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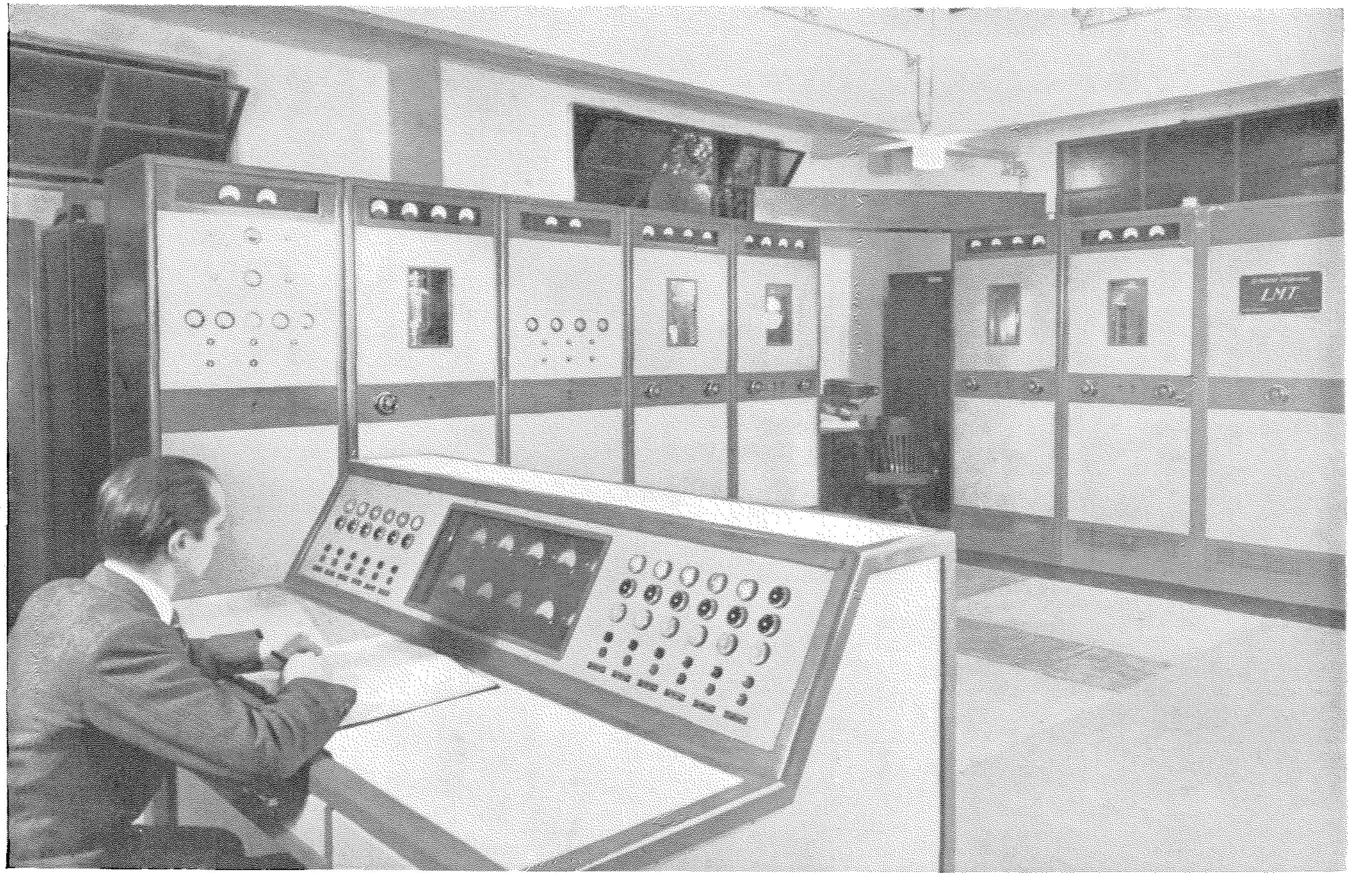

The Eiffel Tower 30 kW Television Transmitter installed for the French P.T.T. by Le Materiel Téléphonique, Paris, was inspected in April by Press Representatives at the invitation of the Minister of the P.T.T.

The Creed No. 10 Tape Teleprinter

Recent Advances in Printing Telegraph Technique

By A. E. THOMPSON,

Creed and Company, Ltd., London, England

INTRODUCTION

At no period in the history of the Telegraph have there been such revolutionary improvements in methods and equipment, or such rapid development and expansion in its service to the community, as during the past decade. This progress was initiated by the introduction of Teleprinter systems which, by providing instruments requiring no telegraphic skill on the part of operators, altered the whole outlook of telegraphy and stimulated development in all its branches.

All the older types of machine telegraph apparatus are rapidly being superseded by Teleprinters ; and even hand Morse working—the backbone of telegraphy for more than a century—has now been entirely abandoned in the British Telegraph service, the bulk of the traffic being handled by Creed Teleprinters. A higher grade of service is consequently being rendered and substantial operating economies are being effected ; furthermore, new and more profitable fields for exploitation have been opened up through the introduction of private wire and Teleprinter Exchange services.

With parallel developments in other countries, a " New Telegraphy " is being evolved, catering more adequately for the manifold needs of modern business. Thus the prestige of the telegraph services throughout the world is being enhanced and their prosperity restored.

The widespread adoption of Teleprinter systems has stimulated research and development, and has profoundly influenced design and construction technique by making possible the introduction of the most modern manufacturing methods for quantity production. By the application of machine tools, jigs and fixtures in the manufacture of apparatus parts, and functional gauges in the assembly of independent apparatus units, a high standard of uniformity and interchangeability has been attained and both manufacturing and maintenance costs have been substantially reduced.

In the design of Teleprinter systems there are many conditions to be fulfilled besides those necessary to meet the high standards of modern telegraph transmission. A machine may be suitable for use in telegraph offices under the constant supervision of a specialised personnel but it may not be adapted for use in ordinary business offices. A subscriber's machine must provide a dependable service at all times without requiring frequent routine adjustments or systematic lubrication. It must also incorporate certain supplementary operating facilities and be quiet in operation. In the case of news and ticker services, operation at high speeds is also required. It is primarily for providing such services with a faster and quieter instrument that the new Creed No. 10 Tape Teleprinter has been introduced.

DESCRIPTION

The No. 10 Teleprinter is a single-magnet receiving instrument, driven by an electric motor and designed for printing messages on a paper tape by means of a typewheel and ink-ribbon (Fig. 1).

It employs the Start-Stop principle for maintaining unison with the transmitting machine and operates on the Teleprinter signalling code in which each permutation of five selecting impulses is preceded by a start impulse and followed by a stop impulse (Fig. 2).

The printing speed has been set at 428 characters per minute (50 bauds), in order to work with other Teleprinter systems, but the printer can be operated at 20 per cent. higher speeds with an ample margin of safety.

In order to provide perfect printing visibility, a new and patented arrangement is used in

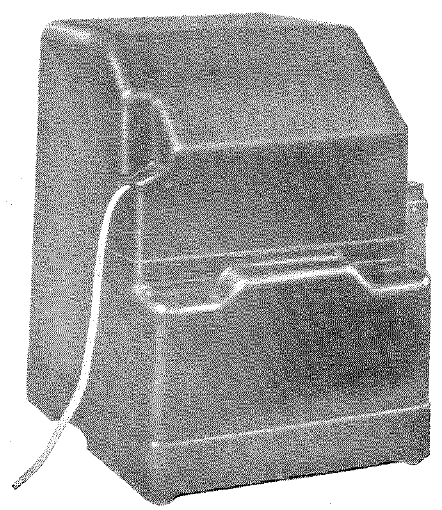

Fig. 1—No. 10 Tape Teleprinter.

which the typewheel is located underneath the paper tape with the printing hammer and typewriter ribbon above and at right angles to the tape. This arrangement also prevents the type becoming clogged since it does not come into contact with the ink-ribbon.

Contacts are provided for operating a calling bell by a predetermined signal permutation; also for controlling an "answer-back" device.

The motor is started by the first incoming signal and automatically stopped after transmission has ceased. It is located beneath the printer; both units are mounted on a patented system of spring suspension (Fig. 3).

Where a regulated frequency A.C. power supply is available, a synchronous motor can be used. With D.C. supply, or where an A.C. supply is not suitable for synchronous motors, the speed of the motor is maintained constant within ± 0.5 per cent. by a centrifugal governor requiring negligible maintenance.

The motor is coupled to the printer by a centrifugal clutch of patented construction, in order to avoid the need for high starting torque. It starts on no load, and reaches approximately half its normal speed before the clutch operates and drives the printer.

The tape roll holder is located outside the cover of the instrument and includes a simple device for preventing paper fluff entering the mechanism.

Where transmitting equipment is required, the motor and printer are mounted side by side on a base to which a keyboard with four rows of keys and an automatic "shift" insertion mechanism* can be fitted. This combined sending and receiving instrument, the Model No. 10-B Teleprinter, is shown in Figs. 4 and 5. Alternatively, a simpler keyboard with only three rows of keys, arranged for manual "shift" operations, can be furnished.

DESIGN AND CONSTRUCTION

The design requirements of the No. 10 Teleprinter were determined by the need for

* "Improved Teleprinter Keyboard Technique," by F. R. Thomas, *Electrical Communication,* July, 1934.

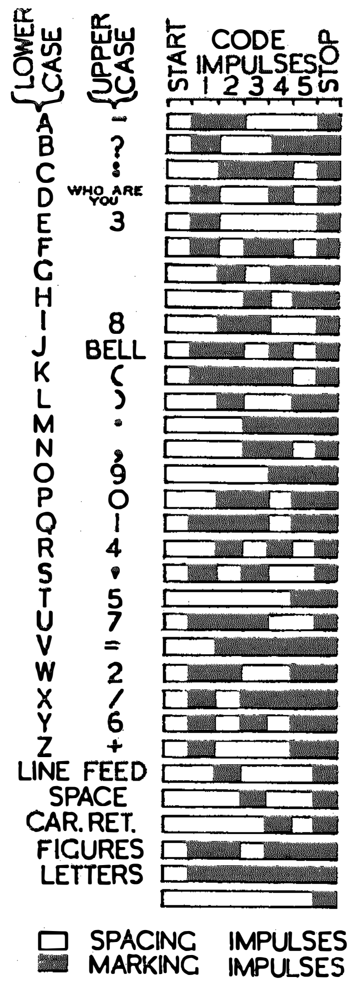

Fig. 2—Teleprinter Signalling Code.

providing a Teleprinter suitable for high speed news and ticker services. The following were among the more important considerations:

(1) Reliable service at a speed of 85 words per minute;
(2) Low manufacturing and maintenance costs;
(3) Quiet operation;
(4) Small dimensions.

The No. 10 printer is radically different in design from other Creed Teleprinter systems and incorporates mechanisms entirely new in the printing telegraph art. It employs the principle of aggregate motion. The difficulties encountered in earlier efforts of telegraph inventors to apply this simple and positive method of controlling a typewheel were successfully overcome by evolving a novel patented system of epicyclic gearwheels, operating by means of ratchet clutches in a manner which imparts harmonic motion to the individual gear-wheels and the typewheel. Hence, comparatively noiseless operation is achieved.

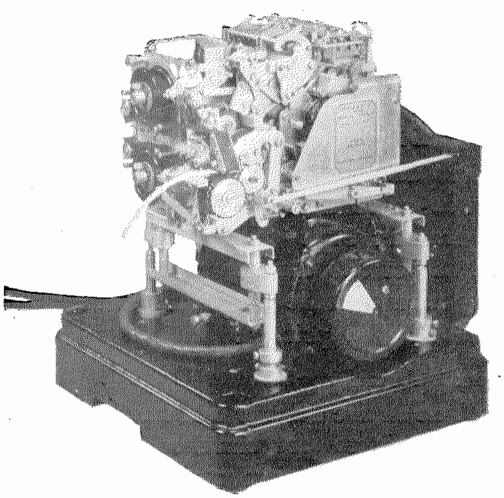

Fig. 3—No. 10 Tape Teleprinter—Cover Removed.

Manufacturing costs have been reduced by the extensive use of stampings, die castings and bakelite mouldings. Maintenance costs have been reduced by evolving mechanisms with a breakdown speed well above 100 words per minute.

The printer is designed to operate continuously for periods of 300 hours without requiring lubrication. Ball bearings and oil impregnated bearings, as well as sliding surfaces lubricated by means of oil reservoirs, are used.

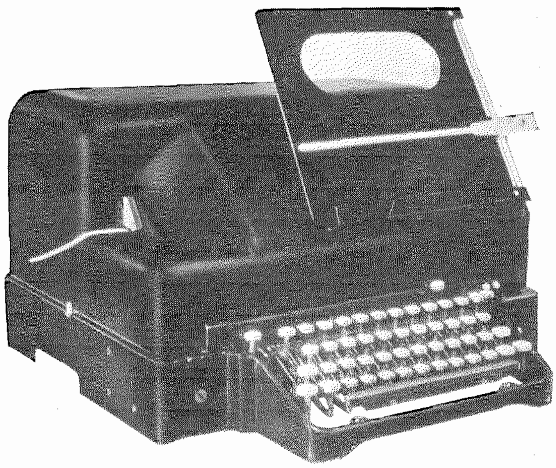

Fig. 4—Combined Sending and Receiving Instrument—10-B Teleprinter.

Seven major apparatus units are involved:

1. The Electro-Magnet;
2. The Main Frame unit, which comprises the Signal-Selecting and Distributing mechanisms;
3. The Translator, or Typewheel-control, unit;
4. The Printing unit;
5. The Ink-ribbon unit;
6. The Motor and Speed Governor;
7. The Main Base.

These units are strictly interchangeable and can be readily taken apart or reassembled. Parts which may require adjustment are easily accessible.

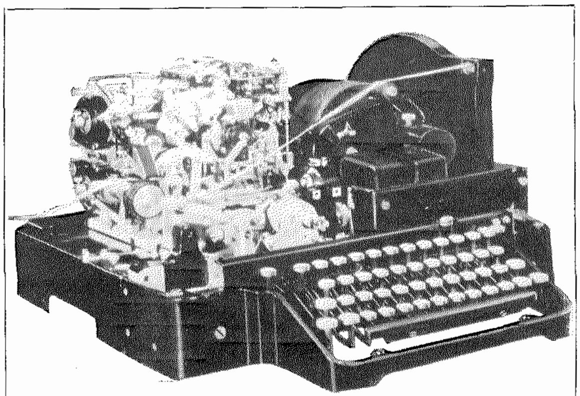

Fig. 5—Combined Sending and Receiving Instrument—10-B Teleprinter—Cover Removed.

The printer in block schematic form is illustrated in Fig. 6.

Operating Margin

In Teleprinter communication the signal elements representing each character are transmitted continuously in accurately timed sequence, but in passing over the telegraph channel they may be deformed by unequal attenuation of the high frequency components or other causes. In order that the receiving instrument may correctly identify deformed signal elements, the selecting mechanism, which tests their polarity, must utilize the smallest possible part of the mid-portion of each element. The smaller the part utilized, the greater is the "margin" of the printer. For example, where the selecting period is 6 milli-seconds, and the signal elements being transmitted are of 20 milli-seconds duration (50 bauds), the margin of the receiving instrument is $\pm$ 35 per cent. This is the value fixed by the C.C.I.T. for the margin of Teleprinter systems operating over international lines.

The No. 10 Teleprinter requires a selecting period of only 3.2 milli-seconds; therefore, its margin is $\pm$ 42 per cent.

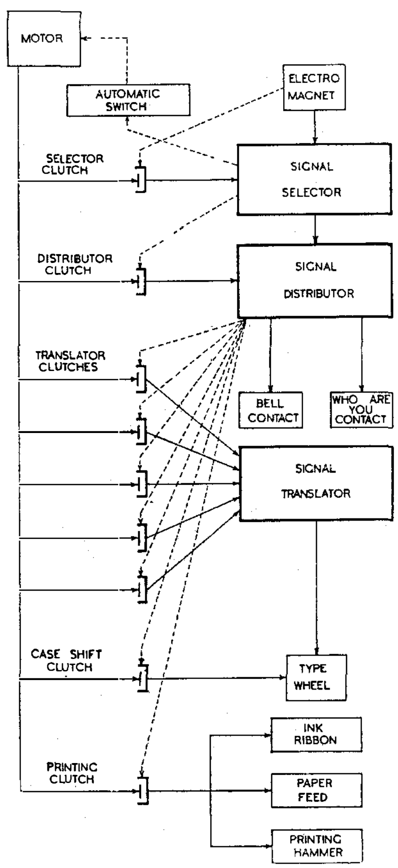

Fig. 6—Block Schematic of No. 10 Teleprinter.

Signal Selector

High precision in identifying the polarity of distorted signal elements is achieved by means of a knife-edge member which, at the instant of selection, strikes a co-operating knife-edge on a fork-shaped extension of the electro-magnet armature and locks it in a position corresponding to the signal element received. A sword selecting member then strikes the fork and is tilted about its pivot in a clockwise or an anti-clockwise direction, according to which arm of the fork it engages. Having thus identified the signal element, the sword rises and transfers its setting to the control member of the distributor.

Signal Distributor

This distributor unit comprises a camshaft with co-operating control levers for positioning five transfer bars. These are set successively into one of two endway positions according to the polarity of the five signal elements of the character to be printed.

Signal Translator

When all five transfer bars have been set, they are caused to act simultaneously upon the translating mechanism composed of five half-revolution ratchet clutches which drive the planet pinions of a system of five epicyclic gear-wheels. The latter are interconnected in a manner such as to form an aggregate-motion system which translates the permutations stored by the transfer bars into corresponding rotary movements of the typewheel, thus bringing the selected character to the printing position.

Printing and Tape Feed

The printing unit is controlled by a cam-operated lever which also actuates a correcting device for ensuring that the selected character on the typewheel is accurately positioned.

The printing hammer is located at the end of a heavy bar. A co-operating helical spring causes the hammer to deliver a sharp and powerful impact.

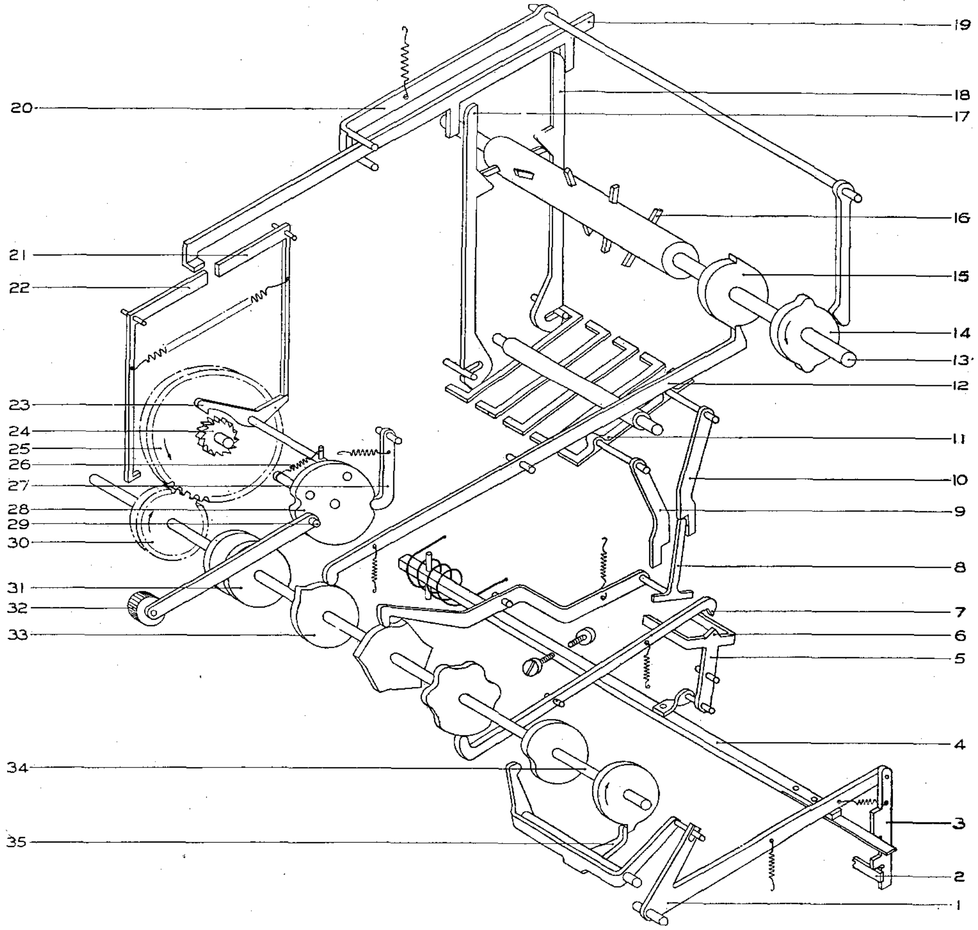

Fig. 7—Selector and Distributor.

During the return movement of the printing unit, a pawl and ratchet wheel are actuated to move the tape forward.

"Shift" and Subsidiary Functions

The upper and lower case characters are arranged in two rows on the typewheel. The means provided for bringing the selected row of type underneath the printing hammer include a half-revolution clutch which causes the typewheel to be moved axially according to the setting of the transfer bars.

Feeding of the paper tape and the operation of the printing hammer do not occur when the typewheel is set in a functional position, such as figure or letter shift.

Closure of the contacts for operating the calling bell is under the direct control of the transfer bars. Similar means are provided for releasing the "answer-back" mechanism incorporated in the keyboard of the No. 10-B model.

Automatic Motor Switch

The mechanism of the motor switch is simple and does not impose any load on the armature of the electro-magnet. It comprises a system of levers, a tripping latch and a control cam. Immediately the first code impulse is received, the releasing bellcrank of the selector control-shaft trips the latch-arm of the motor control unit from its stop and so causes the motor to be switched on. Subsequent "Start" impulses prevent the latch-arm from re-engaging its stop, the motor thus remaining in operation.

When transmission has ceased, the latch-arm re-engages its stop and causes a slowly rotating cam to actuate a lever for switching off the motor.

Ink-ribbon Feed

The ink-ribbon is carried around the printing unit and passes between the printing hammer and the paper tape. It is automatically fed forward, the reversing mechanism being positive in operation. The spools are conveniently located one above the other on the left side of the printer and the ribbon can, therefore, easily be changed.

Cover and Base

The No. 10 Model is totally enclosed in a silencing cover, a small strip of glass being provided between the printing point and the tape outlet. The temperature inside the cover is kept at a low level by fan-driven air circulation, the heat from the motor being driven out through a special chamber in the base of the printer.

The cover of the No. 10-B model is hinged at the back and can be raised or lowered without breaking the tape. It is cut away at the printing point to ensure good visibility and to facilitate tearing off the tape. The rear side of the base provides a convenient apparatus compartment for the wiring, resistances and condensers, and is protected by a metal plate which can be taken off by removing two screws.

PRINCIPLE OF OPERATION

Referring to Fig. 7, the armature 4 of the electro-magnet is connected by a link to the selecting fork 5 and also carries an extension co-operating with the starting latch 3. When a "Start" impulse is received, the armature pushes the starting latch away from the supporting member 2. This causes the lever 1 to withdraw the control detent 35, thus releasing the camshaft 34 which is frictionally driven by the clutch 31 and allowed to make one revolution.

Selection

The movements of the armature rock the selecting fork about its pivot and are identified when the knife-edge 7 strikes the co-operating knife-edge 6, thereby locking the fork until the selecting sword 8 has been set. The sword strikes five times during one revolution of the camshaft. In its downward travel, its horizontal arm engages one arm of the selecting fork, according to the position of the armature, and the vertical arm of the sword is thus deflected either to the left or right.

Distribution

Upon being raised, the sword engages one or other of the distributor control links, 9 or 10, to position the rocking member 11.

When the right-hand side of the rocking member is uppermost, its arms embrace the lower ends of the five operating levers 18 (one only is shown) ; and, when tilted in the opposite direction, its arms embrace the lower ends of the five operating levers 17.

The cam 33, by means of the lever 12 and detent 15, releases the frictionally-driven shaft 13 which turns through half a revolution during the five settings of the rocking member. The cams 16 pass the contact faces of the two sets of operating levers, 17 and 18, in sequence and cause these levers to be spread outwards. When the levers are restrained at their lower ends by the rocking member, they are moved outwards at their upper ends, carrying with them their associated transfer bars 19.

Translation

When the transfer bars have all been positioned, their supporting member 20 is actuated by the cam 14. Each transfer bar thereupon depresses one or other of its associated detents, 21 or 22, to enable the driving pawl 23 to engage the ratchet wheel 24. The latter is fastened to the gear, 25, which is driven continuously by gear 30.

The driving disc 28, carrying the pawl 23, spring 26 and crank pin 29, is mounted coaxially with gear 25 and rotates with it when the pawl and ratchet wheel are in engagement. The pawl 23 is held against the tension of the spring 26 by the retaining lever 27.

In the position shown, this pawl is held out of engagement with the ratchet wheel by the detent 21. Upon the latter being actuated by its transfer bar, the pawl engages the ratchet wheel and the driving disc makes half a revolution. It is stopped when the pawl is thrown out of engagement with the ratchet wheel by striking the opposite detent 22.

The driving discs, 28, actuate the planet-pinions 32 of five epicyclic gear units. Referring to Fig. 8-A, these units each comprise an inner sun-gear, a planet-pinion and an outer sun-gear. The outer sun-gear of each unit is connected to the inner sun-gear of the next unit, the gear ratios being such that the angular movements of the outer sun-gears are halved when transferred from one unit to the next.

The fifth sun-gear is connected to the typewheel by intermediate gears so proportioned that, when driving disc 5 rotates 180 degrees, the typewheel also rotates 180 degrees (Fig. 8-B).

The step down in angular movement at each stage is indicated in Fig. 8-C. Driving disc 4 has rotated 180 degrees but the resulting movements of the planet-pinion and sun-gear of the fifth unit are such that the typewheel has only rotated 90 degrees.

Similarly, when the driving disc No. 3 is actuated, the step down at the succeeding stages results in the typewheel being rotated through 45 degrees (Fig. 8-D).

When driving disc No. 2 is actuated, the typewheel rotates one-sixteenth of a revolution; and, when No. 1 is actuated, the typewheel rotates one-thirty-second of a revolution, or one character. It will be seen, therefore, that by operating the five driving discs in accordance with the permutations of the five-unit code, it is possible to select any one of thirty-two positions of the typewheel.

Among the advantages of this method of controlling the typewheel are the following:

(1) Since the connecting rods of the planet-pinions are driven by rotary clutches, the movements are all harmonic; hence, the typewheel is accelerated and brought to rest without shock;

(2) The typewheel is positioned positively;

(3) In passing from one code permutation to another, the mechanical movements are reduced to a minimum since only the particular code elements actually changed affect the mechanism;

(4) The typewheel can rotate in either direction and so moves directly from one character to the next;

(5) The mechanism is practically noiseless in operation.

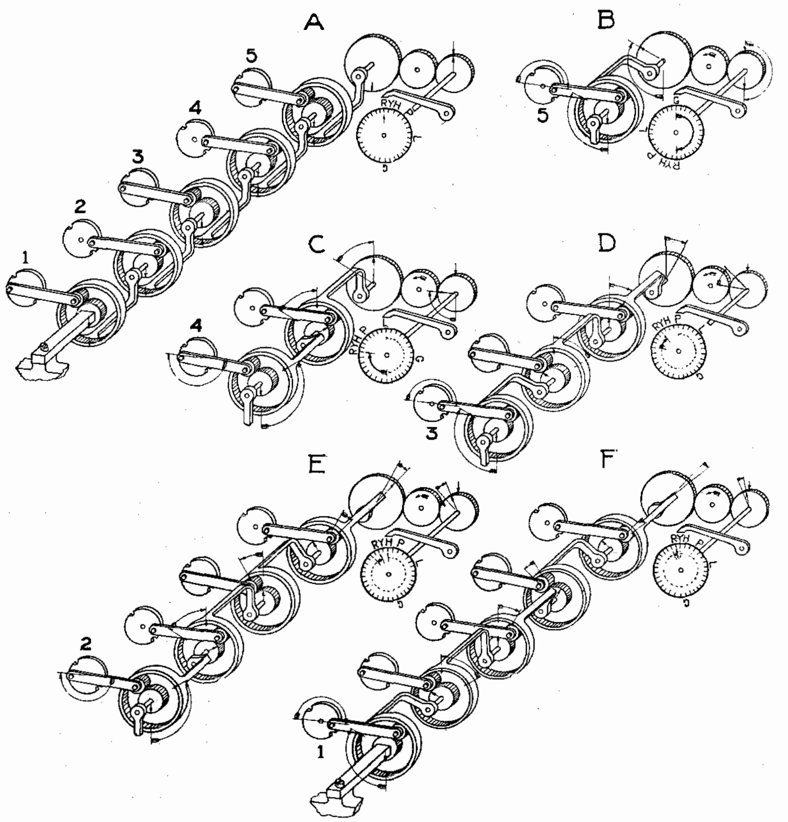

Fig. 8—Epicyclic Translators.

Printing

Referring to Fig. 9, when the selected character on the typewheel has been brought to the printing position, the cam 39 actuates the lever 40. This raises the end of the bellcrank 43, thereby extending the spring 42 and causing

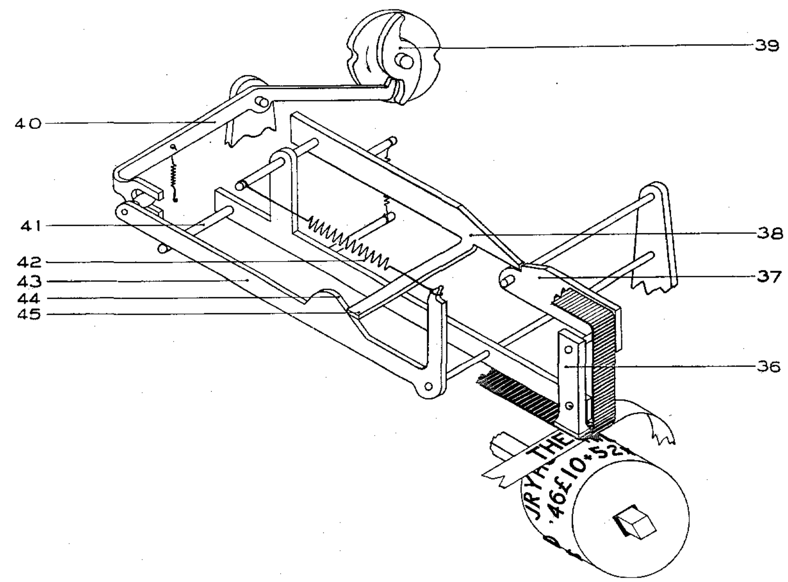

Fig. 9—Printing Hammer.

tion in 130 m.s. and is then arrested by the stop signal. The commencement of rotation of this camshaft is shown near the middle of the start signal, thus allowing for the time required for the armature to release the camshaft detent and for the camshaft to accelerate and reach full speed.

The first cam on the signal selector actuates the knife-edge member which locks the armature while the sword is being positioned. It will be observed that the transit from the free to the locked condition occurs at the mid-portion of each code impulse.

While the armature is locked the sword descends and, having been set in accordance with the polarity of the code impulse, it rises again and positions the distributor rocking member. The upward movement of the sword is so timed that it is just clear of the armature at the moment the latter is unlocked.

The signal selector also carries a cam for effecting the release of the distributor shaft. The latter carries five cams which are timed to position the five transfer bars during the successive periods when the sword is in engagement with the rocking member.

The distributor camshaft carries two other cams: one adapted to actuate the transfer bars to release the clutches of the epicyclic unit immediately after the fifth transfer bar has been set; the other, to release the printing camshaft 20 m.s. later.

The rotation of the typewheel from one printing position to the next occupies 118 m.s. During this period the cam on the printing unit extends the printing hammer spring and retracts the paper feeding pawl, simultaneously advancing the corrector tooth towards the corrector wheel. When the typewheel comes to rest the corrector tooth enters the corrector wheel, thus

the trigger 45 to ride up on the cam surface 44. The latch 38 is thus forced out of engagement with the abutment 37 and the stressed spring instantly causes the hammer 36 to strike the ink-ribbon and the paper tape against the typewheel.

The mechanism is re-set by the depression of the pin 41 upon the return of the bellcrank 43.

TIMING OF OPERATIONS

The sequence of the mechanical operations and their time intervals are indicated in Fig. 10. The movements of the armature of the electromagnet correspond to the letters Y and T, it being assumed that the code impulses are received free of distortion and are therefore square-topped. The effect of distortion is to displace the moment of transit of the armature, either forwards or backwards, and it will be observed that, provided the displacement does not encroach upon the middle portion of the impulses indicated by the shaded vertical lines over a period of 3.2 m.s., the signal selector will correctly identify the polarity of the impulses.

The first movement of the armature, in responding to the start impulse, releases the signal selector camshaft, which completes one revolu-

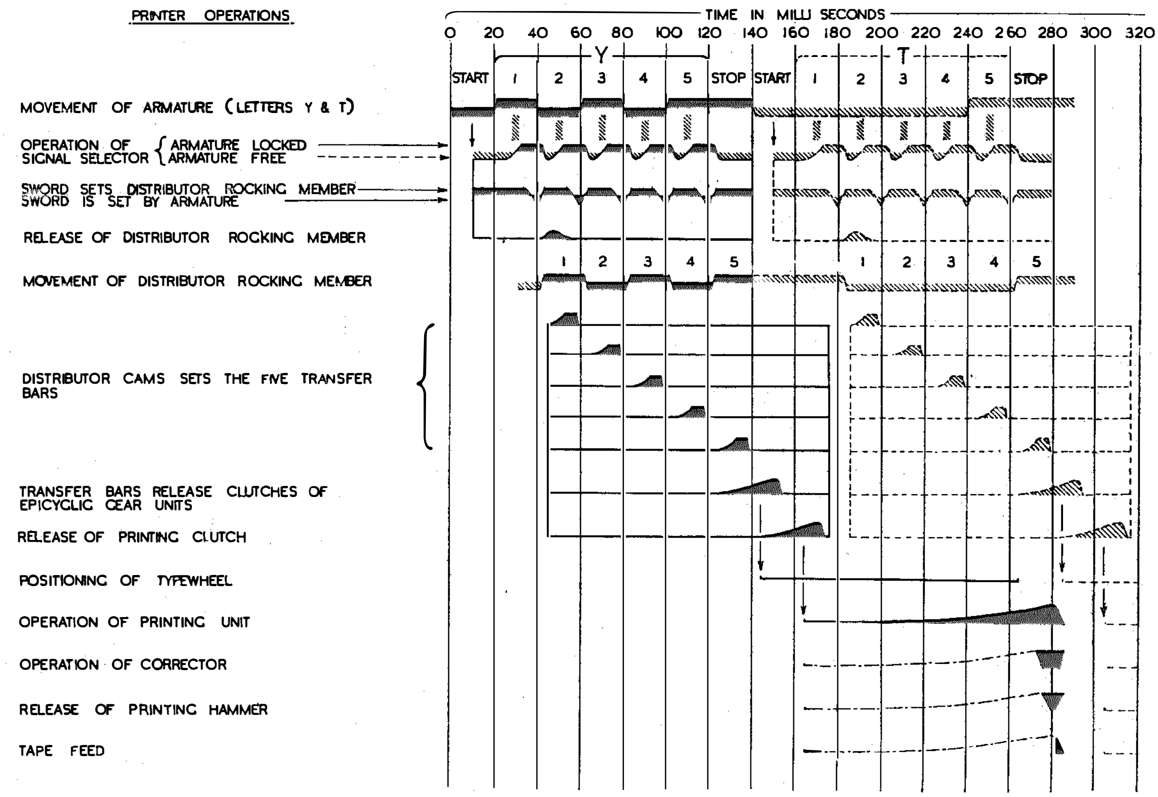

Fig. 10—*Sequence of Mechanical Operations and Time Intervals.*

holding the typewheel in the precise printing position. Whilst the correcting action is taking place, the printing hammer is released to strike the paper immediately the typewheel is accurately positioned. The return movement of the printing hammer takes place under the action of the retractile spring after the cam has come to rest. This movement also acts to feed the paper.

It will be observed that there is complete overlap between the operation of the signal selector and the translator unit. While the epicyclic gears are rotating to advance the letter Y to the printing position, the succeeding letter T is being received and stored by the five transfer bars in readiness to release the clutches of the epicyclic gears immediately the letter Y has been printed.

CONCLUSION

The mechanisms incorporated in the No. 10 Teleprinter are the outcome of several years of continual study and experiment undertaken to evolve the design requirements of a printer that is not only capable of operating at higher speeds than existing types, but also of providing quieter operation and more economical maintenance. Wide factors of safety have been provided throughout the machine, and the reliability and durability of the component mechanisms and parts have been proved by rigorous tests. It may be confidently anticipated, therefore, that this new instrument will fulfil all the requirements of a highly modern Tape Teleprinter, suitable for use by Telegraph Administrations and private companies, as well as by high speed news and ticker services.

Carrier Rediffusion

By A. WIESSNER, Dipl. Ing.,

Rediffusion Laboratories, C. Lorenz A.G., Berlin-Tempelhof, Germany

CARRIER rediffusion, in its present technical embodiment, represents an economical and reliable communication facility, comparable in efficiency and field of application with radio broadcasting itself. It may be expected to find application primarily in locations where difficulty is encountered in providing adequate radio broadcast facilities.

Carrier rediffusion employs the telephone network for the dissemination of radio broadcast programmes, but need not be confined to telephone subscribers. Simple means are available for extending the service to non-telephone subscribers.

Substitution for radio broadcast reception, on the part of the user, does not involve any change in the operation of the receiving set. Nevertheless, improvement in reception and reproduction results.

C. Lorenz, A.G., based on extensive research

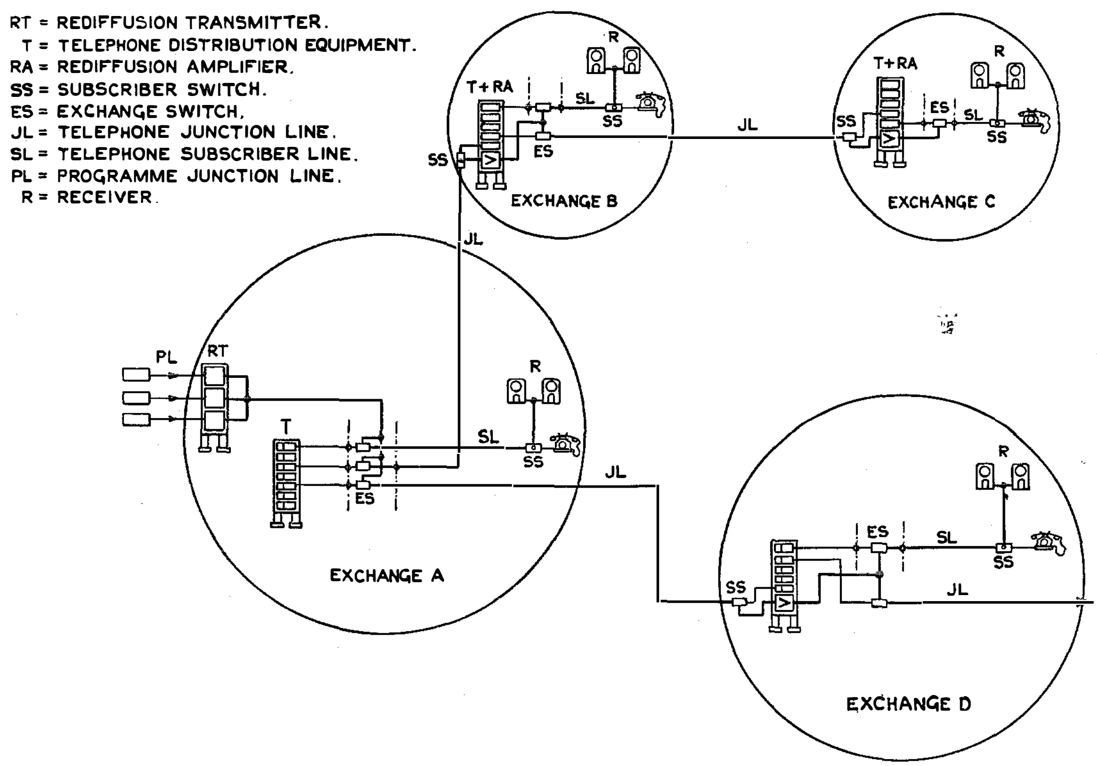

Fig. 1—Schematic of Rediffusion Network.

and development in radio transmission and reception, as well as in wire transmission, has developed carrier rediffusion equipment which has proved eminently suitable for practical use. The equipment and its application are described below.

Rediffusion Transmitter

As compared to radio transmission, particularly in the long wave range, telephone lines provide superior transmission conditions. It is consequently practicable, in the case of carrier rediffusion, to employ comparatively low power transmission and still provide sufficiently high potentials for reliable and interference-free reception. As is generally recognised, interference is dependent on the signal-noise ratio. A few watts has been found to be the appropriate high frequency output for the rediffusion transmitter, and is sufficient to cater for the participating rediffusion subscribers of the associated telephone exchange as well as for the amplifiers located in neighbouring exchanges comprised in the rediffusion distribution network.

