THEORY AND DESIGN OF DIRECTIONAL ANTENNA SYSTEMS

By Carl E. Smith, E. E.

FOREWORD

The NAB Engineering Handbook, 1946 (3rd) edition, presented a section on directional antennas taken in its entirety from the book entitled "Directional Antennas" written and published by Carl E. Smith. Mr. Smith is Vice-President in Charge of Engineering, United Broadcasting Company, and President of the Cleveland Institute of Radio Electronics. The material presented to the radio engineering profession by Mr. Smith has enjoyed wide acceptance as an authoritative reference work for professional engineers concerned with the design, construction and operation of directive arrays.

The author has used as basic sources for the paper which follows, his book "Directional Antennas" and appropriate sections of the home study course on "Advanced Radio and Communication Engineering" prepared and made available to engineers, technicians and operators by the Cleveland Institute of Radio Electronics. The material which follows develops the theory and practical design of the shape and size of directional antenna systems and concludes with a section on feeder system design. As combined, amplified and edited from the two above sources, it is believed that for the first time a coordinated and complete work, authoritative in nature, is made available with respect to a practical approach in the design of directive arrays.

The NAB Department of Engineering presents this material with a twofold objective: first, that professional engineers may find in their work, with respect to directional antenna systems, a common understanding of the basic principles of design; and, second, that the broadcast engineer charged with the operation and maintenance of a directional antenna system, no matter how complex, will better understand its design and operation, which understanding in turn will pay high dividends through more efficient, economical and consistent broadcast service.

Further information with respect to Mr. Smith's book entitled "Directional Antennas" or the college-level review of the principles of radio and communication engineering as presented in the home study course "Advance Radio and Communication Engineering" may be obtained by addressing inquiries to the Cleveland Institute of Radio Electronics, 4900 Euclid Ave., Cleveland 3, Ohio.

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1. FUNDAMENTAL PROPERTIES

a. *INTRODUCTION.*--The purpose of a radio broadcasting station is to transform sound waves into radio waves that can be picked up by radio receiving sets. The utility of this service to the public depends upon. (1) *Signal Intensity*, (2) *Program Content*, and (3) *System Distortion*. Of these factors, the radio broadcasting station engineer is concerned with producing an intense signal that will override noise and undesired signals in the receiving sets being served and with minimizing distortion in the audio and radio facilities of the station.

The antenna is the last point in the system under the control of the radio broadcasting station. Radio waves radiated from the transmitting antenna are propagated through space to the receiving antenna. The only control over these propagated waves is in the selection of the antenna site, the polarization, and the intensity of the signals leaving the transmitting antennas. The selection of the antenna site is determined by many considerations, such as; ground constants, terrain, distance and direction to the populated areas to be served, distance and direction to the areas to be protected, and last but not least is the availability of a suitable land area to install the necessary towers and ground system.

For standard broadcast stations vertical polarization is used because of its superior ground wave propagation characteristics and the simplicity of antenna design. The intensity of the signal from the transmitting antenna, in any given direction, depends upon the output power of the transmitter and the antenna design. Since the output power is regulated by the Federal Communications Commission for the class of station involved, the only factors remaining under the engineer's control are the antenna siting and design. These factors go hand in hand when designing directional antennas for broadcasting purposes.

b. *PURPOSE.*--Directional broadcasting antennas are required for one or more of the following reasons:

(1) Protect the service area of other broadcasting stations by causing the waves to cancel in these directions.

(2) Increase the service area of a broadcasting station, particularly in the direction of densly populated areas, by causing the waves to be reinforced in these directions.

c. *CONTROL OF PATTERN SHAPE.*--The usual problem in broadcast practice is to mould the radiation pattern into the desired shape to cover the service areas and give the required protection to other radio stations. As a matter of economics, it is desirable to do the job with the minimum number of antennas. With severe requirements, the number of antennas must be increased until the radiation pattern can be made to conform to the required shape. As a general rule, two stations can be given the required protection with two towers; three stations can be completely protected with three towers in line or if the protection is not severe, it is possible many times to do the job with two towers. With four towers it is always possible to completely control the nulls toward four stations; however if the job can be done with three towers there is a saving of the cost of one tower.

In controlling the pattern shape, consideration has to first be given to fulfilling the conditions of the required protection to other radio stations in accordance with the Standards of Good Engineering Practice of the Federal Communications Commission. The FCC Standards is an excellent practical guide for allocation work. The Standards of Good Engineering Practice was compiled by FCC from extensive data collected by its Engineering Department over long periods of time. This material is under almost constant revision as the art progresses. It is recommended that in connection with any study of this article the FCC Standards contained in Section One of the NAB Engineering Handbook be referred to for details.

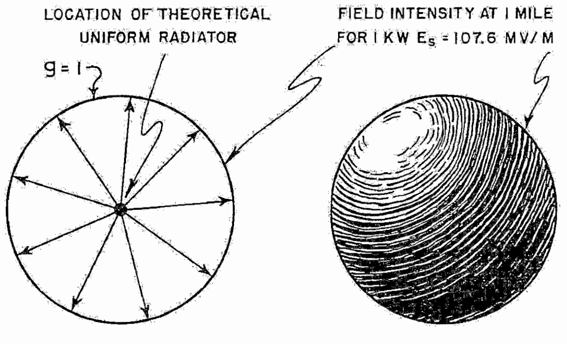
The next consideration is to locate the directional antenna system so that the horizontal lobes will be directed toward the population areas to be served without having too many people within the blanket area, that is, the area near the transmitter where the signal is so strong that other radio stations cannot be received without objectionable interference and still be able to serve the business district with at least 25 millivolts per meter field intensity.

d. *DETERMINATION OF PATTERN SIZE.*--Pattern size is determined by the class of station and efficiency of the antenna system. The power for the class of station is regulated by the Federal Communications Commission as outlined in the Standards of Good Engineering Practice. In general the power ranges from; 0.25 to 50 kilowatts for stations on clear channels, 0.5 to 5 kilowatts for regional channel stations and 100 to 250 watts for local channel stations. However, local channel stations are not authorized to use directional antenna systems. The predominance of directional antenna designs are for regional stations which have to protect co-channel or adjacent channel stations in other regions of the country. Directional antennas are used by a few of the clear channel stations.

The efficiency of the antenna system depends upon the antenna design. It is always desirable to maintain a high efficiency in order that a high percentage of the output power of the transmitter may be radiated, particularly in the horizontal plane. To do this it is sometimes necessary to use a more complicated directional antenna system to give the required degree of protection to other radio stations. In other words, a low efficiency simple antenna system might give the required protection, but would not meet the FCC minimum requirement in millivolts per meter root-mean-square unattenuated field intensity for one kilowatt at one mile in the horizontal plane.

2. GENERAL TREATMENT

a. *STANDARD REFERENCE ANTENNAS*--(1) Uniform Spherical Radiator.--The Uniform, Omnidirectional, or Isotropic radiator, in free space, is taken as THE Standard Reference Antenna because it has no directivity. Such as antenna is illustrated in Fig. 1. It is defined as a theoretical antenna which radiates waves having the same field intensity in all directions. Actually such a radiator of radio waves can not be realized, because all radio antennas have directional properties. In the case of the acoustic waves, this standard is represented by a sphere pulsating radially.



(1)
CROSS-SECTION VIEW OF SPHERICAL PATTERN WHICH HAS A FIELD INTENSITY GAIN OF I, A POWER GAIN OF I (g = I) AND A DECIBEL GAIN OF ZERO

(2)
SPHERICAL RADIATION PATTERN SURFACE OF A UNIFORM OR ISOTROPIC RADIATOR

PATTERN OF A UNIFORM RADIATOR WHICH IS THE THEORETICAL STANDARD REFERENCE ANTENNA

FIG. I

For a 1 kw power source, a uniform radiator will produce a field intensity of

$$E_s = 107.6 \quad (1)$$

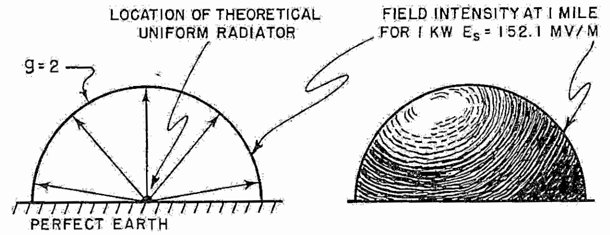
where E_s = millivolts per meter unattenuated field intensity at one mile for one kilowatt.

This standard has come into rather common use in antenna work during and since World War II. The figure of merit of all other antennas can be compared with this basic standard. Although this basic standard can not be realized in the case of electromagnetic waves, if the merit of all antennas are compared to this basic standard, then the relative figures of merit between the other antennas can be obtained immediately. Other secondary standards for free space may be selected and used as convenience demands.

(2) Uniform Hemispherical Radiator.--If a uniform radiator is placed at the surface of a perfectly conducting earth, all of the power must be radiated in the hemisphere above the surface of the earth as shown in Fig. 2. For a given power source, the power flow will have twice the intensity of a uniform radiator in free space, hence the power gain is said to be 2. For this case, the field intensity gain is $\sqrt{2}$, therefore the field intensity is $107.6\sqrt{2}$ or,

$$E_s = 152.1 \quad (2)$$

where E_s = millivolts per meter unattenuated field intensity at one mile for one kilowatt.



(1)
CROSS-SECTION VIEW OF A HEMISPHERICAL PATTERN WHICH HAS A FIELD INTENSITY GAIN OF I.4, A POWER GAIN OF g = 2 AND A DECIBEL GAIN OF 3.0I

(2)
HEMISPHERICAL RADIATION PATTERN SURFACE OF A UNIFORM RADIATOR

PATTERN OF A UNIFORM RADIATOR AT THE SURFACE OF A PERFECT REFLECTING AND CONDUCTING EARTH

FIG. 2

At the present state of the art this antenna has only academic interest, however, it is a standard for antennas at the surface of the earth, such as radio broadcasting antennas. It is particularly useful in the computation of antenna gains. This type of antenna can be considered as a standard for determining the directivity of antennas located on the surface of the earth.

(3) Current Element Antenna in Free Space.--An electric current element in free space, sometimes referred to as an elementary doublet or dipole, consists of a very short conductor (mathematically of infinitesimal length) having a uniform current distribution. This infinitesimal antenna is universally used

in developing the radiation property of an antenna of any configuration. This current element antenna is a mathematical convenience only, because such an antenna in practice for a specified field intensity would require excessive transmitter power because of the high losses due to the radiation resistance being low in comparison to the loss resistance encountered in practice. The field intensity at any distant point in space as shown in Fig. 3(a) is given by:

$$E = \frac{60\pi}{d\,\lambda}\, I\,(\delta G)\cos\theta \qquad (3)$$

or

$$= \frac{60\pi f}{dc}\, I\,(\delta G)\cos\theta$$

where E = field intensity in volts per meter at point P

π = 3.1416

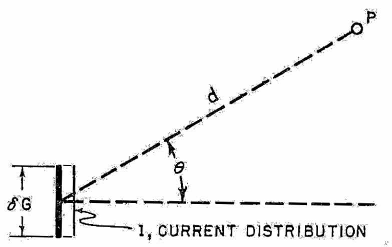
d = distance in meters from current element to the point P

λ = wave length of radiated wave in meters

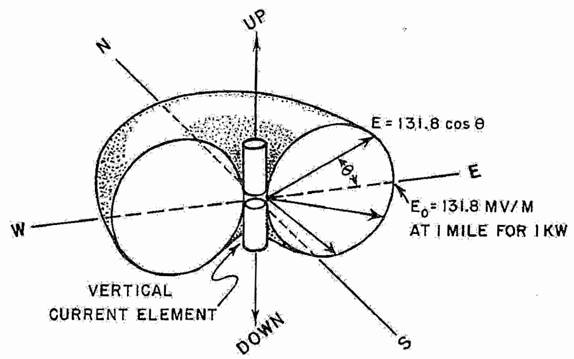
f = frequency of current in cycles per second

c = 3×10^8 meters per second, the velocity of light

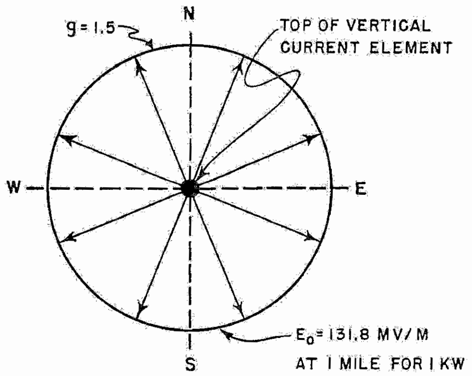
I = effective current in amperes flowing in the conductor



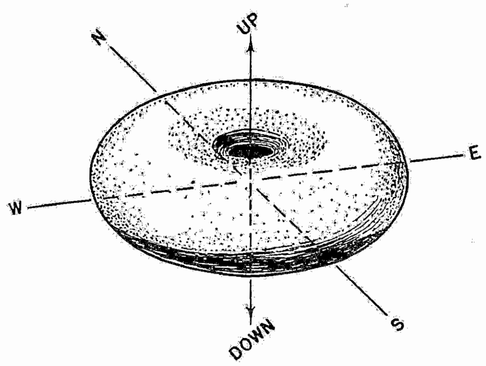
(a) ELECTRIC CURRENT ELEMENT OF LENGTH δG AND CARRYING CURRENT I



(b) SIDE CROSS-SECTION VIEW OF THE TOROIDAL RADIATION PATTERN SHOWING THE VERTICAL DIRECTIVITY.



(c) HORIZONTAL CROSS-SECTION VIEW OF TOROIDAL RADIATION PATTERN SHOWING UNIT HORIZONTAL DIRECTIVITY



(d) SURFACE PATTERN SHOWING TOROIDAL OR DOUGHNUT SHAPE

THE RADIATION PATTERN OF A VERTICAL ELECTRIC CURRENT ELEMENT IN FREE SPACE

FIG. 3

δG = elementary length (or height) of conductor measured in meters

θ = elevation angle of point P measured from a plane perpendicular to the conductor.

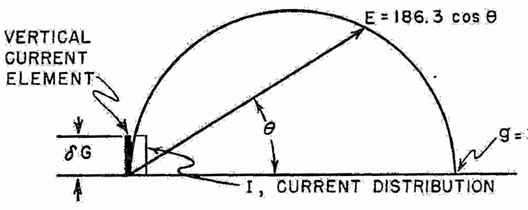
When this elementary antenna radiates one kilowatt of power the field intensity at one mile is

$$E = E_o \cos\theta \qquad (4)$$
$$= 131.8 \cos\theta$$

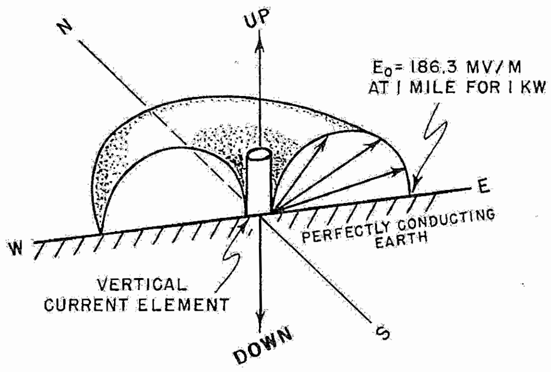
where E = millivolts per meter unattenuated field intensity at one mile for one kilowatt

E_o = millivolts per meter field intensity measured on a plane perpendicular to the conductor and in this case at the distance of one mile for one kilowatt of radiated power

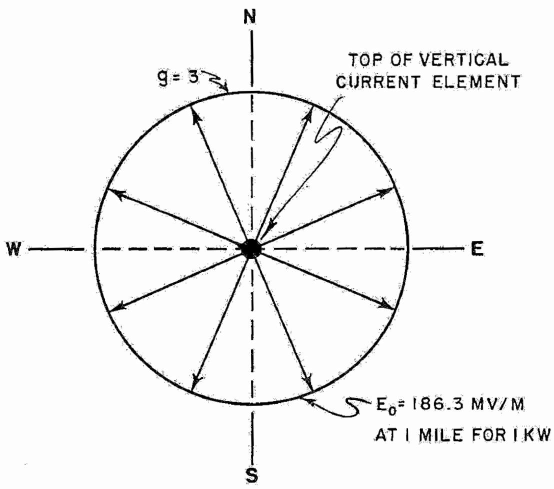
θ = elevation angle as shown in Fig. 3(a).



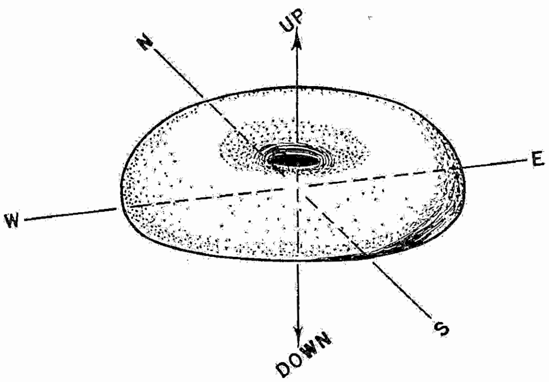
(a) VERTICAL RADIATION PATTERN



(b) SIDE CROSS-SECTION VIEW OF THE TOROIDAL HEMISPHERICAL PATTERN SHOWING THE VERTICAL DIRECTIVITY.



(c) HORIZONTAL CROSS-SECTION VIEW OF TOROIDAL HEMISPHERICAL RADIATION PATTERN SHOWING UNIT HORIZONTAL DIRECTIVITY



(d) SURFACE PATTERN SHOWING TOROIDAL HEMISPHERICAL SHAPE

THE RADIATION PATTERN OF A VERTICAL ELECTRIC CURRENT ELEMENT OVER A PERFECTLY CONDUCTING EARTH

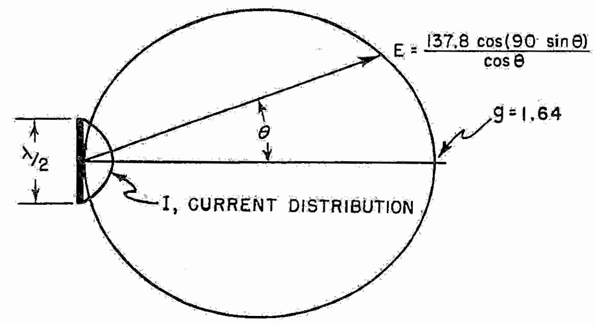
FIG. 4

The value of 131.8 is the maximum field intensity and is a constant in the horizontal plane. See Fig. 3(c). This current element antenna is sometimes used as a secondary standard reference antenna.

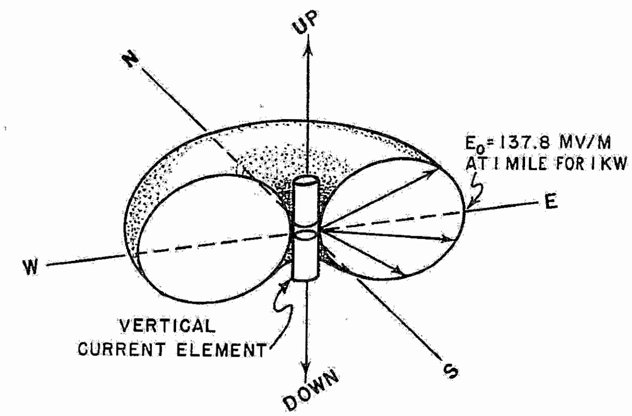
(4) Vertical Current Element Antenna Over a Perfectly Conducting Earth.--If a vertical current element antenna is located at the surface of a perfect earth, the radiation will be hemispherical as shown in Fig. 4.

When this vertical current element radiates 1 kw of power the field intensity at one mile is

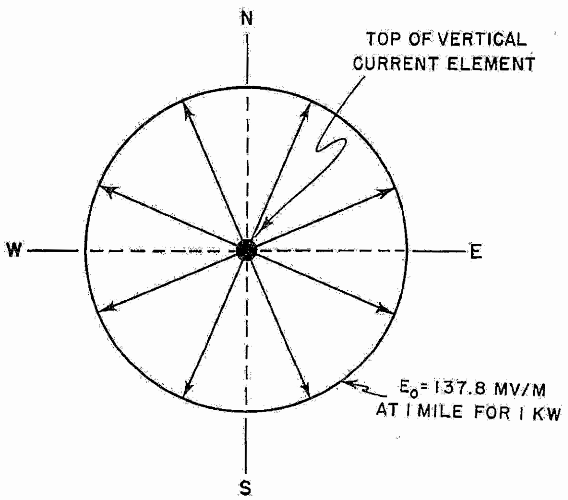
$$E = 131.8\sqrt{2}\cos\theta$$
$$= 186.3\cos\theta \qquad (5)$$



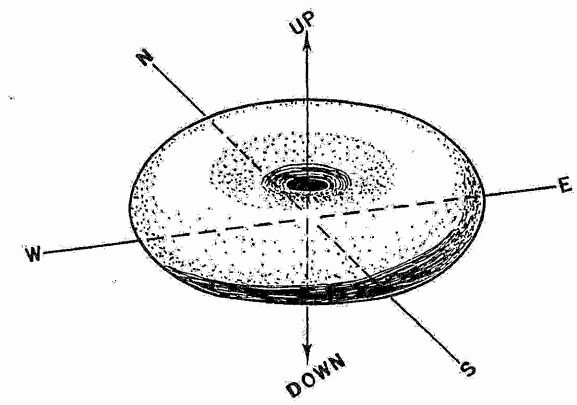
(a) VERTICAL RADIATION PATTERN



(b) SIDE CROSS-SECTION VIEW OF THE RADIATION PATTERN SHOWING THE VERTICAL DIRECTIVITY



(c) HORIZONTAL CROSS-SECTION VIEW OF RADIATION PATTERN SHOWING UNIT HORIZONTAL DIRECTIVITY



(d) SURFACE PATTERN SHOWING DOUGHNUT SHAPE

THE RADIATION PATTERN OF A VERTICAL HALF WAVE RADIATOR IN FREE SPACE

FIG. 5

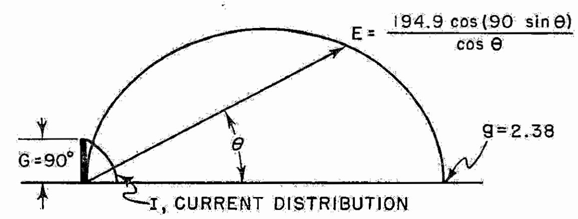
where E and θ are defined in Eq. (4) and Fig. 4(a).

This vertical current element antenna is sometimes used as secondary reference antenna. It gives the vertical pattern of a zero height antenna. See Eq. (10). While one of these infinitesimal antennas itself is of no practical value it is useful in summing up the radiation effects of antennas having practical dimensions.

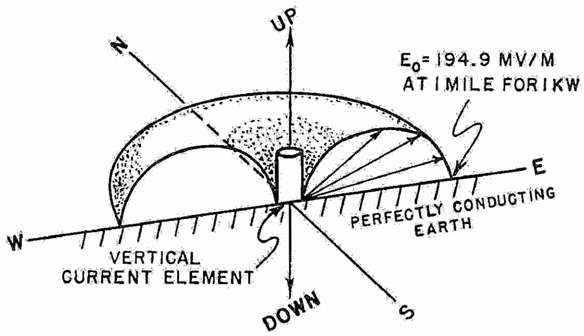
(5) Half Wave Antenna in Free Space.--A half wave antenna in free space will have essentially a sinusoidal current distribution as shown in Fig. 5(a). If the radiation effects of the current elements as given in Eq. (3) are summed up for the whole antenna the field intensity pattern is given by

$$E = \frac{137.8 \cos (90 \sin \theta)}{\cos \theta} \qquad (6)$$

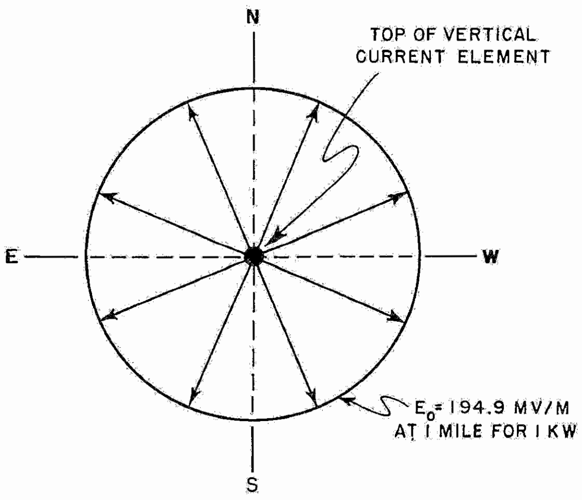
where E and θ are defined in Eq. (3) and Fig. 5(a).



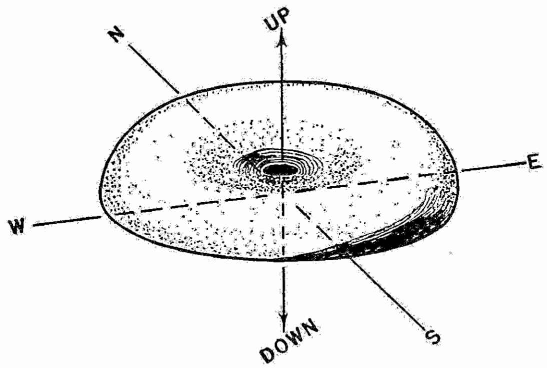
(a) VERTICAL RADIATION PATTERN



(b) SIDE CROSS-SECTION VIEW OF THE RADIATION PATTERN SHOWING THE VERTICAL DIRECTIVITY



(c) HORIZONTAL CROSS-SECTION VIEW OF RADIATION PATTERN SHOWING UNIT HORIZONTAL DIRECTIVITY



(d) SURFACE PATTERN SHOWING DOUGHNUT SHAPE

THE RADIATION PATTERN OF A VERTICAL QUARTER WAVE RADIATOR OVER A PERFECTLY CONDUCTING EARTH

FIG. 6

This type of antenna is often used as a secondary standard reference antenna because it is a practical type of antenna that is easy to set up experimentally. For example, the Federal Communications Commission uses this standard for FM and TV Broadcast Stations.

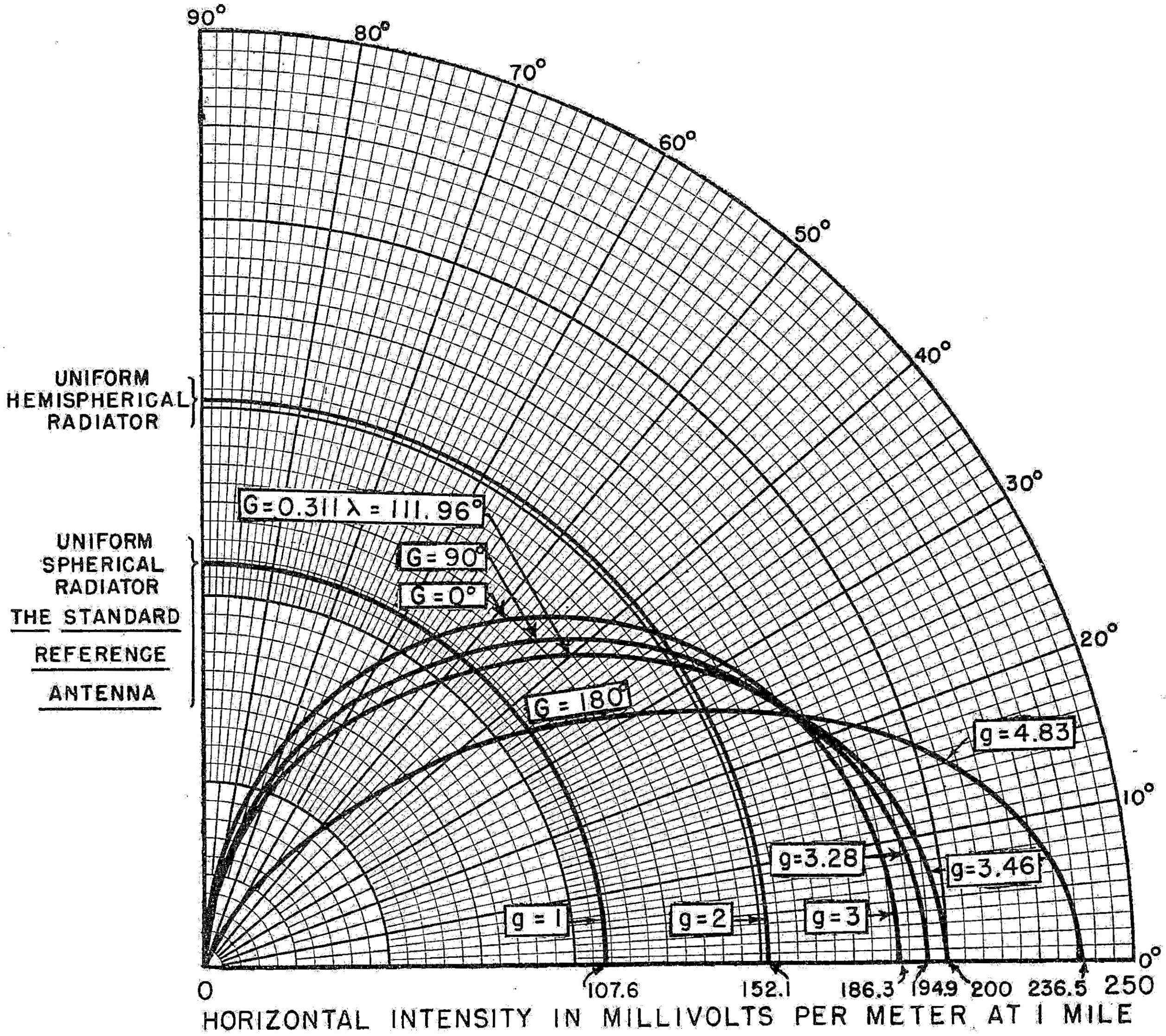
(6) Quarter-Wave Vertical Antenna over a Perfectly Conducting Earth.--This type of antenna is very common and is often used as a secondary standard reference antenna. With a sinusoidal current distribution as shown in Fig. 6(a) the radiation pattern is given by

$$E = \frac{194.9 \cos (90 \sin \theta)}{\cos \theta} \qquad (7)$$

where E and θ are defined in Eq. (3) and Fig. 6(a).

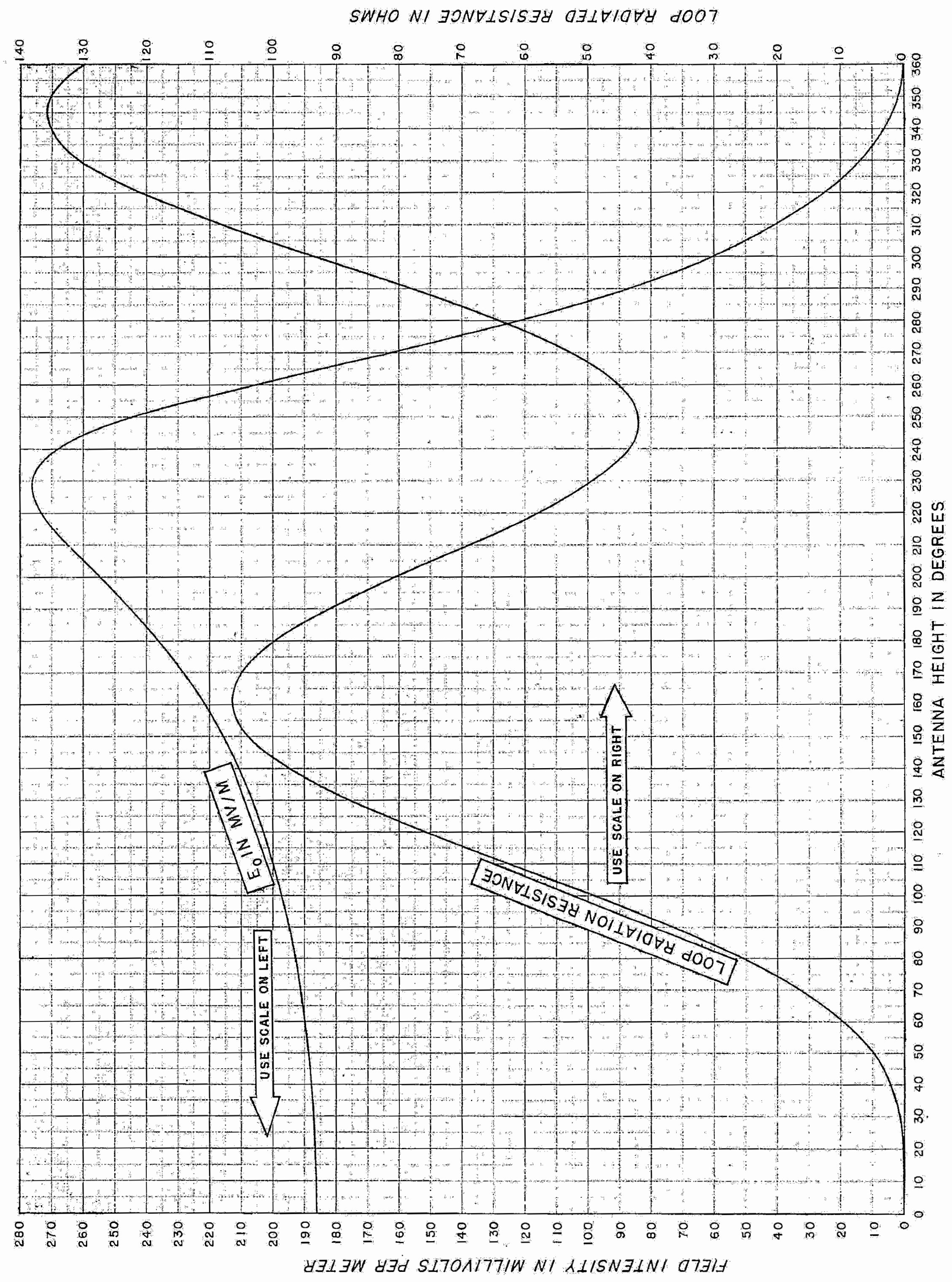
Many directional antenna arrays are designed with quarter wave elements because of the ease of making computations in design and adjustments during the proof of performance.

(7) 0.311λ Vertical Antenna Over a Perfectly Conducting Earth.--The Federal Communications Commission at the present time uses this height antenna (G = 111.96°) as their standard reference antenna. It at one time represented the average height of broadcast antennas in the United States. With sinusoidal current distribution the radiation pattern is given by



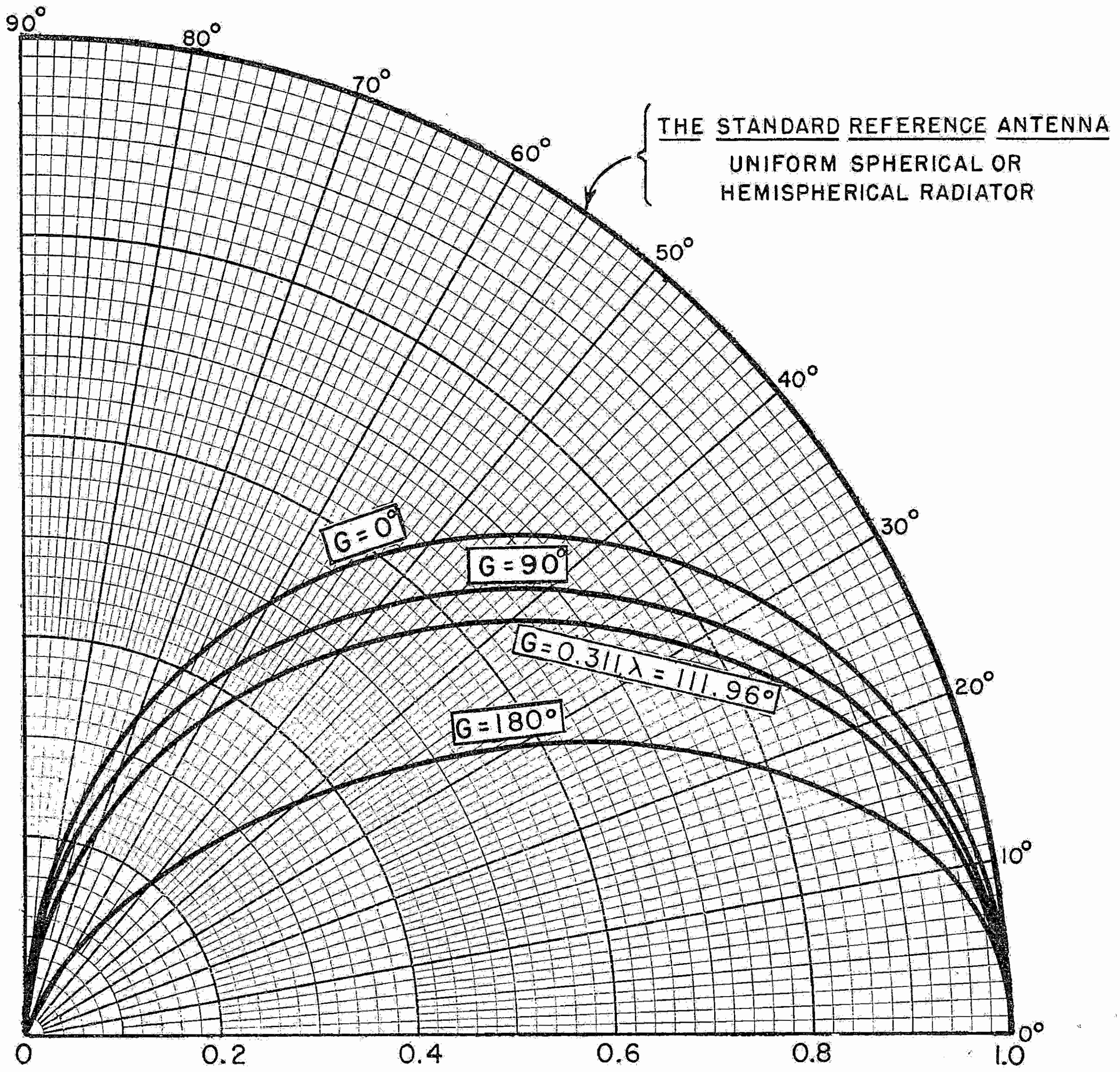
COMPARISON OF VERTICAL RADIATION PATTERN OF STANDARD REFERENCE ANTENNAS WITH A RADIATED POWER OF 1 KILOWATT

FIG. 7



UNATTENUATED FIELD INTENSITY AT ONE MILE FOR ONE KILOWATT AND THE LOOP RADIATION RESISTANCE AS A FUNCTION OF ANTENNA HEIGHT OVER A PERFECTLY CONDUCTING EARTH

FIG. 8



COMPARISON OF THE VERTICAL RADIATION CHARACTERISTICS FOR SEVERAL STANDARD REFERENCE ANTENNAS

FIG. 9

$$E = 200 \frac{\cos(G \sin\theta) - \cos G}{(1 - \cos G)\cos\theta} \quad (8)$$

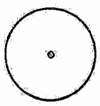
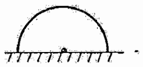
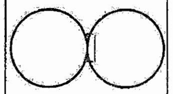
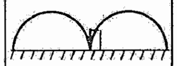
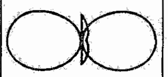
$$= 145.56 \frac{\cos(111.96 \sin\theta) + 0.3740}{\cos\theta}$$

where E and θ are defined in Eq. (3) and G = 111.96° the height of the antenna in electrical degrees.

(8) Half-Wave Vertical Antenna Over a Perfectly Conducting Earth.--For comparison purposes it is of interest to present the radiation pattern of a half-wave antenna. With sinusoidal current distribution the radiation pattern is given by

TABLE I

SUMMARY OF STANDARD REFERENCE ANTENNAS

TYPE OF ANTENNA	VERTICAL PATTERN SHAPE	MV/M FOR 1 WATT AT 1 MILE E_0	MV/M FOR 1 KW AT 1 MILE E_0	POWER GAIN g	db GAIN G	TYPE OF ANTENNA	VERTICAL PATTERN SHAPE	MV/M FOR 1 WATT AT 1 MILE E_0	MV/M FOR 1 KW AT 1 MILE E_0	POWER GAIN g	db GAIN G
UNIFORM SPHERICAL RADIATOR		3.402	107.6	1	0	UNIFORM HEMISPHERICAL RADIATOR		4.811	152.1	2	3.010
CURRENT ELEMENT		4.167	131.8	1.5	1.761	VERTICAL CURRENT ELEMENT		5.893	186.3	3	4.771
HALF WAVE ANTENNA		4.358	137.8	1.641	2.151	QUARTER WAVE VERTICAL ANTENNA		6.163	194.9	3.282	5.161
0.622 λ ANTENNA		4.472	141.4	1.728	2.375	0.311 λ VERTICAL ANTENNA		6.324	200	3.456	5.386
TWO END ON HALF WAVE IN PHASE ANTENNA		5.283	167.1	2.411	3.822	HALF WAVE VERTICAL ANTENNA		7.471	236.2	4.822	6.832

$$E = 236.5 \frac{\cos (G \sin\theta) - \cos G}{(1 - \cos G) \cos \theta} \qquad (9)$$

$$= 118.25 \frac{\cos (180 \sin \theta) + 1}{\cos \theta}$$

Where E and θ are defined in Eq. (3) and G = 180° the height of the antenna in electrical degrees.

(9) Comparison of the Vertical Radiation Patterns of Primary and Secondary Standard Reference Antennas.--Fig. 7 gives a comparison of the radiation patterns of primary and secondary standard reference antennas. The vertical patterns are expressed in millivolts per meter at one mile for one kilowatt of input power. The field intensity in millivolts per meter at one mile as a function of antenna height is given in Fig. 8.

b. *VERTICAL RADIATION CHARACTERISTICS*.--Already we have considered a number of vertical radiation patterns in Fig. 7. If these patterns are made equal to unity in the horizontal plane they are then known as vertical radiation characteristics. If this is done the curves of Fig. 9 result.

For a vertical antenna having a sinusoidal current distribution with a current node at the top, the vertical radiation characteristic takes on the form

$$f(\theta) = \frac{\cos (G \sin \theta) - \cos G}{(1 - \cos G) \cos \theta} \qquad (10)$$

where $f(\theta)$ = vertical radiation characteristic

G = electrical height of the antenna in electrical degrees

θ = elevation angle of the observation point measured up from the horizon in degrees.

The curves in Fig. 10 represent the solution of this equation over the most useful range of antenna heights. The vertical radiation characteristic is plotted as a function of elevation angle for various values of tower height. A representative set of tower heights have been selected for Fig. 10. Fig. 11 represents this same information in a different form; that is, the vertical radiation characteristic is plotted as a function of electrical tower height for various values of elevation angle. In order to prevent reversals of the lines as plotted in

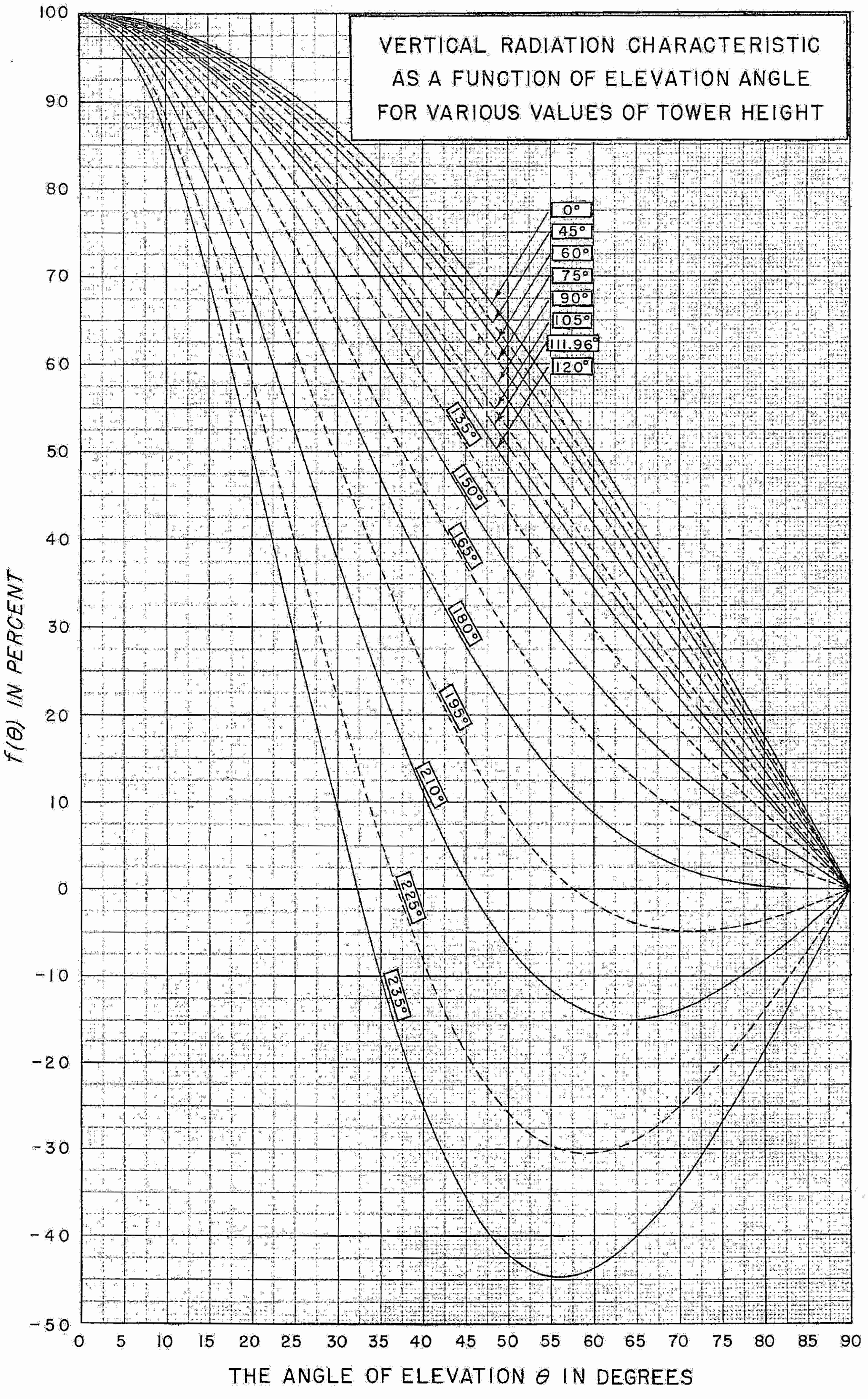

VERTICAL RADIATION CHARACTERISTIC
AS A FUNCTION OF ELEVATION ANGLE
FOR VARIOUS VALUES OF TOWER HEIGHT
0°
45°
60°
75°
90°
105°
111.96°
120°
135°
150°
165°
180°
195°
210°
225°
235°
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
f(θ) IN PERCENT
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
THE ANGLE OF ELEVATION θ IN DEGREES

FIG. 10

Fig. 11, the curves are extended to negative values in Fig. 10. This merely means that the high angle lobe is of opposite phase to the low angle radiation.

The vertical radiation characteristic will be used in the generalized equation for determining the shape of the directional antenna pattern. See Eq. (27). In the horizontal plane this function reduces to unity and simplifies the design equation.

c. *DIRECTIVITY DEFINITIONS*.--(1) On the Basis of Equal Powers.--*Directivity* or *directive gain* of a given antenna can be defined as the ratio of the maximum power flow intensity to the power flow intensity of a uniform radiator when the total power output of both sources are equal. In equation form,

$$g = \frac{P_m}{P_s} \quad \text{(equal powers)} \qquad (11)$$

where g = directivity or power gain

P_m = maximum power flow intensity from the directional antenna radiating 1 kw of power

P_s = uniform power flow intensity from the standard reference antenna radiating 1 kw of power.

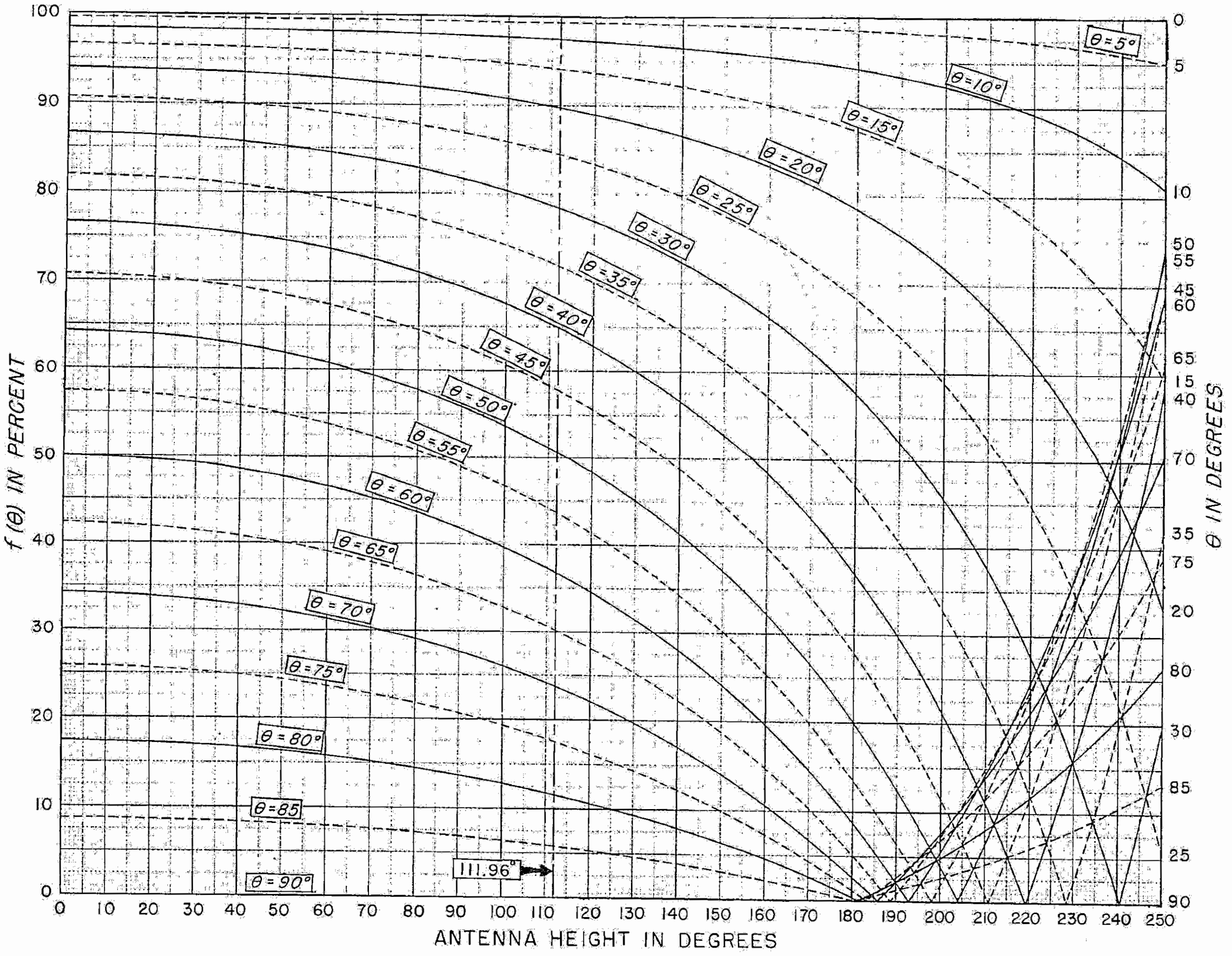
Or, in terms of field intensities power gain can be determined by the following equation,

$$g = \left[\frac{E_m}{E_s}\right]^2 \quad \text{(equal powers)} \qquad (12)$$

where g = directivity or power gain

E_m = maximum field intensity in mv/m from the directional antenna at 1 mile for 1 kw of radiated power

E_s = field intensity (107.6 mv/m) from a uniform spherical antenna at 1 mile for 1 kw of radiated power.



VERTICAL RADIATION CHARACTERISTIC AS A FUNCTION OF ELECTRICAL TOWER HEIGHT FOR VARIOUS VALUES OF ELEVATION ANGLE

FIG. 11

(2) On Basis of Equal Field Intensities.--The directivity can also be defined by taking the ratios of the power radiated when the maximum field intensity of the directional antenna is made equal to the field intensity from a uniform spherical antenna. In equation form,

$$g = \frac{P_s}{P_r} \quad \text{(equal field intensities)} \quad (13)$$

where g = directivity or power gain

P_s = power radiated (1 kw) from a uniform spherical antenna to produce a given field intensity of E_s (107.6 mv/m) at 1 mile

P_r = power radiated from the directional antenna to produce the same given maximum field intensity E_m (107.6 mv/m) at 1 mile.

To illustrate, let the field intensity of a uniform hemispherical antenna be adjusted to produce 107.6 mv/m unattenuated field intensity at 1 mile. The power radiated will be 500 watts, hence,

$$g = \frac{P_s}{P_r} = \frac{1.0 \text{ kw}}{0.5 \text{ kw}} = 2$$

the power gain of a uniform hemispherical antenna. In other words, twice the amount of power has to be supplied to the uniform spherical radiator to produce the same maximum field intensity at one mile. In the uniform spherical radiator, the additional 500 watts is used in the other hemisphere to make the radiation pattern spherical.

(3) On Basis of Decibels.--The directivity can also be computed in terms of decibels by the equation,

$$\text{decibels gain} = 10 \log g \quad (14)$$

where g = directivity or power gain.

d. *SELF BASE IMPEDANCE CHARACTERISTICS.*--The loop radiation resistance as given in Fig. 8 is not very useful because it is for a very thin conductor and the antenna is not driven at the loop point very often. Its primary application is in theoretical calculations. Since most antennas have a reasonable cross-sectional area and are driven at the base, the self base impedance equation along with the self base resistance and reactance curves will be presented. The values of self base resistance and reactance are needed in the solution of the mesh-equations of a directional antenna system.

The self resonance frequency of a tower depends upon its vertical shape as well as its cross-sectional size.[1] The first factor to be considered is the average characteristic impedance Z_o which represents the average cross-sectional size. For a cylindrical antenna

$$Z_o = 60 \left(\ln \frac{2G}{a} - 1 \right) \quad (15)$$

where Z_o = average characteristics impedance, ohms

ln = base of natural logarithms

G = antenna height, degrees or same units as a

a = antenna radius, degrees or same units as G.

EXAMPLE 1: Determine the average characteristic impedance of a 400 foot uniform cross-sectional tower that is 6.5 feet square. Use 4 feet as the equivalent radius.

SOLUTION: Substituting in Eq. (15)

$$Z_o = 60 \left(\ln \frac{2\,(400)}{4} - 1 \right)$$
$$= 60 \,(\ln 200 - 1)$$
$$= 60 \,(2.303 \times 2.301 - 1)$$
$$= 60 \,(5.3 - 1) = 258 \text{ ohms} \qquad \text{ANS.}$$

The second factor to be considered is the vertical shape. If the antenna is nonuniform, the first approximation is to regard it as uniform. With this assumption the error will be small for short antennas, and will become prohibitively large for really tall antennas. In practice, broadcasting antennas are not very tall in terms of wavelengths and we can treat them as transmission lines with slightly variable characteristic impedance.

Applying the theory of nonuniform transmission lines the self base impedance can be expressed by the following first order approximation,[1]

$$Z_b = Z_o \frac{H \sin G + j\,(F-N) \sin G - j\,(2Z_o - M) \cos G}{(2\,Z_o + M) \sin G + (F+N) \cos G - j\,H \cos G} \quad (16)$$

where

$Z_b = R_b + jX_b$ self base impedance, ohms

R_b = self base resistance, ohms

X_b = self base reactance, ohms

Z_o = average characteristic impedance, ohms

G = antenna height in degrees or radians

γ = 0.5772 ... Euler's constant

Si = sine integral function

Ci = cosine integral function

$$F = 60 \;\; \text{Si}\, 2G + 30\,(\text{Ci}\, 4G - \ln G - \gamma) \sin 2G - 30\, \text{Si}\, 4G \cos 2G \quad (17)$$

$$H = 60(\gamma + \ln 2G - \text{Ci}\, 2G) + 30\,(\gamma + \ln G - 2\,\text{Ci}\,2G + \text{Ci}\, 4G) \cos 2G + 30\,(\text{Si}\, 4G - 2\,\text{Si}\, 2G) \sin 2G \quad (18)$$

$$M = 60\,(\ln 2G - \text{Ci}\, 2G + \gamma - 1 + \cos 2G) \quad (19)$$

$$N = 60\,(\text{Si}\, 2G - \sin 2G) \quad (20)$$

---

[1] S. A. Schelkunoff, "Theory of Antennas of Arbitrary Size and Shape", Proc. I.R.E., Vol. 29, pp. 493-521; September, 1941.

If Eq. (16), for a cylindrical vertical antenna, is solved for the base resistance and reactance the family of curves in Fig. 12 and 13 will result for the different values of average characteristic impedance indicated. If the self base impedance is needed for other antenna dimensions the solution can be obtained from Eq. (16).

EXAMPLE 2: Determine the self base input resistance and reactance of a 400 foot uniform cross-section tower that is 6.5 feet square. Use 4 feet as the equivalent radius. The operating frequency is 950 kc.

SOLUTION:

From Example 1, $Z_o = 258$ ohms.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{950 \times 10^3} = 315.8 \text{ meters}$$

$$= 315.8 \times 3.281 = 1038 \text{ feet per wavelength}$$

$$G = \frac{400}{1038} \, 360^\circ = 139^\circ = \frac{139}{57.3} = 2.43 \text{ radians}$$

In Eq. (17)

$$F = 60 \text{ Si } 4.86 + 30 (\text{Ci } 9.72 - \ln 2.43 - 0.5772) \sin 278^\circ - 30 \text{ Si } 9.72 \cos 278^\circ = 132$$

In Eq. (18)

$$H = 60(0.5772 + \ln 4.86 - \text{Ci } 4.86) + 30(0.5772 + \ln 2.43 - 2\text{Ci } 2.43 + \text{Ci } 4.86) \cos 278^\circ + 30(\text{Si } 9.72 - 2\text{Si } 4.86) \sin 278^\circ = 194$$

In Eq. (19)

$$M = 60(\ln 4.86 - \text{Ci } 4.86 + 0.5772 - 1 + \text{Cos } 278^\circ) = 89.7$$

In Eq. (20)

$$N = 60(\text{Si } 4.86 - \sin 278^\circ) = 154$$

Now substituting in Eq. (16)

$$Z_b = 258 \, \frac{(194)(0.656) + j(132-154)(0.656) - j(516-89.7)(-0.755)}{(516+89.7)(0.656) + (132+154)(-0.755) - j(194)(-0.755)}$$

$$= 258 \, \frac{127 + j308}{181 + j146} = 258 \, \frac{333 \angle 67.5^\circ}{233 \angle 38.8^\circ} = 369 \angle 28.7^\circ = 324 + j176 \qquad \text{ANS.}$$

Thus, $R_b = 324$ ohms and $X_b = 176$ ohms inductive reactance. These values can be checked on the appropriate curve of Fig. 12 and 13 respectively.

e. *MUTUAL BASE IMPEDANCE AND PHASE ANGLE CHARACTERISTICS.*--The mutual impedance between vertical antennas is needed by the designer of directional antenna feeder systems. Since, vertical antennas are usually base driven and the antennas may vary in height in the same array it is desirable to have enough mutual impedance information to handle the majority of practical cases. It is therefore desirable to have the mutual base impedance between antennas of unequal as well as equal height for all practical spacings. Sufficient design equations have been developed and presented in the literature[2,3] to solve this problem, however, only a few cases of equal and unequal height antennas for spacings up to 600 degrees has been solved.

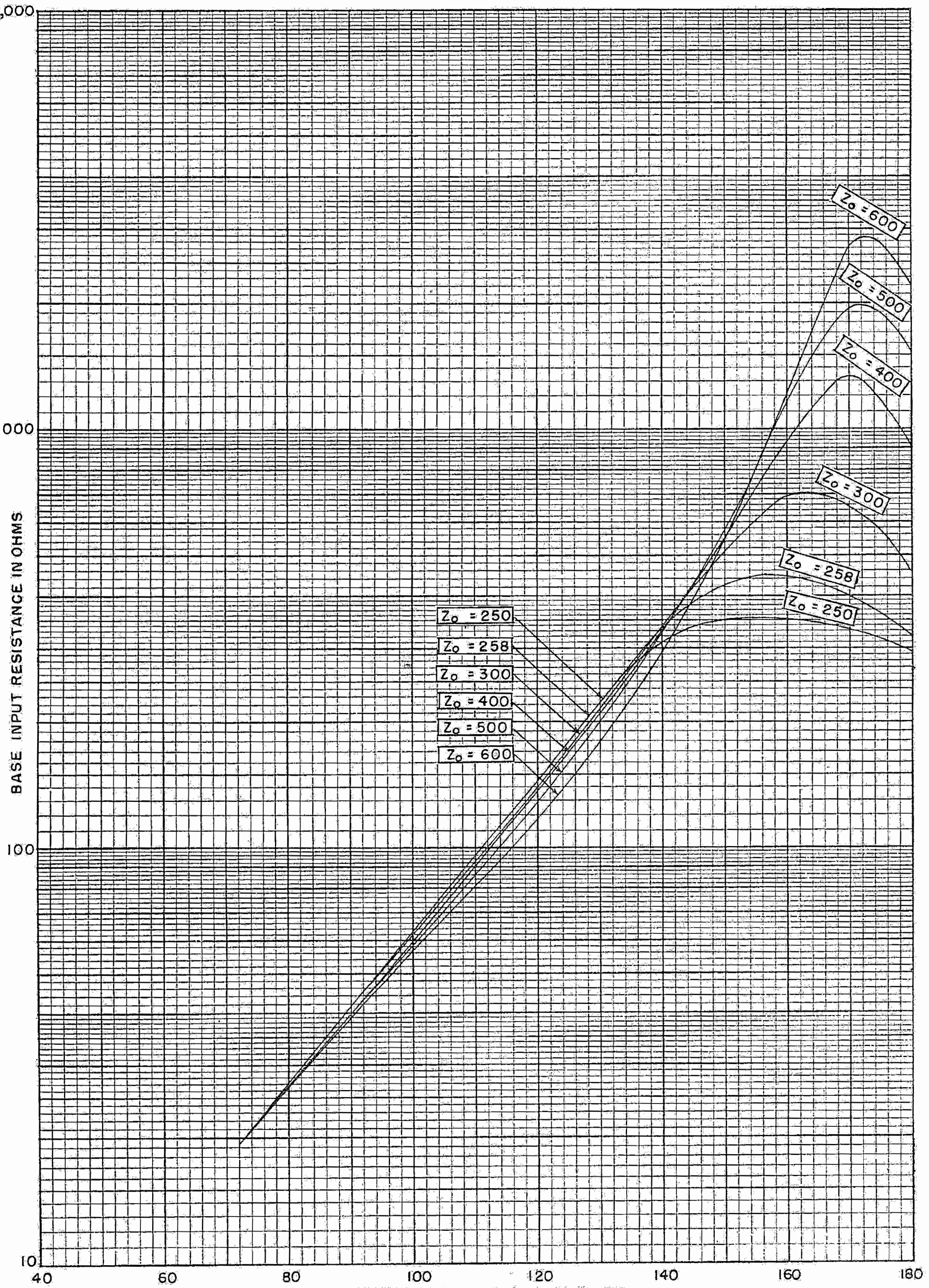
The following mutual impedance curves are presented in Appendix A:

1. Base mutual impedance curves by Chambers and Garrison.
2. Family of loop mutual impedance curves for towers of equal height by D. B. Hutton and R. H. Garrett.
3. Loop mutual impedance curves for $G_1 = G_2 = 90$ and $G_1 = G_2 = 180$ by J. F. Morrison.
4. Loop mutual impedance curves for $G_1 = 90$, $G_2 = 180$ by W. G. Hutton.

The general design equation for the mutual base resistance and reactance is given by,

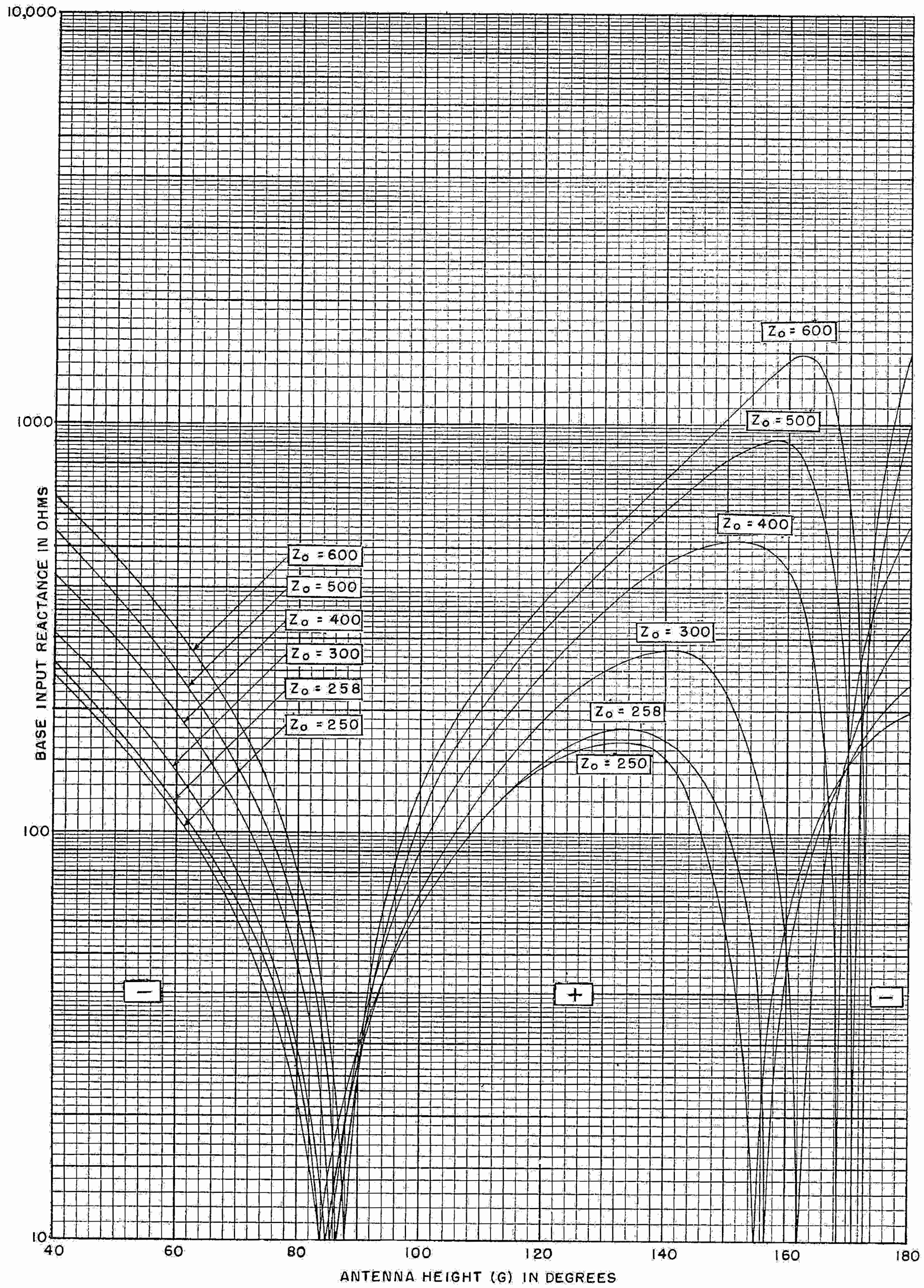
[2] G. H. Brown, "Directional Antennas," Proc. I.R.E., Vol. 25, pp. 81-145; January, 1937.

