DR. S G FOORD

ELECTRICAL COMMUNICATION

Technical Journal of the
International Telephone and Telegraph Corporation
and Associate Companies

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MICROWAVE RELAY LINK FOR TELEVISION

TEST SET FOR RECORDING IMPULSES

ROTARY AMPLIFIERS IN SERVOMECHANISMS

MICROSTRIP—NEW KILOMEGACYCLE TRANSMISSION TECHNIQUE

SIMPLIFIED THEORY OF MICROSTRIP TRANSMISSION SYSTEMS

MICROSTRIP COMPONENTS

RADIO DISPATCHING SYSTEM FOR A LARGE TAXICAB FLEET

RECENT DEVELOPMENT IN COMMUNICATION TECHNIQUE

NOTE ON DELTA MODULATION

INTERNATIONAL · TELEPHONE AND TELEGRAPH

Volume 30 **MARCH, 1953** **Number 1**

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ELECTRICAL COMMUNICATION

Technical Journal of the
International Telephone and Telegraph Corporation
and Associate Companies

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Published Quarterly by the
INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION
67 BROAD STREET, NEW YORK 4, NEW YORK, U.S.A.

Sosthenes Behn, Chairman William H. Harrison, President
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Subscription, $2.00 per year; single copies, 50 cents
Electrical Communication is indexed in Industrial Arts Index
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Volume 30 MARCH, 1953 Number 1

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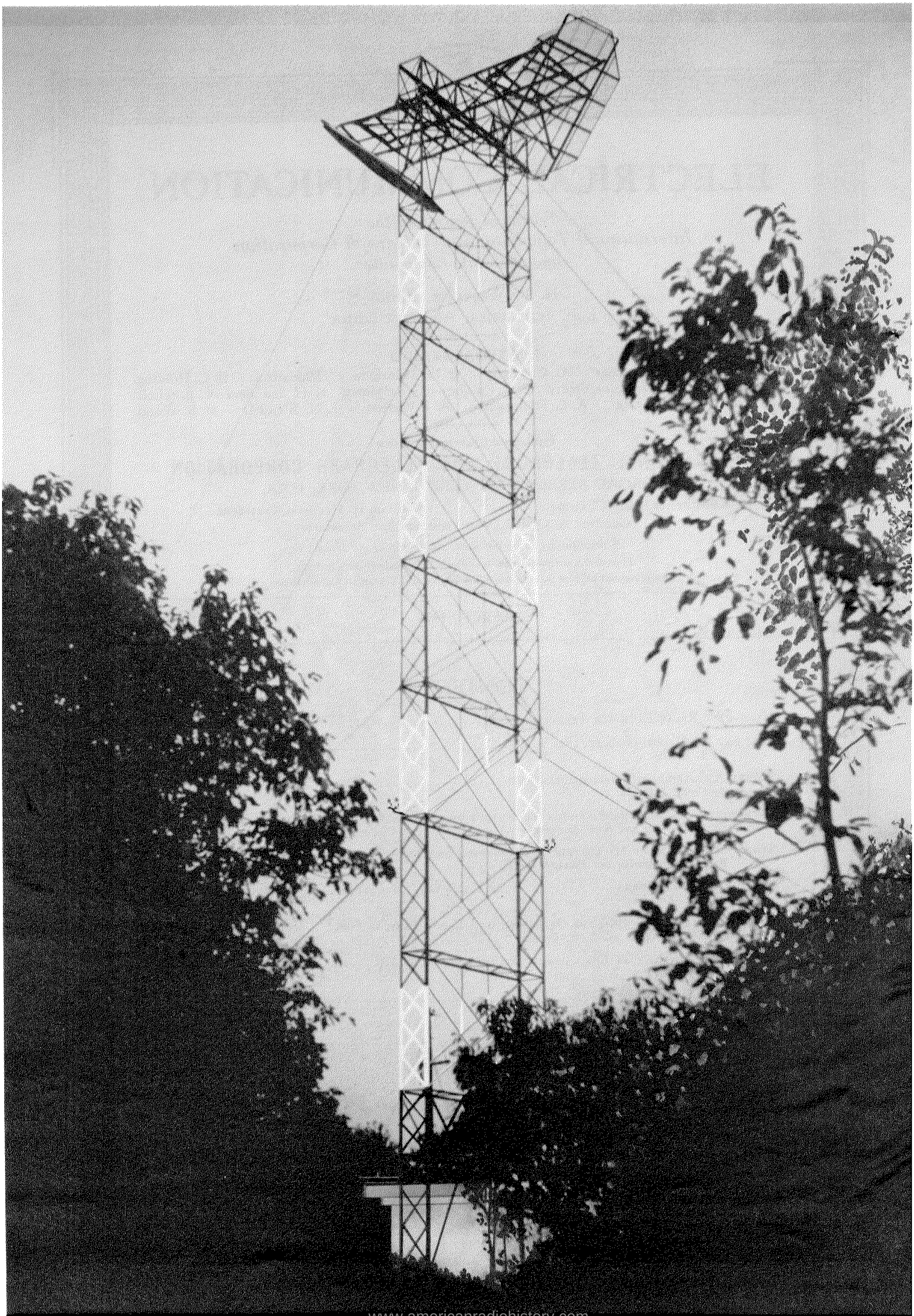

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Microwave Relay Link for Television

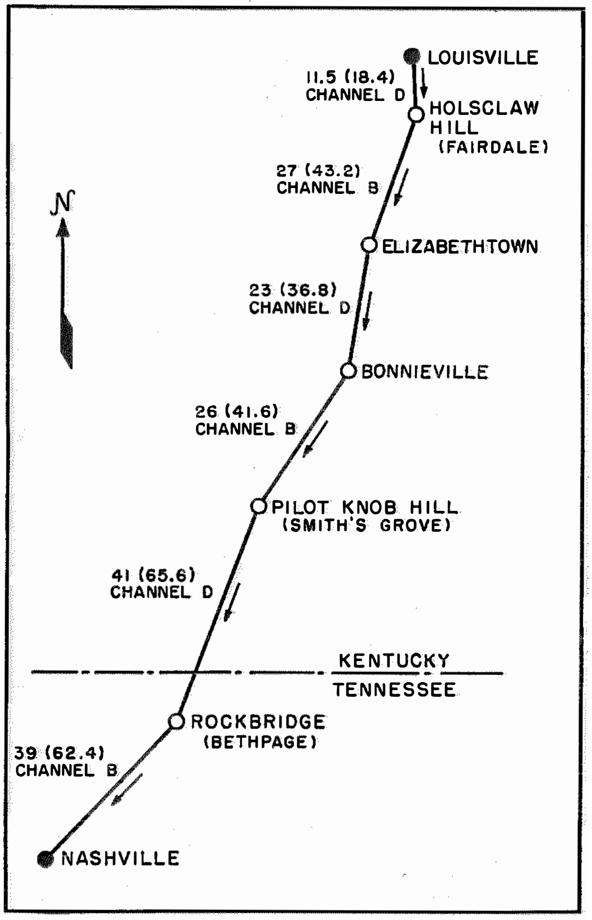
Although the television relay network in the United States now stretches from the Atlantic to the Pacific coasts and covers large portions of the intervening country, there are many television broadcasting stations that are not yet connected to it. To provide such connection and bring nationally important programs to television station WSM–TV in Nashville, Tennessee, a private link has been installed by Federal Telecommunications Laboratories between it and the nearest relay-network station, which is in Louisville, Kentucky. Five repeaters and two terminal stations are required to transmit microwave signals over the geographical path shown on the map. The airline distance between the terminals is 157 miles (251 kilometers), and the route actually covered by the radio beams is a few miles longer.

The link now operates in only one direction, although arrangements can be made easily to add the return path if it is needed. Two transmission frequencies are alternated along the link to avoid the interference that would be certain to occur if all repeaters operated on the same frequency. The entire system will accommodate a channel bandwidth of 5 megacycles, which is ample for the American standard television signal. A unique feature of the system is that the accompanying sound program is transmitted over the link by subcarrier equipment, thus making separate sound-channel facilities unnecessary.

The system is of the frequency-modulation type in which modulation results from applying the signal to the reflector electrode of a 5-watt reflex-type transmitting klystron. The nominal radio-frequency bandwidth of the transmitted signal is about 16 megacycles. This signal is applied to a paraboloidal antenna located at the base of a tower. The resulting beam of microwave energy is pointed directly up along the tower and is reflected toward the receiving antenna of the next repeater by a flat conducting surface mounted at the top of the tower.

Shown on the facing page is the first repeater in the link. The tower is about 100 feet (30 meters) high and is on top of Holsclaw Hill at Fairdale.

The receivers are of the single-superheterodyne type in which tuned cavities select the signal incoming from the antenna and apply it to a crystal detector to be mixed with the frequency of a local oscillator. After intermediate amplifica-



The television link makes available to station WSM–TV in Nashville, Tennessee, all programs carried by the coaxial-cable network presently terminating at Louisville, Kentucky. Two terminals and 5 repeaters are incorporated, transmitting alternately on television-link channels *B* (2008 to 2025 megacycles) and *D* (2042 to 2059 megacycles). The distances are indicated in miles (kilometers).

tion at 120 megacycles, the signal is detected and amplified again. It may then be sent to the transmitter for relaying to the next repeater or at the terminal be passed to the control room of television broadcast station WSM–TV.

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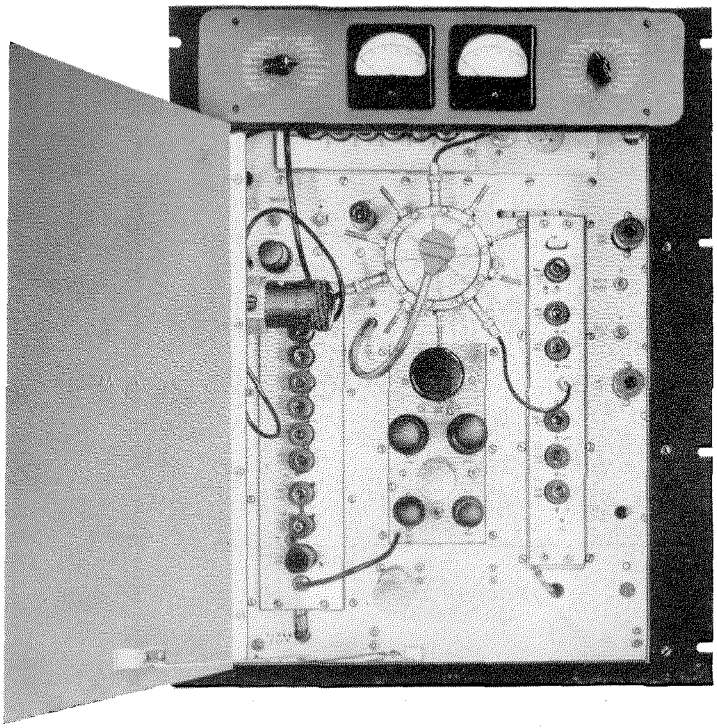
The passive-reflector type of antenna used in the link for both transmission and reception has been found to be very economical as lighter structures can be used and because no radio-frequency equipment need be mounted high on a tower. The paraboloidal antennas for receiving and transmitting are either 6 or 10 feet (1.8 or 3 meters) in diameter, the size being chiefly determined by the distance between stations.

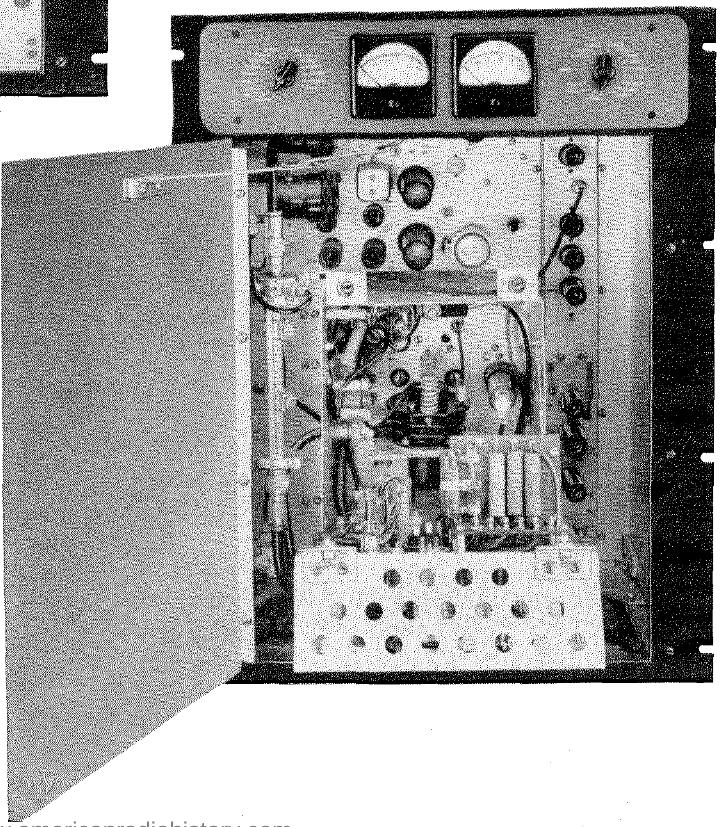
Standby power-generation equipment is included at each repeater and in case of mains failure will immediately supply all power for operation of the equipment. In case of failure of one of the repeaters in the link, a coded supervisory signal is transmitted from the following repeater to the receiving terminal at Nashville, and this signal will indicate where a repairman should be sent. The repeaters are designed for unattended operation.

This *FTL-27* link was installed during the latter part of 1950 and has been in use since then. Its successful performance is in very large part due to the efforts of Colonel J. H. DeWitt and the engineering staff of WSM–TV.



On the opposite page is a tower-mounted plane reflector as seen looking upward from a position just alongside one of the paraboloidal antennas. These reflectors turn the vertically transmitted beam to a horizontal direction aimed exactly at the following repeater or terminal station.

Above, receiver chassis showing *2K28* reflex-klystron local oscillator with slug-tuned external cavity at upper center. Only partly visible along the top are the input tuned-cavity circuits, crystal mixer, and a 6-stage intermediate-frequency preamplifier. Along the left side is the main intermediate-frequency amplifier and the second detector. This circuit feeds the video amplifier on the panel just below the local oscillator. The automatic-frequency-control circuits are in the strip just to the right of the oscillator.



At right is the rack-mounted transmitter chassis. The video amplifier is at top, and the *SRL-7-C* reflex-klystron transmitting tube is at the bottom center. The automatic-frequency-control circuits are along the right side of the chassis, and a two-slug coaxial-line tuner at the left matches the klystron output line to the antenna system.

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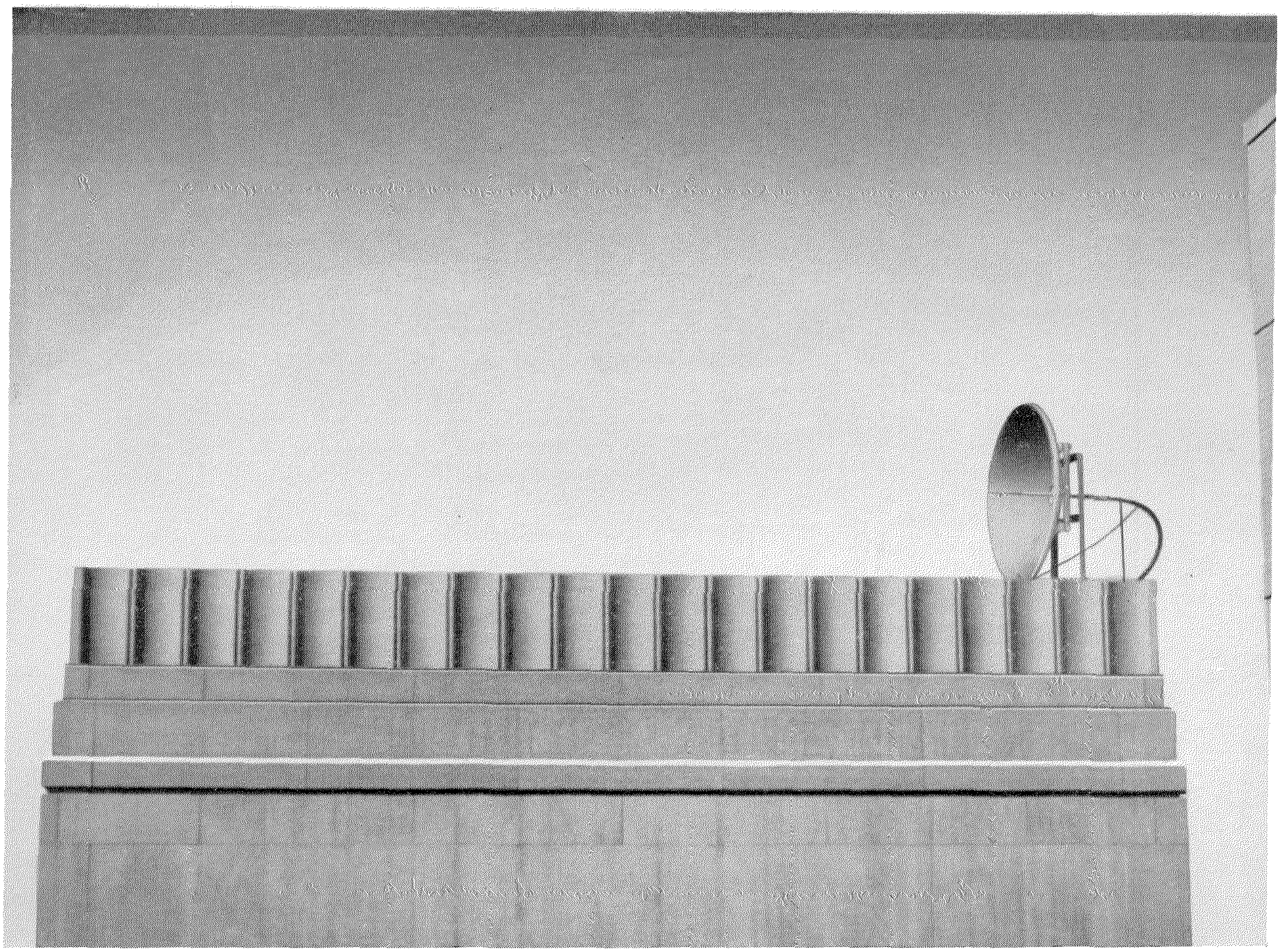
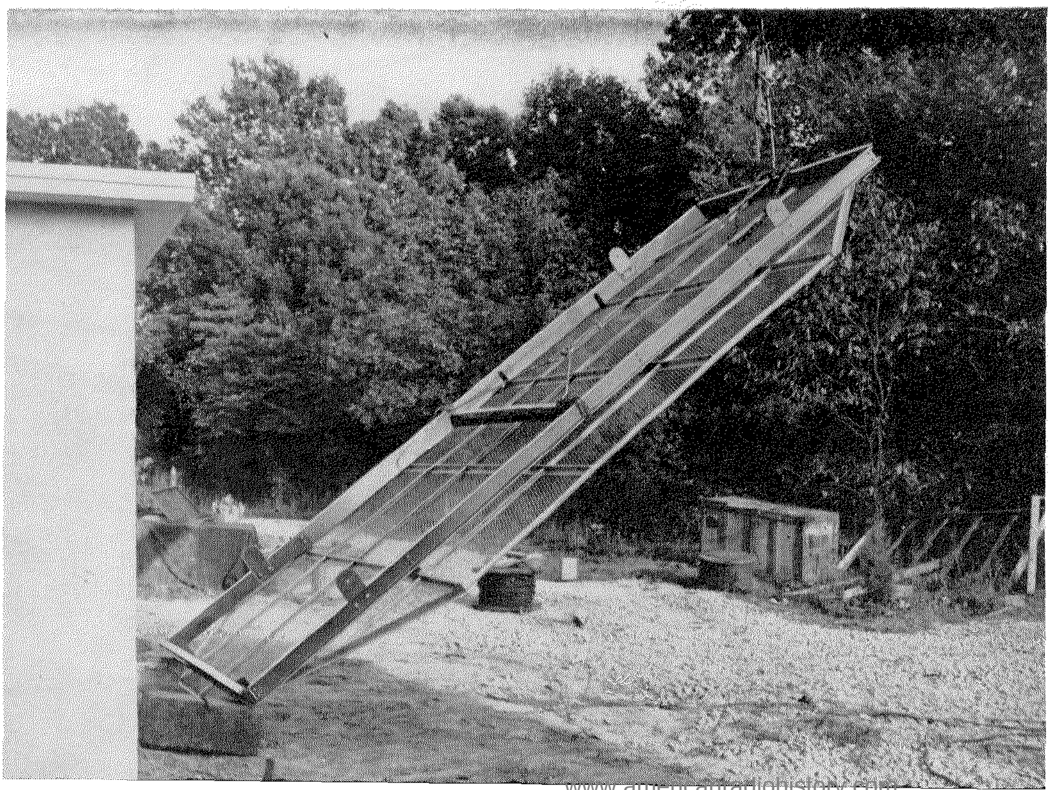


Transmitting 4-foot paraboloidal antenna at Louisville terminal atop the *Courier-Journal* building. This station transmits the signals obtained from the national television coaxial-cable and radio-relay network.



A plane reflector ready for hoisting and mounting on the tower. The reflecting surface is made of wire mesh to reduce wind loading, but appears as a continuous surface to microwave radiation.

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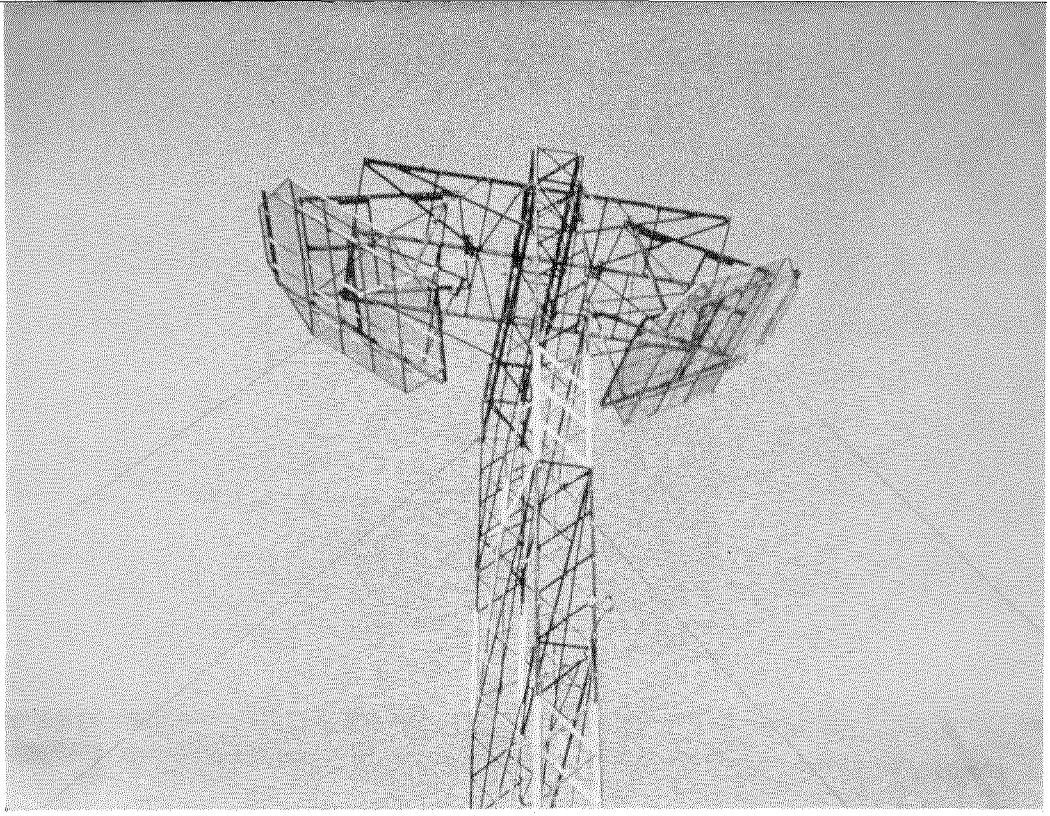
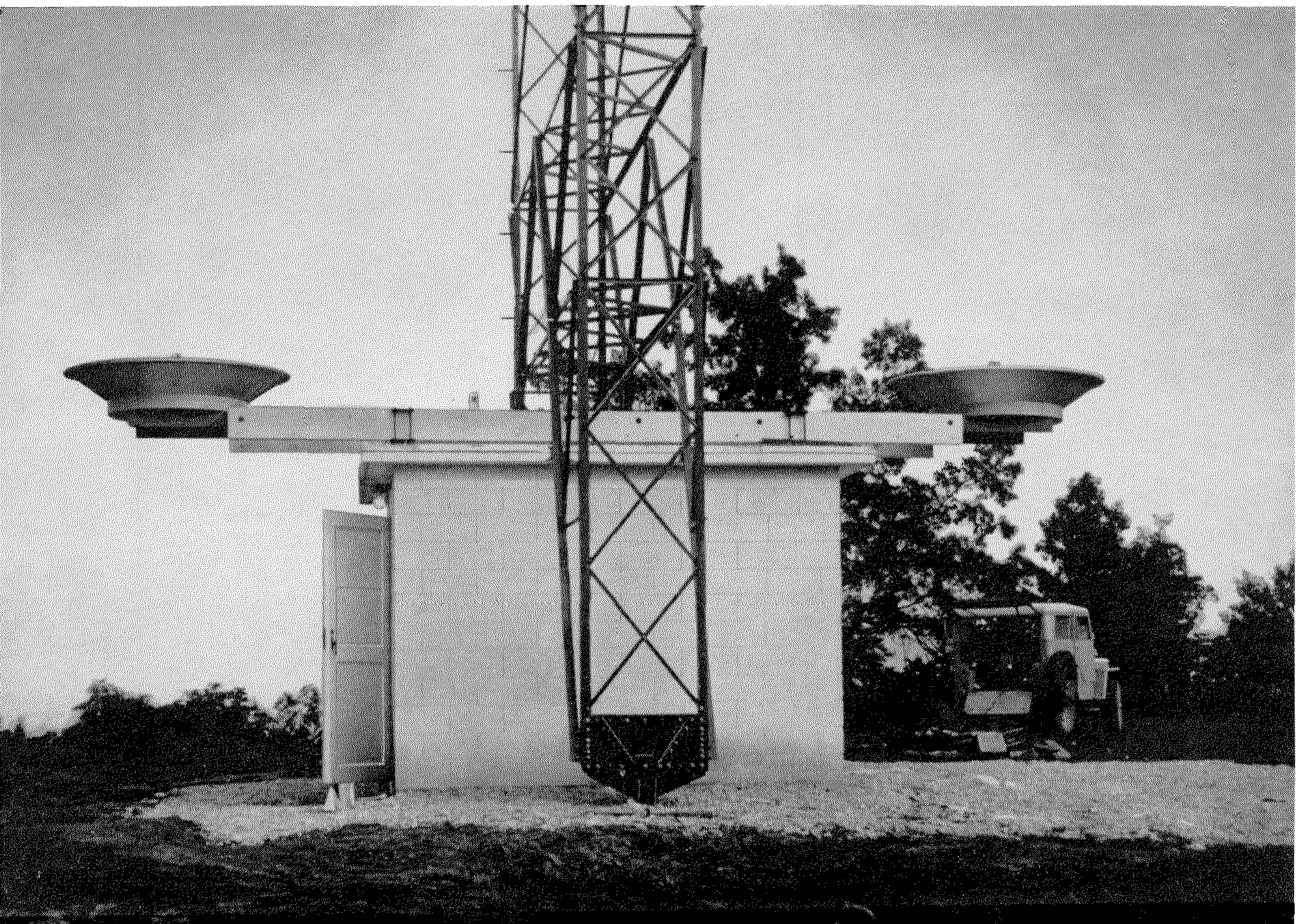
The top of a repeater-station tower showing the two plane conducting surfaces for reflecting the incoming wave to the receiving paraboloid and directing the transmitted wave to the next repeater.



Below is shown the repeater hut at Bonnieville. The two paraboloidal antennas receive and radiate beams in conjunction with their associated flat reflectors. The paraboloids are 9 feet (2.7 meters) above the ground.



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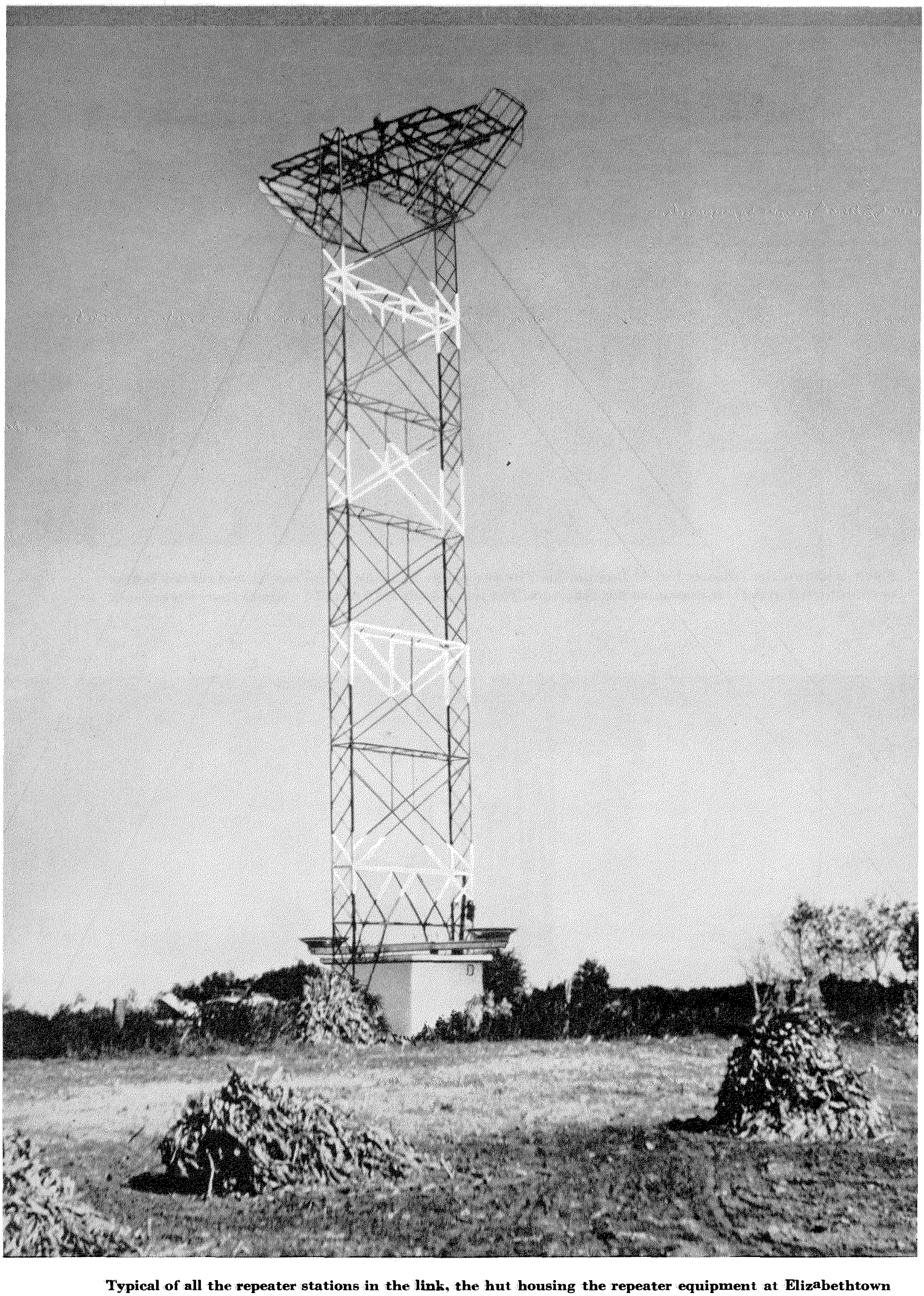

Typical of all the repeater stations in the link, the hut housing the repeater equipment at Elizabethtown is accommodated between the legs of the 100-foot (30-meter) tower.

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Test Set for Recording Impulses

By L. J. NIJS

Bell Telephone Manufacturing Company; Antwerp, Belgium

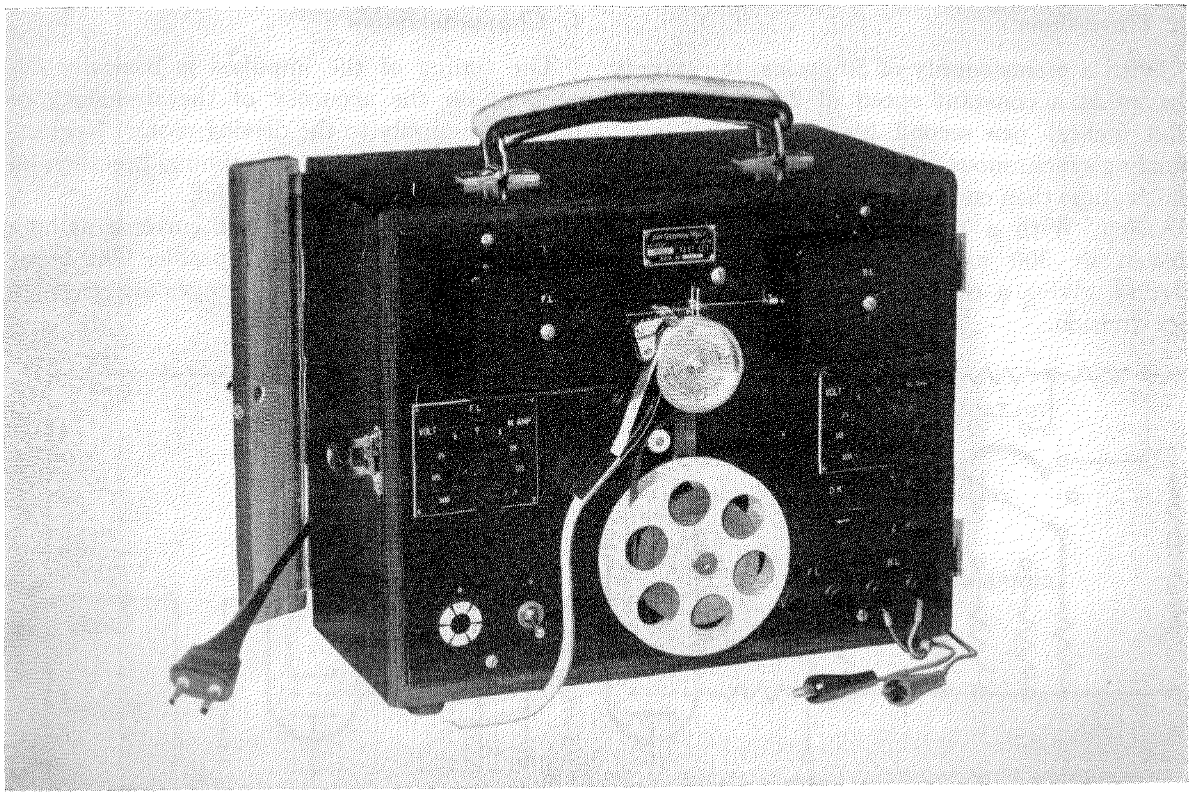
IMPULSE sending, the basic principle of many automatic switching systems, creates the necessity of impulse timing. Everyone engaged in the installation and maintenance of automatic exchanges will appreciate the development of a light portable instrument giving an accurate picture of the kind and length of the impulse sent or received through various stepping circuits in a central office or private branch exchange.

A portable test set that is easy to handle and very robust was recently developed. This instrument is equipped with two separate recording channels and a mechanical time-base-printing device. Two different electrical phenomena may be measured and registered simultaneously on a waxed paper tape 12.7 millimeters (0.5 inch) wide. Each recording channel consists of an electromagnetic recording head preceded by a two-stage amplifier with adjustable gain and very high input impedance. The *7012-A* Test Set may be used on mains supplies of 115, 125, 135, 150, 220, and 250 volts at 50 or 60 cycles per second.

1. Recording Head

The alternating- or direct-current impulses are passed through the coils of a pen motor, which moves the recording needle perpendicularly to the direction of motion of the paper tape. Balance of the magnetic circuit of the pen motor has been achieved by the use of adjustable airgaps to ensure correct response of the needles in both directions.

Due to the light weight of the moving system and the use of sapphire needles, a very fine line



The *7012-A* test set produces recordings of two independent inputs and a timing marking on a waxed paper tape.

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has been obtained and records that are easily readable up to 600 cycles may be obtained.

2. *Direct- and Alternating-Current Amplifier*

The amplifier circuit is shown in Figure 1. It may be operated with either direct- or alternating-current impulses. An adjustable voltage and current divider permits undistorted recordings over a wide range of input signals.

Both stages of each amplifier use twin triodes having common cathodes.

As a result, supply-voltage variations do not affect the amplitude of the deflections of the recording needles. Supply-voltage stabilization is therefore unnecessary. When no signal is received, the coils of the pen motor being connected in opposition, the magnetic field in the coils is balanced and aging of the magnet of the pen motor does not occur. Further, a decrease of cathode emission with time affects both triodes to the same extent thus leaving the balance of the coils in the no-signal position of the needles unchanged, preventing zero drift.

3. *Time Base*

With a mains supply of 50 cycles, the tape is driven at a constant speed of 250 millimeters (9.8 inches) per second by means of a self-starting synchronous motor. The time base marks on the tape then correspond to 4 milliseconds per division. With a 60-cycle source, the tape is driven at 300 millimeters (11.8 inches) per second, giving a time base of 3.33 milliseconds per division.

The time base is mechanically printed along the middle of the tape by teeth on the driving drum. The motor and driving drum run continually when the set is being used; the motion of the paper is started during the recording interval by the operation of an electromagnet that presses the paper tape against the driving drum by means of a roller. The electromagnet is actuated by a push button mounted on the front panel of the instrument. Parallel terminals are provided to connect an external circuit to permit remote control of the paper motion.

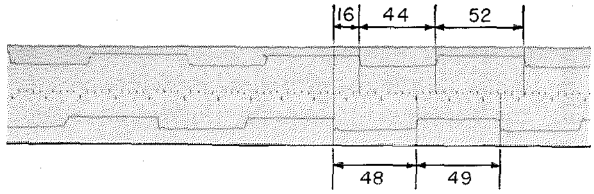

Figure 2—An actual-size sample of tape. The bottom waveform is of impulses incoming to a slow-acting relay and that at the top is of the outgoing impulses. The 4-millisecond time base is at the center, with time going from left to right.