In a telephone cable network, the unamplified rediffusion working range is equivalent to approximately 6 to 8 km. of subscribers' cable, and is sufficient in all cases to serve adequately

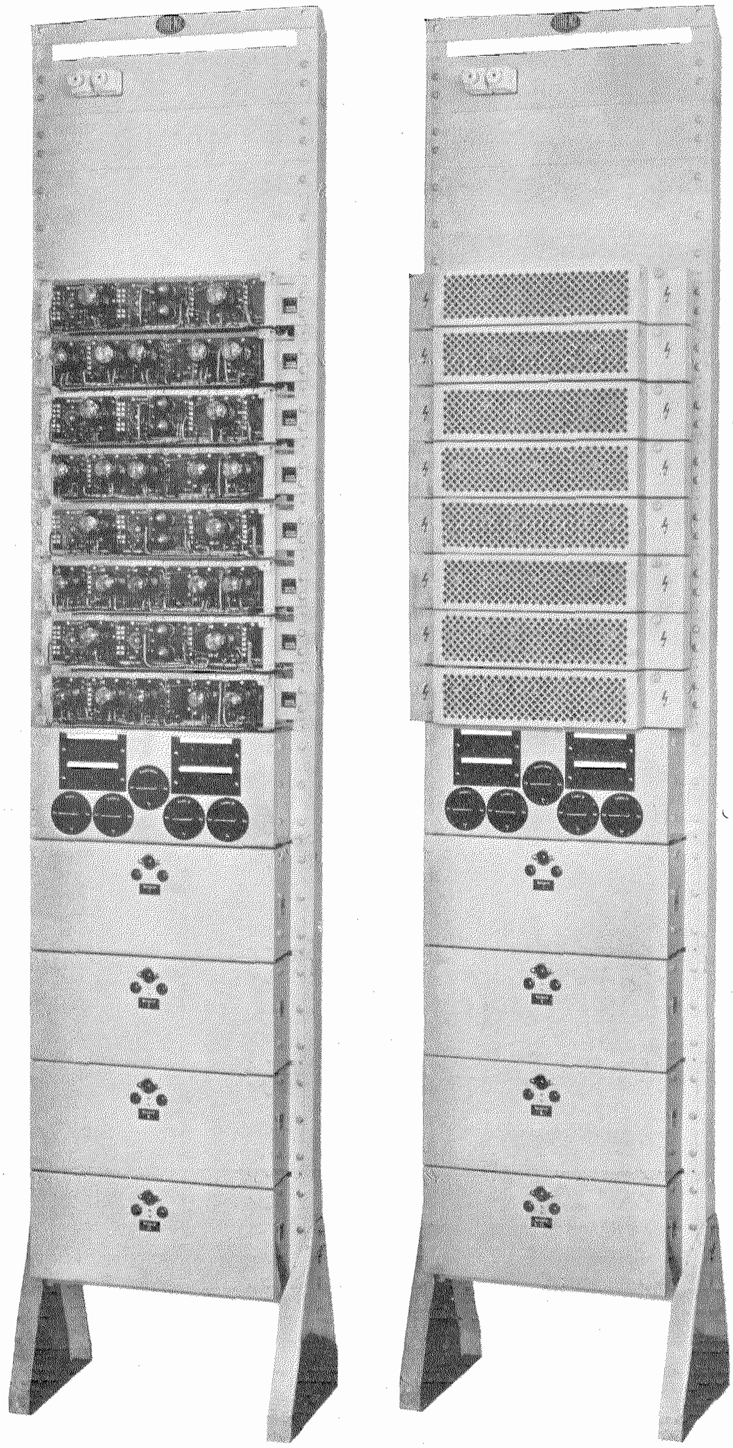

Fig. 2—Rediffusion Transmitter Bays Equipped for 4 Carrier Frequencies.

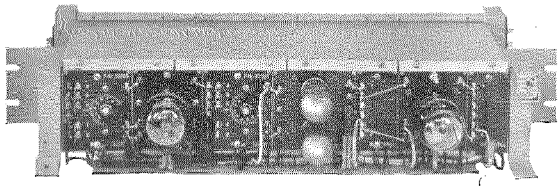

Fig. 3—Rediffusion transmitter.

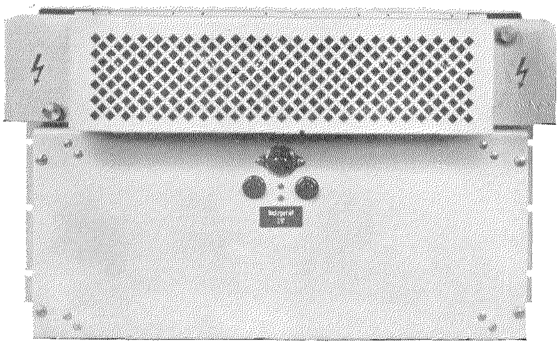

Fig. 4a—Amplifier and Mains Equipment.

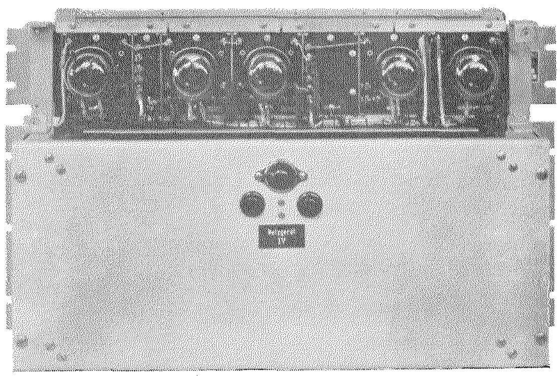

Fig. 4b—Amplifier and Mains Equipment—Cover Removed.

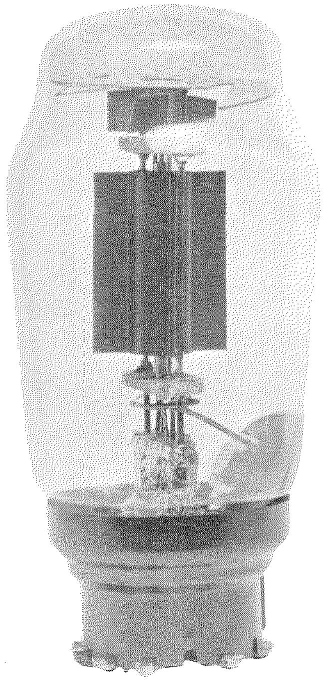

Fig. 5—15-Watt Triode.

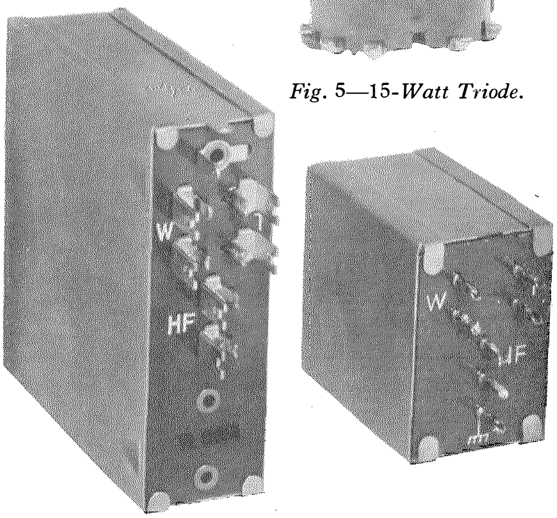

Fig. 6—Two Types of Central Office Filters.

a local central office area. For serving rediffusion subscribers in neighbouring exchanges and providing trunks to more distantly located exchanges, all exchanges are provided with rediffusion amplifiers.

Grouped near the rediffusion transmitter, according to the size and density of the area, are the amplifiers necessary for serving the local networks. A schematic of a rediffusion network is illustrated in Fig. 1. The point of installation of a rediffusion transmitter is chosen primarily with reference to convenience in making arrangements for disseminating the broadcast programmes to the distributing lines.

Carrier rediffusion may be regarded as a supplementary telephone service and, accordingly, the transmitter is designed with a view to installation in a telephone exchange. Depending on the number of programmes to be transmitted, a maximum of four transmitters may be mounted on a transmitter-rack (Fig. 2). Additional transmitters may be mounted on a second rack when the number of programmes to be furnished is greater than four. Experience has shown that the number of programmes that can be furnished simultaneously is not limited by a dearth of suitable carrier frequencies but, rather, by an insufficiency of available broadcast programmes.

Each of the transmitters accommodated on the rediffusion transmitter rack produces a carrier frequency within a range of 150 and 300 kc. The generator and modulator unit of a transmitter is illustrated in Fig. 3. Within the frequency range of 150 and 300 kc., the carrier frequency of the generator stage can be set as required by resoldering coil terminal points and varying the condenser adjustment.

In the modulator stage, which consists of two diodes, the carrier and programme frequencies are superimposed so that the modulator output includes the carrier frequency and two side-bands. Distortion-free modulation up to 80 per cent. is ensured. Assuming a volume range of 1 : 100 (40 db), the percentage modulation will vary from 0.8 per cent. to 80 per cent.

From the modulator stage the carrier fre-

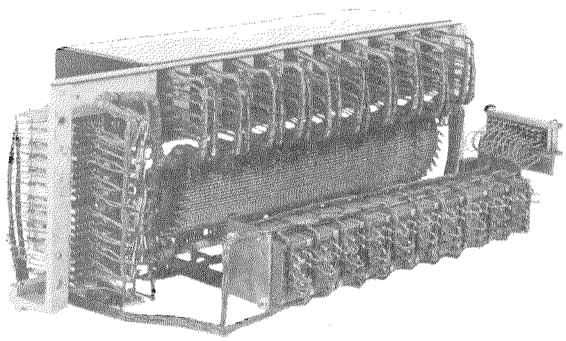

Fig. 7—Preselector Switches with Exchange Filters.

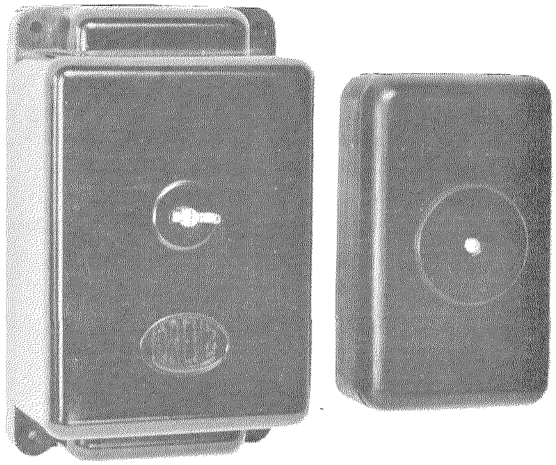

Fig. 8—Subscribers' Rediffusion Filters.

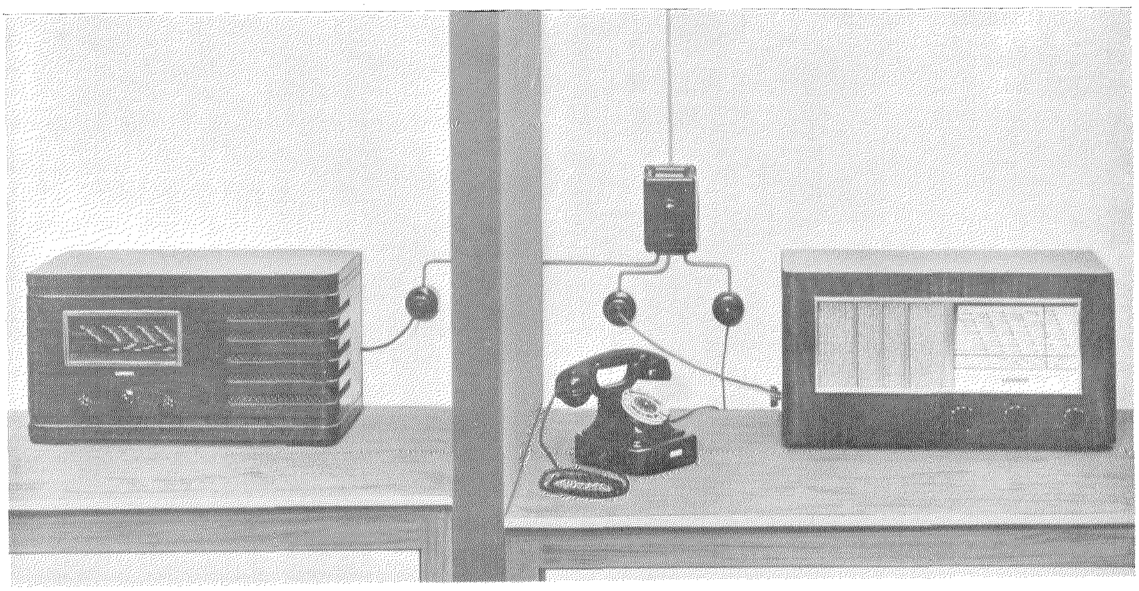

Fig. 9—Example of Arrangement for Rediffusion Reception.

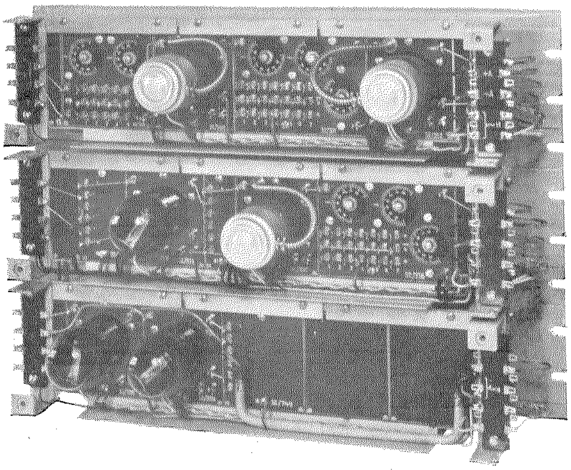

Fig. 10—Single Channel Amplifier (Carrier Output, 2.5 W).

quency with both side-bands is transmitted to the power amplifier (Fig. 4) comprising four voltage amplification stages and one push-pull power stage. The available carrier output is 2.5 watts.

The power supply for the amplifier, as well as for the generator and modulator stages, is furnished through an A.C. mains equipment. The latter draws 180 watts from the mains and is suitable for use in connection with the mains voltages customarily encountered. A separate mains equipment, incorporated in the same rack, is furnished for each transmitter. The same type of valve, a 15-watt triode (Fig. 5), is used uniformly in all the amplification stages of the transmitter.

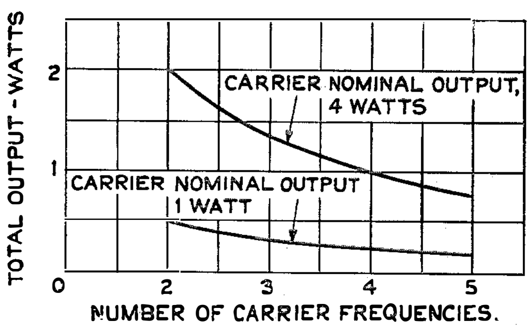

Fig. 11—Total Power Output of the Broad Band Amplifier as Function of the Number of Channels Used.

The carrier frequencies from the power amplifiers are combined by means of low loss filters and impressed on the telephone network.

All normal telephone lines are suitable for the transmission of rediffusion frequencies provided their transmission characteristics have not been artificially restricted by loading. In order to ensure freedom from interference in the case of lines intended for both rediffusion and telephone purposes, all such lines are provided with rediffusion filters. The rediffusion carrier frequencies are transmitted over the telephone line through these filters, types of which are shown in Fig. 6. The larger type is mounted on special racks; the smaller is used as a supplement to the exchange equipment illustrated in Fig. 7.

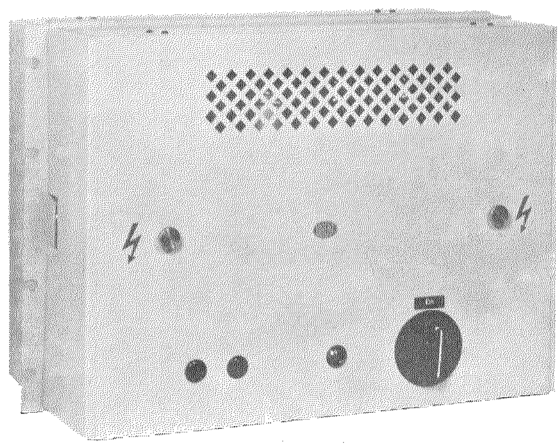

Fig. 12—Broad Band Amplifier (Carrier Output, 1 W).

At the end of a telephone subscriber's line the rediffusion frequencies are passed through a receiving filter and terminal box to one or more receivers. Thus the rediffusion subscribers in a block of flats or in adjacent buildings may be supplied with a rediffusion service over a single telephone line.

Fig. 8 shows a form of subscribers' rediffusion filter which is mounted in a moisture-proof housing. Special wiring must be run from the receiving filter to the various receiving sets. The expense involved in providing this wiring does not, however, exceed that for supplying simple antennae in ordinary radio broadcast reception.

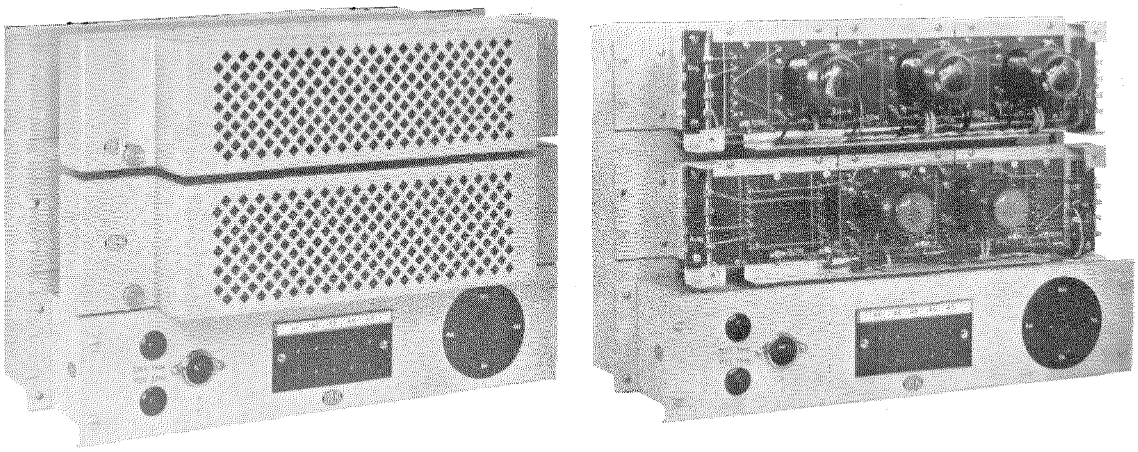

Fig. 13—*Broad Band Amplifier (Carrier Output,* 4 *W).*

This method of multi-reception corresponds in principle to the community antenna employed in radio broadcast reception. The fundamental arrangement involved in rediffusion reception is illustrated in Fig. 9.

Rediffusion Amplifier

Two types of amplifier are available for the amplification of rediffusion carrier frequencies:

1. Channel amplifier, in which the carrier frequencies are amplified separately;
2. Broad band amplifier, in which the carrier frequencies are amplified in combination.

Fig. 10 illustrates a channel amplifier with a carrier output of 2.5 watts. Where channel amplifiers are used, a separate amplifier must be provided for each carrier frequency; each amplifier is capable of frequency adjustment, as

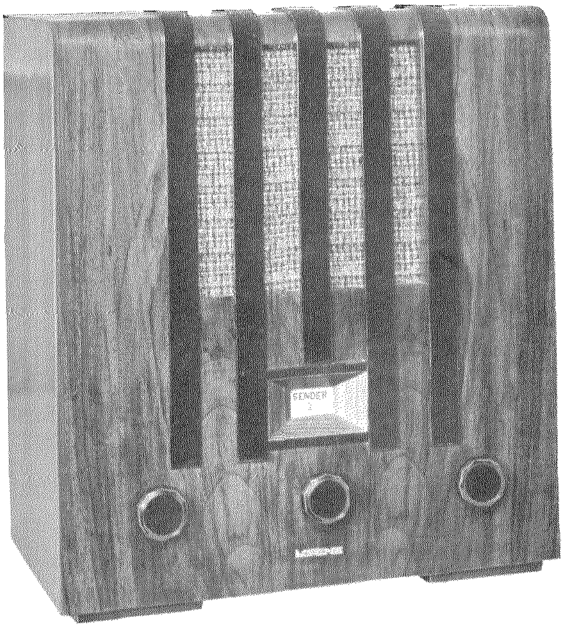

Fig. 14—*Special Rediffusion Receiver.*

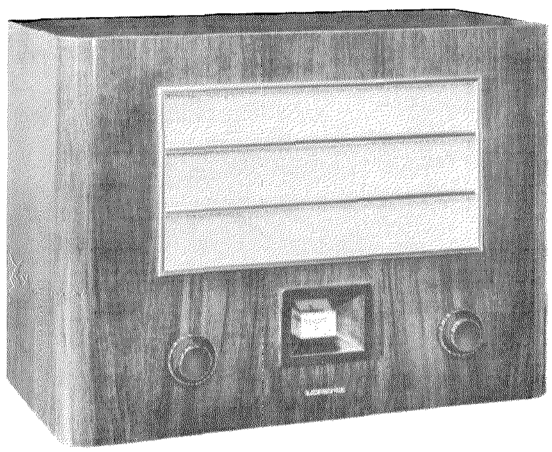

Fig. 15—*Special Rediffusion Receiver.*

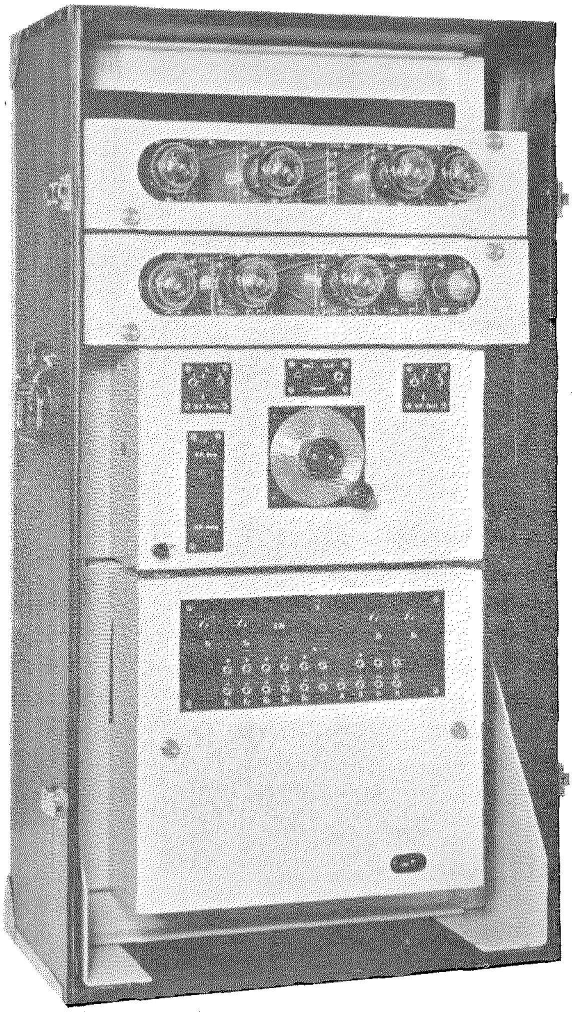

Fig. 16—Rediffusion Test Transmitter.

TABLE I

Rediffusion Transmitter Output for a Carrier Frequency of 250 kc.

Distance (km.)	Transmitter Output for One Line (mW). 0.8 mm. Conductor Cable	1.4 mm. Conductor Cable
1	0.017	0.008
2	0.073	0.015
3	0.310	0.029
4	1.34	0.064
5	5.7	0.10
6	24.4	0.16
7	100.0	0.40
8	400.0	0.78
9		1.53
10		2.94

required, within the rediffusion frequency range. The associated A.C. mains equipment consumes 150 watts.

The output obtainable from a broad band amplifier depends on the number of carrier frequencies and is indicated in the curves of Fig. 11 for two amplifiers of different size (Figs. 12 and 13). The curves show the sum of the outputs for two or more carrier frequencies. The total output may be distributed evenly or—with a view to the line characteristics—differently over the individual carrier frequencies. The broad band amplifiers, according to size, are associated with A.C. mains equipment consuming 100 or 150 watts.

The transmitter output for varying distances and typical cable conductor diameters is shown in Table I. These output values were determined with due regard to freedom from interference as well as the transmission characteristics of the telephone lines. While the output requirements vary greatly with length of line, it will be apparent that even in the extreme case the power consumption is sufficiently low to be economically efficient.

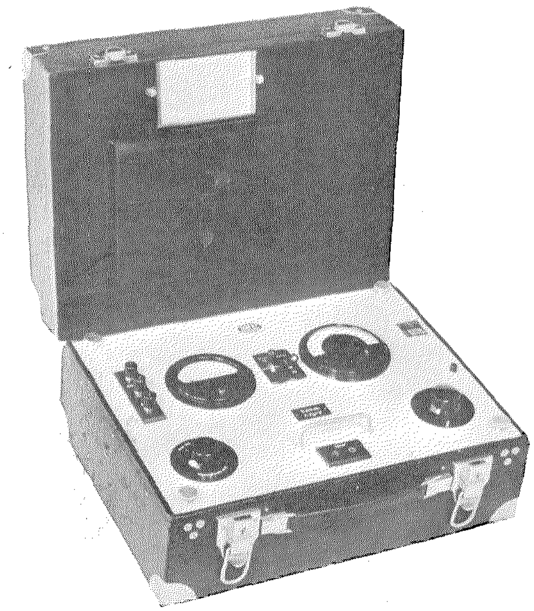

Fig. 17—Rediffusion Test Receiver.

From the number and length of lines to be connected to one transmitter or amplifier the size and number of amplifiers required can readily be determined.

Receivers

The carrier frequencies used in this rediffusion system are such that they can be received on any long wave radio receiver.

The "antenna" and "earth" terminals provided on all broadcast receivers are connected to the rediffusion terminal box. Programmes are selected by tuning as in broadcast reception. A sufficiently large receiving potential ensures freedom from interference. A detector receiver is sufficient for headphone reception.

The development of this carrier rediffusion system included the construction of special "rediffusion receivers," by means of which it is possible to reproduce faithfully the broad modulation band initially impressed on the line. Since it was possible to make the high frequency component of this receiver simpler than is required for high-class broadcast radio receivers, most weight was laid on the low frequency component of the receiver. Figs. 14 and 15 illustrate two different special rediffusion receivers, one embodying a straightforward circuit and the other the heterodyne principle. Both receivers are equipped with a 4-watt loudspeaker and can be employed as control receivers for the additional connection of amplifiers and high-power loudspeakers.

Test Transmitter and Receiver

A portable rediffusion test transmitter (Fig. 16) has been developed for preparatory tests on rediffusion networks as well as for temporary installation. The carrier frequency of this transmitter can be varied within a range of 80 and 300 kc.; its high frequency output is 10 watts maximum. With a high frequency output of 2.5 watts, 80 per cent. modulation can be obtained. The transmitter is arranged for A.C. mains operation.

A rediffusion test receiver, capable of providing direct readings of current and voltage values and suitable also for headphone reception, has been developed. This instrument (Fig. 17) is housed in a convenient carrying case, which includes the filament and anode battery.

MR. FRANK GILL, O.B.E., Hon. M.I.E.E.

THE Council of the Institution of Electrical Engineers have elected Mr. Frank Gill, O.B.E., M.I.E.E., an Honorary Member of the Institution.

The award is made to "a person who is distinguished by his work in Electrical Science or Engineering," and the present number of Honorary Members is 14.

Mr. Gill is a Vice-President of the International Standard Electric Corporation, and is associated with other companies in the I. T. and T. System.

Electrical Properties of Aerials for Medium and Long Wave Broadcasting

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EDITOR'S NOTE : *The exigencies of space necessitate distribution of this article over two issues of ELECTRICAL COMMUNICATION. In the present instalment are given Sections* (1) *to* (13) *inclusive, together with the complete Bibliography ; the remaining sections, which contain closely related theoretical and practical material, will be published in the July issue.*

The object of this paper is to present in collected form information hitherto either unpublished or else scattered through many different publications, some of which are not readily accessible.

The first part of the paper is theoretical, and is devoted chiefly to an exposition of the treatment of an aerial as a dissipative transmission line, and to the statement of formulae whereby the radiation and driving impedance of an aerial can be estimated. Particular attention is given to anti-fading aerials of the mast and loaded-top types. In the case of mast aerials the problems of the current distribution and vertical radiation polar diagram are analysed in detail. An approximate method is given for the derivation of these properties on the basis that the actual current can be replaced by a wattless component as assumed on the conventional theory of sinusoidal distribution, together with an energising current in time quadrature with the wattless current.

The second part is devoted to the citation of measured values of current distribution, impedance, etc., from various sources, on both real and on model aerials, and to the discussion of the light thrown by these measurements on the applicability of the general method of treatment previously described.

PART I: THEORETICAL

(1) General Method of Attack

In considering the electrical suitability of an aerial the engineer requires to know three things :

(*a*) The horizontal " figure of merit," i.e., the field strength set up at 1 km distance in the ground plane for 1 kW input to the aerial ;

(*b*) The vertical polar diagram, for the purpose of estimating the probable fading-free radius ;

(*c*) The impedance at the driving point, by which is determined the aerial coupling circuit.

Before any of this required information can be obtained, it is necessary to know the current distribution over the aerial both in amplitude and in phase.

There has not yet been evolved an exact mathematical solution by which the current distribution can be obtained. Moreover, the problem is so complicated that when a solution is found it will probably be useless for engineering computation purposes. It is possible, however, to obtain a fairly good approximation to the truth by assuming that the aerial can be treated as a dissipative transmission line with appropriate termination. Broadcasting aerials are usually terminated either in an open circuit or in a reactance such as is formed by an end capacity disc alone or in series with an inductance.

Making the assumption that the aerial can be treated as a transmission line the current distribution is then defined by the effective length of the aerial, its characteristic impedance Z_0 and its complex propagation constant $P = A + jB$.

(2) Determination of Characteristic Impedance

For a uniform aerial (that is straight wire, cage, or tower of uniform cross-section) of non-magnetic material, the characteristic impedance $Z_0 \simeq \sqrt{\frac{L_0}{C_0}}$, where L_0 and C_0 are the inductance and capacity per unit length. The phase change component of the propagation constant is

$$B = \omega\sqrt{L_0 C_0} = \frac{2\pi}{\lambda} \text{ radians/unit length.}$$

From these relations it follows that

$$Z_0 = \frac{3\ 333 \text{ ohms}}{C_0\ \mu\mu\text{F/metre}} = \frac{1\ 010 \text{ ohms}}{C_0\ \mu\mu\text{F/foot.}} \quad \ldots . (1)$$

Professor Howe has calculated the value of C_0 for various types of wire aerial and, by means of the relation quoted above, Z_0 may be determined from the capacity figures. Tables I and II are based on the tables in his article on the calculation of aerial capacity.[1]

TABLE I

CHARACTERISTIC IMPEDANCE OF SINGLE WIRE AND FLAT MULTI-WIRE AERIALS

No. of wires	Length/spacing or (single wire only) Length/ (radius of wire × 1 000)				
	20	50	100	150	300
1	571	625	665	687	732
2	366	442	462	486	527
3	289	342	384	408	450
4	246	299	342	365	407
5	220	273	312	336	377
7	184	235	274	298	340
10	153	202	240	262	304
12	138	187	226	247	289

For flat aerials (or portions of aerials, such as a flat top), the characteristic impedance can be estimated from Table I; in the multi-wire case, this assumes that the ratio spacing/wire radius remains constant at 100. For cage aerials

TABLE II

CHARACTERISTIC IMPEDANCE OF CAGE AERIALS

No. of Wires	l/d=20	l/d=50	l/d=100	l/d=150	l/d=200
4	239	279	322	345	360
6	191	238	276	298	313
9	165	210	246	266	282
12	152	198	231	250	264

l = length of cage.
d = spacing between adjacent wires.

[1] For numbered references see Bibliography at end of present instalment.

the characteristic impedance can be estimated by reference to Table II which gives Z_0 for various ratios of length of aerial to spacing between wires, i.e., the chord distance between any two adjacent wires. This table assumes that the ratio of wire spacing to wire diameter is constant at 100.

For tower aerials it is possible to apply the direct formula

$$Z_0 = 60 \left(\log_e \frac{H}{r} - 1\right) \text{ ohms} \quad \ldots\ldots\ldots (2)$$

where H = height and r = mean radius.

It has been pointed out by Professor Howe that the quantities C_0 and L_0, on which the value of Z_0 is based, are extremely difficult to define [2]. Even in the static case, the capacity which we should attribute to a given portion of a line is dependent on the charge distribution over the whole line; while in the A.C. case, there are also to be considered the potential modifications arising from the magnetic field, and the finite time taken for the field set up at one point to reach another point. He comes to the conclusion that in practical calculations we must be content to assume a charge distribution, calculate from this the corresponding potential and capacity distribution, and from the insertion in the appropriate transmission line formulæ of the constants so found verify that the potential distribution given thereby is in reasonable agreement with that given directly by the charge distribution. Following out this line of argument, it is clear that the value of Z_0 to be assigned to any part of a given mast will be to some extent a function of the current distribution or, as it is sometimes described, the mode of oscillation or angular length of the aerial. We must also expect to find variations in the measured value of Z_0 even provided the angular length is kept constant, if we change the geometry of the mast as for example during erection. We must also expect to find variations in the value of B arising from changes in the position and magnitude of any impedance irregularities consequent on discontinuities in the rate of change of mast cross-section with height.

It has been shown by F. G. W. White[3] that the effective inductance per unit length of an aerial is largely independent of the current distribution, and further analysis by Moullin[4], [6] leads to the equation

$$Z_0 \doteqdot 60 \log_e \frac{\lambda}{2r} \text{ ohms} \ldots\ldots\ldots\ldots\ldots \quad (3)$$

which for the case of half-wave operation becomes $Z_0 \doteqdot 60 \log \frac{H}{r}$, i.e., equation (2) less the correction factor for the effect of the image in the ground. The value given by this approximation is high compared with the value deduced from the base impedance measurements.

Another expression for the impedance is given in the Steinmetz equations for a non-return conductor, i.e., a lightning conductor.[15] These equations follow the general principle of calculating the magnetic field as if the length of conductor under examination were part of a very long line, carrying a current of constant amplitude and phase, but allowing in the phase of the flux for the time of propagation of the magnetic field. This leads to the expression :

$$Z_0 = 60 \left(\log_e \frac{\lambda}{2r} - 1.73\right) \text{ ohms} \ldots\ldots \quad (4)$$

which may be compared with Moullin's expression

$$Z_0 = 60 \log_e \frac{\lambda}{2r} \text{ ohms}$$

and with Howe's static expression

$$Z_0 = 60 \left(\log_e \frac{H}{r} - 1\right) \text{ ohms.}$$

This last expression is obviously to be suspected, as it takes no account of the time of propagation or of the wavelength ; but it gives a figure which agrees well with observation for the half-wave mode of oscillation, and could for that case be re-written

$$Z_0 = 60 \left(\log_e \frac{\lambda}{2r} - 1\right) \text{ ohms,}$$

which is more in line with the other two expressions.

Still another expression is that quoted by Dr. Siegel[5]

$$Z_0 = 60 \left(\log_e \frac{H}{r} - 1 - 0.5 \log_e \frac{2H}{\lambda}\right) \text{ ohms.} \quad (5)$$

For half-wave operation ($H = \frac{\lambda}{2}$), this reduces to $Z = 60 \left(\log_e \frac{H}{r} - 1\right)$.

All these expressions are only approximations. They agree, however, in indicating a figure of about 200 ohms for the characteristic impedance of guyed mast radiators, and this is close to the figure actually found in practice for the majority of half-wave narrow-based guyed radiators, either uniform or cigar shaped. None of the expressions is applicable to broad-based self-supporting towers, the cross-section of which varies too rapidly with height for any formula based on uniform cross-section to be even approximately valid.

(3) Determination of Angular Length

For uniform vertical aerials the angular component of the propagation constant is approximately $B = \frac{2\pi}{\lambda}$ and the total phase shift along an aerial of height H terminated in an open circuit at the far end is

$$Bl = \frac{2\pi H}{\lambda} \text{ radians.} \ldots\ldots\ldots\ldots\ldots \quad (6)$$

If the aerial is terminated not in an open circuit but by a reactance such as a non-radiating capacity crown, the assembly can be replaced by an equivalent uniform line of length equal to the actual height of the radiator plus a fictitious length corresponding to that length of transmission line having the same characteristic impedance as the radiator portion and reactance equal to that of the termination. If X is the reactance of the termination and Z_0 the characteristic impedance of the radiator, the actual termination can be replaced by an additional (non-radiating) length b, of aerial, given by

$$b = \frac{\lambda}{2\pi} \cot^{-1} - \frac{X}{Z_0} \ldots\ldots\ldots\ldots\ldots \quad (7)$$

The total length of transmission line then becomes $H + b$; the total phase shift $\frac{2\pi}{\lambda}(H + b)$.

Frequently the termination takes the form of a circular crown or network ; if the diameter of the crown is D_{cm}, the capacity of the crown is approximately

$$C = \frac{D^{cm}}{\pi}\ \mu\mu\text{F},$$

and

$$X = -\frac{530\ \lambda}{C\ \mu\mu F}$$

$$= -\frac{16.6\ \lambda_m}{D_m} \ \ldots\ldots\ldots\ldots\ldots \ (8)$$

It should be noted that a fixed capacity termination has a reactance which is not constant but varies with the operating wavelength, and that the apparent length which it adds to the aerial is correspondingly not a constant.

If the termination takes the form of a capacity ring in series with an inductance L, then the total terminating reactance is

$$X = (\text{reactance of ring} - \text{reactance of } L)$$

$$= -\left(16.6\frac{\lambda_{\,metres}}{D_{metres}} - \frac{1880\ L}{\lambda_{\,metres}}\ \mu H\right) .$$

If the termination consists not of a ring, but of two or more radial conductors each of characteristic impedance Z_{01}, and length l_1, it is frequently convenient to neglect the radiation from the radials, and consider only their reactances. In this case the aerial assembly reduces to the radiating vertical of height H and a non-radiating extension of length b

$$= \frac{\lambda}{2\pi}\cot^{-1} - \frac{X}{Z_0} \text{ where}$$

$$X = -\frac{Z_{01}}{n}\cot\frac{2\pi l_1}{\lambda} \ \ldots\ldots\ldots\ldots \ (9)$$

n = number of radials.

This treatment is applicable even to T aerials, provided that the branches of the T are short compared with the wavelength, say not greater than $\frac{\lambda}{10}$.

(4) *Power Dissipation and Radiation Resistance*

Having fixed the values to be assigned to Z_0 and Bl for a given case, we have now to find a value for A, the real part of the propagation constant. To obtain this value we must know the power radiated, which can only be calculated for a known current distribution. The simplest approximation which we can make to the current distribution so far is to assume that it corresponds to that on a *lossless* transmission line of the same length and phase shift. This is usually referred to as " sinusoidal distribution," and gives the following expression for the current at any point :

$$i_x = \sqrt{2}\,I\sin(\omega t + \varphi)\sin\frac{2\pi}{\lambda}(l - x) \quad (10)$$

where l = total effective length of aerial,

x = distance from base to point at which i_x is measured,

I = current at " loop," i.e., antinode,

$\omega = 2\pi f = 2\frac{\pi c}{\lambda}$,

c = velocity of light.

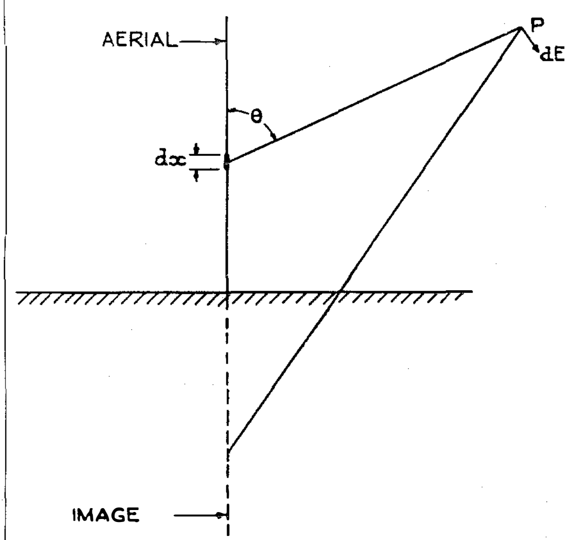

Fig. 1.

Referring to Fig. 1, the element of field strength dE set up at point P distant r from an elementary vertical doublet of length dx carrying current $\sqrt{2}\,I_x\sin(\omega t + \varphi)$ amperes is

$$dE_{\,volts/cm} = \frac{60\pi}{\lambda r}\sin\theta\ .\ I_x\cos\left(\omega t + \varphi - \frac{2\pi r}{\lambda}\right) d_x\ , \ \ldots\ldots\ldots\ldots \ (11)$$

where θ is the zenith angle of the point of observation P relative to the direction of the doublet, and all dimensions are in centimetres. The total field at point P is given by the integral of the above equation over the whole length of the aerial and its image in the ground if the latter is taken as perfectly reflecting. By repeating this integration for a series of points at constant and large distance r from the base of the aerial, the complete polar diagram can be obtained, and we can write

$$E_{millivolts/metre} = \frac{60\ I \text{ amps}}{D_{km}} K_\theta \quad \ldots\ldots(12)$$

where K_θ is the "polar coefficient" and is determined by the reference point in the aerial at which the current I is measured.

