# Proceedings of the I·R·E

SEPTEMBER 1942

VOLUME 30 NUMBER 9



Institute of Radio Engineers

# Proceedings *of the* I·R·E

*Published Monthly by*
The Institute of Radio Engineers, Inc.

VOLUME 30 — *September, 1942* — NUMBER 9



The contents of papers published in the PROCEEDINGS are the responsibilities of the authors and are not binding on the Institute or its members.

Entered as second-class matter October 26, 1927, at the post office at Menasha, Wisconsin, under the Act of February 28, 1925, embodied in Paragraph 4, Section 538 of the Postal Laws and Regulations. Publication office, 450 Ahnaip Street, Menasha, Wisconsin. Editorial and advertising offices, 330 West 42nd St., New York, N. Y. Subscription, $10.00 per year; foreign, $11.00.

# THE INSTITUTE OF RADIO ENGINEERS

INCORPORATED

## New York Meeting—October 7, 1942

### SECTION MEETINGS

ATLANTA
October 16

CLEVELAND
September 25

DETROIT
October 16

LOS ANGELES
October 20

PHILADELPHIA
October 1

PITTSBURGH
October 12

WASHINGTON
October 12

### SECTIONS

**ATLANTA**—Chairman, J. M. Comer; Secretary, G. V. Waldo, 3166 Peachtree Dr., N. E., Atlanta, Ga.

**BALTIMORE**—Chairman, V. D. Hauck; Secretary, G. J. Gross, Pa. Water & Power Co., 1611 Lexington Bldg., Baltimore, Md.

**BOSTON**—Chairman, J. M. Henry; Secretary, R. O. Oberg, Northeastern Univ., Boston, Mass.

**BUENOS AIRES**—Chairman, J. P. Arnaud; Secretary, Alexander Nadosy, Florida St. 1065, Dept. C-6, Buenos Aires, Argentina.

**BUFFALO-NIAGARA**—Chairman, F. H. Scheer; Secretary, A. J. Dybowski, Dymac Radio, Inc., 1531 Main St., Buffalo, N. Y.

**CHICAGO**—Chairman, R. A. Kay; Secretary, Walter Kenworth, 5505 Potomac Ave., Chicago, Ill.

**CINCINNATI**—Chairman, W. L. Schwesinger; Secretary, Howard Leppele, 4432 Raceview Ave., Cincinnati, Ohio.

**CLEVELAND**—Chairman, P. L. Hoover; Secretary, A. S. Nace, 3209 Archwood Ave., Cleveland, Ohio.

**CONNECTICUT VALLEY**—Chairman, F. G. Webber; Secretary, C. I. Bradford, 187 Glenarden Dr., Fairfield, Conn.

**DALLAS-FORT WORTH**—Chairman, Truett Kimzey; Secretary, P. C. Barnes, Radio Station WFAA-WBAP, Grapevine, Texas.

**DETROIT**—Chairman, H. F. Tank; Secretary, Paul Frincke, 219 S. Kenwood Ave., Royal Oak, Michigan.

**EMPORIUM**—Chairman, R. K. Gessford; Secretary, H. D. Johnson, Hygrade Sylvania Corp., Emporium, Pa.

**INDIANAPOLIS**—Chairman, S. E. Benson; Secretary, B. H. Rinehart, 1920 Park Ave., Indianapolis, Ind.

**KANSAS CITY**—Chairman, H. K. Morgan; Secretary G. L. Taylor, Midland Radio School Power and Light Building, Kansas City, Mo.

**LOS ANGELES**—Chairman, C. F. Wolcott; Secretary, E. S. Sievers, Room 567, 417 S. Hill St., Los Angeles, Calif.

**MONTREAL**—Chairman, J. A. Ouimet; Secretary, J. R. Bain, Dept. 912, Northern Electric Co., 1231 Shearer St., Monteal, Que., Canada.

**PHILADELPHIA**—Chairman, C. C. Chambers; Secretary, R. L. Snyder, 103 Franklin Rd., Glassboro, N. J.

**PITTSBURGH**—Chairman, D. A. Myer; Secretary, A. P. Sunnergren, West Penn Power Co., 14 Wood St., Pittsburgh, Pa.

**PORTLAND**—Chairman, R. W. Deardorff; Secretary, B. R. Paul, 5960 S. W. 41st. Ave., Portland, Ore.

**ROCHESTER**—Chairman, O. L. Angevine, Jr.; Secretary, G. R. Town, Stromberg-Carlson Tel. Mfg. Co., Rochester, N. Y.

**ST. LOUIS**—Chairman, O. S. McDaniel; Secretary, H. D. Seielstad, 1017 S. Berry Rd., St. Louis, Mo.

**SAN FRANCISCO**—Chairman, H. E. Held; Secretary, W. G. Wagener, Heintz and Kaufman, South San Francisco, Calif.

**SEATTLE**—Chairman, K. H. Ellerbeck; Secretary, J. F. Johnson, 4316 Whitman Ave., Seattle, Wash.

**TORONTO**—Chairman, T. S. Farley; Secretary, L. C. Simmonds, A. C. Simmonds and Sons, 301 King St., E., Toronto, Ont., Canada.

**TWIN CITIES**—Chairman, R. E. Allison; Secretary, A. G. Peck, Twin-Cities Studios, Columbia Broadcasting System, 625 Second Ave., S., Minneapolis, Minn.

**WASHINGTON**—Chairman, E. M. Webster; Secretary, J. D. Wallace, Radio Division U. S. Naval Research Laboratory, Anacostia Station, D. C.

# A Frequency-Modulation Station Monitor*

H. R. SUMMERHAYES, JR.†, ASSOCIATE, I.R.E.

*Summary*—*This paper describes the development, theory, and characteristics of a frequency-modulation station monitor developed in the general engineering laboratory of the General Electric Company. The monitor measures mean carrier frequency with and without modulation and the percentage of frequency modulation as a percentage of 75-kilocycle deviation. In addition, it provides a flasher and alarm circuit for overmodulation adjustable from 50 to 120 per cent and an audio-frequency fidelity monitoring circuit. The paper begins with a discussion of the requirements for frequency-modulation monitors and compares the problems involved with those involved in standard broadcast monitors. Several schemes which were examined for use as a frequency-modulation monitor are briefly described, including counter and frequency-division systems. The principles and design of the final monitor are discussed.*

*The last part of the paper is devoted to a discussion of the salient development problems encountered, to a detailed description of the circuit, and to a discussion of the over-all specifications of the commercial sample.*

## Requirements for Frequency-Modulation Monitors

It is convenient to introduce the problems involved in a frequency-modulation station monitor by recalling what is required of standard broadcast monitors. The first of these to be considered is the frequency monitor, which is a relatively simple device designed to measure the frequency of the unmodulated carrier. Simplicity is achieved by virtue of the fact that in an amplitude-modulation broadcast transmitter the unmodulated carrier wave is always available for frequency measurement. The frequency of this unmodulated carrier may be measured independently in one part of the circuit while the same carrier is being modulated in another part of the circuit. The measured frequency of the unmodulated wave is the same as that of the carrier component of the radiated band of frequencies resulting from modulation.

In frequency modulation, however, as employed in the systems now in extensive use, no isolated unmodulated carrier component exists during the modulation process. In one system, frequency modulation is accomplished by means of a reactance tube which directly varies the frequency of the master oscillator. Thus, during modulation, there is obviously no isolated carrier component in this system which is available for frequency measurement. In another system, the frequency modulation is accomplished by using a balanced amplitude modulator and reinserting the carrier component after shifting its phase by 90 degrees. Since this system begins with an amplitude-modulation process in which the unmodulated carrier is readily available for frequency measurement, it might appear that no difficulty would exist in obtaining the frequency of the radiated carrier component of the frequency-modulated wave. However, the frequency-modulated wave which is obtained in this manner inherently must have a very small frequency swing for distortionless modulation. This condition is fulfilled by modulating at a lower frequency and subsequently employing frequency multipliers. In practice, these increase the frequency swing by more than two thousand times in order to produce the required standard swing of 75 kilocycles for 100 per cent modulation. This multiplication factor is so high that if it were applied directly to the initial frequency-modulated wave, it would produce a resultant frequency band far above the regular 42- to 50-megacycle band. Therefore, the multiplication is combined with a beating process in order to reduce the frequency of the modulated wave without reducing the swing. As a result of this beating process, the frequency of the final carrier component is a function not only of the frequency of the initial amplitude-modulated carrier but also of the frequency of the heterodyne oscillator.

Thus, in neither of the present widely used systems is the carrier component of the final frequency-modulated wave directly available for measurement. The carrier-frequency component must either be measured without modulation with a conventional frequency meter or it must be measured during modulation with a device which separates it from the side tones.

At present, the Federal Communications Commission requires only that the "center frequency," which is defined as the frequency of the carrier without modulation, be held within 2000 cycles of the assigned value in the high-frequency broadcasting band (42 to 50 megacycles). However, it is desirable that the frequency monitor be capable of indicating the frequency of the carrier component during modulation because it is especially during this time that it is important to keep the transmitter frequency within its tolerance since shifts of radiated signal band may easily cause adjacent-channel interference or receiver distortion. However, in a frequency-modulated wave, the carrier component is a very elusive thing; under some conditions of modulation it disappears entirely, and under fluctuations in level such as exist in normal programs, its amplitude is constantly varying so that in itself it may be regarded as an amplitude-modulated signal having a small band of frequencies which may be spaced even closer than the lowest audio modulating frequency. Thus, its direct separation from the rest of the components of the wave seems hopeless of accomplishment. A more useful quantity, and one which can be measured, is the mean frequency of the wave. The mean frequency is the continuous average of the instantaneous frequency taken over a time which is long compared with the period of the lowest modulating

* Decimal classification: R254×R414. Original manuscript received by the Institute, November 17, 1941. Presented, Summer Convention, Detroit, Michigan, June 23, 1941; Pacific Coast Convention, Seattle, Washington, August 22, 1941.

† General Electric Company, Schenectady, N. Y.

frequency. In an ideal frequency-modulated wave, the mean frequency is the same as the carrier-component frequency. This follows from the definition of a frequency-modulated wave, the equation for which may be written as

$$e = A \sin \left\{ \omega_0 t - \frac{K\omega_0}{p} \sin pt \right\}.$$

In this equation $\omega_0/2\pi$ is the frequency of the carrier component and $p/2\pi$ is the frequency of the audio modulating wave and $A$ and $K$ are constants. The instantaneous frequency is defined as $1/2\pi$ times the rate of change of the phase angle

$$\left( \omega_0 t - \frac{K\omega_0}{p} \sin pt \right)$$

which is $(\omega_0/2\pi)(1-K \cos pt)$. The mean value of the right-hand term of this quantity, taken over a long time compared with $2\pi/p$, approaches zero so that the mean frequency approaches $\omega_0/2\pi$ which is the frequency of the carrier component.

The monitor to be described in this paper measures the mean frequency of the frequency-modulated wave which, as shown above, is the same as the frequency of the carrier component when averaged over a period sufficiently long compared with the period of the lowest modulating frequency.

Returning to the comparison with amplitude-modulation monitoring practice, it will be recalled that a percentage-modulation monitor is also required in a standard broadcast station. This is usually a linear detector with an audio amplifier and peak-reading voltmeter. The percentage modulation is proportional to the ratio between the amplitude of the detected envelope of the carrier and the average carrier amplitude. In frequency modulation, however, the percentage modulation is defined by the Federal Communications Commission for high-frequency broadcasting stations as the percentage of a 75-kilocycle frequency swing or deviation above and below the carrier component which the instantaneous frequency undergoes during modulation. Thus, the measurement of percentage modulation is a measurement of the range of instantaneous frequency with 75 kilocycles as 100 per cent. It will be noted that a device which measures the sideband breadth is not suitable for this purpose since for high modulating frequencies the sidebands are considerably broader than the frequency deviation. The monitor described in this paper measures percentage of frequency modulation and also operates a warning flasher which may be adjusted to flash on peaks of modulation which exceed the value for which the flasher control is set. This value may be adjusted to any desired level between 50 and 120 per cent modulation.

In addition to performing these functions, the monitor also supplies an audio output for monitoring fidelity. In certain cases, this may also be found useful in estimating transmitter distortion and noise level. Thus, the unit monitors four essential characteristics of a frequency-modulated wave as follows: (1) mean frequency of carrier with and without modulation, (2) percentage of frequency modulation with (3) alarm indication for overmodulation, and (4) fidelity of the modulated signal.

## General Description of the Monitor

Several schemes for the frequency monitor were considered. One of them was based on the principle of frequency division. When a frequency-modulated wave is passed through a series of frequency dividers, the ratio of sideband energy to carrier-component energy progressively decreases and when the frequency de-

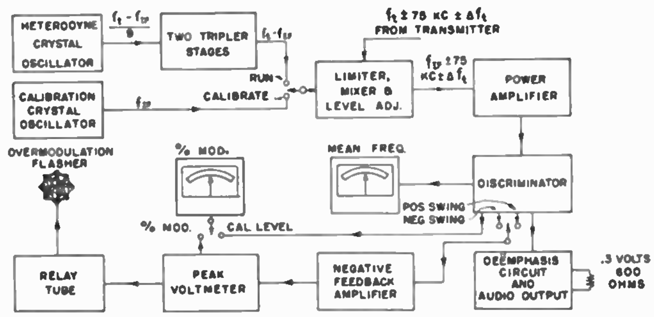



Fig. 1—Block diagram of frequency-modulation station monitor.

viation becomes less than the lowest modulating frequency, practically all the energy resides in the subharmonic of the carrier component. The frequency of this component, which may be easily measured, is then equal to the frequency-modulation carrier component divided by the division factor. However, no sufficiently simple and economical means of accomplishing the large frequency division required was found which would justify the use of this principle in a monitor.

A counter system was examined whereby a large number of individual frequency-modulation carrier cycles would be counted during accurately timed intervals. The indicated number of counted cycles during the intervals would be proportional to mean frequency. This, again, was far too complicated and expensive a system to be practicable.

Due to the multiplicity of sidetones in wide-band frequency modulation and the ever-changing amplitude of the carrier component itself, no means of identifying and measuring the frequency of this component directly was found.

The method finally adopted measures the average of the instantaneous carrier frequency using a circuit which in principle is a modified precision frequency-modulation receiver. A general account of this will be given followed by a more detailed discussion of the problems encountered in the development.

Referring to Fig. 1, the theory of operation is as follows: The frequency-modulated wave $f_t \pm 75$ kc $\pm \Delta f_t$ is fed into the monitor from the transmitter. It is immediately limited and converted to an intermediate frequency of approximately 5 megacycles in the mixer tube. The beat oscillator frequency is derived from a crystal oscillator and multiplier which has an output frequency of $f_t - f_{IF}$. The frequency-modulated intermediate frequency $f_{IF} \pm 75$ kc $\pm \Delta f_t$ is then fed to a frequency-discriminator circuit through a buffer-amplifier stage. The discriminator circuit is designed to deliver a current which is proportional to the instantaneous deviation of the carrier from its assigned value, being zero at the assigned value. The zero-output-current point of the discriminator may be calibrated when necessary by a separate precision crystal oscillator which has a frequency of $f_{IF}$. The discriminator output current is averaged over the audio cycle and the resultant direct current is proportional to the shift of the mean frequency from its assigned value. A zero-center, direct-current instrument with a linear scale marked from $-2000$ to $+2000$ cycles indicates the difference between the mean carrier frequency and its assigned value.

The alternating-current component of the output of the discriminator circuit has the form of the modulating wave because of the linear relation between current and frequency in the discriminator. Since the peak value of the signal is proportional to the peak frequency deviation, the per cent modulation is indicated by a peak voltmeter. This is operated by the discriminator audio output after amplification and it is so calibrated that 100 per cent modulation is indicated when the instantaneous peak frequency deviation is 75 kilocycles. A gas tetrode 2051 tube is arranged with the peak rectifier output applied to its grid so that when the modulation peaks exceed some preset value, which is under the control of the operator, the tube is "fired." Since the plate supply voltage for the gas tube is 60 cycles alternating current, the plate is negative every 1/60 of a second with respect to the cathode. This allows the grid to regain control of the plate current every 1/60 of a second. The "firing" of the tube causes a red flasher lamp to light and also closes a relay circuit which may be connected to an external alarm device or to a counter to record the number of peaks of modulation which exceed the value for which the control is set.

Audio quality monitoring has been provided in this monitor. Since it is standard practice in frequency-modulation broadcasting to pre-emphasize the high frequencies of the audio modulating signal according to a standard frequency characteristic, a de-emphasis is necessary in recovering the original audio signal. In the monitor, as in standard frequency-modulation receivers, this de-emphasis circuit has an impedance function which is the inverse of that of the pre-emphasis circuit in the transmitter. The output of the discriminator is passed through this circuit and thence into a low-distortion output tube designed to feed an external 600-ohm audio system.

## Development Problems

At the outset of the development, it was realized that an averaging circuit, if linear, would deliver a direct output current proportional to mean frequency but it was not evident that such a circuit could be made to maintain sufficiently precise alignment or stability to give a reliable output indication. The circuit must have output-current response which is strictly proportional to frequency over a range greater than 150 kilocycles in order to follow faithfully the full excursion of the instantaneous frequency. However, the total range of the mean frequency indication is only 4 kilocycles.

A simple example will illustrate the point. Suppose that the transmitter is operating at its assigned frequency and is modulated with a square wave (50 per cent pulse width) at full $\pm 75$-kilocycle deviation. During the positive half cycles of the square-wave modulation, the discriminator output current is a steady positive current of $+75$ units, assuming 1 current unit per kilocycle to be the discriminator slope. If the discriminator characteristic is linear, then during the negative half cycles of the square-wave modulation, the discriminator output current is $-75$ units, and the average current, which indicates the mean frequency, is zero. Now suppose that the discriminator characteristic is not strictly linear in such a way that $+75$ current units flow during the positive modulation intervals but only $-74$ instead of $-75$ current units flow during the negative intervals, a nonlinearity of only 1.3 per cent; then the average current is $+1$ unit instead of zero and the indicated mean frequency is 1000 cycles above the actual value, an error of one half of full scale due to only 1.3 per cent nonlinearity.

It must be realized, of course, that with ordinary complex modulation waveforms small deviations from linearity of the discriminator characteristic are not as serious as would seem from the above example since the average current is obtained from an integration of current over the whole discriminator characteristic rather than from two isolated points as is the case for square-wave modulation.

The first major question to be answered in the development, then, was whether or not a sufficiently linear response could be obtained economically in the frequency-discriminator circuit. Rather than find the answer to this question by mathematical analysis of the complicated discriminator circuit, the circuit was set up and adjusted for best linearity of response between output current and input frequency. Holding constant input amplitude, it was found that the characteristic did not deviate from a straight line by more than about 1/2 of 1 per cent and this was the order of accuracy of the output current measurement and of the constancy of the input amplitude.

This result seemed to justify an experimental test of the stability of the output indication of mean frequency since this was the next questionable characteristic of this type of circuit for use in a precision monitor. It was found, for example, that in a typical discriminator circuit a change in secondary capacitance of only 52 parts per million caused a 200-cycle change in indicated frequency out of the possible full-scale indication of 2000 cycles. It was previously mentioned that the discriminator may be instantly calibrated by throwing a switch. This applies a signal from a precision crystal oscillator whose frequency is equal to the crossover frequency of the discriminator. The output indication at this frequency can be adjusted to zero

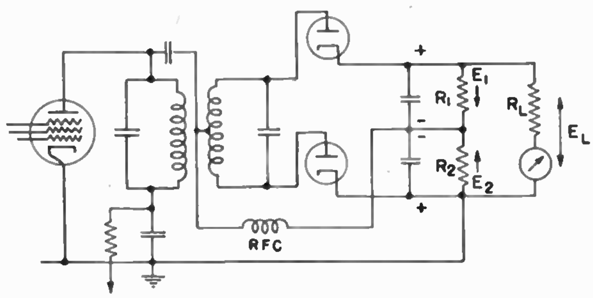



Fig. 2—Conventional frequency-discriminator circuit.

by a vernier secondary trimmer capacitance on the discriminator. Nevertheless, in order that this calibration process be required as infrequently as possible, it is desirable to have as high a stability as possible. For stability of 200 cycles in the output indication of mean frequency, the requirement for secondary discriminator inductance and capacitance is a stability of 0.005 per cent. This requirement was met by using special precautions in the design and manufacture of the coil form and winding and by employing suitable negative-temperature-coefficient tuning capacitors to supply compensation for the positive temperature coefficient of the inductances. With these precautions it has been found possible to omit the temperature control of the discriminator transformer employed in the earlier samples without loss of accuracy in the frequency indication.

The author would like to acknowledge at this juncture that it was largely due to E. D. Cook of the General Electric Company that this type of monitor circuit was investigated at all. It was generally believed that the linearity and stability requirements would be too severe for such a circuit to be practicable for use in a precision mean-frequency monitor. However, once having established by test that the discriminator could be made to have the required characteristics, the decision to concentrate on such a design was easily made and the monitor circuit, as previously described and as illustrated in Fig. 1, was developed.

The intermediate frequency is a function of the frequency of both the beat oscillator and the incoming wave to be monitored. For this reason the highest quality, temperature-controlled crystal available is used in the oscillator-multiplier circuit. The converter has been designed to provide amplitude limiting of the incoming wave from the transmitter in order to reduce to an acceptable degree any amplitude modulation which may be caused by maladjustment of the transmitter or by nonuniformity in the frequency response of the cable used to introduce the transmitter signal into the monitor. Amplitude modulation of this type is synchronous with the frequency modulation; that is, as the instantaneous frequency changes following the modulating wave so does the amplitude of the output change at a rate controlled by the modulating wave and the shape of the frequency response. If such a frequency- and amplitude-modulated wave is supplied to the discriminator, an incorrect mean-frequency indication will in general result because the current output of the discriminator is proportional to amplitude as well as to frequency of the signal supplied to it.

