GF = MS.$$
$$LH + HF = MS.$$

6. *A normal to the curve at any point bisects the angle formed by lines from the foci to that point.*

In fig. 131,

Normal NR bisects angle θ formed by lines LG and FG, from foci L and F, to point G.

TEST QUESTIONS

1. *Give visual proof that the sum of the three angles of a triangle is equal to two right angles or 180°.*
2. *State a few propositions relating to triangles.*
3. *Draw a quadrilateral showing various parts.*
4. *Give visual proof that the sum of the angles of a quadrilateral is equal to four right angles or 360°.*
5. *State a few propositions relating to regular polygons.*
6. *How does a radius, perpendicular to a chord, cut the chord?*
7. *How do the angles compare which have their vertices at the periphery of a circle and are subtended by the same chord?*
8. *State some propositions relating to circles.*
9. *How are the foci of an ellipse located?*
10. *How do the major and minor axes of an ellipse divide the figure?*

CHAPTER 4

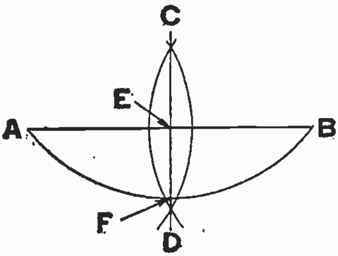
Geometrical Problems

Geometrical Problems.—The following problems illustrating how various geometrical figures are constructed, are to be solved by the use of pencil, dividers, compasses, and scale.

Many of these problems are such as are encountered in sheet metal work in laying out patterns. Proficiency in the solution of these problems will be of value to draughtsmen.

***Problem 1.**—To bisect or divide into two equal parts a straight line or arc of a circle.*

In fig. 132, from the ends AB, as centers, describe arcs cutting each other at C and D, and draw CD, which cuts the line at E, or the arc at F.



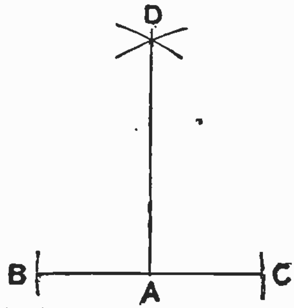


FIG. 132.—***Problems 1 and 2.** To bisect a straight line or arc of a circle and to erect a perpendicular to the line which is a radial line to the arc.*

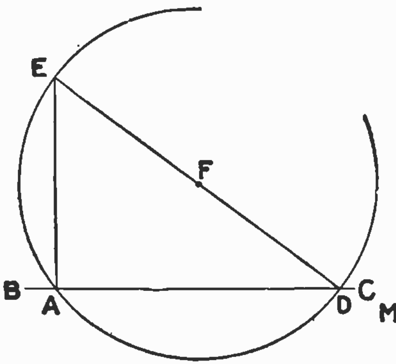
FIG. 133.—***Problem 3** To erect a perpendicular to a straight line, from a given point in that line.*

Problem 2.—To draw a perpendicular to a straight line, or a radial line to an arc.

In fig. 132 the line CD is perpendicular to AB, moreover, the line CD, is radial to the arc AFB.

Problem 3.—To erect a perpendicular to a straight line, from a given point in that line.

In fig. 133 with any radius from any given point A, in the line BC, describe arcs cutting the line at B and C. Next, with a longer radius describe arcs with B and C, as centers, intersecting at D, and draw the perpendicular DA.



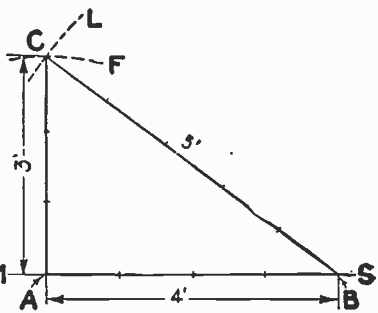


FIG. 134.—*Problem 3. Second method.*

FIG. 135.—*Problem 3. Third method (boat builder's laying down method).*

Second method. In fig. 134. from any center F, above BC describe a circle passing through the given point A, and cutting the given line at D; draw DF, and produce it to cut the circle at E; now draw the perpendicular AE.

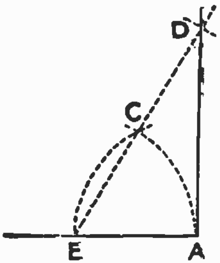
Third method (boat builders' laying down method)—In fig. 135 let MS be the given line and A, the given point. From A, measure off a distance AB, say 4 ft. With centers A and B, and radii of 3 and 5 ft. respectively, describe arcs F and L, intersecting at C. Draw a line through A and C. which will be the perpendicular required.

Fourth method.— In fig. 136, from A, describe an arc EC, and from E, with the same radius, the arc AC. cutting the other at C; through C, draw a line ECD, and set off CD, equal to CE, and through D, draw the perpendicular AD.

Problem 4.—To erect a perpendicular to a straight line from any point without the line.

In fig. 137, from the point A, with a sufficient radius, cut the given line at F and G; and from these points describe arcs cutting at E. Place triangle on points A and E, and from A, draw perpendicular to line GF

Second method.— In fig. 138, from any two points, B,C, at some distance apart, on the given line, and with the radii BA, CA, respectively, describe arcs cutting at A and D. Place triangle on points A and D, and draw the perpendicular AD.



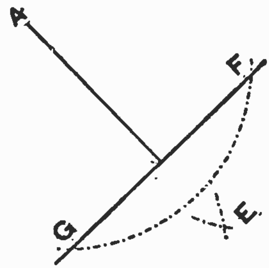
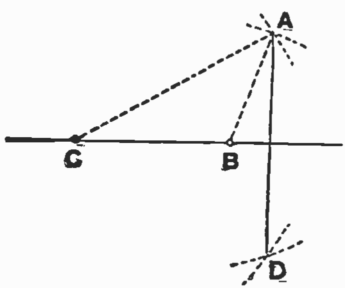


FIG. 136.—*Problem 3. Fourth method.*

FIG. 137.—*Problem 4. To erect a perpendicular to a straight line, from any point without the line.* If there be no room below the line, the intersection may be taken above the line, that is to say, between the line and the given point.



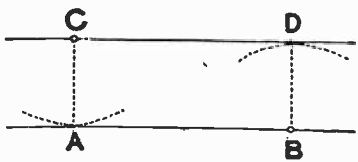


FIG. 138.—*Problem 4. Second method.*

FIG. 139.—*Problem 5. Through a given point to draw a line parallel with a given line.*

***Problem 5.**—Through a given point to draw a line parallel with a given line.*

In fig. 139, with C, as center describe an arc tangent to the given line AB; the radius will then equal distance from given point to the given line. Take a point B, on line remote from C, and with same radius, describe an arc. Draw a line through C, tangent to this arc and it will be parallel to the given line AB.

Second method.—In fig. 140, from A, the given point, describe the arc FD, cutting the given line at F; from F, with the same radius, describe the arc EA, and set off FD, equal to EA. Draw the parallel through the points AD.

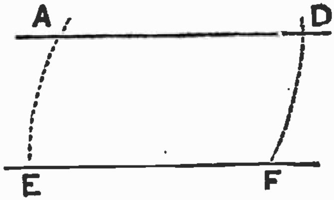


FIG. 140.—*Problem 5. Second method.*

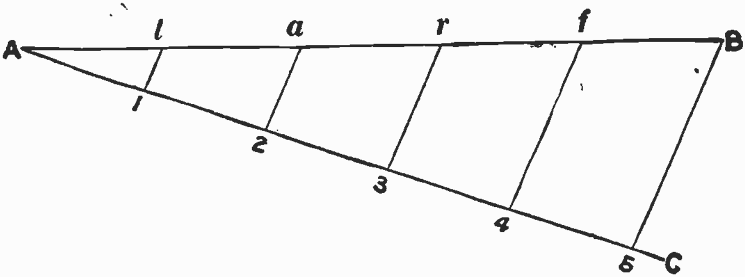


FIG. 141.—*Problem 6. To divide a line into a number of equal parts.*

***Problem 6.**—To divide a line into a number of equal parts.*

In fig. 141 assuming line AB, is to be divided into say 5 parts, draw a diagonal line AC, and space off 5 unit lengths. Join B5, and through the points 1,2,3,4, draw lines 1*l*, 2*a*, etc., parallel to B5, then will AB, be divided into five equal parts, A*l*, *la*, *ar*, *rf*, and *f*B.

***Problem 7.**—Upon a straight line to draw an angle equal to a given angle.*

In figs. 143 and 144 let A, be the given angle and FG, the line.

With any radius from the points A and F, describe arcs DE, and IH, cutting the sides of the angle A, and the line FG.

Set off the arc IH, equal to DE, and draw FH. The angle F, is equal to angle A, as required.

Problem 8.—To bisect an angle.

In fig. 145, let ACB, be the angle; with center C, describe an arc

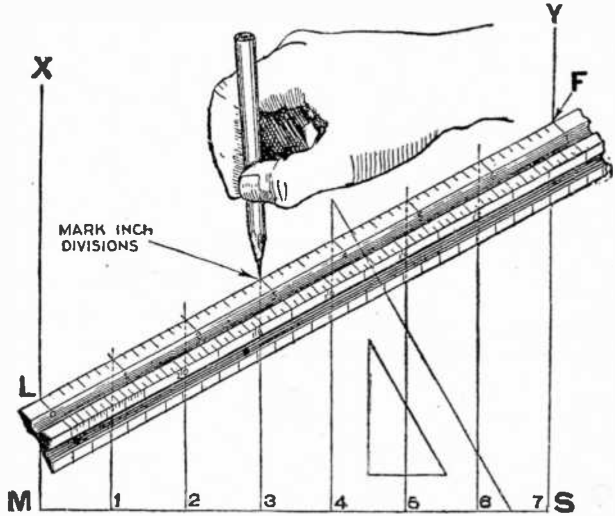
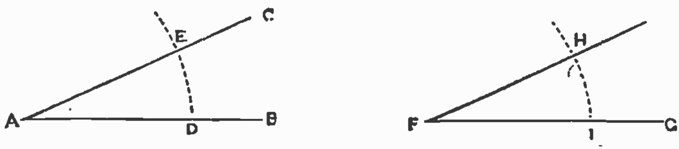


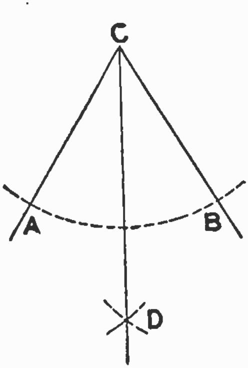
FIG. 142.—To divide a given line into any number of equal parts without dividers. Let M S, be the line and say it is to be divided into seven equal parts. Erect perpendiculars M X and S Y. Lay the O, mark of the scale on the line M X, and place scale at such angle that coincides with line S Y. Draw a light line L F, and mark the inch divisions as shown. With a triangle and T square draw lines from the points on L F, to M S, cutting M S, at 1, 2, 3, etc., which divide M S, into seven equal parts.



FIGS. 143 and 144.—*Problem 7. Upon a straight line to draw an angle equal to a given angle.*

cutting the sides at A and B. On A and B, as centers describe arcs cutting at D. A line through C and D will divide the angle into two equal parts.

Problem 9.—To find the center of a circle



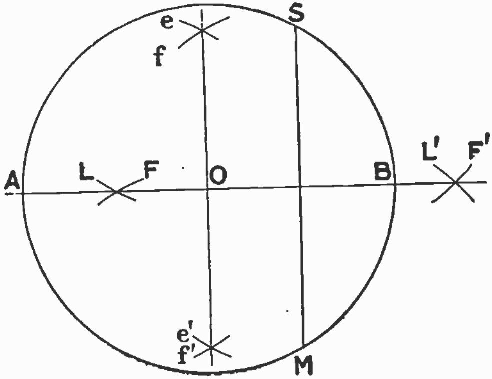
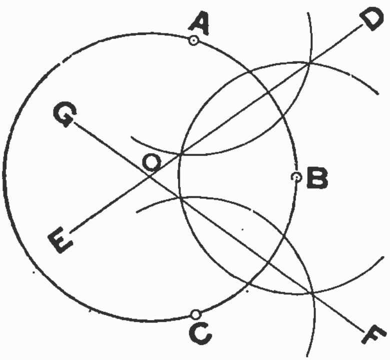


Fig. 145.—***Problem 8.*** *To bisect an angle.*
Fig. 146.—***Problem 9.*** *To find the center of a circle.*



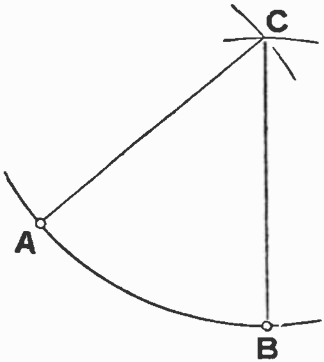


Fig. 147.—***Problem 9.*** *Second method.*
Fig. 147.—***Problem 10.*** *To describe a circle passing through three given points.*
Fig. 148.—***Problem 11.*** *Through two given points to describe an arc of a circle with a given radius.*

In fig. 146, draw any chord as MS. With M and S, as centers and any radius, describe arcs L,F, and L',F', and a line through their intersection, giving a diameter AB. Applying same construction with centers A and B, describe arcs *ef*, and *e' f'*. A line drawn through the intersections of these arcs will cut AB, at O, the center of the circle.

Problem 9.—Second method. To find the center of a circle.

In fig. 147, select three points. A,B,C, in the circumference, well apart: with the same radius describe arcs from these three points cutting each other, and draw two lines DE, FG through their intersections. The point O, where they cut is the center of the circle or arc.

Problem 10.—To describe a circle passing through three given points.

In fig. 147, let A,B,C, be the given points and proceed as in last problem to find the center O, from which the circle may be described. This problem

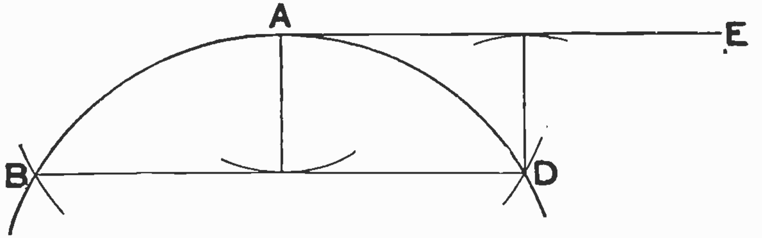


FIG. 149.—***Problem 12. To draw a tangent to a circle from a given point in the circumference.***

is useful in such work as laying out objects of large diameter as an arch, when the span and rise are given.

Problem 11.—Through two given points to describe an arc of a circle with a given radius.

In fig. 148, take the given points A and B, as centers, and, with the given radius, describe arcs cutting at C; from C, with the same radius, describe the required arc AB.

Problem 12.—To draw a tangent to a circle from a given point in the circumference.

In fig. 149 from point A set off equal segments AB, AD; join BD, and draw AE, parallel with it, for the tangent.

Problem 13.—On a given straight line A5, to construct any regular polygon say a pentagon.

In fig. 150 produce the given side A5, say to the left. With center A, and radius A5, describe a semi-circle. Divide the semi-circle into as many equal parts as the polygon is to have sides; in this case 5 equal parts, by trial with

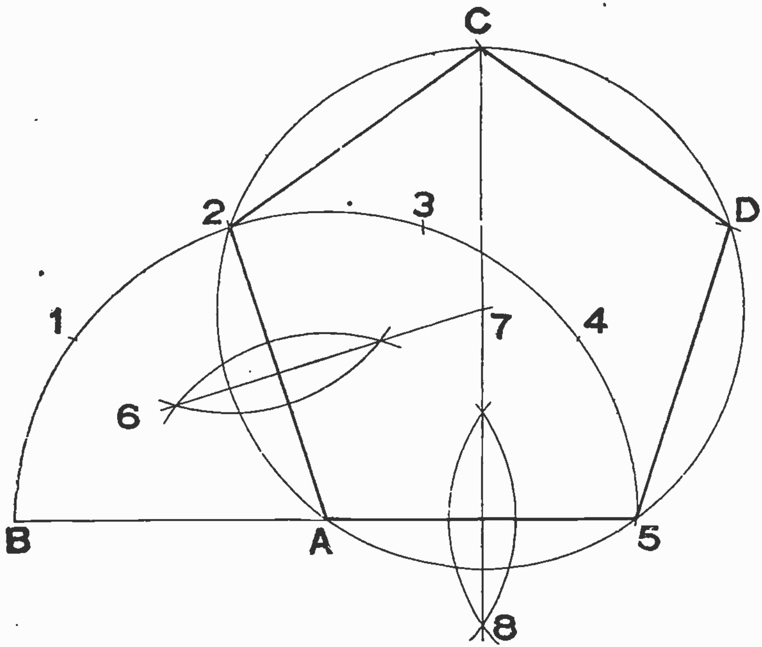


FIG. 150.—*Problem 13. On a given straight line, to construct a regular polygon.*

compasses. From A, draw A2, which gives another side of the polygon; and no matter how many sides the polygon is to have, always draw from A, to the second division on the semi-circle. Bisect the sides 2 A, A 5, by lines 6 7, and 8 7, intersecting at point 7, which is the center of the polygon. With center 7, and radius 7 A, describe the circle. Mark off, on the circumference, the divisions 2C, CD, equal to A5. Joint 2C, CD, D5. Then A2CD5, is the required regular polygon.

Problem 14.—To ascertain approximately, the length of the circumference of a given circle.

In fig. 151, draw a diameter AB. Find center C. Draw AD, perpendicular to AB, and 3 times the length of the radius. Draw BE, perpendicular to AB. With 30° triangle, draw angle BCH = 30°. Mark joint J, on BE. Join J D. Then line J D, is (approximately) equal to half the circumference; and twice J D = the whole circumference. This method is sufficiently accurate for all practical work, because the result is wrong only by about a $\frac{1}{100{,}000}$

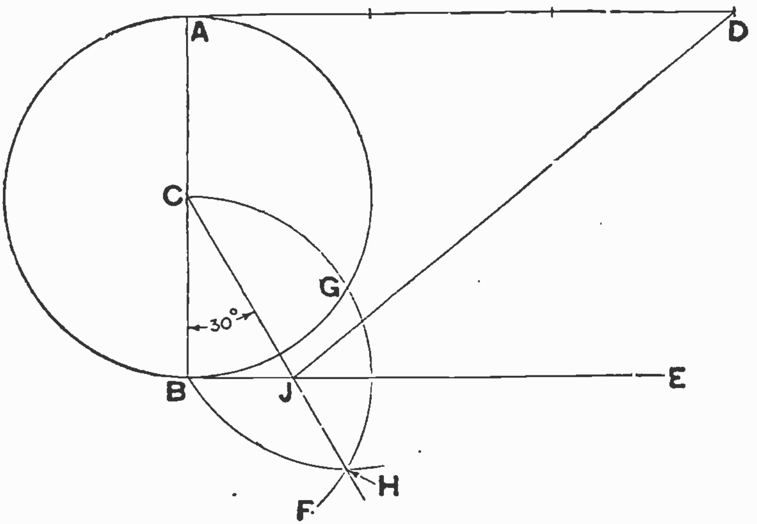


FIG. 151.—*Problem 14. To ascertain approximately, the length of the circumference of a given circle.*

part. This problem helps to ascertain approximately, the length of certain portions of the circumference. Thus ⅓ of JD = ⅙ of the circumference. Archimedes demonstrated that the diameter is to the circumference, within a minute fraction, as 7 is to 22, or 1 to $3^1/_7$. Thus, for all practical purposes, it may be assumed that if the diameter = 1 in., the circumference = $3^1/_7$ ins. To describe a circle having a circumference equal to the circumferences of any number of given equal or unequal circles: *Draw a line equal to the sum of the diameters of the given circles. This line is the diameter of the required circle.*

Problem 15.—To find the center of a given circle, or arc of a circle.

In fig. 152, draw any two chords, 12 and 23. Bisect these chords by perpendiculars 45, and 67, intersecting at A. Point A, is the center of the circle or arc. The chords are not obliged to meet at 2. They may be drawn anywhere in the circle or arc, but it is better, when possible, to let them be at about right angles to each other. The chords may intersect. They should not be made too short.

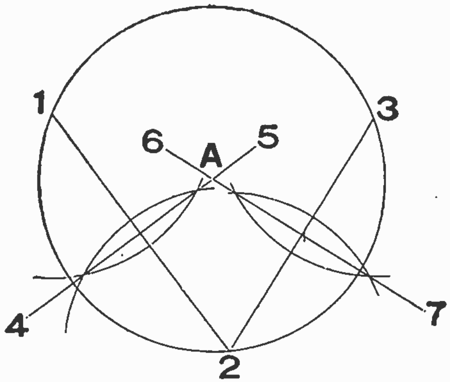


FIG. 152.—*Problem 15. To find the center of a given circle, or arc of a circle.*

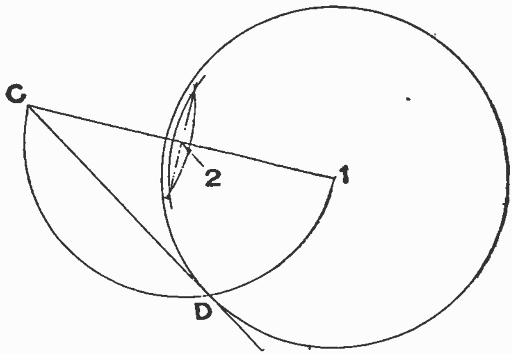


FIG. 153.—*Problem 16. To draw a tangent to a given circle from any given point.*

Problem 16.—To draw a tangent to a given circle from any given point C, outside the circle.

In fig. 153, 1 is the center of the given circle. Join points C1. Bisect C1, at 2; and with center 2, and radius 2 C, or 21, describe a semi-circle, cutting the circle at D. Point D, is the point of contact. Through D, draw CD, which is the required tangent.

CD, is tangent because a line through the point of contact D, and center 1, of the circle makes a right angle with CD. Why?

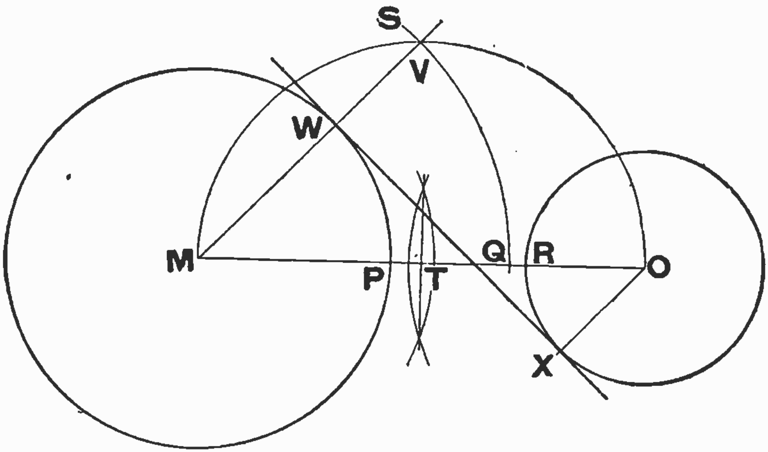


FIG. 154.—*Problem 17. To draw an interior tangent to two unequal circles.*

Problem 17.—To draw an interior tangent to two unequal circles M and O.

In fig. 154, join centers M and O. Bisect MO, at T, and describe a semi-circle on MO. From P, on the larger circle, mark off PQ = OR, the radius of the smaller circle. With center M, and radius MQ, describe arc QS, cutting the semi-circle at V. Join MV, and mark point W. Draw OX, parallel with MV. Through the points of contact W and X, draw the interior tangent WX.

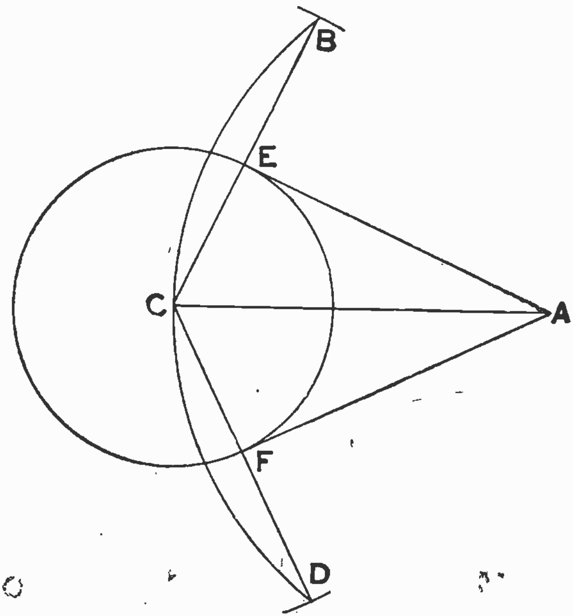


FIG. 155.—*Problem 18. To draw tangents to a circle from points without.*

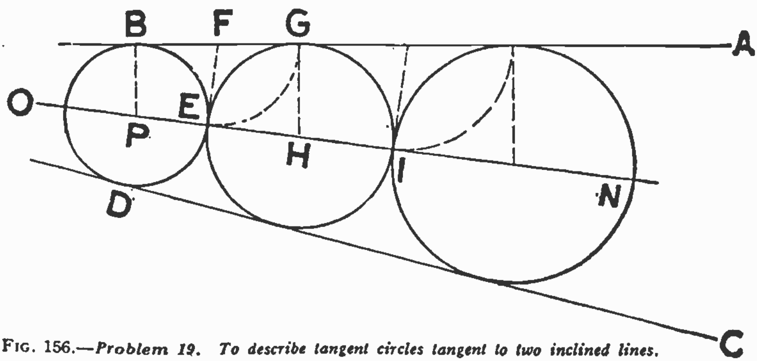


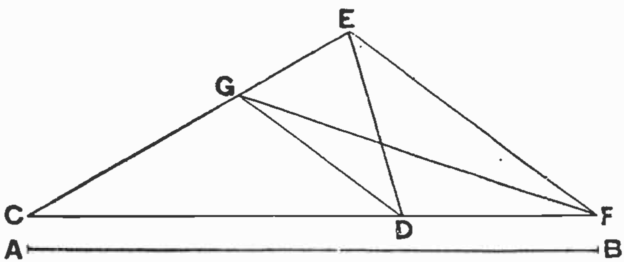
FIG. 156.—*Problem 19. To describe tangent circles tangent to two inclined lines.*

Problem 18.—To draw tangents to a circle from points without.

In fig. 155, from A, and with the radius AC, describe an arc BCD, and from C, with a radius equal to the diameter of the circle, cut the arc at BD; join BC, CD, cutting the circle at EF, and draw the tangents, AF, AE.

Problem 19.—Between two inclined lines to describe a series of circles tangent to these lines and tangent to each other.

In fig. 156, bisect the inclination of the given lines AB, CD, by the line NO. From a point P, in this line, draw the perpendicular PB, to the line AB, and about P, describe the circle BD, touching the lines and cutting the center line at E. From E, draw EF, perpendicular to the center line, cutting



FIGS 157 and 158.—*Problem 20. To construct a triangle having a given base and equivalent to any rectilineal figure*

AB, at F, and about F, describe an arc EG, cutting AB, at G. Draw GH parallel with BP, giving H, the center of the next circle, to be described with the radius HE, and so on for the next circle IN.

Problem 20.—To construct a triangle, having a given base A B, and equivalent to any rectilineal figure, say equal in area to the triangle CDE.

In figs. 157 and 158, produce one side CD, to F, making CF, equal to the given base AB. Join FE. Draw DG, parallel to FE. Join FG. Then CFG, is the required triangle.

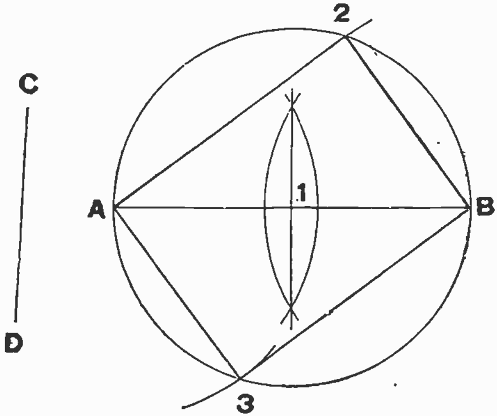
Problem 21.—To construct a rectangle, when each of the diagonals is equal to A B, and each of one pair of opposite sides is equal to C D.

In figs. 159 and 160, bisect AB at 1, and with center 1, and radius 1A, describe a circle. With centers A and B, and radius CD, obtain points 2 and 3. Join A2, 2B, B3, 3A. Then A2B3 is the required rectangle. If the longer side A2 be given, instead of the shorter side, then describe arcs at 2 and 3, with the longer side as radius.

Problem 22.—To construct a square, whose diagonal is given

In fig. 161 Bisect R S, by a perpendicular 2 3. Cut off 1 4, and 1 5, equal to 1R or 1S. Join R5, 5S, S4, 4R. Then R5S4 is the square required, having a given diagonal RS.

Problem 23.—To construct a square equal in area to any number of given squares.



FIGS. 159 and 160.—*Problem 21. To construct a rectangle when each diagonal is equal to a given line and each of one pair of opposite sides is equal to another given line.*

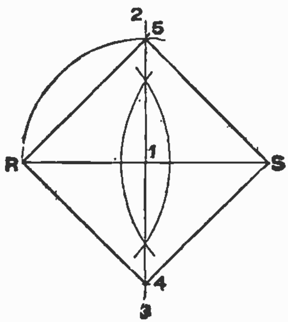
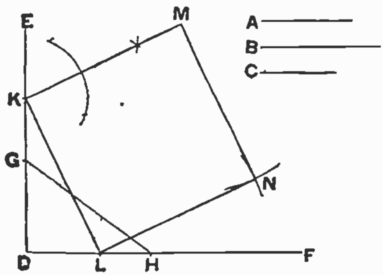


FIG. 161.—*Problem 22. To construct a square with given diagonal.*

In figs. 162 and 163:

Let A, B, C be the side of the three given squares. Make D G = A and D H = B. Join G H. Then the square upon G H equals the squares upon A and B. Make D K = G H and D L = C. Join K L. Then the square K L N M, equals the three squares upon A, B and C.



FIGS. 162 and 163.—*Problem 23. To construct a square equal in area to any number of given squares.*

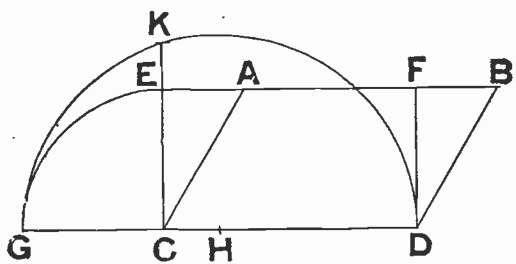


FIG. 164.—*Problem 24. To construct a square equal in area to any parallelogram.*

Problem 24.—To construct a square, equal in area to any parallelogram. Thus, construct a square equivalent to the rhomboid C D B A.

In fig. 164, make the rectangle C D F E, equal to C D B A, by producing E F, and erecting perpendiculars C E, D F. Produce D C. Make C G, = C E. Bisect G D, at H. With center H, and radius H G, describe a semicircle. Produce C E, to K. Then C K, is the mean proportional if G C, C D, and a square constructed with C K as a side is equal in area to the rhomboid C D B A.

Problem 25.—Describe a catenary having given the span and versed sine.

In fig. 165, divide half span, as AB, into any required number of equal parts, as 1, 2, 3 and let fall BC and AO, each equal to versed sine of curve;

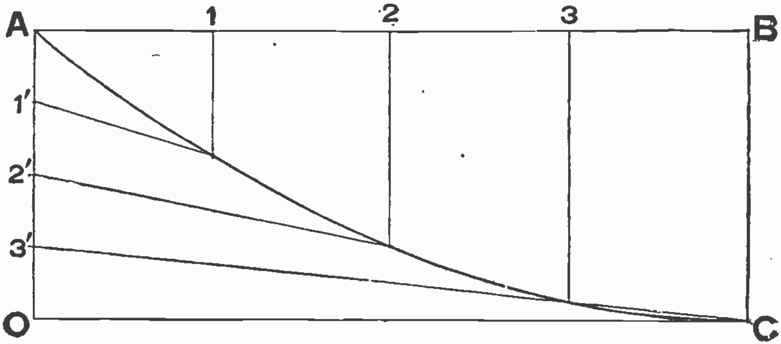


FIG. 165.—*Problem 25. To describe a catenary having given the span and versed sine.*

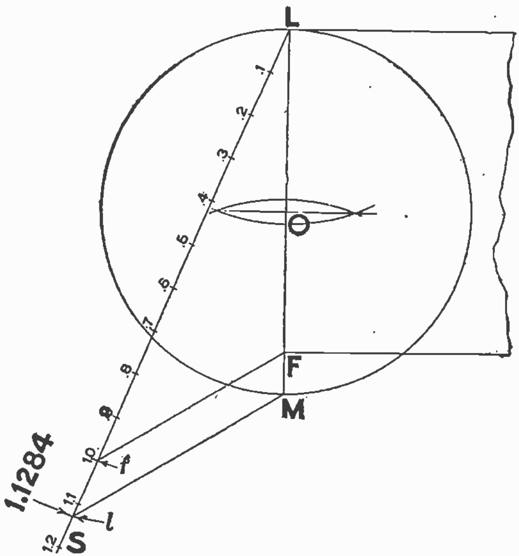


FIG. 166.—*Problem 26. To construct a circle equal in area to a given square.*

divide AO into like number of parts, 1′, 2′, 3′, as AB. Connect C′1, C2′ and C3′ and points of intersection of perpendiculars let fall from AB will give points through which curve is to be drawn.

The catenary is the curve assumed by a perfectly flexible cord when its ends are fastened at two points, the weight of a unit length being constant.

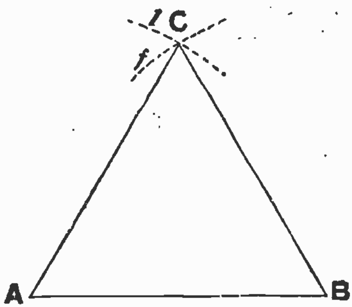


FIG. 167.—*Problem 27. To construct an equilateral triangle on a given base.*

Problem 26.—To construct a circle equal in area to a given square.

In fig. 166, let LF be side of the given square. Through L, draw proportional line L S, and with any convenient scale divide it into 12 equal parts. At the point *f*, or 10th division, draw line *f* F, and at a point *l*, between 11 and 12 draw *l* M parallel with *f* F. LM is the diameter of the required circle. Bisect this diameter at O and with radius O L describe the required circle.

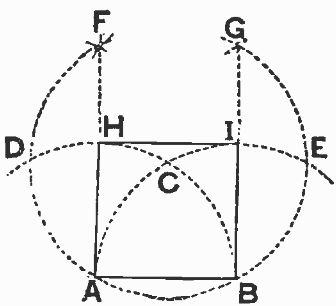


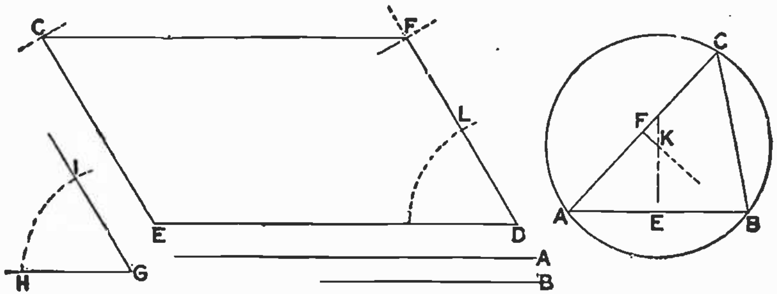
FIG. 168.—*Problems 28 and 29. To construct a square and a rectangle on a given base.*

Problem 27.—To construct an equilateral triangle on a given base.

In fig 167, with A, and B, as centers and radius equal to AB, describe arcs *l* and *j*. At their intersection C, draw lines CA, and CB, sides of the required triangle.

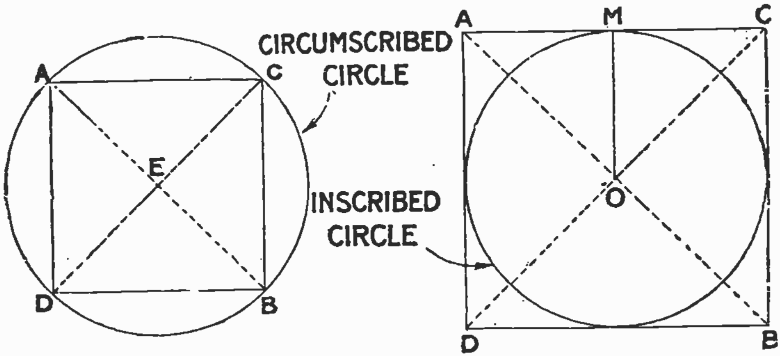
Problem 28.—To construct a square on a given base.

In fig. 168, with end points A and B, of base as centers and radius equal to AB, describe arcs cutting at C; with C as center, describe arcs



FIGS. 169 to 171.—*Problem 30. To construct a parallelogram having given the sides and an angle.*

FIG. 172.—*Problem 31. To describe a circle about a triangle.*



FIGS. 173 and 174.—*Problem 32. To circumscribe about (fig. 173) and inscribe (fig. 174) a circle in a square* cutting the others at DE; and with D and E, cut these at FG. Draw AF, and BG, and join the intersections HI, then ABIH is the required square.

Problem 29.—To construct a rectangle on a given base.

In fig. 168, let AB, be given base. Erect perpendiculars at A and B, equal to altitude of the rectangle, and join their ends H and I, by line HI, ABIH, is the rectangle required.

Problem 30.—To construct a parallelogram having given the sides and an angle.

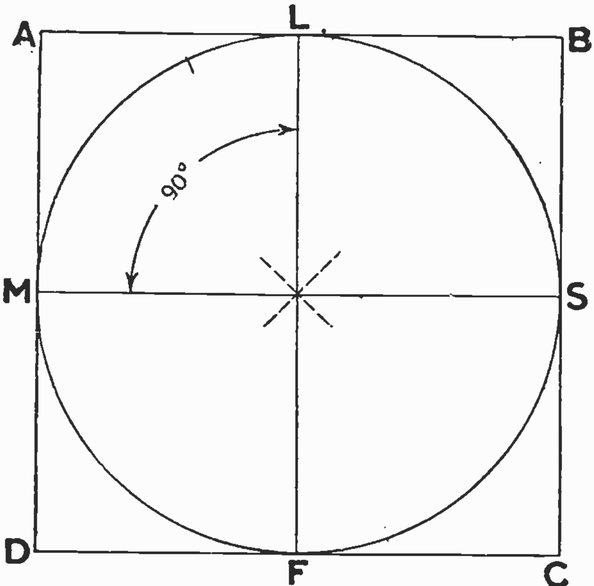


FIG. 175.—*Problem 33. To circumscribe a square about a circle. Second method*

In figs. 169 to 171, draw side DE, equal to the given length A, and set off the other side DF, equal to the other length B, forming the given angle IGH. From E, with DF, as radius, describe an arc, and from F, with the radius DE, cut the arc at C. Draw FC, EC. Or, the remaining sides may be drawn as parallels to DE, DF

Problem 31.—To describe a circle about a triangle.

In fig. 172. bisect two sides AB, AC, of the triangle at E and F, and from these points draw perpendiculars intersecting at K. From the center K, with the radius KA, describe the circle ABC.

***Problem 32.**—To circumscribe about and inscribe a circle in a square.*

In fig. 173, draw the diagonals AB and CD, intersecting at E. With

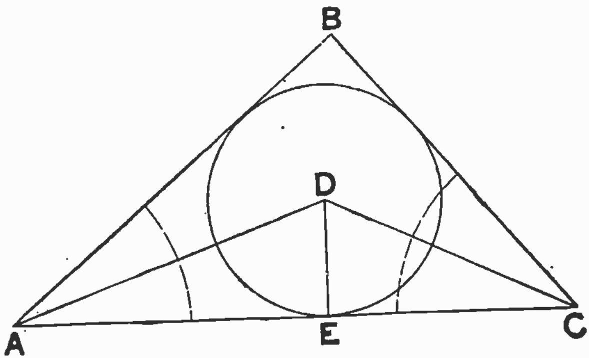


FIG. 176.—*Problem 34. To inscribe a circle in a triangle.*

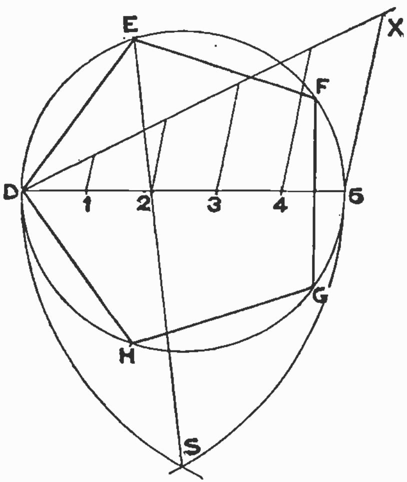


FIG. 177.—*Problem 35. To inscribe any regular polygon in a given circle.*

radius EA, circumscribe the circle. To inscribe a circle let fall from the center (as just found) a perpendicular to one side of the square as OM, in fig. 174. With radius OM, inscribe the circle.

Problem 33.—To circumscribe a square about a circle.

In fig. 175, draw diameters MS and LF, at right angles to each other. At the points M, L, S, F, where these diameters cut the circle, draw tangents that is, lines perpendicular to the diameter, thus obtaining the sides of the circumscribed square ABCD.

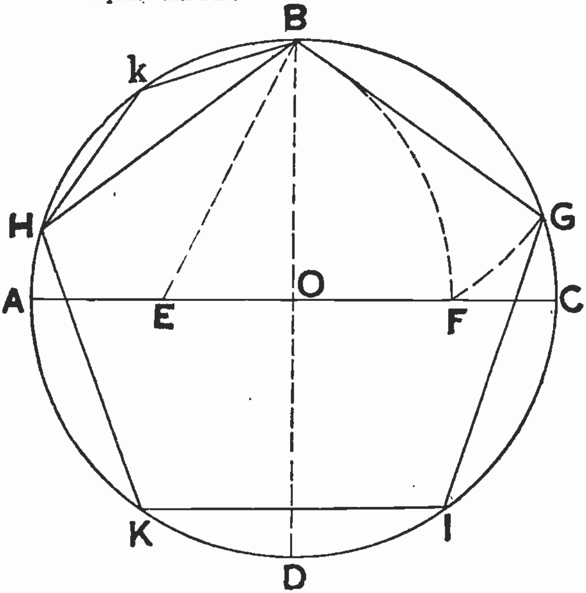


FIG. 178.—*Problem 36. To inscribe a pentagon in a circle.*

Problem 34.—To inscribe a circle in a triangle.

In fig. 176, bisect two of the angles A and C, of the triangle by lines cutting at D; from D, draw a perpendicular DE, to any side, and with DE, as radius, describe a circle.

Problem 35.—To inscribe any regular polygon in a given circle.

In fig. 177, draw a diameter D 5. Divide D 5, into as many equal parts as the polygon is to have sides, in this case, five equal parts. With points D and 5, as centers, and the diameter D 5, as radius, describe arcs intersecting at 6. From 6, draw a line through Point 2 to E. Join D E, which is one side of the required polygon. Make E F, F G, G H, each equal D E. Join E F, F G, G H, H D. Then D E F G H, is the required polygon.

This method is only approximately correct. It is however, sufficiently accurate for all practical work. On the same principle, an arc can (approximately) be divided into any number of equal parts, or a circle into equal sectors. By this method, a regular polygon having any number of sides

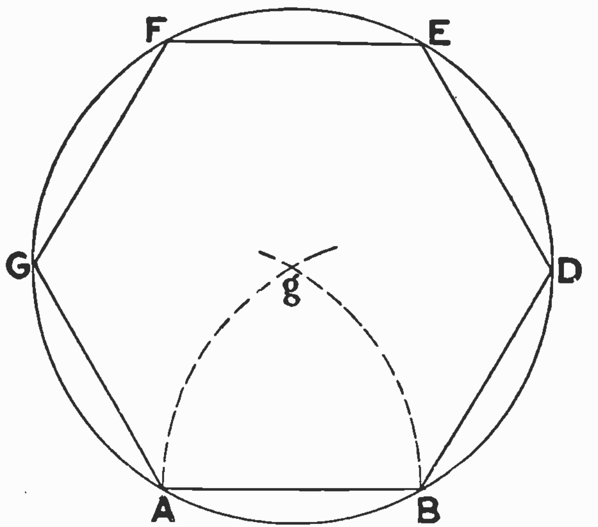


FIG. 179.—*Problem 37. To construct a hexagon upon a given straight line.*

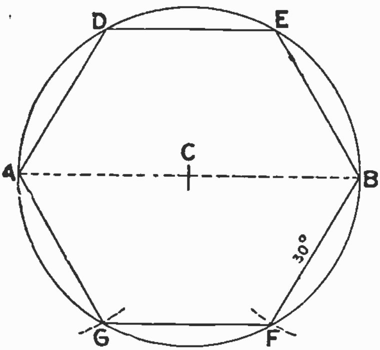
can be inscribed, (approximately) within a given circle. If a nonagon is to be inscribed, divide the diameter into nine equal parts, and then proceed as above. To get the first side of the polygon, always draw a line from point 6; through the 2nd division on the diameter, no matter how many sides the polygon is to have. In a polygon, that has an even number of sides, a line drawn from one angle to the opposite angle (a diagonal) passes through the center. When there is an odd number of sides, a line from an angle through the center, bisects the opposite side. Note these facts as tests for accuracy in the work.

Problem 36.—To inscribe a pentagon in a circle.

In fig. 178, draw two diameters, AC, BD, at right angles intersecting at O; bisect AO, at E, and from E, with radius EB, cut AC, at F, and from B, with radius BF, cut the circumference at G,H, and with the same radius step round the circle to I and K; join the points so found to form the pentagon.

Problem 37.—To construct a hexagon upon a given straight line.

In fig. 179, from A and B, the ends of the given line describe arcs intersecting at *g*; from *g*; with the radius *g*A, describe a circle. With the same radius set off the arcs AG, GF and BD, DE. Join the points so found to form the hexagon.



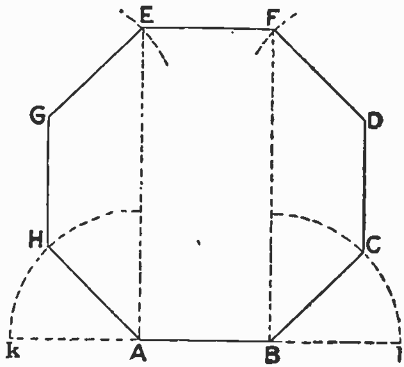


FIG. 180.—***Problem 38. To inscribe a hexagon in a circle.***

FIG. 181.—***Problem 39. To construct an octagon on a given straight line.***

Problem 38.—To inscribe a hexagon in a circle.

In fig. 180, draw a diameter ACB; from A and B, as centers with the radius of the circle AC, cut the circumference at D,.E, F, G, and draw AD, DE, etc., to form the hexagon.

The points DE, etc., may be found by stepping the radius (with the dividers) six times round the circle.

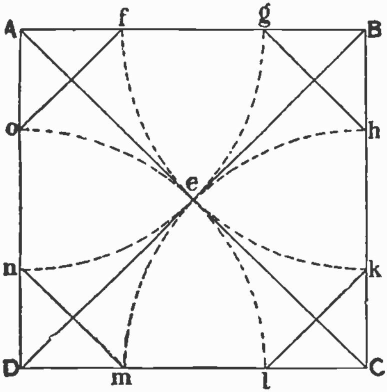
Problem 39.–To construct an octagon on a given straight line.

In fig. 181, produce the given line AB, both ways, and draw perpendiculars AE, BF; bisect the external angles A and B, by the lines AH, BC, which make equal to AB. Draw CD and HG parallel with AE and equal to

AB; from centers G, D, with the radius AB, cut the perpendiculars at EF, and draw EF, to complete the octagon.

Problem 40.—To inscribe an octagon in a square.

In fig. 182 draw the diagonals of the square intersecting at *e;* from the



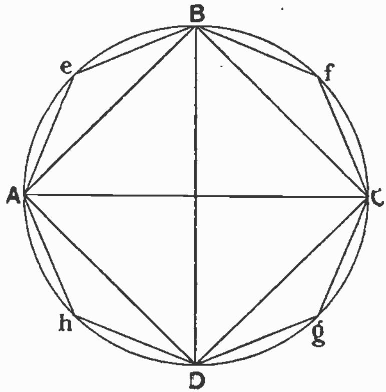
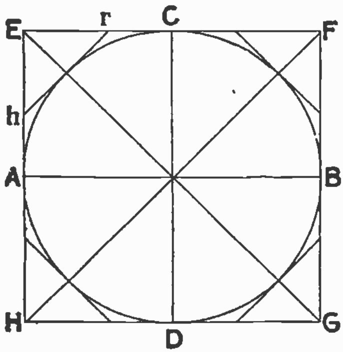


FIG. 182.—*Problem 40. To inscribe an octagon in a square.*
FIG. 183.—*Problem 41. To inscribe an octagon in a circle.*



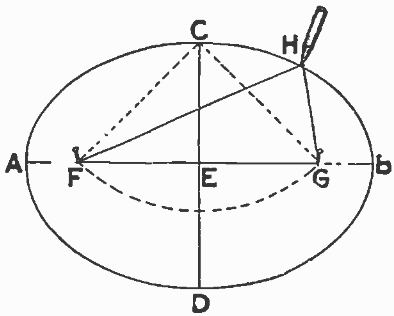


FIG. 184.—*Problem 42. To circumscribe an octagon about a circle.*
FIG. 185.—*Problem 43. To describe an ellipse when the two axes are given.*

corners A,B,C,D, with *Ae*, as radius, describe arcs cutting the sides at *g*, *h*, etc.; and join the points so found to complete the octagon.

Problem 41.—To inscribe an octagon in a circle

In fig. 183, draw two diameters AC, BD, at right angles; bisect the arcs AB, BC, etc., at *e*, *f*, etc., and join the points of division to form the octagon.

Problem 42.—To circumscribe an octagon about a circle.

In fig. 184, circumscribe a square EFGH, about the given circle Draw

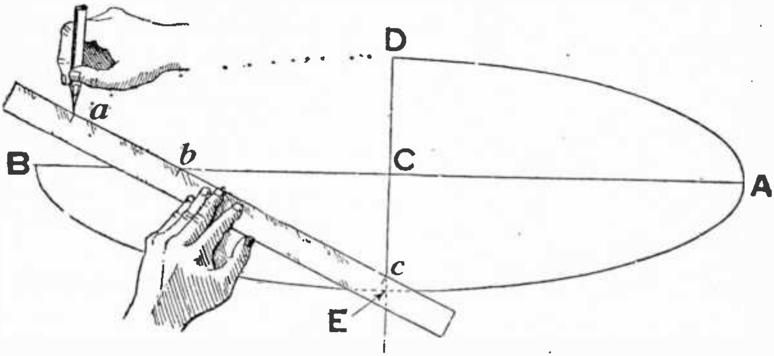


FIG. 186.—*Problem 43. Second method.*

diagonals HF and EG, and tangents *h r*, etc., through points where the diagonals cut the circle to form with the intercepts, the octagon.

Problem 43.—To describe an ellipse when the two axes are given

In fig. 185, draw the major and minor axes AB and CD, at right angles, intersecting at E. On the center C, with AE, as radius, cut the axis AB, at F and G, the *foci;* insert pins through the axis at F and G, and loop a thread or cord upon them equal in length to the axis AB, so that when stretched it reaches the extremity C, of the *minor axis*, as shown in dotted lines. Place a pencil inside the cord, as at H, and guiding the pencil in this way, keeping the cord equally in tension, carry the pencil round the pins FG, and so describe the ellipse.

Second Method.— In fig. 186 along the edge of a piece of paper, mark off a distance *ac*, equal to AC, half the major axis, and from the same point, a distance *ab*, equal to CD, half the minor axis. Place the slip so as to bring the point *b*, on the line AB, or major axis, and the point *c*, on the line DE, or minor axis. Set off the position of the point *a*. Shifting the slip, so that the point *b*, travels on the major axis, and the point *c*, on the minor axis, any number of points in the curve may be found, through which the curve may be traced.

Problem 44.—To construct a spiral or volute, by means of tangential arcs of circles.

Construct a square 1 2 3 4, and produce the sides, fig. 44. With center 2, and radius 2 1, describe arc 1 A; center 3, and radius 3 A, describe arc A B; center 4, and radius 4 B, describe arc B C; center 1, and radius 1 C, describe C D; center 2, and radius 2 D, describe D E. In the same way describe any number of arcs, E F, F G, G H. The curve obtained is a spiral or volute. 1 2 3 4, is the eye of the spiral. The eye can be formed by any regular or irregular rectilineal figure, not having a re-entrant angle. In every case, proceed as above.

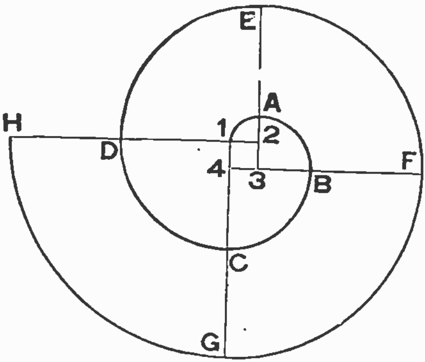


FIG. 187.—***Problem 44. To construct a spiral or volute, by means of tangential arcs of circles.***

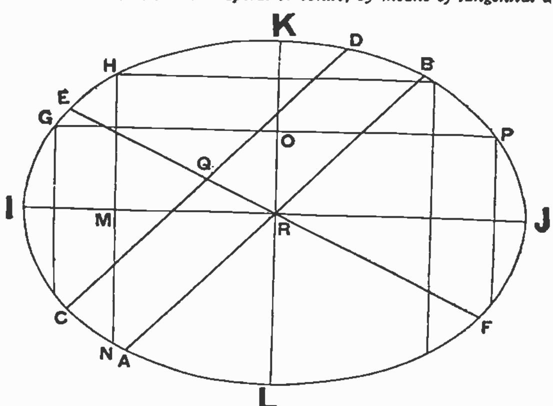


FIG. 188. —***General notes about ellipses.*** If from any points G, H, in the curve of an ellipse, lines parallel to the major axis I J, be drawn, or to the minor axis K L, be drawn, and the distance M N, be made equal to M H, or O P, be made equal to O G, other points, N, P, in the elliptic curve are obtained. A line Q C, or Q D, drawn from any point Q, in a diameter E F, and parallel to a conjugate diameter A B, is called an ordinate. M H, O P, are also ordinates. The whole line C D, H N, or G P, is a double ordinate. Draw any cord C D, parallel to A B. Bisect A B, C D, at Q R. Then E F, drawn through Q R, is a *conjugate diameter* to A B. The minor axis is called "the conjugate axis," because of its relationship to the major axis. The major and minor axes are a *pair* of conjugate diameters.

Problem 45.—To find the foci of an ellipse and then to draw the elliptic curve by means of intersecting arcs, the major axis P Q and minor axis T V being given.

In fig. 189, with T, one end of the minor axis, as center and X Q, half the major axis as radius, describe arc Y, cutting the major axis at F′, F″. These points are the required foci. Between F′ and X, mark any number of points 1, 2, 3, 4. With centers F′, F″, and radius PI, describe arcs *a, a, a, a*. With the same centers and radius Q 1, cut arcs *a, a, a, a*, at *b, b, b, b*. With each focus as center and radius P2, describe arcs, *c, c, c, c*. With the

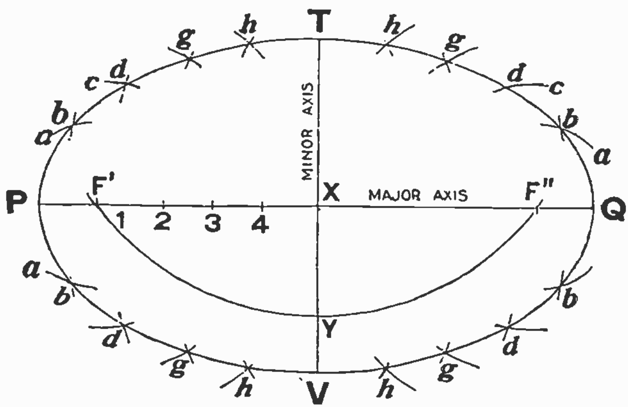


FIG. 189.—*Problem 45. To construct an ellipse having given minor and major axes.*
FIG. 189.—*Problem 46. The major axis and foci of an ellipse being given to find the minor axis.*

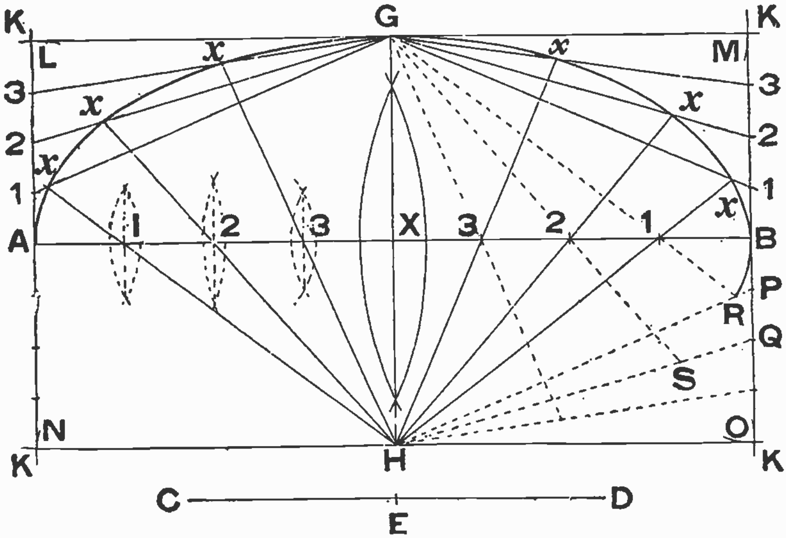
same centers and radius Q 2, cut these arcs at *d, d, d, d*. In the same way use points 3 and 4, to get *g, g, h, h*. Through points *b, d, g, h*, draw the curve of the ellipse. The points 1, 2, 3, 4 may be at any distance apart, but it is more convenient to let the divisions decrease in length toward F′. Do not make the arcs too long, as this causes confusion.

Problem 46.—The major axis and foci of an ellipse being given to find the minor axis.

In fig. 189, bisect P Q, at X. With X P, or X Q, as radius and the foci as centers, strike arcs cutting at T and V. Join T V. Then T V, is the minor axis.

Problem 47.—Draw the curve of an ellipse by means of intersecting lines. The lengths of the major and minor axes AB, and CD, being given.

In figs. 190 and 191 bisect CD at E. Bisect AB, at X, by a perpendicular. Make X G, X H, each equal to E C, or E D. With centers G, H, and radius X A, describe arcs at K, K,K,K. With centers A, B, and radius X G, cut these arcs at L, M, N, O. Join L M, M O, O N, N L. Divide A L, A N, B M, B O, A X, B X, each into the same number of equal parts, say four. Draw lines from G, to 1, 2, 3, on A L, B M. From H, through 1 (on A X), draw a line to meet 1 G, at *x*. Through 2, draw a line from H, to meet 2 G, at



FIGS. 190 and 191.—*Problem 47. Draw the curve of an ellipse by means of intersecting lines.*

x. Through 3, draw a line to meet 3 G, at *x*. In the same way get points *x*,*x*,*x*, on the other side. Also get similar points for the lower half of the ellipse, as shown by dotted lines at R and S. Through *x*,*x*,*x*, R, S, draw the curve of the ellipse. The divisions on A L, A X, may be unequal, provided those on A X, be proportional to those on A L. A French curve may be used for drawing the elliptic curve, through the points *x*,*x*,*x*. By this method, an ellipse may be inscribed in any rectangle. By joining A G, G B, B H, H A, a rhombus is obtained. Therefore an ellipse can be circumscribed about a rhombus, or a rhombus can be inscribed in an ellipse.

Problem 48.—The curve or portion of the curve of an ellipse being given, to find the center and the major and minor axes.

In fig. 192, draw two parallel chords A B, C D. Bisect them at E and F, Through E, F, draw G H, which is a diameter. Bisect it at K, which is the center of the ellipse. With center K, and any convenient radius, describe the arc L M N. With centers L and M, and any radius, describe arcs cutting at O From O, through K, draw P Q, which is the major axis.With centers M and N; and any radius, describe arcs cutting at R. From R, through K, draw T S, which is the minor axis. Instead of describing arcs at O and R, L M, M N, may be joined and the axes through K, parallel with L M, M N, drawn.

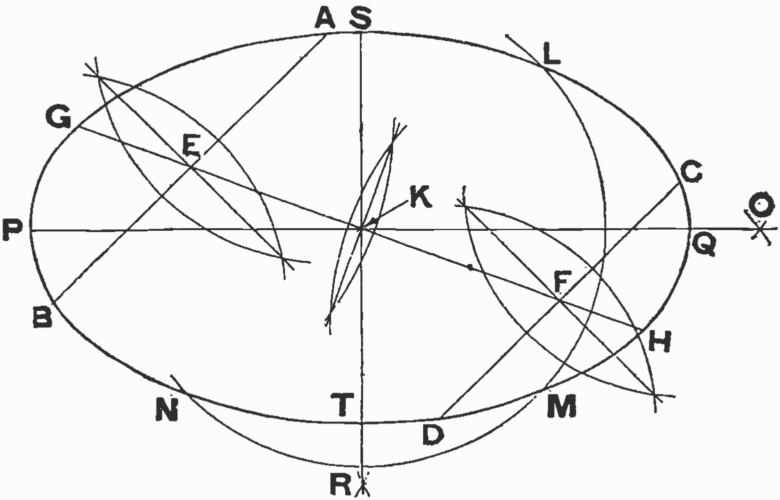


FIG. 192.—*Problem 48. The curve or portion of the curve of an ellipse being given, to find the center and the major and minor axes.*

For convenience, the given ellipse may be drawn with a piece of thread as shown in fig 185, If a small portion of the curve be given, the chords A B, C D, must be drawn closer together, If only one end of G H, meet the curve, draw another pair of parallel chords, and get another diameter, then the intersection of the two diameters gives the center. The portion of the curve given should contain at least one end of each axis.

NOTE.—*To draw an ellipse* when the foci and one point in the curve are given. Draw a line of indefinite length through the foci. Draw a line from each focus to the given point. The sum of these two lines gives the length of the major axis. With half the major axis as radius, and the foci as centers, describe arcs intersecting at points, which give the ends of the minor axis. Obtain the curve of the ellipse.

***Problem 49.**—At any point A, in the curve of an ellipse, to draw a normal; and through any point B in the curve to draw a tangent.*

In fig. 193, draw the ellipse with a piece of thread. From each focus, draw a line through A, to D, and C. Bisect angle D A C, by A E. The line A E, is the required normal (or perpendicular). From the foci draw lines to B. Produce one of the lines, say to G. Bisect the angle G B F″, by H K. Then the line H K is the required tangent. The normal may also be obtained by bisecting the angle F′ A F″. To draw a normal at either

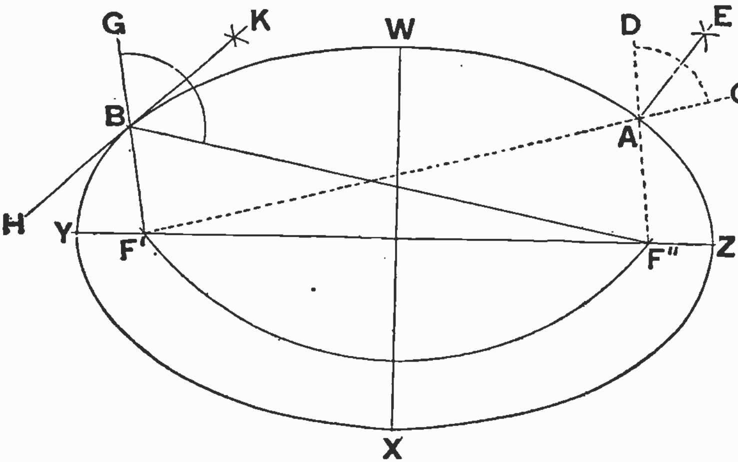


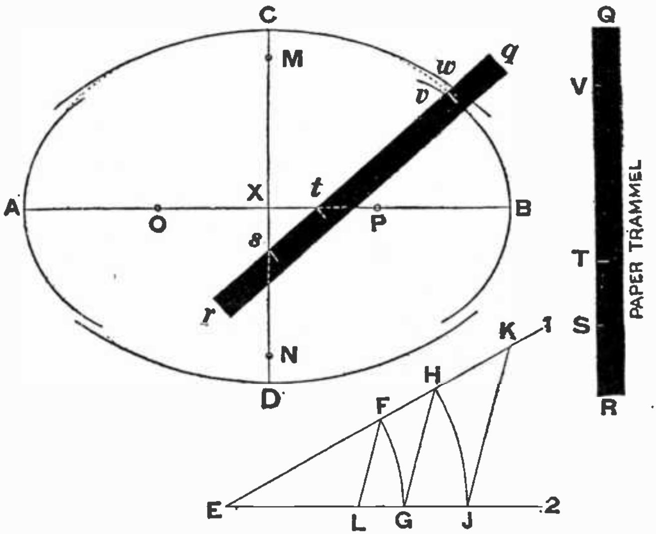
FIG. 193.—***Problem 49.** At any point in the curve of an ellipse to draw a normal, and through any point in the curve to draw a tangent.*

extremity of the major or minor axis, simply produce the axis. A tangent at either extremity of the major or minor axes must be drawn at right angles to the axis.

NOTE.—*To draw tangential lines to an ellipse from a given point outside the curve.* Call given point 1, and place it in any position with regard to ellipse. With 1, as center and the distance to the nearer focus, as radius, describe about half of a circle cutting the ellipse in two places. With the further focus as center and the major axis as radius, cut the arc in points 2 and 3. From points 2 and 3, draw lines to the further focus. These lines cut the ellipse in two points. Call these points 4 and 5; they are the required points of contact. Draw two lines from the given point 1, through points 4 and 5; and these lines are the required tangents.

Problem 50.—To get the curve of an ellipse approximately with arcs of circles, and by the use of a paper trammel.

In figs. 194 to 196, lines A B, C D, are major and minor axes. Drew E 1, E 2, at any angle. Make E G = X C, and E H = X A. Join G H. With center E, and radii E G, E H, strike the arcs G F, H J. Draw F L, J K, parallel with G H. Make D M, C N, equal to E K, and A O, B P, equal to E L. With centers N and M, and radius N C, describe arcs passing through C and D. With centers O, P, and radius O A, describe arcs at A and B. These four arcs give approximately parts of the ellipse. On one edge of a straight



Figs. 194 to 196.—*Problem 50. To get the curve of an ellipse approximately with arcs of circles and by the use of a paper trammel. This method applies only when the minor axis is more than about 3/5 of the major axis. In making a narrow ellipse M and N will fall outside the ellipse.*

slip of paper Q R, set off V S, equal to A X, and V T, equal to C X. Then use Q R, as a trammel. Adjust the trammel Q R, in such a manner, that

NOTE.—*To describe an ellipse,* having one diameter given, similar to any given ellipse. In two similar ellipses, any two conjugate diameters of one ellipse have the same proportion to each other as the corresponding conjugate diameters of the other ellipse have to each other. Therefore find a fourth proportional to the given diameter, and the two diameters of the given ellipse. This fourth proportional gives the length of the other diameter of the required ellipse. Place the two diameters bisecting each other, and at the required angle and describe the ellipse.

point *t*, rests somewhere on the major axis; and point *s*, on the minor axis. Wherever point *v*, comes, will be a point situated in the curve of the ellipse. Mark several points as at *w*, and through these points draw curves connecting the arcs. E L, is a third proportional less, and E K, is a third proportional greater, to the lines E G, E H. A French curve may be used to connect the arcs through the points at *w*. The entire curve can be drawn by means of points obtained with a trammel. When an ellipse has a short minor axis, the points M and N, fall outside the ellipse, on the minor axis produced. This method is exceedingly useful when representing circles in perspective, and also in mechanical drawing when describing ellipses.

TEST QUESTIONS

1. *Draw a perpendicular to a straight line.*
2. *Give the boat builder's laying down method.*
3. *Divide a line into a number of equal parts.*
4. *Bisect an angle.*
5. *Find the center of a given circle.*
6. *Describe a circle passing through three given points.*
7. *Find approximately the length of the circumference of a given circle.*
8. *From a given point draw a tangent to a given circle.*
9. *Construct a square having its diagonal given.*
10. *Construct a square equal in area to any number of given squares.*
11. *Construct a circle equal in area to a given square.*
12. *How is an equilateral triangle constructed on a given base?*
13. *Erect a rectangle on a given base.*
14. *Describe a circle about a triangle.*
15. *Inscribe a circle in a triangle.*
16. *Inscribe any regular polygon in a given circle.*
17. *Inscribe a hexagon in a circle.*
18. *Construct an octagon on a given straight line.*
19. *Describe an ellipse when the two axes are given.*

20. *What is the method of constructing a spiral or volute by means of tangential arcs of circles?*

21. *Give the method of describing an ellipse by means of intersecting arcs.*

22. *Describe an ellipse by the method of intersecting lines.*

23. *Draw a tangent to an ellipse at a given point in the curve of the ellipse.*

CHAPTER 5

Mensuration

Mensuration is *the process of measuring.*

It is that branch of mathematics that has to do with finding the length of lines, the area of surfaces, and the volume of solids. Accordingly the problems which follow will be divided into three groups, as:

1. Measurement of lines.
    a. One dimension, *length*
2. Measurement of surfaces (*areas*).
    a. Two dimensions, *length* and *breadth*
3. Measurement of solids (*volumes*).
    a. Three dimensions. *length, breadth*, and *thickness*

1. Measurement of Lines

(length)

Problem 1.—To find the length of any side of a right triangle, the other two sides being given.

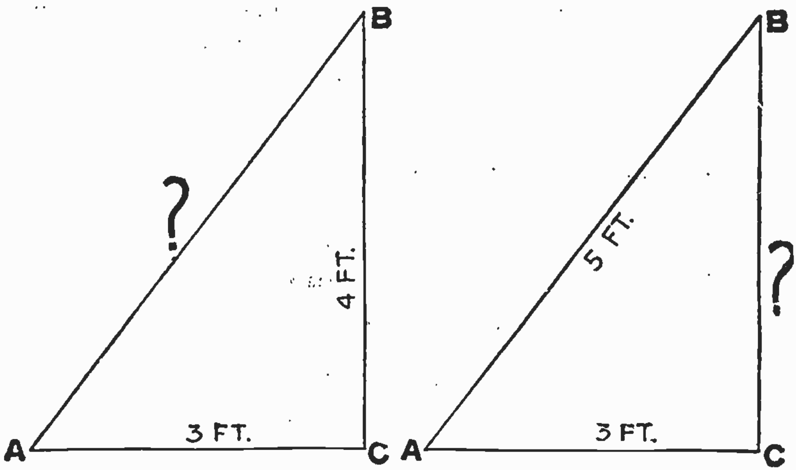
Rule.—*Length of hypotenuse equals square root of the sum of* *the squares of the two legs; length of either leg equals square root of the difference of the square of the hypotenuse and the square of the other leg.*

Example.—The two legs of a right triangle measure 3 and 4 ft.; find length of hypotenuse. If the length of hypotenuse and one leg be 5 and 3 ft. respectively, what is the length of the other leg?

In fig 197

$$AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

In fig. 198 $BC = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4.$



FIGS. 197 and 198:—*Problem 1. To find the length of any side of a right triangle.*

Problem 2.—To find length of circumference of a circle.

Rule.—*Multiply the diameter by* 3.1416.

Example.—What length of moulding strip is required for a circular window 5 ft. in diameter?

$$5 \times 3.1416 = 15.7 \text{ ft.}$$

As the mechanic does not ordinarily measure feet in tenths, the .7 should be reduced to inches; it corresponds to 8⅜ ins. from the table below. That is, the length of moulding is 15 ft. 8⅜ ins. (approx).

Problem 3.—To find the length of an arc of a circle.

Rule.—*As 360° is to the number of degrees of the arc so is the length of the circumference to the length of the arc.*

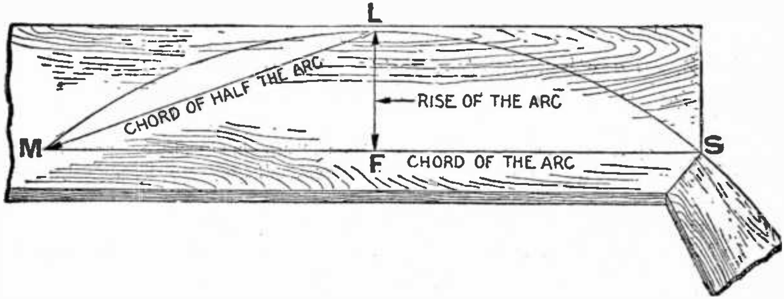


FIG. 199.—*Problem 4. To find width of board required for plate form of circular pattern.*

Decimals of a Foot and Inches

Inch	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"	10"	11"
0	.0	.0833	.1677	.2500	.3333	4167	.5000	.5833	.6667	.7500	.8333	.9167
1-16	.0052	.0885	.1719	.2552	.3385	.4219	.5052	.5885	.6719	.7552	.8385	.9219
1-8	.0104	.0937	.1771	2604	.3437	.4271	.5104	.5937	.6771	.7604	.8437	.9271
3-16	.0156	.0990	.1823	.2656	.3490	.4323	.5156	.5990	.6823	.7656	.8490	9323
1-4	.0208	.1042	.1875	.2708	.3542	.4375	.5208	.6042	.6875	.7708	.8542	.9375
5-16	0260	1094	.1927	.2760	.3594	.4427	.5260	.6094	.6927	.7760	.8594	.9427
3-8	0312	.1146	.1979	2812	.3646	.4479	.5312	.6146	.6979	7812	.8646	.9479
7-16	.0365	.1198	.2031	.2865	.3698	.4531	.5365	.6198	.7031	.7865	.8698	9531
1-2	.0417	.1250	2083	.2917	.3750	.4583	.5417	.6250	.7083	.7917	.8750	.9583
9-16	.0469	.1302	.2135	.2969	.3802	.4635	.5469	.6302	.7135	.7969	8802	.9635
5-8	.0521	.1354	.2188	3021	.3854	.4688	.5521	.6354	.7188	.8021	.8854	9688
11-16	.0573	.1406	.2240	.3073	.3906	.4740	.5573	.6406	.7240	.8073	.8906	.9740
3-4	.0625	.1458	.2292	3125	.3958	.4792	.5625	.6458	.7292	8125	.8958	.9792
13-16	.0677	.1510	.2344	.3177	4010	.4844	.5677	6510	.7344	.8177	.9010	.9844
7-8	.0729	.1562	.2396	.3229	4062	4896	5729	.6562	7396	.8229	.9062	.9896
15-16	.0781	.1615	.2448	.3281	4115	.4948	.5781	6615	7448	.8281	9115	.9948

Example.—If the circumference of a circle be 6 feet, what is the length of 60° arc?

Let X = length of the arc, solving for X.

$$360 : 60 = 6 : X = \frac{60 \times 6}{360} = \frac{360}{360} = 1 \text{ ft.}$$

Problem 4.—To find the rise of an arc.

Rule 1.—*The rise of an arc is equal to the square of the chord of half the arc divided by the diameter.*

Rule 2.—*Length of chord subtending an angle at the center is equal to twice the radius times the sine of half the angle.*

Example.—A circular pattern 10 ft. in diam. has six plate forms. Find width of board required for these forms allowing 3 ins. margin for joints as in fig. 199.

Each plate will subtend an angle of 360 ÷ 6 = 60°
The "chord of half the arc" (mentioned in rule 1) will subtend 60 ÷ 2 = 30°.

Applying rule 2, "half the angle" = 30° ÷ 2 = 15°.

From table of "trigonometrical functions" (page 244); sine of 15° = .259, which with radius of 5 ft., becomes

sin 15° (on 10-ft. circle) = 5 × .259 = 1.295

Applying rule 2 length of chord MS. = 2 × 1.295 = 2.59

Applying rule 1 rise of arc MS, = 2.59^2 ÷ 10 = .671 ft. or $8\frac{1}{16}$ ins. **(approx.)**

Add to this 3 ins. margin for joints and obtain

width of board $8\frac{1}{16}$ + 3 = $11\frac{1}{16}$ Use 12 in. board

2. Measurement of Surfaces

(areas)

Problem 5.—To find the area of a square.

Rule.—*Multiply the base by the height.*

Example.—What is the area of a square whose side is 5 ft. as in fig. 200?

$$5 \times 5 = 25 \text{ sq. ft.}$$

Problem 6.—To find the area of a rectangle.

Rule.—*Multiply the base by the height* (*i. e., width by length*).

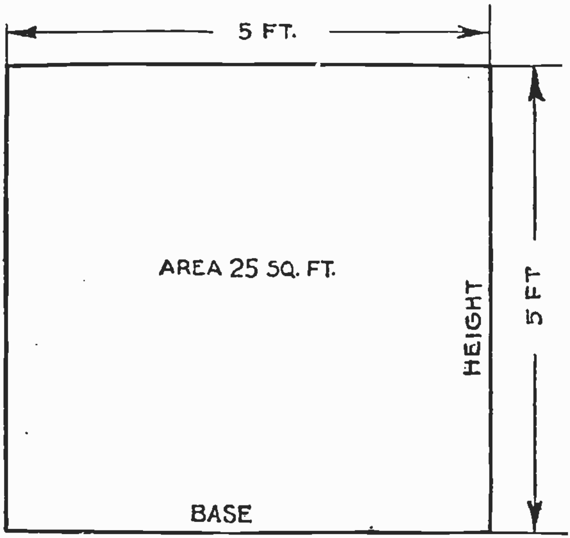


FIG. 200.—***Problem 5.*** *Area of square.*

Example.—What is the area of a rectangle 5 ft. wide and 12 ft. long, as in fig. 201?

$$5 \times 12 = 60 \text{ sq. ft.}$$

Problem 7.—To find the area of a parallelogram.

Rule.—*Multiply base by perpendicular height.*

Example.—What is the area of a parallelogram 2 ft. wide and 10 ft. long?

$$2 \times 10 = 20 \text{ sq. ft.}$$

Problem 8.—To find the area of a triangle.

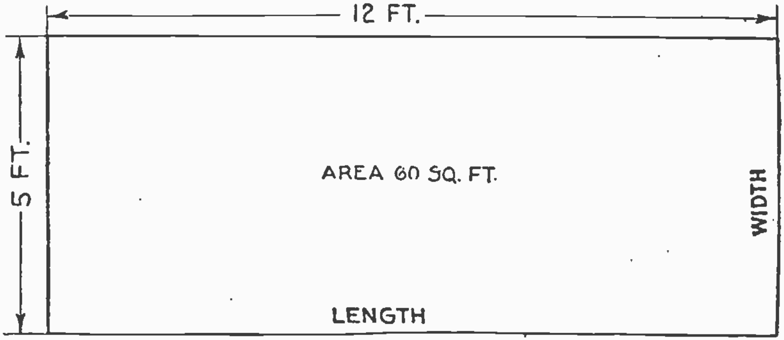


FIG. 201.—*Problem 6. Area of rectangle.*

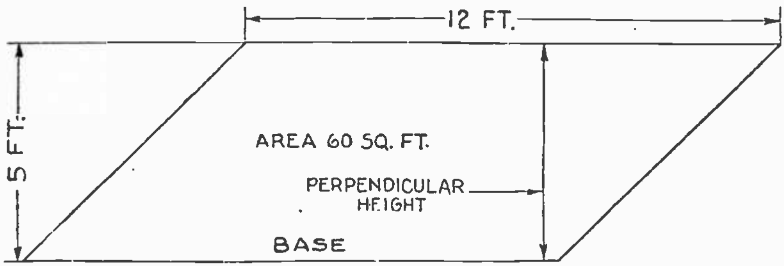


FIG. 202.—*Problem 7. Area of parallelogram.*

Rule.—*Multiply the base by half the altitude.*

Example.—How many sq. ft. of sheet tin are required to cover a church steeple having four triangular sides, measuring 12 ft. (base) × 30 ft. (altitude) as in fig. 203?

$$\tfrac{1}{2} \text{ of altitude} = 15 \text{ ft.}$$

$$\text{area of one side} = 12 \times 15 = 180 \text{ sq. ft.}$$

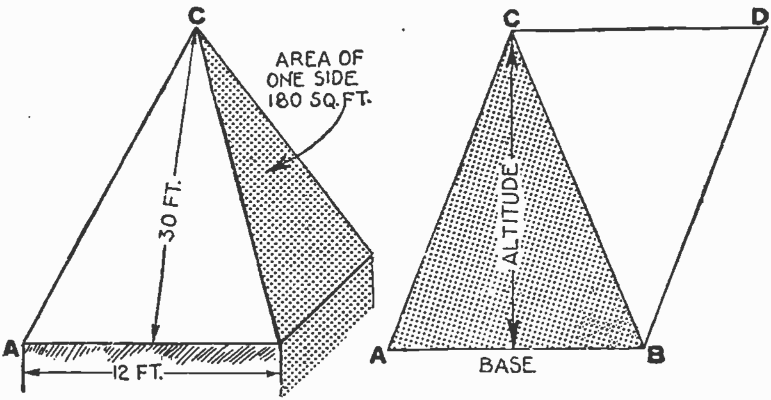
Total area (four sides) 4 × 180 = 720 sq. ft.

Problem 9.—To find the area of a trapezoid.

Rule.—*Multiply one-half the sum of the two parallel sides by the perpendicular distance between them.*

Example.—What is the area of the trapezoid shown in fig. 205?

Here LA and FR, are the parallel sides and MS, the perpendicular dis-



FIGS. 203 and 204.—***Problem 8.*** *Area of triangle.* An inspection of fig. 204 will show that area of triangle = base' × ½ altitude because constructing a parallelogram ABCD, it is made up of two equal triangles and its area = base × altitude. Hence ½ altitude is taken in finding area of a triangle.

tance between them. Applying rule

$$\text{area} = \tfrac{1}{2}(LA + FR) \times MS$$
$$= \tfrac{1}{2}(8 + 12) \times 6 = 60 \text{ sq. ft.}$$

Problem 10.—To find the area of a trapezium.

Rule.—*Draw a diagonal, dividing figure into triangles; measure diagonal and altitudes and find area of the triangles.*

Example.—What is the area of the trapezium shown in fig. 206, for the dimensions given? Draw diagonal LR, and altitudes AM and FS.

area triangle ALR	$= 12 \times 6/2 =$	36 sq. ft.
area triangle LRF	$= 12 \times 9/2 =$	54 sq. ft.
area trapezium LARF	=	90 sq. ft.

Problem 11.—To find the area of any irregular polygon.

Rule.—*Draw diagonals dividing the figure into triangles and find the sum of the areas of these triangles.*

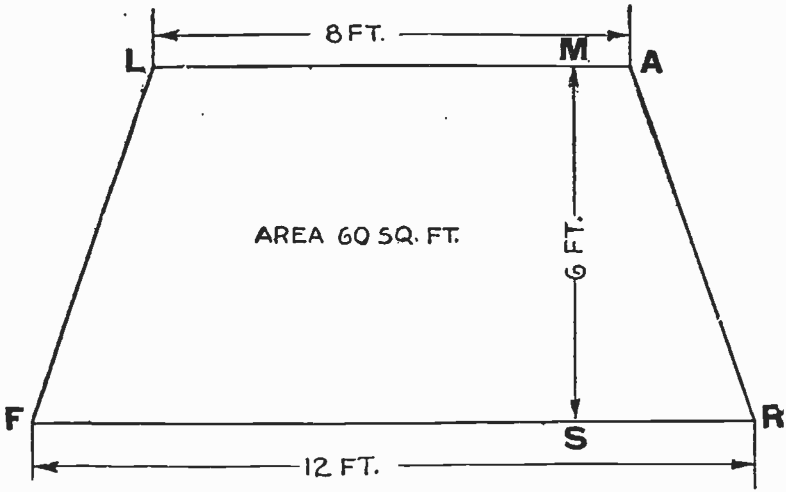


FIG. 205.—*Problem 9. Area of trapezoid.*

Problem 12.—To find the area of any regular polygon when length of side only is given.

Rule.—*Multiply the square of the sides by the figure for "area, when side = 1" in the table following:*

Number of sides	3	4	5	6	7	8	9	10	11	12
Area when side =1..........	.433	1.	1 721	2.598	3.634	4.828	6.181	7 694	9.366	11.196

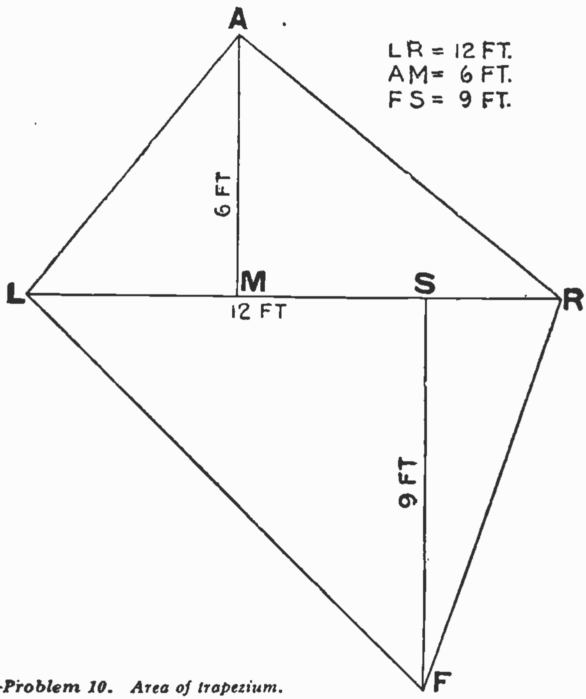


FIG. 206.—*Problem 10. Area of trapezium.*

Example.—What is the area of an octagon (8-sided polygon) whose sides measure 4 ft.

In the above table under 8, find 4.828. Multiply this by the square of one side.

$$4.828 \times 4^2 = 77.25 \text{ sq. ft.}$$

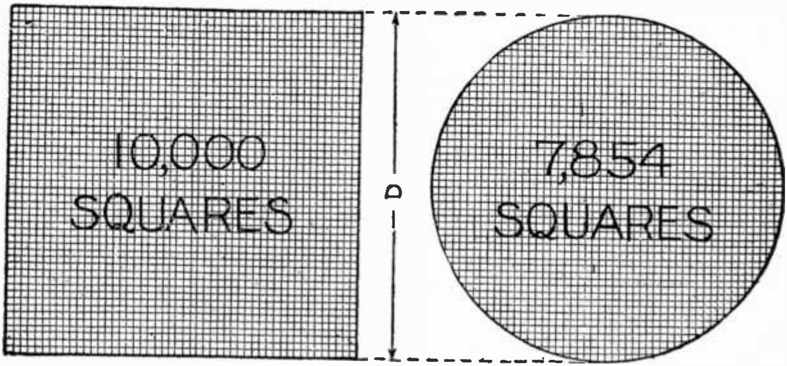
Problem 13.—To find the area of a circle.

Rule.—*Multiply square of diameter by* .7854.

Example.—What is the area of a circle 10 ft. in diameter?

$$10^2 \times .7854 = 78.54 \text{ sq. ft.}$$

Figs. 207 and 208 show why the decimal .7854 is used in finding the area of a circle.



FIGS. 207 and 208.—Showing why the decimal .7854 is used to find the area of a circle. If the square be divided into 10,000 parts or small squares, a circle having a diameter D, equal to a side of the large square will contain 7,854 small squares, hence, if the area of the large square be 1 sq. in., then the area of the circle will be 7854 ÷ 10,000 or .7854 sq. ins., that is, area of the circle = .7854 × D × D = .7854 × 1 × 1 = .7854 sq. ins.

Problem 14.—To find the area of a sector of a circle.

Rule.—*Multiply the arc of the sector by half the radius.*

Example.—How much tin is required to cover a 60° sector of a 10 foot circular deck?

$$\text{length of } 60° \text{ arc} = \frac{60}{360} \text{ of } 3.1416 \times 10 = 5.24 \text{ ft.}$$

The reason for the above operation should be apparent without any explanation.

Applying rule

tin required for 60° sector = 5.24 × ½ of 5 = 13.1 sq. ft.

Problem 15.—To find the area of a segment of a circle.

Rule.—*Find the area of the sector which has the same arc and also the area of the triangle formed by the radii and chord; take the sum of these areas if the segment be greater than* 180°; *take the difference if less.*

Problem 16.—To find the area of a ring.

Rule.—*Take the difference between the areas of the two circles.*

Problem 17.—To find the area of an ellipse.

Rule.—*Multiply the product of the two diameters by* .7854.

Example.—What is the area of an ellipse when the minor and major axes are 6 and 10 ins. respectively?

10 × 6 × .7854 = 47.12 sq. ins.

Problem 18.—To find the circular area of a cylinder.

Rule.—*Multiply* 3.1416 *by the diameter and by the height.*

Example.—How many sq. ft. of lumber are required for the sides of a cylindrical tank 8 ft. in diameter and 12 ft. high; how many pieces 4″ × 12′ will be required?

cylindrical surface 3.1416 × 8 × 12 = 302 sq. ft.

circumference of tank = 3.1416 × 8 = 25.1 ft.

Number 4″ × 12′ pieces 25.1 ÷ $^4/_{12}$ = 25.1 × 3 = 75.3, say 76.

Problem 19.—To find the slant area of a cone.

Rule.—*Multiply* 3.1416 *by diameter of base and by one-half the slant height.*

Example.—A conical spire having a base 10 ft. diameter and altitude of 20 ft. is to be covered. Find area of surface to be covered.

In fig. 210, first find slant height, thus

$$\text{slant height} = \sqrt{5^2 + 20^2} = \sqrt{425} = 20.62 \text{ ft.}$$

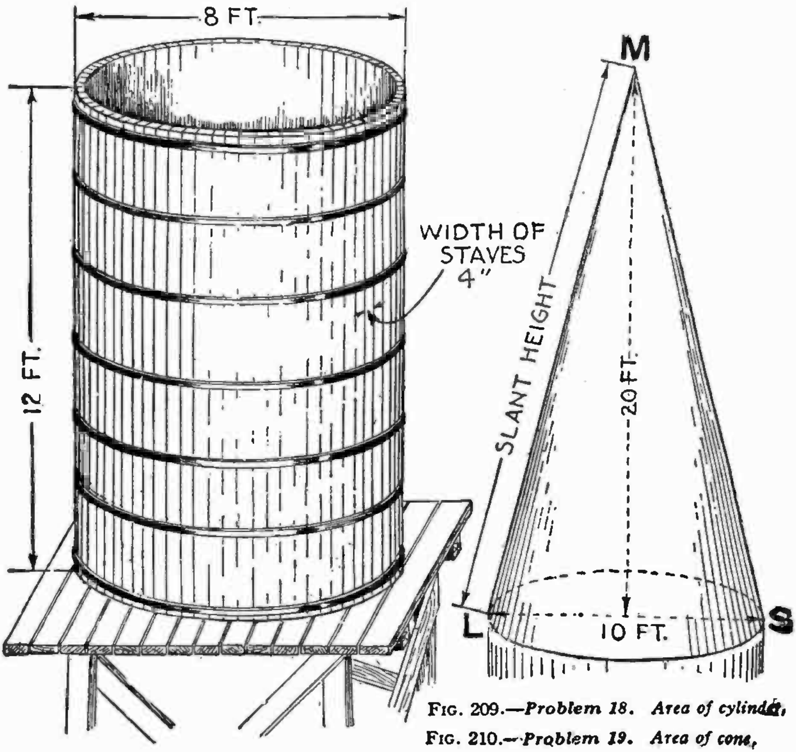


FIG. 209.—*Problem 18. Area of cylinder.*
FIG. 210.—*Problem 19. Area of cone.*

circumference of base = 3.1416 × 10 = 31.42 ft.
area of conical surface = 31.42 × ½ of 20.62 = 324 sq. ft.

Problem 20.—To find the (slant) area of the frustum of a cone.

Rule.—*Multiply half the slant height by the sum of the circumferences.*

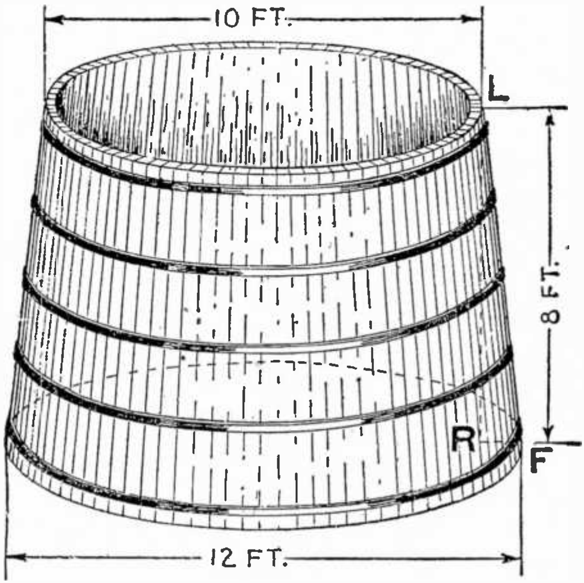


FIG. 211.—***Problem 20. Area of frustum of a cone.*** This is the shape of the ordinary wooden tank seen in wind mill towers. In the figure LR = height of tank. Since the difference between the two diameters is two feet, RF = 1 ft. Hence slant height or LF = $\sqrt{1^2 + 8^2}$ = 8.06.

Example.—A tank is 12 ft in diameter at the base, 10 ft at the top, and 8 ft. high. What is the area of the slant surface?

circumference 10 ft. circle = 3.1416 × 10 = 31.42 ft.
circumference 12 ft circle = 3.1416 × 12 = 37.7 ft
sum of circumferences = 69.1 ft.

$$\text{slant height} = \sqrt{1^2 + 8^2} = \sqrt{65} = 8.06$$

$$\text{slant surface} = \text{sum of circumferences} \times \tfrac{1}{2}\ \text{slant height}$$

$$= 69.1 \times \tfrac{1}{2} \text{ of } 8.06 = 278.5 \text{ sq. ft.}$$

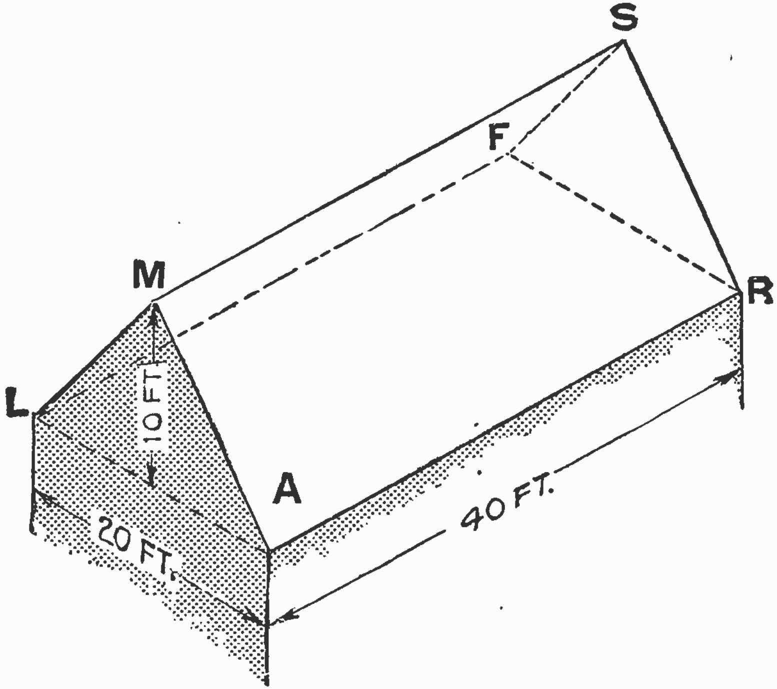


Fig. 212.—*Problem 21. Volume of rectangular wedge.*

3. Measurement of Solids

(volumes)

Problem 21.—*To find the volume of a rectangular wedge.*

Rule.—*Multiply length, breadth and one half height.*

Example.—Find the volume LARFMS of the barn shown in fig. 212.

$40 \times 20 \times \frac{1}{2}$ of $10 = 4000$ cu. ft.

Problem 22.—To find the volume of a cylinder.

Rule.—*Find the area of the base and multiply this by the length.*

Example.—What is the volume of a cylinder whose diameter is 4 ft. and length 7½ ft.?

4	.7854
4	16
16	47124
	7854
	12.5664 = area of base in sq. ft.
	7.5 = length in ft.
	628320
	879648
	Answer, 94.24800 cu. ft.

Problem 23.—To find the volume of a cone.

Rule.—*Multiply the area of the base by ⅓ the altitude and the product will be the volume.*

Example.—What is the volume of a cone whose diameter is 12 ft. and altitude 10 ft.?

Area of a circle $= .7854 \times$ sq. of the diameter

Area of base $= .7854 \times 12^2 = 113.1$ sq. ft.

volume $= 113.1 \times \frac{1}{3}$ of $10 = 377$ cu. ft.

Problem 24.—To find the volume of a sphere.

Rule.—*Multiply the cube of the diameter by* .5236.

Example.—Find the volume of a sphere whose diameter is 5 ft.

Cube of diameter	Diam.$^3 \times .5236$
5	.5236
5	125
25	26180
5	10472
$125 = 5^3$	5236
	65.4500 cu. ft.

Problem 25.—Find the volume of a segment of a sphere.

Rule 1.—*To three times the square of the radius of the segment's base, add the square of the height; then multiply this sum by the height and the product by* .5236.

Example.—How many cu. ins. in a spherical segment, having a base with diameter of 60 ins. and a height of 20 ins.?

Radius $= 60 \div 2 = 30$ ins.

Three times square of radius $= 3 \times 30 \times 30 = 2{,}700$

Add the square of the height

$$2{,}700 + (20 \times 20) = 3{,}100.$$

Multiply this by the height, $3100 \times 20 = 62{,}000$

Multiply by .5236 and obtain

$$62{,}000 \times .5236 = 32{,}463.2 \text{ cu. ins.}$$

Rule 2.—*From three times the diameter of the sphere subtract twice the height of the segment; multiply the remainder by the square of the height, and that product by* .5236 *for the volume.*

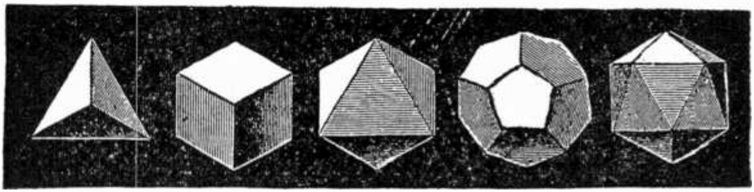
Example.—If the diameter of a sphere be 3 ft. 6 ins. what is the volume of a segment whose height is 1 ft. 3 ins.?

$$3 \times 3.5 = 10.5$$
$$2 \times 1.25 = 2.5$$
$$8$$
$$8 \times 1.25 \times 1.25 \times .5236 = 6.55 \text{ cu. ft.}$$

Problem 26.—To find the volume of an irregular solid.

Rule.—1. *Divide the irregular solid into different figures; the sum of their solidities will be the solidity required.* 2. *To find the* *solidity of a piece of wood or stone that is craggy or uneven, put it into a tub or cistern, and pour in as much water as will just cover it; then take it out and find the contents of that part of the vessel through which the water has descended and it will be the solidity required.*

Problem 27.—To find the surface and volume of any of the five regular solids, figs. 213 to 217.



FIGS. 213 to 217.—*The five regular solids:* Fig. 213, tetrahedron or solid, bounded by four equilateral triangles; fig. 214, hexahedron or cube, bounded by six squares; fig. 215, octahedron, bounded by eight equilateral triangles; fig. 216, dodecahedron, bounded by twelve pentagons; fig. 217, icosahedron, bounded by twenty equilateral triangles.

Rule (surface).—*Multiply the tabular area below, by the square of the edge of the solid.*

Rule (volume).—*Multiply the tabular contents below, by the cube of the given edge.*

Surfaces and Volumes of Regular Solids

Number of Sides	NAME	Area. Edge = 1	Contents. Edge = 1
4	......Tetrahedron........	1.7320	0.1178
6	......Hexahedron...... ..	6.0000	1.0000
8	......Octahedron.........	3.4641	0.4714
12	......Dodecahedron......	20.6458	7.6631
20	......Icosahedron..........	8.6603	2.1817

Mensuration of Surfaces and Volumes

(*Summary*)

Area of rectangle = length × breadth.
Area of triangle = base × ½ perpendicular height.
Diameter of circle = radius × 2.
Circumference of circle = diameter × 3.1416.
Area of circle = square of diameter × .7854.

$$\text{Area of sector of circle} = \frac{\text{area of circle} \times \text{number of degrees in arc.}}{360}$$

Area of surface of cylinder = circumference × length + area of two ends.

To find diameter of circle having given area: Divide the area by .7854, and extract the square root.

To find the volume of a cylinder: Multiply the area of the section in square inches by the length in inches = the volume in cubic inches. Cubic inches divided by 1728 = volume in cubic feet.

Surface of a sphere = square of diameter × 3.1416.
Solidity of a sphere = cube of diameter × .5236.
Side of an inscribed cube = radius of a sphere × 1.1547.

Area of the base of a pyramid or cone, whether round, square or triangular, multiplied by one-third of its height = the solidity

Diam. × .8862 = side of an equal square.
Diam. × .7071 = side of an inscribed square.
Radius × 6.2832 = circumference.
Circumference = 3.5449 × $\sqrt{\text{Area of circle.}}$
Diameter = 1.1283 × $\sqrt{\text{Area of circle}}$
Length of arc = No. of degrees × .017453 radius.
Degrees in arc whose length equals radius = 57°3'
Length of an arc of 1° = radius × .017453
" " " 1 Min. = radius × .0002909
" " " 1 Sec. = radius × .0000048

π = Proportion of circumference to diameter = 3.1415926.
π^2 = 9.8696044
$\sqrt{\pi}$ = 1.7724538
Log. = 0.49715
$1/\pi$ = 0.31831
$\pi/360$ = .008727
$360/\pi$ = 114.59

Lineal feet	×	.00019	= Miles.
" yards	×	.0006	= "
Square inches	×	.007	= Square feet.

Square feet	×	.111	=Square yards.
" yards	×	.0002067	=Acres.
Acres	×	4840	=Square yards.
Cubic inches	×	.00058	=Cubic feet.
" feet	×	.03704	=Cubic yards.
Circular inches	×	.00546	=Square feet.
Cyl. inches	×	.0004546	=Cubic feet.
" feet	×	.02909	= " yards.
Links	×	.22	=Yards.
"	×	.66	=Feet.
Feet	×	1.5	=Links.
Width in chains	×	8	=Acres per mile.
183346 circular in			=1 square foot.
2200 Cylindrical in			=1 cubic foot.
Cubic feet	×	7.48	=U. S. gallons.
" inches	×	.004329	=U. S. gallons.
U. S. gallons	×	.13368	=Cubic feet.
U. S. "	×	231	= " inches.
Cubic feet	×	.8036	=U. S. bushel.
" inches	×	.000466	= " " "
Cyl. feet of water	×	6	=U. S. gallons.
Lbs. Avoir	×	.009	=cwt. (112)
" "	×	.00045	=Tons (2240)
Cubic feet of water	×	62.5	=Lbs. Avoir.
" inch " "	×	.03617	= " "
Cyl. feet water	×	49.1	= " "
Cyl. inch water	×	.02842	= " "
13.44 U. S. gallons of water			=1 cwt.
268.8 U. S. " " "			=1 ton.
1.8 cubic feet of water			=1 cwt.
35.88 cubic feet of water			=1 ton.
Column of water, 12 inches high, and 1 inch in diameter			=.341 Lbs.
U. S. bushel	×	.0495	=Cubic yards.
" " "	×	1.2446	= " feet.
" " "	×	2150.42	=inches.

Problem 28.—To find the volume of a cylindrical ring.

Rule.—*To diameter of body of ring add inner diameter of ring; multiply sum by square of diameter of body, and product by* 2.4674.

Example.—What is volume of an anchor ring. diameter of metal, being 3 ins. and inner diameter of ring. 8 ins.?

$3+8\times 3^2=99$=product of sum of diameter and square of diameter of body of ring.

Then $99\times 2.4674=244.2726$ cu. ins.

Problem 29.—To find the sectional area of a pipe.

Example.—A pipe has an external diameter of 2 ins. and an internal diameter of 1¾ ins. Find its sectional area in sq. ins.

Rule.—*From the area of the greater circle subtract that of the lesser.*

$$\text{area of } 2 \ \odot = 2^2\times .7854 = 3.1416$$

$$\text{area of } 1\tfrac{3}{4} \ \odot = (1\tfrac{3}{4})^2\times .7854 = \underline{2.4053}$$

$$.7363 \text{ sq. ins.}$$

Properties of the Circle

(According to Kent)

Diameter of circle	× .88623	=side of equal square
Circumference of circle	× .28209	=side of equal square
Circumference of circle	×1.1284	=perimeter of equal square
Diameter of circle	× .7071	=side of inscribed square
Circumference of circle	× .22508	=side of inscribed square
Area of circle× .90031	÷diameter	=side of inscribed square
Area of circle	×1.2732	=area of circumscribed square
Area of circle	× .63662	=area of inscribed square
Side of square	×1.4142	=diam. of circumscribed circle
Side of square	×4.4428	=circum.
Side of square	×1.1284	=diam. of equal circle
Side of square	×3.5449	=circum. of equal circle
Perimeter of square	× .88623	=circum. of equal circle
Square inches	×1.2732	=circular inches

TEST QUESTIONS

1. *What is mensuration?*
2. *What are the three divisions of mensuration?*
3. *What is the length of the hypotenuse of a triangle in terms of the legs?*

4. *What is the number 3.1416 used for?*
5. *What is the value of* ¼π*?*
6. *What length of moulding strip is required for a circular window 5 ft. in diameter?*
7. *Give rule for finding the length of an arc of a circle.*
8. *If the circumference of a circle be 6 ft. what is the length of a 60° arc?*
9. *What is the area of a rectangle 5 ft. wide and 12 ft. long?*
10. *How many sq. ft. of sheet tin are required to cover a church steeple having four triangular sides, measuring 12 ft. (base) × 30 ft. (altitude)?*
11. *Give rule for finding the area of a trapezium.*
12. *What is the method of finding the area of any irregular polygon?*
13. *What is the area of an 8 sided polygon whose sides measure 4 ft.?*
14. *Draw a diagram showing the meaning of the much used .7854.*
15. *How much sheet tin is required to cover a 60° sector of a 10 ft. circular deck?*
16. *What is the area of an ellipse whose two diameters are 10 and 6 ins.?*
17. *What is the displacement per minute of a 5×6 engine running 600 r.p.m.?*
18. *Give rule for finding the slant area of the frustum of a cone.*

19. *What is the volume of a cone whose diameter is 12 ft. and altitude 10 ft.?*

20. *Find the volume of a sphere whose diameter is 5 ft.*

21. *How many cu. ins. in a spherical segment having a base whose diameter is 60 ins. and a height of 20 ins.?*

22. *Name five regular solids.*

23. *A pipe has an external diameter of 2 ins. and an internal diameter of 1¾ ins. Find its sectional area in sq. ins.*

CHAPTER 6

Solid Geometry

By definition solid geometry is that branch of geometry which includes *all three dimensions of space in its reasoning*.

It involves the consideration of

1. Planes;
2. Prisms;
3. Polyhedrons;
4. Solids of revolution.

   a. Cylinder;

   b. Cone;

   c. Sphere.

Planes.

A plane or plane surface is *one such that a straight line joining any two points in it will be wholly in the surface*. It has length and breadth but not thickness.

Properties of Planes.—1. *Any three points, one of which does not lie in the same straight line, determines one and only one plane.*

2. *A line and a point not on the line determine a plane.*

3. *Two parallel lines determine a plane.*

4. *Two intersecting lines determine a plane.*

5. *Two intersecting planes cut each other in a straight line.*

6. *From a point outside a plane only one perpendicular can be let fall to the plane.*

7. *From a point within a plane only one perpendicular to the plane can be erected.*

8. *A line parallel with a plane is one in which every point is at the same distance from the plane.*

Definitions.

Altitude.—The perpendicular distance between the vertex, or top, and the base.

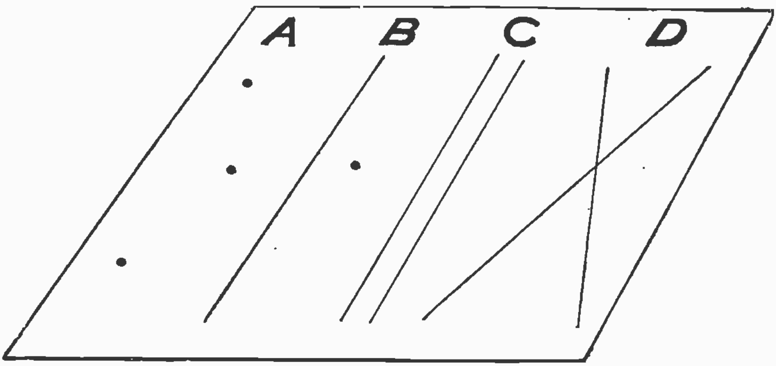


FIG. 218.—Conditions which determine a plane *A* three points; *B*, line and a point; *C*, parallel lines; *D*, intersecting lines. See accompanying text: *Properties of planes*.

Cone.—A solid generated by the revolution of a right triangle on one of its legs as an axis.

Cylinder.—A solid generated by the revolution of a rectangular plane on one of its edges as an axis.

Cylindrical Surface.—A curved surface generated by a moving straight line called the *generatrix* which moves always parallel with itself and constantly passes through a fixed curve called the *directrix*. The generatrix in any one position is called an *element* of the surface.

Dihedral Angle.—The angle formed by the intersection of two planes.

Directrix.—A line which so determines the motion of another line, or of a point that the latter will describe some surface or curve. See cylindrical surface.

Element.—The generatrix in any one position.

Frustum.—That which is left of a cone or pyramid after the upper part has been cut off by a plane *parallel* with the base.

Generatrix.—A line point or figure that generates another figure by its motion.

Lateral Area.—In a pyramid each side, or triangle, is a lateral face; the sum of their areas is the lateral area. The cylindrical surface of a cylinder is called the lateral surface; the conical surface of a cone is the lateral surface.

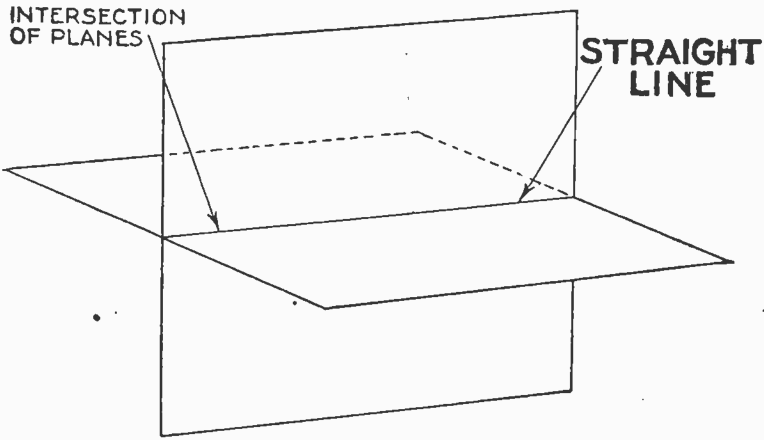


FIG. 219.—Two planes cut each other in a straight line.

Perimeter.—The length of the boundary line of any plane figure.

Plane Angle.—The angle formed by lines drawn in the two faces of a dihedral angle from the same point in the edge of the angle and perpendicular to the edge.

Polyhedron.—A solid bounded by plane faces especially more than four.

Prism.—A solid whose bases or ends are similar plane figures, and whose sides are parallelograms; prisms are called triangular, square, regular, etc., according as the bases are triangles, squares, regular polygons, etc.; and they are right or oblique according to whether the lateral edges are perpendicular or oblique to the bases.

Revolution.—Rotation about an axis.

Sphere.—A solid, every part of whose circumference is equidistant from a point within called the center.

Superpose.—To place one figure upon another so as to demonstrate whether or not they are equal.

Trihedral Angle.—A polyhedral angle having three faces.

Truncated.—Applied to a cone or pyramid whose vertex has been cut off by a plane, *either oblique to or parallel with* the base; and to a prism which has been cut off, usually oblique, to the base.

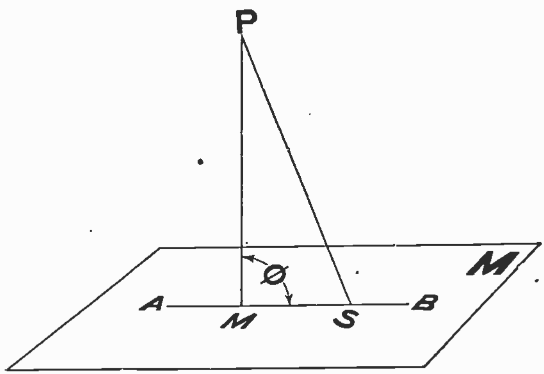


FIG. 220.—*Theorem 1.*

Theorem 1.—*A perpendicular let fall from a point to a plane is the shortest distance from the point to the plane.*

In fig. 220,

Given: PM, a perpendicular from P to plane M.

To Prove: PM shortest distance from P to plane M.

Proof.

If P be ⊥ to the plane it is ⊥ to any line in the plane as AB, passing through its foot M.

$$\therefore \angle \phi = 90°$$

Draw any other line from P to the plane as PS. Then PS is the hypotenuse of a right angled triangle.

$$\therefore PM < PS$$ **Q.E.D.**

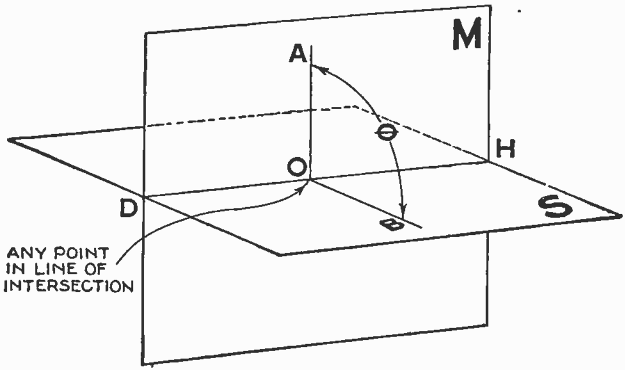


FIG. 221.—Plane angle AOB which measures the dihedral angle M-HD-S made by the intersecting planes.

Dihedral Angles.—When two planes cut each other, a *dihedral angle* is formed. To measure a dihedral angle take any point in the line of intersection and erect in each plane, a line perpendicular to the line of intersection as OA and OB in fig. 221. The angle AOB, then measures the dihedral angle ϕ formed by the intersecting planes M and S.

Theorem 2.—*When a plane bisects a dihedral angle every point in the plane is equidistant from the faces of the angle.*

In fig. 222,

Given: Plane L bisecting the dihedral angle M-HD-S.

To Prove: PA = PB.

Proof.

From any point P in L, drop ⊥s to planes **M** and **S.**

Since plane PAOB is ⊥ to planes M and S, it is ⊥ to the edge HD.

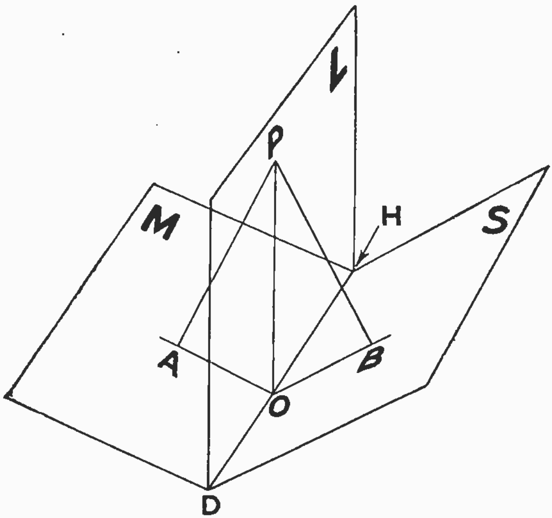


FIG. 222.—*Theorem 2.*

∴ HD ⊥ AO, BO and PO.

The ∡ AOP and BOP are the plane angles of the dihedral angles M-HD-L and S-HD-L.

Since the dihedral angles M-HD-L and S-HD-L are equal,

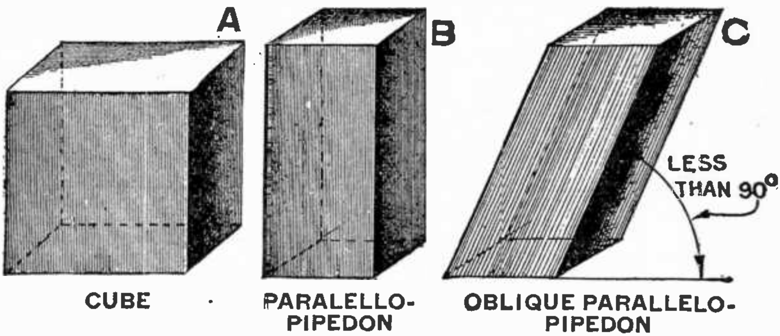
$$\angle AOP = \angle BOP$$

$$\therefore PA = PB$$ **Q.E.D.**

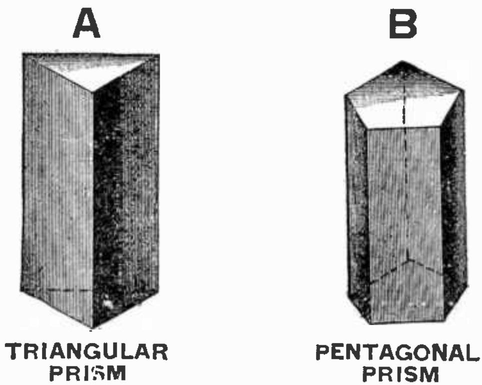
Prisms.

A prism is *a solid whose ends or bases are equal and parallel* polygons and whose sides are parallelograms.

The altitude of a prism is the perpendicular distance between its bases. The accompanying illustrations show the appearance of various prisms.



FIGS. 223 to 225.—*Various solids 1.*

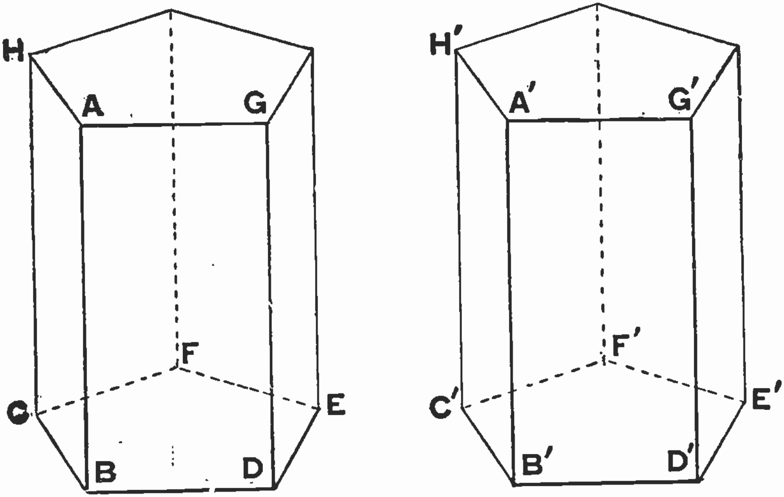


FIGS. 226 and 227.—*Various solids 2.*

Theorem 3.—*If two prisms have the three faces which bound any solid angle in each, equal, similar and similarly placed, the prisms are equal.*

In figs. 228 and 229,

Given: Prisms A – BDEFC and A′ – B′D′E′F′C′ with three faces which bound the solid angles B, B′, equal, similar and similarly placed.



Figs. 228 and 229.—*Theorem 3.*

To Prove: The prisms are equal.

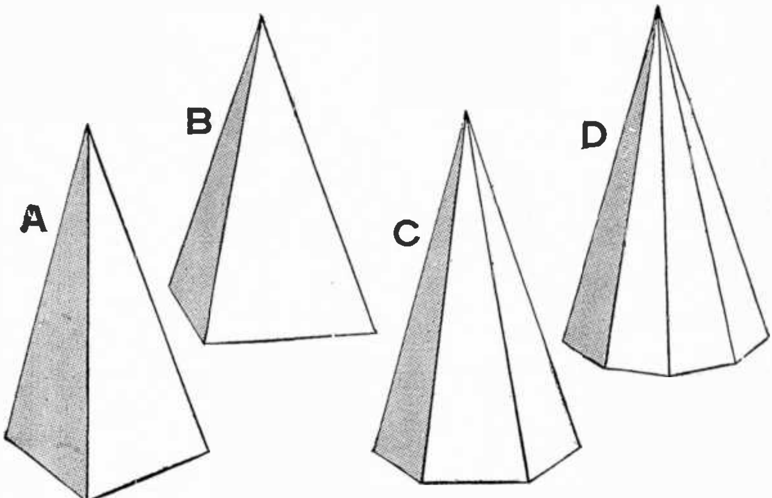
Superpose the two prisms so that B shall be on B′, and the bases coincident. Because the three angles which contain the angle B, are respectively equal to the three which contain the angle B′, their faces are equally inclined to each other.

If the plane angles which constitute one trihedral angle be equal, each to each, to the plane angles which constitute another, the faces of the two ∠s are equally inclined to each other.

∴ faces AD and AC coincide with A′D′ and A′C′

and because they are equal and similar, the lines AG, AH will coincide with A′G′, A′H′.

∴the upper bases being equal and similar to the lower bases and to each other, will coincide throughout and hence the



FIGS. 230 to 233.—Various pyramids. A, triangular; B, quadrilateral; C, hexagonal; D, octagonal.

other faces will coincide. Since the prisms coincide, they are therefore equal to each other.

Pyramids.

A pyramid is *a polyhedron having for its base a polygon and for its other faces three or more triangles which terminate in a common vertex.*

Theorem 4.—*If a plane cut a pyramid parallel with the base: 1, the edges and the altitude will be cut proportionally; 2, the intercepted section is similar to the base.*

In fig. 234,

Given: Pyramid H – ABC and plane cutting the pyramid at A′B′C′ parallel with the base.

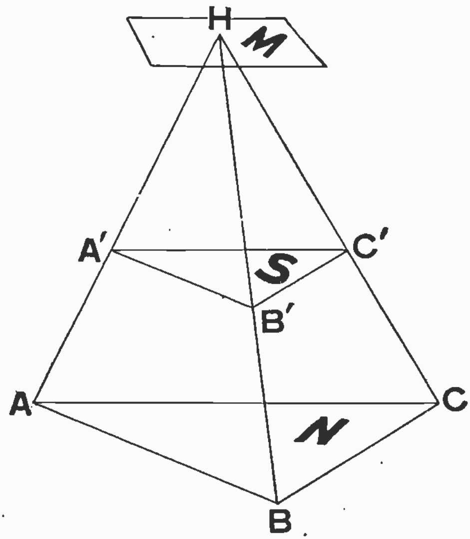


Fig. 234.—*Theorem 4.*

To Prove: 1. Edges and altitude are cut proportionally.

2. A′B′C′ is similar to ABC.

Proof.

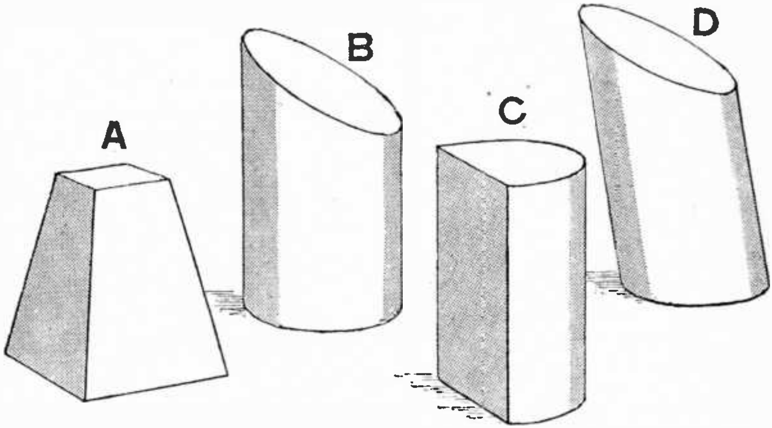
1. The edges and altitude will be cut proportionally, for suppose a plane M, to pass through H, parallel with the plane of

the base; then the edges and the altitude, being cut by three parallel planes, M, S and N, will be cut proportionally, because if two straight lines be cut by parallel planes they will be cut in the same ratio.

2. A′B′C′ will be similar to ABC because AB is ∥ with A′B′ and the triangles HAB and HA′B′ are similar.

$$\therefore HA : HA' = AB : A'B'$$

also $$HA : HA' = AC : A'C'$$



FIGS. 235 to 238.—Various solids. **A**, frustum of a pyramid; **B**, **C** and **D**, ungula.

If two proportions have one ratio in each the same, the remaining terms are in proportion.

$$\therefore AB : AC = A'B' : A'C'$$

Also $$\angle BAC = \angle B'A'C'$$

and the sides about the equal angle are proportional and similarly for any other angle of the figure.

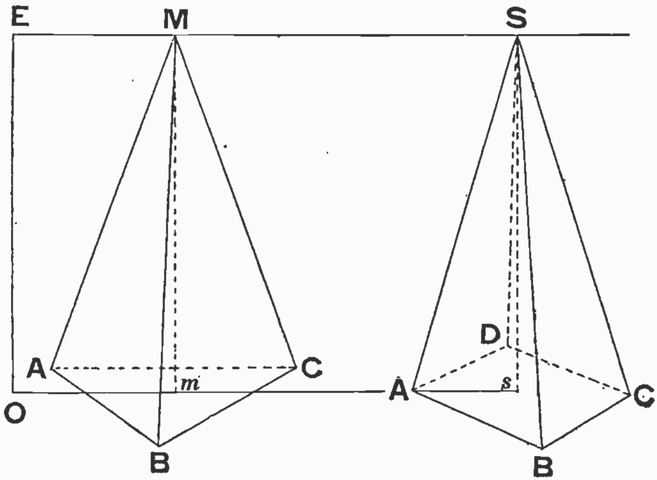
∴ ⧋ ABC and A′B′C′ are similar. **Q.E.D.**

Theorem 5.—*Pyramids on equal bases and of the same altitude are equal to each other.*

In figs. 239 and 240,

Given: *Pyramids M-ABC and S-ABCD having the common altitude OE and whose bases ABC and ABCD are equal in area.*

To Prove: Pyramids equal to each other.



FIGS. 239 and 240.—*Theorem 5.*

OE is the common altitude (=M*m*=S*s*).

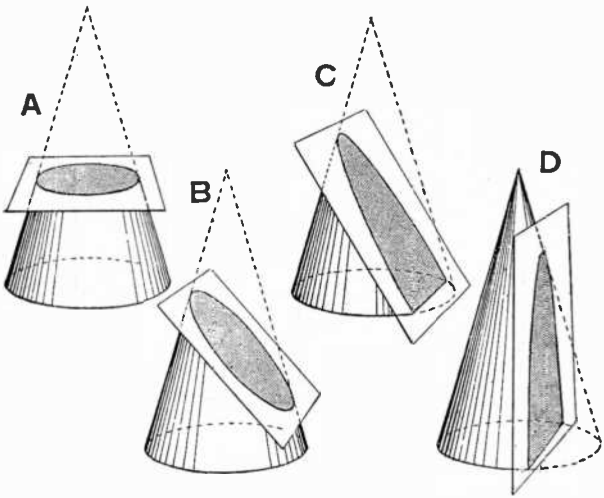
Proof.

Divide altitude OE into any number of equal portions, and through the points of division suppose planes to pass parallel with the bases. They will cut equal sections from the pyramids. On these sections inscribe in each of the pyramids a series of prisms. They will be equal to each other and this will be true whatever the number of divisions in OE. Hence when that number is infinitely increased the series of prisms coincide with the pyramids.

$$\therefore \text{MABC} = \text{SABCD} \qquad \textbf{Q.E.D.}$$

Polyhedrons.

A polyhedron is *a solid bounded by* **plane faces** *especially by more than four.*



FIGS. 241 to 244.—*Conic Sections:* A, circle; B, ellipse; C, parabola; D, hyperbola.

There are numerous classes of polyhedrons, known as:

Conjugate.—Two polyhedra so placed or formed that the faces of one correspond in position to the vertices of the other.

Convex.—A polyhedron in which not more than two faces pass through any edge and where there are no summits on different sides of the plane of a face.

Euler's Theorem.—The sum of the number of faces and vertices of any convex polyhedron exceeds the number of its edges by 2.

Regular Polyhedron.—One whose faces are equal and regular polygons and having all the angles equal and regular that meet at the vertices.

There are but five, if stellated polyhedra be excluded; the regular tetrahedron, hexahedron, octahedron, duodecahedron and icosahedron.

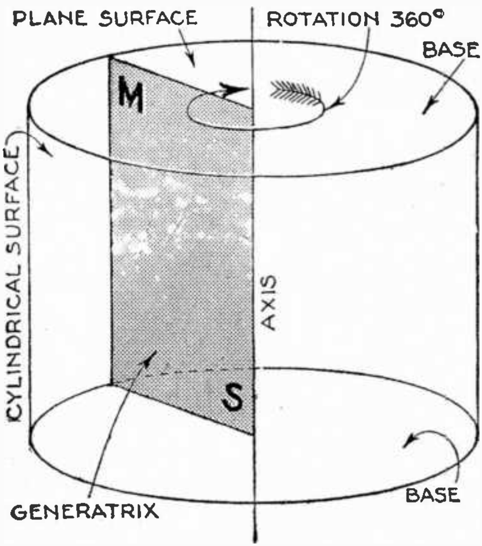


FIG. 245.—***Cylinder*** as generated by the rotation of a rectangular plane about one of its edges.

Semi-regular Polyhedron.—A polyhedron in which all the vertices are alike, but all the angles at a vertex are not equal.

Simple Polyhedron.—A polyhedron whose summits are distinct, none falling within an edge and with no two faces having a common point.

Solids of Revolution.

Under this heading is included such solids as:

1. Cylinder;

2. Cone;
3. Sphere,

being "generated" by the revolution of a plane, having for its boundary:

1. A rectangle;
2. A triangle;
3. A semi-circle,

respectively.

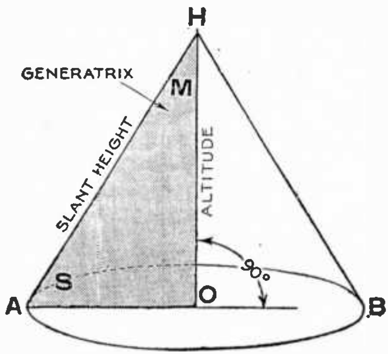


FIG. 246.—Right cone or cone of revolution.

Cylinders.—By definition a cylinder *is a solid bounded by a cylindrical surface and two parallel planes; a solid generated by the revolution of a rectangular plane on one of its edges as in fig.* 245.

Theorem 6.—*The cylindrical surface of a cylinder is equal to the surface of the sides of an inscribed polyhedron having an infinite number of sides.*

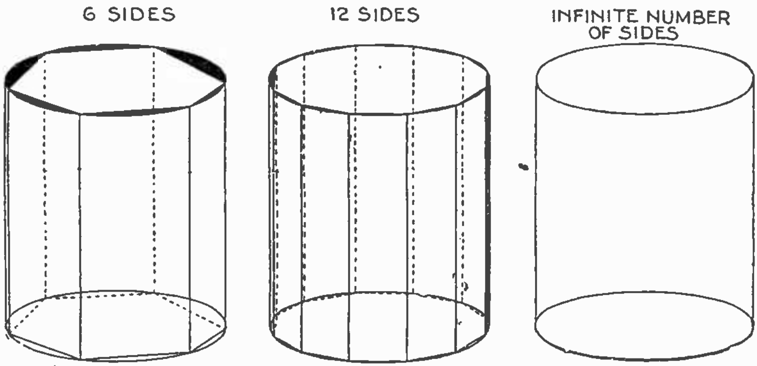
In fig. 247,

Given: A cylinder and inscribed polyhedron.

To Prove: Surfaces of cylinder and of a polyhedron of an infinite number of sides are equal.

Proof.

The area of the side surfaces of a polyhedron or cylindrical surface of a cylinder is equal to perimeter of the base × height.