For a vertical aerial without horizontal radiating elements, as shown in Fig. 1, the power radiated can then be calculated by the Poynting vector method, i.e., integrating the normal component of energy flow over the surface of the hemisphere enclosing the aerial, giving

$$P_{watts} = 60\ I^2_{amp} \int_0^{\frac{\pi}{2}} K^2_\theta \sin\theta d\theta \quad \ldots (13)$$

and the radiation resistance is

$$R_r = 60 \int_0^{\frac{\pi}{2}} K^2_\theta \sin\theta d\theta. \quad \ldots\ldots\ldots\ldots (14)$$

This radiation resistance R_r, as a function of K_θ, is likewise a function of the reference point of current measurement on the aerial.

The power radiated can also be determined by the "induced e.m.f." method, as used by Moullin[4), (6)] and others. This method has the advantage of giving both the real and imaginary components of the aerial impedance, and is sometimes more easily applied than the Poynting vector method, particularly when the aerial is not a simple vertical wire. It leads to the same value as given by equation (13); and, when the polar distribution of the field is required, it is still necessary to evaluate the integral arising from equation (11).

(5) Radiation Resistance and Polar Radiation Curve of Plain Vertical Aerial

Accepting the conventional sinusoidal distribution of aerial current, equations (11), (12) and (14) lead to the following formulæ, in which the reference current I is the loop current, corresponding to $\dfrac{l-x}{\lambda} = \dfrac{1}{4}$:

$$E_{millivolts/metre} = \frac{60\ I \text{ amps}}{D_{km}} K_\theta\ ;$$

$$K_\theta = \frac{\cos\left(\frac{2\pi H}{\lambda}\cos\theta\right) - \cos\frac{2\pi H}{\lambda}}{\sin\theta}, \quad \ldots(15)$$

where H = height of aerial.

Loop Radiation resistance[(7), (8), (9)]

$$R_r = 60 \int_0^{\frac{\pi}{2}} K^2_\theta \sin\theta d\theta$$

$$= 60 \left[\cos^2 a\ S_1(2a) - \tfrac{1}{4}\cos 2a\ S_1(4a) - \tfrac{1}{2}\sin 2a\left\{Si\ (2a) - \tfrac{1}{2}\ Si\ (4a)\right\}\right] \text{ ohms}, \quad \ldots(16)$$

where $a = \dfrac{2\pi H}{\lambda}$,

$$S_1(x) = \log x + .5772 - Ci\ (x)$$

$Si\ (x)$[(16)] is the tabulated integral,

$$\int_0^x \frac{\sin u}{u} du = \frac{\pi}{2} - \text{col}\ (x)^{(15}$$

$Ci\ (x)$[(16)] is the tabulated integral,

$$-\int_x^\infty \frac{\cos u}{u} du = -\text{sil}\ (x)^{(15)}.$$

Here it may be remarked that while the formulae for K_θ are usually straightforward and embody only the ordinary trigonometric functions, the formulae for radiation resistance are usually involved and require tables of integral functions such as those just mentioned, or their substitution by rather cumbersome series. Inasmuch as we generally require to know K_θ in any case, so as to obtain the polar diagram, it is in many cases simpler to obtain the radiation resistance by graphical evaluation of $\int_0^{\frac{\pi}{2}} K^2_\theta \sin\theta d\theta$ rather than by use of the mathematical solution. The superior accuracy of the latter is not vital in view of the fact that we start with an admitted approximation to the current distribution; and cases may arise to which no mathematical solution is readily available, whereas the graphical evaluation is always possible.

Inserting the values of K_θ and R_r, we get the polar diagram in terms of figure of merit:

$$F_\theta = \frac{E_\theta D}{\sqrt{P}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$= \frac{1\,900\;K_\theta}{\sqrt{R_r + R_l}}\; mV/m. \text{ at } 1\; km. \text{ for } 1\; kW \text{ input} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

where R_l = dead loss resistance.

For radiation in the equatorial plane, $\theta = 90°$, equation (18) becomes in this case

$$F = \frac{1\,900\;(1 - \cos \frac{2\pi H}{\lambda})}{\sqrt{R_r + R_l}} \ldots\ldots\ldots\ldots (19)$$

The value of the loss resistance R_l is a matter on which there is little definite information, but experience tends to show that on medium broadcasting wavelengths it is unlikely to be less than five ohms, and that ten ohms is a reasonable average figure to take. Note that both R_r and R_l are referred to the "loop" current.

(6) Radiation Resistance and Polar Radiation Curve of Loaded-top Aerial

For a loaded-top aerial, e.g., an aerial with a top capacity, if H = height of radiating portion, and b = equivalent length of the top-loading at the operating frequency, as explained earlier, the polar coefficient K_θ is given by

$$K_\theta = \frac{1}{\sin\theta}\Big\{\cos\frac{2\pi b}{\lambda}\,.\,\cos\Big(\frac{2\pi H}{\lambda}\cos\theta\Big) - \sin\frac{2\pi b}{\lambda}\,.\cos\theta.\,\sin\Big(\frac{2\pi H}{\lambda}\cos\theta\Big) - \cos\frac{2\pi(H+b)}{\lambda}\Big\}, \ldots\ldots\ldots\ldots (20)$$

while the loop radiation resistance [7], [10] is

$$R_r = 60\int_0^{\frac{\pi}{2}} K_\theta^2 \sin\theta d\theta$$

$$= 60\Big\{\frac{\sin^2\beta}{2}\Big(\frac{\sin 2\gamma}{2\gamma} - 1\Big) + \cos^2\alpha.\,S_1(2\gamma) - \frac{1}{4}\cos 2\alpha.\,S_1(4\gamma) - \frac{\sin 2\alpha}{2}\Big[Si\,(2\gamma) - \frac{1}{2}\,Si\,(4\gamma)\Big]\Big\} \text{ ohms} \ldots\ldots\ldots\ldots (21)$$

TABLE III

Loop Radiation Resistance in ohms of Vertical Aerial with Non-radiating Top Termination

Height of aerial $= H$.
Equivalent length of top $= b$.

H/λ	$b/\lambda = 0$	$b/\lambda = 0.05$	$b/\lambda = 0.10$	$b/\lambda = 0.15$	$b/\lambda = 0.20$	$b/\lambda = 0.30$
0.125	3.2	10	16.5	21.5	22	13
0.150	6.5	15.5	25	30.5	30.5	15.5
0.175	11.5	24	35	40	38	16.5
0.200	18	33	46	50	45	16.5
0.225	27	44	57	58.5	50	15
0.250	36.5	56	67.5	66	52.5	12.5
0.275	48	68	77	71.5	53.5	9.1
0.300	60	79.5	84.5	74.5	51.5	5.9
0.325	72	88.5	89.5	73.5	47	3.3
0.350	84	96.5	91.5	71	41.5	1.7
0.375	93.5	100.5	90.5	65	34	2.0
0.400	100	101.5	86	57.5	27	4.5
0.425	104.5	101	79	49	20.5	9.2
0.450	106	96	71	40.5	15.5	16
0.475	105	88.5	61	32	12	25
0.500	99.5	79.5	51.5	26.5	12.5	36
0.525	92.5	69	43	22.5	17	48
0.550	83	59	36	22	22	60
0.575	72.5	50	32	25	31.5	70
0.600	65.5	44	32	32	41.5	78
0.625	52.5	39.5	35.5	41	54.5	84
0.650	46.5	39.5	41.5	52.5	67	87.5
0.675	42	43	51	65	79	84
0.700	42.5	50	69	77.5	89.5	82.5

where $\alpha = \frac{2\pi (H + b)}{\lambda}$

$\beta = \frac{2\pi b}{\lambda}$

$\gamma = \frac{2\pi H}{\lambda}$

and $S_1(x)$ and $Si(x)$ are the tabulated integrals already mentioned.

In the equatorial plane, $\theta = 90°$,

$$K_\theta = \cos \frac{2\pi b}{\lambda} - \cos \frac{2\pi(H + b)}{\lambda} \quad \ldots\ldots (22)$$

and the horizontal figure of merit is

$$F = \frac{1900 \left\{ \cos \frac{2\pi b}{\lambda} - \cos \frac{2\pi(H + b)}{\lambda} \right\}}{\sqrt{R_r + R_l}} \quad (23)$$

TABLE IV

$S_1 (x) - \lg n (x) + C - Ci (x) = \lg n (x) + 0.5772157 - Ci (x)$

x	S_1 (x)	x	S_1 (x)	x	S_1 (x)	x	S_1 (x)	x	S_1 (x)	x	S_1 (x)
0.0	0.00000	5.0	2.37669	10.0	2.92527	15.0	3.23899	20.0	3.52853	25.0	3.80295
0.1	0.00249	5.1	2.38994	10.1	2.94327	15.1	3.25090	20.1	3.53173		
0.2	0.00998	5.2	2.40113	10.2	2.96050	15.2	3.26308	20.2	3.53535		
0.3	0.02241	5.3	2.41044	10.3	2.97688	15.3	3.27552	20.3	3.53946		
0.4	0.03973	5.4	2.41801	10.4	2.99234	15.4	3.28814	20.4	3.54402		
0.5	0.06185	5.5	2.42402	10.5	3.00688	15.5	3.30087	20.5	3.54905		
0.6	0.08866	5.6	2.42866	10.6	3.02045	15.6	3.31363	20.6	3.55456		
0.7	0.12002	5.7	2.43210	10.7	3.03300	15.7	3.32641	20.7	3.56049		
0.8	0.15579	5.8	2.43452	10.8	3.04457	15.8	3.33911	20.8	3.56687		
0.9	0.19578	5.9	2.43610	10.9	3.05514	15.9	3.35167	20.9	3.57368		
1.0	0.23981	6.0	2.43704	11.0	3.06467	16.0	3.36401	21.0	3.58085		
1.1	0.28766	6.1	2.43749	11.1	3.07323	16.1	3.37612	21.1	3.58840		
1.2	0.33908	6.2	2.43764	11.2	3.08083	16.2	3.38790	21.2	3.59629		
1.3	0.39384	6.3	2.43766	11.3	3.08749	16.3	3.39932	21.3	3.60446		
1.4	0.45168	6.4	2.43770	11.4	3.09322	16.4	3.41032	21.4	3.61288		
1.5	0.51233	6.5	2.43792	11.5	3.09814	16.5	3.42088	21.5	3.62155		
1.6	0.57549	6.6	2.43847	11.6	3.10225	16.6	3.43096	21.6	3.63037		
1.7	0.64088	6.7	2.43947	11.7	3.10561	16.7	3.44050	21.7	3.63935		
1.8	0.70820	6.8	2.44106	11.8	3.10828	16.8	3.44947	21.8	3.64842		
1.9	0.77713	6.9	2.44335	11.9	3.11038	16.9	3.45788	21.9	3.65751		
2.0	0.84739	7.0	2.44643	12.0	3.11190	17.0	3.46568	22.0	3.66662		
2.1	0.91865	7.1	2.45040	12.1	3.11301	17.1	3.47288	22.1	3.67568		
2.2	0.99060	7.2	2.45534	12.2	3.11370	17.2	3.47945	22.2	3.68465		
2.3	1.06295	7.3	2.46130	12.3	3.11412	17.3	3.48543	22.3	3.69348		
2.4	1.13540	7.4	2.46834	12.4	3.11429	17.4	3.49077	22.4	3.70216		
2.5	1.20764	7.5	2.47649	12.5	3.11436	17.5	3.49553	22.5	3.71059		
2.6	1.27939	7.6	2.48577	12.6	3.11437	17.6	3.49969	22.6	3.71879		
2.7	1.35038	7.7	2.49619	12.7	3.11438	17.7	3.50330	22.7	3.72670		
2.8	1.42035	7.8	2.50775	12.8	3.11453	17.8	3.50639	22.8	3.73427		
2.9	1.48903	7.9	2.52044	12.9	3.11484	17.9	3.50895	22.9	3.74153		
3.0	1.55620	8.0	2.53423	13.0	3.11540	18.0	3.51107	23.0	3.74838		
3.1	1.62163	8.1	2.54906	13.1	3.11628	18.1	3.51276	23.1	3.75483		
3.2	1.68511	8.2	2.56491	13.2	3.11754	18.2	3.51404	23.2	3.76089		
3.3	1.74646	8.3	2.56171	13.3	3.11924	18.3	3.51500	23.3	3.76651		
3.4	1.80552	8.4	2.59938	13.4	3.12142	18.4	3.51568	23.4	3.77170		
3.5	1.86211	8.5	2.61786	13.5	3.12414	18.5	3.51610	23.5	3.77644		
3.6	1.91613	8.6	2.63704	13.6	3.12745	18.6	3.51633	23.6	3.78072		
3.7	1.96745	8.7	2.65686	13.7	3.13134	18.7	3.51645	23.7	3.78459		
3.8	2.01600	8.8	2.67721	13.8	3.13587	18.8	3.51648	23.8	3.78801		
3.9	2.06170	8.9	2.69799	13.9	3.14104	18.9	3.51648	23.9	3.79101		
4.0	2.10449	9.0	2.71909	14.0	3.14688	19.0	3.51660	24.0	3.79360		
4.1	2.14438	9.1	2.74042	14.1	3.15338	19.1	3.51661	24.1	3.79582		
4.2	2.18131	9.2	2.76186	14.2	3.16054	19.2	3.51685	24.2	3.79767		
4.3	2.21535	9.3	2.78332	14.3	3.16835	19.3	3.51727	24.3	3.79917		
4.4	2.24648	9.4	2.80468	14.4	3.17677	19.4	3.51790	24.4	3.80036		
4.5	2.27479	9.5	2.82583	14.5	3.18583	19.5	3.51879	24.5	3.80129		
4.6	2.30033	9.6	2.84669	14.6	3.19545	19.6	3.52002	24.6	3.80197		
4.7	2.32317	9.7	2.86713	14.7	3.20564	19.7	3.52156	24.7	3.80243		
4.8	2.34344	9.8	2.88712	14.8	3.21630	19.8	3.52348	24.8	3.80271		
4.9	2.36124	9.9	2.90651	14.9	3.22746	19.9	3.52578	24.9	3.80288	50.0	4.49486

In Table III is given the loop radiation resistance R_r for a series of aerials of different heights and top loadings most likely to occur in practice. Tables IV[10] and V[10] give the definite integrals $S_1(x)$ and $Si(x)$ so that if necessary the radiation resistance of an aerial not covered by Table III may be calculated from the equation given above.

(7) *Radiation Resistance and Polar Radiation Curve of Radiating-top (T and L) Aerials*

When circumstances do not permit the use of an aerial with a non-radiating top, recourse is sometimes had to aerials of the T or inverted L pattern. If the radiation from the horizontal members of such aerials is taken fully into account the equations for K_θ, etc., become too

TABLE V

$$Si(x)=\int_0^x \frac{\sin u}{u}\,du$$

x	Si (x)	x	Si (x)	x	Si (x)	x	Si (x)	x	Si (x)	x	Si (x)
0.0	0.00000	5.0	1.54993	10.0	1.65835	15.0	1.61819	20.0	1.54824	25.0	1.53148
0.1	0.09994	5.1	1.53125	10.1	1.65253	15.1	1.62226	20.1	1.55289		
0.2	0.19956	5.2	1.51367	10.2	1.64600	15.2	1.62575	20.2	1.55767		
0.3	0.29850	5.3	1.49732	10.3	1.63883	15.3	1.62865	20.3	1.56253		
0.4	0.39646	5.4	1.48230	10.4	1.63112	15.4	1.63093	20.4	1.56743		
0.5	0.49311	5.5	1.46872	10.5	1.62294	15.5	1.63258	20.5	1.57232		
0.6	0.58813	5.6	1.45667	10.6	1.61439	15.6	1.63359	20.6	1.57714		
0.7	0.68122	5.7	1.44620	10.7	1.60556	15.7	1.63396	20.7	1.58186		
0.8	0.77210	5.8	1.43736	10.8	1.59654	15.8	1.63370	20.8	1.58641		
0.9	0.86047	5.9	1.43018	10.9	1.58743	15.9	1.63280	20.9	1.59077		
1.0	0.94608	6.0	1.42469	11.0	1.57831	16.0	1.63130	21.0	1.59489		
1.1	1.02869	6.1	1.42087	11.1	1.56927	16.1	1.62291	21.1	1.59873		
1.2	1.10805	6.2	1.41871	11.2	1.56042	16.2	1.62657	21.2	1.60225		
1.3	1.18396	6.3	1.41817	11.3	1.55182	16.3	1.62339	21.3	1.60543		
1.4	1.25623	6.4	1.41922	11.4	1.54356	16.4	1.61973	21.4	1.60823		
1.5	1.32468	6.5	1.42179	11.5	1.53571	16.5	1.61563	21.5	1.61063		
1.6	1.38918	6.6	1.42582	11.6	1.52835	16.6	1.61112	21.6	1.61261		
1.7	1.44959	6.7	1.43121	11.7	1.52155	16.7	1.60627	21.7	1.61415		
1.8	1.50582	6.8	1.43787	11.8	1.51535	16.8	1.60111	21.8	1.61525		
1.9	1.55778	6.9	1.44570	11.9	1.50981	16.9	1.59572	21.9	1.61590		
2.0	1.60541	7.0	1.45460	12.0	1.50497	17.0	1.59014	22.0	1.61608		
2.1	1.64870	7.1	1.46443	12.1	1.50088	17.1	1.58443	22.1	1.61582		
2.2	1.68763	7.2	1.47509	12.2	1.49755	17.2	1.57865	22.2	1.61510		
2.3	1.72221	7.3	1.48644	12.3	1.49501	17.3	1.57285	22.3	1.61395		
2.4	1.75249	7.4	1.49834	12.4	1.49327	17.4	1.56711	22.4	1.61238		
2.5	1.77852	7.5	1.51068	12.5	1.49234	17.5	1.56146	22.5	1.61041		
2.6	1.80039	7.6	1.52331	12.6	1.49221	17.6	1.55598	22.6	1.60806		
2.7	1.81821	7.7	1.53611	12.7	1.49287	17.7	1.55070	22.7	1.60536		
2.8	1.83210	7.8	1.54894	12.8	1.49430	17.8	1.54568	22.8	1.60234		
2.9	1.84219	7.9	1.56167	12.9	1.49647	17.9	1.54097	22.9	1.59902		
3.0	1.84865	8.0	1.57419	13.0	1.49936	18.0	1.53661	23.0	1.59546		
3.1	1.85166	8.1	1.58637	13.1	1.50292	18.1	1.53264	23.1	1.59168		
3.2	1.85140	8.2	1.59810	13.2	1.50711	18.2	1.52909	23.2	1.58772		
3.3	1.84808	8.3	1.60928	13.3	1.51188	18.3	1.52600	23.3	1.58363		
3.4	1.84191	8.4	1.61981	13.4	1.51716	18.4	1.52339	23.4	1.57945		
3.5	1.83313	8.5	1.62960	13.5	1.52291	18.5	1.52128	23.5	1.57521		
3.6	1.82195	8.6	1.63857	13.6	1.52905	18.6	1.51969	23.6	1.57097		
3.7	1.80862	8.7	1.64665	13.7	1.53352	18.7	1.51863	23.7	1.56676		
3.8	1.79339	8.8	1.65379	13.8	1.54225	18.8	1.51810	23.8	1.56262		
3.9	1.77650	8.9	1.65993	13.9	1.54917	18.9	1.51810	23.9	1.55860		
4.0	1.75820	9.0	1.66504	14.0	1.55621	19.0	1.51863	24.0	1.55474		
4.1	1.73874	9.1	1.66908	14.1	1.56330	19.1	1.51967	24.1	1.55107		
4.2	1.71837	9.2	1.67205	14.2	1.57036	19.2	1.52122	24.2	1.54762		
4.3	1.69732	9.3	1.67393	14.3	1.57733	19.3	1.52324	24.3	1.54444		
4.4	1.67583	9.4	1.67473	14.4	1.58414	19.4	1.52572	24.4	1.54154		
4.5	1.65414	9.5	1.67446	14.5	1.59072	19.5	1.52863	24.5	1.53897		
4.6	1.63246	9.6	1.67316	14.6	1.59702	19.6	1.53192	24.6	1.53672		
4.7	1.61101	9.7	1.67084	14.7	1.60296	19.7	1.53357	24.7	1.53484		
4.8	1.58998	9.8	1.66757	14.8	1.60851	19.8	1.53954	24.8	1.53333		
4.9	1.56956	9.9	1.66338	14.9	1.61360	19.9	1.54378	24.9	1.53221	50.0	1.55162

TABLE VI

Base Radiation Resistance in Ohms of Flat-top Antenna.

λ_0=natural wavelength of antenna unloaded and grounded.
λ =wavelength when loaded with inductance at base.

$$\gamma = \frac{\text{length of flat horizontal part of antenna}}{\text{total length of antenna}}.$$

(Electrical length assumed equal to physical length.)

λ/λ_0	Radiation Resistance in ohms for γ equal								
	0	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.0	36.60	33.30	29.70	25.50	20.30	14.70	9.70	4.90	1.200
1.1	28.00	26.00	23.90	20.00	16.00	11.90	7.60	3.80	1.070
1.2	21.80	20.20	18.80	15.80	12.40	9.00	6.00	2.90	0.940
1.3	18.20	16.90	15.10	12.60	11.20	7.20	4.90	2.40	0.810
1.4	15.10	14.00	12.20	10.50	8.60	6.10	4.00	2.00	0.700
1.5	12.80	11.70	10.40	9.00	7.30	5.20	3.30	1.70	0.600
1.6	11.00	10.00	9.00	7.80	6.30	4.40	2.80	1.40	0.500
1.7	9.50	8.60	7.60	6.70	5.40	3.70	2.50	1.20	0.400
1.8	8.30	7.70	6.70	6.00	4.70	3.20	2.20	1.10	0.330
1.9	7.40	6.80	6.20	5.30	4.20	2.90	1.90	0.90	0.240
2.0	6.50	6.10	5.50	4.80	3.80	2.70	1.70	0.75	0.180
2.2	5.20	5.00	4.60	3.90	3.00	2.20	1.40	0.57	0.160
2.4	4.40	4.20	3.80	3.20	2.50	1.80	1.20	0.48	0.140
2.6	3.80	3.50	3.10	2.70	2.10	1.50	1.00	0.42	0.120
2.8	3.30	3.00	2.60	2.30	1.80	1.30	0.86	0.37	0.100
3.0	2.80	2.50	2.20	1.90	1.50	1.10	0.74	0.33	0.090
3.2	2.50	2.30	2.00	1.70	1.30	0.92	0.64	0.29	0.080
3.4	2.20	2.00	1.80	1.60	1.10	0.84	0.55	0.25	0.072
3.6	2.00	1.90	1.60	1.40	1.00	0.77	0.47	0.22	0.066
3.8	1.75	1.70	1.40	1.30	0.94	0.71	0.39	0.19	0.060
4.0	1.62	1.50	1.30	1.10	0.88	0.66	0.31	0.16	0.055
4.5	1.30	1.21	1.05	0.89	0.75	0.54	0.26	0.12	0.042
5.0	1.00	0.93	0.80	0.68	0.63	0.42	0.22	0.09	0.032
5.5	0.78	0.73	0.65	0.56	0.53	0.36	0.19	0.08	0.025
6.0	0.61	0.54	0.49	0.44	0.43	0.29	0.16	0.07	0.019
6.5	0.48	0.45	0.41	0.38	0.35	0.25	0.14	0.07	0.015
7.0	0.38	0.36	0.33	0.32	0.28	0.22	0.12	0.06	0.013
7.5	0.32	0.31	0.29	0.28	0.25	0.19	0.11	0.06	0.013
8.0	0.28	0.27	0.25	0.23	0.22	0.17	0.10	0.05	0.012
8.5	0.26	0.25	0.23	0.21	0.19	0.15	0.09	0.05	0.012
9.0	0.25	0.22	0.20	0.18	0.16	0.13	0.08	0.05	0.012
9.5	0.24	0.20	0.19	0.17	0.15	0.12	0.08	0.05	0.011
10.0	0.22	0.18	0.17	0.15	0.13	0.11	0.07	0.04	0.011
10.5	0.21	0.16	0.15	0.14	0.12	0.10	0.07	0.04	0.010
11.0	0.20	0.14	0.13	0.12	0.11	0.09	0.06	0.04	0.010
11.5	0.19	0.13	0.12	0.11	0.10	0.08	0.06	0.04	0.009
12.0	0.18	0.12	0.11	0.10	0.09	0.07	0.05	0.03	0.009
12.5	0.16	0.11	0.10	0.09	0.08	0.07	0.05	0.03	0.008
13.0	0.15	0.10	0.09	0.09	0.08	0.06	0.05	0.03	0.008
13.5	0.14	0.09	0.08	0.08	0.07	0.06	0.04	0.03	0.007
14.0	0.12	0.08	0.07	0.07	0.06	0.05	0.04	0.02	0.007
14.5	0.11	0.08	0.07	0.06	0.06	0.05	0.04	0.02	0.006
15.0	0.10	0.07	0.06	0.06	0.05	0.04	0.03	0.02	0.006
15.5	0.08	0.06	0.06	0.05	0.05	0.04	0.03	0.02	0.005
16.0	0.06	0.06	0.06	0.05	0.04	0.04	0.03	0.02	0.005

cumbersome for ordinary engineering use. The simplified treatment outlined below will in general give all the accuracy that is really justified.

In the case of the T aerial, radiation from the top portion is small, since the current therein is split into two oppositely directed portions and the spacing between even the ends of the arms is usually so short compared with the wavelength that there is no opportunity for space-phasing to neutralise the antiphasing produced by the opposite directions of the two current components. We may therefore neglect the radiation altogether, and consider the top

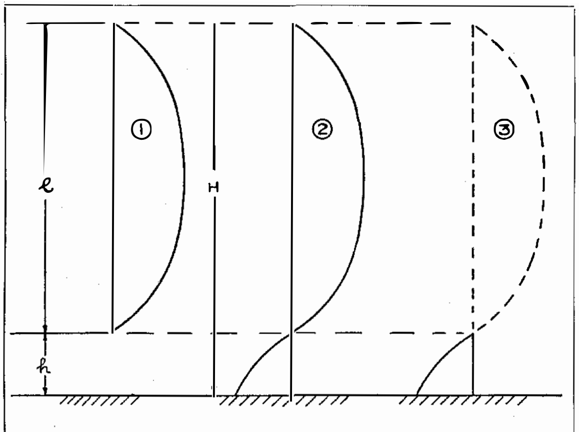

Fig. 2.

as being merely a reactive termination of length b calculated by equation (7). The polar coefficient K_θ, the radiation resistance R_r, and the polar figure of merit may then be calculated exactly as for a loaded-top aerial.

In the case of the inverted L aerial, there is no cancellation effect along the top portion, and there is no sound justification for ignoring the top radiation altogether. In these cases, however, both the vertical and horizontal portions usually have the same impedance, and it becomes possible to apply Table VI which gives directly the *base* radiation resistance (8) of inverted L aerials as a function of the ratios

$$\lambda/\lambda_0 = \frac{\text{working wavelength}}{\text{natural wavelength}}$$

$$\gamma = \frac{b}{\lambda} = \frac{\text{length of horizontal portion of aerial}}{\text{total length of aerial}}$$

$$\lambda_0 = 4 \text{ x total length of aerial} = 4l\,.$$

The polar coefficient K_θ can then be calculated from the formula for the loaded top aerial, using b = actual length of aerial top, and H = aerial height; this is correct for the equatorial plane, $\theta = 90°$, and approximately correct for other values of θ. The figure of merit is then given by

$$F_\theta = \frac{1900\, K_\theta}{\sin \dfrac{2\pi l}{\lambda}\sqrt{\text{base radiation resistance} + \text{base loss resistance}}} \quad \ldots\ldots\ldots\ (24)$$

The value of F_θ so obtained will be theoretically correct for $\theta = 90°$, i.e., for ground wave propagation, and approximately correct for all other angles.

(8) Elevated Vertical Aerial

In the case of a vertical aerial of length l, with its base at a height h above ground, and therefore top at height $H = h + l$ above ground, assuming the usual "lossless" current distribution and velocity of propagation as in air, the effective radiation polar factor may be considered as that of an aerial of height H, with base at ground level, less that of an aerial of height h, base at ground level, with non-radiating top of effective length b, as shown in Fig. 2. This gives

$$K_\theta = \frac{1}{\sin\theta}\left[\cos\frac{2\pi H\cos\theta}{\lambda} - \cos\frac{2\pi l}{\lambda} \times \cos\frac{2\pi h\cos\theta}{\lambda} + \cos\theta\sin\frac{2\pi l}{\lambda}\sin\frac{2\pi h\cos\theta}{\lambda}\right]. \quad (25)$$

Such an aerial would normally be fed at the current antinode through a transmission line. If in order to couple the transmission line it is necessary to insert inductance at the centre of the radiator, this will modify the current distribution, as will also be the case if the halves of the radiator are loaded at the remote ends to give a Hertzian doublet radiator. By analysing the complex system into a number of superposed elements in which the current distribution is defined as occurring along part of a fictitious aerial of a type dealt with previously, it is always possible to arrive at the value of K_θ for the complex structure by algebraic addition of K_θ for the component elements, as illustrated by the simple example quoted above.

In aerials of this type, energised at the current antinode, the same considerations apply as are subsequently dealt with in section (19), and the real distribution of the current does not differ greatly from that given by the conventional "lossless distribution" theory, even if the characteristic impedance of the radiator is low.

(9) "Low Velocity" Aerials

If an aerial is loaded with additional distributed inductance and/or capacity, the velocity of propagation along the aerial is decreased and the distance between successive nodal points becomes shorter, i.e., the aerial is "compressed." Such loading may either be deliberately applied, or may arise from the presence of numerous guy wires, changes in cross-section with length, and so on. As the result of the compression of the standing wave both the polar diagram and the radiation resistance are modified as compared with the values given by a normal aerial operated in the same mode and therefore of the same electrical length or phase shift BH.

If the velocity of propagation along the loaded aerial is related to that in air by the factor $\frac{1}{k}$, the electrical length is k times the physical length, B becomes $\frac{2\pi k}{\lambda}$, and the current distribution on the usual "lossless" basis can be written :

$$I_x = I_{loop} \sin \frac{2\pi k}{\lambda} (l - x) \quad \dots\dots\dots\dots (26)$$

where l is the total length of the aerial and x is the distance from base to point at which Ix is measured. Making the second usual assumption, that the earth behaves as a perfect reflector, we then get the polar coefficient for a plain aerial of height H with no top loading.

$$K_\theta = \frac{k \sin\theta}{k^2 - \cos^2\theta}\left\{\cos\frac{2\pi H\cos\theta}{\lambda} - \cos\frac{2\pi kH}{\lambda}\right\} \quad \dots\dots\dots\dots (27)$$

which for $\theta = 90°$ (radiation in the equatorial plane) simplifies to

$$K_{90°} = \left(1 - \cos\frac{2\pi k H}{\lambda}\right)\Big/ k \quad \dots\dots (28)$$

Knowing K_θ, the loop radiation resistance can be obtained by graphical evaluation of the integral

$$R_r = 60 \int_0^{\frac{\pi}{2}} K_\theta^2 \sin\theta \, d\theta.$$

The figure of merit is given by the usual equation

$$F = \frac{ED}{\sqrt{P}} = \frac{1\,900\, K_\theta}{\sqrt{R_r + R_l}},$$

where E is in millivolts/metre,
D is in $km.$,
P is in kW.

Table VII below shows how the figure of merit (ignoring R_l) in the horizontal plane alters with increasing loading of an aerial whose length is always adjusted to give half-wave operation ($BH = 180°$) no matter what the loading may be. The last column gives the figure of merit for an aerial of the same H/λ but k held constant at unity.

TABLE VII
LOW VELOCITY HALF-WAVE AERIALS

$\frac{H}{\lambda}$	k	$K_{90°}$	R_r	$ED/\sqrt{P}$ Loaded Aerial	$ED/\sqrt{P}$ Normal Aerial
0.5	1	2	99	382	382
0.475	1.05	1.9	93.5	373	371
0.455	1.1	1.82	88	369	362
0.435	1.15	1.74	82	365	354
0.415	1.20	1.67	77	361	350
0.400	1.25	1.60	72	359	355
0.385	1.30	1.54	68	356	337

Table VIII gives values of K_θ and R_r for a number of loaded aerials of various lengths and degrees of loading.

(10) *Radiation Diagram for Two Similar Spaced and Equally Energised Vertical Radiators*

Let λ = wavelength,
θ = zenith angle,
α = horizontal angle relative to the line of the radiators,
β = phase difference between the currents in the two radiators,
l = distance between the radiators,

with the convention that $\alpha = \theta$ corresponds to a point in front of that radiator whose current leads the current in the second radiator.

K_θ = polar coefficient of one radiator by itself, so that

$$E_{mV/m} = \frac{60\, I_{amps}}{D_{km}} \cdot K_\theta$$

= field at distance D and zenith angle θ due to one radiator carrying loop current I_{amps}.

Then for two radiators each carrying I_{amps} loop current the total field at a point distance D,

TABLE VIII

"LOW VELOCITY" AERIALS

H/λ	k	$BH°$	R_r	K_θ for zenith angle									
				$\theta=0$	10°	20°	30°	40°	50°	60°	70°	80°	90°
0.238	1.05	90°	33	0	0.131	0.272	0.399	0.54	0.668	0.783	0.872	0.93	0.955
0.475	1.05	180°	93.5	0	0.044	0.082	0.224	0.45	0.765	1.15	1.52	1.80	1.91
0.455	1.1	180°	88	0	0.047	0.121	0.266	0.483	0.782	1.14	1.475	1.725	1.82
0.435	1.15	180°	82	0	0.057	0.145	0.289	0.50	0.792	1.135	1.43	1.66	1.74
0.403	1.24	180°	73	0	0.076	0.182	0.336	0.535	0.80	1.085	1.35	1.54	1.61
0.38	1.05	144°	82.5	0	0.033	0.178	0.377	0.63	0.89	1.17	1.41	1.57	1.65
0.57	1.05	216°	57	0	− 0.167	− 0.272	− 0.282	− 0.146	+ 0.163	+ 0.63	+ 1.14	+ 1.56	+ 1.72
0.715	1.05	270°	45	0	− 0.662	− 0.773	− 1.10	− 1.25	− 1.13	− 0.62	+ 0.035	+ 0.87	+ 0.95
0.59	1.06	225°	48	0	− 0.197	− 0.351	− 0.412	− 0.315	− 0.045	+ 0.450	+ 0.995	+ 1.44	+ 1.605

zenith angle θ, horizontal angle α, is the sum of two vectors differing in phase by

$$\varphi = \beta + \frac{2\pi l}{\lambda} \cos\alpha \sin\theta \quad \ldots\ldots\ldots\ldots\ldots (29)$$

and $E_{mV/m} = \dfrac{60\, I_{amps}}{D_{km}} K_\theta \,.\, N,$

$$\text{where } N = \sqrt{2(1+\cos\varphi)} = 2\cos\frac{\varphi}{2}\,.$$

The total radiation resistance R of the system, defined by

$$W = I^2 R,$$

where W = power radiated for loop current I in each radiator, is given by

$$R = 30 \int_0^{\frac{\pi}{2}} K_\theta^2 \sin\theta \,.\, 2 \left\{1 + \cos\beta \,.\, \mathcal{J}_0 \left(\frac{2\pi l}{\lambda} \times \sin\theta\right)\right\} d\theta \quad \ldots\ldots\ldots\ldots\ldots (30)$$

in which $\mathcal{J}_0(x)$ is the tabulated Bessel function of the first kind of zero order. For spacing l greater than $\lambda/4$, the approximation may be used:

$$R = 120 \int_0^{\frac{\pi}{2}} K_\theta^2 \sin\theta \, d\theta\,,$$

i.e., the total radiation resistance is approximately twice that of a single radiator by itself.

Taking R_T = total radiation resistance plus total loss resistance, the polar diagram can be expressed in terms of figure of merit by the equation

$$\frac{ED}{\sqrt{P}} = \frac{1900}{\sqrt{R_T}} K_\theta \,.\, 2\cos\frac{\varphi}{2} \quad \ldots\ldots\ldots\ldots (31)$$

where $\varphi = \beta + \dfrac{2\pi l}{\lambda} \cos\alpha \sin\theta.$

It can be seen from this that while for large zenith angles, i.e., in the horizontal plane ($\theta = 90°$), the polar diagram may be very far from circular, for small zenith angles, i.e., for sky wave radiation, the diagram is not nearly so much distorted. In other words, a system which is directional in the ground plane remains almost non-directional for sky wave radiation.

(11) *Note on the "Uniform Aerial Current" Theory*

In the past a great deal of use has been made of equations which assume that the current in the aerial is constant in amplitude and phase along the whole length of the (vertical) radiating portion. Such a current distribution is ***never*** obtained in practice; if, however, the vertical portion is short in comparison with the wavelength, and there is a large non-radiating capacity roof, the error involved is not very serious.

Assuming that the uniform current distribution actually held good, we would get the following equations:

$$K_\theta = \frac{\sin\theta}{\cos\theta} \left\{\cos \frac{2\pi H \cos\theta}{\lambda} - 1\right\} \quad \ldots\ldots (32)$$

where H is the height of the aerial in which the current is supposedly uniform.

The radiation resistance (in terms of the current at any point, since the current is the same at all points) would be given by

$$R_r = 60 \int_0^{\frac{\pi}{2}} K_\theta^2 \sin\theta \, d\theta$$

$$= 30 \left\{ \cos\frac{4\pi H}{\lambda} + \frac{4\pi H}{\lambda} \cdot Si\frac{4\pi H}{\lambda} + \frac{\lambda}{4\pi H} \cdot \sin\frac{4\pi H}{\lambda} - 2 \right\} \text{ohms} \quad \ldots\ldots\ldots(33)$$

$$= 1580 \left(\frac{H}{\lambda}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(34)$$

if $\frac{H}{\lambda}$ is small.

The magnitude of the error which would attend the application of this theory to unterminated vertical radiators of the height used in broadcasting is illustrated by the following:

$$\text{Height of aerial} = \frac{\text{wavelength}}{4},$$

Radiation resistance on transmission line basis = 36.5 ohms,

Radiation resistance on "uniform current" basis = 84 ohms.

In order to avoid such a large discrepancy recourse is sometimes had to the concept of "effective height," in which instead of the real height H use is made of a "mean height" h, equal to between 0.5 H and 0.7 H, according to the type of aerial and the manner in which it is terminated. The whole basis of the theory is however fundamentally unsound, and the use of correction factors merely mitigates the inherent error, and does not really justify the formulae.

(12) Conventional Assumptions Underlying the Radiation Resistance Equations, etc.

In evolving the equations for the polar constant K_θ and the loop radiation resistance R_r it has been assumed that the velocity of propagation along the aerial is the same as in air, except where otherwise stated, and that the earth acts as a *perfect* reflector.

The assumption as to the velocity of propagation has a reasonable theoretical basis; it holds good for an aerial of uniform cross-section free in space, and its validity is disturbed only by the incidents of proximity of one end of the aerial to earth and perhaps changes in cross-section of the aerial along its length together with the effect of stay wires and so on. Except in a few special cases the departure from the theoretical value is not very great.

The assumption that the earth is a perfect reflector, i.e., that it reflects without change of amplitude or phase, is not easy to justify theoretically. It is customary to consider radiation problems in terms of optical plane wave theory, according to which perfect reflection is possible only from a surface of mathematically infinite conductivity; if the conductivity is finite—the practical case—reflection should be accompanied by a phase change of the order of 180°. As pointed out by Feldman [11], however, the application of plane wave theory is justified only at a great distance from a radiator; in the immediate neighbourhood of the latter the wave front is spherical rather than plane, and plane wave methods of analysis are not directly applicable.

From the engineering standpoint the use of the assumption of perfect reflection as in the preceding sections is justified by experience. It has been deliberately checked by Ratcliffe, Vedy, and Wilkins [12], by comparison between the measured and calculated field strengths from an aerial with known current distribution, on a wavelength of 1 000 m.; while, in the broadcasting band [13], the measured fields of a large number of aerials have been found to coincide very closely with that calculated on the "perfect reflection" theory. Even in the short-wave band [17], despite the increasing influence of the soil inductivity, it seems that the assumption of perfect reflection holds sufficiently good for ordinary engineering purposes.

(13) Note on "Natural Wavelength"

In the formulae quoted in the preceding sections little use has been made of the quantity "natural wavelength," usually designated by λ_0, corresponding to the longest wavelength at which the aerial will resonate, i.e., present zero reactance, when connected to ground. This quantity was originally used in early radiation formulae, based on simple wire aerials, because it constituted a convenient way of introducing the velocity of propagation factor which determines the current distribution along the aerial.

The same factor is in our formulae covered by BH and Bl where $B = \frac{2\pi}{\lambda}$. While the term "natural wavelength" does in fact correspond to a useful constant in the case of a simple wire aerial, it is very misleading in the case of a complex aerial such as one with a top, radiating or non-radiating, of different characteristic impedance to the remainder of the aerial. The presence of such an impedance irregularity introduces a reflection factor varying with frequency in such a way that in order to make "λ_0" valid in the current distribution and radiation resistance equations it is necessary to use a different value of "λ_0" for each different operating wavelength. Thus λ_0 as used in the equations has no longer any simple relation with λ_0 as measured, and only confusion is caused by its retention. In many of the formulae use is made of $b =$ "fictitious length" of aerial equivalent to the top-loading; as pointed out in section (4), this quantity is not a "constant," any more than λ_0 would be, but since it has no relation to any directly measurable significant quantity its use is less liable to misinterpretation than is that of the term "natural wavelength."

(*To be concluded.*)

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Part of the Stand of the Bell Telephone Manufacturing Company, Antwerp, at the Brussels International Fair —March 13–27, 1938—including displays of Selenium Rectifiers, Automatic P.B.X's, Telephone Sets, Radio Direction Finders, Ships Radio Equipment and Street Traffic Control Systems.