4. *Characteristics*

The timing of the impulses is basically dependent on the accuracy of the frequency of the power supply to the driving motor. Neglecting this source of error, a timing precision of within 2 milliseconds is obtained.

Input voltages up to 500 and currents as high as 625 milliamperes are acceptable. The input impedances for the various ranges are given in Table 1.

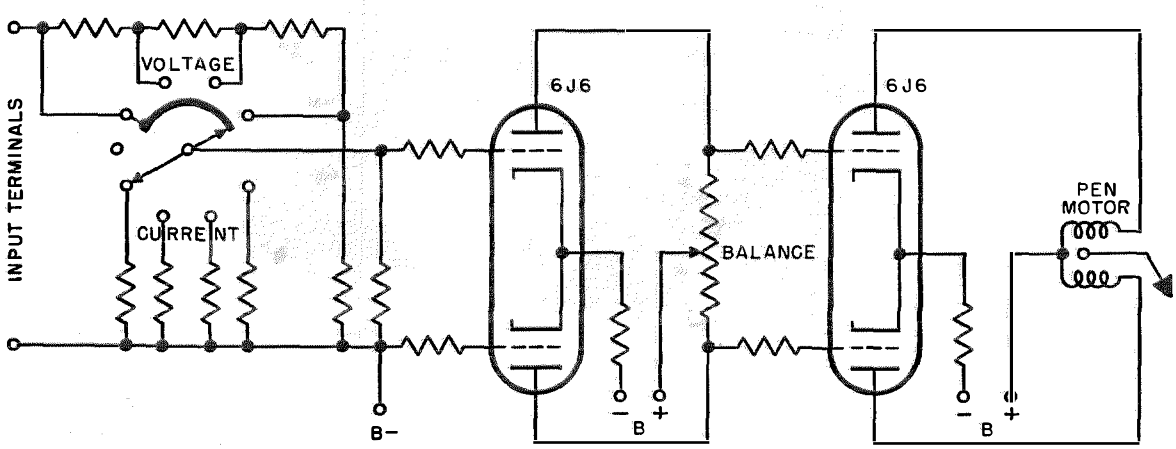

Figure 1—Circuit arrangement for each of the amplifier-recorder systems. Two such channels are employed.

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5. *Applications*

The instrument has a wide range of applications. The provision of two independent amplifier-recorder systems operating on a single tape permits ready comparison of the performance of computing devices or of testing against a standard. Some of the fields in which it is useful include measuring subscriber-line dial impulse ratios and speeds; operating time of relays (Figure 2), step-by-step switches, and markers; and pulse sending time for message registers, ticket printers, train dispatching, and signaling. Records may be made of the distortion of impulse ratios when sending the impulses through various stepping circuits. The time lag between the operation of an impulse sender before and after it has been registered through multiple operation of circuits and the relation between operating and releasing time of two relays may also be studied with this measuring set.

TABLE 1

INPUT VOLTAGE, CURRENT, AND IMPEDANCE

Volts	Impedance in Ohms
1–5	100,000
5–25	180,000
25–125	190,000
125–500	200,000
Milliamperes	
0.01–1	100,000
1–5	1,000
5–25	200
25–125	40
125–625	8

Recent Telecommunication Development

Etching of Lord Kelvin

William Thomson (1824–1907), who became Baron Kelvin of Largs in 1892, is honored in the most recent of the etchings published by the International Telecommunications Union. He was born in Belfast, Ireland. Although he was a research physicist in the broadest sense of the term, having published over 300 original papers bearing on nearly every branch of physical science, Lord Kelvin is perhaps most renowned for his work on submarine telegraphy in cables. He developed the mathematical theory of attenuation in cables, and his practical researches produced the mirror galvanometer and siphon recorder. He suggested the use of stranded conductors.

The etching of Lord Kelvin is the eighteenth in the series that was started in 1935. On a good grade of paper measuring 9 by 6⅝ inches (23 by 17 centimeters) including margins, these etchings are available at 3 Swiss francs each from Secrétariat général de l'Union internationale des Télécommunications, Palais Wilson, 52, rue des Pâquis, Genève, Suisse. The entire series includes etchings of Ampère, Baudot, Bell, Erlang, Faraday, Ferrié, Gauss and Weber, Heaviside, Hertz, Hughes, Kelvin, Marconi, Maxwell, Morse, Popov, Pupin, Siemens, and Tesla.

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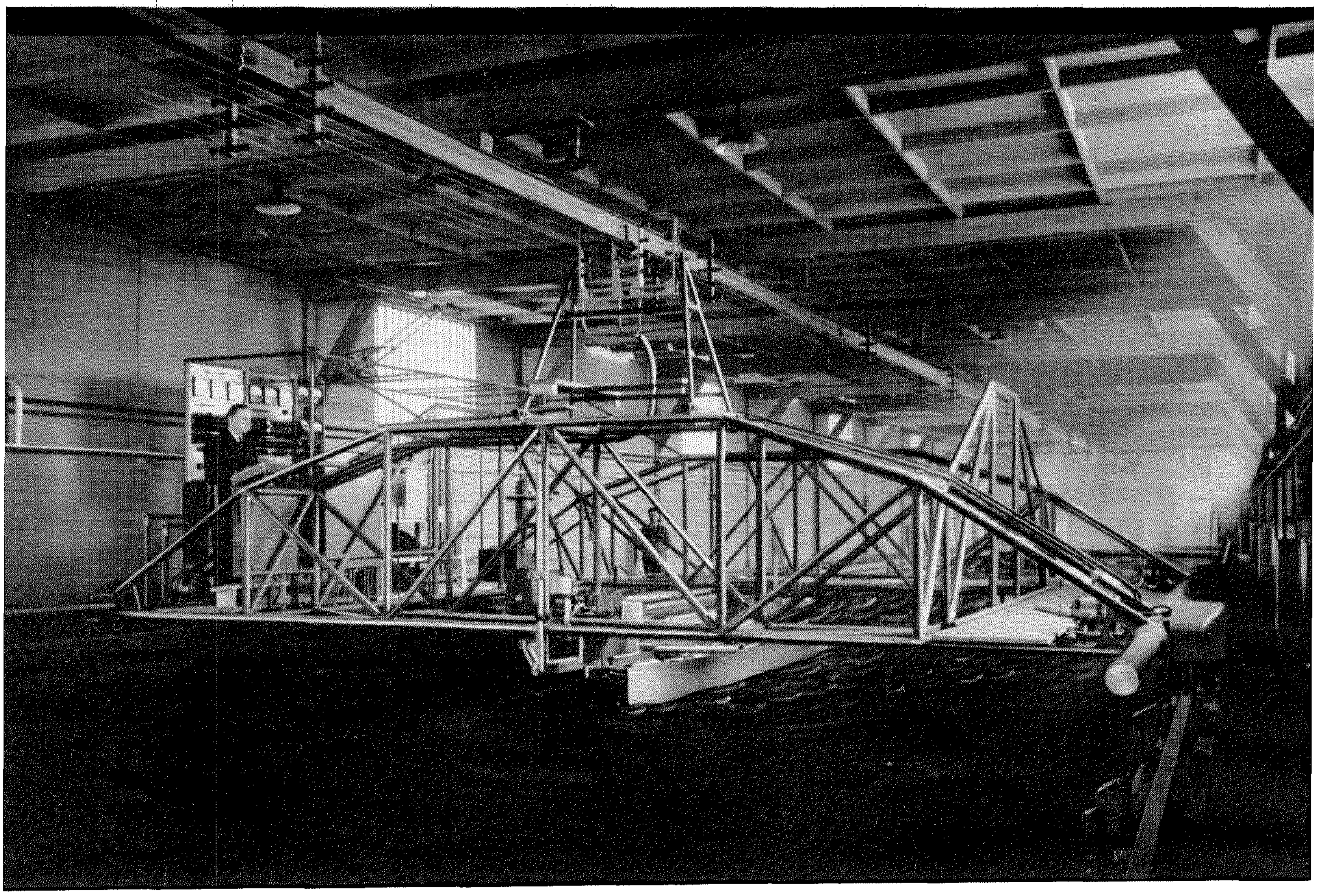

By courtesy of the French Bureau of Naval Construction and Armament.

The towing carriage of this ship-model testing basin is controlled by a high-accuracy servomechanism.

Rotary Amplifiers in Servomechanisms*

By GERARD LEHMANN

Laboratoire Central de Télécommunications; Paris, France

and

Société des Servomécanismes Electroniques; Paris, France

PRINCIPLES and construction of standard direct-current generators are reviewed, as is their operation under transient conditions. Properties of the machine as an amplifier are then described, and response and phase-lag-versus-frequency curves are presented.

It is shown that the time constant of the direct-current generator is not a good measure of its ability to give a quick response. An infinite time constant is sometimes desirable. A few points of comparison between generators incorporating one and two stages of amplification are given.

The effects of hysteresis and eddy currents in the magnetic circuits of the machines are then described.

Finally, the properties of vacuum-tube and rotary-amplifier assemblies are discussed with emphasis on the flexibility of such an arrangement when used in servomechanisms.

* This paper describes work done at Société des Servomécanismes Electroniques of Paris. This company was founded in 1949 by four French manufacturers of electric motors and switchgear. It is concerned with the production of regulators, servomechanisms, and similar equipment in which electric machinery and electronic devices are combined. A cooperative agreement exists between Société des Servomécanismes Electroniques and Laboratoire Central de Télécommunications.

Presented before the Société des Radioélectriciens, December 15, 1951. Published under the title, "Les Dynamos Amplificatrices Leur Emploi dans les Servomécanismes," *L'Onde Electrique*, volume 32, pages 78–88; March, 1952: also, ***Bulletin de la Société Française des Electriciens*, volume 2, pages 198–209; April, 1952.**

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To the radio engineer, the standard amplifier remains the vacuum-tube amplifier, even though other systems of amplification are in common use. For instance, there have been developed pneumatic and hydraulic controlling devices, magnetic amplifiers, and the direct-current generator that is in effect a rotary amplifier. This latter is finding increasing use as the power stage of an amplifying chain consisting mostly of vacuum tubes. The rotary amplifier is very often used in the antenna servomechanism of radar sets.

In view of their wide application, these devices should seemingly be of interest to radio engineers, many of whom may not be familiar with them.

In most present uses, rotary amplifiers are parts of systems incorporating negative feedback; i.e., servomechanisms. Among the servomechanisms in common use are voltage regulators for alternators, current regulators for electromagnets used in atomic physics (cyclotrons and mass spectrographs), remote controls for weapons or radar sets, controls of industrial machines (rolling mills, planing machines, etc.) and elevators and mine hoisting machines.

1. Principles of Construction and Operation

A rotary amplifier, illustrated in Figure 1, consists of a direct-current generator driven at a constant speed by a motor; in this case, a three-phase motor directly connected to the mains.

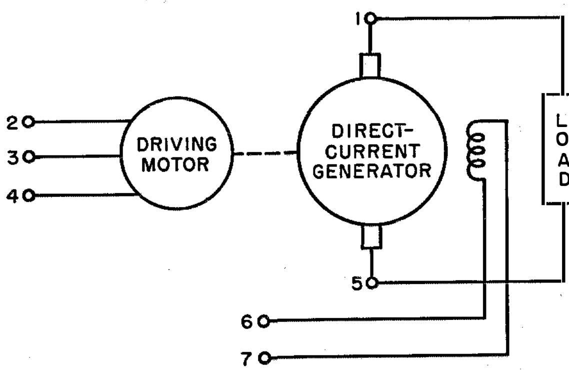

Figure 1—Basic diagram of a rotary amplifier.

The armature winding of the generator is connected to the load impedance Z, through which flows a current I produced by the electromotive force E of the generator. The current i to be amplified flows through the field winding of the generator. The power delivered to the load Z is derived from the mains through the rotating unit.

For this analysis, *6* and *7* are the input terminals of the amplifier and *1* and *5* are the output terminals while *2*, *3*, and *4* are the power-supply terminals.

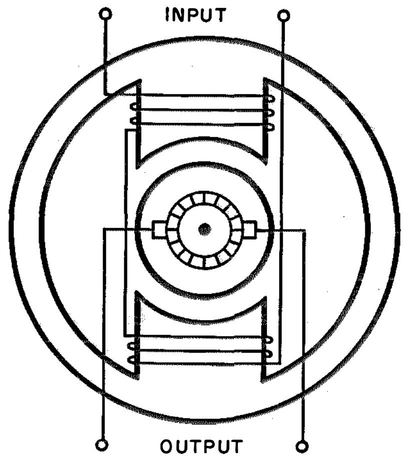

Figure 2—Construction of a rotary amplifier in simplified form.

A direct-current generator of the standard type, Figure 2, constitutes the simplest form of an amplifying machine. As is well known, this machine consists of a rotating armature associated with a set of brushes and commutator segments, and a stationary inductor or field, usually having more than the 2 poles shown.

In the elementary theory of the machine, the following simplifying hypotheses will be assumed for the present.

A. The curve of magnetizing force versus resulting flux for the machine is a straight line passing through the point of origin, i.e., the machine is never saturated and there is no hysteresis.

B. There are no eddy currents produced in the magnetic material when the magnetic field changes.

C. The magnetic flux of the field is independent of the current in the armature.

D. The internal impedance of the armature is negligible.

Our problem is to examine the operation of the machine under variable working conditions, when the output current generated is no longer

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a steady direct current but follows a variable signal applied at the field terminals *6, 7*.

1.1 Conventional Analysis

For an analysis of these machines using standard methods, the excellent article[1] by Valentin is recommended.

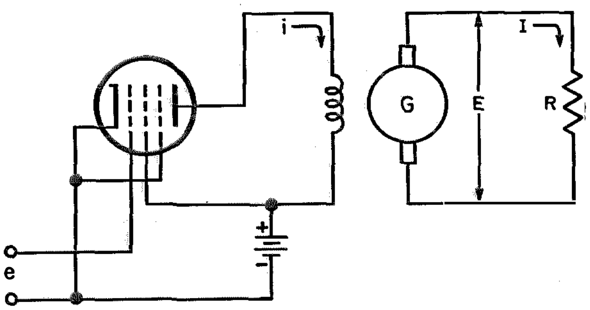

Figure 3—The field winding of a rotary amplifier may be excited by a pentode.

The field circuit includes a series resistance r and has an inductance L defined by

$$L=\phi/i,$$

where ϕ is the flux through the machine and i is the field current. Inductance L is defined within the limits of the hypothesis in A above of a linear magnetization curve.

With the armature revolving at a constant speed, an electromotive force E is generated and at every moment is proportional to the field flux flowing through the machine. It follows that the operation of the machine is characterized by the fundamental equation given below, which is exact at every instant within the limits of the simplifying hypotheses.

$$E=K\times i, \tag{1}$$

where K is a constant. When the machine runs steadily at its nominal rate, the direct-current power available to the load impedance is

$$W=E\times I.$$

The direct-current field excitation power is

$$w=ri^2,$$

and hence the amplification of the machine is

$$G_0=E\times I/ri^2. \tag{2}$$

This expression holds for steady-state direct-current conditions and corresponds to what the radio engineer terms gain G_0 at zero frequency.

If the field is suddenly connected via the input terminals in Figure 2 to a source of continuous voltage of zero internal impedance, the input current i and, consequently, the output voltage E follow an exponential law, the time constant of which is

$$\tau=L/r. \tag{3}$$

A well-designed machine should present a high amplification G_0 and a low time constant τ. To characterize the qualities of a rotary amplifier, Valentin has proposed a dynamic amplification factor

$$D=G_0/\tau. \tag{4}$$

τ is expressed in seconds and can vary from a few hundredths of a second to several seconds; G_0 can vary from 10 to 100 or more; and D varies from 100 to 5000 according to the type of machine.

1.2 Transient-State Conditions

The rotary amplifier being part of a negative-feedback-amplifier chain, its behavior should be studied in the presence of variable-frequency

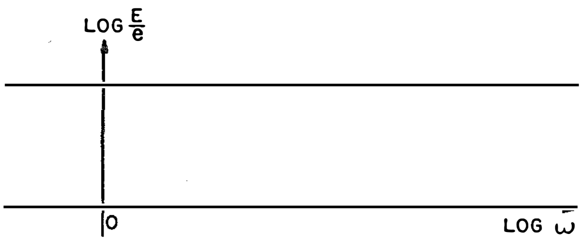

Figure 4—Idealized response curve of the amplifier shown in Figure 3.

sinusoidal inputs. The complex values of the sinusoidal currents and voltages corresponding to the angular frequency ω will be then represented by the parameters i, I, and E.

The modulus and phase of the transmission properties of the machine must be known as a function of frequency. Before proceeding, however, it should be noted that the properties of the

[1] A. Valentin, "Les Dynamos Amplificatrices, l'Amplidyne," *Bulletin de la Société Française des Electriciens*, volume 8, pages 304–328; June, 1948.

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amplifying stage are closely dependent on the internal impedance of the preceding stage and on the load impedance Z.

Let us suppose that Z is a pure resistance (this case is very seldom met in practice). Also let us assume that the stage preceding the generator has infinite internal impedance. This is nearly

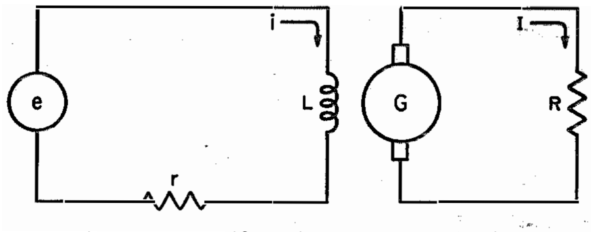

Figure 5—Amplifier driven by a stage having negligible internal impedance.

the case when this stage is a pentode, its plate current being applied to the field (Figure 3).

In this case, the field current i will have constant amplitude at all frequencies (the input voltage applied to the grid of the pentode being kept constant), and the amplification factor between the input voltage e and the output voltage E is constant at all frequencies.

In this case (simplified by hypotheses, but approximately feasible in practice), the time constant τ does not seem to play any particular part; the machine is able to transmit signals having a very wide spectrum of frequencies and no phase difference is introduced by the amplifier, which appears to be nearly perfect, whatever the construction of the machine. The response curve is a horizontal straight line as shown in Figure 4.

Let us now suppose that the preceding stage has negligible internal impedance, as might be found with an amplifying stage using a triode or a thyratron or when two rotating amplifiers are used in series (Figure 5). The field current i is expressed by

$$i = \frac{e}{r + j\omega L}.$$

In terms of (1), the amplification ratio becomes

$$\frac{E}{e} = \frac{K}{r + j\omega L} = \frac{K}{r}\,\frac{1}{1 + j\omega\tau}.$$

Figure 6 shows the shape taken by the response curves. These curves alone are not sufficient to determine the essential characteristics of a complete amplifier when corrective devices and feedback are employed.

We see that the main effect of the time constant τ is to determine the frequency

$$\omega_\tau = 1/\tau$$

beyond which the amplification factor of the machine drops 6 decibels per octave if the impedance of the preceding stage is nil.

This frequency has no direct relation to the useful passband of the complete amplifier and consequently has nothing to do with the response time of the servomechanism incorporating the rotary amplifier. This feature is fundamental.

It is next necessary to find the gain of the machine for a steady-state sinusoidal wave ω.

Whatever the impedances of the elements external to the machine, it is capable of delivering a nominal voltage E and a nominal current I at frequency ω, and consequently its nominal power output is constant at all frequencies. To obtain this power output, the preceding stage of

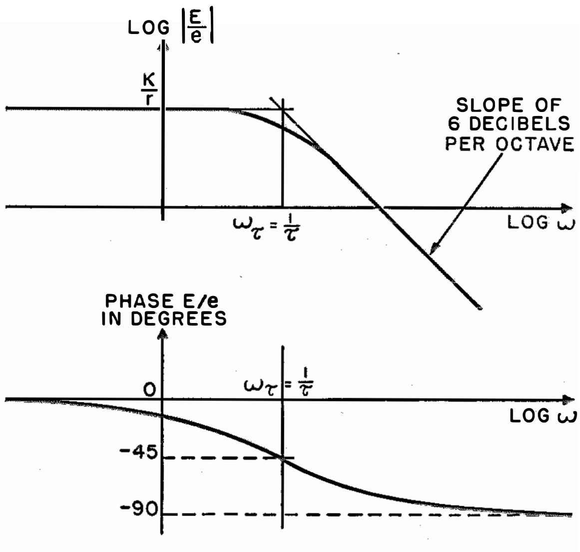

Figure 6—Response curve of amplifier of Figure 5. Amplitude curve is at top and phase below.

the machine must be able to drive a sinusoidal current i through the field winding. A potential difference

$$e = i(r + j\omega L)$$

must then be developed across the terminals of

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this winding. The apparent energy generated by the preceding stage will therefore equal

$$w = i^2|r + j\omega L| = i^2(r^2 + L^2\omega^2)^{1/2}.$$

This energy determines the dimensions of the preceding stage. Hence the equation for the gain at frequency ω is

$$G = \frac{W}{w} = \frac{EI}{ri^2(1+\omega^2\tau^2)^{1/2}},$$

giving the principal equation

$$G = \frac{G_0}{(1+\omega^2\tau^2)^{1/2}}. \qquad (5)$$

Figure 7 shows the variation of gain versus frequency.

The particular frequencies of interest are $\omega_\tau = 1/\tau$ and frequency ω_c for which the gain

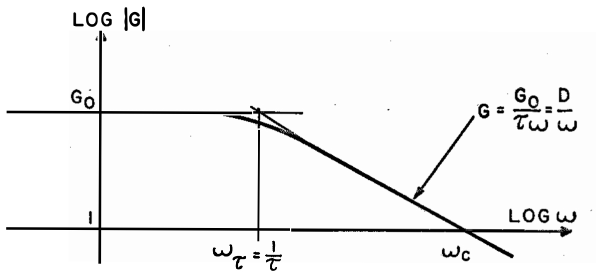

Figure 7—Basic gain-versus-frequency curve.

drops to unity; we shall call this the angular cutoff frequency of the machine. It is the frequency beyond which the machine becomes useless because it introduces a loss instead of a gain.

It can now be shown that ω_c is the dynamic amplification factor of Valentin.

In effect, if G_0 is large compared to unity, which is always the case, we can write

$$1 = G_0/\omega_c\tau,$$

whence

$$\omega_c = G_0/\tau = D.$$

The cutoff frequency of a machine of which the dynamic amplification factor D is known can be readily obtained by the expression

$$F_c = D/2\pi$$

For the rotary amplifiers that are described in this paper, the values of the above factors are close to

$$G_0 = 100,$$
$$D = 2000,$$
$$F_c = 320 \text{ cycles per second.}$$

For $F = 15$ cycles, the gain of the machine is still 21, or 13 decibels.

We must emphasize here the analogy existing between a rotary amplifier and its dynamic amplification factor D, and an amplifying pentode and its figure of merit M. It is well known that when used in a resistance-coupled amplifier, such a tube will amplify a band of frequency from zero to a maximum frequency depending on the figure of merit, which is the quotient of its slope S by the sum of the input and output capacitances C;

$$M = S/C.$$

It appears that this factor M is the frequency at which the gain of the stage cannot be higher than 1. For this frequency, we have

$$1 = C\omega_c/S,$$

whence

$$\omega_c = S/C = M.$$

In an amplifying stage using a vacuum tube, the cutoff frequency may be of the order of 100 megacycles, which limits to about 10 megacycles the band of frequencies available with a gain of about 10.

The same is true for a well-designed rotary amplifier; the cutoff frequency is of the order of 300 cycles, and it is therefore possible to obtain amplification from zero to 30 cycles (these values are given as an example of magnitudes).

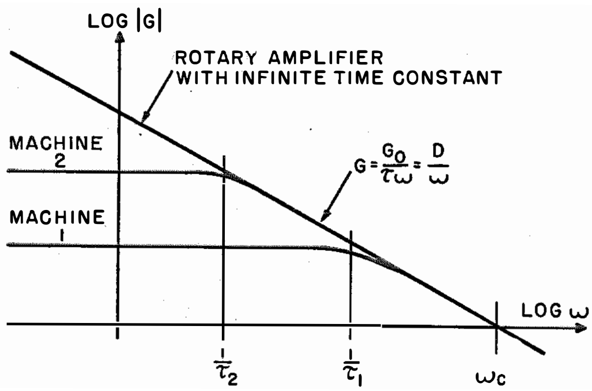

Figure 8—Influence of rotary-amplifier time constant on gain curve.

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There is an analogy between the circuit of the tube (in which the limit results from a parallel capacitance) and the circuit of the direct-current generator (where the limit results from a series inductance).

The time constant τ is a rather misleading concept as there will be a tendency to select a machine having the shortest time constant. If two machines have the same cutoff frequency, their respective gains could be as shown in Figure 8. It is obvious that the total bandwidths of the servomechanisms using the two machines will be the same, controlled only by ω_c. However, machine *2*, which has the greater time constant τ_2, is more useful because it has a higher gain at low frequencies and therefore a larger amount of negative feedback can be used to increase the accuracy of the servomechanism.

In fact, it would be desirable to design servomechanisms with infinite gain at zero frequency. This would be feasible with a rotary amplifier having an infinite time constant.

This result can be obtained by making the total resistance of the field circuit tend to zero. Such an effect can be approximated by means of positive feedback; and some machines such as the Rototrol[2] have been so designed. When positive feedback is added by means of a second field winding, the transfer ratio of the machine takes the form

$$E/e = K/j\omega L,$$

in which K and L do not have the same values as mentioned previously. In symbolic notation, this ratio becomes

$$\frac{E}{e} = \frac{K}{L}\frac{1}{p}.$$

This equation shows that a machine having an infinite time constant constitutes a perfect integrator and permits the use of an infinite percentage of negative feedback at zero frequency.

These conditions are often met in the free-piston amplifier stage of hydraulic speed regulators of turbines. Referring to the diagram in Figure 9, let x be the displacement of the piston, representing the output value of the amplifier; and let α be the angular position of the oil admission valve, representing the input value. We will assume that the oil flow in volume per unit of time is proportional to the angle α of the valve at every instant.

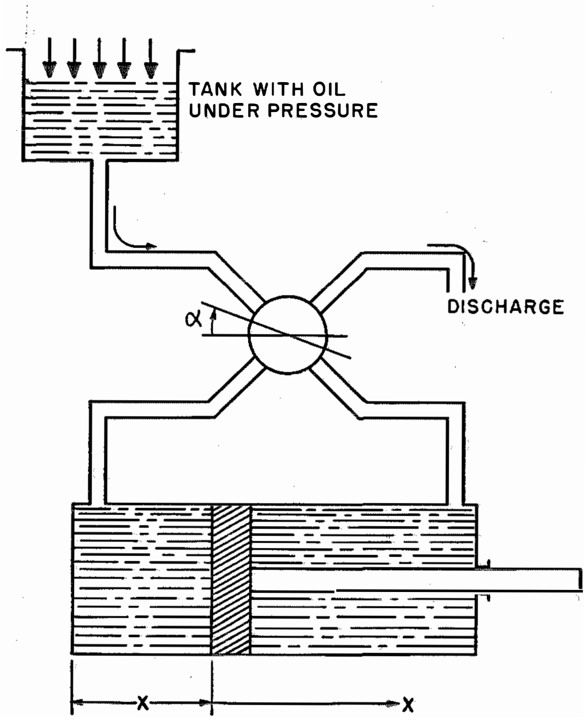

Figure 9—Hydraulic amplifier with free piston.

The speed of the piston is at every moment proportional to the angle $dx/dt = K\alpha$. Hence the transfer ratio of the amplifier is, using classical notation,

$$x = K\int \alpha\, dt.$$

Using symbolic notation,

$$x = K/p.$$

In sinusoidal operation,

$$x = K/j\omega.$$

We have thus a typical example of an amplifier with an infinite time constant; a perfect integrator, the gain of which is infinite at zero frequency. This means that if we give α a sinusoidal movement of constant amplitude with a frequency tending toward zero, the amplitude of the resulting sinusoidal piston movements increases indefinitely.

[2] Jacques Helot, "Le Rototrol," *Bulletin de la Société Française des Electriciens*, volume 9, pages 328–342; July, 1949.

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At the limit, if the valve is permanently open (zero frequency), the piston moves off indefinitely with a uniform velocity until it touches the end of the cylinder, which is the limit of the region in which the equations hold. The characteristics of this amplifier are shown in Figure 8 by an indefinite line having a slope of $1/\omega$.

It can thus be seen that a given time constant can appear simultaneously to be both advantageous and a handicap; this is why it is recommended that it not be used in calculations.

1.3 Operation as an Inverter

An important property of rotary amplifiers is their reversibility from the thermodynamic viewpoint. This means that energy can flow either from the amplifier toward the load circuit or in the opposite direction.

Normally, the machine is a generator and the load circuit is fed from the mains. It can be seen that if a counterelectromotive force greater than the electromotive force of the rotary-amplifier output is developed in the load circuit that the current will change direction. The amplifier then operates as a motor, and the energy it receives from the load circuit is sent back to the mains through the three-phase driving motor, which is also reversible (see Figure 1).

This condition occurs commonly when the load circuit consists of a motor; the direction of energy flow may reverse during periods of deceleration of the motor. This is especially true in remote-control devices and in planing machines where the massive table is given a reciprocating motion. The same thing occurs in the controls of hoisting apparatus during periods of braking.

Reversibility is thus an essential characteristic in many servomechanisms. The rotary amplifier has this quality inherently, but this is not true of most other types of amplifiers. Grid-controlled rectifiers (thyratrons, for example) can be used in reversible amplifiers only by special arrangements that cause them to operate as inverters. In general, operation of thyratrons as inverters requires the use of large inductors to insure proper commutation, and these render the equipment very heavy.

1.4 Power Factor

In a rotary amplifier, there is not necessarily any relation between the output current of the amplifier and the reactive power derived from the three-phase mains.

This is not true of amplifiers using thyratrons or grid rectifiers, or of magnetic amplifiers. When these latter systems deliver a heavy current at low voltage (say, to feed a motor having a high torque at low speed), the reactive energy taken from the mains is important and the power factor drops to a low value.

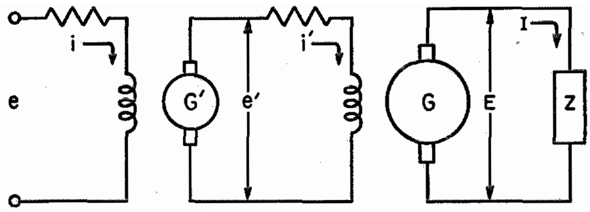

Figure 10—Rotary amplifier with two stages in series.

This comparison shows the usefulness of rotary amplifiers where only limited power is available, such as in ships and aircraft. Furthermore, the addition of a flywheel to the amplifier shaft permits in the case of a sudden violent overload a reduction of the peak current demand from the mains. An example of this is the Illgner sets that feed the motors of the most powerful steel rolling mills.

2. *Rotary Amplifiers with Two Stages in Series*

Many rotary amplifiers in operation at the present time are so built that they constitute in reality a set of two rotary amplifiers connected in series. Particular cases of these are the amplidyne and Metadyne.

The basic diagram of these machines is equivalent to that shown in Figure 10. Actually, these machines have only one armature and one commutator, but the field windings are more complicated than those of standard direct-current generators with a single stage of amplification.

When comparing the characteristics of these twin machines with the single-stage rotary amplifier, some precautions must be taken. In steady operation at zero frequency, it is easy to see that the gain of a two-stage machine is equal to the product of the gains of each stage at that frequency. If the gain of each is 100, the total gain will be 10,000.

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Analysis of the dynamic characteristics is more complicated and radio engineers will plot amplitude and phase response curves for both machines together. It is well known that the complex transfer ratio of two stages in series is equal to the product of the individual transfer ratios of each stage.

The transfer ratio of the dual machine may be defined as follows. The first-stage ratio is

$$\frac{e'}{e}=\frac{K}{r+j\omega L}=\frac{K}{r}\,\frac{1}{1+j\omega\tau},$$

and the second-stage ratio

$$i'=\frac{e'}{r'+j\omega L'}=\frac{e'}{r'}\,\frac{1}{1+j\omega\tau'}.$$

In accordance with the conditions generally met in practice, we assume the time constants of the two stages to be identical and equal to τ, whence

$$i'=e\frac{K}{rr'}\,\frac{1}{(1+j\omega\tau)^2}.$$

But, as previously seen for the output stage, we know that

$$E=K'i'.$$

Therefore, the transfer ratio of the dual machine may be found by

$$\frac{E}{e}=\frac{KK'}{rr'}\,\frac{1}{(1+j\omega\tau)^2}. \tag{6}$$

From this equation, we shall derive the value of the gain as a function of frequency. This gain at a given frequency is, as before, the ratio of the nominal power of the output stage to the apparent power used by the field winding of the input stage. Let E and I represent the output nominal voltage and current, respectively; $|i|$ is the current in the input field; and $|e|$ is the voltage at the terminals of the input field. Then,

$$G=\frac{EI}{|e|\times|i|}=\frac{I}{|i|}\,\frac{E}{|e|}.$$

Let us insert in this equation the voltage ratio calculated as above. We obtain

$$G=\frac{I}{|i|}\,\frac{KK'}{rr'}\,\frac{1}{1+\omega^2\tau^2},$$

whence

$$G=\frac{G_0{}^2}{1+\omega^2\tau^2}=\left[\frac{G_0}{(1+\omega^2\tau^2)^{1/2}}\right]^2, \tag{7}$$

where $G_0{}^2$ is the gain of the dual machine at zero frequency.

From the above relations, we can draw certain conclusions. Equation (6) shows that, with frequencies greater than $\omega_\tau=1/\tau$, the transfer ratio drops 12 decibels per octave (instead of 6 as in a single machine) and the phase lag of the machine reaches 180 degrees (instead of 90 degrees). It follows that the substitution of a dual amplifier for a single amplifier in a servomechanism changes the problem of stability so much that any elementary comparison becomes impossible.

Let us examine (7). The time constant and amplification G_0 at zero frequency for each elementary machine are very nearly the same for both the dual rotary amplifier and the single amplifier of the same quality of manufacture.

Equation (7) shows, therefore, that the cutoff frequency (the frequency at which the power gain of the machine drops to unity) is of the same magnitude for a single-stage rotary amplifier, for each stage of a series amplifier, and for a complete two-stage machine. It will be recalled that for particularly well-designed amplifiers this cutoff frequency is of the order of 100 to 500 cycles.

Figure 11 gives the general appearance of the gain curves of the two types of machines, the internal impedance of the preceding stage being negligible.

The value of the dynamic amplification factor given[1] for the amplidyne is as high as 100,000 to 500,000. It is obvious that if this factor and the cutoff frequency are identical for single-stage

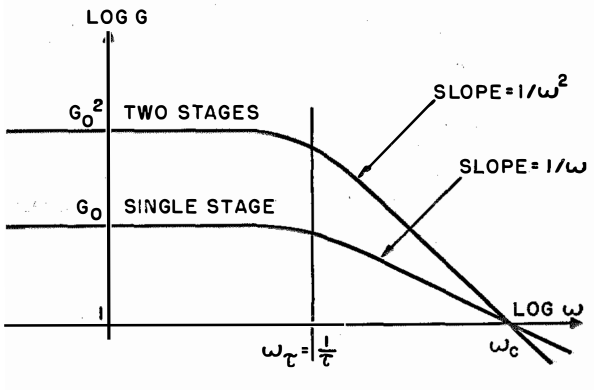

Figure 11—Response curves of single- and two-stage rotary amplifiers.

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direct-current generators, it will no longer be the same for dual machines, and the comparison between the two types made on the dynamic-amplification-factor basis does not give a clear picture of the whole problem.

Moreover, a little further along, we shall see that the dynamic amplification factor of an equipment consisting of a single-stage generator preceded by an electronic amplifier can attain a value that will possibly exceed one thousand million or 10^9.

It is well known that the cost of the last stage of an electronic power amplifier is the dominating factor in the cost of a whole unit. The cost of the preceding stages is low compared to that of the last. The same is true of rotary amplifiers. The driving power required by the field winding of a single-stage rotary amplifier is only 1 to 5 percent of the output power of this generator. Therefore, in the selection of a system of amplification to supply this driving power, the cost is not the only deciding factor.

Let us consider a single-stage rotary amplifier of a power below 100 kilowatts, which covers the general range of existing needs. The power of the penultimate stage will be in all cases below 5 kilowatts, and more likely will be between 100 watts and 1 kilowatt.

Electronic amplifiers of that power (particularly using thyratrons) are reliable and efficient and avoid the 90-degree phase lag inherent in rotary amplifiers.

Perhaps some apprehension was justified a few years ago regarding the reliability of such electronic amplifiers. Today, however, the progress made in this technique has completely removed these doubts.

The discussion so briefly outlined here shows the elements that must be taken into account in the general study of a servomechanism amplifier. Further, an examination of electronic amplifiers proves that, in general, stages with multiple functions (reflex amplification) have been abandoned for other systems using a larger number of single amplifying stages, the cost of the whole being governed by that of the last and most powerful stage.

3. Construction of Rotary Amplifiers

By reason of the above considerations, the Société des Servomécanismes Electroniques in close cooperation with the Sociétés Gramme and Ragonot has designed a range of rotary amplifiers that are to be driven by amplifiers of the electronic type. Only the essential particulars of the problem will be presented here.

As we have seen before, the gain of a machine is given by

$$G=G_0/(1+\omega^2\tau^2)^{1/2}.$$

The design problem involves increasing G_0 and decreasing τ or, in other words, increasing the cutoff frequency to increase to the maximum the accuracy and bandwidth of the servomechanism in which the amplifier is to be inserted.

It has been shown[1] that in a single-stage machine the angular cutoff frequency is given approximately by

$$\omega_c=\frac{C}{e}V^2,$$

where

V = peripheral speed of the armature,

e = air gap of the machine,

C = thickness of the equivalent copper cylinder that would surround the armature core if the active portion of its winding were so altered.

This equation shows the need of using an armature suitably wound and running at a high peripheral speed, but practical considerations impose severe limits on these two parameters. However, appreciable advantage can be obtained in practical construction by using a small air gap.

The minimum air gap should be used because at high frequencies the power required by the input circuit is almost solely the reactive power that generates the alternating flux flowing through the magnetic circuit. This reactive power is directly proportional to the maximum magnetic energy stored in the magnetic circuit of the field during every cycle of oscillation.

Now, as long as the machine is not saturated (which is the case with rotary amplifiers), the greatest amount of this magnetic energy appears across the air gap. It is given by

$$T=\frac{H^2}{8\pi}v,$$

where

H = field in the air gap,

v = total volume of the air gap.