[3] C. Russell Cox, "Mutual Impedance between Vertical Antennas of Unequal Heights," Proc. I.R.E., Vol. 35, pp. 1367-1370; November, 1947.



BASE INPUT RESISTANCE OF CYLINDRICAL ANTENNAS
OVER A PERFECTLY CONDUCTING GROUND PLANE

FIG. 12



BASE INPUT REACTANCE OF CYLINDRICAL ANTENNAS
OVER A PERFECTLY CONDUCTING GROUND PLANE

FIG. 13

$$R_{12} = \frac{15}{\sin G_1 \sin G_2} \Big\{ \cos (G_2 - G_1)\big[Ci\ u_1 - Ci\ u_o + Ci\ v_1 - Ci\ v_o + 2\ Ci\ y_o - Ci\ y_1 - Ci\ s_1\big]$$
$$+ \sin (G_2 - G_1) \big[Si\ u_1 - Si\ u_o + Si\ v_o - Si\ v_1 - Si\ y_1 + Si\ s_1\big]$$
$$+ \cos (G_2 + G_1) \big[Ci\ w_1 - Ci\ v_o + Ci\ x_1 - Ci\ u_o + 2\ Ci\ y_o - Ci\ y_1 - Ci\ s_1\big]$$
$$+ \sin (G_2 + G_1) \big[Si\ w_1 - Si\ v_o + Si\ u_o - Si\ x_1 - Si\ y_1 + Si\ s_1\big] \Big\} \qquad (21)$$

$$X_{12} = \frac{15}{\sin G_1 \sin G_2} \Big\{ \cos (G_2 - G_1) \big[Si\ u_0 - Si\ u_1 + Si\ v_0 - Si\ v_1 + Si\ y_1 - 2\ Si\ y_0 + Si\ s_1\big]$$
$$+ \sin (G_2 - G_1) \big[Ci\ u_1 - Ci\ u_0 + Ci\ v_0 - Ci\ v_1 - Ci\ y_1 + Ci\ s_1\big]$$
$$+ \cos (G_2 + G_1) \big[Si\ v_0 - Si\ w_1 + Si\ u_0 - Si\ x_1 + Si\ y_1 - 2\ Si\ y_0 + Si\ s_1\big]$$
$$+ \sin (G_2 + G_1) \big[Ci\ w_1 - Ci\ v_0 + Ci\ u_0 - Ci\ x_1 - Ci\ y_1 + Ci\ s_1\big] \Big\} \qquad (22)$$

where

R_{12} = mutual base resistance between antennas No. 1 and No. 2, ohms

X_{12} = mutual base reactance between antennas No. 1 and No. 2, ohms

G_1 = height of No. 1 antenna, degrees

G_2 = height of No. 2 antenna, degrees

Si = sine integral function

Ci = cosine integral function

S = spacing between antennas, degrees

$u_0 = \sqrt{S^2 + G_1^2} - G_1$, degrees

$u_1 = \sqrt{S^2 + (G_2 - G_1)^2} + G_2 - G_1$, degrees

$v_0 = \sqrt{S^2 + G_1^2} + G_1$, degrees

$v_1 = \sqrt{S^2 + (G_2 - G_1)^2} - G_2 + G_1$, degrees

$w_0 = v_0$, degrees

$w_1 = \sqrt{S^2 + (G_2 + G_1)^2} + G_2 + G_1$, degrees

$x_0 = u_0$, degrees

$x_1 = \sqrt{S^2 + (G_2 + G_1)^2} - G_2 - G_1$, degrees

$y_0 = S$, degrees

$y_1 = \sqrt{S^2 + G_2^2} + G_2$, degrees

$s_0 = y_0 = S$, degrees

$s_1 = \sqrt{S^2 + G_2^2} - G_2$, degrees.

If the mutual base impedance information needed does not appear in Appendix A then the desired solution can be obtained by solving Eq. (21) and (22).

If the antennas are of equal height ($G = G_1 = G_2$) Eq. (21) and (22) reduce to,

$$R_{12} = \frac{15}{\sin^2 G} \Big\{ 4\ Ci\ u_1 - 2\ Ci\ u_0 - 2\ Ci\ v_0 + \cos 2G \big[Ci\ w_1 - 2\ Ci\ v_0 + Ci\ x_1 - 2\ Ci\ u_0 + 2\ Ci\ u_1\big] + \sin 2G \big[Si\ w_1 - 2\ Si\ v_0 - Si\ x_1 + 2\ Si\ u_0\big] \Big\} \qquad (23)$$

$$X_{12} = \frac{15}{\sin^2 G} \Big\{ - 4\ Si\ u_1 + 2\ Si\ u_0 + 2\ Si\ v_0 + \cos 2G \big[- Si\ w_1 + 2\ Si\ v_0 - Si\ x_1 + 2\ Si\ u_0 - 2\ Si\ u_1\big] + \sin 2G \big[Ci\ w_1 - 2\ Ci\ v_0 - Ci\ x_1 + 2\ Ci\ u_0\big] \Big\} \qquad (24)$$

Now, if antenna No. 2 is 90° in height ($G_2 = 90°$) then Eq. (21) and (22) reduce to,

$$R_{12} = 15 \Big\{ Ci\ u_1 + Ci\ v_1 - Ci\ w_1 - Ci\ x_1 + \cot G_1 \big[Si\ u_1 - Si\ v_1 - 2\ Si\ y_1 + 2\ Si\ s_1 + Si\ w_1 - Si\ x_1\big] \Big\} \qquad (25)$$

$$X_{12} = 15 \Big\{ Si\ w_1 + Si\ x_1 - Si\ u_1 - Si\ v_1 + \cot G_1 \big[Ci\ u_1 - Ci\ v_1 - 2\ Ci\ y_1 + 2\ Ci\ s_1 + Ci\ w_1 - Ci\ x_1\big] \Big\} \qquad (26)$$

EXAMPLE 3: Determine the mutual base impedance between two antennas spaced at 160° with electrical heights of 120° and 90° respectively.

SOLUTION:

$G_1 = 120°$, $G_2 = 90°$ and $S = 160°$

$u_1 = \sqrt{(160)^2 + (30)^2} + 90 - 120 = 132.8°$ or

2.32 radians

$v_1 = \sqrt{(160)^2 + (30)^2} - 90 + 120 = 192.8°$ or

3.366 radians

$w_1 = \sqrt{(160)^2 + (90 + 120)^2} + 90 + 120 = 474°$

or 8.27 radians

$x_1 = \sqrt{(160)^2 + (90 + 120)^2} - 90 - 120 = 54°$

or 0.943 radians

$y_1 = \sqrt{(160)^2 + (90)^2} + 90 \quad = 273.6°$ or 4.77

radians

$s_1 = \sqrt{(160)^2 + (90)^2} - 90 \quad = 93.6°$ or 1.63

radians

Substituting in Eq. (25) and (26)

$$R_{12} = 15 \Big\{ \text{Ci } 2.32 + \text{Ci } 3.366 - \text{Ci } 8.27 - \text{Ci } 0.943$$
$$+ \text{Cot } 120 \left[\text{Si } 2.32 - \text{Si } 3.366 - 2 \text{ Si } 4.77 \right.$$
$$\left. + 2 \text{ Si } 1.63 + \text{Si } 8.27 - \text{Si } 0.943 \right] \Big\}$$
$$= -2.935 \text{ ohms}$$

$$X_{12} = 15 \Big\{ \text{Si } 8.27 + \text{Si } 0.943 - \text{Si } 2.32 - \text{Si } 3.366$$
$$+ \text{Cot } 120 \left[\text{Ci } 2.32 - \text{Ci } 3.366 - 2 \text{ Ci } 4.77 \right.$$
$$\left. + 2 \text{ Ci } 1.63 + 8.27 - \text{Ci } 0.943 \right] \Big\}$$
$$= -28.85 \text{ ohms}$$

Therefore the mutual base impedance is

$Z_{12} = -2.935 - j\ 28.85$ ohms <u>ANS.</u>

<u>EXAMPLE 4:</u> Determine the mutual base impedance between two towers of equal height G = 110° and having a spacing of 200°.

<u>SOLUTION:</u> Since the tables of the Sine Integral and Cosine Integral functions are ordinarily tabulated with the arguments in radians, it has been found simpler to convert G and S to radians before substituting in the formulas.

$G_1 = G_2 = 110° = 1.9199$ radians.

$S \; = 200° = 3.4907$ radians.

Then

$u_o = \sqrt{3.4907^2 + 1.9199^2} - 1.9199 = 2.0639$

$u_1 = \; S = 3.4907$

$v_o = \sqrt{3.4907^2 + 1.9199^2} + 1.9199 = 5.9037$

$w_1 = \sqrt{3.4907^2 + (2 \times 1.9199)^2} + (2 \times 1.9199)$

$= 9.0290$

$x_1 = \sqrt{3.4907^2 + (2 \times 1.9199)^2} - (2 \times 1.9199)$

$= 1.3496$

$\sin^2 G = 0.9397^2 = 0.8830$

Substituting in Eq. (23) yields

$$R_{12} = \frac{15}{0.8830} \Big\{ 4 \text{ Ci } (3.4907) - 2 \text{ Ci } (2.0639)$$
$$- 2 \text{ Ci } (5.9037)$$
$$+ \cos 220° \left[\text{Ci } (9.0290) - 2 \text{ Ci } (5.9037) \right.$$
$$+ \text{Ci } (1.3496) - 2 \text{ Ci } (2.0639)$$
$$\left. + 2 \text{ Ci } (3.4907) \right]$$
$$+ \sin 220° \left[\text{Si } (9.0290) - 2 \text{ Si } (5.9037) \right.$$
$$\left. - \text{Si } (1.3496) + 2 \text{ Si } (2.0639) \right] \Big\}$$
$$= 16.9875 \Big\{ -0.1188 - 0.8180 + 0.1666$$
$$- 0.7660 \left[0.0524 + 0.1666 + 0.4549 \right.$$
$$\left. - 0.8180 - 0.0594 \right]$$
$$- 0.6428 \left[1.6663 - 2.8598 - 1.2203 \right.$$
$$\left. + 3.2672 \right] \Big\}$$
$$= 16.9875 \Big\{ -0.7702 - 0.7660 \; (-0.2035)$$
$$- 0.6428 \; (0.8534) \Big\}$$
$$= 16.9875 \Big\{ -0.7702 + 0.1559 - 0.5486 \Big\}$$
$$= 16.9875 \; (-1.1629) = -19.75 \text{ ohms}$$

For the reactance component of the mutual, substituting in Eq. (24) yields

$$X_{12} = 16.9875 \Big\{ -4 \text{ Si } (3.4907) + 2 \text{ Si } (2.0639)$$
$$+ 2 \text{ Si } (5.9037)$$
$$+ \cos 220° \left[- \text{Si } (9.0290) + 2 \text{ Si } (5.9037) \right.$$
$$- \text{Si } (1.3496)$$
$$\left. + 2 \text{ Si } (2.0639) - 2 \text{ Si } (3.4907) \right]$$
$$+ \sin 220° \left[\text{Ci } (9.0290) - 2 \text{ Ci } (5.9037) \right.$$
$$\left. - \text{Ci } (1.3496) + 2 \text{ Ci } (2.0639) \right] \Big\}$$
$$= 16.9875 \Big\{ - 7.3360 + 3.2672 + 2.8598$$

$$- 0.7660 \left[-1.6663 + 2.8598 - 1.2203 + 3.2672 - 3.6680\right]$$
$$\left. - 0.6428 \left[0.0524 + 0.1666 - 0.4549 + 0.8180\right] \right\}$$
$$= -21.33 \text{ ohms}$$

and $Z_{12} = 29.1 \angle -132.8^\circ$ ohms. ANS.

The loop values of mutual impedance are readily obtained from the above base impedance equations if the sine terms in the denominator of the leading coefficient is omitted. Since in Eq. (25) and (26) sin G_1 does not appear in the denominator the answer can be multiplied by sin G_1 to convert from base to loop impedance.

It should be pointed out that when any of the towers are over about 120° in height, and particularly in the case of self supporting towers, the values of the base mutual impedance given by these equations are considerably in error and therefore should not be relied upon. In such cases the use of loop values of mutual and self impedance will give a better indication of the power division in the directional antenna array.

It is of cardinal importance not to mix measured and theoretical values. If reliable measured values are available they should be used, however do not use measured base self-impedance and theoretical base mutual-impedances to determine driving point base impedance and power division in the directional antenna array. It is usually feasible to measure the self-impedance of each tower with the other towers disabled and then measure the magnitude of the mutual impedance between each pair of towers. Since the mutual impedance phase angles are more difficult to measure it is quite common practice to use the theoretical phase angle since they are usually more reliable than the measured values.

3. CONTROL OF PATTERN SHAPE

a. *INTRODUCTION.*--There are two basic problems in a directional antenna design. First, it is desirable to control the pattern shape by proper selection of pattern parameters and second the size of the pattern for a given amount of power is very useful in many applications. This section will be devoted to the first problem of moulding the pattern shape. The second problem of determining the size or gain of the directional antenna will be treated in Section 4.

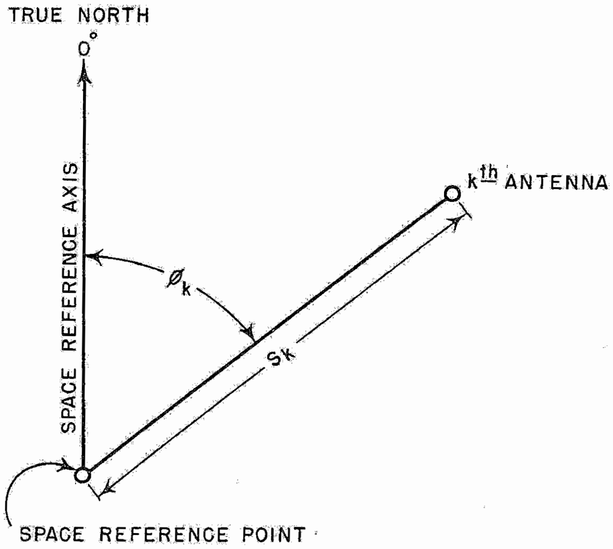
The field intensity at any given point in space is a function of the placement of the current elements along with magnitude and phase of the field intensity produced by each current element. In other words we must know the *space configuration* of the radiating elements and the *individual vectors* of the field intensities produced by these radiating elements. This information can be expressed in equation form as a vector equation or only as the magnitude of the resultant field intensity which is commonly called the pattern formula.

It is common practice to treat each tower as a radiating element over a perfectly conducting earth and select the observation point, P, far enough away from the radiating system to assume that lines joining the radiating elements and the observation point are parallel. This simplifies the mathematics and does not introduce appreciable error except when it is desired to deal with the nearby radiation field. For this case the more general equations must be employed.

b. *SPACE CONFIGURATION.*--In order to establish a system for specifying the location of each tower in such a manner that the information can be used in the design equations let us refer to Fig. 14 which gives the plan view of the space configuration of the k^{th} tower. The primary purpose of this figure is to clearly specify the position of the k^{th} tower on the ground plane. This specification contains the following:

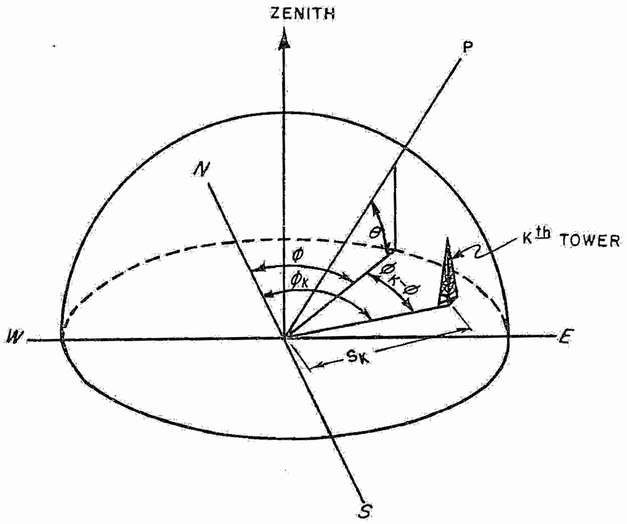
(1) Space reference point, which is the space origin or point from which all distances are measured.

(2) Space reference axis, which is a line passing through the space reference point and has a true north-south orientation. It is the reference line from which all azimuth angles are measured.



PLAN VIEW OF SPACE CONFIGURATION OF k^{th} ANTENNA

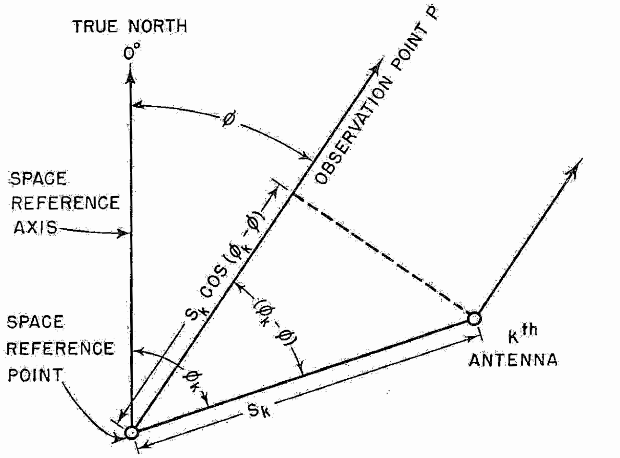
FIG. 14



SPACE VIEW OF OBSERVATION POINT P

AND THE k^{th} TOWER

FIG. 15



PLAN VIEW OF k^{th} ANTENNA SHOWING SPACE PHASING IN THE HORIZONTAL PLANE

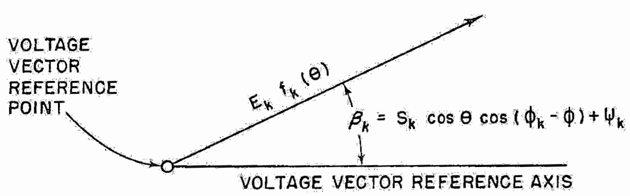
FIG. 17

(3) True azimuth of k^{th} antenna, ϕ_k, as shown in Fig. 14 is measured clockwise in degrees from the space reference axis.

(4) Spacing of k^{th} antenna, S_k, from the space reference point, is measured in electrical degrees along the horizontal plane as shown in Fig. 14.

c. *VECTOR DIAGRAM*.--The field intensity at the point P in space can be expressed as the sum of the voltage vectors in the vector diagram. These vectors depend upon the space configuration, the height of the towers and the vector field intensity radiated from each tower.

For the space configuration of the k^{th} antenna as presented in Fig. 14 and 15 we have the voltage vector at the point P as shown in Fig. 16. The magnitude of this vector is simply E_k in the horizontal plane because the vertical radiation characteristic $f_k(\theta) = 1$ when $\theta = 0$. For a



VOLTAGE VECTOR DIAGRAM FOR THE k^{th} ANTENNA

FIG. 16

tower with sinusoidal current distribution the vertical radiation characteristic is given by Eq. (10).

The vector for the k^{th} tower makes an angle β_k with respect to the voltage vector reference axis which is along the positive x-axis according to the usual mathematical convention. The angle β_k is a function of space phasing and time phasing. The space phasing is represented by

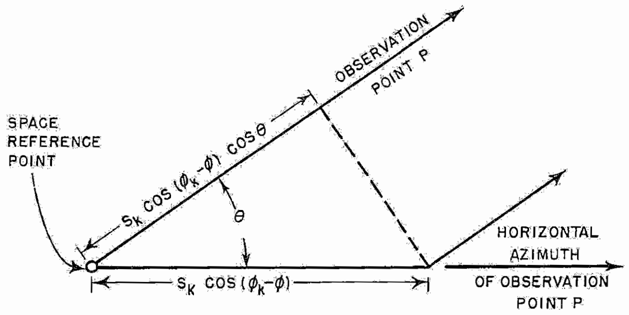
$$S_k \cos\theta \cos(\phi_k - \phi)$$

while the time phase is simply ψ_k.

For illustration, if the tower is placed at the space reference point as shown in Fig. 14 the spacing S_k becomes zero hence the space phasing also is zero. In this case only the time phase ψ_k is involved. For this case the voltage vector in Fig. 16 would make an angle ψ_k with respect to the voltage reference axis. This angle remains constant for all positions of point P.

In many directional antennas one tower is placed at the space reference point and all voltage vectors are referred to this tower. For this elementary case the voltage vector will lie along the voltage vector reference axis as shown in Fig. 16. The other voltage vectors are then referred to this vector which is in the same position as the voltage vector reference axis.

In the space phasing portion of β_k the term $\cos\theta$ reduces to unity in the horizontal plane. In this plane the space phasing is simply the difference in distance from the observation point P to the k^{th} antenna and the space reference point as shown in Fig. 17.



ELEVATION ANGLE θ SHORTENS THE SPACING S_k TO THE VALUE $S_k \cos\theta$

FIG. 18

Dropping a perpendicular from the k^{th} antenna to the straight line connecting the observation point P and the space reference point creates a right triangle the side of which is $S_k \cos(\phi_k - \phi)$. This space phasing quantity is the required difference distance since the observation point P is assumed to be at a great distance and it holds true for any direction in the horizontal plane.

When the observation point is at some elevation angle θ the k^{th} antenna will appear to be closer to the space reference point by the multiplying factor $\cos\theta$. Referring to Fig. 18, it is seen that a right triangle can be formed by dropping a perpendicular to the line connecting the space reference point and the observation point P.

The k^{th} antenna then appears to have space phasing of $S_k \cos\theta \cos(\phi_k - \phi)$ from the space reference point. This is the complete expression for the space phasing of any antenna in the system and for any observation point P in the hemisphere.

Now, if the field intensity vectors of all the towers are added in series the resultant field intensity at the observation point P can be obtained. Such a vector diagram is illustrated in Fig. 19.

d. *GENERALIZED EQUATION*.--The vector equation to express the vectors of Fig. 19 is the generalized equation that can be used to express the pattern shape for a directional antenna array with n towers. The equation in condensed form is,

$$E = \sum_{k=1}^{k=n} E_k f_k(\theta) \;\underline{|\beta_k} \qquad (27)$$

where:

E = the total effective field intensity vector at unit distance (P) for the antenna array with respect to the voltage vector reference axis. This vector makes the angle β with respect to this axis as shown in Fig. 19.

k = the k^{th} tower in the directional antenna system.

n = the total number of towers in the directional antenna array.

E_k = the magnitude of the field intensity at unit distance in the horizontal plane produced by the k^{th} tower acting alone.

$f_k(\theta)$ = vertical radiation characteristic of the k^{th} antenna as given in Eq. (10).

θ = elevation angle of the observation point P measured up from the horizon in degrees.

$$\beta_k = S_k \cos\theta \cos(\phi_k - \phi) + \psi_k \qquad (28)$$

= phase relation of the field intensity at the observation point P for the k^{th} tower taken with respect to the voltage vector reference axis $S_k \cos(\phi_k - \phi)\cos\theta$ = space phasing portion of β_k due to the location of the k^{th} tower.

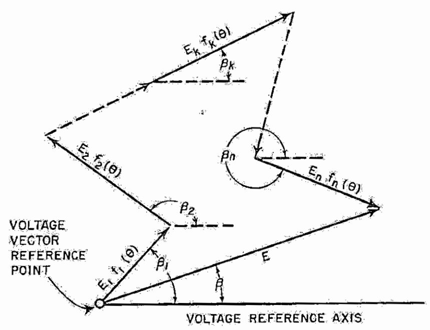
S_k = electrical length of spacing of the k^{th} tower in the horizontal plane from the space reference point.

ϕ_k = true horizontal azimuth, orientation of k^{th} tower with respect to the space reference axis.

ϕ = true horizontal azimuth angle of the direction to the observation point P (measured clockwise from true north).

ψ_k = time phasing portion of β_k due to the electrical phase angle of the voltage (or current) in the k^{th} tower taken with respect to the voltage vector reference axis.

The shape of any directional antenna pattern can be computed by applying the above equations, however, many directional antenna arrays can be designed by simplified versions of this equation. The treatment that is to follow will show how to apply some simplified versions of the above equations.



SUMMATION OF FIELD INTENSITY VECTORS FOR n ANTENNAS IN THE DIRECTIONAL ANTENNA ARRAY

FIG. 19

EXAMPLE 5: The following parameters are furnished. Write the design equation.

PATTERN PARAMETERS

Tower No.	Electrical Height G°	True Tower Orientation ϕ°	Spacing S°	Phasing Ψ°	Horizontal field $E_{mv/m}$
1	45	78.68	185.70	133.3	154.5
2	45	108.97	195.54	226.7	163.8
3	90	178.38	50.00	226.7	327.5
4	90	257.99	178.08	226.7	163.8
5	138	289.58	187.79	133.3	154.5
6	52	358.38	50.00	133.3	309.0

SOLUTION: From Eq. (10), (27) and (28) the design equation is,

$$E = 154.5\,\frac{\cos(45\sin\theta) - \cos 45}{(1-\cos 45)\cos\theta}\;\underline{|185.70\cos\theta\cos(78.68-\phi) + 133.3}$$

$$+ 163.8\,\frac{\cos(45\sin\theta) - \cos 45}{(1-\cos 45)\cos\theta}\;\underline{|195.54\cos\theta\cos(108.97-\phi) + 266.7}$$

$$+ 327.5\,\frac{\cos(90\sin\theta) - \cos 90}{(1-\cos 90)\cos\theta}\;\underline{|50.00\cos\theta\cos(178.38-\phi) + 266.7}$$

$$+ 163.8\,\frac{\cos(90\sin\theta) - \cos 90}{(1-\cos 90)\cos\theta}\;\underline{|178.08\cos\theta\cos(257.99-\phi) + 266.7}$$

$$+ 154.5\,\frac{\cos(138\sin\theta) - \cos 138}{(1-\cos 138)\cos\theta}\;\underline{|187.79\cos\theta\cos(289.58-\phi) + 133.3}$$

$$+ 309.0\,\frac{\cos(52\sin\theta) - \cos 52}{(1-\cos 52)\cos\theta}\;\underline{|50\cos\theta\cos(358.38-\phi) + 133.3}$$

ANS.

e. *DIRECTIONAL ANTENNA PARAMETERS.*--The symbols needed to describe a directional antenna array must include antenna height, true orientation, tower spacing, electric current or electric field phasing, electric field intensity at unit distance and elevation angle. For convenience it is desirable to add a symbol to express the vertical radiation characteristic and the ratio of field intensities.

The symbols chosen are those most widely used, with a few exceptions. The exceptions are necessary in order to make the system consistent in itself and not cause undue conflict with other usages of the terms. The choice of terms and their use was also influenced to a great extent by the present practice of FCC. For example, the true azimuth angle is expressed by ϕ and is measured clockwise from true north. This practice differs from mathematics where angles are measured counterclockwise from the positive x-axis. It also differs from electrical engineering where the symbol ϕ is used to measure phase angle. In another case θ is used to express elevation angle instead of zenith angle as is the usual case in mathematical treatments involving space geometry. On the other hand some consultants in the past have chosen to express the vertical angle or elevation angle by the symbol V.

Another difference of opinion revolved about the symbol to use for electrical height. The letter *h* or *H* is descriptive but the letter G was selected because of its rather general usage. Since the symbol ϕ was used for azimuth the symbol Ψ was selected to express phase angle.

Considerable discussion has revolved around the selection of a term to express field intensity ratio. The term *r* for ratio had been used but caused confusion because of its common use to express resistance. The term *M* had been used to express current ratio, but since this is often different from the field ratio, it was decided to retain *M* to express current ratio. The term *F* is selected to express field ratio, hence it can no longer be used to express field intensity interchangeably with the term *E*.

f. *HORIZONTAL PATTERN SHEET.*--To illustrate the use of the foregoing nomenclature a directional antenna array has been quite completely specified in Fig. 20 using the parameters of Example 5.

In preparing the pattern data, the following factors were considered:

Horizontal plane pattern plot with asimuth angles measured clockwise from true north.

Direction and distance to each protected station, with the azimuth sector indicated if an area is to be protected.

Pattern plotted to the largest scale possible on polar coördinate paper having 10 or 15 major radial divisions, using divisions and subdivisions having values of 1, 2, 5 or 10^x.

Minimum values, where possible, plotted on a 10 x expanded scale.

MEOV (maximum expected operating value) plotted and designated on the expanded scales and on the regular scales where the expanded scales do not apply.

Horizontal *rms* field intensity circle, E_0, drawn on the pattern sheet.

Tower placement sketch showing the location and number of each tower with true north at the top of the diagram, along with dimensions useful in special design equations if such are used to make the pattern computations.

Symbol definitions are as follows:

G the electrical height, should be the value used in the vertical radiation characteristic function,

$$f(\theta) = \frac{\cos (G \sin\theta) - \cos G}{(1 - \cos G) \cos\theta}. \qquad (10)$$

If this is not the case, then the correct vertical radiation characteristic should be furnished.

ϕ true orientation of observation point measured in degrees.

ϕ with a subscript, as ϕ_2 for No. 2 tower, describes the true orientation of each tower in the system.

S with a subscript, as S_2 for No. 2 tower, describes the spacing of each tower from the space reference point measured in degrees.

ψ with a subscript, as ψ_2 for No. 2 tower, describes the phase of the electric field with respect to the voltage reference axis.

E the total field intensity at the angle ϕ and θ.

E with a subscript, as E_2 for No. 2 tower, describes the magnitude of the horizontal field intensity of each tower in the system. It is convenient to express this value in millivolts per meter unattenuated field intensity at one mile.

F the ratio of the horizontal field intensities. Double subscripts can be used if the value is to be expressed outside the table, thus

$$F_{21} = \frac{E_2}{E_1}$$

M tower current ratios which may be considerably different than the field ratios. With subscripts it is written,

$$M_{12} = \frac{I_1}{I_2}$$

This symbol is not needed in the systematization on this sheet

θ elevation angle from the space reference point measured in degrees.

E_θ the magnitude of the *rms* field intensity circle at the elevation angle θ. Subscripts are used, such as E_0 at zero degrees elevation and E_{40} at 40° elevation, to denote the elevation angle of the *rms* field intensity. It is convenient to express this value in millivolts per meter unattenuated field intensity at one mile.

E_s the field intensity of an equivalent standard hemispherical uniform radiator. For 1 kw of radiated power this standard hemispherical uniform radiator will give 152.1 mv/m unattenuated field intensity at one mile.

P_r the amount of radiated power from the proposed directional antenna.

Station data tabulation, with station call letters entered on the first line of station data; if the call letters have not been assigned, line remains blank or the word *new* inserted. The proposed operating frequency in kc and the proposed operating power in kw inserted. Time of operation designated by one of the following:

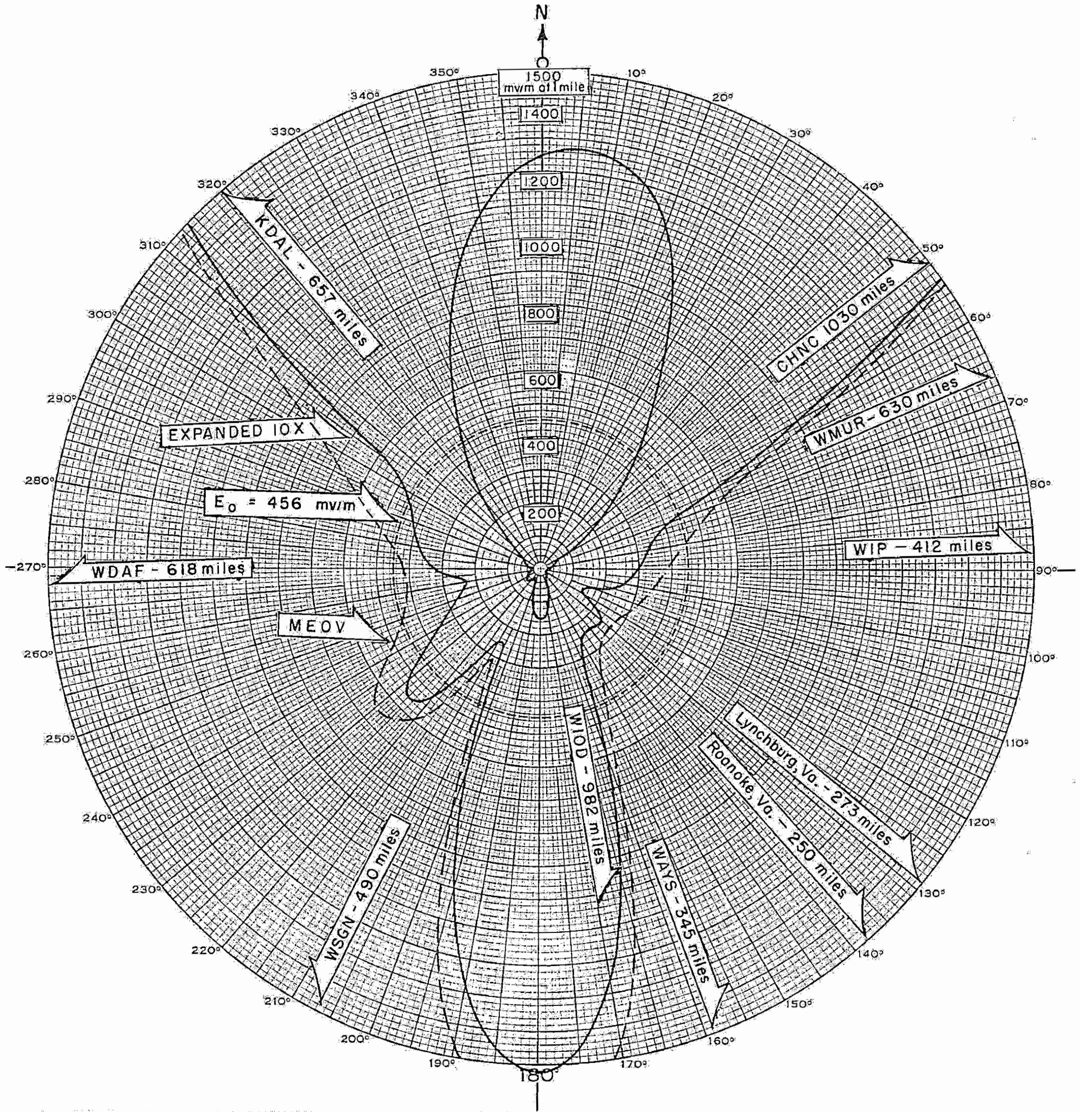
U unlimited.
D daytime.
N night time.
ST shares time.
L limited time with dominant station.
LS local sunset.
SH specified hours.
SA special authorization.

Type of operation specified by one of the following:

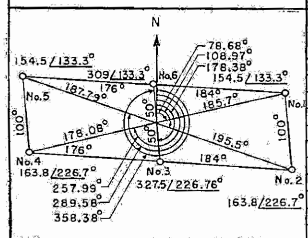
DA-N directional antenna night time.
DA-D directional antenna day time.
DA-1 directional antenna with same power and same pattern day and night.
DA-2 directional antenna with different power and/or different pattern day and night.

Ground system briefly described by the number of radials per tower, the length of these radials and the dimension of the ground screen if used.

Latitude and longitude given for the space reference point of the directional antenna system. Supersedes refers to the previous design pattern, if any. This number can be given as year in two digits, month in two digits, and day in two digits. Pattern number refers to the proposed



TOWER PLACEMENT SKETCH



PATTERN PARAMETERS

TOWER No.	ELECTRICAL HEIGHT $G°$	TRUE TOWER ORIENTATION $\phi°$	SPACING $S°$	PHASING $\psi°$	HORIZONTAL FIELD mv/m E	FIELD RATIO F	ELEV. ANGLE $\theta°$	ELEVATION RMS FIELD mv/m E_θ
1	45	78.68	185.70	133.3	154.5	1.00	0	456
2	45	108.97	195.53	226.7	163.8	1.06	10	416
3	90	178.38	50.00	226.7	327.5	2.12	20	401
4	90	257.99	178.08	226.7	163.8	1.06	30	343
5	138	289.58	187.79	133.3	154.5	1.00	40	300
6	52	358.38	50.00	133.3	309.0	2.00	50	260
7							60	186
8							70	100
9							80	21
10							E_s = 340 mv/m	
DAYTIME OPERATION ☐			NIGHTTIME OPERATION ☑				P_r = 5 kw	

STATION DATA

CALL	WHKC
FREQUENCY	610 kc
POWER	5 kw
TIME OF OPERATION	U
TYPE OF OPERATION	DA-N
DAYTIME TOWERS	No.5
NIGHTTIME TOWERS	1,2,3,4,5,6
No. RADIALS PER TOWER	120
LENGTH OF RADIALS	90° 400 ft.
GROUND SCREEN DIMENSION	
LAT.	39° 52' 26.2"
LONG.	82° 58' 36.1"
SUPERSEDES	470423
PATTERN No.	470919

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TERMINAL TOWER
CLEVELAND 13, OHIO

HORIZONTAL PLANE PATTERN

USING 15 MAJOR CIRCLE POLAR COORDINATE PAPER

FIG. 20

pattern and is given as year in two digits, month in two digits and day in two digits.

The horizontal plane pattern graph paper, as shown in Fig. 20, has a circle diameter of 10" and is printed on standard 11" x 17" sheets which can be reduced to 8.5" x 11" for record purposes. However, the 11" x 17" sheet size should be presented to FCC in order to meet their requirements.[4]

g. *BROADCAST ANTENNA REQUIREMENTS*.--The design of a broadcast antenna system is somewhat more complex than that of the so-called "beam" directional antennas used extensively for transmission in the short and ultra-short wave regions. The latter antenna systems almost invariably are designed to have a more or less concentrated beam of energy in one direction with energy in all other directions suppressed to a greater or less degree, depending upon the gain of the array. These short wave arrays often consist of numbers of stacked elements arranged symmetrically and fed with in-phase currents which can be distributed and phased throughout the array by means of transmission line sections. Either horizontal or vertical elements may be used, as sky wave or direct wave transmission is employed and the ground acts as a reflector rather than an absorber of the incident energy.

Other long-wire arrays of the rhombic or V type are also found in use at the higher frequencies, and operate on a traveling wave principle, in which the fields from each segment of the wires add up in one direction to give a sharp beam.

Broadcast antenna arrays present a rather different problem. They must usually be designed to produce a unique non-symmetrical pattern, as each directional antenna design must fulfill the peculiar requirements imposed upon the station by the proximity, direction and powers of other stations on the same or adjacent channels. Vertical radiating elements are universally used, as most of the useful transmission is by means of the surface wave, which suffers tremendous attenuation when horizontally polarized transmission is employed.

An excellent visualization of the problem involved in broadcast directional antenna design is afforded in Fig. 21, showing a typical regional broadcast channel in the United States. It will be noted that the pattern of each station is arranged so that a minimum amount of energy is directed towards other stations on the same channel. In addition the arrays are located in a position with respect to their primary service area so that the main lobe of energy covers this primary area as completely as possible.

For a complete discussion of skywave interference ratio and selection of the proper range of vertical angles for sky wave transmission reference is made to the FCC Standards of Good Engineer Practice for Standard Broadcasting Stations, issued by the Federal Communications Commission. Here the generally accepted ratio of desired to undesired co-channel signal of 20 to 1 is mentioned, and methods of calculating the interference presented to one station by another, in the primary service area of the first station, are described.

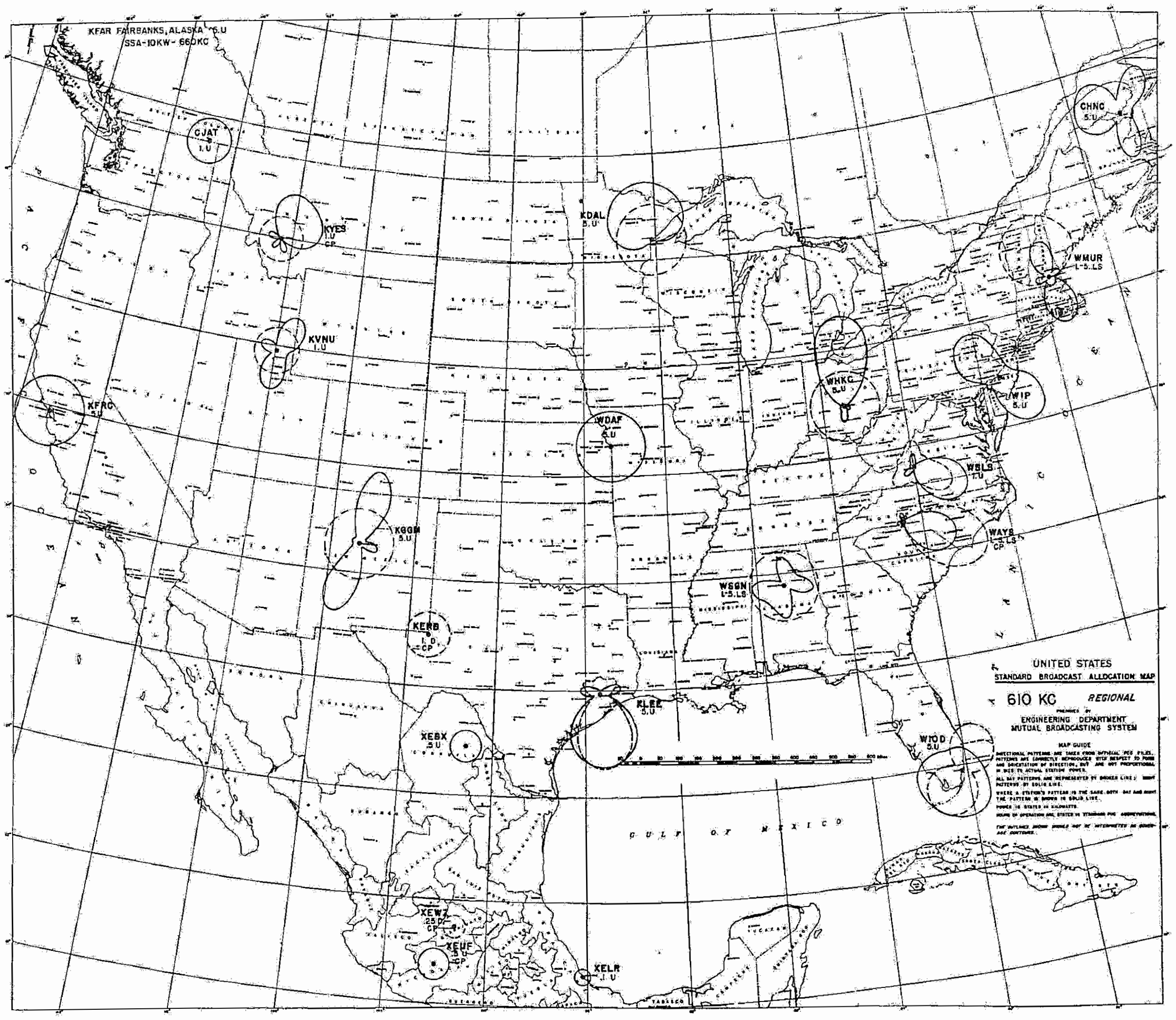
Sky wave interference, for allocation purposes, is determined by use of certain curves in the standards. These curves were prepared after exhaustive tests and surveys had been made of ground and skywave propagation characteristics at broadcast frequencies. However, changing ionospheric conditions indicate that a wholly realistic picture of interference conditions cannot be obtained solely from the curves. Actual measurements with field strength recorders are often necessary to determine absolute existing signal strengths at the points where the interference ratio is to be determined. Equipment is available for measuring the interference ratio while several stations are operating simultaneously on the same frequency.

The protection requirements will impose upon the pattern nulls and minima, at various elevation angles, the number being determined by the number of stations to be protected. The designer strives to provide the required protection with the minimum of towers, as a matter of simple economics. Several different arrangements may be tried out on paper in an attempt to find the optimum solution to the problem.

After the pattern shape has been determined, by methods such as outlined in this section, then the pattern size should be determined by methods such as outlined in the next section. With this information at hand the driving-point impedance of each tower in the array can be determined. Then it is possible to design the directional antenna feeder system which will consist of impedance matching networks between each tower and its respective transmission line, computation of phase shift in the transmission lines, provision of adequate phase shifting networks and a power dividing network at the input to the directional antenna system, which is usually at the output of the transmitter. A typical directional antenna feeder system block diagram is shown in Fig. 22.

After the construction has been completed, in accordance with the construction permit issued

[4] Two types of polar coordinate paper have been prepared by the author, with 50 and 75 complete circles. The paper is available in heavy or light weight 11" x 17" sheets. By using these two types of polar coordinate paper and reasonable scales the pattern can be drawn to at least 67 percent of full scale value.



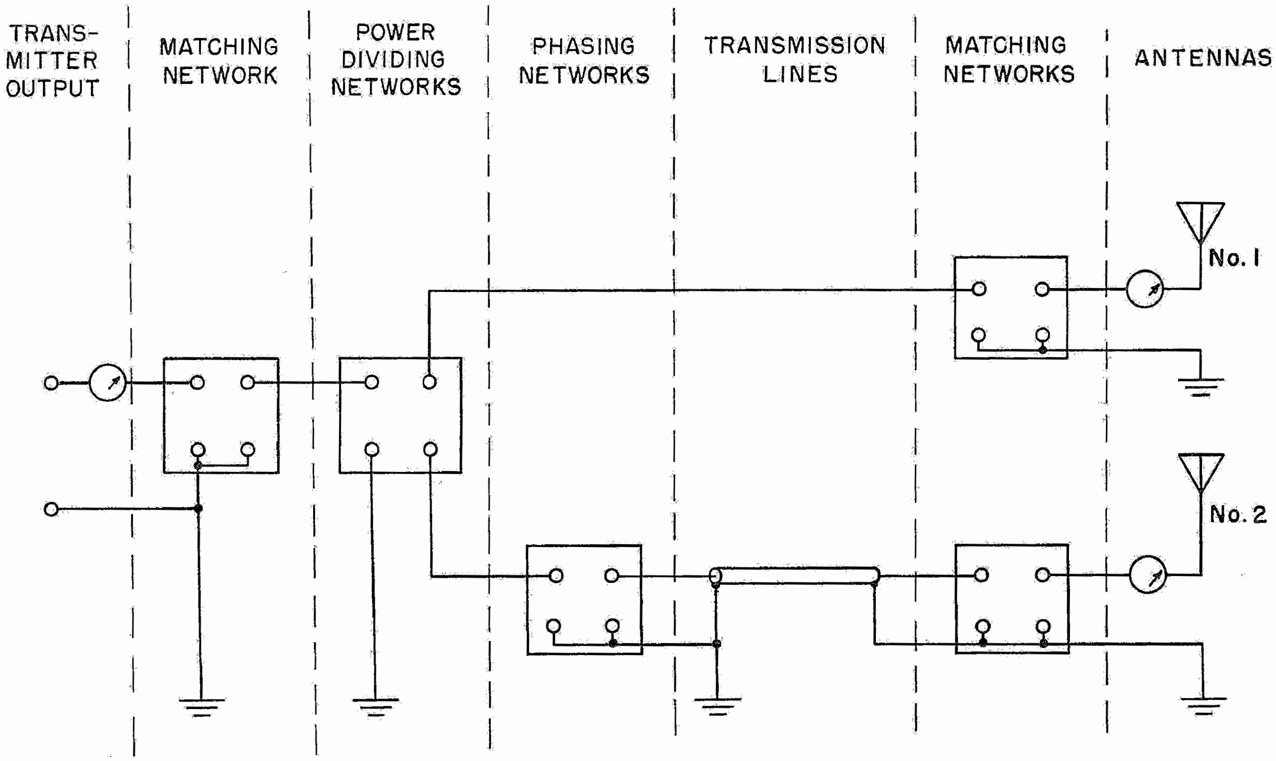
BROADCAST CHANNEL ALLOCATION MAP
SHOWING DIRECTIONAL ANTENNA PATTERNS USED BY STATIONS
TO PROTECT EACH OTHER (COURTESY MUTUAL BROADCASTING SYSTEM)

FIG. 21

by the FCC, equipment test authority is requested of FCC. Operating with this authority the networks are adjusted to meet the design requirements. Actual measurements must be made in the field to confirm that the operating parameters and field intensities are in accordance with the specifications in the construction permit. A report is then prepared and filed with the application for station license with FCC. Concurrently with the filing of this application, program test authority can be requested of FCC, thus permitting the station to go on the air before FCC has an opportunity to grant the station license.

Finally the operation of the antenna is kept under constant surveillance by means of amplitude and phase monitoring of the tower currents and by periodically checking the field strength at selected monitoring points placed in critical directions from the antenna system.

It should be realized that directional antenna design is a most exacting task, fraught with difficulties and not always culminating in unqualified success. Some large arrays for broadcasting stations have actually been designed and placed into operation, after which it was found impossible to tune them up to operate in accordance with FCC efficiency requirements. These design failures were due to neglect to consider high losses caused by excessive circulating currents in the array - another factor which must be carefully evaluated, as the FCC



BLOCK DIAGRAM OF A DIRECTIONAL ANTENNA FEEDER SYSTEM

FIG. 22

requires that a directional antenna produce at least a minimum unattenuated r-m-s field intensity at one mile. For specific details see the FCC Standards in Section One of the NAB Engineering Handbook. Some leeway for losses is afforded by the stipulation that the actual measured common driving point resistance may be multiplied by the factor 0.925 (5 kw and under) or the factor 0.95 (over 5 kw power) to obtain the operating power resistance which, times current squared, equals the licensed radiated power of the station.

h. *TWO TOWER PATTERN USING THE ADDITION FORM*--Perhaps the **simplest** directional antenna design equation is found in the two tower addition form. We will now proceed to develop this equation in considerable detail. Referring to Fig. 23(a) No. 1 tower is initially placed at the space reference point and No. 2 tower is placed directly north on the space reference axis. By substituting in the generalized Eq. (27) and (28) the horizontal pattern equation can be written,

$$E = E_1 \underline{|0^\circ} + E_2 \underline{|S_2 \cos \phi + \psi_2} \tag{29}$$

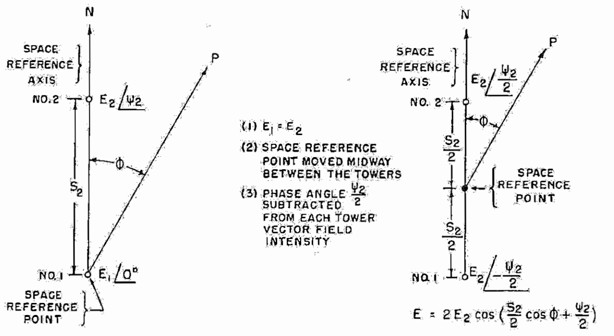
where the values are defined following Eq. (27). The vector diagram for this equation is shown in Fig. 24(a). The problem is to convert this vector equation into the simplest possible pattern formula. This can be done if E_1 is made equal to E_2, the space reference point is moved to a position midway between the two towers, and the phase angle $\psi_2/2$ is subtracted from the phase angle of the vector field intensity of each tower. The resulting space configuration is shown in Fig. 23(c) and the resulting vector diagram is shown in Fig. 24(b).

The purpose of making this change is to remove the imaginary or j component, from the design equation. In Eq. (29) E_1 lays along the voltage vector reference axis, hence does not have a j component, however the position of E_2 will rotate with respect to E_1 and will therefore have a j component for all phase relations, β, other than 0° and 180°.

If the new parameters, as shown in Fig. 23 (c) are substituted in the general Eq. (27) and (28) we have,

$$E = E_2 \underline{\left| \frac{S_2}{2} \cos \phi + \frac{\psi_2}{2} \right.} + E_2 \underline{\left| -\frac{S_2}{2} \cos \phi - \frac{\psi_2}{2} \right.} \tag{30}$$

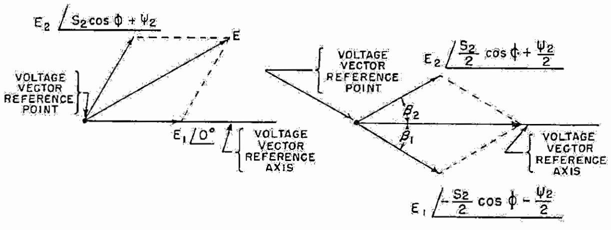
where the values are defined following Eq. (27).



(*a*) Original placement (*b*) Required conditions (*c*) Final placement and design equation

SPACE CONFIGURATION FOR ADDITION METHOD

FIG. 23



(*a*) Original vector diagram (*b*) Final vector diagram

VECTOR DIAGRAM FOR ADDITION METHOD

FIG. 24

Converting these vectors from polar to rectangular form gives,

$$E = E_2 \left[\cos\left(\frac{S_2}{2}\cos\phi + \frac{\Psi_2}{2}\right) + j\sin\left(\frac{S_2}{2}\cos\phi + \frac{\Psi_2}{2}\right)\right] + E_2\left[\cos\left(-\frac{S_2}{2}\cos\phi - \frac{\Psi_2}{2}\right) + j\sin\left(-\frac{S_2}{2}\cos\phi - \frac{\Psi_2}{2}\right)\right]$$

In this equation the j sine terms cancel and the cosine terms add giving the desired answer

$$E = 2E_2 \cos\left(\frac{S_2}{2}\cos\phi + \frac{\Psi_2}{2}\right) \quad (31)$$

where

E = unattenuated horizontal field intensity from the directional antenna, mv/m

E_2 = unattenuated horizontal field intensity from each tower acting alone, mv/m

$S_2/2$ = spacing from the space reference point to each tower, degrees

ϕ = true azimuth angle toward observation point, P, degrees

$\Psi_2/2$ = time phase of No. 2 tower and the negative of the time phase of No. 1 tower, degrees.

The reason this design equation is so simple is that the j components cancel out. This can be seen quite clearly by reference to Fig. 24(b). The space reference point and time phase have been chosen such that the vector E_2 makes a positive angle β_2 that is equal to the negative of β_1 that the vector E_1 makes with the voltage vector reference axis. Since $E_1 = E_2$ the j components are always equal and opposite hence cancel each other leaving the real terms to add along the voltage vector reference axis.

When the space phase plus the time phase, $(\frac{S_2}{2}\cos\phi + \frac{\Psi_2}{2})$, equals zero the cosine term becomes unity and the maximum field intensity is $2E_2$ in Eq. (31). For this condition the vectors E_1 and E_2 lie along the positive x-axis as shown in Fig. 25(a). For illustration if $S_2 = 180°$ and $\Psi_2 = +180°$ then when $\phi = 180°$ we have the maximum positive value due south in the pattern of Fig. 25(b). From Eq. (31) we have,

$$E = 2E_2 \cos(90 \cos\phi + 90)$$

as the pattern equation.

As the azimuth angle ϕ of the observation point P for the above illustration is moved counter clockwise from true south the first vector term in Eq. (30) which represents the No. 2 tower takes on a positive angle as shown in Fig. 24(b) while the second vector term in Eq. (30) which represents No. 1 tower takes on an equal negative angle. The resultant E still lies along the positive x-axis as illustrated in Fig. 24(b) and Fig. 25(a). The magnitude of this resultant decreases as the azimuth angle decreases up to 90° at which point the resultant becomes zero because the vectors E_2 and E_1 point in opposite directions as shown in Fig. 25(a). Then as the value of ϕ decreases from 90° to 0° the resultant becomes negative and increases in value until it reaches a maximum negative value as shown in Fig. 25(a) and (b).

It is of interest to note that one lobe is positive because the resultant vector E takes on positive values as it moves back and forth along the positive x-axis while the other lobe is negative because the vector E moves back and forth along the negative x-axis.

In this type of design equation the vector E always lies along the x-axis in the vector diagram. However, in order to plot the pattern in polar coordinates the magnitude of E is plotted in a polar fashion as a function of the true azimuth angle ϕ. Both ways of representing E are shown in Fig. 25.

If it is desired to fill both nulls an equal amount the field ratio can be made other than unity. In fact with a two tower array both nulls will naturally be filled an equal amount for a given field ratio. The amount of fill increases as the field ratio is increased. When the field ratio becomes infinite all of the radiation is from one tower and the pattern becomes non-directional or circular in the horizontal plane.

EXAMPLE 6: Write the equation of a cardioid pattern having a maximum field of 200 mv/m, to the north, using 90° spacing and the addition method.

SOLUTION: For this condition $S_2 = 90°$ and if the cardioid pattern is to point north then the cosine term of Eq. (31) must be unity when $\phi = 0$. For this condition the parenthesis term in Eq. (31) must be zero, thus

$$\frac{S_2}{2} \cos \phi + \frac{\psi_2}{2} = 0$$

Substituting, $$\frac{90}{2} \cos 0 + \frac{\psi_2}{2} = 0$$

Solving, $$\psi_2 = -90° \text{ or } 270°$$

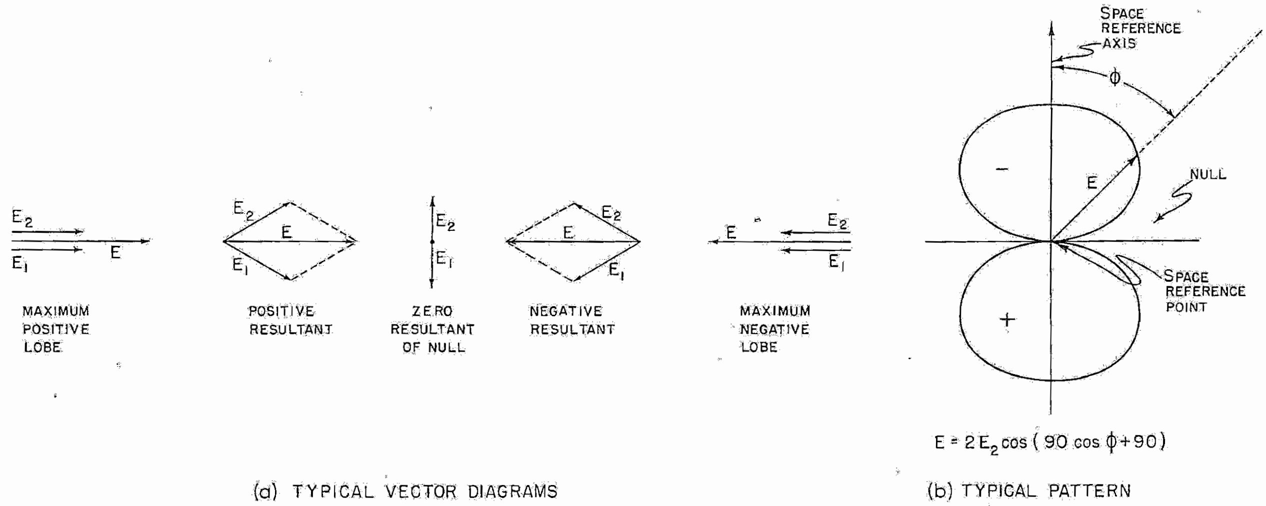
Therefore, the complete design equation can be written, by substituting in Eq. (31),

$$E = 2(100) \cos \left(\frac{90}{2} \cos \phi - \frac{90}{2}\right)$$

$$= 200 \cos (45 \cos \phi - 45) \qquad \text{ANS.}$$

For this pattern the values of E always lie along the positive x-axis, thus the whole pattern is a positive lobe pointing north with a null due south.

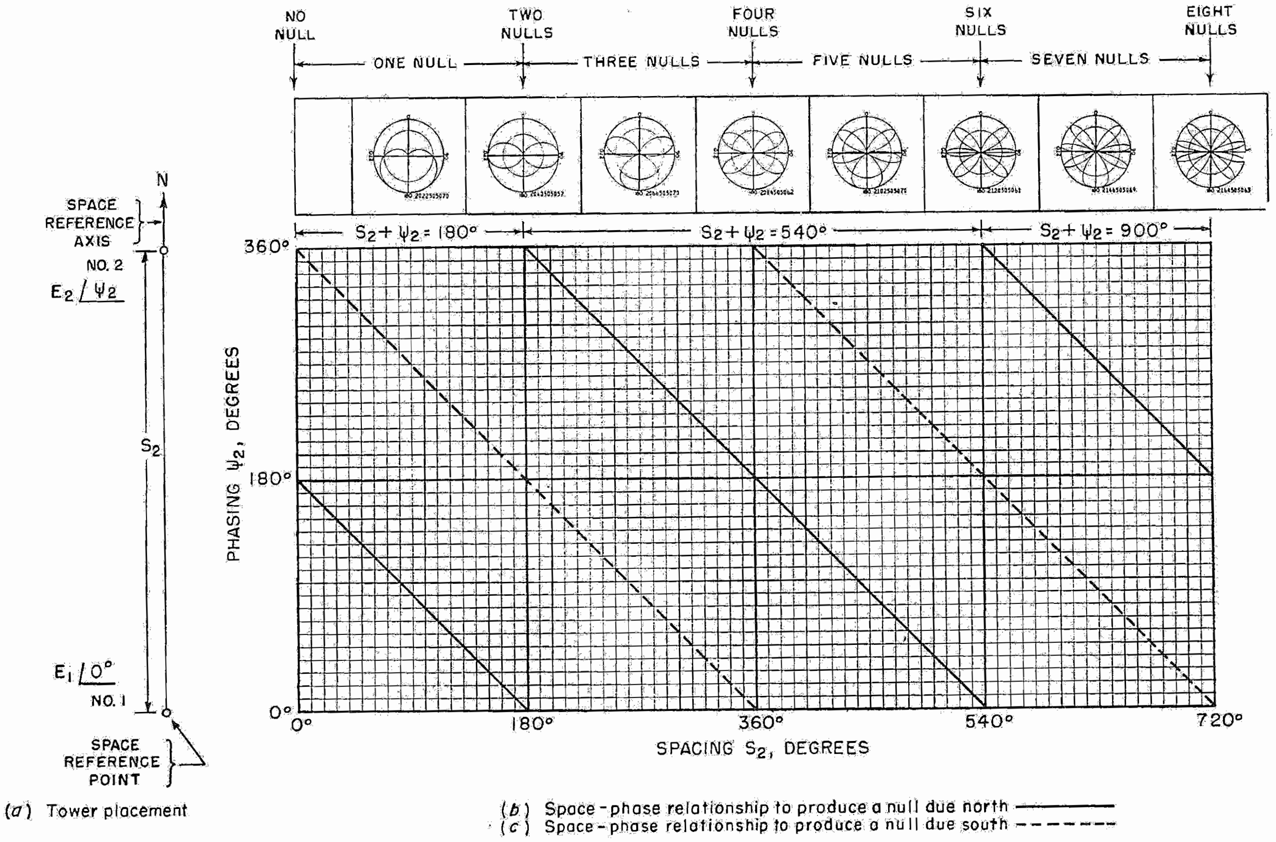
If we had used $\psi_2 = 270°$ instead of $\psi_2 = -90°$ then E would always lie along the negative x-axis, thus the whole pattern would be a negative lobe pointing north with a null due south.



(a) TYPICAL VECTOR DIAGRAMS (b) TYPICAL PATTERN

VECTOR DIAGRAMS AND PATTERN FOR TWO TOWER ADDITION FORM

FIG. 25



TWO TOWER SPACE-PHASE RELATIONSHIP TO PRODUCE A NULL DUE NORTH

FIG. 26

i. *SYSTEMATIZATION OF TWO TOWER PATTERNS.*--Since two tower patterns are so useful, a systematization of 568 patterns is presented in Appendix B. 256 of these patterns are general patterns with spacings out to four wavelengths, while 312 are detailed patterns for spacings out to one wavelength. The general patterns are for spacings in steps of 45 degrees and phasings in steps of 45 degrees. The detailed patterns are for spacings in steps of 15 degrees and phasings in steps of 15 degrees. The phasings are only presented from 0 to 180 degrees since the same patterns, oriented 180 degrees, result for phasings from 180 degrees to 360 degrees. This is readily observed by inspecting the general patterns which present all the phasings.

j. *DIRECTION OF HORIZONTAL NULLS FOR TWO TOWER PATTERNS.*--Whenever the parenthesis term $\left(\frac{S_2}{2}\cos\phi + \frac{\Psi_2}{2}\right)$ in Eq. (31) becomes 90° or 270° the total field intensity vector E becomes zero and produces a null. If only one null is desired it must be either due north or due south since this is in the line of the towers. *A two tower pattern is always symmetrical around the line of towers,* therefore if the null is not in line with the towers there must be at least two nulls. Of course it is possible but not necessary to have two nulls in the line of the towers (due north and due south).

Let us consider the special case of a null due north. Let $\phi = 0$, then the parenthesis term in Eq. (31) must be equal to 90° for spacings up to 180°, hence

$$S_2 + \Psi_2 = 180° \qquad (32)$$

From this equation it is seen that for every spacing S_2 there is one possible phasing Ψ_2 that will produce a null due north. When the spacing goes beyond 180° then the phase angle becomes negative unless 360° is added to the spacing. Referring to Fig. 26(b) the solid diagonal lines graphically gives the relationship between S_2 and Ψ_2 to produce a null due north. The patterns, for illustration at the top of the figure, it will be noted, have been lifted from the general systematization of two tower patterns. It should be observed that for zero spacing no null can exist, for spacings from 0° to 180° only one null can exist and it will be due north for this special case. At a spacing of 180° two nulls will exist, one due north and the other due south. From 180° to 360° spacing three nulls will exist, one of which will be due north and the others will be symmetrically located from due south in the third and fourth quadrants. At 360° spacing four nulls will exist in the exact direction of north, east, south, and west. This process of increasing the number of nulls will continue to increase, in a similar fashion, as shown in Fig. 26(b). A study of the general systematization of two tower patterns will also reveal this relationship.

EXAMPLE 7: Two towers on a North-South line have a spacing of 110°. Determine the phasing required to produce a null directly north.

SOLUTION: Substituting in Eq. (32)

$$110° + \Psi_2 = 180°$$
$$\Psi_2 = 70° \qquad \text{ANS.}$$

A similar special relationship exists for a null due south. The *dashed line* in Fig. 26(b) gives the relationship between S_2 and Ψ_2 for this condition. This relationship is also illustrated in the general systematization of two tower patterns.

For the more general condition of producing a null in the direction ϕ_N, the azimuth angle of the null from the line of towers, we can apply the equation,

$$S_2 \cos \phi_N + \Psi_2 = \pm 180° \qquad (33)$$

where ϕ_N = azimuth angle of the nulls from the line of the towers, degrees
= azimuth angle clockwise and counterclockwise from true north to the nulls, if the towers are on a north-south space reference axis, degrees

Ψ_2 = total phasing of the No. 2 (north tower) with respect to the No. 1 (south tower), degrees

S_2 = total spacing between the towers, degrees.

This equation has been used to prepare the chart in Fig. 27.