Final Stage Class "B" Modulation

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An outstanding feature of the evolution of broadcasting transmitters during recent years has been the conquest of the technical difficulties which, for a long time, stood in the way of efficient operation as viewed from the standpoint of power consumption.

There were several lines of attack and a number of solutions have been evolved, one of which is the system of final stage Class "B" modulation.

The development of this system followed inevitably from the considerations that: (1) plate modulation is inherently efficient; and (2) the difficulty, which in the past prevented its utilisation to full advantage, would be overcome if Class "B" high efficiency audio frequency amplifiers with transmission characteristics necessary for broadcasting service could be constructed for the required high power level.

A broadcasting transmitter using this system of modulation comprises a master oscillator followed by a chain of high frequency amplifiers terminating in a high efficiency plate modulated amplifier and a chain of low frequency amplifiers terminating in a Class "B" push-pull power amplifier. The output voltages from the final low frequency amplifier are applied to the plates of the final high frequency amplifier in series with the D.C. plate supply to the latter. For full modulation the output voltage of the low frequency amplifier has a peak value nearly equal to the D.C. plate voltage on the high frequency amplifier, so that the plates of the latter are swung at audio frequency from

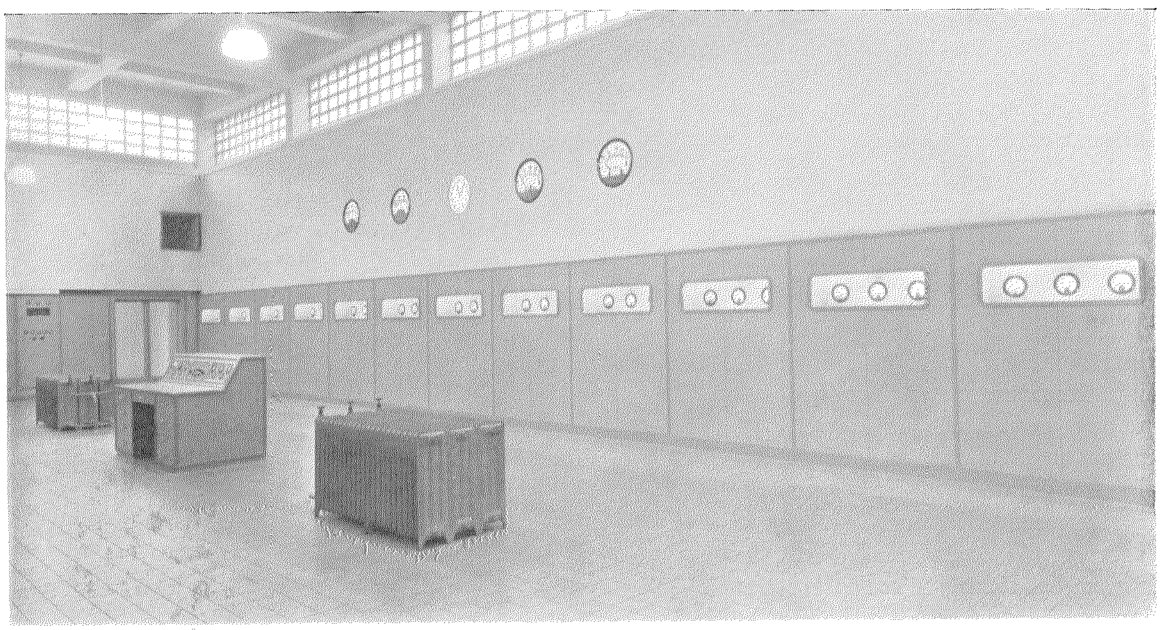


Fig. 1—100 kW Class "B" modulated transmitter at Melnik, Czechoslovakia. Frequency 1 113 *kc. Complete reserve penultimate and final stage valve cubicles are provided and can be switched in at a moment's notice without requiring any tuning readjustments.*

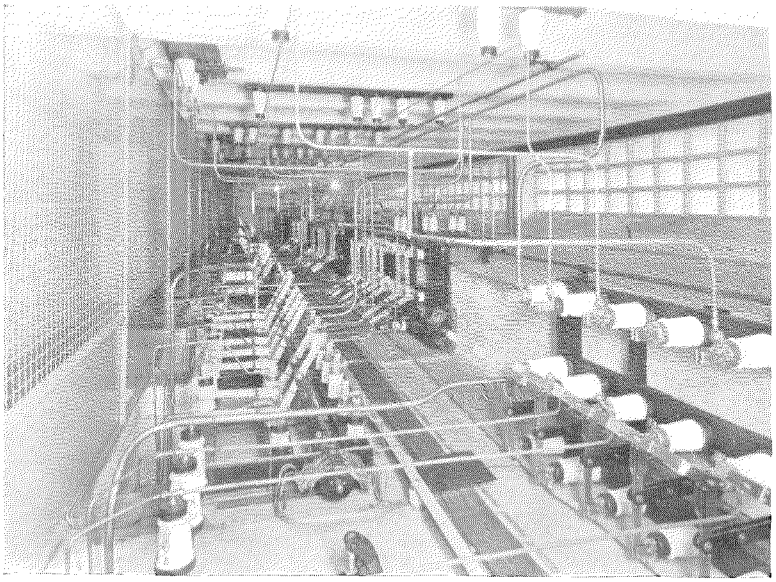


Fig. 2—Cubicle change-over switchgear at Melnik. Cubicles not in service are completely isolated and are accessible while the transmitter is in operation. The switching enclosure is situated directly over the valve cubicles.

approximately zero to twice the D.C. value.

A number of transmitters (Figs. 1 to 3 and 16 to 19 employing this system have been put into operation during the last few years, and the general principles are well known. It may be of interest, however, to review the technique and examine the performance, with particular reference to efficiency and fidelity, being obtained in practice at the present time.

In considering the overall efficiency, it is sufficient for comparison with other systems to focus attention on the overall final stage conversion efficiency which takes account of the combined efficiencies of the final high frequency and low frequency amplifiers. It is well-known that the efficiency of the plate modulated high frequency amplifier alone is high, of the order of 70 per cent., and interest lies mainly in consideration of the performance of the low frequency amplifier.

The useful output power is delivered by the high frequency amplifier and is equal to $P\left(1 + \frac{K^2}{2}\right)$, where P is the carrier power and K is the modulation factor. The power delivered to the amplifier comprises a D.C. component, which is nearly constant for all levels of modulation and equal to $\frac{P}{\eta_1}$; and an A.C. component, delivered by the voice frequency amplifier, varying with the modulation level and equal to $\frac{P}{\eta_1} \cdot \frac{K^2}{2}$, where η_1 is the conversion efficiency of the high frequency amplifier. To obtain the overall efficiency it remains to evaluate the D.C. input required by the low frequency amplifier at various levels of modulation.

In an ideal case the input to the low frequency amplifier would be zero for zero modulation, and the efficiency would be high and constant for all outputs. An approximation to that state of affairs would be obtained in the case of modulation by an alternator. If η_2 were the

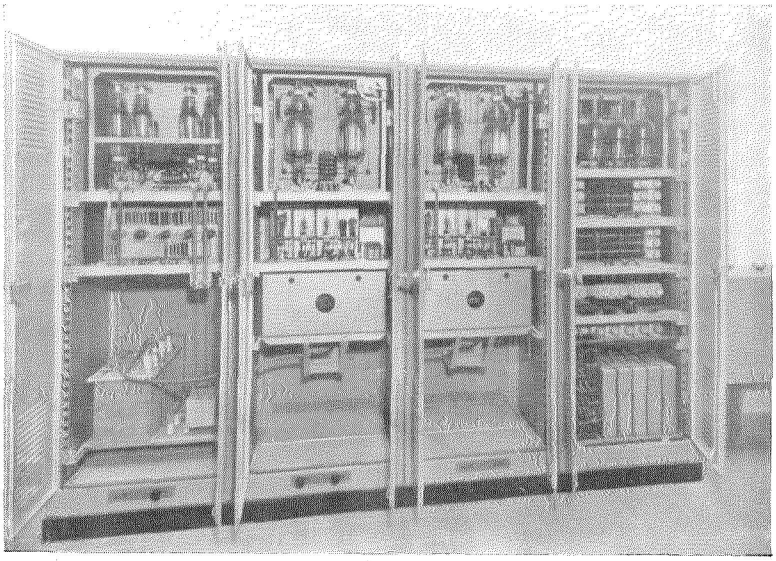


Fig. 3—High frequency and low frequency exciter units with reserve high frequency exciter and exciter power supply and control unit, as utilised in 10 *kW*, 20 *kW and* 100 *kW transmitters. Reserve valves are provided in the final stages of the exciters.*

efficiency of the low frequency source, the overall final stage efficiency would be η_1 for zero modulation and $\dfrac{2\,\eta_1\,\eta_2\left(1+\dfrac{K^2}{2}\right)}{2\,\eta_2+K^2}$ for other levels of modulation.

Thus, if η_1 and η_2 were both equal to 70%, the overall efficiency would be 70% for zero modulation and 61% for full modulation. This represents optimum conditions which may serve as a basis for comparison with the results actually obtained.

In the practical case the overall efficiency for carrier is less than η_1 because it has not yet been found possible to design a high fidelity low frequency amplifier such that the input may be reduced to zero for the quiescent condition. Moreover, the efficiency at intermediate levels of modulation is reduced because the conversion efficiency of the low frequency amplifier falls off as the power delivered is decreased; and, finally, the efficiency for full modulation is less than the above mentioned figure, since it is not possible with existing technique to utilise the low frequency amplifier up to an efficiency of 70% for full output.

The input to the low frequency amplifier for the quiescent condition may not be reduced to zero inasmuch as amplitude distortion would then be introduced due to the curvature of the plate current—grid voltage characteristics of the valves at their lower extremities. It is necessary to work with a residual current, the value of which is relatively critical for the best conditions of fidelity. This will be appreciated from Fig. 4, showing qualitatively how, by proper choice of the residual current, it is possible to arrive at a practically linear overall characteristic for the two valves in push-pull. With existing valves and present technique the residual current for the highest fidelity is of sufficient magnitude to have a pronounced effect on the overall efficiency for zero modulation, reducing it in fact from about 70% to the order of 60%.

The efficiency of the low frequency amplifier when delivering power depends first on the residual current and, secondly, on the ratio of the audio frequency plate voltage to the D.C. plate voltage, rising as the ratio increases

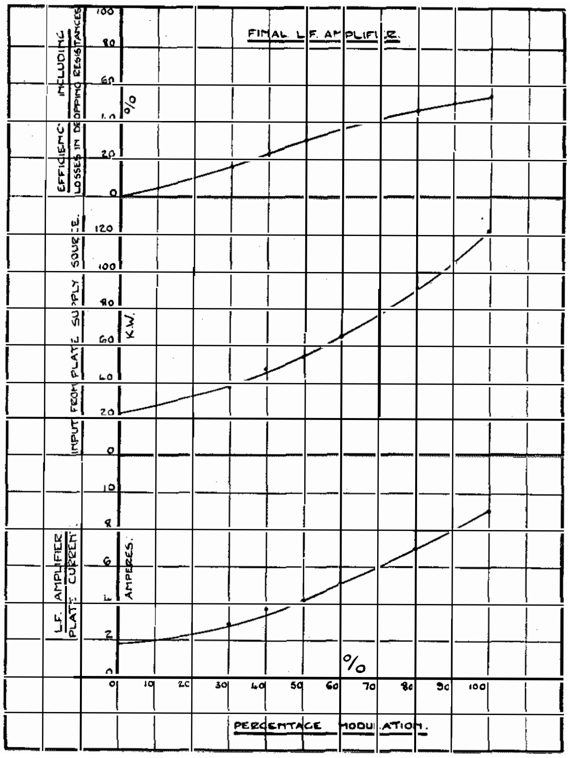


Fig. 5—Curves relating to the final low frequency amplifier of a 100 kW broadcasting transmitter. The conversion efficiency rises to 54% at the load level corresponding to full modulation. The static residual plate current is one-fifth of the full load plate current. The high tension supply voltage is 13 000.

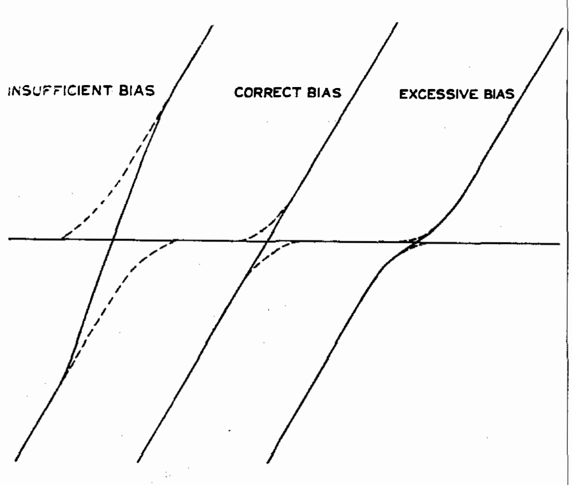


Fig. 4—The grid bias voltage for the final low frequency amplifier is determined so that the overall static plate current-grid voltage characteristic for the two valves in push-pull is rectilinear.

with increasing output. At full load the peak A.C. plate swing should be as high a fraction of the D.C. voltage as is consistent with requisite fidelity. The working conditions which may finally be adopted depend on details of design and on the provisions which may pcssibly be incorporated for distortion correction.

It is convenient to express the value of the residual plate current for the quiescent condition as a fraction of the low frequency amplifier plate current corresponding to the output required for full modulation. If we call that ratio $\frac{1}{R}$ and if η_1 and η_2 are the conversion efficiencies of the high frequency amplifier and of the low frequency amplifier at full load, respectively, the overall final stage efficiencies for zero and full modulation can be simply expressed as follows :

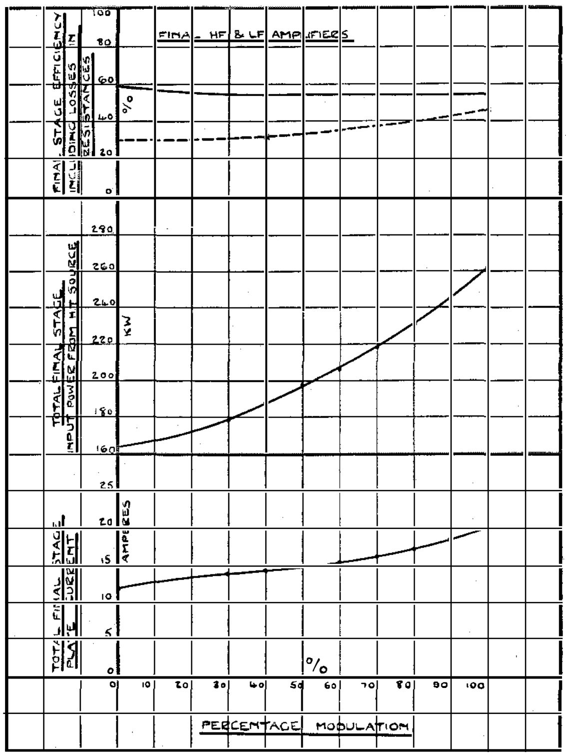


Fig. 6—Curves of input and efficiency for the final low frequency and high frequency amplifiers of a 100 *kW transmitter. The overall conversion efficiency is* 60% *for zero modulation dropping to* 55% *at full modulation. The conversion efficiency of the high frequency amplifier alone is* 71 %. *The final stage efficiency of a grid modulated transmitter is shown for comparison as a dotted curve.*

$$\text{Efficiency for zero modulation} = \frac{2\,R\,\eta_1\,\eta_2}{2\,R\,\eta_2 + 1}\,;$$

$$\text{Efficiency for full modulation} = \frac{3\,\eta_1\,\eta_2}{2\,\eta_2 + 1}\,.$$

By way of example the results, which may be obtained in practice taking figures derived from a 100 kW medium wave brcadcasting transmitter, may now be considered.

The conversion efficiency cf the final high frequency amplifier involving valves and output circuit is 75%, dropping, however, to 71 % when account is taken of the lcsses in short circuit current limiting resistances inserted in the plate feed circuits in the interests of reliability.

In the low frequency amplifier the peak A.C. plate swing for full modulation level amounts to 80% of the D.C. voltage on the valves. For that condition the conversion efficiency is 54%, losses in the limiting resistances being included. Furthermore, the ratio of the plate current at full load to the residual current for the quiescent condition is equal to 5.

Substituting these values in the above expressions :

Efficiency for zero modulation

$$= \frac{2 \times 5 \times 0.71 \times 0.54}{(2 \times 5 \times 0.54) + 1}$$

$$= 60\%\,;$$

Efficiency for full modulation

$$= \frac{3 \times 0.71 \times 0.54}{(2 \times 0.54) + 1}$$

$$= 55\%.$$

The equivalent values for a transmitter of the old-fashioned type would be about 33% and 49% for zero and full modulation, respectively.

The variations of plate current, input power and efficiency for the final stages of a 100 kW transmitter are shown in the curves of Figs. 5 and 6.

Fidelity considerations involve : first, the frequency response characteristic ; and, secondly, the amplitude distortion. The latter may be expressed as the amplitude of introduced harmonics given as a percentage of the fundamental for the case of single tone modulation, and the amplitude of introduced inter-

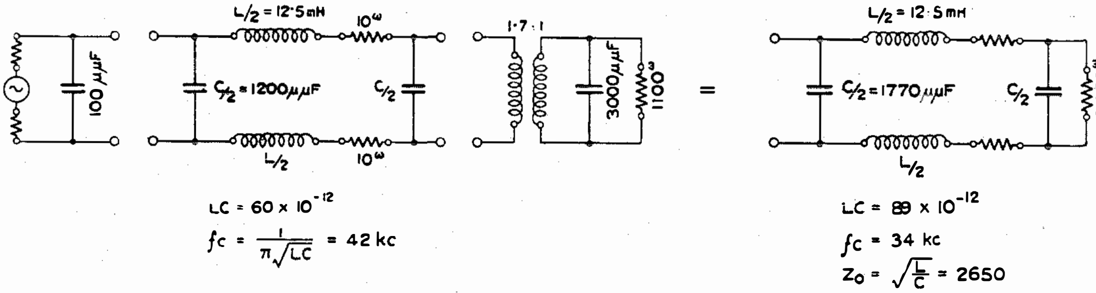


Fig. 7—100 kW transmitter. Approximate equivalent circuit of output transformer at the higher programme frequencies. The cut off frequency exceeds three times the highest frequency to be transmitted. The characteristic impedance is too low to match the load but is satisfactory as indicated by the practical results.

modulation components for the case of two tone modulation.

For broadcasting service the frequency response curve should be flat within about plus or minus two decibels from thirty to ten thousand cycles. No particular difficulty is encountered in meeting this requirement since it is practicable to construct high power inter-stage and output transformers with transmission characteristics equivalent to those of the familiar low power types, as will be evident from consideration of the representative cases illustrated in Figs. 7 and 8. They show the equivalent circuits for the higher transmitted frequencies of the output transformers of 100 kW and 2.0 kW transmitters, both including and excluding the transferred extra capacities equivalent to the unavoidable shunt capacities of the terminating circuits. The networks constitute low pass filter sections having cut-off frequencies well in excess of the highest frequency to be transmitted. The loss resistance gives rise to one or two decibels attenuation below the cut-off frequency. The characteristic impedances of the networks, including the extra shunt capacities, are not so far different from the terminating resistances that loss or gain due to reflection is important. Fig. 9 shows a response curve for a 100 kW transmitter.

Amplitude distortion occurring in practice is due mainly to limitations in the low frequency amplifier, except at levels of modulation above about 90% when the high frequency amplifier contributes a predominating share.

The fidelity of the low frequency amplifier system depends to a marked extent on the characteristics and design of the penultimate stage, particularly with regard to its output

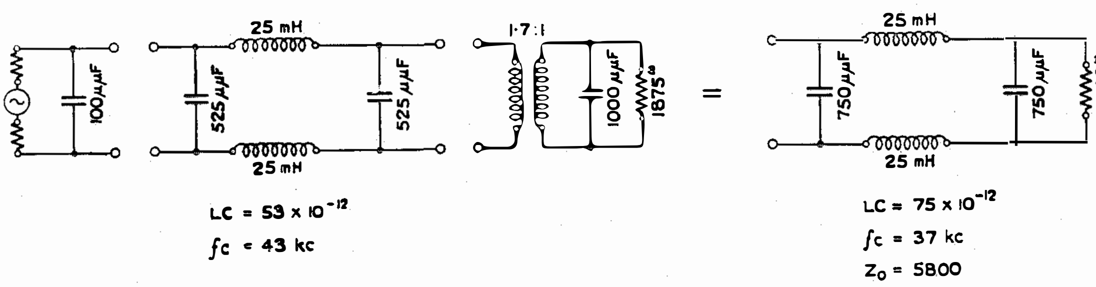


Fig. 8—2.0 kW transmitter. Equivalent circuit of output transformer. The characteristic impedance is nearly equal to the transferred load resistance so that the final low frequency amplifier valves work into a unity power factor load circuit.

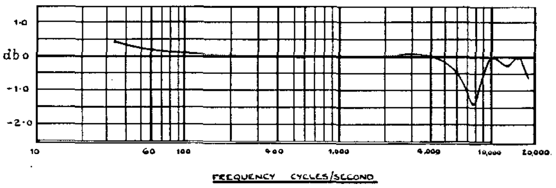


Fig. 9—100 *kW broadcasting transmitter overall frequency response characteristic with feed-back.*

voltage regulation under the varying load presented by the grids of the final stage.

The regulation is determined by the internal resistance of the penultimate valves and the transmission properties of the interstage coupling device which, for convenience, is a step-down transformer. The requirement is that the impedance of the driving stage, as seen from the final stage grids, should be low for all frequencies in the transmitted band.

From considerations of amplitude distortion it is advisable to use valves in push-pull in the penultimate stage, preferably operated in Class "A" rather than in Class "B." Imposing severe requirements on the exciter amplifier driving the stage is thus avoided, even though with given valves a somewhat lower internal resistance could be obtained with Class "B" operation by permitting a larger step-down in the interstage transformer.

In a high power transmitter it is convenient in the penultimate amplifier to use valves working from the same high voltage plate supply as the final amplifiers. Hence, an interstage transformer of adequate step-down ratio is required in order to obtain the lowest possible resistance as seen from the grids of the following amplifier. In the 100 kW transmitter, the valve resistance from plate to plate is 8 000 ohms and the transformer voltage ratio is six to one, reducing the resistance of the driving stage to 222 ohms.

Having obtained the requisite low resistance by proper selection of the valves and transformer step-down ratio, it becomes necessary to consider whether there is any reactance in the coupling system capable of impairing the regulation, particularly at the extremities of the band of frequencies to be transmitted. The transformer cannot be ideal and has leakage reactance and shunt capacity; also, there is shunt capacity in the plate and grid circuits between which it is connected. The result in practice is that, unless appropriate measures are taken, the regulation deteriorates at the higher frequencies, giving rise to progressive increase of amplitude distortion with increasing frequency. This difficulty can be overcome or at least reduced to reasonable proportions by so designing the transformer that its leakage inductance and shunt capacity, together with the shunt terminating capacities, constitute approximately a constant K low pass filter section having a characteristic impedance, considered on the primary side, more or less equal to the plate resistance of the penultimate valves. Under these conditions, if the cut-off

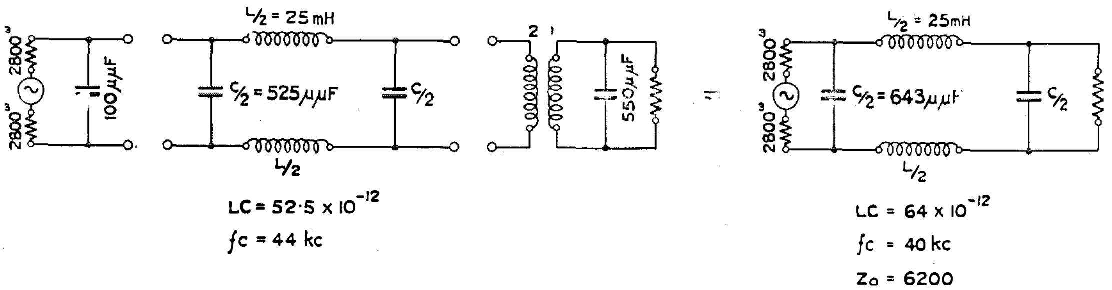


Fig. 10—10 *kW transmitter. Approximate equivalent circuit at high frequencies of the output transformer of the penultimate low frequency amplifier. The characteristic impedance is nearly equal to the plate resistance of the driving valves so that a pure resistance is seen from the grids of the final amplifier.*

frequency of the section is considerably higher than the highest frequency to be transmitted, the impedance seen from the driven stage is a pure resistance nearly constant with frequency, and equal to a value determined only by the valve resistance and the transformer ratio. These design features are realised in practice as far as possible, and are exemplified in Fig. 10 for the penultimate amplifier of a 10 kW transmitter.

The use of a swamping resistance across the grids of the final amplifier can improve the regulation of the drive. It is of no advantage from the point of view of reducing the resistance of the driving stage, rather the reverse, since by loading the valves it reduces the maximum plate swing which can be obtained without

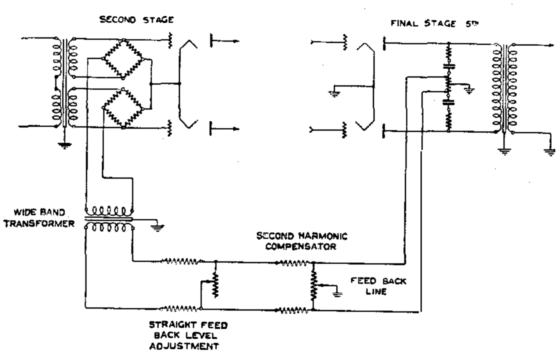


Fig. 12—Diagram showing principle of feed-back system. Voltages are tapped off a potentiometer across the plates of the final L.F. amplifier and introduced in opposing phase into the second stage. Additional second harmonic compensation is obtained by unbalancing the system with respect to ground.

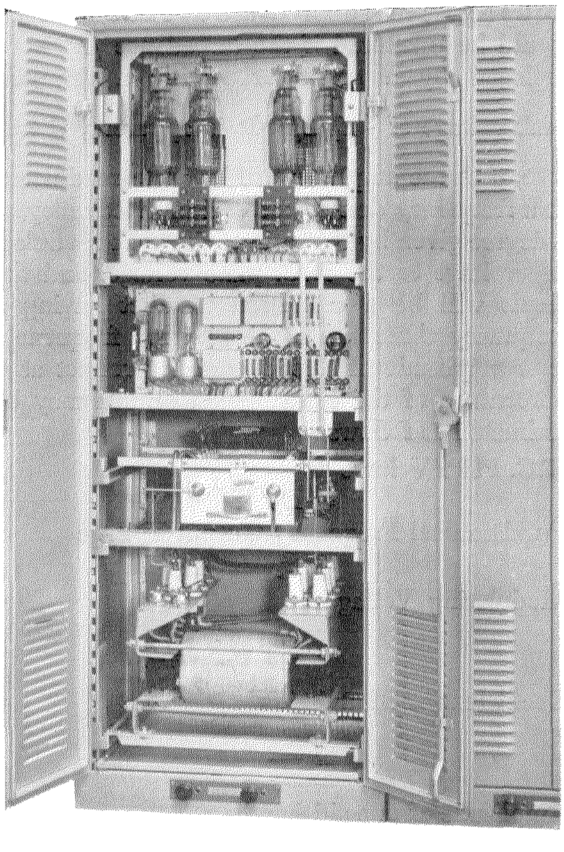


Fig. 11—Low frequency exciter unit driving the penultimate L.F. amplifier in a 100 kW transmitter. It comprises four stages in Class "A" push-pull. Feed-back is introduced into the grid circuit of the second stage and subsequent stages are resistance-capacity or choke-capacity coupled. A neon tube limiter is incorporated in the last stage cutting off sharply at a level above a predetermined value, so avoiding excessive voltages in the power amplifiers due to overmodulation.

grid current and so necessitates reduction of the transformer ratio; but it can help to reduce the effect of reactance if the system is not exactly equivalent to a matched filter. In the 100 kW transmitter a resistance of 800 ohms is connected from grid to grid, the value being so chosen that, with the transformer and valves utilised, the penultimate stage begins to draw grid current at a drive level slightly above that corresponding to the conditions for full modulation.

Another source of possible amplitude distortion arises in the output transformer of the final stage if it is not designed in proper relation to the load resistance and the inevitable shunt capacities of valves and associated circuits. If this relationship is faulty, the input power factor will not be sensibly unity over the transmitted band of frequencies. Assuming adequate primary inductance for proper operation at the low frequencies, the leakage inductance and shunt capacities may then be such as to cause embarrassment at the high frequencies. The proper solution is to design the transformer so that, in association with the terminating shunt capacities, it will be equivalent at the higher frequencies to a constant K low pass filter having a characteristic impedance approximately equal to the load resistance and having a cut-off frequency at least three times the highest frequency to be transmitted. This is the aim, but in high power transmitters, at any rate, it is difficult to avoid arriving at a characteristic

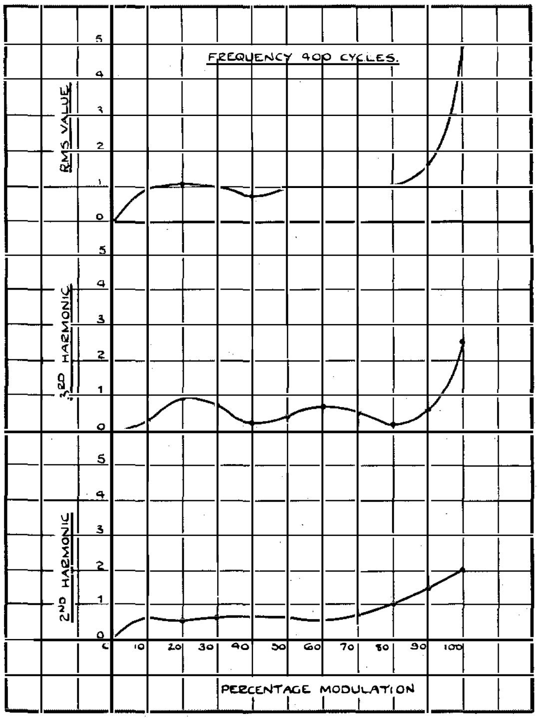


Fig. 13—100 *kW transmitter. Measured values of introduced harmonic content for* 400 *cycles input plotted against modulation level.*

impedance somewhat lower than the load resistance, as shown, for example, in the case illustrated in Fig. 7. Final results, however, indicate that such a mismatch is permissible.

An additional point relating to the output transformer requires careful attention. In the Class "B" final amplifier, there are normally high second harmonic voltages on the primary side of the transformer. These could be transferred to the secondary through the capacity between the windings with the result that important second harmonic distortion components would appear in the output delivered. Interposing a grounded screen between the primary and secondary windings has been found, in practice, to eliminate the trouble, but at the expense of making more difficult the design of the transformer to meet other requirements involving low leakage reactance, etc.

The amplitude distortion is dependent also on the impedances of the plate supply and grid bias supply sources feeding the final low frequency amplifier. The amplifier in operation feeds back, into the supply sources, current at double the modulating frequency and at sum and difference frequencies in the case of multi-tone modulation. If the supply sources present considerable impedance to these currents, there will result voltages which, in general, will remodulate the amplifier and thus give rise to distortion components.

Consider, for example, the plate supply system, if the amplifier is fully driven by a single tone of frequency f with corresponding total plate current I_P. The second harmonic current passing through the plate supply source will have a peak amplitude equal to $\frac{2}{3} I_P$ and, if the impedance of the supply system to the second harmonic be $/Z/$, a voltage of $\frac{2}{3} I_P /Z/$ will be set up across the supply source. The conditions would be unaltered if the impedance $/Z/$ were replaced by a generator having a terminal peak voltage of $\frac{2}{3} I_P/Z/$. It is apparent, therefore, that the amplifier will be plate modulated at a frequency double the exciting frequency to a degree given by the ratio of the peak modulating voltage to the D.C. plate voltage, and the result will be a distortion product having a frequency three times the exciting frequency. The amplitude of this distortion product as a percentage of the fundamental output may be estimated as follows:

D.C. Plate Voltage

$$= E_P$$

Fundamental Frequency Output Voltage

$$= E_o$$

Modulating Voltage

$$= E_m = \frac{2}{3} I_P/Z/$$

Coefficient of Modulation

$$= \frac{E_m}{E_P}$$

Amplitude of one sideband

$$= \frac{E_o}{2} \times \frac{E_m}{E_P}$$

Distortion Content as Percentage of Fundamental Output

$$= \frac{100}{E_o} \times \frac{E_o}{2} \times \frac{E_m}{E_P} = \frac{33\, I_P/Z/}{E_P}$$

In the case of a 100 kW transmitter with a capacity of 50 microfarads in the high tension filter feeding the final low frequency amplifier and a short-circuit limiting resistance of 25 ohms between the filter and the valves, there is obtained, for a modulating frequency of 30 cycles:

$$E_P = 11\,840$$
$$I_P = 9.7$$
$$|Z| = \sqrt{25^2+106^2} = 109 \text{ ohms}$$

Percentage third harmonic distortion at full output

$$= \frac{33 \times 9.7 \times 109}{11\,840} = 2.95\%$$

Similarly, when two tones of frequencies f_1 and f_2 are simultaneously applied, they are modulated by the sum and difference frequencies (f_1-f_2) and (f_1+f_2) to an extent depending on the impedance of the supply system to those frequencies. Modulation by the difference frequencies is the more important, since it is at the lower frequencies that the supply system has appreciable impedance in practice. Distortion products at f_1 plus the difference frequency, and f_2 minus the difference frequency, (if f_1 be the higher of the two frequencies) result.

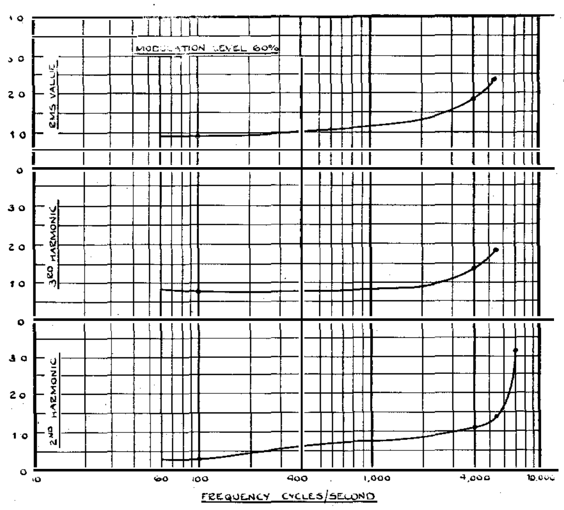


Fig. 14—100 kW medium wave transmitter. Harmonic content for 60% modulation plotted against input frequency.

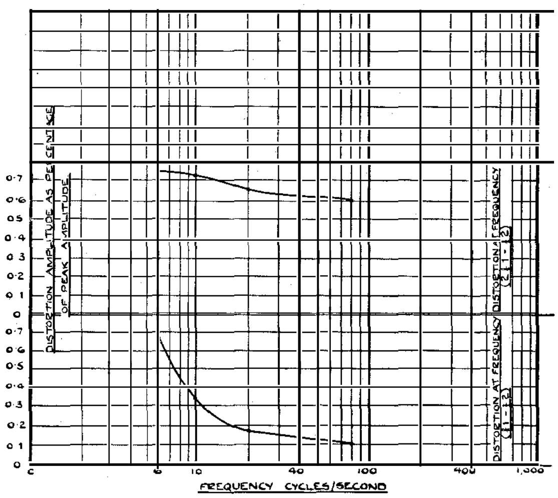


Fig. 15—100 kW transmitter. Curves showing intermodulation distortion. The upper curve shows distortion due to push-pull unbalance and the lower curve shows distortion due to the impedance of the plate supply source. In both cases the values are very low.

The amplitudes of these distortion products may be computed in the manner just indicated for the estimation of the third harmonic distortion, as follows:

Intermodulating product from two tones each of half full amplitude as a percentage of the full load amplitude $= \dfrac{12.5\, I_P|Z|}{E_P}$,

where I_P is the full load plate current.

Thus, if f_1 is 1030 cycles, and f_2 1 000 cycles, and other values are as in the example above:

Percentage Intermodulation component at 1 060 cycles

$$= \frac{12.5 \times 9.7 \times 109}{11\,840} = 1.1\%$$

At the resonant frequency of the filter and the supply unit, whether rectifier or machine, the impedance has a maximum value; and it can be seen from the preceding that, in the case of a 100 kW transmitter working on about 12 000 volts, the impedance should not exceed something of the order of 250 to 300 ohms. The insertion of extra damping resistances in the resonant circuit is thus required. If the

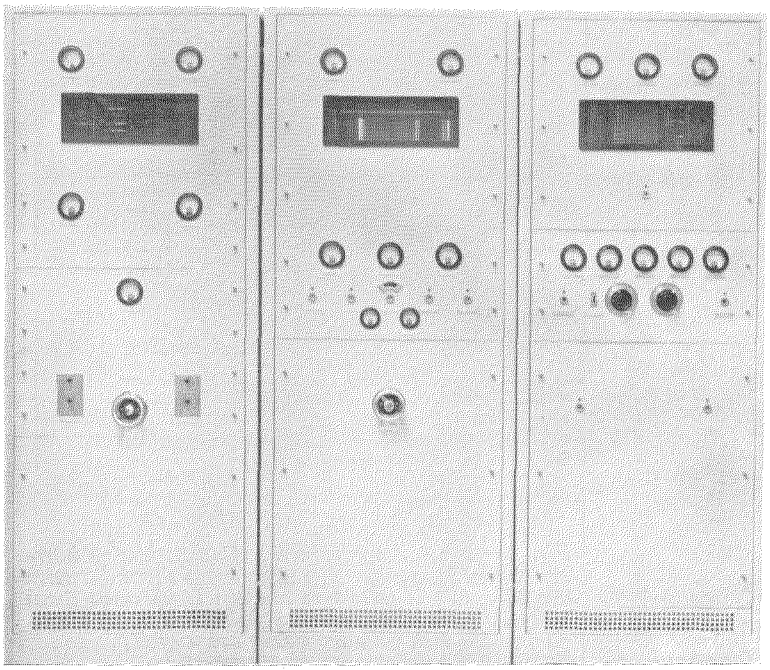


Fig. 16—2.0 kW final stage class "B" modulated broadcasting transmitter. Left to right : Power and control unit—high frequency oscillator and amplifier unit—low frequency amplifier unit.

inductance and capacity of the filter are 1.5 henries and 50 microfarads, respectively, the internal resistance of the supply unit is, say, 60 ohms, and the inserted damping resistance is 100 ohms, the frequency for maximum impedance is 16.6 cycles, the impedance at that frequency being 265 ohms. To that must be added the limiting resistance between the filter and the valves, say 25 ohms, giving 290 ohms altogether.

If the push-pull balance of the low frequency amplifier is not perfect, then in the case of two tone modulation the sum and difference frequency currents flowing in the parallel mode in the primary of the output transformer will give rise to distortion components at frequencies $(f_1 - f_2)$ and (f_1+f_2), the amplitude depending on the degree of unbalance. An increase of second harmonic distortion will of course also occur. Experience shows that it is not necessary, in order to obtain excellent results, to select valves to match each other in the push-pull amplifiers, or to make any differential adjustment of grid bias.

It is evident from the above considerations that it is very advantageous to provide separate high tension filters for the final high frequency and low frequency amplifiers in order to avoid an increase of the second harmonic content due to modulation of the high frequency amplifier by the second harmonic voltages set up across the filter by the action of the low frequency amplifier.

When all has been correctly accomplished, the distortion in a Class "B" modulated transmitter is within the limits at present regarded as acceptable for broadcasting, but still better results can be obtained with inconsiderable outlay by the expedient of applying reverse feed-back in the voice frequency system. It would be troublesome to apply a heavy degree of feed-back, but very little complication and difficulty is involved in applying a degree of

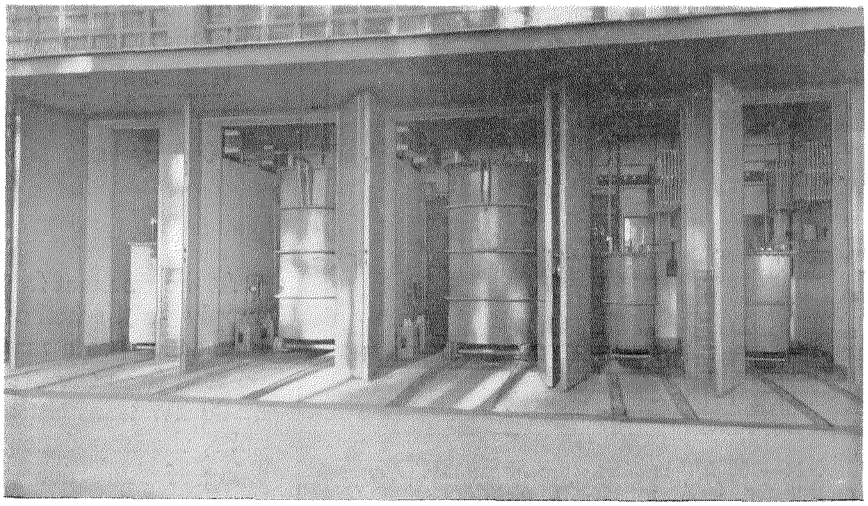


Fig. 17—Penultimate and final stage low frequency transformers with reserves.