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In direct-current generators of standard construction, the width of the air gap is influenced by considerations relating to the transverse flux (armature reactive flux at 90 degrees to the field flux) and to commutation.

Arrangements adopted in Gramme direct-current generators and in Ragonot amplifiers have made it possible to reduce this air gap to a minimum value consistent with mechanical construction.

Specifically, these arrangements involve the use of a smooth stator with notches to house the field winding, and the use of a second field winding in series with the armature that neutralizes the armature reactive flux and allows good commutation. A series of machines with nominal powers ranging from 3 kilowatts upward has been designed by Sociétés Ragonot (small machines) and Gramme (larger machines).

The characteristics of these machines are much alike, the order of magnitude being $G_0=100$, $\tau=0.05$ second, and $\omega_c=2000$ radians per second.

These rotary amplifiers make possible the design of servomechanisms having very high accuracy with cutoff frequencies (for complete servomechanisms) that can exceed 25 cycles, the corresponding response time being 0.02 second. It is extremely rare in practice that the equipment driven by such servomechanisms or regulators can fully utilize these high response speeds. The cutoff frequencies most commonly used are between 5 and 15 cycles, notably higher than those of many remote-control servomechanisms used during the second world war, where cutoffs were of the order of 3 cycles.

4. *Secondary Phenomena*

Certain simplifying hypotheses made in Section 1, and the effects of these factors on the design and operation of rotary amplifiers will now be taken into consideration.

4.1 Influence of Faults in Magnetization Curve

We will first consider the curvature of the magnetization characteristic and its hysteresis.

With respect to the linear hypothesis, these faults produce distortions similar to that caused by curvature of the transfer characteristic of electronic power tubes. By using single-stage rotary amplifiers, these faults are localized in the output stage of the amplifying chain; it is well known that the use of a large percentage of negative feedback (characteristic of accurate servomechanisms) permits an almost complete correction of such distortion.

By constructing stationary and rotating magnetic circuits of thin laminations of high-quality magnetic materials, these faults are minimized, and the application of large amounts of negative feedback reduces them to unimportant values.

No special device is necessary for correcting hysteresis. In the case of a two-stage rotary amplifier, this situation would not be true, for magnetic distortions would then appear in a low-level stage at a point where the corrective action of negative feedback is much less effective. It is well known that negative feedback does not correct for harmful effects that occur at the input of amplifiers, but in the equipments studied here the input stages are of the electronic type and are relatively free of distortion.

4.2 Absence of Eddy Current

It is well evident from Lenz law that in a closed magnetic circuit, induced currents oppose any variation of magnetic flux. Any such losses occurring in the magnetic circuit of a rotary amplifier increase rapidly with frequency and thereby reduce the cutoff frequency of the machine. It follows that the magnetic circuit of a high-quality rotary amplifier should be constructed of the finest transformer-steel laminations.

For the present type of amplifier, the rotor and stator are similar to those of alternating-current motors, permitting the application of motor theory up to frequencies of the order of 100 cycles.

4.3 Independence of Field and Armature Fluxes

Independence of the armature and field fluxes is assured by the auxiliary field winding that in practice neutralizes the 90-degree armature reaction flux. Consequently a small air gap can be used, whatever the current in the armature.

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4.4 Low Output Impedance

The reaction-flux compensation winding also permits an important decrease in the inductive reactance of the armature and hence of the output impedance. This reactance has no effect in standard direct-current power generators under steady working conditions, but it has a bad influence on machines in which a varying signal is amplified.

The inductance of the armature is particularly harmful when the amplifier output is applied to a motor. A typical motor can be likened to a very large capacitance, sometimes[3] reaching values of 1 farad. The presence of an armature inductance in the generator and the motor capacitance gives rise to a resonance in the middle of the transmitted band that gravely complicates the stabilization of servomechanisms. In machines of standard construction, this resonance may occur at frequencies lower than 10 cycles.

By using compensation windings in the generator and motor, it is possible to reduce the reactances considerably and consequently to increase the resonant frequency, perhaps out of the operating-frequency band.

This short description brings out the importance of the often-secondary considerations that must be studied when the rotary amplifier is to be used in precision equipment. These principles demonstrate the significant differences between a rotary amplifier and the ordinary direct-current generator.

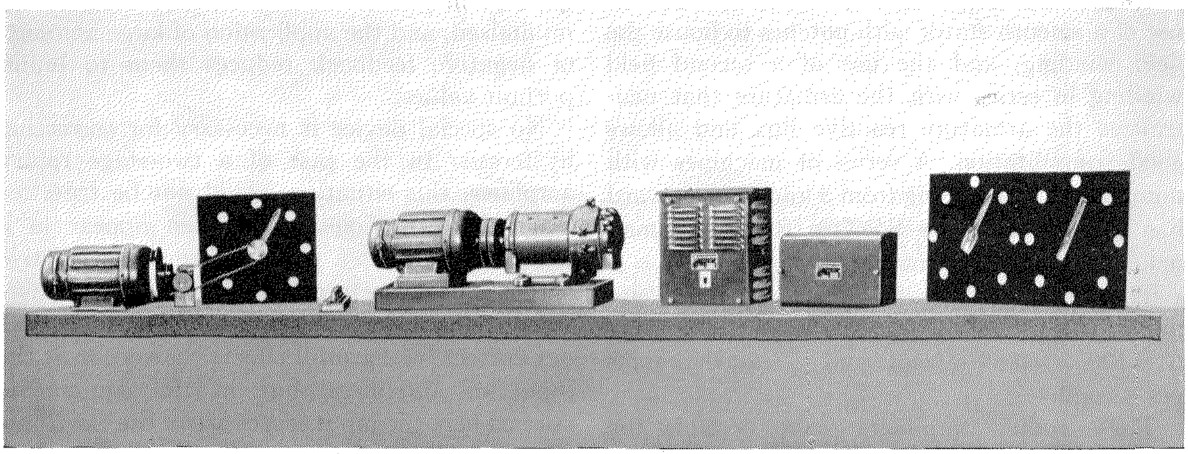

Figure 12—Rapid-response remote-control equipment. The control input at right and working motor at left.

5. Rotary Amplifier Driven by an Electronic Amplifier

Electronic amplifiers used with the rotary type by the Société des Servomécanismes Electroniques consist of two stages equipped with receiving tubes and one stage utilizing gas tetrodes manufactured by Laboratoire Central de Télécommunications. These amplifiers are standardized, preconstructed, and utilized for whatever type of servomechanism is to be built.

At zero frequency, the rotary amplifier delivers its nominal output power when about 0.3 volt is applied at the input terminals of the vacuum-tube amplifier, the input impedance of the latter being 1 megohm. The nominal input power is 10^{-7} watt (0.1 microwatt). For an amplifier of 10 kilowatts nominal power, the gain is then $G_0 = 10^{11}$. The time constant τ of the machine is 0.05 second; that of the electronic amplifier is entirely negligible. The dynamic amplification factor is then $D = 2 \times 10^{12}$. This quantity, a very high value, characterizes results obtained with an integrated set consisting of an electronic and a rotary amplifier.

The workable bandwidth of the set is 15 to 30 cycles, according to the type; it is limited by the 50-cycle frequency of the mains feeding the thyratron stage. Higher bandwidths can be obtained to solve special problems.

[3] R. Aubry, G. Lehmann, and H. Le Boiteux, "Etude et Réalisation d'une Télécommande Electronique d'Artillerie," *L'Onde Electrique*, volume 29, pages 269–370; August, 1949: see page 319.

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The high value of amplification has been chosen to enable the engineer to adjust the response of the set to fit any particular problem, without being inconvenienced by insufficient gain. To obtain the required servomechanism accuracy and stability, the amplifier response is modified by low-level corrective networks in series, by auxiliary feedback loops, or both. It is an easy matter to give to the complete set dynamic properties fundamentally different from those of the rotary machine alone.

For solving the problem of stability, it is a common practice to use corrective networks across the cathode–grid impedance of one of the amplifying tubes. These usually take the form of resistance–capacitance networks comprising resistors of the radio type and capacitors of a few microfarads maximum, the voltage never exceeding 50 volts. Such networks are simple to calculate, build, and adjust—their volume may be about that of a box of kitchen matches.

It is often desirable to limit to a required maximum value some mechanical or electrical quantity, such as armature current, torque, or acceleration, that could rise to dangerous values under unusual working conditions. Such limitations can easily be set by introducing nonlinear components (selenium rectifiers) in the low-level input circuit of the electronic amplifier. Such arrangements are very often used in voltage regulators to avoid voltage surges and in remote-control devices to avoid dangerous mechanical stresses.

6. *Conclusions*

It happens, sometimes, that a problem is amenable to solution by either conventional electrical engineering techniques or by the application of electronics. It is quite usual that the most practical solution will simultaneously employ both techniques since in many cases, these are fittingly complementary.

Particularly, the association of a direct-current generator (discovered by Gramme in 1869) with electronic amplifiers constitutes a solution that in many problems offers extraordinary flexibility.

Thanks to the facilities provided by electronic amplification, it is relatively easy to solve a great variety of problems with a limited range of preconstructed amplifiers and generators and the addition of only minor components such as stabilizing and limiting networks. This possibility is of major importance in France, where the somewhat limited sales make the development of especially designed equipments economically painful.

7. *Acknowledgments*

Gratitude is expressed to both the Société Gramme and its technical director, Mr. Perotin, and to the Société Ragonot and its technical director, Mr. Henry-Baudot, who designed the rotary machines described in this paper. We thank them warmly for the considerate and cooperative spirit in which they have carried out the work.

8. *Appendix*

At a demonstration during the original presentation of this paper before the Société Française des Radioélectriciens, some typical equipments were shown in operation. A brief description of some practical rotary amplifiers as used in industrial synchronized-control equipment may be in order here.

For the purpose of the demonstration, the components illustrated in Figures 12 through 14 were connected to form a servomechanism terminating in a servo drive motor having an output of 5 nominal and 20 peak horsepower. This motor was placed at the left of the set of equipment shown in Figure 12.

At the far right of Figure 12, behind the two large dials, are the controlling selsyns and a tachometer generator (a generator giving a direct-current output directly proportional to speed of rotation).

The outputs of the selsyns were converted to direct current and applied to a preamplifier and power amplifier (respectively, the small and large cabinets in Figure 12). As will be evident from the schematic in Figure 13, a difference voltage representing the angular error between the position of the shaft of selsyn *S2* (corresponding to the desired working-motor-shaft position) and the shaft position of selsyn *S1* (corresponding to the actual working-motor-shaft position) was amplified and applied to the field of the rotary amplifier.

The tachometer output was applied to the stabilizing network. Depending on the speed at which the working motor was running, a greater

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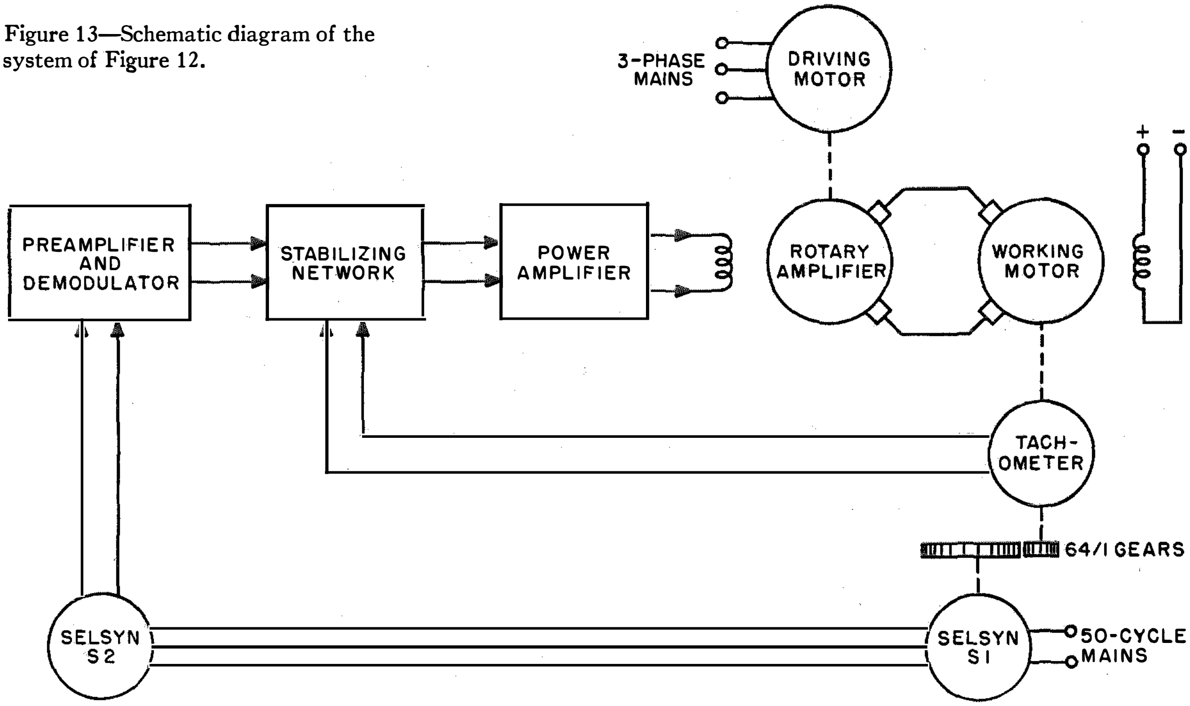
Figure 13—Schematic diagram of the system of Figure 12.



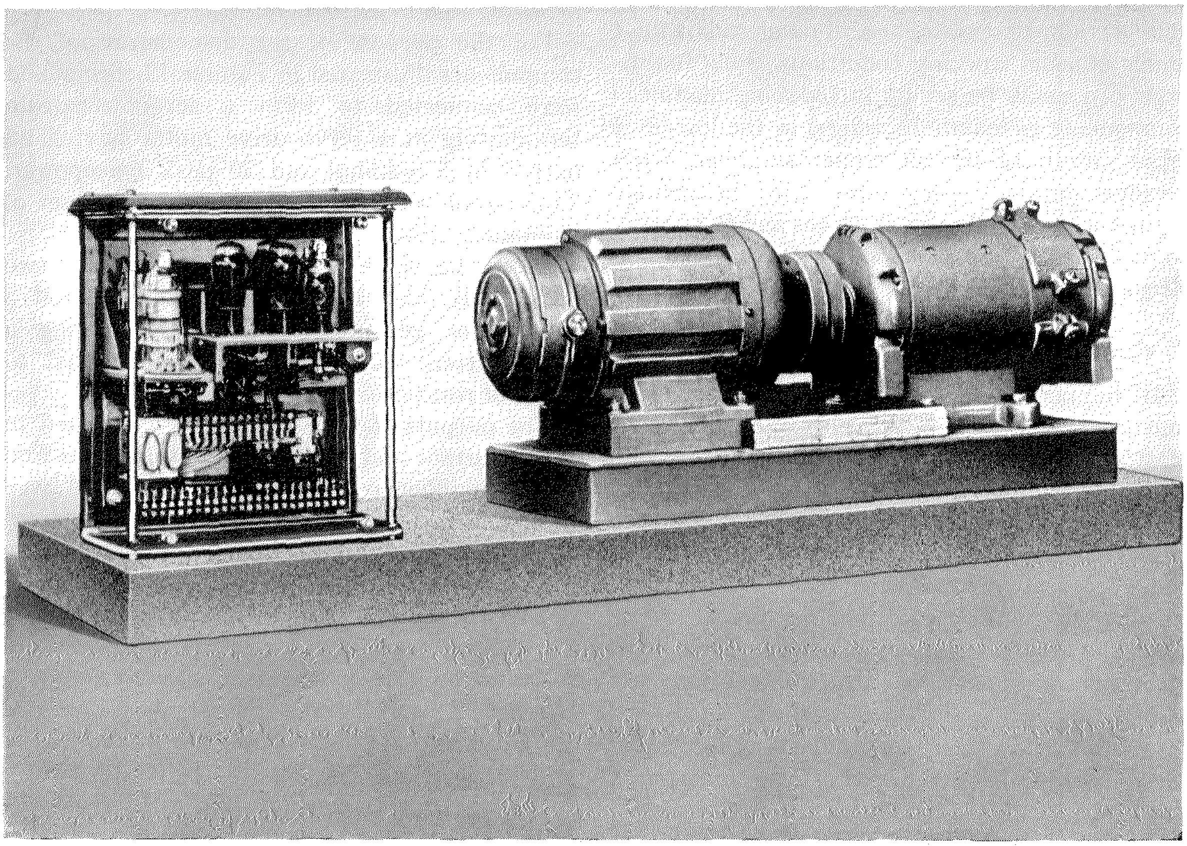


Figure 14—Another view of the electronic and rotary amplifiers of Figure 12.

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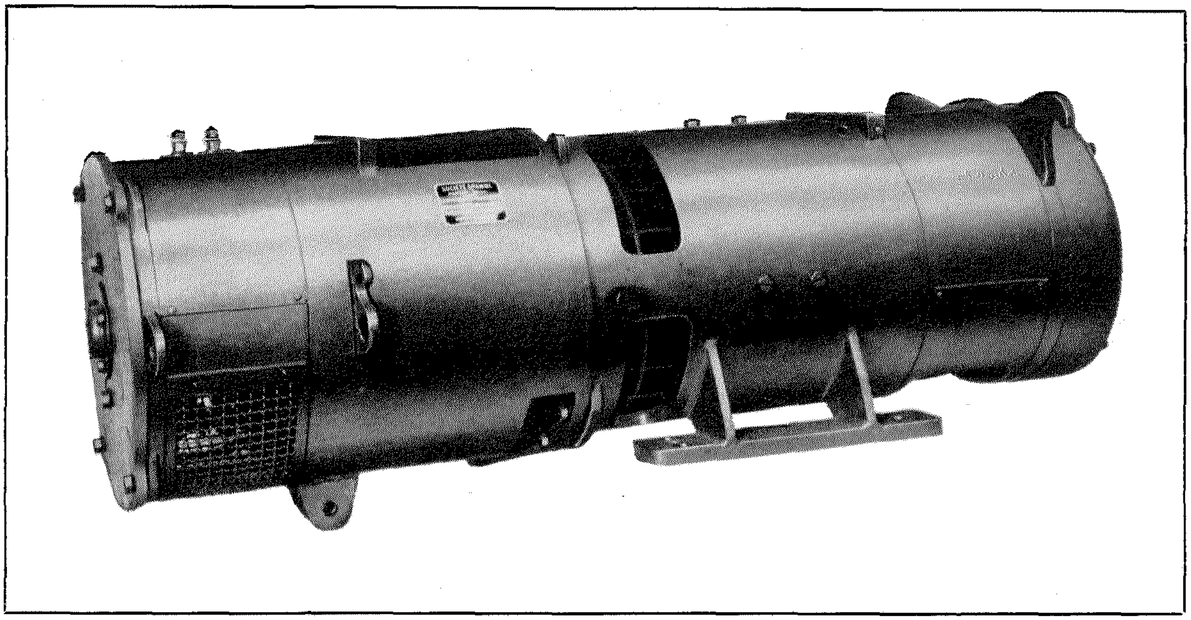

Figure 15—Société Gramme rotary amplifier and driving motor set for 15 nominal and 45 peak horsepower. The length of the machine is about 4 feet (1.2 meters).

or lesser energization of the rotary-amplifier field was necessary to overcome inertia. The stabilizing network provided a rate of change compensation and helped prevent overrun and hunting.

The amplifiers consist of a vacuum-tube amplifier handling the 50-cycle signals of the selsyns followed by a balanced demodulator. Six receiving-type tubes are used here. The power stage comprises two type *3868-A* gas tetrodes and supplies 100 watts to the rotary-amplifier field.

During the first experiment, all the servo loops were broken and the tachometer generator was connected to the input of the amplifier with the working motor at the output. The rotational speed of the tachometer and motor were then approximately proportional with a ratio of 1 to 1000, a speed of 1 revolution per minute of the tachometer producing 1000 revolutions per minute in the 5-horsepower motor. This proportionality was maintained between -3000 and $+3000$ revolutions per minute of the motor.

This illustrated the possibility of obtaining a speed amplification of 1000 within a feedback loop.

During the second experiment, the servo circuits corresponding to Figure 13 for angular synchronization were closed. The angular position of the motor shaft was then controlled by the movements of selsyn *S2* operated by hand. One revolution of the selsyn corresponded to 64 revolutions of the working motor.

The angular accuracy, the great rapidity of response, and the aperiodic damping of the device were evident. The cutoff frequency of the servomechanism was 10 cycles corresponding to a response time of 0.05 second.

A close-up of the rotary amplifier of Figure 12 is given in Figure 14, and a larger rotary amplifier of 15 kilowatts nominal and 45 kilowatts peak power is shown in Figure 15.

The progress made in two years regarding simplicity and general over-all dimensions of servomechanisms will be appreciated when comparing these pictures with those illustrating the article referred to in footnote 3. The accuracy and rapidity of response of the two equipments are similar; the power of the apparatus presented in 1951 is higher than that of 1949.

I acknowledge my indebtedness to Appareillage Electromécanique V.F.B. for designing the electronic equipments and also to the technical staff of the Société des Servomécanismes Electroniques, particularly to Messrs. Csech and Cambornac, for their valuable assistance in the preparation of the experiments that are presented in this appendix.

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Microstrip—A New Transmission Technique for the Kilomegacycle Range*

By D. D. GRIEG and H. F. ENGELMANN

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

A NOVEL APPROACH to microwave transmission and components is described. In place of the more familiar waveguide or coaxial structures, this technique utilizes a single conductor supported above a ground plane. Such a configuration is equivalent to a parallel-wire system for the image of the conductor in the ground plane produces the required symmetry. Losses in this system are approximately equal to those of coaxial structures. The extent of the field spread about the conductor is small, thus making possible compact configuration. Printed-circuit techniques are particularly applicable to the construction of compact, rugged, inexpensive, and noncritical microwave components. A general description, background theory, and laboratory experiments are given in this and other papers.[1,2]

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Three principal types of structures have been utilized in microwave systems: waveguides, coaxial lines, and parallel wires. Waveguides have comparatively low losses and, in common with the coaxial configurations, provide complete shielding and, hence, high Q, if required. Decoupling between components is more readily achieved than with other systems. Waveguide structures, on the other hand, are heavy, bulky, dimensionally critical, and expensive to manufacture. Coaxial components can be made smaller for a given wavelength than the waveguide equivalent, but usually require critical tolerances that make them still more expensive and difficult to manufacture. The third type, the parallel-wire system, while avoiding the disadvantages of the previous systems, possesses severe limitations so far as microwaves are concerned in that rigid symmetry is necessary to avoid perturbations and corresponding radiation and losses.

Other types of systems have also been considered. One type is the so-called G line involving the propagation of a TM mode along a single dielectric-coated conductor.[3,4] The relatively large spread of the fields about the conductor and the wave launching and collecting mechanisms have limited the usefulness of this system to antenna transmission lines and similar applications. More recently, a transmission line that corresponds to a flattened coaxial with the sides removed has been described.[5] This approach, while yielding configurations that are somewhat simpler to fabricate, still requires that close tolerances be maintained as in the case of coaxial construction. In addition, the parallel outer conductors subject the system to problems of transverse modes of transmission.

The relative simplicity of the parallel-line system suggested further study of this type or of some equivalent "open" system. This work has resulted in an interesting variation of the parallel-line system that avoids the requirements for extreme accuracy and dimensional symmetry. Because of the ease of manufacture and the apparent similarity to conventional wiring, the generic name of microstrip has been given to this transmission system. More specific forms,

* Reprinted from *Proceedings of the IRE*, volume 40, pages 1644–1650; December, 1952. Presented at the National Convention of the Institute of Radio Engineers in New York, New York, on March 5, 1952.

[1] Fred Assadourian and Emanuel Rimai, "Simplified Theory of Microstrip Transmission Systems," *Electrical Communication*, volume 30, pages 36–45; March, 1953: also, *Proceedings of the IRE*, volume 40, pages 1651–1657; December, 1952.

[2] J. A. Kostriza, "Microstrip Components," *Electrical Communication*, volume 30, pages 46–54; March, 1953: also, *Proceedings of the IRE*, volume 40, pages 1658–1663; December, 1952.

[3] G. Goubou, "Surface Waves and Their Application to Transmission Lines," *Journal of Applied Physics*, volume 21, pages 1119–1128; November, 1950.

[4] S. S. Attwood, "Surface-Wave Propagation Over a Coated Plain Conductor," *Journal of Applied Physics*, volume 22, pages 504–509; April, 1951.

[5] R. M. Barrett and M. H. Barnes, "Microwave Printed Circuits," *Radio and TV News*, Radio-Electronic Engineering Section, volume 46; September, 1951.

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which are described in this and the accompanying papers,[1,2] have been termed "wire-above-ground," "strip-line" and the corresponding "strip-plumbing," as well as "conductor–ground-plane" configurations.

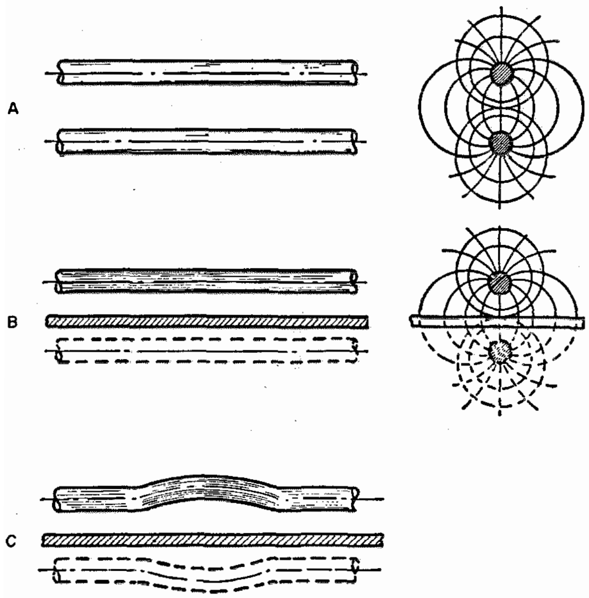

Figure 1—Evolution of line above ground.

1. General Description

The basic principle of this transmission system is illustrated in Figure 1. For reference purposes, a parallel-wire system is shown at A. If a ground plane of theoretically infinite width is inserted between the conductors and if the lower conductor is removed, the configuration at B results. If we assume as a first-order theory that the electostatic case applies, then an image of the upper conductor will exist in the ground plane when a field is present. The significance of the image is represented at C.

If the upper conductor is disturbed by moving it upward, the image obligingly moves downward to compensate and maintain structural symmetry. Similarly, if the conductor diameter is modified or the conductor is disturbed longitudinally, the image compensates in a corresponding manner. While the picture gives, of course, a first-order description only, it does serve to illustrate the basic principle involved. If the perturbations of the conductor are small compared to the wavelength, symmetry will be maintained.

A cross section of the wire-above-ground system, as well as a cross section of a variation of this system using a strip conductor in place of the round wire, is shown in Figure 2.

In the idealized case using a single uniform dielectric and a lossless conductor, the type of transmission corresponds to the TEM mode. This has been confirmed approximately by theoretical work and by measurements performed on practical systems comprising composite dielectrics and finite conductor dimensions.

On the basis of the TEM mode of transmission, the system can be considered as approximating the characteristics of the familiar parallel-wire system. While this assumption neglects such things as fringing effects, in addition to the finite conductor and dielectric characteristics, the equivalence has been found useful as a guide to the design of various components.

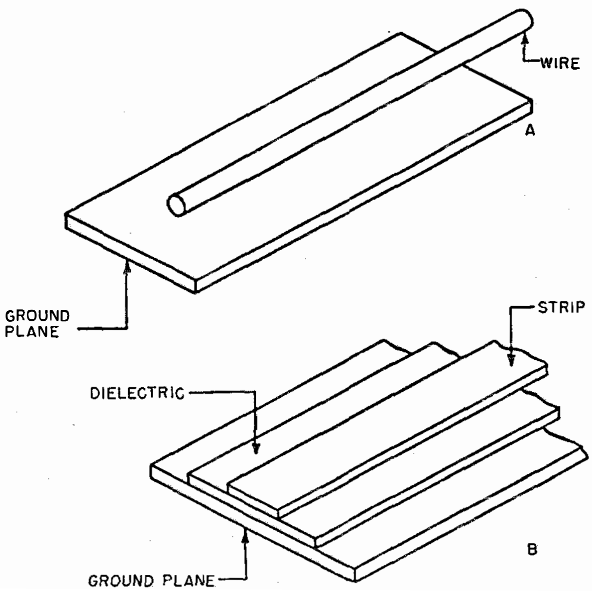

Figure 2—Cross section of transmission system. A shows the wire above ground and B illustrates the strip line.

An important characteristic of the system is the power-flow distribution between the conductor and ground plane. Figure 3 gives the results of calculations of the ratio of power flow in a particular cross section to the total flow of power for a given b/h, where b is the radius of the wire and h is the distance from the center of the wire to the ground plane.

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While the distribution shown is approximate only, the interesting conclusion that can be drawn from the mapping is that most of the power flow is adjacent to the conductor. In the case of the strip line, essentially all of the power

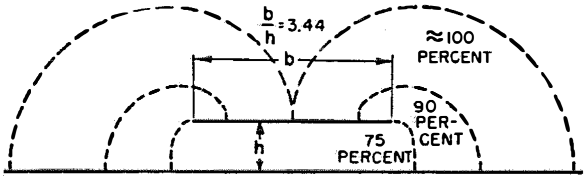

Figure 3—Power-flow distribution.

is confined to a region of the ground plane equal to approximately three times the strip width for large b/h.

A comparison between the calculated losses for the various types of transmission systems and for the conductor–ground-plane system at different frequencies is given in Table 1. Calculations are given for both air and polystyrene dielectrics.

TABLE 1

Calculated Attenuation in Decibels per Foot
Multiply by ≈3 to Obtain Decibels per Meter

Frequency in Megacycles	Rectangular Waveguide		Coaxial Line		Wire Above Ground	
	Air	Polystyrene	3/8-Inch (9.53-Millimeter) Air	*RG-8/U*	Air	Polystyrene
1,000	—	—	0.031	0.085	0.018	0.047
5,000	0.021	0.18	0.070	0.250	0.041	0.155
10,000	0.076	0.48	0.099	0.420	0.058	0.270

As can be seen from the table, the waveguide system using air dielectric yields the lowest attenuation values as would be expected because of the large copper area involved. The conductor-ground-plane system, however, has somewhat lower losses than the coaxial system.

In general, measured values for all the transmission systems are somewhat higher due to errors introduced by the physical condition of the conductors and dielectrics and to the presence of errors involved in the measurement process. Actual measurements have shown reasonable correspondence with the calculated values given. From both the calculated and measured data, the general conclusion can be drawn that this system yields losses that are closely comparable to those obtained with coaxial systems.

In view of the open type of construction of the microstrip structures, the radiation losses are of direct interest. Sterba and Feldman[6] have shown that the power radiated by a terminated parallel-wire line has a magnitude approximately twice that radiated by a doublet antenna of length equal to the line spacing and whose current is equal to the current in the line. In the above, for a line spacing small compared to the wavelength and for a line terminated in its characteristic impedance, the following expression is derived:

$$P_r/I^2 = 160(\pi D/\lambda)^2, \tag{1}$$

in watts per ampere², where P_r is the radiated power, D/λ is the line spacing, and I is the root-mean-square current in the line.

An approximation for the wire-above-ground case can be made by assuming that power is radiated in one hemisphere only, which divides the numerical constant by a factor of 2 and by writing for D the quantity $2h$, h being the height

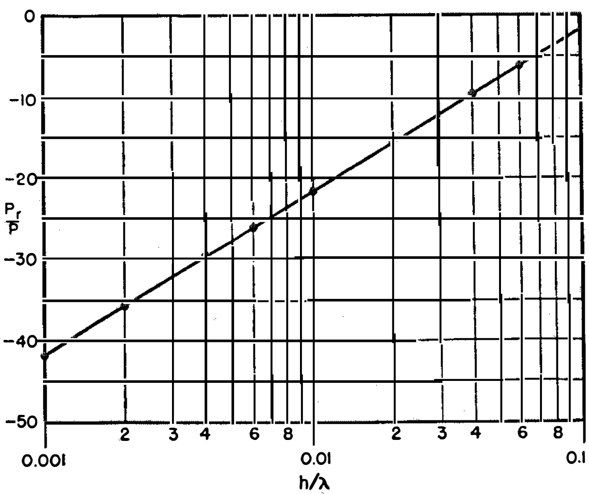

Figure 4—Radiated power ratios as a function of h/λ in accordance with (2) and for a line having a characteristic impedance $Z_0 = 50$ ohms.

above ground. If the characteristic impedance Z_0 is used to determine the power in the line, (1) becomes

$$\frac{P_r}{P} = \frac{320}{Z_0}\left(\frac{\pi h}{\lambda}\right)^2, \tag{2}$$

[6] E. J. Sterba and C. B. Feldman, "Transmission Lines for Short-Wave Radio System," *Proceedings of the IRE*, volume 20, pages 1163–1202; July, 1932.

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where P is the power in the line and λ is the wavelength. Figure 4 illustrates the radiated-power ratios for a 50-ohm line for different conductor--ground-plane spacings and frequencies. It is seen from the data presented that within the assumptions inherent in the use of (2), the radiated power is a small percentage of the total power transmitted for small ratios of h/λ.

2. *Design and Structural Details*

It is apparent that the basic idea of using a wire above a ground plane as a transmission-line system is not new,[7–9] and in fact this general type of system was described in the early literature of radio. The novelty is that it is applicable to microwaves and that particular configurations give optimum electrical and mechanical characteristics.

Several different types of configurations, each having their particular advantages and applications, might be visualized. Two main types of microstrip have been investigated to date. These are:

A. Wire-above-ground, which consists of a cylindrical conductor suspended above a ground plane;

B. Strip line, which consists of a narrow ribbon conductor separated from the ground plane by a dielectric material.

Table 2 gives some of the characteristics of a practical microstrip system. The wire-above-ground structure may be of the air-dielectric type with the central conductor supported by dielectric beads or by equivalent stubs. Alternatively, the wire may be supported above the ground plane by a continuous dielectric strip or be immersed in the dielectric. Because of the absence of sharp corners, the wire-above-ground line possesses the higher power-handling capabilities. In addition, the air-dielectric line yields the lowest losses.

TABLE 2

STRIP-LINE AND WIRE-ABOVE-GROUND STRUCTURES

	Strip Line	Wire Above Ground
Characteristic Impedance	50 Ohms	50 Ohms
Conductor Dimensions	0.220 by 0.002 Inch (5.59 by 0.05 Millimeters)	0.125-Inch (3.18-Millimeter) Diameter
Conductor Material	Copper	Copper
Ground-Plane Width	3 Times Conductor Width	3 Times Conductor Diameter
Conductor Support	0.064-Inch (1.63-Millimeter) Polystyrene Sheet	Polystyrene Bead
Height Above Ground Plane	0.064 Inch (1.63 Millimeters)	0.024 Inch (0.61 Millimeter)

Table 3 gives the ratios h/b for various optimized characteristics. As in the coaxial case,

TABLE 3

DIMENSIONS FOR OPTIMIZED CHARACTERISTICS

	h/b	Characteristic Impedance of Air Line in Ohms
Maximum Voltage Between Conductors	2.45	92.6
Maximum Power Transfer	1.60	62.6
Minimum Attenuation	2.45	92.6
Minimum Resonant Impedance	1.0	0
Maximum Resonant Impedance	5.55	144

equivalent impedances of the order of 50 ohms represent a close-enough compromise between the various characteristics listed.

For the particular wire-above-ground dimensions given, an estimation of the radiation can be made using (2). For $h=0.025$ inch (0.63 millimeter), $Z_0=50$, and $\lambda=7$ centimeters, the ratio of power radiated to power in the line is -23 decibels. If it is assumed that all this power impinges on an adjacent line then, since the efficiency of the line as a receiving antenna is down at least the same 23 decibels, the coupling of the radiation fields between two similar lines is a minimum of -46 decibels.

Since the radiation varies directly with the square of the spacing, radiation coupling can be reduced still further by decreasing h. To maintain the same impedance, the diameter of the wire in this case must also be reduced. While this reduces the copper area and thus increases the line losses, for most applications to microwave circuitry where line lengths are short, the increase in attenuation should not be a determining factor. An additional factor, which is of second-order magnitude and which further increases losses as the spacing of the wire above ground is decreased, is the "proximity effect" and is due to the increased concentration of the field nearest the

[7] "Reference Data for Radio Engineers," Federal Telephone and Radio Corporation, New York, New York, Third Edition; 1949: page 324.

[8] W. H. Timbie and V. Bush, "Principles of Electrical Engineering," John Wiley and Sons, Inc., New York, New York. Third Edition; 1947: pages 299–304.

[9] A. A. Alford, United States Patent 2,159,648; May 23, 1949.

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ground plane.[10] This factor is present to a much-lesser degree in the strip-line type of configuration.

Another item of interest is the power-handling capabilities of the line as a function of spacing. If we assume an air breakdown characteristic of 30 kilovolts per centimeter (76 kilovolts per inch), the breakdown voltage for the example cited is 1.9 kilovolts. For a 50-ohm impedance, this yields a theoretical power-handling capacity of approximately 70 kilowatts. In the actual case, projections in the wire and the condition of the conductors and dielectric can reduce this value considerably. Tests on an experimental sample have given a breakdown voltage of 1.8 kilovolts and a corresponding peak power capability of 65 kilowatts.

On the basis of the above, it is apparent the preferred configuration for the wire-above-ground or equivalent system for application to microwaves should maintain h/λ as small as is practicable. For the more-popular range of microwave frequencies (e.g., approximately 1 to 10 kilomegacycles) this spacing is of the order of a few thousandths of an inch (hundredths of a millimeter).

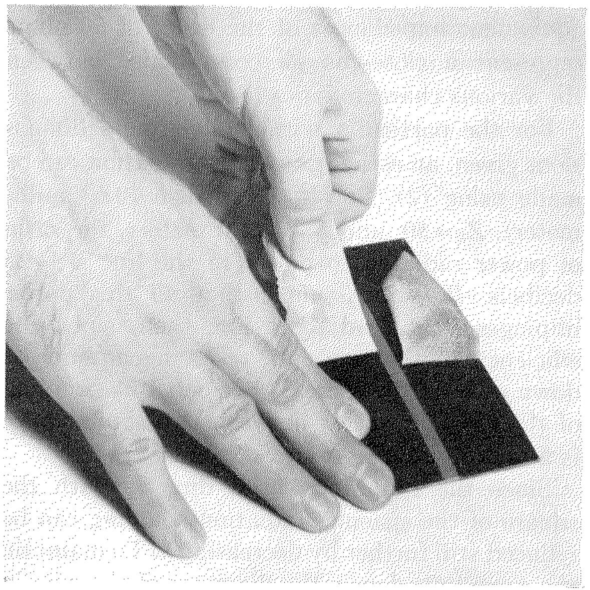

Figure 5—Mechanical stripping process.