## Design of Discriminator

As the development progressed, a problem of sensitivity soon presented itself in regard to obtaining sufficient current from the discriminator for mean-frequency indication. The conventional discriminator circuit illustrated in Fig. 2, as used in receiver applications, is designed primarily to have a high voltage sensitivity; that is, to deliver an audio voltage as large as possible for a given swing in frequency. This is applied to the loudspeaker through a suitable amplifier. In the monitor, however, it is also required that the direct output current, which is proportional to variation of the mean frequency, be available at a useful level. The range of mean-frequency variation is 4000 cycles whereas the range of the instantaneous frequency is 150,000 cycles so that the amplitude of the mean-frequency indicating signal is only 4/150, or 2.7 per cent of the amplitude of the audio signal from the discriminator.

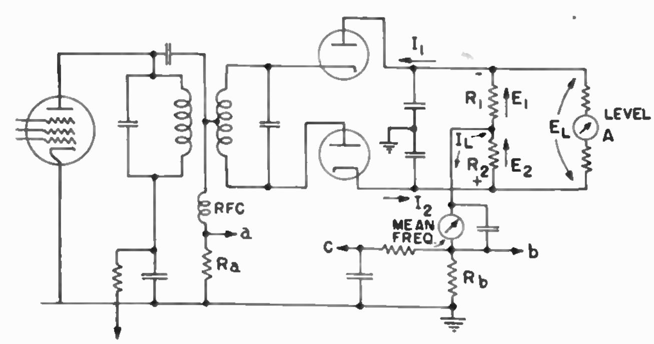



Fig. 3—Current-sensitive frequency-discriminator circuit.

criminator. Furthermore, the mean-frequency indication is a slowly varying direct-current signal, and a stable direct-current amplifier is not easy to build. These considerations led to the development of a modified discriminator circuit, illustrated in Fig. 3, which is especially adapted to giving a high current sensitivity rather than a high voltage sensitivity. Moreover, this modified discriminator is balanced to ground and allows an audio signal of either polarity to be obtained from it. Thus, indication of either positive or negative percentage modulation is available. A direct-current instrument which is by-passed for audio frequencies has been provided to indicate the average output current directly from the discriminator and direct-current amplification is therefore not required.

In the conventional discriminator (see Fig. 2) $E_1$ and $E_2$ are equal and opposite direct voltages when the center frequency is applied to the discriminator. The sum of these voltages $E_L$ is zero. As the frequency deviates to one side of center frequency $E_1$ increases and $E_2$ decreases so that $E_L$ becomes positive; on the other side of center frequency, $E_L$ becomes negative. A direct-current instrument to measure mean frequency must be be placed in series with $R_L$ across $R_1$ and $R_2$, and $R_L$ must be large compared with $R_1$ and $R_2$. This results in relatively low current sensitivity through $R_L$.

In the current-sensitive discriminator circuit (see Fig. 3), on the other hand, the diode voltages $E_1$ and $E_2$ are in the same direction and proportional to $I_1$ and $I_2$, respectively. The difference current $I_L$ between $I_1$ and $I_2$ flows through the center-frequency instrument through a small resistor to ground, and back up to the secondary coil of the discriminator through another resistor and a radio-frequency choke. At center frequency, $I_1 = I_2$ and $I_L = 0$. At frequencies above and below center frequency, $I_L$ has positive and negative values, respectively, and follows a linear characteristic with frequency as does the voltage $E_L$ in a conventional discriminator circuit. The resistors $R_a$ and $R_b$ are inserted to give peak voltages proportional to positive and negative frequency swing during modulation. Voltages from these resistors supply the modulation-monitor circuits.

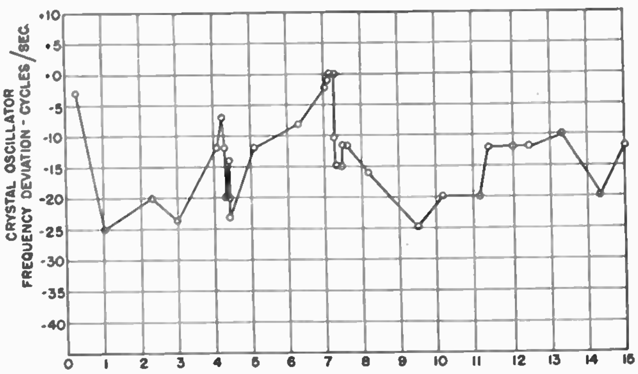



Fig. 4—Frequency-modulation station monitor crystal-oscillator frequency deviation taken during a 15-day continuous run.

Since the center-frequency indication is a function of both frequency and amplitude of the signal at the discriminator, it is required that a particular signal level exist there for correct indication. This may be indicated conveniently by a high-resistance direct-current instrument connected across $R_1$ and $R_2$. For any frequency in the pass band, $E_1$ is as much greater than its value at center frequency as $E_2$ is less than its value at center frequency. Thus, the sum of $E_1$ and $E_2$ is constant over the pass-band and $E_L$ is a direct voltage even during modulation. Provision has been made to employ the percentage-modulation instrument for this purpose. It may be inserted in the level-measuring circuit by means of a front-of-panel push button when calibration is desired.

The de-emphasis circuit employed in this monitor is a resistance and capacitance network having the standard time constant. It is shown between $R_b$ and point $C$ in Fig. 3. This feeds the audio-output tube which delivers approximately 0.3 volt audio in 600 ohms at 100 per cent, 100-cycle modulation with very low distortion.

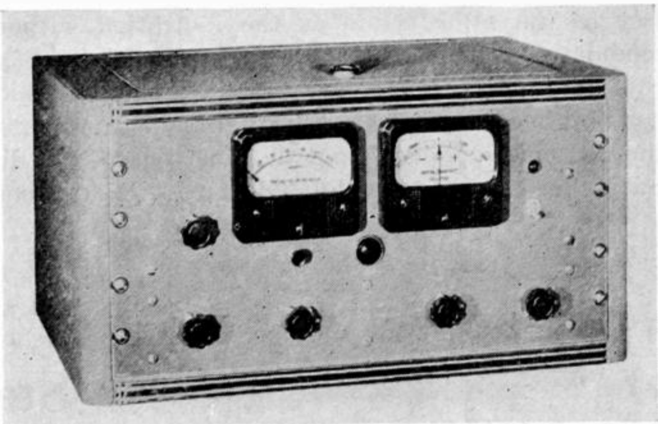


Fig. 5—Frequency-modulation station monitor complete with cabinet.

The amplifier which raises the level across $R_a$ or $R_b$ to a value sufficient for percentage-modulation indication is highly stabilized by negative feedback; the gain of this amplifier is only one thirtieth of what it would be without the feedback. This results in a calibration of the percentage-modulation instrument which is virtually independent of aging or changing of the percentage-modulation-circuit tubes.

No calibration of the monitor beyond adjusting for level and center frequency, as described previously, is necessary. This results from the inherent stability of the slope of the discriminator. Since a given change in the discriminator constants produces about one hundredth as much effect on the slope as it produces on the crossover point and since the stability with respect to crossover point is already on the order of 200 cycles, it is apparent that the stability of the slope of the curve is more than adequate.

## Over-all Accuracy

The over-all accuracy of the center-frequency indication without modulation is mainly dependent on the accuracy of the local oscillator. During a 15-day continuous test run, one of these oscillators held within 25 cycles of its proper frequency. The error produced in center-frequency indication from this cause was a maximum of 225 cycles because of the ninefold multiplication of this frequency preceding the beating process. It should be noted, however, that a 25-cycle error in the calibration oscillator produces only 25 cycles error in the calibration of the discriminator center frequency because this operation is carried out at the fundamental frequency of the calibration oscillator. Rigorous tests have proved that accuracy is well within the 1000-cycle tolerance required by the Federal Communications Commission for frequency-modulation monitors.

The accuracy of the mean-frequency indication during modulation is more difficult to evaluate. It is subject to the same errors as the indication without modulation but, in addition, it is affected by the accuracy of the averaging process. One method which has been used to measure this accuracy is to separate the carrier component from the side tones. In a distortionless frequency-modulated wave the side tones are symmetrically placed about the carrier and the mean frequency of the wave is identical with the carrier-frequency component. When a high modulating frequency is used, the side tones nearest to the carrier are separated from it by frequency intervals equal to this high modulating frequency. Thus, for such a condition, it is easy to separate the carrier-frequency component and to measure its frequency. A test of this kind on the monitor showed that the accuracy of the indicated mean frequency with modulation was within 150 cycles of the accuracy of the measurement of carrier frequency without modulation. This was for 100 per cent, 10-kilocycle modulation. However, a sufficient condition for the accuracy of indication is that the over-all discriminator output be a linear function of the input frequency.

# The Service Area of Medium-Power Broadcast Stations*

P. E. PATRICK†, ASSOCIATE, I.R.E.

*Summary—According to modern standards for amplitude-modulated transmission in the band from 500 to 1500 kilocycles, the field intensities required to provide a good service in urban, residential, and rural areas are 25, 5, and 0.5 millivolts per meter, respectively. It is useful to be able to make a rapid preliminary assessment of the service area that will be obtained from a station in the considered band, for various values of power, frequency, and conductivity. Families of curves are given to show these relationships for the powers 1/5, 1, 5, and 25 kilowatts and for the three field intensities 25, 5, and 0.5 millivolts per meter. When the distribution of population in an area is known, it is then easy to find the station site which will give the most efficient coverage.*

*The main factors and principles affecting the coverage of a station are discussed, and methods of dealing with practical problems are detailed.*

## I. Introduction

The problem of coverage from a medium-power broadcast transmitter in the band from 500 to 1500 kilocycles, is of interest to a large number of radio engineers. Although few are called upon to do detailed design and test work, there are many who have to deal with the preliminary design of stations in this band. Furthermore a clear understanding of the factors and principles involved is of value to all radio engineers.

The subject has, of course, received considerable attention and many excellent papers have been written on specific problems. However the field is wide and certain aspects are very complex. An attempt will be made, therefore, to gather together and present logically the results of available theoretical and practical investigations.

The method of approach to this synthesis of facts has been based on the fundamental idea that the ratio between two quantities is more important than their numerical difference. A simple illustration should suffice to make the meaning clear.

The power of a transmitter is doubled from 25 to 50 kilowatts. The field intensity received by any listener then increases by about 40 per cent. Thus the actual audio signal from the listener's loudspeaker increases by the same percentage. The increase in power at any rate is quite impressive. However the listener's ear, which is the instrument that really matters, works according to Weber's law. This law says that the perceptible increase in stimulus is proportional to the stimulus already existing. This means that any change heard by the listener must be measured in terms of ratios and not in terms of absolute magnitudes. Fortunately engineers are already accustomed to express ratios logarithmically in terms of decibels. In this case an increase in power from 25 to 50 kilowatts gives an increase of 3 decibels. It can easily be proved in any control room that a change of 3 decibels in the level of speech or music is of small account. Therefore a twofold increase in power may have a certain publicity value, but the listener will scarcely notice the change.

If the question of coverage is approached with a clear understanding of the importance of the idea of ratios, it will be much simpler to obtain a picture in true perspective.

## II. Power

A variation of 1 decibel in an audible pure tone is just noticeable, but for speech or music the variation must be some 3 decibels, and about 6 decibels before it is really effective. The effect of power variation in

* Decimal classification: R270. Original manuscript received by the Institute, November 17, 1941.
† South African Broadcasting Corporation, Grahamstown, South Africa.

terms of decibels is shown in Fig. 1. A variation of 7 decibels corresponds to a fivefold change in power. If, therefore, a power of 1 kilowatt is taken as a reference point, fivefold changes give powers of 1/5, 1, 5, and 25 kilowatts. This is a useful series, as standard transmitters are available in these powers. In a later section families of curves for service areas will be given for each of these four powers.

It must be emphasized here that small changes in power are meaningless as far as the listener is concerned. Four- and fivefold changes are about the smallest that should be considered.

## III. Noise Level

The service area of a broadcast station is considerably affected by the prevalent noise levels. Modern standards for the field strengths required to give a good service in urban, residential, and rural areas are 25, 5, and 0.5 millivolts per meter, respectively.

The noise that must be overcome by the signal in urban and residential areas is caused almost entirely

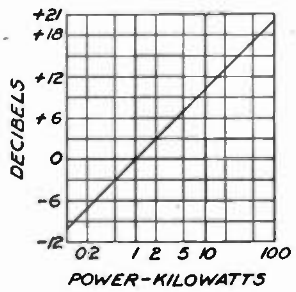



Fig. 1—Power variation in terms of decibels.

by electrical appliances. Although random in effect, the incidence of this noise is practically independent of frequency in the band from 500 to 1500 kilocycles.

In rural areas, however, the noise is mainly due to natural atmospheric disturbances. The available evidence shows that the amplitude of this noise varies inversely as the frequency between 500 and 1500 kilocycles.[1,2] The signal strength required to overcome noise in rural areas is taken as 0.5 millivolt per meter at 750 kilocycles. The signal required at other frequencies is shown in Fig. 2.

The intensity of atmospheric disturbances rises rapidly after nightfall, and is at its highest in midsummer. During summer there are areas where the suggested rural signal will be inadequate. Local measurements over a lengthy period will then show what field intensity will be required. As atmospheric disturbances occur during the peak listening period, it will usually be necessary to provide a signal-to-noise ratio of not less than 20 decibels.

[1] Glenn D. Gillett and Marcy Eager, "Some engineering and economic aspects of radio broadcast coverage," Proc. I.R.E., vol. 24, pp. 190–206; February, 1936.
[2] J. H. Little and F. X. Rettenmeyer, "A five-band receiver for automobile service," Proc. I.R.E., vol. 29, p. 153; April, 1941.

## IV. Fading

Variations in the amplitude of a received signal are easily smoothed out by automatic volume control on the receiver. The distortion due to fading, however, sets a definite limit to the nighttime service area of a

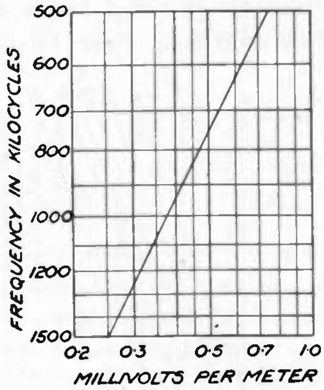



Fig. 2—The field intensity in millivolts per meter required to overcome noise in rural areas.

transmitter. Severe fading and distortion will occur beyond the radius at which the sky wave reaches half the intensity of the ground wave. Inside this "fading radius" distortion usually will be absent. As the radius at which fading commences depends upon the ratio of the ground wave to the sky wave, it is independent of the radiated power. The three factors affecting the fading radius are the frequency, conductivity, and antenna.

The attenuation of the ground wave is determined chiefly by the soil conductivity and the frequency. The intensity of the sky wave depends largely upon the vertical radiation pattern of the antenna. The variation in the radiation pattern with antenna height for sinusoidal distribution of antenna current is given in Fig. 3. The vertical scale shows the ratio between the

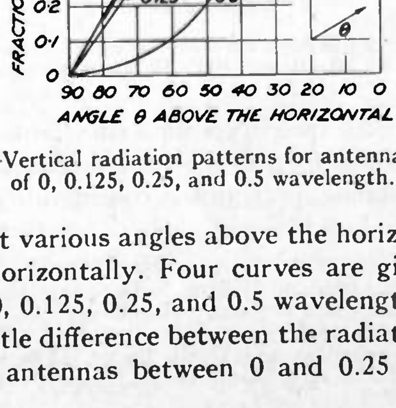



Fig. 3—Vertical radiation patterns for antenna heights of 0, 0.125, 0.25, and 0.5 wavelength.

radiation at various angles above the horizontal to the radiation horizontally. Four curves are given for antennas of 0, 0.125, 0.25, and 0.5 wavelength in height. There is little difference between the radiations at high angles for antennas between 0 and 0.25 wavelength in height. Therefore, as far as fading is concerned, antennas in this range have similar properties.

As will be seen in Fig. 3, the high-angle radiation from a 0.5-wave antenna is much less than that from antennas of 0.25 wavelength or less in height. It is seldom, however, that the additional cost of a 0.5-wave antenna can be justified for a medium-power station. It will be seen later that, for powers up to 5 kilowatts, all or most of the rural service area is within the fading-free area for a short antenna.

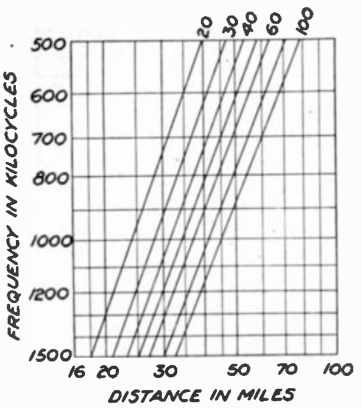



Fig. 4—Fading radius for a short antenna for conductivities of 20, 30, 40, 50, 60, 80, and $100 \times 10^{-15}$ electromagnetic unit.

The fading radius at which the ground wave has twice the intensity of the sky wave has been plotted in Fig. 4 for various values of conductivity. These curves are derived from previously published figures.[3] Although the curves were calculated for a 0.25-wave antenna, they can be used for shorter antennas with only a slight error. Similar curves for a 0.5-wave antenna are shown in Fig. 5. These curves are probably less reliable than the curves for a short antenna, as the measured values for the high-angle radiation from a 0.5-wave antenna may differ considerably from the theoretical values.[4]

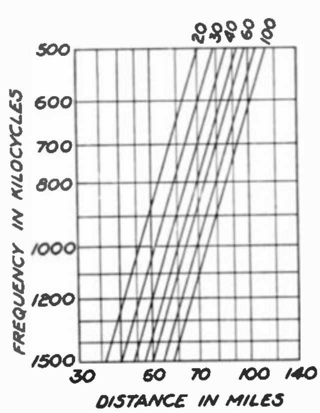



Fig. 5—Fading radius for a 0.5-wave antenna for conductivities of 20, 30, 40, 50, 60, 80, and $100 \times 10^{-15}$ electromagnetic unit.

## V. Conductivity

Accurate determination of the conductivity of an area must be done by measurement. The most satisfactory method is to take a measured curve of field intensity versus distance and fit it to calculated curves as has been described previously.[3,5] However for preliminary work the table of conductivities given in Table I has been found useful.[5]

TABLE I

Determination of Conductivity

| Terrain | Difference in Elevation (Feet) | Soil Type | Remarks | Conductivity $\times 10^{-15}$ Electromagnetic Unit |
|---|---|---|---|---|
| Sea water | | | | 10,000 |
| Fresh water | | | | 5000–8000 |
| Marsh | | Loam and silt | | 1000 |
| Flat or rolling gently | 50 | Black loam | | 150–200 |
| Rolling | 50– 100 | Loams and sandy loams | | 80–100 |
| Rolling | 100– 500 | Sandy loam mostly | | 60–80 |
| Hilly | 500– 800 | Gravelly, sandy and rocky loams | | 40–60 |
| Suburbs and small towns | | | | 30–40 |
| Hilly | 600–1000 | Gravelly, sandy and rocky loams | | 30–40 |
| Flat or hilly | | Sand and shale | | 25–40 |
| Very broken | 300–1000 | Gravelly, stony land | Ravined but not necessarily high | 20–30 |
| Residential sections and towns | | | Not for high steel buildings | 20–30 |
| Mountains | 1000–1500 | Stony land | | 10 |
| Broken mountains | 1000–8000 | Stony land | | 5–7 |

## VI. Variation in Conductivity

The van der Pol curves[3,5] of field intensity versus distance reveal that in the band from 500 to 1500 kilocycles the attentuation approaches a maximum which is independent of the conductivity after a certain distance has been reached. The attentuation then becomes approximately proportional to the 2.3 power of the distance. At short distances the attenuation decreases as the conductivity increases. Therefore a transmitted wave should be given a good start by commencing its journey over ground of the best possible conductivity. To illustrate the importance of this conclusion, curves *A* and *B* have been plotted in Fig. 6. It was assumed that a transmitter had an inverse field intensity of 100 millivolts per meter at 1 mile. It was required to transmit a signal to a point at a distance of 40 miles. In the case of curve *A*, the conductivity for the first 20 miles was $25 \times 10^{-15}$ electromagnetic unit and for the second 20 miles it was $100 \times 10^{-15}$ electromagnetic unit. In the case of curve *B* conditions were reversed and the conductivity for the first 20 miles was $100 \times 10^{-15}$ electromagnetic unit. Points on

[3] William A. Fitch and William S. Duttera, "Measurement of broadcast coverage and antenna performance," *RCA Rev.*, vol. 2, pp. 396–413; April, 1938.

[4] A. B. Chamberlain and W. B. Lodge, "The broadcast antenna," Proc. I.R.E., vol. 24, pp. 11–35; January, 1936.

[5] H. E. Gihring, "A field intensity slide rule," *Broadcast News*, December, 1935.

the curves were obtained by using a field-intensity slide rule.[5]

Fig. 6 reveals that the effect is considerable at the higher frequencies. At 1500 kilocycles the field intensity for curve $A$ is about 17 decibels below that for curve $B$.

## VII. Inverse Field Intensity

Three main factors affect the inverse field intensity which is usually taken as the unattenuated field intensity in millivolts per meter at a distance of 1 mile from the antenna for an input power of 1 kilowatt. The three factors are the number of radial wires in the earth system, the length of these radials, and the height of the antenna.