EXAMPLE 8: Number 2 tower is spaced 140° from No. 1 tower on a true bearing of 40°. Determine the phasing that will produce a null at 110°. In what direction is the other null?

SOLUTION: Substituting in Eq. (33)

$$140° \text{ Cos } 70° + \Psi_2 = 180°$$
$$140° \times 0.342 + \Psi_2 = 180°$$
$$\Psi_2 = 132.1° \qquad \text{ANS.}$$

The two tower pattern being symmetrical around the line of towers the other null will be at 40° - 70° = -30° or 330° from true north. ANS.

A careful inspection of this chart will reveal a number of interesting facts about two tower directional antennas. In general, spacing up to 180° will produce two symmetrical nulls for any given phasing. Of course when the null is at $\phi_N = 0°$ or $\phi_N = 180°$ there is only one null for spacings up to 180°. The positive sign in Eq. (33) is used for this case when $\phi_N = 90°$ the phasing $\Psi = 180°$ for all spacings. In other words, when the towers are exactly out of phase a null is always produced exactly broadside to the array regardless of the spacing.

For spacings from 180° to 360° there are in general two or four symmetrical nulls. The positive and negative sign in Eq. (33) must be used to determine both sets of nulls. Of course two of these nulls degenerate into a single null when $\phi_N = 0°$ or $\phi_N = 180°$ and for this condition there will be three nulls. If the spacing is increased from this value four nulls will exist while if the spacing is decreased only two nulls will exist. When there is a condition of three nulls the phasing can be increased or decreased as shown in the chart to produce either two or four nulls. When there are two nulls there may be one or two other minimums depending upon the pattern parameters. This can be visualized by observing the systematization of two tower directional antenna patterns.

EXAMPLE 9: Determine the phase and spacing to produce one pair of nulls at 80 and 280 degrees, and another pair at 150 and 210 degrees for a two tower directional antenna.

SOLUTION: Referring to Fig. 27 the specified null angles are satisfied for a spacing of approximately 346 degrees and a phasing of approximately 120 degrees. ANS.

If an exact solution is desired Eq. (33) can be applied to give the following simultaneous equations,

$$S_2 \cos 80 + \Psi_2 = +180$$
$$S_2 \cos 150 + \Psi_2 = +180$$

Substituting for the cosine values and subtracting

$$0.1736\ S_2 + 0.8660\ S_2 = 0° \text{ or add } 360°$$
$$S_2 = 346.3° \text{ spacing} \qquad \text{ANS.}$$

Substituting this value of spacing in the first equation

$$\Psi_2 = 180 - (346.3)(0.1736) = 119.9° \qquad \text{ANS.}$$

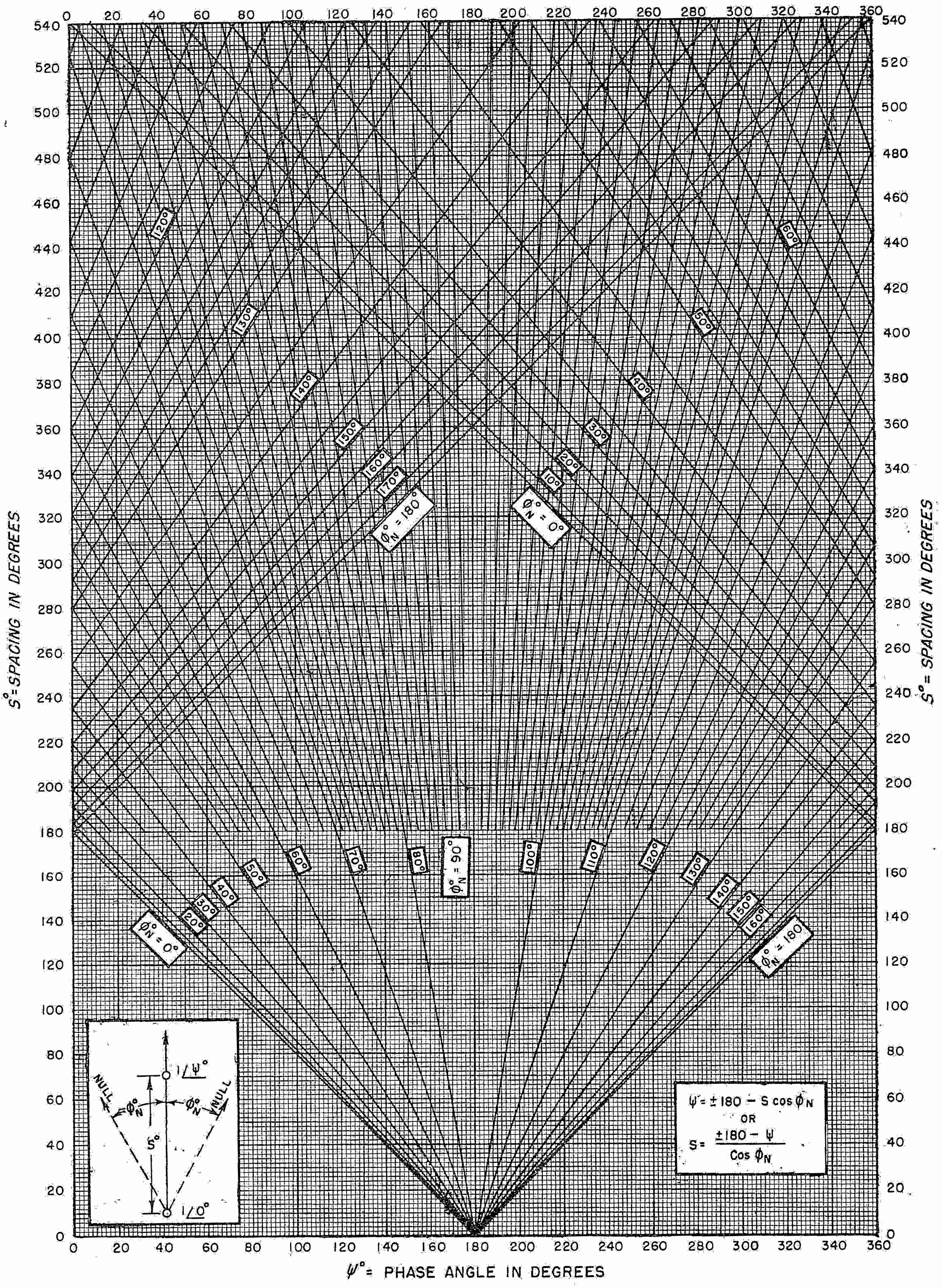


CHART FOR DETERMINING NULL DIRECTIONS OF TWO TOWER ARRAYS

FIG. 27

k. *TWO TOWER PATTERNS USING THE COSINE LAW FORM.* --Consider the generalized equation for two towers in a directional antenna array,

$$E = E_1 f_1(\theta) \ \underline{|\beta_1} + E_2 f_2(\theta) \ \underline{|\beta_2} \tag{34}$$

The terms in this equation are defined in Eq. (27) and (28), and illustrated in Fig. 29. These vectors can be resolved into components along the x-axis and y-axis and by taking the square root of the sum of the squares, the magnitude of the resultant is obtained, thus,

$$|E| = \sqrt{\left[E_1 \ f_1(\theta) \cos\beta_1 + E_2 \ f_2(\theta) \ \cos\beta_2\right]^2 + \left[E_1 \ f_1(\theta) \ \sin\beta_1 + E_2 \ f_2(\theta) \ \sin\beta_2\right]^2} \tag{35}$$

Expanding terms under the radical gives,

$$|E| = \sqrt{\begin{array}{l}\left[E_1 f_1(\theta)\right]^2 \cos^2\beta_1 + \left[E_2 f_2(\theta)\right]^2 \cos^2\beta_2 + 2E_1 f_1(\theta) \ E_2 f_2(\theta) \ \cos\beta_1 \cos\beta_2 \\ + \left[E_1 f_1(\theta)\right]^2 \sin^2\beta_1 + \left[E_2 f_2(\theta)\right]^2 \sin^2\beta_2 + 2E_1 f_1(\theta) \ E_2 f_2(\theta) \ \sin\beta_1 \sin\beta_2\end{array}} \tag{36}$$

Since $\sin^2\beta + \cos^2\beta = 1$ and $\cos\beta_1 \ \cos\beta_2 + \sin\beta_1 \ \sin\beta_2 = \cos(\beta_2 - \beta_1)$ the above equation can be reduced to

$$|E| = \sqrt{\left[E_1 f_1(\theta)\right]^2 + \left[E_2 f_2(\theta)\right]^2 + 2 \ E_1 f_1(\theta) \ E_2 f_2(\theta) \ \cos(\beta_2 - \beta_1)} \tag{37}$$

This is the cosine law form of a two tower pattern equation. To illustrate, consider the application of the cosine law to the triangles of Fig. 28.

If we write: $a = E_1 f_1(\theta)$, $b = E_2 f_2(\theta)$, $c = |E|$, and $C = 180 - (\beta_2 - \beta_1)$. Making these substitutions in the cosine law equation of Fig. 28 results in Eq. (37) the cosine law design equation.

Now if the field intensity ratios are defined by

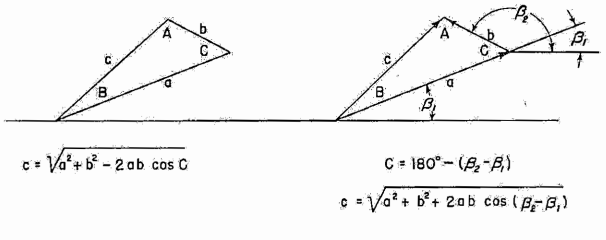
$$F_{21} = \frac{E_2 \ f_2(\theta)}{E_1 \ f_1(\theta)} \tag{38}$$

then the quantity $E_1 f_1(\theta)$ can be removed from under the radical in Eq. (37) and the equation can be written,

$$|E| = E_1 f_1(\theta) \sqrt{1 + F_{21}^2 + 2F_{21} \ \cos(\beta_2 - \beta_1)} \tag{39}$$

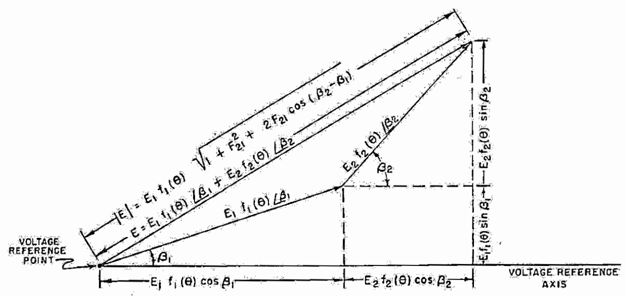
This is the cosine law equation expressed in ratio form. For some types of antenna designs this is a useful form of pattern equation. This equation is shown in Fig. 29.

EXAMPLE 10: If the towers in Example 8 have horizontal field intensities of 120 mv/m and 170 mv/m for No. 1 and No. 2 towers respectively, write the horizontal pattern equation using the cosine law form.



COSINE LAW EQUATION APPLIED TO A TRIANGLE

FIG. 28



VOLTAGE VECTOR DIAGRAM FOR TWO TOWER PATTERN

FIG. 29

SOLUTION: Substituting in Eq. (28)

$\beta_1 = 0$ since No. 1 tower is at the space reference point

$\beta_2 = 140° \times \cos 0° \times \cos(40°-\phi) + 132.1°$

Substituting in Eq. (38)

$F_{21} = \frac{170}{120} = 1.416$

Substituting in Eq. (39)

$|E| = 120\sqrt{1+1.416^2+2 \times 1.416 \cos 140° \cos (40°-\phi)+132.1°}$ ANS.

1. *TWO TOWER PATTERNS USING THE MULTIPLICATION FORM.*--It is a simple matter to convert Eq. (39) to the multiplication form. Let No. 1 tower be located at the space reference point then E_1 will lay along the voltage reference axis. Now if the terms under the radical are divided by 2 F_{21} the equation can be written,

$$|E| = E_1 f_1(\theta)\sqrt{2 F_2}\sqrt{\frac{1 + F_2^2}{2 F_2} + \cos (S_2 \cos\theta\cos\phi + \Psi_2)} \quad (40)$$

where F_2 is the same as F_{21} defined in Eq. (38). The double subscript is dropped because all towers are referred to the reference tower.

For the special case of equal field intensities this equation reduces to

$$|E| = 2 F_1 f_1(\theta) \cos\left[\frac{S_2}{2} \cos\theta \cos\phi + \frac{\Psi_2}{2}\right] \quad (41)$$

EXAMPLE 11: For the directional antenna in Example 10 express the horizontal pattern equation in the multiplication form.

SOLUTION: Substituting in Eq. (38)

$F_{21} = \frac{170}{120} = 1.416$

Substituting in Eq. (40)

$$|E| = 120\sqrt{2}\sqrt{1.416}\sqrt{\frac{1+1.416^2}{2 \times 1.416} + \cos (140° \cos 0° \cos\phi + 132.1°)}$$

$$= 202\sqrt{1.068 + \cos(140° \cos\phi + 132.1°)}$$ ANS.

EXAMPLE 12: Prove the two tower multiplication equation, that is, show the transition from Eq. (40) to (41).

SOLUTION: Equation (41) is for the special case of equal field intensities.

First, for Eq. (40) we have

$$E = E_1 f_1(\theta)\sqrt{2F_2}\sqrt{\frac{1 + F_2^2}{2F_2} + \cos (S_2 \cos\theta \cos\phi + \Psi_2)}$$

And for equal field strengths this first reduces to

$$E = E_1\sqrt{2}\, f_1(\theta)\sqrt{1 + \cos (S_2 \cos\theta \cos\phi + \Psi_2)}$$

A basic trigonometric formula is

$$\cos^2\alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

$$2\cos^2\alpha = 1 + \cos 2\alpha$$

$$\sqrt{2}\cos\alpha = \sqrt{1 + \cos 2\alpha}$$

Then letting

$$2\alpha = S_2 \cos\theta \cos\phi + \Psi_2$$

$$\alpha = \frac{S_2}{2} \cos\theta \cos\phi + \frac{\Psi_2}{2}$$

And

$$E = E_1\sqrt{2}\ f_1(\theta)\ \sqrt{2}\cos\left(\frac{S_2}{2}\cos\theta\cos\phi + \frac{\Psi_2}{2}\right)$$

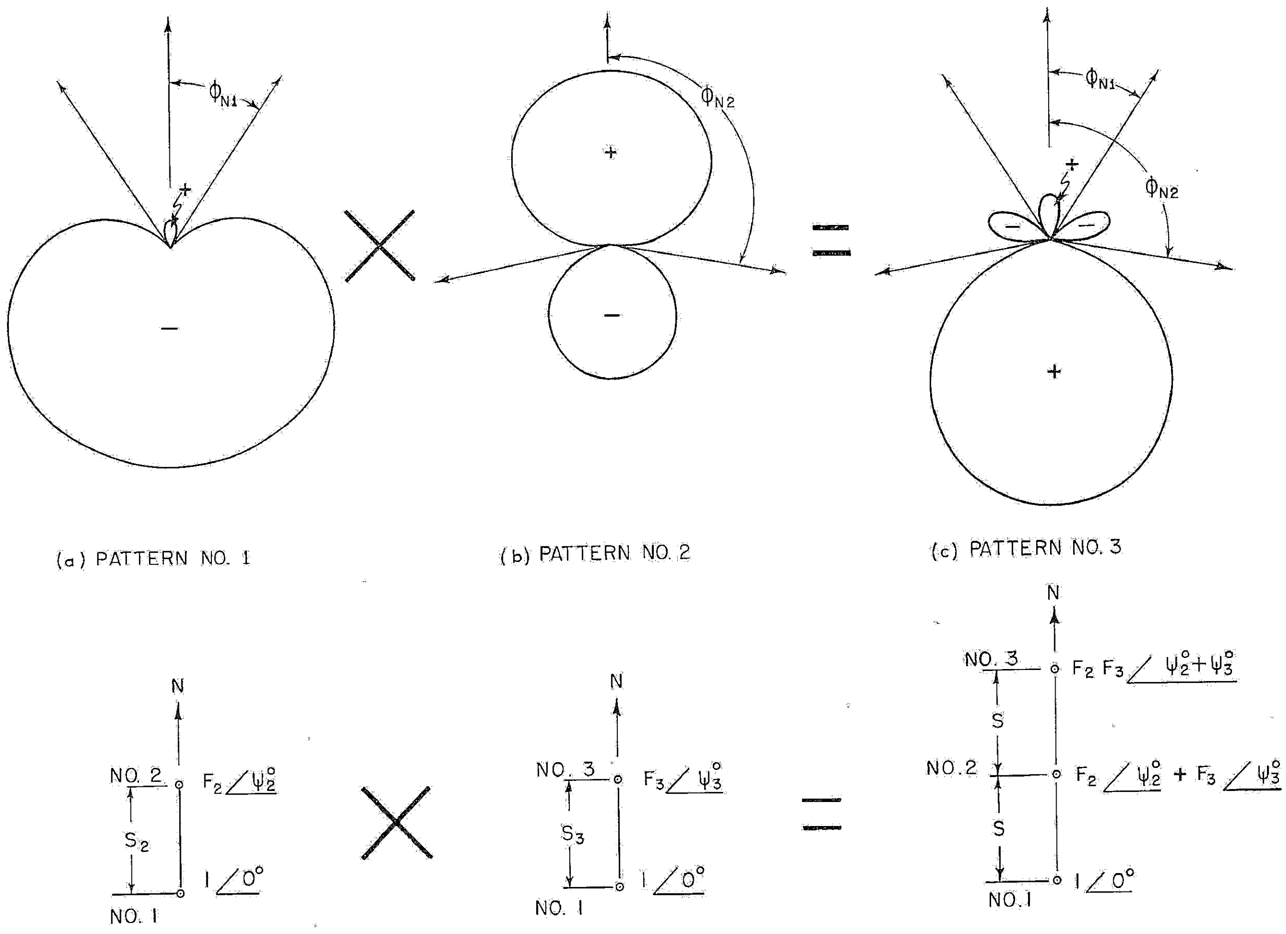
$$E = 2E_1\ f_1(\theta)\ \cos\left(\frac{S_2}{2}\cos\theta\cos\phi + \frac{\Psi_2}{2}\right)$$ ANS.

m. *THREE TOWER IN LINE PATTERNS USING THE MULTIPLICATION FORM.*--If Eq. (40) is used to express pattern No. 1 in Fig. 30 then pattern No. 2 can be expressed by,

$$|E| = E_1 f_1(\theta)\sqrt{2F_3}\sqrt{\frac{1+F_3^2}{2F_3} + \cos(S_3\cos\theta\cos\phi + \Psi_3)} \qquad (42)$$

Now if the radical terms of Eq. (40) and Eq. (42) are multiplied together the multiplied pattern No. 3 of Fig. 30 results, thus,

$$|E| = 2E_1 f_1(\theta)\sqrt{F_2F_3}\sqrt{\left[\frac{1+F_2^2}{2F_2} + \cos(S_2\cos\theta\cos\phi + \Psi_2)\right]\left[\frac{1+F_3^2}{2F_3} + \cos(S_3\cos\theta\cos\phi + \Psi_3)\right]} \qquad (43)$$



MULTIPLICATION OF PATTERNS TO PRODUCE A THREE TOWER IN LINE ARRAY

FIG. 30

This multiplication involved all but the field intensity $E_1 f_1(\theta)$ from the reference tower. $E_1 f_1(\theta)$ is not squared because the shape factors only are multiplied. It will be noted in Fig. 30 that the field intensity of the reference tower is taken as unity, therefore, after performing the pattern multiplication it is necessary to control the pattern size by the field intensity term $E_1 f_1(\theta)$. For the multiplication in Eq. (43) the spacing $S = S_2 = S_3$. Also, it will be noted that the tower numbers have been arranged 1, 2, 3 in Fig. 30(c). This does not mean that tower Nos. 2 and 3 from Fig. 30(a) and (b) are literally moved into these new positions. In the three tower pattern No. 1 tower is the reference tower while towers No. 2 and No. 3 result from the multiplication. Tower No. 2 of the three tower array must produce a field that is the sum of the vector fields of towers No. 2 and No. 3 of the multiplied patterns, while tower No. 3 of the three tower array must produce a field that is the product of the vector fields of towers No. 2 and No. 3 of the multiplied patterns.

EXAMPLE 13: With a three tower in line array having spacings of 90° between towers it is desired to protect radio stations with true azimuth angles of 33.4°, 99.6° and 326.6°. If the field intensity from an end tower is 382 mv/m determine the equation for each pair of towers and then multiply the two patterns to arrive at the final design equation. Plot the pattern for each pair of towers acting alone and then plot the final pattern.

SOLUTION: If the azimuth angle is split between 33.4° and 326.6° it is found that the pair of towers to produce nulls in these azimuth directions must have a true north south bearing. Therefore, the final three tower array will also have this bearing.

Now, from Eq. (33) the phase angle for the first pair of towers is determined as follows,

$$S_2 \cos \phi_N + \psi_2 = \pm 180$$

$$90 \cos 33.4 + \psi_2 = \pm 180$$

$\psi_2 = 105°$ when ϕ_N is measured from true north

And from Eq. (40) the pattern equation for towers No. 1 and 2 is,

$$E_{12} = E_1\sqrt{2}\ \sqrt{1 + \cos\ (S_2 \cos \phi + \psi_2)}$$

$$= 540\sqrt{1 + \cos\ (90 \cos \phi + 105)}$$ ANS

Similarly, from Eq. (33) the phase angle for the second pair of towers is determined as follows,

$$90 \cos 99.6 + \psi_3 = \pm 180$$

$\psi_3 = 195°$ when ϕ_N is measured from true north

And from Eq. (42) the pattern equation for towers No. 1 and 3 is,

$$E_{13} = E_1\sqrt{2}\ \sqrt{1 + \cos\ (S_3 \cos \phi + \psi_3)}$$

$$= 540\sqrt{1 + \cos\ (90 \cos \phi + 195)}$$

Now, if the pattern No. 1 is multiplied by pattern No. 2, only the radical terms are multiplied as shown in Eq. (43) to give the final horizontal pattern equation,

$$E = E_1\ 2\sqrt{[1 + \cos\ (S_2 \cos \phi + \psi_2)] \times [1 + \cos\ (90 \cos \phi + \psi_3)]}$$

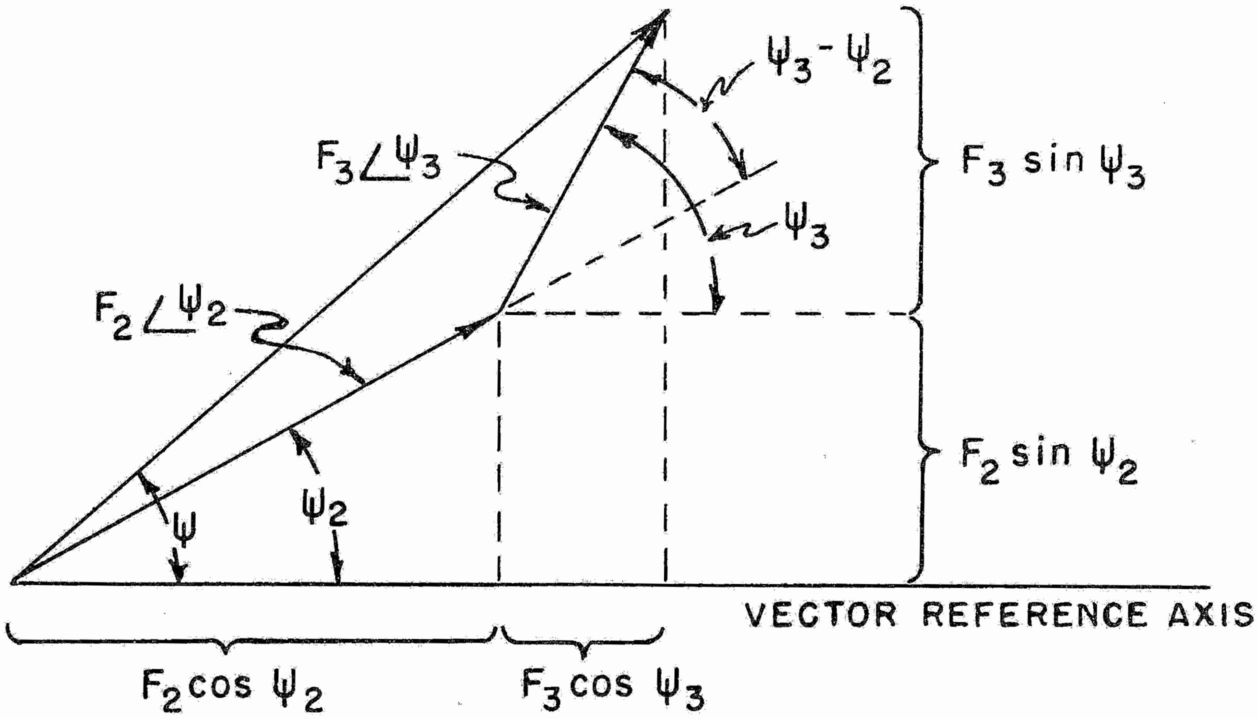
$$= 764\sqrt{[1 + \cos\ (90 \cos \phi + 105)] \times [1 + \cos\ (90 \cos \phi + 195)]}$$ ANS

The pattern for each pair of towers and the final pattern are plotted in Fig. 30. ANS

The sum of the vector field intensities radiated from tower No. 2 can be expressed as a magnitude at an angle. Using the geometry of Fig. 31 we can write,

$$F_2 \underline{|\ \Psi_2} + F_3 \underline{|\ \Psi_3} = \sqrt{2F_2F_3}\sqrt{\frac{F_2^2 + F_3^2}{2F_2F_3} + \cos\ (\Psi_3 - \Psi_2)}\ \underline{/\Psi} \qquad (44)$$

This equation makes it possible to write out the magnitude and phase for No. 2 tower, thus all three field intensity vectors can be expressed as shown in Fig. 32(a). Since the cosine of a positive angle equals the cosine of a negative angle we can write $\cos(\Psi_2 - \Psi_3) = \cos(\Psi_3 - \Psi_2)$ hence the expression for No. 2 tower is correct in Fig. 32(a).



SUM OF VECTOR FIELDS FROM TOWER No. 2

FIG. 31

EXAMPLE 14: Prove Eq. (44).

SOLUTION: Referring to Fig. 31 the vector sum can be written

$$F_2 \underline{|\psi_2} + F_3 \underline{|\psi_3} = \sqrt{(F_2 \cos\psi_2 + F_3 \cos\psi_3)^2 + (F_2 \sin\psi_2 + F_3 \sin\psi_3)^2} \quad \angle \tan^{-1} \frac{F_2 \sin\psi_2 + F_3 \sin\psi_3}{F_2 \cos\psi_2 + F_3 \cos\psi_3}$$

Expanding under the radical

$$F_2 \underline{|\psi_2} + F_3 \underline{|\psi_3} = \sqrt{\begin{matrix} F_2^2 \cos^2\psi_2 + 2F_2F_3 \cos\psi_2 \cos\psi_3 + F_3^2 \cos^2\psi_3 \\ + F_2^2 \sin^2\psi_2 + 2F_2F_3 \sin\psi_2 \sin\psi_3 + F_3 \sin^2\psi_3 \end{matrix}} \quad \underline{|\psi}$$

Since $\cos^2\psi_2 + \sin^2\psi_2 = 1$ and $\cos\psi_2\cos\psi_3 + \sin\psi_2\sin\psi_3 = \cos(\psi_3 - \psi_2)$ the above equation becomes

$$F_2 \underline{|\psi_2} + F_3 \underline{|\psi_3} = \sqrt{F_2^2 + F_3^2 + 2F_2F_3 \cos(\psi_3 - \psi_2)} \quad \underline{|\psi}$$

$$= \sqrt{2F_2F_3} \sqrt{\frac{F_2^2 + F_3^2}{2F_2F_3} + \cos(\psi_3 - \psi_2)} \quad \underline{|\psi} \qquad \underline{\text{ANS}}.$$

where

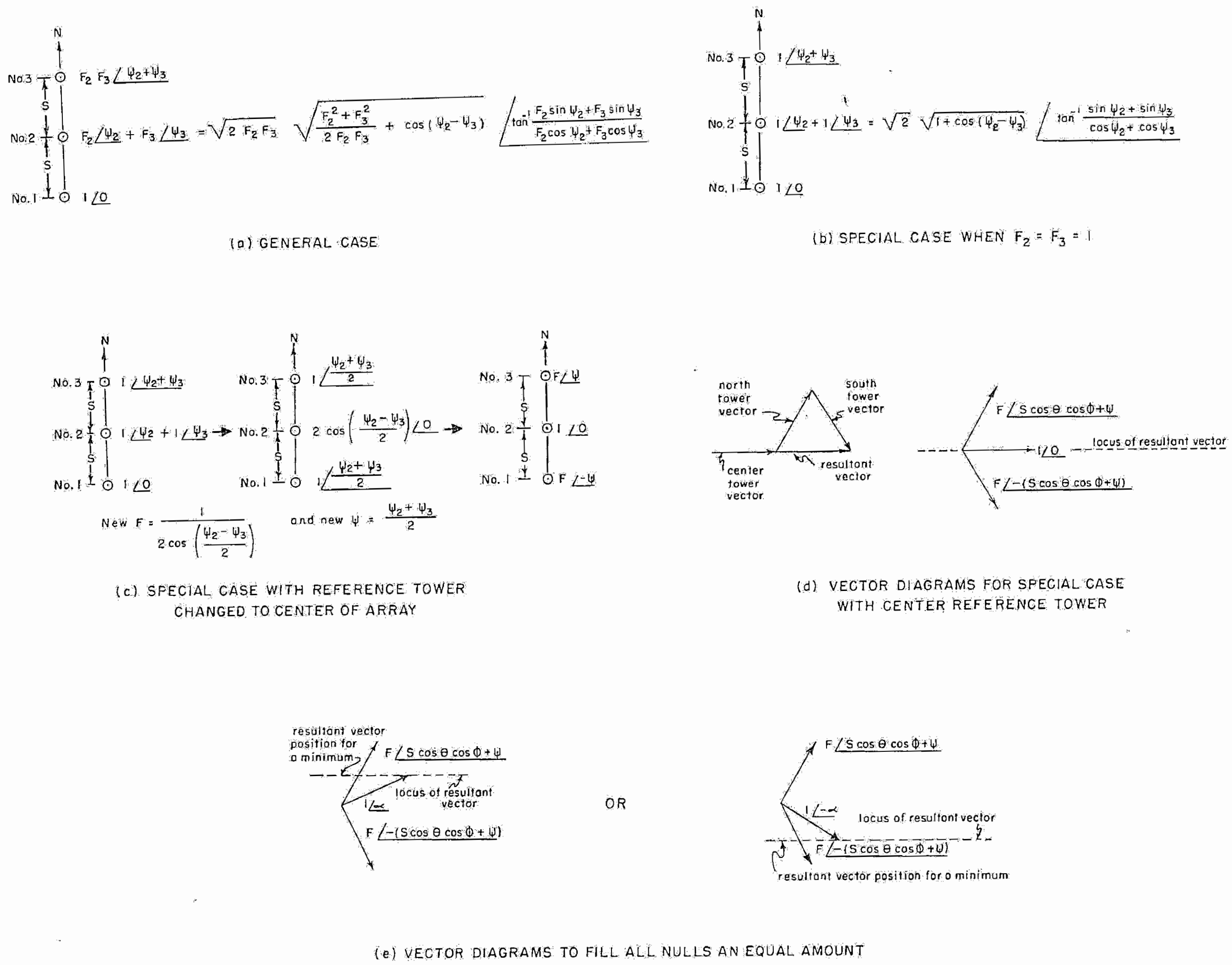
$$\Psi = \tan^{-1} \frac{F_2 \sin \Psi_2 + F_3 \sin \Psi_3}{F_2 \cos \Psi_2 + F_3 \cos \Psi_3} \qquad \text{ANS.}$$

Equation (43) is the general equation for multiplying patterns for a three tower in line array. If the field intensity ratios F_2 and F_3 are made unity a special case of interest results as shown in Fig. 32(b). For this special case Eq. (43) simplifies to read,

$$\left| F \right| = 2E_1 f_1(\theta) \sqrt{\left[1 + \cos(S \cos\theta \cos\phi + \Psi_2)\right] \left[1 + \cos(S \cos\theta \cos\phi + \Psi_3)\right]} \qquad (45)$$

where all three towers are of the same height.

For this special case it is convenient to change the reference tower to the center of the array as shown in Fig. 32(c). This was accomplished by subtracting the phase angle $\frac{\Psi_2 + \Psi_3}{2}$ from each tower field intensity vector. The pattern is not altered by performing this operation with the result that the center tower is now the reference tower with zero phase angle. Design Eq. (44) is still valid.



VECTOR FIELDS FOR THREE TOWER IN LINE ARRAYS USING THE MULTIPLICATION FORM

FIG. 32

Another simplification can be made by changing all of the field magnitudes such that the center tower will have a unit field intensity magnitude. For this condition design Eq. (43) can be simplified to read,

$$\left|E\right| = 2FEf(\theta) \left[\frac{1}{2\ F} + \cos(S \cos\theta \cos\phi + \psi)\right] \quad (46)$$

where the tower arrangement and parameters are shown in Fig. 32(c). This equation is for towers of equal height. The center tower field intensity is E $f(\theta)$. The reason this design equation is so simple is that the resultant vector always lies along the voltage reference axis. Fig. 32(d) shows how the three vectors add to produce this resultant.

EXAMPLE 15: A three tower in line array has a spacing of 135° between each tower and the array has a true north-south bearing. Write the horizontal pattern equation if stations are to be protected at azimuth angles of 10 and 30 degrees and the field from No. 1 tower is 197 mv/m.

SOLUTION: From Eq. (33) the phasing for a null at 10 degrees is

$$\psi_2 = +180 - 135 \cos 10 = 47^\circ$$

and at 30°

$$\psi_3 = +180 - 135 \cos 30 = 63^\circ$$

Applying the formulas for the special case as illustrated in Fig. 32(c),

$$F = \frac{1}{2\cos\left(\frac{\psi_2 - \psi_3}{2}\right)} = \frac{1}{2\cos\left(\frac{47 - 63}{2}\right)} = 0.505$$

$$\psi = \frac{\psi_2 + \psi_3}{2} = \frac{47 + 63}{2} = 55$$

The horizontal pattern equation by substituting in Eq. (46) is,

$$\left|E\right| = (2)(0.505)(197) \left[\frac{1}{(2)\ (0.505)} + \cos(135 \cos\phi + 55)\right]$$

$$= 199 \left[0.99 + \cos(135 \cos\phi + 55)\right] \qquad \text{ANS.}$$

n. *NULL FILLING USING THE MULTIPLICATION FORM.*--If it is desired to fill all nulls an equal amount the phase of the center tower can be shifted from 0° by the angle α as shown in Fig. 32(e). When the reference vector phase is shifted in this manner the resultant vector traverses a locus parallel to the reference vector axis and thus the resultant vector magnitude will never drop to zero. The magnitude of the minimums is proportional to sin α. The minimums will occur when the resultant vector is at right angles to the vector reference axis. For this condition the magnitude of the minimums can be written,

$$\left|E\right|_{min} = E_2\ f(\theta)\ \sin\alpha \quad (47)$$

where

α = the phase angle of the center tower reference vector, and the other values are as defined above.

E_2 = the field of the center tower, mv/m.

When it is desired to fill only one set of nulls, say $\pm\phi_{N1}$ in Fig. 30, then the field ratio F_2 must be made different than unity in the design Eq. (43). If at the same time it is desired to retain nulls $\pm\phi_{N2}$ then the field ratio F_3 must be made unity. If the field intensity does not drop to zero the value is called a minimum rather than a null. Strictly speaking a null is a zero value only and a minimum is a low value that approaches a null value.

EXAMPLE 16: It is desired to fill the horizontal nulls to a minimum of 20 mv/m in a three tower in line array where, $E_1 = 197\angle 0^\circ$, $E_2 = 390\ \angle 55^\circ$ and $E_3 = 197\angle 110^\circ$ and the spacing between consecutive towers is 135°.

Determine α and give the approximate and exact design equations.

SOLUTION: From Eq. (47) and for the horizontal plane we have,

$$/E/_{min} = E_2\ f(\theta)\ \sin\alpha$$

$$20 = 390\ (1)\ \sin\alpha$$

$$\sin\alpha = \frac{20}{390} = 0.05128$$

$$\alpha = 2.94^\circ$$

ANS

Making the center tower No. 2 the reference tower, by Eq. (46) with complete nulls we have,

$$/E/ = 199\left[0.99 + \cos\ (135\ \cos\ \emptyset + 55)\right]$$

Now if the phase of the center tower is shifted $\pm\alpha = \pm\ 2.94^\circ$ we can write approximately

$$E = 199\left[0.99 + \cos\ (135\ \cos\ \emptyset + 55)\right] + j\ 20 \quad \text{ANS}$$

The approximation is that $\pm$ j 20 is added to $/E_2/ = 390$ making $E_2 = 390 \pm j\,20$ rather than the exact solution which is

$$E_2 = 390\ \varepsilon^{\pm j\alpha} = 390\underline{/\pm j\alpha}$$

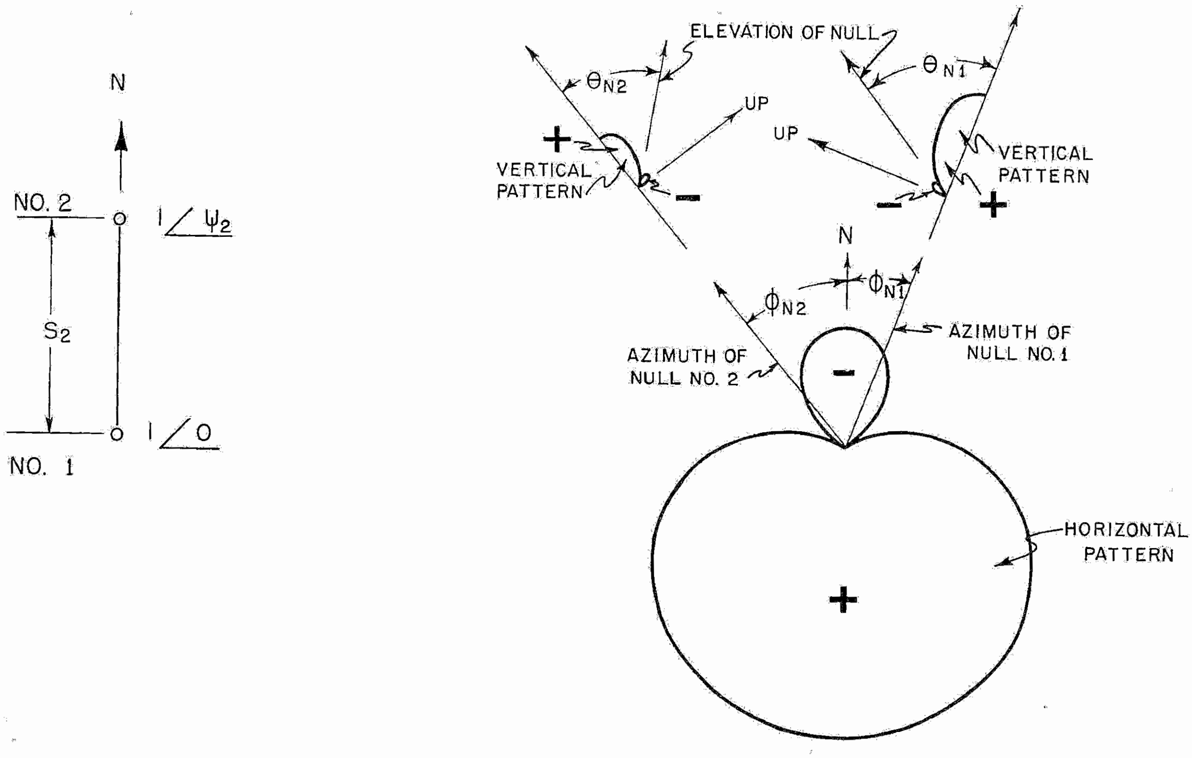
$$= 389.49 \pm j\ 20$$

which would give an exact solution of

$$E = 199\left[0.99 + \cos\ (135\ \cos\ \emptyset + 55)\right] - 0.51 + j20 \quad \text{ANS}$$

o. *DIRECTION OF VERTICAL NULLS FOR TWO TOWER PATTERNS.*--If a null is to be produced in the vertical plane then the space phasing term must also include the elevation angle. In equation form we can write for null No. 1

$$S_2\ \cos\phi_{N1}\ \cos\phi_{N1} + \Psi_2 = \pm\ 180 \qquad (48)$$



(a) TOWER PLACEMENT

(b) HORIZONTAL AND VERTICAL PATTERNS

AZIMUTH AND ELEVATION ANGLES TO LOCATE VERTICAL NULLS

FIG. 33

where S_2 = spacing from No. 1 to No. 2 tower, degrees

ϕ_{N1} = azimuth angle of null No. 1 from line of towers, degrees

θ_{N1} = elevation angle of null No. 1, degrees

Ψ_2 = phase of No. 2 tower with respect to No. 1 tower, degrees

In a similar fashion the equation for null No. 2 can be written

$$S_2 \cos\phi_{N2} \cos\theta_{N2} + \Psi_2 = \pm 180 \tag{49}$$

where the terms are similar to those in Eq. (48) except that the values are for vertical null No. 2.

EXAMPLE 17: A two tower directional antenna array has the following parameters:

No	G°	Ø°	S°	ψ°	E	F
1	90	0	0	0	200	1
2	90	0	90	135	200	1

Determine the azimuth angles of the horizontal nulls, the elevation angle of the vertical nulls for azimuth angles of 20° and 320°. Also, plot the horizontal pattern and vertical patterns for the above azimuth angles of 20° and 320°.

SOLUTION: By Eq. (48), in the horizontal plane

$$S_2 \cos Ø_{N1} \cos\theta_{N1} + \psi_2 = \pm 180$$

$$90 \cos Ø_{N1} + 135 = \pm 180$$

$Ø_{N1} = 60°$ (using the + sign) ANS

$= 300°$ (using the - sign) ANS

For an azimuth angle of 20° the vertical null is determined by Eq. (48), thus,

$90 \cos 20 \cos \theta_{N1} + 135 = \pm 180$

$\theta_{N1} = 58°$ (using the + sign) ANS

For an azimuth angle of 300° the vertical null is determined by Eq. (49), thus,

$90 \cos 300 \cos \theta_{N2} + 135 = \pm 180$

$\theta_{N2} = 49°$ (using the + sign) ANS

The desired horizontal and vertical patterns are plotted in Fig. 33. ANS

It is sometimes desirable to specify two vertical nulls at different bearings when using a two tower array. For illustration it may be desired to protect two stations having an azimuth angle ϕ_{12} between the two stations. For this case the azimuth angle of the two vertical nulls must add to give this angle as shown in Fig. 33, thus

$$\phi_{12} = \phi_{N1} + \phi_{N2} \tag{50}$$

Since the distance to the stations to be protected determines the elevation angle the values θ_1 and θ_2 can be specified. Of course it may be necessary to give protection over a vertical and also a horizontal angle. In such cases an estimate is made and the pattern fit is determined. If the pattern does not quite fit then the misfit will usually indicate what to try next.

If for example + signs are used in Eq. (48) and (49) then by subtracting Eq. (49) from Eq. (48) the spacing, phase, and constant 180° can be eliminated to get

$$\cos\phi_{N1} \cos\theta_{N1} = \cos\phi_{N2} \cos\theta_{N2}$$

Substituting for ϕ_{N2} from Eq. (24) we can write

$$\cos\phi_{N1} = \cos(\phi_{12} - \phi_{N1}) \frac{\cos\theta_{N2}}{\cos\theta_{N1}}$$

This equation can be rearranged to give

$$\tan\phi_{N1} = \frac{\cos\theta_{N1} - \cos\theta_{N2} \cos\phi_{12}}{\cos\theta_{N2} \sin\phi_{12}} \tag{51}$$

where

ϕ_{N1} = azimuth angle from line of towers to null No. 1, which is the value to be determined, and the other values, as defined above are known. This design equation makes it possible to determine the true azimuth of the line of towers when two radio stations are to be protected in vertical nulls.

When this method is used to fix vertical nulls for two towers it is possible to use the multiplication method and protect other stations with another two towers, the pattern of which is to be multiplied by the first pattern. Since the multiplication method is being used a null with one set of towers will not be altered if the pattern is multiplied by the pattern of another set of towers. This is true because a null or zero value multiplied by some value is still a null or zero value. This fact makes the multiplication method very attractive to the directional antenna design engineer.

EXAMPLE 18: The vertical elevation of a null to protect station No. 1 is 20°, the vertical elevation of a null to protect station No. 2 is 30° and the azimuth angle between the two stations is 90°. What is the azimuth angle from the line of towers toward station No. 1?

SOLUTION: Applying Eq. (51) we have

$$\tan \phi_{N1} = \frac{\cos 20 - \cos 30 \cos 90}{\cos 30 \sin 90} = 1.083$$

$$\phi_{N1} = 47.3^\circ$$

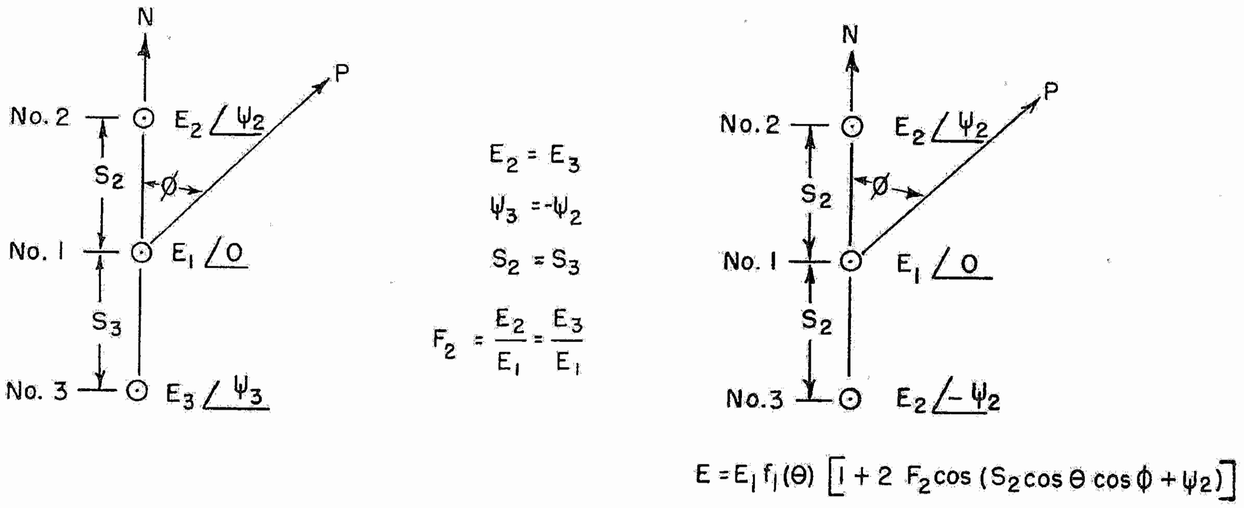
ANS.

p. *THREE TOWER IN LINE PATTERNS USING THE ADDITION FORM.*--Referring back to Fig. 24 a three tower in line array can be obtained by placing another tower at the space reference point as shown in Fig. 34. For this special condition the design equation is

$$\left| E \right| = E_1 f_1(\theta) \left[1 + 2 F_2 \cos (S_2 \cos \theta \cos \phi + \psi_2) \right] \quad (52)$$

where the values are as defined above and specified in Fig. 34(c).

In this case towers No. 2 and No. 3 must be of the same height. If tower No. 1 is different in height F_2 must be determined from Eq. (38). In Eq. (52) the non-directional pattern of the center tower is represented by number one the first term inside the bracket. $E_1 f_1(\theta)$ multiplied by this term gives the horizontal pattern as shown in Fig. 35(a). The second term within the bracket gives the pattern for No. 2 and No. 3 towers as shown in Fig. 32(b). This term is the same as Eq. (31)



(a) Original (b) Required equality. (c) Final values and equation.

THREE TOWER PLACEMENT FOR ADDITION METHOD

FIG. 34

except that f (θ) is multiplied by the magnitude term and the cos θ term is multiplied by the space phasing term in the angle. Also, the spacing is S_2 instead of $S_2/2$ and the phasing is Ψ_2 instead of $\Psi_2/2$. Eq. (31) is for the horizontal pattern while Eq. (52) gives the pattern for the hemisphere.

In Fig. 35 it will be noted that the vector which produces pattern No. 1 is positive and lies along the reference axis. For pattern No. 2 the sum of the two vectors for the end towers produces a positive vector which also lies along the reference axis for the positive lobe of the pattern and lies along the negative reference axis for the negative lobe of the pattern. The sum of these two pattern vectors gives the resultant addition pattern. It will be noted that both vectors lie along the vector reference axis. If it is desired to fill all nulls an equal amount the time phase of No. 1 tower can be shifted in accordance with Eq. (47).

EXAMPLE 19: When the antenna parameters in Fig. 34 are $E_1 = 63.5$ mv/m, $F_2 = F_3 = 1$, $S_2 = S_3 = 90$, and $\psi_2 = -\psi_3 = 90$ write out the horizontal pattern equation for the end towers acting alone and plot the pattern. Then add the center tower, write out the horizontal pattern equation for the 3 towers and plot the pattern.

SOLUTION: By Eq. (52) for the end towers only we have,

$$E = 63.5 \left[2 \cos (90 \cos \phi + 90)\right]$$

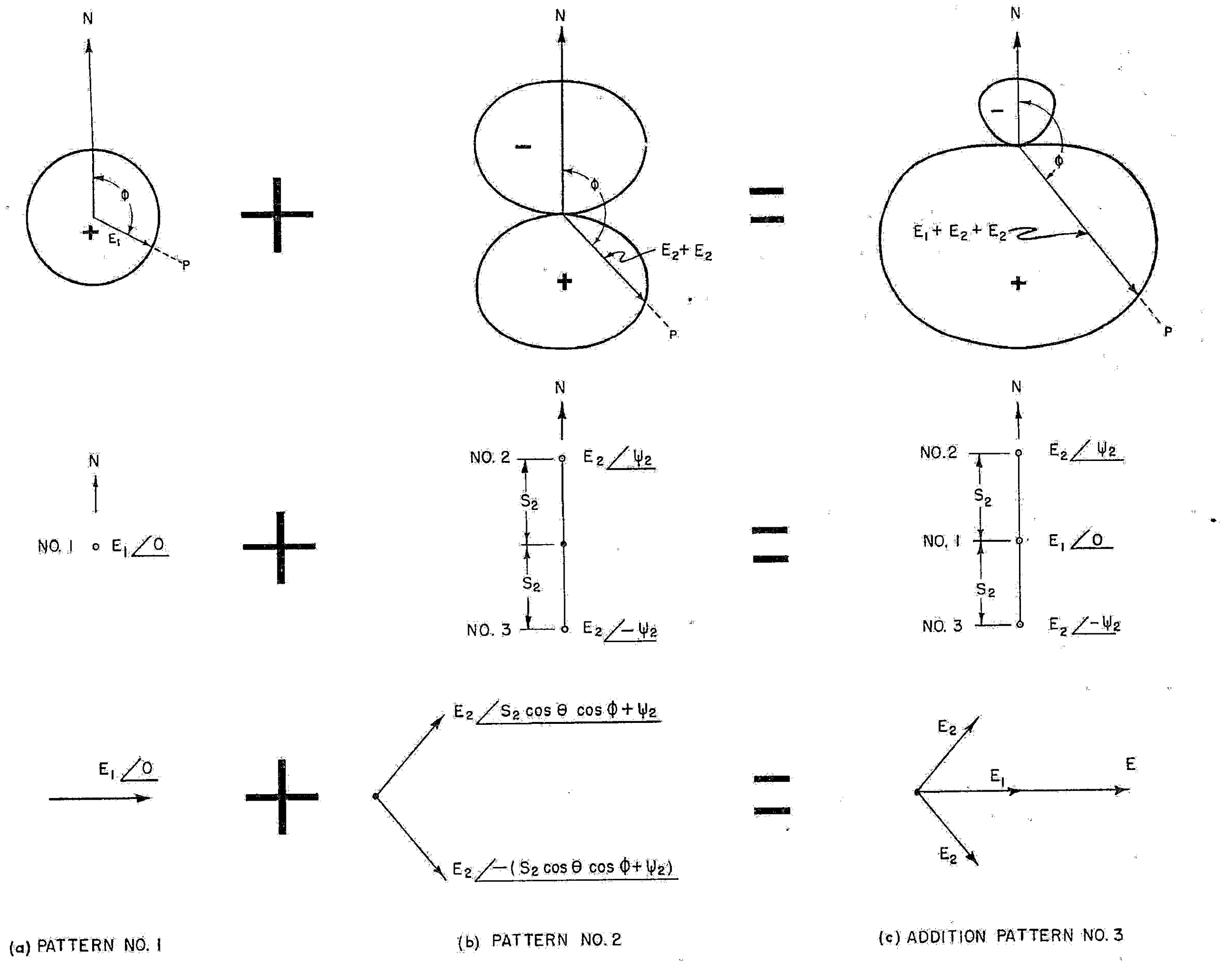
$$= 127 \cos (90 \cos \phi + 90) \qquad \text{ANS}$$

This pattern is plotted in Fig. 35 (b) **ANS**

By Eq. (52) for all three towers we have

$$E = 63.5 \left[1 + 2 \cos (90 \cos \phi + 90)\right] \qquad \text{ANS}$$

And the pattern is plotted in Fig. 35 (c) **ANS**



ADDITION OF PATTERNS TO PRODUCE A THREE TOWER IN LINE ARRAY

FIG. 35

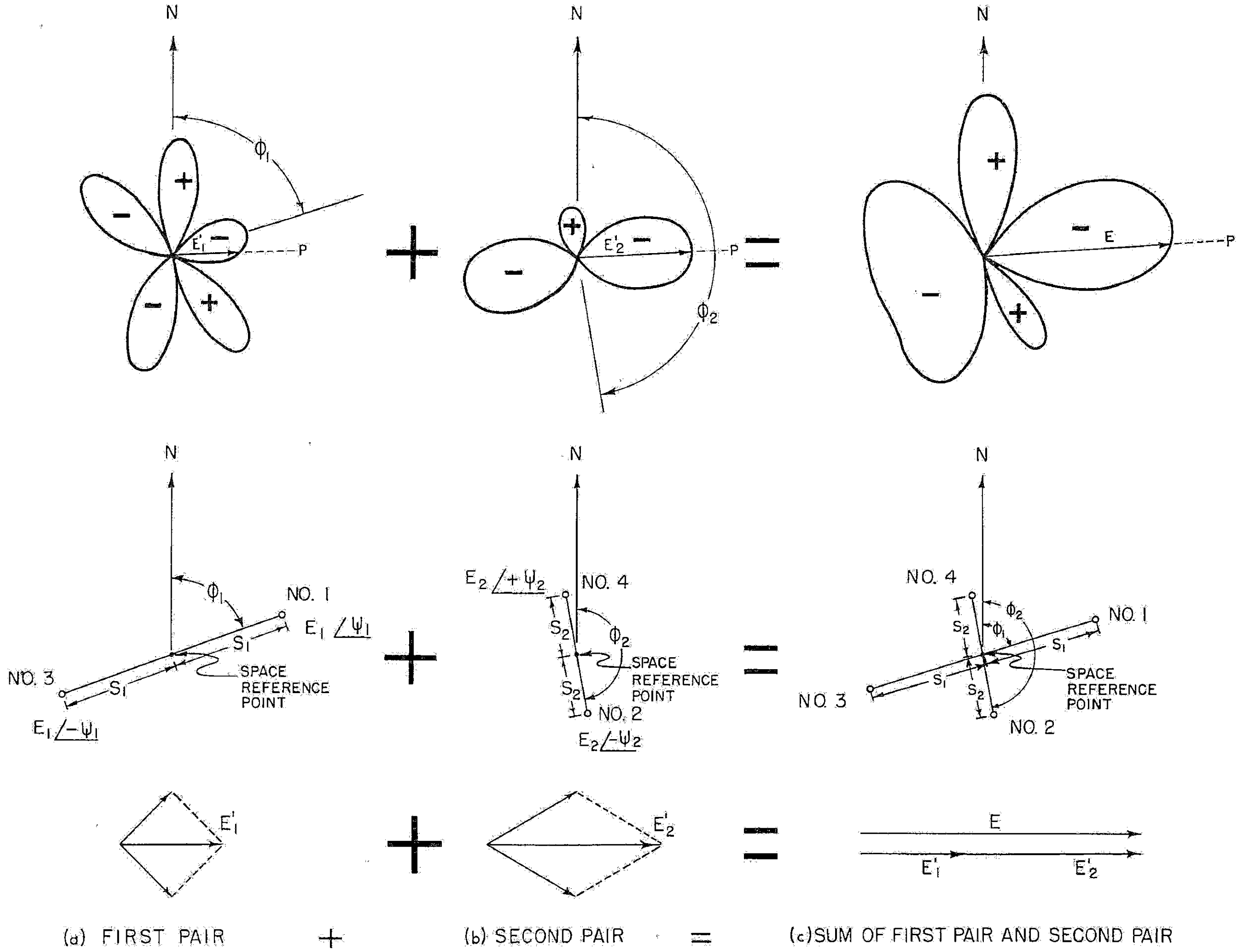
q. *FOUR TOWER PARALLELOGRAM ARRAY USING THE ADDITION FORM*.--The addition method in some respects is quite simple. With this method it is possible to lay one pattern on top of another pattern and make the required addition to produce the sum pattern. In general, to do this requires that the line of towers have different azimuth angles. Consider that towers No. 1 and No. 3 lie along a line making an angle ϕ_1 with respect to the reference axis as shown in Fig. 36(a). Similarly, place towers No. 2 and No. 4 along a line making an angle ϕ_2 with respect to the reference axis as shown in Fig. 36(b).

For the first pair of towers in Fig. 36(a) we can write,

$$E_1' = 2E_1 f_1(\theta) \cos \left[S_1 \cos \theta \cos(\phi_1 - \phi) + \Psi_1 \right] \tag{53}$$

and for the second pair of towers in Fig. 36(b) we can write,

$$E_2' = 2E_2 f_2(\theta) \cos \left[S_2 \cos \theta \cos(\phi_2 - \phi) + \Psi_2 \right] \tag{54}$$



ADDITION OF PATTERNS TO PRODUCE A FOUR TOWER PARALLELOGRAM ARRAY

FIG. 36

where

ϕ_1 = the azimuth orientation of the first pair of towers with respect to true north

ϕ_2 = the azimuth orientation of the second pair of towers with respect to true north

E_1' = field intensity from first pair of towers

E_2' = field intensity from second pair of towers

and the other values are as specified in Fig. 36. The total field intensity from the parallelogram array is then the *sum*,

$$\begin{aligned} E &= E_1' + E_2' \\ &= 2E_1 f_1(\theta) \cos \left[S_1 \cos\theta \cos(\phi_1 - \phi) + \psi_1 \right] \\ &\quad + 2E_2 f_2(\theta) \cos \left[S_2 \cos\theta \cos(\phi_2 - \phi) + \psi_2 \right] \end{aligned} \tag{55}$$

In this equation towers No. 1 and No. 3 must have the same height and towers No. 2 and No. 4 must have the same height because of the required symmetry to make the resultant vectors traverse the x-axis or reference axis. Since the resultant vectors of each pair is simply a positive or negative quantity the sum vector E must also be a simple positive or negative quantity.

EXAMPLE 20: A pair of towers having a spacing of 404° are arranged such that No. 1 tower has a true orientation of 73° with respect to No. 3 tower and a time phasing of -140° with respect to No. 3 tower. Another pair of towers are spaced 210° and arranged such that No. 2 tower has a true orientation of 170° with respect to No. 4 tower and a time phasing of 340° with respect to No. 4 tower. If the field intensity from each tower is unity, write out the addition form of the horizontal pattern design equation for each pair of towers acting alone and then give the design equation when the patterns are added together. Plot the pattern for each pair of towers acting alone and finally the pattern of the total field intensity of all four towers acting together. Also, show the arrangement for each pair of towers and the final arrangement of the four towers.

SOLUTION: From Eq. (53) the design equation for the first pair of towers No. 1 and No. 3 is

$$E_1' = 2 \cos \left[\frac{404}{2} \cos(73-\phi) - \frac{140}{2} \right]$$

$$= 2 \cos \left[202 \cos(73-\phi) - 70 \right] \qquad \underline{\text{ANS}}$$

And from Eq. (54) the design equation for the second pair of towers No. 2 and No. 4 is

$$E_2' = 2 \cos \left[\frac{210}{2} \cos(170-\phi) + \frac{340}{2} \right]$$

$$= 2 \cos \left[105 \cos(170-\phi) + 170 \right] \qquad \underline{\text{ANS}}$$

Finally, the design equation for the four tower array by Eq. (55) is

$$\begin{aligned} E &= E_1' + E_2' \\ &\quad 2 \cos \left[202 \cos(73-\phi) - 70 \right] \\ &\quad + 2 \cos \left[105 \cos(170-\phi) + 170 \right] \end{aligned} \qquad \underline{\text{ANS}}$$

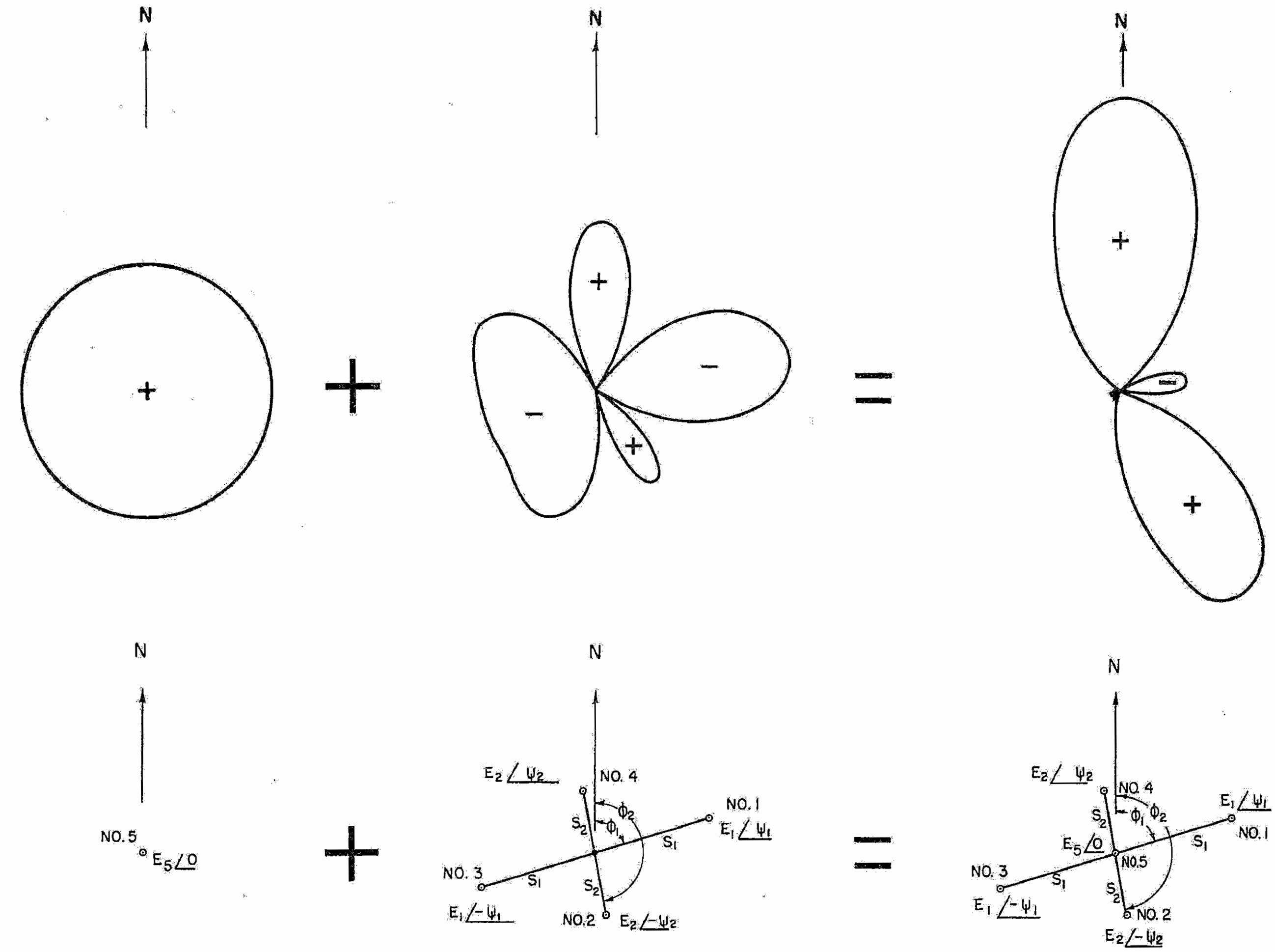
The required patterns along with the tower arrangements are shown in Fig. 36. ANS

r. *MULTI-TOWER ARRAYS USING THE ADDITION FORM*.--First let us consider a five tower array consisting of a parallelogram of four towers and a fifth tower at the space reference point. For this condition we can immediately write down the design equation which is the sum of the field from the center tower plus the sum of the fields from the other two pairs of towers, thus

$$\begin{aligned} E &= E_5 f_5(\theta) \\ &\quad + 2\,E_1 f_1(\theta) \cos \left[S_1 \cos\theta \cos(\phi_1 - \phi) + \psi_1 \right] \\ &\quad + 2\,E_2 f_2(\theta) \cos \left[S_2 \cos\theta \cos(\phi_2 - \phi) + \psi_2 \right] \end{aligned} \tag{56}$$

where the terms are defined as shown in Fig. 37

From the inductive steps taken in dealing with the addition form it should now be evident that more pairs of towers can be added by increasing the terms in Eq. (56).



(a) REFERENCE TOWER + (b) 2 PAIRS OF TOWERS = (c) SUM OF REFERENCE TOWER AND 2 PAIRS OF TOWERS

ADDITION OF PATTERNS TO PRODUCE A FIVE TOWER DIRECTIONAL ARRAY

FIG. 37

<u>EXAMPLE 21:</u> To the four tower array of Example 20 add a fifth tower at the space reference point which has a magnitude of 2.1 and zero time phasing. Write out the horizontal pattern design equation. Plot the single tower pattern, the four tower pattern and the five tower pattern. Show the tower arrangement.

