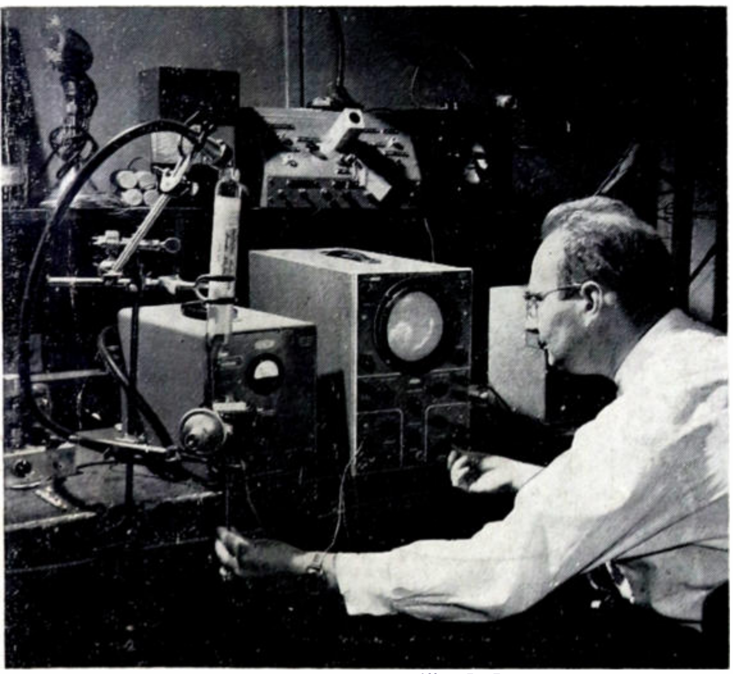
Proceedings of the I·R·E

A Journal of Communications and Electronic Engineering

March, 1952

Volume 40 Number 3



Allen B. DuMont Laboratories, Inc.

INDUSTRIAL OSCILLOGRAPHY

An oscilloscope and transducer are used to study the operation of an automobile distributor.

PROCEEDINGS OF THE I.R.E.

The Electronics Systems Engineer
Magnetic Amplifier Technique
Properties of Transmission Systems
The L-Cathode Structure
AM Rejection in the Ratio Detector
CRT for Recording High-Speed Transients
Accuracy of Differential Analyzers
The Duplex Traveling-Wave Klystron
Electronically Controllable Resistors (Abstract)
Echo Distortion in Frequency-Division Multiplex
Close Channel Spacing at VHF
90-Degree Phase-Shift Networks
Selective RC Bridge
Waves on Cylindrical Structures
Slotted-Cylinder Antennas
Abstracts and References

TABLE OF CONTENTS, INDICATED BY BLACK-AND-WHITE MARGIN, FOLLOWS PAGE 64A

The Institute of Radio Engineers

501R

5604

880

Whatever your Power Requirements at whatever the Frequency there's a standard

Amperex

tube to fit your need!

6077/AX9906

5868/AX-9902

5894/AX9903

5924/AX-9904 R

May we recommend a tube for your purpose?

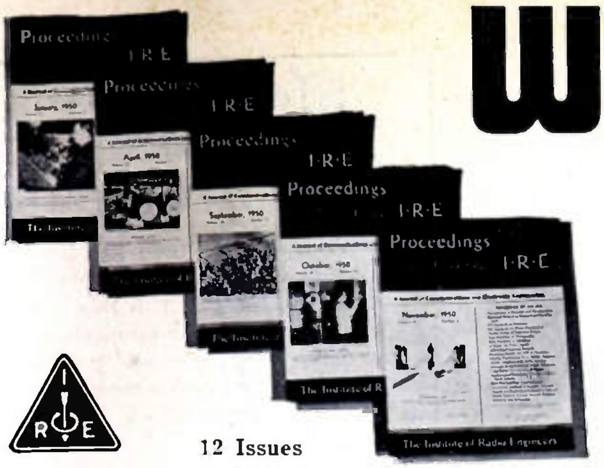
The few tubes listed will give you some indication of the extent of our line

TUBE TYPE	POWER OUTPUT	FREQUENCY AT MAX. RATINGS	PRICE
5894/AX9903	85 Watts	Up to 250 Mc.	$19.00
5868/AX-9902	1.5 Kw.	Up to 100 Mc.	60.00
501R	2 Kw.	Up to 150 Mc.	100.00
5924/AX-9904 R	5.0 Kw.	Up to 220 Mc.	225.00
5604	22.5 Kw.	Up to 25 Mc.	540.00
880	40 Kw.	Up to 25 Mc.	510.00
6077/AX9906	108 Kw.	Up to 30 Mc.	1700.00

Our Newest Plant, Hicksville, New York

AMPEREX ELECTRONIC CORP.

25 WASHINGTON STREET, BROOKLYN 1, N. Y.
In Canada and Newfoundland: Rogers Majestic Limited
11-19 Brentcliffe Rd., Leaside, Toronto 17, Ontario, Canada
Cable: "AMPRONICS"



12 Issues
a year

WHY — 25,627* Radio Engineers have joined THE INSTITUTE OF RADIO ENGINEERS

Value to readers of "Proceedings of the I.R.E."

In 1951 this vital engineering journal published 1594 pages of text, which equals on a word count basis 3188 pages in a standard engineering text, the equivalent of six 540 page books. (This is exclusive of all advertising, product stories and some departmental reading matter.)

Since 1912, "Proceedings of the I.R.E." has been the authoritative source of radio engineering information. In reference indexes it is the most quoted publication in its field. Its research papers are often years ahead of manufacture and point the way to future developments. It is generally acknowledged that no other single medium has contributed so much to the advancement of radio-electronic science.

Every IRE member receives "Proceedings of the I.R.E." as part of his membership. It is a treasured asset, and a working tool in his "equipment" as an engineer — a source of knowledge.

Benefit from IRE Meetings —

IRE Section meetings are held in 74 cities throughout the world. More than 700 such meetings are held a year, for hearing and discussing engineering papers. Wherever there is a concentration of engineers, an active section serves IRE Members on a geographic basis.

In addition, there are 16 Professional Groups which hold meetings on specialized subjects, or branches of radio, ranging from audio to airborne electronics, from broadcasting to nuclear science. These groups provide specialized study into the deep corners of a gigantic technology.

These meetings, together with the regional and national conventions and exhibits, are provided for IRE members, to keep them ever abreast with the advances of their chosen science.

*ABC Statement of Dec. 31, 1951 for all membership from young to senior engineers.

IRE Services to Members and Industry —

Here in the IRE Headquarters Building at 1 East 79th Street, New York City, a busy staff of editorial people work on the magazine, and another group process applications and service correspondence The technical department organizes professional groups, and forty standing technical committees which keep order and establish standards in the world's fastest growing science.

Four rooms are constantly busy with technical meetings, which serve members and industry by coordination and clarification, expressed in "Standards" and the annual "IRE DIRECTORY".

IRE Meetings and Exhibits Speed Electronic Progress!

"SPRING TECHNICAL CONFERENCE" on Color and UHF Television

Sponsored by the Cincinnati Section of the IRE

April 19, 1952 — Cincinnati, Ohio, Engineering Society Building — See Page 2A

PROCEEDINGS OF THE I.R.E. March, 1952, Vol. 40, No. 3. Published monthly by the Institute of Radio Engineers, Inc., at 1 East 79 Street, New York 21, N.Y. Price per copy: members of the Institute of Radio Engineers $1.00; non-members $2.25. Yearly subscription price: to members $9.00; to non-members in United States, Canada and U.S. Possessions $18.00; to non-members in foreign countries $19.00. Entered as second class matter, October 26, 1927, at the post office at Menasha, Wisconsin, under the act of March 3, 1879. Acceptance for mailing at a special rate of postage is provided for in the act of February 28, 1925, embodied in Paragraph 4, Section 412, P. L. and R., authorized October 26, 1927. Printed in U.S.A.

Table of Contents will be found following page 64A

Meetings with Exhibits

● As a service both to Members and the industry, we will endeavor to record in this column each month those meetings of IRE, its sections and professional groups, and some closely related groups *which include exhibits.*

Cincinnati Engineering Building

Δ

Spring Technical Conference
on Television
April 19, 1952
Cincinnati Engineering Society
Building, Cincinnati, Ohio
Adv. & Exhibits: Wynne W. Gulden
3272 Dayton Avenue
Cincinnati 11, Ohio

Δ

NEREM, Saturday, May 10, 1952
Copley Plaza Hotel, Boston, Mass.
New England Radio
Engineering Meeting
Gen. Chairman: Alfred J. Pote
71 West Squantum St.
N. Quincy, Mass.

Δ

National Conference on
Airborne Electronics
May 12, 13 & 14, 1952
Hotel Biltmore, Dayton, Ohio
Exhibits: Paul D. Hauser
1430 Gascho Drive, Dayton 3

Δ

4th Southwestern IRE Conference
May 16, 17, 1952
Rice Hotel, Houston, Texas
Exhibits: Gerald L. K. Miller
1622 W. Alabama
Houston 6, Texas

Δ

Radio Parts Show
Chicago
May 19-24, 1952

Δ

Western Electronic Show and
IRE Regional Convention
August 27, 28 & 29, 1952
Municipal Auditorium
Exhibits: Heckert Parker
215 American Avenue
Long Beach, Calif.

Δ

National Electronic Conference
Sept. 29, 30, Oct. 1, 1952
Hotel Sherman, Chicago, Ill.
Exhibits Manager: Mr. R. M. Krueger, c/o Amphenol, 1830 South 54th Ave., Chicago 50, Ill.

Only One Source gives you Double Duty TV!

Highest Quality Studio Programming Resources—plus

Portable Packaging for Maximum Studio-Field Flexibility!

When you invest in GPL TV *studio* equipment, you're buying *field* equipment as well. Every GPL unit provides unparalleled flexibility, light weight, easy handling, precise control. Let GPL engineer *your* station, from camera to antenna. Have *The Industry's Leading Line*—in *quality*, in *design*.

Camera Unit

Precision-built, lightweight, fast-handling. Push-button turret, remote iris control, remote focus and range selection. Easiest to service.

Camera Control Unit

Touch-identified controls. 8½" monitor tube. Split or single headphone intercom system. CRO views horizontal, vertical, and vertical sync block. Iris control.

Camera Power Unit

Rugged, dependable, compact. Matched to other units in GPL chain. Standard relay panels swing out for maintenance.

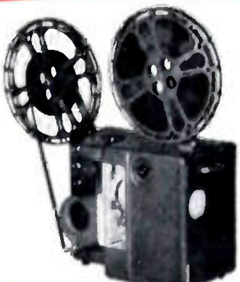
Synchronizing Generator

Affords maximum circuit reliability without operator adjustment. Binary counters and delay lines, stable master oscillator. Built-in power supply.

Complete TV Station Installations from Camera to Antenna

Video Switcher

Full studio flexibility anywhere. Control can view, preview, fade, dissolve, etc. Views any of 5 inputs, 2 remotes, outgoing line. Twin fade levers.

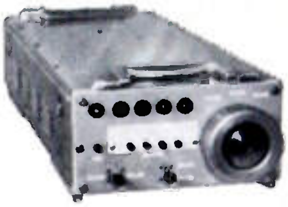


"3-2" Projector

Portable sync unit. No need for special phasing facilities. Projects rear-screen or "direct in." Ideal for remote origination of film. Relieves load on Telecine.

Professional TV Projector

Highest quality 16-mm projector designed specifically for TV. Delivers 100 foot-candles to tube. Sharp, steady pictures from 4000-foot film magazine.



Remote Control Box

Provides revolutionary remote control of camera focus, lens change, pan, tilt. Styled to match other components in the GPL TV line.

WRITE WIRE OR PHONE FOR DETAILS

General Precision Laboratory

INCORPORATED

PLEASANTVILLE NEW YORK

TV Camera Chains • TV Film Chains • TV Field and Studio Equipment • Theatre TV Equipment

JOHNSON *types* C and D CAPACITORS

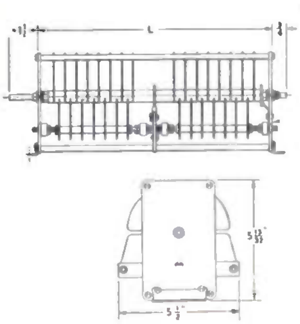
WE'RE PROUD of our C and D capacitors! They're rugged, reliable and simple. Their functional design permits rapid, accurate assembly which results in lower cost to the user. Materials are appropriate for the application and the finest available today. If you're building medium powered radio frequency equipment it will pay you to use JOHNSON C and D capacitors.

CONSTRUCTION

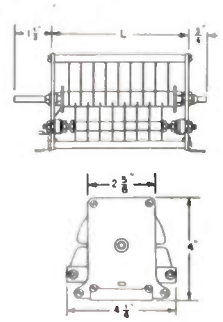
Heavy aluminum end frames, .051" plates and 5/16" tie rods assure extreme rigidity. Rotor contacts are laminated phosphor bronze. Dual models have center rotor contact for electrical symmetry. Low-loss Steatite insulators are located outside the most intense RF fields and used solely to support stator assemblies. Shafts are ¼" diameter, cadmium plated with ¾" rear extensions. Mounting brackets furnished for normal or inverted mounting. End frames drilled and tapped for panel mounting, special brackets or mounting of accessory components.

SPECIAL TYPES

Variations from standards such as special capacitances, ball bearings, dynamically balanced rotors, stainless steel shafts and right angle drive duals can be furnished in production quantities.

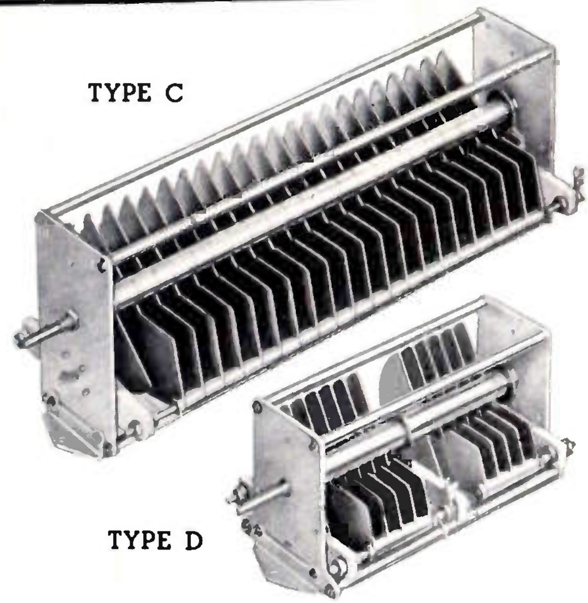


TYPE C DUAL



TYPE D SINGLE

Do you have our newest General Products Catalog 972? Most of this diverse line of electronic material can still be furnished with reasonable delivery. Here, perhaps, are the answers to some of your current production problems.



TYPE C

TYPE D

TYPE C SINGLE SECTION

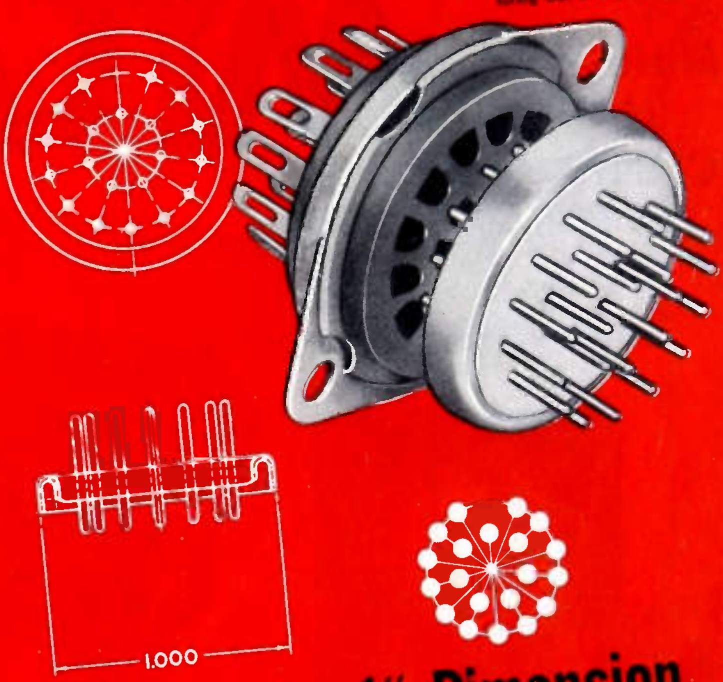
Cat. No.	Max. *Cap. per Sect.	Min. *Cap. per Sect.	Spacing	Plates Per Sec.	L
250C70	252	34	.175"	24	6
500C70	496	56	.175"	47	12
250C90	245	45	.250"	31	12
350C90	337	63	.250"	43	14
50C110	51	19	.350"	8	4
100C110	103	30	.350"	17	8
250C110	251	66	.350"	41	18
50C130	51	24	.500"	10	7
100C130	102	42	.500"	21	13
TYPE C DUAL SECTION					
200CD45	204	21	.125"	15	8
300CD45	290	26	.125"	21	10
200CD70	198	27	.175"	19	12
300CD70	305	37	.175"	29	16
150CD90	147	30	.250"	19	14
50CD110	50	18	.350"	8	10
100CD110	103	32	.350"	17	16
50CD130	51	24	.500"	10	14
TYPE D SINGLE SECTION					
100D35	99	14	.080"	8	2
250D35	252	24	.080"	20	4
500D35	496	36	.080"	39	6
100D45	104	19	.125"	12	4
150D45	146	23	.125"	17	4
50D70	51	17	.175"	7	2
70D70	72	18	.175"	11	4
100D70	98	23	.175"	15	4
150D70	151	31	.175"	23	6
250D70	244	45	.175"	37	10
350D70	351	62	.175"	53	13
50D90	53	20	.250"	10	4
70D90	73	25	.250"	14	5
100D90	99	30	.250"	19	7
150D90	149	43	.250"	29	10
TYPE D DUAL SECTION					
100DD35	95	13	.080"	8	4
150DD35	147	15	.080"	12	5
200DD35	202	19	.080"	16	7
300DD35	291	24	.080"	23	9
500DD35	496	38	.080"	39	13
150DD45	155	24	.125"	18	9
200DD45	198	27	.125"	23	12
50DD70	52	15	.175"	8	5
70DD70	72	17	.175"	11	7
100DD70	97	22	.175"	15	9
150DD70	151	31	.175"	23	13
50DD90	52	19	.250"	10	9
100DD90	97	30	.250"	19	14

*Nominal Values

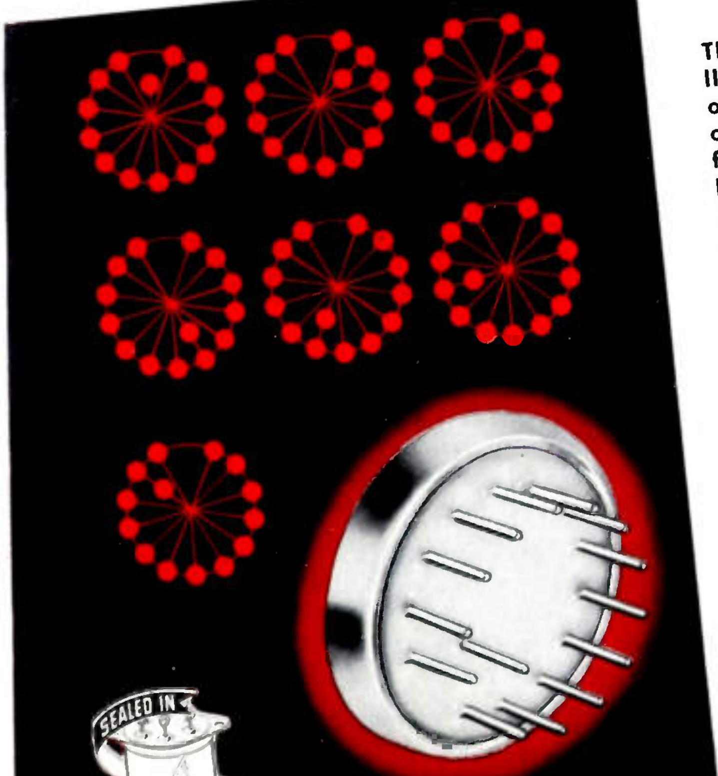
E. F. JOHNSON COMPANY WASECA, MINNESOTA

The socket shown below was made by the Cinch Manufacturing Co. of Chicago, Illinois.

Another Great *Hermetic* FIRST in MINIATURIZATION



☆ 20-Terminal, Plug-In Header In 1″ Dimension



The Electro-Seal Corporation of Des Plaines, Illinois, is an acknowledged leader in the field of hermetically sealed electronic components of exceptional quality. It was natural, therefore, that it should single out HERMETIC SEAL PRODUCTS CO. to develop a needed, polarized 20-terminal, plug-in header in a 1″ maximum dimension. It knew that only HERMETIC, with its vast experience, equipment and engineering staff, could design and develop such a plug . . . one that would be able to withstand the mass spectrometer tests to which it would be subjected for leaks and cracks. Each and every component is thoroughly tested in Electro-Seal's efforts to maintain the quality standard for which it has become famous.

● The 20-terminal, ceramic-metal plug has 7 terminals on the inside circle and 13 in the outer circle. It is also available for other applications as a 14-terminal plug-in with 7 different polarized positions as shown on the print.

Submit your own problems in this highly exacting field to our specialist-engineers. They are eager to be of help.

SEALED IN by HERMETIC HEADERS

Write for your copy of our new 32-page brochure, the most complete and informative presentation ever made on hermetic seals.

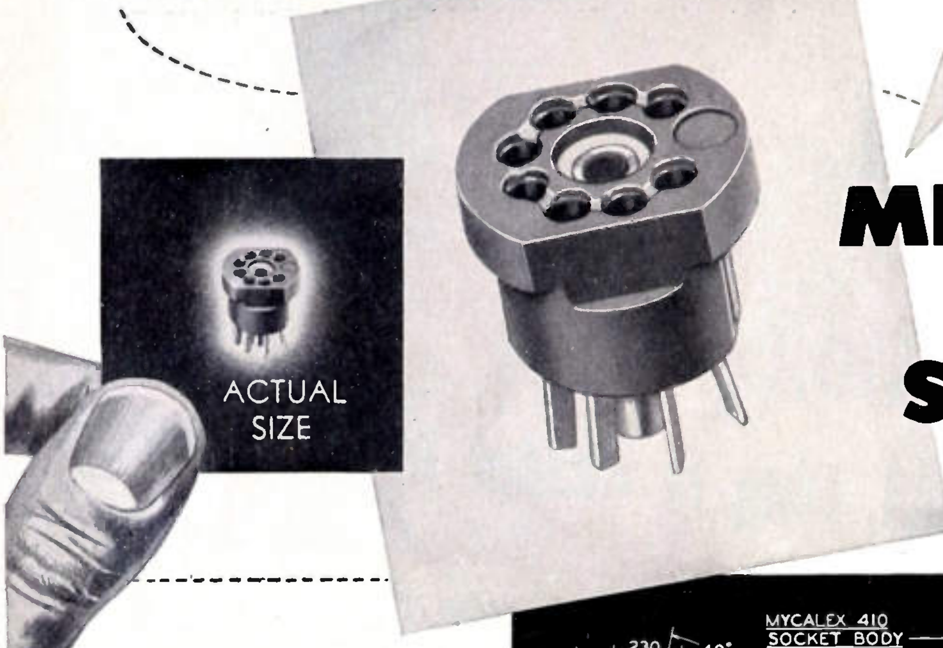
Hermetic Seal Products Co.

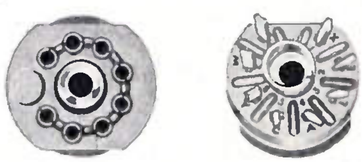
29 South Sixth Street • Newark 7, New Jersey

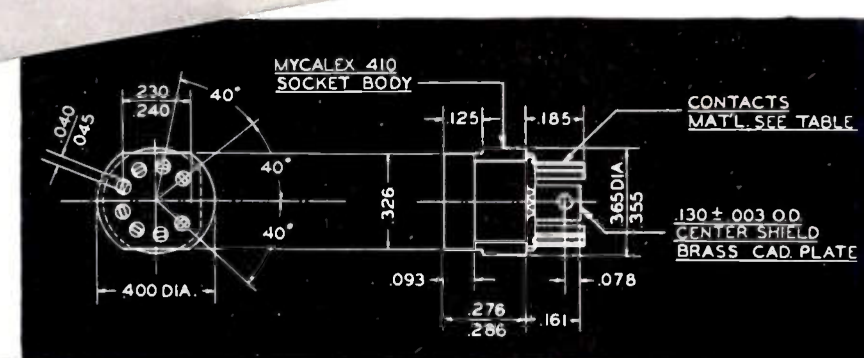
MINUTE IN SIZE... GIANTS IN PERFORMANCE!

MYCALEX 410

8-PIN SUB-MINIATURE TUBE SOCKETS







- LOW LOSS FACTOR OF .015 at 1 MC.
- INSULATION RESISTANCE (min.) 50,000 Megohms
- ARC RESISTANCE, ASTM SECS. 250+
- OPERATING TEMPERATURE 135°C (limitation due to contact metal)
- Q-value at 40°C—50% RH—not less than 1000
- Q-value at 40°C—90% RH min.—not less than 333

WRITE FOR DATA SHEETS

Mycalex 8-Pin Sub-Miniature Tube Sockets are fully described in the new Data Sheets. Other catalogs are available on Mycalex Insulation for every electronic or electrical application.

New MYCALEX 410 Sub-Miniature Tube Sockets are designed for use in electronic and electrical equipment where space is at a premium. Because they are extremely compact, these sockets offer a ready solution to numerous design problems involving spatial limitations. Installation is simple, mounting being accomplished without screws or rivets in shaped chassis holes.

Improved electrical performance and greater mechanical protection for the tube than are available with ordinary insulating materials are afforded by this socket through the use of MYCALEX 410 glass-bonded mica. MYCALEX 410 is rated Grade L-4B insulation under N.M.E.S. JAN-I-10. It offers superior electrical and mechanical properties in combination with practical cost per unit.

MYCALEX TUBE SOCKET CORPORATION
Under Exclusive License of
MYCALEX CORPORATION OF AMERICA
30 ROCKEFELLER PLAZA, NEW YORK 20, N. Y.

SINCE 1919
MYCALEX
THE INSULATOR
TRADE MARK REG. U. S. PAT. OFF.

MYCALEX CORPORATION OF AMERICA
Owners of 'MYCALEX' Patents and Trade-Marks
Executive Offices: 30 ROCKEFELLER PLAZA, NEW YORK 20 — Plant & General Offices: CLIFTON, N.J.

ARE YOU DESIGNING A TRANSFORMERLESS TV CHASSIS?

Additional 15-volt Bonus in B+ Voltage now possible with new G-E Germanium Power Rectifier

- **A B+ reserve that eliminates marginal operation** under low line conditions is now available to television circuit designers. General Electric's G-10, an entirely new rectifier of the junction type, has a forward resistance of *only 3 ohms*—considerably lower than that normally encountered with other type rectifiers.

- **Life tests** conducted on typical samples indicate that a life of 10,000 hours may be expected. Our application engineers are ready to demonstrate important advantages for your consideration.

- **Military applications**—Where extremely low forward resistance and high efficiency are necessary, these rectifiers are being accepted for use in military equipment. *General Electric Company, Electronics Park, Syracuse, New York.*

Specifications

Description and Maximum Ratings
TYPE G-10

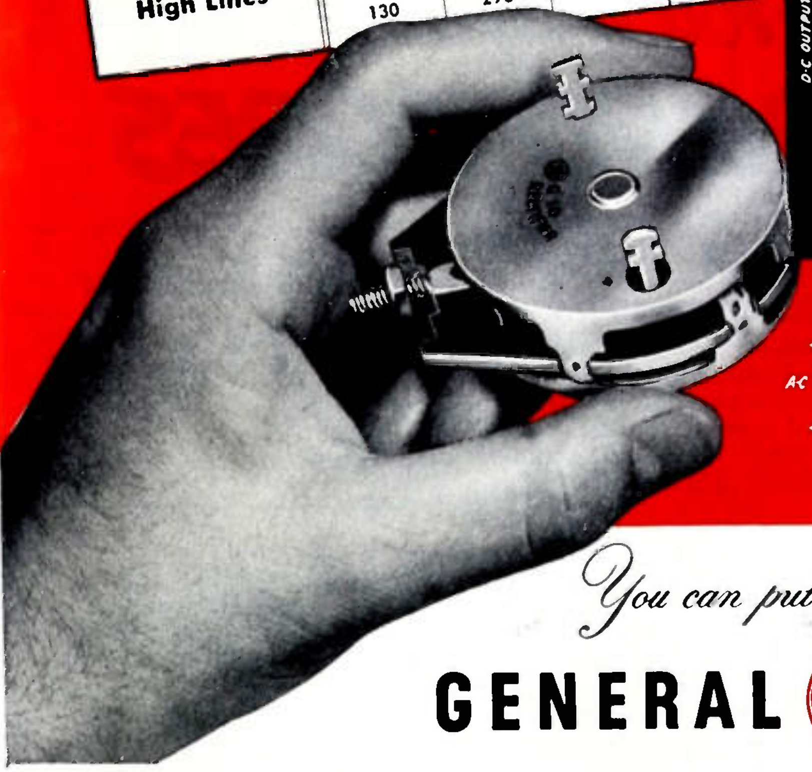
Ambient Temperature	40°C	55°C	65°C
RMS Input Voltage (Max.)	130	130	130 Volts
RMS Current (Max.)	1.2	1.2	.2 Amps
D-C Output Current (Max.)	400	350	50 Ma
D-C Surge Current (Max.)	25	20	2.5 Amps
Peak Forward Current (Max.)	3	3	.5 Amps
Peak Inverse Voltage (Max.)	400	400	400 Volts
Full Load Voltage Drop (Max.)	1.5	1.4	1.3 Volts
Operating Frequency (Max.)	50	50	50 Kc

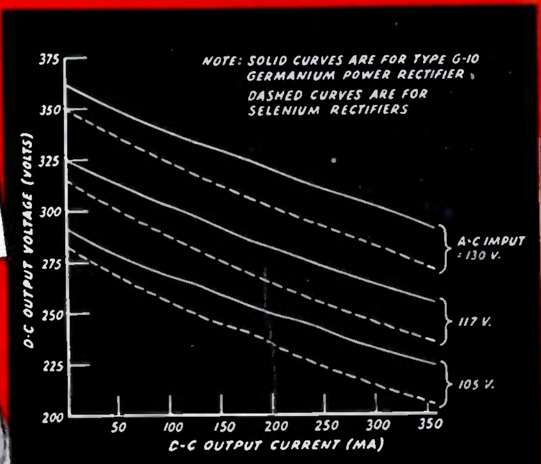
ALSO AVAILABLE

Single Rectifier Types		G-10A	G-10B	G-10C
RMS Input Voltage (Max.)	25°C	32	50	65 Volts
D-C Output Current (Max.)	25°C	200	200	200 Ma
Peak Inverse Voltage (Max.)	25°C	100	150	200 Volts

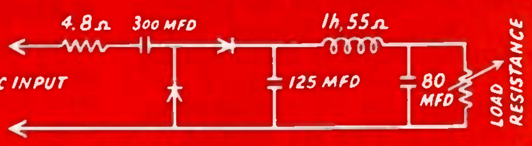
Typical B+ Voltages for Low, Nominal and High Lines

A-C Input Voltage	D-C Output Voltage $R_L = 900$	D-C Output Voltage $R_L = 1000$	D-C Output Voltage $R_L = 1100$
105	240	243	247
117	266	270	273
130	296	302	305





REGULATION CHARACTERISTICS



VOLTAGE DOUBLER POWER SUPPLY

You can put your confidence in—

GENERAL ELECTRIC

Help to Engineers

who are concerned with the future of their careers

ARE YOU IN A "DEAD END" JOB with no chance to move forward?

Would you like work that challenges your creative thinking and skills?

Is your present position limiting your opportunity for the complete expression of your talents in electronics?

Do you and your family worry about your career, or where you live now, or about security and your future?

If the answer is "yes" to one or more of these questions—then you should send for a *free* copy of RCA's new booklet CHALLENGE AND OPPORTUNITY, *The Role of the Engineer in RCA*.

This 36-page, illustrated booklet, just off the press, will show you the splendid opportunities offered by RCA to put your career on the upswing. See how, as part of the RCA team, daily contact with the best minds in various fields of electronics, and with world-renowned specialists will stimulate your creative thinking.

For graduate engineers who can see the challenge of the future, RCA offers opportunities for achievement and advancement that are legion. Send for a copy of CHALLENGE AND OPPORTUNITY, *The Role of the Engineer in RCA*. It is yours *free* for the asking.

To Qualified Engineers:

If you qualify for any of the positions listed below, write us for a personal interview—include a complete résumé of your education and experience. Write to address in coupon.

RADIO CORPORATION of AMERICA

POSITIONS OPEN IN THE FOLLOWING FIELDS:

- ***Television Development***
- ***Electron Tube Development***
- ***Transformer and Coil Design***
- ***Communications***
- ***Radar—Missile Guidance***
- ***Computer Development and Design***
- ***Navigational Aids***
- ***Technical Sales***
- ***Electronic Equipment Field Service***

MAIL COUPON NOW!

MR. ROBERT E. McQUISTON, Manager
Specialized Employment Division, Dept. 94C
Radio Corporation of America
30 Rockefeller Plaza, New York 20, N. Y.

Without obligation on my part, please send me a free copy of CHALLENGE AND OPPORTUNITY

Name________________

Address________________

City__________ State__________

List degree or degrees________________

New developments are essential in resistors, too!

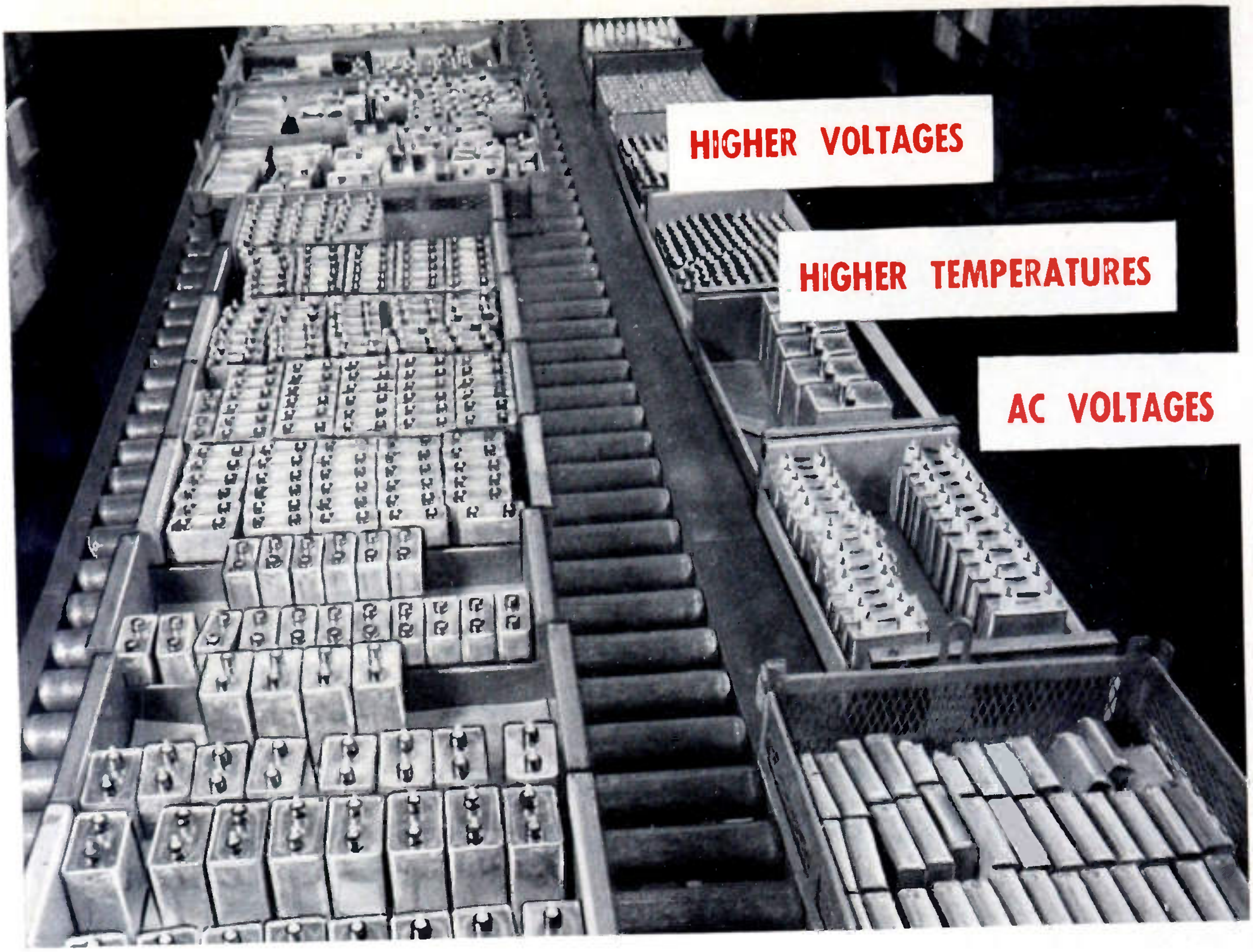
IRC LAUNCHES NEW BORON-CARBON RESISTOR (Type BOC)
LATEST DEVELOPMENT IN STABLE FILM-TYPE RESISTORS

- Reduces temperature-coefficient of conventional deposited carbon resistors . . .
- Provides high accuracy *and* long-time stability...
- Replaces high value wire wound precisions at savings in space and cost!

NO LONGER A LABORATORY ITEM. NOW FULLY AVAILABLE THROUGH IRC'S MASS PRODUCTION TECHNIQUES AND QUALITY CONTROL.

Here's a completely new tool for electronic and avionic engineers— one that's going to make possible higher stability circuits with smaller components. IRC's new Type BOC Boron-Carbon Resistor promises tremendous advantages in military electronic equipment such as gunfire control, radar, communications, telemetering, computing and service instruments. Heretofore strictly a laboratory item, Type BOC is now available to equipment manufacturers. Be sure *you* get full details.


OHMS
VOLTS
ZERO ADJ
A.C.V.
OHMS
MODEL 303



General Electric can show you how to make wider use of JAN-C-25 capacitors

From years of experience in manufacturing paper-dielectric capacitors, General Electric can show you how to make wider use of your JAN capacitors.

These capacitors are used in thousands of applications—primarily d-c at rated voltages and temperatures. However, most JAN units can be operated at other voltages and under widely varying conditions.

For example, actual life tests have shown that a General Electric 1 muf. CP 70 unit rated for a minimum life of 10,000 hours at 1000 v. d-c and 40 C or 700 v. d-c and 85 C, can also be used at:

Higher voltages—1380 v. d-c at 85 C for 500 hours.
1300 v. d-c at 85 C for 1000 hours.

Higher temperatures—105 at 525 v. d-c for 500 hours.

AC voltages—440 volts, 60 or 400 cycles
with normal JAN-C-25 derating.

General Electric has similar data for most of its JAN units, showing how each may be operated under a variety of conditions. For information on how these standard G-E capacitors may be applied in your circuits, consult your Apparatus Sales Office, or write to Specialty Capacitor Sales, General Electric Company, Hudson Falls, N.Y.

GENERAL ELECTRIC

407-107

LITTON INDUSTRIES NEWS

Litton Model 3900 Thermopile

LITTON THERMOPILE WITH STANDARD METER FORMS ACCURATE, LOW COST INDICATOR FOR SMALL DIFFERENTIAL TEMPERATURES

Engineers in increasing numbers are using Litton Model 3900 Thermopile in conjunction with microwave water loads to measure rf power, and in cooling systems to monitor temperature changes.

The Thermopile has 30 pairs of copper-constantan junctions, tapped at 10 and 20 pairs. Junctions protrude into a fluid flow channel milled in a plastic block to which water fittings are mechanically attached. The plastic block is encased in a cast aluminum housing. Binding posts are provided for electrical connection, and ¼" Uniflare fittings for water connection. Internal resistance is approximately 6 ohms.

With rf water loads using appropriate water flow, meter sensitivity and number of junctions, average powers from 10 watts to several kilowatts can be measured conveniently and accurately. For lower power levels, several thermopiles can be used in series.

The 30-junction thermopile generates approximately 1 millivolt per °C differential temperature. To determine water flow rate and indicating meter, the following formula is useful:

(P = power dissipated in watts; Q = water flow in gals. per minute; R = meter internal resistance in ohms; M = meter sensitivity in millivolts for full-scale deflection.)

For full-scale meter deflection, approximately:

$$250M\frac{(R+6)}{R} = \frac{P}{Q}$$

Also, to avoid excessive heat losses, differential temperature should not exceed 20°C, where for pure water

$$T = \frac{P}{246Q}$$

(T being temperature differential in °C.)

Because of stray losses in plumbing and the load, the system is best calibrated by direct dissipation of metered power in a water-cooled resistor in series with the water load.

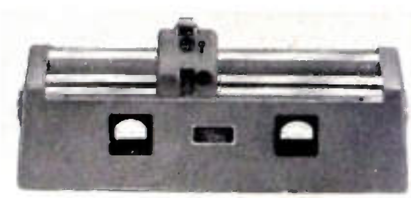
Time of response in minutes is determined by the volume of the system V in gallons divided by Q. (Time constant of thermopile is negligible.) For a typical installation of Litton Model 4000 U-Line, Model 4100 Water Load and Model 3900 Thermopile, operating at the kilowatt level, using a meter with M = 7 millivolts, R = 71 ohms, time of response is approximately 20 seconds. Litton Model 3900 Thermopile, price $75.

WATER LOAD

Litton Model 4100 Water Load is a termination for 1⅝", 50-ohm coaxial lines, and is particularly useful in high-power applications where power output must be accurately measured. The Load is conservatively rated at 2 kilowatts capacity, 950 to 3,000 mc/sec. VSWR is less than 1.2 over full range, less than 1.1 above 2,000 mc/sec. The equipment includes two adjustable-depth probes for sampling rf power. Model 3900 Thermopile is recommended for use with this load. Model 4100 Water Load, price $425.

U-LINE AND STUB COMBINATION

Litton Model 4000 U-Line offers convenience and accuracy in quickly determining VSWR in high- or low-power coaxial lines. The equipment transduces power from a 1⅝" coaxial line to a U-shaped configuration with a rigid central and outer conductor. A traveling probe moves on a precision carriage through the open end of the "U." A 500-millimeter scale with vernier indicates probe position.


Litton Model 4000 U-Line

Model 4000 U-Line offers continuous frequency coverage from 450 to 2,750 mc/sec. with insertion VSWR of less than 1.05. Teflon bead supports permit a CW power rating of 2 kilowatts. Mounting holes are provided for meters. Price $700.

STUB

For use with Model 4000 U-Line. Permits rapid insertion, variation of phase position and withdrawal of mismatch of known VSWR in the U-Line. Calibrated scale permits insertion of known VSWR up to 2.0, at frequencies 950 to 2,750 mc/sec. Thus, equipment may be used as a calibrated mismatch or matching device.

Insertion at any phase position is possible with relative phase readable on millimeter scale on the U-Line.

Model 4200 Stub is a metallically-loaded Teflon rod contoured to fit the U-Line. The stub is suspended from a carriage riding on the U-Line. Price, $100.

Data subject to change without notice. All prices f.o.b. San Carlos, Calif.

LITTON INDUSTRIES
SAN CARLOS, CALIFORNIA, U.S.A.

2438

DESIGNERS AND MANUFACTURERS of:
Glassworking Lathes and Accessories, Vertical Sealing Machines, Burner Equipment, Precision Spotwelders, Oil Vapor Vacuum Pumps, Glass Baking Ovens, Vacuum Tubes and Tube Components, Magnetrons, High Vacuum Molube Oil, Microwave Equipment.

HI-Q* SERVES NATIONAL DEFENSE

Wherever Electronics Guide Them Home

The amazing and intricate science of electronics has provided new eyes and ears to bring our airmen straight home from anywhere . . . to sight a target many horizons beyond the span of human vision. On land and sea, electronics likewise has become a vital keystone in national defense. And wherever you find electronics, you'll find **Hi-Q** . . . Small Ceramic Disk Capacitors, for example, of both the by-pass and temperature compensating types. Tubulars, perhaps . . . Plates or the new High Voltage units. And wherever you find **Hi-Q** you'll find unerring dependability, rigid adherence to specifications and tolerances, and long life. Whether your needs are for standard or specially designed components, **Hi-Q** engineering and production keenness can meet your most exacting requirements.

Specializing in ceramic capacitors, **Hi-Q** has developed a complete line of Temperature Compensating Disk Capacitors with a capacity range from 475 mmf to .3 mmf and standard tolerances of ±5%, 10% or 20%. For applications requiring a large gradient of capacity vs. temperature **Hi-Q** Extended Temperature Compensating Disk Capacitors are available. These together with **Hi-Q** By-pass Disk Capacitors give you one source of supply for all ceramic Disk type capacitors. Write for New Engineering Bulletin on Disks.

**Trade Mark Reg. U.S. Pat. Office*

Electrical Reactance Corp. **Hi-Q*** **OLEAN, NEW YORK**

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PLANTS: Olean, N. Y., Franklinville, N. Y., Jessup, Pa., Myrtle Beach, S. C.

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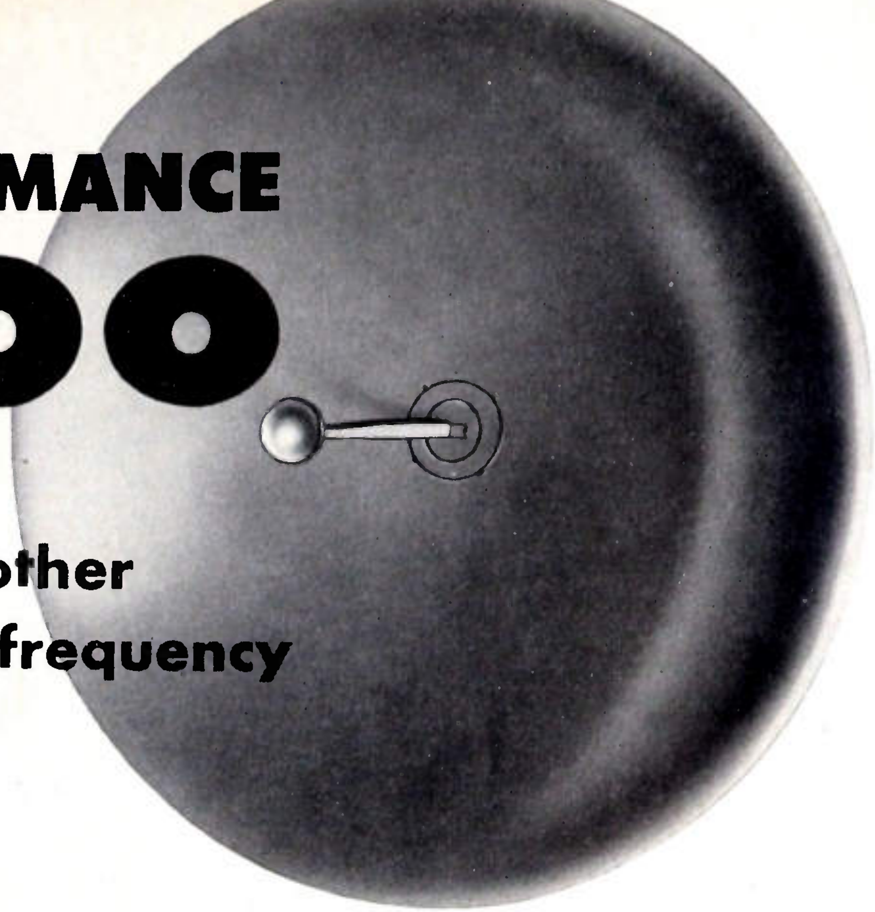
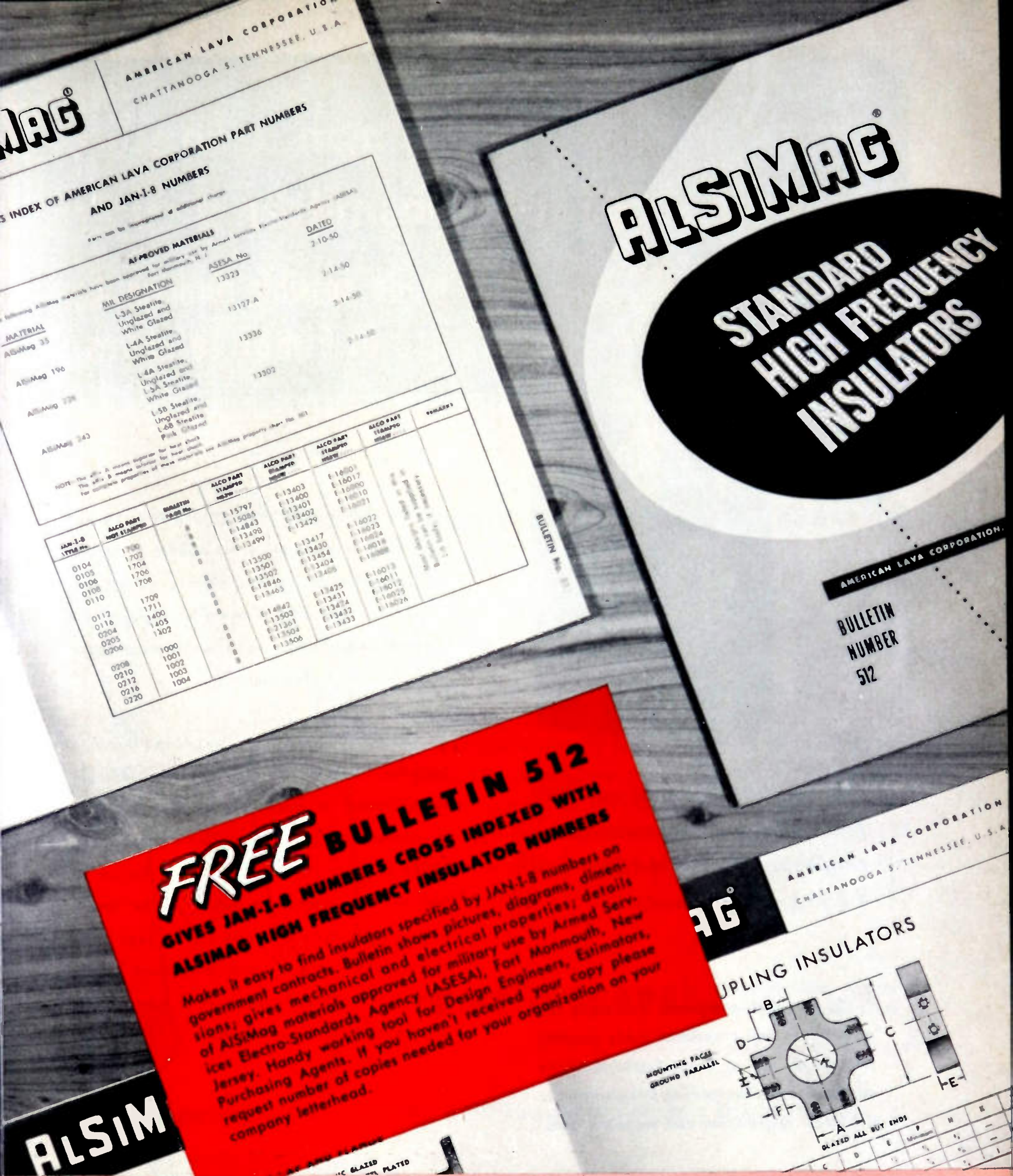


PARABOLIC ANTENNA COMPUTER
THE WORKSHOP ASSOCIATES, INC.



50TH YEAR OF *CERAMIC LEADERSHIP*

AMERICAN LAVA CORPORATION

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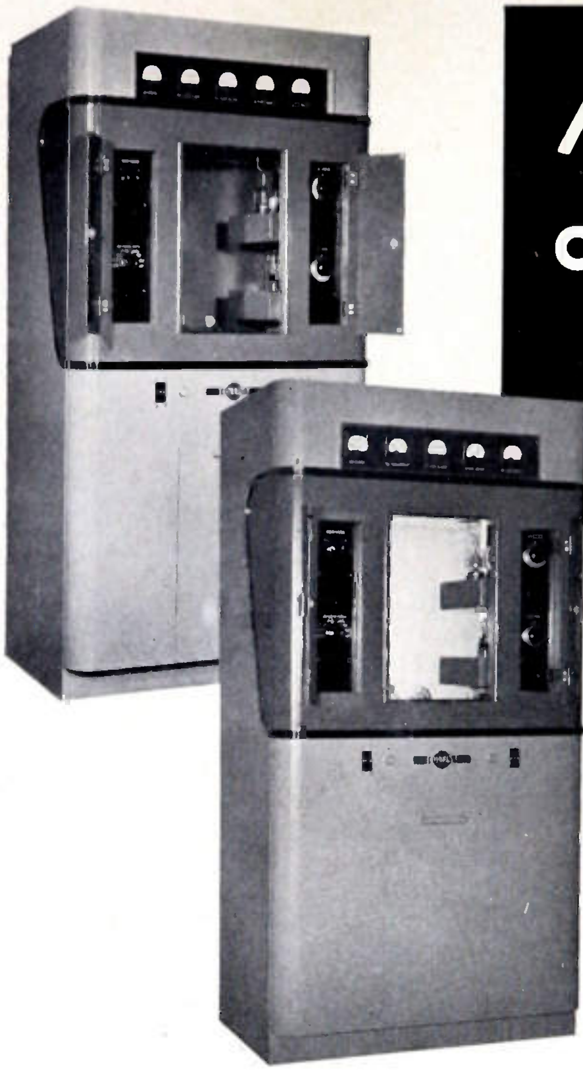
LOS ANGELES, 232 South Hill St., Mutual 9076 • CHICAGO, 228 North LaSalle St., Central 6-1721 • ST. LOUIS, 1123 Washington Ave., Garfield 4959



A triumph in the art of transmitter design..

made possible through the use of Eimac tetrodes.

Collins 300J 250-watt and 20V 1kw AM broadcast transmitters employing Eimac 4-125A and 4-250A power tetrodes.

The Collins 300J 250-watt and 20V 1kw AM broadcast transmitters are a tribute to the art of transmitter design. Performance, circuit simplicity and economy of operation highlight the many features Collins Radio has incorporated in these modern transmitters.

Through the use of high-gain, long-life Eimac tetrodes, Collins has achieved considerable simplification in circuits associated with the modulator and power amplifier stages. These highly efficient tetrodes also permit the use of low drain receiver-type tubes in the driver stages. Spare tube inventory can be kept small and representing a minimum investment. As an example; the 300J employs only 16 tubes of but 7 types in the entire transmitter.

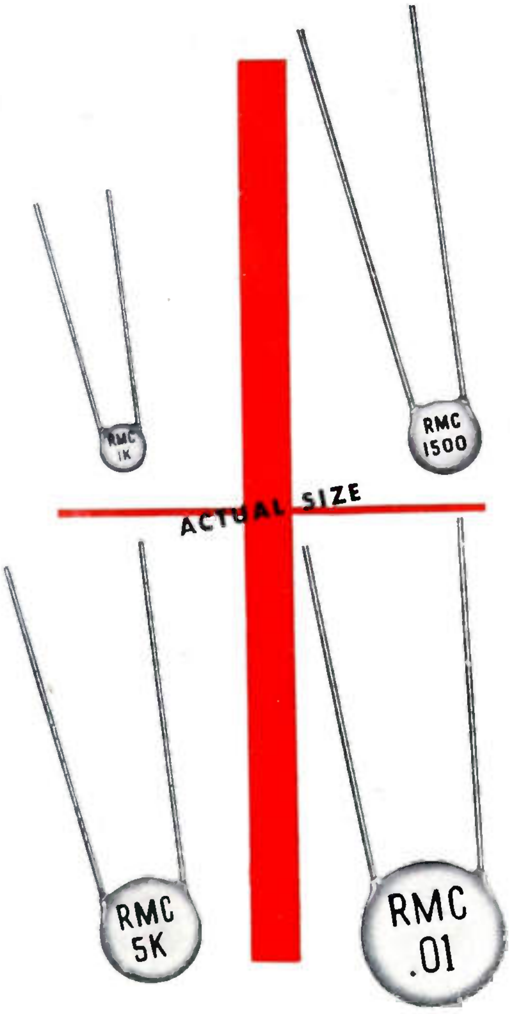
In your own equipment . . . enjoy the advantages and economies made possible through the use of Eimac tetrodes. Write our Application Engineering Department for the latest information and technical data.

EITEL-McCULLOUGH, INC.
San Bruno, California
Export Agents: Frazar & Hansen, 301 Clay St., San Francisco, California

Follow the Leaders to
Eimac TUBES
The Power for R-F

Another RMC First!

RMC "HEAVY DUTY" By-Pass DISCAPS



Modern Engineering Requires This

"HEAVY DUTY" CERAMIC CAPACITOR

The heavier ceramic dielectric element made by an *entirely new process* provides the necessary safety factor required for line to ground applications or any application where a steady high voltage condition may occur. Designed to withstand constant 1000 V.A.C. service.

It is wise to specify RMC "HEAVY DUTY" by-pass DISCAPS throughout the entire chassis because they *cost no more* than ordinary lighter constructed units.

Specify them too, for your own peace of mind, with the knowledge that they can "take it." And if you want proof—request samples.

"RMC DISCAPS" *The Right Way to Say Ceramic Condensers*

A New Development from the RMC Technical Ceramic Laboratories

DISCAP
CERAMIC
CONDENSERS

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GENERAL OFFICE: 3325 N. California Ave., Chicago 18, Ill.
FACTORIES AT CHICAGO, ILL. AND ATTICA, IND.
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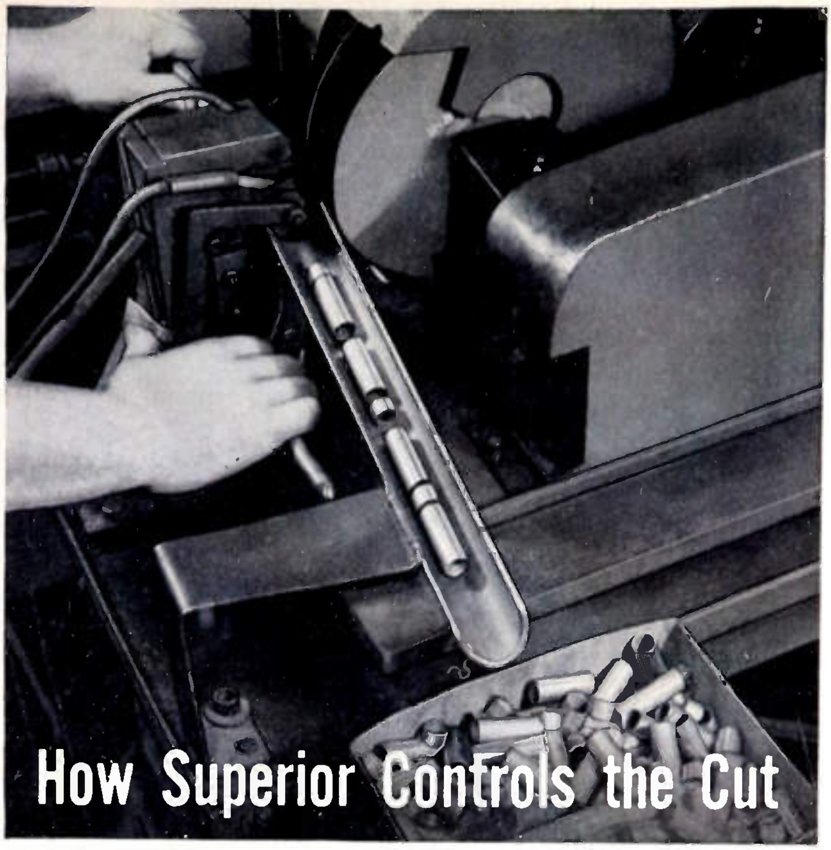


to give you better tubular parts

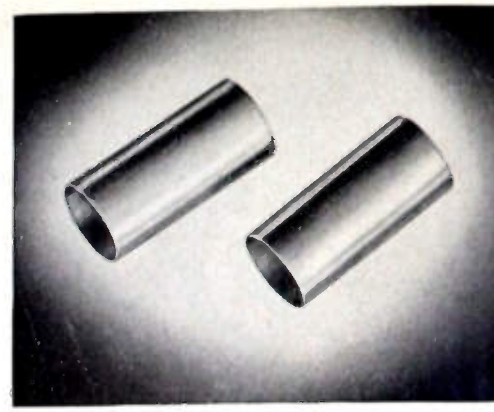
● Cutting tubing into exact lengths as the first step in the fabrication of tubular Electronic parts is a simple operation. Or is it?

Complications set in when the temper of the tubing is changed to meet customer specifications; when the tubing to be cut has a wall .010″ or thinner; when length tolerances as close as .010″ are required; when a 3° to 10° angle cut with a tolerance of ± ½° is called for; and when flattening, denting or other distortion must be prevented.

But overcoming complications in simple operations . . . and finding ways around them in other basically more difficult ones, is a specialty of the Electronics Division of Superior.

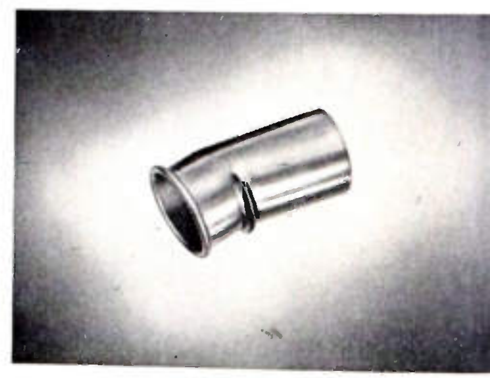
Our customers for Electronics parts have come to expect us to deliver the goods, exactly to specifications, whether standard production or complex experimental parts. What's more, they frequently ask us for suggestions about improvement on their designs and specifications . . . and they get them.

There is nothing unusual about all this—it's our job and we know how to do it. If you are a manufacturer or experimenter in the Electronics Industry and you need a tubular part that presents a problem, tell us about it. We'll probably be able to help and will gladly do so. Write The Superior Tube Company, 2506 Germantown Ave., Norristown, Pennsylvania.



Cutting and Tumbling. Cutting machines and jigs of many types and sizes are combined with extensive tumbling equipment to permit fast accurate production of quantities of parts at Superior.

Fabrication: Parts can be readily rolled at either or both ends, flared, flanged, expanded, or beaded (embossed) as required. The anode above is one of many such parts we produce at high speed and low cost.



The Finished Part. Final stage in the fabrication of the part shown above at three stages of production is a bend nicely controlled for both precise angle and freedom from other, unwanted distortion.

This Belongs in Your Reference File
. . . Send for It Today.

NICKEL ALLOYS FOR OXIDE-COATED CATHODES: This reprint describes the manufacturing of the cathode sleeve from the refining of the base metal. Includes the action of the small percentage impurities upon the vapor pressure, sublimation rate of the nickel base; also future trends of cathode materials are evaluated.

Superior

THE BIG E IN SMALL TUBING

All analyses .010″ to ⅝″ O.D.
Certain analyses (.035″ max. wall) Up to 1⅛″ O.D.

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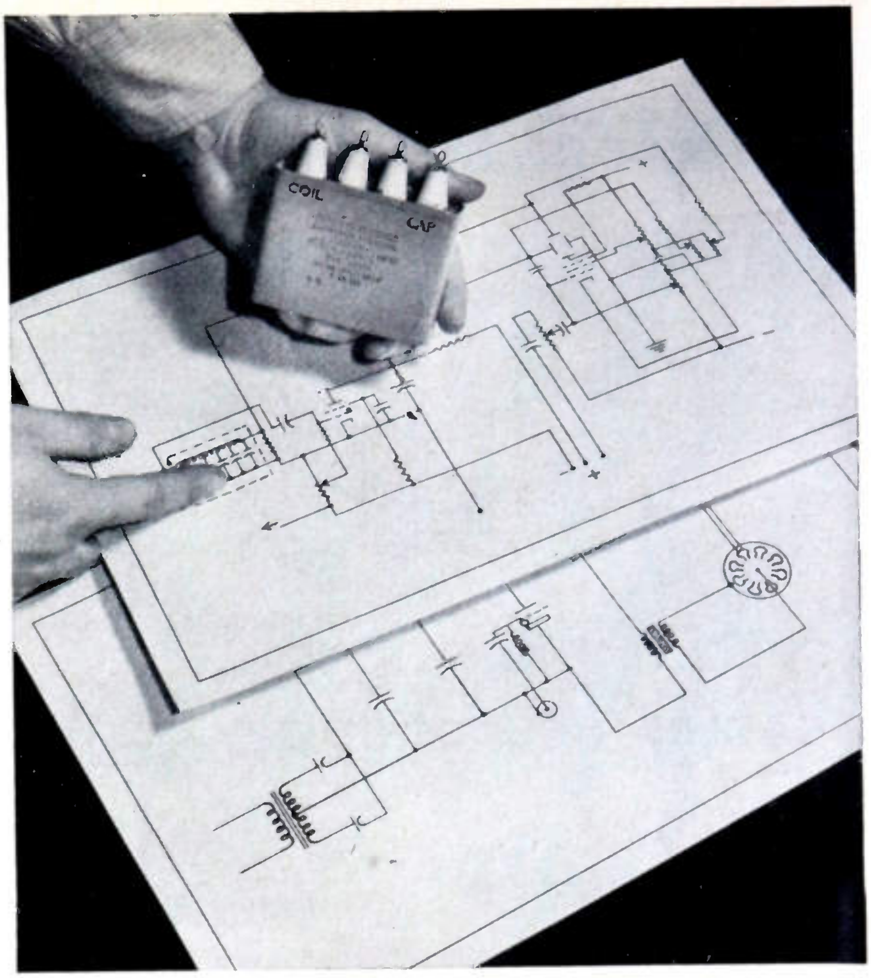
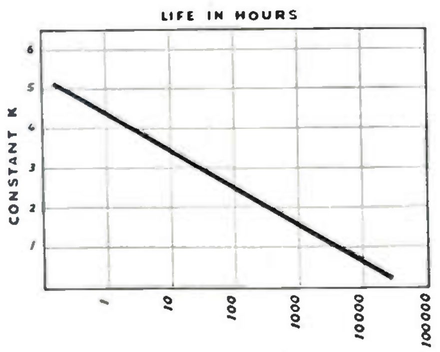
Got a really tough capacitor network problem for us?



...let our network designers help you solve it!



This curve results from actual life tests of capacitor networks under pulsing conditions in a radar modulator. Service life, for varying values of "K", starts at 2 tenths of one hour and stretches out to something over 40,000 hours (more than 4½ years of continuous operation at full-rated voltages.)

It's just one example of actual performance data G-E engineers use in designing capacitor networks for the job they must do.

Whether your problem deals with guided missiles—aircraft—land or sea radar equipments, General Electric application and design engineers can help you solve it. We've designed and built capacitor networks for every type of pulse radar equipment since the inception of radar.

Take service life for example. You can specify a service life of 10,000 hours—or just 60 seconds. And we'll deliver pulse networks to match your requirements. Here's why:

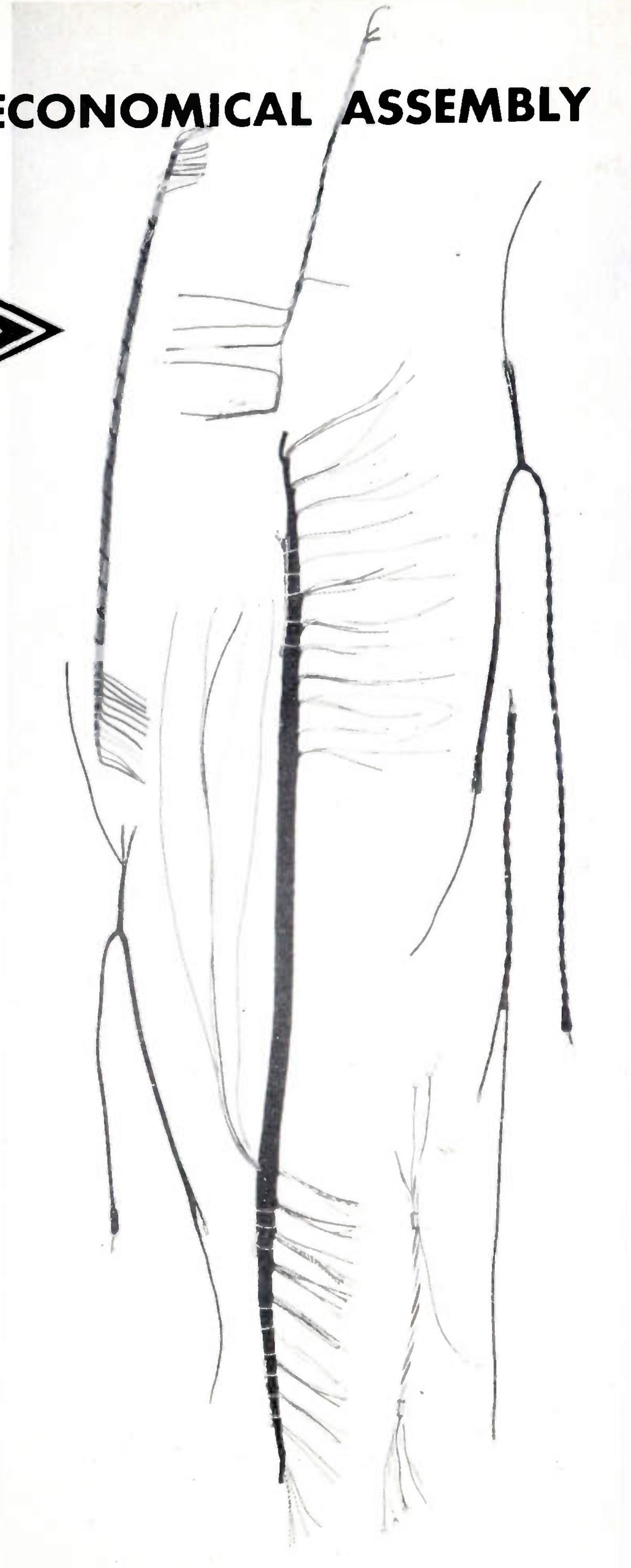
> Since 1944 General Electric has been running continuous life tests on many types of networks. We've established life limitations, under varying conditions of temperature and voltage, for all types of dielectrics, bushings, materials for coil forms and treating processes.

Let us use this store of information and experience to solve your capacitor network problems. Your inquiry addressed to your nearest Apparatus Sales Office, or to Capacitor Sales Division, General Electric Company, Hudson Falls, N. Y. will receive prompt attention.

General Electric Company, Schenectady 5, New York.

GENERAL ELECTRIC

407-308



Industrial Engineering Notes[1]

Mobilization

President Truman has signed an executive order implementing the provisions of the Electromagnetic Radiation Control Act. Under the order the **Federal Communications Commission is delegated authority to control all civilian stations in event of attack** while the head of each government agency operating station is given the same authority in his field. A station is defined as any device capable of emitting electromagnetic radiations between 10 kc and 100,000 meg. suitable as a navigational aid beyond five miles. The order also authorizes the FCC and other government agencies involved "to insure appropriate rules, regulations, orders, and instructions, and to take such other action as may be necessary, to assure the timely and effective operation of the plans and for carrying out their respective functions hereunder, and are authorized to require full compliance with their respective plans." . . . The **pinch on materials for consumer durable goods, including radios and television, will continue into 1953**, Defense Mobilizer Charles Wilson announced in his fourth quarterly report to the President. "The requirements of the military and the atomic energy programs for most materials will either continue at the same levels throughout 1952 or rise slightly," he said. "The production of steel and aluminum should rise somewhat during the year, but the main results of the expansion program will not be felt until late in 1952 and in 1953. Any significant expansion of our copper supplies cannot be expected until even later." Citing some of the problems being encountered in the production of new military equipment, Mr. Wilson pointed out that a B-47 requires 40 miles of wiring compared to 10 miles for the B-29, and that a B-47 contains over 1,500 electronic tubes."

RTMA Activities

RTMA carrying out a program of increased activities and services authorized by the Board of Directors and inaugurated by President Glen McDaniel, **has moved its headquarters office to larger quarters in the Wyatt Building**, at 777 Fourteenth St. N.W., Washington, D. C. President McDaniel, general manager James D. Secrest, and the RTMA headquarters staff will have offices in Suite 800 in the new building. The new RTMA telephone number is NAtional 3902. . . . President Glen McDaniel **has appointed Albert Coumont, formerly sales manager, Electronics Section, International General Electric Company, Inc., as service manager of RTMA.** Mr. Coumont succeeded E. W. Merriam, who served as the first RTMA service manager on a temporary basis. Mr. Coumont has had a diversified experience in the radio and television field and has been with General Electric since 1935. Mr. Coumont's entire business career has been spent in the radio and television

(Continued on page 38A)

NEW

marion "tamper-proof" hermetically sealed running time meter

REGISTERS OPERATING TIME OF MACHINE TOOLS, ELECTRONIC, ELECTRICAL OR GENERAL INDUSTRIAL EQUIPMENT

SPECIFICATIONS

- Registers in 1/10 hour steps to 9999.9 or hour steps to 99999
- Hermetically sealed, glass to metal
- Moisture-proof, dust-proof
- Drawn steel case provides magnetic shielding
- Self-starting synchronous motor
- Moderately priced

Marion's new Running Time Meter is absolutely tamper-proof because it is sealed in a drawn steel case. Designed for a wide range of operating temperatures, it is also ideal for use in hazardous atmospheres. The easy-to-read dial is viewed through tempered glass crystal which is fused directly to the case.

Powered by a durable self-starting synchronous motor, available for 110-125, 220-250 volts . . . 50 or 60 cycle A. C. . . . the Marion Running Time Meter occupies no more panel space than an ordinary 3½" meter.

Demands of our national mobilization program come first, of course, but we will gladly supply further information and serve you to the best of our ability.

MARION ELECTRICAL INSTRUMENT CO., 401 CANAL ST., MANCHESTER, N. H.

marion meters

MANUFACTURERS OF MARION Ruggedized PANEL METERS

Industrial Engineering Notes

(Continued from page 36A)

field, beginning in 1932. His experience includes the operation of his own home radio repair company and an active participation in the home radio service divisions of manufacturing companies. One of the first tasks to be undertaken by Mr. Coumont will be to promote training courses for service technicians in the nation's trade and vocational schools. A recommended agenda for a training course has been prepared by the RTMA Service Committee, under Chairman R. J. Yeranko, and has been distributed among trade and vocational schools.

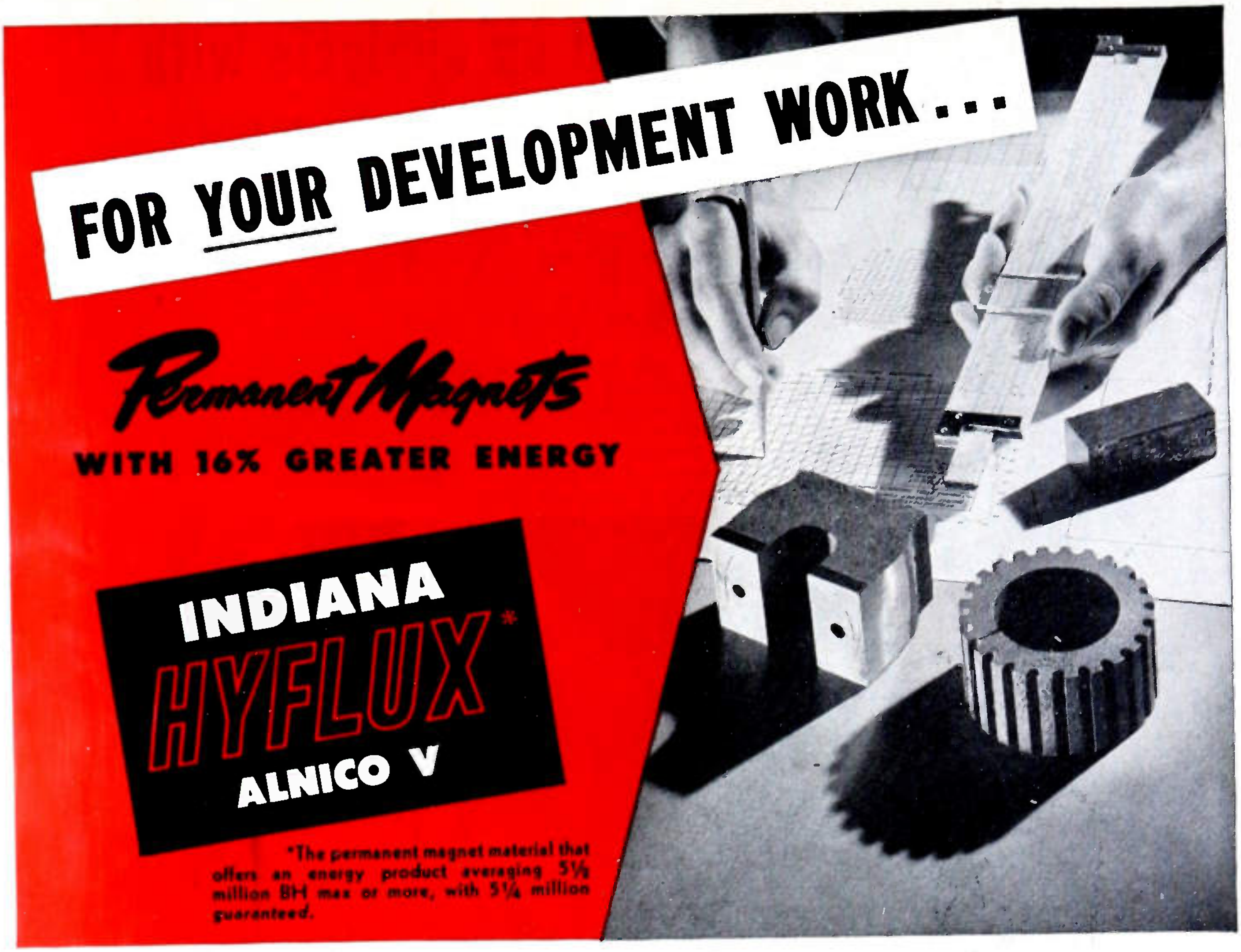
Industry Statistics

Production of radio and television receivers in November, 1951, totalling 747,914 and 415,332 sets, respectively, represented a decrease of 40 per cent under the corresponding month of 1950, according to estimates compiled for RTMA. RTMA's estimates showed a drop of 38 per cent in radio receiver production and 44 per cent in the TV set output. . . . Total revenues of the combined AM, FM, and TV broadcast services passed the half-billion dollar mark in 1950 for the first time, the FCC announced recently. Total revenues were $550.4 million, an increase of 22 per cent above 1949. The 1950 revenues of the aural broadcast stations and networks were reported at $444.5 million, or seven per cent over the preceding year, while the 1950 TV revenues of $106.9 million were more than triple that of 1949. Total revenues consist of income derived from the sale of time, talent, and program material to advertisers. . . . Sales of television picture tubes to receiver manufacturers totaled 4,062,375 units valued at $97,937,583 in the first 11 months of 1951, according to reports of RTMA member companies. November, 1951, sales to manufacturers amounted to 460,566 units values at $9,696,247 compared with 455,636 tubes valued at $9,388,382 in October. Picture tubes with rectangular screens and 16 and 17 inches in size represented 61 per cent of the sales in November. Tubes 18 inches and larger amounted to 35 per cent of the month's sales. All other tube sizes accounted for the remainder.

Research

The establishment of two new Signal Corps Engineering Laboratories Field Stations has been announced. They will be located at Yuma, Ariz., and Milwaukee, Wis. The Yuma field station will provide meteorological service for the Yuma Test Station. In Milwaukee the field station will take over the vehicular suppression and installation responsibilities of the Signal Corps Engineering Laboratories. . . . The Office of Technical Services, has announced that it has available a survey by Navy researchers on the theories of luminescence. The report provides a fairly ex-

(Continued on page 66A)



Whether your problem is new design or product improvement, take advantage of the greater energy product INDIANA HYFLUX Alnico V offers!

These exclusive, new, super strength permanent magnets mean lower production costs, more compact design and higher efficiency for your products.

What's more, INDIANA HYFLUX — with its 16% greater energy product — costs not a penny more than regular Alnico V!

Here's still another bonus you'll enjoy! THE INDIANA STEEL PRODUCTS COMPANY, world's largest producer of permanent magnets, offers free of charge its wealth of experience and "know-how" that has developed more than 30,000 permanent magnet applications.

Let INDIANA engineers help you with your design problems. They can supply — out of stock — many types and sizes of INDIANA HYFLUX Alnico V for your experiments, can suggest those best suited to your product.

INDIANA is the only manufacturer furnishing all commercial grades of permanent magnet alloys. You have a choice of cast, sintered, formed or ductile materials.

Why delay — write or phone INDIANA today. Ask for Catalog No. 11G-3 that describes stock experimental magnets.

INDIANA PERMANENT MAGNETS

THE INDIANA STEEL PRODUCTS COMPANY

VALPARAISO, INDIANA • • • Sales Offices Coast to Coast

SPECIALISTS IN "PACKAGED ENERGY" SINCE 1908

Speed up analysis with these Brush instruments

Direct-coupled Amplifier
Model BL-932

AMPLIFIES VERY LOW VOLTAGES. The Brush Direct-coupled Amplifier features high sensitivity and low drift. When used in conjunction with the Brush Magnetic Oscillograph, it gives one chart millimeter deflection per millivolt input. Design features reduce effects of power line fluctuation. Zero signal drift not more than one chart millimeter per hour. Frequency response essentially uniform from d-c to 100 cycles.

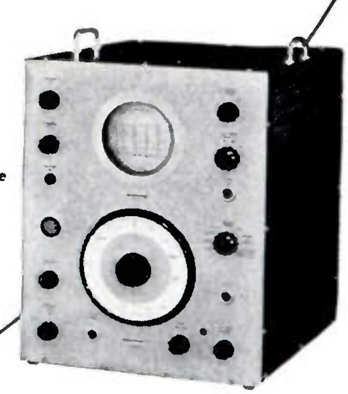
When used with the Brush Magnetic Oscillograph, the Amplifier can be used to record phenomena previously requiring the use of complicated intermediate equipment. Analysis of static or dynamic conditions involving either high or low signal strength is simplified and speeded with this equipment. Below, it is shown recording time constants of a reactor to provide a saturation curve.

PROVIDES IMMEDIATE RECORDING. The Brush Magnetic Oscillograph, used with the proper Brush Amplifier, makes a direct chart recording of physical phenomena which is immediately available. Either direct inking or electric stylus models available. Gear shift provides chart speeds of 5, 25, and 125 mm per second. An auxiliary chart drive is available for speeds of 50, 250, and 1250 mm per hour. Accessory equipment provides event markers where an accurate time base is required, or where it is desirable to correlate events. Photo shows two-channel model for recording of two phenomena simultaneously.

Direct-writing Two-Channel
Magnetic Oscillograph Model BL-202

CHECKS FREQUENCY RESPONSE QUICKLY. The Frequency Response Tracer permits visual examination of frequency response characteristics of radio receivers, amplifiers, transmission lines, filters. Electro-acoustic investigation of loudspeakers, microphones, and telephones can be made. Frequency range is 20 to 20,000 cycles, logarithmic scale. Continuous motor drive scans entire frequency range in 8 seconds.

Write for free copy of Bulletin 618 giving details on these Brush instruments. The Brush Development Company, Dept. F-24, 3405 Perkins Ave., Cleveland 14, Ohio. In Canada: A. C. Wickman Limited, Box 9, Station N, Toronto.



Frequency Response
Tracer Model
BL-4703

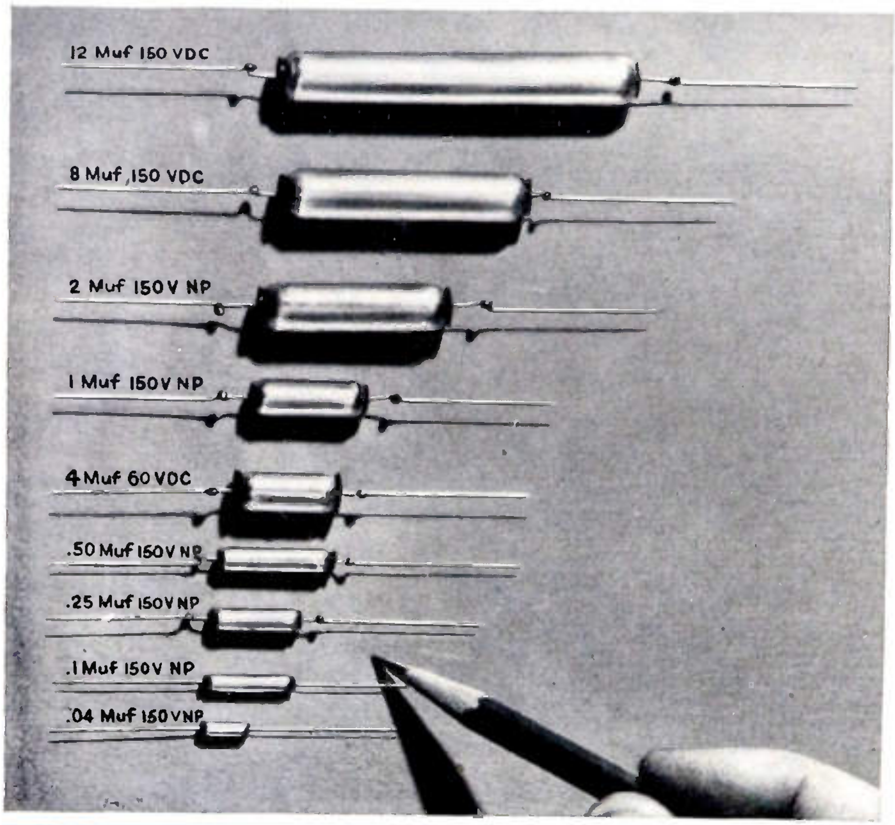
PUT IT IN WRITING WITH A BRUSH RECORDING ANALYZER

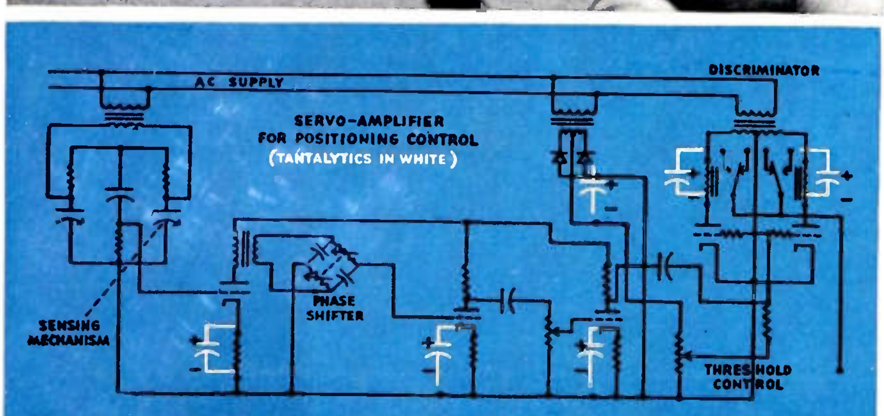
THE Brush DEVELOPMENT CO.

PIEZOELECTRIC CRYSTALS AND CERAMICS • MAGNETIC RECORDING
ACOUSTIC DEVICES • ULTRASONICS • INDUSTRIAL & RESEARCH INSTRUMENTS

DESIGNER'S

FOR SMALL SIZE, SUPERIOR PERFORMANCE IT'S G-E TANTALYTIC CAPACITORS





NEW tantalum-electrolyte units offer excellent low-temperature properties

Superior performance and large capacitance per unit volume make new General Electric Tantalytic capacitors valuable wherever miniaturization is a "must." Designed for low-voltage, direct-current applications, these capacitors excel in low-temperature properties and shock resistance.

Other advantages: Long shelf life • Exceedingly low leakage current • Hermetic sealing • Good stability • Chemically-neutral electrolyte

Operating temperatures range from −55C to +85C, ratings from .02 muf to 12 muf at 150 volts d-c. For further data, send coupon for Bulletin GEA-5753. For specific applications, list temperature range, leakage resistance values, and operating voltage and write *Capacitor Sales Division, General Electric Co., Hudson Falls, N. Y.*

For example: on this gun control system—

Design specifications for the circuit of a gun control servo-amplifier system required capacitors with great stability over a wide temperature range. Airborne equipment was involved, so size and weight were also extremely important. G-E Application Engineers were called in while the design was still on the board. Tantalytic capacitors were recommended because they are small, light, chemically stable. Result: a finished design that meets every requirement.

GENERAL ELECTRIC

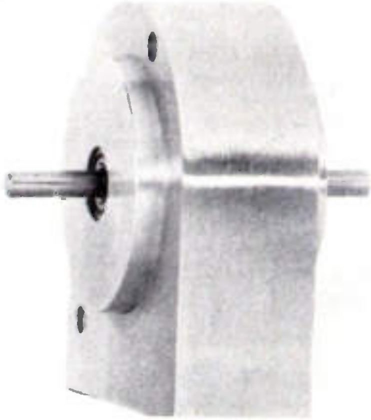
667-19

News—New Products

These manufacturers have invited PROCEEDINGS readers to write for literature and further technical information. Please mention your I.R.E. affiliation.

Gear Trains

Bowmar Instrument Corp., 4214 Leo Rd., Fort Wayne, Ind., announces a new line of precision gear trains for instrument applications. These gear trains are available in ratios up to 15,000:1 in the same general case dimensions. The shaft height from the mounting surface and shaft diameter are standard for use with conventional laboratory servomechanisms, breadboards, and components.



For applications where various reductions are required at different times, the gear train is available with a modified design to incorporate a variety of internal gearing arrangements. Various gear clusters for obtaining a multitude of ratios using the same housing can be supplied.

Precision-hobbed gears and bearings are used exclusively, with the result that maximum efficiency, uniform torque, and minimum backlash are obtained. The mounting faces may be made to hanger all common servo motors, synchros, potentiometers and other electromechanical components.

Signal Generators

A new signal generator, named the Decalator, is announced by **Decade Instrument Co.,** Caldwell, N. J. It has been developed for a range of from 10 kc to 10 mc, and consists, of a series of decade-switched oscillators. Among its features are direct readings for 9,000 separate steps of frequency, all frequency settings with decade-switching, ±2 cps at all frequencies, high accuracy, and ±0.05 per cent, at maximum frequency.

(Continued on page 50A)

Klystrons

AT WORK IN THE LABORATORY

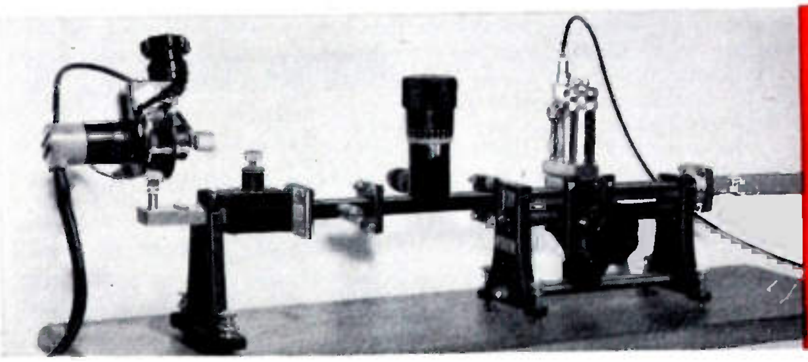
Sperry Klystron tubes are doing heavy duty in the labs where a practical source of continuous microwave energy is needed for general test and measuring work. A complete line of 2K tubes is available for bench oscillator use from 2660 to 10,300 mc.

Stemming from its sponsorship of the development of the klystron in 1939, Sperry has had many years' experience in the manufacture of these tubes. Besides the 2K-series for laboratory use, other Sperry Klystrons include transmitting tubes for microwave relays, radars (both pulsed and cw), radar beacons, aeronautical navigation (DME and ILS), and radio communication systems. Other Sperry Klystrons are used as local oscillators in radar and microwave communication receivers. Klystron multiplier tubes are used in frequency standards and for other applications where crystal control at microwave frequencies is desired.

Sperry's pioneering in microwave measuring techniques has resulted in a complete line of Microline* instruments which includes every type of device essential to precision measurement, in the entire microwave field.

Our Special Electronics Department will be happy to supply you with complete details on Klystrons and Microline equipment.

*T. M. REG. U. S. PAT. OFF.



SPERRY 2K SERIES

	TUNING RANGE	POWER OUTPUT
2K 41 (WITH TUNER)	2660 -3310 mc	250 mw
2K 42	3300 -4200 mc	250 mw
2K 43	4200 -5700 mc	250 mw
2K 44	5700 -7500 mc	250 mw
2K 39	7500 -10,300 mc	250 mw

SPERRY GYROSCOPE COMPANY

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News—New Products

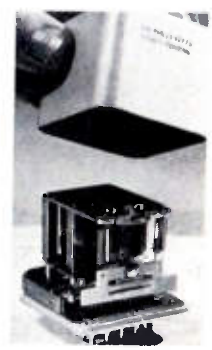
These manufacturers have invited PROCEEDINGS readers to write for literature and further technical information. Please mention your I.R.E. affiliation.

(Continued from page 48A)

Another Decalator, with a frequency range of 10 kc to 100 kc, known as Model #10-100, is now available at $795.00 F.O.B. Caldwell, N. J. Two other models are to be announced. One will be known as Model #100-1, covering 100 kc to 1 mc. The other, Model #1-10, will cover 1 mc to 10 mc. Prices for these last two models on request.

Regulator

A compact, universal, electric-circuit controller with large power-handling capacity is now being produced by **Electric Regulator Corp.**, South Norwalk, Conn. This is a device for converting low-power-level signals into control signals of a much higher power level.



The new plug-in unit, known as Regohm size 3, is a control which can be utilized in voltage, current or speed regulation, and can be adapted to servo systems. Compact design and unusual characteristics make the new Regohm adaptable to rotating machinery, line and load regulation, rectification, battery charging, servo mechanisms, saturable reactors, and close differential relays.

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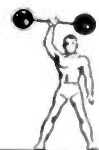
Carbon Microphone

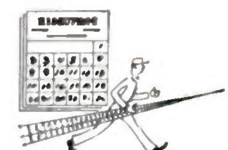
A newly developed single-button carbon microphone is the latest addition to the microphone line of the **Astatic Corp.**, Conneat, Ohio. Ideal response for maximum clarity of speech, 100 to 4,500 cps range, is claimed for the new unit, which has been designated Model 11M5. Sensitivity is rated at one volt for 100 microbar

(Continued on page 58A)

Truscon Engineers have the answer...