The effect of increase in the number of radials on the inverse field intensity at 1 mile for 1 kilowatt is shown in Fig. 7. These curves are derived from experimental results obtained at a frequency of 3000 kilocycles.[6] They will, however, serve to show the nature of the

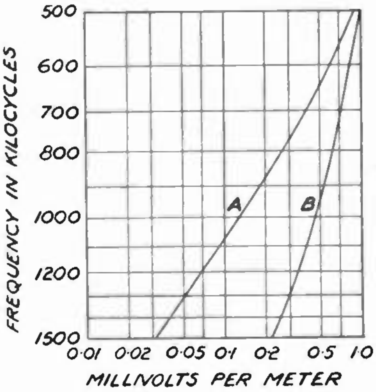



Fig. 6—Field intensity at a point 40 miles from a transmitter with an inverse field intensity of 100 millivolts per meter at 1 mile for the following conductivities:

| Curve | Miles | Electromagnetic Unit |
|---|---|---|
| $A$ | 0–20 | $25\times10^{-15}$ |
| | 20–40 | $100\times10^{-15}$ |
| $B$ | 0–20 | $100\times10^{-15}$ |
| | 20–40 | $25\times10^{-15}$ |

effect. Curves $A$ and $B$ are for 0.412-wave radials and for 0.25- and 0.125-wave antennas, respectively. Curves $C$ and $D$ are for 0.274-wave radials and for 0.25- and 0.125-wave antennas, respectively. The dashed line represents the theoretical maximum inverse field intensity of 194.5 millivolts per meter obtainable from a 0.25-wave antenna. Curve $B$ is suspect because it differs in shape from curves $A$, $C$, and $D$. These curves show that the field intensity obtained from a 0.125-wave antenna is only about half a decibel below that obtained from a 0.25-wave antenna under the same conditions. It is possible to obtain an inverse field intensity of about 180 millivolts per meter from a 0.125-wave antenna at 1 mile for 1 kilowatt.

It is, however, interesting to note that all of the four curves are within 2 decibels of the theoretical maximum inverse field for a 0.25-wave antenna provided the number of radials exceeds 30. It seems therefore that the use of a large number of radials may not always be justifiable on grounds of economy. This conclusion draws the attention to the need for reliable experimental curves showing the relationship between the inverse field intensity and the number of radials for various antenna heights, conductivities, frequencies, and lengths of radials.

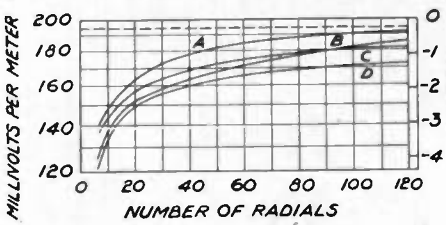



Fig. 7—The inverse field intensity at 1 mile for 1 kilowatt versus the number of radials for various values of antenna height $H$ and radial length $L$.

| | | |
|---|---|---|
| Curve $A$ | $H=0.25$ | $L=0.412$ |
| Curve $B$ | $H=0.125$ | $L=0.412$ |
| Curve $C$ | $H=0.25$ | $L=0.274$ |
| Curve $D$ | $H=0.125$ | $L=0.274$ |

Certain straight-line relations between the inverse field intensity and the length of the radials have been given for various antenna heights.[4] However the inverse field intensity cannot increase in this way as there are theoretical limits above which it should not rise.[6] Curves, therefore, have been derived from experimental results obtained at a frequency of 3000 kilocycles,[6] and these are shown in Fig. 8. Curves $A$ and $C$ are derived for 0.25- and 0.125-wave antennas, and curve $B$ was given in the original investigation for a 0.215-wave antenna. The theoretical maximum inverse field intensity of 194.5 millivolts per meter for a 0.25-wave antenna is also shown as a dashed line in Fig. 8. The curves in Figs. 7 and 8 seem to be reliable

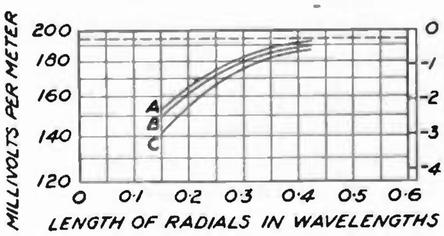



Fig. 8—The inverse field intensity at 1 mile for 1 kilowatt versus the length of radials for 120 radials and for various values of antenna height $H$.

| | |
|---|---|
| Curve $A$ | $H=0.25$ |
| Curve $B$ | $H=0.215$ |
| Curve $C$ | $H=0.125$ |

as they approach but do not exceed the theoretical maximum inverse field intensity of 194.5 millivolts per meter. It may therefore be said that although 0.5-wave radials may be ideal, 0.35-wave radials will be quite satisfactory. This conclusion is important because the latter radials require only half the ground area for installation.

The curves in Fig. 8 also show that the inverse field intensity for the antennas considered is within 2

[6] G. H. Brown, R. F. Lewis, and J. Epstein, "Ground systems as a factor in antenna efficiency," Proc. I.R.E., vol. 25, pp. 753–787; June, 1937.

decibels of the theoretical maximum if the radial length exceeds 0.2 of a wavelength. Although no comprehensive experimental curves for the relation between the inverse field intensity and the radial length are available, it is possible to conclude that, where economy is important, the length of the radials may be considerably reduced without reducing the inverse field intensity appreciably. However the voltage at the

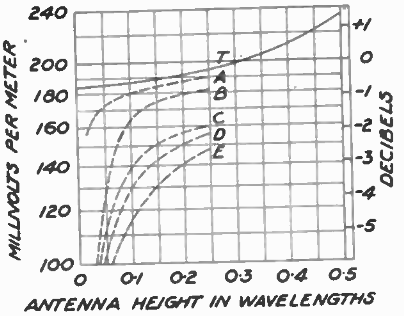



Fig. 9—The inverse field intensity at 1 mile for 1 kilowatt versus the antenna height for various numbers of radials $N$ and radial length $L$.

| | Theoretical Maximum Values | |
|---|---|---|
| Curve $T$ | | |
| Curve $A$ | $N=113$ | $L=0.412$ |
| Curve $B$ | $N=113$ | $L=0.274$ |
| Curve $C$ | $N=15$ | $L=0.412$ |
| Curve $D$ | $N=113$ | $L=0.137$ |
| Curve $E$ | $N=15$ | $L=0.137$ |

base of a short series-fed antenna may be considerable and the total earth current near the antenna may rise to high values.[6,7] The earth system near the base of a short antenna, therefore, must be efficient so that earth losses may be minimized. If all the radial wires do not extend right up to the base of the antenna, a ground screen should be used.

As the area of the ground system is proportional to the square of the radial length, the area of the ground will become very large at low frequencies. A search was made for some legitimate reason for reducing the length of radials with decrease in frequency. A previous investigation discussed the losses in the earth system.[6]

It was assumed that the earth current travels in a layer of earth of thickness

$$s = \frac{1}{\sqrt{\pi p c f}} \tag{1}$$

where $f$ is the frequency in cycles per second, $p$ is the permeability, and $c$ is the conductivity.

Then the power $dP$ lost in a ring of width $dx$ at a distance $x$ from the antenna is

$$dP = \frac{I^2 dx}{2\pi x s c} \tag{2}$$

$$= K_1\sqrt{f}\,dx \tag{3}$$

where $K_1$ is independent of the frequency.

The earth loses are proportional to the square root of the frequency.

[7] H. E. Gihring and G. H. Brown, "General considerations of tower antennas for broadcast use," Proc. I.R.E., vol. 23, pp. 311–356; April, 1935.

It has been assumed so far that

$$L \propto \frac{1}{f} \tag{4}$$

where $L$ is the radial length.

If, however, it is desired to keep the losses in the earth system constant, it might be possible to do so by increasing the length of the radials so that

$$L \propto \sqrt{f}. \tag{5}$$

If (4) and (5) are combined,

$$L \propto \frac{1}{\sqrt{f}}. \tag{6}$$

This means that if 0.5-wave radials are considered sufficient at 1500 kilocycles, 0.29-wave radials will suffice at 500 kilocycles. The use of this principle would economize considerably in the area of ground required at the lower frequencies.

If (2) is examined for the effect of conductivity, the following relation is obtained for the power lost:

$$dP = \frac{K_2}{\sqrt{c}}\,dx \tag{7}$$

where $K_2$ is independent of the conductivity.

It is possible to conclude, therefore, that less-extensive earth systems are satisfactory where the soil conductivity is high.

The theoretical curve for antenna height versus the inverse field intensity at 1 mile for 1 kilowatt is shown as a full line in Fig. 9. The dashed curves show experimental results obtained at 3000 kilocycles for various lengths and numbers of radials.[6] It appears that the use of an efficient earth system becomes more important as the antenna height decreases. It would seem that antennas below 0.1 wavelength in height might be used, but there is a practical difficulty in the way. The average of several curves for the variation in base voltage with the height of a series-fed antenna is given in Fig. 10 for a power of 1 kilowatt. The peak voltage

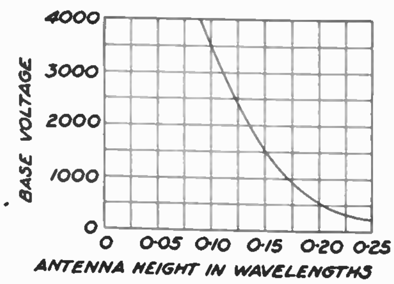



Fig. 10—Base voltage versus antenna height in wavelengths for 1 kilowatt for a series-fed antenna.

for 100 per cent modulation is 2.828 times the figures given. It will be seen that the voltage rises rapidly for antennas below 0.15 wavelength in height. Reasonably efficient antenna-coupling circuits can be economically designed for heights of the order of 0.125 of a wavelength.

According to the curves given in Fig. 9, it is quite easy to obtain an inverse field intensity within 2 decibels of the theoretical maximum even with comparatively small earth systems, as long as the antenna height exceeds 0.125 of a wavelength. Except for very poor earth systems, the inverse field intensity from a 0.25-wave antenna exceeds that from a 0.125-wave antenna by only a fraction of a decibel. It would appear therefore that the most economical antenna is the shortest which can be fed efficiently. The only important consideration, which may justify the use of an antenna much above 0.125 wavelength in height, is the increase in the fading-free area which may be obtained by using an antenna of the order of 0.5 of a wavelength in height.

The effect on the inverse field intensity of capacitive tops and other devices for reducing the height of the antenna is negligible, although the distance at which fading commences may be affected when these devices are added to antennas of the order of 0.4 of a wavelength in height.[7,8]

By the use of a directional antenna array it is possible to reduce the radiation in some desired direction by quite a large amount. It is not, however, possible to increase the radiation in a desired direction by more than a few decibels. The inverse field intensity is increased by 3 decibels for each bisection of the horizontal angle through which the power is radiated. Therefore, as far as field intensity and service area are concerned the additional cost of a directional array will seldom be warranted.

It is unlikely that shunt feeding will facilitate the use of antennas below 0.125 wavelength in height. It has been shown both theoretically and experimentally that the current below the tap point in a shunt-fed antenna may rise to values many times normal.[9,10] As the base of a shunt-fed antenna is at earth potential, there may be a high voltage gradient between this point and the tap point, and a correspondingly large current will flow.

## VIII. Service Area

Service radius curves are given in Figs. 11, 12, 13, and 14 for 1/5, 1, 5, and 25 kilowatts, respectively. These curves are derived from van der Pol's empirical formula for the propagation of radio waves.[3,5] The curves are drawn for the following conditions:

1. Inverse field intensities of 80, 180, 400, and 900 millivolts per meter at 1 mile for 1/5, 1, 5, and 25 kilowatts, respectively.
2. Minimum field intensities of 25 and 5 millivolts per meter for urban and residential areas, respectively.

[8] G. H. Brown, "A critical study of the characteristics of broadcast antennas as affected by antenna current distribution," Proc. I.R.E., vol. 24, pp. 48–81; January, 1936.

[9] Pierre Baudoux, "Current distribution and radiation properties of a shunt-excited antenna," Proc. I.R.E., vol. 28, pp. 271–275; June, 1940.

[10] J. F. Morrison and P. H. Smith, "The shunt-excited antenna," Proc. I.R.E., vol. 25, p. 686; June, 1937.

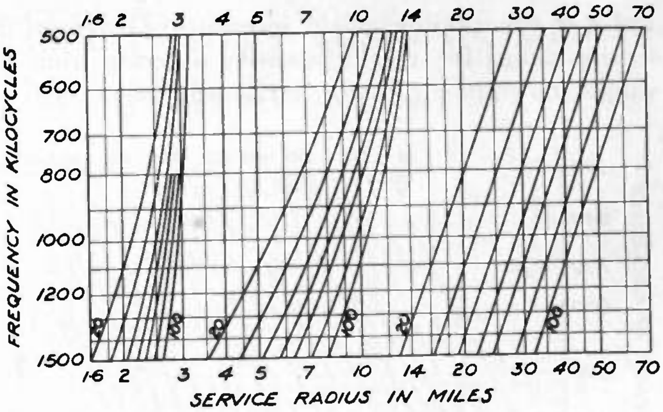



Fig. 11—Service radius in miles for a power of 1/5 kilowatt for urban, residential, and rural areas for conductivities of 20, 30, 40, 50, 60, 80, and $100\times10^{-15}$ electromagnetic unit.

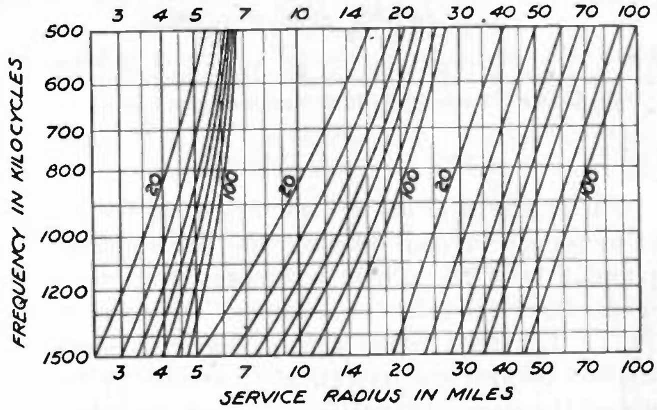



Fig. 12—Service radius in miles for a power of 1 kilowatt for urban, residential, and rural areas for conductivities of 20, 30, 40, 50, 60, 80, and $100\times10^{-15}$ electromagnetic unit.

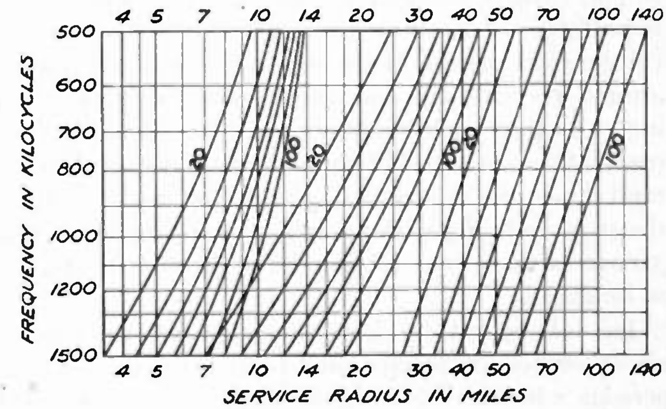



Fig. 13—Service radius in miles for a power of 5 kilowatts for urban, residential, and rural areas for conductivities of 20, 30, 40, 50, 60, 80, and $100\times10^{-15}$ electromagnetic unit.

3. Minimum field intensities as shown in Fig. 2 for rural areas.
4. Conductivities of 20, 30, 40, 50, 60, 80, and $100\times10^{-15}$ electromagnetic unit.

A useful relation applies to the curves for service in rural areas:

$$\frac{r}{R}=\sqrt[2.3]{\frac{e}{E}} \tag{8}$$

where $R$ is the known service radius, $E$ is the known field intensity, while $r$ is the unknown service radius, and $e$ is the unknown field intensity. Therefore, the service radius for a field intensity differing from the values shown in Fig. 2 may be found easily.

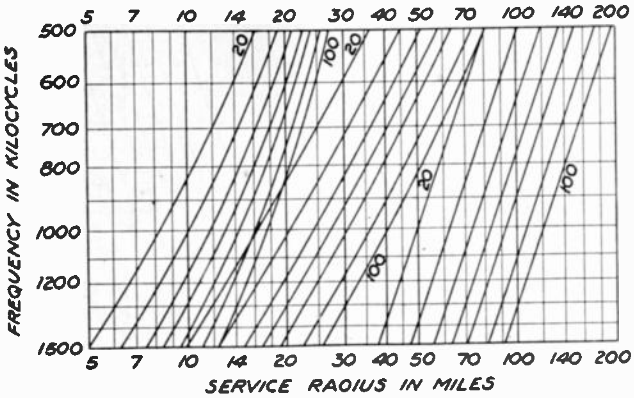



Fig. 14—Service radius in miles for a power of 25 kilowatts for urban, residential, and rural areas for conductivities of 20, 30, 40, 50, 60, 80, and $100\times10^{-15}$ electromagnetic unit.

## IX. Service Efficiency

The first step in the preliminary design of a station is to prepare a map showing the distribution of the population in the urban, residential, and rural areas of the district that has to be served. The curves in Figs. 11, 12, 13, and 14 show that it is generally advisable to place the transmitter as near as possible to the urban areas. However alternative sites may be selected according to well-known principles and local conditions. The contours for 25, 5, and 0.5 millivolts per meter should then be drawn for each site. The fading radius is used for the rural-area contour if it is less than the rural-service radius. If the conductivity is not uniform around the site, the contours must be drawn accordingly, so that each contour need not necessarily be a complete circle, but will be made up of arcs of different radii. Once the contours have been drawn, a count of the persons living within each contour enables a comparison of the service efficiencies of the alternative sites to be made.

The following formula is suggested for the comparison of service efficiencies, as it takes into account those persons who receive a signal less than that which is considered to be effective:

$$S = \frac{a + b/2}{P} \tag{9}$$

where $S$ is the service efficiency, $P$ is the total population of the district to be served, and the number of persons receiving an effective signal is given by $a$. The number of persons receiving a signal one degree lower than normal is given by $b$. Urban listeners receiving a field intensity between 5 and 25 millivolts per meter and listeners in a residential area receiving a field intensity between 0.5 and 5 millivolts per meter are in this class. Urban listeners who receive a field intensity between 0.5 and 5 millivolts per meter are not counted as this class of reception is invariably bad.

The final economic measure of the efficiency of a station is the annual cost per person in the effective service area:

$$C = \frac{A}{a + b/2} \tag{10}$$

where $A$ is the annual cost of operating the station. An evaluation of $C$ makes it possible to compare alternative designs.

## X. Conclusions

The service area and service efficiency of a station in the band from 500 to 1500 kilocycles are mainly determined by four factors; the power, conductivity, frequency, and the placing of the station with respect to the distribution of the population.

1. The power and the field intensity may be increased by 21 decibels through the range from 1/5 to 25 kilowatts.
2. When the conductivity rises from $20\times10^{-15}$ electromagnetic unit to $100\times10^{-15}$ electromagnetic unit, the average increase in the field intensity is of the order of 10 decibels, but the increase may be as much as 18 decibels and as little as a fraction of a decibel.
3. When the frequency decreases from 1500 to 500 kilocycles, the average increase in field intensity is of the order of 15 decibels, and the maximum increase may be as much as 20 decibels.
4. When the distance between the transmitter and the receiver is halved, the average increase in the field intensity is of the order of 10 decibels but the increase will not be more than 14 decibels, nor will it be less than 6 decibels.

By comparison with the above four factors, the design of the antenna and the earth system is of secondary importance. The antenna need not exceed 0.125 of a wavelength in height and the use of a reasonably efficient earth system will ensure that the field intensity will be only a few decibels below the theoretical maximum.

The conductivity is beyond the control of the designer and the station frequency and power are usually chosen for him. The successful design of a station must then rest upon the careful choice of the most suitable site.

# Circuit for Neutralizing Low-Frequency Regeneration and Power-Supply Hum*

WEN-YUAN PAN†, ASSOCIATE, I.R.E.

***Summary***—*In high-gain multistage amplifiers, low-frequency regeneration frequently exists which affects the amplification characteristics of the amplifier and commonly causes oscillations and "motorboating." A simple bridge-balancing circuit has been devised to neutralize this effect. Besides reducing regeneration, it also reduces the effects due to hum present across the plate-supply filter. Reductions in the order of 40 decibels are attainable.*

IN HIGH-GAIN multistage amplifiers, low-frequency regeneration frequently exists which affects the amplification characteristics of the amplifier and commonly causes oscillations and "motorboating." Such low-frequency regeneration is principally caused by the impedance of the power supply, i.e., the impedance measured across the output terminals of the rectifier-filter system looking toward the rectifier. Voltage developed across this impedance by the current in the final amplifier stage is fed back to stages of lower power levels, to cause the regeneration and oscillations.

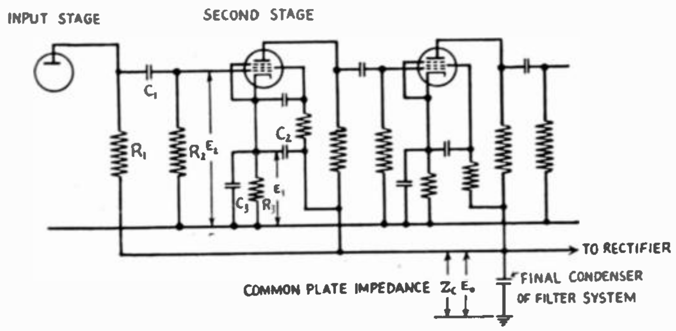



Fig. 1—Schematic of neutralized amplifier.

The usual method of combating this regeneration is to employ resistance-capacitance combinations as decoupling filters. However, the effectiveness of the decoupling filter is less the lower the frequency, whereas it is at the very low frequencies that the regenerative effects are most troublesome. The purpose of this paper is to present a simple circuit of low cost that eliminates the regeneration at all frequencies, and yet has little or no effect on amplifier performance.