FIGS. 247 to 249.—*Theorem 6.*

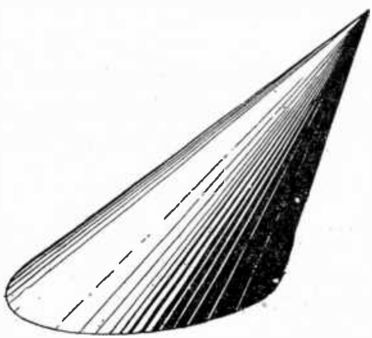


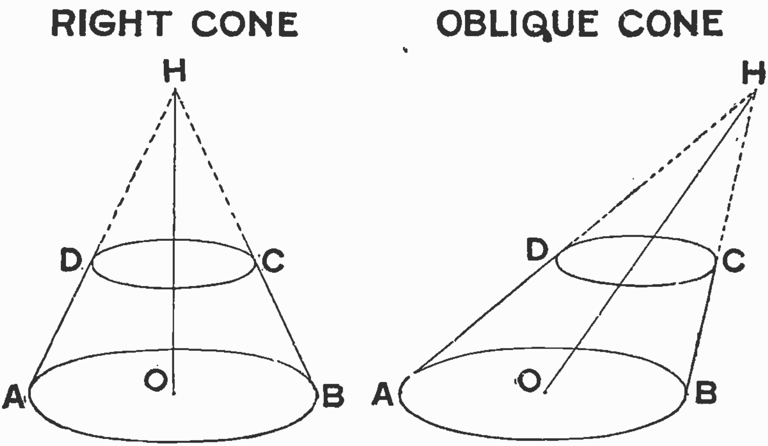
FIG. 250.—Oblique or scalene cone.

In fig. 247 perimeter of the hexagonal prism is considerably less than the circumference of the base of the cylinder.

If the number of sides of the prism be increased, the difference in its perimeter and the circumference of the base of the cylinder becomes less, as in fig. 248.

Accordingly for an infinite number of sides, perimeter of polyhedron = circumference of base of cylinder.

∴ Cylindrical surface of a cylinder = surface of the sides of an inscribed polyhedron having an infinite number of sides.

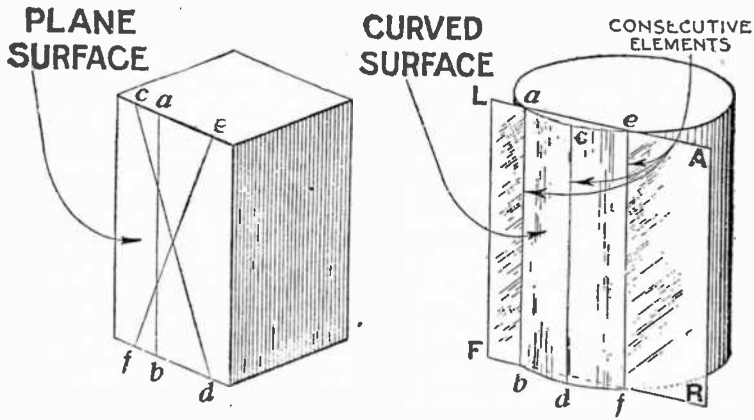


FIGS. 251 and 252.—Frustums of right and oblique cones, showing frustum ABCD and part cut away CDH.

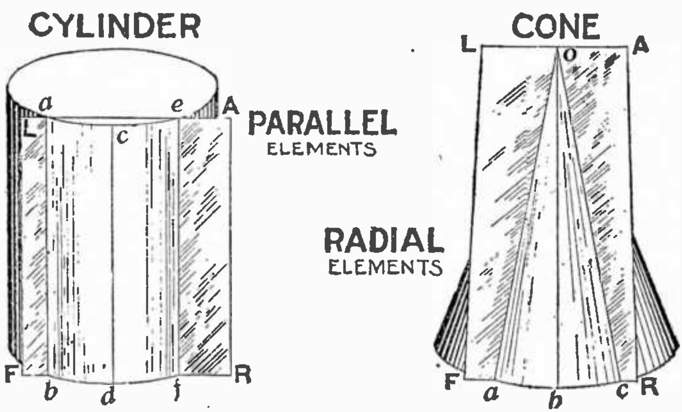
Cones.—A cone is *a solid figure that tapers uniformly from a circular base to a point.*

If the point lie in the perpendicular from the center of the base, the cone is a *right cone,* otherwise an *oblique cone.*

The slant height of a right cone is the length of one of its elements. A right cone is called a cone of revolution, since it may be generated by revolving a right triangle about one of its legs as an axis.



Figs. 253 and 254 —Plane and curved surfaces. Elements of a plane surface may be drawn in the surface in any direction as *ab*, *cd*, *ef*, as in fig. 253. In a curved surface no three *consecutive* elements lie in the same plane as in fig. 254.



Figs. 255 and 256. —Distinction between ***cylindrical*** and ***conical*** surfaces. Fig. 255, elements parallel: fig. 256, elements radial. Both surfaces being curved surfaces, no three consecutive elements lie in the same plane as indicated by plane LARF, passing through the first and third of the three consecutive elements *ab*, *cd*, and *ef*.

The frustum of a cone is that part which is left after cutting off the upper part, the part including the vertex, by a plane parallel with the base.

The lateral surface of a frustum of a cone is the portion of the lateral surface of the cone included between the bases of the frustum.

The slant height of a frustum of a cone of revolution is the portion of any element of the cone included between the bases.

TEST QUESTIONS

1. *Give eight properties of planes.*
2. *What are the four conditions which determine a plane?*
3. *What is the shortest distance from a point to a plane?*
4. *What is a dihedral angle?*
5. *What is the difference between a cube and a parallelopipedon?*
6. *What is a prism?*
7. *What is the proof of the equality of prisms?*
8. *Of what does the base and faces of a pyramid consist?*
9. *How do pyramids having equal bases and the same altitude compare? Give proof.*
10. *What is a polyhedron?*
11. *Make drawings of various polyhedra.*
12. *What is Euler's theorem?*
13. *Distinguish between a semi-regular and a regular polyhedron.*

14. *Name the various solids of revolution.*

15. *Prove that the cylindrical surface of a cylinder is equal to the surface of the sides of an inscribed polyhedron having an infinite number of sides.*

16. *How is a cone generated?*

17. *How do the elements of a cone differ from the elements of a cylinder?*

18. *What is a frustum of a cone?*

CHAPTER 7

Spherical Geometry

A sphere or spherical surface is *a closed surface every point of which is equally distant from a point within called the center* The term sphere is used to denote either the spherical surface or the solid bounded by the spherical surface. In most cases it relates to the surface.

Properties of a Sphere.

1. All points on the surface are equidistant from the center.

2. All radii of a sphere or of equal spheres are equal.

3. Spheres having equal radii or equal diameters are equal.

4. A plane passing through the center of a sphere cuts the sphere in a great circle.

5. All great circles of a sphere are equal.

6. Every great circle divides a sphere into two halves called hemispheres.

7. Any two great circles of a sphere bisect each other in the diameter which is the line of intersection of the planes of the two great circles.

8. One and only one great circle will pass through two points not diametrically opposite.

9. The distance between two points on a sphere is the length of the arc of the great circle joining these points.

10. The great circle arc joining two points on a sphere is the shortest distance between the points, on the surface of the sphere.

11. A sphere may be generated by the revolution of a semi-circle about its diameter.

12. The diameter of a small circle decreases the farther it is from the equator.

13. The arc intercepted by the great circle sides of a spherical angle on the great circle at the distance of a quadrant from the vertex of this angle measures the angle.

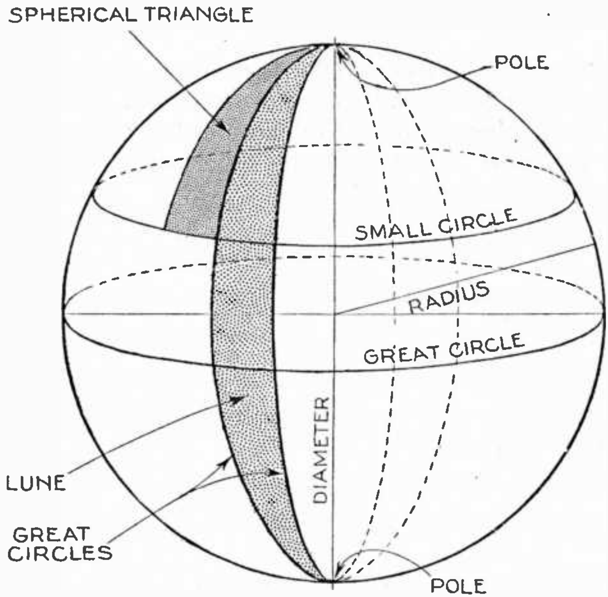


FIG. 257.—Representation of a sphere giving pictorial definitions and illustrating how *not* to draw a spherical triangle. See Fig. 301.

14. The two spherical angles at the extremities of a lune are equal.

15. The spherical angle at the extremity of a lune determines the lune.

16. A tangent to a sphere touches the surface of the sphere in only one point.

Definitions.

Convex Spherical Triangle.—One in which each angle is less than two right angles and each side is less than two quadrants.

Diametrical.—In a direction passing through the center; as if at opposite extremities of a diameter.

Great Circle.—Any section of a sphere made by a plane passing through the center. Two great circles always bisect each other.

Lune.—The portion of the surface of a sphere enclosed between the semi-circumferences of two great circles.

Normal.—A straight line perpendicular to a line or plane which is tangent to a curve and meeting it at the point of tangency.

Polar Triangle.—A spherical triangle whose vertices are the poles of the sides of another triangle.

Small Circle.—Every section of a sphere made by a plane not through the center.

Spherical Angle.—The intersection of two great circle arcs.

Spherical Cord.—The arc of a great circle which subtends an arc of a small circle on the surface of the sphere.

Spherical Conic.—The curve produced by the intersection of a cone of the second degree and a sphere whose center is at the vertex of the cone.

Spherical Harmonic Analysis.—A method by which a function is expressed as distributed over a spherical surface; used in a great variety of physical problems.

Spherical Polygon.—The space on the surface of the sphere bounded by arcs of any number of great circles.

Spherical Sector.—A solid formed by the revolution of a circular sector about any radius of the circle.

Spherical Segment.—The portion of a sphere cut off by a plane.

Spherical Triangle.—The space enclosed on the surface of a sphere by the arcs of three great circles, each arc being less than a semi-circumference. It may be called right angled, equilateral or isosceles.

Spheroid.—A body having nearly the form of a sphere.

Spherics.—The geometry and trigonometry of the sphere.

Spherical Zone.—The surface of a sphere between two parallel planes. Its altitude is the perpendicular distance between the planes.

Theorem 1.—*A plane ⊥ to the extremity of a radius of a sphere is tangent to the sphere.*

In fig. 258,

Given: Plane **MS** ⊥ to radius OD of sphere at D.

To Prove: **MS** tangent to sphere.

Proof.

Connect any point A in the plane with O and join AD.

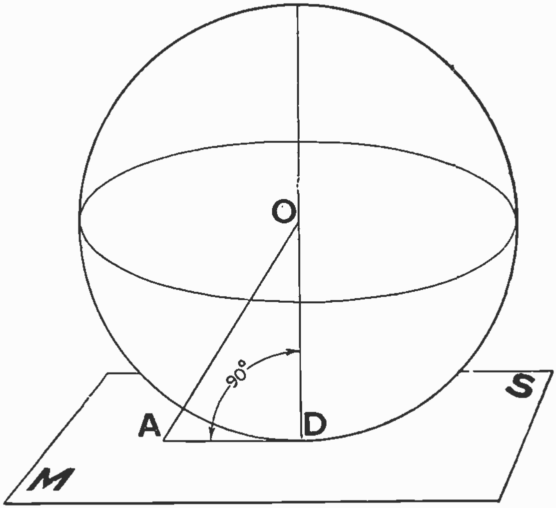


FIG 258.—*Theorem 1.*

In the triangle OAD

$$\angle ODA = 90°$$

$$\therefore OD < OA$$

Since OD is the shortest line from O to the plane, every other line from O to the plane is greater than the radius.

Accordingly, every point in the sphere except D lies without the plane.

∴ MS is tangent to the sphere at D. **Q.E.D.**

Theorem 2.—*Every section of a sphere made by a plane is a circle.*

In fig. 259,

Given: Plane MS cutting a sphere in *m.s.* and O center of sphere.

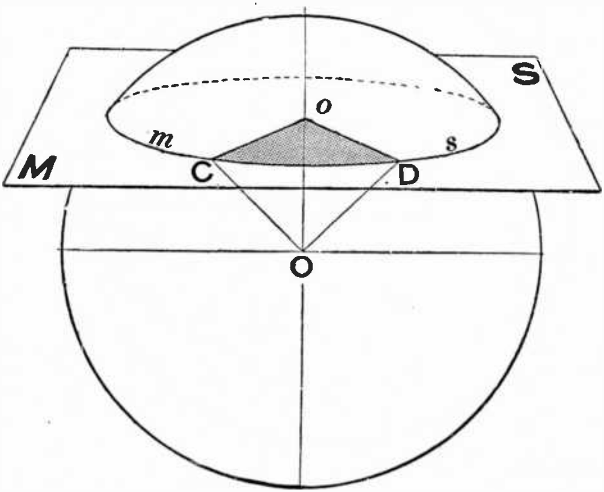


FIG. 259.—*Theorem 2.*

To Prove: The curve of intersection *m s* is a circle.

Proof.

Draw O *o* ⊥ to section cut by MS; join *o* to C and D any two points in the perimeter of the intercepted section; draw OC and OD.

In the rt. △s O *o* C and O *o* D

O *o* = O *o* and OC = OD

∴ △ O *o* C = △ O *o* D

∴ *o* C = *o* D

Since C and D are any two points on the perimeter of the intercepted curve *m s*, all points on the perimeter are equidistant from *o*.

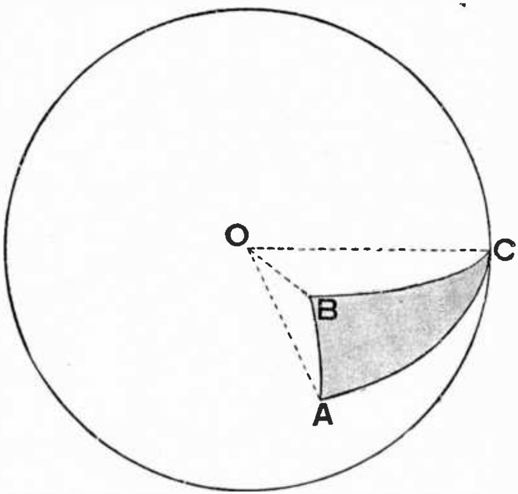


FIG. 260.—*Theorem 3.*

∴ the curve *m s* is a circle **Q.E.D.**

Theorem 3.—*The sum of any two sides of a spherical triangle is greater than the third side.*

In fig. 260,

Given: △ ABC.

To Prove: AB + BC > CA

Proof.

Join A,B,C, with center O.

$$\angle AOB + \angle BOC > \angle COA$$

These angles are measured by the sides of the triangle ABC.

$$\therefore AB + BC > CA$$ **Q.E.D.**

TEST QUESTIONS

1. *Give the properties of a sphere.*
2. *How is the distance between two points on a sphere measured?*
3. *What is a great circle?*
4. *How is a sphere generated?*
5. *What is a lune?*
6. *What is the difference between a great circle and a small circle?*
7. *Prove that a plane ⊥ to the extremity of a radius of a sphere is tangent to the sphere.*
8. *What is the shape of any section of a sphere cut by a plane?*
9. *Prove that the sum of any two sides of a spherical triangle is greater than the third side.*
10. *What does the spherical angle at the extremity of a lune determine?*

11. *Explain spherical harmonic analysis.*

12. *Prove that every section of a sphere made by a plane is a circle.*

CHAPTER 8

Descriptive Geometry

This branch of geometry is concerned with *the graphic methods of representing all geometrical magnitudes and the solution of problems relating to these magnitudes.*

It is based on *parallel projections to a plane by rays perpendicular to the plane.*

If the plane be horizontal the projection is called the *plan* of the figure, and if the plane be vertical, the *elevation*.

The drawings are so made as to present to the eye, situated at a particular point, the same appearance as the object itself, were it placed in the proper position.

The method of representation is known as *orthographic projection*. In this method *the point of sight is at an infinite distance in a perpendicular drawn to the plane of projection.*

In this method the point of sight being at an infinite distance, the projecting lines drawn from any points of an object of finite magnitude to this point, *will be parallel with each other and perpendicular to the plane of projection.*

In the projection shown in fig. 261, two planes are used, at right angles to each other.

1. The horizontal plane, **H**
2. The vertical plane, **V**

These planes form by their intersection four dihedral angles. The *first angle* is above the horizontal and in front of the vertical plane. The *second* is above the horizontal and behind the vertical. The *third* is below the horizontal and behind the vertical. The *fourth* is below the horizontal and in front of the vertical.

Problem 1.—*Given the two projections of a point to find the point.*

In fig. 261,

Let p and p' be the projections of the point in the horizontal and vertical planes respectively. At p and p' erect ⊥s, by drawing lines from p and p' ⊥ to the **H** and **V** planes respectively.

The intersection P of these lines is at the point required.

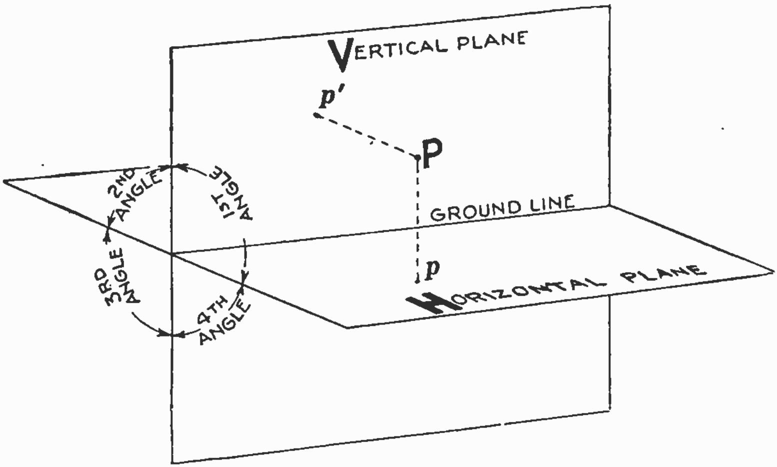


FIG. 261.—*Problem 1.*

Problem 2.—*Given the projections of the extremities of a line to find the line.*

This is simply an extension of problem 1.

In fig. 262,

Let ms and $m's'$ be the projections of the extremities of the line in the **H** and **V** planes respectively.

Project as in problem 1, to locate the extremities M and S, of the line. Join M and S, giving MS, the line required.

Problem 3.—*Given the traces of a line to find the line.*

In fig. 263,

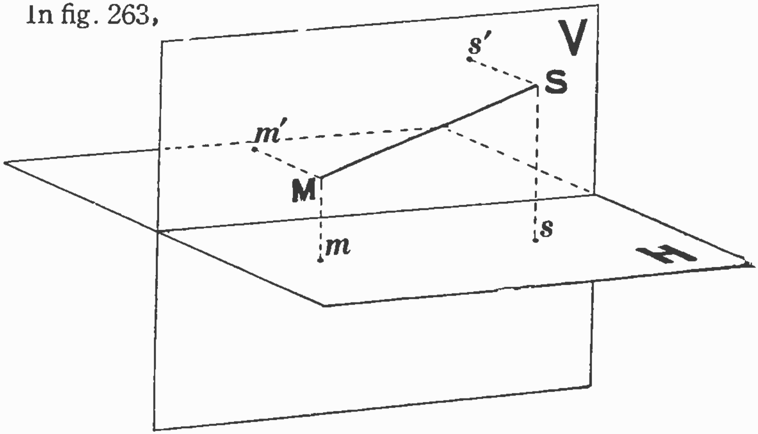


FIG. 262.—*Problem 2.*

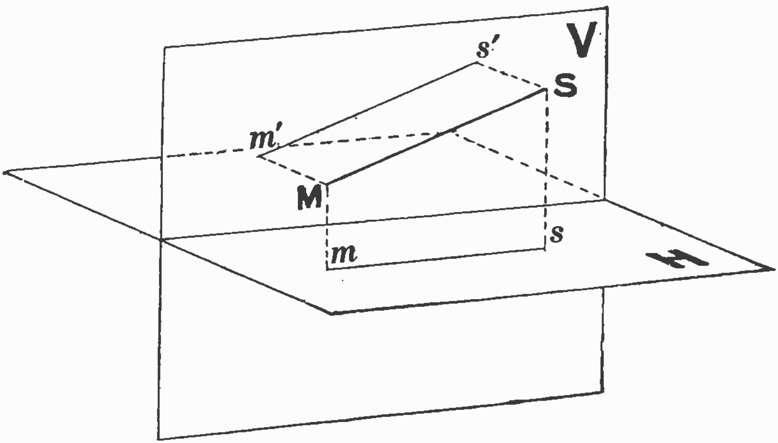


FIG. 263.—*Problem 3.*

Let *ms* and *m's'* be the **H** and **V** traces respectively of the line.

From the points *ms* and *m's'*, the extremities of the traces, project as in problem 2 to locate the extremities M and S of the line.

Join M and S, giving M S the required line.

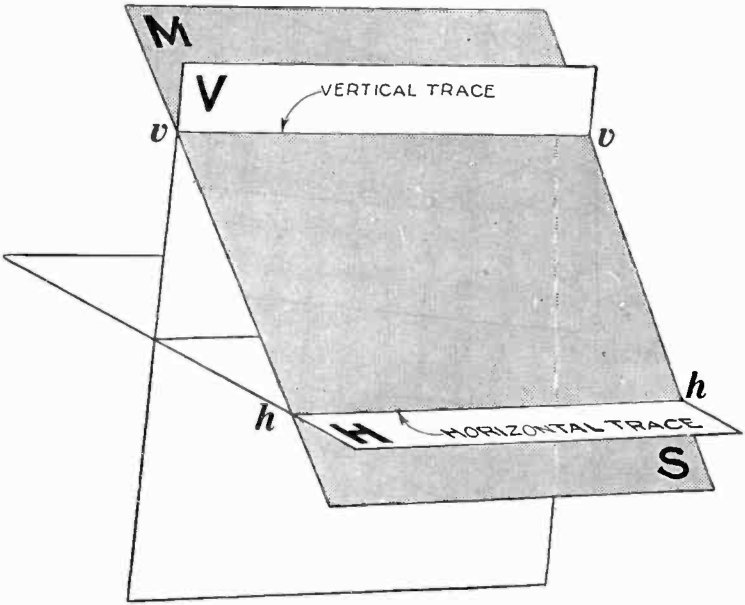


FIG. 264.—Intersection of a plane *MS*, with the *V* and *H* planes, showing horizontal trace *hh* and vertical trace *vv*.

Planes.—A plane is determined by its two *traces*, which are two lines cut on the *projection* planes.

If the plane be parallel with the axis its traces are parallel with the axis. Of these one may be at infinity; then the plane will cut one of the planes of projection at infinity and will be parallel with it. Thus a plane parallel with the plane has only one finite trace, that is the trace made by its intersection with the **V** plane.

If the plane pass through the axis both its traces coincide with the axis. This is the only case in which the representation of the plane by its two traces fails.

Problem 4.—*To locate the point in which a given straight line extended, pierces the planes of projection.*

In fig. 266,

Let *ms* and *m′s′* be the projections of the line.

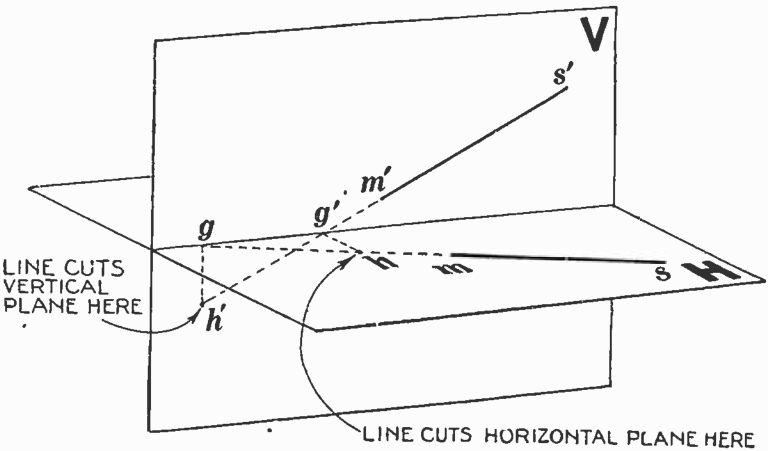


FIG. 265.—*Problem 4.*

Produce the vertical trace *m′s′* until it intersects the ground line at *g′*, and at *g′* erect a ⊥ to the ground line in the **H** plane and produce it until it intersects the horizontal trace extended at *h*, the required point of intersection with the **H** plane.

By a similar construction the intersection *h′* with the **V** plane is obtained.

Problem 5.—*To find the distance between two points* M *and* S *in space.*

In fig. 266,

Let *ms* and *m′s′* be the traces of a line joining the two points M, S.

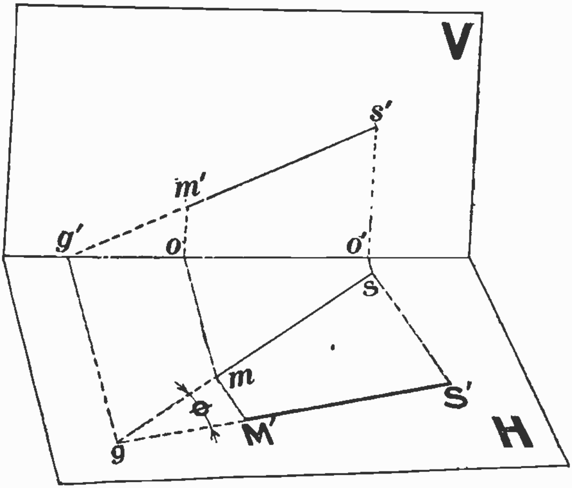


FIG. 266.—*Problems 4 and 5.*

From the ends of the vertical trace *m′s′* erect ⊥s to the ground line *m′o* and *s′o′*, cutting the ground line at o and o′ respectively. On the horizontal projection *ms*, erect ⊥s; *m*M′ = *om′* and *s*S′ = *o′s′*.

Join the points M′ and S′. The length of this line M′S′ is equal to the length of the line MS (not shown) in space.

In order not to complicate the drawing, the line MS in space is not shown.

If the construction be accurate M′S′ extended will cut *ms* extended at *g* the point of intersection with the **H** plane, as determined by projecting over from *g′* the intersection of the vertical trace with the axis.

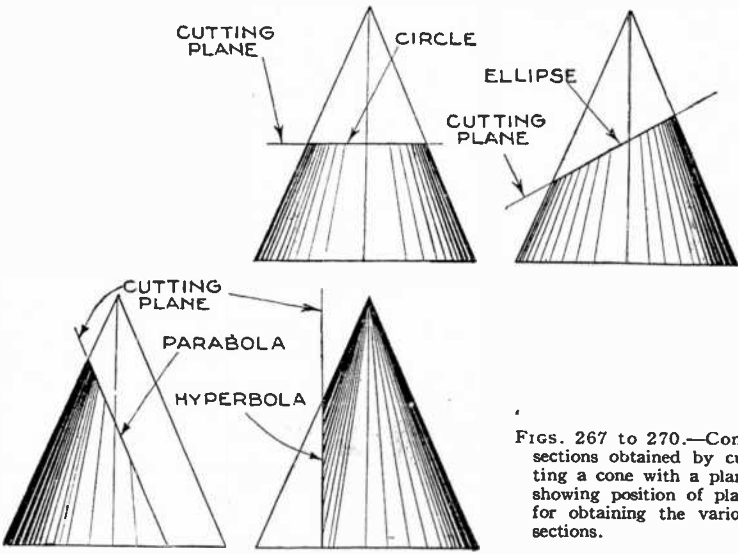
The angle ϕ is the angle made by **MS** with the **H** plane.

Conic Sections.—By definition a conic section is ***a section cut by a plane passing through a cone.***

These sections are bounded by well known curves, and the latter may be any of the following depending upon the inclination or position of the plane with the axis of the cone.

1. Triangle

Plane passes through apex of cone



FIGS. 267 to 270.—Conic sections obtained by cutting a cone with a plane; showing position of plane for obtaining the various sections.

2. Circle

Plane parallel with base of cone

3. Ellipse

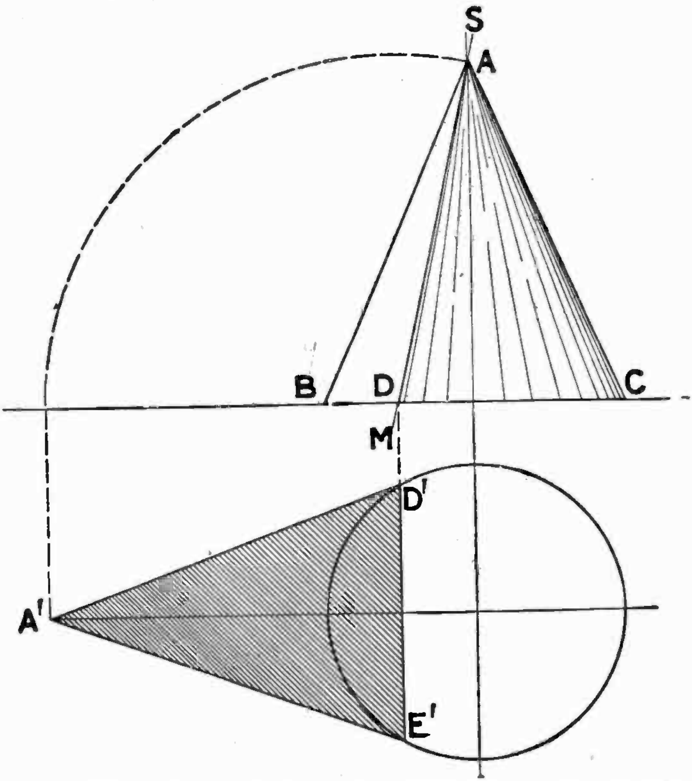
Plane inclined to axis of cone

4. Parabola

Plane parallel with one element of cone

5. Hyperbola

Plane parallel with axis of cone



FIGS. 271 and 272.—Surface cut by plane passing through apex of a cone—*triangle.*

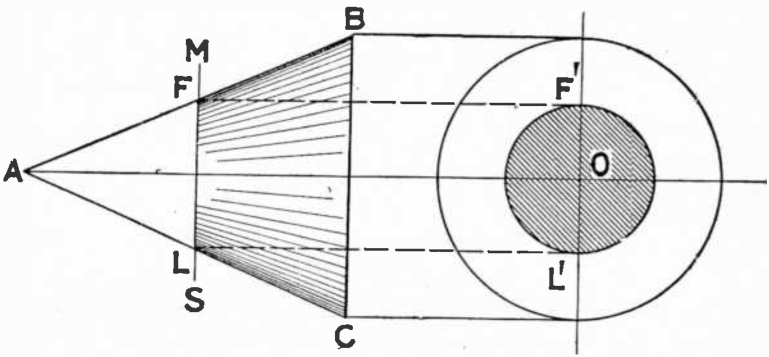
These sections appear as straight lines in elevation, while in plan they appear (with exception of the triangle) as curves.

Problem 12.—Find curve cut by a plane passing through apex of cone as in figs. 271 and 272.

Let ABC, be elevation of cone and MS, cutting plane passing through apex. Project point D, down to plan parallel with axis cutting base of cone at D′ and E′, obtaining line D′E′, base of developed surface.

With D, as center and radius DA, equal to element of cone swing A, around to base line and project down to A′. Join A′ with D′ and E′. Then, A′D′E′ is the developed surface or triangle cut by plane MS, with cone.

Problem 13.—Find surface cut by a plane passing through a cone parallel with its base as in figs. 273 and 274.



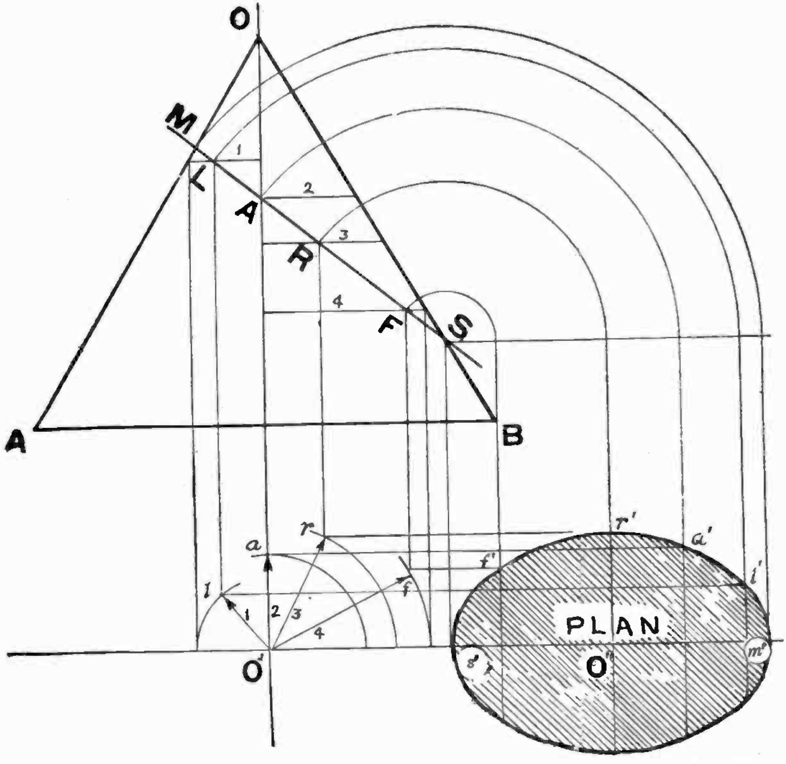
Figs. 273 and 274.—Surface cut by plane passing through a cone parallel with its base—*circle.*

This may be found by simply projecting over to the plan. Where MS, cuts the element AB, as at F, project over to the axis of the plan and obtain point F′.

Similarly point L′ may be found. These points are equidistant from the center O, hence with radius=OF′=OL′, describe a circle which is the curve cut by plane MS, when parallel with base of cone.

Problem 14.—Find the curve cut by a plane passing through a cone inclined to its axis as in fig. 275.

In fig. 275, let the plane cut the elements OA and OB, of the curve at M and S, respectively. Project S, down to *s′*, in plan giving one point on the curve. With S, as center swing M, around and project down to *m′*, in plan giving a second point on the curve, *m′s′*, being the major axis of the curve.



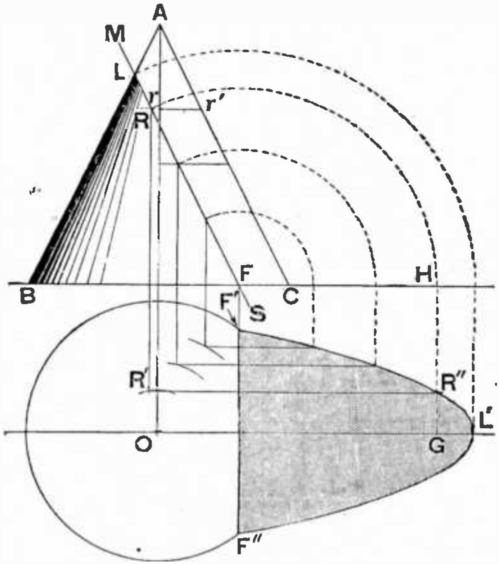
FIGS. 275 and 276.—Surface cut by plane passing through a cone inclined to its axis—*ellipse.*

To find the minor axis of curve, bisect MS, at R, and swing R, around to horizontal with S, as center and project down to plan. Through R, draw radius 3, and describe arc with radius 3, about O′ as center. Where this cuts projection of R at *r*, project over to plan, intersecting the vertical plan projection of R at *r′*. O″*r′*, is half the minor axis.

To find the projection of any other point as L, or F, proceed in similar manner as indicated, obtaining *l′*, or *f′*. The curve joining these points and symmetrical points below the major axis is an ellipse.

Problem 15.—Find the curve cut by a plane passing through a cone parallel with an element of the cone as in figs. 277 and 278.

Let the plane MS, cut element AB, at L, and base at F. Project F, down to plan cutting base at F′ and F″, which are two points in the curve.



FIGS. 277 and 278.—Surface cut by plane passing through a cone parallel with an element of the cone—*parabola*.

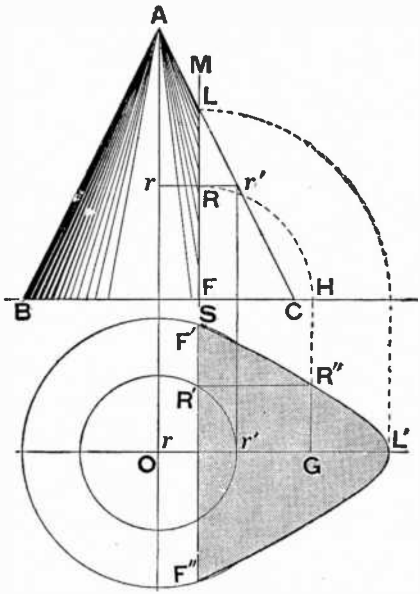
With F, as center and radius LF, swing point L, around and project down to axis of plan, obtaining point L′ in the curve.

Now any other point as R, may be obtained as follows: swing R around with F as center and project down to plan with line HG.

Describe an arc in plan with a radius (=radius *rr′* of cone at elevation of point R), and where such cuts the projection of R at R′; project R′ over to line HG, and obtain point R″, which is a point in the curve.

Other points may be obtained in a similar manner. The curve is traced through points F′,R″,L′, etc., and similar points on the other side of the axis, ending at F″. Such curve is called a *parabola*.

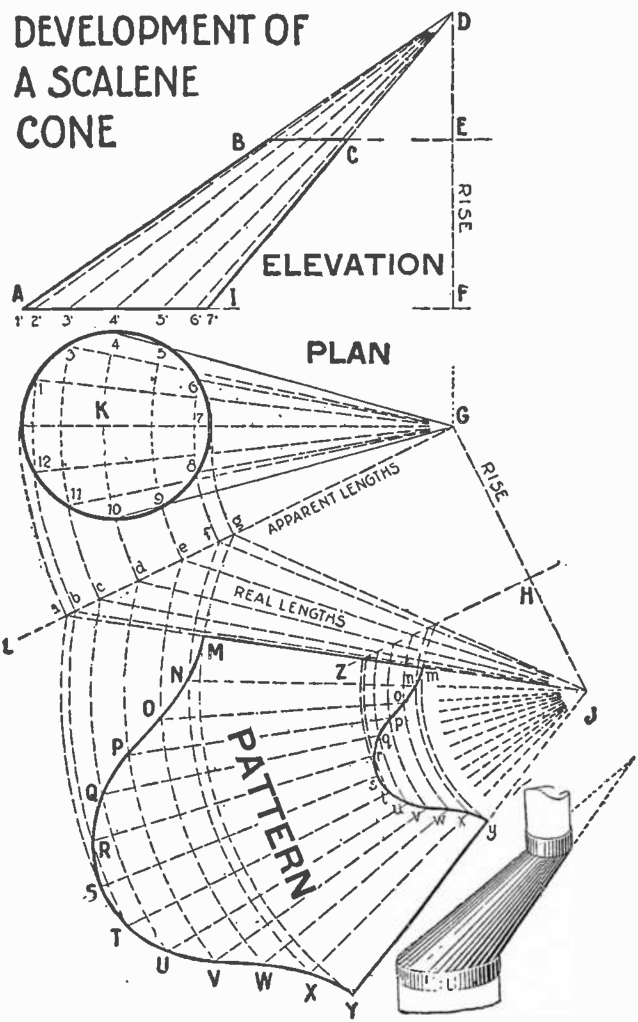
Problem 16.—Find the curve cut by a plane passing through a cone parallel with the perpendicular axis of the cone, as in figs. 279 and 280.



FIGS. 279 and 280.—Surface cut by a plane passing through a cone parallel with the axis of the cone—*hyperbola*.

Let plane MS, cut element AC, at L, and base at F. Project F, down to plan cutting base at F′ and F″, which are two points in the curve. With F, as center and radius FL, swing point L, around and project down to axis of plan obtaining point L′, in the curve.

Now any other point as R, may be obtained as follows: Swing R, around with F, as center and project down to plan with line HG. Describe a circle



FIGS. 281 to 284.—*Problem 40.* Scalene cone and development of its patterns.

in plan with radius rr' (=radius of cone at elevation of point R) and where this circle cuts the projection of R at R′, project over to line HG, and obtain point R″, which is a point in the curve. Other points may be obtained in a similar manner.

The curve is traced through points F′,R″,L′, and similar points on the the other side of the axis ending at F″. Such curve is called a *hyperbola*.

It will be noted that problems 15 and 16 are virtually worked out in the same way. In fact the text of one will apply to the other.

TEST QUESTIONS

1. *What is descriptive geometry?*
2. *How many projection planes are used?*
3. *Find a point whose two projections are given.*
4. *What are the traces of a line?*
5. *How is a plane determined?*
6. *Locate the point in which a given straight line extended, pierces the plane of projection.*
7. *What is a conic section?*
8. *Find the curve cut by a plane passing through the apex of a cone and inclined to the axis.*
9. *What curve is cut by a plane passing through a cone parallel with an element of the cone?*
10. *What is the curve cut by a plane intersecting a scalene cone parallel with the base?*
11. *Draw the development of a scalene cone.*
12. *Find the curve cut by a plane intersecting a scalene cone oblique to the base.*

CHAPTER 9

Analytical Geometry

Analytical geometry may be defined as that branch of geometry in which *position is indicated by algebraic symbols and the reasoning conducted by analytic operations.*

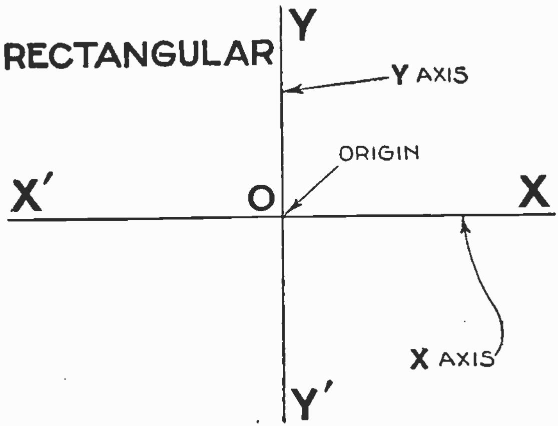


FIG. 285.—Rectangular axes.

Axes and Co-ordinates.—It is assumed that the points, lines and figures here considered *lie in one and the same plane* and accordingly the plane *need not in any way be referred to.*

The position of any point in this plane *is fixed by its distances from two axes.* The axes may be rectangular as in fig. 285, or oblique as in fig. 286.

Rectangular axes are generally used; however, sometimes it is more convenient to use oblique axes.

The position of a point is fixed by two co-ordinates, as shown in fig. 287, known as

1. Abscissa;
2. Ordinate.

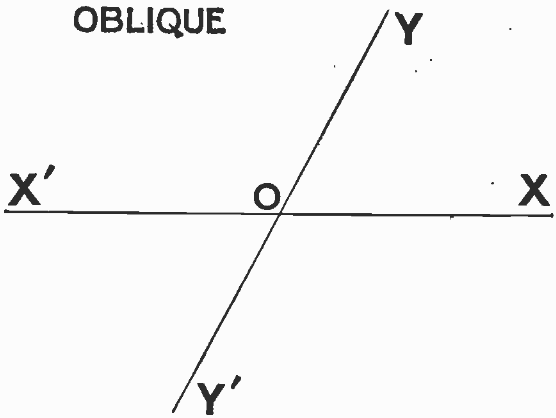


FIG. 286.—Oblique axes.

An abscissa is *the distance of any point from the axis of ordinates* (the Y axis) *measured on a line parallel with the axis of abscissae*, that is, the X axis.

An ordinate is *the distance of any point from the axis of abscissae* (the X axis) *measured on a line parallel with the axis of ordinates*, that is, the Y axis.

In fig. 287, the abscissa of the point P is PH, and the ordinate PD. To distinguish abscissae and ordinates they are represented by the symbols x and y respectively as indicated in the figure.

A point is referred to as the point *a,b*, when its co-ordinates are

$x=a$ (abscissa)

$y=b$ (ordinate)

Note that the abscissa is always mentioned first.

Since the two intersecting axes divide the plane into four quadrants it is necessary to distinguish the quadrant containing a given point. The quadrant is indicated by the signs (positive or negative) given to the co-ordinates as in fig. 288. For instance a point $x=a$ and $y=b$ is in the first quadrant

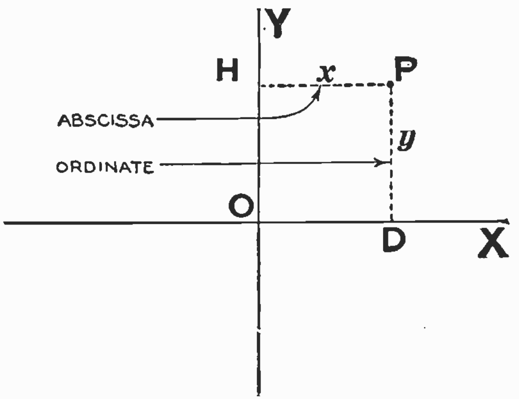


Fig. 287.—Rectangular axes illustrating co-ordinates.

(as shown in fig. 288). If the point were in some other quadrant its position would be expressed as follows:

2nd quad $x=-a;\ y=+b$
3rd quad $x=-a;\ y=-b$
4th quad $x=+a;\ y=-b$

When no sign is given, the value of x or y is $+$.

Equations and Loci.—If x,y in fig. 287 be the point P and assuming that $x=0$ (zero), then P lies on the Y axis at a distance above the origin 0, corresponding to the value given to y.

The equation $x=o$ will be satisfied for all values given to y and accordingly this is the equation of the Y axis.

Similarly $y=o$ indicates that the point lies on the X axis, and accordingly this is the equation of the X axis.

The equation $x=o$, $y=a$, is the equation of a point P on the y axis, as in fig. 289.

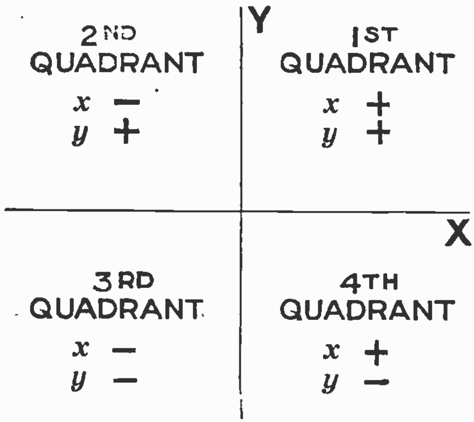


FIG. 288.—Co-ordinate signs for the form quadrant.

An equation $x=a$, where a is a constant, as in fig. 290, expresses that P lies on a parallel with the Y axis through a point m on the axis of X such that $om=a$.

Every line parallel with the Y axis has an equation of this form.

Similarly, every line parallel with the X axis has an equation of the form $y=b$, where b, is some definite constant. These are simple cases of the fact that a single equation, in the current co-ordinates of a variable point (x, y) imposes one limitation on the freedom of that point to vary.

The co-ordinates of a point taken at random in the plane will, as a rule, not satisfy the equation, but infinitely many points, and in most cases infinitely many real ones, have co-ordinates which do satisfy it, and these points are exactly those which lie upon some locus of one dimension, a straight line or more frequently a curve, which is said to be represented by the equation.

Problems.

Problem 1.—*To find the equation of a straight line parallel with one of the axes.*

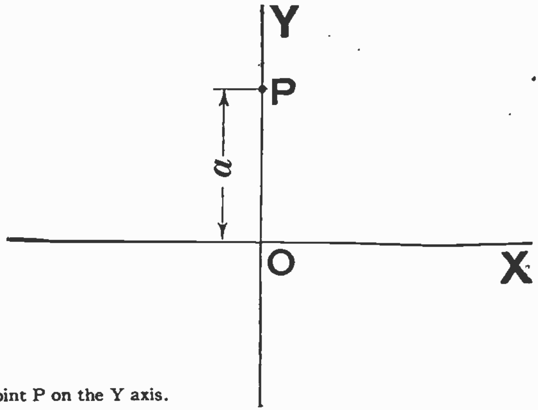


FIG. 289.—Point P on the Y axis.

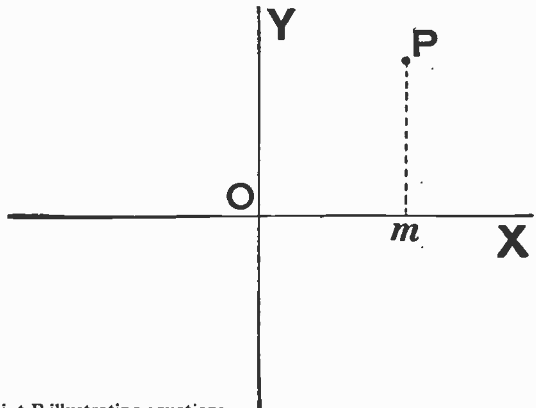


FIG. 290.—Point P illustrating equations.

In fig. 291,

LP is the straight line ‖ with the **X** axis at a distance OL $= b$.

Let x,y be the co-ordinates of any point P on the line. Then the ordinate NP is equal to OL. Hence $y = b$ is the equation of the line.

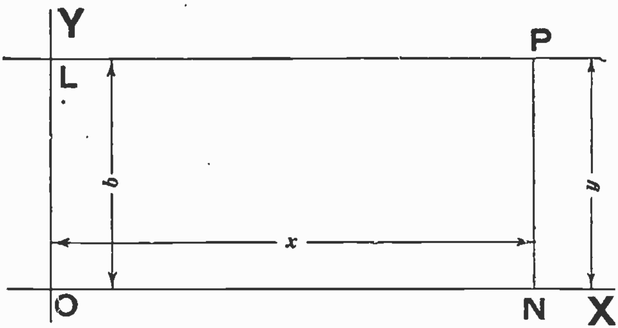


FIG. 291.—*Problem 1.*

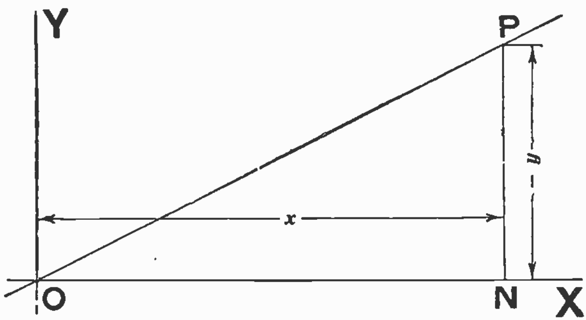


FIG. 292.—*Problem 2.*

Problem 2.—*To find the equation of a straight line which passes through the origin.*

In fig. 292,

OP is the straight line. Let tan NOP $=m$, and x,y the co-ordinates of any point P on the line, then

$$NP = \tan NOP \times ON$$

$$\therefore y = m\,x \text{ is the required equation.}$$

Problem 3.—*To find the equation of any straight line.*

In fig. 293,

Let LMP be the straight line meeting the axes in the points L, M.

Let OM $=c$, and let tan OLM $=m$.

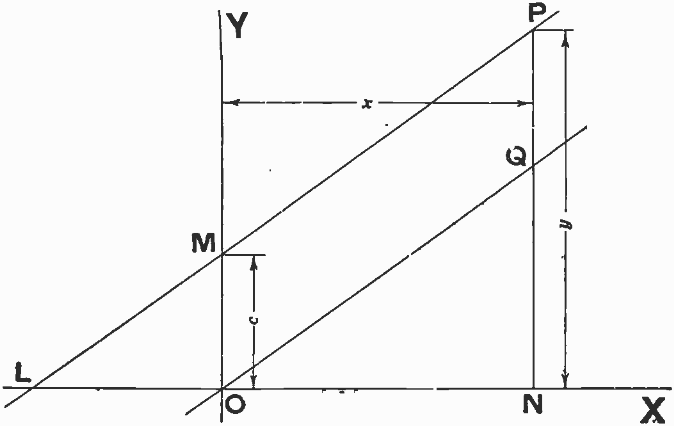


FIG. 293.—*Problem 3.*

Let x,y be the co-ordinates of any point P on the line.

Draw PN parallel with the Y axis, and OQ parallel with the line LMP, as shown.

Then NP = NQ + QP = ON tan NOQ + OM.

But

NP $=y$, ON $=x$, OM $=c$, and tan NOQ = tan OLM $=m$

$$\therefore y = mx + c$$

which is the equation of any straight line.

Problem 4.—*To find the equation of a straight line in terms of the intercepts which it makes on the axes.*

In fig. 294,

Let A, B be the points where the straight line cuts the axes, and let OA $= a$, and OB $= b$.

Let the co-ordinates of any point P, on the line be $x\ y$.

Draw PN parallel with the axis of y, and join OP.

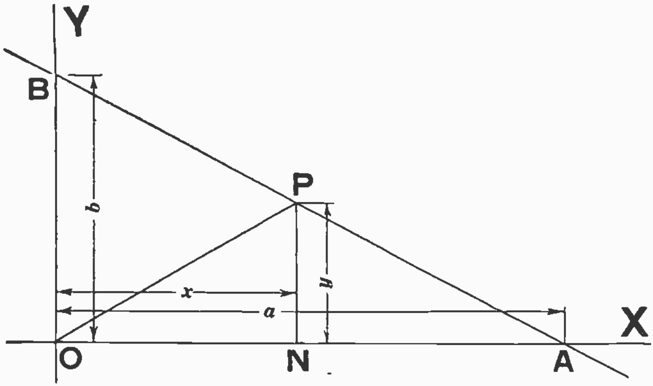


FIG. 294.—*Problem 4.*

Then $\triangle \text{APO} + \triangle \text{PBO} = \triangle \text{ABO}$;

$$ay + bx = ab,$$

or

$$\frac{x}{a} + \frac{y}{b} = 1.$$

This equation may be written in the form

$$lx + my = 1.$$

where l and m are the reciprocals of the intercepts on the axes.

Problem 5.—*To find the equation of a circle referred to any rectangular axes.*

In fig. 295,

Let C be the centre of the circle, and P any point on its circumference.

Let d,e, be the co-ordinates of C; x,y the co-ordinates of P; and let a, be the radius of the circle.

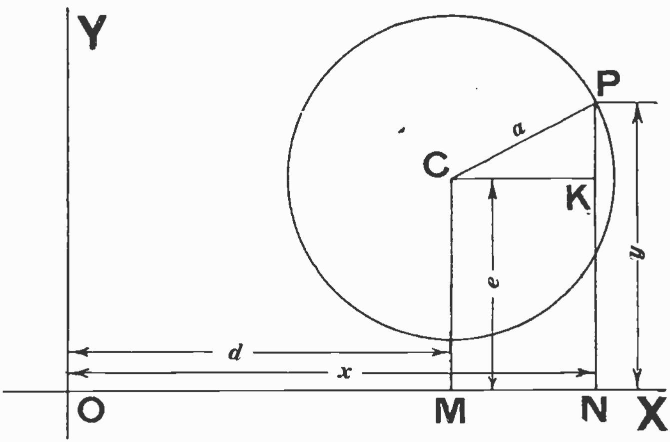


FIG. 295.—*Problem 5.*

Draw CM, and PN, parallel with OY, and CK parallel with OX.

Then $$(CK)^2+(KP)^2=(CP)^2$$

But $$CK=x-d, \text{ and } KP=y-e;$$

$$\therefore (x-d)^2+(y-e)^2=a^2$$

is the required equation.

If the centre of the circle be the origin, *d* and *e* will both be zero, and the equation of the circle will be

$$x^2+y^2=a^2$$

Problem 6.—*To find the equation of a parabola.*

In fig. 296,

Let S be the focus, and YY′ the directrix.

Draw SO perpendicular to YY′ and let OS = 2a.

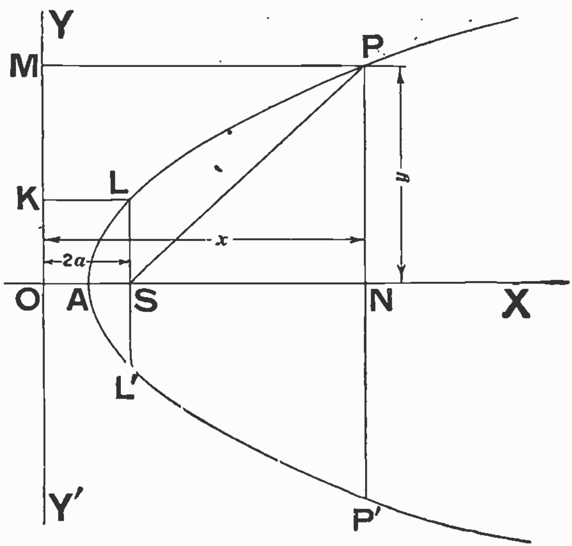


FIG. 296.—*Problem 6.*

Take OS for the axis of x, and OY for the axis of y.

Let P be any point on the curve, and let the co-ordinates of P be x,y.

Draw PN, PM, perpendicular to the axes, as shown, and join SP.

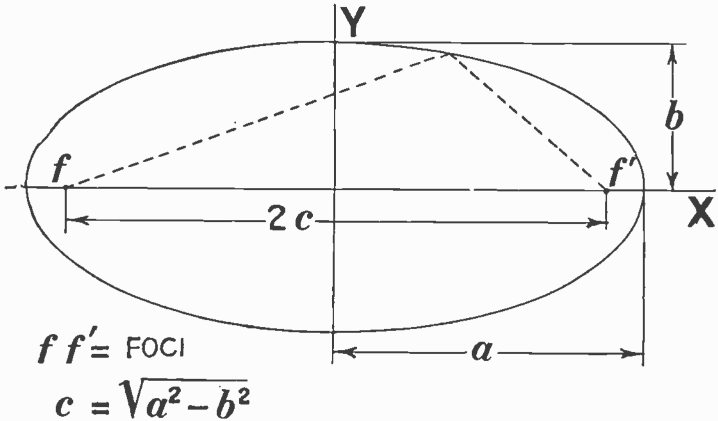


FIG. 297.—*The ellipse.* A curve *generated by a point moving so that the sum of its distances from two fixed points is always constant.* The fixed points are called the *foci.* Equation of ellipse with center at origin:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are half the major and minor axes.

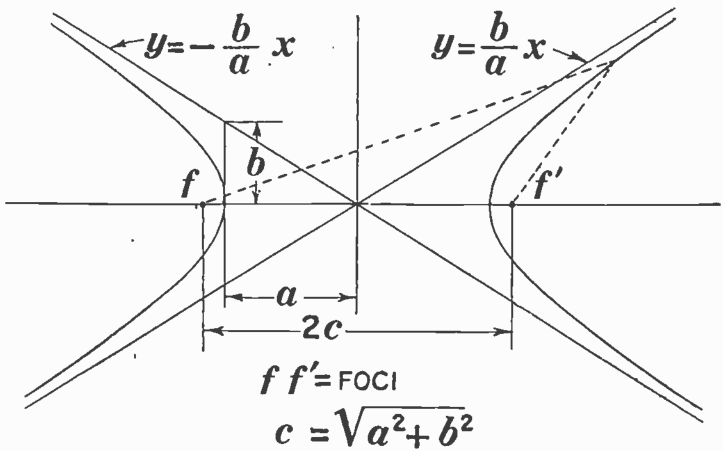


FIG. 298.—*The hyperbola.* A curve *generated by a point moving so that the difference of its distances from two fixed points is always constant.*

Then, by definition, SP = PM;

hence $$(PM)^2 = (SP)^2 = (PN)^2 + (SN)^2$$

that is $$x^2 = y^2 + (x - 2a)^2$$

$$y^2 = 4a\ (x - a)$$

the required equation of the curve.

If the parabola be taken with its axis along the X axis and vertex at origin, the equation becomes

$$y^2 = 4\ ax$$

TEST QUESTIONS

1. *Define analytical geometry.*
2. *How is the position of any point determined?*
3. *What is the difference between an abscissa and an ordinate?*
4. *Which of the two co-ordinates is mentioned first?*
5. *Give the sign of the co-ordinates of the four quadrants.*
6. *In which quadrant are both co-ordinates minus?*
7. *Give the equation for a point which lies in the X axis.*
8. *Find the line whose equation is* $lx + my = 1$.

CHAPTER 10

Trigonometry

Trigonometry is that branch of mathematics *which treats of the measurement of plane and spherical triangles*, that is, *the determination of three of the parts of such triangles when the numerical values of the other three parts are given.*

Since any plane triangle can be divided into right angled triangles, the solution of all plane triangles can be reduced to that of right angled triangles; moreover according to the theory of similar triangles, the ratios between pairs of sides of a right angled triangle depend only upon the magnitude of the acute angles of the triangle, and may therefore be regarded as functions of either of these angles.

Trigonometry is divided into three branches:

1. Plane;
2. Spherical;
3. Analytical.

Plane trigonometry deals with plane triangles, and spherical trigonometry with spherical triangles, the difference being shown in figs. 300 and 301. Evidently the kind of trigonometry the mechanic is interested in is plane trigonometry. Spherical trigonometry is useful in navigation.

This chapter explains plane trigomometry.

Angles.—When two lines meet, as shown in fig. 299, they form an angle with each other. The point where the two lines meet or intersect is called the *vertex* of the angle. The two lines forming the angle are called the *sides* of the angle.

Angles are measured in degrees and sub-divisions of a degree. If the circumference (periphery) of a circle be divided into 360 parts, each part is called *one degree* and the angle formed by two lines from the center of the circle to the ends of this small part of the circumference is a one degree angle.

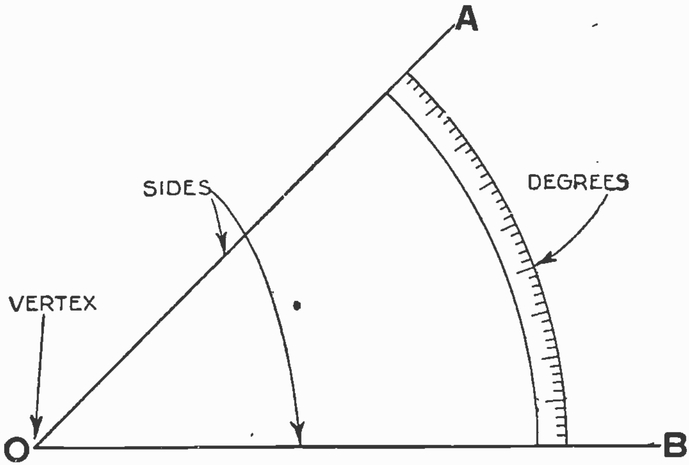


FIG. 299.—Oblique angle showing parts and how measured.

As the whole circle contains 360 degrees, one half of a circle contains 180 degrees, and one quarter of a circle 90 degrees. A 90 degree angle is called a *right* angle.

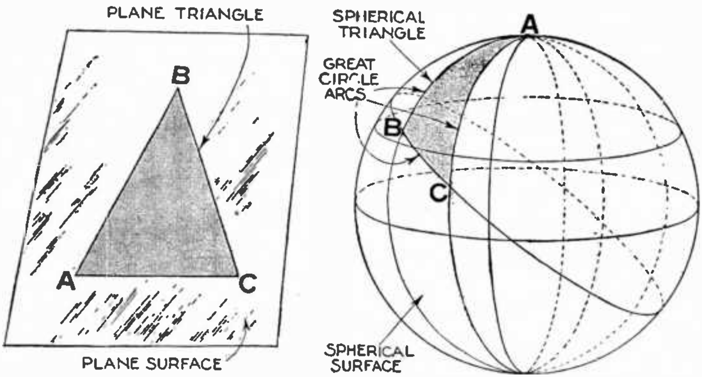
An angle larger than 90 degrees is called an *obtuse* angle, and an angle less than 90 degrees is called an *acute* angle. Any angle which is not a right angle is called an *oblique* angle.

NOTE.—*Angles.* The side of the angle that revolves is called the *terminal line.* The side from which the revolution is measured is called the initial line. An angle is said to be in the quadrant in which its terminal line is located. A *positive angle* is one in which the terminal line revolves in a counter-clockwise direction. A *negative angle* is one in which the terminal line revolves in a clockwise direction.

When two lines form a right or 90 degree angle with each other, one line is said to be *perpendicular* to the other.

Triangles.—Every triangle has six parts:

1. Three angles.
2. Three sides.



FIGS. 300 and 301.—Plane and spherical triangles illustrating *plane* and *spherical* branches of trigonometry.

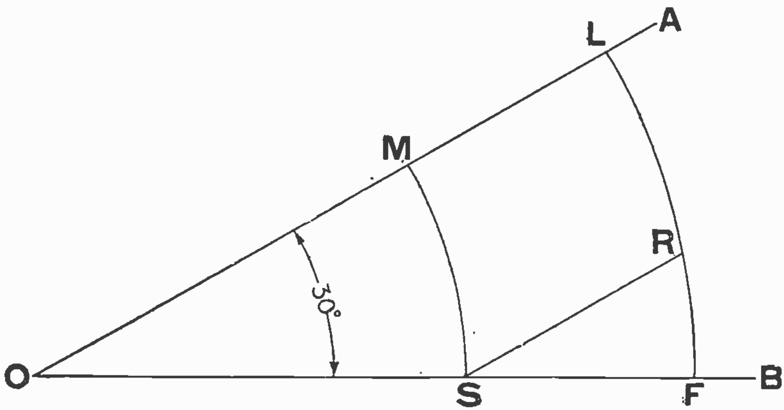
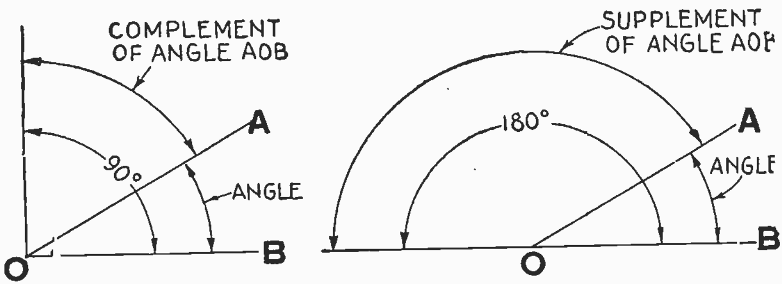


FIG. 302.—Angle showing that the length of a degree depends upon the diameter of the circle on which it is measured.

When any three of these parts are given, provided one of them be a side, the other parts may be determined. The determination of the unknown parts is called the *solution of triangles*.

Measurement of Angles.—In trigonometry the arcs of circles are used to measure angles. The unit of measurement of angles is the *degree* (°). In this system of measurement, the circumference of every circle is supposed to be divided into 360 equal parts, called degrees; thus, a degree is 1/360 of the circumference of any circle. A degree is divided into 60 parts called minutes



FIGS. 303 and 304.—*Complement* and *supplement* of an angle.

expressed by (′), and each minute is divided into 60 seconds, expressed by (″), so that the circumference of any circle contains 21,600 minutes or 1,296,000 seconds.

Evidently, then the length of a degree depends upon the diameter of the circle as shown in fig. 302.

The *complement* of an angle is the difference between 90° and the angle; the *supplement* of an angle is the difference between 180° and the angle. These terms are illustrated in figs. 303 and 304.

Trigonometrical Functions.—A function is a *quantity in* *mathematics so connected with another quantity that if any alteration be made in the latter there will be a consequent alteration in the former.* The dependent quantity is said to be a function of the other. Thus, the circumference of the circle is a function of the diameter.

These functions may consist of:

1. Ratios.
2. Lines ("natural functions").

In the first instance they are defined by referring to a triangle made by

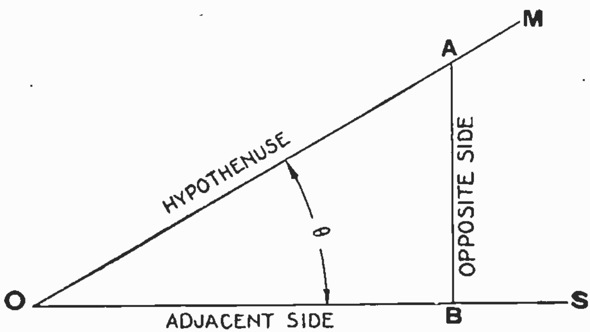


FIG. 305.—Angle θ and constructed triangle OAB for expressing trigonometrical functions as ratios.

drawing (as in fig. 305) a perpendicular from any point A, on one side of a given angle, MOS or θ, to the other side, as AB.

It will be noted that the triangle thus formed is a right triangle, that is, angle ABO = 90°. In this triangle the trigonometrical functions, expressed as ratios are as follows:

Sine of the angle $\theta = \frac{AB}{AO} = \frac{\text{opposite side}}{\text{hypotenuse}}$

Cosine of the angle $\theta = \frac{OB}{CA} = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$$\textit{Tangent} \text{ of the angle } \theta = \frac{AB}{OB} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\textit{Cotangent} \text{ of the angle } \theta = \frac{OB}{BA} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\textit{Secant} \text{ of the angle } \theta = \frac{OA}{OB} = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$\textit{Cosecant} \text{ of the angle } \theta = \frac{OA}{AB} = \frac{\text{hypotenuse}}{\text{opposite side}}$$

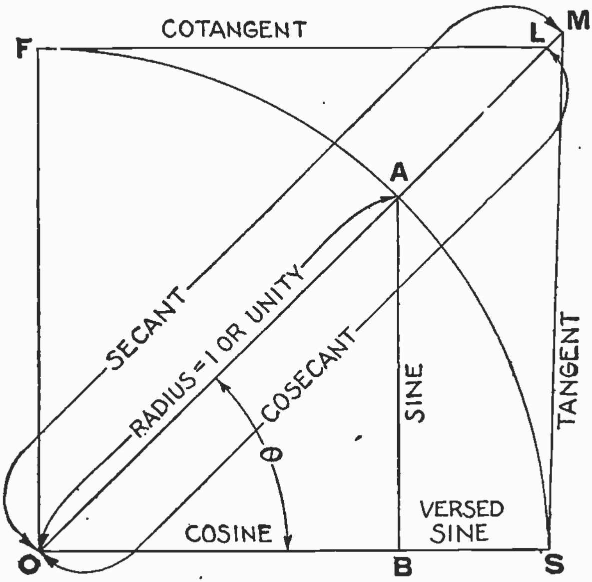


FIG. 306.—*Natural trigonometrical functions, or functions expressed as lines.*

For the sake of brevity the names of the functions are contracted, thus: for *sine θ*, write *sin θ*; for cosine θ, write *cos θ*, etc.

The cosine, cotangent (cot.) and cosecant (cosec) of an angle are respectively the sine, tangent and secant of the complement of that angle.

In the second instance the trigonometrical functions are defined by *certain lines* whose lengths depend upon the arc which measures the angle. These are virtually ratios but by taking what corresponds to the hypotenuse OA, of the triangle AOB, in fig. 305 as a ***radius of unity length*** of a circle the denominators of the ratios become unity or 1, and disappear leaving only the numerators, that is, a line instead of a ratio or function; these lines are the so called ***"natural functions,"*** thus in fig. 306.

$$\textit{Sine}\text{ angle }\theta = \frac{AB}{\text{radius}} = \frac{AB}{1} = AB$$

$$\textit{Cosine}\text{ angle }\theta = \frac{OB}{\text{radius}} = OB$$

$$\textit{Tangent}\text{ angle }\theta = \frac{MS}{OS} = \frac{MS}{\text{radius}} = MS$$

$$\textit{Cotangent}\text{ angle }\theta = \text{tangent of complement of angle }\theta$$

$$= \frac{FL}{OF} = \frac{FL}{\text{radius}} = FL$$

$$\textit{Secant}\text{ angle }\theta = \frac{OM}{OS} = \frac{OM}{\text{radius}} = OM$$

$$\textit{Cosecant}\text{ angle }\theta = \text{secant of complement angle }\theta$$

$$= \frac{OL}{OF} = \frac{OL}{\text{radius}} = OL$$

$$\textit{Versed sine}\text{ angle }\theta = \frac{BS}{OS} = \frac{BS}{\text{radius}} = BS$$

It is these natural trigonometrical functions that are especially useful, rather than the functions expressed as ratios, because, with the aid of the table of natural trigonometrical functions given on page 244, the exact lengths of the functions for an arc of unity radius can be found.

Change of Sign.—Whether the value of a function is + or − (positive or negative) depends upon the quadrant.

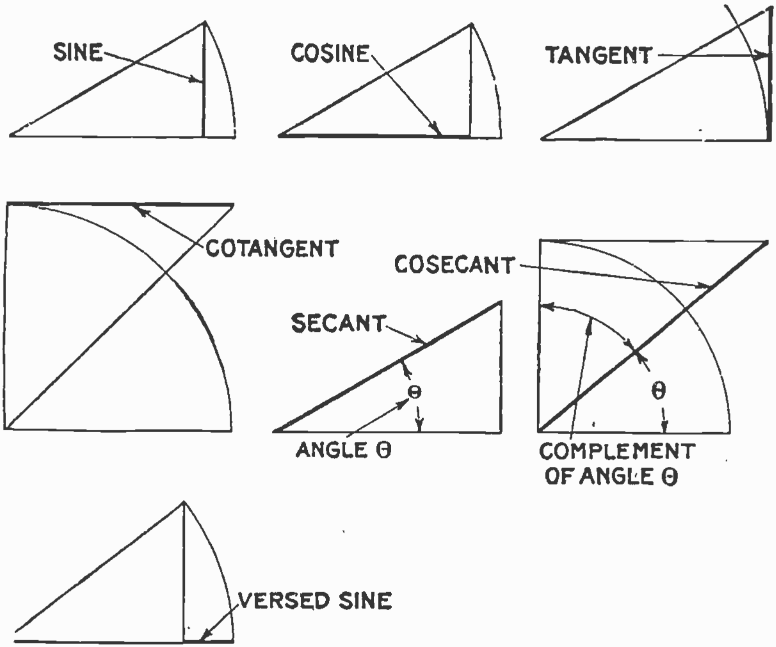
If a circle be divided into four quadrants by a vertical and a horizontal diameter the upper right hand quadrant is called the first, the upper left

Natural Trigonometrical Functions

Degree	Sine	Cosine	Tangent	Secant	Degree	Sine	Cosine	Tangent	Secant
0	.00000	1.0000	.00000	1.0000	46	.7193	.6947	1.0355	1.4395
1	.01745	.9998	.01745	1.0001	47	.7314	.6820	1.0724	1.4663
2	.03490	.9994	.03492	1.0006	48	.7431	.6691	1.1106	1.4945
3	.05234	.9986	.05241	1.0014	49	.7547	.6561	1.1504	1.5242
4	.06976	.9976	.06993	1.0024	50	.7660	.6428	1.1918	1.5557
5	.08716	.9962	.08749	1.0038	51	.7771	.6293	1.2349	1.5890
6	.10453	.9945	.10510	1.0055	52	.7880	.6157	1.2799	1.6243
7	.12187	.9925	.12278	1.0075	53	.7986	.6018	1.3270	1.6616
8	.1392	.9903	.1405	1.0098	54	.8090	.5878	1.3764	1.7013
9	.1564	.9877	.1584	1.0125	55	.8192	.5736	1.4281	1.7434
10	.1736	.9848	.1763	1.0154	56	.8290	.5592	1.4826	1.7883
11	.1908	.9816	.1944	1.0187	57	.8387	.5446	1.5399	1.8361
12	.2079	.9781	.2126	1.0223	58	.8480	.5299	1.6003	1.8871
13	.2250	.9744	.2309	1.0263	59	.8572	.5150	1.6643	1.9416
14	.2419	.9703	.2493	1.0306	60	.8660	.5000	1.7321	2.0000
15	.2588	.9659	.2679	1.0353	61	.8746	.4848	1.8040	2.0627
16	.2756	.9613	.2867	1.0403	62	.8829	.4695	1.8807	2.1300
17	.2924	.9563	.3057	1.0457	63	.8910	.4540	1.9626	2.2027
18	.3090	.9511	.3249	1.0515	64	.8988	.4384	2.0503	2.2812
19	.3256	.9455	.3443	1.0576	65	.9063	.4226	2.1445	2.3662
20	.3420	.9397	.3640	1.0642	66	.9135	.4067	2.2460	2.4586
21	.3584	.9336	.3839	1.0711	67	.9205	.3907	2.3559	2.5593
22	.3746	.9272	.4040	1.0785	68	.9272	.3746	2.4751	2.6695
23	.3907	.9205	.4245	1.0864	69	.9336	.3584	2.6051	2.7904
24	.4067	.9135	.4452	1.0946	70	.9397	.3420	2.7475	2.9238
25	.4226	.9063	.4663	1.1034	71	.9455	.3256	2.9042	3.0715
26	.4384	.8988	.4877	1.1126	72	.9511	.3090	3.0777	3.2361
27	.4540	.8910	.5095	1.1223	73	.9563	.2924	3.2709	3.4203
28	.4695	.8829	.5317	1.1326	74	.9613	.2756	3.4874	3.6279
29	.4848	.8746	.5543	1.1433	75	.9659	.2588	3.7321	3.8637
30	.5000	.8660	.5774	1.1547	76	.9703	.2419	4.0108	4.1336
31	.5150	.8572	.6009	1.1666	77	.9744	.2250	4.3315	4.4454
32	.5299	.8480	.6249	1.1792	78	.9781	.2079	4.7046	4.8097
33	.5446	.8387	.6494	1.1924	79	.9816	.1908	5.1446	5.2408
34	.5592	.8290	.6745	1.2062	80	.9848	.1736	5.6713	5.7588
35	.5736	.8192	.7002	1.2208	81	.9877	.1564	6.3138	6.3924
36	.5878	.8090	.7265	1.2361	82	.9903	.1392	7.1154	7.1853
37	.6018	.7986	.7536	1.2521	83	.9925	.12187	8.1443	8.2055
38	.6157	.7880	.7813	1.2690	84	.9945	.10453	9.5144	9.5668
39	.6293	.7771	.8098	1.2867	85	.9962	.08716	11.4301	11.474
40	.6428	.7660	.8391	1.3054	86	.9976	.06976	14.3007	14.335
41	.6561	.7547	.8693	1.3250	87	.9986	.05234	19.0811	19.107
42	.6691	.7431	.9004	1.3456	88	.9994	.03490	28.6363	28.654
43	.6820	.7314	.9325	1.3673	89	.9998	.01745	57.2900	57.299
44	.6947	.7193	.9657	1.3902	90	1.0000	Inf.	Inf.	Inf.
45	.7071	.7071	1.0000	1.4142		—	—	—	—

NOTE.—*For intermediate values* reduce angles from degrees, minutes and seconds to degrees and decimal parts of a degree (as 40° 21′ 30″ = 40.358°) interpolate or consult a larger table.

the second, the lower left the third, and the lower right, the fourth, as in fig. 314. The signs of the functions in the four quadrants are as follows:



FIGS. 307 to 313 —The natural trigonometrical functions each shown separately for clearness. As elsewhere stated the cos., cot, and cosec. of an angle are respectively the sine, tan, and sec. of the complement of the angle.

	First quad.	Second quad.	Third quad.	Fourth quad.
Sine and cosecant	+	+	[illegible]	−
Cosine and secant	+	−	−	+
Tangent and cotangent	+	−	+	−

Functions of the Supplement of an Angle.—*The sine of an angle is equal to the sine of its supplement, and the cosine, tangent* *and cotangent are each equal to minus the same functions of its supplement.* That is:

$$\sin\,(180° - \phi) = \sin\,\phi$$
$$\cos\,(180° - \phi) = -\cos\,\phi$$
$$\tan\,(180° - \phi) = -\tan\,\phi$$
$$\cot\,(180° - \phi) = -\cot\,\phi$$

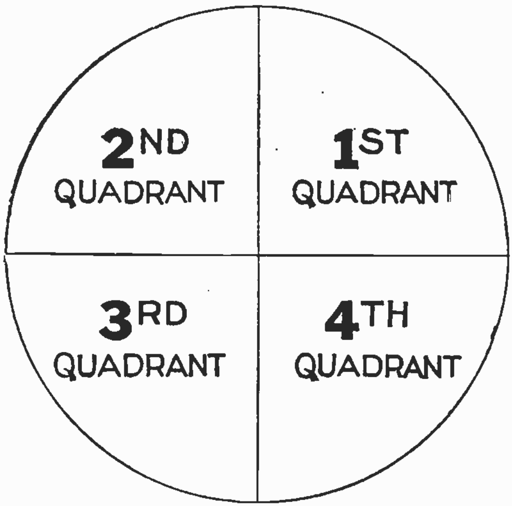


FIG. 314.—The four quadrants.

Law of Sines and Cosines.—All formulae for the solution of triangles are based on:

1. The law of sines;
2. The law of cosines.

together with the fact that *the sum of the three angles of a triangle equals* 180°.

In a triangle, *any side is to any other side as the sine of the angle opposite the first side is to the sine of the angle opposite the other side;* that is, if *a* and *b* be the sides, and A and B the angles opposite them:

$$\frac{a}{b}=\frac{\sin\ A}{\sin\ B} \qquad \text{Law of sines.}$$

In a triangle, *the square of any side is equal to the sum of the squares of the other two sides minus twice their product times the cosine of the included angle;* or if *a*, *b*, and *c*, be the sides and the angle opposite side *a*, be denoted A, then:

$$a^2=b^2+c^2-2\ bc \cos A \qquad \text{Law of cosines.}$$

from which $a=\sqrt{b^2+c^2-2\ bc \cos A}$.

The Right Triangle.—The process of "solving" a triangle consists in finding the parts not given.

In any triangle there are six parts:

1. Three sides;
2. Three angles.

The sides are denoted by italic letters as

a *b* *c*

and the angles opposite these sides by capital letters as

A B C

This arrangement of symbols is shown in fig. 315.

Any triangle can be solved if *three parts, of which one is a side be given.*

Since in a right triangle the right angle is a part of it, a right triangle requires *only two additional parts for solution—two sides; or one side and an acute angle.*

For convenience in solving triangles the formulae should be arranged in tabular form so that any formula to be used can be quickly selected as here given.

In fig. 315,

$$\sin \varphi = \frac{a}{c} \qquad \cos \phi = \frac{b}{c}$$

$$\tan \phi = \frac{a}{b} \qquad \cot \phi = \frac{b}{a}$$

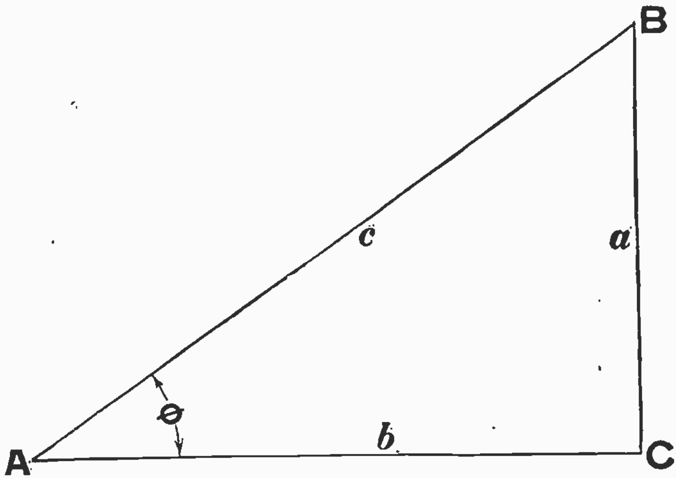


FIG. 315.—Symbols used to denote sides and angles of a triangle.

$$\sec \phi = \frac{c}{b} \qquad \text{cosec } \phi = \frac{c}{a}$$

$$\angle C = 90° \qquad \angle A + \angle B + \angle C = 180°$$

$$a^2 = c^2 - b^2 \qquad b^2 = c^2 - a^2$$

$$c^2 = a^2 + b^2$$

To solve a triangle apply the following rule:

Rule.—*Select a formula containing two of the given parts substituting the given values to find a third part.* Continue *this process until all the parts are found.*

Choose the shortest way to solve a triangle.

Example 1.—The hypotenuse of a right triangle is 1.4142 and one of the sides is 1. Find the remaining parts.

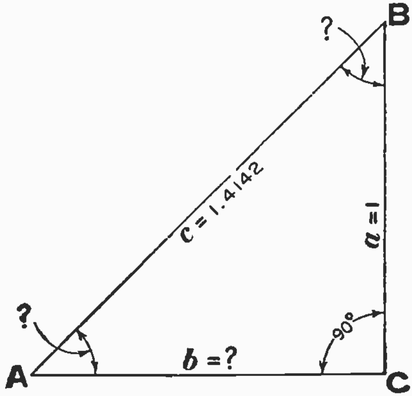


FIG. 316.—Triangle for *example 1.*

In fig. 316,

$$\sin A = \frac{a}{c} = \frac{1}{1.4142}$$

```
1.4142)1.0000000(.7071
        98994
        100600
         98994
          16060
          14142
           1918
```

that is

$$\sin A = .7071$$

This is the natural function and the angle corresponding is given in the table on page 244, and here reproduced in part. Accordingly, to find the angle where sin = .7071 consult the table, which gives the angle A as 45°.

Natural Trigonometrical Functions.

Degree	Sine	Cosine	Tangent	Secant
42				
43				
44				
45	.7071			

ANGLE   NATURAL SINE OF 45° ANGLE

Solving for the other parts

$$\angle A + \angle B + \angle C = 180°$$

but C = 90°. Hence, substituting

$$B + 45 + 90 = 180$$
$$B = 180 - (45 + 90)$$
$$B = 45°$$

Solving for *c*

$$\text{Cos } A = \frac{b}{c}$$

or

$$b = c \cos A$$
$$= 1.4142 \cos A$$
$$\cos A = \cos 45° = .7071$$

Substituting

$$b = 1.4142 \times .7071$$
$$= .999981$$

being a close approximation of 1 by the four place table.

Example 2.—Solve by logarithms a triangle whose hypotenuse is 47.653 and a side 21.34.

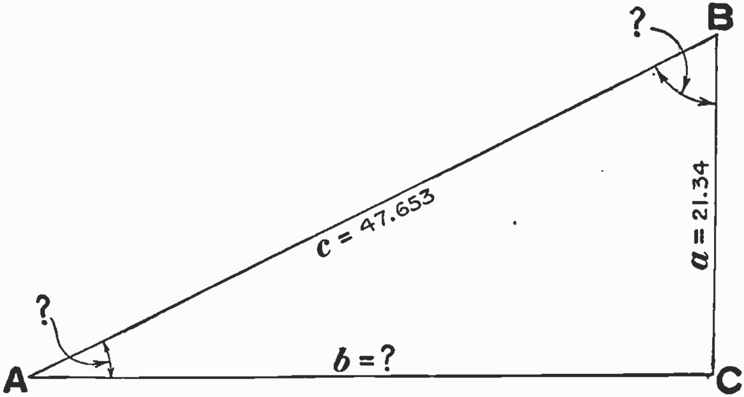


Fig. 317.—Triangle for *example 2.*

In the solution which follows, the logarithms should be obtained from a table of logarithms of trigonometric functions to each minute.

In fig. 317.

$$\sin A = \frac{a}{c}$$

$$\log \sin A = \log a - \log c$$
$$\log 21.34 = 1.32919$$
$$\log 47.653 = 1.67809$$
$$\log \sin A = 9.65110 - 10$$

$$A = 26° \ 36' 14''$$

$$\cos A = \frac{b}{c}$$

$$\log b = \log c + \log \cos A$$

$$\log 47.653 = 1.67809$$

$$\log \cos 26° \ 36' \ 14'' = 9.95140 - 10$$

$$\log b = 1.62949$$

$$b = 42.608$$

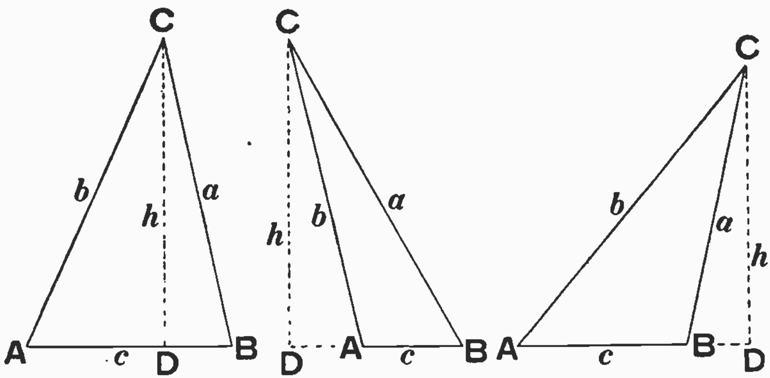


FIG. 318 to 320.—Various oblique triangles.

The Oblique Triangle.—Although oblique triangles may be solved by the use of right triangles, this method is sometimes awkward and the solution is rendered more simple with the special formulae here given.

In figs. 318 to 320 let fall the ⊥ CD and call it *h*.

Then in fig. 318,

$$\sin A = \frac{h}{b}$$

in fig. 319,

$$\sin A = \sin (180° - A) = \frac{h}{b}$$

and in fig. 320,

$$\sin B = (180° - B) = \frac{h}{a}$$

By division

$$\frac{\text{Sin A}}{\text{Sin B}} = \frac{h}{b} \div \frac{h}{a} = \frac{h}{b} \times \frac{a}{h} = \frac{a}{b}$$

Similarly

$$\frac{\text{Sin B}}{\text{Sin C}} = \frac{b}{c}$$

and

$$\frac{\text{Sin C}}{\text{Sin A}} = \frac{c}{a}$$

Evidently these formulae are used for the solution of triangles having given

1. One side and two angles, or
2. Two sides and an angle opposite one of them.

The formulae taking the form

$$a = \frac{\sin A}{\sin B} \times b$$

etc.

Example 3.—Solve the oblique triangle in which is given $a = 36.738$. $A = 36° 55' 54''$, $B = 72° 5' 56''$

In fig. 321,

$$C = 180° - (A+B) = 180° - 109°\ 1'\ 50'' = 70°\ 58'\ 10''$$

To find b	*To find c*
$\frac{b}{a} = \frac{\sin\ B}{\sin\ A}$	$\frac{c}{a} = \frac{\sin\ C}{\sin\ A}$

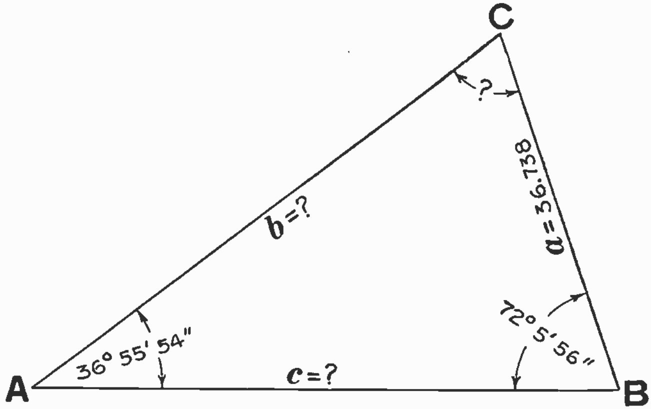


FIG. 321.—Triangle for *example 3.*

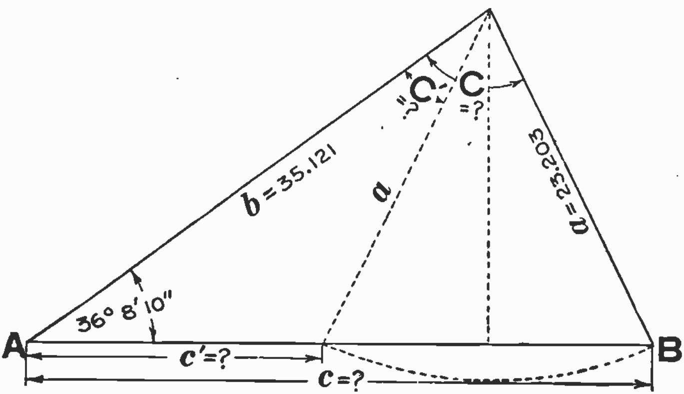


FIG. 322.—Triangle for *example 4.*

$$\log a = 1.56512$$
$$\log \sin B = 9.97845 - 10$$
$$\text{colog} \sin A = 0.22123$$
$$\log b = 1.76480$$
$$b = 58.184$$

$$\log a = 1.56512$$
$$\log \sin C = 9.97559 - 10$$
$$\text{colog} \sin A = 0.22123$$
$$\log c = 1.76194$$
$$c = 57.80$$

Example 4.—Solve the oblique triangle in which is given $a = 23.203, b = 35.121,\ A = 36° 8' 10''$

In fig. 322,

To find B and B'

$$\frac{\sin B}{\sin A} = \frac{b}{a}$$

$$\log b = 1.54556$$
$$\log \sin A = 9.77064 - 10$$
$$\text{colog } a = 8.63445 - 10$$
$$\log \sin B = 9.95065 - 10$$
$$B = 63° 12'$$
$$B' = 180° - B = 116° 48'$$

To find C and C'

$$C = 180° - (A + B) = 80° 39' 50''$$
$$C' = 180° - (A + B') = 27° 3' 50''$$

To find c and c'

$$\frac{c}{a} = \frac{\sin C}{\sin A}$$

$$\log a = 1.36555$$
$$\log \sin C = 9.99421 - 10$$
$$\text{colog} \sin A = 0.22936$$
$$\log c = 1.58912$$
$$c = 38.825$$

$$\log a = 1.36555$$
$$\log \sin C' = 9.65800 - 10$$
$$\text{colog} \sin A = 0.22936$$
$$\log c' = 1.25291$$
$$c' = 17.902$$

Trigonometric Formulae.—A tabulation of the various formulae used is here given in convenient form for reference. The relations are deduced from the properties of similar triangles. Radius = 1.

$\cos A : \sin A :: 1 : \tan A$, whence $\tan A = \dfrac{\sin A}{\cos A}$:

$\sin A : \cos A :: 1 : \cot A$, whence $\text{cotan } A = \dfrac{\cos A}{\sin A}$:

$\cos A : 1 :: 1 : \sec A$, whence $\sec A = \dfrac{1}{\cos A}$:

$$\sin A : 1 :: 1 : \text{cosec } A, \text{ whence cosec } A = \frac{1}{\sin A}$$

$$\tan A : 1 :: 1 : \cot A, \text{ whence } \tan A = \frac{1}{\cot A}$$

Functions of the Sum and Difference of Two Angles

$$\sin (A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos (A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin (A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A-B) = \cos A \cos B + \sin A \sin B$$

By addition and subtraction the following equations are obtained from these formulae.

$$\sin (A+B) + \sin (A-B) = 2 \sin A \cos B$$

$$\sin (A+B) - \sin (A-B) = 2 \cos A \sin B$$

$$\cos (A+B) + \cos (A-B) = 2 \cos A \cos B$$

$$\cos (A-B) - \cos (A+B) = 2 \sin A \sin B$$

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{2 \sin \frac{1}{2} (A+B) \cos \frac{1}{2} (A-B)}{2 \cos \frac{1}{2} (A+B) \sin \frac{1}{2} (A-B)} = \frac{\tan \frac{1}{2} (A+B)}{\tan \frac{1}{2} (A-B)}$$

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \frac{2 \cos \frac{1}{2} (A+B) \cos \frac{1}{2} (A-B)}{2 \sin \frac{1}{2} (A+B) \sin \frac{1}{2} (A-B)} = \frac{\cot \frac{1}{2} (A+B)}{\tan \frac{1}{2} (A-B)}$$

$$\frac{\sin (A+B)}{\sin (A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}$$

$$\frac{\sin (A+B)}{\cos A \cos B} = \tan A + \tan B \qquad \tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\sin (A-B)}{\cos A \cos B} = \tan A - \tan B \qquad \tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\cos (A+B)}{\cos A \cos B} = 1 - \tan A \tan B \qquad \cot (A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\frac{\cos(A-B)}{\cos A \cos B} = 1 + \tan A \tan B \qquad \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Functions of twice an angle

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$$

Functions of half an angle

$$\sin \tfrac{1}{2} A = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \cos \tfrac{1}{2} A = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan \tfrac{1}{2} A = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \qquad \cot \tfrac{1}{2} A = \pm\sqrt{\frac{1+\cos A}{1-\cos A}}$$

TEST QUESTIONS

1. *Of what does trigonometry treat?*
2. *How are angles measured?*
3. *How many parts has a triangle?*
4. *What is the difference between the complement and the supplement of an angle?*
5. *Name two ways of expressing trigonometrical functions?*
6. *Define sine and versed sine of an angle?*
7. *Explain fully how the natural functions are obtained.*
8. *Why is the value of a function positive or negative?*
9. *Give formulæ for the functions of the supplement of an angle.*
10. *Give the law of sines and the law of cosines.*
11. *Of what does the process of solving a triangle consist?*
12. *What must be given in order to solve a triangle?*
13. *Give the formulæ used in the solution of triangles.*
14. *The hypotenuse of a right triangle is* 1.4142 *and one of the sides is* 1. *Find the remaining parts.*
15. *Solve by logarithms a triangle whose hypotenuse is* 47.653 *and a side* 21.34.
16. *Can oblique triangles always be conveniently solved by the use of right triangles?*
17. *Solve the oblique triangle in which is given* $a=36.738$, $A=36°\ 55'\ 54''$, $B=72°\ 5'\ 56''$.
18. *Given* $a=23.203$, $b=35.121$, $A=36°\ 8'\ 10''$; *solve.*
19. *How are trigonometrical formulæ deduced?*
20. *Give functions for sum and difference of two angles.*
21. *Give formulæ obtained by addition and subtraction.*
22. *Give formulæ for functions of twice an angle.*
23. *Give formulæ for functions of half an angle.*

CHAPTER 11

Algebra

Algebra is *that branch of mathematics in which letters, signs and figures are used in making calculations instead of only signs and figures as in arithmetic.*

By the aid of algebra it is possible to express obscure or involved quantities which are set down as *equations* and the problem solved by treating the equations according to certain definite rules.

Success in solving equations involves *a knowledge of the meaning of the various parts which make up the equation so that it can be properly "handled" or manipulated.*

Symbols.—Letters of the alphabet are used *to represent quantities in algebraic equations.* It is usual to represent known quantities by the first three letters of the alphabet as a, b, c, and unknown quantities by the last letters as, x, y, z.

NOTE.—***Algebra.*** Note the following divisions. ***Boolian*** or ***logical algebra.*** A method of dealing with concepts of logic by means of algebraic symbols and operations.—***double a.***, that form of algebra in which the symbols indicate lines, their lengths and their directions.—***graphic a.***, algebra in which curves are used to express the relations of quantities; ***multiple or n-way a.***, that algebra in which heterogeneous units are combined and multiplied; ***pure a,*** algebra in which all the units are definitely related; ***rhetorical a.***, the discussion of problems by algebraic methods, but in ordinary language; algebra without notation.—***universal a.***—a method of reasoning by symbols, of which the general definitions may be applied to any process of addition and others to any process of multiplication; multiple units treated algebraically.

Signs.—These are characters indicating the relation of quantities, or an operation performed upon them.

Thus, as in arithmetic

$+$ (plus), sign of addition.

$-$ (minus), sign of subtraction.

$\times$ (times), sign of multiplication.

$\div$ (divided by), sign of division.

$=$ (equals), sign of equality.

$<$ (is less than) } signs of inequality.
$>$ (is greater than) }

$\sim$ (difference of two quantities). Thus $a \sim b$ denotes that the difference of the quantities a and b is to be found.

( ) parentheses. Thus: $(a-b)\ x$ indicates that the quantity obtained by subtracting b from a is to be multiplied by x.

- - - (continuation). Read: "and so on."

$\therefore$ (therefore).

—— (vinculum), thus: $\overline{a+b}\times y$, same as $(a+b)\ y$.

$\sqrt{\ \ }$ (radical). Indicates that a root of the number to which it is prefixed is to be extracted.

$\pm$ plus or minus.

Since the signs just given have exactly the same meaning as in arithmetic, no further explanation is necessary.

Coefficient.—This is a number prefixed to a quantity to denote how many times the quantity is to be taken. Thus: $3ay$ means $ay+ay+ay$.

Exponent.—A small figure called an *exponent* is placed to the right of and slightly above a numeral or mathematical symbol, to denote the power to which the magnitude is to be raised. Thus: x^2 (x square) means $x \times x$. Hence, the exponent indicates *the number of times the quantity is to be taken as a factor.*

Algebraic Expressions.—Any combination of algebraic symbols, coefficients and signs, as $2ax+4by$ is known as an expression.

A *term* of an algebraic expression is any combination of symbols and coefficients not separated by a sign.

Thus, in the expression $2ax+4by$, the terms are $2ax$ and $4by$.

There are numerous kinds of expressions. Thus:

$x+y$ (two term) binomial expression

$a+x+y$ (three term) trinomial expression

Similar or like terms are those which differ only in their numerical coefficients. Thus:

$$4ay^2+6ay^2$$

Dissimilar or unlike terms are those which are not similar. Thus:

$$4axy^2+2ax^2y$$

The degree of a term is the number of *literal* factors which it contains. Thus:

$2a$ is of the *first* degree, since it contains but *one* literal factor;

ab is of the *second* degree, since it contains *two* literal factors;

$3a^2b^3$ is of the *fifth* degree, since it contains *five* literal factors.

Homogeneous terms are those of the same degree.

Thus: $3abx$ and y^3

Addition

In algebra *the process of collecting the terms of an expression into an equivalent expression* is called addition. It involves the collection of both positive and negative quantities.

Addition may be divided into three cases:

1. Like terms with the same sign;
2. Like terms with different signs;
3. Unlike terms.

Case 1.—*Like terms with the same sign.*

Rule.—*Add the numerical coefficients, prefix the common sign, and annex the common letters.*

Example.—Add $2ax+3ax+12ax$

$$\begin{array}{r} 2ax \\ 3ax \\ 12ax \\ \hline 17ax \end{array}$$

Here it is not necessary to prefix the common sign to the answer, as in the absence of any sign, $+$ *is understood.*

Example.—Add $-2ax-3\,ax-12ax$

$$\begin{array}{r} -\ 2ax \\ -\ 3ax \\ -12ax \\ \hline -17ax \end{array}$$

Note the minus sign is prefixed to the answer.

Case 2.—*Like terms with different signs.*

Here it is necessary to distinguish between quantities which are the exact opposite of each other in condition or quality.

Thus on the thermometer scale, temperatures *above* zero are + temperatures, and *below* zero, − temperatures.

Adding a negative quantity is equivalent to subtracting a positive quantity of the same absolute value.

Thus:

$$x+(-y)$$

is the same as

$$x-y$$

Rule.—*Subtract the less coefficient from the greater; affix the common symbols and prefix the sign of the greater coefficient.*

Example.—Add $2x-12x$

$$\begin{array}{r} -12x \\ 2x \\ \hline -10x \end{array}$$

Example.—Add $2x-4x+3x-10x$.

1st step	*2nd step*
$2x-4x$	$-14x$
$3x-10x$	$5x$
$5x-14x$	$-9x$

Case 3.—*Unlike terms.*

Rule.—*Add together the terms which are like terms by the rule in the second case, and put down the other terms each preceded by its proper sign.*

Example.—Add $2x+3y-8z$, $10y-3x+2z$, $3x+2-y$.

$$\begin{array}{rrrr} 2x & +3y & -8z & \\ -3x & +10y & +2z & \\ 3x & -y & & +2 \\ \hline 2x & +12y & -6z & +2 \end{array}$$

Subtraction

This is the process of *taking one quantity from another.*

The ***subtrahend*** is the quantity to be subtracted.

The ***minuend*** is the quantity from which it is to be subtracted.

The ***remainder*** is the result of the operation.

Rule.—*Change the sign of the subtrahend and add the result to the minuend.*

Example.—Subtract $7a$ from $10a$.

Here $7a$ is the subtrahend and $10a$ the minuend.

$$\begin{array}{r} 10a \\ -\ 7a \\ \hline 3a \end{array}$$

Example.—Subtract $-7a$ from $10a$.

Changing sign of subtrahend $-7a$ becomes $+7a$, hence

$$\begin{array}{r} 10a \\ 7a \\ \hline 17a \end{array}$$

Example.—Subtract $5x^2y-3ab+m^2$ from $3x^2y-2ab+4n$.

Changing the sign of each term of the subtrahend and adding the result to the minuend,

$$\begin{array}{rrrr} 3x^2y & -2ab & +4n & \\ -5x^2y & +3ab & & -m^2 \\ \hline -2x^2y & +\ ab & +4n & -m^2 \end{array}$$

Parentheses and Brackets

In algebra the parentheses are very frequently used and accordingly it is important to know the rules respecting them.

Rule.—*When an expression within parentheses is preceded by the sign + the parentheses may be removed.*

Thus $a+(b-c)$

is the same as

$$a+b-c$$

Rule.—*When an expression within parentheses is preceded by the sign − the parentheses may be removed if the sign of every term within the parentheses be changed.*

Thus the expression

$$a-(b-c)$$

indicates that the quantity $b-c$ is to be subtracted from a.

Accordingly applying the rule, the quantity

$$a-(b-c)$$

is the same as

$$a-b+c$$

Rule.—*Any number of terms in an expression may be put within parentheses and the sign + placed before the whole.*

Rule.—*Any number of terms in an expression may be put within parentheses and the sign − placed before the whole, provided the sign of every term within the parentheses be changed.*

It is often convenient to put two or more terms within brackets.

Brackets have the same meaning as parentheses but are used for complicated expressions, for instance:

$$4x\left\{\frac{(x-3)+a\ (y-4)}{3x\ (a-b)}-23\right\}$$

Multiplication

Algebraic multiplication involves three cases, as in multiplying:

1. A single term by a single term.
2. An expression by a single term.
3. An expression by an expression.

Preliminary to considering these three cases, a clear understanding should be had of the Rules of Signs which follow.

Rule.—Positive × positive = positive.

$$\text{Thus } (+1)\times(+1) = +1$$

Rule.—Negative × negative = positive.

$$\text{Thus } (-1)\times(-1) = +1$$

Rule.—Positive × negative = negative.

$$\text{Thus } (+1)\times(-1) = -1$$

Case 1.—*A single term by a single term.*

Rule.—*Multiply the coefficients and affix the symbols. Make the product + when the factors have the same sign and − when they have different signs.*

Example.—Multiply $2a$ by $3b$

$$2a\times 3b = 6ab.$$

Example.—Multiply $-2a$ by $3b$.

$$(-2a)\times(+3b)=-6ab$$

Law of Exponents.—To multiply two like quantities having exponents add the exponents.

Case 1.—*A single term by a single term.*

Example.—Multiply x^2 by x^3.

$$x^2\times x^3=x^{2+3}=x^5$$

Case 2.—*Single terms with fractional exponents.*

Rule.—*In a fractional exponent the numerator denotes a power and the denominator a root.*

Example.—$x^{2/3}=\sqrt[3]{x^2}$

Case 3.—*An expression by a single term.*

Rule.—*Multiply each term of the multiplicand by the multiplier, and connect the products with the proper signs.*

Example.—Multiply $a+b$ by $2x$.

$$\begin{array}{r} a+b \\ 2x \\ \hline 2ax+2bx \end{array}$$

Example.—Multiply $a+b-c$ by $-2x$.

$$\begin{array}{r} a+b-c \\ -2x \\ \hline -2ax-2bx+2cx \end{array}$$

Case 4.—*An expression by an expression.*

Rule.—*Multiply each term of the multiplicand by each term of the multiplier; if the terms have the same sign, prefix the sign +*

to the product; if they have different signs, prefix the sign $-$; *then collect these results to form the complete product.*

Example.—Multiply M+S by M+S.

$$\begin{array}{lr} & M+S \\ & M+S \\ \hline \text{Multiplying each term by M gives} & M^2+MS \\ \text{Multiplying each term by S gives} & MS+S^2 \\ \hline \text{Adding terms gives} & M^2+2MS+S^2 \end{array}$$

This is shown graphically in fig. 323.

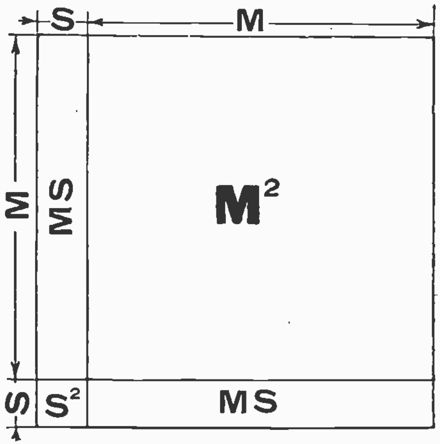


FIG. 323.—Diagram illustrating the product obtained by multiplying M+S by M+S.

Example.—Multiply $-x-x^3+1+x^2$ by $x+1$.

It is convenient to have both multiplicand and multiplier arranged in the same order of powers, and to write the partial products in the same order.

Arranging the expressions according to the ascending powers of x, thus,

$$\begin{array}{l} 1-x+x^2-x^3 \\ 1+x \\ \hline 1-x+x^2-x^3 \\ \quad\; x-x^2+x^3-x^4 \\ \hline 1 \qquad\qquad\quad -x^4 \end{array}$$

Division

The process of division is somewhat more difficult to explain and understand than the foregoing operations, and accordingly division should be studied and thoroughly understood by the student.

Division is *the reverse of multiplication;* that is, in multiplication the product of given factors is to be determined; in division the product and one of the factors is given to determine the other factor.

There are three cases, as, in dividing:

1. One single term by another.
2. An expression by a single term.
3. An expression by another expression.

Law of Signs.—Since the dividend is the product of the divisor and quotient, it follows that

When divisor is	+	and dividend	+,	then quotient is	+
" " "	−	" "	−	" " "	+
" " "	+	" "	−	" " "	−
" " "	−	" "	+	" " "	−

Rule.—*Like signs produce* +; *unlike signs* −.

Rule.—*Write the dividend over the divisor with a line between them; if the expressions have common factors, remove the common factors; prefix the sign + if the expressions have the same sign, and the sign − if they have different signs. One power of any quantity is divided by another power of the same quantity by subtracting the index of the latter power from the index of the former.*

Case 1.—*One single term by another.*

Example.—Divide $12ab$ by $2a$.

$$\frac{12ab}{2a}=6b$$

Example.—Divide a^4 by a^2.

$$\frac{a^4}{a^2}=a^{4-2}=a^2$$

Case 2.—*An expression by a single term.*

Rule.—*Divide each term of the dividend by the divisor, by the rule in Case 1, and collect the results with their proper signs, to form the complete quotient.*

Example.—Divide $12a^3b+6a^2bc-3a^2bc$ by $-3a^2$

$$\frac{12a^3b+6a^2bc-3a^2bc}{-3a^2}=-4ab-2bc+bc=-4ab-bc$$

Case 3.—*An expression by another expression.*

Rule.—1. *Arrange both dividend and divisor according to ascending powers of some common letter, or both according to descending powers of some common letter.* 2. *Divide the first term of the dividend by the first term of the divisor, and put the result for the first term of the quotient.* 3. *Multiply the whole divisor by this term and subtract the product from the dividend.* 4. *To the remainder join as many terms of the dividend, taken in order, as may be required, and repeat the whole operation.* 5. *Continue the process until all the terms of the dividend have been taken down.*

Example.—Divide $m^2+2ms+s^2$ by $m+s$.

$$\begin{array}{r} m+s)\,m^2+2ms+s^2\,(m+s \\ \underline{m^2+ms} \\ ms+s^2 \\ \underline{ms+s^2} \end{array}$$

Example.—Divide $3a^4-10a^3b+22a^2b^2-22ab^3+15b^4$ by $a^2-2ab+3b^2$

$$a^2-2ab+3b^2)3a^4-10a^3b+22a^2b^2-22ab^3+15b^4(3a^2-4ab+5b^2$$

$$\underline{3a^4-6a^3b+9a^2b^2}$$

$$-4a^3b+13a^2b^2-22ab^3$$

$$\underline{-4a^3b+8a^2b^2-12ab^3}$$

$$5a^2b^2-10ab^3+15b^4$$

$$\underline{5a^2b^2-10ab^3+15b^4}$$

The importance of arranging both divisor and dividend according to the same order of some common letter will be seen by attempting to do the above example with the terms arranged without regard to order.

Factoring

Factoring is the process of *finding what expressions will divide a given expression; that is, the process of resolving an expression into its factors.*

Example.—Factor the expression a^3+3a.

Since each term contains a, divide by a, thus:

$$a)\underline{a^3+3a}$$

$$a^2+3$$

that is $$a^3+3a=a(a^2+3)$$

The student should note very carefully the following:

1. x^n-y^n is divisible by $x-y$ if n be *any* whole number.
2. x^n-y^n is divisible by $x+y$ if n be any *even* whole number.
3. x^n+y^n is divisible by $x+y$ if n be any *odd* whole number.
4. x^n+y^n is never divisible by $x+y$ or $x-y$, when n is an *even* whole number.

As an example in the use of the above items consider the expression x^7-y^7. Is it divisible by $x-y$ or $x+y$?

The index 7 is an odd whole number, and the simplest case of this kind is $x-y$, which is divisible by $x-y$, but not by $x+y$. Accordingly x^7-y^7 is divisible by $x-y$ and not by $x+y$.

Again take x^8-y^8, the index 8 is an *even* whole number, and the simplest case of this kind is x^2-y^2, which is divisible both by $x-y$ and $x+y$; hence x^8-y^8 is divisible both by $x-y$ and $x+y$.

Equations

An equation is *a proposition expressed usually by the symbol (=) denoting the equality of two quantities*, that is, an equation *is the expression of the equality of two things.*

Thus $x=y$ indicates that whatever value be given to x is equal to whatever value be given to y.

The expressions on either side of the = symbol are called the *sides* or *members* of the equation.

An *identical* equation is one in which the two sides are equal whatever numbers the symbols represent. An *equation of condition* is one which is not true whatever numbers the symbols represent, but only when the symbols represent some particular number or numbers. Thus $10-x=3$ cannot be true unless $x=7$.

A letter to which a particular value or values must be given in order that the statement contained in an equation may be true is called an *unknown quantity*. Such particular value of the unknown quantity is said to *satisfy the equation*, and is called a *root of the equation*. To solve an equation is to find the root or roots.

In the solution of equations the following axioms should be noted:

1. If equal quantities be added to equal quantities, the sums will be equal.

2. If equal quantities be subtracted from equal quantities, the remainders will be equal.

3. If equal quantities be multiplied by equal quantities, the products will be equal.

4. If equal quantities be divided by equal quantities, the quotients will be equal.

5. If the same quantity be both added to and subtracted from another, the value of the latter will not be changed.

6. If a quantity be both multiplied and divided by another, the value of the former will not be changed.

7. Quantities which are equal to the same quantity are equal to each other.

Equations may be classed as

1. Simple;
2. Simultaneous;
3. Quadratic.

Simple Equations.—An equation that contains the first power of the unknown quantity or quantities *axy* and no higher power is called a *simple equation*. This is an equation of the *first degree*.

The solution of equations will present no difficulty if the following general rule be remembered and applied.

General Rule.—*Whatever be done to one side of an equation must be done to the other side in order not to destroy the equality.*

Thus in the equation $x+3=y$ adding 6 to both sides gives

$$\begin{array}{rcl} x+3 & = & y \\ 6 & & 6 \\ \hline x+9 & = & y+6 \end{array}$$

that is

$$x+3=y \text{ is the same as } x+9=y+6$$

The following examples will illustrate methods used in "handling" or solving equations.

1. Transposition
   a. Single;
   b. Double.

Example.—Solve the equation $x+3a=2c$.

Rule.—*Any term may be transposed from one side of an equation to the other by changing its sign.*

Accordingly,

$$x+3a=2c$$
$$x=2c-3a$$

this is evident from the following:

Subtract 3 *a* from both sides of the equation, thus:

$$\begin{array}{ll} x+3a & =2c \\ \;\;-3a & \;\;-3a \\ \hline x & =2c-3a \end{array}$$

Example.—Solve the equation $3x-2=x+19$.

Single Transposition

$$3x-x=19+2$$
$$2x=21$$
$$x=10\tfrac{1}{2}$$

Double Transposition

$x+19=3x-2$ 1st transposition

$x-3x=-2-19$ 2nd transposition

$$-2x=-21$$
$$x=10\tfrac{1}{2}$$

2. Clearing of fractions.

Rule 1.—*Divide the least common multiple by the denominator of each term of the equation and multiply the quotient by the numerator of the term.*

Example.—Solve the equation $\frac{x}{3}-\frac{3}{4}=\frac{4x}{6}-\frac{3}{8}$

The least common multiple of the denominators is 24.

Dividing least common multiple by denominators and multiplying quotients by numerators:

$$8x-18=16x-9$$

Transposing (double transposition):

$$16x-9\ =8x-18$$
$$16x-8x=9\ -18$$
$$8x=-9$$
$$x=-1\tfrac{1}{8}$$

Rule 2.—*Multiply each term of the equation by the least common multiple.*

Example.—Solve the equation $\frac{x}{3}-\frac{3}{4}=\frac{4x}{6}-\frac{3}{8}$

The least common multiple of the denominators is 24.

Multiplying numerators by least common multiple:

$$\frac{24x}{3}-\frac{72}{4}=\frac{96x}{6}-\frac{72}{8}$$

Clearing of fractions:

$$8x-18=16x-9$$

Collecting and transposing:

$$-9=8x, \text{ that is}$$
$$8x=-9$$
$$x=-1\tfrac{1}{8}$$

Simple Equation Problems

The usefulness of algebra is clearly seen in its application to the solution of problems. In these problems, certain quantities are given and another called the *unknown quantity*. The latter, which is represented by one of the last letters of the alphabet, is to be determined, the solution being possible because of the relation of the unknown to the known quantities.

It is not possible to give general rules for the solution of such problems and the student must rely on his ingenuity.

The following hints will be found helpful:

1. Express the unknown quantity, or one of the unknown quantities, by one of the final letters of the alphabet.

2. From the given conditions, find expressions for the other unknown quantities, if any, in the problem.

3. Form an equation in accordance with the conditions of the problem.

4. Solve the equation thus formed.

Problem.—The sum of two numbers is 475 and their difference 23, find the numbers.

Let x denote the smaller number. Since the difference of the two numbers is 23, then

$$x+23=\text{the greater number}$$

Also since the sum of the two numbers is 475 then

$$x+(x+23)=475$$
$$2x+23=475$$
$$2x=452$$
$$x=226 \text{ the smaller number}$$
$$x+23=249 \text{ the greater number.}$$

Problem.—Stevens is three times as old as Gilbert and eight years ago Stevens was seven times as old as Gilbert. What are their ages at present?

Let x = Gilbert's age

Then $3x$ = Stevens' age

$x-8$ = Gilbert's age 8 years ago

also $3x-8$ = Stevens' age 8 years ago.

According to the relation between ages:

$$3x-8=7(x-8)$$
$$3x-8=7x-56$$

Transposing and collecting:

$$-4x = -48$$
$$x = 12$$

That is, Gilbert's present age (x) is 12 yrs. and Stevens' age ($3x$) $3 \times 12 = 36$ yrs.

Indeterminate Equations.—When a simple equation contains two unknown quantities it is impossible to determine the value of the unknown quantities definitely because if *any* value be assumed for one of the unknown quantities there can be found a value corresponding for the other unknown quantity.

Simultaneous Equations.—Two separate equations considered together are called simultaneous equations when they represent simultaneous relations between the *unknown quantities*.

The solution of simultaneous equations is important, and the student should practice on this subject until he is thoroughly familiar with the methods employed.

Simultaneous equations are solved by elimination of one of the unknown quantities. This may be done by:

1. Addition;
2. Subtraction;
3. Substitution;
4. Comparison.

The following will illustrate the solution of simultaneous equations by these methods:

1. *By Addition.*

Example.—Solve the simultaneous equations:

$$x + y = 7 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$3x - y = 13 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By addition,

$$\begin{array}{r} x+y=\ 7 \\ 3x-y=13 \\ \hline 4x\qquad =20 \\ x=\ 5 \end{array}$$

Substituting this value of x in equation (1),

$$5+y=7$$
$$y=7-5=2$$

2. *By Subtraction.*

Example.—Solve the equations:

$$x+y=\ 7 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$3x-y=13 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Multiply equation (2) by $-$ (minus) and subtract.

$$\begin{array}{r} x+y=\ 7 \\ -3x+y=-13 \\ \hline 4x=20 \\ x=\ 5 \end{array}$$

By subtraction,

Substituting this value of x in equation (1),

$$5+y=7$$
$$y=7-5=2$$

3. *By Substitution.*

Example.—Solve the equations:

$$x+y=\ 7 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$3x-y=13 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solving equation (1) for y

$$y=7-x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting this value of y in equation (2),

$$3x-(7-x)=13$$

Removing parentheses,

$$3x-7+x=13$$

Transposing and collecting,

$$4x=20$$
$$x=5$$

Substituting this value of x in equation (3),

$$y=7-5=2$$

4. *By comparison.*

Rule.—*Find the value of the same unknown from each of the two given equations and set these values equal to each other.*

Example.—Solve by comparison the equations:

$$x+y=7 \quad \dots\dots (1)$$
$$3x-y=13 \quad \dots\dots (2)$$

Find the value of y in each equation.

$$y=7-x \quad \dots\dots (3)$$
$$-y=13-3x \quad \dots\dots (4)$$

Multiply equation (4) by $-$ (minus)

$$y=-13+3x \quad \dots\dots (5)$$

Equating equations (3 and 5):

$$7-x=-13+3x$$
$$-4x=-20$$
$$x=5$$

Substituting this value of x in equation (1),

$$5+y=7$$

Simultaneous Equation Problems

(two unknown quantities)

Problem.—A railway train after traveling an hour is detained 24 minutes, after which it proceeds at six-fifths of its former rate, and arrives 15 minutes late. If the detention had taken place 5 miles further on, the train would have arrived 2 minutes later than it did. Find the original rate of speed of the train, and the distance traveled.

Let $5x$ = original speed of train

y = total distance traveled.

Then will

$y-5x$ = distance to go after detention.

Thus remaining distance at the original speed would consume

$$\frac{y-5x}{5x} \text{ hours}$$

and at the increased rate of speed it would take

$$\frac{y-5x}{6x} \text{ hours}$$

Since the train is detained 24 minutes, and yet is only 15 minutes late at its arrival, it follows that the remainder of the journey is performed in 9 minutes less than it would have been if the rate had not been increased.

9 minutes is $\frac{9}{60}$ of an hour, therefore

$$\frac{y-5x}{6x}=\frac{y-5x}{5x}-\frac{9}{60} \qquad (1)$$

If the detention had taken place 5 miles further on, there would have been $y-5x-5$ miles left to be traveled. Thus,

$$\frac{y-5x-5}{6x}=\frac{y-5x-5}{5x}-\frac{7}{60} \qquad (2)$$

Subtract (2) from (1), thus:

$$\frac{5}{6x}=\frac{5}{5x}-\frac{2}{60}$$

$$50=60-2x$$
$$2x=10$$
$$x=5$$

Substitute this value of x in (1) and it will be found by solving the equation that $y=47\frac{1}{2}$.

Problem.—Gilbert and Steven weigh together 230 lbs. and twice Steven's weight is 60 lbs. more than 3 times Gilbert's weight. Find the weight of each.

Let x=Steven's weight

y=Gilbert's "

Then, weight of Steven and Gilbert together is

$$x+y=230 \ldots\ldots (1)$$

Since twice Steven's weight is 60 lbs. more than 3 times Gilbert's weight

$$2x-60=3y \ldots\ldots (2)$$

The two equations (1) and (2) represent the conditions of the problem.

Multiply (1) by 3 and transpose in (2)

$$\begin{array}{lr} & 3x+3y=690 \\ & 2x-3y=\ \ 60 \\ \hline \text{By addition} & 5x\qquad =750 \end{array}$$

$$x=150 \text{ lbs. (Steven's weight)}$$

Substituting the value of x in equation (1)

$$150+y=230 \text{ lbs.}$$

$$y=230-150=80 \text{ lbs. (Gilbert's weight)}$$

Quadratic Equations

A quadratic equation is *one which contains the square of the unknown quantity but no higher power.*

There are two kinds of quadratic equations:

1. Pure: 2. Affected.

A *pure* quadratic equation is one which contains *only* the square of the unknown quantity, as for instance $2x^2=50$.

An *adfected* quadratic equation is one which contains the first power of the unknown quantity as well as its square, as for instance $2x^2-7x+3=0$.

A pure quadratic is easily solved but in an equation such as

$$4x^2+8x=12$$

it is necessary to find some way of grouping x^2 and x together which will give a single term in x, when the square root of both sides is taken. The process is indicated in the rule which follows for completing the square in x.

Rule.—1. *Group all the terms in x^2 and x on one side of the equation alone, placing the terms containing x^2 first.* **2.** *Divide through by the coefficient of x^2.* **3.** *Add to both sides of the equation the square of one half of the coefficient of the x term.* **4.** *Take the square root of both sides (the left hand side being a perfect square). Then solve as for a simple equation in x.*

Example.—Solve the equation $x^2-10x+24=0$.

Applying the rule just given:

1. Group all the x^2 and x terms on one side of the equation alone, placing those in x^2 first, that is by transposition

$$x^2-10x=-24$$

2. Coefficient of x^2 being 1 in this example no division is necessary.

3. Add to both sides the square of one half the coefficient of the x term.

The coefficient of the x term is 10 and the square of half the coefficient is $\left(\frac{10}{2}\right)^2$.

$$x^2-10x+\left(\frac{10}{2}\right)^2=-24+\left(\frac{10}{2}\right)^2$$

$$x^2-10x+25=-24+25$$

The left hand side of the equation has now been made into a perfect square.

4. Take the square root of both sides.

$$x-5=\pm1$$

transposing

$$x=5\pm1$$

that is

$$x=6 \text{ or } 4.$$

Points on Quadratic Equations.—1. Every quadratic equation can be put in the form $x^2+px+q=0$, where p and q represent some known numbers, whole or fractional, positive or negative.

2. A quadratic equation cannot have more than two roots.

3. When the terms are all on one side, and the coefficient of the square of the unknown quantity is unity, the sum of the roots is equal to the coefficient of the second term with its sign changed, and the product of the roots is equal to the last term.

Problem.—Find two numbers such that their sum is 15, and their product is 54.

Let $x=$ one of the numbers

then

$15-x=$ the other

and by supposition

$$x\,(15-x)=54$$

By transposition

$$x^2-15\,x=-54$$

therefore

$$x^2-15\,x+\left(\frac{15}{2}\right)^2=-54+\left(\frac{15}{2}\right)^2=54+\frac{225}{4}=\frac{9}{4}$$

taking the square root of both sides

$$x-\frac{15}{2}=\pm\frac{3}{2}$$

therefore

$$x=\frac{15}{2}\pm\frac{3}{2}=9 \text{ or } 6.$$

If $x=9$

$$15-x=6$$

If $x=6$

$$15-x=9$$

Thus the two numbers are 6 and 9. Here although the quadratic equation gives two values of x, yet there is really only one solution of the problem.

TEST QUESTIONS

1. *What is the difference between a symbol and a sign!*
2. *Give the various signs used in algebra.*
3. *What is the difference between a coefficient and an exponent?*
4. *Give examples of binomial and trinomial expressions.*
5. *What is the degree of a term?*
6. *Give example of homogeneous terms.*
7. *Name the three cases in addition.*
8. *What is a rule for adding a negative quantity?*
9. *Give the rule for subtraction.*
10. *Subtract* $5x^2y - 3ab + m^2$ *from* $3x^2y - 2ab + 4n$.
11. *Give rule for removing parentheses.*
12. *Multiply* $-2a$ *by* $3b$.
13. *Multiply* $M+S$ *by* $M+S$.
14. *What are the rules of division?*
15. *Give axioms upon which the solution of equations depends.*
16. *Explain the methods of single and double transposition in solving equations.*
17. *Solve the equation* $3x - 2 = x + 19$.
18. *The sum of two numbers is* 475 *and their difference* 23; *find the numbers.*
19. *What are simultaneous equations?*
20. *Solve the equations* $x + y = 7$ *and* $3x - y = 13$.

21. *What is a quadratic equation?*
22. *What is the difference between a pure and an adfected equation?*
23. *Give a few points on quadratic equations.*

CHAPTER 12

Calculus

In higher mathematics, calculus is *a method of calculating which consists in the investigation of the infinitesimal changes of quantities when the relations between the quantities are known.*

There are two divisions of the subject known as:

1. Differential calculus;
2. Integral calculus.

One is the exact opposite of the other.

Differential calculus has to do with *the division of a quantity into infinitesimal small parts,* whereas integral calculus considers *the addition of these small parts to produce the quantity.*

The process employed in these two branches of calculus is called respectively:

1. Differentiation;
2. Integration.

Differential Calculus

In fig. 324 take a line and divide it into such a large number of parts that each one of these parts is *infinitesimally small.* Calling the length of the line x, the length of one of the infinitesimally small parts is represented by the expression

$$\Delta x \text{ or } \delta x \text{ or more frequently } dx$$

which is called the differential of the line x.

In fig. 324 only one dimension was considered,—length. Differentiation may also be applied to areas and volumes.

Thus in fig. 325 let L be a square having sides of the length x.

Now suppose the length of each side be increased by an infinitesimally small amount dx shown greatly exaggerated in the figure. This will give a new and larger square, made up of

old square L+strip M+strip S+little black square E.

The areas of these several parts are

area old square L	$=x \times x = x^2$
" strip M	$=xdx$
" " S	$=xdx$
" little black square E	$=dx \times dx$

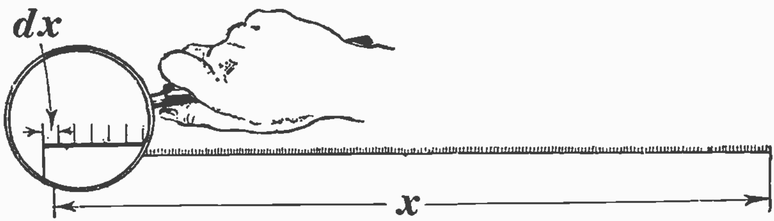


FIG. 324.—Straight line illustrating the *differential*.

from which the area of the addition is

$$M+S+E = xdx + xdx + dx^2$$
$$= 2\,xdx + dx^2$$

In figs. 326 to 328 it will be noted that the *smaller dx becomes, the smaller in more rapid proportion does dx^2 the area of the little black square* E, *become.*

In fact the area of the little black square E decreases so much more rapidly than the strips M and S, that it may be disregarded.

Thus by increasing the length of the sides x of the square L in fig. 325 by the length dx its area is increased by the quantity

$$2xdx$$

that is

$$L+M+S=x^2+2\,x\,d\,x$$

If, instead of increasing the sides of the square by dx as in fig. 325, they be *decreased* by dx as in fig. 329, the area of the square is reduced by the amount $2\,x\,dx$. This infinitesimal area is equal to the differential of the square.

From this the rule follows:

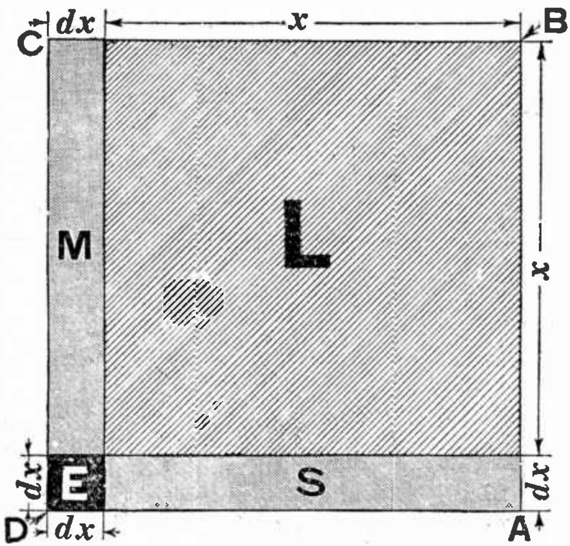
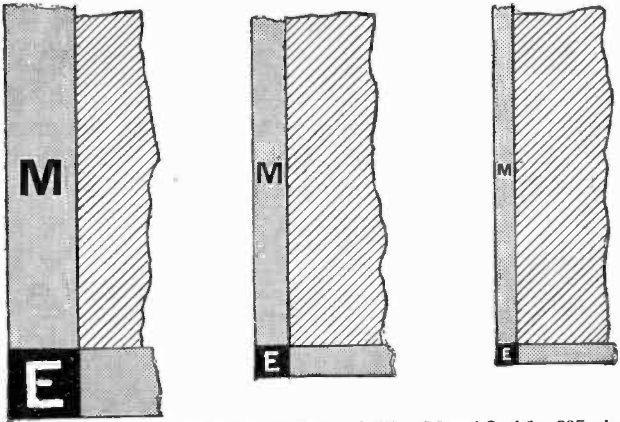


FIG. 325.—Square illustrating the differential. Case 1.