[10] J. R. Carson, "Wave Propagation on Parallel Wires; The Proximity Effect," *Philosophical Magazine*, series 6, volume 41, pages 607–633; April, 1921.

The dielectric-supported wire above ground suggests a second configuration that is more amenable to simple fabrication processes. This is the strip-line system, an experimental design of which is described dimensionally in Table 2.

The strip line can be constructed in many ways. An extremely useful method for experimental work utilizes as the starting material a dielectric of the proper dimensions sandwiched between two high-conductivity metals such as copper and silver. Sandwich material utilizing several different types of dielectric can be purchased on the commercial market. The transmission-line configuration is scribed on one of the conductors and the excess material removed as illustrated in Figure 5. The unscribed metal backing serves as the ground plane and the dielectric is utilized as the support for the transmission-line configuration. It has been found that the tolerances of the dielectric thickness for the commercial product, which may run ±5 percent, is adequate for most of these purposes.

Alternatively, the strip configuration can be printed by photographic methods on one of the conductors and the excess material removed by a chemical process. This latter method, utilizing a ferric-chloride etching process, has been used quite successfully. Figure 6 illustrates the printing of a batch of strip-line components by this process. One group of hybrids and a second group of directional couplers are shown. The individual components are cut apart later.

Various materials and alternate processes may, of course, be used in place of those described above. For low dielectric losses and for good temperature characteristics, a metal-and-dielectric sandwich using teflon may be used.

Printing, evaporating, or embossing methods can, of course, be used instead of removing material in the fabrication process. In general, the strip line lends itself to fabrication by most of the printed-circuit methods.

A variation of the strip line is a construction in which a layer of dielectric is placed over the strip conductor. The strip conductor may be deposited in a fashion similar to that described or, alternatively, may be independently cut from strip stock in the required configuration and sandwiched between the dielectric and the ground plane.

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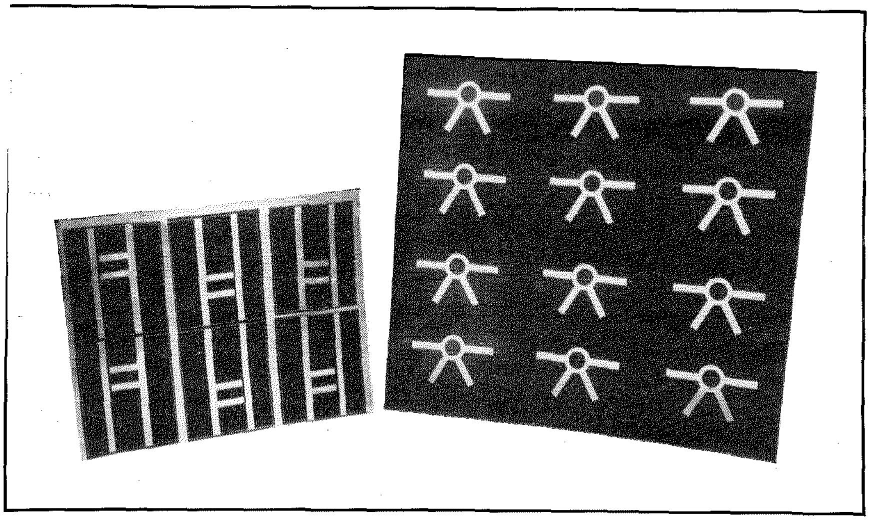

Figure 6—Method of producing printed strip-line components. The larger sheet is about 15 inches (38 centimeters) wide.

determining the materials of construction and the type of system.

The strip line particularly lends itself to satisfactory fabrication processes, and thus many components have been constructed utilizing it. Dielectrics actually used for the development of components have been polystyrene and laminated phenolic plastics. Fibrous sheet materials impregnated with a thermosetting resin have had their use limited to guide-wavelength studies only, because of excessive attenuation.

These products are available commercially in sheet form with either or both sides coated with copper, and have been used with the mechanical stripping, photoengraving, and etching processes. In the case of polystyrene, cementing of metal conductors to the surface has proved successful without introducing appreciable dissipation. Preformed shapes of line conductor can thus be cemented in place to form a particular configuration. Sandwich-type construction is readily accomplished by cementing under pressure.

Components have been successfully operated in several frequency bands, such as 1, 5, and 10 kilomegacycles. In view of the availability of special test equipment, most of the experimental work has been done in the 4400-to-5000-megacycle region. A partial listing of components that have been constructed follows.

It would be expected that radiation effects, power-handling capabilities, and their relation to the spacing of strip would be comparable to the wire-above-ground structure. Experimental evidence has essentially confirmed this. Measurements made on two adjacent strip lines using the configurations of Table 2 yielded coupling values of the order of -35 decibels for separations of approximately $\frac{3}{8}$ wavelength. This value includes not only the coupling of the near fields but leakage introduced by the coaxial transitions used in the experimental setup. Measurements have also been made of the voltage breakdown and power capabilities for 0.05-inch (1.27-millimeter) separation of polystyrene. Maximum voltage has been found to be 15 kilovolts and the corresponding peak power capability, 4×10^6 watts.

3. Components and General Application

From the basic transmission-line elements, many different types of components having application to microwave systems can be designed. In general, it has been found possible to construct most of the applicable components utilizing wire-above-ground or strip-line configurations. For microwave systems, the total length of line required for the components is relatively short. Since insertion losses are therefore small, a greater freedom of choice exists in determining the materials of construction and the type of system.

3.1 Transitions

Wideband transducers have been designed to operate in several regions of the microwave spectrum, including the 5- and 10-kilomegacycle regions. Figure 7 illustrates a strip-to-coaxial transition as compared to the equivalent coaxial-to-waveguide type for the 5000-megacycle range. Voltage standing-wave ratios as low as 1.2 over a 10-percent band utilizing this type of component have been achieved.

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Transitions for waveguides have also been constructed. In general, while it has been found feasible to couple the waveguide directly to the strip line, a more-convenient design has been the use of a cross-bar-feed waveguide-to-coaxial transition and then that of the coaxial-to-strip-line transition of the type described above.

3.2 Crystal Mounts

Wide-band crystal mounts have been designed for the various types of lines. For the strip line, a successful design is the type utilizing a coaxial transition with the crystal holder as an integral part of the coaxial. Figure 8 illustrates a unit

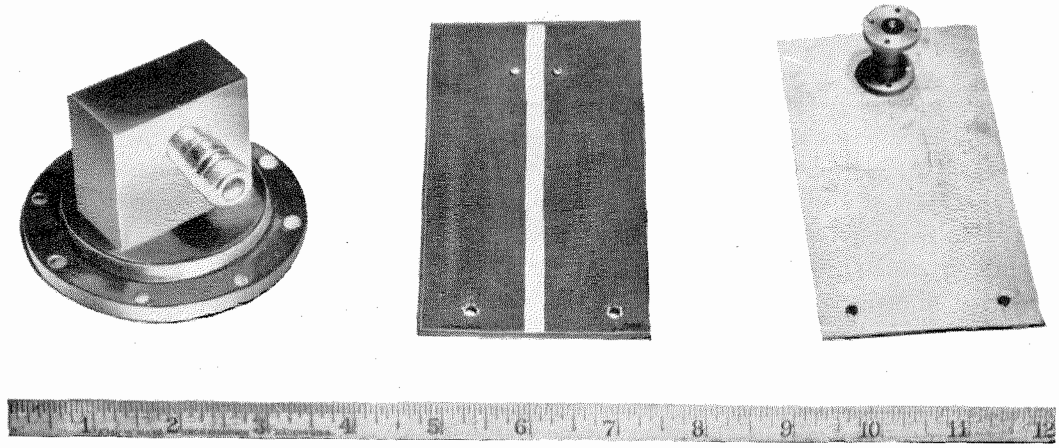

Figure 7—Comparison of a 2-by-1-inch (5-by-2.5-centimeter) waveguide type-*N* transducer and a 3/8-inch (9.53-millimeter) coaxial-line-to-microstrip transducer for the same frequency range.

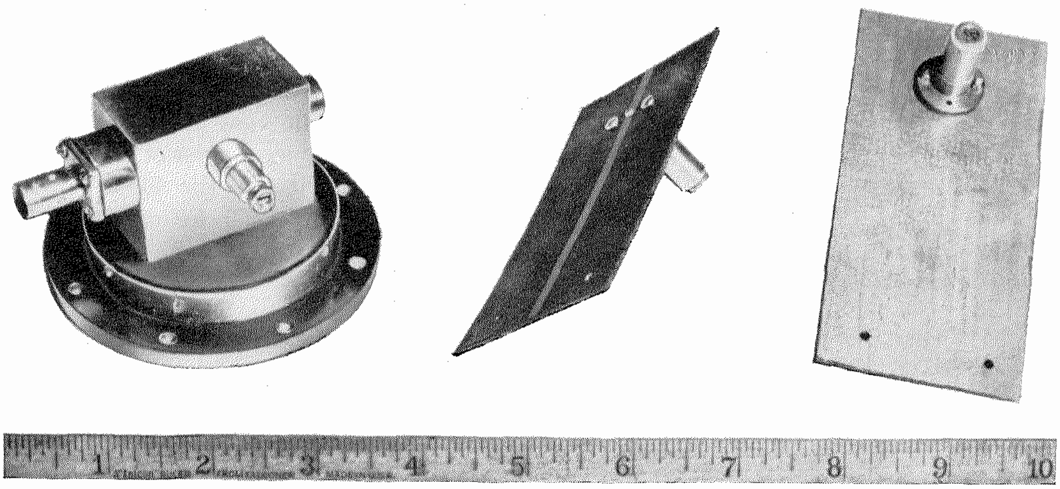

Figure 8—Comparison of crystal mounts on a 2-by-1-inch (5-by-2.5-centimeter) waveguide and on a microstrip-to-coaxial structure using the same type of crystal for a similar frequency band.

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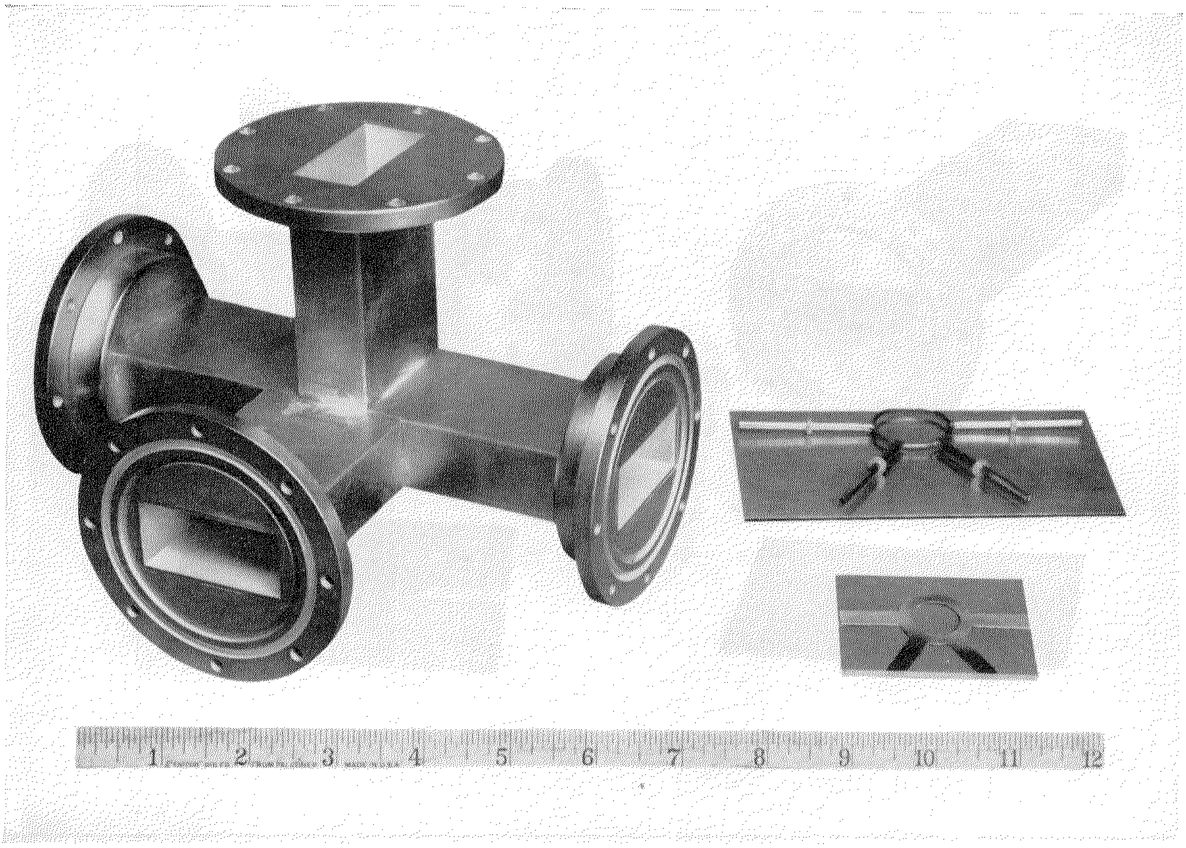

Figure 9—Comparison of waveguide magic tee and microstrip rat races.

designed for operation in the 4400-to-5000-megacycle band. Voltage standing-wave ratios of less than 1.5 have been measured utilizing the *1N21-B* and *1N23-B* types of crystals.

3.3 Magic Tees

The magic tee, which is useful for balanced mixers, automatic-frequency-control circuits, hybrid junctions, and the like, has been constructed in microstrip. Figure 9 illustrates the comparison of microstrip rat races and the waveguide equivalent for operation in the 4400-to-5000-megacycle region. This type of hybrid junction has been utilized with the two arms terminated in crystal holders and the third arm in a matched load. Measurements have shown negligible radiation, balanced crystal response, and extremely low voltage standing-wave ratio over the bandwidth.

3.4 Attenuator Pads and Loads

Figure 10 illustrates a type of fixed load utilizing the microstrip techniques. The line is coated with a lossy dielectric of appropriate characteristics. Graphited paint has been used in some cases. To obtain a proper match, the lossy dielectric can be tapered as shown in the figure. The load illustrated gives a matched termination over a 4000-to-5000-megacycle range.

A variable attenuator, counterpart of the waveguide flap attenuator, is also shown. Variation of attenuation is obtained by rotation of the flap, which adjusts the length of the dielectric run with respect to the strip line. This attenuator has a range of zero to 15 decibels.

3.5 Miscellaneous Components

The preceding components have been described briefly in order to illustrate the various

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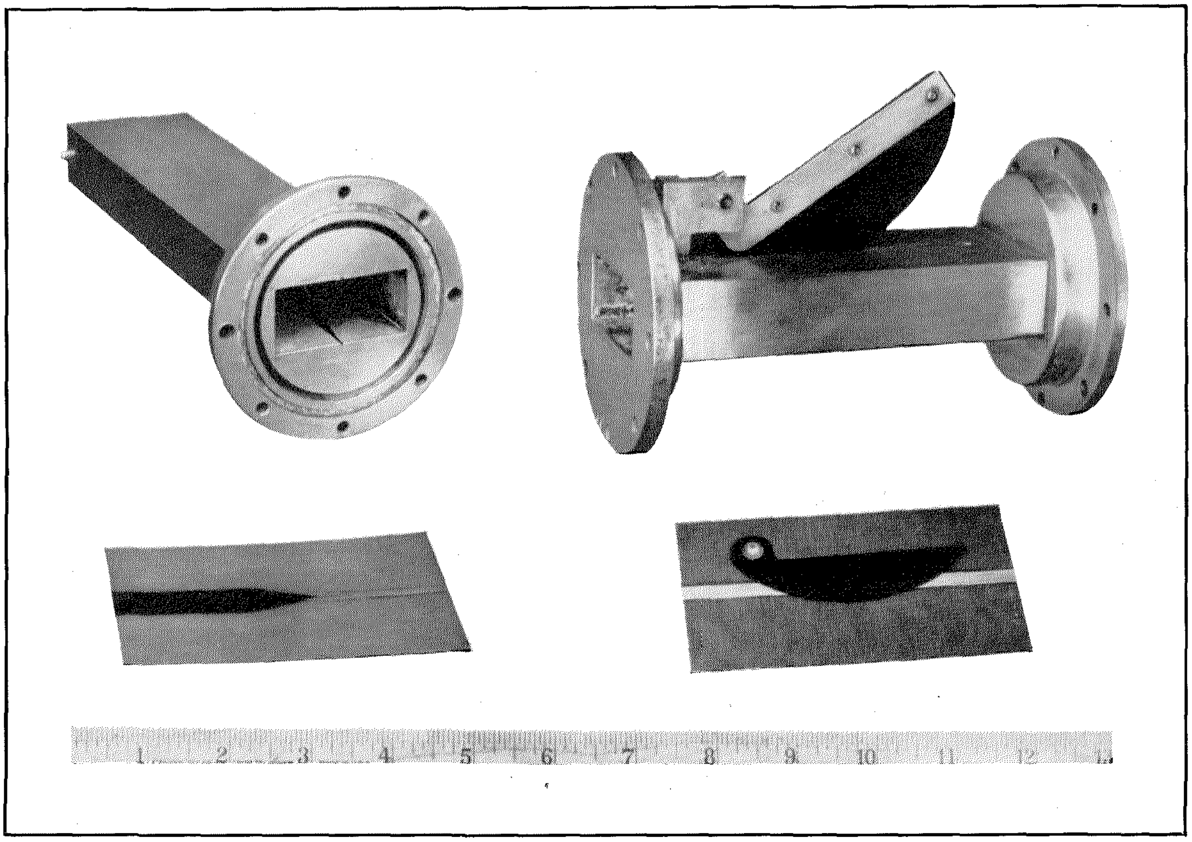

Figure 10—Loads and variable attenuators of rectangular guides and printed strip lines.

possibilities. Other types of components not described have included directional couplers, filter elements, and similar structures. The use of this technique has also been applied to the construction of antennas. By slotting the ground plane at an angle to the strip conductor, radiation in the direction at right angles to the ground plane may be obtained. Various patterns can be built up by increasing the number of slots and varying the length of the strip feed between slots. The ground-plane slot has also been used to couple external cavities and in this manner high-Q configurations can be constructed.

3.6 Microwave Receiver

As an illustration of the general application of the techniques described, the front end of a microwave receiver designed for the 4400-to-5000-megacycle region is shown in Figure 11. The microwave portion of the receiver consists of the following:

A. Coaxial-to-strip-line transition and matching element, which permits its connection to a coaxial antenna feed.

B. Balanced rat-race mixer for low noise figure. Included in the mixer are two balanced crystal holders and matching elements.

C. Coaxial transition for the local oscillator and matching elements.

It should be noted that the ground plane is used as the chassis for the conventional low-frequency stages. The low-noise input stages and the first two stages of the intermediate-frequency amplifier are illustrated in the figures. The top view of the receiver shows the stubs used for matching the various transitions. In practice, an extremely accurate match for these elements can be obtained by successively cutting away portions of the terminating-line capacitance. Receivers of the type shown have yielded noise figures of better than 16 decibels and conversion losses limited only by the crystal characteristics.

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4. Conclusions

While the idea of a conductor above a ground plane is not new and has been used at the lower frequencies, its application to microwaves seems to have been neglected. The fact that simple forms such as the strip line can be utilized at microwaves should prove useful in those applications where the more-bulky and expensive-to-manufacture conventional plumbing is at a disadvantage.

There are, of course, a number of limitations to microstrip. The fact that an open structure is used leads to a somewhat greater coupling between side-by-side configurations as compared to waveguide or coaxial systems. The absolute value of this coupling is small however. Also, the fact that complete circuits can be fabricated in one piece avoids the use of transitions or flanges, which in practice is often the dominant coupling element for the closed transmission systems.

A second factor is the somewhat higher attenuation of the system as compared to waveguide structures. While this item may limit its usefulness in systems where extremely low losses are required, for many applications losses are sufficiently small. In systems such as microwave receivers, where line lengths are small, the insertion losses can be made negligible compared to other losses.

A third factor that should be mentioned is the low resonant impedance inherent in structures of this sort, which limits the magnitude of the obtainable Q. Since it is possible to couple high-Q cavities of either the waveguide or the coaxial type in a simple manner, this limitation for practical designs can be overcome by combining the desirable features of the different systems.

On the basis of the preceding, it is concluded that the techniques described should prove useful for application to microwave systems particularly where a practical compromise must be made between the extremes of maximum electrical performance and optimum physical realizability.

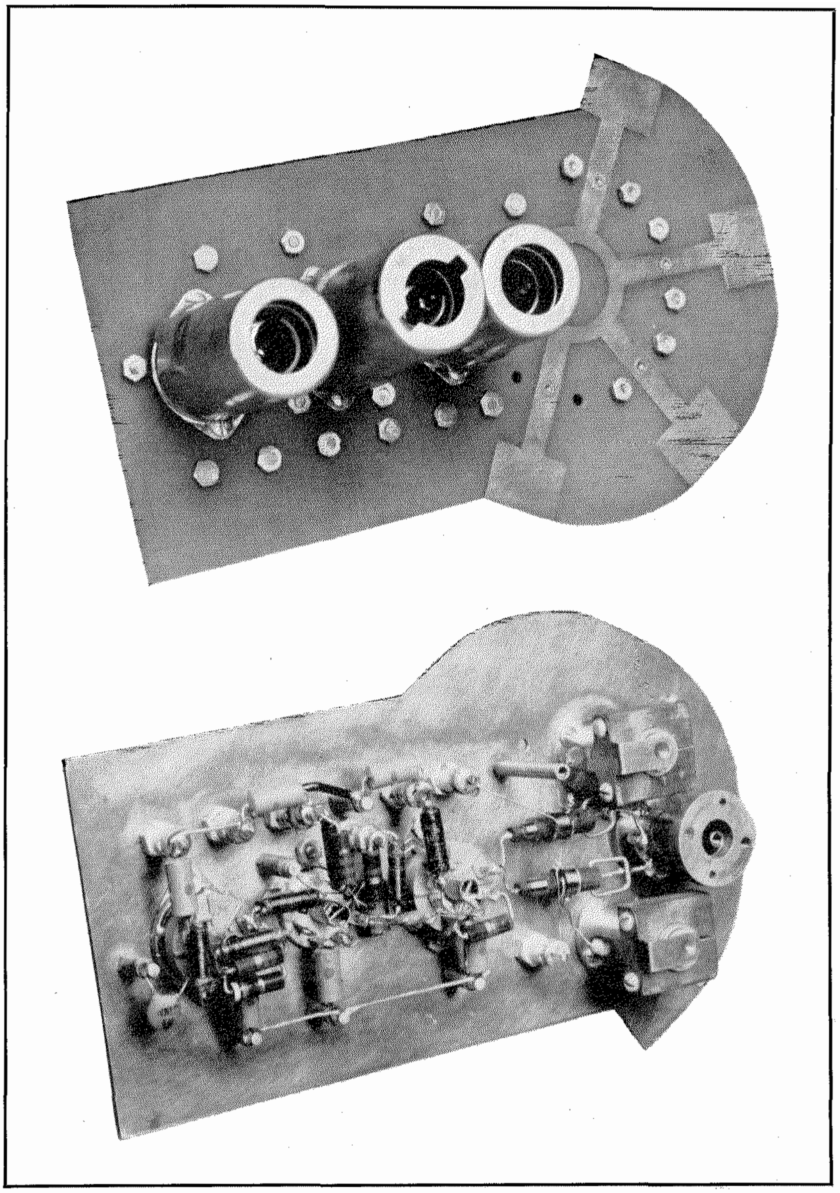

Figure 11—Front end of microwave receiver showing top and bottom views. The unit is 6 inches (15 centimeters) long.

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Simplified Theory of Microstrip Transmission Systems*

By FRED ASSADOURIAN and EMANUEL RIMAI

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

PROPERTIES of *TEM*-mode propagation are examined for a wire or a strip of finite width immersed in a uniform infinite dielectric above a ground plane. Characteristic impedance, power flow, and conductor and dielectric losses are considered. The discussion of the *TEM* mode is in the complex plane.

• • •

This paper is concerned with properties of electromagnetic propagation in transmission systems consisting of an infinitely long circular wire or strip above an infinite ground plane and having uniform cross sections. If the surrounding dielectric (air or solid) is uniform and infinite and the system is lossless, then the transverse electromagnetic (*TEM*) mode can be propagated. However, practical strip lines involve a solid dielectric on which a strip is placed and an air region above the dielectric. Since these strip lines involve composite dielectrics, they cannot support a pure *TEM* mode. Nevertheless, both theory and experiment indicate that the fields and power flow are concentrated in the dielectric between the conductors, so that the assumption of a single infinite dielectric leads to useful results even though it is not rigorous.

It is assumed henceforth that the conductors are immersed in an infinite uniform dielectric. The further assumption of perfect conductors and lossless dielectrics permits computations on the basis of a *TEM* mode. Either a rigorous or a first-order analysis is followed to derive equations for characteristic impedance and concentration of power flow through the cross sections. Transmission losses are calculated wherever possible from the field solutions for the lossless cases.

The analysis of the *TEM* mode is reduced to an investigation of its corresponding electrostatic distribution which is made, not in the original transverse cross-sectional plane of a given configuration, but in a new plane obtained by a conformal transformation. Calculations of desired quantities are carried out more readily in the image plane. Most of the discussion about characteristic impedances has appeared previously in the literature in terms of related capacitances. Much of the discussion about the distribution of power flow and transmission losses may not be well known. Practical meter-kilogram-second units are employed in all equations unless otherwise indicated.

1. Electrostatic Approach to TEM Mode

The electrostatic approach to the analysis of the *TEM* mode is well known.[1] If the electric field is written as $E(x, y)$ multiplied by factors involving time and the direction of propagation ξ, then $E(x, y)$ is an electrostatic field.

One may write $E=-\nabla\phi$, where the scalar potential ϕ satisfies $\nabla^2\phi=0$ and takes specific values, for example ϕ_1 and ϕ_2, at the conductor boundaries.

If q is the charge per unit length on either conductor, then the capacitance C per unit length and the characteristic impedance Z_0 are given by

$$C=q/(\phi_2-\phi_1) \tag{1}$$

and

$$Z_0=(\mu\epsilon)^{1/2}/C. \tag{2}$$

The average power P flowing through an entire cross section is found from

$$\begin{aligned} P&=\frac{1}{2}\left(\frac{\epsilon}{\mu}\right)^{1/2}\int|E|^2\,dS \\ &=(\phi_2-\phi_1)^2/2Z_0, \end{aligned} \tag{3}$$

where dS is an element of area and the limits are the conductor boundaries.

First-order estimates of conductor and dielectric losses are found in the usual way[1] from the previously found field for the lossless case.

* Reprinted from *Proceedings of the IRE*, volume 40, pages 1651–1657; December, 1952. Presented at the National Convention of the Institute of Radio Engineers in New York, New York, on March 5, 1952.

[1] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, New York; 1944.

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One may write

$$\alpha=\frac{P_{c1}+P_{c2}+P_d}{2P}$$
$$=\alpha_{c1}+\alpha_{c2}+\alpha_d=\alpha_c+\alpha_d, \qquad (4)$$

where P_{c1}, P_{c2}, and P_d are power losses per unit length along ξ in the two conductors and the dielectric. The average power dissipated in a conductor per unit distance along ξ is

$$\left.\begin{aligned} P_c&=\frac{\eta}{2}\frac{\epsilon}{\mu}\int|E|^2\,ds,\\ \eta&=(\pi f\mu/\sigma_c)^{1/2}, \end{aligned}\right\}(5)$$

where ds is an element of length along the conductor boundary, f is frequency, and σ_c is conductivity.

The power loss P_d in the dielectric per unit length along ξ is given by

$$P_d=\frac{1}{2}\sigma_d\int|E|^2\,dS, \qquad (6)$$

evaluated over the dielectric portion of a cross section. Since P_d and P involve the same region of integration, one has immediately

$$\alpha_d=\frac{\sigma_d}{2}\left(\frac{\mu}{\epsilon}\right)^{1/2}, \qquad (7)$$

which is independent of the geometry of the conductors. The conductivity σ_d is obtained from the complex dielectric constant

$$\left.\begin{aligned} \epsilon'&=\epsilon-j\epsilon_1=\epsilon(1-j\sigma_d/\omega\epsilon),\\ \sigma_d&=\omega(\epsilon_1/\epsilon)\epsilon, \end{aligned}\right\}(8)$$

where the loss ratio ϵ_1/ϵ is assumed to be small.

2. *Electrostatic Calculations in the Complex Plane*

The quantities listed in the previous section require knowledge of E and ϕ. The method for their calculation followed in this paper uses functions of a complex variable.[2]

A brief summary of the complex-variable method is useful for an understanding of subsequent calculations. If the cross section of a transmission line is drawn in the complex $z=x+jy$ plane, the complex potential $F(z)$ is an analytical function given by

$$F(z)=\phi(x,y)+j\psi(x,y), \qquad (9)$$

where ϕ and ψ satisfy the Cauchy-Riemann and Laplace equations. Curves of constant ϕ are equipotential lines, and the orthogonal curves of constant ψ are flux lines.

The electric intensity vector E can be represented in complex form by

$$E(z)=-(\overline{dF/dz}), \qquad (10)$$

where the bar indicates that the complex conjugate should be taken. The problem of finding $F(z)$ is frequently simplified by using a conformal mapping $z=f(w)$ to map the z plane into an image w plane where $F(w)$ is more readily set up.

If $E(w)$ is the electric field in the w plane, then

$$E(z)=-(\overline{dF/dw})(\overline{dw/dz})$$
$$=E(w)\overline{dw/dz}. \qquad (11)$$

For small elements $|dz|$ and $|dw|$, one has approximately

$$|E(z)|\,|dz|=|E(w)|\,|dw|. \qquad (12)$$

This is the fundamental relation that permits the calculation of all desired quantities in the w plane.

Although the integration of (12) around a conductor boundary and multiplication by ϵ will yield the total charge per unit length along ξ, this charge q is found more quickly from

$$F(w)=\phi(u,v)+j\psi(u,v) \qquad (13)$$

by taking ϵ times the change in ψ over a circuit of either conductor boundary. It is evident from (12) that both q and hence Z_0 are invariant under a conformal mapping if corresponding conductors in the two planes have the same potentials.

Let dz_1 and dz_2 be two orthogonal elements at a point z with orthogonal images dw_1 and dw_2 at the image point w. Then $|dz_1|\,|dz_2|$ and $|dw_1|\,|dw_2|$ are areas of elementary rectangles. If now (12) is written twice for dz_1 and dz_2 and the results are multiplied, one obtains

$$|E(z)|^2\;dS_z=|E(w)|^2\;dS_w. \qquad (14)$$

[2] E. Weber, "Electromagnetic Fields—Theory and Applications," John Wiley and Sons, Inc., New York, New York, Volume I; 1950.

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Reference to (3) shows that $P(z)=P(w)$, so that the total power flow is also invariant under a conformal mapping. Furthermore, the portion of P bounded by the conductors and any pair of flux lines remains the same if their images are considered.

One may write

$$|E(z)|^2\ |dz| = |E(w)|^2\ |dw|\ |dw/dz|. \quad (15)$$

Integration of the left-hand side along conductor boundaries or of the right-hand side along the boundary images and substitution in (5) gives the conductor losses. It should be noted that this calculation gives losses for the original conductors but not for their images.

In summary, it is evident that q, C, Z_0, P, and suitably defined portions of P are invariant, but α_c is not invariant, under conformal mappings. All of these quantities may be calculated in the image plane.

$$E(w) = -\frac{\phi_2-\phi_1}{\ln R}\frac{1}{\bar{w}} \quad (20)$$

and

$$\left.\begin{aligned} q &= 2\pi\epsilon\frac{\phi_2-\phi_1}{\ln R}, \\ C &= \frac{2\pi\epsilon}{\cosh^{-1}(h/b)}, \\ Z_0 &= \frac{1}{2\pi}\left(\frac{\mu}{\epsilon}\right)^{1/2}\cosh^{-1}\frac{h}{b}. \end{aligned}\right\} (21)$$

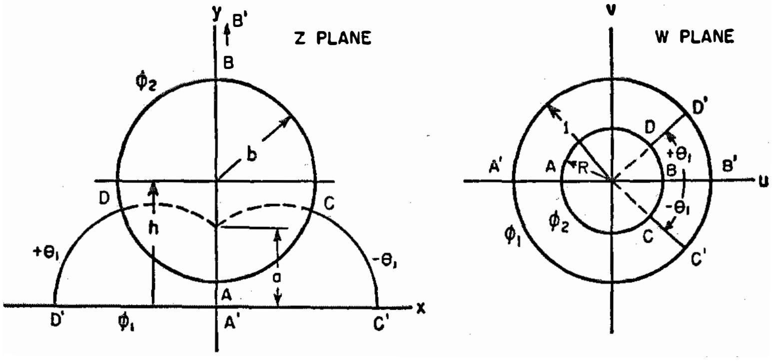

Figure 1—Conformal mapping for a wire above an infinite ground plane.

3. *Circular Wire Above Infinite Ground*

The cross section for a transmission system of a circular wire above an infinitely wide ground conductor is shown in Figure 1. The quantity a is related to height h and radius b by

$$h^2 = a^2 + b^2. \quad (16)$$

The bilinear transformation

$$z = ja(1+w)/(1-w) \quad (17)$$

converts wire and ground plane into a coaxial pair of concentric circles as shown in Figure 1. The complex potential in the image plane is

$$\left.\begin{aligned} F(w) &= (\phi_2-\phi_1)\ln w/\ln R + \phi_1 \\ &= \phi(u,v) + j\psi(u,v), \\ R &= b/(h+a) \end{aligned}\right\} (18)$$

and leads to the well-known results.

$$\left.\begin{aligned} \phi &= \frac{\phi_2-\phi_1}{\ln R}\ln r + \phi_1, \\ \psi &= \frac{\phi_2-\phi_1}{\ln R}\theta,\ w = re^{j\theta}. \end{aligned}\right\} (19)$$

The radial flux lines are images of

$$(x + a\cot\theta)^2 + y^2 = a^2\operatorname{cosec}^2\theta. \quad (22)$$

Some aspects of the following discussion may not be as well known. If k is the fraction of power P flowing through the sectoral region bounded by the conductors and the flux lines $\theta = \pm\theta_1$, then

$$k = (\pi - \theta_1)/\pi. \quad (23)$$

It should be noted again that k is the same in either plane. The choice of several values for θ_1 in (23) yields a picture of power-flow distribution in the *TEM* mode.

The application of (4), (5), and (15) to each conductor in the w plane yields

$$\left.\begin{aligned} \alpha_c &= \frac{1}{2a}\left(\frac{\pi f\epsilon}{\sigma_c}\right)^{1/2}\frac{1+h/b}{\cosh^{-1}(h/b)}, \\ \left(\frac{\pi f\epsilon}{\sigma_c}\right)^{1/2} &= \eta\left(\frac{\epsilon}{\mu}\right)^{1/2}. \end{aligned}\right\} (24)$$

A plot of the variable part of α_c (a fixed) is given in Figure 2. The minimum value of α_c is

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given by

$$\left.\begin{aligned} h/b &= 2.447, \\ (\alpha_c)_{\min} &= \frac{1.117}{a}\left(\frac{\pi f \epsilon}{\sigma_c}\right)^{1/2}, \end{aligned}\right\} \quad (25)$$

in nepers per meter.

For a numerical illustration, consider a wire of radius 0.0625 inch and height 0.0850 inch (1.59 by 2.16 millimeters). Copper conductors are assumed. For a frequency of 5000 megacycles and a normalized dielectric constant of 2.54 with a loss ratio of 4×10^{-4} (polystyrene), one obtains

$a = 0.0576$ inch (1.463 millimeters),

$(\pi f \epsilon/\sigma_c)^{1/2} = 7.80\times10^{-5}$,

$\alpha_c = 0.20$ decibel per foot (0.66 decibel per meter),

$\alpha_d = 0.09$ decibel per foot (0.30 decibel per meter)

$\alpha = \alpha_c + \alpha_d = 0.29$ decibel per foot (0.95 decibel per meter),

$Z_0 = 31$ ohms.

By choosing $\theta_1 = \pm0.25\pi$, $\pm0.10\pi$, and $\pm0.05\pi$ in (23) and using (21), one finds the bounding flux lines for 75, 90, and 95 percent of the power flow in the *TEM* mode. These regions are indicated in Figure 3.

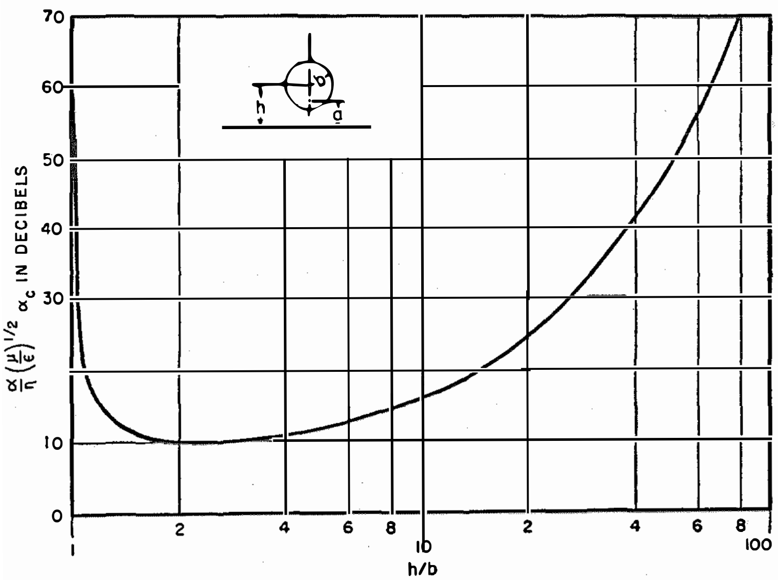

Figure 2—Conductor attenuation for a wire above an infinite ground plane. $\alpha_c = \frac{4.3429}{a}\eta\left(\frac{\epsilon}{\mu}\right)^{1/2}\frac{1+h/b}{\cosh^{-1}(h/b)}$ in decibels per unit length. $\eta = (\pi\mu f/\sigma_c)^{1/2}$.