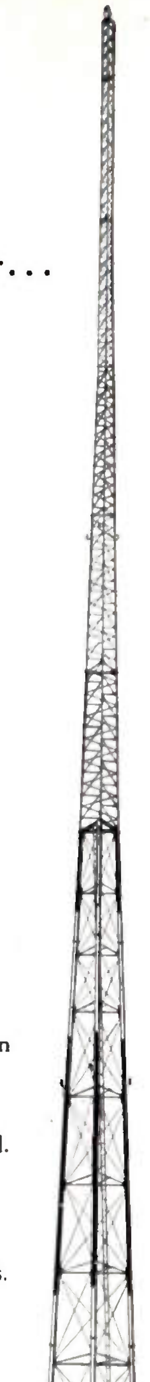


HOW STRONG



HOW SOON

AM-FM-TV-MICROWAVE



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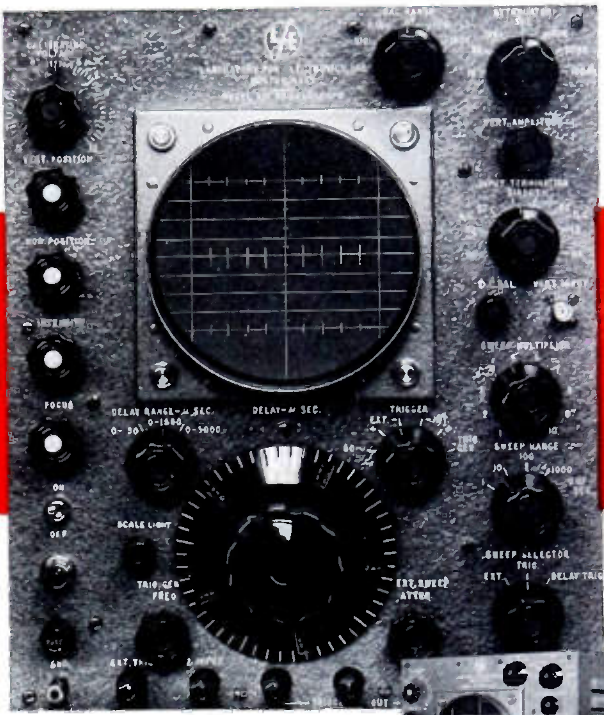

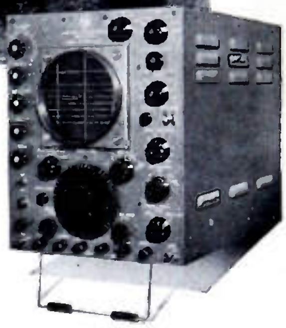

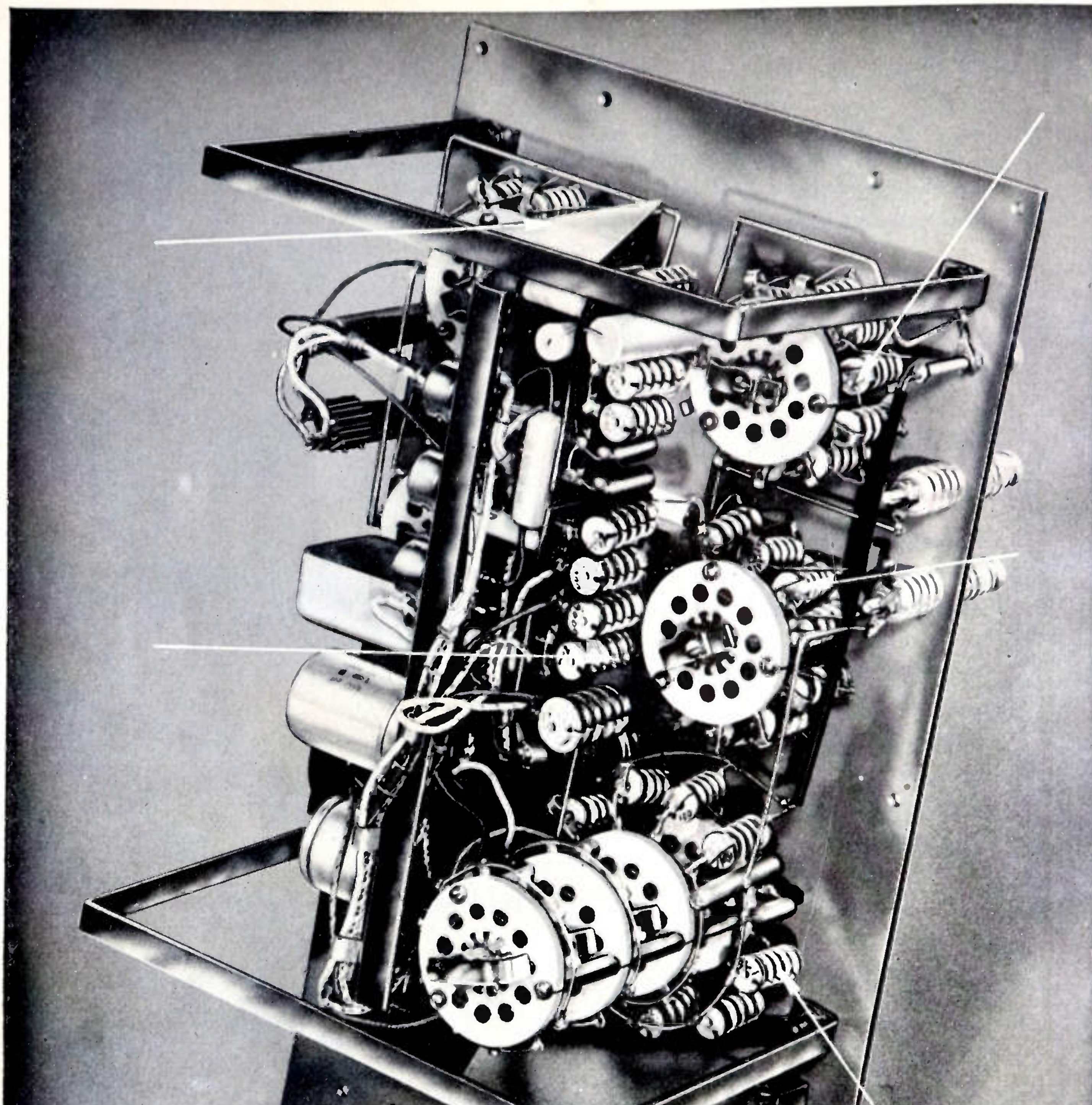
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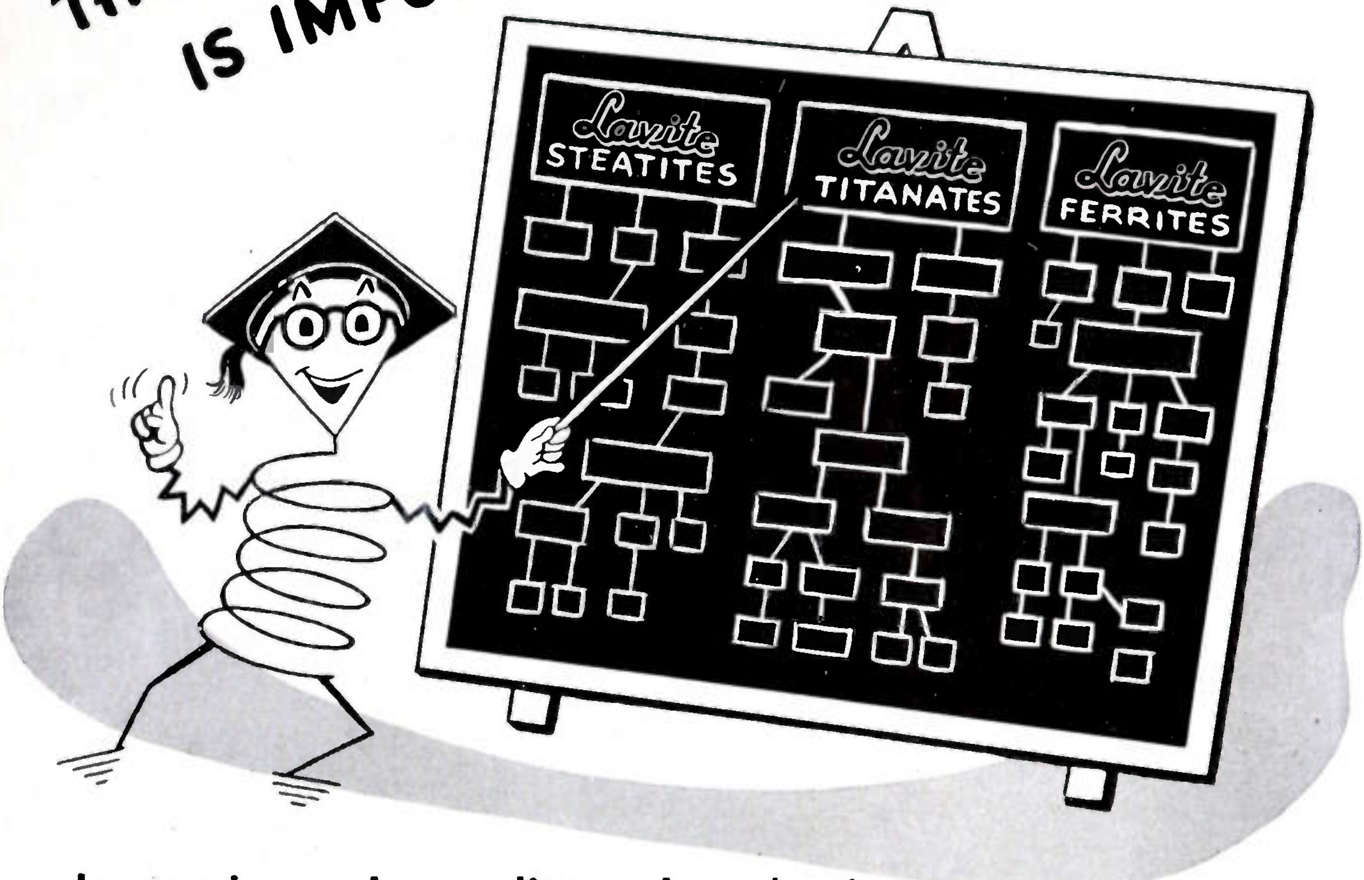

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News—New Products

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(Continued from page 50A)

signal (100 ohm load) (microbar = 1 dyne/cm²).

Designed for convenient hand use, the 11M5 has a double-pole, single-throw switch, with relay and microphone circuits normally open (press-to-talk). It is adaptable to a wide variety of circuits and is recommended for a broad variety of mobile transmitter applications and other hand mike uses. It will work into power output tubes without preamplifier stages, with use of step-up transformer.

Terminal for Hermetic Sealing

The HS-1 for hermetic sealing of all types of electrical components and designed to meet hermetic sealing requirements of MIL-T-27 is available from **International Resistance Co.,** 401 North Broad St., Philadelphia 8, Pa. This is a solder seal type terminal with a molded Polytrifluoromonochloroethylene resin insulating the solder seal ring from the center terminal rod. The molded body, fabricated from M. W. Kellogg's "Kel-F," is

(Continued on page 60A)

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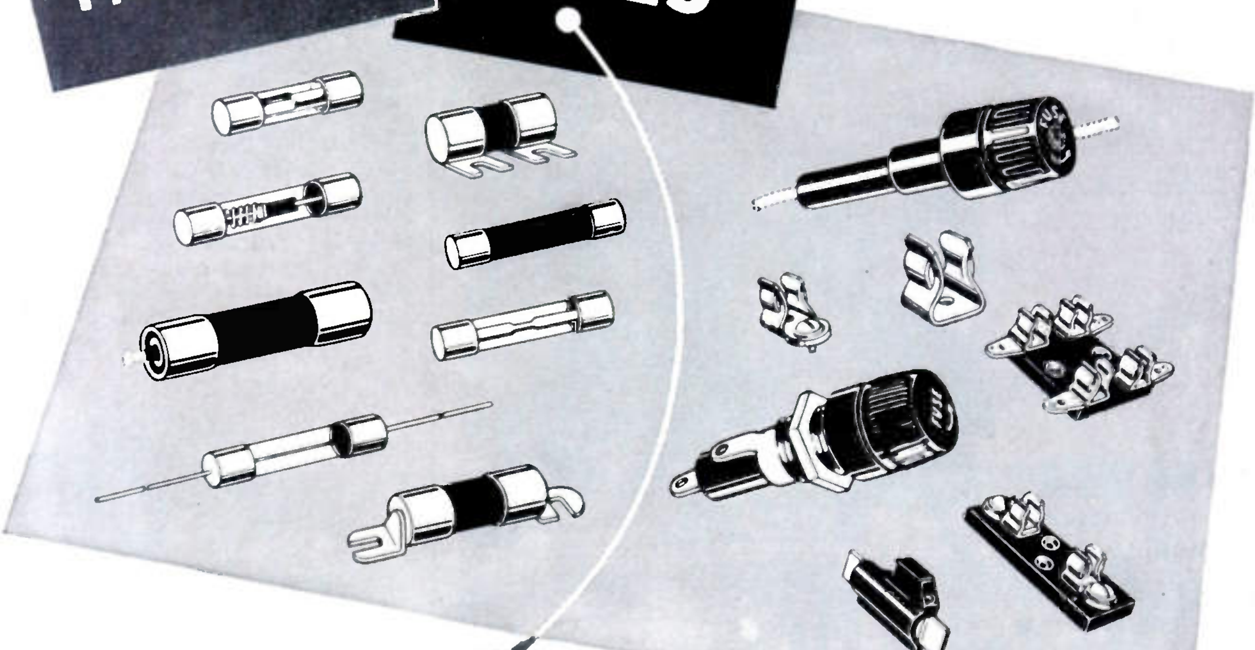

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News—New Products

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(Continued from page 58A)

chemically inert to organic solvents, acids, oils, alkalies, fumes, etc. It is unaffected by high humidity, has zero water absorption and high resistance to thermal shock ($-70°C$ to $+190°C$), is physically tough and will withstand rugged production handling. Overall length is 1 5/16 inches; dielectric strength 5,000 v (rms) 60 cps; Corona starting voltage over 2,000 v (rms) 60 cps.

Coaxial Switch

Designated Part No. 1460-22, a new, compact double-pole double-throw coaxial rf switch to replace the use of two single-pole double-throw switches and provide saving in weight and flexibility of installation has been developed by **Transco Products, Inc.**, 12210 Nebraska Ave., Los Angeles 25, Calif. Switch is motor operated instead of solenoid.

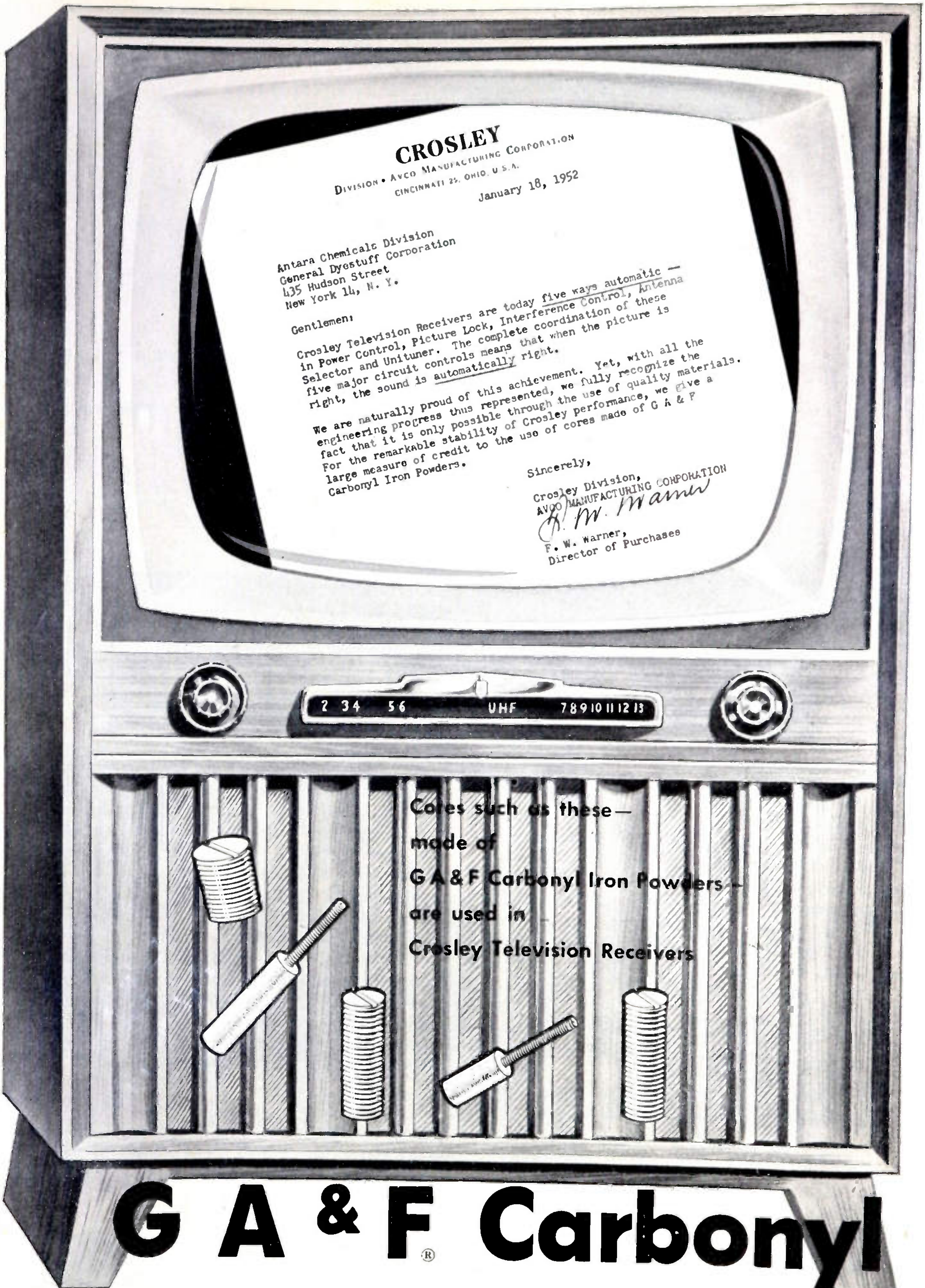
The rf performance characteristics are: frequency range up to 11,000 mc, VSWR less than 1.3 to 1, insertion loss less than 0.5 db throughout operating range, 55 db average, attenuation between unused connectors, power handling capabilities equal to improved type N connectors, and motor driven actuator rating 24–28 v dc.

Various models are available for critical aircraft applications requiring efficient performance under extreme temperature and shock conditions. Switches are precision built to military specifications.

Plant Expansion

The Staver Co., Inc., has announced the completion of a new plant at 41–51 North Saxon Ave., Bay Shore, L. I., N. Y. A new building will house the main office of the company and most of the production activity. Certain manufacturing facilities will continue to be maintained at the former location, 91 Pearl St., Brooklyn, N. Y.

(Continued on page 90A)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Sincerely,
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8 Availability: Manufacturing facilities in two RCA plants insure continuous high-volume supply.

For technical data or design assistance on RCA kinescopes or other types of tubes, write RCA, Commercial Engineering, Section CR47, or contact your nearest RCA field office:—

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ELECTRON TUBES HARRISON, N. J.

PROCEEDINGS OF THE I.R.E.

Published Monthly by

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PROCEEDINGS OF THE I.R.E.

Glen H. Browning, Director, 1952–1953 258
Electrons, Engineers, and Education .... J. D. Ryder 259
4147. The Electronic "Systems" Engineer (Systems Project Director) .... Ralph I. Cole 260
4148. Some Aspects of Magnetic Amplifier Technique .... F. E. Butcher and R. Willheim 261
4149. Some Fundamental Properties of Transmission Systems .... F. B. Llewellyn 271
4150. The L-Cathode Structure .... G. A. Espersen 284
4151. The Theory of Amplitude-Modulation Rejection in the Ratio Detector .... B. D. Loughlin 289
4152. Cathode-Ray Tube for Recording High-Speed Transients .... S. T. Smith, R. V. Talbot, and C. H. Smith, Jr. 297
4153. Some Limitations on the Accuracy of Electronic Differential Analyzers .... A. B. Macnee 303
4154. The Duplex Traveling-Wave Klystron .... T. G. Mihran 308
4155. Electronically Controllable Resistors (Abstract) .... James N. Thurston 315
4156. Echo Distortion in the FM Transmission of Frequency-Division Multiplex .... W. J. Albersheim and J. P. Schafer 316
4157. Correction to "Seven-League Oscillator" .... F. B. Anderson 328
4158. Techniques for Close Channel Spacing at VHF and Higher Frequencies .... Charles F. Hobbs 329
4159. Cascade Connection of 90-Degree Phase-Shift Networks .... Oswald G. Villard, Jr. 334
4160. Correction to "Radio Relay Design Data 60 to 600 mc" .... Richard Guenther 337
4161. A Note on a Selective RC Bridge .... Peter G. Sulzer 338
4162. Waves on Inhomogeneous Cylindrical Structures .... R. B. Adler 339
4163. Radiation Patterns and Conductance of Slotted-Cylinder Antennas .... Obed C. Haycock and Frank L. Wiley 349
4164. Discussion on "Correction of Deflection Defocusing in Cathode-Ray Tubes. Jenny E. Rosenthal, R. G. E. Hutter, and S. W. Harrison 353
4165. Correction to "Color Television and Colorimetry" .... W. T. Wintringham 357
Contributors to the PROCEEDINGS OF THE I.R.E. 358

Correspondence:
4166. "The Equivalent Circuit of the Transistor" .... Marcel J. E. Golay 360
4167. "Resistance Matching and Attenuating Networks" .... Sidney Bertram 361
4168. "An Ultra-High-Frequency Moon Relay" .... P. G. Sulzer, G. F. Montgomery and I. H. Gerks 361

INSTITUTE NEWS AND RADIO NOTES SECTION

Technical Committee Notes 362
Professional Group News 363
IRE People 364

Books:
4169. "Vacuum-Tube Voltmeters," by John F. Rider .... Reviewed by C. E. Atkins 367
4170. "Integral Transforms in Mathematical Physics," by C. J. Tranter .... Reviewed by L. T. DeVore 367
4171. "Synthesis of Electronic Computing and Control Circuits by the Staff of the Computation Laboratory, Harvard University," .... Reviewed by W. H. Boghosian 367
4172. "Electronic Fundamentals and Application," by John D. Ryder .... Reviewed by W. C. White 368
4173. "Lines, Networks, and Filters," by William Breazeale and Lawrence R. Quarles .... Reviewed by F. W. Grover 368
4174. "Audio Amplifiers and Associated Equipment" .... Reviewed by J. J. Baruch 368
4175. "Ultrasonics," by P. Vigoureux .... Reviewed by J. D. Kraus 368
4176. "Electromagnetic Waves and Radiating Systems," by Edward C. Jordan .... Reviewed by S. B. Cohn 369
4177. "Dielectric Breakdown of Solids," by S. Whitehead .... Reviewed by C. W. Carnahan 369
4178. "The Design of Switching Circuits," by William Keister, Alistair E. Ritchie, and Seth H. Washburn .... Reviewed by J. R. Ragazzini 369
Sections and Professional Groups 370
4179. Abstracts and References 372
Meetings with Exhibits 2A
Industrial Engineering Notes 36A
News—New Products 48A
Section Meetings 72A
Student Branch Meetings 81A
Membership 83A
Positions Open 96A
Positions Wanted 100A
Advertising Index 127A

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Glen H. Browning

DIRECTOR, 1952-1953

Glen H. Browning was born in Solon, Iowa, in 1897. He was awarded a scholarship to Cornell College in 1917, and graduated with a B.A. degree in 1921, with the highest scholastic honors in the class. In 1922–1923, he was awarded the Lydia C. Perkins Scholarship for graduate work with the Cruft Laboratory at Harvard University, and in 1923–1924, he was a research fellow at Harvard.

In conjunction with F. H. Drake, Mr. Browning developed a special mathematically designed tuner radio-frequency transformer in 1924, which was later incorporated in a circuit known as the Browning-Drake Circuit. Working as a radio and research engineer he then became the president of the Browning-Drake Corporation in 1926–1937. At this same time, he acted in a consultant capacity to several radio companies. Since 1937, he has been the president of the Browning Laboratories, Incorporated, at Winchester, Mass.

During World War II, Mr. Browning developed a sensitive electronic-alarm system which was widely used by the Armed Services. He is also the holder of patents on numerous devices for power-factor insulation testing, shielding methods, and sensitive watt meters. Numerous technical and semitechnical articles by Mr. Browning have appeared in radio magazines and periodicals.

Mr. Browning joined the Institute of Radio Engineers as an Associate Member in 1924, transferred to Member grade in 1928, and became a Senior Member in 1943. He served as Chairman of the IRE Boston Section from 1946–1947, and was recently elected as Regional Director of the IRE North Atlantic Region. Mr. Browning was awarded an Honorary Doctor of Science Degree from Cornell College, on June 12, 1950.

Electrons, Engineers, and Education

J. D. RYDER

Events and objects are often described, with varying degrees of clarity. But underlying trends and basic philosophies are less often mentioned, and frequently with disturbing incompleteness or inaccuracy.

Communications engineers are usually well acquainted with the phenomena and devices involved in their profession, and generally are thoroughly versed in the underlying theories of their field. Less often they appreciate fully how fundamental and radical a revision of electro-physics and electro-technology was required in the understanding of apparently straightforward communications phenomena. The "classical" electrical engineer of 1901 would have been baffled, and even appalled, by the properties of the elementary particles of modern physics and the governing laws now used in the daily work of communications experts. Information theory, cybernetics, the relativistic behavior of electrons, the uncertainty principle—these and many other aspects of the present art would have been utterly strange and almost incomprehensible to him.

The writer of the following guest-editorial, a Fellow of the IRE and a member of its Board of Directors, has presented a basic and thoughtful description of one major aspect of this intellectual revolution. So significant an analysis is a stimulating document, well meriting careful study by its readers.—*The Editor.*

For an invisible object of indefinite position, indeterminate time, insignificant mass, and infinitesimal charge, the electron has had a major impact on our ways of thought and life. Its apparent existence contributed to the overthrow of certain classical mechanics concepts within the atom, and perhaps we may ultimately conclude that the electron has led to the overthrow of some classical concepts outside the atom as well. Certainly its effect on the many different areas of life has been disproportionate to the size or energy of the particle which produces the effect.

In electrical engineering education, the electron has been given credit for producing a change, but consideration indicates that this change actually was a major upheaval. In fact, comparison of the progress of electrical education in the 20 years since the advent of electronics with a similar period of time in other areas of engineering education, which have not been stimulated by electronic impact, causes the upheaval produced by this impact to take on some of the more ordered aspects of an explosion.

In the 1920's, it was possible to teach the student electrical engineer about a world of synthetic simplicity, with a landscape of almost entirely sinusoidal shapes, change was rare, and all was steady state. Sixty cycles was king, the machine his castle, and the laws of the realm were linear and bilateral.

Into this kingdom of sinusoidal stability was dropped an electron, and just as Cinderella's world dissolved on the stroke of twelve, so did the synthetic paradise of the electrical engineer disappear. Cinderella returned to the cold hard facts of daily existence, but those electrical engineering educators who cared to look found that underlying the ruins of the tight little kingdom were foundations and fundamentals of great stability, on which could be founded not one kingdom but many, each more intriguing and challenging than the one before.

Thus, those willing to dig among the ruins found many relics of truth, from which it was possible to build new and stronger structures. These new structures were founded on a new basic law—*that all things may be nonlinear.* No longer was it always possible to consider a resistor, an inductor, or a capacitor as a circuit constant since many of the new circuits performed purely because of this nonlinearity of the parameters.

The impact of the electron with the electrical world was so severe that even the landscape was changed, no longer was it sinusoidally regular, but it was now found to include cliffs and escarpments, called "steps," and occasional steep-sided mountain ranges known as "pulses." Travel through such a realm required new methods of locomotion and these were found in the archives of the former kingdom as the works of Fourier, Laplace, and others. Thus the sinusoidal configuration became only one special case of many.

The electrical circuit associated with electronic devices was frequently discovered to be complex, however; certain useful laws and theorems had been developed and preserved in the archives. These gave a broader and more fundamental approach to the solution of circuits, and circuit theory increased in importance as the emphasis on the machine was reduced.

Devices in which electrons moved in relative individual freedom had to be studied, and the interactions of electrons with other charges and with fields had to be considered. Such things as the thermodynamics and entropy of electron clouds and bunches have become of importance, calling for a viewpoint on the teaching of thermodynamics somewhat different than that which usually leads to the Carnot or Rankine cycles and heat engines.

The concept of an electric field, coexistent with a time-varying magnetic field, formerly received very little treatment in machinery theory. The magnetic field received the emphasis, and the transfer of energy through the fields, although admitted, was not stressed. This condition, then, was altered by the impact of the electron.

It may be concluded that the education of electrical engineers since the advent of electronics has been and must be more fundamental in nature. Graduates must be prepared to grow with and to go with the field as it progresses into new areas of technical knowledge. The college can prepare the graduate—it is a duty and responsibility of the IRE to assure this growth and professional progress.

The Electronic "Systems" Engineer (Systems Project Director)*

RALPH I. COLE†, SENIOR MEMBER, I.R.E.

I. Introduction

MORE THAN EVER before we are becoming acutely cognizant that the "systems" engineer, identified herein as the "systems project director," must assume a greater and greater responsibility for large scale electronic programs, be they research, development, or operational in character. Partially, this arises from a need to integrate the electronics with other elements, but possibly of greater moment is the fact that the electronics portion within itself is being required to accomplish many new functions, more quickly, more efficiently, and with greater precision than ever before. The mere placing of "systems" responsibilities upon "equipment" engineers without regard to their capabilities does not in itself provide for the foresight, guidance, and engineering management required.

It should be emphasized that the type of systems effort being discussed herein concerns comprehensive or multiple types. It is obvious that the same criteria can apply to systems of lesser complexity by lessening of the standards by which we judge the qualities of the "systems" engineer.

II. Criteria in the Selection of "Systems" Project Directors

The ever constant pressure being exerted for the completion of all phases of present electronic systems nearing final usage, and the simultaneous requirement to initiate the development of many more, immediately brings into sharp focus the problem that management must now face in the selection of additional "systems project directors" to augment existing staffs.

In previous papers, "Management's Role in the Research and Development of Electronic Systems,"[1] and "Management Aspects of Electronic Systems Engineering,"[2] the author has pointed out that successful systems engineering direction involves a broad scale comprehension of engineering problems and an intimate knowledge of the prior art. On the assumption, therefore, that we are now considering utilizing as "system project directors" only experienced component, equipment, or staff engineers, what criteria can be set up to enable judging the most competent personnel available for "systems effort"?

Paramount attributes sought are the following:

(a) Possession of, or ability to acquire a broad scale comprehension of objectives by personal initiative. Engineers who are not self starters can hardly compete in "systems" engineering.

(b) Rapid grasp and understanding of problems that are distinguishable in engineering task assignment. Systems project directors must be able to delineate clearly these engineering tasks, and turn them over to others, often not his subordinates, for execution. This implies that, above all, the systems project director must possess executive management know-how or be able to acquire it quickly through proper grooming.

(c) Ability to win respect of subordinate engineers, above all for engineering and scientific knowledge, as well as for administrative talents.

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It has been elsewhere stated that there is a requirement for a certain degree of proficiency in technical expression and/or technical writing for those engaged in systems engineering. That this can be developed in an otherwise competent, experienced engineer is not open to question providing that one is willing to understand exactly what is expected of him and to realize that these systems engineering duties carry with them much greater co-ordination responsibilities than do those in other branches of electronics. With regard to this co-ordination, it is well to realize that research and development status reporting of "systems" is not merely the summation of progress on separate equipments or elements. Scientific direction of the project, at least through the development stage, implies continuous examination of the technical objectives, rephrasing systems "logic" where required.

IV. Conclusions

An attempt has been made in this paper to derive the criteria for selection of engineers capable of executing direction of comprehensive systems engineering projects, and to point out the basic duties of this engineer which are common to systems engineering projects in general. It is quite apparent that the electronic systems engineer must demonstrate his executive qualities by precept and example. Additionally, he must prove the ability to assimilate an enormous amount of new scientific data. The keenness of his direction results directly from his ability and special training. Attention is invited to the fact that flexibility of this scientific direction is a must, since systems engineering is necessarily evolutionary.

Some Aspects of Magnetic Amplifier Technique*

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This paper has been approved by the Tutorial Papers Subcommittee of the IRE Committee on Education as a part of a planned program of publication of valuable tutorial material.—*The Editor.*

***Summary*—Design problems of magnetic amplifiers have been mastered by a technique which appertains to core construction, arrangement and proportioning of windings, and circuiting. Principles are described which have led to the construction of precision amplifiers adaptable to a wide range of input impedances and output requirements. Performance figures are given and practical applications described. Graphical methods for dimensioning the circuit elements and for predicting the performance of the assembly are set out and applied to push-pull amplifiers. Performance limits imposed by zero stability and response time are reviewed and assessed.**

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Introduction

THE PRESENT PAPER is mainly concerned with the use of magnetic amplifiers for dc-dc amplification at relatively low input levels. Particular attention is devoted to features which have proved essential for precision amplifiers of good linearity and stability. In the interest of a more general treatment, and to conform to practical precision amplifier technique, the twin core reactor with series connected ac windings and separate positive feedback (self-excitation) winding has been singled out as representative, the theory and performance of which is in many ways applicable to the self-saturating magnetic amplifier now so widely employed in servo and power applications.

I. *Magnetic Circuits*

The function of the Magnetic Amplifier has been recognized to be that of a switch,[1] making at a controllable instant of the supply voltage wave, and breaking at or near the zero passage of the current wave. Such a performance is achieved by utilizing the reactor units as circuit elements of alternately very high and very low impedance, an effect facilitated by the existence of a point of rapid change in slope of the magnetizing characteristic. Because the presence of air gaps in the magnetic circuit impairs the initial slope and is undesirable, the inference appears to have been drawn that a completely closed iron path is called for within the individual stampings, or that at least spirally wound cores are required. This is, however, not the case.

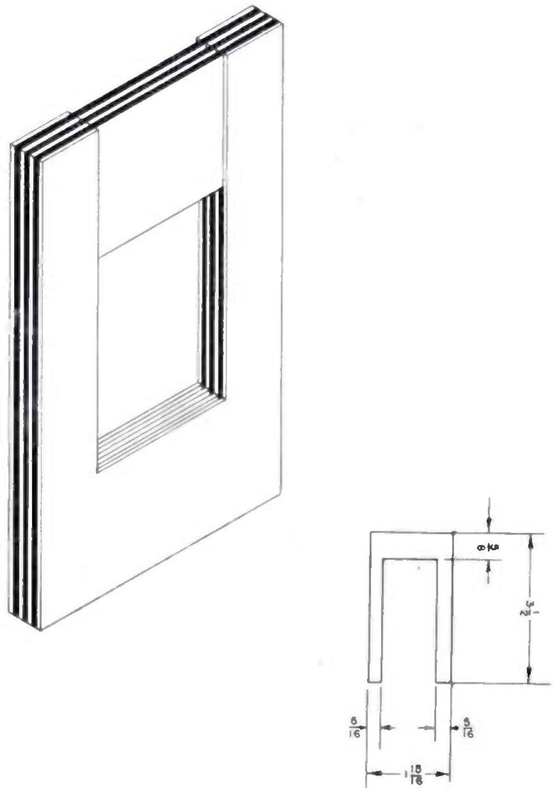


Fig. 1—Core with sideways air gap using U-shaped laminations with reinforced yoke.

True, the conventional technique of interleaved laminations would not do. At the overlap every other lamination is required to carry up to twice the flux apportioned to it elsewhere in the limb ("isthmus effect"), and it is obvious that local saturation sets in when the main part of the magnetic circuit has reached only 50 per cent saturation flux density; a three-zone magnetizing characteristic with two "knees" ensues. The existence of the middle zone spoils the performance, while the third zone is outside the range of practical use. This difficulty is overcome by application of a "sideways" air gap, Fig. 1, in which the flux is transferred from the limbs to yokes reinforced to twice the width of the limbs. There is no isthmus effect, and the reluctance of the gap is minimized by the large area available for flux transfer. The latter advantage is accentuated if the core is built up from thinner laminations, because the flux density in the gap area is proportionately reduced. While this is of importance where very high power gain is desired (say 10^6 in one stage), it may, in passing, be remarked that the low initial permeability of some important core materials seems to come more readily into appearance once the air gap is practically eliminated. The magnetizing characteristic of such a core approximates to a two-zone characteristic not passing through the origin but displaced in the direction of the ampereturn axis.

Of the customary high permeability core materials, Mu-metal (Permalloy) has no preferred direction and leaves full freedom for circular or rectangular shape of the stampings. Grain oriented 50 per cent nickel-iron strip has two preferred directions at right angles to each other, thus favoring the application of a rectangular outline of the stampings.

A core built up from punched laminations using sideways air gaps is, for practical purposes, magnetically equivalent to one of gapless design. The advantage that the coils can be wound on formers and the core finally assembled without being subjected to any mechanical stress, explains the wide appeal of saturable reactors cored with laminations of the reinforced yoke type. Standardized laminations of the "E" type, to be used with only one wound limb, are available in the principal core materials, Mu-metal, and grain oriented 50 per cent nickel-iron. The authors believe that to have a considerable portion of a saturated magnetic path not covered by windings entails the piling up of free magnetomotive forces. It is, hence, less desirable than a uniform distribution of the windings along the saturable core, as ideally represented by a ring shaped core, and practically materialized by the U-core with two wound limbs. The latter is also less prone to respond to external disturbances such as stray fields, the magnetic field of the earth, and proximity of magnetic materials, including the case.

Magnetic circuits with ac coils and dc control windings on different limbs are in many respects simpler than the twin core design, but they lack the close interaction of the two winding systems. Types working with self-saturation, as well as designs using 100 per cent positive feedback (self-excitation) with ac and feedback windings arranged concentrically, are hardly affected by this in view of the very small residual ampereturns on the individual magnetically operative limbs. This point should, however, be watched in designs with moderate positive feedback or with no feedback at all.

To secure uniform performance of individual saturable reactors, and to facilitate their proper matching in pairs capable of operating in push-pull arrangement, it is necessary to test incoming batches of laminations and to classify core stacks with regard to initial permeability,

[1] A. U. Lamm, "Some fundamentals of a theory of the transductor or magnetic amplifier," *Trans. AIEE*, vol. 66, pp. 1078–1085; 1947.

slope of linear portion of the magnetic characteristic, and saturation flux density.

II. *Magnetic Amplifiers with One Saturable Reactor*

Some of the more important circuits are shown in Fig. 2. For the dc controlled saturable reactor unit itself, the short and descriptive term "transductor" has been coined.

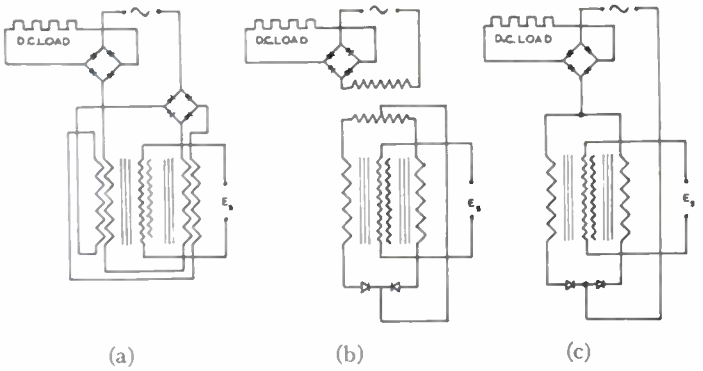


Fig. 2—Magnetic amplifiers with one saturable reactor (dc output). (a) Circuit with self-excitation (ac windings connected in series). (b) Self-saturating reactor, full-wave circuit. (c) Self-saturating reactor, doubler circuit.

A. Basic Analysis. E-I Chart, Load Lines: The magnetic amplifier is closely related to the conventional transformer, inasmuch as the ampereturns of the output winding balance those of the dc windings at any instant, apart from a small residual percentage which produces the useful flux. The dc component of the latter, appropriately visualized as a flux shift displacing the ac flux wave, is the controlling quantity of the amplifier performance, and requires for its excitation only a small fraction of the total dc ampereturns. From Fig. 3 it is seen that for suitable core material the ac current is for a wide range of voltages essentially determined by the dc excitation, and that the influence of the voltage consists in a small deviation of the slope from the vertical direction. If the ac output is rectified, a dc-dc transformation is effected.

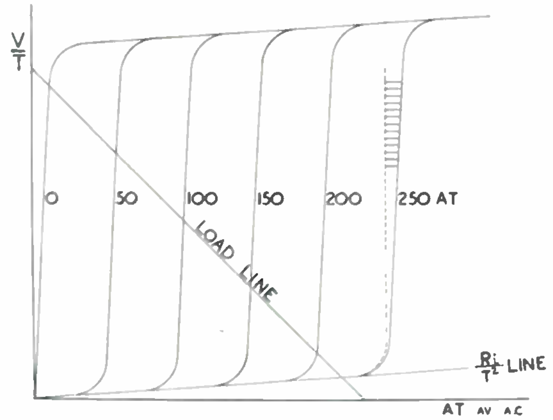


Fig. 3—Family of voltage-current characterics of saturable reactors without feedback for varying dc ampereturns. Load line construction.

Now let part of the ac supply voltage be consumed in a load resistance inserted in series with the twin reactor. By entering a load line in Fig. 3, this can be accounted for. As long as average values of E and I are used, such load lines are not ellipses, as they would be for rms values, but straight lines. To deduce this without elaborate mathematics, it is useful to investigate the variation of currents and fluxes with time. This has been done in Fig. 4. Blocking periods alternate with conducting periods according to whether or not the core is magnetically operative. Of the two-voltage areas the first represents the volt-seconds absorbed by the reactive member; when saturation sets in, the voltage is thrown upon the resistive element[1] (second area). Hence

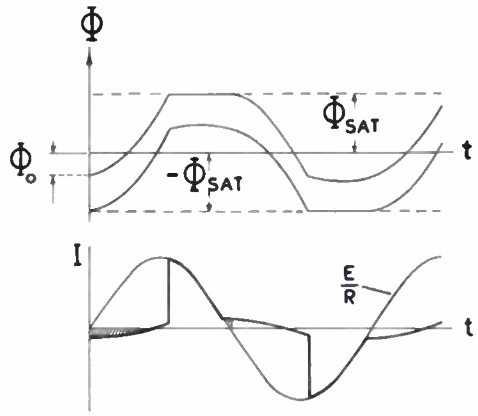


Fig. 4—Wave shapes of fluxes (top) and current (bottom) of series connected saturable reactor. The E/R wave represents the supply voltage.

$$E_{\text{supply av.}} = E_{\text{reactor av.}} + I_{\text{output av.}} \times R_{\text{load}}, \qquad (1)$$

which is indeed the equation of the straight load line in Fig. 3. (It should be realized that in series connected twin reactors, after one core has reached saturation, the other behaves as a short-circuited transformer, causing mainly resistance drop, provided that the short-circuit reactance and the control circuit resistance are both kept low.)

The term, $E_{\text{reactor av.}}$, includes internal resistance drops and rectifier drops. The magnetically absorbed voltage is $E_{\text{reactor av.}} - I_{\text{output av.}} \times R_i$. The slope of the top and bottom sections of all characteristics is determined by the internal resistance R_i, provided that completely saturable core material is employed.

The "ampere-turn balance principle" which, in a modified meaning, the saturable reactor has in common with the conventional transformer, can be written as

$$AT_{\text{output}} = AT_{\text{dc}} + k(E_{\text{reactor}} - R_i AT_{\text{output}}) + C, \qquad (2)$$

valid for average quantities, with E_{reactor} in volts per turn and R_i in ohms per turn.[2]

Equation (2) is evidently the analytical expression for the straight portion of the graphs $AT_{\text{dc}} = \text{const.}$ in Fig. 3. Instead of investigating the factor k connecting the magnetically absorbed reactor volts with the ampere-turn deficiency, we shall be content to explore, by elementary methods, such information as can be derived from the mere existence of these straight portions. In-

[2] S. E. Tweedy, "Magnetic amplifiers," *Electronic Eng.*, (London), vol. 20, pp. 38–43, 84–88; February and March, 1948.

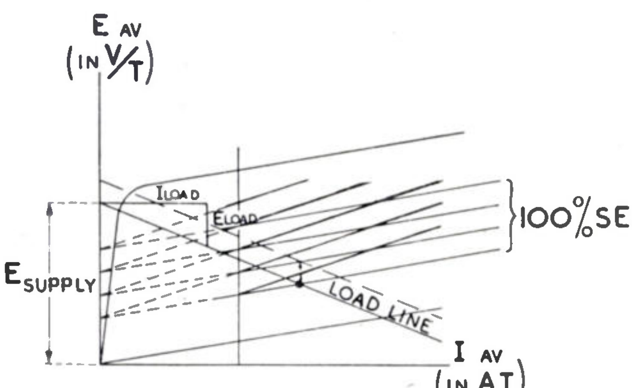
side the lozenge-shaped complete E-I chart Fig. 3, they occupy a region the borders of which are roughly parallel to the top and bottom boundary lines.

Experimentally the family of E-I characteristics is conveniently obtained by tests with externally set dc excitation, but the physical, meaning of AT_{do} is not confined to this narrow interpretation. In particular, there is the possibility of introducing positive feedback, making use of the "equivalence principle"

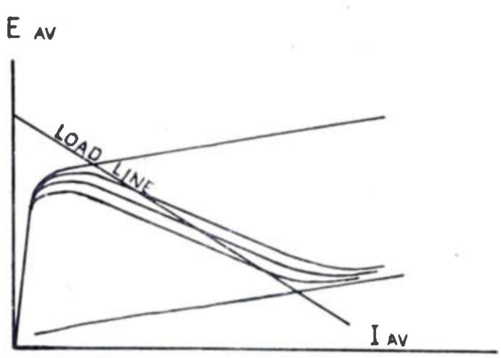
$$AT_{do} = AT_{control} + \alpha AT_{output}, \qquad (3)$$

where α is the per unit feedback factor.

The application of positive feedback relieves the external signal source of all but a small basic excitation current. In this way an ampere-turn gain is achieved and, at the same time, the power gain is increased considerably. The manner in which the rectified output is fed back does not depend on the polarity of the controlling signal. This suggests talking of "self-excitation" rather than of "positive feedback."



(a)



(b)

Fig. 5—E-I charts for self-excited saturable reactor. (a) Self-excitation 100 per cent or less; linear transformation of signal lines. (b) Self-excitation more than 100 per cent.

From (2) and (3) we can now deduce:

$$(1 - \alpha + kR_i)AT_{output} = AT_{control} + kE_{reactor} + C. \quad (4)$$

(The term C may include influences in the nature of bias ampereturns or of basic voltage drops of rectifiers.)

The following conclusions can be drawn (Fig. 5(a)):

1) The "signal lines" $AT_{output} = f(E_{reactor})$ for constant control ampereturns are, in a certain region of the E-I chart, straight lines for any chosen self-excitation α.

2) Within this linear area their slope is $1 - \alpha + kR_i/k$, becoming R_i for $\alpha = 1$. Thus for 100 per cent self-excitation and for self-saturation the E-I characteristics run parallel to the R_i slope, i.e., to the borders of the lozenge containing all working points. (Due to self-excitation winding and rectifier forward resistances, R_i is not the same as in Fig. 2.)

3) The vertical spacing of any two signal lines (i.e. $\Delta E_{reactor}$ for $\Delta AT_{output} = 0$) is a measure of their label difference $\Delta AT_{control}$, independent of the degree of self-excitation.

4) The process of transforming one family of signal lines into another, retaining the steps in the signal labelling, consists in turning the straight lines around their points of intersection with any vertical line AT_{output} = const. (To make the absolute levels of the signals the same, the loss in self-excitation ampere-turns, i.e. $(\alpha_1 - \alpha_2)AT_{output}$, has to be replaced by bias.)

By applying (1), (4) can be transformed into

$$[(1 - \alpha) + k(R_i + R_{load})]AT_{output}$$
$$= AT_{control} + kE_{supply} + C. \quad (5)$$

For a given supply voltage, we obtain for the reciprocal of the ampere-turn gain

$$\frac{\Delta AT_{control}}{\Delta AT_{output}} = (1 - \alpha) + k(R_i + R_{load}). \qquad (6)$$

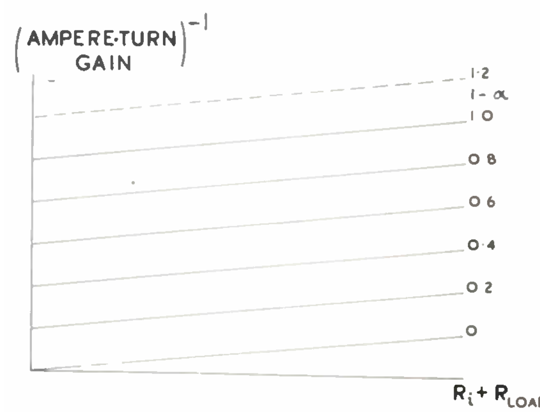


Fig. 6—Milnes' diagram of magnetic amplifier performance.

Milnes[3] has made this relation the basis of the very interesting and experimentally well confirmed diagram, Fig. 6.

In attaching absolute values to the labelling of the signal lines in Fig. 5(a), a difficulty is encountered. Remembering what has been said in section I about the horizontal displacement of the magnetizing curve of the saturable reactor, it should be realized that application of positive feedback results in converting the natural ac ampere-turn displacement into dc ampere-turns acting as

[3] A. G. Milnes, "A new theory of the magnetic amplifier," *Proc. IEE* (London), vol. 97, pp. 460–474; August, 1950.

a self-bias. A compensating negative bias is used to put the working point to where it is wanted for best performance with respect to one or more of the essential features: sensitivity, linearity, stability.

Outside the linear range, the *E-I* characteristics for constant signal are no longer straight, but curved, and no longer equidistant, but crowding in the proximity of top and bottom border.

Drawing a load line through an *E-I* chart supplies the relevant information for yet another description of amplifier performance, viz., the transfer characteristic by which the output current (or voltage) is given as a function of the controlling signal.

The *E-I* charts in conjunction with load lines present a versatile tool, giving at a glance the influence of changes in load, supply voltage, and degree of self-excitation, as well as providing the answer to the question of load matching.

The load line for maximum power at a given signal, i.e., maximum power gain, is the reflection of the *E-I* characteristic on a horizontal line. This gives the maximum area of the triangle formed by E_{load} and I_{load} in Fig. 5(a). For maximum output at very large signals, the load line should be chosen to form the reflection of the bottom border on the horizontal. The two conditions coincide for 100 per cent self-excitation (self-saturation).

The steep slope of the signal lines in Fig. 3 gives good stability against supply voltage variations. In Fig. 5(a) however, this feature is seen to be sacrificed to high gain. Hence, if positive feedback is to be utilized to full advantage, stability considerations demand the use of push-pull arrangements. Another way of improving stability is indicated in Fig. 3. The vertical characteristic shown there as a dotted line is obtained by an auxiliary, voltage-proportional excitation $kE_{reactor}$ opposing the main excitation AT_{dc}. In Fig. 5(a) the stabilizing effect of this method of compounding is shown to hold good for positive feedback. If, for instance, a supply voltage variation should result in shifting the load line upwards into the dotted position, the output can be restored by changing over to a signal line of lower label. With the help of this principle, improved stability is obtained automatically when, e.g., for the purpose of backing off a given positive signal, negative bias is derived from the rectified supply voltage.

The *E-I* diagram presents a way of visualizing the fact that the average value of the magnetically absorbed voltage $E_{reactor} - R_i I_{output}$ is in a simple relation to the flux swing as plotted in Fig. 4, top:

$$\frac{1}{2f}\left(E_{reactor\ av.} - R_i I_{output\ av.}\right)$$

$$= \int_0^{1/2f} (e_{reactor} - R_i i)dt = 2(\Phi_{sat} - \Phi_0). \qquad (7)$$

This is illustrated in Fig. 7, and leads to the result that each working point inside the lozenge is characterized by the initial flux Φ_0 (or, for that matter, the flux shift $\frac{1}{2}(\Phi_{sat}+\Phi_0)=\Phi_d$) and by the output current. All lines for constant Φ_0 are parallel to the borders of the lozenge, and coincide with the *E-I* characteristics for 100 per cent self-excitation. (A negligibly small error is caused by the shaded current areas indicated in Fig. 4, bottom.) In this way we derive from Fig. 7 the simple relation:

$$\Delta I_{output\ av.} \times (R_{load} + R_i) = 4f\Delta\Phi_0, \qquad (8)$$

which is of considerable importance for time constant theory.

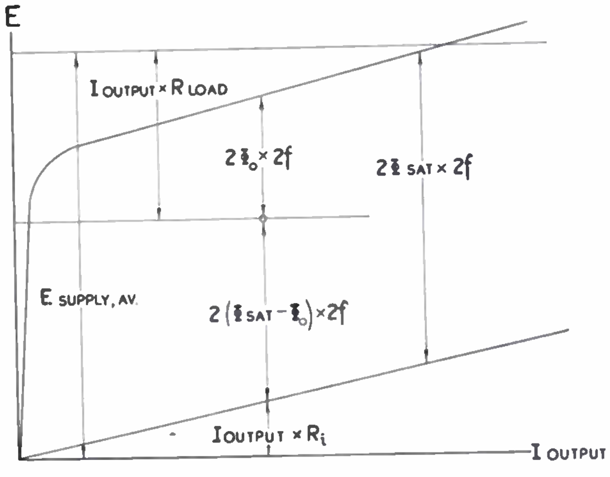


Fig. 7—The position of the working point as a measure of the initial flux Φ_0.

For more than 100 per cent self-excitation, the linear sections of the *E-I* characteristics assume a negative slope, bending back towards the bottom border, Fig. 5(b). Along the falling part of the resulting S-loop, instability is usually expected to occur, but it should be realized that for sufficiently steep load lines there is only one point of intersection. In such cases the sections with negative slope are stable and give high amplification.

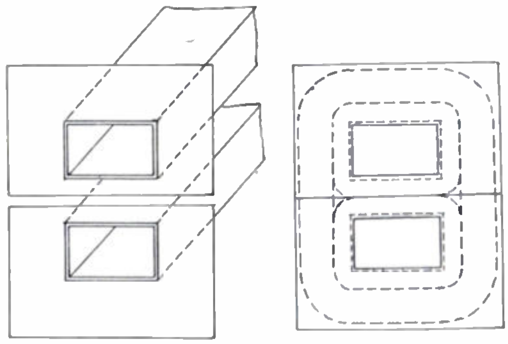


Fig. 8—Bobbins and winding arrangement for twin-core reactors.

B. Electric Circuiting: Twin core reactors are preferably designed with special bobbins on the pattern of Fig. 8, with common dc windings which comprise self-excitation and signal windings, often also bias and balancing windings, compounding and feedback windings, as the case may be. Some of them deserve a few words of explanation.

It is possible to combine the ac windings with a 100 per cent self-excitation winding by an arrangement which has become known as self-saturation,[4] simplified self-excitation,[5] etc. Considerable economy in winding space is gained thereby. Supplementary feedback windings have to be provided for increasing or reducing the effective self-saturation. For precision amplifiers with dc output, the leakage current of rectifiers responsible for the self-saturation is of great influence. In Fig. 2, load rectifier and self-saturation rectifier are shown as separate elements which can be selected in accordance with their functions.

The signal winding or windings usually take up the major part of the winding space available, as this is essential for high power amplification. For the same purpose the input resistance of the amplifier should be reasonably well matched to the impedance of the signal source. However, de-matching by a ratio of 2:1 or 1:2 means a loss of only 11 per cent of the maximum possible input power. By splitting the signal winding into two sections, as are anyway available on a two-limb design, and applying series-parallel connection, a range of source impedance of 1:16 can be catered for with one amplifier. Units with input impedances ranging from 0.1 to 65,000 ohms have been built, the upper limit being set by the requirements of sturdiness and size limitation. When rectified ac signals have to be employed, the basic rectifier drop (approximately 0.7 volts for a single-disk selenium rectifier bridge) has to be overcome. The loss in signal can be reduced by applying a suitable step-up transformer for the original signal. A series resistor must be inserted between rectifier and signal windings, to minimize the possibility of spurious signals being produced by rectification of the second harmonic which is induced in the dc windings. Similarly, if a barrier layer photocell is the signal source, measures are required to prevent that the second harmonic is rectified and converted into a spurious dc signal. A capacitor parallel to the signal source is often used, but the effect on the response time has to be watched.

Bias windings are often supplied from mains through rectifiers and series resistors high enough to prevent them from acting as slugs and impairing the time response. Care should be taken to keep such bias windings free from the intrusion of capacitive currents of mains frequency which depend on the polarity of the ac connections and, by passing through the bias winding every other half cycle, could produce spurious effects.

The gain of a magnetic amplifier incorporating a single saturable reactor could be adjusted by a resistor shunting the self-excitation winding. A disadvantage of this method is the difference in temperature coefficient between winding and shunt. It has been found preferable to shunt only a section of the self-excitation winding. Stability of the resistors is an important requirement.

Balancing windings of small volume are applied for various purposes. If all dc circuits are of very high impedance, a balancing winding, short-circuited on itself, enforces equality of flux variation in the two cores, and ensures a well defined performance. A balancing winding may also be employed for influencing transient performance.

Feedback windings are a feature of some importance for push-pull arrangements where the relevant points will be mentioned.

For servo-applications, further windings may be provided to accept signals from tacho generators etc., or to introduce derivative feedback or other signals of a transient character.

C. Applications: A magnetic amplifier incorporating a single saturable reactor has unidirectional dc output, a rather limited range of linearity, and a certain standing output at zero signal. In addition, good stability of the zero setting is not achieved without special measures. The working point could be shifted (biased) to the linear section of the input-output characteristic, the standing current compensated, and even duo-directional output achieved. Nevertheless, the applications for instrumentation purposes are infrequent and restricted to low gain so as to minimize the stability problem. Output stages for stepping up the power obtainable from push-pull arrangements provide an example. Control applications, however, have been made with particular success, a combination of a magnetic amplifier with a dc relay being the outstanding representative. Intrinsic power amplifications of the order of 10^6 have been obtained in one stage. In the design illustrated by Fig. 9, matching of source impedance and input resistance is intentionally avoided, the latter being designed to amount to, say, one tenth of the former. This means a deliberate sacrifice of two thirds of the maximum available input power, and requires a corresponding increase in gain.

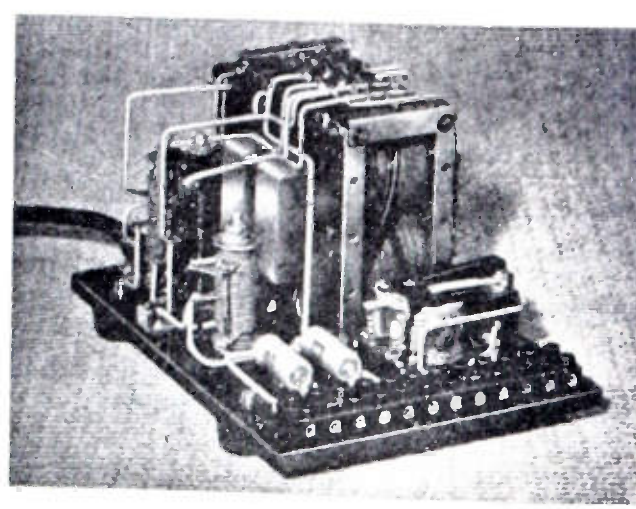


Fig. 9—Magnetic amplifier relay.

[4] W. J. Dornhoefer, "Self-saturation in magnetic amplifiers," *Trans. AIEE*, vol. 68, pp. 835–846; 1949.
[5] L. F. Borg, "Transductors and their application," *Elec. Times*, vol. 117, pp. 269–273; February, 1950.

According to what has been said at the end of section II A, this can be done by judiciously employing more than 100 per cent self-excitation, still avoiding trigger action. The over-all effect is one of obtaining a given output for a given signal, and yet achieving a considerable reduction in response time. In (11) to (11(d)) following it will be seen that for this purpose the signal volts per turn should be made as high as possible by designing the signal winding with a low number of turns and providing a high ampere-turn gain.

Magnetic amplifier relays have been produced for operation from leakage currents, barrier layer photocells, resistance thermometer bridges, and thermocouples. Other applications are the surface or interface level control of highly inflammable liquids, and the operation of power devices by electric signals from intrinsically safe circuits situated in explosive atmospheres.

III. *Push-Pull Amplifiers*

For a number of reasons, push-pull amplifiers are the answer to problems requiring high precision of performance:

a) They provide duo-directional output, giving zero output for zero signal;

b) They are intrinsically stable, the effect of voltage variations cancelling out by the combination of the two units;

c) The linear range is extended, as can be deduced from the deviation from linearity of a continuously curved input-output characteristic. The output current of a magnetic amplifier biased by a current I_b and receiving a controlling signal I_i is

$$f(I_b + I_i) - f(I_b).$$

For push-pull connection this is modified to

$$f(I_b + I_i) - f(I_b - I_i).$$

By expansion it can be seen that in the second case linearity is improved by a full order of magnitude.

From the graphical treatment in section III B it will become apparent that, compared with the individual magnetic amplifier, the output available in the load has been halved, although the amount of material has been doubled. This is the price paid for improved performance.

It is common practice to use two identical saturable reactors conveniently arranged in a common case together with supply transformer, mixing circuit, and potentiometers for zero setting and gain control.

A. Circuitry: The various windings of the individual transductors are essentially the same as those described in section II B. Fig. 10 gives a principal circuit diagram.

The bias windings and the signal windings are connected so as to act in the same sense in one unit and to oppose each other in the other unit. Hence, the bias windings can be used to fix the position of the so-called "working point" for zero signal. A potentiometer can be arranged to trim the bias ampereturns and to set zero. It may be necessary to reset zero when the gain is changed over a wide range.

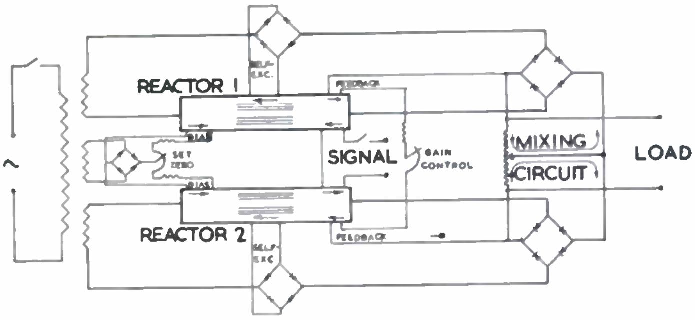


Fig. 10—Basic circuit diagram of self-excited push-pull amplifier for dc output.

In both units the self-excitation windings have the same sense with respect to the bias windings and can be supplemented by feed-back windings which are excited from the output circuit. The directional effect of the current through these latter windings, which act in sympathy with the signal input, justifies the use of the term feed-back for the function of such windings. It is feasible to provide more than 100 per cent self-excitation, and to apply negative feed-back for gain control which can be effected by potentiometer excitation of the feed-back winding.

There remains the choice of deriving the feed-back from the load current or the load volts. Both methods have their merits and their range of application (S. E. Tweedy). The application of feedback results in a linear transformation of the E-I chart, the straight sections of which turn round their points of intersection with the vertical or horizontal axis through the working point. The new slope and, hence, the dynamic resistance (which, by the way, is indicative of the load resistance giving highest power gain) can be made high or low, in accordance with the feed-back arrangement chosen.

The dominating problem of duo-directional dc amplifier circuitry is that of the mixing circuit for the two transductor output currents. There are two basic solutions, both requiring the absence of direct electric connections between the two units on the ac side, and, accordingly, the use of supply transformers with two separate, though closely linked, secondary windings. (A tertiary winding is provided for the bias circuit.) Suitable circuits are shown in Fig. 11 and are readily recognized as current mixing, or parallel mixing, or Y-circuit on the one hand, and as voltage mixing, or series mixing, or Δ-circuit on the other. It should not be assumed that the two mixing resistances R_m in the parallel mixing circuit (which may be assumed to include the rectifier forward resistance) are dispensable. They secure that over a given output current range the load resistance is not shunted by the one rectifier which receives the smaller ac current. This would be the case if $I_2 R_m$ would not exceed $(I_1 - I_2)R_{load}$. It will be seen that the parallel mixing circuit, in order to have its linear range extended to higher loads or higher currents, requires a sacrifice in mixing efficiency. A fraction $R_m/R_m + 2R_{load}$ of the output voltage is lost. Similarly, the mixing resistors of the series mixing circuit divert the fraction $R_{load}/R_{load} + 2R_m$ of the differential current from the useful load. This suggests a high value for R_m, which in turn results in high supply voltage and high standing losses. The performance analysis with the help of load lines is the subject of section III B.

The parallel mixing circuit automatically limits the current output into a given load, a feature which is sometimes desirable for thermal reasons (motor field currents) or for early stages of multistage amplifiers.

B. Graphical treatment in design stage: From the equations entered into the circuit-diagrams, Fig. 11(a) and 11(b), it is easy to derive the relations

$$R_m \frac{I_1 + I_2}{2} = E_{supply} - \frac{E_{a_1} + E_{a_2}}{2}$$

$$(R_m + 2R_{load}) \frac{I_1 - I_2}{2} = \frac{E_{a_2} - E_{a_1}}{2}, \qquad (9)$$

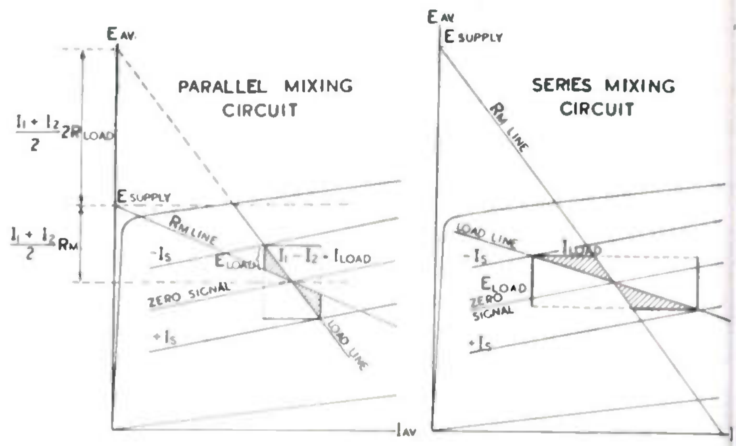
valid for the parallel mixing circuit,
and

$$R_m \frac{I_1 + I_2}{2} = E_{supply} - \frac{E_{a_1} + E_{a_2}}{2}$$

$$\frac{I_1 - I_2}{2} = \left(\frac{1}{R_m} + \frac{1}{\frac{R_{load}}{2}} \right) \frac{E_{a_2} - E_{a_1}}{2} \qquad (10)$$

valid for the series mixing circuit, all magnitudes representing average values. The points (E_{a_1}, I_1) and (E_{a_2}, I_2) are somewhere on the signal lines labelled $+I_s$ and $-I_s$.

The interpretation of (9) and (10) can be found in Figs. 12(a) and 12(b), respectively. The following procedure can be adopted:



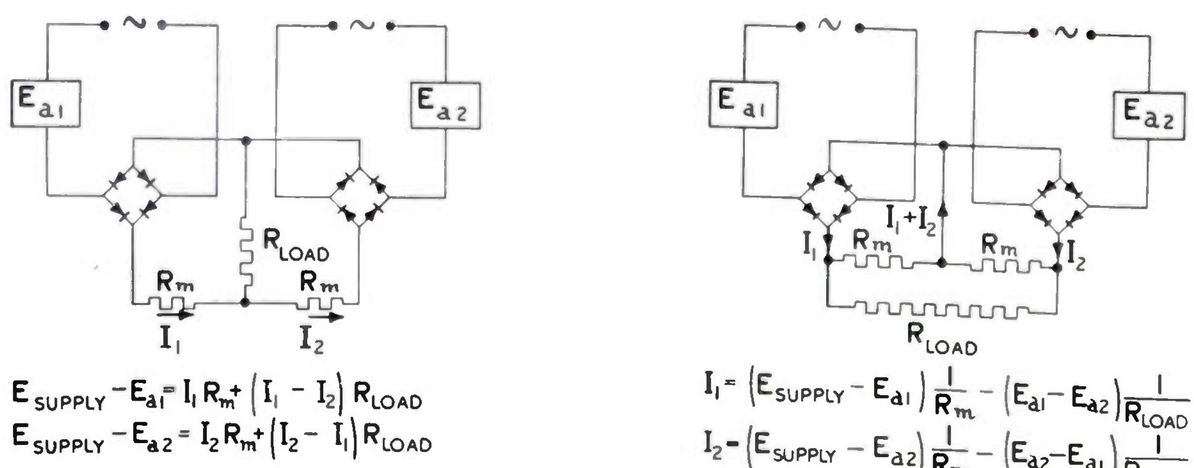
(a) (b)

Fig. 12—Graphical design procedure for push-pull amplifiers. (a) Parallel mixing circuit; (b) Series mixing circuit.

1) Choose a working point in the linear part of the lozenge, preferably well to the right of the voltage axis. This implies a given dc excitation consisting of a combination of self-excitation and bias. Draw a load mixing line under a slope corresponding to R_m through the working point. The point of intersection with the ordinate axis indicates the required supply volts. The primary resistance of the supply transformer contributes to the effective mixing resistance.

2) It will usually suffice to consider the signal lines for 100 per cent self-excitation, without resorting to the linear transformation indicated in Fig. 5(a) or those mentioned in section IIIA. Negative feed-back can be relied upon for final trimming and gain control.

3) Draw a load line through the zero-signal working point with a slope corresponding to the series connection of R_m and $2R_{load}$ for a parallel mixing circuit, or to



(a) (b)

Fig. 11—Mixing circuits for dc output. (a) Parallel mixing circuit; (b) Series mixing circuit.

the parallel connection of R_m and $\frac{1}{2}R_{load}$ for a series mixing circuit.

4) The points of intersection between that load line and two symmetrically labelled signal lines of the *E-I* chart determine the actual working points of the individual saturable reactors on application of signal. The position of these points inside the experimentally known limits of the linear range of the lozenge has to be confirmed.

5) Preferably, the load line should intersect with the ordinate axis just below the saturation voltage of the saturable reactor.

6) The saturation output can be found on the other end of the same load line, and should satisfy the specification.

7) The lozenge should then be checked with respect to the assumed resistance of windings and rectifiers which can now be established more accurately.

8) Load voltage and load current are in Figs. 11(a) and 11(b) marked with bold lines.

This procedure has to be slightly modified if the two lines for positive and negative signal are not symmetrically positioned with regard to the zero signal line. A load line of the aforementioned slope has to be drawn between the two signal lines so that it is bisected by the R_m-line.

The method described in this section has proved a safe guide in designing push-pull amplifiers and devising suitable component values for mixing circuits. Its range of applicability includes, for instance, integrating motors which are essentially devices drawing a constant current. Their load line is a straight line parallel to the R_m-line, displaced by a constant current.

C. Performance Figures for Push-Pull Amplifiers: Representative performance figures for a design which has originated in Britain and has been introduced in the United States are tabulated below:

- Input impedance: 0.5 to 135,000 ohms; preferred values: 2, 200, 20,000 ohms.
- Output impedance: preferred value, 10 to 15 ohms.
- Nominal output current: 5 milliamperes.
- Linear output: 100 milliwatts. Maximum output: 0.8 watt.
- Power gain: Controllable from 700 to 45,000.
- Zero stability: Short-term 10^{-12} watts, long-term 3×10^{-11} watts.
- Amplification stability: $\pm\frac{1}{2}$ per cent of readings for a power gain of 1,000 (including the effect of supply voltage fluctuations by ± 10 per cent).
- Time Constant: 2.5 seconds at a power gain of 15,000.
- Supply: 115 volts ± 10 per cent; consumption 10 va.
- Weight: 22 lbs.
- Dimensions: $8\times12\times6\frac{1}{2}$ inches.

D. Some Examples of Practical Applications of Push-Pull Amplifiers: Power sources of mains frequency, 400, and 1,000 to 2,000 cps are prevalent.

The more frequent signal sources are: electronic; photo-electric (e.g., photometer, input for full scale deflection 10^{-8} watts); bridge unbalances (e.g., strain gage bridges for stress monitoring and horsepower recording, also inductance bridges for recording of turbine eccentricity); thermocouples, thermopiles (integrating solarimeter), lead sulphide cells.

For signals characterized by an emf (thermocouples) rather than a current (barrier layer photocells, high impedance sources) the temperature dependence of the input resistance requires attention. Swamping resistances, signal cancellation by resistance feedback, and signal correction derived from a bridge network in the output circuit are means of redress, given in ascending order of efficiency.

Push-pull amplifiers have been used as stable preamplifiers for the magnetic amplifier relay combination described in section II C, increasing the sensitivity by a factor of 1,000, at the same time permitting of continuous indication. (Example: Detection of oil in water by measuring opacity; full scale deflection for 2×10^{-8} watts corresponding to two parts in a million; relay response at 50 per cent full scale deflection, release at 45 per cent. Similar application: pH supervision with antimony-calomel electrode as detecting element, operation for 20 millivolts deviation from an adjustable basic signal of 200 to 400 millivolts.)

On their output side, push-pull amplifiers for dc loads are used for metering, recording, telemetering, or for control of saturable reactors, fhp motors, etc. The combination with integrating motors has increased the scope of both devices.

Oscillographic recording of amplified low-frequency signals requires conversion of the output consisting of sequences of voltage or current areas of twice supply frequency into average magnitudes by means of elementary filter arrangements or RC circuits.

The zero balancing of a magnetic amplifier is greatly simplified by dc output which for ac output is a rather difficult proposition unless the output circuit is selective for the active component of the fundamental wave. If ac output is required, cascading with a first dc output stage has the advantage of providing a superior solution for the problem of zero balance and zero stability.

IV. *Transient Performance*

The basic concept is that of a change in flux shift developing over a number of half cycles of the supply frequency. The analogy with the process of building up a flux from a dc source through a resistance has been applied. This leads, with the help of (8), to the formula[6]

$$T = \frac{1}{4f} \times \text{(over-all volts-per-turn gain).} \qquad (11)$$

A number of equivalent expressions can readily be derived by simple arithmetic:

[6] S. E. Hedstrom and L. F. Borg, "Transductor fundamentals," *Electronics*, vol. 21, pp. 88–93; September, 1948.

$$T = \frac{1}{4f} \times (\text{ampere-turn gain}) \times \frac{R_{\text{ac circuit}}}{R_{\text{control circuit}}}, \quad (11(a))$$

$$T = \frac{1}{4f} \times \frac{\text{over-all power gain}}{\text{ampere-turn gain}} \quad (11(b))$$

$$T = \frac{1}{4f} \sqrt{\text{over-all power gain}} \sqrt{\frac{R_{\text{ac circuit}}}{R_{\text{control circuit}}}}. \quad (11(c))$$

All resistances are actual resistances divided by the square of the number of turns per core of the respective circuit. "Over-all" gains are computed in such a way that in the power circuit all internal resistance drops and power losses are counted as output, and that the internal resistance of the signal source counts as input resistance.

An alternative formula is

$$T = \frac{1}{4f} \sqrt{\text{useful power gain}} \times \sqrt{\frac{R_{\text{control winding}}}{R_{\text{load}}}} \times \frac{R_{\text{ac circuit}}}{R_{\text{control circuit}}}. \quad (11(d))$$

There is no conflict between statements such as "the time constant is proportional to the power gain" and "the time constant is proportional to the square root of the power gain", provided the full expressions (11(b)) or (11(c)) are taken into account. It is just a matter of what is kept constant.

Dc windings coupled with the control winding participate in the transient, acting as slugs. The time constant is increased by a factor

$$1 + \frac{R_{\text{control circuit}}}{R_{\text{coupled circuits}}}.$$

This applies to saturable reactors with parallel connected ac windings. Self-excitation windings are an example of cases in which rectifiers would appear to provide a continuous unidirectional path for transient currents which tend to oppose a decrease in signal. The effect is, however, confined to insignificance by the ac current retaining control of the rectifier commutation.

If the signal itself is derived from an ac source through a rectifier, reduction of the signal emf results in a transient current bypassing the ac resistance via the rectifier.

The interpretation, often given to the expression for the time constant, that increased supply frequency reduces the time constant is, according to (11), correct with the qualification that the overall volts per turn gain has to be kept constant. However, as the amplifying properties could be increased proportional to the frequency, any resultant advantage regarding time constant goes at the expense of the potential service obtainable from a given frame size.

If a low-frequency signal of circular frequency ω is applied, the output is not in phase with the signal emf but is lagging by an angle

$$\phi = \tan^{-1} \omega T, \quad (12)$$

(T being the time constant), and the volts per turn gain is reduced from A_{dc} to

$$A = A_{dc} \cos \phi. \quad (13)$$

With increasing signal frequency the attenuation approaches 6 decibels per octave.

For a push-pull amplifier the expressions given for T are still valid, each transductor sensing a load composed of mixing resistor and useful load.

If two amplifiers are connected in cascade, their transient response is described by a magnitude T which can be computed from the time constants T_1 and T_2 of the individual stages:

$$\frac{T_2}{T} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{T_2}{T_1}}. \quad (14)$$

There are damped oscillatory as well as double-exponential transients, and features similar to those of forcing circuits can be produced. By way of an analogue, the first stage can be visualized as a capacitance, the second stage as an inductance, the two being connected in series with a resistance. The critical ratio $T_2/T_1 = \frac{1}{4}$ gives a single exponential transient with $T = 2T_2 = \frac{1}{2}T_1 = \sqrt{T_1 T_2}$, illustrating the considerable advantage in response time obtainable by cascading.

V. *Conclusion: Scope and Limitations*

Magnetic amplifiers have come to stay in applications requiring sturdiness and longevity, and should also be considered where filament heating circuits are unsatisfactory from the aspects of starting-up time, heat dissipation, or power consumption. For dc amplification they provide an advanced solution. Basically they are current amplifiers as opposed to voltage amplifiers, and serve a range from very low to moderately high input impedances, say up to 50 or 100 kilo-ohms.

Increased supply frequency (limited by the efficiency of the magnetic circuit) results in higher output or shorter response time, or in reduced size and weight. Judicious selection of circuits and careful matching of components is required for push-pull arrangements, the use of which is indispensable for precision amplifiers of high sensitivity. The latter is determined by the short-term and long-term zero wander which for existing push-pull designs are of the orders of 10^{-12} and 10^{-11} watts, respectively. As to linearity and stability of gain, push-pull amplifiers satisfy average accuracy requirements, provided power gains of 40,000 per stage (for ac supply with mains frequency) are not exceeded. Power gain control ratios of 50:1 are obtained by negative feedback. Response time problems are fully understood; higher supply frequency or cascading are the more obvious means of shortening. True reproduction of oscillatory signals ceases at very low signal frequencies inversely proportional to the time constant.

Acknowledgment

The authors wish to express appreciation of the pleasant and clarifying discussions with S. E. Tweedy.

Some Fundamental Properties of Transmission Systems*

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Summary—The problem of the minimum loss in relation to the singing point is investigated for generalized transmission systems that must be stable for any combination of passive terminating impedances. It is concluded that the loss may approach zero db only in those cases where the image impedances seen at the ends of the system are purely resistive. Moreover, in such cases, the method of overcoming the transmission loss, whether by conventional repeaters or by series and shunt negative impedance loading, or otherwise, is quite immaterial to the external behavior of the system as long as the image impedances are not changed. The use of impedance-correcting networks provides one means of insuring that phase of the image impedance of the over-all system approaches zero.

General relations are derived which connect the image impedance and the image gain of an active system with its over-all performance properties.

SINCE THE TIME when amplifiers first were introduced into the telephone plant, the properties of two-way repeaters have been subjected to extensive analysis. From this it might be inferred that further study is likely to uncover very little that is not already known. Nevertheless, it frequently happens that new types and permutations of repeater and loading circuits are proposed, and current methods of analysis are found to be quite difficult.

In the face of this situation, the present paper is intended to review the underlying fundamentals, and to present them in what is hoped to be a form that will allow them to be simply and easily applied in determining the over-all performance. In a wider sense, what is attempted is to state certain basic physical properties and limitations in a way that allows one to say, "Regardless of detail, if these rules are violated, it follows that the circuit cannot perform as predicted," or, on the other hand, to say, "The ideal performance of such-and-such a system is so-and-so. If the proposed plan does not approximate this ideal, it must be possible to find a better one."

As sometimes happens, this review of the properties of transmission systems has led to several concepts which are thought to be new. Their importance becomes more pronounced in connection with the current tendency to reduce the net operating loss of telephone systems to lower values than were customary in the past.

In the case of the telephone repeater, the extent of the various combinations and permutations that are encountered in practice has made difficult the statement of generalizations in simple terms. The present attempt is based on the development of linear network theory in respect to active four-poles that has been progressing perhaps quietly but nonetheless steadily in the past years. Like most mathematical generalizations, the solution of one problem is really the solution of a class of problems, and it will be found that in their broadest form, the generalizations which are now presented are just as applicable to the case of four-wire telephone and radio systems as they are to the conventional two-way repeater.

The system to be considered may contain repeaters of the 22-type, such as is illustrated in Fig. 1, or it may contain any of the other varieties. Moreover, there is no restriction placed on whether the gain is the same in both directions or not, and sections of line or of other

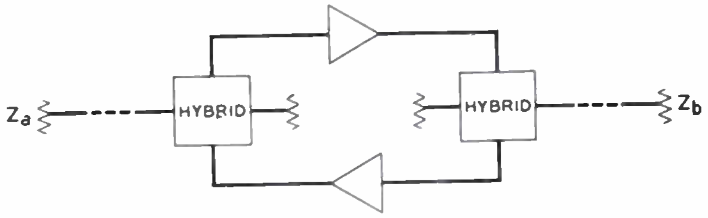

Fig. 1—Schematic of 22-type repeater.

circuit networks may be included as part of the unit under consideration. Even more broadly, the unit considered may consist either of a single repeater section, or of an unlimited number of repeater sections in tandem comprising an entire system. Restrictions are placed on these broad limits only in dealing with specific applications.

The analysis then directs itself to the general linear four pole such as is illustrated in Fig. 2 where the rectangular box may contain as much or as little as meets the needs of the particular situation. When terminations are added, the diagram illustrates the situation.

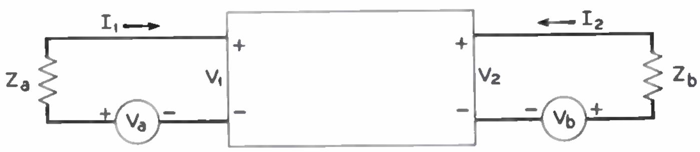

Fig. 2—Diagram of linear four-pole with terminations.

The equations describing Fig. 2 may be written:

$$\begin{aligned} Z_{11}I_1 + Z_{12}I_2 &= V_1 \\ Z_{21}I_1 + Z_{22}I_2 &= V_2 \end{aligned} \tag{1}$$

where the Z's are characteristic of the four-pole only, and do not involve the terminations. More will be said later about their properties and how they are derived.

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The corresponding equations including the terminations may be written down immediately by noting that the terminations Z_a and Z_b are related to the currents and voltages by the formulas

$$\begin{aligned} V_1 &= V_a - Z_a I_1 \\ V_2 &= V_b - Z_b I_2. \end{aligned} \tag{2}$$

When combined with (1) these give

$$\begin{aligned} (Z_{11} + Z_a)I_1 + Z_{12}I_2 &= V_a \\ Z_{21}I_1 + (Z_{22} + Z_b)I_2 &= V_b, \end{aligned} \tag{3}$$

which may be solved for the currents,

$$I_1 = \frac{V_a(Z_{22} + Z_b) - V_b Z_{12}}{\Delta} \tag{4}$$

$$I_2 = \frac{(Z_a + Z_{11})V_b - Z_{21}V_a}{\Delta} \tag{5}$$

where

$$\Delta = Z_{11}Z_{22} - Z_{12}Z_{21} + Z_a Z_{22} + Z_b Z_{11} + Z_a Z_b \tag{6}$$

is the determinant of the system of (3). It will be noted, and later use will be made of the fact, that the determinant of (1) does not depend on the terminations, and is given by

$$\Delta_0 = Z_{11}Z_{22} - Z_{12}Z_{21}, \tag{7}$$

and, consequently, that (6) may be written

$$\Delta = \Delta_0 + Z_a Z_{22} + Z_b Z_{11} + Z_a Z_b. \tag{8}$$

When the four-pole of Fig. 2 is driven from the left, V_b may be set equal to zero in (4) and (5). Under these conditions, the generator V_a sees the internal impedance Z_a in series with the impedance presented by the four-pole. From (4) we have then

$$V_a = I_1 \frac{\Delta}{Z_{22} + Z_b} \cdot \tag{9}$$

But we can write

$$V_a = I_1(Z_a + Z_A) \tag{10}$$

where Z_A is the input impedance of the four-pole when it is terminated by Z_b. It results from (9) and (10) that

$$Z_A = \frac{\Delta}{Z_{22} + Z_b} - Z_a. \tag{11}$$

or, by (8),

$$Z_A = \frac{\Delta_0 + Z_b Z_{11}}{Z_{22} + Z_b} \cdot \tag{12}$$

An exactly similar procedure based on driving the four-pole from the right instead of from the left, gives the impedance seen looking into that end when the left-hand termination is Z_a. The result is:

$$Z_B = \frac{\Delta_0 + Z_a Z_{22}}{Z_{11} + Z_a} \cdot \tag{13}$$

From these last two relations, it is possible to find the impedance values for the terminations Z_a and Z_b that would simultaneously match the impedances Z_A and Z_B. These are the so-called image impedances, and are found by putting

$$\begin{aligned} Z_a &= Z_A = Z_{\mathrm{I}} \\ Z_b &= Z_B = Z_{\mathrm{II}}, \end{aligned}$$

and solving (12) and (13) simultaneously. The result is:

$$Z_{\mathrm{I}} = \sqrt{\frac{Z_{11}}{Z_{22}}\Delta_0} \tag{14}$$

$$Z_{\mathrm{II}} = \sqrt{\frac{Z_{22}}{Z_{11}}\Delta_0}. \tag{15}$$

When the terminations have these values, there are no reflections from the terminating impedances (although there may be internal reflections within the four-pole) and, in the cases where the image impedances (14) and (15) are pure resistances, the gain in power resulting from the presence of the four-pole is a maximum.

Concerning these power relationships, there is a good deal more that needs to be said. In the first place, it turns out to be more convenient to deal in terms of "virtual" power rather than real power. The difference is that the former is given by I^2Z in general, even when the currents are represented by complex numbers involving imaginaries, while the latter is equal to the product of the square of the magnitude of the current times the resistive component of the impedance. Consequently, the writing is greatly simplified by the concept of virtual power, whereas the real power may be found from it when that is required, and the phase of the terminations is known.

When the four-pole is driven from the left, so that V_b may be put equal to zero in (5), the virtual power in the output termination Z_b is given by

$$I_2^2 Z_b = V_a^2 \frac{Z_{21}^2}{\Delta^2} Z_b = V_a^2 \frac{Z_{21}}{Z_{12}} \frac{Z_{12}Z_{21}}{\Delta^2} Z_b. \tag{16}$$

The operating gain is defined as the ratio of this to the virtual power that the generator V_a would deliver directly to a matched load, Z_a. Thus, when the generator V_a is connected to a matched load, the current is $V_a/2Z_a$, and the virtual power in the load is $V_a^2/4Z_a$. The operating gain[1] is therefore

$$\Gamma_{21} = \frac{Z_{21}}{Z_{12}} \cdot \frac{Z_{12}Z_{21}}{\Delta^2} \cdot 4Z_a Z_b \tag{17}$$

where symbol Γ_{21} indicates that the gain is from left to right. In the opposite case, where the four-pole is driven from V_b on the right while the virtual output power is absorbed in Z_a on the left, the corresponding expression for operating gain is

[1] The insertion gain may be found by multiplying the operating gain by

$$\frac{(Z_a + Z_b)^2}{4Z_a Z_b} \cdot$$

$$\Gamma_{12} = \frac{Z_{12}}{Z_{21}} \frac{Z_{12}Z_{21}}{\Delta^2} 4Z_a Z_b. \tag{18}$$

It is obvious, therefore, that the ratio of the gains in the two directions is $(Z_{21}/Z_{12})^2$. When Z_{12} is equal to Z_{21}, the gain (or loss, which is the reciprocal of the gain) in the two directions is likewise the same.

The expressions (17) and (18) are not particularly complicated, but for physical interpretation they may be put into very much better shape. This requires a little algebra, but, to make our proofs complete, it is worth outlining the procedure in some detail, rather than merely stating the final result.

The first two steps may probably be combined into one without impossing undue difficulties. Thus, from (7) the expression Z_{12}/Z_{21} may be replaced by $Z_{11}Z_{22}-\Delta_0$. This is the first step. The next one is a matter of definition, and merely eliminates Z_a and Z_b by introducing the ratios

$$\begin{aligned} a &= Z_a/Z_I \\ b &= Z_b/Z_{II}. \end{aligned} \tag{19}$$

When these substitutions are made in (17), remembering that Δ is given by (8), we have, with the help of (14) and (15),

$$\begin{aligned} \Gamma_{21} &= \frac{Z_{21}}{Z_{12}} \frac{[Z_{11}Z_{22}-\Delta_0]4ab\Delta_0}{[\Delta_0(1+ab)+(a+b)\sqrt{Z_{11}Z_{22}\Delta_0}]^2} \\ &= \frac{Z_{21}}{Z_{12}} \frac{\left[1-\frac{\Delta_0}{Z_{11}Z_{22}}\right]4ab}{\left[a+b+(1+ab)\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}\right]^2} \\ &= \frac{Z_{21}}{Z_{12}} \frac{1-\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}{1+\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}} \left[\frac{1+\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}{a+b+(1+ab)\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}\right]^2 4ab. \end{aligned} \tag{20}$$

In the event that the terminations on input and output sides are matched to the image impedances, so that a and b are both equal to unity, the gain from (20) is given by

$$\Gamma_{21}' = \frac{Z_{21}}{Z_{12}} \frac{1-\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}{1+\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}, \tag{21}$$

which is often written in the alternative form

$$\Gamma_{21}' = \frac{Z_{21}}{Z_{12}} \frac{1-\tanh\theta}{1+\tanh\theta}$$

where θ is the propagation constant of the four-pole. It is convenient to write this matched gain in the more condensed form

$$\Gamma_{21}' = \frac{Z_{21}}{Z_{12}} \Gamma_0$$

where the image gain Γ_0 is defined by the relation

$$\Gamma_0 = \frac{1-\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}{1+\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}}}. \tag{22}$$

The quantities under the radical may then be written as follows by solving (22):

$$\sqrt{\frac{\Delta_0}{Z_{11}Z_{22}}} = \frac{1-\Gamma_0}{1+\Gamma_0}. \tag{23}$$

For its physical meaning, note that in the matched condition, Γ_0 is the geometric mean between the gains in the two directions.

Substitution of (23) into (20) gives:

$$\Gamma_{21} = \frac{Z_{21}}{Z_{12}} \Gamma_0 \frac{4a}{(1+a)^2} \cdot \frac{4b}{(1+b)^2} \cdot \frac{1}{\left(1-\Gamma_0 \frac{1-a}{1+a} \cdot \frac{1-b}{1+b}\right)^2}. \tag{24}$$

The expression (24) is now in the form which we were seeking. Its advantage is the physical interpretation which may be given to factors of the form

$$\frac{4a}{(1+a)^2} \quad \text{and} \quad \frac{1-a}{1+a}.$$

The first of these might be called the "mismatch" factor, and expresses the ratio of the virtual power which a generator puts into a load connected directly across its terminals to the virtual power it would put into a matched load similarly connected. The situation is well known for the case where a is a real number, and calculation illustrates how slowly the gain departs from its matched value as the impedance ratio departs from unity. For example, a two-to-one impedance mismatch means a loss of 0.5 db only. Even a ten-to-one mismatch gives only 4.8 db loss. Note, too, the curve is symmetrical about the value of unity for the impedance ratio.

The other factor is the ratio of the reflected to the incident current at the end of a line terminated by an impedance mismatch. Its reciprocal is thought to constitute a more precise definition of "return loss" than is usually given in current literature. Note also that the two factors are related through the equation

$$\frac{4a}{(1+a)^2} + \left(\frac{1-a}{1+a}\right)^2 = 1, \tag{25}$$

which states the physical fact that the sum of the absorbed power and the reflected power is equal to the incident power.

With these relations in mind, it is possible now to interpret the various factors in (24) in connection with the diagram of Fig. 2. Imagine the generator V_a to send a wave into the four-pole represented by the rectangle in the drawing. Disregarding the factor Z_{21}/Z_{12} for the moment, we can visualize the wave as progressing from the generator V_a toward the right until it meets the impedance discontinuity between Z_a and Z_I, the image impedance of the four-pole seen from the left. Of the virtual power in the incident wave, the fraction $4a/(1+a)^2$ progresses on into the four-pole while the remainder is reflected and lost in the generator impedance. Having entered the four-pole, the current wave is amplified by the factor $\sqrt{\Gamma_0 Z_{21}/Z_{12}}$, and emerges from the right-hand end of the rectangle. Here another impedance discontinuity is encountered and the fraction $4b/(1+b)^2$ of the power enters the load, while the fraction $(1-b)/(1+b)$ of the current is reflected and progresses back toward the left through the four-pole. The current is amplified by the amount $\sqrt{\Gamma_0 Z_{21}/Z_{12}}$, is reflected in part by the factor $(1-a)/(1+a)$ at the left-hand termination, and moves once more toward the right. Thus, within the four-pole there is set up a to-and-fro surging which, each time the wave arrives at the right, contributes a little more to the power in the output.

In a single round trip through the four-pole, the wave of current or voltage is modified by the factor

$$\sqrt{\Gamma_0 \frac{Z_{21}}{Z_{12}}}\sqrt{\frac{Z_{12}}{Z_{21}}\Gamma_0}\,\frac{1-a}{1+a}\cdot\frac{1-b}{1+b} = \Gamma_0\frac{1-a}{1+a}\cdot\frac{1-b}{1+b},$$

and the sum of an infinite number of round trips assumes the form

$$S = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \tag{26}$$

where

$$x = \Gamma_0\frac{1-a}{1+a}\cdot\frac{1-b}{1+b},$$

and when $|x| < 1$. The square of the sum must be taken in (24) because S represents a current, while (24) represents a power ratio. It is thus seen then that all of the factors in (24) may be accounted for on a physical basis, and the whole action may consequently be thought of in pictorial perspective. The usefulness of introducing the image gain Γ_0, which is the geometrical mean of the forward and reverse gains, has also been demonstrated in this connection.

However, its usefulness does not stop with (24), and the impedances presented by the four-pole may also be expressed in terms of Γ_0. Thus (12) and (13) may be written respectively, with the help of (23):

$$\frac{Z_A}{Z_I} = \frac{1-\Gamma_0\dfrac{1-b}{1+b}}{1+\Gamma_0\dfrac{1-b}{1+b}} \tag{27}$$

$$\frac{Z_B}{Z_{II}} = \frac{1-\Gamma_0\dfrac{1-a}{1+a}}{1+\Gamma_0\dfrac{1-a}{1+a}}. \tag{28}$$

These show immediately that the impedance presented by the four-pole becomes the same as the image impedance whenever $a=1$ in the one case, or $b=1$ in the other. This is, of course, axiomatic. A much more striking property is shown by noting that the intrinsic algebraic sign of the impedance must perforce be the same as that of the image impedance whenever the magnitude of $\Gamma_0(1-b)/(1+b)$ in the one case, and of $\Gamma_0(1-a)/(1+a)$ in the other, is less than unity. The converse is true when the magnitudes are greater than unity, so that whenever the image gain is sufficiently large, the input impedance is the negative of the image impedance unless a or b, as the case may be, is identically unity.

When $\Gamma_0=1$, it is interesting to note that $Z_A=Z_bZ_I/Z_{II}$, and when $\Gamma_0=-1$, that $Z_A=Z_IZ_{II}/Z_b$.

Having dealt now with the derivation and discussion of expressions for impedances and gains, we come to the very important question of stability, that is, freedom from oscillation. This may be approached in several ways, but the most rigorous is probably to return to the general equations (3), and their solutions given by (4) and (5). From (4) and (5) it is seen that the currents I_1 and I_2 may be different from zero even in the absence of the driving sources V_a and V_b whenever $\Delta=0$. But Δ is a function of all of the internal network impedances as well as of the terminations Z_a and Z_b. In turn, all of these impedances are functions of $j\omega$. For purposes of analysis, $j\omega$ may be replaced by the more general variable $p=\alpha+j\omega$, so that currents and voltages of the form $e^{j\omega t}$ now become $e^{pt}=e^{(\alpha+j\omega)t}$. The significance of α then becomes apparent. When it is positive, the currents and voltages increase indefinitely with time. When it is zero, they are the usual sinusoids of constant amplitude, and when it is negative, the currents and voltages decrease with time and eventually die away altogether.

For stability it is evident that the relation $\Delta=0$ must be satisfied for negative values of α only, and not for positive values, as otherwise the currents in (4) and (5) would increase indefinitely with time, even in the absence of the driving sources V_a and V_b. If the equation $\Delta=0$ is satisfied only for negative values of α, the currents die away when the sources are removed and the system is stable, except in the contingency that one of the coefficients of V_a or V_b in (4) or (5) should become infinite for some positive value of α while, at the same time, Δ itself remained finite. Since Δ may be written

$$\Delta = (Z_{11}+Z_a)(Z_{22}+Z_b) - Z_{12}Z_{21},$$

and consequently involves all of the aforementioned coefficients, Δ can remain finite when one of the coefficients becomes infinite only if the coefficient with which it is paired in the above expression for Δ becomes zero simultaneously or (a more usual situation) is identically zero for all values of p. That is, either $(Z_{11}+Z_a)$ is infinite for the same value of p that causes $(Z_{22}+Z_b)$ to become zero, or vice versa, or else Z_{12} is infinite for the same value of p that causes Z_{21} to become zero. In either event, instability would require that the real part of p should be positive. This alternative contingency seldom occurs in bilateral systems, but is not infrequently encountered in unilateral cases. One particular example that is illustrative happens when the interstage coupling circuit between two unilateral amplifier stages contains negative impedances and, when isolated, is unstable. Connecting it between two vacuum tubes does not cause it to become stable, and it will be found that the four-pole equations for the system show that Z_{12} is zero for all frequencies, but that Z_{21} may become infinite for a positive value of α.

Whenever (3) is derived by first writing the mesh equations for the entire multi-mesh network, one equation for each mesh, and from these equations eliminating all currents but the two corresponding to the input and output meshes, the stability conditions are completely determined either by the vanishing of Δ, or by the simultaneous vanishing of one of a pair of factors forming Δ together with the vanishing of the reciprocal of the other.

Possibility of trouble occurs, however, when approximations are made. For example, when a vacuum tube with feedback is considered, the impedance looking into a pair of terminals may become negative in certain frequency ranges. There is then a strong inclination to simplify by replacing the complete details of the circuit which produced the negative impedance by the negative impedance itself. Actually, there is no objection to doing this providing that the negative impedance is completely and accurately specified over the whole frequency range.

This point is very important. For example, note that a negative impedance which was the exact negative of some passive impedance over the whole frequency range from zero to infinity, could not possibly be unstable on either open or short circuit. This is at once evident when it is considered that the values of p which satisfy the passive equation

$$Z(p) = 0$$

are identical with those that satisfy the active equation

$$-Z(p) = 0$$

and hence, if the α for the one is always negative, so also is the α for the other. From this it may further be concluded that a negative impedance which is unstable on either open or short circuit cannot possibly be the exact negative of any passive impedance whatever over the whole frequency range. One can go even further, however, and invoke some of the methods of complex function theory to show that such a negative impedance cannot even be the exact negative of any passive impedance over any finite frequency band, no matter how small.

The point of this discussion is to bring out the fact that stability or lack of it in systems involving negative impedances is often determined by the departure of the negative impedances from being the negatives of passive impedances, and hence that any disregard of this fundamental fact is likely to lead to trouble. These departures may, and in fact often do, exist at frequencies outside of the band that is of interest from the standpoint of normal use. Their effect reflects back into that band nonetheless.

How then should one proceed? Is the device of using the concept of negative impedances of no practical value? The answer to this is supplied in part by Crisson, who, some years ago, introduced the concept of series and shunt types of negative resistances. By definition, the series type is unstable on short circuit, and the shunt type is unstable on open circuit. Interpreted in the light of the foregoing discussion, these definitions may be rephrased somewhat as follows:

A negative resistance is one which behaves very nearly like the negative of a positive resistance over a fairly large frequency range. Outside of that range, however, a series type negative resistance departs from that approximation in such a way that the circuit element is unstable on short circuit, and a shunt type negative resistance departs in such a way that the circuit element is unstable on open circuit. Graphically, this would imply that, if the imaginary part of the negative impedance were plotted against the real part for all values of frequency, that is for all values of $p = \alpha + j\omega$ where $\alpha = 0$, the graph of a series type would look something like Fig. 3, and the graph of a shunt type would look something like Fig. 4. They both encircle the origin, but in different directions. It is obvious that this approximation, useful as it is, has certain limitations, and that the safest way of dealing with new or untried circuits is to be sure that the negative impedance is, in fact, specified to a sufficient extent over the whole pertinent

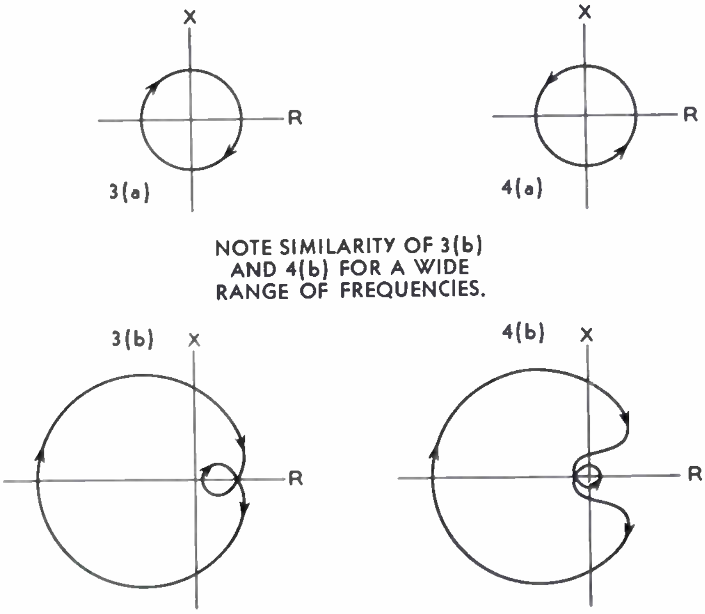

Fig. 3 (a) and (b)—Examples of graphs of series type negative impedance.