Rule.—*To find the differential of any power of x, reduce the power of x by one, multiply by dx and place before the quantity a coefficient which is the same number as the power of x being differentiated;* thus the differential of $x^7=7\,x^6\,dx$.

By similar investigation it is found that the "differential of"



FIGS. 326 to 328.—Progressive reduction of width of strips M and S of fig. 325, showing that the little black square E may be disregarded.

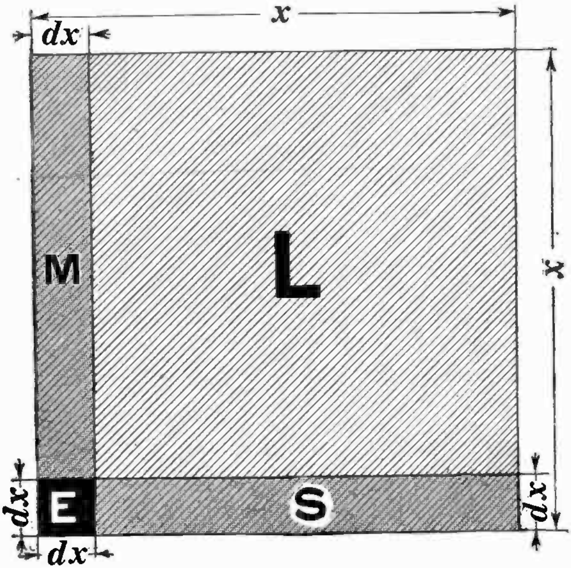


FIG. 329.—Square illustrating the differential. Case 2.

the following quantities, expressed by enclosing the quantity in parenthesis, is

$$d(x^3)=3x^2dx$$
$$d(x^4)=4x^3dx$$

The object of differentiating any quantity is *to get the value of the infinitesimals which make up the quantity.*

Finding the value of the infinitesimal, that is the differential whose gradual addition makes the quantity, is the same as finding out the *rate of growth.* That is, differentiation gives the "rate of growth" or acceleration.

A constant quantity, *since it has no rate of change cannot be differentiated.* Hence, its differential is zero, but if a constant quantity such as 12 be multiplied by a variable quantity such as x, the differential of $12\,x=12\,dx$, and the differential of $2x^2=2$ times $2xdx=4xdx$. Also the differential of

$$2\,x^2+12-3\,x^3=4xdx+0-9x^2dx$$
$$=4xdx-9x^2dx$$

Two Variables.—Sometimes there are two variable quantities dependent upon each other and it is required to find the rate of change of one with respect to the rate of change of the other; when the rate of change of one is known.

Take for instance the equation

$$x=y+2$$

If a definite value be given to one of the variables, a corresponding value can be found for the other, thus:

If $x=0$, then $y=-2$
If $y=0$, $x=2$ etc.

Now in fig. 330 draw the line MS through the points $x=2$ and $y=-2$.

Select any point on the line as A, and another point C very close to A. The co-ordinates of the point A will be x and y and of the point C

$$x+dx$$

$$y+dy$$

Hence in the little triangle ABC

$$AB=dx$$

$$BC=dy$$

and

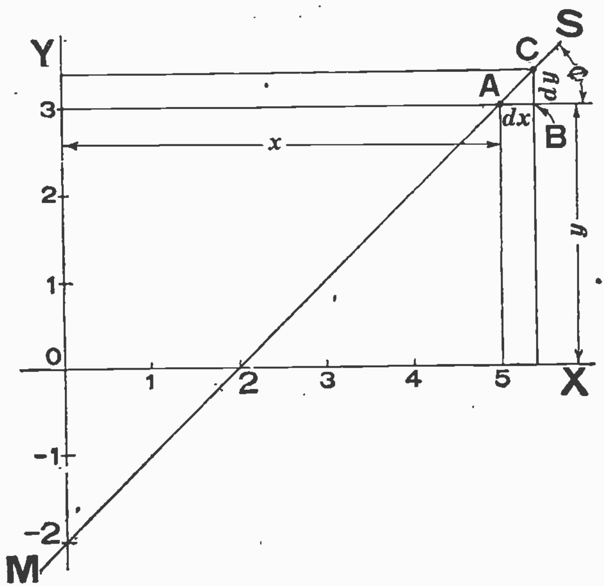


FIG. 330.—Line corresponding to the equation $x=y+2$.

$$\tan\angle BAC=\tan\phi=\frac{CB}{AB}=\frac{dy}{dx}$$

Accordingly if either dx or dy and tan ϕ be known the other can be found.

Sometimes tan ϕ or $\dfrac{dy}{dx}$ is expressed by y_x and $\dfrac{dx}{dy}$ by x_y.

To differentiate a fraction take *the differential of the numerator times the denominator minus the differential of the denominator times the numerator all divided by the square of the denominator*. Thus:

$$d\left(\frac{u}{v}\right)=\frac{vdu-udv}{v^2}$$

Square Root.—*The differential of the square root* of a quantity is equal *to the differential of the quantity divided by* 2 *times the square root of the quantity*. Thus if

$$v=u^{1/2},\text{ that is, } v=\sqrt{u},\ d(v)=\frac{du}{2\sqrt{u}}=\tfrac{1}{2}u^{-1/2}\,du$$

Miscellaneous.—Formulae for differentiating algebraic functions are here given for reference:

1. $d(a)=0.$
2. $d(ax)=a\,dx.$
3. $d(x+y)=dx+dy$
4. $d(x-y)=dx-dy$
5. $d(xy)=x\,dy+y\,dx$
6. $d\left(\frac{x}{y}\right)=\frac{y\,dx-x\,dy}{y^2}$
7. $d(x^m)=mx^{m-1}\,dx$
8. $d(\sqrt{x})=\frac{dx}{2\sqrt{x}}$

Mediate Differentiation.—This is the process of differentiating a variable with respect to some other variable. It is used when it is necessary to find the differential of several terms, some containing x, and some y.

To differentiate a variable x with respect to some other variable y apply the rule which follows.

Rule.—*Differentiate the expression first as to x and then multiply by x_y or vice versa.*

Maxima and Minima.—To find the maximum and minimum value of x in an equation containing x and y:

1. Differentiate the equation with reference to y.

2. Solve for the value of x_y.

(See end of paragraph on Two Variables, page 292.)

3. Make the result obtained in 2 equal to 0 and then solve for the value of y in the resulting equation.

Example.—To find the value of x which will render the function y a maximum or minimum in the equation of the circle $y^2+x^2=R^2$.

$$\frac{dy}{dx}=-\frac{x}{y}; \text{ making } -\frac{x}{y}=0 \text{ gives } x=0.$$

The second differential coefficient is $\frac{d^2y}{dx^2}=-\frac{x^2+y^2}{y^3}$

When $x=o$, $y=R$; hence $\frac{d^2y}{dx^2}=-\frac{1}{R}$, which being negative, y is a maximum for R positive.

In applying the rule to practical examples, first find an expression for the function which is to be made a maximum or minimum.

If in such expression a constant quantity be found as a factor, it may be omitted in the operation; for the product will be a maximum or a minimum when the variable factor is a maximum or a minimum.

Any value of the independent variable which renders a function a maximum or a minimum will render any power or root of that function a maximum or minimum; hence square both members of an equation to free it of radicals before differentiating.

Integral Calculus

In the process of differentiation, a quantity is divided into infinitesimal parts, the opposite process is applied in integration, that is the infinitesimals are *added*. Hence integration is the opposite of differentiation. By definition, an integral is *a functional expression derived from a differential.*

An integral is indicated by the sign:

$$\int$$

which is read "*the integral of.*"

For instance:

$$\int 4x^3\,dx = x^4$$

is read "the integral of $4x^3\,dx = x^4$."

Consider the line in fig. 324 of indefinite length x, to be made up of an infinite number of small parts, as dx.

The integral of these parts could be expressed as:

$$dx+dx+dx+dx\ldots\ldots\ldots\ldots$$

continued to infinity but the integral sign expresses the same thing thus:

$$\int dx = x$$

meaning the *summation* of all the parts which make up the line of the length x.

Now the length x being indefinite and in order to sum up any definite portion of the line x it is integrated *between* limits.

These limits are placed at the top and bottom of the integral sign thus

$$\int_{\text{lower limit}}^{\text{upper limit}}$$

Thus if the upper limit $=5$ and the lower limit 2 it would be expressed:

$$\int_{x=2}^{x=5} dx$$

These two limits indicate exactly between what two points the length of the line is to be determined.

In order to solve the expression:

$$\int_{x=2}^{x=5} dx = (x)_{x=2}^{x=5}$$

substitute inside of the parentheses the upper limit and subtract the lower limit, thus:

$$(x)_{x=2}^{x=5} = 5-2=3$$

Accordingly 3 is the length of the line between the points 2 and 5.

Finding the value of an area involved *integrating one quantity with respect to another.*

The integral

$$\int_{x=a}^{x=b} y\,dx$$

cannot be easily solved but the expression is simplified by replacing the y by some term containing x. This is illustrated in the following example.

Example.—Find the area included between a curve the x axis and the ordinates $x=a$ and $x=b$.

$$\text{Area} = \text{A} = \int_{a}^{b} y\,dx$$

The value of y in terms of x is found from the given equation and substituted in the formula. The initial value of x is a, and the final value b. Similarly, the area included between a curve, the Y axis, and the horizontal lines $y=c$ and $y=d$ is

$$\text{area} = \text{A} = \int_{c}^{d} x\,dy$$

where c and d are the limits of y.

Example.—Find the area bounded by the parabola $y^2=4x$, the x axis and the ordinates $x=4$ and $x=9$.

Fig. 331 shows the area to be found.

The required area is indicated by the expression

$$A = \int_4^9 y\,dx \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Substitute in (1) the value of y in terms of x thus

$$y^2 = 4x \qquad y = 2\sqrt{x}$$

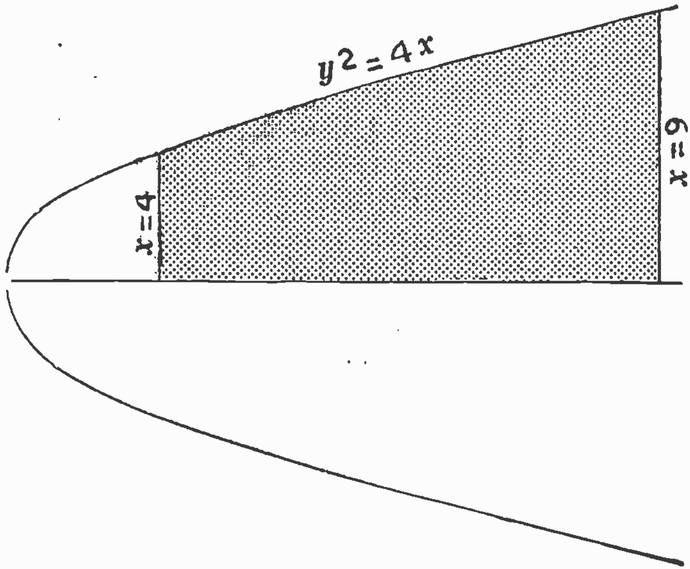


FIG. 331.—Diagram illustrating accompanying example.

Substituting in (1)

$$A = \int_4^9 2\sqrt{x}\,dx$$

$$= \frac{4}{3} x^{3/2} \Big]_4^9 = 25\tfrac{1}{3}$$

TEST QUESTIONS

1. *What is calculus?*
2. *What is the difference between differential and integral calculus?*
3. *What processes are employed in differential and integral calculus?*
4. *Give an example illustrating differential calculus.*
5. *Give various symbols used for expressing the differential.*
6. *If the sides of a square are decreased by dx how much is the area of the square reduced?*
7. *Give rule for finding the differential of any power of x.*
8. *What is the differential of* x^2 *and of* x^4*?*
9. *What is the object of differentiating any quantity?*
10. *Can a constant quantity be differentiated?*
11. *What methods are employed in differentiating two variables?*
12. *Give rule for differentiating a fraction.*
13. *How is the square root of a quantity differentiated?*
14. *What is the differential of* $\sqrt{u}$*?*
15. *Give formulæ for differentiating algebraic functions.*
16. *What is mediate differentiation?*
17. *Explain maxima and minima.*
18. *Define integral calculus.*
19. *What does the sign* $\int$ *indicate?*

20. *Explain the process of integral calculus.*
21. *How is a variable integrated between limits?*
22. *Find the area included between a curve, the x axis and the ordinates.*
23. *Find the area bounded by the parabola* $y^2 = 4x$, *the x axis and the ordinates* $x = 4$ *and* $x = 9$.

SECTION B

PRACTICAL ELECTRICAL CALCULATIONS

Electrical Calculations

(Direct Current)

OHM'S LAW

SYMBOLS	MEANING OF SYMBOLS
$I = \frac{E}{R}$	CURRENT = $\frac{\text{PRESSURE}}{\text{RESISTANCE}}$ THAT IS AMPERES = $\frac{\text{VOLTS}}{\text{OHMS}}$
$R = \frac{E}{I}$	RESISTANCE = $\frac{\text{PRESSURE}}{\text{CURRENT}}$ THAT IS OHMS = $\frac{\text{VOLTS}}{\text{AMPERES}}$
$E = IR$	PRESSURE = CURRENT X RESISTANCE THAT IS VOLTS = AMPERES X OHMS

Example.—A circuit having a resistance of 5 ohms is under a pressure of 110 volts. How much current will flow?

From Ohm's law, amperes = volts ÷ resistance = 110 ÷ 5 = 22 amperes.

Example.—If the resistance of a circuit be 10 ohms, what voltage is necessary for a flow of 20 amperes?

From Ohm's law, volts = amperes × resistance = 20 × 10 = 200 volts.

Example.—On a 110 volt circuit what resistance is necessary to obtain a flow of 15 amperes?

From Ohm's law, resistance = volts ÷ amperes = 110 ÷ 15 = 7⅓ ohms.

OHM TABLE*

	Date	International Ohm	Legal Ohm	B. A. Ohm	Siemens' Ohm
International Ohm	1893–4	1.	1.0028	1.0136	1.0630
Legal Ohm	1884	.9972	1.	1.0107	1.0600
B. A. Ohm	1864	.9866	.9894	1.	1.0488
Siemens' Ohm	....	.9407	.9434	.9535	1.

Example.—Two cells of a battery are connected in series and in opposition. One cell tests 1.05 volts and the other 1.79. What is the resultant voltage in the circuit?

$$1.79 - 1.05 = .74 \text{ volt.}$$

Example.—If an arc lamp require a current of 8 amperes, how much electricity does it consume per hour?

Since one coulomb = one ampere second, the quantity of electricity consumed per hour is

$$\left.\begin{matrix}\text{amperes}\\ 8\end{matrix}\right\} \times \left\{\begin{matrix}\text{seconds}\\ 60\times 60\end{matrix}\right\} = 28{,}800 \text{ coulombs.}$$

*NOTE.—*In the above table* to reduce, for instance, British Association ohms to International ohms, multiply by .9866, or divide by 1.0136; to reduce legal ohms to International ohms, multiply by .9972, or divide by 1.0028, etc.

A current of one ampere will deposit .0003286 gramme of copper in a copper voltameter, or .0003386 gramme of zinc in a zinc voltameter.

Ohm's law shows that the strength of the current falls off in proportion as the resistance in the circuit increases.

Divided Circuits.—The relative strength of current in several branches in parallel is proportional to their conductivities.

Example.—If, in 2 branches the resistance of R = 10 ohms, and R′ = 20 ohms, the current through R will be to the current through R′, as $\frac{1}{10}$ to $\frac{1}{20}$; or, as 2:1, or, in other words, ⅔ of the total current will pass through R, and ⅓ through R′. The joint resistance of the two branches between A and B, will be less than the resistance of either branch singly, because the current has increased facilities for travel. In fact, the joint conductivity will be the sum of the two separate conductivities.

Example.—Two branches in parallel have a resistance of 100 ohms each. What is the joint resistance?

$$\frac{1}{100}+\frac{1}{100}=\frac{2}{100}\text{; the reciprocal is }\frac{100}{2}=50\text{ ohms.}$$

The following formula may be applied for joint resistance

$$\text{Joint resistance}=\frac{R\times R'}{R+R'}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Example.—One of two parallel branches has a resistance of 10 ohms and the other 20 ohms. What is the joint resistance?

Substituting in formula (1):

$$\text{Joint resistance } \frac{10 \times 20}{10+20} = \frac{200}{30} = 6\tfrac{2}{3} \text{ ohms.}$$

This rule *cannot* be employed for more than two branches at a time.

Example.—A current of 42 amperes flows through three conductors in *parallel* of 5, 10 and 20 ohms resistance respectively. Find the current in each conductor.

Solution.—Joint conductance $= \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{7}{20}$

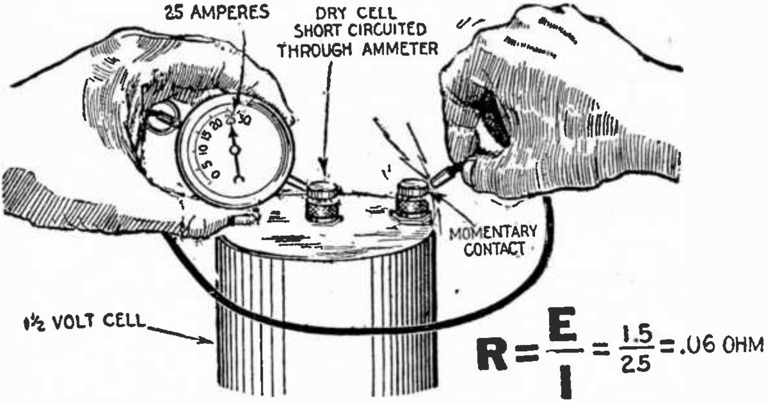


FIG. 1.—Application of Ohm's law to find internal resistance of a dry cell.

Supposing the current to be divided into 7 parts, 4 of these parts would flow in the first conductor 2 in the second and 1 in the third.

The whole current is 42 amperes.

$\frac{4}{7}$ of 42 = 24.
$\frac{2}{7}$ of 42 = 12.
$\frac{1}{7}$ of 42 = 6.

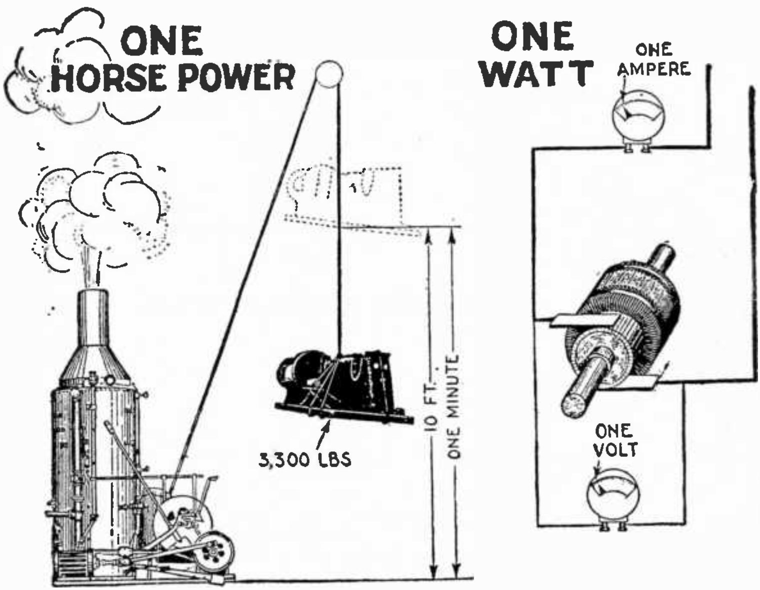
Current in first conductor = 24 amperes.		Ans.
" " second " = 12 "		
" " third " = 6 "		

Power.—The electric unit of power is the watt which is equal to **one volt × one ampere**, that is

$$\text{Power or watts} = IE \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

but by Ohm's law

$$E = I\,R$$



FIGS. 2 and 3.—Examples illustrating one horse power and one watt. ***Rules:* One horse** power = 33,000 ft. lbs. per minute. One watt = one ampere × one volt.

Substituting this value of E in equation (1)

$$\text{Power} = I^2R \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Example.—A current of 12 amperes flows through a resistance of 23 ohms. What is the power?

Substituting in equation (2):

$$\text{Power} = 12^2 \times 23 = 3{,}312 \text{ watts}$$

Electrical Horse Power.—One watt is equivalent to one joule per second or 60 joules per minute. One joule, in turn, is equivalent to .7374 ft. lbs., hence 60 joules equal:

$$60 \times .7374 = 44.244 \text{ ft. lbs.}$$

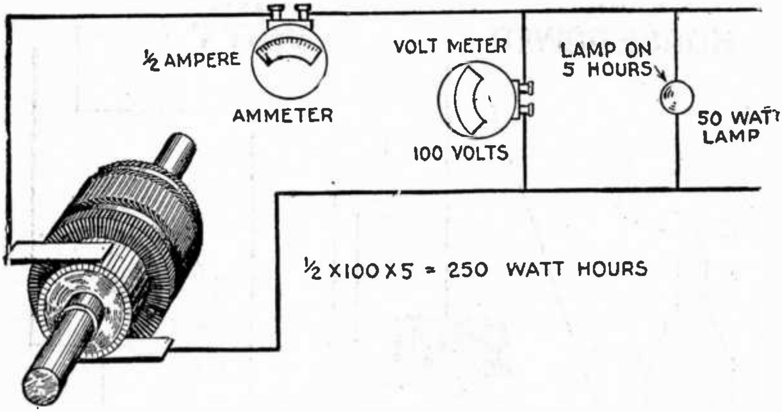


Fig. 4.—Example illustrating watt hours Rule: Watt hours = amperes × volts × hours.

Since one horse power = 33,000 ft. lbs. per minute, the electrical equivalent of one horse power is

$$33{,}000 \div 44.244 = 746 \text{ watts.}$$

or,

$$\frac{746}{1{,}000} = .746 \text{ kilowatt } (kw.)$$

Again, one kilowatt or 1,000 watts is equivalent to

$$1{,}000 \div 746 = 1.34 \text{ horse power}$$

Example.—What is the horse power delivered by a dynamo which furnishes 23 amperes at 475 volts?

$$\text{horse power} = \frac{23 \times 475}{746} = 14.6$$

Joule's Law.—*The* **heat** *generated in a conductor by an electric current is proportional to the resistance of the conductor, the time during which the current flows, and the square of the strength of the current.*

Calories per second = ohms × amperes² × seconds × .24 expressed by symbols

$$H = .24\ I^2\ R\ T$$

in which

H = heat or calories per second.

R = ohms.

I = amperes.

T = time in seconds.

Example.—How much heat is produced by a current of 20 amperes flowing for ½ hour in a circuit whose resistance is 6 ohms?

$$H = .24 \times 6 \times 20^2 \times (30 \times 60) = 1{,}036{,}800 \text{ calories}$$

Example.—How much greater is the heating effect of a current of 32 amperes than that of one of 8 amperes, in a wire of 10 ohm resistance? No time is mentioned.

$$32 \times 32 \times 10 = 10{,}240$$

$$8 \times 8 \times 10 = 640$$

$$10{,}240 \div 640 = 16 \text{ times as great}$$

Example.—A current of 10 amperes passes through a wire one centimeter in circumference. The wire has a resistance of .3 ohm per 100 meters. How many degrees will its temperature be increased?

The watts per second due to the passage of the current are given by the expression $I^2R = 100 \times .3 = 30$.

The surface area of 100 meters of the wire is 10,000 square centimeters.

The calories expended on 100 meters are $30 \times .24 = 7.2$.

The calories per square centimeter are the product of 7.2 by the area of the wire, or $7.2 \times 10^{-4} = .00072$ calorie. Dividing, .00072 by .00025 gives 2.88° C., the degrees C. above the temperature of the air to which the wire would be heated by such a current.

Thermo-Voltage.—When two dissimilar conductors are connected and the joint heated, *a difference of pressure is produced.*

The following table gives the difference of voltage set up by contact of some of the metals.—

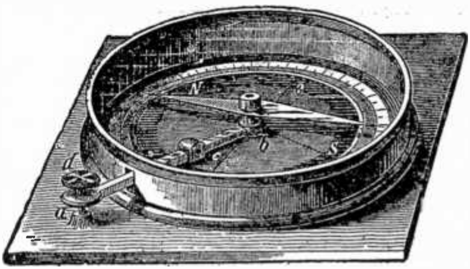


FIG. 5.—Simple compass. *It consists of* a magnetic needle resting on a steel pivot, protected by a brass case covered with glass, and a graduated circle marked with the letters N, E, S, W, to indicate the cardinal points. *a,b*, is a lever which arrests the needle by pushing it against the glass when the button *d*, is pressed.

Zinc / Lead	.210	Iron / Copper	.146
Lead / Tin	.069	Copper / Platinum	.238
Tin / Iron	.313	Platinum / Carbon	.113

Magnetism

Magnetism.—*If two points such as minute magnet poles act upon each other the action will vary inversely with the square of the distance separating them and with the product of the strength of the poles.*

Expressed by a formula

$$F = N \frac{m\,m'}{l^2}$$

in which

F = force

N = a constant

m and m' = magnetic quantities acting upon each other

l = distance between m and m'

Example.—Two equal magnet poles act upon each other with a force of 3.4 dynes. The distance between them is 2.5 cm. What is the strength of each?

By the conditions of the problem, $m = m'$, and the formula becomes

$$F = \frac{m^2}{l^2}$$

Substituting for F and l their values

$$3.4 = \frac{m^2}{2.5^2} \text{ or } m^2 = 2.5^2 \times 3.4 = 21.25 \text{ and } m = 4.6$$

Each pole, therefore, is equal to 4.6 unit magnet poles.

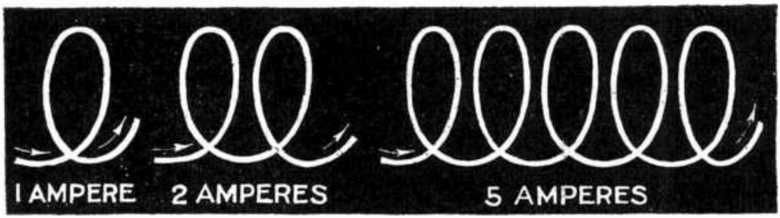
Rule.—*The number of lines of force (called induction, per square centimeter) set up in an empty helix (air core solenoid), equals the product of the number of turns (per centimeter in length of the coil) times the number of amperes multiplied by* 1.257. *When the inch is used the constant is* 3.2.

Example.—How many turns of a solenoid to the inch are required to set up an induction of 400 lines with a current of 5 amperes?

One ampere turn gives 3.2 lines of force; $400 \div 3.2 = 125$ ampere turns; $125 \div 5 = 25$.

Example.—A current of 5 amperes flows through a 24 in. helix of 192 turns. How many lines of force per inch inside the coil?

$192 \div 24 = 8$ turns per inch. $8 \times 5 \times 3.2 = 128$.



FIGS. 6 to 8.—***Ampere turns.*** ***By definition*** **the ampere turns are equal to the** ***product of the current passing through a coil multiplied by the number of turns in the coil.*** **Thus, in fig. 6, 1 ampere ×1 turn = 1 ampere turn; in fig. 7, 2 amperes ×2 turns = 4 ampere turns; in fig. 8, 5 amperes ×5 turns =25 ampere turns.**

Rule.—*One ampere turn sets up* 1.2566 *units of magnetic pressure. Accordingly*

$$\text{magnetic pressure} = 1.2566 \times \text{turns} \times \text{amperes}$$

The unit of magnetic pressure is the *gilbert* (named after William Gilbert, the English physicist) and is equal to

$$1 \div 1.2566 \text{ ampere turn} = .7958 \text{ ampere turn}$$

Example.—How many *gilberts* generated by 23 amperes flowing in a solenoid of 475 turns?

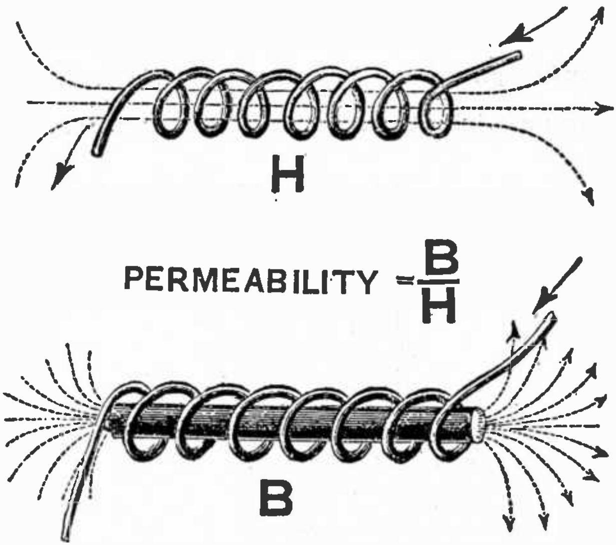
Number of ampere turns $= 23 \times 475 = 10{,}925$

Since 1 gilbert $= .7958$ ampere turn

Number of gilberts $= 10{,}925 \div .7958 = 13{,}728$.

Rule.—*The traction or weight a magnet will lift when attached to its armature is equal to the square of the number of lines of force per sq. in. multiplied by the area of contact and divided by* 72,134,000.

Example.—A bar of iron is magnetized to 12,900 lines per sq. in.; its cross section is 3 sq. ins. What weight can it sustain, assuming the armature not to change the intensity of magnetization?



Figs. 9 and 10.—Illustrating the effect of introducing an iron core into a solenoid. Few lines pass through the air core, while many pass through the iron core. The number of lines B, passing through a unit cross section of the iron core divided by the number of lines H, passing through a unit cross section of the air core is called the *permeability* and designated by the Greek letter μ

$$\frac{12{,}900^2 \times 3}{72{,}134{,}000} = 6.92 \text{ lbs.}$$

The reluctance of a material is *the inverse of its permeability*, and is expressed as follows:

$$\text{reluctance} = \frac{1}{\text{permeability}} \times \frac{\text{length in ins. (or cm.)}}{\text{cross section in sq. in. (or cm.)}}$$

Equivalents.—The following equivalents will be found useful.—

To change	*To*	*Multiply or divide*
Lines of force per sq. cm.	Lines of force per sq. in.	×6.45
Lines of force per sq. in.	Lines of force per sq. cm.	÷6.45
Magnetizing force per sq. cm.	Ampere turns per in. in length	÷3.2 and ×6.45
Ampere turns per in. in length	Magnetizing force per sq. cm.	×3.2 and ÷6.45

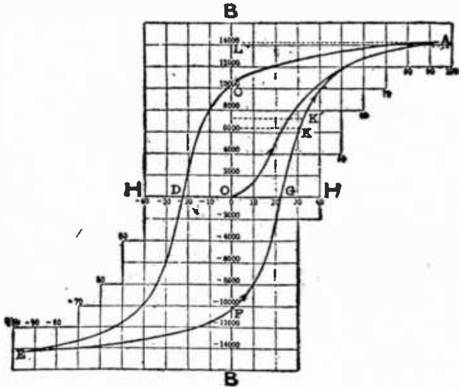


FIG. 11.—Hysteresis loop or curve showing how *B*, changes when H, is periodically varied. *In the figure* H = number of lines of force per sq. cm. (strength of field) and B = number of lines of induction per sq. cm. If now H, be gradually diminished to zero, it is found that the value of B, for any given value of H, is considerably greater when that value of H, was reached by *decreasing* H, from a higher value, than when the same value was reached by *increasing* H, from a lower value.

There is a loss of energy due to hysteresis.

Ewing gives the value for the energy in ergs dissipated per cubic centimeter, for a complete cycle of doubly reversed strong magnetization for a number of substances as follows:

Ergs per Cubic Centimeter

(According to Ewing)

Substance	Energy dissipated (ergs)
Very soft annealed iron	9,300
Less " " "	16,300
Hard drawn steel wire	60,000
Annealed " "	70,000
Same steel glass hard	76,000
Piano steel wire annealed	94,000
" " " normal temper	116,000
" " " glass hard	117,000

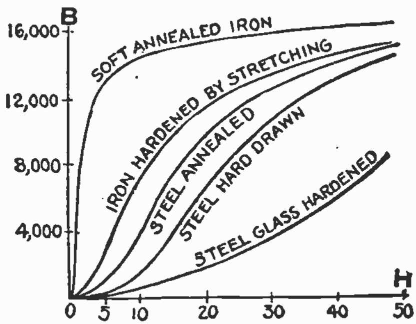


FIG. 12.—*BH* curves for iron and steel

Approximately 28 foot pounds of energy are converted into heat in making a double reversal of strong magnetization in a cubic foot of iron.

Example.—A ring of iron 200 cm. long with a cross section of 30 sq. cm. and a permeability of 700 when 50,000 lines of force pass through it, has wound upon it a coil of 400 turns. How much current is required to set up this magnetization?

Number of lines = magnetic pressure ÷ reluctance.

$$\text{Reluctance} = \frac{1}{700} \times \frac{200}{30} = \frac{200}{21.000} = \frac{2}{210}$$

$$\text{Number of lines: } 50{,}000 = 1.257 \times 400 \times I \div \frac{2}{210}$$

$$= \frac{210 \times 1.257 \times 400 \times I}{2}$$

$$= 105 \times 1.257 \times 400 \times I$$

$$= 42{,}000 \times 1.257 \times I$$

$$= 52{,}794 \times I$$

$$I = 50{,}000 \div 52{,}794$$

$$I = .95 \text{ ampere.}$$

Electro-magnetic Induction.—The unit of electric pressure, called the *volt*, is the electric pressure produced by cutting 100,000,000 lines per second, usually expressed 10^8.

Example.—If a coil of wire of 50 turns cut 100,000 lines in 1/100 of a second, what will be the induced voltage?

The number of lines cut per second per turn of the coil is

$$100{,}000 \times 100 = 10{,}000{,}000$$

The total number of lines cut by the coil of 50 turns is

$$10{,}000{,}000 \times 50 = 500{,}000{,}000$$

which will induce a pressure of

$$500{,}000{,}000 \div 10^8 = 5 \text{ volts}$$

Table of Sparking Distances in Air*

Volts.	Distance. (Inches.)	Volts.	Distance. (Inches.)
5000	.225	60000	4.65
10000	.47	70000	4.85
20000	1.00	80000	7.1
30000	1.625	100000	9.6
35000	2.00	130000	12.95
45000	2.95	150000	15.00

*NOTE.—These values are correct for effective sinusoidal voltages.

Table of Induction Coil Dimensions

Length of spark......	$\frac{3}{8}$ inch	$\frac{1}{2}$ inch	1 inch	2 inches
Size of bobbin ends...	$2\frac{1}{8}\times1\frac{1}{4}$	$2\frac{1}{2}\times\frac{5}{16}$	$3\times\frac{3}{8}$	$4\times2\frac{3}{4}\times\frac{3}{8}$
Length of bobbin.....	4	$5\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$
Length and diameter of core............	$4\frac{1}{4}\times\frac{7}{16}$	$6\times\frac{5}{8}$	$6\frac{1}{2}\times\frac{3}{4}$	—
Size of base..........	$7\frac{1}{4}\times3\frac{1}{4}\times1\frac{1}{2}$	$9\times5\times2$	$14\frac{1}{2}\times6\times1\frac{3}{4}$	$12\times7\frac{1}{2}\times3\frac{3}{4}$
Size of tinfoil sheets...	4×2	$5\frac{1}{2}\times3\frac{1}{4}$	6×4	6×6
Number of tinfoil sheets.............	36	40	40	60
Size of paper sheets...	5×3	$6\frac{1}{2}\times4\frac{1}{4}$	9×5	—
Primary coil.........	No. 18	No. 18	2 layers No. 16, silk covered.	2 layers 14B W. G. silk covered.
Secondary coil........	$\frac{3}{4}$ lb. No. 40	1 lb. No. 40	$1\frac{1}{4}$ lbs. No. 38	$2\frac{1}{2}$ lbs. No. 36.

Coil Winding Calculations.—The following formulæ are being given without the usual individual illustrations, the relations being sufficiently clear to those requiring their use. It should be noted that it is impossible to accurately state the value of turns per sq. in. ohms per cu. in., etc. These values are dependent upon winding conditions, and will therefore, vary considerably between different types of machines, and even between different sizes of coils. The tables as given are average values and results derived therefrom do not under ordinary conditions vary more than 5% either way. All resistance data is based on 68° F. or 20° C.

Referring to fig. 13 the following is the notation:

L = Length of winding space
D = Outside diameter of winding
d = Diameter of insulated core
M = Mean diameter
T = Thickness of winding
V = Winding volume
R = Total resistance
ρ = Resistance per cu. in. (Table B)
s = Resistance per lineal in. (Table A
p = Resistance per lb.
N = Total number of turns
n = Turns per sq. in. (Table D)
W = Total weight of insulated wire
w = Weight per cu. in. (Table E)
m = Weight per 1000 ft. (Table F)

Resistance Per Inch

Table A

B. & S.	OHMS	B. & S.	OHMS	B. & S.	OHMS
8	.0000552	19	.0006698	30	.008583
9	.0000659	20	.0008450	31	.01082
10	.0000831	21	.001065	32	.01365
11	.0001047	22	.001343	33	.01722
12	.0001322	23	.001693	34	.02171
13	.0001666	24	.002136	35	.02736
14	.0002101	25	.002692	36	.03452
15	.0002649	26	.003396	37	.04352
16	.0003341	27	.004281	38	.05487
17	.0004212	28	.005399	39	.06920
18	.0005312	29	.006809	40	.08725

Formulæ showing the relations between given factors.

$$T = \frac{D-d}{2}$$

$$R = Vr = \pi MLTr = \pi MNs = Wp$$

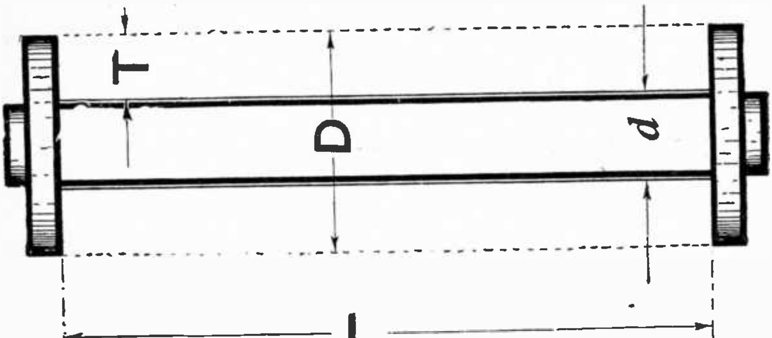


FIG. 13.—Diagram of coil to accompany coil winding calculations.

$$M = \frac{D+d}{2} = T+d \qquad N = LTn = \frac{R}{\pi Ms}$$

$$D = \sqrt{\frac{4V+\pi Ld^2}{\pi L}} \qquad W = \frac{R}{p} = Vw = \pi MLTw = \frac{\pi MNm}{12{,}000}$$

$$V = \pi MLT = \frac{\pi L\,(D^2-d^2)}{4} = \frac{R}{r} = \frac{W}{w}$$

To find size of wire, take the size having in the tables following a value nearest corresponding to that determined by either of the following formulæ:

Ohms Per Cubic Inch

Table B

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	.00315	.00293	.00265	....	....	.00287	....
9	.00475	.00435	.00388	....	....	.00422	....
10	.00748	.00698	.00631	....	....	.00664	....
11	.01183	.01088	.00974	....	....	.01047	....
12	.01878	.01718	.01519	....	....	.01651	....
13	.0295	.0266	.0233	....	....	.0251	....
14	.0464	.0416	.0359	....	....	.0393	....
15	.0734	.0650	.0551	....	....	.0609	....
16	.1162	.1042	.0869	.1172	.1092	.0966	.1089
17	.1840	.1613	.1331	.1840	.1705	.1508	.1718
18	.2910	.2508	.2008	.2910	.2672	.2326	.2682
19	.4560	.3890	.3048	.4565	.4145	.3560	.4165
20	.7200	.6008	.4605	.7165	.6430	.5440	.6500
21	1.134	.9240	.6920	1.123	.9960	.8310	1.007
22	1.800	1.515	1.162	1.766	1.545	1.354	1.578
23	2.820	2.320	1.744	2.743	2.370	2.066	2.438
24	4.488	3.557	2.596	4.293	3.642	3.150	3.790
25	7.080	5.440	3.822	6.645	5.570	4.820	5.867
26	11.27	8.300	5.740	10.05	8.510	7.318	9.100
27	17.75	12.52	8.330	15.75	12.89	11.08	13.92
28	28.34	18.90	12.15	24.83	19.54	16.74	21.75
29	44.32	28.05	17.30	37.65	29.08	24.91	33.12
30	70.15	42.08	25.15	58.45	43.75	37.08	50.56
31	110.4	62.45	36.05	89.40	65.08	55.40	77.60
32	172.6	91.45	50.76	134.7	95.40	81.35	116.8
33	279.0	134.0	71.30	208.0	140.5	120.8	179.1
34	433.2	195.6	99.77	309.5	205.8	175.	265.
35	684.5	281.8	138.7	459.6	297.3	251.7	396.7
36	1094.	405.7	191.6	685.6	429.	364.2	597.3
37	1721.	576.7	263.	1014.	613.5	522.	817.5
38	2693.	817.7	357.	1497.	875.8	735.5	1294.
39	4332.	1148.	480.	2193.	1235.	1048.	1927.
40	6770.	1605.	650.	3202.	1736.	1461.	2791.

Ohms Per Pound

Table C

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	.0124	.0124	.0123	....	....	.0122	....
9	.0197	.0197	.0197	....	....	.0194	....
10	.0314	.0313	.0310	....	....	.0310	....
11	.0497	.0498	.0492	....	....	.0488	....
12	.0791	.0788	.0778	....	....	.0773	....
13	.1255	.1250	.1232	....	....	.1227	....
14	.2005	.1960	.1953	....	....	.1958	....
15	.3180	.3150	.3080	....	....	.3100	....
16	.5050	.4990	.4885	.5080	.5040	.4890	.5000
17	.8051	.7900	.7730	.8075	.8005	.7785	.7960
18	1.278	1.253	1.218	1.283	1.270	1.250	1.292
19	2.032	1.990	1.915	2.038	2.015	1.950	2.002
20	3.239	3.145	3.050	3.235	3.190	3.100	3.183
21	5.138	4.980	4.760	5.140	5.070	4.870	5.030
22	8.186	7.865	7.435	8.150	8.035	7.720	8.000
23	12.97	12.41	11.78	12.94	12.75	12.16	12.63
24	20.60	19.65	18.35	20.58	20.15	19.20	20.00
25	32.70	31.10	28.60	32.58	31.75	30.15	31.72
26	51.95	48.90	44.50	51.50	50.25	47.60	50.30
27	82.55	77.15	68.00	81.75	79.60	74.00	79.60
28	131.2	121.5	106.3	129.6	125.9	118.2	126.3
29	208.7	192.0	167.	205.8	198.0	187.0	198.8
30	331.5	303.0	260.5	326.0	310.0	289.5	312.2
31	526.5	471.0	400.0	517.0	486.0	451.5	493.0
32	836.5	735.0	607.5	816.0	758.5	699.8	776.0
33	1332.	1135.	910.0	1280.	1180.	1085.	1227.
34	2118.	1762.	1351.	2020.	1850.	1680.	1931.
35	3352.	2730.	2050.	3175.	2905.	2550.	3020.
36	5340.	4170.	3040.	5035.	4500.	3907.	4755.
37	8460.	6360.	4340.	7885.	7050.	5750.	7420.
38	13490.	9400.	6290.	12350.	10780.	8465.	11520.
39	21450.	13800.	8875.	19550.	16500.	12370.	18000.
40	34100.	20150.	12500.	31500.	24380.	18350.	27810.

$r = \frac{R}{V}$ $n = \frac{N}{LT}$

The following examples show the easiest method of working out the three principal forms of coil winding problems:

(By Slide Rule)

Example.—Given bobbin and wire—to find the winding data.

L=4″ D=4″ d=1⅛″ No. 24 Enamel.

Given:
D =4
d =1.125 subtracting
2)2.875 dividing
T =1.438 $\left(\frac{D-d}{2}\right)$

d =1.125 adding

M=2.563 (T+d)
πM=3.1416×2.563
=8.038
V =8.038×4×1.438
=46.23 cu. in. (πMLT)

R =46.23×4.488 (Vr)
=207.4 ohms.

N =4×1.438×2,100 (LTn)
=12,090

$\left(\frac{R}{\pi Ms}\right)$

Also N =207.4÷(8.038×.002136)
=12,060 turns
W=46.23×.2178 (Vw)
=10.07 lb.

Also W=207.4÷20.60 $\left(\frac{R}{P}\right)$
=10.07 lb.

Example.—Given bobbin, resistance and insulation of wire—to find size of wire.

Given: L=2¾″ D=1½″ d=½″ 400 ohms. Silk enamel

By the above method:
V=8.635 cu. in.

r=400÷8.635 $\left(\frac{R}{V}\right)$
=46.29 ohms.

Table B indicates No. 30 Silk enamel as being nearest in value to that required.

Example.—Given winding length and diameter of insulated core, resistance and wire—to find the number of turns.

Given: L=2″ d=.4″ R=125 ohms. No. 30 S. S.

Turns Per Square Inch

Table D

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	57	53	48	......	......	52	......
9	72	66	59	......	......	64	......
10	90	84	76	......	......	80	......
11	113	104	93	......	......	100	......
12	141	129	114	......	......	124	......
13	177	160	140	......	......	151	......
14	221	198	171	......	......	187	......
15	277	245	208	......	......	230	......
16	348	312	260	351	327	289	326
17	437	383	316	437	405	358	408
18	548	472	378	548	503	438	505
19	681	581	455	682	619	532	622
20	852	712	545	848	761	644	769
21	1065	868	650	1055	935	780	946
22	1340	1128	865	1315	1150	1008	1175
23	1665	1370	1030	1620	1400	1220	1140
24	2100	1665	1215	2010	1705	1475	1775
25	2630	2020	1420	2470	2070	1790	2180
26	3320	2445	1690	3005	2510	2155	2680
27	4145	2925	1945	3680	3010	2590	3275
28	5250	3500	2250	4600	3620	3100	4030
29	6510	4120	2560	5530	4270	3660	4865
30	8175	4900	2930	6810	5100	4320	5890
31	10200	5770	3330	8260	6010	5120	7170
32	12650	6700	3720	9870	6990	5960	8560
33	16200	7780	4140	11850	8160	7020	10400
34	19950	9010	4595	14250	9480	8060	12200
35	25000	10300	5070	16800	10870	9200	14500
36	31700	11750	5550	19850	12430	10550	17300
37	39600	13250	6045	23300	14100	12000	20400
38	49100	14900	6510	27300	15960	13400	23600
39	62600	16600	6935	31700	17850	15150	27850
40	77600	18400	7450	36700	19900	16750	32000

Pounds Per Cubic Inch

Table E

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	.2540	.2362	.2154	......	......	.2352	......
9	.2411	.2208	.1969	......	......	.2175	......
10	.2382	.2230	.2036	......	......	.2142	......
11	.2381	.2185	.1980	......	......	.2145	......
12	.2374	.2180	.1953	......	......	.2131	......
13	.2350	.2128	.1891	......	......	.2045	......
14	.2314	.2122	.1838	......	......	.2007	......
15	.2308	.2063	.1789	......	......	.1964	......
16	.2301	.2089	.1777	.2318	.2150	.1975	.2178
17	.2287	.2042	.1722	.2279	.2130	.1937	.2158
18	.2277	.2001	.1648	.2268	.2103	.1860	.2075
19	.2262	.1955	.1592	.2240	.2058	.1826	.2062
20	.2224	.1912	.1510	.2215	.2015	.1754	.2041
21	.2208	.1856	.1454	.2186	.1965	.1706	.2002
22	.2198	.1926	.1563	.2167	.1923	.1754	.1973
23	.2173	.1869	.1481	.2121	.1858	.1712	.1930
24	.2178	.1810	.1414	.2116	.1814	.1641	.1896
25	.2165	.1748	.1337	.2040	.1754	.1598	.1849
26	.2170	.1697	.1290	.1952	.1693	.1537	.1810
27	.2151	.1624	.1223	.1927	.1619	.1479	.1749
28	.2160	.1556	.1141	.1915	.1552	.1416	.1725
29	.2128	.1461	.1036	.1830	.1468	.1312	.1666
30	.2121	.1388	.0966	.1793	.1411	.1281	.1619
31	.2097	.1326	.0901	.1729	.1341	.1227	.1574
32	.2064	.1244	.0836	.1651	.1257	.1162	.1505
33	.2094	.1181	.0784	.1625	.1191	.1113	.1458
34	.2045	.1111	.0738	.1532	.1112	.1042	.1372
35	.2041	.1032	.0677	.1447	.1023	.0987	.1313
36	.2049	.0973	.0630	.1361	.0954	.0933	.1256
37	.2032	.0907	.0606	.1286	.0870	.0908	.1196
38	.1996	.0870	.0568	.1212	.0812	.0869	.1123
39	.2019	.0832	.0541	.1122	.0749	.0847	.1070
40	.1985	.0297	.0520	.1017	.0712	.0796	.1003

Weight Per 1000 Feet in Pounds

Table F

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	50.55	50.60	51.15	....	....	51.25	....
9	40.15	40.15	40.60	....	....	40.70	....
10	31.80	31.85	32.18	....	....	32.26	....
11	25.25	25.30	25.60	....	....	25.66	....
12	20.05	20.10	20.40	....	....	20.48	....
13	15.90	15.99	16.20	....	....	16.32	....
14	12.60	12.73	12.91	....	....	12.90	....
15	10.00	10.10	10.33	....	....	10.27	....
16	7.930	8.025	8.210	7.890	7.955	8.180	8.010
17	6.275	6.395	6.540	6.260	6.315	6.480	6.340
18	4.980	5.080	5.235	4.970	5.015	5.160	5.040
19	3.955	4.035	4.220	3.940	3.990	4.120	4.010
20	3.135	3.218	3.373	3.132	3.173	3.275	3.190
21	2.490	2.561	2.685	2.488	2.520	2.625	2.545
22	1.970	2.048	2.168	1.976	2.006	2.118	2.045
23	1.565	1.635	1.727	1.570	1.593	1.668	1.608
24	1.245	1.304	1.398	1.247	1.272	1.335	1.283
25	.988	1.039	1.129	994	1.018	1.071	1.018
26	.7845	.8335	.9140	.7905	.8100	.8570	.8100
27	.6220	.6660	.7560	.6280	.6450	.6845	.6445
28	.4940	.5325	.6075	.4980	.5140	.5480	.5140
29	.3915	.4255	.4890	.3970	.4130	.4375	.4120
30	.3105	.3400	.3955	.3160	.3330	.3555	.3295
31	.2465	.2762	.3257	.2517	.2678	.2874	.2635
32	.1960	.2230	.2700	.2100	.2170	.2348	.2115
33	.1550	.1816	.2270	.1611	.1750	.1904	.1683
34	.1230	.1478	.1928	.1290	.1412	.1551	.1348
35	.0980	.1202	.1600	.1035	.1130	.1286	.1085
36	.0776	.0994	.1361	.0823	.0920	.1062	.0872
37	.0616	.0822	.1204	.0663	.0740	.0908	.0704
38	.0488	.0702	.1049	.0534	.0623	.0778	.0572
39	.0387	.0602	.0937	.0424	.0504	.0669	.0463
40	.0307	.0519	.0838	.0345	.0429	.0571	.0376

Outside Diameters*

Table G

B. & S.	Beld-enamel	Single Cotton	Double Cotton	Single Silk	Double Silk	Cot-enamel	Silk-enamel
8	.1306	.1355	.1415	....	....	.1376	....
9	.1165	.1214	.1274	....	....	.1235	....
10	.1040	.1079	.1129	....	....	.1100	....
11	.0927	.0967	.1017	....	....	.0987	....
12	.0828	.0868	.0918	....	....	.0888	....
13	.0740	.0780	.0830	....	....	.0800	....
14	.0661	.0701	.0751	....	....	.0721	....
15	.0591	.0631	.0681	....	....	.0651	....
16	.0528	.0558	.0608	.0528	.0546	.0578	.0548
17	.0470	.0503	.0553	.0473	.0491	.0520	.0498
18	.0421	.0453	.0503	.0423	.0441	.0471	.0441
19	.0377	.0409	.0459	.0379	.0397	.0427	.0397
20	.0337	.0370	.0420	.0340	.0358	.0387	.0357
21	.0302	.0335	.0385	.0305	.0323	.0352	.0322
22	.0269	.0293	.0333	.0273	.0291	.0309	.0289
23	.0241	.0266	.0306	.0246	.0264	.0281	.0261
24	.0215	.0241	.0281	.0221	.0239	.0255	.0235
25	.0192	.0219	.0259	.0199	.0217	.0232	.0212
26	.0171	.0199	.0239	.0179	.0197	.0211	.0191
27	.0153	.0182	.0222	.0162	.0180	.0193	.0173
28	.0136	.0166	.0206	.0146	.0164	.0176	.0156
29	.0122	.0153	.0193	.0133	.0151	.0162	.0142
30	.0109	.0140	.0180	.0120	.0138	.0149	.0129
31	.0097	.0129	.0169	.0109	.0127	.0137	.0117
32	.0087	.01195	.01595	.00995	.01175	.0127	.0107
33	.0077	.01108	.01508	.00908	.01088	.0117	.0097
34	.0069	.01030	.01430	.00830	.01010	.0109	.0089
35	.0062	.00961	.01361	.00761	.00941	.0102	.0082
36	.0055	.00900	.01300	.00700	.00880	.0095	.0075
37	.0049	.00845	.01245	.00645	.00825	.0089	.0069
38	.0044	.00796	.01196	.00596	.00776	.0084	.0064
39	.0039	.00753	.01153	.00553	.00733	.0079	.0059
40	.0035	.00714	.01114	.00514	.00694	.0075	.0055

*These figures in process of revision at time of printing.

$$V = 125 \div 58.45 \left(\frac{R}{r}\right)$$
$$= 2.140 \text{ cu. in.}$$

$$D = \frac{\sqrt{(4 \times 2.140) + (3.1416 \times 2 \times .4^2)}}{3.1416 \times 2}$$
$$= 1.235''$$

Then by the first method:

$$T = .4175$$
$$N = 2 \times .4175 \times 6810 \qquad (LTn)$$
$$= 5{,}693 \text{ turns}$$

Armature Calculations

For valuable assistance in the preparation of this section, the author is indebted to Mr. P. E. Chapman of St. Louis, Mo., noted authority on armature windings and manufacturer of armature winding machines.

Example in Design.—Determine size of wire, number of turns, etc., for an 8×8 in. armature, for a flux of 70,000 lines per sq. in., 110 volts, 1,200 *r.p.m.*, 5 horse power.

Cross sectional area of armature = 8×8 = 64 sq. ins.

Deduct teeth and shaft from diameter before figuring core area. Thus teeth say ⅞ in. deep, shaft 1½, a total of 3¼ off of 8 in. or net 4¾×8 = 38 sq. ins. core area. Flux in this size core should be 70,000 to 75,000 lines—say 70,000×38 = 2,660,000 lines.

Now since it requires 10^8 or 100,000,000 lines of force cut per second to generate one volt, the number of times this flux must be cut per second to generate the given 110 volts is

$$\frac{\text{required rate of cutting}}{\text{total flux}} = \frac{110 \times 100{,}000{,}000}{2{,}660{,}000} = 4{,}135 \text{ times per sec.}$$

The number of inductors (wires) necessary to place on the armature to cut 4,135 times per second will depend on the speed, thus

$$\text{number of inductors} = \frac{\text{total times per wire per sec.}}{\text{revolutions per sec.}} = \frac{4{,}135}{1/60 \text{ of } 1{,}200} = 207$$

from brush to brush. As there are two inductors per turn, there are two paths in parallel from brush to brush, hence the total number of turns on the armature will be

$$\frac{207}{2} \times 2 = 207$$

Approximate Turns Per Sq. In.

(American Enameled Magnet Wire Co.)

WIRE B. & S. Gauge	Diam. Inches	10 mil Double Cotton	5 mil Single Cotton	4 mil Double Silk	2 mil Single Silk	Enamel	Bare Wire
16	.0508	270	321			359	387
17	.0453	328	396			447	487
18	.0403	395	487			567	616
19	.0359	476	599			715	776
		8 mil	4 mil				
20	.0319	627	775	775	869	885	980
21	.0284	752	950	950	1075	1126	1240
22	.0253	900	1160	1160	1335	1400	1560
23	.0226	1070	1420	1420	1665	1736	1970
24	.0201	1270	1725	1725	2050	2160	2470
25	.0179	1490	2090	2090	2520	2770	3170
26	.0159	1740	2500	2500	3090	3460	3940
27	.0141	2025	3020	3020	3810	4270	4950
28	.0126	2350	3630	3630	4690	5400	6250
29	.0112	2700	4270	4270	5650	6600	7900
30	.0100	3080	5100	5100	6940	8260	10000
31	.0089	3470	5920	5920	8420	10830	12620
32	.0079	3910	6940	6940	10100	13430	16020
33	.0070	4380	8110	8110	12120	16830	20400
34	.0063	4880	9430	9430	14510	21000	25200
35	.0056	5400	10850	10850	17270	26010	31900
36	.0050	5920	12350	12350	20400	31820	40000
				3 mil	1½ mil		
37	.0044			18000	27780	43400	51600
38	.0039			20580	33060	54080	65700
39	.0035			23450	39530	69400	81600
40	.0031			26450	46250	86500	104000

For five horse power at 110 volts

$$\text{total watts} = 746 \times 5 = 3{,}730; \ \text{amperes} = \frac{\text{watts}}{\text{volts}} = \frac{3730}{110} = 34$$

to which must be added about 10% to allow for armature losses in a motor, or field losses in a dynamo, say 38 amperes.

Since there are two paths through the armature in parallel,

$$\text{amperes per circuit} = 38 \div 2 = 19$$

There are many factors that affect the size of the wire selected for any particular winding. In any event the maximum permissible temperature is the final controlling factor. In well proportioned motors and dynamos (not including turbines) 40° C. rise above the room temperature is indicated by good practice as a maximum. Temperatures higher than this should be avoided. Among those factors which increase the temperature are the wattage loss in the wire itself, loss in the iron core of the armature and stator, loss in commutator friction and brush resistance and bearing losses.

Outside Diameters of Wires

(American Enameled Magnet Wire Co.)

Number B. & S. G.	KIND OF INSULATION D. C. C.	Enameled	S. C. C.	D. S. C.	S. S. C.	ENAMEL AND S. C. C.	ENAMEL AND S. S. C.
12	.0908	.0827	.0858	.0848	.0828	.0878	.0848
13	.0810	.0738	.0765	.0760	.0740	.0785	0760
14	.0731	.0658	.0686	.0681	.0661	.0705	.0680
15	.0661	.0587	.0616	.0611	.0591	.0633	.0608
16	.0598	.0523	.0553	.0548	.0528	.0569	.0544
17	.0543	.0468	.0498	.0493	.0473	.0513	.0488
18	.0493	.0417	.0448	.0443	.0423	.0462	.0437
19	.0444	.0372	.0399	.0394	.0374	.0413	.0388
20	.0410	.0333	.0365	.0360	.0340	.0378	.0353
21	.0365	.0298	.0325	.0325	.0305	.0338	.0318
22	.0334	.0266	.0294	.0294	.0274	.0306	.0286
23	.0306	.0237	0266	.0266	.0246	.0277	.0257
24	.0281	.0212	.0241	.0241	.0221	.0252	.0232
25	.0259	.01895	.0219	.0219	.0199	.0229	.0209
26	.0239	.0169	.0199	.0199	.0179	.0209	.0189
27	.0222	.01515	.0182	.0182	.0162	.0192	.0172
28	.0206	.01355	.0166	.0166	.0146	.0175	.0155
29	.0193	.01215	.0153	.0153	.0133	.0162	.0142
30	.0180	.01075	0140	.0140	.0120	.0148	.0128
31	.0169	.00965	0129	.0129	.0109	.0137	.0117
32	.0160	.00865	.0120	.0120	.0100	.0127	.0107
33	.0151	.00765	.0111	.0111	.0091	.0117	.0097
34	.0143	.00685	.0103	.0103	.0083	.0109	.0089
35	.0136	.00605	.0096	.0096	.0076	.0101	.0081
36	.0130	.00545	.0090	.0090	.0070	.0095	.0075
37	.0125	.0049	.0085	.0085	.0065	.0089	.0069
38	.0120	.0044	.0080	.0080	.0060	.0084	.0064
39	.0115	.0038	.0075	.0075	.0055	.0078	.0058
40	.0112	.0034	.0071	.0071	.0051	.0074	.0054

D. C. C. = Double cotton covered.
S. C. C. = Single cotton covered.
D. S. C. = Double silk covered.
S. S. C. = Single silk covered.

The only factor that decreases the temperature is ventilation.

This is markedly influenced by the speed and freedom with which the air plays over the various parts, as for instance, the windage of the armature, any fans, or whether ventilation is restricted by more or less enclosure, etc. Temperature calculations, therefore, become very complex, so much so that they are usually ignored and the following rule of thumb, which will be found very accurate, is used for motors and dynamos of moderate size and of the somewhat freely ventilated types.

Rule.—*For continuous duty, armatures, rotating fields and other moving parts,* **400** *circular mils per ampere of current. For stationary parts, whether stators, fields or armatures, about* **600** *circular mils per ampere. For extra large size machines of the totally enclosed type or when run in hot places, these allowances must be increased.*

Example.—What size wire should be used on the armature of a five horse power, 110 volt motor, assumed to take 20.6 amperes per circuit?

$$20.6 \times 400 = 8{,}240 \text{ circular mils.}$$

The nearest size to this is No. 11 or 8,234 circular mils.

To calculate the size wire for a slotted armature a single slot should be considered, and the wire chosen if possible with reference as to how it will fit in the slot, that is, the size should be such as to fill the slot with the least amount of waste space. ***In design,*** the approximate width of the slot is obtained by *multiplying the diameter of the wire over insulation by the number of turns per layer* and adding double thickness of slot insulation and an allowance for clearance.

The depth of the slot may be obtained by multiplying the diameter of the wire by the number of layers of the winding, adding thickness of the slot insulation and making an allowance for slot closing sticks, which on small very tightly wound armatures may be as thin as 3/32 in. but are usually about 1/8 in. up to 1/4 in. thick on 5 or 10 *h.p.* armatures, and seldom as thick as 3/8 in. on any size armature.

Where the wires are of sufficient size to warrant layer winding owing to the narrow width of the slot, hexagonal bedding seldom occurs, for the narrow width of the slot dictates that only a few turns will occur in a layer. Even where this bedding occurs there will be lost space at the end of each layer, so that in calculations the bedding factor should be ignored.

To find the number of inductors per slot when the speed and flux are fixed, the following formula may be used:

$$\text{inductors per slot} = \frac{10^8 \times \text{volts}}{\text{flux} \times \text{slots} \times \text{rev. per sec.}} \quad \ldots\ldots\ldots (1)$$

Example.—How many inductors per slot are required, to generate 110 volts, with a total flux of 1,920,000 lines, 24 slots and 1,200 revolutions per minute, two pole?

10^8 = 100,000,000
and
1,200 rev. per minute =
1,200 ÷ 60 = 20 rev. per sec.
Substituting in (1)

$$\text{inductors per slot} = \frac{100{,}000{,}000 \times 110}{1{,}920{,}000 \times 24 \times 20} = 12$$

Example.—If the slots of a 24 slot, 24 coil armature be ½ in. wide and there be 12 inductors per ½ coil per slot arranged as a three layer coil winding, what is the maximum size wire that can be used, and current capacity for a four pole machine? If flux be provided to generate 110 volts what horse power will be developed?

FIG. 14.—Fairbanks Morse Type TR machine armature construction.

The table on page 20 gives the outside diameter of variously insulated wire. Double cotton covered should be selected for this armature.

About 1/16 in. in width would be allowed for slot insulation and a few thousandths more for clearance, leaving the net slot width say .42 in. In the example, since there are 12 conductors per ½ coil and the wire is in 3 layers, the turns per layer would be 12 ÷ 3 = 4; which would give .42 ÷ 4 = .105, the maximum outside diameter of the wire that can be used. No. 11 wire .1017 outside diameter is the nearest size.

The area of the No. 11 wire is 8,234 circular mils (page 112) which at 400 mils per ampere gives a capacity of 20.6 amperes.

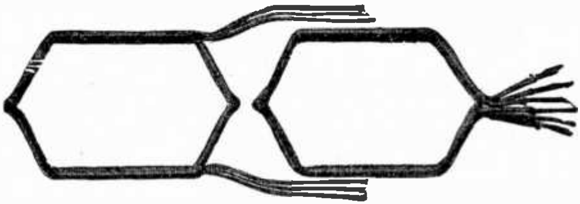
Four pole machines of this size usually have wave wound armatures which have two paths, therefore the carrying capacity of the machine would be 2 times 20.6 amperes or 41.2 amperes.

At 110 volts:

$$\text{watts} = 41.2 \times 110 = 4{,}532$$

Horse power would therefore $= 4{,}532 \div 746 = 6.1$

from which must be subtracted the armature losses which may run as high as 15%, making the net answer say 5 *h.p.* (Field losses are figured separately.)



FIGS. 15 and 16.—Fairbanks Morse wire wound armature coils of type TR machine. *In construction*, the coils are form wound and are thoroughly insulated and baked before assembling in the slots. Material of great mechanical strength as well as high insulating value is used, and the coils are subjected to repeated dippings in insulating compound and to repeated bakings, thus thoroughly driving out all moisture and making a coil which is practically water proof and which will withstand rough handling. These coils when completed, are placed in the slots, where they are retained by bands on the three smaller sizes and by hardwood wedges on the larger sizes. Cores of all sizes are provided with ventilating spaces, running from the surface to the central opening of the core, so that air is drawn through the core and blown out over the windings by the revolution of the armature.

After having determined the size of wire, number of turns per coil, the drop or voltage loss due to the resistance of the winding should be determined to see if this loss be within limit.

Example.—If the average length per turn of the coils in the armature of the previous example be 2 ft., what is the drop or loss of voltage in the armature?

For 12 turns per coil

$$\text{length of each coil} = 12 \times 2 = 24 \text{ ft.}$$

Now while the machine has 4 poles, being wave wound, there are two paths in parallel, hence only half of the coils or 12 coils need be considered in determining the drop.

Accordingly,

$$\text{length of 12 coils} = 24 \times 12 = 288 \text{ ft.}$$

According to the table on page 112, the resistance of No. 11 wire at 140° Fahr. is .00126 ohm per foot, hence

$$\text{resistance of 12 coils} = 288 \times .00126 = .36 \text{ ohm}$$

According to Ohm's law

$$\text{current} = \frac{\text{volts}}{\text{ohms}}, \text{ or volts} = \text{current} \times \text{ohms}$$

Substituting in the expression for volts,

$$\text{volts or "drop"} = 20.6 \times 36 = 7.42$$

which is within standard practice.

Magnet Calculations.—In figuring field magnets, the unit ampere turn is frequently employed and is defined as *the magnetic force due to a current of one ampere flowing through one turn of a magnet winding;* numerically it is equal to the *product of one turn multiplied by one ampere.*

Thus, one ampere flowing through 10 turns, gives 1 × 10 = 10 ampere turns. Again, 10 amperes flowing through 10 turns gives 10 × 10 = 100

NOTE.—***To find the speed*** when the volts, flux, and number of inductors are fixed, use this formula:

$$\text{rev. per sec.} = \frac{100{,}000{,}000 \times \text{volts}}{\text{flux} \times \text{number of slots} \times \text{inductors per slot}}$$

NOTE.—***To find the strength of field*** when the volts, inductors and speed are, fixed, use the formula:

$$\text{flux} = \frac{100{,}000{,}000 \text{ volts}}{\text{inductors per slot} \times \text{number of slots} \times \text{rev. per sec.}}$$

NOTE.—***To find the volts*** when the inductors, flux, and speed are fixed use the formula:

$$\text{volts} = \frac{\text{flux} \times \text{inductors per slot} \times \text{number of slots} \times \text{rev. per sec.}}{100{,}000{,}000}$$

ampere turns. Having fixed the voltage and size of wire it makes no difference in the magnetic effect how many turns are contained in the winding, that is, for a given voltage and size of wire ***the ampere turns remain the same regardless of the number of turns in the winding***.

Thus, if 10 amperes flow through 10 turns of the winding the result is $10 \times 10 = 100$ ampere turns. Now, if the number of turns be doubled, the resistance of the winding will be doubled which will cut down the current one half, that is, 5 amperes $\times$ 20 turns = 100 ampere turns. Of course, this is not strictly true where the magnet is made up of more than one layer, because the diameter of an outer turn being greater than that of an inner turn, its length and resistance is greater, the resulting effect being to slightly decrease the ampere turns as each layer is added. The reason then for increasing the number of turns in a magnet winding is *to cut down the current sufficiently to prevent overheating of the winding.*

Example.—If the winding on a spool 8 ins. in diameter be one inch thick, what is the average diameter of the turns?

The diameter of the inner layer turns is 8 ins., and the outer layer turns, $8 + 2 = 10$ ins., hence,

$$\text{average diameter of the turns} = \tfrac{1}{2}(8 + 10) = 9 \text{ ins.}$$

Example.—If the magnet of the previous example contain 500 turns, what is the length of the winding?

The average diameter of the turns, as obtained, being 9 ins.,

$$\text{length of winding} = \frac{9 \times 3.1416 \times 500}{12} = 1{,}178 \text{ ft.}$$

Example.—If a winding one inch deep be placed on an 8 in. spool, what is the smallest size wire that will give 10,000 ampere turns with 110 volts?

$$\text{average diameter of turns} = \tfrac{1}{2}(8 + 10) = 9 \text{ ins.}$$

$$\text{length of average turn} = \frac{9 \times 3.1416}{12} = 2.36 \text{ ft.}$$

The sectional area of the smallest wire (in circular mils) is obtained from the formula

$$\text{*area wire} = \frac{12 \times \text{length average turn in feet} \times \text{ampere turns}}{\text{volts}} \quad \text{........} \quad (1)$$

Substituting

$$\text{area wire} = \frac{12 \times 2.36 \times 10{,}000}{110} = 2{,}575 \text{ circular mils}$$

*NOTE.—In the formula, 12 is the resistance of 1 mil foot of copper at 130° Fahr nearest size wire from table is No. 16 B. & S. gauge.

Having determined the minimum size of wire, the next step is to find how many turns must be placed on the spool to prevent undue heating.

The watts lost by the current heating the winding *is equal to the square of the current multiplied by the resistance*, that is

$$watts\ lost = amperes^2 \times ohms.$$

Table of Constants

B. & S. Gauge.	*k* Constant for Length.			*c* Constant for Weight.			*a* Constant for Resistance.		
	Double Cotton	Single Cotton	Single Silk	Double Cotton	Single Cotton	Single Silk	Double Cotton	Single Cotton	Single Silk
20	40.9	50 4	56 7	.137	.162	177	415	.512	.576
21	50.4	64.1	72.7				.638	.812	.920
22	60.2	78.0	89.7				.97	1.257	1.445
23	68.3	89.7	104.7				1.387	1.82	2.08
24	83.6	113.6	135.	.1115	.149	169	2.14	2.91	3.46
25	97.2	185.	163.				3.14	4.36	5.27
26	114.	163.	202.				4.65	6 65	8.24
27	135.	202.	255.				6.94	11.75	13.1
28	148.	226.	291.	.0845	.122	148	9.60	14.62	18.82
29	182.	291.	387.				14.85	23.7	31.6
30	201.	334.	454.				20.7	34.4	46.8
31	226.	387.	542				29.36	50.25	70 4
32	255.	454.	655.	.0687	.1045	132	41.8	74.4	107 2
33	291.	542.	812.				60.33	114.5	168.
34	334.	655.	1023.				87.1	170.5	266.5
35	354.	712.	1140				116.2	234.	374.8
36	387	811.	1340	.0492	.0825	1115	160.	335.5	555.
37	422.	897.	1582.				220.5	468.	806.
38	457.	1023.	1825.				366.	674.	1192.
39	496.	1170.	2165.				412.	972.	1795.
40	532.	1300.	2525.	.038	.0615	.0888	557.	1360.	2645.

In proportioning the winding for depth and length, the depth of the winding must be such that there will be from 1 to 2 sq. ins. of surface per watt. With 1 sq. in. per watt, the magnet in operation will be "hot," and with 2 sq. ins., "warm."

Example.—How much radiating surface (neglecting the ends) on a magnet whose outside dimensions are 9 ins. diameter, 6 ins. long

$$\text{area outer cylindrical surface} = 9 \times 3.1416 \times 6 = 169.6 \text{ sq. ins.}$$

Example.—An 8 in. spool is to be wound with No. 16 wire to a depth of 1 in., which, as calculated in a previous example, is the smallest size wire that will give a required 10,000 ampere turns with 110 volts. How many turns of wire must be wound on the spool to prevent undue heating?

For winding magnets what is known as *magnet wire* is used, the wire generally having a single covering of insulation as enamel, silk, cotton or paper.

By reference to the table on page 19 the number of turns per sq. in. of cross sectional area is obtained. Taking a portion of the winding covering an inch length of spool 1 in. deep the sectional area of this portion is 1 sq. in. Referring to the table of magnet wire on page 19, No. 16 wire single covered, will wind 312 turns per sq. in., that is, per inch length of spool. The length of the average turn being 2.36 ft. (as calculated in a previous example)

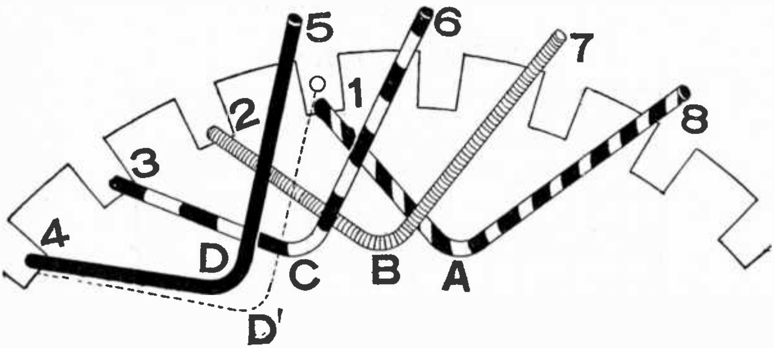


FIG. 17.—Method of placing two layer lap winding coils in armature slots. ***In a two layer*** winding one side of a coil will be at the bottom of a slot and the other at the top of another slot. To place coils in slot, put in the lower sides first as, 1, 2, 3, 4, of coils A, B, C, D, leaving the other side of each coil outside its slot. Evidently when enough coils to make up the inner layer have been placed this way, the upper layer side of the last coil so placed can be put into the slot. Thus, after lower layer side 4, of coil D, is put in the slot, the upper layer side 5, may be put in position on top of side 1, of coil A, thus moving the last coil from point D, to D′, indicated by the dotted line.

$$\text{length of winding per inch of spool} = 312 \times 2.36 = 736 \text{ ft.}$$

and from table its resistance being 4.016 ohms per 1,000 ft.

$$\text{resistance of winding per in. of spool} = \frac{736}{1,000} \text{ of } 4.016 = 3 \text{ ohm.}$$

The outside diameter of the winding being 10 ins.,

radiating surface per inch of spool = 10 × 3.1416 = 31.4 sq. ins.

Now, in any electric circuit, the energy lost by heating the wire, or

$$\text{watts} = \text{amperes}^2 \times \text{ohms} \quad \ldots\ldots\ldots\ldots (1)$$

but by Ohm's law

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}$$

Substituting this value for amperes in equation (1)

$$\text{watts lost} = \frac{\text{volts}^2}{\text{ohms}^2} \times \text{ohms} = \frac{\text{volts}^2}{\text{ohms}}$$

And if the coil be designed for "warm" working by allowing 2 sq. in. radiating surface per watt, then it must be so proportioned that

$$\text{radiating surface} = 2 \times \text{watts lost} = 2 \times \frac{\text{volts}^2}{\text{ohms}} \ldots\ldots\ldots\ldots\ldots \quad (2)$$

In order to determine the length of the coil, first find what resistance would be necessary if the winding were to consist of only the one inch portion just considered. To do this, solve equation (2) for resistance. thus

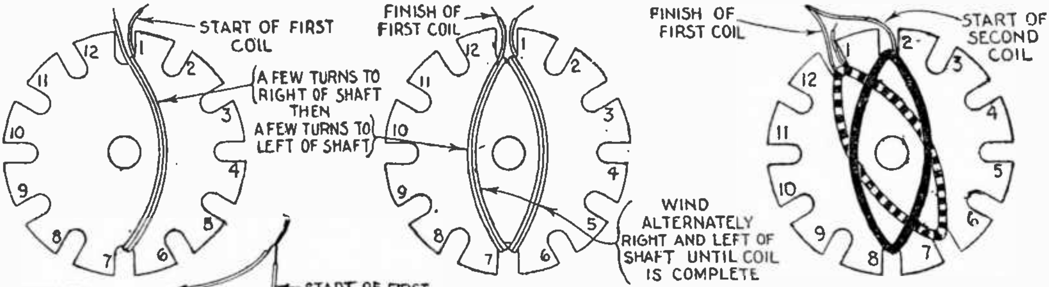
$$\text{ohms} = \frac{2 \times \text{volts}}{\text{radiating surface}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

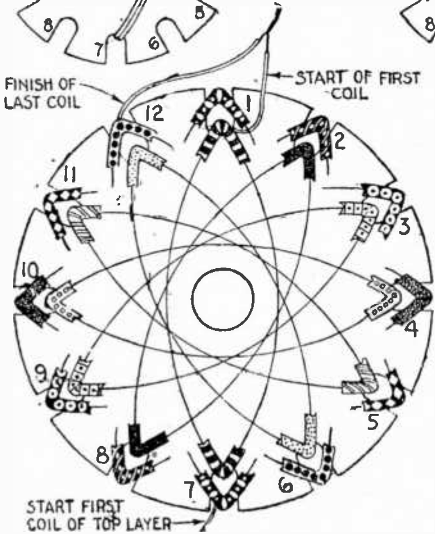
This will give a resistance much greater than the 3 ohms as calculated for that portion of the winding, hence, the spool length of the winding must be increased until the resistance of the winding has a value as obtained by equation (3). Thus, substituting in equation (3), 110 volts, and 31.4 sq. ins. radiating surface in equation (3), the necessary resistance of the winding for "warm" working, is

$$\text{ohms} = \frac{2 \times 110}{31.4} = 7$$

Accordingly, since the resistance of the winding is proportional to its length

$$\text{length of winding} = 1 \text{ in.} \times \frac{7}{3} = 2\tfrac{1}{3} \text{ ins.}$$





BOTTOM LAYER			TOP LAYER		
COIL	SLOTS		COIL	SLOTS	
1	1	7	7	7	1
2	2	8	8	8	2
3	3	9	9	9	3
4	4	10	10	10	4
5	5	11	11	11	5
6	6	12	12	12	6

Figs. 18 to 21.—Operation in winding a ***diametrically split winding.*** In this winding each coil is split around the shaft, the top layer starts where the bottom layer finishes, in front of slot 1. The winding table given above refers to slots and order of winding of each coil. ***In winding*** the two half coils, care must be taken that they are both in the same direction. As the armature is usually rolled over to wind the second half, the direction of winding will be backward to the operator on the second half coil in order that it will be in the same direction on the armature. Loops may be thrown out between coils for the leads.

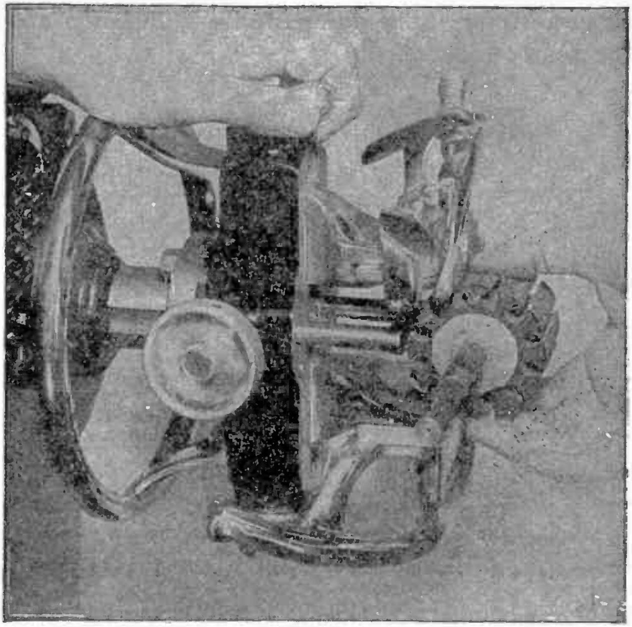


FIG. 22 —Method of placing armature in Chapman machine. Point the commutator end of the core toward you and select a pair of slots which give the correct span for the coils "spotting" them with the thumb and fingers as shown. Set the core in the jaws with their edges aligned, the jaws overhanging the slot about .005 to .010 of an in. the slots projecting. See that both sides are clear and free. Tighten up with the left hand. Let the larger part of the armature project in cord winding. (It will hold them.) The illustration shows the winding machine or head proper, equipped with 3 in. jaws, carrying the lead former and twister. When setting open slot cores, set about ½ of the width of the slot under the jaws, that is, allow about ½ of the slot to project. This is especially desirable where the core is small in diameter.

NOTE.—*Number of armature slots.* As a rule there are not less than ten slots per pole. In multi-polar machines there are at least three or four slots in the space between adjacent pole tips. The area per slot on machines above five horse power is approximately one sq. in. and roughly the capacity of a slot of this area is about 1,000 ampere turns for machines designed to work on less than 500 volts.

Power of a Motor.—The difference between power and work should be clearly understood.

$$\text{work} = \text{distance} \times \text{resistance (lbs.)}$$

$$\text{power} = \frac{\text{work}}{\text{time}} = \frac{\text{distance} \times \text{resistance (lbs.)}}{\text{time}}$$

$$\text{horse power} = \frac{\text{foot pounds per minute}}{33{,}000}$$

(33,000 foot pounds per minute = 1 horse power)

NOTE.—*Number of commutator bars.* This depends on the voltage between the bars. The number of bars may be a multiple of the number of slots. A large number of commutator bars improves the commutation but this advantage is offset by increased difficulties encountered in construction.

NOTE.—*Current density in armature inductors.* In determining the intensity of current much depends upon the provision for ventilation and operating conditions. *In general* 400 cir. mils per ampere is safe for overloads or for operation in hot engine rooms a greater area may be used. For fields and stators use 600 mils per ampere up.

NOTE.—*Magnetic densities.* In small machines the density in the air gap is rarely over 32,000 lines per sq. in.; in large machines the density may be as high as 60,000 lines per sq. in. Density in teeth is usually about 100,000 lines per sq. in. being somewhat less in very small machines. *Density in magnet core:* cast iron may be worked up to about 40,000 or 50,000 lines per sq. in.; wrought iron and cast steel being about 95,000 to 105,000 or more lines per sq. in. Density in yoke: for cast iron the density should be about 30,000 lines per sq. in.; for cast steel, about 75,000 lines, and for wrought iron forgings about 85,000 lines. *Density in armature core:* this may be taken at from 85,000 to 90,000 lines per sq. in. for drum armatures

NOTE.—*Dynamo losses.* These are the mechanical losses due to friction, and electrical losses in the core, field and armature. *Friction loss.* This ranges from 3 to 5% in, respectively, small and large machines of good design. Core loss. In well designed machines this should not exceed 2% of the output at full load. *Field loss.* A portion of the electrical energy generated in the armature is lost in exciting the field magnets. *Armature loss.* This is usually termed the copper loss since it is due to the resistance of the winding; it is a very variable quantity and is equal to the square of the current multiplied by the resistance of a section of the winding between brushes.

NOTE.—*Armature paths in wave and lap windings.* A wave winding has but two paths through the armature, regardless of the number of poles; whereas a lap winding has as many paths as there are poles. This distinction is important in figuring the size of wire for the winding to carry the current without undue heating.

Example.—If the armature pull of a motor having a two foot pulley be such that a weight of 500 lbs. attached to the rim, is just balanced, and the speed be 1,000 revolutions per minute, what is the horse power?

Here, the distance that the pull acts from the center of the shaft is one foot, hence for each revolution the resistance of 500 pounds is overcome through a distance equal to the circumference of the pulley or

$$\pi \times \text{diameter} = 3.1416 \times 2 = 6.2832 \text{ feet.}$$

The *work done* in one minute is expressed by the following equation:

$$\left\{\begin{matrix}\text{work}\\ \text{per}\\ \text{minute}\end{matrix}\right\} = \left\{\begin{matrix}\text{weight}\\ \text{in lbs.}\end{matrix}\right\} \times \left\{\begin{matrix}\text{circumference}\\ \text{of pulley}\\ \text{in feet}\end{matrix}\right\} \times \left\{\begin{matrix}\text{revolutions}\\ \text{per minute}\end{matrix}\right\} = \text{foot pounds}$$

$$= 500 \times 6.2832 \times 1{,}000 = 3{,}141{,}600$$

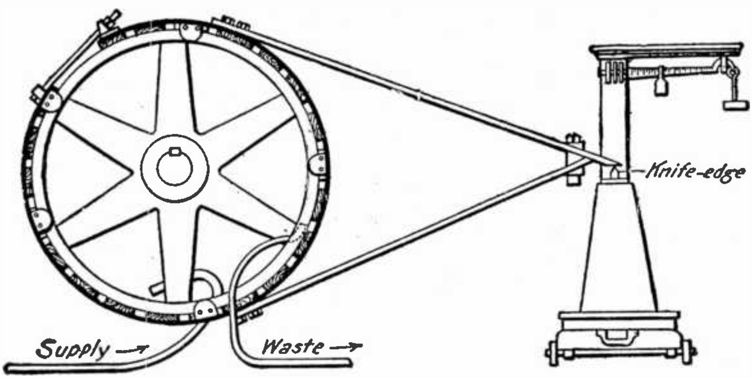


FIG. 23.—Prony brake for determining brake horse power. *It consists of* a friction band ring which may be placed around a pulley or fly wheel, and attached to a lever bearing upon the platform of a weighing scale in such a manner that the friction between the surfaces in contact will tend to rotate the arm in the direction in which the shaft revolves. This thrust is resisted and measured *in pounds* by the scale. In setting up the brake the distance between the center of the shaft and point of contact (knife edge) with the scales must be accurately measured, *the knife edge being placed at the same elevation as the center of the shaft.* An internal channel permits the circulation of water around the interior of the rim as shown. to prevent overheating.

Hence, the power developed is

$$31{,}416 \div 33{,}000 = .952\ h.p.$$

The formula for brake horse power is

$$B.H.P. = \frac{2\pi\ L\ N\ P}{33{,}000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which

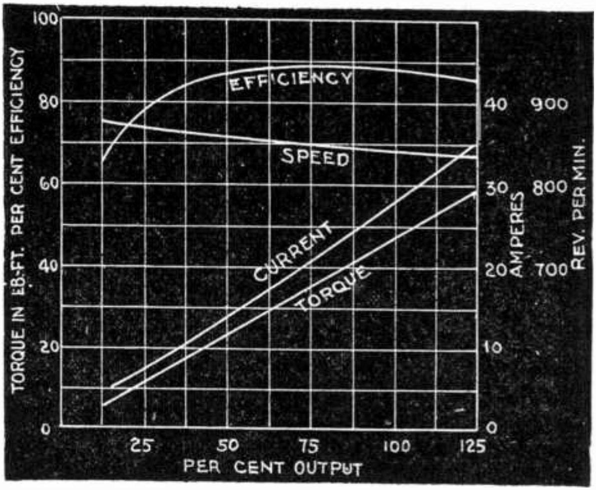


Fig. 24.—Characteristic curves for a General Electric type RC, 7½ horse power 230 volt commutating pole shunt wound motor of open construction.

P = unbalanced pressure or weight in pounds, acting on the lever arm at a distance L;

L = length of lever arm in feet from center of shaft;

N = number of revolutions per minute.

Simplifying, equation (1) becomes

$$B.H.P. = \frac{2\pi\ L\ N\ P}{33,000} = .0001904PLN \ldots\ldots (2)$$

It should be noted in equation (1) that if $L = 33 \div 2\pi$ the equation becomes

$$B.H.P. = \frac{2\pi}{33,000} \times \frac{33}{2\pi} \times N\ P = \frac{NP}{1,000} \ldots\ldots (3)$$

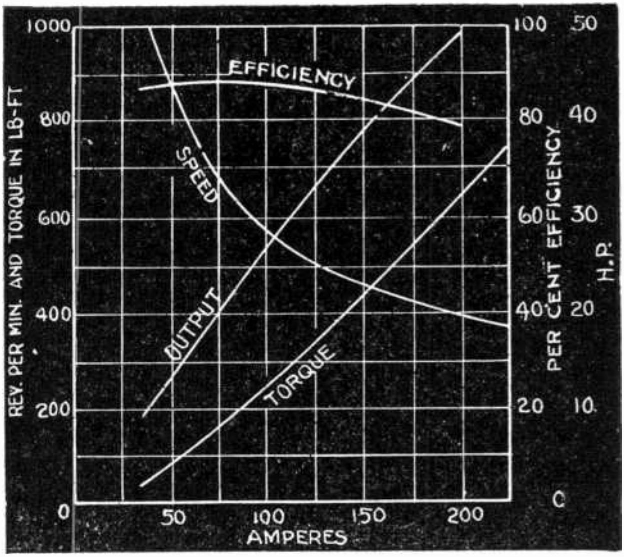


FIG. 25.—Characteristic curves for a Westinghouse type MC-50 230 volt series wound en closed motor for a steel mill crane and hoist service.

Accordingly, in order to use the simplified formula (2) the arm L is made $33 \div 2\pi$ or 5.285 feet, very approximately 5 ft. 3 7/16 inches.

$$B.H.P. = \frac{2\pi \times 3 \times 1,000 \times 30}{33,000} = 17.1$$

Using formula (2)

$$B.H.P. = .0001903 \times 30 \times 3 \times 1{,}000 = 17.1$$

$$\text{Input} = E.H.P. = \frac{220 \times 65}{746} = 19.17.$$

and since the output is 17.1 horse power,

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{brake horse power}}{\text{electrical horse power}} = \frac{17.1}{19.16} = 89\%.$$

Batteries

Batteries.—The voltage of a battery is equal to *the voltage of a single cell × the number of cells in series.*

Example.—What is the voltage of a battery consisting of four 1½ volt cells in series?

$$\text{voltage} = 1\tfrac{1}{2} \times 4 = 6$$

The amperage of a cell depends upon *its voltage, internal resistance and resistance of the external circuit.*

The internal resistance of a *primary* cell is much greater than that of a *secondary* cell.

Example.—If a 1½ volt dry cell show 25 amperes on short circuit what is its internal resistance?

According to Ohm's law

$$R = E \div I$$

Substituting in the formula

$$\text{internal resistance} = 1.5 \div 25 = .06 \text{ ohm}.$$

Example.—If a 1½ volt storage battery show 250 amperes on short circuit what is its internal resistance?

internal resistance = 1.5 ÷ 250 = .006 ohm.

Voltage of a Secondary Cell.—This depends on the density of the electrolyte, the character of the electrodes and condition of the cell; it is independent of the size of the cell.

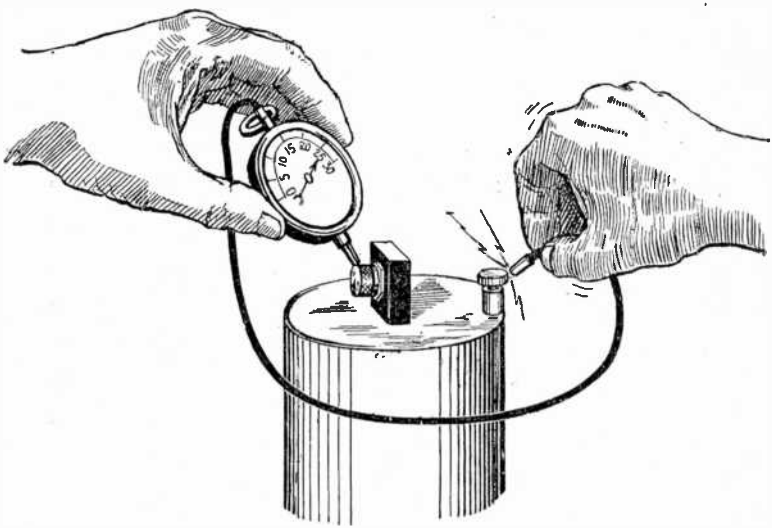


FIG. 26 —How to test a dry cell. *Place terminal* of ammeter on proper pole of cell and *momentarily touch* the other terminal with the ammeter lead. Cell should "kick" 25 to 30 amperes if fresh. Don't buy a dry cell without testing it yourself. In testing, don't hold the connection any longer than necessary to read the ammeter. If a dealer objects to cells being tested it is evidence that they are no good and that the dealer is dishonest.

The voltage of a lead sulphuric acid cell when being charged is from 2 to 2.5 volts. While the cell is being discharged, it decreases from 2 to 1.7 volts. The voltage due to the density of the electrolyte may be calculated from the following formula:

$$V = 1.85 + .917\ (S - s)$$

in which

V = voltage;
S = specific gravity of the electrolyte;
s = specific gravity of water at the temperature of observation.

Example.—If a storage battery require about 3 amperes for charging, how is this current obtained from a 110 volt circuit?

Each 16 candle power carbon filament lamp in the lamp bank would give approximately ⅓ ampere with the cells in series in the lamp circuit. Therefore, 3×3 or 9 lamps should be used in parallel to give 3 amperes.

Specific Gravity Table

Sulphuric acid (Per cent.).	Water (Per cent.).	Specific gravity of Mixture.
50	50	1.398
47	53	1.370
44	56	1.342
41	59	1.315
38	62	1.289
35	65	1.264
32	68	1.239
29	71	1.215
26	74	1.190
23	77	1.167
20	80	1.144
17	83	1.121
14	86	1.098
10	90	1.068

The amperes obtained will be slightly less due to the opposition offered by the internal resistance of the battery.

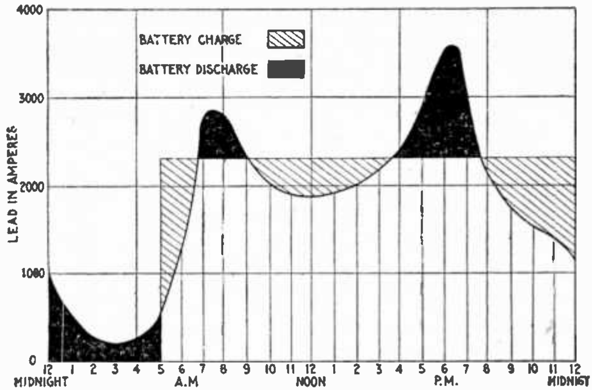


FIG. 27 —Load curve showing use of storage battery as an aid to the generating machinery. In the diagram, it is seen that the battery discharges at minimum and maximum loads and is charged at other times, the battery furnishing current for the entire minimum load and part of the maximum load.

In the selection of a storage battery *the number of cells is determined by the voltage of the system.* Thus, according to Gould:

Voltage of System	Number of Cells	Voltage of System	Number of Cells
110	60	220	120
115	64	230	126
125	70	250	138

NOTE.—The size of a 110 volt battery can be determined thus, assuming that the battery will be charged at any time during the day convenient to operate the dynamo and that the battery will be able to furnish current for lamps as follows:

Time	Number of Lamps	3 Amperes	4 Number of hours	Ampere Hours col. 3 × col. 4
5. p.m. to 10 p.m.	Twenty 16 *c. p.*	10	5	50
10 p.m. to 6 a.m.	Two 8 *c. p.*	½	8	4
6 a.m. to 8 a.m.	Six 16 *c. p.*	3	2	6
				Total 60

Comparison of the Baumé and Specific Gravity Scales at 60° Fahrenheit

Degrees Baume	Specific Gravity	Degrees Baume	Specific Gravity	Degrees Baume	Specific Gravity	Degrees Baume	Specific Gravity
0	1.000	17	1.133	34	1.306	51	1.542
1	1.007	18	1.142	35	1.318	52	1.559
2	1.014	19	1.151	36	1.330	53	1.576
3	1.021	20	1.160	37	1.342	54	1.593
4	1.028	21	1.169	38	1.355	55	1.611
5	1.036	22	1.179	39	1.368	56	1.629
6	1.043	23	1.188	40	1.381	57	1.648
7	1.051	24	1.198	41	1.394	58	1.666
8	1.058	25	1.208	42	1.408	59	1.686
9	1.066	26	1.218	43	1.421	60	1.707
10	1.074	27	1.229	44	1.436	61	1.726
11	1.082	28	1.239	45	1.450	62	1.747
12	1.090	29	1.250	46	1.465	63	1.768
13	1.098	30	1.261	47	1.479	64	1.790
14	1.107	31	1.272	48	1.495	65	1.812
15	1.115	32	1.283	49	1.510	66	1.835
16	1.124	33	1.295	50	1.526		

Strength of Dilute Sulphuric Acid of Different Densities at 59° Fahr.

**Strength of Dilute Sulphuric Acid
of
Different Densities at 59° Fahr.**

Per cent. of Sulphuric Acid	Specific Gravity	Per cent. of Sulphuric Acid	Specific Gravity
100	1.842	23	1.167
40	1.306	22	1.159
31	1.231	21	1.151
30	1.223	20	1.144
29	1.215	19	1.136
28	1.206	18	1.129
27	1.198	17	1.121
26	1.190	16	1.116
25	1.172	15	1.106
24	1.174	14	1.098

Size of Storage A Battery required.—The proper size battery for use with a radio set depends upon

1. Type of tube used
2. Number of tubes
3. Average number of hours used each night
4. Facilities for recharging together with the convenience desired.

Properties of Tubes

(According to Westinghouse Union Battery Co.)

Type of Tube	Filament		Plate Voltage		Neg. "C" Bat'ry Volts	Plate Millia. Norm. Opatn.§
	Volts	Amp.	Detector	Ampl.		
UV-199, UX-199, C-299 CX-299	3.0	06	45	90	4.5	2.5
UV-200, UX-200, C-300 CX-300	5.0	1.	16–22.5	.....	.....	.....
UV-201-A, UX-201-A, C-301-A, CX-301-A	5.0	0.25	45	90	4.5	3.0
				135	9.0	4.0
UX-112, CX-112	5.0	0.5	22.5–45	157.5	10.5	7.9
				135	9.0	5.8
				112.5	7.5	2.5
				90	6.0	2.4
UX-120, CX-120	3.0	0.125	....	135	22.5	6.5
UX-210, CX-210	7.5	1.25	....	425	35	22.0
	7.5	1.25	....	350	27	18.0
	7.5	1.25	....	250	18	12.0
	6.0	1.10	....	157.5	10.5	6.0
	6.0	1.10	....	135	9.0	4.5
	6.0	1.10	....	112.5	7.5	3.0
	6.0	1.10	....	90	4.5	1.0
C-11, C-12, WD-11 WD-12	1.1	.25	....	.....	.....	.....
DV-2	4.5	.26	45	90	4.5	3.0
DV-3	3.0	.07	45	90	4.5	2.5

Example.—A given set uses 5 UV201-A tubes. Each tube requires ¼ ampere, 5 tubes require 1¼ amperes and a 6 volt battery is needed. The 6BRO-7 battery has a capacity of 60 ampere hours. 60 divided by 1¼ gives 48 hours. 48 divided by 3 equals 16. If used 3 hours a night, the battery would give 16 nights of service per charge. The 6BRO-11 would give about 27 nights service or about twice as much service per charge.

Alternating Current

The alternating current is represented by the sine curve as shown in fig. 30.

The equation of the sine curve is

$$y = \sin \phi$$

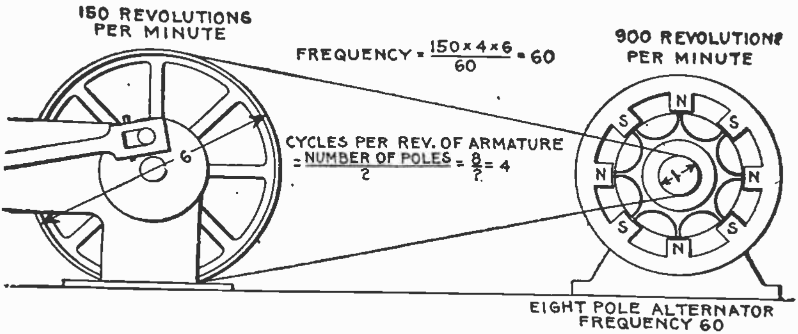


FIG. 28.—Diagram of an alternator and engine, illustrating *frequency*. The frequency or *cycles per second* is equal to *the revolution of armature per second multiplied by one-half the number of poles per phase*. In the figure the armature makes 6 revolutions to one of the engine; one-half the number of poles = 8 ÷ 2 = 4, hence frequency = (150 × 4 × 6) ÷ 60 = 60. The expression in parentheses gives the cycles per minute, and dividing by 60, the cycles per *second*.

in which y, is any ordinate, and ϕ, the angle of the corresponding position of the coil in which the current is being generated.

The symbol ~ is read "cycles per second."

Thus, in an 8 pole machine, there will be four cycles per revolution. If the speed be 900 revolutions per minute, the frequency is

$$\frac{8}{2} \times \frac{900}{60} = 60\sim$$

Form Factor.—This denotes *the ratio of the virtual value of an alternating wave to the average value.* That is

$$\text{form factor} = \frac{\text{virtual value}}{\text{average value}} = \frac{.707}{.637} = 1.11$$

Inductance.—The unit of inductance is called the *henry.*

The formula for the henry is:

$$L = \frac{N \times T}{10^8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

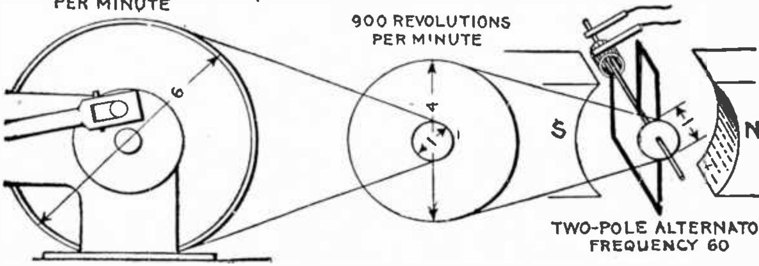


FIG. 29.—Diagram answering the question: Why are alternators always built multipolar? They are made multipolar because it is desirable that the frequency be high. It is evident from the figure that to obtain high frequency would require too many revolutions of the armature of a bipolar machine for mechanical safety—especially in large alternators. Moreover, a double reduction gear in most cases would be necessary, adding complication to the drive. Comparing the above illustration with fig. 28 shows plainly the reason for multipolar construction.

NOTE.—***The henry and the C.G.S. lines.*** One volt is produced *when a conductor cuts* 100.000,000, that is, 10^8 *C.G.S. lines per second.* The formula for the henry is frequently given for a single layer air core coil expressed in C.G.S. units, as 4π area $\times$ total amperes $\times$ turns $\times$ K $\div$ (length $\times 10^9$) and unfortunately it gives a correct answer. Various authorities differ as to whether 10^8 or 10^9 C.G.S. lines should be used and do not always agree with themselves. 1 C.G.S. unit pole at 1 *c.m.* distance produces a flux density of 1 C.G.S. line per sq. *c.m.* One unit pole produces 4π lines = 12.5662 lines, hence C.G.S. current unit = 12.5662 lines under unit conditions. This C.G.S. current unit *is not* an ampere, and here's where the confusion starts. It is constantly confused with the ampere—it is 10 amperes and a good name for it is deka-ampere. The henry is not 10^9 C.G.S. lines, but 10^8. Now 10^9 is simply a corruption caused by confusing the C.G.S. current unit (deka-ampere) with the practical ampere and combining the deka-ampere to ampere magnetic correction divisor (10) with the voltage constant of divisor 10^8.—*P. E. Chapman.*

where

L = coefficient of self induction in henrys;

N = total number of lines of force threading a coil when the current is one ampere;

T = number of turns of coil.

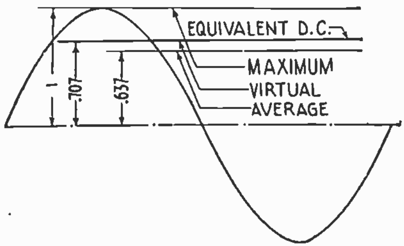


FIG. 30 —Maximum, *virtual* and average volts *The virtual value of an alternating pressure or current is equivalent to that of a direct pressure or current which would produce the same effect.* If a Cardew volt meter be placed on an alternating current circuit in which the volts are oscillating between maxima of +100 and −100 volts, it will read 70 7 volts, though the arithmetical mean is really only 63.7; not withstanding this, 70 7 steady volts would be required to produce an equal reading. The word *effective is commonly*, yet *erroneously used for virtual.*

The inductance of a coil is calculated from the formulæ:

$$L = .4\pi^2 r^2 n^2 \div (l \times 10^8) \ldots\ldots (1)$$

or $\left(L = 4\pi^2 r^2 n^2 \div (l \times 10^9) \right)$

for a thin coil with air core, and

$$L = .4\pi^2 r^2 n^2 \mu \div (l \times 10^8) \ldots\ldots (2)$$

for a coil having an iron core. In the above formulæ:

L = inductance in henrys;

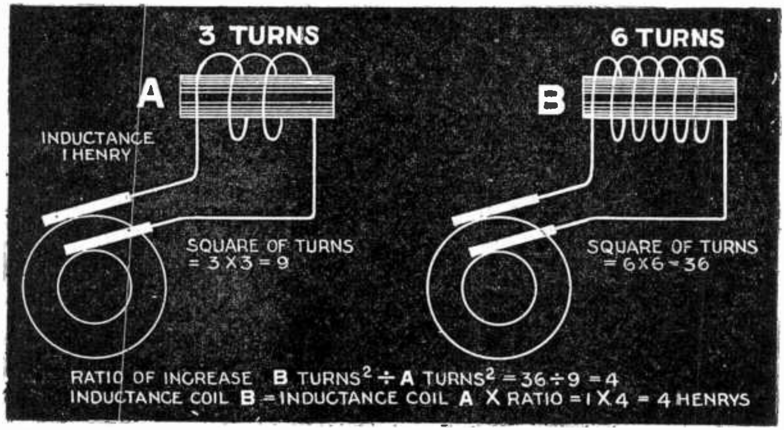
$\pi = 3.1416$;

r = average radius of coil in centimeters;
n = number of turns of wire in coil;
μ = permeability of iron core;
l = length of coil in centimeters.

Example.—An air core coil has an average radius of 10 centimeters and is 20 centimeters long, there being 500 turns, what is the inductance?

Substituting these values in formula (1)

$$L = .4 \times (3.1416)^2 \times 10^2 \times 500^2 \div (20 \times 10^8) = .00494 \text{ henry.}$$



FIGS. 31 and 32.—Diagrams illustrating relation of number of turns of an inductive coil and the inductances.

The henry being a very large unit, it is the custom to express inductance in thousandths of a henry, that is, in *milli-henrys*. The answer then would be $.04935 \times 1{,}000 = 49.35$ milli-henrys.

Example.—An air core coil has an inductance of 50 milli-henrys; if an iron core, having a permeability of 600 be inserted, what is the inductance?

The inductance of the air core coil will be multiplied by the permeability of the iron; the inductance then is increased to

$$50 \times 600 = 30{,}000 \text{ milli-henrys, or } 30 \text{ henrys.}$$

Ohmic Value of Inductance.—The ohmic equivalent of the inductance reactance of an alternating circuit is expressed by the formula

$$X_i = 2\pi f L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which

X_i = ohmic equivalent of inductance
f = frequency
L = inductance

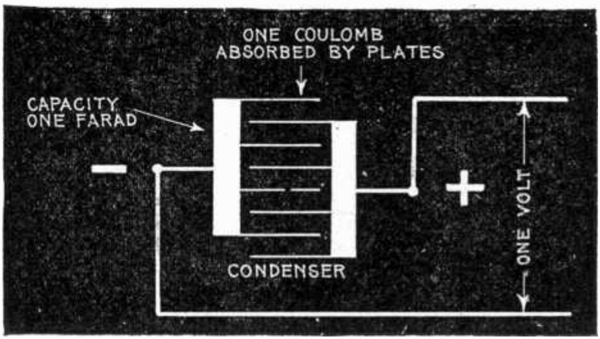


FIG. 33.—Diagram illustrating a *farad*. A condenser is said to have a capacity of one farad if it will absorb one coulomb of electricity when subjected to a pressure of one volt. The farad is a very large unit, and accordingly the microfarad or one millionth of a farad is often used, though *this must be reduced to farads before substituting in formulæ.*

Example.—A coil of wire is of such inductance that a current changing at the rate of one ampere per second induces a reverse pressure of .025 volt. An *a.c.* having a frequency of 100 passes through it. Neglecting the ohmic resistance, what is the ohmic equivalent of inductance?

Substituting the given value in equation (1)

$$X_i = 2\pi \times 100 \times .025 = 15.7 \text{ ohms (equivalent).}$$

Capacity

Capacity.—A condenser is said to have *a capacity of one farad if one coulomb (that is, one ampere flowing one second), when stored on the plates of the condenser will cause a pressure of one volt across its terminals.*

The farad being a very large unit, the capacities ordinarily encountered in practice are expressed in millionths of a farad, that is, in *microfarads* —a capacity equal to about three miles of an Atlantic cable.

Ohmic Value of Capacity.—The ohmic equivalent of capacity is expressed by the formula

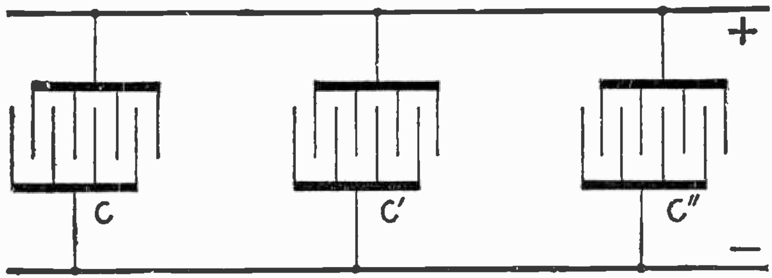


FIG. 34.—Parallel connection of condensers. Like terminals are joined together. The capacity of such arrangement is equal to *the sum of the respective capacities*, that is $C=c+c'+c''$

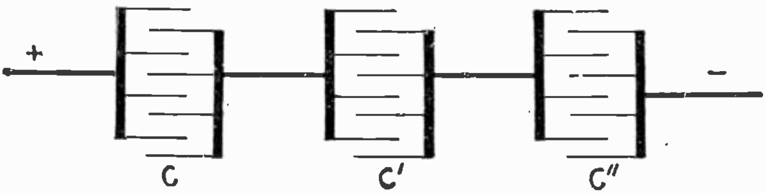


FIG. 35.—Series or cascade connection of condensers. Unlike terminals are joined together as shown. The total capacity of such connection is equal to *the reciprocal of the sum of the reciprocals of the several capacities*, that is, $C = 1 \div \left(\frac{1}{c} + \frac{1}{c'} + \frac{1}{c''}\right)$

$$X_c = \frac{1}{2\pi f C} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which X_c = ohmic value of capacity; C = capacity; f = frequency.

Example.—What is the resistance equivalent of a 50 microfarad condenser to an alternating current having a frequency of 100?

Substituting the given values in the expression for ohmic value

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.1416 \times 100 \times .000050} = \frac{1}{031416} = 31.8 \text{ ohms}$$

Lag and Lead

Lag.—If there be inductance in an *a.c.* circuit, the current will *lag;* if there be capacity, the current will *lead* in phase.

The formula for angle of lag is

$$\tan \phi = \frac{\text{reactance}}{\text{resistance}} = \frac{2\pi f L}{R} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Example 1.—A circuit through which an alternating current is passing has an inductance of 6 ohms and a resistance of 2.5 ohms. What is the angle of lag?

Substituting these values in equation (1),

$$\tan \phi = \frac{6}{2.5} = 2.4$$

Referring to the table of natural sines and tangents on page 153 the corresponding angle is approximately 67°.

Lead.—If there be capacity in an *a.c.* circuit, the current will *lead*. The formula for angle of lead

$$\tan \phi = \frac{\text{reactance}}{\text{resistance}} = -\frac{\frac{1}{2\pi f C}}{R} \quad\ldots\ldots\ldots\ldots\ldots (1)$$

Reactance

The term "reactance" means simply *reaction.*

The term *reactance*, alone, that is, unqualified, is generally understood to mean *inductance reactance.*

Inductance reactance is simply *inductance measured in ohms.*

Example. —An alternating current having a frequency of 60 is passed through a coil whose inductance is .5 henry. What is the reactance?

Here $f=60$ and $L=.5$: substituting these in formula for inductive reactance,

$$X_i=2\pi f L=2\times 3.1416\times 60\times .5=188.5 \text{ ohms}$$

The quantity $2\pi f L$ or reactance being of the same nature as a resistance, is used in the same way as a resistance. Accordingly, since, by Ohm's law

$$E=RI \quad \ldots\ldots (1)$$

an expression may be obtained for the volts necessary to overcome reactance by substituting in equation (1) the value of reactance previously given, thus

$$E=2\pi f L I \quad \ldots\ldots (2)$$

Example. —How many volts are necessary to force a current of 3 amperes with frequency 60 through a coil whose inductance is .5 henry? Substituting in equation (2) the values here given

$$E=2\pi f L I=2\pi\times 60\times .5\times 3=566 \text{ volts.}$$

Example. —In a circuit containing only capacity, what is the reactance when current is supplied at a frequency of 100, and the capacity is 50 microfarads?

$$50 \text{ microfarads}=50\times\frac{1}{1{,}000{,}000}=.00005 \text{ farad}$$

capacity reactance, or

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.1416 \times 100 \times .00005} = 31.83 \text{ ohms}$$

Impedance

Impedance.—This term means *the total opposition in an electric circuit to the flow of an alternating current.* All power circuits for *a.c.* are calculated with reference to impedance.

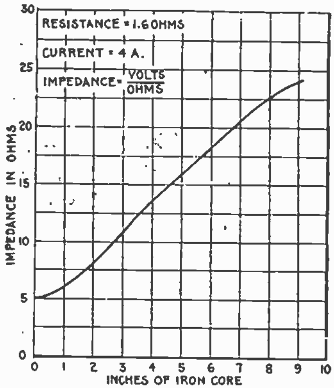


FIG. 36.—Impedance curve for coil with variable iron core. The impedance of an inductive coil may be increased by moving an iron wire core into the coil. In making a test of this kind, the current should be kept constant with an adjustable resistance, and volt meter readings taken, first without the iron core, and again with 1, 2, 3, 4, etc., inches of core inserted in the coil. By plotting the volt meter readings and the positions of the iron core on section paper as above, the effect of inductance is clearly shown.

For a circuit which does not contain capacity

$$\textit{impedance} = \sqrt{\text{resistance}^2 + \text{reactance}^2} \ldots . (1)$$

Example.— If an alternating pressure of 100 volts be impressed on a coil of wire having a resistance of 6 ohms and inductance of 8 ohms, what is the impedance of the circuit and how many amperes will flow through the coil? In the example here given, 6 ohms is the resistance and 8 ohms the reactance. Substituting these in equation (1)

$$\text{Impedance} = \sqrt{6^2+8^2} = \sqrt{100} = 10 \text{ ohms.}$$

The reactance is not always given but instead in some problems the frequency of the current and inductance of the circuit. An expression to fit such cases is obtained by substituting $2\pi fL$ for the reactance as follows: (using symbols for impedance and resistance)

$$Z = \sqrt{R^2+(2\pi fL)^2} \quad \text{.......................... (2)}$$

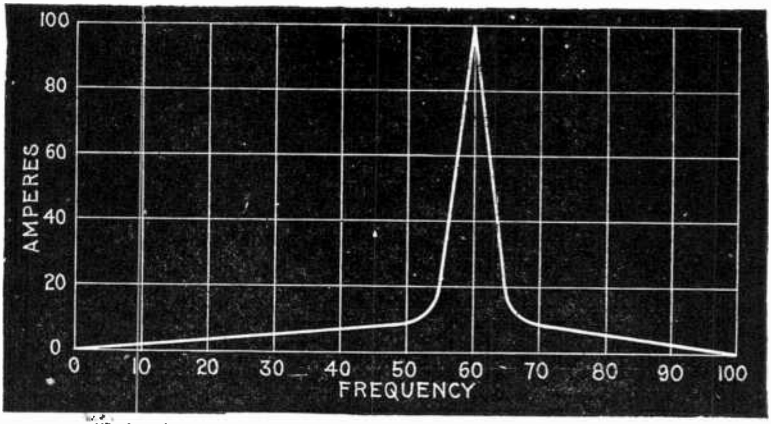


FIG. 37.—Curve showing variation of the current by increasing the frequency in a circuit having inductance and capacity. The curve serves to illustrate the "critical frequency" or frequency producing the maximum current. The curve is obtained by plotting current values corresponding to different frequencies, the pressure being kept constant.

Example.— If an alternating current, having a frequency of 60, be impressed on a coil whose inductance is .05 henry and whose resistance is 6 ohms, what is the impedance?

Here $R=6$; $f=60$, and $L=.05$; substituting these values in (2)

$$Z = \sqrt{6^2+(2\pi \times 60 \times .05)^2} = \sqrt{391} = 19.8 \text{ ohms.}$$

For a circuit having resistance, inductance and capacity.

$$impedance = \sqrt{resistance^2 + (inductance\ reactance - capacity\ reactance)^2}$$

or using symbols,

$$Z = \sqrt{R^2 + (X_l - X_c)^2} \quad \text{.....................} (3)$$

Example.—A current has a frequency of 100. It passes through a circuit of 4 ohms resistance, of 150 milli-henrys inductance, and of 22 microfarads capacity. What is the impedance?

a. The ohmic resistance R, is 4 ohms.

b The inductance reactance, or

$$X_l = 2\pi f L = 2 \times 3.1416 \times 100 \times .15 = 94.3 \text{ ohms.}$$

note that 150 milli-henrys are reduced to .15 henry before substituting in the above equation.

c. The capacity reactance, or

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.1416 \times 100 \times .000022} = 72.4 \text{ ohms}$$

note that 22 microfarads are reduced to .000022 farad before substituting in the formula.

Substituting values as calculated in equation (3)

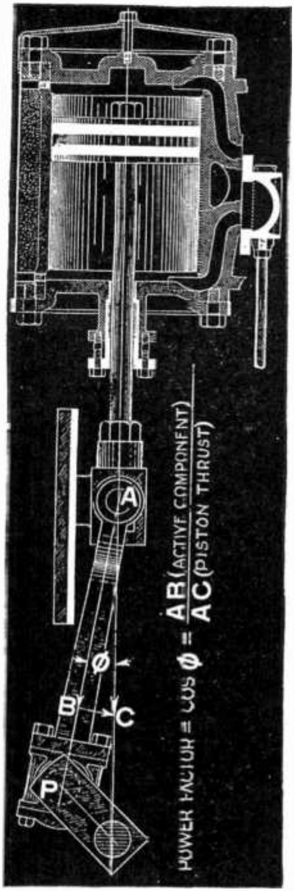
$$Z = \sqrt{4^2 + (94.3 - 72.4)^2} = \sqrt{496} = 22.3 \text{ ohms.}$$

Power Factor

The term power factor may be defined as: *The number of watts indicated by a watt meter, divided by the apparent watts*, the latter being the *watts as measured by a volt meter and ammeter.*

The power factor may be expressed as being equal to

$$\frac{true\ power}{apparent\ power} = \frac{true\ watts}{apparent\ watts} = \frac{true\ watts}{volts \times amperes}$$



The power factor is *that quantity by which the apparent watts must be multiplied in order to give the true power.*

That is,

true power = apparent watts × power factor . . . . (5)

Numerically, *the power factor is equal to the cosine of the angle of phase difference between current and pressure.* that is

$$\text{power factor} = \cos \phi$$

Electrical equipment cost varies inversely (approximately) as the power factor so that a system designed for 70 per cent power factor costs about 40 per cent more than one designed for unity power factor.

In the case of a manufacturer who purchases current his power rate must contain all of the generating cost as well as a profit on these costs so that if the power factor of the generating system be low, the manufacturer in his power rate pays the interest charges upon the extra investment caused by low power factor.

FIG. 38.—Angularity of the connecting rod, analogy of power factor. Pressure due to steam acting on the piston is applied to the wrist pin in the axial direction AC. Let distance AC, represent this pressure. Draw CB, perpendicular to connecting rod AP. Then will AC, represent the *apparent* pressure applied to P, the crank pin; AB, the active component or actual pressure applied to P, and BC, the no energy component. Power factor = AB ÷ AC, = Cosϕ. *Example.*—If 1,000 lbs. pressure be applied by piston and AB ÷ AC = .9, then the actual pressure applied at P = 1,000 × .9 = 900 lbs.

Example.— If in an alternating current circuit, the volt meter and ammeter readings be 110 and 20 and the angle of lag 45°, what is the apparent power and true power?

The apparent power is simply the product of the current and pressure readings or

$$\text{apparent power} = 20 \times 110 = 2{,}200 \text{ watts}$$

The true power is the product of the apparent power multiplied by the cosine of the angle of lag. Cos 45° = .707 hence

$$\text{true power} = 2{,}200 \times .707 = 1{,}555.4 \text{ watts}$$

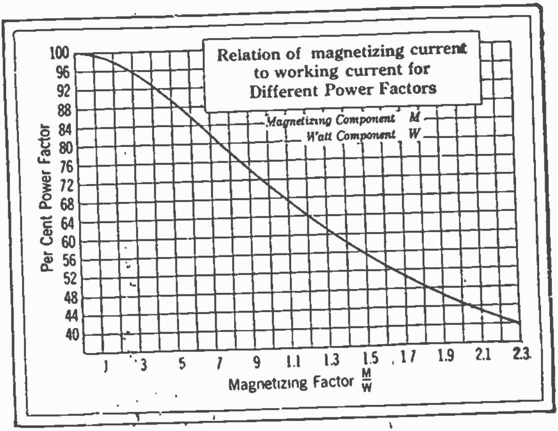


FIG. 39.—Power factor curve. By aid of this curve the magnetizing *kva.* can be determined for any power factor. *For instance,* a line with 60 per cent. power factor and a load of 150 *kw.* requires 150 multiplied by 1.3 = 195 magnetizing *kva.*

Example.— A circuit having a resistance of 3 ohms, and a resultant reactance of 4 ohms, is connected to a 100 volt line. What is: 1, the impedance, 2, the current, 3, the apparent power, 4, the angle of lag, 5, the power factor, and 6, the true power?

1. *The impedance of the circuit.*

$$Z = \sqrt{3^2 + 4^2} = 5 \text{ ohms.}$$

2. *The current.*

$$\text{current} = \text{volts} \div \text{impedance} = 100 \div 5 = 20 \text{ amperes.}$$

3. *The apparent power.*
apparent power = volts × amperes = 100 × 20 = 2,000 watts.

4. *The angle of lag.*
tan ϕ = reactance ÷ resistance = 4 ÷ 3 = 1.33. From table of natural tangents (page 153) $\phi = 53°$.

5. *The power factor.*
The power factor is equal to the cosine of the angle of lag, that is, power factor = cos 53° = .602 (from table).

6. *The true power.*
The true power is equal to the apparent watts multiplied by the power factor, or

$$\text{true power} = \text{volts} \times \text{amperes} \times \cos\phi$$

$$= 100 \times 20 \times .602 = 1{,}204 \text{ watts.}$$

Example.—An alternator delivers current at 800 volts pressure at a frequency of 60, to a circuit of which the resistance is 75 ohms and .25 henry.

Determine: *a*, the value of the current; *b*, angle of lag; *c*, apparent power; *d*, power factor; *e*. true power.

a. Value of current

$$\text{current} = \frac{\text{pressure}}{\text{impedance}} = \frac{E}{\sqrt{R^2 + (2\pi f L)^2}}$$

$$= \frac{800}{\sqrt{75^2 + (2 \times 3.1416 \times 60 \times .25)^2}} = 6.64 \text{ amperes}$$

b. The angle of lag

$$\tan\phi = \frac{\text{reactance}}{\text{resistance}} = \frac{2\pi f L}{R} = \frac{2 \times 3.1416 \times 60 \times .25}{75} = 1.25$$

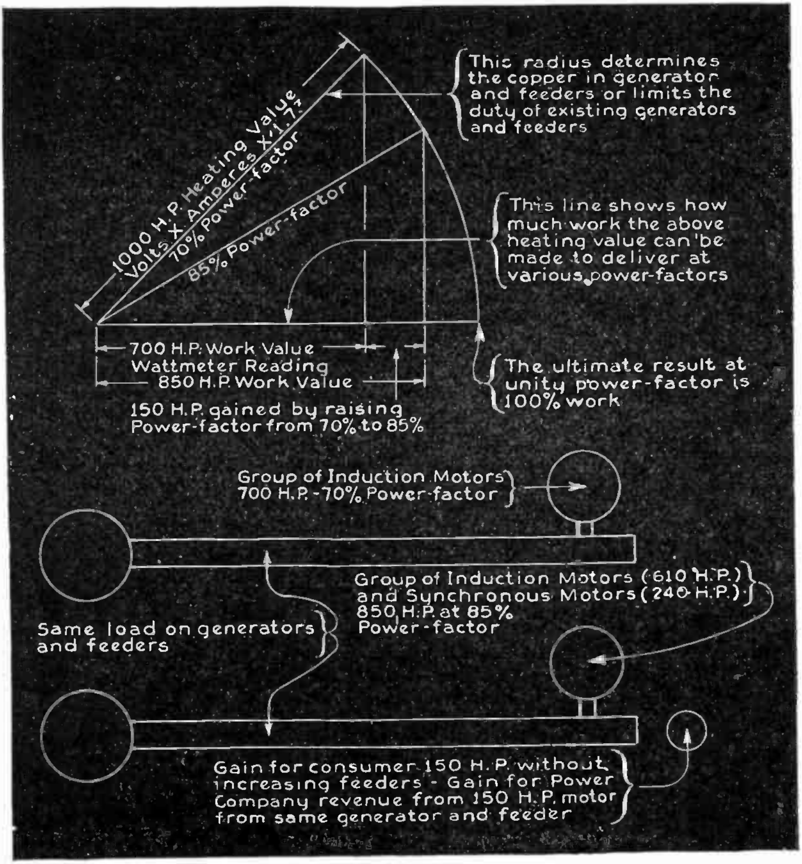
ϕ = angle of lag = 1.25 = 51° 15′ (interpolating from table, page 153).

c. The apparent power
apparent power = volts × amperes = 800 × 6.64 = 5,360 watts
= 5.36 kva.

d. The power factor
power factor = cosine of the angle of lag
= cos 51° 15′ = .626.

e. The true power

$$\text{true power} = \text{apparent power} \times \text{power factor}$$
$$= 5{,}360 \times .626 = 3{,}355 \text{ watts.}$$



FIGS. 40 to 42.—Typical power factor diagram showing gain by raising the power factor; fig. 42 shows benefits to consumer and power company on the basis shown in fig. 41.

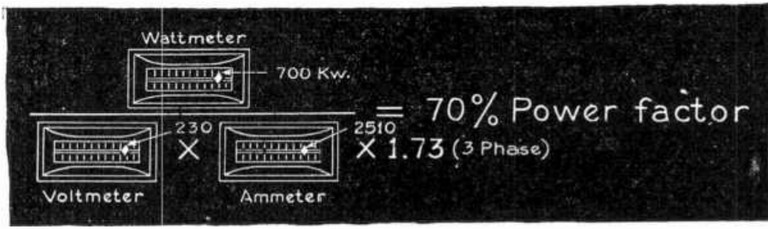


FIG. 43.—Meter reading on three phase circuit 70% power factor.

Alternators

The voltage of an alternator at no load is expressed by the formula

$$E = 4.44 \times N \times f \times \phi \times 10^{-2}$$

This expression gives the mean effective voltage. Should the voltage curve have a sine wave form, then the maximum voltage is equal to the mean effective voltage multiplied by $\sqrt{2}$ that is, by 1.41.

In the above formula:

E = the no load voltage at the terminals of one phase of the winding;

N = number of turns in series per phase,

f = number of magnetic cycles per second;

ϕ = magnetic flux, in megalines simultaneously linked with the N turns. (One megaline = 1,000,000 *c.g.s.* lines.)

Voltages Generated in Various Single Phase Windings

	Correction factor for voltage of variously distributed windings
Single coil winding	E=1.000
Double coil winding	E=.707×single coil winding
Triple coil winding	E=.667× " " "
Quadruple coil winding	E=.654× " " "
Multi-coil or thoroughly distributed winding	E=.637× " " "

The terms single, double, triple coil, etc., in the accompanying table indicate whether the inductors are arranged in one, two, three, etc., equally spaced single coils per pole piece, the single coil being constituted by the group of inductors lying in one slot. The conditions are equivalent to the component voltages generated in each group being in one, two, three, etc., different phases, irrespective of the number of resultant windings into which they are combined.

In any alternator the number of magnets required

$$= 2 \times \frac{\text{frequency} \times 60}{\text{revolutions per minute}} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Example.—How many magnets required for a single phase alternator running at 900 *r.p.m.* to give a frequency of 60?

Substituting in formula (1)

$$\text{Number of magnets} = 2 \times \frac{60 \times 60}{900} = 8$$

The value of the armature ampere turns which tend to distort and to diminish or augment the effect of the ampere turns on the field magnet is sometimes calculated as follows:

$$A = \frac{.707 \times I \times T \times P}{s}$$

in which

A = armature ampere turns;
I = current per phase;
T = turns per pole per phase;

P = number of phases;
s = product of the distribution and pitch factors of the winding.

This value of ampere turns, combined at the proper phase angle with the field ampere turns gives the value of the ampere turns available for producing useful flux.

Example.—How many field magnets are required on a two phase alternator direct connected to an engine running 240 revolutions per minute, for a frequency of 60?

An engine running 240 revolutions *per minute* will turn

$$240 \div 60 = 4 \text{ revolutions } \textit{per second}.$$

A frequency of 60 requires

$$60 \div 4 = 15 \text{ cycles per phase per revolution, or}$$
$$15 \times 2 = 30 \text{ poles per phase.}$$

Hence for a two phase alternator the total number of poles required is

$$30 \times 2 = 60.$$

It is thus seen that a considerable length of spider rim is required to attach the numerous poles, the exact size depending upon their dimensions and clearance.

The flux that must enter the armature from each pole at full load

$$= \frac{\text{full load voltage per phase} \times 10^8}{\text{frequency} \times \begin{pmatrix}\text{No. of inductors in}\\ \text{series per phase}\end{pmatrix} \times K}$$

K is a multiplier depending on the relative values of the width of the winding on each side of a coil, the span of the polar arc and pitch of the poles.

Transformers

For any transformer or reactive coil:

Let $E = \sqrt{\overline{\text{mean}^2}}$ of the induced voltage;

ϕ = total flux;

B'' = lines of force per sq. in.

A = section of magnetic circuit in sq. ins.

N = frequency in cycles per second;

T = total turns of wire in series.

$$4.44 = \frac{2\ \pi}{\sqrt{2}} = \sqrt{2} \times \pi$$

Then

$$E = \frac{4.44\ N\ \phi\ T}{10^8} \quad \text{(1)}$$

This equation is based on the assumption of a sine pressure wave and is the most important of the formulae used in the design of an alternating current transformer.

By substituting and transposing, an equation can be obtained for any unknown quantity.

Thus if the volts, frequency and turns be known, then

$$\phi = \frac{E \times 10^8}{4.44 \times N \times T} \quad \text{(2)}$$

but

$$\phi = B''A \quad \text{(3)}$$

Therefore

$$A = \frac{E \times 10^8}{4.44 \times N \times T \times B''} \quad \text{(4)}$$

which equation gives at once the cross section of iron necessary for the magnetic circuit after the total primary turns, and the density at which it is desired to work the iron have been decided.

Again, if the volts, frequency, cross section of core, and density be known, then transposing equation (4)

$$T = \frac{E \times 10^8}{4.44 \times N \times B'' \times A}$$

The core loss is generally neglected in the measurements.