Fig. 1 is a schematic diagram showing the neutralizing of feedback from high-level stages back to the plate circuit of the first stage as a result of the plate-supply impedance $Z_c$ that is common to several stages. The only difference between this and the ordinary circuit is the addition of a condenser $C_2$. By properly adjusting the circuit constants used in the grid and cathode

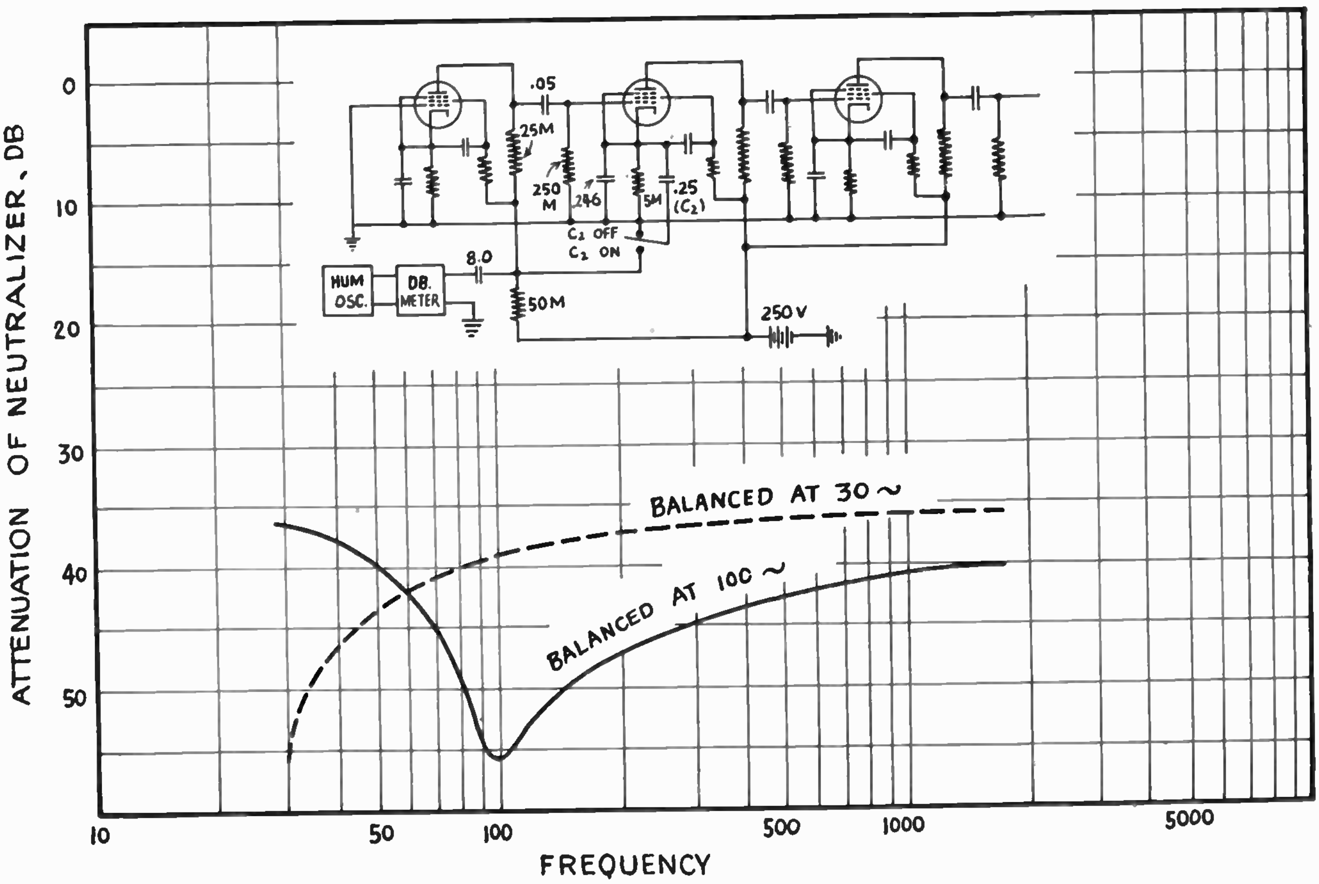



Fig. 2—Attenuation characteristics of neutralized amplifier.

* Decimal classification: R363.2. Original manuscript received by the Institute, November 26, 1941.
† China Defense Supplies, Inc., Washington, D. C., and Universal Trading Corporation, New York, N. Y.

circuits of the second amplifier stage the condenser $C_2$ can be made to introduce a voltage $E_1$ across the cathode resistor of this stage that exactly balances out the voltage $E_2$ applied to the grid of the same tube because of the presence of voltage across the common plate impedance $Z_c$.

Let $E_0$ = voltage across the common plate impedance

$E_1$ = voltage introduced across the cathode of second amplifier stage due to the presence of $E_0$

$E_2$ = voltage existing on the grid of second amplifier stage due to the presence of $E_0$

$\omega = 2\pi \times$ frequency.

Then,

$$E_1 = E_0 \frac{1}{1 + \frac{C_3}{C_2} + \frac{1}{j\omega C_2 R_3}} \tag{1}$$

$$E_2 = E_0 \frac{1}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_p} + \frac{1 + \frac{R_1}{R_p}}{j\omega C_1 R_2}} \tag{2}$$

where,

$R_p$ = plate resistance of the input amplifier tube

For perfect neutralization, the voltages $E_1$ and $E_2$ should be equal in magnitude and in phase, which gives

$$\frac{C_3}{C_2} = \frac{R_1}{R_2}\left(1 + \frac{R_2}{R_p}\right) \tag{3}$$

$$\frac{C_1}{C_2} = \frac{R_3}{R_2}\left(1 + \frac{R_1}{R_p}\right). \tag{4}$$

Equations (3) and (4) are independent and both must be satisfied. It will be noted that the conditions for balance are independent of frequency.

This neutralizing circuit reduces power-supply hum to the same extent as it does regeneration. This is because such hum is caused by a voltage fed back from the power supply to the input stages, just as regeneration is caused by a voltage fed back in the same way. The only difference in the two cases is that the voltage across the output of the power supply arises from different causes.

# Formulas for the Skin Effect*

HAROLD A. WHEELER†, FELLOW, I.R.E.

*Summary*—*At radio frequencies, the penetration of currents and magnetic fields into the surface of conductors is governed by the skin effect. Many formulas are simplified if expressed in terms of the "depth of penetration," which has merely the dimension of length but involves the frequency and the conductivity and permeability of the conductive material. Another useful parameter is the "surface resistivity" determined by the skin effect, which has simply the dimension of resistance. These parameters are given for representative metals by a convenient chart covering a wide range of frequency. The "incremental-inductance rule" is given for determining not only the effective resistance of a circuit but also the added resistance caused by conductors in the neighborhood of the circuit. Simple formulas are given for the resistance of wires, transmission lines, and coils; for the shielding effect of sheet metal; for the resistance caused by a plane or cylindrical shield near a coil; and for the properties of a transformer with a laminated iron core.*

THE "skin effect" is the tendency for high-frequency alternating currents and magnetic flux to penetrate into the surface of a conductor only to a limited depth. The "depth of penetration" is a useful dimention, depending on the frequency and also on the properties of the conductive material, its conductivity or resistivity and its permeability. If the thickness of a conductor is much greater than the depth of penetration, its behavior toward high-frequency alternating currents becomes a surface phenomenon rather than a volume phenomenon. Its "surface resistivity" is the resistance of a conducting surface of equal length and width, and has simply the dimension of resistance. In the case of a straight wire, the width is the circumference of the wire.

Maxwell[1] discovered that the voltage required to force a varying current through a wire increases more than could be explained by inductive reactance. He explained this as caused by a departure from uniform current density. This discovery was followed up by Heaviside, Rayleigh, and Kelvin. It came to be called the "skin effect," because the current is concentrated in the outer surface of the conductor. The ratio of high-frequency resistance to direct-current resistance for a straight wire was computed in terms of Bessel functions and was reduced to tables.[2–7]

* Decimal classification R144×R282.1. Original manuscript received by the Institute, May 13, 1942. Presented, Rochester Fall Meeting, November 10, 1941.

† Hazeltine Service Corporation, Little Neck, N. Y.

[1] J. C. Maxwell, "Electricity and Magnetism," on page 385. 1873/1937, vol. 2, section 690, p. 322.

[2] Lord Rayleigh, *Phil. Mag.*, vol. 21, p. 381; 1886.

[3] C. P. Steinmetz, "Transient Electric Phenomena and Oscillations," pp. 361–393, 1909/1920.

[4] S. G. Starling, "Electricity and Magnetism," 1912/1914, pp. 364–369.

[5] E. B. Rosa and F. W. Grover, "Formulas and tables for the calculation of mutual and self-inductance (revised)," Bureau of Standards, S-169, pp. 172–182, 1916.

[6] "Radio Instrumentsand Measurements," Bureau of Standards, C-74, pp. 299–311, 1918/1924.

[7] J. H. Morecroft, "Principles of Radio Communication," 1921, pp. 114–136.

Steinmetz defined the "depth of penetration" without restriction as to the shape of the conductor. He applied this concept to laminated iron cores, as well as to conductors. Unfortunately, he gave two definitions which differ slightly, one for iron cores and another for conductors. The latter definition has been generally adopted, as in the Steinmetz tables on page 385.

More recent writers have reduced the treatment of the skin effect to simple terms and have generalized its application.[8–16] Schelkunoff and Stratton have given the most comprehensive treatment of the subject, including the depth of penetration in all kinds of problems involving conductors. They have introduced the concept of surface impedance, from which the surface resistivity is a by-product.

In spite of this active history of the skin effect, there is still a need for a simple and direct summary which will facilitate its appreciation and its application to simple problems. That is the purpose of this presentation.

Following Harnwell and Stratton, the mks rationalized system of units is employed for all relations, except where inches are specified. The properties of materials are taken for room temperature (20 degrees centigrade or 293 degrees absolute). The following list gives the principal symbols used herein.

$d$ = depth of penetration (meters)
$R_1$ = surface resistivity (ohms)
$\sigma$ = conductivity (mhos per meter)
$\rho = 1/\sigma$ = resistivity (ohm-meters)
$\mu$ = permeability (henrys per meter)
$\mu_0 = 4\pi \cdot 10^{-7}$ = permeability of space
$f$ = frequency (cycles per second)
$\omega = 2\pi f$ = radian frequency (radians per second)
$j = \sqrt{-1}$
$e = 2.72$ = base of logarithms
$\exp x = e^x$ = exponential function
$z$ = depth from the surface into the conductive medium (meters)
$w$ = width (meters)
$l$ = length (meters)
$a$ = thickness or radius (meters)
$b$ = distance, length or width (meters)
$c$ = distance (meters)
$r$ = radius (meters)
$A$ = area (square meters)
$I$ = current (amperes)
$i$ = current density at a depth $z$ (amperes per square meter)
$i_0$ = current density at the surface ($z=0$)
$H$ = magnetic field intensity at a depth $z$ (amperes per meter)
$H_0$ = magnetic field intensity at the surface ($z=0$)
$E$ = electromotive force (volts)
$P$ = power (watts)
$P_1$ = power dissipation per unit area (watts per square meter)
$Z = R+jX$ = impedance (ohms)
$X$ = reactance (ohms)
$R$ = resistance (ohms)
$G$ = conductance (mhos)
$L$ = inductance (henries)
$L_0$ = inductance in space outside of conductive medium
$m$ = number of laminations
$n$ = number of turns
$r$ = ratio of resistivity
$x$ = ratio of radii
$Q$ = ratio of reactance to resistance

Fig. 1 is a chart[17] giving the surface resistivity $R_1$ and the depth of penetration $d$ for various metals, over a wide range of frequency $f$. The depth is plotted in parts of an inch, since this aids in practical application and introduces no confusion with the mks electrical units. Each sloping line represents one metal, depending on its resistivity $\rho$ or conductivity $\sigma$ and its permeability $\mu$ at room temperature (20 degrees centigrade or 293 degrees absolute). The heavy lines are for copper, which is the logical standard of comparison. Additional lines can be drawn to meet special requirements, shifting them from the copper line in accordance with the properties of the metal.

Fig. 2 shows a slab of conductive material to be used in describing the skin effect. The current $I$ is concentrated in the upper surface. From Harnwell, the alternating-current density $i$ in the surface of a conductor decreases with depth $z$ according to the formula

$$\begin{aligned}\frac{i}{i_0} &= \exp - z\sqrt{j\omega\mu\sigma} \\ &= \exp - (1+j)z\sqrt{\frac{\omega\mu\sigma}{2}} \\ &= \exp - \frac{z}{d}\exp - j\frac{z}{d}. \qquad (1)\end{aligned}$$

[8] E. J. Sterba and C. B. Feldman, "Transmission lines for short-wave radio systems," PROC. I.R.E., vol. 20, pp. 1163–1202; July, 1932; *Bell Sys. Tech. Jour.*, vol. 11, pp. 411–450; July, 1932. (Convenient formulas.)

[9] S. A. Schelkunoff, "The electromagnetic theory of coaxial transmission lines and cylindrical shields," *Bell Sys. Tech. Jour.*, vol. 8, pp. 532–579; October, 1934. (The most complete theoretical treatment.)

[10] S. A. Schelkunoff, "Coaxial communication transmission lines," *Elec. Eng.*, vol. 53, pp. 1592–1593; December, 1934. (A brief description of the physical behavior.)

[11] E. I. Green, F. A. Leibe, and H. E. Curtis, "The proportioning of shielding circuits for minimum high-frequency attenuation," *Bell Sys. Tech. Jour.*, vol. 15, pp. 248–283; April, 1936.

[12] August Hund, "Phenomena in High-Frequency Systems," 1936, pp. 333–338.

[13] S. A. Schelkunoff, "The impedance concept and its application to problems of reflection, refraction, shielding and power absorption," *Bell Sys. Tech. Jour.*, vol. 17, pp. 17–48; January, 1938.

[14] G. P. Harnwell, "Principles of Electricity and Electromagnetism," pp. 313–317, 1938.

[15] W. R. Smythe, "Static and Dynamic Electricity," 1939, pp. 388–417.

[16] J. A. Stratton, "Electromagnetic Theory," pp. 273–278, 500–511, and 520–554, 1941, (mks units.)

[17] This chart has been reprinted in the report of the Rochester Fall Meeting in *Electronics*, December, 1941. More recently, a similar chart has appeared in the following reference, together with other valuable formulas and curves: J. R. Whinnery, "Skin effect formulas," *Electronics*, vol. 15, pp. 44–48; February, 1942.

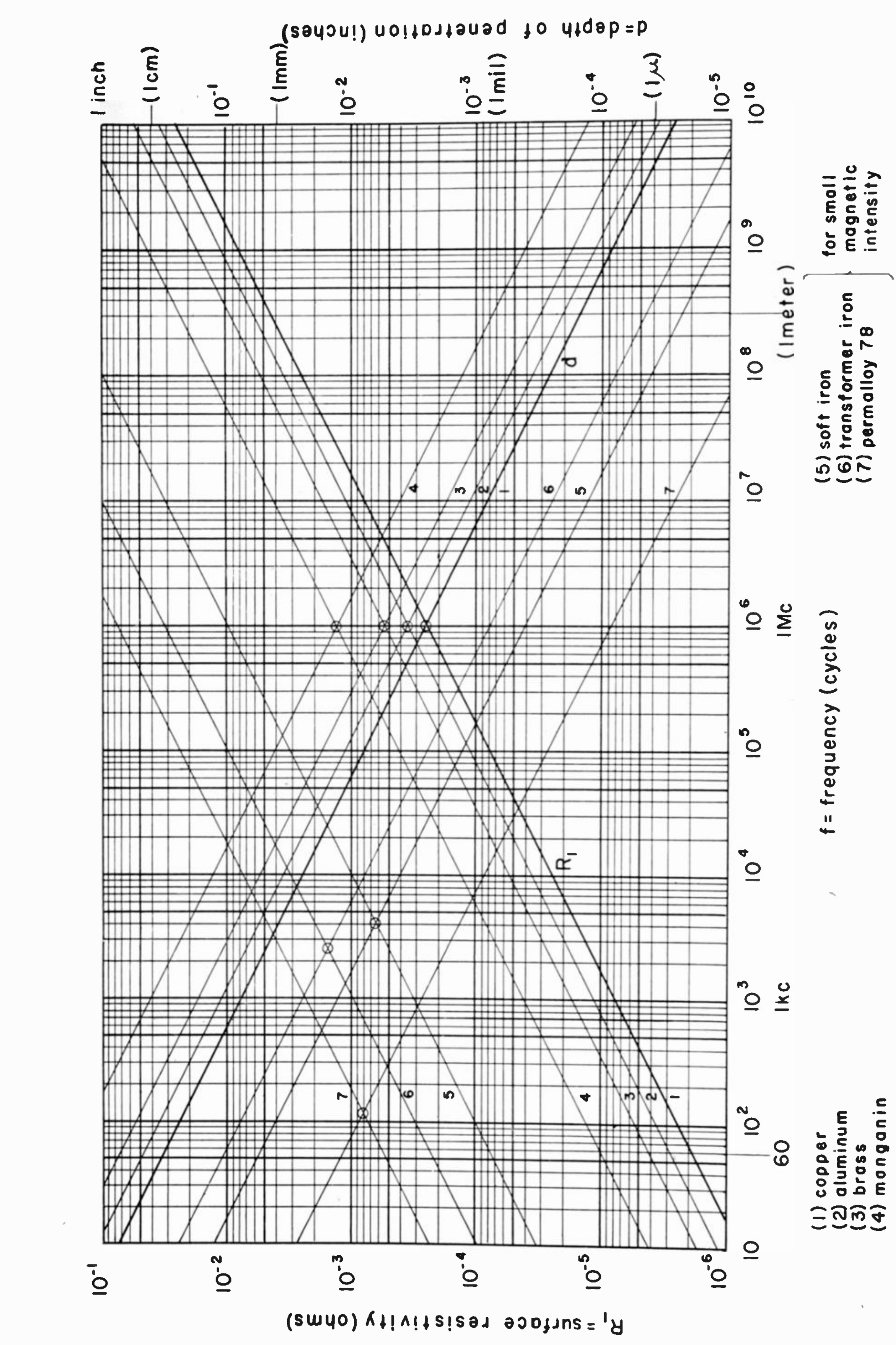



Fig. 1—Surface Resistivity and Depth of Penetration.

This decay of current density is shown by the shaded area plotted on the side of the slab.

The depth of penetration is defined by the last formula, as the depth at which the current density (or magnetic flux) is attenuated by 1 napier (in the ratio $1/e = 1/2.72$, or $-8.7$ decibels). At the same depth, its phase lags by 1 radian, so $d$ is $1/2\pi$ wavelength or 1 radian length in terms of the wave propagation in the conductor.

The depth of penetration, by this definition, is

$$d = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f \mu\sigma}} \qquad \text{meters.} \quad (2)$$

It is noted that the $\sqrt{2}$ factor arises when the $\sqrt{j}$ is resolved into its real and imaginary components in the exponent in (1).

The total current is the integral of the current density in the conductive medium. This integral from the surface into the medium is a decaying spiral in the complex plane, which rapidly approaches its limit if the thickness is much greater than the depth of penetration. The total current is therefore given by the integral for infinite depth, over the width $w$:

$$I = w\int_0^\infty i \cdot dz$$

$$= i_0 w \int_0^\infty \exp - (1+j)\frac{z}{d}\, dz$$

$$= \frac{i_0 w d}{1+j} \qquad \text{amperes.} \quad (3)$$

The voltage $E$ on the surface along the length of the conductor is obtained from the current density and the volume resistivity.

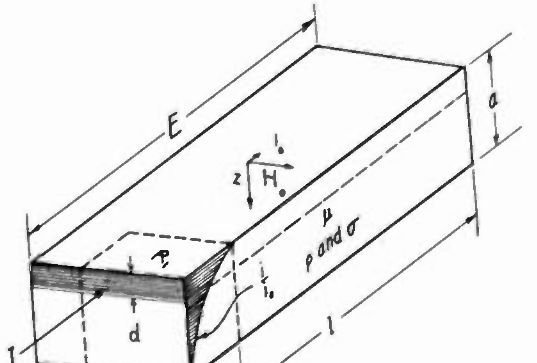



Fig. 2—The skin effect on the surface of a conductor.

$$E = i_0 l \rho \qquad \text{volts.} \quad (4)$$

If this voltage were to be measured, the return circuit would have to be adjacent to the surface so as not to include any of the magnetic flux in the near-by space.

The "internal impedance" or "surface impedance" is computed from this voltage $E$ and the current $I$.

$$Z = \frac{E}{I} = (1+j)\frac{\rho l}{wd}$$

$$= (1+j)\frac{l}{w}\sqrt{\pi f \mu \rho} = \frac{l}{w}\sqrt{j\omega\mu\rho} \qquad \text{ohms.} \quad (5)$$

Its real and imaginary components are the resistance

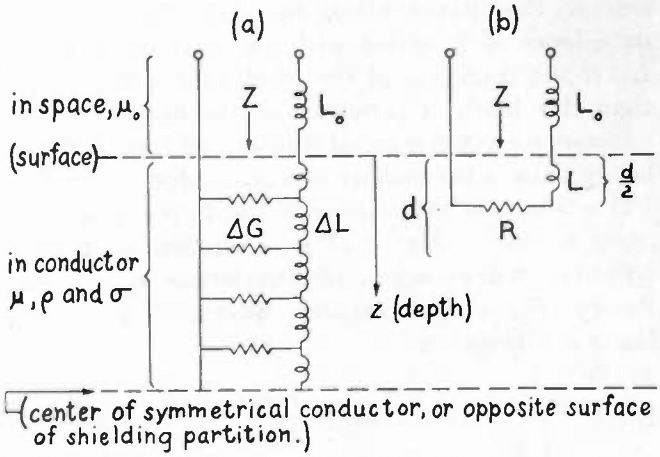



Fig. 3—The internal impedance of a conductor, in terms of distributed circuit parameters (a) and equivalent lumped parameters (b).

and the internal reactance, which are equal.

$$Z = R + jX,$$

$$R = X = \frac{l}{w}\sqrt{\pi f \mu \rho} \qquad \text{ohms.} \quad (6)$$

The surface resistivity $R_1$, given in the chart, is defined as the resistance of a surface of equal length and width.

$$R_1 = \frac{\rho}{d} = \sqrt{\pi f \mu \rho}$$

$$R = X = \frac{l}{w} R_1 \qquad \text{ohms.} \quad (7)$$

For example, $R_1$ is the resistance of the unit square surface in Fig. 2.[18]

The internal inductance is the part of the total inductance which is caused by the magnetic flux in the conductive medium. It is computed from the internal reactance.

$$L = \frac{X}{\omega} = \frac{l}{w}\left(\mu\frac{d}{2}\right) \qquad \text{henries.} \quad (8)$$

This is the inductance of a layer of the conductive material having a thickness of $d/2$, one half the depth of penetration. This merely means that the mean depth of the current is one-half the thickness of the conducting layer.