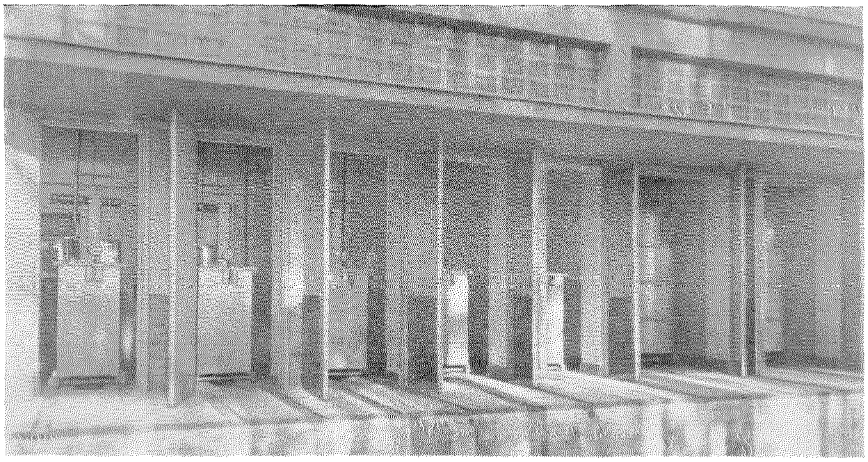


Fig. 18—*Final high frequency amplifier plate feed reactors.*

feed-back sufficient to halve the amplitude of the distortion products originally present within the programme band of frequencies.

The principle of the feed-back system as incorporated in a 100 kW transmitter is illustrated in Fig. 12. The voltages to be fed back are tapped off from a potentiometer connected across the primary of the output transformer of the low frequency amplifier, and are introduced into the grid circuit of the second low frequency amplifying stage across resistance bridges which reduce phase shift due to the output transformer of the first stage. The stages following the point at which the feed-back is introduced are resistance-capacity or choke-capacity coupled with the exception of the penultimate stage, which is transformer-coupled.

The limit to the degree of feed-back which can be applied is determined by phase shift. This occurs mainly in the penultimate stage transformer, which is designed on that account to have the slowest possible phase rotation with frequency.

As the feed-back line is of low impedance, a small transformer is used at the termination to match the impedance of the bridge circuit. This

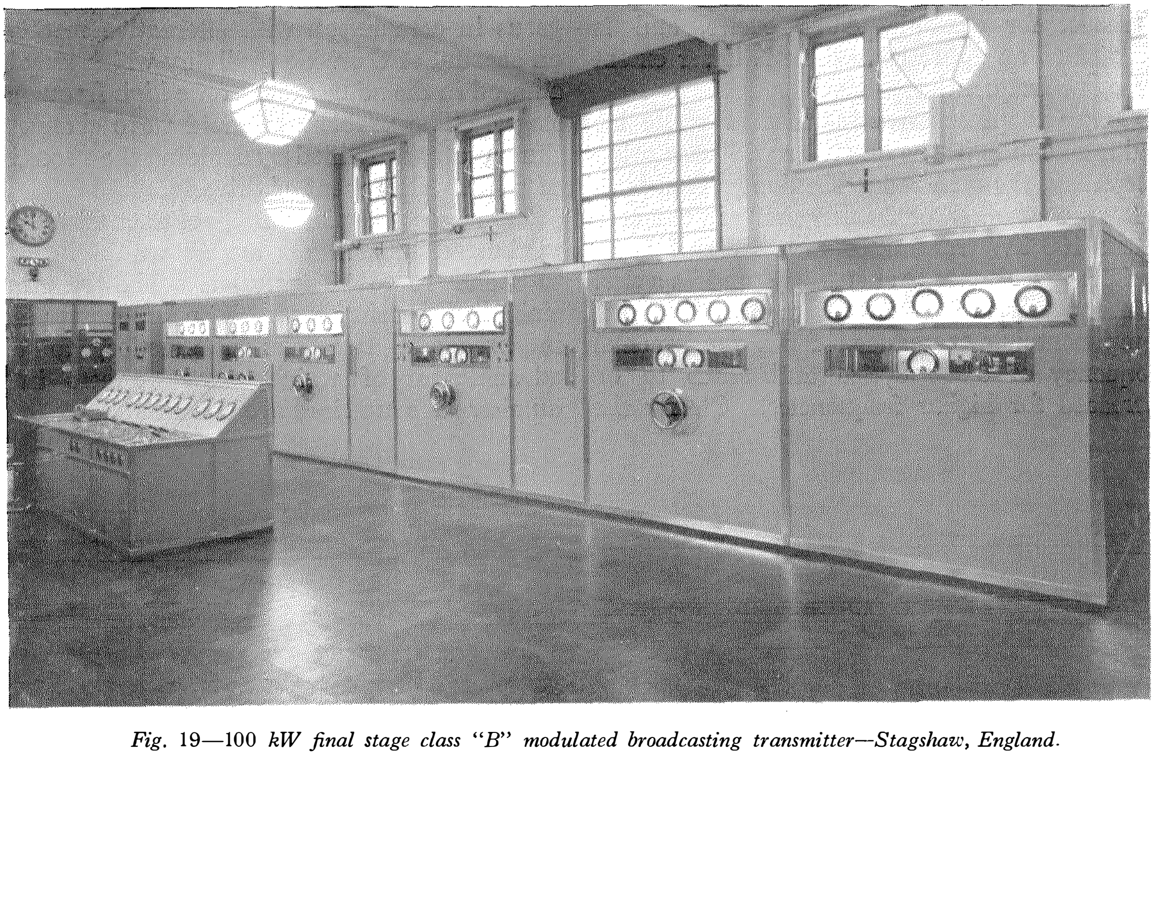


Fig. 19—*100 kW final stage class "B" modulated broadcasting transmitter—Stagshaw, England.*

transformer is specially designed to have negligible phase shift compared to that in the penultimate stage transformer. The transformer serves also to reduce parallel current in the feed-back line.

The feed-back introduced is adjusted to that value which will cause a reduction of 6 db in the overall gain, for which condition the percentage distortion components are also reduced by 6 db, that is to say, their amplitude is halved.

The voltage fed back is less than half of that which could cause instability. Since there are no valves in the feed-back circuit, the adjustments, once made, are permanent.

In addition to the distortion compensation obtained by the direct feed-back, additional compensation of the second harmonic distortion components can be obtained by unbalancing, with respect to ground, the tappings on the feed-back potentiometer, or, more simply, by adjusting the position of the centre-point tapping, thus inducing into the feed-back circuit a suitable fraction of the second harmonic current delivered by the final low frequency amplifier valves into the potentiometer in the parallel mode. In this way the second harmonic distortion can be very considerably reduced for frequencies around the one for which the balance is made, for example, 4 000 cycles, and is reduced to an appreciable though lesser extent for other frequencies.

Owing to progressive increase of phase lag and attenuation in the amplifier with increasing frequency, the gain reduction due to feed-back falls off at the higher frequencies. In the case of the 100 kW transmitter with 6 db of feed-back, it reaches zero at about eighteen kilocycles. Consequently, the third harmonic distortion component is not reduced for a fundamental frequency of 6 000 cycles and tends to be increased for higher frequencies. Clearly the harmonics of these high frequencies do not constitute distortion since they fall outside the received band but, if very exaggerated, they could give rise to interference in adjacent channels. For high input frequencies at which frequency response limitations begin to apply, there is a considerable increase in the harmonic content amplitudes for high levels of modulation but not to a marked extent for levels of modulation below 50 per cent. Consequently, in view of the fact that for normal programmes the modulation at the higher frequencies does not attain high levels—not, for example, exceeding about 10 per cent. at 8 000 cycles—adjacent channel interference effects are not to be expected and, in fact, are not detected in practice.

The fidelity finally attained in Class "B" modulated transmitters is of a very high order as may be appreciated by reference to Figs. 13, 14 and 15, showing harmonic content and intermodulation values achieved in practice. Moreover, the performance is very consistent, the characteristics measured on successive occasions reproducing themselves with remarkable precision.

In its application to short wave transmitters, the Class "B" system is particularly advantageous in comparison with alternative methods on account of the wide flexibility which may be allowed in the adjustment of the high frequency stages. The problem of rapid wave change is thus greatly simplified by avoiding the necessity of critical adjustment of drive levels.

Views in connection with final stage Class "B" modulated transmitters are shown in Figs. 1 to 3 and 16 to 19 inclusive. The technique embodied therein now holds an important place in broadcast transmission practice, to the advancement of which many workers in a number of countries have contributed.

Telephone Traffic in Shanghai *

By A. B. WILKINSON, Assoc.I.E.E.,

Shanghai Telephone Company, Shanghai, China

This article gives an idea of the scope of the system of the Shanghai Telephone Company and brings out a number of points concerned with its traffic, engineering and operating problems.

THE city of Shanghai is divided into three political areas, namely, the International Settlement, the French Concession, and the contiguous Chinese administered districts of Nantao, Chapei and Pootung. Since the inception in 1882 of telephone service in Shanghai the first two areas have been commonly served by successive, non-Chinese controlled telephone companies, and the Chinese area, with the qualification mentioned hereinafter, by the Chinese Shanghai Telephone Administration. On the borders of the foreign settlements, there are also two so-called "extra-settlement" zones which are served by the telephone company due to a situation arising many years ago through special political characteristics then obtaining.

TELEPHONE DEVELOPMENT

The foreign settlements contain the city's main business centre—in appearance little different from that of an occidental city,—Chinese business and residence areas, and high grade "Western" residential areas. The Chinese administered districts contain Chinese businesses and residences in the main, and the two "extra-settlement" zones comprise, respectively, a high-grade "Western" residential area and a mixture of Chinese and non-Chinese businesses and residences. The approximate population and telephone density of the foreign and "extra-settlements" combined and of the Chinese administered areas, as of December, 1936, were as shown in the adjoining Table.

	Population	
	"Foreign and Extra Settlements"	"Chinese Administered Areas"
Non-Chinese	60 000	—
Chinese	1 645 000	2 089 000
Total	1 705 000	2 089 000

	Telephones per 100 of Population	
	Foreign Settlements	Chinese Areas
Non-Chinese	33·7	—
Chinese	2.02	.29
Combined	3.14	.29

Shanghai has a low overall telephone development. Notwithstanding its importance as the chief port of China, its population figures are somewhat misleading with respect to actual and potential telephone development. The average "wealth per capita" is extremely low judged by Western standards; more than half of the population exists in poverty. It is evident from the telephone development figures that this adverse factor is least prominent in the foreign and extra-settlement areas. With the development of Shanghai as a transit centre of commerce linking China with the rest of the world, however, a slow improvement in the standard of living is taking place along with growing acceptance by the Chinese of the Western conception of living and the appurtenances of business. This trend is reacting on the telephone business and it is reasonable to believe that, under favourable circumstances, a greatly increased rate of telephone expansion will materialise within the next two decades.

It is interesting to note, incidentally, that the Shanghai foreign settlements emphasise, due to an unusual language situation, the advantages

* Manuscript submitted June, 1937.

of automatic switching over manual working for the local services. For economic and other reasons Chinese operators must be used on manual "A" boards; yet, because of the numerous languages and varied dialects encountered, these native operators are unable to give the quick and accurate service achievable with manual service in Western countries and demanded in large business centres such as Shanghai. Approximately fifteen foreign nations are represented in Shanghai in considerable numbers, and at least four distinct versions of Chinese are spoken.

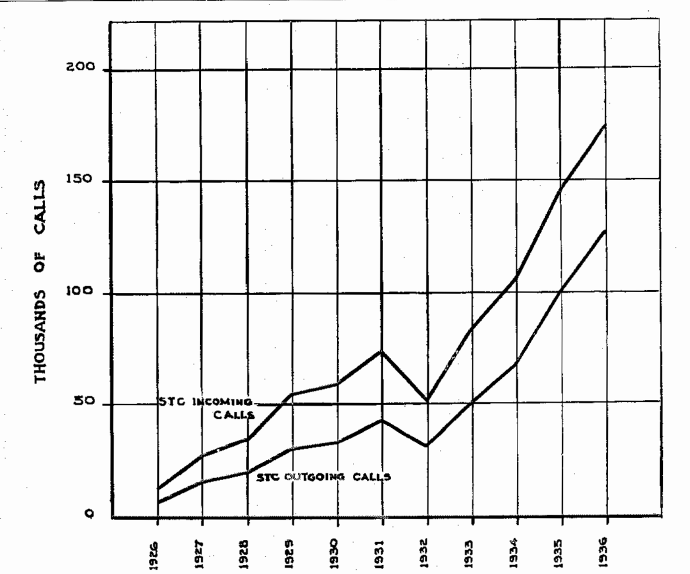


Fig. 2—Long Distance Toll Calls (Yearly) Between Shanghai Telephone Co. and Chinese Government Toll System.

EQUIPMENT

Shanghai Mutual Telephone Company, Ltd.

This predecessor Company (1900 to 1930) of the Shanghai Telephone Company (SHANTELCO), installed 1 000 lines of Ericsson gear-driven equipment in 1924 to serve a lower river front section of the city. Subsequently, this office was extended to 2 000 lines. In September, 1927, 5 000 lines of Rotary 7–A1 equipment were cut-over to serve a section of the city's main business centre, this office being enlarged to 10 000 lines in 1929. It was evident, however, that public satisfaction could not be assured with old magneto and two types of automatic equipment operating in the Company's relatively small system, and subscribers naturally were intolerant of the wide divergence between the grade of local service provided by the automatic and by the manual central offices. Moreover, the problem of interworking between offices of different types was difficult to solve economically in a manner suitable for long term operation.

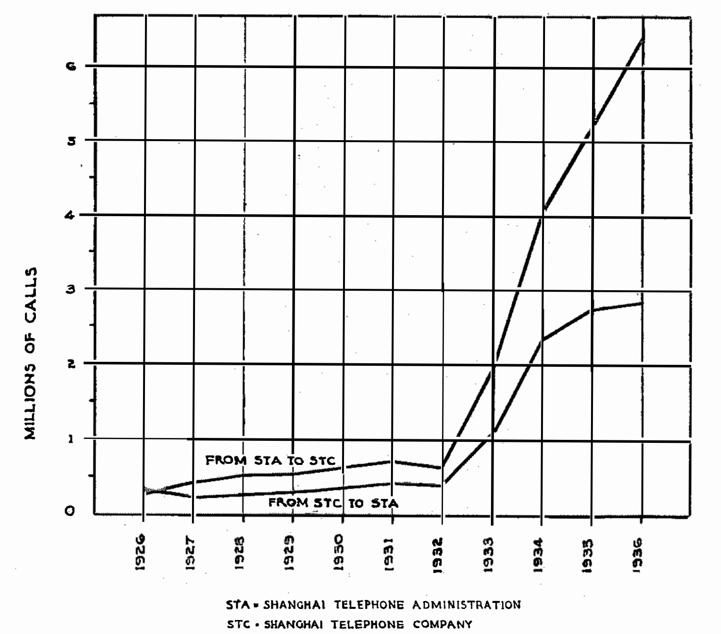


Fig. 1—Intercommunication Toll Calls Between Shanghai Telephone Co. and Shanghai Telephone Administration.

Shanghai Telephone Company

Following the investigation in 1929 by Mr. B. O. Anson of the British Post Office, the properties of the Shanghai Mutual Telephone Company, Ltd., were acquired by the International Telephone and Telegraph Corporation. By March 26, 1932, within a period of twenty

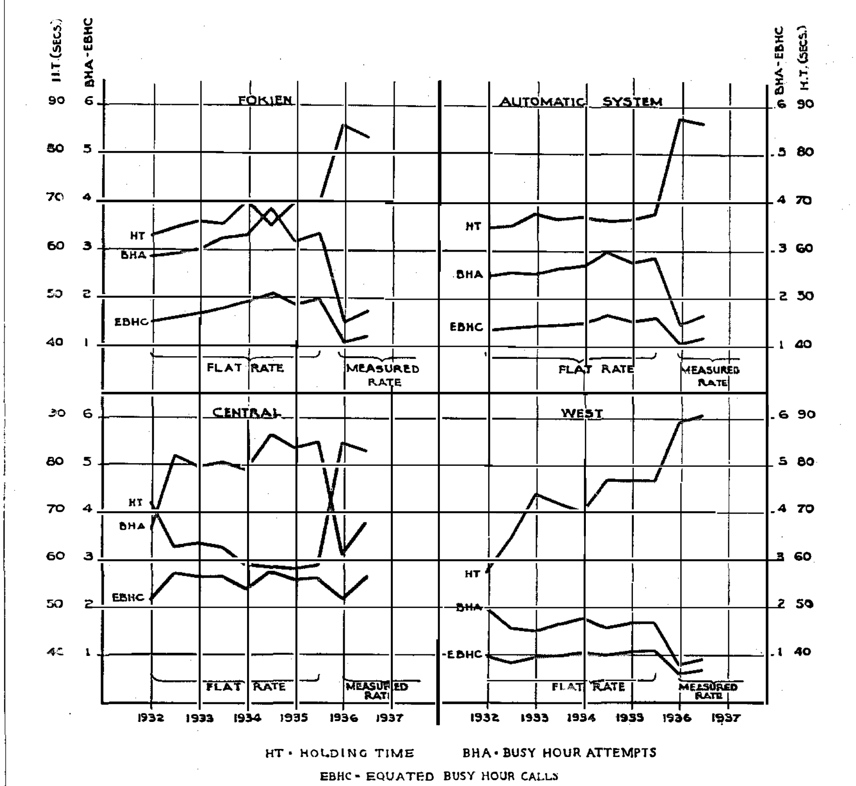


Fig. 3—*Representative Calling Rate Data.*

months, the new Shanghai Telephone Company had provided dial telephone service to all subscribers within the foreign settlements through the installation of 31 400 lines of automatic equipment.* By May, 1937, the foreign settlements and one extra settlement were served by seven No. 7–A1 Rotary automatic offices with a total capacity of 48 400 lines.

The other extra-settlement zone is served by a common battery office of 3 000 lines capacity and a satellite office of 300 lines capacity.

The Chinese administered areas are served by two step-by-step automatic offices with capacities of 3 000 and 1 500 lines (cut-over in 1933), and a common battery office of 600 lines capacity. The working subscribers' exchange lines, as of April 30, 1937, were :

Shanghai Telephone Company's system : 39 692.

Shanghai Telephone Administration's system : 4 500 (approx.).

INTERCONNECTION AND LONG DISTANCE SERVICES

Up to January, 1926, no intercommunication existed between the telephone systems of the foreign settlements and of the Chinese areas in Shanghai. After an interconnecting service was established, development was relatively slow until the cut-over of the two step-by-step offices in the Chinese administered area in 1933 ; thereafter considerable expansion of the Chinese Shanghai Telephone Administration's system took place. Traffic between the two systems is now handled via a manual C.L.R. dial-trunking plan, each network having direct access to that of the other.

In May, 1926, the then somewhat limited long distance lines and associated distant communities became accessible to the telephones of Shanghai's foreign settlements. In the interim, the long distance network has been considerably expanded by the joint efforts of the Chinese Central and Provincial Governments. Figs. 1 and 2 represent the development of traffic to and from the system of the foreign settlements.

The Shanghai Telephone Company, operating in the foreign and extra-settlements, furnishes long distance service solely as a tributary system, traffic in both directions going through the Chinese Government Shanghai Toll Centre. The latter's terminating toll lines provide access to towns and cities in the provinces of Anhwei, Chekiang, Fokien, Honan, Hunan, Hupeh, Kiangsu, Kiangsi ; and, also to the radio links : Shanghai-Tokyo (1 100

* "The Reconstruction of the Shanghai Telephone System," by J. Haynes Wilson, M.C., *Electrical Communication,* July, 1932.

air miles, opened February 15, 1936), Shanghai-Hankow (430 air miles, opened September 1, 1936, supplementing a metallic circuit), Shanghai-Canton (750 air miles, opened December 5, 1936) and Shanghai-San Francisco (7 000 air miles, opened May 19, 1937). Tests are being conducted by the Chinese Government engineers on prospective radio circuits linking Shanghai with London, Berlin and Rome.

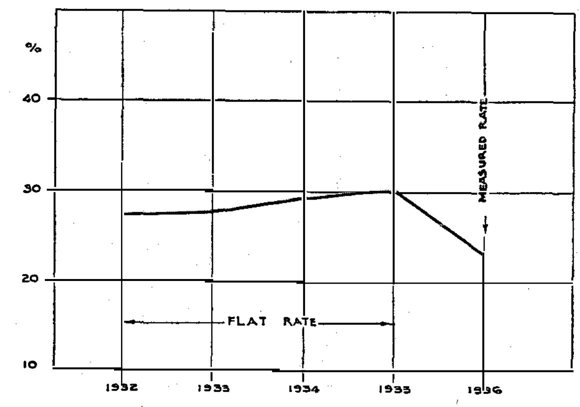


Fig. 5—Proportion of "Non-conversation Time" in 1st Selector Total Holding Time.

CALLING RATES

Subscribers' calling rates, because of their many ramifications, are worthy of special study. Their trend, also, was a major factor leading to the change from flat rate to measured rate service in the SHANTELCO system.

In the definition of calling rates in relation to equipment requirements in the Rotary system, the registers required per line vary approximately in direct proportion to changes in the line busy hour machine attempts; whereas (as in all systems), in respect to speech equipment, the average holding time must also be studied. The holding time of the register remains practically a constant figure whereas, as will be noted from Fig. 3, there may be considerable variation in speech equipment holding time with variation of line busy hour attempts.

Fig. 3 gives calling rate data for three automatic offices and for all SHANTELCO automatic offices consolidated. Central and Fokien serve what may be termed "Foreign" and "Chinese" type business areas, respectively, whilst West is a "mixed" area containing approximately 35% business and 65% residence telephones. Each office, prior to the introduction of measured rate service, shows individual trends of busy hour originating attempts and their holding time. In Central, comparable with a European area in the transaction of business, the situation was exceptional in that whilst the line demand for speech equipment rose by 23%, the line demand for registers, used only in setting up connections, rose by over 51% between April, 1932, and December, 1935. The register is the most expensive unit of Rotary automatic equipment. West experienced a marked upward trend in holding time and, during the years 1933 to 1936, fair stability of line busy hour originating attempts, resulting in a rising trend in the equated calling rate. Fokien showed a rising trend in both factors and the result was a very pronounced upward trend in the equated calling rate. In the automatic system comprising seven dial offices, during the years of 1932 to 1935 when the flat rate was in effect, the average line busy hour originating attempts rose by 15.6% (peak in 1934 of 22.2%) and the equated calling rate by 20.6% (peak in 1934 of 24.4%), resulting in equivalent increases in the line demands for registers and speech equipment, respectively.

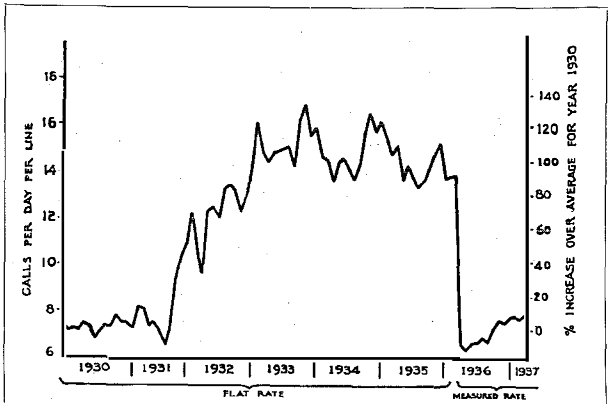


Fig. 4—Line Completed Calls per Average Day, 1930–1937.

The factors influencing the local calling rate are multifarious. It is possible, however, from the experience of SHANTELCO in the years 1930 to 1937, to identify the following four major elements:

(1) Kind of service (i.e., flat or measured) and the rate structure ;
(2) Quality of the telephone service ;
(3) Characteristics of subscribers (i.e., the proportion of business and residence, of P.B.X. and of regular lines, as well as national characteristics) ;
(4) The type of service (manual or automatic).

(1) is obvious: Figs. 3 and 4 clearly show changes in the calling rate occasioned by the transfer from flat to measured service; and also that, during four years of flat rate service, the calling rate did not stabilise but continued to increase, indicating that in a large community a flat rate must tend towards an increasing calling rate until the overloading of lines halts the calling rate development. At this point the system will be burdened with a high proportion of abortive busy calls, and the telephone rates, supporting a high average line equipment investment, will be prohibitive of development in the less wealthy strata of the community.

It is convenient to refer to Fig. 4 to illustrate the joint influences of factors (2) and (4). In August 1931 the first offices in the dial conversion programme were cut into service, resulting in improved plant and operation and the switching of more traffic by automatic or semi-automatic means. Subscriber appreciation is evident by the almost immediate rise in line-day completed calls.

Referring to factor (3), it is obvious that a wholly business area will have a higher calling rate than a wholly residence area; and, in a specific case, an increasing proportion of business lines will result in greater overall line calling rates. Similarly, since P.B.X.'s are the result of actual traffic demand and since groups of lines associated therewith carry more traffic per line than regular lines at equivalent grades of service, it follows that an increasing proportion of P.B.X. lines in an area will usually indicate an increasing originating calling rate. In calling rate forecasts for business areas the influence of P.B.X. lines may usually be provided for by using the *station* calling rate as the base figure, whereas in residence areas the *line* calling rate is the better base.

Furthermore, bearing on factor (3): (*a*) Chinese shops fronting the streets are usually "open house" to itinerants in so far as flat rate utilities are concerned; (*b*) several small and separate businesses, conducted in adjacent rooms, share the same telephone; (*c*) the traditional Chinese business day is at least twelve hours, most Chinese operating small businesses residing on their business premises—the Western conception of hours of work is far from being accepted even in Shanghai; and (*d*) the Chinese "family" is more extensive in scope, frequently including a wide variety of relatives and having correspondingly greater social contacts.

As to the holding time factor, SHANTELCO's experience has been the usual one, i.e., increased local holding time will result from a greater proportion of social traffic (often accompanied by a shift in the office or group busy hours) and from change from flat to measured rates.

TABLE I

	F.R.	M.R.	
		A	B
Business Line Day Completed Calls System	18·33	8·84	10·80
,, ,, ,, ,, ,, Central	18·91	10·78	13·09
,, ,, ,, ,, ,, Fokien	21·96	8·91	11·91
Residence ,, ,, ,, ,, System	7·82	3·82	4·36
,, ,, ,, ,, ,, Pichon	7·53	4·18	4·64
,, ,, ,, ,, ,, Montigny	8·40	3·68	4·12
H.T., Busy Hour Originating Attempts System	69″		88″
,, ,, ,, ,, ,, Central	59″		83″
,, ,, ,, ,, ,, Fokien	69″		83″
,, ,, ,, ,, ,, Pichon	76″		89″
,, ,, ,, ,, ,, Montigny	69″		88″

Notes: 1. "A" data for period March to June, 1936.
"B" ,, ,, ,, January to March, 1937.
2. H.T. (holding time) for F.R. and M.R. derived in period July–December, 1935, and July–December, 1936, respectively.

Decreased holding time, in general, will result from the substitution of automatic for manual service.

CHANGE FROM FLAT TO MEASURED RATES

It would probably be helpful to many "flat rate" companies if, from the experience of SHANTELCO, formulae could be quoted for assessing the effect on the calling rate of a change from a flat to a measured rate for various types of measured rate structures. It seems to the author, however, that the constituent factors are too complex to permit of specific evaluation. Not only is a forecast required of the reduction in calls per day per line ; but, also, the change in the line busy hour attempts and the average holding time need to be estimated, in order that the future investment in central office equipment per line may be assessed. It may be possible, nevertheless, when the calling rate data (day calls, busy hour attempts and holding time) have reached stability under the measured rate, to determine certain approximately quantitative principles. The data in Table I, illustrating the results of SHANTELCO's rate change to date, will be of interest.

The approximate classification of the subscribers in the territory served by SHANTELCO and of four of its central offices are given in Table II.

TABLE II.
DIRECT EXCHANGE LINES (%)

	Business	Residence	Chinese	Non-Chinese
System ..	55	45	68	32
Central ..	96	4	45	55
Fokien ..	88	12	95	5
Pichon ..	18	82	43	57
Montigny ..	57	43	86	14

The greater uniformity of the holding time data under the measured rate is noteworthy, as is also the larger reduction in the calling rate of the primarily "Chinese" areas.

It is of interest to consider the breakdown of the average holding time into conversation and non-conversation time. Obviously, the higher the proportion of conversation time, the more effectively the machines are working. SHANTELCO observations have shown that the first selector average "non-conversation time," i.e., time required for building up a connection to the required line, wait-answer or hold-busy interval, switch clearing time, etc., is approximately 21.5 seconds. Fig. 5, based upon service observations, gives the relationship between first selector "non-conversation time" and total holding time from 1932 to 1937, and shows a situation progressively worse under the flat rate but a marked improvement under the measured rate.

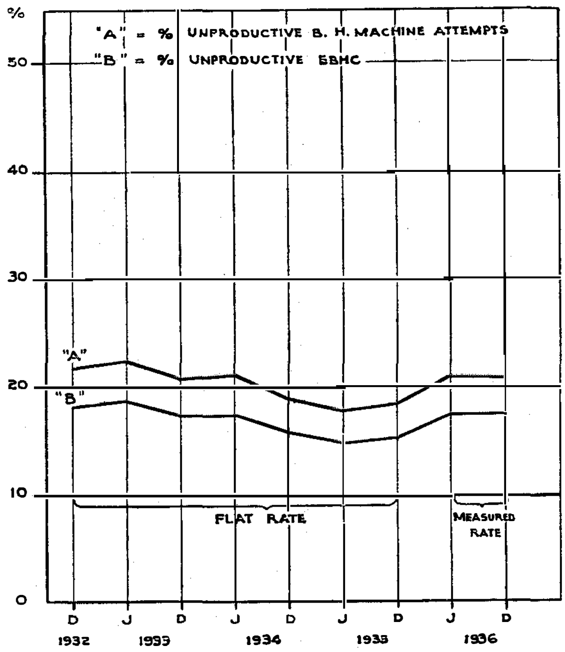


Fig. 6—Percentage Unproductive Register Traffic.

Any tendency on the part of subscribers to lift and replace handsets without dialling, to dial incomplete numbers, etc., will reduce the traffic efficiency of the plant, particularly that of the registers. A large proportion of such operations occurs in the Shanghai system, usually more pronounced in business areas and most evident of all in Central. The latter contains the highest proportion of P.B.X's from which, characteristically due to extensions "flashing" for the operator after connection to the central office, many non-productive actuations of central office equipment result.

Fig. 6 records the percentage of non-productive machine attempts as measured on the registers of SHANTELCO's system since conversion to automatic working took place. It is interesting to note that an up-turn occurred with the introduction of measured rate service, attributable partly to increased care by subscribers in verifying numbers before completion of dialling, and partly to enhanced station gain, bringing in more inexperienced telephone users.

Noteworthy is the fact that the per cent. register non-productive equated calls practically represents the per cent. registers necessary to operate the unproductive traffic. This non-productive traffic is apt to be overlooked when designing an office. Its major contributory factors in Shanghai are:

(*a*) "Curiosity" usage by subscribers;
(*b*) Careless usage by subscribers, necessitating re-dialling;
(*c*) P.B.X. extension users "flashing," as noted above;
(*d*) Subscribers lifting handset, replacing, and then verifying number in directory;
(*e*) Subscribers "dialling before tone."

No certain and quick means appear to be available of materially reducing these misusages to a low level; nevertheless, it is reasonably certain that, over a period of years and with a greatly expanded system, there will be considerable improvement in the precision of subscribers' operation.

Multi-Office Area Traffic Distribution

Due to the numerous extensions to offices in SHANTELCO's system since the dial conversion, and to the Annual Budget requirement of a five-year trunk forecast, ample scope for the testing of methods of calculating traffic distribution has been afforded. Despite unfavourable factors such as the generally unstable calling rate and station development, the decision has been reached that the "community of interest" factor method gives results within the expected variation of a forecast from actual realisation.

This method is based upon the theory that the busy hour originating calls of an office will be distributed to all offices in the area in proportion to the ratio of each office busy hour originating calls to the sum of the busy hour originating calls of all offices, i.e.,

$$EBHC\text{, office A to office B } = \frac{t_a \times t_b}{T}$$

where $t_a = BH$ originating equated calls, office A,
$t_b = BH$ originating equated calls, office B,
and T = sum of BH originating equated calls, all offices.

Actually, of course, this relationship between two offices usually varies from the above expression. The variation is evaluated by dividing the actual calls by the theoretical calls, thus:

$$K = \frac{t_{ab} \times T}{t_a \times t_b},$$

where t_{ab} = actual $EBHC$, office A to office B
and K = "community of interest" factor.

In applying this theory for trunk traffic forecasting, provided it be assumed that the relationship between any two offices will remain

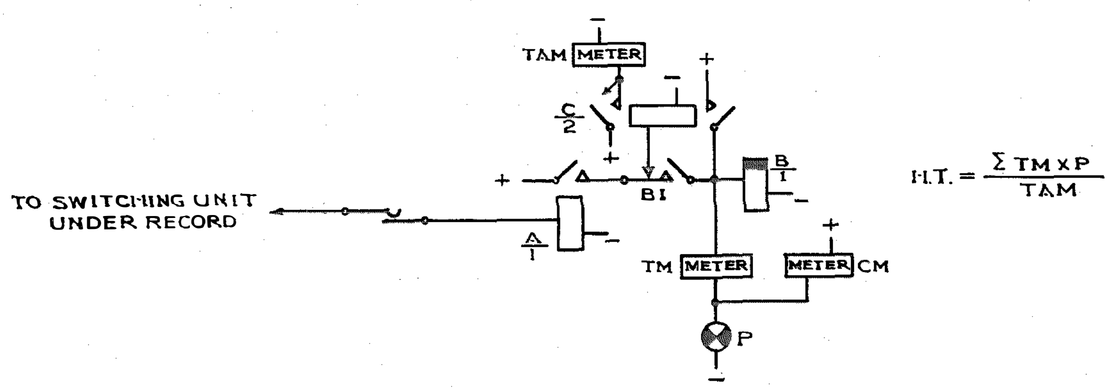


Fig. 7—Holding Time Recorder (Meter Type).

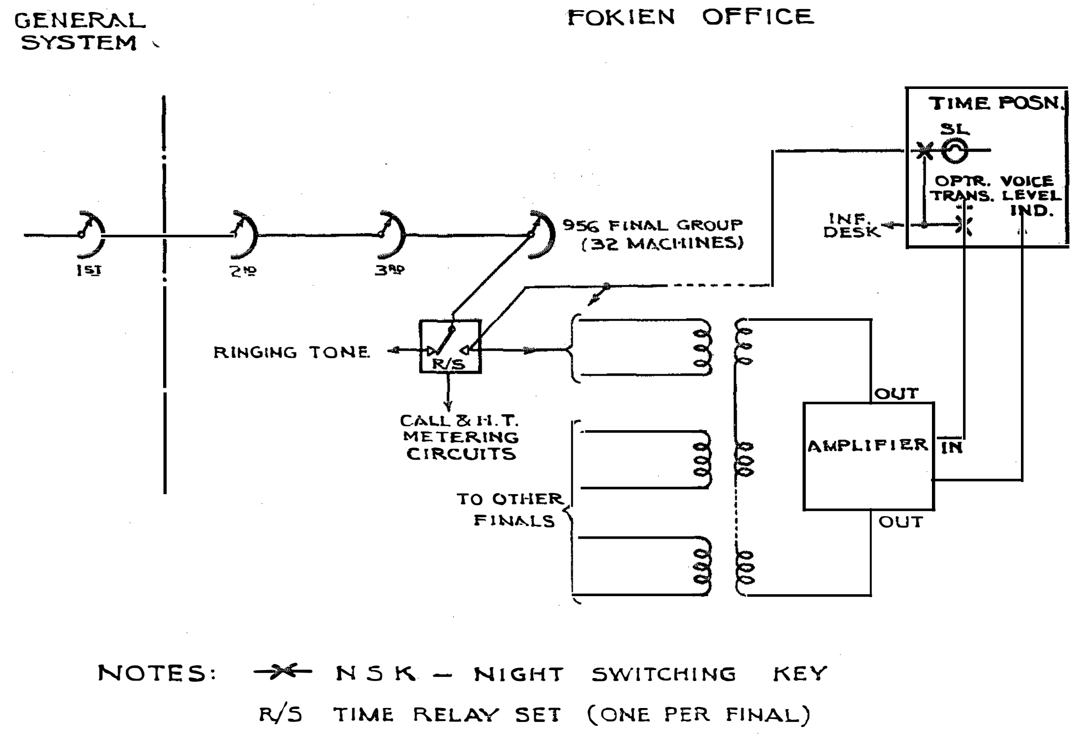


Fig. 8—General Trunking Plan—"Time" (Code 9–5678)

unchanged, the traffic forecast becomes a simple proportion calculation:

$$t'_{ab} = t_{ab}\left(\frac{t'_a \times t'_b \times T}{t_a \times t_b \times T'}\right),$$

where t'_{ab} = future *EBHC*, office A to office B,
t'_a = future *BH* originating equated calls, office A,
t'_b = future *BH* originating equated calls, office B,
and T' = sum of future *BH* originating equated calls, all offices.

If it be thought, however, that the "community of interest" factor between two offices will change within the period of the forecast, owing perhaps to an alteration in the composition of the areas or to a rate change, a new "community of interest" factor must be estimated and incorporated in the calculation:

$$t'_{ab} = t_{ab}\left(\frac{t'_a \times t'_b \times T}{t_a \times t_b \times T'}\right)\frac{K'}{K}$$

where K' = estimated future "community of interest" factor.

Estimation of changes in "community of interest" factors requires considerable care and knowledge of the areas concerned, and downward alteration of one factor may necessitate upward changes in some or all of the remaining factors of the outgoing traffic of the offices involved. When a new office is being engineered, establishment of a considerable number of new "community factors" may be necessary, particularly if the new office is not intended to provide relief on a random basis.

Holding Time Measurements

The derivation of average holding times for the purposes of traffic engineering of subscribers', central office and miscellaneous equipment is a problem requiring considerable attention, since such data are of fundamental importance. Methods employed by SHANTELCO vary according to the facilities available on or at the circuits involved.

For holding time studies on subscribers' private branch exchanges, stop watch records have been largely used. For studies on equipment located in central office buildings, multi-pen chart recording instruments, recording ammeters and meter-type holding time recorders have been employed. From extensive experience with these three methods, it has been found that the meter-type recorder, which employs the well-known principles embodied in Fig. 7, is the most satisfactory from the view-points of accurate results and cost of integration of records as well as maintenance of equipment. The requirement of an impulse medium tends to restrict this type of recorder to central office plant but, with some auxiliary arrangements, it may also be used for records at subscribers' premises.

Multi-pen recorders have been used extensively for manual office recording, but the expense of integrating records by manual means is somewhat high. However, such recorders are highly useful for traffic engineering purposes since they can conveniently be associated with a variety of circuits.

Recording ammeters, SHANTELCO's experience indicates, are not entirely satisfactory for holding time purposes inasmuch as only approximately accurate results are obtainable because of pen lag troubles, integration inaccuracies, and the difficulty of identifying errors occasioned by faults in auxiliary equipment.

TABLE III
SURNAME CHARACTERS WITH OTHER MEANINGS.

Characters (1) (2) (3)	Phonetic Equivalent	Meaning (if any)
周 鄒	CHOW	(1) Around (2) ———
劉 柳	LIU	(1) ——— (2) Willow
史 施 石	SHIH	(1) History (2) Philanthropy (3) Stone
李 黎 利	LI	(1) Plum Tree (2) People (3) Profit
吳 伍 武	WU	(1) ——— (2) Five (3) Martial
陸 魯 盧	LU	(1) Land (2) Vulgar (3) ———
魏 韋 衛	WEI	(1) Elevated (2) Leather (3) Protect
朱 諸 褚	CHU	(1) Red (2) All (3) ———
符 傅 富	FU	(1) Amulet (2) Teacher (3) Rich

INFORMATION SERVICE

This service is mainly of interest because of the bilingual files employed. They are of the Rotary type; at present, they contain 46 500 listings in each "English" file and 37 000 listings in each "Chinese" file. Foreign residents, in general, are not listed in the "Chinese" files. The disposition of the listings in the "Chinese" files follows the plan used in the Chinese Standard Dictionary of the reign of Emperor K'ang Hsi of the Ching Dynasty (approximately 300 years ago), the characters being arranged according to the number of "strokes" which vary from one to twenty-seven per character. Thus, the surname with the simplest character appears at the beginning of the file, corresponding to names beginning with "A" in the "English" file, and that with the most complex character at the end of the file. The sub-index system under similar names follows the same plan, listings with the simplest "personal" name characters appearing first in each block of surnames. No abbreviation of characters by means of initials is possible.

When a Chinese subscriber calls "09" (Information) and makes his request, the operator has to visualise the surname character and how many "strokes" it has before referring to the file. Many enquiries are complicated by the existence of two or more entirely dissimilar characters having the same phonetic equivalent; thus the two characters 張 and 章 are both pronounced "Chang," but the operator is able to paraphrase the first into "Long Bow" Chang and the second into "Stand Early" Chang. Further examples, involving similar indirect indentification of characters, are shown in Table III.

It is worthy of note that the average time taken to deal with an Information call requiring reference to a "Chinese" file is no longer than that required to deal with an "English" file call.