The voltage and current relations are approximately as follows:

primary voltage: secondary voltage = primary turns: secondary turns

primary current: secondary current = secondary turns: primary turns

The hysteresis loss is caused by the reversals of the magnetism in the iron core, and differs with different qualities of iron. With a given quality of iron, this loss varies as the 1.6 power of the voltage with constant frequency.

Steinmetz gives a law or equation for hysteresis as follows:

$W_H = \eta B^{1.6}$

W_H = Hysteresis loss per cubic centimeter per cycle, in ergs ($= 10^{-7}$ joules)

η = constant dependent on the quality of iron.

If N = the frequency,

V = the volume of the iron in the core in cubic centimeters,

P = the power in watts consumed in the whole core,

then

$P = \eta N V B^{1.6} 10^{-7}$

and

$$\eta = \frac{P}{NVB^{1.6}10^{-7}}$$

The *copper losses* in a transformer are the sum of I^2R losses of both the primary and secondary coils, and the eddy current loss in the conductors. In any well designed transformer, however, the eddy current loss in the conductors is negligible, so that the sum of I^2R losses of primary and secondary can be taken as the actual copper loss in the transformer.

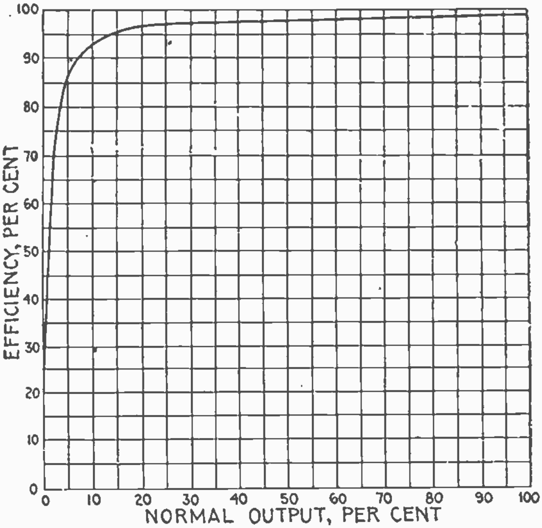


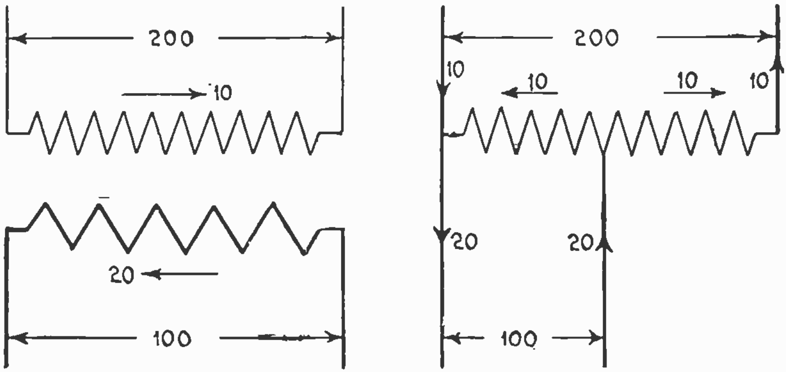
FIG. 44.—Efficiency curve of Westinghouse 375 *kw.* transformer. Pressure 500 to 15,000 volts; frequency 60. Efficiencies at different loads: full load efficiency, 98%; ¾ full load efficiency, 98%; ½ full load efficiency, 97.6%; ¼ full load efficiency, 96.1%; regulation non-inductive load, 1.4%; load having .9 power factor, 3.3%.

The efficiency of a transformer is the ratio of the output watts to the input watts. Thus

$$\text{Efficiency} = \frac{\text{Output watts}}{\text{Input watts}} = \frac{\text{Output}}{\text{Output} + \text{Core loss} + \text{Copper loss}}$$

The core loss, which is made up of the hysteresis loss and eddy current loss, remains constant in a constant potential transformer at all loads, while the copper loss, or I^2R loss, varies as the square of the current in the primary and secondary.

$$\text{All-day efficiency} = \frac{\text{Full load} \times 5}{\text{Core loss} \times 24 + I^2R \times 5 + \text{full load} \times 5}$$



FIGS. 45 and 46.—Two winding transformer and a single winding or auto-transformer. Fig. 45 shows a 200:100 volt transformer having a 10 amp. primary and a 20 amp. secondary, the currents being in opposite directions. If these currents be superposed by using one winding only, the auto-transformer shown in fig. 46 is employed where the winding carries 10 amp. only and requires only one-half the copper (assuming the same mean length of turn). If R, be the ratio of an auto-transformer, the relative size of it compared with a transformer of the same ratio and output is as $\frac{R-1}{R}$: 1. *For example*, a 10 *kw*. transformer of 400 volts primary and 300 volts secondary could be replaced by an auto-transformer of $10 \times \frac{1\ 33-1}{1\ 33} = 2.5$ *kw*.; or, in other words, the amount of material used in a 2½ *kw*. transformer could be used to wind an auto-transformer of 400 : 300 ratio and 10 *kw*. output.

Transformers for Two and Three Phase Motors.

Delivered voltage of circuit	Single phase transformer voltages			
	110 volt motor		220 volt motor	
	Primary	Secondary	Primary	Secondary
1,100	1,100	122	1,100	244
2,200	2,200	122	2,200	244

Very small transformers should not be used, even when the motor is large compared to the work it has to do, as the heavy stärting current may burn them out.

The following table gives the proper sizes of transformer for three types of induction motor.

Capacities of Transformers for Induction Motors.

Size of motor horse power	Kilowatts per transformer		
	Two single phase transformers	Three single phase Transformers	One three phase transformer
1	0.6	0.6	
2	1.5	1.0	2.0
3	2.0	1.5	3.0
5	3.0	2.0	5.0
7	4.0	3.0	7.5
10	5 0	4.0	10.0
15	7.5	5.0	15.0
20	10.0	7.5	20.0
30	15.0	10.0	30.0
50	25.0	15.0	50.0
75	40.0	25.0	75.0
100	50.0	30.0	100.0

A. C. Motors

Synchronous Motors.—The speed of a synchronous motor is that at which it would have to run, if driven as an alternator, to deliver the number of cycles which is given by the supply alternator.

The following simple formula gives the speed relations between alternators and motors connected to the same circuit and having different numbers of poles.

$$r = \frac{P \times R}{p}$$

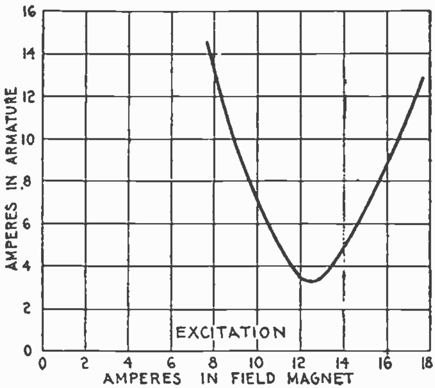


FIG. 47.—Diagram illustrating the method of representing the performance of synchronous motors. The *V shaped curve* is obtained by plotting the current taken by motor under different degrees of excitation, the power developed by the motor remaining constant. The current may be made to lag or lead while the load remains constant, by varying the excitation. A certain value may be reached by varying the excitation, which will give a minimum current in the armature; this is the condition of unity power factor. If now the excitation be diminished the current will lag and increase in value to obtain the same power; if the excitation be increased the current will *lead* and increase in value to obtain the same power The results plotted for several values of the excitation current will give the V curve as shown. This is an actual curve obtained by Morley on a 50 *kw.* machine running unloaded as a motor. Other curves situated above this one may be obtained for various loadings of the motor.

in which
r = Revolutions per minute of the motor;
p = Number of poles of the motor;
R = Revolutions per minute of the alternator;
P = Number of poles of the alternator.

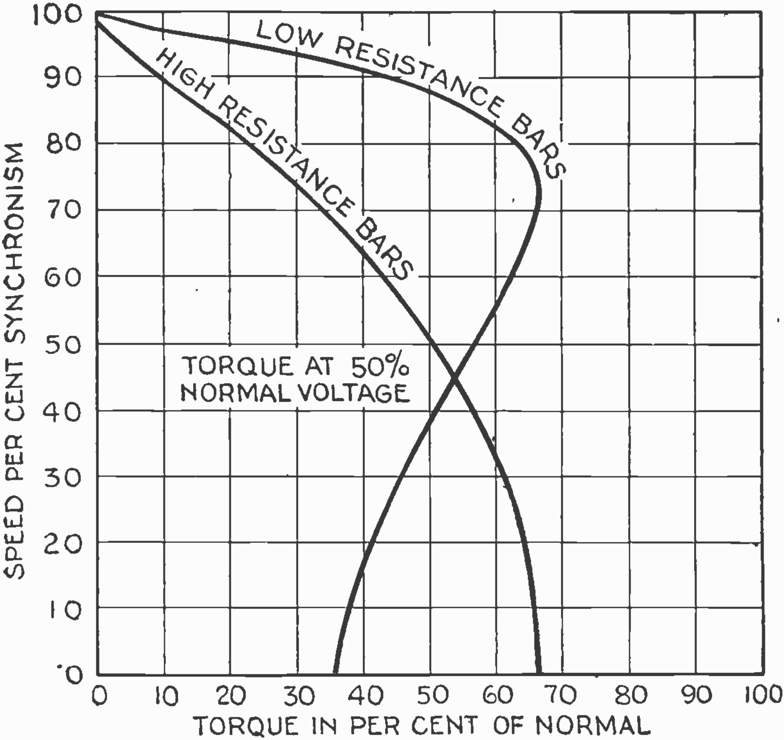


FIG. 48.—Starting torque curves for a synchronous motor with *low and high resistance squirrel cages* on the armature. *In order that* a synchronous motor may pull into step, it is necessary to accelerate it until synchronous speed is quite closely approached. The nearer this approach, the more powerful is the synchronizing action. The slip of an induction motor depends upon the load and also upon the resistance in the cage winding. If a high cage resistance be used in order to provide a high starting torque, it is quite possible that the slip will be so great that sufficient synchronizing action cannot be developed to pull the motor into step. The curves show the difference in effect of high resistance brass bars and low resistance copper bars.

Squirrel Cage Motors.—The slip of a squirrel cage induction motor varies from 2 to 5% of synchronous speed depending upon the size.

The slip is obtained from the following formula:

$$\text{Slip (rev, per sec.)} = S_f - S_a$$

or, expressed as a percentage of synchronism, that is, of the synchronous speed,

$$\text{Slip (\%)} = \frac{(S_f - S_a \times 100}{S_f}$$

Maximum Starting Current for Squirrel Cage Motors

(According to Century Electric Co.)

Number of Poles	Maximum Starting Current In Percentage of Full Load Current	Minimum Static Torque In Percentage of Full Load Torque
	Fractional Horsepower 60 and 50 cycle	
4	450%	250%
6	415%	200%
	Fractional Horsepower 25 cycle	
2	450%	250%
4	450%	200%
	All other motor sizes not included above. 60, 50, 40 and 25 cycle	
2	450%	175%
4	450%	175%
6	415%	150%
8	415%	135%
10	375%	125%
12	375%	125%

where

S_f = synchronous speed, or *r.p.m.* of the rotating magnetic field;

S_a = speed of the armature.

The synchronous speed is determined the same as for synchronous motors by use of the following formula:

$$S_f = \frac{2f}{P} \times 60$$

where

S_f = synchronous speed or *r.p.m.* of the rotating magnetic field;
P = Number of poles;
f = frequency.

The following table gives the synchronous speed for various frequencies and different numbers of poles:

Table of Synchronous Speeds

Frequency	R.P.M. of the rotating magnetic field, when number of poles is					
	2	6	10	16	20	24
25	1,500	500	300	188	150	125
60	3,600	1,200	720	450	360	300
80	4,800	1,600	960	600	480	400
100	6,000	2,000	1,200	750	600	500
120	7,200	2,400	1,440	900	720	600
125	7,500	2,500	1,500	938	750	625

Example.—A 60 cycle, sixteen pole, three phase motor has a slip at full load of 6 per cent; at what speed does the armature turn at full load?

Synchronous speed = 450 *r.p.m.*

Slip = 450 × 6 per cent = 27 *r.p.m.*

Armature speed = 450 − 27 = 423 *r.p.m.*

Check

$$\text{Slip } (=\%) = \frac{\text{synchronous speed} - \text{armature speed}}{\text{synchronous speed}} \times 100$$

$$= \frac{450-423}{450} \times 100 = \frac{27}{450} \times 100 = 6 \text{ per cent}$$

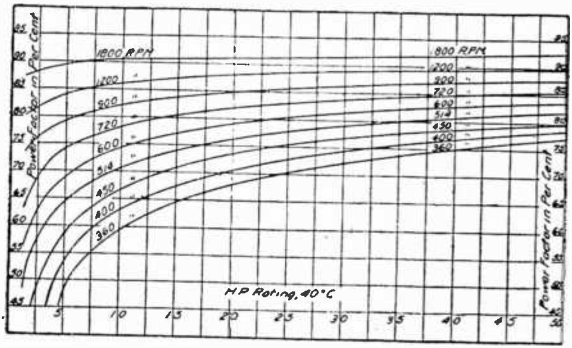


Fig. 49.—Efficiency of 60 cycle, squirrel cage induction motors at 100% load, 3 to 50 *h.p.*

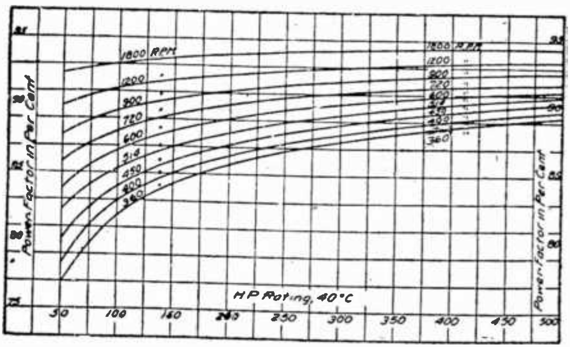


Fig. 50.—Power factor of 60 cycle, squirrel cage induction motors at 100% load, 50 to 500 *h.p.*

Internal Resistance Induction Motors.—This type motor has *an armature so constructed as to obtain a high resistance* (ohmic or spurious) *while starting and a low resistance while running without external connections, to avoid the very heavy starting current and to increase the starting torque.*

In order to give maximum starting torque, total armature resistance should be

$$r_1 = \sqrt{r^2 + (x_1 + y)^2}$$

Where r_1 = rotor resistance per circuit reduced to field system

x_1 = rotor reactance per circuit reduced to field system

r = resistance per field circuit

y = reactance per field circuit.

External Resistance or Slip Ring Motors.—In this type the starting torque and the starting current are under the control of the operator and may be varied at his will. The slip ring

NOTE —*Effect of changes in voltage and frequency on induction motor operation.* According to B. G. Lamme, some variations from normal voltage and frequency are generally permissible with any induction motor, but such variations are always accompanied by changes from normal performance. With either the voltage or the frequency differing from normal the following performance changes must be expected:

Conditions	Power Factor	Torque	Slip
Voltage high	Decreased	Increased	Decreased
Voltage low	Increased	Decreased	Increased
Frequency high	Increased	Decreased	Per cent slip unchanged
Frequency low	Decreased	Increased	Per cent slip unchanged

Usually a variation of either voltage or frequency not exceeding 10% is permissible and within this limit the efficiency remains approximately unchanged. The voltage and frequency should not be varied simultaneously in opposite directions, that is, one decreased and the other increased. If an induction motor must operate on frequency other than standard, the performance will be better if the voltage be changed in proportion to the square root of the frequency. Thus a 400 volt, 60 cycle motor operating on 66⅔ cycles will have very nearly its normal operating characteristics if the voltage be raised to $400 \times \sqrt{66\frac{2}{3} \div 60} = 420$ volts. Decreasing the voltage much below normal is seldom permissible on account of resulting increased temperature rises. An increase in the frequency results in a considerable reduction in the maximum load which an induction motor can carry.

motor accordingly permits the heaviest loads to be started slowly and smoothly with no objectionable line disturbances.

The Heyland Diagram.—By aid of this diagram it is possible to calculate horse power output, *kva* input, amperes per

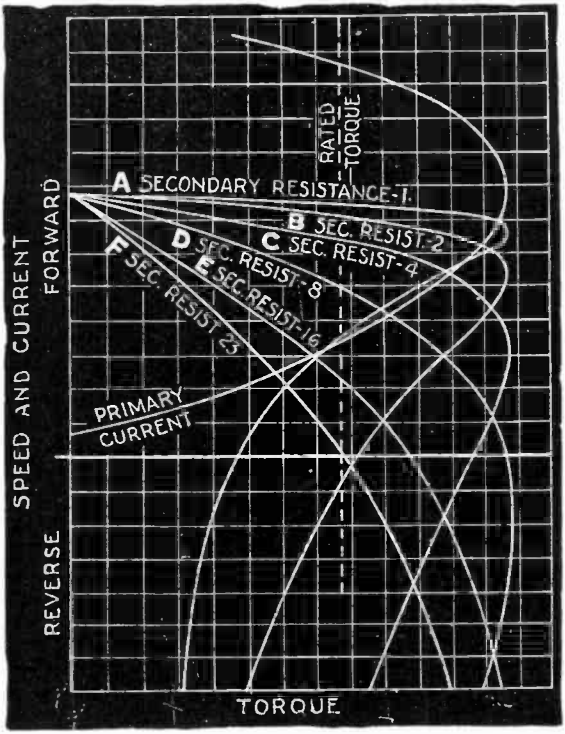


Fig. 51.—Speed torque and current curves of a polyphase induction motor with different values of secondary resistance. For constant torque any variation in the armature resistance requires a proportionate variation in the slip. If the slip with a given torque be 10%, for instance, it must be 20% for double the resistance. The armature resistance may be in the windings themselves as in internal resistance motors, or it may be entirely separate from the machine and connected to the windings by suitable means, as shown in external resistance or slip ring motors.

terminal, per cent power factor for different loads, per cent inrush at starting under full voltage, per cent torque at starting, maximum or pull out torque, per cent slip of motor at different loads and actual *r.p.m.* of motor at different loads.

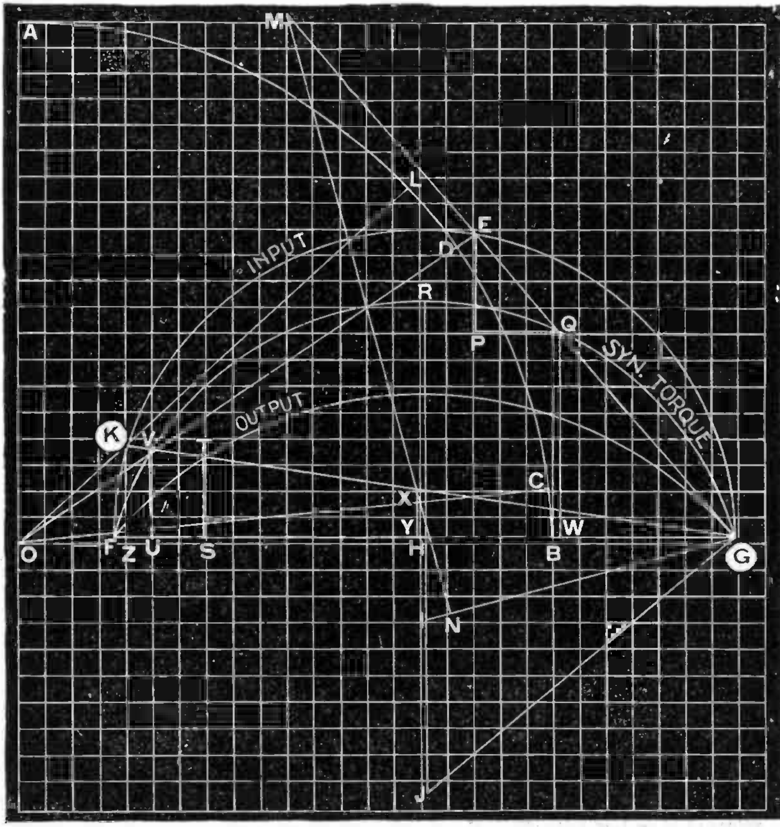


FIG. 52.— Heyland diagram for calculating horse power output, *kva* input, amperes per terminal, per cent power factor for different loads, per cent inrush at starting under full voltage, per cent torque at starting, maximum or pull out torque, per cent slip of motor at different loads, and actual *r.p.m.* of motor at different loads.

All of this data may be obtained from the diagram, after obtaining from test under no load conditions, and with the armature blocked, the volts, amperes and *kw* input to the motor.

The following example indicates plainly just how the data is obtained.

Example.—In a test of a 5 *h.p.* 220 volt, 3 phase, 60 cycle, 1,200 *s.r.p.m.* slip ring motor, 17 amperes per terminal, the no load and blocked armature tests gave the following results:

	NO LOAD					BLOCKED ARMATURE				
	Volts	Amperes	Kva	Kw.	P.f	Volts	Amperes	Kva.	Kw	P f
Test...	230	12	4.770	.550	11 5	114.3	35.5	7.020	3.980	56 7
Ratioed	220	11.5	4.370	.....	....	220.	68.3	26.000	.....	....

With scale of 250 *va* to one division, 4,370 *kva* = 17.5 div.

$$\frac{\text{Resistance between terminals} \times 3}{2} = .97 = \text{total resistance of armature, hot.}$$

Short circuited full voltage blocked $I^2R = 68.3^2 \times .97 = 4{,}520$ *va* = 18.1 divisions.

Full load of 5 *h.p* ×746 = 3,730 *va* = 14.9 divisions.

With O, as center, and a radius of 100 divisions, strike arc AB, which is the power factor arc. Draw OC, through 11.5 power factor and lay off OF, equal to 17.5 divisions (no load condition).

Draw OD, through 56.7 power factor and lay off OE, equal to 104 divisions. (Full voltage blocked armature condition.)

Through F and E, draw arc FKEG, with center at H. This is input arc of motor. Connect E and G, draw JG, perpendicular to EG.

With center at J, draw arc through F and G. This is output arc of motor.

Lay off ST, equal to full load, which equals 14.9 divisions. Draw GT, through to K. OK, equals *kva*, input full load, from which full load amperes is calculated to equal 17.8 amperes. (OK, equals 27.2 divisions, multiplied by scale of 250, equals *kva*; this, divided by 220 volts and 1.73 for three phase, equals 17.8 amperes.) OK, extended to L, gives a power factor of 68% for full load.

Draw EP, equal to 18.1 divisions, as per second paragraph above, and PQ, perpendicular to intersection of EP.

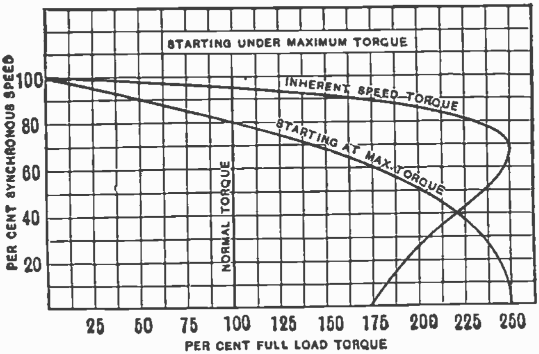


FIG. 53.—Speed torque curves for Wagner slip ring motors with armature short circuited (inherent speed torque) and with resistance inserted to give maximum torque at standstill.

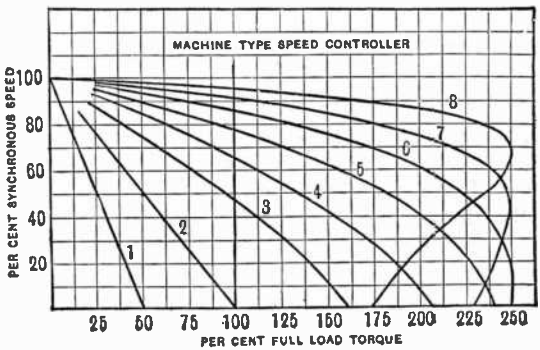


FIG. 54.—Speed torque curves of Wagner slip ring motor equipped with machine type speed controller having eight steps of resistance. The diagram shows the speed torque curves of a slip ring motor used with a machine type controller. If the machine driven require full load torque at all speeds, the intersections of the speed torque curves with the 100% torque line show the motor speeds with the controller handle on the various steps.

With scale of 250 *va*, to one division, 26 *kva* = 104 div. With center I, on HJ, draw arc FQG. This is synchronous torque arc.

Extend GE, to M, the latter being 100 divisions above line FG. Draw MN, perpendicular to IG, XY, then in divisions is per cent slip; in this case 9.2% Then *r.p.m.* is 90.8% of synchronous *r.p.m.* or 1,090 *r.p.m.*

Per cent inrush equals

$$\frac{OE}{OK} \text{ or substituting} = \frac{104}{27.2}$$

which is 383% of full load; with power factor read at D, as 56.7%. Maximum, or pull out torque equals

$$\frac{RH}{TS} \text{ which} = \frac{45}{15} \text{ or } 300\% \text{ of full load}$$

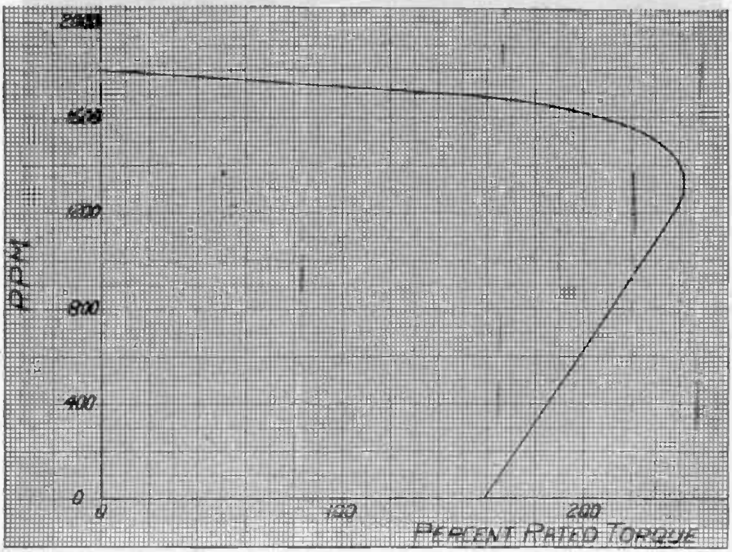


FIG. 55.—Domestic split phase induction motor speed torque curve. Operation and characteristics. The speed of an induction motor depends upon the number of poles of the motor and the frequency of the current upon which it is operated. This is why only certain definite speeds are obtainable. The speed changes very slightly from no load to full load.

$\frac{QW}{TS} = \frac{39.5}{15}$ or 263%, which is per cent starting torque of full load when armature rings are short circuited, in the case of a slip ring motor FK, is secondary amperes. $\frac{TS}{KZ}$ = per cent efficiency.

$\frac{\text{synchronous } h.p. \times 33.000}{2\,\pi \times \text{synchronous } r.p.m.}$ = torque in pounds at 1 foot radius.

Single Phase Induction Motors.—This type is generally called *split phase.*

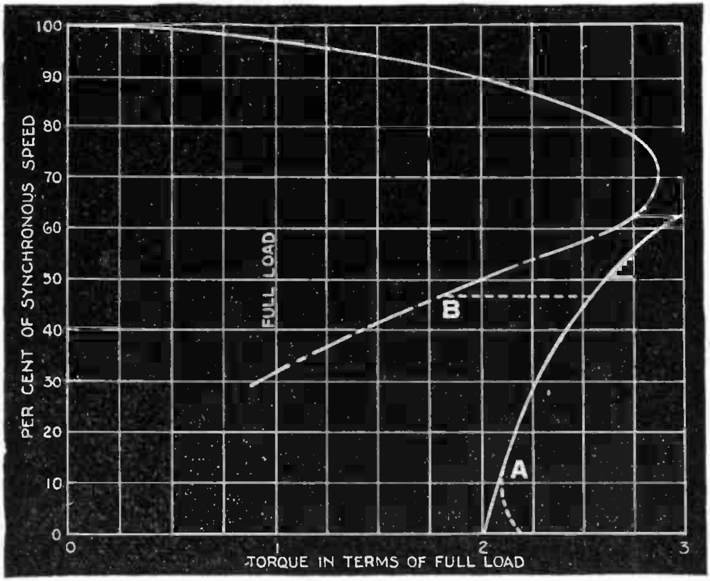


FIG. 56. —Speed torque curve of split phase induction motor. The pull up torque is the greatest load torque the motor will pull through and go on up to speed, that is, it is the lowest torque point on the motor curve below pull out. In the above curve it coincides with the starting torque; if the motor had a higher starting torque, as shown dotted, the pull up torque would be at A; or if the centrifugal switch opened below its best operating speed as shown by the horizontal dotted line, the pull up torque would occur at B.

A.C. and Voltage in Terms of D.C.—(According to Steinmetz)

	DIRECT CURRENT	SINGLE PHASE	TWO PHASE	THREE PHASE	SIX PHASE	TWELVE PHASE	n PHASE
VOLTS BETWEEN COLLECTOR RING AND NEUTRAL POINT	1	$\frac{1}{2\sqrt{2}}=.354$	$\frac{1}{2\sqrt{2}}=.354$	$\frac{1}{2\sqrt{2}}=.354$	$\frac{1}{2\sqrt{2}}=.354$	$\frac{1}{2\sqrt{2}}=.354$	$\frac{1}{2\sqrt{2}}=.354$
VOLTS BETWEEN ADJACENT COLLECTOR RINGS	1	$\frac{1}{\sqrt{2}}=.707$	$\frac{1}{2}=.5$	$\frac{\sqrt{3}}{2\sqrt{2}}=.612$	$\frac{1}{2\sqrt{2}}=.354$	.183	$\frac{\sin\frac{\pi}{n}}{\sqrt{2}}$
AMPERES PER LINE	1	$\sqrt{2}=1.414$	$\frac{1}{\sqrt{2}}=.707$	$\frac{2\sqrt{2}}{3}=.943$	$\frac{\sqrt{2}}{3}=.472$	.236	$\frac{2\sqrt{2}}{n}$
AMPERES BETWEEN ADJACENT LINES	1	$\sqrt{2}=1.414$	$\frac{1}{2}=.5$	$\frac{2\sqrt{2}}{3\sqrt{3}}=.545$	$\frac{\sqrt{2}}{3}=.472$	.455	$\frac{\sqrt{2}\sin\frac{\pi}{n}}{n}$

Its torque is due to the product of the quadrature speed field flux and that component of the secondary current which is in time phase with such field flux and occupies the mechanical space position along the air gap opposite such flux.

Converters.—The ordinary range of size of rotaries is from 3 *kw*. to 3,000 *kw*.

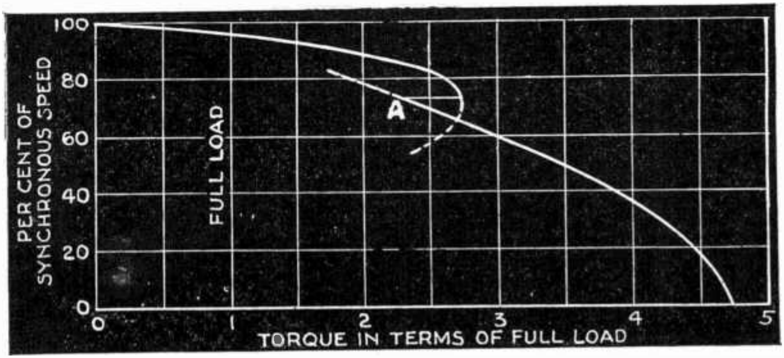


FIG. 57.—Speed torque curve of repulsion start induction motor.

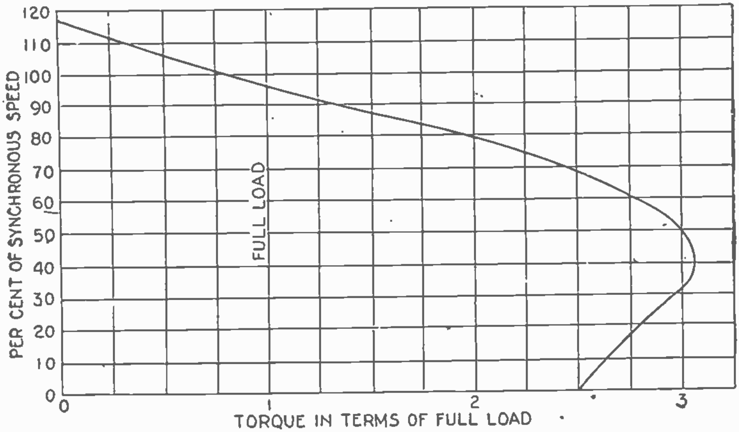


FIG. 58.—Speed torque curve of repulsion induction motor. The starting torque is about two and one-half times full load torque and the starting current is about two and one-half to three times full load current. The speed torque curve is very similar to that of a compound wound *d.c.* motor. As the motor speeds up, the torque increases until the maximum torque point is reached, which insures that the motor will bring up to full speed any load that it will start. Due to the somewhat drooping speed torque characteristic, this motor tends to throw off its load as it is overloaded. As the applied voltage is decreased, the motor speed decreases and the power required to drive a constant torque load decreases. The current on overload, therefore, does not increase as rapidly as in the case of the induction motor or the repulsion start induction motor At light loads, the speed of this motor may be above synchronism.

For single phase or two phase machines it is 1 to .7; for three phase, 1 to .612, or six phase, 1 to .7 or 1 to .613 depending upon the kind of connection used for the transformer.

Example.—A two phase rotary receiving alternating current at 426 volts will deliver direct current at 600 volts, while a three phase rotary receiving alternating current at 367 volts will deliver direct current at 600 volts.

Motor Generator Sets.—The combination of a motor and a dynamo or alternator is used in preference to rotary converters when it is desirable that the generating element be independent of the *a.c.* line voltage so that any degree of voltage regulation can be obtained.

Some of these sets are used for frequency changing.

The frequency obtained at the slip rings will depend on the speed of rotation, the number of poles for which the changer is wound and the frequency of the supply circuit.

The old formula holds:

$$N = \frac{r.p.m. \times \frac{P}{2}}{60} + \text{line frequency}$$

or excitation frequency.

N = slip ring frequency

P = number of poles

r.p.m—revolutions per minute of rotor

Example.—Assume 60 cycle excitation and a 6 pole machine at 1,200 *r.p.m.* then

$$\frac{1{,}200 \times \frac{6}{2}}{60} + 60 = 120 \text{ cycles.}$$

The maximum speed for standard frequency changers is 2,200 *r.p.m.* When frequencies are required which would demand higher speeds for any particular machine, it will be necessary to select a frequency changer, having a greater number of poles, with a correspondingly lower basic speed.

The power input to the stator from the commercial lines for an induction frequency changer at full load is approximately equal to the kilowatt rating of the machine at the frequency of the commercial circuit plus the excitation losses of approximately 10 per cent.

A. C. Windings

In the operation of an alternator the maximum pressure generated may be expressed by the following equation:

$$E_{max} = \frac{\pi f Z N}{10^8} \quad \ldots\ldots (1)$$

in which

E = volts;
f = frequency;
Z = number of inductors in series in any one magnetic circuit;
N = magnetic flux, or total number of magnetic lines in one pole or in one magnetic circuit.

The maximum value of the pressure, as expressed in equation (1), occurs when $\theta = 90°$.

The virtual value of the volts is equal to the maximum value divided by $\sqrt{2}$, or multiplied by $\frac{1}{2}\sqrt{2}$, hence,

$$E_{virt} = \frac{\frac{1}{2}\sqrt{2} \times \pi f Z N}{10^8} = \frac{2.22 f Z N}{10^8} \quad \ldots\ldots (2)$$

This is usually taken as the fundamental equation in designing alternators. It is, however, deduced on the assumptions that the distribution of the magnetic flux follows a sine law, and that the whole of the loops of active inductors in the armature circuit act simultaneously, that is the winding is concentrated.

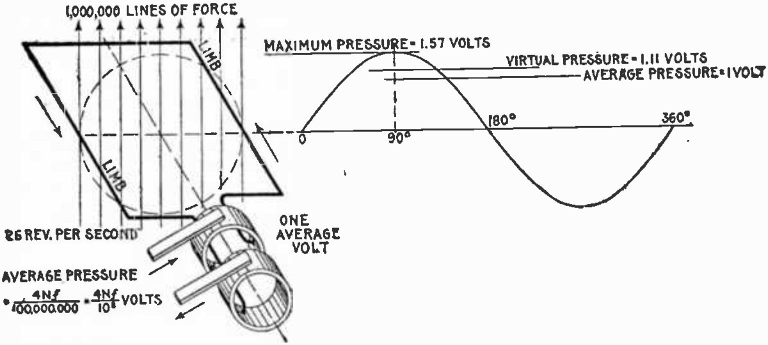


FIG. 59.—Elementary alternator, developing one average volt. If the loop make one revolution per second, and the maximum number of lines of force embraced by the loop in the position shown (the zero position) be denoted by N, then each limb will cut 2N lines per second, because it cuts every line during the right sweep and again during the left sweep. Hence each limb develops an average pressure of 2N units (C.G.S. units), and as both limbs are connected in series, the total pressure is 4N units *per revolution*. Now, if the loop make f revolutions *per second* instead of only one, then f times as many lines will be cut *per second*, and the average pressure will be 4N f units. Since the C.G.S. unit of pressure is so extremely small, a much greater practical unit called the *volt* is used, which is equal to 100,000,000, or 10^8 C.G.S. units is employed. Hence average voltage $= 4Nf \div 10^8$. The value of N, in actual machines is very high, being several million lines of force. The illustration shows one set of conditions necessary to generate one average volt. The maximum pressure developed is $1 \div .637 = 1.57$ volts; virtual pressure $= 1.57 \times .707 = 1.11$ volts.

The Kapp Coefficient.—In practice, the coils are often more or less distributed, that is, they do not always subtend an exact pole pitch; moreover, the flux distribution, which depends on the shaping and breadth of the poles, is often quite different from a sine distribution. Hence, the coefficient 2.22 in equation (2) is often departed from, and in the general case equation (2) may be written

$$E_{virt} = \frac{k f ZN}{10^8} \qquad (3)$$

where k, is a number which may have different values, according to the construction of the alternator. This number k, is called the *Kapp coefficient* because its significance was first pointed out by Prof. Gisbert Kapp.

The value of k, is further influenced by a "breadth coefficient" or spread or span "factor."

NOTE.—*The values of the spread factor* as given in the table below are based upon the same number of inductors being placed in each of the slots and all of the slots being used.

Slots per phase per pole	Spread Factor		
	Single phase	Two phase	Three phase
1	1.000	1.000	1.000
2	.707	.924	.966
3	.663	.911	.960
4	.653	.906	.958
6	.644	.903	.956

NOTE.—*Voltages generated in various single phase windings.* The terms single, double, triple coil, etc., in the table below, indicate whether the inductors are arranged in one, two, three, etc., equally spaced single coils per pole piece, the single coil being determined by the group of inductors lying in one slot. The conditions are equivalent to the component voltages generated in each group being in one, two, three, etc., different phases, irrespective of the number of resultant windings into which they are combined.

Type of Winding	Correction factor for voltage of variously distributed windings
Single coil winding	E = 1.000
Double coil winding	E = .707 × single coil winding
Triple coil winding	E = .667 × " " "
Quadruple coil winding	E = .654 × " " "
Multi-coil or thoroughly distributed winding	E = .637 × " " "

NOTE.—*Spread factor.* To correct for the distortion due to spreading, a factor, known as the spread factor, is introduced into voltage calculations. For practical estimation this reduction factor may be taken as .96 in the case of a distributed three phase winding; .90 for a distributed two phase winding and .84 for a single phase winding distributed over two thirds of the pole pitch.

Windings for turbines (on account of the high speed) must be quite different from those driven by steam engines. Accordingly, in order that the frequency be not too high, turbine driven alternators must have very few poles—usually two or four, but rarely six.

Table of Frequency and Revolutions

FREQUENCY	REVOLUTIONS		
	2 POLE	4 POLE	6 POLE
25	1,500	750	500
60	3,600	1,800	1,200
100	6,000	3,000	2,000

Reconnecting A. C. Windings

Voltage Changes.—Nearly all commercial motors are arranged so that they can be reconnected for two voltages.

To make these changes, the polar groups are connected in series for the higher voltage and in parallel for the lower voltage.

In changing to higher voltages it should be noted that motors as manufactured are provided with insulation good for 550 volts or for 2,500 volts.

The capacity of the insulation should accordingly be considered and no change be made beyond the capacity of the insulation.

In making a voltage change, *the voltage per coil or per turn must be approximately the same after reconnection as before.*

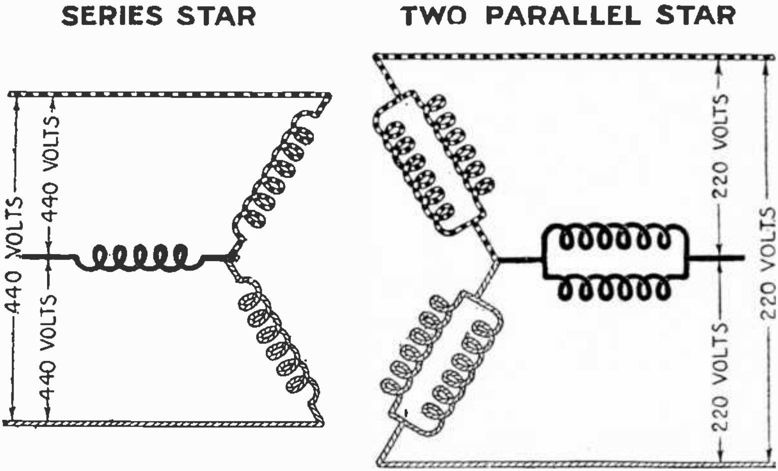
Example.—A motor is connected series-star for three phase 440 volts. How should it be connected for 220 volts?

Motor Voltages with Various Connections and Phases

Form of connection	Three phase series star	Three phase 2 parallel star	Three phase 3 parallel star	Three phase 4 parallel star	Three phase 5 parallel star	Three phase 6 parallel star	Three phase series delta	Three phase 2 parallel delta	Three phase 3 parallel delta	Three phase 4 parallel delta	Three phase 5 parallel delta	Three phase 6 parallel delta	Two phase series	Two phase 2 parallels	Two phase 3 parallels	Two phase 4 parallels	Two phase 5 parallels	Two phase 6 parallels
Three phase series star........	100	50	33	25	20	17	58	29	19	15	12	10	81	41	27	20	16	14
Three phase 2 parallel star.....	200	100	67	50	40	33	116	58	39	29	23	19	162	81	54	40	32	27
Three phase 3 parallel star	300	150	100	75	60	50	173	87	58	43	35	29	243	122	81	60	48	41
Three phase 4 parallel star.....	400	200	133	100	80	67	232	116	77	58	46	39	324	163	108	80	64	54
Three phase 5 parallel star.....	500	250	167	125	100	83	289	144	96	72	58	48	405	203	135	100	80	68
Three phase 6 parallel star.....	600	300	200	150	120	100	346	173	115	87	69	58	486	243	162	120	96	81
Three phase series delta........	173	86	58	43	35	29	100	50	33	25	20	17	140	70	47	35	28	23
Three phase 2 parallel delta....	346	173	115	87	69	58	200	100	67	50	40	33	280	140	94	70	56	47
Three phase 3 parallel delta....	519	259	173	130	104	87	300	150	100	75	60	50	420	210	141	105	84	70
Three phase 4 parallel delta...	692	346	231	173	138	115	400	200	133	100	80	67	560	280	188	140	1 1	93
Three phase 5 parallel delta....	865	433	288	216	173	144	500	250	167	125	100	83	700	350	233	175	140	117
Three phase 6 parallel delta...	1,038	519	346	260	208	173	600	300	200	150	120	100	840	420	280	210	168	140
Two phase series.............	125	63	42	31	25	21	72	37	24	18	15	12	100	50	33	25	20	17
Two phase 2 parallels.........	250	125	84	63	50	42	144	73	49	37	29	24	200	100	67	50	40	33
Two phase 3 parallels.........	375	188	125	94	75	63	216	111	73	55	44	37	300	150	100	75	60	50
Two phase 4 parallels.........	500	250	167	125	100	84	288	148	97	73	58	49	400	200	133	100	80	67
Two phase 5 parallels.........	625	313	208	156	125	105	360	165	122	91	73	61	500	250	167	125	100	84
Two phase 6 parallels.........	750	375	250	188	150	125	433	217	144	108	87	72	600	300	200	150	120	100

Look along the horizontal line corresponding to *three phase series star* and under the intersecting vertical column headed *three phase series star* is found the number 100. This means that the motor as it stands on 440 volts is considered 100 per cent. The new voltage is to be 220, which is 50 per cent of 440. Hence, the same horizontal line in the table, namely *three phase series star*, is followed along until the desired figure, 50 is found, which is under the vertical heading "*three phase 2 parallel star*." This is the correct answer: that is, if a motor be connected three phase series star for operation on 440 volts, it must be connected three phase 2 parallel star, as shown in fig. 61, to operate correctly on 220 volts.

Frequency Changes.—For the same number of poles a change in frequency will cause the speed to vary directly as the frequency.



FIGS. 60 and 61.—Winding diagrams illustrating a 440 volt series star connected motor and reconnection for operation at 220 volts as described in the accompanying text.

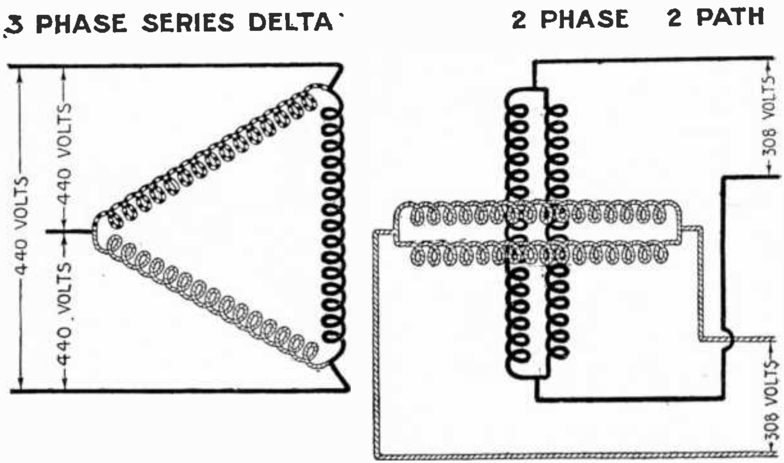
In making a frequency change if the speed is to remain the same, *the number of poles must be changed in the same ratio as the frequency, or approximately so*:

Example.—If a motor have four poles and be operated on 25 cycles, it will have a synchronous speed of $3{,}000 \div 4 = 750$ *r.p.m.* If the motor is to have the same speed on 60 cycles, the nearest possible pole number is 10 and the synchronous speed will be $7{,}200 \div 10 = 720$ *r.p.m.* It is apparent that in very few cases of this kind is it possible to re-connect the same winding.

Phase Changes.—The change most frequently desired is from two to three phases, or from three to two phases.

Example.—A three phase 440 volt motor is to be reconnected for two phase 440 volts. What changes must be made?

In the table on page 86 it is seen that the winding as it stands on 440 volts is four pole three phase series delta. Select the horizontal column



FIGS. 62 and 63.—Winding diagrams illustrating a 440 volt series star connected motor and reconnection for two phase operation indicated in the table on page 86, the voltage is reduced to 308 volts.

in the table marked three phase series delta and follow it across, looking for a vertical column showing the value 100, since the desired two phase voltage is the same as the present three phase voltage or 100 per cent. Inspection shows that there is no 100 under any two phase connection. This indicates at once that a three phase series delta connected motor which is normally operated on 440 volts cannot be changed and operated on two phase 440 volts, without rewinding.

Speed Changes.—The speed of an induction motor may be changed by regrouping the field coils for a different number of coil groups. In this connection it should be noted that *an increase in the number of poles will decrease the speed, whereas, a decrease in the number of poles will increase the speed.*

To reverse a two phase four wire induction motor, interchange the connections of the two leads on either phase.

For a two phase three wire motor, interchange the two outside leads.

For a three phase machine, interchange the connections of any two leads.

Condensers

It may be said that economical operation of a generating and distributing system is dependent on the maintenance of a relatively high power factor.

In a plant the power factor may be improved to some extent *by re-arrangement of the motors so that they will operate more nearly at full load.*

There are two types of equipment available for correcting low power factor:

1. Synchronous condenser;
2. Static condenser.

Low power factor operation results in *increased losses in alternators, exciters, distribution lines, transformers, and in the consumer's plant.*

NOTE.—The running fuse sizes given in contemporary tables are too small, and therefore a menace to polyphase motors. They should be eliminated, or if an over zealous inspection department insist that running fuses must be used, they should be at least 2½ times the rating of the motor or more. The most satisfactory running fuses, or, those that will do the motor the least damage, are the so called "starting" fuses of these same tables, although they are possibly a little large.—*Chapman.*

The following formula gives the currents at various power factors that will be required to carry a 75 *kw.*, 550 volt, single phase motor load:

$$1 = \frac{\text{Watts}}{\text{E (\%) P.F.}}$$

These currents for power factor 100 to 50 are given in the following table:

Currents for Various Power Factors

% Power Factor..................	100	90	80	70	60	50
Current.........................	136.3	151.5	170.5	195	228	273
Kva...........................	75	83	94	107	125	150

Another factor that this table shows well, is that at lower power factors, there is considerable line drop, which necessitates impressing over voltage at the supply end; making the voltage regulation poor.

The regulation of transformers is approximately 1% at unity power factor, and 3% at 70% power factor.

The effect of low power factor on the lines can best be shown by examples.

Example.—Assume a distance of two miles and a load of 100 *kw.* It is desired to deliver this load at about 2,300 volts, 3 phase, 60 cycles, with an energy loss of 10%.

Each conductor at unity power factor would require an area of 25,000 cir. mils; at .9 power factor, 30,820 cir. mils, while at .6 power factor, 69,500 cir. mils would be necessary. From this is will be seen that the investment in copper will have to be nearly 2.8 times as much at .6 power factor as at unity. If the same size wire were used at .6 as at unity, the energy loss would be 2.8 times the loss at unity, or 28%. Low lagging power factor on a system, therefore, will generally mean limited output of the prime movers, greatly reduced kilowatt capacity of alternators, transformers and lines, as well as increased energy losses. The regulation of the entire system will also be poor.

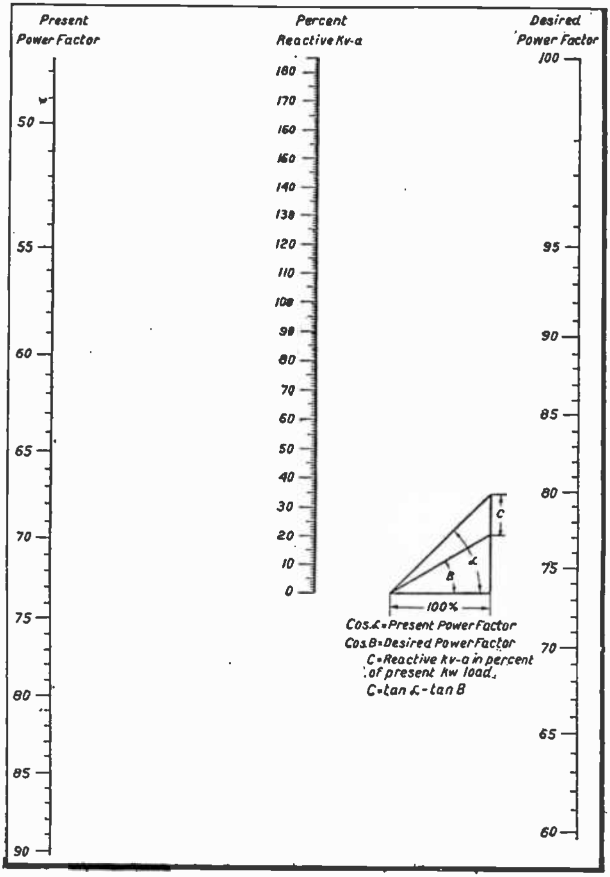
Power Factor Improvement—Table 1

(The figures below × kilowatt input = leading *kwa.* required to improve from one power factor to another.)

Original Power Factor %	Desired Power Factor 100 %	95 %	90 %	85 %	80 %	Original Power Factor %	Desired Power Factor 100 %	95 %	90 %	85 %	80 %
20	4.899	4.570	4.415	4.279	4.149	61	1.299	.970	.815	.679	.549
21	4.656	4.327	4.171	4.036	3.906	62	1.266	.937	.781	.646	.515
22	4.433	4.104	3.949	3.813	3.683	63	1.233	.904	.748	.613	.482
23	4.231	3.902	3.747	3.611	3.481	64	1.201	.872	.716	.581	.450
24	4.045	3.716	3.561	3.425	3.295	65	1.169	.840	.685	.549	.419
25	3.873	3.544	3.389	3.253	3.123	66	1.138	.810	.654	.518	.388
26	3.714	3.385	3.229	3.094	2.964	67	1.108	.779	.624	.488	.358
27	3.566	3.238	3.082	2.946	2.816	68	1.078	.750	.594	.458	.328
28	3.429	3.100	2.944	2.809	2.679	69	1.049	.720	.565	.429	.298
29	3.300	2.971	2.816	2.680	2.550	70	1.020	.691	.536	.400	.270
30	3.180	2.851	2.696	2.560	2.420	71	.992	.663	.507	.372	.241
31	3.067	2.738	2.583	2.447	2.317	72	.964	.635	.480	.344	.214
32	2.961	2.632	2.476	2.341	2.211	73	.936	.608	.452	.316	.186
33	2.861	2.532	2.376	2.241	2.111	74	.909	.580	.425	.289	.158
34	2.766	2.437	2.282	2.146	2.016	75	.882	.553	.398	.262	.132
35	2.676	2.347	2.192	2.056	1.926	76	.855	.527	.371	.235	.105
36	2.592	2.263	2.107	1.972	1.842	77	.829	.500	.344	.209	.078
37	2.511	2.182	2.027	1.891	1.761	78	.802	.474	.318	.182	.052
38	2.434	2.105	1.950	1.814	1.684	79	.776	.447	.292	.156	.026
39	2.361	2.032	1.877	1.741	1.611	80	.750	.421	.266	.130	
40	2.291	1.963	1.807	1.671	1.541	81	.724	.395	.240	.104	
41	2.225	1.896	1.740	1.605	1.475	82	.698	.369	.214	.078	
42	2.161	1.832	1.676	1.541	1.410	83	.672	.343	.188	.052	
43	2.100	1.771	1.615	1.480	1.349	84	.646	.317	.162	.026	
44	2.041	1.712	1.557	1.421	1.291	85	.620	.291	.136		
45	1.985	1.656	1.501	1.365	1.235	86	.593	.265	.109		
46	1.930	1.602	1.446	1.310	1.180	87	.567	.238	.082		
47	1.877	1.548	1.392	1.257	1.128	88	.540	.211	.056		
48	1.828	1.499	1.343	1.208	1.077	89	.512	.183	.028		
49	1.779	1.450	1.395	1.159	1.029	90	.484	.155			
50	1.732	1.403	1.248	1.112	.982	91	.456	.127			
51	1.687	1.358	1.202	1.067	.936	92	.426	.097			
52	1.643	1.314	1.158	1.023	.892	93	.395	.066			
53	1.600	1.271	1.116	.980	.850	94	.363	.034			
54	1.559	1.230	1.074	.939	.808	95	.329				
55	1.518	1.189	1.034	.898	.768	96	.292				
56	1.479	1.150	.995	.859	.729	97	.251				
57	1.442	1.113	.957	.822	.691	98	.203				
58	1.405	1.076	.920	.785	.654	99	.142				
59	1.368	1.040	.884	.748	.618	100					
60	1.333	1.004	.849	.713	.583						

Example.—Total *kw.* input of plant from watt meter reading 100 *kw.* at a power factor of 60%. The leading reactive *kva* necessary to raise the power factor to 90% is found by multiplying the 100 *kw.* by the factor found in the table which is .849. 100 *kw.* ×.849 = 84.9 *kva.* If static condensers be used, choose the standard unit nearest to 84.9. If synchronous motors be used, see example under the table on page 93.

Power Factor Chart



FIGS. 64 to 67.—Chart for use in determining the per cent reactive *kva.* required to raise the power factor to a desired value. *Example:* To find the "per cent reactive *kva.*" necessary to raise the power factor from present power factor to desired power factor, lay a straight edge across the chart connecting these two values. Read the reactive *kva.* in per cent of the present *kw.* load on the middle scale.

Example.—Assume a load of 450 *kw.* at .65 power factor. It is desired to raise the power factor to .9. What will be the rating of the condenser?

Referring to the diagram, fig. 69., it is necessary to start with 450 *kw.* At .65 power factor, or 692 *kva.*, this has a wattless lagging component of $\sqrt{692^2-450^2}=525$ *kva.* With the load unchanged and the power factor raised to .9, there will be 500 apparent *kva.*, which will have a wattless component of $\sqrt{500^2+450^2}=218$ *kva.*

It is obvious that the condenser must supply the difference between 525 *kva.* and 218 *kva.* or 307 *kva.* A 300 *kva.* condenser would, therefore, meet the requirements.

Power Factor Improvement—Table 2

P.F.	Reactive Kv-a.	P.F.	Reactive Kv-a.	P.F.	Reactive Kv-a.
1.00	.000	.83	.672	.66	1.138
.99	.142	.82	.698	.65	1.169
.98	.203	.81	.724	.64	1.201
.97	.251	.80	.750	.63	1.233
.96	.292	.79	.776	.62	1.266
.95	.329	.78	.802	.60	1.299
.94	.363	.77	.829	.60	1.333
.93	.395	.76	.855	.59	1.368
.92	.426	.75	.882	.58	1.405
.91	.456	.74	.909	.57	1.442
.90	.484	.73	.936	.56	1.479
.89	.512	.72	.964	.55	1.508
.88	.540	.71	.992	.54	1.559
.87	.567	.70	1.020	.53	1.600
.86	.593	.69	1.049	.52	1.643
.85	.620	.68	1.078	.51	1.687
.84	.646	.67	1.108	.50	1.732

NOTE.—The figures in the table show the amount of reactive *kva.* for each *kw.* energy load at various power factors. For synchronous motors, the figures show the leading reactive *kva.* per *kw.* input. For induction motors or a load with lagging power factor, the figures show the lagging reactive *kva.* per *kw.* energy load.

Example.—Refer to the table on page 91. Assume improvement desired by substitution .8 power factor synchronous motor for induction motors. For each *kw.* load driven by induction motors operating at average power factor of .6, the table shows there is 1.333 lagging reactive *kva.* For each *kw.* input in .8 power factor synchronous motor, the table shows a leading reactive *kva.* of .75. If .8 power factor synchronous motors replace .6 power factor induction motors, each *kw.* in synchronous motors reduces the lagging reactive *kva.* 1.333 ± .75 = 2.083 *kva.* The total reduction necessary to improve the power factor is shown by the table on page 91 to be 84.9 *kva.* The capacity in .8 power factor synchronous motor required is 84.9 ÷ 2.083 = 40.8 *kw.* A 50 *h.p.*—.8 power factor motor should be recommended.

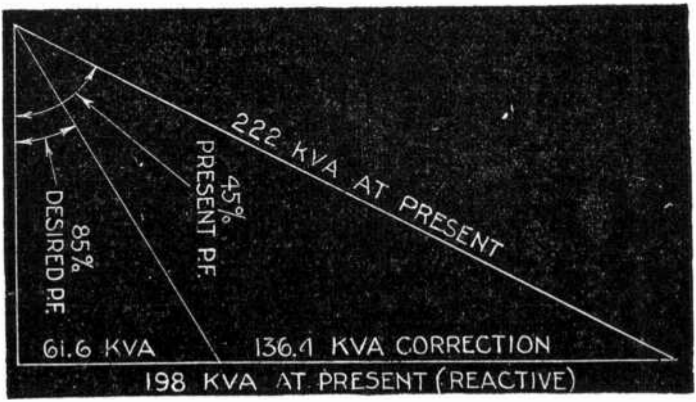


FIG. 68.—Diagram for power factor correction as explained in the accompanying example.

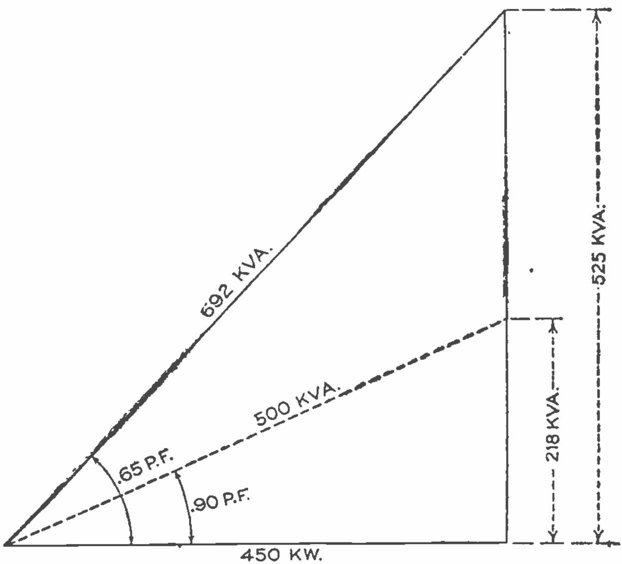


FIG. 69.—Diagram for synchronous condenser calculations.

Power Factor Correction Table

Kw. Load	Original Power Factor %	Kv-a. of Condenser to Raise to 85%	Variations in Power Factor (%) with Change in Load, the New Load Having the Same Power Factor without Condenser: 50 Kw.	75 Kw.	100 Kw.	125 Kw.	150 Kw.
100	40	167.3	68*	99.9*	85	72	66
100	45	136.8	81*	99.8	85	74	68
100	50	115	90*	97	85	77.5	72
100	55	90	96*	95	85	78	73.5
100	60	71	99*	93.5	85	79	76
100	65	55	99.5	92	85	81	78
100	70	40	97.7	90	85	82	79.5
100	75	26	94	88	85	83	81.5
100	80	13	89.5	86.5	85	84	83.5
100	85	0	85	85	85	85	85

*Leading power factor.

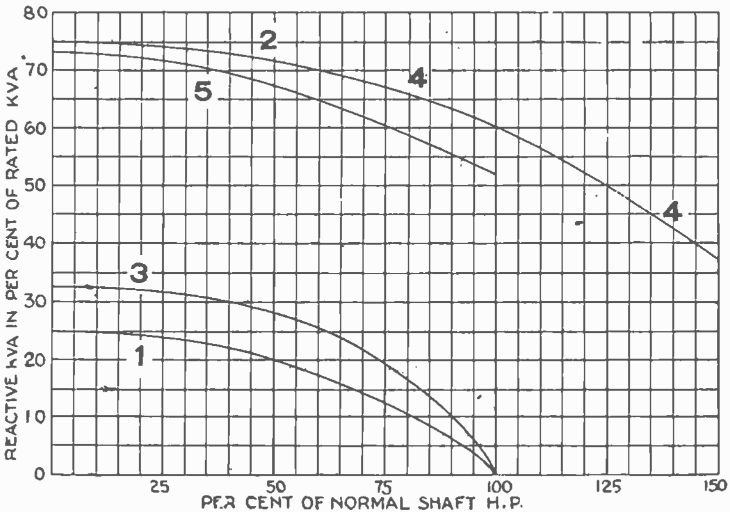


FIG. 70.—Curves for synchronous motors; reaction *kva.* available for power factor correction at different *h.p.* outputs. Assume motor field current is held constant at normal value. These curves are based on average values and therefore are approximate. 1, 1 *p.f.* belt driven motors; 2, .8 *p.f.* belt driven motors; 3, 1 *p.f.* air compressor motors; 4, .8 *p.f.* motors of 50% overload MG sets; 5, .85 *p.f.* motors of continuous rated motor generator sets.

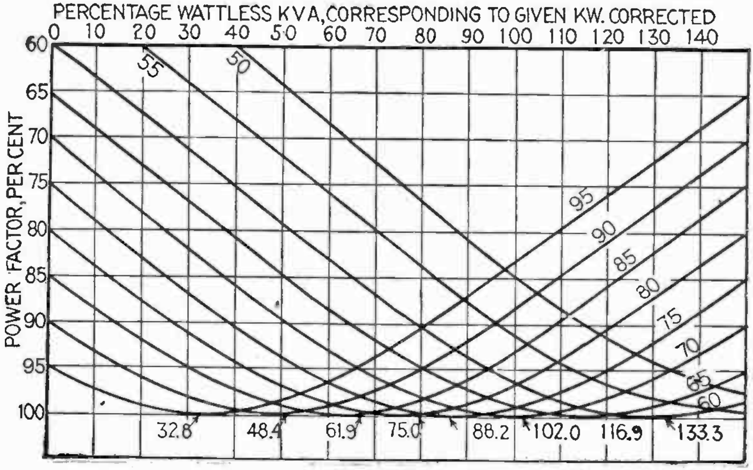


FIG. 71.—Curves showing the amount of wattless component required to raise the power factor of a given *kw.* load to required higher value. The wattless components are expressed as percentages of the original *kw.* load. ***The numbers at the right*** which indicate the points of tangency of the power factor curves to the 100 per cent. line, show the amount of wattless component required to raise a given *kw.* load of given lagging power factor to unity power factor. Obviously the addition of further wattless component in a given case would result in a leading power factor less than unity.

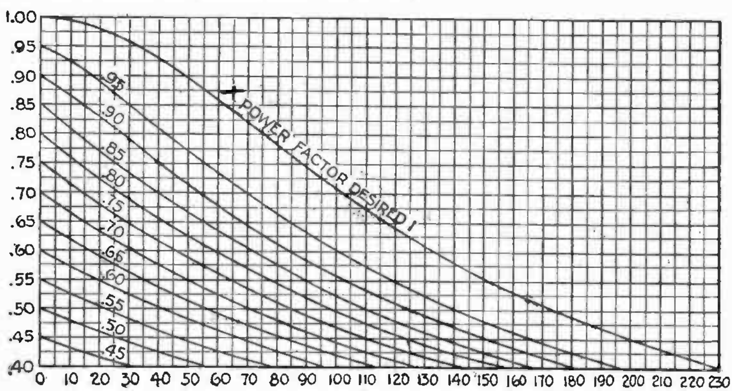


FIG. 72.—Curves showing determination of static condenser required to give desired correction in power factor. Follow horizontal line corresponding to present power factor of

Calculations for Static Condensers.—The *kva.* of static condensers required to correct any given power factor to any desired power factor is *entirely dependent on the kw. load of the plant.*

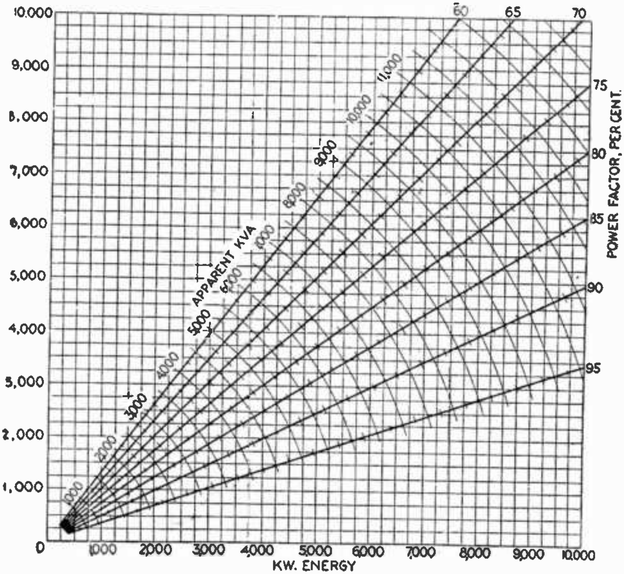


FIG. 73.—Chart showing the relation of energy load to apparent load and wattless components at different power factors.

Example.—Present load = 100 *kw.* at 45% power factor. Desired power factor = 85%.

$$\text{Present } kva. = \frac{100}{.45} = 222 \ kva.$$

$$\text{Present reactive } kva. = \sqrt{222^2 - 100^2} = 198 \ kva.$$

FIG. 72.—*Text continued.*

load until it intersects curve representing power factor desired. The vertical projection of this intersection on the base gives the size of condenser required in per cent of *kw.* load. *Example.*—Load 300 *kw.* Present power factor 60 per cent; power factor desired 90 per cent. Projection of intersection of 60 per cent power factor line with 90 per cent power factor curve gives desired condenser as 84.9 per cent of 300 *kw.* or 255 *kva.*

$Kva.$ at desired power factor $= \frac{100}{.85} = 117.8$ *kva.*

Reactive *kva.* at desired power factor $= \sqrt{117.8^2 - 100^2} = 61.6$ *kva.*

Corrective effect needed $= 198 - 61.6 = 136.4$.

Size of standard condenser $= 150$ *kva.*

Capacity in Micro-Farads per Mile of Circuit for Three Phase System

Size B. & S.	Diam. in inch.	Distance A in inches.	Capacity C in micro-farads	Size B. & S.	Diam. in inch.	Distance A in inches.	Capacity C in micro-farads
0000	.46	12	.0226	4	.204	12	.01874
		18	.0204			18	.01726
		24	.01922			24	.01636
		48	.01474			48	.01452
000	.41	12	.0218	5	.182	12	.01830
		18	.01992			18	.01690
		24	.01876			24	.01602
		48	.01638			48	.01426
00	.365			6	.162	12	.01788
		18	.01946			18	.01654
		24	.01832			24	.01560
		48	.01604			48	.0140
0	.325	12	.02078	7	.144	12	.01746
		18	.01898			18	.01618
		24	.01642			24	.01538
		48	.01570			48	.01374
1	.280	12	.02022	8	.128	12	.01708
		18	.01952			18	.01586
		24	.01748			24	.01508
		48	.0154			48	.01350
2	.258	12	.01972	9	.114	12	.01660
		18	.01818			18	.01552
		24	.01710			24	.01478
		48	.01510			48	.01326
3	.229	12	.01938	10	.102	12	.01636
		18	.01766			18	.01522
		24	.01672			24	.01452
		48	.01480			48	.01304

Distribution Systems.—The inductive effect of one line upon another is equal to the algebraic sum of the fluxes due to the different conductors of the first line, considered separately, which link the secondary line.

Inductance per Mile of Three Phase Circuit

Size B. & S.	Diam. in inch.	Distance *d* in inches.	Self inductance L henrys.	Size B. & S.	Diam. in inch.	Distance *d* in inches.	Self inductance L henrys.
0000	.46	12	.00234	4	.204	12	.00280
		18	.00256			18	.00300
		24	.00270			24	.00315
		48	.00312			48	.00358
000	.41	12	.00241	5	.182	12	.00286
		18	.00262			18	.00307
		24	.00277			24	.00323
		48	.00318			48	.00356
00	.365	12	.00248	6	.162	12	.00291
		18	.00269			18	.00313
		24	.00285			24	.00329
		48	.00330			48	.00369
0	.325	12	.00254	7	.144	12	.00298
		18	.00276			18	.00310
		24	.00293			24	.00336
		48	.00331			48	.00377
1	.289	12	.00260	8	.128	12	.00303
		18	.00281			18	.00325
		24	.00308			24	.00341
		48	.00338			48	.00384
2	.258	12	.00267	9	.114	12	.00310
		18	.00288			18	.00332
		24	.00304			24	.00348
		48	.00314			48	.00389
3	.229	12	.00274	10	.102	12	.00318
		18	.00294			18	.00340
		24	.00310			24	.00355
		48	.00351			48	.00396

INDUCTANCE IN MILLIHENRIES

PER 1000 FEET OF CONDUCTOR

SOLID CONDUCTORS

Size B. & S. Gauge	Resistance at 20°C. Ohms per 1000 ft.	Distance between centres (inches)									
		6	9	12	18	24	30	36	42	60	72
10	1.0259	.3052	.3299	.3473	.3720	.3895	.4031	.4141	.4236	.4452	.4563
8	.6452	.2911	.3158	.3332	.3579	.3754	.3890	.4000	.4095	.4311	.4422
6	.4058	.2770	.3017	.3191	.3438	.3613	.3749	.3859	.3954	.4170	.4281
4	.2552	.2629	.2876	.3050	.3297	.3472	.3608	.3718	.3813	.4029	.4140
2	.1605	.2488	.2735	.2909	.3156	.3331	.3467	.3577	.3672	.3888	.3999
1	.1272	.2418	.2665	.2839	.3086	.3261	.3397	.3507	.3602	.3818	.3929
1/0	.1009	.2347	.2594	.2768	.3015	.3190	.3326	.3436	.3531	.3747	.3858
2/0	.08003	.2276	.2523	.2697	.2944	.3119	.3255	.3365	.3460	.3676	.3787
3/0	.06347	.2206	.2453	.2627	.2874	.2949	.3185	.3295	.3390	.3606	.3717
4/0	.05033	.2135	.2382	.2556	.2803	.2978	.3114	.3224	.3319	.3535	.3646

STRANDED CONDUCTORS

Size Circular Mils	Resistance at 20°C. Ohms per 1000 ft.	Distance between centres (inches)									
		6	9	12	18	24	30	36	42	60	72
4 B.&S.	.2598	.2604	.2851	.3025	.3272	.3447	.3583	.3693	.3788	.4004	.4115
2 "	.1633	.2464	.2711	.2885	.3132	.3307	.3443	.3553	.3648	.3864	.3975
1 "	.1294	.2381	2688	.2812	.3059	.3234	.3370	.3480	.3575	.3791	.3902
1/0 "	.1027	.2318	.2565	.2739	.2986	.3161	.3297	.3407	.3502	.3718	.3829
2/0 "	.08164	.2248	.2495	.2669	.2916	.3091	.3227	.3337	.3432	.3648	.3759
3/0 "	.06470	.2178	.2425	.2599	.2846	.3021	.3157	.3267	.3362	.3578	.3689
4/0 "	.05125	.2106	.2353	.2527	.2774	.2949	.3085	.3195	.3290	.3506	.3617
250,000	.04344	.2057	.2304	.2478	.2725	.2900	.2036	.3146	.3241	.3457	.3568
300,000	.03625	.2001	.2248	.2422	.2669	.2844	.2980	.3090	.3185	.3401	.3512
350,000	.03102	.1954	.2201	.2375	.2622	.2797	.2933	.3043	.3138	.8354	.3465
400,000	.02722	.1914	.2161	.2335	.2582	.2757	.2893	.3003	.3098	.3314	.3425
450,000	.02413	.1877	.2124	.2298	.2545	.2720	.2856	.2966	.3061	.3277	.3388
500,000	.02177	.1847	.2094	.2268	.2515	.2690	.2826	.2936	.3031	.3247	.3358

Resistances above are for Hard-drawn Copper having a conductivity of 97.3 per cent. of the International Annealed Copper Standard. These resistances do not allow for Skin Effect.

INDUCTIVE REACTANCE IN OHMS

PER 1000 FEET OF CONDUCTOR

SOLID CONDUCTORS **FREQUENCY 60 CYCLES PER SECOND**

Size B.&S. Gauge	Distance between centres (inches)									
	6	9	12	18	24	30	36	42	60	72
10	.11505	.12438	.13093	.14025	.14685	.15198	.15612	.15970	.16785	.17203
8	.10973	.11906	.12561	.13493	.14153	.14666	.15080	.15438	.16253	.16671
6	.10441	.11374	.12029	.12961	.13621	.14134	.14548	.14906	.15721	.16139
4	.09909	.10842	.11497	.12429	.13089	.13602	.14016	.14374	.15189	.15607
2	.09377	.10310	.10965	.11897	.12557	13070	.13484	.13842	.14657	.15075
1	.09111	.10044	.10699	.11631	.12291	.12804	.13218	.13576	.14391	.14809
1/0	.08845	.09778	.10433	.11365	.12025	.12538	.12952	.13310	.14125	.14543
2/0	.08579	.09512	.10167	.11099	.11759	.12272	.12686	.13044	.13859	.14277
3/0	.08313	.09246	.09901	.10833	.11493	.12006	.12420	.12778	.13593	.14011
4/0	.08047	.08980	.09635	.10571	.11227	.11740	.12154	.12512	.13327	.13745

STRANDED CONDUCTORS **FREQUENCY 60 CYCLES PER SECOND**

Size Circular Mils	Distance between centres (inches)									
	6	9	12	18	24	30	36	42	60	72
4 B.&S.	.0982	.1076	.1141	.1234	.1300	.1351	.1393	.1429	.1511	.1553
2 "	.0929	.1023	.1088	.1181	.1247	.1298	.1340	.1376	.1458	.1500
1 "	.0901	.0995	.1060	.1153	.1219	.1270	.1312	.1348	.1430	.1472
1/0 "	.0874	.0968	.1033	.1126	.1192	.1243	.1285	.1321	.1403	.1445
2/0 "	.0847	.0941	.1006	.1099	.1165	.1216	.1258	.1294	.1376	.1418
3/0 "	.0821	.0915	.0980	.1073	.1139	.1190	.1232	.1268	.1350	.1392
4/0 "	.0794	.0888	.0953	.1046	.1112	.1163	.1205	.1241	.1323	.1365
250,000	.0775	.0869	.0934	.1027	.1093	.1144	.1186	.1222	.1304	.1346
300,000	.0754	.0848	.0913	.1006	.1072	.1123	.1165	.1201	.1283	.1325
350,000	.0736	.0830	.0895	.0988	.1054	.1105	.1147	.1183	.1265	.1307
400,000	.0721	.0815	.0880	.0973	.1039	.1090	.1132	.1168	.1250	.1292
450,000	.0707	.0801	.0866	.0959	.1025	.1076	.1118	.1154	.1236	.1278
500,000	.0696	.0790	.0855	.0948	.1014	.1065	.1107	.1143	.1225	.1267

INDUCTIVE REACTANCE IN OHMS

PER 1000 FEET OF CONDUCTOR

SOLID CONDUCTORS — **FREQUENCY 25 CYCLES PER SECOND**

Size B.&S. Gauge	Distance between centres (inches)									
	6	9	12	18	24	30	36	42	60	72
10	.04796	.05185	.05458	.05846	.06121	.06335	.06507	.06657	.06996	.07170
8	.04574	.04963	.05236	.05624	.05899	.06113	.06285	.06435	.06774	.06948
6	.04352	.04741	.05014	.05402	.05677	.05891	.06063	.06213	.06552	.06726
4	.04130	.04519	.04792	.05180	.05455	.05669	.05841	.05991	.06330	.06504
2	.03908	.04297	.04570	.04958	.05233	.05447	.05619	.05769	.06108	.06282
1	.03797	.04186	.04459	.04847	.05122	.05336	.05508	.05658	.05997	.06171
1/0	.03686	.04075	.04348	.04736	.05011	.05225	.05397	.05547	.05886	.06060
2/0	.03575	.03964	.04237	.04625	.04900	.05114	.05286	.05436	.05775	.05949
3/0	.03464	.03853	.04126	.04514	.04789	.05003	.05175	.05325	.05664	.05837
4/0	.03353	.03742	.04015	.04403	.04678	.04892	.05064	.05214	.05553	.05728

STRANDED CONDUCTORS — **FREQUENCY 25 CYCLES PER SECOND**

Size Circular Mils	Distance between centres (inches)									
	6	9	12	18	24	30	36	42	60	72
4 B.&S.	.0409	.0448	.0476	.0514	.0542	.0563	.0580	.0595	.0629	.0647
2 "	.0387	.0426	.0454	.0492	.0520	.0541	.0558	.0573	.0607	.0625
1 "	.0375	.0414	.0442	.0480	.0508	.0529	.0546	.0561	.0595	.0613
1/0 "	.0364	.0403	.0431	.0469	.0497	.0518	.0535	.0550	.0584	.0602
2/0 "	.0353	.0392	.0420	.0458	.0486	.0507	.0524	.0539	.0573	.0591
3/0 "	.0342	.0381	.0409	.0447	.0475	.0496	.0513	.0528	.0562	.0580
4/0 "	.0331	.0370	.0398	.0436	.0464	.0485	.0502	.0517	.0551	.0569
250,000	.0323	.0362	.0390	.0428	.0456	.0477	.0494	.0509	.0543	.0561
300,000	.0314	.0353	.0381	.0419	.0447	.0468	.0485	.0500	.0534	.0552
350,000	.0307	.0346	.0374	.0412	.0440	.0461	.0478	.0493	.0527	.0545
400,000	.0301	.0340	.0368	.0406	.0434	.0455	.0472	.0487	.0521	.0539
450,000	.0295	.0334	.0362	.0400	.0428	.0449	.0466	.0481	.0515	.0533
500,000	.0290	.0329	.0357	.0395	.0423	.0444	.0461	.0476	.0510	.0528

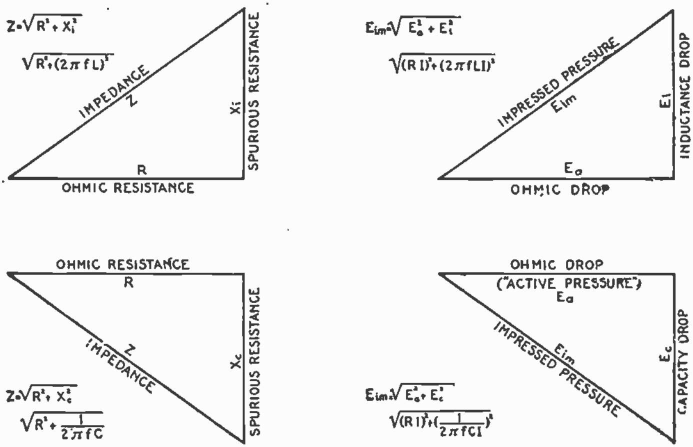
A three phase three wire transmission line spaced at the corners of an equilateral triangle as regards capacity acts precisely as though the neutral line were situated at the center of the triangle.

The capacity of circuits is readily calculated by applying the following formulæ:

$$C = \frac{38.83\ sc\ 10^{-3}}{\log (D \div d)}$$ per mile, insulated cable with lead sheath;

$$C = \frac{38.83 \times 10^{-3}}{\log (4h \div d)}$$ per mile, single conductor with earth return;

$$C = \frac{19.42 \times 10^{-3}}{\log (2A \div d)}$$ per mile of parallel conductors forming metallic circuit;



Figs. 74 to 77.—Triangles for obtaining graphically the impedance, impressed pressure, etc., in alternating current circuits. For a full explanation of this method the reader is referred to Vol. 4, Chapter 48, on Alternating Current Diagrams *in Audel's Engineers and Mechanics Guide.*

in which

C = Capacity in micro-farads; for a metallic circuit, C = capacity between wires

sc = Specific inductive capacity of insulating material; = 1 for arc, and 2.25 to 3.7 for rubber;

D = Inside diameter of lead sheath;

d = Diameter of conductor;

h = Distance of conductors above ground;

A = Distance between wires.

Frequency.—The number of cycles per second, or the frequency, has a direct effect upon the inductance reactance in an alternating current circuit, as is plainly seen from the formula

$$X_l = 2\pi f L$$

The natural period of a line, with distributed inductance and capacity, is approximately given by

$$P = \frac{7{,}900}{\sqrt{LC}}$$

where L, is the total inductance in milli-henrys, and C, the total capacity in micro-farads.

Skin Effect.—In a conductor carrying alternating current, *the current tends to flow near the circumference rather than through the center of the conductor. This is called skin effect.*

Approximate values of the effective resistance of straight copper conductors at 68° F. can be obtained by multiplying the actual ohmic resistance by factors given in the following table.

Factors to Obtain Effective Resistance from Ohmic Resistance

Diameter Bare Copper Conductor Inches	Approximate Area in Circular Mils	Frequency			Diameter Bare Copper Conductor Inches	Approximate Area in Circular Mils	Frequency		
		25	60	130			25	60	130
2.00	4,000,000	1.265	1.826	2.560	1.000	1,000,000	1.020	1.111	1.397
1.75	3,062,500	1.170	1.622	2.272	.75	563,500	1.007	1.040	1.156
1.50	2,500,000	1.098	1.420	1.983	.50	250,000	1.002	1.008	1.039
1.25	1,562,500	1.053	1.239	1.694	46	211,600	1.001	1.006	1.027
1.125	1,265,625	1.035	1.168	1 545					

To calculate skin effect, *its area in circular mils multiplied by the frequency gives the ratio of the wire's ohmic resistance to its combined resistance.*

That is to say, the factor thus obtained multiplied by the resistance of the wire to direct current, will give its combined resistance or resistance to alternating current.

The following table gives these ratio factors for large conductors.

RATIO FACTOR FOR COMBINED RESISTANCE

Circular mils × frequency	Factor	Circular mils × frequency	Factor
10,000,000	1.000	80,000,000	1.158
20,000,000	1.008	90,000,000	1.195
30,000,000	1.025	100,000,000	1.230
40,000,000	1.045	125,000,000	1.332
50,000,000	1.070	150,000,000	1.443
60,000,000	1.096	175,000,000	1.530
70,000,000	1.126	200,000,000	1.622

Wires should be so spaced as *to lessen the tendency to leakage and to prevent the wires swinging together or against towers.*

The following spacing is in accordance with average practice.

SPACING FOR VARIOUS VOLTAGES

Volts	Spacing	Volts	Spacing	Volts	Spacing
5,000	28 ins.	45,000	60 ins.	90,000	96 ins.
15,000	40 ins.	60,000	60 ins.	105,000	108 ins.
30,000	48 ins.	75,000	84 ins	120,000	120 ins.

The Three Circuits.—The transmission of alternating current power involves three separate circuits, one of which is composed of the wires forming the transmission line, while the others lie in the medium surrounding the wires. The constants of these circuits are interdependent; although any one may vary greatly from the others in magnitude.

The following table gives a comparison of the three circuits.

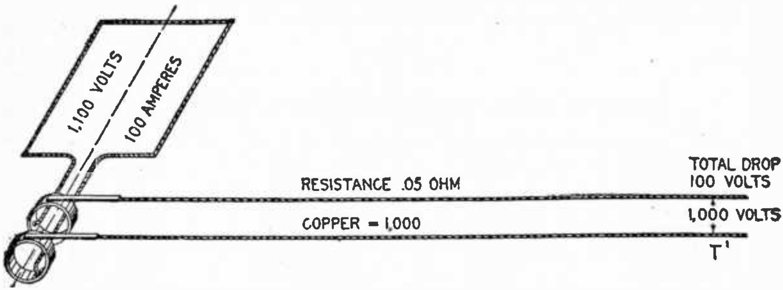
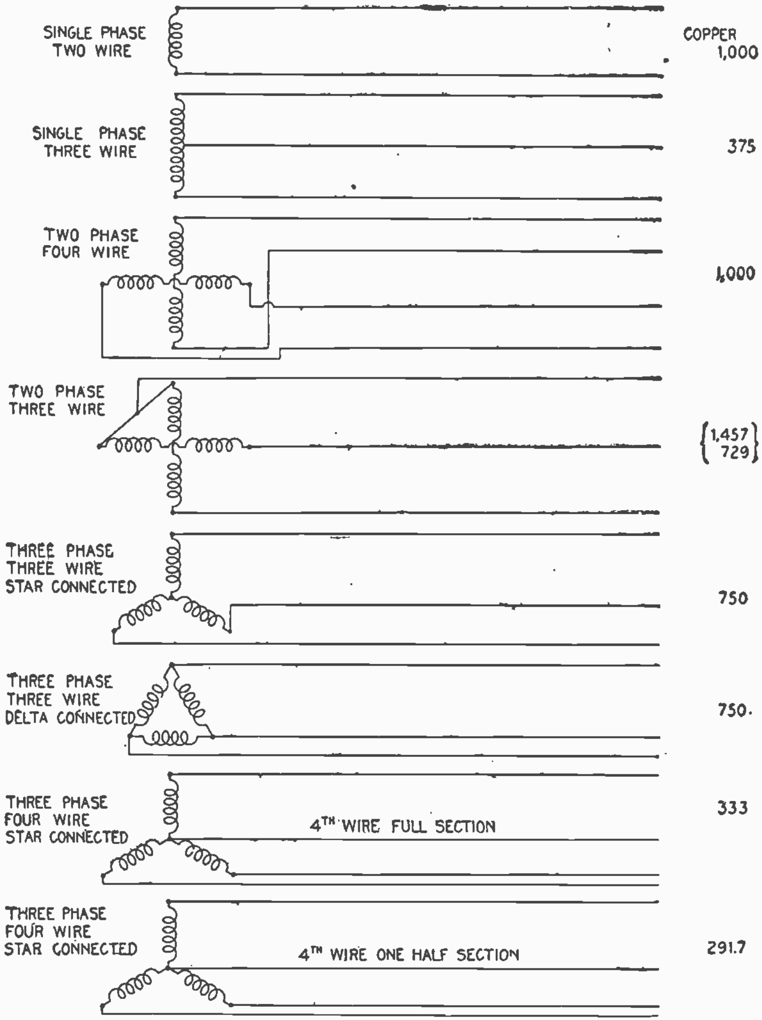


FIG. 78.—Single phase line, used as basis of comparison in obtaining the relative weights of copper required by polyphase systems, as indicated in figs. 79 to 86.

The Three Circuits

The electric circuit	The magnetic circuit	The dielectric circuit
Current I Voltage $E = RI$ Electric power	Magnetic flux ϕ Magnetomotive force $F = ni$ Magnetic energy	Dielectric flux ψ Electromotive force $E = Q/C$ Dielectric energy
Resistivity Resistance $R = W/I^2$	Reluctivity Reluctance R Inductance $L = \phi/i$	Elastivity $1/K$ Elastance S Capacitance $C = \psi/E$
	Reactance $x = wL - 1/wC$	
Impedance $z = \sqrt{r^2 + x^2}$		
Conductivity γ Conductance $g = W/E^2$ Conductance $g = r/z^2$	Permeability $\mu = B/H$ Permeance $M = \phi/4\pi F$	Permittivity K Permittance (Capacitance) C
	Susceptance $b = x/z^2$	
Admittance $y = 1/z = g \pm jb = \sqrt{g^2 + b^2}$		



FIGS. 79 to 86.—Circuit diagrams showing relative copper economy of various alternating current systems.

Choice of Voltage.—The most economical voltage for a transmission line depends on the length of the line and the cost of apparatus.

No fixed rule can be established for proper voltage based on the length, but the following table will serve as a guide:

Usual Transmission Voltages

Length of line in miles	Voltage
1	500 to 1,000
1 to 2	1,000 to 2,300
2 to 3	2,300 to 6,600
3 to 10	6,600 to 13,200
10 to 15	13,200 to 22,000
15 to 20	22,000 to 44,000
20 to 40	44,000 to 66,000
40 to 60	66,000 to 88,000
60 to 100	88,000 to 110,000

D. C. Wiring Calculations

D. C. Wiring Calculations.—The Board of Underwriters specifies that the carrying capacity of a conductor is safe when the wire will conduct a certain current without becoming painfully hot.

Circular mil=area of a circle one mil (.001 in.) in diameter.

Square mil=$.001\times.001=.000001$ in.

Mil foot=a volume one mil in diameter and one ft. long.

Lamp foot=one 16 candle power lamp at a distance of one ft. from the point of supply.

Ampere foot=the product of one ampere multiplied by one ft.

The formula for size of wire is

$$\textbf{circular mils} = \frac{\textbf{amperes} \times \textbf{ft.} \times 21.6}{\text{``drop''}} \ldots\ldots\ldots(1)$$

Example.—What size wire should be used on a 250 volt circuit to transmit a current of 200 amperes a distance of 350 feet to a center of distribution with a loss of three per cent. under full load?

The volts lost or drop is equal to $250\times.03=7.5$ volts.

Substituting the given value in formula (1)

$$\text{circular mils} = \frac{350\times200\times21.6}{7.5} = 201{,}600$$

$$\text{Diameter} = \sqrt{201{,}600} = 449 \text{ mils or } .449 \text{ in.}$$

From the table (on page 111) the nearest (*larger*) size of wire is 0000 B. W.G. or 0000 B. & S. gauge.*

*CAUTION.—The size thus obtained should be compared with the table of carrying capacity of wires as given on page 125 to see if the wires would have to carry more than the allowable current.

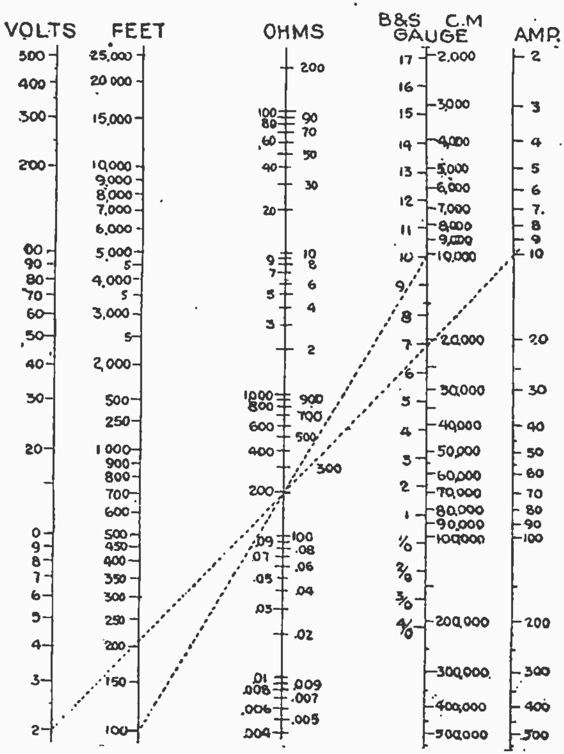


FIG. 87.—Chart for calculating *d.c.* or *a.c.* wiring circuits of ordinary length and high power factor, such as for incandescent lamps.

Example.—To find the size of wire to carry 10 amperes a distance of 100 feet with a 2 volt drop.

Solution.—First—Place a straight edge on 2 volts on volts scale and 10 amperes on amperes scale. Read at point of intersection on ohms scale —which reading is 1.2 ohms.

Second—Place straight edge on 1.2 on ohms scale and 100 feet on feet scale and read at point of intersection on B. & S., C. M. Gauge—which reading is No. 10—the size of wire desired.

After the wire size has been calculated it is necessary to determine whether this wire will safely carry the current. Safe carrying capacities of wires, as given in the "National Electrical Code" of the National Board of Fire Underwriters, will be found in the table on page 125.

From this table we find that 10 amperes is permissible for No. 10 wire with any type of insulation.

Properties of Copper Wire

Giving weights, length and resistances of wires of Matthiessen's Standard Conductivity for both B. & S. G. (Brown & Sharpe Gauge) and B. W. G. (Birmingham Wire Gauge) from Transactions October 1903, of the American Institute of Electrical Engineers.

Gauges.	To the nearest fourth significant digit.			Weight. Lbs. per 1,000 feet.	Length.	Resistance.
		Diameter.	Area.		Feet per lb.	Ohms per 1,000 feet.
B. & S.	B. W. G.	Inches.	Circular mils			@ 68° F.
0000		0.460	211,600	640.5	1.561	.04893
	0000	0.454	206,100	623.9	1.603	.05023
	000	0.425	180,600	546.8	1 829	.05732
000		0.4096	167,800	508.0	1.969	.06170
	00	0.380	144,400	437.1	2.288	.07170
00		0.3648	133,100	402.8	2.482	.07780
	0	0.340	115,600	349.9	2.858	.08957
0		0.3249	105,500	319.5	3.130	.09811
	1	0.3000	90,000	272.4	3.671	.1150
1		0.2893	83,690	253.3	3.947	.1237
	2	0.2840	80,660	244.1	4.096	.1284
	3	0.2590	67,080	203.1	4.925	.1543
2		0.2576	66,370	200.9	4.977	.1560
	4	0.2380	56,640	171.5	5.832	.1828
3		0.2294	52,630	159.3	6.276	.1967
	5	0.2200	48,400	146.5	6.826	.2139
4		0.2043	41,740	126.4	7.914	.2480
	6	0 2030	41,210	124.7	8.017	.2513

Properties of Copper Wire

(Continued)

Gauges.	To the nearest fourth significant digit.			Weight. Lbs. per 1,000 feet.	Length. Feet per lb.	Resistance. Ohms per 1,000 feet. @ 68° F.
B. & S.	B. W. G.	Diameter. Inches.	Area. Circular mils			
5		0.1819	33,100	100.2	9.980	.3128
	7	0.1800	32,400	98.08	10.20	.3196
	8	0.1650	27,230	82.41	12.13	.3803
6		0.1620	26,250	79.46	12.58	.3944
	9	0.1480	21,900	66.30	15.08	.4727
7		0.1443	20,820	63.02	15.87	.4973
	10	0.1340	17,960	54.35	18.40	.5766
8		0.1285	16,510	49.98	20.01	.6271
	11	0.1200	14,400	43.59	22.94	.7190
9		0.1144	13,090	39.63	25.23	.7908
	12	0.1090	11,880	35.96	27.81	.8715
10		0.1019	10,380	31.43	31.82	.9972
	13	0.0950	9,025	27.32	36.60	1.147
11		0.09074	8,234	24.93	40.12	1.257
	14	0.08300	6,889	20.85	47.95	1.503
12		0.08081	6,530	19.77	50.59	1.586
	15	0.07200	5,184	15.69	63.73	1.997
13		0.07196	5,178	15.68	63.79	1.999
	16	0.06500	4,225	12.79	78.19	2.451
14		0.06408	4,107	12.43	80.44	2.521
	17	0.0580	3,364	10.18	98.23	3.078
15		0.05707	3,257	9.858	101.4	3.179
16		0.05082	2,583	7.818	127.9	4.009
	18	0.04900	2,401	7.268	137.6	4.312
17		0.045260	2,048	6.200	161.3	5.055
	19	0.042000	1,764	5.340	187.3	5.870
18		0.040300	1,624	4.917	203.4	6.374
19		0.035890	1,288	3.899	256.5	8.038
	20	0.035000	1,225	3.708	269.7	8.452
	21	0.032000	1,024	3.100	322.6	10.11
20		0.031960	1,022	3.092	323.4	10.14
21		0.028460	810.1	2.452	407.8	12.78
	22	0.028000	784.0	2.373	421.4	13.21
22		0.025350	642.4	1.945	514.2	16.12
	23	0.025000	625.0	1.892	528.6	16.57

Properties of Copper Wire

(Concluded)

Gauges.	To the nearest fourth significant digit.			Weight. Lbs. per 1,000 feet.	Length.	Resistance.
		Diameter.	Area.		Feet per lb.	Ohms per 1,000 feet.
B. & S.	B. W. G.	Inches.	Circular mils			@ 68° F.
23		0.022570	509.5	1.542	648.4	20.32
	24	0.022000	484.0	1.465	682.6	21.39
24		0.020100	404.0	1.223	817.6	25.63
	25	0.020000	400.0	1.211	825.9	25.88
	26	0.018000	324.0	.9808	1,020	31.96
25		0.017900	320.4	.9699	1,031	32.31
	27	0.016000	256.0	.7749	1,290	40.45
26		0.015940	254.1	.7692	1,300	40.75
27		0.014200	201.5	.6100	1,639	51.38
	28	0.014000	196.0	.5933	1,685	52.83
	29	0.013000	169.0	.5116	1,955	61.27
28		0.012640	159.8	.4837	2,067	64.79
	30	0.012000	144.0	.4359	2,294	71.90
29		0.011260	126.7	.3836	2,607	81.70
30		0.010030	100.5	.3042	3,287	103.0
	31	0.010000	100.0	.3027	3,304	103.5
	32	0.009000	81.0	.2452	4,078	127.8
31		0.008928	79.70	.2413	4,145	129.9
	33	0.008000	64.0	.1937	5,162	161.8
32		0.007950	63.21	.1913	5,227	163.8
33		0.007080	50.13	.1517	6,591	206.6
	34	0.007000	49.0	.1483	6,742	211.3
34		0.006305	39.75	.1203	8,311	260.5
35		0.005615	31.52	.09543	10,480	328.4
36	35	0.005000	25.0	.07568	13,210	414.2
37		0.004453	19.83	.06001	16,660	522.2
	36	0.004000	16.	.04843	20,650	647.1
38		0.003965	15.72	.04759	21,010	658.5
39		0.003531	12.47	.03774	26,500	830.4
40		0.003145	9.888	.02993	33,410	1047.

NOTE—It should be noted that the Underwriters prohibit the use of wire smaller than No. 14 B. & S. gauge, except as allowed for fixture work and pendant cord.

Table of Various Wire Gauges

Number of Wire Gauge	American, or Brown & Sharpe (B. & S.)	Birmingham, or Stubs (B. W. G.)	Washburn & Moen Mfg. Co., Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	G. W. Prentiss, Holyoke, Mass.	Old English, From Brass Mfrs' List	British Standard (S. W. G.)	Number of Wire Gauge
0000000							.500	0000000
000000			.460				.464	000000
00000			.430	.450			.432	00000
0000	.46000	.454	.393	.400			.400	0000
000	.40964	.425	.362	.360	.3586		.372	000
00	.36480	.380	.331	.330	.3282		.348	00
0	.32486	.340	.307	.305	.2994		.324	0
1	.28930	.300	.283	.285	.2777		.300	1
2	.25763	.284	.263	.265	.2591		.276	2
3	.22942	.259	.244	.245	.2401		.252	3
4	.20431	.238	.225	.225	.2230		.232	4
5	.18194	.220	.207	.205	.2047		.212	5
6	.16202	.203	.192	.190	.1885		.192	6
7	.14428	.180	.177	.175	.1758		.176	7
8	.12849	.165	.162	.160	.1605		.160	8
9	.11443	.148	.148	.145	.1471		.144	9
10	.10189	.134	.135	.130	.1351		.128	10
11	.090742	.120	.120	.1175	.1205		.116	11
12	.080808	.109	.105	.1050	.1065		.104	12
13	.071961	.095	.0920	.0925	.0928		.0920	13
14	.064084	.083	.0800	.0800	.0816	.08300	.0800	14
15	.057068	.072	.0720	.0700	.0726	.07200	.0720	15
16	.050820	.065	.0630	.0610	.0627	.06500	.0640	16
17	.045257	.058	.0540	.0525	.0546	.05800	.0560	17
18	.040303	.049	.0470	.0450	.0478	.04900	.0480	18
19	.035890	.042	.0410	.0400	.0411	.04000	.0400	19
20	.031961	.035	.0350	.0350	.0351	.03500	.0360	20
21	.028462	.032	.0320	.0310	.0321	.03150	.0320	21
22	.025347	.028	.0280	.0280	.0290	.02950	.0280	22
23	.022571	.025	.0250	.0250	.0261	.02700	.0240	23
24	.020100	.022	.0230	.0225	.0231	.02500	.0220	24
25	.017900	.020	.0200	.0200	.0212	.02300	.0200	25
26	.015940	.018	.0180	.0180	.0194	.02050	.0180	26
27	.014195	.016	.0170	.0170	.0182	.01875	.0164	27
28	.012641	.014	.0160	.0160	.0170	.01650	.0148	28
29	.011257	.013	.0150	.0150	.0163	.01550	.0136	29
30	.010025	.012	.0140	.0140	.0156	.01375	.0124	30
31	.008928	.010	.0130	.0130	.0146	.01225	.0116	31
32	.007950	.009	.0120	.0120	.0136	.01125	.0108	32
33	.007080	.008	.0110	.0110	.0130	.01025	.0100	33
34	.006305	.007	.0100	.0100	.0118	.00950	.0092	34
35	.005615	.005	.0095	.0095	.0109	.00900	.0084	35
36	.005000	.004	.0090	.0090	.0100	.00750	.0076	36
37	.004453		.0085	.0085	.0095	.00650	.0068	37
38	.003965		.0080	.0080	.0090	.00575	.0066	38
39	.003531		.0075	.0075	.0083	.00500	.0052	39
40	.003145		.0070	.0070	.0078	.00450	.0048	40
41							.0044	41
42							.0040	42

NOTE.—*The sizes of wire* are ordinarily expressed by an arbitrary series of numbers. Unfortunately there are several independent numbering methods, so that it is always necessary to specify the method or wire gauge used. The above table gives the numbers and diameters in decimal parts of an inch for the various wire gauges in general use.

To facilitate finding the equivalent sizes the accompanying table of wire equivalents has been prepared.

Table of Wire Equivalents

GAUGE B. & S.	NUMBER OF WIRES 2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384
0000	0	3	6	9	12	15	18	21	24	27	30	33	36	39
.000	1	4	7	10	13	16	19	22	25	28	31	34	37	40
00	2	5	8	11	14	17	20	23	26	29	32	35	38	0+6
0	3	6	9	12	15	18	21	24	27	30	33	36	39	1+7
1	4	7	10	13	16	19	22	25	28	31	34	37	40	2+8
2	5	8	11	14	17	20	23	26	29	32	35	38	3+9	4+6
3	6	9	12	15	18	21	24	27	30	33	36	39	4+10	5+7
4	7	10	13	16	19	22	25	28	31	34	37	40	5+11	6+8
5	8	11	14	17	20	23	26	29	32	35	38	6+12	7+9	
6	9	12	15	18	21	24	27	30	33	36	39	7+13	8+10	
7	10	13	16	19	22	25	28	31	34	37	40	8+14	9+11	
8	11	14	17	20	23	26	29	32	35	38	9+15	10+12		
9	12	15	18	21	24	27	30	33	36	39	10+16	11+13		
10	13	16	19	22	25	28	31	34	37	40	11+17	12+14		
11	14	17	20	23	26	29	32	35	38	12+18	13+15			
12	15	18	21	24	27	30	33	36	39	13+19	14+16			
13	16	19	22	25	28	31	34	37	40	14+20	15+17			
14	17	20	23	26	29	32	35	38	15+21	16+18				
15	18	21	24	27	30	33	36	39	16+22	17+19				
16	19	22	25	28	31	34	37	40	17+23	18+20				
17	20	23	26	29	32	35	38	18+24	19+21					
18	21	24	27	30	33	36	39	19+25	20+22					
19	22	25	28	31	34	37	40	20+26	21+23					
20	23	26	29	32	35	38	21+27	22+24						
21	24	27	30	33	36	39	22+28	23+25						
22	25	28	31	34	37	40	23+29	24+26						
23	26	29	32	35	38	24+30	25+27							
24	27	30	33	36	39	25+31	26+28							
25	28	31	34	37	40	26+32	27+29							
26	29	32	35	38	27+33	28+30								
27	30	33	36	39	28+34	29+31								
28	31	34	37	40	29+35	30+32								
29	32	35	38	30+36	31+33									
30	33	36	39	31+37	32+34									
31	34	37	40	32+38	33+35									
32	35	38	33+39	34+36										
33	36	39	34+40	35+37										
34	37	40	36+39											
35	38	37+39												
36	39	38+40												
37	40													

Figuring in Watts Instead of Amperes.—*The power required to operate most electrical appliances is marked in watts.*

The proper size of wire for a 660 watt circuit will depend upon the voltage for which the lamps are made. For example: a 16 candle power lamp which consumes 56 watts on 110 volt circuit will take, 56÷110=.5 or ½ ampere of current, while the same lamp, if made for 220 volts, will take only 55÷220=.25 or ¼ ampere. Therefore, eleven 16 candle power 56 watt lamps will require a current of 5½ amperes at 110 volts, or 2¾ amperes at 220 volts.

TABLE FOR TAPS, BRIDGES OR OTHER WIRES AT NEGLIGIBLE DROP

Wire Nos.	0	1	2	3	4	5	6	7	8	10	12	14	16	18
Lamp Feet { 52 v.	300	260	200	160	130	100	80	65	50	38	24	15	9	6
Lamp Feet { 110 v.	1,280	1,085	860	680	560	435	345	280	220	160	100	60	40	25

NOTE.—In using this table, it is only necessary to calculate the lamp feet of the tap and take the size of wire corresponding to the nearest greater number of lamp feet in the table. The lamp feet specified by this table should not be exceeded by more than 10 per cent. Thus, if a tap measure 108 lamp feet, in 110 volt lamps, No. 12 wire would be used, but if it measure 115 lamp feet, it would be advisable to use No. 10 wire.

In calculating a three wire installation proceed as follows:—

Example.—If in calculating a three wire feeder, the over all voltage be 220 volts, the drop=4.4 volts, twice the distance=400 feet, and the current=20.5 amperes, then,

$$\frac{4.4 \text{ volts}}{400 \text{ feet} \times 20.5 \text{ amperes}} = .0005366 \text{ ohms per foot.}$$

In the table of the properties of copper wire which gives the resistance of various sizes of wire, it will be noted that at all of the given temperatures No. 8 wire has a resistance greater than the value just calculated, therefore, No. 6 B. & S. gauge wire should be used for the outer wires of the feeder. In the table the resistance is given per 1,000 feet, hence it is only necessary to move the decimal point to obtain the resistance per foot. This table is shown on pages 111 to 113.

If the calculation be based on the lamp voltage, 110 volts, the formula (1) must be modified to

$$\frac{\text{drop} \times 4}{2 \times \text{distance} \times \text{amperes}} = \text{resistance} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In this case, drop=2.2 volts, 2×distance=400 feet, and current=41 amperes, then.

$$\frac{2.2 \text{ volts} \times 4}{400 \text{ feet} \times 41 \text{ amp.}} = \frac{8.8}{16{,}400} = .0005366 \text{ ohm.}$$

Size of the Neutral Wire.—In three wire circuits, the size of the neutral wire will depend to a great extent upon operating conditions. In the case of installations which occasionally have to be worked as two wire systems, the cross section of the neutral wire should be equal to the combined cross section of the two outer wires.

Figuring in Watt Feet.—By definition a *watt foot* is *the product of one watt multiplied by one foot*; it is a convenient unit for quick calculation with the aid of tables.

30 Volts

Rule.—To find the size of wire to carry a given number of watts a given distance, on 30 volt circuits, *multiply the distance in feet by the total number of watts to be carried (thus obtaining the watt feet), and use the size of wire in the table specified for the nearest number of watt feet.*

Table of Watt Feet for 30 Volts

Between	0	and	18,870	watt	ft.	use	No.	12	wire
"	18,870	"	29,000	"	"	"	"	10	"
"	29,000	"	46,545	"	"	"	"	8	"
"	46,545	"	73,018	"	"	"	"	6	"
"	73,018	"	116,363	"	"	"	"	4	"
"	116,363	"	186,180	"	"	"	"	2	"
"	186,180	"	232,727	"	"	"	"	1	"
"	232,727	"	290,000	"	"	"	"	0	"
"	290,000	"	372,362	"	"	"	"	00	"

Example.—Ten 20 watt lamps are to be installed in a barn 200 ft. distant. What size wire is required for a 30 volt circuit?

Load = 10×20×200 = 40,000 watt ft. Nearest size wire from table is No. 8.

110 Volts

***Rule.**—In using the table just given for* 110 *volts multiply either the watts or the distance by* 3⅔, because for a given load the current is reduced 110÷30 = 3⅔ times as compared with 30 volts.

A. C. Wiring Calculations

The formula for size of wire for *a.c.* circuits is

$$\text{circular mils} = \frac{\text{watts} \times \text{feet} \times \text{M}}{\% \text{ loss} \times \text{volts}^2} \quad ..(1)$$

in which M, is a coefficient which has various values according to the kind of circuit and value of the power factor. These values are given in the following table:

Values of M.

SYSTEM	POWER FACTOR.									
	1.00	.98	.95	90	85	.80	.75	.70	.65	.60
Single phase	2,160	2,249	2,400	2,660	3,000	3,380	3,840	4,400	5,112	6,000
Two phase (4 wire)	1,080	1,125	1,200	1,330	1,500	1,690	1,920	2,200	2,556	3,000
Three phase (3 wire)	1,080	1,125	1,200	1,330	1,500	1,690	1,920	2,200	2,556	3,000

NOTE.—*The above table* is calculated as follows: For ***single phase*** M = 2,160 ÷ power factor² × 100; for ***two phase*** four wire, or three phase three wire, M = ½ (2,160 ÷ power factor²) 100. Thus the value of M, for a single phase line with power factor .95 = 2,160 ÷ .95² × 100 = 2,382.

It must be evident that when 2,160 is taken as the value of M, formula 1 applies to a two wire direct current circuit and also to a single phase alternating current circuit when the power factor is unity.

In the table the value of M, for any particular power factor is found by dividing 2,160 by the square of that power factor for single phase and twice the square of the power factor for two phase and three phase.

Since the two phase system is virtually two single phase systems, the four wires of the two phase system are half the size of the two wires of the single phase system, and accordingly, the weight is the same for either system, when the load, voltage and power factor are the same in each case.

***Values of T.**

System	Power Factor				
	1.00	.98	90	.80	.70
Single phase..........	1.00	.98	.90	.80	.70
Two phase, 4 wire.....	2.00	1.96	1.80	1.60	1.40
Three phase, 3 wires...	1.73	1.70	1.55	1.38	1.21

Although there is no saving in copper in using two phases, the two phase system has the advantage over the one phase system in that it is more desirable on power circuits, because two phase motors are self-starting.

*NOTE.—*This table* is for finding the value of the current in line, using the formula, $I = W \div (E \times T)$, in which I = current in line; E = voltage between main conductors at receiving or consumers' end; W = watts. For instance, what is the current in a two phase line transmitting 1,000 watts at 550 volts, power factor .80? $I = 1{,}000 \div (550 \times 1.60) = 1.13$.

Example.—What size wires must be used on a single phase circuit 200 feet in length to supply 30 *kw*. at 220 volts with loss of 4%, the power factor being .9?

The formula for circular mils is

$$\text{circular mils} = \frac{\text{watts} \times \text{feet} \times M}{\% \text{ loss} \times \text{volts}^2} \quad \text{.......... (1)}$$

Substituting the given values and the proper value of M from the table in (1)

$$\text{circular mils} = \frac{30{,}000 \times 200 \times 2{,}660}{4 \times 220^2} = 82.438$$

Referring to the table of the properties of copper wire, on pages 111 to 113 the nearest *larger* size wire is No. 1 B. & S. gauge having an area of 83.690 circular mils.

Drop.—In order to determine the drop or volts lost in the line, the following formula may be used:

$$\textbf{drop} = \frac{\% \textbf{ loss} \times \textbf{volts}}{\textbf{100}} \times \textbf{S} \quad \text{........ (1)}$$

in which the % loss is a percentage of the applied power, that is, the power delivered to the consumer and not a percentage of the power at the alternator. "Volts" is the pressure at the consumer's end of the circuit.

Example.—A circuit supplying current at 440 volts, 60 frequency, with 5% loss and .8 power factor is composed of No. 2 B. & S. gauge wires spaced one foot apart. What is the drop in the line?

According to the formula

$$\text{drop} = \frac{\% \text{ loss} \times \text{volts}}{100} \times S$$

Substituting the given values, and value of S, as obtained from the table for frequency 60

$$\text{drop} = \frac{5 \times 440}{100} \times 1 = 22 \text{ volts}$$

Value of "S" for 60 Cycles

Size of wire B. & S. gauge	Area in circular mils.	.96 power factor					.90 power factor					.80 power factor					.70 power factor				
		Spacing of conductors					Spacing of conductors					Spacing of conductors					Spacing of conductors				
		1"	3"	6"	12"	24"	1"	3"	6"	12"	24"	1"	3"	6"	12"	24"	1"	3"	6"	12"	24"
500,000	500,000	1.21	1.45	1.61	1.77	1.92	1.32	1.80	2.11	2.44	2.75	1.27	1.89	2.25	2.64	3.03	1.14	1.72	2.12	2.53	2.92
300,000	300,000	1.15	1.29	1.38	1.48	1.57	1.19	1.47	1.66	1.84	2.02	1.11	1.46	1.68	1.90	2.12	1.00	1.33	1.56	1.78	2.01
0,000	211,600	1.12	1.22	1.28	1.34	1.41	1.13	1.33	1.45	1.58	1.63	1.63	1.27	1.43	1.58	1.75	1.00	1.14	1.29	1.45	1.69
000	167,800	1.09	1.18	1.22	1.28	1.29	1.08	1.23	1.33	1.44	1.53	1.00	1.16	1.28	1.41	.153	1.00	1.02	1.15	1.28	1.50
00	133,100	1.07	1.14	1.18	1.21	1.25	1.03	1.16	1.24	1.32	1.40	1.00	1.07	1.17	1.27	1.36	1.00	1.00	1.03	1.13	1.21
0	105,500	1.05	1.10	1.14	1.17	1.20	1.00	1.09	1.16	1.22	1.28	1.00	1.00	1.07	1.15	1.22	1.00	1.00	1.00	1.01	1.09
1	83,690	1.04	1.08	1.10	1.13	1.15	1.00	1.05	1.09	1.14	1.19	1.00	1.00	1.00	1.05	1.11	1.00	1.00	1.00	1.00	1.00
2	66,370	1.02	1.05	1.08	1.10	1.12	1.00	1.00	1.04	1.08	1.12	1.00	1.00	1.00	1.00	1.02	1.00	1.00	1.00	1.00	1.00
3	52,630	1.02	1.04	1.06	1.07	1.09	1.00	1.00	1.00	1.03	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
4 5	41,740 33,100	1.00	1.02	1.03	1.04	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6 7	26,250 20,820	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8 9 10	16,510 13,090 10,380	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Additional Wiring Formulæ.—For most practical purposes the following formulæ can be used to determine the size of copper conductors, current per wire, and weight of copper per circuit for any system of electrical distribution.

$$\text{Area of wire in circular mils} = \frac{D \times W}{P \times E^2} K$$

$$\text{Current in main wire} = \frac{W}{E} T. \quad P = \frac{D \times W}{cm \times E^2} K$$

Value of "S" for 25 Cycles

Size of wire B. & S. gauge	Area in circular mils.	.98 power factor					.90 power factor					.80 power factor					.70 power factor				
		Spacing of conductors					Spacing of conductors					Spacing of conductors					Spacing of conductors				
		1″	2″	6″	12″	24″	1″	3″	6″	12″	24″	1″	3″	6″	12″	24″	1″	8″	6″	12″	24″
500,000	500,000	1.01	1.17	1.23	1.29	1.36	1.02	1.22	1.35	1.43	1.61	1.00	1.15	1.30	1.47	1.62	1.00	1.00	1.16	1.33	1.49
300,000	300 000	1.04	1.10	1.13	1.18	1.21	1.00	1.08	1.16	1.25	1.31	1.00	1.00	1.09	1.16	1.25	1.00	1.00	1.00	1.02	1.12
0,000	211,600	1.03	1.07	1.09	1.11	1.14	1.00	1.02	1.07	1.13	1.15	1.00	1.00	1.00	1.03	1.1[illegible]	1.00	1.00	1.00	1.00	1.00
000	167,800	1.00	1.05	1.06	1.09	1.10	1.00	1.00	1.02	1.07	1.11	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00
00	133,100	1.00	1.03	1.05	1.06	1.08	1.00	1.00	1.00	1.02	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0	105,500	1.00	1.01	1.02	1.03	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1 2 3	83,690 66,370 52,630	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
4 5 6	41,740 33,100 26,250	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7 8 9 10	20,820 16,510 13,090 10,380	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$$\text{Weight of copper} = \frac{D^2 \times W \times K \times A}{P \times E^2 \times 1{,}000{,}000} \text{ pounds.}$$

In these equations the symbols used denote the following quantities:

W = total watts delivered

D = distance of transmission, one way in feet

E = voltage between main wires at the receiving or consumers' end of circuit

P = loss in line in per cent. of power delivered, *i.e.* of W, this being a whole number. K, T and A, are constants given in the following table:

Wiring Formulæ Constants

System	Values of A	Values of K					Values of T				
		Per Cent. Power Factor					Per Cent. Power Factor				
		100	95	90	85	80	100	95	90	85	80
1-phase, & D C	6.04	2160	2400	2660	3000	3380	1.00	1.05	1.11	1.17	1.25
2-phase-4 wire.	12.08	1080	1200	1330	1500	1690	.50	.53	.55	.59	.66
3-phase-3 wire.	9.06	1080	1200	1330	1500	1690	.58	.61	.64	.68	.72

Physical Properties of Copper, Aluminum, Iron and Steel Wire

Physical Properties		Copper Annealed	Copper Hard Drawn	Aluminum 99 Per Cent. Pure	Iron (Ex. B. B.)	Steel (Siemens Martin)
Conductivity, Matthiessen's Standard		99 to 102	96 to 99	61 to 63	16.8	8.7
Ohms per mil-foot at 68° F. = 20° C. (K)		10.36	10.57	16.7	62.9	119.7
Ohms per mile at 68° F. = 20° C.		54,600 / cir. mils	55,700 / cir. mils	88,200 / cir. mils	332,000 / cir. mils	632,000 / cir. mils
Pounds per mile-ohm at 68° F. = 20° C.		875	896	424.0	4700	8900
Temperature co-efficient per degrees F. Mean values		.00233	.00233	.0022	.0028	....
Temperature co-efficient per degrees C. Mean values		.0042	.0042	.0040	.0050	....
Specific gravity. Mean values		8.89	8.94	2.68	7.77	7.85
Pounds per 1,000 feet per circular mil		.003027	.003049	.000909	002652	002671
Weight, in pounds per cubic inch		.320	.322	.0967	.282	.283
Specific heat. Mean values		.093	.093	.214	.113	.117
Melting point in degrees F. Mean values		2012	2012	1157	2975	2480
Melting point in degrees C. Mean values		1100	1100	625	1635	1360
Mean co-efficient of linear expansion. Degrees F.		.00000950	.00000950	.00001285	.00000673	.00000662
Mean co-efficient of linear expansion. Degrees C.		.0000171	.0000171	.0000231	.000120	.000118
Solid Wire Pounds per square inch	Tensile strength	30,000 to 42,000	45,000 to 68,000	20,000 to 35,000	50,000 to 55,000	100,000 to 120,000
	Elastic limit	6,000 to 16,000	25,000 to 45,000	14,000	25,000 to 30,000	50,000 to 72,000
	Modulus of elasticity	7,000,000 to 17,000,000	13,000,000 to 18,000,000	10,500,000 to 11,500,000	22,000,000 to 27,000,000	22,000,000 to 27,000,000
Concentric Strand Pounds per square inch	Tensile strength	29,000 to 37,000	43,000 to 65,000	25,800	....	98,000 to 118,000
	Elastic limit	5,800 to 14,800	23,000 to 42,000	13,800	....	45,000 to 55,000
	Modulus of elasticity	5,000,000 to 12,000,000	12,000,000 to 14,000,000	Approx. 10,000,000	....	16,000,000 to 22,000,000

Power Wiring Calculations

D. C. Motors.—The proper size of wire may be readily determined by means of the following formula:

$$\text{circular mils} = \frac{H.P. \times 746 \times L \times 21.6}{E \times D \times K}$$

in which

H.P. = horse power of motor:

746 = watts per H.P.;

L = length of motor circuit from fuse block to motor;

21.6 = ohms per foot run in circuit where wires are one mil in diameter;

E = voltage at the motor;

D = drop in percentage of the voltage at the motor;

K = efficiency of the motor expressed as a decimal.

The average values for K, are about as follows: 1 H.P., .75; 3 H.P., .80; 5 H.P., .80; 10 H.P. and over, 90 per cent.

Example.—What is the proper size of wire for a 10 *h.p.*, 220 volt motor with 2% drop on 200 ft. circuit?

Substituting the given values in the above formula:

$$\text{circular mils} = \frac{10 \times 746 \times 200 \times 21.6}{220 \times 4.4 \times .9} = 36{,}992$$

The nearest larger value to this result, in the table of carrying capacities of copper wire (page 125), is 41,740, corresponding to No. 4 wire, B. & S. gauge.

A. C. Motors.—If the efficiency and power factor of an *a.c.* motor at a given horse power load be known, the current in amperes per phase which will be required to drive the motor at rated voltage is given by the formula:

$$\text{amperes} = \frac{\text{horse power} \times 746}{K \times \text{volts} \times \text{efficiency} \times \text{power factor}}$$

in which

K = 1 for single phase;

= 2 for two phase, four wire system,

= $\sqrt{3}$ for three phase, three wire system.

Example.—A 50 horse power, 440 volt motor has a full load efficiency of .9 and power factor of .8. How much current is required: 1, for single phase motor; 2. for three phase, three wire motor?

Allowable Carrying Capacities of Wires

(According to the National Electrical Code)

Gage No.	Diameter of Solid Wires in Mils	Area in Circular Mils	Column A Rubber Insulation Amperes	Column B Varnished Cambric Insulation, Amperes	Column C Other Insulation Amperes
18	40.8	1,624	3		5
16	50.8	2,583	6		10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	25
10	101.9	10,380	25	30	30
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325.	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460.	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1,000
		1,100,000	690	830	1,080
		1,200,000	730	880	1,150
		1,300,000	770	920	1,220
		1,400,000	810	970	1,290
		1,500,000	850	1,020	1,360
		1,600,000	890	1,070	1,430
		1,700,000	930	1,120	1,490
		1,800,000	970	1,160	1,550
		1,900,000	1,010	1,210	1,610
		2,000,000	1,050	1,260	1,670

1 Mil = 0.001 inch.

Standardized Stranding

(According to the National Electrical Code)

Strands			Cable		Allowable Carrying Capacities in Amperes		
No. of Strands	Mils Dia.	Gage No.	Area in Cir. Mils	Outside Dia. over Copper	Column A Rubber Insulation	Column B Varnished Cambric Insulation	Column C Other Insulation
7/	25	22	4,490	.075	15	18	20
7/	32	20	7,150	.096	20	25	25
7/	40	18	11,370	.120	25	30	35
7/	51	16	18,080	.153	35	40	50
7/	64	14	28,740	.192	50	60	70
7/	81	12	45,710	.253	70	85	90
7/	91	11	58,000	.273	80	95	110
7/	102	10	72,680	.306	90	110	130
19/	64	14	78,030	.320	100	120	150
19/	72	13	98,380	.360	125	150	175
19/	81	12	124,900	.405	150	180	210
19/	91	11	157,300	.455	175	210	250
19/	107	*	217,500	.540	225	270	325
19/	114	9	248,700	.570	250	300	350
37/	91	11	306,400	.637	275	330	400
37/	97	*	347,500	.679	300	360	450
37/	102	10	381,200	.714	325	390	500
37/	116	*	484,300	.798	400	480	600
61/	102	10	633,300	.918	475	565	700
61/	107	*	698,000	.963	500	600	750
61/	114	9	798,300	1.030	550	660	825
61/	121	*	893,100	1.090	600	720	900
61/	128	8	1,007,000	1.150	650	780	1000
91/	114	9	1,191,000	1.250	725	870	1125
91/	128	8	1,502,000	1.410	850	1020	1350
127/	114	9	1,660,000	1.480	900	1100	1460
127/	128	8	2,097,000	1.660	1100	1300	1700

*These individual strands are odd sizes not listed in the American Wire Tables.

Amperes and Wire Sizes for D.C. Motors

Horse-power.	Amperes at Full Load.			Size of Wire.								
				Rubber Insulation.			Varnished Cloth Insulation.			Other Insulation.		
	115 Volts.	230 Volts.	550 Volts.	115 Volts.	230 Volts.	550 Volts.	115 Volts.	230 Volts.	550 Volts.	115 Volts.	230 Volts.	550 Volts.
0.5	5	2.5	1.1	14	14	14	14	14	14	14	14	14
1	8.8	4.4	1.8	14	14	14	14	14	14	14	14	14
2	16	8	3.4	12	14	14	12	14	14	14	14	14
3	24	12	5	8	14	14	10	14	14	10	14	14
5	40	20	8.4	6	10	14	6	12	14	8	12	14
7.5	58	29	12.1	3	6	14	4	8	14	5	8	14
10	76	38	15.9	1	6	12	2	6	12	3	8	14
15	112	56	23.4	00	4	8	0	4	8	1	0	10
20	146	73	30.5	0,000	1	6	000	2	8	0	3	8
25	182	91	38.1	0,000	0	6	0,000	0	6	00	2	8
30	216	108	45.2	300,000	00	4	0,000	0	6	000	1	6
35	252	126	52.6	400,000	000	4	300,000	00	4	0,000	0	6
40	288	144	60.2	500,000	0,000	3	400,000	000	4	300,000	0	5
50	356	178	74.4	600,000	0,000	1	500,000	0,000	2	350,000	00	3
60	428	214	89.5	800,000	350,000	0	600,000	0,000	2	500,000	000	2
75	532	266	111	1,100,000	450,000	00	750,000	300,000	0	600,000	250,000	1
100	710	355	148	1,700,000	600,000	0,000	1,200,000	500,000	00	900,000	350,000	0
125	886	443	185	Two 850,000	850,000	300,000	1,700,000	600,000	0,000	1,200,000	500,000	00
150	1076	638	224	Two 1,100,000	1,100,000	400,000	Two 900,000	900,000	250,000	1,600,000	700,000	0,000

Based on 40-degree motors and 25 per cent overload. For 50-degree motors the fusing would be for about 15 per cent overload.

Note that varnished cloth wires smaller than No. 6 may be used only by special permission.

1. *For single phase motor.*

Substituting in the formula and taking K = 1

$$\text{amperes} = \frac{50 \times 746}{1 \times 440 \times .9 \times .8} = 117.6$$

2. *For three phase, three wire motor.*

Substituting in the formula and taking $K = \sqrt{3} = 1.73$

$$\text{amperes} = \frac{50 \times 746}{1.73 \times 440 \times .9 \times .8} = 67.8$$

Table of Amperes per Motor

H. P.	Per Cent. Eff.	Watts Input.	50 Volts.	100 Volts.	220 Volts.	500 Volts.
¾	70	800	16	7	4	2
1½	70	1600	32	15	7	3
3	75	2980	60	27	14	6
5	80	4660	93	42	21	9
7½	85	6580	132	60	30	13
10	85	8780	176	80	40	18
15	85	13200	264	120	60	26
20	85	17600	352	160	80	35
25	85	21900	438	199	100	44
30	90	24900	498	226	113	50
40	90	33200	664	301	151	66
50	90	41400	828	376	188	83
60	90	49700	994	452	226	99
70	90	58000	1160	527	264	116
80	90	66300	1330	603	302	133
90	90	74600	1490	678	339	149
100	90	82900	1660	755	377	166
120	90	99500	1990	905	453	199
150	90	124000	2480	1130	564	248

Table of Amperes per Dynamo

K. W.	125 Vs.	250 Vs.	500 Vs.	Appx. H. P.	K. W.	125 Vs.	250 Vs.	500 Vs.	Appx. H. P.
1.	8	4	2	1.3	30.	240	120	60	40.
2.	16	8	4	2.7	37.5	300	150	75	50.
3.	24	12	6	4.0	40.	320	160	80	53.
5.	40	20	10	6.7	50.	400	200	100	67.
7.5	60	30	15	10.	60.	480	240	120	80.
10.	80	40	20	13.	75.	600	300	150	100.
12.5	100	50	25	17.	100.	800	400	200	134.
15.	120	60	30	20.	125.	1000	500	250	167.
20.	160	80	40	27.	150.	1200	600	300	201.
25.	200	100	50	34.	200.	1600	800	400	268.

Example.—A 50 horse power single phase 440 volt motor, having a full load efficiency of .92 and power factor of .8, is to be operated at a distance of 1,000 feet from the alternator. The wires are to be spaced 6 inches apart and the frequency is 60, and % of loss 5. Determine: **A,** *electrical horse power;* **B,** *watts;* **C,** *apparent load;* **D,** *current;* **E,** *size of wires;* **F,** *drop;* **G,** *voltage at the alternator.*

A. *Electrical horse power*

$$\text{E.H.P.} = \frac{\text{brake horse power}}{\text{efficiency}} = \frac{50}{.92} = 54.3$$

or,

$$54.3 \times 746 = 40{,}508 \text{ watts}$$

B. *Watts*

$$\text{watts} = \text{E.H.P.} \times 746 = 54.3 \times 746 = 40{,}508$$

C. *Apparent load*

$$\text{apparent load or } kva = \frac{\text{actual load or watts}}{\text{power factor}} = \frac{40{,}508}{.8} = 50{,}635$$

D. *Current*

$$\text{current} = \frac{\text{apparent load or } kva}{\text{volts}} = \frac{50{,}635}{440} = 115 \text{ amperes}$$

E. *Size of wires*

$$\text{cir. mils} = \frac{\text{watts} \times \text{feet} \times M}{\% \text{ loss} \times \text{volts}^2} = \frac{40{,}508 \times 1{,}000 \times 3{,}380}{5 \times 440^2} = 141{,}443$$

From table page 111, nearest size *larger* wire is No. 000 B. & S. gauge

F. *Drop*

$$\text{drop} = \frac{\% \text{ loss} \times \text{volts}}{100} \times S = \frac{5 \times 440}{100} \times 1.17 = 25.74 \text{ volts.}$$

NOTE.—Values of S are given on page 121

G. *Voltage at alternator*

alternator pressure = volts at motor + drop = 440 + 25.74 = 465.7 volts

The following table gives minimum size wire for *d.c.* motor wiring when wires are concealed or partly concealed, also good practice for open wires.