Fig. 3 illustrates the concept of internal impedance in terms of electric circuit elements. In the diagram (a), the inductance $L_0$ is that caused by the magnetic flux in the space adjacent to the conductor. Each part

[18] Schelkunoff (footnote 9, p. 550) calls $R_1$ the "intrinsic resistance" of the material.

of the current meets additional inductance in proportion to its depth from the surface of the conductor. This inductance is $\Delta L$ per element of depth. The conductance of the material is $\Delta G$ for the same element of depth. The conductive slab behaves as a transmission line with paths of shunt conductance in layers parallel to the surface, and series inductance between layers. This hypothetical line presents the internal impedance $Z$ in series with the external inductance $L_0$. If the thickness of the conductor is much greater than the depth of penetration, the impedance is unaffected by conditions at the far end of the line, or beyond the other surface of the conductor.

The internal impedance of the hypothetical transmission line in Fig. 3(a) is computed from its distributed inductance and conductance, by circuit theory. Since the magnetic flux path has an area $l\Delta z$ and a length $w$

$$\Delta L = \frac{\mu l \Delta z}{w} \qquad \text{henries. (9)}$$

Since the current path has an area $w\Delta z$ and a length $l$,

$$\Delta G = \frac{\sigma w \Delta z}{l} = \frac{w \Delta z}{\rho l} \qquad \text{mhos. (10)}$$

The impedance of a long line with these properties is

$$Z = \sqrt{j\omega \Delta L / \Delta G} = \frac{l}{w}\sqrt{j\omega\mu\rho} \qquad \text{ohms. (11)}$$

This is an independent complete derivation of (5), without recourse to electromagnetic-wave equations.

The components of internal impedance are shown in Fig. 3(b) as $R$ and $L$. The resistance $R$ is that of a layer whose thickness is equal to the depth of penetration $d$. The internal inductance is that of a layer whose thickness is $d/2$, one half the depth of penetration.

Some inductance formulas carry the assumption that the current travels in a thin sheet on the surface of the conductor, as if the resistivity were zero. Such assumptions are usual for transmission lines, wave guides, cavity resonators, and piston attenuators. Such formulas can be corrected for the depth of penetration by assuming that the current sheet is at a depth $d/2$ from the surface. This is the same as assuming that the surface of the conductor recedes by the amount

$$\frac{d}{2} \cdot \frac{\mu}{\mu_0}. \qquad (12)$$

The second factor has an effect only if the conductive material has a permeability $\mu$ differing from that of space $\mu_0$. The same correction is applicable to shielding partitions, regarding their effect on the inductance of near-by circuits.

There is sometimes a question which surface of a conductor will carry the current. The rule is, that the current follows the path of least impedance. Since the impedance is mainly inductive reactance, in the common cases, the current tends to follow the path of least inductance. In a ring, for example, the current density is greater on the inner surface. In a coaxial line, the current flows one way on the outer surface of the inner conductor and returns on the inner surface of the outer conductor.

In determining whether the thickness is much greater than the depth of penetration, the effective thickness corresponds to the length of the hypothetical line in Fig. 3(a). In a symmetrical conductor with penetration from both sides, as in a strip or a wire, the effective thickness is the depth to the center of the conductor. In a shielding partition with penetration into the surface on one side and with open space on the other side, the effective thickness is the actual thickness. If the effective thickness exceeds twice the depth of penetration, the accuracy of the above impedance formulas is sufficient for most purposes, within two per cent for a plane surface.

The shielding effect of a conductive partition depends not only on the material and thickness of the partition, but also on its location. For example, two layers of metal have more shielding effect if they are separated by a layer of free space than if they are close together. If a shielding partition carries current on one surface ($z=0$) and is exposed to free space at the other surface ($z=a$) the current density has a definite ratio between one surface and the the other. For the thickness $a$, much greater than the depth of penetration, as in Fig. 2, this ratio is

$$\frac{i_a}{i_0} = 2 \exp - \frac{a}{d} \qquad (a \gg d)$$
$$= 0.69 - \frac{a}{d} \qquad \text{napiers}$$
$$= 6 - 8.7\frac{a}{d} \qquad \text{decibels. (13)}$$

The factor 2 is caused by reflection at the far surface. The space on either side of a shield usually adds to the attenuation indicated by this formula.

The shielding ability of a given metal at a given frequency is best expressed as the attenuation for a convenient unit of thickness, disregarding the reflection factor. The unit of thickness may be 1 millimeter ($10^{-3}$ meter) or 1 mil ($2.54\cdot 10^{-5}$ meter). In copper at 1 megacycle, for example, it is 132 decibels per millimeter or 3.3 decibels per mil. In iron, it is much greater and depends also on the magnetic flux density, since that affects the permeability.

The power dissipation in the surface of a shield is determined by the magnetic field intensity at its surface. The same is true of current conductors or iron cores but in those cases there are more direct methods of computation in terms of current and effective resistance. Since the magnetic flux path has a length equal to the width $w$ of the conductor, and since the magnetomotive force is equal to the current $I$, the magnetic intensity at the surface is

$$H_0 = \frac{I}{w} \qquad \text{amperes per meter.} \quad (14)$$

The power dissipation is

$$P = I^2R = (wH_0)^2 \frac{l}{w} R_1$$
$$= lwH_0{}^2R_1 = lwP_1 \qquad \text{watts} \quad (15)$$

in which the power dissipation per unit area of surface is

$$P_1 = H_0{}^2R_1 \qquad \text{watts per square meter.} \quad (16)$$

For most purposes, the power dissipation is more readily computed by the following method, in terms of effective resistance in a circuit.

The "incremental-inductance rule" is a formula which gives the effective resistance caused by the skin effect, but is based entirely on inductance computations. Its great value lies in its general validity for all metal objects in which the current and magnetic intensity are governed by the skin effect. In other words, the thickness and the radius of curvature of exposed metal surfaces must be much greater than the depth of penetration, say at least twice as great. It is equally applicable to current conductors, shields, and iron cores.

This rule is a generalization of (7) which states that the surface resistance $R$ is equal to the internal reactance $X$ as governed by the skin effect. The internal reactance is the reactance of the internal inductance $L$ in (8). This inductance is the increment of the total inductance which is caused by the penetration of magnetic flux under the conductive surface. This change of inductance is the same as would be caused by the surface receding to the depth given in (12). Starting with a knowledge of this depth, the reverse process of computation gives the increment of inductance caused by the penetration, and from that the effective resistance as governed by the skin effect.

The incremental-inductance rule is stated, that the effective resistance in a circuit is equal to the change of reactance caused by the penetration of magnetic flux into metal objects. It is valid for all exposed metal surfaces which have thickness and radius of curvature much greater than the depth of penetration, say at least twice as great.

The application of the incremental-inductance rule involves the following steps:

(a) Select the circuit in which the effective resistance is to be evaluated, and identify the exposed metal surfaces in which the skin effect is prevalent.

(b) Compute the rate of change of inductance of this circuit with recession of each of the metal surfaces, $\sigma L_0/\sigma z$, assuming zero depth of penetration.[19]

(c) Note that the increment of inductance caused by penetration into each surface is

$$L = \frac{\mu}{\mu_0} \cdot \frac{d}{2} \cdot \frac{\partial L_0}{\partial z} \qquad \text{henries.} \quad (17)$$

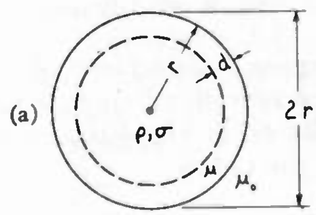
(d) Compute the effective resistance contributed by each surface,

$$R = \omega L = \frac{1}{\mu_0} \cdot \frac{\partial L_0}{\partial z} R_1 \qquad \text{ohms.} \quad (18)$$

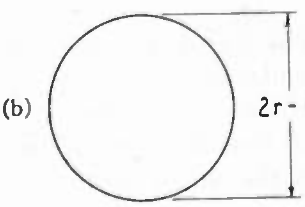
For a surface carrying the current of the circuit, this is identical with (7). For the effect of near-by metal objects, such as shields, this formula is easily applied in many practical cases. It is most useful in cases of nonuniform current distribution, which otherwise would require special integrations.

A straight wire has its current concentrated in a tubular surface layer as shown in Fig. 4(a). The depth of this layer is $d$. The radius of the wire is $r$ but the mean radius of the current tube is $r - d/2$. The resistance ratio of the wire is the ratio of the alternating-current resistance $R$ of the direct-current resistance $R_0$. It is the inverse ratio of the effective cross-sectional areas,

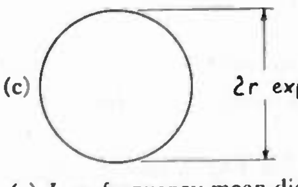
$$\frac{R}{R_0} = \frac{\pi r^2}{\pi(2r - d)d}$$
$$= \frac{r}{2d} + \frac{1}{4} + \cdots \qquad (r > 2d). \quad (19)$$




(a) High-frequency current tube.




(b) High-frequency mean diameter.




(c) Low-frequency mean diameter.

Fig. 4—The current distribution in a straight wire.

[19] A second-order approximation is secured if $\delta L_0/\delta z$ is computed assuming that the surface is below the actual surface by the amount given in (12).

Since the assumptions are an approximation at best, only the first two terms of this series deserve attention. They give a close approximation if the radius exceeds twice the depth of penetration, or if the resistance ratio exceeds 5/4.[20–26]

The inductance of a straight wire is determined by the mean diameter of the current path. Fig. 4(b) shows the equivalent current sheet for the case in which the radius is very much greater than the depth of penetration. A perfect conductor, to have the same inductance with zero depth of penetration, has a radius which is less by the amount given in (12). This rule is reliable only if the equivalent radius is greater than 7/8 the actual radius.

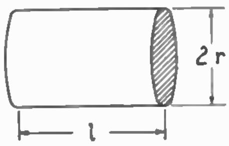



Fig. 5—Straight wire.

The low-frequency inductance of a straight wire, with uniform current distribution, is its maximum inductance. As shown in Fig. 4(b), the equivalent current sheet has the radius

$$r \exp - \frac{\mu}{4\mu_0} \qquad (20)$$

in which the factor exp-1/4 is the "geometric-mean distance" of a circular area.[27]

The straight wire of Fig. 5, assuming a depth of penetration very much less than the radius, has its resistance expressed by the simple formula

$$R = \frac{l}{2\pi r} R_1 \qquad \text{ohms.} \quad (21)$$

This neglects the second term in the series of (19). It is on this simple basis that the following cases are described.[28,8,11]

The coaxial line of Fig. 6 has its current flowing one way on the lesser radius $r_1$ and returning on the greater radius $r_2$. The total resistance is

$$R = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \frac{l}{2\pi} R_1 \qquad \text{ohms.} \quad (22)$$

The inductance in the space between the conductors is[29]

$$L_0 = \frac{\mu_0 l}{2\pi} \log \frac{r_2}{r_1} \qquad \text{henries.} \quad (23)$$

For a given value of the greater radius $r_2$, minimum attenuation in this line requires minimum $R/L_0$, and this is obtained with $r_2/r_1 = 3.59$, approximately.[30,31] With

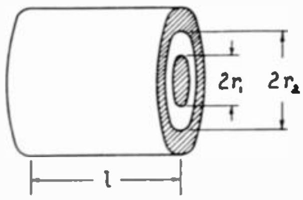



Fig. 6—Coaxial conductors.

this shape, the resistance of inner and outer conductors is divided as 78 per cent and 22 per cent of the total. Since the optimum ratio satisfies the equation

$$\log \frac{r_2}{r_1} = 1 + \frac{r_1}{r_2}, \qquad (24)$$

the ratio of reactance to resistance for this shape is, for a nonmagnetic conductor,[32]

$$Q = \frac{2r_1}{d} = \frac{r_2}{1.8\, d} \qquad (\mu = \mu_0). \quad (25)$$

This is the ratio of the diameter of the inner conductor to the depth of penetration. In general,

$$Q = \frac{2r_1}{d} \cdot \frac{\log \dfrac{r_2}{r_1}}{1 + \dfrac{r_1}{r_2}}. \qquad (26)$$

This value is reduced slightly by end effects.

If a coaxial line is used as the inductance of a resonant circuit, maximum impedance at parallel resonance may be desired. This is obtained with maximum $L_0^2/R$, which determines the condition

$$\frac{1}{2} \log \frac{r_2}{r_1} = 1 + \frac{r_1}{r_2}. \qquad (27)$$

The required shape is $r_2/r_1 = 9.2$, approximately.[33] If the length of the line is much less than one-quarter wavelength, so its shunt capacitance is negligible, this optimum shape has the following resistance at parallel resonance: $(\mu = \mu_0)$.

$$R' = Q^2 R = 0.307 \frac{l r_2}{d^2} R_1 \qquad \text{ohms.} \quad (28)$$

For given frequency and material, this resistance is proportional to the area of the conducting surfaces.

[20] Morecroft, (footnote 7, p. 116), curves of resistance ratio.
[21] E. Jahnke and F. Emde, "Tables of Functions," B. G. Teubner, Berlin, Germany, 1933, chapter 18, p. 314, Fig. 165, curve $rb_0/2b_1$.
[22] August Hund, "High-Frequency Measurements," 1933. pp. 263–266. Series expansions.
[23] Schelkunoff, footnote 9, pp. 551–553, formulas and curves for resistance and reactance ratio.
[24] August Hund, "Phenomena in High Frequency Systems," 1936. p. 338. Series expansions.
[25] J. H. Miller, "R−F resistance of copper wire," *Electronics*, vol. 9, no. 2, p. 338; February, 1936. Curves and formula.
[26] Stratton, footnote 16, p. 537, series expansions.
[27] Rosa and Grover, footnote 5, p. 167.
[28] Alexander Russell, "The effective resistance and inductance of a concentric main," *Phil. Mag.*, sixth series, vol. 17, pp. 524–552; April, 1909.
[29] Harnwell, footnote 14, p. 304.
[30] Sterba and Feldman, footnote 8, p. 419.
[31] Green, Leibe, and Curtis, footnote 11, p. 253.
[32] In all cases, $Q$ is expressed on the assumption of a nonmagnetic conductor.
[33] F. E. Terman, "Resonant lines in radio circuits," *Elec. Eng.*, vol. 53, pp. 1046–1053; July, 1934.

A pair of straight parallel wires is shown in cross section in Fig. 7. The same current flows in opposite directions in the two wires, and is concentrated on the surface. If the wire diameter 2a is much less than the center-to-center separation 2b, the resistance of each wire is given by (21) for Fig. 5. As the diameter in Fig. 7 approaches equality with the separation, the proximity of wires causes greater current density on the inner sides.[34,35] This effect is easily evaluated by the incremental-inductance rule. The approximate and

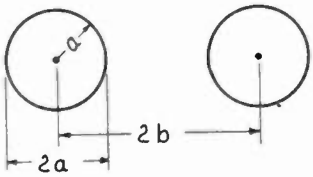



Fig. 7—Parallel wires.

exact formulas for the external inductance of this pair of wires, of length $l$, are

$$L_0 = \frac{\mu_0 l}{\pi} \log \frac{2b}{a} \quad (a \ll b)$$

$$= \frac{\mu_0 l}{\pi} \operatorname{anticosh} \frac{b}{a}$$

$$= \frac{\mu_0 l}{\pi} \log \left[\frac{b}{a}\left(1 + \sqrt{1 - \left(\frac{a}{b}\right)^2}\right)\right]. \tag{29}$$

This first formula neglects the proximity of the inner sides of the wires. The third formula shows in the parenthetical factor, by the amount the factor departs from 2, the reduction of inductance by the extra concentration of current on the inner sides. Since the penetration $\partial z$ corresponds to $-\partial a$, the effect of surface recession is

$$\frac{\partial L_0}{\partial z} = -\frac{\partial L_0}{\partial a} = \frac{\mu_0 l}{\pi a} \cdot \frac{1}{\sqrt{1 - (a/b)^2}} \tag{30}$$

in which the last factor is the proximity factor. From this formula and (18), the resistance is

$$R = \frac{l}{\pi a \sqrt{1 - (a/b)^2}} R_1 \quad \text{ohms.} \tag{31}$$

The proximity factor appears as a reduction of the effective circumference of the wire, because the current is concentrated toward one side of each wire. Otherwise, this formula is the same as for a single wire of length $2l$.

A ring of wire is shown in Fig. 8, with $r_2$ as the radius of the ring and $r_1$ as the much smaller radius of the wire, both being much greater than the depth of penetration $d$. The resistance is

$$R = \frac{2\pi r_2}{2\pi r_1} R_1 = \frac{r_2}{r_1} R_1 \quad \text{ohms.} \tag{32}$$

[34] J. R. Carson, "Wave propagation over parallel wires: The proximity effect," *Phil. Mag.*, vol. 41, p. 627; April, 1921.
[35] Green, Leibe, and Curtis, footnote 11, pp. 267–268.

On the same assumption, $r_1 \ll r_2$, the inductance is[36]

$$L_0 = \mu_0 r_2 \left(\log \frac{8r_2}{r_1} - 2\right)$$

$$= \mu_0 r_2 \log \frac{8r_2}{e^2 r_1} \quad \text{henries.} \tag{33}$$

For a given ring diameter $2r_2$, the maximum ratio of reactance to resistance is obtained with approximately

$$\frac{r_2}{r_1} = \frac{e^3}{8} = 2.5 \tag{34}$$

in which case the inductance and the ratio of reactance to resistance are

$$L = \mu_0 r_2 \quad \text{henries.}$$
$$Q = \frac{2r_1}{d} \tag{35}$$

This ratio is the same for the ring as for the coaxial line. Only the simple approximate formulas are given for the ring because no exact formula is known. In the absence of an exact inductance formula, it is also impossible to find easily the effect of current concentra-

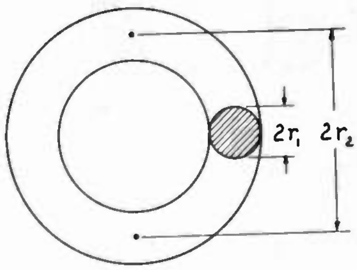



Fig. 8—Circular ring.

tion on the inner side of the ring conductor. The added resistance of the ring caused by radiation and near-by objects is neglected.

A shielding wall near the ring of Fig. 8 is shown in Fig. 9, the wall being a metal sheet parallel to the ring at a distance $c$. The added resistance caused by this shield is computed by the incremental-inductance rule. Assuming first that the shield is a perfect conductor, the effective inductance of the coil is reduced by an amount equal to the mutual inductance with its image (shown in dotted lines) at a distance $2c$. Therefore, the change of inductance is[37]

$$L_0' = -\frac{\pi \mu_0 r_2^4}{16c^3} \quad \text{henries.} \tag{36}$$

To obtain the effect of penetration in the shild, $\partial c$ corresponds to $\partial z$, so

$$\frac{\partial L_0'}{\partial z} = \frac{\partial L_0'}{\partial c} = \frac{3\pi \mu_0 r_2^4}{16c^4}. \tag{37}$$

[36] Harnwell, footnote 14, p. 305.
[37] Harnwell, footnote 14, pp. 304–305.

From this formula and (18), the added resistance is

$$R' = \frac{3\pi r_2^4}{16c^4} R_1' \text{ ohms} \tag{38}$$

in which $R_1'$ is the surface resistivity of the shield. This is equal to the change of reactance which would be caused if the shield were moved further back by the displacement $(d/2)(\mu/\mu_0)$ as shown in Fig. 9. Comparing the added resistance with the resistance $R$ of the ring alone, formula (32), the relative change of resistance is

$$\frac{R'}{R} = \frac{3\pi r_1 r_2^3 R_1'}{16c^4 R_1}. \tag{39}$$

This ratio is independent of the frequency, so long as the depth of penetration is the controlling factor. As an example, a copper ring with the optimum shape

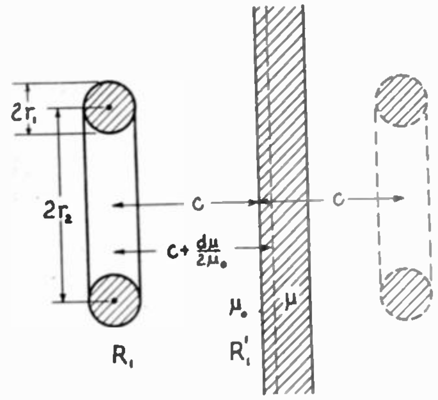



Fig. 9—A ring near a shielding wall.

($r_2 = 2.5r_1$) at a distance of 1 diameter from a soft-iron shield ($R_1' = 40R_1$) would suffer about 59 per cent increase of resistance caused by the shield. In this location, a slightly smaller wire diameter would be optimum, because the inductance of the ring would increase in a greater ratio than the total resistance. The reduction of inductance (36) by the shield varies with the inverse cube of the distance, whereas the added resistance (38) varies with the inverse fourth power.

A ring perpendicular to the shield, instead of parallel as in Fig. 9, and with its center at the same distance, would suffer only one half as much change of inductance and resistance. This follows from the fact that the mutual inductance with its image would be one half as great. This is a striking example of the utility of the incremental-inductance rule, since the departure from axial symmetry would make this problem very difficult of solution by field-integration methods.