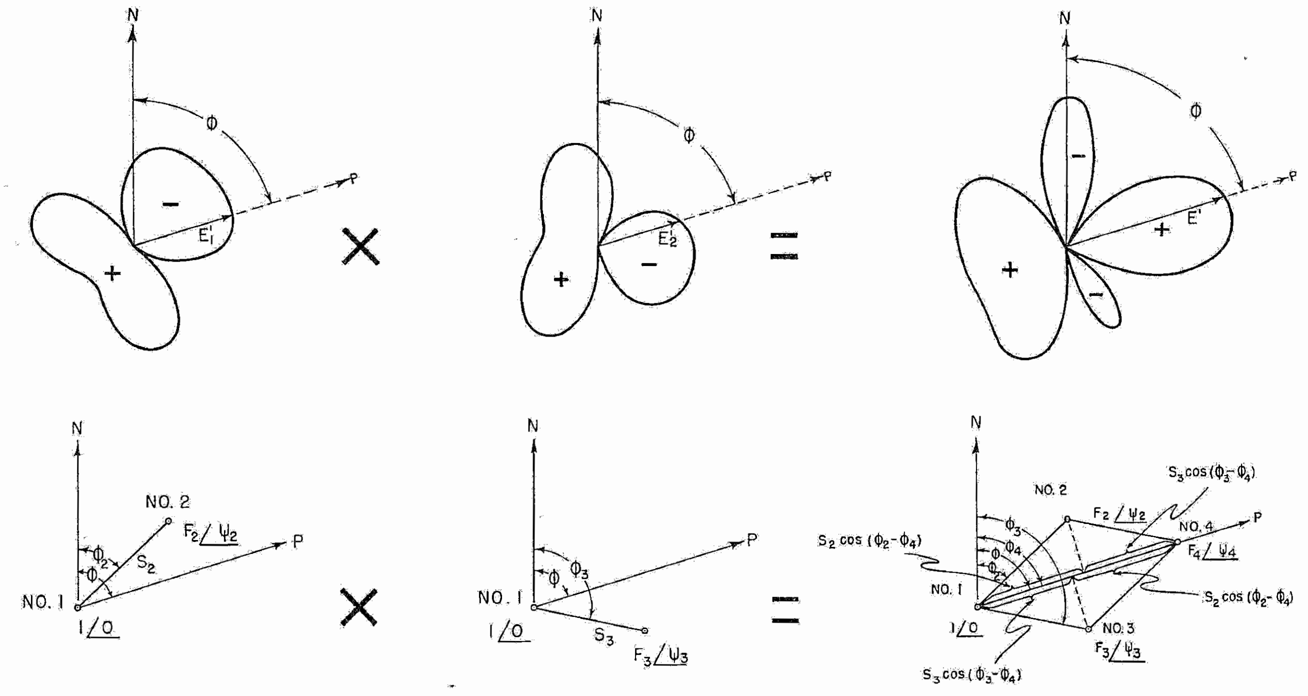
<u>SOLUTION:</u> From Eq. (56) and the answer in Example (20) we can write,

$$E = 2.1 + 2 \cos \left[202 \cos (73-\phi) - 70\right]$$

$$+ 2 \cos \left[105 \cos (170-\phi) + 170\right] \qquad \underline{\text{ANS}}$$

The required patterns and tower arrangement are shown in Fig. 37 <u>ANS</u>

s. *FOUR TOWER PARALLELOGRAM ARRAY USING THE MULTIPLICATION FORM.*--The multiplication of two patterns which results in a four tower parallelogram array is one of the most useful tools in designing a directional antenna which must afford protection to a number of radio stations. The three tower in line array is actually a special case of the four tower parallelogram array. When the spacings are made equal and the azimuth angles of the tower pairs are identical the four tower parallelogram becomes a three tower in line array. In this case the center tower performs the function of two towers.



(a) 1ST PAIR OF TOWERS (b) 2ND PAIR OF TOWERS (c) 4 TOWER PARALLELOGRAM ARRAY

MULTIPLICATION OF PATTERNS TO PRODUCE A FOUR TOWER PARALLELOGRAM ARRAY

FIG. 38

In order to better understand the parallelogram array consider the multiplication applied to two pairs of towers as shown in Fig. 38. The normalized field (reference tower has unity as its field intensity) for the first pair of towers can be written,

$$E_1' = 1\angle 0 + F_2\angle \beta_2 \tag{57}$$

And the normalized field for the second pair of towers can be written

$$E_2' = 1\angle 0 + F_3\angle \beta_3 \tag{58}$$

By multiplying these two patterns together we get the total normalized field

$$\begin{aligned} E' &= E_1' E_2' = 1\angle 0 + F_2\angle \beta_2 + F_3\angle \beta_3 + F_2F_3\angle \beta_2 + \beta_3 \\ &= 1\angle 0 + F_2\angle \beta_2 + F_3\angle \beta_3 + F_4\angle \beta_4 \\ &\text{where } F_2F_3 = F_4 \text{ and } \beta_2 + \beta_3 = \beta_4 \end{aligned} \tag{59}$$

This multiplication shows that the multiplication of two pairs of towers results in a four tower array. No. 1 tower is located at the space reference point, No. 2 tower is located where No. 2 tower of the first pair of towers is located with respect to the reference tower, No. 3 tower is located where No. 3 tower of the second pair of towers is located with respect to the reference tower of that pair, and the location of No. 4 tower is yet to be determined. Its location is controlled by the space phasing terms in β_4 which in turn is equal to the sum of the space phasing and time phasing terms of β_2 and β_3. This means that the sum of the space phasing terms No. 2 and No. 3 towers must equal the space phasing term of No. 4 tower, that is,

$$S_2 \cos(\phi_2 - \phi) + S_3 \cos(\phi_3 - \phi) = S_4 \cos(\phi_4 - \phi) \tag{60}$$

as shown in Fig. 39 and the sum of the time phasing terms of towers No. 2 and No. 3 must equal the time phasing term of tower No. 4, thus

$$\psi_2 + \psi_3 = \psi_4 \tag{61}$$

If the multiplication of the patterns is to hold, tower No. 4 must be placed such that Eq. (60) is satisfied. Consider one case by making $\phi = \phi_4$ and place No. 4 tower at a point to complete the parallelogram. For these conditions we have Eq. (60) reduce to

$$S_2 \cos(\phi_2 - \phi_4) + S_3 \cos(\phi_3 - \phi_4) = S_4 \qquad (62)$$

An inspection of the parallelogram in Fig. 38(c) reveals that this equation is true if the perpendicular lines are drawn from towers No. 2 and No. 3 to the line joining No. 1 and No. 4 towers.

Although this is not as rigorous a proof as could be furnished it is hoped that the geometrical picture makes the results more realistic.

Equation (43) can be generalized for the parallelogram array by simply orienting each pair of towers appropriately, thus,

$$E = 2E_1 f_1(\theta)\sqrt{F_2F_3}\sqrt{\left[\frac{1+F_2^2}{2F_2} + \cos\left\{S_2 \cos\theta \cos(\phi_2 - \phi) + \Psi_2\right\}\right] \times \left[\frac{1+F_3^2}{2F_3} + \cos\left\{S_3 \cos\theta \cos(\phi_3 - \phi) + \Psi_3\right\}\right]} \qquad (63)$$

where

E = magnitude of the resultant field, mv/m
$E_1 f_1(\theta)$ = field intensity from reference tower, mv/m
$F_2 = \frac{E_2}{E_1}$ ratio of field intensities of No. 2 and No. 1 towers
$F_3 = \frac{E_3}{E_1}$ ratio of field intensities of No. 3 and No. 1 towers
S_2 = spacing of No. 2 tower from No. 1 tower, degrees
θ = elevation angle of observation point, degrees
ϕ_2 = azimuth angle of tower No. 2 with respect to tower No. 1 degrees
ϕ = azimuth angle of observation point, degrees
Ψ_2 = time phase of No. 2 tower with respect to No. 1 tower, degrees
S_3 = spacing of No. 3 tower from No. 1 tower, degrees
ϕ_3 = azimuth angle of No. 3 tower with respect to No. 1 tower, degrees
Ψ_3 = time phase of No. 3 tower with respect to No. 1 tower, degrees

Although this equation does not specify No. 4 tower it is necessary to have No. 4 tower placed such that it will complete the parallelogram. Its field must be equal to

$$E_4 = E_1F_2F_3 \qquad (64)$$

and its time phase must be

$$\Psi_4 = \Psi_2 + \Psi_3 \qquad (65)$$

For the special case of equal field intensities from each tower Eq. (63) reduces to

$$E = 4E_1 f_1(\theta) \cos\left\{\frac{S_2}{2} \cos\theta \cos(\phi_2 - \phi) + \frac{\Psi_2}{2}\right\} \cos\left\{\frac{S_3}{2} \cos\theta \cos(\phi_3 - \phi) + \frac{\Psi_3}{2}\right\} \qquad (66)$$

EXAMPLE 22: The first pair of towers No. 1 and No. 2 have the parameters, $\emptyset_2 = 47.10°$, $S_2 = 238.74°$ and $\psi_2 = 240°$. The second pair of towers No. 1 and No. 3 have the parameters, $\emptyset_2 = 101.8°$, $S_3 = 216.01°$ and $\psi_3 = 100°$. Tower No. 1 is at the space reference point for both pairs of towers. For unity field intensity give the design equation for each pair of towers acting alone and then give the design equation for the multiplied patterns. Finally, plot the patterns for each pair of towers acting alone and then plot the pattern for the four tower multiplied pattern. Also show the tower placement plans.

SOLUTION: Substituting in Eq. (40), after modification for the orientation of the array, for the first pair of towers results in

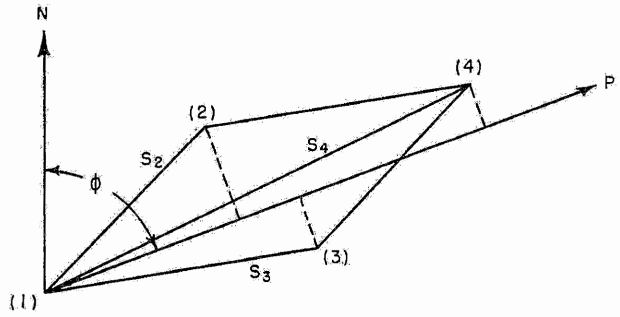


DIAGRAM TO ILLUSTRATE Eq. (60)

FIG. 39

$$E_{12} = E_1\ f_1(\theta)\sqrt{2\ F_2}$$

$$X\sqrt{\frac{1 + F_2^2}{2\ F_2} + \cos\ S_2\ \cos\ \theta\ \cos\ (\emptyset_2 - \emptyset)\ + \psi_2}$$

For the horizontal pattern

$$E_{12} = E_1\ \sqrt{2}$$

$$X\sqrt{1 + \cos\ [S_2\ \cos\ (\emptyset_2 - \emptyset)\ + \psi_2]}$$

$$= 2\ E_1\ \cos\ \left[\frac{S_2}{2}\ \cos\ (\emptyset_2 - \emptyset)\ + \frac{\psi_2}{2}\right]$$

And for the parameters specified

$E_{12} = 2\ \cos\ [119.37^\circ\ \cos\ (47.1^\circ - \emptyset) + 60^\circ]$ ANS

In a similar fashion by Eq. (42) for the second pair of towers

$E_{13} = 2\ \cos\ [108^\circ\ \cos\ (101.8^\circ - \emptyset) + 50^\circ]$ ANS

The design equation for the multiplied pattern by Eq. (66) is,

$$E = E_{12}\ X\ E_{13}$$

$$= 4\ \cos\ [119.37^\circ\ \cos\ (47.1^\circ - \emptyset) + 120^\circ]$$

$$X\ \cos\ [108^\circ \cos\ (101.8^\circ - \emptyset) + 50^\circ]$$ ANS

The required patterns and tower placement plans are shown in Fig. 38 ANS

t. *SIX TOWER PARALLELOGRAM ARRAY USING THE MULTIPLICATION FORM*.--When simpler designs will not afford the required protections it is often possible to apply a six tower design to get the desired result. If the six towers are arranged in a parallelogram array the pattern is the result of multiplying three two-tower patterns. Two of the patterns must be the result of a three tower in line array with equal spacings between the towers. Often this type of array is referred to as a two by three tower multiplication.

If the three towers are located at an angle ϕ_2 with respect to the space reference axis and the two tower pair make an angle of ϕ_4 with respect to the space reference axis as shown in Fig. 40(a) the design equation can be written,

$$E = E_1 f_1(\theta)\ \sqrt{8F_2F_3F_4}\ \left\{\left[\frac{1 + F_2^2}{2F_2} + \cos\left\{S_2\ \cos\ \theta\ \cos(\phi_2 - \phi) + \Psi_2\right\}\right]\right.$$
$$X\left[\frac{1 + F_3^2}{2F_3} + \cos\left\{S_2\ \cos\ \theta\ \cos(\phi_2 - \phi) + \Psi_3\right\}\right]$$
$$\left. X\left[\frac{1 + F_4^2}{2F_4} + \cos\left\{S_4\ \cos\ \theta\ \cos(\phi_4 - \phi) + \Psi_4\right\}\right]\right\}^{\frac{1}{2}} \qquad (67)$$

In this equation it will be noted that the multiplication consists of three patterns two of which have a spacing of S_2 and an orientation of ϕ_2 since three of the towers are equally spaced and are in line.

For this design equation the nulls can be specified in the horizontal and elevation angles by the following equations in terms of phase angle and spacing, thus

$$\Psi_2 = \pm\ 180\ -\ S_2\ \cos(\phi_2\ \pm \phi_{N2})\ \cos\ \theta_{N2}$$

where

Ψ_2 = phase angle to use in Eq. (67)

S_2 = spacing to use in Eq. (67).

ϕ_2 = orientation of No. 2 and No. 3 towers. (68)

$\pm\phi_{N2}$ = orientation of nulls in horizontal plane with respect to the line of the 3 towers.

$\phi_2 \pm \phi_{N2}$ = orientation of nulls in horizontal plane of the two or more nulls for one pair of towers in the three tower lineup.

θ_{N2} = elevation angle of the nulls of the two or more nulls for one pair of towers in the three tower lineup.

The directional antenna parameters for the second pair of towers in the three tower lineup can be specified in terms of the orientation angle ϕ_{N3} and elevation angles θ_{N3} thus,

$$\psi_3 = \pm 180 - S_2 \cos(\phi_2 \pm \phi_{N3}) \cos\theta_{N3} \quad (69)$$

and the other terms are similar to those defined in Eq. (68).

In a similar fashion the nulls for the third pair of towers can be written,

$$\psi_4 = \pm 180 - S_4 \cos(\phi_4 \pm \phi_{N4}) \cos\theta_{N4} \quad (70)$$

With these last three equations it is possible to specify at least 6 nulls, two with each equation. If the spacing is greater than 180 degrees then more than 2 nulls can be specified for each pair of towers, or each equation.

For unity field ratios Eq. (67) reduces to the following simple form,

$$E = 8 E_1 f_1(\theta) \cos\left[\frac{S_2}{2}\cos\theta\cos(\phi_2 - \phi) + \frac{\psi_2}{2}\right] \cos\left[\frac{S_2}{2}\cos\theta\cos(\phi_2 - \phi) + \frac{\psi_3}{2}\right]$$
$$\times \cos\left[\frac{S_4}{2}\cos\theta\cos(\phi_4 - \phi) + \frac{\psi_4}{2}\right] \quad (71)$$

For the horizontal plane pattern this equation reduces to

$$E = 8 E_1 \cos\left[\frac{S_2}{2}\cos(\phi_2 - \phi) + \frac{\psi_2}{2}\right] \cos\left[\frac{S_2}{2}\cos(\phi_2 - \phi) + \frac{\psi_3}{2}\right]$$
$$\times \cos\left[\frac{S_4}{2}\cos(\phi_4 - \phi) + \frac{\psi_4}{2}\right] \quad (72)$$

This short form is very useful in locating nulls in the horizontal plane. After making this computation Eq. (41) or (45) can be used to determine the vertical radiation patterns.

EXAMPLE 23: By inductive reasoning write out the general design equation for a 9 tower array and sketch the tower layout.

SOLUTION: By referring to Eq. (63) and (67) it should be apparent that the required 9 tower array pattern design equation is,

$$E = 4E_1 f_1(\emptyset) \sqrt{F_2 F_3 F_4 F_7}$$
$$\times \Bigg\{ \left[\frac{1+F_2^2}{2F_2} + \cos\{S_2\cos\theta\cos(\emptyset_2-\emptyset) + \psi_2\}\right]$$
$$\times \left[\frac{1+F_3^2}{2F_3} + \cos\{S_2\cos\theta\cos(\emptyset_2-\emptyset) + \psi_3\}\right]$$
$$\times \left[\frac{1+F_4^2}{2F_4} + \cos\{S_4\cos\theta\cos(\emptyset_4-\emptyset) + \psi_4\}\right]$$
$$\times \left[\frac{1+F_7^2}{2F_7} + \cos\{S_4\cos\theta\cos(\emptyset_4-\emptyset) + \psi_7\}\right]\Bigg\}^{1/2}$$

ANS

And the tower layout sketch is shown in Fig. 40 (b) ANS

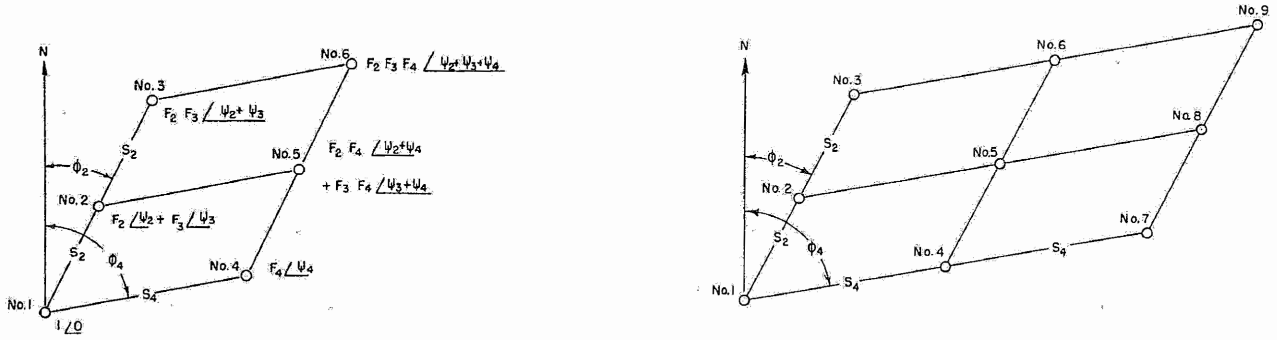
u. *FOUR TOWER IN LINE ARRAY USING THE MULTIPLICATION FORM.*--A special case of the 6 tower array which results in a 4 tower in line array is very useful because it gives the flexibility of multiplying three patterns. This design is limited to the same azimuth angle for all pairs of towers so in some cases a null may go to waste by giving protection where it is not needed.

In Eq. (67) if we let $S = S_2 = S_4$ and $\phi_2 = \phi_4$ the 6 tower array can be represented by 4 towers since tower No. 2 and No. 4 of Fig. (40(a) are the same and towers No. 3 and No. 5 are the same The resulting four tower array is given in Fig. 41(a) and the design equation is,

$$E = E_1 f_1(\theta)\sqrt{8F_2F_3F_4}\left\{\left[\frac{1+F_2^{\,2}}{2F_2} + \cos\left\{S\cos\theta\cos(\phi_2-\phi)+\psi_2\right\}\right]\right.$$

$$\times\left[\frac{1+F_3^{\,2}}{2F_3} + \cos\left\{S\cos\theta\cos(\phi_2-\phi)+\psi_3\right\}\right]$$

$$\left.\times\left[\frac{1+F_4^{\,2}}{2F_4} + \cos\left\{S\cos\theta\cos(\phi_2-\phi)+\psi_4\right\}\right]\right\}^{\frac{1}{2}} \qquad (73)$$



(a) SIX TOWER ARRAY (b) NINE TOWER ARRAY

SIX AND NINE TOWER PARALLELOGRAM ARRAYS

FIG. 40

This equation, for unity field ratios, reduces to

$$E = 8\,E_1 f_1(\theta)\cos\left[\frac{S}{2}\cos\theta\cos(\phi_2-\phi)+\frac{\psi_2}{2}\right]\cos\left[\frac{S}{2}\cos\theta\cos(\phi_2-\phi)+\frac{\psi_3}{2}\right]$$

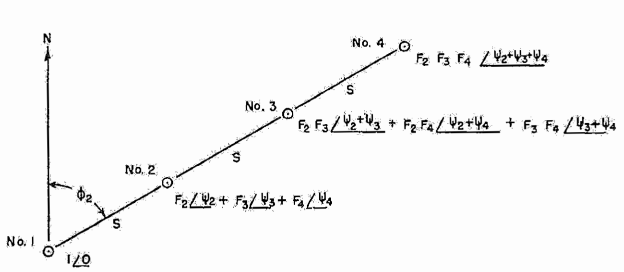
$$\times\cos\left[\frac{S}{2}\cos\theta\cos(\phi_2-\phi)+\frac{\psi_4}{2}\right] \qquad (74)$$

and for the horizontal pattern is further simplified to give

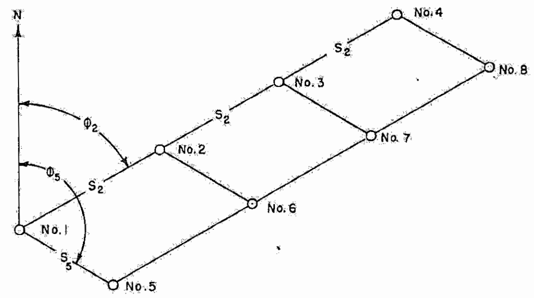
$$E = 8\,E_1\cos\left[\frac{S}{2}\cos(\phi_2-\phi)+\frac{\psi_2}{2}\right]\cos\left[\frac{S}{2}\cos(\phi_2-\phi)+\frac{\psi_3}{2}\right]$$

$$\times\cos\left[\frac{S}{2}\cos(\phi_2-\phi)+\frac{\psi_4}{2}\right] \qquad (75)$$

This short form is very simple considering that this equation actually represents an 8 tower array. This can be visualized by inspecting the array in Fig. 41(a). No. 1 and No. 4 towers represent single vectors. $1\angle 0$ and $F_2F_3F_4\angle\psi_2+\psi_3+\psi_4$ respectively while No. 2 and No. 3 towers represent three vectors each making a total of 8 vectors or 8 towers for the complete array.



(a) FOUR TOWER IN LINE ARRAY



(b) EIGHT TOWER PARALLELOGRAM ARRAY

(a) FOUR AND EIGHT TOWER ARRAYS

FIG. 41

EXAMPLE 24: By inductive reasoning write out the general design equation for an 8 tower array and sketch the tower layout.

SOLUTION: By referring to Eq. (67) and (73) it should be apparant that the required 8 tower array pattern design equation is,

$$E = 4\ E_1\ f_1(\phi)\ \sqrt{F_2\ F_3\ F_4\ F_5}$$

$$\times\left\{\left[\frac{1+F_2^2}{2\ F_2} + \cos\ \{S_2\ \cos\ \theta\ \cos\ (\phi_2-\phi)+\psi_2\}\right]\right.$$

$$\times\left[\frac{1+F_3^{\ 2}}{2\ F_3} + \cos\ \{S_2\ \cos\ \theta\ \cos\ (\phi_2-\phi)+\psi_3\}\right]$$

$$\times\left[\frac{1+F_4^{\ 2}}{2\ F_4} + \cos\ \{S_2\ \cos\ \theta\ \cos\ (\phi_2-\phi)+\psi_4\}\right]$$

$$\left.\times\left[\frac{1+F_5^{\ 2}}{2\ F_5} + \cos\ \{S_5\ \cos\ \theta\ \cos\ (\phi_5-\phi)+\psi_5\}\right]\right\}^{1/2}$$

ANS

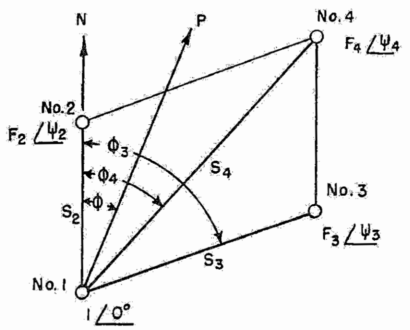
And the tower layout sketch is shown in Fig. 41 (b) ANS

v. *PROOF OF THREE AND FOUR TOWER MULTIPLICATION EQUATIONS.*--The multiplication Eq. (43) and (63) for three and four tower patterns must, by basic definition, be derived from the fundamental pattern formula, which by Eq. (27) is as follows

$$E = \sum_{k=1}^{k=n} E_k\ f_k(\theta)\ \underline{\big|\beta_k}$$

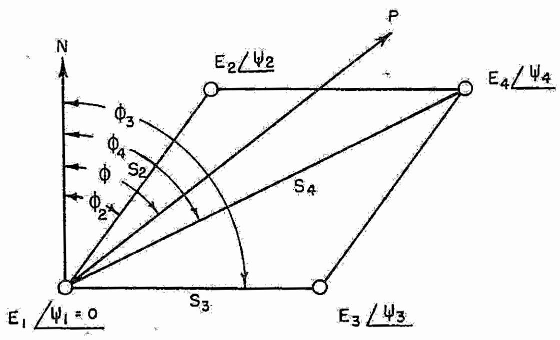
For those interested in a more rigorous proof of this fact the following development made by D. B. Hutton should be carefully studied, in order that the validity of the multiplication equation may be seen.

First of all for the four tower array shown in Fig. 42 the general pattern formula is



SIMPLIFIED FOUR TOWER PARALLELOGRAM ARRAY

FIG. 43



GENERAL FOUR TOWER CONFIGURATION

FIG. 42

$$E = E_1 f_1(\theta) \underline{\lfloor 0^\circ} + E_2 f_2(\theta) \underline{\lfloor \beta_2} + E_3 f_3(\theta) \underline{\lfloor \beta_3} + E_4 f_4(\theta) \underline{\lfloor \beta_4}$$

which, expanded for the horizontal plane pattern, becomes

$$E = E_1 \underline{\lfloor 0^\circ} + E_2 \underline{\lfloor S_2 \cos(\phi_2 - \phi) + \Psi_2} + E_3 \underline{\lfloor S_3 \cos(\phi_3 - \phi) + \Psi_3} + E_4 \underline{\lfloor S_4 \cos(\phi_4 - \phi) + \Psi_4}$$

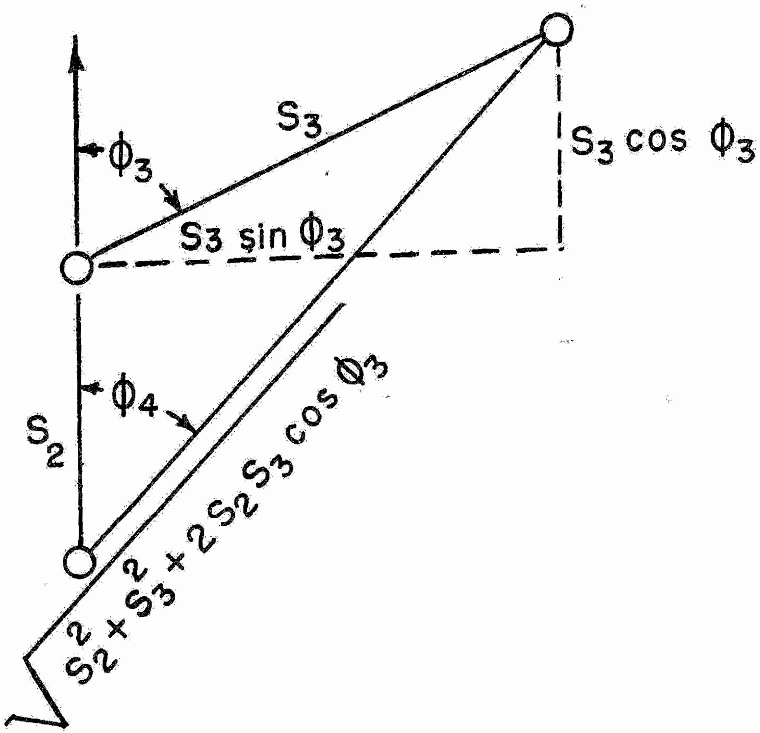
This expression can be simplified by placing No. 2 tower north of the reference point, making $\phi_2 = 0$, and by using field strength ratios giving

$$E = E_1 \left[1 + F_2 \underline{\lfloor S_2 \cos \phi + \Psi_2} + F_3 \underline{\lfloor S_3 \cos(\phi_3 - \phi) + \Psi_3} + F_4 \underline{\lfloor S_4 \cos(\phi_4 - \phi) + \Psi_4} \right.$$

The appearance of the array is now as in Fig. 43. Making use of identity of Eq. (37) the length of S_4 is given by the cosine law equation, and is

$$S_4 = \sqrt{S_2^2 + S_3^2 + 2S_2S_3 \cos \phi_3}$$

By constructing Fig. 44, below, the value of ϕ_4 can be determined in terms of the other parameters as shown.



DETERMINATION OF VALUE OF Φ_4

FIG. 44

Therefore,

$$\tan \phi_4 = \frac{S_3 \sin \phi_3}{S_2 + S_3 \cos \phi_3}$$

$$\cos \phi_4 = \frac{S_2 + S_3 \cos \phi_3}{\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3}}$$

$$\sin \phi_4 = \frac{S_3 \sin \phi_3}{\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3}}$$

returning to the basic pattern formula on page 2-1-54 the angle $S_4 \cos(\phi_4 - \phi) + \Psi_4$ can now be rewritten as

$$\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3} \quad \cos (\phi_4 - \phi) + \Psi_4$$

inserting the terms above, and employing the identity

$$\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

we have for the above angle

$$\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3} \left[\frac{S_2 + S_3 \cos \phi_3}{\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3}} \cos \phi \right.$$

$$\left. + \frac{S_3 \sin \phi_3 \cos \phi}{\sqrt{S_2{}^2 + S_3{}^2 + 2S_2S_3 \cos \phi_3}} \right] + \Psi_4 .$$

$$= S_2 \cos \phi + \left[S_3 \cos \phi_3 \cos \phi + S_3 \sin \phi_3 \sin \phi \right] + \Psi_4$$

and returning the terms in brackets to the form given above the angle is

$$S_2 \cos \phi + S_3 \cos (\phi_3 - \phi) + \Psi_4$$

A vector diagram for the array of Fig. 43 then appears as in Fig. 45.

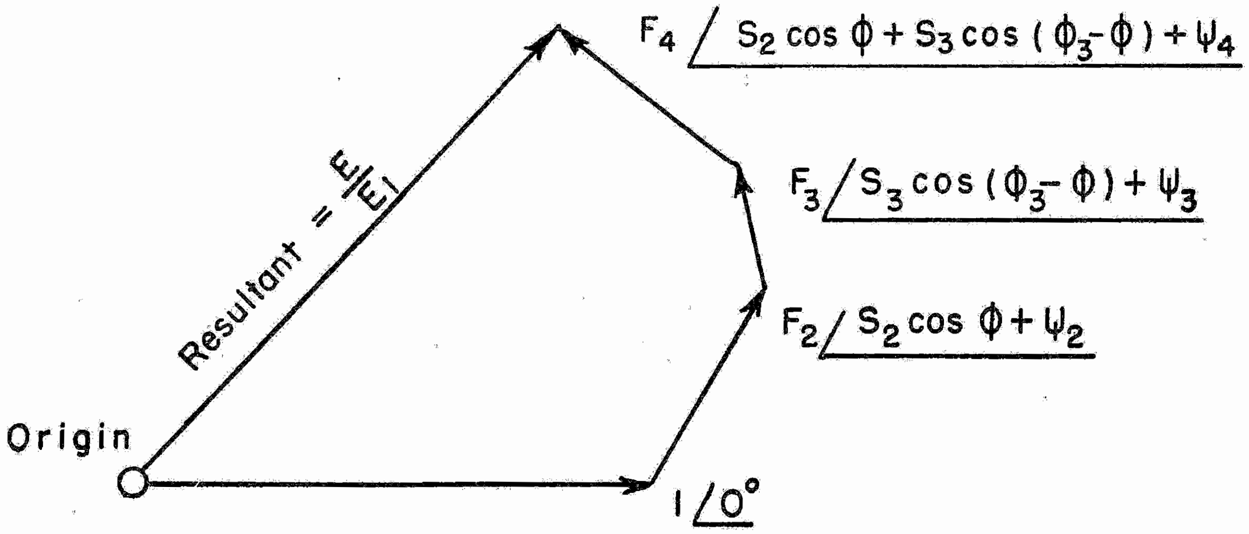
To simplify the angles above let

$$\beta_2 = S_2 \cos \phi + \Psi_2$$

$$\beta_3 = S_3 \cos (\phi_3 - \phi) + \Psi_3$$

So that

$$S_2 \cos \phi + S_3 \cos (\phi_3 - \phi) + \Psi_4 = \beta_2 + \beta_3 + \Psi_4 - \Psi_2 - \Psi_3 = \beta_4$$



VECTOR DIAGRAM OF ARRAY

FIG. 45

The vector diagram above can be resolved into its horizontal and vertical components and the resultant found by use of the expression

$$E = E_1 \sqrt{\left[1 + F_2 \cos\beta_2 + F_3 \cos\beta_3 + F_4 \cos\beta_4\right]^2 + \left[F_2 \sin\beta_2 + F_3 \sin\beta_3 + F_4 \sin\beta_4\right]^2}$$

The terms under the radical can be expanded by squaring and the resulting expression is then simplified by use of the trigonometric formulas

$$\sin^2\beta + \cos^2\beta = 1$$

$$\cos\beta_1 \cos\beta_2 = \frac{1}{2}\left[\cos(\beta_1 + \beta_2) + \cos(\beta_1 - \beta_2)\right]$$

$$\sin\beta_1 \sin\beta_2 = \frac{1}{2}\left[-\cos(\beta_1 + \beta_2) + \cos(\beta_1 - \beta_2)\right]$$

After making use of the above formulas to reduce terms in the expanded equation the field strength is written as

$$E = E_1 \left[1 + F_2^2 + F_3^2 + F_4^2 + 2F_2 \cos\beta_2 + 2F_3 \cos\beta_3 + 2F_4 \cos\beta_4 + 2F_2F_4 \cos(\beta_2 - \beta_4) + 2F_2F_3 \cos(\beta_2 - \beta_3) + 2F_3F_4 \cos(\beta_4 - \beta_3)\right]^{\frac{1}{2}}$$

In the next step F_4 is placed equal to F_2F_3 and ψ_4 is made equal to $\psi_2 + \psi_3$, whereupon $\beta_4 = \beta_2 + \beta_3$. Moreover, by making use of the trigonometric formula

$$\cos(\beta_1 + \beta_2) + \cos(\beta_1 - \beta_2) = 2\cos\beta_1 \cos\beta_2$$

the field strength can be written

$$E = E_1 \Big[1 + F_2^{\,2} + F_3^{\,2} + F_2^{\,2}F_3^{\,2} + 2F_2 \cos \beta_2 + 2F_3 \cos \beta_3$$

$$+ 4F_2F_3 \cos \beta_2 \cos \beta_3 + 2F_2^{\,2}F_3 \cos \beta_3 + 2F_2F_3^{\,2} \cos \beta_2 \Big]^{\frac{1}{2}}$$

This, it can readily be demonstrated, is the expansion of the equation

$$E = E_1 \left[(1 + F_2^{\,2} + 2F_2 \cos \beta_2)(1 + F_3^{\,2} + 2F_3 \cos \beta_3)\right]^{\frac{1}{2}}$$

or, dividing by F_2 and F_3

$$E = 2E_1\sqrt{F_2F_3} \left[\left(\frac{1 + F_2^{\,2}}{F_1} + \cos \beta_2\right)\left(\frac{1 + F_3^{\,2}}{F_3} + \cos \beta_3\right)\right]^{\frac{1}{2}} ,$$

And, as a final step, the values of β_2 and β_3 which have been given above, are substituted in the equation, giving the design Eq. (63) (also returning f (θ) and cos θ to the equation)

$$E = 2E_1\, f_1(\theta) \sqrt{F_2F_3} \sqrt{\left[\frac{1 + F_2^{\,2}}{2F_2} + \cos \left\{ S_2 \cos \theta \cos (\phi_2 - \phi) + \psi_2 \right\}\right] \times \left[\frac{1 + F_3^{\,2}}{2F_3} + \cos \quad S_3 \cos \theta \cos (\phi_3 - \phi) + \psi_3 \Big\}\right]}$$

And for the three tower in-line array on a north-south axis this equation reduces to that of Eq. (43).

$$E = 2E_1\, f_1\, (\theta) \sqrt{F_2F_3} \sqrt{\left[\frac{1 + F_2^{\,2}}{2F_2} + \cos (S_2 \cos \theta \cos \phi + \psi_2)\right] \times \left[\frac{1 + F_3}{2F_3} + \cos (S_3 \cos \theta \cos \phi + \psi_3)\right]}$$

4. CONTROL OF PATTERN SIZE

a. ***GENERAL DESIGN EQUATIONS FOR POWER FLOW INTEGRATION METHOD***.--(1) General Treatment-- The total power radiated from a directional antenna array can be computed by integrating the energy flow outward through an imaginary spherical surface surrounding the directional antenna array. This method does not give information regarding the distribution of power to various towers of the directional antenna array, however, it is very useful for making comparisons of pattern size.

The rate of energy flow in watts per square meter at a given point P in space can be expressed by Poyntings vector, thus

$$= E \times H \qquad (76)$$

where p = watts per square meter energy flow
E = volts per meter electric field intensity
X = cross product sign
H = ampere-turns per meter magnetic field intensity

Out in free space the vectors E and H are orthogonal and have the following magnitude relationship,

$$H = \sqrt{\frac{\epsilon_0}{\mu_0}}\, E \qquad (77)$$

where $\mu_o = 4\pi 10^{-7}$ henries per meter the permeability of free space

$\epsilon_o = \frac{1}{\mu_o c^2}$ farads per meter the permittivity of free space,

$c = 2.99776 \times 10^8$ meters per second the velocity of light.[5]

From these values the characteristic resistance of free space can be determined. The characteristic resistance of free space has a value of resistance such that no energy will be reflected. In other words, energy leaving the directional antenna array and flowing into free space will never return. In equation form

$$R_c = \mu_o c = \sqrt{\frac{\mu_o}{\epsilon_o}} \qquad (78)$$

where R_c = 376.710 ohms the characteristic resistance of free space.

Substituting Eq. (77) and (78) in Eq. (76) the power flow can be expressed

$$p = \frac{E^2}{R_c} \qquad (79)$$

If this power flow is integrated over an imaginary spherical surface surrounding the directional antenna array the total power radiated is

$$P_r = \int_{\text{spherical surface}} p\, dS = \frac{1}{R_c} \int_{\text{spherical surface}} E^2\, dS \qquad (80)$$

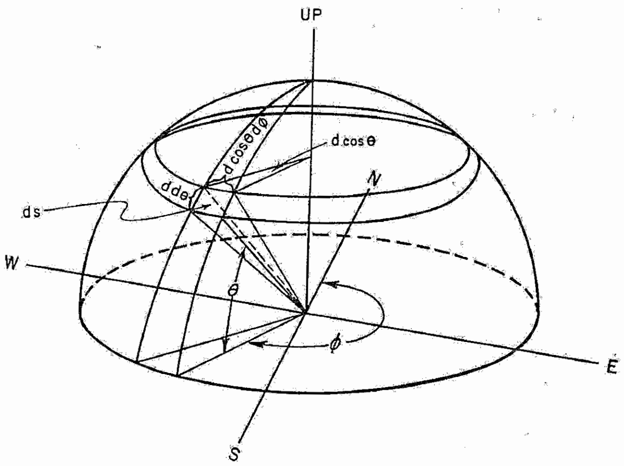
where P_r = watts, total power radiated
R_c = 376.7 ohms the characteristic resistance of free space
E = volts per meter, total field intensity at the surface of the sphere
dS = square meters in the element of area on the surface of the sphere.

If the energy flow outward through the surface of the sphere is integrated as illustrated in Fig. 46 we can write

$$dS = d^2 \cos\theta\, d\theta\, d\phi \qquad (81)$$

where d = meters, radius of spherical surface and the other values are defined above. Substituting this value of dS in Eq. (80) gives

[5]This figure c = (2.99776 + 0.00004)10^{10} cm. sec^{-1} is from Raymond T. Birge, University of California published in Review of Modern Physics, Vol. 14, No. 4, p. 233, October 1941.



SPHERICAL SURFACE INTEGRATION

FIG. 46

$$P_r = \frac{1}{R_c} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} E^2\, d^2 \cos\theta\, d\theta\, d\phi \qquad (82)$$

= total power radiated from an antenna system in free space.

(2) <u>Field Intensity From a Standard Reference Antenna Which Has a Uniform Spherical Pattern.</u>--If the field intensity is the same in all directions the E in Eq. (82) can be replaced by the constant E_s. Moving the constant E_s and d^2 outside of the integral signs and performing the indicated integrations gives

$$P_r = \frac{E_s^2 d^2}{R_c} \int_0^{2\pi} \int_{-\pi/2}^{+\pi/2} \cos\theta\, d\theta\, d\phi$$

$$= \frac{2 E_s^2 d^2}{R_c} \int_0^{2\pi} d\phi = \frac{4\pi E_s^2 d^2}{R_c} \qquad (83)$$

Solving this equation for the solid rms field intensity of a uniform spherical radiator

$$E_s = \sqrt{\frac{P_r R_c}{4\pi d^2}} \qquad (84)$$

For one kilowatt of radiated power the field intensity at one mile is

$$E_s = \sqrt{\frac{(1000)\ (376.710)}{4(3.14159)\ (1609.347)^2}}$$

$$= 107.584 \qquad (85)$$

$\doteq$ 107.6 mv/m the field intensity at one mile for 1 kw from The Standard Reference Spherical Radiator.

(3) Field Intensity From a Standard Reference Antenna Which Has a Uniform Hemispherical Pattern.--In Eq. (82) the integration of θ is from 0 to $\pi/2$, that is, from zero elevation to straight up, therefore,

$$P_r = \frac{1}{R_c} \int_0^{2\pi} \int_0^{+\pi/2} E^2 \, d^2 \cos\theta \, d\theta \, d\phi \qquad (86)$$

Substituting the constant E_s for E and solving for E_s in a fashion similar to the procedure in Eq. (83) and (84) results in,

$$E_s = \sqrt{\frac{P_r \; R_c}{2\pi \; d^2}} \qquad (87)$$

For one kilowatt of radiated power the field intensity at one mile is,

$$E_s = 152.147 \qquad (88)$$

$=$ 152.1 mv/m the field intensity at one mile for 1 kw from a standard reference hemispherical radiator.

This value can also be obtained by multiplying

$$107.584 \sqrt{2} = 152.147$$

(4) Maximum Field Intensity From a Current Element Antenna in Free Space.--For a current element in free space it is of academic interest to determine the maximum field intensity at one mile when one kilowatt of power is radiated. As given in Eq. (4) the field intensity pattern is a cosine function which if substituted in Eq. (82) gives

$$P_r = \frac{1}{R_c} \int_0^{2\pi} \int_{-\pi/2}^{+\pi/2} (E_o \cos\theta)^2 \, d^2 \cos\theta \, d\theta \, d\phi$$

$$= \frac{2\pi E_o^2 \, d^2}{R_c} \int_{-\pi/2}^{+\pi/2} \cos^3\theta \, d\theta = \frac{8\pi E_o^2 \, d^2}{3R_c} \qquad (89)$$

Solving for the maximum field intensity for one kilowatt of radiated power at a distance of one mile

$$E_o = \sqrt{\frac{3 \, P_r \, R_c}{8\pi \, d^2}} = 131.763 \qquad (90)$$

$\doteq$ 131.8 mv/m the maximum field intensity at one mile for 1 kw from a current element antenna in free space.

(5) Horizontal Field Intensity For a Vertical Current Element at the Surface of a Perfect Earth.--Substituting the cosine functions of Eq. (4) into Eq. (86) results in,

$$P_r = \frac{1}{R_c} \int_0^{2\pi} \int_0^{\pi/2} (E_o \cos\theta)^2 \, d^2 \cos\theta \, d\theta \cos\phi \, d\phi$$

$$= \frac{4\pi E_o^2 \, d^2}{3R_c} \qquad (91)$$

Solving for E_o the horizontal field intensity for one kilowatt of power radiated at a distance of one mile

$$E_o = \sqrt{\frac{3 \, P_r \, R_c}{4\pi \, d^2}} = 186.341 \qquad (92)$$

$\doteq$ 186.3 mv/m the maximum field intensity at one mile for one kilowatt from a vertical current element at the surface of a perfect earth.

(6) Field Intensity From a Center Fed Conductor of Length 2G in Free Space.--With a sinusoidal current distribution on the conductor as shown in Fig. 47, the field intensity at any angle from a plane passing through the center and perpendicular to the conductor is

$$E = E_o \, f(\theta) = E_o \frac{\cos(G \sin\theta) - \cos G}{(1-\cos G)\cos\theta} \qquad (93)$$

Substituting this value of E in Eq. (82) and integrating with respect to ϕ gives,

$$P_r = \frac{2\pi \, d^2 \, E_o^2}{R_c \, (1-\cos G)^2} \int_{-\pi/2}^{\pi/2} \frac{[\cos(G\sin\theta) - \cos G]^2}{\cos\theta} d\theta \qquad (94)$$

Performing the indicated integration[6],

$$P_r = \frac{2\pi \, d^2 \, E_o^2}{R_c \, (1-\cos G)^2} \left[(\gamma + \ln 2G - \mathrm{Ci}\, 2G) \right.$$

$$\left. + 1/2 \sin 2G (\mathrm{Si}\, 4G - 2\, \mathrm{Si}\, 2G) \right.$$

[6]See page 434 Ramo and Whinnery "Fields and Waves in Modern Radio," John Wiley and Sons - 1944.

$$+ 1/2 \cos 2G(\gamma + \ln G - 2 \, Ci \, 2G + Ci \, 4G)] \qquad (95)$$

Solving this equation for E_o results in,

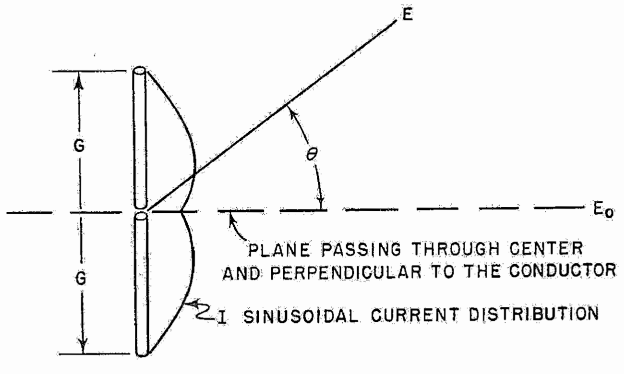
$$E_o = \frac{(1-\cos G)\sqrt{\dfrac{P_r R_c}{2\pi d^2}}}{\sqrt{\begin{array}{l}(\gamma + \ln 2G - Ci \, 2G) \\ + 1/2 \, (Si \, 4G - 2 \, Si \, 2G) \sin 2G \\ + 1/2 \, (\gamma + \ln G - 2 \, Ci \, 2G + Ci \, 4G) \cos 2G\end{array}}} \qquad (96)$$

where E_o = volts per meter, the field intensity on a plane passing through the center and perpendicular to the conductor
P_r = watts, total power radiated
R_c = 376.710 ohms, the characteristic resistance of free space.
d = meters from the antenna
= 1609.347 meters in one mile
γ = 0.57721566 Euler's Constant
ln = natural logarithm to base ϵ = 2.71828
Ci = cosine integral
Si = sine integral
2G = radians or degrees length of the conductor.

Substituting Eq. (96) in Eq. (93) gives the field intensity at any point in space, thus,

$$E = \frac{\dfrac{\cos (G \sin \theta) - \cos G}{\cos \theta}\sqrt{\dfrac{P_r R_c}{2\pi d^2}}}{\sqrt{\begin{array}{l}(\gamma + \ln 2G - Ci \, 2G) \\ + 1/2 \, (Si \, 4G - 2 \, Si \, 2G) \sin 2G \\ + 1/2 \, (\gamma + \ln G - 2 \, Ci \, 2G + Ci \, 4G) \cos 2G\end{array}}} \qquad (97)$$

where θ = the elevation angle from the plane and the other terms are defined following Eq. (96)



CONDUCTOR OF LENGTH 2G IN FREE SPACE

FIG. 47

(7) Field Intensity From a Half Wave Antenna in Free Space.--For a half wave antenna radiating one kilowatt of power the field intensity at one mile, by substituting in Eq. (97) is

$$E = \frac{152.147 \dfrac{\cos (90 \sin \theta)}{\cos \theta}}{\sqrt{1.21884}}$$

$$= 137.813 \frac{\cos (90 \sin \theta)}{\cos \theta} \text{ mv/m} \qquad (98)$$

The space pattern resulting from this equation is shown in Fig. 5.

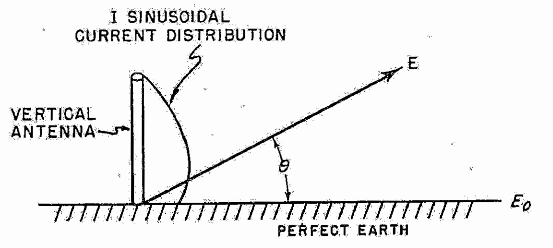
(8) Field Intensity From a Vertical Antenna of Height G Over a Perfectly Reflecting Earth.--Assuming sinusoidal current distribution the limits of integration in Eq. (94) are changed to cover the hemisphere above the surface of the earth, thus

$$P_r = \frac{2\pi d^2 E_o^2}{R_c (1-\cos G)^2} \times \int_0^{\pi/2} \frac{[\cos (G \sin \theta) - \cos G]^2}{\cos \theta} d\theta \qquad (99)$$

Performing the indicated integration and solving for E results in,

$$E = \frac{\dfrac{\cos (G \sin \theta) - \cos G}{\cos \theta}\sqrt{\dfrac{P_r R_c}{\pi d^2}}}{\sqrt{\begin{array}{l}(\gamma + \ln 2G - Ci \, 2G) \\ + 1/2 \, (Si \, 4G - 2 \, Si \, 2G) \sin 2G \\ + 1/2 \, (\gamma + \ln G - 2 \, Ci \, 2G + Ci \, 4G) \cos 2G\end{array}}} \qquad (100)$$

where the terms are defined in Eq. (96) and (97) except E_o is the horizontal field intensity, G is the height of the antenna and θ is the elevation angle as shown in Fig. 48.



VERTICAL ANTENNA OF HEIGHT G OVER A PERFECT EARTH

FIG. 48

Along the horizontal plane Eq. (100) reduces to

$$E_o = \frac{(1-\cos G)\sqrt{\dfrac{P_r R_c}{\pi d^2}}}{\sqrt{\begin{array}{l}(\gamma + \ln 2G - Ci\ 2G) \\ + 1/2\ (Si\ 4G - 2\ Si\ 2G)\ \sin 2G \\ + 1/2\ (\gamma + \ln G - 2\ Ci\ 2G \\ + Ci\ 4G)\ \cos 2G\end{array}}} \quad (101)$$

If this theoretical field intensity E_o is plotted as a function of antenna height for one kilowatt of power at a distance of one mile the curve of Fig. 8 results.

As a matter of interest the loop radiation resistance for a thin vertical conductor is,

$$R_r = 29.99776 \left[(\gamma + \ln 2G - Ci\ 2G) + 1/2\ (Si\ 4G - 2\ Si\ 2G)\ \sin 2G + 1/2\ (\gamma + \ln G - 2\ Ci\ 2G + Ci\ 4G)\ \cos 2G \right] \quad (102)$$

where R_r = ohms, loop radiation resistance of a thin vertical conductor over a perfectly conducting earth. This curve has also been plotted in Fig. 8.

For a vertical quarter wave antenna the loop radiation resistance is

$R_r = (29.9978)\ (1.21884) = 36.5626$ ohms

The base resistance can be determined from the approximate equation,

$$R_b \doteq \frac{R_r}{\sin^2 G} \quad (103)$$

base resistance, ohms.

(9) Field Intensity From a Vertical Quarter Wave Antenna Over a Perfect Earth.--Substituting for $G = 90° = \pi/2$ radians in Eq. (100) when the power is one kilowatt and the distance is one mile the field intensity is,

$$E = 194.897 \frac{\cos (90 \sin \theta)}{\cos \theta} \text{ mv/m} \quad (104)$$

The vertical pattern resulting from this equation is shown in Fig. 7.

The field intensity along the surface of the earth from Eq. (104) is,

$E_o = 194.897$
$= 194.9$ mv/m the theoretical horizontal field intensity at one mile for one mile for one kw of power.

(10) Application of Power Flow Integration Method to Directional Antennas.--Thus far the power flow integration method has been applied to standard reference antennas, secondary standard reference antennas, and to the size of pattern produced by sinusoidal current distribution on a conductor in free space or a vertical radiator of various heights. The method is not confined to these special cases but can be applied to any directional antenna system so long as the field distribution is known from the individual elements that make up the system.

Considerable space has been devoted to determining the pattern size on a non-directional antenna, that is, an antenna which produces a constant field intensity at all azimuth angles in the horizontal plane and at any given elevation angle the field intensity is constant for all azimuth angles. The reason for this approach is that any directional antenna can be transformed to its equivalent non-directional antenna pattern. This can be accomplished by determining the root-mean-square of the pattern at each elevation angle, then it is only necessary to integrate this equivalent non-directional pattern with respect to the elevation angle since the determination of the rms values is the result of integrating the pattern with respect to the azimuth angle.

In order to develop this method consider the horizontal field intensity from two towers, one at the origin and one at the spacing S_2 due north. The total vector field is then

$$E = E_1 \angle 0 + E_2 \angle S_2 \cos \phi + \psi_2 \quad (105)$$

changing to rectangular form,

$$E = E_1 + E_2 \cos (S_2 \cos \phi + \psi_2) + j\ E_2 \sin (S_2 \cos \phi + \psi_2)$$

The magnitude of E squared is

$$\begin{aligned} E^2 &= E_1^2 + E_2^2 \cos^2 (S_2 \cos \phi + \psi_2) \\ &\quad + E_2^2 \sin^2 (S_2 \cos \phi + \psi_2) \\ &\quad + 2E_1 E_2 \cos (S_2 \cos \phi + \psi_2) \\ &= E_1^2 + E_2^2 + 2E_1 E_2 \cos (S_2 \cos \phi + \psi_2) \end{aligned} \quad (106)$$

From Eq. (86) performing the azimuth integration, we substitute the value of E^2 from Eq. (106) to get

$$\int_0^{2\pi} E^2\ d\phi = \int_0^{2\pi} \left[E_1^2 + E_2^2 \right.$$

$$+ 2E_1 E_2 \cos (S_2 \cos \phi + \Psi_2)] \; d\phi$$

$$= 2\pi \left\{ E_1^2 + E_2^2 + 2E_1 E_2 \frac{1}{2\pi} \right.$$

$$\times \int_0^{2\pi} [\cos (S_2 \cos \phi) \cos \Psi_2$$

$$\left. - \sin (S_2 \cos \phi) \sin \Psi_2] \; d\phi \right\}$$

The second term in the integral is zero as can be demonstrated by plotting the function. Therefore,

$$\int_0^{2\pi} E^2 \, d\phi = 2\pi \; [E_1^2 + E_2^2$$

$$+ 2E_1 E_2 \cos \Psi_2 \frac{1}{2\pi} \int_0^{2\pi} \cos (S_2 \cos \phi) \, d\phi]$$

$$= 2\pi \; [E_1^2 + E_2^2 + 2E_1 E_2 \cos \Psi_2 \; J_o(S_2)]$$

where $J_0(S_2)$ = a Bessel function of the first kind of a zero order. In this equation the phase Ψ and spacing S is between elements 1 and 2 hence a more general form is to write

Ψ_{12} = difference in electrical phase angle of the field from the 1st and the 2nd antenna,
S_{12} = electrical length of spacing between the 1st and 2nd antenna.

With this change in nomenclature the above equation becomes

$$\int_0^{2\pi} E^2 \, d\phi = 2\pi [E_1^2 + E_2^2 + 2E_1 E_2 \cos \Psi_{12} \; J_o(S_{12})] \quad (107)$$

In this equation the integration is performed for only two antennas in the horizontal plane. This equation can be generalized to account for any elevation angle θ if the field intensities are multiplied by $f(\theta)$ and the spacing is multiplied by $\cos \theta$. Making these modifications Eq. (107) can be written,

$$E_\theta^2 = \frac{1}{2\pi} \int E^2 \, d\phi = E_1^2 \; f_1(\theta)^2 + E_2^2 \; f_2(\theta)^2$$

$$+ 2E_1 \; f_1(\theta) \, E_2 \; f_2(\theta) \; \cos \Psi_{12} \; J_o(S_{12} \; \cos \theta) \quad (108)$$

where E_θ = root-mean-square field intensity radiated at the elevation angle θ. (In this case for only 2 antennas.)

If Eq. (108) is generalized for any number of elements in the directional antenna array the r-m-s field intensity at the elevation angle θ can be written

$$E_\theta = \sqrt{\sum_{p=1}^{p=n} \sum_{q=1}^{q=n} E_p \; f_p(\theta) \; E_q \; f_q(\theta) \times \cos \Psi_{pq} \; J_o(S_{pq} \; \cos \theta)} \quad (109)$$

where E_θ = root-mean-square effective field intensity at the elevation angle θ.
p = p^{th} antenna in the system.
q = q^{th} antenna in the system.
n = number of elements in the complete directional antenna array.
E_p = horizontal magnitude of the field intensity produced by the p^{th} antenna.
$f_p(\theta)$ = vertical radiation characteristic of the p^{th} antenna.
E_q = horizontal magnitude of the field intensity produced by the q^{th} antenna.
$f_q(\theta)$ = vertical radiation characteristic of the q^{th} antenna
Ψ_{pq} = difference in electrical phase angle of the voltage (or current) between the p^{th} and q^{th} antennas in the directional antenna array.
S_{pq} = spacing in degrees or radians between the p^{th} and q^{th} antennas.
$J_o(S_{pq} \cos \theta)$ = Bessel function of the first kind and zero order of the apparent spacing between the p^{th} and the q^{th} antennas.

Analyzing Eq. (109) for a two element directional antenna reveals how Eq. (108) is deducted. When p = 1 and q = 1 we have the term $E_1 \; f_1(\theta) \; E_1 f_1(\theta) \; \cos \Psi_{11} \; J_0(S_{11} \cos \theta)$. Since $\Psi_{11} = 0$, $\cos \Psi_{11} = 1$ and since $S_{11} = 0$, $J_o(S_{11} \cos \theta) = 1$, thus resulting in the first term of Eq. (108) that is $E_1^2 f_1(\theta)^2$. In a similar fashion the second term of Eq. (108) results when p = 2 and q = 2, that is $E_2^2 f_2(\theta)^2$. When p = 1 and q = 2 we have the term $E_1 f_1(\theta) E_2 f_2(\theta) \cos \Psi_{12} J_o (S_{12} \cos \theta)$ which is one half of the third term in Eq. (108). For the condition p = 2 and q = 1 we have $E_2 f_2(\theta) \; E_1 f_1(\theta) \cos \Psi_{21} J_o (S_{21} \cos \theta)$. Since Ψ_{21} is the negative of Ψ_{12} and the cosine of a given positive or negative angle is the same, we can write $\cos \Psi_{12} = \cos \Psi_{21}$. The magnitude of $S_{12} = S_{21}$ because the distances are measured between the same two points. Therefore, when p = 2 and q = 1 the same value is obtained as when p = 1 and q = 2. The sum of these two terms results in the last term of Eq. (108) that is $2 \; E_1 f_1(\theta) \; E_2 f_2(\theta) \cos \Psi_{12} J_o(S_{12} \cos \theta)$. Since n = 2 in this case, all of the terms in the summation have been accounted for in the results given by Eq. (108).

To clarify the application of Eq. (109) consider a 3 element directional antenna system. In this case n = 3 and the equation can be written.

$$E_\theta = \sqrt{\begin{aligned}&E_1^2 f_1(\theta)^2 + E_2^2 f_2(\theta)^2 + E_3^2\ f_3(\theta)^2 \\ &+2E_1 f_1(\theta)\ E_2\ f_2(\theta)\ \cos\psi_{12}\ J_o(S_{12}\ \cos\theta) \\ &+2E_1 f_1(\theta)\ E_3\ f_3(\theta)\ \cos\psi_{13}\ J_o(S_{13}\ \cos\theta) \\ &+2E_2 f_2(\theta)\ E_3\ f_3(\theta)\ \cos\psi_{23}\ J_o(S_{23}\ \cos\theta)\end{aligned}} \qquad (110)$$

For a directional antenna with more elements the number of terms will increase. It will be noted that the general equation results in the following terms under the radical; first, the square of all the individual antenna field intensities and second, the terms written as twice the product of the field intensity of each individual antenna multiplied by the field intensity of each other individual antenna in the system and then multiplied by the cosine of the phasing of the currents between the antennas and the Bessel function of the apparent spacing between the antennas.

For the horizontal plane $\theta = 0$, $\cos\theta = 1$, $f_1(\theta) = 1$ and $f_2(\theta) = 1$ with the result that Eq. (109) can be reduced to

$$E_o = \sqrt{\sum_{p=1}^{p=n}\ \sum_{q=1}^{q=n}\ E_p E_q\ \cos\psi_{pq}\ J_o(S_{pq})} \qquad (111)$$

where E_o = root-mean-square effective field intensity in the horizontal plane, and the other values are defined in Eq. (109).

In terms of E_θ^2 as given in Eq. (108) it is possible to rewrite Eq. (86) as follows:

$$P_r = \frac{1}{R_c} \int_0^{2\pi} \int_0^{\pi/2} E^2\ d^2\ \cos\theta\ d\theta\ d\phi$$

$$= \frac{2\pi\ d^2}{R_c} \int_0^{\pi/2} E_\theta^2\ \cos\theta\ d\theta\ d\phi \qquad (112)$$

The standard hemispherical field intensity produced by the directional antenna system can be obtained by substituting Eq. (112) in Eq. (87), thus,

$$E_s = \sqrt{\int_0^{\pi/2} E_\theta^2\ \cos\theta\ d\theta} \qquad (113)$$

This is the *exact* formula for determining the size of the directional antenna pattern, however, to perform the integration for the general case would be quite tedious.

A practical and very useful solution is to determine the value of E_θ at a number of elevation angles and to replace the integral with the summation, thus, for intervals of 10 degrees of elevation the approximate equation by application of the trapezoidal rule[7] can be written

$$E_s \doteq \sqrt{\frac{\pi}{18}\left[\frac{E_o^2}{2} + \sum_{n=1}^{n=8} E_{10n}^2\ \cos 10n\right]} \qquad (114)$$

where E_s = the standard hemispherical field intensity produced by the directional antenna system. (152.1 mv/m for 1 kw at 1 mile).

E_o = root-mean-square effective field intensity in horizontal plane.

E_{10n} = root-mean-square effective field intensity at the specified elevation angle.

n = integers from 1 to 8, which multiplied by 10 gives the elevation angle θ in degrees.

EXAMPLE 25: A four tower directional antenna system has the following parameters:

No.	G°	ϕ°	S°	ψ°	F
1	90	0	0	0	1.0
2	90	274.22	176	2	0.786
3	90	302.34	211.3	275	0.841
4	90	358.38	100	260	0.786

Determine the values of horizontal field intensity from each tower for 5 kw operation assuming the system loss is 405 watts according to FCC standards. What is the value of r-m-s field intensity for each 10° elevation angle?

SOLUTION: Initially solving for E_θ at every 10° elevation angle by Eq. (109) in terms of field ratios F results in column 2 of the following table

θ°	E_θ for $E_1 = 1$	E_θ for $E_1 = 275$
0	1.435	395
10	1.432	394
20	1.382	380
30	1.313	361
40	1.212	333
50	1.087	299
60	0.899	247
70	0.657	181
80	0.031	86

[7] Page 192, Smith's Applied Mathematics, published by McGraw-Hill Book Co. - 1945.

Now, by Eq. (114) we have,

$E_s \doteq 1.236 \qquad \text{for } E_1 = 1$

For 5 kw radiated the standard hemispherical field intensity will be

$E_s = 152.1\sqrt{5} = 340$ mv/m

Therefore, the required values of field intensity are:

$E_1 = 340/1.236 = 275$ mv/m   ANS

$E_2 = 275(0.786) = 216$ mv/m   ANS

$E_3 = 275(0.841) = 231$ mv/m   ANS

$E_4 = 275(0.786) = 216$ mv/m   ANS

The final r-m-s field intensity for each 10° elevation angle is tabulated in column 3 of the above table.   ANS

The input power for this problem according to FCC standards is 5,405 watts and the radiated power is 5 kw.

The solution of this problem is quite tedious by the method followed in this example, however with the help of a directional antenna pattern calculator which is capable of drawing the patterns at the various elevation angles the r-m-s values can be quickly measured from these patterns to get the required values of E_θ to use in Eq. (114).

b. ***MUTUAL RESISTANCE METHOD OF DETERMINING PATTERN SIZE.***--(1) General Statement.--The power flow integration method can be used to determine the self and mutual resistance components of the elements in the directional antenna system. Since this method does not give information in regard to the reactance components it is not useful in determining the driving point impendance; however, because of its simplicity it is believed to be a very practical and useful method for determining pattern size. This method can be considered as an extension and simplification of the power flow integration method for some purposes.

(2) Relation of Field Intensity to Antenna Current.--First, let us write the equation for the field intensity E caused by a current I flowing in a single vertical conductor of height G over a perfect earth, thus,

$$E = \frac{R_c I \left[\cos(G \sin\theta) - \cos G\right]}{2\pi d \qquad \cos\theta} \tag{115}$$

where E = field intensity, volts per meter
R_c = 376.710 the characteristic resistance of free space, ohms
I = current at antenna current loop, amp.
d = distance from the antenna, meters
G = electrical height of antenna, degrees
θ = elevation angle, degrees.