SHANTELCO gives only number information to the general public. The Central Information Bureau rotary files, "English" and "Chinese," are used for the semi-annual telephone directories through the medium of photography.

TIME SERVICE

SHANTELCO inaugurated an official "Time" service in April, 1934. Previously, confirming

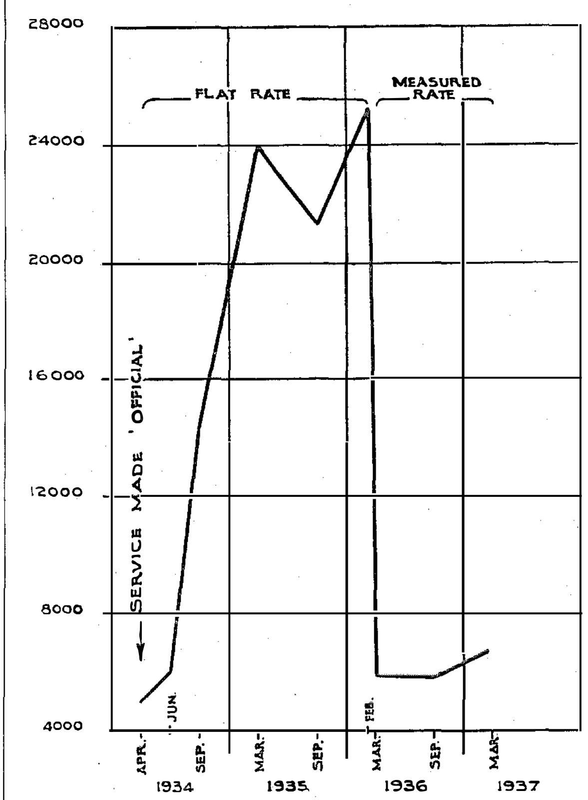


Fig. 9—Time Service (Code 9–5678)—Average Day Calls.

the experience of most telephone companies, subscribers had made increasingly numerous requests for the time of day to the "A" operator, the repair service operator, the information operator, or the toll recording operator. With measured service in view, SHANTELCO decided to establish an official Time Service which would meet the public's requirements and which also would eliminate the practice of dealing with Time Calls individually on miscellaneous positions intended for other purposes.

In the SHANTELCO Time Service, use is made of the trunking scheme shown in Fig. 8. Subscribers dial 9-5678 and calls are routed via the regular channels to the 956 final group, in which each machine is provided with a special switching arrangement which, when the 10's digit is 7, connects the incoming talking wires to a transformer coupling on the output side of an amplifying system associated with the single "Time" position. No level is tripped on the final selector on a "Time" call and the brush carriage returns home whilst the subscriber is listening, thus shortening the machine average holding time by approximately 2 seconds. The 956 final group also contains subscriber lines.

The "Time" position is provided with a Veeder-type electric clock (maintained correct by regular reference to the local foreign observatory), a lamp to indicate that one call at least is connected to the position, a lamp to signal the cutting-in of a reserve amplifier, and a voice level indicator for maintaining uniform signal strength. A maximum of 32 subscribers may be connected simultaneously to "Time." The signal is given to the minute (e.g., twelve—one, three = 12.13) in English and Chinese (Shanghai dialect) consecutively, approximately every ten seconds. The operator is provided with a transmitter only.

Fig. 9 illustrates the development of "Time" traffic since the introduction of code 9-5678. Statistics relating to "Time" are :

Busy hour of day	11–12 noon
Other busy periods	9–11 a.m. and 4–5 p.m.
Highest recorded day calls under flat rate	25 940
Highest recorded busy hour calls under flat rate	2 337
Average day calls, April, 1937 ..	6 713
Average busy hour calls, April, 1937	606
Average holding time per call (flat rate)	21″
Average holding time per call (measured rate)	31″

The rise in call holding time under the measured rate is partially attributable to an increased proportion of P.B.X. subscribers who, on one of their lines, hold connections to "Time" for long periods, in some instances over one hour, in order to avoid repeat calls. Under the measured rate approximately 7% of calls have a duration exceeding one minute. No timed breakdown feature has as yet been incorporated in SHANTELCO's "Time" system.

MISCELLANEOUS TRAFFIC PRACTICES

It may be of interest to review briefly a few practices and standards adopted by SHANTELCO and proved in operation to be thoroughly desirable.

Numerical Margins

Each office is provided with an excess of call numbers over originating equipment (line and cut-off relays). This excess provides P.B.X. series reserves, etc., and ensures new subscribers not being assigned numbers recently vacated by old subscribers. SHANTELCO's numerical margins provide for a minimum number re-assignment period of four months. The percentage margin, of course, varies in each office, and is higher in business than in residence offices ; the overall system per cent. numerical margin at present is 7.5. The system exchange line disconnections in an average period of four months in the year ending April 30, 1937, were 2 930, and the connections 4 230.

Local standards for P.B.X. reserves are :

2–3 initial working lines,	1 reserved	number
4–6 ,, ,, ,,	2 ,,	numbers
7–10 ,, ,, ,,	3 ,,	,,
11–16 ,, ,, ,,	4 ,,	,,

All final groups are traffic engineered to allow of 100% line fill, but 3rd selectors (except when a small group is involved) are engineered for an overall total fill equal to the originating equipment line capacity.

The required numerical capacity of an office with a minimum number re-assignment period of four months is the sum of :

The required net line capacity of the office ;

The estimated changed numbers in seven months ;

The estimated P.B.X. reserves ;

The estimated disconnections per period of four months.

All the above figures are considered as applying when the subsequent relief is cut into service.

P.B.X. Final Groups

Excluding Central, which is an exceptionally P.B.X. developed area (28% P.B.X. lines) and which involves special bay capacity problems, it has been possible and convenient to standardise the number of machines per group at 26 for all offices, suitable load balance being obtained by proper proportioning of P.B.X. and single line assignments to P.B.X. groups. It is always desirable, of course, for load smoothing and terminal economy reasons, to assign a proportion of single lines to P.B.X. groups.

The "slipping" of the appropriate 20-outlet 3rd selector levels has also been modified to permit of good grading, the "slip" now being to the ninth contact only.

Interception Practices

Three kinds of interception circuits are installed. To the dead line interception circuits are connected vacated numbers, these circuits being provided with number unobtainable tone (four short and one long pulse) at 400 i.p.s. supplied by the dialling tone interrupter. Dead line circuits are engineered on the basis of the office terminating traffic and rate of disconnections; numbers remain connected to dead line circuits for approximately six months.

Changed number circuits are used for intercepting calls to the old number in cases of two-office number changes (when old and new numbers are in the same office, e.g., single to P.B.X. service, the two numbers are "strapped"). Calls are routed to an interception operator who advises callers of the new number; numbers remain on the changed number circuit until the new number appears in a directory.

Where two subscribers are listed in error under the same number, that number is connected to a special interception circuit (portable) which routes all incoming calls to an interception operator. Calls for the correctly-listed subscriber are switched through by the depression of a button; callers for the incorrectly-listed subscriber are notified of the correct number.

Load Guide Routine

This routine is an invaluable practice auxiliary to number assignment work; it is designed to check the loading of all final groups and to prescribe an assignment policy for each group, the aim being to assure uniform loading of final groups. The routine includes a bi-monthly two-day call and line count on every final group and an analysis system giving a picture of the relative loads on each group, allowance being made for varying numbers of lines and group traffic capacities. From this picture, a loading, or assignment policy, is determined for the ensuing two months, prescribing that during this period new subscribers with potentially High, Average or Low terminating calling rates should be assigned to specific groups.

This routine is not applied to 1st line finder groups, since overloads on these groups may easily be eliminated by I.D.F. jumper changes. Overflows on these groups, i.e., originating calls finding all finders occupied, are recorded by group overflow meters.

In the foregoing, the author has endeavoured to present in a practical manner the less prosaic problems encountered by the traffic engineering and operating personnel of the Shanghai Telephone Company.

EDITOR'S NOTE.—*Tribute is paid by the "North China Herald" of December* 1, 1937, *to the manner in which tbe Shanghai Telephone Company kept the Shanghai communications going under conditions of great difficulty during the crisis. The use made of the telephone in the emergency is reflected by the statement that in the first week calls exceeded the normal daily volume by* 150%. *Despite the extra work involved in handling exceptionally heavy traffic and restoring communication when certain exchange plant was damaged, the Company found time to develop a new kind of service, known as the Watchman's Checking Service, so that subscribers were able to keep in constant touch, through trained operators, with watchmen or with servants in temporarily vacated premises. Two special systems were installed for private intercommunication between the various authorities, public utilities, fire brigades and police. During the emergency period over* 6 500 *applications for service were handled by the installation staff, as compared with about* 4 000 *during a normal period of similar duration.*

ALBERT FRANÇOIS JOSEPH DAMOISEAUX

On the 2nd February, 1938, a valued and esteemed member of the staff of Le Matériel Téléphonique passed away.

Albert François Joseph Damoiseaux was born on the 23rd March, 1870 at Mons, Belgium, where his father was Principal of the Athénée Royal for a period of forty-four years.

Having remarkable talents, young Damoiseaux intended to pursue a literary career but had to abandon these plans and completely change the direction of his life. On the 1st December, 1887, he entered the Ecole Militaire. He emerged brilliantly as a 2nd Lieutenant in the Engineers. He soon attained the rank of Captain and assisted in the development of the fortified system which was to withstand the first shock of the Great War.

In 1913, he became associated with the Bell Telephone Manufacturing Company. When the War came, although under no military obligation, he resigned and joined the Belgian Army voluntarily. After the fall of Antwerp he was sent to England, but it was not long before he was called to France to take up work with Le Matériel Téléphonique.

When hostilities ceased, he returned to the Bell Telephone Manufacturing Company and participated actively in the reconstruction of the Belgian telephone system.

In connection with the consideration of the automatisation of the Paris telephone network, he again became associated with Le Matériel Téléphonique. He took an important part in the research which resulted in this automatic system becoming one of the finest in the world.

In addition to his high talents as an engineer and as a specialist in machine switching, Mr. Damoiseaux was esteemed for his modesty and industry, loyalty and kindness, and for his lovable personality. He had the rare gift of imparting his knowledge to others, and aided numerous young engineers in their comprehension of the intricacies of modern communication systems. By his associates and many friends he will be held in grateful memory, not only because of his high character and fine personality, but because of his unfailing inclination to help "the other fellow."

General Properties of Dielectric Guides

By J. SAPHORES,

Professor at the Ecole de Physique et de Chimie Industrielles de Paris,
Les Laboratoires, Le Matériel Téléphonique, Paris, France

THE existing methods of transmission of electrical signals belong to two categories:

(1) Systems of radio communications in which, leaving out of consideration the conductance of the ground and of the upper atmosphere, the energy may travel freely in space without being appreciably guided:

(2) Systems of transmission lines in which the energy does not travel freely but is confined to the path determined by the conductors of the line.

Recently a new conception has appeared, that of dielectric guides, employing new means for determining the path of the signals and seemingly able to provide new methods of transmission.

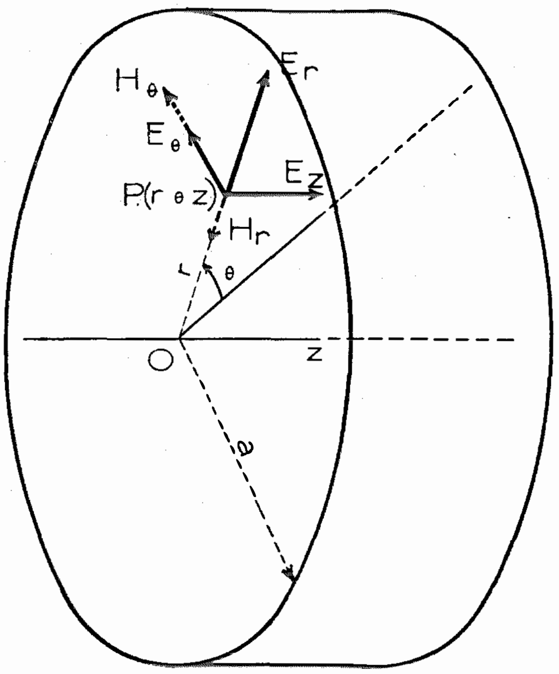
In its simplest aspect, the dielectric guide consists of a metallic tube to one end of which high frequency signals are applied. These travel along the tube to the receiver at the other end, remaining wholly inside the space confined within the tube. The tube forms around the signal a conducting cover, the screening effect of which excludes external perturbations. This confinement of energy to a given path makes dielectric guides, to some extent, similar to ordinary cables, but they have the peculiar characteristic of lacking a second conductor. Another important difference, the origin of which will be examined later on, results from the fact that only very short waves can be used for this kind of transmission; the wavelength cannot exceed certain limits, variable in different cases, but of the same order of magnitude as the diameter of the tube. The technique of dielectric guides for this reason depends upon the development of short waves. The results obtained during the last few years encourage the hope that the difficulty of producing hyperfrequencies is not, however, practically insurmountable. It is necessary to add that the critical wavelengths can, in principle, be shortened by filling the interior of the tube with an insulating material of very high dielectric constant. This aspect of the question must lead inevitably to experimental studies of the properties of dielectrics at very high frequencies, a subject of which little is as yet known.

The theory shows further that, under certain conditions, dielectric guides may consist of the dielectric alone without any metallic envelope. In the sequel, however, this rather complex aspect of the method will be entirely neglected.

The fundamental problem of dielectric guides which physicists will have to solve is the determination of the characteristics and properties of the types of waves capable of being propagated along metallic tubes, which may or may not contain a dielectric material.

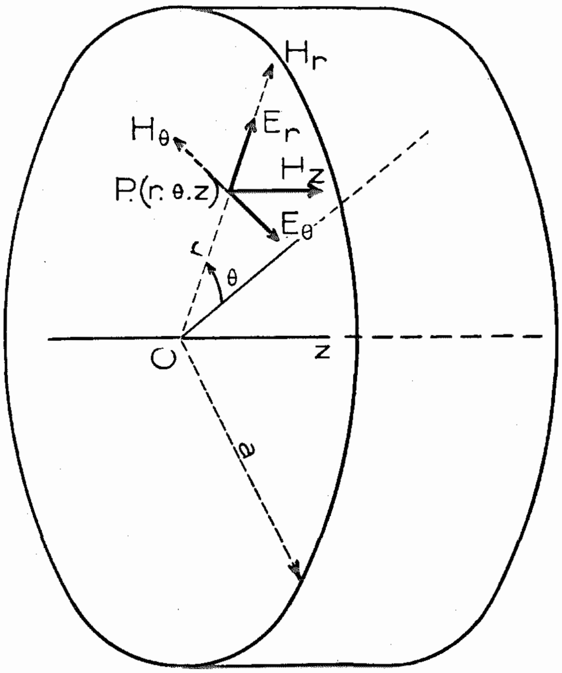
The first detailed analysis of the phenomenon was published by Carson, Mead and Schelkunoff.[1] These authors examined the case of a very long straight tube of circular section and of infinite conductivity. They sought to discover what types of waves, depending periodically upon time, could be propagated along the direction of the axis, with a finite phase velocity, and with the electric intensity E and the magnetic intensity H satisfying the laws of electro-magnetism as expressed by Maxwell's equations. This investigation showed that all possible solutions involved two main classes:

[1] "Hyper-Frequency Wave Guides—Mathematical Theory," *Bell System Technical Journal*, April, 1936, p. 310.



E Waves

Fig. 1.



H Waves

Fig. 2.

The first class consists of those distributions in which the electric intensity E has a component E_z parallel to the axis of the tube, while the magnetic intensity H has no such component ($H_z = 0$). This type was called conventionally E–waves (Fig. 1).

The second class consists of H waves in which, on the contrary, it is the magnetic intensity which has an axial component H_z while the electric intensity has none, viz., $E_z = 0$ (Fig. 2).

Both classes, E-waves and H-waves, contain an infinity of solutions satisfying all the necessary conditions. For each of these solutions the five possible components E_r, E_θ, H_r, H_θ, and E_z or H_z (according to whether the wave is of class E or class H) are expressible in terms of the geometrical and physical constants of the guide by a Bessel's function of the first kind, $J_n(kr)$, of its derivative $J'_n(kr)$, and of the angle $n\theta$.

By considering the functions J_0, J_1, J_2 . . . J_n the E waves are found to consist of the distinct types E_0, E_1, E_2 . . . E_n . . . and the H waves of the types H_0, H_1, H_2 . . . . H_n . . .

The coefficient k which appears in $J_n(kr)$ and $J'_n(kr)$ has the general form :

$$k = 2\pi f \sqrt{\frac{1}{V^2} - \frac{1}{v^2}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where f is the frequency, V the speed of free propagation of waves in a medium identical with that which fills the tube, and v is the phase velocity of the waves within the tube.

By applying the condition that, near the surface of the perfectly conducting tube, the tangential intensity vanishes ($E_z = 0$ and $E_\theta = 0$ for $r = a$), the authors found the following conditions for E_n waves :

$$J_n(ka) = 0$$

which expresses the condition that k must be such that the product ka makes the function J_n under consideration vanish.

In the case of E_0 waves, for instance, ka must be chosen in such a way as to make J_0 vanish. On referring then to tables of the function J_0, the possible values of ka are found to be :

$$ka = 2.4,\ 5.5,\ 8.6,\ 11.8, \text{ etc.}$$

The first of these gives the value $k = \frac{2.4}{a}$ which, substituted in equation (1), yields the result :

$$\frac{2.4}{a} = 2\pi f \sqrt{\frac{1}{V^2} - \frac{1}{v^2}} \quad \ldots\ldots\ldots \quad (2)$$

Thus for each value of f, equation (2) gives the phase velocity of the E_0 wave of this type (E_{01}).

But this equation shows that, for a certain critical frequency f_{oc}, v becomes infinite. For frequencies lower than the critical frequency, waves of this particular type cannot travel along the tube ; thus, by making v infinite in equation (2), there follows :

$$f_{oc} = V\frac{2.4}{2\pi a} \quad \ldots\ldots\ldots\ldots \quad (3)$$

This equation can, however, be replaced by another which gives the critical wavelength for the medium of which the velocity is V :

$$\lambda_{oc} = \frac{V}{f_{oc}} = \frac{2\pi}{2.4}a \quad \ldots\ldots\ldots\ldots \quad (4)$$

This shows, for example, that for a tube of radius $a = 5$ *cm.*, filled with air, the E_0 waves within it cannot have a wavelength greater than

$$\frac{2\pi}{2.4} \times 5 = 13 \textit{ cm.}$$

The other roots of the equation $J_o(ka) = 0$ define E_{02}, E_{03} . . . and represent waves with even shorter critical wavelengths.

The similar consideration of H waves leads to the general condition $J'_n(ka) = 0$, which then defines k and therefrom the critical frequencies of the H waves.

The table below gives the various critical wavelengths for a tube containing only air, in terms of the diameter $D = 2a$:

Types of waves ..	E_0	E_1	E_2	H_0	H_1	H_2
Critical wave-lengths ..	1.31D	0.84D	0.61D	0.84D	1.74D	1.05D

This table shows clearly that there is no hope of employing this method of transmission with tubes of reasonable dimensions without a practical method of producing very short waves. For H_1 waves the critical wavelength is relatively long, and in this comparatively favourable case the tube of diameter 2 cm. requires wavelengths less than 3.5 cm.

It may be mentioned, however, (equation 3) that the critical frequency is proportional to V, or to $\frac{1}{\sqrt{K}}$, K being the dielectric constant of the material within the tube. This relation would indicate the possibility of considerably reducing the critical frequencies by filling the tube with a material of high dielectric constant, but it is evident that the usefulness of this resource will probably be considerably limited by high losses at high frequencies in the dielectric, and also by considerations of cost.

The second problem dealt with by the same authors was the determination of the attenuation due to the dissipation of energy in the metallic tube. These results are qualitatively as follows :

For each of the E_0, E_1, H_1 waves in a given tube, the attenuation has a minimum for some frequency. In particular for H_1 waves, this minimum is very flat and the attenuation varies very little over a wide range of frequency about the optimum frequency. The H_0 wave, however, has a curious peculiarity: the attenuation, infinite at the critical frequency as for the other types, diminishes indefinitely as the frequency increases.

As a whole, this investigation indicates the existence of numerous types of waves having different properties, both in regard to critical frequencies and variation of attenuation. This perhaps suggests that attempts should be made to produce certain types of waves rather than others, but, owing to the simplifying conditions used in the analysis, there are some important points not dealt with in the preceding theory; in particular, the tube is assumed to be straight so that the effect of reflections at bends is neglected. The section of the tube is supposed also to be circular which, in practice, can be the case only to a certain degree of approximation. It is reasonable to ask whether, when a deformation of the section occurs, certain of the fundamental types of waves persist more than others. Thus, the remarkable work of Carson, Mead and Schelkunoff should be followed by studies of the stability of the various types of waves.

This is the essential aim of the theory developed by Professor Léon Brillouin in the article which follows this short summary.

Theoretical Study of Dielectric Cables

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and

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(1) *Introduction*

VARIOUS studies, both experimental and theoretical, have been made on the problem of electro-magnetic wave propagation in hollow conductor tubes. Following articles published in the Bell System Technical Journal,[1, 2] the author investigated the conditions of wave propagation in rectangular tubes[3], and was surprised to find that certain waves were formed only in tubes with a high degree of symmetry (square or circular section); but were split up into different types of waves in tubes of other cross-sections. An interesting study by Barrow[4] confirmed certain of these indications, but left the problem of tubes with flattened cross-sections undetermined. Additional articles by Leigh Page and N. I. Adams, Jr.,[5] as well as by Schelkunoff[6] make very important contributions to this subject, but do not seem to touch on the particular point in question, viz., the detailed determination of the types of waves and propagations in rectangular or elliptical tubes in order to learn how the results vary with gradual deviation from square or circular cross-sections.

Investigation shows that certain types of wave are little affected by changes in cross-section, whilst others are greatly influenced both in their structure and in their laws of propagation. It may, therefore, be assumed that the former types of wave will be "stable" and will propagate without variation in tubes which are only slightly irregular; the latter, on the other hand, show modifications (reflections, changes of wave types with loss of energy) for each irregularity in the form of the tube, or for each bend in the tube, and their use, therefore, will necessitate much more rigidly prescribed operating conditions.

[1] For numbered references see end of article.

(2) *Propagation Equations, General Form*

Maxwell's equations can be written in any system of co-ordinates; orthogonal curvilinear co-ordinates only are used in this article. At a point M, three mutually orthogonal co-ordinate axes Mx_1, x_2, x_3, intersect and give units of local length e_1, e_2, e_3 (Fig. 1) which are functions of x_1, x_2, x_3. Their significance is as follows: a variation dx_1 of the co-ordinate x_1 corresponds to a segment whose actual length is:

$$dl_1 = e_1\, dx_1 ; \quad \ldots\ldots\ldots\ldots \quad (1)$$

similarly for x_2 and x_3. It is accordingly possible to write [7,8] the usual operations of divergence, rotation, gradient, and the Maxwell equations can readily be established. Calling E_1, E_2, E_3, the components of the electric intensity (in C.G.S. electro-static units), and H_1, H_2, H_3, the components of the magnetic intensity (in C.G.S. electro-magnetic units):

$$e_2\, e_3\, \frac{\varepsilon}{c}\, \frac{\partial E_1}{\partial t} = \frac{\partial}{\partial x_2}\,(e_3\, H_3) - \frac{\partial}{\partial x_3}\,(e_2\, H_2)$$

$$e_2\, e_3\, \frac{\mu}{c}\, \frac{\partial H_1}{\partial t} = -\frac{\partial}{\partial x_2}\,(e_3\, E_3) + \frac{\partial}{\partial x_3}\,(e_2\, E_2) \quad \ldots (2)$$

$$\operatorname{div} E = \frac{\partial}{\partial x_1}\,(E_1\, e_2\, e_3) + \frac{\partial}{\partial x_2}\,(E_2\, e_1\, e_3) + \frac{\partial}{\partial x_3}\,(E_3\, e_1\, e_2) = 0$$

$$\operatorname{div} H = 0.$$

In order to study propagation in a tube of any cross-section, the axis x_3 may be taken in the direction of the axis of the tube. Then x_3 is an ordinary rectilinear co-ordinate,

$$e_3 = 1 ; \quad \ldots\ldots\ldots\ldots \quad (3)$$

moreover, e_1 and e_2 will not be dependent on x_3.

The co-ordinates x_1 and x_2 will be curvilinear co-ordinates in the plane of the section of the tube, and may be chosen in a manner such as to

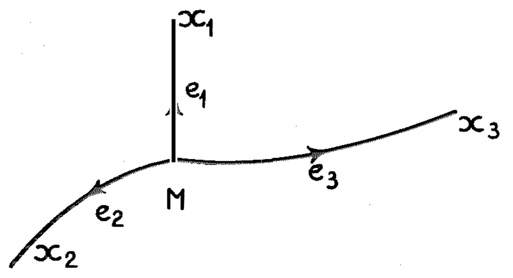

Fig. 1.

simplify the conditions on the surface of the tube as far as possible. (Fig. 2) Thus,

(1) Rectangular cartesian co-ordinates, for a tube with a rectangular cross-section;
(2) Polar co-ordinates, for a tube with a circular section;
(3) Confocal co-ordinates, for a tube with an elliptic section.

Triangular equilateral cross-sections may be dealt with by means of a special system of co-ordinates indicated by Mathieu.

The above general formulae also enable propagation to be studied in a cone-shaped tube of varying cross-section, but it is then necessary to take three curvilinear co-ordinates, including x_3.

(3) *The Particular Form of Progressive Waves*

A general wave can always be resolved into sinusoidal waves, so that wave propagation may be studied with all the quantities E and H depending on the time t and x_3 in the manner

$$exp\ (i\omega t - \gamma x_3) \quad \ldots\ldots\ldots\ldots \quad (4)$$

Equations (2) thus assume the following form :

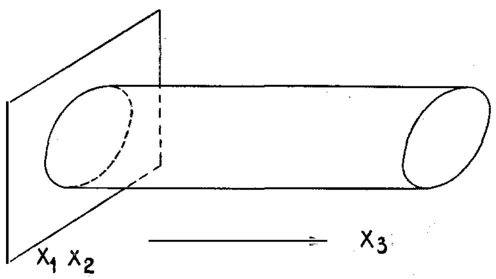

Fig. 2.

$$\left\{\begin{aligned} i\omega e_2 \frac{\varepsilon}{c} E_1 &= \frac{\partial H_3}{\partial x_2} + \gamma e_2 H_2 \\ i\omega e_1 \frac{\varepsilon}{c} E_2 &= -\gamma e_1 H_1 - \frac{\partial H_3}{\partial x_1} \\ i\omega e_1 e_2 \frac{\varepsilon}{c} E_3 &= \frac{\partial}{\partial x_1}(e_2 H_2) - \frac{\partial}{\partial x_2}(e_1 H_1) \end{aligned}\right.$$

$$\left\{\begin{aligned} i\omega e_2 \frac{\mu}{c} H_1 &= -\frac{\partial E_3}{\partial x_2} - \gamma e_2 E_2 \\ i\omega e_1 \frac{\mu}{c} H_2 &= \gamma e_1 E_1 + \frac{\partial E_3}{\partial x_1} \\ i\omega e_1 e_2 \frac{\mu}{c} H_3 &= -\frac{\partial}{\partial x_1}(e_2 E_2) + \frac{\partial}{\partial x_2}(e_1 E_1) \end{aligned}\right.$$

$$\ldots\ldots\ldots\ldots \quad (5)$$

$$\frac{\partial}{\partial x_1}(E_1 e_2) + \frac{\partial}{\partial x_2}(E_2 e_1) - \gamma e_1 e_2 E_3 = 0$$

$$\frac{\partial}{\partial x_1}(H_1 e_2) + \frac{\partial}{\partial x_2}(H_2 e_1) - \gamma e_1 e_2 H_3 = 0.$$

$$\ldots\ldots\ldots\ldots \quad (6)$$

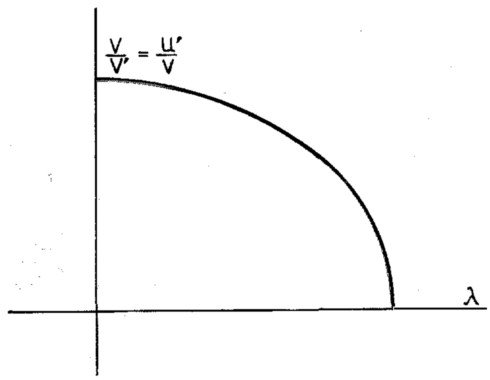

Fig. 3.

The divergence equations (6) result from the rotational equations (5); it will be better not to use the last of the rotational equations and to replace them by divergence equations which are simpler. It is then possible to express the transverse intensities E_1, E_2, H_1, H_2 in terms of the longitudinal intensities E_3, H_3 in the direction x_3 of the axis of the tube.

Taking $v^2 = \dfrac{c^2}{\varepsilon\mu} \quad k^2 = \gamma^2 + \dfrac{\omega^2}{v^2} = \gamma^2 + \dfrac{\omega^2\,\varepsilon\mu}{c^2}$

$$\ldots\ldots\ldots\ldots \quad (7)$$

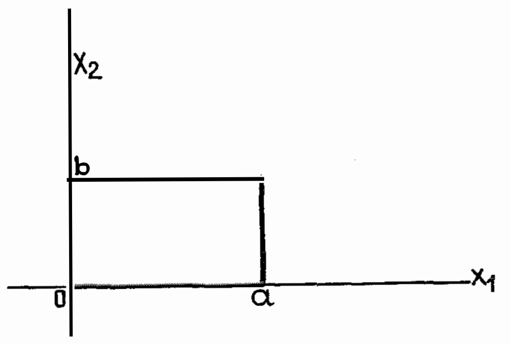

Fig. 4.

(λ^2 is used by Carson, Mead and Schelkunoff, instead of k^2);

$$k^2 E_1 = -\frac{i\omega\mu}{e_2 c}\frac{\partial H_3}{\partial x_2} - \frac{\gamma}{e_1}\frac{\partial E_3}{\partial x_1}$$

$$k^2 E_2 = \frac{i\omega\mu}{e_1 c}\frac{\partial H_3}{\partial x_1} - \frac{\gamma}{e_2}\frac{\partial E_3}{\partial x_2} \quad \ldots\ldots\ldots (8)$$

$$k^2 H_1 = -\frac{\gamma}{e_1}\frac{\partial H_3}{\partial x_1} + \frac{i\omega\varepsilon}{e_2 c}\frac{\partial E_3}{\partial x_2}$$

$$k^2 H_2 = -\frac{\gamma}{e_2}\frac{\partial H_3}{\partial x_2} - \frac{i\omega\varepsilon}{e_1 c}\frac{\partial E_3}{\partial x_1}.$$

By way of verification, employing polar co-ordinates,

$$x_1 = \rho \quad e_1 = 1 \quad x_2 = \theta \quad e_2 = \rho$$

Carson, Mead and Schelkunoff's equations (1) are recovered; now the values (8) of E_1 and E_2 in the first divergence equation (6) are inserted,

$$\frac{1}{e_1 e_2 k^2}\left[\frac{\partial}{\partial x_1}\left(\frac{e_2}{e_1}\frac{\partial E_3}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{e_1}{e_2}\frac{\partial E_3}{\partial x_2}\right)\right] + E_3 = 0; \ldots\ldots\ldots (9)$$

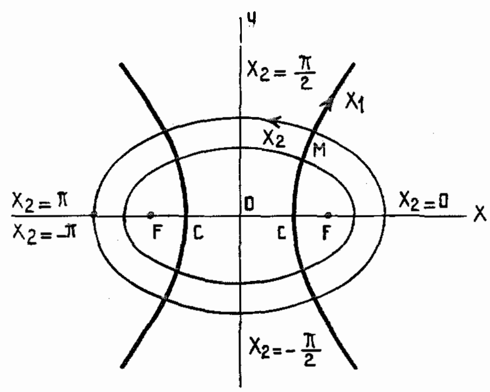

Fig. 5.

and, similarly, for the magnetic intensities:

$$\frac{1}{e_1 e_2 k^2}\left[\frac{\partial}{\partial x_1}\left(\frac{e_2}{e_1}\frac{\partial H_3}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{e_1}{e_2}\frac{\partial H_3}{\partial x_2}\right)\right] + H_3 = 0 \ldots\ldots\ldots (10)$$

(4) *Waves of the Electric and Magnetic Type*

Thus the equations in E_3 and H_3 separate; they are both of the same type and may be written with the two-dimensional Laplace operator in plane curvilinear co-ordinates x_1 and x_2; the expression which enters into the two equations (9) and (10) is the Laplacian Δ; they have therefore the same form,

$$\Delta\varphi + k^2\varphi = 0 \text{ with } \varphi = E_3 \text{ or } H_3;$$

$$\Delta\varphi = \frac{1}{e_1 e_2}\left[\frac{\partial}{\partial x_1}\left(\frac{e_2}{e_1}\frac{\partial\varphi}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{e_1}{e_2}\frac{\partial\varphi}{\partial x_2}\right)\right]. \ldots\ldots\ldots (11)$$

In view of the linear character of equations (8) and the separation of equations (9) and (10), the following procedure may be adopted:

Separate solutions of (9) or (10) must be obtained; a general wave will result from superposition of individual waves.

First of all are derived waves of the *electric type* with a longitudinal magnetic intensity H_3, which is zero in all cases, whilst the longitudinal electric intensity E_3 is determined by (9).

The boundary condition is therefore the annulment of the longitudinal electric intensity E_3 at the boundary.

Electric type: $H_3 \equiv 0$, $E_3 = \varphi$, whilst $\varphi = 0$ at the boundary, ... (12)

that is, the boundary represents the section of a perfectly conducting tube on which the tangential electric intensity E_3 should be zero. This condition is sufficient in itself.

Suppose also that the boundary be defined by the condition $x_1 = 0$ (Fig. 1). Since it was assumed that the tangential component E_3 in the direction x_3 is always zero, it is furthermore necessary that the second tangential component E_2 in the direction of x_2 be zero; this is defined by:

$$k^2 E_2 = -\frac{\gamma}{e_2}\frac{\partial E_3}{\partial x_2}.$$

E_3 being constant and zero on the whole of the boundary, the derivative also is zero, so that the second condition is automatically satisfied.

Moreover, solutions are obtained of the *magnetic type* for which the longitudinal electric intensity E_3 is zero throughout the cross-section, whilst H_3 by means of equations (8) determines the components E_1, E_2, H_1 and H_2.

Magnetic type $E_3 \equiv 0$, $H_3 = \varphi$, whilst $\frac{\partial\varphi}{\partial n} = 0$ at the boundary .... (13)

The boundary condition to be applied to the curve C is now the annulment of the normal derivative $\frac{\partial\varphi}{\partial n}$. In short, assuming that, as in the foregoing, the boundary corresponds to $x_1 = 0$, the tangential components E_2 and E_3 of the electric field must be cancelled. For E_3 this has already been done; E_2 remains:

$$k^2 E_2 = \frac{i\omega\mu}{e_1 c} \quad \frac{\partial H_3}{\partial x_1} = 0 \ .$$

Hence $\frac{\partial H_3}{\partial x_1}$ must equal zero; moreover x_1 is a co-ordinate perpendicular to the boundary. Condition (13) has thus been established.

The electric or magnetic waves consequently correspond to the solutions of equation (11), for which the function φ or its derivative $\frac{\partial\varphi}{\partial n}$ are zero at the inner surface of the conductor tube.

It is also possible, as indicated by Mr. A. G. Clavier in a lecture, delivered on the 15th of April, 1937, at Les Laboratoires L.M.T., to find solutions for which E_3 and H_3 are simultaneously zero; it is therefore necessary to take k^2 equal to zero if it be not desired to annul all the fields represented by equations (8). The rotational equations (8) or (5) are, therefore, automatically satisfied, and the fields E_1, E_2, H_1, H_2 are only governed by the zero divergence equations (6):

$$\frac{\partial}{\partial x_1} \ (E_1 e_2) + \frac{\partial}{\partial x_2} \ (E_2 e_1) = 0$$

$$\frac{\partial}{\partial x_1} \ (H_1 e_2) + \frac{\partial}{\partial x_2} \ (H_2 e_1) = 0 \ . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The divergences of the electric fields E and the magnetic fields H are zero in the plane $x_1 x_2$ of a section of the tube. These conditions will be satisfied only if there are one or more auxiliary conductors inside the tube; if not, all the fields are zero. Clavier has shown how the problem of the coaxial cable occurs again here.

(5) *General Results*

It is very important to notice the general nature of the separation of the waves into electric types and magnetic types for any cross-section. It will be found that in a rectilinear tube, continuous variations in the contour of the cross-section may transform the various types of electric waves or magnetic waves into their respective corresponding forms, but a magnetic wave can never be converted into an electric wave or vice versa.

Transformations of this kind demand special devices: curved tubes or curved electrodes in the tubes. Certain general formulæ should be noted:

Magnetic type :

$$E_3 \equiv 0 \quad \begin{cases} E_1 = \dfrac{i\omega\mu}{\gamma c} \quad H_2 = \dfrac{v'}{v} \sqrt{\dfrac{\mu}{\varepsilon}} \ H_2 \\ E_2 = -\dfrac{i\omega\mu}{\gamma c} \quad H_1 = -\dfrac{v'}{v} \sqrt{\dfrac{\mu}{\varepsilon}} \ H_1 \end{cases} \qquad \ldots\ldots\ldots\ldots (15)$$

Electric type :

$$H_3 \equiv 0 \quad \begin{cases} E_1 = \dfrac{\gamma c}{i\omega\varepsilon} \quad H_2 = \dfrac{v}{v'} \sqrt{\dfrac{\mu}{\varepsilon}} \ H_2 \\ E_2 = -\dfrac{\gamma c}{i\omega\varepsilon} \quad H_1 = -\dfrac{v}{v'} \sqrt{\dfrac{\mu}{\varepsilon}} \ H_1 \end{cases}$$

The expression v' represents the velocity of wave propagation along the axis of the tube; according to the above hypothesis, equation (4).

$$\gamma = \frac{i\omega}{v'} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

These formulæ, due to Clavier, may be compared with the following relating to the co-axial type cable:

$$E_3 = H_3 \equiv 0 \qquad E_1 = \sqrt{\frac{\mu}{\varepsilon}} \ H_2 \qquad k^2 = 0$$

$$E_2 = -\sqrt{\frac{\mu}{\varepsilon}} \ H_1 \qquad v = v'$$

Reverting to electric or magnetic waves for hollow tubes, it has been shown in section 4 how the boundary conditions play a role in each case, and *fix the characteristic value* k^2 for each type of wave.

Once this actual value k^2 has been determined, all the features of the wave will follow directly; equation (7), according to equation (16), then becomes:

$$k^2 = \gamma^2 + \frac{\omega^2}{v^2} = \omega^2 \left(\frac{1}{v^2} - \frac{1}{v'^2} \right) \quad \ldots\ldots (17)$$

The velocities of *propagation* v' along the axis of the tube are phase velocities, which may have any value from v to infinity; the following equation will then be obtained:

$$\left(\frac{v}{v'}\right)^2 + \left(\frac{kv}{\omega}\right)^2 = 1 \qquad \frac{v}{\omega} = \frac{\lambda}{2\pi} \quad \ldots\ldots (18)$$

where λ = wavelength in vacuum, radio type waves, v = velocity of waves in vacuum, radio type waves, v' = phase velocity in the direction of the axis of the tube.

The results will, therefore, be represented by an elliptic diagram. While given for circular section or rectangular section tubes, they apply to all types of sections.

An equation of type (18) automatically leads (Barrow,[4] L. Brillouin[3]) to the definition of group velocity, or *velocity of energy transmission*, u', by the well-known equation:

$$\frac{1}{u'} = \frac{d}{d\omega} \left(\frac{\omega}{v'} \right),$$

giving

$$\frac{v}{v'} = \frac{u'}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Thus the phase-velocity v' is always greater than the velocity of light, but, on the contrary, the energy transmission velocity u' is less than the velocity of light; the product $u'v'$ remains constant. These results are valid for tubes of any shape.

In every case the characteristic value k^2 determines all the features of the wave; Fig. 3 and formula (17) enable k^2 to be evaluated by associating it with the *minimum frequency* ω_m which can be attained with this type of wave:

$$k^2 = \frac{\omega_m^2}{v^2} \quad \ldots\ldots\ldots\ldots (17 \text{ bis})$$

$v' = \infty$ and $u' = o$, for ω_m.

Finally, it will be noted that k^2 is inversely proportional to the square of a length; accordingly, if the tube be modified by increasing all its lateral dimensions in a single ratio ρ:

$$l' = \rho l \quad x'_1 = \rho x_1 \quad x'_2 = \rho x_2 \quad k'^2 = \rho^{-2}\, k^2, \quad \ldots\ldots\ldots\ldots\ldots (20)$$

the characteristic value of k^2 will be multiplied by ρ^{-2}.

Accordingly, study of propagation may be confined to tubes of various shapes and of a *constant area* of cross-section. If the area S of the cross-section be modified without changing the form, the value k^2 is changed so that:

$$k^2\, S = C^{te} \quad \ldots\ldots\ldots\ldots (20 \text{ bis})$$

(6) Tube with Rectangular Cross-Section

The author had studied waves transmitted along a tube with a rectangular cross-section,[3]. The present purpose is to show how, in the first instance, the waves considered in his former paper are to be derived according to the newer viewpoint, and to discuss their properties in greater detail. This problem is interesting in itself; and, in particular, its comparison with the case of tubes with elliptic cross-sections supplies very valuable information.