A coil of $n$ turns near a shield has its inductance and resistance changed by $n^2$ times as much as the ring, that is, by $n^2L_0'$ and $n^2R'$, formulas (36) and (38).

The air-core toroidal coil of Fig. 10 has $n$ turns on a coil radius of $r_1$ and a ring radius of $r_2$. The following simple formulas are based on the asumptions that the coil radius is much less than the ring radius ($r_1 \ll r_2$) and that the current is concentrated on the inner surface with uniform distribution in a layer of depth very much less than the coil radius ($d \ll r_1$).

$$R = \frac{2\pi r_1 n}{2\pi r_2/n} R_1 = \frac{r_1}{r_2} n^2 R_1 \qquad \text{ohms} \tag{40}$$

$$L_0 = \mu_0 n^2 \frac{r_1^2}{2r_2} \qquad \text{henries} \tag{41}$$

$$Q = \frac{r_1}{d}. \tag{42}$$

These values are realized in a "one-turn" air-core toroid of a continuous metal sheet. They are closely approximated in a coil of round wire wound with a pitch only slightly greater than the wire diameter.

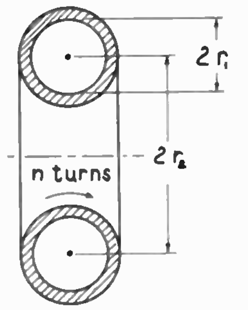



Fig. 10—Toroidal coil.

The preferable shapes of the air-core toroid of Fig. 10 involve a coil radius comparable with the ring radius, a departure from the above assumptions. This is of interest only in the "one-turn" case ($n = 1$) since a layer of wire cannot be wound with optimum pitch over the entire surface of the coil. The exact formula for the inductance is[38]

$$L_0 = \mu_0(r_2 - \sqrt{r_2^2 - r_1^2}) = \frac{\mu_0 r_1^2}{r_2 + \sqrt{r_2^2 - r_1^2}} \qquad \text{henries.} \tag{43}$$

Since the penetration $\partial z$ corresponds with $\partial r_1$,

$$\frac{\partial L_0}{\partial z} = \frac{\partial L_0}{\partial r_1} = \frac{\mu_0 r_1}{\sqrt{r_2^2 - r_1^2}}. \tag{44}$$

From this formula and (18), the resistance is

$$R = \frac{r_1}{\sqrt{r_2^2 - r_1^2}} R_1 \tag{45}$$

and the ratio of reactance to resistance is

$$Q = \frac{r_1}{d} \cdot \frac{2\sqrt{r_2^2 - r_1^2}}{r_2 + \sqrt{r_2^2 - r_1^2}}. \tag{46}$$

There is an optimum design for this case, with the coil diameter slightly less than the ring diameter, but the practical optimum is affected by so many factors that a theoretical optimum is of little value. If the ring diameter $2r_2$ is given, the optimum shape happens to be $r_1 = 0.78r_2$, in which case

$$Q = 0.60 \frac{r_2}{d} = 0.77 \frac{r_1}{d}. \tag{47}$$

[38] Harnwell, footnote 14, p. 302.

There is another optimum design for a given outside diameter, $2(r_1+r_2)$:

$$\frac{r_1}{r_1+r_2} = 0.41; \quad \frac{r_2}{r_1+r_2} = 0.59$$

$$\frac{r_1}{r_2} = 0.70 \tag{48}$$

$$Q = 0.343\,\frac{r_1+r_2}{d} = 0.83\,\frac{r_1}{d}\,.$$

The solenoidal coil of Fig. 11 has $n$ turns wound on a radius $a$ in an axial length $b$. If such a coil has a length much greater than its radius and is wound closely with rectangular wire of thickness much greater than the

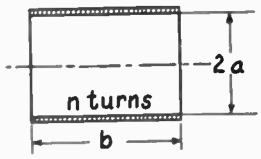



Fig. 11—Solenoidal coil.

depth of penetration, the current flows in a sheet on the inner surface of the wire, and the resistance is

$$R = \frac{2\pi a}{b}\,n^2 R_1 \quad \text{ohms.} \tag{49}$$

In a practical coil of many turns of round wire, there is an optimum diameter of wire slightly less than the pitch of winding. This formula is a rough approximation for practical coils with optimum wire diameter. It corresponds to a coil resistance slightly less than $\pi$ times as great as the resistance of a straight wire of the same length and diameter. (The effect of distributed capacitance and dielectric resistance is omitted.) The inductance is approximately,[39] for $b>0.8a$

$$L_0 = \frac{\mu_0 \pi a^2 n^2}{b+0.9\,a} \quad \text{henries.} \tag{50}$$

The corresponding ratio of reactance to resistance is approximately

$$Q = \frac{a}{d}\cdot\frac{1}{1+0.9\,a/b}\,. \tag{51}$$

These simple formulas are applicable to coils in which the length is greater than the radius, the optimum wire diameter exceeds $4d$, and the number of turns exceeds about 4. In comparison with some recent measurements, these formulas check fairly well the component of resistance caused by the skin effect as distinguished from capacitance effects.[40]

A solenoidal coil in a coaxial tubular shield is shown in Fig. 12. The radius of coil and shield $a_1$ and $a_2$ determines the relative distribution of current on the inner and outer surfaces of the coil. The theoretical relations are based on the ideal long coil and shield, closely wound of rectangular wire, but the conclusions are approximately correct for practical coils. The magnetic intensity $H_1$ inside the coil and $H_2$ between coil

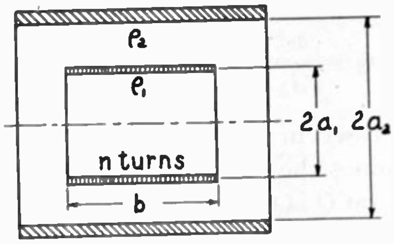



Fig. 12—Solenoidal coil in a coaxial tubular shield.

and shield are in the inverse ratio of the cross-sectional areas because all the flux inside the coil has to return in the space between coil and shield.

$$\frac{H_2}{H_1} = \frac{a_1^2}{a_2^2-a_1^2} = \frac{1}{a_2^2/a_1^2-1}\,. \tag{52}$$

The power dissipation is divided among the inner and outer surfaces of the coil and the inner surface of the shield. By (15), the total is

$$P = 2\pi a_1 b H_1^2 R_1 + 2\pi a_1 b H_2^2 R_1 + 2\pi a_2 b H_2^2 R_2$$
$$= 2\pi a_1 b H_1^2 R_1\left(1+\frac{H_2^2}{H_1^2}+\frac{a_2^2 H_2^2 R_2}{a_1^2 H_1^2 R_1}\right) \quad \text{watts} \tag{53}$$

in which $R_1$ and $R_2$ are the respective values of surface resistivity for the metals of coil and shield. By (14), the total current on both surfaces of the coil is

$$I = (H_1+H_2)b/n \quad \text{amperes.} \tag{54}$$

Therefore, the effective resistance of the coil is

$$R = \frac{P}{I^2}$$
$$= \frac{2\pi a_1 n^2 R_1}{b}\cdot\frac{\left(1+\frac{H_2^2}{H_1^2}+\frac{a_2^2 H_2^2 R_2}{a_1^2 H_1^2 R_1}\right)}{\left(1+\frac{H_2}{H_1}\right)^2}\,. \tag{55}$$

The last factor gives the effect of the shield. It may actually reduce the resistance, by redistribution of surface currents, but not as much as it reduces the inductance. The effective inductance of the coil in the shield is

$$L_0 = \frac{\mu_0}{\dfrac{b}{\pi a_1^2}+\dfrac{b}{\pi(a_2^2-a_1^2)}}$$
$$= \frac{\pi a_1^2 \mu_0}{b}\left(1-\frac{a_1^2}{a_2^2}\right) \quad \text{henries} \tag{56}$$

[39] H. A. Wheeler, "Simple inductance formulas for radio coils," Proc. I.R.E., vol. 16, pp. 1398–1400; October, 1928.
[40] F. E. Terman, "Radio Engineering," 1932/1937, pp. 37–42.

in which the last factor gives the reduction of inductance by the shield. With these substitutions,

$$x = \frac{a_1}{a_2}, \qquad r = \frac{R_2}{R_1} \tag{57}$$

and the ratio of reactance to resistance is

$$Q = \frac{a_2}{d} \cdot \frac{x(1 - x^2)}{1 - (2 - r)x^2 + 2x^4}. \tag{58}$$

This is expressed in terms of the shield radius $a_2$ since that determines the space in which the coil is located. The maximum $Q$ is obtained if $x$ satisfies the equation

$$0 = 1 - (1 + r)x^2 - (4 + r)x^4 + x^6. \tag{59}$$

This is most easily solved by trial. The solutions for the optimum design in several cases are as follows:

| $r = \frac{R_2}{R_1}$ | $x^2$ | $x = \frac{a_1}{a_2}$ | $Q$ (60) |
|---|---|---|---|
| 0 | 0.41 | 0.64 | $0.72 \frac{a_2}{d} = 1.14 \frac{a_1}{d}$ |
| 1 | 0.30 | 0.55 | $0.44 \frac{a_2}{d} = 0.80 \frac{a_1}{d}$ |
| 2 | 0.23 | 0.48 | $0.33 \frac{a_2}{d} = 0.70 \frac{a_1}{d}$ |
| $\infty$ | $\frac{1}{r}$ | $\frac{1}{\sqrt{r}}$ | $\frac{1}{2\sqrt{r}} \frac{a_2}{d} = 0.50 \frac{a_1}{d}$. |

An approximate formula for the optimum ratio of radii is given by the relation,

$$x^2 = \frac{1}{2.3 + r}$$
$$\frac{a_1}{a_2} = \frac{1}{\sqrt{2.3 + R_2/R_1}}. \tag{61}$$

This formula is exact for $r = 1, 2, \infty$. In the first two rows of the table (60), the coefficient in the last column indicates that the reduction of inductance by the shield is not accompanied by a proportionate reduction of $Q$. Therefore, the shield reduces the effective resistance in these cases. In practice, the shield always decreases the $Q$.

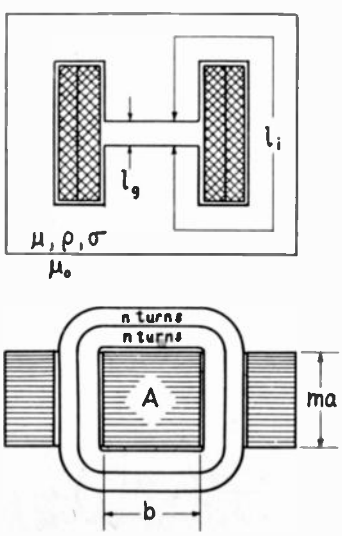



Fig. 13—Transformer with laminated iron core.

The transformer of Fig. 13 has a laminated iron core of cross-sectional area $A$. The flux path in the iron has a length $l_i$ while that in the air gap has a length $l_g$. For simplicity of analysis, the two coils have the same number of turns $n$. If the actual number of turns is $n_1$ and $n_2$, the respective self-impedances and mutual impedance are obtained by letting

$$n^2 = n_1^2,\ n_2^2,\ n_1 n_2. \tag{62}$$

Fig. 14 shows the impedance network which is the equivalent of this transformer. The upper part represents the coil resistance and the part of the inductance caused by magnetic flux in the space outside the core, as if the core space had zero permeability. The lower

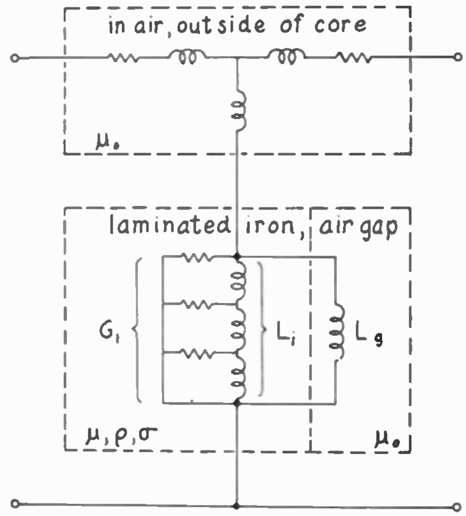



Fig. 14—The distributed-impedance network equivalent to the iron-core transformer.

part represents the impedance caused by the core, including the air gap. The inductance which would be caused by the flux in the iron core of permeability $\mu$, with no air gap, is

$$L_i = \frac{\mu n^2 A}{l_i} \quad \text{henries.} \tag{63}$$

The inductance which would be caused by the flux in the air gap, if the iron core had infinite permeability, is

$$L_g = \frac{\mu_0 n^2 A}{l_g} \quad \text{henries.} \tag{64}$$

The inductance effective at low frequencies is that of $L_i$ and $L_g$ in parallel,

$$L_0 = \frac{L_i L_g}{L_i + L_g} = \frac{\mu_0 n^2 A}{l_g + l_i \mu_0/\mu} \quad \text{henries.} \tag{65}$$

The eddy currents and skin effect depend on the division of the core area into laminations,

$$A = mab \quad \text{square meters} \tag{66}$$

in which $m$ is the number of laminations of thickness $a$ and width $b$. The current paths in the laminations cause an apparent distributed conductance, associated with the iron inductance $L_i$, which has the value

$$G_i = \frac{\sigma a l_i}{4n^2 m b} \qquad \text{mhos (67)}$$

in which $\sigma$ is the conductivity of the iron. The effect of this distributed conductance is least at low frequencies and merges into the skin effect at high frequencies.

Fig. 15 shows a simplified equivalent network in which the shunt resistance $R$ and inductance $L$ have values depending on the frequency. These parallel components are used rather than series components, because the effective shunt resistance varies less with frequency than the effective series resistance.

At low frequencies, the apparent shunt conductance approaches the constant value

$$G = \tfrac{1}{3} G_i \qquad \text{mhos. (68)}$$

The corresponding value of shunt resistance is

$$R = \frac{12 n^2 m b d}{a l_i} R_1 \qquad \text{ohms. (69)}$$

in which $R_1$ is the surface resistivity of the iron. The inductance $L$ has its low-frequency value $L_i$. This is based on the assumption that the alternating flux within the lamination suffers only a small phase lag and no appreciable reduction in magnitude, which is true if the depth of penetration is greater than the thickness of laminations.[41] The corresponding ratio of shunt susceptance to conductance, is

$$Q = \frac{R}{\omega L} = 6 \frac{d^2}{a^2} \qquad (d \gg a). \quad (70)$$

At frequencies so high that the depth of penetration is less than 1/4 the thickness of laminations, the skin effect governs the impedance caused by the iron core. The effective impedance of $R$ and $L$ in parallel is the impedance of the line with distributed series $L_i$ and shunt $G_i$:

$$Z = \frac{1}{1/R + 1/j\omega L} = \sqrt{\frac{j\omega L_i}{G_i}}$$

$$= \frac{4n^2 m b}{(1-j) l_i} R_1 \qquad \text{ohms. (71)}$$

The shunt components of this impedance have the value

$$R = \omega L = \frac{4n^2 m b}{l_i} R_1 \qquad \text{ohms. (72)}$$

The apparent shunt inductance is

$$L = \frac{2d}{a} L_i \qquad \text{henries. (73)}$$

This is the inductance based on twice the depth of penetration as the effective thickness of each lamination.

The air gap sometimes increases the ratio of reactance to resistance in the impedance of an iron-core inductor. This question involves the series resistance $R_c$ of each coil, while the inductance in the space outside the coil is usually negligible. Increasing the air gap

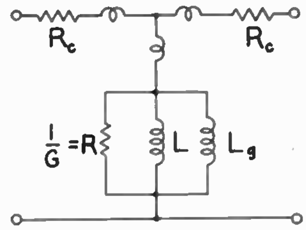



Fig. 15—The lumped-impedance network equivalent to the iron-core transformer.

decreases $L_g$, thereby causing more dissipation in the coils ($R_c$) and less in the core ($R$). The optimum length of air gap is approximately that which divides the dissipation equally between coil and core. The optimum condition is

$$\frac{1}{\omega L} + \frac{1}{\omega L_g} = \sqrt{\frac{1 + R_c/R}{R R_c}} \qquad (74)$$

For this condition, the maximum ratio is[42]

$$Q = \frac{1}{2}\sqrt{\frac{R/R_c}{1 + R_c/R}}. \qquad (75)$$

This is nearly independent of the number of turns. Its value is expressed in terms of three properties of the coil; $\rho_c$ is the resistivity of the copper wire, $l_c$ is the average length of wire per turn, and $A_c$ is the total cross-sectional area of the turns of wire on the winding in question. In a self-inductor of one coil, $A_c$ is somewhat less than the area of each window. The following formulas are simplified on the assumption that $R \gg R_c$ so the optimum air gap gives $Q \gg 1$. At the higher frequencies, where the skin effect predominates, the optimum air gap gives

$$Q = \sqrt{\frac{A A_c \rho}{a d l_i l_c \rho_c}} \qquad (a > 4d). \quad (76)$$

At the lower frequencies, where eddy currents are induced by nearly uniform flux in the laminations, the optimum air gap gives[43]

$$Q = \sqrt{\frac{3 A A_c \rho}{a^2 l_i l_c \rho_c}} \qquad (a < d). \quad (77)$$

These maximum values cannot be realized if the optimum length of air gap is negative. This is true at very low frequencies where the eddy currents are negligible, in which case the air gap is reduced to zero, giving

$$Q = \frac{\omega L_i}{R_c} \qquad (\omega L_i \ll \sqrt{R R_c})$$

$$= \frac{2 A A_c \rho}{d^2 l_i l_c \rho_c} = \frac{2 A A_c \mu}{d_c^2 l_i l_c \mu_0} \qquad (78)$$

in which $d$ and $d_c$ are the depths of penetration in iron

[41] V. E. Legg, "Survey of magnetic materials and applications in the telephone system," *Bell. Sys. Tech. Jour.*, vol. 18, pp. 438–464, July 1939. (In Fig. 7, $\theta$ is the ratio of thickness to depth of penetration.)

[42] L. A. Arguimbau, "Losses in audio-frequency coils," General Radio Experimenter, vol. 11, no. 6, pp. 1–4, November, 1936.

[43] P. K. McElroy and R. F. Field, "How good is an iron-cored coil?" General Radio Experimenter, vol. 16, no. 10, pp. 1–12, March, 1942.

and copper, the materials of the core and the coil. In these formulas, the depth of penetration includes the frequency dimension, enabling the expression entirely in terms of ratios. The value of (78) is actually independent of the $\rho$ of the iron and the $\mu_0$ of the copper, those being involved also in the depth of penetration.

Since copper is the usual material for conductors, it is useful to remember the depth of penetration in copper at a certain frequency and room temperature (20 degrees centigrade):

$$\begin{aligned} At \quad f &= 10^6 \text{ cycles} = 1\ Mc, \\ d_c' &= 66\cdot 10^{-6} \text{ meter} = 0.066 \text{ mm} = 66 \text{ microns} \\ &= 2.6\cdot 10^{-3} \text{ inch} = 2.6 \text{ mils.} \end{aligned} \tag{79}$$

The values for copper and other materials (at a temperature of 0 degrees centigrade) are found in the Steinmetz[3] table, p. 385. The essential properties of copper are (at 20 degrees centigrade):

$$\begin{aligned} \mu_c = \mu_0 &= 4\pi\cdot 10^{-7} \text{ henry per meter} \\ &= 1.257 \text{ microhenrys per meter} \\ \sigma_c &= 5.80\cdot 10^7 \text{ mhos per meter} \\ &= 58 \text{ megamhos per meter} \\ \rho_c &= 1.724\cdot 10^{-8} \text{ ohm-meter} \\ &= 1/58 \text{ microhm-meter} \\ \mu_0\sigma_c &= 72.8 \text{ seconds per square meter} \\ \mu_0\rho_c &= 2.17\cdot 10^{-14} \text{ ohm}^2\text{-second} \end{aligned} \tag{80}$$

in which $\mu_0$ is the permeability of space. The other important value for copper is the surface resistivity, still at 1 megacycle:

$$\begin{aligned} R_{1c}' &= 2.60\cdot 10^{-4} \text{ ohm} \\ &= 0.260 \text{ milohm.} \end{aligned} \tag{81}$$

In order to convert $d_c$ and $R_{1c}$ for other materials, it is necessary to know only their permeability and resistivity relative to copper:

$$\begin{aligned} d &= d_c' \sqrt{\frac{1\text{ Mc}}{f}\,\frac{\mu_0}{\mu}\,\frac{\rho}{\rho_c}} \\ R_1 &= R_{1c}' \sqrt{\frac{f}{1\text{ Mc}}\,\frac{\mu}{\mu_0}\,\frac{\rho}{\rho_c}}\,. \end{aligned} \tag{82}$$

The chart of Fig. 1 gives the depth of penetration $d$ and the surface resistivity $R_1$ plotted against frequency. Each pair of crossed lines is for one material. Some of the materials shown are chosen for their extreme properties (at least, among the common materials). Copper has the least resistivity. The permalloy shown (78 per cent nickel) is used for loading submarine telegraph cables and for shielding against alternating magnetic fields; it has the least depth of penetration, by virtue of its high permeability and small resistivity:

$$\begin{aligned} \mu &= 9000\ \mu_0 \text{ (at small flux density)} \\ \rho &= 9.3\ \rho_c = 0.16 \text{ microhm-meter} \\ &At\ 1 \text{ Mc} \\ d' &= 2.1\cdot 10^{-6} \text{ meter} = 0.084 \text{ mil} \\ R_1' &= 75 \text{ milohms} \end{aligned} \tag{83}$$

Manganin is the material usually used in resistance standards; it has about the highest resistivity compatible with minimum permeability, and therefore the greatest depth of penetration:

$$\begin{aligned} \mu &= \mu_0 \\ \rho &= 25.5\ \rho_c = 0.44 \text{ microhm-meter} \\ &At\ 1 \text{ Mc} \\ d' &= 0.33 \text{ mm} = 13 \text{ mils} \\ R_1' &= 1.3 \text{ milohms.} \end{aligned} \tag{84}$$

Most of the ordinary materials fall within the limits of these three cases.