Fig. 4 (a) and (b)—Examples of graphs of shunt type negative impedance.

frequency range. Such a range would have to be sufficient to insure that the combination of the negative element with the remainder of the circuit did not have a resistive component that became negative at any higher or any lower frequency.

The practical effect of all of this is to point out that certain combinations of series and shunt type negative resistances may be quite stable, while others may not. The general stability criterion, when all things are taken into account, is the determination of the values of p that cause Δ in (4) or (5) to become zero. The alternative condition that results in instability can usually be detected by general inspection of the circuit, or may be tested for each of the four possible contingencies separately.

The investigation of Δ itself turns out to be rather cumbersome, and an easier alternative arises when it is noticed that the expression for gain Γ, given by (17), contains Δ in the denominator. It follows that Γ has an infinity whenever Δ has a zero. Also, Γ has no infinities that are not contributed by zeros of Δ. This may be verified by inserting (6) into (17) and noting that infinities of Z_a and Z_b contribute only zeros to Γ, while infinities of Z_{12} and Z_{21} contribute neither zeros nor infinities to Γ. Consequently, except for the case mentioned before where Z_{21}/Z_{12} has an infinity while $Z_{12}Z_{21}$ does not the zeros of Δ are uniquely determined by the infinities of Γ and, when Δ has no zeros with positive real parts, Γ has no infinities with positive real parts. For every zero of Δ that does have a positive real part, Γ has an infinity with a positive real part.

It may be taken then that, leaving aside the exceptions mentioned, (17) can be used as a basis for determining stability, and therefore that (24), which is merely (17) written in another form, can likewise be used. The infinities of (24) must be investigated to determine whether the real parts of any of them are positive.

The possible infinities of Γ are all determined by the equation

$$\left(1 - \Gamma_0 \frac{1-a}{1+a} \cdot \frac{1-b}{1+b}\right) = 0, \tag{29}$$

as may be seen from (24) by trying all of the other alternatives; namely $1/a=0$, $1/b=0$, $\Gamma_0=0$, $(1+a)=0$, and $(1+b)=0$. None of these others yields infinities.

There is a striking similarity between the form of (29) and the famous equation for the stability of feedback amplifiers, usually written

$$(1 - \mu\beta) = 0.$$

In fact, the similarity goes further than one of form only, and the discussion leading to (26) shows that the physical meaning of the factors involved is quite analogous. This at once suggests the possibility of applying the Nyquist stability criterion and plotting

$$\Gamma_0 \frac{1-a}{1+a} \cdot \frac{1-b}{1+b}$$

on the complex plane as a function of the frequency ω, and seeing whether the plot encircles the point $(1, j0)$. The trouble is that encirclement of the point $(1, j0)$ would indicate instability only under certain special conditions, and cannot be applied with complete generality. It happens that those conditions are fulfilled in the standard type of feedback amplifier, but very often are not in the more general cases which it is now attempted to discuss.

This fact is so important, and the appreciation of it seems so limited in extent, that a brief explanation of the fundamentals involved appears to be in order. The key to the situation is furnished by the realization that, in the conventional feedback amplifier, both μ and β are of the nature of constants multiplied by the ratio of output to input voltage across passive impedance functions (either self or transfer) and hence that neither of them has infinities whose real parts are positive. In the generalized repeater case of (29), where negative impedance elements may be involved, there is no assurance that this is so. In fact, it is readily seen that the reflection coefficient $(1-a)/(1+a)$ may become infinite when a negative resistance is connected facing a positive one, for then a is negative. This seems at first to be very discouraging to an attempt to draw simple conclusions and rules relating to the more general case. The situation is helped only by limiting the problem and being content, not with complete generality, but with an amount sufficient to cover the particular class of problem that is encountered in considering the telephone repeater. Here this analysis requires broadly, not only that the system be stable with a given pair of terminations, but that it be stable when its end terminations are either open circuited or short circuited in any possible combination of the terminations, and, moreover, that it shall be stable for any values of passive terminations in between these two extremes.

Further, it is evident in such a system that the ultimate terminations at the final terminals must consist of passive impedances. This at once implies that the image impedances of the four-pole representing the entire system must likewise have the properties of a passive impedance, as otherwise it may be shown, from (12) for example, that there always exists a value of passive termination that will result in instability. This is really a very important conclusion for it says that, in the design of systems involving negative impedances, care must be taken that the image impedances must have these passive properties at all frequencies if stability is to be guaranteed. This means that their resistive components must be positive at all frequencies from zero to infinity.

If the image impedances were entirely resistive at all frequencies, while the terminations were restricted to being passive, the greatest as well as the least magnitude that could be attained by factors of the form $(1-a)/(1+a)$ would occur when the termination approached a pure reactance, either positive or negative. The magnitude of the factor would then be unity for any value of terminating reactance. Its phase, however, could lie in any of the four quadrants of the complex impedance plane, depending upon the value of terminating reactance. Hence, when the magnitude of the image gain passed through unity, and in the event that the gain factor Γ_0 had even the smallest phase angle, it always would be possible to find values of terminating reactance that would cause the graph of

$$\Gamma_0 \frac{1-a}{1+a} \cdot \frac{1-b}{1+b}$$

to pass through the point $(1, j0)$ on the complex impedance plane. Minute changes in terminating reactance would then cause the graph to pass on one side or the other of the point $(1, j0)$ and consequently change the system from a stable one to an unstable one, or vice versa.

For this case, where the image impedances are pure resistances at all frequencies, it is clear that the system will be stable for any values of passive terminating impedances if and only if the magnitude of the image gain is less than unity. This is the situation that can be approached by the 22-type repeater of Fig. 1 when its image impedances are pure resistances, though even here departures of the hybrid coils from the ideal can introduce phase into the image impedances and create the more general situation which must now be discussed.

In this more general case, the restriction that the resistive components of the image impedances must be positive at all frequencies is still retained, as otherwise passive terminations which will cause singing can always be found. However, no restriction is now placed on the reactive component of the image impedances. Under these conditions, it can be shown from a theorem in functions of complex variables[2] that factors of the form $(1-a)/(1+a)$ attain their greatest and their least magnitudes as well as their greatest and smallest real and imaginary components when the terminations are pure reactances. For this condition, we can write:

$$\frac{1-a}{1+a} = \frac{Z_I - Z_a}{Z_I + Z_a} = \frac{R_I + j(X_I - X_a)}{R_I + j(X_I + X_a)}$$

$$= \sqrt{\frac{1 + y^2 - 2y \sin \phi_I}{1 + y^2 + 2y \sin \phi_I}}\, e^{-j\tan^{-1}(2y\cos\phi_I/(1-y^2))}, \quad (30)$$

where

$$y = X_a/|Z_I| \quad \text{and} \quad \phi_I = \tan^{-1}\frac{X_I}{R_I}.$$

This attains its greatest magnitude when $|X_a| = |Z_I|$. and the algebraic sign of X_a is opposite to that of X_I, In that event, the magnitude becomes

$$\left|\frac{1-a}{1+a}\right|_{\max} = \sqrt{\frac{1 + |\sin \phi_I|}{1 - |\sin \phi_I|}}, \quad (31)$$

and the phase is $\pm\pi/2$ depending upon the phase of Z_I.

[2] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand, 1945; p. 169.

The minimum magnitude is the reciprocal of (31), or

$$\left|\frac{1-a}{1+a}\right|_{\min} = \sqrt{\frac{1 - |\sin \phi_I|}{1 + |\sin \phi_I|}}, \quad (32)$$

and occurs at an angle of π with respect to that for the maximum. The real component of (30) attains its maximum value when

$$y = -\frac{1 \mp \cos \phi_I}{\sin \phi_I}. \quad (33)$$

A graph of (30) for the four cases where the phase of Z_I is 0, 30°, 45° and 60°, respectively, is shown on Fig. 5 which illustrates the locus of the function as X_a takes on all values from $-\infty$ to $+\infty$. Further study of this figure, and the equations above relating to it, shows that the curves are true circles and that the distance from the origin to the center of a given circle is equal to $\tan \phi$, where ϕ is the phase of the corresponding image impedance.

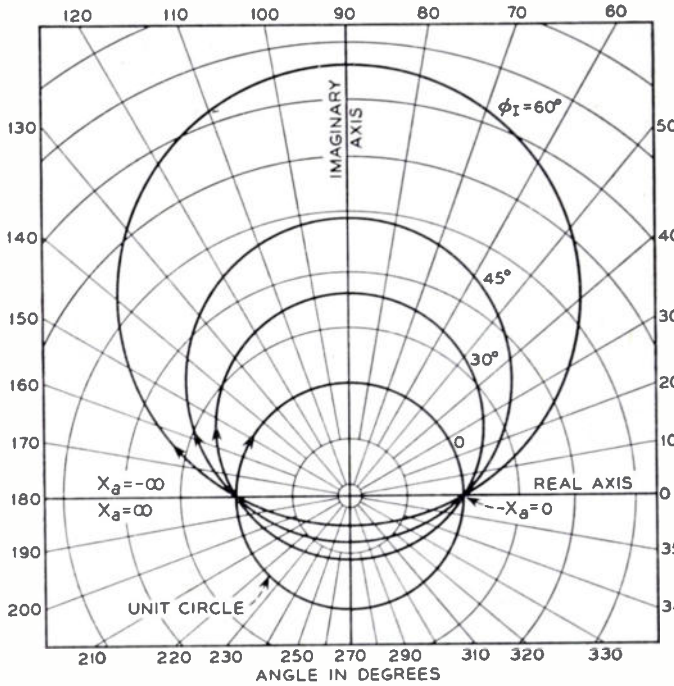

Fig. 5—Locus of $(1-a)/(1+a)$ for increasing values of X_a, where $a = jX_a/Z_I$.

At the same frequency as that for which the graph of $(1-a)/(1+a)$ has been drawn in Fig. 5, the graph of $(1-b)/(1+b)$ may be constructed from the properties of Z_{II}, the image impedance at the output terminals of the network. Where the two image impedances Z_I and Z_{II} are the same, the graph of $(1-b)/(1+b)$ is a duplicate of that of $(1-a)/(1+a)$. For any combination of terminating reactance values X_a and X_b, the product of the two graphs always falls within the envelope obtained by letting $X_a = X_b$. For this condition, Fig. 6 shows the product curve for several values of the phase of the image impedance, and it will be noted that the external envelope of the complete surface is always equal to or greater than unity.

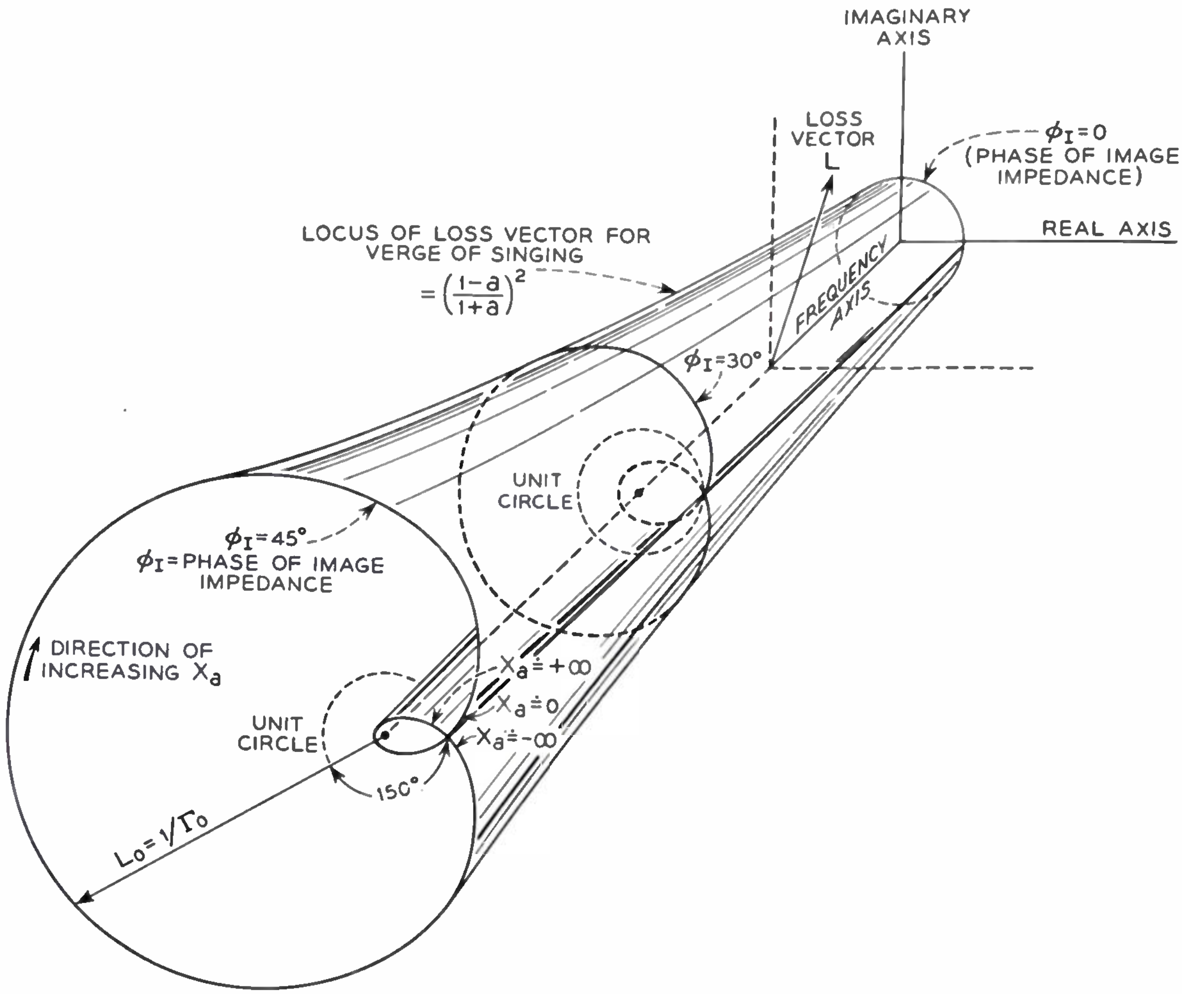

Fig. 6—Locus of stability for the loss vector for different frequencies.

Suppose, at the frequency for which the largest curve in Fig. 6 is drawn, that the gain Γ_0 has, for example, a phase of 150°. The image loss, which is the reciprocal of Γ_0, then has a phase of $-150°$. If it had a magnitude equal to that of the vector shown on the figure, the product

$$\Gamma_1 \frac{1-a}{1+a} \cdot \frac{1-a}{1+a}$$

would be exactly equal to unity, and hence, according to (29), the system would be on the verge of singing. A smaller gain at the same phase would be needed for stability or else, for this case, the same gain at a lesser phase. Fig. 6 therefore sets the relation between the allowable phase and magnitude of the gain at the particular frequency it represents, and for the symmetrical case where the image impedances at both ends of the system are the same. A curve analogous to the one considered above must be constructed for every frequency, and the three-dimensional envelope of all of them determines the allowable relationship between the maximum magnitude and phase of the gain over the frequency range. Several such curves are shown on Fig. 6 for different values of the frequency and the phase of the image impedance. The only cases in which the magnitude of the gain can approach unity are first, those for which the image impedances are both pure resistances and, second, those for which the phase of the gain is exactly zero. In all other cases, the magnitude of the gain must be less than unity to avoid singing. When the phase of the gain is 180°, its magnitude must be less than

$$\frac{1 - |\sin \phi_I|}{1 + |\sin \phi_I|}.$$

When the image impedances are pure resistances, the gain can approach unity regardless of its phase. The three-dimensional surface shown in Fig. 6 can then be regarded as setting the lower limit on the loss. The end of the loss vector must always fall outside of this surface at every frequency.

In many practical cases, the phase of the gain is not under control. For example, the phase changes very rapidly with frequency in a circuit several hundred miles long, and it would not be feasible to attempt to keep it within narrow limits over the speech band. For these usual cases, the curve on Fig. 7, which is plotted from the above expression for the magnitude of the gain, gives the allowable operating condition. For example, when the phase angle of the image impedance is 60°, the loss must exceed 11.3 db. The required loss is, of course, in addition to the allowance of a margin to take care of such things as changes in amplifier gains and in line losses.

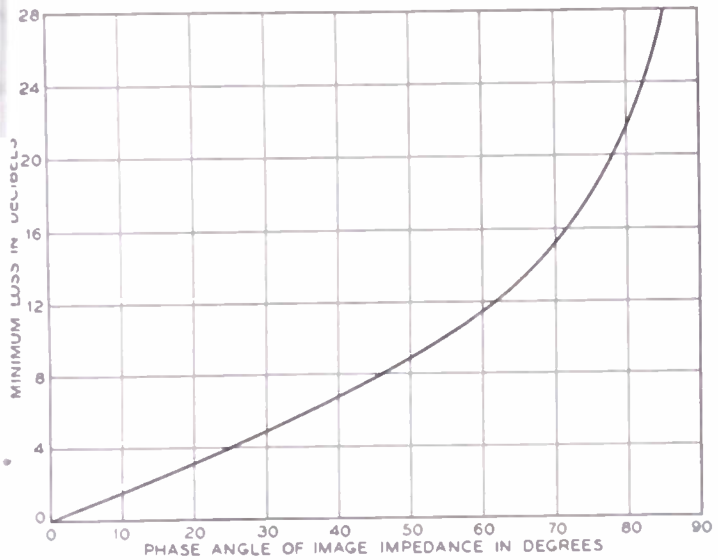

Fig. 7—Relation of minimum loss to phase angle of image impedance.

Of course, a magnitude of gain approaching unity is greatly to be desired in repeatered systems. In those which are composed of similar sections in tandem, and where, in addition, it is desired that the individual sections be stable when isolated and terminated with any combination of passive impedances, the above conditions become very important for they apply to the individual sections and severely restrict the freedom of design. Where, however, the sections need not be stable individually when subjected to all combinations of passive termination, but the system as a whole must nonetheless be stable, a good deal more freedom in design is permissible. The individual sections can actually have gains greater than unity, providing that it is removed before the final terminals are encountered.

For example, Fig. 8 shows a possible system in which repeaters or negative-resistance loading may be used quite freely in a transmission line, with the result that the system would sing if terminated at Z_I, with certain combinations of passive impedances. In general, also, the image impedances of the line will have a reactive component. At each end of the line there is placed an impedance-equalizing four-pole which matches the line on the one side, but presents a purely resistive impedance on the other. Such a network unavoidably introduces a certain loss if composed of passive elements only. An active network, however, such as the 22-type repeater circuit, can accomplish the impedance transformation without loss, or even with gain. In any event, the overall system, now having purely resistive image impedances, can be adjusted to have an overall gain that, with ideal impedance matches, approaches unity as closely as desired, and still will be completely stable for all combinations of passive termination. Any gain greater than unity, however, will result in singing, and the margin needed in practical design is a matter of how constant with time the values of the components of the system can be made, and how accurately the impedances may be matched.

In the event that the phase equalizers or converters of Fig. 8 are to be composed of passive circuit elements, the minimum loss required to consummate the impedance transformation can be found from Fig. 5. It is necessary to note that,

$$\Gamma_0 \frac{1-a}{1+a} \frac{1-b}{1+b}$$

can never pass through the point $(1, j0)$. For a reactive image impedance on the input side of the network, Fig. 5 would show the graph of the factor $(1-a)/(1+a)$ as one of the off-center circles. The graph of $(1-b)/(1+b)$ would be a unit circle, however, because of the resistive image impedance on the output side of the network, and the factor could have its phase anywhere in the four quadrants. The envelope of the product of the two factors would therefore be a circle whose radius vector was equal in magnitude to the maximum value of $(1-a)/(1+a)$, and whose phase could lie anywhere in the four quadrants. The minimum loss possible in a passive phase converter network would therefore be the

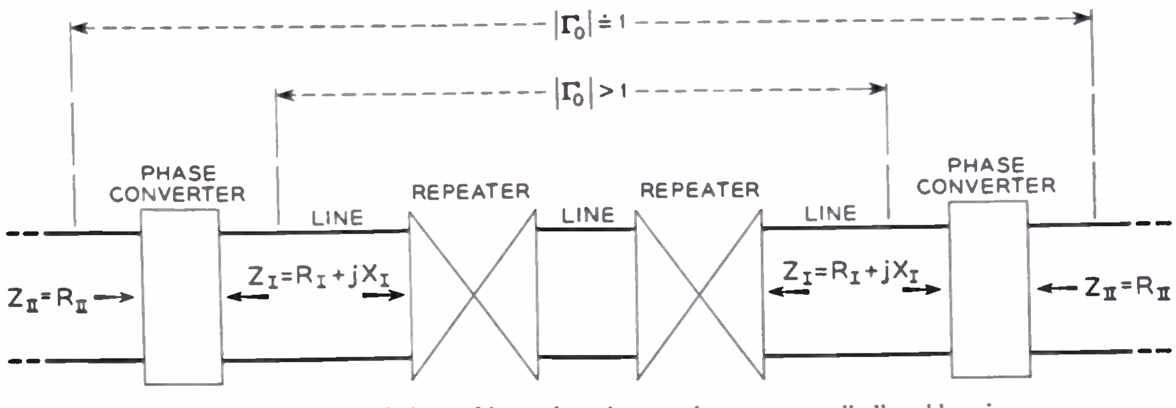

Fig. 8—Correction of phase of image impedance to increase over-all allowable gain.

reciprocal of this value, or

$$\sqrt{\frac{1+|\sin \phi_I|}{1-|\sin \phi_I|}}.$$

For systems in general, when the phase of the image impedances differ on the two ends, the stability conditions are in a sense more severe than for the symmetrical case. Here the criterion may be visualized by referring to the two curves on Fig. 5 corresponding to phases of the image impedances 30° and 60°. The first may be thought of as the reflection factor for the image impedance Z_I, and the second as the reflection factor for the image impedance Z_{II}. What is desired is an envelope analogous to Fig. 6, giving at each phase the maximum possible value of the product of the two factors. When a point on the envelope has a certain phase, ψ, the sum of the phases of the individual factors must be equal to ψ. Thus if ρ_a represents the magnitude of $(1-a)/(1+a)$ and ϕ_a represents its phase, and if ρ_b represents the magnitude of $(1-b)/(1+b)$ and ϕ_b represents its phase, the envelope at the angle ψ has the magnitude $\rho_a\rho_b$ for which $\phi_a+\phi_b=\psi$. The problem is to determine the maximum magnitude of this product for each value of ψ.

The easiest approach seems to be to deal, not with ρ_a and ρ_b directly, but with their logarithms, so that we have

$$\log \rho_a\rho_b = \log \rho_a + \log \rho_b,$$

and the problem is shifted from that of finding the maximum value of a product to the somewhat easier one of finding the maximum value of a sum, subject, however, to the same condition concerning phase, namely,

$$\phi_a + \phi_b = \psi.$$

Fig. 9 is constructed from Fig. 5, and shows $\log \rho_a$ or $\log \rho_b$ plotted against ϕ_a or ϕ_b, as the case may be, for different values of the phase ϕ_I or ϕ_{II} of the respective image impedance. The curves resemble sine waves somewhat, but are not true sinusoids although, for a rough approximation, the assumption that they are would not give large errors for moderately small value of ϕ_I.

$$\rho = \frac{1-a}{1+a}$$

$$a = jX_a/Z_I$$

$$Z_I = \text{image impedance}$$

$$= |Z_I|\, e^{j\phi_I}.$$

To illustrate the use of Fig. 9, assume for example that we are dealing with a system which has a phase of $\phi_I=30°$ for the image impedance seen from the left-hand end, and of $\phi_{II}=60°$ seen from the right-hand end. We deal then with the corresponding curves on Fig. 9, and the lower one on the left corresponds to $\log \rho_a$ and ϕ_a, and the higher to $\log \rho_b$ and ϕ_b. The envelope curve which takes the place of Fig. 6 for this case of image impedances of different phases is then constructed by finding the two ordinates $\log \rho_a$ and $\log \rho_b$, which correspond to the two angles ϕ_a and ϕ_b, such

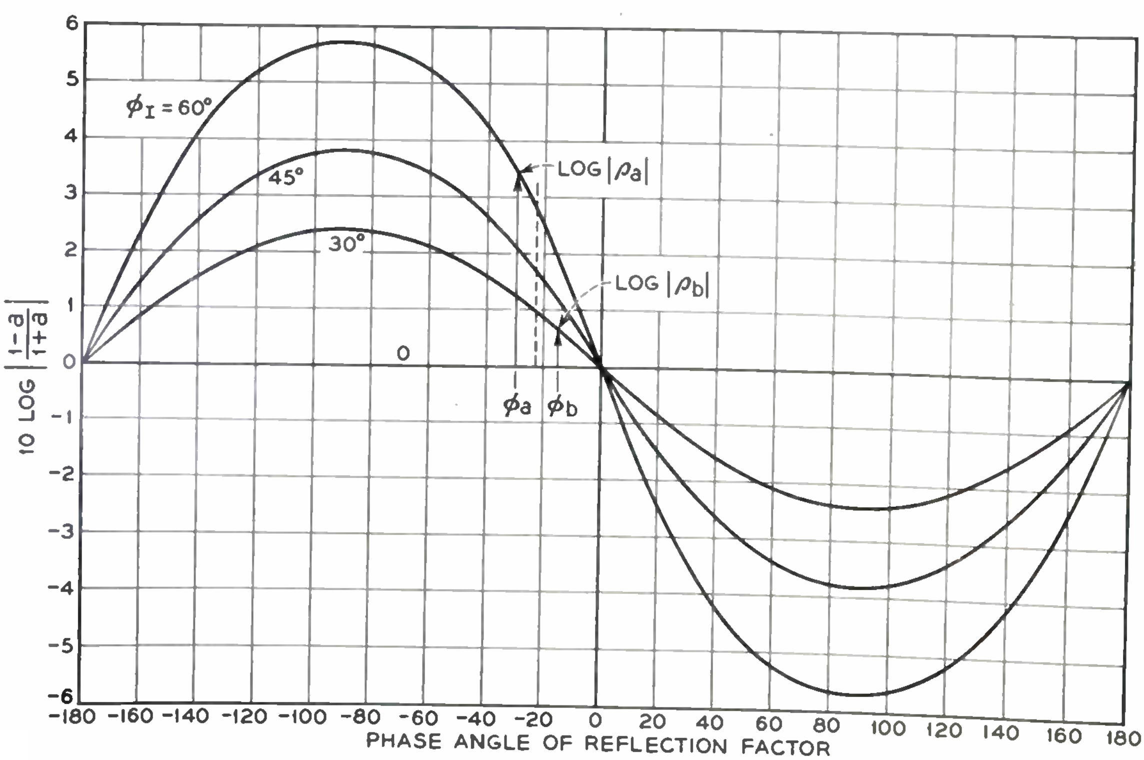

Fig. 9—Graph of logarithm of reflection factor

that the sum of the two ordinates is a maximum when $\phi_a+\phi_b$ equals the envelope angle ψ.

Algebraically this is equivalent to requiring:

$$y_a + y_b = \text{maximum when } x_a + x_b = \text{constant}.$$

Hence, for the maximum

$$\frac{dy_a}{dx_a}dx_a + \frac{dy_b}{dx_b}dx_b = 0.$$

But,

$$dx_b = -dx_a,$$

so that

$$\frac{dy_a}{dx_a} = \frac{dy_b}{dx_b}.$$

This gives the clue to the graphic solution. Suppose that $\psi=-45°$. Starting then with ϕ_a and ϕ_b, both equal to $-22.5°$, we note that the slope of the curve for $\phi_I=60°$ is much greater than that for $\phi_I=30°$. Consequently, we take a value for ϕ_a greater in magnitude than $-22.5°$, and for ϕ_b an equal amount less in magnitude than $-22.5°$ so that their sum again equals 45°. We note whether the slope of the $\phi_I=60°$ curve corresponding to ϕ_a still remains greater than that of the $\phi_I=30°$ curve corresponding to ϕ_b. If so, the departures of ϕ_a and ϕ_b from the mean of $-22.5°$ should be increased further until the two slopes are equal. When such a pair of values has been found, the envelope of the product curve for $\psi=45°$ has the value

$$\log \rho_a + \log \rho_b$$

for its logarithm.

The process is tedious and would have to be repeated for each value ψ of the envelope phase, and besides, the whole graphical construction would have to be repeated for each frequency in order to find the entire three-dimensional contour outside of which the loss must remain if stability is to be insured. Straightforward analytical solution does not offer much hope, either, for the difficulties appear to become even greater. It may happen, of course, that some change of variable or other algorithm will be found, but the probability is not at present very favorable.

In some cases, and particularly when attention is directed toward a general philosophical approach rather than to operating criteria, an extension of the stability conditions along the line proposed by Gewertz[3] has proved useful. In this extension of the work of Gewertz, the coefficients of (1) are written in the matrix form

$$\begin{vmatrix} R_{11}+jX_{11} & R_{12}+jX_{12} \\ R_{21}+jX_{21} & R_{22}+jX_{22} \end{vmatrix}.$$

It may then be shown by the argument given above, that the system is stable for any passive termination, providing that the following conditions are satisfied at all frequencies:

$$R_{11} > 0$$

$$R_{22} > 0$$

$$4(R_{11}R_{22}+X_{12}X_{21})(R_{11}R_{22}-R_{12}R_{21}) - (R_{12}X_{21}-R_{21}X_{12})^2 > 0.$$

In the event that $Z_{12}=Z_{21}$, it will readily be recognized that the last of these three relations reduces to the form

$$(R_{11}R_{22}-R_{12}^2) > 0,$$

which has come to be known as the Gewertz condition. On the other hand, in the event that $Z_{21}=-Z_{12}$ we have the alternative

$$(R_{11}R_{22}-X_{12}^2) > 0.$$

In the symmetrical case, where $Z_{11}=Z_{22}$, and where $Z_{12}=Z_{21}$, it may be shown that the Gewertz condition becomes

$$R_{11}-R_{12} > 0,$$

whence it follows that the real part of

$$Z_I\frac{1-\sqrt{\Gamma_0}}{1+\sqrt{\Gamma_0}}$$

must be greater than zero. This means that the resistive component of the short-circuit impedance of a hypothetical network of half the electrical length of the actual network, should always be positive. From this it is easy to deduce the relationship

$$(1-|\Gamma_0|)\cos\phi - 2\sqrt{|\Gamma_0|}\sin\phi\sin\beta > 0,$$

where

$$Z_I = |Z_I|e^{j\phi}$$

$$\Gamma_0 = |\Gamma_0|e^{-2j\beta}.$$

From this equation it follows that, in case $\sin\beta=1$, we have

$$|\Gamma_0| < \frac{1-|\sin\phi|}{1+|\sin\phi|},$$

which agrees with the results previously attained, and shows the connection between the two methods of approach, namely the consideration of the matrix components on the one hand, and of the image parameters on the other.

It is important to point out that, in the present extended form of the Gewertz relations, all that is assured is that the network shall remain passive regardless of what passive terminations are attached. It does not follow that the network has all of the properties of a passive system, in the sense that it may be imbedded in a general network system involving passive feedback from the output to the input and still remain completely stable. A system composed entirely of passive ele-

[3] C. M. Gewertz, "Network Synthesis," The Waverly Press, 1933; pp. 45–63.

ments would, of course, be stable under these conditions.

Notwithstanding the difficulties of the general case where the image impedances at the two ends of the system are different from each other, some of the conclusions which have been pointed out are of broad validity, and in the more restricted case where the two image impedances are the same, quantitative results may be computed with moderate ease for any phase angle of the image impedances. Expressed in terms of the image gain and the image impedances, the relations are so important and general that a few examples may make their meaning clearer.

For the first one, the unilateral amplifier will be considered in order to show that, even in this case, the general principles and method of analysis apply. The schematic circuit diagram is shown in Fig. 10, and the four-pole equations are given on the figure. It is important to note that a feedback admittance Y_x is included for purposes of analysis, but that this is ultimately allowed to become so small that feedback disappears.

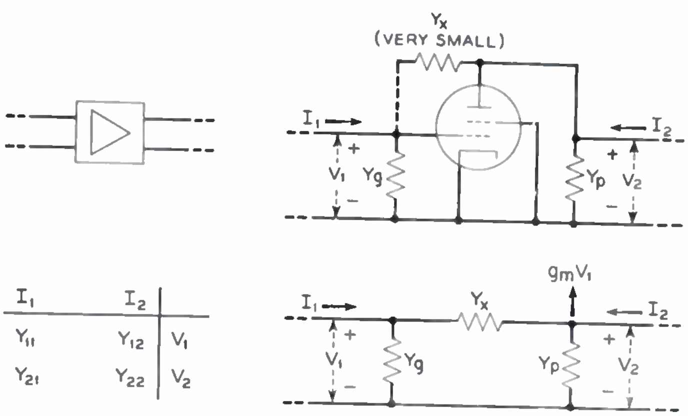

Fig. 10—Development of unilateral amplifier.

It happens that admittances are more convenient than impedances to deal with here, but the form of the various relations is not changed thereby. Application of (14), (15), and (22), gives the image parameters Y_I, Y_{II}, and Γ_0 as follows:

$$Y_I = (Y_g + Y_x)\sqrt{1 - \frac{Y_x(g_m - Y_x)}{(Y_g + Y_x)(Y_p + Y_x)}}$$

$$Y_{II} = (Y_p + Y_x)\sqrt{1 - \frac{Y_x(g_m - Y_x)}{(Y_g + Y_x)(Y_p + Y_x)}}$$

$$\Gamma_0 = \frac{1 - \sqrt{1 - \dfrac{Y_x(g_m - Y_x)}{(Y_g + Y_x)(Y_p + Y_x)}}}{1 + \sqrt{1 - \dfrac{Y_x(g_m - Y_x)}{(Y_g + Y_x)(Y_p + Y_x)}}}.$$

As the feedback admittance Y_x is allowed to become very small, (impedance very high), the image admittances easily and gracefully approach the values Y_g and Y_p, respectively. At the lower frequencies before transit-time effects enter, these are ordinary passive admittances. The image gain approaches zero, but the way it does this can best be seen by using the binomial theorem to expand the radical in the numerator. This gives

$$\Gamma_0 \rightarrow \frac{1}{4}\,\frac{Y_x(g_m - Y_x)}{(Y_g + Y_x)(Y_p + Y_x)} \rightarrow \frac{1}{4}\,\frac{g_m Y_x}{Y_g Y_p}. \quad (34)$$

In this form, it can be seen from (24) that the operating gain from left to right with matched terminations becomes

$$\Gamma_{21}' \doteq \frac{g_m - Y_x}{Y_x}\,\frac{1}{4}\,\frac{g_m Y_x}{Y_g Y_p} \rightarrow \frac{1}{4}\,\frac{g_m^2}{Y_g Y_p}, \quad (35)$$

which is recognizable as the conventional expression for this gain. On the other hand, Γ_0 itself approaches zero, and the matched gain Γ_{12}' from right to left likewise approaches zero, and in such a way that the image gain Γ_0 is the geometric mean between Γ_{12}' and Γ_{21}'. Whenever the feedback admittance Y_x is not quite zero, the circuit may yet be stable for all passive impedance terminations, but only providing that the image gain Γ_0, multiplied by the reflection coefficients, does not encircle the point $(1, j0)$.

This rather extreme illustration was chosen first to demonstrate the generality of the analysis, and to show how it applies in the unilateral case.

As an example of the bilateral case, the properties of the 22-type repeater will be considered. Fig. 1 shows the general schematic and, when the impedances seen by the hybrid coils on the network side, the transmitting side, and the receiving side are completely balanced, the image impedances are equal to the impedance of the passive balancing networks and are independent of the repeater gain. When the over-all image impedance of a system containing 22-type repeaters is a pure resistance, the repeaters may be adjusted until the gain of the system approaches unity before singing can take place. In the more usual case, the image impedances of the individual repeaters are adjusted to match that of the connecting line, which has an appreciable phase angle. Consequently the gain of the system must be held to a flat value of

$$\frac{1 - |\sin\phi_I|}{1 + |\sin\phi_I|},$$

or else must be tailored to fit the conditions discussed in connection with Fig. 6. However, the expedient of providing initial and terminating repeaters, whose input and output hybrids are matched to a pure resistance, will allow the system gain to be brought up to unity even in this case. With ideal impedance matches, the margin which must be allowed in practical design then depends upon the variations in repeater gains and line losses under operating conditions, and not upon the number of sections in the system or upon the over-all line loss. Extra margins are required for unavoidable impedance mismatches resulting from line irregularities.

With other types of repeaters, such as the 21-type illustrated in Fig. 11, the image gain and the image impedances are not so easy to adjust independently. However, there seems to be nothing fundamental to prevent supplying the terminals of the system with networks to provide a purely resistive image impedance. Practically, there are many cases where the construction of such networks offer excessive complexity, though in others their use may be quite feasible. The gain of the system can then be made unity before singing will occur, with any possible combination of passive impedances attached to the terminals.

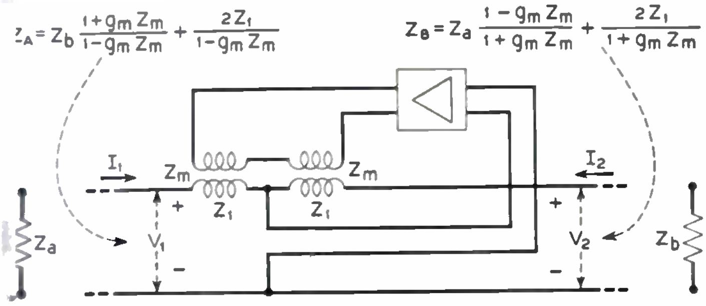

Fig. 11—Schematic of 21-type repeater.

This brings out the point that it is perfectly possible to have a system that is quite stable when terminated at mid-section points, but may be quite unstable when terminated at half-load points, and vice versa. This can happen, for example, when the image impedance is a pure resistance in the one case, but not in the other. It can happen more generally, however, whenever the phase of the image impedance is different in the two cases, and consequently the allowable gain is different.

The analysis also shows that the attempt to improve the singing margin of a system by the addition of pads of various kinds is quite futile in some cases but not in others. If a system has an image impedance that is a pure resistance, its gain may be made unity before singing can occur. The addition of a resistive pad will allow the repeater gains to be increased until the over-all gain is gain unity, but no more, and the system is back again where it started. A reactive pad added to such a system will do harm because it will produce image impedances with reactive components, and the over-all gain must remain correspondingly less than unity. However, as shown before, a phase-correcting network, or pad, applied to a system having an appreciable phase angle for its image impedance, will be helpful. We are thus led to the conclusion that an advantageous terminating four-pole is one that transforms the image impedance into a pure resistance in those cases where it initially has a reactive component.

This observation also gives the key to the best design objective for repeatered systems in general. That objective is to cause the image impedance of the system to be purely resistive to as close a degree as possible, while bringing the gain as nearly to unity as is consistent with safe singing margins.

These examples also illustrate a general conclusion that may be stated as follows:

The external stability of all systems depends only upon the phase of the image impedances and magnitude of the image gain, and not at all upon the details of the internal arrangements of the system by which these quantities are attained.

It does not follow, however, that all systems are alike in terms of the percentage change of voltage on the vacuum tubes which provide the repeater gain or the negative impedance loading, or in terms of the complexity of the equalizing and phase-correcting networks required to give the desired image-impedance terminations. The image gain of a 22-type repeater without feedback is proportional almost directly to the effective voltage of the dc supply source, while the image impedances are almost independent of this voltage. The image gain of a line with negative impedance loading may, under some conditions, vary much less rapidly with supply voltage to the tubes that furnish the negative impedance. Also, systems vary greatly in the amount of trouble resulting from line impedance irregularities.

Consequently, rather than regarding the theory here presented as saying that all systems having the same image parameters will behave alike, it may be more useful to turn the statement around and regard the theory as saying what has to be done to a given system in order that it shall be capable of operating as well as some other system. Conversely, the theory also tells how much more loss the given system perforce must have than a reference system in order to remain unconditionally stable, and it sets up specific and definite standards for the reference system. The present paper has stressed the applicability of the image parameter concept to the determination of singing conditions in telephone systems. However, the methods developed are also capable of dealing with such other properties as talker and listener echo, which are equally important in some applications. These have not been discussed in detail because the paper already is fairly long and because, with the fundamental background as presented, the reader is in a position to carry out a number of extensions for himself.

As a closing word, a few remarks concerning bibliography references are in order. It will be noticed that very few occur in the text. This is because the writer is aware of very few that have a specific and direct bearing on the mode of development of the subject which was employed. He wishes however to express appreciation of the helpful and stimulating conversations he has had with many of his colleagues on the technical staff of the Bell Telephone Laboratories. As general background to the use of image parameters in active circuit analysis, the following may be mentioned in addition to the standard modern text books:

1. H. A. Wheeler, "Wide-band amplifiers for television," Proc. I.R.E., vol. 27, pp. 429-438; July, 1939.
2. A. J. Ferguson, "Termination effects in feedback amplifier chains," *Canad. Jour. Phys.*, Section A, vol. 24, pp. 56-278; July, 1946.

The L-Cathode Structure*

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***Summary*—A new dispenser-type emitter, known as the L cathode, is described and compared with three common types of emitters. A discussion of the methods of measuring the rate of barium evaporation is included, as well as the life performance of the L cathode in a number of diversified types of electron tubes.**

Introduction

THIS PAPER is concerned with a high-emission density thermionic emission cathode known as the L cathode and described in the *Philips Technical Review*, June, 1950, by Messrs. Lemmens, Jansen, and Loosjes.

We will briefly review some of the basic features of this cathode and report some recent progress in its development.

In existing commercial tubes now available, three types of cathodes are being widely used, namely, the oxide-coated cathode, the thoriated tungsten cathode, and tungsten cathodes. The oxide-coated cathode having the best thermal efficiency and requiring the least heating power for a given electronic emission has been widely used in receiving tubes. Its undesirable properties are susceptibility to poisoning through traces of oxygen or other gases, and evaporation of barium causing grids and anodes to emit electrons. For transmitting and X-ray tubes, barium evaporation is very objectionable. In addition, these types of tubes require a cathode capable of withstanding the electrostatic forces of attraction of the anode, which is at a high potential. Oxide-coated cathode coatings peel or are pulled off by these strong electrostatic forces. For these applications, thoriated tungsten and tungsten cathodes are usually preferred.

In microwave tubes, none of the above types of cathodes displays all the desired properties of high-emission density, freedom from damage by sparking, ability to withstand the electrostatic forces of the anode, and poisoning effects of gases or vapors that result from operating the tube. Some improvements have been made in oxide-coated cathodes by reinforcing the oxide coatings with metal, as is accomplished in mesh-and "mush"-type constructions.

The L cathode, the development of which was started at the Philips Laboratories in Eindhoven, overcomes the difficulties of the above-mentioned cathodes. Its great mechanical strength, combined with high-emission density, long life, high resistivity against poisoning, and low noise characteristics, should render it suitable for a large number of tube applications.

Before considering the mechanical structure of the L cathode, we will briefly summarize the properties of the cathodes mentioned.

In Table I we note the emission capabilities of the most generally accepted cathodes used in commercially available electron tubes. In order of emission per square centimeter, the tungsten cathode has the lowest yield, and the L cathode indicates the greatest yield, particularly under dc conditions. As for efficiency, expressed in amperes per watt, the oxide cathode appeared most efficient under pulsed conditions, while the L cathode is best under dc conditions. In comparison to oxide-coated cathodes, the L cathode excels in its resistance to poisoning, to high-voltage electrostatic field, and to high-speed gas ions.

TABLE I[1]

	Maximum Useful Thermionic Emission A/cm²	*Maximum Useful Thermionic Yield A/watt*	*Poisonability*	*Resistance to High Voltage*	*Resistance to High Speed Gas Ions*
Tungsten —dc Tungsten —pulsed	1	0.006	Small	Good	Good
Thoriated W —dc Thoriated W —pulsed	2	0.070	Large	Good	Poor
Oxide cathode —dc Oxide cathode —pulsed	0.5 50	0.25 20.00	Large Large	Poor Fair	(Good for a short time)
L Cathode —dc L Cathode —pulsed	300	10.0	Small	Good	Good

Cathode Construction

In Fig. 1 we show cross-sectional views of two types of planar L cathodes. The essential parts are the molyb-

* Decimal classification: R331. Original manuscript received by the Institute, March 20, 1951; revised manuscript received, October 2, 1951.

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[1] H. J. Lemmens, M. J. Jansen, and R. Loosjes, "A new thermionic cathode for heavy loads," *Philips Tech. Rev.*, vol. 11, pp. 341–350; June, 1950.

lenum body *A*, the porous tungsten plug *B*, the barium-arbonate emission material *P*, and the heater *F*. The tructure on the left-hand side is a mechanically fitted lesign, in which the molybdenum body *A* is peened lightly over the porous tungsten plug to hold it in place and form a tight joint. In the structure on the ight-hand side, the porous tungsten plug is welded irmly to the molybdenum body. It has been found that he welded construction has been in general more satis-actory than the mechanically fitted construction.

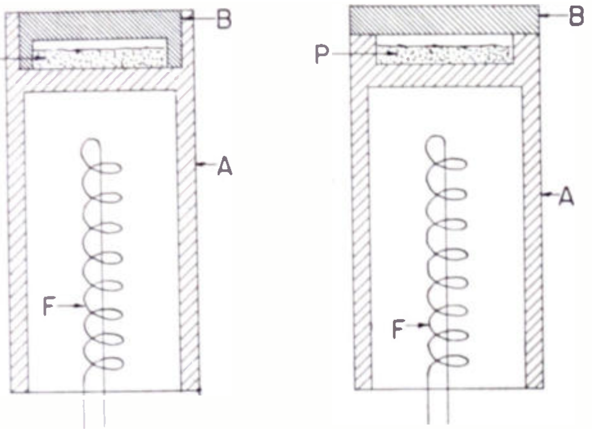

Fig. 1—L cathodes, planar types.

Fig. 2 shows cross-sectional views of two different types of cylindrical L cathodes commonly used on magnetrons, the cathode indicated on the left being of the indirectly heated type, while the one on the right is of the directly heated construction. For cathodes having small diameters, in the order of a millimeter, the cathode on the right-hand side is sometimes preferable. The indirect heater design is recommended for use where the cathode diameter is greater than 2 millimeters.

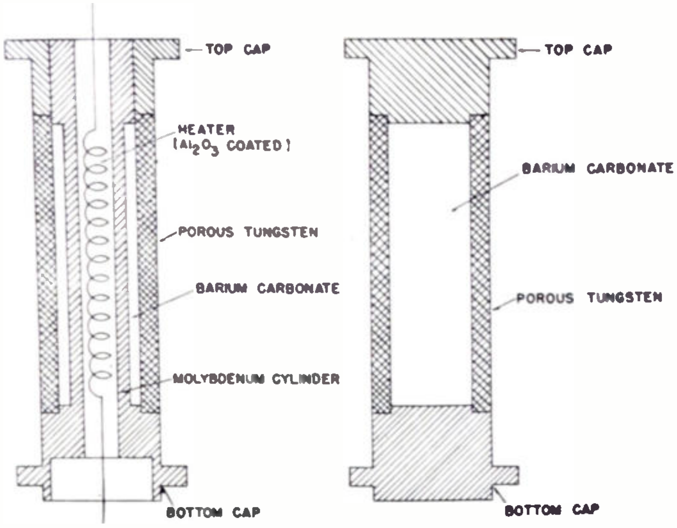

Fig. 2—L cathodes, cylindrical types.

The porous tungsten part for the types of cathodes shown is made by compressing tungsten powder and then sintering it at a high temperature.

Activation

In Table II we show a typical activation schedule. Steps 1 and 2 are more or less standard procedures for most evacuation schedules. In step 3, the barium carbonate is reduced to barium oxide. Step 4 is a preliminary step prior to the anode heating which is accomplished in step 5. Approximately 5 minutes are required to outgas the anode. Step 6 is the actual activation step in which the barium oxide is partially reduced to barium, which diffuses through the porous plug, reaching the outer surface in about 15 minutes to form a uniform emitting surface. An emission of 250 ma corresponds to 3.0 amp per cm² for a 3-mm diameter planar cathode.

TABLE II

1—Seal diode to vacuum system and check for leaks.
2—When vacuum is better than 1×10^{-4} mm of Hg, bake diode at 400 to 450°C for approximately 30 minutes.
3—Remove oven and glow heater, raising cathode temperature gradually to 1,130°C and maintaining a vacuum better than 1.0×10^{-4} mm of Hg.
4—Glow cathode at 1,270°C[2] brightness for one minute. Vacuum is better than 10^{-6} mm of Hg at end of this step.
5—Heat anode by radio-frequency induction to maximum recommended value (1,200°C brightness for zirconium-coated molybdenum) until vacuum is better than 1.0×10^{-6} mm of Hg.
6—Activate cathode at 1,270°C brightness, apply anode voltage adjusting anode voltage to 200 volts dc. Stop activation when emission current reaches 250 ma. Vacuum is better than 1×10^{-6} mm of Hg.
7—Seal from vacuum system.

Emission Characteristics

The factor of primary interest when comparing different cathodes is the temperature dependency of the emission. Fig. 3 shows how the L cathode compares with other conventional cathodes.

Here the saturation emission measured under pulsed conditions in amperes per square centimeter is plotted against the true temperature in degrees C. The vertical dotted line indicates approximately the maximum temperature at which the cathode can be used to give a life in the order of some hundreds of hours. Under dc conditions, the oxide-coated cathode cannot be used continuously for more than one hundredth part of the saturation current at 900 degrees C since the emitting area is damaged if this value is exceeded. The other types of cathodes can operate continuously with an emission close to the saturation value.

The termination point of the curves is at the maximum value of temperature at which the respective emitters can be operated. Operation at temperatures beyond these points usually results in destruction of the cathode. It is noted that for an oxide-coated cathode 120 amp per cm² can be reached under pulsed conditions, while for an L cathode about 300 amp per cm² can be attained. This value is not limited to pulsed conditions, but can be realized under cw conditions. How-

[2] All temperatures listed, with the exception of the oven temperature, are brightness temperatures measured on the molybdenum part.

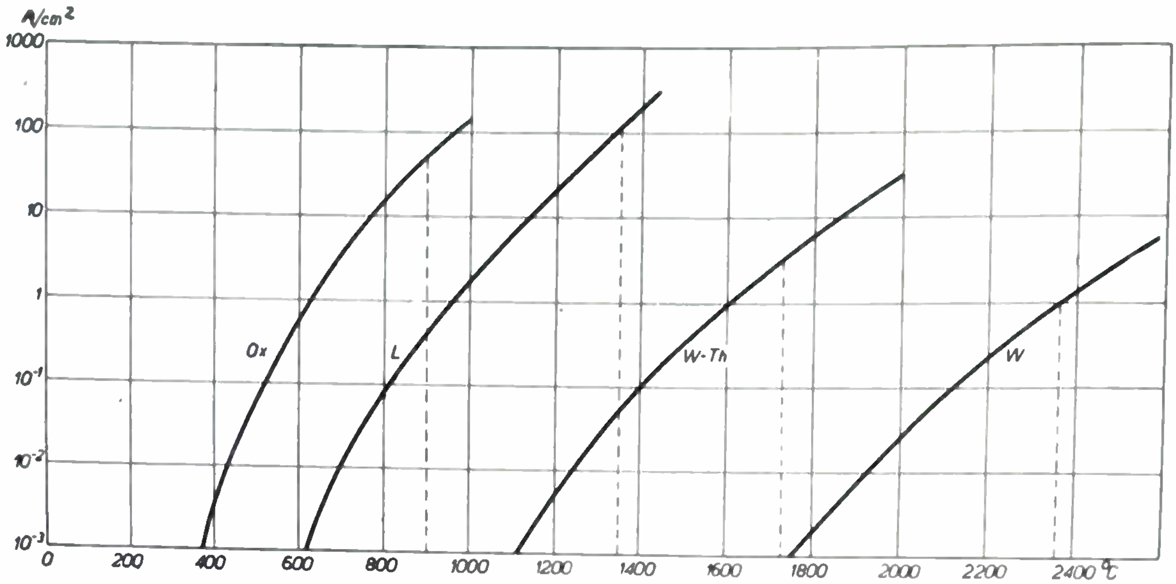

Fig. 3—Saturation emission J_s (amp per cm²) as a function of true temperature (degrees C). This curve has been taken from an article, entitled "A new thermionic cathode for heavy loads." See footnote 1.

ever, operating an L cathode under these conditions imposes other restrictions, namely, the problem of dissipating the anode power. In one test, a planar cathode having 6 mm² of surface area was assembled into a diode structure having a water-cooled anode spaced 0.5 mm from the cathode. The emission obtained was 40 amp per cm² at an anode voltage of 400 volts. Anode dissipation was in the order of 1 kw. The power input into the cathode had to be increased to maintain it at a temperature of 1,300 degrees C brightness since electron cooling at this current density is appreciable.

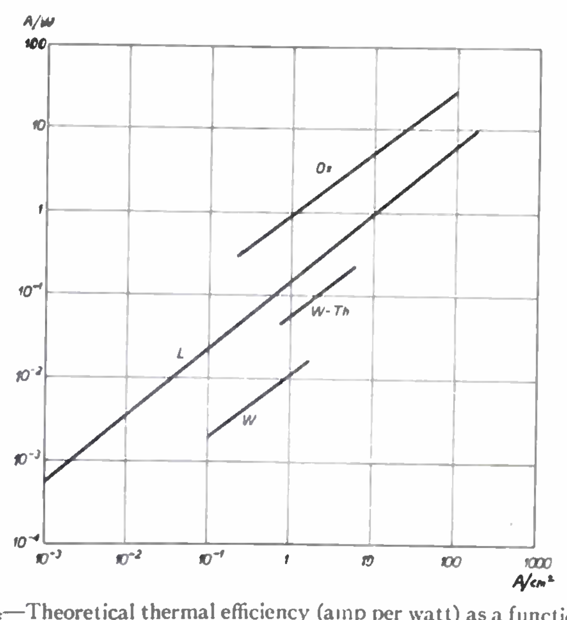

Fig. 4—Theoretical thermal efficiency (amp per watt) as a function of saturation emission (amp per cm²). (Taken from article entitled, "A new thermionic cathode for heavy loads.") See footnote 1.

In Fig. 4 we plot the theoretical thermal efficiency (in amp per watt heating power) as a function of the saturation emission expressed in amp per square cm. Here we note that the theoretical efficiency of the L cathode is greater than that of the tungsten and thoriated tungsten cathodes, but is less than that of the oxide-coated cathode. This is to be expected since the temperature of the L cathode is higher than that of the oxide-coated cathode with equal saturation emission. We appreciate that the oxide-coated cathode is the most efficient, and is highly desirable in applications where the current densities are less than 0.25 amp per cm²; but where higher densities and life are desired, we must scrutinize the thermal efficiency more closely before drawing any conclusions. If we consider an L cathode operating at a dc emission of 40 amp per cm² and multiply this value by 1.8 volts, which is in the nominal work function for the L cathode, the electrons emitted carry 72 watts per cm² out of the cathode. The heating power must therefore be increased by a like amount in order to keep the cathode surface at the temperature required for the emission. Without emission, a power of 20 watts per cm² would be adequate for this temperature.

As for the surface of the L cathode, it displays a smooth metallic emitting surface. The structure possesses great mechanical strength, and is not easily damaged in assembly. There are no particles to peel off under the influence of electrostatic attraction forces, as is the case for oxide-coated cathodes. The surface of the cathode can be turned on a lathe or molded, and it can be machined perfectly flat to exact dimensions within tolerances of a few microns. For disk-seal tubes, where cathode to grid distances are small (tens of microns) in order to shorten the transit times of the electrons, this feature offers excellent construction possibilities, even better than the finest grained oxide-coated cathodes which must be handled with care to prevent injury to the surface. For L cathodes, the surfaces can be made very small because of the high-emission density, thus making possible smaller interelectrode capacitances. The higher density emission cathode requires a high-control grid voltage in the case of grounded grid tubes, which shortens the transit time of the electrons and thus reduces the transit-time damping effect. The L cathode has no cross resistance of any consequence, in contrast to the emitting layer of the oxide-coated cathode.

Some of the inherent weaknesses of the oxide-coated cathodes are also prevalent to some extent in L cathodes, for instance, barium evaporation and susceptibility to poisoning by oxygen or oxygen compounds.

Under exposure to high-velocity gas ions or sparking, the emission of an L cathode recovers within approximately 250 milliseconds to its initial value because of a replenishment of barium to the emitting surface, while the oxide-coated cathode is permanently destroyed.

The most important feature of the L cathode is its life. Unlike the oxide-coated cathode, the life of which depends upon the emission current drawn as well as on the temperature at which it is operated, the life of an L cathode depends only upon the temperature at which it is operated and the quantity of barium oxide contained behind the porous tungsten wall. For a given planar-type cathode 3 mm in diameter, having a 7 mm² emitting area operating at 1,050 degrees C brightness, the amount of barium evaporation is 0.1 mg in 1,000 hours. Therefore, 1 mg of barium oxide should yield 10,000 hours of life at this temperature. A number of these diodes have run longer than 5,000 hours, drawing an emission current of 2.0 amp per cm² dc. Later we will discuss the methods of measuring the barium evaporation, as well as the life obtained to date, on some typical tubes.

Regarding the emission mechanism of the L cathode, let us consider the reaction which takes place. First, the barium carbonate pellet in the enclosed cavity is heated to 1,130 degrees C brightness until the carbonates are reduced to their corresponding oxides, the carbon dioxide being pumped out. Next, the cathode is heated to 1,270 degrees C brightness, and the barium oxide is partly reduced to barium. Thus barium vapor will collect in the closed chamber under a certain small equilibrium pressure. The barium vapor passes through the pores of the tungsten, and forms in those pores as well as on the surface a monatomic layer on the tungsten, bound to it by oxygen present on the surface. The formation of this layer results in a considerable reduction in the work function of the tungsten, from 4.5 to 1.8 volts. Thoriated tungsten cathodes have a work function of 2.75 volts. In Table III we illustrate some of the values of work functions and *A* values determined.

In Table III we note that the range of work function of the L cathode is between 1.6 and 2.0 volts. Experiments run whereby the barium-carbonate pellet was omitted in the L-cathode structure, and the external surface was coated by evaporating a monatomic layer of barium (using Ba azide). This resulted in a work function of 1.66 volts as shown. However, when the normal BaSr carbonates were sprayed on the outside surface (omitting the pellet), a work function of 1.1 was obtained. This is noticeably different from the L cathode. Hence we feel that barium plays the same role in the L cathode as does thorium in the thoriated tungsten cathode, and therefore, perhaps, should be called a "bariated" tungsten cathode.

TABLE III

Type Cathode	*Work Function (Electron Volts)*	*A (A/cm²/deg²)*
Tungsten	4.44 to 4.63	22 to 210
Thoriated tungsten	2.6 2.9	3 15
Oxide cathode	1.0 1.5	0.01 5
L Cathode	1.6 2.0	1 15
Barium evaporated on porous tungsten plug	1.66	—
Thick BaO layer on porous tungsten plug	1.1	—

Barium Evaporation

Before discussing the applications of the L cathode in various types of tubes, we will consider the methods used to investigate the rate of evaporation of barium.

Two methods were employed: The first method involved measuring a fixed quantity of barium carbonate

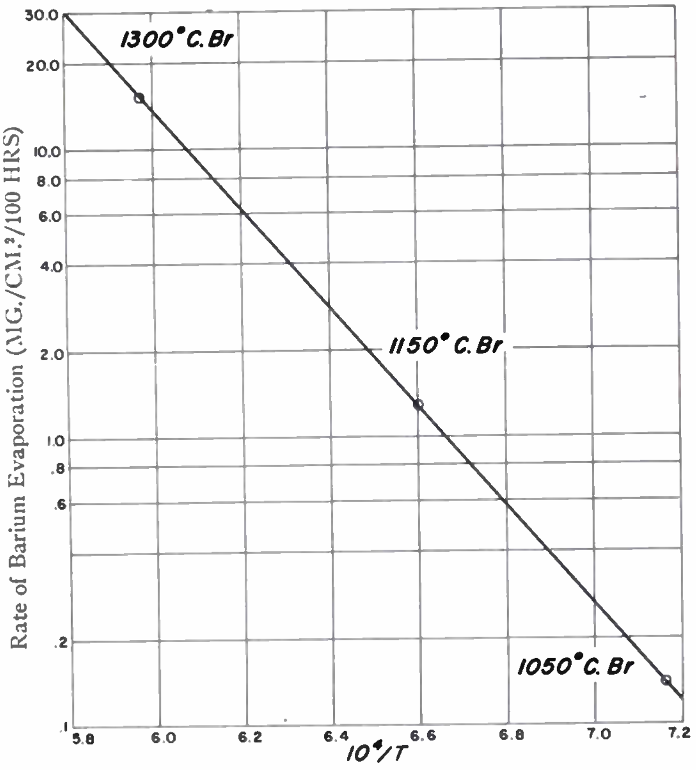

Fig. 5—Barium evaporation curve.

which was inserted into the dispenser of a 3-mm diameter planar cathode prior to activation and life testing. At the end of life, zero emission resulted, and only a trace of barium oxide was present in the dispenser as well as in the porous tungsten plug. Fig. 5 indicates the results obtained for different values of temperatures. The log of rate of barium evaporation was plotted against the reciprocal of the absolute temperature.

These data are for a particular cathode. It is possible to fabricate cathodes having a very much lower rate of barium evaporation.

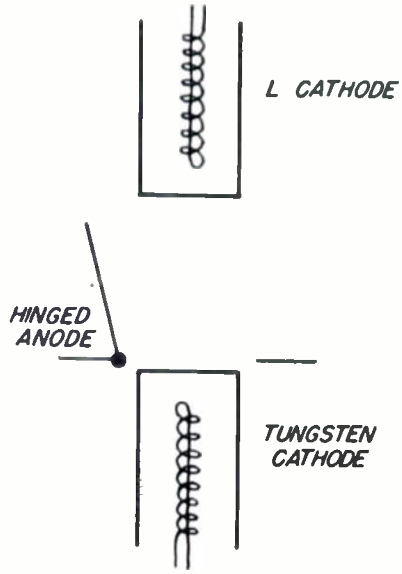

Fig. 6—Schematic of test diode to study barium evaporation.

In the second method, the rate of evaporation of barium was measured by using a simple diode as shown in Fig. 6. Here we show a simplified version of the diode construction. The tungsten cathode shown consists of a highly polished 100-per cent density, 3-mm diameter tungsten cathode, welded to a molybdenum cylinder and heated indirectly by the filament shown. The hinged anode is spaced approximately 0.025 inch from the tungsten cathode. The L cathode is of the standard 3-mm diameter type, and is mounted approximately 0.750 inch from the tungsten cathode. The walls of the glass envelope are approximately the same distance away. No getter is used in this tube, in order to facilitate taking brightness temperature of both cathodes. This diode is processed in the usual manner. The anode is rf bombarded, and both cathodes are heated to 1,300 degrees C brightness until the vacuum is better than 1.0×10^{-7} mm of Hg before sealing off.

After tip off, in order to remove all traces of barium, the tungsten cathode is glowed at 1,300 degrees C with the hinged anode closed and emission voltage applied until the anode current reads zero emission. At this point, the tungsten-cathode heater is turned off, and the hinged anode is opened by tilting the tube. The L-cathode anode is then glowed at any desired temperature, usually between 1,000 degrees C and 1,300 degrees C brightness. At periodic times, the L-cathode heater is turned off, the hinged anode is closed, and the emission is measured from the tungsten cathode operated at 800 degrees brightness, at which temperature barium sticks well to the tungsten surface. When the emission current from the tungsten-cathode stabilizes after successive exposures to the L cathode, a Richardson plot is taken from which the work function is computed. Successive exposures to the L cathode are continued to insure that stable work function has been reached. The rate of barium evaporation by this method appears to be in close agreement with the barium-depletion method described above.

Life

As mentioned previously, the most important contribution of the L cathode, aside from the fact that it can be operated at high-current density, is its lifetime, which is a function of the temperature of operation as well as the amount of barium oxide available from the reservoir. Unlike the oxide-coated cathode, its life does not depend upon the value of the emission current drawn from the cathode. The life can be computed from Fig. 5. For a given 3-mm diameter planar-type cathode having 2 mg of barium oxide in the dispenser, a life of approximately 200 hours is obtained at a cathode temperature of 1,270 degrees C brightness. Operation of the same cathode at 1,050 degrees C brightness results in a life of 20,000 hours, drawing an emission current of 2.0 amp per cm². Life test failures indicate that only a trace of barium remains in the dispenser as well as in the porous tungsten part.

One of the main causes of failure of L cathodes at true temperatures from 1,250 to 1,350 degrees C is the heater. Heaters coated with aluminum oxide apparently have satisfactory life if operated at true temperatures less than 1,200 degrees C.

In Table IV we list a number of experimental tubes in which the L cathode was used as an emitting source.

TABLE IV
EXPERIMENTAL L CATHODE TUBES

	Life (hours)
Cathode-ray 25-kv TV projection	1,000
Klystron two-resonator 2.8 to 3.5 cm 100 watts	5,000
Magnetron, 3 cm, pulsed 1,065-kw 45 per cent eff.	1,000
Rectifier, air filled 3.0 amp per cm² / Rectifier, H_2 filled, no emission	3,000
Triode, disk seal 10 cm	1,600

The first tube listed is a cathode-ray tube of the 3NP4 type. A 3-mm diameter cathode with tantalum heat shields was used. It is important in the cathode design that the cathode is not assembled in intimate contact with ceramic spacers which tend to reduce to metals and poison the cathode surface.

The two-resonator klystron indicated also uses the 3-mm diameter cathode with tantalum heat shields and focussing assembly. Its construction is of the gridless type, having a beam diameter of approximately 0.086 inch. The life indicated has been obtained on a number of such tubes in which oxide-coated cathodes failed after 10 hours. No appreciable loss of Q or power output was experienced through life. Analysis of completely life-tested tubes indicates that the bulk of the evaporated barium is deposited on the side walls of the drift space, and very little barium is present inside the cavities.

The life of magnetrons using L cathodes is equally satisfactory. The magnetron listed is of the "rising-sun" type. Several cw magnetrons are now being constructed for noise-measurement investigations.

Regarding the rectifiers indicated, it is interesting to note that the hydrogen-filled rectifier indicated no emission, which is perhaps attributable to hydride formation.

Conclusions

In conclusion, let us summarize the essential features and properties of the L cathode. It consists essentially of barium oxide contained behind a wall of porous tungsten. During the operation of the cathode, the barium oxide is reduced gradually, and free barium resulting from this reduction escapes through the pores and forms on the surface a monatomic layer of barium bound to the surface of the tungsten by oxygen. Hence the work function of the tungsten is reduced from 4.5 to 1.8 volts.

The essential advantages of the L cathode are its capacity to produce high-thermionic emission, combined with long life. The life of an L cathode is dependent entirely upon the temperature of operation and the amount of barium oxide available in the dispenser. It is particularly suited for application in microwave tubes since it can be machined to exact tolerances.

Acknowledgment

Some of the discussion has been taken from the paper of H. J. Lemmens, M. J. Jansen, and R. Loosjes, *Philips Tech. Rev.*, vol. 11, pp. 341–350; June 1950, as well as from private communications with R. Loosjes.

The author wishes to express his gratitude to O. S. Duffendack, F. K. du Pré, and E. S. Rittner, for their interest and their many valuable suggestions.

The Theory of Amplitude-Modulation Rejection in the Ratio Detector

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Summary—The procedure for a complete mathematical analysis of the AM-rejecting properties of the ratio detector is presented. The operation with 100-per cent efficient diodes is first treated, and it is shown that in this case compensating resistors which reduce the effective efficiency of the diodes must be used to obtain optimum AM rejection. The operation with practical diodes is then treated and design charts for optimum AM rejection are presented. From the theory, the effect of variations in ratio-detector transformer parameters upon the AM-rejection properties is predicted. Unbalanced effects and the manner in which they can be made to cancel each other mutually are briefly described. It is pointed out that the degree of apparent limiting action within the ratio-detector circuit is incidental and unrelated to its AM-rejection properties, and thus represents an inadequate design basis for the ratio detector.

I. Introduction

In order to obtain the full benefits of FM reception, an FM receiver should include an FM-detector system which is insensitive to amplitude modulation. A few years ago a new FM detector, called the "ratio detector," which did not require the use of a limiter, was introduced in receivers being marketed. Since then this detector has been rather widely use in FM receivers and in the sound ends of television receivers. However, in spite of wide usage, the design of the ratio detector is still almost entirely empirical. Several papers have been presented giving a general story on the operating characteristics of this device, but an accurate mathematical analysis has not been presented and the AM-rejection properties have not been adequately explained. The purpose of this paper is to outline a procedure for a complete mathematical analysis of the AM-rejection properties of the ratio detector, giving the results of the complete analysis in the form of graphs to be used for designing ratio-detector circuits. Since a description of the general operating characteristics has been given in the literature,[1] it will not be repeated here.

The complete mathematical analysis of the AM-rejection properties of the ratio detector is lengthy and involved. Presentation of all steps is therefore beyond the scope of this treatment.[2] Instead, the procedure followed will be outlined and the essential background material presented. The complete analysis produces results which are in agreement with experiment. These results will be presented.

II. The Ratio Detector and Its Equivalent Circuit

The mathematical analysis of the ratio detector is simplified through the use of an equivalent circuit. The development of this equivalent circuit from a ratio detector, using either a phase-variation discriminator transformer or a side-tuned transformer, is shown in Fig. 1. The rf circuit components to the left of terminals

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[1] B. D. Loughlin, "Performance characteristics of FM detector systems," *Tele-Tech*, vol. 7, p. 33; January, 1948.

[2] The complete mathematical analysis is given in the Hazeltine Electronics Report #7096.

1, 2, and 3 (Figs. 1(a) and 1(b)) can, with complete generality, be represented by two generators (either current or voltage) and three impedances (connected either as a Y or Δ). For convenience, the form using two current generators and a delta connection of impedance is used here, giving the equivalent rf circuit shown in Fig. 1(c).

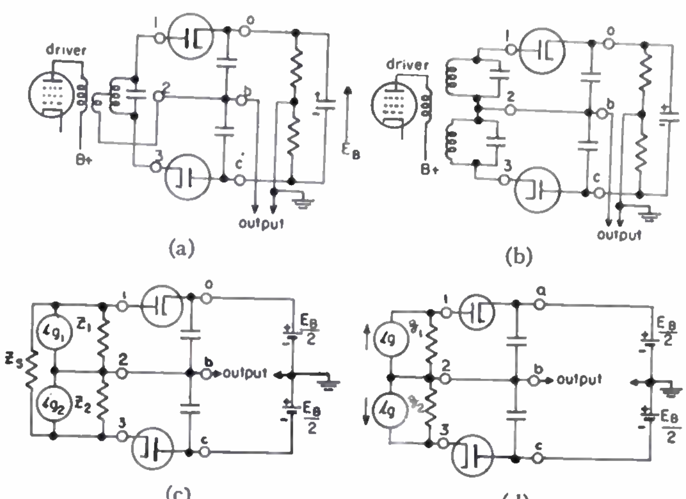


Fig. 1—Development of equivalent circuit for ratio detector. (a) Phase variation. (b) Side-tuned. (c) Equivalent. (d) Approximate.

To further develop the equivalent circuit, the dc circuit components to the right of terminals a, b, and c can be simplified. For a study of dynamic (audio-frequency) AM-rejection properties, this combination of two resistors and a large condenser can be replaced by a center-tapped battery since the large condenser holds the voltage between terminals a and c constant, as far as dynamic variations are concerned. However, the magnitude of this battery voltage is determined by the average operating level.[3] This gives the equivalent circuit as shown in Fig. 1(c).

From the equivalent circuit it is seen that the ratio detector consists fundamentally of two diode rectifier circuits whose dc output terminals are connected in series across a battery. Thus, the circuit as viewed from the dc side is much like two dc generators connected in series across a battery, and the net dc output voltage between terminal b and the ground is thus a function of the regulation characteristics of the dc generators. However, the equivalent dc-generator regulation characteristics are, in general, nonlinear and thus not too easy to handle.

Some further simplifications in the equivalent circuit can be made. For the well-balanced case, it can be shown that the two current generators (i_{g1} and i_{g2}) are identical in amplitude. Also, the shunt impedance Z_s can be effectively eliminated by paralleling half of it with Z_1 and the other half with Z_2. This latter simplification can be shown to involve no approximation in regard to finding the conditions for optimum AM rejection. The approximate circuit of Fig. 1(d) is now obtained.

By comparing Fig. 1(d) with the side-tuned circuit of Fig. 1(b), it can be seen that z_1 and z_2 are of the nature of side-tuned resonant circuits. This equivalent circuit (with side-tuned resonant circuits) applies even for the phase-variation transformer, as will be apparent when it is realized that the rf circuits to the left-hand side of terminals 1, 2, and 3 in Figs. 1(a) and 1(b) produce substantially similar voltage variations with frequency.

By considering these impedances z_1 and z_2 to consist of parallel conductances (g) and susceptances (b_1 and b_2), the equivalent circuit of Fig. 2 is obtained. In the usual balanced case, the conductances (g) are equal and are relatively independent of frequency near the center frequency of the detector, while the susceptances b_1 and b_2 are opposite in sign and equal in magnitude at center frequency, but vary oppositely in magnitude with frequency. That is

$$b_1 = +(b + \Delta b)$$
$$b_2 = -(b - \Delta b), \tag{1}$$

where b is the magnitude of the susceptance at center frequency and Δb is the variation with frequency.

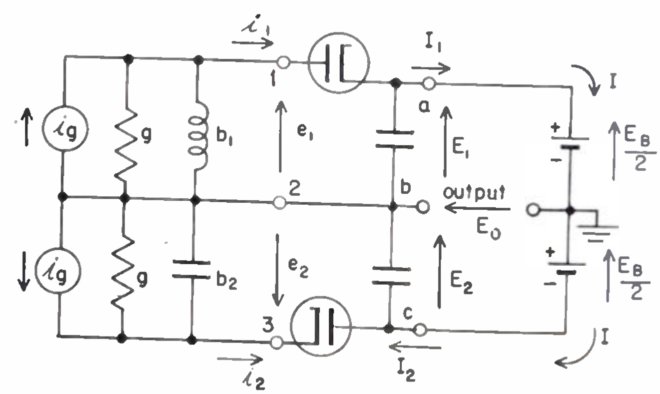


Fig. 2—Ratio-detector equivalent circuit.

It should be noted at this point that the impedance of the ratio-detector transformer at harmonic frequencies is rather low compared to the fundamental frequency impedance. Thus as a first-order approximation, the ac voltages are sinusoidal in shape. When ac voltages (e's) and currents (i's) are mentioned, it will be understood that these e's and i's refer to the peak values of the fundamental frequency ac components of voltage or current. This ignoring of the harmonic currents and voltages is permissible for a first-order approximation; however, the presence of harmonic currents when the harmonic impedance is not zero does result in some reactive effects. These reactive effects will be discussed later under "unbalanced effects."

[3] This difference between static and dynamic operating characteristics of the ratio detector has been explained in the literature. See footnote reference 1, Figs. 10 and 11.

III. Operation with 100-per Cent Efficient Diodes

4. Detector Output Voltage

It is useful to study the performance of the ratio-detector circuit with ideal 100-per cent efficient diodes before proceeding to the practical case involving inefficient diodes.

On the dc side of the equivalent circuit of Fig. 2 there are two apparent relations, namely,

$$I_1 = I_2 = I \tag{2}$$

$$E_B = E_1 + E_2. \tag{3}$$

However, to determine the division of E_B between E_1 and E_2, further relations involving the regulation characteristics are required. These regulation characteristics are obtained from writing the mesh equations involving the ac side of the circuit, namely,

$$ge_1 + jb_1e_1 + i_1 = i_g = ge_2 + jb_2e_2 + i_2, \tag{4}$$

where e_1 and e_2 are the fundamental frequency ac voltages across the two input circuits and i_1 and i_2 are the fundamental frequency ac components of the diode input currents (as marked in Fig. 2). The above equation can be simplified if i_1 and i_2 are in phase with e_1 and e_2, respectively, that is, if the input loading due to the diodes is purely resistive. This is true as a first-order approximation, and will be assumed in the present part of the analysis. Actually, in a practical case reactive components of i_1 and i_2 can result from incomplete by-passing of the diode output circuit, from insufficient stored energy in the input tank circuits, and from a change in diode capacitance with efficiency. These reactive effects will be treated in a later section.

As a next step, the ac equation can be written in terms of an equality of magnitudes instead of a complex equation if it is remembered that e_1 and i_1 (also e_2 and i_2) are in phase.

$$(ge_1 + i_1)^2 + b_1^2e_1^2 = (ge_2 + i_2)^2 + b_2^2e_2^2 = i_g^2. \tag{5}$$

Now this is an equation of magnitudes, and the ac voltages and currents can be replaced by their absolute magnitudes, that is, e_1 can be replaced by $|e_1|$, and so on. These absolute magnitudes can be related to the dc voltages and currents by the well-known relations for a 100-per cent efficient diode, namely, that the dc voltage equals the peak of the applied ac voltage and that the fundamental frequency ac component of the input current is twice the dc current.[4]

$$|e| = E; \qquad |i| = 2I. \tag{6}$$

The following equation results:

$$(gE_1 + 2I)^2 + b_1^2E_1^2 = (gE_2 + 2I)^2 + b_2^2E_2^2 \doteq |i_g|^2. \tag{7}$$

This equation gives the interrelation between the dc voltages and currents and the ac circuit impedances, and can be manipulated to derive the output dc voltage E_0. The algebraic steps required involve the substitution of (1) for b_1 and b_2 and the substitution of

$$\frac{E_B}{2} - E_0 \text{ for } E_1 \text{ and } \frac{E_B}{2} + E_0 \text{ for } E_2.$$

In addition, it is assumed that consideration is limited to conditions near center frequency, that is, $\Delta b^2 \ll b^2$ and $E_0^2 \ll (E_B/2)^2$. The resulting relation for E_0 is

$$E_0 = \frac{E_B}{2}\frac{\Delta b}{b}\frac{q^2}{\left[(1+q^2) + \dfrac{4I}{gE_B}\right]}, \tag{8}$$

where $q = b/g$ and $\Delta b/b$ represents the detuning of the applied signal from center frequency.

Now the important thing to note is that the above relation states that the output voltage E_0 *decreases* when the dc current I is *increased*. It is relatively simple to show that I increases as the input current $|i_g|$ is increased. To show this, I versus $|i_g|$ can be found *at center frequency* from (7);

$$4I = \sqrt{4|i_g|^2 - b^2E_B^2} - gE_B, \quad [I > 0]. \tag{9}$$

Now since I increases with i_g, the output voltage E_0 must decrease with a dynamic increase in input signal level i_g. This is illustrated by Fig. 3, where E_0 is plotted versus $|i_g|$ for some assumed values of circuit constants.

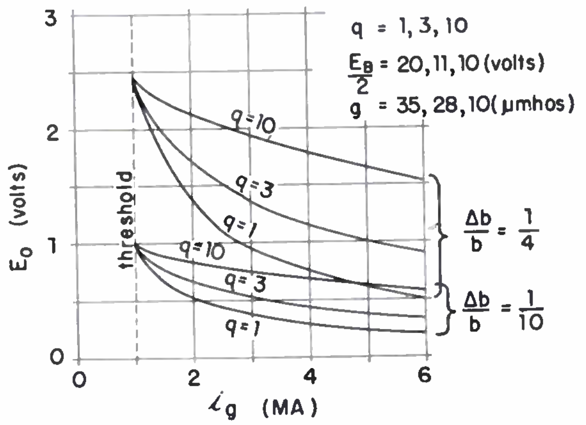


Fig. 3—Output of ratio detector with ideal diodes.

The above leads to the rather important conclusion that a ratio detector with *100-per cent efficient* diodes does not reject amplitude modulation, but instead is *over-compensated* so that a dynamic increase in signal amplitude always results in a reduction of output voltage. From (8) it can also be seen that circuit constants *cannot* be selected to give complete AM rejection. This characteristic should be contrasted with the limiting characteristic of a diode limiter, in which case a 100-per cent efficient diode can give perfect limiting.

It can be seen that as i_g is increased $I=0$ until i_g exceeds $(E_B/2)\sqrt{g^2+b^2}$, that is, until the open-circuit ac voltages exceed the back bias of the battery voltage. This level may be thought of as the dynamic threshold above which the ratio detector operates and below which the diodes are cut off. This threshold is a dynamic threshold since in an actual ratio detector using a

[4] Fundamental property of a short pulse.

long time-constant load the effective battery voltage E_B changes with the average input signal level and the diodes cannot be cut off on a long time basis. Above this threshold the diode current I increases as the input signal level i_g is increased.

B. AM-Reduction Factor

In order to talk about AM rejection in quantitative terms, the quantity "AM-reduction factor," which has been used by workers in the ratio-detector field, is introduced here. This term is equal to the per cent variation of the output signal divided by the per cent amplitude modulation of the input signal producing this output variation; in the other words, calling the AM-reduction factor equal to R,

$$R = \frac{\dfrac{dE_0}{E_0}}{\dfrac{d|i_g|}{|i_g|}}. \tag{10}$$

It is apparent from the definition of AM-reduction factor that perfect AM rejection corresponds to $R=0$. Also, it is apparent that an ideal balanced discriminator with linear diode detectors gives $R=1$.

As a matter of mathematical convenience it has been found easiest to evaluate this AM-reduction factor in two parts, namely,

$$R = R_1R_2 = \frac{\dfrac{dE_0}{E_0}}{\dfrac{dI}{I}} \cdot \frac{\dfrac{dI}{I}}{\dfrac{d|i_g|}{|i_g|}}. \tag{11}$$

As center frequency is approached, R_1 approaches a 0/0 form, and thus needs to be effectively evaluated just slightly off center frequency. However, R_2 can be evaluated at center frequency. Thus R_1 is obtained by differentiation of (8), which gives E_0 versus I *slightly off* center frequency while R_2 is obtained by differentiation of (9), giving I versus $|i_g|$ *at* center frequency. By this procedure the over-all AM-reduction factor can be obtained as

$$R = R_1R_2 = -\frac{(D+1)^2+q^2}{(D+1)(D+1+q^2)}, \tag{12}$$

where D is defined as the ratio of input conductance g_d of the diode to the conductance g of the circuit. From (6) therefore

$$D = \frac{g_d}{g} = \frac{\dfrac{|i|}{|e|}}{g} = \frac{4I}{gE_B}.$$

It can be seen that R is a negative number (implying the opposite polarity effect previously mentioned) with a magnitude less than unity but always greater than 0 (implying imperfect AM rejection). The relation for R is shown by the curves given in Fig. 4.

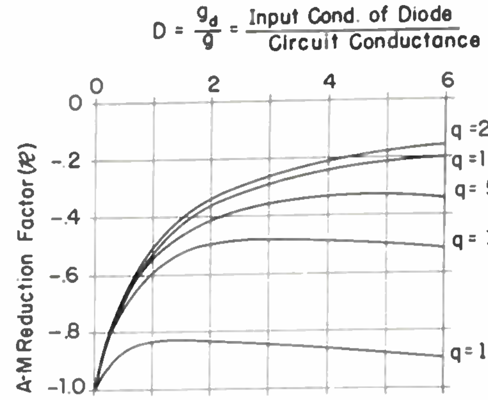
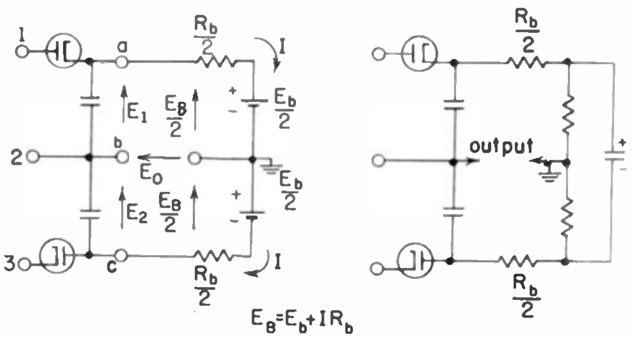


Fig. 4—AM-reduction factor of ratio detector with ideal diodes.

C. Compensating Resistors

In the above-described arrangement with 100-per cent efficient diodes a dynamic increase in signal level produced a reduction in output voltage E_0. If the effective "battery" voltage were to increase as the input signal level increases, the previously mentioned decrease in output voltage could be compensated to give an output that is independent of signal-level variations (over a certain range). This can be accomplished by including compensating resistors in series with the battery, as shown in Fig. 5. The effect of these compensating resistors is illustrated in Fig. 6, where the output voltage is plotted versus the input signal current for several values of compensating resistors and certain of the assumed circuit values of Fig. 3.



(a) (b)

Fig. 5—Ratio detector-load circuit with compensating resistors. (a) Equivalent. (b) Actual.

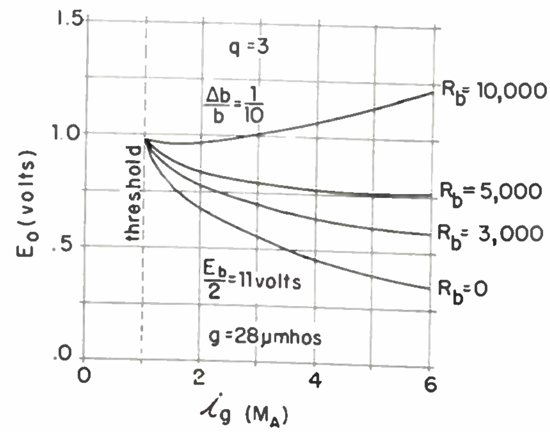


Fig. 6—Effect of compensating resistors in ratio detector with ideal diodes.

The AM-reduction factor for this case, using compensating resistors, can be solved by a similar procedure to that used before. To evaluate R_1, (8) is again differentiated; but this time E_B is not independent of I (instead $dE_B/dI = R_b$). Differentiating and collecting terms R_1 can be written as

$$R_1 = \frac{IR_b}{E_B}\left(\frac{2D+1+q^2}{D+1+q^2}\right) - \frac{D}{D+1+q^2}. \tag{13}$$

From (13) R_1 is seen to be a difference of two terms, and under certain conditions these two terms can be made to cancel giving R_1 equal to zero. When R_1 is zero, the over-all AM-reduction factor R is also zero (actually, $R_2 > 1$). Thus the conditions for perfect AM rejection are obtained by setting $R_1 = 0$. This is obtained when

$$\frac{IR_b}{E_B} = \alpha = \frac{D}{2D+1+q^2}. \tag{14}$$

This (14) specifies the fraction of the total load or battery voltage [$\alpha = IR_b/E_B$] that must be unstabilized in order to obtain perfect AM rejection with 100-per cent efficient diodes.

IV. Operation with Practical Diodes

To get an approximate picture of the effect of using practical diodes, the inefficiencies in the diodes might be thought of as being approximately equivalent to a

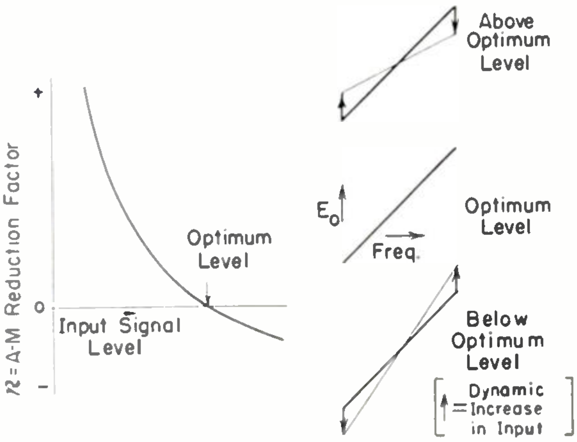


Fig. 7—AM-reduction factor versus signal level for a practical ratio detector.

series resistor inserted in the output circuit of each diode, with the diodes being thought of as having 100-per cent efficiency. Upon insertion of these "inefficiency resistors" the circuit appears like that using compensating resistors, such as shown in Fig. 5. Again, as the input signal level is increased, the effective battery voltage increases. This increased effective battery voltage tends to increase the output voltage E_0 and compensate for the drop in output obtained with high-efficiency diodes. Thus it can be seen that an *appropriate amount of inefficiency* can just cancel the tendency toward over-compensation obtained with 100-per cent efficient diodes, giving perfect AM rejection.

From the above reasoning it would be expected that for a given circuit arrangement there is a particular diode efficiency that would give optimum AM rejection. This means that a particular operating level will give optimum AM rejection since practical diodes have an efficiency that varies with operating level. Operation at a higher than optimum signal level (i.e., at greater than optimum efficiency) will result in an over-compensation effect (approaching that of the 100-per cent efficient diode case), giving a negative AM-reduction factor. On the other hand, operation at lower signal levels will result in a positive AM-reduction factor, that is, it will result in an output signal having an in-phase AM effect. Such a variation of the AM-rejection properties is obtained with a practical ratio detector and is illustrated in Fig. 7.[5]

The AM-rejection properties of the practical ratio detector might be calculated using the above approximation of a compensating resistor in place of diode inefficiencies. However, following this procedure, significant errors are obtained in the calculated level for optimum AM rejection compared to practical operation. The reason for this is that with practical diodes which follow closely the three-halves power law (after correcting for "initial or contact" potential error) the effective value of compensating resistor is a nonlinear element being a function of the instantaneous load current I. The use of an equivalent inefficiency resistor is adequate for most diode circuit analysis, but in the ratio detector the nonlinear nature of the diode is important since significant changes in efficiency occur with dynamic changes in input signal level. This is true because the diode works into a load of approximately constant voltage instead of constant resistance. As a practical matter, in order to obtain results that even approximately agree with experimental data, it has been necessary to consider the diode as a three-halves power nonlinear element.

The interrelation between the ac and dc circuits of a three-halves power diode detector can be found using an approximate solution developed by Wheeler and described in an unpublished Hazeltine report. This solution is based on the assumption that the top part of the applied sine-wave voltage, which causes diode conduction, can be represented by a parabola. The following relation is obtained:

$$|i| = \frac{5+\gamma}{3} I, \tag{15}$$

where γ is the efficiency, that is,

$$E = \gamma |e|.$$

The output voltage E_0 for the practical ratio detector can now be calculated. Equation (5) is applicable since it specifies the interrelation between the magnitudes of the various ac voltages. Then the relations for the practical diode are substituted in (5), giving

[5] When a ratio detector is operated at a very low signal level, the diode detectors become square-law detectors, the AM-rejection property disappears, and the AM-reduction factor will approach 2.

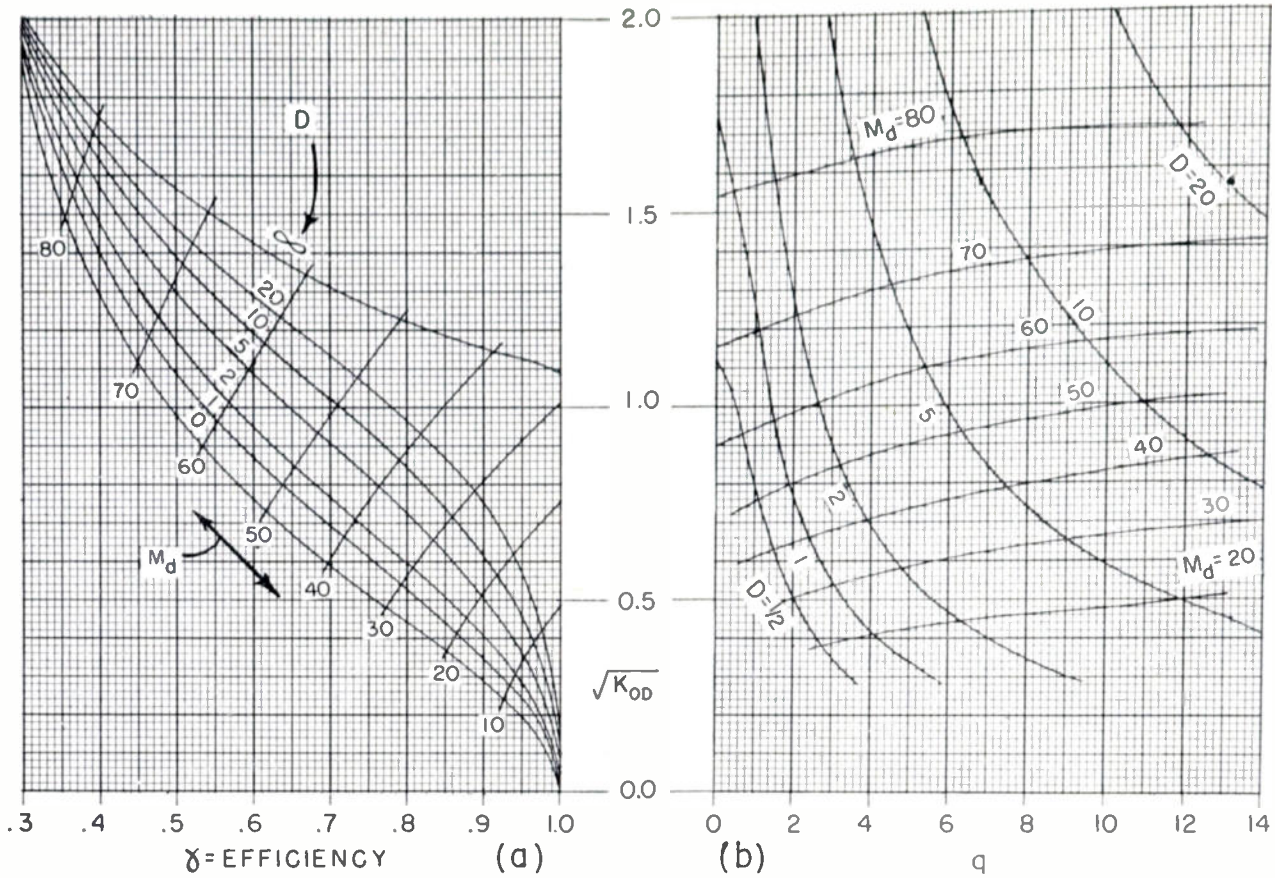


Fig. 8—Conditions for optimum AM rejection in a practical ratio detector.

$$\left(g\frac{E_1}{\gamma_1}+\frac{5+\gamma_1}{3}I\right)^2+b_1^2\frac{E_1^2}{\gamma_1^2}$$
$$=\left(g\frac{E_2}{\gamma_2}+\frac{5+\gamma_2}{3}I\right)^2+b_2^2\frac{E_2^2}{\gamma_2^2}. \qquad (16)$$

The relation finally obtained for the output voltage, after lengthy but straightforward algebra, is

$$E_0=\frac{E_B{}^3b\Delta b(3+\gamma)}{8g^2E_B{}^2\gamma(1+q^2)+32gE_BI\gamma^2+16I^2\gamma^3(1-\gamma)}, \qquad (17)$$

where γ is the value of diode efficiency *at* center frequency when $\gamma_1=\gamma_2$.

While (17) again indicates that E_0 varies inversely with I, the circuit constants can be adjusted to make E_0 independent of I over a certain range because the efficiency γ varies inversely with the diode current I. Thus, circuit parameters in a practical ratio detector can be adjusted to give perfect AM rejection *at some specific diode efficiency.*

By further straightforward algebraic operations, (17) can be used to find the conditions for optimum AM rejection in a practical ratio detector. The results have been plotted as a series of graphs shown in Fig. 8. In these graphs, the quantity M_d is the per cent downward amplitude modulation which can be handled by the ratio detector before the diodes cut off and the ratio detector ceases to function. The quantity D is the ratio of the diode input conductance to the circuit conductance, that is,

$$D=\frac{g_d}{g}=\frac{\frac{i}{e}}{g}=\frac{(5+\gamma)}{3}\times\frac{2I\gamma}{gE_B}.$$

The quantity $\sqrt{K_{OD}}$ is merely a transfer constant used when going between the two graphs.

One important distinctive feature of the ratio detector that is quite apparent from the curves of Fig. 8 is that optimum AM rejection with a *specified downward AM capability* can only be obtained over a *narrow range* of diode efficiency. This limited region of permissible interrelations between circuit constants and diode efficiency is bounded by the curves for $D=0$ and $D=\infty$ (that is, 0 and infinite diode input loading). Of course, this limitation in permissible diode efficiency only applies when compensating resistors are *not* used. It is possible to operate with higher diode efficiencies than those indicated in Fig. 8(a) by the inclusion of compensating resistors (as has been explained with reference to 100-per cent efficient diodes). It appears that *optimum* AM rejection could *not* be obtained by use of *less efficient* diodes than indicated by Fig. 8(a) nor by use of *higher values* of *compensating resistors* than those indicated in (14). Thus, by reference to (14) and Figs. 8(a) and 8(b) the complete range of possible operating conditions are given for optimum AM rejection in the ratio detector.

V. Practical Use of Mathematical Analysis

A. Calculation of Design

Equations have been developed giving the interrelations between D, q, the circuit parameters, and the peak spacing of the FM-detector characteristics of the ratio detector.[6] Using these relations and the graphs of Fig. 8 it is possible to develop a reasonably rational design procedure for the ratio detector which gives answers that agree adequately with empirically obtained design data. Of course, the exact steps to be followed depend upon the known limitations placed upon the design, and the details of the design procedure are beyond the scope of the present paper.

B. Adjustment of Existing Ratio Detector

Frequently the engineer is faced with the problem of adjusting an existing empirically designed device which does not perform in the desired optimum manner either because of incorrect construction or because of stray effects. From the mathematical analysis, a general knowledge of the effect of various adjustments can be obtained so that an existing ratio detector can be adjusted empirically for optimum AM rejection.