4. *Wide Strip of Zero Thickness Above Infinite Ground*

This section considers a strip of zero thickness, width b, and height h above an infinitely wide plane conductor, as shown in Figure 4, with $b \gg h$. For a semi-infinite strip parallel to an infinitely wide ground conductor, it is known[2] that the electric field is infinite at the strip edges and, as one proceeds along the underside of the strip, approaches the homogeneous field that would result from two infinite conductors with the same spacing. The actual field is within 1 percent of the homogeneous field beyond a distance approximately equal to the spacing.

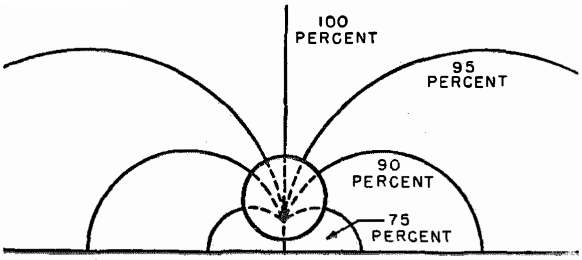

Figure 3—Distribution of power flow for a wire above an infinite ground plane.

The preceding suggests that the effect of the strip edges should become negligible as the center of the underside of the strip is approached in Figure 4. It appears reasonable therefore to calculate the electrostatic distribution for the right half of the configuration by assuming that the strip extends to infinity at the left. The field to the left of the line of symmetry through A then follows by symmetry.

The conformal mapping given by

$$\pi z/h = 1 + w + \ln w \quad (26)$$

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transforms the revised upper z plane into the upper w plane. The images of the semi-infinite strip and infinite ground are the two halves of the u axis. The regions of interest are those bounded by $BCAA'C'B'B$ in both planes. The

For large b/h, it is seen that

$$\left.\begin{aligned} \ln r_A &\approx -1-\pi b/2h, \\ r_B &\approx \ln\,(r_B/r_A) \\ &\approx 1+\pi b/2h+\ln\,(1+\pi b/2h). \end{aligned}\right\} \quad (31)$$

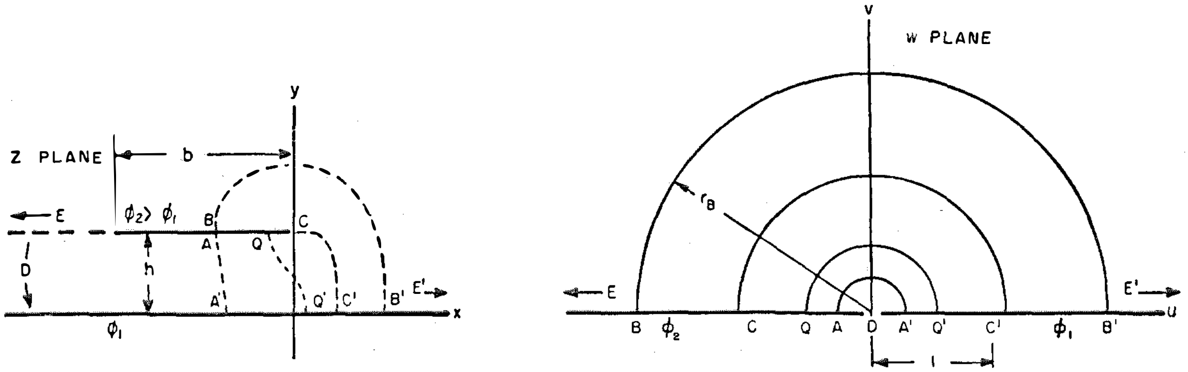

Figure 4—Conformal mapping for a wide strip of zero thickness above an infinite ground plane.

fact that the flux line leaving B in the z plane should be vertical but is not, reveals that the mapping is only approximate.

The worst inaccuracy is in the region of lowest field intensity.

The complex potential in the image plane is

$$\begin{aligned} F(w) &= -j\frac{\phi_2-\phi_1}{\pi}\ln w+\phi_1 \\ &= -j\frac{hE_0}{\pi}\ln w+\phi_1 \\ &= \phi+j\psi, \end{aligned} \quad (27)$$

where E_0 is the constant field that would result if the strip were infinitely wide. By using $w=r\exp(j\theta)$ in (26), one sees that the equipotential and flux lines are given by

$$\left.\begin{aligned} \pi x/h &= 1+r\cos\theta+\ln r, \\ \pi y/h &= \theta+r\sin\theta. \end{aligned}\right\} \quad (28)$$

From (27) one obtains for the electric fields

$$\left.\begin{aligned} E(w) &= -jhE_0/\pi\bar{w}, \\ E(z) &= -jE_0/\overline{w+1}. \end{aligned}\right\} \quad (29)$$

If r_A and r_B are the radii of flux lines in the w plane through A and B, they are found from (28) to satisfy

$$\left.\begin{aligned} -\pi b/2h &= 1-r+\ln r, \\ \ln\,(r_B/r_A) &= r_B-r_A. \end{aligned}\right\} \quad (30)$$

The charge q_{AB} between A and B is

$$q_{AB}=\epsilon(hE/\pi)\ln r_B/r_A.$$

The total charge on the actual strip is $q=2q_{AB}$. The characteristic impedance is

$$\left.\begin{aligned} Z_0 &= \frac{\pi}{2}\left(\frac{\mu}{\epsilon}\right)^{1/2}\frac{1}{\ln\,(r_B/r_A)} \\ &\approx Z_0'\Big/\left\{1+\frac{2h}{\pi b}\left[\ln\left(1+\frac{\pi b}{2h}\right)+1\right]\right\}, \\ Z_0' &= \frac{h}{b}\left(\frac{\mu}{\epsilon}\right)^{1/2}, \end{aligned}\right\} \quad (32)$$

where Z_0' corresponds to a constant field in the absence of fringing and leakage flux. This result has been derived by Maxwell[3] in terms of capacitance.

A rigorous mapping for the entire strip[4] has been previously used to find the accurate electrostatic distribution and, for $b\gg h$, gives rise to the result

$$Z_0\approx Z_0'/\{1+(2h/\pi b)[1+\ln\,(\pi b/h)]\}. \quad (33)$$

For large b/h, (32) and (33) approach each other.

[3] J. C. Maxwell, "Electricity and Magnetism," Third Edition; 1904: page 309.

[4] H. B. Palmer, "Capacitance of a Parallel Plate Capacitor by the Schwartz-Christoffel Transformation," *Transactions of the American Institute of Electrical Engineers*, volume 56, page 363; March, 1937.

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A plot of both expressions is given in Figure 5.

If P represents the power flow bounded by pairs of flux lines starting at A and B, and if P_Q is the power bounded by a pair of flux lines through A and any other symmetrical pair of which one starts at the point Q, then P_Q/P is given by

$$\frac{P_Q}{P}=\frac{\ln(r_Q/r_A)}{\ln(r_B/r_A)}, \tag{34}$$

where $r_Q<r_B$ corresponds to the second pair of flux lines.

For example, $r_c=1$ for flux lines leaving the strip edges and P_c/P is then given by

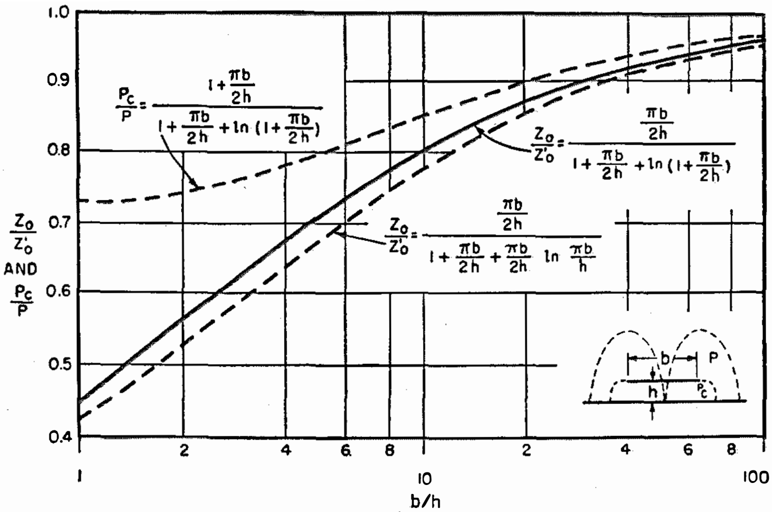

Figure 5—Characteristic impedance and power flow for a wide strip of zero thickness above an infinite ground plane. $Z_0'=(b/h)(\mu/\epsilon)^{1/2}$.

$$\frac{P_c}{P}=\frac{-\ln r_A}{\ln r_B-\ln r_A}$$
$$\approx\frac{1+\pi b/2h}{1+\pi b/2h+\ln(1+\pi b/2h)}. \tag{35}$$

A plot of P_c/P versus b/h for $r_Q=1$ is given in Figure 5. It is apparent from Figure 5 that, for large b/h, most of the power flow is confined between the lower surface of the strip and the ground conductor and that fringing and leakage fields become negligible.

5. *Wide Strip of Small Thickness Above Infinite Ground*

The procedure of the previous section is now altered to take into account a small strip thickness d, where $b\gg h\gg d$. The mapping (26) is replaced by

$$\frac{\pi z}{h}=\frac{p+1}{p^{1/2}}\tanh^{-1}R+\frac{p-1}{p^{1/2}}\frac{R}{1-R^2}+\ln\frac{Rp^{1/2}-1}{Rp^{1/2}+1}, \tag{36}$$

where R and p are defined by

$$\left.\begin{aligned}R&=\left(\frac{w+1}{w+p}\right)^{1/2},\\ p&=-1+2\beta^2+2\beta(\beta^2-1)^{1/2},\\ \beta&=1+d/h.\end{aligned}\right\}\tag{37}$$

The geometry of the mapping is indicated in Figure 6. The electrostatic problem is again

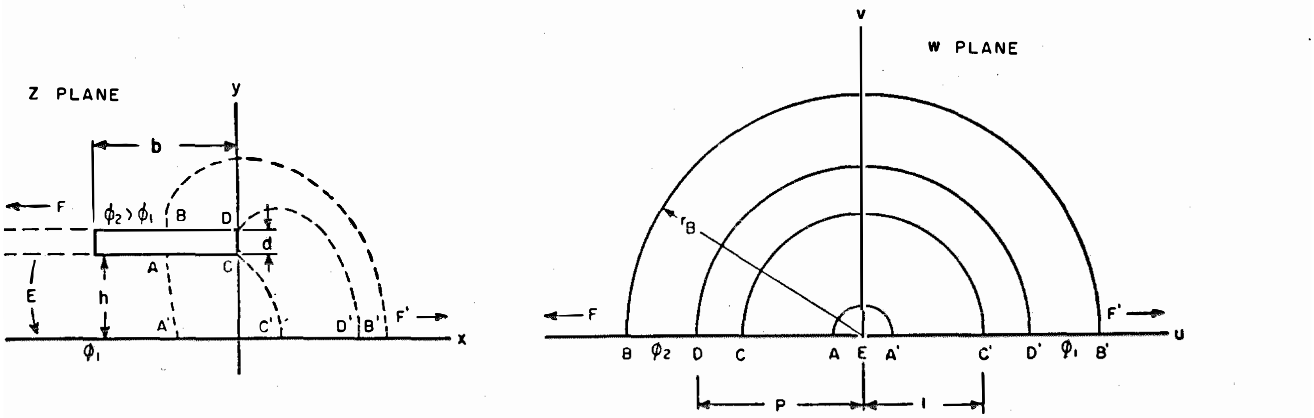

Figure 6—Conformal mapping for a wide strip of small thickness above an infinite ground plane.

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solved for the right half of a cross section on the assumption that the left half is infinite.

Since the w plane is the same as in the previous section, the expressions for $F(w)$, $E(w)$, Z_0, and P_Q/P remain unaltered. For $E(z)$, one now has

$$E(z) = -jE_0p^{1/2}[(w+p)(w+1)]^{1/2}. \qquad (38)$$

For $h \gg d$, one may write $p = 1+\delta$ in (36) and obtain

$$\left.\begin{aligned} \pi z/h &\approx 1+w+\ln w-(w+1)\delta/2, \\ \delta &= p-1. \end{aligned}\right\} \qquad (39)$$

The equations for the equipotential and flux lines are

$$\left.\begin{aligned} \pi x/h &= (1-\delta/2)r\cos\theta+1+\ln r-\delta/2, \\ \pi y/h &= \theta+r(1-\delta/2)\sin\theta, \end{aligned}\right\} \qquad (40)$$

which indicate that the electric field for the zero-thickness case is pulled into the region between the conductors.

The quantities r_A and r_B now satisfy the common equation

$$-\pi b/2h \approx (1-\delta/2)(1-r)+\ln r. \qquad (41)$$

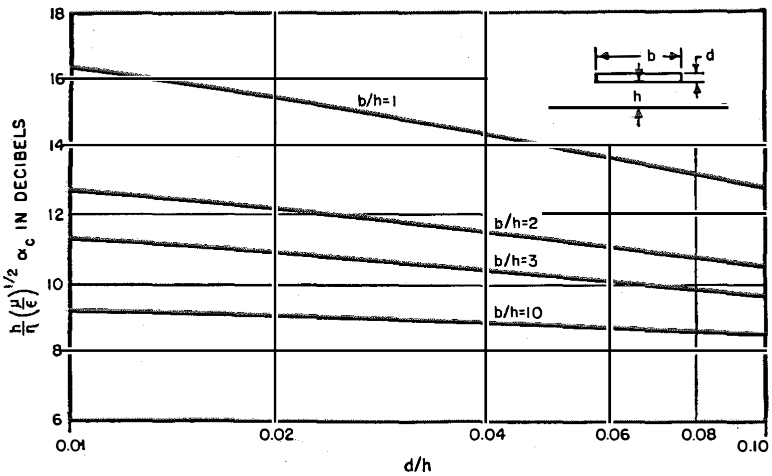

Figure 7—Conductor attenuation for a wide strip of small thickness above an infinite ground plane.

$$\alpha_c = 8.6859\frac{\eta}{h}\left(\frac{\epsilon}{\mu}\right)^{1/2}\frac{1+\dfrac{\pi b}{2h}+\dfrac{\pi}{2}+\ln\dfrac{4}{\delta}}{1+\dfrac{\pi b}{2h}+\ln\left(1+\dfrac{\pi b}{2h}\right)} \text{ in decibels per unit length.}$$

$$\eta \equiv \left(\frac{\pi\mu f}{\sigma_0}\right)^{1/2}, \quad \delta = p-1, \quad p = -1+2\beta^2+2\beta(\beta^2-1)^{1/2}, \quad \beta = 1+d/h.$$

If r' is a root of (30) and r is a root of (41), then

$$r \approx r'(1+\delta/2). \qquad (42)$$

Relations (32) and (34) indicate that Z_0 does not change while P_Q/P changes slightly to a first order.

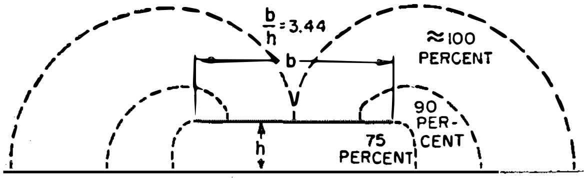

Figure 8—Distribution of power flow for a wide strip of zero thickness above an infinite ground plane.

A calculation of conductor losses using (5) and (15) yields

$$\left.\begin{aligned} \alpha_{c1} &\approx \frac{\eta}{2h}\left(\frac{\epsilon}{\mu}\right)^{1/2}\frac{1+\pi b/2h}{1+\pi b/2h+\ln(1+\pi b/2h)}, \\ \alpha_{c2} &\approx \frac{\eta}{2h}\left(\frac{\epsilon}{\mu}\right)^{1/2}\frac{\pi+1+\pi b/2h-2\ln\delta/4}{1+\pi b/2h+\ln(1+\pi b/2h)}. \end{aligned}\right\} \qquad (43)$$

Figure 7 shows plots of $\alpha_c = \alpha_{c1}+\alpha_{c2}$ against d/h for various values of b/h. For copper conductors, polystyrene dielectric, and a frequency of 5000 megacycles, the value of $\eta(\epsilon/\mu)^{1/2}$ is 7.80×10^{-5}. As a numerical illustration, consider a laboratory setup consisting of a strip of width 0.110 inch (2.79 millimeters), height 0.032 inch (0.813 millimeter), and thickness 0.001 inch (0.025 millimeter), copper conductors, polystyrene dielectric, and a frequency of 5000 megacycles. Calculations yield

$r_A \approx 2.30\times10^{-3}$,
$r_B \approx 10.92$,
$Z_0 \approx 45.0$ ohms.
$\alpha_{c1} \approx 0.10$ decibel per foot (0.33 decibel per meter),
$\alpha_{c2} \approx 0.20$ decibel per foot (0.66 decibel per meter),
$\alpha_d = 0.09$ decibel per foot (0.30 decibel per meter).
$\alpha = \alpha_c+\alpha_d = 0.39$ decibel per foot (1.28 decibels per meter),

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The use of (34) for $P_Q/P=0.75$ and 0.90 gives rise to the regions shown in Figure 8. These regions indicate the approximate distribution of power flow in the *TEM* mode for the present case. As mentioned previously, the 100-percent region should rigorously be infinite, but is not because the mapping used to derive Figure 8 is only approximate.

$$\left.\begin{aligned} -\pi\tau x/h &= (r^2/2)\cos 2\theta+(1-\tau)r\cos\theta \\ &\qquad -\tau\ln r+1/2-\tau \\ -\pi\tau y/h &= (r^2/2)\sin 2\theta+(1-\tau)r\sin\theta-\tau\theta. \end{aligned}\right\} \quad (47)$$

Both r_A and r_B satisfy

$$\pi\tau b/2h=1/2-\tau+r^2/2+(\tau-1)r-\tau\ln r, \quad (48)$$

where τ is first found from (45).

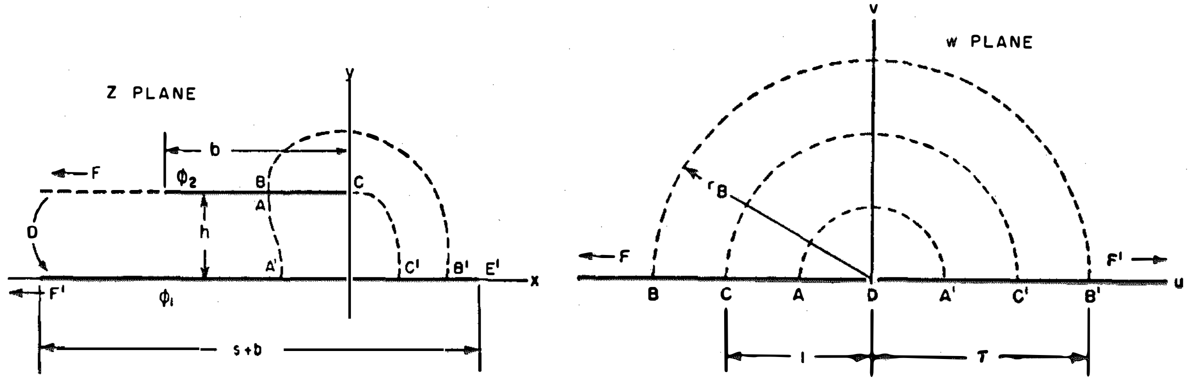

Figure 9—Conformal mapping for a wide strip of zero thickness above a finite ground plane.

6. *Wide Strip Above Finite Ground, Zero Thickness*

The present section differs from Section 5 in that the ground conductor is now assumed to have a finite width $s+d$. The conformal mapping to be used is indicated in Figure 9 and is given by

$$\pi\tau z/h=-w^2/2-(1-\tau)w +\tau\ln w-1/2+\tau, \quad (44)$$

where τ satisfies

$$\pi s/2h=\ln\tau+\tau/2-1/2\tau, \quad \tau\geqslant 1. \quad (45)$$

The region of the w plane that is used for final calculations corresponds to $r_A\leqslant r\leqslant r_B$.

The expressions for $F(w)$, $E(w)$, Z_0, and P_Q/P are the same as in the two preceding sections. For $E(z)$ one now has

$$E(z)=jE_0\tau/\overline{(w+1)(w-\tau)}. \quad (46)$$

The equipotential and flux lines are given by

The magnitude of $E(z)$ along the conductors ($\epsilon=0$ or π) is

$$|E(z)|=E_0\tau/|\pm r-1||\pm r-\tau|. \quad (49)$$

The effect of finite s on Z_0/Z_0' and P_c/P, where the latter is determined from flux lines starting at A, B, and the strip edges, is shown

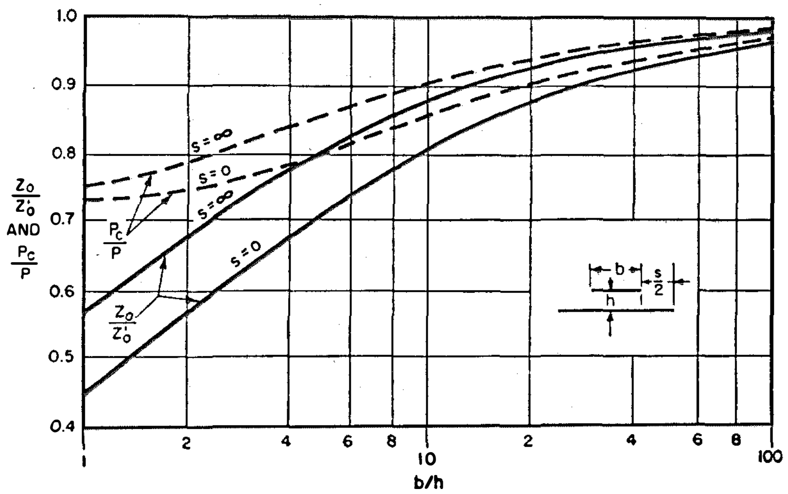

Figure 10—Characteristic impedance and power flow for a wide strip of zero thickness above a finite ground plane. Z/Z_0' is the solid line, P_c/P is the broken line, and $Z_0'=(b/h)(\mu/\epsilon)^{1/2}$.

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graphically in Figure 10. Curves are given for $s=0$ and $s=\infty$. Corresponding curves for nonzero finite s are not shown but fall between those for $s=0$ and ∞. Figure 10 indicates that there is not much change in Z_0/Z_0' and P_c/P as s is varied from 0 to ∞. This change is negligible for s greater than $2b$.

By writing $w=\rho \exp (j\theta)$, one sees that the equipotential and flux lines are given by

$$\left.\begin{aligned} x &= -A\rho(\sin\theta)(A_1+B_1) \\ y &= \frac{-B^2-(A+H)^2\rho^2+2H\rho(A+H)\cos\theta}{2(A+H)} \\ &\qquad \times(A_1-B_1), \end{aligned}\right\} \quad (53)$$

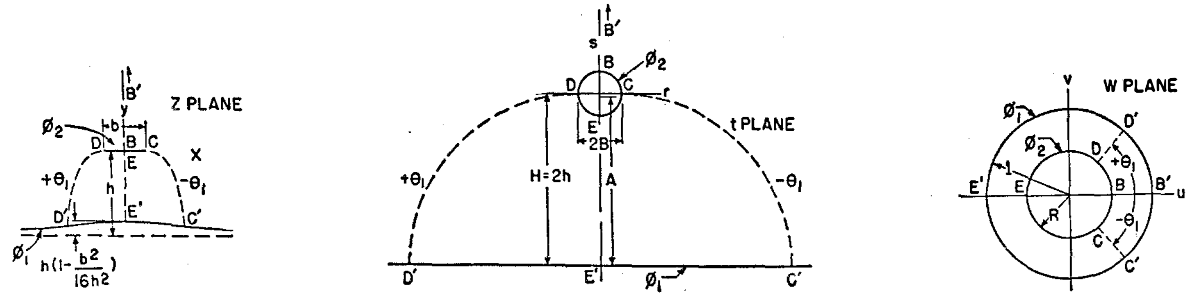

Figure 11—Conformal mapping for a narrow strip above an infinite ground.

7. *Narrow Strip of Zero Thickness Above Infinite Ground*

It is now assumed that $b \ll h$. The present case is treated by applying limiting considerations to Figure 11. The two mappings that are used are given by

$$\left.\begin{aligned} z &= t/2+B^2/2t, \\ t/j &= A(1+w)/(1-w)-H, \end{aligned}\right\} \quad (50)$$

where $B=b/2$, $H=2h$, and $A^2=H^2-B^2$.

The $t\rightarrow z$ mapping transforms the configuration for wire above ground into that for a strip of zero thickness above a curved ground plane. The $t\rightarrow w$ mapping converts the first configuration into a coaxial system. One sees from (50) that the minimum and maximum vertical distances between strip and ground are

$$h(1-b^2/16h^2) \text{ and } h.$$

If now b/h is made small, the ground conductor becomes straight.

The point $t=j(A-H)$ goes into $w=0$ and the circle $|t|=B$ goes into the circle $|w|=R$ with

$$R=(H-A)/B=B/(H+A). \quad (51)$$

The points C or D in the z or t plane go into corresponding points in the w plane with

$$\left.\begin{aligned} w_D &= R\exp(j\theta_1), \\ w_C &= R\exp(-j\theta_1), \\ \tan\theta_1 &= A/B. \end{aligned}\right\} \quad (52)$$

where A_1 and B_1 are defined by

$$\left.\begin{aligned} A_1 &= \frac{1}{1+\rho^2-2\rho\cos\theta}, \\ B_1 &= \frac{B^2(A+H)^2}{B^4+\rho^2(A+H)^4-2B^2\rho(A+H)^2\cos\theta}. \end{aligned}\right\} \quad (54)$$

The flux line from point D leaves horizontally and meets the ground conductor at the point D' for which

$$\left.\begin{aligned} x_{D'} &= -\frac{(H+B)(H^2+HB+B^2)}{2H^2+2BH+B^2}, \\ y_{D'} &= -\frac{H^2(H+B)}{2H^2+2BH+B^2}. \end{aligned}\right\} \quad (55)$$

For narrow strips, $H \gg B$ and (55) reduces to

$$x_{D'} \approx -H/2, \qquad y_{D'} \approx -H/2. \quad (56)$$

Since the characteristic impedance Z_0 is invariant under conformal mappings, one has

$$\begin{aligned} Z_0 &= \frac{1}{2\pi}\left(\frac{\mu}{\epsilon}\right)^{1/2}\cosh^{-1}\frac{H}{B} \\ &\approx \frac{b}{2\pi h} Z_0' \ln\frac{8h}{b}, \end{aligned} \quad (57)$$

where Z_0' corresponds to a uniform field between parallel conductors of width b and separation h. This result has been published[5] in terms of capacitance.

[5] F. E. Terman, "Radio Engineers' Handbook," McGraw-Hill Book Company, New York, New York; 1943: page 113.

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The terminations of flux lines on the ground conductor are given by

$$x \approx -H \cot \theta/2, \qquad y \approx -H/2. \tag{58}$$

The fraction of power flow bounded by flux lines corresponding to $\pm\theta_1$ is given by $k = (\pi - \theta_1)/\pi$ as in Section 4.

Flux lines leaving the strip edges have ground terminations at $x \approx -H/2$ and, since $\theta \approx -\pi/2$ in this case, bound 50 percent of the power flow. For $k = 0.75$, θ is $\pm\pi/4$ and the bounding flux lines have ground terminations at $x \approx \pm 4.8\ h$. Finally, for $k = 0.90$, one obtains $\theta = \pm 0.1\pi$ and $x \approx \pm 12.6\ h$.

8. Conclusions

In the case of a wire above an infinitely wide ground conductor, all results are exact except those for conductor and dielectric losses.

Calculations for a wide strip of zero thickness above an infinitely wide plane conductor reveal that the characteristic impedance and distribution of power flow differ to a first order from those that a uniform field would yield. The introduction of a small strip thickness or a finite ground width (conductor widths large compared to their spacing) alters these results slightly. If the ground width is greater than three times the strip width, it may be considered to be infinite. The introduction of a small strip thickness permits the calculation of both conductor losses. The case of a narrow strip of zero thickness above an infinitely wide plane conductor is also considered.

The above results are not rigorous but remain useful for transmission systems that are imbedded in a solid dielectric of finite height. Numerical calculations indicate that many of the transmission systems discussed in this paper do not have excessive losses, have reasonable characteristic impedances, and have most of their power flow confined to the region between the conductors. They appear to be feasible at microwave frequencies.

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Microstrip Components*

By J. A. KOSTRIZA

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

BROAD-BAND microwave components of many and varied types may be realized in microstrip construction. Complete radio-frequency systems in line-above-ground design are shown to be entirely feasible and offer significant advances in economy, size, and weight.

• • •

Heretofore radio-frequency components in the microwave region have been limited to the metallically enclosed waveguide type of transmission line. Open systems such as lecher wires, dielectric-rod guides, parallel planes, and *G* lines have been limited to just a few special applications.

Studies were initiated to develop a new type of microwave transmission line that would be expressly adaptable to development of wide-band communication-power-level components at a reasonable cost.

Microstrip successfully employs an open system consisting of two parallel conductors, one as a line and the other as a ground plane.

The preceding papers[1,2] of this series have presented a general descriptive and comparative viewpoint stressing the fundamental simplicity of design and ease of construction as well as offering a first-order theoretical analysis of the transmission properties of these lines.

This paper concludes the series with corroborative experimental data obtained prior to and during the development of microstrip components.

1. Electrical Characteristics

1.1 Geometry of the Transmission System

Three types of lines have been examined in some detail. They are:

A. Cylindrical conductor suspended above a ground plane (wire-above-ground).

B. Narrow ribbon conductor separated from the ground plane by a dielectric material (strip line).

C. Similar to *B* but with the strip immersed in the dielectric (dielectric-sandwich line).

On the basis of simple skin-depth considerations, printed-circuit techniques may be used to produce a conducting ribbon of sufficient thickness. Transverse and longitudinal cross sections of these types appear in Figure 1.

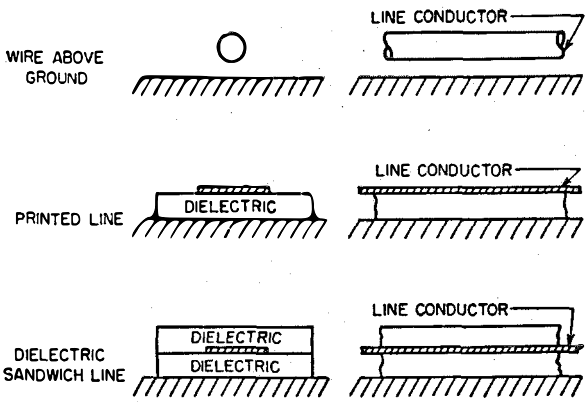

Figure 1—Open transmission lines of the microstrip type.

The ground conductor is a sheet of high-conductivity metal such as copper, silver, or aluminum. In wire lines, the line conductor is a cylinder that may be completely or partially immersed in a dielectric. If the dielectric is solid and is in contact with the ground plane and the wire, the preservation of proper spacing is assured. When air spacing is required, bead supports may be used.

In the case of strip lines, the line conductor is a thin narrow ribbon of metal either cut from

* Reprinted from *Proceedings of the IRE,* volume 40, pages 1658–1663; December, 1952. Presented at the National Convention of the Institute of Radio Engineers in New York, New York, on March 5, 1952.

[1] D. D. Grieg and H. F. Engelmann, "Microstrip—A New Transmission Technique for the Kilomegacycle Range," *Electrical Communication,* volume 30, pages 26–35; March, 1953: also, *Proceedings of the IRE,* volume 40, pages 1644–1650; December, 1952.

[2] Fred Assadourian and Emanuel Rimai, "Simplified Theory of Microstrip Transmission Systems," *Electrical Communication,* volume 30, pages 36–45; March, 1953: also, *Proceedings of the IRE,* volume 40, pages 1651–1657; December, 1952.

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sheet or deposited. Conductor thicknesses of approximately 1 to 5 mils (0.025 to 0.127 millimeter) have proved to be adequate.

The dielectric, which is used as a separating medium between the ground plane and the ribbon or strip, should have fairly flat surfaces; otherwise, the homogeneity of the dielectric may be upset by inclusion of air pockets. Slight irregularities are not too serious as regards power transfer. However, in making voltage standing-wave-ratio measurements, erratic readings are directly traceable to such a condition and it becomes rather difficult to obtain extremely good correlation in phase-wavelength measurements, for example, over a wide frequency band because, in general, the voltage minima positions change with frequency.

The dielectrics actually used have been polystyrene and laminated phenolic plastics (Dilecto *XXXP-22*). Fibrous sheet materials impregnated with a thermosetting resin (Synthane *GLF* and others) have had limited use because of excessive attenuation.

1.2 Measuring Instruments

The fundamental piece of microwave test equipment is the standing-wave detector. Two different models were made for this work, the slot type and the overhead type.

The slot type consists of an accurately ground plate with a narrow longitudinal slot to receive an exploring coaxial probe. The plate serves as the ground plane for wire and strip lines. In use, it is assumed that the slot does not appreciably alter the external radio-frequency field structure nor does it much influence the impedance or phase wavelength of the line. To a first order, this is undoubtedly true provided the ratio of slot width to line-conductor width is small.

Figure 2 shows the overhead type of detector used in conjunction with a dielectric-sandwich test line. The probe is accurately positioned in air in the vicinity of the line conductor, the line section under test being provided with its own

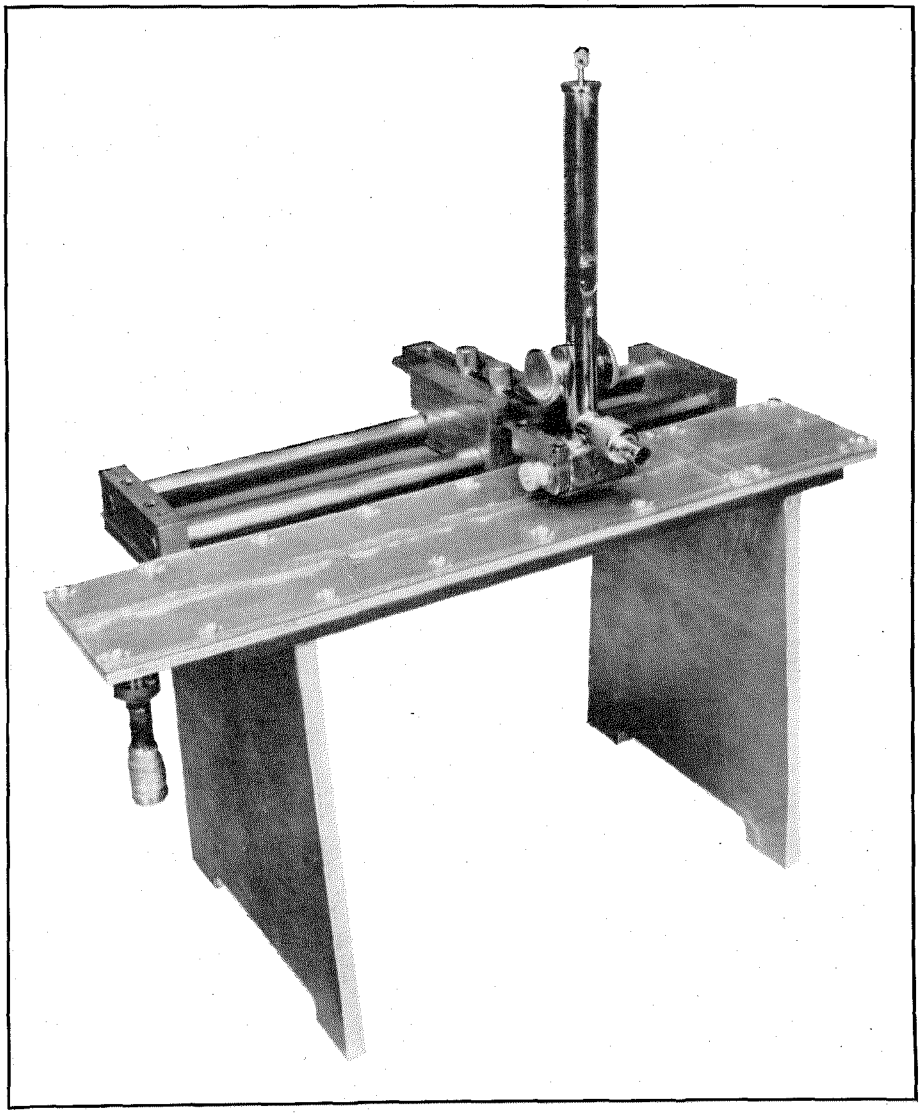

Figure 2—Overhead type of standing-wave detector.

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ground plane. An advantage of this type of instrument is that the ground plane is not mutilated in the least.

Both types of instruments were used for phase-wavelength studies and impedance measurements. Precise comparative data have not been taken. In connection with relative advantages or disadvantages of one type over the other, it should be mentioned that queer effects have been observed when the slot is not symmetrically disposed with reference to the line conductor. Air pockets in strip lines, presumably unsymmetrically disposed to the slot, may cause radiation out of the slot. In attenuator designs, it was observed that dissymmetry above a slot may cause leakage or radiation, whereas the same type of dissymmetry over a solid ground plane may not necessarily produce such effects.

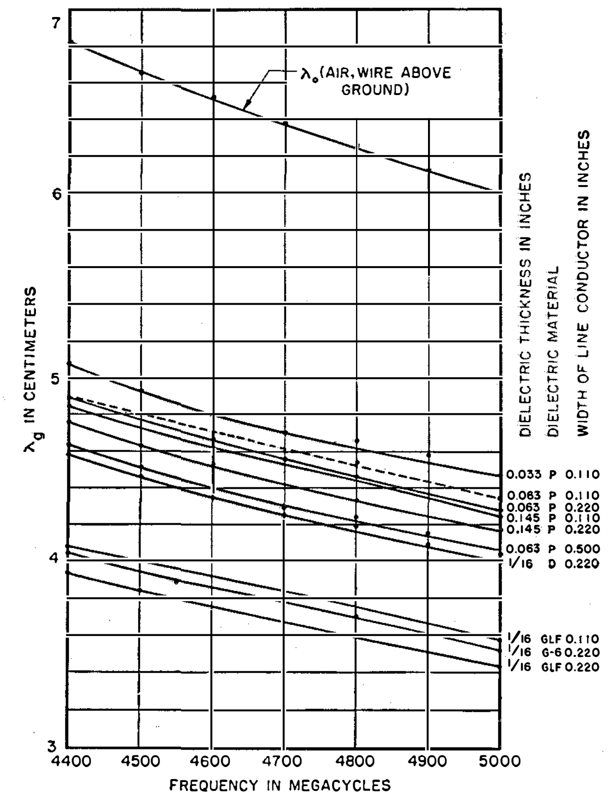

Figure 3—Wavelength of the propagation along a printed line (phase wavelength) plotted against frequency. The upper curve is for a wire above ground plane in air and corresponds to λ_0. The dielectrics are polystyrene = *P*, *XXXP-22* Dielecto = *D*, *G-6* Formica = *G6*, and Synthane = *GLF*.