If we define

$$h = \frac{\cos(G \sin\theta) - \cos G}{\sqrt{\cos\theta}} \tag{116}$$

later expressions can be simplified and Eq. (115) can be written.

$$E = \frac{R_c I}{2\pi d} \quad \frac{h}{\sqrt{\cos\theta}} \tag{117}$$

In the horizontal plane $\theta = 0$ and this equation reduces to

$$E = \frac{R_c I (1 - \cos G)}{2\pi d}. \tag{118}$$

(3) Self and Mutual Power Flow due to Self and Mutual Radiation Resistance Respectively.--By Eq. (79) the power flow vector in the direction of azimuth angle ϕ and elevation angle θ is

$$p = \frac{E^2}{R_c} \tag{79}$$

If the total vector field intensity E is the result of the two antennas with No. 1 antenna at the origin we can write

$$E = E_1 \, f_1(\theta) \angle 0 + E_2 \, f_2(\theta) \angle S_2 \cos(\phi_2 - \phi) \cos\theta + \Psi_2 \tag{119}$$

Changing to rectangular form

$$E = E_1 \, f_1(\theta) + E_2 \, f_2(\theta) \cos\left[S_2 \cos(\phi_2 - \phi) \cos\theta + \Psi_2\right] + j\, E_2 \, f_2(\theta) \sin\left[S_2 \cos(\phi_2 - \phi) \cos\theta + \Psi_2\right] \tag{120}$$

The magnitude of E squared is

$$E^2 = E_1^2 \, f_1(\theta)^2 + E_1 \, f_1(\theta)\, E_2 \, f_2(\theta) \cos\left[S_2 \cos(\phi_2 - \phi) \cos\theta + \Psi_2\right] + E_2 \, f_2(\theta)\, E_1 \, f_1(\theta) \cos\left[S_2 \cos(\phi_2 - \phi) \cos\theta + \Psi_2\right] + E_2^2 \, f_2(\theta)^2 \tag{121}$$

Making this substitution in Eq. (79) gives

$$p = p_{11} + p_{12} + p_{21} + p_{22} \qquad (122)$$

where $$p_{11} = \frac{E_1^2 \; f_1(\theta)^2}{R_c} \qquad (123)$$

= the self power flow due to the self resistance of antenna No. 1.

$$p_{12} = p_{21} = \frac{E_1 \; f_1(\theta) \; E_2 \; f_2(\theta) \; \cos [S_2 \cos (\phi_2 - \phi) \cos \theta + \Psi_2]}{R_c} \qquad (124)$$

= one half of the mutual power flow due to the mutual resistance between antennas No. 1 and No. 2.

$$p_{22} = \frac{E_2^2 \; f_2(\theta)^2}{R_c} \qquad (125)$$

= the self power flow due to the self resistance of antenna No. 2.

The total power radiated by integrating the power flow of p over the hemisphere is

$$P_r = \int_{\substack{\text{hemispherical}\\ \text{surface}}} p \; dS$$

$$= \int_{\substack{\text{hemispherical}\\ \text{surface}}} (p_{11} + 2p_{12} + p_{22}) \; d\,S \qquad (126)$$

(4) Integration of Power Flow to Determine the Self and Mutual Radiation Resistance.--Performing the integration on p_{11} by applying Eq. (86) results in the self-resistance radiated power from antenna No. 1.

$$P_{11} = \int_{\substack{\text{hemispherical}\\ \text{surface}}} \frac{E_1^2 \; f_1(\theta)^2}{R_c} \; d\,S$$

$$= \frac{E_1^2 d^2}{R_c} \int_0^{2\pi} \int_0^{\pi/2} f_1(\theta)^2 \cos \theta \; d\theta \; d\phi$$

$$= \frac{2\pi E_1^2 \; d^2}{R_c} \times$$

$$\int_0^{\pi/2} \left[\frac{\cos (G_1 \sin \theta) - \cos G_1}{(1 - \cos G_1) \cos \theta} \right]^2 \cos \theta \; d\theta$$

$$= \frac{2\pi \; E_1^2 \; d^2}{R_c (1 - \cos G_1)^2} \times$$

$$\int_0^{\pi/2} \frac{[\cos (G_1 \sin \theta) - \cos G_1]^2}{\cos \theta} \; d\theta$$

$$= \frac{2\pi E_1^2 \; d^2}{R_c (1 - \cos G_1)^2} \int_0^{\pi/2} h_1^2 \; d\theta \qquad (127)$$

where h is defined in Eq. (116).

Now by Eq. (115) in the horizontal plane,

$$E_1^2 = \frac{R_c^2 \; I_1^2 \; (1 - \cos G)^2}{4\pi^2 \; d^2} \qquad (128)$$

which if substituted in Eq. (127) gives

$$P_{11} = I_1^2 \; R_{11} = I_1^2 \; \frac{R_c}{2\pi} \int_0^{\pi/2} h_1^2 \; d\,\theta \qquad (129)$$

$$R_{11} = \frac{R_c}{2\pi} \int_0^{\pi/2} h_1^2 \; d\,\theta \qquad (130)$$

= the self-loop radiation resistance of antenna No. 1.

In a similar fashion the self-resistance radiated power from antenna No. 2 is

$$P_{22} = I_2^2 \; R_{22} = I_2^2 \frac{R_c}{2\pi} \int_0^{\pi/2} h_2^2 \; d\,\theta \qquad (131)$$

where $$R_{22} = \frac{R_c}{2\pi} \int_0^{\pi/2} h_2^2 \; d\,\theta \qquad (132)$$

= the self-loop radiation resistance of antenna No. 2.

Performing the integration on p_{12} by applying Eq. (86) results in one half of the mutual-resistance radiated power, thus,

$$P_{12} = \int_{\substack{\text{hemispherical}\\ \text{surface}}} \frac{E_1 \; f_1(\theta) \; E_2 f_2(\theta) \cos [S_2 \cos(\phi_2 - \phi) \cos \theta + \Psi_2] \; d\,S}{R_c}$$

$$= \frac{E_1 \; E_2 \; d^2}{R_c}$$

$$\times \int_0^{2\pi} \int_0^{\pi/2} f_1(\theta) \; f_2(\theta) \cos [S_2 \cos (\phi_2 - \phi) \cos \theta + \Psi_2] \cos \theta \; d\,\theta \; d\,\phi$$

$$= \frac{2\pi E_1 E_2 d^2 \qquad \cos\Psi_2}{R_c(1 - \cos G_1)(1 - \cos G_2)}$$

$$\times \int_0^{\pi/2} h_1 h_2 J_o(S_2 \cos\theta)\, d\theta \qquad (133)$$

Substituting for E_1 and E_2 in the horizontal plane by applying Eq. (118) gives

$$E_1 E_2 = \frac{R_c^2 I_1 I_2 (1 - \cos G_1)(1 - \cos G_2)}{4\pi^2 d^2} \qquad (134)$$

Substituting Eq. (134) in Eq. (133) yields

$$P_{12} = I_1 I_2 \cos\Psi_2 R_{12}$$

$$= I_1 I_2 \cos\Psi_2 \frac{R_c}{2\pi}$$

$$\int_0^{\pi/2} h_1 h_2 J_o(S_2 \cos\theta)\, d\theta \qquad (135)$$

where $R_{12} = \frac{R_c}{2\pi} \int_0^{\pi/2} h_1 h_2 J_o(S_2 \cos\theta)\, d\theta \qquad (136)$

= the mutual loop radiation resistance between antennas No. 1 and No. 2.

The self and mutual resistance terms can be determined quite readily by the following graphical method developed by Edgar F. Vandivere. Referring to Eq. (136) the following approximate equation can be written

$$R_{12} = \frac{R_c}{2\pi} \int_0^{\pi/2} h_1 h_2 J_o(S_{12} \cos\theta)\, d\theta$$

$$\doteq \frac{R_c}{36} \Big[\frac{(1 - \cos G_1)(1 - \cos G_2) J_o(S_{12})}{2}$$

$$+ \sum_{n=1}^{n=8} (h_1)_{\theta = 10n} (h_2)_{\theta = 10n} J_o(S_{12} \cos 10n) \Big] \qquad (137)$$

By using curves of these three functions as drawn in Fig. 49 and Fig. 50 it is possible to determine the mutual resistance values in a relatively short time. This equation is valuable in that it will give the mutual resistance between towers of different heights.

EXAMPLE 26: Determine the mutual loop and base resistances between towers having heights of 90° and 120° when S = 200°

SOLUTION: Values of the height function h and the Bessel function J_o (S cos θ) are tabulated for various elevation angles.

n	θ	h_1 for G_1 = 90°	h_2 for G_2 = 120°	$J_o(S \cos\theta)$ for S = 200°	$h_1 h_2 J_0(S \cos\theta)$
0	0			-0.38	
1	10	0.97	1.45	-0.37	-0.52
2	20	0.89	1.30	-0.34	-0.39
3	30	0.76	1.07	-0.27	-0.22
4	40	0.61	0.83	-0.13	-0.07
5	50	0.45	0.58	0.09	0.02
6	60	0.29	0.57	0.37	0.06
7	70	0.16	0.19	0.68	0.02
8	80	0.06	0.07	0.91	0.003

Substituting in Eq. (137) the mutual loop resistance is

$$R_{12} \doteq \frac{376.7}{36} \left[\frac{(1-\cos 90°)(1-\cos 120°)(-0.38)}{2} - 1.1 \right]$$

$$\doteq 10.46\ (-0.285 - 1.1) = -14.5 \text{ ohms} \qquad \underline{\text{ANS}}$$

The mutual base resistance is obtained by dividing by ($\sin G_1 \sin G_2$), thus,

$$R_{12} \doteq \frac{-14.5}{(1.0)(0.866)} = -16.75 \text{ ohms} \qquad \underline{\text{ANS}}$$

This approximate value can be compared with -17 ohms as obtained from the mutual base impedance curves in Appendix A.

The total radiated power from the 2 element directional antenna system by Eq. (126), (129), (131), and (135) can be written

$$P_r = I_1^2 R_{11} + I_1 I_2 \cos\Psi_2 R_{12}$$

$$+ I_2 I_1 \cos\Psi_2 R_{21} + I_2^2 R_{22} \qquad (138)$$

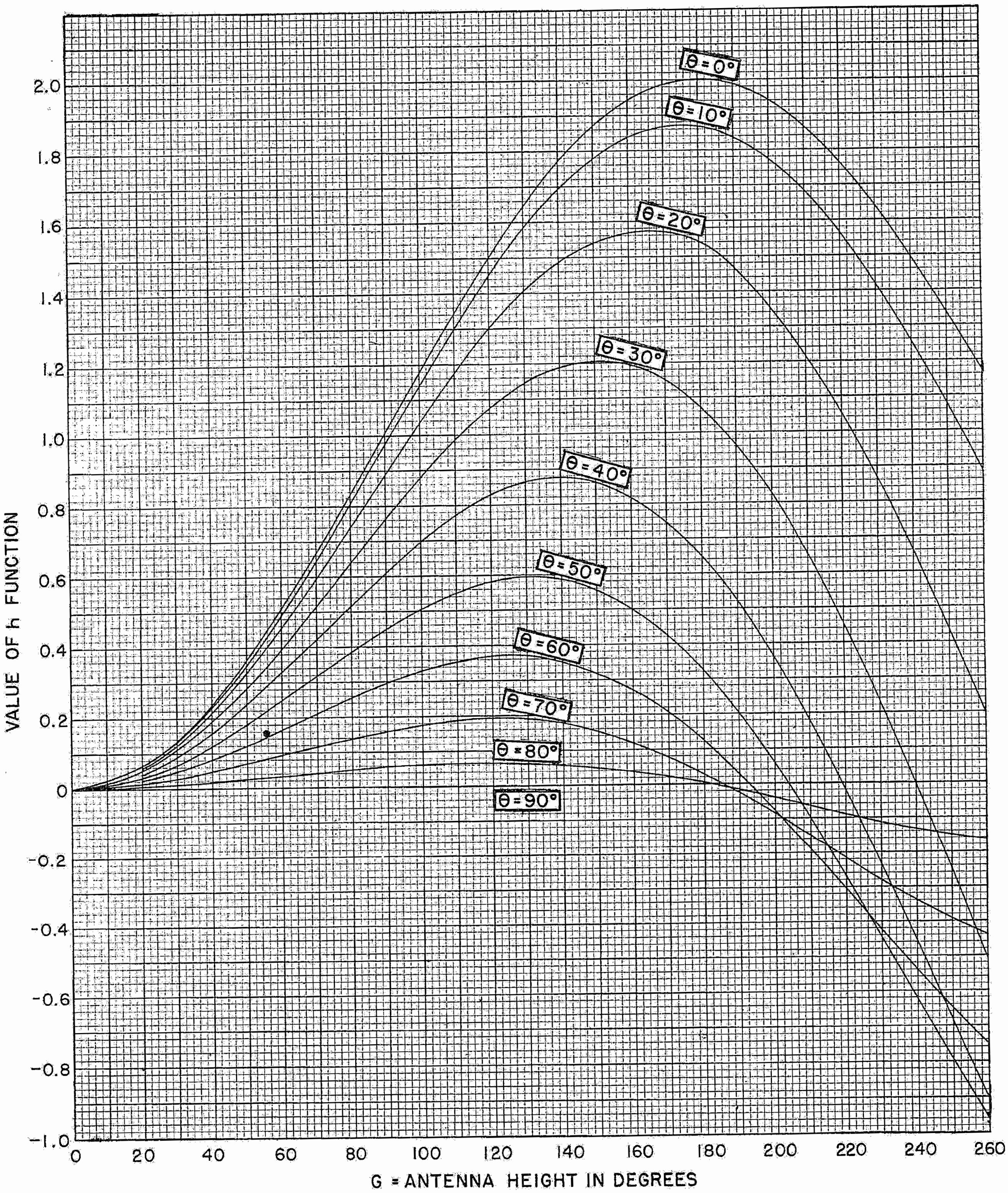
or generalizing for any number of elements in the directional antenna system we can write

$$P_r = I_1^2 R_{11} + I_2^2 R_{22} + I_3^2 R_{33} + \cdots$$

$$+ 2I_1 I_2 \cos\psi_{12} R_{12} + 2I_1 I_3 \cos\psi_{13} R_{13}$$

$$+ 2 I_2 I_3 \cos\psi_{23} R_{23} + \cdots$$

$$= \sum_{p=1}^{p=n} \sum_{q=1}^{q=n} I_p I_q \cos\psi_{pq} R_{pq} \qquad (139)$$



VALUE OF h FUNCTION PLOTTED FOR VARIOUS VALUES OF ELEVATION ANGLE θ AS A FUNCTION OF ANTENNA HEIGHT G

FIG. 49

where $p = p^{th}$ antenna in the system

$q = q^{th}$ antenna in the system

n = number of elements in the complete directional antenna array

R_{pq} = the self-loop radiation resistance if $p = q$ and if $p \neq q$ the mutual-loop radiation resistance is expressed, ohms

I_p = loop current in the p^{th} antenna, amp.

I_q = loop current in the q^{th} antenna, amp.

ψ_{pq} = difference in electrical phase angle of the voltage (or current) between the p^{th} and q^{th} antennas in the directional antenna array, degrees.

(5) Expression for Determining Pattern Size and Resulting Horizontal rms Field Intensity.--The standard hemispherical field intensity E_s produced by the directional antenna system can be obtained by substituting Eq. (139) in Eq. (87), thus,

$$E_s = \sqrt{\frac{P_r R_c}{2 \pi d^2}}$$

$$= \sqrt{\frac{R_c}{2 \pi d^2} \sum_{p=1}^{p=n} \sum_{q=1}^{p=n} I_p I_q \cos \psi_{pq} R_{pq}} . \quad (140)$$

This is an *exact* formula for determining the size of the directional antenna pattern. It gives the same results as Eq. (113) but with the application of a summation instead of an integration. The integration of terms in Eq. (113) results in the mutual and self resistance terms R_{pq} of Eq. (140).

It is now desired to express the horizontal rms field intensity E_o as a function of the antenna heights, radiated power, field intensity ratios, phasings of the antenna currents, spacing, mutual and self resistances. For the purpose of this development let us consider only 2-elements in the directional antenna array.

By Eq. (111) the rms field intensity in the horizontal plane is

$$E_o = \sqrt{E_1^2 + E_2^2 + 2 E_1 E_2 \cos \psi_{12} J_o(S_{12})}$$

$$= E_1 \sqrt{1 + \left(\frac{E_2}{E_1}\right)^2 + 2 \left(\frac{E_2}{E_1}\right) \cos \psi_{12} J_o (S_{12})} .$$

Solving for E_1 we have

$$E_1 = \frac{E_o}{\sqrt{1 + \left(\frac{E_2}{E_1}\right)^2 + 2 \left(\frac{E_2}{E_1}\right) \cos \psi_{12} J_o(S_{12})}} . \quad (141)$$

Solving for I_1 in Eq. (138) results in

$$I_1 = \frac{\sqrt{P_r}}{\sqrt{R_{11} \left[1 + \left(\frac{I_2}{I_1}\right)^2 \frac{R_{22}}{R_{11}} + 2 \left(\frac{I_2}{I_1}\right) \cos \psi_{12} \frac{R_{12}}{R_{11}} \right]}}$$

For equal height antennas $R_{11} = R_{22}$ and by Eq. (118) the current ratios can be replaced by field intensity ratios and I_1 can be replaced by E_1, thus the above equation becomes

$$E_1 = \frac{R_c (1 - \cos G)}{2 \pi d \sqrt{R_{11}}} \sqrt{\frac{P_r}{1 + \left(\frac{E_2}{E_1}\right)^2 + 2 \left(\frac{E_2}{E_1}\right) \cos \psi_{12} \frac{R_{12}}{R_{11}}}} \quad (142)$$

Equating the right hand members of Eq. (141) and (142) and solving for the horizontal rms field intensity results in

$$E_o = \frac{R_c (1 - \cos G)}{2 \pi d} \sqrt{\frac{P_r}{R_{11}}} \times \sqrt{\frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2 \left(\frac{E_2}{E_1}\right) \cos \psi_{12} J_o(S_{12})}{1 + \left(\frac{E_2}{E_1}\right)^2 + 2 \left(\frac{E_2}{E_1}\right) \cos \psi_{12} \frac{R_{12}}{R_{11}}}} \quad (143)$$

Or by applying Eq. (118) again and in doing so if the field intensity is called E_{1s} and defined as the horizontal field intensity from antenna No. 1 acting alone as a secondary reference antenna and radiating P_r then I_{1s} is the current in antenna No. 1 when the power is P_r. By making this substitution in Eq. (143) the first radical reduces to unity thus,

$$E_o = E_{1s}\sqrt{\frac{P_r}{I_{1s}^2 R_{11}}}$$

$$\times \sqrt{\frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12}\, J_o(S_{12})}{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12}\frac{R_{12}}{R_{11}}}}$$

$$= E_{1s}\sqrt{\frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12}\, J_o(S_{12})}{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12}\frac{R_{12}}{R_{11}}}} \quad (144)$$

In Eq. (143) the horizontal rms field intensity for a 2-element directional antenna array with towers of equal height is expressed in terms of the independent directional antenna parameters; G, P_r, voltage (or current) ratio $\frac{E_2}{E_1}$, ψ_{12}, S_{12}, R_{12} and R_{11}. Eq. (144) expresses this same value in terms of E_{1s} instead of P_r.

It is interesting to note that the second radical in both Eq. (143) and (144) have numerators and denominators that are very similar, the only difference being that the numerator contains the Bessel function $J_o(S_{12})$ and the denominator contains the resistance ratio $\frac{R_{12}}{R_{11}}$.

Generalizing in Eq. (143) and (144) for any number of antennas, when the towers have the same height, results in the following very useful equations.

$$E_o \doteq \frac{R_c(1 - \cos G)}{2\pi d}\sqrt{\frac{P_r}{R_{11}}} \times \sqrt{\frac{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p E_q \cos\psi_{pq} J_o(S_{pq})}{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p E_q \cos\psi_{pq}\frac{R_{pq}}{R_{11}}}} \quad (145)$$

$$E_o = E_{1s}\sqrt{\frac{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p E_q \cos\psi_{pq} J_o(S_{pq})}{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p E_q \cos\psi_{pq}\frac{R_{pq}}{R_{11}}}} \quad (146)$$

where E_o = horizontal rms field intensity millivolts per meters.

R_c = 376.71 ohms the characteristic resistance of free space.

G = electrical height of each antenna, degrees.

d = radial distance of the observation point from the directional antenna system, meters.

P_r = total power radiated from the directional antenna system, watts.

R_{11} = self-loop radiation resistance, ohms, of each antenna since they are all of the same height.

E_{1s} = horizontal field intensity from antenna No. 1 acting alone as a secondary standard reference antenna and radiating P_r, millivolts per meter.

n = number of antennas in the system.

E_p = field intensity from the p^{th} antennas, millivolts per meter.

E_q = field intensity from the q^{th} antennas, millivolts per meter.

ψ_{pq} = difference in electrical phase angle of the voltage (or current) between the p^{th} and q^{th} antennas, electrical degrees.

S_{pq} = electrical length of the spacing between the p^{th} and q^{th} antennas, electrical degrees.

R_{pq} = mutual loop resistance between the p^{th} and q^{th} antennas, ohms.

As an illustration let us substitute in Eq. (145) and (146) to express the horizontal rms field intensity for a three tower directional antenna system.

For this case

$$E_o = \frac{R_c(1-\cos G)}{2\pi d}\sqrt{\frac{P_r}{R_{11}}}\sqrt{\frac{E_1^2 + E_2^2 + E_3^2 + 2E_1E_2\cos\Psi_{12} J_o(S_{12}) + 2E_1E_3\cos\psi_{13} J_o(S_{13}) + 2E_2E_3\cos\Psi_{23} J_o(S_{23})}{E_1^2 + E_2^2 + E_3^2 + 2E_1E_2\cos\Psi_{12}\frac{R_{12}}{R_{11}} + 2E_1E_3\cos\psi_{13}\frac{R_{13}}{R_{11}} + 2E_2E_3\cos\Psi_{23}\frac{R_{23}}{R_{11}}}} \quad (147)$$

Or in terms of E_{1s}

$$E_o = E_{1s}\sqrt{\frac{E_1^2 + E_2^2 + E_3^2 + 2E_1E_2 \cos\psi_{12} J_o(S_{12}) + 2E_1E_3 \cos\psi_{13} J_o(S_{13}) + 2E_2E_3 \cos\psi_{23} J_o(S_{23})}{E_1^2 + E_2^2 + E_3^2 + 2E_1E_2 \cos\psi_{12}\frac{R_{12}}{R_{11}} + 2E_1E_3 \cos\psi_{13}\frac{R_{13}}{R_{11}} + 2E_2E_3 \cos\psi_{23}\frac{R_{23}}{R_{11}}}} \qquad (148)$$

where the terms are defined following Eq. (146).

EXAMPLE 27: A three tower directional antenna system has the following parameters:

No	G°	Ø°	S°	ψ°	F
1	90	0	0	4	1.00
2	90	0	287	-49	0.56
3	90	180	287	49	0.56

Determine the theoretical value of E_0, E_1, E_2 and E_3 when the radiated power is 1 kw.

SOLUTION: Eq. (148) if expressed in terms of field intensity ratios can be re-written,

$$E_0 = E_{1s}\sqrt{\frac{1 + F_2^2 + F_3^2 + 2F_2 \cos\psi_{12} J_0(S_{12}) + 2F_3 \cos\psi_{13} J_0(S_{13}) + 2F_2F_3 \cos\psi_{23} J_0(S_{23})}{1 + F_2^2 + F_3^2 + 2F_2 \cos\psi_{12}\frac{R_{12}}{R_{11}} + 2F_3 \cos\psi_{13}\frac{R_{13}}{R_{11}} + 2F_2F_3 \cos\psi_{23}\frac{R_{23}}{R_{11}}}}$$

where:

$E_{1s} = 194.9$ mv/m by Eq. (104)

$F_2 = 0.56$, $F_2^2 = 0.3136$

$F_3 = 0.56$, $F_3^2 = 0.3136$

$F_2F_3 = 0.3136$

$\cos\psi_{12} = \cos(-53°) = 0.6018$

$\cos\psi_{13} = \cos 45° = 0.7071$

$\cos\psi_{23} = \cos 98° = -0.1392$

$J_0(S_{12}) = J_0(S_{13}) = J_0(287°) = -0.1776$

$J_0(S_{23}) = J_0(574°) = -0.2460$

$R_{11} = R_{22} = R_{33} = 36.56$ ohms by Fig. 8 or Eq. (102)

$R_{12} = R_{13} = -9.5$ ohms by Appendix A

$R_{23} = -3.68$ ohms by Eq. (25)

Substituting in the above equation

$$E_0 = 194.9\sqrt{\frac{1 + 0.3136 + 0.3136 + 2(0.56)(0.6018)(-0.1743) + 2(0.56)(0.7071)(-0.1743) + 2(0.3136)(-0.1392)(-0.2467)}{1 + 0.3136 + 0.3136 + 2(0.56)(0.6018)(-0.26) + 2(0.56)(0.7071)(-0.26) + 2(0.3136)(-0.1392)(-0.101)}}$$

$$= 194.9\sqrt{\frac{1.6272 - 0.1172 - 0.1381 + 0.0215}{1.6272 - 0.1752 - 0.2059 + 0.0087}}$$

$$= 194.9\sqrt{\frac{1.3934}{1.2548}} = 194.9\sqrt{1.111} = 194.9(1.055)$$

$= 205.6$ mv/m ANS

By Eq. (110) for E_1 when $\theta = 0$ we have

$$E_1 = \frac{E_0}{\sqrt{1 + F_2^2 + F_3^2 + 2 F_2 \cos\psi_{12} J_0(S_{12}) + 2 F_3 \cos\psi_{13} J_0(S_{13}) + 2 F_2F_3 \cos\psi_{23} J_0(S_{23})}}$$

$$E_1 = \frac{205.6}{\sqrt{1.3934}} = 174 \text{ mv/m} \qquad \underline{\text{ANS}}$$

$$E_2 = E_1 F_2 = 174\ (0.56) = 97.5 \text{ mv/m} \qquad \underline{\text{ANS}}$$

$$E_3 = E_1 F_3 = 174\ (0.56) = 97.5 \text{ mv/m} \qquad \underline{\text{ANS}}$$

c. *DRIVING POINT IMPEDANCE METHOD OF DETERMINING PATTERN SIZE.*--(1) Derivation of Driving Point Impedance.--The total power radiated from a directional antenna system is the sum of the radiation power fed to the respective antennas. If the driving point resistance and input current to each antenna is known, the total radiated power is given by

$$P_r = \sum_{k=1}^{k=n} I_k^2 R_k$$

$$= I_1^2 R_1 + I_2^2 R_2 + \cdots + I_k^2 R_k + \cdots + I_n^2 R_n \qquad (149)$$

where P_r = total power radiated from the directional antenna, watts.

I_k = effective current at the input terminals of the k^{th} antenna amp.

R_k = driving point or radiation resistance at the input terminals of the k^{th} antenna, ohms.

n = number of antennas in the directional antenna array.

The driving point or radiation resistance R_k is the resistance component of the driving point reactance Z_k of the k^{th} antenna. The driving point impedance can be determined by using mesh circuit equations when the self-impedance of each antenna, mutual impedance between each pair of antennas and current input to each antenna is known. To properly solve the mesh circuit equations, all quantities must be handled as vectors. The mesh circuit equations for n antennas in the directional antenna system is given by

$$V_1 = I_1 Z_{11} + I_2 Z_{12} + \cdots + I_k Z_{1k} + \cdots + I_n Z_{1n}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} + \cdots + I_k Z_{2k} + \cdots + I_n Z_{2n}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$V_k = I_1 Z_{k1} + I_2 Z_{k2} + \cdots + I_k Z_{kk} + \cdots + I_n Z_{kn}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$V_n = I_1 Z_{n1} + I_2 Z_{n2} + \cdots + I_k Z_{nk} + \cdots + I_n Z_{nn} \qquad (150)$$

where V_k = vector effective voltage at the input terminals of the k^{th} antenna, volts.

I_k = vector effective current, at the input terminals of the k^{th} antenna, amp.

Z_{kk} = self impedance of the k^{th} antenna, ohms.

Z_{kn} = mutual impedance between the k^{th} and n^{th} antennas, ohms.

All of the values given in Eq. (150) refer to the antenna terminals specified by the subscripts. If all of the currents and impedances are known then all of the input voltages can be determined by solving the set of simultaneous equations given in Eq. (150).

From this set of simultaneous equations it is possible to define the driving point of impedance of the k^{th} antenna as follows

$$Z_k = \frac{V_k}{I_k} = \frac{I_1}{I_k} Z_{k1} + \frac{I_2}{I_k} Z_{k2} + \cdots + Z_{kk} + \cdots + \frac{I_n}{I_k} Z_{kn} \qquad (151)$$

where Z_k = ohms, driving point impedance of the k^{th} antenna and the other quantities are defined following Eq. (150). The resistance component R_k of Z_k is a pure radiation resistance only if there is no loss in the system. The directional antenna system can be calculated on this theoretical basis and then from a knowledge of the system the losses can be estimated with fair accuracy.

The problem now resolves itself to a determination of the self and mutual impedance and then with the current magnitude and phase specified for each antenna, Eq. (150) and (151) can be used to determine the respective driving point impedances. Since Eq. (151) will yield both the resistance and reactance components of the driving point impedance it is useful in designing the impedance matching networks between the respective transmission lines and antennas in the directional antenna array.

To determine the driving point resistance of the k^{th} antenna, the real part of Eq. (151) can be written

$$R_k = \left|\frac{I_1}{I_k}\right| \left|Z_{1k}\right| \cos(\psi_{1k} + \Upsilon_{1k}) + \left|\frac{I_2}{I_k}\right| \left|Z_{2k}\right| \cos(\psi_{2k} + \Upsilon_{2k}) + \cdots + \left|Z_{kk}\right| \cos \Upsilon_{kk} + \cdots + \left|\frac{I_n}{I_k}\right| \left|Z_{nk}\right| \cos(\psi_{nk} + \Upsilon_{nk}) \qquad (152)$$

where R_k = driving point resistance (or radiation resistance; if the system has no loss) at the input terminals of the k^{th} antenna, ohms

$\left|\frac{I_1}{I_k}\right| \left|Z_{1k}\right| \cos(\psi_{1k} + \gamma_{1k}) =$ resistance at the input terminals of the k^{th} antenna due to the effect of antenna No. 1, ohms.

$\left|\frac{I_1}{I_k}\right| \underline{\left|\psi_{1k}\right.} = \frac{I_1}{I_k}$, the vector value of the current ratio for the current at the input terminals of No. 1 and the k^{th} antenna

$\left|Z_{1k}\right| \underline{\left|\gamma_{1k}\right.} = Z_{1k}$, the vector value of the mutual impedance between the No. 1 and the k^{th} antenna, ohms.

$\left|Z_{kk}\right| \cos\gamma_{kk} = R_{kk}$, self resistance at the input terminals of the k^{th} antenna, ohms

$\left|Z_{kk}\right| \underline{\left|\gamma_{kk}\right.} = Z_{kk}$, the vector value of the self impedance of the k^{th} antenna, ohms.

In a similar fashion, the reactance component is

$$X_k = \left|\frac{I_1}{I_k}\right|\left|Z_{1k}\right| \sin(\psi_{1k} + \gamma_{1k}) + \left|\frac{I_2}{I_k}\right|\left|Z_{2k}\right| \sin(\psi_{2k} + \gamma_{2k}) + \cdots + \left|Z_{kk}\right| \sin\gamma_{kk} + \cdots + \left|\frac{I_n}{I_k}\right|\left|Z_{nk}\right| \sin(\psi_{nk} + \gamma_{nk}) \qquad (153)$$

where X_k = driving point reactance at the input terminals of the k^{th} antenna, ohms, and the other values are defined following Eq. (152).

EXAMPLE 28: Determine the driving point impedance for the directional antenna in Example 27 by Eqs. (151), (152) and (153).

SOLUTION: Using the mutual impedance curves or Eq. (25) and (26) we can write,

$$Z_{12} = Z_{13} = -9.5 + j6.0 = 11.24 \underline{|147.72^\circ} \text{ ohms}$$

$$Z_{23} = -3.675 - j4.8 = 6.045 \underline{|232.56^\circ} \text{ ohms}$$

and the self impedance can be written,

$$Z_{11} = Z_{22} = Z_{33} = 36.56 + j21 = 42.18 \underline{|29.87^\circ} \text{ ohms}$$

since the towers are all the same height the current ratios can be replaced by field intensity ratios in Eq. (151) thus,

$$Z_1 = \frac{I_1}{I_1} Z_{11} + \frac{I_2}{I_1} Z_{21} + \frac{I_3}{I_1} Z_{31}$$

$$= Z_{11} + F_2 Z_{21} + F_3 Z_{31}$$

$$= 42.18 \underline{|29.87^\circ} + 0.56 \underline{|-53^\circ}\ 11.24 \underline{|147.72^\circ} + 0.56 \underline{|45^\circ}\ 11.24 \underline{|147.72^\circ}$$

$$= 42.18 \underline{|29.87^\circ} + 6.294 \underline{|94.72^\circ} + 6.294 \underline{|192.72^\circ}$$

$$= 36.56 + j21 - 0.5178 + j6.272 - 6.1394 - j1.386$$

$$= 29.903 + j25.886 = 39.59 \underline{|40.88^\circ} \text{ ohms} \quad \underline{\text{ANS}}$$

$$Z_2 = \frac{I_1}{I_2} Z_{12} + \frac{I_2}{I_2} Z_{22} + \frac{I_3}{I_2} Z_{32}$$

$$= \frac{Z_{12}}{F_2} + Z_{22} + \frac{F_3}{F_2} Z_{32}$$

$$= \frac{11.24 \underline{|147.72^\circ}}{0.56 \underline{|-0.53^\circ}} + 6.045 \underline{|232.56^\circ} + \frac{0.56 \underline{|45^\circ}\ 6.045 \underline{|232.56^\circ}}{0.56 \underline{|-0.53^\circ}}$$

$$= 20.071 \underline{|200.72^\circ} + 42.18 \underline{|29.87^\circ} + 6.045 \underline{|330.56^\circ}$$

$$Z_2 = -18.772 - j7.101 + 36.56 + j21 + 5.264 - j2.971$$

$$= 23.052 + j10.928 = 25.52 \underline{|25.36^\circ} \text{ ohms} \quad \underline{\text{ANS}}$$

$$Z_3 = \frac{I_1}{I_3} Z_{13} + \frac{I_2}{I_3} Z_{23} + \frac{I_3}{I_3} Z_{33}$$

$$= \frac{Z_{13}}{F_3} + \frac{F_2}{F_3} Z_{23} + Z_{33}$$

$$= \frac{11.24 \underline{|147.72^\circ}}{0.56 \underline{|45^\circ}} + \frac{0.56 \underline{|-53^\circ}}{0.56 \underline{|45^\circ}}\ 6.045 \underline{|232.56^\circ} + 42.18 \underline{|29.87^\circ}$$

$$= 20.071 \underline{|102.72^\circ} + 6.045 \underline{|134.56^\circ} = 42.18 \underline{|29.87^\circ}$$

$$= -4.419 + j19.578 - 4.241 + j43.071 + 36.56 + j21$$

$$= 27.90 + j83.649 = 88.25 \underline{|71.56^\circ} \text{ ohms} \quad \underline{\text{ANS}}$$

By Eq. (152)

$$R_1 = |Z_{11}| \cos \gamma_{11} + \left|\frac{I_2}{I_1}\right| |Z_{21}| \cos (\Psi_{21} + \gamma_{21})$$

$$+ \left|\frac{I_3}{I_1}\right| |Z_{31}| \cos (\Psi_{31} + \gamma_{31})$$

$$= 42.18 \cos (29.87)$$
$$+ (0.56)(11.24) \cos (-53 + 147.72)$$
$$+ (0.56)(11.24) \cos (45 + 147.72)$$

$= 36.56 - 0.5178 - 6.1394 = 29.903$ ohms <u>ANS</u>

$$R_2 = \left|\frac{I_1}{I_2}\right| |Z_{12}| \cos(\Psi_{12} + \gamma_{12}) + |Z_{22}| \cos \gamma_{22}$$

$$+ \left|\frac{I_3}{I_2}\right| |Z_{32}| \cos(\Psi_{32} + \gamma_{32})$$

$$= \frac{11.24}{0.56} \cos (53 + 147.72) + 42.18 \cos (29.87)$$

$$+ \frac{(0.56)(6.045)}{(0.56)} \cos (45 + 53 + 232.56)$$

$= -18.772 + 36.56 + 5.264 = 23.05$ ohms <u>ANS</u>

$$R_3 = \left|\frac{I_1}{I_3}\right| |Z_{13}| \cos(\Psi_{13} + \gamma_{13})$$

$$+ \left|\frac{I_2}{I_3}\right| |Z_{23}| \cos (\Psi_{23} + \gamma_{23})$$

$$+ |Z_{33}| \cos \gamma_{33}$$

$$= \frac{11.24}{0.56} \cos(-45 + 147.72 + \frac{(0.56)(6.045)}{(0.56)}$$

$$\cos(-53 - 45 + 232.56) + 42.18 \cos(29.87)$$

$= -4.419 - 4.241 + 36.56 = 27.90$ ohms <u>ANS</u>

By Eq. (153)

$$X_1 = |Z_{11}| \sin \gamma_{11} + \left|\frac{I_2}{I_1}\right| |Z_{21}| \sin(\Psi_{21} + \gamma_{21})$$

$$+ \left|\frac{I_3}{I_1}\right| |Z_{31}| \sin (\Psi_{31} + \gamma_{31})$$

$$= 42.18 \sin (29.87)$$
$$+ (0.56)(11.24) \sin (-53 + 147.72)$$
$$+ (0.56)(11.24) \sin (45 + 147.72)$$

$= 21 + 6.272 - 1.386 = 25.886$ ohms <u>ANS</u>

$$X_2 = \left|\frac{I_1}{I_2}\right| |Z_{12}| \sin(\Psi_{12} + \gamma_{12}) + |Z_{22}| \sin \gamma_{22}$$

$$+ \left|\frac{I_3}{I_2}\right| |Z_{32}| \sin(\Psi_{32} + \gamma_{32})$$

$$= \frac{11.24}{0.56} \sin(53 + 147.72) + 42.18 \sin(29.87)$$

$$+ \frac{(0.56)(6.045)}{0.56} \sin(45 + 53 + 232.56)$$

$= -7.101 + 21 - 2.971 = 10.928$ ohms <u>ANS</u>

$$X_3 = \left|\frac{I_1}{I_3}\right| |Z_{13}| \sin (\Psi_{13} + \gamma_{13})$$

$$+ \left|\frac{I_2}{I_3}\right| |Z_{23}| \sin (\Psi_{23} + \gamma_{23})$$

$$+ |Z_{33}| \sin \gamma_{33}$$

$$= \frac{11.24}{0.56} \sin (-45 + 147.72)$$

$$+ \frac{(0.56)(6.045)}{(0.56)} \sin (-53 - 45 + 232.56)$$

$$+ 42.18 \sin (29.87)$$

$= 19.578 + 43.071 + 21 = 83.649$ ohms

(2) <u>Coupled Field Intensity From Reference Antenna</u>.--For the purpose of development, let us consider only two antennas. By Eq. (149) the power radiated is

$$P_r = I_1^2 R_1 + I_2^2 R_2.$$

Solving the equation for I_1, gives

$$I_1 = \sqrt{\frac{P_r}{R_1 + \left(\frac{I_2}{I_1}\right)^2 R_2}} \qquad (154)$$

Now by Eq. (118) we know that the field intensity from an antenna is proportional to the current in the antenna. This equation is true, regardless of whether the antenna is in a directional antenna system or not. Therefore the following ratios can be equated

$$\frac{E_{1s}}{I_{1s}} = \frac{E_1}{I_1} \tag{155}$$

where E_{1s} = horizontal field intensity from antenna No. 1 acting alone as a secondary standard reference antenna, millivolts per meter.

I_{1s} = loop current of antenna No. 1 acting alone as a secondary standard reference antenna, amp.

E_1 = horizontal field intensity from antenna No. 1 operating in the directional antenna array, millivolts per meter.

I_1 = loop current of antenna No. 1 operating in the directional antenna array, amp.

Solving Eq. (155) for E_1, and substituting for the value of I_1 from Eq. (154) results in

$$E_1 = \frac{E_{1s}}{I_{1s}} I_1 = \frac{E_{1s}}{I_{1s}} \sqrt{\frac{P_r}{R_1 + (\frac{I_2}{I_1})^2 R_2}} \tag{156}$$

If E_{1s} and I_{1s} are for the same amount of power as radiated from the directional antenna, then

$$\frac{P_r}{I_{1s}^2} = R_{11} \tag{157}$$

the self loop radiation resistance of antenna No. 1. Making this substitution in Eq. (156) yields

$$E_1 = E_{1s} \sqrt{\frac{R_{11}}{R_1 + (\frac{I_2}{I_1})^2 R_2}} \tag{158}$$

This equation says that the coupled field intensity E_1 from the reference antenna is equal to the non-directional field intensity E_{1s} from this antenna, when it is radiating P_r, the power radiated from the directional antenna multiplied by the square root of the ratio of the self-loop radiation resistance of antenna No. 1 to the quantity made up of the driving point resistance of antenna No. 1 plus the square of the current ratio between antenna No. 2 and antenna No. 1, multiplied by the driving point radiation resistance of antenna No. 2. In other words, by this equation it is possible to determine the coupled field intensity E_1 from antenna No. 1 in terms of the secondary standard reference field intensity E_{1s} from antenna No. 1 when it is radiating P_r, the self-loop radiation resistance R_{11} of antenna No. 1, the driving point radiation resistances of all the antennas in the array and the current ratios. This means that with a specified radiated power from the directional antenna system, the field intensity from antenna No. 1 can be calculated directly. If the resistance values are due solely to radiation, then the resulting field intensity from antenna No. 1 is theoretically the field that would result if the system had no loss. The value of this field intensity can be reduced to a reasonable practical value by inserting appropriate loss resistances in series with the driving point resistances for each antenna. This method has some merit in that the losses for a given tower and ground system can be taken care of at the driving point, thus, if one of the towers has a high driving point current, the loss power will be proportional to the square of the current in that tower. This method, however, does not take into account losses in the rest of the feeder system which usually is quite low.

If Eq. (158) is generalized for any number of elements in the directional antenna system we can write

$$E_1 = E_{1s} \sqrt{\frac{R_{11}}{R_1 + (\frac{I_2}{I_1})^2 R_2 + \cdots + (\frac{I_k}{I_1})^2 R_k + \cdots + (\frac{I_n}{I_1})^2 R_n}} = \frac{E_{1s}\, I_1 \sqrt{R_{11}}}{\sqrt{\sum_{k=1}^{k=n} I_k^2 R_k}} \tag{159}$$

where E_1 = horizontal field intensity from antenna No. 1 operating in the directional antenna array, mv/m.

E_{1s} = horizontal field intensity from antenna No. 1 acting alone as a secondary standard reference antenna and radiating P_r, mv/m.

R_{11} = self-loop radiation resistance of antenna No. 1, ohms.

R_k = driving point resistance at the input terminals of the k^{th} antenna. If these values are pure radiation resistance, E_1 will be theoretical, however, if they contain loss, E_1 can be made to approach actual operating conditions, ohms.

I_k = loop current of the k^{th} antenna, amp.

I_1 = loop current of antenna No. 1, amp.

EXAMPLE 29: Determine the theoretical horizontal field intensity of No. 1 tower of the three tower directional antenna of Example 27 using the driving point impedance method when the radiated power is 1 kw.

SOLUTION: For 1 kw operation E_{1s} = 194.9 mv/m. Using the values of driving point resistances in Example 28 substituted in Eq. (159) gives,

$$E_1 = E_{1s}\sqrt{\frac{R_{11}}{R_1 + F_2^2 R_2 + F_3^2 R_3}}$$

$$= 194.9\sqrt{\frac{36.56}{29.9 + (0.3136)(23.05) + (0.3136)(27.9)}}$$

$$= 194.9\sqrt{\frac{36.56}{29.9 + 7.23 + 8.75}}$$

$$= 194.9\sqrt{\frac{36.56}{45.88}}$$

$$= 194.9\sqrt{0.7968} = (194.9)(0.894)$$

$$= 174 \text{ mv/m}$$ ANS

It should be noted that the driving point impedance method solution is in agreement with the mutual resistance method solution of Example 27.

It is also of interest to determine the horizontal rms field intensity in terms of the driving point and self resistances rather than in terms of the mutual and self resistances as given in Eq. (145) and Eq. (109). For only two elements,

$$E_o = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\psi_{12} J_o(S_{12})}$$

$$= E_1\sqrt{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12} J_o(S_{12})} \qquad (160)$$

Substituting for the value of E_1 from Eq. (158) results in

$$E_o = E_{1s}\sqrt{\frac{R_{11}}{R_1 + \left(\frac{I_2}{I_1}\right)^2 R_2}}$$

$$\times\sqrt{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12} J_o(S_{12})}$$

$$= E_{1s}\frac{\sqrt{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12} J_o(S_{12})}}{\sqrt{\frac{R_1}{R_{11}} + \left(\frac{I_2}{I_1}\right)^2\frac{R_2}{R_{11}}}}$$

$$= E_{1s}\sqrt{\frac{R_{11}}{R_1}}$$

$$\times\sqrt{\frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1}\cos\psi_{12} J_o(S_{12})}{1 + \left(\frac{I_2}{I_1}\right)^2\frac{R_2}{R_1}}}$$

$$= \frac{E_{1s} I_1\sqrt{R_{11}}}{E_1}$$

$$\times\sqrt{\frac{E_1^2 + E_2^2 + 2E_1E_2\cos\Psi_{12} J_o(S_{12})}{R_1 I_1^2 + R_2 I_2^2}} \qquad (161)$$

Generalizing this equation for any number of antennas

$$E_o = \frac{E_{1s} I_1\sqrt{R_{11}}}{E_1}$$

$$\times\sqrt{\frac{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p E_q \cos\psi_{pq} J_o(S_{pq})}{\sum_{k=1}^{k=n} I_k^2 R_k}} \qquad (162)$$

where E_o = horizontal rms field intensity from the directional antenna array, mv/m.

E_{1s} = horizontal field intensity antenna No. 1 acting alone as a secondary standard reference antenna and radiating P_r, mv/m.

I_1 = loop driving point current in antenna No. 1, amp.

R_{11} = self loop radiation resistance of antenna No. 1, ohms.

E_1 = horizontal field intensity of antenna No. 1 operating in the directional antenna array, mv/m.

E_p = horizontal field intensity of antenna No. p operating in the directional antenna array, mv/m.

E_q = horizontal field intensity of antenna No. q operating in the directional antenna array, mv/m,

ψ_{pq} = difference in phase angle of the voltage (or current) between the p^{th} and the q^{th} antenna, electrical degrees.

S_{pq} = length of the spacing between the p^{th} and the q^{th} antennas, electrical degrees.

I_k = loop driving point current in the k^{th} antenna, amp.

R_k = loop driving point resistance of the k^{th} antenna, ohms.

For the special case of equal height elements the current ratios can be replaced by the corresponding field intensity ratios, resulting in

$$E_o = E_{1s} \sqrt{R_{11}} \times \sqrt{\frac{\sum_{p=1}^{p=n} \sum_{q=1}^{q=n} E_p E_q \cos \psi_{pq} J_o(S_{pq})}{\sum_{k=1}^{k=n} E_k^2 R_k}} \quad (163)$$

where the values are as defined for Eq. (162). This equation corresponds to Eq. (146) with the exception that driving point resistance values are used in place of mutual resistance values.

(3) Mutual Impedance Curves.--In order to use the driving point impedance method for determining pattern size it is necessary to know the values of mutual impedance, which includes both the resistance and reactance components or expressed in polar form as a magnitude at an angle. A number of mutual impedance curves are presented in Appendix A. (See page 2-1-16.) Values for these curves can be substituted in Eq. (152) for determining the driving point resistance which in turn is used in Eq. (162) to determine the horizontal rms field intensity.

d. *HORIZONTAL RMS POWER GAIN OF A DIRECTIONAL ANTENNA ARRAY.*--The coverage of a radio broadcasting station can be increased if for the same power input the horizontal field intensity is increased. It is therefore worth while to investigate the conditions in a directional antenna system that controls the suppression of high angle radiation and increases the low angle radiation along the ground. The merit of a directional antenna in terms of improving the coverage can be expressed in terms of horizontal rms power gain. This horizontal rms power gain is a comparison of the horizontal rms field intensity for the directional antenna array with the horizontal field intensity from the reference element in the array on the basis of equal radiated powers.

Mathematically, the horizontal rms power gain of a directional antenna by Eq. (12) can be defined

$$g_o = \left[\frac{E_o}{E_{1s}}\right]^2 \quad \text{(equal powers)} \quad (164)$$

where g_o = horizontal rms power gain of a directional antenna array

E_o = the horizontal field intensity when the directional antenna is radiating P_r watts, mv/m.

E_{1s} = the horizontal field intensity from No. 1 antenna acting as a secondary reference antenna and is radiating P_r watts, mv/m.

The horizontal rms power gain for a two element directional antenna array, by Eq. (144) and (161) is given by

$$g_o = \frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1} \cos \psi_{12} J_o(S_{12})}{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1} \cos \psi_{12} \frac{R_{12}}{R_{11}}}$$

$$= I_1^2 R_{11} \frac{1 + \left(\frac{E_2}{E_1}\right)^2 + 2\frac{E_2}{E_1} \cos \psi_{12} J_o(S_{12})}{I_1^2 R_1 + I_2^2 R_2} \quad (165)$$

By applying Eq. (146) and (162) this equation can be generalized for any number of antennas, thus

$$g_o = \frac{\sum_{p=1}^{p=n} \sum_{q=1}^{q=n} E_p E_q \cos \psi_{pq} J_o(S_{pq})}{\sum_{p=1}^{p=n} \sum_{q=1}^{q=n} E_p E_q \cos \psi_{pq} \frac{R_{pq}}{R_{11}}}$$

$$= \frac{I_1^2 R_{11}}{E_1^2} \frac{\sum_{p=1}^{p=n} \sum_{q=1}^{q=n} E_p E_q \cos \psi_{pq} J_o(S_{pq})}{\sum_{k=1}^{k=n} I_k^2 R_k} \quad (166)$$

EXAMPLE 30: Determine the horizontal r-m-s power gain of the directional antenna in Example 27.

SOLUTION: Using the first form of Eq. (166) the terms under the radical in Example 27 can be written,

$$g_o = \frac{1.3934}{1.2548} = 1.111 \text{ the horizontal r-m-s power gain.}$$ ANS

e. *EFFICIENCY OF DIRECTIONAL ANTENNAS*--(1) Definitions.--The horizontal rms field intensity of a directional antenna system will depend upon the amount of high angle radiation as compared to the horizontal radiation. For a directional antenna system with no loss this value can be expressed,

$$E_o = E_{1s}\sqrt{g_o} \tag{167}$$

where E_o = horizontal rms field intensity without loss, mv/m

E_{1s} = horizontal field intensity of antenna No. 1 acting alone as a secondary standard reference antenna and radiating P_r, mv/m

g_o = Horizontal rms power gain of directional antenna system compared to the secondary standard reference antenna as defined in Eq. (164).

It is to be noted that the power gain g_o is defined, in Eq. (164), with no loss in either the directional antenna array or the secondary reference standard antenna. Therefore, the horizontal r-m-s field gain, $\sqrt{g_o}$, represents the inherent property of the directional antenna array as to whether it is a gainer or losser of horizontal r-m-s field intensity when compared with the secondary standard reference antenna.

Now, if the directional antenna system has loss the horizontal rms field intensity must be multiplied by the square root of the power efficiency, thus

$$E_{0L} = E_o\sqrt{\eta} = E_{1s}\sqrt{g_o}\ \sqrt{\eta} \tag{168}$$

where E_{0L} = horizontal rms field intensity with loss, mv/m and the power efficiency of the directional antenna system can be defined by,

$$\eta = \frac{P_r}{P_r + P_L} \tag{169}$$

where η = power efficiency of directional antenna system (Multiply by 100 to get efficiency in percent)

P_r = power radiated from directional antenna system, watts

P_L = power lost in directional antenna system, watts.

(2) Two Tower Case.--For a two tower directional antenna system Eq. (138) can be rewritten,

$$P_r = I_1^2 R_{11}\left(1 + M_2^2 \frac{R_{22}}{R_{11}} + 2 M_2 \cos\psi_2 \frac{R_{12}}{R_{11}}\right) \tag{170}$$

where P_r = power radiated from directional antenna system, watts

I_1 = base current magnitude in antenna No. 1, amp

M_2 = I_2/I_1, ratio of base current magnitudes

R_{11} = base self resistance of antenna No. 1, ohms

R_{22} = base self resistance of antenna No. 2, ohms

ψ_2 = time phase of voltage (or current) in antenna No. 2 with respect to antenna No. 1, degrees

R_{12} = base mutual resistance between antenna No. 1 and antenna No. 2, ohms.

And since the power lost in the directional antenna system[8] can be assumed to be in series with the base impedance of each tower it can be written,

$$P_L = I_1^2 R_{L1} + I_2^2 R_{L2} = I_1^2 R_{11}\left(\frac{R_{L1}}{R_{11}} + M_2^2 \frac{R_{L2}}{R_{11}}\right) \tag{171}$$

where P_L = power lost in directional antenna system, watts

R_{L1} = series base loss resistance of antenna No. 1, ohms

R_{L2} = series base loss resistance of antenna No. 2, ohms

I_1 = base current magnitude in antenna No. 1, amp

I_2 = base current magnitude in antenna No. 2, amp.

Substituting Eq. (170) and (171) in Eq. (169) the efficiency can be written,

$$\eta = \frac{1}{1 + \dfrac{\dfrac{R_{L1}}{R_{11}} + M_2^2 \dfrac{R_{L2}}{R_{11}}}{1 + M_2^2 \dfrac{R_{22}}{R_{11}} + 2M_2 \cos\psi_2 \dfrac{R_{12}}{R_{11}}}} \tag{172}$$

[8] Carl E. Smith and Earl M. Johnson, "Performance of Short Antennas" Proc. I.R.E., Vol. 35, No. 10, pp 1026-1038, October, 1947.

The power gain of Eq. (165) for two towers of unequal height can be written,

$$g_o = \frac{1 + F_2^2 + 2F_2 \cos \psi_2 \, J_o \,(S_2)}{1 + M_2^2 \dfrac{R_{22}}{R_{11}} + 2\, M_2 \cos \psi_2 \dfrac{R_{12}}{R_{11}}} \qquad (173)$$

where g_o = horizontal rms power gain of directional antenna system compared to the secondary standard reference antenna

$F_2 = E_2/E_1$, ratio of field intensity magnitudes

S_2 = space phase of antenna No. 2 with respect to antenna No. 1, degrees

and the other terms are defined as following Eq. (170).

Substituting Eq. (172) and (173) in Eq. (168) results in,

$$E_{OL} = E_{1s} \sqrt{\frac{1 + F_2^2 + 2F_2 \cos \psi_2 \, J_o \,(S_2)}{1 + M_2^2 \dfrac{R_{22}}{R_{11}} + 2M_2 \cos \psi_2 \dfrac{R_{12}}{R_{11}} + \dfrac{R_{L1}}{R_{11}} + M_2^2 \dfrac{R_{L2}}{R_{11}}}} \qquad (174)$$

Now if $R_{L1} = R_{L2} = 0$ the *no loss case* results, thus

$$E_o = E_{1s} \sqrt{\frac{1 + F_2^2 + 2F_2 \cos \psi_2 \, J_o \,(S_2)}{1 + M_2^2 \dfrac{R_{22}}{R_{11}} + 2M_2 \text{ Cos } \psi_2 \dfrac{R_{12}}{R_{11}}}} \qquad (175)$$

For the special case of equal height towers $R_{11} = R_{22}$, $R_{L1} = R_{L2} = R_L$ and the current ratios M_2 can be replaced by field ratios F_2, thus Eq. (172) for the efficiency can be written

$$\eta = \frac{R_{11}}{R_{11} + R_L \dfrac{1 + F_2^2}{1 + F_2^2 + 2F_2 \cos \psi_2 \dfrac{R_{12}}{R_{11}}}} \qquad (176)$$

EXAMPLE 31: If each tower in Example 27 has a series base loss resistance of 2 ohms determine the efficiency of the directional antenna and the horizontal r-m-s field intensity.

SOLUTION: If Eq. (176) is written for a 3-tower directional antenna we have,

$$\eta = \frac{R_{11}}{R_{11} + R_L \dfrac{1 + F_2^2 + F_3^2}{\left(1 + F_2^2 + F_3^2 + 2\,F_2 \cos \psi_{12} \dfrac{R_{12}}{R_{11}} + 2\,F_3 \cos \psi_{13} \dfrac{R_{13}}{R_{11}} + 2\,F_2\,F_3 \cos \psi_{23} \dfrac{R_{23}}{R_{11}}\right)}}$$

Now from Example 27 and the above conditions,

$$\eta = \frac{36.56}{36.56 + 2 \dfrac{1.6272}{1.2548}} = 93.4\% \text{ efficiency}$$

By Eq. (168), the solution of Example 27 and the above η the horizontal r-m-s field intensity with loss will be,

$$E_{OL} = E_{1S} \sqrt{g_o} \quad \sqrt{\eta}$$

$$= 194.9 \ \sqrt{1.111} \ \sqrt{0.934} = 198.5 \text{ mv/m} \quad \underline{\text{ANS}}$$

This field intensity is based upon an input power of 1,081 watts to the common point of the directional antenna system according to FCC standards, with 81 watts being lost in the feeder system and 66 watts being lost in the ground system. The power radiated in this case is 934 watts.

If g_o of Eq. (173) is multiplied by the fraction in the denominator of Eq. (176) we can rewrite Eq. (176) as follows after taking the square root of both sides,

$$\sqrt{\eta} = \frac{R_{11}}{R_{11} + g_o \dfrac{1 + F_2^2}{1 + F_2^2 + 2F_2 \cos \psi_2 \, J_o(S_2)} R_L} \qquad (177)$$

Glenn D. Gillett has independently arrived at this form of the efficiency equation for the special case of equal height towers.

(3) Generalized case for any number of towers.--In order to generalize Eq. (174) let us replace the ratios with the original field intensity values and use double subscripts, thus for two towers,

$$E_{0L} = E_{1s}\sqrt{\frac{\frac{R_{11}\,I_1^2}{E_1^2}\left[E_1^2 + E_2^2 + 2E_1E_2 + 2E_1E_2\cos\psi_{12}\,J_o\,(S_{12})\right]}{R_{11}\,I_1^2 + R_{22}\,I_2^2 + 2I_1\,I_2\cos\psi_{12}\,R_{12} + R_{L1}\,I_1^2 + R_{L2}\,I_2^2}} \qquad (178)$$

Referring to Eq. (144), (146) and (149) the same method can be used to generalize this equation, thus

$$E_{0L} = \frac{E_{1s}\,I_1\sqrt{R_{11}}}{E_1}\sqrt{\frac{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} E_p\,E_q \times \cos\psi_{pq}\,J_o\,(S_{pq})}{\sum_{p=1}^{p=n}\sum_{q=1}^{q=n} I_p\,I_q \times \cos\psi_{pq}\,R_{pq} + \sum_{k=1}^{k=n} I_k^2\,R_{Lk}}} \qquad (179)$$

where E_{0L} = horizontal rms field intensity with loss, mv/m

E_{1s} = horizontal field intensity of antenna No. 1 acting alone as a secondary standard reference antenna and radiating P_r, mv/m

I_1 = base current magnitude in antenna No. 1, amp

R_{11} = base self resistance of antenna No. 1, ohms

E_1 = horizontal magnitude field intensity from antenna No. 1 operating in the directional antenna array, mv/m

n = number of towers in the array

E_p = horizontal field intensity magnitude of p^{th} antenna, mv/m

E_q = horizontal field intensity magnitude of q^{th} antenna, mv/m

ψ_{pq} = time phase of voltage (or current) between the p^{th} and q^{th} antennas in the directional antenna array, degrees

S_{pq} = space phase between p^{th} and q^{th} antennas in degrees or radians depending on the values in the Bessel function tables.

R_{pq} = mutual base resistance between the p^{th} and q^{th} antennas, ohms
= self base resistance when p = q, ohms

I_p = base current magnitude in p^{th} antenna, amp

I_q = base current magnitude in q^{th} antenna, amp

I_k = base current magnitude in k^{th} antenna, amp

R_{Lk} = base series loss resistance in k^{th} antenna, amp.

The k subscripts could be replaced with either p or q subscripts without altering the final answer. The k subscript terms are the power loss terms in the directional antenna system.

EXAMPLE 32--Write out the equation for E_{0L} for a three tower array.

SOLUTION--Using general Eq. (179) we have for a three tower array.

$$\begin{aligned}\text{The Numerator} = {} & E_1\,E_1\cos\psi_{11}\,J_o\,(S_{11}) \\ & + E_2\,E_1\cos\psi_{21}\,J_o\,(S_{21}) \\ & + E_3\,E_1\cos\psi_{31}\,J_o\,(S_{31}) \\ & + E_1\,E_2\cos\psi_{12}\,J_o\,(S_{12}) \\ & + E_2\,E_2\cos\psi_{22}\,J_o\,(S_{22}) \\ & + E_3\,E_2\cos\psi_{32}\,J_o\,(S_{32}) \\ & + E_1\,E_3\cos\psi_{13}\,J_o\,(S_{13}) \\ & + E_2\,E_3\cos\psi_{23}\,J_o\,(S_{23}) \\ & + E_3\,E_3\cos\psi_{33}\,J_o\,(S_{33})\end{aligned}$$

since $\psi_{11} = \psi_{22} = \psi_{33} = 0$, $S_{11} = S_{22} = S_{33} = 0$
$\psi_{13} = -\psi_{31}$ $\psi_{21} = -\psi_{12}$, $\psi_{23} = -\psi_{32}$,
$S_{12} = S_{21}$, $S_{31} = S_{13}$ and $S_{23} = S_{32}$

With these substitutions we can write the numerator =

$$\begin{aligned}& E_1^2 + E_2^2 + E_3^2 + 2E_1\,E_2\cos\psi_{12}\,J_o\,(S_{12}) \\ & + 2E_2\,E_3\cos\psi_{23}\,J_o\,(S_{23}) \\ & + 2E_3\,E_1\cos\psi_{31}\,J_o\,(S_{31})\end{aligned}$$

Similarly, the Denominator =

$$\begin{aligned}& I_1\,I_1\cos\psi_{11}\,R_{11} + I_2\,I_1\cos\psi_{21}\,R_{21} \\ & + I_3\,I_1\cos\psi_{31}\,R_{31} + I_1\,I_2\cos\psi_{21}\,R_{12} \\ & + I_2\,I_2\cos\psi_{22}\,R_{22} + I_3\,I_2\cos\psi_{32}\,R_{32} \\ & + I_1\,I_3\cos\psi_{13}\,R_{13} + I_2\,I_3\cos\psi_{23}\,R_{23} \\ & + I_3\,I_3\cos\psi_{33}\,R_{33} + R_{L1}\,I_1^2 + R_{L2}\,I_2^2 \\ & + R_{L3}\,I_3^2\end{aligned}$$

also $R_{12} = R_{21}$, $R_{23} = R_{32}$ and $R_{31} = R_{13}$

With these substitutions we can write the denominator =

$$I_1^2 R_{11} + I_2^2 R_{22} + I_3^2 R_{33} + 2I_1 I_2 \cos \psi_{12} R_{12} + 2I_2 I_3 \cos \psi_{23} R_{23} + 2I_3 I_1 \cos \psi_{31} R_{31} + I_1^2 R_{L1} + I_2^2 R_{L2} + I_3^2 R_{L3}$$

Therefore, the full equation is,

$$E_{OL} = \frac{E_{1s} I_1 \sqrt{R_{11}}}{E_1} \sqrt{\frac{E_1^2 + E_2^2 + E_3^2 + 2E_1 E_2 \cos \psi_{12} J_o (S_{12}) + 2E_2 E_3 \cos \psi_{23} J_o (S_{23}) + 2E_3 E_1 \cos \psi_{31} J_o (S_{31})}{I_1^2 R_{11} + I_2^2 R_{22} + I_3^2 R_{33} + 2I_1 I_2 \cos \psi_{12} R_{12} + 2I_2 I_3 \cos \psi_{23} R_{23} + 2I_3 I_1 \cos \psi_{31} R_{31} + I_1^2 R_{L1} + I_2^2 R_{L2} + I_3^2 R_{L3}}}$$

ANS

5. FEEDER SYSTEM DESIGN

a. *INTRODUCTION*.--A directional antenna feeder system can be designed in many ways. The problem facing the design engineer is to make a simple economical design with adequate controls having sufficient range to make the necessary adjustments. The feeder sustem must be adjusted, during the equipment test period, such that the directional antenna will produce the required pattern in accordance with the specifications of the construction permit issued by FCC.

In general a directional antenna feeder system will consist of power dividing networks, phase shifting networks, transmission lines, and impedance matching networks. A block diagram of a typical directional antenna feeder system is illustrated in Fig. 22. In this feeder system the transmitter output impedance is transformed to match the input impedance of the power divider. The power divider in turn splits the power in the proper proportion between the two antennas. Antenna No. 1 is fed directly through an impedance matching network while antenna No. 2 is fed through a phase shifting network, transmission line and impedance matching network.

Before starting to design the feeder system it is necessary to have some idea about the driving point impedances. As a first approximation the theoretical design values can be used. If the towers are base fed, have a thin uniform cross-section and are not too tall this approximation should be satisfactory. If not, then more tuning range should be provided in the feeder system, particularly in the impedance matching networks at the base of the towers.

b. *GENERAL DESIGN PROCEDURE*.--One general procedure is to start with each tower and determine the possible phase shifts in all networks back to the power divider. Then various combinations of networks can be considered in view of making a simple, efficient design with adequate control.

The *first step* in following this procedure is to determine the driving point impedance and current phase at the input terminals at the base of each tower. Methods outlined in the previous section can be used to determine these values.

The *second step* is to transform the driving point impedance at the base of the tower to match the characteristic impedance of the transmission line. This can be done by means of an L-section, T or π section or an indirectly coupled circuit.

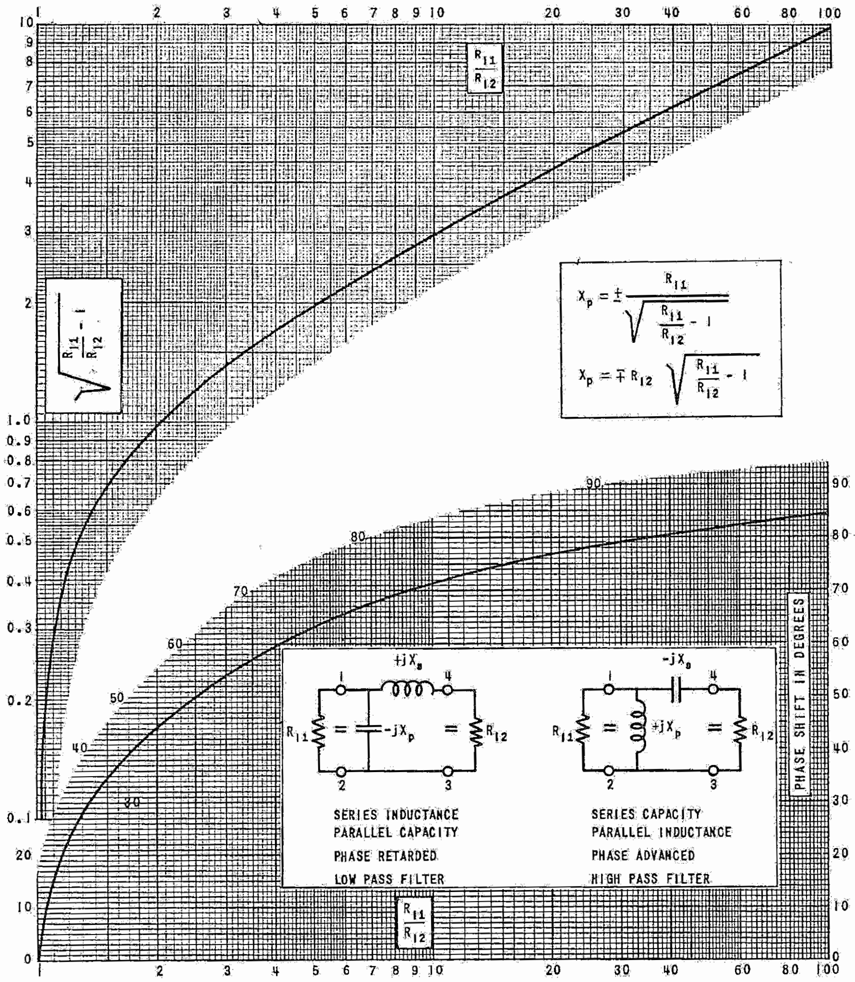
This subject has been covered thoroughly by W. L. Everitt, a brief resume of which is presented in Section 5-3 of the NAB Engineering Handbook by Arthur C. Stewart.