Going back to the curves of Fig. 4, it is seen that for 100-per cent efficient diodes an increase in $q(q=b/g)$ reduces the magnitude of the negative AM-reduction factor. This means that in the practical case a higher diode efficiency (or higher operating level) is required for optimum AM rejection as q is increased. It can be shown that the *optimum* AM-rejection point is moved in the direction of a *higher diode efficiency* or a *higher operating level* by making the following adjustments (since these adjustments increase q):

For the phase-variation transformer

1. Increasing the primary to secondary coupling.
2. Decreasing the tertiary turns.
3. Increasing the secondary Q.
4. Decreasing the primary Q.
5. Increasing the secondary inductance.
6. Decreasing the primary inductance.

For the side-tuned transformer

1. Increasing the fractional detuning of each side-tuned circuit.
2. Increasing the Q of each side-tuned circuit.

The above effects of the variations in circuit parameters upon the AM-rejection properties have been substantiated by a considerable volume of experimental observations on both types of ratio detectors.

[6] The complete interrelations, together with a sample calculated design, are given in the Hazeltine Electronics Report #7096.

VI. Unbalanced Effects

So far unbalanced effects in the ratio-detector circuit have been ignored, that is, all effects discussed have resulted in a perfect balance of both the dc and AM output at center frequency. However, apparent unbalance in the ratio detector should be carefully considered since it may severely reduce the over-all AM-rejection properties of the device.

In the ratio detector the diodes may draw a reactive current, or *appear* to draw a reactive current, which varies with dynamic changes in the input signal level. If this is true, there will appear to be a dynamic shift in center frequency of the detector characteristic (that is, a detuning of the transformer) as illustrated in Fig. 9. This results in an audible output due to amplitude modulation even when the applied signal is at center frequency and is not frequency modulated. This unbalanced effect will result in a dynamic crossover, or *dynamic* (AM) *balance* point, which is *not* at center frequency even though the *dc balance* may be at center frequency. These reactive effects are severe in a ratio-detector circuit and must be carefully controlled.

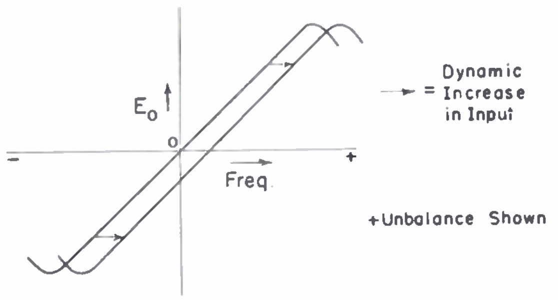


Fig. 9—Effect of apparent frequency shift in a ratio detector.

There are at least three causes for changes in apparent reactive diode currents with dynamic changes in applied signal level. These causes are: first, a change in diode input capacity with change in diode conduction, which produces a positive unbalance; second, insufficient stored energy in the tuned circuits feeding the diodes, which produces a negative unbalance; and third, insufficient by-passing across the detector load, which also produces a negative unbalance. In a practical ratio-detector design these unbalanced effects are made to cancel each other at the desired operating level, generally by adjusting the capacity of diode-load by-pass condensers.

In certain designs it may be found that the negative unbalanced effect due to insufficient stored energy exceeds the positive unbalance due to diode capacity change so that even very large by-pass capacities do not give cancellation of dynamic unbalanced effects. This results in a practical limitation upon the *minimum* ratio-detector transformer *capacity* that can be used with a particular type diode at a particular frequency.

This empirical data on tank capacity in conjunction with empirical data on practical Q's for coils generally gives a starting basis for a ratio-detector transformer design. The remaining circuit constant interrelations can then be determined by the requirements for optimum "balanced" AM rejection.

VII. Inadequacy of Simplified Analysis for Designing a Ratio Detector

The mathematical analysis presented above is certainly rather involved, and might lead one to look for a simplified analysis of the AM-rejection properties of the ratio detector. One type of simplified analysis, which has been considered by a number of engineers, is based upon the fact that if the ac voltages between the various terminals of a ratio-detector transformer are observed on an oscilloscope some apparent limiting action will be seen. It can be demonstrated that the degree of this limiting action is incidental and unrelated to the AM-rejection properties of the ratio detector, and thus provides an inadequate design basis which can result in substantial errors.

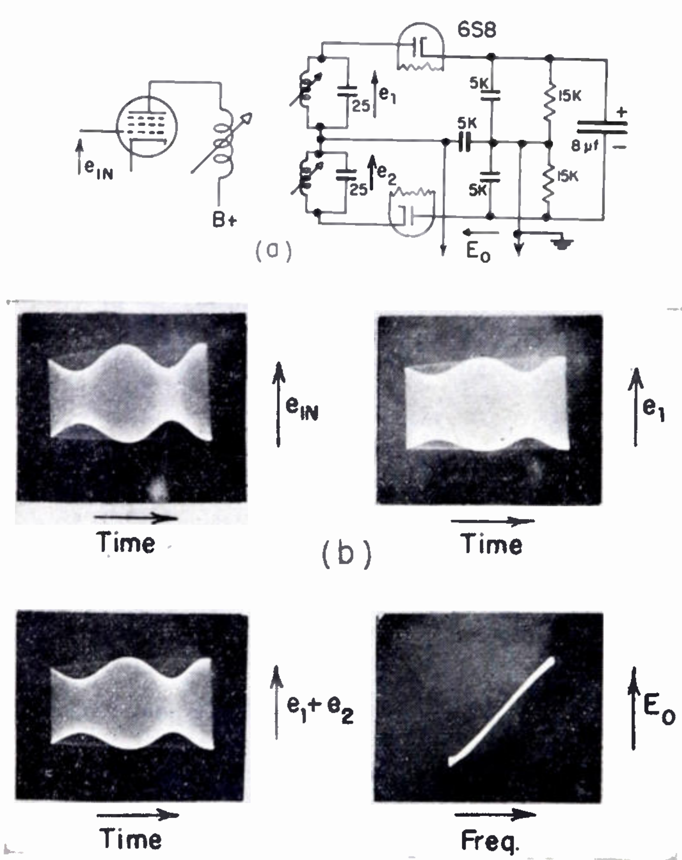


Fig. 10 (a) and (b)—Ratio detector with low-efficiency diodes.

An analysis of the ratio detector using a phase-variation transformer will show that an apparent perfect limiting action can occur across the *secondary* winding and that this *might* actually occur at a signal level which is close to that giving optimum AM rejection for a *particular design* of ratio detector. However, if the phase-variation transformer is replaced by an *equivalent* side-tuned transformer, this apparent perfect limiting action *may* or *may not* be obtained across the terminals 1–3 of Fig. 1(b) (equivalent to secondary of phase-variation transformer), *depending upon* the relative *direction of winding* of the two side-tuned circuit coils. While the direction of winding will substantially change the apparent limiting action, it will have no effect upon the ability to obtain optimum AM *rejection* in the ratio detector since reversing the *phase* of the two input-current generators of Fig. 2 does *not* affect the conditions for optimum AM rejection.

To illustrate the point that this simplified analysis is inadequate and that the apparent limiting on the ac side of a ratio detector is completely incidental and unrelated to the AM-rejection properties, a side-tuned circuit ratio detector with low-efficiency diodes was constructed. As shown by Fig. 10, this device had optimum AM rejection as a ratio detector but very small limiting of amplitude modulation on the ac side.

The oscillogram of E_0 versus frequency was taken with simultaneous AM (30 per cent) and FM (± 75 kc), the AM and FM being nonsynchronous, and shows that the ratio detector has relatively good AM rejection.[7]

VIII. Conclusions

A ratio detector using 100-per cent efficient diodes has been shown to be overcompensated, and compensating resistors or inefficient diodes must then be used to obtain optimum AM rejection. In a practical ratio detector the detector transformer parameters must be correctly designed with respect to the diode efficiency in order to obtain optimum AM rejection at a desired operating level. In addition, it has been shown that the degree of apparent limiting action within the ratio detector is incidental and unrelated to the AM-rejection properties of the ratio detector, and does not provide an adequate design basis.

IX. Acknowledgment

Due credit must be given to N. P. Salz for his assistance in the mathematical analysis presented and to M. Aron for his assistance in collecting the "volumes" of experimental data to substantiate the theory presented.

[7] However, this design using a 6S8 is not suggested for a *production* design of ratio detector because of the large variation in characteristics of various 6S8's.

Cathode-Ray Tube for Recording High-Speed Transients*

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Summary—**A traveling-wave cathode-ray tube has been developed which will record single nonrecurrent transients of 0.5 millimicrosecond rise time. The development was directed toward improvement of writing speed, deflection factor, and shielding between horizontal and vertical deflectors. Distortions due to transit-time effects, and frequency dependent impedance mismatch of the deflector have been minimized. A deflection factor of approximately 0.5 volt per trace width (33 volt per cm) in the vertical direction has been obtained with a reduction of sensitivity of approximately 4 per cent at 1,000 mc. A recorded writing speed in excess of 1×10^{11} trace widths per second (1.5×10^{7} meters per second) has been realized. Oscillograms of recorded traces are shown.**

Introduction

PRESENT CATHODE-RAY tubes have several limitations when used for recording high-speed nonrecurrent transients. Among these are low writing speed, transit-time distortion, high deflection factor, insufficient shielding between horizontal and vertical deflectors, and distortion of the signal due to impedance mismatch at the deflection plates.

Improvement in writing speed can be obtained by using higher beam velocities. However, any increase in beam velocity through the deflector results in an increased deflection factor. This has led to the use of post-deflection acceleration,[1,2] and to smaller trace widths, in order to register more information in a given amount of deflection.

Deflection distortion of cathode-ray tubes at high frequencies, due to finite transit time of the electron beam through the deflector, has been analyzed by Hollman and others.[3] This distortion has been reduced in several cases[4–6] by decreasing the physical length of the deflection plates. Lee[6] has described an oscillograph capable of recording single traces of 10,000-mc signals, but it requires the use of a photographic plate within the evacuated tube itself.

Recently,[7,8] lower deflection factors and lower transit-time distortion have been achieved by the use of traveling-wave deflectors.[9] These systems are designed to match the phase velocity of the signal along the tube axis to the electron-beam velocity. Traveling-wave deflectors are now being applied to the Lee oscillograph.[10] However, the continuously pumped vacuum system becomes undesirable in certain applications where it is necessary to observe and analyze the signals directly on a luminescent screen, or where the auxiliary equipment required by a vacuum system becomes inconvenient.

Need for a cathode-ray tube capable of recording millimicrosecond nonrecurrent transients arose in connection with a special project at the Naval Research Laboratory. As a result, development of such a tube was undertaken. It was desired that the final tube should lend itself to manufacture in fair quantity.

Design Considerations

The design was directed toward both visual observation of nonrecurrent transients of millimicrosecond duration and ability to obtain a permanent photographic record. It was necessary not only to reproduce the applied signal with a minimum of distortion, but to record permanently as much information as possible during the process. One of the problems was, then, that of obtaining the maximum possible resolution on the photographic record. Thus it became desirable to analyze many of the performance characteristics in terms of the recorded spot size or trace width. This is defined, ideally, as the distance between two traces that can just be resolved. In practice, however, there will be some uncertainty as to the exact trace width because of the nonuniform distribution of current within the electron beam, the diffusion effects of the luminescent screen, and the exposure characteristics of the recording film. The recorded trace width also will be found to be a function of the writing speed because of the nonuniform brightness across the trace width and consequent variation in exposure. However, in spite of this uncertainty, the recorded trace width is the chief factor which deter-

* Decimal classification: R138.31. Original manuscript received by the Institute, April 9, 1951; revised manuscript received, August 1, 1951.

† Naval Research Laboratory, Washington, D. C.

[1] P. S. Christaldi, "Cathode-ray tubes and their applications," Proc. I.R.E., vol. 33, pp. 373–381; June, 1945.

[2] J. R. Pierce, "After-acceleration and deflection," Proc. I.R.E., vol. 29, pp. 28–31; January, 1941.

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[4] M. von Ardenne, "Der elektronen-mikrooszillograph," *Hochfreq. und Elektroak.*, vol. 54, pp. 181–188; December, 1939.

[5] K. A. Egerer, H. Ganswindt, and H. Pieplow, "Neuere Ergebnisse der Hochleistungsoszillographie," *Zeit. fur tech. Phys.*, vol. 24, pp. 3–8; January, 1943.

[6] G. M. Lee, "A three-beam oscillograph for recording at frequencies up to 10,000 megacycles," Proc. I.R.E., vol. 34, pp. 121W–127W; March, 1946.

[7] J. R. Pierce, "Traveling-wave oscilloscope," *Electronics*, vol. 22, pp. 97–99; November, 1949.

[8] K. Owaki, S. Terahata, T. Hada, and T. Nakamura, "The traveling-wave cathode-ray tube," Proc. I.R.E., vol. 38, pp. 1172–1180; October, 1950.

[9] A. V. Haeff, U. S. Patent No. 2,064,469.

[10] Navy Contract N173s-15167 has been placed with Central Research Laboratories, Inc., for modification of the Lee Micro-oscillograph to include traveling-wave deflectors.

mines the amount of information obtained, and therefore the performance characteristics should be measured in terms of this factor.

For this application, the deflection factor is defined as the voltage across the deflector required to produce a spot displacement at the screen equal to one trace width, while the field is defined as the total deflection measured in trace widths. To illustrate these factors, Fig. 1 shows

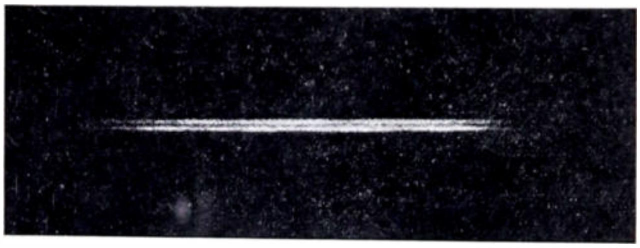

Fig. 1—Two traces separated by a deflection voltage of 1.0 volt. Trace width, about 0.015 cm. Trace spacing, center to center, about 0.03 cm. Horizontal deflection, 2.0 cm.

two traces recorded on the tube. The deflection voltage required to give the trace separation shown is 1.0 volt. From the separation and trace width, a deflection factor of less than 0.6 volt per trace width is established. The total length of each trace, or total field, is measured from Fig. 1 to be approximately 125 trace widths.

The recorded writing speed of the tube is measured in trace widths per second instead of the usual inches per second. For a constant writing speed in trace widths per second, the brightness of the trace does not depend on the trace width. However, as trace width is increased, the speed in inches per second must be increased if the resolution in elements per second is to be maintained. Thus, writing speed expressed in inches per second has no significance in terms of recorded information unless the trace width is also considered.

In terms of the above definitions, the design requirements were as follows:

1. Light output sufficient to obtain a recorded writing speed in excess of 1×10^{11} trace widths per second with a maximum voltage above ground not to exceed 25 kv.
2. A deflection factor of approximately 0.5 volt per trace width on the vertical axis, and not to exceed 1 volt per trace width on the horizontal axis.
3. A total field of approximately 125 trace widths on the vertical axis and 100 trace widths on the horizontal axis.
4. Transit-time distortion in both deflectors corresponding to a reduction of sensitivity of approximately 4 per cent at 1,000 mc.
5. Deflector impedance of 50 ohms at frequencies up to at least 1,000 mc in order to match conventional transmission lines.

Writing Speed

The writing speed required is considerably in excess of that obtained with the conventional cathode-ray tube. It can be attained by the use of high beam velocities, high beam-current density, efficient screen phosphors, and the best photographic lens system and film. The high beam velocity can be obtained by the use of high accelerating voltage. However, the deflection factor requirements limit the beam velocity permissible through the deflector system. The use of post-deflection acceleration is then desirable in order to obtain maximum writing speed while maintaining a low deflection factor. The luminescent screen of highest known photographic efficiency[11,12] is type P-11 phosphor with an evaporated aluminum layer to reflect back radiated light. The most efficient lens system[11] for a recording camera is one with a large numerical aperture and a low magnification of the image from screen to film. A number of commercial films are available for such recording.

The desired high current density in the beam, together with a low deflection factor, requires the use of a high pulse-emission cathode such as the oxide type, and an electron gun designed to produce a narrow and compact beam. This can be achieved by a combination of low magnification from cathode to screen and the use of beam acceleration both before and after deflection.

Deflection Factor

The deflection factor, as defined in volts per trace width, depends upon the gun design as well as on the deflector. Because of transit-time limitations, the use of conventional electrostatic plates would be restricted to short plates which would lead to a large deflection factor. However, the traveling-wave deflector permits a reduction in the deflection factor by an increase in the effective length of the deflector system. In this application, it was considered desirable to provide a low deflection factor both horizontally and vertically. Therefore, traveling-wave systems were considered necessary for both deflectors.

The lowest deflection factor for a given trace width will be obtained with a long deflector, close spacing between deflector elements and ground plane, a large distance to the screen, and a low electron velocity. These factors conflict with the requirements for obtaining high beam current density, and a compromise must be established. Both the length of the system and the spacing affect the total field, while the spacing influences the characteristic impedance. Hence, the deflector must be designed with these other factors in mind. Increasing

[11] R. Feldt, "Photographing patterns on cathode-ray tubes," *Electronics*, vol. 17, pp. 130–137, 262–266; February, 1944.
[12] "DuMont Oscillographer," Allen B. DuMont Laboratories, Inc., Clifton, N. J.; April, June, 1950.

the distance from deflector to screen will increase the trace width and reduce the available writing speed. The distance to the screen, obviously, must be large enough to permit inclusion of the post-deflection accelerating bands.

Field

The size of the field required is determined by the detail desired in the record. It is possible to make measurements on the record to at least one half a trace width. Thus, with the desired field of 125 trace widths, the record can be read to an accuracy of at least 0.5 per cent of full scale. The field is limited by the beam striking the edges of the deflector, and there will be a reduction in writing speed at the edges of the field. This effect can be observed at the ends of the trace in Fig. 1. To obtain a large field for a given size deflector, it is desirable to maintain a small beam diameter within the deflector system. The size of the beam within the deflector also partly determines the current density in the beam.

Transit-Time Distortion

The transit time of the beam passing a deflector element is approximately equal to the ratio of the distance between deflector elements to the beam velocity. When this transit time is an appreciable fraction of the period of the highest frequency component of the applied signal, the deflection is no longer proportional to the signal applied to the deflector plates, and transit-time distortion results. If the phase velocity of the signal along the tube axis in the direction of the electron beam is properly matched to the electron-beam velocity, the transit time-distortion for the system is very nearly the same as that for one element (see Appendix). Geometrical considerations indicate that transit time for a single deflector element and characteristic impedance of the deflector system are closely related.

Deflector Impedance

Although it is possible to use a deflector with an impedance differing from that of the signal line, in general it is more desirable to maintain the same impedance in order to avoid distortion due to frequency dependent impedance mismatch. Since this tube was to be used with a 50-ohm transmission line, a deflector impedance of 50 ohms was selected. For small signal levels it is often necessary to use amplifiers in the signal line. If the capabilities of the cathode-ray tube are to be realized, very wide band amplifiers must be used. Amplifiers having such band widths will be found to have output impedances not far different from 50 ohms. However, special applications may permit the use of higher impedance sources. In this case the deflector should be designed to match this impedance, since for the same input power more deflection voltage will be available at a higher impedance.

Physical Description

A sketch of the tube is shown in Fig. 2. It consists of a cathode, control grid G_1, shield grid G_2, and first anode cylinder followed by the second anode, deflectors, intensifier bands, and luminescent screen.

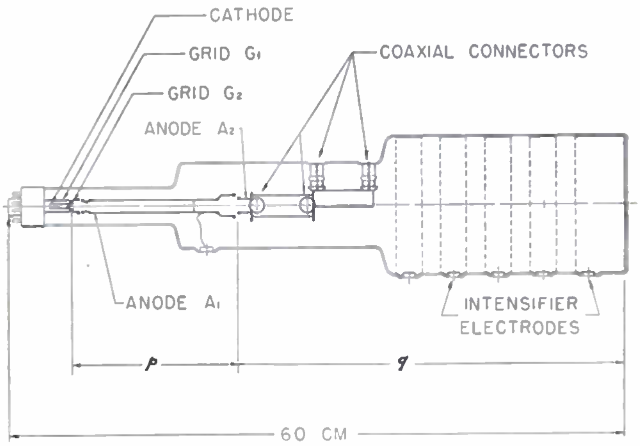

Fig. 2—Diagram of the cathode-ray tube.

The deflectors and second anode are held at ground potential with the cathode at minus 10 kv and the first anode at plus 25 kv with respect to ground. After passing through the deflectors at 10-kv energy, the electrons are accelerated by a series of intensifier bands to a bombarding energy of approximately 35 kv.

The electron focusing lens between the first and second anode is made large to minimize spherical aberration and to provide a suitable focusing voltage ratio. An aperture is placed in the anode cylinder near the focusing lens to limit the size of the beam through the focusing lens and deflector system. This aperture also aids in reducing the number of electrons with high transverse thermal velocities from reaching the screen. A large percentage of the beam current is intercepted by the first anode aperture.

The design of the traveling-wave deflector was based on the selection of the beam size and velocity within the deflector. The beam size and velocity through the deflector were made as small as practicable from electron optical considerations, while still maintaining sufficient current density at the screen. An electron-beam size of approximately 0.15 cm and a beam velocity of 10 kv within the deflector system were found to be suitable values. The length and spacing of the deflectors were then determined by the desired deflection factor and field.

Many types of experimental deflectors were built. These included folded wires, folded strips, semilumped sections of series inductance and shunt capacity, loaded and nonloaded helices, all above a ground plane. The deflectors were made unbalanced (one-sided deflection) since the input signal is derived from a 50-ohm unbalanced transmission line. The electron beam passes between the ground plane and deflector elements.

A spacing above the ground plane of 0.2 cm and a total length of 5 cm were selected for each deflector system.

For the transit time and impedance required, the distributed helix was found to be the most suitable design. It is easily constructed and supported, as well as being fairly rugged. It consists (Fig. 3) of a metal strip wound

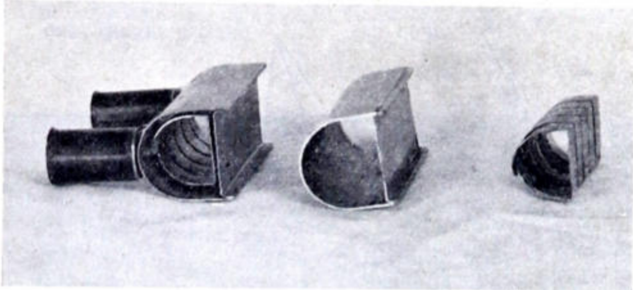

Fig. 3—Photograph of the traveling-wave helical deflector and shield.

in the form of a helix but flat on one side. A metal covering which serves as a shield and ground plane encloses the helix with a constant spacing between helix and covering. As a result of the constant spacing and width, the helix acts as a transmission line with distributed series inductance and shunt capacity. The circumference of the helix is adjusted such that the phase velocity of signals along the axis of the helix equals the electron-beam velocity. The transit time of the electron beam under each deflector element is determined by the distance between deflector elements, or pitch of the helix, divided by the beam velocity.

Each traveling-wave deflector consists of five turns of the distributed helix transmission line. Since the spacing and conductor width are uniform around the circumference, the helix has a uniform phase velocity and characteristic impedance which is close to 50 ohms at frequencies up to and exceeding 1,000 mc. The effective axial length of each deflector element is 1 cm. The electron beam passes between the flat side of the helix and the metal covering, or ground plane (see Fig. 4). The structure has a phase velocity in the direction of the beam equal to the velocity of a 10-kv beam. The transit time through each element is 1.7×10^{-10} seconds, which corresponds to a reduction in sensitivity of 4 per cent at 940 mc for a single deflector element, neglecting fringing. The transit-time distortion in the distributed system will be nearly the same as this if the velocities are properly matched and the deflector is short compared to the distance from the center of the system of the screen.

The tube and deflectors have been designed in order that a deflector can be inserted in series with a 50-ohm line without producing an appreciable discontinuity. Thus the signals on the line can be observed and recorded without disturbing the normal function of the line. In order to do this, both input and output connectors for the structure have an impedance as near as possible to 50 ohms. Small discontinuities, such as glass beads in the glass-to-metal seals do not produce an appreciable effect at the frequencies involved.

The helical inner conductor of the transmission line is supported by the coaxial connectors. Mechanical alignment of the gun and deflector is quite critical because of the gun length and deflector spacing. The alignment is checked after attaching the gun and connectors to the envelope. The neck of the envelope is made of such a dimension as to permit mounting a modified television deflection yoke over the first anode cylinder. The yoke is used to compensate for mechanical misalignments and extraneous magnetic fields. This method of centering the beam is essential if maximum beam current is to be obtained through the limiting aperture.

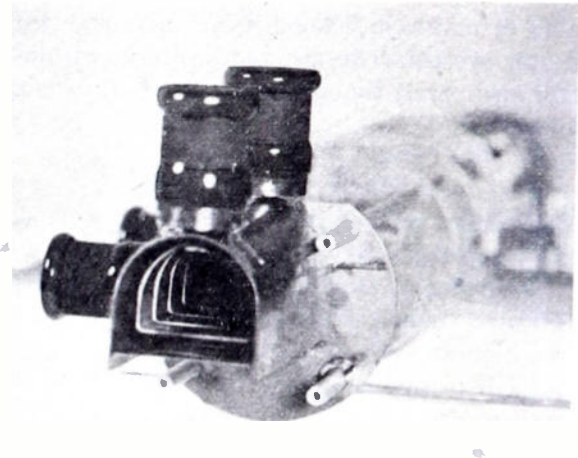

Fig. 4—Photograph of assembled deflector.

The center section of the tube has a diameter sufficient to enclose the deflector structures and the coaxial connectors. The front section of the tube is similar to

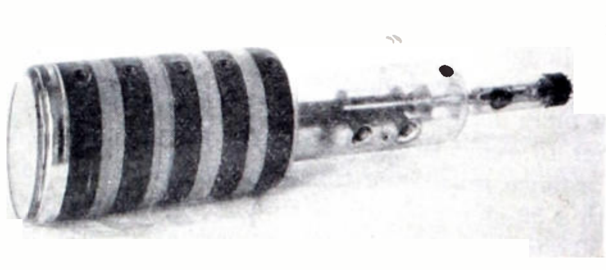

Fig. 5—Photograph of the final model of the traveling-wave cathode-ray tube. Length, 60 cm; diameter, 13 cm. (Manufactured by Electronic Tube Corporation.)

a type 5RP bulb. Only the center portion of the screen is used. Thus the beam passes through only the center portion of the post-deflection accelerating field, and deflection distortion is thereby reduced. Fig. 5 shows a photograph of the final model of the tube.

Applications

The tube can be used in many of the same applications as a conventional cathode-ray tube. However, it is best adapted to the purpose for which it was designed, the observation and recording of nonrecurrent transients. The tube and deflector is designed to operate with a cathode potential of 10 kv. However, lower cathode voltages can be used, with a corresponding reduction in the deflection factor, if the increased transit-time distortion can be tolerated.

A total post-deflection accelerating voltage of from 20 to 25 kv is normally used. Lower voltages can be used, with some increase in field and reduction in deflection factor, but with a decrease in writing speed.

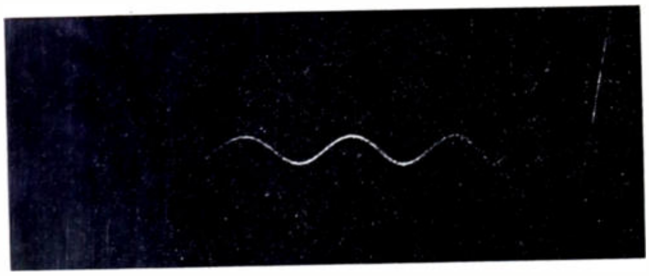

Fig. 6—Photograph of a single trace of a 3,000-mc sine wave. Trace width, approximately 0.015 cm. Sine wave amplitude, 0.21 cm peak to peak. Time axis deflection, 2.7 cm.

For nonrecurrent sweeps, the first grid is normally biased beyond visual cutoff. An unblanking pulse is applied immediately before the signal is allowed to reach the deflector.

In Fig. 6, a single fast-rising wave front is displayed on one axis against a 3,000-mc sine wave on the other axis. It is seen that the total time during which the transient is visible is approximately 1×10^{-9} seconds.

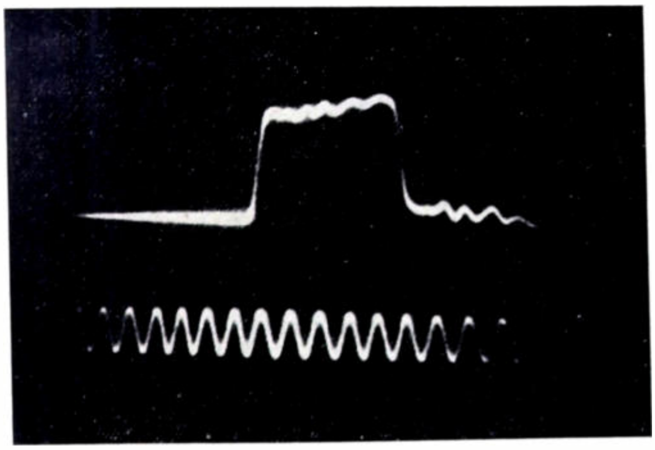

Fig. 7—Upper trace shows photograph of a single pulse with 0.5×10^{-9} seconds rise time and 5×10^{-9} seconds duration. Lower trace displays a single trace of 1,000-mc sine wave on the same time base for calibrating purposes. Sine-wave amplitude, 0.18 cm, peak to peak. Pulse amplitude, 0.53 cm. Horizontal deflection, 2.3 cm.

On the upper trace of Fig. 7, a single pulse is displayed on a linear time base. On the lower trace, a 1,000-mc sine-wave voltage is displayed on the same time base. The rise time of the pulse is seen to be approximately 5×10^{-10} seconds, and the duration 5×10^{-9} seconds. In each case the tube deflectors were in series with terminated 50-ohm lines. The amplitude of the pulse displayed in Fig. 7 is 20 volts at the cathode-ray tube.

The tube is very useful as a monitor device since the signal on a 50-ohm line can be observed without disturbing the line electrically. Both deflectors have a 50-ohm impedance. Hence, signals on two separate lines can be used in making time or amplitude comparison. Many other applications are, of course, possible.

Acknowledgment

Design and development of this tube was carried out at the Naval Research Laboratory under the technical and administrative direction of Drs. E. H. Krause and A. V. Haeff. Much assistance and co-operation were given by members of the Vacuum Tube Engineering Section in construction and processing of first models of the tubes. Drs. E. P. Epstein and F. H. Nicoll of R.C.A. suggested the grid-cathode structure of the electron gun. Electronic Tube Corporation made several experimental models, and manufactured the final tube.

Appendix

The deflection distortion, due to the finite transit time of the electron beam under the deflector, is commonly referred to as transit-time distortion. This distortion comes about in the following manner: A voltage $V(t)$ applied to a deflector element produces an electric field within the deflecting space. The resulting force causes acceleration, and by integration during the transit time, a transverse velocity component of electrons leaving the deflector element results. The tangent of the deflection angle, and to a good approximation, the deflection at the screen, are proportional to this transverse velocity component. Therefore, the deflection at the screen is very nearly proportional to the integral of the input voltage $V(t)$ where the integration is taken over the electron transit time through the deflector element. This expression is

$$y = k \int_{t-T/2}^{t+T/2} V(t) dt, \qquad (1)$$

where y is the beam displacement at the screen, k is a proportionality constant, t is the time when the electron is midway through the deflector element, and T is the transit time of an electron through the deflector element. The time t is measured from the midway point for convenience.

For slow nonrecurrent transients, V is practically a constant during the transit-time T, and the integration yields the product VT, which causes the deflection y to be directly proportional to the signal voltage $V(t)$.

However, during a rapid transient in which the signal variation is comparable to the transit time, a time integration of the signal occurs during the time interval T. The resulting inability to measure short times accurately, or loss in time resolution, has been called "time smearing."

In the case of sine waves, the integration distorts amplitude only, and the usual factor of

$$\frac{\sin wT/2}{wT/2}$$

is obtained for the relative deflection. In the case of a signal with an exponential rise, a factor

$$\frac{\sinh \alpha T/2}{\alpha T/2}$$

has been derived where α is the time constant of the exponential.

The most desirable traveling-wave deflection system would be one which is continuously distributed in the direction of the electron beam so that the signal is propagated with the same direction and velocity as the electron beam. Such a system might consist of a pair of plates used in a parallel-plane transmission line with the velocity of propagation along the line matching the beam velocity. Disregarding end effects, this line would produce no signal distortion due to transit-time effect. The simplest line of this type requires continuous loading, and does not appear practical at the impedance and beam velocity used in this tube.

Another type of deflector may have the signal propagated along a transmission line which crosses the axis of the beam periodically. This type has a finite transit time during which the beam is under one element of the line. The line itself may consist of either lumped or distributed elements.

In the case of the lumped line, there will be additional distortion of the signal due to the frequency response of the line. The lumped line with series inductive and shunt capacitive elements has the characteristics of a low-pass filter with velocity.

$$v = \frac{1}{N\sqrt{LC}} \tag{2}$$

and cutoff frequency,

$$f_c = \frac{Nv}{\pi}, \tag{3}$$

where v is the velocity in meters per second, N the number of sections per meter, L the inductance per section in henries, and C the capacity per section in farads.

The buildup time of a step function applied through a low-pass filter is approximately:[13]

$$T_a = \frac{\pi}{2Nv}. \tag{4}$$

When each capacity section acts as a deflector element for the electron beam, then the time for the electron beam to pass from one section to the next, or transit time per deflector element, is

$$T = \frac{1}{Nv}, \tag{5}$$

and from (4)

$$T_a = \frac{\pi}{2} T. \tag{6}$$

Both the buildup time and the transit time produce distortion of the signal in the lumped line. Equation (6) indicates that the buildup time is nearly equal to the transit time under the conditions when each capacity element is used as a deflector. It can be seen from (4) that for a fixed velocity the buildup time can be reduced only by increasing the number of sections N. One way of increasing the number of sections, and thereby reducing the buildup time, is to make the line distributed with sections of the line crossing the beam axis at regular intervals. Thus, in the distributed line, the buildup time can be made small, leaving only the transit-time distortion.

In case the beam velocity is not matched to the axial velocity of the deflector system, an additional distortion will be introduced. This distortion is due to the beam being deflected by different parts of the wave front as the beam travels through the deflection system. The mismatch distortion is different from the transit time distortion previously discussed, although it can be represented as an equivalent loss in time resolution, or time smearing. The mismatch distortion will cause a reduction in sensitivity to sine waves, in addition to the reduction due to transit-time distortion. A minimum in the response curve will occur when the mismatch is such that the phase difference of the signal at each deflector element produces a minimum resultant deflection of the beam. When the beam velocity is below the design value, both the transit time and mismatch distortion increase; but when the beam velocity is above the design value, the transit-time distortion decreases while the mismatch distortion increases. Thus, distortions are more serious for velocities lower than the design value.

The tube described in this paper has been operated with a beam velocity of 5 kv instead of the design value of 10 kv. The total equivalent time smearing is approximately 5×10^{-10} seconds. At 5-kv beam velocity the first minimum in the response curve due to mismatch distortion occurs at approximately 3,000 mc.

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Some Limitations on the Accuracy of Electronic Differential Analyzers*

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Summary—Electronic differential analyzers, or simulators, employ computing elements which must be physically realizable electric circuits, and this fact introduces unavoidable errors into the differential-equation solutions. These errors are investigated mathematically for the general case of differential equations with constant coefficients. It is found that in certain cases an error of 1 per cent can be introduced by an adder bandwidth 2,000 times the highest frequency in the differential-equation solution.

I. Introduction

During World War II there was a rapid development of computing circuits of various types. Since the war the application of electrical and electronic computing circuits has been expanded into many fields of engineering and science. A most interesting application of these circuits is the solution of ordinary differential equations by a group of computing machines variously called "analogue computers," "simulators," and "differential analyzers."[1-7]

Mechanical differential analyzers have been used for many years, and the errors encountered in their operation have been studied.[8,9] This paper considers some of the errors found in the operation of an electronic differential analyzer. Most of today's electronic analyzers employ time as the independent variable and voltages as the dependent variables; only this type of analyzer is considered here. The transient response of two of the most important differential analyzer components is presented, and the errors in the differential-equation solutions (which will be observed when a differential analyzer uses such components) are studied. It is shown, for example, that in certain extreme cases an error of 1 per cent can be produced by an adding unit having a bandwidth two thousand times the highest frequency present in the differential-equation's solution; in such cases it may be necessary to control the frequency characteristic of an integrating unit out to frequencies at which the magnitude of the integrator gain is -60 decibels. Failure to recognize the importance of this source of errors can result in the solution of a differential equation radically different from the desired differential equation.

II. Unavoidable Limitations of Computing Elements

This section is concerned with the unavoidable limitations of electronic differential-analyzer[10] computing elements, such as adders, integrators, and differentiators, which must be physically realizable electric circuits. Attention here will be restricted to the two most common computing elements, namely, integrators and adders, and to differential equations which can be solved with these elements—linear differential equations with constant coefficients.

A. Adders

If voltages $e_1, e_2, \cdots, e_n$ are connected to the input of an ideal adding unit, the output voltage will be

$$e_{\text{out}} = (e_1 + e_2 + \cdots + e_n). \tag{1}$$

This ideal expression implies a device having a perfect transfer characteristic; it introduces zero attenuation and phase shift for all frequencies. Because of unavoidable stray capacities in any electric circuit, such a characteristic is not physically realizable.[11] The response to sinusoidal steady-state voltages of the simplest physically realizable adder will be of the form

$$E_{\text{out}(\omega)} = K \frac{(E_1 + E_2 + \cdots + E_n)}{1 + j\omega T_1}. \tag{2}$$

The attenuation and phase characteristics of such an adder are plotted in Fig. 1(a). The step responses of an ideal and a physically realizable adder are shown in Fig. 2(a). An adder can have a much more complicated response than that given by (2), but even the characteristic of a complicated adding circuit can be approximated closely enough by (2) to validate using this for further analysis.

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10 For the balance of this paper, the term "electronic differential analyzer" will be applied to machines used to solve differential equations, with time as the independent variable.

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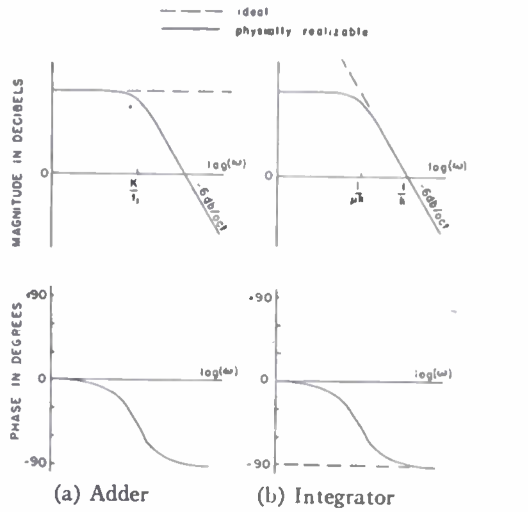
B. Integrators

The sinusoidal steady-state response of an ideal integrating circuit would be

$$\frac{E_{out}}{E_{in}}(\omega) = \frac{1}{j\omega k}. \tag{3}$$

Equation (3) cannot be realized with physical circuits since it requires an infinite gain at zero frequency. The best that can be achieved at zero frequency is a large, but finite, gain μ. A physically realizable integrator can approximate (3) over a wide frequency range, but at some low-frequency the attenuation must leave the -6 decibel per octave slope and approach a constant; this change of slope must also be accompanied by a reduction of the phase shift to zero degrees.[11] The simplest physically realizable integrator characteristic will therefore be



(a) Adder (b) Integrator

Fig. 1—Ideal and physically realizable attenuation and phase characteristics of computing elements.

$$\frac{E_{out}}{E_{in}}(\omega) = \frac{1}{j\omega k}\,\frac{1}{(1 + 1/j\omega\mu k)}. \tag{4}$$

This is the solid line curve of Fig. 1(b). It will be noted that, at least in theory, it is possible to have a physically realizable integrating circuit with a perfect high-frequency response—something that is impossible for an adding circuit. Comparing the magnitude and phase curves in Fig. 1 for the physically realizable adder and integrator, it may be noted that they are of exactly the same form. The only difference lies in the frequencies $1/k$ and $1/T_1$. A proper computer design requires that $1/k$ be much lower, and $1/T_1$ be much higher, than any frequency encountered in the differential equation solution.

For a practical differential analyzer a perfect integrator high-frequency response is not usually available; it is then necessary that the high-frequency response of the integrators employed be much better than the high-frequency response of the adders. This means that care must be taken in integrator design to control the high-frequency characteristics out to frequencies far beyond those encountered in the normal computer operation. Equation 4(a) gives a typical integrator response with a single high-frequency time constant.

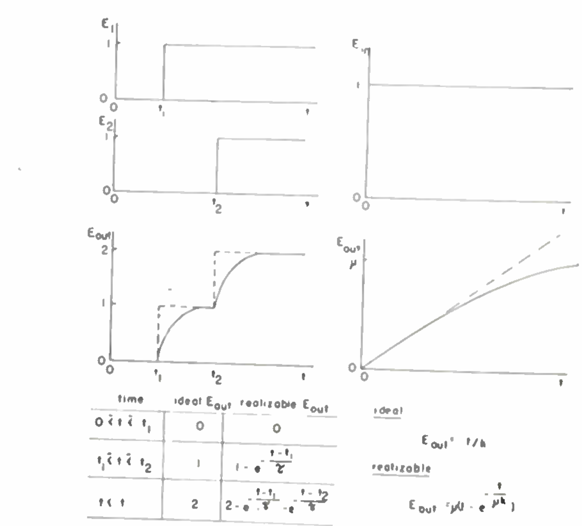
$$\frac{E_{out}}{E_{in}}(\omega) = \frac{1}{j\omega k}\,\frac{1}{(1 + 1/j\omega\mu k)}\,\frac{1}{(1 + j\omega T)}. \tag{4a}$$

The step-function response of an integrator circuit having the transfer characteristic of (4) is shown in Fig. 2(b).

C. Summary

Figs. 1 and 2 summarize the steady-state frequency and step-function response characteristics of physically realizable adders and integrators.

It is important to bear in mind that the physically realizable adder and integrator characteristics given by (2) and (4), and plotted in Figs. 1 and 2, are the best possible approach to ideal characteristics. With practical circuits one can hope to approach these characteristics, but in general computing elements will show additional deviations from the ideal characteristics, which *can* have effects equally as important as those considered here. These limiting characteristics are simple enough to be tractable, and a study of their influence on the differential equation solutions indicates what may be expected from more complicated cases.



(a) Adder (b) Integrator

Fig. 2—Ideal and physically realizable step-function responses of computing elements.

III. Errors in the Solution of Ordinary Differential Equations with Constant Coefficients

A. Analysis of Differential-Analyzer Error

The general ordinary differential equation with constant coefficients,

$$\sum_{n=0}^{m} A_n \frac{d^n y}{dt^n} = F(t), \tag{5}$$

can be solved on an electronic differential analyzer by the setup of Fig. 3; this requires integrating, adding, and function-generating units.[12] The solution of the reduced equation, obtained from (5) by making $F(t)=0$, is

$$y = \sum_{n=1}^{m} C_n e^{s_n t}, \tag{6}$$

where the C_n are constants depending upon the initial conditions of the particular solution desired, and $s_1, s_2, \cdots, s_m$ are the roots of the characteristic equation

$$C(s) = \sum_{n=0}^{m} A_n s^n = 0. \tag{7}$$

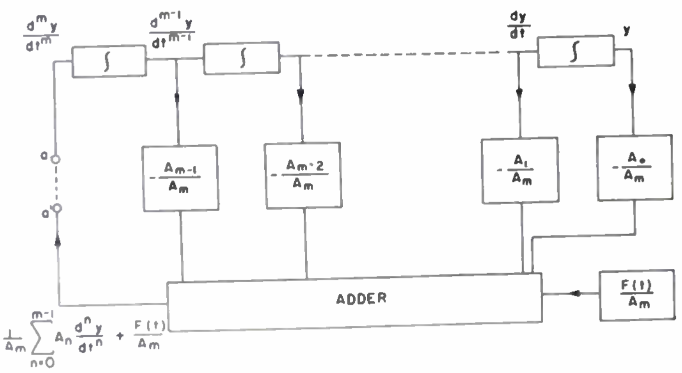


Fig. 3—Block diagram of setup for the solution of an ordinary differential equation of order m with constant coefficients.

If the components in the setup of Fig. 3 are ideal, the solution given by (6) is observed on the differential analyzer. Physically realizable components are not ideal; integrators must deviate from the ideal at low frequencies (and usually also will deviate at high frequencies), and adders *must* deviate at high frequencies. To find the effect of these deviations upon the solution of (6), consider that all integrators in Fig. 3 have the same time constants T_0 and T_1, and the adder has a bandwidth $\Delta f = 1/2\pi T_2$. Under these conditions the frequency characteristic of the m-th integrator in Fig. 3 is

$$\left(\frac{E_{\text{out}}}{E_{\text{in}}}\right)_m = -\frac{A_{m-1}}{A_m}\,\frac{T_0}{1+j\omega T_0}\,\frac{1}{1+j\omega T_1}, \tag{8}$$

and the adder has a characteristic

$$E_{\text{out}} = \frac{E_1 + E_2 + \cdots + E_{m-1}}{1+j\omega T_2}. \tag{9}$$

When components having characteristics given by (8) and (9) are used in the setup of Fig. 3, it can be shown with the help of the Laplace transform[13] that the electronic differential analyzer solves a differential equation whose characteristic roots are solutions of the equation.

$$C\left\{T_1 s^2 + s\left(1+\frac{T_1}{T_0}\right) + \frac{1}{T_0}\right\} = -T_2 s\left[T_1 s^2 + s\left(1+\frac{T_1}{T_0}\right)+\frac{1}{T_0}\right]^m. \tag{10}$$

If the errors are to be small, this equation must have m roots $s_1', s_2' \cdots, s_m'$, differing only slightly in value from the roots of (7). For such roots,

$$s_n' = s_n + e_n \tag{11}$$

can be written, where $e_n \ll s_n$. Assuming further $T_0 \gg s_n \gg T_1$ and T_2 (also a necessary condition if the errors are to be small),[14] it can be shown that

$$e_n = -\frac{1}{T_0} - T_1 s_n^2 - \frac{T_2 s_n^{m+1}}{C'(s_n)}, \quad n = 1, 2, \cdots, m, \tag{12}$$

where $C'(s_n)$ indicates the derivative of $C(s)$ evaluated at the point $s = s_n$.

Equations (11) and (12) give m roots of (10), which differ by an amount e_n from the roots of the desired characteristic equation (7). Since (10) is of order $2m+1$, there will be $m+1$ additional roots. Under the assumption already made that $T_0 \gg s_n \gg T_1$ or T_2, m of these roots will all lie in the vicinity of the point $s = -1/T_1$, and one root will occur near $s = -1/T_2$.

The important result of the preceding analysis is the fact that the characteristic roots of the differential equation being solved suffer perturbations if the equation is solved by a differential analyzer using physically realizable adders and integrators. There will also be additional terms in the solution due to the $m+1$ additional roots introduced; however, because these roots have large negative real parts, these additional terms will damp out very rapidly. Equation (12) permits the perturbation of any characteristic root s_n to be determined provided the value of the root and the various time constants of the computing elements are known. It is perhaps worthwhile to consider these perturbations in a little more detail.

B. Low-Frequency Integrator Errors

The first term in (12) is caused by the low-frequency cutoff of the integrators of Fig. 3, and is independent of the characteristic roots being considered. It can be immediately said that the effect of this error is to multiply every term in (6) by a damping factor e^{-t/T_0}. Failure of the integrators to have infinite time constants results in a compression of the differential-equation solution increasing exponentially with time! Because the same factor multiplies the entire differential-equation solution, its effect could be compensated for in the output element.

C. High-Frequency Integrator Errors

The second error term in (12), which is the result of the high-frequency cutoff of the integrating units, varies with the position of the characteristic root s_n. Where the desired term of the solution is $C_n e^{s_n t}$, the solution ob-

[12] The use of differentiators is not considered here since it can be shown that they cannot be used to solve differential equations if the characteristic roots have positive real parts.

[13] See Appendix.

[14] This inequality states that the roots of the characteristic equation should be remote from both the high- and low-frequency cutoffs of the computing elements.

served will be $C_n e^{s_n(1-T_1 s_n)t}$. The percentage error in the root position is $100\ T_1 s_n$. As the examples of the next section show, the percentage error in the root position gives a good measure of the error to be encountered in the term of the differential-equation solution, provided the roots do not lie too close to the imaginary axis in the s plane.

D. High-Frequency Adder Error

The last term in (12) gives the error in the root position caused by the high-frequency cutoff of the adding unit in Fig. 3. This term, which will be designated as e_{n_3}, can be written with the help of (7), as

$$e_{n3} = \frac{-T_2 s_n^{m+1}}{mA_m s_n^{m-1} + (m-1)A_{m-1}s_n^{m-2} + \cdots + A_1}. \quad (13)$$

IV. Examples of Errors Encountered in Typical Cases

This section is devoted to the application of (12) of the previous section to some second-order differential equations.

A. Simple Harmonic Equation

The equation for simple harmonic motion

$$\frac{d^2y}{dt^2} + \omega_0^2 y = 0 \quad (14)$$

is particularly sensitive to the errors we have been considering. As its characteristic roots $s = \pm j\omega_0$ lie directly on the axis of imaginaries in the s plane, any small motion of these roots produces a marked change in the character of the solution. For this case (12) gives the perturbation of the root position as

$$e_1 = e_2 = \omega_0^2\left(T_1 + \frac{T_2}{2}\right) - \frac{1}{T_0}. \quad (15)$$

This illustrates clearly the statement made previously that the integrator high-frequency time constant T_1 is equally as important as the adder time constant T_2.

For the case of $y=1$ and $dy/dt=0$ at $t=0$, the solution of (14) is

$$y = \cos \omega_0 t; \quad (16)$$

the solution obtained on an electronic differential analyzer employing computing elements which have characteristics given by (8) and (9) is

$$y = e^{+[\omega_0^2(T_1+T_2/2)-1/T_0]t} \cos \omega_0 t. \quad (17)$$

Depending upon the relative values of ω_0, T_0, T_1 and T_2, the amplitude of the sinusoidal oscillation, which should be constant, can either increase or decrease with time. For a sufficiently high natural frequency ω_0 the amplitude will always increase with time.

For example, assuming perfect integrators ($T_1=0$ and $T_0=\infty$), an adder bandwidth $1/2\pi T_2 = 10$ kc and a natural frequency $\omega_0/2\pi = 5$ cps; the solution amplitude will increase 1 per cent after only $4/\pi$ seconds! It should be noted that in this example the ratio of the solution's natural frequency to the adder bandwidth is 1:2,000.

For small errors (17) shows that the percentage error is maximum at the end of the differential-analyzer solution time, and is given by

Maximum error

$$\stackrel{\Delta}{=} E = 100\left[\omega_0^2\left(T_1 + \frac{T_2}{2}\right) - \frac{1}{T_0}\right]T, \quad (18)$$

where T is the solution time. For a given maximum magnitude of error (18) can be used to determine the maximum allowable solution angle $\omega_0 T$.

$$\omega_0 T\Big|_{\max.} = \frac{|E|}{50}\sqrt{\frac{T_0}{2T_1 + T_2}}, \quad (19)$$

and

$$|E| = 100\frac{T}{T_0}. \quad (20)$$

Equation (19) shows the interesting result that for a given allowable percentage error E, the number of cycles of the sinusoidal solution which can be observed is determined only by the ratio of the integrator low-frequency time constant to the adder and integrator high-frequency time constants.

B. Hyperbolic Equation

The equation

$$\frac{d^2y}{dt^2} - \alpha^2 y = 0 \quad (21)$$

has characteristic roots $s = \pm\alpha$. For the case of $y=1$ and $dy/dt=0$ at $t=0$, the solution of (21) is

$$y = \frac{e^{+\alpha t} + e^{-\alpha t}}{2} = \cosh \alpha t. \quad (22)$$

The solution observed on an analyzer employing physically realizable computing elements is

$$y = e^{-\left[\alpha^2(T_1+T_2/2)+1/T_0\right]t}\left\{\frac{e^{\alpha t} + e^{-\alpha t}}{2}\right\}. \quad (23)$$

In this case the observed solution is always smaller than the desired solution. For this equation, the maximum allowable value for αt probably will not exceed 6 because of the finite dynamic range of the computing elements. Therefore, the errors encountered will be much smaller than in the previous case. Assuming perfect integrators, an adder bandwidth of $1/2\pi T_2 = 10$ kc, and an $\alpha = 1/2$, the solution error will be less than 0.01 per cent after 10 seconds ($\alpha t = 5$). In this case the solution error is of the same order of magnitude as the percentage perturbation of the root position.

V. Conclusions

It has been shown there will be inevitable errors in the operation of the adders and integrators of an electronic differential analyzer. This follows directly from the requirement that these computing elements be physically realizable electric currents.

Because of the errors in the operation of its components, an electronic differential analyzer solves a differential equation which differs from the equation inserted by the analyzer operator. The errors in the solution can be computed by comparing the characteristic roots of the differential equation in question with the characteristic roots of the equation the machine solves. If these errors are to be small, the equation solved by the analyzer must have characteristic roots very nearly equal to the characteristic roots of the given equation, and any additional roots must be remote from these desired roots and have negative real parts.[15]

The case of a differential analyzer using integrators and adders to solve differential equations with constant coefficients has been analyzed. The results of this analysis show that the errors are most important for equations having characteristic roots lying near the imaginary axis. For the case of a sinusoidal function, an error of 1 per cent can occur after 6.3 cycles of the solution even though the adder bandwidth is 2,000 times the natural frequency of the equation being solved.

This analysis shows that the greatest precautions should be taken to obtain the best possible frequency response in computer elements. In particular the high-frequency cutoff of an integrator is equally as important as the bandwidth of an adder.

which has roots

$$s = s_n,\ n = 1, 2 \cdots, m. \tag{26}$$

To determine the characteristic equation of the differential equation solved by the differential-analyzer setup of Fig. 3, one denotes the Laplace transform of the voltage at the input to the first integrator by $E_1(s)$. The response of the m^{th} integrator to complex frequencies is assumed to be

$$\left(\frac{E_{\text{out}}}{E_{\text{in}}}\right)_m = \frac{1}{s}\,\frac{A_{m-1}}{A_m}\left[\frac{sT_0}{1+sT_0}\right]\left[\frac{1}{1+sT_1}\right], \tag{27}$$

which can be written

$$\left(\frac{E_{\text{out}}}{E_{\text{in}}}\right)_m = \frac{A_{m-1}}{A_m}\,\frac{1}{\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]}. \tag{28}$$

The adder output is assumed to be

$$E_{\text{out}} = \frac{E_1 + E_2 + \cdots + E_m}{1 + sT_2}. \tag{29}$$

If the Laplace transform of the voltage at the input of the first integrator is $E_1(s)$, the Laplace transform of the voltage at the adder output is

$$E_2(s) = \frac{-\frac{A_{m-1}}{A_m}\,\frac{E_1(s)}{\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]} - \cdots - \frac{A_0}{A_m}\,\frac{E_1(s)}{\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^m}}{1 + sT_2}. \tag{30}$$

The method of analysis employed here can be applied directly to the problem of simultaneous differential equations with constant coefficients.

Acknowledgment

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Appendix

A. Derivation of Characteristic Equation

The ordinary differential equation with constant coefficients

$$\sum_{n=0}^{m} A_n \frac{d^n y}{dt^n} = F(t) \tag{24}$$

is solved on an electronic differential analyzer by the setup of Fig. 3. The characteristic equation of (24) is

$$C(s) = \sum_{n=0}^{m} A_n s^n = 0, \tag{25}$$

[15] This last requirement rules out the *general* use of differentiators.

But in Fig. 3 one forces $E_1(s)$ to be equal to $E_2(s)$. The characteristic equation of this setup is therefore

$$1 + sT_2 = -\frac{A_{m-1}}{A_m}\,\frac{1}{\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]} - \cdots - \frac{A_0}{A_m}\,\frac{1}{\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^m}. \tag{31}$$

Multiplying through by the factor

$$\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^m$$

and rearranging terms, (31) can be rewritten thus:

$$\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^m + \frac{A_{m-1}}{A_m}\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^{m-1} + \cdots + \frac{A_0}{A_m} = -sT_2\left[\frac{1}{T_0} + s\left(1+\frac{T_1}{T_0}\right) + s^2T_1\right]^m. \tag{32}$$

The right-hand member of this equation is recognized as simply

$$C\left\{\frac{1}{T_0} + s\left(1 + \frac{T_1}{T_0}\right) + s^2 T_1\right\},$$

where $C(s)$ is the characteristic equation of the differential equation being solved, (25) above. Equation (32) is therefore (10) of the text, which is rewritten here

$$C\left\{T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0}\right\}$$
$$= -T_2 s\left[T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0}\right]^m. \qquad (33)$$

B. Approximate Solution of Characteristic Equation

If the errors in the solution of (24) by the setup of Fig. 3 are to be small, m roots of (33) must lie near the roots of (25). For these roots

$$s_n' = s_n + e_n, \qquad n = 1, 2, \cdots, m \qquad (34)$$

can be written, where $e_n \ll s_n$. From (25) and (26)

$$C\left(T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0}\right) = 0 \qquad (35)$$

for

$$T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0} = s_n. \qquad (36)$$

Near its zeros $C(s)$ can be expanded in a power series; therefore for s near s_n

$$C\left(T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0}\right)$$
$$\approx \left[T_1 s^2 + s\left(1 + \frac{T_1}{T_0}\right) + \frac{1}{T_0} - s_n\right] C'(s_n) + \cdots, \qquad (37)$$

where $C'(s_n)$ is the derivative of $C(s)$ evaluated at $s = s_n$.[16] Substituting (37) in (33), letting $s = s_n + e_n$, and recognizing that if the errors are to be small,

$$\frac{1}{T_0} \ll s_n \ll \frac{1}{T_1} \quad \text{or} \quad \frac{1}{T_2}; \qquad (38)$$

one can write

$$\left[T_1(s_n + e_n)^2 + e_n + \frac{1}{T_0}\right] C'(s_n) \approx -T_2 s_n^{m+1}. \qquad (39)$$

Solving (39) for e_n, remembering $e_n \ll s_n$,

$$e_n = -\frac{1}{T_0} - T_1^2 s_n - \frac{T_2 s_n^{m+1}}{C'(s_n)}, \quad n = 1, 2, \cdots, m, \qquad (40)$$

which is (12) of Section III.

Equation (33) is of degree $2m+1$ in s and, therefore, has $2m+1$ roots. Equations (34) and (40) together give the location of m roots. The additional $m+1$ roots of (33) can be approximately located by assuming $s \gg s_n$; then (33) can be rewritten

$$(T_1 s^2 + s)^m (1 + T_2 s) = 0. \qquad (41)$$

This equation has a simple root for

$$s = -\frac{1}{T_2}, \qquad (42)$$

and an m^{th}-order root for

$$s = -\frac{1}{T_1}. \qquad (43)$$

Under the assumptions of (38) these roots are much larger than the roots of the equation being solved; since they are also *negative*, error terms associated with these roots will damp out rapidly.

[16] Equation (37) assumes $C(s)$ has a simple zero at $s = s_n$. If multiple zeros are encountered, the higher power terms in the power series expansion must be included.

The Duplex Traveling-Wave Klystron*

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Summary—By combining the properties of the klystron and the distributed amplifier, the duplex traveling-wave klystron allows wide-band amplification of high-level microwave power with an efficiency approaching that of an ordinary klystron. The construction and operation of an experimental model are described briefly.

I. Introduction

THE DUPLEX TRAVELING-WAVE klystron is a microwave power amplifier combining the properties of the klystron[1] and the distributed amplifier.[2] The exact microwave analogue of the distributed amplifier would consist of a large number of two-cavity klystrons inserted into a pair of propagating circuits; however, the duplex traveling-wave klystron goes one step further and combines the many lumped klystrons into a single distributed klystron. The resultant structure, shown in Fig. 1, consists of two

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[1] R. H. Varian and S. F. Varian, "A high frequency oscillator and amplifier," *Jour. Appl. Phys.*, vol. 10, pp. 321–327; May, 1939.

[2] E. L. Ginzton, W. R. Hewlett, J. H. Jasberg, and J. D. Noe, "Distributed amplification," Proc. I.R.E., vol. 36, pp. 956–969; August, 1948.

parallel waveguides coupled by an electron beam of rectangular cross section. The words "buncher" and "catcher" refer to the input and output waveguides, respectively. In the present case these waveguides are identical, and are terminated in their characteristic impedance, hence the "traveling-wave" designation. It should be noted that this device resembles the familiar "traveling-wave tube"[3] in name only.

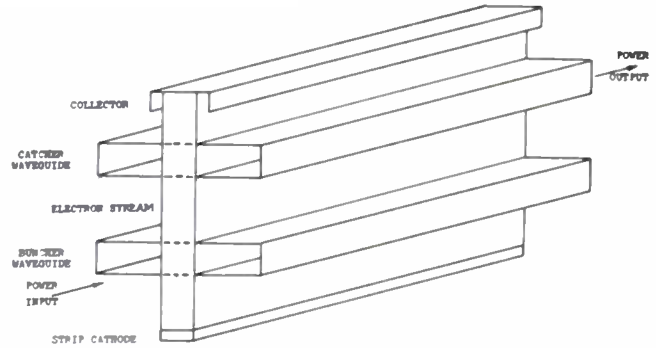

Fig. 1—Duplex traveling-wave klystron.

Electrically, the operation of the duplex traveling-wave klystron is as follows: The input signal, fed into one end of the buncher guide, propagates down the guide in the TE_{10} mode and velocity-modulates the electron stream. In the drift space, diagonal ridges of current form and proceed to "break" upon the catcher guide in much the same way as skewed water waves break upon a beach. The rf energy removed from the bunched beam by the catcher guide is found to propagate toward the output end of the tube in the form of a linearly increasing voltage wave. Because of destructive interference, negligible power appears at the opposite end of the output guide. The gain of a duplex traveling-wave klystron is a linear function of length rather than an exponential function as in traveling-wave tubes. This is true because in the latter there is bilateral interaction between the electron beam and the growing wave, whereas in the former the interaction is unilateral.

In common with the lumped klystron, the functions of electron emission, beam formation, power conversion, and heat dissipation are physically separated in the duplex traveling-wave klystron. In addition, the translational symmetry of this tube provides a new degree of freedom, in that, at a given beam voltage, beam current may be increased without limit simply by extending the tube in the z-direction. Because of this geometrical flexibility, cathode-emission and heat-dissipation problems can be kept to a minimum. This greatly increases the power-handling capacity of the tube. It is perhaps the only microwave tube that could be made to deliver power continuously at extremely high levels.

[3] J. R. Pierce and L. M. Field, "Traveling-wave tubes," Proc. I.R.E., vol. 35, pp. 108–111; February, 1947.

Another feature which distinguishes the duplex traveling-wave klystron is its nonresonant operation. Not only does this permit high power output with relatively low beam voltage, but it also allows extremely wide-band operation. Roughly speaking, the frequency range is limited on one side by the low-frequency cutoff of the propagating structure and on the other side by the advent of higher order modes. When ridge waveguide[4,5] is used, this range is often as high as four-to-one. There exists, however, a relatively slowly-varying dependence of gain upon frequency because of velocity modulation, beam coupling, and impedance considerations.

Because of the absence of resonance, the impedance level into which the electron beam works is extremely low; consequently, to get reasonable amplification, an extremely high perveance beam must be used. To obtain this, a rather lengthy cathode is required. Cathodes several feet in length are necessary for even marginal gain; hence, it is evident that the duplex traveling-wave klystron will not be used for amplification at lower power levels in competition with present devices. Rather, this tube offers many advantages for extremely high-power work, especially where a large bandwidth is desired.

A small-signal analysis of the duplex traveling-wave klystron shows that if a tube is of sufficient length for the catcher voltage to build up to beam voltage, efficiency is 29 per cent. A tube operating in this manner is called a build-up section. Further extension of the tube requires that catcher voltage be kept in the vicinity of beam voltage, while the electron beam continually introduces additional power into the catcher. This condition may be satisfied in several ways, such as, (a) tapering the impedance of the catcher guide inversely with length, (b) tapering the catcher coupling coefficient in a similar manner, or (c) continually bleeding off the arriving power, thus using the catcher chiefly as a power-removing device rather than as a power-carrying structure. Neglecting beam loading, the efficiency of such a constant-catcher-voltage section is 58 per cent. When a build-up section is combined with a constant-catcher-voltage section, the over-all efficiency of the compound tube approaches this higher figure. However, while beam loading has negligible effect in a build-up tube, both the power output and efficiency of a compound tube are considerably reduced by its action.

II. Theory

In this section, equations determining voltage and current in the catcher guide are set up, and their solution discussed. Expressions for power output and efficiency are derived. The effect of beam loading on the power output and efficiency of build-up and compound tubes is considered.

[4] S. B. Cohn, "Properties of ridge waveguide," Proc. I.R.E., vol. 35, pp. 783–788; August, 1947.

[5] T. G. Mihran, "Open- and closed-ridge waveguide," Proc. I.R.E. vol. 37, pp. 640–644; June 1949.

1. Analysis of the Catcher Wave in a Buildup Section

All voltages and currents are assumed to be sinusoidal functions of time with the factor $e^{j\omega t}$ understood. Series resistance and shunt conductance are assumed to be negligible. The effect of shunt beam-loading conductance is considered in a later section. Buncher and catcher waveguides are assumed to be physically identical.

Consider the output waveguide of a duplex traveling-wave klystron. Assume it to be a parallel-strip transmission line with no fringing fields, subject to an externally provided rf shunt current per unit length. This shunt current is set up in the electron stream by a wave traveling in the $+z$ direction with phase velocity $c=\omega/\beta$ along the buncher line. Hence the total z variation of the shunt current is given by the factor $e^{-j\beta z}$. The usual transmission-line equations may be written for the catcher line, however, augmented by this shunt current, which leads the displacement current by 90 degrees.

$$\frac{dV_2}{dz} + j\omega L_l I_2 = 0 \tag{1}$$

$$\frac{dI_2}{dz} + j\omega C_l V_2 = I_l' e^{-j\beta z}, \tag{2}$$

where

V_2 = catcher voltage
I_2 = catcher current
I_l' = magnitude of externally introduced rf shunt current per unit length
L_l = inductance per unit length
C_l = capacitance per unit length
ω = angular frequency
$c=(L_lC_l)^{-1/2}$ = phase velocity of buncher and catcher waves.

These equations, subject to boundary conditions representing termination of the catcher line in its characteristic impedance at $z=0$ and $z=L$, are solved in the Appendix. The solutions for V_2 and I_2 are

$$V_2 = \frac{1}{2} I_l' Z_0 \left[z e^{-j\beta z} - \frac{1}{\beta} e^{-j\beta L} \sin \beta(z-L) \right] \tag{3}$$

$$I_2 = \frac{1}{2} I_l' \left[z e^{-j\beta z} + \frac{1}{\beta} e^{-j\beta L} \sin \beta(z-L) \right],$$

where

$Z_0=(L_l/C_l)^{1/2}$ = characteristic impedance of buncher and catcher lines.

In these expressions, two kinds of waves appear, a traveling wave and a standing wave. The standing wave may be neglected if $z \gg 1/\beta$, a condition which is always satisfied at the output of practical tubes. The important component of (3) is the traveling wave, for it shows a linear amplification with distance. The phase angle may be dropped and for all practical purposes (3) may be written as

$$\begin{aligned} V_2 &= \tfrac{1}{2} I_l' z Z_0 \\ I_2 &= \tfrac{1}{2} I_l' z = V_2/Z_0. \end{aligned} \tag{4}$$

This analysis may be extended quite simply to include waveguides, providing the shunt current is confined to the region of strong and relatively uniform electric field. In this case (4) still holds, except that Z_0 must be replaced by Z_{pv}, the power-voltage definition of characteristic impedance. This substitution may be justified by the following reasoning: An element of rf shunt current, $I_l'dz$, upon being decelerated by the catcher voltage, V_2, gives up to the guide an increment of power, dP_2, given by

$$dP_2 = \tfrac{1}{2} V_2 I_l' dz. \tag{5}$$

Since, by definition,

$$P_2 = \frac{1}{2} \frac{V_2^2}{Z_{pv}}, \tag{6}$$

(5) may be written

$$d\left(\frac{1}{2} \frac{V_2^2}{Z_{pv}}\right) = \frac{1}{2} V_2 I_l' dz,$$

from which

$$dV_2 = \tfrac{1}{2} I_l' Z_{pv} dz. \tag{7}$$

Integration of the forward-traveling increments of voltage gives an expression identical with (4) except for the impedance definition.

Thus the equation specifying voltage buildup in the catcher waveguide of a duplex traveling-wave klystron is

$$V_2 = \tfrac{1}{2} I_l' z Z_{pv}, \tag{8}$$

or more simply

$$V_2 = \tfrac{1}{2} I_0' Z_{pv}, \tag{9}$$

where I_0' is the total rf shunt current.

2. RF Shunt Current

The case of small-signal bunching in a parallel electron beam of rectangular cross section has been investigated for the two cases of grid coupling and gap coupling.[6] With certain restrictions which are nearly always observed in practice, the shunt current seen by the catcher guide may be written in the following form:

$$I_0' = \begin{bmatrix} \text{basic} \\ \text{klystron} \\ \text{current} \end{bmatrix} \begin{bmatrix} \mu_b\text{:} \\ \text{buncher} \\ \text{coupling} \\ \text{factor} \end{bmatrix} \begin{bmatrix} \mu_c\text{:} \\ \text{catcher} \\ \text{coupling} \\ \text{factor} \end{bmatrix} \begin{bmatrix} \text{de-} \\ \text{bunching} \\ \text{factor} \end{bmatrix}.$$

With gap coupling this becomes

$$I_0' = \left[I_0 \pi N \frac{V_1}{V_0} \right] \left[\mu_0 \frac{\sinh C}{C \cosh A} \right] \left[\mu_0 \frac{\cosh C}{\cosh A} \right]$$

[6] E. Feenberg and D. Feldman, "Theory of small signal bunching in a parallel electron beam of rectangular cross section," *Jour. Appl. Phys.*, vol. 17, pp. 1025–1037; December, 1946.

$$\left[\frac{\sin h'l}{h'l}\right], \tag{10}$$

where

I_0 = total beam current
I_0' = total rf shunt current
V_0 = beam voltage
V_1 = peak rf buncher voltage
v_0 = dc beam velocity
f = frequency
$\lambda' = v_0/f$ = beam wavelength (average distance traveled by the beam in one cycle)
$K' = 2\pi/\lambda'$ = beam wave number
a_2 = drift-space width
b_2 = gap spacing
c_2 = beam width at gap
l = drift length
$A = K'a_2/2$ = one-half normalized drift-space width
$B = K'b_2/2$ = one-half normalized gap spacing
$C = K'c_2/2$ = one-half normalized beam width
$N = l/\lambda'$ = drift length in cycles
$\mu_0 = (\sin B)/B$ = coupling coefficient with grids
μ_b = buncher coupling factor
μ_c = catcher coupling factor
$\epsilon_0 = 1/36\pi \times 10^{-9}$ farad/meter = dielectric constant of free space
$h = (i_0/2\epsilon_0 v_0 V_0)^{1/2}$ = debunching wave number
h' = modified debunching wave number.

The buncher and catcher coupling factors are not the same because for small signals the modulation of the beam appears at the catcher in the form of a ripple on the surface of the beam, the volume charge density of the beam remaining unchanged. Since fields die off as the hyperbolic cosine, the catcher coupling factor is simply the gridded coupling factor reduced by the ratio of the field at the edge of the beam to the field at the edge of the gap. However, the buncher field must couple to the entire cross section of the beam; hence an average coupling factor is obtained by integration. The simple debunching wave-number h is reduced by the presence of conducting drift-space walls. The modified debunching wave-number h' ranges from $0.2h$ to $0.7h$ for typical geometries. The exact correction factor may be obtained from Fig. 2 of Feenberg and Feldman.[6]

The basic klystron current, $I_0\pi NV_1/V_0$, is a first approximation to the more accurate first-order Bessel function representation,[7]

$$I_0' = 2I_0J_1(\pi NV_1/V_0). \tag{11}$$

Since efficient power amplification requires the use of a large bunching parameter, $\pi NV_1/V_0$, it is expedient to use the Bessel function representation for current, (11), but to include in it the coupling and debunching factors of (10). This cannot be justified rigorously, but it results in a useful first approximation to large signal theory. The buncher coupling factor and the debunching factor effectively reduce the buncher voltage; hence, V_1 in (11) is replaced by $V_1\mu_b(\sin h'l)/h'l$. The effect of the beam current at the catcher is reduced by the catcher coupling coefficient; hence, in (11) I_0 is replaced by $\mu_c I_0$. Thus the approximate expression for rf shunt current becomes

$$I_0' = 2I_0\mu_c J_1\left(\mu_b\pi N\frac{V_1}{V_0}\frac{\sin h'l}{h'l}\right). \tag{12}$$

3. Power Output and Efficiency

By definition power output is given by (6). From (9) and (12) this may be written

$$P_2 = \tfrac{1}{2}V_2I_0\mu_c J_1(x), \tag{13}$$

where x is the argument of the Bessel function in (12) dc power input is V_0I_0; hence, efficiency is

$$\eta = \frac{V_2\mu_c J_1(x)}{2V_0}. \tag{14}$$

To obtain maximum efficiency, the Bessel function in (14) should equal its maximum value of 0.58, and the catcher voltage should be as large as possible. The maximum allowable catcher voltage is approximately V_0/μ_c since with higher rf voltages an excessive number of electrons will be turned back in the catcher gap. With these substitutions, (14) shows that the maximum efficiency of a duplex traveling-wave klystron in which the catcher voltage builds up linearly to V_0/μ_c is 29 per cent.

To obtain increased power output and efficiency, it is possible to extend the tube further and to use either of the previously discussed power-bled or impedance-tapered schemes. In either case the catcher voltage remains constant at V_0/μ_c, while the catcher power increases linearly with additional length. The power output of the compound tube is the sum of the power from the build-up section, P_2, and the power delivered to the constant-catcher-voltage section.

$$P_4 = P_2 + \tfrac{1}{2}I_i'(z - L_1)V_0/\mu_c. \tag{15}$$

It is apparent that the efficiency of a compound tube approaches 58 per cent as z becomes large compared to L_1. This figure of 58 per cent for efficiency must be used with caution for two reasons: (a) It is reduced considerably by beam loading, and (b) kinematic analysis of ordinary klystron action shows that large-signal effects reduce theoretical efficiency to 35 or 40 per cent.[8]

4. The Effect of Beam Loading upon Power Output and Efficiency

Velocity modulation of the electron stream by power propagating down the buncher guide requires a transfer of energy from the guide to the stream, and hence leads to an attenuation of buncher power. This phenomenon is called "beam loading," and is present not only in the buncher guide but in the catcher guide as well. This

[7] D. L. Webster, "Cathode ray bunching," *Jour. Appl. Phys.* vol. 10, pp. 501–508; July, 1939.

[8] L. Marton, "Advances in Electronics," Academic Press Inc., New York, N. Y., vol. III, pp. 62–64; 1951.

effect obviously sets a limit on maximum power output and efficiency.

The extent to which an electron beam loads a waveguide depends upon the ratio of guide impedance to beam impedance and upon the coupling between the beam and the guide. Feenberg[9] has solved the problem of beam loading for the grid-coupled sheet beam and the gap-coupled sheet beam. His equations specify a lumped conductance, but in reality this conductance is distributed along the entire length, L, of the buncher and catcher waveguides. Such a distributed conductance per unit length shunted across a transmission line, if it is small compared to β/Z_{pv}, results in the exponential attenuation of a wave propagating down the line, namely,

$$V_1 = V_{10}e^{-\alpha z},$$

where

$$\alpha = \frac{g_b Z_{pv}}{2}, \tag{16}$$

g_b being the shunt conductance per unit length. For a beam-loaded length, L,

$$V_1 = V_{10}e^{-Z_{pv}g_bL/2} = V_{10}e^{-Z_{pv}G_b/2},$$

where G_b is the total shunt conductance.

In the following discussion it is assumed that the dc current through both buncher and catcher guides is the same, although in practice it is quite probable that some current would be lost in the drift space. It is assumed that buncher and catcher guides are identical in impedance and coupling to the beam. Consider the increment of voltage which is added to the accumulating wave of voltage in the catcher as a result of the arrival of the rf current $I_t'dz$ at point z. This was given by (7). In this expression I_t' is given by (12), except that because of beam loading V_1 attenuates as $e^{-\alpha z}$. Let x_1 be the argument of the Bessel function at $z=0$, then

$$dV_2 = Z_{pv}\mu_c I_t J_1(x_1 e^{-\alpha z})dz.$$

In traveling to the end of a catcher whose over-all length is L_1, this increment of voltage suffers an attenuation $e^{-\alpha(L_1-z)}$; hence, it gives rise to an increment of voltage at $z=L_1$ given by

$$dV_L = Z_{pv}\mu_c I_t e^{-\alpha L_1}e^{\alpha z}J_1(x_1e^{-\alpha z})dz.$$

The integration from 0 to L_1 may be indicated

$$V_L = Z_{pv}\mu_c I_t e^{-\alpha L_1}\int_0^{L_1} e^{\alpha z}J_1(x_1e^{-\alpha z})dz. \tag{17}$$

This integral cannot be expressed in closed form, but can be evaluated by a series expansion and term-by-term integration.

For maximum power output, V_L must build up to V_0/μ_c, in which case (17) may be written

$$\frac{1}{2r\mu_c^2} = \alpha L_1 e^{-\alpha L_1}\frac{1}{L_1}\int_0^{L_1} e^{\alpha z}J_1(x_1e^{-\alpha z})dz, \tag{18}$$

where $r = G_0/G_b = g_0/g_b$.

If the drive at the beginning of the tube is sufficient to cause optimum bunching of current at that point, $x_1 = 1.84$, and (18) expresses αL_1 as a function of $1/2r\mu_c^2$. The αL_1 determined by the solution of this transcendental equation may be used to obtain the efficiency as a function of $1/2r\mu_c^2$,

$$\eta = \frac{1}{2}\,\frac{1}{2r\mu_c^2}\,\frac{1}{\alpha L_1}$$

a relationship plotted in Fig. 2. This graph gives the efficiency of the build-up section of a duplex traveling-wave klystron as a function of a parameter depending upon the ratio of dc beam conductance to beam-loading conductance, r, and the catcher coupling coefficient, μ_c. For typical values of $r=10$ and $\mu_c^2=0.5$, η is found to be 26 per cent rather than 29 per cent. Hence it is evident that beam-loading effects are not very pronounced in a typical build-up section. Beam loading would have even less effect on efficiency if the tube were overdriven somewhat at the input to the buncher.

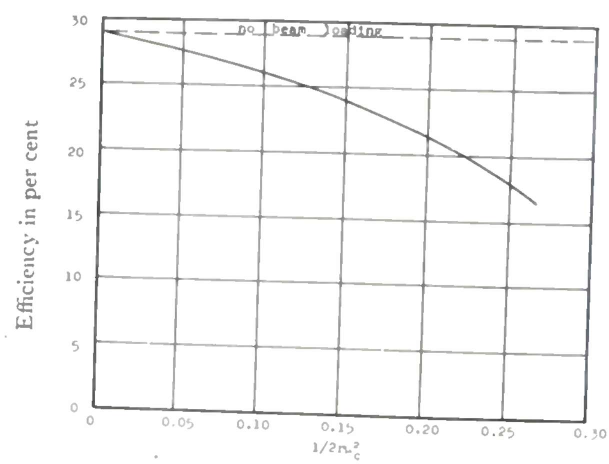

Fig. 2—Efficiency of a build-up section with beam loading.

The small-signal case is of sufficient interest to warrant mention. If x_1 is small, the Bessel function may be replaced by its linear approximation, and (17) becomes simply

$$V_L = Z_{pv}\mu_c I_t L_1 \frac{x_1}{2} e^{-\alpha L_1}. \tag{19}$$

This simple result shows that for a tube with considerably less than optimum drive, the output voltage is attenuated $e^{-\alpha L_1}$ by beam loading. This is a reasonable result because all signal paths through the tube are exactly of length L_1 insofar as beam-loaded travel is concerned, although varying portions of the paths are in the buncher and catcher. Incidentally, (19) may be used safely in the large-signal case since it would give a pessimistic result.

As mentioned previously, increased power output may be obtained by following the build-up section by a power-bled or impedance-tapered section. These alternatives are quite similar in their behavior, therefore, only the first will be considered in detail. The catcher

[9] S. Feenberg, "Theory of Resonator-Beam Interaction," Sperry Gyroscope Co., Report #5220.12.137.

voltage is maintained at $V_L = V_0/\mu_c$ in the power-bled catcher, and the power that may be removed from the catcher in length dz is the difference between the power that is delivered and the power necessary to overcome beam loading in this length.

$$dP_3 = \frac{1}{2} I_l' V_L dz - \frac{1}{2} V_L^2 \frac{2\alpha}{Z_{pv}} dz.$$

Measuring z from the beginning of the power-bled section, the rf shunt current is given by

$$I_l' = 2I_l\mu_c J_1(x_2 e^{-\alpha z}),$$

where $x_2 = x_1 e^{-\alpha L_1}$. From the definition of the attenuation constant, (16),

$$dP_3 = \frac{1}{2}\frac{V_L^2}{Z_{pv}}\left[4r\mu_c^2\alpha J_1(x_2e^{-\alpha z}) - 2\alpha\right]dz. \qquad (20)$$

Integration from 0 to L_2 gives the total bled power to which the straight-through power, $\frac{1}{2}(V_L^2/Z_{pv})$, must be added in order to obtain the total power output.

$$P_4 = \frac{1}{2}\frac{V_L^2}{Z_{pv}}\left[1 - 2\alpha L_2 + 4r\mu_c^2\alpha\int_0^{L_2} J_1(x_2e^{-\alpha z})dz\right] \qquad (21)$$

dc power input is given by $V_0I_l(L_1+L_2)$; therefore, efficiency becomes

$$\eta = \frac{1}{2}\frac{1}{2r\mu_c^2}\frac{1}{\alpha L_1 + \alpha L_2}\left[1 - 2\alpha L_2 + 4r\mu_c^2\alpha\int_0^{L_2} J_1(x_2e^{-\alpha z})dz\right]. \qquad (22)$$

The integrals of (21) and (22) must be evaluated by series expansion and term-by-term integration. The variation with distance of the power output and efficiency of a compound tube for a typical case, namely, $r=10$ and $\mu_c^2=0.5$ (and with optimum bunching at the beginning of the build-up section), is illustrated in Fig. 3. In addition, power output and efficiency are presented neglecting beam loading. It is evident from these curves that beam loading severely restricts maximum power output and efficiency of compound tubes.

Power output will be maximum when (20) is zero, i.e., when the power delivered to the catcher is just sufficient to make up the catcher beam-loading loss. Let this length be called L_m, then from (20)

$$J_1(x_2e^{-\alpha L_m}) = \frac{1}{2r\mu_c^2}.$$

Since the right-hand side is small, the linear approximation for $J_1(x)$ may be used, and an expression for L_m obtained.

$$L_m = \frac{1}{\alpha}\ln(x_2 r\mu_c^2).$$

This is the maximum length a power-bled section should be made without renewing buncher power.

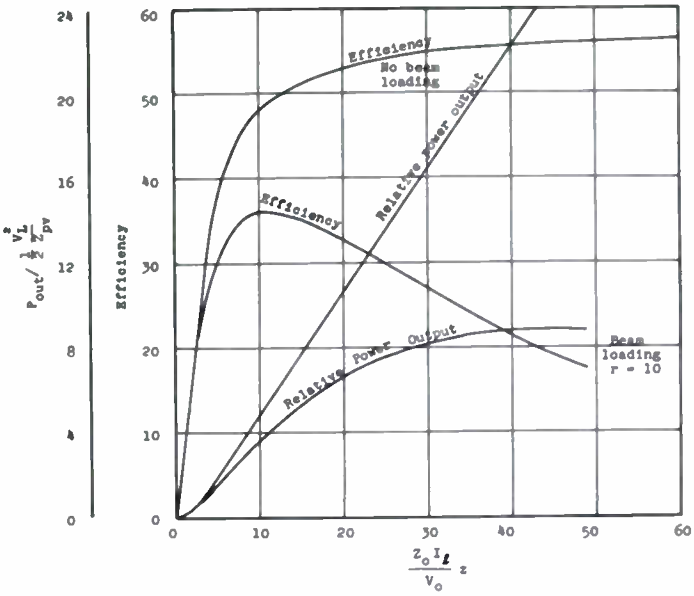

Fig. 3—Effect of beam loading on power output and efficiency.

III. The Experimental Model

The construction of a simple build-up duplex traveling-wave klystron was undertaken in order to verify the basic theory of the tube. The frequency of operation was chosen to be around 3,000 mc. The tube was designed to receive directly the output of a 10-megawatt pulser, the M.I.T. Radiation Laboratory Model 16 modulator. This unit delivered 1-microsecond pulses of voltage, 20 kv in amplitude. Pulse current was 500 amps and the repetition rate was 350 cps.

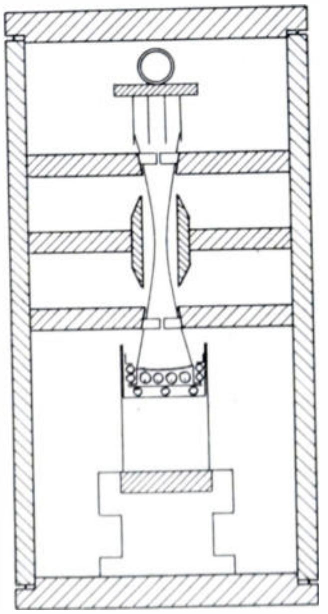

Fig. 4—Cross section of experimental model.

Open-ridge waveguide was used for both buncher and catcher in order to minimize beam-current interception. The wide dimensions of these guides coincided with that of standard $1\frac{1}{2}$ inch×3 inch S-band waveguide. Gap spacing was 0.6 cm; drift space width was 1.0 cm. The active length of the tube, i.e., the z dimension of the electron stream, was chosen to be 75 cm. Drift-space length was restricted to 4.0 cm (not including gap spacing) in order to avoid excessive dc beam spread and rf debunching. Beam spread was calculated and a Pierce cathode was designed to give the beam the property entry angle. A cross-sectional view of the tube is shown in Fig. 4.

Physically, the model consisted of two adjacent open-ridge waveguides set into a brass box, the top, bottom, and end plates of which were removable. Four half-wavelength matching sections were also built into this main structure, one at each end of the buncher and catcher waveguides. Fuse-wire gaskets provided vacuum-tight seals, and four resonant-iris glass windows allowed rf entry and exit. The tube was continuously evacuated. A photograph of the tube completely assembled is shown in Fig. 5. The electron gun was made in three sections, each 10 inches long and three-fourths inch wide. An internally water-cooled collector provided

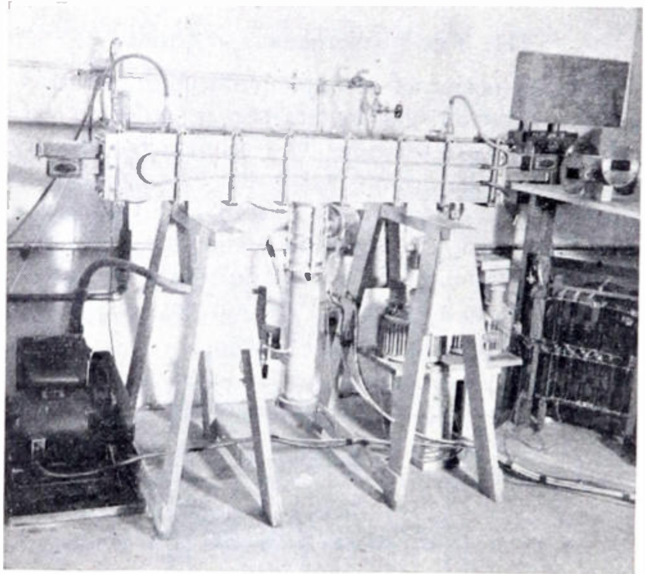

Fig. 5—Photograph of assembled tube.

fins to trap secondary electrons during low-voltage tests. For details of the electron gun and collector see Fig. 6.

The rf input was supplied by a 720BY magnetron capable of delivering 200-kw rf pulses. These pulses were synchronized with the modulator output. Since all power measurements were made with an average-power meter, synchronization was extremely important.

This experimental model had one serious deficiency which persisted despite all efforts to overcome it: Only 40% of the beam current arrived at the collector. The theoretical power gain of the tube was 5.6; but since power gain is dependent upon the square of current, a power gain of only 0.9 could be expected from the actual tube. This is very nearly what was measured. In one instance, with a beam voltage of 22 kv, unity gain was measured. It is interesting to note that the ratio of power output with beam on, to power output with beam off, was over 40 db. Hence it was concluded that the theory of the tube was substantiated.

Acknowledgment

Without the encouragement and guidance of E. L. Ginzton, this work would not have been done. Likewise, M. Chodorow and H. J. Shaw gave many valuable theo-

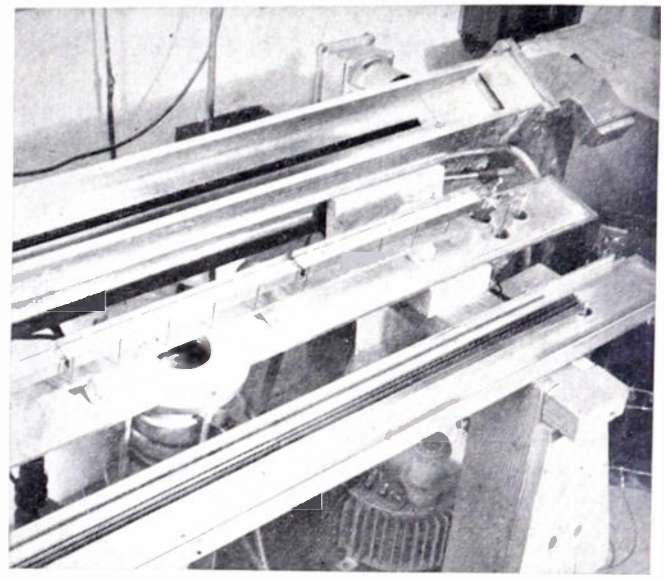

Fig. 6—Close-up photograph of collector (front), electron gun (center), and grid (rear).

retical and practical suggestions during its course. The work was sponsored in the most part by the Navy Department (Office of Naval Research) and the U. S. Army (Signal Corps) under ONR Contract n6-ONR-251 Consolidated Task No. 7.

Appendix

1. Solution of Catcher Line Equations

In this section (3) is obtained by solving (1) and (2) simultaneously, subject to suitable boundary conditions. By differentiation and substitution, (1) and (2) yield two second-degree equations.

$$\begin{aligned} \frac{d^2V_2}{dz^2} + \beta^2 V_2 &= -j\omega L_l I_l' e^{-j\beta z} \\ \frac{d^2 I_2}{dz^2} + \beta^2 I_2 &= -j\beta I_l' e^{-j\beta z}. \end{aligned} \tag{23}$$

These equations are members of a well-known class since the right-hand side is a solution of the homogeneous equation. In such a case the general solution is known to be

$$\begin{aligned} V_2 &= Ae^{j\beta z} + Be^{-j\beta z} + Cze^{-j\beta z} \\ I_2 &= De^{j\beta z} + Ee^{-j\beta z} + Fze^{-j\beta z}. \end{aligned} \tag{24}$$

The constants C and F may be determined by substituting (24) into (23). The expressions for V_2 and I_2 become

$$\begin{aligned} V_2 &= Ae^{j\beta z} + Be^{-j\beta z} + \tfrac{1}{2} I_l' Z_0 z e^{-j\beta z} \\ I_2 &= De^{j\beta z} + Ee^{-j\beta z} + \tfrac{1}{2} I_l' z e^{-j\beta z}, \end{aligned} \tag{25}$$

where $Z_0 = \sqrt{L_l/C_l}$.

This equation completely describes the desirable wave n the catcher, i.e., the linearly amplified wave. This vave propagates in the same direction as the voltage vave in the buncher, and with the same velocity. Two of the four remaining constants may be eliminated y substituting (25) into either of the two original dif- erential equations (1) or (2), from which it is found hat $A = -Z_0D$, and $B = Z_0(E - jI_i'/2\beta)$. With this implification, (25) becomes

$$V_2 = Z_0\left[\tfrac{1}{2}I_i'ze^{-j\beta z} + \left(E - j\frac{I_i'}{2\beta}\right)e^{-j\beta z} - De^{j\beta z}\right] \qquad (26)$$

$$I_2 = \tfrac{1}{2}I_i'ze^{-j\beta z} + Ee^{-j\beta z} + De^{j\beta z}.$$

The boundary conditions completely determine the onstants D and E. At $z=0$, it is assumed a perfect ermination is present; no energy has previously been lelivered to the catcher, hence only a backward wave can exist at this point, for which, $V_2/I_2 = -Z_0$. The insertion of this boundary condition into (26) allows solution for E in terms of known quantities, namely, $E = jI_i'/4\beta$. Only D remains to be determined, and this is done by substituting into (26) the boundary condition at $z=L$, namely, $V_2/I_2 = Z_0$. This is true because of the absence of any backward traveling wave at this point. This gives $D = -jI_i'e^{-j2\beta L}/4\beta$. The two non-amplified terms may be put into neater form by the following manipulation:

$$\frac{j}{2\beta}\left[e^{-j\beta z} - e^{j\beta(z-2L)}\right] = \frac{j}{2\beta}e^{-j\beta L}\left[e^{-j\beta(z-L)} - e^{j\beta(z-L)}\right]$$

$$= \frac{1}{\beta}e^{-j\beta L}\sin\beta(z-L).$$

This completes the particular solutions of (1) and (2) in the case where Z_0 terminations are used.

Electronically Controllable Resistors*

JAMES N. THURSTON†, SENIOR MEMBER, IRE

IN THE FIELD of electrical measurement and control, the need frequently arises for a resistance whose value can be adjusted rapidly and conveniently over a certain limited range. For some applications, mechanically controlled resistive elements are too sluggish or too complicated to satisfy the requirements. An electronically controllable resistor appeared to be a possibility, and the present summary outlines some of the work done on this subject.

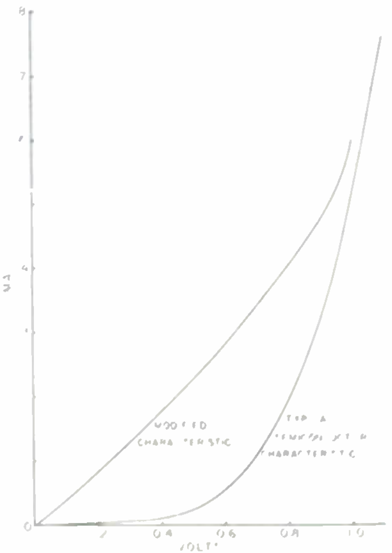

Fig. 1—Typical nonlinear volt-ampere characteristics.

Methods now available for the electronic control of resistance include use of the nonlinear relationship between the plate current and plate voltage of a high-vacuum triode. Another possibility is the use of feedback to control the input impedance of a vacuum-tube circuit.[1] Both of these methods suffer from a lack of stability.

A somewhat different approach to the problem is to employ a nonlinear resistor, such as a silicon carbide or germanium semiconductor, with a vacuum tube used to control the bias current, and hence, the dynamic resistance of the element. The approximate volt-ampere characteristic of a typical semiconductor, for normal direction of current flow is shown in Fig. 1.

If the characteristic of Fig. 1 is too nonlinear, it can be modified by linear resistors in series and parallel combination with the semiconductor. As an example of what can be done in this manner, assume that the following specifications have been set up for the controllable resistor as applied to a strain-gage measurement problem.

Quiescent current $I_0 = 2.0$ ma

$R_{ac} = \dfrac{de}{di} = 200$ ohms at $I_0 = 2.0$ ma

$R_{ac} = bi + c$, where b and c are constants to be determined

$\dfrac{dR_{ac}}{di} = -25$ ohms per ma

From these specifications, the required expression for the volt-ampere characteristic becomes $e = -12{,}500\, i^2 + 250\, i$, where i is in amperes and e is in volts. The calculated volt-ampere characteristic is shown in Fig. 1.

A circuit which approximates the desired characteristic, and which can be used to provide a convenient control of the dynamic resistance, R_{ac}, is shown in Fig. 2. Measured characteristics of this circuit are shown in Fig. 3.

A small voltage, having the same frequency as the control voltage, e_1, appears at the terminals across which R_{ac} is measured. If this is undesirable, a more complicated circuit can be devised in which most of the voltage is eliminated.

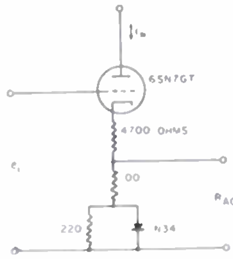

Fig. 2—Modified nonlinear element with control circuit.

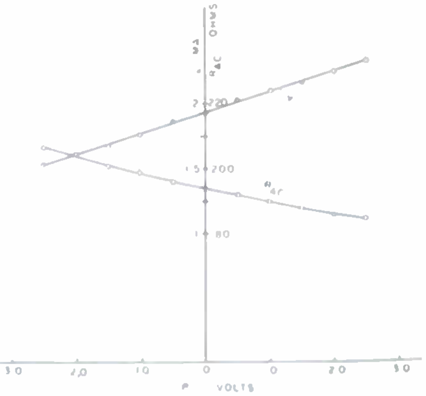

Fig. 3—Measured characteristics of modified nonlinear element.

* Decimal classification: 621.375.9×R383.2. Original manuscript received by the Institute, December 27, 1950; abstract received, July 25, 1951. The work reported here was done at the Research Laboratory of Electronics, MIT, Cambridge, Mass., under Navy Bureau of Ordnance Contract NOrd 9661.

† Electrical Engineering Department, University of Florida, Gainesville, Fla.

[1] H. J. Reich, "The use of vacuum tubes as variable impedance elements," Proc. I.R.E., vol. 30, pp. 288–293; June, 1942.