1.3 Phase Wavelength

In the theory paper,[2] *TEM* propagation is assumed in which case low-frequency approximations are admissible. A basic prerequisite for such realizability is that the dielectric in which the conductor is imbedded be completely homogeneous.[3] The printed and dielectric-sandwich lines are obviously nonhomogeneous and, therefore, must depart from *TEM* behavior.

A practical complication is introduced by the fact that in a composite-dielectric case, very little is known about the value to be assigned to the "equivalent" dielectric constant. Consequently, experimental procedures were resorted to in evaluating the phase wavelength (or guide wavelength) of these lines as a function of operating frequency, line conductor width, dielectric constant, and the separation between line and ground conductors.

Figure 3 shows the variation of λ_g for some printed lines over the 4400-to-5000-megacycle band.

Comparison of the curves yields the following information:

A. For an air-dielectric wire-above-ground line, $\lambda_g = \lambda_0$.

B. For a given width of line conductor, λ_g decreases as the thickness of the dielectric increases.

C. For a given thickness of dielectric, λ_g decreases as the width of the line conductor increases.

D. For a given thickness of dielectric and line-conductor width, λ_g decreases as the dielectric constant of the material increases.

E. The ratio of λ_0/λ_g is not equal to $\epsilon^{1/2}$, where ϵ is the value of the dielectric constant of the dielectric layer (measured either at low or high frequency).

The printed line then departs from pure *TEM* behavior. But to what extent? A rough comparison may be made on the basis of percent departure of observed λ_g to the theoretical norm or ideal λ_{TEM}. In the case of an infinite-extent polystyrene dielectric, the phase wavelength λ_g for *TEM* transmission would be as shown in the *TEM* curve of Figure 4. This curve, serving as the norm for the 4400-to-5000-megacycle band, holds irrespective of the conductor geometry. On this basis, the printed-line characteristic is 14 percent off.

[3] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, New York, New York; 1944: page 353.

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From other sources,[4] if a composite-dielectric coaxial line may be used for pattern-behavior predictions for the printed line, it would appear

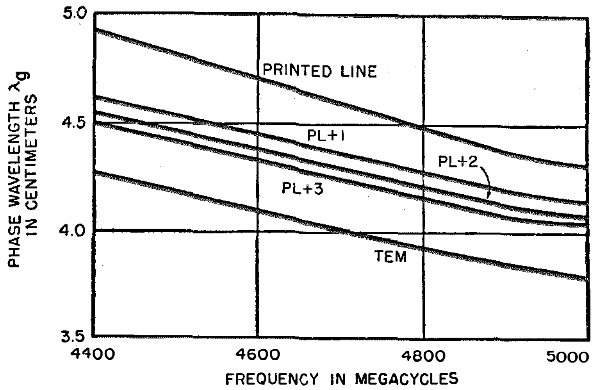

Figure 4—Phase wavelength plotted against frequency for printed lines having a strip conductor 0.220-inch (5.59-millimeters) wide and a dielectric spacing of 0.063 inch (1.60 millimeters). The computed equivalent dielectric constants $(\lambda_0/\lambda_g)^2$ are $\epsilon(PL) \approx 1.97$, $\epsilon(PL+1) \approx 2.16$, $\epsilon(PL+2) \approx 2.21$, and $\epsilon(PL+3) \approx 2.25$.

that as the composite-dielectric region is filled more and more with dielectric, the ratio of $(\lambda_0/\lambda_g)^2$ should approach an ϵ value determined by the dielectric layer only, i.e., the wave structure should approximate *TEM* behavior closer and closer. This tendency was verified by observations on the dielectric-sandwich type of line. Again referring to Figure 4, the curve marked *(PL+3)* shows 6-percent departure from *TEM* behavior. (*PL+3* means the original printed line plus 3 layers of polystyrene on top to form a dielectric-sandwich line.)

Identical conclusions are reached on the basis of an equivalent dielectric constant for a composite-dielectric line. If ϵ (equivalent) is defined as $(\lambda_0/\lambda_g)^2$, Figure 4 shows that it increases with increasing over-lying layers of dielectric and presumably would eventually approach ϵ (polystyrene), a value of 2.55.

1.4 Dispersion

An item of interest is dispersion,[5] a measure of frequency rate of change of λ_0/λ_g for printed and dielectric-sandwich lines. To this end, a series of measurements were made over a 5-to-1 frequency range for strips having widths of 0.110, 0.220, and 0.500 inch (2.79, 5.59, and 12.7 millimeters). One set of curves is shown in Figure 5. For all practical purposes, the variation may be considered a constant to a first order over a band of frequencies extending from 2000 to 10,000 megacycles.

1.5 Radio-Frequency Impedance

Another item of importance is the relative radio-frequency impedance of a section of line, especially for construction of hybrids, tees, and the like. No means has as yet been devised to measure this quantity directly. Indirect methods,[6] which are used in evaluating sharply defined discontinuities, may lead to profitable results. Preliminary attempts have not led to reliable data, however.

So far for printed lines, the relative radio-frequency impedance has been assumed to be inversely proportional to line-conductor width. Hybrid rings have been successfully fashioned,

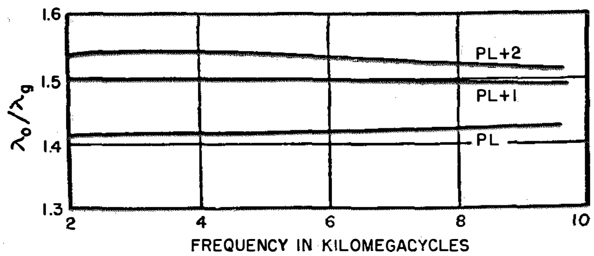

Figure 5—Phase wavelength plotted against frequency for a strip line having a conductor 0.220-inch (5.59 millimeters) wide and polystyrene spacing of 0.063 inch (1.60 millimeters).

but observations are limited to a conductor-width range from 0.110 to 0.220 inch (2.79 to 5.59 millimeters). The behavior for other widths cannot be predicted with certainty.

In the case of a wire-above-ground line completely imbedded in air, radio-frequency component designs based on the characteristic-impedance formula given by electrostatic considerations have proved successful, although observations are limited to 50-to-70-ohm line

[4] J. A. Kostriza, "The Composite Coaxial Line," Master's Thesis, Brooklyn Polytechnic Institute, Brooklyn, New York; 1949.

[5] L. Page, "Introduction to Theoretical Physics," D. Van Nostrand Company, Incorporated, New York, New York, Second Edition; 1935: page 250.

[6] N. Marcuvitz, "Representation and Measurement of Waveguide Discontinuities," *Proceedings of the IRE*, volume 36, pages 728–735; June, 1948.

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sections with $\frac{1}{8}$- and $\frac{1}{16}$-inch (3.18- and 1.59-millimeter) line conductors.

In connection with Figure 5, a word might be added about the absolute characteristic impedance of strip lines at radio frequencies. As is well known, the quantity λ_g/λ_0 plays an important part in determining Z_0, the characteristic impedance. Since λ_0/λ_g versus frequency is a constant (over the band shown), then Z_0 may be expressed as some constant divided by C, where C is the capacitance per unit length. It yet remains to be definitely established that C remains invariant as a function of frequency. Therefore, the characteristic impedance evaluated at low frequency and by low-frequency techniques may be held in question at high frequency.

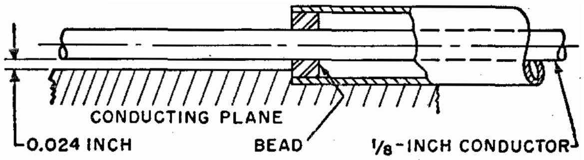

Figure 6—Straight-through transition for a wire-above-ground structure and a coaxial line.

1.6 Attenuation versus Width of Ground Plane

To study the effects of width of ground plane, a 50-ohm $\frac{1}{8}$-inch (3.18-millimeter) wire-above-ground air line with brass conductors was selected. Since the attenuation of a 12-inch (30-centimeter) length of line could not be measured with any degree of accuracy at 4700 megacycles, only the percent change in transmitted power was recorded as a function of width of ground plane. The transmitted power increased by approximately 2.5 percent when the width of ground plane was changed from 2 to 5 times the line-conductor diameter.

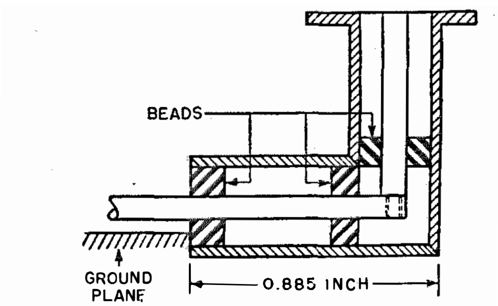

Figure 7—Right-angle transition for a wire-above-ground and a coaxial structure.

1.7 Summary

At this point, the highlights may be summarized.

A. Air-dielectric wire-above-ground type of line operates essentially in the *TEM* mode.

B. Strip and dielectric-sandwich lines for some applications may be approximated to *TEM* behavior.

C. The width of the ground plane required is not excessive.

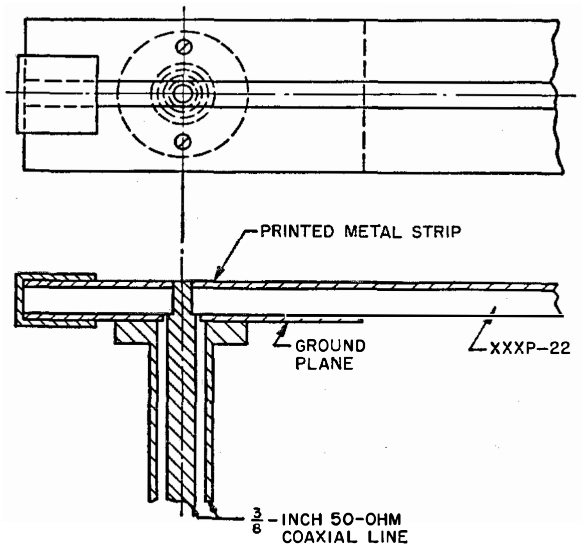

Figure 8—Transition for a printed line and a coaxial line.

2. *Radio-Frequency Components*

The components to be discussed have been developed for communication-power levels in the 5000-megacycle region. Transitions to coaxial line, crystal holders, loads and pads, directional couplers, and hybrid structures are practical in wire, printed, and dielectric-sandwich types of line. Only representative structures are presented, since space does not permit illustration of each type. It should be noted that the results presented are not necessarily the optimum that may be realized.

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2.1 Transitions to $\frac{3}{8}$-Inch (9.53-Millimeter) 50-Ohm Coaxial Line

Wide-band transitions have been designed to operate in the 4400-to-5000-megacycle band from $\frac{3}{8}$-inch (9.53-millimeter) coaxial line to wire, printed, and sandwich lines.

Figure 6 shows a straight-through transition from a 50-ohm wire line. When the coaxial line is terminated in a matched load, the input voltage standing-wave ratio on the wire system is less than 1.5 over the band, the average value being about 1.2. Preliminary measurements indicate that the discontinuity susceptance at the junction of the two lines is essentially inductive and that transmission-line matching techniques may be used. The dielectric bead at the junction of the two lines is used as a rigid support and as a matching element.

A second type of transition from wire to coaxial is shown in Figure 7. In this unit, the coaxial line forms a right angle with the wire line. This transition has a maximum voltage standing-wave ratio of 1.33 over the complete band.

Two types of transitions have been developed for printed lines, both using *XXXP-22* material as the dielectric.

The first type, which contains an integral direct-current return path between the line and ground conductor, is shown in Figure 8. This unit was developed for narrow-band applications and has not been checked over a wider band than required. From 4750 to 5000 megacycles, the measured voltage standing-wave ratio is less than 1.2.

A second type, shown in Figure 9, was tested over the total band. The two types differ in the manner in which the short section of printed line

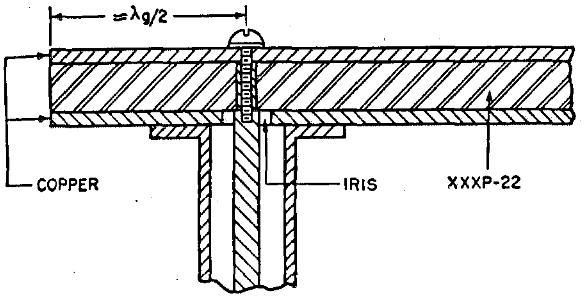

Figure 9—Transition for a printed line and a coaxial line.

is terminated past the junction of the printed line and the coaxial line. In this case, the printed line is open-circuited. The $\lambda_g/2$ termination and iris dimensions in the ground plane are the broad-banding elements. For a 0.220-inch-wide

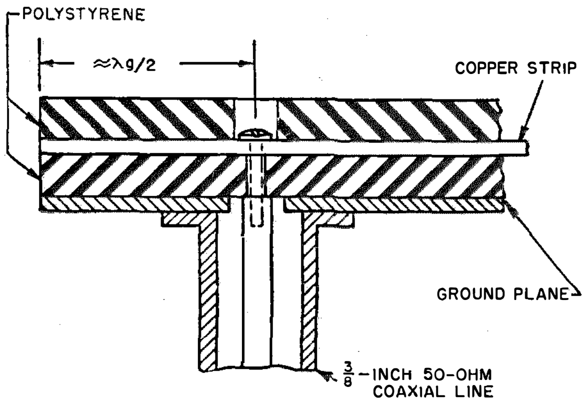

Figure 10—Transition for a sandwich line and a coaxial line.

(5.59-millimeter) strip, the voltage standing-wave ratio is less than 1.20 over the band. Similar constructions are used and similar results are realized for polystyrene printed-type transitions.

A transition to a polystyrene sandwich line is shown in Figure 10. This unit varies its voltage standing-wave ratio from 1.08 to 1.35 at the band edges.

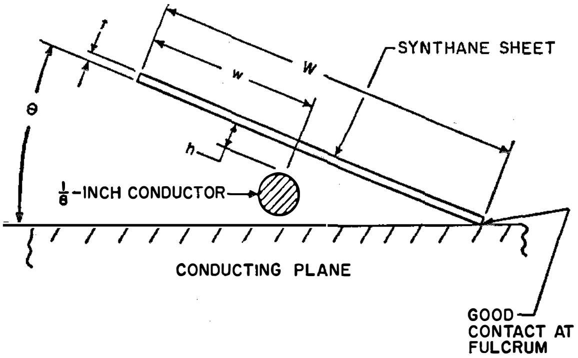

Figure 11—Variable attenuator for a wire-above-ground transmission line.

2.2 Attenuator Pads and Loads

Specific designs are given for wire-above-ground applications to pads and loads because observations were made over a wide band of

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frequencies for only this type. Printed-line pads and loads have been made, but observations are limited to spot frequencies and will therefore not be included.

The dielectric used as the lossy medium is Synthane *L-564*, which is obtainable in a wide variety of sheet thicknesses.

A transverse cross-sectional view of a variable attenuator is shown in Figure 11. Synthane *L-564* standard sheet thicknesses of $\frac{1}{16}$, $\frac{1}{8}$, and $\frac{1}{4}$ inch (1.54, 3.18, and 6.35 millimeters) were found adaptable. The flap-type attenuator depicted may be made variable by controlling the dimension h. For unit longitudinal length, maximum attenuation is a function of t, w, W, and θ, when $h=0$. Increasing the longitudinal length of the sheet increases the attenuation.

In general, this type of attenuator may have a tendency to radiate out of the open end of the triangle, radiation decreasing with decreasing angle θ. A shield may be used to minimize the effect on neighboring circuits.

Increasing the longitudinal length of attenuators will naturally produce matched loads. For strip lines, a much easier method of fashioning a load is simply to taper the leading edge of a thin sheet of Synthane. This is such an obvious solution that sketches are entirely extraneous.

2.3 Crystal Mounts

Wide-band crystal mounts have been designed for wire-above-ground, printed, and sandwich-type lines.

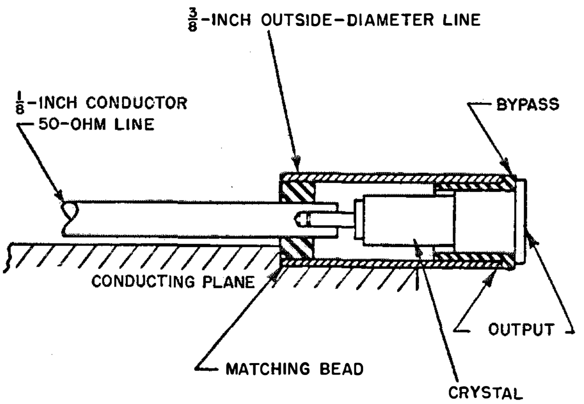

Figure 12—Wide-band fixed-tuned crystal mount for a wire-above-ground transmission line.

A crystal holder or mount for wire-above-ground was designed to operate over the 4400-to-5000-megacycle band with an input voltage standing-wave ratio of less than 2, using Sylvania *1N21-B* crystals. The operating conditions are a 50-to-100-ohm load resistance and a rectified current of 0.5 milliampere. A longitudinal cross section of the unit is shown in Figure 12. The direct-current return path must be provided in the radio-frequency circuit external to the crystal mount.

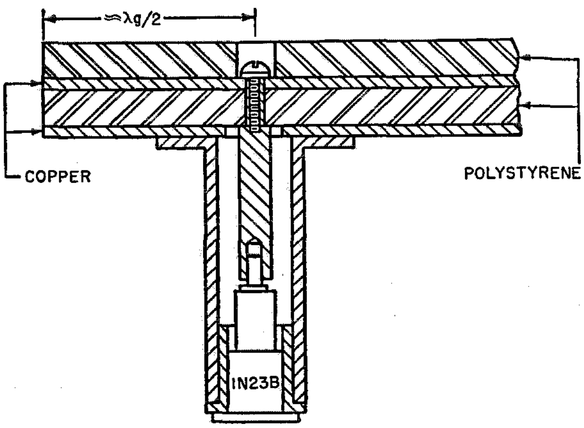

Figure 13—Crystal mount for a dielectric-sandwich line.

A right-angle unit, similar to Figure 7 in external appearance only, has a frequency characteristic similar to that of the mount shown in Figure 12.

Figure 13 illustrates a longitudinal section of a crystal mount in a 0.220-inch (5.59-millimeter) dielectric-sandwich line. As in transitions, $\lambda_g/2$ terminations and iris dimensions are useful broad-banding elements. The voltage standing-wave ratio is less than 1.5 over the complete band for a dozen somewhat-selected *1N23-B* crystals. This is a fixed-tuned unit.

2.4 Directional Couplers

Two basic types of directional couplers have been examined to a limited extent, specifically, a two-probe and a long-slot equivalent. Work on directional couplers has been limited to establishing feasibility of use of structures only and not to optimizing directivity or design for a particular value of attenuation.

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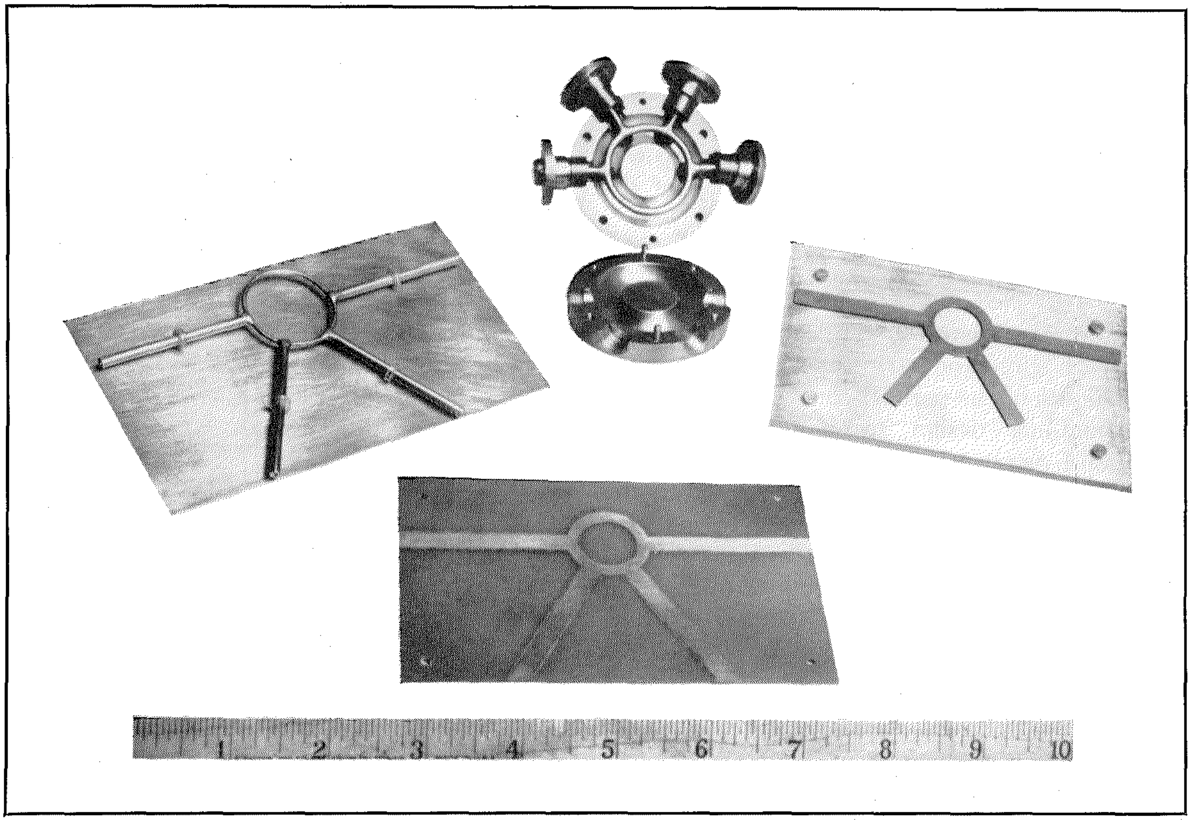

Figure 15—The familiar rat race in wire-above-ground and strip forms compared with the coaxial type.

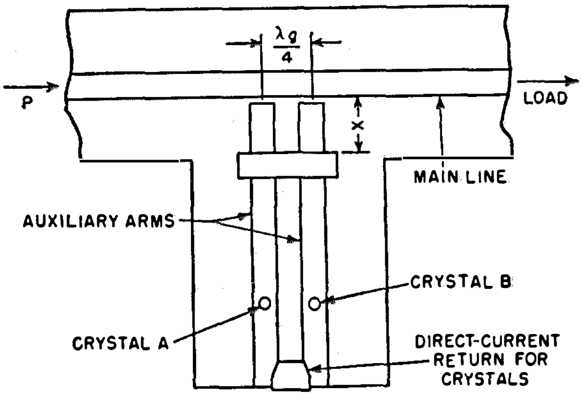

Figure 14—Two-probe type of directional coupler.

2.4.1 *Two-Probe Type*

Very briefly, a two-probe coupler[7] is an arrangement whereby two probes, when similarly point coupled into a transmission line but separated by some fraction of a wavelength, are further so interconnected that the resultant destructive wave interference in the probe lines results in directional properties.

The plan view of a quarter-wave double-probe directional coupler is shown in Figure 14. It is a forward-coupling type. With the dimension X set for maximum directivity at 4700 megacycles, crystal current A was substantially zero from 4400 to 4800 megacycles, indicating a substantial value of directivity.

2.4.2 *Long-Slot Type*

A long-slot coupler is formed when two transmission lines are coupled uniformly along their longitudinal dimension, either by a rather long slot (as a waveguide) or by leakage or mutual coupling. Theoretically, a half-wave uniformly coupled line should display directional properties. This was verified on a wire-above-ground line and on a polystyrene printed line.

[7] C. G. Montgomery, "Technique of Microwave Measurements," Massachusetts Institute of Technology Series, Volume 11, McGraw-Hill Book Company, New York, New York; 1946: chapter 14.

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The half-wave coupler was extended to a length approximately 6 wavelengths, a long-line coupler. A polystyrene printed line was used. When tested over the entire band (4400 to 5000 megacycles) the lowest directivity observed was 6 decibels, the highest directivity being of the order of 10 decibels.

2.5 Hybrid Ring Structures

Figure 15 illustrates the comparative simplicity of wire- and strip-type construction of ring circuits of the parallel-connection type.[8]

Coaxial-line design techniques have been successful, as regards quarter-wave spacings and ratios of arm and ring impedances. Note the narrower width of ring strip in comparison with the side arms.

In the wire case, with the ring terminated in crystal mounts and load as in a balanced mixer, the crystal response was strikingly close as the frequency was varied over the band. Strip units have not been checked over the complete band. Some at center design frequencies gave voltage standing-wave ratios of the order of 1.05 to 1.10.

2.6 Shunt-T Junction

Junctions with three arms in which the branching takes place in the H plane is termed an H-plane T junction. At low frequencies, the junction is a shunt junction.

Figure 16 illustrates a right-angle type of power divider. When power is fed into A or B, with the other arms terminated, it was found that both arms received power though, in general, not in equal amounts. This division of power can be controlled for a given thickness t of the auxiliary ground plane by varying the radius of opening R.

When R was adjusted for approximately equal power split between B and C or A and C, a movable short-circuit on B could be so positioned that the power in C was either a maximum or very nearly zero. For the condition of (almost) zero power transfer into arm C, an impedance measurement in the two sections, A-to-junction and junction-to-B, gave an almost coincident short circuit at the junction, indicating that the junction is an equivalent shunt-T junction. The measurement was performed at 4700 megacycles. The location of actual planes for the equivalent circuit has not been attempted nor has the frequency sensitivity of the structure been examined.

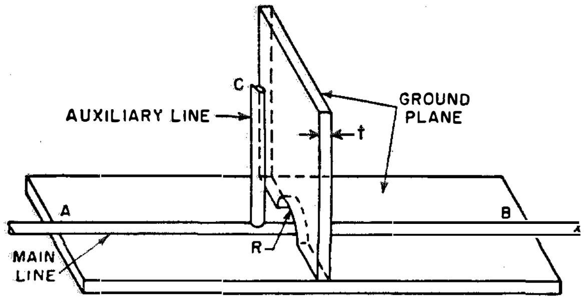

Figure 16—Power divider.

3. *Acknowledgment*

Appreciation is expressed to H. Seidel for his work on directional couplers and to P. Terranova for the exhaustive experimental work entailed.

[8] W. A. Tyrrell, "Hybrid Circuits for Microwaves," *Proceedings of the IRE*, volume 35, pages 1294–1306; November, 1947.

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Radio Dispatching System for Operation of a Large Taxicab Fleet*

By A. R. VALLARINO and S. W. LEWINTER

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

DISPATCHING of taxicabs by radio has proved to be practical and economical but due to the shortage of radio-frequency channels assigned to this service by the Federal Communications Commission, interference difficulties have been experienced. Solutions to this problem are offered by means of frequency and space allocations.

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The shortage of radio-frequency channels has been one of the major impediments to use of radio dispatching in large taxicab fleet operations. There is an upper limit to the number of taxicabs that can be handled as a group on one radio-frequency channel. The exact number depends on the type of control used in the system.

To use radio successfully for a large taxicab fleet, the communication load must be divided into noninterfering groups. If the division is attempted by assignment of separate frequencies to each group, then the number of frequencies required usually exceeds the number presently assigned to taxicab radio service by the Federal Communications Commission (FCC). Methods of dividing the communication load of a single radio-frequency channel on a geographic basis by means of directional antennas are known. These are never entirely satisfactory since the directivity patterns that can be obtained are never sharp or stable enough, and interference in boundary regions is inevitable. Moreover, reflections from neighboring objects aggravate this interference.

This article develops methods of frequency and space allocations whereby a taxicab fleet of any size can be served by an interference-free radio system using not more than four radio-frequency channels. In most cases, only three channels are required and in many cases two are adequate. These solutions are practical from both technical and economic considerations. It is believed that the methods described are of fundamental importance and of sufficient generality to be of value in dealing with communication problems of any large taxicab company.

Four practical solutions requiring either two or three radio-frequency channels and combining geographic and frequency subdivision of the communication load are described. The particular solution that would be most appropriate will depend on local conditions.

Each mobile equipment is designed for the number of frequencies required by the system so that the driver can select any one of them by means of a switch. The city is divided into areas so arranged that no interference zones exist when each area is assigned the proper frequency.

* Reprinted from *Electrical Engineering*, volume 71, pages 232–235; March, 1952. Presented before the American Institute of Electrical Engineers Winter General Meeting, New York, New York; January 23, 1952.

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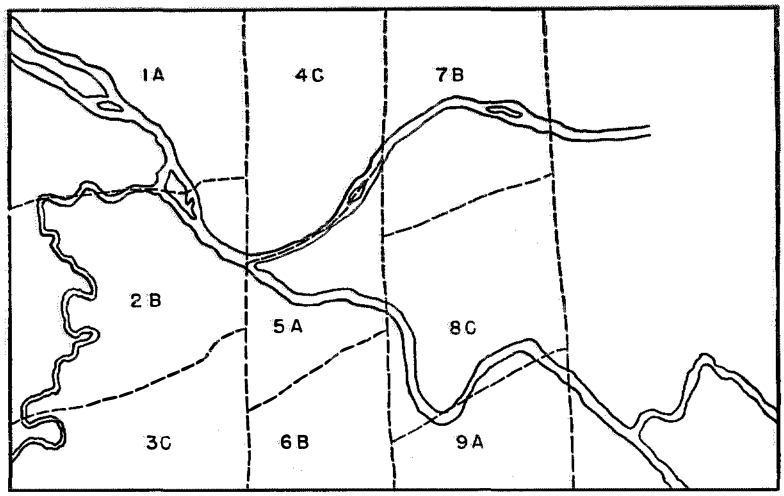

Figure 1—Division of an area for operation with three radio-frequency channels. The sectors are numbered and the channels are lettered.

See Figures 1 and 2. Each area contains a base-station transmitter and receiver with their antennas. Land-lines connect these stations to the main dispatching office. Whenever a driver crosses an area boundary, he switches to the correct channel for the new area. As many radio dispatchers as there are areas can be used during busy periods. The force can be contracted during slow periods by combining the operation of several areas under one dispatcher.

As examples of the potential opportunity for radio in the taxicab field, there are the 600 fleet cabs in Pittsburgh, 2000 fleet taxicabs in Philadelphia, approximately 13,000 fleet taxicabs in New York City, and about 50,000 fleet taxicabs in the country.

1. *Limitations of Present Radio Dispatching Systems*

A study was initiated about a year ago to determine whether we could propose for the large taxicab fleet operators a radio dispatching system using the limited number of radio-frequency channels allocated to the taxicab service by the Federal Communications Commission. It was intended that the study should bring out clearly the differences between the communication problems of the small taxicab operators now using radio dispatching and the largest taxicab fleets not so doing at the time the study was begun. The results were expected to prove of assistance in the design of radio dispatching and control systems for the largest taxicab fleets.

At the present time, radio is used mostly by taxicab companies having fleets of fewer than 125 cabs. Beyond this point, the difficulties of dispatching cabs by radio multiply rapidly. The technical problems posed by these larger operations center about the dispatcher's capacity to retain large amounts of continually changing information and the shortage of radio-frequency channel space allocated by the Federal Communications Commission to the taxicab industry.

A description of the operating methods of a typical small radio-controlled taxicab fleet will show why it is not possible to extrapolate directly to obtain operating procedures for the largest fleet operators. This company operates a fleet of 50 radio-controlled cabs. The dispatching is handled by two operators using a single radio-frequency channel. One of them receives incom-

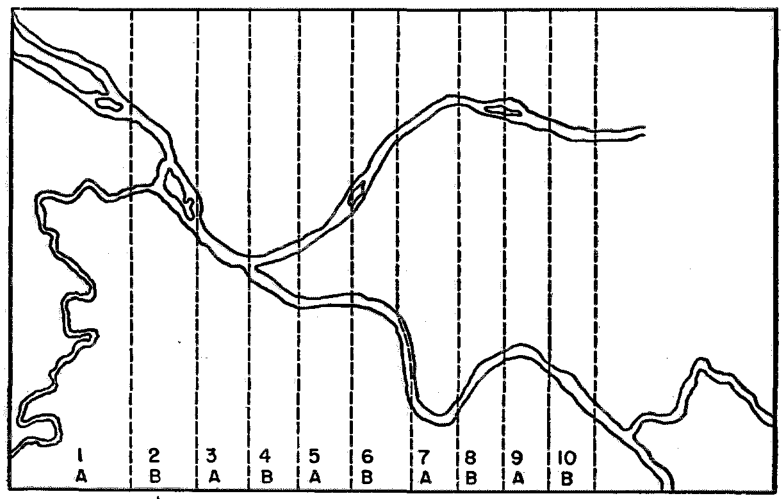

Figure 2—Division of area of Figure 1 for operation with two radio-frequency channels which are lettered and the sectors numbered.

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ing telephone requests for taxis and relays the information on slips of paper to the radio operator. The radio dispatcher calls the vacant cab that is closest to the location of the pickup and records the information on a slip of paper of another color. Both slips are filed on a board which contains numbered receptacles corresponding to the cab numbers. These slips are a record of the position and status of every cab in the fleet. The number of cabs that can be handled with this system is limited by the ability of the dispatcher to avoid a nervous breakdown on rainy days when everyone wants a cab. Either human or channel load capacity or a combination of both will limit the extension of such a system to fleets of several hundred cabs.

A radio communication system to meet the needs of a metropolitan cab company of 350 cabs will be outlined. The experience of several taxicab operators using radio systems in which the detailed movements of the fleet are controlled will be used in planning the proposed system. Under normal traffic conditions, one unassisted dispatcher is capable of controlling a maximum of about 70 cabs; at the busiest times, this number shrinks to about 35 cabs. If the dispatchers work independently, each assigned to a sector having approximately the same number of cabs, ten dispatchers and ten channels will be required to handle peak loads. The term "channel" as used here does not necessarily mean ten 2-way frequencies. A channel is any device by which calls from base stations to mobile units and mobile units to base stations can be segregated into mutually noninterfering groups. In considering the general problem of radio dispatching to taxicabs, channels can be obtained either by frequency or space allocation, the latter to be accomplished by directive antennas. Combinations of the two methods are often the most practical solutions to the channel problem. In any of these methods, ten channels require ten base-station installations.

2. *Design of Interference-Free System*

The radio communication load for the given example must be divided into ten approximately equal parts to permit the handling of the necessary volume of messages. One way of accomplishing this is to use frequency allocation and to transmit over the entire city from ten base stations on ten radio frequencies and have one-tenth the cabs receive a given frequency. To divide the incoming messages, ten cab transmitting frequencies are needed with ten receivers used at the base station. In calling a given cab the dispatcher must transmit at the frequency to which the receiver in that cab is tuned and he must expect an answer from the proper base-station receiver. This method is impractical since it requires ten duplex channels, more than the total number presently assigned to the taxicab radio service by the Federal Communications Commission.

The other extreme in dividing the communication load is space allocation. One duplex channel is used with the city divided into ten areas in which the volume of cab business is approximately equal. By the use of selected transmitting sites and directional antennas, an attempt is made to restrict the coverage of each base station to the assigned area. The same antennas are used to receive the taxicab transmissions. By this means and also by restricting the power radiated by the cabs to as small a value as is consistent with reliable operation, the messages from the cabs are divided into ten groups. This method always results in a certain amount of interference in the boundary zones between areas. The directive patterns of the antennas cannot be made sharp enough to prevent overlap; moreover, there are reflections from buildings and other topographical features that produce interference zones.

A combination of frequency and space allocation can result in a system that is efficient in the use of radio-frequency channels and yet does not create zones in which the transmissions from the various areas interfere with each other. The region served by the taxicab company is divided into ten areas. Each area has a base-station transmitter with antenna pattern and power adjusted to confine the radiation into that area plus a small amount of overlap into adjacent areas. The base stations in adjacent areas transmit on different frequencies and these stations are also designed to cover their areas plus some overlap. A mobile receiver in the boundary zone between adjacent areas can receive either base station, but they will be received on different frequencies. The cab receiver and transmitter must be

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designed for multichannel operation so that the driver merely throws a switch the moment he crosses the boundary line.

Each base station also contains a receiver that is tuned to the channel used by the taxicab transmitters when they are in that area. The base receivers are connected to the same antennas as the base transmitters through an antenna relay, or they are connected permanently to separate antennas having the same directive properties.

The boundaries between areas are marked off on a map and the drivers can be required to memorize the boundary locations. A driver crossing an area boundary would notify his dispatcher and then switch over to his new frequency. The dispatcher then would transfer his number to the dispatcher for the new area.

3. *Allocation Patterns*

There remains the problem of setting up the ten areas and determining how many frequency channels are required. The frequencies used by the base stations in any two areas having a common boundary must be different. The area boundaries are to be drawn in such a way that the number of frequencies required is at a minimum.

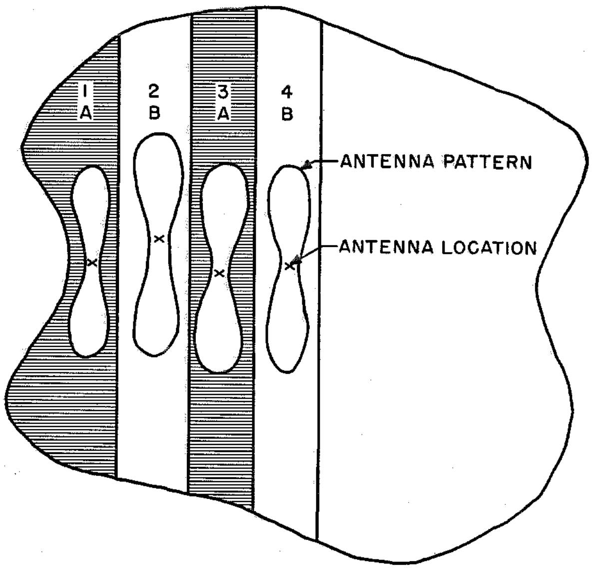

Figure 3—Trapezoidal pattern of space with numbered sectors and frequency allocations, which are lettered, using two radio-frequency channels.