Minimum Wire Sizes for D.C. Motors

(Western Electric Co.)

H.P.	Size wire B. & S. Gauge 30 volts	110 volts	220 volts	H.P.	Size wire B. & S. Gauge 30 volts	110 volts	220 volts
1/16	14	..	.	3	..	10	14
¼	14	..	..	4	..	8	12
½	10	14	14	5	..	6	10
1	8	14	14	7½	..	4	8
2	..	12	14	10	..	3	6

This table shows only safe size of wire to avoid overheating. If motor be over 30 ft. from dynamo, larger wire must be used. Find amperes in the table which follows and select size of wire by using the table of watt ft., page 117, dividing the watt ft., by the proper factor for voltages other than 30.

Approximate Amperes Taken at 30 Volts

Motors: Horsepower	Amperes	Mazda Lamps: Watts	Amperes
1/16	2½	10	.3
⅛	5	20	7
¼	9	40	1.3
½	16	75	2.5
1	30	...	...

	Amperes		Amperes
8 in. 4 blade fan motors	.80	Coffee percolator	14.0
12 in., 6 blade fan motors	1.00	Water heater	15.6
Flatiron	16.4	Soldering iron	10.0
Toaster	15.0	Cleaner	4.0
Disc heater	15.0	Washing machine	8.0
Water heater	9.4	Sewing machine	.66

Data Relating to Standard Annealed & Cleaned Bare Copper Cable Stranded

Approximate Values

Circular Mils.	Number of Wires in Strand	Diameter Each Wire, Inches	Diameter of Strand, Inches	Weight per 1,000 Foot Strand, Pounds	Weight per Mile, Pounds	Area Strand Square, Inches	Resistance per 1,000 Feet at 68° F. or 20° C.
2,000,000	91	.1482	1.6302	6164.	32546.	1.56874	.00530
1,750,000	91	.1386	1.5257	5394.	28480.	1.36494	.00607
1,500,000	91	.1284	1.4124	4623.	24409.	1.17831	.00707
1,250,000	91	.1172	1.2892	3853.	20344.	98170	.00852
1,000,000	61	.1280	1.1520	3081.	16268.	.78494	.01060
950,000	61	.1248	1.1232	2927.	15455.	.74618	.01115
900,000	61	.1215	1.0935	2773.	14641.	.70724	.01179
850,000	61	.1181	1.0629	2619.	13828.	.66852	.01247
800,000	61	.1145	1.0305	2465.	13015.	.62810	.01325
750,000	61	.1109	.9981	2311.	12202.	.58922	.01413
700,000	61	.1071	.9639	2157.	11389.	.54954	.01514
650,000	61	.1032	.9288	2003.	10576.	51020	.01630
600,000	61	.0992	.8928	1849.	9763.	47146	.01767
550,000	37	.1219	.8533	1694.	8944.	.43181	.01925
500,000	37	.1162	.8134	1540.	8131.	.39237	.02116
450,000	37	.1103	.7721	1386.	7318.	.35234	.02349
400,000	37	.1040	.7280	1232.	6505.	.31431	.02648
350,000	37	.0973	.6811	1078.	5692.	.27512	.03026
300,000	19	.1257	.6285	923.	4873.	.23591	.03531
250,000	19	.1147	.5738	769.	4060.	.19635	.04233
211,600	19	.1055	.5275	647.1	3416.7	.16609	.04997
167,772	19	.094	.4700	513.2	2709.7	.13187	.06293
133,079	7	.1378	.4134	405.9	2143.2	.10429	.07935
105,625	7	.1228	.3684	321.7	1698.6	.08303	.10007
83,694	7	.1093	.3279	255.2	1347.5	.06559	.12617
66,358	7	.0973	.2919	202.4	1068.7	.05205	15725
52,624	7	.0867	.2601	160.5	847.4	.04132	.19827
41,738	7	.0772	.2316	127.3	672.1	.03276	25000
26,244	7	.0612	.1836	80.0	422.4	.02059	39767
16,512	7	.0485	.1458	50.3	265.6	.01298	.62686
10,384	7	.0385	.1155	31.6	166.8	.00815	1 00848
6,528	7	.0305	.0915	19.9	105.1	.00511	1.59716
4,108	7	.0242	.0726	12.5	66.0	.00322	2.54192

Construction of Stranded Copper Conductors

To ascertain the diameter of the wires in a cable of any given capacity, divide the circular mils. capacity of the cable by the number of wires in the strand and extract the square root of the quotient. The result thus obtained gives the diameter in mils. of the wires composing the strand.

To ascertain the diameter of a concentric strand of 7, 19, 37, 61, 91, 127 or 169 wires:

7 wire strand, diameter equals 3 times diameter wires composing strand
19 wire strand, diameter equals 5 times diameter wires composing strand
37 wire strand, diameter equals 7 times diameter wires composing strand
61 wire strand, diameter equals 9 times diameter wires composing strand
91 wire strand, diameter equals 11 times diameter wires composing strand
127 wire strand, diameter equals 13 times diameter wires composing strand
169 wire strand, diameter equals 15 times diameter wires composing strand

The diameter of a 49-wire conductor, rope lay (7x7), equals 9 times the diameter of the individual wires, and the diameter of a conductor of 133 wires (7x19) equals 15 times the diameter of the individual wires.

To ascertain in circular mils. the capacity of a cable of which the number and diameter of the component wires are given, square the diameter (in mils.) of the individual wires and multiply the product by the number of wires in the strand.

All rules and data here given are based upon strands in which all of the individual wires are of the same size. The use of two or more different sizes of wire in the same strand complicates the subject to such an extent as to prevent giving specific instruction or rules.

Calculation Tables for D.C. and A.C. Machines

To Find	Direct Current	ALTERNATING CURRENT		
		Single-Phase	Two-Phase* Four-Wire	Three-Phase
Amperes when Horse-power is Known	$\frac{H.P.\times 746}{E\times \%\,Eff.}$	$\frac{H.P.\times 746}{E\times \%\,Eff.\times P.F.}$	$\frac{H.P.\times 746}{2\times E\times \%\,Eff.\times P.F.}$	$\frac{H.P.\times 746}{1.73\times E\times \%\,Eff.\times P.F.}$
Amperes when Kilowatts is Known	$\frac{K.W.\times 1000}{E}$	$\frac{K.W.\times 1000}{E\times P.F.}$	$\frac{K.W\times 1000}{2\times E\times P.F.}$	$\frac{K.W.\times 1000}{1.73\times E\times P.F.}$
Amperes when K.V.A. is Known		$\frac{K.V.A.\times 1000}{E}$	$\frac{K.V.A.\times 1000}{2\times E}$	$\frac{K.V.A.\times 1000}{1.73\times E}$
Kilowatts	$\frac{I\times E}{1000}$	$\frac{I\times E\times P.F.}{1000}$	$\frac{I\times E\times 2\times P.F.}{1000}$	$\frac{I\times E\times 1.73\times P.F.}{1000}$
K. V. A.		$\frac{I\times E}{1000}$	$\frac{I\times E\times 2}{1000}$	$\frac{I\times E\times 1.73}{1000}$
Horse-power (Output)	$\frac{I\times E\times \%\,Eff.}{746}$	$\frac{I\times E\times \%\,Eff.\times P.F.}{746}$	$\frac{I\times E\times 2\times \%\,Eff.\times P.F.}{746}$	$\frac{I\times E\times 1.73\times \%\,Eff.\times P.F}{746}$

I = Amperes; E = Volts; % Eff. = per cent Efficiency; P. F. = Power Factor.
K.W. = Kilowatts; K. V. A. = Kilo-Volt-Amperes; H. P. = Horse-power.
* For three-wire, two-phase circuits the current in the common conductor is **1.41** times that in either of the other two conductors.

Example.—What size wire is required for a 1 *h.p.* 220 volt motor located 200 feet from the dynamo?

From the table of approximate amperes at 30 volts, a 1 *h.p.* motor would require 30 amperes. Load at 30 volts = 30×30×200 = 180,000 watt feet, and for 220 = 180,000×30÷220 = 24,545 watt feet. Size wire for table of watt feet, page 117, for 30 volts is No. 10.

Current per Phase in Motor Circuits

H.P.	110 Volts			220 Volts			440 Volts		
	1-ph.	2-ph.	3-ph.	1-ph.	2-ph.	3-ph.	-ph.	2-ph.	3-ph.
1	12.72	5.57	6.43	6 36	2.78	3.22	3 18	1 39	1 61
2	23.80	10.10	11.54	11.90	5.05	5.77	5 95	2 52	2 89
3	34 30	14 24	16.44	17.15	7.12	8.22	8 53	3 56	4 11
5	52.30	22.92	26.50	26.15	11.46	13.25	13 07	5 73	6 63
7½	68.75	34.42	39.70	34.37	17.21	19.85	17 19	8 60	9 93
10	90.60	45.30	52.40	45.30	22.65	26.20	22.65	11 32	13 10
15	132.8	66.40	76 80	66.4	33 20	38.40	33 2	16 60	19.20
20	175.2	87.4	101.3	87 6	43 70	50.70	43.8	21 85	25.35
25	219.0	109.6	126 7	109 5	54.8	63.4	54 7	27.4	31.70
30	263.0	131.5	152.0	131.5	65 8	76 0	65 8	32.9	38.0
35	321.0	160.5	185.8	160 5	80.2	92.9	80 0	40.1	46.4
40	350.0	175.0	202.1	175.0	87.5	101.0	87 5	43.7	50.5
45	394.0	197.0	227.6	197.0	98.5	113.8	98 5	49.3	56.9
50	428.0	214.0	247.2	214.0	107.0	123.6	107 0	53.5	61.8
60	513.0	256.5	296.2	256.5	128.2	148.1	128.2	64.1	74.1
70	611.0	306.0	353.0	305.5	153.0	176.5	152 7	76.3	88 3
75	656.0	328.0	379.1	328.0	164.0	189.5	164 0	82 0	94.7

Elevators

Roping.—The simplest roping or hitch is 1:1, shown in fig. 88. Here the peripheral velocity of the rope sheave is equal to the elevator speed. With 2:1 roping the peripheral velocity of the rope sheave is twice the elevator speed.

2:1 roping gives, usually, twice the capacity at half the speed. Thus an elevator rated 2,500 lbs. at 300 *f.p.m.* 1:1 roping could be roped 2:1 to carry 5,000 lbs. at 150 *f.p.m.* See fig. 89.

Gearless machines are 1:1 roped for 600 *f.p.m.* and above. For 500 or 450 *f.p.m.* they may be either 2:1 or 1:1. For 400 *f.p.m.* they are almost always 2:1 roped. The selection of one or the other depends on the speed and duty. 1:1 roping is sometimes called "direct hitch" or "straight hitch."

The object of using 2:1 roping instead of 1:1 roping in certain cases is to get the advantage of the lower first cost of the higher speed driving unit.

Thus, for example, an elevator rated 10,000 lbs. at 100 *f.p.m.* would have a lower first cost if it used a hoisting machine with a combination of gear ratio and sheave diameter suitable for 5,000 lbs. at 200 *f.p.m.* and roped 2:1 than if the hoisting machine were built with a gear ratio suitable for 100 *f.p.m.* and a load of 10,000 lbs. Such a machine could not be built economically with a single worm and gear.

A compound geared machine, that is a worm geared machine with an additional spur gear reduction, roped 1:1, could be used instead of the single geared machine roped 2:1, and there are certain cases, especially where the elevator speed does not exceed 100 *f. p.m.* where a compound geared arrangement is preferred.

Extension of Elevator Ropes.—This acts in the case of sudden stopping or starting of a load attached to a wire rope to ease the stress a little. In the case of a short rope the elastic extension is small, but it increases proportionally to the increase in length of rope. This needs to be given due weight

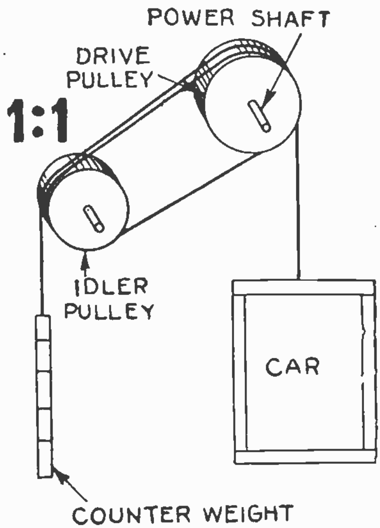


FIG. 88.—Direct (1:1) traction drive or method of transmitting power from the power unit to the car by means of frictional contact of the rope in passing one or more times around the drive pulley. This arrangement, since it does not employ a drum where size has to be considered, can be used for lifts of any height, and is the prevailing type today.

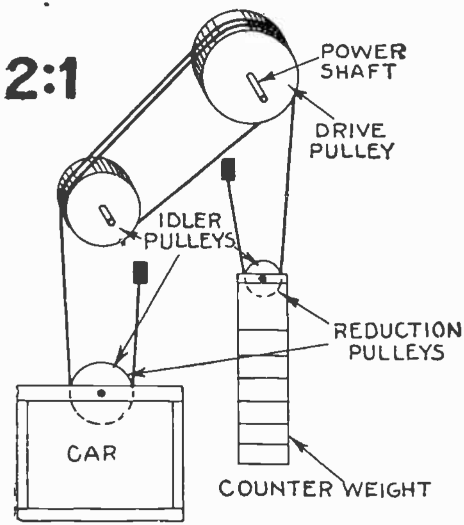


FIG. 89. — Geared, or 2:1 friction drive, or frictional contact transmission with reduction gear pulleys—a type used for moderate speed elevators.

Extension of Elevator Ropes

(According to American Steel & Wire Co.)

Length of rope in feet	Extension for cast steel rope in feet	Extension for plow steel in feet
500	0.833	1.000
1000	1.667	2.000
1500	2.500	3.000
2000	3.333	4.000
2500	4.167	5.000
3000	5.000	6.000
3500	5.833	7.000
4000	6.667	8.000
4500	7.500	9.000
5000	8.333	10.000

in any problem involving fast acceleration, jerks or shocks on a wire cable and the final safety factor should take into consideration these points.

Horse Power.—The horse power required for an elevator motor is

$$\text{horse power} = \frac{L \times S}{E \times 33{,}000}$$

in which

L = unbalanced load in pounds;

S = speed of elevator in feet per minute;

E = efficiency of the system generally taken at 50%.

Example—What size motor will be required for an elevator having an unbalanced load of 2,000 lbs. 400 ft. per minute speed and overall efficiency of 50%?

$$\text{horse power} = \frac{2{,}000 \times 400}{.5 \times 33{,}000} = 48$$

Example—A certain elevator has a capacity of 3,000 lbs.; car speed 200 ft. per minute and with 1,000 lbs. over counter-weight; 65% overall efficiency. Determine from chart the size of motor required.

The net load to be lifted will be 2,000 lbs. The diagonal line for 2,000 lbs. cuts the 200 ft. per minute vertical line at the horizontal line which

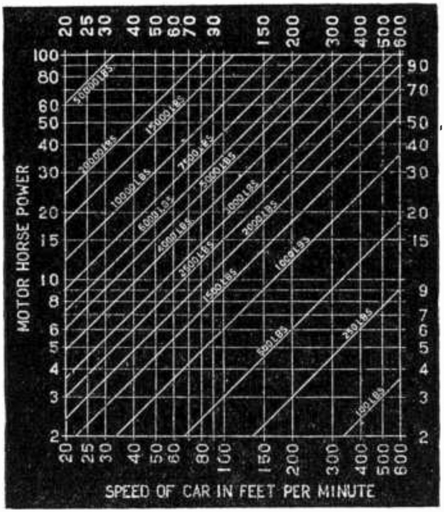


FIG. 90.—Elevator motor horse power diagram. Three factors determine the horse power of the motor that should be used, namely, the weight to be hoisted, the speed of travel and the efficiency of the elevator. *In the diagram*, the efficiency of the elevator is assumed to be 50 per cent. To determine the proper size motor to use in any case follow the diagonal line corresponding to the unbalanced load up to the point where it crosses the vertical line corresponding to the speed desired. The horizontal line at this point will indicate the horse power of motor required.

corresponds to about 25 *h.p.* This will be the size of the motor required if the overall efficiency of the elevator be 50%. If 65% be the known efficiency, the required horse power will be less.

$$\text{horse power} = \frac{25}{2} \times \frac{1}{.65} = 19.2, \text{ say } 20.$$

In the following formula for the determination of torque required to start an elevator, the starting efficiency has been assumed at 20 per cent as outlined below.

If this efficiency in any case be lower, the elevator will not start with the torque as derived by the formula.

If the efficiency be greater and allowance be not made in the formula, the derived torque may be so great that the start will be abrupt unless some control arrangement, external to the motor, be included to reduce the initial torque.

$$T=\frac{5{,}252\times L\times S\times 2\frac{1}{2}}{33{,}000\times .5\times r.p.m.}$$

$$T=\frac{.8\times L\times S}{r.p.m.}$$

in which

T = Torque in pounds at one ft. radius on the motor shaft.

L = Net load in pounds.

S = Full speed of elevator in ft. per min.

r.p.m. = Full load speed of motor selected.

NOTE.—***Power required for traveling cranes and hoists.*** Ulrich Peters, in *Machinery*, November, 1907, develops a series of formulæ for the power required to hoist and to move trolleys on cranes. The following is a brief abstract. Resistance to be overcome in moving a trolley or crane bridge. P_1 = rolling friction of trolley wheels, P_2 = journal friction of wheels or axles, P_3 = inertia of trolley and load. P = sum of these resistances = $P_1+P_2+P_3=(T+L)\left(\frac{F_1+F_2d}{D}+\frac{V}{1{,}932t}\right)$ in which T = weight of trolley, L = load, F_1 = coefficient of rolling friction, about .002 (.001 to .003 for cast iron or steel); F_2 = coefficient of journal friction, = .1 for starting and .01 for running, assuming a load on brasses of 1,000 to 3,000 pounds per square inch; (F_2 is more apt to be .05 unless the lubrication be perfect); d = diameter of journal; D = diameter of wheels; V = trolley speed in feet per minute; t = time in seconds in which the trolley under full load is required to come to the maximum speed. Horse power = sum of resistances × speed in feet per minute ÷ 33,000. Force required for hoisting and lowering: F^h = actual hoisting force, F^o = theoretical force or pull, L = load, V = speed in feet per minute of the rope or chain, c = hoisting speed of the load L, c ÷ V = transmission ratio of the hoist, e = efficiency = $F^o \div F^h$. The actual work to raise the load per minute = $F^hV = Lc = F^oV \div e$. The efficiency e, is the product of the efficiencies of all the several parts of the hoisting mechanism, such as pulleys, windlass, gearing, etc. Methods of calculating these efficiencies, with examples, are given at length in the original paper by Mr. Peters.

Telpher Motors.—The sizes of motor for telphers and hoists will depend upon the class of work to be done; the motors for telpher tractors vary from 5 to 15 *h.p.* and for the hoists, from 3 to 75 *h.p.*, the loads being from 500 lbs. to 30,000 lbs. The load factor for the tractor motor is .25 and for the hoisting motor .16. The driving wheels and the motors may be connected by gears or by chain drive. The maximum service efficiency of the motors is that corresponding to the efficiency obtained between one-half and three-quarters full load. The motors are of slow or medium speed.

Radio

Wave Length.—The formula for wave length is

$$\text{wave length} = \frac{300{,}000{,}000}{\text{wave frequency in cycles per sec.}} \quad \ldots\ldots\ldots(1)$$

From the formula it is seen that *the shorter the wave length the higher the frequency.*

Examples.—What are the frequencies for wave lengths of 10 and 200 meters? From formula 1

$$\text{wave frequency} = \frac{300{,}000{,}000}{\text{wave length}} \quad \ldots\ldots\ldots(2)$$

substituting 10 in formula 2

$$\text{wave frequency} = \frac{300{,}000{,}000}{10} = 30{,}000{,}000 \text{ cycles or } 30{,}000 \text{ kilocycles}$$

substituting 200 in formula 2

$$\text{wave frequency} = \frac{300{,}000{,}000}{200} = 1{,}500{,}000 \text{ cycles or } 1{,}500 \text{ kilocycles}$$

that is, for a 10 meter wave, the frequency is (30,000 ÷ 1,500) or 20 greater than for a 200 meter wave.

Combination of Resistances.—Circuits may contain resistances in *series* or in *parallel*, equal or unequal. If the resistances be connected in series as in fig. 91, the total resistance is the sum of all the resistors in the circuit. Thus the equation may be written

$$R \text{ eff.} = R_1 + R_2 + R_3 + R_4 + R_5 \text{ etc.}$$

It will be noted on examination of the diagram that in series circuits the current is the same through all of the resistors but that the voltage drop across the resistors depends upon the value of the individual resistor.

In some circuits there are resistances in parallel. If the numerical values of the resistances be equal then the effective circuit resistance can be obtained from the following equation:

$$R\ \text{eff.} = \frac{R}{N} \qquad (1)$$

in which

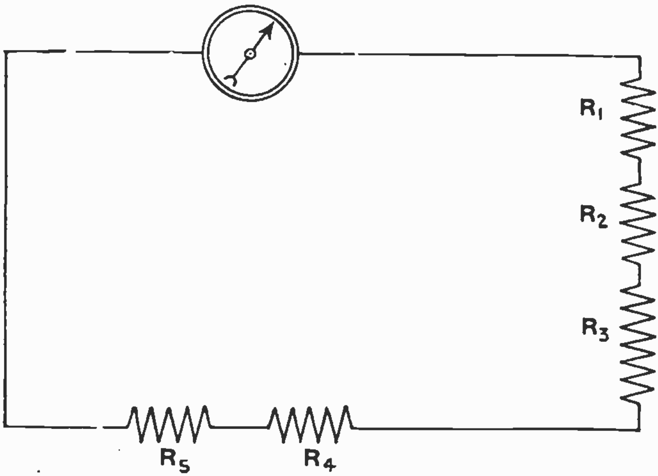


FIG. 91.—Circuit with resistances *in series*.

R = Value of one of the resistors
N = Number of resistors in the circuit.

Example.—A circuit contains 6 resistors in parallel and the resistance of each is 12 ohms. What is the effective resistance?

Substituting in formula 1

$$\text{effective resistance} = \frac{12}{6} = 2 \text{ ohms.}$$

Formula (1) is to be used only when the resistors are all of equal value.

Fig. 92 shows a circuit having several resistors in parallel and of unequal value. For such circuits the following formula should be used:—

$$R \text{ effective} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\text{ etc.}} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Example.—The circuit in fig. 92 contains resistors in parallel whose resistances are: 10, 6, 5, and 7 ohms. What is the effective resistance?

Substituting in formula (2)

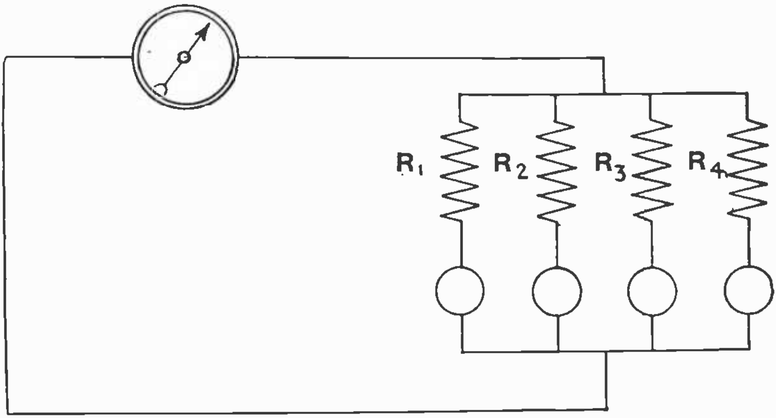


FIG. 92.—Circuit with unequal resistances *in parallel.*

$$R \text{ effective} = \frac{1}{\frac{1}{10}+\frac{1}{6}+\frac{1}{5}+\frac{1}{7}}$$

Solving: $\frac{1}{10}=.1;\ \frac{1}{6}=.166;\ \frac{1}{5}=.2;\ \frac{1}{7}=.14$

Adding: $.1+.166+.2+.14=.606.$

Substituting .606 for the denominator in the formula

$$R \text{ effective } \frac{1}{.606}=1.6 \text{ ohms.}$$

Resistances in Series and Parallel.—In networks having resistors in series and parallel the solution of the effective value of resistance is obtained by breaking up the circuit into its local circuits, solving each portion consisting of parallel circuits and then resolving them into simple series circuits.

Example.—Find the effective value of the resistance in the circuit shown in fig. 93.

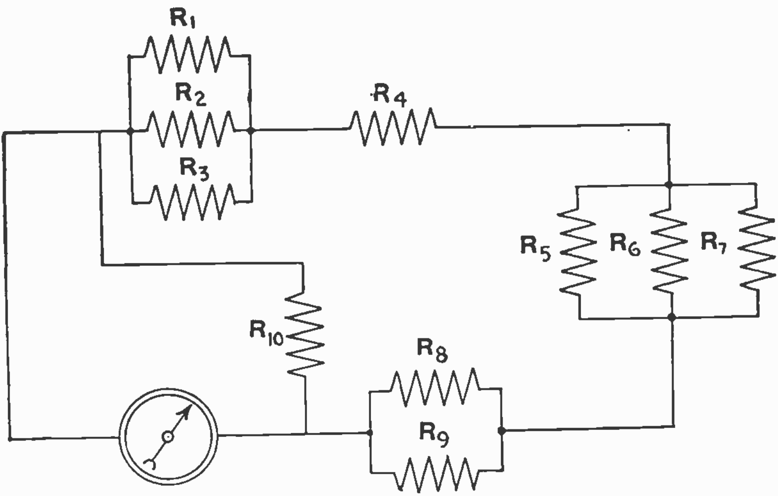


FIG. 93.—Circuit with resistances *in series* and *in parallel.*

First solve all the branch circuits thus:

Circuit R_1: R_2: R_3 has an effective resistance of 3 ohms.

Circuit R_5: R_6: R_7 has an effective resistance of 2.5 ohms.

Circuit R_8: R_9 has an effective resistance of 2.2 ohms.

As the above parallel circuits are in series with resistor R_4, the effective value of resistance is found by adding 10, 3, 2.5 and 2.2 together. This totals 17.7 ohms.

Resistor R_{10} is connected across the voltage supply and the effective value of the resistance network R_1 to R_9 is in turn connected across R_{10}.

Thus R_{10} is in parallel with the 17.7 ohm resistance of the network. Solving for parallel circuits 50×17.7/50+17.7 the effective total circuit resistance is 12.8 ohms.

Knowing that the voltage applied is 100 volts and the effective resistance is 12.8 ohms and applying Ohm's law

$$I = \frac{100}{12.8} = 7.8 \text{ amperes.}$$

Impedance and Reactance.—The formula for reactance of a circuit due to capacity is

$$X_c = \frac{1000000}{6.28fC} \text{ ohms}$$

in which

f = frequency

C = capacity in microfarads.

The formula for reactance due to inductance is

$$X_i = 6.28fL \text{ ohms}$$

in which

L = inductance in henries.

Resonance.—A circuit is tuned to resonance when the reactance due to inductance is equal to the reactance due to capacity. In a resonant circuit the two reactances cancel each other and the frequency at which any combination achieves this condition may be determined from the equation

$$f = \frac{159200}{\sqrt{L \times C}} \quad \text{(1)}$$

in which

L = inductance in microhenries.

C = capacity in microfarads.

Example.—A coil having an inductance of 203 microhenries is tuned with a condenser set at .0005 microfarad. To what frequency does the combination tune?

Substituting in formula (1)

$$f = \frac{159,200}{\sqrt{203 \times .0005}} \text{ c.p.s.}$$
$$= 500,000.$$

meaning 500,000 cycles per second or as usually expressed 500 kilocycles per second.

Electric Welding

Welding Currents.—It is difficult to give universally applicable figures covering current, speed, etc., for electric arc welding because of the effect of conditions under which the work is done, the character of the work, and the varying skill of operators.

The following figures for bare metallic electrodes, are based on favorable working conditions and a skilled operator However, they are approximations only and are given merely as a general guide.

Electrode Diameter in Inches	Amperes Hand Welding	Corresponding Plate Thickness in Inches
1/16	50–100	Up to 3/16
3/32	100–150	Up to 1/4
1/8	125–175	Above 1/8
5/32	150–200	Above 1/4
3/16	175–350	Above 3/8
1/4	225–400	Above 3/8

Electric Heating

The application of electrical energy to domestic and industrial heating has numerous advantages. The choice of material for a heating unit depends upon temperature conditions.

The average use of water is from 20 to 125 gallons per family per day; temperature 104° Fahr. for bath purposes; 150° Fahr. for dish washing.

If water be heated as required for use, a large demand, 2 to 5 *k.w.* is created for a short time and under usual conditions, does not secure a sufficiently low energy rate to be economical.

Efficiency and Gallons per 24 Hours of Water Heated to 104° Fahr.

(36 gal. tank covered with 1 in. hair felt insulation on tank and 1 in. magnesia covering circulation piping. Cold water 39° Fahr. Faucet close to tank.)

Kind of system	Kind of equipment	Watts	Efficiency per cent.	No. gal. hot water available (at 104° Fahr. per 24 hrs.)
Storage	Outside circulation	600	82	75
Storage	Outside circulation	1,000	76	117
Storage	"Clamp on"	750	78	89
Intermittent	Outside circulation	3,000	73	330
Intermittent	Outside circulation	5,000	69	525

Space Heaters.—As its name implies a space heater is for diffused instead of concentrated heat, such as room heating.

The electric energy required to heat an ordinary sized room when the outside air is near the freezing point ranges from about 1 to 2 watts per cu. ft.

Loss of Heat per Sq. Ft. of Surface

Kind of Surface	B. t. u. per hour	Kind of Surface	B. t. u. per hour
4 in. brick wall.......	.68	Window, single glass....	.776
8 in. brick wall......	.46	Window, double glass...	.518
12 in. brick wall.......	.32	Skylight, single glass....	1.118
16 in. brick wall.......	.26	Skylight, double glass...	.621
20 in. brick wall.......	.23	Ceilings, fire proof......	145
Floors, fire proof......	.124	Ceilings, wooden beams.	.104
		Ordinary wooden wall, lathed and plastered..	.1

In the table above, is shown loss of heat per sq. ft. of window and wall surface, for one degree Fahr., difference of inside and outside temperature, the loss being expressed in heat units per hour.

Example.—What will be the loss of heat per hour in a single room, wooden structure when the temperature inside is maintained at 70° Fahr., while the outside is at 32°. Size of room 10×10×10, having three 3×6 windows. Here all surfaces must be considered.

Area of windows = 3 (3×6) = 54 sq. ft.
Area of walls = 4 (10×10) − 54 = 346 sq. ft.
Area of floor = 10×10 = 100.
B.t.u. lost through windows = (70−32)×.776× 54 = 1,592.4
B.t.u. lost through walls = (70−32)×.1 ×346 = 1,314.8
B.t.u. lost through floor = (70−32)×.083×100 = 315.4

Total loss of heat per hour.................. = 3,222.6 *B.t.u.*

The maximum safe operating temperature of the space heater is between 700° and 800° Fahr.

Building Heating, Thumb Rule.—As a rough approximation, a rule of thumb is as follows:

.35 watts per cu. ft. (See *A*)
Plus 3.5 watts per sq. ft. of wall area (See *B*)
Plus 35. watts per sq. ft. of glass area (See *C*).

A. This takes care of the heat required for raising the temperature of the air approximately one complete change of air per hour. For more frequent changes, increase the wattage proportionately.

B. This takes care of the loss of heat through the walls. In figuring the wall area, the area of the four sides of the rooms and the ceiling and the floor are all included and a deduction is made for the glass area. The rule assumes good building construction, such as a good 12 in. brick wall or well made, double frame wall.

C. For measuring glass area, the overall area of the frame is measured and this area is deducted from the total wall area.

The above rule assumes a temperature elevation of 70° Fahr. or, in other words, external temperature of zero, room temperature of 70° Fahr. If the room adjoin other heated rooms, allowance must be made, based on the difference in temperature between the room under consideration and the adjoining rooms.

This thumb rule is for rough estimates only. It will agree quite closely with more complicated calculations in some cases, but on the other hand, there may sometimes be a considerable error so that it must be used cautiously The following two examples will illustrate how the above rule is applied:

Example.—Small house 12 ft.×10 ft.×9 ft. high, good construction, 12 in. brick walls, with 3 windows each 3 ft.×5 ft., is to be heated to 70° Fahr. with zero temperature outdoors and one complete air change per hour.

$$.35\,(12\times10\times9)$$
$$\text{Plus } 3.5\,[(12\times10\times2)+(12\times9\times2)+(10\times9\times2)-(3\times5\times3)]$$
$$\text{Plus } 35(3\times5\times3)$$

Equals 4,022 watts. Eight 500 watt space heaters should be used, which will give a total of 4,000 watts.

Voltage Factors on Basis of 110 Volts

Voltage..........	100	120	150	200	220	240	32
Factor...........	91	1.09	1.36	1.82	2.0	2.18	0.29

Nichrome Wire—110 Volts

Watts	Amperes	B. & S. Size	Ohms 75° F	Length	
250	2.27	25	43 6	21 Ft.	2 Ins.
300	2.72	24	36.4	22	0
350	3.2	23	31.0	24	0
400	3.64	23	27.2	21	0
450	4.10	22	24.1	23	5
475	4.32	22	23.0	22	4
500	4.55	22	21.8	21	0
550	5.0	21	19.8	24	5
575	5.23	21	18.9	23	2
600	5.46	21	18.1	22	4
615	5.6	20	17.7	27	6
640	5.82	20	17.0	26	6
660	6.0	20	16.5	25	8
700	6.36	20	15.6	24	4
750	6.81	19	14.5	28	5

Nichrome Ribbon—110 Volts—1/16″ Width

Watts	Amperes	Thickness	Ohms 75° F	Length	
400	3.64	.003	27.2	9 Ft.	10 Ins.
425	3.87	.0035	25.6	10	9
440	4.00	.0035	24.7	10	5
450	4.1	.004	24.1	11	7
475	4.32	.0045	22.9	12	5
500	4.55	.0045	21.8	11	8
525	4.77	.005	20.8	12	6
550	5.0	.005	19.8	11	11
575	5.23	.0056	18.9	12	10
600	5.46	.0056	18.1	12	3
625	5.68	.0063	17.4	12	11
650	5.91	.0071	16.7	13	11
660	6.00	.0071	16.5	13	10
675	6.15	.008	16.1	15	0
700	6.36	.008	15.6	14	7
750	6.82	.0089	14.5	15	1

Electro-Plating

A boiler is necessary in large plating rooms where hot water is not available, or where close temperature regulation of hot solutions is desired. In general practice, one boiler *h.p.* will heat 25 gals. of water or its equivalent from 60° Fahrenheit in one hour, to a temperature rise of 150°. Using this as a basis gives the following formula for determining the *h.p.* required for a given job:

Properties of Electro-Plating Elements

Element	Symbol	Specific Gravity	Atomic Weight	Common Valences	Electro-Chemical Equivalent Grams Per Ampere Second	Ampere Hours Per Grams Deposited
Potassium	K.	0.870	39.10	1	.000406	0.685
Sodium	NA.	0.971	23.00	1	.000239	1.163
Barium	BA.	3 80	137.37	2	.000713	0.390
Calcium	CA.	1.54	40.07	2	.000208	1.338
Magnesium	MG.	1.74	24.32	2	.000126	2.202
Aluminum	AL.	2.70	27.1	3	.000094	2.969
Chromium	IC CR.	6.92	52.0	3	.000180	1.544
	OUS CR.	6.92	52.0	2	.000270	1.030
Manganese	IC MN.	7.42	54.93	3	.000189	1.463
	OUS MN.	7.42	54.93	2	.000284	0.975
Zinc	ZN.	7.00	65.37	2	.000339	0.820
Cadmium	CD.	8.65	112.4	2	.000582	0.477
Iron	IC FE.	7.28	55.9	3	.000193	1.439
	OUS FE.	7.28	55.9	2	.000289	0.196
Cobalt	IC CO.	8.72	58.97	3	.000203	1.365
	OUS CO.	8.72	58.97	2	.000305	0.909
Nickel	IC NI.	8.80	58.68	3	.000202	1.371
	OUS NI.	8.80	58.68	2	.000304	0.914
Tin	IC SN.	7.30	118.7	4	.000308	0.901
	OUS SN.	7.30	118.7	2	.000616	0.451
Lead	IC PB.	11.4	207.2	4	.000535	0.518
	OUS PB.	11.4	207.2	2	.001070	0.259
Hydrogen	H.	0.0695 Compared to air	1.008	1	.00001044	26 60
Antimony	IC SB.	6.70	120 2	5	.000249	1 115
	OUS SB.	6.70	120.2	3	.000415	0.669
Bismuth	IC BI.	9.78	208.0	5	.000430	0.645
	OUS BI.	9.78	208.0	3	.0007185	0.387
Arsenic	IC AS.	5.73	74.96	5	.000155	1.790
	OUS AS.	5.73	74.96	3	.000259	1.073
Copper	IC CU.	8.90	63.57	2	.000329	0.843
	OUS CU.	8.90	63.57	1	.000659	0.422
Mercury	IC HG.	13.595	200.6	2	.001039	0.268
	OUS HG.	13.595	200.6	1	002079	0.134
Silver	AG.	10.5	107.88	1	.001118	0.249
Platinum	IC PT.	21.37	94.8	4	.000505	0.550
	OUS PT.	21.37	94.8	2	.001010	0.275
Gold	IC AU.	19.3	197.2	3	000681	0.408
	OUS AU.	19.3	197.2	1	.002040	0.136

$$\frac{\text{Temperature rise}\times\text{gallons of solution to be heated}}{25\times150\times\text{time in hours}}=h.p.$$

Temperature rise in this formula is the difference between 60° Fahr. and the temperature to which solution is to be heated. Time in hours is the time allowed to bring the solution to the required temperature. It is usually not practical to figure more than 3 hrs. to heat a solution.

Rate of Deposit.—It has been found that *a current of one ampere will deposit* .017253 *grain, or* .001118 *gramme, of silver per second* on one of the plates of a silver voltameter, the liquid employed being a solution of silver nitrate containing from 15 to 20% of the salt. The rate of hydrogen similarly set free by a current of one ampere is .00001044 gramme per second.

Therefore, knowing the amount of hydrogen thus set free, and the chemical equivalents of the constituents of other substances, the weight of their elements that will be set free or deposited in a given time by a given current, can be calculated.

The rate of deposit is proportional to current.

However, since there is a certain amount of hydrogen liberated in the plating process, there is also a partial solution of the metal, so that there is always a deduction to be made from the theoretical value. Thus:

Gold	gives about		80 to 90%
Nickel	"	"	80 to 95%
Silver	"	"	90 to 95%
Copper	"	"	98%

An ampere of current maintained for one hour, which serves as a unit of quantity called the "ampere hour" represents:

Gramme	.0376	Grain	.58
Ounce Troy	.00121	Ounce Avoirdupois	.00132

which multiplied by the chemical equivalent will furnish the weight of any substance deposited

The rate of deposit should be varied to suit the nature and form of the surface of the object.

Large smooth surfaces take the greatest rate of deposit, while other more rough and irregular surfaces require a slower rate.

The length of time for plating depends on the current rate.

Amperes Required.—The amount of current required for plating articles with various metals is given in the table following:

Amperes required to plate one square foot.

Solution of Metal.	Average amperes
Nickel	4
Brass	6 to 8
Bronze	6 to 8
Copper	6 to 8
Acid copper	10 to 12
Silver	2
Gold	1½
Zinc	10
Cadmium	6 to 8
Chromium	1 per sq. in.

Example. If the plater figure on plating with nickel, about 20 sq. ft. of surface, by referring to the table, it will be seen that each sq. ft. requires about four amperes, which would make it necessary to use approximately 80 amperes.

Again, to plate about 10 sq. ft. with copper, note each sq. ft. requires between six and eight amperes, which would mean about 70 additional amperes, or the total for the two would approximate 150. If this be the maximum output, a 150 ampere dynamo would be sufficient.

Dynamo Wiring.—The following table shows the proper size of conductors for the different size dynamos and distances between dynamo and tank.

The sizes of bars are shown in round and flat. If flat bars be used, they need not be of the exact dimensions given, but should be of equal or greater cross section.

Size of Main Conductors.

Dynamo Amperes	5 to 20 Feet		20 to 35 Feet		35 to 50 Feet		50 to 65 Feet	
	Round Bars	Flat Bars	Round Bars	Flat Bars	Round Bars	Flat Bars	Round Bars	Flat Bars
100	5/16	....	3/8	....	7/16	....	1/2	....
200	1/2	....	9/16	....	5/8	....	3/4	....
300	9/16	....	11/16	....	3/4	....	1	1 5/8 x 1/2
500	3/4	....	7/8	....	1	1 5/8 x 1/2	1 1/8	2 x 1/2
750	7/8	....	1 1/8	2 x 1/2	1 3/16	2 1/4 x 1/2	1 3/8	3 x 1/2
1,000	1"	1 5/8 x 1/2	1 1/4	2 1/2 x 1/2	1 3/8	3 x 1/2	1 1/2	3 1/2 x 1/2
1,500	1 1/4	2 1/2 x 1/2	1 1/2	3 1/2 x 1/2	1 3/4	4 x 5/8	1 7/8	3 3/4 x 3/4
2,000	1 1/2	3 1/2 x 1/2	1 3/4	4 x 5/8	2	4 1/4 x 3/4	2 1/8	4 x 7/8
2,500	1 9/16	3 x 5/8	1 7/8	3 3/4 x 3/4	2 3/16	5 x 3/4	2 3/8	4 1/2 x 1
3,000	1 3/4	4 x 5/8	2 1/8	4 x 7/8	2 3/8	4 1/2 x 1	2 5/8	4 3/8 x 1 1/4
4,000	2	4 1/4 x 3/4	2 1/2	5 x 1	2 3/4	4 3/4 x 1 1/4	3	4 3/4 x 1 1/2
5,000	2 1/4	4 1/2 x 7/8	2 3/4	4 3/4 x 1 1/4	......	5 1/4 x 1 3/4	3 3/4	5 1/2 x 2
7,500	2 3/4	4 3/4 x 1 1/4		5 1/2 x 1 3/4	......	5 1/2 x 2	4 1/4	7 x 2

Natural Trigonometrical Functions

Degree	Sine	Cosine	Tangent	Secant	Degree	Sine	Cosine	Tangent	Secant
0	.00000	1.0000	.00000	1.0000	46	.7193	.6947	1.0355	1.4395
1	.01745	.9998	.01745	1.0001	47	.7314	.6820	1.0724	1.4663
2	.03490	.9994	.03492	1.0006	48	.7431	.6691	1.1106	1.4945
3	.05234	.9986	.05241	1.0014	49	.7547	.6561	1.1504	1.5242
4	.06976	.9976	.06993	1.0024	50	.7660	.6428	1.1918	1.5557
5	.08716	.9962	.08749	1.0038	51	.7771	.6293	1.2349	1.5890
6	.10453	.9945	.10510	1.0055	52	.7880	.6157	1.2799	1.6243
7	.12187	.9925	.12278	1.0075	53	.7986	.6018	1.3270	1.6616
8	.1392	.9903	.1405	1.0098	54	.8090	.5878	1.3764	1.7013
9	.1564	.9877	.1584	1.0125	55	.8192	.5736	1.4281	1.7434
10	.1736	.9848	.1763	1.0154	56	.8290	.5592	1.4826	1.7883
11	.1908	.9816	.1944	1.0187	57	.8387	.5446	1.5399	1.8361
12	.2079	.9781	.2126	1.0223	58	.8480	.5299	1.6003	1.8871
13	.2250	.9744	.2309	1.0263	59	.8572	.5150	1.6643	1.9416
14	.2419	.9703	.2493	1.0306	60	.8660	.5000	1.7321	2.0000
15	.2588	.9659	.2679	1.0353	61	.8746	.4848	1.8040	2.0627
16	.2756	.9613	.2867	1.0403	62	.8829	.4695	1.8807	2.1300
17	.2924	.9563	.3057	1.0457	63	.8910	.4540	1.9626	2.2027
18	.3090	.9511	.3249	1.0515	64	.8988	.4384	2.0503	2.2812
19	.3256	.9455	.3443	1.0576	65	.9063	.4226	2.1445	2.3662
20	.3420	.9397	.3640	1.0642	66	.9135	.4067	2.2460	2.4586
21	.3584	.9336	.3839	1.0711	67	.9205	.3907	2.3559	2.5593
22	.3746	.9272	.4040	1.0785	68	.9272	.3746	2.4751	2.6695
23	.3907	.9205	.4245	1.0864	69	.9336	.3584	2.6051	2.7904
24	.4067	.9135	.4452	1.0946	70	.9397	.3420	2.7475	2.9238
25	.4226	.9063	.4663	1.1034	71	.9455	.3256	2.9042	3.0715
26	.4384	.8988	.4877	1.1126	72	.9511	.3090	3.0777	3.2361
27	.4540	.8910	.5095	1.1223	73	.9563	.2924	3.2709	3.4203
28	.4695	.8829	.5317	1.1326	74	.9613	.2756	3.4874	3.6279
29	.4848	.8746	.5543	1.1433	75	.9659	.2588	3.7321	3.8637
30	.5000	.8660	.5774	1.1547	76	.9703	.2419	4.0108	4.1336
31	.5150	.8572	.6009	1.1666	77	.9744	.2250	4.3315	4.4454
32	.5299	.8480	.6249	1.1792	78	.9781	.2079	4.7046	4.8097
33	.5446	.8387	.6494	1.1924	79	.9816	.1908	5.1446	5.2408
34	.5592	.8290	.6745	1.2062	80	.9848	.1736	5.6713	5.7588
35	.5736	.8192	.7002	1.2208	81	.9877	.1564	6.3138	6.3924
36	.5878	.8090	.7265	1.2361	82	.9903	.1392	7.1154	7.1853
37	.6018	.7986	.7536	1.2521	83	.9925	.12187	8.1443	8.2055
38	.6157	.7880	.7813	1.2690	84	.9945	.10453	9.5144	9.5668
39	.6293	.7771	.8098	1.2867	85	.9962	.08716	11.4301	11.474
40	.6428	.7660	.8391	1.3054	86	.9976	.06976	14.3007	14.335
41	.6561	.7547	.8693	1.3250	87	.9986	.05234	19.0811	19.107
42	.6691	.7431	.9004	1.3456	88	.9994	.03490	28.6363	28.654
43	.6820	.7314	.9325	1.3673	89	.9998	.01745	57.2900	57.299
44	.6947	.7193	.9657	1.3902	90	1.0000	Inf.	Inf.	Inf.
45	.7071	.7071	1.0000	1.4142		—	—	—	—

NOTE.—*For intermediate values* reduce angles from degrees, minutes and second [illegible] degrees and decimal parts of a degree and interpolate or consult a larger table.

Powers, Roots, etc.

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1	1	1	1.00000	1.00000	1.00000
2	4	8	1.41421	1.25992	.50000
3	9	27	1.73205	1.44224	.33333
4	16	64	2.00000	1.58740	.25000
5	25	125	2.23606	1.70997	.20000
6	36	216	2.44948	1.81712	.16666
7	49	343	2.64575	1.91293	.14285
8	64	512	2.82842	2.00000	.12500
9	81	729	3.00000	2.08008	.11111
10	100	1000	3.16227	2.15443	.10000
11	121	1331	3.31662	2.22398	.09090
12	144	1728	3.46410	2.28942	.08333
13	169	2197	3.60555	2.35133	.07602
14	196	2744	3.74165	2.41014	.07142
15	225	3375	3.87298	2.46621	.06666
16	256	4096	4.00000	2.51984	.06250
17	289	4913	4.12310	2.57128	.05882
18	324	5832	4.24264	2.62074	.05555
19	361	6859	4.35889	2.66840	.05263
20	400	8000	4.47213	2.71441	.05000
21	441	9621	4.58257	2.75892	.04761
22	484	10648	4.69041	2.80203	.04545
23	529	12167	4.79583	2.84386	.04347
24	576	13824	4.89897	2.88449	.04166
25	625	15625	5.00000	2.92401	.04000
26	676	17576	5.09901	2.96249	.03846
27	729	19683	5.19615	3.00000	.03703
28	784	21952	5.29150	3.03658	.03571
29	841	24389	5.38516	3.07231	.03448
30	900	27000	5.47722	3.10723	.03333
31	961	29791	5.56776	3.14138	.03225
32	1024	32768	5.65685	3.17480	.03125
33	1089	35937	5.74456	3.20753	.03030
34	1156	39304	5.83095	3.23961	.02941
35	1225	42875	5.91607	3.27106	.02857
36	1296	46656	6.00000	3.30192	.02777
37	1369	50653	6.08276	3.33222	.02702
38	1444	54872	6.16441	3.36197	.02631
39	1521	59319	6.24499	3.39121	.02564
40	1600	64000	6.32455	3.41995	.02500
41	1681	68921	6.40312	3.44821	.02439
42	1764	74088	6.48074	3.47602	.02380
43	1849	79507	6.55743	3.50339	.02325
44	1936	85184	6.63324	3.53034	.02272
45	2025	91125	6.70820	3.55689	.02222
46	2116	97336	6.78233	3.58304	.02173
47	2209	103823	6.85565	3.60882	.02127
48	2304	110592	6.92820	3.63424	.02083
49	2401	117649	7.00000	3.65930	.02040
50	2500	125000	7.07106	3.68403	.02000
51	2601	132651	7.14142	3.70842	.01960
52	2704	140608	7.21110	3.73251	.01923
53	2809	148877	7.28010	3.75628	.01886
54	2916	157464	7.34846	3.77976	.01851
55	3025	166375	7.41619	3.80295	.01818
56	3136	175616	7.48331	3.82586	.01785
57	3249	185193	7.54983	3.84850	.0·754
58	3364	195112	7.61577	3.87087	.01724

SECTION

C

PRACTICAL MECHANICAL CALCULATIONS

FOR

Engineers, Draughtsmen
Machinists, Builders

AND ALL

Mechanics

Mechanics

Mechanical Powers.—By definition the mechanical powers are *mechanical contrivances that enter into the composition or formation of all machines.*

They are:

1. The lever.
2. The wheel and axle.
3. The pulley.
4. The inclined plane.
5. The screw.
6. The wedge.

These can in turn be reduced to three classes:

1. A solid body turning on an axis.
2. A flexible cord.
3. A hard and smooth inclined surface.

They all depend for their action upon what is known as the *principle of work*, that is: *The applied force, multiplied by the distance through which it moves, equals the resistance overcome, multiplied by the distance through which it is overcome.*

The principle of work may be also stated as follows:

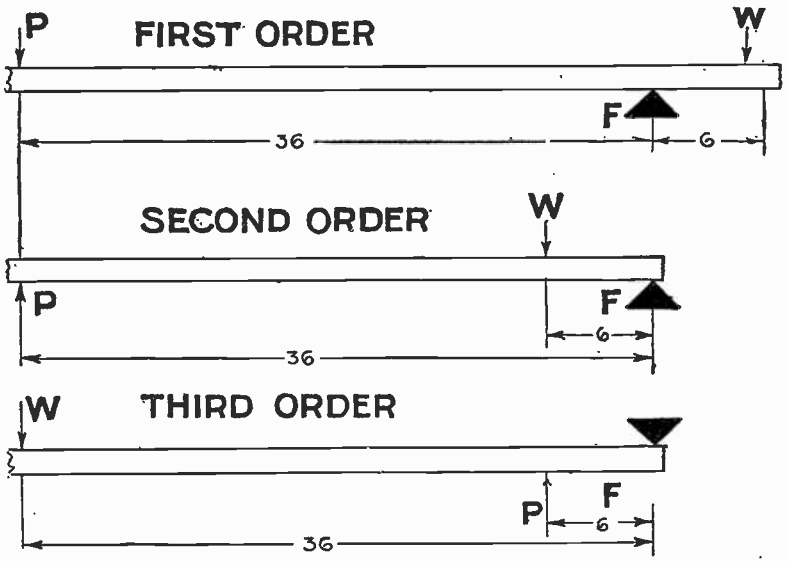
Work put into machine = lost work + work done by machine.

The Lever.—The following general rule holds for all classes of lever:

Rule. *The force P, multiplied by its distance from the fulcrum, is equal to the load W; multiplied by its distance from the fulcrum.* That is:

$$\textit{Force} \times \textit{distance} = \textit{load} \times \textit{distance} \ldots\ldots\ldots (1)$$

Example.—What force applied at 3 ft. from the fulcrum will balance



FIGS. 1 to 3.—Diagrams of the *three orders of lever* illustrating the accompanying examples.

a weight of 112 lbs. applied at 6 ins. from the fulcrum? Here the distances or "leverages" are 3 feet and 6 inches.

The distance must be of the same denomination; hence reducing ft. to ins., $3 \times 12 = 36$ ins.

Applying the rule

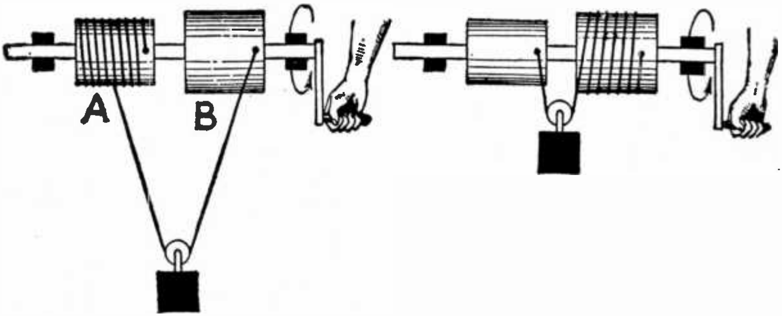
$$Force \times 36 = 112 \times 6$$

Solving

$$Force = \frac{112 \times 6}{36} = 18.67 \text{ or } 18\tfrac{2}{3} \text{ lbs.}$$

This solution holds for all levers as illustrated in figs. 1 to 3.

Wheel and Axle.—Comparison of the wheel and axle with a 1st order lever shows that in principle they are the same thing. The general equation (1) on page 2 applies to the wheel and axle.



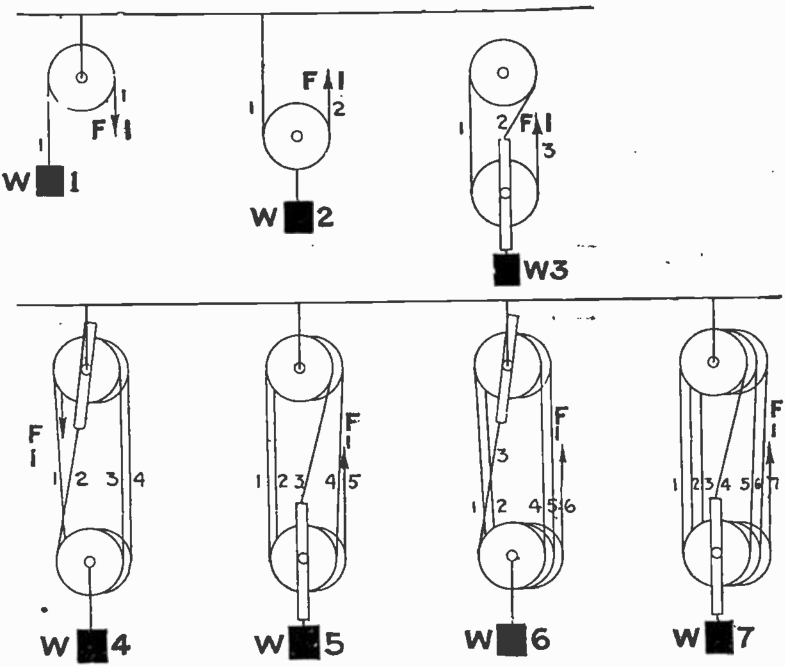
FIGS. 4 and 5.—Principle of the differential hoist. As the crank is turned clockwise the cable winds on B, and unwinds on A, and since B is larger in diameter, the length of cable between the two drums and load is gradually taken up, thus lifting the load. Evidently by making the difference in diameter of the two drums very small an extremely large leverage is obtained, thus enabling very heavy weights to be lifted with little effort. The load will remain suspended at any point, *because the difference in the diameter of the two drums is too small to overbalance the friction of the parts*. Fig. 5 shows the end of the lifting operation.

Chinese Wheel and Axle.—This is a modification of the wheel and axle and is used for obtaining extreme degree of leverage. Its principle and construction are shown in figs. 4 and 5.

The Pulley.—Pulleys are classed as *fixed* or *movable*.

In the fixed pulley no mechanical advantage is gained, but its use is of the greatest importance in accomplishing the work appropriate to the pulley, such as raising water from a well.

The *movable* pulley, by distributing the weights into separate parts, is attended by mechanical advantages proportional to the number of points of support.



FIGS. 6 to 12.—Elementary pulley combinations illustrating accompanying rule for relation between force applied and load lifted and showing how the load may be increased from 1 to 7 times per unit of force applied. Of course a greater range may be secured by additional pulleys, but there is a limit in practice to which it is mechanically expedient.

The following rule expresses the relation between the force and load.

Rule.—*The load capable of being lifted by combination of pulleys is equal to the force × the number of ropes supporting the lower or movable block.*

The Inclined Plane.—By such substitution of a sloping path for a direct upward line of ascent, a given weight can be raised by another weight weighing less than the weight to be raised.

The inclined plane becomes a *mechanical power* in consequence of its supporting part of the weight, and of course leaving only a part to be supported by the power.

Rule.—*As the applied force* P, *is to the load* W, *so is the height,* H, *to the length of the plane* W.

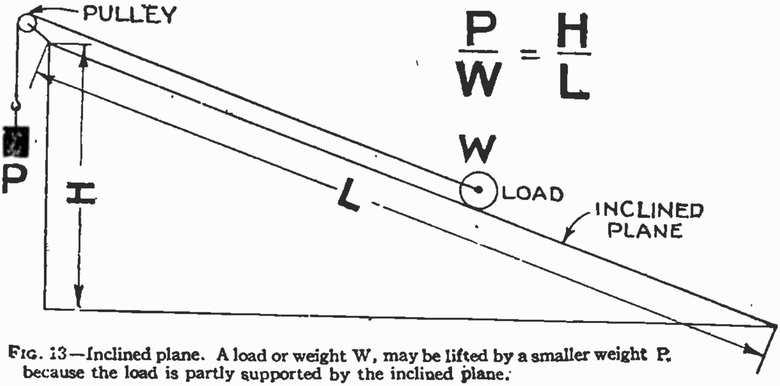


FIG. 13—Inclined plane. A load or weight W, may be lifted by a smaller weight P. because the load is partly supported by the inclined plane.

That is:

Force : load = height : plane length. . . . .(2)

Example.—What force (P) is necessary to raise a load of 10 lbs. if the height be 2 ft., and plane 12 ft.?

Substitute in equation (2)

$$P : 10 = 2 : 12$$

$$P \times 12 = 2 \times 10$$

$$P = \frac{10 \times 2}{12} = \frac{20}{12} = 1\tfrac{2}{3} \text{ lbs.}$$

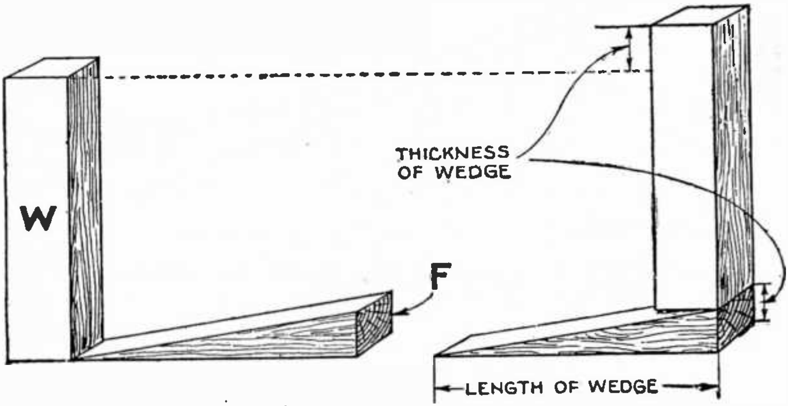
The Screw.—This is simply *an inclined plane wrapped around a cylinder.*

The screw is generally employed when severe pressure is to be exerted through small spaces; being subject to great loss from friction it usually exerts but a small power of itself, but derives its principal efficacy from the lever or wheel work with which it is very easily combined.

Rule.—*As the applied force is to the load so is the pitch to the length of thread per turn*, that is:

Applied force : load = pitch : length of thread per turn........(3)

Example:—If the distance between the threads or *pitch* be ¼ in. and a force of 100 lbs. be applied at the circumference of the screw, what weight will be moved by the screw, the length of thread per turn of the screw being 10 ins.



FIGS. 14 and 15.—Application of the wedge in raising a heavy load.

Substituting in equation (3)

$$100 : \text{load} = \tfrac{1}{4} : 10$$
$$\text{load} \times \tfrac{1}{4} = 10 \times 100$$
$$\text{load} = \frac{10 \times 100}{\tfrac{1}{4}} = 4{,}000 \text{ lbs.}$$

The Wedge.—This is *virtually a pair of inclined planes in contact along their bases or back to back.*

Rule.—*As the applied force is to the load so is the thickness of the wedge to its length;* that is:

Applied force : load = thickness : length of wedge. . . . .(4)

Example.—What force is necessary to apply to a wedge 20 ins. long and 4 ins. thick to raise a load of 2,000 lbs.?

Substituting in equation (4)

$$\text{Applied force} : 2{,}000 = 4 : 20$$

$$20 : 4 = 2000 : \text{applied force}$$

$$\text{applied force} \times 20 = 4 \times 2000$$

$$\text{applied force} = \frac{4 \times 2000}{20} = 400 \text{ lbs.}$$

Principle of Moments.—*When two or more forces act upon a rigid body and tend to turn it about an axis, then equilibrium will exist if the sum of the moments of the forces which tend to turn the body in one direction equals the sum of the moments of those which tend to turn it in the opposite direction about the same axis.*

The lever safety valve when at the point of blowing is a good illustration of the above principle.

Newton's Laws of Motion.—***1st Law.*** *If a body be at rest, it will remain at rest, or if in motion it will move uniformly in a straight line until acted upon by some force.*

2nd Law. *If a body be acted on by several forces, it will obey each as though the others did not exist, and this whether the body be at rest or in motion.*

3rd Law. *If a force act to change the state of a body with respect to rest or motion, the body will offer a resistance equal and directly opposed to the force; or to every action there is opposed an equal and opposite reaction.*

Physics

Mechanical Equivalent of Heat.—*One pound falling through a distance of one foot represents one foot pound of work.*

In 1843 Dr. Joule of Manchester, England, determined by numerous experiments that when 772 foot pounds of energy had been expended on one pound of water, the temperature of the latter had risen one degree, and the relationship between heat and mechanical work was found; the value 772 foot pounds is known as Joule's equivalent. More recent experiments give higher figures, the value 778, is now generally used but according to Kent 777.62 is probably more nearly correct. ***Marks and Davis*** in their steam tables have used the figure 777.52.

Pressure Scales.—The term vacuum is defined as *a space devoid of matter*. This is equivalent to saying *a space in which the pressure is zero*. According to common usage it means *any space in*

Fig. 16.—Elementary boiler or closed vessel illustrating the difference between *gauge*, and *absolute pressure*.

which the pressure is less than that of the atmosphere. This gives rise to two scales of pressure:

Gauge pressure is expressed as absolute pressure by adding 14.74, or for ordinary calculation, 14.7 lbs.

Thus 80 lbs. gauge pressure = 80 + 14.74 = 94.74 lbs. absolute pressure.

Absolute pressure is expressed as gauge pressure by subtracting 14.7.

Thus 90 lbs. absolute pressure = 90 − 14.7 = 75.3 lbs. gauge pressure.

The pressures below atmospheric pressure are usually expressed in lbs. per sq. in. when making calculations or "inches of mercury" in practice.

Barometer.—By definition a barometer is an instrument for measuring the pressure of the atmosphere.

Pressure in lbs. per sq. in. is obtained from the barometer reading by multiplying by .49116.

Thus, a 30 inch barometer reading signifies a pressure of

$$.49116 \times 30 = 14.74 \text{ lbs. per sq. in.}$$

Pressure of the atmosphere per square inch for various readings of the barometer.

Barometer (ins. of mercury)	Pressure per sq. ins., lbs.	Barometer (ins. of mercury)	Pressure per sq. ins., lbs.
28.00	13.75	**29.921**	**14.696**
28.25	13.88	30.00	14.74
28.50	14.00	30.25	14.86
28.75	14.12	30.50	14.98
29.00	14.24	30.75	15.10
29.25	14.37	31.00	15.23
29.50	14.49		
29.75	14.61		

The above table is based on the standard atmosphere, which by definition = **29.921** ins. of mercury = **14.696** lbs. per sq. in., that is **1** in. of mercury = 14.696 ÷ 29.921 = **.49116** lbs. per sq. in.

Energy.—By definition, *energy is stored work*, that is, the ability to do work, or in other words, to move against resistance.

The measure of actual energy is the product of the weight of the body into the height from which it must fall to acquire its actual velocity. If v = the velocity in ft. per sec. according to the principle of falling bodies, h, the height due to the velocity $= v^2 \div 2g$; and if w = the weight, the energy $= \frac{1}{2}\, mv^2 = wv^2 \div 2g = wh$. Since energy is the capacity for performing work, the units of work and energy are equivalent, or $FS = \frac{1}{2}\, mv^2 = wh$. Energy exerted = work done.

Conservation of Energy.—The doctrine of physics, that energy can be transmitted from one body to another or transformed in its manifestations, but *may neither be created nor destroyed*.

Work.—*The overcoming of resistance through a certain distance*. The unit of work is the *foot pound*.

Power.—By definition, power is *the* **rate** *at which work is done;* in other words, it is *work divided by the* **time** *in which it is done.*

The unit of power in general use is the **horse power*** which is defined as 33,000 *foot pounds per minute.*

Expansion and Contraction.—Practically all substances expand with increase in temperature and contract with decrease of temperature. The expansion of solid bodies in a longitudinal direction is known as *linear expansion;* the expansion in volume is called the *volumetric expansion.*

The following example will illustrate the use of the table on next page:

Example.—How much longer is a 36 in. rod of aluminum when heated from 97 to 200° Fahr.?

*NOTE.—The term "horse power" is due to James Watt, who figured it to represent the power of a strong London draught horse to do work during a short interval, and used it as a power rating for his engines

Increase in temperature 200 − 97 = 103°

Coefficient of expansion for aluminum from table = .00001234.

Increase in length of rod = 36 × .00001234 × 103 = .046 in.

Linear Expansion of Common Metals

(Between 32 and 212 degrees Fahr.)

	Linear expansion per unit length per degree Fahr.
Aluminum	.00001234
Antimony	.00000627
Bismuth	.00000975
Brass	.00000957
Bronze	.00000986
Copper	.00000887
Gold	.00000786
Iron, cast	.00000556
Iron, wrought	.00000648
Lead	.00001571
Nickel	.00000695
Steel	.00000636
Tin	.00001163
Zinc, cast / Zinc, rolled	.00001407

Volumetric expansion = 3 × linear expansion.

Melting Point of Solids.—The temperatures at which a solid substance changes into a liquid is called the melting point.

The following table gives the melting point for commercial metals:

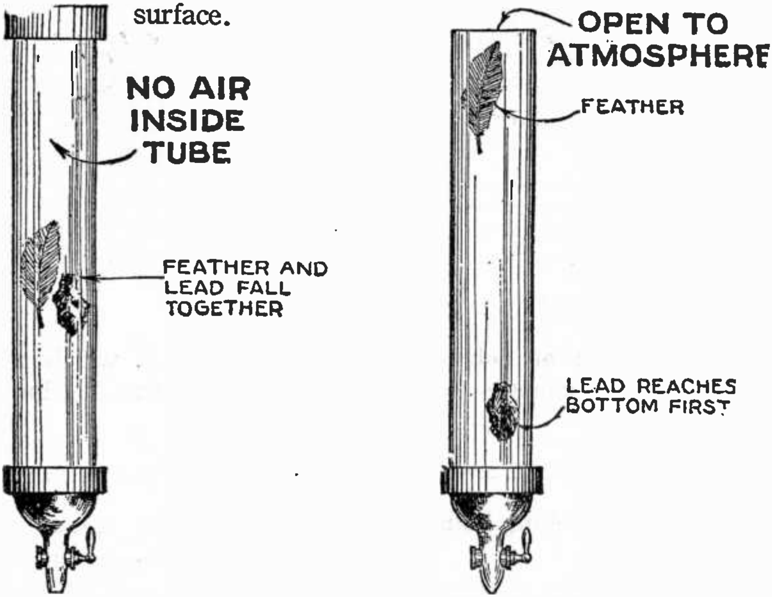
Melting Points of Commercial Metals

	Degrees Fahr.
Aluminum	1,200
Antimony	1,150
Bismuth	500
Brass	1,700–1,850
Copper	1,940

Table—*Continued*

	Degrees Fahr.
Cadmium	610
Iron, cast	2,300
Iron, wrought	2,900
Lead	620
Mercury	−38
Steel	2,500
Tin	446
Zinc, cast	785

Gravity.—By definition gravity is *the force that attracts* ..ies*, at or near the surface of the earth, toward the center of* ..arth. This force varies at different points on the earth's surface.



Figs. 17 and 18.—Experiments with falling bodies. Place a feather and a piece of lead in a long tube and pump out the air. If the tube be suddenly inverted it will be found that the two objects fall side by side from top to bottom as in fig. 17. If the top be left open so that the objects are surrounded by air, when the tube is inverted, as in fig. 18., it will be found that the lead reaches the bottom before the feather.

Velocity.—In physical problems, velocity is generally expressed *in feet per second*, and in engineering work, *in feet per minute*.

The formula is

$$V = \frac{S}{t}$$

in which

S = space, or distance;
V = velocity;
t = time.

Falling Bodies.—A body falling freely from rest to the earth *acquires during the first second a velocity of* 32.174 *ft. per second per second; at the end of the second second, a velocity of* 32.174 + 32.174 = 64.348 *ft. per second, and so on.*

Mass.—The formula for mass is

$$\text{Mass} = \frac{\text{weight}}{g}$$

in which

g = acceleration due to gravity. If the weight and g be taken at the same place their ratio will be constant for all places.

Example.—The mass of a 100 lb. weight equals $\frac{100}{32.16} = 3.11$ lbs. On the surface of the sun, where the force of gravity is 28 times as great as on the earth, the same object would weigh 2,800 lbs., but its mass would be $\frac{28 \times 100}{28 \times 32.16} = 3.11$ lbs. as before.

It will be observed that both mass and weight are taken in pounds. This double use of the word pound is customary, although somewhat ambiguous. Mass is an important factor in the study of motion.

Center of Gravity.—Briefly, the center of gravity of a body is *that point of the body about which all its parts are balanced, or which being supported, the whole body will remain at rest, though acted upon by gravity.*

The center of gravity may be found by calculation, and in some cases, more conveniently by experiments, as in fig. 19.

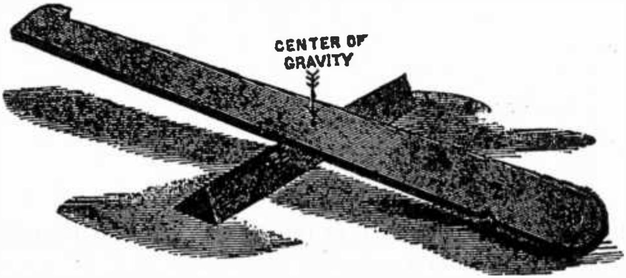


FIG. 19.—Method of finding the center of gravity of the lever. The center of gravity of the lever is the point where the bar would be in equilibrium if balanced over a knife edge or any other support with a sharp corner placed at right angles to the lever, as shown in the figure.

Momentum.—In popular language momentum may be defined as *the power of overcoming resistance as possessed by a body by virtue of its motion;* that which makes a moving body hard to stop. Numerically it is equal to the product of mass of the body multiplied by its velocity.

$$momentum = mass \times velocity$$

using symbols

$$" \quad = m \times v$$

$$= \frac{w}{g} \times v$$

in which

w = weight
g = acceleration due to gravity = 32.17

Friction.—By definition, friction is *that force which acts between two bodies at their surface of contact so as to resist their sliding on each other;* it is the resistance to motion when one body is moved upon another.

STRENGTH OF BEAMS

Beam. (For notation see page 300.)	Rectangular Beam. Breaking Load.	Rectangular Beam. Deflection for Load P or W	Beam of any Section. Maximum Moment of Stress.		Beam of any Section. Moment of Rupture.	Beam of any Section. Deflection Δ
Fixed at one end, load at the other	$P = \frac{1}{6}\frac{Rbd^2}{l}$	$\frac{4Pl^3}{Ebd^3}$	Pl	=	$\frac{RI}{c}$	$\frac{1}{3}\frac{Pl^3}{EI}$
Same with load distributed uniformly	$W = \frac{1}{3}\frac{Rbd^2}{l}$	$\frac{3}{2}\frac{Wl^3}{Ebd^3}$	$\frac{1}{2}Wl$	=	$\frac{RI}{c}$	$\frac{1}{8}\frac{Wl^3}{EI}$
Supported at ends, loaded in middle	$P = \frac{2}{3}\frac{Rbd^2}{l}$	$\frac{Pl^3}{4Ebd^3}$	$\frac{1}{4}Pl$	=	$\frac{RI}{c}$	$\frac{1}{48}\frac{Pl^3}{EI}$
Same, loaded uniformly	$W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{5}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{8}Wl$	=	$\frac{RI}{c}$	$\frac{5}{384}\frac{Wl^3}{EI}$
Same, loaded at middle, and also with uniform load, ...	$2P + W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{4}\left(P + \frac{1}{8}W\right)\frac{l^3}{Ebd^3}$	$\left(\frac{1}{4}P + \frac{1}{8}W\right)l$	=	$\frac{RI}{c}$	$\frac{1}{48}\left(P + \frac{5}{8}W\right)\frac{l^3}{EI}$
Fixed at both ends, loaded in middle	$P = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{1}{16}\frac{Pl^3}{Ebd^3}$	$\frac{1}{8}Pl$	=	$\frac{RI}{c}$	$\frac{P}{192}\frac{l^3}{EI}$
Same. Barlow's Experiments	$P = \frac{Rbd^2}{l}$		$\frac{1}{6}Pl$	=	$\frac{RI}{c}$	
Same, uniformly loaded	$W = \frac{2Rbd^2}{l}$	$\frac{1}{32}\frac{Wl^3}{Ebd^3}$	$\frac{1}{12}Wl$	=	$\frac{RI}{c}$	$\frac{W}{384}\frac{l^3}{EI}$
Fixed at one end, supported at the other, loaded at $0.634l$ from fixed end, ...		$\frac{0.1148Pl^3}{Ebd^3}$	$\frac{3}{8}\left(2\sqrt{3} - 3\right)Pl$	=	$\frac{RI}{c}$	$\frac{P}{105}\frac{l^3}{EI}$ (nearly)
Same, uniformly loaded	$W = \frac{4}{3}\frac{Rbd^2}{l}$	$\frac{0.0648Wl^3}{Ebd^3}$	$\frac{1}{8}Wl$	=	$\frac{RI}{c}$	$\frac{W}{185}\frac{l^3}{EI}$ (nearly)

NOTE.—In the above table: P = load at middle; W = total load, distributed uniformly; l = length, b = breadth, d = depth in ins., E = modulus of elasticity; R = modulus of rupture, or stress per sq. in. of extreme fibre; I = moment of inertia; c = distance between neutral axis and extreme fibre. For breaking load of circular section, replace bd^2 by $.59d^3$. The value of R at rupture, or the modulus of rupture is about 60,000 for structural steel, and about 110,000 for strong steel (Merriman). For cast iron the value of R varies greatly according to quality. Thurston found 45.740 and 67.980 in No. 2 and No 4 cast iron, respectively. For beams fixed at both ends and loaded in the middle, Barlow, by experiment, found the maximum moment of stress = $\frac{1}{6}Pl$ instead of $\frac{1}{8}Pl$, the result given by theory. Prof. Wood (Resist. Matrls. p. 155) says of this case: The phenomena are of too complex a character to admit of a thorough and exact analysis, and it is probably safer to accept the results of Mr. Barlow in practice than to depend upon theoretical results.

Belts and Pulleys

To calculate size of pulleys the following formula is used

$$D = \frac{dr}{R} \qquad (1)$$

in which

D = diameter of driver pulley

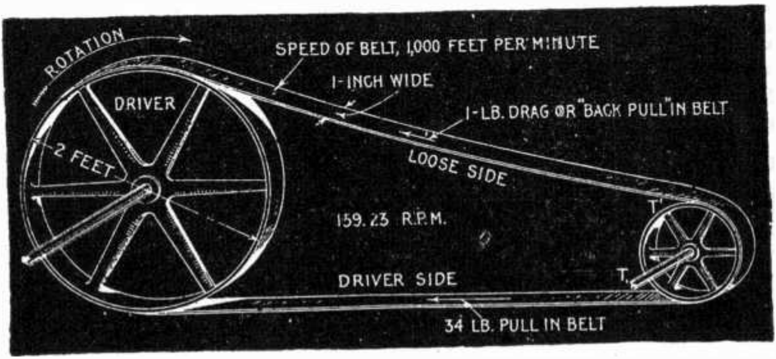


FIG. 20.—One horse power transmitted by belt illustrating the rule: *A single belt one inch wide and travelling 1,000 feet per minute will transmit one horse power: a double belt under the same conditions will transmit two horse power.*

d = " " driven "

R = revolutions per minute of driver pulley

r = " " " " driven "

The following formulae are derived from equation (1)

$$d = \frac{DR}{r} \qquad (2)$$

$$R = \frac{dr}{D} \qquad (3)$$

$$r=\frac{DR}{d}\quad\text{.............................}\quad(4)$$

Example.—Find the diameter of pulley required on engine to run a dynamo at a speed of 1,450 revolutions per minute, the dynamo pulley being 10 ins. in diameter and the speed of engine is 275 revolutions per minute.

Substituting in formula (1)

$$D=\frac{10\times1450}{275}=53 \text{ ins. nearly}$$

Example.—If the speed of engine be 325 revolutions per minute, diameter of engine wheel 42 ins. and the speed of the dynamo 1,400 revolutions per minute, how large a pulley is required on dynamo?

Substituting in formula (2)

$$d=\frac{42\times325}{1400}=9\tfrac{3}{4} \text{ ins.}$$

Example.—What will be the required speed of an engine having a belt wheel 46 ins. in diameter to run a dynamo 1,500 revolutions per minute, the dynamo pulley is 11 ins. in diameter?

Substituting in formula (3)

$$R=\frac{11\times1500}{46}=359 \text{ nearly.}$$

Example.—If a steam engine, running 300 revolutions per minute, have a belt wheel 48 ins. in diameter, and is belted to a dynamo having a pulley 12 ins. in diameter, how many revolutions per minute will the dynamo make?

Substituting in formula (4)

$$r=\frac{48\times300}{12}=1200$$

Shop Calculations

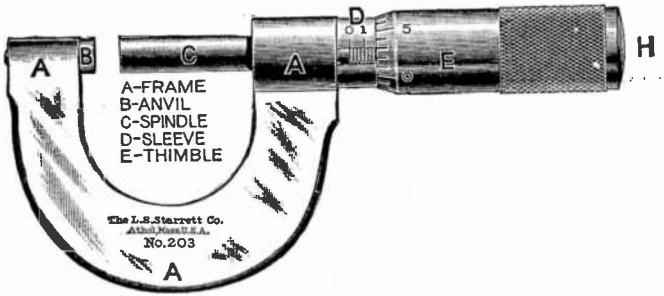


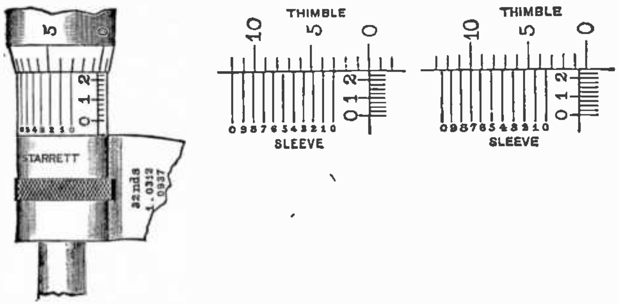
FIG. 21.—Starrett micrometer caliper.

How to Read a Micrometer Caliper.—Readings in ten thousandths of an inch are obtained by the use of a vernier, so named from Pierre Vernier, who invented the device in 1631. As applied to a caliper this consists of ten divisions on the adjustable sleeve, which occupy the same space as nine divisions on the thimble.

The difference between the width of one of the ten spaces on the sleeve and one of the nine spaces on the thimble is therefore one-tenth of a space on the thimble. In fig. 23 the third line from 0 on thimble coincides with the first line on the sleeve. The next two lines on thimble and sleeve do not coincide by one-tenth of a space on thimble; the next two, marked 5 and 2, are two-tenths apart, and so on.

In opening the tool, by turning the thimble to the left, each space on the thimble represents an opening of one thousandth of an inch. If, therefore, the thimble be turned so that the lines marked 5 and 2 coincide, the caliper will be opened two-tenths of one thousandth or two ten thousandths.

Turning the thimble further, until the line 10 coincides with the line 7 on the sleeve as in fig. 24, the caliper has been opened seven ten thousandths, and the reading of the tool is .2507.



Figs. 22 to 24.—How to read Starrett's ten-thousandths micrometer caliper.

To read a ten thousandths caliper, first note the thousandths as in the ordinary caliper, then observe the line on the sleeve which coincides with a line on the thimble. If it be the second line, marked 1, add one ten thousandth; if the third marked 2, add two ten thousandths, etc.

Formulae for Speed of Gears.—When three factors are known, the fourth can be found by using one of the following formulae:

$$\text{Revs. of driver} = \frac{\text{Revs. of follower} \times \text{teeth on follower}}{\text{teeth on driver}}$$

$$\text{Revs. of follower} = \frac{\text{Revs. of driver} \times \text{teeth on driver}}{\text{teeth on follower}}$$

$$\text{Teeth on driver} = \frac{\text{Revs. of follower} \times \text{teeth on follower}}{\text{Revs. of driver}}$$

$$\text{Teeth on follower} = \frac{\text{Revs. of driver} \times \text{teeth on driver}}{\text{Revs. of follower}}$$

As in the case of pulleys, great speed changes are made by trains of gears in place of a pair Examples are found in hoists, clocks, lathes, etc.

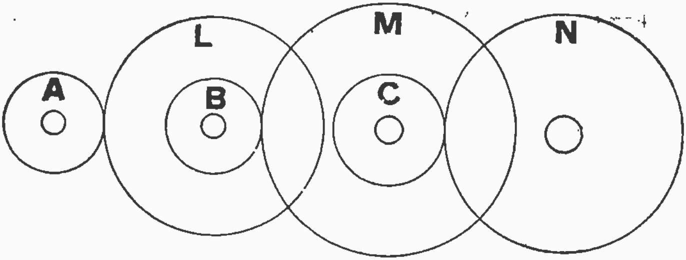


FIG. 25.—Diagram to accompany formulae for speed of gears.

Each pair in the train has its driver and follower, and if the shafts be parallel it is usual to get the speed change by keying two gears of unequal size on every shaft, except the first and last.

Rule.—*The product of the number of teeth on all the drivers divided by the product of the number of teeth on all the followers is the velocity ratio.*

Example.—Assume that a train of gears has three drivers, A,B,C, and three followers L, M, and N, as in fig. 25. A has 14 teeth and drives L having 70 teeth. Pinion B, on same shaft with L, has 13 teeth and drives M, having 104 teeth. Pinion C, has 15 teeth, and is on the same shaft with M; C drives N having 75 teeth. What is the velocity of A to N?

$$\text{Velocity ratio} = \frac{\text{teeth on A} \times \text{teeth on B} \times \text{teeth on C}}{\text{teeth on L} \times \text{teeth on M} \times \text{teeth on N}}$$

$$= \frac{14 \times 13 \times 15}{70 \times 104 \times 75} = \frac{1}{200}$$

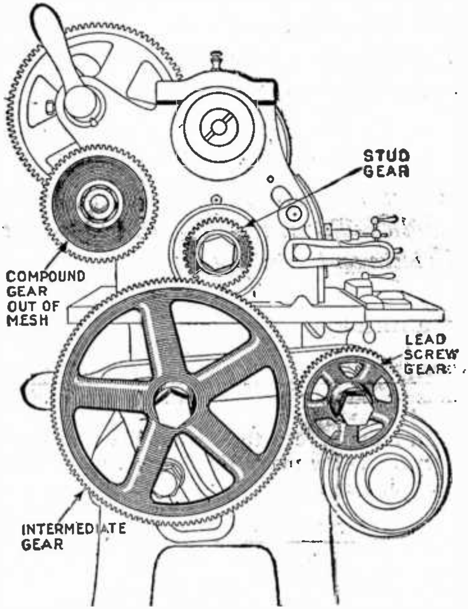


FIG. 26.—Simple train of lathe gears for thread cutting.

Knowing the velocity ratio of the train, it is easy to find the speed of N, if the speed of A, be known. If A run at 1800 revolutions per minute, N will make only 9 revolutions; for $1800 \div 200 = 9$.

When the speed of the first driver or the last follower is also known, the speed may be figured from the rule following.

Rule.—*Multiply the revolutions per minute of the first driver by the continued product of the number of teeth on all drivers, and divide by the continued product of the teeth on all followers. The quotient will be the revolutions per minute of the last follower.*

Lathe Gearing.—In figuring change gears, *the number of threads per inch to be cut corresponds to the revolutions of the driver, and the number of turns on the lead screw to move the carriage one inch corresponds to the speed of the follower.*

Simple Gears

Rule.—*The number of threads to be cut multiplied by the teeth on the spindle stud equals the number of threads on the lead screw multiplied by the teeth on the lead screw gear.*

Expressed as a formula:

$$\frac{\text{threads to be cut}}{\text{threads on lead screw}} = \frac{\text{teeth on lead-screw gear}}{\text{teeth on spindle stud}}$$

Example.—If a lathe have 6 threads on the lead screw and 40 teeth on the lead screw gear, how many threads will be cut if a 24 tooth gear be placed on the spindle stud?

$$\frac{\text{threads to be cut}}{6} = \frac{40}{24}$$

$$\text{threads to be cut} = \frac{40}{24} \times 6 = 10$$

The above assumes that the lathe is geared 1:1; that is, the lathe screw constant is equal to the number of threads per inch on the lead screw. If the lathe be not so geared, the lathe screw constant should be used in place of the threads per inch on the lead screw.

This example shows how the figuring can be done when the gears are on the spindle stud and lead screw; but the problem is usually one of finding out what gears to use.

Example.—Assume seven threads are to be cut, and there are five threads per inch on the lead screw. What gears must be used?

$$\frac{\text{threads to be cut}}{\text{threads on lead screw}} = \frac{\text{teeth on lead screw gear}}{\text{teeth on stud gear}}$$

$$\frac{7}{5} = \frac{\text{teeth on lead screw gear}}{\text{teeth on stud gear}}$$

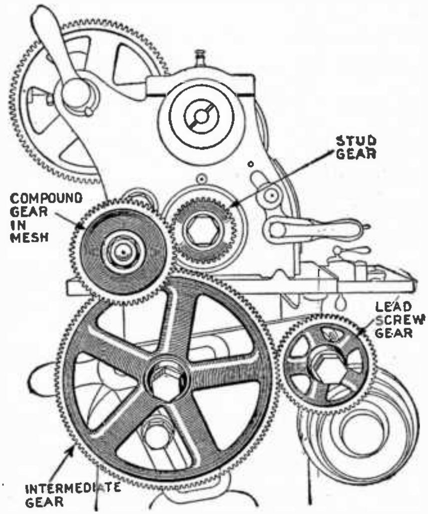


FIG. 27.—Compound train of lathe gears for thread cutting.

The ratio of the gears is as 7:5. By multiplying both 7 and 5 by any number, such as 6, gives

$$\frac{42}{30} = \frac{\text{teeth on lead screw gear}}{\text{teeth on stud gear}}$$

If gears with 30 and 42 teeth be not available multiply by some other number as for instance multiplying by say 4 gives

$$\frac{4 \times 7}{4 \times 5} = \frac{28}{20} = \frac{\text{teeth on lead screw gear}}{\text{teeth on stud gear}}$$

Compound Gears

Example.—Assume that a gear having 90 teeth is needed upon the lead screw to cut a given number of threads. If the set of gears furnished fail to provide a 90 gear, but provides one of 45 teeth, placing this on the lead screw and meshing the two to one compound stud into the train completes the desired ratio, and advances the tool as if the 90 tooth gear had been used.

Speed for Drills.—The following peripheral cutting speed is recommended where high speed cutting tools are used:

Cast iron	55 ft. per min.
Machine steel	35 " " "
Wrought iron	40 " " "
Tool steel, annealed	25 " " "
Brass	100 " " "
Bronze	70 " " "
Grey or red fibre	50 " " "

To find the cutting speed of a revolving shaft:

Rule.—*Multiply the diameter of the shaft in inches by* 3.1416 *and multiply the product by the r.p.m. of the shaft, and divide by* 12, *which will give the peripheral speed in ft. per min.*

Example.—A shaft 1 in. in diameter revolves 134 *r.p.m.* What is the cutting speed?

$$\frac{1 \times 3.1416 \times 134}{12} = 35 \text{ ft. per min.}$$

Milling Machine Indexing.—The calculations by which the index tables are produced and which must be followed for determining the circle and moves for indexing numbers not given in the tables can, perhaps, be best understood by taking several practical examples which follow.

Example.—Indexing less than 40 divisions. Assume that a piece of work mounted between centers is to be divided into 20 equal parts. This will require $^{1}/_{20}$ of a turn of the spindle for each division, and since the ratio between worm and worm wheel is 40 to 1, this will require $^{40}/_{20}$ or two turns of the worm and, therefore, two turns of the index crank. The gears connecting the worm shaft and the index crank shaft are equal in size.

Example.—Indexing more than 40 divisions. It is desired to divide the circle into 80 divisions. This time the worm wheel will make $^{1}/_{80}$ of a turn, while the worm and index crank will make $^{40}/_{80}$ or ½ a turn. In both of the above cases the index pointer always engages the same hole in the index plate, consequently it is immaterial which one of the even number circles of holes it is set to.

Example.—Indexing 152 divisions. From the above two examples it is evident that the ratio between worm and worm wheel is 40 to 1. Note the following rules:

Rule 1.—*Forty divided by the number of divisions required will determine the number of turns or the fractional part of a turn to be made by index pointer, which was two turns for 20 divisions, and ½ a turn for 80 divisions. Now, following this rule, divide 40 by 152, which, expressed in the form of a fraction, is $^{40}/_{152}$, of which:*

Rule 2.—*The denominator represents the circle to be used and the numerator represents the number of holes in this circle over which the index pin must be passed for each division.*

Applying these rules to the first example gives the fraction $^{40}/_{20}$. If the pin were in the 20 hole circle, it would pass over 40 holes, or two turns for each division. Now, referring to the example in question, the index plate does not have a circle containing 152 holes.

It is therefore necessary to transform this fraction into an equivalent fraction whose denominator will be the same number as the number of holes in one of the circles of the index plate. It does contain a 38 hole circle. Hence, transform the fraction $^{40}/_{152}$ to the equivalent fraction or $^{10}/_{38}$ by dividing both the numerator and denominator by 4.

Applying Rule 2 to this new fraction 38 is the circle to which the index pin must be adjusted, and it must move over a series of 10 holes for each of the 152 divisions into which the work is to be divided.

Example.—Indexing 33 divisions. The fraction now takes the form of $^{40}/_{33}$. The plate does not contain a 33 hole circle, neither does it contain an 11 hole circle nor a 3 hole circle, and since these are the only numbers which can be evenly divided into 33, make the transformation by multiplying instead of dividing. It is found that the plate does contain a 66 hole circle; therefore, by multiplying both numerator and denominator by 2, gives the equivalent fraction or $^{80}/_{66}$, in which 66 is the circle and 80 is the number of holes over which the pin must pass for each division; but since 80 holes are more than the 66 hole circle contains, divide 80 by 66, and find that it is contained once with 14 left over; therefore, the pointer must make one complete turn and 14 holes in addition.

Example.—Indexing 395 divisions. The fraction is $^{40}/_{395} = {}^{80}/_{790} = {}^{8}/_{79}$, in which case use the 79 hole circle and index over eight holes. The highest number that can be obtained with a high number indexing attachment is 7960. The fraction is $^{40}/_{7960} = {}^{1}/_{199}$. Here use the 199 hole circle and index one hole for each of the 7960 divisions.

Differential Indexing.—In differential indexing the spindle or driven shaft and the index plate are connected by a train of gearing which causes the plate to turn either in the same or opposite direction to that in which the crank is turned.

The total movement of the crank at every indexing is, therefore, equal to its movement relative to the plate, *plus* the movement of the plate, when the plate revolves in the same direction as the crank, or *minus* the movement of the plate, when the plate revolves in the opposite direction to the crank.

N = number of divisions required;

H = number of holes in index plate;

n = number of holes taken at each indexing;

V = ratio of gearing between index crank and spindle;

x = ratio of the train of gearing between the spindle and the index plate;

S = gear on spindle
G_1 = 1st gear on stud } drivers;

G_2 = 2nd gear on stud
W = gear on worm } driven;

$$x = \frac{HV - Nn}{H} \text{ if HV be greater than } Nn;$$

$$x = \frac{Nn - HV}{H} \text{ if HV be less than } Nn;$$

$$x = \frac{S}{W} \text{ (for simple gearing);}$$

$$x = \frac{S\,G_1}{G_2 W} \text{(for compound gearing);}$$

As applied to the spiral head of a milling machine made by Brown & Sharpe Mfg. Co. V is equal to 40 and the index plates furnished have the following numbers of holes:—(15, 16, 17, 18, 19, 20), (21, 23, 27, 29, 31, 33), (37, 39, 41, 43, 47, 49).

The gears furnished have the following numbers of teeth:—24 (2) 28, 32, 40, 44, 48, 56, 64, 72, 86, 100. These index plates and gears provide for the indexing of all divisions up to 382.

In selecting the index circle to be used, it is best to select one with a number having factors that are contained in the change gears on hand, for if H contain a factor not found in the gears, *x* cannot usually be obtained, unless the factor be cancelled by the difference between HV and N*n*, or unless N contain the factor.

Multiplying the numbers of holes in the plates by 40 gives all the values of HV that can be obtained with the regular index plates. Following is a table of these products, which will be found convenient to use, especially when many combinations are to be obtained.

15 × 40	600	21 × 40	840	37 × 40	1480
16 × 40	640	23 × 40	920	39 × 40	1560
17 × 40	680	27 × 40	1080	41 × 40	1640
18 × 40	720	29 × 40	1160	43 × 40	1720
19 × 40	760	31 × 40	1240	47 × 40	1880
20 × 40	800	33 × 40	1320	49 × 40	1960

When HV is greater than Nn and gearing is simple, use 1 idler.

When HV is greater than Nn and gearing is compound, use no idlers.

When HV is less than Nn and gearing is simple, use 2 idlers.

When HV is less than Nn and gearing is compound, use 1 idler.

Select n, so that the ratio of gearing will not exceed 6; 1 on account of the excessive stress upon the gears.

Example.—N=59. Required H, n and x.

Assume H=33 $n=22$

$$x=\frac{(33\times40)-(59\times22)}{33}=\frac{22}{33}=\frac{2}{3}$$

Select gears giving this ratio, as 32 and 48, the 32 being the gear on spindle and the 48 the gear on worm. HV is greater than Nn and the gearing is simple, requiring one idler.

Example.—N=319. Required H, n and x.

Assume H=29 $n=4$

$$x=\frac{(319\times4)-(29\times40)}{29}=\frac{116}{29}=\frac{4}{1}$$

When the ratio is not obtainable with simple gearing, try compound gearing. $\frac{4}{1}$ can be expressed as follows: $\frac{3\times4}{1\times3}$ or $\frac{72\times64}{24\times48}$ for which there are available gears.

HV is less than Nn and the gearing is compound, requiring one idler.

Example.—Spacing for quarter degrees. Required H, n and x for spacing ¼ degree or 1440 divisions.

Assume H=33 $n=1$

$$\frac{(1440\times1)-(33\times40)}{33}=\frac{120}{33}\text{ or }\frac{64\times100}{40\times44}$$

One idler is required.

Aliquant or fractional spacing.

Example.—Required: A vernier to read to 1-12 degree or 5 minutes, the scale being divided to degrees.

Each Vernier space can equal 11-12 degree.

$$\frac{11}{12} \times \frac{1}{360} = \frac{11}{4320} \text{ or } \frac{4320}{11} \text{ spaces in whole circle} = 392\frac{8}{11} \text{ spaces.}$$

Assume H = 18 $\quad n = 2$

Then
$$\frac{(392\frac{8}{11} \times 2) - (18 \times 40)}{18} = \frac{\frac{720}{11}}{18} = \frac{720}{11} \times \frac{1}{18} = \frac{40}{11} = \frac{64 \times 100}{40 \times 44}.$$

One idler is required.

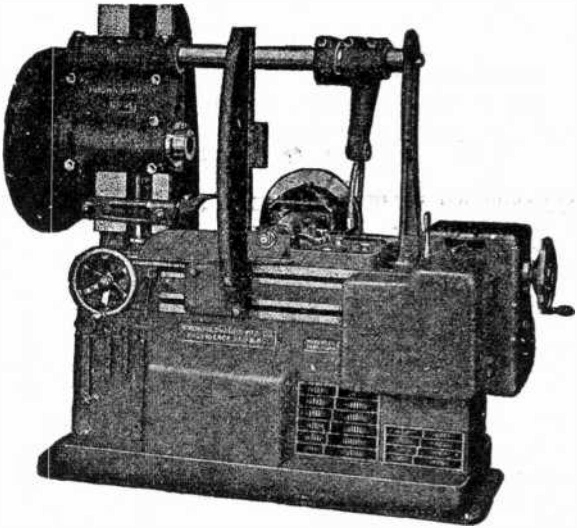


Fig. 28.—Brown & Sharpe automatic spur gear cutting machine. The cutter spindle is supported in heavy bearings and is provided with a balance wheel, keyed to the tapered end of spindle, to steady the cutting action. The spindle has keys for driving cutters with either new or old size keyways. It is driven from a worm sliding in a splined horizontal driving shaft, through a bronze driving gear, which is fitted to integral keys on the spindle itself. Positive clutches and integral keys are used to eliminate the possibility of backlash in the spindle driving mechanism. The spindle can be removed and smaller sizes substituted and the spindle outer bearing may be adjusted for wear. The work spindle slide has a tapered gib on the inside of the way which provides an added guide when the slide is loosened for adjustment. The slide has a positive bearing on both sides of the flat and dove-tail ways to take end pressure. A central oiling station located on the cutter slide assures positive lubrication of the spindle bearings and the cutter slide ways. The indexing mechanism is also lubricated from a central station within the case itself. The cooling piping is arranged to avoid sharp bends and air pockets.

Gears

> *The author is indebted to the Brown & Sharpe Mfg. Co., Providence, R. I., for the formulae relating to spur, bevel, worm and worm wheel gears.*

SPUR GEARING.

Two spur gears in action are comparable to two corresponding plain rollers whose surfaces are in contact, these surfaces representing the pitch circles of the gears.

Pitch of Gears.

For convenience of expression the pitch of gears may be stated as follows:

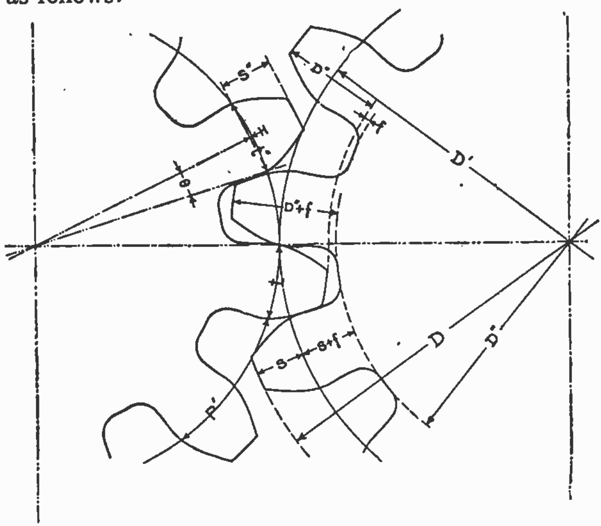


Fig. 29.—Spur gears with reference letters to accompany formulae.

Circular pitch is the distance from the centre of one tooth to the centre of the next tooth, measured on the pitch line.

Diametral pitch is the number of teeth in a gear per inch of pitch diameter. That is, a gear that has, say, six teeth for each inch in pitch diameter is six diametral pitch, or, as the expression is universally abbreviated, it is "six pitch." This is by far the most convenient way of expressing the relation of diameter to number of teeth.

Module is the pitch diameter of a gear divided by the number of teeth.

FORMULAS.

N = number of teeth.
s = addendum and module.
t = thickness of tooth on pitch line.
t'' = chordal thickness of tooth.
f = clearance at bottom of tooth.
D'' = working depth of tooth.
D'' + f = whole depth of tooth.
D' = pitch diameter.
D = outside diameter.
D''' bottom diameter.
P' = circular pitch.
P = diametral pitch.
H = height of arc.
s'' = distance from chord to top of tooth.
θ = ¼ the angle subtended by circular pitch.

$$P = \frac{N+2}{D}; \text{ or } = \frac{\pi}{P'}$$

$$P' = \frac{\pi}{P}; \text{ or } = D' \pi \frac{\theta}{90^\circ}; \text{ or } = \frac{D' \pi}{N}; \text{ or } = \frac{D\pi}{N+2}$$

$$s = \frac{1}{P}; \text{ or } = \frac{P'}{\pi}; \text{ or } = .3183\, P'; \text{ or } = \frac{D'}{N}; \text{ or } = \frac{D}{N+2}$$

$$t = \frac{P'}{2};\ \text{or} = \frac{\pi}{2P} = \frac{1.5708}{P}$$

$$f = \frac{t}{10}$$

$$s + f = \frac{1.157}{P};\ \text{or} = .3683\ P'$$

$$D'' = 2\,s;\ \text{or} = \frac{2}{P}$$

$$D'' + f = \frac{2.157}{P};\ \text{or} = .6866\ P'$$

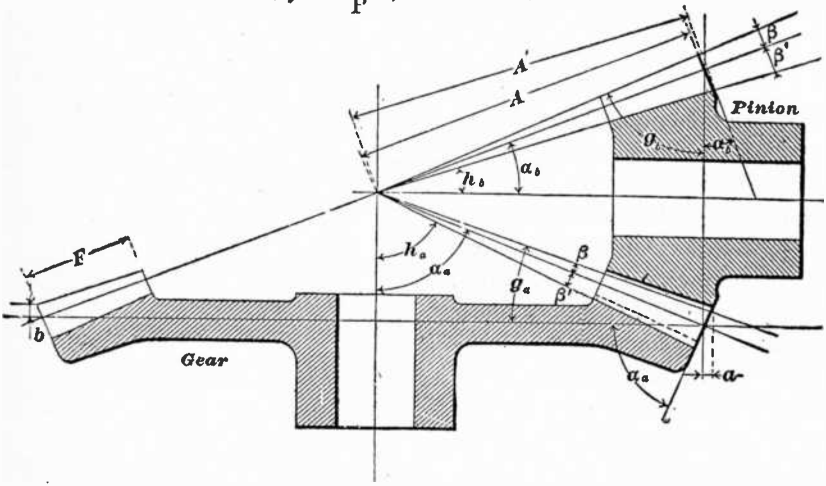


Fig. 30.—Bevel gears with reference letters to accompany formulae—axes at right angles.

$$D' = \frac{N}{P},\ \text{or} = \frac{N\,P'}{\pi};\ \text{or} = .3183\ P'\ N;\ \text{or} = \frac{D\,N}{N+2}$$

$$D = D' + 2\,s;\ \text{or} = \frac{N+2}{P}$$

$$\theta = \frac{90^\circ}{N}$$

$$t'' = D' \sin \theta$$

$$N = D'\,P;\ \text{or} = D\,P - 2$$

$$H = \frac{D'(1 - \cos \theta)}{2}$$

$$s'' = s + H$$

$$D''' = D - 2\,(D'' + f);\ \text{or} = \frac{N - 2.31416}{P}$$

BEVEL GEARS—AXES AT RIGHT ANGLES.

FORMULAS.

$N_a =$ } Number of teeth { gear.
$N_b =$ } Number of teeth { pinion.

P = diametral pitch. P′ = circular pitch.

$\alpha_a =$ } centre angle = angle of edge or pitch angle { gear.
$\alpha_b =$ } centre angle = angle of edge or pitch angle { pinion.

β = angle of top. β' = angle of bottom.

$g_a =$ } angle of face { gear.
$g_b =$ } angle of face { pinion.

$h_a =$ } cutting angle { gear.
$h_b =$ } cutting angle { pinion.

A = apex distance from pitch circle.

A′ = apex distance from large bottom of tooth.

D′ = pitch diameter. D = outside diameter.

s = addendum and module.

t = thickness of tooth at pitch line.

f = clearance at bottom of tooth.

D″ = working depth of tooth.

D″ + f = whole depth of tooth. $2\,a$ = diameter increment.

b = distance from top of tooth to plane of pitch circle.

F = width of face.

$$\tan\ \alpha_a = \frac{N_a}{N_b};\quad \tan\ \alpha_b = \frac{N_b}{N_a};$$

$$\tan \beta = \frac{2 \sin \alpha}{N};\ \text{or} = \frac{s}{A}.$$

$$\tan \beta' = \frac{2.314 \sin \alpha}{N}; \text{ or } = \frac{s+f}{A};$$

$$g_a = 90° - (\alpha_a + \beta); \quad g_b = 90° - (\alpha_b + \beta)$$

$$h = \alpha - \beta'$$

$$A = \frac{1}{2P}\sqrt{N_a^2 + N_b^2}$$

$$A = \frac{N}{2 P \sin \alpha}$$

$$A' = \frac{A}{\cos \beta'}; \text{ or } = \frac{N}{2 P \sin \alpha \cos \beta'}$$

$$A = \frac{\frac{1}{2} D}{\sin (\alpha + \beta)} \cos \beta$$

$$P = \frac{N}{2 A \sin \alpha}; \text{ or } = \frac{N + 2 \cos \alpha}{D}; \text{ or } = \frac{\pi}{P'}$$

$$P' = \frac{\pi}{P}$$

$$D' = \frac{N}{P}; \text{ or } = \frac{N P'}{\pi}; \text{ or } = \frac{D N}{N + 2 \cos \alpha}; \text{ or } = D - \frac{2 \cos \alpha}{P}$$

$$D = D' + 2a$$

$$2a = 2s \cos \alpha$$

$$b = a \tan \alpha \begin{cases} a \text{ for gear } = b \text{ for pinion} \\ a \text{ for pinion} = b \text{ for gear} \end{cases}$$

$$s = \frac{1}{P}; \text{ or } = \frac{P'}{\pi}; \text{ or } = .3183\, P'; \text{ or } = A \tan \beta$$

$$s + f = .3683\, P'; \text{ or } = A \tan \beta'; \text{ or } = \frac{1.157}{P}$$

$$D'' = 2s$$

$$D'' + f = \frac{2.157}{P}; \text{ or } = .6866\, P'$$

$$t = \frac{P'}{2}; \text{ or } = \frac{\pi}{2P}$$

$$*F = \frac{A}{3}; \text{ or } = \frac{5\,P'}{2} \qquad f = \frac{t}{10}$$

NOTE.—Formulas containing notations without the designating letters *a* and *b* apply equally to either gear or pinion. If wanted for one or the other, the respective letters are simply attached.

*The formula giving the lesser value of F should always be used.

BEVEL GEARS WITH AXES AT ANY ANGLE.

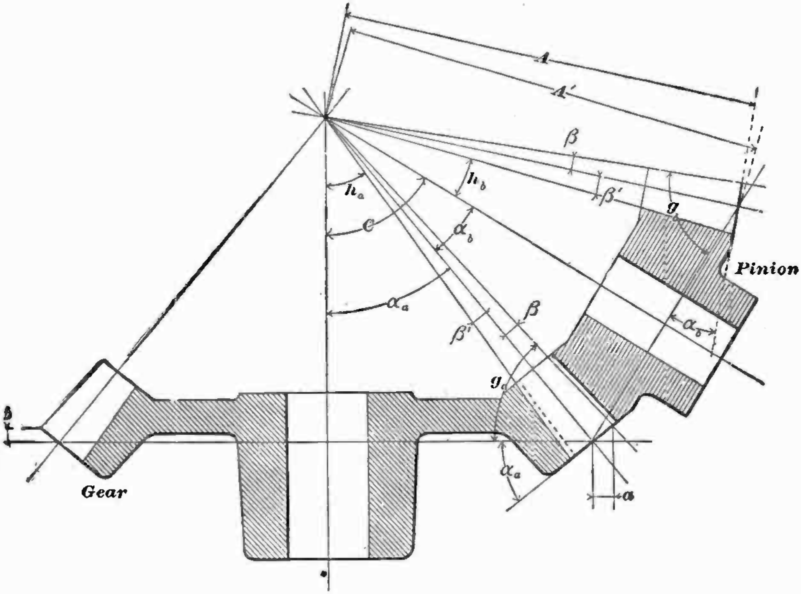


FIG. 31.—Bevel gears with reference letters to accompany formulae—axes at any angle.

FORMULAS.

C = angle formed by axes of gears.

N_a = } number of teeth { gear
N_b = } { pinion.

P = diametral pitch

P' = circular pitch.

$\left.\begin{array}{l}\alpha_a = \\ \alpha_b = \end{array}\right\}$ angle of edge = pitch angle $\left\{\begin{array}{l}\text{gear.} \\ \text{pinion.}\end{array}\right.$

β = angle of top.

β' = angle of bottom.

$\left.\begin{array}{l}g_a = \\ g_b = \end{array}\right\}$ angle of face $\left\{\begin{array}{l}\text{gear.} \\ \text{pinion.}\end{array}\right.$

$\left.\begin{array}{l}h_a = \\ h_b = \end{array}\right\}$ cutting angle $\left\{\begin{array}{l}\text{gear.} \\ \text{pinion.}\end{array}\right.$

A = apex distance from pitch circle.

A′ = apex distance from large bottom of tooth.

D = outside diameter.

D′ = pitch diameter.

$2a$ = diameter increment.

b = distance from top of tooth to plane of pitch circle.

NOTE.—The formulae for tooth parts as given on page 31 apply equally to these cases

$$\tan \alpha_a = \frac{\sin C}{\frac{N_b}{N_a} + \cos C}, \qquad \cot \alpha_a = \frac{N_b}{N_a \sin C} + \cot C$$

$$\tan \alpha_b = \frac{\sin C}{\frac{N_a}{N_b} + \cos C}; \qquad \cot \alpha_b = \frac{N_a}{N_b \sin C} + \cot C$$

NOTE.—The following formulae are correct only for values of C less than 90°. If C be greater than 90° see pages 37 and 38.

$$\tan \beta = \frac{2 \sin \alpha}{N}; \text{ or} = \frac{s}{A};$$

$$\tan \beta' = \frac{2.314 \sin \alpha}{N}; \text{ or} = \frac{s+f}{A};$$

$g_a = 90° - (\alpha_a + \beta)$ for cases I and II ; or $= \beta$, for case III;
or $= 90° - (\alpha_a - \beta)$ for case IV.

$g_b = 90° - (\alpha_b + \beta)$

$h = \alpha - \beta'$

$$A = \frac{N}{2 P \sin \alpha}$$

$$A' = \frac{A}{\cos \beta'}$$

$$D' = \frac{N}{P} \text{ or } = \frac{N P'}{\pi}$$

$$^*D' = \frac{D N}{N + 2 \cos \alpha}; \quad \text{or} = D - \frac{2 \cos \alpha}{P}$$

$*D = D' + 2\,a$

D $= D'$, for gear in case III; or $= D' - 2\,a$, for gear in Case IV.

$2\,a = 2\,s \cos \alpha$

$b = s \sin \alpha$

NOTE.—Formulas containing notations without the designating letters *a* and *b* apply equally to either gear or pinion. If wanted for one or the other, the respective letters are simply attached.

* For cases I and II and for pinions in cases III and IV

The formulas given for α_a and α_b (when C, N_a and N_b are known) undergo some modifications for values of C greater than 90°.

For bevel gears at any angle but 90° we may distinguish four cases; C, N_a, N_b being given.

I. Case. See pages 35 to 37.

II. Case. C is greater than 90°.

$$\tan \alpha_a = \frac{\sin (180 - C)}{\frac{N_b}{N_a} - \cos (180 - C)}; \quad \tan \alpha_b = \frac{\sin (180 - C)}{\frac{N_a}{N_b} - \cos (180 - C)}.$$

III. Case. $\alpha_a = 90°$; $\alpha_b = C - 90°$

IV. Case.

$$\tan \alpha_a = \frac{\sin E}{\cos E - \frac{N_b}{N_a}}; \quad \tan \alpha_b = \frac{\sin E}{\frac{N_a}{N_b} - \cos E}$$

For an example to apply to Case III., the following condition must be fulfilled:

$$N_a \sin (C - 90°) = N_b$$

To distinguish whether a given example belongs to Case II. or Case IV., we are guided by the following condition:

Is: $N_a \sin (C - 90°)$ { *smaller* than N_b, we have Case II, *larger* than N_b, we have Case IV.

UNDERCUT IN BEVEL GEARS.

By undercut in gears is understood a special formation of the tooth, which may be explained by saying that the elements of the tooth below the pitch line are nearer the centre line of the tooth than those on the pitch line. Such a tooth outline is to be found only in gears with few teeth. In a pair of bevel gears where the pinion is low-numbered and the ratio high, we are apt to have undercut. For a pair of running gears this condition presents no objection. Should, however, these gears be intended as patterns to cast from, they would be found useless, from the fact that they would not draw out of the sand.

If a pair of bevel gears with teeth constructed on this basis have undercut, we can nearly eliminate the undercut—and for the practical working this is quite sufficient—by taking as a basis for the construction of the tooth outline a pressure angle of 20°.

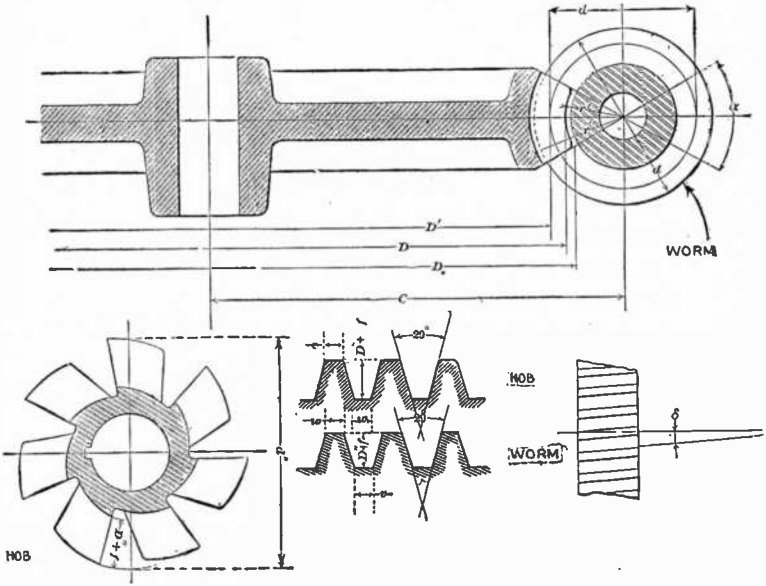
The question now is: When do we and when do we not have undercut? Let there be:

N = number of teeth in gear.
n = number of teeth in pinion.

$$\frac{n\sqrt{N^2+n^2}}{N}=p$$

where we have undercut for p less than 30.

This formula is strictly correct for epicycloidal gears only. It is, however, used as a safe and efficient approximation for the involute system.



FIGS. 32 to 35.—Worm and worm wheel gears with reference letters to accompany formulae.

Worm and Worm Wheel

L = lead of worm.
N = number of teeth in gear.
m = turns per inch of worm.
d = diameter of worm.
d' = pitch diameter of worm.
d'' = diameter of hob.
D = throat diameter.
D′ = pitch diameter of worm wheel.
B = blank diameter (to sharp corners).
C = distance between centres.
P = diametral pitch.
P′ = circular pitch for worm wheels or axial pitch for worms.
r', r'' } See figs. 32 to 35.
s = addendum and module.
t = thickness of tooth at pitch line.
t^n = normal thickness of tooth.
f = clearance at bottom of tooth.
D″ = working depth of tooth.
D″ + f = whole depth of tooth.
b = pitch circumference of worm.
v = width of worm thread tool at end.
w = width of worm thread at top and width of hob tool at end.
δ = angle of tooth of worm wheel with its axis, or the angle of thread with a line at right angles to its axis.

If the lead is for single, double, triple, etc., thread, then

$$L = P',\ 2\ P',\ 3\ P',\ \text{etc.}$$

In multiple threaded worms and their mating wheels, if the angle δ is more than 18° the tooth parts should be figured on the normal as for spiral gears. In using the formulas for spiral gears, it should be borne in mind that while P′ is the axial pitch for worms it is the circular pitch for spiral gears.

$\alpha = 60° \text{ to } 90°$

$$L = \frac{1}{m}$$

$$P' = \frac{\pi D}{N + 2}$$

$$D' = \frac{N P'}{\pi}; \quad \text{or} = \frac{N}{P}$$

$$D = \frac{N}{P} + 2s$$

$$b = \pi (d - 2s); \quad \text{or} = \pi d'$$

$\tan \delta = \dfrac{L}{b}$ { Practical only when width of wheel on wheel pitch circle is not more than ⅔ pitch diameter of worm.

$$t^n = t \cos \delta$$

$$r' = \frac{d}{2} - 2s$$

$$r'' = r' + D'' + f$$

$$C = \frac{D' + d}{2} - s; \quad \text{or} = \frac{D' + d'}{2}$$

$B = D + 2\left(r' - r' \cos \dfrac{\alpha}{2}\right)$ A measurement of sketch is generally sufficient.

$$d'' = d + 2f$$

$v = .3095\ P'$ NOTE.—Hob and worm should be marked, as per example:

$w = .3354\ P'$

4 turns per 1″ single .25 P′; .25 R. H.
2 turns per 1″ double .25 P′; .50 L. H.

NOTE.—The notations and formulas referring to tooth parts, given on pages 31 and 32, for spur gears, apply to worm wheels and are here used.

Heat

Thermometer Scales.—Three scales are in general use

1. Fahrenheit.

2. Centigrade.

3. Reaumur.

The Fahrenheit thermometer is generally used in English speaking countries, and the Centigrade or Celsius thermometer in countries that use the metric system. In many scientific treatises in English, however, Centigrade readings are also used, either with or without their Fahrenheit equivalents. The Reaumur thermometer is used to some extent on the continent of Europe.

Fahrenheit Scale.—The number of degrees between the two fixed points is 180. The freezing point is 32° above zero, hence the boiling point is 32°+180°=212°.

Centigrade Scale.—The number of degrees between the two fixed points is 100. The freezing point is zero, hence the boiling point is 100°.

Reaumur Scale.—The number of degrees between the two fixed points is 80. The freezing point is zero, and accordingly, the boiling point, 80°.

The following conversion fractions will be found convenient to obtain equivalent readings.

1 degree	Fahrenheit	= 5/9	degree	Centigrade	= 4/9	degree Reaumur
1 "	Centigrade	= 9/5	"	Fahrenheit	= 4/5	" "
1 "	Reaumur	= 9/4	"	"	= 5/4	" Centigrade

Temperature	Fahrenheit	$= 9/5 \times$ temp. C $+ 32° = 9/4$ R $+ 32°$
"	Centigrade	$= 5/9 \times$ (temp. Fahr. $- 32) = 5/4$ R
"	Reaumur	$= 4/5$ temp. C $= 4/9$ (Fahr. $- 32°$)

Specific Heat.—Defined as the *ratio of the quantity of heat needed to raise its temperature one degree to the amount needed to raise the temperature of the same weight of water one degree;* expressed as a formula,

$$\text{Specific heat} = \frac{B.t.u.\text{ required to raise temperature of substance } 1°}{B.t.u.\text{ required to raise temperature same weight water } 1°}$$

from this it follows that,

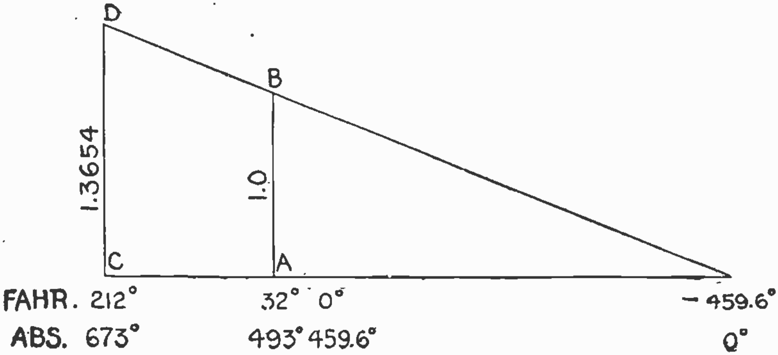


FIG. 36.—Graphical method of determining the absolute zero. *In construction* draw a horizontal line to represent temperatures to any scale and mark on it points representing the freezing point and boiling point of water, marked 32° and 212° respectively. From 32° set out, at right angles to the line of temperature, a line of pressure AB, = 1 atmosphere to any scale, and at 212° a line CD = 1.3654 atmospheres to the same scale. Join the extremities DB, of these lines to intersect the line of temperatures. It is assumed by physicists that, *since the pressures vary regularly per degree of change of temperature between certain limits within the range of experiment, they vary also at the same rate beyond that range,* and, therefore, that the point of intersection of the straight line DB, produced gives the point at which the pressure is reduced to zero, this point being known as the *absolute zero.*

Specific heat = B.t.u. required to heat one lb. of a substance 1° F.

One of the simplest methods of determining specific heat is by mixing the substance with water.

Example. Suppose that six pounds of mercury at 100° C, be poured into two pounds of water at 0° C, and that the resulting temperature of the "mixture" is 9°. The specific heat S, of the mercury can then be found as follows:

In falling from 100° to 9° the six pounds of mercury give out 6 × (100—9) × S, or 546 S heat units. These have gone to heat two pounds of water from 0° to 9°, which requires 2 × 9, or 18 heat units. Hence, we may write,

$$546\ S = 18$$

Therefore,

$$S = 18 \div 546 = .033$$

As given by Röntgen, the specific heat of various substances is as follows:

Specific Heat of Various Substances

Solids

Copper	.0951
Wrought iron	.1138
Glass	.1937
Cast iron	.1298
Lead	.0314
Tin	.0562
Steel, Sott	.1165
Steel, Hard	.1175
Brass	.0939
Ice	.504

Liquids

Water	1.
Sulphuric Acid	.335
Mercury	.0333
Alcohol (nnn)	.7
Benzine	.95
Ether	.5034

Gases

	Constant pressure	Constant volume
Air	.23751	.16847
Oxygen	.21751	.15507
Hydrogen	3.409	2.41226
Nitrogen	.2438	.17273
Ammonia	.508	.299
Alcohol	.4534	.399

Absolute Temperature.—According to various experiments that have been made with pure gases with the use of air thermometers, it has been found that *air expands approximately*

$\frac{1}{459.6}$ *of its volume per degree increase in temperature at zero F.*

$\frac{1}{273.1}$ *of its volume at* 0° *C.*

NOTE.—*Specific heat of gases.* Experiments by Mallard and Le Chatelier indicate a continuous increase in the specific heat at constant volume of steam, carbon dioxide, and even the perfect gases, with rise of temperature. The variation is inappreciable at 212° F., but increases rapidly at the high temperatures of the gas engine cylinder.

Accordingly, *by cooling the air below zero, the reverse process should be true;* that is to say, for each degree F. decrease in temperature, the volume at zero would be contracted $\frac{1}{459.6}$. It must be evident then, if a volume of a *perfect* gas could be cooled to −459.6° F. it would cease to exist, giving the theoretical point known as the *absolute zero*. However, all gases assume the liquid form at very low temperature, and accordingly do not obey the law of contraction of gases at and near the absolute zero.

High Temperature Judged by Color.—The following table has been generally accepted, giving the colors and their corresponding temperature as below:

	Deg. C	Deg. F
Incipient red heat..	525	977
Dull red heat......	700	1,292
Incipient cherry red heat.............	800	1,472
Cherry red heat....	900	1,652
Clear cherry red heat	1,000	1,832
Deep orange heat..	1,100	2,021
Clear orange heat..	1,200	2,192
White heat.......	1,300	2,372
Bright white heat..	1,400	2,552
Dazzling white heat	1,500 to 1,600	2,732 to 2,912

Conductivity.—This is *the relative value of a material, as compared with a standard, in affording a passage through itself or over its surface for heat.*

Latent Heat.—Defined as *that quantity of heat which disappears or becomes concealed in a body causing a* "change of state," as in changing ice into water or water into steam. Latent heat causes a change of state without a change in temperature.

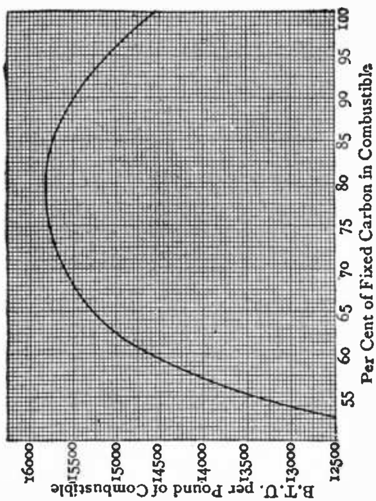


FIG. 37.—Curve of relation between heat value per pound of combustibles and fixed carbon in combustion as deduced by Kent.

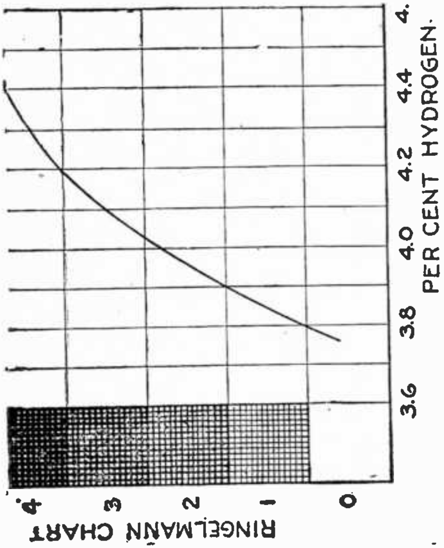


FIG. 38.—Ringelman chart showing how the density of smoke varies with the percentage of hydrogen in the coal.

Combustion

The chemical compounds formed by the combinations of carbon and hydrogen called *hydro-carbons*, are: Methane or marsh gas (CH_4) having a heat value of 23,616 *B.t.u.*; ethylene or olefiant gas (C_2H_4), having a value of 21,344 *B.t.u.*; acetylene (C_2H_2), having a heat value of about 18,196 *B.t.u.*; benzole (C_6H_6), having a heat value of about 18,000 *B.t.u.*

If these gases be completely consumed so as to develop the number of heat units given, the products will be carbon dioxide (CO_2) and water (H_2O). The igniting temperature of these gases varies from 580° to 667° C.

Air Required.—The theoretical amount of air per pound of coal varies between 7 and a little over 11 pounds.

Air Required for Different Fuels

Fuel	Air theoretically required per pound of coal	Air theoretically required per 10,000 *B.t.u.*, generated
Illinois bituminous, poor quality	7.0	7.6
Illinois bituminous, good quality	9.4	7.55
Anthracite, average	10.2	7.65
Semibituminous, Pocahontas	11.2	7.5
Liquid fuel	14.24	7.04

Let the ultimate analysis be as follows:

	Per cent
Carbon	74.79
Hydrogen	4.98
Oxygen	6.42
Nitrogen	1.20
Sulphur	3.24
Water	1.55
Ash	7.82
	100.00

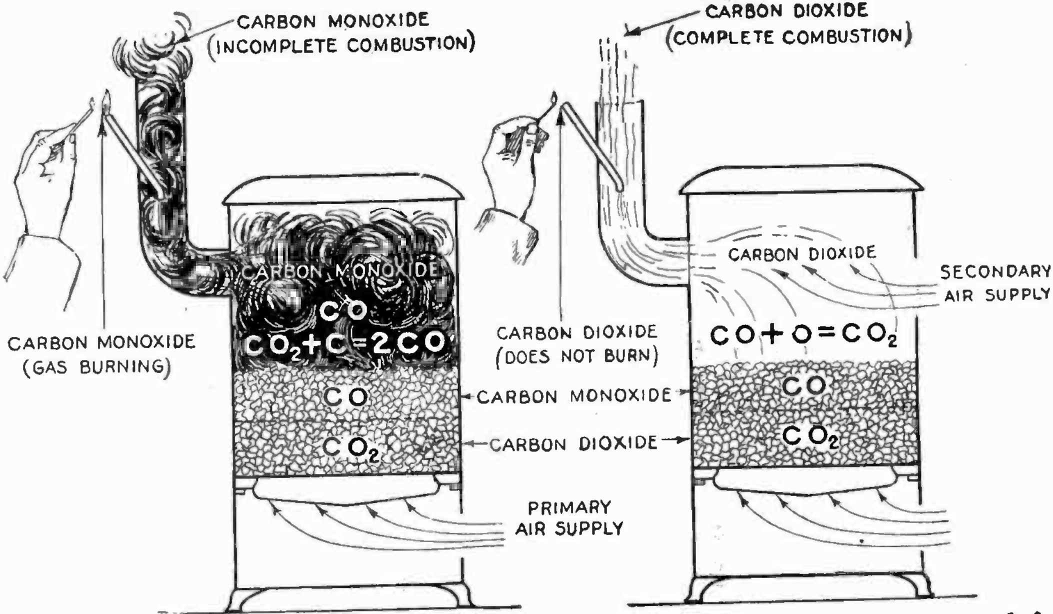
Expressed numerically, the theoretical amount of air for the above analysis is as follows:

$$.7479\ C \times 2\tfrac{2}{3} = 1.9944\ O \text{ needed}$$

$$\left(.0498 - \frac{.0642}{8}\right) H \times 8 = .0418\ \text{“} \quad \text{“}$$

$$.0324\ S \times 1 = .0324\ \text{“} \quad \text{“}$$

$$\text{Total } 20.686\ \text{“} \quad \text{“}$$



FIGS. 39 and 40.—Combustion in an ordinary stove illustrating incomplete (fig. 39) and complete (fig. 40) combustion. In fig. 39, air enters from below the grate only, and on passing through the lower layer of incandescent coal gives up its oxygen to the carbon of coal forming carbon *di*oxide, thus:

$$\text{carbon} + \text{oxygen} = \text{carbon dioxide}$$
$$C + 2O = CO_2$$

As this carbon dioxide passes up through the upper layer of the coal where there is but little oxygen it gives up one-half of its oxygen to the carbon of the coal forming carbon *mon*oxide, thus:

$$\text{carbon dioxide} + \text{carbon} = \text{carbon monoxide}$$
$$CO_2 + C = 2CO$$

One pound of oxygen is contained in 4.32 pounds of air.

The total air needed per pound of coal, therefore, will be $2.361 \times 4.32 = 10.2$.

The weight of combustible per pound of fuel is .7479+.0498*+.0324+.012=.84 pounds, and the air theoretically required per pound of combustible is 10.2÷.83=12.14 pounds.

The above is equivalent to computing the theoretical amount of air required per pound of fuel by the formula:

$$\text{Weight per pound} = 11.52\ C + 34.56\left(H - \frac{O}{8}\right) + 4.32\ S$$

where C, H, O and S, are proportional parts by weight of carbon, hydrogen, oxygen and sulphur by ultimate analysis.

Ques. Is it possible in practice to obtain perfect combustion with the theoretical amount of air?

Ans. No.

Heating Value of Fuels.—In calculating the heating value of a fuel the result is brought down to a comparative *basis of evaporation from and at* 212 *degrees Fahrenheit*, and mean atmospheric pressure.