It is suggested that in determining the phase shifts initially an L-section with a phase advance or retard be specified wherever only impedance matching is required. An L-section has two elements and in all cases can be made to theoretically satisfy the impedance transformation requirement. The amount of phase shift is completely controlled by the resistance ratio of the terminating resistances at the end of the L-section. See Fig. 51. The phase can be advanced or retarded a given number of degrees depending on whether the L-section is a high or low pass type of filter section.

The *third step* is to determine the phase shift in the transmission lines. Usually, the physical layout of the towers and transmitter building will control the length of each transmission line. The amount of phase shift in the line will be determined by the physical length of the line, specified in degrees, and multiplied by the velocity of propagation which is usually specified by the manufacturer in percent of the free space velocity. See Section 5-1 of the NAB Engineering Handbook.

The transmission lines usually emanate from a common point either in the transmitter building or a tuning house centrally located such as to minimize the feeder system losses. At this common point, where the input impedance to the directional antenna system is measured, it is necessary to provide power dividing and phase shifting networks.

The *fourth step* is to specify 90° phase advancing or retarding networks at the common point end of each transmission line. See Section



L - SECTION REACTANCE NETWORK
FIG. 51

5-3 of the NAB Engineering Handbook. This is a widely used type of phase shifter because practically a pure phase shift without impedance transformation can be achieved by varying the series reactance elements of the T-section in unison and leaving the shunt reactance element fixed in value. The shunt reactance element controls the characteristic impedance of the network while the series elements control the phase shift. Variation of the shunt impedance value can be used to advantage in some applications as a power division control. If the series elements are inductive and the shunt element is capacitive the network will retard the phase. The network can be made to advance the phase if capacitors of suitable value are placed in series with the series variable inductors and an inductor is used as the shunt arm. The usual practice is to use variable inductors rather than variable capacitors in this particular type of application.

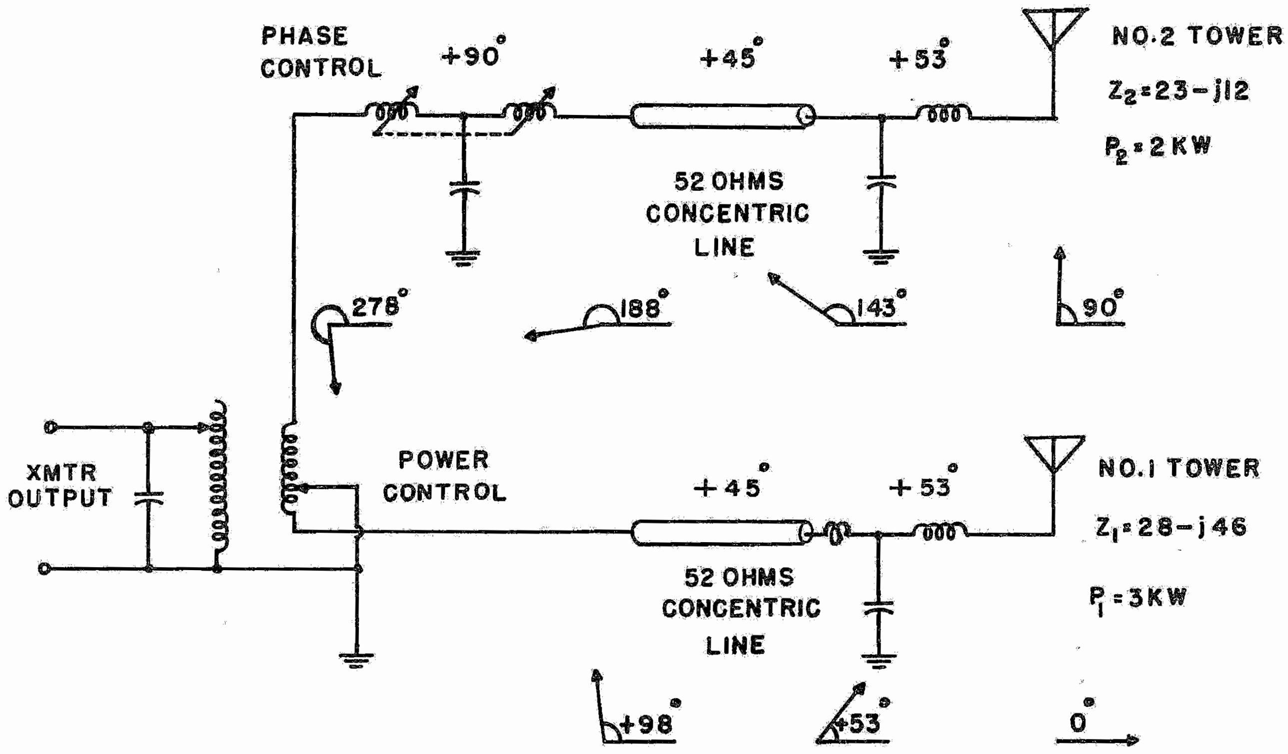
It is also possible to achieve the same results with an equivalent π-section that will either advance or retard the phase. In a π-section the shunt arms have practically pure phase control while the series arm will control the characteristic impedance or power division.

Often the feeder system can be simplified and made more efficient by making the networks as simple and direct as possible from the common point to the tower taking the most power. It is often possible to eliminate the power divider and phase shifting network in this line. For this case the impedance matching networks can be simple L-sections. Then in the feeder lines to towers requiring less power, the power division and phase shifting networks can be inserted for supplying the necessary control. If the phase shifts do not work out properly with L-sections used for impedance matching then the matching network must be expanded into T or π sections which will give the necessary phase shift to make the phase shifters operate at or near 90°.

The *fifth step* is to review the total phase shift from the power divider to each tower for various combinations of phase advancing and retarding networks in an effort to select a combination which will give a total phase shift in each line such that all vectors are of the proper phase at the power divider. Depending upon the type of power divider to be used the phases may all be equal, 90° or 180° out of phase.

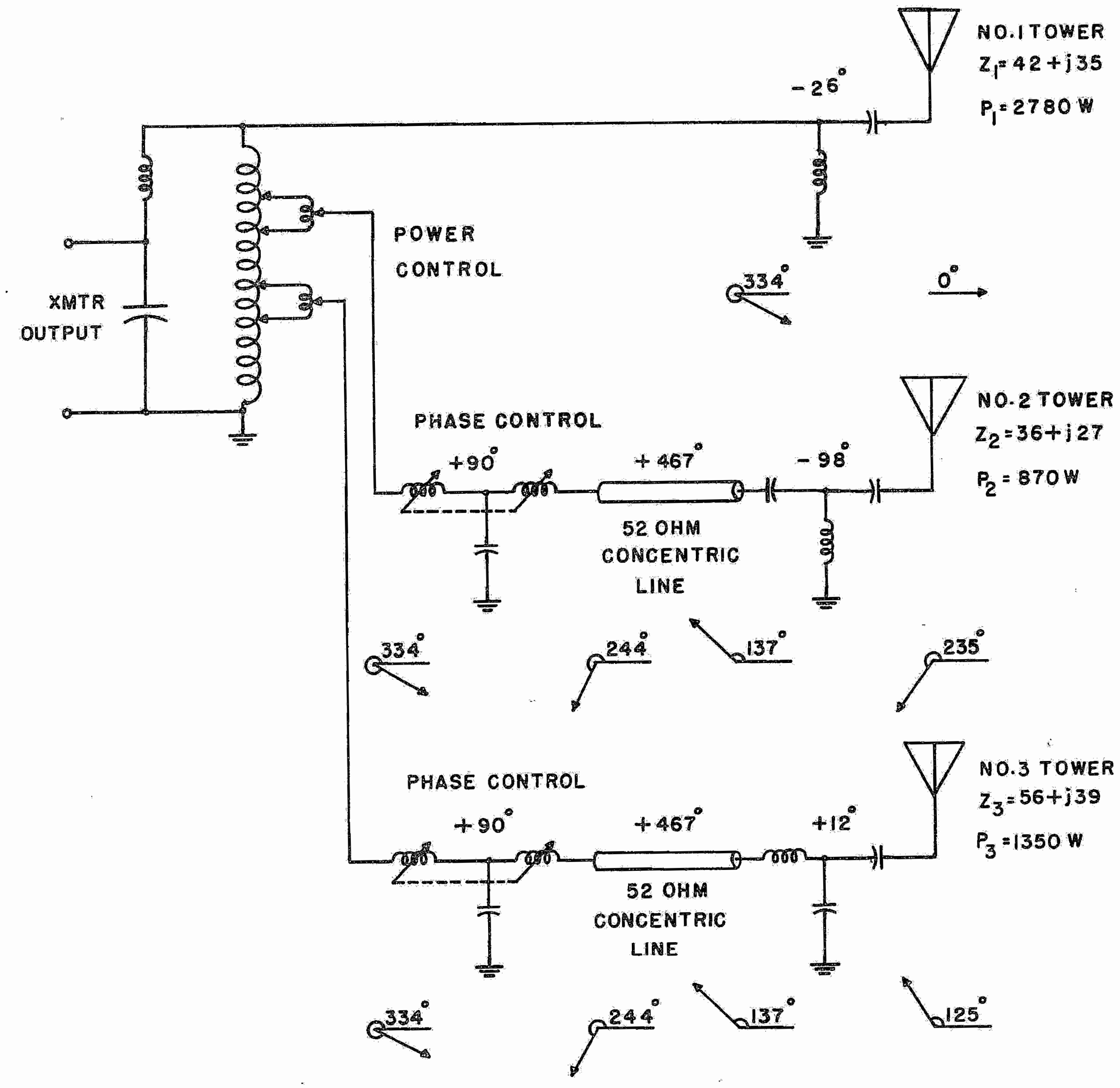
In order to minimize possible confusion in following through with this 5 step procedure it is helpful to line up the driving point impedance values and current or field vector positions along the right hand side of a work sheet as shown in Fig. 52. In this figure No. 2 tower has a driving point resistance of 23 ohms to be transformed to 52 ohms with an L-section. Referring to Fig. 51 the phase shift will be approximately ± 53°. Since the driving point current vector has a phase of +90° a lagging L-section will have an input vector of 143° = 90° + 53°. Adding the phase lag of the transmission line we have an input vector to the line with a phase of 188° = 45° + 143°. Finally, the 90° lagging phase shifter will have an imput vector phase of 278° = 90° + 183°.

The phase shifts in the No. 1 tower feeder circuit are as follows. *First*, the driving point resistance is 28 ohms and the phase is 0°. *Second*, the phase shift in an L-section is approximately ± 48° from Fig. 52, giving an L-section input phase of 48° for a lagging network. *Third*, the input phase to No. 1 transmission line will be 93°. *Fourth*, since No. 2 tower has a phase shifter the phase of No. 2 tower can be controlled with respect to No. 1 tower hence it is desirable to feed power into this line directly from the power divider. Furthermore, this would be desirable from an efficiency standpoint since No. 1 tower takes the largest amount of power. *Fifth*, a review of the phase shifts shows that 278° the phase of the current input to No. 2 network is almost 180° out of phase with 93° the phase of the current input to No. 1 network. This phase correction can be made in the phase shifter, however, it is better practice to change one or both of the L-sections to T or π sections. If the phase lag in the matching network to No. 1 tower is increased from 48° to 53° by adding a series inductance in the arm of the T-section facing the transmission line the desired results will be obtained as shown in Fig. 52. Then a transformer type of power divider can be used



TWO TOWER NETWORK WITH LOADS 180° OUT OF PHASE

FIG. 52.



THREE TOWER NETWORK WITH IN PHASE LOADS

FIG. 53

with the center tap of the secondary variable and grounded while the two opposite ends which are 180° out of phase are connected to the respective loads.

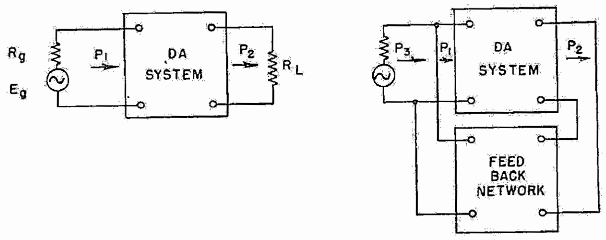
Another way to correct the phasing problem in the feeder system of Fig. 52 is to add a capacitor in series with the transmission line and thus make a T-section with a smaller phase lag. Then the original L-section matching No. 1 tower to the concentric line can be used.

Now that the terminating impedances and phase shifts in each section is specified the only thing left to do is to determine the values of capacity and inductance needed. The usual practice is to order inductors having larger values and use a clip lead to short out turns in the field when tuning the directional antenna system. With regard to the capacitors a common practice is to place a small coil in series with the capacitor such that the combination will give the required value of capacity at the operating frequency. This makes it easy to make trimming adjustments in the field when the directional antenna system is being tuned up.

Another feeder system, this time for three towers, is illustrated in Fig. 53. In this case the No. 1 tower is located near the common feed point hence no transmission line is needed to this tower. The L-section matches the antenna to 52 ohms. Actually, this L-section could be replaced with a -j35 ohm series capacitor and eliminate the shunt inductor. This would cause No. 1 tower to present 42 ohms of pure resistance to the power divider and would require 26° change in phase shift in the matching networks to the other two towers.

The power divider in this case is of the auto-transformer type with vernier controls to make small changes in the power division. The taps on the main control are for making large initial adjustments. It will be noted that the output phase of current is the same in this type of power divider for every feeder line. This type of power divider is attractive in many installations because it is not limited as to the number of towers that can be fed from the same transformer.

c. *TOWERS WITH NEGATIVE RESISTANCE*.--In many directional antenna systems one or more of the towers have a negative resistance when operated to produce the correct directional antenna pattern. This means that power must be removed from the antenna terminals of the negative resistance tower. In other words the resistance component of the driving point impedance is negative. While the initial adjustments are made this power can be dissipated but for the final operation this power must be fed back into the system in order to maintain high efficiency.



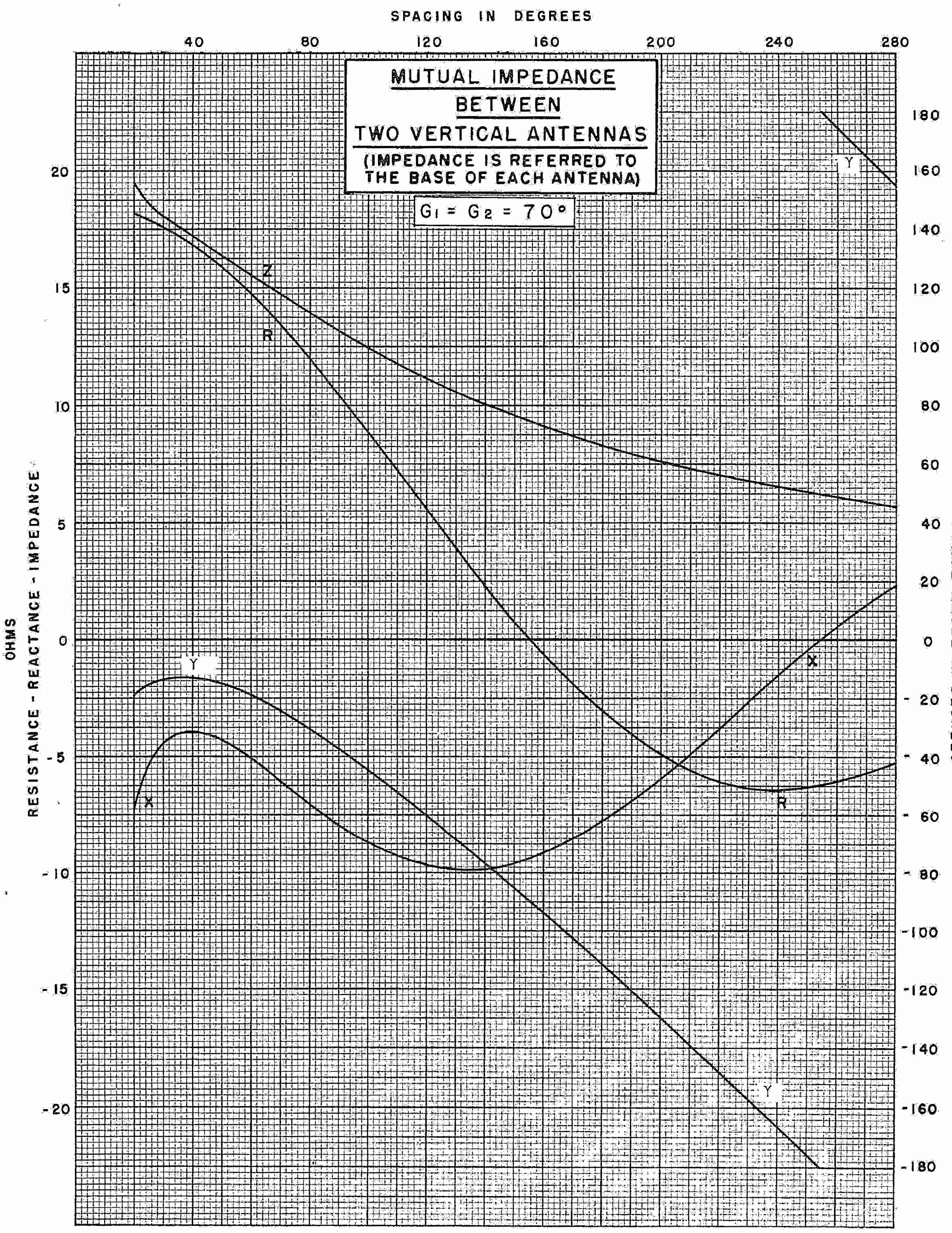
(a) Negative tower connected to dissipative load R_L

(b) Negative tower power fed back into system

ADDITION OF POWER IN DIRECTIONAL ANTENNA SYSTEM

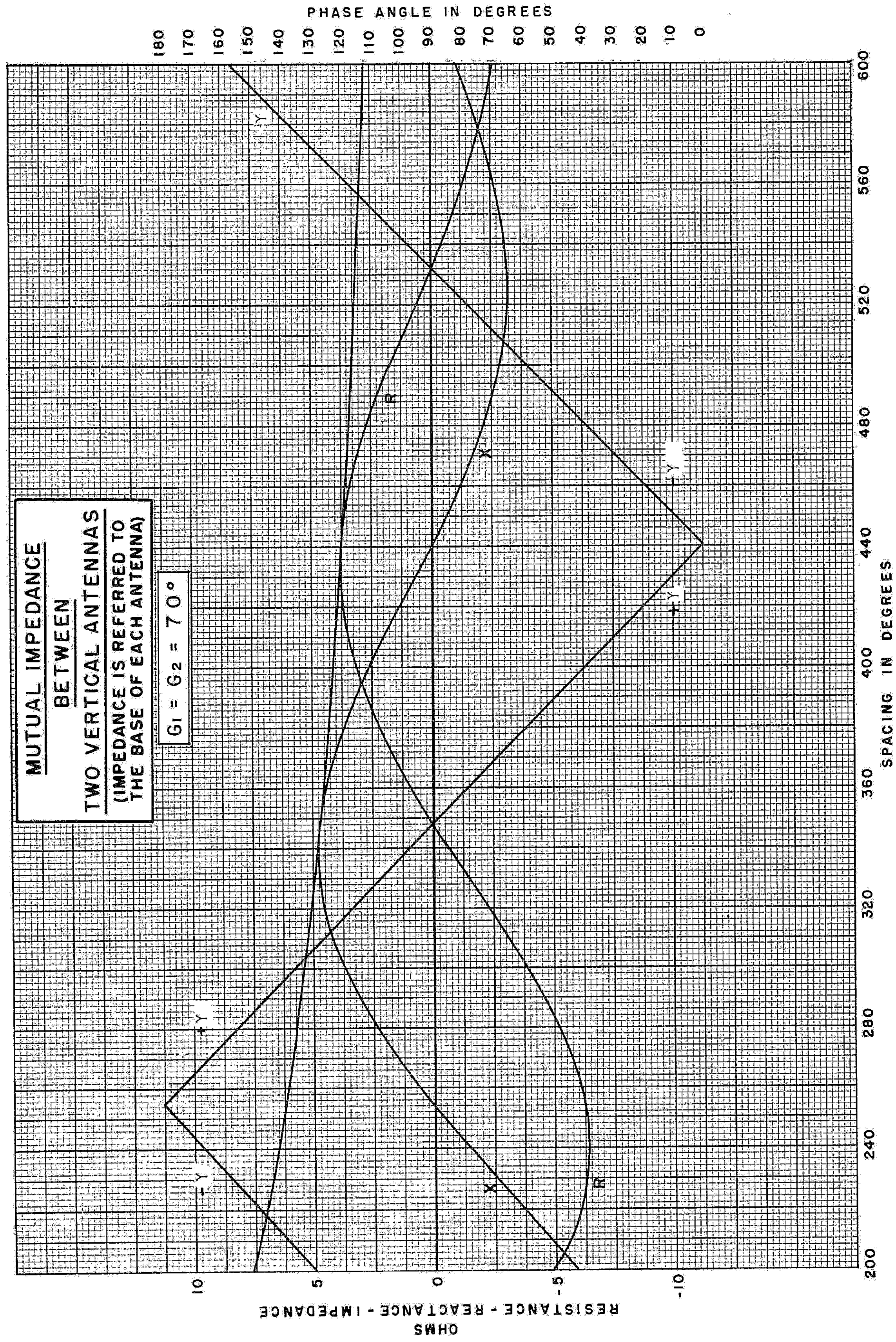
FIG. 54

In Fig. 54(a) we have a directional antenna system properly adjusted to give the required pattern but the power from a negative tower is being dissipated in R_L. If the impedance of the negative tower is matched into a transmission line this power can be carried back to the common point input junction. Now if the phase and magnitude of the feedback voltage is properly adjusted it can be made to correspond with the phase and magnitude of the input voltage. With this condition the two circuits can be connected in parallel as shown in Fig. 54(b). The feedback network is used to adjust the phase and magnitude of the feedback voltage. Then when the feedback network is connected across the input terminals the input resistance will increase because a negative resistance in parallel with a positive resistance will have a resistance value larger than the positive resistance value alone. After the system adjustment is completed the input power will change from P_1 to $P_3 = P_1 - P_2$ where P_2 is the power fed back as shown in Fig. 54(b). Since the input resistance will increase for this condition it may be necessary to re-match the output circuit of the transmitter to the new common point impedance where the input power to the directional antenna system is measured.

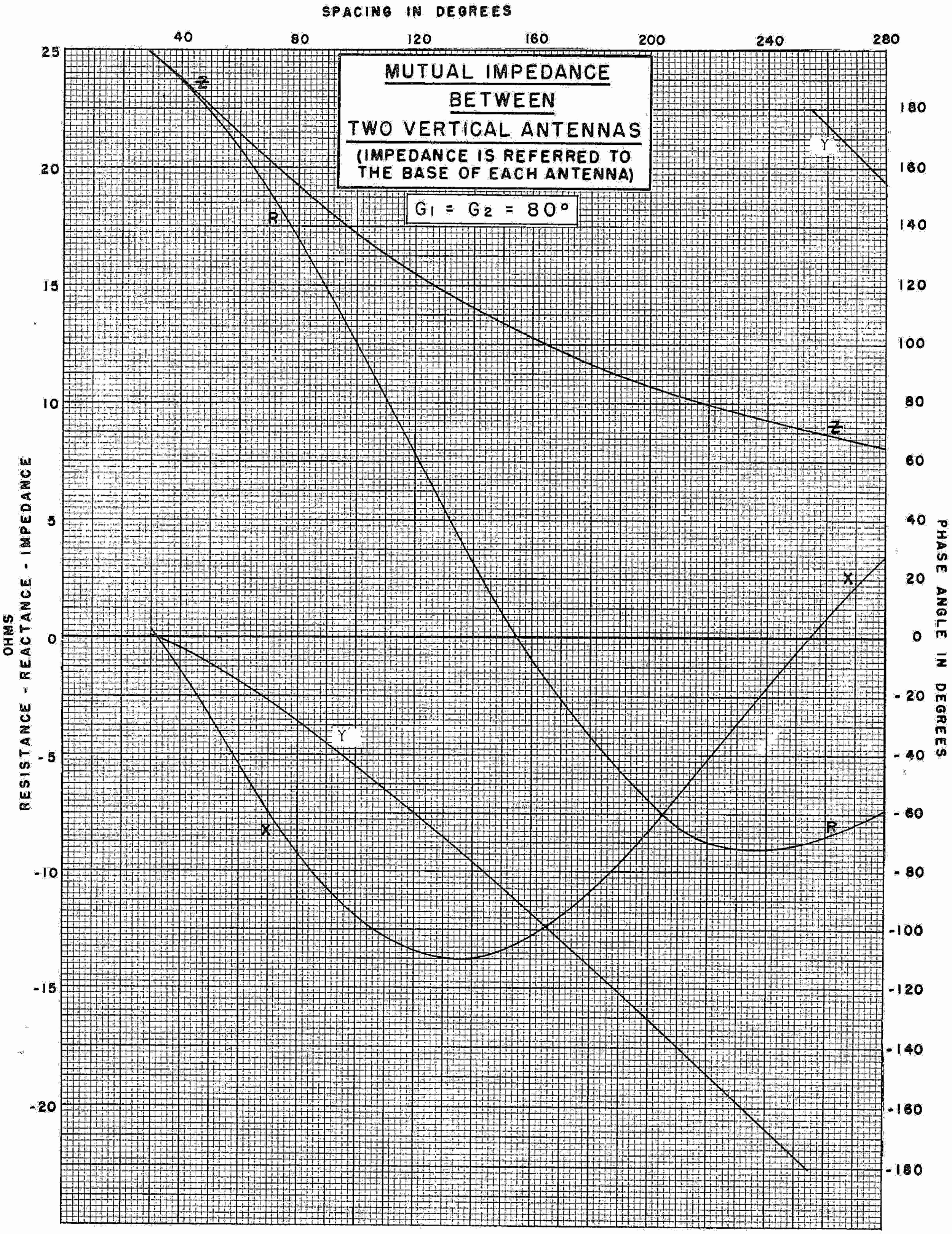


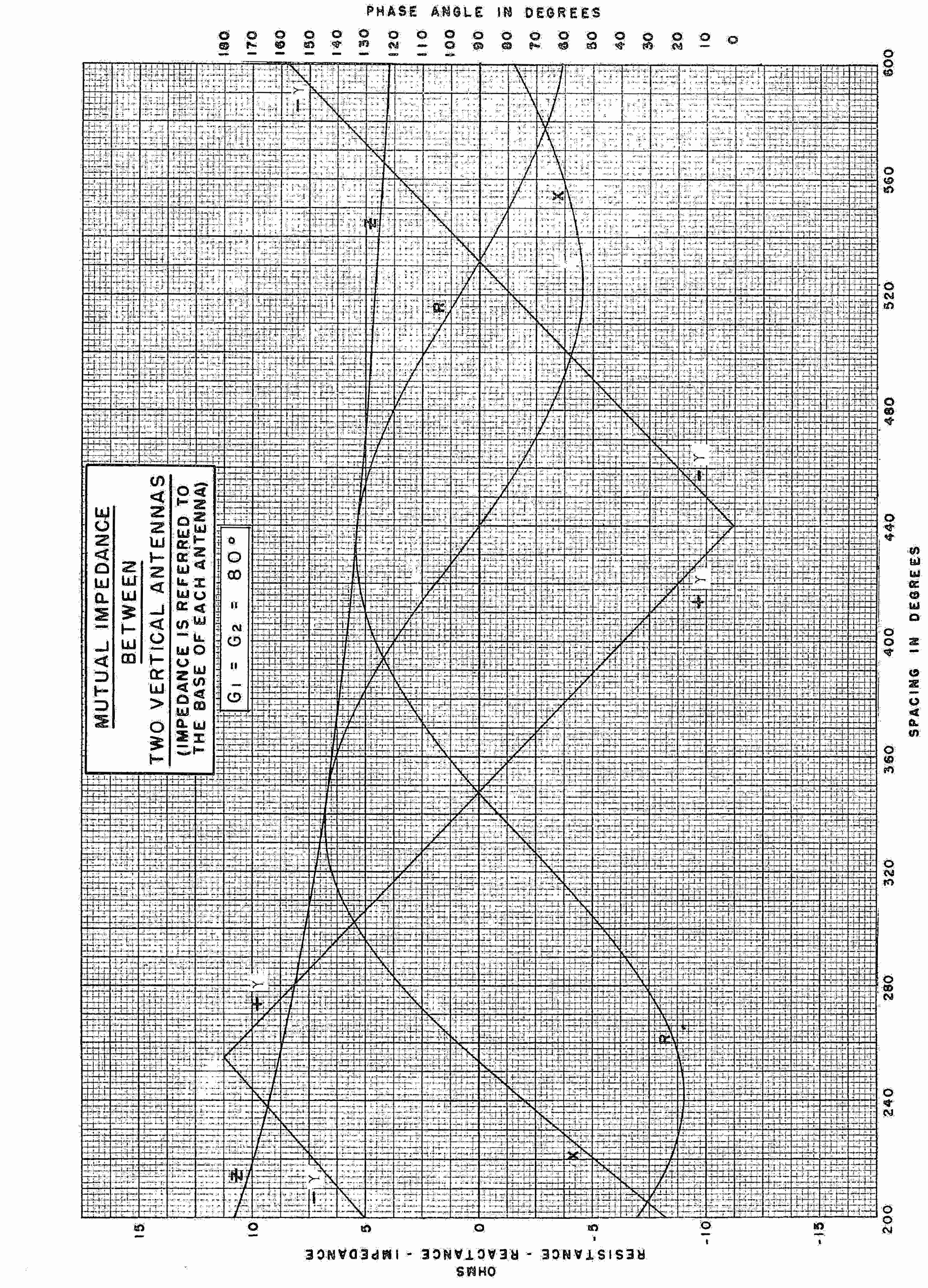
SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = G2 = 70°
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
20
15
10
5
0
-5
-10
-15
-20
PHASE ANGLE IN DEGREES
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Z
R
X
Y

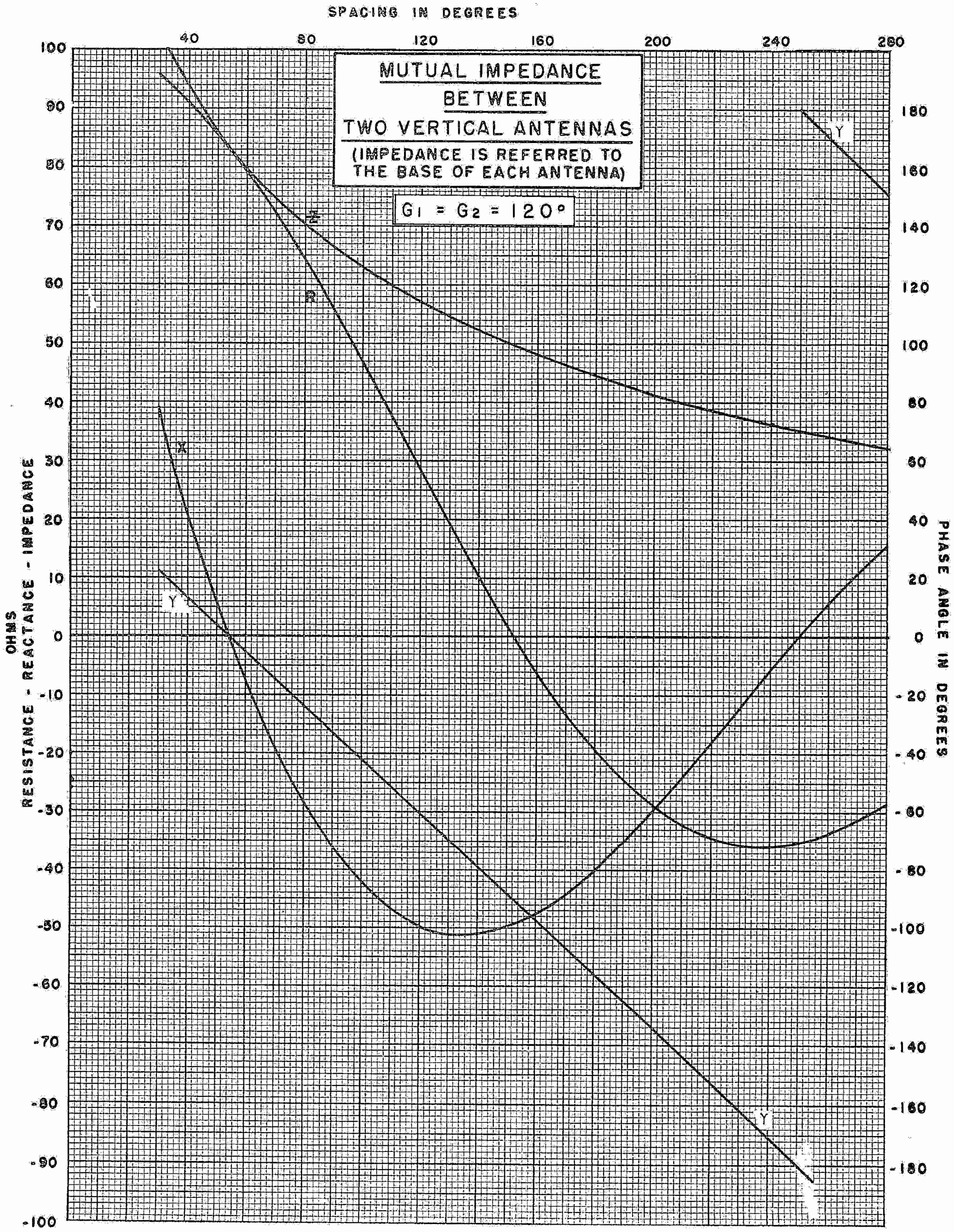
Appendix A-1

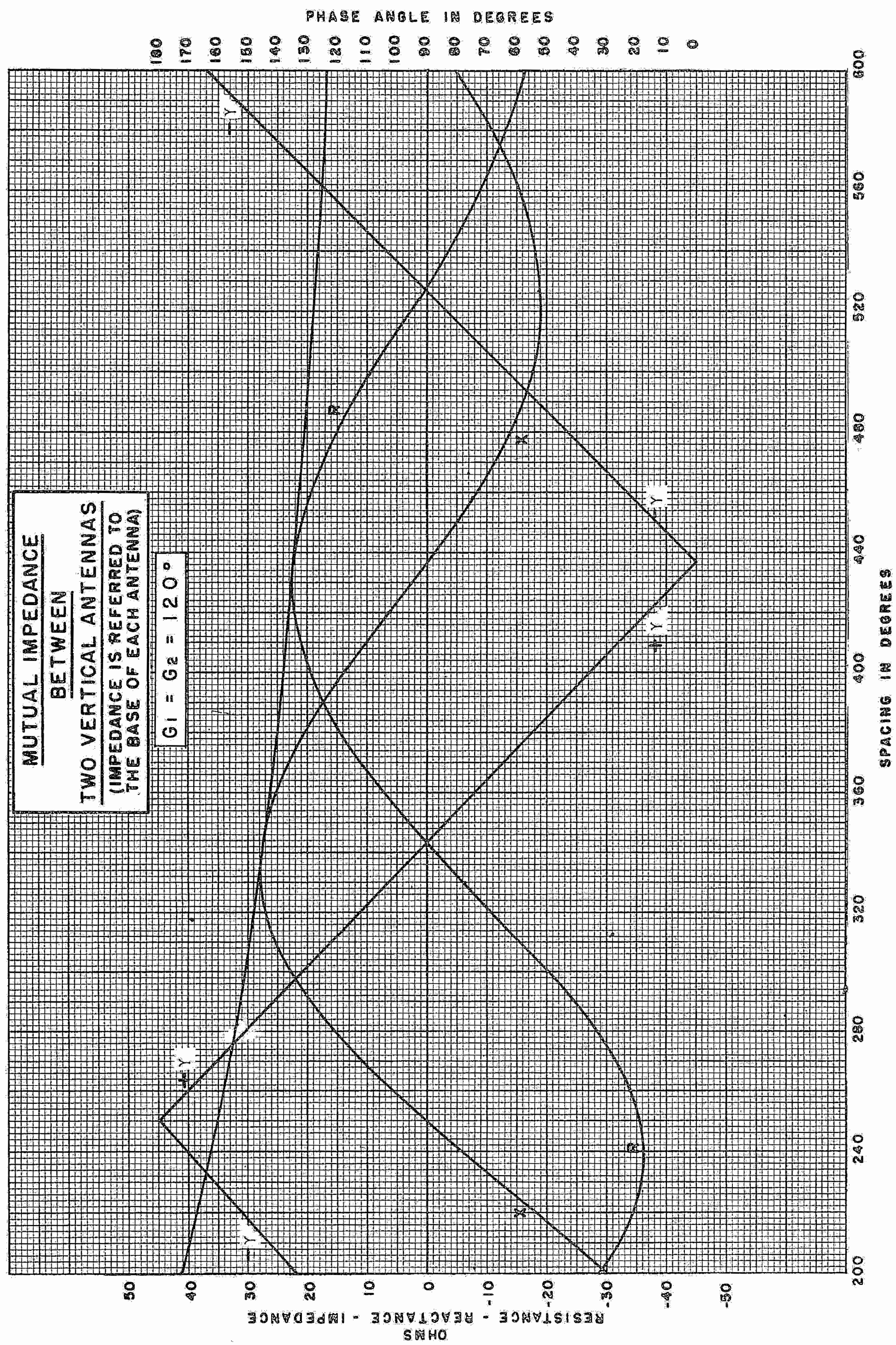


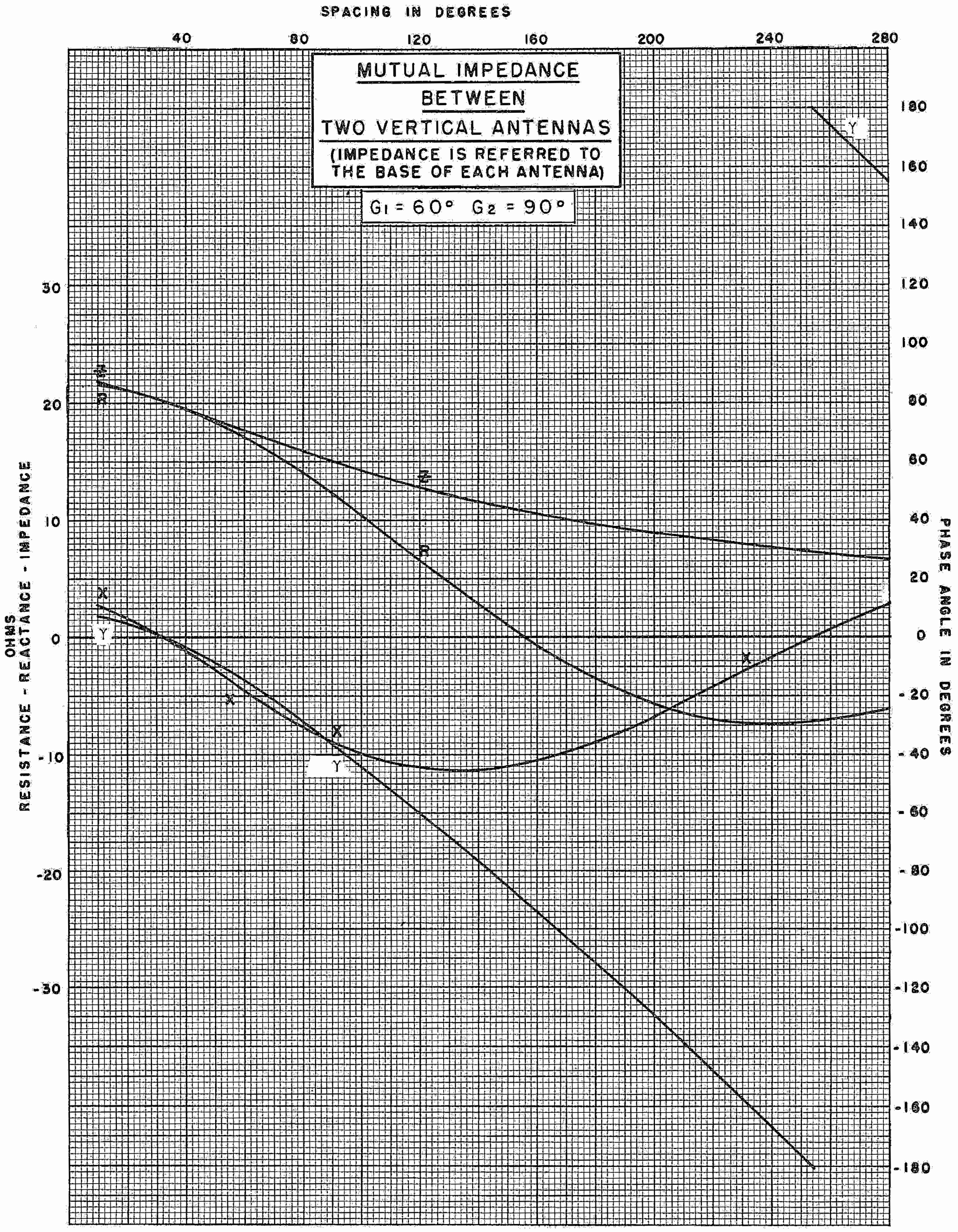
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
$G_1 = G_2 = 70°$
PHASE ANGLE IN DEGREES
SPACING IN DEGREES
RESISTANCE - REACTANCE - IMPEDANCE
OHMS


SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = G2 = 80°
25
20
15
10
5
0
-5
-10
-15
-20
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
PHASE ANGLE IN DEGREES
Z
R
Y
X

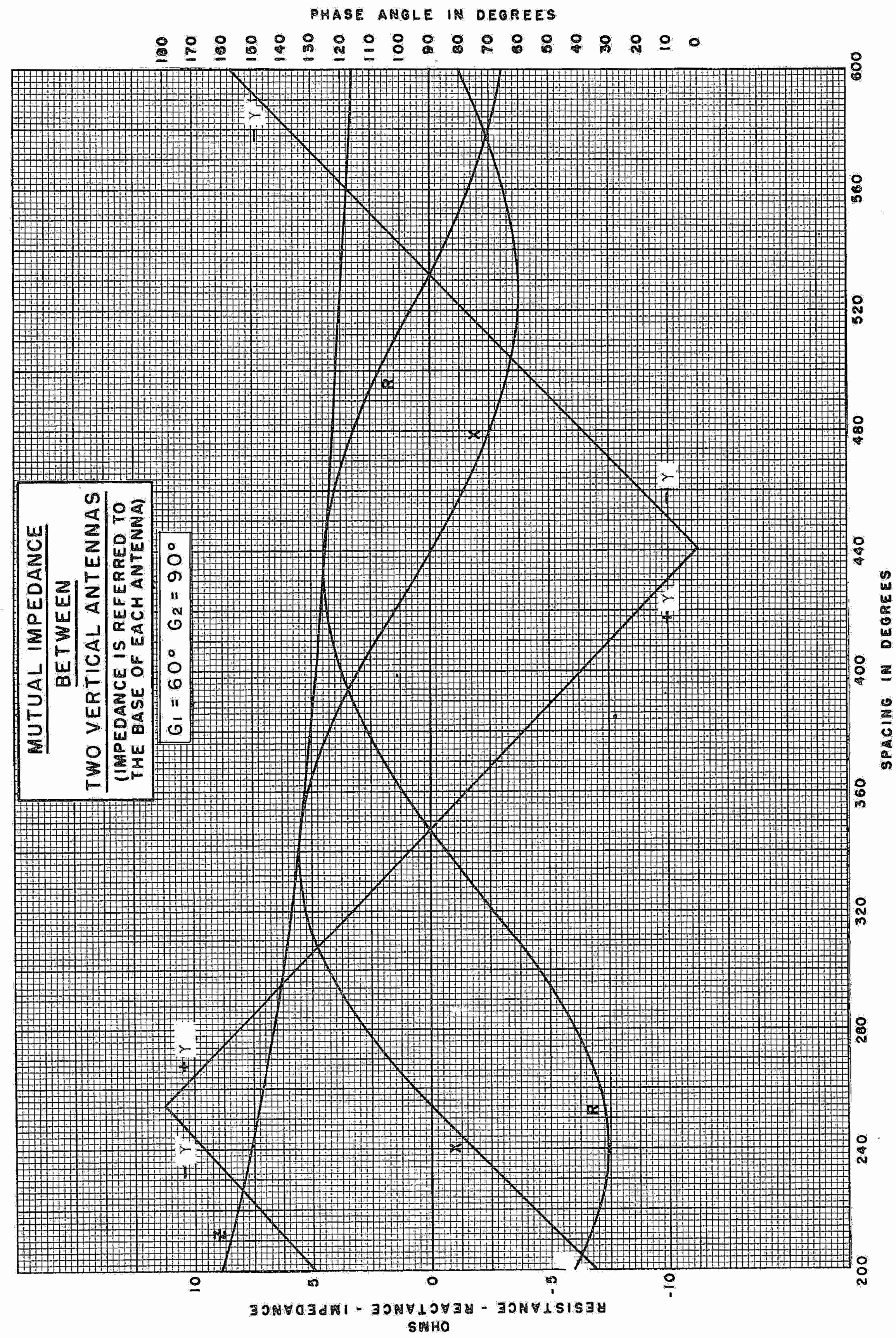

MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = G2 = 80°
PHASE ANGLE IN DEGREES
SPACING IN DEGREES
RESISTANCE - REACTANCE - IMPEDANCE
OHMS


MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = G2 = 120°
SPACING IN DEGREES
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
PHASE ANGLE IN DEGREES
Z
R
X
Y
40
80
120
160
200
240
280
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180

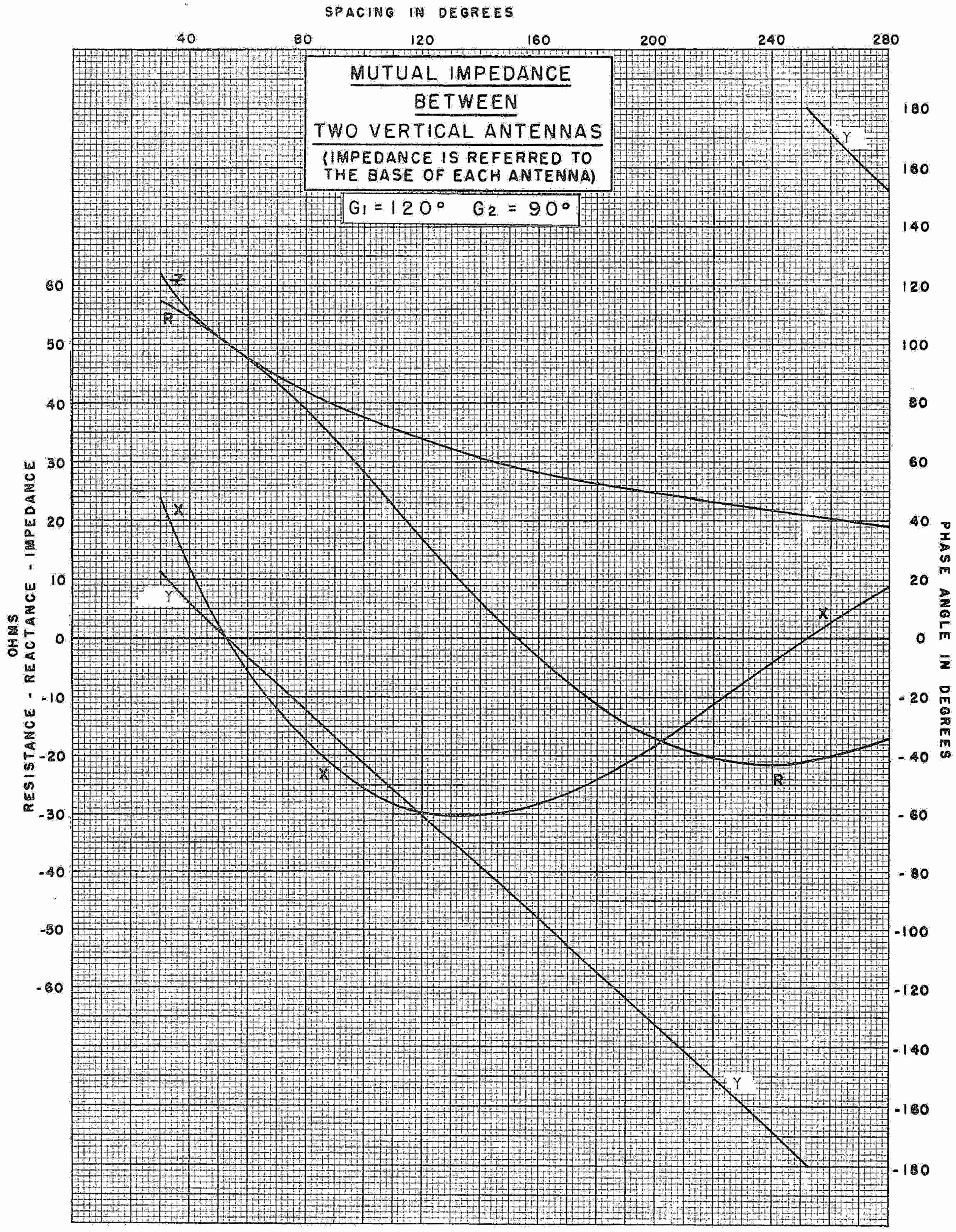

MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = G2 = 120°
PHASE ANGLE IN DEGREES
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0
SPACING IN DEGREES
200 240 280 320 360 400 440 480 520 560 600
RESISTANCE - REACTANCE - IMPEDANCE
OHMS
50 40 30 20 10 0 -10 -20 -30 -40 -50
R
X
Y
+Y
-Y



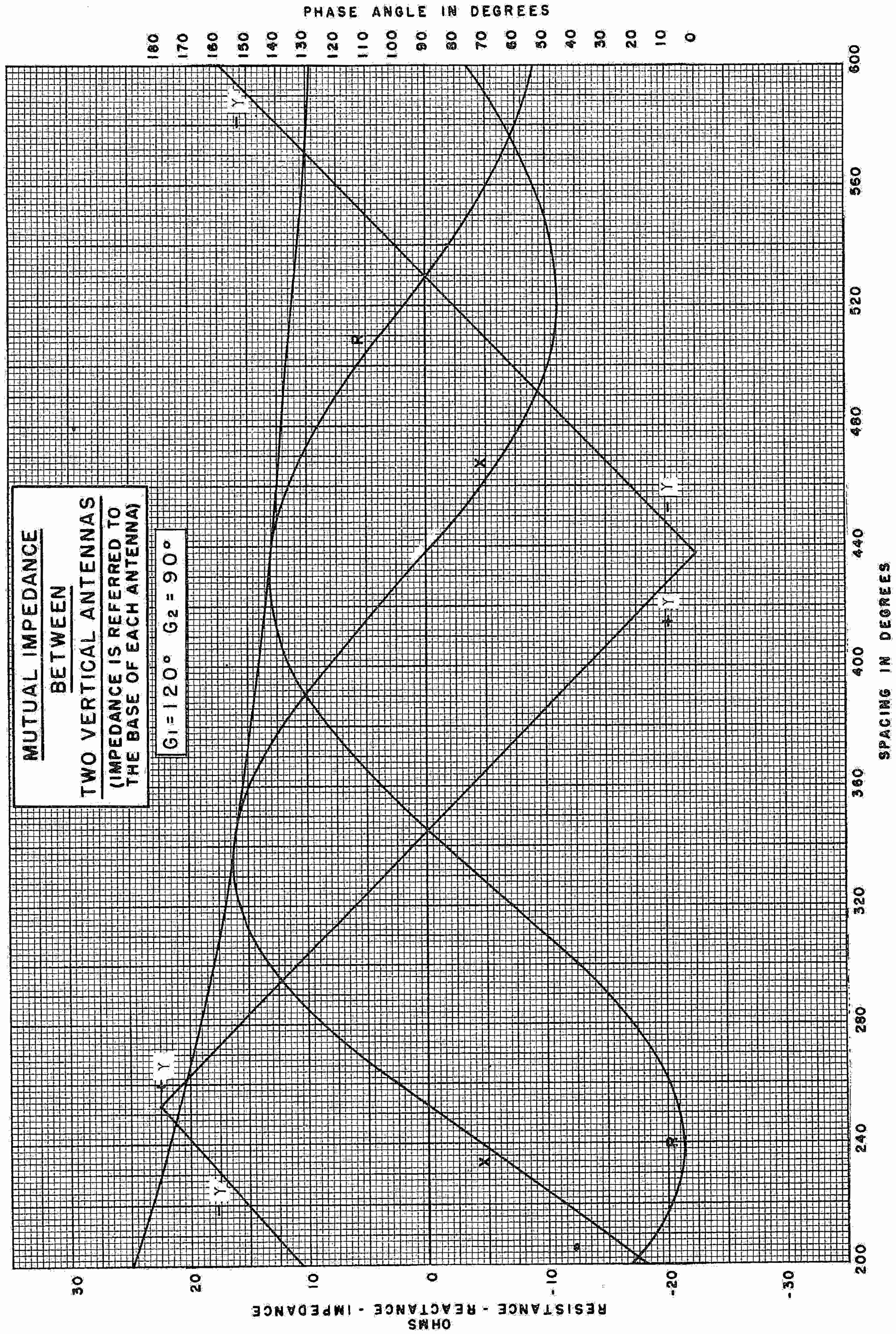
SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 60° G2 = 90°
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
30
20
10
0
-10
-20
-30
PHASE ANGLE IN DEGREES
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
R
Z
X
Y

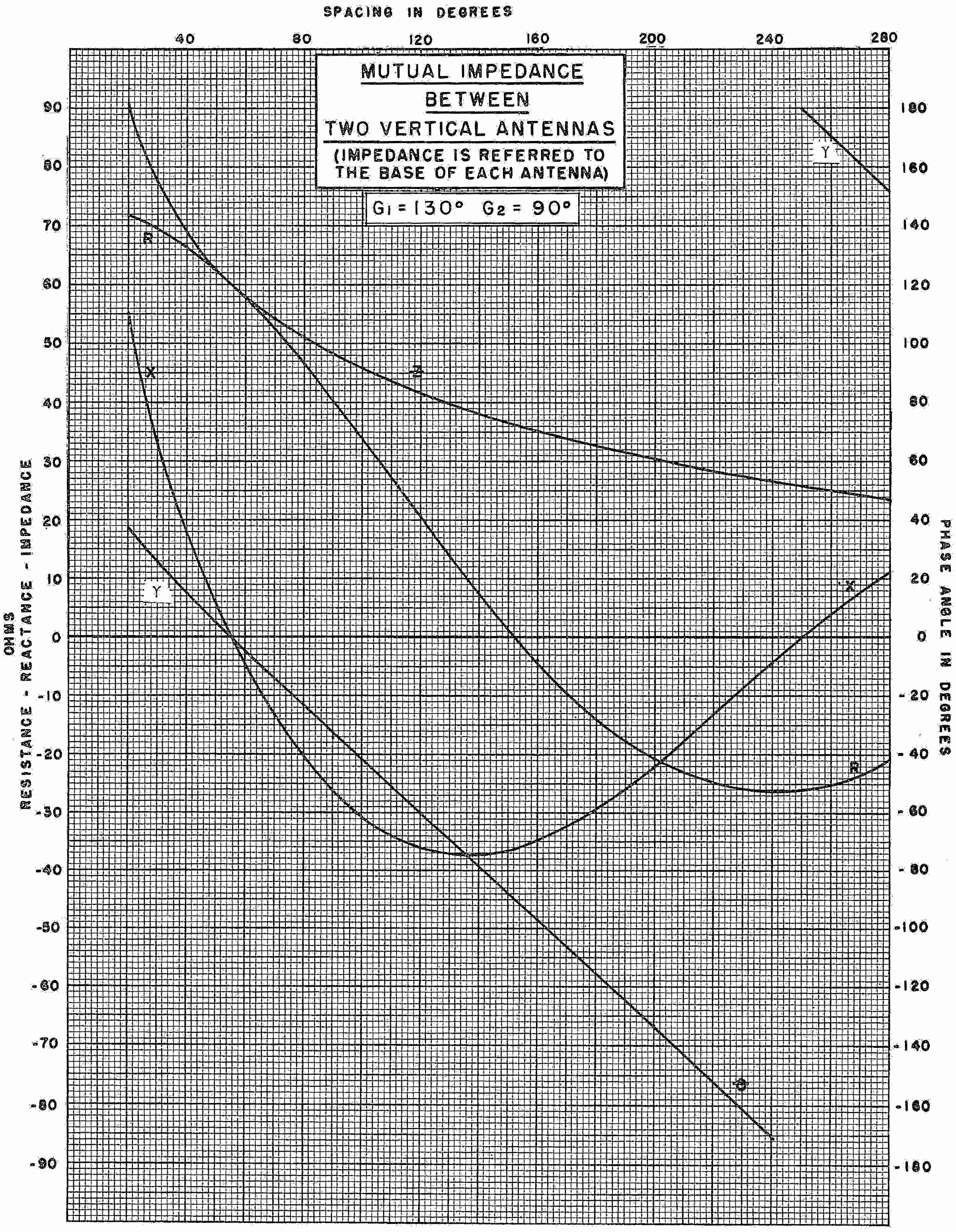


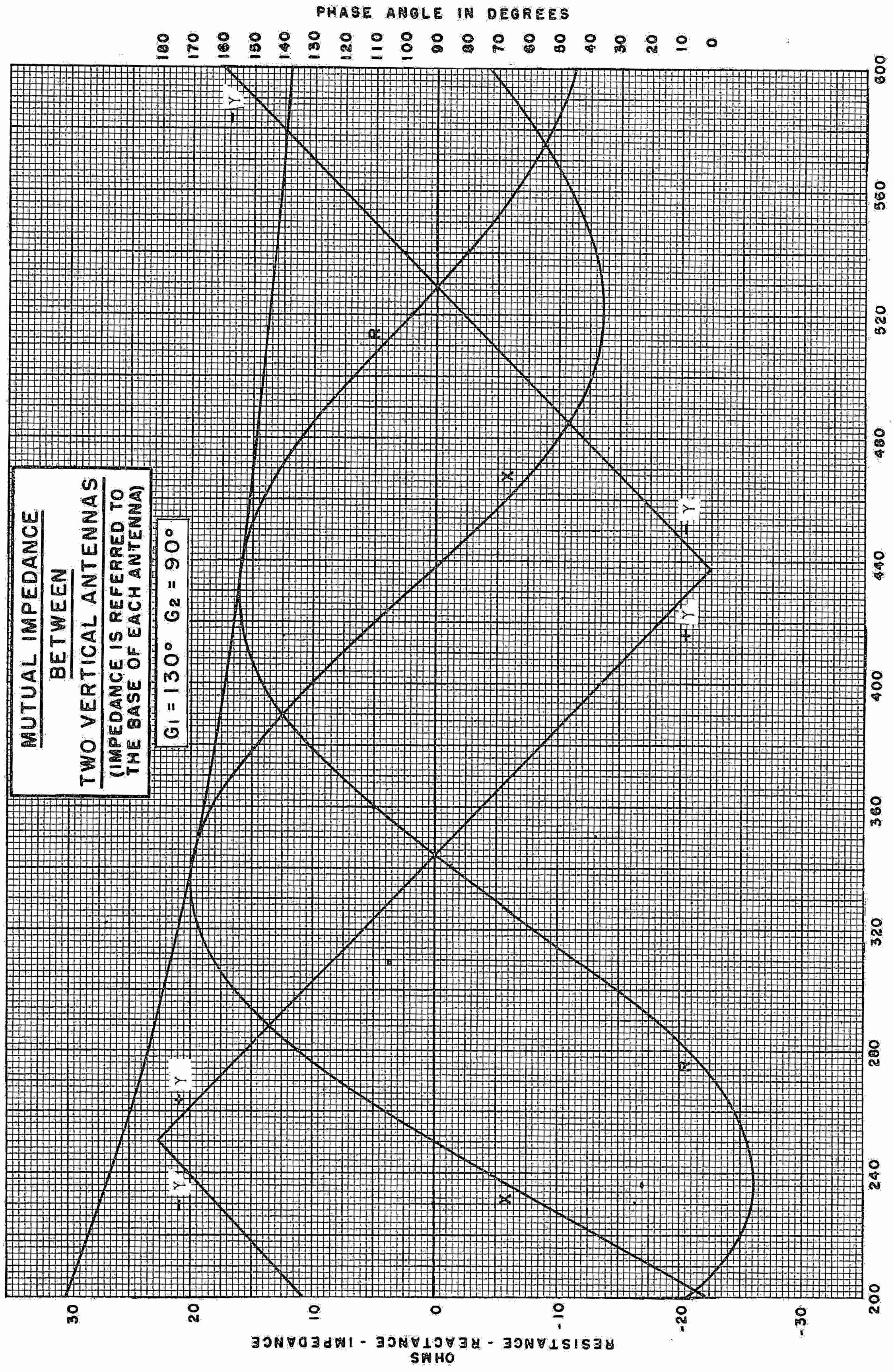
PHASE ANGLE IN DEGREES
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO THE BASE OF EACH ANTENNA)
G1 = 60° G2 = 90°
R
X
Z
+Y
-Y
SPACING IN DEGREES
200 240 280 320 360 400 440 480 520 560 600
RESISTANCE - REACTANCE - IMPEDANCE
OHMS
10 5 0 -5 -10

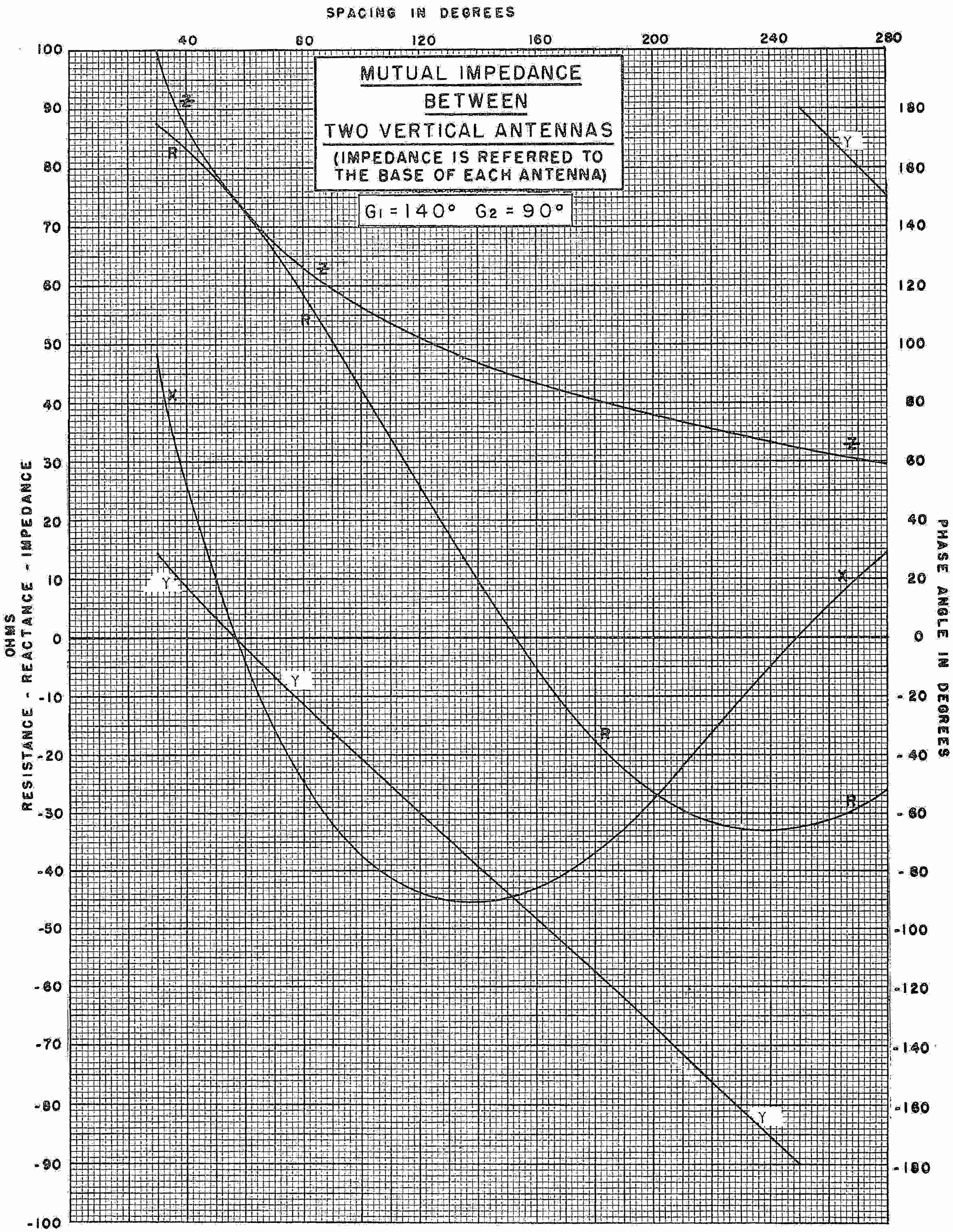


SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 120° G2 = 90°
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
PHASE ANGLE IN DEGREES
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Z
R
X
Y