This problem is very similar to the classical map problem in topology. The map makers wanted to know how many colors at most were needed to contrast adjacent countries in a

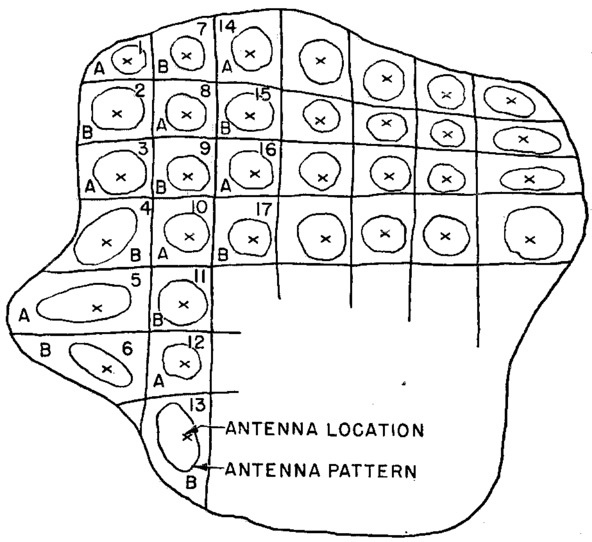

Figure 4—Modified rectangular pattern of space and frequency allocations using two radio-frequency channels, which are lettered.

mythical continent. The continent was to be of an arbitrary shape and the subdivision into countries was to be performed in any desired manner. The answer is that no more than four colors are ever required, no matter how complicated the map. If the four colors are used properly, no two countries having a common boundary will be shown with the same color.

The solution to this problem tells us that no more than four frequencies are ever needed if we divide the city into areas of any desired size and shape. Perhaps four frequencies is the answer to very large cities like New York and Chicago if a taxicab operator of a major fleet wishes to provide a radio call service over the entire region. However, in most situations only two or three frequencies are needed if the geometric layout of the areas is properly made. There are two basic patterns employing two frequencies. In one pattern, the city is divided into a series of trapezoidal strips with alternate strips using the same frequency, Figure 3. The base stations are located at the approximate center of each strip. The second 2-frequency pattern is the modified rec-

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tangular grid of Figure 4. Adjacent boxes use different frequencies. Some interference can be expected in corner regions since the fields must overlap to some extent. Nevertheless, it may be possible sometimes to use this kind of pattern by varying the size and shape of the boxes so that the corners are placed at natural barriers. Figure 4 shows how this can be done.

Where corner interference cannot be tolerated, the 3-frequency pattern of Figure 5 may be used. The basic pattern resembles an arrangement of bricks in a wall but this can be modified as is shown in the figure.

A second 3-frequency method is shown in Figure 6. Here the pattern resembles the spokes of a wheel with alternate sectors being assigned the same frequencies, and the hub assigned the third frequency.

Four basic patterns of areas and frequency allocation have been presented. In designing a radio dispatching system for a specific case, the choice from among the four patterns will be determined by local factors. Some of these are:

A. Street layout and traffic flow pattern.

B. Topography, including hills, cliffs, tall buildings, and open spaces such as rivers and parks.

C. Availability of base-station sites.

D. Number of radio-frequency channels available.

E. Comparative costs.

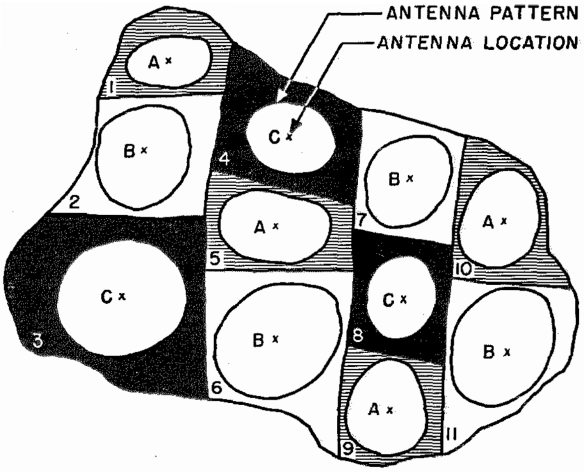

Figure 5—Three-frequency pattern based on rectangular partitioning in which the sectors are numbered and the channels are lettered.

As an example of the advantages and disadvantages of a particular pattern, consider the radial pattern with the central hub. A centralized location for the base stations and antennas is

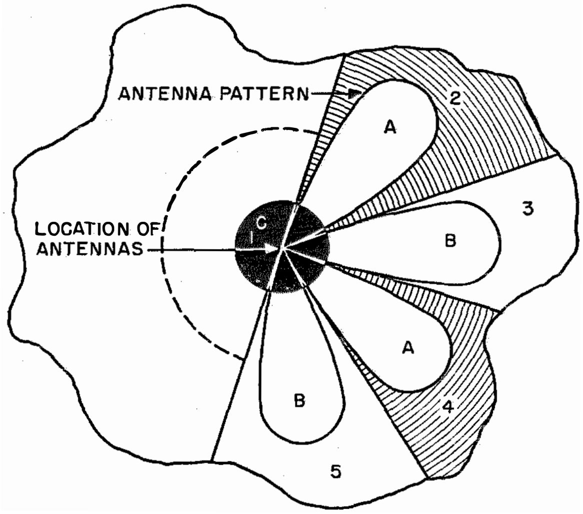

Figure 6—Three-channel system based on sectoral divisions with a circular area at the center which is served by the third channel.

required. This raises certain technical problems. The distances to be covered along the radii are much greater than the distances in a plan using decentralized base stations. Dead spots and distortion of the sector shapes by reflections from cliffs and tall buildings must be avoided. Care must be taken to minimize the directive antenna secondary-lobe interference, which can be expected; see Figure 6. This plan may be very useful in a city that is relatively flat, that does not have many tall structures, and that is built around a central point where a number of important traffic arteries converge. Washington, District of Columbia, is an example.

This plan would be a very poor choice for a large sprawling city composed of a number of distinct and almost self-contained districts. Philadelphia is such a city. The type of pattern illustrated in Figure 5 may be an excellent choice for Philadelphia. This pattern has one distinct advantage in that it permits use of omnidirectional antennas in place of the highly directional type of the radial pattern.

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4. *Selective Calling*

Selective-calling equipment permits a dispatcher to call any one cab, any group of cabs, or all cabs by operating a push-button control board. The call is heard only by the cab or cabs to which it is directed.

Selective-calling equipment is not essential to the operation of the communication system that has been outlined, but its use offers certain distinct advantages. These are summarized briefly.

4.1 Elimination of Nuisance Interference

The area and frequency allocations have been set up so that the base station within a given area is always stronger than interfering signals from other areas on the same frequency. In the presence of both signals, the receiver will respond to the stronger one and no appreciable interference will be heard. However, in some places the weaker station may be audible when the strong station is not transmitting. Selective calling would eliminate this nuisance interference as well as all other calls not directed to a particular cab or group of cabs.

4.2 Reduction in Driver Fatigue

Without selective calling, a driver must pay constant attention to all dispatched calls. Undoubtedly, this will affect his efficiency and may make him miss a call once in a while.

4.3 Increased Passenger Comfort

Some taxicab operators report that passengers often object to the radio chatter and they request the driver to turn off his receiver.

4.4 Speed of Calling

Many taxicab operators objected to the slowness of the old selective-calling equipment. Recently developed equipment puts through a call in less than one-half second. Provision can be made for individual calling, group calling, and general calling.

Low-Noise Traveling-Wave Tube

By A. G. PEIFER, PHILIP PARZEN, and J. H. BRYANT

Correction for Volume 29, Number 3, Pages 234–237, September, 1952

The authors gratefully acknowledge a note from W. W. Mumford of the Bell Telephone Laboratories pointing out that (3) in our paper, which may be written as

$$F_a = F_{abc} - \frac{F_c - 1}{G_a G_b},$$

should be

$$F_a = F_{abc} - \frac{F_c - G_b}{G_a G_b}.$$

This follows from the fact that the noise figure of a matched attenuator (network b) at room temperature is equal to its attenuation. The correction is therefore greater, and hence the measured noise figure lower, but only by a small fraction of a decibel in the described experiment.

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Recent Development in Communication Technique*

By C. W. EARP

Standard Telephones and Cables, Limited; London, England

THE PAPER provides a short introduction to a new system of communication. An attempt is made to show how the system was evolved as a logical development from earlier systems, and to suggest how it may find useful practical application.

A short review of the growth of technique and theory of communication demonstrates the continued search for new systems of high efficiency and leads inevitably to the conception that the ultimate theoretical efficiency of utilization of frequency bandwidth has now been established by Shannon. The view is taken, however, that practical systems still need to be developed in order to approach more nearly to ideal performance.

A discussion of features of pulse-code modulation (pcm)—the most efficient system in current use—shows how suspected deficiencies have led to suggestions for a new system of communication. Examination of this system, tentatively called the "ambiguous-index system," demonstrates that the suspected deficiencies were not, in fact, very real and that the purely theoretical performance of pulse-code modulation has not been improved upon. However, certain practical advantages are revealed, and it is suggested that one variation of the new system, which is very nearly equivalent to the theoretical 2-digit multiple-level pulse-code-modulation system, could be of great value when insufficient frequency bandwidth is available for binary coding by as many as 6 digits.

It is shown that the new system permits a new and satisfactory classification of the system commonly known as "pulsed frequency modulation" that represents a special case of the ambiguous-index system, which, in turn, is very closely related to signal-coding systems. Hence, the very high theoretical performance of pulsed frequency modulation is not in any sense anomalous.

A modification of the pulsed-frequency-modulation system probably represents an almost ideal system of transmission for the case where the bandwidth available for transmission is only about twice that which would be required for single-sideband working.

* Reprinted from *Proceedings of the Institution of Electrical Engineers*, volume 99, part III, pages 181–186; July, 1952.

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1. Review of Progress of Communication Theory and Technique

Since about 1936, when the performance of frequency modulation was clearly established by Armstrong,[1] communication engineers have been interested by the phenomenon that frequency bandwidth can be exchanged for signal-to-noise ratio. The practical development of pulse-width modulation (pwm), pulse-position modulation (ppm), and still later pulse-code modulation (pcm) drew attention to the fact that communication theory needed a considerable clarification or revision.

In a paper[2] that was written in original form in 1946, but not published till 1948, an attempt was made to present the facts of the problem, particularly stressing the merits of pulse-code modulation. Meanwhile, Shannon produced his elegant new statistical theory[3,4] of communication, which is now well established.

Shannon was able to specify the maximum possible efficiency of utilization of bandwidth for any system and to point out that the most efficient practical system, namely pulse-code modulation, as devised by Reeves,[5] definitely

[1] E. H. Armstrong, "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation," *Proceedings of the IRE*, volume 24, pages 689–740; May, 1936.

[2] C. W. Earp, "Relationship Between Rate of Transmission of Information, Frequency Bandwidth, and Signal-to-Noise Ratio," *Electrical Communication*, volume 25, pages 178–195; June, 1948.

[3] C. E. Shannon, "Communication in the Presence of Noise," *Proceedings of the IRE*, volume 37, pages 10–21; January, 1949.

[4] C. E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal*, volume 27, pages 379–423 and pages 623–656; July and October, 1948.

[5] A. H. Reeves, French patent 852 183; October, 1938: British patent 535 860.

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falls short of the theoretical ideal. Furthermore, he was able to devise a theoretical method of transmission that could provide ideal performance, but this does not represent a solution for practical application, since it involves storage of messages in the transmitter before encoding them into the form of continuous-wave functions.

As a result of Shannon's work, it may be that abstract theory of communication has reached substantial finality. The engineer, however, is left with a considerable programme of development work such as the following:—

A. With the knowledge that pulse-code-modulation systems do not represent ideal utilizations of frequency bandwidth, practical systems of at least slightly improved performance may still be found.

B. Practical pulse-code-modulation systems appear to be confined at present to the principle of binary coding and hence, when applied to telephony, require about six separate digital transmissions for each signal value and therefore a considerable frequency bandwidth. Since single-sideband working still represents an ideal method of transmission in the particular case where transmission bandwidth must not exceed signal bandwidth, there must be a strong case for development of pulse-code modulation or some equivalent system that requires a frequency bandwidth of not more than 2 to 3 times the frequency bandwidth of the signal.

C. Whatever frequency bandwidth may be available for transmission, continued circuit development must lead to new standards of economy and simplicity of equipment used.

2. *Possible Deficiencies of Pulse-Code Modulation*

The following discussion represents the author's thoughts before attempting to devise a new and improved communication system. Although later considerations show that very little indeed can be gained by avoiding suspected deficiencies of pulse-code modulation, the argument prompted suggestions for new and practical systems of high efficiency.

2.1 Dissimilarity of the Separate Digit Signals

In pulse-code-modulation systems, there is a suggestion of possible inefficiency imposed by the considerable dissimilarity of function of the various digital transmissions. For example, in decade coding with two digit signals, in which ten possible values of each digital transmission are able to define a total of 100 different signal values, the laws of variation between digit and signal values are very different for the two separate digits. Figure 1 shows the coding pattern suggested by Reeves for such a 2-digit system, and demonstrates that the first digit may be considered as defining the signal value with imperfect accuracy but entirely without ambiguity of value, whereas any particular value of the second digit corresponds to ten different possible values of signal spread over a very wide range. This point is brought out still more clearly if the principle of quantization of digital values is omitted, when the equivalent diagram of representation of signal values may be drawn as Figure 2.

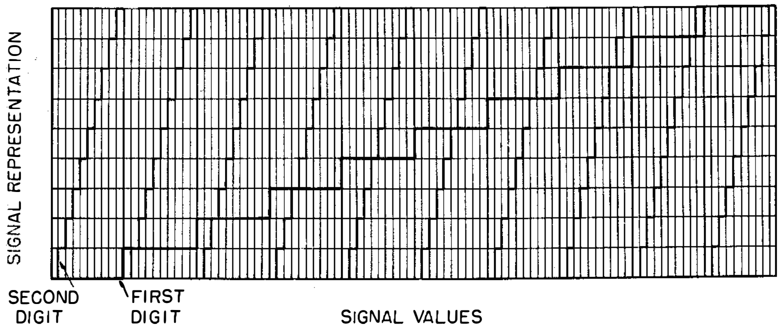

Figure 1—Two-digit decade coding pattern.

There is an apparent weakness in the above argument in that if the signal wave is of such random form that any signal value is equally likely to occur at any sampling instant, then digital transmissions will be very similar indeed. Furthermore, although the radio engineer is more particularly concerned with transmission of waveforms such as speech, when the digital transmissions of pulse-code modulation would certainly be dissimilar, it is difficult to see how modified coding—other than by non-linear

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representation of signal values—could provide an advantage selective to such a case. According to this first aspect, therefore, an improved performance for transmission of speech might be obtained by the use of a coding pattern that shows more rapid changes at small signal levels than for higher ones. Improvement would presumably be identical to that obtained from the technique of signal compression before coding and expansion after decoding, which is already applied in practical pulse-code-modulation systems. In other words, a new coding process could automatically include the compressing operation, giving an effective signal-to-noise gain, or reduced distortion, for exceptionally low-level signals.

From a second aspect, if we neglect the relative susceptibility of transmission systems to breakdown by noise-barrier conditions, signal-to-noise performance for any system is proportional to percentage sideband content for a given very small percentage of full modulation, and this suggests that advantage might be gained by variations of all signal representations corresponding to the range of smallest signal values. Considering this aspect in the case of the coding pattern of Figure 1, only the first digit is modulated for all values of signal up to 10 per cent of maximum value; hence, under these conditions, only one-half of the power available is used for production of sidebands. Is it not possible, therefore, to devise a simple code in which each digit is equally highly modulated and equally ambiguous in representation of signal values? In this case, would a revised 2-digit system benefit in signal-to-noise performance by 3 decibels? Would it be possible to devise a 6-digit binary system in which the whole signal power could be used effectively for each change of signal level, hence yielding an improvement factor of 8 decibels over pulse-code modulation?

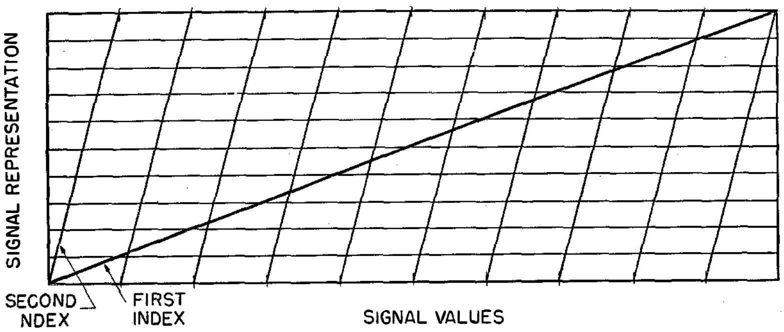

Figure 2—Dual representation without quantization.

Shannon's ideal system depends upon an effective noise-average over the period of time taken to transmit a coded continuous-wave function, this function representing a long succession of signal samples. Therefore, can a system be devised that effectively averages noise carried by the various digit signals corresponding to a single sample? If so, some improvement should be obtained over pulse-code modulation without the necessity for signal storage before encoding.

2.2 Effect of Quantization of Signal Values

Modern information theory is built very largely upon the basis of quantization of signal values, but it is difficult to see why maximum efficiency should result from the process. It is true that nature appears to use the principle frequently, e.g. in the human nervous system, brain, and sense organs, but this is no proof of an inherent advantage. It may be remembered that pulse-code modulation was first known as "step modulation." Although admittedly not a perfect analogy, there is a suggestion of quantization by nature in the conception that animal movement takes place in discrete steps, yet modern transport has derived great advantage from the unnatural phenomenon of smoothly rotating wheels.

In Shannon's ideal system, binary coding of successive signal values is used only as a method for storage of information. The actual transmission is in the form of a continuous-wave function, not a succession of quantized values.

A practical coding system must of necessity have only a finite number of code units, and so cannot transmit signals on the basis of continuous variability of value; hence, some form of quantization must be used. However, as has

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been pointed out,[6] the "most-ambiguous" digit of a pulse-code-modulation transmission (ambiguous in the sense already described with reference to Figures 1 and 2) could be transmitted as the exact signal-value discrepancy necessarily

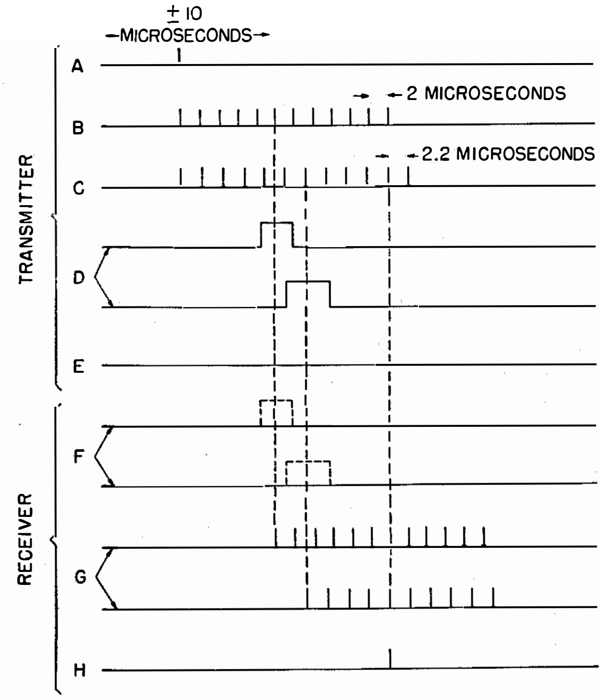

Figure 3—Twin-index pulse-position-modulation transmission. *A*—High-deviation pulse-position modulation. *B*—First comb; modulated ±10 microseconds. *C*—Second comb; modulated ±10 microseconds. *D*—First and second fixed gates. *E*—Transmitted pulses. *F*—First and second index pulses selected. *G*—First and second combs. *H*—Pulse selected by mutual gating.

imposed by the other digits. This modified system does not suffer in signal-to-noise performance. Furthermore, the theoretical performance of pulsed frequency modulation—i.e., the system of transmission of short wave-packets gated from a continuous-wave frequency-modulation transmission of high deviation—appears to be very high,[7–9] yet there is no quantization. Thus, in seeking an improved system it should not be expected that quantization will be necessary. On the other hand, it must be expected that the principle of sampling will be maintained, for without this, coding does not appear to be possible. Possibly there has been some confusion of thought between the essential functions of sampling and quantizing.

3. *Ambiguous-Index System*

The ambiguous-index system[10] will first be described with reference to the embodiment first conceived. It represents a preferred way for using two pulse-position-modulation trains to constitute a single communication channel, the particular method being prompted by considerations of Section 2.

3.1 Transmitter

The following description will be made clearer by reference to Figure 3, in which corresponding lettering has been used.

A. A train of pulses is position modulated by, say, ±11 microseconds.

B. Each pulse of this pulse-position-modulation train is converted to a "pulse-comb" of duration rather greater than 22 microseconds by application to a resonant circuit tuned to 500 kilocycles per second and by squaring and differentiating the resulting damped train. Adjacent pulse spacing of this comb is therefore equal to 2 microseconds, and a total of, say, 15 pulses make up the comb, which corresponds to a single signal sample. Modulation causes, of course, position modulation of the successive pulse combs.

C. A second modulated train of pulse-combs is derived in similar manner from the same highly modulated pulse-position-modulation train, pulse spacing in this case being 2·2 microseconds.

D. Approximately adjacent fixed time gates of 2- and 2·2-microsecond duration are used to select one pulse from each pulse-comb of corresponding pulse spacing, the gate positions corresponding roughly to the centre portions of the combs when the latter are unmodulated.

E. The pulses selected from the combs are transmitted, together with a fixed reference pulse for synchronizing purposes, and if desired, other pairs of pulses corresponding to additional communication channels.

[6] Footnote reference 2, Section 9.3.
[7] Footnote reference 2, Section 5.4.
[8] Frank Rockett, "Effect of Modulation on Transmission Efficiency" (comment on a paper by A. G. Clavier), *Electronics*, volume 21, page 136; January, 1948.
[9] E. M. Deloraine, "Pulse Modulation," *Electrical Communication*, volume 26, pages 222–227; September, 1949: also, *Proceedings of the IRE*, volume 37, pages 702–705; June, 1949.

[10] C. W. Earp and Standard Telephones and Cables, Limited, British patent application 29 464; November, 1950.

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3.2 Receiver

F. Gating pulses derived from the transmitted synchronizing pulse, of 2- and 2·2-microsecond duration (plus practical safety margins) are used to select the two transmitted index pulses into separate circuits.

G. The selected index pulses are used to produce new pulse-combs, each of at least 22-microsecond duration. Adjacent pulse spacing for these combs is 2 and 2·2 microseconds, exactly as in the transmitter combs from which the respective index pulses were derived.

By making a phase adjustment, if necessary, it may be arranged that two pulses near the middle of unmodulated combs, one from each comb, are made coincident in time. In the particular example shown in Figure 3, no such adjustment is necessary, for in all combs the first pulse has been produced in coincidence with the pulse from which the comb was developed, but this is not a general case.

A study of Figure 3 will now reveal that the twelfth pulse of the first transmitter comb and the eleventh pulse of the second are coincident, occurring exactly 22 microseconds later than the pulse that initiated the combs. Furthermore, it will be seen that two receiver-comb pulses must also occur at this same instant, since all pulses represent marking intervals of 2 or 2·2 microseconds from the original single pulse from which the transmission was derived. These two coincident pulses can, of course, be the only coincident pair over a length of comb equal to 22 microseconds.

It is important to realize now that, no matter how far the initiating pulse is modulated or deflected with respect to the fixed time gates used in transmitter and receiver, the two receiver pulse-combs have a pair of pulses, one from each, that occur 22 microseconds later.

H. One receiver comb is used to gate the other by application of the comb waves to separate electrodes of a coincidence valve, hence permitting only the coincident pulse to pass. When the transmitter is modulated, the position deviation of the single pulse produced by mutual gating of combs now bears the full modulation excursion of the transmitter, up to ± 11 microseconds, without ambiguity.

I. Demodulation of this fully modulated pulse train now takes place according to any well-known method.

3.3 Significant Features of System

The particular example of the ambiguous-index system just described is very similar to a 2-digit pulse-code-modulation system, in that the signal wave is sampled and from each sample value two distinct representations of the signal are transmitted. The following significant differences may, however, be noticed. First, the two index signals undergo rapid changes of amplitude for all small changes of signal level, and as a result both represent signal values with a high degree of ambiguity. Figure 4 shows the code of signal representation for each of the index transmissions and demonstrates a much greater similarity between them than between the two pulse-code-modulation digit representations of Figure 1. (The use of two very similar indices suggests the convenient term of twin-index transmission.) Secondly, no process of quantization has been used, although this could easily be added, if required.[11]

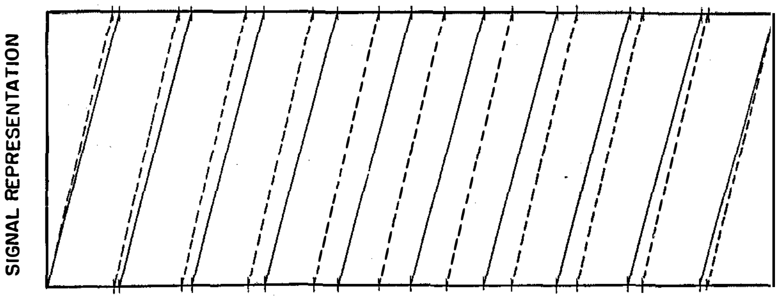

Figure 4—Signal representation for twin-index system.

Among features that are common to pulse-code-modulation systems, it may be noticed that the receiving mechanism breaks down completely when noise exceeds a certain critical value. In the case of 2-digit decade coding by pulse-code

[11] C. W. Earp and Standard Telephones and Cables, Limited, British patent application 31 025; December, 1950.

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modulation, very high noise levels must arise when coding levels cannot be recognized without ambiguity; in the present case, excessive spurious position modulation of pulses may cause incorrect recognition of coincident pulses in the receiver, hence giving excessive noise.

It may be noticed that receiver pulse-combs could be filtered at 500 and 456 kilocycles, and beating together these waves would yield a train of 44-kilocycle wave-packets bearing an unambiguous phase modulation from which the signal wave could be extracted. Signal-to-noise ratios resulting from such a linear process would not be satisfactory, however, being representative only of the differential modulation of the two index transmissions.

The efficiency of utilization of bandwidth for the particular example chosen is not superlative, owing to the fact that signal representations are transmitted by the imperfect pulse-position-modulation system. Assuming, however, that noise-threshold conditions are satisfied, i.e. that no breakdown of the receiving mechanism is possible through false recognition of coincident pulses, the system gives a performance approximately 23 decibels better than that which could be derived from the pulse-position-modulation train of a single index. This figure arises from an approximate tenfold (20-decibel) magnification of time deviation from ± 1 or $\pm 1{\cdot}1$ microseconds to ± 11 microseconds and a further gain of 3 decibels from the fact that noise jitter of the pulse derived by the mutual-gating process in the receiver is reduced by averaging of the noise jitters of the transmitted index pulses.

4. *General Forms of Ambiguous-Index System*

It is not proposed to give details of the many possible forms of the ambiguous-index system, as so little practical work has been undertaken to prove them, but the opportunity will be taken to give a broad definition of the whole range of systems that it comprises.

Ambiguous-index transmission may be defined as the transmission of a signal wave by repetitive sampling of the wave and transmission of two or more independent representations of each signal-sample value on a continuously variable basis, at least one of the signal representations bearing ambiguity of signal values.

Potentially practical systems have been proposed in which representations of signal values may be transmitted in terms of envelope phase, including pulse-position modulation, high-frequency phase modulation, and frequency modulation. In theory, of course, amplitude modulation also could be used.

Subdivided from the above, a parameter such as phase may be transmitted with reference to a fixed standard or as the relative phase between two successive wave-packets.

In a system using two representations of each signal value, the two index signals do not necessarily bear equal time or phase deviations corresponding to small signal values. For example, one index pulse of a 2-index pulse-position-modulation system may be modulated lightly, bearing no ambiguity of representation. The coding pattern of such a system is represented by Figure 2. In this case, the receiving mechanism must first translate one of the index trains to have the same time deviation as the other before the principle of mutual gating may be applied.

The ambiguous-index system may evidently be adapted for the use of any number of indices, although for radio transmission it is possible that the greatest value will be derived from the use of two indices only, owing to economy of both apparatus and frequency bandwidth.

Up to the time of writing, experimental work has been confined to the use of two indices only, and the mutual gating of pulse-combs has always been used as an essential part of the receiving technique.[12] Mutual gating, however, is not necessarily restricted to the application of waves to separate electrodes of a gating device. Effective gating may be achieved by simple addition of waves derived from the various indices. When the number of indices is greater than three, ambiguity would almost certainly be resolved by the simple addition of sine waves, or possibly symmetrical sawtooth waves. In all cases, the gating or adding process yields a single prominent pulse, which is position modulated by the signal waves.

[12] C. W. Earp and Standard Telephones and Cables, Limited, British patent application 29 465; November, 1950.

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5. *Signal-to-Noise Performance of Ambiguous-Index System*

Practical pulse-code-modulation systems usually occupy at least twice the theoretical minimum bandwidth; this is necessary unless digital signal values can be transmitted as video-frequency pulses or by single-sideband transmission. Furthermore, the theoretical power required must be exceeded, even for the video-frequency case, unless both positive and negative values can be transmitted as such. The same practical limitations apply to the new system, although—in theory, at least—index values could be transmitted as values of amplitude, both positive and negative, either on a video-frequency basis or by single-sideband technique.

It seems unlikely that apparatus for transmission of index values in terms of amplitude will ever become practicable, but in so far as radio transmission is concerned, transmission of indices in terms of radio-frequency phases should require similar bandwidth as for transmission of pulse-code-modulation digital values by amplitude modulation and not quite so much power.

5.1 Twin-Index and 2-Digit Pulse-Code-Modulation Systems

So far as is known to the author, the 2-digit decade coding system mentioned in Section 2.1 has never been developed, owing to practical difficulty in defining ten separate levels with adequate precision. Such a system is, of course, within the scope of pulse-code modulation and in some circumstances, e.g., when sufficient frequency bandwidth is not available for binary coding, might prove to be of value.

The nearest ambiguous-index equivalent of this system would be a 2-index system having a coding pattern according to Figure 2. One index would be modulated without ambiguity and the other would be an ambiguous representation of the signal, ten possible signal values corresponding to any particular representation. Signal-to-noise performance for this system would be determined substantially by the highly modulated ambiguous index only, and would be very similar to that of the equivalent pulse-code-modulation system, in which signal-to-noise performance is effectively determined by the ambiguous second digit.

The twin-index system described in Section 3 bears multiple ambiguity of signal representation on each index, and the two indices contribute more or less equally to the production of sidebands, even at the lowest level of modulation. The principle of mutual gating of pulse combs in the receiver permits equal and coherent contributions by the two indices towards definition of signal value and two random additions of noise. This results in a signal-to-noise improvement factor of 3 decibels. However, the consideration is not quite complete, for susceptibility to breakdown by excessive noise has been modified.

For a completely satisfactory comparison between the two systems, it is convenient to use Shannon's concept of a "signal mapping line." Consideration of the mapping line not only permits a study of threshold performances, but also represents a satisfactory basis for comparison of quantized and unquantized systems. Since it is not intended to claim advantage or disadvantage for a system because of substitution of fixed distortion for noise, comparison between signal outputs based upon length of mapping line, which is exactly equivalent to translation of pulse-code modulation to the type in which the most-ambiguous digit is transmitted on a continuously variable basis, should be completely fair.

The reader is referred, therefore, to a diagram[13] that was used by Shannon and described as "efficient mapping of a line into a square," and this is reproduced in essentials as Figure 5 of the present paper. The line length can be considered to represent a range of signal values that can be transmitted by two line co-ordinates or index signals, values of these indices being recorded by horizontal and vertical displacements, respectively. The particular mapping used by Shannon corresponds roughly to the 2-digit coding system of normal type, in which the vertical co-ordinate is unambiguous and the horizontal co-ordinate has multiple ambiguity.

In the coding arrangement of the new system, the mapping line traverses horizontal and

[13] Footnote reference 3, Figure 4.

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vertical axes at an equal rate, and the new mapping line corresponds roughly to the pattern shown as Figure 6. Comparison between Figures 5 and 6 shows that, when ambiguity factors of horizontal co-ordinates are equal, the mapping line for Figure 6 is the longer by a factor of $2^{1/2}$. Unfortunately, adjacent lines of Figure 6 are closer together by the same factor. The small circles drawn on these figures represent areas of uncertainty due to noise, and these demonstrate clearly that improved signal-to-noise performance of the system of Figure 6 has been obtained at the cost of a threshold condition which manifests itself for a smaller noise power. Overlapping areas of uncertainty would, of course, produce ambiguity of signal value and errors due to false recognition of coincident pulses in the receiver.

Remapping according to Figure 6, but with adjacent-line spacing equal to that of Figure 5, has the effect of producing the same length of mapping line. Thus, performance of the system of Figure 5 is very similar to that of the modified Figure 6. A very small advantage can be claimed

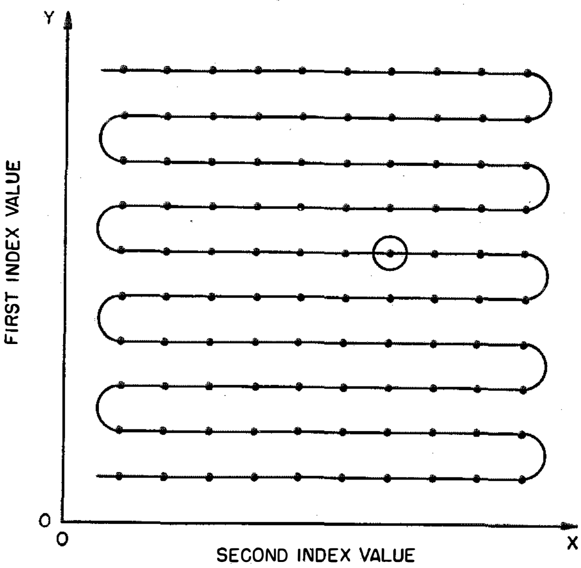

Figure 5—Two-dimensional mapping line.

for the ambiguous-index system, for in the case of quantized pulse-code modulation both positive and negative errors can occur on either digit, giving four distinct possibilities of error, whereas in the new system ambiguity arises only when either of two combinations of noise from two indices causes deflection of the mapping line at right angles to itself. Thus, when operated near to the noise threshold, ambiguities of signal value occur only half as often. However, this represents an insignificant improvement, so the hypothesis that symmetrical indices should have some advantage appears to have failed almost completely. Perhaps it can be considered that the new system has a practical advantage in that the same performance is obtained without resorting to the higher ambiguity factor that must be associated with the horizontal axis of Figure 5.

5.2 Multiple-Index and Binary-Coded Pulse-Code-Modulation Systems

If provided with about six separate index channels, the new system becomes closely equivalent to binary-coded pulse-code modulation. The possibilities of such a system have not yet been fully explored.

At first sight, it might be expected that the new system, which transmits signals in a form very similar to white noise, should show some improvement over pulse-code modulation. However, if as seems likely from our consideration of the 2-index system, any lengthening of the signal mapping line (by inclining its direction of progress to all the indicial axes of the mapping space) is accompanied by a nearer approach to the noise barrier, there can be no arrangement of ambiguity factors that provides a significant improvement in signal-to-noise performance over binary pulse-code modulation.

If the index signals of a multiple-index system are transmitted in the form of phase angles of a high-frequency wave, signal-to-noise performance should be very similar to that of binary pulse-code modulation in which positive and negative values are used. However, if the new system transmits index values by positive and negative amplitudes, its behaviour is very different. A considerable advantage could be claimed on the grounds that no power need be consumed except during active modulation. On the other hand, there is a serious disadvantage from another standpoint.

In binary pulse-code-modulation, digital signal-to-noise ratio in the transmission path may fall

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very nearly to unity without production of errors. No information is removed from a digital transmission until the noise peak is equal to the signal. In the new system, peak index values would not be inverted in sense, but small index values tend to lose information-bearing properties; in the limit, a zero level may be turned into any other level, positive or negative. This appears to represent a very definite advantage for binary pulse-code modulation, an advantage that is derived directly from the process of quantization. Strangely enough, this advantage is not manifested to any marked degree except in the case of binary coding. In the more general case of multiple coding levels, only the top and bottom levels, where limiters may be used, are affected by noise less than intermediate levels.

Against this apparent advantage for binary coding, in the multiple continuously variable-index system all indices are used for estimation rather than absolute determination of a signal value, and the complete failure of a single index does not necessarily produce an error. In other words, it may be considered that all but two indices are merely held in reserve for noise conditions that could cause breakdown.

6. *Classification of Pulsed Frequency Modulation*

A very high efficiency has been predicted[7–9] for the system of transmission known as pulsed frequency modulation, or time-sharing frequency modulation, as mentioned and defined in Section 2.2. The author's own prediction[7] was based upon the concept that when a very small degree of modulation is used, sideband power is linear with modulation power so that according to the earliest principles of linear transmission, signal-to-noise performance should be determined solely by sideband power. The remarkable rate of growth of sidebands on a continuous-wave frequency-modulation signal chopped into very short pulses (in terms of rate of increase of bandwidth) therefore indicated that, if a receiver could be constructed that provided ideal linear performance for a small degree of modulation and also responded without distortion to a high degree of modulation, very high signal-to-noise performance would be achieved. No physical receiver was proposed.

No satisfactory explanation could be offered for such a simple uncoded transmission having superlative signal-to-noise characteristics, and the system could not be classified logically among others.

The first clue to logical classification of pulsed frequency modulation is that high performance cannot be expected for frequency modulation of an oscillator by an amplitude-modulated pulse train, even though the constant-frequency portions of the wave are gated out. The very rapid growth of sidebands for small modulation is present only in the case of short pulses gated from a continuously frequency-modulated oscillator; furthermore, an ideal receiver would have to make use of the parameter high-frequency phase, either by referring each wave-packet to some fixed reference, or by comparison of phase of successive wave-packets.