Under this condition one pound of water is turned into steam by the addition of 970.4 heat units. The quantity of water which can be evaporated under these conditions by one pound of pure and dry carbon is 14.94 pounds. As a heat unit is equal to 778 foot pounds, and as a pound of carbon contains about 14,500 heat units, the heat it contains would be equal to 14,500 multiplied by 778=1,281,000 foot pounds.

In the case of hydrogen, one pound of the fuel would evaporate about 65 pounds of water.

Dulong's formula for the heating value per pound of dry fuel is

$$B.t.u. = 14{,}600\ C + 62{,}000\left(H - \frac{O}{8}\right) + 4{,}000\ S$$

where C, H, O and S are the proportionate parts by weight of carbon, hydrogen, oxygen and sulphur.

*NOTE.—Available hydrogen.

"Available" Heating Value.—*If 9 lbs. of water which would result from the burning of one pound of hydrogen and the giving off of 62,000 heat units, the water being cooled to the temperature of the air, be passed into a hot furnace, it will be decomposed into eight pounds of oxygen and one pound of hydrogen.*

Example.—If one pound of hydrogen be burned in just enough air to supply 8 pounds of oxygen the hydrogen and air be supplied at 62°, and the products of combustion escape at 212°F., what is the net available heating value?

	B.t.u.	*B.t.u.*
Total heating value of 1 pound of hydrogen.......		62,000
Heat lost, latent of 9 pounds of water at 212°F.= 970.4×9..	8,733.6	
Nine pounds of water heated from 62°F. to 212°F..	1,440	
Nitrogen with 8 pounds oxygen heated from 62°F. to 212°F.=8×3.32×150×.2438 (specific heat)......	971	11,144.6
Net available heating value.....................	11,144.6	50,855.4

Example.—If the air supply be double that required to effect the combustion of the hydrogen, the other conditions being the same as in the first example, what is the net heating value?

		B.t.u.
Net available heating value (from example 1).....		50,855.4
Excess air 8×4.32 pounds		
Heat loss due to excess air 4.32*×150×.2375†=...	153.9	153.9
Net heating value (including loss by excess air)...............		50,701.5

*NOTE.—4.32 is the proportion of air to oxygen by weight;

†NOTE.—.2375 is the specific heat of air.

Calculation from Average Analysis

	Available H.	Total C.	Difference	Fixed Carbon	Difference
Pocahontas........	3.89×3=11.67	84.87	73.20	74.84	+1.64
Thacker...........	4.27×5=21.35	78.65	57.30	56.57	− .63
Pittsburgh.........	4.15×5=20.75	75.24	54.49	53.81	− .68
Darlington.........	4.01×5=20.05	75.19	55.14	54.69	− .45
Mahoning..........	3.71×5=18.55	71.13	52.58	50.95	−1.63
Upper Freeport.....	3.94×5=19.70	72.65	52.95	51.63	−1.32
Jackson............	3.22×5=16.10	70.72	54.62	52.78	−1.84
Hocking Valley.....	3.34×5=16.70	68.03	51.33	49.64	−1.69

Heating Value of Gaseous Fuels.—The accompanying table gives the calorific value of the more common combustible gases, together with the theoretical amount of air required for their combustion.

Weight and Heating Value of Various Gasses at 32° F. and Atmospheric Pressure with Theoretical Amount of Air Required for Combustion

Gas	Symbol	Cubic feet of gas per pound	*B.t.u.* per pound	*B.t.u.* per cubic foot	Cubic feet of air required per pound of gas	Cubic feet of air required per cubic foot of gas
Hydrogen..........	H	177.90	62000	349	428.25	2.41
Carbon monoxide...	CO	12.81	4450	347	30.60	2.3
Methane...........	CH^4	22.37	23550	1053	214.00	9.57
Acetylene	C_2H_2	13.79	21465	1556	164.87	11.93
Olefiant gas........	C^2H^4	12.80	21440	1675	183.60	14.33
Ethane.............	C^2H^6	11.94	22230	1862	199.88	16.74

Example—Assume a natural gas, the analysis of which in percentages by volume is oxygen = .40, carbon monoxide = .95, carbon dioxide = .34, olefiant gas (C^2H^4) = .66, ethane (C^2H^6) = 3.55, marsh gas (CH^4) = 72.15 and hydrogen = 21.95. All but the oxygen and the carbon dioxide are combustibles, and the heat per cubic foot will be,

From CO	= .0095 ×	339	=	3.22
C^2H^4	= .0066 ×	1,675	=	11.05
C^2H^6	= .0355 ×	1,859	=	65.99
$C H^4$	= .7215 ×	1,050	=	757.58
H	= .2195 ×	346	=	75.95
			B.t.u. per cubic foot	913.79

Calculated Theoretical Amount of Air Required per pound of Various Fuels

Fuel	Weight of constituents in one pound dry fuel			Air required per pound of fuel pounds
	Carbon per cent	Hydrogen per cent	Oxygen per cent	
Coke....................	94.	...	...	10.8
Anthracite coal..........	91.5	3.5	2.6	11.7
Bituminous coal.........	87.	5.	4.	11.6
Lignite..................	70.	5.	20.	8.9
Wood....................	50.	6.	43.5	6.
Oil......................	85.	13.	1.	14.3

Example—Assume a blast furnace gas, the analysis of which in percentages by weight is, oxygen = 2.7, carbon monoxide = 19.5, carbon dioxide = 18.7, nitrogen = 59.1. Here the only combustible gas is the carbon monoxide, and the heat value will be,

$$.195 \times 4.450 = 867.8 \textit{ B.t.u.} \text{ per pound.}$$

The *net* volume of air required to burn one pound of this gas will be,

$$.195 \times 30.6 = 5.967 \text{ cubic feet.}$$

Oxygen and Air Required for Combustion of 32 Degrees and 29.29 Inches. By Weight

1	2	3	4	5	6	7	8	9	10
Oxidizable Substance or combustible	Chemical symbol	Atomic or combining weight	Chemical reaction	Product of combustion	Oxygen per pound of column 1 pounds	Nitrogen per pound of column 1 = 332* x C pounds	Air per pound of column 1 = 4.32† x O pounds	Gaseous product per pound of column 1 = 1+ column 8 pounds	Heat value per pound of column 1 *B.t.u.*
Carbon..........	C	12	$C+2O=CO_2$	Carbon dioxide...	2.667	8.85	11.52	12.52	14,600
Carbon..........	C	12	$C+O=CO$	Carbon monoxide.	1.333	4.43	5.76	6.76	4,450
Carbon monoxide.	CO	28	$CO+O=CO_2$	Carbon dioxide...	.571	1.9	2.47	3.47	10,150‡
Hydrogen........	H	1	$2H+O=H_2O$	Water...........	8	26.56	34.56	35.56	62,000
Methane........	CH_4	16	$CH_4+4O=CO_2+2H_2O$	Carbon dioxide and water......	4	13.28	17.28	18.28	23,550
Sulphur.........	S	32	$S+2O=SO_2$	Sulphur dioxide...	1	3.32	4.32	5.32	4,050

Oxygen and Air Required for Combustion at 32 Degrees and 29.92 Inches. By Volume

1	2	11	12	13	14	15	16	17	18
Oxidizable substance or combustible	Chemical symbol	Volumes of column 1 entering combination volume	Volumes of oxygen combining with column 11 volume	Volumes of product formed volume	Volume per pound of column 1 in gaseous form cubic feet	Volume of oxygen per pound of column 1 cubic feet	Volume of products of combustion per pound of column 1 cubic feet	Volume of nitrogen per pound of column 1 = 3.782§ x column 15 cubic feet	Volume of gas per pound of column 1 = column 16+ column 17 cubic feet
Carbon	C	1C	2	$2CO_2$	14.95	29.89	29.89	112.98	142.87
Carbon..........	C	1C	1	2CO	14.95	14.95	29.89	56.49	86.38
Carbon monoxide.	CO	2CO	1	$2CO_2$	12.80	6.40	12.80	24.20	37.00
Hydrogen........	H	2H	1	$2H_2O$	179.32	89.66	179.32	339.09	518.41
Methane.........	CH_4	1C4H	4	$1CO_2$ $2H_2O$	22.41	44.83	67.34	169.55	236.89
Sulphur..........	S	1S	2	$1SO_2$	5.60	11.21	11.21	42.39	53.60

* Ratio by weight of O to N in air.
† 4.32 pounds of air contains one pound of O.
‡ Per pound of C in the CO.
§ Ratio by volume of O to N in air.

The *net* air required for combustion of one cubic foot of the gas will be,

$$\begin{aligned}
CO &= .0095 \times 2.39 = .02 \\
C^2H^4 &= .0066 \times 14.33 = .09 \\
C^2H^6 &= .0355 \times 16.74 = .59 \\
CH^4 &= .7215 \times 9.57 = 6.90 \\
H &= .2195 \times 2.41 = .53
\end{aligned}$$

Total net air per cubic foot 8.13

CO_2 and Fuel Losses.

(for pure carbon and 500°F. stack temperature

According to Hays

Pct. CO_2	Pct. preventable Fuel Loss	Pct. CO_2	Pct. preventable Fuel Loss	Pct. CO_2	Pct. preventable Fuel Loss
15.	.0	10.	5.69	5.	22.79
14.8	.148	9.8	6.04	4.8	24.21
14.6	.305	9.6	6.4	4.6	25.76
14.4	.47	9.4	6.78	4.4	27.44
14.2	.635	9.2	7.18	4.2	29.29
14.	.808	9.	7.58	4.	31.28
13.8	.99	8.8	8.02	3.8	33.58
13.6	1.17	8.6	8.47	3.6	36.08
13.4	1.36	8.4	8.95	3.4	38.87
13.2	1.54	8.2	9.44	3.2	42.01
13.	1.75	8.	9.66	3.	45.28
12.8	1.95	7.8	10.51	2.8	49.64
12.6	2.16	7.6	11.09	2.6	54.34
12.4	2.38	7.4	11.7	2.4	60.32
12.2	2.6	7.2	12.34	2.2	66.3
12.	2.84	7.	13.02	2.	74.
11.8	3.08	6.8	13.74	1.8	83.56
11.6	3.33	6.6	14.49	1.6	95.45
11.4	3.59	6.4	15.3	1.4	
11.2	3.86	6.2	16.16	1.2	
11	4.13	6.	17.09	1.	
10.8	4.43	5.8	18.06	.8	
10.6	4.72	5.6	19.12	.6	
10.4	5.03	5.4	20.25	.4	
10.2	5.35	5.2	21.47	.2	

CO_2 AND AIR EXCESS

(According to Hays)

Percentage CO_2	Percentage air excess	Percentage CO_2	Percentage air excess
15	38	7	158.7
14	47.8	8	195.7
13	59.2	6	245
12	72.5	5	314
11	88.1	4	417
10	107.	3	590
9	130.	2	935
		1	1970

Air

Air is a mechanical mixture composed of 78% by volume of nitrogen, 21% of oxygen, and 1% of argon. The weight of pure air at 32° Fahr. and atmospheric pressure (29.92 ins. of mercury or 14.7 lbs. per sq. in.) is .08073 lb. per cu. ft.

The volume of a pound of air at the same temperature and pressure is 12.387 cu. ft.

The weight of air at any temperature or pressure is:

$$W = \frac{1.325 \times B}{T} \qquad (1)$$

in which

W = weight in lbs. per cu. ft.

B = height of barometric pressure in ins. of mercury.

T = absolute temperature Fahr.

The absolute zero from which all temperatures must be counted when dealing with the weight and volume of gases is assumed to be −459.6° Fahr. Hence, to obtain the absolute temperature T used in the formula above, *add to the temperature observed on a regular Fahrenheit thermometer the value* 459.6.

In obtaining the value of B, 1 in. of mercury at 32° Fahr. may be taken as equal to a pressure of .491 lb. per sq. in.

Example.—What is the weight of a cubic foot of air at atmospheric pressure (29.92 ins. of mercury) at 100° Fahr.?

Substituting in formula (1),

$$W = \frac{1.325 \times 29.92}{100 + 459.2} = .0709 \text{ lb.}$$

The specific heat of air, that is, the heat units required to raise the temperature of 1 lb. of air 1° Fahr. equals, at constant pressure .2375 *B.t.u.* and, at constant volume .1689 *B.t.u.*

The pressure of the atmosphere varies with the elevation. The pressure decreases *approximately one-half pound for every* 1,000 *feet of ascent*, being measured by a barometer in inches of mercury.

To obtain the pressure in lbs. per sq. in. from the barometer reading, *multiply the barometer reading by* .49116.

Thus, a 30 inch barometer reading signifies a pressure of

$$.49116 \times 30 = 14.74 \text{ lbs. per sq. in.}$$

The following table gives the pressure of the atmosphere in pounds per square inch for various readings of the barometer.

Pressure of the atmosphere per square inch for various readings of the barometer:

Rule.—*Barometer in inches of mercury* × .49116 = *lbs. per sq. in.*

Barometer (ins. of mercury)	Pressure per sq. ins., lbs.	Barometer (ins. of mercury)	Pressure per sq. ins., lbs.
28.00	13.75	29.921	14.696
28.25	13.88	30.00	14.74
28.50	14.00	30.25	14.86
28.75	14.12	30.50	14.98
29.00	14.24	30.75	15.10
29.25	14.37	31.00	15.23
29.50	14.49		
29.75	14.61		

Weight and Volume of Air

1 cu. ft. of air at 32° F. and atmospheric pressure weighs .080728 lb.

1 ft. in height of air at 32° F. = $\begin{cases} .0005606 \text{ lb. per sq. in.} \\ .015534 \text{ in. of water at } 62° \text{ F.} \end{cases}$

For air at any other temperature T° Fahr. multiply by 492÷(460+T).

1 lb. pressure per sq. ft.=12.387 ft. of air at 32° F.

1 " " " sq. in.=1784. " " " " " "

1 in. of water at 62° F. =64.37 " " " " " "

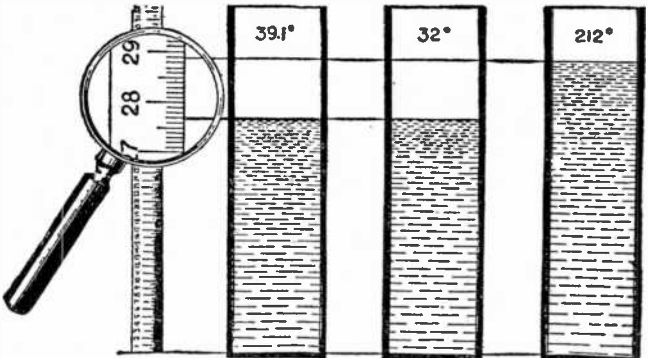
For air at any other temperature multiply by (460+T)÷492

At any fixed temperature the weight of a given volume is proportional to the absolute pressure.

Water

Water freezes at 32° and boils at 212° Fahr. when the barometer reads 29.921 ins.

The boiling point of water will lower as the altitude increases; at an altitude of 5,000 feet, water will boil at a temperature of 202° Fahr.



FIGS. 41 to 44.—The most remarkable characteristic of water: *expansion below and above its temperature or "point of maximum density"* 39.1° *Fahr.* Imagine one pound of water at 39.1° F. placed in a cylinder having a cross sectional area of 1 sq. in. as in fig. 42. The water having a volume of 27.68 cu. ins., will fill the cylinder to a height of 27.68 ins. If the liquid be cooled it will expand, and at say the *freezing point* 32° F., will rise in the tube to a height of 27.7 ins., as in fig. 43 before freezing. Again, if the liquid in fig 42 be heated, it will also expand and rise in the tube, and at say the *boiling point* (for atmospheric pressure 212° F.), will occupy the tube to a height of 28.88 cu. ins. as in fig. 44.

At its maximum density (39.1° F.) water will expand as heat is added, and it will also expand slightly as the temperature falls from this point. An increase of pressure elevates the boiling point.

The weight of a cu. ft. of water at maximum density is 62.425 lbs.

Water contains, mechanically mixed with it, about 5% of its volume of air, hence the necessity of air pumps for surface condensers.

The compressibility of water is from .000040 to .000051 for one atmosphere decreasing with increase of temperature.

1 lb. of water = 27.464 cu. ins.

= .12 U. S. gallon.

1 cu. in. of water = .03607 lb. at 62° Fahr.

Expansion of Water.—The following table gives the relative volumes of water at different temperatures, compared with its volume at 4° C. according to Kopp, as corrected by Porter.

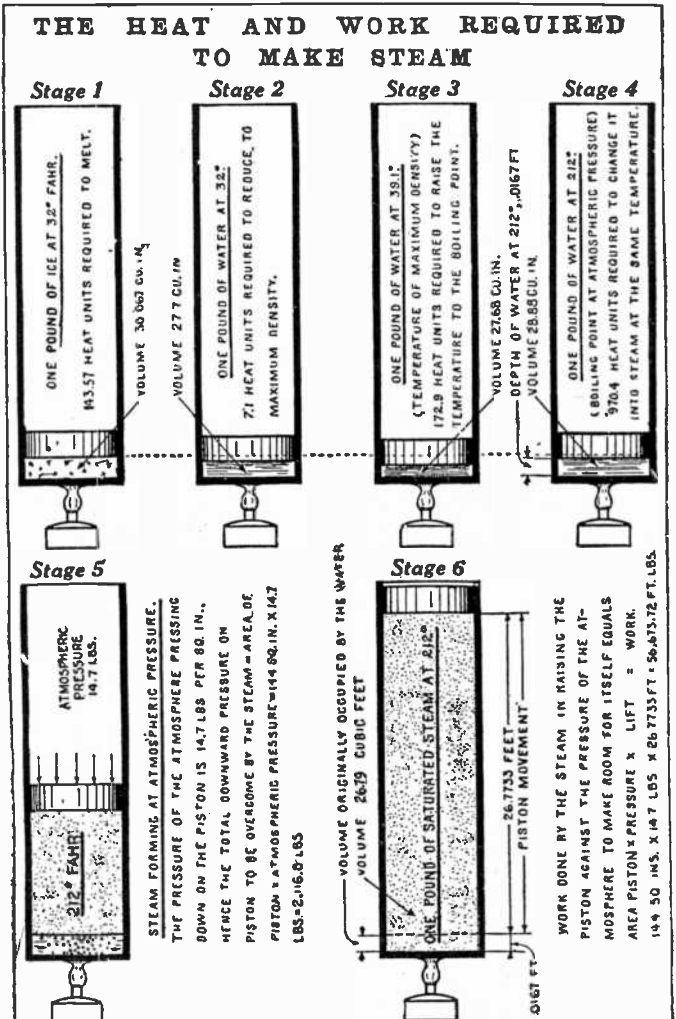
Expansion of Water

Cent.	Fahr.	Volume.	Cent.	Fahr.	Volume.	Cent.	Fahr.	Volume.
4°	39.1°	1.00000	35°	95°	1.00586	70°	158°	1.02241
5	41	1.00001	40	104	1.00767	75	167	1.02548
10	50	1.00025	45	113	1.00967	80	176	1.02872
15	59	1.00083	50	122	1.01186	85	185	1.03213
20	68	1.00171	55	131	1.01423	90	194	1.03570
25	77	1.00286	60	140	1.01678	95	203	1.03943
30	86	1.00425	65	149	1.01951	100	212	1.04332

Steam

Steam is the *invisible* vapor of water and is classified as: *wet, dry, saturated, superheated* or *gaseous*.

NOTE.—The white cloud seen issuing from an exhaust pipe, and ignorantly called steam, is not steam, but in reality, a fog of minute liquid particles produced by *condensation*.



FIGS. 45 to 50.—*From ice to steam*, illustrating the *six stages* in the making of steam from ice at 32° Fahr.

The author does not agree with the generally accepted calculation for the external latent heat, or external work of vaporization and holds that it is wrong in principle. The common method of calculating this work is based on the assumption that the amount of atmosphere displaced per pound of steam, is equal to the volume of one pound of saturated steam at the pressure under which it is formed; it is just this point wherein the error lies, as will now be shown. The volume of one pound of water at 212° atmospheric pressure, is 28.88 cu. ins. Now, if this water be placed in a long cylinder, having a cross sectional area of 144 sq. ins. it will occupy a depth of .0167 ft.

If a piston (assumed to have no weight and to move without friction) be placed on top of the water as in stage 4 (fig. 48), and heat applied, vaporization will begin, and when all the water has been changed into saturated steam, the volume has increased to 26.79 cu. ft., as in stage 6, (fig. 50); that is, the volume of one pound of saturated steam at atmospheric pressure is 26.79 cu. ft.

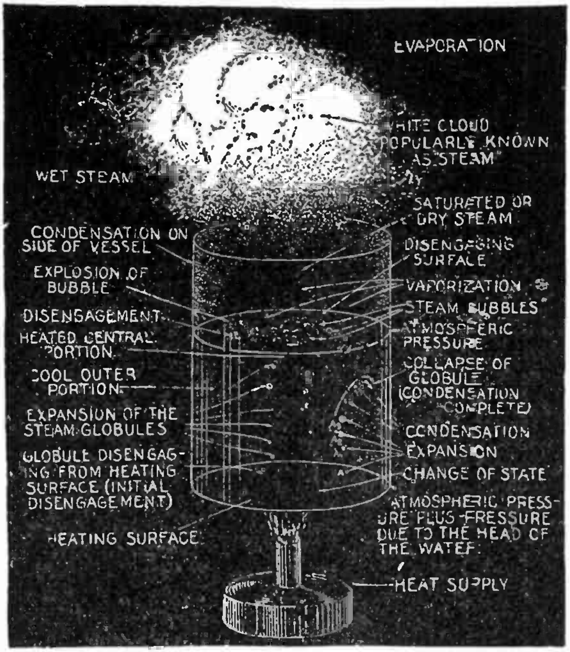


FIG. 51.—The phenomena of *vaporization* or process of boiling.

Since the area of the piston is 1 sq. ft., the linear distance from the bottom of the cylinder to the piston is 26.79 ft., but *the piston has not moved this distance.* The initial position of the piston being .0167 ft. above the bottom of the cylinder, its actual movement is

$$26.79 - .0167 = 26.7733 \text{ ft.}$$

Accordingly, the work done by the steam in moving the piston against the pressure of the atmosphere to make room for itself or

external work = *area piston* × *pressure of atmosphere* × *movement of piston*
= 144 sq. ins. × 14.7 lbs. per sq. ins. × 26.7733 ft.
= 56,673.72 ft. lbs.

The erroneous method of making this calculation is to consider the movement of the piston equal to the distance between the bottom of the cylinder and the piston, or 26.79 ft., which would give for the external work

$$144 \times 14.7 \times 26.79 = 56{,}709.07 \text{ ft. lbs.}$$

being in excess of the true amount by

$$56{,}709.07 - 56{,}673.72 = 35.35 \text{ ft. lbs.}$$

or

$$.0167 \text{ ft.} \times 144 \text{ sq. ins.} \times 14.7 = 35.35 \text{ ft. lbs.}$$

Motion is purely a relative matter, and accordingly something must be regarded as being stationary as a basis for defining motion; hence the question:

Is the movement of the piston in stage 6 (fig. 50) *to be referred to a **stationary** water level or to a **receding** water level?*

The author holds that the movement of the piston referred to a stationary water level gives the true displacement of the air and is accordingly the proper basis for calculating the external work. It must be evident that since the water already existed at the beginning of vaporization, the atmosphere was already displaced to the extent of the volume occupied by the water, and therefore this displacement must not be considered as contributing to the external work done by the steam during its formation. The amount of error (35.35 ft. lbs.) of the common calculation, though very small, is an appreciable amount; its equivalent in heat units is

$$35.35 \div 777.52 = .0455 \text{ B. t. u.}$$

The thermal equivalent of the external work is

$$56{,}673.72 \div 777.52 = 72.89 \text{ B. t. u.}$$

Steam Table

giving

Properties of Saturated Steam

VACUUM INCHES OF MERCURY	ABSOLUTE PRESSURE LBS. PER SQ. IN.	TEMPERATURE FAHRENHEIT	TOTAL HEAT ABOVE 32° F.		LATENT HEAT, L = H−h HEAT UNITS	VOLUME CU. FT. IN 1 LB OF STEAM	WEIGHT OF 1 CU. FT. STEAM LB.	ENTROPY OF THE WATER	ENTROPY OF EVAPORATION
			IN THE WATER h HEAT UNITS	IN THE STEAM H HEAT UNITS					
28.50	.696	90	58.	1099.2	1041.2	469.3	.002131	.1114	1.8944
GAUGE	14.70	212	180.	1150.4	970.4	26.79	.03732	.3118	1.4447
3.3	18	222.4	190.5	1154.2	963.7	22.16	.04512	.3273	1.4127
7.3	22	233.1	201.3	1158	956.7	18.37	.05445	.3430	1.3811
75.3	90	320.3	290.5	1184.4	893.9	4.89	.2044	.4644	1.1461
80.3	95	324.1	294.5	1185.4	890.9	4.65	.2151	.4694	1.1367
95.3	110	334.8	305.5	1188	882.5	4.047	.2472	.4834	1.1108
101.3	116	338.7	309.6	1189	879.3	3.848	.2599	.4886	1.1014
135.3	150	358.5	330.2	1193.4	863.2	3.012	.3320	.5142	1.0550
200.3	215	388	361.4	1199.2	837.9	2.138	468	.5513	.9885

The Steam Table.—By definition *the properties of steam given in tabular form*. A few values are given above to illustrate how to use the steam table.

For table giving values from 29.74 in. vacuum to over 5,000 lbs. absolute pressure, see ***Audel's Engineers and Mechanics Guide*** Vol. I.

The following examples illustrate how to use the steam table:

Example.—How many heat units are saved in heating 25 lbs. of feed water from 90° to 212°?

In column 4, total heat in the water at 212° = 180
In column 4, total heat in the water at 90° = 58
Heat units saved per lb. of feed water = 122
Total heat units saved = 122×25 = 3,050 B. t. u.

Example.—What is the weight of 20 cu. ft. of steam at 150 lbs. absolute pressure?

The weight of 1 cu. ft. steam at 150 lbs. abs. is given in column 9 at .332 lb. Twenty cu. ft. then will weigh: .332×20 = 6.64 lbs.

Example.—How much more heat is required to generate 26 lbs. of steam at 150 lbs. abs., than at 90 lbs. abs.

In column 5 total heat in steam at 150 lbs. abs. = 1,193.4
In column 5 total heat in steam at 90 lbs. abs. = 1,184.4
Excess heat required per pound (weight) = 9 B. t. u.
Total for 26 lbs. = 9×26 = 23.4 B. t. u.

Example.—How much heat is absorbed by the cooling water, if a condensing engine exhaust 17 lbs. of steam per hour at a terminal pressure of 18 lbs. absolute into a 28½ inch vacuum.

In column 5, total heat in the steam at 18 lbs. abs. = 1,154.20
In column 4, total heat in the water with 28½″ vacuum = 58.00
Heat to be absorbed per lb. of steam .... = 1,096.2
Total heat absorbed by the cooling water per hour

1,096.2×17 = 18,635.4 B. t. u.

Factor of Evaporation.—Defined as *the quantity which when multiplied by the amount of steam generated at a given pressure from water at a given temperature, gives the equivalent evaporation from and at 212° Fahr.*

Factors of Evaporation

PRESSURE IN POUNDS PER SQUARE INCH ABOVE THE ATMOSPHERE.

Temperature Feed Water. Fahrenheit. Degr's	0	5.	15.	25.	35.	45.	55.	65.	75.	85.	95.	105.	115.	125.	135.	145.	155.	165.	175.	185.	200.	Temperature Feed Water. Fahrenheit. Degr's
32	1.187	1.192	1.199	1.204	1.209	1.212	1.216	1.218	1.221	1.223	1.226	1.228	1.230	1.231	1.233	1.235	1.236	1.238	1.239	1.240	1.241	32
35	1.184	1.189	1.196	1.201	1.206	1.209	1.213	1.215	1.218	1.220	1.223	1.225	1.227	1.228	1.230	1.232	1.233	1.235	1.236	1.237	1.238	35
40	1.179	1.181	1.191	1.196	1.201	1.204	1.208	1.219	1.213	1.215	1.218	1.220	1.222	1.223	1.225	1.227	1.228	1.230	1.231	1.232	1.233	40
45	1.173	1.178	1.185	1.190	1.195	1.198	1.202	1.204	1.207	1.209	1.242	1.214	1.216	1.217	1.219	1.221	1.222	1.224	1.225	1.226	1.227	45
50	1.168	1.173	1.180	1.185	1.190	1.193	1.197	1.199	1.202	1.204	1.207	1.209	1.211	1.212	1.214	1.216	1.217	1.219	1.220	1.221	1.222	50
55	1.163	1.168	1.175	1.180	1.185	1.188	1.192	1.194	1.197	1.199	1.202	1.204	1.206	1.207	1.209	1.211	1.212	1.214	1.215	1.216	1.217	55
60	1.158	1.163	1.170	1.175	1.180	1.183	1.187	1.189	1.192	1.194	1.197	1.199	1.201	1.202	1.204	1.206	1.207	1.209	1.210	1.211	1.212	60
65	1.153	1.158	1.165	1.170	1.175	1.178	1.182	1.184	1.187	1.189	1.192	1.194	1.196	1.197	1.199	1.201	1.202	1.204	1.205	1.206	1.207	65
70	1.148	1.153	1.160	1.165	1.170	1.173	1.177	1.179	1.182	1.184	1.187	1.189	1.191	1.192	1.194	1.196	1.197	1.199	1.200	1.201	1.202	70
75	1.143	1.148	1.155	1.160	1.165	1.168	1.172	1.174	1.177	1.179	1.182	1.184	1.186	1.187	1.189	1.191	1.192	1.194	1.195	1.196	1.197	75
80	1.137	1.142	1.149	1.154	1.159	1.162	1.166	1.168	1.171	1.173	1.176	1.178	1.180	1.181	1.183	1.185	1.186	1.188	1.189	1.190	1.191	80
85	1.132	1.137	1.144	1.149	1.154	1.157	1.161	1.163	1.166	1.168	1.171	1.173	1.175	1.176	1.178	1.180	1.181	1.183	1.184	1.185	1.186	85
90	1.127	1.132	1.139	1.144	1.149	1.152	1.156	1.158	1.161	1.163	1.166	1.168	1.170	1.171	1.173	1.175	1.176	1.178	1.179	1.180	1.181	90
95	1.122	1.127	1..34	1.139	1.144	1.147	1.151	1.153	1.156	1.158	1.161	1.163	1.165	1.166	1.168	1.170	1.171	1.173	1.174	1.175	1.176	95
100	1.117	1.122	1.129	1.134	1.139	1.142	1.146	1.148	1.151	1.153	1.156	1.158	1.160	1.161	1.163	1.165	1.166	1.168	1.169	1.170	1.171	100
105	1.111	1.116	1.123	1.128	1.133	1.136	1.140	1.142	1.145	1.147	1.150	1.152	1.154	1.155	1.157	1.159	1.160	1.162	1.163	1.164	1.165	105
110	1.106	1.111	1.118	1.123	1.128	1.131	1.135	1.137	1.140	1.142	1.145	1.147	1.149	1.150	1.152	1.154	1.155	1.157	1.158	1.159	1.160	110
115	1.101	1.106	1.113	1.118	1.123	1.126	1.130	1.132	1.135	1.137	1.140	1.142	1.144	1.145	1.147	1.149	1.150	1.152	1.153	1.154	1.155	115
120	1.096	1.101	1.108	1.113	1.118	1.121	1.125	1.127	1.130	1.132	1.135	1.137	1.139	1.140	1.142	1.144	1.145	1.147	1.148	1.149	1.150	120
125	1.091	1.096	1.103	1.108	1.113	1.116	1.120	1.122	1.125	1.127	1.130	1.132	1.134	1.135	1.137	1.139	1.140	1.142	1.143	1.144	1.145	125
130	1.085	1.090	1.097	1.102	1.107	1.110	1.114	1.116	1.119	1.121	1.124	1.126	1.128	1.129	1.131	1.133	1.134	1.136	1.137	1.138	1.139	130
135	1.080	1.085	1.092	1.097	1.102	1.105	1.109	1.111	1.114	1.116	1.119	1.121	1.123	1.124	1.126	1.128	1.129	1.131	1.132	1.133	1.134	135
140	1.075	1.080	1.087	1.092	1.097	1.100	1.104	1.106	1.109	1.111	1.114	1.116	1.118	1.119	1.121	1.123	1.124	1.126	1.127	1.128	1.129	140
145	1.070	1.075	1.082	1.087	1.092	1.095	1.099	1.101	1.104	1.106	1.109	1.111	1.113	1.114	1.116	1.118	1.119	1.121	1.122	1.123	1.124	145
150	1.065	1.070	1.077	1.082	1.087	1.090	1.094	1.096	1.099	1.101	1.104	1.106	1.108	1.109	1.111	1.113	1.114	1.116	1.117	1.118	1.119	150
155	1.059	1.064	1.071	1.076	1.081	1.084	1.088	1.090	1.094	1.095	1.098	1.100	1.102	1.103	1.105	1.107	1.108	1.110	1.111	1.112	1.113	155
160	1.054	1.059	1.066	1.071	1.076	1.079	1.083	1.085	1.088	1.090	1.093	1.095	1.097	1.098	1.100	1.102	1.103	1.105	1.106	1.107	1.108	160
165	1.049	1.054	1.061	1.066	1.071	1.074	1.078	1.080	1.083	1.085	1.088	1.090	1.092	1.093	1.095	1.097	1.096	1.100	1.101	1.102	1.103	165
170	1.044	1.049	1.056	1.061	1.066	1.069	1.073	1.075	1.078	1.080	1.083	1.085	1.087	1.088	1.090	1.092	1.093	1.095	1.096	1.097	1.098	170
175	1.039	1.044	1.051	1.056	1.061	1.064	1.068	1.070	1.073	1.075	1.070	1.080	1.082	1.083	1.085	1.087	1.088	1.090	1.091	1.092	1.093	175
180	1.033	1.038	1.045	1.050	1.055	1.058	1.062	1.064	1.067	1.069	1.072	1.074	1.076	1.077	1.079	1.081	1.082	1.084	1.085	1.086	1.087	180
185	1.028	1.033	1.040	1.045	1.050	1.053	1.057	1.059	1.062	1.064	1.067	1.069	1.071	1.073	1.074	1.076	1.077	1.079	1.080	1.081	1.082	185
190	1.023	1.028	1.035	1.040	1.045	1.048	1.052	1.054	1.057	1.059	1.062	1.064	1.066	1.067	1.069	1.071	1.072	1.074	1.075	1.076	1.077	190
195	1.018	1.023	1.030	1.035	1.040	1.043	1.047	1.049	1.052	1.054	1.057	1.059	1.061	1.062	1.064	1.066	1.066	1.069	1.070	1.071	1.072	195
200	1.013	1.018	1.025	1.030	1.035	1.038	1.042	1.044	1.047	1.049	1.052	1.054	1.056	1.057	1.059	1.061	1.062	1.064	1.065	1.066	1.067	200
205	1.001	1.012	1.019	1.024	1.029	1.032	1.036	1.038	1.041	1.043	1.048	1.048	1.050	1.051	1.053	1.055	1.056	1.058	1.059	1.060	1.062	205
210	1.002	1.007	1.014	1.019	1.024	1.027	1.031	1.033	1.036	1.038	1.041	1.043	1.045	1.046	1.048	1.050	1.051	1.053	1.054	1.055	1.057	210
212	1.000	1.005	1.012	1.017	1.022	1.025	1.029	1.031	1.034	1.036	1.039	1.041	1.043	1.044	1.046	1.048	1.049	1.051	1.052	1.053	1.056	212

Expressed as a formula:

$$F=\frac{H-h}{H'-h'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in which F = Factor of Evaporation.
H = Heat above 32° Fahr. in the steam at given pressure.
h = Heat above 32° Fahr. in water at given pressure.
H' = Heat above 32° Fahr. in steam at atmospheric pressure.
h' = Heat above 32° Fahr. in water at atmospheric pressure.

Formula (1) just given is expressed in the simplest form as

$$F=\frac{H-h}{970.4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Here $970.4 = H' - h' = 1150.4 - 180$ (see steam table)

Example:—What is the factor of evaporation for steam at 200 pounds pressure when the feed water is delivered to the boiler at a temperature of 150° Fahr.? From the steam table, the heat H, in the steam at 200 pounds pressure = 1,199.2 *B.t.u.* The heat h, in the feed water *above 32°* at 150° Fahr. is 150−32=118 *B.t.u.* Substituting these values in formula 2

$$F=\frac{1{,}199.2-118}{970.4}=1.1142$$

The meaning of it is that if a boiler were generating, say 1,000 pounds of steam per hour at 200 pounds pressure, from feed water at 150° Fahr. it would absorb the same amount of heat from the fire as when generating

$$1{,}000 \times 1.1142 = 1114.2 \text{ lbs.}$$

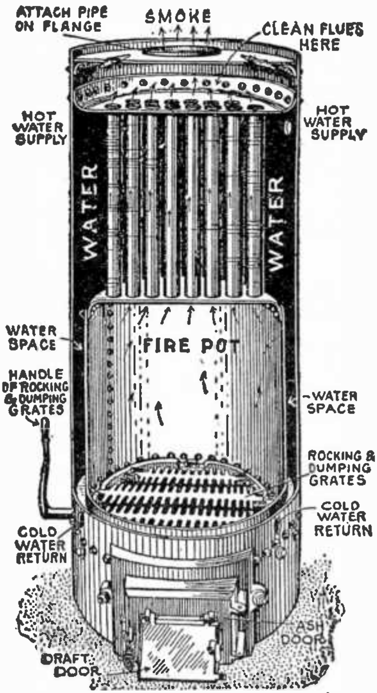
of steam "*from and at 212°*", that is generating steam at atmospheric pressure from feed water at 212°.

Example.—A boiler evaporates 1,000 pounds of steam at 95 pounds gauge pressure and the feed water is heated to 110°. How much steam will it evaporate *from and at 212°*?

Referring to the table on page 64 , the factor of evaporation given for steam at 95 lbs. pressure with feed water at 110°, is 1.145.

Steam and Hot Water Heating

Loss of Heat from Buildings.—To determine the size of a heating plant it is necessary to first estimate the loss of heat from the building. This is figured on a basis of *B.t.u.* lost per hour.



Example.—What will be the loss of heat in *B.t.u.* per hour in a single room of a brick residence if the temperature inside be maintained at 72 degrees F., when the temperature outside is at 30 degrees, the construction being as follows: 16-inch brick walls; four 3×6 windows, northern and western exposure; fire proof flooring and ceiling; size of room 15×20×14 (high). Area of windows 4×(3×5) = 60.

FIG. 52.—Andrews vertical tubular boiler. An example of a heating boiler which has adequate heating surface. Most heating boilers have a ridiculously small amount of heating surface and are accordingly very wasteful of fuel. See page 73.

Loss of Heat per Square Foot of Surface

Material	*B.t.u.* per hour	Material	*B.t.u.* per hour
Masonry		**Partitions, Floors and Ceilings**	
4 inch brick wall	.68	Stud partition, lath and plaster one side	.26
8 " " "	.46	Stud partition, lath and plaster both sides	.15
12 " " "	.32	Ordinary lath and plaster ceiling separating unheated space from heated rooms	.26
16 " " "	.26	Floor, single, thickness ¾ inch, warm air above and cold space below:	
20 " " "	.23	A. No plaster beneath joists	.20
For reinforced concrete add 20 per cent to brick values		B. Lath and plaster beneath joists	.12
12 inch stone wall, block masonry	.45	Floor, double, thickness 1½ inches, warm room above and cold space below:	
16 inch stone wall, block masonry	.4	A. No plaster beneath joists	.13
20 inch stone wall, block masonry	.36	B. Lath and plaster beneath joists	.08
24 inch stone wall, block masonry	.3	**Miscellaneous**	
28 inch stone wall, block masonry	.27	Wood as flooring	.83
36 inch stone wall, block masonry	.25	" " ceiling	.104
44 inch stone wall, block masonry	.2	" " wall	.22
Planks		Fire proof flooring	.124
1½ inch pine planks	.3	" " ceiling	.145
2 " " "	.26	Cement as flooring	.31
2½ " " "	.23	Dirt " "	.23
3 " " "	.2	Wood under slate or composition roof	.3
Windows, Skylights, and Outside Walls		Wood, under iron	.17
Single window	1.10	Tile (no boards underneath)	1.25
" " double glass	.62	Cement roof	.6
Double "	.50		
Single skylight	1.16		
¾ inch sheathing and clapboards	.30		
¾ inch sheathing, paper and clapboards	.23		

Area of walls $2(15+20)\times 14-60=920$ square feet
" " floor $15\times 20=$ 300 " "
" " ceiling $15\times 20=$ 300 " "

The values given for heat losses in the table should be increased as follows:

For northeastern, northwestern, western or northern exposure....................20 to 30%.
For rooms 13 to 14½ feet high...........6½%.
For rooms 14½ to 18 feet high............10%.
When building is heated during the day only 30%.
When building remains for long periods without heat.................................50%.

B.t.u. lost through	windows	$=60\times 1.1\times(72°-32°)$	$=2,640$
" " "	walls	$=920\times .27\times(72°-32°)$	$=9,936$
" " "	floor	$=300\times .124\times(72°-32°)$	$=1,488$
" " "	ceiling	$=300\times .145\times(72°-32°)$	$=1,740$

Total loss of heat per hour, normal conditions 15,804 *B.t.u.*

This loss is increased: 30 % by northern and western exposure; 6½ % for high ceiling, that is, total loss of heat per hour under the special condition is:

$$15{,}804\times 1.365=21{,}573\ B.t.u.$$

Estimating Radiation.—The amount of heat given off by ordinary radiators is commonly taken as

1. For steam

250 *B.t.u.*

2. For hot water

150 *B.t.u.*

The following table gives values for the various systems when the air is at 70° Fahr.

Heat Given Off by Radiators

System	Working pressure	Temperature degrees F.	*B.t.u.* per square foot per hour
Steam (low pressure).......	5 lbs. gauge	228	250
" (atmospheric).......	0 lbs. "	212	227
" (so called vapor)....	5 in. vacuum	203	213
" (vacuum)..........	10 " "	192	195
" "	15 " "	179	174
" "	20 " "	161	146
Water..................	....	160	150

The values **250** *and* **150** *for low pressure steam and hot water respectively although only approximate, are standard values for ordinary calculations.*

Radiation Tables
(For steam and hot water)

Dwellings	Cu. ft. of space using steam	Cu. ft. of space using hot water
Living rooms, one side exposed.........	50 to 55	25 to 30
" " two sides "	45 " 50	20 " 25
" " three " "	40 " 45	15 " 20
Sleeping "	50 " 70	30 " 35
Halls and bath rooms.	40 " 50	20 " 30
Public Buildings		
Offices...............................	50 " 25	30 " 40
Schools..............................	40 " 60	20 " 30
Factories and stores..................	70 " 100	40 " 60
Assembly halls and churches............	100 " 150	60 " 80

Example.—How many sq. ft. of radiator surface are required to heat a room for low pressure steam and hot water radiators when the heat loss is 22,293 *B.t.u.* per hour?

Total heat loss per hour is 22,293 *B.t.u.*

For steam sq. ft. radiator surface = 22,293 ÷ 250 = 89
" hot water " " " " = 22,293 ÷ 150 = 149

Size of Mains.—The steam main can be determined by taking the total amount of direct radiation to which add 25 per cent for piping, and from this total extract the square root, dividing same by 10, which gives the size of main to use. This is for one pipe work. For two pipe work, one size less is sufficient, and the return can be one or two sizes less than the supply. A steam main should not decrease in size according to the area of its branches, but very much slower.

Example.—Having 500 feet of direct radiation add to it 25 per cent or 125, which equals 625. The square root of this is 25, which divided by 10 gives 2½, or the size of the pipe. For handy reference and practical use the following table can be used, though not not exactly in accord with the foregoing.

Size of Steam Mains

Radiation	One-pipe work	Two-pipe work
125 square feet	1½ inch	1¼×1 inch
250 " "	2 "	1½×1¼ "
400 " "	2½ "	2 ×1½ "
650 " "	3 "	2½×2 "
900 " "	3½ "	3 ×2½ "
1,250 " "	4 "	3½×3 "
1,600 " "	4½ "	4 ×3½ "
2,050 " "	5 "	4½×4 "
2,500 " "	6 "	5 ×4½ "
3,600 " "	7 "	6 ×5 "
5,000 " "	8 "	7 ×6 "
6,500 " "	9 "	8 ×6 "
8,100 " "	10 "	9 ×6 "

Heating Surface Factors for Green House Heating

Temperature of air in house	Temperature of water in heating pipes				Steam
	140°	160°	180°	200°	Three lbs. pressure
	Square feet of glass and its equivalent proportion to one square foot of surface in heating pipes or radiator.				
40°	4.33	5.25	6.66	7.69	8.
45°	3.63	4.65	5.55	6.66	7.5
50°	3.07	3.92	4.76	5.71	7.
55°	2.63	3.39	4.16	5.	6.5
60°	2.19	2.89	3.63	4.33	6.
65°	1.86	2.53	3.22	3.84	5.5
70°	1.58	2.19	2.81	3.44	5.
75°	1.37	1.92	2.5	3.07	4.5
80°	1.16	1.63	2.17	2.73	4.
85°	.99	1.42	1.92	2.46	3.5

* Sizes of Hot Water Mains

Radiation	Pipe
75 to 125 square feet	1¼ inch
125 " 175 " "	1½ "
175 " 300 " "	2 "
300 " 475 " "	2½ "
475 " 700 " "	3 "
700 " 950 " "	3½ "
950 " 1,200 " "	4 "
1,200 " 1,575 " "	4½ "
1,575 " 1,975 " "	5 "
1,975 " 2,375 " "	5½ "
2,375 " 2,850 " "	6 "

In hot water, flow mains may be reduced in size in proportion to the branches taken off. They should, however, have as large area as the sum of all branches beyond that point. It is advisable that the horizontal branches be one size larger than the risers. Returns should be same as flows.

* NOTE—As recommended by the William Page Boiler Co., New York.

Table of Mains and Branches

Main			Branch
1 in. will supply	...2	....	¾ in.
1¼ in. " "	...2	....	1 in.
1½ in. " "	...2	....	1¼ in.
2 in. " "	...2	....	1½ in.
2½ in. " "	...2, 1½ in. and 1, 1¼ in., or 1, 2 in. and 1, 1¼ in.		
3 in. " "	...1, 2½ in. and 1, 2 in., or 2, 2 in. and 1, 1½ in.		
3½ in. " "	...2, 2½ in. or 1, 3 in., and 1, 2 in. or 3, 2 in.		
4 in. " "	...1, 3½ in. and 1, 2½ in., or 2, 3 in. and 4, 2 in.		
4½ in. " "	...1, 3½ in. and 1, 3 in., or 1, 4 in. and 1, 2½ in.		
5 in. " "	...1, 4 in. and 1, 3 in., or 1, 4½ in. and 1, 2½ in.		
6 in. " "	...2, 4 in. and 1, 3 in., or 4, 3 in. or 10, 2 in.		
7 in. " "	...1, 6 in. and 1, 4 in., or 3, 4 in. and 1, 2 in.		
8 in. " "	...2, 6 in. and 1, 5 in., or 5, 4 in. and 2, 2 in.		

Properties of Radiators

Peerless Two-Column Radiators
For Steam and Water

No. of Sections	Length 2½ in. per Sec.	Heating Surface—Square Feet: 45-in. Height 5 Sq.Ft. per Sec.	38-in. Height 4 Sq.Ft. per Sec.	32-in. Height 3⅓ Sq.Ft. per Sec.	26-in. Height 2⅔ Sq.Ft. per Sec.	23-in. Height 2⅓ Sq.Ft. per Sec.	20-in. Height 2 Sq.Ft per Sec
2	5	10	8	6⅔	5⅓	4⅔	4
3	7½	15	12	10	8	7	6
4	10	20	16	13⅓	10⅔	9⅓	8
5	12½	25	20	16⅔	13⅓	11⅔	10
6	15	30	24	20	16	14	12
7	17½	35	28	23⅓	18⅔	16⅓	14
8	20	40	32	26⅔	21⅓	18⅔	16
9	22½	45	36	30	24	21	18
10	25	50	40	33⅓	26⅔	23⅓	20
11	27½	55	44	36⅔	29⅓	25⅔	22
12	30	60	48	40	32	28	24
13	32½	65	52	43⅓	34⅔	30⅓	26
14	35	70	56	46⅔	37⅓	32⅔	28
15	37½	75	60	50	40	35	30
16	40	80	64	53⅓	42⅔	37⅓	32
17	42½	85	68	56⅔	45⅓	39⅔	34
18	45	90	72	60	48	42	36
19	47½	95	76	63⅓	50⅔	44⅓	38
20	50	100	80	66⅔	53⅓	46⅔	40
21	52½	105	84	70	56	49	42
22	55	110	88	73⅓	58⅔	51⅓	44
23	57½	115	92	76⅔	61⅓	53⅔	46
24	60	120	96	80	64	56	48
25	62½	125	100	83⅓	66⅔	58⅓	50
26	65	130	104	86⅔	69⅓	60⅔	52
27	67½	135	108	90	72	63	54
28	70	140	112	93⅓	74⅔	65⅓	56
29	72½	145	116	96⅔	77⅓	67⅔	58
30	75	150	120	100	80	70	60
31	77½	155	124	103⅓	82⅔	72⅓	62
32	80	160	128	106⅔	85⅓	74⅔	64

Peerless Three-Column Radiators
For Steam and Water

No. of Sections	Length 2½ in. per Sec.	Heating Surface—Square Feet: 45-in. Height 6 sq. ft. per Sec.	38-in. Height 5 sq. ft. per Sec.	32-in. Height 4½ sq. ft. per Sec.	26-in. Height 3¾ sq. ft. per Sec.	22-in. Height 3 sq. ft. per Sec.	18-in. Height 2¼ sq. ft. per Sec.
2	5	12	10	9	7½	6	4½
3	7½	18	15	13½	11¼	9	6¾
4	10	24	20	18	15	12	9
5	12½	30	25	22½	18¾	15	11¼
6	15	36	30	27	22½	18	13½
7	17½	42	35	31½	26¼	21	15¾
8	20	48	40	36	30	24	18
9	22½	54	45	40½	33¾	27	20¼
10	25	60	50	45	37½	30	22½
11	27½	66	55	49½	41¼	33	24¾
12	30	72	60	54	45	36	27
13	32½	78	65	58½	48¾	39	29¼
14	35	84	70	63	52½	42	31½
15	37½	90	75	67½	56¼	45	33¾
16	40	96	80	72	60	48	36
17	42½	102	85	76½	63¾	51	38¼
18	45	108	90	81	67½	54	40½
19	47½	114	95	85½	71¼	57	42¾
20	50	120	100	90	75	60	45
21	52½	126	105	94½	78¾	63	47¼
22	55	132	110	99	82½	66	49½
23	57½	138	115	103½	86¼	69	51¾
24	60	144	120	108	90	72	54
25	62½	150	125	112½	93¾	75	56¼
26	65	156	130	117	97½	78	58½
27	67½	162	135	121½	101¼	81	60¾
28	70	168	140	126	105	84	63
29	72½	174	145	130½	108¾	87	65¼
30	75	180	150	135	112½	90	67½
31	77½	186	155	139½	116¼	93	69¾
32	80	192	160	144	120	96	72

The respiration of one adult person will vitiate hourly about 500 cu. ft. of air, to which should be added vitiation from other sources, such as moisture from the body, methods of illumination, etc., making a requirement of about 1,000 cubic feet per hour of fresh air for each adult person in average living rooms and places of assembly.

The fresh air from the outside passes through registers at a velocity from 200 to 300 feet per minute. The clear openings of a register will be approximately two-thirds of its full area; thus a 12 by 15 register would have an available area of 120 inches. The fresh, warm air, passing at this rate per minute, would supply from 10,000 to 15,000 cubic feet an hour, and meet the requirements of a family of from 10 to 15 persons

The requirement of Massachusetts Laws in the ventilation of school rooms is 30 cubic feet of fresh air per minute for each pupil. Thus, the average room providing for 50 pupils would require 1,500 cubic feet per minute, or 90,000 cubic feet per hour Contemplating a movement of the air at the rate of 5 feet per second, and supply and exhaust registers—2 by 2½ feet each—or an area of 5 square feet will insure the desired result.

For churches and general assembly halls, the requirement is 15 cubic feet per minute for each person.

Steam Heating Boilers

25:1

The Heating Surface.—The author after a laborious examination of about one hundred boiler catalogues found that while nearly all gave the grate area, very few gave the area of heating surface (for obvious reasons).

While, for example, he found that in one size of the Vance boiler 38 square feet of heating surface per square feet of grate is provided, in another make boiler (the name ought to be printed in large letters) only 8.3 square feet of heating surface is provided per square foot of grate.

Where there is any regard for economy an ***adequate amount*** *of heating surface will be provided.*

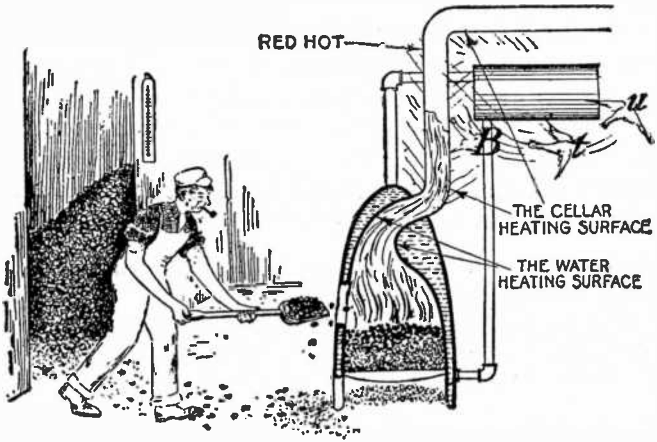


FIG. 53.—The principal reason why the tenants get no hot water. It's not the fault of the manufacturer, he simply builds what the public is willing to pay for and does not worry about the coal bills.

According to the American Society of Heating and Ventilation Engineers: "The grate surface to be provided depends on the rate of combustion and this, in turn, on the attendance and draught, and on the size of the boiler. Small boilers are usually adapted for intermittent attention and a slow rate of combustion. The larger the boiler the more attention is given to it and the more heating surface is provided per square foot of grate."

"The following rates of combustion are common for internally fired heating boilers":

Sq. ft. of grate....................	4 to 8	10 to 18	20 to 30
Lbs. coal per sq. ft. of grate per hour	4	6	10

The following table from Kent gives some proportions and results that should be obtained:

Proportions and Performance of Heating Boilers

	Low boiler	Medium boiler	High boiler
1 square foot of grate should burn.....	3	4	5 pounds coal per hour
" " " " " " develop.	30,000	40,000	50,000 *B.t.u.* per hour
" " " " " *will require....*	*15*	*20*	*25 square feet heating surface*
" " " " " " supply.....	120	160	200 square feet radiating surface

Steam Heating Boiler Tests

Number of boiler	Fuel anthracite pounds per square foot of grate	Area of grate square feet	Number of sections including dome	Steam produced per pound of coal	8 hour rating square feet
0	4.39	1.23	1	7.5	200
1	5.12	1.23	2	8.	250
1½	5.28	1.23	3	8.5	275
2	5.44	1.23	4	9.	300

The above tests (as given in one manufacturer's catalogue), of several cast iron boilers all having the same size grate, but with different number of sections, show the importance of an adequate amount of heating surface. It is simply a matter of whether the purchaser prefers ***a cheap boiler and big coal bills, or the expensive boiler and small coal bill***—that is for him to decide.

Plumbing

The plumber in his work comes in contact with many articles such as pipes, fittings, fixtures, etc., made of various materials such as lead, iron, brass, etc.

Varieties of Modern Babbitts

Reference Numbers	75 per cent Aluminized Nickel	Copper	Antimony	30 per cent Aluminized Zinc	Tin	Lead	Cadmium	Metallic Phosphoro
1	....	....	12.	...	....	85.	....	3.
2	....	....	15.	....	....	82.	....	3.
3	....	....	18.	...	....	78.	....	4.
4	....	....	15.	....	5.75	76.	....	3.25
5	....	....	12.50	....	15.	70.	....	2.50
6	....	....	10.	....	25.	61.	....	4.
7	....	....	8.	....	45.	41.	....	6.
8	....	4.	6.	....	86.	....	....	4.
9	....	6.	6.	....	82.	....	....	6.
10	1.50	3.50	....	7.	87.	....	....	1.
11	3.	2.	....	5.	88.	....	1.	1.
12	4.	....	...	3.	90.	....	2.	1.
13	....	7.	5.	....	83.	....	....	5.
14	....	2.	10.	....	20.	66.	....	2.
15	....	3.	9.	....	30.	55.	....	3.
16	....	4.	8.	....	40.	44.	....	4.
17	....	5.	7.	....	50.	33.	....	5.
18	....	6.	6.	....	60.	22.	....	6.

Properties of Lead and Tin

Ingredients	Melting Point	Specific Gravity	Weight	
			Per cu. in.	Per cu. ft
Lead	620° Fahr.	11.07 to 11.44	.4106	709.7
Tin	449° Fahr.	7.297 to 7.409	.2652	458.3

Sheet Metal and Wire Gauges

No.	United States Steel and Sheet Iron	British Imperial Standard	London	Washburn & Moen or United States Steel Wire	Birmingham or Stubbs	Brown & Sharps or American Wire Gauge
0000000	.500	.500	....	....	....	......
000000	.46875	.464	....	....	....	......
00000	.4375	.432	....	....	....	......
0000	.40625	.400	.454	.3938	.454	.460
000	.375	.372	.425	.3625	.425	.40964
00	.34375	.348	.380	.3310	.380	.36480
0	.3125	.324	.340	.3065	.340	.32495
1	.28125	.300	.300	.2830	.300	.28930
2	.265625	.276	.284	.2625	.284	.25763
3	.25	.252	.259	.2437	.259	.22942
4	.234375	.232	.238	.2253	.238	.20431
5	.21875	.212	.220	.2070	.220	.18194
6	.203125	.192	.203	.1920	.203	.16202
7	.1875	.176	.180	.1770	.180	.14428
8	.171875	.160	.165	.1620	.165	.12849
9	.15625	.144	.148	.1483	.148	.11443
10	.140625	.128	.134	.1350	.134	.10189
11	.125	.116	.120	.1205	.120	.09074
12	.109375	.104	.109	.1055	.109	.08081
13	.09375	.092	.095	.0915	.095	.07196
14	.078125	.080	.083	.0800	.083	.06408
15	.0703125	.072	.072	.0720	.072	.05707
16	.0625	.064	.065	.0625	.065	.05082
17	.05625	.056	.058	.0540	.058	.04525
18	.05	.048	.049	.0475	.049	.04030
19	.04375	.040	.040	.0410	.042	.03589
20	.0375	.036	.035	.0348	.035	.03196
21	.034375	.032	.0315	.0317	.032	.02846
22	.03125	.028	.0295	.0286	.028	.025347
23	.0281	.024	.027	.0258	.025	.022571
24	.025	.022	.025	.0230	.022	.0201
25	.021875	.020	.023	.0204	.020	.0179
26	.01875	.018	.020[illegible]	.0181	.018	.01594
27	.0171875	.0164	.0187	.0173	.016	.014195
28	.015625	.0148	.0165	.0162	.014	.012641
29	.0140625	.0136	.0155	.0150	.013	.011257
30	.0125	.0124	.0137	.0140	.012	.010025
31	.0109375	.0116	.0122	.0132	.010	.008928
32	.01015625	.0108	.0112	.0128	.009	.00795
33	.009375	.0100	.0102	.0118	.008	.00708
34	.0085937	.0092	.0095	.0104	.007	.0063
35	.0078125	.0084	.009	.0095	.005	.00561
36	.0070312	.0076	.0075	.0090	.004	.005
37	.0066406	.0068	.0065	.0085	....	.00445
38	.00625	.0060	.0057	.0080	....	.003965

Solder Required for Wiped Joints.

(According to Hutton)

Size of pipe ins.	½	¾	1	1¼	1¼ water	1½ waste
Ounces of Solder	9	12	16	16	18	18

Size of pipe ins.	1½ water	2 waste	3 waste	4 waste	4 vertical
Ozs. of Solder	20	20	24	34	28

Sheet Metal and Wire Gauges.—The thicknesses of sheet metals and the diameters of wires conform to various gauging systems.

Much confusion has resulted from the use of gauge numbers, and in ordering materials, it is preferable to give the *exact dimensions in decimal fractions of an inch.*

While the dimensions thus specified should conform to the gauge ordinarily used for a given class of material, any error in the specification due, for example, to the use of a table having "rounded off" or approximate equivalents, will be apparent to the manufacturer at the time the order is placed. Furthermore, the decimal method of indicating wire diameters and sheet metal thicknesses has the advantage of being self-explanatory, whereas arbitrary gauge numbers are not. The difference in various gauges will be apparent from the table on page 77.

Table of Brazing Solders

Description	Copper	Zinc	Tin	Lead
Coppersmiths' strong spelter	75	25		
Coppersmiths' spelter	58	42		
Ordinary refractory spelter	50	50		
Hard white solder	57½	28	14½	
Half white, easily fusible	44	50	4½	1½
Spelter, readily fusible	33⅓	66⅔		

Brazing.—Brass or gun metal united by this process will produce a joint as hard as the metal pieces united.

Iron and steel, especially small pieces of finished work, may be united by the same means.

Miscellaneous Brazing Solders

PERCENTAGE				Characteristics	Color
Copper	Zinc	Tin	Lead		
58	42			Very strong	Reddish yellow
53	47			Strong	Reddish yellow
48	52			Medium	Reddish yellow
54.5	43.5	1.5	0.5	Medium	Reddish yellow
34	66			Easily fusible	White
44	50	4	2	Easily fusible	Gray
55	26	15	4	White solder	White

Wrought Pipe

Wrought Pipe.—The term "wrought iron pipe" is often *erroneously* used to refer to pipes made to Briggs' standard sizes rather than of the material, hence, in ordering pipe, if iron pipe be wanted instead of steel, care should be taken to specify *genuine wrought iron*, or *guaranteed wrought iron* pipe.

For calculating heating plants, etc., the following table will be found useful:

Properties of Standard Wrought Pipe

Size	Diameters		Nominal Thickness	Circumference		Transverse Areas			Length of Pipe per Square Foot of		Length of Pipe Containing One Cubic Foot	Nominal Weight per Foot		Number of Threads Per Inch of Screw
	External	Approximate Internal		External	Internal	External	Internal	Metal	External Surface	Internal Surface		Plain Ends	Threaded and Coupled	
Inches	Inches	Inches	Inches	Inches	Inches	Sq. Ins.	Sq. Ins.	Sq. Ins.	Feet	Feet	Feet			
⅛	.405	.269	.068	1.272	.845	.129	.057	.072	9.431	14.199	2533.775	.244	.245	27
¼	.540	.364	.088	1.696	1.144	.229	.104	.125	7.073	10.493	1383.789	.424	.425	18
⅜	.675	.493	.091	2 121	1.549	.358	.191	167	5.658	7.747	754 360	.567	.568	18
½	.840	.622	.109	2.639	1.954	.554	.304	250	4.547	6 141	473.906	.850	.852	14
¾	1.050	.824	113	3.299	2 589	.866	.533	333	3.637	4.635	270.034	1 130	1 134	14
1	1.315	1.049	.133	4.131	3.296	1 358	.864	494	2.904	3.641	166.618	1 678	1.684	11½
1¼	1 660	1.380	.140	5.215	4.335	2 164	1 495	.669	2.301	2.767	96 275	2.272	2.281	11½
1½	1 900	1 610	.145	5 969	5.058	2.835	2 036	.799	2.010	2.372	70.733	2.717	2 731	11½
2	2.375	2.067	.154	7.461	6.494	4.430	3.356	1.075	1.608	1.847	42 913	3 652	3.678	11½
2½	2.875	2 469	203	9.032	7 757	6 492	4.788	1.704	1.328	1.547	30.077	5.793	5 819	8
3	3.500	3 068	216	10.996	9.638	9.621	7.393	2 228	1 091	1.245	19.479	7 575	7.616	8
3½	4.000	3.548	.226	12.566	11.146	12 566	9 886	2 680	.954	1 076	14.565	9.109	9 202	8
4	4.500	4.026	.237	14.137	12.648	15.904	12.730	3.174	.848	.948	11.312	10.790	10.889	8
4½	5.000	4.506	.247	15.708	14.156	19.635	15.947	3.688	.763	.847	9.030	12.538	12.642	8
5	5 563	5.047	.258	17.477	15.856	24.306	20.006	4.300	.686	.756	7.198	14.617	14.810	8
6	6.625	6.065	.280	20.813	19.054	34.472	28.891	5.581	.576	.629	4 984	18.974	19.185	8
7	7.625	7.023	301	23.955	22.063	45.664	38.738	6.926	.500	.543	3.717	23.544	23 769	8
8	8.625	8.071	277	27.096	25.356	58.426	51.161	7.265	.442	473	2 815	24.696	25 000	8
8	8.625	7.981	.322	27.096	25 073	58.426	50 027	8.399	.442	478	2.878	28.554	28.809	8
9	9.625	8.941	.342	30.238	28.089	72.760	62 786	9 974	396	427	2 294	33.907	34 188	8
10	10.750	10.192	279	33.772	32.019	90.763	81.585	9.178	.355	.374	1 765	31.201	32 000	8
10	10.750	10 136	307	33.772	31.843	90.763	80.691	10.072	355	.376	1 785	34.240	35 000	8
10	10 750	10 020	.365	33 772	31.479	90.763	78 855	11.908	355	381	1.826	40 483	41 132	8
11	11 750	11 000	.375	36.914	34.558	108.431	95 033	13.401	325	.347	1.515	45 557	46 247	8
12	12.750	12 090	330	40.055	37.982	127.676	114 800	12.876	299	.315	1.254	43.773	45.000	8
12	12.750	12 000	.375	40 055	37.699	127 676	113 097	14 579	299	.318	1 273	49.562	50.706	8

Average Weight of Cast Iron Pipe

(According to Abendroth)

Diam. in ins.	2	3	4	5	6	8	10	12	15
	Weight in lbs. per 5 ft. length								
Standard..........	17½	22½	32½	42½	52½	85	115	165	225
Extra heavy........	27½	47½	65	85	100	170	225	270	375

Cast Iron Pipe.—In packing the annular space or socket of bell and spigot pipe, the amount of material necessary is given in the following table:

Oakum and Lead for Caulked Joint

Material	Size of Pipe							
	2	3	4	5	6	7	8	10
Oakum......... (Ft. reqd.)......	3	4½	5	6½	7½	8½	9½	12
Lead........... (lbs. reqd.).....	1½	2¼	3	3¾	4½	5¼	6	7½

Soil Pipe.—The regular length of pipe shall be such as to lay 5 ft. Including hub the average weights shall be not less than the following:

Weight of Soil Pipe

Size, Inches	Single Hub		Double Hub
	Per 5′ Length Pounds	Per Foot, Including Hub, Pounds	Per 5′ Length Pounds
2	27½	5½	27½
3	47½	9½	47½
4	65	13	65
5	85	17	85
6	100	20	100

Individual lengths of pipe and fittings may weigh 5% less than designated above, but only when the average weight of a given size and make of pipe and fittings selected at random is not less than above shown.

Radius of Fittings.—The standard radii for bends, offsets and traps are as given in the following table:—

Offsets.—The radii of the bends on offsets, both regular and ⅛ bend offsets, shall be as follows:

Diameter,	2″	3″	4″	5″	6″
Radius,	2″	2½″	3″	3½″	4″

The angle of the bends of regular offsets shall not be more than 76 degrees.

The angle of bends of eighth bend offsets shall be 45 degrees.

Traps.—The radii of all traps shall be as follows:

Diameter,	2″	3″	4″	5″	6″
Radius,	2″	2½″	3″	3½″	4″

The caulking room at spigot end shall not be less than 5″.
The seal to be not less than 2½″.

Laying Lengths.—The laying length as given in the tables on page 83 is *the overall length less the telescoping.*

Plumber's Brass Tubing.—For bars, waste piping, railings, etc., nickel plated brass tubing is largely used, because of its appearance and the light duty it receives permits a lighter tube than regular wrought pipe sizes.

The usual stock lengths of the above tubes are 3 and 6 ft. Owing to the thinness of the tubes finer threads are used than on regular wrought

Comparison of Threads

Diameter	½	⅝	¾	⅞	1	1¼
	Threads per inch					
Tubing	28	20	20	18	18	18
Pipe	14		14		11½	11½

Pipe is not made in the ⅝ and ⅞ in. sizes.

BENDS

All Degrees	Size				
	2″	3″	4″	5″	6″
Radius, Regular..........	3″	3½″	4″	4½″	5″
" Short Sweep......	5″	5½″	6″	6½″	7″
" Long Sweep......	8″	8½″	9″	9½″	10″

BRANCH FITTINGS

All Laying Lengths are in Inches	Size					All sizes with Branches of Diameter			
	2″	3″	4″	5″	6″	2″	3″	4″	5″
Tees...	9	10	11	12	13	9	10	11	12
Tapped Tee	9	9	9	9	9	9			
Sanitary Tee.................	9	10	11	12	13	9	10	11	12
Tapped Sanitary Tee..........	9	9	9	9	9	9			
Y.........................	9	10½	12	13½	15	9	10½	12	13½
½ Y.......................	9	10	11	12	13	9	10	11	12
Inverted Y.................	11	12½	14	15½	17	11	12½	14	15½
Tapped Inverted Y...........	11	11	11	11	11	11			
Comb. Y and ⅛ Bend........	9	10½	12	13½	15	9	10½	12	13½
Upright Y	9	10½	12	13½	15	9	10½	12	13½
Vent Branch.................	9	10	11	12	13	9	10	11	12
Tapped Vent Branch..........	9	9	9	9	9	9			

MISCELLANEOUS FITTINGS

All Laying Lengths are in Inches	Size				
	2″	3″	4″	5″	6″
Reducers................	5	5	5	5	5
Increasers...............	9	9	9	9	9
Tapped Increasers.........	9	9	9	9	9
Single Hub...............	½	½	½	½	½
Double Hub...............	1	1	1	1	1

pipe. Note very carefully the difference in the number of threads on plumbers' tubing and on wrought pipe, as given in the table on page 82.

The size of a tube as given corresponds to its ***outside diameter*** rather than a nominal diameter as with wrought pipe. The following are the sizes and thicknesses generally used:

Nickel Plated Brass Tubing

Outside diameter.......	⅜	½	⅝	¾	⅞	1	1⅛
Gauge................	20	20	20	20	20	20	20
Outside diameter.......	1¼	1⅜	1½	2	1¼	1⅜	1½
Gauge................	20	20	20	20	17	17	17

Sizes of Range Storage Tanks

Approximate Capacity Gallons	Diameter, Inside, of Shell Inches	Length of Shell Feet
18	12	3
21	12	3½
24	12	4
24	14	3
27	12	4½
28	14	3½
30	12	5
32	14	4
36	14	4½
40	14	5
42	16	4
47	16	4½
52	16	5
53	18	4
66	18	5
82	20	5
100	22	5
120	24	5
144	24	6
168	24	7
192	24	8

Hot Water Storage Tank. — The error of calling this fixture a "boiler" is inexcusable, and it would require a stretch of the imagination to consider it as such.

The ordinary sizes are given in the table;

The maximum length of pipe from the tank to the most remote fixture that can be satisfactorily used without circulation is as follows:

Pipe Size	Linear Ft.
½ in.	150
¾ "	90
1 "	50
1¼ "	30

The value of insulating all hot surfaces cannot be too strongly emphasized. The following minimum coverings are recommended for various temperatures:

Tank Temperature	Thickness of Covering	Approx. Sav. cu. ft. of Gas per mo. per sq. ft.
120° F.	1 in.	85
140° F.	1½ "	125
160° F.	2 "	160
180° F.	3 "	200

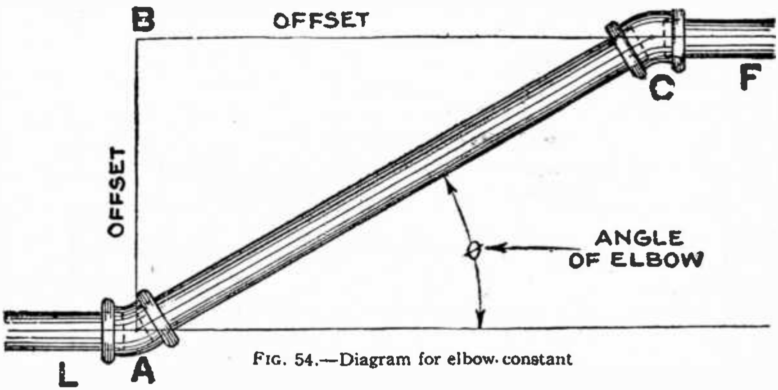


FIG. 54.—Diagram for elbow constant

Elbow Constants

Angle of Elbow	Elbow Centers AC	Offset AB
60°	1.15	.58
45°	1.41	1.00
30°	2.00	1.73
22½°	2.61	2.41
11¼°	5.12	5.02
5⅝°	10.20	10.15

NOTE.—In the above table the letters refer to fig. 54.

Friction of Water in Pipes

According to Prof. Geo. E. Russell there is wide misunderstanding in the plumbing trade concerning the laws governing rates of discharge of water from faucets, and the relations of pressure and discharge. This misunderstanding produces great waste.

Prof. Russell has prepared the accompanying table based on his experiments and which will be found of great value in calculating the loss of pressure due to friction in pipes of various sizes which deliver various volumes of water per minute.

TABLE 1

Small Wrought Iron Pipe

Loss in Pressure by Friction, per 100 ft. of Length. Figures are in Pounds per Square Inch

Gallons per Minute	Nominal Diameter in Inches					
	½	¾	1	1¼	1½	2
1	0.9	0.2	....	....	....	....
				Pounds	per squar	e inch
2	3.2	0.8	....	....	....	....
3	6.9	1.8	0.6	....	....	....
4	11.7	3.2	0.9	0.3	0.1	....
5	17.8	4.5	1.4	0.4	0.2	....
6	24.8	6.4	2.0	0.5	0.3	0.1
8	42.6	10.9	3.4	0.9	0.4	0.1
10	64	16.5	5.1	1.4	0.6	0.2
12	90	23.1	7.1	1.9	0.9	0.3
14	120	30.4	9.6	2.5	1.2	0.4
16	....	40	12.2	3.2	1.5	0.5
18	....	48.7	15.2	4.0	1.8	0.7
20	....	59.2	18.3	4.8	2.3	0.8
30	....	120	38.7	10.2	4.8	1.7
40	....	....	66	17.4	8.2	2.9
50	....	....	98	26.1	12.4	4.3

In addition to the friction encountered in pipes there is *additional friction in faucets*. This varies more or less according to the type and make of faucet.

Prof. Russell in his experiments on a ½ in. faucet, obtained results upon which he based the following table:

TABLE 2

Friction of Water Through Small Faucet

(½ in. Commercial Faucet)

Rate of Flow Gals. per min.	Loss in Pressure Lbs. per sq. in.
4	2
5	3½
8	9
10	15
15	33
20	60

It should be remembered that this table relates to *one faucet only*. Different makes give different loss, hence allowance should be made.

The plumber should be able to estimate the losses in pressure due to the several factors that enter into the problem, and be able in this way to determine the pressure required at fixtures to deliver water at a certain rate of flow.

The following examples show how the problem will illustrate the methods of calculation.

Example.—What pressure is required for a flow of 10 gals. per minute through a ¾ in. pipe line 350 ft. long?

In table 1, the drop in pressure is 16.5 lbs. per 100 ft. of ¾ in. pipe for a flow of 10 gals. per min. and for 350 ft. the required pressure is

$$\frac{16.5 \times 350}{100} = 16.5 \times 3.5 = 57.8 \text{ lbs.}$$

Example.—The height of water in a wind mill tank is 60 ft. How many gals. per min. will flow through 500 ft. of 1½ in. pipe with a vertical rise of 23 ft.?

2.3 ft. head = 1 lb. pressure

Hence

Pressure due to elevation of tank is

= 60 ÷ 2.3 = ... 26.1 lbs.

Pressure due to 23 ft. rise

23 ÷ 2.3 = ... 10 lbs.

Pressure available to cause flow = ... 16.1 lbs.

Pressure available per 100 ft. of pipe is

16.1 ÷ 5 = ... 3.2 lbs.

In table 1, two nearest values per 100 ft. 1½ in. pipe are given.

Pressure	Gals. per min.
2.3 lb.	20
4.8 lb.	30

from which 3.2 lbs. corresponds to a flow of $20+\frac{3.2-2.3}{4.8-2.3}$ of $10=23.6$ gals. per minute.

Example.—A ¾ in. pipe runs 75 ft. with a vertical rise of 23 ft. What pressure will be required at the mains in the street in order to deliver 10 gals. per minute through a ½ in. faucet?

From Table 1

Loss through 75 ft. of pipe, due to friction

$=16.5\times\frac{75}{100}=$ .... 12.5 lbs.

Loss due to rise of 23 ft.

= 23 ÷ 2.3 = ... 10 lbs.

From Table 2

Loss in flowing through faucet at 10 gals. per minute = ... 15 lbs.

Total pressure required .... 37.5 lbs.

In most installations the loss due to flowing through the meter, must be allowed, for Prof. Russell's experiments show the meter loss to average about as follows:

TABLE 3

Meter Loss

Size of Meter	Gals. per min.	Loss in Pressure Lbs.
⅝ in.	10	2 to 7
⅝ in.	30	16 to 65
¾ in.	30	14 to 30
1 in.	30	4 to 7

At the same rate of flow, 30 gals., it will be seen that as the size of meter increases, the loss in pressure in passing through the meter drops off rapidly.

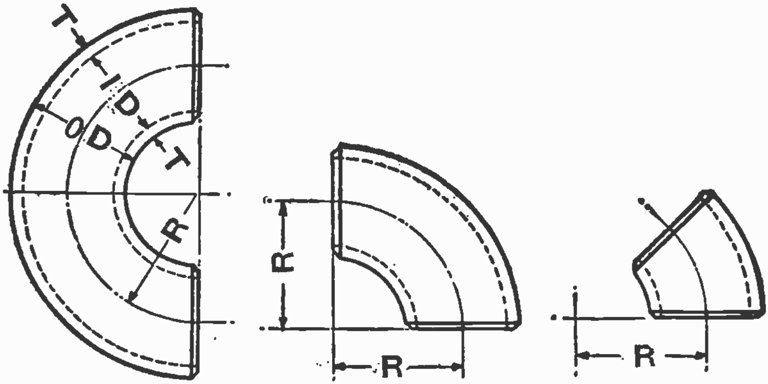
Example.—A ¾ in. pipe passes through a meter, runs 125 ft. from the main to a faucet. The faucet is 23 ft. above the street main, and must deliver 10 gals. per min. What pressure will be required at the main to obtain this rate of discharge?

From Table 1,

Loss due to friction in 125 ft. of pipe $=16.5\times1.25=$	20.6 lbs.
Loss due to height $=23\div2.3=$	10 lbs.
Loss in passing through faucet = (From Table 2)	15 lbs.
Loss through meter arbitrarily taken as	3 lbs.
Pressure required at street main	48.6 lbs.

In these calculations no account has been taken of the loss in pressure due to the use of fittings. Unless these are somewhat numerous, the loss may sometimes be overlooked or estimated.

According to Prof. Russell: "A rough rule for calculating the loss in pressure due to an elbow in small pipes is *to allow from ¼ to ½ lb.* which points to the desirability of using *as few fittings as possible.* The reaming of pipe is advisable for the same reason."



FIGS. 55 to 57.—180°, 90° and 45° tube turn fittings. The dimension letters refer to the dimensions in the accompanying table.

Properties of Tube Turns

Standard Thickness—IRON PIPE SIZES—*Series 1½R*

Nominal Pipe Size	Radius Inches (R)	PIPE DIAMETER O. D.	PIPE DIAMETER I. D.	Wall Thickness (T)	Weight Pounds (180° Type)	Bursting Pressure Pounds
1"	1½	1.315	1.048	.134	¾	10,600
1¼"	1⅞	1.660	1.380	.140	1	8,770
1½"	2¼	1.900	1.610	.145	1½	7,940
2"	3	2.375	2.067	.154	2¾	6,355
2½"	3¾	2.875	2.469	.203	5¾	6,640
3"	4½	3.500	3.068	.216	9	5,800
3½"	5¼	4.000	3.548	.226	12½	5,310
4"	6	4.500	4.026	.237	17	4,950
5"	7½	5.563	5.047	.258	29	4,360
6"	9	6.625	6.065	.280	45	3,970
8"	12	8.625	7.981	.322	90	3,435
10"	15	10.750	10.020	.365	159	3,120
12"	18	12.750	12.000	.375	234	2,710
14"OD	21	14.000	13.250	.375	300	2,465
16"OD	24	16.000	15.250	.375	393	2,160
18"OD	27	18.000	17.182	.409	544	2,090
*20"OD	30	20.000	19.182	.409	672	1,880

Boilers

Boiler "Horse Power."—By definition one boiler horse power is the *evaporation of* 30 *pounds of water from an initial temperature of* 100° *Fahr. to steam at* 70 *pounds gauge pressure,* which (as accepted by the A. S. M. E. Power Plant Code Committee) is equivalent to 34.5 *pounds of water evaporated per hour from a feed water temperature of* 212° *into dry steam at the same temperature.*

Example.—A 250 horse power triple expansion marine engine requires 15 pounds of steam per hour per horse power at 165 pounds pressure. If the feed water be delivered to the boiler at 122° Fahr., how much boiler horse power is required, allowing 10 per cent. of the total steam for auxiliaries?

Steam required by engine	=250×15	=3,750 lbs
Steam required by auxiliaries	=10 per cent. of 3,750	= 375 "
Total steam required per hour		=4,125 "

From the table of "factors of evaporation" (page 64), factor for evaporation at 165 pounds pressure for feed water at 122° by interpolation =1.145.

Equivalent evaporation from and at 212° = 4,125×1.145 = 4,723, or

4,723 ÷ 34.5 = 137 boiler horse power

Example.—A 250 horse power tug engine requires 50 pounds of steam per horse power hour, at 70 pounds pressure. If the feed water be delivered at 100° Fahr., what boiler horse power is required, allowing 10 per cent. of the total steam for auxiliaries?

Steam required by engine	=250×50	=12,500 lbs.
Steam requred by auxiliaries	=10 per cent. of 12,500	= 1,250 "
Total steam required per hour		=13,750 "

Since one boiler horse power = 30 pounds evaporation from feed at 100° into steam of 70 pounds pressure,

Boiler capacity = 13,750 ÷ 30 = 458 horse power

The two examples just given show that engines of the same horse power may require boilers of widely different capacities, hence, *engine horse power is no index of boiler horse power.*

Some manufacturers calculate the boiler horse power on the basis of 12 *sq. ft. of heating surface per horse power for* 36 *in. and* 42 *in. fire tube boilers*, 11 *sq. ft. of heating surface for* 48 *in. boilers and* 10 *sq. ft. of heating surface for larger boilers.*

For fire tube boilers it is customary to assume 12 to 15 sq. ft. of heating surface as representing one boiler horse power; in water tube boilers 10 sq. ft. of heating surface is equivalent to one boiler horse power.

Fuel Required.—Upon the heating value of the fuel and efficiency of the boiler will depend the amount necessary to burn in a given time to generate the required amount of steam.

Example.—If the heating value of a certain coal and efficiency of boiler be such as to give an evaporation of 8 to 1, from and at 212° Fahr., how many pounds of coal are required per hour for an evaporation of 500 pounds of steam at 80 pounds per hour with feed water at 100° Fahr.

Factor of evaporation for 80 pounds pressure and 110° feed water is (from table), 1.142.

Equivalent evaporation at 80 pounds and 110° = 8 ÷ 1.142 = 7.01.

Coal required = 500 ÷ 7.01 = 71.3 pounds per hour

Rate of Combustion.—The size of the grate will depend upon the rate of combustion.

Example.—What size grate is required for the boiler of the preceding example if the rate of combustion be 15 pounds of coal per square foot of grate per hour?

Size of grate = 71.3 ÷ 15 = 4.75 sq. ft.

Example.—A locomotive when developing 1,000 horse power, requires 30 pounds of steam per horse power hour at 175 pounds pressure, feed water 70° Fahr. What size grate is required if the evaporation be 7 to 1, and rate of combustion be 125 pounds of coal per square foot per hour?

Total feed water = 1,000 × 30 = 30,000 lbs. per hour.

Factor of evaporation (from table), for 175 lbs and 70° = 1.200.

Equivalent evaporation from and at 212° is

$$30{,}000 \times 1.1936 = 35{,}808 \text{ lbs.}$$

Coal per hour $= 35{,}808 \div 7 = 5{,}115$ lbs.

Size of grate $= 5{,}115 \div 125 = 40.9$ sq. ft.

Efficiency.—As stated by Kent, the efficiency of a boiler is *the percentage of the total heat generated by the combustion of the fuel which is utilized in heating water and in raising steam.*

Example.—With anthracite coal the heating value of the combustible portion is very nearly 14,800 *B.t.u.* per pound, equal to an evaporation from and at 212°, of $14{,}800 \div 970.4 = 15.25$ pounds of water.

A boiler which when tested with anthracite coal shows an evaporation of 12 pounds of water per pound of combustible, has an efficiency of

$$12 \div 15.26 = 78.6 \text{ per cent.}$$

a figure which is approximated, but scarcely ever reached in practice.

Safe Working Pressure.—This is *the maximum pressure safe to carry on a boiler consistent with the factor of safety used in the design;* the maximum pressure at which the safety valve should blow. The safe working pressure is sometimes called "working pressure."

The formula is

$$\text{working pressure} = \frac{\text{tensile strength} \times \text{thickness of plate} \times \text{efficiency of joint}}{\text{radius of shell} \times \text{factor of safety}}$$

or using the usual symbols

$$\textbf{Working Pressure} = \frac{T \times t \times E}{R \times F}$$

in which

T = ultimate tensile strength stamped on shell plates, pounds per square inch.

t = minimum thickness of shell plates in weakest course, inches.

E = efficiency of longitudinal joint or of ligaments between tube holes (whichever is the least).

R = inside radius of the weakest course of the shell or drum, inches.

F = factor of safety, or the ratio of the ultimate strength of the material to the allowable stress.

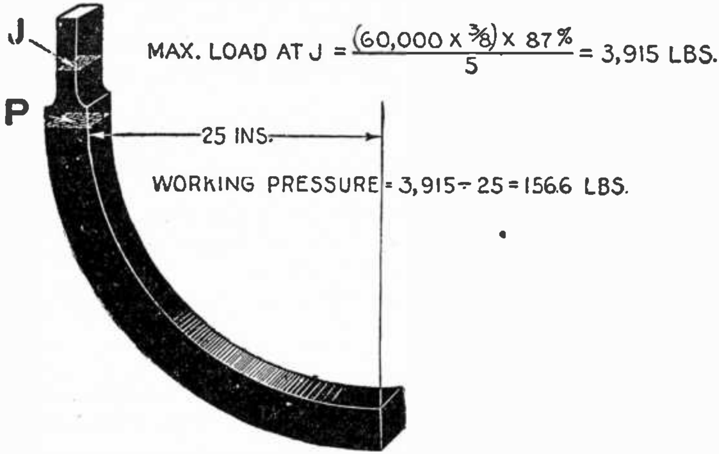


FIG. 58.—Half section of shell, illustrating method of determining the *working pressure.*

Example.—What is the maximum allowable working pressure to be carried on a boiler 50 inches in diameter, tensile strength 60,000 pounds, plates $\frac{3}{8}$ inch thick, efficiency of joint 87 per cent, factor of safety 5.

A tensile strength of 60,000 pounds corresponds to a stress of

$$60{,}000 \times \tfrac{3}{8} = 22{,}500 \text{ pounds}$$

in a $\frac{3}{8}$-inch plate per inch length of section, and for a factor of safety of 5 the maximum load allowable on the solid metal of the shell is

$$22{,}500 \div 5 = 4{,}500 \text{ pounds}$$

Considering the efficiency of 87 per cent of the joint, this load must be reduced to

$$87 \text{ per cent of } 4{,}500 = 3{,}915 \text{ pounds}$$

not pounds per square inch, but the **maximum** *allowable force* tending to pull the metal of the shell apart. Since this force is distributed over the radius of the shell or $50 \div 2 = 25$ inches (that is, 25 square inches, considering 1 inch length of shell), the maximum allowable working pressure is

$$3{,}915 \div 25 = 156\tfrac{1}{2} \text{ pounds}$$

Substituting values in formula (1),

$$\text{safe working pressure} = \frac{60{,}000 \times \tfrac{3}{8} \times .87}{(\tfrac{1}{2} \text{ of } 50) \times 5} = 156\tfrac{1}{2} \text{ lbs.}$$

The efficiency of riveted joints is about as follows: single riveted lap .56; double riveted lap .7; double riveted butt and double strap .82; triple riveted butt and double strap .85 to .87; quadruple riveted butt and double strap .94.

Heating Surface.—The method of calculating the heating surface of a horizontal steam tubular boiler is as follows:

Rules. Take the dimensions in inches. **Shell:** *Multiply length by fraction of circumference in contact with hot gases* **Tubes:** *Multiply the sum of the* **inside** *circumferences of all of the tubes by their common length.* **Tube Sheets:** *Multiply twice the diameter squared by* .7854 *and subtract twice the sum of the internal areas of all the tubes (similarly obtained).* **Total Heating Surface:** *Divide the sum of the several areas just obtained by* **144.**

A. Shell

Sq. ins.

$$area = 54 \times 3.1416 \times \frac{200}{360} \times (16 \times 12) = 18{,}096$$

NOTE.—3.1416 or π is the number which multiplied by the diameter of a circle gives the circumference. 200 ÷ 360 is the fractional part of the shell in contact with the hot gases. (16 × 12) is length of shell *in inches.*

NOTE.—*Boiler calculation.*—For complete calculation of all parts of a boiler according to the Amer. Soc. of Mech. Eng. Boiler Code. See chapter on "How to Design a Boiler" in volume 5, *Audel's Engineers and Mechanics Guides.*

B. Tubes

$$area = 40 \times (3.732 \times 3.1416) \times (16 \times 12) = 90{,}004$$

NOTE.—40 is the number of tubes. 3.732 is the inside diameter of a standard 4 in. tube obtained from the table on page 97 (3.732 × 3.1416) is the inside circumference. (16 × 12) is the length of each tube *in inches*.

C. Tube Sheets

$$\begin{aligned}
&\textit{gross area of sheets} = 2 \times 54^2 \times .7854 && = 4{,}580 \\
&\textit{twice area of tubes} = 2 \times 40 \times 3.732^2 \times .7854 = && 875 \\
&\textit{net area of sheets} = \tfrac{2}{3} \times \textit{gross area sheets} \\
&-\textit{twice area tubes} = \tfrac{2}{3} \times 4{,}580 - 875 && = \quad 2{,}178 \\
&\textit{total heating surface of boiler in sq. ins.} && = \quad 102{,}814
\end{aligned}$$

NOTE.—**Gross area of sheets:** This is the area not including that cut away by holes for tubes. 54 = diameter boiler; .7854 is that fraction which multiplied by the diameter squared gives the gross area of one sheet. **Area of tubes:** This means the cross sectional area. 40 = number of tubes; 3.732 = inside diameter of a tube, the inside diameter being taken because the thickness of each tube is here considered as forming a part of the tube sheet, as it is regarded as sheet heating surface. **Net area of tubes:** The factor ⅔ is introduced because ordinarily only ⅔ of the sheet area is exposed to the hot gases—this is an approximation and to be exact the exposed portion of sheet area must be measured. The factor 2 is used in both equations because there are two sheets.

D. Total Heating Surface

$$\textit{total area} = \frac{A+B+C}{144} = \frac{18{,}096 + 82{,}540 + 2{,}178}{144} = 714 \quad \textit{sq. ft.}$$

In finding the heating surface of the tubes it should be noted that the sizes of boiler tubes as given are the *outside diameters*, whereas, in calculating the heating surface, in the case of **fire tube** boilers, the **inside** diameter is taken, in order to determine the surface **exposed** *to the fire and hot gases*.

It is the common practice of boiler makers to use the **external** instead of the **internal** diameter of fire tubes, for greater convenience in calculation. This is an error and gives more than the true amount of heating surface. The purchaser should therefore note this fact where the builder agrees to furnish a given area of heating surface.

The following table gives not only the inside diameters of tubes but other items or "properties" as they are called which greatly facilitate the calculations for heating surface, etc.

Properties of Standard Lap Welded Boiler Tubes

External Diameter	Internal Diameter	Standard Thickness		Internal Circumference	External Circumference	Internal Area	External Area	Length of Tube per Sq. Ft. of Inside Surface	Length of Tube per Sq. Ft. of Outside Surface	Weight per Lineal Foot
ins.	ins.	ins.	B.W.G.	ins.	ins.	sq. ins.	sq. ins.	ft.	ft.	lbs.
1	.810	.095	13	2.545	3.142	.515	.785	4.479	3.820	.90
1¼	1.060	.095	13	3.330	3.927	.882	1.227	3.604	3.056	1.15
1½	1.310	.095	13	4.115	4.712	1.348	1.767	2.916	2.547	1.40
1¾	1.560	.095	13	4.901	5.498	1.911	2.405	2.448	2.183	1.679
2	1.810	.095	13	5.686	6.283	2.573	3.142	2.110	1.910	1.932
2½	2.282	.109	12	7.169	7.854	4.090	4.909	1.674	1.528	2.783
3	2.782	.109	12	8.740	9.425	6.079	7.069	1.373	1.273	3.365
3½	3.260	.120	11	10.242	10.996	8.347	9.621	1.172	1.091	4.331
4	3.732	.134	10	11.724	12.566	10.939	12.566	1.024	.995	5.532
4½	4.232	.134	10	13.295	14.137	14.066	15.904	.903	.849	6.248
5	4.704	.148	9	14.778	15.708	17.379	19.635	.812	.764	7.669
6	5.670	.165	8	17.813	18.850	25.250	28.274	.674	.637	10.282

The following examples will illustrate the great convenience of the table.

Example.—How many sq. ft. of inside heating surface in 40, four inch tubes, each 16 feet long?

From the table, the length of a 4 in. tube per sq. ft. of inside surface is 1.024 ft., hence

$$\textit{total heating surface} = \frac{40 \times 16}{1.024} = 625 \textit{ sq. ft.}$$

Example.—How many feet of one inch tube is required for 140 sq. ft. of inside heating surface, and what is the weight?

From the table the length of 1 in. tube per sq. ft. of inside surface is 4.479 ft., hence

$$\textit{amount of } 1 \textit{ in. tube required} = 140 \times 4.479 = 627 \textit{ ft.}$$

From the table 1 in. tube weighs .9 lb., per lineal foot hence,

$$\textit{weight of } 627 \textit{ ft. of } 1 \textit{ in. tube} = 627 \times .9 = 564 \textit{ lbs.}$$

Boiler Fixtures

The Safety Valve.—The simple equation from which any lever safety valve problem can be solved is

$$Sv = Vv + Gg + Bb.$$

in which

S = *total* pressure due to the steam tending to raise the valve;

This is equal to the steam pressure indicated by the steam gauge multiplied by the area of the valve. The area of the valve is equal to its diameter squared, multiplied by .7854.

V = weight of valve and spindle;
G = " " lever;
B = " " ball.

The distances at which these forces act are:

v = distance from fulcrum to center of the valve;
g = " " " " center of gravity of the lever;
b = distance from fulcrum to the ball.

The weights are measured in pounds, and the distances in inches.

The weight of the lever is considered as acting at its center of gravity g, distance from the fulcrum.

The center of gravity of the lever is that point where it would be in equilibrium if balanced over a knife edge or any other support with an edge, as in fig. 19.

Example:—What weight ball must be put on a 3″ safety valve so that it will blow at 100 lbs., if the weight of valve and spindle be 8 lbs., lever, 24 lbs., distance of valve from fulcrum 4″; distance of center of gravity from fulcrum 16″; distance from fulcrum to ball 38″.

S. the total pressure tending to raise the valve is equal to the steam pressure multiplied by the area of the valve in square inches = 100 × diam. × diam. × .7854 = 100 × 3 × 3 × .7854 = 706.9 lbs., say 707 lbs.

Now write out the equation and substitute the values given in the example and value just found for S, under the proper letters, thus:

$$\underbrace{Sv}_{707\times4} = \underbrace{Vv}_{8\times4} + \underbrace{Gg}_{24\times16} + \underbrace{Bb}_{B\times38}$$

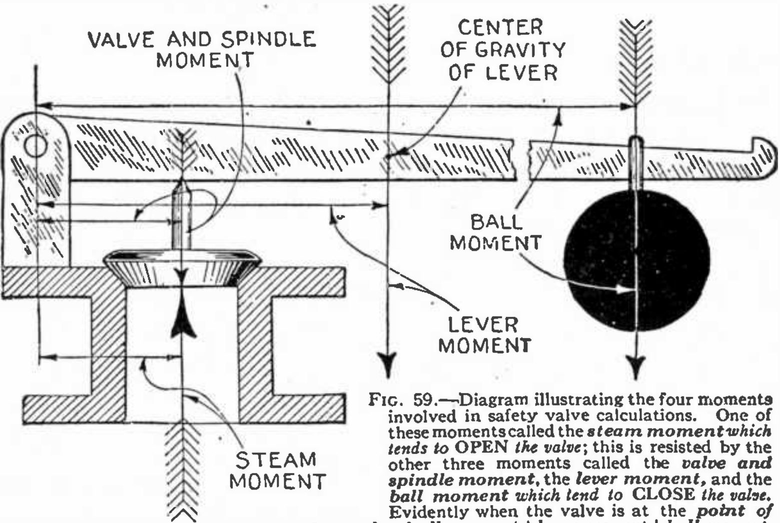


FIG. 59.—Diagram illustrating the four moments involved in safety valve calculations. One of these moments called the *steam moment which tends to* OPEN *the valve*; this is resisted by the other three moments called the *valve and spindle moment*, the *lever moment*, and the *ball moment which tend to* CLOSE *the valve*. Evidently when the valve is at the *point of blowing off*, the *steam moment = valve and spindle moment + lever moment + ball moment*. Evidently if the steam pressure increase a very small amount sufficient to cause the steam moment to overcome the *friction of the mechanism*, the valve will open and blow off.

multiplying

$$2,828 = 32 + 384 + 38B$$

and adding

$$2,828 = 416 + 38B$$

The equation must be "solved for B," which means that everything must be transferred to the left hand side of the equality sign except the B. The first step then is to get the 416 on the left hand side; to do this, subtract 416 from both sides, thus:

$$\begin{array}{r} 2{,}828 = 416 + 38B \\ 416 \quad 416 \qquad \\ \hline 2{,}412 = \qquad 38B \end{array}$$

As it now stands, 2,412 = 38B, or in other words, 38B = 2,412:

Now, divide both sides by 38, thus:

$$\frac{38B}{38} = \frac{2{,}412}{38}, \text{ hence:}$$

$$B = \frac{2{,}412}{38} = 63.5 \text{ lbs., weight of ball.}$$

Problem.—

Given: Area valve 7.07 sq. ins. Weight of valve and spindle = 8 lbs.; weight of lever, 24 lbs.; distance fulcrum to spindle, 4 ins.; distance fulcrum to center of gravity of lever, 16 ins.; distance fulcrum to ball, 20.2 ins. Steam pressure 60 lbs.

Find: *Weight of ball.*

A. First step

7.07	sq. ins. area of valve
60	lbs. per sq. in., steam pressure
424.20	lbs.

*This gives the **total pressure due to the steam** acting on the valve.*

B. Second step

424.2	lbs. total pressure on valve
4	ins. distance valve to fulcrum
1,696.8	

This is the ***steam moment tending to open the valve.***

C. Third step

24	lbs. weight of lever
16	ins. distance from fulcrum to center of gravity of lever
144	
24	
384	

This is the ***lever moment tending to close the valve.***

D. Fourth step

8	lbs. weight of valve and spindle
4	ins. distance fulcrum to spindle
32	

This is the ***valve and spindle moment tending to close the valve.***

E. Fifth step

384 lever moment from C
32 valve and spindle moment from D
416

This is the sum of the *lever moment* and the *valve and spindle moment.*

F. Sixth step

1,696.8 steam moment from B
416 lever moment+valve and spindle moment
1,280.8

20.2)1280.8(63.41
1212
688
606
820
808
120

This is the required weight necessary to put on the lever at the given distance for the valve to blow at 60 lbs.

Injectors.—Increasing the distance through which the feed water must be lifted decreases the capacity of the injector. Very low or very high steam pressures also influence the capacity.

U. S. Marine Rule.—

RULE II, 23.—The areas of all safety valves on boilers contracted for or the construction of which commenced on or after June 1, 1904, shall be determined in accordance with the following formula and table:

$$\text{Formula: } a = .2074 \times \frac{W}{P} \quad \ldots\ldots (1)$$

Where a = area of safety valve, in square inches, per square foot of grate surface.
W = pounds of water evaporated per square foot of grate surface per hour.
P = absolute pressure per square inch = working gauge pressure + 15.

When this calculation results in an odd size of safety valve, use next larger standard size.

EXAMPLE.

Boiler pressure = 75 pounds per square inch (gauge).

2 furnaces: Grate surface = 2(No.) × 5 feet 6 inches (long) × 3 feet (wide) = 33 square feet.

Water evaporated per pound of coal = 8 pounds.

Coal burned per square foot grate surface per hour = 12½ pounds.

Evaporation per square foot grate surface per hour or W, = 8 × 12½ = 100 pounds. P = 75 + 15 = 90 lbs. absolute.

From equation (1)

$$a = .2074 \times \frac{100}{90} = .23 \text{ cu. ins.}$$

Therefore area of safety valve = 33 × .23 = 7.59 square inches.

For which the diameter is 3⅛ inches nearly.

For example, an injector designed to operate with pressure ranging from 60 to 110 lbs. steam pressure has its capacity decreased about one third when operating on 30 lbs. steam pressure; at 125 lbs. steam pressure the capacity is lowered about one tenth; and at a pressure of 140 lbs. the decrease is about the same as for 30 lbs. steam pressure.

Unless an injector be designed to handle water with a temperature about 76° Fahr. its capacity will be decreased as the temperature is increased above this point.

The equation of the injector is

$$S=\frac{W\left[(t_2-t_1)d+.1851\,p\right]}{\left[H-(t_2-32°)\right]d-.1851\,p}$$

in which

S = number of lbs. of steam used.

W = number of lbs. of water lifted and forced into the boiler.

t_1 = temperature of the water before it enters the injector.

t_2 = temperature of the water after leaving the injector;

d = weight of 1 cu. ft. of water at temperature t_2;

p = absolute pressure of steam lbs. per sq. in.;

H = total heat above 32° F. in one lb. of steam in the boiler, in heat units.

Auxiliaries

Boiler Feed Pumps.—In determining the size of a boiler feed pump, it must supply *an amount of water equal to the maximum capacity of the boiler plus an additional allowance for refill when the water level in the boiler becomes low.*

Example.—A certain boiler, at maximum load, evaporates 1,000 lbs. of steam per hour. If, when the feed pump is not running, 3 "cocks" of water is evaporated in 10 minutes, what would be the capacity of a feed pump to supply water equal to the maximum evaporation + a refill in 5 minutes?

Water used in 10 minutes = $\frac{1}{6}$ of 1,000 = 167 lbs.

Since this amount is to be pumped in 5 minutes,

capacity of pump = 1,000 + (2 × 167) = 1,334 lbs. per hour

or, 1,334 ÷ $8\frac{1}{3}$ = 160 gals. per hour.

Formulae for theoretical capacity of a pump as given by Kent are as follows:

Let Q = cu ft. per min., Q′ = U. S. gals. per min. = 7.4805 Q; d = diam. of pump in ins., l = stroke in ins.; N = number of single strokes per min.

Capacity in cu. ft. per min.

$$Q = .0004545\ Nd^2l$$

Capacity in U. S. gals. per min.

$$Q^1 = .0034\ Nd^2l$$

Capacity in gals. per hour.

$$Q^2 = .204\ Nd^2l$$

Diameter required for a given capacity per min.

$$d = 46.9\sqrt{\frac{Q}{Nl}} = 17.15\sqrt{\frac{Q'}{Nl}}$$

If v = piston speed in ft. per min.

$$d = 13.54\sqrt{\frac{Q}{v}} = 4.95\sqrt{\frac{Q'}{v}}$$

If the piston speed be 100 ft. per min.:

$$Nl = 1{,}200 \text{ and } d = 1.354\sqrt{Q} = .495\sqrt{Q^1};\ Q^1 = 4.08\ d^2 \text{ per min.}$$

The actual capacity will be from 60% to 95% of the theoretical, according to the tightness of the piston, valves, suction pipe, etc.

Feed Water Heaters.—Heating boiler feed water by means of exhaust steam effects a considerable saving in fuel.

The heat imparted to the water in a feed water heater takes the place of an equivalent amount of heat from fuel on the grates. The amount of saving for any given condition can be determined by a simple calculation.

The formula for saving due to feed water heater is

$$S = \frac{h_2 - h_1}{H - h_1}$$

in which

S = saving due to heating the feed water;

H = total heat of 1 lb. of steam at the boiler pressure;

h_1 = total heat of 1 lb. feed water before entering the heater;

h_2 = total heat of 1 lb. feed water after passing through the heater.

Example.—If the temperature of the feed water be 60° Fahr., before entering the feed water heater, and the heater raise its temperature to 210°, what is the saving with boiler pressure at 100 pounds?

Total heat in one pound of steam at 100 pounds pressure = 1,188.7 *B.t.u.*

Heat in 1 pound of feed water before entering heater, 60 − 32 = 28 *B.t.u.*

Heat required to form 1 pound of steam = 1,160.7 *B.t.u.*

Heat saved by feed water heater, 210 − 60 = 150 *B.t.u.*

Percentage of saving = 150 ÷ 1,160.7 = .129, that is, 12.9 per cent., which is equal to 1 per cent. for each 150 ÷ 12.9 = 11.6 degrees that the temperature of the feed water is raised by the heater. No saving of fuel is realized where the water is heated by an injector or by a live steam purifier, since both devices first draw from the boiler the heat which is to be imparted to the water, whereas in the exhaust steam a waste product is utilized.

Condensers.—According to Barrus, "It is held in the popular mind that the economy of condensing is, in round numbers, 25%. This percentage usually relates to simple engines and it refers to the economy as measured by the difference in the coal consumption produced by a condenser."

The evidence of some of Barrus' tests shows that "this belief is not well founded except in special cases." "If the feed water be heated by the exhaust steam of the non-condensing engine to a temperature of 100°, which is that of the ordinary hot well, to a temperature of 210°, the non-condensing engine can be credited with about 11% less coal consumption, which should be considered in determining condenser economy."

The average of a number of Barrus' tests gives a saving produced by condensing of 22.3%. "If we allow for the steam or power used by an economical condenser, it would be seen that the net economy of condensing is at best, not much over 20%, based on steam consumption. If furthermore, we allow for the difference produced by heating the feed water to the extent above mentioned, the *saving of fuel* would be reduced to about 10%."

1. Jet Condensers.

In a jet condenser the amount of injection water is given by the formula

$$Q=\frac{1114°+.3\times T_1-T_2}{T_2-T_0} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

In the formula

T_1 = temperature of the steam whose latent heat is L;

T_0 = temperature of the cooling water, whose quantity in pounds is Q;

T_2 = temperature after condensation or that of the hot well;

T_1+L = total heat of the steam.

$(T_1+L)-T_2$ also $Q(T_2-T_0)$ } Heat absorbed by the cooling water.

Example.—To find the amount of injection water required for an engine, the steam at exhaust being at a pressure of 10 lbs. absolute; the temperature of the sea is 60° and it is required to keep the hot well at 120°.

The temperature corresponding to 10 lbs. is 193°.

$$Q = \frac{1114° + .3 \times 193° - 120°}{120° - 60°} = 17.53 \text{ lbs.}$$

That is, the amount of injection water is 17.53 times the weight of steam for this particular case. The allowance made for the injection water of engines working in the temperate zone is usually 27 to 30 times the weight of steam, and for the tropics 30 to 35 times; 30 times is sufficient for ships which are occasionally in the tropics, and this is what was usually allowed for general traders.

Another formula is:

$$Q = \frac{WH}{R}$$

In which

W, is the weight of steam condensed; H, the units of heat given up by 1 lb. of steam in condensing, and R, the rise in temperature of the cooling water. This is applicable both to jet and to surface condensers.

2. Surface Condensers.

In a surface condenser the amount of *cooling* water is given by the formula:

$$Q = \frac{1114 + .3\,T_1 - T_3}{T_2 - T_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

T_3 being the temperature of the feed, the other letters meaning the same as in formula (1).

1. When the cooling water is 60°

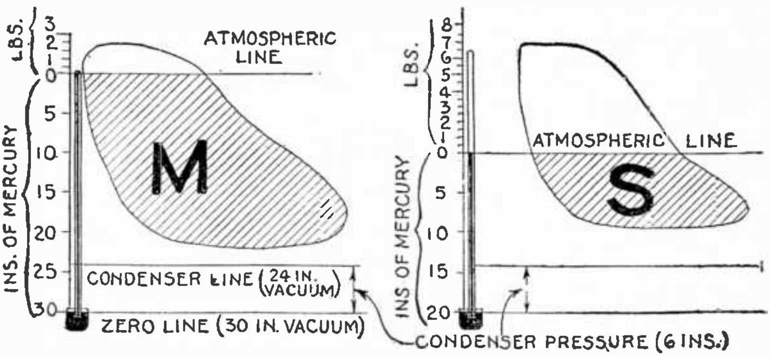
$$Q=\frac{1114+.3\times183-120}{100-60}=26.22$$

That is, the water required is 26.22 times the weight of steam. If this had been a jet condenser, the quantity would be only 17.48 times.

2. When the cooling water is 75°

$$Q=\frac{1114+.3\times183-120}{100-75}=41.95 \text{ times}$$

Evidently the quantity of cooling water will depend on its initial temperature, which in actual practice may vary from 40° in the winter of temperate zones to 80° of the West Indies and other sub-tropical seas. In the latter case a pound of water requires only 20 thermal units to raise it to 100°, while 60 are required in the former. From this it is seen that the quantity of circulating water required in the tropics is three times that of the North Atlantic in the spring of the year.



FIGS. 60 and 61.—Indicator cards illustrating effect of barometer changes on condensing engine operation. In fig. 60, with 30 inch barometer a large proportion of the work is done below atmospheric pressure as indicated by the shaded portion M, of the card. Now if the barometer were to fall to say 20 ins., as in fig. 61, considerably less work would be done below atmospheric pressure, as indicated by the area S, which is much smaller than M.

Horse Power.—The term horse power is defined as *33,000 foot pounds of work done in one minute.* There are, however, several kinds of horse power and it is important to clearly understand the differences between these and their application to avoid confusion and errors. They are:

1. *Nominal horse power.
2. Indicated horse power.
3. Brake horse power.
4. Effective horse power.
5. Hydraulic horse power.
6. Boiler horse power.
7. Electrical horse power.

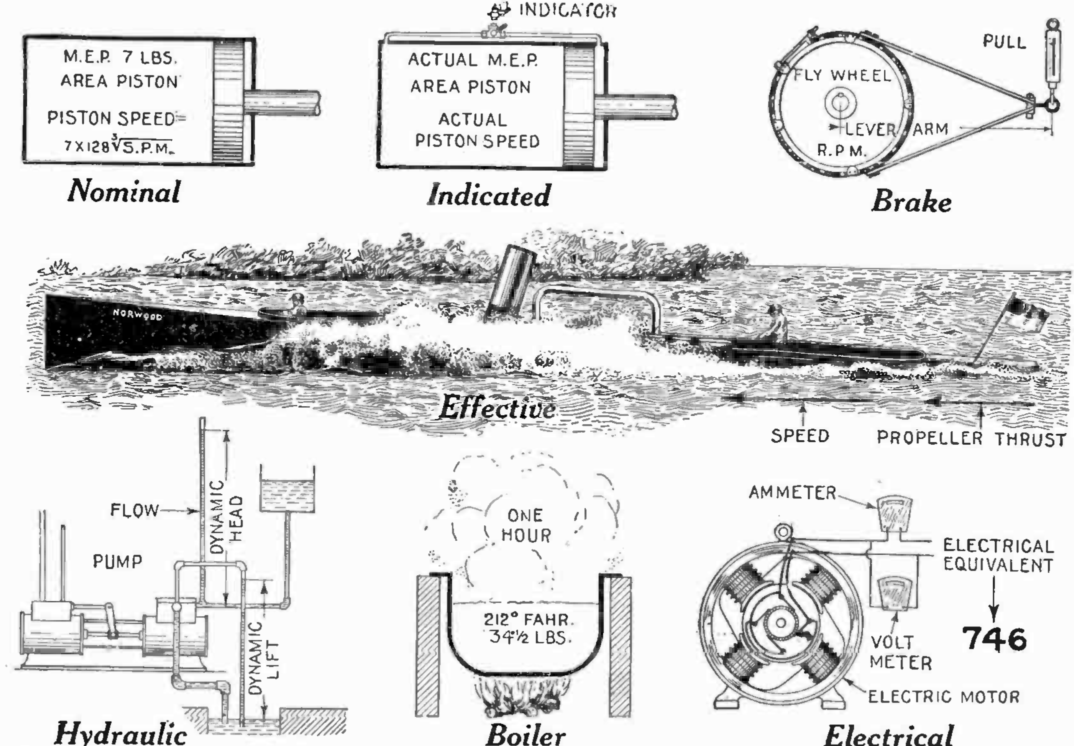
Indicated Horse Power.—This represents the power developed in the engine cylinder as measured by the indicator. It does not, however, represent the useful power delivered by the engine.

Brake Horse Power.—This is the *actual* horse power delivered by the engine to the shaft, being equal to the *indicated horse power minus the friction of the engine.* It is measured by applying a dynamometer or friction brake to the fly wheel (hence the name); though usually known as *brake* horse power it is sometimes called *delivered* horse power.

Effective Horse Power.—In any system of which the engine is only one element, the effective horse power is *the actual horse power given by or to the system.* Thus, in a locomotive, the effective horse power given by the system (locomotive) to the cars is:

$$\frac{\textit{pull on tender draw link} \times \textit{speed}}{33{,}000}$$

*NOTE.—***Nominal horse power.**—This term, which is due to Watt, has long been obsolete but it should, however, be understood. Watt found that the mean pressure usually obtained in the cylinders of his engines was 7 pounds per square inch. He had also fixed the proper piston speed at $128 \times \sqrt[3]{\text{strokes}}$ from which he calculated the power which would be developed = area of piston $\times 7 \times 128 \times \sqrt[3]{\text{strokes per minute}} \div 33{,}000$. With subsequent increases in boiler pressures with resulting increases in mean pressures, the difference between nominal horse power and the actual horse power developed by the engine has become so great as to render Watt's rating useless.



FIGS. 62 to 68.—Principal factors involved in the calculation of various kinds of horse power, as described in detail in the accompanying text.

Hydraulic Horse Power.—This may be defined as *the horse power required to elevate or force water against pressure;* it is as usually measured the indicated horse power at the water end of a pump. In calculations of hydraulic horse power required to elevate water, it is necessary to understand the difference between:

1. Static head,
2. Dynamic head,
3. Static lift,
4. Dynamic lift.

Static head is the difference in elevation between the pump and height to which the water is raised.

Dynamic head is the *equivalent* head allowing for increase in pressure due to the friction of the water in flowing through the pipes.

Static lift is the difference in elevation between the pump and the level of the water supply.

Dynamic lift is the equivalent lift allowing for the increase (due to friction) in pressure necessary to force the water from the supply to the pump.

Electrical Horse Power.—It is desirable to establish the relation between *watts* and *foot pounds* in order to determine the *capacity* of an electric generator or motor in terms of *horse power.*

Since one *watt* is equivalent to 44.244 foot pounds, the electrical equivalent of one horse power is:

$$33{,}000 \div 44.244 = 746 \text{ watts, or } .746\ k.w.$$

Again, one kilowatt or 1,000 watts is equivalent to

$$1{,}000 \div 746 = 1.34 \text{ horse power.}$$

How to Calculate Horse Power.—The old formula is

$$\text{H.P.} = \frac{2 \times P \times L \times A \times N}{33{,}000} = \frac{2\,(.7854D^2)\,PLN}{33{,}000} \quad \text{.......................(1)}$$

in which

P = mean effective pressure in lbs. per sq. ins.;
L = length of stroke in *feet;*
A = area of piston in sq. ins. = .7854 × diameter of piston squared:
N = number of *revolutions* per minute;
D = diameter of piston

Example.—What is the horse power of a 5×6 engine running at 500 revolutions per minute and 50 lbs. mean effective pressure?

Substituting these values in the formula, and remembering that the area A of the piston = .7854 × its diameter squared,

$$\text{H. P.} = \frac{2 \times (.7854 \times 5^2) \times 50 \times \frac{6}{12} \times 500}{33{,}000} = 14.87$$

It is a ridiculous waste of time to use this formula and attention is called to the following:

$$\textbf{H. P.} = .000004\ D^2\ L\ N\ P \quad . \ . \ . \quad (2)$$

This is simply formula (1) reduced to its lowest terms.

Example.—What is the horse power of the engine in the previous example (running under the same conditions), as calculated by formula (2)?

Substituting the given values in (2)

$$\text{H. P.} = .000004 \times 5^2 \times 6 \times 500 \times 50 = 15.$$

Comparing the two formulæ, 15 h. p. is here obtained instead of 14.87, the error introduced by using the constant .000004 instead of .000003966, being only

$$15 - 14.87 = .13 \text{ horse power or } \frac{86}{100} \text{ of } 1\%.$$

This short formula (2) is very valuable to those who have frequent occasions to calculate horse power. The power of any engine on a basis of of 500 revolutions and 50 lbs. mean effective pressure can be very quickly found with this formula and the method of using it, as given below, will firmly fix it in mind, though as before stated, the author does not recommend memorizing formulæ but instead, the acquirement of a knowledge of principles upon which they depend.

Now, for a quick calculation of the horse power of the engine in the previous example,

1. Write down the cylinder dimensions, squaring the diameter

$$(5^2 \times 6)$$

2. Disregard the decimal point and write 4 instead of .000004

$$4 \times (5^2 \times 6)$$

3. Insert the revolutions per minute and the mean effective pressure

$$4 \times (5^2 \times 6) \times 500 \times 50$$

The product of these factors is the horse power when the decimal point is inserted in the right place.

4. Since the product of the first and last two factors is 100,000, disregarding the ciphers, only the factors inside the parenthesis need be considered to obtain the horse power, thus

$$5^2 \times 6 = 5 \times 5 \times 6 = 150 \quad \ldots\ldots \quad (3)$$

It remains only to insert the decimal point, which is determined from the sense of proportion, that is, any one familiar with engines would know that a 5×6 engine running at 500 R. P. M. and 50 lbs. mean effective pressure does not develop 150 h. p. as written in equation (3); neither does it develop only 1.5 h. p.; it must then develop 15 h. p.

Short Formula.—

$$\text{horse power} = \frac{\text{diameter}^2}{2}$$

This is correct whenever the product of mean effective pressure and piston speed = 21,000, as in the following combinations:

Mean effective pressure.........	30	35	38 2 (Approx.)	42
Piston speed...................	700	600	550	500

Formula for Brake Horse Power.—The net work of the engine or horse power delivered at the shaft is determined as follows:

Let W = power absorbed per minute;

P = unbalanced pressure or weight in pounds, acting on the lever arm at a distance L;

L = length of lever arm in feet from center of shaft;

N = number of revolutions per minute;

V = velocity of a point in feet per minute at distance L, if arm were allowed to rotate at the speed of the shaft $= 2\pi L N$

$$\text{Since brake horse power} = \frac{P V}{33{,}000}$$

substituting for V,

$$\text{B. H. P. (brake horse power)} = \frac{2\pi L N P}{33{,}000} \quad \ldots\ldots \quad (1)$$

Simplifying, equation (I) becomes

$$B.H.P. = .0001903PLN$$

It should be noted in equation (I) that if $L = 33 \div 2$ the equation becomes

$$B.H.P. = \frac{2\pi}{33,000} \times \frac{33}{2\pi} \times N\,P = \frac{NP}{1,000} \quad \ldots\ldots (2)$$

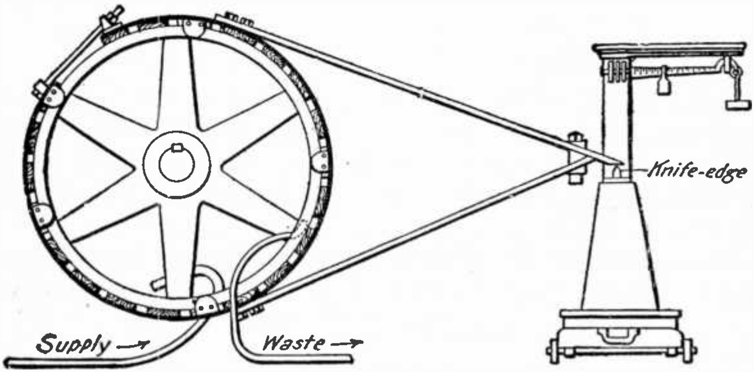


FIG. 69.—Prony brake. *It consists* of a friction band which may be placed around the fly wheel or the crank shaft, and attached to a lever bearing upon the platform of a weighing scale, as shown. A brake used for testing purposes should be self-adjusting to a certain extent, so as to maintain, automatically, a constant resistance at the rim of the wheel. For comparatively small engines, various forms of rope brake, satisfy this requirement very well. In such cases, a weight is hung to one end of the rope and a spring scale to the other end. The wheel should be provided with interior flanges, holding water for keeping the rim cool. For very high speeds, some form of water friction brake should be employed, as they have the advantage of being self-cooling.

Accordingly, in order to use the simplified formula (2) the arm L is made $33 \div 2\pi$ or 5.285 feet, very approximately 5 ft. $3\frac{1}{16}$ inches.

Size of Cylinder.—The first step in calculating the cylinder dimension is *to arrange the horse power formula in the proper form for obtaining the value of the unknown quantity.*

Thus, starting with the formula

$$H.\ P. = .000004\ D^2\ L\ N\ P \quad \ldots\ldots\ldots \quad (1)$$

the quantities to be found are D, the diameter of cylinder or piston, and L, the length of stroke. Accordingly, solving for these quantities

$$D^2 = \frac{H.\ P.}{.000004\ L\ N\ P},\ \text{or}\ D = \sqrt{\frac{H.\ P.}{.000004\ L\ N\ P}} \quad \ldots \quad (2)$$

$$L = \frac{H.\ P.}{.000004\ D^2\ N\ P} \quad \ldots\ldots\ldots \quad (3)$$

Example.—Find the size of cylinder of a Corliss engine to develop 85 horse power when running under the following conditions: Initial pressure, 80 lbs.; ¼ cut off; mean back pressure, 2 lbs. (non-condensing) diagram factor .9; piston speed 600 ft. per minute.

The solution consists of three steps, viz.: finding, 1, the mean effective pressure; 2, the stroke, and 3, the diameter of cylinder.

CASE 1. DIAGRAM FACTOR GIVEN

1. Mean effective pressure.

1. Find total number of expansions (neglecting clearance).†

Rule.—One divided by the reciprocal* of the cut off.

$$1 \div \frac{1}{\frac{1}{4}} = 1 \times 4 = 4$$

2. Find mean forward pressure.

Rule.*—Multiply initial pressure by 1+hyp. log. of expansions, and divide by number of expansions.*

†NOTE.—It should be understood that in the example the expression one-quarter cut off relates to the point of stroke at which steam is cut off by the valve gear; it does not represent the *real* cut off, with respect to the expansion of steam, because clearance must be considered, and on this account is, strictly speaking, called the *apparent* cut off. The economical range of horse power being considerable, correction for the apparent cut off need not ordinarily be made.

*NOTE.—The *reciprocal* of the cut off means one divided by the cut off.

From table page 229, hyp. log. of 4 = 1.3863.

$1 + \text{hyp. log. } 4 = 1 + 1.3863 = 2.3863$

initial pressure absolute $= 80 + 14.7 = 94.7$

$$\text{mean forward pressure} = \frac{94.7 \times 2.3863}{4} = 56.5 \text{ lbs. per sq. in.}$$

3. Find mean effective pressure.

Rule.—Subtract mean back pressure absolute from mean forward pressure, and multiply the difference by the diagram factor.

2 lbs. mean back (gauge) pressure = 2 + 14.7 = 16.7 lbs. absolute

$$(56.5 - 16.7) \times .9 = 35.8 \text{ lbs. per sq. in.}$$

2. *Choice of Stroke.*

The length of stroke must be such as will give a desirable number of revolutions, and bear a proper relation to the cylinder diameter. The Corliss engine is a slow speed or long stroke type, usual ratio of stroke to diameter being about 2 : 1 or more, hence of the several lengths of stroke that could be used, one should be selected that will come within the ratio limits and also give the proper speed in developing the rated power. Ordinarily the revolutions may be from 100 to 125, and with valve gears especially designed for high speed, 150 r. p. m., or higher. The revolutions or

$$\text{R. P. M.} = \text{piston speed} \div 2 \times \text{stroke (in feet)}$$

thus, for say, 24″ stroke and given piston speed of 600 feet

$$\text{R. P. M.} = 600 \div 2 \times \frac{24}{12} = 150$$

Similarly, the following table is obtained:

R. P. M. for 600 ft. piston speed

Stroke	24	30	36
R. P. M.	150	120	100

3. *Diameter of cylinder.*

The m. e. p. obtained in *1*, is 35.8 lbs. per sq. ins.; now inspecting the table in *2*, a trial may be made with the 36″ stroke which gives 100 r. p. m. Substituting the values in formula (2) page 114.

$$D = \sqrt{\frac{85}{.000004 \times 36 \times 100 \times 35.8}} = 12.8 \quad \ldots\ldots (a)$$

For the given power, this diameter of cylinder may be used with any stroke in the table in *2* at the revolutions given, that is the cylinder dimension may be

12.8×36 for 100 r. p. m.
12.8×30 for 120 r. p. m.
12.8×24 for 150 r. p. m.

calling the diameter 13 ins., in each case, the stroke diameter ratios are 2.77, 2.3, and 1.87 respectively, the first two being within limits and the last two small.

In the case of a growing plant where more power will be soon required the 13×36 would be desirable, as the r. p. m. could be increased considerably to increase the power. Ordinarily, the 13×30 would be desirable, as it would cost less, and would run at a more desirable r. p. m.

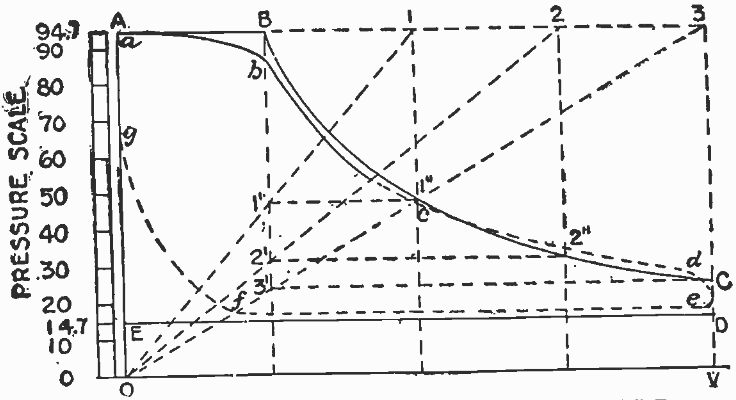


FIG. 70.—*The expected diagram.* Having drawn the theoretical card A B C D E, success in obtaining the proper diagram factor depends upon the experience and judgment of the designer which guides him in sketching in the expected diagram *a b c d e f g*, which he "expects" will represent the actual performance of the engine when built and operating under the specified conditions.

CASE 2. DIAGRAM FACTOR NOT GIVEN

A graphical solution of the example just given consists in drawing the theoretical card corresponding to the given values of initial pressure, cut off, etc., and inscribing in this diagram a card drawn to represent the "expected" performance of the actual engine. This card is drawn after considering a large number of actual cards of similar engines operating under similar conditions. Accordingly, the more experienced the designer, the nearer can he come to drawing a card that will represent the actual performance of the engine.

The steps in this graphical method are: 1, drawing the theoretical card; 2, drawing the expected card; 3, finding the expected m. e. p.; 4, finding the cylinder dimensions.

1. The theoretical and expected cards.

The appearance of these cards is shown in fig. 70. It will be noted that the area of the expected card is less than the area of the theoretical card, the difference representing the losses.

2. The expected M. E. P.

This is determined from the area of the expected card, fig. 70, by means of the following formula:

$$\textit{expected m. e. p.} = \frac{\textit{area of card}}{\textit{length of card}} \times \textit{pressure scale} \quad . \ . \ . \ . \ . \ (1)$$

The area is best obtained by use of a planimeter, or approximately by ordinates (figs. 75 and 76). In this case the area is by planimeter; length of card 4 ins., and pressure scale 40. Substituting these values in (1)

$$\textit{expected m. e. p.} = \frac{3.42}{4} \times 40 = 34.2 \text{ lbs. per sq. ins.}$$

3. Cylinder dimensions.

In Case I, a 36″ stroke was selected for future excess power demands, and a 30″ stroke where such provision was not made. Substituting the value 34.2 lbs. per sq. ins. *expected m. e. p.* in the formula for the 36 ins. stroke

$$D = \sqrt{\frac{85}{.000004 \times 36 \times 100 \times 34.2}} = 13.1 \text{ ins. say 13 ins.}$$

As in Case I, the cylinder dimensions could be either 13×36, or 13×30 according to conditions and judgment of the designer.

Steam Engines

Initial Pressure.—*The effect of increasing the initial pressure is to increase the economy* **provided** *that* **proper values** *be given to the* **other factors,** such as quality of the steam, degree of expansion, condensation, terminal pressure, temperature range, number of expansion stages, clearance, piston speed, etc.

Expansion of the Steam.—The equilateral or rectangular hyperbola referred to its asymptotes is taken to represent the expansion of steam.

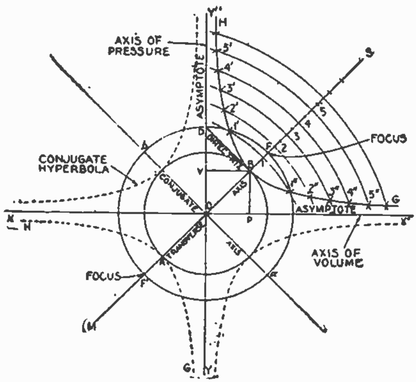
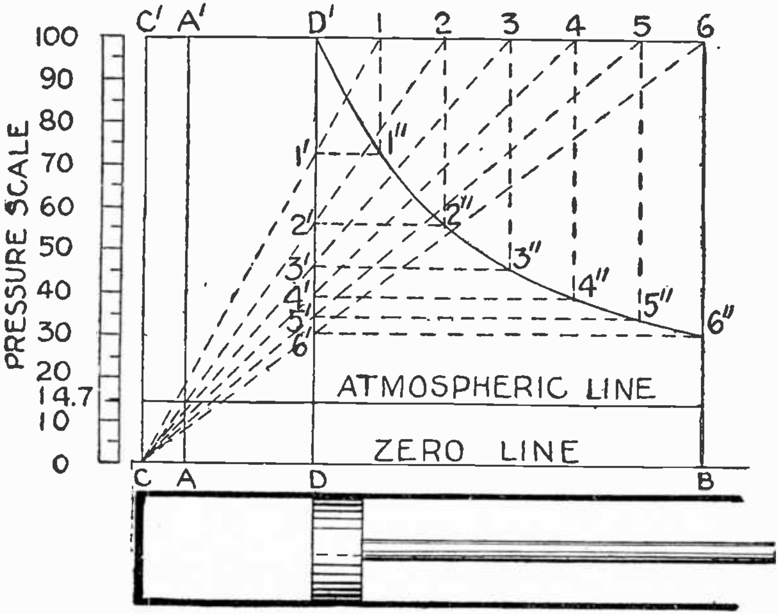


FIG. 71.—To describe an equilateral or rectangular hyperbola referred to its rectangular asymptotes. General method: Draw the axis of volumes, or horizontal asymptote X X′, and the axis of pressures, or vertical asymptote Y Y′, cutting X X′, at O, or hyperbolic center. Through O, draw M S, at 45° to X X′. Take any point on M S, as B, and with radius O B, describe a circle, cutting M S, in B and A, giving A B, the transverse axis. At B, erect a perpendicular cutting Y Y′, at D, giving O D, the directrix. With O D, as radius describe a circle cutting M S, at F and F′; these points are the *foci* of the hyperbola. On M S, take any number of points 1, 2, 3, etc., and from F, and F′ as centers, with A1, B1, A2, B2, etc., as radii, describe arcs cutting each other in 1′, 2′, 3′, etc., and 1″, 2″, 3″, etc., through which points the branch H B G, of the hyperbola is described. Similarly, the other branch H A G, and conjugate hyperbola shown in dotted lines may be described, but for the purpose in view, only one branch H B 6 need be described. It is a property of the hyperbola referred to its rectangular asymptotes, as above, that if one asymptote as X X′, be taken as an axis of volumes and the other an axis of pressures measured from the intersection O, then for any point on the curve as B, the product of its distance from Y Y′, multiplied by its distance from X X′, =constant, that is B V×B P =constant, or pressure×volume =constant which is in accordance with Boyle's law.