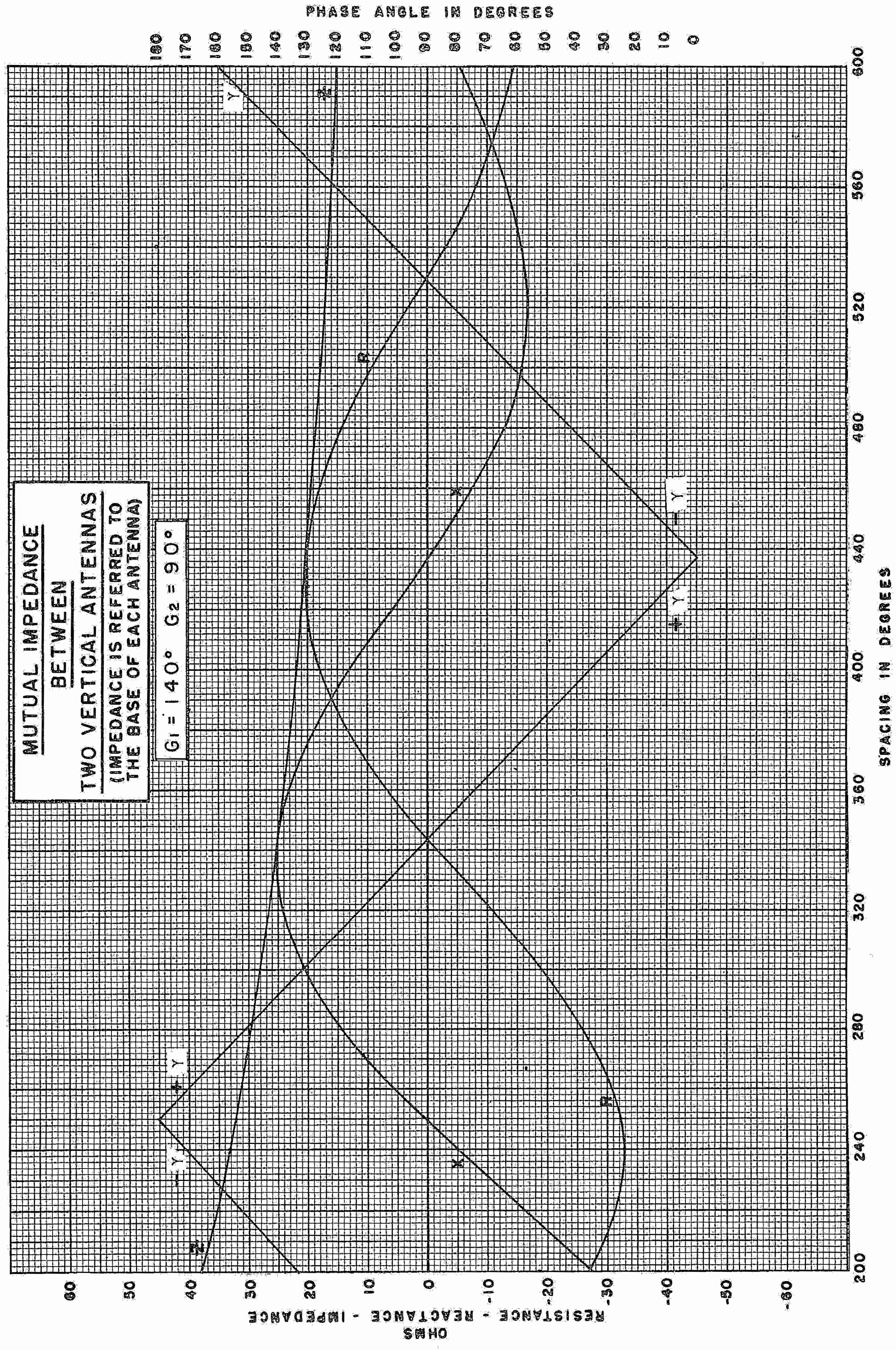

MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO THE BASE OF EACH ANTENNA)
G1 = 120° G2 = 90°
PHASE ANGLE IN DEGREES
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0
SPACING IN DEGREES
200 240 280 320 360 400 440 480 520 560 600
RESISTANCE - REACTANCE - IMPEDANCE
OHMS
30 20 10 0 -10 -20 -30
R
X
+Y
-Y

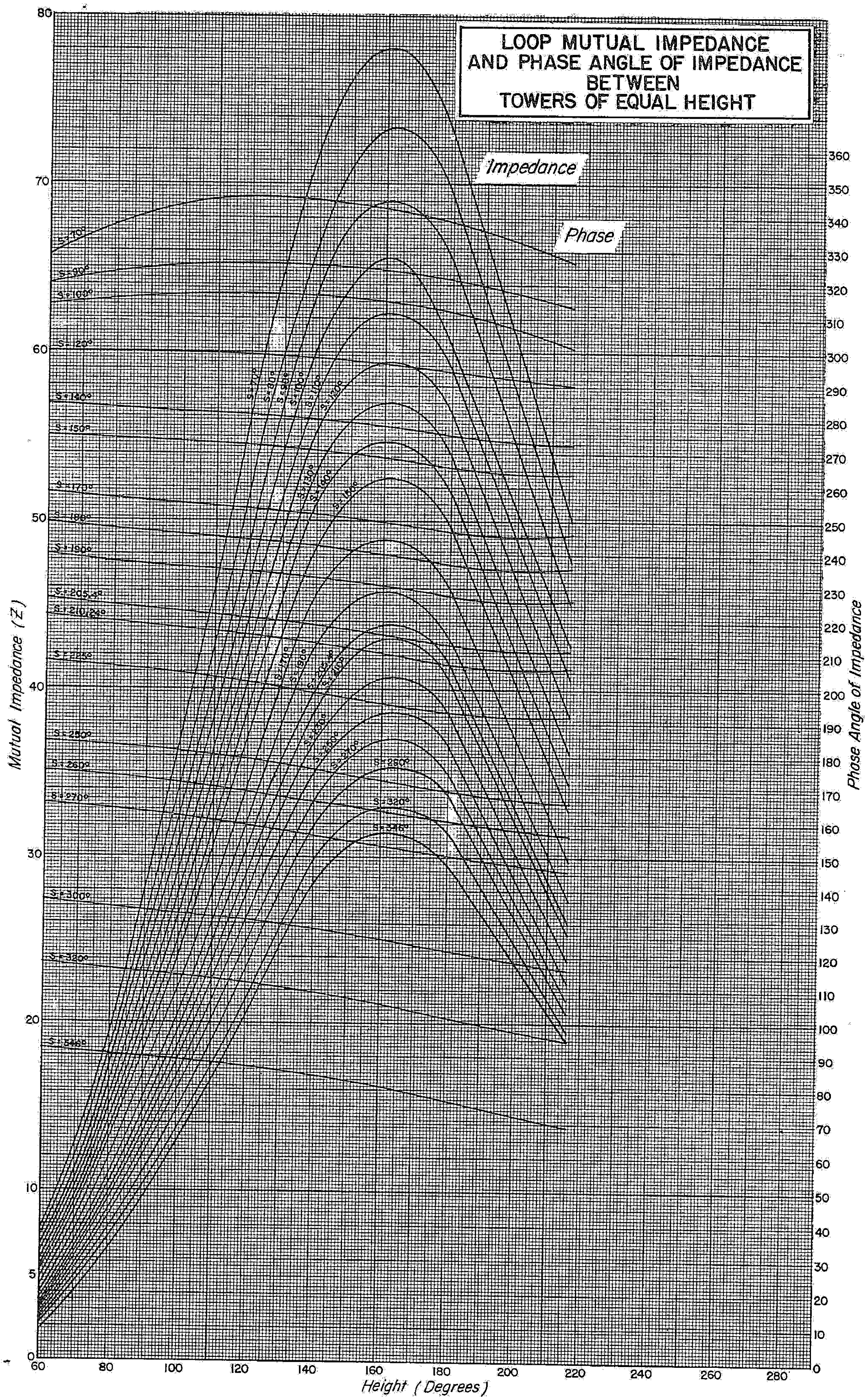

SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 130° G2 = 90°
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
PHASE ANGLE IN DEGREES
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
R
X
Z
Y


MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 130° G2 = 90°
PHASE ANGLE IN DEGREES
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0
SPACING IN DEGREES
200 240 280 320 360 400 440 480 520 560 600
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
30 20 10 0 -10 -20 -30
R
X
+Y
-Y

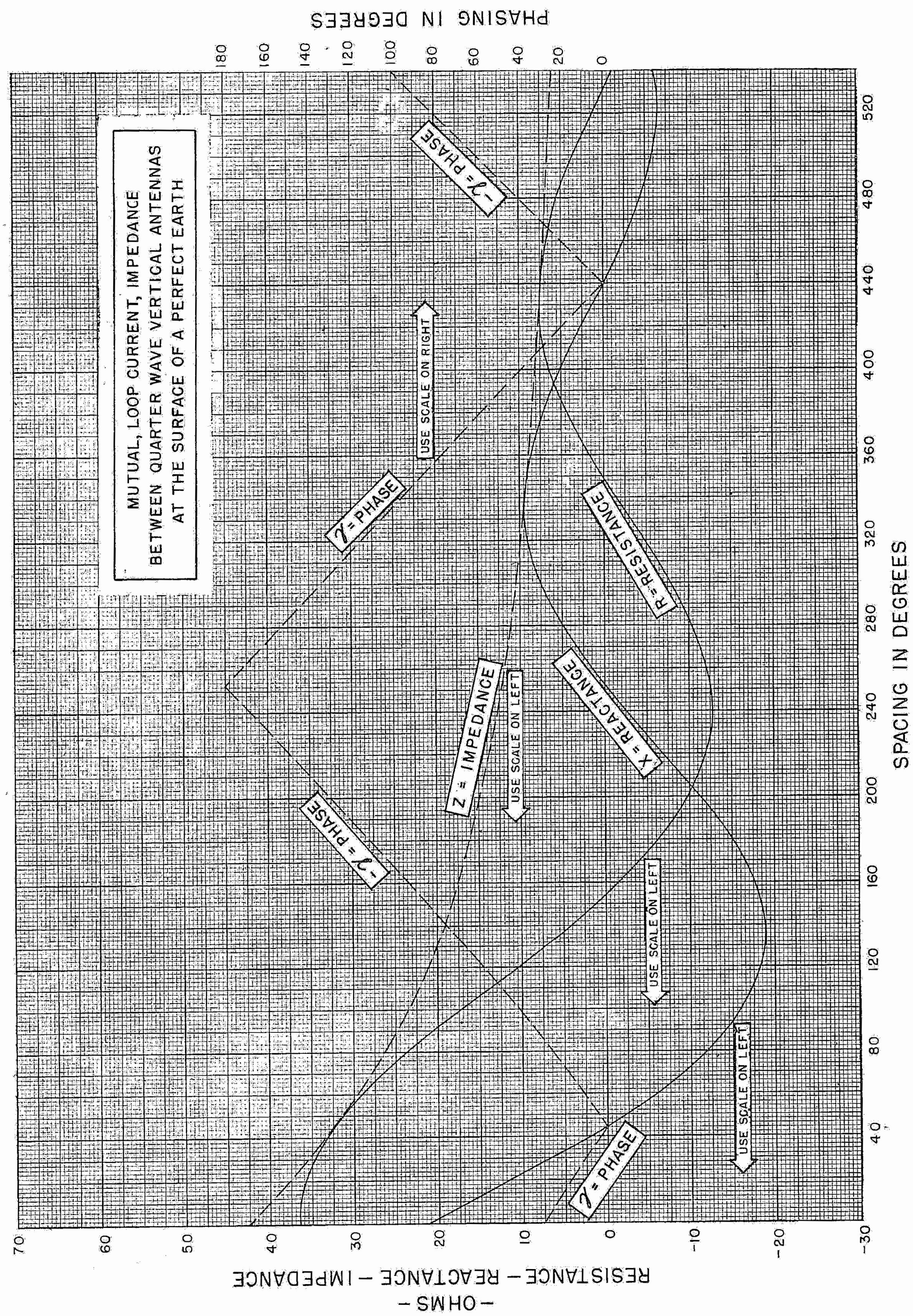


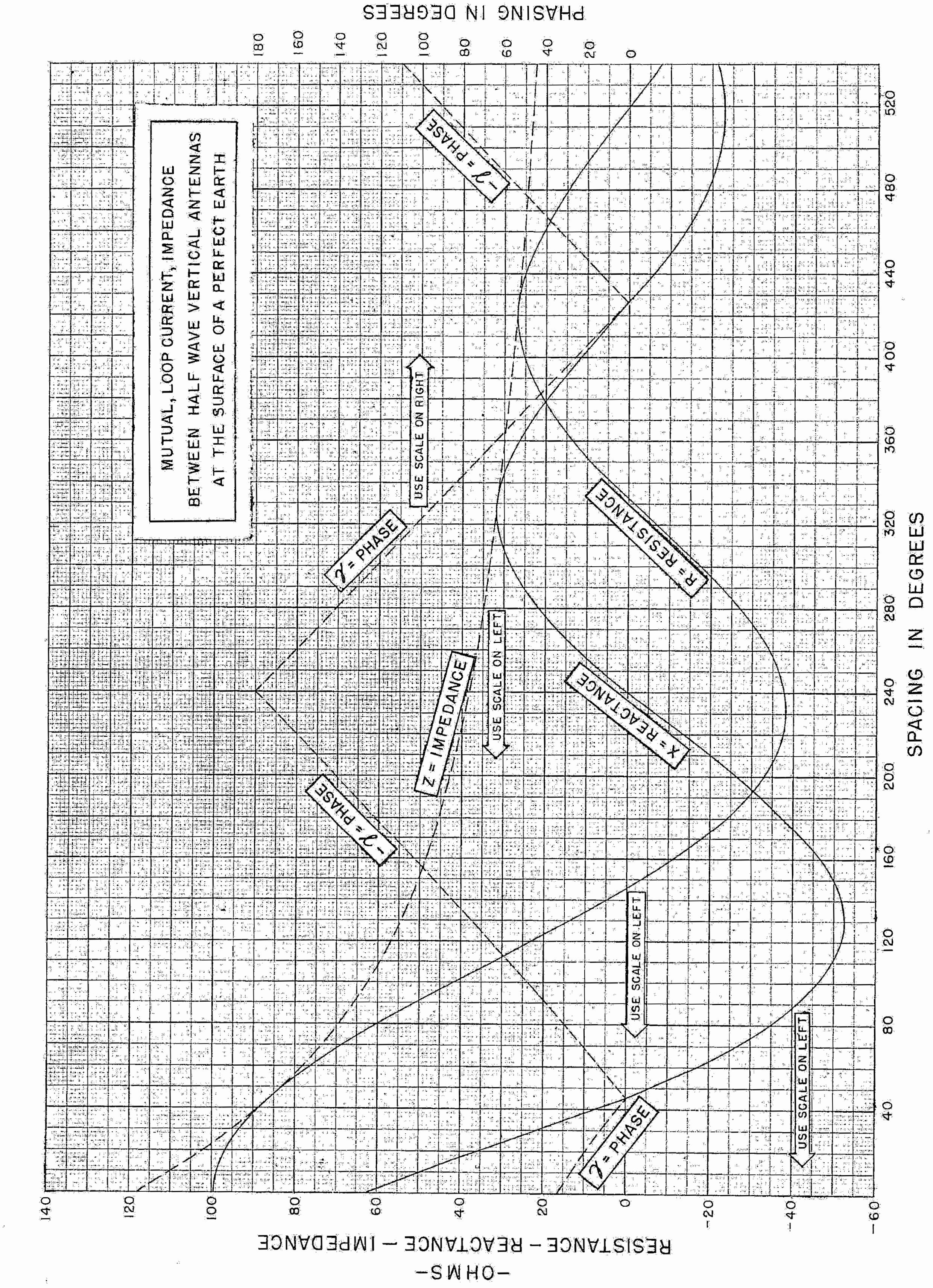
SPACING IN DEGREES
40
80
120
160
200
240
280
MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 140° G2 = 90°
OHMS
RESISTANCE - REACTANCE - IMPEDANCE
PHASE ANGLE IN DEGREES
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
Z
R
X
Y

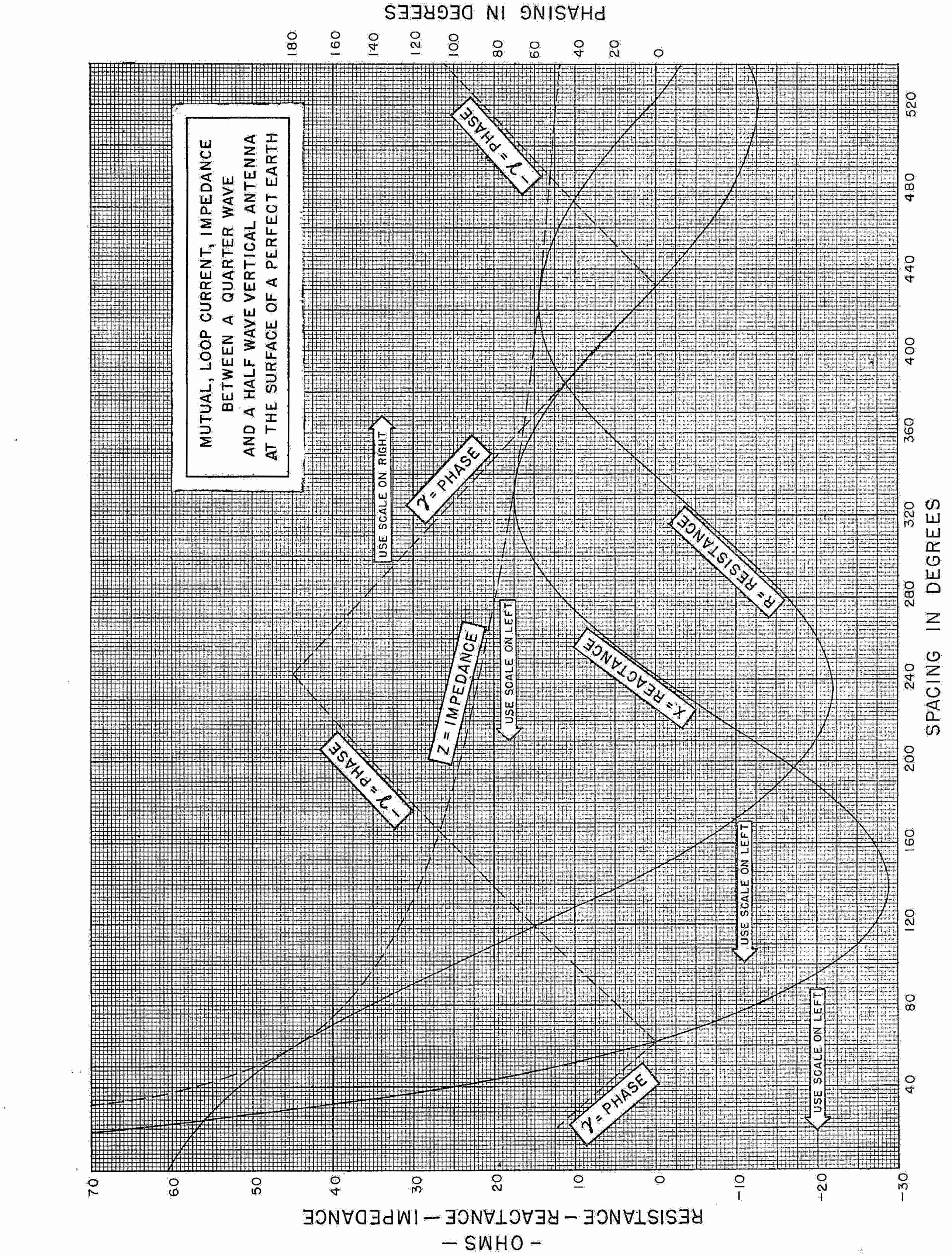

MUTUAL IMPEDANCE
BETWEEN
TWO VERTICAL ANTENNAS
(IMPEDANCE IS REFERRED TO
THE BASE OF EACH ANTENNA)
G1 = 140° G2 = 90°
PHASE ANGLE IN DEGREES
SPACING IN DEGREES
RESISTANCE - REACTANCE - IMPEDANCE
OHMS

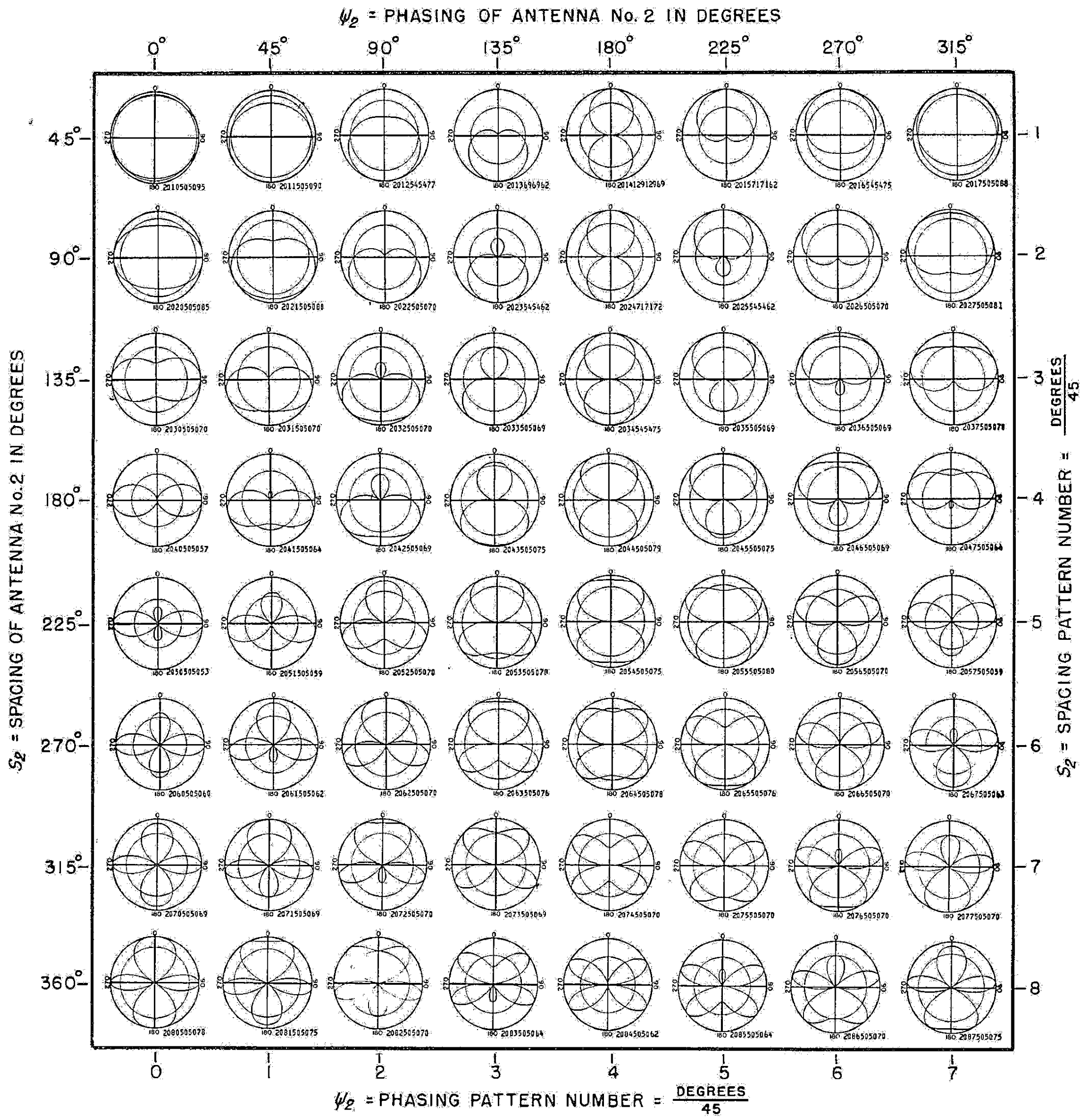


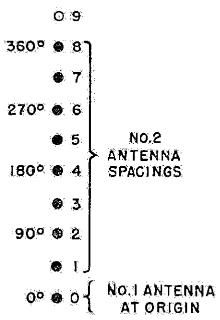
LOOP MUTUAL IMPEDANCE
AND PHASE ANGLE OF IMPEDANCE
BETWEEN
TOWERS OF EQUAL HEIGHT
Impedance
Phase
Mutual Impedance (Z)
Phase Angle of Impedance
Height (Degrees)


MUTUAL, LOOP CURRENT, IMPEDANCE
BETWEEN QUARTER WAVE VERTICAL ANTENNAS
AT THE SURFACE OF A PERFECT EARTH
PHASING IN DEGREES
SPACING IN DEGREES
RESISTANCE – REACTANCE – IMPEDANCE
– OHMS –
R = RESISTANCE
X = REACTANCE
Z = IMPEDANCE
γ = PHASE
-γ = PHASE
USE SCALE ON RIGHT
USE SCALE ON LEFT
180
160
140
120
100
80
60
40
20
0
40
80
120
160
200
240
280
320
360
400
440
480
520
70
60
50
40
30
20
10
0
-10
-20
-30


MUTUAL, LOOP CURRENT, IMPEDANCE
BETWEEN HALF WAVE VERTICAL ANTENNAS
AT THE SURFACE OF A PERFECT EARTH
PHASING IN DEGREES
180 160 140 120 100 80 60 40 20 0
SPACING IN DEGREES
40 80 120 160 200 240 280 320 360 400 440 480 520
RESISTANCE – REACTANCE – IMPEDANCE
-OHMS-
140 120 100 80 60 40 20 0 -20 -40 -60
R = RESISTANCE
X = REACTANCE
Z = IMPEDANCE
γ = PHASE
-γ = PHASE
USE SCALE ON RIGHT
USE SCALE ON LEFT


MUTUAL, LOOP CURRENT, IMPEDANCE
BETWEEN A QUARTER WAVE
AND A HALF WAVE VERTICAL ANTENNA
AT THE SURFACE OF A PERFECT EARTH
PHASING IN DEGREES
180 160 140 120 100 80 60 40 20 0
SPACING IN DEGREES
40 80 120 160 200 240 280 320 360 400 440 480 520
RESISTANCE — REACTANCE — IMPEDANCE
— OHMS —
70 60 50 40 30 20 10 0 −10 −20 −30
R = RESISTANCE
X = REACTANCE
Z = IMPEDANCE
γ = PHASE
−γ = PHASE
γ = PHASE
−γ = PHASE
USE SCALE ON RIGHT
USE SCALE ON LEFT
USE SCALE ON LEFT
USE SCALE ON LEFT



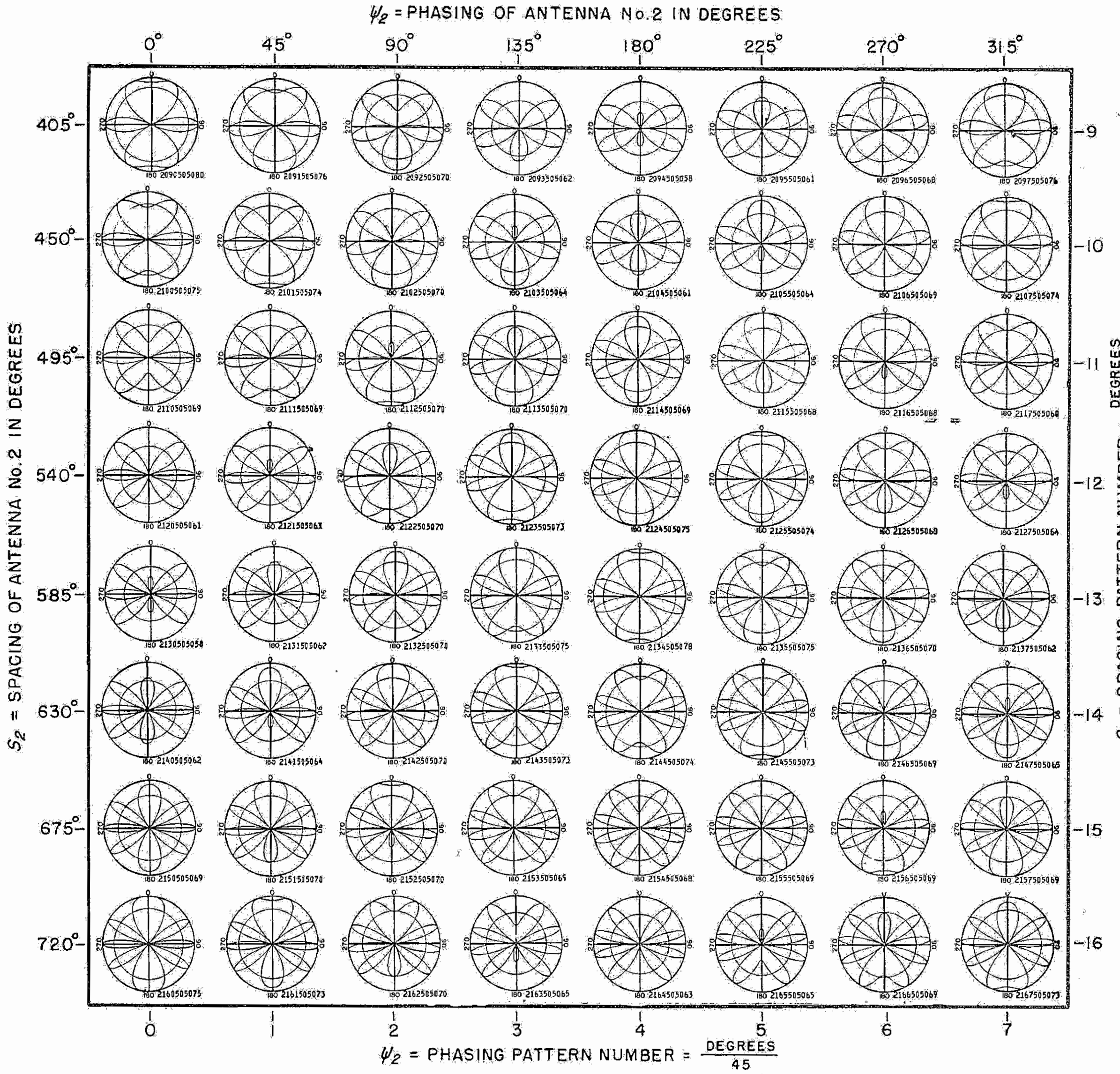


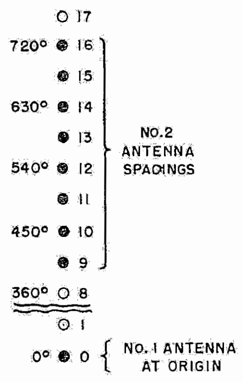
SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS.

NUMBER OF ANTENNAS IN THE SYSTEM.	SPACING OF NO. 2 ANTENNA = DEGREES / 45	PHASING OF NO. 2 ANTENNA = DEGREES / 45	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 1 ANTENNA.	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 2 ANTENNA.	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM.
No.	S_2	ψ_2	E_1	E_2	E_0
2	0	0	50	50	95

PATTERN NOMENCLATURE

CLEVELAND INSTITUTE OF RADIO ELECTRONICS
TERMINAL TOWER, CLEVELAND 13, OHIO.





SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS.

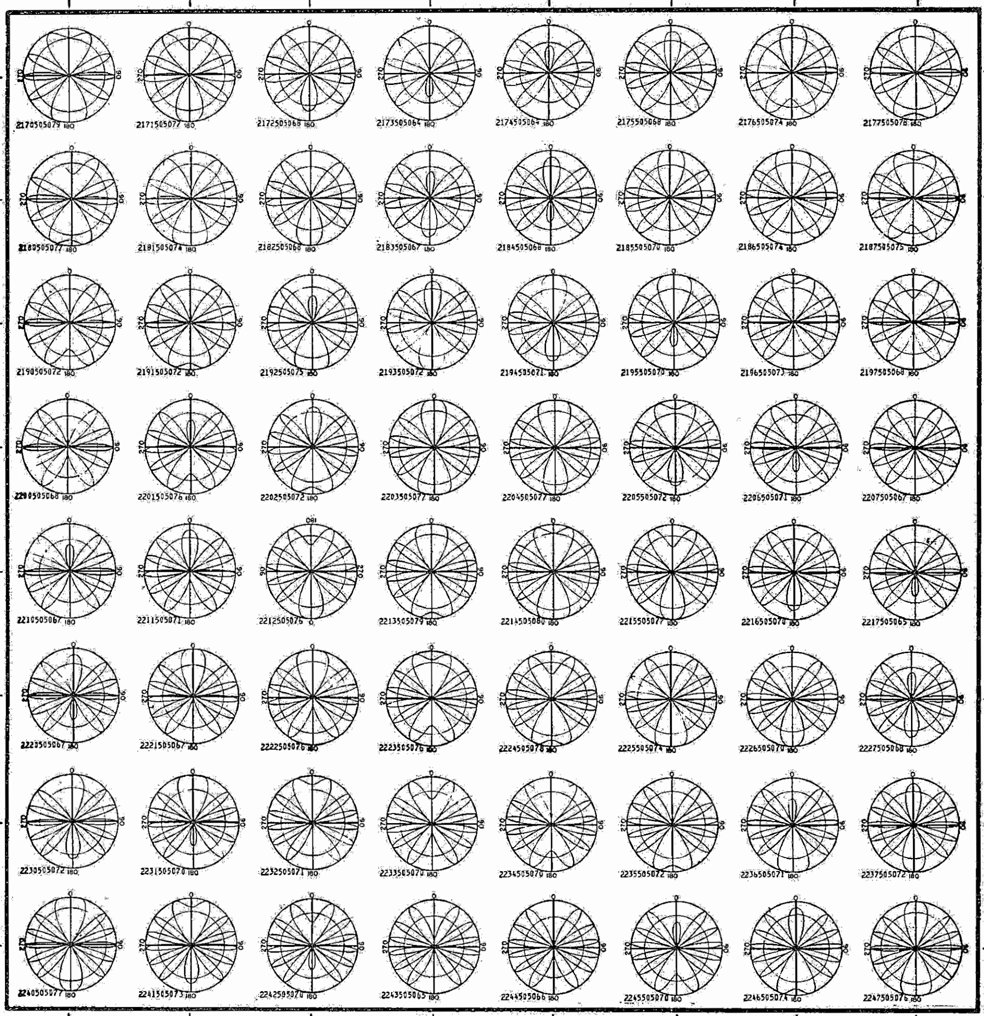
NUMBER OF ANTENNAS IN THE SYSTEM	SPACING OF NO. 2 ANTENNA = DEGREES/45	PHASING OF NO. 2 ANTENNA = DEGREES/45	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 1 ANTENNA.	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 2 ANTENNA.	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM.
NO.	S_2	ψ_2	E_1	E_2	E_0
2	09	0	50	50	80

PATTERN NOMENCLATURE

ψ_2 = PHASING OF ANTENNA No. 2 IN DEGREES

0° 45° 90° 135° 180° 225° 270° 315°

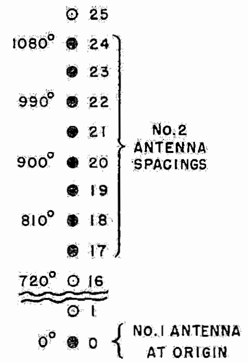
S_2 = SPACING OF ANTENNA No. 2 IN DEGREES: 765° 810° 855° 900° 945° 990° 1035° 1080°



S_2 = SPACING PATTERN NUMBER = $\frac{\text{DEGREES}}{45}$: 17 18 19 20 21 22 23 24

0 1 2 3 4 5 6 7

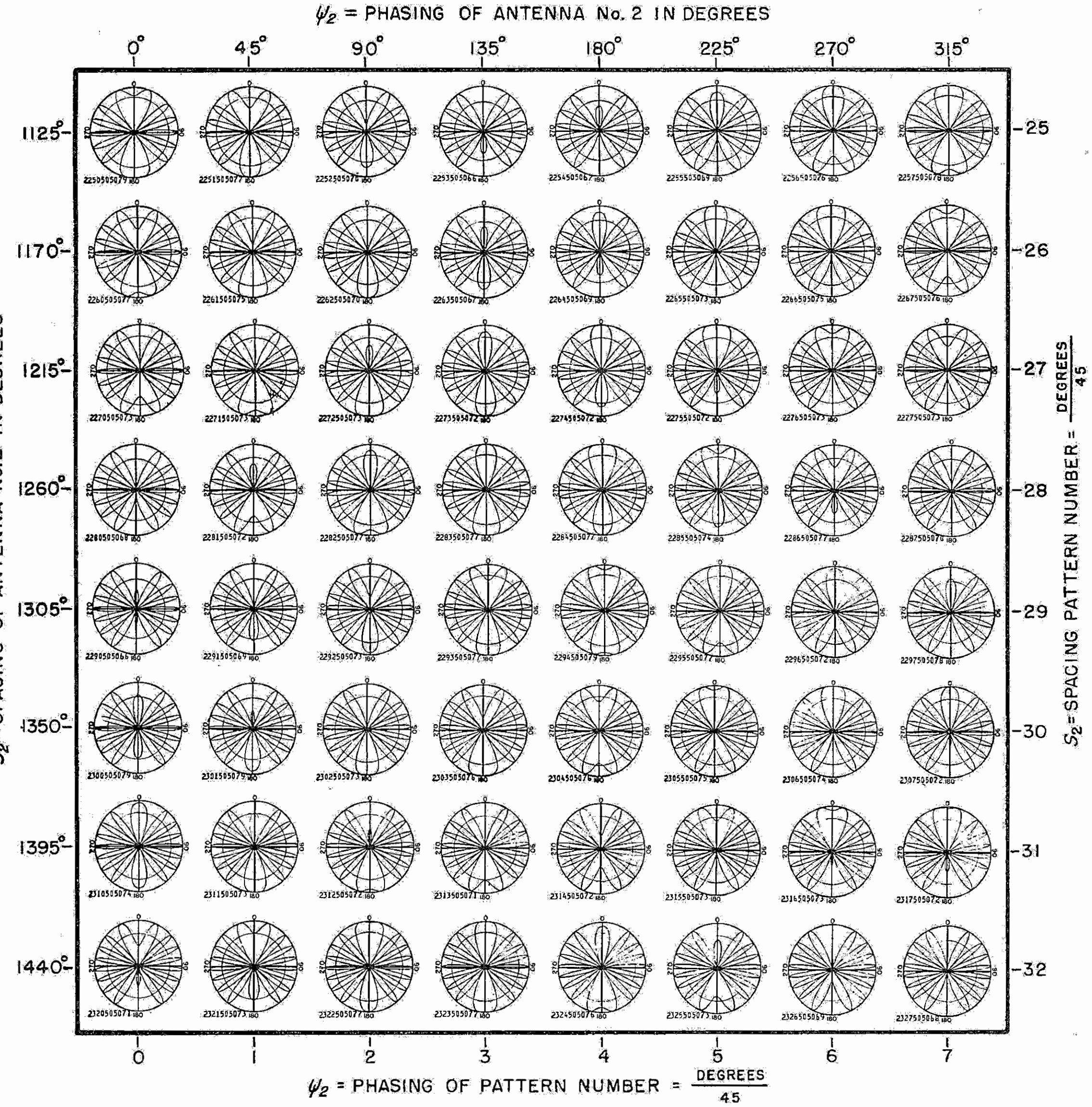
ψ_2 = PHASING PATTERN NUMBER = $\frac{\text{DEGREES}}{45}$

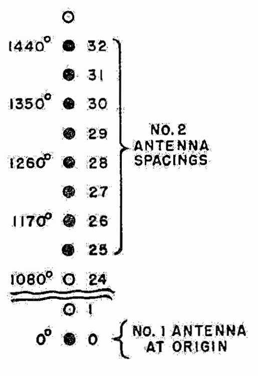


SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS.

NUMBER OF ANTENNAS IN THE SYSTEM.	SPACING OF No. 2 ANTENNA = $\frac{\text{DEGREES}}{45}$	PHASING OF No. 2 ANTENNA = $\frac{\text{DEGREES}}{45}$	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 1 ANTENNA.	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 2 ANTENNA.	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM.
No.	S_2	ψ_2	E_1	E_2	E_0
2	17	0	50	50	79

PATTERN NOMENCLATURE





SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS.

NUMBER OF ANTENNAS IN THE SYSTEM.	SPACING OF NO. 2 ANTENNA = DEGREES / 45	PHASING OF NO. 2 ANTENNA = DEGREES / 45	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 1 ANTENNA.	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM NO. 2 ANTENNA.	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM.
No.	S_2	ψ_2	E_1	E_2	E_0
2	25	0	50	50	79

PATTERN NOMENCLATURE

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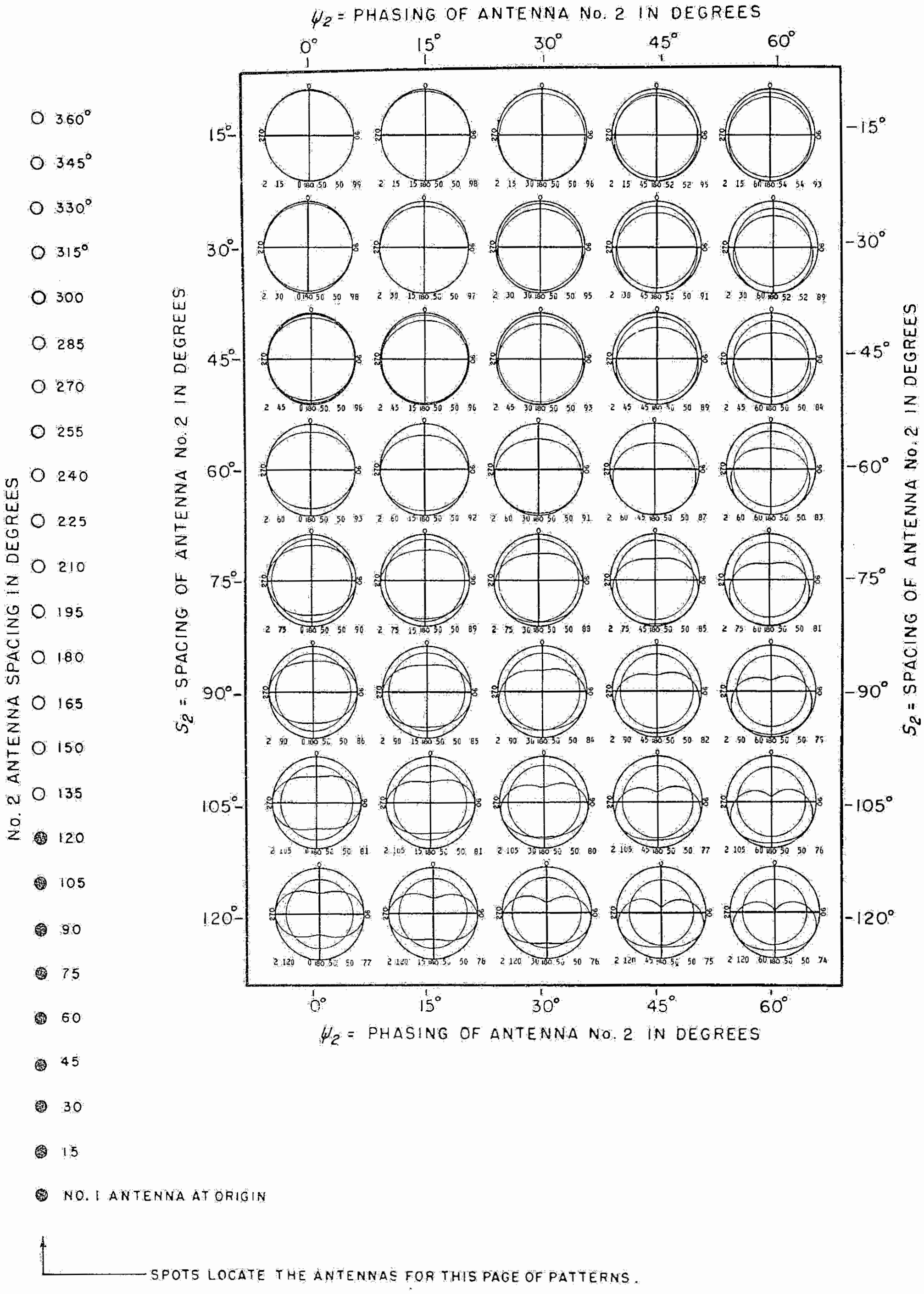
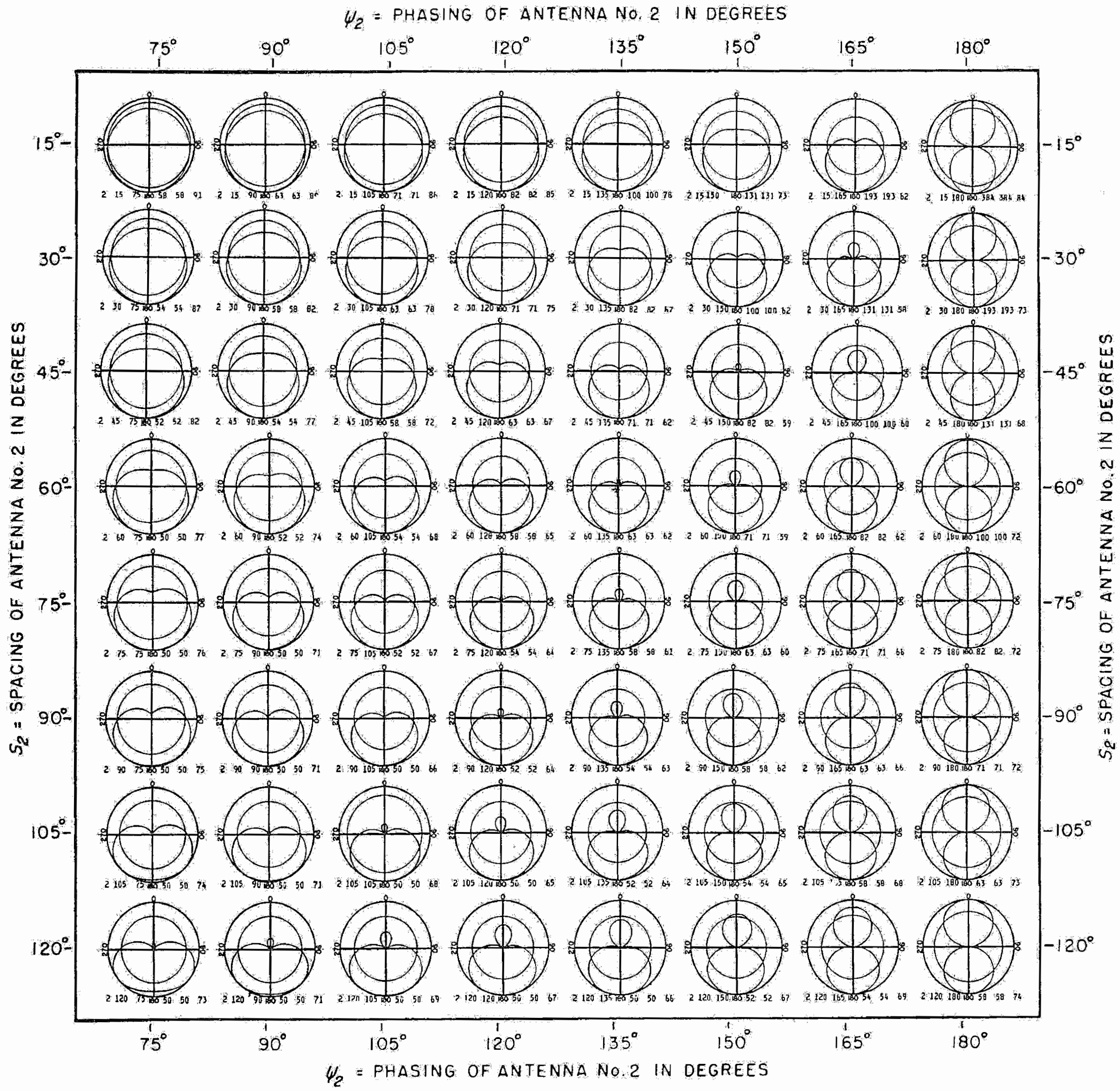
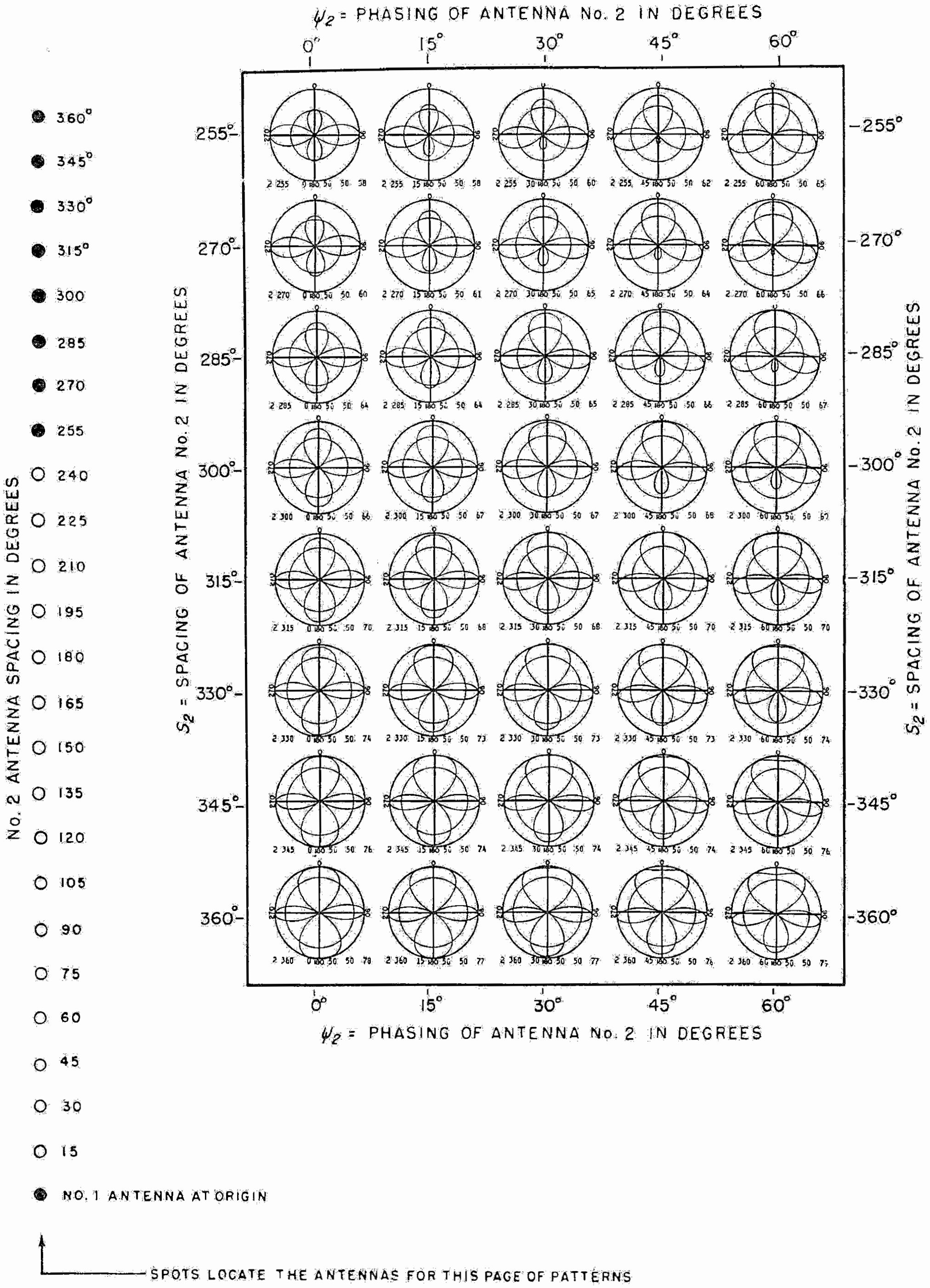

ψ2 = PHASING OF ANTENNA No. 2 IN DEGREES
0° 15° 30° 45° 60°
S2 = SPACING OF ANTENNA No. 2 IN DEGREES
15° 30° 45° 60° 75° 90° 105° 120°
-15° -30° -45° -60° -75° -90° -105° -120°
No. 2 ANTENNA SPACING IN DEGREES
360° 345° 330° 315° 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60 45 30 15
NO. 1 ANTENNA AT ORIGIN
SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS.



CLEVELAND INSTITUTE OF RADIO ELECTRONICS
TERMINAL TOWER, CLEVELAND 13, OHIO.

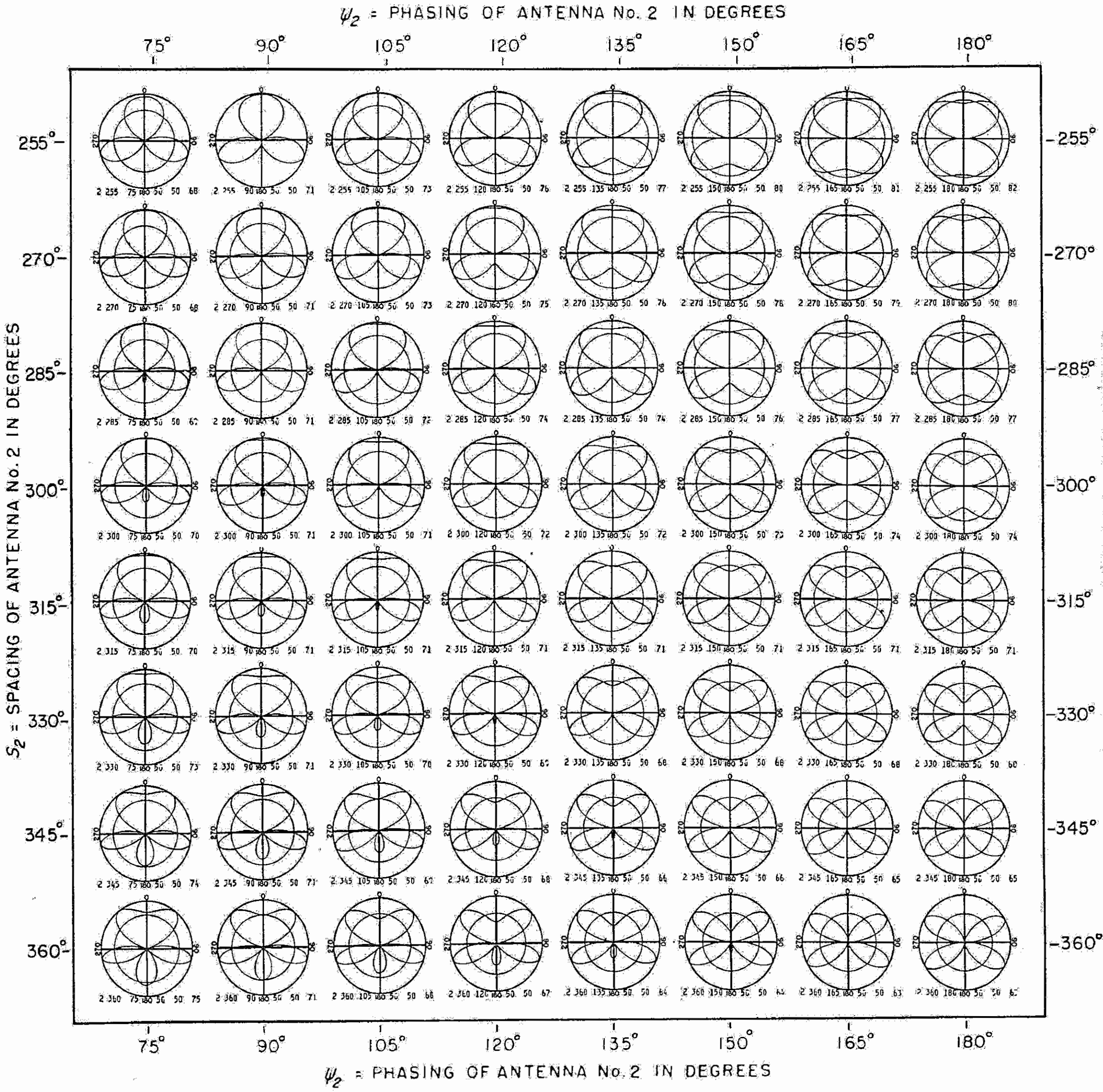
NUMBER OF ANTENNAS IN THE SYSTEM	SPACING OF No. 2 ANTENNA IN DEGREES	PHASING OF No. 2 ANTENNA IN DEGREES	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 1 ANTENNA	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 2 ANTENNA	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM
No.	S_2	ψ_2	E_1	E_2	E_0
2	15	75	58	58	91

PATTERN NOMENCLATURE



ψ_2 = PHASING OF ANTENNA No. 2 IN DEGREES
0° 15° 30° 45° 60°
S_2 = SPACING OF ANTENNA No. 2 IN DEGREES
255° 270° 285° 300° 315° 330° 345° 360°
No. 2 ANTENNA SPACING IN DEGREES
360° 345° 330° 315° 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60 45 30 15
NO. 1 ANTENNA AT ORIGIN
SPOTS LOCATE THE ANTENNAS FOR THIS PAGE OF PATTERNS

CLEVELAND INSTITUTE OF RADIO ELECTRONICS
TERMINAL TOWER, CLEVELAND 13, OHIO.



No.	S_2	ψ_2	E_1	E_2	E_r
NUMBER OF ANTENNAS IN THE SYSTEM	SPACING OF No. 2 ANTENNA IN DEGREES	PHASING OF No. 2 ANTENNA IN DEGREES	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 1 ANTENNA	% FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM No. 2 ANTENNA	% RMS FIELD INTENSITY RADIATED IN HORIZONTAL PLANE FROM ANTENNA SYSTEM
2	255	75	50	50	68

PATTERN NOMENCLATURE

DIRECTIONAL ANTENNAS - THEIR CARE AND MAINTENANCE

By Dixie B. McKey
Consulting Radio Engineer, Washington, D. C.

Standard band broadcast stations employing complex multi-element arrays, present a series of maintenance problems for the station engineering personnel. The handling of these problems by established scheduled routine tests will pay dividends in many forms. The radio engineer in attempting to plan methods whereby his plant will operate in as orderly a manner as possible, very definitely has the job of applying science to a practical business and in fulfilling it, shoulders the responsibility of bringing the results of technical achievement and business together.

Regular scheduled operating maintenance routines are a well established practice in a number of larger broadcast stations. These routines vary considerably, from a visual equipment inspection after cleaning, to elaborate tests designed to furnish recorded data that would permit the station engineer to anticipate failures and remove questionable equipment. These routines produce results in two ways; first, they reduce lost program time due to equipment failure, to a minimum, and, secondly, they familiarize the operating personnel with the location and functions of the individual apparatus units, which they would have been unable to acquire during the regular operating period.

The worth of such routines is not debatable and the long record of uninterrupted program operation of radio broadcast stations using these routine maintenance practices represents a dollar value far in excess of the small maintenance costs.

Any directional antenna system is dependent for satisfactory operation on a large number of individual component parts, carefully related and adjusted to produce a specified radiation pattern; electrical or mechanical failure of individual units will result in maladjustment of the array.

A large number of standard broadcast stations are operating full time or night directional antenna arrays that, due to the station's location with respect to other co-channel stations, must radiate low values of field strength in the direction of the co-channel stations. An array of this type as originally installed and adjusted will meet the values specified in the construction permit with an allowable tolerance for small day to day or seasonal variations providing the equipment is maintained properly. Failure of individual components in such an array will produce serious trouble.

In a number of cases non-directional operation on a single tower with reduced power is impracticable if not impossible, due to the difficulty in reducing the transmitter output to a sufficiently low value to produce the required minimum signal toward the co-channel station to prevent any objectionable interference. Under these conditions failure of any of the directional antenna array component units means a shutdown and a serious and costly loss of program air time. Consequently, any method or procedure that will reduce such loss to a minimum can be justified on a cost basis.

While speech input equipment and transmitters are fairly well standardized as to type and methods of operation, each and every directional antenna array is a custom-built job designed and constructed to meet certain specified requirements. In view of this fact, it is not possible to set up a complete specific maintenance routine that can be directly applied to any and all arrays.

However, a general set of maintenance practices based on a typical modern antenna array system can be used as a guide for the preparation of similar routines designed to fit individual array requirements.

A set of recommended schedules and forms based on a regional station operating with a four tower in line array is used as a typical example.

Before discussing a definite maintenance routine practice, it might be well to consider briefly some of the maintenance problems which generally have a considerable bearing on the satisfactory operation of a directional antenna array and yet are of such a type that they do not fit into a specified routine. For instance, in the case of many antennas, it has been established through considerable experience that in order to maintain the array as originally adjusted, all the conditions which were present during the period of Proof of Performance Tests must be maintained as closely as possible and, while maintenance routines as later suggested will perform this function as regards the individual apparatus unit, it is these factors which are not readily adaptable to a routine and yet may have a considerable bearing on the successful operation of your system. One of the most important factors in the holding of array adjustments is the maintenance of the ground system and periodic checks should be made at regular intervals by the supervisor or the chief engineer.

These checks should consist of an examination of the ground screens, radials, and the bonding for a loose or bad connection and corrosion. This is of particular importance in some locations where heavy corrosion due to the particular soil content has caused a considerable shift in the array pattern in a period of six months to a year after installation. The area immediately adjacent to the towers should be kept free of high grass and weeds and where the installation is in rolling terrain the entire ground system area should be planted with a soil holding type of grass or crop, in order to prevent erosion due to heavy rainfall. The latter is particularly important in installations where a fill has been necessary at either the tower locations or within the ground system area.

Another factor which enters into the matter of maintaining proper adjustment of an array is lightning hits, which, in many locations, are responsible for the largest percentage of lost program time. These hits or surges will cause not only carrier interruption but in some cases destruction of the component equipment parts. In the majority of cases, multiple vertical radiators make very effective lightning rods and can be expected to receive several direct hits and numerous induced surges during the summer static season. In many locations this condition is not truly seasonable as a considerable amount of difficulty has been experienced due to surges that are built up by friction from rain, snow, sand, and wind. Consequently, whenever possible, protective equipment should be utilized to minimize or eliminate potential failures due to these hits or surges. A large number of our modern transmitters contain protective circuits both for the transmitter and the transmission line. However, field experience indicates that it may be necessary to provide additional protective devices and equipment in order to secure maximum protection.

This subject has received considerable treatment by a number of engineers, and particular reference is made to an article by Mr. H. V. Tollison[1] which presents a rather complete summary of a number of methods and circuits which may be employed in the solution of this problem.

The maintenance tests as outlined have been arranged so that they can be made by regular station personnel using test and measuring equipment generally found in any modern radio station, with the addition of a field intensity meter and a radio frequency bridge together with its associated oscillator and detector units. The latter test equipment becomes a necessity for any station operating a complex directional antenna array system under present day conditions.

The preparation of a station maintenance routine for a directional antenna system should be based on the Proof of Performance Report as prepared and filed at the time of the original antenna installation. This report is a part of a station's permanent Federal Communications Commission's file and contains a complete description of the array, circuit diagram, circuit components, meter readings, and the field intensity measurement data. The importance of this report cannot be overemphasized as it provides a source of information that is of greatest importance in the day to day operation of a station.

With this report as the basis for a routine maintenance the first step in the preparation of such a routine will be to prepare a combined maintenance report book or log.

This report book should be divided into three sections. The first section shown in Fig. 1 will consist essentially of a complete description of the individual component parts arranged in four subdivisions consisting of inductance coils, capacitors, resistors, and relays. The listing in each of the subdivisions should carry a description of the unit, its location, or function in the circuit, its type number, replacement ordering information and circuit designation. The listing should also cover the number of turns in use of each inductance coil, the value of each of the capacitors, and the dial settings of the capacitors, if variable.

The second subdivision (Fig. 2) should furnish complete information concerning all meters used in the operation of the array and should consist of a listing of these units showing the circuit designations, the location and function of the unit, its range, type and serial number, and the current reading for the required output. The required phase monitor reading for normal operation should be listed in this subdivision.

The third subdivision (Fig. 3) should contain a listing of the monitoring points as designated by the Federal Communications Commission's license and should show the number of the monitoring points, a complete description for reaching these points, the bearing and distance, the specified unattenuated field value at one mile, the obtained unattenuated field value at one mile, together with the actual received field.

The fourth and last subdivision (Fig. 4) should contain a schematic diagram and, if available, a wiring diagram of the complete antenna array equipment together with the series of curves from the Proof of Performance Report showing the dividing network or driving point impedance of the array (Fig. 5) equipment and operating tower impedance (Fig. 6) if a single tower is used for non-directional daytime opera-

[1]Electronic Industries, Nov. 1946.

tion. Additional pertinent information that would be of assistance to the maintenance engineer, such as, transmission line and capacitor gas pressures, etc., should be included. While it may appear that some of this information will be a duplicate of the Proof of Performance Report, it is believed that by rearranging the data in this form and making it part of the maintenance routine, the constant reference to this information by the station engineering personnel will acquaint them with the equipment and its functions far better than a casual reference to the Proof of Performance Report.

The second section of the maintenance report book should contain a complete list and schedules of the daily work to be done and where necessary, complete instruction covering the methods and equipment to be used. A suggested list is shown in Table I. In the case of the example used the antenna maintenance work is divided into two main classifications. The first classification is designed to cover the daily routine inspection and work that is to be handled by the "late trick" station engineer after sign-off. This work has been arranged in such a manner that all parts and circuits will be cleaned and inspected at least once each week. The second classification already mentioned has been designed to cover a series of special maintenance routines for checking the array operation at regular specified intervals.

The third section of the maintenance report book contains the weekly station maintenance log. This portion of the routine is of extreme importance as it will provide this station supervisor or chief engineer with a method for checking the work done, as well as the data necessary to determine the operating conditions of the array and its equipment at all times. The log shown in Fig. 7 as set up for the example station is arranged to cover one week's complete maintenance information divided into daily sections.

Each daily section provides space for recording the temperature, weather, array meter readings, phase monitor readings, transmission line pressures, routine maintenance performed, routine tests results, other pertinent data, and the signature of the duty engineer.

The recorded data obtained from the daily and special maintenance tests after a period of six months can be plotted to show the actual operating conditions of the array, variations from the normal conditions will undoubtedly appear in the graphs. By a careful analysis of the records, it will be possible to identify any variations due to seasonal or weather conditions. This accumulated information properly evaluated should provide the station engineer with a complete and thorough understanding of his directional antenna array operating and maintenance problems.

It is, of course, realized that the suggestions and recommendations described above are not the total and complete answer to all and every directional antenna. However, it is believed the need for establishing schedules similar to the suggested program is well demonstrated and may be fitted to the individual station requirements, the results of which will be of mutual benefit to both the engineering staff and management.