Considering a tube of rectangular cross-section (Fig. 4), with sides a and b; the conditions of equation (11) must be met:

$$\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_2^2} + k^2\varphi = 0,$$

together with the auxiliary conditions imposed by equation (12) or (13) on the rectangular boundary. The solutions are easy to find:

Electric Type:

$$E_3 = \varphi = \sin \frac{n_1 \pi x_1}{a} \; \sin \frac{n_2 \pi x_2}{b} \quad \ldots\ldots (21)$$

Magnetic Type:

$$H_3 = \varphi = \cos \frac{n_1 \pi x_1}{a} \; \cos \frac{n_2 \pi x_2}{b} \quad \ldots (22)$$

$$k^2 = \left(\frac{\pi n_1}{a}\right)^2 + \left(\frac{\pi n_2}{b}\right)^2.$$

$k^2_{n1\ n2}$ has the same value in the two cases; n_1 and n_2 are integers.

To each set of two integers n_1 and n_2 *there correspond two waves, an electric and a magnetic,* having the same characteristic value k^2; and, accordingly, the same velocity of propagation v', along the axis of the tube, when ω is fixed, as is evident from the observations in the preceding section.

It must, however, be remembered that, if one of the two values n_1 or n_2, is zero, the electric wave (21) will disappear entirely, so that: *the waves* $0, n_2$ or $n_1, 0$ can only be of magnetic type.

Considering, therefore, the case of a rectangle which is deformed in a continuous manner whilst keeping its area of cross-section constant and equal to 1,

$$S = ab = 1,$$

and seeking the variation of k^2 for the different types of wave;

$$\frac{k^2}{\pi^2} = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2 = \left(\frac{n_1}{a}\right)^2 + \left(n_2 a\right)^2 : \qquad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

k^2 has, for $a = 1$, the value of $n_1^2 + n_2^2$; —it is the case of the *square cross-section* tube. The minimum value of k^2 is obtained for:

$$a = \sqrt{\frac{n_1}{n_2}} \qquad \left(\frac{k^2}{\pi^2}\right)_{\text{minimum}} = 2n_1 \, n_2 \ \ldots\ldots (24)$$

The different curves intersect as shown on Plate I.* This clearly shows the dotted curves, representing a single H wave alone, and the solid curves corresponding simultaneously to a wave E and a wave H. These two waves, of the same character and same propagation, are polarised in directions at right angles.

The reflections on the parallel orthogonal mirrors, which comprise the rectangular section tube, do not separate these two polarised waves at right angles. They continue to propagate in the same manner.

In tubes with elliptic or circular section, on the other hand, these two waves will be separated; the curves of the graph which correspond to them will split into two. Further, one may predict that the different curves will join each other in the centre of the figure in a slightly different way, whilst the distant branches corresponding to flattened tubes remain practically the same for rectangular or elliptical tubes.

Plate I shows the distribution of the nodal lines in a cross-section for the different types of waves. For the waves E, the magnitude considered (axial intensity E_3) is always nil at the boundary, and has no, one or more nodal lines where E_3 is within the cross-section.

In the case of waves H, the magnetic axial intensity H_3 is the quantity under consideration. It is maximum on the boundary, and has one or two nodal lines in the inside of the tube; H_3 vanishes on these nodal lines.

(7) *Elliptic Cross-Section Tube: Elliptic Co-ordinates*

The above discussion applies to a rectangular cross-section; it is interesting to study a similar problem for the case of elliptic tubes. On the one hand, it will show how the properties of the waves vary when the circular tube is flattened; on the other hand, the types of waves which may be transformed one into the other under these conditions. In accordance with the conditions imposed, it might be interesting to produce these transformations at will or, conversely, to try to prevent them in order to avoid loss of energy.

In the case of an elliptic cross-section, it is necessary to use plane confocal co-ordinates which consist of a set of ellipses and hyperbolæ (Fig. 5); hence:

$$x = c \cosh x_1 \cos x_2,$$
$$y = c \sinh x_1 \sin x_2 \ \ldots\ldots\ldots\ldots (25)$$

The foci are on the x axis and their distance apart is $2c$. With x_1 constant and x_2 *varying from* 0 *to* 2π, an *ellipse* is described by

$$\left(\frac{x}{c \cosh x_1}\right)^2 + \left(\frac{y}{c \sinh x_1}\right)^2$$
$$= \cos^2 x_2 + \sin^2 x_2 = 1 \ldots\ldots (26)$$

Its minor and major semi-axes are, respectively, b and a, where

$$b = c \sinh x_1 \quad a = c \cosh x_1 \quad \frac{b}{a} = \tanh x_1 \qquad \ldots\ldots\ldots (27)$$

Leaving x_2 constant and *varying* x_1 from 0 *to infinity*, a *hyperbola* will be described. These ellipses and hyperbolae always intersect at right angles, and the local units of length (see section 2) have the value:

$$e_1 = e_2 = c \sqrt{\cosh^2 x_1 - \cos^2 x_2} \ \ldots\ldots (28)$$

* For plates I to IV see end of article.

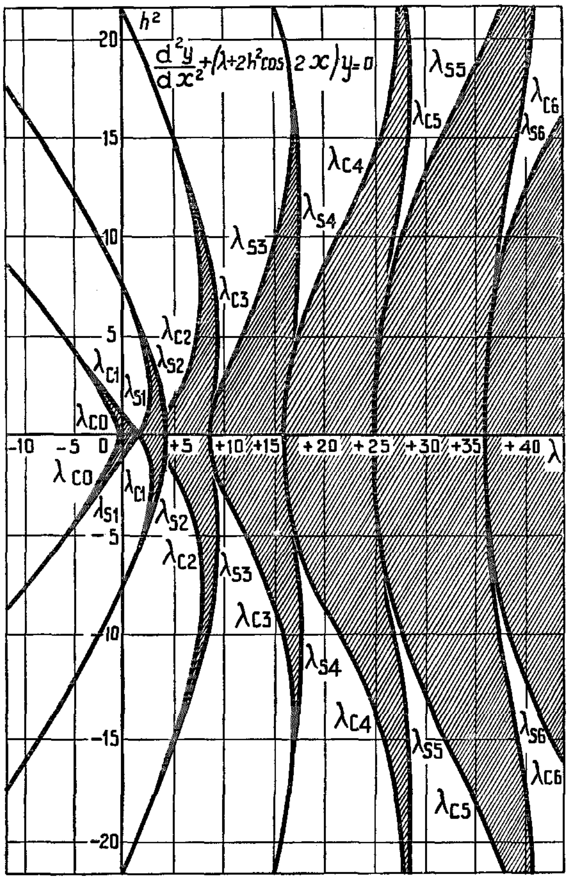

Fig. 6.

which permits direct conversion of all the equations of sections 2, 3 and 4 into confocal coordinates. Accordingly, the general equation (11), involving the waves E and H simultaneously, as already seen, becomes :

$$\frac{1}{c^2(\cosh^2 x_1 - \cos^2 x_2)}\left[\frac{\partial^2 \varphi}{\partial x_1{}^2}+\frac{\partial^2 \varphi}{\partial x_2{}^2}\right] + k^2\varphi = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

This equation may be solved by separating the function φ into two functions φ_1 and φ_2, each with a single variable :

$$\varphi = (x_1\, x_2) = \varphi_1\,(x_1)\,\varphi_2\,(x_2). \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

The general equation (29) will be satisfied if φ_1 φ_2 are determined by :

$$\left\{\begin{aligned} &\frac{d^2\varphi_1}{dx_1{}^2} + \varphi_1\,(-R + 2h^2\cosh 2x_1) = 0,\\ &\frac{d^2\varphi_2}{dx_2{}^2} + \varphi_2\,(R - 2h^2\cos 2x_2) = 0\,; \end{aligned}\right. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

R is arbitrary and $4h^2 = k^2 c^2$.

These two equations are of the *Mathieu type* and have been systematically studied by a number of authors, including Strutt [9, 10, 11, 12, 13,] and the present writer. Reference is made more particularly to the Mathieu equation with φ_2, where an ordinary cosine appears ; the equation with φ_1, reduces to the first type by taking ix_1 as variable. The constants R and h^2 should have the same values in the two equations ; further, h^2 is explicitly given, but R is determined by the condition that the solution presents the period 2π with respect to x_2, so as to supply a uniform solution in the plane of Fig. 5. This condition can only be realised if R varies as a function of h^2, describing one of the curves of Fig. 6. Each of the branches corresponds to the solutions :

$$Ce_0\,(x_2)\;\; Se_1\,(x_2)\;\; Ce_1\,(x_2)\;\; Se_2\,(x_2)\;\; Ce_2\,(x_2)\;\; \ldots.$$

which are the actual *Mathieu functions.* These functions have been calculated and expansions in terms of h^2 are available. When h^2 approaches zero, they reduce, respectively, to :

$$1 \quad \sin x_2 \quad \cos x_2 \quad \sin 2x_2 \quad \cos 2x_2 \quad \ldots\ldots$$

They always form a complete system of orthogonal functions (not normalised).

As indicated, the values of R and h^2 should be the same in both equations, so that the solutions φ of equation (11) become :

$$\varphi = Ce_n(ix_1)\; Ce_n(x_2) \text{ or } Se_n(ix_1)\; Se_n(x_2)\,,\; . \; . \quad (32)$$

the functions Ce_n or Se_n having the value

$$h^2 = \frac{1}{4}\; k^2 c^2 = \frac{c^2}{4}\left(\gamma^2 + \frac{\omega^2}{v^2}\right), \; \ldots. \quad (33)$$

which determines the geometrical conditions and the type of wave considered.

The solutions with n are characterised by the presence of n nodal branches in the plane of the ellipse or cross-section of the cylinder. These nodal branches are differently distributed for functions Ce and Se (Fig. 7).

In section 6 is calculated the vibration of a rectangular tube of section ab. Curves giving $\frac{k^2}{\pi^2}$ as a function of a with a constant section $ab = 1$ also are shown.

These curves give a preliminary indication of what happens in a tube with an oval cross-section, provided a and b are the semi-axes of the ellipse of area πab. Consider the same area of cross-section in the two cases. Since k^2 is proportional to the reciprocal of an area (by virtue of equation 20 bis), it is necessary to multiply by π^{-1} the values of $\frac{k^2}{\pi^2}$ shown on Plate I.

On the other hand, the formulae of the elliptic problem yield $\frac{k^2}{4} = \frac{h^2}{c^2}$ directly from equation (33). As an initial indication, the curves giving

$$\frac{\pi^2}{4} \cdot \frac{1}{\pi} \left(\frac{k^2}{\pi^2}\right)_{\text{Rectang.}} = \frac{\pi}{4} \left(\frac{k^2}{\pi^2}\right)_{\text{Rectang.}} \approx \left(\frac{k^2}{4}\right)_{\text{Elliptic}}$$

are drawn as a function of a (Plate II).

This correspondence, for the circular case of radius ($a = 1$), yields the following values with which the direct values are compared :

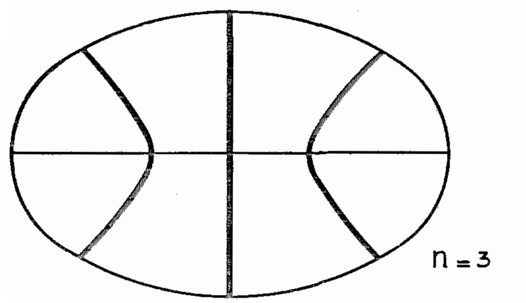

Fig. 7.

8. *The Perfect Elliptic Cylinder Conductor: Electric Solutions, Type E; Magnetic Solutions, Type H*

The electric solutions are characterised by the fact that the longitudinal components H_3 of the magnetic field are zero, whilst the longitudinal components E_3 of the electric field are represented by a function (32).

$\left(\frac{k^2}{4}\right)_{\text{Square}} \sim$	0.785	1.57		3.14	3.93		
Wave type	H	E or H		H	E or H		
$\left(\frac{k^2}{4}\right)_{\text{Circular}} =$	0.85	1.44	2.3	3.66	3.66	4.5	.... (34)
Wave type	H_1	E_0	H_2	H_0	E_1	H_3	
$k =$	1.84	2.4	3.05	3.83		4.25	

The values of k relating to the waves E_0, E_1, H_0, H_1, H_2, and H_3, have been given by Carson, Mead and Schelkunoff. The regrouping of the curves is, therefore, sufficiently different, in the circular case, from that found in rectangular tubes ; *the coincidence of waves H_0 and E_1* is typical as is also the separation of the waves E and H, which have the same characteristics for the case of rectangular section.

The continuous transition from rectangular to elliptic sections will be discussed hereinafter.

For this electric intensity E_3, the condition to be fulfilled is that it must be zero on the surface of the conducting cylinder (12). This surface must correspond to a certain value X_1 of the variable x_1 such that :

$$Ce_n\,(iX_1) = 0 \quad \text{or} \quad Se_n\,(iX_1) = 0 \ldots\ldots.\ (35)$$

These conditions can be fulfilled making use of the first, second .... and the mth root of the function Ce_n or Se_n ; therefore, the solutions may be characterised by means of :

(1) The indices c, n or s, n — indicating the function Ce_n or Se_n; or

(2) by another index m — determining the order of the root.

This will yield waves of the types

$$E_{c,\,n,\,m,} \text{ or } E_{s,\,n,\,m.}$$

Each of these waves involves geometrical quantities:

distance $2c$ of the foci,
value X_1,

and the value of the parameter h; with equations (35), h as a function of $2c$ and X_1 is derived:

$$h = h_{cnm}\,(2c,\ X_1) \text{ or } h = h_{snm}\,(2c,\ X_1)\,. \quad \ldots (36)$$

The value of h is thus fixed when the elliptic section and the type of wave have been assigned; hence, there results an equation (33) which yields the characteristic value k^2:

$$\frac{k^2}{4} = \frac{h^2}{c^2} \quad \ldots\ldots\ldots\ldots\ldots (37)$$

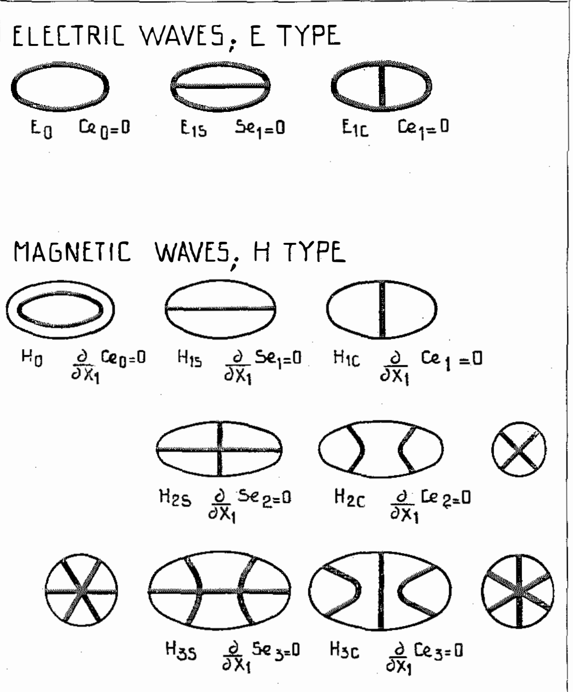

Fig. 8.

and which supplies all the useful information, as shown in section 5.

All, therefore, depends on solving equations (35); that is, of determining the roots of the Mathieu functions having integral indices.

The *magnetic waves* correspond to the case where the longitudinal electric field E_3 is identically zero whilst the longitudinal magnetic field H_3 is represented by a function (32). The condition to be fulfilled on the elliptic curve is annulment (13) of the normal derivative; that is,

$$\frac{\partial}{\partial x_1} Ce_n\left(iX_1\right) = 0 \text{ or } \frac{\partial}{\partial x_1} Se_n\left(iX_1\right) = 0 \ldots (38)$$

In connection with the waves $H_{c,\,n,\,m,}$ and $H_{s,\,n,\,m,}$ the functions Ce_n or Se_n of the integral index n and the mth root of equations (38) will be employed. These equations will determine the values of the parameter h and, in consequence, the characteristic values of k^2 (37).

It merely remains, therefore, to determine the root h of equations (35) or (38). Let us determine only the geometrical meaning of the variables c and X_1 which fix the outer elliptic boundary; when $2a$ and $2b$ are the two axes, equations (27) give:

$$\left.\begin{aligned} c \cosh X_1 &= a, \\ c \sinh X_1 &= b, \\ \tanh X_1 &= \frac{b}{a} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (39)$$

$2c$ = distance apart of foci.

Consider a series of ellipses of the same area S,

$$S = \pi\, ab = \pi\,; \quad ab = 1.$$

If t equal the value of the hyperbolic tangent of X_1, formulae result:

$$a = \frac{1}{\sqrt{t}};\ b = \sqrt{t}\,;\ c = \frac{a}{\cosh X_1} = \frac{1}{\sqrt{\sinh X_1 \cosh X_1}};$$

$$t = \tanh X_1. \quad \ldots\ldots\ldots\ldots (40)$$

An ellipse which differs only slightly from a circle gives a high value of X_1, so that t is nearly 1. The small values of X_1 give very flat ellipses:

TABLE I

X_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5
t	0	0.1	0.2	0.291	0.38	0.462	0.537	0.604	0.663	0.716	0.762	0.905
a	∞	3.16	2.24	1.85	1.62	1.47	1.365	1.29	1.23	1.18	1.145	1.05
b	0	0.316	0.447	0.54	0.616	0.68	0.732	0.777	0.815	0.845	0.873	0.95
c	∞	3.16	2.24	1.78	1.5	1.305	1.15	1.02	0.92	0.825	0.742	0.446

9. Approximations to the Mathieu Functions: Small Values of h.

To the writer's knowledge, no table of Mathieu's functions exists. Only approximate expansions, which can be used under certain conditions, are available.

The Mathieu expansions are valid for small values of the *parameter h* (Strutt, page 33)[9]. Those of interest in connection with the present discussion follow; directly introduced is the variable ix_1:

$$Ce_0\,(ix_1) = 1 - \frac{h^2}{2}\cosh 2x_1 + \frac{h^4}{32}\cosh 4x_1 + \frac{h^6}{128}\left(7\cosh 2x_1 - \frac{1}{9}\cosh 6x_1\right)$$

$$Ce_1\,(ix_1) = \cosh x_1 - \frac{h^2}{8}\cosh 3x_1 + h^4\left[-\frac{\cosh 3x_1}{64} + \frac{\cosh 5x_1}{192}\right] + h^6\left[-\frac{\cosh 3x_1}{1536} + \frac{\cosh 5x_1}{1152} - \frac{\cosh 7x_1}{9216}\right]$$

$$i\,Se_1\,(ix_1) = \sinh x_1 - \frac{h^2}{8}\sinh 3x_1 + h^4\left[-\frac{\sinh 3x_1}{64} + \frac{\sinh 5x_1}{192}\right] - h^6\left[\frac{\sinh 3x_1}{1536} + \frac{\sinh 5x_1}{1152} + \frac{\sinh 7x_1}{9216}\right]$$

$$Ce_2\,(ix_1) = \cosh 2x_1 + h^2\left[\frac{1}{4} - \frac{\cosh 4x_1}{12}\right] + \frac{h^4}{384}\cosh 6x_1 + h^6\left[-\frac{5}{192} + \frac{43}{13824}\cosh 4x_1 - \frac{\cosh 8x_1}{23040}\right] + \ldots\ldots \quad (41)$$

The expression for Se_1 is obtained from the relation:

$$Se_1(h^2, \xi) = Ce_1\left(-h^2, \xi + \frac{\pi}{2}\right) \quad \xi = ix_1 \quad \ldots. (42)$$

Strutt gives the co-efficients of the expansions of Ce_2, Ce_3, Ce_4, Ce_5; the functions Se_n can be deduced from them by the equation:

$$Ce_{2n+1}\,(h^2, \xi) = (-1)^n\, Se_{2n+1}\left(-h^2, \xi + \frac{\pi}{2}\right).$$

These expansions may only be used if h^2 be small. Equation (37) shows that h^2 is equal to $\frac{k^2c^2}{4}$, and the values of c^2 in Table I run from 0.2 to 10 for the field which interests us. Thus the Mathieu expansions yield only the *low branches* of the curves representing k^2 as a function of a, in the regions where the actual values of k^2 are small. Reference to Plate II reveals that this case arises for curves H of the downward type; they are the magnetic waves of the types H_{oc}, H_{1c}, H_{2c}, H_{3c}, etc.

These downward branches are not very numerous. The majority curve upward on both sides from a minimum. For the upward branches, the high values of k^2 correspond to the high values of h^2. These curves can be calculated by means of asymptotic expansions of the Mathieu functions (see section 10).

It would be highly interesting to obtain accurate curves of k^2 for nearly circular ellipses (a nearly equal to unity); some special expansions of the Mathieu functions in series of Bessel functions (Strutt, pp. 45–48)[9] might be used. Calculation of the roots h^2 of equations (33) or (38) by this process does not, however, seem practicable.

The expansions (41) determine the rule

indicated in section 7, and according to which the Mathieu functions correspond (for h^2 very small) to the trigonometrical functions.

$$\begin{array}{cccccc} Ce_0\,(x) & Se_1\,(x) & Ce_1\,(x) & Se_2\,(x) & Ce_2\,(x), \text{ etc.} \\ 1 & \sin x & \cos x & \sin 2x & \cos 2x, \text{ etc.} \end{array}$$

Let us recall that the waves are represented by the products (32) of two Mathieu functions, one relating to the angular variable X_2, which represents a motion on the ellipse; the other, relating to iX_1 and corresponding to a motion on a hyperbola (Fig. 5).

These indications suffice to show the distribution of the nodal lines in the various possible cases. The curves of Fig. 8 summarise these results. The E waves always have a nodal line on the outer elliptic curve; the waves E_{1s} and E_{1c}, in the circular case, yield practically the same configuration with a nodal diameter. The corresponding curves cross one another for the value $a = 1$, which represents a circle.

The H waves never have the nodal line on the outer ellipse; the waves H_{1s} and H_{1c} yield the same configuration for the circular case ($a = 1$); their curves, therefore, intersect similarly. The waves H_{2s} and H_{2c}, as do also H_{3s} and H_{3c}, give the same figure and join up again for the circle $a = 1$.

(10) Asymptotic Formulae for h^2 Large

S. Goldstein[15] and Ince[14] have developed a system of asymptotic formulae representing approximately the Mathieu functions for large values of h^2; formulae are given for real values of the variable x (Strutt, p. 38)[9] in two alternative

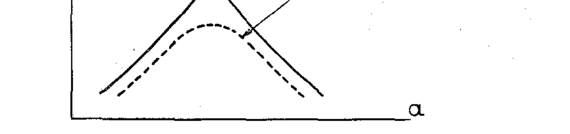

Fig. 9

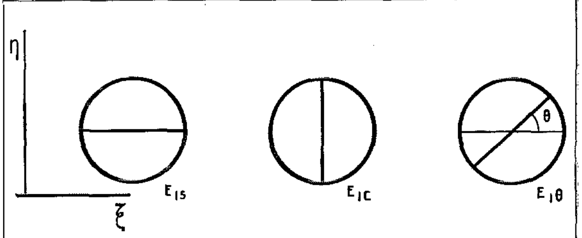

Fig. 10

forms, according to whether the variable is in the neighbourhood of $2\,k\pi$ or $(2\,k+1)\,\pi$. An expression which can be used for purely imaginary values, $x = ix_1$, of the variable is required. Referring to Goldstein's original article, it is apparent that it is necessary in this case to employ the development corresponding to x in the neighbourhood of 0 (or $2k\,\pi$). The formulae, accordingly, are:

$$\left.\begin{array}{l}(+)\,Ce_m\,(ix_1)\\(-)\,Se_{m+1}\,(ix_1)\end{array}\right\} = \frac{1}{(\cos\,ix_1)^{m+1}}$$

$$\left\{ e^{2h\sin ix_1}\left[\cos\left(\frac{\pi}{4}+\frac{ix_1}{2}\right)\right]^{2m+1} \pm e^{-2h\sin ix_1}\left[\sin\left(\frac{\pi}{4}+\frac{ix_1}{2}\right)\right]^{2m+1} \right\} \quad \ldots\ldots\ldots\ldots\ (43)$$

The functions Ce_m are even in x_1 and are distinguished by the plus sign, whilst the odd functions Se_{m+1} correspond to the minus sign. Some elementary calculations are necessary to separate the real and imaginary components and to regroup the terms accordingly. The results are:

$$\varphi = 2h\sinh x_1 \quad \ldots\ldots\ldots\ldots\ (44)$$

$$Ce_0\,(ix_1) = \frac{\sqrt{2}}{\cosh x_1}\left[\cosh\frac{x_1}{2}\cos\varphi + \sinh\frac{x_1}{2}\sin\varphi\right]$$

$$Se_1\,(ix_1) = \frac{i\sqrt{2}}{\cosh x_1}\left[\cosh\frac{x_1}{2}\sin\varphi - \sinh\frac{x_1}{2}\cos\varphi\right]$$

$$Ce_1\,(ix_1) = \frac{1}{\sqrt{2}\cosh^2 x_1}$$

$$\left[\cosh^3\frac{x_1}{2}\cos\varphi+3\cosh^2\frac{x_1}{2}\sinh\frac{x_1}{2}\sin\varphi\right.$$

$$\left.-3\cosh\frac{x_1}{2}\sinh^2\frac{x_1}{2}\cos\varphi-\sinh^3\frac{x_1}{2}\sin\varphi\right]$$

$$=\frac{1}{\sqrt{2}\cosh^2 x_1}$$

$$\left[\cos\varphi\cosh\frac{x_1}{2}(2-\cosh x_1)+\sin\varphi\sinh\frac{x_1}{2}\right.$$

$$\left.(2+\cosh x_1)\right]$$

$$Se_2\,(ix_1)=\frac{i}{\sqrt{2}\cosh^2 x_1}$$

$$\left[\cosh^3\frac{x_1}{2}\sin\varphi-3\cosh^2\frac{x_1}{2}\sinh\frac{x_1}{2}\cos\varphi\right.$$

$$\left.-3\cosh\frac{x_1}{2}\sinh^2\frac{x_1}{2}\sin\varphi+\sinh^3\frac{x_1}{2}\cos\varphi\right]$$

$$=\frac{i}{\sqrt{2}\cosh^2 x_1}$$

$$\left[\sin\varphi\cosh\frac{x_1}{2}(2-\cosh x_1)-\cos\varphi\sinh\frac{x_1}{2}\right.$$

$$\left.(2+\cosh x_1)\right].$$

Use will be made of these formulae to determine the asymptotic parts of the rising curves for the high values of h^2 and, consequently, of k^2. They are valid as soon as $\frac{k^2}{4}$ exceeds approximately 3.

(11) *E Waves*

For the E waves are required the roots of the equations

$$Ce_0 = 0 \quad Se_1 = 0 \quad Ce_1 = 0 \text{ etc.,}$$

giving the asymptotic conditions

$$\text{Wave } E_0 \quad \tan\varphi = -\coth\frac{x_1}{2}$$

$$\text{Wave } E_{is} \quad \tan\varphi = \tanh\frac{x_1}{2}$$

$$\text{Wave } E_{ic} \quad \tan\varphi = -\coth\frac{x_1}{2}\left(\frac{2-\cosh x_1}{2+\cosh x_1}\right) \quad \ldots\ldots\ldots\ldots\ldots (45)$$

These equations determine φ from its tangent, thus permitting a certain latitude. Calling φ_0 the smallest value of φ in accordance with equations (45), and taking,

$$\varphi = n\pi+\varphi_0 \qquad h=\frac{n\pi+\varphi_0}{2\sinh x_1}, \quad \ldots\ldots (46)$$

h (according to 44) may be positive or negative since the formulae only contain its square. In practice, it is necessary to include the solutions $n = -1, 0+1$; the values of n furthest distant produce too high values of k^2, which are certain to be unacceptable.

In order to make a choice of these three determinations, it is necessary to be guided by the relationship of the curves to those already traced for the rectangular sections and by the necessity of reconciliation with the points corresponding to the circular section. This

TABLE II

Sinh X_1	0	0.1	0.202	0.304	0.41	0.52	0.637	0.758	0.888	1.027	1.176	2.125	
X_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5	
$\tanh\frac{X_1}{2}$	0	0.05	0.1	0.15	0.198	0.245	0.291	0.336	0.38	0.422	0.462	0.635	Wave E_{1s} for the circle 3.66
φ_0	0	0.05	0.1	0.15	0.195	0.24	0.284	0.324	0.36	0.4	0.43	0.56	
$h=\frac{\pi+\varphi_0}{2\sinh X_1}$	∞	15.9	8	5.4	4.07	3.25	2.68	2.28	1.97	1.73	1.51	0.87	
$\frac{k^2}{4}=h^2/c^2$	∞	25.5	12.8	9.2	7.35	6.2	5.42	5	4.6	4.4	4.15	3.8	
$\coth\frac{X_1}{2}$	∞	20	10	6.7	5.05	4.07	3.43	2.97	2.63	2.37	2.16	1.575	Wave E_0 for the circle 1.44
φ_0	1.57	1.52	1.47	1.422	1.376	1.33	1.287	1.247	1.21	1.17	1.14	1.07	
$h=\frac{-\pi+\varphi_0}{2\sinh X_1}$	$-\infty$	−8.1	−4.14	−2.83	−2.15	−1.74	−1.46	−1.25	−1.09	−0.96	0.853	−0.49	
$k^2/_4=h^2/c^2$	∞	6.55	3.41	2.52	2.05	1.78	1.61	1.5	1.41	1.35	1.32	1.20	
$\cosh X_1$	1	1.0051	1.02	1.045	1.081	1.127	1.185	1.255	1.337	1.43	1.543	3.75	Wave E_{1c} for the circle 3.66
$\frac{\cosh X_1-2}{\cosh X_1+2}=\Gamma$	−0.333	−0.330	−0.324	−0.313	−0.298	−0.279	−0.256	−0.229	−0.197	−0.165	−0.132	+0.3	
$\coth\frac{X_1}{2}.\Gamma$	$-\infty$	−6.6	−3.24	−2.1	−1.505	−1.135	−0.877	−0.68	−0.517	−0.391	−0.285	+0.39	
φ_0	−1.57	−1.42	−1.27	−1.127	−0.985	−0.834	−0.72	−0.597	−0.477	−0.378	−0.278	+0.39	
$h=\frac{\pi+\varphi_0}{2\sinh X_1}$	∞	8.6	4.64	3.31	2.63	2.22	1.9	1.68	1.49	1.35	1.22	0.83	
$k^2/_4=h^2/c^2$	∞	7.4	4.3	3.46	3.07	2.9	2.73	2.71	2.63	2.68	2.72	3.45	

method obviously may give rise to some error; a more certain choice would require tracing the beginnings of the curves for ellipses very little different from the circle, but the corresponding calculations seem rather cumbersome. Values of a and c corresponding to x_1 are obtainable from Table I.

For the curve E_{1s}, the validity of the approximate expressions extends very nearly to the case of the circle; for the wave E_0, these latter estimates are clearly too low. As to the wave E_{1c}, the form of the curve, with its minimum, is clearly indicated, but this minimum is produced for values of n^2 and k^2 fairly low, and appears too sharp. The author, therefore, decided to trace the curve fairly clearly above the points calculated by this method. The curves E_{1c} and E_{1s} are related and intersect for the circular section ($a = 1$). Plate III contains a summary of the results relating to the E waves.

Table II gives some details of the calculations, which were made on a slide rule and represent only a rather crude approximation, but suffice for indicating the configuration of the curves.

(12) *H Waves – H_0 Wave*

Study of the asymptotic formulae of section 10 discloses how the curves may be reconciled with the lower branches calculated by another method.

Let us first deal with the *wave* H_0; it is governed by the function Ce_0, the derivative of which must be equated to zero. Differentiating formula (44):

$$\frac{\partial}{\partial x_1} Ce_0(ix_1) = \frac{\sqrt{2}}{\cosh x_1} \left\{ \begin{array}{l} \cos\varphi \left[-\tanh X_1 \cosh \frac{X_1}{2} + \frac{1}{2}\sinh\frac{X_1}{2} + 2\,h \cosh X_1 \sinh \frac{X_1}{2} \right] \\ +\sin\varphi \left[-\tanh X_1 \sinh\frac{X_1}{2} + \frac{1}{2}\cosh\frac{X_1}{2} - 2\,h\cosh X_1 \cosh\frac{X_1}{2}\right] \end{array} \right\} = 0 \,. \quad \ldots\ldots (47)$$

Substituting $\varphi = 2\,h \sinh X_1$,

$$-\tan\varphi = \frac{-\tanh X_1 \cosh\frac{X_1}{2} + \frac{1}{2}\sinh\frac{X_1}{2} + 2h\cosh X_1 \sinh\frac{X_1}{2}}{-\tanh X_1 \sinh\frac{X_1}{2} + \frac{1}{2}\cosh\frac{X_1}{2} - 2h\cosh X_1\cosh\frac{X_1}{2}} . \quad \ldots . (48)$$

The complication arises from the fact that h enters simultaneously on both sides of the equation. The curve H_0 includes high branches with a sufficiently *high value of h*; *low values of* x_1 correspond to flat ellipses. The numerator of the fraction (48) is always small since all the terms include $\sinh x_1$ or $\sinh \frac{1}{2}x_1$ as a factor; the denominator contains $\cosh x_1$ and $\cosh \frac{1}{2}x_1$, which remain nearly equal to unity. If x_1 is sufficiently small to enable one to limit the development of the hyperbolic functions to their first terms, it will be found that:

$$\tan\varphi = x_1 \frac{h - \frac{3}{4}}{2\,h - \frac{1}{2}}, \text{ where } x_1 \text{ is very small.} \ldots (49)$$

A solution φ_0 in the neighbourhood of zero is therefore always possible, and h is given by:

$$h = \frac{n\pi + \varphi_0}{2\sinh x_1}, \ldots\ldots\ldots\ldots\ldots (50)$$

where $n = +1, 0, -1$

It is necessary to choose the integer n. Consider first of all, from this point of view, the case of very small values of x_1, by taking $\sinh x_1 = x_1$ in (50); moreover, x_1 being small in (49), the angle φ_0 also will be small. Hence

$$\varphi_0 = x_1 \frac{h - \frac{3}{4}}{2h - \frac{1}{2}} = 2hx_1 - n\pi,$$

which determines h by means of the equation:

$$4h^2x_1 - 2h(x_1 + n\pi) + \frac{n\pi}{2} + \frac{3}{4}x_1 = 0. \ldots . (51)$$

The solutions are :

$$h = \frac{1}{4x_1}\left[x_1 + n\pi \pm \sqrt{(x_1 + n\pi)^2 - 2n\pi x_1 - 3x_1^2}\right].$$

If we take $n = 0$, an unusable imaginary expression results ; a choice of $n = \pm 1$ remains. The minus sign in front of the radical, however, gives the value of h in the neighbourhood of $\frac{1}{4}$ and approximations will not be correct unless h is large ; this, consequently, is a false solution. Taking the plus sign in front of the radical and developing the radical with due regard to the smallness of x_1, the result is :

$$h = \frac{1}{4} + \frac{n\pi}{2x_1},$$

where $n = \pm 1$; h has a very high value in both cases.

$$h^2 = \frac{\pi^2}{4x_1^2} \pm \frac{\pi}{4x_1}. \quad \ldots\ldots\ldots\ldots\ldots \quad (52)$$

For small values of x_1, on the other hand, from (Table I) :

$$a = c = \frac{1}{\sqrt{x_1}}\ .$$

Accordingly, $\dfrac{k^2}{4} = \dfrac{h^2}{c^2} = \dfrac{\pi^2}{4}\ a^2 \pm \dfrac{\pi}{4}$ ....(53)

The uncertainty of the double sign, therefore, makes a difference of only $\pm \dfrac{\pi}{4} = \pm\ 0.785$ in the value of $\dfrac{k^2}{4}$. It is clearly difficult to choose between these two possibilities ; the plus sign, which corresponds to $n = +1$, would nevertheless appear preferable. To decide, it is necessary to include a point corresponding to a value of a in the neighbourhood of unity with a view to ensuring reconciliation with the circular section. By a series of successive approximations, the author has calculated the value of h for $x_1 = 0.8$, $a = 1.23$ and $c = 0.92$. The result, $h = 1.93$, gives :

$$\frac{h^2}{c^2} = \frac{k^2}{4} = 4.4$$

By employing the minus sign it is found that $h = 1.55$ and $\dfrac{k^2}{4} = 2.85$, a value which seems much too low since the circular section yields $\dfrac{k^2}{4} = 3.65$. The curve has been drawn tentatively under these conditions with $n = +1$, but its correctness is not absolutely certain.

13. Waves H_{1s} and H_{1c}

The rising branches of the curves corresponding to the waves H_{1s} will first be calculated by a method similar to the method used in the preceding section. Subsequently, the relation of these curves to the descending branches H_{1c}, calculated by Dr. Jeffreys in connection with the problem of the normal modes of vibration of an elliptic lake, will be indicated.

The asymptotic form of the function Se_1 is indicated in (44). The derivative may be written in the following form :

$$\frac{1}{i\sqrt{2}}\ \frac{\partial}{\partial x_1} Se_1(ix_1) = \frac{1}{\cosh x_1}\left\{\begin{aligned}&\sin\varphi\left[-\tanh x_1 \cosh\frac{x_1}{2} + \frac{1}{2}\sinh\frac{x_1}{2} + 2h\cosh x_1 \sinh\frac{x_1}{2}\right]\\ &+\cos\varphi\left[+\tanh x_1 \sinh\frac{x_1}{2} - \frac{1}{2}\cosh\frac{x_1}{2} + 2h\cosh x_1\cosh\frac{x^1}{2}\right]\end{aligned}\right\} \quad \ldots\ldots \quad (54)$$

When this derivative equals zero :

$$\tan\varphi = \frac{\tan x \sinh\dfrac{x_1}{2} - \dfrac{1}{2}\cosh\dfrac{x_1}{2} + 2h\cosh x_1 \cosh\dfrac{x_1}{2}}{\tan x_1 \cosh\dfrac{x_1}{2} - \dfrac{1}{2}\sinh\dfrac{x_1}{2} - 2h\cosh x_1\ \sinh\dfrac{x_1}{2}} \quad \ldots\ldots \quad (55)$$

Assuming h large, x_1 small, and replacing $\sinh x_1$ by x_1 and $\cosh x_1$ by 1 :

$$\tan\varphi = -\frac{1}{x_1}\ \frac{2h - \frac{1}{2}}{h - \frac{3}{4}}$$

$$2h\ x_1 = \varphi = n\pi + \frac{\pi}{2} + x_1 \frac{h - \frac{3}{4}}{2h - \frac{1}{2}}$$

and :

$$4h^2 - 2h\left(1 + \frac{A}{x_1}\right) + \frac{3}{4} + \frac{A}{2x_1} = 0. \quad \ldots\ldots (56)$$

where $A = n\pi + \frac{\pi}{2}$

Accordingly,

$$h = \frac{1}{4}\left[1 + \frac{A}{x_1} \pm \sqrt{\left(1 + \frac{A}{x_1}\right)^2 - \frac{2A}{x_1} - 3}\right]$$

$$= \frac{1}{4}\left[1 + \frac{A}{x_1} + \sqrt{\left(\frac{A}{x_1}\right)^2 - 2}\right].$$

The plus sign in front of the radical results in a high value of h; by expending the radical :

$$h \approx \frac{2A}{x_1} + \frac{1}{4}.$$

Hence,

$$h^2 = \frac{A^2}{4x_1^2} + \frac{A}{4x_1} \ldots \text{ with } A = \pm\frac{\pi}{2} \quad \ldots\ldots (57)$$

The interesting values are those which correspond to $n = 0$ or -1; that is, $A = \frac{\pi}{2}$ or $-\frac{\pi}{2}$.

Taking the first value, which gives the best results :

$$h^2 = \left(\frac{\pi}{4x_1}\right)^2 + \frac{\pi}{8x_1} \qquad a^2 \approx c^2 \approx \frac{1}{x_1} \quad \ldots\ldots (58)$$

$$\frac{k^2}{4} = \frac{h^2}{c^2} = \left(\frac{\pi}{4}\right)^2 a^2 + \frac{\pi}{8},$$

giving the following values :

a = 1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
$\frac{k^2}{4}$ = 1.6	1.97	2.39	2.87	3.39	3.94	4.54	5.22	5.94	6.69

The curve H_{1s} traced by means of these values corresponds very well to the points $\left(\frac{k^2}{4} = 0.85\right)$ relating to the circle and to the curves H_{1c} calculated by Dr. Jeffreys.

This author dealt with *normal modes of vibration of an elliptic lake*, leading to equations relating to the H waves of our problem. He has given some numerical indications on the waves H_1, but has not been able to obtain any result

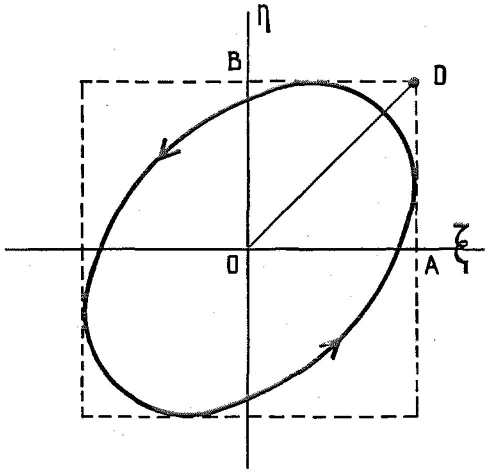

Fig. 11

on the waves H_0 and H_2.