Rule. *The terminal pressure equals the initial pressure divided by the number of expansions.*

Thus, if the initial pressure be 100 lbs. absolute, and the number of expansions 4,

terminal pressure = 100 ÷ 4 = 25 lbs. absolute.



FIGS. 72 and 73.—To describe hyberbolic curve for expansion, 2nd method: Draw the zero line A B = stroke, and extend it to C, making A C = clearance. Take the distance C C′ = admission pressure, and at the point of cut off D, erect the perpendicular D D′, giving the admission line A′ D′. Extend C′ D′, and lay off any number of points 1, 2, 3, etc. From C, draw radial lines, C1, C2, C3, etc. Draw horizontal lines through 1′, 2′, 3′, etc., and vertical lines through 1, 2, 3, etc. Their intersections 1″, 2″, 3″, are points on the hyperbolic curve.

Example.—If the initial pressure be 100 lbs. gauge, and the number of expansions be 4, what is the terminal gauge pressure?

100 lbs. gauge = 100 + 14.7 = 114.7 lbs. absolute.

114.7 ÷ 4 { = 28.68 lbs. absolute, or
= 28.68 − 14.7 = 13.98 gauge.

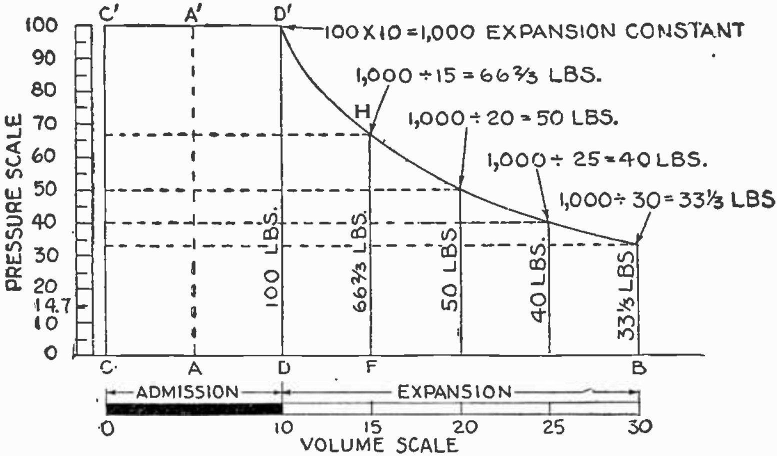


FIG. 74.—Diagram illustrating the expansion constant *and its use.* According to Boyle's law, *pressure×volume=constant.* If, as indicated in the diagram, steam be admitted to a cylinder during 10 inches of the stroke and expanded to 30 inches, the expansion constant = 100×10=1,000, from which the pressure at any other point =*constant÷volume*, that is, when the piston is at

15 ins.	20 ins.	25 ins.	30 ins.

of the stroke, the *expansion constant÷volume* is

1,000÷15	1,000÷20	1,000÷25	1,000÷30

which is equal to

66⅔ lbs.	50 lbs.	40 lbs.	33⅓ lbs.

Similarly *volume=constant÷pressure*, that is, when the pressure due to the expansion is

66⅔ lbs.	50 lbs.	40 lbs.	33⅓ lbs.

the *expansion constant÷pressure* is

1,000÷66⅔	1,000÷50	1,000÷40	1,000÷33⅓

which is equal to

15 ins.	20 ins.	25 ins.	30 ins.

Expansion Constant.—To determine the pressure at any point of the stroke, use is made of a constant found by multiplying the volume of steam at cut off by the initial pressure.

For instance, if steam at 80 lbs. absolute pressure be cut off when th piston has moved 10 inches of the stroke, then

volume×pressure=constant

substituting the above values

$$10 \times 80 = 800$$

Rule. *The pressure at any point of the stroke equals the expansion constant divided by the volume at that point.*

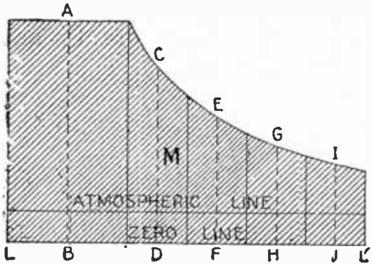
Thus, when the piston has passed through 20 inches of the stroke the pressure at that point is

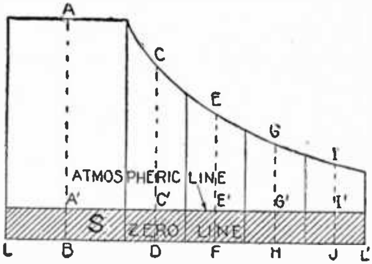
$$800 \div 20 = 40 \text{ lbs. absolute.}$$

Rule. *The volume corresponding to any pressure is equal to the expansion constant divided by the pressure at that point.*

Thus when the pressure has decreased to 40 lbs. absolute, the volume corresponding as measured by the piston movement is

$$800 \div 40 = 20 \text{ inches}$$





FIGS. 75 and 76.—Theoretical cards illustrating ***mean forward pressure*** and ***back pressure.*** The two cards are the same as the card in fig. 74. If in fig. 74 an ordinate be drawn through the middle of each of the areas C D C′ D′, D′ H F D, etc., they will appear in fig. 75 as the dotted vertical lines A B, C D, E F, etc. ***The mean forward pressure*** represented by the area of M ÷ its length L L′ × the pressure scale, is equal to the average of the ordinates, that is, their sum divided by the number, and multiplied by the pressure scale, or $\frac{(A B + C D + E F + G H + I J)}{5}$ × pressure scale. Similarly in fig. 76, ***the back pressure,*** = area S ÷ its length L L′ × pressure scale = $\frac{A' B + C' D + E' F + G' H + I' J}{5}$ × pressure scale, or since in this case all are of the same length, back pressure = height of S × pressure scale.

Mean Effective Pressure.—Horse power calculations are based upon this calculation and it is equal to

mean forward pressure − mean back pressure

Rule.—*To find the mean effective pressure, multiply the initial pressure* **in lbs. absolute** *by 1 + hyp. log. of the number of expansions, and divide by the number of expansions. From the quotient, subtract the* **absolute** *back pressure.*

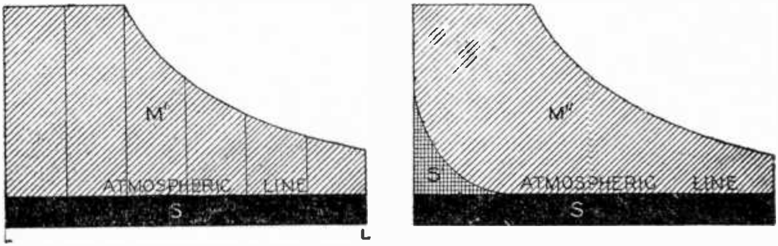
In the form of an equation the mean effective pressure or

$$\text{M.E.P.} = \frac{\textit{initial pressure abs.} \times 1 + \textit{hyp.log.no. of expansions}}{\textit{number of expansions}} - \textit{back pressure abs.}$$

or, expressed in the usual symbols

$$\text{M. E. P.} = \frac{\text{P} \times 1 + \text{hyp. log. } r}{r} - \text{B. P.}$$

It should be remembered that the initial pressure P, and back pressure B. P., are taken *in lbs.* **absolute.**



FIGS. 77 and 78.—Theoretical cards illustrating ***mean effective pressure:*** fig. 77, constant back pressure; fig. 78, variable back pressure. Since the back pressure directly opposes the forward pressure, evidently the net or actual pressure tending to move a piston, or mean effective pressure is the difference between these two pressures, that is, *M. E. P. = mean forward pressure — back pressure.* In fig. 75 the mean forward pressure is figured from the area M, and in fig. 76 the back pressure from the area S, hence, the mean effective pressure must depend on the difference of these two areas, that is M — S, or M′ as shown in fig. 77, *when the back pressure is constant.* Where compression is taken into account, as in fig. 78, evidently *M. E. P. = mean forward pressure — mean back pressure,* but in the figure, the mean back pressure is figured from area S + area S′, hence, in this case M. E. P. depends on the difference between area M in fig. 75 and areas S + S′ in fig. 78, and giving the area M″ where *average ordinate × pressure scale = M. E. P.*

Example.—What is the mean effective pressure, with 80 lbs. initial gauge pressure, one-third cut off, 16 lbs. absolute back pressure?

Initial pressure absolute $= 80 + 14.7 = 94.7$ lbs.

Number of expansions $= 1 \div {}^1/_3 = 1 \times {}^3/_1 = 3$.

Hyp. log. of 3 (from table page 229) $= 1.0986$.

$1 + \text{hyp. log } 3 = 1 + 1.0986 = 2.0986$.

$$\text{Mean effective pressure} = \frac{94.7 \times 2.0986}{3} - 16 = 50.2 \text{ lbs.}$$

It is not possible in steam engines to convert all the energy of the steam into useful work, see fig. 70. There are various losses

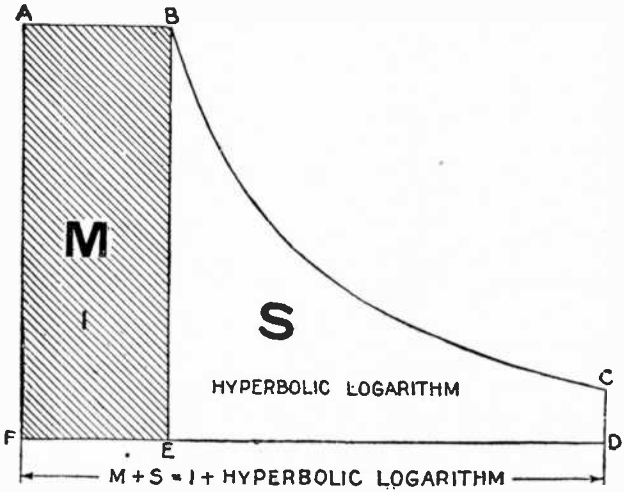


FIG. 79.—Theoretical card showing application of the *hyperbolic logarithm* in finding the *mean effective pressure*. Total area = M + S = 1 + *hyperbolic logarithm.*

due to leakage, radiation, condensation and other causes, all of which tend to make the actual mean effective pressure obtained in an engine less than that calculated.

Diagram Factor.—Due to losses the indicator card is always *less* in area than the theoretical card and the ratio of the two areas is called the diagram factor, that is

$$\textbf{\textit{diagram factor}} = \frac{\textit{area of actual card}}{\textit{area of theoretical card}}$$

Table for Finding Mean Pressure

Number of expansions	$\frac{1+\text{hyp. log. } r}{r}$	Number of expansions	$\frac{1+\text{hyp. log. } r}{r}$
1.0	1.00	11.0	0.309
1.5	0.937	12.0	0.290
2.0	0.847	13.0	0.274
2.5	0.766	14.0	0.260
3.0	0.700	15.0	0.247
3.5	0.644	16.0	0.236
4.0	0.597	17.0	0.226
4.5	0.556	18.0	0.216
5.0	0.522	19.0	0.208
5.5	0.492	20.0	0.200
6.0	0.465	21.0	0.192
7.0	0.421	22.0	0.186
8.0	0.385	23.0	0.180
9.0	0.355	24.0	0.174
10.0	0.330	25.0	0.169

Diagram Factors

Particulars of Engine	Diagram Factor	
Expansive engine, special valve gear, or with a separate cut off valve, cylinders jacketed........	.94	.9
Expansive engine having large ports, etc., and good ordinary valves, cylinders jacketed..............	.9 to .92	.86 to .88
Expansive engines with the ordinary valves and gear as in general practice and unjacketed.............	.8 to .85	.77 to .82
Compound engines, with expansion valve to H. P. cylinder; cylinders jacketed, and with large ports, etc..................................	.9 to .92	.86 to .88
Compound engines, with ordinary slide valves, cylinders jacketed, and good ports, etc..	.8 to .85	.77 to .82
Compound engines as in general practice in the merchant service, with early cut off in both cylinders without jackets and expansion valves.............	.7 to .8	.67 to .77
Triple expansion engines, with ordinary slide valves, good ports, unjacketed, moderate piston speed....	.65 to .7	.62 to .67
Fast running engines of the type and design usually fitted in war ships............................	.6 to .7	.58 to .67

NOTE.—If the theoretical mean pressure be calculated and the necessary corrections made for *clearance* and *compression*, according to Seaton the expected mean effective pressure may be found by multiplying the results by the factor in the first column of the diagram factor table.

Table Showing Effect of Changes in Lap, Throw, Angular Advance, Etc.

Event	Period	Increasing *outside* lap	Increasing *inside* lap	Increasing *angular advance*	Increasing *throw*	Increasing *angular advance*, decreasing *throw*: *Shifting eccentric*	*Swinging eccentric*: Pin on crank	*Swinging eccentric*: Pin opposite crank	*Swinging eccentric*: Offset pin on crank
lead	....	reduced	unchanged	increased	increased	constant	increased	decreased	small increase
....	pre-admission	reduced	unchanged	increased	increased	unchanged	increased	decreased	small increase
port opening ...	....	reduced	unchanged	unchanged	increased	reduced	reduced	decreased	decreased
cut off	....	earlier	unchanged	earlier	later	earlier	earlier	earlier	earlier
....	admission	reduced	unchanged	unchanged	increased	reduced	reduced	reduced	reduced
....	expansion	increased	increased	unchanged	reduced	increased	increased	increased	increased
pre-release	....	unchanged	later	earlier	unchanged	earlier	earlier	earlier	earlier
....	exhaust	unchanged	reduced	unchanged	unchanged	unchanged	unchanged	unchanged	unchanged
exhaust opening	....	unchanged	reduced	unchanged	increased	reduced	reduced	reduced	reduced
compression	....	unchanged	earlier	earlier	unchanged	earlier	earlier	earlier	earlier

Valve Gears

The accompanying table shows the effect of changes in design.

Valve gear problems are best solved by use of the ***Bilgram diagram*** as shown in fig. 81.

Problem.—A 7×7 engine is to be run at 450 revolutions per minute. What are the principal dimensions of the slide valve and ports for a steam velocity of 8,000 feet per minute through the port opening and 6,000 feet through the ports? Lead 1/16 inch, cut off ¾, release .9 stroke, length of ports .8 the diameter of the cylinder, and length of connecting rod 2½ times the stroke.

NOTE.—***Zeuner diagram.*** The leading data that are given in designing a valve motion are the point of cut off, the port opening, and the lead of the valve (not the lead angle of the crank, as is often conveniently assumed). It is the radical defect of the Zeuner diagram that none of these dimensions can be laid off from known points. The lead must be laid off from an unknown point of the center line, and the port opening from an unknown point on an unknown line. Finally, through these unknown points and the center of the shaft the valve circle is to be drawn from an unknown center and with an unknown radius. Under these circumstances the result sought is found only through blind trial.

1. Find area and dimensions of port opening and port.

$$\textit{area port opening in sq. ins.} = \frac{\textit{area piston in sq. ins.} \times \textit{piston speed in feet}}{8{,}000}$$

$$= \frac{(7^2 \times .7854) \times (450 \times 7/12 \times 2)}{8{,}000} = 2.53 \text{ sq. in.}$$

Since the velocity of the steam through the port is reduced to 6,000 feet per minute, it is made larger than the port opening in the proportion of 8,000 ÷ 6,000, or

$$\textit{area of port} = 2.53 \times \frac{8}{6} = 3.37 \text{ sq. in.}$$

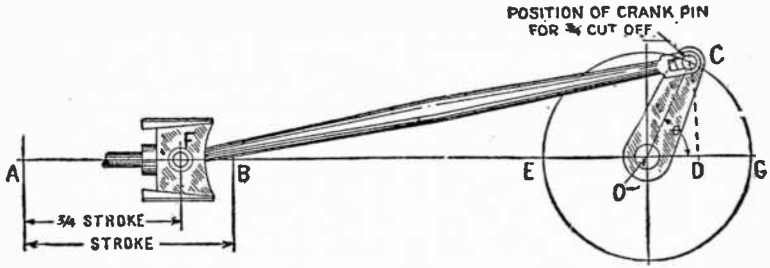


Fig. 80.—Diagram for finding the crank position corresponding to a given piston position. Since the piston and cross head move in unison, the latter is used. With the wrist pin as center, and connecting rod as radius, an arc D C, is described cutting the crank pin circle in C. A line connecting the center of the circle with C, will give the corresponding crank position.

The length of the ports is made .8 the cylinder diameter, or

Length of ports = 7 × .8 = 5.6, say 5½ ins.

Width of steam ports = area ÷ length

or

3.37 ÷ 5½ = .612, say ⅝ in.

Width of port opening =

$$\frac{5}{8} \times \frac{6{,}000}{8{,}000} = \frac{15}{32}, \text{ say } \frac{1}{2} \text{ in.}$$

2. Find the position of the crank for ¾ cut off.

In fig. 80 draw a horizontal line, and near one end describe a circle with any radius as O E. Measure off the length of the connecting rod A E = 5 × O E, and A B = 2 × O E. A B, then is the length of the stroke.

Find the position of the cross head F, when the piston is at ¾ stroke by taking A F = ¾ A B. With F, as center and radius F D = A E, describe an arc cutting the circle in C. This is the position of the crank pin when the piston is at ¾ stroke.

3. Find the lap, linear advance and travel of the valve.

Use is here made of the *Bilgram diagram,* and the successive steps in its application are as follows:

a. A horizontal line K N, is drawn, and the crank position for ¾ cut off transferred from fig. 80 to fig. 81.

b. Draw a line M S parallel to K N at a distance above ($\frac{1}{16}$ in.) equal to the lead; M S, then is the lead line.

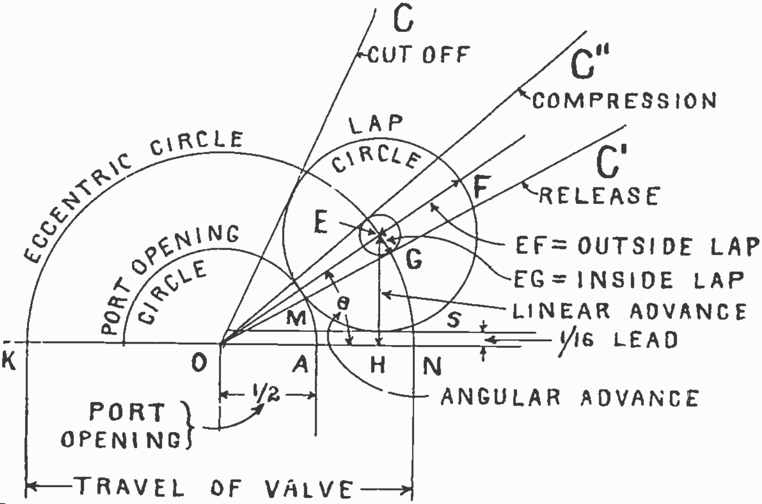


FIG. 81.—*Bilgram diagram* for finding the lap, angular advance travel, etc., of the slide valve. With this diagram, any valve problem may be easily and quickly solved.

c. With a radius O A, equal to the port opening (½ in.), describe the port opening circle.

d. Now, find by trial the radius E F, and center E, of a circle that shall be tangent to the lead line M S, the port opening circle, and the cut off line O C. *The radius* E F, *of this circle is the outside lap.*

e. Draw E H perpendicular to K N, then *the distance* E H *is the linear advance.*

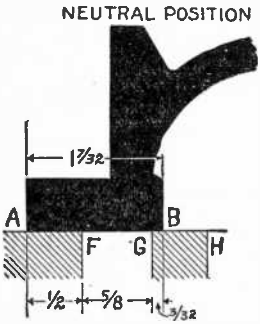
f. Now by the method of fig. 80, find the crank position for release at .9 stroke. In fig. 81, draw this crank position O C′, and a circle tangent to it with center E G. *The radius* E G, *of this circle is the inside lap.*

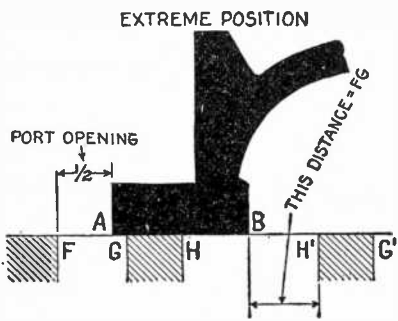
g. A second tangent O C″, to this circle gives the crank position for compression.

Measuring the diagram, the dimensions for the valve are

Outside lap = ½ *in.*	*Linear advance* = 9/16 *in.*
Inside lap = 3/32 *in.*	*Travel of valve* = 2 *ins.*

With the dimensions just obtained and the given data, the valve and ports may be laid down in the following manner:





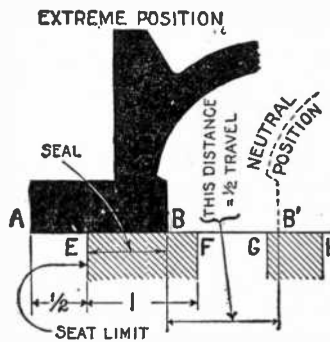


FIG. 82.—*How to lay out the slide valve.* I. With the dimensions obtained by the Bilgram diagram the length of the face is determined by sketching one end of the valve in its central or *neutral* position as here shown.

FIG. 83.—*How to lay out the slide valve.* II. The width of the exhaust port is obtained by sketching one end of the valve in extreme position for admission. Evidently H', must be so located that B H' = F G, in order not to choke the exhaust.

FIG. 84.—*How to lay out the slide valve.* III. The seat limit is obtained by sketching one end of the valve in extreme position for exhaust as shown and locating the seat limit E between A B, at some point which will give sufficient contact E B, for a tight joint, this distance being called the *seal.*

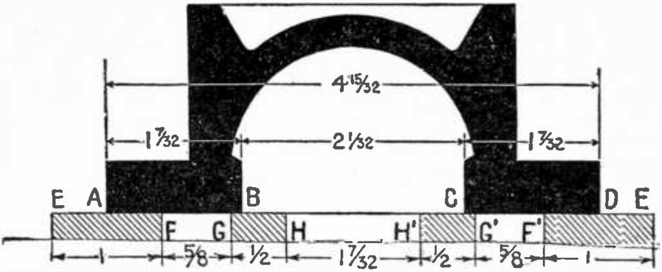
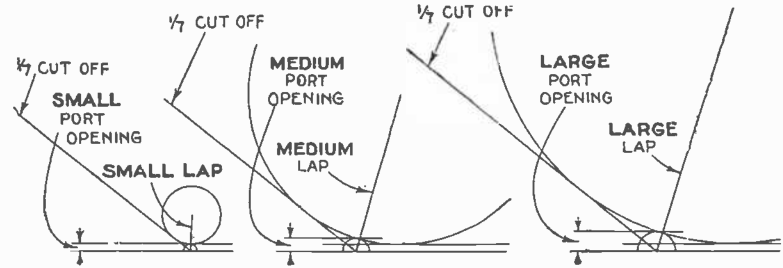
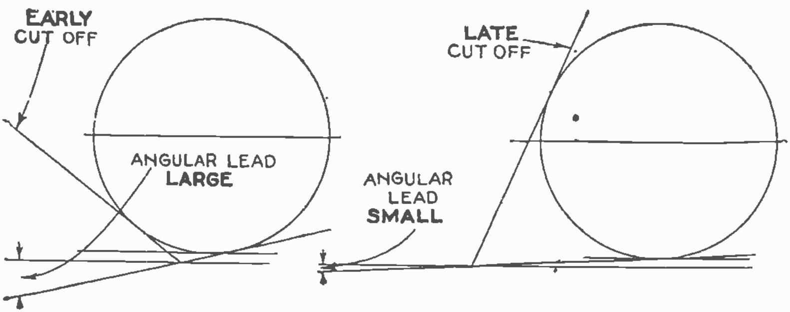


FIG. 85.—Dimensioned drawing of slide valve from measurements obtained in the diagram on page 127. In the drawing all the longitudinal dimensions necessary for proportioning the steam features are given. The thickness of valve walls will depend on the type of engine, steam pressure, etc.



FIGS. 86 to 88.—Bilgram diagrams illustrating influence of lap on the port opening for early cut-off. It is evident from the diagrams that *the greater the lap the greater the port opening at early cut off*.



FIGS. 89 and 90.—Effect on the lead of variable cut off by shifting eccentric: *As the cut off is shortened by shifting the eccentric, the lead is increased*, that is, the point of pre-admission occurs earlier.

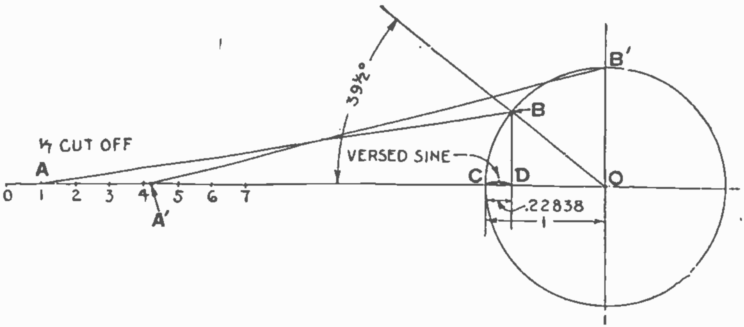


FIG. 91.—Instantaneous velocity of the piston at the point of 1/7 cut off (connecting rod ratio 2:1). Lay off connecting rod position A B for 1/7 cut off giving crank angle COB = 39½° as measured by protractor, and position A′ B′ for crank angle of 90°. OB′ is crank position of maximum piston velocity because at this instant the crank pin and wrist pin are moving in parallel paths, and the actual velocity of the piston at this instant = velocity of crank pin = (34.558 × 250) ÷ 12 = 720 ft. per min. neglecting angularity of the connecting rod. Now assuming the velocity of the piston to vary as the versed sine of the crank angle, then piston velocity at point A, or 1/7 cut off = maximum velocity × ver. sin. 39½° = 720 × .22838 = 164.4 ft. per min.

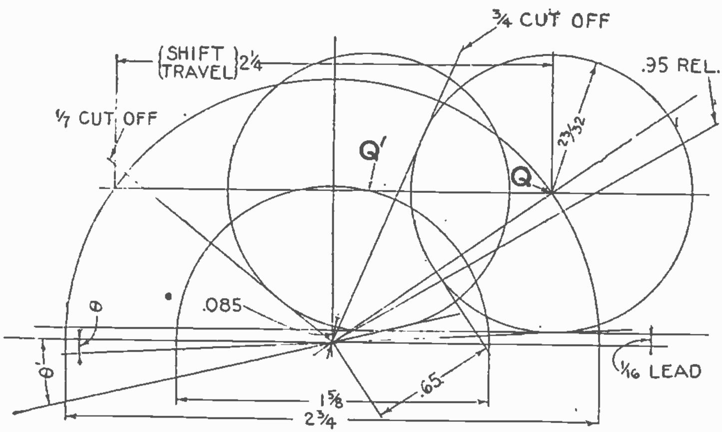
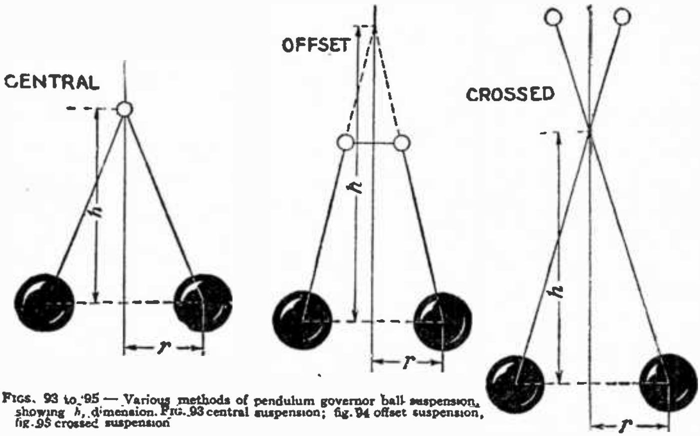


FIG. 92.—Bilgram diagram for straight slot shifting eccentric. *Given:* maximum cut off ¾; minimum cut off 1/7; lead; 1/16. *By measurement:* Lap 23/32; port opening at ¾ cut off, .65; port opening at 1/7 cut off, .085. It should be noted that in the operation of the straight slot eccentric under the conditions of the above diagram, the angle of pre-admission changes from θ to θ' as the center of the eccentric moves from Q to Q′ in shortening the cut off from ¾ to 1/7.

Governors

Centrifugal Governors.—The action of governors of this type depends upon *the change of centrifugal force when the rate of rotation changes.*

In designing a governor the first step is naturally to assume an allowable speed variation and then compute *h* (figs. 93 to 95) for maximum and minimum speed. Then by knowing the rise and fall of the balls, the movement of the collar upon the stem can be determined, and the desired movement of the regulating device be obtained by interposing the proper gearing.



FIGS. 93 to 95 — Various methods of pendulum governor ball suspension, showing *h*, dimension. FIG. 93 central suspension; fig. 94 offset suspension, fig. 95 crossed suspension

The expression for the total centrifugal force horizontally outward is:

$$C = 12\ W\ v^2 \div gr \qquad (1)$$

in which

C = total centrifugal force in lbs.;
W = weight of both balls in lbs.;
v = tangential velocity of balls in feet per second;

g = acceleration due to gravity = 32.16;
r = radius of rotation of the balls, in inches.

Now

$$Wr = Ch \quad \ldots \ldots \ldots \ldots \ldots (2)$$

Substituting for C, its value as in (1),

$$Wr = 12\ W\ v^2\ h \div gr \quad \ldots \ldots \ldots \ldots \quad (3)$$

from which

$$h = gr^2 \div 12\ v^2 \quad \ldots \ldots \ldots \ldots \quad (4)$$

Now the tangential velocity v, of the balls for any rotative speed R. is

$$v = 2\ \pi\ r\ R \div (12 \times 60) = \pi\ r\ R \div 360$$

and since $g = 32.16$, substituting in (4)

$$h = 32.16\ r^2 \div 12\ (\pi\ r\ R \div 360)^2 = 35{,}191.7 \div R^2 \quad \ldots \ldots \ldots \quad (5)$$

Example.—In an engine running at 200 r.p.m., the governor is to run at half that speed and have a "regulation" within 4 per cent.

Four per cent regulation means that the maximum speed of the engine is to be

$$200 \times 1.02 = 204 \text{ r. p. m}$$

and the minimum speed

$$200 \times .98 = 196 \text{ r. p. m.}$$

Then the corresponding maximum and minimum speeds of the governor will be half these values or 102 and .98 r. p. m. respectively. Substituting these values in (4), for maximum speed

$$h = 35{,}191.7 \div 102^2 = 3.38$$

and for minimum speed

$$h = 35{,}191.7 \div 98^2 = 3.66$$

which means that the balls must rise and fall vertically a distance of

$$3.66 - 3.38 = .28 \text{ inch}$$

for a total variation of 4 per cent in the speed of the engine, that is for 4 per cent regulation. The corresponding movement of the collar upon the governor shaft will depend upon the lengths of the different arms and connecting levers and may be determined graphically.

Efficiency

Efficiency.—In general the term efficiency may be defined as *the ratio of the useful work performed by a prime mover to the energy expended*, that is, *the* **output** *divided by the* **input.**

There are several kinds of efficiency.

a. Thermal·

b. Mechanical;

c. Combined;

d. Net or commercial;

e. Volumetric.

Thermal Efficiency.—A *perfect* engine converting all the heat energy of the steam into work would require:

$$33{,}000 \text{ foot pounds} \div 777.54 = 42.44 \; B.t.u.$$

per minute per indicated horse power. This figure 42.44 divided by the number of *B.t.u.* per minute per *i.h.p.*, consumed by an engine gives its thermal efficiency, that is, its efficiency compared with a perfect engine; expressed as an equation:

$$\text{thermal efficiency} = \frac{42.44}{B.t.u. \text{ per min. per } i.h.p.}$$

In practice the thermal efficiency is always less than 1, the difference representing the per cent losses, as by radiation, condensation, wire drawing, leakage, drop, etc.

Mechanical Efficiency.—*In any engine the power delivered to the shaft* of the moving parts. Accordingly ***the power delivered to the shaft will always be less than the power applied to the piston,*** the difference being in amount equal to the power lost by friction. Thus, in a certain engine 5 per cent of the power is lost by friction, accordingly the power delivered to the shaft is 100% — 5% = 95% of the power supplied to the piston.

This percentage, 95%, or .95, as it is usually expressed, represents the ratio of the power delivered to the power applied and is known as the mechanical efficiency.

By definition then, ***mechanical efficiency*** is the *ratio of the power delivered to the power applied*, that is, the power delivered to the shaft.

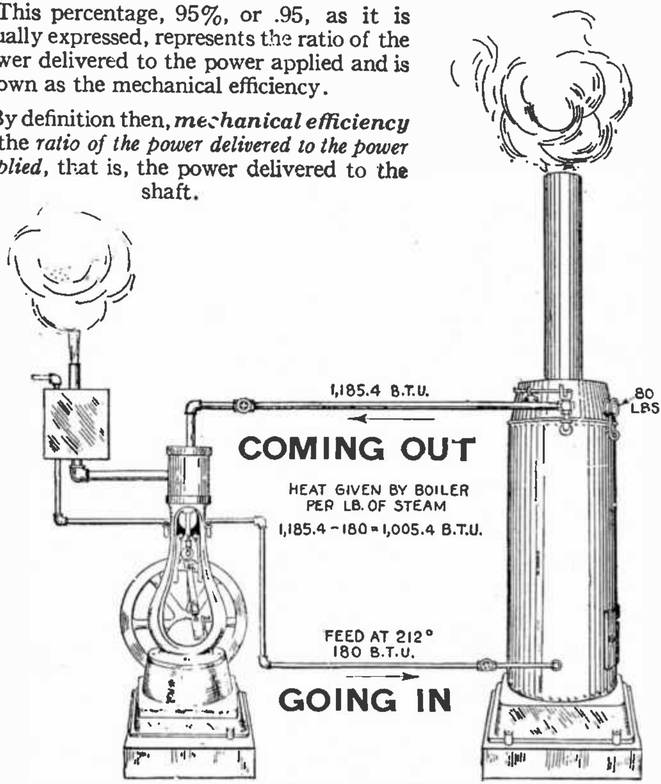


FIG. 96.—***Thermal efficiency.*** The heat chargeable to an engine per lb. of steam is the difference between the total heat in a lb. of steam (***coming out***) at the boiler pressure and that in a lb. of the feed water ***going in*** the boiler. In the above case then, heat given by the boiler per lb. of steam = 1005.4 *B.t.u.* Now the *B.t.u.* per ***minute*** used by the engine per lb. of steam per *i.h.p.* hour = 1005.4 + 60 = 16.76 and if the engine require 50 lbs. of steam per *i.h.p.* hour, then *B.t.u.* per *minute* per *i.h.p.* = 16.76 × 50 = 838, and the ***thermal efficiency***—42.44 + 838 = .05, that is 5%.

divided by the power applied to the piston—in other words, the brake horse power divided by the indicated horse power. Evidently then, the steam consumption of an engine as usually stated is only the *apparent* consumption and to obtain the true consumption the mechanical efficiency must be considered.

Example.—A certain engine developing 100 indicated horse power, requires 25 pounds of steam per indicated horse power hour and its mechanical efficiency is .87. What is the real steam consumption?

The mechanical efficiency being .87, the power delivered from the shaft will be reduced to

$$100 \times .87 = 87 \text{ horse power}$$

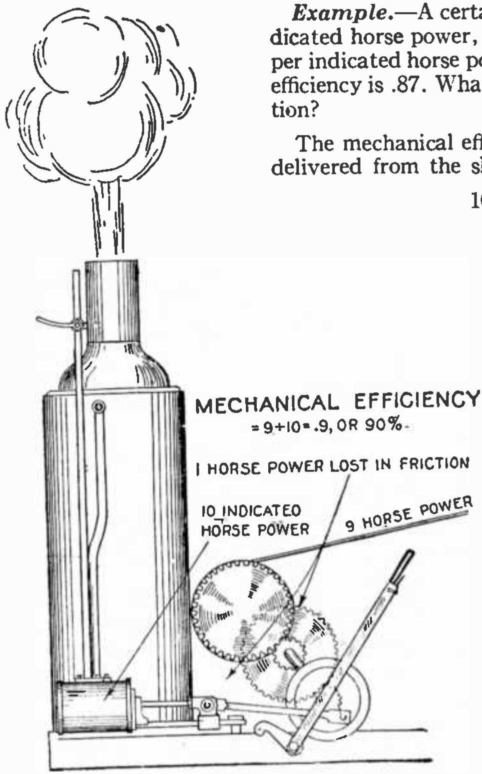


FIG. 97.—Hoisting engine illustrating mechanical efficiency. The power delivered at the drum is always *less* than the power applied to the piston by the steam; this difference, in amount being equal to the friction of all the working parts from the piston to the drum.

but the total steam supplied to the engine per hour is

$$100 \times 25 = 2{,}500 \text{ pounds}$$

and as only 87 horse power is delivered to the shaft, the true steam consumption is

$$2{,}500 \div 87 = 28.7 \text{ pounds}$$

instead of 25 pounds.

In general the mechanical efficiency of engines varies from .85 to .95.

Boiler Efficiency.—In general, the efficiency of the boiler is defined as *the percentage of the total heat generated by the combustion of the fuel which is utilized in heating the water and generating the steam.* This is evidently the combined or overall efficiency, being the product of:

1. Efficiency of the grate.
2. Efficiency of the furnace.
3. Efficiency of the heating surface.
4. Efficiency of the boiler covering or insulating material.

In determining the "efficiency" of a boiler (meaning the combined efficiency of the 4 above factors) the ratio is found between *the heat absorbed per pound of dry coal and the heat (calorific) value of one pound of dry coal*, or the ratio of *the two based on coal as fired.**

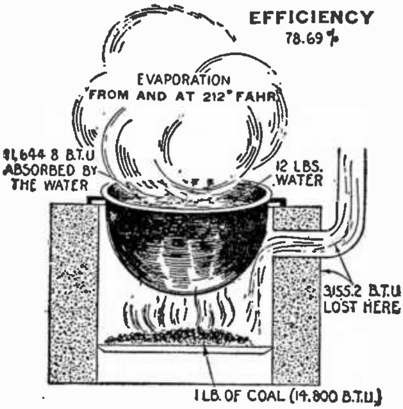


FIG. 98.—Evaporation *"from and at 212° Fahr."*, illustrating **boiler efficiency** as explained in the text above. Complete combustion is assumed.

Example:—The equivalent steam consumption (from and at 212°) of a certain engine of 100 *i.h.p.* is 20 pounds per horse power hour. Anthracite coal having a heating value of 14,800 *B.t.u.* per pound is used. Assuming 15% passes through the grate as ash and unburned coal, how much coal is required per hour to run the engine?

Total steam required to run the engine is

$$20 \times 100 = 2{,}000 \text{ pounds per hour}$$

Since 15% of the coal passes through the grate the percentage available for combustion is

$$100\% - 15\% = 85\%$$

Since the heating value 14,800 *B.t.u.* is reduced to

$$14{,}800 \times .85 = 12{,}580 \; B.t.u. \text{ per pound of coal fired}$$

and the theoretical evaporation (from and at 212°) under these conditions is

$$12{,}580 \div 970.4 = 12.96 \text{ pounds per pound of coal fired,}$$

the amount of coal required to run the engine is

$$2{,}000 \div 12.96 = 15.43 \text{ pounds per hour}$$

*NOTE.—The weight of *combustible* is sometimes used instead of weight of coal. The term *combustible* means the portion of the fuel that is burned—the total weight of coal fired less that which passes through the grate. Accordingly efficiency based on combustible furnishes an approximate means for comparing the results of different tests, when the losses of unburned coal due to grates, cleaning, etc., are eliminated. In coal analysis combustible is defined as *the fixed carbon and volatile matter, the moisture and ash being excluded.* By some writers it is called *coal dry and free from ash*, and by others, *pure coal.*

Combined Efficiency.—In any system comprising several elements, as an engine and boiler, an engine connected to a dynamo, or to a pump, etc., the combined efficiency (sometimes called the overall efficiency) is the factor which includes the efficiency of each element. By definition, the ***combined efficiency*** of a system composed of several elements is ***the product of the efficiencies of each element.*** Thus in a system composed of two elements, engine and boiler, the combined efficiency is the product of the engine efficiency multiplied by the boiler efficiency.

Example.—In a certain pumping plant, as shown in fig. 99, there is a power pump connected by belt drive to an engine. If the efficiency of the various elements be: boiler 73%; engine: thermal 15%, mechanical 95%, belt drive 98%, pump 95%, what is the combined or overall efficiency?

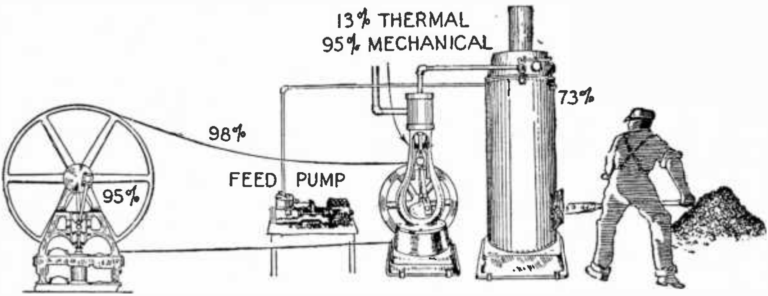


FIG. 99.—Diagram of simple steam plant illustrating combined efficiency. In the diagram 73% represents the efficiency of the boiler, including the pump.

The product of the above efficiencies is the combined efficiency, that is

$$\underset{\text{boiler}}{.73} \times \underbrace{.15 \times .95}_{\text{engine}} \times \underset{\text{belt}}{.98} \times \underset{\text{pump}}{.95} = .097 \text{ or } 9.7\%$$

Example.—In a 25 horse power 110 volt direct connected generator set the mechanical efficiency of the engine is .92 and efficiency of dynamo .9. What is the energy and current output of the dynamo?

As usually rated the value for power refers to the engine and here means 25 indicated horse power.

The combined efficiency then of the two elements is

$$.92 \times .9 = .83$$

Hence the energy output of the dynamo is

$$25 \times .83 = 20.8 \text{ horse power or}$$
$$.746 \times 20.8 = 15.5 \text{ kw}$$

The current output is

$$15.5 \times 1.000 \div 110 = 140.9 \text{ amperes.}$$

Net or Commercial Efficiency.—If the entire plant be regarded not merely from the standpoint of *combined efficiency* of the power system but also takes into consideration its *cost, depreciation, maintenance, etc.*, a true idea of the actual cost of the power is obtained. A factor may be found which takes into account all these items and which is called the *net efficiency*. It must be evident that this is a most important point, for upon it depends *the* **actual** *cost of the power*.

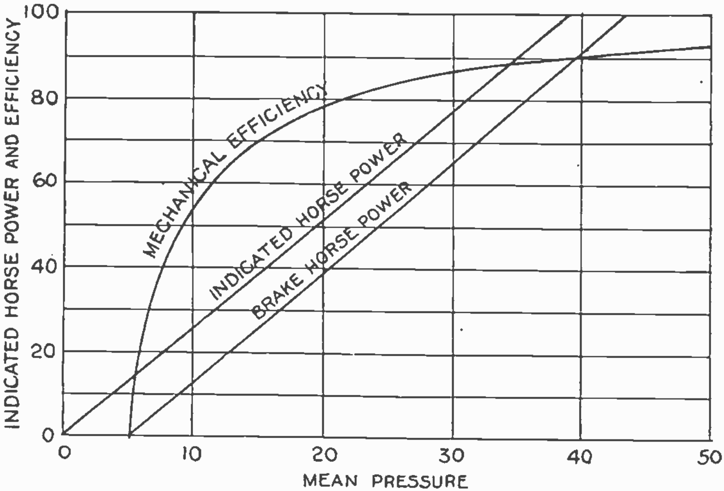


FIG. 100 —Mechanical efficiency curve. *In construction*, it is drawn for any engine by taking the value *brake horse power* ÷ *indicated horse power* for various mean effective pressures and setting off the value of the fraction to a vertical scale of percentage, and joining the points thus found. It will thus be evident that, so far as the mechanical efficiency is concerned, an engine should be worked up to its full load to obtain the maximum efficiency. The friction of an engine does, no doubt, to some extent increase with the load, but the proportional increase is so small as practically not to affect the result. The above remarks assume perfect efficiency of lubrication. Thurston gives the following values for the relative distribution of the friction in an engine with a balanced slide-valve: Main bearings, 47 per cent.; piston and rod, 32.9; crank pin 6.8; cross head and wrist pin, 5.4; valve and rod, 2.5; eccentric strap, 5.3 per cent. The frictional resistance of engines in general varies from about 8 per cent. to 20 per cent. of the full power.

Example.—Two types of power plants are to be considered for a certain proposed installation. ***Plant A:*** Cost $1,000; depreciation and maintenance, 5%; coal per horse power hour, 5 pounds. ***Plant B:*** Cost $5,000 depreciation and maintenance, 8%; coal, per horse power per hour, 1.5 pounds. With coal at $3.00 per ton, and interest at 6% on the investment, which plant is the more efficient *commercially* each operating 10 hours per day, 300 days per year?

Plant **A.** Coal used ***per horse power per year.***

$$5 \times 10 \times 300 = 15,000 \text{ pounds or } 7.5 \text{ tons.}$$

Cost of fuel per year = 7.5 × 5 = ...	37 5
Depreciation and maintenance, 1,000 × .05..........	50
Interest on the investment, 1,000 × .06..............	60
Actual cost of the power per horse power year.........	$147.5 (1)

Plant **B.** Coal used ***per horse power per year.***

$$1.5 \times 10 \times 300 = 4,500 \text{ pounds or } 2.25 \text{ tons.}$$

Cost of fuel per year, 2.25 × 5 = ...	11.25
Depreciation and maintenance, 5,000 × .08 = ...	400
Interest on the investment, 5,000 × .06..	300
Actual cost of the power per horse power year........	$711.25 (2)

Comparing items (1) and (2), plant **A** is 711.25 ÷ 147.5 = 4.8 times more efficient *commercially* than plant **B.** That is, calling the net efficiency of plant **A,** 100%, then the net efficiency of plant **B,** is 147.5 ÷ 711.25 = .207, that is, 20.7% as efficient as plant **A.**

Consumption of Condensing Engines

(Approximate steam consumption according to Kent)

Type of engine	Steam Pressure							
	400	300	250	200	150	100	75	50
	Lbs. of steam per hour per horse power							
Ideal engine (Rankine cycle).........	6.95	7.5	7.9	8.45	9.2	10.5	11.4	12.9
Quadruple expansion wastes 20 per cent	8.75	9.15	9.75	10.5	11.6	13.0	14.0	15.6
Triple expansion wastes 25 per cent....	9.25	9.95	10.5	11.15	12.3	14.0	15.1	16.7
Compound wastes, 33 per cent........	10.5	11.25	11.8	12.7	13.9	15.6	16.9	18.9
Simple engine wastes, 50 per cent......	14.0	15.0	15.8	16.8	18.4	20.4	22.7	25.2

Pumping Engines

The practical limit of *lift* is from 20 to 25 ft.

When the water is warm, the height to which it can be lifted decreases, on account of the increased pressure of the vapor. That is to say, for illustration, a boiler feed pump taking water at say 153° Fahr., could not produce a vacuum greater than 21.78 ins., because at that point

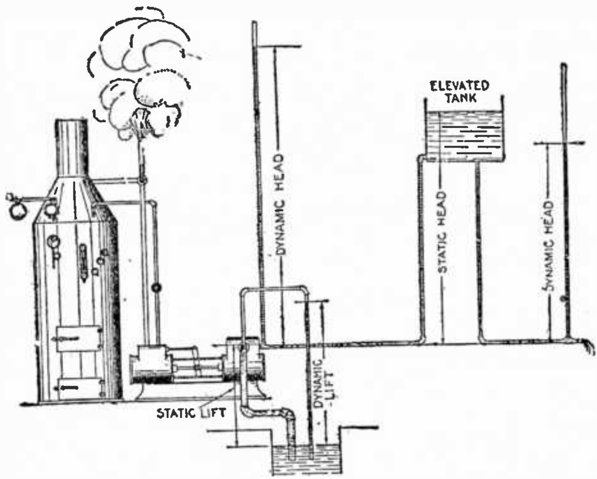


FIG. 101.—View of elevated tank with pump in operation maintaining the supply which is being drawn upon as shown, illustrating the terms static lift, dynamic lift, static head, and dynamic head.

the water would begin to boil and fill the pump chamber with steam. Accordingly, the theoretical lift corresponding would be

$$34 \times \frac{21.78}{30} = 24.68 \text{ ft. approximately.}$$

The result is approximate because no correction has been made for the 34 which represents a 34 foot column of water at 62°; of course, at

153° the length of such column would be slightly increased.

It should be noted that the figure 24.68 ft. is the *approximate* theoretical lift for water at 153°; the *practical* lift would be considerably less.

The following table shows the theoretical maximum lift for different temperatures, leakage not considered.

Theoretical Lift for Various Temperatures

Temp. Fahr.	Absolute pressure of vapor lbs. per sq. ins.	Vacuum in inches of mercury	Lift in feet	Temp. Fahr.	Absolute pressure of vapor lbs. per sq. ins.	Vacuum in inches of mercury	Lift in feet
102.1	1	27.88	31.6	182.9	8	13.63	15.4
126.3	2	25.85	29.3	188.3	9	11.6	13.1
141.6	3	23.83	27	193.2	10	9.56	10.8
153.1	4	21.78	24.7	197.8	11	7.52	8.5
162.3	5	19.74	22.3	202	12	5.49	6.2
170.1	6	17.70	20	205.9	13	3.45	3.9
176.9	7	15.67	17.7	209.6	14	1.41	1.6

How to Figure Capacity.—*RULE: Multiply the area of the piston in sq. ins. by the length of the stroke in ins, and by the number of delivery strokes per minute, divide the product by* **1,728** *to obtain the theoretical capacity in* **cu. ft.,** *or by* **231** *to obtain theoretical capacity in* **U. S. gallons.** *The result thus obtained is to be multiplied by an assumed* **factor** *representing the efficiency of the pump to obtain the* **approximate** *net capacity.*

The rule expressed as a formula is

$$\text{Approximate net capacity} = \frac{.7854 \times D^2 \times L \times N}{1{,}728} \times (1 - f) \text{ cu. ft.,}$$

or

$$= \frac{.7854 \times D^2 \times L \times N}{231} \times (1 - f) \text{ gallons}$$

in which

Table of Theoretical Horse Power Required to Raise Water to Different Heights

Gallons per Minute	5 feet	10 feet	15 feet	20 feet	25 feet	30 feet	35 feet	40 feet	45 feet	50 feet	60 feet	75 feet	90 feet	100 feet	125 feet	150 feet	175 feet	200 feet	250 feet	300 feet	350 feet	400 feet	Gallons per Minute
5	.006	.012	.019	.025	.031	.037	.044	.05	.06	.06	.07	.09	.11	.12	.16	.19	.22	.25	.31	.37	.44	.50	5
10	.012	.025	.037	.050	.062	.075	.087	10	.11	.12	.15	.10	.22	.25	.31	.37	.44	.50	.62	.75	.87	1.00	10
15	.019	.037	056	.075	.094	.112	.131	.15	.17	19	.22	.28	.34	.37	.47	.56	.66	.75	.94	1.12	1.31	1.50	15
20	.025	.050	.075	100	.125	.150	175	.20	.22	.25	.30	.37	.45	50	.62	.75	.87	1.00	1.25	1.50	1.75	2.00	20
25	.031	.062	.093	.125	.156	.187	.219	.25	.28	.31	.37	.47	.56	.62	78	.94	1.09	1.25	1.56	1.87	2.19	2.50	25
30	.037	.075	.112	.150	.187	.225	.262	.30	.34	.37	.45	.56	67	.75	.94	1.12	1.31	1.50	1.87	2.25	2.62	3.00	30
35	.043	.087	.131	.175	.219	.262	.306	.35	.39	.44	.52	.66	.79	87	1.08	1.31	1.53	1.75	2.19	2.62	3.06	3.50	35
40	.050	.100	.150	.200	.250	.300	.350	.40	.45	.50	.60	.75	90	1.00	1.25	1.50	1.75	2.00	2.50	3.00	3.50	4.00	40
45	.056	.112	.168	.225	.281	.337	.394	.45	.51	.56	.67	.84	1.01	1.12	1.41	1.69	1.97	2.25	2.81	3.37	3.94	4.50	45
50	.062	.125	.187	.250	.312	.375	.437	.50	.56	.62	.75	.94	1.12	1.25	1.56	1.87	2.19	2.50	3.12	3.75	4.37	5.00	50
60	.075	.150	.225	.300	.375	.450	.525	.60	.67	.75	.90	1.12	1.35	1.50	1.87	2.25	2.62	3.00	3.75	4.50	5.25	6.00	60
75	.093	.187	.281	.375	.469	.562	.656	.75	.84	.94	1.12	1.40	1.69	1.87	2.34	2.81	3.28	3.75	4.69	5.62	6.56	7.50	75
90	.112	.225	.337	.450	.562	.675	.787	.90	1.01	1.12	1.35	1.68	2.02	2.25	2.81	3.37	3.94	4.50	5.62	6.75	7.87	9.00	90
100	.125	.250	.375	.500	.625	.750	.875	1.00	1.12	1.25	1.50	1.87	2.25	2.50	3.12	3.75	4.37	5.00	6.25	7.50	8.75	10.00	100
125	.156	.312	.469	.625	.781	.937	1.094	1.25	1.41	1.56	1.87	2.34	2.81	3.12	3.91	4.69	5.47	6.25	7.81	9.37	10.94	12.50	125
150	.187	.375	.562	.750	.937	1.125	1.312	1.50	1.69	1.87	2.25	2.81	3.37	3.75	4.69	5.62	6.56	7.50	9.37	11.25	13.12	15.00	150
175	.219	.437	.656	.875	1.093	1.312	1.531	1.75	1.97	2.19	2.62	3.28	3.94	4.37	5.47	6.56	7.66	8.75	10.94	13.12	15.31	17.50	175
200	.250	.500	.750	1.000	1.250	1.500	1.750	2.00	2.25	2.50	3.00	3.75	4.50	5.00	6.25	7.50	8.75	10.00	12.50	15.00	17.50	20.00	200
250	.312	.625	.937	1.250	1.562	1.875	2.187	2.50	2.81	3.12	3.75	4.69	5.62	6.25	7.81	9.37	10.94	12.50	15.72	18.75	21.87	25.00	250
300	.375	.750	1.125	1.500	1.875	2.250	2.625	3.00	3.37	3.75	4.50	5.62	6.75	7.50	9.37	11.25	13.12	15.00	18.75	22.50	26.25	30.00	300
350	.437	.875	1.312	1.750	2.187	2.625	3.062	3.50	3.94	4.37	5.25	6.56	7.87	8.75	10.94	13.12	15.31	17.50	21.87	26.25	30.62	35.00	350
400	.500	1.000	1.500	2.000	2.500	3.000	3.500	4.00	4.50	5.00	6.00	7.50	9.00	10.00	12.50	15.00	17.50	20.00	25.00	30.00	35.00	40.00	400
500	.625	1.250	1.875	2.500	3.125	3.750	4.375	5.00	5.62	6.25	7.50	9.37	11.25	12.50	15.62	18.75	21.87	25.00	31.25	37.50	43.75	50.00	500

This table gives the actual water horse power. When selecting motors, turbines allowance must be made for pipe friction and loss in the pump, gears, belts, etc. One foot head equals .43 pounds pressure to the square inch.

D^2 = square of piston or plunger diameter in sq. ins.;
L = length of stroke in ins.;
N = number of delivery strokes per minute;
f = factor representing assumed slip in per cent. of displacement;
1,728 = cu. ins. in one cu. ft.;
231 = cu. ins. in one U. S. gallon.

Example.—What is the approximate net capacity of a 3×5 double acting power pump running at 75 revolutions per minute with an assumed slip of 5 per cent., applying this formula?

$$\text{Approximate net capacity} = \frac{.7854 \times 3^2 \times 5 \times 150}{1{,}728} \times (1 - .05) = 2.92 \text{ cu. ft.}$$

$$= \frac{.7854 \times 3^2 \times 5 \times 150}{231} \times (1 - .05) = 21.8 \text{ galls.}$$

Theoretical Horse Power at the Water End.—The theoretical horse power required to raise water at a given rate to a given elevation is obtained by the following formula:

$$\text{T. H. P.} = \frac{V \times W \times (L + H)}{33{,}000}$$

in which

V = volume in cu. ft. per minute;
W = weight of one cu. ft. of water;
L = lift in ft.;
H = head in ft.

Example.—What is the theoretical horse power required to raise 100 cu. ft. of water 200 ft. with a 10 ft. lift when the water is at a temperature of 75° Fahr., and when at 35° Fahr.?

For a temperature of 75°, one cu. ft. of water weighs 62.28 lbs. Substituting this and the other data in the formula,

$$\text{T.H.P.} = \frac{100 \times 62.28 \times (10 + 200)}{33{,}000} = 39.63$$

Now if the water have a temperature of only 35°, as might be in very cold weather, the weight of one cu. ft. will increase to 62.42, and the horse power would accordingly increase in proportion to the ratio of the two weights, or

$$\text{T.H.P. (at } 35^\circ \text{ Fahr.)} = 39.63 \times \frac{62.42}{62.28} = 39.7$$

By observing the very slight difference in the two results it will be seen that for ordinary calculation, the temperature need not be considered, taking the usual value 62.4 lbs.

Horse Power Absorbed at the Water End.—The actual horse power required at the water end of a pump (not including slip or mechanical efficiency) is equal to *the theoretical horse power plus an allowance for the friction of the water through the pipes and pump passages.*

There is also friction of water in the elbows which is usually taken into account. Values for these two items are obtained by consulting tables of friction of water in pipes from which the virtual head to be used is easily found and which when inserted in the T.H.P. formula will give the "actual horse power" as above defined.

Duty of Pumps.—The word "duty" is used in engineering to express the efficiency of a steam pumping engine as measured by the work done by a certain quantity of fuel, or steam. Duty, then, stands for foot pounds or work done, and means the number of pounds of water lifted one foot, or its equivalent, by 100 pounds of coal, or 1,000 pounds of saturated steam.

Another unit of duty is, the *foot pounds of work at the water end per million heat units furnished by the boiler.* This is the equivalent of 100 pounds of coal where each pound imparts 10,000 heat units to the water in the boiler, or where the evaporation is 10,000 ÷ 965.7 = 10.355 pounds of water from and at 212°, per pound of coal.

The last mentioned unit which was reported in 1891 by a committee of the *A. S. M. E.* (*Trans.* XII, 530), reaffirmed it in 1915 as the standard unit and defined it as follows: *the duty per million heat units is found by dividing the number of foot pounds of work done during the trial by the total number of heat units consumed, and multiplying the quotient by* 1,000,000.

Air Compressors

The Compression of Air.—When the space occupied by a given volume of air is changed, both its pressure and temperature are changed in accordance with the following laws:

Boyle's law: *At constant temperature, the* **absolute** *pressure of a gas varies* **inversely** *as its volume.*

Charles' law: *At constant pressure, the volume of a gas is proportional to its* **absolute** *temperature.*

In the ordinary process of air compression, therefore, two elements are at work toward the production of a higher pressure:

1. The reduction of volume by the advancing piston;
2. The increasing temperature due to the increasing pressure corresponding to the reduced volume.

The application of ***Boyle's*** and ***Charles'*** laws may be illustrated by a cylinder fitted with an air tight piston. If the cylinder be filled with air at atmospheric pressure (14.7 lbs. per sq. in. ***absolute***) represented by volume A, and the piston be moved to reduce the volume, say to ⅓ A, as represented by B, then according to Boyle's law the pressure will be trebled or $=14.7\times3=44.1$ lbs. absolute, or $44.1-14.7=29.4$ ***gauge*** pressure.

In reality, however, a pressure gauge on the cylinder would at this time show a higher pressure than 29.4 gauge pressure because of the ***increase in temperature*** *produced in compressing the air.*

Now, in the actual work of compressing air, it should be carefully noted that *the extra work which must be expended to overcome the excess pressure due to rise of temperature is lost, because after the compressed air leaves the cylinder* ***it cools,*** *and the* ***pressure drops*** *to what it would have been if compressed at constant temperature.*

The relationship of pressure, temperature and volume of air is expressed by the formula:

$$\frac{P \times V}{T} = 53.3 \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which

P = absolute pressure in lbs. per sq. ft.;

V = volume in cu. ft. of 1 lb. of air at the given pressure and temperature;

T = absolute temperature in degrees F.

Example.—What is the volume of 1 lb. of air at a pressure of 24.7 lbs. per sq. in. and at a temperature of 210° F.?

$$\frac{24.7 \times 144 \times V}{210 + 459.6} = 53.3$$

or

$$V = \frac{53.3 \times 669.2}{24.7 \times 144} = 10.03 \text{ cu. ft.}$$

Adiabatic Compression.—Formulae are:

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{.71} \qquad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.41} \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{.41}$$

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{2.46} \qquad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{3.46} \qquad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.29}$$

in which

P_1 = initial absolute pressure in lbs. per sq. ft.;

V_1 = initial volume in cu. ft.;

T_1 = initial absolute temperature in degrees F.;

P_2=absolute pressure in lbs. per sq. ft. after compression;

V_2=volume in cu. ft. after compression;

T_2=absolute temperature in degrees F. after compression.

The formulae just given hold if all pressures are in lbs. per sq. in., or if all volumes are in cu. ins.

Isothermal Compression.—Formula:

$$P_1 \times V_1 = P_2 \times V_2 = C.$$

in which

P_1=initial pressure in lbs. per sq. ft.;

V_1=initial volume in cu. ft.;

P_2=absolute pressure in lbs. per sq. ft. after compression;

V_2=volume in cu. ft. after compression;

C =constant depending on the temperature.

For a temperature of 32° F. constant C equals 26,200 ft. lbs. and for other temperatures it may be found from the formula $C=53.3\,T$, in which T is the absolute temperature which is maintained during the expansion or compression.

Example.—A volume of 165 cu. ft. of air, at a pressure of 15 lbs. per sq. in. is compressed adiabatically to a pressure of 80 lbs. per sq. in. What will be the volume at this pressure?

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{.71} = 165\left(\frac{15}{80}\right)^{.71} = 50 \text{ cu. ft. approx.}$$

Example.—A volume of 165 cu. ft. of air is compressed isothermally from 15 to 80 lbs. per sq. in. What will be the volume after compression?

$$V_2 = \frac{P_1 \times V_1}{P_2} = \frac{165 \times 15}{80} = 31 \text{ cu. ft.}$$

The work required to compress air is given in the following tables.

Horse Power Required for Compression.—The following table No. 1 according to Kent, gives horse power required to compress and deliver one cu. ft. of free air per minute to a given pressure with no cooling of the air during the compression; also the horse power required, supposing the air to be maintained at constant temperature during the compression.

Table No. 1

Gauge pressure	Air not cooled	Air constant temperature
5	.0196	.0188
10	.0361	.0333
20	.0628	.0551
30	.0846	.0713
40	.1032	.0843
50	.1195	.0946
60	.1342	.1036
70	.1476	.1120
80	.1599	.1195
90	.1710	.1261
100	.1815	.1318

Table No. 2

Gauge pressure	Air not cooled	Air constant temperature
5	.0263	.0251
10	.0606	.0559
20	.1483	.1300
30	.2573	.2168
40	.3842	.3138
50	.5261	.4166
60	.6818	.5266
70	.8508	.6456
80	1.0302	.7700
90	1.2177	.8979
100	1.4171	1.0291

The above table No. 2 according to Kent gives the horse power required to compress and deliver one cu. ft. of compressed air per minute at a given pressure (the air being measured at the atmospheric temperature) with no cooling of the air during the compression; also supposing the air to be maintained at constant temperature during the compression.

The horse power given in these tables is the theoretical power, no allowance being made for friction of the compressor or other losses, which may amount to 10% or more.

Calculation of Horse Power.—The following illustrates how to calculate the horse power required for compressors.

Example.—What horse power is required to compress 475 cu. ft. of free air per minute into a receiver against a pressure of 78.3 lbs. per sq. in., assuming air enters compressor at 14.5 lbs. absolute and air maintained at constant temperature?

One method of solving this problem is as follows:

Compression ratio = 78.3 ÷ 14.5 = 5.4.

Hyperbolic logarithm of 5.4 (from table page 229) = 1.6864.

1 + hyp. log. of 5.4 = 2.6864.

$$\text{M.E.P.} = \left[\frac{P_r\ abs. \times 1 + hyp.\ log.\ C}{C} - P_i\ abs.\right] \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which

M.E.P. = mean effective pressure in lbs. per sq. in.

P_r = receiver pressure.

C = ratio of compression.

P_i = intake pressure.

Substituting in formula (1)

$$\text{M.E.P.} = \left[\frac{93 \times 2.6864}{5.4} - 14.5\right] = 32.1 \text{ lbs.}$$

The size of the compressor piston depends on conditions, but for calculation of horse power, any size piston may be assumed. Accordingly, for simplicity, assume piston to have an area of 1 sq. ft., that is, 144 sq. ins. so that 475 ft. per min. will correspond to the piston speed.

Now,

$$\text{horse power} = \frac{\text{piston area} \times m.e.p. \times \text{piston speed}}{33{,}000} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting in formula (2)

$$\text{horse power} = \frac{144 \times 32.1 \times 475}{33{,}000} = 66.5$$

It should be understood that this is the power required for compressing the air and to which must be added extra power to overcome the friction between the air piston and the drive piston, to obtain the power required for the driver.

Hoists

Capacity of Hoisting Engines.—The horse power required to raise a load at a given speed is equal to

$$\frac{\textit{gross weight in lbs.} \times \textit{speed in feet per minute}}{33{,}000}$$

To this there should be added from 25 to 50 per cent for friction, contingencies, etc. The gross weight includes the weight of the cage, load, and rope. In a shaft with two cages balancing each other, the net load is taken.

Limit of Depth in Hoisting.—Taking the weight of a hoisting rope, 1⅛ inches in diameter, at two pounds per foot, and its breaking strength at 84,000 lbs., it should, theoretically, sustain itself until 42,000 feet long before breaking from its own weight. But taking the usual factor of safety of 7, then the safe working length of such a rope would be only

$$42{,}000 \div 7 = 6{,}000 \text{ feet.}$$

NOTE.—***Power required for traveling cranes and hoists.*** Ulrich Peters, in ***Machinery*** November, 1907, develops a series of formulæ for the power required to hoist and to move trolleys on cranes. The following is a brief abstract. Resistance to be overcome in moving a trolley or crane bridge. P_1 = rolling friction of trolley wheels, P_2 = journal friction of wheels or axles, P_3 = inertia of trolley and load. P = sum of these resistances =

$$P_1 + P_2 + P_3 = (T+L)\left(\frac{F_1 + F_2 d}{D} + \frac{V}{1{,}932t}\right)$$

in which T = weight of trolley, L = load, F_1 = coefficient of rolling friction, about .002 (.001 to .003 for cast iron on steel); F_2 = coefficient of journal friction, = .1 for starting and .01 for running, assuming a load on brasses of 1,000 to 3,000 pounds per square inch; (F_2 is more apt to be .05 unless the lubrication be perfect); d = diameter of journal; D = diameter of wheels; V = trolley speed in feet per minute; t = time in seconds in which the trolley under full load is required to come to the maximum speed. Horse power = sum of resistances × speed, feet per minute ÷ 33,000. Force required for hoisting and lowering: F_h = actual hoisting force, F° = theoretical force or pull, L = load, V = speed in feet per minute of the rope or chain, c = hoisting speed of the load L, c ÷ V = transmission ratio of the hoist, e = efficiency = $F_h \div F^\circ$. The actual work to raise the load per minute = $F_h V = Lc = F^\circ V \div e$. The efficiency e is the product of the efficiencies of all the several parts of the hoisting mechanism, such as pulleys, windlass, gearing, etc. Methods of calculating these efficiencies, with examples, are given at length in the original paper by Mr. Peters.

If now a weight of three tons, which is equivalent to that of a cage of moderate capacity with its loaded cars, be hung to the rope the maximum length at which such a rope could be used with the factor of safety of 7, is

$$6{,}000-\frac{6{,}000}{2}=3{,}000 \text{ feet}$$

The limit may be considerably increased, by using: 1, special steel rope of greater strength; 2, a smaller factor of safety, and 3, taper ropes.

Pile Drivers

Pile Driver Proportions

For quick work, the following are suitable proportions of engine, boiler and hammer.

Engine	Weight hoisted single rope, lbs.	Boiler dimensions in inches		Weight of monkey lbs.
		Diam.	Height	
5×8	2,000	32	75	1,250
6¼×8	3,000	36	75	1,600
6¼×10	4,000	38	81	1,800
7×10	5,000	40	84	2,500
8¼×10	8,000	42	90	3,000
9×10	9,000	48	102	3,500
10×10	10,500	50	102	4,200
10×12	12,000	53	102	5,000
12×12	16,000	56	120	6,500

Transmission Rope

Properties of Transmission Rope

Diameter of Rope	Square of Diameter	??rox. Shipping Wgt per ft.	Breaking Strength	Maximum Allowable Tension	Length of Splice, feet			Smallest Diam. of Sheaves, inches	Maximum Number of Revolutions per Minute
					3 Strands	4 Strands	6 Strands		
¾	5625	21	3,950	112	6	8		28	760
⅞	7656	27	5,400	153	6	8		32	650
1	1	36	7,000	200	7	10	14	36	570
1⅛	1 2656	45	8,900	253	7	10	16	40	510
1¼	1 5625	56	10,900	312	7	10	16	46	460
1⅜	1 8906	68	13,200	378	8	12	16	50	415
1½	2 25	80	15,700	450	8	12	18	54	380
1⅝	2 6406	92	18,500	528	8	12	18	60	344
1¾	3 0625	1 08	21,400	612	8	12	18	64	330
2	4	1 40	28,000	800	9	14	20	72	290
2¼	5 0625	1 80	35,400	1,012	9	14	20	82	255
2½	6 25	2 20	43,700	1,250	10	16	22	90	230

Weight of rope $= .34 \times \text{diam.}^2$
Breaking strength $= 7{,}000 \times \text{diam.}^2$
Maximum tension $= 200 \times \text{diam.}^2$

Diam. smallest practicable pulley $= 36 \times \text{diam.}$
Velocity of rope (assumed) $= 5{,}400$ ft. per min.

Horse Power.—For convenience in determining the horse power that single ropes of different diameters will transmit, the following table is given:

Horse Power of Manila Rope

Diameter of Rope	Velocity, Feet per Minute										
	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000
¾	2.3	3.3	4.3	5.2	6.0	6.6	7.2	7.3	7 4	7.3	6.9
⅞	3.0	4.5	5.9	7.0	8.2	9.0	9.6	9 8	10 0	9.6	9.0
1	4.0	5 9	7 7	9 2	10.6	11.8	12 7	12 9	13.0	12.7	12.0
1⅛	5.0	7.5	9 7	11 6	13.5	14.9	16.0	16.3	16.7	16.5	15.3
1¼	6.3	9 1	12.0	14 3	16 7	18 5	20 0	20.2	20 7	20.1	18.9
1⅜	7.5	10.8	14.4	17 4	20.0	22.1	23 7	24.5	24 6	24 0	22.3
1½	9.0	13.5	17.4	20 7	23.0	26.3	28 7	29 0	29.5	28.6	26.7
1⅝	10.5	15.5	20.1	24.3	27 9	30 8	32.9	34 1	34.3	33.3	31.0
1¾	12.3	18.0	23.6	28.2	32.7	36.4	38.5	39.4	40.5	38.7	36.0
2	16.6	23.2	30.6	36.8	42.5	46.7	50.0	51.7	52.8	50.6	47.3
2¼	20 0	29.6	38.6	46.6	53.6	59.2	63.6	65.8	66.3	64.4	60.3
2½	25.0	36.6	47.7	57 5	66 0	71.2	73 0	80.0	81.0	79.0	73.8

The following table gives the sag on driving and slack sides.

Sag

Distance between Pulleys, Feet	Sag on Driving Side, All Speeds, Feet	Sag on Slack Side Velocity, Feet per Minute				
		3,000	4,000	4,500	5,000	5,500
30	.19	.45	.39	.36	.33	.30
40	.34	.80	.69	.64	.59	.53
50	.53	1.2	1.1	1.0	.92	.84
60	.76	1.8	1.7	1.4	1.3	1.2
70	1.0	2.4	2.1	1.9	1.7	1.6
80	1.4	3.2	2.9	2.5	2.3	2.1
90	1.7	4.0	3.5	3.2	3.0	2.7
100	2.1	5.0	4.3	4.0	3.7	3.3
120	3.0	7.2	6.2	5.7	5.3	4.8
140	4.1	9.9	8.5	7.8	7.2	6.6
160	5.4	12.9	11.1	10.2	9.5	8.6

Steam Hammers

Rating.—The capacity or rating of a steam hammer is *the weight of the ram and its attached parts, such as the piston and rod.* The steam pressure behind the piston is not considered as far as the rating is concerned.

Example.—A 1,000 pound hammer has reciprocating parts of that weight.

Rule.*—Multiply the area of the largest cross section to be worked by* 80, *if of steel, or* 60, *if of iron, and the product will be required rating of the hammer in pounds.*

Example.—The capacity of a hammer for working steel billets 5 inches square would be determined as follows: 5×5=25 and 25×80=2,000, which is the rating of the hammer in pounds. A hammer rated according to this rule is an economical size to use, although it can, of course, be employed for heavier work.

For general blacksmith work the following table of sizes may be followed, with modifications according to conditions:*

Diameter of stock	Size of hammers	Diameter of stock	Size of hammers
3½ inches	250 to 350 pounds	5 inches	800 to 1,000 pounds
4 inches	350 to 600 pounds	6 inches	1,100 to 1,500 pounds
4½ inches	600 to 800 pounds		

The capacity of most of the board drop hammers in use varies from 800 to 1,500 pounds; the steam hammers found in drop forging plants usually range from 2,000 to 5,000 pounds capacity, for handling average work. It does not seem practicable to build board drops larger than 3,000 pounds falling weight, and where the forgings are heavy enough to require a capacity over 1,500 or 2,000 pounds, steam hammers are usually preferred. The latter type is also preferred in some forge shops for all classes of work. It is generally conceded that the cost of operation and repairs is greater for steam hammers, but the latter has a greater output for a given capacity.

Boiler Capacity.—For operating steam hammers the size of boiler depends upon the number of hammers in the equipment and the service. It may vary from one boiler horse power for each 100 pounds falling weight, as in a forge where many hammers are in use, to three horse power per hundred pounds falling weight where a single hammer is installed at hard service.

Hammers should be operated with steam at a pressure of from 75 pounds to a more efficient pressure of 100 pounds per square inch. They also may be operated by compressed air, in which case the operating valve should be fitted to suit

For average conditions the boiler horse power can be determined approximately as follows:

*NOTE.—As recommended by the Niles-Bement-Pond Co.

***Rule.**—Divide the rated capacity of the hammer in pounds by 100, and the quotient will be the boiler horse power required for continuous operation.*

Example.—If a hammer be rated at 2,000 pounds, the boiler horse power would be 2,000 ÷ 100 = 20.

The rule also applies in cases where the hammer is not used continually, by estimating the amount of idle time and making suitable allowance, but the boiler capacity must not be reduced to such an extent that there is a decided diminution in the pressure during the working period.

Sizes of Road Rollers.—The usual sizes of tandem rollers are 2½, 3, 5, 8, and 10 tons, and of the three wheel rollers, 8, 10, 12, and 15 tons. This range of sizes has been found to meet the varied requirements of road construction.

Proportions of Road Rollers.—The following dimensions will serve to illustrate the general practice in the design of machinery of this class:

Erie 5 ton tandem roller.

General: Weight, 11,000 lbs.; length, 13 ft. 6 ins.; width, 4 ft. 2 ins.; wheel base, 9 ft. 1 in.; height to top of stack, 8 ft. 8 ins.

Boiler: Diameter, 30½ ins.; height, 54 ins.; 108 one and one-half inch tubes.

Engine: Two cylinder high pressure, diameter 6 ins.; stroke, 6 ins.

Rear: Rollers, diameter, 48 ins.; width, 39 ins.; compression, 210 lbs.

Case 10 ton three wheel roller.

General: Rolling width, 81 ins.; wheel base, 113 ins.; length, 18 ft., height, 10 ft.; ground clearance, 16 ins.; speed, 2¼ miles per hour at normal speed of engine.

Engine: 6⅛ and 9 by 10 ins. compound; 36 brake horse power; Hackworth radial valve gear; fly wheel 36 diameter, 9½ face; revolution, 250 per minute.

Boiler: Locomotive type; diameter, 26 ins.; grate area, 6.14 sq. ft.; heating surface, 130.2 sq. ft.; pressure, 130 lbs.

Locomotives

Capacity.—The capacity of a locomotive or the load it will haul is limited to the adhesion or traction due to the weight on the driving wheels. It is usually taken as 1/5 to 1/4 of the weight on the drivers.

The formula for traction is

$$F=\frac{4\pi d^2pS}{4\pi D}=\frac{d^2pS}{D}$$

in which

F = indicated tractive force in lbs.

p = average effective pressure in cylinder in lbs. per sq. in.

S = stroke of piston in ins.

d = diameter of cylinders in ins.

D = diameter of driving-wheels in ins.

The Baldwin Locomotive Works gives the following formula for compound engines of the Vauclain four cylinder type:

$$T=\frac{C^2S\times\frac{2}{3}P}{D}+\frac{c^2S\times\frac{1}{4}P}{D}$$

in which

T = tractive force in lbs.

C = diameter of high pressure cylinder in ins.

c = diameter of low pressure cylinder in ins.

P = boiler pressure in lbs.

S = stroke of piston in ins.

D = diameter of driving-wheels in ins.

For a two cylinder or cross compound engine it is only necessary to consider the high pressure cylinder, allowing a sufficient decrease in boiler pressure to compensate for the necessary back pressure. The formula is

$$T = \frac{C^2 S \times \frac{2}{3} P}{D}$$

The above formulae are for speeds of from 5 to 10 miles an hour, or less; above that the capacity of the boiler limits the cut off which can be used, and the available tractive force is rapidly reduced as the speed increases.

Diameter of Cylinder.—Since the length of stroke is usually fixed to harmonize with the arrangement and diameter of the driving wheels, the determination of the size of the cylinder usually consists in a calculation of the diameter.

In order to make the calculation of diameter of the driving wheels, the weight on the driving wheels, the boiler pressure and the stroke of the piston must be known. With this data the diameter of the cylinder can be calculated by the formula:

$$A = \frac{.218WD}{PS}$$

in which

A = area of each piston in sq. ins.;

W = weight on driving wheels;

C = circumference of driving wheels = 3.1416 D;

D = diameter of driving wheels in ins.;

P = boiler pressure in lbs. per sq. in.;

S = stroke of piston in ins.

Example.—Take an engine in which the weight on the driving wheels is 95,000 pounds; the diameter of the driving wheels is 62 inches; the boiler pressure 180 pounds per sq. in. and the piston stroke is 24 ins. The last formula then becomes:

$$A = \frac{.218 \times 95{,}000 \times 62}{180 \times 24} = 297.2 \text{ sq. in.}$$

or the diameter of the cylinder should be 19.44 ins. Such a cylinder would in practice probably be made 19 ins. in diameter.

Size of Boiler.—The conditions of locomotive design are such that it is practically impossible to make the boiler too large.

The weight and dimensions of locomotive boilers are in nearly all cases determined by the limits of weight and space to which they are necessarily confined. It may be stated generally that *within these limits a locomotive boiler cannot be made too large.*

Horse Power.—Maximum horse power is usually attained at speeds of from 25 to 35 miles per hour in freight locomotives, and at speeds of from 50 to 60 miles per hour in passenger locomotives.

At higher speeds the power decreases in saturated steam locomotives; but is maintained nearly constant in those using superheated steam.

A formula for horse power given by Kent is

$$h.p. = \frac{pd^2SM}{375D}$$

in which

p = mean effective pressure;

d = diameter of piston in ins.;

S = length of stroke in ins.;

M = speed of train in miles per hour;

D = diameter of driving wheels in ins.

R.P.M. for Various Speeds and Driving Diameter

(Kent)

Diameter of Wheel.	Miles per Hour.							
	10	20	30	40	50	60	70	80
50 in.	67	134	201	268	336	403	470	538
56 in.	60	120	180	240	300	360	420	480
60 in.	56	112	168	224	280	336	392	448
62 in.	54	108	162	217	271	325	379	433
66 in.	51	102	153	204	255	306	357	408
68 in.	49	99	148	198	247	296	346	395
72 in.	47	93	140	187	233	279	326	373
78 in.	43	86	129	172	215	258	301	344
80 in.	42	84	126	168	210	252	294	336
84 in.	40	80	120	160	200	240	280	320
90 in.	37	75	112	150	186	224	261	299

Marine Engines

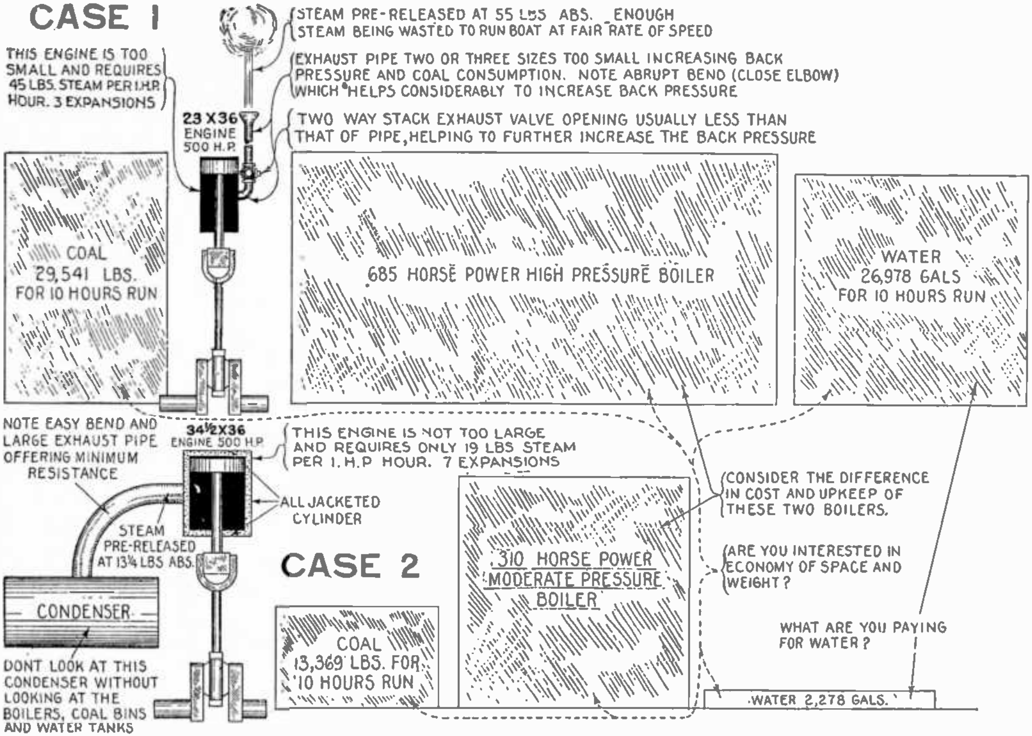
A very common mistake, and one which tends to prejudice boat owners against the use of condensers, is the installation of engines too small for the load making it necessary to use initial pressures anywhere from 100 to 150 lbs. gauge.

An engine operating under these conditions cutting off at one-third or one-fourth stroke will have a terminal pressure considerably above atmosphere and to obtain a 24 or 26 inch vacuum would require an abnormally large (and expensive) condenser; moreover, the excessive temperature range will cause excessive condensation in the cylinder and considerably reduce the saving due to the condenser.

The following example will illustrate the difference in size of a single cylinder engine proportioned according to the usual practice and according to the author's views.

Stern Wheel Steam Boat Proportions

Size	Length of hull feet	Length over all feet	Beam of hull feet	Width on deck feet	Draft unloaded inches	Capacity		Towing capacity	Approx. fuel consumption ten (10) hours		Double engines	
						Tons	Draft inches		Soft coal tons	Wood cords	Diam. of cylinder and stroke	Indicated horse power 150 lbs. pressure
A	50	59	14	15½	12	10	20	30	1	2	5 × 20	30
B	65	76½	15	17	12	17	20	55	1¼	2¼	6 × 30	45
C	70	82½	16	18	14	20	21	80	1½	2½	7 × 32	62
D	80	94	18	20	15	25	24	110	2	3½	8 × 36	75
E	90	103	22	25	16	30	24	150	2½	4	9 × 42	111
F	110	125	25	28	18	50	28	225	3	5	10 × 48	157
G	135	154	30	33	22	100	32	300	4	7	12 × 60	214



FIGS. 102 and 103.—Two steam lighter power plants illustrating usual practice (*case 1*) and the author's system (*case 2*). For details of a special engine designed by the author, see *Audel's Engineers and Mechanics Guide*, Vol. 4, pages 1,677 to 1,732*b*.

Example.—Find the cylinder dimensions of a 500 horse power, single cylinder, lighter engine operating under the following conditions: *Case I.* Initial pressure, 125 lbs.; cut off ⅓ stroke; back pressure, 3 lbs. abs. (23.8″ vacuum); 110 revolutions; unjacketed cylinder, diagram factor .8. *Case II.* Initial pressure, 80 lbs.; cut off, 1/7 stroke; back pressure, 2 lbs. abs. (25.9″ vacuum); 125 revolutions; jacketed cylinder, diagram factor .9.

Case I. Usual practice

$$125+15=140 \text{ lbs. abs. initial pres.}$$

$$\text{M. E. P.} = \left(\frac{140\times2.099}{3} - 3\right)\times.8=76 \text{ lbs.}$$

For 36″ stroke

Size
23 × 36

$$\text{piston area} = \frac{500}{000004\times36\times110\times76}=415 \text{ sq. ins.}$$

$$\text{piston diameter} = \sqrt{\frac{415}{7854}} = 22.9, \text{ say, } 23''.$$

Case II. Author's proportions

$$80+15=95 \text{ lbs. init. pres.}$$

$$\text{M. E. P.} = \left(\frac{95\times2.946}{7} - 2\right)\times.9=34 \text{ lbs.}$$

Size
34½ × 36

For 36″ stroke

$$\text{piston area} = \frac{500}{.000004\times36\times110\times34}=929 \text{ sq. ins.}$$

$$\text{piston diameter} = \sqrt{\frac{929}{.7854}} = 34.4, \text{ say, } 34\tfrac{1}{2}''.$$

According to tests* a saving as high as 40.4 per cent has been made by the use of steam jackets*f* on a single cylinder Meyer cut off engine.

Case II, furnishes ideal conditions for the use of a jacket, and having

*NOTE.—Special attention is called to these tests, which were made by Bryan Donkin. For data on the value of steam jackets comprising an elaborate series of experiments see *Pro. Inst. Mech. Eng.* 1892, page 464. ***It should be noted,*** as stated above, that the steam jacket saving of 40.4% was obtained with an engine fitted with the Meyer cut off gear—a gear not adapted to obtaining best results possible with a very early cut off because of the large clearance. This large clearance may be reduced to almost zero by the four valve engine shown in fig. 102. As shown, a shifting eccentric operates the admission valves, and a fixed eccentric the exhaust valves. With this arrangement the best economy possible for a given cut and temperature range in a single cylinder counter flow engine with non-releasing valve gear should be obtained.

f NOTE.—The diversity of opinion which still exists as to the value of the steam jacket is due to its mis-application, the tests in such cases being misleading except to the better informed. A jacket should only be used with a very short cut off, and to obtain the full economy the clearance, as mentioned above, should be reduced to a minimum. For maximum effect heads and piston as well as the cylinder should be jacketed and *proper provision for drainage made.* See Prof. Prosser's tests.

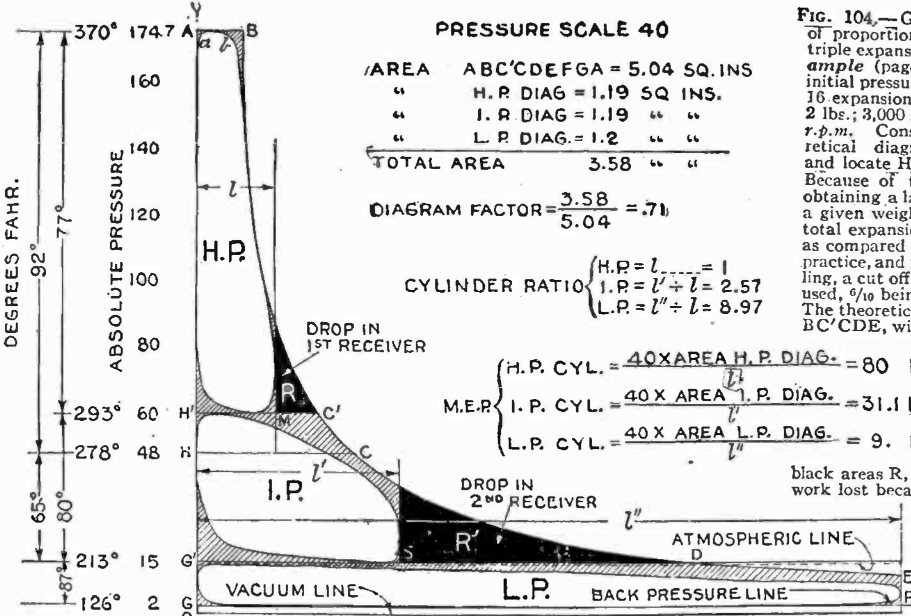


FIG. 104.—Graphical method of proportioning cylinders in triple expansion engines. *Example* (page 164): Given initial pressure 160 lbs. gauge; 16 expansions; back pressure 2 lbs.; 3,000 horse power; 140 *r.p.m.* Construct the theoretical diagram ABCEFG, and locate H'C' and G'D. Because of the necessity of obtaining a large power with a given weight of engine, the total expansions (16) are low as compared with stationary practice, and for ease of handling, a cut off later than ½ is used, 6/10 being the standard. The theoretical expansion line BC'CDE, will be interrupted by *drop* or free expansion in the receivers giving the triangular solid black areas R, R', representing work lost because of the late cut off. *In construction*, assuming 6/10 cut off in *h. p. cyl.*, find point M, such that AB = 6/10 of its distance *l*, from OY, and draw expected card. For equal cylinder diameters *l*, = stroke of *h.p. cyl.*, but for equal strokes *l*, represents relatively area of *h.p. piston*. The theoretical volume exhausted into *i. p. cyl.*, is represented by H'C'. Similarly with 6/10 cut off, take S, at a distance *l'*, from OY, such that H'C' = 6/10 *l'*, and draw *expected card*, then *l'*, with equal strokes, represents relatively, area of *i. p. piston*. The relative size of the low pressure piston is represented by *l''*. The *l.p.* expected card is now drawn assuming a cut off (within limits) that will give most desirable receiver pressure. Calling *l*, unity, then *l*, *l'*, and *l''*, represent the *cylinder ratio*. By measurement *l*, *l'*, and *l''* = .5921 1.53, and 5.34 ins. repectively, and calling *l*, unity, the cylinder ratio = 1, 2.57 and 8.97, as tabulated above. The areas of the cards are preferably obtained by planimeter; these and the *m.e.p.* for each cylinder are also tabulated above. See accompanying text.

in mind the large clearance of the Meyer cut off and the effect of clearance on economy, it would seem possible to still further reduce the feed water consumption by the use of rocking valves located in the heads. In regard to clearance, it should be noted that its detrimental effect is not measured in terms of per cent of displacement of full stroke but per cent of displacement up to point of cut off, and at very short cut off, as one-seventh, this becomes considerable; accordingly with the ordinary clearance, the saving due to the short cut off is largely offset.

The author believes that the feed water consumption of the ordinary single cylinder marine engine can be reduced at least 50 per cent, with the following design and working conditions: cylinder and heads steam jacketed, four rocking valves in the heads (reducing clearance to almost zero); variable cut off by shifting eccentric; separate eccentric for exhaust valves; 80 lbs. initial pressure; 6 to 7 expansions, 26 inch vacuum; speed somewhat higher than usual practise to avoid an abnormally larger engine for the power delivered.

The following example is given to illustrate the method of *finding the cylinder sizes* in the design of a triple expansion engine.

Example.—Find the cylinder dimensions of a high speed triple expansion engine for a passenger steamer to run as follows: initial pressure 160 lbs.; number of expansions, 16; 2 lbs back pressure; 140 revolutions per minute.

The graphical method consists in drawing a theoretical diagram from the given data of initial pressure, total number of expansions, etc., and inscribing within, diagrams for the various cylinders of such contour as will represent the expected actual performance of the engine. It is evident, then, that the success of this method depends upon the experience and judgment of the designer in drawing these "expected diagrams," their forms being obtained by first examining a large number of indicator diagrams from engines similar to the one to be designed and running under the same conditions with open column frame, and built up bed plate.

Assuming the expected diagrams in fig. 104 to represent approximately the actual diagrams, the cylinder dimensions are found as follows:

Distribution of power

Total area expected diags. = 1.19 + 1.19 + 1.2 = 3.58 sq. in.

Power	h. p. cyl	= 3,000 × 1.19 ÷ 3.58 =	997	horse power
"	i p. "	= " × " ÷ " =	997	" "
"	l. p. "	= 3,000 × 1.2 ÷ 3.58 =	1,006	" "
		Total	3,000	" "

Stroke

The length of stroke should be such that will give a proper piston speed

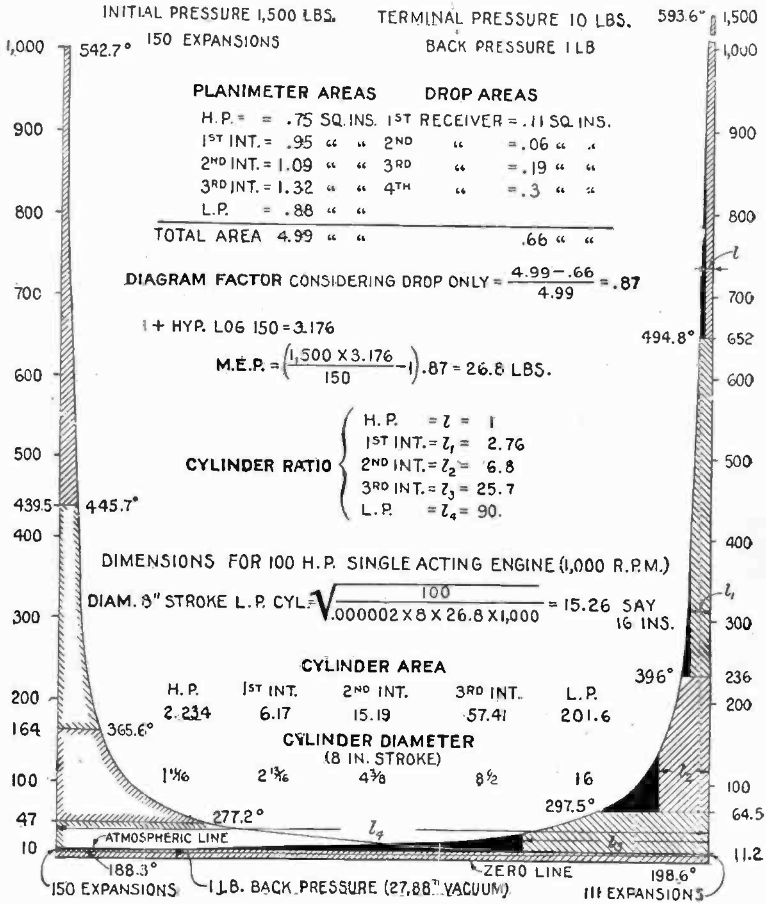


FIG 105.—Diagrams illustrating ***quintuple*** or ***five stage*** expansion at initial pressures of 1,000 and 1,500 lbs. absolute. The calculations are for the diagram at the right for 1,500 lbs., 150 expansions, and indicate the cylinder ratio and cylinder dimensions for 100 horse power single acting engine suitable for a high speed boat. In a proposition of this kind the aim should be to obtain equal temperature ranges rather than equal powers for each cylinder, because the main object sought is to reduce the steam consumption to a minimum for the given initial pressure and expansion ratio, in order to reduce the size of the boiler and thus develop the given power with the least weight possible. With proper construction 100° to 150° of superheat could be used, and the net weight per *h.p.* probably further reduced by means of reheating receivers between the *2nd int* and *l.p.* cylinders.

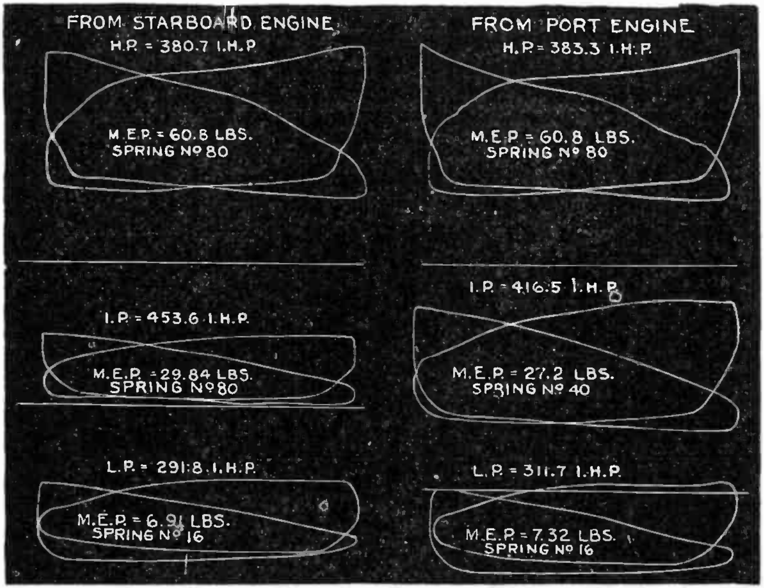
for the service required. For 140 revolutions per minute a stroke of 3 feet will give a piston speed of

$$2\times3\times140=840 \text{ ft.}$$

which represents a medium speed and may be adopted, assuming that this will give a *h. p.* cylinder diam. somewhat less than the stroke.

High Pressure Cylinder

M. E. P. (as in fig. 104) = 80 lbs.



FIGS. 106 to 111.—Indicator cards from the triple expansion engines of the twin screw steamer *Monmouth*, C. R. R. of N. J. (Sandy Hook Route), size of engine 19¼″, 30″, and 50″ by 30″ stroke. Cards taken during a speed trial at Delaware Breakwater. Length of course 1.261 nautical miles. The data is as follows:

Starboard Engine Steam Pressures						Total I. H. P.	Time over course	Speed in knots	Speed in miles
Steam Boiler	1st receiver	2nd receiver	Vacuum	R.P.M.	I. H. P.				
155	55	7	26	142	1,126.1	2,237.6	3–58	19.07	22
Port Engine									
	56	5	26	143	1,111.5				

$$\text{area piston} = \frac{997 \times 33{,}000}{80 \times 840} = 489.59, \text{ say } 490 \text{ sq. ins.}$$

$$\text{diameter corresponding} = \sqrt{\frac{490}{.7854}} = 24.98, \text{ say } 25 \text{ ins.}$$

Intermediate Pressure Cylinder

$$\text{area piston} = \frac{997 \times 33{,}000}{31.1 \times 840} = 1{,}259.2, \text{ say } 1{,}260 \text{ sq. ins.}$$

$$\text{diameter corresponding} = \sqrt{\frac{1{,}260}{.7854}} = 40.1, \text{ say } 40 \text{ ins.}$$

Low Pressure Cylinder

$$\text{area piston} = \frac{1{,}006 \times 33{,}000}{9 \times 840} = 4{,}391 \text{ sq. ins.}$$

$$\text{diameter corresponding} = \sqrt{\frac{4{,}391}{.7854}} = 74.76, \text{ say } 74\tfrac{3}{4} \text{ ins.}$$

Steam Turbines

Effect of Vacuum.—The steam consumption *varies greatly with the degree of vacuum.*

For high pressure units there is a decrease in steam consumption of about 5% for each inch of vacuum secured between 24 and 27 in. vacuum, a saving of 6 to 7% between 27 and 28 in. and of 7 to 8% between 28 and 29 in.

For low pressure units the gain due to increase of vacuum is greater.

If the velocity of the steam as it leaves the last rotating buckets reaches the velocity of about 1,400 ft. per sec., further increase of vacuum has no effect on the steam economy.

Effect of Superheat.—Turbines have been designed for temperatures as high as 800° Fahr.

High temperature steam should be kept away from the wheel casing and turbine blading on account of its destructive effect on cast iron and other metals, and also because the great expansion interferes with small clearances.

Velocity of the Steam.—The formula is

$$S = \frac{WV}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (I)$$

in which

S = velocity of the steam in ft. per sec.;

W = weight of steam per second to be passed through the turbine to generate the required power;

V = cu. ft. of steam per lb.;

A = net cross sectional area of passage in sq. ft.

Example.—A 1000 *h.p.* turbine using 18 lbs. of steam per hour per horse power has a net annular steam space between the blades of 50 sq. ins. at a point where the steam passes the cross section at a pressure of 60 lbs. per sq. in. Find the velocity of the steam in the direction of the axis of the turbine in order to pass the weight of steam required. (Specific volume of steam at 60 lbs. absolute pressure = 7 cu. ft. per lb.).

Substituting in formula (I),

$$S = \frac{1000 \times 18 \times 7 \times 144}{60 \times 60 \times 50}$$

$$= 100.8 \text{ ft. per sec.}$$

To maintain this velocity approximately constant throughout the whole range of guide and wheel blades, there must be an approximately constant ratio between the volume of the steam at any given cross section at which the pressure is known and the cross sectional area of the passage through which the steam at the given pressure is passing.

Internal Combustion Engines

Efficiency.—The working cycles of practicable heat engines are always imperfect, that is, the operations are such that. although perfectly carried out, the maximum efficiency possible could not be attained by them. The formula for efficiency is

$$E = \frac{T_1 - T_2}{T_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which

E = efficiency;

T_1 = quantity of heat given to an engine;

T_2 = quantity of heat discharged by the engine after performing work.

Example.—What is the theoretical thermal efficiency of a gas engine in which the initial temperature is 2,800° and the final temperature 80° Fahr.?

In the case of the gas engine, the initial temperature means the explosion temperature, and the final temperature, that of the exhaust. Reducing the given values to the absolute scale.

$$T' = 2{,}800 + 459.6 = 3{,}259.6°$$
$$T'' = 80 + 459.6 = 539.6°$$

substituting in (1)

$$\text{Theoretical thermal efficiency} = \frac{3{,}259.6 - 539.6}{3{,}259.6} = 83.5 \text{ per cent.}$$

From the equation (1) evidently efficiency is expressed by a fraction whose numerator is the energy utilized and whose denominator is the total available energy. When this is expressed in symbols for a given weight W, and a given value for the specific heat C, it takes the form

$$\text{Efficiency} = \frac{WCT_1 - WCT_2}{WCT_1} \quad \ldots\ldots\ldots\ldots (2)$$

because the engine rejects at the exhaust an energy WCT_2 and can therefore only have utilized the difference between the energy at the beginning and at the end of the stroke.

Eliminating the common factors for the weight and specific heat, the equation appears,

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1} \dots\dots\dots\dots\dots\dots (3)$$

This is an expression for the efficiency of a heat engine first deduced by Carnot, and is the expression for the best result theoretically obtainable from an engine operating between the two temperatures T_1 and T_2. To have it apply to the gas engine, the temperature T_1, which is the maximum of the Carnot cycle, must become the temperature at the end of the compression phase. It emphasizes the fact that the efficiency increases as it is possible to increase the amount of energy in the gas at its initial stage, and that the efficiency can never reach unity unless T_2 becomes zero on the absolute scale.

Losses.—The principal sources of loss in the operation of a gas engine are:

Radiation, circulating water, exhaust, and friction.

Some idea of the distribution of these losses is afforded by the following table, which shows the disposition of 100 heat units absorbed by the working substance of a four cycle engine.

Heat lost by radiation and conduction	11
Heat lost to cylinder walls	53
Heat lost in exhaust gases	14
Heat converted into mechanical energy	22
	100

The actual thermal efficiency in this case is 22 per cent.

Example.—If an engine have 33 per cent clearance space, and the charge be admitted at 13 lbs. absolute pressure, and 90° Fahr., what is the final pressure and temperature of compression under the usual working conditions?

The accompanying table has been prepared for determining the compression pressure and temperatures which usually occur in gas engines.

Table of Pressure and Temperature and Coefficients

(Compression)

C	P	T	C	P	T
3.	4.407	146.89	4.	6.498	162.45
3.05	4.506	147.74	4.1	6.718	163.86
3.1	4.606	148.58	4.2	6.940	165.25
3.15	4.707	149.42	4.3	7.164	166.62
3.2	4.808	150.25	4.4	7.390	167.96
3.25	4.910	151.06	4.5	7.618	169.29
3.3	5.011	151.87	4.6	7.847	170.59
3.35	5.115	152.67	4.7	8.078	171.88
3.4	5.217	153.47	4.8	8.311	173.17
3.45	5.322	154.25	4.9	8.546	174.40
3.5	5.462	155.03	5.	8.783	175.64
3.55	5.531	155.80	5.1	9.020	176.87
3.6	5.637	156.57	5.2	9.260	178.07
3.65	5.742	157.32	5.3	9.501	179.26
3.7	5.848	158.08	5.4	9.744	180.44
3.75	5.956	158.82	5.5	9.988	181.60
3.8	6.064	159.56	5.6	10.234	182.75
3.85	6.171	160.29	5.8	10.730	185.01
3.9	6.280	161.02	6.	11.233	187.22

These pressures and temperatures are obtained by means of the table as follows:

Final compression pressure = initial pressure × P . . . . (1)

Final compression temperature = intial temperature abs. $\times \frac{T}{100}$ (2)

In these two formulæ:

P = pressure factor for the given ratio of compression C;

T = temperature factor for the given ratio of compression C;

For a clearance space of 33⅓ per cent of the piston displacement the compression ratio, C, is:

$$\frac{1+33\frac{1}{3}}{33\frac{1}{3}}=4;$$

the pressure factor corresponding to C=4, from table is given as 6.498. Substituting these values in (1)

Final compression pressure=13×6.498=84½ lbs.

Similarly, substituting in (2),

$$\text{Final compression temperature}=(90+459.6)\times\frac{162.45}{100}=893°$$

The accompanying table has been prepared for obtaining the usual pressures and temperatures of exhaust corresponding to various degrees of expansion.

Table of Pressure and Temperature Coefficients

(Terminal)

E	P.	T	E	P	T
3.	23.975	719.22	4.	16.494	659.75
3.05	23.464	715.66	4.1	15.973	654.88
3.1	22.973	712.18	4.2	15.480	650.17
3.15	22.5	708.75	4.3	15.014	645.55
3.2	22.045	705.43	4.4	14.572	641.16
3.25	21.605	702.15	4.5	14.152	636.85
3.3	21.180	698.95	4.6	13.745	632.66
3.35	20.770	695.80	4.7	13.374	628.60
3.4	20.374	692.72	4.8	13.013	624.63
3.45	19.991	689.68	4.9	12.669	620.8
3.5	19.621	686.72	5.	12.340	617.03
3.55	19.262	683.80	5.1	12.027	613.37
3.6	18.915	680.94	5.2	11.727	609.81
3.65	18.580	678.14	5.3	11.440	606.34
3.7	18.253	675.36	5.4	11.166	602.95
3.75	17.937	672.66	5.5	10.903	599.64
3.8	17.631	669.99	5.6	10.650	596.41
3.85	17.334	667.36	5.8	10.175	590.15
3.9	17.046	664.79	6.	9.737	584.19

To ascertain the exhaust pressure corresponding to a given explosion pressure, the given explosion pressure is multiplied by the number in the column marked P, opposite the number in the column marked E corresponding to the given expansion ratio, and the product divided by 100. The result is the absolute exhaust pressure required.

Example.—If the explosion pressure be 225 lbs., and the expansion ratio 3.8, what is the possible exhaust pressure?

Applying the rule to the table

$$\text{Probable exhaust pressure} = \frac{(225+14.7)\times 17.631}{100} = 42.3 \text{ lbs. absolute}$$

or $42.3-14.7=27.6$ lbs. gauge pressure.

To ascertain the exhaust temperature corresponding to a given explosion temperature, the given explosion temperature is multiplied by the number in the column marked T, opposite the number in the column marked E, corresponding to the given expansion ratio, and the product divided by 1,000.

Fuel Oils

Gravity of Oils.—The gravity of oils should be taken at a temperature of 60° Fahr.

A rough rule for determining the gravity at any other temperature is: *For every* 10° *Fahr. above* 60°, *subtract one degree from the Baume reading, and for every* 10° *below* 60°, *add one degree.*

Example.—If the hydrometer indicate a gravity of 27.5° B. and the temperature of the oil for this reading be 75° Fahr., the temperature then is 15° above 60°, and the correct gravity is

$$27.5^\circ - \frac{15}{10} = 26^\circ \text{ B.}$$

In case the hydrometer indicate a gravity of 24° B. at a temperature of 40° Fahr. Then the correct gravity will be

$$24 + \frac{60-40}{10} = 26^\circ \text{ B.}$$

Formula for reducing Baumè reading (liquids lighter than water) to specific gravity:

$$\text{specific gravity} = \frac{140}{130 + \text{degrees Baumè}}$$

Thus:

gravity Baumè = 26 at 60° F.

or

$$\text{specific gravity} = \frac{140}{130 + 26} = .897$$

Diesel Engines

Diesel Cycle.—In the pressure volume diagram, fig. 112, assume the piston to be at its extreme outward position of travel, point 1, and the space between the piston and the cylinder head is filled with air at the pressure and temperature indicated by point 1. The air is compressed adiabatically until point 2 is reached.

Point 2 is at the end of the stroke and the compression has increased the pressure until it reaches the value indicated by point 2.

The piston now starts to move in the reverse direction and heat is added at a rate such that the pressure of the air behind the piston remains constant. This heat addition continues until point 3 is reached. Although the pressure has remained constant during the addition of heat, the temperature at point 3 is higher than that at point 2.

From 3 to 4 the piston is forced out by the expansion of the air behind it. This expansion takes place adiabatically and the internal energy is converted into work. The pressure and temperature drop to the values indicated by point 4.

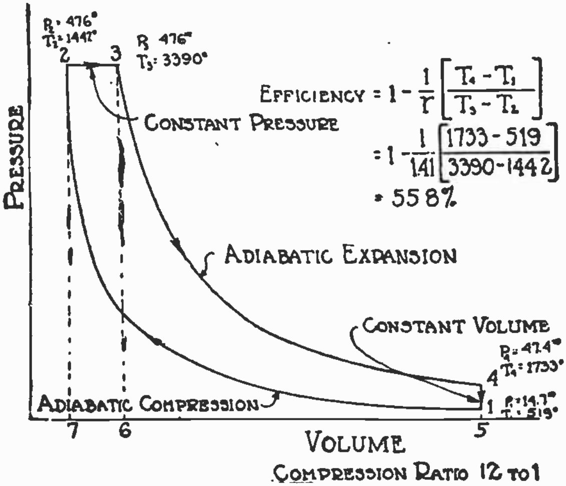


FIG. 112.—Diesel theoretical pressure volume diagram

Point 4 is at the extreme end of the stroke and with the piston fixed at this point sufficient heat is absorbed to bring the air back to its initial pressure and temperature. This completes the cycle.

$$\text{Efficiency} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \quad \ldots\ldots\ldots\ldots (1)$$

in which

C_v = specific heat at constant volume.

C_p = specific heat at constant pressure.

The relationship between the specific heat at constant pressure and the specific heat at constant volume is usually called r.

Substituting r for $C_v - C_p$ equation (1) becomes

$$\text{Efficiency} = 1 - \frac{1}{r} \times \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad \text{(2)}$$

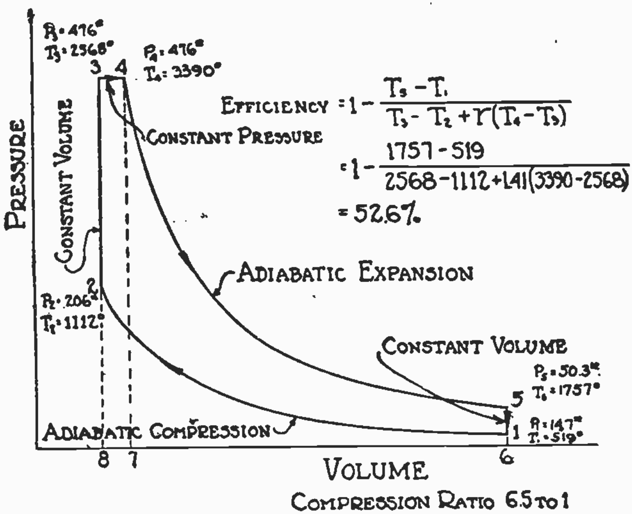


Fig. 113.—***Semi-Diesel*** theoretical pressure volume diagram.

Semi-Diesel Engines

The Semi-Diesel Cycle.—In the pressure volume diagram assume the piston to be at point 1 and the cylinder filled with air.

This air is compressed adiabatically until point 2 is reached

From 2 to 3 heat is added, while the volume remains constant, and from 3 to 4 heat is added with the pressure remaining constant.

From 4 to 5 the air expands adiabatically and converts internal energy into work and from 5 to 1 heat is absorbed from the air with the volume remaining constant.

$$\text{Efficiency} = 1 - \frac{T_5 - T_1}{T_3 - T_2 + r\,(T_4 - T_3)} \quad \ldots\ldots\ldots\ldots (3)$$

Horse Power

Internal Combustion Engine Horse Power.—The short formula for indicated horse power of a steam engine is

$$h.p. = .000004D^2LNP \quad \ldots\ldots\ldots\ldots\ldots (1)$$

In the steam engine there are two power strokes per revolution, but only one power stroke every two revolutions in a four cycle engine and one power stroke per revolution for a two cycle engine.

Accordingly, formula (1) becomes

1. For four cycle engine

$$\text{horse power} = .000001D^2LNP$$

2. For a two cycle engine

$$\text{horse power} = .000002D^2LNP$$

Refrigeration

The following tables will be found useful in refrigeration calculations.

Absolute Pressure Exerted by Various Refrigerants at Different Temperatures

Temp. Deg. Fahr.	Anhy Ammonia.	Ethyl Chloride.	Carbon Dioxide.	Sulphur Dioxide.
0	30.8	4.1	314.5	10.3
5	34.2	4.7	335.0	11.9
10	38.3	5.4	362.8	13.4
15	42.9	6.1	391.7	15.3
20	48.0	6.9	422.5	17.1
25	53.4	7.8	455.7	19.4
30	59.4	8.7	490.0	21.6
35	65.9	9.7	526.2	24.3
40	73.0	10.8	565.2	27.0
45	80.7	12.0	607.2	30.1
50	89.0	13.3	650.0	33.4
55	97.9	14.8	696.7	37.0
60	107.6	16.3	745.0	41.0
65	118.1	18.0	795.5	45.2
70	129.2	19.9	849.3	49.7
75	141.2	22.0	906.5	54.6
80	154.1	24.2	967.0	59.9

Vacuum and Corresponding Brine Temperatures

Inches vacuum	Approximate temperature of outgoing brine	Inches vacuum	Approximate temperature of outgoing brine
22	4°	14	31°
21	8°	13	34°
20	12°	12	36°
19	16°	11	39°
18	19°	10	41°
17	23°	9	43°
16	26°	8	46°
15	29°	7	48°

The amount of ammonia ordinarily taken to charge a compression plant is:

Tons of ice per 24 hours:	5	10	15	25	50	100
Pounds ammonia:	100	250	500	1000	2000	4000

TABLE OF HEAT VALUES AND BOILING POINT.

SUBSTANCE	BOILING POINT DEGREES F.	LATENT HEAT B. T. U.	SPECIFIC HEAT
Water	212.0	966.0	1.00
Sulphur Dioxide	14.0	168.7	0.41
Ether	—10.0	——	——
Anhydrous Ammonia	—28.5	573.0	1.0058
Carbon Dioxide	—109.0	141.0	0.955

Specific and Latent Heat of Various Food Products

Substance	Composition		Specific Heat Above Freezing in Heat Units	Specific Heat Below Freezing in Heat Units	Latent Heat of Freezing in Heat Units
	Water	Solids			
Lean beef	72.00	28.00	0.77	0.41	102
Fat beef	51.00	49.00	.60	.34	72
Veal	63.00	37.00	.70	.39	90
Fat Pork	39.00	61.00	.51	.30	55
Eggs	70.00	30.00	.76	.40	100
Potatoes	74.00	26.00	.80	.42	106
Cabbage	91.00	9.00	.93	.48	129
Carrots	83.00	17.00	.87	.45	118
Milk	87.50	12.50	.90	.47	124
Oysters	80.38	19.62	.84	.44	114
Whitefish	78.00	22.00	.82	.43	111
Eels	62.07	37.93	.69	.38	88
Lobster	76.62	23.38	.81	.42	108
Pigeon	72.40	27.60	.78	.41	102
Chicken	73.70	26.30	.80	.42	105

The figures in the last column showing the latent heat of freezing, have been obtained by multiplying the latent heat of freezing water, which is 142 heat units, by the per cent. of water contained in the different materials considered, for as the solid constituents remain in their original condition, only the liquid or watery portion of these materials are conuerned in the solidification or freezing of them.

Working Pressure of Ethyl Chloride

Gauge pressure lbs.	Approximate temperature of outgoing condensing water
10	60°
15	71°
20	81°
25	91°
30	96°
35	106°

Compressor.—A rule of thumb method of calculating the size of compressor required is to assume that a certain number of cubic inches of displacement per minute is equivalent to a ton of duty per 24 hours and to allow for ice making from 12,000 to 14,000 cu. ins. displacement per ton, and for refrigerating from 7,000 to 8,000 cu. ins.

Condensers.—The ordinary allowance for cooling surface is 24 sq. ft. per ton of refrigerating capacity, when operating with a cooling water temperature of from 58° to 70° Fahr., which is equivalent to approximately 38 lineal feet 2-inch pipe per ton of duty.

As the temperature of water is never the same in any two plants the amount of piping in the atmospheric condenser must be proportioned to suit the local conditions. The following table applies to atmospheric condensers, giving feet per ton capacity.

TABLE.

For water at Deg. F.	1 inch pipe.	1¼ inch pipe.	1½ inch pipe.	2 inch pipe.
50	60 feet	50 feet	40 feet	31 feet
55	65 feet	55 feet	45 feet	33 feet
60	75 feet	60 feet	50 feet	36 feet

TABLE—*Continued*

65	80 feet	65 feet	55 feet	40 feet
70	85 feet	70 feet	60 feet	43 feet
75	92 feet	76 feet	67 feet	46 feet
80	98 feet	82 feet	73 feet	50 feet

The amount of water required per minute per ton of refrigeration for atmospheric condensers is

TABLE.

At 50 degrees F. allow	½ gallon per minute
At 55 degrees F. allow	⅝ gallon per minute
At 60 degrees F. allow	¾ gallon per minute
At 65 degrees F. allow	⅞ gallon per minute
At 70 degrees F. allow	1 gallon per minute
At 75 degrees F. allow	1¼ gallon per minute
At 80 degrees F. allow	1½ gallon per minute

These quantities are based on water leaving the condenser at 95 degrees F.

To find the amount of water required per day multiply the amount given for the temperature of the water by minutes in a day of 24 hours and by the number of tons refrigeration required.

Brine.—The strength of brine solution generally used is from 40 to 90° salinometer. The following table gives strength of the brine for various readings of the salinometer.

In a salt brine circulation system one running foot of two inch pipe will take care of forty feet of space if it is to be maintained at 32° Fahr.

On a still more liberal basis it is frequently assumed that one ton of refrigerating capacity will take care of 4500 cu. ft. of cold storage space from 32 to 35 degrees Fahr., and that from 200 to 250 feet of 1¼ inch pipe should be used for that purpose.

TABLE—CHLORIDE OF SODIUM (SALT) BRINE.		
DEGREES ON SALOMETER.	POUNDS SALT PER GALLON.	FREEZING POINT DEGREES FAHR.
0	0	32
5	0.105	30.3
10	0.212	28.6
15	0.321	26.9
20	0.433	25.2
25	0.548	23.6
30	0.664	22
35	0.78	20.4
40	0.897	18.7
45	1.019	17.1
50	1.142	15.5
55	1.265	13.9
60	1.389	12.2
65	1.522	10.7
70	1.656	9.2
75	1.791	7.7
80	1.928	6.1
85	2.067	4.6
90	2.207	3.1
95	2.347	1.6
100	2.488	0

Freezing Mixtures

Composition of Freezing Mixtures in Parts by Weight	Reduction of Temperature in Deg. Fahr.		Amount of Fall in Deg. Fahr.
	From	To	
Snow 2 parts; common salt 1 part		—5	
Snow 5; common salt 2; ammonium chloride 1		—12	
Snow 14; common salt 10; ammonium chloride 5; potassium nitrate 5		—18	
Snow 12; common salt 5; ammonium nitrate 5		—25	
Ammonium chloride 5; potassium nitrate 5; water 16	+50	+4	46
Ammonium nitrate 1; water 1	+50	+4	46
Ammonium chloride 5; potassium nitrate 5; sodium sulphate 8; water 16	+50	+4	46

Freezing Mixtures

Continued

Composition of Freezing Mixtures in Parts by Weight	Reduction of Temperature in Deg. Fahr.		Amount of Fall in Deg. Fahr.
	From	To	
Ammonium chloride 5; potassium nitrate 5; sodium sulphate 8; water 16	+50	+4	46
Sodium sulphate 5; dilute sulphuric acid 4	+50	. 3	47
Sodium sulphate 8; hydrochloric acid 9	+50	0	50
Sodium nitrate 3; dilute nitric acid 2	+50	—3	53
Ammonium nitrate 1; sodium carbonate 1; water 1	+50	—7	57
Sodium sulphate 6; ammonium chloride 4; potassium nitrate 2; dilute nitric acid 4	+50	—10	60
Snow 8; dilute sulphuric acid 3; dilute nitric acid 3	—10	—56	46
Sodium sulphate 6; ammonium nitrate 5; dilute nitric acid 4	+50	—14	64
Snow 1; common salt 1	+32	0	32
Snow 3; dilute sulphuric acid 2	. 32	—23	55
Snow 3; hydrochloric acid 5	+32	—27	59
Snow 7; dilute nitric acid 4	+32	—30	62
Snow 4; calcium chloride 5	. 32	—40	72
Snow 8; calcium chloride 5	+32	—40	72
Snow 2; calcium chloride, crystalized 3	+32	—50	82
Snow 3; potassium 4	. 32	—51	83
In the Following Mixtures Materials Must be Previously Cooled:			
Snow 3; calcium chloride 4	+20	—4	24
Sodium phosphate 5; ammonium nitrate 3; dilute nitric acid 4	0	—34	34
Snow 3; dilute nitric acid 2	0	—46	46
Snow 1; calcium chloride, crystalized 2	0	—66	66
Snow 2; dilute sulphuric acid 1; dilute nitric acid 1	—10	—56	46
Sodium phosphate 9; dilute nitric acid 4	+50	—12	62
Snow 2; calcium chloride 3	—15	—68	53
Snow 12; common salt 5; ammonium nitrate 5	—18	—25	7
Snow 1; dilute sulphuric acid 1	—20	—60	40
Sodium Phosphate 3; ammonium nitrate 2; dilute mixed acid	—34	—50	16
Snow 1; calcium chloride 3	—40	· 73	33
Snow 2; calcium chloride, crystalized 3	—40	—73	33
Snow 8; dilute sulphuric acid 10	—68	—91	23

When no snow is to be had, finely crushed or grated ice will answer.
Dilute acid is 1 part strong commercial acid and 9 parts water.

HORSE POWER REQUIRED TO PRODUCE ONE TON OF REFRIGERATION.

CONDENSER PRESSURE AND TEMPERATURE.

REFRIGERATOR PRESSURE AND TEMP.

	P	103	116	127	139	153	168	164	200	218
p	t	65°	70°	75°	80°	85°	90°	95°	100°	105°
4	—20°	1.0584	1.1304	1.2051	1.2832	1.3611	1.4427	1.5251	1.6090	1.6910
6	—15°	.9972	1.0692	1.1450	1.2221	1.3001	1.4101	1.4609	1.5458	1.6300
9	—10°	.9026	.9777	1.0453	1.1183	1.1926	1.2602	1.3471	1.4352	1.5093
13	— 5°	.8184	.8833	.9537	1.0230	1.0935	1.1679	1.2437	1.3209	1.3964
16	0°	.7352	.8008	.8648	.9328	1.0019	1.0718	1.1467	1.2194	1.2547
20	5°	.6665	.7312	.7946	.8593	.9278	.9978	1.0656	1.1381	1.2121
24	10°	.5915	.6629	.7257	.7894	.8545	.9205	.9911	1.0595	1.1294
28	15°	.5410	.5998	.6641	.7276	.7924	.8553	.9224	.9943	1.0603
33	20°	.4745	.5340	.5923	.6716	.7148	.7796	.8420	.9031	.9736
39	25°	.4103	.4659	.5227	.5804	.5992	.7022	.7667	.8289	.8922
45	30°	.3509	.4056	.4612	.5178	.5755	6353	.6944	.7590	.8172
51	35°	.3005	.3546	.4101	.4666	.5214	5804	.6398	.7009	.7629

Cold Storage Temperatures

Articles	Degr. Fahr.
Fruit	
Apples	32-36
Bananas	34
Berries, fresh	36
Cranberries	33-36
Cantaloupes	40
Dates, figs, etc.	50-55
Fruits, dried	35-40
Grapes	34-36
Lemons	33-36
Oranges	34-36
Peaches	34-36
Pears, watermelons	34-36
Meats	
Brined	33
Beef, fresh	33
Beef, dried	36-40
Calves	32-33
Hams, ribs, shoulders (not brined)	20
Hogs	29-33
Lard	38
Livers	20-30
Sheep, lambs	33
Ox-tails	30
Sausage casings	20
Tenderloin, butts, etc.	33

Articles	Degr. Fahr.
Fish	
Fresh fish	20
Dried fish	36
Oysters in shell	30-35
Oysters in tubs	25
Canned Goods	
Sardines	35-40
Fruits	35-40
Meats	35-40
Butter, Eggs, etc.	
Butter	18-20
Butterine	18-20
Cheese	34
Eggs	31
Liquids	
Beer, ale, porter, etc.	33
Cider	30
Ginger ale	36
Wines	40-45
Flour and Meal	
Buckwheat flour	36-40
Corn meal	36-40
Oat meal	36-40
Wheat flour	36-40

Articles	Degr. Fahr.
Vegetables	
Asparagus	34-35
Cabbage	34-35
Carrots	34-35
Celery	34-35
Dried beans	32-40
Dried corn	35
Dried peas	35-40
Onions	36
Parsnips	34-35
Potatoes	36-40
Sauerkraut	35
Miscellaneous	
Cigars, tobacco	35
Furs, woolens, etc.	35
Honey	45
Hops	40
Maple syrup, sugar	40-45
Oils	35
Poultry, dressed, iced	28-30
Poultry, dry picked	26-28
Poultry, scalded	20
Game, to freeze	15-18
Game, after frozen	25-28
Poultry, to freeze	15-18
Poultry, after frozen	25-28
Nuts in shells	35-40
Chestnuts	33

Carpentry

Nails.—The raw material of the modern wire nail factory is ***hand drawn wire*** in bundles, just as it comes from the wire drawing block.

One explanation for the *penny system* is that one thousand ten penny nails weigh ten pounds. The most generally used nails are called *common nails*, and are regularly made in sizes from 1 in. (2d) to 6 in. (60d) according to the following table:—

Common Nails

PLAIN				COATED			
Size	Length Ins.	Gauge No.	Approximate No. to Lb.	Length Ins.	Gauge No.	No. of Nails in Keg	Net Wgt. Lbs.
2*d*	1	15	876	1	16	85,700	79
3*d*	1¼	14	568	1⅛	15½	54,300	64
4*d*	1½	12½	316	1⅜	14	29,800	61
5*d*	1¾	12½	271	1⅝	13½	25,500	70
6*d*	2	11½	181	1⅞	13	17,900	65
7*d*	2¼	11½	161	2⅛	12½	15,300	72
8*d*	2½	10¼	106	2⅜	11½	10,100	71
9*d*	2¾	10¼	96	2⅝	11½	8,900	68
10*d*	3	9	69	2⅞	11	6,600	63
12*d*	3¼	9	63	3⅛	10	6,200	80
16*d*	3½	8	49	3¼	9	4,900	80
20*d*	4	6	31	3¾	7	3,100	83
30*d*	4½	5	24	4¼	6	2,400	84
40*d*	5	4	18	4¾	5	1,800	82
50*d*	5½	3	14	5¼	4	1,300	79
60*d*	6	2	11	5¾	3	1,100	82

Holding Power of Nails and Spikes

(Values given are in lbs. per in. of penetration)

Size	Cut Nails					
	Parallel to grain			Cross grain		
	Yellow Pine	White Pine	White Oak	Yellow Pine	White Pine	White Oak
6*d*	89	....	....	154	77	317
8*d*	206	89	520	327	211	630
10*d*	222	108	580	324	181	650
20*d*	320	148	692	407	298	800
50*d*	439	170	820	570	316	991
60*d*	445	200	950	639	324	1,040

Size	Wire Nails				
	Parallel to grain	Cross grain			
	White Pine	Cedar (dry)	White Oak	Yellow Pine	White Pine
6*d*	30	....	129	108	60
10*d*	50	....	390	132	70
60*d*	....	....	731	465	....
⅜ in.	370	283	1,188	590	450
7/16 in.	344	....	....	....	436
½ "	113	338	744	700	364

Spikes.—A spike is *a stout piece of metal from 3 to 12 ins. in length and thicker in proportion than a common nail.* Spikes are made in sizes from 3 to 12 ins., according to the following table:

Ordinary Spikes

Size	Length	Gauge No.	Deg. of Counter-sunk	Diam. Head	Head Rad. (oval)	Approximate No. to Pound
10*d*...........	3 in.	6	123	13/32	7/16	41
12*d*...........	3¼ "	6	....	....	......	38
16*d*...........	3½ "	5	123	7/16	7/16	30
20*d*...........	4 "	4	123	15/32	7/16	23
30*d*...........	4½ "	3	123	½	7/16	17
40*d*...........	5 "	2	123	17/32	7/16	13
50*d*...........	5½ "	1	....	....	......	10
60*d*...........	6 "	1	123	9/16	7/16	9
7 inch........	7 "	5/16 inch	123	⅝	⅝	7
8 "	8 "	⅜ "	123	¾	¾	4
9 "	9 "	⅜ "	....	....	......	3½
10 "	10 "	⅜ "	....	....	......	3
12 "	12 "	⅜ "	....	....	......	2½

Holding Power of Nails.—Tests show the holding power of nails according to the table on page 186:

Nail Table

For 1,000 shingles, allow 5 lb. four penny nails or 3½ lb. three penny.
" 1,000 laths, 7 lb. three penny fine, or for 100 sq. yd. of lathing, 10 lb. three penny fine.
" 1,000 sq. ft. of beveled siding, 18 lb. six penny.
" 1,000 sq. ft. of sheathing, 20 lb. eight penny or 25 lb. ten penny.
" 1,000 sq. ft. of flooring, 30 lb. eight penny or 40 lb. ten penny.
" 1,000 sq. ft. of studding, 15 lb. ten penny and 5 lb. twenty penny.
" 1,000 sq. ft. of 1 by 2½ in furring, 12 in centers, 9 lb. eight penny or 14 lb. ten penny.
" 1,000 sq. ft. of 1 by 2½ in furring, 16 in centers, 7 lb. eight penny or 10 lb. ten penny.