CIRCUIT INFORMATION

Circuit Designation	Circuit Function	Adjustment Information	Replacement Information
L1	Shunt Input Coil	20-1/2 turns	Continuously Variable Coil - Type 4224 MS4 - 26 uh.
L2	Series Input Coil	9-1/2 turns	Variable Tap Coil - Type 322-4N4 - 26 uh.
L3	No. 1 Tank Coil	8 turns	Variable Tap Coil - Type 3208NT10 - 38 uh.
L4	No. 2 Tank Coil	7 turns	Variable Tap Coil - Type 3208NT10 - 38 uh.
L5	No. 1 Line Feed Coil	1/2 turn	Continuously Variable Coil - Type 4103 - HMS5 - 7 uh.
L6	No. 2 Line Feed Coil	7-3/4 turns	Continuously Variable Coil - Type 4103 - HMS5 - 7 uh.
L7	No. 3 Line Feed Coil	7 turns	Continuously Variable Coil - Type 4103 - HMS5 - 7 uh.
L8	No. 4 Line Feed Coil	5-1/2 turns	Continuously Variable Coil - Type 4103 - HMS5 - 7 uh.
L9	No. 1 Line Series Phase Coil	13-1/4 turns	Continuously Variable Coil - Type 4224 MS4 - 26 uh.
L12) L13)	No. 2 Line Series Phase Coil	8-1/4 turns	Continuously Variable Coil - Type 4224 MS4 - 26 uh.
L14	No. 2 Line Shunt Phase Coil	8-1/2 turns	Variable Tap Coil - Type 4164-N5 - 16 uh.
L15) L16)	No. 3 Line Series Phase Coil	11-1/3 turns	Continuously Variable Coil - Type 4224 MS4 - 26 uh.
L17	No. 3 Line Shunt Phase Coil	5-3/4 turns	Variable Tap Coil - Type 4164-N5 - 16 uh.
L18) L19)	No. 4 Line Series Phase Coil	8-1/4 turns	Continuously Variable Coil - Type 4224 MS4 - 26 uh.
L20	No. 4 Line Shunt Phase Coil	6 turns	Variable Tap Coil - Type 4164-N5 - 16 uh.
L21	No. 1 Tower Series Input Coil	11 turns	Variable Tap Coil - Type 3164-N5 - 16 uh.
L22	No. 1 Tower Series Output Coil (D)	7 turns	Variable Tap Coil - Type 4165-N5 - 22 uh.
L23	No. 1 Tower Shunt Coil	D10 turns ND 3-1/2 turns	Variable Tap Coil - Type 3106-NT12 - 10 uh.
L25	No. 2 Tower Series Input Coil	9 turns	Variable Tap Coil - Type 4165-N5 - 22 uh.
L26	No. 2 Tower Series Output Coil	4-1/2 turns	Variable Tap Coil - Type 4105-N5 - 12 uh.
L27	No. 2 Tower Shunt Coil	10 turns	Variable Tap Coil - Type 4164-N5 - 16 uh.
L29	No. 3 Tower Series Input Coil	1 turn	Variable Tap Coil - Type 4143-N5 - 8 uh.
L30	No. 3 Tower Series Output Coil	4 turns	Variable Tap Coil - Type 4165-N5 - 22 uh.
L31	No. 3 Tower Shunt Coil	11 turns	Variable Tap Coil - Type 4164-N5 - 16 uh.
L33	No. 1 Tower Series Output Coil (ND)	10-3/4 turns	Variable Tap Coil - Type 4165-N5 - 22 uh.
L34	No. 4 Tower Series Input Coil	11-1/4 turns	Variable Tap Coil - Type 3164-N5 - 16 uh.
L35	No. 4 Tower Series Output Coil	5-1/2 turns	Variable Tap Coil - Type 3165-N5 - 22 uh.
L36	No. 4 Tower Shunt Coil	12 turns	Variable Tap Coil - Type 3164-N5 - 16 uh.
L24	No. 1 TC Choke Coil		RF Choke
L28	No. 2 TC Choke Coil		RF Choke
L32	No. 3 TC Choke Coil		RF Choke
L37	No. 4 TC Choke Coil		RF Choke
L38	No. 1 Tower Static Drain		
L39	No. 2 Tower Static Drain		
L40	No. 3 Tower Static Drain		
L41	No. 4 Tower Static Drain		
	CONDENSERS	SETTING	
C1	Shunt Input Capacity		Type 750FBA90 - 750 uuf.
CA	Series Input Capacity		Type 1000 FBA90 - 1000 uuf.
C2	No. 1 Tank Tuning Capacity	45	Type 750FVSP250 - 750 uuf.
C3	No. 2 Tank Tuning Capacity	85	Type 750FVSP250 - 750 uuf.
C4	No. 1 Line Series Phase Capacity		Type 1500 FBA90 - 1500 uuf.
C5	No. 2 Line Shunt Phase Capacity		Type 1000 FBA90 - 1000 uuf.
C6	No. 3 Line Shunt Phase Capacity		Type 1000 FBA90 - 1000 uuf.
C7	No. 4 Line Shunt Phase Capacity		Type 1000 FBA90 - 1000 uuf.
C8	No. 1 Tower Series Output Capacity		Type 1000 FBA90 - 1000 uuf.
C9	No. 1 Tower Shunt Capacity		Type 1250FD150 - 1200 uuf.
C11	No. 2 Tower Series Output Capacity		Type 1000 FBA90 - 1000 uuf.
C12	No. 2 Tower Shunt Capacity		Type 1000 FBA90 - 1000 uuf.
C14	No. 3 Tower Shunt Output Capacity		Type 1000 FBA90 - 1000 uuf.
C16	No. 4 Tower Shunt Capacity		Type 1000 FBA90 - 1000 uuf.
C17	No. 3 Tower Series Output Capacity		Type 1000 FBA90 - 1000 uuf.
C19	No. 4 Tower Series Output Capacity		Type 1000 FBA90 - 1000 uuf.
C20	No. 1 Tower Series Output Capacity (D)		Type 1000 FBA90 - 1000 uuf.
C10	No. 1 TC Shunt Capacity		Type CD 2-MFD
C13	No. 2 TC Shunt Capacity		Type CD 2-MFD
C15	No. 3 TC Shunt Capacity		Type CD 2-MFD
C17	No. 4 TC Shunt Capacity		Type CD 2-MFD
	RELAYS		
S1	Antenna Array Transfer Relay		RF Contactor
S4	No. 1 Tower Antenna Ammeter Switch		MBB Switch
S5	No. 1 Tower Antenna Transfer Relay		RF Contactor
S6	No. 2 Tower Antenna Ammeter Switch		MBB Switch
S7	No. 2 Tower Antenna Transfer Relay		RF Contactor
S8	No. 3 Tower Antenna Ammeter Switch		MBB Switch
S9	No. 3 Tower Antenna Transfer Relay		RF Contactor
S12	No. 4 Tower Antenna Ammeter Switch		MBB Switch
S13	No. 4 Tower Antenna Transfer Relay		RF Contactor

Figure 1

METER INFORMATION

Meter	Range	Model	No.	Current D	Current ND	Unit
M1	0-8	640	2845	4.58	-	Transmitter Input to Divider Network
M2	0-15	640	2843	1.0	9.9	Transmission Line Current - Tower No. 1
M3	0-8	640	2835	3.9	-	Transmission Line Current - Tower No. 2
M4	0-8	640	2820	3.8	-	Transmission Line Current - Tower No. 3
M5	0-8	640	2830	2.5	-	Transmission Line Current - Tower No. 4
M6	0-15	640	2810	1.0	9.45	Transmission Line Coupling Unit - Tower #1
M9	0-8	640	2815	2.6	-	Transmission Line Coupling Unit - Tower #2
M12	0-8	640	2816	3.85	-	Transmission Line Coupling Unit - Tower #3
M15	0-5	640	2819	2.5	-	Transmission Line Coupling Unit - Tower #4
M8	0-15	640	2821	3.0	10.39	Antenna Current - Tower #1
M11	0-8	640	2822	5.4	-	Antenna Current - Tower #2
M14	0-8	640	2823	5.35	-	Antenna Current - Tower #3
M17	0-5	640	2924	3.11	-	Antenna Current - Tower #4
*M7	0-15	425	3130	3.0	10.39	Remote Antenna Current - Tower #1
*M10	0-8	425	3131	5.4	-	Remote Antenna Current - Tower #2
*M13	0-8	425	3133	5.35	-	Remote Antenna Current - Tower #3
*M16	0-8	425	3134	3.11	-	Remote Antenna Current - Tower #4
MP1	0-150%	743	3638	100%	100%	Phase Monitor - Tower #1
MP2	0-150%	743	3624	100%		Phase Monitor - Tower #2
MP3	0-150%	743	3636	100%		Phase Monitor - Tower #3
MP4	0-150%	743	3621	100%		Phase Monitor - Tower #4

NOTE: All meters Weston Electric Company

*External Heater Type - See Figure 4 for Location in Circuits

Phase Monitor Readings for Directional Operation

Tower #1 Leads Tower #2 by 230°
Tower #1 Leads Tower #3 by 84°
Tower #1 Leads Tower #4 by 302°

Figure 2

MONITOR POINTS AND FIELD INTENSITY INFORMATION

Point No. 4

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
12°	1.89	30	20	3.95

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 14

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
40°	2.75	90	76	6.5

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 25

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
63°	1.33	70	25	8.55

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 62

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
181°	1.36	72	40	10.75

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 82

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
219°	2.18	52	40	5.6

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 104

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
302°	.86	40	17	7.6

Insert Location of Measuring Point as Described in Proof of Performance

Point No. 118

Azimuth Angle	Distance (Miles)	MV/M 1 Mile Specified	MV/M 1 Mile Obtained	MV/M Measured
337°	1.15	61	40	13.4

Insert Location of Measuring Point as Described in Proof of Performance

Figure 3

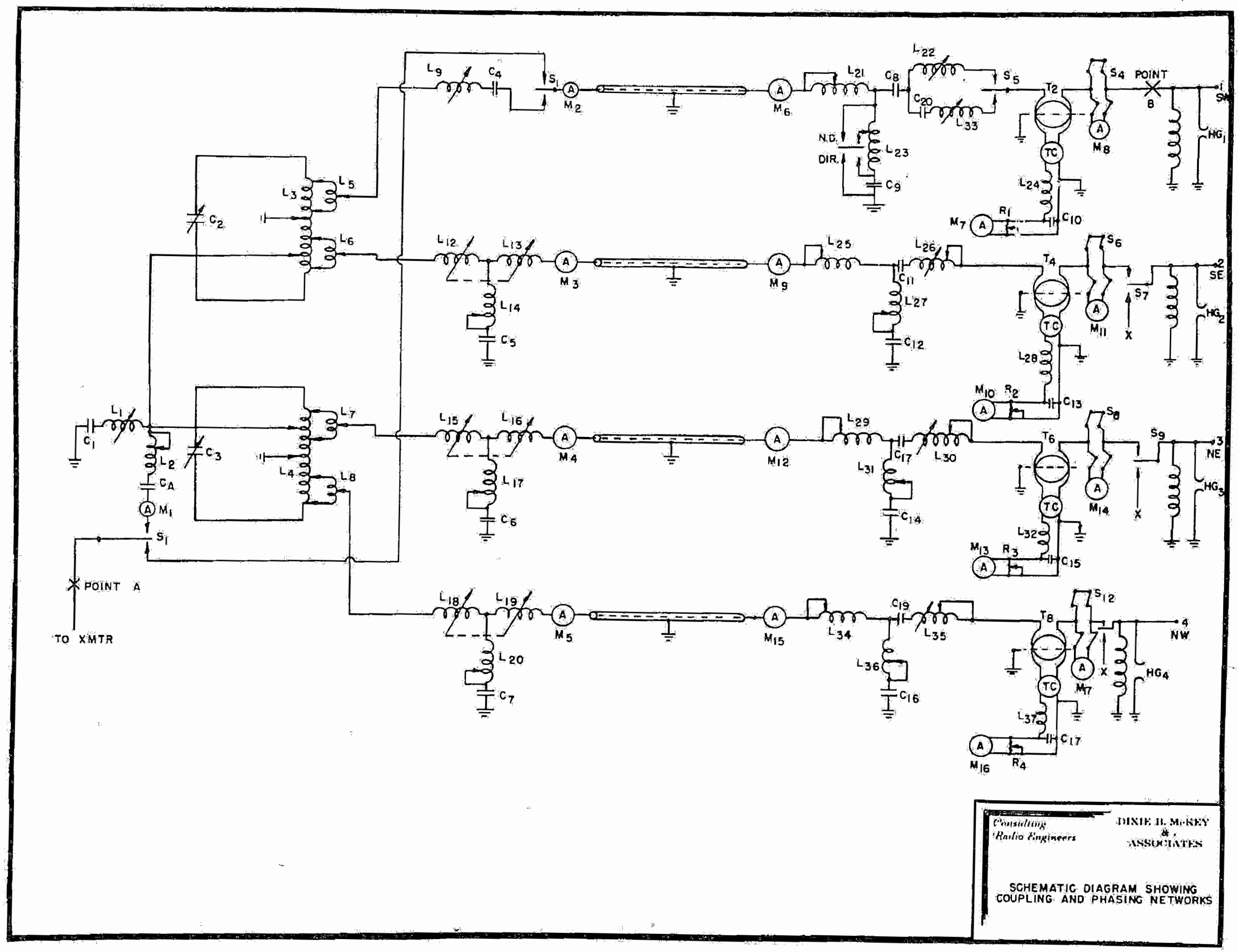


Figure 4

TABLE I

MAINTENANCE SCHEDULES

Sunday

1. Transfer all towers and antenna phasing units to emergency transmission lines and operate at full power for five minutes.

2. Restore all equipment to regular transmission lines. Check operation under full power.

3. Clean and check all connections, remote meter, phase monitor, meter panels, and phase monitor.

4. Check all phase monitor tubes with tube check. Check all transmission line protective circuits.

5. Check and record all transmission line gas pressures.

Monday

1. Check all condensers and other equipment in antenna phasing unit immediately after sign-off for over heating.

2. Clean and check all transmission line end seals.

3. Clean interior all sections antenna phasing units.

4. Clean contacts and check alignment antenna transfer relay.

5. Check and tighten all connections in antenna phasing unit.

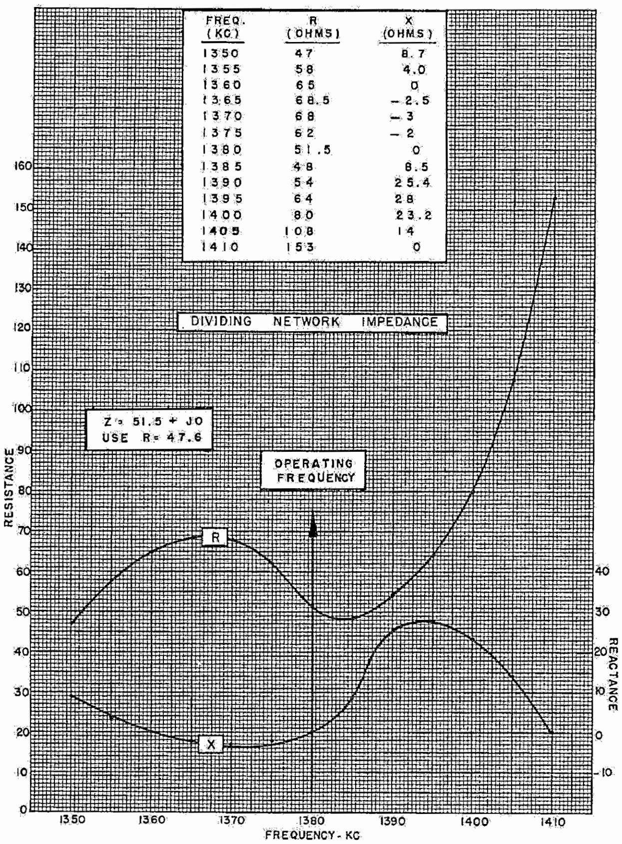
6. Check gas filled condenser pressures.

Tuesday

1. With array set for directional operation check drive point impedance at X with radio frequency bridge at operating frequency. (First and Third Tuesdays)

2. With array set for non-directional operation check drive point impedance at X with radio frequency bridge at operating frequency. (First and Third Tuesdays)

3. Set up array for normal full power directional operation. Compare readings all antenna and remote antenna meters. Make any necessary adjustments.



FREQ. (KC)	R (OHMS)	X (OHMS)
1350	47	8.7
1355	58	4.0
1360	65	0
1365	68.5	− 2.5
1370	68	− 3
1375	62	− 2
1380	51.5	0
1385	48	8.5
1390	54	25.4
1395	64	28
1400	80	23.2
1405	108	14
1410	153	0

Figure 5

4. Make complete set field intensity readings at indicated monitor points. (Second and Fourth Tuesdays)

5. Check and record all transmission line gas pressures.

Wednesday

Antenna Coupling Unit No. 1

1. Check all condensers and equipment in coupling house for overheating immediately after sign-off

2. Check spacing and clean antenna and transmission line horn gaps.

3. Check and clean all antenna lead in insulators.

4. Check and clean all transmission line end seals.

5. Clean contacts and check alignment antenna relay.

6. Clean contacts and check alignment antenna ammeter switch.

7. Check and tighten all connections of inductance coils and condensers.

8. Clean all meters.

Transmitter Building

Read and record all transmission line gas pressures.

Thursday

Antenna Coupling Unit No. 2

1. Check all condensers and equipment in coupling house for overheating immediately after sign-off.

2. Check spacing and clean antenna and transmission line horn gaps.

3. Check and clean all antenna lead in insulators.

4. Check and clean all transmission line end seals.

5. Clean contacts and check alignment antenna relay.

6. Clean contacts and check alignment antenna ammeter switch.

7. Check and tighten all connections of inductance coils and condensers.

8. Clean all meters.

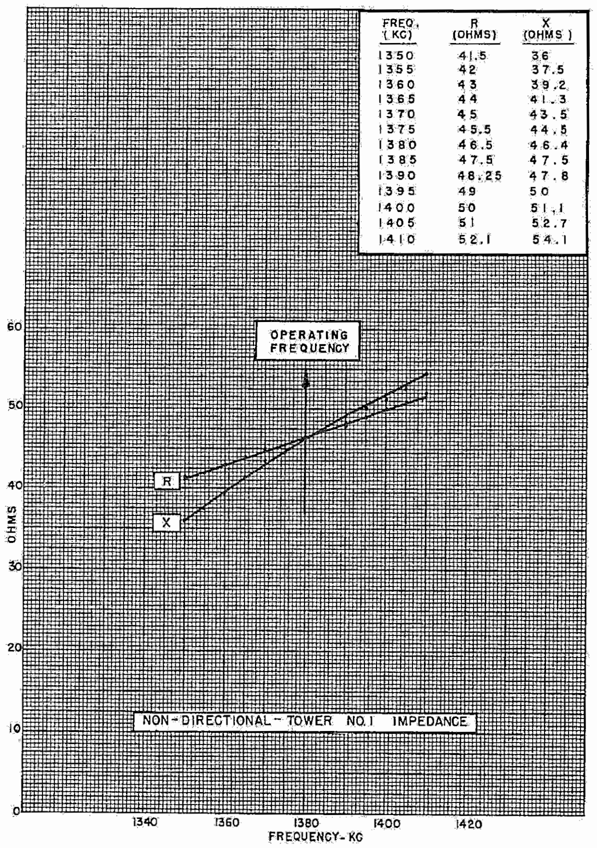
Transmitter Building

Read and record all transmission line gas pressures.

Friday

Antenna Coupling Unit No. 3

1. Check all condensers and equipment in coupling house for overheating immediately after sign-off.



FREQ. (KC)	R (OHMS)	X (OHMS)
1350	41.5	36
1355	42	37.5
1360	43	39.2
1365	44	41.3
1370	45	43.5
1375	45.5	44.5
1380	46.5	46.4
1385	47.5	47.5
1390	48.25	47.8
1395	49	50
1400	50	51.1
1405	51	52.7
1410	52.1	54.1

Figure 6

WEEKLY MAINTENANCE REPORT

Month		Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Remarks	Eng.	
Date											
Weather											
Temperature											
M1 - Dir.											
M2 - Non-Dir.											
M7 - Dir.											
M7 - Non-Dir.											
M10											
M13											
M16											
#1 - #2 Phase											
#1 - #3 Phase											
#1 - #4 Phase											
TX Line #1 Press.											
TX Line #2 Press.											
TX Line #3 Press.											
TX Line #4 Press.											
Phase Monitor											
Prot. Circuits											
EMG TX Lines											
Main Phase Unit											
Non-Dir. Imp.											
Dir. Imp.											
FS Mon. Pt. #4											
FS Mon. Pt. #14											
FS Mon. Pt. #25											
FS Mon. Pt. #62											
FS Mon. Pt. #82											
FS Mon. Pt. #104											
FS Mon. Pt. #118											
Overall Dir. Test											
#1 Coupling Unit											
#2 Coupling Unit											
#3 Coupling Unit											
#4 Coupling Unit											
E-207-A (7-41)											

Figure 7

2. Check spacing and clean antenna and transmission line horn gaps.

3. Check and clean all antenna lead in insulators.

4. Check and clean all transmission line end seals.

5. Clean contacts and check alignment antenna relay.

6. Clean contacts and check alignment antenna ammeter switch.

7. Check and tighten all connections of inductance coils and condensers.

8. Clean all meters.

Transmitter Building

Read and record all transmission line gas pressures.

Saturday

Antenna Coupling Unit No. 4

1. Check all condensers and equipment in coupling house for overheating immediately after sign-off.

2. Check spacing and clean antenna and transmission line horn gaps.

3. Check and clean all antenna lead in insulators.

4. Check and clean all transmission line end seals.

5. Clean contacts and check alignment antenna relay.

6. Clean contacts and check alignment antenna ammeter switch.

7. Check and tighten all connections of inductance coils and condensers.

8. Clean all meters.

Transmitter Building

Read and record all transmission line gas pressures.

NOTES ON TELEVISION ANTENNA EQUIPMENT

Reprinted from the RCA Manual for Television Technical Training Program through courtesy of the Radio Corporation of America

The following notes are a summary of lectures on antenna equipment for television systems. They present, in outline form, the various pieces of equipment required in the television station beyond the transmitter. Each piece is discussed as to its purpose, requirements, basic principles, construction, and performance.

SUMMARY OF ITEMS OF ANTENNA EQUIPMENT

NAME	LOCATION	FUNCTION
Balun (BALanced to UNbalanced)	Part of transmitter	Converts double-ended line to single-ended line and maintains bandwidth.
Vestigial Sideband Filter (VSBF)	Station, behind transmitter	Reduces energy in portion of lower sideband (portion outside channel) in accordance with FCC requirements.
Diplexer	Station, behind transmitter	Permits simultaneous use of a same antenna for visual and aural transmission.
Triplexer	Station, behind transmitter	Permits simultaneous use of a same antenna for FM broadcasting in the 88-108 mc band.
Antennas (Standard Types)	Top of tower	Radiates visual and aural signals as well as FM signal, if triplexing is used.
Antennas (Special Types)	Top of tower	Same as above, except fulfills special requirements as higher gain, simultaneous use of one site for several television stations, directional effects, etc.
Antennas (Combination Types)	Top of tower	Allows FM broadcasting by use of Pylon.
Transmission Line	In station and between station and antenna	Transmits power with minimum reflection and loss.

GENERAL PRINCIPLES

TRANSMISSION LINES

THE INFINITE LINE is a uniform line of infinite length. For practical purposes, a line terminated in its characteristic impedance (also called surge impedance) behaves like an infinite line because there are then no reflections from the far end of the line.

SURGE IMPEDANCE - The surge (or characteristic) impedance of a uniform line is the impedance

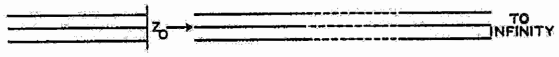


Figure 7-1 - Infinite Line and Surge Impedance

which terminates any length of the line without causing any reflections.

STANDING WAVES - Standing waves on a transmission line are caused by reflections from a mistermination at the far end or by discontinuities on the line itself. The reflected wave is alternately in phase and out of phase with the incident wave. Voltage at a point on the line measures less than the incident wave if the reflected wave is out of phase with the incident wave; it measures more than the incident wave if the reflected wave is in phase with the incident wave.

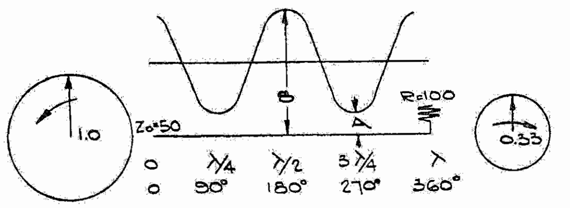


Figure 7-2 - Standing Waves on Transmission Line

VOLTAGE STANDING WAVE RATIO (VSWR) - The VSWR is defined as the ratio of maximum-to-minimum volts appearing along the transmission line. These voltage maxima and minima are the resultants of the addition and subtraction, respectively, of the incident and reflected waves. The surge impedance of the line being designated by Z_0 and the terminating impedance being called Z, the voltage reflection coefficient K is

$$K = \frac{Z - Z_0}{Z_0 - Z}$$

Thus for $Z_0 = 50$ ohms and $Z = 100$ ohms we have $K = 0.33$.

If A and B are the minimum and maximum voltages along the line, respectively, then

$$VSWR = \frac{B}{A} = \frac{1 + |K|}{1 - |K|} \; (= 2 \; for \; K = 0.33).$$

From experimental data it is found that the VSWR of the antenna system should be 1.1 or better over the band, to avoid the appearance of echoes or multiple images in the picture.

BANDWIDTH - The bandwidth is the frequency range over which the VSWR is within certain defined limits.

General Principles of Maintaining Bandwidth.

a. *Constant-impedance Network* - This is a parallel-resonant circuit having resistive components in each branch. When

$$R_C = R_L = \sqrt{L/C}$$

the circuit has constant resistance at all frequencies. Such a circuit is used in modulator and sideband filters.

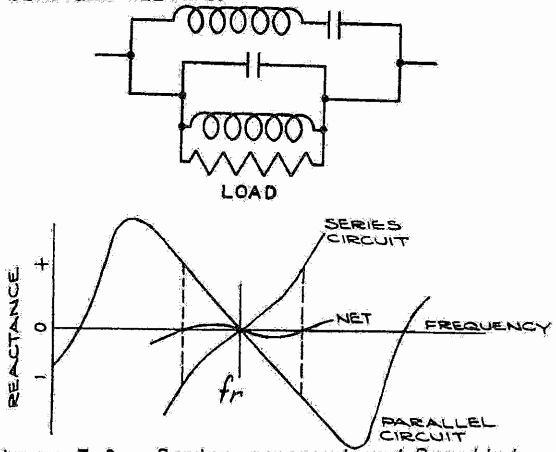


Figure 7-3 - Series-resonant and Parallel-resonant Circuit Combination

b. Series-resonant circuits, used in conjunction with parallel-resonant circuits, provide complementary reactances off resonance frequency. This principle is applied in the balun, diplexer, and antenna. It is illustrated in Figure 7-3, which shows the circuit arrangement as well as the series-circuit and parallel-circuit reactances as functions of frequency. The overall, or net, reactance is the difference of the series-circuit and parallel-circuit reactances. This net reactance is seen to be nearly zero within the range of frequencies (marked off in dotted lines on the diagram) which defines the bandwidth of the system.

BALUN

This name is derived from the words "BALanced to UNbalanced" which define the purpose and function of the device, namely, to convert the output of the 8D21 tube to a single 72-ohm line, on both aural and visual sides.

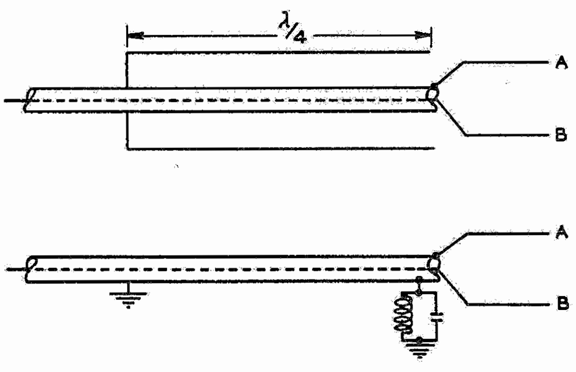


Figure 7-4 - Quarter-wave Transformer

The principle is illustrated in the first diagram of Figure 7-4. This shows a coaxial line, the end of which is surrounded by a quarter-wave sleeve connected to the outer conductor. The equivalent circuit of this "quarter-wave transformer" is shown in the second diagram of Figure 7-4 as a parallel-resonant circuit connected between the outer conductor *A* and ground, but not across the inner conductor *B*.

As a remedy to this unsymmetrical arrangement, the device of Figure 7-5 is used; it is here shown with its equivalent circuit. This is an application of the second principle discussed previously for broadbanding (see paragraph b under the heading BANDWIDTH). The actual circuit shown in Figure 7-5 has been used in the diplexer. For the balun unit, the loads and generator should be interchanged.

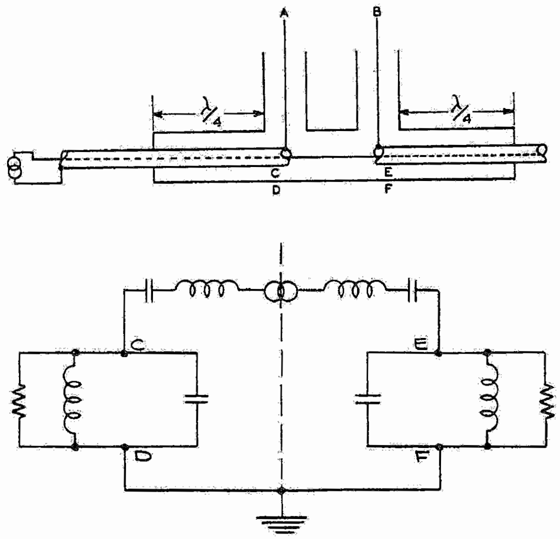


Figure 7-5 - Principle of Diplexer and Balun Units

VESTIGIAL SIDEBAND FILTER

PURPOSE - The vestigial sideband filter reduces the energy in the lower sideband in accordance with FCC requirements, while still presenting constant resistance to the transmitter over the required band.

REQUIREMENTS

(1) 20 db in addition to the average 40 db inherent in typical picture.

(2) Attenuation occurs in 1/2 mc.

(3) Constant VSWR of 1.1, or better, over the band.

(4) Small power absorption.

Typical characteristics of the vestigial sideband filter are shown by the curves of Figure 7-6.

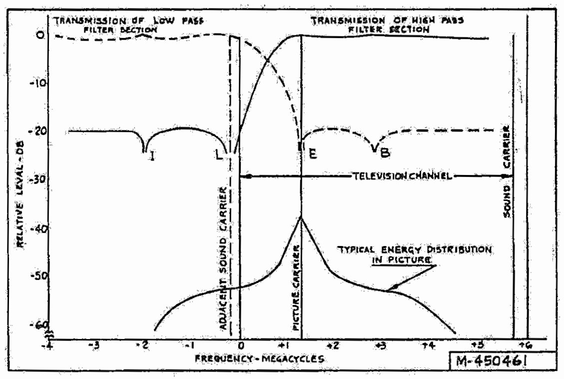


Figure 7-6 - Characteristics of Vestigial Sideband Filter

PRINCIPLE - A simple filter circuit is shown in Figure 7-7. This filter, however, does not satisfy the second of the requirements listed in the preceding paragraph. It is thus necessary to use a four-leg filter network, such as the

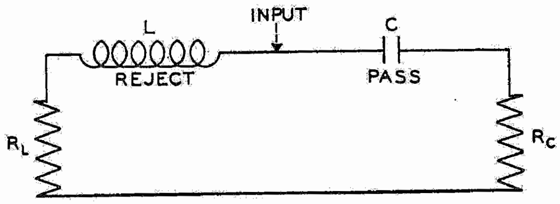


Figure 7-7 - Elementary Filter Circuit
$(R_L = R_C = \sqrt{L/C}$

one shown in the upper diagram of Figure 7-8. Depending on the frequency considered, this network appears with a low-pass or a high-pass characteristic, and is equivalent to the circuits shown in the lower diagram of Figure 7-8.

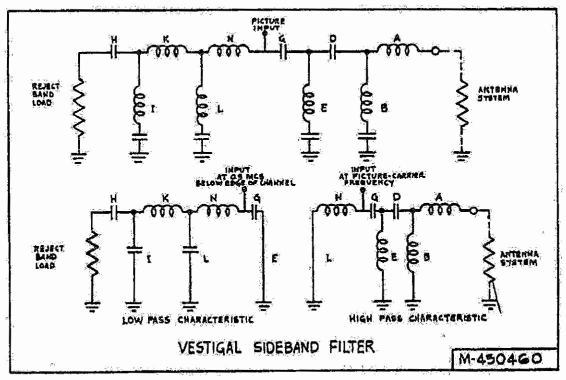
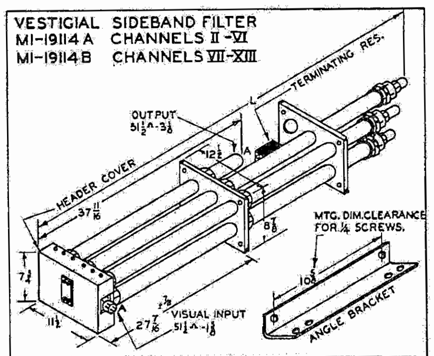


Figure 7-8 - Vestigial Sideband Filter Circuit

CONSTRUCTION - See Figure 7-9 Four shunt lines are made re-entrant to limit the length to a value of 92 to 121 inches, depending on the channel considered. Series capacitors and inductors are line sections less than one-quarter wavelength long. These sections are telescoped into the shunt sections. The entire unit is 92 to 121 inches long, 11 1/2 inches wide, and 7 3/4 inches high, and is designed for ceiling mounting.



CHANNEL	L	CHANNEL	L	CHANNEL	L
II	121"	III	108"	IV	100"
V	88"	VI	80"	VII	110"
VIII	107"	IX	103"	X	100"
XI	97"	XII	95"	XIII	92"

NAME	VESTIGIAL SIDEBAND FILTER
MI	19114A & 19114B
FREQUENCY MC	54-216
WEIGHT LBS.	200
RF INPUT & OUTPUT	51½^ SEE K-8889742
TYPE OF MTG.	HORIZONTAL SUSPENDED FROM CEILING ABOVE OR BESIDE XMTR.

Figure 7-9 - Vestigial Sideband Filter Construction

PERFORMANCE - All units are preset at the factory and require no field adjustment. The power absorbed in the load resistor never exceeds 30 watts, out of 5 kw, and then only with unusual detail energy beyond 1 1/4 mc. Normally, negligible power is absorbed in a typical picture. The unit meets all requirements outlined previously. Power absorption is 3.37% at 100 mc.

R-F LOAD AND WATTMETER

PURPOSE - The purpose of the R-F Load and Wattmeter is to determine the operating power in accordance with FCC requirements. The unit isolates the antenna system from the transmitter when shooting trouble.

REQUIREMENTS

(1) Dissipates black-level power of 3 kw.
(2) Accurately terminates output of transmitter with no reflection.

PRINCIPLE - The principle of the R-F Load is illustrated in Figure 7-10. A small resistance R_1 can be inserted at section A. Section B should then be terminated for a surge impedance of $Z_0 - R_1$, which results in a smaller diameter. The process is continued until the required surge impedance is zero. This is merely a means of using a resistor of finite size when an infinitesimally small resistor would otherwise be needed for proper termination, since this is, practically, not feasible. The shape of the outer conductor, theoretically, follows an exponential curve, but a straight line is close enough an approximation for practical cases.

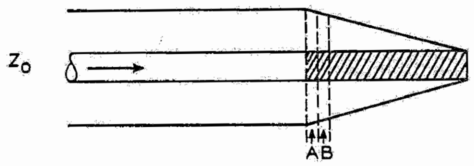


Figure 7-10 - Principle of R-F Load

The resistor is cooled by immersing it in a light grade of oil. With the unit in vertical position, the oil flows by convection to the top of the tank, where it is cooled by tap water while in use. Power is measured by measuring the voltage across the resistor, using a crystal detector for rectification.

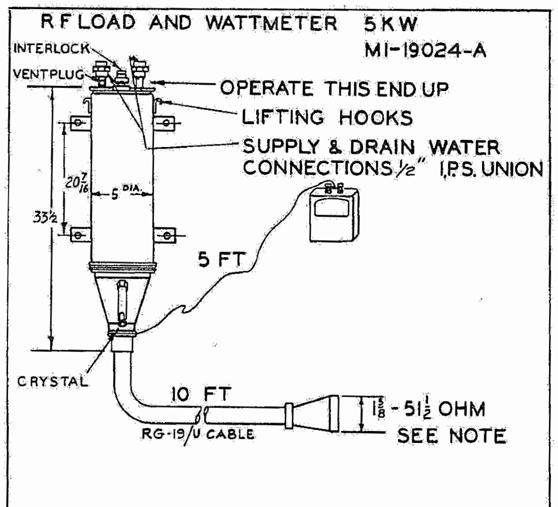
CONSTRUCTION - The load consists of a tank 5 inches in diameter and 33 1/2 inches overall in height. Connections are made to the output of the transmitter by 10-foot lengths of flexible RG-19/U cable. The unit terminates a 51.5-ohm line and, if used with the 75-ohm vestigial sideband filter, requires a quarter-wave transformer for conversion. See Figure 7-11.

DIPLEXER

PURPOSE - The diplexer permits the use of the same antenna for visual and aural transmission.

REQUIREMENTS

(1) Low cross-talk.
(2) VSWR of 1.1, or better, over the band.
(3) Minimum power absorption.



NAME	RF LOAD & WATTMETER 5 KW
MI	19024-A
FREQUENCY MC	44-216
WEIGHT LBS.	46
RF INPUT	1⅝ 51½ Ω
RF OUTPUT	
NON RF CONNECTIONS	TAP WATER 1 GALLON PER MINUTE
TYPE OF MOUNTING	VERTICAL SURFACE

Figure 7-11 - R-F Load and Wattmeter Construction

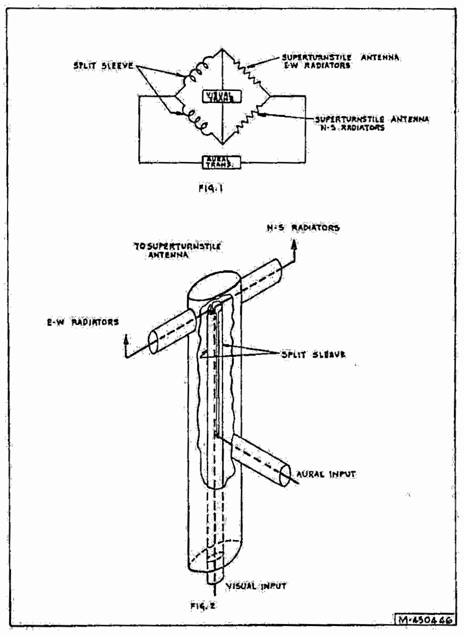


Figure 7-12 - Diplexer Schematic Diagram

PRINCIPLE - Because of the turnstiling principle, which will be discussed under the heading ANTENNAS, the antenna can be considered as two separate loads - a N-S radiator and an E-W radiator - which do not interact (see Figure 7-12). The diplexer is a bridge circuit, as shown in diagram 1 of Figure 7-12.

The visual transmitter output is single-ended and must be converted to a double-ended output. This is done with another balun unit, applying the same principle as discussed previously.

The use of a combination of sideband filter and diplexer is illustrated in Figure 7-13. The energy from the picture transmitter goes through the vestigial sideband filter, where the unwanted part of the lower sideband is suppressed. The energy to be transmitted then passes through the diplexer, where the sound energy joins in. The combined television signal (visual and aural energy) is then fed to the antenna over twin transmission lines, of which one is made one-quarter wavelength longer than the other to provide proper quadrature-phasing in the antenna. This last point is discussed in a later paragraph concerning the turnstiling principle.

CONSTRUCTION - The construction of the diplexer is apparent from Figure 7-14 and is not further described here.

PERFORMANCE

(1) No cross-talk has ever been encountered in the many checks that have been made.
(2) The VSWR is 1.1, and usually better, over the band.
(3) The power absorbed is a fraction of a decibel.

ANTENNA

REQUIREMENTS (ELECTRICAL)

(1) Horizontal polarization.
(2) Operation in 54-88 mc and 174-216 mc bands.
(3) Low VSWR over channel.
(4) Gain sufficient to provide good coverage.
(5) Pattern to be circular.

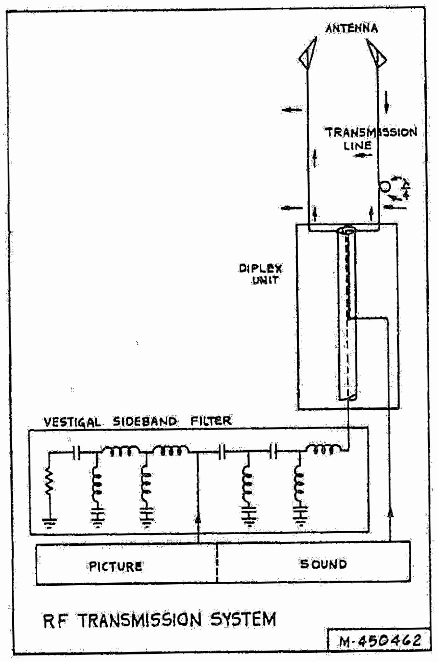
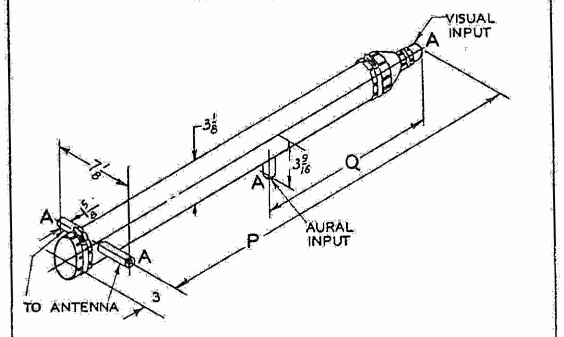


Figure 7-13 - R-F Transmission System

DIPLEXER MI-19028



CHANNEL	P	Q	CHANNEL	P	Q
I	133 15/16	68 7/8	VIII	36 5/16	20 1/16
II	107 7/16	55 5/8	IX	35 5/16	19 9/16
III	97 9/16	50 11/16	X	34 5/16	19 1/16
IV	89 7/16	46 5/8	XI	33 9/16	18 11/16
V	78 9/16	41 3/16	XII	32 9/16	18 3/16
VI	73 5/16	38 9/16	XIII	31 13/16	17 13/16
VII	37 9/16	20 11/16			

NAME	DIPLEXER
MI	19028
FREQUENCY MC	54-216
WEIGHT LBS.	12 TO 30
RF INPUT AND OUTPUT	1⅝-5½"(SEE K-8889742)
TYPE OF MOUNTING	HORIZONTAL ALONG TRANSMISSION LINE (CAN BE SUSPENDED ABOVE VESTIGIAL SIDE-BAND FILTER OR TRANSMITTER.)
MAXIMUM POWER	5 KW
RF EFFICIENCY	99%

Figure 7-14 - Diplexer Construction

(6) Ability to handle visual and aural power of transmitter, and additional FM transmitter in some cases.

(7) Must lend itself to transmission of visual and aural energy from the same antenna.

(8) Transmission of FM from the same antenna is also desirable.

REQUIREMENTS (MECHANICAL)

(1) Low wind resistance.
(2) Mechanically rugged.
(3) Sleet melting provision.
(4) Minimum number of end seals.
(5) Not to be vulnerable to lightning.
(6) Easily erected.

PRINCIPLE

(1) For low VSWR response over the band (1.1 or better), use is made of the second general principle, previously discussed under the heading BANDWIDTH, of combining a parallel-resonant with a series-resonant circuit. Such a combination circuit, using lumped parameters, was shown in Figure 7-3, together with its reactance-frequency characteristic curve. The distributed-parameter equivalent, which forms an antenna, is shown in Figure 7-15a. Its reactance-frequency curve is given in Figure 7-15b.

Addition of a second stub to the arrangement of Figure 7-15a provides a second ground point and adds rigidity; see Figure 7-15c.

In general, radiators with large diameters tend to have a more constant impedance over a given band since the ratio X_L/R, and hence the "Q", is smaller. Since a large diameter is not suitable for mechanical reasons, the same advantages are obtained by using a large flat sheet. This sheet can be visualized as a series of dipoles, each carrying a current proportional to the current distributed along the two stubs. See Figure 7-15d.

By notching-in the sides, the current in the upper and lower edges of the radiator, also called "batwing", is increased, which flattens the vertical pattern and increases the gain. One radiator has the same gain as two dipoles spaced one-half wavelength apart. See Figure 7-15e.

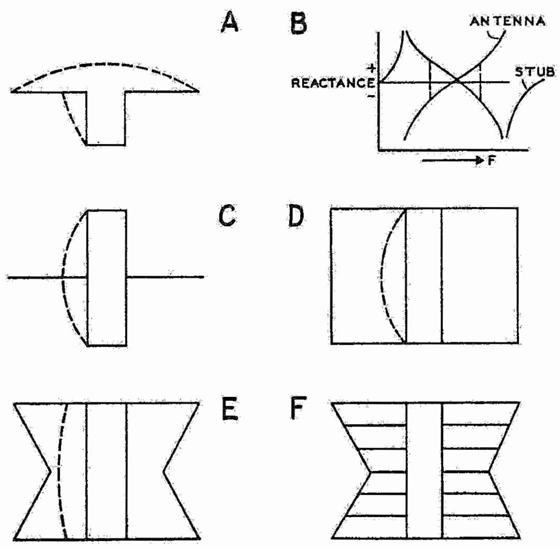


Figure 7-15 - Development of TV Antenna

By experimentally determining the minimum number of rods that can be used instead of a solid sheet, wind resistance is considerably reduced. See Figure 7-15f.

(2) *The Turnstiling Principle* - We now have a dipole with wide-band characteristics. To obtain a circular pattern, turnstiling is applied.

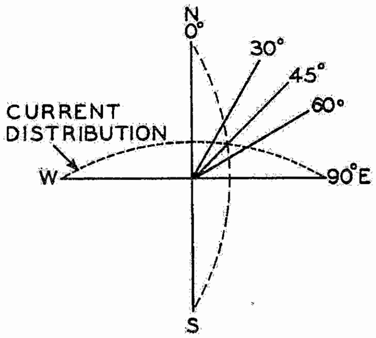


Figure 7-16 - Turnstiling Principle (Field Magnitude is Proportional to the Angle Between Direction of Radiator and Direction of Observation)

A circular radiation pattern can be obtained from cross dipoles arranged in the shape of a turnstile as follows (see Figure 7-16):

Angular Bearing	0	30°	45°	60°	90°
Field from N-S Radiator	0	0.5	0.707	0.866	1.0
Field from E-W Radiator	1.0	0.866	0.707	0.5	0
Vector Addition	1.0	1.0	1.0	1.0	1.0

To achieve this effect, the phase difference between the currents in the N-S and E-W radiators must be 90°, which is accomplished by making one of the lines 90 degrees (or one-quarter wavelength) longer than the other.

(3) *Stacking Sets of Radiators to Increase Gain* - This principle (see Figure 7-17) is an old one and has been used for years in communications antennas. If the currents in radiators *A*, *B*, *C* have the same phase (same number of wavelengths from the source) and if distances *AP*, *BP*, and *CP* are substantially alike, the field at *P* will be greater than that produced by a single radiator. Similarly, sets of batwing radiators are stacked to increase the gain. The length of the feed-line for each set, back to junction point, is kept the same. The centers of the successive radiators are spaced one wavelength apart.

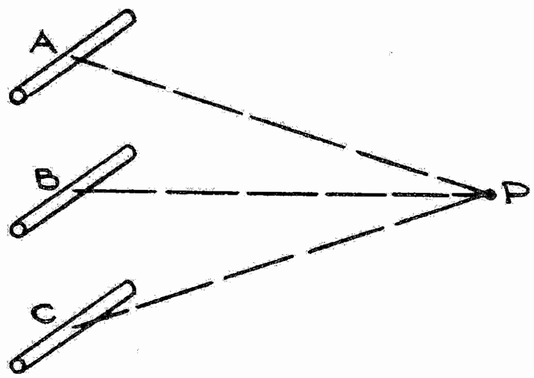


Figure 7-17 - Stacking of Antenna Radiators

CONSTRUCTION - The sets of crossed super-turnstile radiators are mounted on flag poles. The number of radiators that can be stacked is limited by the flag-pole taper and mechanical requirements.

The following types have been made:

FREQ. (MC)	STANDARD	SPECIAL
54-66 66-88 174-216	3-bay 3-bay 6-bay	4-bay, 5-bay

In the 3-bay antenna, each radiator has a 154.5-ohm impedance from the feed point at the center to ground. For three radiators in parallel, this becomes 51.5 ohms, which is the surge impedance of the main transmission line. Each branch line has an impedance of 154.5 ohms. For other types, transformers are used.

PERFORMANCE (ELECTRICAL)

(1) The antenna has extremely wide bandwidth. One set of radiators covers the 54-66 mc band -22%; another the 66-88 mc band -33%; and a third set covers the 174-216 mc band -25%. The pole-to-radiator spacing is varied slightly for each channel by means of pads or adjustments. An SWR of 1.1, or better, can always be achieved.

(2) The gain approaches the ideal that can be achieved and is 1.2 times the number of sections. This gain is obtained by measuring the vertical field pattern and taking into account losses in the radiators and feed system.

An exact tabulation of gain is shown:

	3-BAY		3-BAY			6-BAY						
TV Channel	2	3	4	5	6	7	8	9	10	11	12	13
Approximate Gain	3.3	3.7	3.3	3.8	4.1	6.4	6.6	6.7	6.8	6.9	7.0	7.1

(3) Theoretically the pattern is circular, but actually it departs from this slightly because the super-turnstile radiator is not a point source. This produces a square circle which, under ideal conditions, deviates by less than ±0.5 db. At no frequency within the specified TV channel will the pattern deviate by more than ±1.5 db.

(4) The antenna will handle a power of 20 kw, which is limited by the feed lines. The actual power for a 5-kw TV transmitter is 3 kw of visual power (black level) and 2 1/2 kw of aural power. This power is multiplied by the VSWR to allow for local heating at the current peaks. For TV, this is only 1.1. Hence, the total TV power is 6 kw. When triplexing is used, FM power is then limited to 14 kw divided by the maximum VSWR that may occur in the feed lines. This amounts to 10 kw for the 66-88 mc antenna, and 3 kw for the 54-66 mc antenna.

(5) By using the turnstiling principle and the bridge circuit of the diplexer, both visual and aural power can be transmitted over the same antenna. In addition, a triplexer enables the antenna, because of its response in the 88-108 mc band, also to be used to radiate FM power.

PERFORMANCE (MECHANICAL)

(1) Low wind resistance is achieved as a result of the open super-turnstile radiators.

(2) Ruggedness is achieved by:

(a) The inherent stiffness of triangular structures. Both top and bottom are firmly grounded and no weight is supported on insulators.

(b) The pole is made from Bessemer seamless steel tubing which has a U.T.S. of 60,000, and has shown as much as 70,000, 80,000, and even 120,000 on actual test. Since the material is stressed only to 20,000 lbs./square-inch, a high safety factor is obtained.

(c) Calculating 20 lbs. per square-foot of wind loading on rounds, a wind velocity of 95 miles per hour is required to stress the material to 20,000 lbs. per square-inch without ice. With 1/2 inch of radial ice, a velocity of 85 miles per hour is required to reach this same stress value.

(d) All brackets and radiator clamps are bonderized and zinc plated.

(3) De-icing is done for electrical reasons only, and not to reduce wind loading. Hence, only the section adjustment to the main pole is de-iced, which is the only portion affected by ice.

Heaters are used as follows:

ANTENNA TYPE	WATTS PER RADIATOR	KILOWATTS PER BAY
54-66	750	3
66-88	500	2
174-216	250	1

(4) Minimum end-seals are needed for feeding this antenna.

NUMBER PER BAY	ANTENNA TYPE
2	54-66 and 66-88
4	174-216
8	Dipole Construction

TOWER DESIGN - RCA has published the projected area of the antenna as well as other pertinent dimensions. From this information the tower manufacturer can design his tower. A standard specification is now being considered by RMA Committees, which will put all tower designs on the same basis.

The tower should be designed to take the standard guide flange and pole sockets provided (see Figure 7-18).

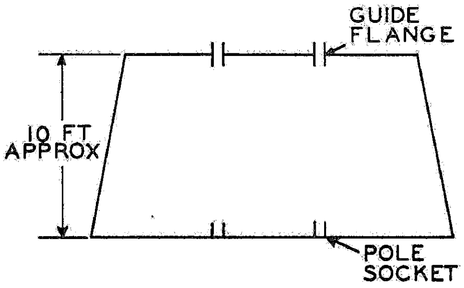


Figure 7-18 - Antenna Tower Layout

INSTALLATION

SHIPMENT - The pole can be shipped broken down as follows:

	ANTENNA	NUMBER OF POLE SECTIONS	LENGTHS - FT. AND INCHES		
3-bay	54-66	3	17'6"	23'0"	24'0"*
3-bay	66-88	2	21'6"	30'0"*	
4-bay	66-88	2	36'2"	32'6"	
5-bay	66-88	3	28'8"	31'0"	29'0"
6-bay	174-216	2	26'8"	22'1" */**	

* *For Pylon mounting, the sections listed on the right are 3 to 6 ft. shorter.*

** *On most installations to date, this antenna, because of its greater number of parts, higher frequency, and more critical assembly, has been assembled in Camden by workmen and engineers familiar with this work. The antenna has then been shipped to the customer in one piece.*

ASSEMBLY - If the antenna is assembled on the customer's premises, it should be assembled on the ground in a horizontal position, with the pole resting on saw horses. This assembly should be done under engineering supervision. The feed system should be put on the antenna. The bending tool provided should be used. Kinks and flats on the line must be avoided.

TEST ON GROUND - After assembly, the RCA Service Co. should be requested to make the following checks:

(1) Gas leaks
(2) D-C resistance
(3) Electrical leakage tests
(4) VSWR check over the band, with panoramic sweep equipment.

The RCA Service Co. has experienced men and suitable equipment, and is prepared to render this service at a reasonable cost. These tests should be made, whether the antenna is assembled in Camden or in the field. Simple corrections become extremely complex to make after the antenna is erected.

ERECTION

IN ONE PIECE - When the antenna is to be erected in one piece, the pole should be welded before assembly. Quite a number of installations have been successfully made in this way.

PARTIAL DISASSEMBLY AND REASSEMBLY - The disassembly should be done by the riggers who will reassemble the antenna. Feed lines should be carefully removed and kept in the original position. The pole need not be welded if this is done, but can be caulked after erection.

RIGGERS - It is most economical to use the same riggers who are erecting the tower. The same gin poles, donkey engines, etc., can be used. This requires close scheduling.

PAINTING - The pole and radiators are shipped with one coat of red lead only, for protection in transit and during erection. They should be painted with CAA colors after erection. All portions, except the end seals, can be painted.

SPECIAL ANTENNAS

4-BAY AND 5-BAY - These antennas have been built for the 66-38 mc band. None have been built for the 54-66 mc band, although there is no technical reason to prevent it. It is not feasible to carry flag-pole construction to greater gains, because the pole taper causes a wider spacing between the lower radiators, resulting in a different impedance than for the upper sections.

SUPER-GAIN ANTENNAS - These antennas will take the form of a tower type. One type would consist of dipoles mounted on the four faces of a tower one-half wavelength square. Such antennas are used for three reasons:

(1) *Higher Gain* - The allocation scheme of FCC is based on 50 kw radiated power. A 5-kw transmitter, with a standard 3-bay or 6-bay antenna and nominal line loss, will radiate about 15 kw in the 54-88 band, and 25 to 30 kw in the 174-216 band.

Super-gain antennas will permit gains of 10 and 12, so that 50 kw radiated power can be achieved.

(2) *Stacked Antenna Systems* - If one site is desirable for several television stations, the lower antenna can be a tower type and the upper a standard type. Propositions have been worked up for three television stations with additional FM Pylons. Since receiving antennas are directive and generally aimed at a central cluster of stations, it is highly advisable to be in this central cluster. The use of the stacked antenna makes this possible.

(3) *Directional Antennas* - Radiators can be left off on one or two faces of the tower, to produce a directional pattern. Directional patterns are not recommended as a rule, since the total area covered is less. See Figure 7-19.

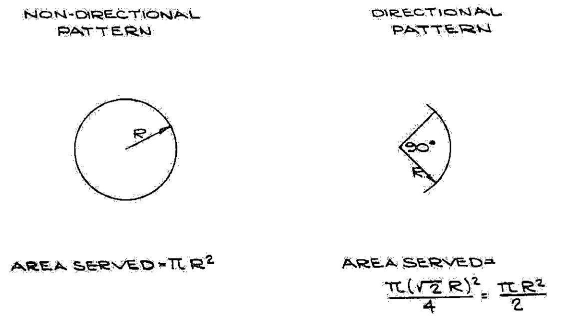


Figure 7-19 - Antenna Radiation Patterns - With the directional pattern shown, the area served is one-half the area covered with the circular pattern. Also, four times the power is concentrated in a same area. Hence, the field strength is doubled and the radius is extended by $\sqrt{2}R$ (as a general approximation, the service radius varies as the fourth root of the power).

In some cases, of course, where mountains behind the station cause serious ghosts or where protection to other stations is necessary, directional antennas must be used.

COMBINATION ANTENNAS

a. COMBINATION WITH FM BY USE OF PYLON ANTENNA

A 2- or 4-section Pylon underneath a TV antenna can be placed below any 3- or 4-bay antenna in the 54-88 band, or a 6-bay antenna in the 174-216 band.

Pylons have a gain of 1.5 per section midband and will take up to 50 kw of FM. They are 20 inches in diameter and 14 feet high. The FM feed system is inside. The TV system is mounted on the back of the pylon, opposite the slot. All FM-pylon and TV-antenna combinations have been checked for structural soundness by independent consultants.

b. COMBINATION WITH FM BY USE OF TRIPLEXING

Purpose - To use TV antennas for FM radiation.

Requirements

(1) Must keep FM out of TV and vice versa.
(2) Must not disturb VSWR of either service.
(3) Must have low insertion loss.

Principle - The TV and FM energies are kept flowing in their proper paths by suitable notch filters, as shown in Figure 7-20.

Construction - The external dimensions are the same as those of the 54-88 mc diplexer.

Performance - Installations made at stations WNBW, WEWS, WLWT, are all working satisfactorily.

Power Limitations - Because of limitations in the feed system of the super-turnstile antenna discussed previously, FM power is limited to 10 kw for the 66-88 mc antenna and 3 kw for the 54-66 antenna.

c. USE OF TV SUPER-TURNSTILE FOR FM ONLY -

This implies that the TV installation will be made at a later date.

Principle - See Figure 7-21.

d. COMBINATION WITH AM

Purpose - Use of AM tower for TV antenna support.

Principle - TV line must be isolated from ground at AM frequency.

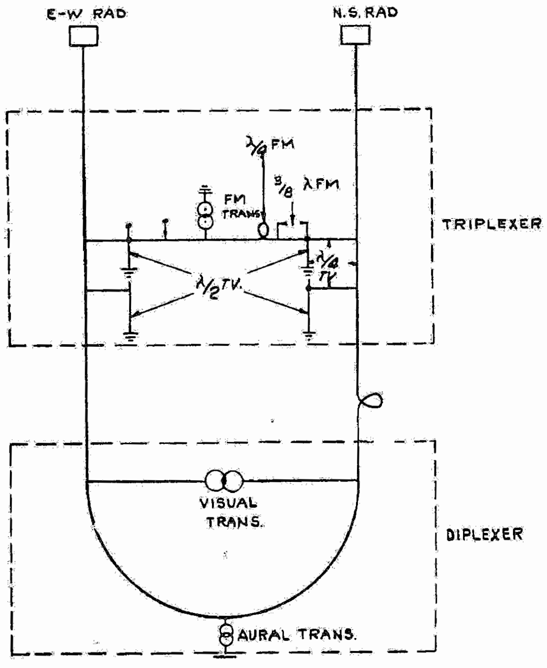


Figure 7-20 - Principle of Triplexing

As shown in Figure 7-22a, the tower acts as the outer conductor of a balun. The capacitance at the end *A* loads the balun to resonance if the tower is not high enough to furnish one-quarter wavelength at the AM frequency.

Referring to Figure 7-22b, a balun of wires is constructed around the TV line approaching the AM tower for a distance of one-quarter wavelength, or equivalent.

TRANSMISSION LINE

PRINCIPLE - Without insulators, the surge impedance is

$$Z_o = 138 \log \frac{D\ (outer)}{d\ (inner)}.$$

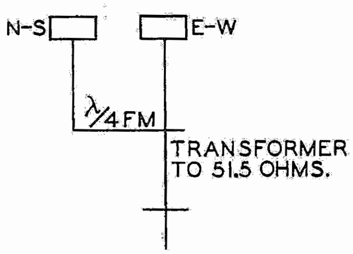


Figure 7-21 - TV Superturnstile Antenna

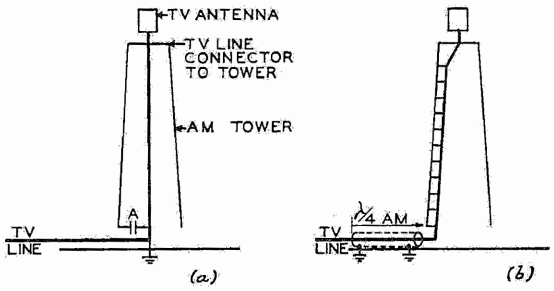


Figure 7-22 - Use of AM Tower for TV Antenna

In practice, insulators are always required. The line must then be treated as made up of recurrent T-sections, with each insulator considered as a lumped capacity. See Figure 7-23.

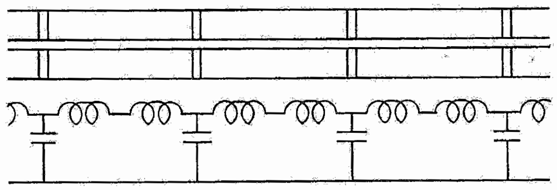


Figure 7-23 - Transmission Line Principle

This brings about two additional considerations:

(1) The line must always be cut half-way between insulators, to avoid discontinuities.

(2) The line is really a filter and has a cutoff frequency. On standard RMA line, the cutoff point is above 216 mc, due to a proper choice of insulator capacity and spacing.

RMA SIZES - Standard sizes are 7/8, 1-5/8, 3-1/8 and 6-1/8 inches. Of these, the 1-5/8- and 3-1/8-inch sizes are the most widely used for 5-kw TV installations.

SURGE IMPEDANCE - The value is 51.5 ohms for all sizes. The primary consideration in this choice was the I^2R loss, with a slight compromise on attenuation. The odd value of 51.5 ohms was chosen because this is the actual impedance of solid-dielectric line. Hence, the two types of line can be used in the same run.

POWER RATING - This is one-half the power required to raise the temperature of the outer conductor 40° C.

SIZE (IN.)	50 MC	100 MC	200 MC
7/8	4.5 kw	3 kw	2 kw
1 5/8	16	10	7
3 1/8	64	42	27
6 1/8	235	166	118

LOSS - Loss takes into account copper loss, insulation loss, and some derating due to connections. The average power transmitted at 100 mc over a 500-foot run of line is 75% for the 1-5/8-inch line and 85% for the 3-1/8-inch line. See published curves in the RCA equipment specification.

GAS - Dehydrated air, obtained from a commercial dehydrator, is probably best. Only a *small* positive pressure is needed; 5 lbs. is adequate.

CONSTRUCTION - See Figure 7-24. The primary purpose of this construction is two-fold:

(1) To prevent electrical discontinuities in the line.
(2) To make the installation as nearly foolproof as possible.

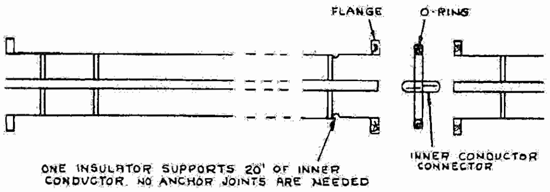


Figure 7-24 - Transmission Line Construction

No soldering or brazing is needed in the field. If a length other than 20 feet is needed, the line is cut and a flange adapter is used, which requires no brazing. Flange joints are pulled up tight and require no "feel".

FITTINGS - Fittings must be designed to prevent electrical discontinuities. This means that diameters must remain the same and insulators must continue at the same spacing. Elbows must have a wide sweep. See Figure 7-25.

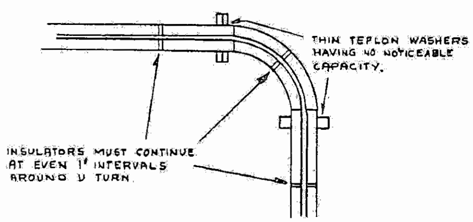


Figure 7-25 - Transmission Line Elbow Fitting

TELEVISION STANDARDS - While lines intended for TV and FM have the same approximate dimensional standards and external appearance, the surge impedance limits on TV line are tighter because the reflection problem is highly important in television. In ordering lines, be sure to specify it for TV standards, or better yet, order it through RCA.

CUTTING POINTS OTHER THAN HALF-WAY BETWEEN INSULATORS - If this is absolutely necessary, a point should be chosen further back, where the line can be cut half-way between insulators. The odd balance is then filled out with a line containing no insulators and having a diameter that will give a surge impedance of 51.5 ohms.

EXTRA QUARTER-WAVE FOR DIPLEXING - The equipment specifications (AS-5979) show several methods of introducing this at the diplexer. It also gives a table, for each channel, of the difference that should exist, taking into account the actual velocity of propagation along the line.

ATTACHING LINE TO TOWER

ELECTRICAL REQUIREMENTS - Both lines should be as identical as possible all the way down the tower. Flanges and elbows should be located in the same places from the tower top to the diplexer in the station, where the extra quarter-wave is inserted. This is because each elbow can insert a slight mismatch. If the two lines are exactly alike as to mismatch, the latter is balanced out by the extra quarter-wave section, which inverts the reactance.

MECHANICAL REQUIREMENTS - The primary requirement is to provide allowance for expansion. The following should be noted.

(1) Expansion of the inner conductor is made possible by the sliding of the spring-loaded inner-conductor connector.

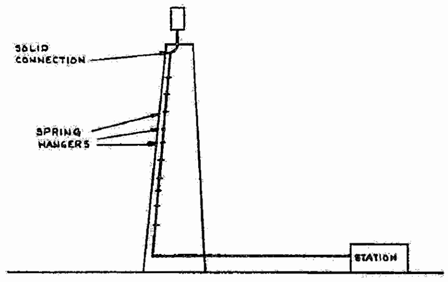


Figure 7-26 - Transmission Line Installation Layout

(2) Expansion of the outer conductor is permitted by spring hangers. The line is connected solidly at the top of the tower; and then

it is connected to the tower every 10 feet (see Figure 7-26). These hangers are pre-loaded in accordance with the temperature conditions at the time of erection. Under ordinary climatic conditions, expansions of 1 inch in 100 feet will occur, and this will be accommodated by the hangers. An elbow at the bottom of the tower should be arranged to take this movement.

If the tower is more than 500 feet high, the arrangement can follow the layout shown in Figure 7-27.

CHECKING LAYOUTS - Transmission line layouts will be checked by RCA for accuracy and completeness, if submitted.

NOTE - *Any pains taken in establishing a good transmission line and antenna installation pays enormous dividends. Here, if anywhere, a slight effort in time will prevent expensive and lengthy delays at a later date.*

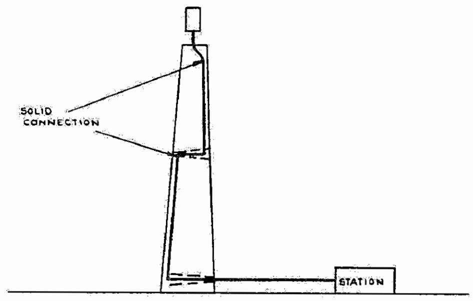


Figure 7-27 - Transmission Line Installation Layout