Echo Distortion in the FM Transmission of Frequency-Division Multiplex*

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Summary—**The composite multiplex signals generated by frequency-division methods long standard in telephone communication, can be transmitted by the new transcontinental broad-band FM radio relays. Signal intermodulation by echoes must be minimized. Such intermodulation is investigated in this paper experimentally and analytically. Two types of echoes are considered: (1) Weak echoes with delays exceeding 0.1 microseconds, caused mainly by mismatched long lines; and (2) Powerful echoes with delays shorter than 0.01 microseconds, caused by multipath transmission, and leading to selective fading. Using random noise signals, the distortion is evaluated as a function of various parameters of the echo, the baseband and the rf modulation.**

Introduction

THE BROAD-BAND long-distance radio repeater system which is rapidly spreading over this country was designed for the double purpose of transmitting either one television channel or, by multiplex telephony, several hundred voice frequency channels. In the system employing coaxial cable, the multiplex method used is to assign to each voice channel a band four kilocycles wide, the whole forming a block occupying a bandwidth of the order of two megacycles. If it were possible to employ this same frequency-division technique of telephony over the radio circuits, the identity in type of terminal apparatus would obviously be advantageous in the system as a whole. The possibility of using the radio system in this way has therefore been carefully studied in the Bell System.

Multiplex signals transmitted over nonlinear circuits are subject to intermodulation. In FM radio, such as is here used, the nonlinearities are not caused primarily by amplitude characteristics of electronic devices, but by inequalities in the propagation constant over the frequency band of those transmission circuits which are traversed by the frequency-modulated carrier. These disturbances may be thought of as amplitude and phase distortions, but for present purposes it is easier to interpret them in terms of transmission irregularities producing reflections and echoes.

In this paper we have attempted, both by experiment and theory, to determine the intermodulation of multiplex signals as a function of echo parameters. Such information makes it possible to establish the limits of echoes which can be tolerated in high grade commercial telephone transmission, this work being part of a larger program occasioned by the first large scale installation of a system of this sort on a country-wide basis.

Some experimental and theoretical work on echoes in FM circuits is found in the literature,[1–7] but the quantitative data available relates mostly to single intermodulation products of sinusoidal two-tone signals. The wave shapes of multichannel carrier telephone signals are highly irregular, approaching random noise in character, and their distortion products are due to complex mixtures of second, third, and higher order modulation. These modulation products had previously been analyzed for AM only. Thus, it became necessary to extend to FM the theory of nonlinear distortion by noise-like multiplex signal bands, and to check and supplement the theory by extensive experimental research.

To achieve clear-cut test conditions, it was deemed impractical to cope with the irregularities of live telephone conversations or with the vagaries of the echoes encountered in actual long distance transmission. Multiplex speech was therefore simulated by a random noise band extending over the intended subcarrier frequencies, in accordance with a method proposed by E. Peterson,[2] and echoes of the desired types were artificially produced under controlled laboratory conditions.

Two distinct classes of echo interference were investigated. The first comprises relatively weak echoes with delays ranging upward from 0.1 μ second. This is the type caused by impedance mismatch at the ends of long lines such as the wave guide connections to the antenna towers, and in some cases, by reflections from buildings near the transmission path. These are designated as line echoes. The second comprises powerful echoes approaching the level of the main signal, with very short time delays (usually less than 0.01 μ second) and caused by atmospheric multi-path transmission or by ground reflections. They may be called transmission path echoes.

I. Line Echoes

Experimental Procedure

Although most of the actual echoes originate in the rf path, the experimental echoes were produced in the 70

* Decimal classification: R630.3×R480. Original manuscript received by the Institute, April 4, 1951; revised manuscript received, November 21, 1951.

† Bell Telephone Laboratories, Inc., Deal, N. J.

[1] Murray G. Crosby, "Frequency modulation propagation characteristics," Proc. I.R.E., vol. 24, pp. 898–913; June, 1936.

[2] E. Peterson, "Gas tube noise generator for circuit testing," *Bell Lab. Rec.*, vol. 18, no. 3, p. 81; November, 1939.

[3] W. R. Bennett, "Cross modulation requirements on multichannel amplifiers below overload," *Bell Sys. Tech. Jour.*, vol. 19, pp. 587–610; October, 1940.

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[5] S. T. Meyers, "Nonlinearity in frequency modulation radio system due to multipath propagation," Proc. I.R.E., vol. 34, pp. 256–265; May, 1946.

[6] B. VanderPol, "The fundamental principles of frequency-modulation," *Jour. IEE* (London), vol. 93, pp. 153–158; May, 1946.

[7] S. O. Rice, "Statistical properties of a sine wave plus random noise," *Bell Sys. Tech. Jour.*, vol. 27, pp. 109–157; January, 1948.

mc if circuits which permit easier control of path length, reflection coefficient, and phase. The net result, as far as distortion is concerned, is the same.

Two means of generating echoes were used: direct connected two-path transmission, and reflection from a misterminated spur line (Fig. 1). The first method was

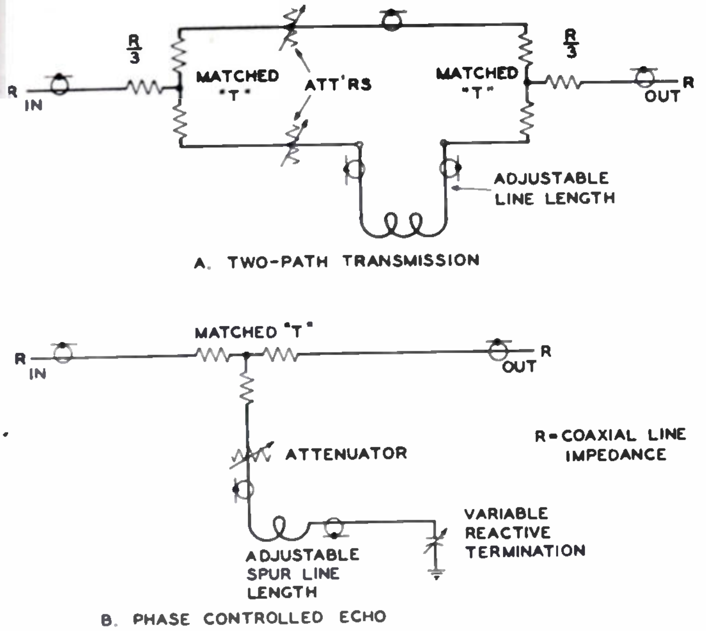

Fig. 1—Methods of echo generation.

used only when strong echoes were needed. In all other cases the reflection method was employed, as it has the experimental advantage of requiring shorter line length, and lends itself more easily to continuously adjustable phase control by means of a variable terminating reactance.

The signal source was amplified fluctuation noise of vacuum tubes, filtered to the desired bandwidth and equalized to a flat frequency characteristic, or to the desired pre-emphasis of high baseband frequencies. In order to be able to measure distortion products falling inside the modulating frequency band, narrow gaps were cut into it by sharp band elimination filters at the transmitter input. The received cross modulation was observed at these filter frequencies. By comparing the reception level with and without the band elimination filters, the distortion-to-signal ratio was established.

The complete test assembly is shown in Fig. 2.

Test Results

The distorting effects of echoes were investigated as functions of the following parameters: a. Echo Characteristics: amplitude, phase, delay, and number of echoes; b. Signal Characteristics: frequency deviation, bandwidth, band location, subcarrier location, and amplitude characteristic of the baseband signal.

a.1 Echo Amplitude: Theory[8] indicates that unless the echo approaches the main signal amplitude, the distortion-to-signal amplitude ratio D/S is proportional to the echo-to-signal amplitude ratio r.[9] This was verified by test over a 40 db range of r, as shown in Fig. 3, on the following page.

a.2 Echo Phase: An echo can be divided into two vector components: one in phase with the main signal at the carrier frequency, and one in quadrature. Analysis shows that for small echoes the in-phase component

[8] See Analytical Appendix to this paper and (3) following.

[9] If an echo exceeds the amplitude of the first signal, it may be interpreted as the main signal and the actual signal, as a weaker echo with negative time delay.

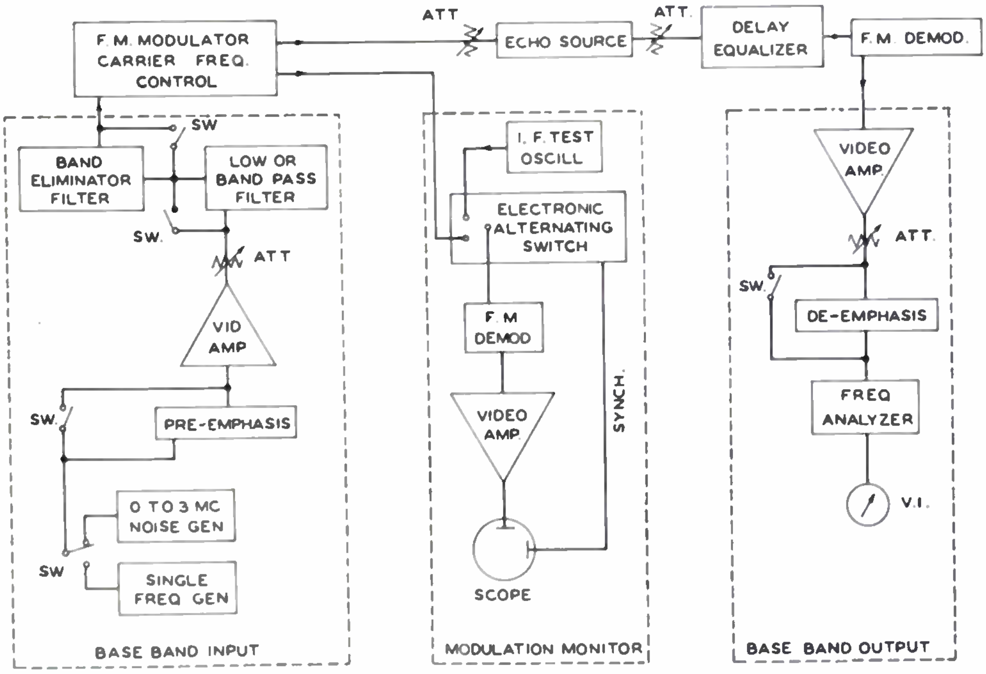

Fig. 2—FM distortion by line echoes. Test assembly.

produces only odd order distortion products in the baseband, and the quadrature component only even order products.

Let r = relative echo amplitude

τ = echo delay in seconds

s = FM carrier frequency deviation (radians per second)

p_m = maximum frequency of the signal noise band (radians per second). Also bandwidth of signal if the signal noise band starts from zero frequency

p = frequency of observed signal or distortion product

$k = p/p_m$ signal frequency relative to maximum

D/S = distortion-to-signal amplitude ratio.

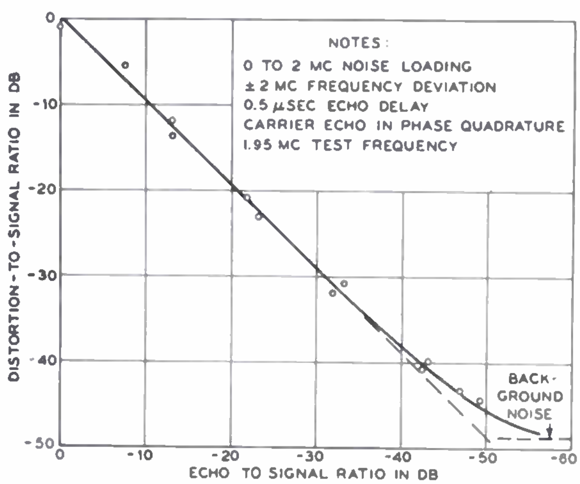

Fig. 3—Distortion versus echo strength.

One may define a "short delay range" in which the delay time is shorter than the reciprocal of the highest baseband frequency and shorter than the reciprocal of the FM deviation:

$$\tau p_m < 1 \tag{1}$$

$$\tau s < 1. \tag{2}$$

In this short delay range the distortion can be computed for any given signal wave form, for instance, for a flat band of random noise extending upward from zero frequency.

Then, as derived in the Appendix, the quadrature echo component r_q produces a second order distortion ratio

$$\frac{D_2}{S} = 0.2 r_q \tau^2 s p \sqrt{1 - 0.5k}, \tag{3}$$

and the in-phase echo component r_p, a third order ratio

$$\frac{D_3}{S} = 0.019 r_p \tau^3 s^2 p \sqrt{1 - 0.333k^2}. \tag{4}$$

Equations (3) and (4) show that in the short delay region the quadrature echo component produces the predominant distortion, since the numerical factor of D_2/S is over 20 db larger than that of D_3/S, and since D_3/S contains an extra factor τs which, according to (2), is <1.

At the other extreme, in the "long delay range," when

$$\tau s \gg 1, \tag{5}$$

the instantaneous carrier sweeps through both in-phase and quadrature regions and the level difference corresponding to the two phase conditions of the unmodulated carrier tend to disappear.

Experiments have confirmed all these relations and shown close quantitative agreement with (3) and (4) in the low modulation region, Fig. 4.

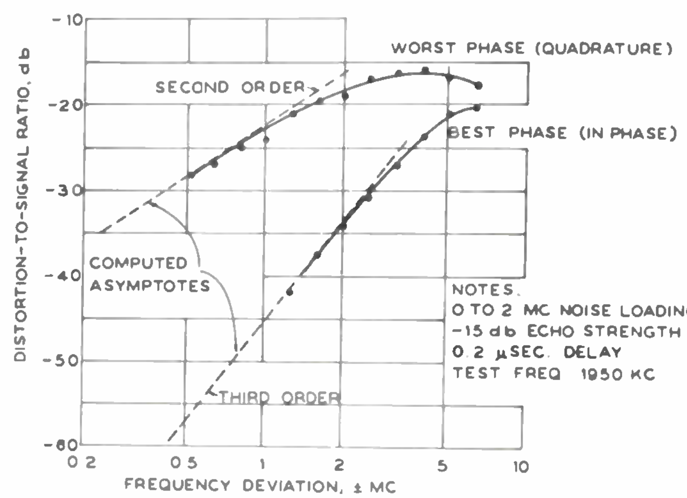

Fig. 4—Distortion versus echo phase.

a.3 Echo Delay: Equation (3) indicates that in the short delay region the predominant second order distortion increases with the square of delay time, Fig. 5, shows a smooth transition from the computed straight

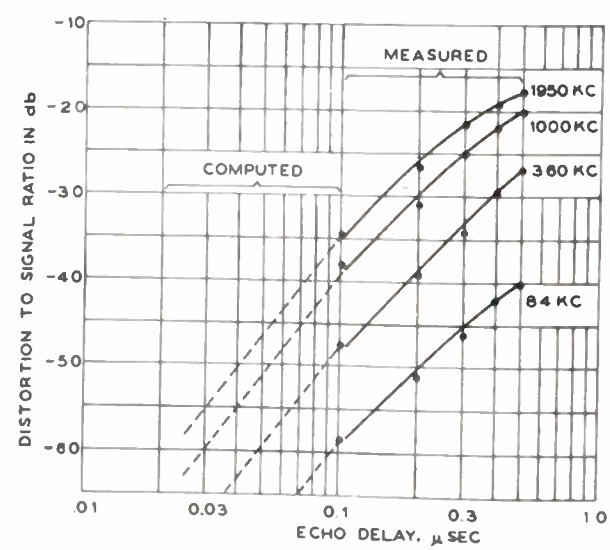

Fig. 5—Distortion versus echo delay. Echo strength: −15 db; noise input: 0–2 mc; frequency deviation: ±1 mc.

line short delay asymptotes computed from (3), to the measured values at longer delay times. In the long de-

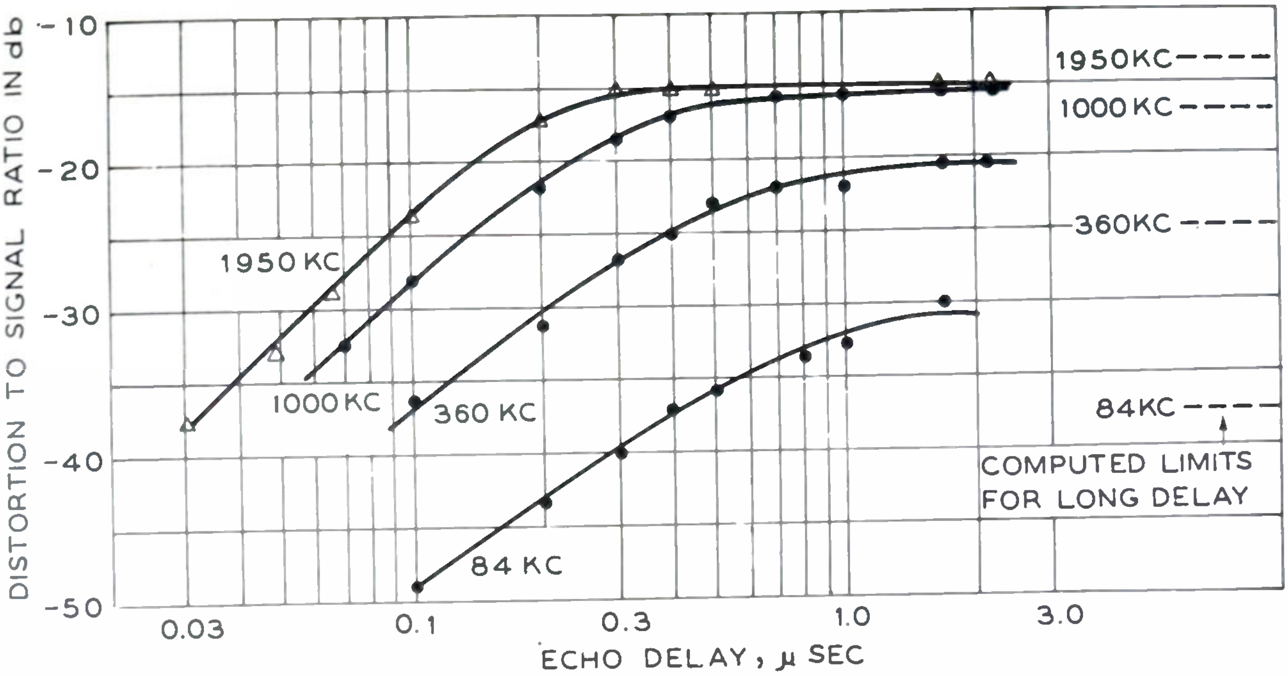

Fig. 6—Distortion versus echo delay. Echo strength: −15 db; noise input: 0–2 mc; frequency deviation: ±4 mc.

lay region the phase of the echo becomes practically unrelated to that of the signal, and the echo acts much like noncoherent noise. The relative distortion level therefore approaches a constant limit independent of delay. This flattening of the distortion-delay characteristic at large delays is shown in Fig. 6.

The limiting distortion-to-signal ratio for long delays can be computed if the frequency modulation index is large:

$$s/p_m \gg 1,$$

in which case the energy distribution of the interference approaches an error function. Assuming that all but a negligible fraction of the echo energy is contained between the nominal frequency deviation limits, one arrives at the approximate formula

$$\frac{D}{S_a} \doteq 5.8 r p_m{}^{0.5s-1.5} p \epsilon^{-2.88p^2 s^{-2}} \qquad (7)$$

as shown in the Analytical Appendix.

The measured distortion asymptotes from Fig. 6 agree within experimental accuracy with the values computed from (7) for the high-frequency distortion products. For the much smaller low-frequency distortion products, the measured values exceed the computed asymptotes. This may indicate that at low frequencies there was still some phase correlation between the signal and the echo of longest delay.

a.4 Multiple Echoes: A single echo produces an approximately sinusoidal ripple in the received phase characteristic. If

ω = rf signal frequency
r = echo amplitude
τ = echo delay and
θ = echo phase at the carrier frequency, then the phase deviation of the received signal

$$\phi_d = \tan^{-1}\frac{r \sin(\omega\tau + \theta)}{1 + r\cos(\omega\tau + \theta)} \doteq r \sin(\omega\tau + \theta). \qquad (8)$$

As an harmonic function of frequency, the phase error has the three characteristics of a rotating vector: amplitude, frequency, and phase.

If two echoes of equal time delay are superimposed, they interfere like vectors of equal rotational speed; they reinforce each other when in phase and cancel one another when equal and opposite.

The cancellation obtained experimentally by two echo lines of equal length is shown in Fig. 7, case A; it ap-

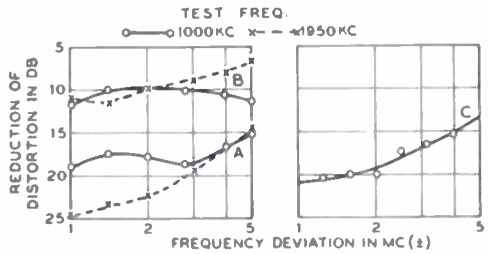

Fig. 7—Reduction of distortion by compensation echo. Noise loading: 0–2 mc; A—compensator delay = distorter delay; B—compensator delay = 0.84 of distorter delay; C—compensator delay = average of 3 distorter delays.

proximates 20 db. The slight residual distortion may be attributed to experimental errors and to the deviation of the phase ripples from pure sine-wave shape.

If a radio relay system contains many antenna towers of equal height, their accidental mismatch echoes will add in random phase relation. Hence, their most probable combined amplitude increases with the square root of the number of repeater stations. It is also possible to cancel the cumulative distortion due to such echoes if methods are provided for recognizing the phase ripple or its distorting effect. Tests show that the cancellation of echo ripples is possible even if amplitude limiters are interposed between distorting and correcting echoes.

A partial cancellation of plural echoes by a single correcting line is feasible even if the time delays of the interfering echoes spread over a limited range. This is explained by the equation

$$r \sin \omega\tau_1 - r \sin \omega\tau_2 = 2r \sin 0.5\omega(\tau_1 - \tau_2) \cos 0.5\omega(\tau_1 + \tau_2). \quad (9)$$

If τ_1 and τ_2 differ by a relatively small amount, the factor $\sin 0.5\omega(\tau_1 - \tau_2)$ is $\ll 1$ over the entire useful radio frequency band, and reduces the ripple caused by the cosine term which corresponds to the mean of the two echo delays.

This analytical result is verified by the curves of Fig. 7, case B, which show a 10 db average reduction of echo distortion by a correcting echo with a 16 per cent shorter delay. Fig. 7, case C, shows that the distortion caused by three echoes with delays extending from 0.16 to 0.23 μ second was reduced about 20 db by a single correcting echo with 0.20 μ second delay.

b.1 Peak Frequency Deviation: The frequency deviation s enters all three approximate equations (3), (4), and (7) for the distortion-to-signal ratio. It has already been shown in section *a.2* that in the short delay region the highest relative distortion, which occurs at phase quadrature, increases proportionally to the FM deviation in accordance with (3).

In the long delay region, (7) indicates that the distortion ratio decreases 1.5 db per db increase of frequency deviation.

The computed short-delay and long-delay asymptotes are compared with measured values in Fig. 8. All four curves of this figure show the distortion ratio near the upper limit of a flat-noise signal band extending from 0 frequency up. The long delay asymptotes are closely approached by all curves except by that for the narrowest signal band. (See comment to Fig. 5, section *a.3*.) The short delay asymptotes agree fairly well with the curves for the 0 to 67 kc and 340 kc bands, but the computed distortions are much too high for the broader bands to which they should not be expected to apply because inequality (1) is not satisfied. Note, however, that even in these wide bands the increase of the low-level distortion ratio is proportional to the deviation as long as inequality (2) is satisfied.

For the transition region between the asymptotes the experimental rule applies that the maximum distortion-to-signal ratio at the upper edge of a flat noise band may in some cases reach, but not exceed, the echo-to-signal ratio.

b.2 Bandwidth: Equations (3), (4), and (7) show that for a given peak frequency deviation and signal frequency, the distortion-to-signal ratio increases only slightly with the signal band width in the regions subject to analysis. The increased numbers of modulation products is offset by their reduced amplitudes. In the intermediate region the distortion may even be lowered by a broadening of the signal band, as illustrated by the crossing of curves 3 and 4 in Fig. 8. This occurs at low FM index, when inequality (2) is satisfied but not inequality (1).

b.3 Band Location (Spread Band): If the telephone subcarrier channels do not occupy the lowest possible frequency band but are shifted upward by at least their collective bandwidth so that the entire rf baseband is contained within one octave, all second order products fall outside of the signal band and cannot cause distortion. This results in a lower distortion ratio in the region which satisfies inequality (2) (short delay and low modulation) because in this region only the third order products are appreciable and, according to (4), the distortion ratio decreases with the square of deviation.

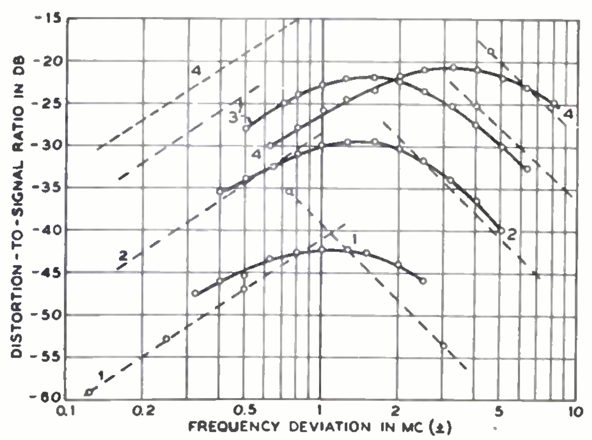

Fig. 8—Distortion versus frequency deviation. Flat-noise loading. Dashed straight lines: computed asymptotes.

Curve	1	2	3	4
Noise Band in kc	67	340	1,000	1,800
Test Freq. in kc	84	360	1,000	1,950

Echo Strength: −21 db; Echo Delay: 0.5μ sec.

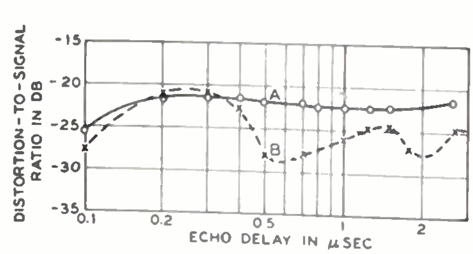

Fig. 9—Distortion versus band location. A—modulating noise band 0–1 mc, test freq. 1 mc; B—modulating noise band 1–2 mc, test freq. 1.95 mc (spread band). Echo strength: −15 db; frequency deviation: ±4 mc.

In the region of long delay and large modulation index, however, computations indicate that the asymptotic value for the distortion ratio for the highest frequency voice carrier should be increased 6 db by the frequency shifts, and the relative distortion ratio becomes even more unfavorable in the lower channels. In the intermediate range the experimental spread band distortion ratio varies irregularly, as shown in Fig. 9.

For the conditions which would apply in the TD-2[10] Radio Relay System, if the voice carrier band were limited to 1 mc and the predominant echoes had 0.5 μ-second delay, the spread band distortion ratio is somewhat lower than the nonspread. However, the ratio of background noise to signal amplitude increases with the baseband frequency, and thus lowers the safety margin of the system during fades. Furthermore, the required rf bandwidth increases with the baseband frequency. For these reasons spreadband operation does not seem to be advisable in the TD-2 system.

b.4 Channel Location in Signal Band: Both (3) and (7) indicate that with flat noise loading the distortion ratio increases approximately in linear proportion to the frequency of the signal channel. This is in agreement with the well known fact that noise and distortion in FM circuits approximate a triangular frequency spectrum. The solid curves of Fig. 10 show the distortion ratio as a function of baseband frequency for two echo delays.

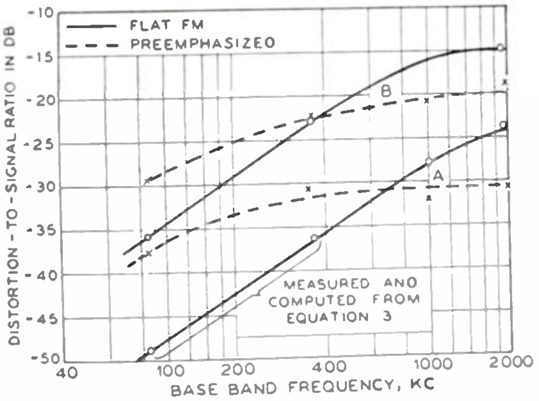

Fig. 10—Distortion versus signal frequency. Noise loading: 0–2 mc; FM deviation: ±4 mc. Echo strength: −15 db; echo delay $\begin{cases} A: 0.1\ \mu \text{ sec} \\ B: 0.5\ \mu \text{ sec} \end{cases}$

b.5 Amplitude Characteristic (Pre-emphasis): The above mentioned piling up of noise and distortion in the high baseband frequencies can be reduced by pre-emphasizing the high frequencies in the transmitting modulator, and correspondingly de-emphasizing them after FM demodulation. If this pre-emphasis were carried to the extreme of 6 db per octave, the characteristic would be flat on a phase modulation scale. However, analysis shows that there would then be excessive distortion-to-signal ratio at low baseband frequencies. The pre-emphasis actually used, Fig. 11, varied from 0 to 13 db in passing from the lowest to the highest channel (from 60 kc to 2 mc). This reduced the distortion ratio at the highest baseband frequency by about 5 db, and increased the much smaller distortion ratio in the lowest channel by 10 db, as shown by the dashed curves of Fig. 10.

[10] TD-2 is the Western Electric code for the type of equipment used on the New York-Chicago and other radio relay systems of the American Telephone and Telegraph Co.

II. Transmission Path Echoes

Experimental Procedure

Observation shows that selective fading is caused by destructive interference between a microwave signal and one or more echoes lagging, at most, a few millimicroseconds. Two-path transmission was simulated in the present investigation by reflection in an air-filled coaxial line, and the echo phase was regulated by a sliding probe. It was realized that in nature a great variety of combinations of three and more echoes probably occur. However, the interference patterns due to such combinations can usually be related to two-path patterns of similar amplitude and phase characteristics, at least in the regions of sharp minima which produce nonlinear distortion.

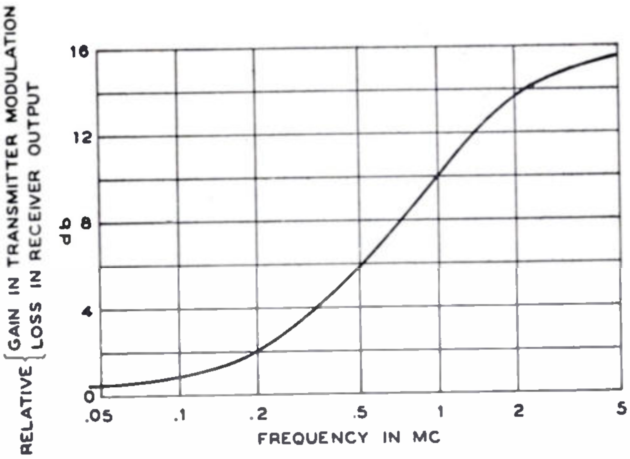

Fig. 11—Pre-emphasis / De-emphasis Characteristic.

The signal-source, modulating, receiving, and analyzing means were the same as described in section I. The test assembly used for the fading tests is shown in Fig. 12, on the following page.

Test Results

Since selective fades are interpreted in terms of short time echoes, their distorting effects were investigated as functions of parameters similar to those listed in I, that is:

a. Echo characteristics: depth of fade, delay, deviation of carrier from frequency of deepest fading,[11] and number of echoes.
b. Signal characteristics: frequency deviation, bandwidth, band location, subcarrier location, and amplitude characteristic of the baseband signal.
c. Apparatus characteristics: The influence of imperfect limiting on fading distortion and background noise was examined experimentally.

Analytical Discussion

In two-path transmission with short delays no serious selective fading can occur unless the reflection coefficient

[11] The deepest fading occurs when signal and echo are in opposite phase.

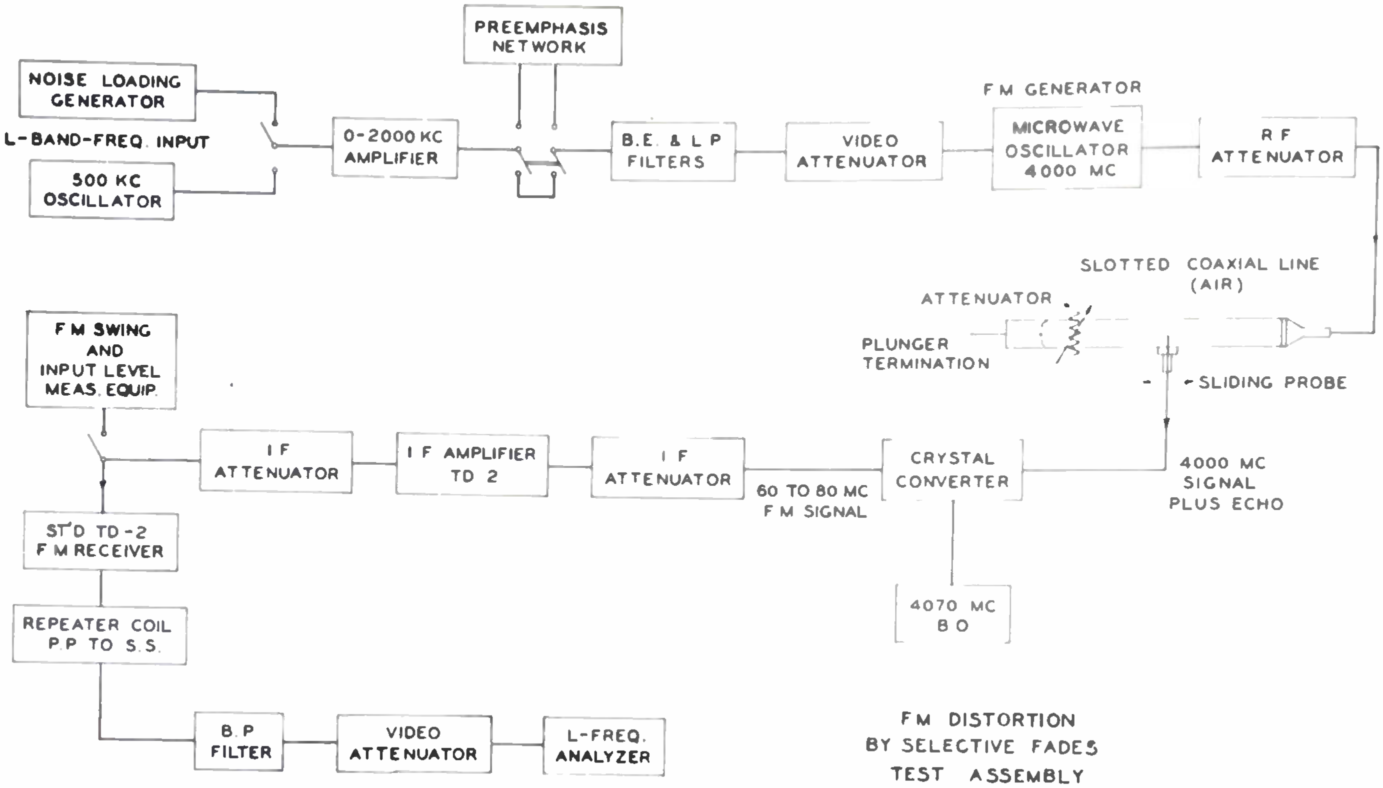

Fig. 12—Test assembly used for fading tests.

r approaches unity so that

$$1 - r = \delta \ll 1. \tag{10}$$

$1/\delta$, which may be called the fading depth, is the amplitude ratio between the signal amplitude in free space and that at the frequency of deepest fading. In the range defined by (10) the theory developed in the Appendix states the distortion of FM baseband signals does not depend on r and τ separately, but on combined function

$$T = \frac{\tau}{1 - r} = \frac{\tau}{\delta}. \tag{11}$$

It can be shown that T equals the maximum of the envelope delay distortion caused by the fade, which occurs at the frequency of deepest fading. All other symbols are the same as listed in section I.

In the "low distortion" range defined by

$$Ts \ll 1, \tag{12}$$

only the lowest orders of distortion products need be considered. At the frequency of deepest fade, all even order distortions are zero. The third order distortion ratio is highest at the frequency of deepest fade, where it has the value

$$\frac{D_3}{S} \doteq 0.038T^3s^2p\sqrt{1 - 0.333k^2}. \tag{13}$$

The second order distortion increases with the deviation β of the carrier from the frequency of deepest fade up to a maximum which is located at

$$\beta_{\max} = \pm\frac{0.58}{T}. \tag{14}$$

At this frequency deviation

$$\frac{D_2}{S} = 0.13T^2sp\sqrt{1 - 0.5k}. \tag{15}$$

In the former approximation range $D_3 \ll D_2$ so that only the second order distortion need be considered. Since tolerance limits for long distance telephone conversations require that D/S be smaller than 0.1 or -20 db, (15) was a suitable starting point for investigation.

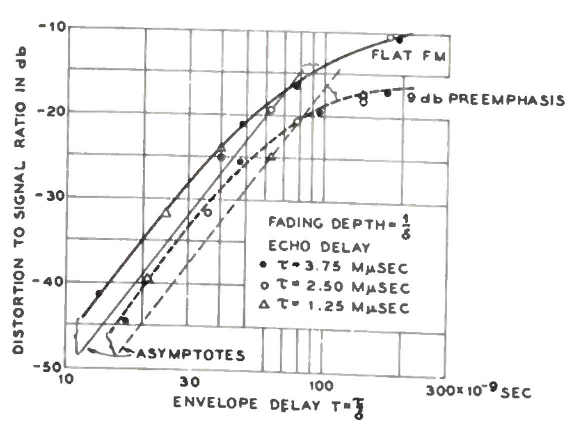

Fig. 13—Distortion versus product of fading depth and echo delay. Noise input: 0–2 mc; test frequency: 1,950 kc; FM deviation: ±4 mc.

a.1 Depth of Fade and Delay: Equation (15) indicates that the distortion ratio should increase with T^2 which is the square of the product of fading depth and echo delay as shown in (11). The upper curve of Fig. 13 shows

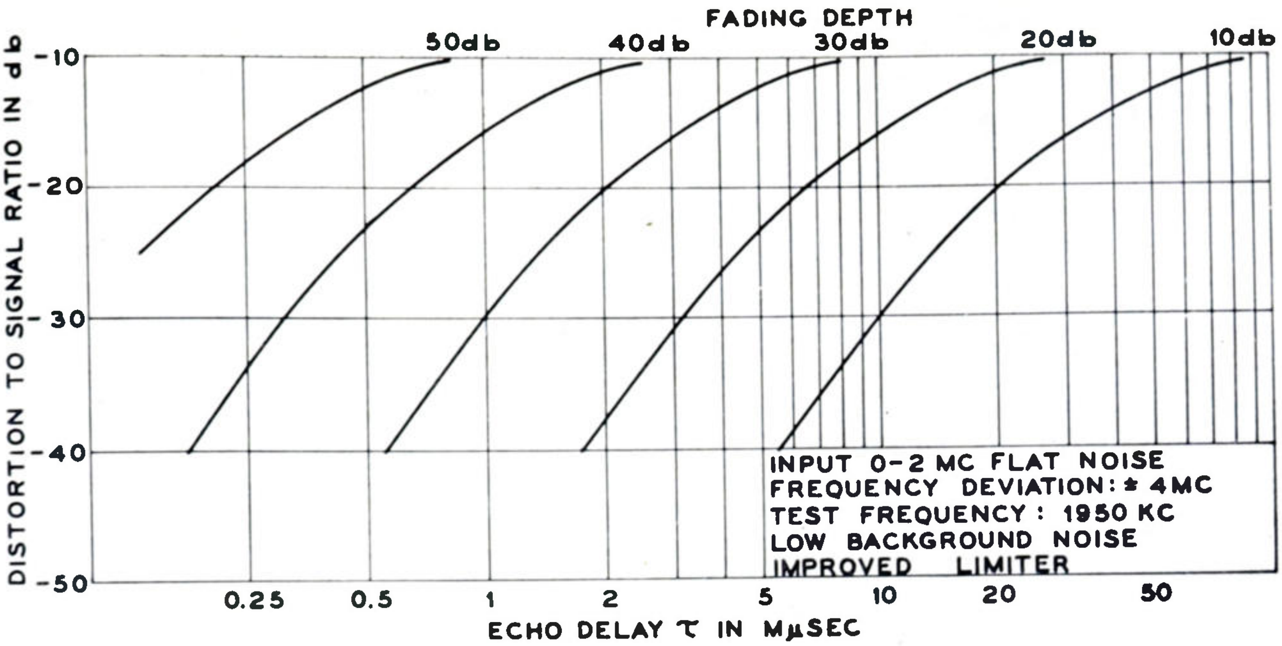

Fig. 14—Distortion versus fading depth and echo delay.

that this relation is reasonably well maintained in the high frequency channels as long as D/S is <-20 db, which means that T is $<6\cdot10^{-8}$. seconds. At larger values of T, the relative distortion increases more slowly and approaches a limiting value of about -10 db. The effect of T can be separated into its components τ and δ by a family of curves shown in Fig. 14, which help to visualize the distortion under specified echo conditions.

a.2 Deviation from Frequency of Deepest Fading: When signal and echo are in phase opposition, there results only a weak third order distortion defined by (13). The second order distortion increases with change of phase angle in either direction up to a maximum which occurs at a phase angle of $\pi\pm0.58\ \delta$ radians. For a stationary fading pattern this corresponds to a frequency shift of $\beta=\pm(0.58/T)$ radians per second.

In our test equipment the phase change was brought about by moving the pickup probe in either direction a small distance from the standing wave node. The minimum of distortion between the maxima was observed, but it was rather shallow unless the following conditions were painstakingly obtained:

1. Sufficient carrier power to prevent the FM limiter from breaking at the bottom of the fade.
2. Very low residual distortion in the test setup.
3. An amplitude limiter far better than needed for normal receiving conditions. This latter precaution was needed because near the bottom of a selective fade the carrier suffers considerable amplitude distortion which produces nonlinear baseband distortion if a fraction of it passes the limiter.

When the carrier frequency of maximum distortion is passed, the distortion ratio quickly drops to the residual distortion of the test equipment.

b.1 FM Deviation: In the low distortion range, the distortion ratio is proportional to the FM deviation s according to (15). Above this range it increases more slowly and seems to approach a limiting value. In Fig. 15 the distortions observed with deviations of 2, 4, and 8 mc are compared under otherwise equal conditions. The spread of relative distortion is greatest in the low distortion range. It does not quite reach the 12 db difference between 2 mc and 8 mc swings predicted by (15), presumably because of residual noise and distortion in the test equipment.

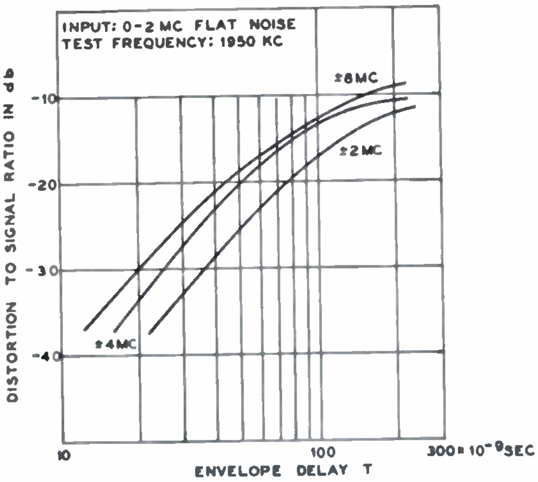

Fig. 15—Distortion versus frequency deviation.

b.2 Bandwidth: Equations (13) and (15) show that for transmission path echoes as well as line echoes, the distortion-to-signal ratio increases only slightly with the signal bandwidth, for a given peak frequency deviation and signal frequency.

b.3 Baseband Location (Spread Band): Theory indicates that second order distortion can be avoided by spread band modulation. However, no tests were conducted because the use of spread band was ruled out for reasons given in section I.

b.4 Channel Location: Equations (13) and (15) indicate that the distortion-to-signal ratio increases in nearly linear proportion with the frequency of the signal. The theoretical difference between the extreme test channel frequencies of 1,950 kc and 84 kc is 24 db. With large delay distortion T, and with carefully adjusted amplitude limiters, the greatest difference obtained in our experiments was 18 db, Fig. 16. With lower values of T, the small intermodulation due to delay distortion falling into the low frequency channels, was masked by intermodulation due to spurious amplitude modulation; hence, the observed difference between low-frequency and high-frequency channel distortions was smaller.

b.5 Amplitude Characteristic (Pre-Emphasis): In discussing distortion (and background noise) resulting from weak echoes of long delay, a desirable pre-emphasis characteristic was arrived at in section I. Using the same characteristic under conditions of deep selective fades, both noise and distortion were again found to be reduced, and by an amount slightly greater than in the first experiment. However, permissible pre-emphasis depends on available carrier power. If during fades the radio frequency noise approaches carrier power, occasional sharp noise peaks will cause "breaking" in the FM detector such that its output will approach a flat noise distribution instead of a triangular one, and the "de-emphasis" circuits will then bring the noise in the lower channels up above distortion in the high channels.

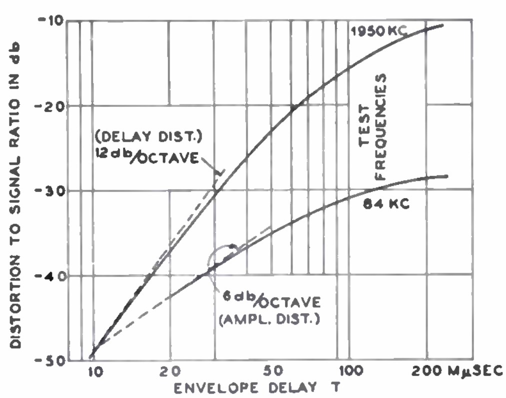

Fig. 16—Distortion versus signal frequency. Improved limiting. Input: 0–2 mc flat noise; freq. deviation: ±4 mc.

For this reason the experimental amount of applied preemphasis was limited to about 10 db; the reduction of the high frequency distortion ratio thus obtained was about 5 db, in close agreement with theory. (Compare upper and lower curves of Fig. 13.)

c.1 Influence of Limiting on Distortion: If a small fraction of the amplitude frequency variation associated with a sharp fade penetrates the amplitude limiters of the FM receiver, nonlinear baseband distortion intermodulation results. Theory indicates that when the carrier frequency differs by the small value

$$\beta = \frac{1}{T} \tag{16}$$

from the frequency of deepest fade, the maximum second order distortion due to an imperfect limiter occurs. In the low distortion range it equals

$$\frac{D_2}{S} \doteq \frac{0.2Ts}{C}\sqrt{1 - 0.5k}. \tag{17}$$

In this equation, C is the factor by which amplitude variations of the received carrier are reduced by the limiter. Comparing (17) with (15) one finds that it indicates a distortion ratio nearly independent of the baseband channel frequency (3 db higher at the extreme low frequency) and directly proportional to the delay distortion T, whereas the distortion ratio per (15) is proportional to the square of T. Hence, the influence of (17) should make itself felt in the low frequency channels and at low values of T.

Experiments with a variety of limiter conditions verified that the low frequency channels were most responsive to improved limiting; but the improvement was noticeable for all values of T, indicating that even for the highest T obtainable, the amplitude outweighed the delay distortion in the low frequency channels, Fig. 17.

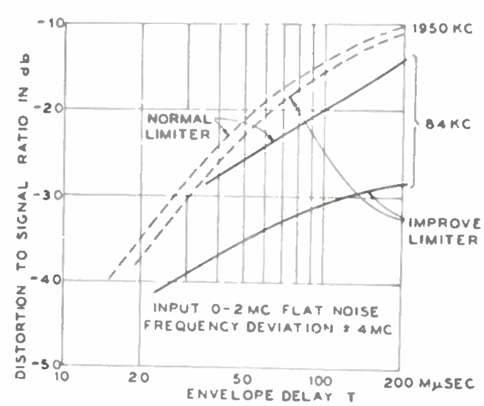

Fig. 17—Distortion versus limiter action.

c.2 Influence of Limiting on Background Noise: At high reception levels, conventional theory indicates that the background-noise amplitude ratio increases in proportion to baseband frequency ("triangular" FM noise spectrum). A more detailed theory[7] indicates that even with an ideal FM receiver, the noise spectrum flattens out when the total background noise energy in the received frequency band approaches the carrier level so closely that occasional noise peaks may cancel the carrier completely by interference.

This flattening out of the noise spectrum occurs at much lower background noise energies if a fraction of the amplitude fluctuations due to background noise penetrates the amplitude limiters in the FM receiver.

With imperfect limiters such flat noise effects may outweigh the distortion effects and prevent the use of pre-emphasis at fading depths which are still safe when the limiter action is improved.

Experiment showed that with a limiter which under normal-level conditions gave excellent reception, the noise difference between extreme baseband channels was reduced to 10 db when the background noise was 18 db below carrier level. By improving the limiter action, the low frequency noise was lowered 17 db (see Fig. 18).[12]

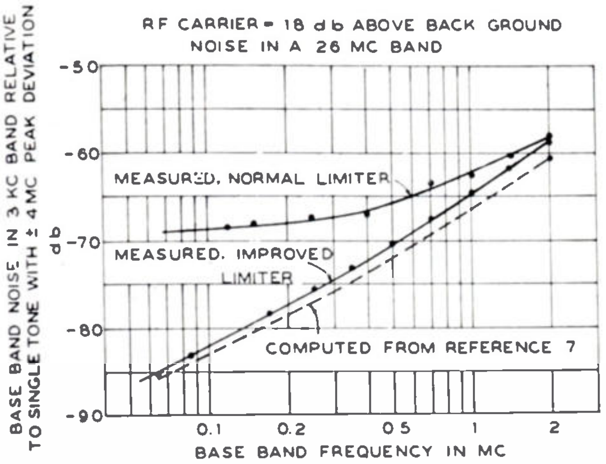

Fig. 18—Baseband noise versus limiter action.

Summary of Results

A. Line Echoes

1. The distortion-to-signal amplitude ratio increases in proportion to the echo strength.

2. In the "short delay" region the distortion varies sharply with the phase relation between the signal carrier and its echo. The maximum distortion (second order) occurs at phase quadrature, the minimum distortion (third order) at phase conjunction and phase opposition. Equations are derived giving the distortion ratios for noise-modulated FM transmission as a function of echo strength, delay, modulation amplitude, baseband width, and the baseband test frequency.

3. In the short delay region the distortion ratio increases with the square of delay; for large delays it approaches a constant value independent of echo phase.

4. In the short delay region the distortion ratio increases with modulation; for large delays it passes through a maximum and finally decreases with the 3/2 power of modulation.

5. The distortion-to-signal ratio increases only slightly with the baseband width for a given peak frequency deviation and signal frequency.

6. The distortion-to-signal ratio for flat noise loading increases nearly linearly with the baseband frequency.

7. In the short delay region the predominant second order distortion products can be excluded by confining the baseband loading frequencies within one octave (spread band loading). Considerations of background noise and rf bandwidth requirements reduce the practical value of this transmitting method.

8. The predominant distortion of high baseband frequencies can be reduced about 5 db by a moderate amount (10 db) of high frequency pre-emphasis in modulation.

[12] The theoretical curve of Fig. 18 was computed[7] by Mr. H. E. Curtis.

B. Transmission Path Echoes

1. For deep fades caused by a single echo, the baseband distortion is a function of the peak delay distortion $T=\tau/1-r$, which equals the echo delay divided by the amplitude ratio between the signal amplitude in free space and that at the frequency of deepest fading.

2. In the "low distortion" region defined by $Ts \ll 1$, the distortion-to-signal ratio varies sharply with the phase relation between the signal carrier and its echo. The maximum distortion (second order) occurs when the echo phase differs from opposition by the small angle $\pm\beta_{max}=0.58\ (1-r)$ radians. A minimum of distortion consisting of odd order products only, occurs at exact phase opposition. At phase difference exceeding β_{max}, the distortion falls off rapidly. Equations are derived giving the distortion-to-signal ratios for noise-modulated FM transmission as functions of peak delay distortion, peak frequency deviation, baseband width, and the baseband test frequency.

3. In the low distortion range the distortion-to-signal ratio is approximately proportional to the frequency deviation, and to the square of the peak delay distortion, but increases only slightly with signal bandwidth.

4. Above the low distortion range the distortion-to-signal ratio approaches a constant value.

5. The distortion ratio and background noise for flat noise modulation increase nearly linearly with the baseband frequency.

6. The predominant distortion of high baseband frequencies can be reduced about 6 db by a moderate amount (10 db) of high frequency pre-emphasis in modulation.

7. The distortion ratio due to multiple echo interference can usually be approximated by that of a two-path interference causing a similar frequency characteristic of phase and of level in the pass band of the FM receiver.

8. The distortion ratio and background noise in the low frequency channels may greatly exceed the low values per 5 unless the amplitude limiter of the FM receiver is carefully designed for a wide range of baseband frequencies and reception levels.

Echo Distortion
Analytical Appendix

I. Line Echoes

Asymptotic Equation for Short Delays: Let the signal voltage be

$$e_1 = e_0 \cos(\omega t + \mu_t) = e_0 \cos\phi_1 \tag{18}$$

where μ_t is the phase modulation at the time t, and let the echo voltage be

$$e_2 = re_0 \cos(\omega t - \omega\tau + \mu_{t-\tau}) = re_0 \cos\phi_2. \tag{19}$$

Then the phase error due to the echo

$$\mu_d = -\tan^{-1}\frac{r\sin(\phi_1-\phi_2)}{1+r\cos(\phi_1-\phi_2)}. \tag{20}$$

For $r \ll 1$,

$$\mu_d \doteq -r \sin(\phi_1 - \phi_2), \tag{21}$$

which indicates that the phase error, and hence the frequency error, is approximately proportional to r.

From (18) and (19)

$$\phi_1 - \phi_2 = \omega\tau + \mu_t - \mu_{t-\tau}. \tag{22}$$

By means of a Taylor series expansion, the foregoing becomes

$$\phi_1 - \phi_2 = \omega\tau + \dot{\mu}\tau - \frac{\ddot{\mu}\tau^2}{2!} \cdots \tag{23}$$

where the dots indicate differentiation with regard to time.

Let μ be a band spectrum with the highest frequency component p_m. Then, if

$$p_m\tau \ll 1 \text{ (inequality (1) of section I)}, \tag{24}$$

$$\phi_1 - \phi_2 \doteq \omega\tau + \dot{\mu}\tau = \omega\tau + M\tau, \tag{25}$$

where M is the frequency modulation of the carrier.

Combining (21) and (25),

$$-\mu_d \doteq r \sin \omega\tau \cos \tau M + r \cos \omega\tau \sin \tau M. \tag{26}$$

The first term on the right side has a maximum when the echo at the carrier frequency is in quadrature to the signal; it is called the quadrature component and produces even order distortion. If the frequency deviation

$$\tau M \ll 1 \tag{27}$$

(which is equivalent to inequality (1) of section I) the even order distortion consists mainly of the second order distortion.

$$\mu_2 \leqq 0.5 r_q \tau^2 M^2, \tag{28}$$

where $r_q = r \sin \omega\tau$ is the quadrature component of the echo.

The second term has a maximum when the carrier echo phase is equal or opposite to that of the signal; it is called the in-phase component and produces odd order distortion. For $\tau M \ll 1$ the main part of the small distortion is the third order product,

$$\mu_3 \leqq \tfrac{1}{6} r_p \tau^3 M^3, \tag{29}$$

where $r_p = r \cos \omega\tau$ is the in-phase component of the echo.

From (26) one derives the second order distortion-to-signal ratio for noise loading as follows. Let the broad band noise modulation M be approximated by a great number m of small sine waves of equal amplitude a and random phase and spaced at unit frequency intervals.

$$M = a \sum_{p=1}^{p_m} \cos(pt + \phi_p). \tag{30}$$

According to (28)

$$\mu_2 = 0.5 r\tau^2 a^2 \sum_n \cdot \sum_q \cos(nt + \phi_n) \cdot \cos(qt + \phi_q). \tag{31}$$

All the error components falling into the same frequency add on a power basis because of their random phase relation. Hence the amplitude

$$\overline{\mu_{2,p}} = 0.5 r\tau^2 a^2 \sqrt{N_2} \tag{32}$$

where N_2 is the number of different combinations n, q for which

$$n \pm q = p. \tag{33}$$

From reference 3, Table II, one finds that for large numbers of signals beginning at zero frequency

$$N_2 \doteq p_m - 0.5p. \tag{34}$$

The phase deviation of the signal at frequency p has the amplitude

$$\overline{\mu_{1,p}} = \frac{a}{p}. \tag{35}$$

Hence, the distortion-to-signal ratio

$$\frac{D_2}{S} = \overline{\mu_{2p}}/\overline{\mu_{1p}} = 0.5 r_q \tau^2 a p \sqrt{p_m - 0.5p}. \tag{36}$$

a is found as follows:

From (30) the amplitude of a sine wave having an rms value equal to that of the composite signal

$$\overline{M} = a\sqrt{p_m}, \tag{37}$$

because the randomly phased components add on a power basis. Assuming an 11 db peak-to-rms ratio for random noise, the amplitude of a sine wave having an rms value equal to that of noise would be 8 db smaller than the peak noise. Since we are dealing with loading by random noise which gives a peak deviation s, this equivalent sine wave will have an amplitude 8 db below s. Equating this amplitude to (37) gives

$$a = 0.4 s p_m^{-0.5}, \tag{38}$$

and by substituting (38) into (36),

$$\frac{D_2}{S} = 0.2 r_q \tau^2 s p \sqrt{1 - 0.5k}, \tag{39}$$

which is (3) of section I.

In an analogous manner one finds from (29),

$$\overline{\mu_{3,p}} = \tfrac{1}{6} r_p \tau^3 a^3 \sqrt{N_3} \tag{40}$$

where N_3 is the number of different combinations m, n, q, for which

$$m \pm n \pm q = p. \tag{41}$$

From the literature,[3] Table II, one finds

$$N_3 \doteq \frac{p_m^2}{2} - \frac{p^2}{6}. \tag{42}$$

Combining (35), (38), (40) and (42) one finds

$$\frac{D_3}{S} \doteq 0.019 r_p \tau^3 s^2 p \sqrt{1 - 0.333k^2} \tag{43}$$

in agreement with (4) of section I.

Asymptotic Equation for Long Delay: A distant echo is assumed to have no phase correlation with the original signal and thus acts like an unrelated noise. In order to find the distortion ratio one must derive the energy distribution of the echo. The instantaneous frequency modulation of the carrier by a random noise band follows an error function, and for a large modulation index it is permissible to assume that the energy spectrum also approaches an error function.

The total energy of the echo

$$W = r^2 W_0 \tag{44}$$

where W_0 is the signal energy and r the relative echo amplitude. The energy distribution

$$dW = \frac{1}{\sqrt{\pi}} W \epsilon^{-x^2} dx, \tag{45}$$

with

$$x = q(\omega - \omega_0). \tag{46}$$

ω_0 is the carrier frequency, and q a proportionality factor. The energy fraction contained in the interval between $-x$ and x

$$W_x = \frac{W}{\sqrt{\pi}} \int_{-x}^{x} \epsilon^{-x^2} dx. \tag{47}$$

Energy function W_x spreads over an infinite frequency range, but 99.93 per cent of the energy is contained between $-2.4 < x < 2.4$. We know from physical considerations that most of the echo energy must fall within the modulation limits of the FM signal.

If we assume accordingly that the energy contained between the peaks of the frequency excursion is 99.93 per cent of the total, we find from probability integral tables that

$$q = \frac{2.4}{s}, \tag{48}$$

and the energy falling into a small interval between frequencies p and $p+dp$

$$dW_p = \frac{2.4}{\sqrt{\pi}} s^{-1} r^2 \epsilon^{-5.76 p^2 s^{-2}} dp W_0. \tag{49}$$

If this energy is received together with the main signal through an amplitude limiter and linear frequency discriminator, only that half of the energy which is in phase quadrature to the main signal is detected. The amplitudes of the two sidebands $\pm p$ add in phase, and the resulting baseband distortion amplitude is proportional to the frequency p. This makes the relative distortion energy per unit frequency

$$W_d = \frac{4.8}{\sqrt{\pi}} s^{-1} r^2 p^2 \epsilon^{-5.76 p^2 s^{-2}}. \tag{50}$$

The frequency modulation of the carrier has the amplitude a and its energy after FM detection, in view of (38)

$$W_s = \frac{a^2}{2} = 0.08 s^2 p_m^{-1}. \tag{51}$$

The distortion amplitude ratio

$$\frac{D}{S} = \sqrt{W_d / W_s} = 5.8 r p p_m^{0.5} s^{-1.5} \epsilon^{-2.88 k^2 p_m^2 s^{-2}} \tag{52}$$

in accordance with (7) of section I.

II. Selective Fades

Distortion of Baseband Signal by Envelope Delay: Since the deep fades observed are caused by echoes with very short delays so that (24) and (25) apply, one may combine equation (20) with (25). Hence,

$$\mu_d = \tan^{-1} \frac{r \sin \psi}{1 - r \cos \psi} \tag{53}$$

with

$$\psi = \tau(\omega - \omega_0 + M) = \tau\beta + \tau M. \tag{54}$$

Differentiating with regard to time, one finds after some rearranging

$$M_d = \dot{\mu}_d = -0.5\tau \dot{M} + \tau \dot{M} \frac{\delta = 0.5\delta^2}{\delta^2 + 2(1 - \cos \psi)} \cdot \tag{55}$$

The first term on the right hand side is a small linear distortion which cannot produce any interchannel crosstalk, and may be neglected.

In the second term both $\delta \ll 1$ and $\psi \ll 1$, hence

$$M_d \doteq \tau \dot{M} \frac{\delta}{\delta^2 + \psi^2} = \frac{T \dot{M}}{1 + T^2(\beta + M)^2}$$

with

$$T = \frac{\tau}{\delta} \cdot \tag{56}$$

The nonlinear distortion products of the n^{th} order are found by developing (56) into a series according to powers of M, and evaluating the products $\dot{M} M^{n-1}$.

For $\beta = 0$

$$| M_3 | \doteq T^3 \dot{M} M^2 \quad \text{or,} \tag{57}$$

by integration,

$$\mu_3 \doteq \int M_3 dt = \tfrac{1}{3} T^3 M^3. \tag{58}$$

By comparing (58) with (29) one finds by analogy

$$\frac{D_3}{S} = 0.038 T^3 s^2 p \sqrt{1 - 0.333 k^2} \tag{59}$$

in accordance with (13) of section II. The second order term has the approximate value

$$| M_2 | \doteq \frac{2 \dot{M} M T^3 \beta}{(1 + \beta^2 T^2)^2} \cdot \tag{60}$$

By differentiation with regard to β, one finds

$$\beta_{\max} = \frac{\sqrt{3}}{T} \tag{61}$$

in accordance with (14) of section II, and

$$M_{2\max} = 0.65T^2\dot{M}M, \tag{62}$$

or, by integration

$$\mu_{2\max} = 0.325T^2M^2. \tag{63}$$

By comparing (63) with (28), one finds by analogy

$$\frac{D_2}{S} = 0.13T^2sp\sqrt{1 - 0.5k} \tag{64}$$

in accordance with (15) of section II.

Distortion of Baseband Signal by Incomplete Amplitude Limiting: Near the frequency of deepest fade, the received rf carrier amplitude

$$A = \sqrt{r^2 \sin^2 \beta\tau + (1 - r \cos \beta\tau)^2}$$
$$= \sqrt{\delta^2 + 2r(1 - \cos \beta\tau)}. \tag{65}$$

For deep fades,

$$A \doteq \delta\sqrt{1 + T^2\beta^2}. \tag{66}$$

Since the average amplitude of the received carrier is held practically constant by agc and by the dc component of the limiter action, spurious amplitude modulation is the product of frequency excursion by logarithmic slope of amplitude characteristic:

$$\frac{d \log A}{d\omega} = \frac{d \log A}{d\beta} = \frac{T^2\beta}{1 + T^2\beta^2} \tag{67}$$

at

$$\beta_{\max} = T^{-1} \tag{68}$$

this slope reaches its maximum

$$\frac{d \log A}{d\omega}_{(\max)} = 0.5T. \tag{69}$$

If the carrier swings back and forth because of the FM signal M, the resultant amplitude modulation is

$$A = \text{const.}\ (1 + 0.5TM). \tag{70}$$

After passing through an imperfect limiter the amplitude modulation is not entirely suppressed but only reduced by a compression factor C. Hence, at the input to the FM discriminator, the amplitude is

$$A_d = \text{const.}\ (1 + 0.5TC^{-1}M). \tag{71}$$

This is multiplied by the linear frequency characteristic of the discriminator so that the output if amplitude, and after linear detection, also the baseband amplitude is

$$A_{\text{out}} = \text{const.}\ (M + 0.5TC^{-1}M^2). \tag{72}$$

The second order component of amplitude modulation

$$\mu_2 = 0.5TC^{-1}M^2. \tag{73}$$

If M is a noise loading which can be approximated by a great number of small sine waves, as discussed above, one arrives by an analogous method of adding all the distortion components which fall into one frequency at

$$\frac{D_2}{S} = 0.2TsC^{-1}\sqrt{1 - 0.5k}, \tag{74}$$

which is the same as (17) of section II.

Correction

F. B. Anderson, author of the paper, "Seven-League Oscillator," which appeared on pages 881–890 of the August, 1951 issue of the PROCEEDINGS OF THE I.R.E., has brought the following errors to the attention of the editors:

f should be added to (2) on page 883, the equation thus reading

$$\frac{V_1}{V} = \frac{b + j\dfrac{f}{f_{c1}}}{1 + j\dfrac{f}{f_{c1}}}, \text{ where } f_{c1} = \frac{1}{2\pi C_1 aR(1 - b)b}.$$

j should be omitted from (12) on page 890. It should be inserted instead in the ninth line of the second column of page 890, the line thus reading

$$\frac{1}{1 - A_2} = \frac{\cos \phi_2}{|1 - A_2|} + \frac{j \sin \phi_2}{|1 - A_2|}.$$

Techniques for Close Channel Spacing at VHF and Higher Frequencies*

CHARLES F. HOBBS†, MEMBER, IRE

Summary—**Physically separated radio transmitters can be frequency-division multiplexed to obtain a great economy of bandwidth utilization at vhf and higher frequencies, by use of a common reference signal at both transmitter and receivers in lieu of locally generated high-frequency oscillations. In an individual transmitter, a modulated low-frequency subcarrier would be used to modulate a harmonic of the reference signal, and the resultant sum (or difference) frequency-band selected for transmission. Amplitude-modulated voice signals spaced only 5 kc apart at frequencies above 100 mc, have been separated with ease in a two-channel laboratory model of such a system by use of a standard low-frequency communication receiver preceded by a wide-band mixer, the local oscillator for which was synchronized with the transmitter reference signal.**

Introduction

As the need for additional communications channels increases, use is made of higher and higher frequency bands. At the same time, new techniques are sought which will allow more channels to be assigned to a given frequency band. Present techniques provide almost complete utilization of the low-frequency end of the radio spectrum, although some improvement could be obtained by a wider use of single-sideband modulation and more efficient coding methods. On the other hand, in the vhf and higher frequency bands, a large part of the spectrum is used as guard bands to separate the channels. These guard bands are necessary to prevent interference between signals on adjacent channels under the most adverse conditions. Factors involved are oscillator drift in the transmitters and receivers, drift in the resonant frequency of the tuned circuits in the receiver, and the inherent departure of receiver selectivity characteristics from the ideal. This paper suggests a method of virtually eliminating oscillator drift as a factor in determining the separation that is necessary between adjacent channels. The use of this method should result in channel spacings at the higher frequencies equal to those now used at the low-frequency end of the radio spectrum, and should encourage the use of band-reducing techniques and more efficient coding methods.

Use of Frequency-Division Multiplexing

When a number of messages are to be transmitted between two given points, a large increase in spectrum utilization can be obtained by the use of frequency-division multiplex systems.[1-4] The improvement results from the use of comparatively low-frequency subcarriers to produce adjacent channel signals of high relative stability. An illustrative example is given in Fig. 1. In

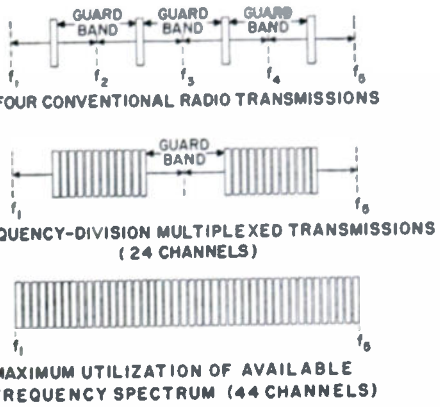

Fig. 1—Example illustrating spectrum utilization.

(a), four single channel transmissions are shown in a frequency band f_1 to f_5, while in (b), two twelve-channel frequency-division multiplexed transmissions occupy this same band. Note that the same guard bands are used between transmissions in both (a) and (b). In (c) it is seen that maximum utilization of the spectrum could be obtained by use of frequency-division multiplexing if the guard bands were eliminated. Fig. 2 illustrates a method of achieving this maximum utilization of the available frequency spectrum. Carriers having a high degree of relative stability are obtained by use of the several harmonics of a reference frequency f_s in lieu of locally controlled oscillations. The messages are impressed on groups of subcarrier signals so that each group of resultant signals covers a bandwidth equal to the reference frequency f_s. The band f_s to $2f_s$ is convenient from the standpoint of receiver tuning range and image rejection. The several groups of subcarrier signals are then used to amplitude modulate their respective carriers, and the upper sidebands of this final modulation process are selected for transmission,

* Decimal classification: R460×R007.1. Original manuscript received by the Institute, February 21, 1951; revised manuscript received, September 26, 1951.

† Air Force Cambridge Research Center, Cambridge, Mass.

[1] R. A. Heising, "Production of single sideband for trans-atlantic radio telephony," Proc. I.R.E., vol. 13, p. 291; June, 1925.

[2] F. A. Polkinghorn and N. F. Schlaack, "A single sideband short wave system for trans-atlantic telephony," Proc. I.R.E., vol. 23, pp. 701–719; July, 1935.

[3] F. A. Polkinghorn, "A single sideband musa receiving system for commercial operation on trans-atlantic radio telephone circuits," *Bell Sys. Tech. Jour.*, vol. 19, pp. 306–335; April, 1940.

[4] N. F. Schlaack and A. C. Dickieson, "Cape Charles-Norfolk ultra-short wave multiplex system," Proc. I.R.E., vol. 33, pp. 78–83; February, 1945.

thus producing a maximum number of information channels in a given frequency band.

The nature of an individual signal is determined by the manner in which the message is impressed on the subcarrier at the first modulator. Although the second

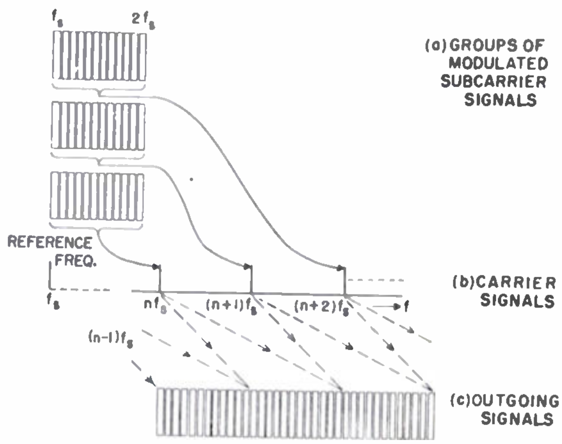

Fig. 2—Use of reference-frequency signal and frequency-division multiplexing to obtain maximum utilization of frequency spectrum.

modulator is a single-sideband suppressed-carrier type of modulator, it produces no change in the type of modulation; it merely changes the frequency of the signal. Fig. 3 illustrates the use of three different types

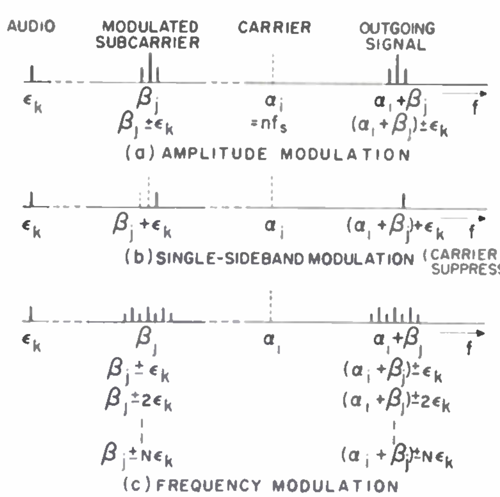

Fig. 3—Nature of outgoing signal.

of modulation. In (a), an audio signal of frequency ϵ_k is used to amplitude modulate the subcarrier of frequency β_j, thus producing the sidebands of frequencies $\beta_j+\epsilon_k$ and $\beta_j-\epsilon_k$. In the final modulation process, the amplitude-modulated signal is translated in frequency by the amount of the carrier frequency α_i. In (b), the first modulation process produces the single-sideband signal of frequency $\beta_j+\epsilon_k$, which is then translated in frequency to an amount α_i as before. The use of frequency modulation is illustrated in (c).

It is not necessary to transmit an entire group of messages from a single transmitter. A number of isolated transmitters may use the same carrier frequency. In the extreme case, all transmitters of a group would use the identical carrier frequency, but each would use a different subcarrier, that is, each station would transmit but a single signal. Thus the method is suitable for broadcasting, ship-to-shore radio, aircraft and mobile communications, and other applications requiring single channel transmissions.

The Transmitter

A block diagram of a transmitter using the proposed stabilization method is shown in Fig. 4. The reference-frequency signal stabilizes a local oscillator, the output of which is used to generate the desired carrier frequency

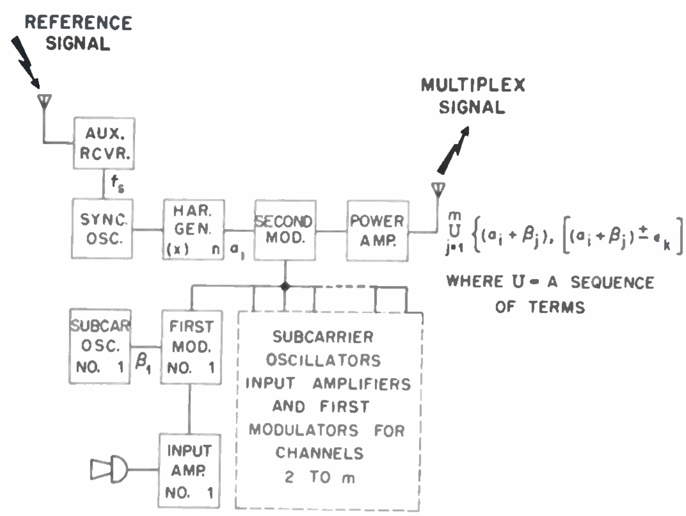

Fig. 4—Block diagram of transmitter.

α_i by means of harmonic amplification. Each subcarrier of frequency β_j is first modulated in the desired manner, after which the signal is translated along the frequency axis in the second modulator. The resultant signals of carrier frequency $\alpha_i+\beta_j$ are amplified and radiated. Sideband components are shown for amplitude modulation in the first modulators.

The block diagram of a single-sideband generator, which might be used as the second modulator in the transmitter of Fig. 4, is shown in Fig. 5. This type of circuit has been used by O. G. Villard[5] and others to generate conventional single-sideband suppressed-carrier signals. The phase shifts are so arranged that the desired sidebands are in phase and the undesired sidebands are out of phase. In order to give complete cancellation of the unwanted sideband components, their amplitudes must be equal at the outputs of the two modulators and their relative phases must be exactly π radians over the entire band. If the amplitudes are equal, a total phase-shift error of approximately 1 degree will result in only 40-db attenuation of an undesired sideband component.

Detuned resonant circuits are suitable for use as the carrier phase shifters. For example, if the circuit stability is 1 part in 100,000, and the Q is 100, the maxi-

[5] O. G. Villard, Jr., "A high-level single-sideband transmitter," Proc. I.R.E., vol. 36, pp. 1419–1426; November, 1948.

mum cumulative phase error in the two circuits would be less than 18 minutes.

Special circuits having constant amplitude and constant phase-shift characteristics over the appropriate frequency band are required for the subcarrier phase

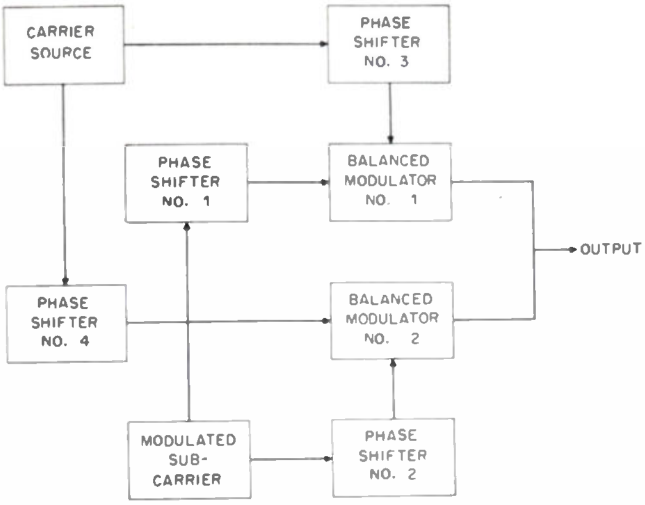

Fig. 5—Block diagram of single-sideband generator.

shifters. Dome has designed circuits which give a constant phase shift to within ± 4 degrees over a frequency range of 28 to 1 in the audio-frequency range, and has pointed out that this range can be increased by connecting a number of stages in cascade.[6] No attempt has been made to extend Dome's technique to the application suggested in this paper because the experimental work outlined below was designed to demonstrate the principles of the proposed method, rather than to obtain quantitative data to facilitate the design of such a system. For this purpose, detuned resonant circuits were used for both the carrier and subcarrier phase shifters. However, the requirements for the subcarrier phase shifters can be stated as follows: For 40-db attenuation of the undesired sideband, the phase shift in each subcarrier phase shifter must be constant to within approximately ± 0.5 degree over a frequency band ratio of 1.0022 to 1 for a single-channel 5-mc subcarrier, and over a frequency-band ratio of approximately 2 to 1 for a subcarrier band covering the frequency range 5 to 10 mc.

The more conventional method of obtaining single-sideband suppressed-carrier signals, as used in carrier telephony[7–10] and radio-telephone[1–4] circuits, is also suitable for use in the final modulation process. In this method, successive stages of modulation are used to obtain the desired increase in frequency, while at the same time, filters following each modulator are used to attenuate the undesired modulation products.

The Receiver

The basic requirements for the reception of a signal generated by one of the methods outlined above are no different than those for the reception of any radio signal. In a conventional vhf or uhf receiver, the pass band is made large enough to allow for anticipated instabilities of the local oscillator, change in circuit tuning due to thermal effects, and drift in the transmitter frequency. If usual channel spacings were used, vhf and uhf signals, generated by the methods previously outlined, could be received by conventional receivers. Because of the close channel spacing, however, much more selective receivers are needed.

If reception were attempted with a conventional superheterodyne receiver having adequate selectivity but using locally generated oscillations, the desired signal would not remain within the pass band of the receiver. However, by synchronizing the local oscillator frequency with the carrier frequency, or harmonically relating the two, their relative instability may be reduced to zero and the desired signal kept within a very narrow pass band. Such synchronization requires the availability of the reference signal, or a signal harmonically related thereto, at the point of reception, and the use of an auxiliary receiver and harmonic generator to provide the stabilizing signal. The auxiliary receiver may be fixed tuned and relatively simple. In the case of two-way communications, a single auxiliary receiver would serve both the transmitter and the receiver.

Fig. 6 is a simplified block diagram which illustrates, in principle, the proposed method of reception. The auxiliary receiver, synchronized oscillator, and harmonic generator are used to reproduce the carrier-frequency signal in the same manner as it was generated at

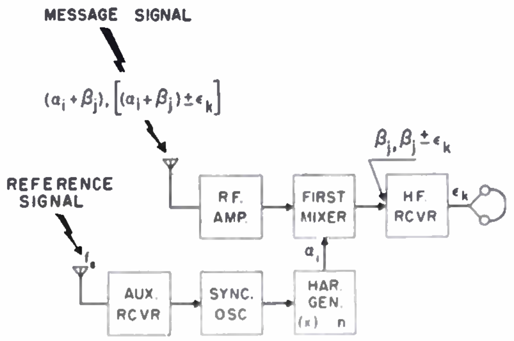

Fig. 6—Simplified block diagram of receiver.

the transmitter. The carrier frequency α_i is subtracted from the frequency of the incoming signal, leaving the desired signal of subcarrier frequency β_j to be selected and demodulated by the conventional receiver represented by the last block in the diagram.

[6] R. B. Dome, "Wideband phase shift networks," *Electronics*, vol. 19, pp. 112–115; December, 1946.

[7] R. E. Crane, J. T. Dixon, and G. H. Huber, "Frequency Division Techniques for a Coaxial Cable Nework," AIEE Technical Paper, pp. 47–258; September, 1947.

[8] H. J. Fisher, M. L. Almquist, and R. H. Mills, "A new single-channel carrier telephone system," *Elec. Eng.*, vol. 57, pp. 25–33, 356; January, 1938. Also appears in *Bell Sys. Tech. Jour.*, vol. 17, pp. 162–184; January, 1938.

[9] C. W. Green and E. I. Green, "A carrier telephone system for toll cables," *AIEE Transactions*, pp. 227–236; 1938.

[10] E. S. Willis, "A new crystal channel filter for broadband carrier systems," *AIEE Transactions*, pp. 134–138; 1946.

Experimental System

An experimental laboratory system using the proposed techniques, the block diagram for which is shown in Fig. 7, has demonstrated the feasibility of close channel spacing in the vhf band. The addition of the subcarrier frequency to the carrier frequency was accomplished in each channel by use of two balanced modulators and phase-shifting circuits, as explained above.

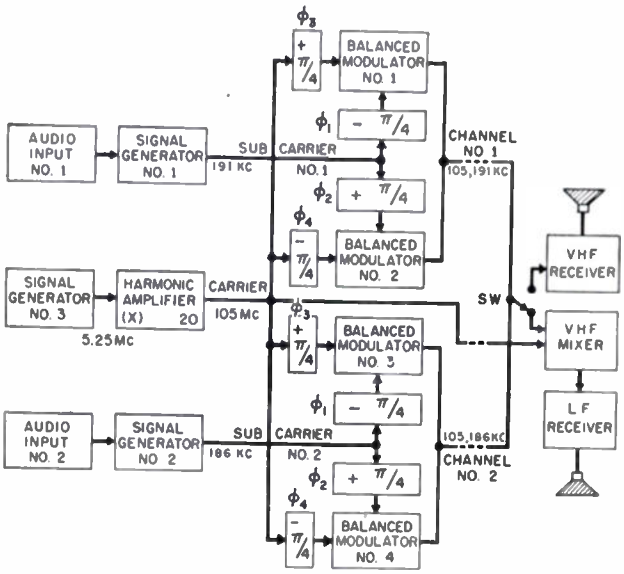

Fig. 7—Block diagram of two-channel laboratory system.

It will be recalled that resonant circuits detuned to their half-power points produce phase shifts of $\pi/4$ radians. Hence, detuning of the input circuits to each pair of balanced modulators to produce phase shifts of $\pi/4$ radians, as shown in Fig. 7, maintained amplitude balance at the inputs, and satisfied the condition for phase opposition of the difference-frequency components in the output of the two modulators. Adjusting the gains of the two modulators to equality resulted in cancellation of the difference-frequency components.

By operation of the switch *SW*, both channels could be connected simultaneously to a conventional vhf receiver, or to a vhf mixer in which the local oscillator signal was of the same frequency as that of the carrier used to generate the communication signals. The output of the mixer went to a conventional low-frequency receiver. When the two channels were connected to the vhf receiver, both programs were received simultaneously because of the large pass band of the receiver; but when the two channels were switched to the vhf mixer, either channel could be selected at will by tuning the low-frequency receiver, even though the two channels were separated by only 5 kc.[11]

Spurious Responses

Receivers for use with the proposed system must provide sufficient adjacent channel selectivity, and must give adequate suppression of spurious signals. At the higher frequencies this requires the use of multiple-superheterodyne receivers of suitable design. Fig. 8 is the block diagram of a triple-superheterodyne receiver that can be designed to operate satisfactorily at fre-

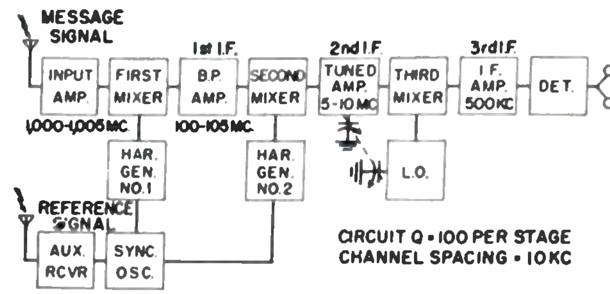

Fig. 8—Block diagram of triple superheterodyne receiver.

quencies of the order of 1,000 mc. Table I gives the adjacent channel selectivity and spurious response to be expected from such a receiver operating at 1,000 mc. It is assumed that channel spacings are 10 kc, and the Q of a single-tuned circuit used in any stage of the receiver is 100. The total intermodulation distortion produced by each mixer is the rms summation of all distortion products formed by a given mixer that fall in a single message channel. A method for making such a summation where large numbers of components are involved, is given in another paper.[12]

Multiplex transmitters are also subject to intermodulation distortion in the final modulator and power amplifier. Means of reducing this distortion include the use of the spread-sideband technique,[13] negative feedback,[14] and high-level modulation.

TABLE I

Spurious Response of Triple-Superheterodyne Receiver

Channel Spacing = 10 kc
Circuit Q = 100 per stage

	Freq.	*No. Stages*	*Adj. Chan. Response*	*Image Response*	*Maximum Total Intermodulation Distortion*
Input Amp.	1,000–1,005 mc	2	—	−62.6 db	−39.1 db
1st IF	100–105 mc	3	—	−76.8 db	−43.8 db
2nd IF	5–10 mc	3	—	−76.8 db	−54.6 db
3rd IF	500 kc	5	−62 db	—	—

Application

The adoption of the proposed system should result in greatly increasing narrow-band facilities of all types in the vhf and higher-frequency bands, and at the same

[11] The effective audio band pass of the system was limited to 2,500 cps.

[12] C. F. Hobbs, "Intermodulation Distortion in Mixers," to be published.

[13] E. H. Ullrich, "Ultra-short-wave communication," *Elec. Commun.* (London), vol. 16, pp. 64–86; July, 1937.

[14] C. R. Burrows and A. Decino, "Ultra-short-wave multiplex," Proc. I.R.E., vol. 33, pp. 84–94; February, 1945.

time make more channels available for FM and television broadcasting, and other wide-band applications.

One possible application is to vhf or uhf commercial broadcasting. A particular transmitter could be used to transmit a single program or several programs could be multiplexed on a single transmitter, as shown in Fig. 4. The reference signal would be transmitted as a public service to permit the carrier frequency α_i, used by the transmitter, to be synthesized at the receivers, as shown in Fig. 6. A large number of channels could be provided. For example, the receiver shown in block diagram form in Fig. 6 might be used to receive more than 150 high-fidelity AM channels occupying a total bandwidth of only 5 mc, with a fixed tuned front end, and the hf rcvr portion tunable over the range 5 to 10 mc. By making the front end, including the rf amp and har. gen., adjustable in 5-mc steps, the number of channels could be increased manyfold. This assumes $f_s = 5$ mc.

An application to which the system appears to be particularly suited is that of providing individual channels for air-ground and ground-air communications for air-traffic control. For this application, air-ground communications would be carried on in one block of frequencies, and ground-air communications in another. This would result in a flexible system in which simultaneous two-way communications could be carried on when necessary. The block diagram of an individual channel net is shown in Fig. 9.

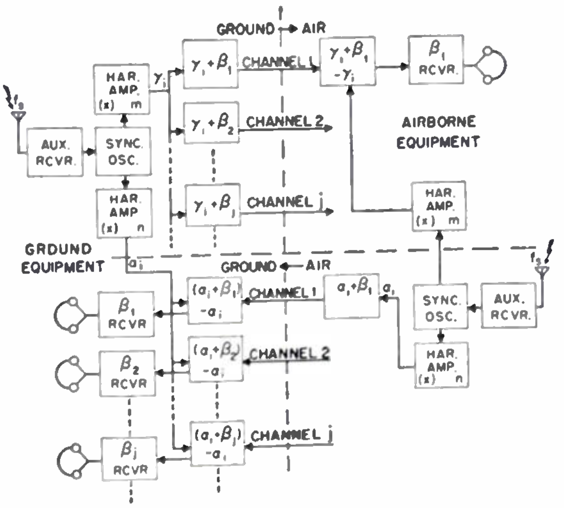

Fig. 9—Simplified block diagram of typical individual channel net.

Other possible applications of the proposed system include radio-telephone for commercial, public service, and emergency uses.

Doppler Effect

In any communications system in which the transmitter and/or receiver are in motion, consideration must be given to the apparent change in frequency resulting from the relative velocity between the two. The doppler effect may be a major factor in determining the minimum channel spacing that can be used for communications involving high-speed aircraft. For example, a 1,000 mc signal will be received with an error of 800 cps when the relative velocity between the transmitter and receiver is 240 meters per second (approximately 535 mph). Under certain conditions, the frequency error of the derived subcarrier signal is doubled if the receiver local oscillator signal and the transmitter carrier are synchronized by the techniques proposed in this paper. This is true when the apparent change in the frequency of the reference signal is maximum in one direction, while the apparent change in the signal frequency is maximum in the opposite direction. That is, the aircraft is between the ground transmitter and the reference signal transmitter, and is flying directly toward one and away from the other. Even if all other factors are neglected, doppler effect alone would require guard bands of approximately[15]

$$4\Delta f = \frac{v f_{mc}}{75}, \tag{1}$$

where

Δf = frequency shift due to doppler in cps

v = maximum aircraft velocity in meters per second

f_{mc} = frequency in mc.

Thus, if $v = 240$ meters per second, and $f_{mc} = 1{,}000$ mc, the doppler effect makes necessary guard bands of 3,200 cps.

Channel Spacing

Minimum channel spacing, consistent with the bandwidth of the information to be transmitted, can be attained at vhf and higher frequencies by use of a synchronized system such as described in this paper. For fixed services, channel spacings comparable to those now used at the low-frequency end of the radio-frequency spectrum, are possible. For such applications, the bandwidth of the information to be transmitted, and the type of modulation used, are the major factors in determining the channel spacings, but for mobile applications, particularly those involving high-speed aircraft, the frequency shift due to the doppler effect is a major consideration also.

Single-sideband modulation may be used with the proposed system and fixed installations to obtain channel spacings of the order of the bandwidth of the information to be transmitted. It is impracticable to use single-sideband modulation with complete suppression of the carrier component for air-to-air or air-to-ground communications because of the doppler effect. The frequency of the carrier component as restored to the signal in the receiver must follow, to within a few cps, the change in frequency caused by the doppler effect. This can be accomplished if a residual carrier component

[15] The fractional frequency change due to doppler is

$$\frac{\Delta f}{f} = \frac{v}{c},$$

where

c = velocity of propagation = 3×10^8 meters per second

f = frequency in cps

or

$$\Delta f = \frac{vf}{3 \times 10^8} = \frac{v f_{mc}}{300}.$$

of sufficient amplitude to synchronize an oscillator over the required frequency range, or to provide reconditioned carrier, is transmitted along with the single-sideband signal.

Conclusions

The use of a common-reference frequency signal in lieu of locally generated oscillations to stabilize radio transmitters and receivers appears to offer a good possibility of obtaining minimum spacing of communications channels in the vhf and higher frequency bands. This technique should provide a large increase in the number of radio channels without changing the essential nature of the transmissions, and at the same time should offer the opportunity for a further increase through the use of single-sideband modulation and other band-reducing techniques. The adoption of this method of communication should result in the release of additional frequency spectra for wide-band applications, such as FM and television broadcasting, in addition to greatly increasing narrow-band facilities of all types in the higher frequency bands.

Acknowledgments

The author wishes to express his appreciation to Walton B. Bishop for his assistance in the theoretical study which preceded the preparation of this report, and to Frank Malloy for planning and supervising the experimental work.

Cascade Connection of 90-Degree Phase-Shift Networks*

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Summary—**A method of connecting 90-degree audio phase-difference networks for use in selective-sideband transmission and reception is shown, whereby an over-all performance is obtained which is analogous to the cascade operation of conventional filters. Three networks are required to obtain twice the rejection in decibels of one. For best results two of the networks must be accurately matched. The cascade connection may either be used with three identical networks to deepen the rejection obtainable with one network over the design range, or it may be used with two similar and one dissimilar network to extend the frequency range over which a given rejection may be obtained.**

Introduction

Ninety-degree audio phase-shift networks may be used in radio transmission and reception to achieve by modulation methods a selectivity equivalent to that customarily obtained by means of passive filters.[1] Two basic types of these networks have been disclosed. In one, the phase difference between the two output voltages is for all practical purposes 90 degrees, and the relative amplitudes are made as nearly equal as possible;[2,3] in the other, the relative amplitudes are inherently equal, and the desired phase difference is approximated.[4]

In ordinary filter practice, when adequate selectivity cannot be obtained in a single unit, two or more may be connected (with suitable isolation) in series. Thus, if one filter has a 20-decibel rejection in its stopband, a second will provide an over-all rejection of 40 decibels.

While the cascade connection is obvious in the case of conventional filters, it is not readily evident how an equivalent result may be obtained with selective systems using audio phase-shift networks. The problem is relatively easier with the first type of network mentioned above, and one possible approach has been published.[5] It is the purpose of this paper to describe how the other, or difference-phase, type of network may effectively be connected in cascade.

Basic Method

A block diagram is shown in Fig. 1. The material within the upper dotted enclosure represents a single-sideband generator employing audio phase-shift networks. Expressions describing the voltages at each point in the circuit are shown. It will be assumed for simplicity in the following that the radio-frequency phase shift is always exactly 90 degrees and that the audio networks have unity transmission and a phase shift which fails to be 90 degrees by some value $\pm\alpha$,

* Decimal classification: R423.52×R143. Original manuscript received by the Institute, July 20, 1950; revised manuscript received September 10, 1951.

This paper is based on work carried out under Contract W-28(099) Ac-131, between Stanford University and the Air Material Command.

† Department of Electrical Engineering, Stanford University, Stanford, California.

[1] I. F. Macdiarmid and D. G. Tucker, "Polyphase modulation as a solution of certain filtration problems in tele-communications," *Proc. IEE* (London), pt 3, vol. 97, pp. 349–358; September, 1950.

[2] R. V. L. Hartley, U. S. Patent No. 1,666,206; April 17, 1928.

[3] B. Lenehan, "A new single-sideband carrier system for power lines," *Elec. Eng.*, vol. 66, pp. 549–592; June, 1947.

[4] R. B. Dome, "Wideband phase shift networks," *Electronics*, vol. 19, pp. 112–115; December, 1946.

[5] H. Chireix, U. S. Patent No. 1,946,274; February 6, 1934.

corresponding to networks of the difference-phase type. To simplify presentation further, all the phase shift is shown lumped in one side of each branch.

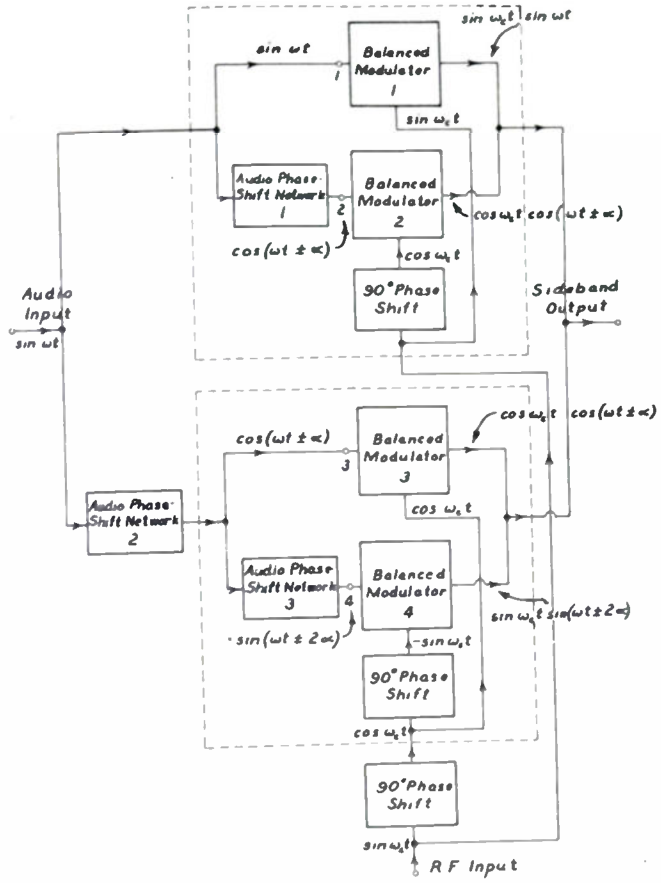


Fig. 1—Connection for obtaining equivalent of cascade operation in single-sideband transmission.

In the upper enclosure of Fig. 1, the output of the first balanced modulator is

$$\sin \omega_c t \sin \omega t = \tfrac{1}{2} \cos (\omega_c - \omega)t - \tfrac{1}{2} \cos (\omega_c + \omega)t, \quad (1)$$

where ω_c is the radio frequency and ω is the audio frequency. That of the second is

$$\cos \omega_c t \cos (\omega t \pm \alpha) = \tfrac{1}{2} \cos [(\omega_c - \omega)t \mp \alpha] + \tfrac{1}{2} \cos [(\omega_c + \omega)t \pm \alpha]. \quad (2)$$

When $\alpha = 0$, the upper sidebands cancel, and the lowers add exactly. When $\alpha \neq 0$, a resultant upper sideband R_1 appears whose magnitude and phase is

$$R_1 = \sin \frac{\alpha}{2} \cos \left[(\omega_c + \omega)t \pm \left(90° + \frac{\alpha}{2} \right) \right]. \quad (3)$$

Now the resulting unwanted sideband, caused by imperfect performance of the phase-shift network, may itself be balanced against a similar resultant derived from a second audio network and pair of balanced modulators. The arrangement is shown in the remainder of Fig. 1. The lower dotted enclosure is similar to the upper, but the audio- and radio-frequency voltages fed to it have each received an additional 90-degree phase shift. Let it be assumed that the three audio-frequency networks are identical. The sideband output of balanced modulator number 3 is

$$\cos \omega_c t \cos (\omega t \pm \alpha) = \tfrac{1}{2} \cos [(\omega_c - \omega)t \mp \alpha] + \tfrac{1}{2} \cos [(\omega_c + \omega)t \pm \alpha]. \quad (4)$$

That of number 4 is

$$\sin \omega_c t \sin (\omega t \pm 2\alpha) = \tfrac{1}{2} \cos [(\omega_c - \omega)t \mp 2\alpha] - \tfrac{1}{2} \cos [(\omega_c + \omega)t \pm 2\alpha]. \quad (5)$$

The resultant of the two upper sidebands is

$$R_2 = \sin \frac{\alpha}{2} \cos [(\omega_c + \omega)t \mp (90° - \alpha)]. \quad (6)$$

The combination of (3) and (6) yields

$$\text{Undesired sidebands} = R_1 + R_2 = 2 \sin^2 \frac{\alpha}{2} \cos (\omega_c + \omega)t. \quad (7)$$

$$\text{Desired sidebands} \approx 2 \cos (\omega_c - \omega)t.$$

The ratio of magnitudes of undesired to desired sidebands in the output is approximately $\sin^2 \alpha/2$, whereas in one enclosure alone it is $\sin \alpha/2$. Thus, three phase-shift networks, connected as shown, can provide twice the rejection (in decibels) of one network alone.