On the chart, the intersection of each pair of lines moves upward with increasing resistivity and toward the left with increasing permeability. (It is purely coincidental that the intersection is at 1 megacycle for nonmagnetic materials.)

In this collection of formulas, the properties of the conductive materials are usually expressed in terms of depth of penetration $d$ and surface resistivity $R_1$, both of which involve also the frequency. The former appears in ratios with other "length" dimensions. The latter appears in impedance formulas, where it brings in the "resistance" dimension. Other quantities usually appear in ratios so they do not complicate the dimensions. The two parameters $d$ and $R_1$ are most useful because they have not only dimensional simplicity but also obvious physical significance.

## Discussion on
# "The Distribution of Amplitude with Time in Fluctuation Noise"*

Vernon D. Landon

**K. A. Norton**[1]: In this paper the author finds that the distribution with time of the instantaneous amplitude of fluctuation noise is represented by the normal distribution. His derivation of this analytical result is not entirely clear and is apparently incorrect. If fluctuation noise may be represented analytically by a large number of unit impulses, each occurring at randomly different times, then, at any given frequency, the fluctuation noise may be considered to consist of a large number of oscillations (at that frequency) with random relative phases. Many years ago Lord Rayleigh[2] determined the probable distribution with time of the instantaneous amplitude of a large number $n$ of oscillations with equal unit amplitudes and with random relative phases. The probability $dp$ of a resultant amplitude between $V$ and $V+dV$ was found to be

$$dp = \frac{2}{n} e(\exp - V^2/n)VdV \tag{1a}$$

so that, after a simple integration, we find that the instantaneous amplitude $V$ will be exceeded for the percentage of time $P$ given by

$$P = 100p = 100e(\exp - V^2/n), \tag{2a}$$

The above result may be generalized to apply to the case of a large number $n$ of waves with random relative phases and with *unequal* amplitudes $E_1, E_2, \cdots, E_n$ such that $E_i^2 \ll E_1^2+E_2^2+ \cdots +E_n^2$ for $i=1$ to $n$. In this case (1a) and (2a) become

$$dp = \frac{2}{E_1^2+E_2^2+\cdots+E_n^2} e(\exp - V^2/(E_1^2+E_2^2 + \cdots + E_n^2))VdV \tag{1b}$$

$$P = 100e(\exp - V^2/(E_1^2+E_2^2+\cdots+E_n^2)). \tag{2b}$$

If we write $E$ for the root-mean-square value of the resulting disturbance, then

$$E^2 = \int_0^\infty V^2 dp = E_1^2 + E_2^2 + \cdots + E_n^2 \tag{3}$$

and we obtain the interesting result that the root-mean-square value of the resulting disturbance is equal to the root-sum-square value of the amplitudes of the individual oscillations. Thus we see that (1b) and (2b) may be expressed in terms of the root-mean-square value $E$ of the resulting disturbance as follows:

$$dp = \frac{2}{E^2} e(\exp - (V/E^2))VdV \tag{1c}$$

$$P = 100e(\exp - (V/E^2)). \tag{2c}$$

Equation (2c) is given graphically in Fig. 1. We see by (2c) that $V>4E$, for a percentage of time $P=0.0000113$ per cent; $V>3E$, for a percentage of time $P=0.0123$ per cent; and $V>2E$, for a percentage of time $P=1.83$ per cent.

The average value of the disturbance may also be easily determined. Thus

$$\overline{V} = \int_0^\infty V dp = \frac{\sqrt{\pi}}{2} E = 0.886E. \tag{4}$$

This value lies somewhat nearer the value 0.85 $E$ as determined experimentally by Jansky[3] than the theoretical value 0.798 $E$ determined by Landon from the normal distribution.

It is obvious from (1b) and (2b) that the instantaneous amplitude of two disturbances, each with amplitudes distributed according to the Rayleigh law and with root-mean-square values $E_a$ and $E_b$ would also be distributed according to the Rayleigh distribution and would have a root-mean-square value equal to $\sqrt{E_a^2+E_b^2}$.

Having established the above theoretical relations, it is desirable to examine further the postulates on which these relations are based. In deriving the above equations, it was tacitly assumed that the amplitudes of the individual oscillations $E_i$ remain constant over

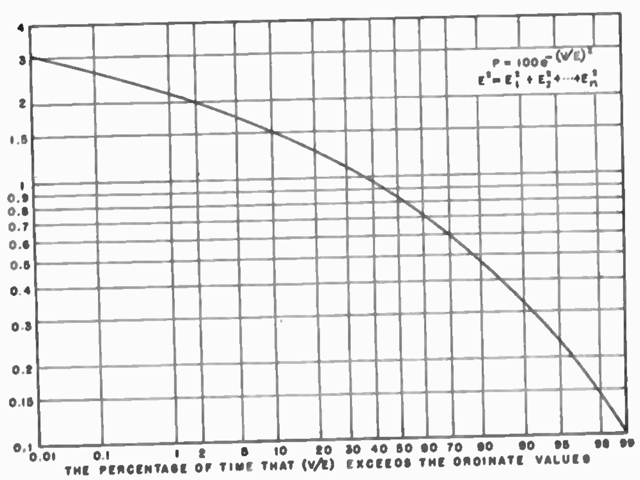



Fig. 1—The Rayleigh distribution.

the period of time required for a measurement of $E$, $\overline{V}$, or of $P$; actually, a necessary and sufficient condition for the applicability of (1c), (2c), and (4) to fluctuation noise is the constancy of $E$ over the period of

* Proc. I.R.E., vol. 29, pp. 50–55; February, 1941.
[1] Office of the Chief Signal Officer, War Department, Washington D. C.
[2] Lord Rayleigh, "On the resultant of a large number of vibrations of the same pitch and of arbitrary phase," *Phil. Mag.*, vol. 10, pp. 73–78; 1880; see also the book "Theory of Sound", 1894, second edition, paragraph 42a.
[3] Karl G. Jansky, "An experimental investigation of the characteristics of certain types of noise," Proc. I.R.E., vol. 27, pp. 763–768; December, 1939.

time in question. If $E$ also varies with time, then its distribution will be superimposed upon that given by the Rayleigh distribution.

It may be mentioned in passing that this same distribution applies to the instantaneous intensity of ionospheric waves (sky waves), irrespective of the radio frequency, but does not apply to tropospheric waves. In the study of the variation of ionospheric-wave intensities with time, since the root-mean-square value $E$ of the disturbance varies with changes in the ionosphere absorption, the Rayleigh distribution will only apply for periods of time which are short enough not to include any large variations of ionospheric absorption; for frequencies in the standard broadcast band this period of time is of the order of one hour on the average.

**Vernon D. Landon**[4]: The mathematics in Mr. Norton's discussion appears to be substantially correct (with minor exceptions). Paradoxically, the mathematics in my paper also is believed to be correct. The apparent discrepancy is due to the fact that we are not talking about exactly the same thing. The point can be cleared up by reference to Fig. 2. In this figure the value of a noise voltage at a certain instant is represented by the point $V$. The instantaneous value of the envelope corresponding to $V$ is represented by the point $A$. The $V$ of my mathematics is the $V$ of the figure. However, the $V$ of Mr. Norton's mathematics is apparently the $A$ of the figure.

Perhaps the choice of words in the title to my paper could have been improved upon. Had the title been "The Distribution of Voltage with Time in Fluctuation Noise" there would have been less opportunity for misinterpretation. Instantaneous amplitude can perhaps be interpreted as meaning the value of the envelope. Actually, what was meant was the instantaneous voltage.

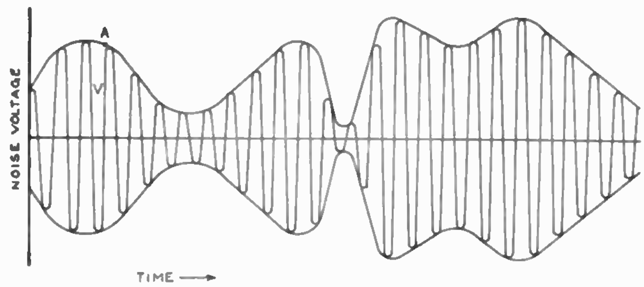


Fig. 2—Noise voltage wave showing envelope.

As I proved in my paper, the probability that the instantaneous voltage lies between $V$ and $V+dV$ is

$$dp_V = \frac{-1}{E\sqrt{2\pi}} \exp\left(-\frac{V^2}{2E^2}\right)dV. \tag{5}$$

Experimental proof is provided by Dunn and White[5] (Fig. 14 of their paper). On the other hand, it can be shown that the probability that the instantaneous value of the envelope lies between $A$ and $A+dA$ is[6]

$$dp_A = \frac{A}{E^2} \exp\left(-\frac{A^2}{2E^2}\right)dA. \tag{6}$$

If the symbol $V$ is substituted for $A$ and the symbol $n/2$ for $E^2$, the result is Mr. Norton's equation (1a). Mr. Norton bases his equations on Lord Rayleigh's work. Lord Rayleigh was concerned with the resultant amplitude when a large number of sine waves of the same frequency are added with random phase. By "amplitude" he meant the peak value of the resultant sine wave. When this peak value varies from moment to moment, it evidently traces out the envelope of the wave.

I have defined $E$ as the root-mean-square value of the noise voltage. In Mr. Norton's work, $E$ is the root-mean-square summation of the *peak* values of a number of waves. Hence, his value of $E$ is larger than mine by a factor $\sqrt{2}$.

The relationship between (5) and (6) can be made clear by reference to Fig. 3 where $dp_V/dV$ and $dp_A/dA$ are plotted as functions of $V$ and $A$. It should be noted that the probability of the envelope $A$ going to zero is infinitesimal, while zero is the most probable value of the instantaneous voltage $V$. The reasonableness of this can be seen by reference to Fig. 2.

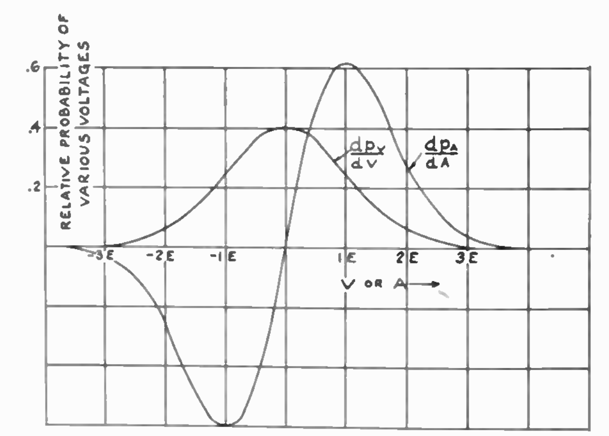


Fig. 3—Relative probability curves for instantaneous voltage and envelope.

It should be pointed out that the concept of an envelope to the noise voltage has a definite meaning only for noise which is confined to a relatively narrow frequency spectrum. If the noise has a bandwidth comparable to the mean frequency, or if the band includes zero frequency, then the fluctuations of the envelope occur about as rapidly as the fluctuations of the voltage wave itself. For this case the concept of an envelope has little meaning.

It is of interest to note the similarity between the equations for molecular motion in a perfect gas and (5)

[4] RCA Manufacturing Company, Inc., Camden, New Jersey.
[5] H. K. Dunn and S. D. White, "Statistical measurements on conversational speech," *Jour. Acous. Soc. Amer.*, vol. 11, pp. 278–288; January, 1940.

[6] This relationship was first pointed out to me by Dr. D. O. North of the Research Laboratories of the RCA Manufacturing Company, Harrison, N. J.

and (6) of the present discussion. The distribution of molecular velocities in one, two, and three dimensions is as follows:

$$dp_u = -\sqrt{\frac{a}{\pi}}\exp(-au^2)du$$
$$= \frac{-1}{\sqrt{2\pi}\,E}\exp\left(-\frac{u^2}{2E^2}\right)du \qquad (7)$$

$$dp_r = 2ar\exp(-ar^2)dr = \frac{r}{E^2}\exp\left(-\frac{r^2}{2E^2}\right)dr \qquad (8)$$

$$dp_s = 4\sqrt{\frac{a^3}{\pi}}\,s^2\exp(-as^2)ds$$
$$= \sqrt{\frac{2}{\pi}}\,\frac{s^2}{E^3}\exp\left(-\frac{s^2}{2E^2}\right)ds \qquad (9)$$

where $u$, $r$, and $s$ are, respectively, the magnitudes of the velocities on the $x$ axis, in the $xy$ plane, and in space. $E$ is the root-mean-square velocity along the $x$ axis in each case and $a=1/2E^2$.

It can be seen that the expressions for $dp_u$ and $dp_r$ are identical to the expressions for $dp_V$ and $dp_A$ of (5) and (6).

The following additional facts about the envelope may be of some interest.

*The Probability that the Envelope Will Exceed the Value A*

The probability that the envelope will be greater than a certain voltage $A$ at a given instant is

$$p_A = \int_A^\infty \frac{A}{E^2}\exp\left(-\frac{A^2}{2E^2}\right)dA$$
$$= \exp\left(-\frac{A^2}{2E^2}\right). \qquad (10)$$

Mr. Norton omits the 2 in the denominator of the exponent and, hence, his values are in error. Actually, if $A=4E$, then $p_A=e^{-8}=0.00034$. If $A=3E$, then $p_A=e^{-9/2}=0.011$. If $A=2E$, then $p_A=2^{-2}=0.135$. As would be expected, these values are from ten times to three times the corresponding values for $p_V$. The increased values would be expected because the envelope encloses more area than the wave itself.

*The Most Probable Value of A*

The most probable value of the envelope is where $d^2p_A/dA^2$ goes to zero.

$$\frac{d^2p_A}{dA^2} = \frac{A}{E^2}\left(\frac{-A}{E^2}\right)\exp\left(-\frac{A^2}{2E^2}\right)$$
$$+\frac{1}{E^2}\exp\left(-\frac{A^2}{2E^2}\right) = 0 \qquad (11)$$

$$\frac{A_p^2}{E^4} = \frac{1}{E^2}$$

$$A_p = \pm E. \qquad (12)$$

Thus, the most probable value of the envelope of a radio-frequency noise wave is equal to the root-mean-square value of the wave.

*The Mean Value of A*

The mean value of $A$ is the integral from 0 to $\infty$ of $A$, times the probability of occurrence of $A$.

$$\bar{A} = \int_0^\infty A\,\frac{A}{E^2}\exp\left(-\frac{A^2}{2E^2}\right)dA$$
$$= E\sqrt{\frac{\pi}{2}} = 1.252E. \qquad (13)$$

(Mr. Norton's value of $0.886E$ is in error by the factor $\sqrt{2}$ because in (1b), $E_1$, $E_2$, etc., are peak values not root-mean-square values. The mean absolute value of $V$ is $|\overline{V}|=0.798E$ as proved in my paper.)

The value of $\overline{A}$ is of some importance because it is the value of the direct-current output voltage from an ideal diode detector (when the ideal diode detector is defined as one in which the output voltage follows the envelope of the wave exactly).

*The Root-Mean-Square Deviation of A from its Mean Value*

The root-mean-square deviation of $A$ from its mean value is the root-mean-square audio output of an ideal diode detector with noise but no signal applied. It is obtained by a simple integration as follows:

$$A_r = \sqrt{\int_0^\infty \left(A - E\sqrt{\frac{\pi}{2}}\right)^2 \frac{A}{E^2}\exp\left(-\frac{A^2}{2E^2}\right)dA}$$
$$= E\sqrt{2-\frac{\pi}{2}} = 0.655E. \qquad (14)$$

*The Root-Mean-Square Value of A*

The root-mean-square value of $A$ may be obtained directly by integration or from the following:

$$A_{rms} = \sqrt{\bar{A}^2 + A_r^2} = \sqrt{2}\,E. \qquad (15)$$

CORRECTION

I would like to take this opportunity to correct an error in my paper. On page 52, equation 13, the limits of integration should be 0 to $\infty$.

**K. A. Norton**[1]: As Mr. Landon admits, his equations are for the distribution of voltage rather than amplitude with time. The equations given in my discussion give the correct distribution for the amplitude with time in fluctuation noise. There is no error of $\sqrt{2}$ as Landon states; this may be seen by reference to (3) which proves that the root-mean-square value of the instantaneous amplitudes $V$ of the disturbance averaged over a long period of time is equal to $E$. Landon was apparently confused by my statement that "the root-*mean*-square value of the resulting disturbance is equal to the root-*sum*-square value of the amplitudes of the individual oscillations"; as far as the experimenter is concerned, only the resulting disturbance can be observed and its root-mean-square value is just $E$.

Mr. Landon states that my equations are for the distribution of the envelope; that is correct to the extent that any useful meaning may be attached to the envelope of a noise disturbance in which the fluctuations are varying just as rapidly as the fluctuations of the voltage wave itself. My equations, however, are always applicable to the distribution of the amplitude of the fluctuation noise; this amplitude, which I have denoted by $V$ (but which Landon denotes by $A$), may be defined by the following equation for the instantaneous voltage of a disturbance of angular frequency $\omega$ and phase $\phi$.

$$v = V \sin (\omega t + \phi) \tag{16}$$

where $v$ denotes the instantaneous voltage and $\phi$ is the phase of the instantaneous value of the disturbance. Note that $V$ has positive values only while $v$ will be negative for half of the time.

It is important to note that in Jansky's experimental work the average value of the envelope $V$, rather than the average value of the voltage $v$, was measured; this probably explains the close agreement between his experimental value and my theoretical value.

It is instructive to determine the distribution of the voltage $v$ of fluctuation noise directly from (16). Since $V$ is distributed between 0 and $\infty$ in accordance with the probability law

$$dp/dV = \frac{2V}{E^2} e(\exp - (V/E))^2 \tag{17}$$

and since $U \equiv \sin (\omega t + \phi)$ is independently distributed between $-1$ and $+1$ in accordance with the probability law

$$dp/dU = \frac{1}{\pi\sqrt{1 - U^2}}. \tag{18}$$

Then, using a theorem due to Huntington,[7] we may write for the distribution of the product $UV = v$

$$dp/dv = \frac{2}{\pi E^2} \int_{-\infty}^{+\infty} \frac{V}{\sqrt{V^2 - v^2}} e(\exp - (V/E))^2 dV. \tag{19}$$

If we set $V^2 - v^2 = X^2$, the integration may be performed and we obtain

$$dp/dv = \frac{1}{E\sqrt{\pi}} e(\exp - (V/E))^2. \tag{20}$$

This equation is not directly comparable to (5) for the probability of a voltage between $v$ and $v + dv$ since the $E$ in (20) is the root-mean-square value of $V$. If we write $E_v$ for the root-mean-square value of $v$ we obtain

$$E_v^2 = \int_{-\infty}^{+\infty} v^2 dp = \frac{1}{E\sqrt{\pi}} \int_{-\infty}^{+\infty} v^2 e(\exp - (V/E))^2 dv = \frac{E^2}{2} \tag{21}$$

and we see that the root-mean-square value of $V$ is $\sqrt{2}$ times the root-mean-square value of $v$.

When $E_v$, as determined by (21), is substituted for $E$ in (20) and (17) we obtain Landon's equations (5) and (6). Thus there is no discrepancy in the mathematics. However, since it is the distribution of the envelope $V$, which was measured experimentally by Jansky rather than the distribution of the voltage $v$, my relation $\overline{V} = 0.886\, E$ is the one applicable to the experimental results instead of Landon's relation $\bar{v} = 0.798 E_v$.

**Vernon D. Landon**[4]: It now appears that Mr. Norton and I are in almost perfect agreement providing we both charitably agree to use the other fellow's latest definitions in reading his earlier work.

Thus in the title of my original paper, "The Distribution of Amplitude with Time in Fluctuation Noise," the word amplitude must be interpreted as the instantaneous voltage, as I intended. Fortunately, a more accurate wording is used in many places in the body of the paper.

In Mr. Norton's discussion when he speaks of "the root-mean-square value of a wave," he means not the root-mean-square voltage but something larger by the factor $\sqrt{2}$. (See equation (21).) Similarly, when he speaks of the average value of the wave he means not the average voltage but the average magnitude of the envelope.

Of the three of us, Jansky alone was quite lucid on these definitions. His use of the words, "effective, or root-mean-square voltage," and his defining equations (1) and (2) leave no room for doubt. He was trying to measure the ratio of the average voltage to the effective voltage, not the ratio of the average value of the envelope to the root-mean-square value of the envelope. What he actually measured is something else again. From his circuit diagrams, it appears that his measure of the average voltage might be nearer to the average value of the envelope, while his measure of the effective voltage might be accurate. If this were true the expected value of the ratio would be 1.252, which is still further from a check. Perhaps the point is not worth belaboring further. There seems to be no question now as to the theoretical values of the various ratios.

I, too, have derived the voltage distribution from the envelope distribution by a method similar to that of Mr. Norton. It should be pointed out that the proof depends on the assumption that the phase angle is random when the value of the envelope is given. This assumption appears to be quite justified.

[7] E. V. Huntington, "Frequency distribution of product and quotient," *Annals Math. Statistics*, vol. 10, pp. 195–198; June, 1939.

It is a curious fact that the reverse derivation cannot be carried out by this method. Given the value of the instantaneous voltage, the phase angle is not random! This derivation has been carried out by another method but it is somewhat involved and need not be repeated here.

In spite of the initial misunderstanding I believe that the discussion has brought out important material not previously published. It is to be hoped that the controversy will not becloud the issue. I believe that (accepting the above definitions) either Mr. Norton's work or my own is an accurate treatment. Certain of the details brought out in the discussion should find important practical applications.

## Discussion on

# "Distortion Tests by the Intermodulation Method"*

John K. Hilliard

**Benjamin F. Miessner**[1]: This paper is especially interesting to me, inasmuch as I have used the identical method for at least ten years in my researches on electronic musical instruments.