Approximate Number of Wire Nails per Pound

American Steel & Wire Co.'s Steel Wire Gauge	LENGTH 3/16	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3	3 1/2	4	4 1/2	5	6	7	8	9	10	11	12
2/0	.....	.....	.....	.....	.....	.....	.....	39	26	23	20	17	15	15	12	11	11	8.9	7.9	7.1	6.4	5.2	4.5	4.0	3.4	3.2	2.9	2.7
1/0	.....	.....	.....	.....	.....	.....	.....	43	38	34	29	25	22	20	18	16	15	13	11	10	9.0	7.6	6.5	5.7	5.0	4.5	4.1	3.8
1	.....	.....	.....	.....	.....	.....	.....	47	44	40	34	29	26	23	21	20	18	16	14	12	11	9.3	8.0	7.0	6.3	5.7	5.2	4.7
2	.....	.....	.....	.....	.....	.....	.....	60	54	48	41	36	31	28	25	23	21	18	16	14	13	11	9.3	8.1	7.2	6.6	6.1	5.6
3	.....	.....	.....	.....	.....	.....	.....	67	60	55	47	41	36	32	29	27	25	21	18	16	15	12	11	9.4	8.3	7.6	7.1	6.6
4	.....	.....	.....	.....	.....	.....	.....	81	74	66	55	48	41	37	34	31	29	25	22	20	18	15	13	11	9.8	8.9	8.1	7.5
5	.....	.....	.....	.....	.....	.....	.....	90	81	74	61	52	45	41	38	35	32	28	24	22	21	18	16	14	12	12	11	9.6
6	.....	.....	.....	213	174	149	128	113	101	91	76	65	58	52	47	43	39	34	29	26	24	20	18	16	15	13	11	11
7	.....	.....	.....	250	208	174	148	132	120	110	92	78	70	61	55	51	47	40	35	31	28	24	19	18	16	14	13	12
8	.....	.....	.....	272	238	198	174	153	139	126	106	93	82	74	66	61	56	48	42	38	34	28	24	21	19	17	15	14
9	.....	.....	.....	348	286	238	213	185	170	152	128	112	99	87	79	71	67	58	50	45	41	34	29	25	23	21	19	17
10	.....	.....	.....	469	373	320	277	242	216	196	165	142	124	111	100	91	84	71	62	55	49	42	36	31	27	25	23	21
11	.....	.....	.....	510	417	366	323	285	254	233	200	171	149	136	122	111	103	87	77	69	61	52	44	39	35	31	29	26
12	.....	.....	.....	740	603	511	442	397	351	327	268	229	204	182	161	149	137	118	103	95	87	71	63	56	50	45	40	36
13	.....	.....	1356	1017	802	688	590	508	458	412	348	297	260	232	209	190	175	153	138	123	110	93	.....	.....	.....	.....	.....	.....
14	.....	2293	1664	1290	1037	863	765	667	586	536	459	398	350	312	278	256	233	201	176	157	140	117	.....	.....	.....	.....	.....	.....
15	.....	2899	2213	1619	1316	1132	971	869	787	694	578	501	437	390	351	317	290	246	220	196	177	145	.....	.....	.....	.....	.....	.....
16	.....	3932	2720	2142	1708	1414	1229	1099	973	872	739	635	553	496	452	410	370	318	277	248	226	.....	.....	.....	.....	.....	.....	.....
17	.....	5316	3890	2700	2306	1904	1581	1409	1253	1139	956	831	746	666	590	532	486	418	360	322	295	.....	.....	.....	.....	.....	.....	.....
18	.....	7520	5072	3824	3130	2608	2248	1976	1760	1590	1338	1150	996	890	820	740	680	585	507	448	412	.....	.....	.....	.....	.....	.....	.....
19	.....	9920	6860	5075	4132	3508	2816	2556	2284	2098	1772	1500	1390	1205	1060	970	895	800	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
20	18620	14050	9432	7164	5686	4795	4230	3596	3225	2893	2412	2070	1810	1620	1450	1315	1215	1035	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
21	23260	17252	12000	8920	7232	6052	5272	4576	4020	3640	3040	2665	2310	2020	1830	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
22	28528	21508	14676	11776	9276	7672	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
23	36586	27089	18026	13519	10815	9013	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
24	44936	34018	22678	17008	13607	11339	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
25	57357	43243	28828	21622	17297	14414	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

These approximate numbers are an *Average* only, and the figures given may be varied either way, by changes in the dimensions of the heads or points. Brads and no-head nails will run more to the pound than table shows, and large or thick-headed nails will run less.

Nails for Soft Wood Boxes

1/4	in. thickness use	4*d* cement coated nails
3/8	" "	5*d* " " "
7/16 or 1/2	" "	6*d* " " "
9/16 or 5/8	" "	7*d* " " "
7/8	" "	8*d* " " "

1 1/4 in. thickness use special large 3*d* or regular 4*d* cement coated nails

Nails for Hard Wood Boxes

3/8	in. thickness use	4*d* cement coated nails
7/16 or 1/2	" "	5*d* " " "
9/16 or 5/8	" "	6*d* " " "
7/8	" "	7*d* " " "

Screws.—A wood screw is *a screw nail, having a right handed coarse thread to give a good grip, a gimlet point to enter the wood and a slotted head for the reception of a screw driver.*

The following table gives the standard proportions.

Standard Wood Screw Proportions

(Asa S. Cook Co.)

Screw Numbers	*A*	*B*	*C*	*D*	Number of Threads per Inch	Screw Numbers	*A*	*B*	*C*	*D*	Number of Threads per Inch
0	....	....	...	0.0578	30	13	0.4427	0.1286	0.055	0.2289	11
1	....	....	...	0.0710	28	14	0.4790	0.1362	0.057	0.2421	10
2	0.1631	0.0454	0.030	0.0841	26	15	0.5053	0.1437	0.059	0.2552	9.5
3	0.1894	0.0530	0.032	0.0973	24	16	0.5316	0.1513	0.061	0.2684	9
4	0.2158	0.0605	0.034	0.1105	22	17	0.5579	0.1589	0.064	0.2815	8.5
5	0.2421	0.0681	0.036	0.1236	20	18	0.5842	0.1665	0.066	0.2947	8
6	0.2684	0.0757	0.039	0.1368	18	20	0.6368	0.1816	0.070	0.3210	7.5
7	0.2947	0.0832	0.041	0.1500	17	22	0.6895	0.1967	0.075	0.3474	7.5
8	0.3210	0.0908	0.043	0.1631	15	24	0.7421	0.2148	0.079	0.3737	7
9	0.3474	0.0984	0.045	0.1763	14	26	0.7421	0.1967	0.084	0.4000	6.5
10	0.3737	0.1059	0.048	0.1894	13	28	0.7948	0.2118	0.088	0.4263	6.5
11	0.4000	0.1134	0.050	0.2026	12.5	30	0.8474	0.2270	0.093	0.4546	6
12	0.4263	0.1210	0.052	0.2158	12	...	....	....	....	....	...

Safe Loads for Wood Screws

(Inserted across the grain)

Kind of Wood	Gauge Number							
	4	8	12	16	20	24	28	30
White oak	80	100	130	150	170	180	190	200
Yellow pine	70	90	120	140	150	160	180	190
White pine	50	70	90	100	120	140	150	160

Bolts.—A bolt is *a pin or rod used for holding anything in its place, and often having a permanent head on one end.*

Properties of U. S. Standard Bolts

(U. S. Standard Threads)

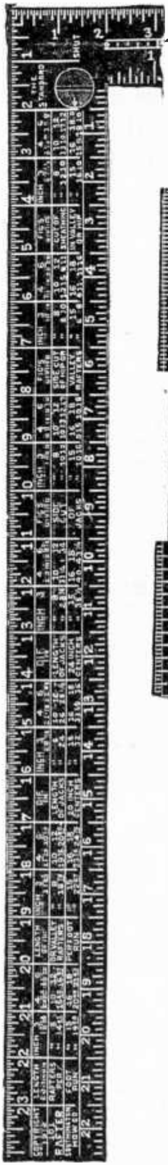
The tap drill diameters in the table provide for a slight clearance at the root of the thread, in order to facilitate tapping and reduce tap breakages. If full threads are required, use the diameters at the root of the threads for the tap drill diameters.

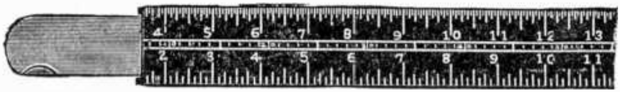
Diameter	Number of Threads per Inch	Diameter at Root of Thread	Diameter of Tap Drill	Area in Square Inches		Tensile Strength at Stress of 6000 Pounds per Square Inch	Dimensions of Nuts and Bolt Heads				
				Of Bolt	At Root of Thread						
1/4	20	0.185	13/64	0.049	0.026	160	1/2	0.578	0.707	1/4	1/4
5/16	18	0.240	1/4	0.076	0.045	270	19/32	0.686	0.840	5/16	19/64
3/8	16	0.294	5/16	0.110	0.068	410	11/16	0.794	0.972	3/8	11/32
7/16	14	0.345	23/64	0.150	0.093	560	25/32	0.902	1.105	7/16	25/64
1/2	13	0.400	27/64	0.196	0.126	760	7/8	1.011	1.237	1/2	7/16
9/16	12	0.454	15/32	0.248	0.162	1,000	31/32	1.119	1.370	9/16	31/64
5/8	11	0.507	17/32	0.307	0.202	1,210	1 1/16	1.227	1.502	5/8	17/32
3/4	10	0.620	41/64	0.442	0.302	1,810	1 1/4	1.444	1.768	3/4	5/8
7/8	9	0.731	3/4	0.601	0.419	2,520	1 7/16	1.660	2.033	7/8	23/32
1	8	0.838	55/64	0.785	0.551	3,300	1 5/8	1.877	2.298	1	13/16
1 1/8	7	0.939	31/32	0.994	0.694	4,160	1 13/16	2.093	2.563	1 1/8	29/32
1 1/4	7	1.064	1 3/32	1.227	0.893	5,350	2	2.310	2.828	1 1/4	1
1 3/8	6	1.158	1 7/32	1.485	1.057	6,340	2 3/16	2.527	3.093	1 3/8	1 3/32
1 1/2	6	1.283	1 11/32	1.767	1.295	7,770	2 3/8	2.743	3.358	1 1/2	1 3/16
1 5/8	5 1/2	1.389	1 27/64	2.074	1.515	9,090	2 9/16	2.960	3.623	1 5/8	1 9/32
1 3/4	5	1.490	1 17/32	2.405	1.746	10,470	2 3/4	3.176	3.889	1 3/4	1 3/8
1 7/8	5	1.615	1 21/32	2.761	2.051	12,300	2 15/16	3.393	4.154	1 7/8	1 15/32
2	4 1/2	1.711	1 49/64	3.142	2.302	13,800	3 1/8	3.609	4.419	2	1 9/16
2 1/4	4 1/2	1.961	2 1/64	3.976	3.023	18,100	3 1/2	4.043	4.949	2 1/4	1 3/4
2 1/2	4	2.175	2 15/64	4.909	3.719	22,300	3 7/8	4.476	5.479	2 1/2	1 15/16
2 3/4	4	2.425	2 31/64	5.940	4.620	27,700	4 1/4	4.909	6.010	2 3/4	2 1/8
3	3 1/2	2.629	2 11/16	7.069	5.428	32,500	4 5/8	5.342	6.540	3	2 5/16
3 1/4	3 1/4	2.879	2 15/16	8.296	6.510	39,000	5	5.775	7.070	3 1/4	2 1/2
3 1/2	3 1/4	3.100	3 11/64	9.621	7.548	45,300	5 3/8	6.208	7.600	3 1/2	2 11/16
3 3/4	3	3.317	3 3/8	11.045	8.641	51,800	5 3/4	6.641	8.131	3 3/4	2 7/8
4	3	3.567	3 5/8	12.566	9.963	59,700	6 1/8	7.074	8.661	4	3 1/16
4 1/4	2 7/8	3.798	3 27/32	14.186	11.340	68,000	6 1/2	7.508	9.191	4 1/4	3 1/4
4 1/2	2 3/4	4.028	4 3/32	15.904	12.750	76,500	6 7/8	7.941	9.721	4 1/2	3 7/16
4 3/4	2 5/8	4.255	4 5/16	17.721	14.215	85,500	7 1/4	8.374	10.252	4 3/4	3 5/8
5	2 1/2	4.480	4 9/16	19.635	15.760	94,000	7 5/8	8.807	10.782	5	3 13/16
5 1/4	2 1/2	4.730	4 13/16	21.648	17.570	105,900	8	9.240	11.312	5 1/4	4
5 1/2	2 3/8	4.953	5 1/32	23.758	19.260	116,000	8 3/8	9.673	11.842	5 1/2	4 3/16
5 3/4	2 3/8	5.203	5 9/32	25.967	21.250	127,000	8 3/4	10.106	12.373	5 3/4	4 3/8
6	2 1/4	5.423	5 1/2	28.274	23.090	138,000	9 1/8	10.539	12.903	6	4 9/16

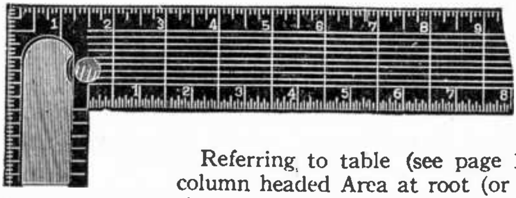
Referring to table above find in the line for 1 in. bolt 3,300 lbs. corresponding to a stress on the bolt of 6,000 lbs. per sq. in.

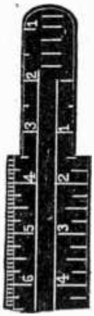
Example.—What size bolt is required to support a load of 2,500 lbs. for a stress of 6,000 lbs. per sq. in.?

$$\text{Area at root of thread} = \text{given load} \div 6{,}000$$
$$= 2{,}500 \div 6{,}000 = .417 \text{ sq. in.}$$









Referring to table (see page 190) in the column headed Area at root (or bottom) of thread, find .419 sq. in. nearest area; this corresponds to a $^7/_8$ in. bolt.

The "Steel" Square.—In buying a *framing* square (erroneously called *steel* square) it is advisable to get one with all the markings rather than a cheap square with some of the scales and tables omitted. Thus

Markings of Cheap Square Tables

Rafter—Essex—brace

Graduations

$^1/_{16}$, $^1/_{12}$, $^1/_8$, $^1/_4$

Complete Markings

Rafter, Essex, brace, octagon, polygon cuts

$^1/_{100}$, $^1/_{32}$, $^1/_{16}$, $^1/_{12}$, $^1/_{10}$, $^1/_8$, $^1/_4$

Figs. 114 to 117.—Southington standard take-down square, Fig. 114, face of body; fig. 115, face of tongue; fig. 116 back of body; fig. 117, back of tongue. *In construction* the tongue fits easily and locks with an anchored cam. The cam lock may be turned by a screw driver or coin. The long bearing joint gives maximum strength and insures the truth of the square.

The accompanying table gives values for measurements on tongue and body of the square such that by joining the points corresponding to the measurements any angle may be laid out from 1 to 45°.

Angle Table for Square

Angle	Tongue	Body	Angle	Tongue	Body	Angle	Tongue	Body
1	.35	20.	16	5.51	19.23	31	10.28	17.14
2	.7	19.99	17	5.85	19.13	32	10.6	16.96
3	1.05	19.97	18	6.58	19.02	33	10.89	16.77
4	1.4	19.95	19	6.51	18.91	34	11.18	16.58
5	1.74	19.92	20	6.84	18.79	35	11.47	16.38
6	2.09	19.89	21	7.17	18.67	36	11.76	16.18
7	2.44	19.85	22	7.49	18.54	37	12.04	15.98
8	2.78	19.81	23	7.8	18.4	38	12.31	15.76
9	3.13	19.75	24	8.13	18.27	39	12.59	15.54
10	3.47	19.7	25	8.45	18.13	40	12.87	15.32
11	3.82	19.63	26	8.77	17.98	41	13.12	15.09
12	4.16	19.56	27	9.08	17.82	42	13.38	14.89
13	4.5	19.49	28	9.39	17.66	43	13.64	14.63
14	4.84	19.41	29	9.7	17.49	44	13.89	14.39
15	5.18	19.32	30	10.	17.32	45	14.14	14.14

TABLE FOR INSCRIBED POLYGONS.

Number of Sides	3	4	5	6	7	8	9	10	11	12
Radius	56	70	74	60	60	98	22	89	80	85
Side	97	99	87	60	52	75	15	95	46	44

Example.—Lay off a pentagon with a side of 6 ins. Set the bevel to the figures in column 5, the lesser number on the tongue. In this case $^{74}/_{8} = 9\frac{1}{4}$ on the tongue, and $^{87}/_{8} = 10\frac{7}{8}''$ on body of the square. Sliding the bevel to 6 upon the body, the length of the radius, $5^{3}/_{32}$ will be read upon the tongue.

Pitch.—By definition pitch is the *proportion that the rise bears to the whole width of the building (or the span)**.

The pitch expressed as an equation is:

$$\text{pitch} = \frac{\text{rise}}{\text{span}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The following pitch table will be found useful.

Pitch Table

Pitch................	1	11/12	5/6	3/4	2/3	7/12	1/2	5/12	1/3	1/4	1/6	1/12
Run.................	12	12	12	12	12	12	12	12	12	12	12	12
Rise................	24	22	20	18	16	14	12	10	8	6	4	2

The following table gives the points on square for top and bottom cuts of various rafters.

Square Points for Top and Bottom Cuts

<table>
<tr><th colspan="2">PITCH</th><th>1</th><th>11/12</th><th>5/6</th><th>3/4</th><th>2/3</th><th>7/12</th><th>1/2</th><th>5/12</th><th>1/3</th><th>1/4</th><th>1/6</th><th>1/12</th></tr>
<tr><td rowspan="3">Tongue</td><td>Common</td><td colspan="12">12</td></tr>
<tr><td>Octagon</td><td colspan="12">13</td></tr>
<tr><td>Hip or Valley</td><td colspan="12">17</td></tr>
<tr><td colspan="2">Body</td><td>24</td><td>22</td><td>20</td><td>18</td><td>16</td><td>14</td><td>12</td><td>10</td><td>8</td><td>6</td><td>4</td><td>2</td></tr>
</table>

*NOTE.—Where rafters rise to a deck instead of a ridge, it is necessary to subtract the width of the deck from the total span.

Example.—A building 24 ft. wide has a roof with a rise of 8 ft. What is the pitch of the roof?

Substituting in (1)

$$\text{pitch} = 8/24 = 1/3$$

Board Measure Rule.—*Multiply length in ft. by width in ft. of the board and multiply this product by 1 for board an inch or less than an inch in thickness, and by the thickness in inches and fractions of an inch for board over 1 in. in thickness.*

TABLE

Board Measure

1 board 1 in. thick × 1 ft. wide × 1 ft. long = 1 ft. board measure (B. M.)
1 board 2 in. thick × 1 ft. wide × 1 ft. long = 2 ft. board measure
1 board ½ in. thick × 1 ft. wide × 1 ft. long = 1 ft. board measure
etc.

Example.—How many feet board measure (B. M.) in a board 12 ft long by 18 ins. wide by ½ in. thick?; by 1¾ in. thick?

$$18 \text{ ins.} = 18 \div 12 = 1\tfrac{1}{2} \text{ ft.}$$

$$\text{board } \tfrac{1}{2} \text{ in. thick} = 12 \times 1\tfrac{1}{2} \times 1 = 18 \text{ ft. B. M.}$$

$$\text{board } 1\tfrac{3}{4} \text{ in. thick} = 12 \times 1\tfrac{1}{2} \times 1\tfrac{3}{4}$$

$$= 12 \times 1.5 \times 1.75 = 31.5 \text{ ft. B. M.}$$

Strength of Timbers.—The various mechanical properties of woods have been determined by many tests.

The following working stresses and loads from the New York Building Code will indicate limits of stress to which wood should be subjected in proportioning the parts.

Working Stresses for Timbers
(lbs. per sq. in. of cross section)

	Tension	Compression	
		with grain	across grain
Oak	1,200	1,400	1,000
Yellow pine, long leaf	1,200	1,600	1,000
Spruce and Douglas fir	800	1,200	800
Douglas fir	800	....	....
White pine, short leaf yellow pine, N. C. pine and fir	....	1,000	300
White pine	700	....	....
Locust	....	1,200	1,000
Hemlock	600	800	800

Example.—A tank 10 ft. in diameter and 16 ft. high is to be supported by four rectangular posts. Find size of these posts for a working stress of 800 lbs. per sq. in. neglecting weight of the structure.

Cubic feet of water contained in the tank when full

$$= \left\{\begin{matrix}\text{area bottom}\\ .7854 \times 10^2\end{matrix}\right\} \times \left\{\begin{matrix}\text{height}\\ 16\end{matrix}\right\} = 1{,}256.6 \text{ cu. ft.}$$

Taking weight of water at 62½ lbs. per cu. ft., then

$$\text{weight of water} = 1256.6 \times 62.5 = 78{,}538 \text{ lbs.}$$

For four posts and working stress of 800 lbs. per sq. in:

$$\text{cross area of each post} = \frac{78{,}538}{4 \times 800} = 24.5 \text{ ins.}$$

nearest rectangular post is 4 × 6

Working Stresses for Columns*

(for various length diameter ratios)

Lbs. per sq. in. cross section.

Length ÷ diameter. or least side	Long leaf yellow pine	Spruce
30	600	390
25	700	475
20	800	560
15	900	645
12	960	695
10	1,000	730

The following values are given in *U. S. Forestry Circular* No. 15 for crushing strength across the grain of various timbers in lbs. per sq. in. (3% deformation):

Long leaf pine.............................. 1,000
Short leaf pine..............................900
White pine.................................. 700
Spruce......................................1,200
White cedar.................................700
Douglas spruce.............................. 800
White oak.................................. 2,200
Shagbark hickory........................... 2,700
White elm...................................1,200
White ash.................................. 1,900

*NOTE.—For columns of short leaf yellow pine, N. C. pine or Douglas fir, the working stresses shall not exceed ¾ of the corresponding values given for long leaf yellow pine, for columns of white pine or fir, the working stresses shall be taken the same as for spruce; for columns of white oak, the working stresses shall be taken the same as for long leaf yellow pine.

Shearing Stresses.—There are two kinds of shearing stresses according as the specimen is in *single* or *double* shear.

The following table gives safe shearing stresses for timbers.

Safe Shearing Stresses for Timbers

(lbs. per sq. in. of shearing surface)

	With grain	Across grain
Oak	200	1,000
Yellow pine, long leaf	150	1,000
Yellow pine, short leaf	150	1,000
N. C. pine, Douglas fir	100	1,000
White pine, spruce and fir	100	500
Hemlock	100	600

Shingles.—Ordinary wood shingles are furnished in random widths, but 1,000 shingles are equivalent to 1,000 shingles each 4 ins. wide. Dimension shingles are sawed to a uniform width, being either 4, 5, or 6 ins. wide.

The following table will be of value in estimating shingles.

Shingle Table

Distance laid to weather	Actual number per square		
	Without waste	Plain roof 8% waste	Hip roof 12% waste
4	900	972	1008
4½	800	864	896
5	720	778	806
5½	655	707	734
6	600	648	672

Stock Shingle Sizes

Length..................	16	18	20	24
Thickness at butt..........	5/16	7/16	1/2	1/2

Spacing of Shingles

Length of shingle........	16	18	20	24
Exposure to weather.....	4 7/8	5 1/2	6 1/8	7 1/2

Space Covered by 1,000 Shingles

Exposure to weather..	4 1/4	4 1/2	4 3/4	5	5 1/2	6
Area covered in sq. ft.	118	125	131	138	152	166

Nail Table

Materials	Unit	Quantity nails required in lbs.	Size	Kind
Joists and sills......	per 1,000 ft. B. M.	25	20d	common
Studding...........	" " " "	15	10"	"
Rafters.............	" " " "	15	10"	"
Sheathing, siding....	" " " "	20	8"	"
Cornice.............	" " lineal feet	25	8"	finish
Shingling...........	" M (1,000)	4	4"	common
Bevel siding........	" 1,000 ft. B. M.	18	6"	"
Ceiling, wainscoting.	" " " "	20	6"	"
Floors, pine.........	" " " "	30	8"	"
Floors, hard wood...	" " " "	30	6"	"
Base board.........	" " " "	12	8"	finish
Window trim one side		3/4	8"	"
Door trim one side..		3/4	8"	"
Lath..............	" 1,000	8	3"	common pine
Lattice for porches..	" 1,000 sq. ft.	20	3"	"
Balustrade.........	" " lineal ft.	18	6"	casing

Studding, Corner Posts and Plate.—Calculate total length of 2×4 lumber required for studs, corner posts and plate.

Count one stud under each window.

Sheathing.—*Calculate the exact surface to be covered, that is, calculate total surface and deduct openings for doors and windows; then, add* $^1/_{12}$ *for 12″ boards,* $^1/_{10}$ *for 10″ boards,* ⅛ *for 8″ boards, etc.*

These additions are due to the fact that on account of seasoning and dressing, a 12″ board becomes about 11½″, a 10″ board, 9⅝″, an 8″ board, 7¾″, etc. The additions specified approximately allow for waste.

Lap Siding.—For bevel or lap siding *calculate the exact surface,*

Rafters and Gables

Width of Building	Fourth Pitch					Third Pitch					Half Pitch				
	Length of Rafter		From Plate to Comb		Area of Two Gables	Length of Rafter		From Plate to Comb		Area of Two Gables	Length of Rafter		From Plate to Comb		Area of Two Gables
	ft.	in.	ft.	in.		ft	in.	ft.	in.		ft.	in.	ft.	in.	
6	3	4	1	6	9	3	7	2	0	12	4	3	3	0	18
7	3	11	1	9	12	4	0	2	4	16	5	0	3	6	25
8	4	6	2	0	16	4	10	2	8	21	5	8	4	0	32
9	5	0	2	3	20	5	5	3	0	27	6	5	4	6	41
10	5	7	2	6	25	6	0	3	4	33	7	1	5	0	50
12	6	8	3	0	36	7	2	4	0	48	8	6	6	0	72
14	7	10	3	6	49	8	5	4	8	65	9	11	7	0	98
16	9	0	4	0	64	9	7	5	4	85	11	4	8	0	128
18	10	1	4	6	81	10	10	6	0	108	12	9	9	0	162
20	11	2	5	0	100	12	0	6	8	133	14	2	10	0	200
22	12	4	5	6	121	13	2	7	4	161	15	7	11	0	242
24	13	5	6	0	144	14	5	8	0	192	17	0	12	0	288
26	14	6	6	6	169	15	7	8	8	225	18	6	13	0	338
28	15	8	7	0	196	16	10	9	4	261	19	11	14	0	392
30	16	9	7	6	225	18	0	10	0	300	21	4	15	0	450
32	17	11	8	0	256	19	2	10	8	341	22	9	16	0	512

To the lengths of rafters above given, must be added the desired projection for cornice.. Add also to make stock lengths.

For length of rafter on one-way roofs, take the rafter given for double the width thus: The rafter for a one-way roof on a building 10 feet wide, 4th pitch is that given for 20 feet wide or 11 feet, 2 inches.

In area of gable above given no allowance is made for waste or laps.

To verify above or obtain length of rafters for buildings of other widths than above given multiply the width of building by .559 for 4th pitch; by .6 for 3d pitch and by .71 for half pitch.

deducting for window and door openings; this gives net surface to be coverėd by the siding. Now since part of each siding plank overlaps the adjoining plank, add 25% for 6″ siding when laid 4½″ to the weather; 50% for 4″ siding.

Rafters.—The table on page 199 will be found useful in figuring rafters.

Roof Boards.—*Area to be covered* = 2 (*length of rafter* + *length of building* + 2 × *overhang*).

Paint.—For three coats of paint on wood *one gallon of paint should cover from* 175 *to* 200 *sq. ft., of surface.*

If dark colored paint, such as gray, tan, buff, drab, etc., be used in three coat work on weather boarding or matched boards, one gallon of paint should cover from 200 to 225 sq. ft.

Mitring Polygonal Figures.—The following table gives square setting for various mitres.

Properties of Polygons

NAME	NUMBER OF SIDES	SETTING		EXTERNAL ANGLE	INTERNAL ANGLE	MITRE ANGLE	AREA FOR UNITY SIDE
TRIANGLE	3	7	4	120	60	30	.433
SQUARE	4	12	12	90	90	45	1.
PENTAGON	5	13¾	10	72	108	54	1.72
HEXAGON	6	4	7	60	120	60	2.6
HEPTAGON	7	12½	6	51.43	128.57	64.29	3.63
OCTAGON	8	18	7½	45	135	67.5	4.82
NONAGON	9	22½	9	40	140	70	6.18
DECAGON	10	9½	3	36	144	72	7.69
UNDECAGON	11	10¾	3	32.7	147.3	73.65	9.37
DUODECAGON	12	11¼	3	30	150	75	11.2

Actuary's Estimate Tables—To find Quantities of Lumber Required

STUDDING on 16 inch centers. Estimate 1 to the lineal foot: This allows for doubling at openings and at corners.

JOISTS AND RAFTERS on 16 inch centers. To ¾ of the length of the building add 1; thus: For a building 16x32, ¾ of 32=24, to which add 1, or 27, being the joists required, or the rafters for 1 side. Add 1 or 2 for each bearing partition.

ROOF SHEATHING LAID SOLID. To full area of roof add 10 per cent for waste. If laid 2 inches apart ¾ of above will be required.

ARTICLE	Count Width	Face Width	Loss in Matching	To area to be covered add
Shiplap	12 inch	11¼	7 %	1-12
"	10 "	9¼	8⅓ "	1-10
"	8 "	7¼	11 "	1-8
"	6 "	5¼	12½ "	1-5
Flooring	6 "	5¼	12½ "	1-5
"	4¼ "	3¼	18 "	¼
"	4 "	3¼	19 "	[illegible]
"	3 "	2¼	25 "	[illegible]
"	2¾ "	2	27 "	[illegible]
"	2½ "	2	20 "	[illegible]
"	2 "	1½	25 "	⅓

Drop siding, ceiling, and partition same as above.

ARTICLE	Size	Exposed	To area to be covered add
Siding, beveled	½x4	3¼ inch	¼
" "	½x4	3 "	⅓
" "	½x4	2¾ "	[illegible]
" "	½x5	4¼ "	1-5
" "	½x5	4 "	¼
" "	½x5	3¾ "	[illegible]
" "	½x6	5¼ "	1-5
" "	½x6	5 "	9-40
" "	½x6	4¾ "	⅙

SHINGLES

When exposed 4 inches to the weather require 9 to the square foot.
" " 4½ " " " " " 8 " " " "
" " 5 " " " " " 7 1-5" " " "
" " 5½ " " " " " 6½ " " " "
" " 6 " " " " " 6 " " " "
} Add 1-10 for Waste

CORNICES. Multiply the total lineal feet, by the combined width of planceer, frieze, and fascia thus: If the planceer is 12 inches, the frieze 8 inches, and the fascia 4 inches, the combined width is 24 inches or 2 feet b. m. to the lineal foot of cornice.

CORNER BOARDS AND OUTSIDE BASE. Estimate on same plan as cornices and then add ¼ if of 1¼ or ½ if of 1½ stuff.

BRIDGING. Multiply the total lineal feet, measuring each string in a straight line by the following:

For 2x6 2x8 or 2x10 on 16 inch centers by 2
" 2x12 16 " " " 2¼
" 2x14 16 " " " 2½
" 2x6 and 2x8 12 " " " 2
" 2x10 and 2x12 12 " " " 2¼
" 2x14 12 " " " 2⅜

LATTICE

1⅛ wide multiply the area by 12 for the lineal feet required.
1⅜ " " " " " 10 " " " " "
1¾ " " " " " 8 " " " " "

Lath

Lath when laid ⅜ inch apart, as for lime, require 1½ to the square foot, or 13½ to the square yard to which add 4% for waste, making practically 14 to the square yard. So to find the lath required increase the square feet to be lathed by ½ thus: 900 square feet require 900+½ of 900 or 1350 lath plus 4%=1404.

When laid ¼ inch apart, as for cement plasters, require 7 per cent more lath.

When there are no openings add 10 per cent to amount obtained by above.

Mill Made Doors.—Stock sizes of doors cover a wide range but those most commonly used are,

2′ 6″×6′ 6″
2′ 8″×6′ 8″
2′ 10″×6′ 10″
3′ ×7′

These sizes either 1⅜ or 1¾ inches thick.

Single Inside Door Jambs.—On architects' plans figures for doors are generally given so that it will be easy to get the sizes.

For example, to measure or copy: commence by listing all the different widths of doors; say, 2′ 4″, 2′ 6″, and 3′ 0″. Then take the different heights, as, 6′ 6″, 6′ 8″, 6′ 10″, etc., as may be shown on plans, and make a list as on page 203.

Sizes of Four Light Sash

Size of Glass	Size of Window	Size of Glass	Size of Window
10″ × 20″	2′ 1″ × 3′ 10″	14″ × 26″	2′ 9″ × 4′ 10″
10″ × 22″	2′ 1″ × 4′ 2″	14″ × 28″	2′ 9″ × 5′ 2″
10″ × 24″	2′ 1″ × 4′ 6″	14″ × 30″	2′ 9″ × 5′ 6″
10″ × 26″	2′ 1″ × 4′ 10″	14″ × 32″	2′ 9″ × 5′ 10″
10″ × 28″	2′ 1″ × 5′ 2″	14″ × 34″	2′ 9″ × 6′ 2″
10″ × 30″	2′ 1″ × 5′ 6″	14″ × 36″	2′ 9″ × 6′ 6″
10″ × 32″	2′ 1″ × 5′ 10″	14″ × 38″	2′ 9″ × 6′ 10″
10″ × 34″	2′ 1″ × 6′ 2″	14″ × 40″	2′ 9″ × 7′ 2″
10″ × 36″	2′ 1″ × 6′ 6″	14″ × 42″	2′ 9″ × 7′ 6″
12″ × 20″	2′ 5″ × 3′ 10″	14″ × 44″	2′ 9″ × 7′ 10″
12″ × 22″	2′ 5″ × 4′ 2″	14″ × 46″	2′ 9″ × 8′ 2″
12″ × 24″	2′ 5″ × 4′ 6″	14″ × 48″	2′ 9″ × 8′ 6″
12″ × 26″	2′ 5″ × 4′ 10″	15″ × 24″	2′ 11″ × 4′ 6″
12″ × 28″	2′ 5″ × 5′ 2″	15″ × 26″	2′ 11″ × 4′ 10″
12″ × 30″	2′ 5″ × 5′ 6″	15″ × 28″	2′ 11″ × 5′ 2″
12″ × 32″	2′ 5″ × 5′ 10″	15″ × 30″	2′ 11″ × 5′ 6″
12″ × 34″	2′ 5″ × 6′ 2″	15″ × 32″	2′ 11″ × 5′ 10″
12″ × 36″	2′ 5″ × 6′ 6″	15″ × 34″	2′ 11″ × 6′ 2″
12″ × 38″	2′ 5″ × 6′ 10″	15″ × 36″	2′ 11″ × 6′ 6″
12″ × 40″	2′ 5″ × 7′ 2″	15″ × 38″	2′ 11″ × 6′ 10″
12″ × 42″	2′ 5″ × 7′ 6″	15″ × 40″	2′ 11″ × 7′ 2″
12″ × 44″	2′ 5″ × 7′ 10″	15″ × 42″	2′ 11″ × 7′ 6″
12″ × 46″	2′ 5″ × 8′ 2″	15″ × 44″	2′ 11″ × 7′ 10″
12″ × 48″	2′ 5″ × 8′ 6″	15″ × 46″	2′ 11″ × 8′ 2″
14″ × 24″	2′ 9″ × 4′ 6″	15″ × 48″	2′ 11″ × 8′ 6″

Door Jambs

2′ 4″	2′ 6″	3′
2—6′ 6″	10—6′ 10″	25—7′
7—6′ 10″	5—7′ 4″	15—7′ 6″
10—7′	12—7′ 2″	20—7′ 8″
15—7′ 6″	9—7′ 6″	24—7′ 10″
9—7′ 8″	6—7′ 8″	

Window Sash.—The list on page 202 is for four light windows, all $1\frac{3}{8}$ or $1\frac{3}{4}$ in. in thickness. An allowance of 5 in. in width and 6 in. in height is made for the outside sizes of the sash; which is the size of frame (between stiles, and between jamb and sill).

Lath.—Wood lath are sold by the 1,000, in bundles usually containing 100 lath.

Wood lath vary in dimensions, the common size being $1\frac{1}{2}'' \times \frac{1}{4}'' \times 4'$. This allows proper nailing to studding spaced either 12 or 16 ins. on centers.

Panel Strips.—These may be had or made in a great variety of patterns. In stock they are usually plain, $\frac{1}{4}'' \times 1\frac{1}{8}''$ and should not be less than $\frac{4}{16}'' \times 1\frac{1}{2}''$; $\frac{3}{8}'' \times 2''$ is better.

Matched Ceilings.—*The standard (nominal) thicknesses of yellow pine ceiling are $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{4}$ in., the actual thickness of each being $\frac{1}{16}$ in. less.*

The $\frac{3}{8}$ in. ceiling is dressed one side only, the other thicknesses being dressed on both sides.

Mill Work.—There are various kinds of mills.

They may be classed as the ordinary saw mill where the cut logs are sawed up into lumber of various sizes; sash and door mills, furniture, stair mills and various other mills specializing in some particular product. Such machines as band and circular saws, lathes, planers, joiners, sanders, etc., are to be found in nearly all mills.

According to Dodge the approximate power required for circular rip saws running at from 7,000 to 9,000 ft. per minute is

$$\text{approx. horse power} = \frac{(\text{diam. saw})^2}{40} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Example.—What is the approximate horse power required for a 50-in rip saw running under ordinary conditions?

Substituting in the formula (1)

$$\text{approximate horse power} = \frac{50 \times 50}{40} = 62$$

Of course the actual power required will depend largely upon: thickness of cut; rate of feed; kind of wood and its condition; condition of saw.

Cutting Speeds for Circular Saws

(Speeds given in feet per minute)

Circular rip saws for hard cutting wood	3,000 to 6,000
Circular rip saws for oak	4,000 to 8,000
Circular rip saws for soft woods	6,000 to 12,800
Circular cross cut saws	3,000 to 6,000

Example.—The size of tight and loose pulleys on countershaft of a circular saw is 10 in. diameter. If the countershaft speed must be 650 *r. p. m* to give 3,000 *r. p. m.* of the saw, what diameter of line shaft pulley must be used for a line shaft speed of 250 *r. p. m.*?

The relation is expressed in the following proportion:

speed of line shaft: speed of counter shaft = diameter of counter pulley: diameter of line pulley

Substituting $250 : 650 = 10 : x$

solving for x

$$250x = 6{,}500$$

$$x = \frac{6{,}500}{250} = 26 \text{ ins.}$$

Speed of Machines.—The following are the ordinary speeds:

Light portable mills.................. 450 to 650 *r.p.m.*
High speed steam feed mills.......... 600 to 900 *r.p.m.*
Small circular bench saws............2000 to 2500 *r p.m.*

Be careful not to overspeed.

The speed must be known before a saw can be hammered to the right tension.

Wood lathe speed *should range from* 2,400 *to* 3,000 *when the belt is at the smallest step of the cone.*

At this speed stock up to 3 ins. diameter can be safely turned. Use second step of cone for stock up to 6 ins. diameter and largest step for all stocks over 6 ins. diameter. Glued work should not be run at too high speed owing to danger of it being thrown from lathe by centrifugal force.

Brick Work

The standard sizes of brick are

Common brick
Textured face brick } .... $2\frac{1}{4} \times 3\frac{3}{4} \times 8$
Smooth face brick .... $2\frac{1}{4} \times 3\frac{7}{8} \times 8$

Fire brick are made in various sizes (and shapes) to suit the requirements. The ordinary size of fire brick is $9 \times 4\frac{1}{2} \times 2\frac{1}{2}$.

Strength of Brick.—The ratio of the crushing strength of brick and of walls built from them varies with the strength of the brick, the workmanship and character of the mortar joints.

Properties of Brick

Name of Grade	Absorption Limits, per cent.		Compressive Strength (on edge), lb. per sq. in.		Modulus of Rupture, lb. per sq. in.	
	Mean of 5 Tests	Individual Maximum	Mean of 5 Tests	Individual Minimum	Mean of 5 Tests	Individual Minimum
Vitrified Brick........	5 or less	6.0	5000 or over	4000	1200 or over	800
Hard Brick..........	5 to 12	15.0	3500 or over	2500	600 or over	400
Medium Brick........	12 to 20	24.0	2000 or over	1500	450 or over	300
Soft Brick..........	20 or over	No limit	1000 or over	800	300 or over	200

Volume of Masonry.—To ascertain the volume of bricks in masonry, proceed as follows:

Rule—1. To face dimensions of particular brick used, add thickness of mortar joint.

2. Compute area in sq. ft. and divide this by area of face of wall;
3. Multiply quotient thus obtained by number of rows of brick across wall which will give number of brick in wall.*

Example.—A 12 in. wall is 4 ft. high and 80 ft. long; size of brick $8 \times 2\frac{1}{4} \times 3\frac{3}{4}$; ½ in. joints. How many brick does it contain?

*NOTE.—In case of a header row consider header row as two rows.

1. face dimen. + joint

$8 + \frac{1}{2} = 8\frac{1}{2}$

$2\frac{1}{4} + \frac{1}{2} = 2\frac{3}{4}$

2. face area brick and mortar $\frac{8\frac{1}{2}\times 2\frac{3}{4}}{144} = .162$ sq. ft.

face area wall $=4\times 80=320$ sq. ft.
No. of brick in face of wall $=320\div .162=1975$

3. Number of brick in wall $1{,}975\times 3=5{,}925$

Mortar.—The tables following give the amount of cement and sand required for laying brick and tile for joints of average three-eighths inch thickness.

Materials for Cu. Yd. of Mortar

(According to Universal Atlas Cement Co.)

Proportions by Parts Cement	Sand	*Barrels of Cement*	*Cubic Yards of Sand*
1	0	8.31	
1	1	4.88	.72
1	1½	3.87	.86
1	2	3.21	.95
1	2½	2.74	1.01
1	3	2.39	1.06
1	4	1.90	1.13

Materials for Laying 1000 Brick

Width of Joint Inches	Cu. Ft. of Mortar	1:2 Mortar		1:2½ Mortar		1:3 Mortar		1:3½ Mortar	
		Cement Bbls.	Sand Cu. Yds.	Cement Bbls.	Sand Cu. Yds.	Cement Bbls.	Sand Cu. Yds.	Cement Bbls.	Sand Cu. Yds.
¼	10	1.2	0.35	1.0	0.37	0.9	0.39	0.8	0.41
⅜	15	1.8	0.53	1.5	0.56	1.3	0.59	1.2	0.61
½	18	2.1	0.63	1.8	0.67	1.6	0.71	1.4	0.73
⅝	22	2.6	0.77	2.2	0.82	1.9	0.86	1.7	0.90

Note:—In using this table, bear in mind that quantities for brick work are only approximate, because there is sure to be considerable variation in thickness of joints and in the size of brick.

The covering capacity of mortar given in the table following is calculated for average sand and with no allowance for waste.

In estimating, allowance should be made for waste. When figured for stucco, the loss of mortar in forming the keys behind the lath for the first coat should be taken into account.

Covering Capacity of Mortar and Stucco

Area Covered by One Barrel of Cement in Various Mixes

MIX Parts by Volume		THICKNESS OF COAT				
		¼ Inch	⅜ Inch	½ Inch	¾ Inch	1 Inch
Cement	Sand	Sq. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.
1	1	266	177	133	89	66
1	1½	336	226	168	112	84
1	2	404	270	202	135	101
1	2½	472	314	236	157	118
*1	3	542	362	271	181	136
1	3½	612	408	306	204	153
1	4	682	455	341	227	171

Area Covered by One Cubic Yard of Sand in Various Mixes

MIX Parts by Volume		THICKNESS OF COAT				
		¼ Inch	⅜ Inch	½ Inch	¾ Inch	1 Inch
Cement	Sand	Sq. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.
1	1	1800	1200	900	600	450
1	1½	1508	1006	754	503	377
1	2	1364	910	682	455	341
1	2½	1282	.855	641	427	321
*1	3	1222	815	611	407	306
1	3½	1178	785	589	393	294
1	4	1148	765	574	383	287

*1:3 is the mix most used for stucco work.

Note:—The above areas are calculated for average sand and with no allowance for waste. In estimating, allowance should be made for waste. When figured for stucco, the loss of mortar in forming the keys behind the lath for the first coat should be taken into account.

Foundations.—If foundation footings were made large enough much expense of cracked walls due to settling would be avoided. The ordinary allowances for footings are given in the following table.

Safe Loads on Foundation Beds

Kind of foundation bed		Safe load in Tons per sq. ft.
Rock		15 to 30
Sand, gravel and boulders		5
Fine sand		2
Hard pan		10
Clay	ordinary	2
	large % sand	4

To be conservative these loads could be reduced at least 25%.

Example.—The weight to be transmitted by the footings of a small 10×20 garage is 500 lbs. per sq. ft. of floor area. What size footings should be provided if the foundation bed be of fine sand?

$$\text{Floor area} = 10 \times 20 = 200 \text{ sq. ft.}$$

$$\text{total weight to be carried} = 200 \times 500 = 100{,}000 \text{ lbs.,}$$
$$\text{or } 100{,}000 \div 2{,}000 = 50 \text{ tons.}$$

From the table the safe load for fine sand is 2 tons per sq. ft., hence

$$\text{Area of footing} = 50 \div 2 = 25 \text{ sq. ft.}$$
$$\text{Approximate length of footing} = (2 \times 10) + 2 \times 20 = 60 \text{ ft.}$$
$$\text{Width of footing} = (25 \div 60) \times 12 = 5 \text{ ins.}$$

In a case like this the footing would be made not less than the thickness of the wall or 8 ins.

Chimneys.—According to the American Chimney Corp. Depth of foundation 1/26 height of chimney.

$$\text{thickness of walls (bottom)} = \frac{\text{height of chimney}}{9} + 7''$$

decreasing to 7 ins. (at top) for 7 ft. diameter; 10 ins. for diameter over 10 ft.

Taper approximately 4′ per 100′.

Flue width .6 D; height 1.45 D, where D = clear diameter of chimney at top.

Theoretical Draft in Inches of Water at Sea Level.—

Let

D = Theoretical draft.
H = Distance from top of chimney to grates.
T = Temperature of air outside of chimney.
T_1 = Temperature of gases in the chimney.

Then

$$D = 7.60\,H\left(\frac{1}{461+T} - \frac{1}{461+T_1}\right)$$

The results obtained represent the theoretical draft at sea level.

For higher altitudes they are subject to correction, as follows:

For altitudes of	Multiply with
1000′	0.966
2000′	0.932
3000′	0.900
5000′	0.840
10000′	0.694

Shoring.—A shoring timber acts as a strut.

Shore Table

For walls from	Inches	Inches
15 to 20 feet in height	4×4 to	6×6
20 " 30 " "	4×8 "	6×8
30 " 40 " "	6×8 "	8×10
40 " 50 " "	8×8 "	10×10
50 " 75 " "	10×12 "	12×14

Strength of Brickwork.—The compressive strength of brick is an important item and it will be seen in the following table to vary greatly for different brick.

Compressive Strength of Individual Brick

(Tested Flat)

Brick	Lbs. per sq. in.	Tons per sq. ft.
Arkansas		
Red grade 1	12,253	953
" " 2	11,966	860
" " 3	5,620	406
Illinois		
Shale building brick	10,690	770
Underburned common	3,920	280
Kentucky		
Dark gray	20,030	1,442
Gray	16,793	1,210
Dark green	7,243	521
Red	5,290	380

Strength of Mortar.—The Bureau of Standards tests showed practically no difference in strength of straight 1 – 3 Portland cement mortar and similar mortar in which 35% by volume of cement was replaced by lime.

Cement lime mortar is cheaper and saves bricklayers' time because of its greater plasticity. Lime mortar, while naturally weaker than cement or cement lime mortar, produces brickwork strong enough for many purposes.

Figuring Brickwork.—The only way to arrive at the exact cost of the brick and mortar, and the time to lay them, is to figure the actual number of bricks to be used.

The following tables will be helpful:

Number of Brick required for every square foot of Brick Wall 4″ to 4½″ thick

Size of Brick	⅜″ joints. No. of brick in each square foot.	½″ joints. No. of brick in each square foot.	⅝″ joints. No. of brick in each square foot.
8¼ × 4 × 2¼......	6½	6	5¾
8¼ × 4 × 2½......	5¾	5½	5¼
8½ × 4⅛ × 2½......	5¾	5¼	5
8¾ × 4⅜ × 2¾......	5	4¾	4½

Number of Brick required for every square foot of Brick Wall 8″ to 9″ thick

Size of Brick	⅜″ joints. No. of brick in each square foot.	½″ joints. No. of brick in each square foot.	⅝″ joints. No. of brick in each square foot.
8¼ × 4 × 2¼.....	13	12	11½
8¼ × 4 × 2½.....	11½	11	10½
8½ × 4⅛ × 2½.....	11½	10½	10
8¾ × 4⅜ × 2¾.....	10	9½	9

Number of Brick required for every square foot of Brick Wall 12″ to 13″ thick

Size of Brick	⅜″ joints. No. of brick in each square foot.	½″ joints. No. of brick in each square foot.	⅝″ joints. No. of brick in each square foot.
8¼ × 4 × 2¼.....	19½	18	17¼
8¼ × 4 × 2½.....	17¼	16½	15¾
8½ × 4⅛ × 2½.....	17¼	15¾	15
8¾ × 4⅜ × 2¾.....	15	14¼	13½

Number of Brick required for every square foot of Brick Wall 16″ to 18″ thick

Size of Brick	⅜″ joints. No. of brick in each square foot.	½″ joints. No. of brick in each square foot.	⅝″ joints. No. of brick in each square foot.
$8\frac{1}{4} \times 4 \times 2\frac{1}{4}$	26	24	23
$8\frac{1}{4} \times 4 \times 2\frac{1}{2}$	23	22	21
$8\frac{1}{2} \times 4\frac{1}{8} \times 2\frac{1}{2}$	23	21	20
$8\frac{3}{4} \times 4\frac{3}{8} \times 2\frac{3}{4}$	20	19	18

Number of Brick per sq. ft.

(Four inch wall; no headers)

Joint	⅛	¼	⅜	½	⅝	¾
No. of Brick	7½	7	6½	6⅛	5¾	5½

Example.—Estimate by the area method the number of brick (of size $8\frac{1}{4} \times 4 \times 2\frac{1}{2}$ including mortar) for a garage size $12' \times 20' \times 9'$ high having five 3×6 windows and one $7\frac{1}{2} \times 9$ door.

Gross area of walls = 2 (12 × 9) + 2 (20 × 9) = 576 sq. ft.

Deduct:
Area of five 3 × 6 windows = 5 (3 × 6) = 90 sq. ft.
" " one 7½ × 9 door = 7½ × 9 = 67.5 " "
total deduction = 157.5 " " 157.5 " "
Net wall area 418.5 " "

In the table on this page look under ¼ in. joint and find 7 brick required per sq. ft. Hence, total number of brick required for garage is

$$(418.5 \times 7) \times 2 = 5{,}859 \text{ brick*}$$

It should be noted that this is for an 8 in. wall with stretcher bond without any header cornices.

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*NOTE.—5,859 brick as here obtained checks very closely with the number (5,842) obtained by the volume method. The reason for the difference in the result is because the figure 7 in the table is an approximation to avoid fraction and unnecessary figuring.

Mortar required per 1,000 Brick

(⅜ joints)

PROPORTIONS	QUANTITIES		
	Cement	*Lime*	*Sand*
Cement Mortars			
1 part cement 2 parts sand	1¾ bbls.	¼ bbl. optional	½ cu. yd.
1 part cement 2½ parts sand	1⅜ bbls.	¼ bbl. optional	½ cu. yd.
1 part cement 3 parts sand	1⅛ bbls.	¼ bbl. optional	½ cu. yd.
Lump Lime Mortars			
1 part lime 2 parts sand		⅞ bbl.	½ cu. yd.
1 part lime 2½ parts sand		¾ bbl.	½ cu. yd.
1 part lime 3 parts sand		⅝ bbl.	½ cu. yd.
Hydrated Lime Mortar			
1 part lime 2 parts sand		3½ sacks	½ cu. yd.
1 part lime 2½ parts sand		3 sacks	½ cu. yd.
1 part lime 3 parts sand		2½ sacks	½ cu. yd.
Cement-Lime Mortars			
1 part cement 1 part lime 6 parts sand	½ bbl.	1 sack hydrated, or ¼ bbl. lump lime	½ cu. yd.
Grout for $\frac{3}{16}$"-¼" Joints			
1 part cement 3 parts sand	approx. ¾ bbl.		approx. ⅓ cu yd.

Example.—How much cement lime mortar is required for the small garage brick laid in common bond?

The number of brick required for common bond as found is 6,818. Hence the quantities given in the table for cement lime mortar must be multiplied by $\frac{6,818}{1,000}$ or 6.82. Thus

Cement	= ½ bbl.	× 6.82 = 3.41	say	3½ bbl.
Lime	= ¼ "	× 6.82 = 1.71	"	1¾ "
Sand	= ½ cu. yd.	× 6.82 = 3.41	"	3½ cu. yds.

Double Shell Wall Tile.—A 6×12×5 tile is used for 6 in. walls, and an 8×12×5 tile for 8 in. walls.

Allow ⅜ in. for mortar joints.

The following table shows the strength of double shell tile:

***Strength of Double Shell Tile.**

Number of Specimen	Nominal Size	Net Area (Sq. In.)	Maximum Load Total (Lbs.)	Maximum Load Units (Lbs per Sq. In.)
1	8" x 12" x 5"	44.25	299450	6770
2	8" x 12" x 5"	44 25	258580	5840
3	8" x 12" x 5"	44.25	285280	6450
4	6" x 12" x 5"	39.75	238000	5990
5	6" x 12" x 5"	39 75	311650	7840
6	6" x 12" x 5'	39.75	270510	6810
7	8" x 12" x 5"	44.25	224760	5080
8	6" x 12"/x 5"	39 75	252050	6340

*NOTE.—These tests were made on Natco tile by Carnegie Institute of Technology, Pittsburg, Pa., July, 1918. Specimens 7 and 8 were glazed. Specimen 7 showed a detail failure at one end due likely to improper bedding, which no doubt explains the low result obtained. All tile were tested on end and were bedded in plaster of Paris on top and bottom, the plaster of Paris cap extending over the webs so that the full cross section of the tile was in bearing. The unit loads were bared on the net area.

NOTE.—*Portland cement* is packed in bags of 94 lbs. net weight. Four bags make a barrel of 376 lbs. net. For ease of calculation, Portland cement is often assumed to weigh 400 lbs. gross or 380 lbs. per barrel and when proportioning, is figured to weigh 100 lbs. per cu. ft. Lump lime is sold in bulk by the bushel, the bushel varying from 75 to 85 lbs. net. It is also sold by the barrel. A 180 lb. barrel contains 3.1 cu. ft. and a 280 lb. barrel contains 4.7 cu. ft. A cu. ft. of lime weighs from 60 to 75 lbs. net.

Concrete

In mixing concrete *satisfactory results are obtained only by proper proportioning.*

To make dense concrete, the cement, sand and stone must be proportioned so that the voids in the coarse aggregate are filled with the finer particles of sand and cement, and so that the voids in the sand are filled and bound together with the particles of cement.

For instance, 7 cu. ft. of material when mixed make only 4½ cu. ft. of concrete instead of 7 cu. ft. This point should be remembered in order to avoid mistakes in estimating the quantity of materials to make a given quantity of concrete.

Volume of Concrete Mixtures

Proportions of Mixture			Volume of concrete
Cement	Sand	Gravel or stone	
1 bag	1½ cu. ft.	3 cu. ft.	3½ cu. ft.
1 "	2 "	3 "	$3^{9}/_{10}$ "
1 "	2 "	4 "	4½ "
1 "	2½ "	5 "	$5^{2}/_{5}$ "
1 "	3 "	5 "	$5^{4}/_{5}$ "

Mixing Water Required for Concrete

(Approximate quantities)

Mixture			Water Required Gals. per sack of Cement	
Cement	Aggregate			
	Fine	Coarse	Minimum	Maximum
1	1¼	2½	5	5½
1	1½	3	5½	6
1	2	3	5¾	6¼
1	2	4	6	6½
1	2½	5	7¼	7¾
1	3	6	8¼	8¾

Strength of Concrete

(For various proportions of water)

Gallons of water for one bag batch	28 days compressive strength lbs. per sq. in.
5¾	2,770
6	2,600
6¼	2,400
6½	2,250
7	1,950
7½	1,670
8	1,470
9	1,100
10	830
12	480
15	200

Strength of Concrete.—An excess of water weakens the concrete, an insufficient amount prevents thorough mixing.

Output of Concrete.—For conservatively estimating output, an average time for each batch should be from two to three minutes. This means actual mixing of at least one minute. Note results given in table.

Example.—A mixer turned out 164 two-sack batches of 1:2½:5 concrete. What is the yardage? The table on page 219 gives .202 cu. yds. for a one-sack batch, so multiply .202 by 2 to get the quantity for a two-sack batch, and this by 164 batches to get the total yardage. Thus: $.202 \times 2 \times 164 =$ 66.2 cu. yds.

Concrete Blocks.—Sand, gravel, cement and water are the materials generally used for making blocks. The thickness of concrete block walls is often regulated by state or local building codes.

The following table represents acceptable practice for walls of residences and apartment buildings (always only lightly loaded), except that the 10 in. block has been almost entirely replaced by the 8 in. block.

Total Safe Loads for Reinforced Beams

(In pounds, uniformly distributed)

Depth of Beam in Inches	SPAN—IN FEET 5	6	7	8	9	10	Depth to Center Line of Steel	Depth Below Center Line of Steel	Steel Area *
	For Beams 6 Inches Wide								
8	2760	2160	1764	1440	1188	960	7.00	1.00	.258
9	3390	2736	2226	1824	1512	1260	7.75	1.25	.282
10	4380	3528	2900	2400	2000	1680	8.75	1.25	.318
11	5490	4428	3654	3072	2592	2220	9.75	1.25	.354
12	6720	5472	4536	3840	3240	2760	10.75	1.50	.396
	For Beams 8 Inches Wide								
8	3680	2880	2352	1920	1584	1280	7.00	1.00	.344
9	4520	3648	2968	2432	2016	1680	7.75	1.25	.376
10	5840	4704	3867	3200	2667	2240	8.75	1.25	.424
11	7320	5904	4872	4096	3456	2960	9.75	1.25	.472
12	8960	7296	6048	5120	4320	3680	10 75	1.50	.528
	For Beams 10 Inches Wide								
10	7300	5880	4760	4000	3330	2800	8.75	1.25	.530
11	9150	7380	6090	5120	4320	3700	9.75	1.25	.590
12	11200	9120	7560	6400	5400	4600	10.75	1.50	.660
13	13000	10440	8680	7360	6300	5400	11.50	1.50	.700
14	15300	12480	10430	8800	7560	6500	12.50	1.50	.760

*See Table page 220, for size of rods.

Based on information contained in "Concrete, Plain and Reinforced," by Taylor and Thompson.

Unit stress for concrete considered as 500 pounds per square inch in compression; for steel as 14.000 pounds per square inch in tension, extra conservative design.

Reinforced Concrete.—Because of adhesion and shrinkage of the concrete during the process of setting or hardening the concrete firmly grips the reinforcing metal.

In proportioning reinforced concrete beams note the safe loads given in the table.

CONCRETE

Hourly Output of Concrete

Proportions 1 sack batch	Average time per batch 2 minutes	Average time per batch 3 minutes	Average time per batch 4 minutes
	Cu. Yds.	Cu. Yds.	Cu. Yds.
1:1½:3	4.0	2.6	2.0
1:2 :3	4.3	2.9	2.2
1:2 :4	5.0	3.3	2.5
1:2½:4	5.4	3.6	2.7
1:2½:4½	5.7	3.8	2.9
1:2½:5	6.0	4.0	3.0

Proportions	*Yardage of Concrete* Amount of Concrete in a 1 sack batch	
	Cu. Ft.	Cu. Yds.
1:1½:3	3.53	.131
1:2 :3	3.90	.145
1:2 :3½	4.22	.156
1:2 :4	4.50	.167
1:2½:4	4.88	.181
1:2½:4½	5.17	.192
1:2½:5	5.44	.202
1:3 :5	5.81	.215
1:3 :5½	6.11	.226
1:3 :6	6.38	236

Equivalent Spacing of Reinforcing Bars for Concrete Slabs

(To be used in substituting one sized bar for another size)

⅜" Bars		½" Bars		⅝" Bars		¾" Bars		⅞" Bars		1" Bars	
Rd.	Sq.	Rd.	Sq.	Rd.	Sq.	Rd.	Sq.	Rd.	Sq.	Rd.	Sq.
										3"	3¾"
									3⅜"	3½	4½
								3"	3¾	4	5
								3½	4½	4½	5¾
								3¾	4¾	5	6¼
						3"	3¾"	4¼	5¼	5⅜	6¾
						3¼	4⅛	4½	5⅝	6	7½
					3⅜"	3¾	4¾	5	6½	6½	8½
					3½	4	5	5½	7	7	9
				3"	3¾	4¼	5½	6	7½	7½	9½
				3¼	4⅛	4¾	6	6½	8	8¼	
				3½	4½	5	6½	7	8¾	9	
			3"	3¾	4¾	5½	6⅞	7⅜	9¼	9½	
			3¼	4	5	5¾	7⅜	8	10		
			3½	4¼	5½	6	7¾	8¼			
			3⅝	4½	5¾	6½	8¼	8¾			
		3"	3¾	4¾	6	6¾	8½	9¼			
		3¼	4⅛	5¼	6½	7½	9½				
		3½	4½	5½	7	8	10				
		3¾	4¾	6	7½	8½					
		4¼	5½	6½	8½	9½					
		4½	5¾	7	9	10					
		4¾	6	7½	9½						
	3⅜"	5	6⅜	8	10						
3"	3¾	5¼	6¾	8¼							
3⅛	4	5½	7	8¾							
3⅜	4¼	6	7⅝	9⅜							
3⅝	4⅝	6½	8¼	10							
4	5⅛	7	9								
4¼	5⅜	7½	9⅝								
4½	5¾	8	10								

Thickness of Walls (ins.) *Load 60 lbs. per sq. ft.*

No. of stones	Basement	First story	Second story	Third story	Fourth story
1	8	8	..	..	..
2	10	8	8	..	..
3	12	12	10	8	..
4	16	12	12	10	8

Materials for Small Quantities of Concrete

Cubic feet of Concrete	1: 1½: 3 Mixture			1: 2: 3 Mixture			1: 2: 4 Mixture			1: 2½: 5 Mixture			1: 3: 6 Mixture		
	Bags Cement	Cu. Ft. Sand	Cu. Ft. Stone	Bags Cement	Cu. Ft. Sand	Cu. Ft. Stone	Bags Cement	Cu. Ft. Sand	Cu. Ft. Stone	Bags Cement	Cu. Ft. Sand	Cu. Ft. Stone	Bags Cement	Cu. Ft. Sand	Cu. Ft. Stone
100	28	42	84	25 4/5	51 3/5	77 3/4	22	44	88	18	45	90	16	48	96
90	25 1/5	37 4/5	75 3/5	23 1/5	46 2/5	69 3/5	19 4/5	39 3/5	79 1/5	16 1/5	40 1/2	81	14 2/5	43 1/5	86 2/5
80	22 2/5	33 3/5	67 1/5	20 2/3	41 1/3	62	17 3/5	35 1/5	70 2/5	14 2/5	36	72	12 4/5	38 2/5	76 4/5
70	19 3/5	29 2/5	58 4/5	18	36	54	15 2/5	30 4/5	61 3/5	12 3/5	31 1/2	63	11 1/5	33 3/5	67 1/5
60	16 4/5	25 1/5	50 2/5	15 1/2	31	46 1/2	13 1/5	26 2/5	52 4/5	10 4/5	27	54	9 3/5	28 4/5	57 3/5
50	14	21	42	13	26	39	11	22	44	9	22 1/2	45	8	24	48
40	11 1/5	16 4/5	33 3/5	10 1/3	20 2/3	31	8 4/5	17 3/5	35 1/5	7 1/5	18	36	6 2/5	19 1/5	38 2/5
30	8 2/5	12 3/5	25 1/5	7 3/4	15 1/2	23 1/4	6 3/5	13 1/5	26 2/5	5 2/5	13 1/2	27	4 4/5	14 2/5	28 4/5
20	5 3/5	8 2/5	16 4/5	5 1/5	10 2/5	15 3/5	4 2/5	8 4/5	17 3/5	3 3/5	9	18	3 1/5	9 3/5	19 1/5
10	2 4/5	4 1/5	8 2/5	2 3/5	5 1/5	7 4/5	2 1/5	4 2/5	8 4/5	1 4/5	4 1/2	9	1 3/5	4 4/5	9 3/5
9	2 1/2	3 4/5	7 3/5	2 1/3	4 2/3	7	2	4	8	1 3/5	4	8	1 1/2	4 1/3	8 2/3
8	2 1/4	3 3/8	6 3/4	2	4 1/6	6 1/4	1 3/4	3 1/2	7	1 2/5	3 3/5	7 1/5	1 1/4	3 7/8	7 3/4
7	2	3	6	1 4/5	3 1/2	5 2/5	1 1/2	3	6	1 1/4	3 1/8	6 1/4	1 1/8	3 3/8	6 3/4
6	1 2/3	2 1/2	5	1 3/5	3 1/5	4 3/5	1 1/3	2 2/3	5 1/3	1 1/10	2 3/4	5 1/2	1	3	6
5	1 2/5	2 1/10	4 1/5	1 1/3	2 2/3	4	1 1/10	2 1/5	4 2/5	9/10	2 1/4	4 1/2	4/5	2 2/5	4 4/5
4	1 1/8	1 3/4	3 3/8	1	2	3 1/5	7/8	1 3/4	3 1/2	7/10	1 4/5	3 3/5	5/8	2	4
3	4/5	1 1/4	2 1/2	3/4	1 1/2	2 1/3	2/3	1 1/3	2 2/3	1/2	1 1/3	2 2/3	1/2	1 1/2	3
2	9/16	7/8	1 11/16	1/2	1	1 1/2	7/16	7/8	1 3/4	1/3	9/10	1 4/5	5/16	1	2
1	9/32	7/16	7/8	1/4	1/2	3/4	7/32	7/16	7/8	1/5	1/2	1	5/32	1/2	1

Example:—To find the materials for 291 cubic feet of 1:2:4 mixture, look in the columns headed "1:2:4 mixture," copy out the figures for 100 cubic feet and multiply them by two, to make 200. This gives 44 *bags cement*, 88 *cubic feet of sand, and* 176 *of stone.* Then look under the same head, opposite 90 cubic feet; 19 4/5 *bags cement*, 39 3/5 *cubic feet sand, and* 79 1/5 *cubic feet stone.* Then look opposite 1 cubic foot and there is found 7/32 *bag cement*, 7/16 *cubic foot sand, and* 7/8 *cubic foot stone.* Add these three, and it will be found that for 291 cubic feet of concrete in 1:2:4 mixture, there will be needed 64 bags cement, 128 cubic feet sand, and 256 cubic feet stone or gravel.

Building Code Requirements for Live Load

In Pounds Per Square Foot

STRUCTURE	Baltimore	Boston	Buffalo	Chicago	Cincinnati	Indianapolis	Milwaukee	Minneapolis	New Orleans	New York	Philadelphia	Pittsburgh	St. Louis	San Francisco	Seattle	Washington
Apartments	60	50	70	40	40	50	30	50	40	...	70	...	50	60	40	50
Assembly Halls	...	...	100	100	100	125	...	125	...	100	120	150	100	...	...	...
Dwellings	60	50	40	40	40	50	30	50	40	40	70	70	50	60	40	50
Hospitals	...	...	70	50	...	50	30	50	...	...	70	...	50	60	50	...
Hotels	60	...	70	50	40	75	30	50	40	...	70	...	50	60	40	50
Manufacturing	175	...	...	...	150	200	...	...	...	...	150	200	150	250	...	...
Light Manufacturing	125	125	120	100	100	100	100	100	125	...	120	...	100	125	125	...
Heavy Storehouses	250	250	...	...	150	200	...	...	...	...	150	...	...	250	...	150
Warehouses	...	250	150	...	150	200	...	...	200	...	150	200	150	250	...	150
Offices	75	100	70	50	50	75	40	75	70	60	100	...	60	60	50	75
Schools—Class Rooms	75	60	...	...	60	100	40	100	60	75	...	...	75	75	50	75
Stairways, General	...	70	...	100	80	...	60	...	70	...	...	...	...	...	100	...
Roofs—Slope Under 20°	...	40	40	25	25	30	30	50	30	40	30	50	30	30	40	25
Wind Pressure	30	...	30	20	20	...	30	30	...	30	30	...	30	20	...	30

*Table of Proportions and Quantities for One Cubic Yard of Concrete using various sizes of aggregates

Based upon laboratory investigations, using approved materials, compressive strength, 28 days, with workable plasticity, 6 by 12-inch cylinders, 3,000 pounds per square inch.

Sizes		Fine Aggregates, Screen Openings per Inch														
Coarse Aggregates Inches	Cement in bbls. Aggregates in Cubic Yards	0-28			0-14			0-8			0-4			0-3/8-in.		
		Cement	Fine	Coarse	Cement	Fine	Coarse	Cement	Fine	Coarse	Cement	Fine	Coarse	Cement	Fine	Coarse
No. 4 Screen to 3/4	Proportions....	1	1.3	2.4	1	1.6	2.4	1	1.8	2.3	1	2.0	2.3	1	2.7	1.5
	Quantities.....	1.96	.37	69	1.85	.44	.66	1.82	.48	.62	1.75	.52	.59	1.79	.72	40
No. 4 Screen to 1	Proportions....	1	1.3	2.7	1	1.6	2.6	1	1.8	2.6	1	2.0	2.5	1	2.6	1.8
	Quantities....	1 90	.36	76	1 77	.42	68	1.72	.46	.66	1.67	.50	.62	1.72	66	.46
No. 4 Screen to 1 1/2	Proportions..	1	1.2	3.1	1	1.6	3.2	1	1.7	3.1	1	2	3	1	2.4	2.4
	Quantities ..	1.82	32	.84	1.68	.40	.79	1.63	.41	75	1.61	.47	.72	1.62	57	.57
3/8 to 3/4	Proportions.	1	1.3	2.3	1	1.7	2.3	1	1.9	2.3	1	2.2	2.2	1	2.8	1.4
	Quantities	1 96	37	.67	1.85	.46	.63	1.82	.51	.62	1.75	.57	57	1 79	.75	37
3/8 to 1	Proportions..	1	1.3	2.6	1	1.7	2.6	1	1.9	2.5	1	2.2	2.4	1	2.7	1.7
	Quantities	1 90	.36	.74	1.77	.44	.68	1.72	.48	.64	1.67	.54	.59	1.72	.68	43
3/8 to 1 1/2	Proportions ..	1	1.3	3.0	1	1.7	3.0	1	1.9	3.0	1	2.1	2.9	1	2.6	2.2
	Quantities.	1 82	.35	80	1.68	.43	.75	1.63	.46	.73	1.61	.50	.68	1.62	.63	.53
1/2 to 3/4	Proportions..	1	1.5	2.3	1	1.9	2.2	1	2.1	2.2	1	2.3	2.1	1	2.8	1.3
	Quantities	1 96	44	.67	1 85	52	61	1 82	.56	.59	1.75	.59	.54	1.79	.75	.34
1/2 to 1	Proportions..	1	1.5	2.5	1	1.9	2.5	1	2.1	2.4	1	2.3	2.4	1	2.8	1.6
	Quantities...	1 90	.42	.70	1.77	50	.66	1 72	.53	.61	1 67	.57	.59	1.72	72	.41
1/2 to 1 1/2	Proportions..	1	1.4	2.8	1	1.9	2.9	1	2.1	2.9	1	2.2	2.8	1	2.7	2.1
	Quantities....	1 82	37	.75	1.68	47	73	1.63	51	69	1 61	.52	.66	1.62	65	.51
1/2 to 2	Proportions..	1	1.4	3.3	1	1.9	3.8	1	2.0	3.4	1	2.2	3.3	1	2.7	2 7
	Quantities...	1.75	36	.86	1 63	46	79	1.55	.46	78	1 52	50	.74	1 53	62	.62
3/4 to 1	Proportions..	1	1.7	2.4	1	2.1	2.4	1	2.4	2.1	1	2.6	2.2	1	3.1	1.5
	Quantities.....	1 90	48	68	1 77	55	.63	1 72	.61	53	1 67	.64	.55	1.72	79	.39
3/4 to 1 1/2	Proportions..	1	1.7	2.7	1	2.0	2.8	1	2.3	2.7	1	2.5	2.7	1	3.0	2.0
	Quantities..	1.82	.46	.73	1.79	.50	70	1.63	.55	.65	1.61	.59	64	1.62	.73	48
3/4 to 2	Proportions....	1	1.7	3.1	1	2.0	3.1	1	2.3	3.1	1	2.5	3.0	1	3.0	2 4
	Quantities.	1 75	44	80	1 63	.48	.75	1 55	.53	.72	1.52	.56	.67	1.53	68	55
1 to 1 1/2	Proportions....	1	1.7	2.8	1	2.0	2.9	1	2.3	2.7	1	2.6	2.6	1	3.1	2 0
	Quantities..	1 82	46	75	1.68	.50	.73	1.63	.55	.65	1.61	.62	.62	1.62	.75	48
1 to 2	Proportions..	1	1.5	3.2	1	1.9	3.5	1	2.2	3.3	1	2.4	3.3	1	3.0	2.6
	Quantities.	1 75	39	.83	1.63	.46	.85	1 58	51	.76	1.52	.51	.74	1 53	68	59

**Compiled by Prof. Duff A. Abrams of Structural Materials Research Laboratory, Lewis Institute, Chicago.*

Materials for Small Foundation Walls of Concrete

Wall 7 ft. high—Material needed for each 10 ft. length.

Thickness	1:2:4 Mixture			1:2½:5 Mixture			1:3:6 Mixture		
	Bags Cement	Cubic Feet Sand	Cubic Feet Stone	Bags Cement	Cubic Feet Sand	Cubic Feet Stone	Bags Cement	Cubic Feet Sand	Cubic Feet Stone
8 in.	10⅕	20⅗	41⅕	8⅖	21	42	7½	22⅗	44⅘
9 in.	11⅘	23⅘	46⅗	9⅗	23⅗	47⅗	8⅗	25⅕	50⅗
10 in.	12⅘	25⅗	51⅗	10½	26⅗	52½	9⅗	28	56
12 in.	15⅗	30⅗	61⅗	12⅗	31½	63	11⅕	33⅗	67⅕
18 in.	23	46⅕	92⅗	18⅘	47⅕	94⅗	16⅘	50⅗	100

Wall 8 ft. high—Material needed for each 10 ft. length.

8 in.	11⅘	23½	47	9⅗	24	48	8½	25½	51⅕
9 in.	13⅕	26⅗	52⅘	10⅘	27	54	9⅗	28⅘	57⅗
10 in.	14⅗	29⅗	58½	12	30	60	10⅗	32	64
12 in.	17⅗	35⅕	70⅗	14⅖	36	72	12⅘	38⅖	76⅘
18 in.	26⅖	52⅘	106	21⅗	54	108	19⅕	57½	115

Wall 9 ft. high —Material needed for each 10 ft. length.

8 in.	13⅕	26½	53	10⅘	27	54	9⅗	28⅘	57½
9 in.	14⅘	29⅗	59⅕	12⅕	30⅗	60⅘	10⅘	32⅗	64⅘
10 in.	16½	33	66	13½	33⅘	67½	12	36	72
12 in.	19⅘	39⅗	79⅕	16⅕	40½	81	14⅗	43⅕	86⅗
18 in.	29¾	59½	119	24⅖	60⅘	121	21⅗	64⅘	130

Material for each 10 ft. of length of Footings 1: 3: 6 Mixture

Size (height x width)	Cement Bags	Sand Cu. Ft.	Stone Cu .Ft.
6 in. x 12 in.	⅘	2⅗	4⅘
7 in. x 14 in.	1⅛	3⅜	6¾
8 in. x 16 in.	1½	4⅓	8⅔
9 in. x 18 in.	1⅘	5⅓	10⅗
10 in. x 20 in.	2⅕	6½	13
12 in. x 24 in.	3⅙	9⅓	18⅗
15 in. x 30 in.	5	14⅗	29⅖

Atlas Table for Determining Quantity of Materials

Cubic feet of Concrete	1 : 1½ : 3 Mixture			1 : 2 : 3 Mixture			1 : 2 : 4 Mixture			1 : 2½ : 5 Mixture			1 : 3 : 6 Mixture		
	Bags Cement	Cubic feet Sand	Cubic feet Stone	Bags Cement	Cubic feet Sand	Cubic feet Stone	Bags Cement	Cubic feet Sand	Cubic feet Stone	Bags Cement	Cubic feet Sand	Cubic feet Stone	Bags Cement	Cubic feet Sand	Cubic feet Stone
100	28	42	84	25 4/5	51 3/5	77 2/5	22	44	88	18	45	90	16	48	96
90	25 1/5	37 4/5	75 3/5	23 1/5	46 2/5	69 3/5	19 4/5	39 3/5	79 1/5	16 1/5	40 1/2	81	14 2/5	43 1/5	86 2/5
80	22 2/5	33 3/5	67 1/5	20 2/3	41 1/3	62	17 3/5	35 1/5	70 2/5	14 2/5	36	72	12 4/5	38 2/5	76 4/5
70	19 3/5	29 2/5	58 4/5	18	36	54	15 2/5	30 4/5	61 3/5	12 3/5	31 1/2	63	11 1/5	33 3/5	67 1/5
60	16 4/5	25 1/5	50 2/5	15 1/2	31	46 1/2	13 1/5	26 2/5	52 4/5	10 4/5	27	54	9 3/5	28 4/5	57 3/5
50	14	21	42	13	26	39	11	22	44	9	22 1/2	45	8	24	48
40	11 1/5	16 4/5	33 3/5	10 1/3	20 2/3	31	8 4/5	17 3/5	35 1/5	7 1/5	18	36	6 2/5	19 1/5	38 2/5
30	8 2/5	12 3/5	25 1/5	7 3/4	15 1/2	23 1/4	6 3/5	13 1/5	26 2/5	5 2/5	13 1/2	27	4 4/5	14 2/5	28 4/5
20	5 3/5	8 2/5	16 4/5	5 1/5	10 2/5	15 3/5	4 2/5	8 4/5	17 3/5	3 3/5	9	18	3 1/5	9 3/5	19 1/5
10	2 4/5	4 1/5	8 2/5	2 3/5	5 1/5	7 4/5	2 1/5	4 2/5	8 4/5	1 4/5	4 1/2	9	1 3/5	4 4/5	9 3/5
9	2 1/2	3 4/5	7 3/5	2 1/3	4 2/3	7	2	4	8	1 3/5	4	8	1 1/2	4 1/3	8 2/3
8	2 1/4	3 3/8	6 3/4	2	4 1/6	6 1/4	1 3/4	3 1/2	7	1 2/5	3 3/5	7 1/5	1 1/4	3 7/8	7 3/4
7	2	3	6	1 4/5	3 1/2	5 2/5	1 1/2	3	6	1 1/4	3 1/8	6 1/4	1 1/8	3 3/8	6 3/4
6	1 2/3	2 1/2	5	1 3/5	3 1/5	4 3/5	1 1/3	2 2/3	5 1/3	1 1/10	2 3/4	5 1/2	1	3	6
5	1 2/5	2 1/10	4 1/5	1 1/3	2 2/3	4	1 1/10	2 1/5	4 2/5	9/10	2 1/4	4 1/2	4/5	2 2/5	4 4/5
4	1 1/8	1 3/4	3 3/8	1	2	3 1/5	7/8	1 3/4	3 1/2	7/10	1 4/5	3 3/5	5/8	2	4
3	4/5	1 1/4	2 1/2	3/4	1 1/2	2 1/3	2/3	1 1/3	2 2/3	1/2	1 1/3	2 2/3	1/2	1 1/2	3
2	9/16	7/8	1 11/16	1/2	1	1 1/2	7/16	7/8	1 3/4	1/3	9/10	1 4/5	5/16	1	2
1	9/32	7/16	7/8	1/4	1/2	3/4	7/32	7/16	7/8	1/5	1/20	1	5/32	1/2	1

NOTE.—*In building a square tank* or a room or anything hollow for which the proportioning of materials is the same in all its parts. Figure on the whole thing as if it were solid and then subtract the volume of the hollow part in the center. For instance, find out the number of cu. ft. of concrete needed for a tank 10 ft. long and 10 ft. wide and 8 ft. high with walls, floor and roof all 6 ins. thick, by calculating first the volume of the whole thing as if it were solid. That would be 10 × 10 × 8, or 800 cu. ft. Then figure the volume of the hollow inside. Allowing for the 6 in. walls, the inside measurements will be 9 ft. long, 9 ft. wide and 7 ft. high. So the volume of the hollow part will be 9 × 9 × 7, or 567 cu. ft. The number of cu. ft. in the walls, floor and roof will be the difference between 800 and 567 or 233 cu. ft.

NOTE.—*To calculate* for anything circular there is one rule to remember: the area of a circle is found by multiplying the diameter by the diameter. then multiplying the result by 3¹/₇ and dividing by 4. So that a circular slab of concrete 7 ft. in diameter would have an area 7 times 7, 3¹/₇, divided by 4, that is 38½ sq. ft. Multiply this area by the thickness in ft. or fraction of a ft. and obtain the number of cu. ft. in the slab. If this slab be 6 ins. thick, the number of cu. ft. in it will be 38½ multiplied by ½, or 19¼.

NOTE.—*In building a circular tank,* figure the total volume as if it were solid. Then calculate the volume of the hollow space that is not to be filled with concrete and subtract this from the first total.

Stone Masonry.—The resistance to crushing or compression strength of stone varies to an enormous extent for various stones, from a little over 60 tons per sq. ft. which is the limit for a weak limestone, up to 1,300 tons for the hardest granites.

The following table gives average data for building stones of good quality.

Properties of Stone

(Average values according to R. P. Miller)

Kind of Stone	Weight lbs. per cu. ft.	Crushing strength lbs. per sq. in.	Shearing strength lbs. per sq. in.
Sandstone..........	150	8,000	1,500
Granite............	170	15,000	2,000
Limestone..........	170	6,000	1,000
Marble............	170	10,000	1,400
Slate..............	175	15,000	
Trap Rock.........	185	20,000	

Steel Joists.—The following table gives the safe uniform square foot loading and safe concentrated loadings applied between joists.

Total Safe Uniform Loads in Lbs. on Truscon Steel Joists

Deflection Greater than 1/360 of Span for Loads Below Heavy Line

Size of Steel Joists	4"	5"	6"	7"	8"	9"	10"	11"	12"
Wt. lbs. per ft. Plate Girder	3.7	4.2	4.9	5.4	5.9	6.6	7.6	9.0	10.0
Wt. lbs. per ft. Double Chan'l	3.7	4.2	4.7	5.5	6.1	7.0	8.0	9.5	10.5
Clear Span in Feet 6	2311								
7	1981	2667							
8	1733	2333	3067						
9	1541	2074	2726	3793					
10	1387	1867	2453	3413					
11	1261	1697	2230	3103					
12	1155	1555	2045	2845	3733				
13		1436	1887	2626	3446	4348			
14		1333	1752	2438	3200	4038			
15			1635	2275	2986	3769	4729	5973	7111
16			1533	2133	2800	3533	4433	5600	6666
17				2008	2635	3325	4172	5271	6275
18				1896	2489	3141	3940	4978	5926
19					2358	2975	3733	4716	5614
20					2240	2827	3546	4480	5333
21					2133	2692	3378	4267	5079
22						2570	3224	4073	4848
23						2458	3084	3896	4638
24							2955	3733	4445
25							2837	3584	4267
26							2728	3446	4102
Maximum Safe Load			3940	4890	3840	4600	5900	6624	8050

Joists braced laterally as in standard floor construction. Bending moment ⅛ WL. Maximum stress 16,000 lbs. per sq. inch.

Deflection Limited to 1/360 of Span for All Loads Below

Size of Steel Joists	4"	5"	6"	7"	8"	9"	10"	11"	12"
Wt. lbs. per ft. Plate Girder	3.7	4.2	4.9	5.4	5.9	6.6	7.6	9.0	10.0
Wt. lbs. per ft. Double Chan'l	3.7	4.2	4.7	5.5	6.1	7.0	8.0	9.5	10.5
Clear Span in Feet 6	2311								
7	1981	2667							
8	1733	2333	3067						
9	1380	2074	2726	3793					
10	1118	1867	2453	3413					
11	922	1550	2230	3103					
12	773	1305	2045	2845	3733				
13		1113	1754	2626	3446	4348			
14		958	1511	2438	3200	4038			
15			1318	2135	2986	3769	4729	5973	7111
16			1159	1880	2800	3533	4433	5600	6666
17				1669	2500	3325	4172	5271	6275
18				1481	2225	3141	3940	4978	5926
19					2000	2844	3733	4716	5614
20					1800	2562	3546	4480	5333
21					1637	2322	3250	4267	5079
22						2120	2942	4073	4848
23						1937	2700	3752	4638
24							2475	3446	4445
25							2285	3181	4133
26							2115	2930	3820
Maximum Safe Load			3940	4890	3840	4600	5900	6624	8050

Joists braced laterally as in standard floor construction. Bending moment ⅛ WL. Maximum stress 16,000 lbs. per sq. in.

Total Safe Uniform Loads on Truscon Steel Joists in Pounds per Square Foot of Floor Area

Joists spaced 24" center to center

Clear Span in Ft.	Depth of Joists in Inches								
	4	5	6	7	8	9	10	11	12
6	193								
7	142	191							
8	108	146	192						
9	77	115	152	211					
10	56	94	123	171					
11	42	70	102	141					
12	32	54	85	119	156				
13		43	67	101	133	167			
14		34	54	87	115	144			
15			44	71	100	126	158	199	237
16			37	59	88	111	139	175	209
17				49	74	98	123	155	185
18				41	62	88	110	139	165
19					53	75	99	124	148
20					45	64	89	112	134
21					39	56	78	102	121
22						48	67	93	110
23						42	59	82	101
24							52	72	93
25							46	64	83
26							41	57	74

Joists spaced 19" center to center

Clear Span in Ft.	Depth of Joists in Inches								
	4	5	6	7	8	9	10	11	12
6	244								
7	179	240							
8	137	184	241						
9	97	146	191	266					
10	71	118	155	216					
11	53	89	128	178					
12	40	69	107	149	196				
13		54	85	127	167	211			
14		43	68	110	145	182			
15			55	90	126	159	199	252	299
16			46	74	110	140	175	221	263
17				62	93	124	155	196	233
18				52	78	110	138	175	208
19					66	95	124	157	186
20					57	81	112	141	169
21					49	70	98	128	153
22						61	85	117	139
23						53	74	103	127
24							65	91	117
25							57	80	104
26							51	71	93

Joists spaced 16" center to center

Clear Span in Ft.	Depth of Joists in Inches								
	4	5	6	7	8	9	10	11	12
6	289								
7	212	286							
8	163	219	287						
9	116	173	227	316					
10	84	140	184	256					
11	63	106	152	212					
12	48	82	128	178	234				
13		64	101	152	199	251			
14		51	81	130	172	216			
15			66	107	150	188	236	299	356
16			54	88	131	166	208	262	313
17				74	110	147	184	232	277
18				62	93	131	164	208	247
19					79	113	147	186	221
20					68	96	133	168	200
21					58	83	116	152	182
22						72	101	139	165
23						63	88	123	151
24							77	108	139
25							68	95	124
26							61	85	110

Joists spaced 12" center to center

Clear Span in Ft.	Depth of Joists in Inches								
	4	5	6	7	8	9	10	11	12
6	385								
7	283	381							
8	217	292	383						
9	154	231	303	421					
10	112	187	245	341					
11	84	141	203	282					
12	64	109	170	237	311				
13		86	134	202	265	334			
14		68	108	174	229	288			
15			88	142	199	251	315	398	474
16			73	118	175	221	277	350	417
17				98	147	196	246	310	369
18				82	124	175	219	277	329
19					105	150	197	248	295
20					90	128	177	224	267
21					78	111	155	203	242
22						96	134	185	220
23						84	117	164	202
24							103	144	185
25							91	127	165
26							81	113	147

Above loads include dead weight of floor construction and ceiling, averaging about 40 pounds per square foot. To find the safe live load, deduct the dead load from these values. The maximum deflection is less than 1/360 of the span. All values are for joists braced laterally as in standard floor construction. Tables are based on bending moment of ⅛ WL and maximum fibre stress of 16,000 lbs. per square inch.

Table of Hyperbolic Logarithms

No.	Hyp. log.	No.	Hyp. log.	No.	Hyp. log.	No.	Hyp. log.
1.1	0.0953	4.5	1.5041	7.9	2.0669	19.0	2.9444
1.2	0.1823	4.6	1.5261	8.0	2.0794	20.0	2.9957
1.3	0.2624	4.7	1.5476	8.1	2.0919	21.0	3.0445
1.4	0.3365	4.8	1.5686	8.2	2.1041	22.0	3.0910
1.5	0.4055	4.9	1.5892	8.3	2.1163	23.0	3.1355
1.6	0.4700	5.0	1.6094	8.4	2.1282	24.0	3.1781
1.7	0.5306	5.1	1.6292	8.5	2.1401	25.0	3.2189
1.8	0.5878	5.2	1.6487	8.6	2.1518	26.0	3.2581
1.9	0.6419	5.3	1.6677	8.7	2.1633	27.0	3.2958
2.0	0.6931	5.4	1.6864	8.8	2.1748	28.0	3.3322
2.1	0.7419	5.5	1.7047	8.9	2.1861	29.0	3.3673
2.2	0.7885	5.6	1.7228	9.0	2.1972	30.0	3.4012
2.3	0.8329	5.7	1.7405	9.1	2.2083	31.0	3.4340
2.4	0.8755	5.8	1.7579	9.2	2.2192	32.0	3.4657
2.5	0.9163	5.9	1.7750	9.3	2.2300	33.0	3.4965
2.6	0.9555	6.0	1.7918	9.4	2.2407	34.0	3.5263
2.7	0.9933	6.1	1.8083	9.5	2.2513	35.0	3.5553
2.8	1.0296	6.2	1.8245	9.6	2.2618	36.0	3.5835
2.9	1.0647	6.3	1.8405	9.7	2.2721	37.0	3.6109
3.0	1.0986	6.4	1.8563	9.8	2.2824	38.0	3.6376
3.1	1.1312	6.5	1.8718	9.9	2.2925	39.0	3.6636
3.2	1.1632	6.6	1.8871	10.0	2.3026	40.0	3.6889
3.3	1.1939	6.7	1.9021	10.5	2.3513	41.0	3.7136
3.4	1.2238	6.8	1.9169	11.0	2.3979	42.0	3.7377
3.5	1.2528	6.9	1.9315	11.5	2.4430	43.0	3.7612
3.6	1.2809	7.0	1.9459	12.0	2.4849	44.0	3.7842
3.7	1.3083	7.1	1.9601	12.5	2.5262	45.0	3.8067
3.8	1.3350	7.2	1.9741	13.0	2.5649	46.0	3.8286
3.9	1.3610	7.3	1.9879	13.5	2.6027	47.0	3.8501
4.0	1.3863	7.4	2.0015	14.0	2.6391	48.0	3.8712
4.1	1.4110	7.5	2.0149	15.0	2.7081	49.0	3.8918
4.2	1.4351	7.6	2.0281	16.0	2.7726	50.0	3.9120
4.3	1.4586	7.7	2.0412	17.0	2.8332		
4.4	1.4816	7.8	2.0541	18.0	2.8904		

NOTE.—Hyperbolic or Naperian logarithms are common logarithms multiplied by 2.3025851.

Natural Trigonometrical Functions

Degree	Sine	Cosine	Tangent	Secant	Degree	Sine	Cosine	Tangent	Secant
0	.00000	1.0000	.00000	1.0000	46	.7193	.6947	1.0355	1.4395
1	.01745	.9998	.01745	1.0001	47	.7314	.6820	1.0724	1.4663
2	.03490	.9994	.03492	1.0006	48	.7431	.6691	1.1106	1.4945
3	.05234	.9986	.05241	1.0014	49	.7547	.6561	1.1504	1.5242
4	.06976	.9976	.06993	1.0024	50	.7660	.6428	1.1918	1.5557
5	.08716	.9962	.08749	1.0038	51	.7771	.6293	1.2349	1.5890
6	.10453	.9945	.10510	1.0055	52	.7880	.6157	1.2799	1.6243
7	.12187	.9925	.12278	1.0075	53	.7986	.6018	1.3270	1.6616
8	.1392	.9903	.1405	1.0098	54	.8090	.5878	1.3764	1.7013
9	.1564	.9877	.1584	1.0125	55	.8192	.5736	1.4281	1.7434
10	.1736	.9848	.1763	1.0154	56	.8290	.5592	1.4826	1.7883
11	.1908	.9816	.1944	1.0187	57	.8387	.5446	1.5399	1.8361
12	.2079	.9781	.2126	1.0223	58	.8480	.5299	1.6003	1.8871
13	.2250	.9744	.2309	1.0263	59	.8572	.5150	1.6643	1.9416
14	.2419	.9703	.2493	1.0306	60	.8660	.5000	1.7321	2.0000
15	.2588	.9659	.2679	1.0353	61	.8746	.4848	1.8040	2.0627
16	.2756	.9613	.2867	1.0403	62	.8829	.4695	1.8807	2.1300
17	.2924	.9563	.3057	1.0457	63	.8910	.4540	1.9626	2.2027
18	.3090	.9511	.3249	1.0515	64	.8988	.4384	2.0503	2.2812
19	.3256	.9455	.3443	1.0576	65	.9063	.4226	2.1445	2.3662
20	.3420	.9397	.3640	1.0642	66	.9135	.4067	2.2460	2.4586
21	.3584	.9336	.3839	1.0711	67	.9205	.3907	2.3559	2.5593
22	.3746	.9272	.4040	1.0785	68	.9272	.3746	2.4751	2.6695
23	.3907	.9205	.4245	1.0864	69	.9336	.3584	2.6051	2.7904
24	.4067	.9135	.4452	1.0946	70	.9397	.3420	2.7475	2.9238
25	.4226	.9063	.4663	1.1034	71	.9455	.3256	2.9042	3.0715
26	.4384	.8988	.4877	1.1126	72	.9511	.3090	3.0777	3.2361
27	.4540	.8910	.5095	1.1223	73	.9563	.2924	3.2709	3.4203
28	.4695	.8829	.5317	1.1326	74	.9613	.2756	3.4874	3.6279
29	.4848	.8746	.5543	1.1433	75	.9659	.2588	3.7321	3.8637
30	.5000	.8660	.5774	1.1547	76	.9703	.2419	4.0108	4.1336
31	.5150	.8572	.6009	1.1666	77	.9744	.2250	4.3315	4.4454
32	.5299	.8480	.6249	1.1792	78	.9781	.2079	4.7046	4.8097
33	.5446	.8387	.6494	1.1924	79	.9816	.1908	5.1446	5.2408
34	.5592	.8290	.6745	1.2062	80	.9848	.1736	5.6713	5.7588
35	.5736	.8192	.7002	1.2208	81	.9877	.1564	6.3138	6.3924
36	.5878	.8090	.7265	1.2361	82	.9903	.1392	7.1154	7.1853
37	.6018	.7986	.7536	1.2521	83	.9925	.12187	8.1443	8.2055
38	.6157	.7880	.7813	1.2690	84	.9945	.10453	9.5144	9.5668
39	.6293	.7771	.8098	1.2867	85	.9962	.08716	11.4301	11.474
40	.6428	.7660	.8391	1.3054	86	.9976	.06976	14.3007	14.335
41	.6561	.7547	.8693	1.3250	87	.9986	.05234	19.0811	19.107
42	.6691	.7431	.9004	1.3456	88	.9994	.03490	28.6363	28.654
43	.6820	.7314	.9325	1.3673	89	.9998	.01745	57.2900	57.299
44	.6947	.7193	.9657	1.3902	90	1.0000	Inf.	Inf.	Inf.
45	.7071	.7071	1.0000	1.4142		—	—	—	—

NOTE.—*For intermediate values* reduce angles from degrees, minutes and seconds to degrees and decimal parts of a degree and interpolate or consult a larger table

Logarithms of Numbers, from 0 to 1000

For 1 and for all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number. For example, the characteristic of the logarithm of either 587 or 587.6 is 2, and of 2146 is 3.

For numbers smaller than 1, that is for numbers wholly decimal, the characteristic is negative and its numerical value is one more than the number of ciphers between the decimal point and the first figure which is not a cipher. For example, the characteristic of the logarithm of 0.045 is (—2), and the characteristic of 0.0004 is (—4). The minus sign is usually written thus: (—) as the minus sign refers only to the characteristic and not to the mantissa, which is always positive.

No.	0	1	2	3	4	5	6	7	8	9
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52633	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58093	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810

Logarithms of Numbers, from 0 to 1000—Concluded

No.	0	1	2	3	4	5	6	7	8	9
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81756	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87680	87737	87795	87852	87910	87967	88024
76	88081	88138	88196	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88763	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98623
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957

Circumferences and Areas of Circles

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
1/64	0.0491	0.0002	3	9.4248	7.0686	8	25.1327	50.265	16	50.2655	201.06
1/32	0.0982	0.0008	1/16	9.6211	7.3662	1/8	25.5254	51.849	1/8	50.6582	204.22
1/16	0.1964	0.0031	1/8	9.8175	7.6699	1/4	25.9181	53.456	1/4	51.0509	207.39
3/32	0.2945	0.0059	3/16	10.0138	7.9798	3/8	26.3108	55.088	3/8	51.4436	210.60
1/8	0.3927	0.0123	1/4	10.2102	8.2958	1/2	26.7035	56.745	1/2	51.8363	213.82
5/32	0.4909	0.0192	5/16	10.4065	8.6179	5/8	27.0962	58.426	5/8	52.2290	217.08
3/16	0.5890	0.0276	3/8	10.6029	8.9462	3/4	27.4889	60.132	3/4	52.6217	220.35
7/32	0.6872	0.0376	7/16	10.7992	9.2806	7/8	27.8816	61.862	7/8	53.0144	223.65
1/4	0.7854	0.0491	1/2	10.9956	9.6211	9	28.2743	63.617	17	53.4071	226.98
9/32	0.8836	0.0621	9/16	11.1919	9.9678	1/8	28.6670	65.397	1/8	53.7998	230.33
5/16	0.9817	0.0767	5/8	11.3883	10.321	1/4	29.0597	67.201	1/4	54.1925	233.71
11/32	1.0799	0.0928	11/16	11.5846	10.680	3/8	29.4524	69.029	3/8	54.5852	237.10
3/8	1.1781	0.1105	3/4	11.7810	11.045	1/2	29.8451	70.882	1/2	54.9779	240.53
13/32	1.2763	0.1296	13/16	11.9773	11.416	5/8	30.2378	72.760	5/8	55.3706	243.98
7/16	1.3745	0.1503	7/8	12.1737	11.793	3/4	30.6305	74.662	3/4	55.7633	247.45
15/32	1.4726	0.1726	15/16	12.3700	12.177	7/8	31.0232	76.589	7/8	56.1560	250.95
1/2	1.5708	0.1964	4	12.5664	12.566	10	31.4159	78.540	18	56.5487	254.47
17/32	1.6690	0.2217	1/16	12.7627	12.962	1/8	31.8086	80.516	1/8	56.9414	258.02
9/16	1.7672	0.2485	1/8	12.9591	13.364	1/4	32.2013	82.516	1/4	57.3341	261.59
19/32	1.8653	0.2769	3/16	13.1554	13.772	3/8	32.5940	84.541	3/8	57.7268	265.18
5/8	1.9635	0.3068	1/4	13.3518	14.185	1/2	32.9867	86.590	1/2	58.1195	268.80
21/32	2.0617	0.3382	5/16	13.5481	14.607	5/8	33.3794	88.664	5/8	58.5122	272.45
11/16	2.1598	0.3712	3/8	13.7445	15.033	3/4	33.7721	90.763	3/4	58.9049	276.12
23/32	2.2580	0.4057	7/16	13.9408	15.466	7/8	34.1648	92.886	7/8	59.2976	279.81
3/4	2.3562	0.4418	1/2	14.1372	15.904	11	34.5575	95.033	19	59.6903	283.53
25/32	2.4544	0.4794	9/16	14.3335	16.349	1/8	34.9502	97.205	1/8	60.0830	287.27
13/16	2.5525	0.5185	5/8	14.5299	16.800	1/4	35.3429	99.402	1/4	60.4757	291.04
27/32	2.6507	0.5591	11/16	14.7262	17.257	3/8	35.7356	101.62	3/8	60.8684	294.83
7/8	2.7489	0.6013	3/4	14.9226	17.721	1/2	36.1283	103.87	1/2	61.2611	298.65
29/32	2.8471	0.6450	13/16	15.1189	18.190	5/8	36.5210	106.14	5/8	61.6538	302.49
15/16	2.9452	0.6903	7/8	15.3153	18.665	3/4	36.9137	108.43	3/4	62.0465	306.35
31/32	3.0434	0.7371	15/16	15.5116	19.147	7/8	37.3064	110.75	7/8	62.4392	310.24
1	3.1416	0.7854	5	15.7080	19.635	12	37.6991	113.10	20	62.8319	314.16
1/16	3.3379	0.8866	1/16	15.9043	20.129	1/8	38.0918	115.47	1/8	63.2246	318.10
1/8	3.5343	0.9940	1/8	16.1007	20.629	1/4	38.4845	117.86	1/4	63.6173	322.06
3/16	3.7306	1.1075	3/16	16.2970	21.135	3/8	38.8772	120.28	3/8	64.0100	326.05
1/4	3.9270	1.2272	1/4	16.4934	21.648	1/2	39.2699	122.72	1/2	64.4026	330.06
5/16	4.1233	1.3530	5/16	16.6897	22.166	5/8	39.6626	125.19	5/8	64.7953	334.10
3/8	4.3197	1.4849	3/8	16.8861	22.691	3/4	40.0553	127.68	3/4	65.1880	338.16
7/16	4.5160	1.6230	7/16	17.0824	23.221	7/8	40.4480	130.19	7/8	65.5807	342.25
1/2	4.7124	1.7671	1/2	17.2788	23.758	13	40.8407	132.73	21	65.9734	346.36
9/16	4.9087	1.9175	9/16	17.4751	24.301	1/8	41.2334	135.30	1/8	66.3661	350.50
5/8	5.1051	2.0739	5/8	17.6715	24.850	1/4	41.6261	137.89	1/4	66.7588	354.66
11/16	5.3014	2.2365	11/16	17.8678	25.406	3/8	42.0188	140.50	3/8	67.1515	358.84
3/4	5.4978	2.4053	3/4	18.0642	25.967	1/2	42.4115	143.14	1/2	67.5442	363.05
13/16	5.6941	2.5802	13/16	18.2605	26.535	5/8	42.8042	145.80	5/8	67.9369	367.28
7/8	5.8905	2.7612	7/8	18.4569	27.100	3/4	43.1969	148.49	3/4	68.3296	371.54
15/16	6.0868	2.9483	15/16	18.6532	27.688	7/8	43.5896	151.20	7/8	68.7223	375.83
2	6.2832	3.1416	6	18.8496	28.274	14	43.9823	153.94	22	69.1150	380.13
1/16	6.4795	3.3410	1/8	19.2423	29.465	1/8	44.3750	156.70	1/8	69.5077	384.46
1/8	6.6759	3.5466	1/4	19.6350	30.680	1/4	44.7677	159.48	1/4	69.9004	388.82
3/16	6.8722	3.7583	3/8	20.0277	31.919	3/8	45.1604	162.30	3/8	70.2931	393.20
1/4	7.0686	3.9761	1/2	20.4204	33.183	1/2	45.5531	165.13	1/2	70.6858	397.61
5/16	7.2649	4.2000	5/8	20.8131	34.472	5/8	45.9458	167.99	5/8	71.0785	402.04
3/8	7.4613	4.4301	3/4	21.2058	35.785	3/4	46.3385	170.87	3/4	71.4712	406.49
7/16	7.6576	4.6664	7/8	21.5984	37.122	7/8	46.7312	173.78	7/8	71.8639	410.97
1/2	7.8540	4.9087	7	21.9911	38.485	15	47.1239	176.71	23	72.2566	415.48
9/16	8.0503	5.1572	1/8	22.3838	39.871	1/8	47.5166	179.67	1/8	72.6493	420.00
5/8	8.2467	5.4119	1/4	22.7765	41.282	1/4	47.9093	182.65	1/4	73.0420	424.56
11/16	8.4430	5.6727	3/8	23.1692	42.718	3/8	48.3020	185.66	3/8	73.4347	429.13
3/4	8.6394	5.9396	1/2	23.5619	44.179	1/2	48.6947	188.69	1/2	73.8274	433.74
13/16	8.8357	6.2126	5/8	23.9546	45.664	5/8	49.0874	191.75	5/8	74.2201	438.36
7/8	9.0321	6.4918	3/4	24.3473	47.173	3/4	49.4801	194.83	3/4	74.6128	443.01
15/16	9.2284	6.7771	7/8	24.7400	48.707	7/8	49.8728	197.93	7/8	75.0055	447.69

Circumferences and Areas of Circles—*Continued*

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
24	75.3982	452.39	32	100.531	804.25	40	125.664	1256.6	48	150.796	1809.6
1/8	75.7909	457.11	1/8	100.924	810.54	1/8	126.056	1264.5	1/8	151.189	1819.0
1/4	76.1836	461.86	1/4	101.316	816.86	1/4	126.449	1272.4	1/4	151.582	1828.5
3/8	76.5763	466.64	3/8	101.709	823.21	3/8	126.842	1280.3	3/8	151.975	1837.9
1/2	76.9690	471.44	1/2	102.102	829.58	1/2	127.235	1288.2	1/2	152.367	1847.5
5/8	77.3617	476.26	5/8	102.494	835.97	5/8	127.627	1296.2	5/8	152.760	1857.0
3/4	77.7544	481.11	3/4	102.887	842.39	3/4	128.020	1304.2	3/4	153.153	1866.5
7/8	78.1471	485.98	7/8	103.280	848.83	7/8	128.413	1312.2	7/8	153.545	1876.1
25	78.5398	490.87	33	103.673	855.30	41	128.805	1320.3	49	153.938	1885.7
1/8	78.9325	495.79	1/8	104.065	861.79	1/8	129.198	1328.3	1/8	154.331	1895.4
1/4	79.3252	500.74	1/4	104.458	868.31	1/4	129.591	1336.4	1/4	154.723	1905.0
3/8	79.7179	505.71	3/8	104.851	874.85	3/8	129.983	1344.5	3/8	155.116	1914.7
1/2	80.1106	510.71	1/2	105.243	881.41	1/2	130.376	1352.7	1/2	155.509	1924.4
5/8	80.5033	515.72	5/8	105.636	888.00	5/8	130.769	1360.8	5/8	155.902	1934.2
3/4	80.8960	520.77	3/4	106.029	894.62	3/4	131.161	1369.0	3/4	156.294	1943.9
7/8	81.2887	525.84	7/8	106.421	901.26	7/8	131.554	1377.2	7/8	156.687	1953.7
26	81.6814	530.93	34	106.814	907.92	42	131.947	1385.4	50	157.080	1963.5
1/8	82.0741	536.05	1/8	107.207	914.61	1/8	132.340	1393.7	1/8	157.472	1973.3
1/4	82.4668	541.19	1/4	107.600	921.32	1/4	132.732	1402.0	1/4	157.865	1983.2
3/8	82.8595	546.35	3/8	107.992	928.06	3/8	133.125	1410.3	3/8	158.258	1993.1
1/2	83.2522	551.55	1/2	108.385	934.82	1/2	133.518	1418.6	1/2	158.650	2003.0
5/8	83.6449	556.76	5/8	108.778	941.61	5/8	133.910	1427.0	5/8	159.043	2012.9
3/4	84.0376	562.00	3/4	109.170	948.42	3/4	134.303	1435.4	3/4	159.436	2022.8
7/8	84.4303	567.27	7/8	109.563	955.25	7/8	134.696	1443.8	7/8	159.829	2032.8
27	84.8230	572.56	35	109.956	962.11	43	135.088	1452.2	51	160.221	2042.8
1/8	85.2157	577.87	1/8	110.348	969.00	1/8	135.481	1460.7	1/8	160.614	2052.8
1/4	85.6084	583.21	1/4	110.741	975.91	1/4	135.874	1469.1	1/4	161.007	2062.9
3/8	86.0011	588.57	3/8	111.134	982.84	3/8	136.267	1477.6	3/8	161.399	2073.0
1/2	86.3938	593.96	1/2	111.527	989.80	1/2	136.659	1486.2	1/2	161.792	2083.1
5/8	86.7865	599.37	5/8	111.919	996.87	5/8	137.052	1494.7	5/8	162.185	2093.2
3/4	87.1792	604.81	3/4	112.312	1003.8	3/4	137.445	1503.3	3/4	162.577	2103.3
7/8	87.5719	610.27	7/8	112.705	1010.8	7/8	137.837	1511.9	7/8	162.970	2113.5
28	87.965	615.75	36	113.097	1017.9	44	138.230	1520.5	52	163.363	2123.7
1/8	88.357	621.26	1/8	113.490	1025.0	1/8	138.623	1529.2	1/8	163.756	2133.9
1/4	88.750	626.80	1/4	113.883	1032.1	1/4	139.015	1537.9	1/4	164.148	2144.2
3/8	89.143	632.36	3/8	114.275	1039.2	3/8	139.408	1546.6	3/8	164.541	2154.5
1/2	89.535	637.94	1/2	114.668	1046.3	1/2	139.801	1555.3	1/2	164.934	2164.8
5/8	89.928	643.55	5/8	115.061	1053.5	5/8	140.194	1564.0	5/8	165.326	2175.1
3/4	90.321	649.18	3/4	115.454	1060.7	3/4	140.586	1572.8	3/4	165.719	2185.4
7/8	90.713	654.84	7/8	115.846	1068.0	7/8	140.979	1581.6	7/8	166.112	2195.8
29	91.106	660.52	37	116.239	1075.2	45	141.372	1590.4	53	166.504	2206.2
1/8	91.499	666.23	1/8	116.632	1082.5	1/8	141.764	1599.3	1/8	166.897	2216.6
1/4	91.892	671.96	1/4	117.024	1089.8	1/4	142.157	1608.2	1/4	167.290	2227.0
3/8	92.284	677.71	3/8	117.417	1097.1	3/8	142.550	1617.0	3/8	167.683	2237.5
1/2	92.677	683.49	1/2	117.810	1104.5	1/2	142.942	1626.0	1/2	168.075	2248.0
5/8	93.070	689.30	5/8	118.202	1111.8	5/8	143.335	1634.9	5/8	168.468	2258.5
3/4	93.462	695.13	3/4	118.596	1119.2	3/4	143.728	1643.9	3/4	168.861	2269.1
7/8	93.855	700.98	7/8	118.988	1126.7	7/8	144.121	1652.9	7/8	169.253	2279.6
30	94.248	706.86	38	119.381	1134.1	46	144.513	1661.9	54	169.646	2290.2
1/8	94.640	712.70	1/8	119.773	1141.0	1/8	144.906	1670.9	1/8	170.039	2300.8
1/4	95.033	718.69	1/4	120.166	1149.1	1/4	145.299	1680.0	1/4	170.431	2311.5
3/8	95.426	724.64	3/8	120.559	1156.6	3/8	145.691	1689.1	3/8	170.824	2322.1
1/2	95.819	730.62	1/2	120.951	1164.2	1/2	146.084	1698.2	1/2	171.217	2332.8
5/8	96.211	736.62	5/8	121.344	1171.7	5/8	146.477	1707.4	5/8	171.609	2343.5
3/4	96.604	742.64	3/4	121.737	1179.3	3/4	146.869	1716.5	3/4	172.002	2354.3
7/8	96.997	748.69	7/8	122.129	1186.9	7/8	147.262	1725.7	7/8	172.395	2365.0
31	97.389	754.77	39	122.522	1194.6	47	147.655	1734.9	55	172.788	2375.8
1/8	97.782	760.87	1/8	122.915	1202.3	1/8	148.048	1744.2	1/8	173.180	2386.6
1/4	98.175	766.99	1/4	123.308	1210.0	1/4	148.440	1753.5	1/4	173.573	2397.5
3/8	98.567	773.14	3/8	123.700	1217.7	3/8	148.833	1762.7	3/8	173.966	2408.3
1/2	98.960	779.31	1/2	124.093	1225.4	1/2	149.226	1772.1	1/2	174.358	2419.2
5/8	99.353	785.51	5/8	124.486	1233.2	5/8	149.618	1781.4	5/8	174.751	2430.1
3/4	99.746	791.73	3/4	124.878	1241.0	3/4	150.011	1790.8	3/4	175.144	2441.1
7/8	100.138	797.98	7/8	125.271	1248.8	7/8	150.404	1800.1	7/8	175.536	2452.0

SECTION

D

HOW TO USE

A

SLIDE RULE

The Slide Rule

The principle of the slide rule is based on the fact that *the addition of logarithms multiplies the numbers that they represent and the subtraction of logarithms divides the numbers.*

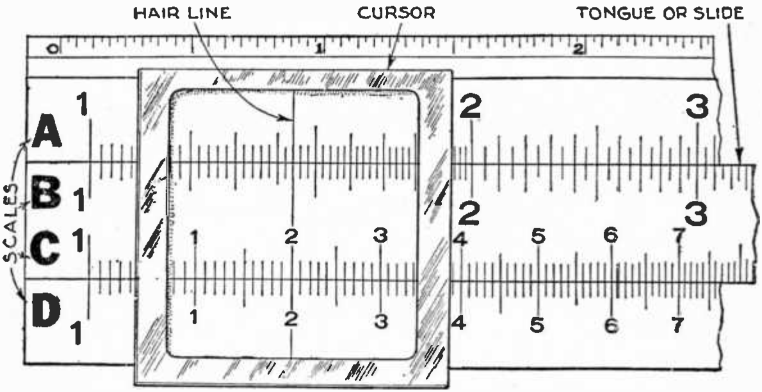


FIG. 1.—Detail of slide rule showing scales and parts. Note especially the scales as they are constantly referred to by letter, as scale C, scale D, etc.

By use of the slide rule the operations of multiplication, division, the finding of powers and the extraction of roots, may be performed rapidly and with an approximation to accuracy which is sufficient for many purposes.

With a good 10 inch rule the results obtained are usually accurate to ¼ of 1 per cent.

A slide rule has four scales as shown in fig. 1 and designated by the letters A,B,C,D. The two outer scales A and D, are stationary, but the two inner scales B and C, are on a tongue arranged to slide between the two outer scales.

The chief difficulty in learning to use the slide rule is to learn to read the graduations on the scales.

Taking first part of the D scale and beginning at the left hand end it will be seen in fig. 2, that between the large 1 and 2 there are ten divisions indicated by the small figures 1, 2, 3, etc.

Now, in fig. 3 the large 1 is read 1 and since there are 10 large divisions between large 1 and large 2, the reading at small 2 for instance is 1.2, 12,

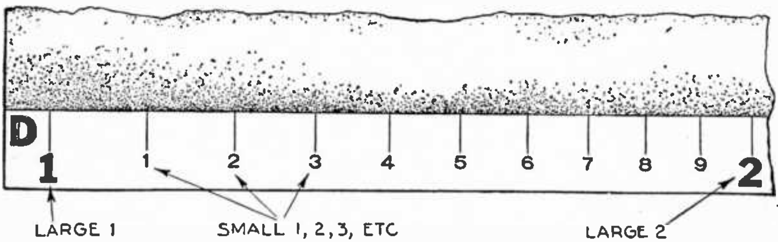


Fig. 2.—Detail of D scale showing large and small figures.

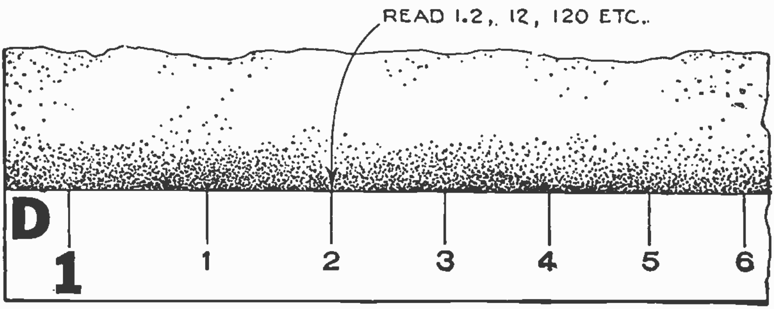


Fig. 3.—Detail of D scale showing how to read 1.2, 12, 120, etc.

120, etc. Hence it will be noted that the slide rule does not place the decimal point.

It will be further noted that between each of the divisions designated by small 1, 2, etc., there are 10 smaller divisions (fig. 1). Hence each one of these is one hundredth of the large division from large 1 to large 2. This permits reading accurately a number of three figures and by "guess" a number of four figures. Thus to read 1 1 3, read the big 1 (for 100), then the small 1 (for 10) and finally the third hundredths division (for 3) counting to the right from the little 1 division as indicated in fig. 4.

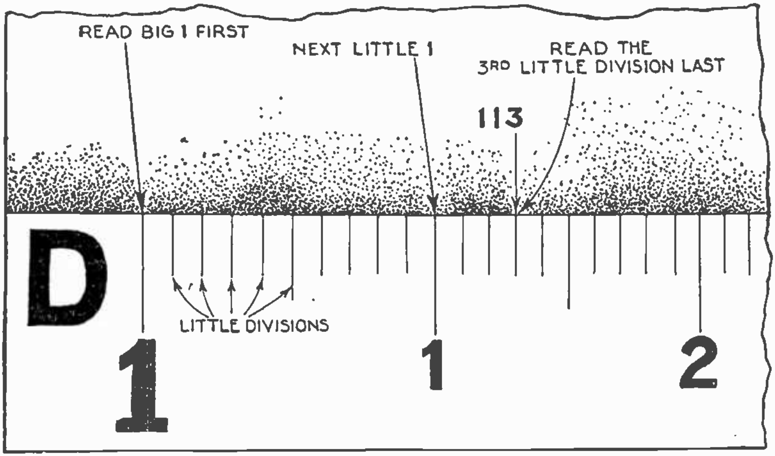
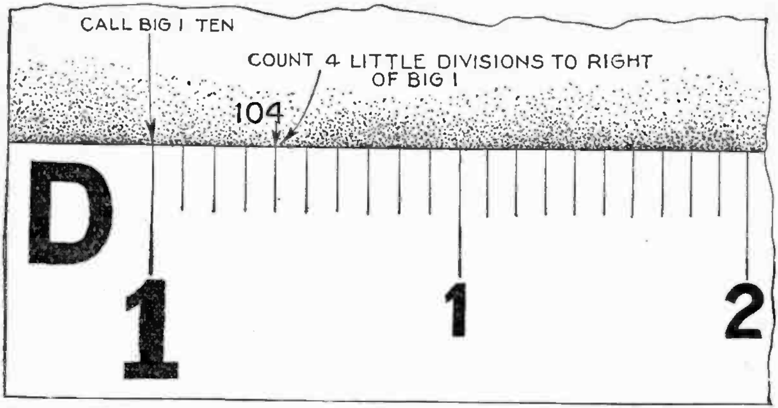


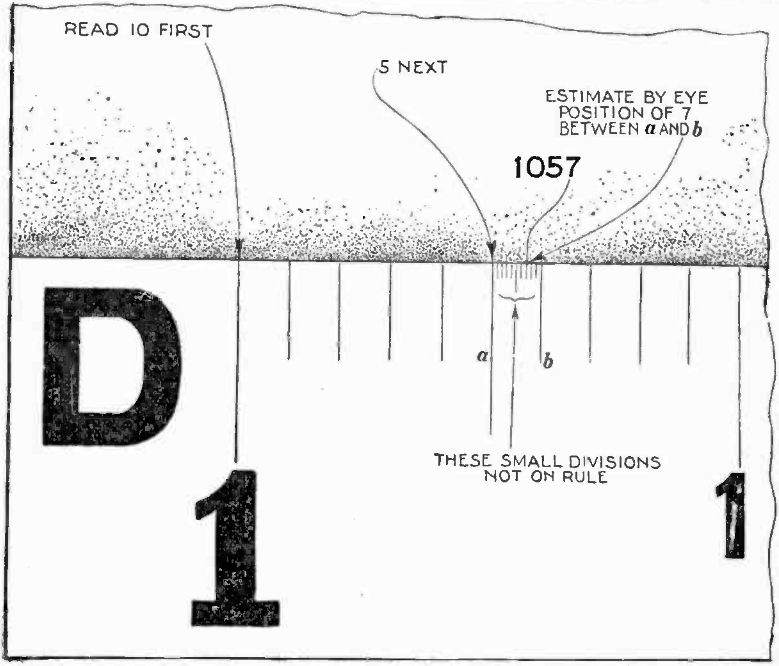
FIG. 4.—Detail of C scale showing how to read numbers such as 113.

To read a number such as 104, remember that the big 1 can be read either 1 or 10. Hence call the big 1 ten and count 4 of the little divisions to the right as in fig. 5.

The accuracy with which a number of four figures can be read depends upon the ability of the eye to gauge the position of the 4th figure between two of the small divisions. Thus, in fig. 6, to read the number 1,057 take big 1 for the first two figures, that is 10; count to right from big 1 five of the little divisions indicated by *a*, for the third figure.

Now the fourth figure is found between the two adjacent little divisions *a* and *b*. The illustration which is greatly magnified shows *a*,*b*, divided


CALL BIG 1 TEN
COUNT 4 LITTLE DIVISIONS TO RIGHT OF BIG 1
104
D
1
1
2


READ 10 FIRST
5 NEXT
ESTIMATE BY EYE POSITION OF 7 BETWEEN a AND b
1057
D
a
b
THESE SMALL DIVISIONS NOT ON RULE
1
1

into 10 divisions, but these divisions are not on the rule itself and must be judged or guessed at "by the eye" and upon the precision with which this is done depends the accuracy of the fourth figure or in this case the 7 of the number 1057.

To read numbers beginning with 2, 3, etc., start with the big 2, big 3, etc., and proceed similarly as in the foregoing explanations.

Thus to read the number 275, read first in fig. 7, the big 2; count off seven small divisions to right, that is to division *a*, and then gauge by eye half the distance or 5 invisible divisions between *a* and *b* obtaining the number 275.

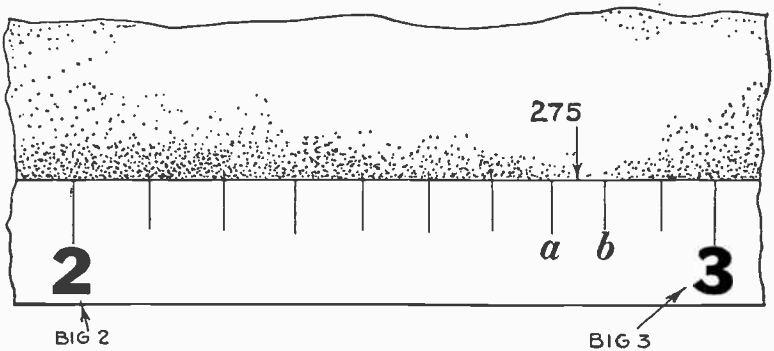


FIG. 7.—Detail of D scale showing how to read numbers of three figures beginning with 2, 3, etc.

Example.—Multiply 2×7.

In fig. 8 slide last division or "index" at right end of C scale opposite big 2 on D scale. Move "cursor" (the sliding glass with hair line) opposite big 7 on C scale and read 14 on D scale indicated by the hair line.

The "index" at either end may be used, but if the second factor come "off the scale" take the index at the other end of the tongue.

Refer to illustrations on page 4.

FIG. 5.—Detail of D scale showing how to read numbers containing a cipher (or ciphers) such as 104.

FIG. 6.—Detail of D scale showing how to read a number of four figures.

Example.—Divide 4 by 2.

In fig. 9 slide cursor till hair line registers with large 4 on **D** scale; slide tongue to register large 2 (on **C** scale) with hair line. At large 1 on the end of tongue (scale **C**) read 2 on scale **D**, which is the answer.

Example.—Divide 475 by 23.

In fig. 10 slide cursor till hair line registers with 475 on scale **D**; slide tongue until 23 on scale **C**, registers with 475 on scale **D**. At large 1 on end

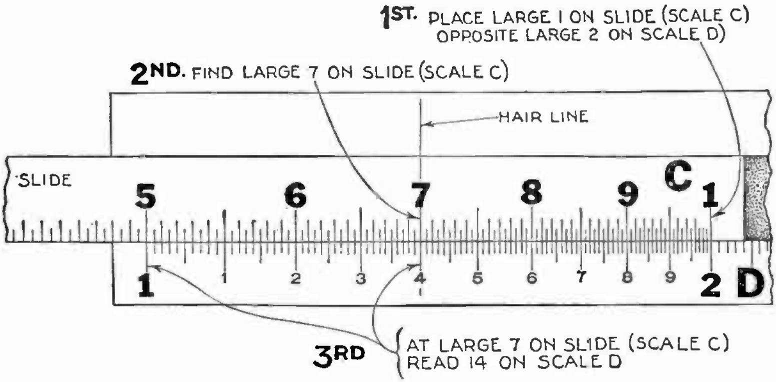


FIG. 8.—Detail of **D** and **C** scales showing how to multiply.

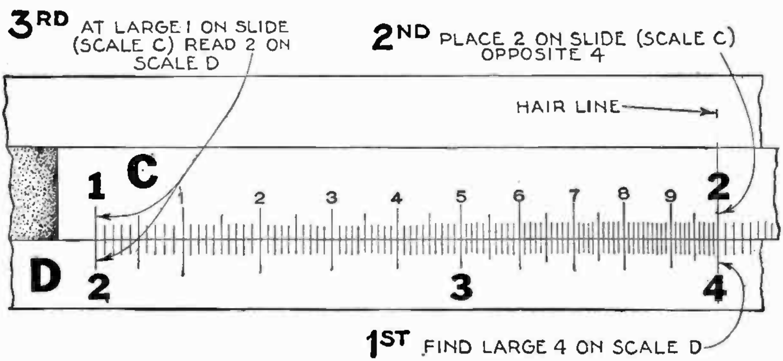


FIG. 9.—Detail of **D** and **C** scales showing how to divide numbers of one figure each.

of tongue read 2,065 (scale **D**) and place the decimal point to read 20.65. As the slide rule does not give the decimal point this has to be done

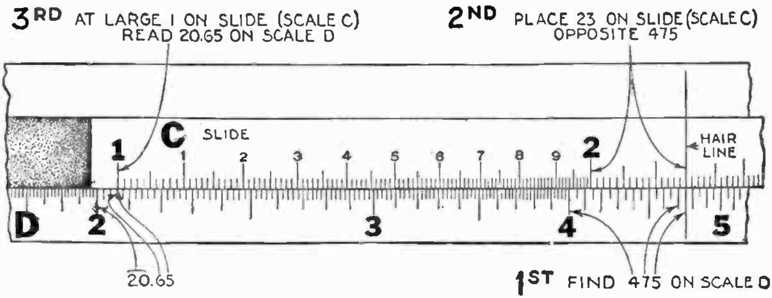


FIG. 10.—Detail of **D** and **C** scales showing how to divide numbers of three figures each.

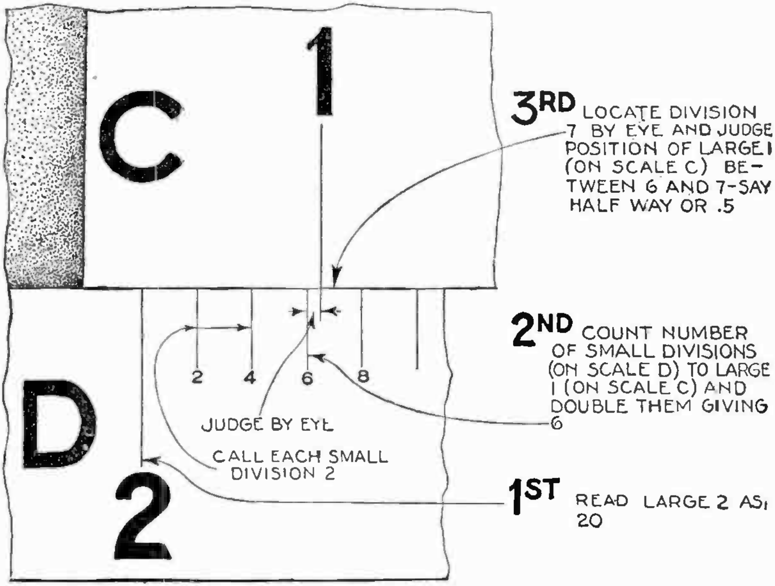


FIG. 11.—Magnified detail of **D** and **C** scales showing how to read the answer in the example as given in fig. 10.

independently. This example illustrates how to read a number of four figures as explained under figs. 10 and 11.

Continued Multiplication.—When there are more than two factors proceed as follows:

Rule.—*Multiply the first two factors in the regular way; set hair line to register first product and continue by placing index to register with hair line.*

Squares and Square Roots.—These are found by using the A and D scales. Thus:

Rule.—*Set the cursor so that the hair line stands over the number on the D scale and the square of that number will be found on the A scale under the hair line.*

In finding the square root of a number, note that the number is located on the A scale and the root is found directly below, on the D scale, the hair line being used to make the readings accurately.

Cubes and Cube Roots.—Finding the cube of a number requires the use of all four scales. The index of the C scale is set to the given number on the D scale and opposite the same number on the *left hand* B scale, the cube is read from the A scale. Three cases may arise, as follows:

Case 1.—*The slide may project to the left and the cube be found on the left hand A scale.*

Case 2.—*The slide may project to the right and the cube be found on the right hand A scale.*

Case 3.—*The slide may project to the right and the cube be found on the left hand A scale.*

Cube Root.—The operation of finding the cube root is *the reverse of finding the cube*, and the method of determining the number of figures in the root, preceding or following the decimal point, is the same as that used in arithmetic; that is, the number is pointed off into periods of three figures each, beginning at the decimal point, and the root then contains one figure for each period or part of a period.

If the number contain a decimal, add ciphers to the right of it, if necessary, so that the number of figures in the decimal part will be exactly divisible by 3, without a remainder.

The scales and indexes to be used in finding the cube root depend upon the number of figures in the first period at the left, not counting ciphers when they *immediately* follow the decimal point.

The rules to be followed are:

Case 1.—*If there be three figures in the first period, use the left hand A scale and the right hand index of the C scale.*

Case 2.—*If there be two figures in the first period, use the right hand A scale and the left hand index of the C scale.*

Case 3.—*If there be one figure in the first period, use the left hand A scale and the left hand index of the C scale.*

TEST QUESTIONS

1. *What is the principle of the slide rule?*
2. *What mathematical operations can be done on the slide rule?*
3. *What do the large and small figures on the slide rule indicate?*
4. *Name the chief difficulty in learning to use the slide rule.*
5. *How is a number such as* 104 *read?*
6. *What is the cursor used for?*
7. *What is the "index"?*
8. *If a factor come "off the scale," what should be done?*
9. *Multiply* 6 *by* 23.
10. *What is the object of the hair line on the cursor?*
11. *Explain how* 475 *is divided by* 23.
12. *Explain how to proceed when there are several factors.*
13. *What scales are used for squares and square root?*
14. *On what scale is the number to be squared located?*
15. *Find the cube root of* 12,623.
16. *Upon what do the scales and indexes to be used in finding the cube root, depend?*

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