The actual connection for difference-phase networks is shown in Fig. 2. The branches A_1, B_1, and the like represent the component lattice networks whose constant-magnitude output voltages differ in phase by approximately 90 degrees.

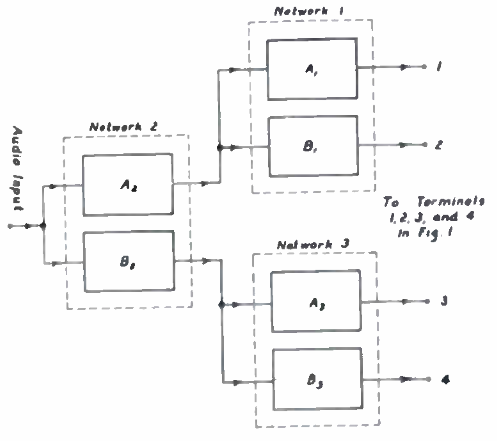


Fig. 2—Connection for networks of the difference-phase type.

It will be found that it is not mandatory to secure the improved undesired-sideband rejection on the output side of the balanced modulators, although this may in some cases be desirable; the improvement may also be obtained directly at audio frequencies. One is led to the circuit of Fig. 3 by analogy to Fig. 1. The two output voltages in Fig. 3 are now found to differ in phase by exactly 90 degrees, but the imperfect phase shift in the networks results in the voltages differing in magnitude by an amount equal to $(1 - \cos \alpha)$. When two such voltages are fed to a balanced modulator, the ratio of desired to undesired sidebands may be shown to be approximately $\sin^2 \alpha/2$. Thus, the circuit of Fig. 3 leads to the same result as Fig. 2, but in a slightly different way.

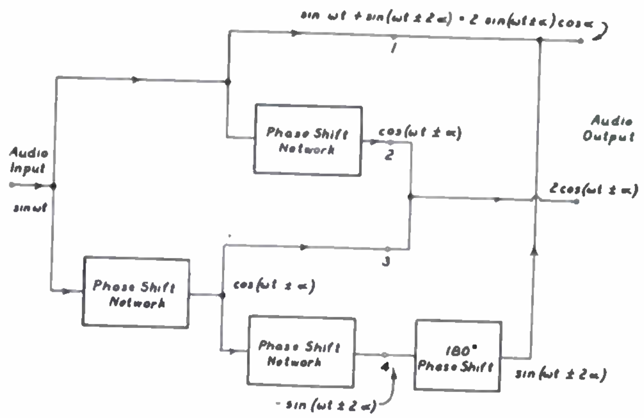


Fig. 3—Audio-frequency equivalent of Fig. 1.

These same circuits are readily adaptable to single-sideband reception.[6,7]

Matched-Performance Requirement

The success of the cascade connection depends on the equality of the phase-shift of networks 1 and 3 in Fig. 1. If these networks are identical, the undesired-sideband output of the first pair of balanced modulators will have the correct phase relationship with respect to that of the second pair to permit best cancellation with the aid of audio network 2. If the shift in network 1 does not match that of network 3, the over-all rejection obtainable will be seriously reduced.

Phase-difference networks may be realized in two ways, either as lattice filters incorporating passive elements only or as half-lattice filters with isolation and phase inversion supplied by vacuum tubes.[4]

With passive networks, the requirements of cascade operation can be met by matching each component of network 3 to the corresponding component in network 1 irrespective of the exact absolute values.

Networks incorporating vacuum tubes may be adjusted for equality of performance to a high degree of accuracy with the aid of connection illustrated in Fig. 4.

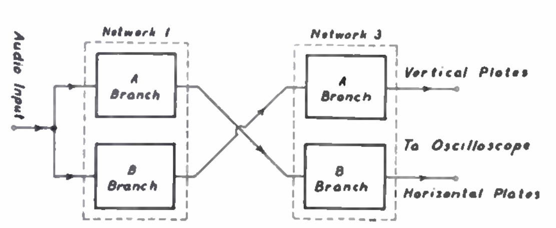


Fig. 4—Connection for making performance of two difference-phase networks virtually identical.

[6] E. I. Green, U. S. Patent No. 2,020,409; November 12, 1935.
[7] O. G. Villard, Jr., "Simplified single-sideband reception," *Electronics*, vol. 21, pp. 82–85; May, 1948.

If both have equal phase shifts, the phase difference between the outputs is zero and is displayed as a straight line on the oscilloscope. One network may readily be matched to the other by trimming its time constants until a straight line is obtained at all frequencies of interest. This indication is, of course, much more accurate than the circular 90-degree counterpart.

Extension of Frequency Range

In compensation for the requirement that networks 1 and 3 in Fig. 1 be matched is the circumstance that network 2 may be entirely different. Thus the cascade connection may also be used to give a wider total band over which acceptable performance is obtained, rather than an improvement in performance in a given band. For example, the phase-shift networks of the General Electric type YRS-1 single-sideband adapter afford a rejection of the order of 30 decibels from 70 to 7,000 cycles, a frequency ratio of 100 to 1. Three such networks in cascade should deepen the rejection over this range to something of the order of 60 decibels. However, if network 2 is redesigned to operate from 7,000 to 700,000 cps, the three could then be connected in cascade to give a 30-decibel rejection between 70 and 700,000 cps, a frequency ratio of 10,000 to 1, provided only that the low-frequency networks are able to maintain uniform transmission and similar phase characteristics up to the highest frequency.

Laboratory Test

A laboratory test was performed using the phase-shift and summing stages of three identical YRS-1 adapters connected in the receiving equivalent of Fig. 3. Undesired-sideband rejection was measured by feeding in two test audio voltages having equal amplitudes and a very accurate 90-degree phase difference. The transmission, when all three sideband selector switches were thrown to the passed "sideband" (or phase rotation), was compared with that when the three switches were thrown to the rejected "sideband." The results are shown in Fig. 5.

Networks 1 and 3 were adjusted for equal performance in accordance with the procedure of Fig. 4. Care must be exercised in setting the amplitude balance controls in networks 1 and 3, since an incorrect amplitude setting can sharply affect the phase as well as the amplitude of the partially suppressed "sideband" passed by these units, and thus the over-all rejection.

Conclusion

An arrangement has been shown whereby systems deriving their selectivity from the use of modulation techniques and 90-degree audio phase-difference networks of finite quality may be so connected that an improved over-all performance is obtained in a manner similar to the series connection of conventional filters.

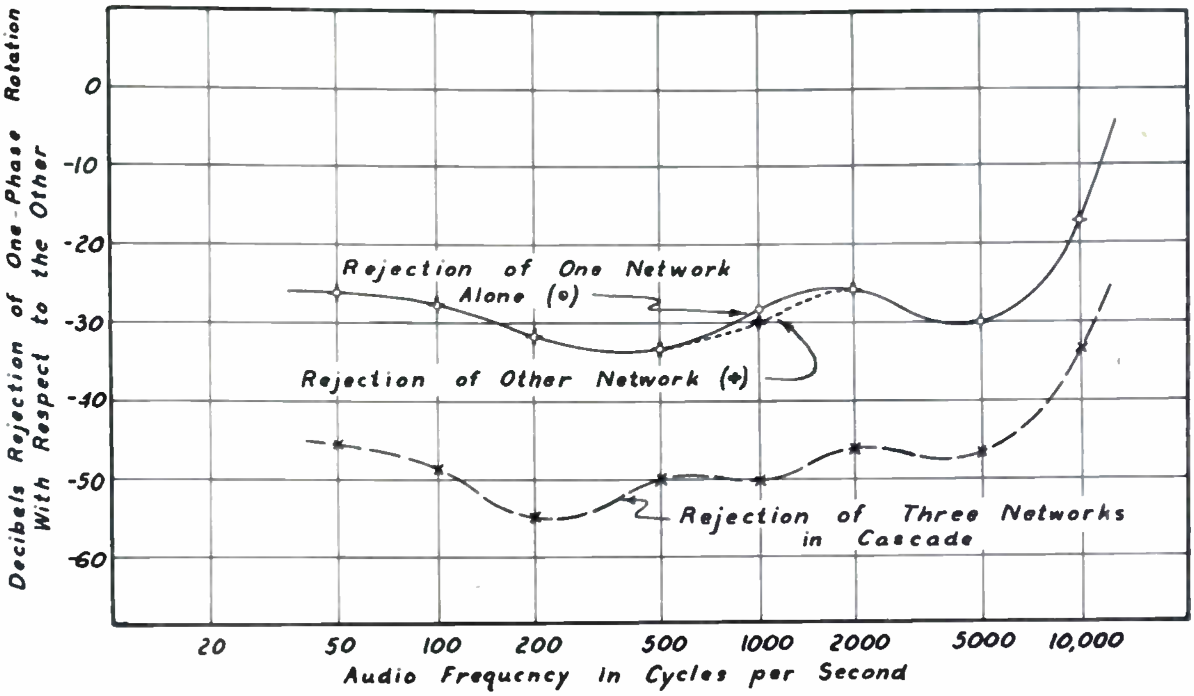


Fig. 5—Rejection obtainable with cascaded commercial 90-degree phase-shift networks. (Individual units part of General Electric Type YRS-1 selector. Nominal frequency range 70 to 7,000 cps.)

Now that the design of RC phase-difference networks for any desired performance is well understood,[8,9] one of the few remaining problems in obtaining high selectivities by this means has been the difficulty of obtaining network components of the desired tolerance. Since resistors and condensers are relatively compact and inexpensive, it may prove desirable in some cases to attain the end result by means of cascaded networks employing relatively lower-tolerance components.

[8] S. Darlington, "Realization of a constant phase difference," *Bell Sys. Tech. Jour.*, vol. 24, pp. 94–104; January, 1950.

[9] H. J. Orchard, "Synthesis of wideband two-phase networks," *Wireless Eng.*, vol. 27, pp. 72–81; March, 1950.

Acknowledgment

The assistance of J. P. Paddock in carrying out the experimental work is gratefully acknowledged.

Correction

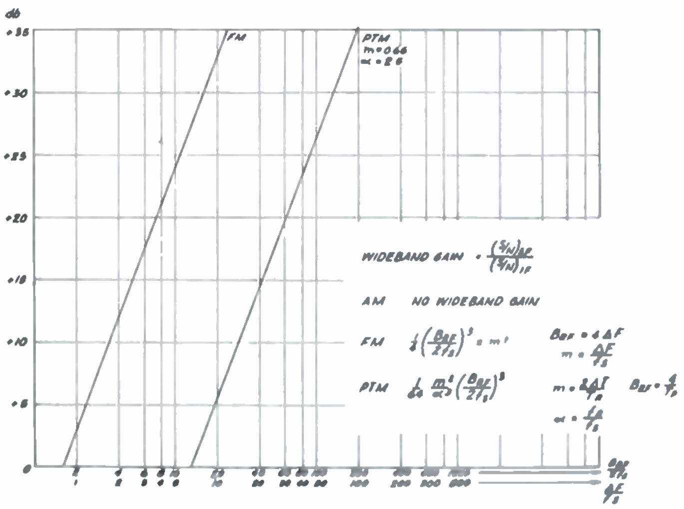
Richard Guenther, author of the paper, "Radio Relay Design Data 60 to 600 mc," which appeared on pages 1027–1034 of the September, 1951 issue of the Proceedings of the I.R.E., has brought the following corrections to the attention of the editors:

In Table I, at line "PM or FM," the wide-band gain should be $1/4(B_{rf}/2f_s)^3 = m^3$ instead of $1/4(B_{rf}/2f_s)^2 = m^2$.

The curve for FM in Fig. 7, therefore, runs parallel to the PTM curve according to the corrected figure.

This modifies the results for the two FM examples in Table II (b). According to the increase of the wide-band gain from 6 to 12 db, the last line should read:

S/N per channel 46 db (instead of 40 db), 36 db (instead of 30 db), 48 db (instead of 42 db), 13 db (instead of 7 db).



A Note on a Selective RC Bridge*

PETER G. SULZER†, ASSOCIATE, IRE

Summary—**A simple resistance-capacitance bridge is described The circuit is capable of providing higher selectivity than the Wien bridge, although it employs the same number of circuit elements.**

RESISTANCE-CAPACITANCE bridges have been used for the measurement of frequency,[1] resistance, and capacitance,[2] and have also been employed as selective elements in feedback amplifiers.[3] The Wien bridge and the parallel-T are commonly used in such applications. It is the purpose of this note to describe a simple RC bridge capable of providing a moderate increase in selectivity without requiring an increase in the number of circuit elements. The circuit is an RC form of a generalized six-arm bridge described by Anderson.[4]

Before describing the subject bridge, let us consider the Wien bridge of Fig. 1(a) as a basis for comparison.

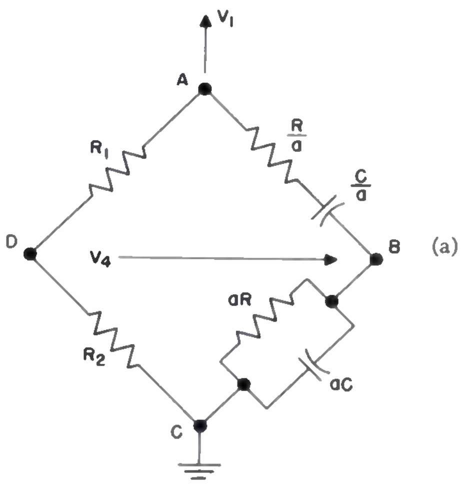


Fig. 1(a)—The Wien bridge.

In this circuit, provision has been made for modifying the values of resistance and capacitance in arms $A-B$ and $B-C$ to obtain the maximum selectivity. Applying Kirchoff's laws, it is found that

$$\beta = \frac{-ju}{b(b+ju)}, \tag{1}$$

and therefore

$$|\beta| = \frac{u}{b\sqrt{b^2+u^2}}, \tag{2}$$

where

$$\beta = \frac{V_4}{V_1},$$

$$u = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega},$$

$$\omega_0 = \frac{1}{RC},$$

$$b = 1 + a^2 + \frac{1}{a^2},$$

and $R_1 = (b-1)R_2$ (the balance condition).

The voltage V_1 and V_4 are defined by the figure, as is the parameter a; ω is the angular frequency.

It is convenient to work in terms of the frequency variable u. Defining the selectivity S (at balance) in terms of this quantity,

$$S = \left.\frac{d|\beta|}{du}\right|_{u=0}. \tag{3}$$

For the Wien bridge under consideration,

$$S = \frac{1}{b^2} = \frac{1}{\left(1 + a^2 + \frac{1}{a^2}\right)^2}. \tag{4}$$

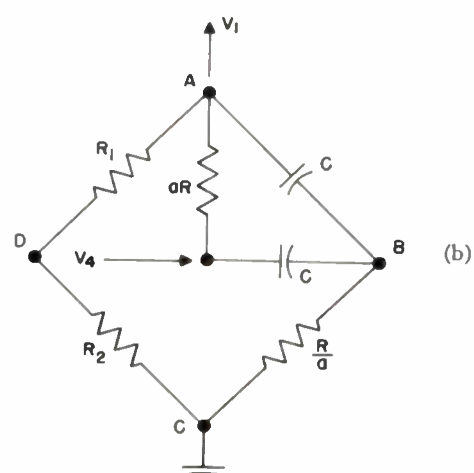


Fig. 1(b)—An RC bridge providing higher selectivity than the Wien bridge.

Examination of (4) shows that the maximum selectivity, numerically equal to 1/9, occurs for $a=1$, that

* Decimal classification: R207.2. Original manuscript received by the Institute, March 9, 1951; revised manuscript received, August 10, 1951.
† National Bureau of Standards, Washington 25, D. C.
[1] F. E. Terman, "Radio Engineer's Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., p. 957; 1943.
[2] J. G. Ferguson and B. W. Bartlett, "The measurement of capacitance in terms of resistance and frequency," *Bell Sys. Tech. Jour.*, vol. 7, p. 420; July, 1928.
[3] G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 384–408; 1948.
[4] B. Hague, "Alternating Current Bridge Methods," Pitman Publishing Corp., New York, N. Y., pp. 54–56; 1938.

is, with equal resistances and capacitances in arms A-B and B-C.

Consider, now, the circuit of Fig. 1(b), which contains the same number of components as Fig. 1(a). Here,

$$\beta = \frac{jua}{\left(\frac{2}{a}+a\right)^2 + ju\left(\frac{2}{a}+a\right)}, \tag{5}$$

and therefore

$$|\beta| = \frac{ua}{\left(\frac{2}{a}+a\right)\sqrt{\left(\frac{2}{a}+a\right)^2+u^2}}, \tag{6}$$

where a is defined by the figure. The null condition requires

$$R_1 = \frac{a^2}{2} R_2.$$

It is interesting to note that for $a=1$, (6) is identical with (2), indicating that, with equal resistances and capacitances in the right-hand arms, the bridges of Figs. 1(a) and 1(b) have identical properties, including selectivity as defined. The selectivity of the bridge of Fig. 1(b) is given by

$$S = \frac{a^3}{(2+a^2)^2}. \tag{7}$$

As expected from the above, $S=1/9$ when $a=1$. However, S increases with a to a maximum at $a=\sqrt{6}$, where $S_{max}=3\sqrt{6}/32$, or approximately 0.23. Consequently, the maximum selectivity is more than double that of the Wien bridge, permitting more accurate measurements of frequency, resistance or capacitance.

The bridge under discussion employs equal capacitances as frequency-determining elements. The alternate form shown in Fig. 1(c) employs equal resistances, which is convenient when variable-frequency operation is to be obtained with a dual variable resistor.

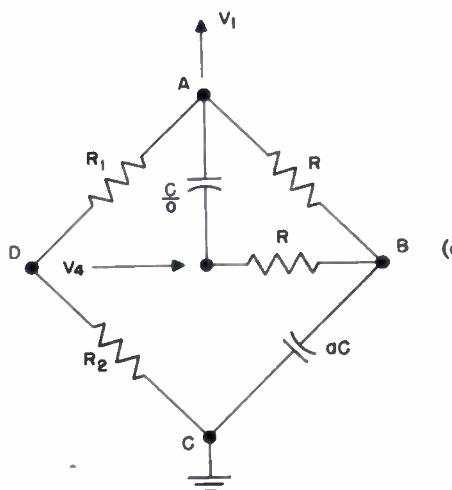


Fig. 1(c)—Another RC bridge providing higher selectivity than the Wien bridge.

The bridges described have been employed by the writer as frequency-determining elements in audio-frequency oscillators. It is thought that they should also be useful in some other applications requiring measurement of resistance, capacitance, or frequency.

Waves on Inhomogeneous Cylindrical Structures*

R. B. ADLER†, ASSOCIATE, IRE

Summary—An analysis is given of some of the basic properties of exponential modes on passive cylindrical structures in which the material constants of the medium vary over the cross section. The bounding surface is assumed to be opaque, in the form of an electric or a magnetic wall; it is therefore always nondissipative. Major consideration is given to structures in which the internal medium is also nondissipative.

Each mode is usually a $TE=TM$ mixture. Some of the conventional orthogonality conditions no longer remain valid. In certain circumstances, however, the instantaneous and vector power that flow along the system are still additive among the various modes. Stored and dissipated energies per unit length generally are not additive. The propagation constant for modes on a nondissipative structure cannot be complex. The relation between the direction of the time-average Poynting vector at any point of the cross section, and that of the phase and group velocities, is no longer necessarily conventional, and the space angle between the transverse electric and magnetic fields may vary over the cross section. The field distribution of each mode varies with frequency in a manner which is clarified by physical interpretation.

* Decimal classification: R118.2. Original manuscript received by the Institute, June 2, 1950; revised manuscript received, September 18, 1951. This work has been supported in part by the Signal Corps, the Air Materiel Command, and the ONR. It is based on a doctoral thesis in the Electrical Engineering Dept., M.I.T., Cambridge, Mass., and was published as Technical Report No. 102, Research Laboratory of Electronics, M.I.T., May 27, 1949.

† Research Laboratory of Electronics, M.I.T., Cambridge, Mass.

I. Introduction

THE MOST COMMON waveguide structures consist of a perfectly conducting metal tube, uniformly filled with a dissipationless dielectric material. Under these physical conditions, the analysis of the steady-state electromagnetic fields in the guide can be carried out completely in terms of the set of familiar modes appropriate to the geometric form of the structure.[1,2,3,4] These modes represent waves which can exist on the system in the absence of sources within the guide. They constitute a set of solutions for fields having harmonic time dependence $e^{j\omega t}$ and exponential behavior $e^{-\gamma z}$ in the longitudinal (or z) direction. Although there are no sources within any finite length of the structure, these solutions, or "free modes," may often conveniently be thought of as originating from sources located at $z=\pm\infty$.

The utility of this mode point of view springs from a number of general mode properties that are well understood for the simple configurations described above. When the dielectric is not distributed uniformly in the transverse plane of the guide, it is still possible in many cases to find exponential solutions to the source-free problem. Solutions of this type have been found for "closed" structures (like the partially filled waveguide[5]) as well as for "open" ones (like the dielectric rod[6,7]). The properties of these solutions have been investigated for individual cases,[8] with the result that certain of the familiar mode properties appear to be preserved under the new circumstances while others apparently are lacking.

This paper outlines a more general analysis of significant similarities and differences between free modes on conventional (or homogeneous) waveguides, and those encountered on somewhat more complicated (or inhomogeneous) systems. Consideration will be limited to the properties of exponential modes on cylindrical structures in which the material constants of the (passive) medium may vary in the transverse plane, and the cross section is bounded entirely by an electromagnetically opaque tube.

II. Formulation of the Problem

A. Co-ordinates and Notation

With reference to Fig. 1, the following notation will be clear:

$\hat{E}(t)$, $\hat{H}(t)$, $\hat{B}(t)$, $\hat{D}(t)$—Real field vectors, functions of (x, y, z, t)

$\hat{E}$, $\hat{H}$, $\hat{B}$, $\hat{D}$—Complex field vectors, functions of (x, y, z, ω)

E, H, B, D—Complex field vectors, functions of (x, y, ω) only

$\hat{E}_T(t)$, $\hat{E}_T$, E_T, and so on.—Vector functions as above but having space components only in the transverse (T) plane (x, y)

$\hat{E}_n$, $\hat{E}_\tau$, $\hat{E}_z$, and so on.—Complex scalar components, functions of (x, y, z, ω)

E_n, E_τ, E_z, and so on.—Complex scalar components, functions of (x, y, ω) only.

The presence of a caret (^) over either a scalar or a vector therefore merely means that the z dependence is present. When the caret is omitted, the quantity represents what is left after the z dependence has been separated out. This notation, while not standard, greatly simplifies the equations which appear in the subsequent text, and is justified only on that basis.

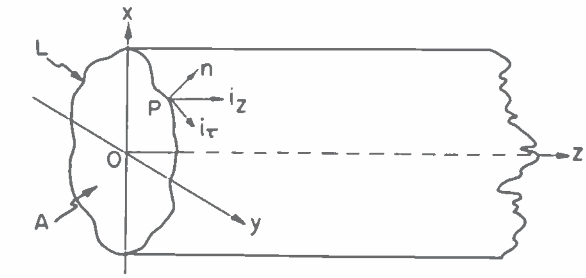


Fig. 1—Co-ordinate system for cylindrical structure.

In Fig. 1, P is any point on the bounding wall, A is any cross-sectional area of guide, and L is any bounding contour line of the guide wall; n, i_τ and i_z are real unit co-ordinate vectors.

Additional detailed notation will be introduced as required, with mks rationalized units employed throughout.

B. Reduction of Maxwell Equations to Cylindrical Form

When the time variation of the fields is taken to be harmonic ($e^{j\omega t}$), the appropriate form of the Maxwell equations applicable to the cylindrical system of Fig. 1, in the absence of sources, is

$$\nabla \times \hat{E} = -j\omega\mu\hat{H} \tag{1}$$

$$\nabla \times \hat{H} = j\omega\epsilon'\hat{E}, \tag{2}$$

with $j\omega\epsilon' = \sigma + j\omega\epsilon$. It is to be recalled that ϵ, μ, and σ, the (real) dielectric, permeability, and conductivity constants of the medium within the guide, may be functions of the transverse co-ordinates (x, y), but not functions of z. For the sake of simplicity, these parameters have also been taken to be independent of frequency ω.

Since the problem is cylindrically symmetric, it is natural to search for solutions which have the behavior

$$\hat{E} = Ee^{-\gamma z}, \qquad \hat{H} = He^{-\gamma z}. \tag{3}$$

The complex "propagation constant" γ will presumably be determined at any frequency ω from the bound-

[1] J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., New York, N. Y.; 1941.
[2] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," Rad. Lab. Ser. 8, McGraw-Hill Book Co., New York, N. Y.; 1948.
[3] J. C. Slater, "Microwave electronics," *Rev. Mod. Phys.*, vol. 18, p. 441; 1946.
[4] S. A. Schelkunoff, "Transmission theory of plane electromagnetic waves," Proc. I.R.E., vol. 25, pp. 1457–1492; November, 1937.
[5] L. Pincherle, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," *Phys. Rev.*, vol. 66, p. 118; 1944.
[6] H. Bondi and M. H. L. Pryce, "Dielectric cylinders as waveguides," *A. S. E. Rep.* (London), M434; August, 1942.
[7] M. Abele, "Teoria della propagazione di un campo elettromagnetico lungo una guida dielettrica a sezione circolare," *Consiglio Nazionale Delle Richerche* (Roma), Tipografia Del Senato; 1948.
[8] R. B. Adler, "Properties of Guided Waves on Inhomogeneous Cylindrical Structures," Technical Report No. 102, Research Laboratory of Electronics, M.I.T., Cambridge, Mass.; 1949.

ary conditions; but it should be emphasized that γ is a function of frequency.

The transverse parts of the Maxwell equations (1) and (2) may be rewritten in a new form, appropriate to the exponential z dependence:

$$H_T = \frac{\gamma}{p^2} \nabla_T H_z + \frac{j\omega\epsilon'}{p^2} i_z \times \nabla_T E_z \tag{4a}$$

$$E_T = \frac{\gamma}{p^2} \nabla_T E_z - \frac{j\omega\mu}{p^2} i_z \times \nabla_T H_z, \tag{4b}$$

where ∇_T is the transverse part of the gradient operator. The function p^2 introduced in (4) is defined by the relations

$$p^2 = -(\gamma^2 + k^2), \tag{5a}$$

where

$$k = \omega\sqrt{\epsilon'\mu} \tag{5b}$$

or

$$k^2 = \omega^2\epsilon'\mu = -j\omega\mu(\sigma + j\omega\epsilon). \tag{5c}$$

By reason of the dependence of ϵ' and μ upon the transverse co-ordinates, k^2 is also a function of position in the guide cross section.

Equation (4) constitutes a restatement of the transverse parts of the two Maxwell equations in a form which is particularly applicable to cylindrical systems.

The equations governing the behavior of E_z and H_z may be obtained by eliminating E_T and H_T from (1) and (2), using (3), (4), and (5). The results are

$$\nabla_T^2 E_z - p^2 E_z = \frac{1}{p^2}\left\{\left[\gamma^2 \frac{\nabla_T \epsilon'}{\epsilon'} - k^2 \frac{\nabla_T \mu}{\mu}\right] \cdot \nabla_T E_z + j\omega\mu\gamma i_z \cdot \left[\frac{\nabla_T k^2}{k^2} \times \nabla_T H_z\right]\right\} \tag{6a}$$

$$\nabla_T^2 H_z - p^2 H_z = \frac{1}{p^2}\left\{\left[\gamma^2 \frac{\nabla_T \mu}{\mu} - k^2 \frac{\nabla_T \epsilon'}{\epsilon'}\right] \cdot \nabla_T H_z - j\omega\epsilon'\gamma i_z \cdot \left[\frac{\nabla_T k^2}{k^2} \times \nabla_T E_z\right]\right\}. \tag{6b}$$

These last equations between E_z and H_z replace the longitudinal parts of the Maxwell equations, just as (4) replaces the transverse parts.

The boundary conditions to which the solutions of (6) and (4) must be subjected will be taken in either of the forms

$$n \times E = 0 \text{ on the boundary } L \tag{7a}$$

or

$$n \times H = 0 \text{ on the boundary } L. \tag{7b}$$

These conditions are appropriate for a "closed" structure; for an "open" structure they would be replaced by continuity conditions at the bounding interface.[7,9]

[9] S. Sensiper, "Electromagnetic Wave Propagation on Helical Conductors," Sc.D. Thesis, M.I.T. E.E. Dept., Cambridge, Mass.; May, 1951.

In order to solve any particular problem, the solutions of (6) must be expressed in terms of the transverse co-ordinates (x, y) and the unknown value of γ. Equation (4) determines the transverse fields, and application of the boundary conditions leads to a functional equation which will select the appropriate values of γ at each frequency. Among the boundary conditions must be included the usual finiteness, single-valuedness, and continuity conditions which are always imposed upon solutions to the source-free field equations.

It should be emphasized that according to (5) p^2 is a function of the transverse co-ordinates. As a result, it does not have the significance of an eigenvalue in these inhomogeneous problems. For any particular frequency, the set of allowed values of γ form the eigenvalues. In general, the functional equations determining γ will be transcendental, and the various branches of the functions will designate the "modes." Since k^2 is a function of both the frequency ω and the co-ordinates (x, y) and since γ^2 is only a function of ω, it is to be anticipated that p^2 will be a function of x, y, and ω. Hence for each mode, the field distribution in the transverse plane, governed by (6), will in general change with frequency. This fact is in marked contrast with the situation in homogeneous guides, where p^2 is a constant for each mode, and (6) does not contain coefficients dependent upon ω. In such cases, the field distribution for any particular mode remains the same over the entire frequency range $0 < \omega < \infty$.

When the problem is not homogeneous, the variation of the field distribution with frequency makes it much harder to identify the different modes.

It is not the function of the following portions of this paper either to solve (6), or to prove that allowed values of γ must exist under the boundary conditions prescribed by (7). Rather, an investigation will be conducted to determine some of the general properties of those modes which do exist so that some insight may be gained to guide the search for solutions to any given problem. In particular, the existence of some propagation constants and associated modes will simply be assumed in this discussion. Moreover, the difficult questions about the completeness of the entire set of modes (for the purpose of representing any given transverse field distribution, for example) will not be touched upon.

III. Basic Properties of the Modes

A. Combined TE-TM Character of the Modes

When the guide is uniformly filled with material, $\nabla_T\epsilon' = \nabla_T\mu \equiv 0$. Then (6) reduces to the familiar scalar Helmholtz equations in E_z and H_z, separately. Therefore, as far as the medium inside is concerned, two independent solutions are possible, one with $H_z \equiv 0 (TM)$ and one with $E_z \equiv 0 (TE)$. Similarly, the transverse fields given by (4) can be split into two corresponding groups, the forms of which are evident. The resulting equations are the conventional set for ordinary waveguides.[1,2,3,4]

It is apparent from (6), however, that E_z and H_z are generally tied together by the variations in medium parameters. A study of (4), (6), and (7) shows[8] that a highly restricted relationship between the shape of the boundaries and the variations of ϵ, μ, σ is required if any TE or TM modes are to be solutions of the problem. Occasionally, these conditions do arise,[5,6,7] but then only some of the modes have TE or TM character, the majority being TE-TM combinations.

A complete set of modes (if it exists at all) must be composed almost completely of TE-TM waves.

B. Incident and Reflected Waves

An elementary, but useful, symmetry property of the boundary-value problem posed by the guide structure amounts to the fact that for every mode solution there is always a second one which travels in the opposite direction. This alternate wave may be referred to as the "reflected" wave corresponding to the "incident" wave given originally. The phase velocity of the reflected field is along the z axis in a direction opposite to that of the incident field. Moreover, only the longitudinal component of the complex Poynting vector S reverses upon "reflection."

The proof of these statements is immediate because there is nothing in the present structure boundary conditions or in the internal medium which distinguishes one longitudinal direction from the other. A modified theorem of this type can be proved for less obvious cases like the helix,[9] but then a more detailed type of proof, along the lines given elsewhere,[8,9] is required.

In the present case, the main conclusions from the symmetry are the following:

(a) *The eigenvalue equation always has solutions γ and $-\gamma$. It may therefore be said to determine only γ^2.*

(b) *The difference between the field solutions corresponding to γ and $-\gamma$ is merely a reversal in sign of E_T and H_z, or H_T and E_z.*

C. Orthogonality Conditions

In conventional waveguide problems, a number of orthogonality relations are known to hold. If the subscripts 1 and 2 refer to any two exponential modes for which $\gamma_1 \pm \gamma_2 \neq 0$, then it is true[3] that at any particular frequency

$$\int_A \epsilon' E_{z1} E_{z2} da = \int_A \epsilon' E_{T1} \cdot E_{T2} da = \int_A \epsilon' E_1 \cdot E_2 da = 0. \quad (8)$$

Also,

$$\int_A i_z \cdot (E_{T1} \times H_{T2}) da = 0. \quad (9)$$

In these equations the integral is taken over the cross-sectional area A of the guide (Fig. 1), with the recollection that all the quantities concerned are functions of only the transverse co-ordinates. Also, H may be interchanged with E everywhere.

As long as the wall remains opaque, and therefore lossless, the validity of (8) and (9) is not impaired by the presence of losses in the internal medium, provided such losses are uniformly distributed in the cross section.

It is interesting that under the same conditions (including possible uniform loss in the medium) the fields in an homogeneous problem also have the properties

$$\int_A \epsilon'^* E_{z1} E_{z2}^* da = \int_A \epsilon'^* E_{T1} \cdot E_{T2}^* da = \int_A \epsilon'^* E_1 \cdot E_2^* da = 0 \quad (10)$$

as well as

$$\int_A i_z \cdot (E_{T1} \times H_{T2}^*) da = 0, \quad (11)$$

where the asterisk (*) represents the complex conjugate. Again, H and E may be interchanged everywhere

With reference to (8) and (10), it is convenient to refer to the properties described by them as "energy-orthogonality" conditions, while the properties expressed in (9) and (11) may be referred to simply as "power-orthogonality" conditions. The proofs of these various orthogonality properties are usually given from the nature of the scalar Helmholtz equation and the boundary conditions.

It is a matter of experience[8] that the energy-orthogonality conditions of (8) and (10) do not necessarily hold when the problem is inhomogeneous. Of course, the standard procedures for proving them cannot be applied to (6) and (4).

The standard procedures for proving (9) and (11) break down under the same conditions. Nevertheless, it is possible to show that the latter equations remain true, with certain additional limitations.

The reciprocity theorem[2] forms the basis of the required proof; it may be written in two convenient ways for any region in which there are no sources. It is supposed that (ϵ, μ, σ) are reasonable functions of the co-ordinates, and that two linearly independent fields ($\widehat{E}_1$, $\widehat{H}_1$) and ($\widehat{E}_2$, $\widehat{H}_2$) are solutions to the Maxwell equations at the same frequency ω. Then

$$\nabla \cdot (\widehat{E}_1 \times \widehat{H}_2 - \widehat{E}_2 \times \widehat{H}_1) = 0 \quad (12)$$

$$\nabla \cdot (\widehat{E}_1 \times \widehat{H}_2^* + \widehat{E}_2^* \times \widehat{H}_1) = -2\sigma \widehat{E}_1 \cdot \widehat{E}_2^*. \quad (13)$$

Application[8] of (12) is now made to a pair of exponential modes on a cylindrical structure of the type shown in Fig. 1. The resulting equation is integrated over the guide cross section, and use is made of the symmetry property discussed in Sec. III-B. These manipulations lead to the conclusion that as long as $\gamma_1 \pm \gamma_2 \neq 0$

$$\int_A i_z \cdot (E_{T1} \times H_{T2}) da = 0. \quad (14)$$

Equation (14) constitutes an orthogonality condition between any two different exponential modes on an inhomogeneous cylindrical structure of the "closed" variety. The only exclusions occur when both waves have the same γ (and hence are degenerate modes), or if either is the "reflected" counterpart of the other.

When the entire system is lossless ($\sigma=0$), (13) becomes

$$\nabla\cdot(\widehat{E}_1 \times \widehat{H}_2^* + \widehat{E}_2^* \times \widehat{H}_1) = 0. \qquad (15)$$

By steps similar to those outlined above, the resulting new orthogonality condition

$$\int_A i_z\cdot(E_{T1} \times H_{T2}^*)da = 0 \qquad (16)$$

follows readily when $\gamma_1 \pm \gamma_2 \neq 0$.

Emphasis must be placed upon the fact that (14) holds for structures with both dissipative and nondissipative internal media. When the structure is entirely nondissipative, (14) and (16) become valid together.

Since condition (14) holds more generally than (16), it is the one which acts most effectively as an orthogonality condition. Equation (16) is useful primarily for the purpose of understanding energy relations in a dissipationless cylindrical guide on which several modes are present together.

The usefulness of (14) as an orthogonality condition arises in the familiar problem of finding the coefficients in a transverse-field expansion.[3] While a determination of these coefficients by no means proves the completeness of the set of free modes for the expansion of given transverse fields, it is an aid to such expansions once the completeness of the set is known.

D. Nature of the Propagation Constant

One of the most important facts about the modes in homogeneous problems is that the propagation constant γ must be either pure real or pure imaginary when the structure is nondissipative.

To extend this theorem to the inhomogeneous problem, it must first be observed that in general $\widehat{E}(\omega)$ is the Fourier transform of $\widehat{E}(t)$ [and $\widehat{H}(\omega)$ is the Fourier transform of $\widehat{H}(t)$]. Since $\widehat{E}(t)$ is real, the properties of Fourier transforms require

$$\widehat{E}(-\omega) = \widehat{E}^*(\omega) \qquad (17)$$

for all values of (x, y, z) in the system.

Applying (17) to an exponential wave leads to the conclusion that

$$E(-\omega) = E^*(\omega) \qquad (18a)$$

$$\gamma(-\omega) = \gamma^*(\omega). \qquad (18b)$$

If $\gamma(\omega)=\alpha(\omega)+j\beta(\omega)$, (18) shows that $\alpha(\omega)=\alpha(-\omega)$ and $\beta(-\omega)=-\beta(\omega)$.

Under the assumption that the structure is lossless, consider the complex Maxwell equations and boundary conditions from (4), (6), and (7). From the properties of these equations under the transformation $\omega\rightarrow-\omega$, it can be shown[7] that the eigenvalue equation is invariant to this change of variable. According to Sec. III-B, therefore

$$\gamma^2(-\omega) = \gamma^2(\omega)$$

or

$$\gamma(-\omega) = \pm\,\gamma(\omega). \qquad (19)$$

Equations (18b) and (19) show that

$$\gamma(\omega) = \pm\,\gamma^*(\omega). \qquad (20)$$

The propagation constant for a lossless cylindrical structure with opaque walls must be either pure real or pure imaginary. It cannot be complex.

IV. Physical Characteristics of the Modes

A. Power and Energy Consequences of the Orthogonality Conditions

It is profitable to point out the consequences of (14) and (16) in terms of energy propagation when two modes are present simultaneously on the given structure.

If the system is lossless, both (14) and (16) are valid together. As a result:

Both the total instantaneous and vector longitudinal power flow down the guide are the simple sums of the corresponding flow for each mode alone, provided the structure is nondissipative.

When the mediun contains loss, the orthogonality condition (14) gives no information about the vector power. It is to be expected, therefore, that cross terms will appear in both the average (or active) power flow and the reactive power flow.

With regard to the energy orthogonalities for an inhomogeneous structure, the complex Poynting and reciprocity theorems, the symmetry property discussed in Sec. III-B, and (14), (16), and (7) can be used[7] to prove the following relations, provided $\gamma_1 \pm \gamma_2 \neq 0$:

$$\int_A (\epsilon' E_1\cdot E_2 + \mu H_1\cdot H_2)da = 0; \qquad (21)$$

and if $\sigma=0$,

$$\int_A (\epsilon E_1\cdot E_2^* - \mu H_1\cdot H_2^*)da = 0. \qquad (22)$$

Equation (21) is valid whether the internal medium is dissipative or not; (22) is true for nondissipative media only. It is also possible to show in a similar way that all the energy orthogonalities could be proven if only the scalar functions E_z and H_z were orthogonal with respect to the weight functions ϵ and μ, respectively. Unfortunately, particular examples[8] show that such a result cannot be obtained in general, nor can a corresponding result with weight-function unity be guaranteed. While it is possible that some other weight function would be appropriate, the physics of the problem appears to be of little help in finding it.

In summary, then, an extension of the power-orthogonality conditions found in Sec. III-C to the various energy orthogonalities mentioned there cannot generally be accomplished.

It appears that the power orthogonalities are properties of the Maxwell equations and symmetries of the structure; in particular, they are consequences of the reciprocity theorem. They are therefore common to both homogeneous and inhomogeneous problems. The energy orthogonalities, however, depend essentially upon the scalar functions (E_z, H_z), through the particular differential equations and boundary conditions to which they are solutions.

B. Power and Energy in a Single Mode

The results of Secs. III-B and D may be combined with the complex Poynting theorem and applied to free modes on a lossless structure.[7] The conclusions are summarized in the following equations:

If $\gamma=\alpha\neq 0$,

$$\int_A Re\, i_z\cdot(E_T \times H_T^*)da = 0. \tag{23}$$

If $\gamma=j\beta\neq 0$,

$$\int_A Im\, i_z\cdot(E_T \times H_T^*)da = 0 \tag{24a}$$

and

$$\int_A (\mu H\cdot H^* - \epsilon E\cdot E^*)da = 0. \tag{24b}$$

Equations (23) and (24a) together show that

on a lossless inhomogeneous guide a single wave below cutoff carries no total active power, while above cutoff it carries no total reactive power.

On the other hand, (24b) and (22) show that

no matter how many purely propagating modes are present at once the total time-average electric and magnetic stored energies per unit length of lossless guide must be equal.

The behavior of the lossless inhomogeneous guide below cutoff appears, for the moment, to be quite conventional. Nevertheless, it should not be assumed that

$$Re\,[i_z\cdot(E_T \times H_T^*)]$$

must be zero at each point of the cross section merely because the wave is below cutoff. It is true that the integrated value must vanish, according to (23). It is also true that when either *TE* or *TM* modes exist alone the longitudinal component of *S* does become imaginary at every point of the cross section. But when the problem is inhomogeneous, *TE* and *TM* modes are generally mixed, and extrapolation from the properties of the total power flow to those of the Poynting vector at a point may no longer be possible.

In these more general circumstances, the longitudinal component of the Poynting vector for a wave below cutoff may be calculated from (4) to yield

$$Re\, S_z = \frac{1}{2}\,\frac{Re\,[i_z\cdot(\nabla_T E_z \times \nabla_T H_z^*)]}{(\alpha^2 + k^2)}. \tag{25}$$

It is shown later (Sec. IV-E) that it is always possible to choose E_z and H_z 90° out of phase below cutoff. If this choice is elected, the $Re\, S_z$ will vanish everywhere, along with its integrated value. It can also be shown[7] that in many symmetrical problems such a choice is not necessary. Then (23) and (25) show that $Re\, S_z$ must necessarily be positive over some portions of the cross section and negative over others, otherwise the integrated power could not vanish. In the Appendix there appears a very simple example of a mixed *TE-TM* mode illustrating this behavior below cutoff.

It will also be observed from the Appendix that when the *TE-TM* mode is above cutoff the $Re\, S_z$ may still be negative over some portions of the cross section and positive over others. The latter phenomenon is not surprising, because for a wave above cutoff

$$2\, Re\, S_z = \frac{1}{|p|^4}\left[(\beta^2 + k^2)\, Re\, i_z\cdot(\nabla_T E_z \times \nabla_T H_z^*) + \omega\beta(\mu\|\nabla_T H_z\|^2 + \epsilon\|\nabla_T E_z\|^2)\right], \tag{26}$$

where the double magnitude ($\|A\|$) of a complex vector A represents $\sqrt{A\cdot A^*}$. Therefore, when $\beta>0$, for example, $Re\, S_z$ will become negative at any point where the first term becomes negative and simultaneously exceeds the second term in magnitude. The example in the Appendix merely shows that this situation may occur.

It is a consequence of the essentially vector character of the TE-TM modes on inhomogeneous lossless structures that the correlation between the direction of active power flow at a point[10] *and the algebraic sign of β is no longer necessarily unique. Moreover, there may be active power flow in either direction at various points of the cross section, even when a mode is below cutoff.*

C. Frequency Dependence of the Propagation Constant

The energy theorem[2] may be applied to a purely propagating mode ($\gamma=j\beta$) on a lossless structure, with the result[8] that

$$2\frac{\partial\beta}{\partial\omega}\int_A Re\, i_z\cdot(E_T \times H_T^*)da = \int_A (\epsilon E\cdot E^* + \mu H\cdot H^*)da. \tag{27}$$

[10] It is recognized that the phrases "power flow at a point" or "over an open surface" are loose terminology. Nevertheless, they are convenient. Strictly, only the integral of the Poynting vector over a closed surface may be considered as power flow.

Therefore,

$(\partial\beta/\partial\omega)$ and the total time-average longitudinal power flow along the guide have the same algebraic sign.

In fact, the equation shows that the group velocity $(\partial\beta/\partial\omega)^{-1}$ is also, in a sense, the velocity of energy propagation since it is merely the real power flow divided by the time-average total energy stored per unit length of guide. In this respect, modes on homogeneous and inhomogeneous structures are alike.

It is profitable to continue the investigation of mode properties by examining them at cutoff and at high frequencies. First, let it be supposed that a cutoff exists where $\gamma=0$ and $\omega=\omega_c>0$. At such a cutoff, (6) becomes free of E_z-H_z cross terms. So far as the internal medium is concerned, the *TE* and *TM* waves, which normally form a single mode, are now completely independent. The boundary conditions in (7) may also be satisfied by *TE* or *TM* waves alone. The entire problem of the guide reduces to one in only two dimensions. There is no z dependence for any field component and no total vector power flowing along the guide. The *TE* and *TM* modes (now completely independent solutions to the problem) are really "*TEM*" waves with respect to some axis in the (x, y) plane, the direction of this axis depending upon the particular point in question. This follows from (4), which shows that the *TE* wave has only transverse E and longitudinal H, while the *TM* wave has only transverse H and longitudinal E. The mechanism of cutoff is seen to be similar to the familiar picture in simpler cases. "*TEM*" waves, or "fans" of plane waves, are spreading out in the transverse plane, but now are refracted by the variations in ϵ and μ, as well as being reflected from the enclosing wall. Both polarizations of the "plane" waves are available, but which one is actually present at cutoff will depend upon the particular mode in question. It should be emphasized that any mode which is mixed *TE-TM* at other frequencies must degenerate to either pure *TE* or pure *TM* at cutoff. It is indeed commonly found[5,7] that the *TE-TM* modes can be split into two groups, which might be called "primarily *TE*" and "primarily *TM*." The former assume *TE* character at cutoff, while the latter degenerate into *TM* waves at cutoff.

If either the wall or the internal medium is dissipative, it is to be expected that γ will remain complex over the whole range of frequencies. It will not become zero (except possibly at $\omega=0$) since the source-free problem evidently cannot become two-dimensional ($\gamma=0$) when any losses are present.

An open structure, even though dissipationless, suffers from a similar difficulty because power can leave the guide system through the walls. It is not surprising then to find that the concept of cutoff, as outlined above, simply breaks down for the free modes on these open structures.[8]

Whenever a propagating mode on a lossless closed structure approaches cutoff, there will still be fields in the guide (solutions to (4), (6), and (7) with $\gamma=0$). The right side of (27) therefore remains finite, while the longitudinal power flow becomes zero. Hence, $(\partial\beta/\partial\omega)$ must increase without limit. At cutoff, the phase velocity becomes infinite ($\beta\to 0$), while the group velocity $(\partial\omega/\partial\beta)$ becomes zero. The cutoff frequency is therefore a branch point of $\gamma(\omega)$, and incidentally also of the fields (E, H) (Section IV-E).

At higher frequencies, above cutoff, the picture of mode behavior on nondissipative structures becomes quite different. It is to be kept clearly in mind that, at any frequency, ω, $k=\omega\sqrt{\epsilon\mu}$ is a function of position in the guide cross section. The values of $(\epsilon\mu)$ range from a minimum $(\epsilon\mu)_{\min}$ to a maximum $(\epsilon\mu)_{\max}$. In general, there will be certain areas of the cross section in the vicinity of which $k\sim k_{\max}$, and others where $k\sim k_{\min}$. Remaining portions of the cross section can be considered as transition regions. This concept becomes most striking when either $k_{\max}$ or $k_{\min}$, or both, occur within the guide boundary; because if k is any reasonable function of the transverse co-ordinates, $\nabla_T k=0$ at its maxima and minima.

An understanding of mode behavior at high frequencies can be gained from a review of the Maxwell equations in the limiting instance $\omega\to\infty$. It is well known[1] that when $\lambda|\nabla\mu/\mu|$ and $\lambda|\nabla\epsilon/\epsilon|$ everywhere in the cross section become $\ll 1$ the percentage changes in dielectric properties per local wavelength are small enough to make the governing wave equations differ only slightly from those in a homogeneous medium. The "average value" of k^2 must, however, still be considered to change from region to region of the cross section. Therefore, as $\omega\to\infty$, the wave equation for the rectangular components of $\hat{E}$ becomes approximately

$$\nabla^2\hat{E}+k^2\hat{E}=0, \tag{28}$$

in which k^2 is still a function of the transverse co-ordinates.

As applied to the z component of an exponential wave, (28) may be written

$$\nabla^2E_z-p^2E_z=0,\ (\omega\to\infty). \tag{29}$$

By similar reasoning, an identical equation holds for H_z. At very high frequencies, therefore, the *TE-TM* coupling due to continuous variations of ϵ and μ becomes negligible. Since the present considerations are limited to guides with opaque walls, any mode actually does begin to take on either a purely *TE* or purely *TM* character at sufficiently high frequencies. Both the differential equations and the boundary conditions can be satisfied (in the limiting case) by such separated fields.

Manipulation[8] of (29) yields the conclusion that at sufficiently high frequencies,

$$k_{\max}>\beta>k_{\min}. \tag{30}$$

Additional information about the behavior of $\beta(\omega)$ for large ω may be obtained from (27) and (24b) by expressing all quantities in terms of E_z. In particular, at high frequencies

$$\left|\frac{\partial\beta}{\partial\omega}\right| > \left|\frac{\beta}{\omega}\right|. \tag{31}$$

It is to be observed that (31) is also valid at frequencies slightly above cutoff, because there, too, the modes become approximately either pure TE or pure TM, as indicated previously. Of course, (30) is not valid near cutoff, where in fact $\beta \to 0$.

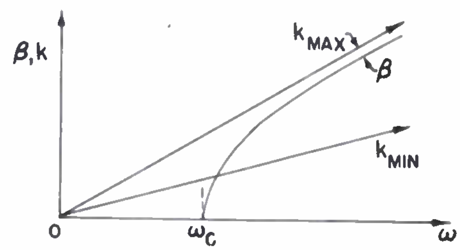


Fig. 2—$\beta(\omega)$ for a lossless inhomogeneous structure with opaque walls.

The results given in this section suggest that the general form of the β versus ω curve for any mode will appear as shown in Fig. 2. This figure should be regarded merely as a convenient summary of the facts presented by the analysis, rather than as an example of the detailed nature of the curve.

D. Frequency Dependence of the Transverse Field Distribution

The preceding section makes it possible to give a physical interpretation to the frequency dependence of the field distribution in lossless systems. The factor which is of primary importance in this connection is the parameter p. Now $p^2 = -\gamma^2 - k^2$ is surely negative at all points of the cross section when $\omega \leqq \omega_c$, because γ is real (or zero). The significant fact illustrated by Fig. 2 is that when the frequency is sufficiently far above cutoff p^2 eventually becomes positive in at least some regions of the cross section. At any such high frequency, the figure shows that p^2 will, loosely, be positive where k is "small" and negative where k is "large."

Equation (29) is of the form

$$\nabla^2\phi = p^2\phi, \tag{32}$$

in which p^2 is a real function of position (x, y). Therefore, $\phi(x, y)$ may be taken as real, also. It is well known, however, that the Laplacian of ϕ at a certain point represents the difference between the average value of ϕ over a small contour surrounding the point in question and the value of ϕ at that point. As a result, in regions of space where p^2 is negative, or $\nabla^2\phi$ and ϕ have opposite signs, the general trend is to make $|\phi_{av}|$ over neighboring points less than $|\phi|$ at a given point. In other words, a negative value of p^2 in a region of the cross section causes ϕ to oscillate up and down, assuming alternately positive and negative values. It has been observed (Fig. 2) that p^2 is negative in regions where k is "large" (near k_{max}). Hence the function $\phi(=E_z$ or $H_z)$ has oscillatory behavior there at high frequencies.

In those regions of the cross section where k is "small" (near k_{min}) Fig. 2 shows that p^2 becomes large and positive at sufficiently high frequencies. Then $|\phi_{av}|$ over neighboring points tends to be $>|\phi|$ at a given point, and ϕ has monotonic behavior over such regions.

Finally, where $p^2 \sim 0$ (somewhere in the "transition" regions of the cross section), it is necessary that $\nabla\phi$ $(=\nabla_T E_z$ or $\nabla_T H_z) \sim 0$ as well as $\nabla^2\varphi\phi \sim 0$. This restriction on $\nabla\phi$ comes from (4), with the stipulation that the transverse fields E_T and H_T remain finite at all points of the cross section (for all finite frequencies). These transition regions, then, must form the parts of the cross section where ϕ has essentially "flat" behavior, connecting those regions where it is monotonic with those in which it becomes oscillatory.

It is a familiar fact that a plane wave which attempts to pass through a discontinuity, from a lossless medium of uniformly high k into one of uniformly lower k, will suffer total reflection when the angle of incidence is sufficiently far from the normal. On the high-k side of the discontinuity, the reflected and incident waves will set up oscillatory standing waves in planes normal to the boundary, while on the low-k side there will be a monotonic decrease of all field components in similar planes.

The behavior of the lossless inhomogeneous waveguide with opaque walls at high frequencies can now be seen to present a very similar picture. In the transverse plane, the waves become "trapped" in regions of high k, and fall off monotonically in other parts of the cross section. To be sure, the trapping is not quite a result of critical reflection but rather of an excessive refraction where k varies rapidly with position. If the transition regions between those of highest and lowest k are squeezed down almost to lines of discontinuity, the trapping phenomena become more pronounced. But in any case, the fields are always crowded into the regions of highest k when the frequency becomes sufficiently high. As a consequence, the curve of $\beta(\omega)$ in Fig. 2 may be expected to become asymptotic to k_{max} as $\omega \to \infty$ since more and more of the field becomes crowded into corresponding regions of the guide. That is, the propagation constant k_{max} eventually controls virtually all of the field when the frequency becomes sufficiently high.

E. Polarization of the Fields

The questions of polarization to be discussed in this section are reasonably clear cut only in the lossless problems. Attention should therefore be focused on (4), (6), and (7), with the recognition that $\epsilon' = \epsilon$ is pure real.

If the mode under consideration is propagating $(\gamma = j\beta)$, then all the coefficients in (6) are entirely real. It will therefore always be possible to choose solutions for E_z and H_z which are entirely real functions. It appears from (4) that E_T and H_T are pure imaginary since they each become just j times a real vector function. Therefore, each represents a linearly polarized vector in the time domain.

When the mode is below cutoff ($\gamma=\alpha$), the situation is slightly different. Reference to (6) shows that if a new function $H_z'=-jH_z$ is substituted therein, all the coefficients again become real. In other words, below cutoff, a possible solution is that E_z shall be real, while H_z is pure imaginary. On the other hand, the substitution $E_z'=-jE_z$ also accomplishes a similar reduction of the coefficients in (6). Therefore, below cutoff, it may occur that either E_z is real and H_z imaginary, or E_z is imaginary and H_z is real. In either case, it is not hard to show, by reasoning similar to the above, that E_T and H_T represent linearly polarized vectors, although now they are out of time phase by 90°. It follows that

in a lossless problem it is always possible to choose modes in such a way that the transverse fields will be linearly polarized over the entire frequency range.

The significance of the two possible choices for the fields (E_z, H_z) below cutoff can be further elucidated. The reasoning upon which the real and/or imaginary character of (E_z, H_z) has just been based hinges upon the nature of the cross terms in (6). Above cutoff, E_z and H_z can always be chosen as pure real, regardless of the mode, i.e., regardless of $\beta(\omega)$. Suppose such a choice has been made for a particular $\beta_\nu(\omega)$, defining a particular mode ν. As the frequency is decreased through cutoff, $\beta_\nu(\omega)$ passes into $\alpha_\nu(\omega)$. But exactly at cutoff it has been shown (Sec. IV-C) that *TE* or *TM* character alone is sufficient to describe the fields. Now, in addition, it has just been shown that below cutoff there are two possibilities (if any exponential field exists at all):

(a) E_z remains real, H_z becomes imaginary

or

(b) E_z becomes imaginary, H_z remains real.

Surely only one transition is possible for a single (continuous) mode with a particular $\gamma_\nu(\omega)$. Since apparently both situations (a) and (b) are compatible with all the conditions of the problem, it follows that there must be two groups of modes, one corresponding to transition "a," and the other to transition "b." Moreover, if transition "a" takes place continuously, H_z must pass continuously from pure real to pure imaginary, whence $H_z=0$ at cutoff. The "a" modes are then the "primarily *TM*" modes. Similarly, the "b" modes will have $E_z=0$ at cutoff, and are therefore the "primarily *TE*" modes.

The discussion of polarization given above indicates that

if linearly polarized transverse fields are chosen in defining a mode the Re $S_z\equiv 0$ *at frequencies below cutoff.*

Similarly, it is not difficult to show that

the choice of linearly polarized transverse fields leads to modes for which the component of the time average Poynting vector in any direction in the transverse plane is zero at all frequencies.

It has been shown that the choice of linearly polarized transverse fields certainly avoids the appearance of positive and negative "power flows" at various points of the cross section when a mode is below cutoff. The example in the Appendix shows that this same choice of linear polarization does not necessarily avoid power reversal when the mode is above cutoff. Therefore,

it is possible for a TE-TM mode with linearly polarized transverse fields to exist above cutoff in such a way that Re S_z *changes sign at certain points of the cross section.*

While an appropriate definition of the modes may always be made which will guarantee linear polarization of $\widehat{E}_T(t)$ and $\widehat{H}_T(t)$, it is still not possible to assume that the space angle between them is the same at each point of the cross section. An examination of (4) will show that, even with linear polarization, $(\|E_T\|)(\|H_T\|)$ is not in general the same function of (x, y) as $(E_T\cdot H_T)$; hence the space angle between $\widehat{E}_T(t)$ and $\widehat{H}_T(t)$ is generally a function of position in the transverse plane. This variation of angle makes the process of visualizing and utilizing mode behavior much more difficult than is the case with homogeneous systems.

V. Conclusion

Whatever simplicity does remain in the properties of modes on lossless structures is generally lost when dissipation is present, either in the bounding wall or the internal medium.[8] Equations (6) develop complex coefficients as a result of the fact that ϵ' is complex. Then p^2 is also a complex function of (x, y) in the general case. The real and imaginary parts of E_z (and/or H_z) can no longer be taken as constant multiples of each other since the real and imaginary parts no longer satisfy the same differential equations. There is no guarantee that the polarization of the transverse fields can always be made linear, and it may necessarily be forced to vary in type from point to point of the cross section. The mode structure (if it still exists) becomes very difficult to visualize, despite the fact that the orthogonality condition of (14) still remains to help separate one mode from another.

It must be emphasized that the validity of the orthogonality condition (14) in no way implies the completeness of the set of exponential modes. In fact, the general study presented here has been extended[8] to include lossless "open" structures like the dielectric rod. It has been found that many of the conclusions apply directly. The primary differences occur because the propagation constant in such circumstances must be purely imaginary ($\gamma=j\beta$). It is further limited by lying between that of plane waves in the surrounding medium and that of plane waves in a medium having the intrinsic propagation constant $\omega\sqrt{(\epsilon\mu)_{max}}$. Each of the modes, therefore, simply ceases to exist below that frequency at which β becomes zero. At any specific frequency there may be only a finite number of free exponential modes which can satisfy both the boundary value problem and the symmetry conditions imposed by a given source. These free modes cannot account for any radiation because of the restrictions on the propagation constant. Therefore, they cannot constitute a complete set. Still, as long as the frequency is such that several such modes can exist simultaneously, the orthogonality conditions (14) and (16) continue to remain valid, provided that the cross section A is interpreted as the entire transverse plane. Additional extensions[8,9] of this study to structures with impedance or helix boundary conditions have also borne out the conclusion that the orthogonality conditions presented here can remain valid in many circumstances where the set of free modes is not, or may not be, complete.

Appendix

"TE-TM" Waves in a Rectangular Waveguide

Fig. 3 represents a conventional rectangular waveguide with perfectly conducting metal walls. It is well known that when m, $n \neq 0$, the $TE_{m,n}$ and $TM_{m,n}$

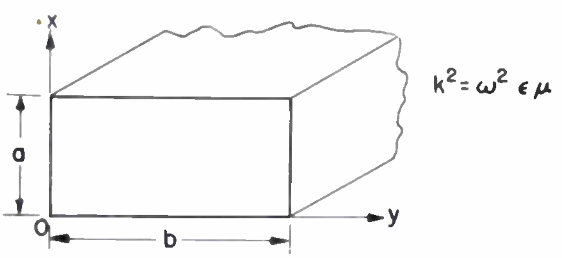


Fig. 3—Co-ordinates for rectangular guide.

waves are degenerate. Consider a mixture of the two waves below cutoff, and let A_{mn} and B_{mn} represent respectively the amplitudes of H_{zmn} and E_{zmn}. Then (25) of the text yields

$$2\,Re\,S_z = \frac{Re\,(A^*_{mn}B_{mn})}{|p_{mn}|^2}\left(\frac{mn\pi^2}{ab}\right)\left[\sin\pi\left(\frac{mx}{a} - \frac{ny}{b}\right)\sin\pi\left(\frac{mx}{a} + \frac{ny}{b}\right)\right]. \tag{33}$$

Equation (33) shows that $Re\,S_z$ reverses sign in certain parts of the cross section. This can be appreciated more easily by considering the special case $m = n = 1$, and $Re(A^*_{11}B_{11}) > 0$. Then $Re\,S_z = 0$ along the lines

$$x = \left(\frac{a}{b}\right)y \tag{34a}$$

and

$$x = a - \left(\frac{a}{b}\right)y. \tag{34b}$$

It is not zero along the lines $y = 0$ and $y = b$. Thus the sign distribution of $Re\,S_z$ will be as shown in Fig. 4.

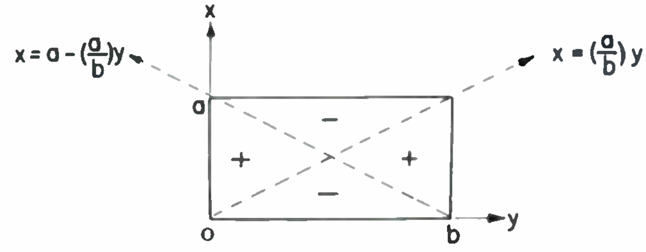


Fig. 4—Sign distribution of $Re\,S_z$ for a $(TE\text{-}TM)_{1,1}$ wave below cutoff.

By expanding (33) trigonometrically, it can be verified that

$$\int_0^a \int_0^b Re\,S_z dxdy = 0. \tag{35}$$

If the mode is above cutoff, (26) of the text applies. Now E_{zmn} and H_{zmn} are independent of frequency, provided that A_{mn} and B_{mn} are not functions of frequency. Therefore, if $\beta > 0$, it must be possible to find a frequency sufficiently near cutoff ($\beta \simeq 0$) such that the second term in the numerator of (26) becomes arbitrarily small. If $Re\,(A^*_{mn}B_{mn}) > 0$, however, (33) and Fig. 4 show that the first term of the numerator in (26) is always negative over certain portions of the cross section. In view of the fact that $k^2 \neq 0$, even when β becomes zero, it follows that in these regions of the cross section the $Re\,S_z$ will necessarily become negative at frequencies sufficiently near (but nevertheless above) cutoff.

It should be recognized that the Poynting vector reversals indicated in the foregoing can take place above cutoff when A_{mn} and B_{mn} are entirely real, under which conditions the transverse fields would be linearly polarized.

Acknowledgment

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Radiation Patterns and Conductance of Slotted-Cylinder Antennas*

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Summary—A narrow slot cut in the wall of a hollow cylinder is excited by a transmission line and produces radiation. A theoretical solution to the radiation patterns and conductance is obtained by solving Maxwell's equations for the fields in the far zone, and requiring them to satisfy the known boundary conditions at the surface of the cylinder. Advantage is taken of published data on potential distribution across the slot.

Discussion

THIS PAPER deals with a theoretical means of calculating the conductance and radiation patterns of an antenna comprised of a narrow slot cut in the wall of a hollow circular cylinder of infinite conductivity.

Such a slot, having a length of $2L$ and subtending an angle of ϕ_0 at the z axis, is shown in Fig. 1 (at the right). A transmission line is shown supplying power to the slot.

The subject of a slotted cylinder antenna has been treated recently by several authors. The problem of radiation from an infinitely long slot in an infinite cylinder has been solved and used, as an approximation, to calculate the radiation pattern in a plane normal to the cylinder axis through the center of the slot.[1,2,3] Silvers and Saunders have shown in their papers[4,5] that this pattern is correctly given by this solution, and independent of the axial field distribution in the slot.

Silvers and Saunders have solved the problem of radiation from an arbitrary slot in an infinite cylinder, and have obtained the radiation patterns in the far zone for a sinusoidal axial field distribution with a wavelength equal to that of the exciting source.

Jordan and Miller[6] have measured the field distribution along axial slots of various lengths and show that the wavelength is a function of the cylinder diameter in wavelengths, equaling the excitation wavelength with large diameters and increasing to infinity at some smaller cutoff diameter. This cutoff diameter is quite different from that predicted for a circular waveguide. The cutoff diameter is also influenced by the slot width and cylinder thickness.

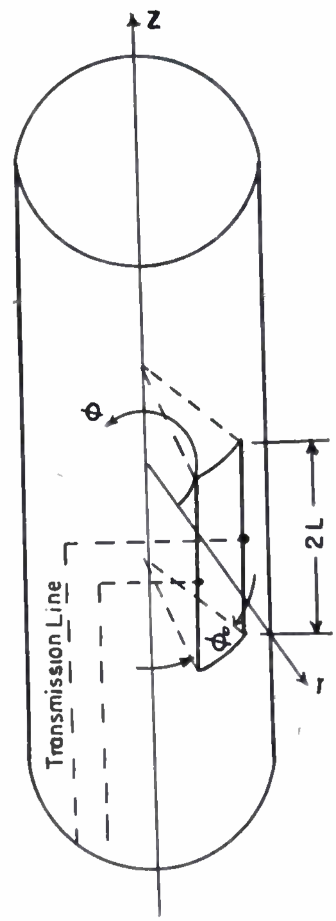

Fig. 1—Slotted cylinder.

The cylinder will be assumed to be of such proportions that reflections and radiation from the ends of the cylinder may be considered to be negligible. This will be true if the cylinder diameter is less than the cutoff value for any mode that may be transmitted with the cylinder acting as a circular waveguide, and if the cylinder is long enough so that any propagated mode will be attenuated sufficiently to be neglected.

It will be assumed that, by the manner of excitation, $E_z = 0$, and that for $r = a$, the radius of the cylinder, E_ϕ

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[1] G. Sinclair, E. C. Jordan, and E. W. Vaughan, "Measurement of aircraft-antenna patterns using models," PROC. I.R.E., vol. 35, pp. 1451–1462; December, 1947.

[2] C. H. Papas and R. W. P. King, *Quart. Appl. Math.*, no. 2, July, 1949.

[3] G. Sinclair, "Patterns of slotted-cylinder antennas," PROC. I.R.E., vol. 36, pp. 1487–1492; December, 1948.

[4] S. Silvers and W. K. Saunders, "The external field produced by a slot in an infinite circular cylinder," *Jour. Appl. Phys.*, vol. 21, p. 153; February, 1950.

[5] S. Silvers and W. K. Saunders, "The radiation from a transverse rectangular slot in a circular cylinder," *Jour. Appl. Phys.*, vol. 21, p. 745; August, 1950.

[6] E. C. Jordan and W. E. Miller, "Slotted-cylinder antennas," *Electronics*, vol. 20, pp. 90–93; February, 1947.

must reduce to the excitation field across the slot, and to zero over the conducting cylinder, thus:

Let

$$E_\phi = E_0 \sin k'(L - |z|) \text{ when } |\phi| < \frac{\phi_0}{2} \text{ and } |z| < L \quad (1)$$

$$E_\phi = 0 \text{ when } |\phi| > \frac{\phi_0}{2} \text{ or } |z| > L.$$

Where ϕ_0 is the angle subtended at the z axis by the edges of the slot,

$$k' = \frac{2\pi}{\lambda s}, \qquad k = \frac{2\pi}{\lambda}$$

λs is the wavelength along the slot,
λ is the wavelength of the source of excitation,
$2L$ is the length of the slot.

This distribution of E_ϕ in (1) may be represented by a double summation in ϕ and z. However, due to the infinite period of the distribution in the z direction, a Fourier integral will be used.

In the ϕ direction E_ϕ is periodic, and may be represented by a Fourier series in ϕ, where A_n will also be a function of z.

$$E_\phi = \frac{A_0(z)}{2} + \sum_{n=1}^{\infty} A_n(z) \cos n\phi$$

$$A_n(z) = \frac{1}{\pi} \int_0^{2\pi} E_\phi \cos n\phi d\phi.$$

Now $A_n(z)$ may in turn be represented by a Fourier integral:

$$A_n(z) = \frac{1}{\pi} \int_0^\infty dh \int_{-L}^{+L} A_n(x) \cos hz \cos hx \, dx.$$

Performing this operation we have

$$E_\phi = \frac{E_0\phi_0}{2\pi^2} \int_0^\infty \frac{(2k')}{(k'^2 - h^2)} (\cos hL - \cos k'L) \cos hz \, dh$$
$$+ \sum_{n=1}^{\infty} \frac{2E_0}{n\pi^2} \sin \frac{n\phi_0}{2} \cos n\phi \int_0^\infty \frac{(2k')}{(k'^2 - h^2)}$$
$$\cdot (\cos hL - \cos k'L) \cos hz \, dh. \quad (2)$$

A solution to Maxwell's equations in cylindrical coordinates will give the fields outside the cylinder. The wave equation for H may be expanded into three scalar equations with a sine wave time-variation $e^{j\omega t}$ assumed; the equation in H_z then permits direct solution by separation of variables. This solution and $E_z = 0$ permit solving for the other four components of the field. Stratton[7] also outlines a method. The resultant equations are:

[7] J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Company, New York, N. Y.; 1941.

$$E_\phi = j\omega\mu \sum_{n=0}^{\infty} \cos n\phi \int_0^\infty \alpha_n(h) \frac{H_n'^{(2)}(pr)}{\sqrt{k^2 - h^2}} \cos (hz) dh, \quad (3)$$

$$H_z = \sum_{n=0}^{\infty} \cos n\phi \int_0^\infty \alpha_n(h) H_n^{(2)}(pr) \cos (hz) dh \quad (4)$$

where $p = \sqrt{k^2 - h^2}$, $k = \dfrac{2\pi}{\lambda}$ and $\alpha_n(h)$ is to be determined by the boundary condition.

A double summation has been formed by summing over all n's and h's. This is done to place the solution from the field equations in a form similar to (2).

Now when $r = a$, the cylinder radius, E_ϕ from Maxwell's equation must reduce to the prescribed distribution on the cylinder, (2).

By equating corresponding terms of the two series, (2) and (3), $\alpha_n(h)$ may be determined, and the boundary condition satisfied, giving

$$E_\phi = \int_0^\infty \frac{H_0'^{(2)}(pr)(2k')(E_0)}{H_0'^{(2)}(pa)(k'^2 - h^2)(2\pi^2)}$$
$$\cdot (\cos hL - \cos k'L) \cos hz \, dh$$
$$+ \sum_{n=1}^{\infty} \frac{2E_0}{n\pi^2} \sin \frac{n\phi_0}{2} \cos n\phi \int_0^\infty \frac{H_n'^{(2)}(pr)(2k')}{H_n'^{(2)}(pa)(k'^2 - h^2)}$$
$$\cdot [\cos hL - \cos (k'L)] \cos hz \, dh \quad (5)$$

$$H_z = \frac{1}{j\omega\mu} \int_0^\infty \frac{H_0^{(2)}(pr)\sqrt{k^2 - h^2}\,(2k')(E_0\phi_0)}{H_0'^{(2)}(pa)(k'^2 - h^2)(2\pi^2)}$$
$$\cdot (\cos hL - \cos k'L) \cos hz \, dh$$
$$+ \frac{1}{j\omega\mu} \sum_{n=1}^{\infty} \cos n\phi \sin \frac{n\phi_0}{2} \frac{(2E_0)}{(n\pi^2)}$$
$$\cdot \int_0^\infty \frac{H^{(2)}{}_n(pr)\sqrt{k^2 - h^2}\,(2k')}{H_n'^{(2)}(pa)(k'^2 - h^2)}$$
$$\cdot (\cos hL - \cos k'L) \cos hz \, dh. \quad (6)$$

The remaining components, E_r, H_ϕ, and H_r, may be determined in a like manner. It will be found that E_r is of the order of $1/r$ times E_ϕ, so only E_ϕ must be considered for the patterns in the far zone.

When $r \gg 1$, $H_n'^{(2)}(pr)$ may be replaced by its asymptotic expression. Also, when ϕ_0 is small, $\sin n\phi_0/2 \cong n\phi_0/2$, and

$$E_\phi = \frac{E_0\phi_0}{\pi^2} \sqrt{\frac{2}{\pi r}} (2k') \exp\left(-j\frac{\pi}{4}\right)$$
$$\cdot \left\{\frac{I_0}{2} + \sum_{n=1}^{\infty} j^n I_n \cos n\phi\right\}. \quad (7)$$

where

$$I_n = \int_0^k \frac{1}{\sqrt{p}} \frac{\exp(-jpr)(\cos hL - \cos k'L) \cos hz \, dh}{H_n'^{(2)}(pa)(k'^2 - h^2)}.$$

When ka is small, the series converges quite rapidly, and only a few terms will be necessary. When ka is greater than one, the series will converge slowly. This equation may be used to calculate vertical as well as horizontal patterns of the vertical cylinder.

By integrating Poynting's vector over the surface of the cylinder, the conductance at the center of the slot may be determined.

$$P=\frac{1}{2}\int_{-\phi_0/2}^{\phi_0/2}\int_{-L}^{+L} E_\phi H_z a\, d\phi\, dz$$

$$=R_e\frac{V^2Y}{2}=R_e\frac{a^2\phi_0^2E_0^2(\sin^2 k'L)Y}{2}.$$

$$G=R_eY$$

$$=R_e\frac{1}{j\omega\mu}\int_0^\infty\frac{H_0(pa)\sqrt{k^2-h^2}\,(2k')^2(\cos hL-\cos k'L)^2dh}{H_0'^{(2)}(pa)(2\pi^2)(k'^2-h^2)^2(a)(\sin^2 k'L)}$$

$$+R_e\frac{1}{j\omega\mu}\sum_{n=1}^{\infty}\frac{4\sin^2\frac{n\phi_0}{2}}{a\phi_0^2(n\pi)^2\sin^2 k'L}$$

$$\cdot\int_0^\infty\frac{H_n^{(2)}(pa)\sqrt{k^2-h^2}\,(2k')^2}{H_n'^{(2)}(pa)(k'^2-h^2)^2}(\cos hL-\cos k'L)^2dh.$$

To determine the conductance, we shall be concerned with integrals of the general type.

$$I_n=R_e\frac{1}{j\omega\mu}\int_0^\infty\frac{H_n^{(2)}(pa)\sqrt{k^2-h^2}\,(2k')^2}{H_n'^{(2)}(pa)(k'^2-h^2)^2}(\cos hL-\cos k'L)^2dh.$$

The integrands of these integrals will be real only while $h\leqq k$, and the integration need only be taken to k. When $h>k$, $H_n(pa)/H_n'(pa)$ is totally imaginary, $\sqrt{k^2-h^2}$ will contribute another j and the odd number of j's will give a net imaginary result. For small ϕ_0's,

$$\frac{4}{n^2\phi_0^2}\sin^2\frac{n\phi_0}{2}\cong 1$$

and

$$G=\text{Real } Y=\frac{R_e}{\pi^2 a\sin^2 k'L}\left\{\frac{I_0'}{2}+\sum_{n=1}^{\infty}\frac{I'_n}{2}\right\}$$

$$I'_n=\frac{1}{\omega\mu}\int_0^k\left\{\text{Imag}\frac{H_n^{(2)}(pa)}{H_n'^{(2)}(pa)}\right\}\cdot\frac{\sqrt{k^2-h^2}\,(2k')^2(\cos hL-\cos k'L)^2}{(k'^2-h^2)^2}dh$$

$$\text{Imag}\frac{H_n^{(2)}}{H_n'^{(2)}}$$

$$=\text{Imag}\left[\frac{J_nJ_n'+N_nN_n'+j(J_nN_n'-N_nJ_n')}{J_n^2+N_n^2}\right]$$

$$=\frac{J_n\left(\frac{-n}{pa}N_n+N_{n-1}\right)-N_n\left(\frac{-n}{pa}J_n-J_{n-1}\right)}{J_n^2+N_n^2}.$$

Since

$$N_n'=-\frac{n}{pa}N_n+N_{n-1}\quad\text{and}$$

$$J_n'=-\frac{p}{pa}J_n+J_{n-1},\ \frac{J_nN_{n-1}-J_{n-1}N_n}{J_n^2+N_n^2}$$

$$=\frac{\frac{2}{\pi pa}}{J_n^2+N_n^2}=\text{Imag}\frac{H_n^{(2)}}{H_n'^{(2)}}.$$

Also

$$\omega\mu=k\eta,\qquad p=\sqrt{k^2-h^2}$$

$$I_n'=\frac{1}{k\eta}\int_0^k\frac{\frac{2}{\pi a}(2k')^2(\cos hL-\cos k'L)^2\,dh}{(J_n^2+N_n^2)(k'^2-h^2)^2}$$

Let

$$h=kx,\qquad pa=ka\sqrt{1-x^2}$$

$$I_n'=\frac{8a}{(ka)^2\pi\eta}\int_0^1\frac{\alpha^2(\cos kLx-\cos k'L)^2\,dx}{[J_n^2(pa)+N_n^2(pa)](\alpha^2-x^2)^2}$$

where $\alpha=k'/k$ and thus,

$$G=\frac{8}{\eta(ka)^2\pi^3\sin^2 k'L}\left\{\frac{A_0}{2}+\sum_{n=1}^{\infty}A_n\right\}\qquad(8)$$

where

$$A_n=\int_0^1\frac{\alpha^2(\cos kLx-\cos k'L)^2\,dx}{[J_n^2(ka\sqrt{1-x^2})+N_n^2(ka\sqrt{1-x^2})](\alpha^2-x^2)^2}.$$

As the cylinder diameter in wavelengths increases, α will approach unity.

The integrals of (8) were evaluated by numerical integration. For any desired accuracy, more terms will be required as ka increases.

Results

The results are shown in Figs. 2, 3, 4, and 5. Figs. 2 and 3 show vertical and horizontal patterns for two

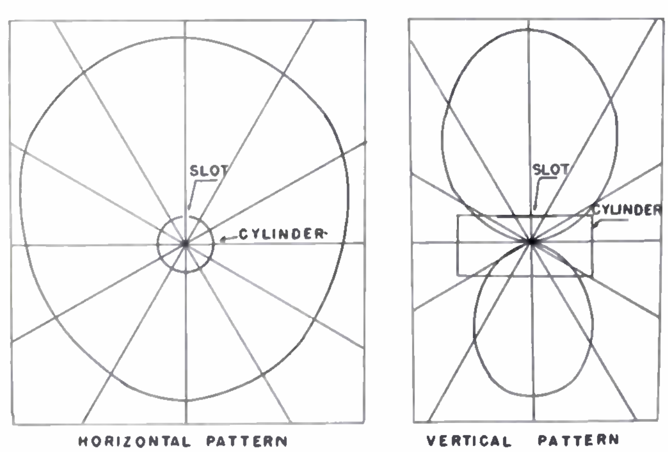
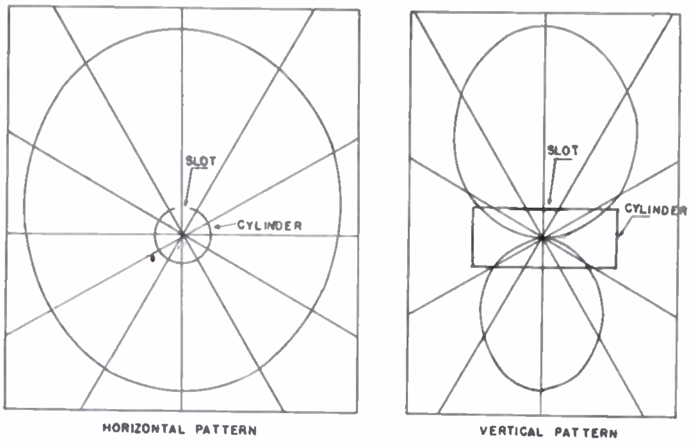


Fig. 2—Radiation patterns of slotted-cylinder antenna calculated from (7). ($D=0.13\lambda$, $\phi_0=0.124$, $2L=0.12\lambda$)

different cylinder diameters. It is seen that the pattern resembles that of a dipole except that greater radiation is produced on the slot side of the cylinder than on the reverse side.

Experimental data are not available for comparison with these patterns. However, their general shape might be expected from some of the results published by Jordan and Miller.[6]



(a) (b)

Fig. 3—Radiation patterns of slotted-cylinder antenna calculated from (7). ($D=0.125\lambda$, $\phi_0=0.124$, $2L=0.12\lambda$)

The curves of Fig. 4 show the effects of the cylinder diameter on the horizontal patterns, the patterns being circular for small diameters and nonsymmetrical for large diameters. In this case, the greatest radiation is on the slot side of the cylinder.

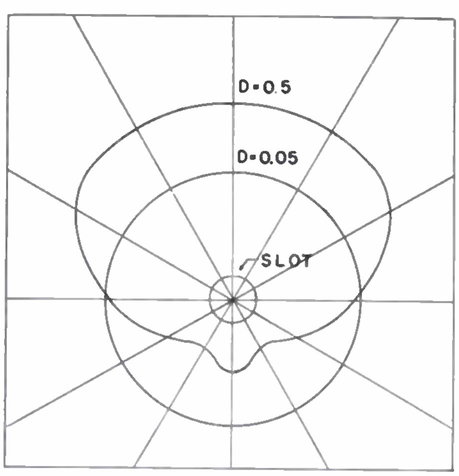


Fig. 4—Effect of cylinder diameter on horizontal radiation patterns. (D in wave lengths as shown, $\phi_0=0.124$, $2L=0.12\lambda$)

Fig. 5 shows the measured and calculated conductance. The measured curve is from Jordan and Miller,[6] and the calculated from (8). The former requires the reading of impedance from curves and conversion to admittance. It is difficult to interpolate the curves closely; hence, some error is expected in the conversion. Another source of error is in the determination of k' by measurement. The dip in the curve may result from this, or it may result from the resonance that occurs at this size slot.

Except for this dip, the two curves are similar and of the same order of magnitude.

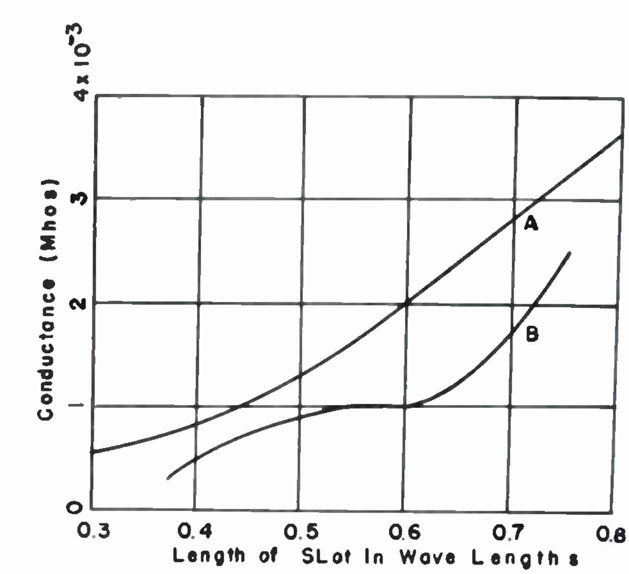


Fig. 5—Input conductance for slotted antenna $D=0.130$ wave lengths, $\phi_0=0.124$. (A calculated curve, B measured curve by Jordan and Miller[6])

CONCLUSION

Equations have been developed which can be used to calculate both radiation patterns and conductance of a slotted-cylinder antenna. The equations involve series which converge rapidly when $ka \ll 1$, and more slowly as ka increases.

The series contained in the equation by Silvers and Saunders[5] would do the same. However, they would be easier to apply for the calculation of patterns, as they do not involve numerical integration.

APPENDIX

The calculation of conductance[8] is as follows for $D=0.13\lambda$:

$$G = \frac{8\left(\frac{1}{2}I_0 + \sum_{n=1}^{\infty} I_n\right)}{\eta(ka)^2\pi^3 \sin^2 k'L}$$

where

$$I_n = \int_0^1 \frac{\alpha^2(\cos kxL - \cos k'L)^2}{(J_n{}^2 + N_n{}^2)(\alpha^2 - x^2)^2}\,dx.$$

$2L$	I_0	I_1	I_2
.3	.02588	.009238	.0003590
.4	.07763	.02783	.001089
.5	.1773	.0638	.002516
.6	.3474	.1260	.005016
.7	.5672	.2072	.008347

[8] Eugene Janke and Fritz Emde, "Table of Functions," Dover Publishing Company, New York, N. Y.; 1943.

Discussion on

"Correction of Deflection Defocusing in Cathode-Ray Tubes"*

JENNY E. ROSENTHAL

R. G. E. Hutter and S. W. Harrison:[1] The paper, "Correction of Deflection Defocusing in Cathode-Ray Tubes," by Jenny E. Rosenthal includes an analysis of deflection-defocusing effects produced by two-dimentional electrostatic deflection systems. Since this subject has been treated in the literature a number of times,[2,3,4,5] it seems desirable to compare the method used in her paper with that of earlier investigators.

Any analysis of the behavior of such deflection fields must use methods of approximation in determining the electron path since the equations of motion cannot be integrated directly for a general potential distribution. The method used in the papers referred to above—the so-called path method[4]—is to expand the potential about the axis of symmetry of the deflection system and, making use of this expansion, to integrate the Euler-Lagrange equations of motion by a series of successive approximations. The same general method is used by Miss Rosenthal with slight differences in application. The potential function is again expanded about an axis but, in this case, an axis parallel to the axis of symmetry. The expansion is then used in Lagrange's equations of motion, time being kept as an implicit variable. The path is again found by a series of approximations.

To show the parallel between the two methods more clearly, (23) and (24) of the Rosenthal paper may be derived in a somewhat different way. Using her notation, the potential $V(x, y)$ is expanded about (x_0, y_0) which, according to her equations (14) and (20), represents the axis $x=vt$, $y=\eta$, where v and η are constant. Thus

$$
\begin{aligned}
V(x, y) = V(x_0y_0) &+ (x - x_0)\left(\frac{\partial V}{\partial x}\right)_{x_0y_0} \\
&+ (y - y_0)\left(\frac{\partial V}{\partial y}\right)_{x_0y_0} \\
&+ \frac{(x - x_0)^2}{2}\left(\frac{\partial^2 V}{\partial x^2}\right)_{x_0y_0} \\
&+ \frac{(y - y_0)^2}{2}\left(\frac{\partial^2 V}{\partial y^2}\right)_{x_0y_0} \\
&+ (x - x_0)(y - y_0)\left(\frac{\partial^2 V}{\partial x\partial y}\right)_{x_0y_0} + \cdots. \qquad (1)
\end{aligned}
$$

Making use of this expansion and that given by her equation (19), which may be written

$$
\left.\begin{aligned} x &= x_0 + \delta x_1 + \delta^2 x_2 + \cdots \\ y &= y_0 + \delta y_1 + \delta^2 y_2 + \cdots \end{aligned}\right\}, \qquad (2)
$$

the equations of motion for x and y may be combined to give

$$
\begin{aligned}
\left(\frac{m}{e}\right)\frac{d^2}{dt^2}&[(x_0 + iy_0) + \delta(x_1 + iy_1) + \cdots] \\
&= \left(\frac{\partial V}{\partial x} + i\,\frac{\partial V}{\partial y}\right)_{x_0y_0} \\
&+ [(x - x_0) - i(y - y_0)]\left(\frac{\partial^2 V}{\partial x^2} + i\,\frac{\partial^2 V}{\partial x\partial y}\right)_{x_0y_0} + \cdots \qquad (3)
\end{aligned}
$$

since

$$
\frac{\partial^2 V}{\partial y^2} = -\frac{\partial^2 V}{\partial x^2}.
$$

Changing the independent variable from t to u and substituting for v, $(x-x_0)$, and $(y-y_0)$, (3) may be written

$$
\begin{aligned}
2U(1 - 2r\delta)\,\frac{d^2}{du^2}&[(x_0 + iy_0) + \delta(x_1 + iy_1) + \cdots] \\
&= \left(\frac{\partial V}{\partial x} + i\,\frac{\partial V}{\partial y}\right)_{x_0y_0} \\
&+ \delta(x_1 - iy_1)\left(\frac{\partial^2 V}{\partial x^2} + i\,\frac{\partial^2 V}{\partial x\partial y}\right)_{x_0y_0} + \cdots. \qquad (4)
\end{aligned}
$$

Since

$$
\left.\begin{aligned}
\left(\frac{\partial V}{\partial x} + i\,\frac{\partial V}{\partial y}\right)_{x_0y_0} &= -\,iV_0\,\frac{df(z_0^*)}{du} \\
\left(\frac{\partial^2 V}{\partial x^2} + i\,\frac{\partial^2 V}{\partial x\partial y}\right)_{x_0y_0} &= -\,iV_0\,\frac{d^2 f(z_0^*)}{du^2}
\end{aligned}\right|, \qquad (5)
$$

* J. E. Rosenthal, "Correction of deflection defocusing in cathode-ray tubes," Proc. I.R.E., vol. 49, pp. 10–15; January, 1951.

[1] Sylvania Electric Products Inc., Bayside. L. I., N. Y.

[2] J. Picht and J. Himpan, *Ann. der Phys.*, vol. 39, p. 409; 1941; vol. 43, p. 53; 1943.

[3] G. Wendt, *Die Telefunkenröhre*, vol. 15, p. 100; 1939; *Zeit. für Phys.*, vol. 118, p. 593; 1942.

[4] R. G. E. Hutter, "Electron beam deflection," *Jour. Appl. Phys.*, vol. 18, pt. I, pp. 740–758; August, 1947.

[5] R. G. E. Hutter, "Electron beam deflection," *Jour. Appl. Phys.*, vol. 18, pt. II, pp. 797–810; September, 1947.

equation (4) reduces to

$$(1 - 2r\delta)\frac{d^2}{du^2}(z_0 + \delta z_1 + \cdots)$$

$$= -i\delta\frac{df(z_0^*)}{du} - i\delta^2 z_1^*\frac{d^2f(z_0^*)}{du^2} + \cdots. \qquad (6)$$

Setting the coefficients of various powers of δ equal to zero gives the desired equations from which z_1 and z_2, and hence the first- and second-order paths, can be found by integration. As noted above, this gives x and y as functions of u or t. It should also be remarked that the dependent variables x and y have been expanded in powers of $\delta = V_0/2U$, and the order of any approximation is determined by the power of this quantity.

With a notation similar to that used above, the path-differential equation, given as (26) in reference 4, may be written

$$\frac{d}{dx}\left[y' - \frac{1}{2}\frac{E(x)}{U}yy' - \cdots\right]$$

$$-\left[\frac{1}{2}\frac{E(x)}{U} - \frac{1}{4}\frac{E^2(x)}{U^2}y - \cdots\right] = 0, \qquad (7)$$

where

$$E(x) = -\left(\frac{\partial V}{\partial y}\right)_{y=0} \qquad (8)$$

and the prime indicates differentiation with respect to x. In this case the variable y is not expanded in a power series. A first-order path is found by taking

$$\left|\frac{1}{2}\frac{E(x)}{U}y\right| \ll 1. \qquad (9)$$

Higher order paths are then found by substituting this path back into the terms which were neglected in the first solution and by integrating directly.

With the condition (9), the first-order solution is

$$\frac{d^2y}{dx^2} = -\frac{1}{2}\frac{E(x)}{U} \qquad (10)$$

or

$$y = y_0 - \frac{1}{2U}\int_0^x d\zeta\int_0^\zeta E(x)dx$$

$$= y_0 + \frac{1}{2U}\int_0^x d\zeta\int_0^\zeta\left(\frac{\partial V}{\partial y}\right)_{y=0}dx. \qquad (11)$$

The lower limit in the integration is taken as zero since the Rosenthal paper considers the fields only in the region of the deflection plates which have their entrance edge at $x=o$. The electron is also considered as entering the field with zero slope.

It can be shown from (23) of the Rosenthal paper, making use of the relation given in (5) above, that in her method

$$\frac{d^2y_1}{du^2} = \frac{1}{V_0}\left(\frac{\partial V}{\partial y}\right)_{x_0y_0}. \qquad (12)$$

Thus $y = y_0 + \delta y_1$, the first-order deflection, may be written

$$y = y_0 + \frac{1}{2U}\int_0^{x_0} d\zeta\int_0^\zeta\left(\frac{\partial V}{\partial y}\right)_{x_0y_0}dx_0. \qquad (13)$$

Since $x_0 = u = vt$, (13) is actually an integration in time; but, essentially, it represents an integration over a field-strength distribution along the axis $x = vt$, $y = \eta$. Thus the first-order path differs only slightly from that found in the path method where the field strength along the $y = o$ axis is integrated.

The second-order paths are more difficult to compare since, in this case, the time variable in the Rosenthal equations is not, in general, easily eliminated. It is instructive, perhaps, to apply the results of both methods to a simple deflection field, namely, a uniform deflection field in which edge effects are neglected. Since, in this case, there is no x-component of the field, $x = vt$ and y can easily be expressed as a function of x.

In the complex-plane analysis of this field, $f(z) = z/d$ where $2d$ is the distance between the plates. Equations (20), (26), and (28) in the Rosenthal paper then give

$$\begin{aligned} z_0 &= u + i\eta \\ z_1 &= -\frac{i}{2}\frac{u^2}{d} \\ z_2 &= -i\eta\frac{u^2}{d^2}; \end{aligned} \qquad (14)$$

and hence, within the field,

$$\begin{aligned} x &= vt \\ y &= \eta - \frac{1}{4}\frac{V_0}{U}\frac{x^2}{d} - \frac{1}{4}\frac{V_0^2}{U^2}\frac{x^2}{d^2}\eta. \end{aligned} \qquad (15)$$

This means that the method yields only one correction term to the first-order deflection which, as is well known, is given by the expression

$$\left(\eta - \frac{1}{4}\frac{V_0}{U}\frac{x^2}{d}\right).$$

Referring now to the equations in reference 4 and using the notation above, the axial field is given by $E(x) = V_0/d$, and hence the double integration in (11) gives the first-order solution

$$y = \eta - \frac{1}{4}\frac{V_0}{U}\frac{x^2}{d}. \qquad (16)$$

The two first-order solutions are the same in this case, of course, since the field is constant and equal along any axis. The deviation from the first-order path is given in the path method by

$$\Delta y = \beta_{0100\eta}. \qquad (17)$$

This is the only term left in (33) of reference 4 if the initial slope of the electron path is taken as zero and if terms involving the quantity $(V_0/2U)$ to a higher power than the square are neglected. This means that the results will be given to the same degree of approximation as in the previous case. The expression for β_{0100} is considerably simplified because of such approximations, and may be written

$$\beta_{0100} = \left[\frac{1}{2} - \left(-\frac{1}{2U}\int_0^x E(s)ds\right)^2 - \frac{1}{4U^2}E(x)\int_0^x d\zeta\int_0^\zeta E(s)ds + \frac{1}{2U^2}\int_0^x E^2(s)(s-x)ds\right], \quad (18)$$

s being simply an integration variable. Substituting for $E(x)$ gives

$$\Delta y = \frac{1}{4}\frac{V_0^2}{U^2}\frac{x^2}{d^2}\eta. \quad (19)$$

Thus it is seen that both methods yield the same results if terms of the same order are retained. It should be noted, however, that the second-order solution given by the path method includes many other correction terms which are neglected in the other treatment.

The discussion above has been limited to a comparison of the two methods in regions where fields are assumed to exist, and it is seen that comparable results are obtained. The analysis of deflection-defocusing effects given by Miss Rosenthal, however, involves many other assumptions which must be examined. The most important of these is that edge effects can be ignored. The electron is assumed to move in a straight path outside the region of the deflection plates and even though discontinuities occur in the potential field, the refractive effect of such discontinuities on the path is not taken into account by a suitable change in slope. The method can, of course, be extended to fields with edge effects if $f(z)$ is so determined that the field is zero at entrance and exit of the deflection systems. When this condition is not imposed on $f(z)$, further error is made by considering finite plates to be represented by a continuous curve. In such a case we do not believe that the field considered is a sufficiently close approximation to the actual field for practical purposes.