In short, he systematically uses the developments by Mathieu of section 9, which are not usable save for very small values of h^2 or k^2. The waves H_{1c} calculated by Dr. Jeffreys are those where this method applies best, as they give the lowest branches; he indicates a value of

$$h = 0.748 \qquad \frac{k^2}{4} = \frac{h^2}{c^2} = 0.538$$

$$\text{for } \frac{b}{a} = \frac{3}{5} \qquad a = \sqrt{\frac{5}{3}} = 1.29 \qquad b = \sqrt{\frac{3}{5}} = 0.775 \quad \ldots\ldots (59)$$

He then calculates the wave H_{1s} for the same dimensions and finds :

$$\text{Wave } H_{1s}\ h = 1.19 \qquad \frac{k^2}{4} = \frac{h^2}{c^2} = 1.36$$

$$a = 1.29 \qquad b = 0.775. \quad \ldots\ldots\ldots (60)$$

These points have been plotted on Plate IV (small circles); the relationships with the asymptotic branches just calculated are very good.

For ellipses of great eccentricity, Dr. Jeffreys calculates a limiting value of:

$$h = 0.943\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (61)$$

another method gives him a slightly higher value:

$$h = 0.943\,\frac{194}{189} = 0.968\,.$$

The present author has calculated the descending branches with the value (61), giving:

$$\frac{k^2}{4} = \frac{(0.943)^2}{c^2}$$

a =	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
c =	1.2	1.5	1.72	1.95	2.2	2.4	2.6	2.8	3	3.2
$\frac{k^2}{4}$ =	0.618	0.395	0.3	0.233	0.184	0.154	0.131	0.113	0.099	0.087
$\frac{h^2}{a^2}$ =	0.45	0.345	0.27	0.22	0.184	0.154	0.131	0.113	0.099	0.087

The points $\frac{h^2}{c^2}$ are too high for small values of a; but, taking $\frac{h^2}{a^2}$, the crosses indicated in the plate are obtained, and the curve coincides very well with the preceding values. Dr. Jeffreys' approximation combines a and c in such a way as to leave these points uncertain.

Curves H_{1s} and H_{1c}, for elongated and flat ellipses, coincide very well as might be foreseen by the examination of the nodal lines.

The important consequences of this intersection of curves are discussed in section 16.

(*14*) *Waves* H_2

Waves H_2 show two branches H_{2s} and H_{2c} which join, for the circular section, at the value:

$$\frac{k^2}{4} = 2,\, 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (62)$$

shown in equation (34) of section 7. In Fig. 8 are shown the forms of the nodal curves, the two axes for H_{2s}, and the two hyperbolæ for H_{2c}. For the circle, in the two cases, two orthogonal diameters are obtained.

In (44), the asymptotic expression of Se_2 is given. As in the previous section, it will be used to estimate the climbing branches. Se_2 will be developed for small values of x_1 whilst keeping the terms to the second order:

$$\frac{\sqrt{2}}{i}\,Se_2\,(ix_1) = \sin\varphi\left(1 - \frac{11}{8}\,x_1{}^2\right) - \frac{3}{2}\,x_1\cos\varphi \quad \ldots\ldots\ldots (63)$$

$$\frac{\sqrt{2}}{i}\,\frac{\partial}{\partial x_1}\,Se_2 = \sin\varphi\left[-\frac{11}{4}\,x_1 + 3h\,x_1\cosh x_1\right] + \cos\varphi\left[-\frac{3}{2} + 2h\cosh x_1\right] = 0\;.$$

By annulling the derivative the wave H_{2s} is found and φ and h are determined by the condition:

$$\tan\varphi = -\frac{1}{x_1}\,\frac{2h - 3/2}{3h - \frac{11}{4}}\,; \quad \ldots\ldots\ldots\ldots (64)$$

$$\text{hence } \varphi = 2h\,x_1 = n\pi + \frac{\pi}{2} + x_1\,\frac{3h - \frac{11}{4}}{2h - 3/2}\,,$$

Taking $A = n\pi + \frac{\pi}{2} = \frac{3\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}$ (for $n = 0, \pm 1$)

$$\left(2h - \frac{A}{x_1}\right)(2h - 3/2) - 3h + \frac{11}{4} = 0 \quad .. \quad (65)$$

in terms of h.

The solutions are:

$$h = \frac{1}{4}\left[3 + \frac{A}{x_1} \pm \sqrt{\left(\frac{A}{x_1}\right)^2 - 2}\,\right].$$

The sign in front of the radical must be plus, and

$$h \approx \frac{A}{2x_1} + \frac{3}{4} \qquad h^2 = \frac{A^2}{4\,x_1{}^2} + \frac{3A}{4x_1} \quad \ldots. (66)$$

The two possible values are:

$$A = \pm\frac{\pi}{2} \qquad h^2 = \left(\frac{\pi}{4\,x_1}\right)^2 \pm \frac{3\pi}{8\,x_1}$$

The minus sign represents a branch situated below the curve H_{1s}, which is not reasonable; the plus sign, a branch above H_{1s} and parallel to it:

$$\frac{k^2}{4} = \frac{h^2}{c^2} = \left(\frac{\pi}{4}\right)^2 a^2 + \frac{3\pi}{8} \quad \ldots\ldots (67)$$

It is a curve situated at $\frac{\pi}{4}$ ($= 0.785$) above the curve H_{1s} as will be seen by comparing (67) and (58). The continuity between the two branches H_{2s} of the one and other part of the circle ($a = 1$) necessitates passing through the value $a = 1$ with a horizontal tangent, and drawing the curve as indicated on Plate IV.

The wave H_{2c} should give the two descending branches which are related to the horizontal tangent of the preceding curve. It should be possible to estimate the limiting value of h^2 for great eccentricities by taking x_1 as very small in the formula for Ce_2 (equation 41).

$$\frac{\partial}{\partial x_1} \cosh px_1 = p \sinh px_1 \approx p^2 x_1;$$

the condition $\frac{\partial}{\partial x_1} Ce_2 = 0$, therefore, gives:

$$4 - h^2 \frac{4^2}{12} + h^4 \frac{6^2}{384} + h^6 \ldots + h^8 \ldots = 0.$$

This series is cumbersome of solution; the terms above h^8, h^{10}, etc., play an important part. Dr. Jeffreys has calculated a limiting value by a different process and has obtained:

$$2h = 3.24; \quad h = 1.62.$$

Drawing the curve $\frac{k^2}{4}$ with this value:

$$\frac{k^2}{4} = \frac{(1.62)^2}{a} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

It would seem that calculation of the curves H_3 would show that their arrangement round the point $\frac{k^2}{4} = 4.25$ (relating to the circle) would be similar to that of curves H_1, the continuity of the modes of vibration being similar; the two branches would, therefore, cut one another at an acute angle.

15. *Comparison of Graphs for Elliptic and Rectangular Sections*

This network of curves gives rise to several observations. If the curves E and H are superposed, various coincidences will be noticed.

For the circle, H_0, E_{1s}, E_{1c} have the same characteristic values; similarly, for H_{1s}, H_{1c}; H_{2s}, H_{2c}; H_{3s}, H_{3c}.

For flat ellipses, the coincidences are between H_0 and E_{1s}, between H_{2s} and E_{1c} and also between H_{1s} and E_0.

Finally, the curves H_{1c}, H_{2c} . . . are the only ones which give descending branches. Comparison of Plates II, III and IV shows that some of the curves have similar forms for elliptic or rectangular sections:

elliptic: E_0, E_{1s}, E_{1c}, H_{1s}, H_{1c}, H_{2s}.
rectangular: E_{11}, E_{12}, E_{21}, H_{10}, H_{01}, H_{11}.

The two distinct curves H_0 and H_{2c} take the place of the two intersecting curves H_{20} and H_{02}. Here a type of transformation is involved similar to that which one meets, in various problems of perturbation, in the evolution of curves representing characteristic values (k^2) as a function of the parameter (a).

Similar problems have been studied in wave mechanics, where the stable values of the energy of an atomic system are given by the characteristic values of a certain differential equation; in this case, the perturbation is characterised by a supplementary term in the equation defining the characteristic value k^2, whilst the form of the boundary remains unchanged. In the problems considered herein, the equation remains unchanged, but it is the form of the boundary which is modified. It is to be expected that these two types of problems will be closely related. Moreover, when such a perturbation problem is met in wave-mechanics the result is well known[17]. Suppose that two curves representing k^2 as a function of a parameter a intersect in a certain problem; if, then, a slight disturbance is introduced into the system, the curves separate as shown in Fig. 9. This effect is produced if the disturbance establishes a coupling between the modes of vibration represented by the two initial curves. The author has recently presented a general proof that the two problems of perturbation, either by a variation of the equation, or by a deformation of the limiting curve, are exactly equivalent[18].

The present example comprises two curves H_{02}

and H_{20} which intersect in the case of a rectangular section; if the angles of the rectangle are rounded off, the two curves separate as in Fig. 9. When the reshaping of the boundary is continued until it becomes an ellipse, the higher branch gives the curve H_0; the lower branch forms the curve H_{2c} and is related to H_{2s} as shown above.

It is therefore possible to closely follow the deformation of the curves from a rectangle to an ellipse.

16. *Stability for Small Conductor Tube Deformations*

The networks of curves summarising this study supply very important information on the various types of waves realisable in a circular tube.

Certain waves are represented by curves showing a maximum or a minimum for the circular section. *These waves are stable*; for, if the tube be made slightly oval, their propagation is not affected: such is the case for the waves E_0 and H_0.

The waves H_{2s} and H_{2c} are also more or less stable, but the coincidence of the actual values in the case of the circle may to some extent play a role similar to that next discussed.

The waves E_{1s} and E_{1c}, H_{1s} and H_{1c}, and probably H_{3s} and H_{3c}, as a result of intersection of the curves, are not stable. Only the case of E_{1s} and E_{1c} is considered, the results being entirely similar for H_{1s} and H_{1c}.

The wave E_{1s} has a horizontal nodal line; the wave E_{1c}, a vertical nodal line (Fig. 10); these two waves have the same k^2 and, accordingly, the same velocity of propagation in the tube. If two waves E_{1s} and E_{1c} *having the same phase* be superposed, a wave with a nodal line inclined at an arbitrary angle θ will result.

But two waves E_{1s} and E_{1c} of *different phase* also can be superposed. A more complex wave which will keep its form whilst propagating will be obtained, due to a cause similar to that involved in the propagation of polarised light. Assume two vibrations ξ and η of the same phase superposed; a beam with linear polarisation *OD* will result. If two beams of different phase (Fig. 11) be superposed, a beam with *elliptic polarisation* will be formed.

Propagation of the waves E_1 in a circular

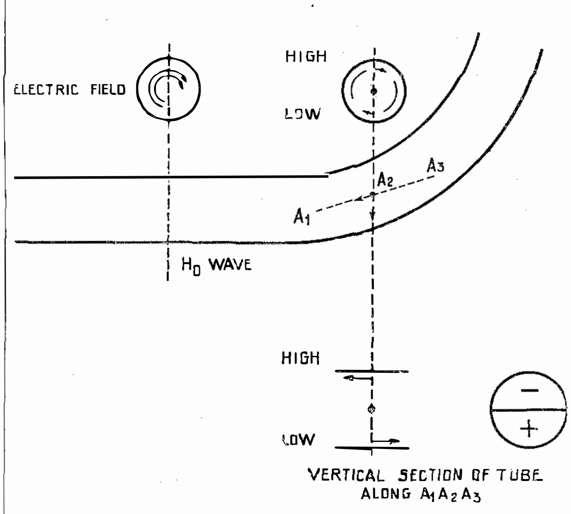

Fig. 12.

tube, therefore, may occur with linear or elliptic polarisation.

Assume that a wave $E_{1\theta}$ (Fig. 10) with oblique linear polarisation has been transmitted through a tube, and that the latter is slightly flattened in the direction $O\eta$. The wave E_1 will split into two components: the components E_{1c} and E_{1s} will be propagated with different velocities; after having travelled through the elliptic constriction, the two waves will reassemble themselves but will no longer be in phase. They, therefore, will produce an *elliptically polarised wave.*

This change of wave form involves undesirable possibilities in propagation and reception. Furthermore, the rapid change in the values of k^2 for the elliptic sections produces a *correlative change in the cut-off frequency*. If, therefore, a wave with a frequency which is a little higher than the cut-off frequency be used, one of the two components may be impeded and reflected. After a certain number of reflections of this kind, at constrictions in various directions, the amplitude of the wave may be decreased enormously.

The wave H_0 is stable with regard to flattened cross-sectional contours, but it cannot be stable in relation to irregularities in the curvatures of the conductor tube; this wave naturally possesses a longitudinal magnetic field and electrical transverse fields in the cross-section

(Fig. 12). If the tube bends, these transverse electric fields will yield small longitudinal components which may produce a wave of the electric type, E_1, as is shown by the alteration of the directions of the two opposite edges. The wave E_1, having the same k^2, and accordingly the same velocity of propagation as H_0, the effects may accumulate and produce the creation of a wave E_1 which is sufficiently noticeable. One must ask, therefore, whether the waves H_0 will not, at each bend of the tube, lose a little energy in the form of E_1 parasitic waves, which are absorbed by virtue of their high attenuation. This would cause an attenuation which would greatly reduce the interest in H_0 waves.

The coincidences of the actual values are only produced for two limiting cases: a circular tube and a very much flattened tube; it may, therefore, be found advisable to avoid disturbances due to this factor by using tubes with elliptic sections and moderate eccentricity. These tubes, moreover, for certain types of wave, enable the value of k^2 to be lower with consequent diminution of the cut-off frequency. A study of the attenuation will be required to determine the practical value of elliptic tubes.

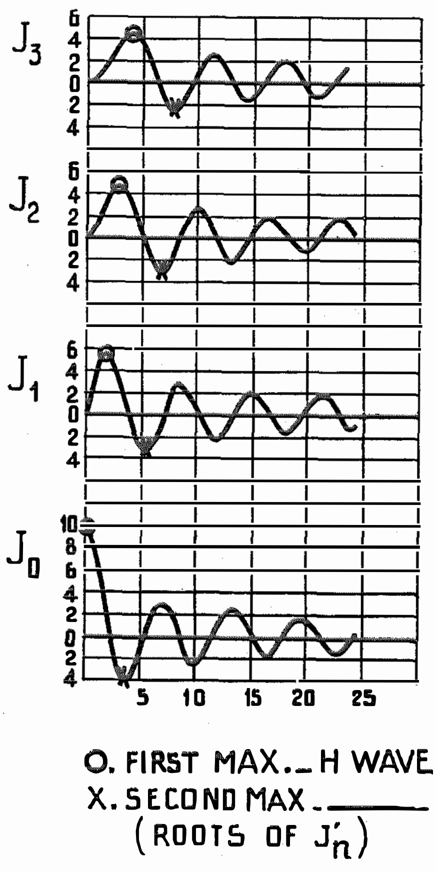

Fig. 13.

17. *Comparison with Similar Acoustic Problems*

The disposition of the characteristic values of k^2, in the case of a circular section, is a little surprising in the first instance; one unexpectedly finds that the fundamental type wave H_0, has a characteristic value which is above that of waves H_1 and H_2. This result is explained by the fact that the problem of *electro-magnetic waves necessitates the elimination of all the zero characteristic values for* k^2. The conditions of zero divergence, equation (6), exclude solutions for which H_3 (or E_3) would be constant over the whole cross-section of the tube; a constant value cannot be other than zero—and cannot correspond to any wave. Moreover, the curves representing the Bessel functions $\mathcal{J}_0$, $\mathcal{J}_1$, $\mathcal{J}_2$, $\mathcal{J}_3$ . . . appear as represented in Fig. 13. It will be seen therefrom that the functions supply, in addition to the above-mentioned characteristic values k^2, a series of zero characteristic values, corresponding to the following types of waves:

Bessel function:

$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$ . . .	$\mathcal{J}_n$	(70)

Type of waves:

H	E	E and H	E and H	E and H

with *characteristic values* $k^2 = 0$.

Thus, the wave H_0 is represented by the *second root* of $\mathcal{J}'_0$; the wave H_1 corresponds to the first root of $\mathcal{J}'_1$, which was zero for the case of $\mathcal{J}'_0$.

In the cases corresponding to $\mathcal{J}'_2$, $\mathcal{J}'_3$ . . . etc., there are introduced supplementary zero roots, but the roots employed herein form a continuous wave and are regularly displaced, increasing when the order of Bessel functions increases.

The zero roots do not correspond to any usable solution in electro-magnetism, but they can supply solutions to other problems, such as in acoustics or hydrodynamics. An acoustic wave is defined by its velocity potential Φ, the partial derivatives supplying the components of the velocity of the particles ; in a rigid tube, the potential Φ is governed by equation (11), subject to the condition of a non-normal component of velocity at the boundary. Consequently, $\frac{\partial \Phi}{\partial n}$ must be zero as in (13). *All the acoustic solutions correspond to the magnetic waves herein considered*, but it is necessary *to add the solutions with characteristic values k^2 zero* ; in acoustics, therefore, the supplementary H solutions shown in (70) apply. For zero functions of a higher order, J_2, J_3, etc. . . ., when the derivative J'_n is zero—at the origin ($k^2 = 0$)—the function J_n is also zero. The corresponding waves disappear entirely. But in the case of J_0, the derivative is zero without the function being zero. Its amplitude Φ is constant along the whole section of the tube ; the velocities of the particles are parallel to the axis of the tube and constant in the whole section. This type of wave corresponds to a portion of plane wave limited by the rigid tube and is the one always considered in elementary considerations on sound pipes.

Thanks to the existence of this wave, the speaking tube can transmit all the acoustic frequencies from the lowest ; but, in the course of a methodical study, higher types of waves with a minimum frequency, corresponding to the waves H_0, H_1, H_2 and H_3 of the magnetic case herein discussed, will be found to appear. It would be interesting to examine the anomalies of the properties of acoustic tubes at frequencies which permit the transmission of waves H_0 and H_1. If a speaking tube were equipped with acoustic transmitting and receiving systems insensitive to the ordinary wave, and regulated for the wave H_1, for example, it would work as a high-pass acoustic filter with a cut-off frequency towards the lower portion of the band, exactly the same as the electro-magnetic type of transmission tubes herein considered.

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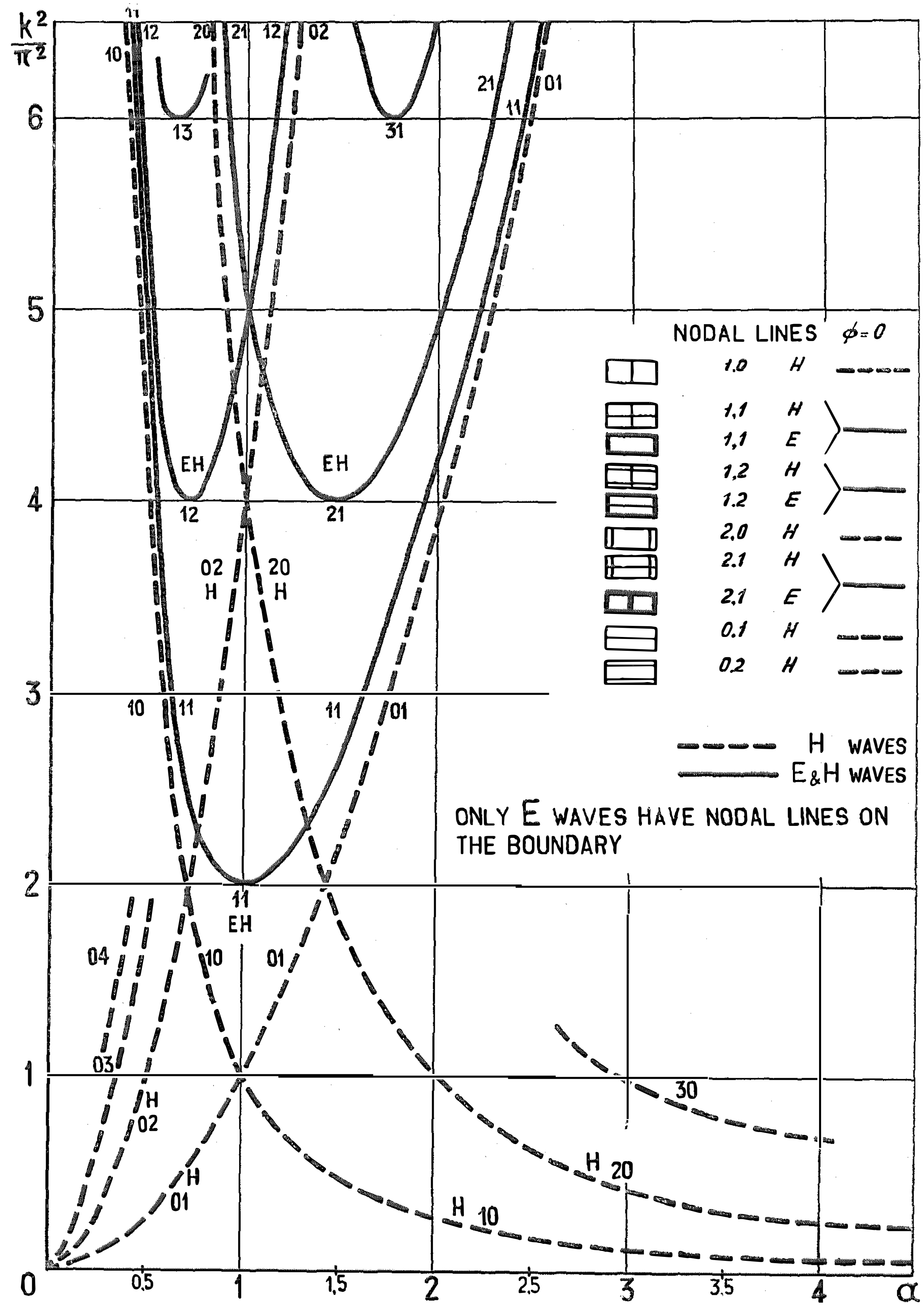
* These authors' $1/h_1$, $1/h_2$, $1/h_3$, correspond to e_1, e_2, and e_3 as used in the present article.

† The following are notations in the various papers :

Strutt.	Mathieu and present papers.	L. Brillouin Strutt earlier papers.
$\frac{\lambda}{h^2}$	$\frac{R}{h^2}$	$\frac{R}{-h^2}$

L. Brillouin Quanten Statistik.	Whittaker P. Humbert.
$\frac{\eta \text{ or } \lambda}{-\gamma}$	$\frac{a}{-8q}$



k²/π²
6
5
4
3
2
1
0
0,5
1
1,5
2
2,5
3
3,5
4
α
NODAL LINES φ=0
1,0 H
1,1 H
1,1 E
1,2 H
1,2 E
2,0 H
2,1 H
2,1 E
0,1 H
0,2 H
H WAVES
E&H WAVES
ONLY E WAVES HAVE NODAL LINES ON THE BOUNDARY
EH 12
EH 21
11 EH
13
31
02 H
20 H
H 02
H 01
H 10
H 20
30
03
04

Plate I.

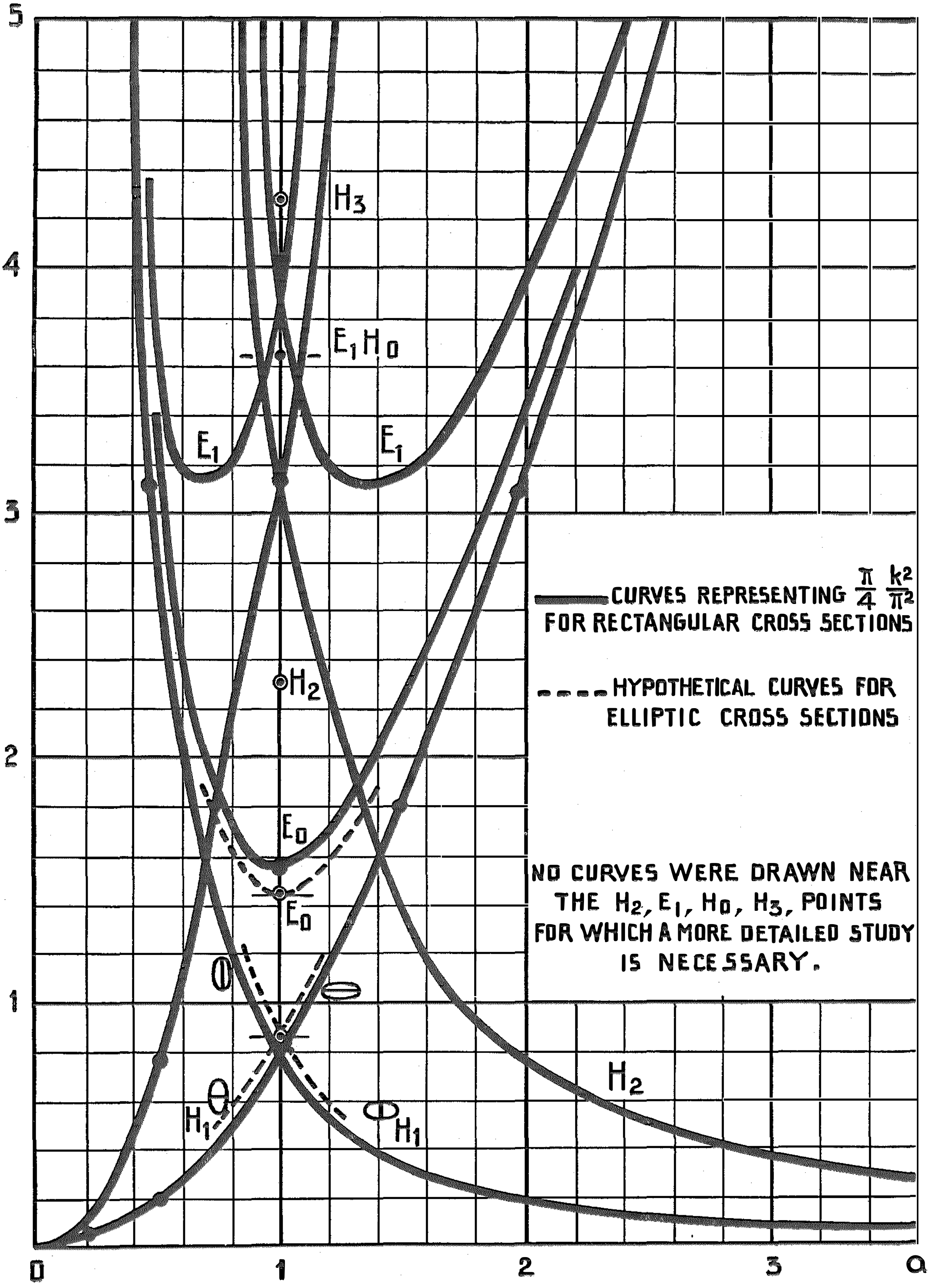

5
4
3
2
1
0
1
2
3
a
H3
E1 H0
E1
E1
H2
E0
E0
H1
H1
H2
CURVES REPRESENTING $\frac{\pi}{4}\frac{k^2}{\pi^2}$
FOR RECTANGULAR CROSS SECTIONS
HYPOTHETICAL CURVES FOR
ELLIPTIC CROSS SECTIONS
NO CURVES WERE DRAWN NEAR
THE H2, E1, H0, H3, POINTS
FOR WHICH A MORE DETAILED STUDY
IS NECESSARY.

PLATE II.

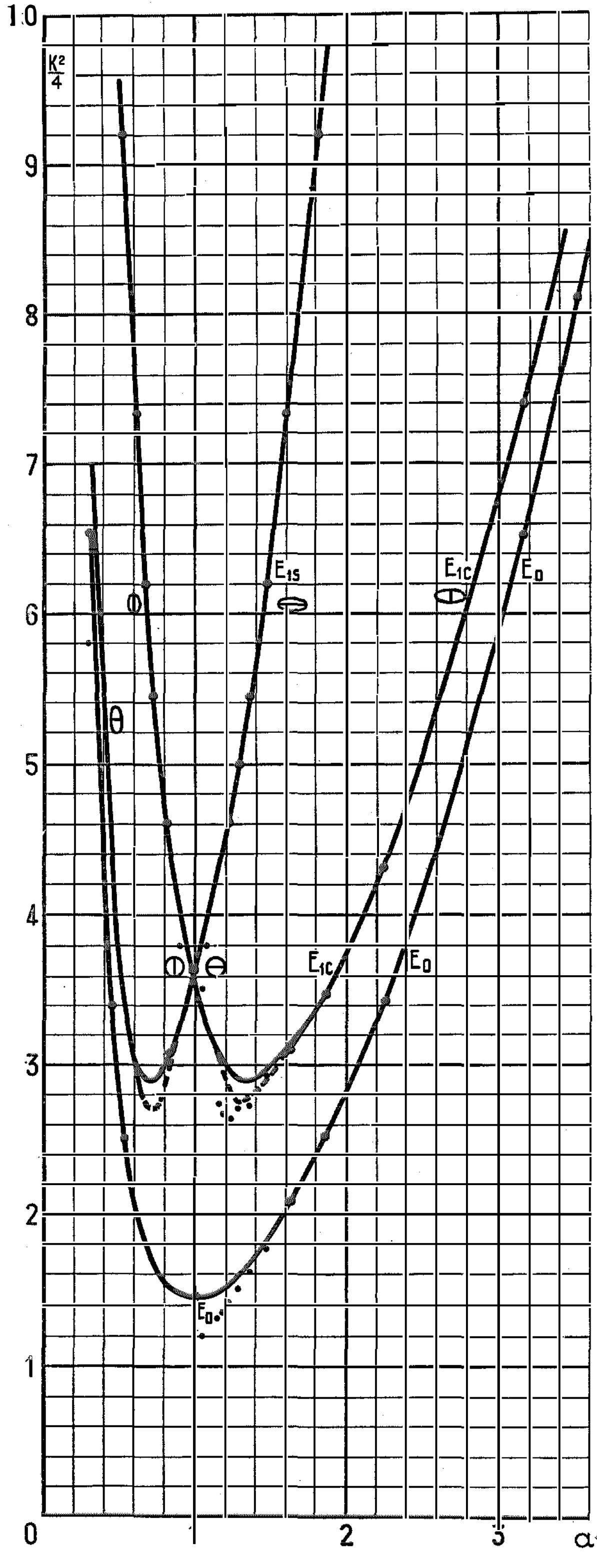


Plate III.

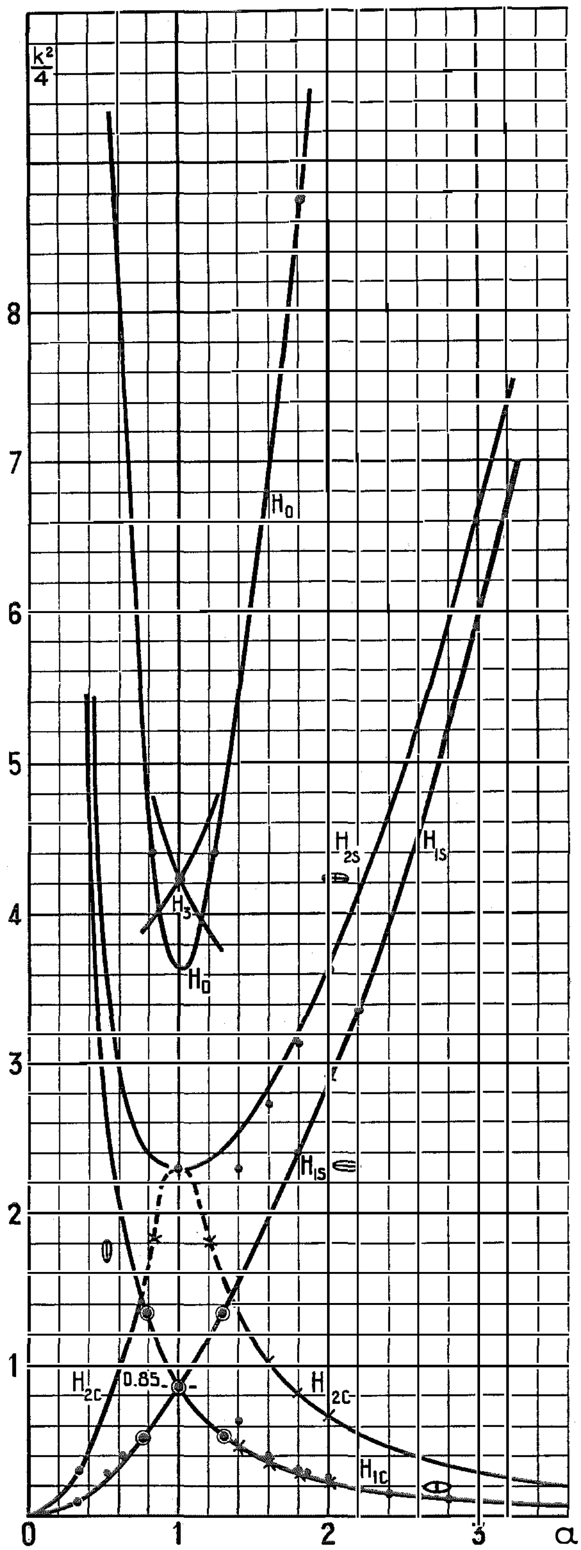


Plate IV.

Recent Telecommunication Developments of Interest

R.9 Aircraft Direction Finding Receiver.—The R.9 Aircraft Direction Finding Receiver has been developed by Standard Telephones and Cables (London) laboratories to meet the demand for a compact, light, highly sensitive radio receiver, which is so simple in operation as to require little effort on the part of the pilot for its satisfactory manipulation. In addition to meeting these requirements, the receiver provides accurate direction finding facilities with either aural or visual indication.

The receiver, with single dial tuning, covers a frequency range of 1200-150 kc. (250-2000 metres), which includes the medium and long wave broadcast bands as well as the commercial aviation band. The only other necessary operation is the rotation of the loop aerial when taking bearings.

For direction finding, any deviation from the true bearing is shown immediately on a course indicator. The needle of the indicator swings left or right of a centre zero point as the machine veers off its course to port or starboard, thus providing an indication of the course. The indicator may be mounted at any convenient position on the pilot's instrument panel.

A special feature of this receiver is the extremely small loop aerial employed for direction finding. The loop rotates inside a fixed streamlined housing, approximately 20 inches (508 mm.) high and 8 inches (203 mm.) in maximum diameter, creating only negligible wind resistance even at high speeds (1.8 lb. at 150 m.p.h.—243 km.p.h.).

R.9 Aircraft Direction Finding Receiver.

High tension supply is obtained from a rotary transformer, while low tension is taken direct from the 12 or 24-volt aircraft battery.

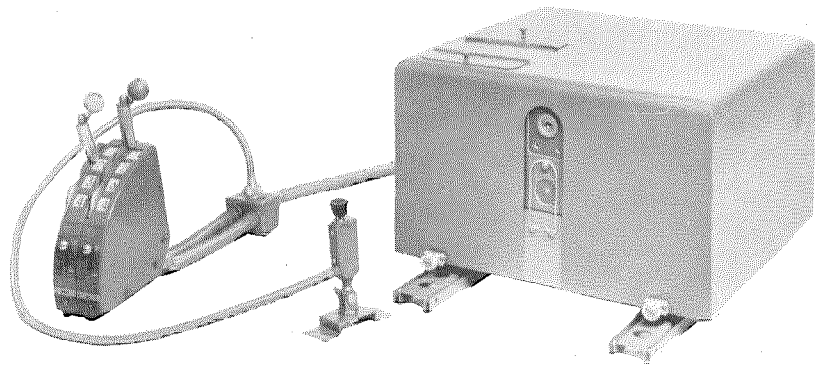
The total weight of the equipment, including the power supply unit, is 37 lb. (16.78 kg.) maximum.

• • •

Type R.10 Short Wave Radio Equipment for Fighter Aircraft.—Much effort has been directed towards producing a really simple, reliable and satisfactory radio equipment for fighter aircraft, so designed that its operation involves practically no effort or attention on the part of the pilot. The R.10 short wave radio equipment, a recent development of Standard Telephones and Cables (London) laboratories, fulfils these requirements. Notwithstanding the facilities provided, its total weight, including transmitter, receiver, rotary transformers and remote controls, does not exceed 51 lb. (23.13 kg.).

The R.10 equipment operates on four predetermined and independent communication channels within the frequency range of 7.5-2.5 megacycles (40-120 metres). Any one of these four may be selected instantaneously by movement of a single lever which automatically changes over the wavelengths of the transmitter and receiver. These four channels may be reserved, respectively, for different purposes, such as communication with the Squadron Leader, communication with the ground, etc.

Complete flexibility of mounting, together with simplicity of operation, have been achieved by the use of an electrical system of remote controls. The only control manipulations necessary on the part of the pilot are: (*a*) the wave change lever; (*b*) an on-off switch; and (*c*) a *press to talk* push button controlling the change-over from receiving to sending. The latter feature obviates the danger of the pilot



Type R.10 Short Wave Radio Equipment for Fighter Aircraft.

leaving his transmitter in operation when not speaking, thus avoiding the jamming of communications from other machines in the vicinity.

Certainty of communication has been attained by the use of crystal control on both transmitter and receiver, thus ensuring that the equipment will function on the exact wavelength selected by the wave-change lever. No tuning adjustments need be carried out by the pilot.

Units of the equipment comprising transmitter, receiver and two rotary transformers are each mounted on separate sub-chassis which fit on to a parent chassis. This arrangement provides for rapid and easy inspection and maintenance.

The power supply is obtained from the above mentioned rotary transformers, designed to be fed from the main battery of the aircraft. Either 12 or 24-volt batteries may be used as desired.

Use may be made of any fixed aerial system, such as a vertical strut with wire to tail, a single wire beneath the fuselage or a "wing tip to tail."

The distance over which communication can be maintained depends to a considerable extent on the wavelength used, the type of country and the height at which the aircraft is flying; but, at a height of 10 000 feet or over, an air to ground range of 120 miles (200 km.) can be obtained. Communication between aircraft in flight is practicable up to distances of 12½ miles (20 km.).

• • •

Type BSZ.237 Micro-Relay.—Standard Telephon und Radio A.G., Zurich, recently developed an extremely sensitive relay combination, intended especially for use with selenium photo-electric cells. The sensitivity is such that it will operate reliably from thermocouples or small radio receivers.

The sensitive or primary element of the relay consists of a micro-ammeter operating a central contact located between two semi-fixed contacts. Both "on" and "off" operation may be set for any desired operating value within a wide range down to a minimum current of 2 micro-amperes or a minimum voltage of 1 millivolt. This is equivalent to a sensitivity of 1/500th of a microwatt.

Contact pressure under any operating conditions is positive and independent of the current for which the relay has been adjusted. This result is achieved by equipping each of the semi-fixed contacts with a small permanent magnet which causes rapid and positive movement of the central contact when it approaches either of the two extremes.

The primary relay operates a secondary or heavy duty locking relay combination, capable of controlling a load of 4 amperes and 250 volts or 1 kW. Simultaneously with the operation of the secondary relay combination, an auxiliary electro-magnetic mechanism is energised and disengages the moving contact from the semi-fixed contact, whereupon the primary relay is ready to operate in the reverse direction.

By means of the sensitive relay with permanent magnet contacts, the secondary heavy duty locking relay combination, and an automatic mechanism for restoring the sensitive relay, a system has been evolved which combines the following features:

(1) Extremely high sensitivity;
(2) Positive operation;
(3) Freedom from vibration effects;
(4) Speed of 30 operations per minute;
(5) Load controlling capacity, 1 kW;
(6) Power consumption during a two-second operating period—only 13.5 watts.

Applications of the device include:

(*a*) *Photo-electric:*

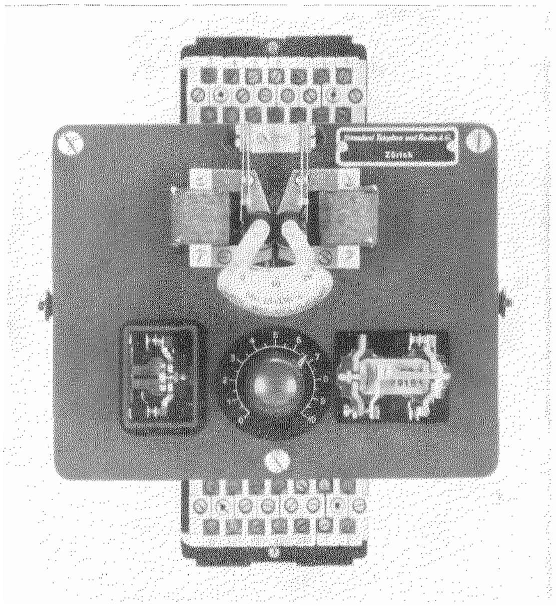
Control of indoor and outdoor lighting;
Smoke and fire detection;
Product control or inspection, resulting from reflection or transparency characteristics of materials, including liquids.

(*b*) *Thermal:*

Measurement and control of temperatures in rooms, ovens, furnaces, etc., over a 0 to 1500 degree centigrade range. The relay in this case is fitted with a scale calibrated in degrees of temperature instead of micro-amperes, and is used in conjunction with a thermocouple.

(*c*) *Radio and Carrier Rediffusion:*

Remote control, signalling, etc., in which case the relay may be operated direct from a small radio or carrier rediffusion receiver.



Type BSZ.237 Micro-Relay with Cover Removed.

(*d*) *Research and Scientific:*

In the electro-medical field, for example, as a protective device in connection with electro-medical treatment apparatus.

In addition to the normal type, the micro-relay is available in dust- and water-proof form for outdoor or industrial use.

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