It is now easy to see that pulsed frequency modulation is very near to an ambiguous-index system using two indices. It may be regarded as having one unambiguous index, the frequency

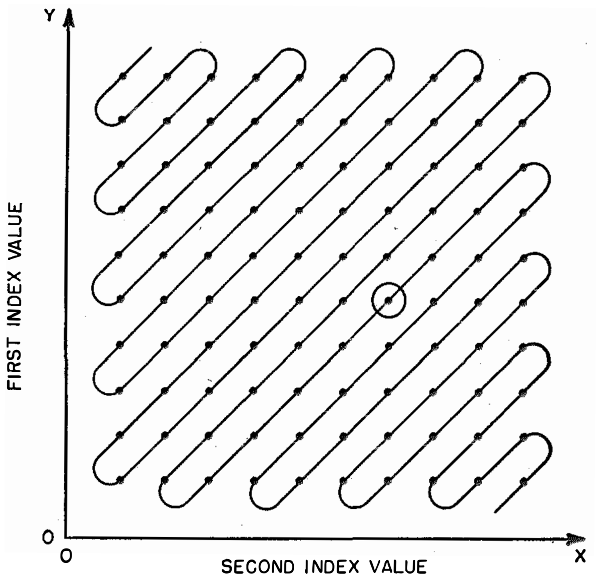

Figure 6—Modified mapping line.

of wave-packets, and one ambiguous index, the relative phase between successive packets. The only difficulty is that the two transmitted indices represent two different functions of the signal. The parameter frequency gives values of the

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signal at sampling instants, and the parameter relative phase gives values of signal integrated over the periods between sampling instants. Receiving technique has now been proposed[14] in which the index values for different functions of the signal are first translated to index values of the same function, after which the usual mutual-gating principle is able to extract the original signal wave without ambiguity and with high signal-to-noise performance.

Thus, the system of pulsed frequency modulation is no longer a peculiar misfit in the range of known systems, but represents a special form of 2-index transmission and therefore derives all the advantages of a coding system.

7. *Modified System of Pulsed-Frequency Modulation*

It seems improbable that any serious attempt will be made to develop for the normal pulsed-frequency-modulation transmission a practical receiver that could provide an approximation to the theoretical performance. A small modification of the normal transmission has now been proposed[15] that permits much simpler receiving technique.

In this system, an oscillator is frequency modulated by a waveform that steps abruptly from one sampling value of the signal to the next value, the level being kept constant for the whole period between sampling instants. The transmitted wave-packets are derived by gating short wave-packets from the oscillator immediately before steps in the modulating wave. By this means, differential phase between successive wave-packets represents the integral of a steady signal value over a definite period; hence, both relative phase of successive packets and the frequency of a wave-packet represent the same signal sample value. This modified transmission is a true 2-index system, and normal receiving technique may be used.

If a number of different channels are interlaced to yield a time-sharing multiplex system, the effective bandwidth of a single channel needs to be very little more than twice the bandwidth of the signal wave. It seems certain that, among practical systems having such restricted bandwidth, the new system must have a higher signal-to-noise performance than any other.

8. *Conclusions*

The ambiguous-index system, the basis of which is the sampling of a signal wave and transmission of two or more representations of each sample on a continuously variable basis, represents a whole new range of possible communication systems. These systems are closely related to pulse-code-modulation systems, but are not true coding systems.

In a new classification of communication systems, it would seem preferable to introduce the new system before pulse-code modulation. The new system depends vitally upon the principle of sampling of the signal wave, and upon extending the signal mapping line by efficient mapping of spaces of two or more dimensions. However, this is achieved entirely without the process of quantization of signal values.

The new system appears to demonstrate that there is very little inherent advantage in quantization. Except in the case of binary coding by pulse-code modulation, it can possibly be considered advantageous to leave out the process of quantization.

In future developments of radio systems, the large bandwidth that must be consumed by binary pulse-code modulation may be considered excessive. The new system opens up the practical possibility of development of systems of very high efficiency in which the required frequency bandwidth is very little greater than that required for simple amplitude modulation. In particular, the use of the parameters phase and frequency for dual representations of a signal value on a single wave-packet appears to permit a higher signal-to-noise performance than for any previously known system of similar bandwidth requirements.

9. *Acknowledgment*

The author is indebted to Mr. J. S. MacMullan for assistance by helpful discussion.

[14] C. W. Earp and Standard Telephones and Cables, Limited, British patent application 29 466; November, 1950.

[15] Footnote reference 10, Figure 17; and footnote reference 14, Figure 3.

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Note on Delta Modulation*

ATTENTION has been directed in two articles[1,2] to a type of coded modulation that has been called by recent authors delta modulation (Δ modulation) and is quite attractive because of the simplicity of the apparatus utilized to produce it.

In view of this renewed interest, it may be useful to present the following considerations to provide a better understanding of the operation of such a system. They will disclose among other things how to calculate distortion with accuracy and how to produce delta modulation by a new method that is simpler than those previously used.

1. Quantization in Time

Impulses of delta modulation corresponding to a given audio-frequency signal can be shown graphically as in Figure 1. (It can be easily seen that this construction is equivalent to those given in previously published material.[1,3])

Let $f(t)$ be the signal to be encoded and plot a curve of $y=f(t)+\alpha t$, where α is a constant equal to the maximum value of $f'(t)$ as a function of t. On the same graph, draw a series of equidistant lines parallel to the axis of t. The curve $y=f(t)+\alpha t$ cuts these lines at the instants t, $t_1, t_2, \cdots t_n$.

A series of reference pulses are produced at regular time intervals that are shorter than the briefest time interval separating two instants t_n and t_{n+1}. The pulses of this series will be transmitted if they immediately follow one of the instants $t_1 \cdots t_n$. If not, they will be suppressed. Thus, delta modulation can be considered as a system quantized in time, the corresponding nonquantized system being one in which the pulses would be transmitted at times $t_1, t_2, \cdots t_n$. If this nonquantized system is of a known type, it will provide a key to methods of producing delta modulation and demodulation.

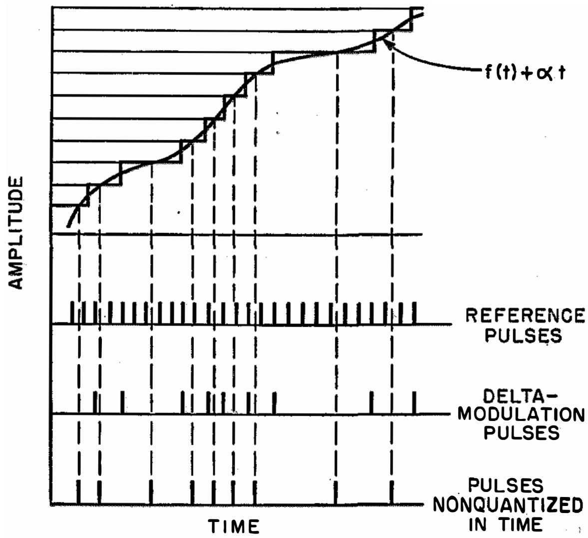

Figure 1—Graphical representation of delta modulation. The modulating signal is quantized in amplitude and a pulse is transmitted for each quantized-unit change. Pulses may occur at only predetermined time intervals, thus producing time quantization.

2. Nature of Nonquantized System

The nature of the nonquantized system can be estimated from the method utilized for decoding delta modulation: pulses are passed through an integrator and a low-pass filter.

If these two operations were reversed, the result would not be changed. If the output is $f(t)$, the signal at the input of the integrator should be $f'(t)$. The pulses nonquantized in time should therefore be such that after passing through a low-pass filter, they deliver $f'(t)$: they are consequently pulses modulated in density by a signal $f'(t) \pm$ a constant.

This result of inductive reasoning can be proved directly. To do so, consider two consecutive nonquantized pulses at the instants t_n and t_{n+1}. We have

$$[f(t_{n+1})+\alpha(t_{n+1})]-[f(t_n)+\alpha t_n]=C.$$

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* Prepared at Laboratoire Central de Télécommunications; Paris, France.

[1] L. J. Libois, "Un Noveau Procédé de Modulation Codée (La Modulation en Δ)," *L'Onde Electrique*, volume 32, pages 26–31; January, 1952.

[2] J. S. Schouten, F. de Jager, and J. A. Greefkes, "Delta Modulation, A New Modulation System for Telecommunication," *Philips Technical Review*, volume 13, pages 237–268; March, 1952.

[3] E. M. Deloraine, S. Van Mierlo, and B. Derjavitch, French patent 932,140; August 10, 1946: United States patent 2,629,857; February 24, 1953.

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Multiplying the two members by the repeating mean frequency

$$\varphi=\frac{1}{t_{n+1}-t_n}$$

of the pulses between t_n and t_{n+1}, we obtain

$$\varphi=\frac{1}{C}\frac{[f(t_{n+1})+\alpha(t_{n+1})]-[f(t_n)+\alpha t_n]}{t_{n+1}-t_n}$$

$$=\frac{1}{C(t_{n+1}-t_n)}\int_{t_n}^{t_{n+1}}[f'(t)+\alpha]dt$$

$$=\frac{1}{C}\left[\overline{f'(t)+\alpha}\right]_{t_n}^{t_{n+1}},$$

where the line over $[f'(t)+\alpha]$ indicates the mean value between the designated limits of t_n and t_{n+1}. Actually, the frequency φ of the nonquantized pulses is proportional to $f'(t)+\alpha$.

3. *Simplified Coder*

Since delta modulation is a pulse-density modulation quantized in time, it can be produced by combining a pulse-density modulator with a quantizer in time. A practical circuit is indicated in Figure 2.

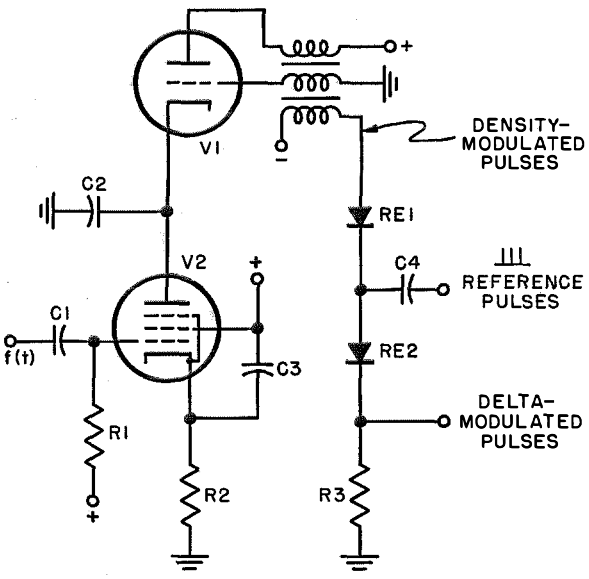

Figure 2—Simplified encoding device to produce delta-modulated signals.

The signal $f(t)$ is differentiated by the circuit *C1–R1* and applied to the grid of *V2*, which constitutes the discharging circuit of capacitor *C2*. This capacitor is a part of a blocking oscillator *V1*. The combination constitutes a density-pulse modulator, the output of which can be quantized in time by the circuit consisting of rectifiers *RE1*, *RE2*, and capacitor *C4*. A pulse train of fixed frequency is applied to *C4*, and only those pulses of this train that are preceded by a pulse coming from *V1* can pass through *R3* and produce output. Rectifiers *RE1* and *RE2* are biased through the output winding of the feedback transformer of *V1*, and *RE2* will conduct only when the reference pulse is added to the density-modulated pulse from *V1*.

Such a coder is remarkably simple compared with an ordinary feedback coder. As long as the signal is not saturating, that is $|f'(t)|\leqslant\alpha$, the two coders are exactly equivalent, but for saturating signals the new one has a better behavior than the conventional coder. In effect, in this new type of coder the transmission is disturbed only during the time when $|f'(t)|>\alpha$ and not for a substantial period of time afterward, as is the case with feedback coders. Furthermore, it is possible to introduce after *C1–R1* an "instantaneous compressor" to improve the dynamic characteristics of the system.

Two conditions must be met if an absolute identity between the output of this coder and that of a conventional delta modulator is to be obtained.

A. Frequency F of the reference-pulse train must be at least twice the mean frequency of the nonquantized pulses.

B. The instantaneous frequency of the nonquantized pulses must vary between essentially zero and F, when $f'(t)$ varies between $-\alpha$ and $+\alpha$. The constants of the circuit should be adjusted accordingly.

4. *Calculation of Distortion*

It is possible by elementary methods[1] to calculate approximatively to a correct order of magnitude the distortion of a delta-modulation system. It is, however, very useful to proceed to a more-accurate computation because the results of rough calculations often depend on the soundness of the approximations, in other words on the intuition of the computer.

The method followed here is based on the unique approximation that a system nonquantized in time does not introduce distortion when the signal is demodulated by the com-

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bination of the low-pass filter and integrator utilized in delta modulation.

Because of the linearity of this combination, the distortion will be due entirely to the process

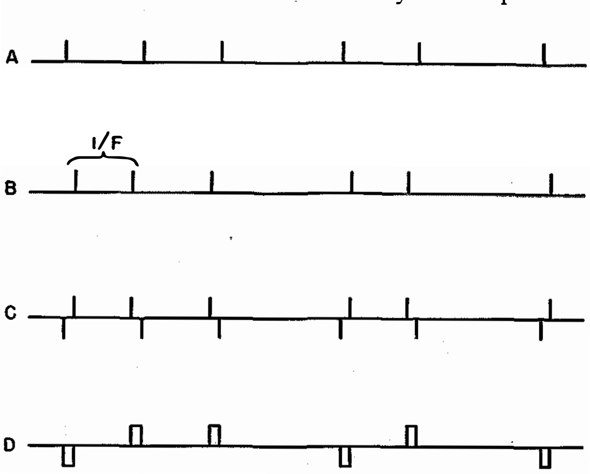

Figure 3—Waveforms in which A = nonquantized impulses delayed by $1/(2F)$, B = quantized impulses, $C = B - A$, and $D = \int C\,dt$.

of quantizing and will be equivalent to the difference between the quantized signal and the nonquantized signal. It should be realized, however, that the nonquantized signal is delayed by $1/(2F)$ second, which is the mean delay to which all the pulses are subjected in the process of quantizing. This is not a distortion.

Distortion will be calculated on the basis of $f'(t)$ rather than $f(t)$. Technically, the result will be just as useful as if based on $f(t)$ because it is always possible to integrate or to differentiate a signal before transmission, if it is so desired. Consequently

$$D = \overline{[f'r(t) - f'(t)]^2},$$

where the line over the terms again indicates the mean value. The relative distortion

$$\frac{D}{P} = \frac{\overline{[f'r(t) - f'(t)]^2}}{\overline{[f'(t)]^2}}.$$

The denominator can easily be estimated as a function of $f'_{\max}(t) = \alpha$ and of the form factor K of $f'(t)$.

$$K^2 = \frac{[f'_{\max}(t)]^2}{\overline{[f'(t)]^2}}$$

whence, $P = \alpha^2/K^2$.

As for D, this is the output power of the low-pass filter when the input consists of the difference between the pulses quantized in time and the nonquantized pulses delayed by $1/(2F)$. The waveforms are indicated in Figure 3.

By assuming that the low-pass filter is preceded by an integrator and followed by a differentiator, which introduces no alteration in the result, the computation is simplified. The output $g(t)$ of the integrator would be given by Figure 3D.

To obtain D, it is necessary to calculate the power spectrum of this signal; to deduce from it that of the distortion signal, which is its derivative; and to integrate this latter power spectrum from 0 to f.

It should be noted that the power spectrum of the derivative of a signal is $4\pi^2 f^2$ times the power spectrum of the signal. Therefore

$$D = 4\pi^2 \int_0^f f^2 df \times [\text{spectrum of } g(t)].$$

The power spectrum of $g(t)$ is computed by means of the theorem of Wiener-Khintchin, according to which it is the Fourier transform

$$\varphi(\tau) = \overline{g(t)g(t+\tau)}.$$

$\varphi(\tau)$ is easily computed when it is remembered that the contributions to $\varphi(\tau)$ of the products of

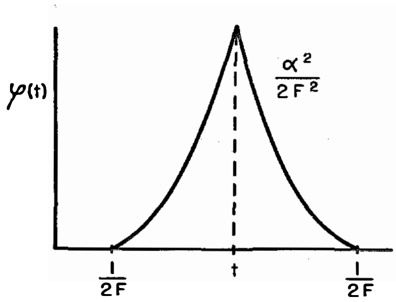

Figure 4—$\varphi(t)$ as a function of t.

the two different pulses of the signal (3D) are zero on an average.[4] Thus there is obtained for $\varphi(\tau)$, the function represented in Figure 4. The scale of amplitude is so determined that to an input signal applied to the low-pass filter (signal consisting of pulses of F frequency) there must be a corresponding output voltage of 2α.

$\varphi(\tau)$ has the form of a very-short pulse. Therefore, its Fourier transform at frequencies lower than f and, consequently, very-much

[4] This approximation might not be good when F is very great.

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lower than F is the same as that of an infinitely short pulse of the same area $\alpha^2/6F^3$, that is to say, a spectrum of uniform density. Whence

$$D=4\pi^2\int_0^f f^2\,df\frac{\alpha^2}{6F^3}=\frac{2\pi^2}{g}\left(\frac{f}{F}\right)^3\alpha^2$$

and

$$\frac{D}{P}=\frac{2\pi^2}{g}K^2\left(\frac{f}{F}\right)^3.$$

Again, we find here the law $(f/F)^3$ predicted by Libois.[1]

It is of interest to compare this result with that giving the distortion of a pulse-code-modulation system with N levels.

$$\frac{D}{P}=\frac{K^2}{12N^2}.$$

This permits the definition of the "equivalent number of pulse-code-modulation levels" for a delta-modulation system.

$$N=\frac{1}{4\pi}\left(\frac{6F}{f}\right)^{3/2}.$$

(Say, 70 levels for $F=25$.)

5. *Conclusions*

The identity of the delta-modulation system with a system quantized in time has been shown; a new coder and a method for computing the distortion of such a system have been deduced.

It should be noted that in practical operation this distortion is introduced into the delta-modulation system on the derivative of the signal, while that for pulse-code modulation is on the signal itself. Certain special statistical properties of audio-frequency signals indicate that there will be less damage if the distortion is introduced on the derivative of the signal rather than on the signal directly. However, this does not present an advantage of delta modulation over the pulse-code method because there is no reason why the audio-frequency signal cannot be differentiated before encoding in the latter system. Propositions along that line were made in 1947 by Messrs. Lair and Aigrain. However, this is a factor that should be taken into account when comparing delta and pulse-code modulations subjectively.

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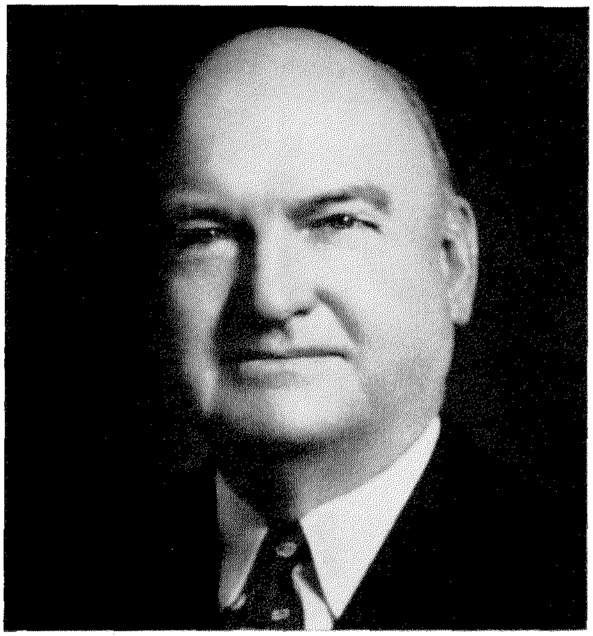
In Memoriam



LUKE McNAMEE

THE RECIPIENT of many honors that included the United States Navy Cross and the French Legion of Honor, Admiral McNamee was loved and respected by his many friends and associates throughout the world. He was an esteemed gentleman, firm in principle, modest and unselfish, with a warm and genial personality. In his death, this country lost a staunch citizen and the International System a beloved friend.

Luke McNamee was born in Mount Hope, Wisconsin on April 4, 1871. He began his illustrious career in the United States Navy when he entered the Naval Academy at Annapolis in 1888. Graduating in 1892, he served in the Spanish-American war as executive officer of the *U.S.S. Princeton.*

During the first world war, he served on the staffs of the commander-in-chief of the Pacific fleet and of the commander of the United States naval forces in European waters.

He was director of naval intelligence in 1922 and 1923. In 1925–1926, he was naval attaché to the American Embassy in London.

Promoted to Rear Admiral in 1925, he commanded destroyer squadrons, was director of fleet training, and commanded the battleships of a battle force. He took command of the battle force in 1932, with the rank of Admiral. He served as president of the Naval War College from 1933 until his retirement in 1934. He received the permanent rank of Admiral (retired) in 1942.

In 1937, he became president of Mackay Radio and Telegraph Company, where he directed an extensive program of expansion and modernization of its overseas and ship-to-shore services. He became chairman of the board in 1950 and a director of International Telephone and Telegraph Corporation in 1943, posts that he relinquished on his retirement in May, 1951.

Admiral McNamee died after a protracted illness on December 30, 1952, in his 81st year.

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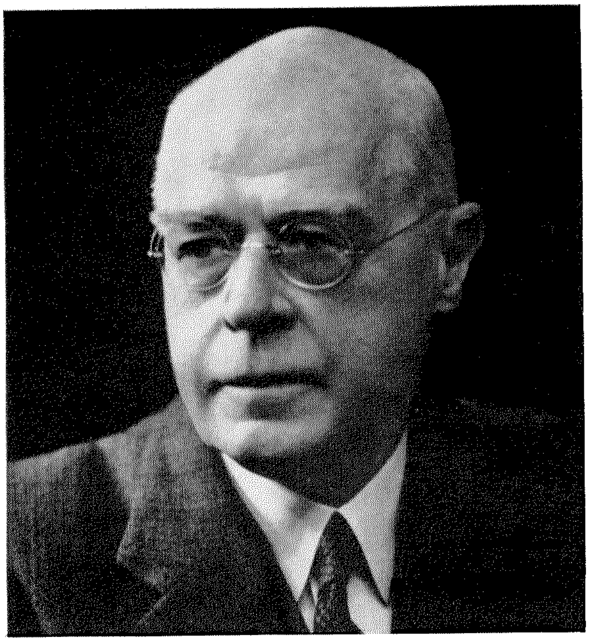
In Memoriam



PER E. ERIKSON

PER E. ERIKSON was born in 1880 in Malmö, Sweden. The Royal Institute of Technology in Stockholm conferred on him the degree of electrical engineer in 1903. He then joined the Western Electric Company in New York City to do developmental work on loading coils and balanced toll cables.

He was transferred to London in 1909 as European transmission engineer. He had charge of the systems planning of the first loaded long-distance cable installed in Europe, which linked London and Birmingham. In 1916, he returned to New York City to continue with transmission studies and design work.

During 1918, he supervised the reconstruction of the telephone transmission line between Rio de Janeiro and São Paulo, the first in Brazil to be equipped with repeaters and loaded toll entrance cables.

He returned to London in 1919 to what was then the International Western Electric Company and was to become the International Standard Electric Corporation in 1925. In 1928, he became assistant vice president in charge of the technical division of the European commercial department. From 1930 to 1932, he was European chief engineer.

In 1940, Mr. Erikson was transferred to Sweden as liaison officer between New York and affiliated companies in Scandinavia. He returned to London in 1946 as technical adviser to the European commercial department.

From 1938 to his retirement in 1950, Mr. Erikson served as a director of A. B. Standard Radiofabrik of Stockholm.

Starting in 1924, he was active in the International Consultative Committee on Telephony (C.C.I.F.). Since 1929, he served as a delegate for the operating companies of the International System, becoming secretary of its C.C.I.F. committee in 1934.

He was a Fellow of the American Institute of Electrical Engineers and a Member of the Institution of Electrical Engineers and of the Svenska Teknologföreningen. He published numerous papers and one written jointly with R. A. Mack received the Fahie premium of the Institution of Electrical Engineers.

Mr. Erikson died on December 7, 1952 in the town in which he was born, 73 years before.

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In Memoriam

WILLIAM THOMAS GIBSON

William Thomas Gibson was born in 1899, in Northampton, England. Educated at Trinity College, Cambridge, he received the M.A. (Cantab) and B.Sc. (London) degrees. Returning to the university after service in the Royal Engineers during the first world war for which he received the O.B.E. award, he took first-class honours in natural sciences in 1921. He then did a year of research work in Cavendish Laboratory under Professor J. J. Thompson and Professor Rutherford.

His entire professional career was with the International System. He started on the staff of the Western Electric Company in London to initiate the manufacture of radio valves and became chief valve engineer of Standard Telephones and Cables, Limited, and manager of the Ilminster laboratories and factory.

He set up the valve laboratory of Le Matériel Téléphonique in Paris in 1928, of the Federal Telegraph Company in the United States in 1932, and was a consultant on similar activities by the Australian, Spanish, and Swedish associated companies.

Mr. Gibson produced as laboratory samples the helical-grid triodes used in the first demonstration of microwave radiotelegraphy, which linked Dover and Calais by 17-centimetre beam in 1931. He was among the first to manufacture transmitting valves using thoriated-tungsten filaments. He pioneered in the development of thermistors. During the second world war, he was responsible for the development of the first valve that could be tuned over a wavelength range from 9 to 20 centimetres and which aided materially in the design of signal generators for research work.

He was a Member of the Institution of Electrical Engineers. He was a gifted amateur photographer and an accomplished pianist. Mr. Gibson died in Bridgewater Hospital on December 27, 1952.

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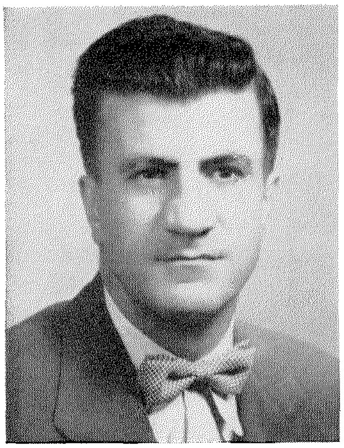
Contributors to This Issue



FRED ASSADOURIAN

FRED ASSADOURIAN was born on April 13, 1915, in Panderma, Turkey. He attended New York University from which he received a B.S. degree in 1935, M.S. degree in 1936, and Ph.D. degree in mathematics in 1940.

Dr. Assadourian was an instructor in mathematics at New York University from 1937 to 1942. He was an associate professor of mathematics at Texas Technological College from 1942 to 1944. He then went to the Westinghouse Research Laboratories as a research engineer and worked on pulse transformers from 1944 to 1946.

Since 1946, he has been employed by Federal Telecommunication Laboratories where he has worked on various problems in electronics and communication. He is now a project engineer in charge of the theory and research group in the special projects laboratory.

Dr. Assadourian is a member of the American Mathematical Society and of Phi Beta Kappa.

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C. W. EARP

C. W. EARP was born at Cheltenham, Gloucestershire, England, on July 14, 1905. He received the B.A. degree with First Class Honours in 1927 from Cambridge University.

In September, 1927, he joined the International Standard Electric Corporation at New Southgate and until 1929 assisted in the study of high-frequency transatlantic transmission.

In 1929, he joined the Paris laboratory as a radio development engineer, where he was chiefly concerned with receiver design, including ship–shore telephone equipments and single-side-band technique.

In 1933, he joined the broadcast receiver development group, which was then transferred from Paris to the Kolster Brandes factory at Sidcup, England, where he became chief of the advance development section.

In 1935, he joined Standard Telephones and Cables at New Southgate as section head for development of radio receivers and direction finders. In 1940, he became head of the newly formed advance development section of the radio division.

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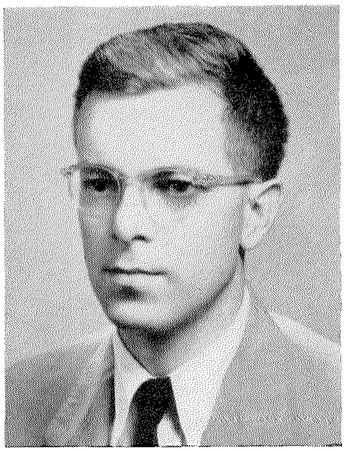

HERBERT F. ENGELMANN

HERBERT F. ENGELMANN was born on June 21, 1918 in Wilmington, Delaware. He received the B.S. degree in engineering physics from Lehigh University in 1940. During the following year, he did graduate work in the Physics Department of Cornell University

From 1941 through 1944, Mr. Engelmann worked at the Naval Research Laboratory as a member of the radio-consultant division. Since 1944, he has served as a research engineer at Federal Telecommunication Laboratories and at the present time is a department head. He is the author of several technical papers that have been published. Mr. Englemann is a Senior Member of the Institute of Radio Engineers.

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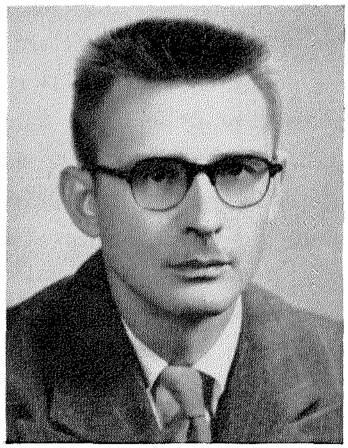
J. A. KOSTRIZA was born on Staten Island, New York, on October 10, 1912. He received the B.S. degree in electrical engineering from Cooper Union Institute of Technology in 1936 and the



J. A. KOSTRIZA

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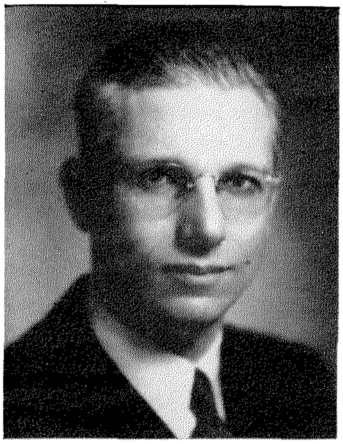

GERARD J. LEHMANN

M.E.E. degree from the Polytechnic Institute of Brooklyn in 1949. From 1938 to 1944, he taught applied mathematics and science in the New York City school system.

In 1944, Mr. Kostriza did research work on glass-vane attenuators for microwaves and taught undergraduate electrical engineering at the Polytechnic Institute of Brooklyn. Starting in 1945, he is now project engineer in charge of the microwave-components group in the special-projects laboratory of Federal Telecommunication Laboratories.

Mr. Kostriza is an Associate Member of the American Institute of Electrical Engineers, a Member of the Institute of Radio Engineers, and an Associate Member of Sigma Xi.

S. W. LEWINTER

GERARD J. LEHMANN was born in Paris in 1909. He graduated from Ecole Centrale des Arts et Manufactures in 1931.

On graduation, he joined Sadir Company in Paris and became its technical director. While working on aerial navigation and radar in 1937, he invented the folded-dipole antenna.

Mr. Lehmann came to Laboratoire Central de Télécommunications in 1940 and is now its director of research. He serves also as the general manager of Société des Servomécanismes Electroniques. He was assigned to Federal Telecommunication Laboratories in New York City from 1943 to 1945.

He has had charge successively of courses in radio navigation, radar, servomechanisms, and, now, information theory at Ecole Supérieure d'Electricité in Paris. He has published over 50 scientific and technical papers.

Mr. Lehmann was recently elected chairman of the French national committee of the International Scientific Radio Union. He was made a Chevalier of the Legion of Honor in 1948 by the French Government.

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SIDNEY W. LEWINTER was born on January 1, 1922, in New York, New York. He received the B.E.E. degree from the College of the City of New York in June, 1943.

From 1943 to 1948, he was employed by the Philco Corporation, where he participated in the development of airborne radar devices. From 1948 to the present, he has been associated with Federal Telecommunication Laboratories, where he has been active in the development of frequency-modulation and pulse communication systems.

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L. J. G. NIJS was born at Perwez, Belgium, on August 4, 1902. He was graduated as a mining engineer from the University of Liege in 1925.

After three years as an engineer in the Belgian coal mines, he joined the Bell Telephone Manufacturing Company in 1928. From 1930 to 1937, he was in charge of raw materials, metal-

L. J. G. NIJS

lurgical, and chemical studies in the manufacturing division. He was then transferred to the apparatus division, becoming its head in 1950.

Mr. Nijs served as an officer in the corps of engineers of the Belgian army for a year starting in September, 1939.

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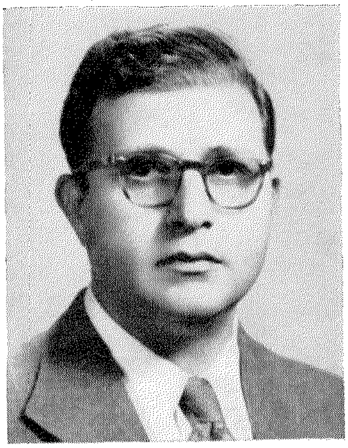
EMANUEL RIMAI was born in New York, New York, on September 24, 1911. He attended the College of the City of New York, from which he received the B.S. degree in 1932 and the M.S. degree in education in 1933.

From 1933 to 1940, Mr. Rimai was employed in several miscellaneous



EMANUEL RIMAI

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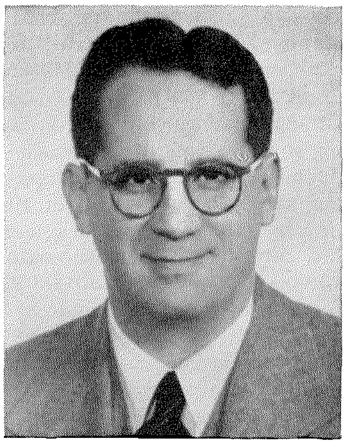
jobs. He served as a laboratory assistant in Brooklyn Technical High School from 1940 to 1942 and as a junior engineer in the Army Signal Corps for a year.

Since 1943, Mr. Rimai has been employed at Federal Telecommunication Laboratories, where he is now a senior engineer in the microwave-components group of the special-projects laboratory. He has worked on pulse and video circuits and on the theoretical analysis of various problems in electronics.



A. R. VALLARINO

A. R. VALLARINO received his B.E.E. degree from Stanford University in 1939. Since that time, he has been associated with Federal Telecommunication Laboratories, where he has done extensive work on frequency-modulation and pulse communication systems for both commercial and military applications.

He is now department head in charge of the development of multiplex terminal equipment. Mr. Vallarino is a Member of the Institute of Radio Engineers.

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INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION

Associate Manufacturing and Sales Companies

United States of America

Capehart-Farnsworth Corporation, Fort Wayne, Indiana
Flora Cabinet Company, Inc., Flora, Indiana
Thomasville Furniture Corporation, Thomasville, North Carolina
The Coolerator Company, Duluth, Minnesota
Federal Telephone and Radio Corporation, Clifton, New Jersey
Federal Electric Corporation, Clifton, New Jersey
International Standard Electric Corporation, New York, New York
International Standard Trading Corporation, New York, New York
I. T. & T. Distributing Corporation, New York, New York
Kellogg Switchboard and Supply Company, Chicago, Illinois

British Commonwealth of Nations

Standard Telephones and Cables, Limited, London, England
Creed and Company, Limited, Croydon, England
International Marine Radio Company Limited, Croydon, England
Kolster-Brandes Limited, Sidcup, England
Standard Telephones and Cables Pty. Limited, Sydney, Australia
Silovac Electrical Products Pty. Limited, Sydney, Australia
Austral Standard Cables Pty. Limited, Melbourne, Australia
New Zealand Electric Totalisators Limited, Wellington, New Zealand
Federal Electric Manufacturing Company, Ltd., Montreal, Canada

North America

Standard Electrica de Mexico, S.A., Mexico City, Mexico.

South America

Compañía Standard Electric Argentina, Sociedad Anónima, Industrial y Comercial, Buenos Aires, Argentina
Standard Electrica, S.A., Rio de Janeiro, Brazil
Compañía Standard Electric, S.A.C., Santiago, Chile

Continental Europe

Vereinigte Telephon- und Telegraphenfabriks Aktiengesellschaft Czeija, Nissl & Co., Vienna, Austria
Bell Telephone Manufacturing Company, Antwerp, Belgium
Standard Electric Aktieselskab, Copenhagen, Denmark
Compagnie Générale de Constructions Téléphoniques, Paris, France
Le Matériel Téléphonique, Paris, France
Les Téléimprimeurs, Paris, France
C. Lorenz, A.G. Stuttgart, Germany
Mix & Genest Aktiengesellschaft and Subsidiaries, Stuttgart, Germany
G. Schaub Apparatebau G.m.b.H., Pforzheim, Germany
Süddeutsche Apparatefabrik Gesellschaft m.b.H., Nuremberg, Germany
Fabbrica Apparecchiature per Comunicazioni Elettriche, Milan, Italy
Nederlandsche Standard Electric Maatschappij N.V., The Hague, Netherlands
Standard Telefon og Kabelfabrik A/S, Oslo, Norway
Standard Electrica, S.A.R.L., Lisbon, Portugal
Compañía Radio Aérea Maritima Española, Madrid, Spain
Standard Eléctrica, S.A., Madrid, Spain
Aktiebolaget Standard Radiofabrik, Stockholm, Sweden
Standard Telephone et Radio S.A., Zurich, Switzerland

Telephone Operating Systems

Companhia Telefônica Nacional, Rio de Janeiro, Brazil
Compañía de Teléfonos de Chile, Santiago, Chile
Cuban American Telephone and Telegraph Company, Havana, Cuba
Cuban Telephone Company, Havana, Cuba
Compañía Peruana de Teléfonos Limitada, Lima, Peru
Porto Rico Telephone Company, San Juan, Puerto Rico

Radiotelephone and Radiotelegraph Operating Companies

Compañía Internacional de Radio, Buenos Aires, Argentina
Compañía Internacional de Radio Boliviana, La Paz, Bolivia
Companhia Radio Internacional do Brasil, Rio de Janeiro, Brazil
Compañía Internacional de Radio, S.A., Santiago, Chile
Radio Corporation of Cuba, Havana, Cuba
Radio Corporation of Porto Rico, San Juan, Puerto Rico

Cable and Radiotelegraph Operating Companies

(Controlled by American Cable & Radio Corporation, New York, New York)

The Commercial Cable Company, New York, New York[1]
Mackay Radio and Telegraph Company, New York, New York[2]
All America Cables and Radio, Inc., New York, New York[3]
Sociedad Anónima Radio Argentina, Buenos Aires, Argentina[4]

[1] Cable service. [2] International and marine radiotelegraph services.
[3] Cable and radiotelegraph services. [4] Radiotelegraph service.

Laboratories

Federal Telecommunication Laboratories, Inc., Nutley, New Jersey
International Telecommunication Laboratories, Inc., New York, New York
Laboratoire Central de Télécommunications, Paris, France
Standard Telecommunication Laboratories, Limited, London, England

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