Further approximations are made in the assumption that large values of x, mean smaller spot distortions. This is a condition to be met only if the beam is to be kept parallel. Extension of results to the case of a point-focused beam, which is a truer counterpart of electron beams in cathode-ray tubes, may be dangerous. It might be pointed out that the quantity β_{0100} calculated above for the uniform field can be completely ignored in the case of a point-focused beam since the quantity which it multiplies in (33) of reference 4 is zero while other terms in Δy become important. The path method gives a quantitative description of deflection defocusing at the screen for any shape of the beam, and no further assumptions are necessary.

Miss Rosenthal's paper gives directly the shape of the deflection plates. If similar approximations were made in the path method, the expression for Δy would certainly be simplified. We believe that the condition $\Delta y=0$ with constant deflection could then theoretically be met by a proper choice of $E(x)$. Only one further step would be necessary to determine finite plate shapes which would give such an axial field distribution, namely, the use of an electrolytic tank. In such a process, edge effects would not be neglected and beams of any shape could be treated.

In conclusion, the authors wish to point out that the path method is applicable to much more general types of fields, such as crossed superimposed two-dimensional electric deflection fields, single three-dimensional electric fields, and crossed magnetic deflection fields.[6] To treat the problem of correcting deflection defocusing in cathode-ray tubes realistically, such fields as these must be considered.

J. E. Rosenthal:[7] The preceding note by Hutter and Harrison seems to call for a number of comments, which will be divided into two groups, the first one dealing with the mathematical methods and the second one with physical assumptions.

All the methods under discussion for finding the deflection of electrons by an electrostatic field represent merely different approximation techniques for solving the classical equations of motion. Thus for any preassigned potential distribution and for any desired degree of approximation the various methods, if correct, should give the same result. The value of any one of them lies in the ease with which it leads to the solution of any given problem. The chief claim put forward for the method presented in my article on the correction of deflection defocusing is that it gives a solution to any desired degree of approximation for *any potential*, no matter what its degree of complexity, satisfying Laplace's equation and the prescribed symmetrical boundary conditions. While my article gives results derived for a parallel electron beam, it is a simple matter to transform the calculations to the case of a converging beam. In my method the potential is *not* expanded. The Hutter and Harrison assertion to the contrary ignores the physical meaning of the quantities involved. In my treatment the space co-ordinates of the electron are the dependent variables to be determined as functions of time, which is the independent variable. The parameter δ, in terms of which I expand the *solution*, is the ratio of one

[6] R. G. E. Hutter, "The deflection of beams of charged particles," "Advances in Electronics," Academic Press, Inc., New York, N. Y., vol. I; 1948.

[7] 226 Bath Ave., Long Branch, N. J.

fourth the potential difference between the plates to the energy of the electron when it leaves the electron gun. It is obviously a fallacy to speak of the expansion of a given steady potential in terms of a time variable and of a variable parameter depending on an electron which will eventually be subjected to the potential in question.

One of the difficulties in solving electrostatic deflection problems is the abstruseness of the solution of Laplace's equation for condenser plates of finite length.[8] Until this solution is brought into a more easily understandable and manageable form (a problem which is being studied at present), we have to depend on approximations to the potential function. If the plates are long as compared to the spacing between them, the potential distribution due to infinitely long plates is usually assumed to hold, except near the edges of the plates. This is the origin of the expression "edge effects." No edge effects would appear in the exact solution for the potential distribution because of plates of finite length.

When it comes to physical assumptions concerning the nature of the potential distribution, their validity—or lack of it—could be argued abstractly *ad infinitum*. The only way to settle this type of argument is to compare with experiment the results obtained on the basis of any particular assumption.

If we apply this criterion to a number of statements made by Hutter and Harrison, we find that their assertions are not supported by either experimental evidence or mathematical proof. Thus they state, "It should be noted, however, that the second-order solution given by the path method includes many other correction terms which are neglected in the Rosenthal treatment." The comment applies specifically to results obtained for a parallel-plate deflection system. Since my mathematical method is rigorous, the additional correction terms can only arise from different assumptions as to the potential distribution. Equation (15) of the Hutter and Harrison note, which they obtain on the basis of my method, represents Deserno's results, which were verified experimentally. Thus the additional correction terms obtained by the path method can only destroy this agreement with experiment.

Another series of sweeping unsupported—and I believe untenable—statements proceeds as follows: "The method can, of course, be extended to fields with edge effects if $f(z)$ is so determined that the field is zero at entrance and exit of the deflection systems. When this condition is not imposed on $f(z)$, further error is made by considering finite plates to be represented by a continuous curve. In such a case we do not believe that the field considered is a sufficiently close approximation to the actual field for practical purposes. Further approximations are made in the assumption that large values of x_S mean smaller spot distortions. This is a condition to be met only if the beam is to be kept parallel. Extension of results to the case of a point-focused beam, which is a truer counterpart of electron beams in cathode-ray tubes, may be dangerous."

My work on the correction of deflection defocusing was started in order to find a method which would lead to an actual plate design since it appeared impossible to get any practical results from the path method as reported by Hutter.[9] Since, as stated above, the Deserno approximation gives experimentally verified results for the parallel-plate system, I took it as a reasonable basis for further work.

The validity of the assumptions made in my paper is proved by experimental data taken on the 7-inch electrostatic P4 tube (Navy contract NObsr 39149). These tests show that the tube whose deflection plates approximate one of my plate designs (lighter curve in Fig. 3 of my article) has smaller spot distortion than tubes whose deflection plates do not follow this pattern.

R. G. E. Hutter and S. W. Harrison:[1] In replying to Miss Rosenthal's comments on our discussion of her paper, we wish to point out once again that our main purpose was simply to compare the two different methods of analyzing deflection-defocusing effects. Since the path method was based on an expansion of the potential about the axis of symmetry, we tried to show that Miss Rosenthal's results could also be derived by expanding the potential about an axis. We believe that the expansion of $V(x, y)$ about the undeflected path is legitimate. The fact that x_0 is a function of time does not cause any difficulties in the equations of motion, if handled properly. Since the use of our expansion leads to the same results as those obtained by Miss Rosenthal, any "fallacy" in our approach must also exist in her derivation. This applies particularly to her expansion of $df(z^*)/dz^*$ in the neighborhood of $z^*=z_0^*$.

Once the parallel between the two methods had been established, an attempt was made to compare the results obtained. Since edge fields were dealt with in entirely different ways, this comparison was limited to deflection regions where similar assumptions could be made about the fields. As an example, it was shown that the two methods give identical results for a parallel beam *within* a uniform field if the *same* approximations are made. This result is contained in (15).

Miss Rosenthal states that this equation represents Deserno's results which were verified experimentally. This remark seems to call for some clarification. The Deserno reference, which was given in Miss Rosenthal's original paper, compares experimental measurements of spot distortion with values computed from a theoretical formula credited to Wallraff and subsequently published by him.[10] Wallraff, incidentally, states that this formula originated with Rogowski.

[8] A. E. H. Love, *Proc. London Math. Soc.*, vol. XXII, p. 337; 1924.

[9] See references 4, 5, and 6 of preceding note.

[10] A. Wallraff, "Verzerrungen am Kathodenstrahlfleck bei grösseren Ablenkungen," *Arch. für Electrotech.*, vol. 29, pp. 351–355; 1935.

Wallraff's derivation is for a point-focused beam in a uniform deflection field, but assumes that the slope of the undeflected path is small so that the x-velocity of the electron is determined by the potential at the point where the electron enters the deflection field. He further assumes that the difference between this potential and the accelerating potential is small compared with the accelerating potential. With these assumptions, the deviation from the first-order deflection is given by an expression equivalent to the last term in (15).

In our discussion, Δy was computed by the path method for a parallel beam in a uniform field. With a point-focused beam, the defocusing *within* the deflection field is given by

$$\Delta y = \beta_{0100} y_{iu} + \beta_{0001} y_{iu}'$$

if the slope of the path is small. The second assumption made by Wallraff can be made here if higher-order terms in $(V_0/2U)$ are neglected in the expressions for β_{0100} and β_{0001}. For an electron entering the deflection field at $y=\eta$, this again gives

$$\Delta y = -\frac{1}{4}\frac{V_0^2}{U^2}\frac{x^2}{d^2}\eta.$$

Additional correction terms given in the path method do not arise from making different assumptions regarding the potential distribution; rather, they arise from including higher-order terms in the analysis.

Returning to Deserno's paper, we find a statement that there was a discrepancy of 35 per cent between the measured results and those predicted by Wallraff's formula and that only by modifying the theoretical results was he able to obtain agreement.

J. E. Rosenthal:[7] As I see it, the crux of the whole discussion is whether a mathematical method does or does not lend itself readily to the solution of practical design problems. While the validity of electrostatic boundary conditions is in itself a highly important problem, Hutter and Harrison only becloud the issue by introducing arguments concerning it instead of answering my contention that Hutter's method is of little if any value in tube design.

Correction

W. T. Wintringham, author of the paper, "Color Television and Colorimetry," which appeared on pages 1135–1172 of the October, 1951 issue of the Proceedings of the I.R.E., has brought the following error to the attention of the editors:

The first entry in Table III should correspond to the quantity

$$K_L(1+4.5907+0.0601).$$

This sum is 5.6508 K_L rather than the figure 4.6508 K_L shown in the table.

This error reacts on two other numbers in Table III as well as on two numbers in the succeeding paragraph. When corrected, Table III would read

TABLE III

Luminances of Certain Colors

Equal-Energy White	5.6508 K_L	72,660 candles sq.m.
Ill. C (Fig. 1)	5.6305 K_L	72,400 "
Red Paper and Ill. C (Fig. 1)	0.8125 K_L	10,450 "

Similarly, when corrected, the next to the last sentence in the succeeding paragraph would read, "The value of K_L must therefore be 72,660/5.6508 or 12,860."

Contributors to Proceedings of the I.R.E.

Richard B. Adler (A'44) was born in New York City on May 9, 1922. He was a member of the Co-operative Course in Electrical Engineering, and received a B.S. degree in Electrical Engineering from the Massachusetts Institute of Technology in 1943.

RICHARD B. ADLER

During the years 1944–46, he was in the United States Naval Reserve assigned as an instructor at the M.I.T. Radar School. Following his discharge, he became a staff member and graduate student at the Research Laboratory of Electronics, M.I.T., and received the Sc.D. degree in Electrical Engineering in 1949.

Since that time he has remained at M.I.T. in both a teaching and research capacity. At present, Mr. Adler is Assistant Professor of Electrical Communication at M.I.T.

Mr. Adler is a member of Sigma Xi.

❖

Walter J. Albersheim (A'43–SM'48) was born in Cologne, Germany, on April 22, 1897. He received the degree of Master in electrical engineering from the Institute of Technology in Aachen, Germany, in 1922, and the doctorate degree from the same institution in 1924.

W. J. ALBERSHEIM

During his graduate studies in 1923, he was associated with Telefunken Radio Corporation in Berlin. In 1924 he immigrated to the United States, and since 1931 has been a citizen of this country.

Until 1929 Dr. Albersheim was engaged in the testing and designing of broadcast receivers, and from 1929 to 1941 had been with Electrical Research Products, Inc., doing research in sound recording and reproducing. In 1937 he became a licensed professional engineer in the state of New York.

From 1941 until the present, Dr. Albersheim has been engaged in radio research with Bell Telephone Laboratories. He is an associate of SMPTE, and a member of the New York Academy of Science. He has been granted more than forty U. S. patents for safety devices, flow meters, vacuum tubes, oscillators, sound recording, radio navigation, and radar systems.

F. E. Butcher was born on February 19, 1908, and received his training at the School of Engineering of his native town of Ipswich, England, and with Bull Motors Ltd. After training, and a period as junior designer, he continued with Bull Motors Ltd. as chief designer, from 1931 until 1935.

F. E. BUTCHER

From 1935 to 1943 Mr. Butcher was with Black and Decker Ltd. (England) as chief electrical engineer, and from 1943 to 1945 with Rotax Ltd. (London) as chief development engineer. After serving as chief engineer to Electro Methods Ltd. (London) from 1945 to 1951, he is now group designs manager at Joseph Lucas (Electrical Ltd.), Birmingham.

Mr. Butcher is a member of the American Institute of Electrical Engineers, Associate of the Institution of Electrical Engineers (London), Member of the Society of Instrument Technology, and Member of the Association of Supervising Electrical Engineers. He is serving on a number of Technical Committees, including the British Standards Institution, and Institution of Electrical Engineers.

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For a photograph and biography of RALPH I. COLE, see page 1568 of the December, 1951, issue of the PROCEEDINGS OF THE I.R.E.

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George A. Espersen (A'34–SM'51) was born in Jersey City, New Jersey, on May 17, 1906. He received the B.S. degree in physics in 1931 from New York University. From 1932 to 1939, Mr. Espersen was employed by Hygrade Sylvania Corporation as a development engineer engaged in the development and production of receiving type tubes. He was tube-development engineer for National Union Radio Corporation in 1939 and 1940.

G. A. ESPERSEN

From 1940 to 1942, Mr. Espersen was a project engineer engaged in research and development of klystron tubes at the Sperry Gyroscope Company.

From 1942 to 1945, he was a research engineer with North American Philips Company. Since 1945 he has been section chief of the Microwave Section of Philips Laboratories, Inc.

Mr. Espersen is a member of the American Physical Society.

Obed C. Haycock (A'40–SM'47) was born in Panguitch, Utah, on October 5, 1901. He received the degree of B.S. in Electrical Engineering from the University of Utah in 1925, and the M.S. degree in Electrical Engineering from Purdue University in 1931.

OBED C. HAYCOCK

After graduation, Professor Haycock joined the Westinghouse Electric and Manufacturing Company, and later returned to the University of Utah as Instructor in Electrical Engineering. He has remained more or less permanently with the University since that time, and was promoted to the rank of Professor in 1947. During the war he was given a leave of absence to do government research in Panama.

In addition to being Professor of Electrical Engineering, he is now Associate Director of "Physics of the Upper Atmosphere," an Air Corps sponsored program connected with White Sands Proving Ground.

Professor Haycock is a member of AIEE, Tau Beta Pi, Sigma Xi, and ASEE

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Charles F. Hobbs (A'41–M'45) was born in Williston, N. D., on February 3, 1909. He received the B.S. degree in Electrical Engineering from the University of North Dakota in 1930, and the Sc.M. degree in Electrical Engineering from the Massachusetts Institute of Technology in 1948.

CHARLES F. HOBBS

From 1930 to 1931, Mr. Hobbs was employed by General Electric Company as a student engineer. He held several positions in water conservation work with the Civilian Conservation Corps, the U. S. Bureau of Biological Survey, and the North Dakota State Planning Board, during the depression years. From 1937 to 1938, he was employed by Western Electric Company in the Engineer of Manufacture Department of their Hawthorne plant.

In 1938, Mr. Hobbs went into government service as a radio instructor in the Army Air Force Technical Schools, and in 1942, he became Chief Instructor of the Sioux Falls, S. D., unit. During 1944 and 1945, he was employed at Harvard University, first as a Training Associate in the employ of the University of California Division of War Research, and later, as a

Contributors to Proceedings of the I.R.E.

Special Research Associate in the Harvard Underwater Sound Laboratory and the Systems Research Laboratory. Since 1945, he has been Assistant Chief of the Communications Laboratory at the Air Force Cambridge Research Center.

Mr. Hobbs is a member of Sigma Tau, honorary engineering society.

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Frederick B. Llewellyn (A'23–F'38) was born September 16, 1897, in New Orleans, Louisiana. Between 1915 and 1922 he spent a total of three years as a radio operator with the United States Navy and on ships of the merchant marine. In 1922 he was graduated from Stevens Institute of Technology with the degree of Mechanical Engineer, and in 1928 received the degree of Doctor of Philosophy in Physics from Columbia University.

F. B. Llewellyn

Joining the engineering department of the Western Electric Company in 1923, he was transferred to the Bell Telephone Laboratories when that company was formed in 1925, and has remained with them ever since.

Dr. Llewellyn has been concerned with radio and circuit research which has extended to the analysis of the electronic behavior of vacuum tubes at high frequencies. More recently his attention has turned to systems engineering. He has attended a number of international radio conferences as a member of the U.S.A. delegation and has served as consultant in the Department of Defense. In 1951 he was Executive Secretary of the Science Advisory Committee in the Office of Defense Mobilization. Several papers by Dr. Llewellyn have appeared in the Proceedings of the I.R.E., and in 1935 he was awarded the Morris Liebmann prize for his outstanding original work on constant-frequency oscillators and on vacuum tube electronics at high frequencies. In 1946 he served as President of the Institute.

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For a photograph and biography of B. D. Loughlin, see page 1337 of the October, 1951 issue of the Proceedings of the I.R.E.

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Alan B. Macnee (S'42–A'45) was born on September 19, 1920, in New York, N. Y. He studied electrical engineering at the Massachusetts Institute of Technology, where he received the B.S. and M.S. degrees in 1943 and the D.Sc. degree in 1948. From 1943 to 1946 he was a staff member in the receiver group at the M.I.T. Radiation Laboratory, and specialized in the noise performance of intermediate-frequency amplifiers. From 1946 to August, 1949 he was engaged in research on high-speed electronic computation at the M.I.T. Research Laboratory of Electronics. He spent one year at the Chalmers Institute of Technology, Gothenburg, Sweden.

Alan B. Macnee

In 1950 Dr. Macnee joined the staff of the University of Michigan; he is now an associate professor of electrical engineering.

Dr. Macnee is a member of Sigma Xi and Eta Kappa Nu.

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T. G. Mihran (S'43–A'50) was born in Detroit, Mich., on June 28, 1924. He received the A.B. degree in electrical engineering from Stanford University in 1944. The following two years were spent in the Navy, where Dr. Mihran's radio-technician duties took him to Japan as a member of the occupation forces.

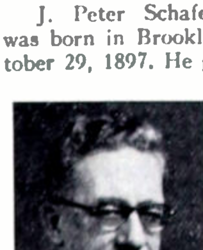
T. G. Mihran

Following his discharge, Dr. Mihran returned to Stanford University, where he received the M.S. degree in 1947 and the Ph.D. degree in electrical engineering with a physics minor in 1950. He is now in the Electron-Tube Section of the General Electric Research Laboratory.

Dr. Mihran is a member of Phi Beta Kappa and Tau Beta Pi, and is an associate member of Sigma Xi.

❖

J. Peter Schafer (A'24–M'30–SM'43) was born in Brooklyn, New York, on October 29, 1897. He graduated from Cooper Union with the B.S. degree in electrical engineering in 1921, and received a post graduate degree in electrical engineering in 1925.



J. Peter Schafer

Mr. Schafer has been associated with the research department of the Western Electric Company and with Bell Telephone Laboratories from 1915 to date. He has worked in many fields for radio research including long and short wave transoceanic telephone transmitter, studies of the ionosphere, special radar projects, and, since 1945, has been concerned with problems associated with FM multiplex microwave relay systems.

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Carl H. Smith, Jr. (A'42) was born in Morgantown, West Virginia, on July 31, 1913. He received the B.S. degree in electrical engineering in 1939 from the George Washington University.

Carl H. Smith, Jr.

Mr. Smith has been employed at the Naval Research Laboratory since 1940. Early in 1945, he was sent to Europe to participate in the Naval Technical Mission to Europe as a representative of the Naval Research Laboratory on guided missile electronics. From 1941 to 1944, he was also a part-time instructor in electrical engineering at George Washington University.

During 1946 and 1947, he took an active part in the upper atmosphere research program of the Rocket Sonde Branch of the laboratory. Since that time he has been associated with the Nucleonics Division in charge of a section devoted to research and development of electronic equipment for nuclear measurements. He participated in the 1948 and 1951 Atomic Weapons Tests.

Mr. Smith is a member of Sigma Tau, and an associate member of AIEE.

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Sidney T. Smith (S'38–A'44) was born on May 27, 1918, in Montezuma, Georgia. He received the B.S. degree in electrical engineering from the Georgia Institute of Technology in 1939, and the Doctor of Engineering degree from Yale University in 1942.

Sidney T. Smith

Since 1942 Dr. Smith has been employed by the Naval Research Laboratory in Washington, D. C. From 1942 to 1945, he did investigative analysis of radar receivers. Since 1945 he has been engaged in research on storage and special tubes.

Dr. Smith is chairman of the Subpanel on Special Tubes of the Research and Development Board.

Contributors to Proceedings of the I.R.E.

For a photograph and biography of PETER G. SULZER, see page 1570 of the December, 1951, issue of the PROCEEDINGS OF THE I.R.E.

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Richard V. Talbot was born on November 26, 1914, in New Plymouth, Idaho. He received the B.S. degree in electrical engineering from the University of Idaho in 1941.

RICHARD V. TALBOT

During 1941 and 1942, Mr. Talbot was employed by the Mountain States Telephone and Telegraph Company as a transmission engineer. Since 1942, he has been associated with the Naval Research Laboratory in Washington, D. C., at first as an electronic engineer doing research and development work on transmission lines and fittings for radar systems.

Since 1947, Mr. Talbot has been a member of the Nucleonics Division, where he has been engaged in the development of electronic measuring and recording equipment for nuclear physics. He participated in the 1948 and 1951 Atomic Weapons Tests.

Mr. Talbot is a member of Sigma Xi.

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Oswald G. Villard, Jr. (S'38–A'41–SM'51) was born at Dobbs Ferry, N. Y., on September 17, 1916. He received the B.A. degree in English literature from Yale University in 1938, and the E.E. and Ph.D. degrees from Stanford University, in 1943 and 1949, respectively.

Dr. Villard has been a member of the staff of the Department of Electrical Engineering at Stanford University since 1941, with the exception of the years 1942 to 1946, when he served as a special research associate at the Radio Research Laboratory, Harvard University.

OSWALD G. VILLARD

At present, Dr. Villard is an assistant professor engaged in undergraduate instruction, and research. He is a member of Sigma Xi and Phi Beta Kappa.

❖

Frank L. Wiley (S'49–A'51) was born in Salt Lake City, Utah, on May 19, 1927. He received the B.S. degree in Electrical Engineering in 1949, and the M.S. degree in 1950, both from the University of Utah. During the war, which interrupted his education, Mr. Wiley served as radar technician in the Navy, and was assigned to the Bikini experiments.

FRANK L. WILEY

After completing his Master's Degree, he joined the Electromechanical Department at North American Aviation, Inc., and is, at present, concerned with the subminiaturization of electronic circuits.

Mr. Wiley is a member of Phi Kappa Phi, Sigma Xi, and Tau Beta Pi.

❖

R. Willheim was born in Austria on November 21, 1892. He is a graduate of the Vienna University of Technology, Department of Electrical Engineering (1919), and received the degree of Doctor of Technical Science from the same University in 1921.

R. WILLHEIM

Dr. Willheim joined the staff of the Allgemeine Elektricitaets Gesellschaft (A.E.G.) in 1920. He was promoted to head of the department for Central Stations and System Protection of the Vienna establishment of the firm, and was called to the main establishment in Berlin in 1929 to take charge of research and development of system protection. In 1931 he became chief electrical engineer of the A.E.G. works manufacturing transformers and high-tension switchgear.

In 1938 Dr. Willheim moved to England, where he became a consultant on switchgear and transformer problems for A. Reyrolle and Company and C. A. Parsons and Company. Since World War II he has been engaged on work in the field of magnetic amplifiers in association with Electro Methods Ltd., London, taking charge of their development. He is now working in London as an independent consulting engineer in the fields of power transformers, saturable reactors, and magnetic amplifiers.

Dr. Willheim is a Member of the American Institute of Electrical Engineers and an Associate Member of the Institution of Electrical Engineers in London.

Correspondence

The Equivalent Circuit of the Transistor*

Published papers[1] on the equivalent circuit of the transistor have indicated that this device can be treated as a passive T-network, provided a positive emf is introduced in the collector branch, numerically equal to $r_m i_e$, where i_e designates the emitter current flowing toward the transistor.

With this convention, the resistance within the collector branch, r_c, is nearly equal to r_m, and the difference $r_c - r_m$, which has greater physical significance than r_c, does not appear explicitly in the equivalent circuit. This must be contrasted with the circumstance that r_c need never appear without $-r_m$ in the discussion of the circuits with grounded base or grounded emitter. It must be noted also that the analogy between the roles played by the grid of the thermionic valve and the base of the transistor makes it appear desirable to express the emf induced in the collector branch in terms of the base current.

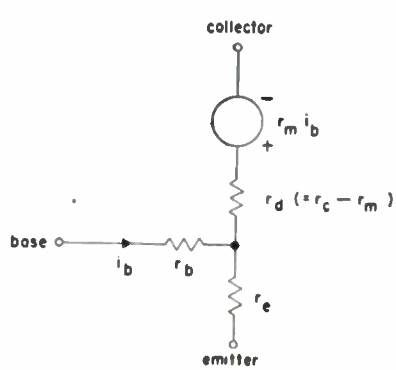

Fig. 1—Equivalent circuit of the transistor.

Fig. 1 illustrates an alternate which is suggested here for the equivalent circuit of the transistor. This circuit is functionally identical to the circuits suggested previously. It expresses the emf induced within the collector branch as $-r_m i_b$, where i_b designates the base current and permits the resistance of this branch to be written as r_d (d for difference) $= r_c - r_m$. With this convention, the important factor of merit, $\alpha/1-\alpha$ (Wallace and Pietenpol, *loc. cit.*), can be expressed simply as $r_m + r_b/r_d$.

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Signal Corps Engineering Laboratories
Fort Monmouth, N. J.

* Received by the Institute, October 18, 1951.

[1] R. M. Ryder and R. J. Kircher, "Some circuit aspects of the transistor," *Bell Sys. Tech. Jour.*, vol. 28, p. 367; 1949.
R. L. Wallace and W. J. Pietenpol, "Some circuit properties and applications of n-p-n transistors," PROC. I.R.E., vol. 39, p. 753; 1951.

Resistance Matching and Attenuating Networks*

The object of this note is to show that the m-derivation of a simple prototype network will yield the equations for resistance attenuating and matching networks. The

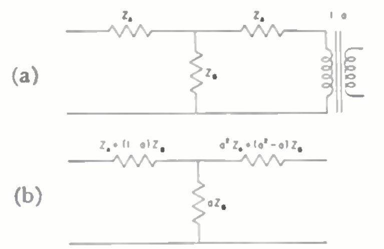

Fig. 1—Equivalent unsymmetrical networks.

result is a generalization of a corresponding result for symmetrical attenuating networks given in a previous note.[1]

The m-derivation of an unsymmetrical T-network can be obtained by considering the unsymmetrical network to be equivalent to a symmetrical T followed by an ideal transformer. The two networks of Fig. 1 are easily shown to be equivalent. An m-derivation of the symmetrical T (Fig. 1 (a)) now yields a second pair of equivalent networks, shown in Fig. 2, having the same image impedances but propagation characteristics different from the original set.

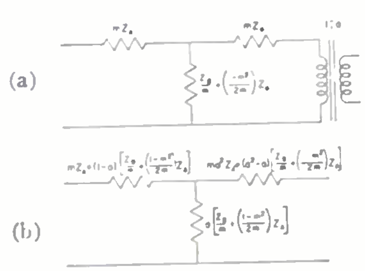

Fig. 2—Networks having the same image impedances as the networks of Fig. 1.

The network of Fig. 1 with $Z_B=0$, $Z_A=Z_{in}$, and $a^2Z_A=Z_{out}$, so that $a=\sqrt{Z_{out}/Z_{in}}$ is taken as the prototype network (the terminating impedances Z_{in} and Z_{out} are assumed to be resistive). The resulting network obviously has infinite attenuation. The m-derived matching network, obtained from the prototype, is shown in Fig. 3; its attenuation is:

$$\text{attenuation} = 20 \log \left(\frac{1+m}{1-m}\right) \text{decibels.}$$

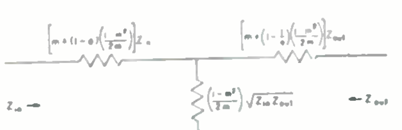

Fig. 3—General resistance matching and attenuating pad.

(Since the attenuation is independent of the transformation ratio, a, it is convenient to let $a=1$ to find the loss.)

The network required to match two impedances with a minimum of loss is often desired. If the output impedance is considered to be higher than the input ($a>1$), the minimum loss will be obtained

Fig. 4—Minimum loss matching pad.

for that value of m, making the input-series arm zero; this is $m=\sqrt{a-1/a+1}$ (for minimum loss). The resulting network is shown in Fig. 4.

Sidney Bertram
The Rand Corporation
Santa Monica, Calif.

* Received by the Institute, January 7, 1952.

[1] S. Bertram, "Resistance attenuating networks," Proc. I.R.E., vol. 37, p. 1446; December, 1949.

An U-H-F Moon Relay*

On last October 28 and November 8, we successfully relayed a continuous-wave 418-mc radio signal from Cedar Rapids, Iowa to Sterling, Va. by using the moon as a reflector. We believe that this is the first successful moon-relay transmission at ultra-high frequency and the first complete message to be received by moon reflection.

On both dates, the signal was received at Sterling over a half-hour period. During the November 8 experiment, the transmitted signal was interrupted periodically on a prearranged time schedule, and a transmission delay time of about 2.5 seconds was measured aurally at the receiver. This measurement checks with the theoretical delay. The direction of arrival of the received signal, found by rotating the receiving antenna in azimuth and elevation, coincided closely with the calculated position of the moon. On November 8, a slowly hand-keyed telegraph message consisting of the words "What hath God wrought?" was sent over the circuit. The message was repeated several times. It was possible to copy all of the message by ear from a tape recording made at the receiver during the message interval.

The transmitting equipment was made up of an experimental resnatron amplifier driven through a multiplier chain from a crystal oscillator. This amplifier was connected by means of a special waveguide to a pyramidal horn of optimum design about 70 feet long and with an aperture of 20 by 24 feet. Except for an aluminum section about 10 feet long near the throat, this horn consisted of "chicken wire" with 1-inch hexagonal mesh supported from telephone poles. The calculated gain was not realized, probably because of leakage through the walls resulting from poor conductivity. The axis of the horn was tilted about 7 degrees above the horizontal to avoid earth reflection and to reduce atmospheric refraction. Polarization was vertical. The signal was modulated by shifting the frequency about 15 kc.

The transmitting antenna was fixed in direction, and the experiment could be made only at those times, occurring about twice monthly, when the moon passed through the transmitting beam.

The receiving equipment consisted of a low-noise receiver that was fed by a half-wave dipole and plane reflector mounted at the focus of a 31-foot diameter paraboloid. The position of the receiving antenna was variable in azimuth and elevation so the moon could be tracked during the experiment.

The estimated performance of the system is specified in the following parameters:

Frequency: 418.0 mc
Transmitter output power: 20 kw
Transmitting antenna gain referred to an isotropic radiator: 21 db
Receiving antenna gain referred to an isotropic radiator: 30 db
Receiver noise figure: 4 db
Receiver bandwidth: 1 kc

Substituting these values in equations (5) and (8)[1] we obtained a theoretical received power of 7.25×10^{-17} watts. With a receiver noise figure of 4 db and bandwidth of 1 kc, this received power corresponds to a signal-to-noise ratio of 8.6 db. Our actual received signal was in good agreement with this figure, although we found, in common with earlier experimenters,[2,3] that the signal was subject to severe fading. In our case, the signal varied from the noise level to occasional peaks as high as 10 db above the noise. Fig. 1 shows the signal-plus-noise level.

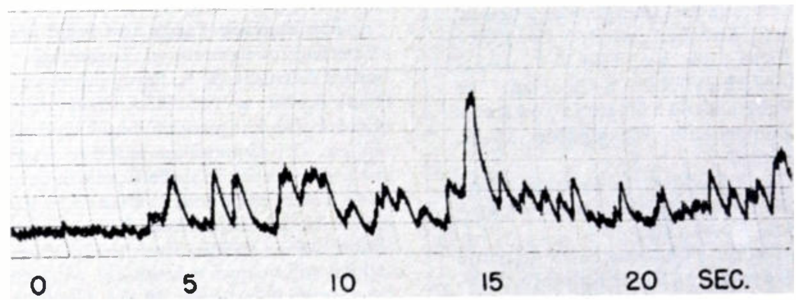

Fig. 1—Pen recording of an unmodulated, moon-reflected signal. The transmitter was switched on at 4 seconds. The peak at 15 seconds corresponds to a signal-to-noise ratio of approximately 10 decibels.

Although the received signal-to-noise ratio was too low for us to examine multiple-reflection effects on the telegraph modulation, we believe an increase of 10 to 20 db in the system gain would probably result in a satisfactory hand-keyed telegraph circuit.

Peter G. Sulzer
G. Franklin Montgomery
National Bureau of Standards
Washington, D. C.
Irvin H. Gerks
Collins Radio Co.
Cedar Rapids, Iowa

* Received by the Institute, January 3, 1952.

[1] D. D. Grieg, S. Metzger, and R. Waer, "Considerations of moon-relay communication," Proc. I.R.E., 36, pp. 652–663; May, 1948.

[2] J. H. DeWitt, Jr., and E. K. Stodola, "Detection of radio signals reflected from the moon," Proc. I.R.E., 37, pp. 229–242; March, 1949.

[3] F. J. Kerr and C. A. Shain, "Moon echoes and transmission through the ionosphere," Proc. I.R.E., 39, pp. 230–242; March, 1951.

Institute News and Radio Notes

Calendar of COMING EVENTS

IRE Connecticut Valley Section, Electronics Industry Day, Storrs, Conn., April 5

Annual British Radio Component Show, Grosvenor House, London, England, April 7–9

Modern Network Synthesis Symposium, Engineering Societies Building, New York, N. Y., April 16–18

Radio and Television Show, Manchester, England, April 23–May 3

IRE Cincinnati Section, Spring Technical Conference, Cincinnati Engineering Societies Building, Cincinnati, Ohio, April 19

URSI-IRE Spring Meeting, National Bureau of Standards, Washington, D. C., April 21–24

IEE Television Convention, London, England, April 28–May 3

Electronic Computer Symposium, University of California, Los Angeles, Calif., April 30–May 1

Association for Computing Machinery Meeting, Mellon Institute, Pittsburgh, Pa., May 2–3

IRE-AIEE-RTMA Symposium on Progress in Quality Electronic Components, Washington, D. C., May 5–7

IRE New England Radio Engineering Meeting, Copley-Plaza Hotel, Boston, Mass., May 10

IRE National Conference on Airborne Electronics, Hotel Biltmore, Dayton, Ohio, May 12–14

The Society for Experimental Stress Analysis Meeting, Indianapolis, Ind., May 14–16

4th Southwestern IRE Conference and Radio Engineering Show, Rice Hotel, Houston, Tex., May 16–17

Radio Parts and Electronic Equipment Show, Conrad Hilton Hotel, Chicago, Ill., May 19–22

1952 IRE Western Convention, Municipal Auditorium, Long Beach, Calif., August 27–29

National Electronics Conference, Chicago, Ill., September 29–October 1

IRE-RTMA Radio Fall Meeting, Syracuse, N. Y., October 27–29

Technical Committee Notes

The **Standards** Committee convened on January 10, 1952, under the Chairmanship of A. G. Jensen. J. R. Ragazzini, W. A. Lynch, and W. M. Pease were nominated to represent the IRE on the ASA Committee Y10.14, Nomenclature for Feedback Control Systems. The Chairman proposed that J. G. Brainerd be appointed as Chairman of the Subcommittee on Basic terms. A. F. Pomeroy reported on liaison with the Munitions Board Standards Agency, pointing out that co-ordination between the Armed Services and industry could now be accomplished more efficiently through this agency. He also gave a brief account of his trip to Europe, where he attended the IEC Conference on Graphical Symbols. The Radiation Counter Tube Definitions and Methods of Test submitted by the **Electron Devices** Committee were given final approval by the Standards Committee. The Committee then turned its attention to a consideration of the Proposed Standards on Receivers: Definitions of Terms, as revised by the Task Group on Receiver Definitions. Further discussion on the Receiver material will continue at the next meeting of the Standards Committee.

The **Antennas and Waveguides** Committee, Chairman A. G. Fox presiding, met on December 12, 1951, and January 15. In conjunction with these two meetings, work has now been completed on "Definitions of Waveguide Terms."

On December 7, 1951, the **Circuits** Committee met under the Chairmanship of C. H. Page. There was continued discussion of tentative definitions submitted by Subcommittee 4.7.

The **Electron Tubes and Solid State Devices** Committee met on November 20, 1952, with Chairman A. L. Samuel presiding. The final report of the 1951 Electron Devices Committee was presented by Chairman H. J. Reich. The Committee voted to separate its 1952 conference into the Conference on Electron Tubes to be held at Ottawa, Canada, on June 16–17, 1952, and the Conference on Solid State Devices, the location and date of which will be decided soon. R. M. Ryder was appointed Chairman of the Conference on Solid State Devices, and W. J. Dodds as Chairman of the Conference on Electron Tubes. The Noise Definitions were given preliminary approval, and a preliminary discussion was held of the Noise Measurements, as revised by H. J. Reich. Final approval of these standards was deferred to give Task Group Chairman Ryder an opportunity to consider the suggested revisions.

The **Sound Recording and Reproducing** Committee convened on December 19, 1951, under the Chairmanship of H. E. Roys. The Committee reviewed the tentative standards on sound recording and reproducing, "Methods of Measurement of Noise in Sound Recording and Reproducing Systems, 1951." Some suggested changes were incorporated and the Committee recommended that the material be sent to the Standards Committee. Lincoln Thompson reported on the activities of his Subcommittee on Mechanical Calibration. Material on record calibration is expected soon. The scope of the Sound Recording and Reproducing Committee, as listed in the compilation of July, 1949, was reviewed and approved.

Under the Chairmanship of W. M. Pease, the **Servo-Systems** Committee convened on November 13, 1951, and decided to change its name to **Feedback Systems** Committee. Action will be taken by the Standards Committee. A further study was made on the scope of the Committee. It was agreed to form a Subcommittee on Terminology. At the Committee meeting on November 27, 1951, it was reported that work had started on compiling a bibliography on feedback control systems. The Committee's relation to the Computer Committee on the business of definitions was discussed.

The **Video Techniques** Committee convened on January 8, 1952, under the Chairmanship of W. J. Poch. Reports were submitted by J. L. Jones, Chairman of Subcommittee 23.4, R. L. Garman, Chairman of Subcommittee 23.2, and A. J. Baracket, Chairman of Subcommittee 23.3. Chairman Poch reported that R. H. Daugherty had indicated that he would be unable to continue as Chairman of the Definitions Subcommittee, 23.1. J. F. Wiggin will be asked to accept the Chairmanship of this Committee.

The Proceedings of the **Joint Technical Advisory** Committee, Volume VII, (Section I, Official Correspondence Between the Federal Communications Commission and the Joint Technical Advisory Committee with other Items of Correspondence Pertinent to the Actions of the JTAC, July 1, 1950 to June 30, 1951; Section II, Approved Minutes of Meetings of the Joint Technical Advisory Committee, July 1, 1950 to June 30, 1951, is now available at the IRE Headquarters for the price of $5.00. A Subcommittee on Land Mobile Allocations under the Chairmanship of F. T. Budelman has been organized by the JTAC. The membership is comprised of industrial representatives who will act as consultants to the JTAC in the report to the FCC, in reply to its request for information on the subject of channel allocations for land mobile communication services. A paper on "The Conservation of the Radio Spectrum," which has been in preparation by the JTAC, is expected to be available for distribution this spring.

The National Association of Radio and Television Broadcasters has accepted an invitation to join the IRE, the RTMA, and the Society of Motion Picture and Television Engineers, on the Steering Committee which will hence forth be known as the Joint Committee for Inter-Society Co-ordination (JCIC). This Committee was formerly known as the IRS Steering Committee, set up to avoid duplicated efforts among the Technical Committees of IRE, RTMA, and SMPTE, working in fields associated with television. The NARTB has recently renewed its standards activity on slides and opaques for use in television. At its recent meeting the JCIC reviewed the general problem of national standards on sound recording being set up by the State Department's participation in CCIR's (International Radio Consultative Committee) international recording standardization program. The need for American Standards on sound recording was discussed, and it was voted to reactivate ASA Sectional Committee Z57 on sound recording. Neal McNaughten was asked to assume the acting Chairmanship of this committee.

Electronics Centennial Program in Progress

Plans are now underway for the convocation of national and international engineering societies to be held in Chicago, September, 1952, as part of the celebration of the Centennial of Engineering.

Meeting rooms of six of Chicago's downtown hotels have been reserved for the sessions, with additional space available for meetings scheduled at the Museum of Science and Industry. Individual symposia and technical meetings on problems encountered in the various phases of engineering will be conducted. Over a hundred nationally known speakers will be present, in addition to prominent laymen who will participate to discuss what the general public expects from the engineering profession in the advance of American social and economic standards.

Notice to Members

IRE membership dues may be deducted as a business expense on Federal income tax returns.

Network Symposium Scheduled for April

A Modern Network Synthesis Symposium (Audio to Microwaves) is to be held on April 16–18, 1952, at the Engineering Societies Building Auditorium, 33 W. 39 Street, New York, N. Y. The symposium, jointly sponsored by the Polytechnic Institute of Brooklyn and the Office of Naval Research, will summarize the progress to date in the various fields of network synthesis, highlighting new developments of current interest to engineers and physicists. American and European authorities, who have made original contributions to the art, will participate.

Copies of the detailed program, hotel accommodation information, and registration forms are available on request. No registration fee is required. All correspondence should be addressed to: Polytechnic Institute of Brooklyn, Microwave Research Institute, 55 Johnson Street, Brooklyn 1, N. Y. A proceedings of the symposium will be published by September, 1952, at a cost of $4.00 per copy. Orders for the proceedings accompanied by check or money order made out to, "Treasurer Network Symposium," will be accepted in advance. Orders should be sent to the Brooklyn address.

Notice! Members

The Professional Group on Electronic Computers is now conducting its membership campaign and application blanks are available from IRE Headquarters. Interested IRE members are requested to send a card to Headquarters. At an Administrative Committee meeting in December, 1951, a constitution was approved and the following officers were elected: Chairman, M. M. Astrahan; Vice-Chairman, H. D. Huskey; Secretary-Treasurer, J. H. Howard. A $2.00 assessment has been voted for publication and mailing expenses.

Professional Group News

Executive Committee Approval

At the January 8, 1952, meeting of the IRE Executive Committee, changes were approved in the Constitutions of the Professional Groups on Airborne Electronics and Broadcast and Television Receivers. The Committee approved an increase from 12 to 15 members in the Administrative Committees of these groups. The Executive Committee also decided that Professional Groups could publish "Transactions" in any form they desire, as long as advertising is excluded. Action is being taken by the Professional Groups Committee to standardize the "Transactions."

Antennas and Propagation

Once again, the USA National Committee of the International Scientific Radio Union (URSI) is sponsoring a spring Technical Meeting jointly with the IRE Professional Group on Antennas and Propagation. The meeting will be held on April 21–24, 1952, at the National Bureau of Standards, Washington, D. C.

Audio

The November, 1951, issue of the bimonthly Newsletter, of the Professional Group on Audio, is available for sale at the IRE Headquarters, for $.36. Included in this issue are two technical editorials entitled, "Magnetic Recording in 1970," by Marvin Camras of the Armour Research Foundation, and, "The Imitation of Natural Sounds," by D. W. Martin, of the Baldwin Company. Also appearing in this issue is an article by R. E. Troxel, of Shure Brothers, Inc., describing the formation of a PGA chapter in the Chicago Section, and an article by A. B. Jacobsen, of the University of Washington, reporting on the use of recorded papers by his Section. A first hand report of the 1951 IRE Western Convention is given by Vincent Salmon of the Stanford Research Foundation.

Circuit Theory

A Committee on Special Problems in the Field, set up by the Professional Group on Circuit Theory, has proposed a project for members of the Group whereby they would like to introduce discussion via correspondence, on novel network problems of interest to the Group. In this manner, they hope to stimulate new ideas and thinking. Those interested in this project are invited to contact the Chairman of the Group, J. G. Brainerd, Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia 4, Pa.

Electronic Computers

On April 30 to May 1, 1952, the Los Angeles Chapter of the IRE Professional Group on Electronic Computers is sponsoring an Electronic Computer Symposium on the campus of the University of California, Los Angeles, Calif.

Industrial Electronics

The Professional Group on Industrial Electronics is planning a joint IRE-AIEE Components Symposium to be held May 22–23, 1952 with the theme, "Electronics and Machines."

Professional Groups Committee

The membership list of the 1952 IRE Committee on Professional Groups is as follows: W. R. G. Baker, Chairman; Austin Bailey, Vice Chairman; W. L. Everitt, R. F. Guy, R. A. Heising, Harner Selvidge, J. R. Steen, L. C. Van Atta, J. J. Fiske, B. B. Bauer, and J. K. Hilliard.

Transactions of IRE Professional Groups

The following issues of Transactions have been published by the IRE Professional Groups and are available from the Institute of Radio Engineers, Inc., 1 East 79 Street, New York 21, N. Y., at the prices listed below.

As additional issues are published, a notice of their availability and cost will appear in these pages.

Sponsoring Group	Publication	Members of Sponsoring Group	IRE Members	Non-member
Audio PGA-1	"An Electronic Music Box," by E. L. Kent	$0.15	$0.23	$0.45
Audio PGA-2	"Horn Loaded Loudspeakers" by D. J. Plach and P. B. Williams	0.18	0.27	0.54
Audio PGA-3	"A Selective Automatic Phonograph Mechanism," by J. C. Kiefer and A. G. Bodoh	0.29	0.44	0.87
Audio PGA-4	"Wear of Phonograph Needles," by B. B. Bauer	0.23	0.35	0.69
Airborne Electronics PGAE-1	"Systems Considerations in Aircraft Antenna Design" by J. V. N. Granger	0.39	0.59	1.17
Airborne Electronics PGAE-2	"The Electrical Characteristics of Shaker Systems Used for Vibration Testing of Equipment," by R. C. Lewis and D. J. Fritch	0.41	0.71	1.41

Institute News and Radio Notes

Section Rebates Increased; IRE Directors Appointed For 1952 Term

The annual meeting of the 1952 IRE Board of Directors, on January 9, 1952, conducted by the new IRE President, D. B. Sinclair, was attended by twenty Directors.

An action taken by the Board which is of particular interest to Section officers and members was the adopting of an amendment to Bylaws Section 70. It was approved that rebates to Sections will be increased from $1.00 to $1.10 for each member, except Students, up to and including seven hundred, and from $1.15 to $1.25 for each member, except Students, over seven hundred.

The meeting also included the appointments of officers for the 1952 term, with the reappointment of Treasurer W. R. G. Baker, Secretary Haraden Pratt, and Editor A. N. Goldsmith. S. L. Bailey, K. C. Black, and A. V. Loughren were selected as Appointed Directors for the year, and Directors I. S. Coggeshall, R. F. Guy, and J. W. McRae were appointed to the 1952 Executive Committee. The other four members of the Executive Committee are the Officers: President Sinclair, Treasurer Baker, Secretary Pratt, and Editor Goldsmith.

The 1952 IRE Committee Chairmen were also selected at the meeting with the following appointments: Administrative Committee of the Board of Editors, A. N. Goldsmith; Admissions, H. P. Corwith; Awards, K. C. Black; Board of Editors, A. N. Goldsmith; Constitution and Laws, R. A. Heising; Education, J. D. Ryder; Membership, R. M. Krueger; Nominations, W. L. Everitt; Papers Review, G. F. Metcalf; Policy Advisory, W. R. Hewlett; Professional Groups, W. R. G. Baker; Public Relations, Lewis Winner; Sections, E. T. Sherwood; and Tellers, J. Z. Millar.

Fellow Grade Nominations

Each year, the IRE Board of Directors confers the Grade of Fellow upon a limited number of nominees, based upon recommendations by the IRE Awards Committee, which selects nominees from candidates submitted by Sections or by individual members.

The Awards Committee requests that recommendations of candidates for Fellow Grade be submitted on Form AC-1. Any member wishing to recommend candidates may obtain AC-1 forms from the IRE Headquarters office, and send them, properly completed, to his Section Chairman for transmittal to Headquarters.

Recommendations to be considered by the Awards Committee for 1953 Fellow Grade nominations must be received at Headquarters prior to April 30, 1952.

Airborne Cooling Equipment Conference Slated

The first Conference on Cooling of Airborne Electronic Equipment will be held March 20–21, 1952, at The Ohio State University, Columbus, Ohio, in co-operation with the United States Air Force. Technical papers will be contributed by the Air Force, the electronics industry, the aircraft industry, and research organizations. Subjects, such as design of airborne cooling systems, application of blowers, component cooling data, and cooling problems in miniaturized construction, will be covered.

Particulars of the conference will be supplied to interested organizations upon request. Please address inquiries to: Walter Robinson, The Ohio State University Research Foundation, Columbus 10, Ohio.

Electronics Fellowships Offered

A number of graduate and advanced research fellowships are offered by the Massachusetts Institute of Technology for study and research in the field of electronics. They are known as Industrial Fellowships in Electronics and are sponsored jointly by a group of industrial organizations concerned with the advancement of electronics and its applications.

Those interested in further information regarding application and stipulations of the fellowships should communicate with the Director, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass. Application should be made at least four months prior to the intended date of entrance.

Airborne Electronics Conference Set for May in Dayton

The 1952 National Conference on Airborne Electronics to be held May 12–15, Hotel Biltmore, Dayton, Ohio, will be the fourth annual conference of its kind. Jointly sponsored by the Dayton IRE Section and the IRE Professional Group on Airborne Electronics, the keynote of the conference will be, "Electronics—Key to Air Supremacy." Added emphasis will be on "reliability," with several outstanding sessions on servicing, maintenance, and reliability of airborne electronic equipment.

Abstracts of papers presented at the various sessions will be distributed, and the Airborne Electronics Group will publish complete notes of several outstanding papers at a later date. This year there will be larger space available for more exhibits.

The social program, which will feature luncheons and dances, will be highlighted by the annual banquet, at which time one of the outstanding pioneers in the field of airborne electronics will be honored.

IRE People

Saul Decker (A'45) has been appointed as chief television engineer of CBS-Columbia Incorporated, Brooklyn, N. Y. In this capacity he will be responsible for the design and development of all television and radio chassis, including circuitry and component specifications.

Mr. Decker attended the Polytechnic Institute of Brooklyn, and during World War II, he served with the Signal Corps in the Joint Communication Command. Prior to joining CBS-Columbia, he held the position of assistant chief engineer at Garod Radio Corporation, and has been employed in a consultant capacity by several noted electronic companies.

Harald T. Friis (A'18–M'26–F'29) has been appointed Director of Research in high frequency and electronics, for the Bell Telephone Laboratories. He has been Director of Radio Research for Bell Laboratories since 1946.

Harald T. Friis

Dr. Friis, who is a native of Denmark, received the electrical engineering degree from the Royal Technical College in Copenhagen, in 1916, and the Ph.D. degree in 1938. During 1916 he served as an assistant to Professor P. O. Pedersen, and in 1917–1918, he was a technical advisor at the Royal Gun Factory in Copenhagen. In 1919, Dr. Friis was made a Fellow of the American Scandinavian Foundation and did graduate work at Columbia University. He then joined the Western Electric Company, which later became a part of the Bell Laboratories. Since 1930, he has been in charge of short wave research studies, with Bell Telephone, and, more recently, transmission studies of waves in the centimeter range.

Dr. Friis was the recipient of the IRE Morris Liebmann Memorial Prize, in 1939, and served as a Director of the Institute in 1941–1944.

Sir Robert A. Watson-Watt (SM'46–F'47), the man who gave Great Britain's Air Force its magic eye, has been presented £50,000 by the Royal Commission on Awards to Inventors for his "initiation of radar and his contribution to the development of radar installations," before and during World War II. This is the largest sum ever awarded by the Commission to an individual.

Sir Robert Watson-Watt

Sir Robert was born in Scotland on April 13, 1892. He graduated, with special distinction in electrical engineering, from the University College, Dundee, at the University of St. Andrews, in 1912, and immediately became assistant to the professor of natural philosophy in the University College, Dundee. He was later appointed to the British Weather Service, and in 1917, he became meteorologist-in-charge of the Branch Meteorological Office at the Royal Aircraft Establishment. In 1934, when the work of the Radio Research Station was merged with radio program of the National Physical Laboratory, Sir Robert was appointed the first superintendent of the Radio Department, National Physical Laboratory. He is considered England's foremost radar authority, and has held many notable government positions in the capacity of scientific advisor and consultant.

His honors include his knighthood, conferred in 1942, and the Companion of the Order of the Bath, awarded in 1941, Fellowship of the Royal Society, the U. S. Medal for Merit, the Valdemar Poulsen Gold Medal of the Danish Academy of the Technical Sciences, and the Hughes Medal of the Royal Society for his pioneer researches in radio-telegraphy.

Sir Robert was the 1950 IRE Vice President, and also served as an IRE Director at that time. He is the president of the Royal Meteorological Society and of the Institute of Navigation.

❖

W. H. Doherty (A'29–M'36–SM'43–F'44), who has been the director of electronics and television research at the Bell Telephone Laboratories, Inc., has recently been appointed director of research in electrical communications.

W. H. Doherty

Mr. Doherty was born in Cambridge, Mass., on August 21, 1907. He received the B.S. and M.S. degrees in electrical engineering, at Harvard University, in 1927 and 1928, respectively. He later received an honorary doctorate degree from the Catholic University. In 1929, he joined the technical staff of the Bell Telephone Laboratories, Inc., at Whippany, N. J., and served in the development of high-power radio transmitters for transoceanic service and broadcasting. Later he participated in pioneering work in the fire-control radar field, and throughout World War II, he supervised a radar development group which was responsible for the design of a number of radars widely used on naval surface ships and submarines for fire and torpedo control. He continued in military electronics work until 1949.

Mr. Doherty is a member of Tau Beta Pi and Sigma Xi, and received the IRE Morris Liebmann Memorial Prize in 1937.

❖

Frank E. LaFetra (S'48) has been appointed as sales engineer of the Palo Alto office of the Carl A. Stone Associates. Mr. LaFetra, a native of California, attended Stanford University in 1947–1951, where he graduated with a B.S. degree in electrical engineering.

❖

Louis G. Caldwell (M'29–SM'43), one of the nation's outstanding authorities in the communications-radio legal field and in administrative law, died recently at his home in Washington, D. C. He was 60 years of age.

Mr. Caldwell was a leader in communications and radio broadcasting law in American Bar Association activities, and was chosen as chairman of the American Bar Association's first committee on radio law in 1928 and 1929, and chairman of the committee on communications from 1929–1933. He was also a pioneer in the creation of communications organizations such as Press Wireless for newspapers and press associations and Aeronautical Radio for airlines. He was the representative for leading groups of international telegraph users in a number of major proceedings before the FCC. A leading figure in the creation of the Federal Communications Bar Association in 1936, he served as that body's first president and on the FCBA's executive committee from 1937–1940. He was the author of many articles on radio and was the editor of the *Journal of Radio Law* in 1931–1932.

Mr. Caldwell received his B.A. degree from Amherst College, and the M.A. and LL.B. degrees from Northwestern University in Evanston, Ill. He lectured there in 1916, and 1919–1926. He also taught law at the John Marshall Law School from 1916–1919. He was the Washington resident partner of his law firm from 1930 to the time of his death.

Irving Wolff (A'27–VA'39–F'42) has been appointed Director of Research of the RCA Laboratories Division at the David Sarnoff Research Center at Princeton, N. J. Dr. Wolff, a pioneer scientist in radar and a specialist in ultra-high radio frequencies, has served with the Radio Corporation of America research staff since 1928. Prior to his recent appointment he was the Director of Radio Tube Research for the RCA Laboratories.

Irving Wolff

Dr. Wolff, who is a native of New York, received his B.S. degree in 1916, from Dartmouth College. He was an instructor at Dartmouth College, from 1916–1919, and at Iowa State College from 1919–1920. He also taught at Cornell University from 1920–1923, where he received his Ph.D. degree in 1923. He then served with the Heckscher Research Council, in 1923–1924, doing research on polarization capacity, and later became affiliated with RCA.

Dr. Wolff has been active on numerous IRE Committees, and was the Chairman of the IRE Philadelphia Section in 1936–1937. He was recently elected as one of the Institute's Directors. He is a fellow member of the American Association for the Advancement of Science and the Acoustical Society of America, and a member of the American Physical Society.

❖

Richard W. Sanford (A'46) has been appointed as chief engineer of Aircrafts Armaments, Incorporated, it was recently announced.

A native of California, Mr. Sanford graduated from the Drexel Institute of Technology with a B.S. degree in electrical engineering in 1941. He also studied at Harvard University and M.I.T., for advanced work in electronics and radar, and served in the Armed Forces as a Radar Project Officer at Wright Field. He later joined the Glen L. Martin Company as chief of the fire control section. Mr. Sanford is a member of the American Ordnance Association.

❖

Lewis Gordon (S'50) has been appointed managing director of the International Division of Sylvania Electric Products Incorporated. Previous to this appointment Mr. Gordon has served in various management capacities for the Sylvania Company.

Mr. Gordon is a native of Rockport, Mass., and a graduate of Harvard College, in 1924. He is a member of the Export Advisory Committee of the Department of Commerce, and a member of the Export Committee of the Radio and Television Manufacturers Association.

IRE People

Albert W. Friend (A'34–M'39–SM'43) has become the director of engineering and development of the Magnetic Metals Company, in Camden, New Jersey.

A. W. Friend

Formerly on the physics and communication engineering staffs of West Virginia and Harvard Universities, Dr. Friend was also a staff member of M.I.T., with the Radiation Laboratory, and the Heat Research Laboratory, and at the same time, a consultant on guided-missile control with Division 5 of the National Defense Research Committee. Since 1944, he has been with the research department of the RCA Laboratories, where his work in the television field and other branches of communications has been of great importance. Most recently, Dr. Friend has been developing radar gun fire control apparatus as director of engineering of the instrument division of Daystrom, Inc.

For a biography of Dr. A. W. Friend see page 1338, of the October, 1951, issue of the Proceedings of the I.R.E.

❖

Thomas J. Killian (F'51) has been appointed Chief Scientist of the Office of Ordnance Research, at Duke University. Dr. Killian, who recently has been Science Director of the Research Divisions of the Office of Naval Research, in Washington, D. C., assumed his duties at the University in Durham, N. C., in February. He heads a group of scientists in directing and co-ordinating the U.S. Army Ordnance Corps' basic research program.

T. J. Killian

Dr. Killian was born on August 4, 1904, in Schenectady, N. Y. He received the B.S. and M.S. degrees from the Massachusetts Institute of Technology, and the M.A. and Ph.D. degrees from Princeton University, in 1927 and 1929, respectively.

Dr. Killian received the Regional Prize of the American Institute of Electrical Engineering in 1936, and the IRE Fellow Award in 1951, for his, "enlightened guidance of basic scientific research." He has served on the Institute's Nuclear Studies Committee in 1947–1950.

Lauriston C. Marshall (SM'50) has been appointed director of the Link-Belt Company's new physical testing and research laboratory at Indianapolis, Ind. As director, he will be responsible for an extensive program of original research.

L. C. Marshall

Dr. Marshall was born in China, and is a graduate of Park College. He received the Ph.D. degree, in physics, from the University of California, followed by a postdoctorate study as a National Research Fellow in physics at Princeton University.

Dr. Marshall spent six years with the U. S. Department of Agriculture, working in biophysical research and as superintendent of the agricultural experiment station at La Jolla, Calif., where he developed the first completely air conditioned greenhouse. In 1937 he joined the staff of the University of California in the electrical engineering department, in charge of the high voltage laboratory, where he helped develop the resnatron high power microwave oscillator, which holds the record for the largest continuous power output, and which during the war was used effectively in jamming German airborne radar.

During World War II, he was a member of the radiation laboratory staff at the Massachusetts Institute of Technology, and served as head of the division responsible for development of radar systems for coastal defense, air search against submarines, and shipborne installations. In the latter part of the war, he served as Chief of the Operational Research Section of the United States Armed Forces, Pacific Ocean Areas.

Dr. Marshall has been professor of electrical engineering at the University of California, and is currently head of the microwave laboratory operated at Berkeley, under the joint sponsorship of the U. S. Air Forces and the Research Corporation. He also is an active member of the university's radiation laboratory staff which has been carrying on nuclear research for the Atomic Energy Commission.

Dr. Marshall is a member of Sigma Xi, Eta Kappa Nu, the American Institute of Electrical Engineers, and the American Physical Society.

❖

Major William S. Dawson (A'49) has been designated Director of Plans and Requirements for Headquarters 1808th Airways and Air Communications Service Wing. In this position he is responsible for supervision of engineering and planning, and for the obtaining of authorization for personnel and equipment for airways communications and electronic navigational aids throughout the entire Pacific and Asia, including Korea.

Major Dawson had been Deputy Director since the recent creation of the new directorate. Prior to this assignment he was Commanding Officer of the 1951st AACS Squadron, which provided airways facilities in Southern Honshu. He is a commercial, multi-engine pilot and has been an amateur radio operator since 1934. He is vice president of the Far East Amateur Radio League as well as a member of the Military Amateur Radio System.

Major Dawson is a member of numerous professional organizations including, American Geophysical Union, American Institute of Physics, the American Meteorological Society, the American Association for Advancement of Science, and the Institute of Aeronautical Sciences.

❖

Ralph Bown (M'22–F'25) has been appointed vice president in charge of research for the Bell Telephone Laboratories. Prior to this appointment, Mr. Bown has been the Director of Research for Bell Laboratories, since 1946, and a member of the Bell System for more than 30 years.

Ralph Bown

Dr. Bown, who has been internationally recognized for his pioneering research and development work in the field of communications engineering, received the IRE Morris Liebmann Memorial Prize in 1926, and served as President of the Institute in 1927. In 1949, he received the IRE annual Medal of Honor.

Dr. Bown received the M.E., M.M.E., and Ph.D., degrees from Cornell University, where he also taught in the physics department. He then served as a captain in the Signal Corps in World War I, prior to joining the development and research department of the American Telephone and Telegraph Company. With that department, he went to the Bell Laboratories in 1934. Much of Dr. Bown's work has been concerned with various aspects of radio broadcasting and ship-to-shore and overseas telephony. He was a division member and consultant of the National Defense Research Committee, specializing in radar, and in 1941 visited England to study radar operations under combat conditions. He also served as an expert consultant to the Secretary of War.

Books

Vacuum-Tube Voltmeters by John F. Rider

Published (1951) by the John F. Rider Publishers, Inc., 480 Canal St., New York 13, N. Y. 403 pages +10-page index+9-page bibliography+ix pages. 215 figures. 5½×8½. $4.50.

John F. Rider is president and editor of the John F. Rider Publishers, Inc., New York, N. Y.

This is the second edition of a book first published in 1941. It contains valuable information on the myriad techniques and procedures for measuring all types of voltages, currents, and resistance by means of vacuum-tube circuitry.

Considerable attention is given to the many applications of vacuum-tube voltmeters. The theory of operation is developed in a simple and forthright manner requiring a minimum of technical background to follow and understand.

While the book is written primarily on the technician level, it should be helpful to the engineer, primarily for the thorough coverage of the current art. There is an excellent chapter on commercial instruments with many helpful circuit diagrams, tables, and graphs, and a section on maintenance and repair.

The book contains an adequate index, and a comprehensive bibliography is included.

C. E. Atkins
Tung-Sol Electric Inc.
Bloomfield, N. J.

Integral Transforms in Mathematical Physics by C. J. Tranter

Published (1951) by John Wiley & Sons, Inc. 440 Fourth Ave., New York 16, N. Y. 110 pages+5-page index+ix pages. 7 figures. 4×6½. $1.50.

C. J. Tranter is an associate professor of mathematics, Military College of Science, Shrivenham, Eng.

This particular Methuen Monograph gives an outline of the use of integral transforms in solving boundary value problems in partial differential equations. The integral transforms are employed to reduce the partial differential education in n variables to one in $(n-1)$ variables. Successive operations of this type can often reduce the problem to the solution of an ordinary differential equation which may be handled easily. The ultimate utility of the transform technique depends upon the existence and our ability to evaluate the transforms and the finding of the inverse transforms which restore the original variables.

The first four chapters of the seven chapters included in this text discuss and apply the Laplace, Fourier (sine, cosine, and complex), Hankel, and Mellin transforms and their inversion formulas to the solution of boundary value problems. Chapter five is concerned with the numerical evaluation of integrals in solutions. It considers formulas which give values for the integrals at the end points, usually the origin and infinity. Also included are some asymptotic formulas which, if applicable, prove useful for moderately large values of the variable involved. A method, due to Filon for the numerical evaluation of integrals containing a trigonometrical term, is also discussed. Chapter six is devoted to a discussion of the application of finite transforms. Here we find a brief outline of Doetsch's work with finite transforms containing sine or cosine kernels, and Sneddon's finite transforms with Bessel function kernels. Also, following Sneddon's suggestion that transforms can be defined and their inversion formulas derived when the kernels are orthogonal functions, a Legendre transform is defined and its application illustrated. It is shown that, while the use of finite transforms employing orthogonal function kernels does not solve problems which are incapable of solution by the classical methods of orthogonal function series, it does facilitate their solution by using the same drill as has already been given for transforms with an infinite range of integration. The following chapter is concerned with the combined use of relaxation methods and integral transforms. Here it is explained how the method of integral transforms may be employed to reduce the number of independent variables in a problem to two, in order that Southwell's relaxation method may be employed to replace the partial differential equation and boundary conditions by their finite difference approximations, and the approximate satisfaction of these at the nodal points of a regular two dimensional network covering the region in question.

The material is presented in such a manner as to illustrate the application of integral transforms to a selection of typical problems encountered in boundary value problems of mathematical physics. Each chapter contains solved examples and a selection of problems which serve as exercises for the reader. Not many of the theorems are rigorously derived nor are the limitations of the techniques discussed at any length, but references are made to specialized treatises and original papers, and a bibliography is included.

It is the opinion of this reviewer that the author of this monograph has succeeded in presenting his material in a very readable and appealing form. He has made an outstanding contribution to a growing list of fine survey monographs.

Lloyd T. DeVore
General Electric Co.
Syracuse, N. Y.

Synthesis of Electronic Computing and Control Circuits by the Staff of the Computation Laboratory, Harvard University

Published (1951) by the Harvard University Press, Cambridge, Mass. 230 pages+49-page appendix+10 pages. 150 figures. 7¾×10¼. $8.00.

This volume (27) in the Harvard Computation Laboratory Series is devoted to an exposition of the techniques developed under the leadership of Professor Howard H. Aiken, for the analysis and synthesis of the functional behavior of electronic digital computing and control circuits, together with the application of these techniques to the derivation of typical components of electronic digital computing machines.

The essential contribution of this work lies in the mathematical methods proposed and applied to the analysis of electronic circuits of the nature employed in digital computers, in which the variable signal voltages are of the on-off type, for example, restricted to two values. The simple algebraic approach in which the manipulations obey the rules of ordinary arithmetic appears excellently justified, by comparison with the use of the less familiar symbolism and rules of symbolic logic in such problems. The reader will especially appreciate the ease and rapidity with which facility is gained in the analysis of computer and control circuits by Professor Aiken's algebraic approach. As might be expected, this ease of analysis benefits considerably the inevitable cut and dry phase of the synthesis problem.

Following a brief introductory, chapter 2 takes up the analysis of the functional behavior of vacuum tube circuits by introducing the concept of switching functions and vacuum tube operators. The specification of the functions that the circuit is required to perform is expressed in terms of a switching function. The switching function is next converted straightforwardly into the form of vacuum tube operators, which express the functional performance of basic vacuum tube configurations and allow directly the derivation of circuit configurations capable of generating the required outputs.

In the next three chapters, general switching functions are studied of several input variables and transformations for minimizing the corresponding number of vacuum tubes. Extensive tables and charts with detailed instructions are provided to deal with the two, three, and four variable case. Chapters 6 and 7 extend the analysis to multiple output circuits. In chapter 9, problems involving variables that change with time are treated with no essential change in the mathematical procedures. Chapters 8, 11, 12, and 13 cover in considerable detail the application of the mathematical techniques to such computer components as triggers, rings, digit counters, coding systems, adders, accumulators, and multipliers. The use of rectifiers as logical circuit elements is discussed in chapter 10. The analysis of rectifier networks proceeds in exactly the same way as vacuum tube circuits, with rectifier operators replacing vacuum tube operators in the analysis.

The authors are to be particularly commended for producing a book which should provide a valuable introduction of the field to students, and, in addition, one of comprehensive character that is certain to be a useful reference. The liberal use of interesting examples in the development of the theory should prove especially welcome.

W. H. Boghosian
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Philadelphia, Pa.

Electronic Fundamentals and Applications by John D. Ryder

Published (1950) by Prentice-Hall, Publishers, 70 Fifth Avenue, New York 11, N. Y. 785 pages+10-page appendix+10-page index+xiv pages. 460 figures. 5½×8½. $9.00.

John D. Ryder is professor and head of the electrical engineering department, University of Illinois, Urbana, Ill.

This is primarily a textbook and is of value and interest to students who have had basic courses in physics, electrical engineering, and mathematics. In other words, it assumes familiarity with ac circuit analysis, calculus, and the handling of simple differential equations.

There are 20 chapters. Five of these are devoted to the fundamentals of the physical principles incorporated in vacuum tubes; seven chapters are devoted to the characteristics of the several classes of tubes; and seven chapters to tube circuits and applications. There is a final 24-page chapter on the newest frontier in the field—solid state electronics. In the chapters on circuits and applications, only the fundamentals are included. Schematic circuits and diagrams plus textual material on such applications as motor control, welding control, and saturation reactors are included, but there are no descriptions, diagrams, or pictures of commercial equipments. The chapter on solid state electronics includes material on blocking layer rectifiers, thermistors, photo conductors, point-contact rectifiers such as the germanium type, and transistors.

Each subject is treated briefly, with only five pages devoted to magnetrons and four pages to klystrons; however, this is necessary in order to cover the field of electronics in a single volume.

Although the style is very readable and the figures well chosen and clear, it is not light reading, as a considerable number of mathematical formulas are included. At the end of each chapter, there is a good list of references and a set of problems for the student. There is also a comprehensive index.

This book is not for tube or equipment designers but for those who have a knowledge of general science and electrical engineering and who wish to obtain specialized information in the modern broad field of electronics.

This reviewer wishes that he had had a text book of this sort available during his student days.

W. C. White
Research Laboratory
General Electric Co.
Schenectady, N. Y.

Lines, Networks, and Filters by William Breazeale and Lawrence R. Quarles

Published (1951) by The International Textbook Company, Scranton, Pa. 268 pages+7-page index +17-page appendix+viii pages. 164 figures. 6×9. $6.50.

W. M. Breazeale is principal physicist in the reactor technology division of the Oak Ridge National Laboratory, and L. R. Quarles is chief development engineer at the Oak Ridge National Laboratory, on leave of absence from the University of Virginia.

This textbook on line communication theory is designed for third-year college students who have had a course on alternating current theory plus two years of college mathematics. In an introductory chapter, the necessary hyperbolic function relations are given. The treatment of transmission theory and reflection and standing waves follows the usual lines, but attention is paid to the vector relations of the voltages and currents. Practical considerations such as loading to correct for distortion and transpositions to avoid cross talk are adequately discussed.

Conventional filter theory for ladder networks is very well presented with the theory for lattice networks also given attention. More unusual is the chapter on uhf and microfrequency lines which is timely and is developed at some length. The derivation, construction, and use of some of the recent transmission line charts are included with problems illustrative of their use. The authors have also paid particular attention to methods of measurements on lines at these frequencies. Other chapters take up impedance matching and the use of sections of transmission line for reactive elements in networks.

From the nature of the text, such a work has to be largely mathematical and the successive steps are clearly indicated. One advantage to the student in such a course is that he should fill in the details of the mathematical transformations. In certain sections of the book, particularly in the latter half, it seems to the reviewer that a little more assistance to the reader is to be desired. This need may be supplied to the student by the teacher, otherwise some sections may be difficult to follow. Proof reading of the book is, on the whole, very good, but a few confusing misprints have been noted.

The authors are to be commended on the choice and arrangement of their material and on their success in giving the discussion unity. The book should be useful not only as a college textbook, but also as a reference work for communications engineers.

Frederick W. Grover
Union College
Schenectady, N. Y.

Audio Amplifiers and Associated Equipment, A Specialized Volume of Photofact Folders

Published (1951) by the Howard W. Sams and Company, Inc., Indianapolis, Ind. 352 pages. Bound volume of 12 folders. 8½×11. $3.95.

Mr. Sam's new volume in his audio amplifiers series is, like the previous two, an invaluable aid to anyone engaged in the repair, servicing, or installing of audio amplifier equipment. In addition, the photographs may be used to good effect in explaining and demonstrating to students the problems which must be faced when converting from a circuit diagram to a piece of hardware. Demonstrations of the techniques used in industry for solving the problem of achieving compactness while maintaining serviceability are especially easy with the aid of the photographs and diagrams.

For the broadcast engineer, or the engineer who must specify which equipment is to be installed on a job, the new book has limited utility. Although the engineer can check such questions as servicibility of the equipment, he is given no information on the performance. The addition of performance data, such as a frequency response curve and the results of a distortion analysis on each unit, would increase the value of the volume immeasurably and would increase the clientel to whom it is of interest by more than a factor or two.

Despite criticism, the new volume represents a complete, up-to-date coverage of the audio amplifier field. The idea of including FM tuners with the audio amplifiers represents to this reviewer an extremely intelligent approach and one which makes the volume of even greater value to the people for whom it is designed to serve.

Jordan J. Baruch
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Ultrasonics by P. Vigoureux

Published (1951) by John Wiley & Sons Inc., 440 Fourth Ave., New York 16, N. Y. 141 pages+5-page index+2-page appendix+13-page bibliography+vi pages. 73 figures. 6×9¼. $4.00.

P. Vigoureux is with the British Royal Navy Scientific Service.

Sound waves higher than 15,000 cps are called "ultrasonic" waves and the general subject of sound in this frequency range is known as "ultrasonics." With present day techniques, ultrasonic waves as high as many megacycles per second can be studied.

For more than a century after the pioneer experiments of Chladni, around the year 1800, acoustical studies were largely restricted to the audible range of frequencies (15 to 15,000 cps). However, the advent a quarter of a century ago of vacuum-tube oscillators and amplifiers and of magnetostriction and piezoelectric transducers has stimulated an ever increasing tempo of research in the ultrasonic realm. The anomalous behavior of ultrasonic waves in fluids has attracted many investigators and although the literature has become extensive, the books are few.

The new book "Ultrasonics" by P. Vigoureux, introduces one to the techniques of ultrasonics and to the theories of ultrasonic propagation in gases and liquids. The dependence of such quantities as velocity and absorption on frequency, temperature, etc. are treated. Numerous references are given to recently published data, bringing up to date our state of knowledge of ultrasonic propagation. The inadequacy of our knowledge is evident in many cases and this should be both a stimulus to the old workers and an invitation to new ones.

Two chapters are devoted to equipment for the generation and detection (observation) of ultrasonic waves. Chapter 3 introduces some of the basic theory of propagation. The remaining two chapters treat both the theory and the experimental measurements of ultrasonic waves in gases and in liquids. An extensive bibliography lists articles published since 1939.

Although the book treats a specialized subject, it is written in a simple manner with only a moderate amount of mathematics. It should be particularly useful to those interested in learning something of both the theoretical and practical aspects of ultrasonic transmission.

John D. Kraus
Dept. of Elec. Eng.
Ohio State University
Columbus 10, Ohio

Electromagnetic Waves and Radiating Systems by Edward C. Jordan

Published (1950) by Prentice-Hall, Inc. Publishers. 70 Fifth Ave., New York 11, N. Y. 688 pages+10-page index+12-page appendix+x pages. 275 figures. 5¼×8¼. $10.50.

E. C. Jordan is a professor of electrical engineering, University of Illinois, Urbana, Ill.

In this new book Professor Jordan has covered an unusually wide range of topics in electromagnetic theory. The first part of the book which is suitable for senior or beginning graduate students deals with basic electromagnetic theory, guided waves, transmission lines, and waveguides. The latter part suitable for beginning and more advanced graduate students, deals with antennas and propagation.

The discussion of antennas and propagation is particularly thorough with 395 of the 688 pages of text devoted to these subjects. This is obviously the author's field of interest and specialization, and, consequently, it is presented with considerable detail and clarity. The many subtle, confusing points of antenna theory are in most cases lucidly explained, often from several complementary viewpoints to enhance comprehension of the material. A large number of illustrative examples of various antenna problems are worked out as a further aid to the reader.

The earlier parts of the book are less thorough; however, if only a superficial knowledge of their subjects is desired, they are satisfactory. The portion on waveguides is particularly brief, containing only 34 pages, and resonant cavities are neglected entirely.

Therefore, this book may be recommended as a fine and complete text on the theory of antennas. It is not recommended for use in intensive courses on electromagnetic theory, transmission lines, or waveguides, but may be used in a short survey course covering these topics.

Seymour B. Cohn
Sperry Gyroscope Co.
Great Neck, L. I., N. Y.

Dielectric Breakdown of Solids by S. Whitehead

Published (1951) by the Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. 267 pages+4-page index+xv pages. 75 figures. 5¼×8¼. $5.00.

S. Whitehead is the Director of Research, Electrical Research Association, London, Eng.

This is the eighth volume in a series of ten entitled, "Monographs on the Physics and Chemistry of Materials." The series is intended to summarize, in a form useful to physicists in universities and in government and industrial laboratories, the recent results of academic or long-range research in materials and allied subjects.

The book is a study of the dielectric strength of dielectrics, and its scope may be indicated by the chapter headings: Intrinsic Breakdown; Thermal Breakdown; Breakdown Caused by Discharges; Electrochemical Deterioration; and Dielectric Breakdown in Practice. Although theories are given in sufficient detail for the expert, mathematical demonstrations are preceded by qualitative accounts, and experimental results and practical conclusions are summarized at intervals.

By far the most interesting treatment, and the longest, is that of intrinsic breakdown. With suitable precautions, it is possible to determine a value of the electric strength of most dielectrics, which is independent, over substantial ranges, of the particular experimental circumstances other than temperature and pressure. This was first realized and measured more than fifteen years ago by von Hippel using alkali halide crystals, and was followed by work on other crystalline and amorphous materials by other workers. Von Hippel also introduced the concept of electronic breakdown, which was elaborated by Fröhlich into a complete and quantitative theory. The knowledge of solid state physics gained during the last decade is used extensively in Fröhlich's theory, and the author gives an excellent story of the experimental confirmations.

The rest of the book summarizes and recapitulates theories of other types of breakdown. Principles and their discussion form the declared aim of the volume, and a great field of development and information has been omitted which is essential to an engineer in his actual practice. The great number and variety of insulating materials now available precludes any detailed description. Indeed, the author feels that possibly the best approach to insulation engineering practice is by way of the equipment in which the insulation is to be used, granted a fundamental background to which this study may contribute.

As usual with books of this series, the references are extensive and adequate.

C. W. Carnahan
Sandia Corporation
3169 41st Place,
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Albuquerque, N. M.

The Design of Switching Circuits by William Keister, Alistair E. Ritchie, and Seth H. Washburn

Published (1951) by the D. Van Nostrand Company, Inc., 250 Fourth Ave., New York 3, N. Y. 546 pages+10-page index+xviii pages. 429 figures. 6×9. $6.00.

William Keister, Alistair E. Ritchie, and Seth H. Washburn are members of the technical staff of the Bell Telephone Laboratories, Murray Hill, N. J.

As indicated by its title, this is a text which describes the techniques used in the design of switching circuits. The basic tool used in such circuits is the two-state switch as exemplified by the familiar electromechanical relay or by the electronic flip-flop circuit. Multiple-state systems are generally synthesized by interconnecting such two-state devices into relay contact networks. Such networks can be designed to make their operation conform to a logical pattern of control. Complex control functions can thus be exercised by relay networks so that machines may be made to "think" in a manner which reproduces the original plans of the designer. It is natural that the first textbook in this field should have authors who are associated with the Bell Telephone Laboratories, since they have been concerned with switching and control problems in connection with automatic telephone systems for many years. In the past decade, however, automaticity involving the mechanization of complicated logic has been of importance in many other fields, particularly that of automatic digital computation. This book is therefore timely, if not overdue, and should prove of interest to all engineers engaged in designing control systems which must conform to a logical pattern.

The book is carefully written and is pedagogically sound. It is aimed at readers who have no familiarity with switching systems, although its level is not elementary. The material is based on notes used for the instruction of switching engineers at the Bell Telephone Laboratories. An edition of these notes was used as a text in a graduate course given at the Massachusetts Institute of Technology by the authors, and the present volume was revised in the light of this academic experience. For those who teach or who want to learn, problems are included at the end of most chapters. The one real weakness noted is the lack of adequate references to the literature. The only references found are those in the chapter on switching (Boolean) algebra. Even this list is not complete as it includes papers by C. E. Shannon and two British authors but fails to mention work carried out in Japan by Nakashima (1938) or in Germany by Piesch (1939).

The book covers a broad range of subjects from descriptions of electromechanical and electronic relays to a variety of functional circuits. Considerable space is devoted to contact network configurations, application of switching algebra, network manipulations, and the design of combinational and sequential relay circuits. Functional circuits such as those used for counting, coding and translating, selecting, connecting, finding, timing, and checking are described with liberal use of examples.

Of particular interest to those in the computing field is a chapter on calculating circuits. Although the authors make a conscious effort to generalize their presentation, the material is heavily flavored with telephone practice. For this there should be no apology since the telephone art represents an example of the most advanced applications of automatic switching and control. Readers interested in such far reaching fields as biology, nervous diseases, and insurance, as well as computers and controls will supply sufficient examples and applications of switching techniques once they have familiarized themselves with the basic concepts as given in this book.

In conclusion, this book fulfills an important need, because it starts from the beginning and describes the switching art in an organized manner. It makes possible the understanding of switching logic and techniques not only by engineers but also by others who have a logical mind and an understanding of elementary electrical circuit theory.

John R. Ragazzini
Engineering Building
Columbia University
New York 27, N. Y.

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Acoustics and Audio Frequencies	372
Antennas and Transmission Lines	373
Circuits and Circuit Elements	374
General Physics	376
Geophysical and Extraterrestrial Phenomena	376
Location and Aids to Navigation	378
Materials and Subsidiary Techniques	378
Mathematics	379
Measurements and Test Gear	379
Other Applications of Radio and Electronics	379
Propagation of Waves	380
Reception	381
Stations and Communication Systems	381
Subsidiary Apparatus	382
Television and Phototelegraphy	382
Tubes and Thermionics	383
Miscellaneous	384

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The Annual Index to these Abstracts and References, covering those published in the PROC. I.R.E. from February, 1951, through January, 1952, may be obtained for 2s.8d. postage included from the *Wireless Engineer*, Dorset House, Stamford St., London, S.E., England. This index includes a list of the journals abstracted together with the addresses of their publishers.

ACOUSTICS AND AUDIO FREQUENCIES

016:534 293

References to Contemporary Papers on Acoustics—R. T. Beyer. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 595–602; September, 1951.) Continuation of 2608 of 1951.

534.23-14 294

Effect of Temperature Inhomogeneities in the Ocean on the Propagation of Sound—I. Liebermann. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 563–570; September, 1951.) Temperature measurements at different points at a given level exhibit variations of average magnitude 0.05°C over distances of the order of 60 cm. Reflection, scattering, and focusing effects resulting from these inhomogeneities are investigated analytically by using the autocorrelation function, and also experimentally.

534.231 295

Discussion of Papers by Pachner and by Stenzel on Radiation from a Circular Emitter—R. L. Pritchard. (*Jour. Acous. Soc. Amer.*, vol. 23, p. 591; September, 1951.) Stenzel's method of calculation (2707 of 1942), which was criticized by Pachner (1816 of 1951), is shown to be basically correct.

534.231 296

Measurements on an Acoustic Wave Propagated along a Boundary—R. B. Lawhead and I. Rudnick. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 541–545; September, 1951.) Rudnick's theoretical analysis (3387 of 1947) was checked by measurements of amplitude and phase in the sound field of a point source located at a plane boundary between air and Fiberglas. The measured values agree well with calculated values based on measurements of the impedance and propagation constant of Fiberglas.

534.231 297

Acoustic Wave Propagation along a Constant Normal Impedance Boundary—R. B. Lawhead and I. Rudnick. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 546–549; September, 1951.) An expression is obtained for the amplitude and phase in the sound field due to a point source on or near the boundary between air as upper medium and a lower nonisotropic medium with a constant normal impedance. The expression is developed in the same form as that for isotropic media (see 296 above and 3387 of 1947). A medium obeying the condition of constant normal impedance was constructed of tightly packed drinking straws. Measurements at various positions along and above the boundary agreed well with values given by the theory. The approximations introduced are such that the sound field is adequately represented at distances greater than one wavelength from the source.

534.231.3 298

On the Generalization of the Concept of Impedance in Acoustics—O. K. Mawardi. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 571–576; September, 1951.) Present definitions of acoustic impedances are valid only when the specific impedance is constant on a wave front. This restriction is removed by extending the notion of vector fields to specific impedances. A definition based on energy concepts is proposed for acoustic impedance.

534.232 299

Synchronisation of Air-Jet Generators, with an Appendix on The Stem Generator—J. Hartmann and E. Trudsø. (*Dan. Mat. Fys. Medd.*, vol. 26, no. 10, 39 pp.; 1951.)

534.232 300

The Air-Jet Generator as a Means for Setting Up Waves in a Liquid Medium—J. Hartmann and F. Larris. (*Dan. Mat. Fys. Medd.*, vol. 26, no. 11, 26 pp.; 1951.)

534.232 301

On the Radiation Impedance of a Rectangular Plate with an Infinitely Large Fixed Baffle—Y. Nomura and Y. Aida. (*Sci. Rep. Res. Inst. Tohoku Univ.*, *Ser. B*, vol. 1/2, pp. 337–347; March, 1951.) Formulas for calculating the radiation impedance of a plate vibrating normally to the plane of the baffle are obtained from the pressure distribution on the plate. Numerical calculation indicates how the radiation resistance and reactance vary with the shape and size of the plate.

534.232:538.652 302

Performance Theory and Design Procedure for Laminated Magnetostriction Vibrators—Y. Kikuchi and K. Fukushima. (*Sci. Rep. Res. Inst. Tohoku Univ.*, *Ser. B*, vol. 1/2, pp. 141–189; January, 1951.) Theoretical investigations are made of the performance of laminated magnetostriction vibrators for emitting ultrasonic waves in water and other liquids. Formulas for the calculation of the resonance frequency and the mechanical and electrical constants of rectangular- and ring-type vibrators are given. The effects of lamination space-factor and various material constants on the transduction efficiency are carefully studied, and design methods for this type of vibrator are determined.

534.232:538.652:621.3.017.32 303

On the Magnetic-Hysteresis Losses in Magnetostriction Vibration—Y. Kikuchi and H. Shimizu. (*Sci. Rep. Res. Inst. Tohoku Univ.*, *Ser. B*, vol. 1/2, pp. 365–379; March, 1951.)

534.26 304

The Diffraction of Sound by Circular Apertures—T. Nimura. (*Sci. Rep. Res. Inst. Tohoku Univ.*, *Ser. B*, vol. 1/2, pp. 381–389; March, 1951.) Spheroidal wave functions are used in a rigorous theory of the diffraction of plane sound waves by a circular aperture with radius of the same order of magnitude as the wavelength. Numerical and graphical results are given for the sound fields in the neighborhood of the aperture, the directional characteristics, acoustic impedances, and the power transmitted through the aperture.

534.321.9:549.514.51 305

Mechanical Breakdown of Quartz Transducers at Resonance—T. F. Hueter. (*Jour. Acous. Soc. Amer.*, vol. 23, p. 590; September, 1951.) Values given by Epstein, Anderson and Harden (2442 of 1947) for the maximum ultrasonic intensity attainable with a quartz crystal are discussed, and more recently determined values are given.

534.6:621.395.623 306

The Artificial Ear for Calibrating Telephony Receivers—I. Barducci. (*Ann. Télécommun.*, vol. 6, pp. 165–169; June, 1951.) See 1565 of 1950.

534.7:611.85 307

D.C. Potentials and Energy Balance of the Cochlear Partition—G. v. Bekesy. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 576–582; September, 1951.)

534.84 308

Wide [sound] Distribution from Radiator Groups—F. Bergtold. (*Fernmeldetech. Z.*, vol. 4, p. 325; July, 1951.) Note complementary to the paper on sound reproduction in halls and open spaces (2890 of 1951).

534.845.1 309

A Free Field Method of Measuring the Absorption Coefficient of Acoustic Materials—U. Ingård and R. H. Bolt. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 509–516; September, 1951.) The pressure and phase of an incident plane wave are measured first at a point on the absorptive surface, and then at the same point in space with a perfectly reflecting material substituted for the absorptive material. Using the analysis for an infinite plane boundary, the absorption coefficient and the normal impedance for the particular angle of incidence are hence determined. Charts show the relations between these quantities and the measured phase difference and pressure ratio. Experimental results for materials of known properties confirm the validity of the method.

534.845.2 310

Absorption Coefficients of Fir Plywood Panels—R. W. Kenworthy and T. D. Burnam. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 531–532; September, 1951.) Measurements made by the reverberation-chamber method, using variously shaped panels, all of which exhibited greater absorption at 128 and 256 cps than at higher frequencies, are reported.

534.845.2 311

Absorption Characteristics of Acoustic Material with Perforated Facings—U. Ingård and R. H. Bolt. (*Jour. Acous. Soc. Amer.*, vol. 23, pp. 533–540; September, 1951.) An analysis of various sound absorptive structures consisting basically of a layer of porous material separated from a rigid wall by an air cavity. Equations and design charts are given for the impedance and the absorption coefficient. Reverberation measurements give results in agreement with the calculated coefficients.

621.395.623.7 312

Amplitude and Phase Measurements on Loudspeaker Cones—M. S. Corrington and M. C. Kidd. (Proc. I.R.E., vol. 39, pp. 1021–1026; September, 1951.) IRE 1951 National Convention paper. Measurements of the motion of different points on a conical diaphragm were made at various critical frequencies. From these results the cause of various peaks and dips in the sound-pressure curve can be determined, making possible an improved design of loudspeaker.

621.395.625.3 313

On the True Frequency Response in Recording and Reproduction by Magnetic Methods—R. Bierl. (*Z. angew. Phys.*, vol. 3, pp. 161–165; May, 1951.) Consideration of reluctance in the magnetic circuit of the recording and reproducing head leads, respectively, to an inverse logarithmic and a hyperbolic function expressing the flux distributions. The flux in the reproducing head is then proportional to the differential of the magnetization, thus producing a frequency response which compensates that of the recording head. The sharp drop in high-frequency response previously attributed to a demagnetization effect, and the rise and final drop in the over-all frequency response at low frequencies, are easily explained by this theory.

ANTENNAS AND TRANSMISSION LINES

538.566 314

On the Propagation of Electric Waves from a Horizontal Dipole over the Surface of the Earth Sphere—Nomura. (*See* 467.)

621.392:621.396.662 315

The Design of Transmission-Line Tuning Elements for Minimum Dissipation—R. W. Klopfenstein. (Proc. I.R.E., vol. 39, pp. 1089–1094; September, 1951.) "A design procedure is described by which a coaxial transmission-line tuning stub may be designed for minimum energy dissipation when the input susceptance or reactance is specified. The problem has been formulated for air dielectric only where dielectric losses are ordinarily small compared to copper losses. The results are presented in curve form."

621.392.012.8 316

Transmission Line Equivalent Circuit—L. A. Ware. (*Wireless Eng.*, vol. 28, pp. 287–288; September, 1951.) The development of a RLC circuit which simulates a $\lambda/4$ short-circuited line over a narrow band of frequencies is presented.

621.392.09 317

Propagation of Electromagnetic Waves along a Conducting Wire with Thin Dielectric Covering—A. Fromageot and B. Louis. (*Bull. Soc. franç. Élect.*, vol. 1, pp. 291–302; June, 1951.) The case is first considered in which the wire is perfectly conducting and the dielectric covering and surrounding medium are both without loss. The field and the propagation constants are determined, and the field concentration and phase distortion due to the presence of the dielectric layer are calculated. The attenuations due to losses in the conductor, the dielectric, and the surrounding medium are then evaluated, and the complex constant of propagation is calculated, taking account of losses, so as to obtain an expression for the phase constant. The concentration of the field round the conductor facilitates matching of the line to terminal equipment, and reduces the disturbing effects due to metal masses near the wire. The values found for attenuation and phase distortion compare favorably with those of coaxial lines and waveguides.

621.392.26†:621.39.09 318

General Theory of Asymmetrical Waves in a Circular Waveguide with an Open End—L. A. Vainshtein. (*Zh. tekh. Fiz.*, vol. 21, pp. 328–345; March, 1951.) A rigorous solution is obtained of the problem of a circular waveguide within which an asymmetrical electric or magnetic wave travels towards the open end. Owing to diffraction effects, the solution is more complex than in the case of symmetrical waves. Formulas are derived for the coefficient of reflection of an incoming wave from the open end, and for the coefficients of transformation of this wave into others (including the transformation of electric waves into magnetic, and vice versa). Approximate formulas are also derived for the radiation field. See also 2336 of 1951.

621.392.26†:621.39.09 319

Numerical Results obtained from the Theory of Asymmetrical Waves in a Circular Waveguide with an Open End (E_1 and H_1 Modes)—L. A. Vainshtein. (*Zh. tekh. Fiz.*, vol. 21, pp. 346–357; March, 1951.) For asymmetrical modes E_1 and H_1 in a circular waveguide, graphs are plotted of the absolute values and phases of the coefficients of reflection from the open end and the coefficients of transformation from one mode to another. Graphs are also plotted for the radiation characteristics of modes E_{11} and H_{11}, and the results of the rigorous theory are examined in relation to Huyghens' principle. The excitation of various modes in the waveguide by a plane wave incident on the open end is also discussed.

621.392.26†:621.39.09 320

The Propagation of Waves in Cylindrical Waveguides and the Hertzian Solution as Special Cases of the Propagation of Waves in Horns—H. Kleinwächter. (*Arch. elekt. Übertragung*, vol. 5, pp. 231–236; May, 1951.) Propagation in conical and pyramidal horns is analyzed, using spherical co-ordinates. The Bromwich method is used for solving the wave equation, and the solution function is explained by examining the propagation of a particular mode. The method is further elucidated by deriving (a) the familiar solution for a cylindrical waveguide as the limiting case of a cone with zero aperture angle, and (b) the Hertzian solution for radiation from a dipole above a perfectly conducting plane as the limiting case of a cone with 180° aperture angle.

621.392.43 321

The Reactance of Mismatched Resonant Lines—A. Ruhrmann. (*Arch. elekt. Übertragung*, vol. 5, pp. 219–230; May, 1951.) The problem of matching an antenna to a feeder over a wide band of frequencies is treated analytically. For slight detuning of the operating frequency from the line resonance frequency, the input impedance contains a reactive component proportional to the detuning. As a result, the input circuit can be represented by a damped oscillatory circuit with lumped parameters whose reactance at resonance is easily determined. Various impedance-compensation and transformation circuits are analyzed, and equivalent lumped-parameter circuits based on band-pass filter sections are established. The application of the theory for designing resonant-line circuits with predetermined properties is indicated.

621.392.43.012.3 322

Network Design Charts—T. U. Foley. (*Electronics*, vol. 24, pp. 132, 134; September, 1951.) Sets of curves are provided which facilitate the determination of matching reactance limits of T, Π, and L networks over a wide range of both phase shift and load resistance.

621.396.67 323

Theory of the Plane Annular-Slot Aerial—J. Meixner and W. Kloepfer. (*Z. angew. Phys.*, vol. 3, pp. 171–178; May, 1951.) The radiation pattern, radiated power, and impedance of the antenna are determined as functions of λ, slot width, and field distribution in the slot, for the case of a slot in an infinite plane. Numerical results are discussed. The radiation field plotted for a circular disk with a concentric annular slot in one face shows good agreement qualitatively with experimental results of Rhodes (1594 of 1949).

621.396.67 324

A Note on Super-gain Antenna Arrays—N. Yaru. (Proc. I.R.E., vol. 39, pp. 1081–1085; September, 1951.) Numerical calculations have been made for linear broadside super-gain arrays. Using arrays having an over-all length of a quarter wavelength as an example, it is shown that as the required directive gain is increased, tremendous currents are required to produce only a small radiated field, so that such arrays become quite impractical.

621.396.67 325

Radiation Patterns of Arrays on a Reflecting Cylinder—J. E. Walsh. (Proc. I.R.E., vol. 39, pp. 1074–1081; September, 1951.) Simple approximate formulas are derived for the horizontal and vertical patterns of directional arrays of dipoles arranged on an arc of a circle or on the surface of a right circular cylindrical segment and backed by a reflecting cylinder. The dipoles may be aligned axially or circumferentially and the amplitude distribution on them may be uniform or cosinusoidal. The results are valid in the vicinity of the radiation maximum, but are less certain at wide angles unless the dipole spacing is substantially less than half a wavelength. The array patterns compare favorably with those of comparable linear arrays, and in certain cases a greater gain is obtained.

621.396.671 326

Theoretical Calculation of the Input Impedances of Two Parallel Antennae—S. Uda and Y. Mushiake. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 91–104; January, 1951.) Hallén's method for the calculation of the impedance of a single antenna is extended to a system of two linear antennas, and the results of numerical calculations and their applications are given.

621.396.677 327

Dielectric-Lens Aerial for Marine Navigational Radar—D. G. Kiely. (*Wireless Eng.*, vol. 28, pp. 299–304; Oct., 1951.) "The design and performance of a dielectric-lens antenna for marine-navigational radar are described. The antenna has a fan-beam radiation pattern, and is designed for horizontal polarization. Its aperture and focal length are 4 feet, and the maximum sidelobes are some 30 db below the main-beam level over the frequency band 9,320 to 9,500 mc. This low sidelobe performance makes the antenna particularly suitable for marine-navigational radar application where the suppression of "ghost" echoes due to sidelobes is important."

621.396.677 328

Aerials for Beam Stations—K. O. Schmidt. (*Fernmeldetech. Z.*, vol. 4, pp. 313–315; July, 1951.) Supplement to a former paper (2352 of 1951, in which the title should be as above) giving a simple relation between antenna dimensions and beam angles.

621.396.677:621.397.6 329

Horn Antennas for Television—D. O. Morgan. (*Electronics*, vol. 24, pp. 84–85; October, 1951.) An 8-foot equilateral horn antenna, with sides of wire mesh, has proved very effective in the reception of television and FM signals. Since for these signals horizontally polarized waves are of primary importance in the U.S.A., the top and bottom sides of the horn can be omitted. The antenna matches 300-Ω line, one conductor being connected to each sector at the apex.

621.396.679.4 330

Nomographic Determination of Cable Efficiency in Feeding H.F. Energy from Transmitter to Aerial—H. Geschwinde. (*Frequenz*, vol. 5, pp. 67–69; March, 1951.) Formulas for the transmission of hf energy along a feeder cable are embodied in a nomogram whose use is illustrated by several numerical examples. The influence of the terminating impedance on the cable efficiency is shown.

CIRCUITS AND CIRCUIT ELEMENTS

621.3.015.7† 331

Variable Pulse Delay for Radar Ranging—J. F. Gordon. (*Electronics*, vol. 24, pp. 100–103; October, 1951.) Based on paper presented at the 1950 National Electronics Conference. (*Proc. Nat. Electronics Conference* (Chicago), vol. 6, pp. 94–102; 1950.) An electromechanical pulse-delay unit gives continuously variable or fixed pulse delays useful for radar ranging, navigation, propagation studies, and similar techniques. Delay is obtained locally or remotely with range from a few microseconds to several milliseconds with a maximum error of 0.3 μs. A description is given of the circuits and their operation.

621.3.016.35 332

Note on Stability—M. Parodi. (*Jour. Phys. Radium*, vol. 10, pp. 200–201; June, 1949.) The sufficient conditions for a determinantal equation to have the real part of all its roots negative are stated, and the distribution of these roots in the neighborhood of the origin is studied.

621.3.016.35 333

Supplement to a Note on Stability—M. Parodi. (*Jour. Phys. Radium*, vol. 12, pp. 665–666; June, 1951.) Generalization of the results obtained in 332 above.

621.3.016.35 334

Transmission Matrix Stability Criterion—P. M. Honnell. (*Elec. Eng.*, vol. 70, p. 580; July, 1951.) Summary of 1951 AIEE Winter General Meeting paper. An analytical method is presented for predetermining the stability of a transmission network. The stability formulas apply to a network cut at any arbitrary point, being thus more general than previous formulas.

621.3.016.35 335

Extension of Nyquist's Theory to the Case of Nonlinear Characteristics—A. Blaquière. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 345–347; July 30, 1951.) The stabilized amplitude, the variation of frequency with amplitude, and the conditions for the frequency to be independent of amplitude to the first order, are calculated for oscillators with slightly non linear characteristics.

621.3.018.78† 336

A New Method of Measuring and Analyzing Intermodulation—C. J. Le Bel. (*Audio Eng.*, vol. 35, pp. 18–31; July 1951.) Two frequencies are mixed, without intermodulation, in a suitable circuit, passed through the system under test, and then through a high-pass filter. The output of the latter is observed on a cro whose sweep is synchronized with the low-frequency tone. The resulting pattern can be analyzed quantitatively to give the intermodulation percentage.

621.314.3† 337

Analysis and Design of Self-Saturable Magnetic Amplifiers—S. B. Cohen. (Proc. I.R.E., vol. 39, pp. 1009–1020; September, 1951.) IRE 1950 National Convention paper. The theory of operation of the control element of a magnetic amplifier, the self-saturable reactor, is given, and the concepts of extinction angle and firing angle are introduced. By assuming a characteristic for the core, the two angles are related, thus making possible current and power calculations for the circuit. Applications of magnetic amplifiers are discussed, and the merits of electronic and magnetic amplifiers are compared.

621.314.58 338

The Magnetic Modulator—R. Feinberg. (*Wireless Eng.*, vol. 28, pp. 281–286; September, 1951.) A theoretical study of the modulator with sinusoidal transductor voltage and no-load or short-circuit output. Optimum performance is obtained with a core material whose magnetization curve has a narrow hysteresis loop, a sharp bend at the knee, and a high initial permeability, and with a transductor voltage giving a peak ac flux-density in the core equal to the flux-density at the knee of the magnetization curve.

621.314.6.011.1 339

The Principles of Linear R.M.S.—Value Rectifiers—O. Schmid. (*Arch. elekt. Übertragung*, vol. 5, pp. 241–247; May, 1951.) The operation of the rectifier circuit described by Boucke (687 of 1951) is analyzed. Methods are given for determining the appropriate values of circuit components. The magnitude of the residual wave form error is investigated and compared with that for area and peak rectifiers. For simplicity, consideration is restricted to operation with square-pulse, sawtooth-pulse, and sine-wave voltages.

621.314.634+621.314.2 340

The New Siemens Selenium Rectifier for Broadcasting Equipment, and the Design of Suitable Transformers and Chokes—Kühn. (*See* 505.)

621.316.86:621.396.822 341

Flicker Effect in Very Thin Carbon Films—A. Blanc-Lapierre, M. Perrot, and N. Nifontoff. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 241–243; July 16, 1951.) Experimental results are given relating to the noise voltage occuring in resistors formed of thin carbon deposits, and in some commercial carbon resistors, due to the flow of a continuous current $\underline{I}$. For a given resistance, the spectral density e_ν^2 of the noise voltage is a function of I and of the frequency, ν. The effect has been studied for I $\leq$ 1ma and ν between 600 and 800 cps. Deviations of the results from Ohm's Law and from an I^2 law are discussed.

621.318.4 342

Internal Capacitance of a Multilayer Coil—K. Jekelius. (*Frequenz*, vol. 5, pp. 70–77; March, 1951.) Approximate formulas developed previously for calculating the uniformly distributed capacitance between two adjacent windings are reviewed, and a method is developed in which uncertainty due to estimating the mean dielectric constant is avoided, the calculation being based on the values of the dielectric constants for the winding insulation and the air spacing. A numerical example is worked out. The exact assessment and separation of internal and external coil capacitance is important in relation to the investigation of coil resonances.

621.319.4:517.35 343

A Solution for $\int e^{b(x+a\cos x)}dx$—Emden: Rutishauser. (*See* 431.)

621.319.4.011.5 344

Harmonics of Current in [capacitor] Dielectrics—B. Lavagnino. (*Alta Frequenza*, vol. 20, nos. 3/4, pp. 101–112; June–August, 1951.) Harmonic components of current passing through capacitors with imperfect dielectric are nearly always due to the presence of harmonics in the applied voltage. Introduction or intensification of harmonics as a result of dielectric phenomena is to be expected only when the electric field is strong enough to produce ionization. The problem is discussed in relation to the sensitivity and accuracy of bridge measurements.

621.319.4.012.3 345

Temperature-Compensating Capacitor Nomograph—T. T. Brown. (*Electronics*, vol. 24, pp. 132, 134; October, 1951.) An abac which gives, with one setting of a celluloid setsquare, the capacitance values required when two temperature-compensation capacitors are connected in parallel.

621.392 346

Extension of the Reciprocity Concept to Valve Circuits—J. L. Bordewijk. (*Tijdschr. ned. Radiogenoot.*, vol. 16, pp. 137–153; May, 1951.) Propositions of reciprocity theory previously shown to be valid for passive networks [3381 of 1948 (Bode)] are expressed so as to be valid also for active networks (i.e., including tubes) by applying the concept of "reversal." The method is illustrated by comparison of the properties of the cathode-follower and the grounded-grid circuit, and of oscillator arrangements with the resonant circuit connected in the one case to the anode and in the other case to the grid, and by examination of various amplifier arrangements.

621.392 347

The Fundamental Theorem of Electrical Networks—J. L. Synge. (*Quart. Appl. Math.*, vol. 9, pp. 113–127; July, 1951.) An explanation for electrical engineers of Kron's method for the analysis of networks. The method of extracting "meshes" and "trees" from any network is discussed, and Kirchhoff's laws are stated. The theorem relating the mesh currents to the branch currents is developed, and the method of solution of a network using Kron's transformation matrix set out.

621.392 348

The Realization of a Transfer Ratio by means of a Resistor-Capacitor Ladder Network—J. T. Fleck and P. F. Ordung. (Proc.

I.R.E., vol. 39, pp. 1069–1074; September, 1951.) IRE 1951 National Convention paper. A method is described for the solution of this problem, assuming that (a) poles of the transfer ratio $H(p)$ occur at real negative values of p and are simple, (b) zeros of $H(p)$ occur at real negative values of p, but may be multiple, (c) $H(p)$ is finite for $p=j\omega$, where $-\infty \leq \omega \leq \infty$. The resulting network usually has a gain factor considerably greater than can be obtained with a network synthesized on the lattice basis. To illustrate the method, the circuit parameters corresponding to a specified transfer function are determined.

621.392.5 349
The Dynamic Transfer Parameters of a Quadripole in Frequency Modulation—G. Bosse. (*Arch. elekt. Übertragung*, vol. 5, pp. 237–240; May, 1951.) Previously proposed methods for determining the dynamic transfer coefficient (i.e., the coefficient for the case of varying-frequency input voltage) are discussed. Spectral analysis of the input into components of constant frequency is laborious because of the large number of components involved. The input voltage can be considered as a series of pulses of infinitely short duration. The integral of the transient responses is related to the usual static transfer coefficient by a Laplace transform. The approximate evaluation of the dynamic coefficient may then be performed by graphical, numerical, or analytical methods, depending on the particular frequency to time relation of the applied voltage.

621.392.5 350
The Transfer Function of an RC Ladder Network—A. Fialkow and I. Gerst. (*Jour. Math. Phys.*, vol. 30, pp. 49–72; July, 1951.) Analysis of four-terminal networks with two kinds of elements(mutual inductance excluded), including unbalanced networks having a common ground terminal. The topics considered are the zeros and poles of the transfer function, the realizability and the synthesis of the L-network, and of the ladder network, with an illustrative example of the latter. The extension of the RC analysis to RL and LC networks is indicated.

621.392.5:517.432.1 351
Time-Dependent Heaviside Operators—Zadeh. (*See* 432.)

621.392.52:621.317.3 352
Fundamental-Wave Filter for Instruments and Measurement Circuits—H. Poleck. (*Frequenz*, vol. 5, pp. 77–84 and 107–112; March and April, 1951.) Analysis of T-type and Π-type low-pass filters, particularly their properties at frequencies close to the fundamental and their attenuation of harmonics. The treatment is largely mathematical and the case in which the limiting frequency is $\sqrt{2}$ times the fundamental is particularly considered. Application of such filters in a rectifier-type voltmeter, a phase indicator with mechanical rectifier, and a wattmeter is described.

621.392.52:621.392.26† 353
Tunable Waveguide Filters—W. Sichak and H. Augenblick. (Proc. I.R.E., vol. 39, pp. 1055–1059; September, 1951.) IRE 1951 National Convention paper. The bandwidth of a conventional microwave filter using resonant cavities tuned by irises is critically dependent on the resonance frequency. Another type of filter is discussed in which the cavities are tuned by changing the constants of the guide so as to maintain the wavelength in the guide constant at resonance. A filter using this principle has been built in which a strip of dielectric is inserted in a longitudinal slot in the broad face of the waveguide. The relation between resonance frequency and insertion depth was essentially linear between 4.4 and 5.0 kmc.

621.392.52:621.396.611.21 354
The Narrow-Band-Pass Crystal Filter for Short Waves—Y. Watanabe and Z. Abe. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 421–426; March, 1951.) A method is described for eliminating adjacent frequencies in filter crystals by coating part of the crystal face with a damping medium such as Canada balsam.

621.392.6.012.8 355
Equivalent Circuits for 6-Pole and Higher-Order Multipole Networks—G. Schmitt. (*Arch. Elektrotech.*, vol. 40, No. 3, pp. 177–192; 1951.) Two types of six-pole network are considered: (a) with three free pairs of terminals or (b) with two free pairs. For case (a) equations and matrices are given and, by a simple extension of the Π-equivalent for a quadripole, that for the six-pole network is developed, and from this, the double-T equivalent. The physical significance of the elements of the matrices is explained. If a resistor is connected across one terminal pair of a six-pole network, a quadripole results across each of the free terminal pairs. The matrices and equivalent circuits for this quadripole are investigated. It is also shown how the Π-equivalents of any higher-order multipole with more than three free terminal pairs can be derived from that of a six-pole network. Case (b) is treated similarly.

621.396.611.3 356
Control Coupling between Two Oscillatory Circuits—H. Wigge. (*Frequenz*, vol. 5, pp. 63–67; March, 1951.) Various arrangements are illustrated and analyzed in which two oscillatory circuits, connected to tubes, are linked without feedback, each circuit acting on the other through an intermediate (control) element. In addition to three known coupling modes, each with two natural frequencies, two novel coupling modes are possible, each with three natural frequencies. The new couplings can be discussed in terms of classical theory by introducing imaginary values of coupling coefficients, inductances, resistances, and capacitances.

621.396.611.4 357
Resonance Frequency of Spheroidal Cavity Resonator—T. Nimura. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 73–90; January, 1951.) Spheroidal co-ordinates and wave functions are used in analysis of the em fields of oblate and prolate spheroidal cavity resonators. Resonance frequencies are determined for the case of rotationally symmetrical fields.

621.396.611.4 358
Coaxial Resonators Loaded by a Capacitance—M. E. Zhabotinski. (*Zh. tekh Fiz.*, vol. 21, pp. 358–362; March, 1951.) Resonators of this type as used in usw wavemeters and high-Q circuits in oscillators, filters, etc., are discussed. An equivalent transmission line, shorted at one end and loaded by a capacitance at the other, is considered, and the main relations determining the operation of the resonator are derived. From equation (5) the resonance frequencies can be determined, and formula (8) gives the Q. Using this formula, the optimum configuration of a resonator can be found.

621.396.615 359
Wide-Range Variable-Frequency Oscillator—A. Cormack. (*Wireless Eng.* vol. 28, No. 336, pp. 266–270; September, 1951.) The basic circuit of the phase-shift oscillator described comprises an amplifier stage and four cathode followers. Frequency range is up to about 180 mc. Bandwidths >1 octave are obtainable; amplitude is substantially constant over most of the range. Frequency is controlled electronically.

621.396.615.029.3 360
Wide-Range Sweeping Oscillator—L. A. Rosenthal. (*Electronics*, vol. 24, pp. 114–116; October, 1951.) A wobbled af output, or a steady tone of variable frequency, is obtained from a Wien-bridge type of circuit including SiC nonlinear resistors. The oscillation frequency covers a range of over 20 to 1. A thermistor element in the bridge stabilizes the oscillation amplitude and ensures good sinusoidal wave form.

621.396.622.71 361
Numerical Treatment of the Limiting Effect of the Ratio Detector—Behling. (*See* 485.)

621.396.645 362
Amplifiers with Slightly Variable Gain—J. J. Zaalberg van Zelst. (*Tijdschr. ned. Radiogenoot.*, vol. 16, pp. 117–133; discussion, pp. 134–135; May, 1951.) Review of methods of stabilizing the gain of amplifiers, particularly the two types of circuit described in 3063 of 1947. The stability limit of amplifiers, including elements which are not quite constant, is discussed.

621.396.645.015.7† 363
On Design Theory of Pulse-Amplifier—K. Nagai. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 51–72; January, 1951.) The design of pulse amplifiers is described, particular attention being paid to the correction of response at high and low frequencies by inductive load compensation and RC circuits, respectively. Experimental measurements on an amplifier designed for 1-μs pulses are discussed.

621.396.645.015.7† 364
The Problem of the 'Best' Pulse Receiver—L. Huber and K. Rawer. (*Arch. elekt. Übertragung*, vol. 5, p. 254; May, 1951.) Correction to paper noted in 1866 of 1951.

621.396.645.029.3 365
Audio Amplifier Damping—R. M. Mitchell. (*Electronics*, vol. 24, pp. 128–131; September, 1951.) Oscillations generated in the load of an amplifier can be damped out by controlling the output impedance by means of feedback. Methods of measuring the damping factor, defined as the ratio of the load impedance to the effective generator impedance, are described and its values for beam-tetrode and triode power tubes are compared. Tests on the Williamson Type-W20 amplifier are described to illustrate the practical application of feedback.

621.396.645.37 366
Universal Direct-Coupled Differential Amplifier—L. Goldberg. (*Electronics*, vol. 24, pp. 128–131; October, 1951.) Analysis is presented for a basic circuit capable of providing constant closed-loop gain and low output impedance, with or without sign inversion, for applications requiring a high-gain differential amplifier. Mathematical operations of many kinds, which can be effected with the aid of suitable external circuits, are summarized.

621.396.645.37:621.3.016.35 367
The Stability Problem in Feedback Amplifiers—W. A. Lynch. (Proc. I.R.E., vol. 39, pp. 1000–1008; September, 1951.) The criterion for stability is formulated in terms of fundamental restrictions on amplifier characteristics at complex frequencies. From this, a simplified criterion placing equivalent restrictions on the characteristics at real frequencies leads directly to Nyquist's criterion. A survey of important relations between gain and phase shift in minimum-phase-shift networks leads to discussion of the principles by which stability can be effected.

621.396.645.371.011 368
The Exact Equations for Current and Voltage Negative-Feedback—W. Reichardt. (*Frequenz*, vol. 5, pp. 139–143; May, 1951.) Analy-

sis of basic feedback circuits taking account of the alteration of the tube characteristic due to the addition of the feedback loop. The equations derived are of significance in a phase-reversal stage which can be regarded either as a voltage-feedback or a current-feedback circuit.

621.396.662:621.392 369
The Design of Transmission-Line Tuning Elements for Minimum Dissipation—Klopfenstein. (*See* 315.)

621.397.645 370
R. F. Amplifier for U.H.F. Television Tuners—B. F. Tyson and J. G. Weissman. (*Electronics*, vol. 24, pp. 106–109; October, 1951.) A concentric-line amplifier circuit is described which uses a disk-seal planar triode with grounded grid. The effect on gain and bandwidth of the position of the output loop along the concentric line is illustrated, and the optimum position found. A gain of 12.5 db in the band 470 to 890 mc is obtained with the amplifier ahead of a typical crystal mixer, and the power required from the local oscillator is very much reduced.

621.3.012.2 371
Die Ortskurventheorie der Wechselstromtechnik (The Theory of Circle Diagrams in Alternating-Current Technique) [Book Review]—G. Oberdorfer. Publishers: Deuticke, Vienna, Austria, 2nd edn 1950, 100 pp., 15 Swiss francs. (*Bull. schweiz. elektrotech. Ver.*, vol. 42, p. 456; June 16, 1951.) A comprehensive textbook for students and practising engineers.

621.3.016.35:621.3.012.2 372
Praktische Stabilitätsprüfung mittels Ortskurven und numerischer Verfahren (Practical Stability Testing using Circle Diagrams and Numerical Methods) [Book Review]—F. Strecker. Publishers: Springer, Berlin, Germany, 1950, 189 pp., cardboard covered 15 DM, bound 18 DM. (*Arch. elekt. Übertragung*, vol. 5, p. 252; May, 1951.) Familiarity with modern ideas on network theory is necessary for an understanding of this book, but function theory and matrix calculations are not used.

GENERAL PHYSICS

53.05:518.4 373
The Analysis of an Experimental Curve is the Ideal Method of Determining a Physical Characteristic: Examples—P. Vernotte. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 230–232; July 16, 1951.)

535.422:538.56 374
On the Diffraction of Electric Waves by a Perfectly Reflecting Wedge—Y. Nomura. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 1–23; January, 1951.) Mathematical analysis of the diffraction of the spherical waves from an oscillating dipole by a perfectly reflecting wedge of arbitrary angle, with detailed discussion of the field in the geometrical shadow, and numerical examples.

536.491:621.3.032.44 375
Stationary Temperature of Current-Carrying Wires of Moderate Length—J. Fischer. (*Arch. Elektrotech.*, vol. 40, no. 3, pp. 141–171; 1951.) A comprehensive mathematical treatment for the case of a uniform wire extended between two relatively massive blocks in air, or in a liquid, or in vacuo. A solution for small temperature rises is first obtained, and then the general solution is considered. Approximate solutions are derived, and a numerical example is worked out for the case of a Pt wire, 4 cm. long and 0.5 mm. in diameter, carrying a current of 10a in air.

537.311.31:539.23 376
Tentative Interpretation of the Mechanism of Electrical Conduction in Very Thin Metal Films—N. Mostovetch. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 360–364; July 30, 1951.)

537.311.33 377
Internal Barriers in Semiconductors—H. K. Henisch. (*Phil. Mag.*, vol. 42, pp. 734–738; July, 1951.) "The temperature dependence of conductivity is calculated for a semiconducting specimen containing internal barriers of various heights. This leads to a new interpretation of activation energies as deduced from conduction measurements."

537.525 378
High-Frequency Discharges: Part 1—Breakdown Mechanism and Similarity Relationship—F. L. Jones and G. D. Morgan. (*Proc. Phys. Soc.*, vol. 64, pp. 560–573; July 1, 1951.) An experimental investigation covering the range of frequencies $f=3.5$–70 mc is reported. Geometrically similar systems break down at the same potential when parameters $a\,p$ and f/p are constant, a being a linear dimension and p the gas pressure. For small electron-cloud oscillations, breakdown between coaxial cylinders depends only on the inner cylinder radius. The significance of the similarity relation is discussed.

537.525 379
High Frequency Discharges: Part 2—Similarity Relationship for Minimum Maintenance Potentials—F. L. Jones and G. D. Morgan. (*Proc. Phys. Soc.*, vol. 64, pp. 574–578; July 1, 1951.) The similarity relation applying to breakdown (see part 1, 378 above) applies also to the maintenance of hf currents, but the dependence of the minimum maintenance potential on the parameters $a\,p$ and f/p is not so well defined, because of discharge instability.

537.525.6 380
The Ion-Surge Discharge, a Cause of Arc-Back in the Mercury-Vapor Rectifier—T. Wasserrab. (*Arch. Elektrotech.*, vol. 40, no. 3, pp. 171–177; 1951.)

537.562 381
Theory of Ionized Media with Translational Symmetry—M. Hoyaux. (*Rev. gén. élect.*, vol. 60, pp. 279–291 and 317–328; July and August, 1951.) The analysis presented was made in relation to mercury-vapor rectifier arcs, but is of more general application. The statistical properties of ionized media are studied; generalized equations of mobility are derived and extended to the case of bipolarized media. An experimental verification of the theoretical results was made using Ledrus' plasmograph.

537.562:537.311.37 382
Conductivity of Ionized Air in a High-Frequency Alternating Field—A. Székely. (*Acta Phys. austriaca*, vol. 3, pp. 22–37; June, 1949.) Measurements at pressures down to 0.1 mm Hg and frequencies of 3 to 5 mc show that with sufficiently low measurement voltages the conductivity increases considerably with increased frequency. The corresponding increase of the dielectric constant is only slight.

538.56:537.525.6 383
Longitudinal and Transverse Electrical Waves in Homogeneous Moving Plasma—W. O. Schumann. (*Z. angew. Phys.*, vol. 3, pp. 178–181; May, 1951.) For an em oscillation propagated in the plasma in the direction of its motion, the charge density and velocity of the two types of wave generated are simply determined by using the relativistic transformation from the stationary state. For this method to be applicable, both plasma and phase velocity must be sufficiently low. See also 717 of 1951.

538.652 384
Magnetostriction in a Rotating Field—T. Delourmel and A. Herpin. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 239–241; July 16, 1951.) Theory previously developed for the case of a magnetizing field in a given direction is extended to cover the case of magnetization by a transverse rotating field, the distribution of magnetization of the individual domains being then asymmetrical, and maximum distortion occurring in a direction at an angle to the mean direction of magnetization. This theory is illustrated by the Wiedemann effect, and explains the sensitivity of the effect to mechanical influences.

GEOPHYSICAL AND EXTRATERRESTRIAL PHENOMENA

523.5:621.396.9 385
The Summer Daytime Meteor Streams of 1949 and 1950: Part 1—Measurement of the Radiant Positions and Activity—A. Aspinall and G. S. Hawkins. (*Mon. Not. R. Astr. Soc.*, vol. 111, no. 1, pp. 18–25; 1951.) Radio echo observation of summer daytime meteor streams during 1949 and 1950 have been made with improved continuously recording equipment. Simultaneous photographic records were obtained using two narrow-beam antennas directed along different azimuths. From these observations the right ascension and declination of a major stream may be determined to within ±1.5°. Three permanent streams, seen each year, have been identified. Other streams of a sporadic or long-period character were observed. Part 2: 386 below (Davies and Greenhow). Part 3: 387 below (Almond).

523.5:621.396.9 386
The Summer Daytime Meteor Streams of 1949 and 1950: Part 2—Measurement of the Velocities—J. G. Davies and J. S. Greenhow. (*Mon. Not. R. Astr. Soc.*, vol. 111, no. 1, pp. 26–36; 1951.) Observations of meteor velocities made simultaneously with the directional observations described in part 1 are discussed, and tables and histograms given. The velocities of meteors belonging to each stream have a considerable spread. Part 1: 385 above (Aspinall and Hawkins). Part 3: 387 below (Almond).

523.5:621.396.9 387
The Summer Daytime Meteor Streams of 1949 and 1950: Part 3—Computation of the Orbits—M. Almond. (*Mon. Not. R. Astr. Soc.*, vol. 111, no. 1, pp. 37–44; 1951.) The orbits of four daytime meteor streams have been computed from the observational data given in parts 1 and 2. The orbit of the main day-time stream indicates an association with a known visual night-time stream, while another orbit appears to be coincident with that of Encke's comet. Part 1: 385 above (Aspinall and Hawkins). Part 2: 386 above (Davies and Greenhow).

523.746 388
Forecasting Sunspot Variations to 1957—A. F. Wilkins. (*Wireless Eng.*, vol. 28, pp. 298–299; Oct., 1951.) "Published forecasts of the variation of twelve-month running averages of relative sunspot numbers for the period succeeding the 1947 maximum, and up to the next maximum, are presented. Although the trend of sunspot number in the immediate future has been obscured by an unexpected variation in the observed values following the 1947 maximum, it is considered that the forecasts should be suitable for the purposes of planning radiocommunication services for most of the remainder of the present sunspot cycle."

523.75:550.38 389
Geometric Solar-Flare Effects at Lerwick and Eskdalemuir, and Relationship with Allied Ionospheric Effects—D. H. McIntosh. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 315–342; 1951.) Geomagnetic data for the period 1936 to 1949 are tabulated, and annual and daily variations are discussed in relation to solar flares and sw radio fade-outs.

523.854:621.396.822 390
The Origin of Galactic Radio-Frequency Radiation—J. H. Piddington. (*Mon. Not. R. Astr. Soc.*, vol. 111, no. 1, pp. 45–63; 1951.) "The results of observations of the intensity and distribution of radio-frequency radiation from the galaxy at frequencies from 9.5 to 3,000 mc have been collected. Some of these data are used to determine spectrum curves of the radiation from chosen regions of the galaxy. Using the equations defining the propagation of radio waves in an ionized gas, the forms of spectrum curves from distributions of gas are determined. It is shown that the power flux of radio energy from a thermally radiating gas cloud increases with frequency except under special conditions which are defined. From a comparison of the observed and the theoretical spectrum curves, it is shown that galactic radiation at radio frequencies probably originates partly in hot ionized interstellar gas and partly in stellar atmospheres. The ionized gas provides most of the radiation at the higher frequencies and evidences itself by absorption at the lower. The properties of the stellar sources are discussed."

550.38 "1951.01/.07" 391
Indices of Geomagnetic Activity of the Observatories Abinger (Ab), Eskdalemuir (Es) and Lerwick (Le), January to July 1951—(*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 376–380; 1951.)

551.51 392
The Thermal Excitation of Atmospheric Oscillations—M. V. Wilkes. (*Proc. Roy. Soc. A*, vol. 207, pp. 358–370; July 6, 1951.)

551.510.52:551.524.7 393
Mean Meridional Temperature Profile in the Troposphere as affected by Vertical and Horizontal Exchange Processes and the Heat of Condensation—F. Defant. (*Arch. Met. Geophys. Bioklimatol. A*, vol. 2, pp. 184–206; March 28, 1950.)

551.510.52:621.396.11.029.62 394
The Effective Height of Tropospheric Inversion Layers in Ultra-Short-Wave Propagation—Scholz (*See* 476.)

551.510.535 395
Magnetic and Ionospheric Storms—E. Appleton. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 113–119; November 15, 1950. In English.) The characteristic ionospheric-storm variation of F_2-layer ionization at a medium-latitude station is compared with corresponding effects for other latitudes. The well known marked variability of the F_2-layer critical frequency is attributed in part to the operation of the ionospheric storm mechanism. Lack of correlation between variations of F_2 ionization at widely separated sites is due to variations of sign and magnitude of the ionospheric storm manifestation with geomagnetic latitude, and to a certain dependence on local time.

551.510.535 396
A Critical Survey of Ionospheric Temperatures—N. C. Gerson. (*Rep. Progr. Phys.*, vol. 14, pp. 316–365; 1951.) The various methods available for the evaluation of temperature from an ionospheric parameter are discussed in detail, and the relative order of importance is given. The probable temperature/altitude relation is tabulated for the range 100 to 400 km. The electron and gas temperatures are essentially identical. Experimental evidence does not indicate a totally ionized atmosphere below 400 km. Thermal equilibrium, and therefore a Maxwellian velocity distribution, exists among the gaseous constituents from 100 km to 400 km, with no large diurnal temperature variations. It is considered that a temperature maximum, probably arising from the various absorption or heating processes together with a sufficient number density of particles, may occur in the ionosphere at about 400 km.

551.510.535 397
The phase and Group Paths of Radio Waves Returned from Region E of the Ionosphere—J. W. Findlay. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 353–366; 1951.) Equipment is described which has been used to measure the changes of phase path of radio waves of frequency 2.4 mc returned from region *E* of the ionosphere. The daily variation of phase path cannot be explained quantitatively on the assumption that region *E* behaves as a simple Chapman region, although there is qualitative agreement with the theory. The seasonal changes of noon values of the group path are studied and found to disagree with those predicted for a Chapman region. Some evidence for the existence of a solar tide in the height of region *E* is given.

551.510.535 398
The Determination of the Collision Frequency of Electrons in the Ionosphere from Observations of the Reflection Coefficient of the Abnormal *E* Layer—B. H. Briggs. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 345–348; 1951.) The reflection and transmission coefficients of the *E* layer are deduced from simultaneous measurements of the amplitudes of echoes returned from the *E* and *F* layers. The results are consistent with the assumption of a layer about 5 km thick in which the collision frequency is about 2×10^6 per second at a well defined height.

551.510.535 399
Results of Ionospheric Observations made near Adélie Land—M. Barré and K. Rawer. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 311–314; 1951. In French.) Photographic *h'f* traces recorded 470 km from the South magnetic pole are reproduced. Echoes from all regions, and particularly from the F_2 region, are often very scattered. "High level" *E* echoes and other unusual records are probably due to oblique-incidence reflection from ionic clouds. See also 112 and 2997 of 1951.

551.510.535 400
Solar Tides in the Ionosphere Over Calcutta—A. P. Mitra. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 286–295; 1951.) The data for the F_2 region over the four-year period January 1946 to January 1950 are examined, using an extension of Martyn's analysis which includes the effects of both recombination and vertical transport of ions. Numerical values of the resultant vertical drift velocity and of the recombination coefficients at Calcutta are deduced. The anomalous changes in F_2-layer ionization are explained.

551.510.535 401
Effects of Temperature Variations of the Upper Atmosphere on the Formation of Ionospheric Layers: F_2 Layer Anomaly: F_2 and F_1 Separation: F_2 Layer Behaviour during Magnetic Storms—D. Lepechinsky. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 278–285; 1951.) An increase in temperature (*T*) causes a rise in the height of a Chapman layer, accompanied by a fall in the rate of ion production (*I*). Assuming a diurnal or seasonal temperature cycle, the *I* curve at constant height may show a maximum or a minimum at the time when *T* is highest, depending on the height chosen. The changes in height of the F_2 layer at Slough can be explained by a 46 per cent rise in *T* at summer noon as compared with winter noon. The corresponding rise between 0600 and noon in summer would be 21 per cent. The splitting of the *F* layer into F_1 and F_2 layers and certain storm characteristics can also be explained by this theory. See also 2995 of 1951 (Lejay and Lepechinsky).

551.510.535 402
Studies of the Tidal Effect in the Ionosphere—O. Burkard. (*Jour. Atmos. Terr. Phys.*, vol. nos. 5/6, pp. 349–352; 1951. In German.) The F_2-layer critical frequency at Huancayo exhibits a luni-solar variation of the same nature as the variations of the geomagnetic components. The semidiurnal lunar variation is pronounced only during the daytime, and is practically nonexistent during the night.

551.510.535:537.562 403
Plasma Resonance in Ionospheric Irregularities—N. Herlofson. (*Ark. Fys.*, vol. 3, no. 15, pp. 247–297; 1951. Reprint.) A critical review of the experimental and theoretical deductions from meteor data. A wave theory of reflection is developed which includes the theory of Lovell and Blackett as a limiting case. The effects of induced currents are fully discussed and expressions for the scattering cross section as a function of radius/wavelength are deduced for uniform, parabolic, exponential, and Gaussian distributions fo electron density. The physical causes of the differences between parallel and sagittal scattering are discussed in detail, and the conditions under which the scattering power can be greatly modified by plasma oscillations are found. Plasma oscillations can also be generated by small irregularities in the ionosphere at levels near the point of reflection of radio waves. The presence of these oscillations should be detectable experimentally. Suggestions for testing the full theory experimentally are included.

551.510.535:621.396.11 404
An Investigation of Sudden Radio Fadeouts on a Frequency near 2 Mc/s—J. W. Findlay. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 367–375; 1951.) Experiments to determine the height at which the extra ionization occurs during fade-outs are described. Observations were made of phase-path variations during 105 fade-outs. The collision frequency at the height of greatest extra ionization is estimated to be of the order of 4×10^5 per second, the corresponding height being <101 km.

551.510.535:621.396.6 405
Echo Sounding Equipment for Ionosphere and Weather Research—R. Schrott. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 109–112; November 15, 1950.) Description of pulse transmitter and receiver developed at Graz for measurement of ionosphere critical frequencies and layer heights, etc.

551.510.535:621.396.822 406
Ionospheric Thermal Radiation at Radio Frequencies—J. L. Pawsey, L. L. McCready, and F. F. Gardner. (*Jour. Atmos. Terr. Phys.*, vol. 1, nos. 5/6, pp. 261–277; 1951.) Radio noise on medium frequencies includes a component due to thermal radiation from the ionosphere. Methods used to identify and measure this radiation are described; observations are made far from towns and during midday hours, on frequencies between 1 and 2 mc, so as to avoid both man-made noise and atmospherics as far as possible. The magnitude of the thermal radiation observed corresponds to an effective temperature of about 300°K, or a field strength of about 10^{-9} v/m in a frequency band of 1 kc. The method is useful for studying the electron temperature in the absorbing (and emitting) region of the ionosphere, which is at a height of 70 to 80 km for the frequencies used. The measured temperatures, 240° to 290°K, agree with other observations of temperatures at these heights.

551.594 407
Studies of the Atmospheric Potential Gradient: No. 4—Current Theory of the Electric Field in the Air—H. W. Kasemir. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3,

pp. 84–97; November 15, 1950.) Benndorf's theory, according to which atmospheric electric currents are due to meteorological phenomena, is applied particularly to investigate the thunderstorm as energy source for these currents.

551.594 408
Air-Earth Current Observations in Various Localities—A. R. Hogg. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 40–55; November 15, 1950. In English.) Observations at a number of stations indicate a statistical relation between the conductivity of the atmosphere near the ground and the air-earth conduction current. An estimate of the annual variation of air-earth current over the whole globe gives general support to the thunder-storm theory of the maintenance of the earth's electric charge.

551.594 409
The Development of Fundamental Ideas on Atmospheric Electricity—H. Israël. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 1–16; November 15, 1950.)

551.594.12 410
Variations of Ion Concentration near the Ground—H. Norinder and R. Siksna. (*Arch. Met. Geoph. Bioklimatol., Ser. A*, vol. 3, pp. 29–39; November 15, 1950.) Measurements made at Uppsala indicate time variations of ion concentration unrelated to meteorological phenomena. In some cases they may be due to accumulation of radioactive substances near the ground.

551.594.13 411
The Control of the Electrical Conductivity of the Lower Atmosphere—J. J. Nolan. (*Arch. Met. Geoph. Bioklimatol., Ser. A*, vol. 3, pp. 17–28; November 15, 1950. In English.) Methods are described for counting the condensation nuclei, and for determining their sizes and masses. The difficulty of establishing a simple relation between atmospheric conductivity, rate of ion production, and nucleus concentration is enhanced by the tendency to an increase of airborne radioactive matter with increase of nucelus concentration.

551.594.14 412
New Studies on the Radioactivity of the Atmosphere—V. F. Hess. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 56–63; November 15, 1950. In English.) Report of methods used and values found in an experimental investigation of the ionization due to the γ radiation from radioactive substances near the ground, and of the total ionization produced by these substances.

551.594.21 413
The Distribution of Electricity in Thunderclouds—B. F. J. Schonland and D. J. Malan. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 64–69; November 15, 1950. In English.)

551.594.22 414
Measurements by Airplane of Electric Charge passing Vertically through Thunderstorms to Ground—G. R. Wait. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 70–76; November 15, 1950. In English.)

551.594.6 415
Progress in World Radio Meteorology and Weather Prediction—J. Lugeon. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 120–138; November 15, 1950. In French.) Investigations of the relations between atmospherics and synoptical meteorology from 1920 onwards are summarized. An account is given of up-to-date methods of observation, and some results are discussed. Lines are indicated for the organization of radiometeorological networks.

551.594.6 416
Analysis of Audio-Frequency Atmospherics—R. K. Potter. (PROC. I.R.E., vol. 39, pp. 1067–1069; September, 1951.) Audio-frequency atmospherics have been reported by long-wave-radio and ocean-cable engineers, and are known as whistlers, swishes, tweeks, and rumblers. These have been studied by means of the sound spectrograph which displays a graph of frequency against time. The different types of frequency and overtone variation with time are described.

LOCATION AND AIDS TO NAVIGATION

621.396.9:523.5 417
The Summer Daytime Meteor Streams of 1949 and 1950. (*See* 385–387.)

621.396.933 418
Miniature Radar Transponder Beacon—R. S. Butts. (*Electronics*, vol. 24, pp. 104–107; September, 1951.) Description of aircraft equipment which, on receipt of radar signals, automatically transmits a reply to the interrogating station, thus extending the radar range obtainable with passive reflection alone and making possible the determination of aircraft azimuth as well as range. The equipment weighs under 4 pounds and comprises receiver, temperature-compensated coaxial cavity resonator, and transmitter with pulse modulator. Potting of receiver and modulator in resilient casting resin reduces shock effects.

MATERIALS AND SUBSIDIARY TECHNIQUES

533.5:.535.61–31 419
Application of Optical Absorption in the Far Ultraviolet to the Detection of Leaks in Vacuum Apparatus and to the Measurement of Low Pressures—J. Romand, V. Schwetzoff and B. Vodar. (*Le Vide*, vol. 6, pp. 1046–1051; July/September, 1951.) Devices are described in which a beam of ultraviolet rays, of wavelength in the range 1,200 to 1,850 Å, traverses the gas under examination before impinging on a photoelectric target. Preliminary test results are presented.

535.215 420
Energy Levels of F-Centres—L. Pincherle. (*Proc. Phys. Soc.*, vol. 64, pp. 648–657; July 1, 1951.) The possible association between the existence of F centers and photoconductivity in ionic crystals is investigated.

535.37 421
Inhibiting Action of Iron on the Luminescence of Zinc Sulphide—N. Arpiarian. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 387–389; July 30, 1951.) Specimens of ZnS containing known amounts of Cu activator and of Fe were exposed to ultraviolet light at a temperature of 20°C. Curves are given showing the decay of the luminescence at various time intervals after removal of the excitation, for various compositions. The results are compared with those obtained using Ni or Co as inhibitor, and the theory of the effect is discussed.

537.228.1:548.0:547.476.3 422
Theory and Measurement of the Piezoelectricity of Rochelle Salt—S. Honda. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 117–140; January, 1951.) In order to study the characteristics and applications of crystals such as Rochelle salt, it is convenient to use the polarization theory. The general piezoelectric equations according to this theory are deduced. The electric, elastic, and piezoelectric constants of Rochelle salt X-cut plates were measured, the last by a newly developed direct dynamic method.

537.311.33 423
Change of Activation Energy with Impurity Concentration in Semiconductors—L. Pincherle (*Proc. Phys. Soc.*, vol. 64, pp. 663–664; July 1, 1951.) Analysis of the effect of screening by free carriers of the field around a trapping center, resulting in a potential of the form $V=(e^2/\epsilon r)\exp(-r/r_0)$. The case of Si is considered quantitatively for various impurity concentrations and temperatures; comparison of theoretical and experimental values of activation energy suggests that although screening may not be the main cause of the variation, it can be significant.

537.311.33 424
The Theory of Contact between Two Semiconductors of the Same Type of Conductivity—A. I. Gubanov. (*Zh. tekh. Fiz.*, vol. 21, pp. 304–315; March, 1951.) The theory of the rectifying effect of the contact between a metal and a semiconductor is extended to cover the case of two semiconductors with the same type of conductivity. The effect in the latter case is smaller, for the same contact potential difference, than that of the contact between a metal and a semiconductor. It is particularly small with a large current, when, under certain conditions, rectification in the opposite direction may also appear. In the limiting case when the conductivity of one of the semiconductors is considerably greater than that of the other, all the formulas of the proposed theory are reduced to the well known formulas for the case of the contact between a metal and a semiconductor. This enables the limits of applicability of the theory to be established.

537.311.33:546.816.221 425
Lead Sulphide—an Intrinsic Semiconductor—E. H. Putley and J. B. Arthur. (*Proc. Phys. Soc.*, vol. 64, pp. 616–618; July 1, 1951.) Measurements of the Hall coefficient and conductivity of PbS over wide ranges of temperature and effective purity show that p-type and n-type specimens give the same results at sufficiently high temperatures. PbS shows intrinsic conductivity with an energy gap of about 1.2 ev, resembling PbTe.

538.221 426
Torque Curves and other Magnetic Properties of Alcomax—K. Hoselitz and M. McCaig. (*Proc. Phys. Soc.*, vol. 64, pp. 549–559; July 1, 1951.)

538.221 427
Study of Macroscopic Magnetic Texture—I. Épelboin. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 358–360; July 30, 1951.) It is possible by thermal and electrolytic treatments to influence separately the various features determining the macroscopic magnetic texture, even of relatively thick specimens of high-permeability alloys. This affords a method of investigating the composition and the magnetic behavior of such alloys in weak alternating fields.

621.316.993 428
Impulse Resistance of Different Earth Electrodes and Embedding Materials—H. Norinder and O. Salka. (*Bull. schweiz. elektrotech Ver.*, vol. 42, pp. 321–327; May 19, 1951. In German.)

621.318.4.042.15 429
Loss Angle of Manganese-Ferrite Cores—A. Weis. (*Elektrotechnik* (Berlin), vol. 5, pp. 213–216; May, 1951.) Curves are given showing the loss angle of Mn-ferrite and MnZn-ferrite as dependent on (a) frequency to 100 kc, (b) field strength to 90 millioersted. An ultrasonic resonance effect was observed in a Mn-ferrite ring core at 44.5 kc. This was investigated both with and without an es screen round the core.

621.396.611.21:549.514.51 430
Crystal-Plates without Overtones—J. J. Vormer. (PROC. I.R.E., vol. 39, pp. 1086–1087;

September, 1951.) For a similar account see 1169 of 1951.

MATHEMATICS

517.35:621.319.4 431
A Solution for $\int e^{b(x+a\cos x)}dx$—K. Emden: H. Rutishauser. (*Z. angew. Math. Phys.*, vol. 2, pp. 289–293; July 15, 1951.) The integral occurs in the expression for the charge on a capacitor, in series with a resistor and battery, when the plate distance is varied periodically. The integral $\int e^{b(y+a\sin y)}dy$ is easily derived by the substitution $y=x+\pi/2$. Rutishauser gives a simpler method of determining the Fourier coefficients in the solution.

517.432.1:621.392.5 432
Time-Dependent Heaviside Operators—L. A. Zadeh. (*Jour. Math. Phys.*, vol. 30, pp. 73–78; July, 1951.) Discussion of some of the basic properties. See also 1617 of 1950.

517.512.2 433
A Series Expansion of the Fourier Integral—W. A. Whitcraft, Jr. (Proc. I.R.E., vol. 39, pp. 1087–1089; September, 1951.) "Evaluation of the Fourier integral by parts leads to two different series representations of a time function in the frequency domain. These series involve powers of the frequency variable and either derivatives or integrals of the function, evaluated at the upper time limit only."

681.142:531.764.5 434
A Digital-Computer Timing Unit—R. M. Goodman. (Proc. I.R.E., vol. 39, pp. 1051–1054; September, 1951.) Description of the time-reference equipment for the EDVAC computer, and discussion of the application of electronic matrices to time-pulse production.

501.517.944.3 435
Integral Transforms in Mathematical Physics [Book Review]—C. J. Tranter. Publishers: Methuen, London, Eng. and J. Wiley and Sons, New York, N. Y., 1951, 118 pp., 5s. (*Wireless Eng.*, vol. 28, p. 315; Oct., 1951.) One of Methuen's Monographs on Physical Subjects. "The object has been to show the reader what techniques are worth trying for the solution of various types of problem."

517.516 436
Fastperiodische Funktionen (Nearly Periodic Functions) [Book Review]—W. Maak. Publishers: Springer-Verlag, Berlin, Germany, 1950, 240 pp., 24.60 DM. (*Z. angew. Math. Phys.*, vol. 2, pp. 306–307; July 15, 1951.) Concerned exclusively with the generalizations of Bohr's theory of nearly periodic functions.

MEASUREMENTS AND TEST GEAR

621.317.001.5:519.2 437
An Outline of the Principles of Design and Analysis of Experiments—D. L. Richards. (*P.O. Elect. Eng. Jour.*, vol. 44, pp. 55–60; July, 1951.) Discussion of concepts underlying statistical techniques used at the P.O. Research Station in connection with investigations involving human subjects.

621.317.3:550.372 438
Ground Conductivity Measurements in Italy—G. Galligioni. (*Alta Frequenza*, vol. 20, pp. 119–127; June/August, 1951.) A conductivity map of the whole of Italy has been prepared by Radio Italiana, based on about 3,000 measurements of the field strength of Italian broadcasting stations. The method is described, and the reliability of the results is discussed. They can be used to predict field strength for different sites, frequencies, power values, and antenna types.

621.317.3:621.392.52 439
Fundamental-Wave Filter for Instruments and Measurement Circuits—Poleck. (*See* 352.)

621.317.321 440
The Measurement of High Alternating Voltages by means of Capacitive Voltage Dividers (C-Measurement)—P. Böning. (*Arch. tech. Messen*, no. 184, pp. T49–T52; May, 1951.) The development of the basic circuits particularly considered is the insertion of an inductor in the meter circuit connected across one of the two capacitors comprising the divider. Three conditions, depending on choice of the value of the inductor, are considered, and vector diagrams showing voltage and current distribution are constructed to determine the optimum values of the components in the measurement circuit.

621.317.335.3†:621.317.755 441
Investigation of Dielectrics by Means of a Cathode-Ray Oscillograph—N. S. Novosil'tsev. (*Zh. tekh. Fiz.*, vol. 21, pp. 369–374; March, 1951.) A description is given of an oscillograph with which the dielectric constant and leakage can be determined for substances with a single relaxation time. With a little practice it is possible to determine directly from the image on the screen the amount of absorption, the presence of several mechanisms, the speed of their operation, and the magnitude of losses. The effects of temperature and humidity on the dielectric properties are shown very clearly.

621.317.335.3.029.64† 442
New Method for Determination of Dielectric Constants in the cm-Wave Range—E. Ledinegg and E. Fehrer. (*Acta Phys. austriaca*, vol. 3, pp. 82–110; June, 1949.) A comprehensive mathematical treatment of the method in which a plate of the material to be tested is inserted in a cylindrical cavity resonator. Field contours for fundamental and harmonic resonance are shown graphically and useful approximate formulas are given.

621.317.35 443
The Experimental Representation of Modulated Currents and their Comparative Examination by means of a Simple Test Assembly—M. Kreuzritter. (*Fernmeldetech. Z.*, vol. 4, pp. 320–324; July, 1951.) Description of multi-purpose instrument for analysis of AM and FM signals. It is mains-operated, and incorporates an electromechanical modulation device and a vibrating-reed indicator.

621.317.7 444
Moving-Coil Meters. External and Core-Type Magnetic System. Pivot Bearing and Tension-Strip Suspension—A. Hug. (*Bull. schweiz. elektrotech. Ver.*, vol. 42, pp. 385–389; June 2, 1951. In German.) Practical requirements are discussed, and the advantages and disadvantages of the different types are compared. The possibility of combining their good features is briefly considered.

621.317.73 445
Impedance Measurements in the 50–2000 Mc/s Range—R. A. Soderman. (*Radio & Telev. News, Radio Electronic Eng. Sect.*, vol. 46, pp. 3–6, 25; July, 1951.) A direct-reading admittance meter, which can be balanced like a bridge and is suitable for the frequency range 50 to 1,000 mc, comprises a T arrangement of three coaxial lines fed at their common junction. When used together with a special constant-impedance adjustable-length line, the upper frequency limit is raised to 2,000 mc.

621.317.733 446
The Wheatstone Bridge with Load-Dependent Resistances: Part 1—Fundamentals—G. Nidetzky. (*Arch. tech. Messen*, no. 184, pp. T55–T56; May, 1951.) Analysis of the effect of input voltage fluctuations in bridge circuits including resistors such as gas-filled tungsten-filament lamps, iron-in-hydrogen barretters, carbon-filament lamps, semiconductors, and gas discharge lamps.

621.317.75:621.397.5 447
Television Waveform Display—K. R. Sturley. (*Wireless Eng.*, vol. 28, pp. 261–265; September, 1951.) The construction of a simple unit for the visual examination of television wave forms is described. The 50-cps mains supply is used as a driver source, and 1 to 20 lines of both frames may be selected for separate and simultaneous display. Slow drift of the wave form, due to afc of the frame-synchronizing pulse at the transmitter, can be corrected manually, or the frame-scan generator of the receiver may be used to provide the driver source. The required additional equipment is described.

621.317.755:621.396.67.012 448
Oscillograph Field Plotter—C. Susskind and A. R. Perrins. (*Electronics*, vol. 24, p. 140; September, 1951.) The position of a probe moving in the field of a microwave radiator is correlated by a potentiometer arrangement with that of the spot on the screen of a cro. The intensity of the spot is controlled by the signal picked up by the probe, and the field pattern is recorded photographically with a time exposure.

621.317.784 449
Power Meter and Mismatch Indicator—A. F. Boff. (*Wireless Eng.*, vol. 28, pp. 278–281; September, 1951.) A simple directional power meter is described for the frequency band 150 to 250 mc. The instrument is based on a directional coupler consisting of a loop inserted into a short coaxial line and loosely coupled to both the electric and magnetic fields in such a way that components cancel for waves in one direction and add in the other direction. Forward and reflected power in a transmission line are indicated simultaneously on two meters reading up to 10w, with negligible loading of the line.

621.319.4 (083.74) 450
R.F. Standard Capacitors for Minute Increments—J. A. Conner. (*Electronics*, vol. 24, pp. 250, 262; October, 1951.) The errors inherent in coaxial-cylinder capacitors are analyzed, with emphasis on the design of incremental standards with variations of less than 10 pF per inch. Design details are given for a differential standard with a linear capacitance variation of 0.2 pF per inch of inner-electrode insertion, accurate to within 1 per cent over the range of movement, and with a residual uncertainty of only 0.000 15 pF.

621.396.615.029.3 451
Wide-Range Sweeping Oscillator—Rosenthal. (*See* 360.)

621.396.677.5.089.6† 452
Checking Calibration of Loop Antennas—P. C. Gardiner. (*Electronics*, vol. 24, pp. 140, 250; October, 1951.) Description of a laboratory method of calibrating loop antennas for field-strength meters, in which the calibrating field is provided by a signal generator feeding a loop through a high impedance. This loop is concentric and coplanar with the receiving loop to be calibrated, has a smaller radius, and should have a smaller inductance. The number of turns in the receiving loop must be known, and the mutual inductance between the loops must be calculated or measured. Design data are discussed.

OTHER APPLICATIONS OF RADIO AND ELECTRONICS

612.8:621.39.001.11 453
A Particular Problem of Signal Transmission [in the nervous system]—A. Goudot. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 290–292; July 23, 1951.) The nervous system is compared with normal signalling circuits. Various analogies exist. Selective trans-

mission by certain nerve fibres is compared with propagation in waveguides.

620.179.14 454
Crawler detects Gun-Barrel Cracks—R. D. Kodis and R. Shaw. (*Electronics*, vol. 24, pp. 92–95; September, 1951.) After the gun barrel has been circularly magnetized, its inner surface is scanned by a pickup coil that rotates at 900 or 1,800 rpm round a central core. A servo system keeps the motion of the coil in step with the recording equipment.

620.179.16 455
Metal Wall Thickness Measurement from One Side by the Ultrasonic Method—N. G. Branson. (*Elec. Eng.*, vol. 70, pp. 619–623; July, 1951.) 1951 AIEE Winter General Meeting paper. The method is based on setting up standing waves in the metal. The thickness is deduced from the measurement of two frequencies at which the metal is in thickness resonance. The circuit of a practical instrument, normally using frequencies in the range 1 to 20 mc is given. Limitations of the method are discussed.

621.316.7.076.7 456
The Electro-analogue, an Apparatus for Studying Regulating Systems: Part 2—The Electrical Execution—J. M. L. Janssen and L. Ensing. (*Philips tech. Rev.*, vol. 12, pp. 319–335; May, 1951.) Detailed discussion of various parts, including the process analogues, the universal amplifier, the combining stages, the integrator and differentiator, models of discontinuously acting controllers, the oscilloscope, the supply, and the mechanical construction. Part 1: 2782 of 1951.

621.365.54/.55† 457
Economics and Applications of High-Frequency Heating—G. Lang. (*Bull. schweiz. elektrotech. Ver.*, vol. 42, pp. 328–342; May 19, 1951. In German.) Illustrated review of development of industrial heating techniques, with extensive bibliography.

621.365.55†:621.396.615.141.2 458
Magnetrons for Dielectric Heating—Nelson. (*See* 556.)

621.385.833 459
Resolving Power and Magnification Limit of the Magnetic Electron-Microscope as dependent on the Accelerating Voltage and Magnetic Field Strength—W. Glaser. (*Acta Phys. austriaca*, vol. 3, pp. 38–51; June, 1949.)

621.385.833 460
New Formulae for Third-Order Aberrations in Electrostatic Lenses—P. Sturrock. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 243–245; July 16, 1951.)

621.385.833 461
The Electrostatic Lens as Velocity Filter with High Resolving Power—G. Möllenstedt and O. Rang. (*Z. angew. Phys.*, vol. 3, pp. 187–189; May, 1951.)

621.385.833 462
Graphical Method for Approximate Determination of [charge] Carrier Paths in Electrostatic Lenses, taking account of the Effects of Space Charge—W. Walcher. (*Z. angew. Phys.*, vol. 3, pp. 189–190; May, 1951.)

621.385.833 463
Calculation of the Optimum Focusing of a Beam by an Electron Lens, taking account of Space Charge—M. D. Gabovich. (*Zh. tekh. Fiz.*, vol. 21, pp. 363–368; March, 1951.)

621.387.424† 464
A Directive Effect in Geiger-Müller Counters—A. Rogozinski. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 426–428; July 30, 1951.) The rate of rise of the pulse in a G-M counter is most rapid when the angle between the trajectory of the discharging particle and the axis of the counter is zero. Thus a counter circuit which selects the most sharply rising pulses will register only particles traveling parallel to the counter axis.

621.387.424† 465
Factors affecting the Stability of Geiger-Müller Counters with Thin End Window (Argon-Alcohol-Filled Bell-Shaped Counters)—A. Papineau. (*Jour. Phys. Radium*, vol. 12, pp. 667–670; June, 1951.)

621.398:621.315 466
Electronic Equipment for Telecontrol in High-Voltage Networks—F. Burlando. (*Bull. schweiz. elektrotech. Ver.*, vol. 42, pp. 352–356; May 19, 1951. In French.) Description of a system in which a 900-VA Wien-bridge oscillator is inductively coupled to a 25-MVA line, thus superimposing a control signal, variable from 50 to 100 kc, on the mains frequency. The oscillator output is automatically controlled by a thyratron circuit suitably adjusted to the impedance of the line.

PROPAGATION OF WAVES

538.566 467
On the Propagation of Electric Waves from a Horizontal Dipole over the Surface of the Earth Sphere—Y. Nomura. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 25–49; January, 1951.) The electromagnetic field generated by a horizontal electric or magnetic dipole on or over a finitely conducting spherical surface is considered, and formulas for the magnitude of each of the spherical-coordinate components of the electric and magnetic forces are obtained. The field of the horizontal dipole is calculated from that of the vertical dipole. An approximate expression based on optical ray theory is also given.

538.566 468
Surface Waves 'ad infinitum'—H. Ott. (*Arch. elekt. Übertragung*, vol. 5, pp. 343–346; July, 1951.) A reply to criticism of the author's theory by Kahan and Eckart (2517 of 1951), pointing out that in their treatment they have extended to all values of $\pm$ and any loss angle a result which is only valid for propagation over an absorption-free earth as $r \rightarrow \infty$. With correct treatment, results previously given by the author are confirmed.

538.566 469
Surface Wave in Dipole Radiation over Plane Earth—T. Kahan and G. Eckart. (*Arch. elekt. Übertragung*, vol. 5, pp. 347–348; July, 1951.) A reply to Ott (468 above) maintaining the correctness of their conclusions, which are shown to be confirmed by the results of several other investigators.

621.396.11 470
Investigation of Ionospheric Propagation and RF Interaction by means of Electrical Models—M. Carlevaro. (*Alta Frequenza*, vol. 19, pp. 185–210; August, 1950.) On the basis of the well-known analogy between the propagation of plane em waves in a homogeneous medium and the propagation of voltage or current waves along a transmission line, models for demonstrating propagation phenomena in ionized and magnetoactive media are constructed from transmission lines or symmetrical ladder networks. The limits of validity of the method are examined, and various examples are presented. A detailed description is given of a proposed ladder network for demonstrating the Italian experiments on gyrointeraction carried out during the last three years. The method may be useful in eliminating algebraic calculations not only in ionospheric propagation problems but also in the study of electromagnetic screens.

621.396.11 471
The Effective Velocity of Propagation of Short Radio Waves—J. Fuchs. (*Arch. Met. Geoph. Bioklimatol, A*, vol. 3, pp. 139–152; November 15, 1950.) For making the necessary corrections to time signals transmitted by short waves, their propagation time between transmitter and receiver must be known accurately. A theory is developed according to which the effective propagation velocity v_{eff} is a function of angle of incidence and apparent height of reflection; values of v_{eff} corresponding to angles of 30°, 20°, 10°, and 0° are, respectively, 246,000 km, 268,000 km, 281,000 km and 290,500 km. The principles of methods used for calculating v_{eff} for any distances and ionization conditions are discussed with the aid of world maps of critical frequencies, and a new method of calculating apparent height of reflection is proposed.

621.396.11 472
Wave Propagation in the Ionosphere at Oblique Incidence, taking account of the Earth's Magnetic Field: Part 2—K. Försterling. (*Arch. elekt. Übertragung*, vol. 5, pp. 209–215; May, 1951.) Continuation of 3518 of 1949. The equations of propagation are derived and discussed for large angles of incidence, assuming dielectric constant to be a known function of height.

621.396.11:551.510.535 473
Time-Delay Measurements on Radio Transmissions. Results on Medium Frequencies—R. Naismith and E. N. Bramley. (*Wireless Eng.*, vol. 28, pp. 271–277; September, 1951.) The time delay of the first ionospheric echo with respect to the ground wave, and the variation of this delay, were measured at distances from zero up to 1,200 km, at frequencies between 0.7 and 2.0 mc. The equivalent height of the main reflecting region was in the range 90 to 97 km, but other regions at 70 to 76 km, 105 to 110 km, and 120 to 130 km were observed under certain conditions. These results indicate that stratification and partial-reflection phenomena have an important influence on oblique propagation at medium frequencies.

621.396.11:621.317.353.3† 474
Ionospheric Cross-Modulation—G. R. Mather. (*Electronics*, vol. 24, pp. 252, 260; September, 1951.) Controlled experiments on the Luxemburg effect were carried out on five consecutive nights, using two broadcasting transmitters with frequencies around 1.1 mc, the disturbing power radiated being over 100 kw. Measurements at a distance of 555 miles from the wanted transmitter, using a wave analyzer in conjunction with a receiver, showed that the carrier of the wanted transmission was modulated to a depth of 0.6 to 0.75 per cent by the unwanted transmission. This was a smaller transfer of modulation that had been expected from theory.

621.396.11.029.55:523.745 475
Variation of Angle of Arrival of Short Waves in Transatlantic Communications due to the Influence of the Sunspot Cycle—H. Neyer and K. Rawer. (*Arch. elekt. Übertragung*, vol. 5, pp. 215–218; May, 1951.) Angle-of-arrival measurements made in 1939 to 1940 by Kotowski, Schüttlöffel, and Vogt (434 of 1951) and repeated in 1944, are discussed in relation to opposing views on the mechanism of long-distance propagation, viz., whether the waves are guided or suffer zig-zag reflection. The observations were made at Brück a M. on transmissions from 15 North American stations operating on frequencies from 6.10 to 17.83 mc. The observed variation of angle of arrival with frequency supported the theory of zig-zag reflections. Compared with the earlier measurements, the 1944 figures show a marked de-

crease of angle of arrival for a given frequency. This is due to the reduction of F_2-layer critical frequency corresponding to the changed phase of the sunspot cycle.

621.396.11.029.62:551.510.52 476

The Height of Tropospheric Inversion Layers Effective in Ultra-Short-Wave Propagation—W. Scholz. (*Fernmeldetech. Z.*, vol. 4, pp. 287–293; July, 1951.) Report and discussion of measurements made from 1938 to 1945 over a 64-km path in Germany. A 50-w transmittter radiated a signal on 206 mc, with 800-cps AM, from an antenna 135 m above ground. Received field strength 43 m above ground and temperature gradient up to a height of 240 m, measured by twelve thermometers at different heights at about the mid-point of the path, were recorded.

621.396.81 477

Reception of N.B.S. Standard Signals—C. Egidi. (*Alta Frequenza*, vol. 20, pp. 113–118; June to August, 1951.) Field-strength measurements made at Turin of the WWV 10-mc standard signals are reported. The measurements were made between 0700 and 0900 C.E.T. over the period 1947 to 1950.

621.396.11.029.6 478

Propagation of Short Radio Waves [Book Review]—D. E. Kerr (Ed.). Publishers: McGraw-Hill, New York, N. Y., 1951, 728 pp., $10.00. (*Electronics*, vol. 24, pp. 320, 324; October, 1951.) M.I.T. Radiation Laboratory Series, vol. 13. ". . . covers its field in a comprehensive fashion."

RECEPTION

519.272.15:621.39.001.11:621.396.621 479

Application of Correlation Analysis to the Detection of Periodic Signals in Noise—Y. W. Lee, T. P. Cheatham, Jr., and J. B. Wiesner. (Proc. I.R.E., vol. 39, pp. 1094–1096; September, 1951.) Discussion on paper abstracted in 730 of 1951.

621.396.621:621.396.712.2 480

Remote-Pickup Broadcast Receiver—A. A. Kelley. (*Electronics*, vol. 24, pp. 102–103; September, 1951.) A circuit diagram and details are given of a station crystal-controlled 26-mc receiver for programs from a remote transmitter. Simplicity ensures reliability and ease of operation and servicing.

621.396.621.015.7† 481

Superregenerative Receivers for Pulse Reception—S. Marmor. (*Ann. Télécommun.*, vol. 6, pp. 170–180; June, 1951.) Three superregenerative receivers are described for reception of pulses of the order of 2 μs duration, on a frequency of 170 mc. They illustrate alternative methods for damping the trains of oscillations of the detector tube on application of the quench voltage, and for effectively increasing the quench frequency.

621.396.621.54 482

Tracking of Superheterodyne Receivers—H. S. de Koe. (*Wireless Eng.*, vol. 28, pp. 205–315; Oct., 1951.) Tracking frequencies are calculated for various criteria defining the best tracking-error curve. The capacitances and inductances in the oscillator circuit are then calculated by means of formulas containing dimensionless variables. Wiring capacitances, etc., necessitate the use of a domain of possible combinations of inductance and capacitance represented by nearly linear curves. Calculations are illustrated by examples. A curve published by M. Wald (3313 of 1941) is shown to be wrong.

621.396.622 483

Characteristics of A.M. Detectors—W. E. Babcock. (*Audio Eng.*, vol. 35, pp. 9–11, 27; July, 1951.) The operation and distortion characteristics of various diode and triode detectors are discussed. The "infinite-impedance" triode detector combines the low distortion of the diode with the high input impedance of the triode, and minimum distortion is produced even with large grid signals. A disadvantage of the infinite-impedance detector, and of all the other detectors considered except the diode, is that if avc is desired, a separate channel must be used.

621.396.622 484

The Dynamic Characteristics of Frequency Detectors—G. Bosse. (*Arch. elekt. Übertragung*, vol. 5, pp. 314–320; July, 1951.) Simple approximate formulas are derived for the dynamic characteristics of the conventional phase discriminator, from which the distortion arising from transient effects can be calculated. Means of reducing even-harmonic distortion occurring with greater relative bandwidth are described. Experimental results confirm the theory.

621.396.622.71 485

Numerical Treatment of the Limiting Effect of the Ratio Detector—H. Behling. (*Frequenz*, vol. 5, pp. 89–97; April, 1951.) Making use of a graphical representation of the diode characteristics and equivalent circuits, a method is developed for determination of the amplitude limitation effected in the detector circuit. A complete analysis is presented in a numerical example showing how coupling, damping, diode load resistance, etc., may be chosen for optimum limiting effect.

621.396.662 486

'Physiological Volume-Control' in Broadcast Receivers—F. Bergtold and S. Sawade, (*Telefunken Ztg.*, vol. 24, pp. 48–50; March, 1951.) Discussion of the volume adjustments for the different audio frequencies required to obtain faithful reproduction. Tests carried out on ten observers indicated that with reduced output level, not only should the lower frequencies be boosted, but the higher frequencies should be reduced in comparisons with the values acceptable at a normal output level.

621.396.82 487

A Theoretical Investigation of the Elimination of Whistle Interference in Superheterodyne Receivers—F. Wetzorke. (*Fernmeldetech. Z.*, vol. 4, pp. 315–319; July, 1951.) A method is developed for determination of the optimum IF for avoidance of harmonic whistles and image signals. From a classification of interfering stations as local, near, or distant, according to an empirical formula, and from a series of equations derived, optimum values of IF can be calculated. A diagram is constructed representing the number of whistle interferences as a function of IF over the range 350 to 530 kc. The optimum IF can be found directly from inspection of the contour of the diagram.

STATIONS AND COMMUNICATION SYSTEMS

621.39.001.11 488

Basis of Information Theory—S. Malatesta. (*Alta Frequenza*, vol. 20, pp. 128–159; June/August, 1951.) The fundamental principles of modern communication theory are outlined. Essential elements of statistical calculus are presented and used to examine the time functions representing signal and noise. Quantity of information is defined; the magnitude of the quantity of information contained in a signal in the presence of noise is discussed. The generalized Hartley Law is established which relates quantity of information to signal bandwidth, signal duration, and signal/power-to-noise power ratio.

621.39.001.11 489

On the Word 'Cybernetics'—A. A. (*Onde élect.*, vol. 31, p. 257; May, 1951.) The word "cybernetics" has been used before in senses quite different from that now given to it by Wiener. A passage from Plato is quoted and translated, in which κυβερνητική clearly means "the art of navigation." Ampère, in his "Essai sur la philosophie des sciences, ou exposition naturelle de toutes les connaissances humaines," published in 1834, defines cybernetics as the study of how it may be made possible for citizens to enjoy secure peace.

621.394/.396 490

Evolution of the Technique of Long-Distance Lines during the Last 15 years—R. Sueur. (*Ann. Télécommun.*, vol. 6, pp. 146–164; June, 1951.) Developments in overhead lines, underground and submarine cables, and radio links are described, together with regulating apparatus, repeaters, filters, and related apparatus. A comprehensive bibliography, referring mainly to the French literature, is appended.

621.395/.396].5 491

New Long-Distance Telephony Systems—J. Schniedermann. (*Frequenz*, vol. 5, pp. 125–137; May, 1951.) Analytical review of the development of Siemens and Halske equipment for the German telephone service. Block diagrams of cable and radio systems are shown. Choice of frequency and channel capacity are discussed. SW ssb transmission and reception apparatus for transcontinental and oversea services, and directional vhf and uhf multichannel equipment are described, with illustrations. Unit-type construction and miniaturization of components are discussed in considering the economics of the service.

621.396.5 492

A Subscriber's Battery-Operated Single-Channel V.H.F. Radio-Telephone Equipment—J. H. H. Merriman, R. Hamer, and B. R. Horsfield. (*P.O. elect. Eng. Jour.*, vol. 44, pp. 75–80; July, 1951.) "This article describes experimental equipment operating in the 70 to 90 mc range, using phase modulation, and powered by primary batteries, which, together with signalling units, provides a duplex radio channel for connection in a subscriber's line circuit. The characteristics of this equipment which make it suitable for providing telephone service to isolated communities are explained, and mention is made of the performance obtained on experimental links using such equipment."

621.396.619.11 493

Transient Response of Asymmetrical Carrier Systems—G. M. Anderson & E. M. Williams (Proc. I.R.E., vol. 39, pp. 1064–1066; September, 1951.) A time-domain study is made of the transient response of asymmetrical AM carrier systems. A vector integral method is used for determining the system response to arbitrary modulation. Nonlinearity of the envelope transfer function is examined as dependent on modulation depth. The time delay associated with the response to an applied step voltage is found to be always less than the corresponding delay on cutting off the voltage. Improvement of the transient response of asymmetrical systems as a consequence of detuning appears to be accompanied by a decrease in signal-to-noise ratio.

621.396.65 494

940–960-Mc/s Communications Equipment for Industrial Applications—F. B. Gunter. (*Elec. Eng.*, vol. 70, pp. 573–578; July, 1951.) Factors influencing the design of a multichannel system are discussed, particularly the choice and stability of frequency, method of modulation, and antenna and receiver design. Details of one system are given.

621.396.65 495

Radio Relay Design Data, 60 to 600 Mc/s—R. Guenther. (Proc. I.R.E., vol. 39, pp. 1027–

1034; September, 1951.) IRE 1950 National Convention paper. General design data for a complete radio-relay transmission system are shown graphically. From these data, signal and noise levels can be determined. The ratio of the theoretical to the practical wide-band gain is a measure of the deficiency of conversion in the demodulator. Comparison of different modulation systems must be based on measured values of wide-band gain. Numerical data for FM and PTM systems are derived and tabulated.

621.396.65:621.396.41 496
U.H.F. Multiplex Radiotelephony Communications—H. Chireix. (*Ricerca Sci.*, vol. 21, pp. 1105–1126; July, 1951. In French.) The subject is examined from the theoretical viewpoint, and a calculation is made of the threshold value of receiver input power above which the modulation system used can provide an improvement in signal-to-noise ratio. The insertion loss due to radio links with or without passive relays is calculated, and hence, the required transmitter power. System gain is determined for the case of FM on carrier and for the case of PTM with FM or AM transmission. Equipment for terminal and relay stations for FM and PM systems is considered. Problems relating to systems with a large number of channels, and to television relaying, are discussed.

621.396.65:621.396.41.029.64 497
Multiplexing Klystrons—W. L. Firestone. (*Electronics*, vol. 24, pp. 112–116; September, 1951.) Possible methods of combining several modulated rf signals in a single microwave transmission line involve the use of hybrid-T junctions, directional couplers, line stretchers, tunable stubs, multiple-feed antennas, bandpass or band stop cavities, and other special devices. These are discussed with particular reference to crosstalk, signal loss, bandwidth, simplicity of tuning, and interference in receiver circuits. In the final system developed, three transmitters and three receivers are connected to a common main line by means of suitably tuned cavities.

621.396.65:629.135.4 498
Siting Microwave Antennas by Helicopter—B. I. McCaffrey. (*Electronics*, vol. 24, pp. 82–84; September, 1951.) Two machines, one with a transmitter and the other with a receiver, hover over proposed sites for microwave relay stations and, by means of observations at gradually decreasing heights, determine optimum heights for the antennas that are to be used.

621.396.712 499
German Broadcasting Transmitters—(*Funk-Technik* (Berlin), vol. 6, p. 256; May, 1951.) A list of existing and proposed lw, mw, sw and usw transmitters showing location, frequency, and power.

621.396.7 500
Radio Installations [Book Review]—W. E. Pannett. Publishers: Chapman and Hall, London, Eng., 444 pp., 45s. (*Wireless Eng.*, vol. 28, pp. 289–290; September, 1951.) "... concerned with the practical design, construction, and maintenance of modern radio transmitting and receiving installations for broadcasting and commercial communications purposes."

SUBSIDIARY APPARATUS

621–526 501
Graphical Synthesis of Networks for A.C. Servos—G. A. Bjornson. (*Elec. Eng.*, vol. 70, p. 571; July, 1951.) Summary of 1951 AIEE Winter General Meeting paper. The performance of a servomechanism is often improved by modifying the data signal. A method is given for determining the transfer function of a compensation network which modifies the signal in the desired manner.

621–526:621.3.016.35 502
A General Criterion for the Stability of Servomechanisms which are Centres of Hereditary Phenomena—J. Loeb. (*Compt. Rend. Acad. Sci.* (Paris), vol. 233, pp. 344–345; July 30, 1951.)

621.314.1.082.72 503
Electrostatic D.C. Transformer—G.W.O.H. (*Wireless Eng.*, vol. 28, pp. 291–293; Oct., 1951.) Discussion of the principles, construction, and possible applications of the apparatus described by Malpica (270 of February).

621.314.5 504
Comparative Representation of Various D.C./A.C. Converters—H. Tigler. (*Arch. elekt. Übertragung*, vol. 5, pp. 335–342; July, 1951.) Formulas are derived from which the performance of a converter can be assessed. Application to various types commonly used in power and communication engineering shows that the best results are obtained with arrangements with parallel oscillatory circuits, dc chokes, and square-wave modulation.

621.314.634+621.314.2 505
The New Siemens Selenium Rectifier for Broadcasting Equipment, and the Design of Suitable Transformers and Chokes—R. Kuhn. (*Funk u. Ton*, vol. 5, pp. 449–465; September, 1951.) Details of the various types available in these vacuum rectifiers are given. Their power, current, and voltage characteristics are tabulated and shown graphically, so that the technical data required for the design of low-power rectifier transformers and chokes can be determined. Four examples illustrate the method of calculation.

621.316.722.078.3 506
Characteristics of Glow Lamps in Voltage-Stabilization Circuits—B. Schumacher. (*Frequenz*, vol. 5, pp. 121–124; May, 1951.) Analysis of simple, compensated, and bridge-type circuits, and discussion of the operating characteristics of different lamps.

621.316.933 507
Protection of Low-Voltage Systems and Equipment—M. Witzig. (*Brown Boveri Rev.*, vol. 38, pp. 115–119; April, 1951.) An account of methods and equipment developed to reduce overvoltage effects on consumers' premises.

621.316.933 508
The Overvoltage Protection of Electrical Systems—C. Degoumois. (*Brown Boveri Rev.*, vol. 38, pp. 92–95; April, 1951.) The mode of operation of various types of protector is discussed, with particular reference to conversion of surge energy to heat, conversion of es to em energy, screening, and surge-energy storage.

621.316.933.3 509
The Function of Lightning Arresters in Modern Overvoltage Protection Schemes—H. Bossi. (*Brown Boveri Rev.*, vol. 38, pp. 96–105; April, 1951.) Discussion with particular reference to "Resorbit" arresters, which include quenched spark gaps in series with a resistor having a nonlinear characteristic, so that a low resistance is offered to surge voltages while the resistance to normal voltages is much greater.

621–526 510
An Introduction to Servomechanisms [Book Review]—A. Porter. Publishers: Methuen and Co., London, Eng., 1950, 154 pp., 7s. 6d. (*Ann. Télécommun.*, vol. 6, p. A362; June, 1951.) One of the Methuen series of Monographs on Physical Subjects. The fundamental nature of closed control systems is discussed, and modern techniques for the analysis and synthesis of servomechanisms are presented.

621–526 511
Dynamik selbsttätiger Regelungen (The Dynamics of Automatic Regulation), Vol. I [Book Review]—R. C. Oldenbourg and H. Sartorius. Publishers: R. Oldenbourg, Munich, Germany, 2nd edn 1950, 258 pp., 26 DM. (*Arch. elekt. Übertragung*, vol. 5, p. 252; May, 1951.) The broad presentation and the mathematical introduction to regulating processes make this book particularly useful for private study.

TELEVISION AND PHOTOTELEGRAPHY

621.397.262 512
Multiplexed Broadcast Facsimile—J. V. L. Hogan and J. W. Smith. (*Electronics*, vol. 24, pp. 97–99; October, 1951.) Facsimile newspaper pictures and print are relayed from New York to Ithaca by ultrasonic modulation superposed on normal audio modulation in FM transmissions. Filters ensure no interference between the facsimile reproduction and the audio program.

621.397.5 513
Dot Arresting Improves TV Picture Quality—K. Schlesinger. (*Electronics*, vol. 24, pp. 96–101; September, 1951.) A method of receiving dot-interlace television without using a separate synchronous detector is described. The method involves a process of dot arresting, or deflection sampling. An anastigmatic deflection yoke and dot-arrestor coil assembly is described. Figures are given for the brightness and sampling merit of sine-wave arresting and gating, and practical circuits and comparison photographs are shown. Selective dot-coverage circuits remove the objectionable dot structure from large areas and leave the dots in regions of fine detail.

621.397.5 514
Problems in Mobile TV—E. B. Pores. (*Electronics*, vol. 24, pp. 136, 168; September, 1951.) Transmission of a 7-kmc signal to a fixed receiver from a ship moving at 18 knots was effected by manual tracking of both the transmitting and the receiving antenna. Methods of reducing to a minimum the frequency variations of the engine-driven generator are described, and equipment installation problems in television broadcasts from remote locations are discussed.

621.397.5:535.62 515
Crispening Circuit for Color TV—D. G. F. (*Electronics*, vol. 24, pp. 85–87; September, 1951.) Designed for improved resolution in field-sequential color television, the "crispening" technique sharpens the vertical edges of extended objects by combining the video wave form and its derivative, modified nonlinearly. Details are given of practical circuits for use with domestic field-sequential receivers, and at the end of a coaxial cable network. Oscillograms and photographs show results obtained.

621.397.5:621.317](083.74) 516
New American Directions for the Measurement of Television Transmission Quantities—W. Reichel. (*Fernmeldetech. Z.*, vol. 4, pp. 311–313; July, 1951.) Summary of IRE Standard 23.S1, noted in 2035 of 1950, and comment on its applicability to prospective television in Germany.

621.397.5:621.317.75 517
Television Waveform Display—Sturley. (*See* 447.)

621.397.6 518
Television Apparatus for a 625-Line Service, shown in Belgium—J. C. Nonnekens. (*HF, Brussels*, no. 9, pp. 237–254; 1951.) The general principles involved in the provision of a television service are stated, and apparatus and techniques demonstrated at the Salon de la

Radio et de la Télévision, September, 1950, are described.

621.397.6:535.62 519
Picture Generator for Color Television—R. P. Burr, W. R. Stone, and R. O. Noyer. (*Electronics*, vol. 24, pp. 116–119; August, 1951.) A simple color-pattern generator is obtained by modifying the circuit of an existing black-and-white-pattern source. The hue and saturation of each bar of the pattern are under the operator's control. The equipment is suitable for testing either simultaneous, dot-sequential or field-sequential systems.

621.397.611.2 520
[Television] Camera Equipment—R. Aschen. (*Radio franç.*, no. 3. pp. 8–15; March, 1951.) Technical description of apparatus ranging from Zworykin's original iconoscope to the most modern equipment.

621.397.611.2 521
Impedance Changes in Image Iconoscopes—J. E. Cope and R. Theile. (*Wireless Eng.*, vol. 28, pp. 239–247; August, 1951.) To obtain a good signal-to-noise ratio, a high value of load resistance must be used, and because of the capacitance across it, this gives rise to signal integration which must be compensated by differentiation in the following amplifier. The degree of compensation must be varied according to the average brightness of the televised scene, otherwise "streaking" effects occur due to change in impedance of the camera tube. This impedance varies with the secondary-emission current from the storage surface, which in turn depends on the changing stream of photoelectrons. Two methods of automatic compensation of this effect are discussed. A suitable feedback to the signal plate provides a practical solution.

621.397.611.2 522
Television Camera Tubes—L. Bedford: J. D. McGee and E. L. C. White. (*Wireless Eng.*, vol. 28, pp. 288–289; September, 1951.) Further discussion on 1264 and 2556 of 1951.

621.397.611.21 523
Marconi Orthicon Camera—J. Sánchez-Cordovés. (*Rev. Telecomun.*, Madrid, vol. 6, pp. 42–49; June, 1951.) Detailed description of the construction and properties of this image tube, with particular reference to sensitivity, coefficient of merit, light flux required, relative output, response to color, resolution, effect of spurious signals, and limitation due to image persistence.

621.397.62 524
Modified-Butterfly U.H.F.-TV Converter—M. W. Slate, J. P. Van Duyne and E. F. Mannerberg. (*Electronics*, vol. 24, pp. 92–96; October, 1951.) Highly stable noncontacting resonators of the semi-butterfly type constructed from silvered glass-bonded mica are used for the oscillator and mixer tuned circuits. Coupling from the oscillator and antenna to the mixer circuit is by bifilar coiled transmission lines giving balanced to unbalanced transformation. A cascode IF amplifier feeds the vhf television receiver.

621.397.62 525
Flywheel Synchronizing Circuit for Television-Receiver Time-Bases—A. B. Starks-Field. (*Wireless Eng.*, vol. 28, pp. 293–297; Oct., 1951.) In the normal triggering method of synchronizing television line time-bases, random variations in the firing time may be produced by the effect of noise on the synchronizing signal, giving rise to irregularities in any vertical edges in the picture. A "flywheel" synchronizing circuit is described in which the time base phase is compared with that of the incoming synchronizing signal, the difference being used to generate a control voltage which governs the time-base recurrence frequency.

621.397.62:621.385.2 526
A New Damper Diode—Bareiss. (*See* 551.)

621.397.621 527
Efficiency Line-Scan Circuits—W. T. Cocking. (*Wireless World*, vol. 57, pp. 302–305, 347–350 and 425–430; August–October, 1951.) To design "efficiency" line-scan circuits successfully, proper allowance must be made for departures from ideal characteristics of the transformer. The directly fed deflector-coil circuit is examined first, as being not only simpler to analyze but also a practical alternative to the transformer-coupled circuit. Control of the "efficiency" diode by means of a damped tuned circuit with a phase-reversing winding is explained, and the relative merits of operation with and without such control are discussed. The reduction of efficiency inherent in the use of a transformer is outweighed when hv boost is used by recovering energy from the diode autobias circuit. Differences in U. S. and British practice are illustrated and explained.

621.397.621 528
Theory of Deflection Systems for Television Tubes—F. Kirschstein. (*Funk u. Ton*, vol. 5, pp. 177–189; April, 1951.) Description, with circuit diagrams and performance curves, of the theory and operation of magnetic deflection circuits used currently in receivers.

621.397.621.2 529
General Characteristics of Cathode Ray Tubes [for television]—(*Radio franç.*, no. 3, p. 24; March, 1951.) Characteristics of six tubes are tabulated, one using electrostatic deflection, with maximum anode voltage 6 kv, and the others having electromagnetic deflection, with maximum anode voltages ranging from 12 to 19 kv.

621.397.645 530
R. F. Amplifier for U.H.F. Television Tuners—Tyson and Weissman. (*See* 370.)

621.397.645.37 531
Television Picture Amplifier with Negative Feedback for the Reduction of Microphony and Input-Circuit Noise—W. Dillenburger (*Funk u. Ton*, vol. 5, pp. 113–123; March, 1951.) The use of a high-resistance load for the camera tube and subsequent selective feedback in the amplifier circuit is discussed. The design of the circuit is considered, taking account of inter-electrode and stray capacitances. The circuit and typical response curves of a single-stage and three-stage amplifier are shown. While the design of the former is simple to calculate, the latter is best designed empirically. Component values are critical, and the circuit tends to self-oscillation. An advantage is that nonselective feedback can be applied to the individual stages hence tube replacement is simplified.

621.397.8 532
Long-Distance Reception of Television—(*Radio-Revue TV*, no. 6, p. 231; June, 1951.) Dates and times in May, 1951 are quoted when good reception of television broadcasts was achieved in Belgium over ranges as great as 2,200 km, mainly from Leningrad. Such reception is attributed to propagation by reflection from sporadic-E clouds.

621.397.5(43) 533
Einführung in die neue deutsche Fernsehtechnik (Introduction to Modern German Television Technique) [Book Review]—W. Dillenburger. Publishers: Schiele and Schön, Berlin, Germany, 1950, 210 pp., 12.50 DM. (*Z. angew. Phys.*, vol. 3, p. 200; May, 1951.) Written for technicians and engineers, but excluding difficult mathematical treatment.

621.396.61:621.396.619.13 534
High-Power F.M. Broadcasting Transmitters—L. Rohde, N. Nitsche, and A. Pfefferl. (*Frequenz*, vol. 5, p. 52; February, 1951.) Corrections to 764 of 1951.

621.396.619.13 535
Characteristics of Frequency Modulation with Reactance Valves—G. Bronzi. (*Alta Frequenza*, vol. 19, pp. 175–184; August, 1950.) A theoretical-experimental method of investigation is proposed based on measurement of two frequencies, viz., the center (carrier) frequency, and that obtained with the modulator tube off. The applicability of the method is confirmed by experimental results.

621.396.619.13:621.396.61 536
Arrangement for Frequency Modulation of Transmitters by means of Premagnetized H.F. Iron-Cored Coils—H. Boucke. (*Fernmeldetech. Z.*, vol. 4, pp. 201–206; May, 1951.) A method was investigated in which the inductance of a frequency-determining hf iron-cored coil was varied in accord with the modulation frequency. The center frequency used was 80 mc with a frequency swing of about 1 mc, and the highest modulation frequency was 160 kc. Details are given of the magnet winding and core, and of the oscillator coil, and the various possible sources of nonlinear distortion are investigated.

621.396.619.2 537
'FMQ'—W. S. Mortley. (*Wireless World*, vol. 57, pp. 399–403; October, 1951.) The Marconi "frequency-modulated quartz" system was devised to give a reliable circuit with adequate center-frequency stability and low distortion. The modulator is of balanced susceptance type, and a crystal with a quarter-wave network attached is used in the oscillator. A circuit is described which gives the required frequency modulation with audio input.

621.396.67 538
V.H.F. Common-Aerial Working—E. G. Hamer. (*Electronic Eng.*, vol. 23, pp. 244–249; July, 1951.) The problems involved are reviewed briefly and the design of suitable filters using lengths of cable or lumped elements is discussed. The special difficulties encountered in common-antenna working using transmitters whose frequency difference is between 5 per cent and 0.6 per cent, or <0.6 per cent are also dealt with.

621.396.97:621.396.61 539
Improving Program-Limiter Performance—D. W. Howe, Jr. (*Electronics*, vol. 24, pp. 108–111; September, 1951.) Circuit development, adjustment, operation, and performance data are given for a delay limiter which prevents side-band spread on AM broadcasting channels. Fringe-area reception is improved, overmodulation peaks are reduced in number, and greater total modulation energy is obtained.

TUBES AND THERMIONICS

621.383.42 540
New Photocells—J. Stuke. (*Elektron Wiss. Tech.*, vol. 5, pp. 112–114; April, 1951.) Discussion of improvements obtained in sensitivity and durability, particularly under tropical conditions, in the new design of Se photocell in which a semiconducting layer with enhanced conductivity replaces the thin semitransparent metallic coating usually applied over the Se layer.

621.385:621.317.723 541
Receiving Valves suitable for Electrometer Use—G. A. Hay. (*Electronic Eng.*, vol. 23, pp. 258–261; July, 1951.) Ordinary receiving tubes of certain types can be used as electrometer tubes after selection and special treatment. The characteristics of these are compared with those of special tubes.

621.385.017.72 542
Cooling of Electronic Valves by Vaporization of Water—C. Beurtheret. (*Onde élect.*, vol. 31, pp. 271–281; June, 1951.) Anode cooling of high-power tubes is greatly improved by the use

of thick anodes, with correctly shaped fins, enclosed in a water jacket. The water is vaporized, thus abstracting heat from the anode, and the steam is condensed and the water re-used. With a smooth anode, small bubbles of vapor adhere to the surface and form a nonconducting layer, leading to the destruction of the tube. By using the finned surface, the power dissipated by the anode may be increased by a factor of three without risk. Tubes are illustrated having anode dissipations of 40 to 50 kw.

621.385.029.63/.64 543

Amplification and Generation of Microwave Oscillations with Travelling-Wave Valves—H. Schnitger. (*Funk u. Ton*, vol. 5, pp. 143–145; March, 1951.) Data on the noise factor of the helix-type tube and the double-beam tube, concluding the paper noted in 3004 of 1950.

621.385.029.63/.64 544

The Calculation of Traveling-Wave-Tube Gain—C. C. Cutler. (Proc. I.R.E., vol. 39, pp. 914–917; August, 1951.) A summary of available relevant information presented in the form of graphs and nomograms.

621.385.029.63/.64 545

Production of Directional—Dependent Electronic Attenuation in Travelling-Wave Valves—H. Schnitger. (*Fernmeldetech. Z.*, vol. 4, pp. 301–305; July, 1951.) Investigation of the attenuation of the normal reflected wave effected by interaction with a second wave injected into the tube and traveling in the opposite direction. This is achieved by suitably decreasing the beam-current intensity and making the beam voltage 10 to 20 per cent lower than for optimum amplification. Quantitative measurements show general agreement with calculated values of attenuation. For a helix 13 cm long and mean wavelength 30.5 cm, the attenuation was >2 neper over 9 per cent of the waveband.

621.385.029.63/.64:621.396.822 546

Noise in Traveling-Wave Tubes—F. N. H. Robinson and R. Kompfner. (Proc. I.R.E., vol. 39, pp. 918–926; August, 1951.) Gain noise factor, and attenuation in traveling-wave tubes are discussed; the design and performance of a tube with a dispersive helix are described. Much of the noise observed may be due to beam partition effects. These are analyzed, and approximate expressions are given for their magnitude. Noise due to electron-wave-type amplification and thermal noise originating in the resistive part of the helix are also discussed, and experiments are described.

621.385.029.64/.65 547

A Spatial-Harmonic Traveling-Wave Amplifier for Six-Millimeters Wavelength—S. Millman. (Proc. I.R.E., vol. 39, pp. 1035–1043; September, 1951.) An amplifier is described in which the electron beam interacts with a spatial harmonic of an em wave propagated along an array of resonator slots. This construction makes possible a considerable reduction of beam voltage for a given physical separation of the circuit elements. With a tube of this type operating with a beam voltage of 1,200, net power gains of about 18 db have been obtained at wavelengths of about 6 mm. A magnetic field of 1,600 gauss is sufficient for proper beam focusing. Apart from small variations of gain with frequency caused by internal reflections, the bandwidth is of the order of 3 per cent.

621.385.032.21 548

New Cathode Design improves Tube Reliability—D. R. Hill. (*Electronics*, vol. 24, pp. 104–106; August, 1951.) See also 773 of 1951.

621.385.032.216 549

Thermionic Emission from (Ba-Sr)O Cathodes Illuminated by the Incandescent Lamp—T. Hibi and K. Ishikawa. (*Phys. Rev.*, vol. 83, pp. 659–660; August 1, 1951.) The emission current was examined as a function of intensity and duration of illumination. From the experimental results it is concluded that the trapping energy of an electron in an isolated impurity center is greater than 1·32 ev.

621.385.032.216 550

Mechanism of Thermionic Emission from an Oxide Cathode in Pulsed Operation—Ya. E. Pokrovski. (*Zh. Eksp. Teor. Fiz.*, vol. 21, pp. 423–428; March, 1951.) The main phenomena of pulsed electron emission from an oxide cathode are discussed on the basis of the energy-level theory of semiconductors. Reasons for inaccuracies in earlier theories of electron emission from semiconductors are pointed out, and a formula is derived representing the process of fatigue of the oxide cathode. The physical meaning of the constant characterizing the speed of the fatigue process is established, and the effect of a barrier layer on the emission properties of the oxide cathode is taken into account. The theoretical results obtained are in agreement with experimental data.

621.385.2:621.397.62 551

A New Damper Diode—M. Bareiss. (*Electronics*, vol. 24, pp. 262–270; October, 1951.) Description of the design of an experimental tube similar to Type 6W4GT, but incorporating a new heater assembly and with an increased anode-cathode spacing. Heating time is 16 seconds. Sample tubes withstood a short inverse peak-voltage test of 6,000.

621.385.3 552

The Amplification Factor of a Triode: Part 1—A Parallel Plane Triode with Arbitrary Electrode Dimensions—M. Wada. (*Sci. Rep. Res. Inst. Tohoku Univ., Ser. B*, vol. 1/2, pp. 399–420; March, 1951.) A general theory is developed for determining the characteristics of planar triodes. This is used to determine the amplification factor of uhf triodes with small grid-cathode spacing.

621.385.3:546.289 553

The Germanium Triode—H. Salow. (*Z. angew. Phys.*, vol. 3, pp. 231–239; June, 1951.) Discussion of the construction, characteristics, operation, and applications of the transistor and related devices.

621.385.832 554

Charge Storage in Cathode-Ray Tubes—C. V. Parker. (Proc. I.R.E., vol. 39, pp. 900–907; August, 1951.) "The charging process in cathode-ray tubes used for static storage of information is analyzed for both a stationary spot and a linear scan, with and without redistribution of secondary electrons. Approximate equations are derived for the surface potentials and charging current as functions of time and other parameters, such as primary beam current, writing speed, and initial potentials. The results are presented graphically in special cases for comparison with photogrraphs of experimental wave forms."

621.396.615.14 555

New U.H.F. Resnatron Designs and Applications—D. B. Harris. (*Electronics*, vol. 24, pp. 86–89; October, 1951.) Resnatrons have been developed for uhf amplifiers with high power output. Gain, bandwidth, efficiency, frequency stability, and noise level are satisfactory for television transmission.

621.396.615.141.2:621.365.55† 556

Magnetrons for Dielectric Heating—R. B. Nelson. (*Elec. Eng.*, vol. 70, pp. 627–633; vol. 70, July, 1951.) The use of centimeter wavelengths for dielectric heating leads to a high rate of heating, but entails disadvantages in respect of the difficulty of screening the equipment, nonuniform heating due to the production of interference patterns, and attenuation of waves as they penetrate the dielectric. A suitable compromise is achieved at the commonly used frequencies of 915 and 2,450 mc. A 5-kw magnetron and associated circuits for use at 915 mc are described.

621.396.615.142.2 557

Wide-Band 5-kW Klystron Amplifier—P. Guénard, B. Epsztein and P. Cahour. (*Ann. Radioélect.*, vol. 6, pp. 109–113; April, 1951.) The inherently narrow pass-band of the klystron amplifier is expanded, at the expense of over-all gain, by using a three-cavity klystron in which the resonance frequencies of the first two cavities are appropriately spaced. A cw power of 5 kw at 900 mc is obtained, with a gain of 20 db and a variation of less than 1 db in a bandwidth of 6 mc.

621.396.615.142.2 558

High-Power U.H.F.-TV Klystron—Engineering Staff of Varian Associates. (*Electronics*, vol. 24, pp. 117–119; October, 1951.) A two-stage tripie-cavity klystron amplifier for use in the band 470 to 890 mc is described. The cathode, of tantalum, is heated by electron bombardment, and can be replaced to extend the life of the tube. The cavities are stagger tuned to provide a bandwidth of 5.6 mc between 1-db points. By using a water-cooled collector, cw output powers of 5 kw ware obtained, the gain being 24 db. See also (A) 2304 of 1951, No. 46 (Abraham et al.).

621.396.822 559

Induced Grid Noise and Noise Factor—R. L. Bell. (Proc. I.R.E., vol. 39, pp. 1059–1063; September, 1951.) An approximate treatment of narrow-band triode noise factor is developed, taking account of induced grid noise by a new method. Minimal values of noise factor predicted theoretically are compared with experimental values. From a knowledge of circuit losses, shot noise, mutual conductance, space-charge input capacitance, and dc grid current on the tube, noise factor and optimum source conductance can be predicted with accuracy over a wide range of operating conditions. Lead inductance effects and normal induced grid effects (grid noise, space-charge capacitance, transit-time damping) have no influence on noise performance, but merely affect tube admittances. Direct compensation in the grid circuit of components of input admittance to which these effects give rise, leads to a deterioration of noise factor usually attributed to grid noise. The optimum grid-circuit detuning may be provided automatically by the input space-charge capacitance.

MISCELLANEOUS

001.891 560

Scientific Research of Philips' Industries from 1891 to 1951—W. de Groot. (*Philips Tech. Rev.*, vol. 13, pp. 3–48; July/August, 1951.) An account written to commemorate the jubilee of the Philips organization at Eindhoven. It was not possible to celebrate this event suitably at the due time, in 1941. Research is organized under five broad headings: (a) light and its production, including gas discharges; (b) electrotechnics, acoustics, and radio; (c) chemistry, including metallurgy; (d) X-ray investigations; (e) mathematics and mathematical physics. The historical development in all these fields is outlined.

621.396:061.4 561

18th National Radio Exhibition—(*Wireless Eng.*, vol. 28, pp. 316–320; Oct., 1951.) A number of developments in evidence at the exhibition are discussed, chief attention being paid to television receivers. Methods of station selection and various improved scanning circuits are described. Other items discussed include new models of test equipment, and trends in sound-broadcasting receiver design. Component developments are briefly reviewed.