The May, 1939, issue of *Radio-Craft* contains an article by A. C. Shaney on this same subject and the August, 1939, issue contains a letter of mine concerning the Shaney article.

For qualitative determination of such distortion in chordal instruments such as piano and organ, and for setting the maximum allowable gain on the instrument's amplifier by its internal volume control, I always used the two-note, maximum-intensity test. Any such distortion quickly showed up as a third beat frequency clearly defined, particularly if the two notes chosen were in the 2000- to 4000-cycle region where the harmonic content is poor.

For qualitative measurements two sine-wave audio-frequency signals were sent through the system, picked up by a high-quality microphone close to the speaker, and analyzed by a General Radio wave analyzer. I have long considered this the ideal distortion-measuring system since it measures directly what the ear itself hears as the objectionable element in sound reproduction. With one pure tone the ear has great difficulty in determining the presence of harmonics introduced by distortion, simply because these do fall naturally in a Fourier series. The two-tone beat method, with the frequencies chosen for a maximum discordant beat tone having a frequency within the region of maximum ear sensitivity, and representing the worst possible actual operating condition for the sound-reproducing system, immediately and unmistakably indicates to the ear any disturbing distortion, and the ear is the final criterion anyway.

In practice I have found that amplifier or whole-system power ratings obtained by the single-tone harmonic-amplitude content method gives values which are considerably above tolerable ratings for high-quality performance.

* Proc. I.R.E., vol. 29, pp. 614–620; December, 1941.
[1] Miessner Inventions, Inc., Millburn, New Jersey.

# Institute News and Radio Notes

## Executive Committee

Meetings of the Executive Committee were held on August 7 and August 11. At both meetings all members were present, the attendance consisting of A. F. Van Dyck, chairman; I. S. Coggeshall, Alfred N. Goldsmith, R. A. Heising (guest), F. B. Llewellyn, Haraden Pratt, B. J. Thompson, and H. P. Westman, secretary.

Approval was granted of 122 applications for Associate, 113 for Student, 9 for Junior, and 5 for transfer to Associate grade.

Applications for admission to Member numbering 14 and a similar number for transfer to Member were approved.

A number of matters pertaining to the operation of the office were considered.

At the request of the chairman of the Membership Committee, an interpretation of the requirements for Student membership was made. It is the view of the Committee that a Student is eligible for Student membership if he is devoting a major proportion of his time as a registered Student in a course in engineering or science in a school that offers four-year courses and degrees. The Student need not be following one of these four-year courses nor be working for a degree.

A request for the use of the Institute mailing list for advertising purposes was denied.

The Sections Committee at its Annual Meeting on June 29, 1942, adopted a new section to be added to the Constitution for Sections. The matter was referred to the chairman of the Sections Committee and Dr. Llewellyn, that member of the Executive Committee responsible for section activities.

## Selective Service Deferment

Under date of June 18, 1942, Occupational Bulletin (No. 10) was issued by General Lewis B. Hershey, the Director of the Selective Service System. It concerns the deferment of students in various engineering and scientific courses among which radio engineering is included.

It points out that such students require several years of training and that personnel shortages now exist in these fields.

Careful consideration for occupational classification (possibly involving deferment) is recommended for undergraduate students who have completed two years of college work and who intend to continue, and for graduate students who are doing instructional work or are engaged in scientific research related to the war effort which is supervised by a recognized federal agency. In addition, it is recommended that graduates on leaving school be allowed approximately sixty days to become engaged in a critical occupation essential to the war effort.

Colin B. Kennedy

1885–1942

*Colin Bruce Kennedy was born in Teeswater, Ontario, Canada, on February 6, 1885. His primary education was obtained in the Canadian Public Schools. He learned telegraphy while working as an errand boy for a small-town drugstore which was also the telegraph office.*

*Leaving home when about fourteen years of age, he spent the next ten years as a telegraph operator in many cities throughout Canada. The last two of these years he was in charge of two radio stations on the west coast of Vancouver Island in the Canadian Government wireless service.*

*His next seven years were spent with the Federal Telegraph Company at Palo Alto, California, as a radio operator, station engineer, and research and development engineer. This included the entire period of World War I during which the company built many large and small radio transmitters for the Government.*

*In 1919, he organized and became the President of the Colin B. Kennedy Company. Although originally established for the manufacture of radio receiving equipment for experimenters, at the advent of radio broadcasting, the company was among the first to produce receivers for home use. This business continued until 1933 when economic conditions forced a suspension of operation.*

*Mr. Kennedy continued in the radio field, predominantly in a merchandising capacity. In February, 1942, he entered the service of his country, becoming a civilian employee at the Signal Corps inspection depot in Chicago. In his death on June 16, radio has lost another of the pioneering spirits whose vigorous and constructive work in the early days did much to advance the art and the science.*

## Membership

The following indicated admissions and transfers of memberships have been approved by the Board of Directors.

### Transfer to Member

Aston, J. P., 1336 Crawford Bridge Ave., Verdun, Que., Canada
Felch, E. P., 44 Elmwood Ave., Chatham, N. J.
Hoffman, E. J., 409 E. Third St., Emporium, Pa.
Hogencamp, H. C., 492 Prospect St., Maplewood, N. J.
Katzin, M., 3538 A St., S.E., Washington, D. C.
LeBel, C. J., 370 Riverside Dr., New York, N. Y.
McCoy, J. C., Bell Telephone Laboratories, Inc., 180 Varick St., New York, N. Y.
McKechnie, J. S., 124 Lincoln Ave., Little Falls, N. J.
Morris, R. M., 22 Mountain View Rd., Millburn, N. J.
Nichols, W. A., Canadian Broadcasting Corp., 1012 Keefer Bldg., Montreal, Que., Canada
Olmstead, N. C., Bell Telephone Laboratories, Inc., Whippany, N. J.
Schlesinger, L., c/o Naval Research Laboratory, Washington, D. C.
Smith, J. P., 39 Harrison Ave., Erlton, N. J.
Somers, R. M., c/o Thomas A. Edison, Inc., Lakeside Ave., West Orange, N. J.

### Admission to Member

Anderson, F. B., 188 Elmwood Rd., Oakhurst, N. J.
Black, H. S., Bell Telephone Laboratories, Inc., 180 Varick St., New York, N. Y.
Bollman, J. M., Bell Telephone Laboratories, Inc., 180 Varick St., New York, N. Y.
Bond, W. L., Bell Telephone Laboratories, Inc., 180 Varick St., New York, N. Y.
Dorsey, J. W., University of Manitoba, Winnipeg, Manit., Canada
Edson, J. O., 10 Rustic Pl., Great Kills, S. I., N. Y.
Kinzer, J. P., Bell Telephone Laboratories, Inc., 180 Varick St., New York, N. Y.
Krist, H. K., 100 Lake Dr., Mountain Lakes, N. J.
Maggio, J. B., 548 Springfield Ave., Summit, N. J.
McCurdy, R. G., Bell Telephone Laboratories, Inc., 463 West St., New York, N. Y.
Page, E. C., 504 Munsey Bldg., Washington, D. C.
Stratton, J. A., Massachusetts Institute of Technology, Cambridge, Mass.
Van Tassel, E. K., 419 S. Chesnut St., Westfield, N. J.
Young, C. H., Bell Telephone Laboratories, 463 West St., New York, N. Y.

### Admission to Associate

Alter, J. M., 301½ Coleridge Ave., Altoona, Pa.
Anagnost, A. L., 2839 Decatur Ave., New York, N. Y.
Armstrong, D. E., 79 Shaver Ave., Shavertown, Pa.
Baird, M. N., General Electric Company, Fort Wayne, Ind.
Balwanz, W. W., 2001 Commonwealth Ave., Alexandria, Va.
Bauman, H. W., 1704 Surf Ave., Belmar, N. J.

Beebe, W. J., 2109 Napa St., Vallejo, Calif.
Bueffel, B. H., Jr., 90 Clinton St., White Plains, N. Y.
Chang, D. C., 1117-A Clio St., Honolulu, T. H.
Clarke, R. L., 328 Manheim St., Philadelphia, Pa.
Cohen, A. A., c/o Department 679, RCA Manufacturing Company, Harrison, N. J.
Creedon, H. T., 121 Seacord Rd., New Rochelle, N. Y.
Denoncourt, J. L. E., R.C.A.F. Detachement, McGill University, Montreal, Que., Canada
Flynn, G., Radio Station WOW, Omaha, Neb.
Garrahan, C. J., Swarthmore College, Swarthmore, Pa.
Gunn, C. J., Airway Comm. Sta., Box 1689, Fort Worth, Tex.
Hancock, J. W., 21 W. Mt. Airy Ave., Philadelphia, Pa.
Hannaford, J. C., Signal Section, Fourth Air Force, San Francisco, Calif.
Hastings, A. E., Naval Research Laboratory, Anacostia, Washington, D. C.
Hedrich, A. L., 3018 Ninth St., S.E., Washington, D. C.
Johnson, C. F., National Bureau of Standards, Washington, D. C. (Transfer)
Kelmis, K. J., 153 W. 66th St., New York, N. Y.
Kimball, R. E., 44 Lincoln St., Stoneham, Mass.
Koch, R. F., 432 Broadway, Cedarhurst, N. Y. (Transfer)
Kreutter, H. R., 545 W. Taft Ave., Bridgeport, Conn.
Lambert, R. O., 30 Manor Vale, Boston Manor Rd., Brentford, Middx., England
Lee, F. N., 222 Oakland Ave., Methuen, Mass.
McGlashan, A. R., 10 Hill Crest Grove, Sherwood, Notts., England
Miller, R. H., Jr., 1328—106 Ave., Oakland, Calif. (Transfer)
Miller, W. T., 37 Newburg St., Roslindale, Mass.
Milton, V. A., 4627 Saratoga Ave., Ocean Beach, Calif.
Morey, A. M., 9525 N. Kellogg St., Portland, Ore.
Moses, E. C., Box 437, Lake Charles, La.
Naftel, F. E., C.A.P.O. No. 3, RCAF Overseas
Newbegin, J. A., RCAF #10 Repair Depot, Calgary, Alta., Canada
Oker, W. A., 3306 Renfro Ave., Cincinnati, Ohio
Peake, H. J., 715 S. Washington St., Alexandria, Va., (Transfer)
Petrich, L. G., R.F.D. 2, Morris Plains, N.J.
Poole, H. H., Royal Dublin Society, Dublin, Ireland
Rathjen, P., Box 5, Montville, N. J.
Robertson, R. D., 29 Lexington Ave., Bloomfield, N. J.
Rogers, G. L., 8330 Third Ave., Inglewood, Calif. (Transfer)
Rowley, H., 179 Mcgillivray St., Ottawa, Ont. Canada
Russell, A. C., Box 371B, GPO, Hobart, Tasmania, Australia
Sales, L., 145 W. 188th St., New York, N Y.
Satterlee, H. A., 2234 Carabel Ave., Cleveland, Ohio
Schlegelmilch, R. O., 206 Tenth Ave., Belmar, N. J.
Schmeisser, K. R., 13117 LaSalle Blvd., Detroit, Mich.
Schramm, E. O., 48-05—14 St., N. W., Washington, D. C.
Sievers, W. C., 619 Bixby Ave., Bellflower, Calif.
Sisk, J. J., General Delivery, Excelsior Springs, Mo.
Sivin, L., F.A.S. (W), Fort Sill, Okla.
Smullin, W. B., Box 43, Eureka, Calif.
Somerville, F., "Duncruin," Copley Way, Tadworth, Surrey, England
Springer, P. W., 36 Klee Ave., Dayton, Ohio
Sproul, S. S., Magnetic Survey Range, Newport, R. I.
Stewart, R. D., No. 10 The Garrison, Barbados, B.W.I.
Tetley, W. H., Box 156, Pebble Beach, Monterey, Calif.
Tibbs, C. E., 33 Colcokes Rd., Banstead, Surrey, England
Tyler, S. R., Jr., 4505 Wornall Rd., Kansas City, Mo.
Uecke, E. H., 2652 Workman St., Los Angeles, Calif.
Volkner, J. R., Jr., ACRM (AA) USN, Radio Materiel School, Naval Research Laboratory, Bellevue, D. C.
Walker, F. R., General Electric Company, Schenectady, N. Y.
White, A. W., Box 4588, Johannesburg, South Africa
Whyte, T. R., c/o Midland Bank Ltd., New St., Birmingham, England
Wolf, S. H., U.S.S. Jovett, D396, c/o Postmaster, New York, N. Y.
Zimmerman, S., 1037 Manor Ave., Bronx, New York, N. Y.

# Books

## Television Broadcasting, by Lenox R. Lohr

Published by McGraw-Hill Book Company, 330 West 42 Street, New York, N. Y., 1940. 274 pages. 88 illustrations Price $3.00.

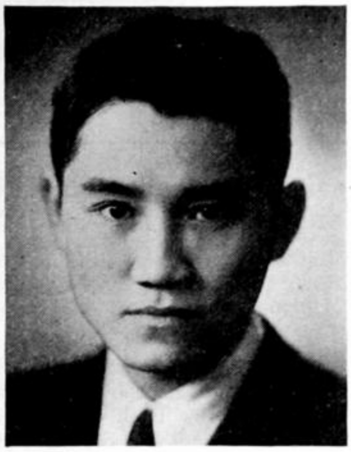
For those whose interest in television is general, as well as for the serious workers in the field, this book on television broadcast production will be interesting and helpful. The author was president of the National Broadcasting Company up to early 1940, while this organization was carrying on the most extensive experiments in programming in the United States. He writes, with a clear, readable style, the answers to the large number of questions asked by the man in the street about this new form of home entertainment. Simple, accurate descriptions are given of the technical portion of the equipment too.

To give an idea of how "Television Broadcasting" is subdivided, some of the chapter headings are listed: The Television System; Putting it to Work; Television Programming—Basic Considerations; Studio, Motion-Picture and Outdoor Programs; Network Broadcasting; Basic Economic Factors; Legal Aspects; The Technical Elements of the Television System; Summary of Regular Service Operations; and A Television Script, "The Three Garridebs."

Mr. Lohr's book, covering television broadcasting technique up to 1940, can well form a very useful starting point for future developments in this field when the present world-upsetting events are over.

Albert F. Murray
National Defense Research Committee
Washington, D. C.

# Contributors



Wen-Yuan Pan

Wen-Yuan Pan (A'41) was born in China on July 15, 1912. He received an engineer's degree in electrical engineering in 1939 and the Ph.D. degree in 1940 from Stanford University. He was a teaching and research assistant at Stanford in 1939 and has been responsible for the procurement of electrical-communication equipment and materials for the Chinese Government under the Lend-Lease Act and the American loans to China since 1940. Mr. Pan is a member of Sigma Xi and the American Military Engineers.

❖

Percy E. Patrick (A'34) was born in the Union of South Africa in 1911. He received the B.Sc. degree in 1934 and the M.Sc. degree in 1936 from the University of Cape Town. He joined the African Broadcasting Company in 1933 and was sent to the Marconi Works, England, in 1936 to watch the construction of studio equipment for Johannesburg, where he was appointed engineer-in-charge in 1937. He is now branch manager for the South African Broadcasting Corporation at Grahamstown.

Note: The African Broadcasting Company, a commercial concern, was changed by act of Parliament into a public utility company, the South African Broadcasting Corporation, in August, 1936.

❖

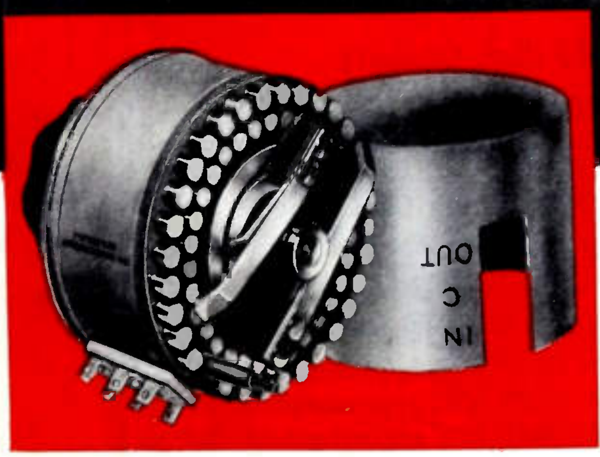
Percy E. Patrick

Harry R. Summerhayes

Harry R. Summerhayes, Jr., (A'36) was born at Schenectady, New York, on February 19, 1914. He received a B.S. degree in physics from Union College in 1935 and after a year of graduate work he received an M.S. degree in science from the same institution. Since that time he has been employed in the General Engineering Laboratory of the General Electric Company where he has been engaged in various television and radio engineering projects. Mr. Summerhayes is an associate member of Sigma Xi.

❖

For a biographical sketch of Harold Alden Wheeler, see the Proceedings for January, 1942.



# BUILT TO ENDURE!

**HK 854**
450 Watt Plate diss.
Max. output 1800 Watts

**MAIL THIS COUPON TODAY FOR NEW FREE BULLETIN**
(Attach coupon to your letterhead)

Gentlemen:

Please send me your new free bulletin, "THIRTEEN WAYS TO PROLONG TUBE LIFE."

The four colossal heads carved in granite on the rugged heights of Mount Rushmore will weather the storm and fury of ages to come.

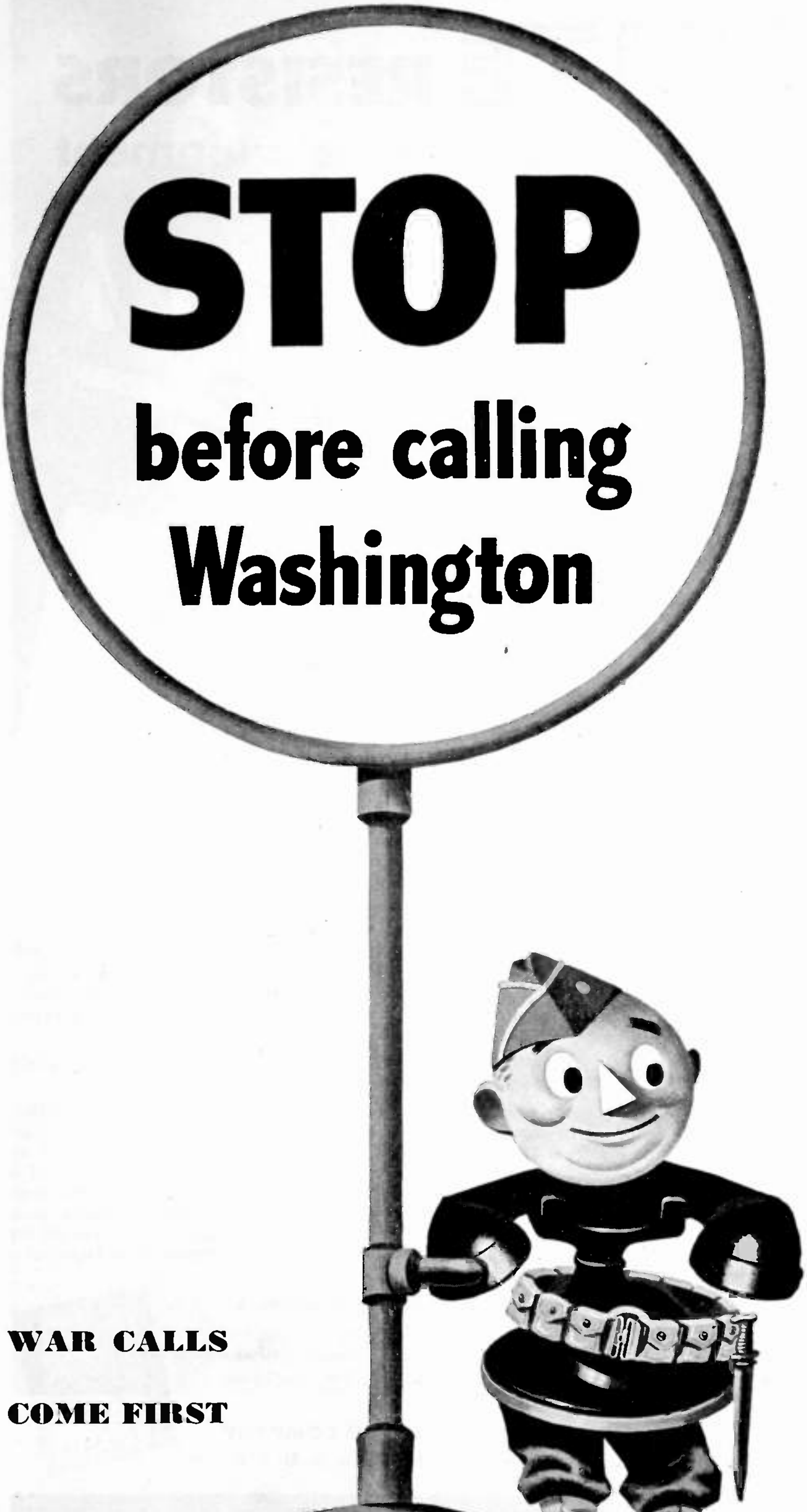
Like this mightiest sculpture of man, GAMMATRON tubes are built to endure terrific punishment. Tantalum plate and grid construction, elimination of all internal insulators, and a special exhaust process positively prohibit the release of gas even at tremendous overloads.

GAMMATRON low voltage type filaments have a large reserve emission supply. They are conservatively rated, have an ample safety factor, and are operated at a point which provides the largest number of watt hours per dollar.

In these days the *extra* stamina of long-lasting GAMMATRONS is of greater-than-ever importance.

HEINTZ AND KAUFMAN LTD.
SOUTH SAN FRANCISCO CALIFORNIA U·S·A·

# GAMMATRONS OF COURSE!




**B**usiest place in the world right now is Washington, D. C. The war effort centers there.

Long Distance telephone calls in and out of Washington alone are about 70,000 a day and the number is growing.

We can't keep building ahead of this traffic because the materials have gone to war.

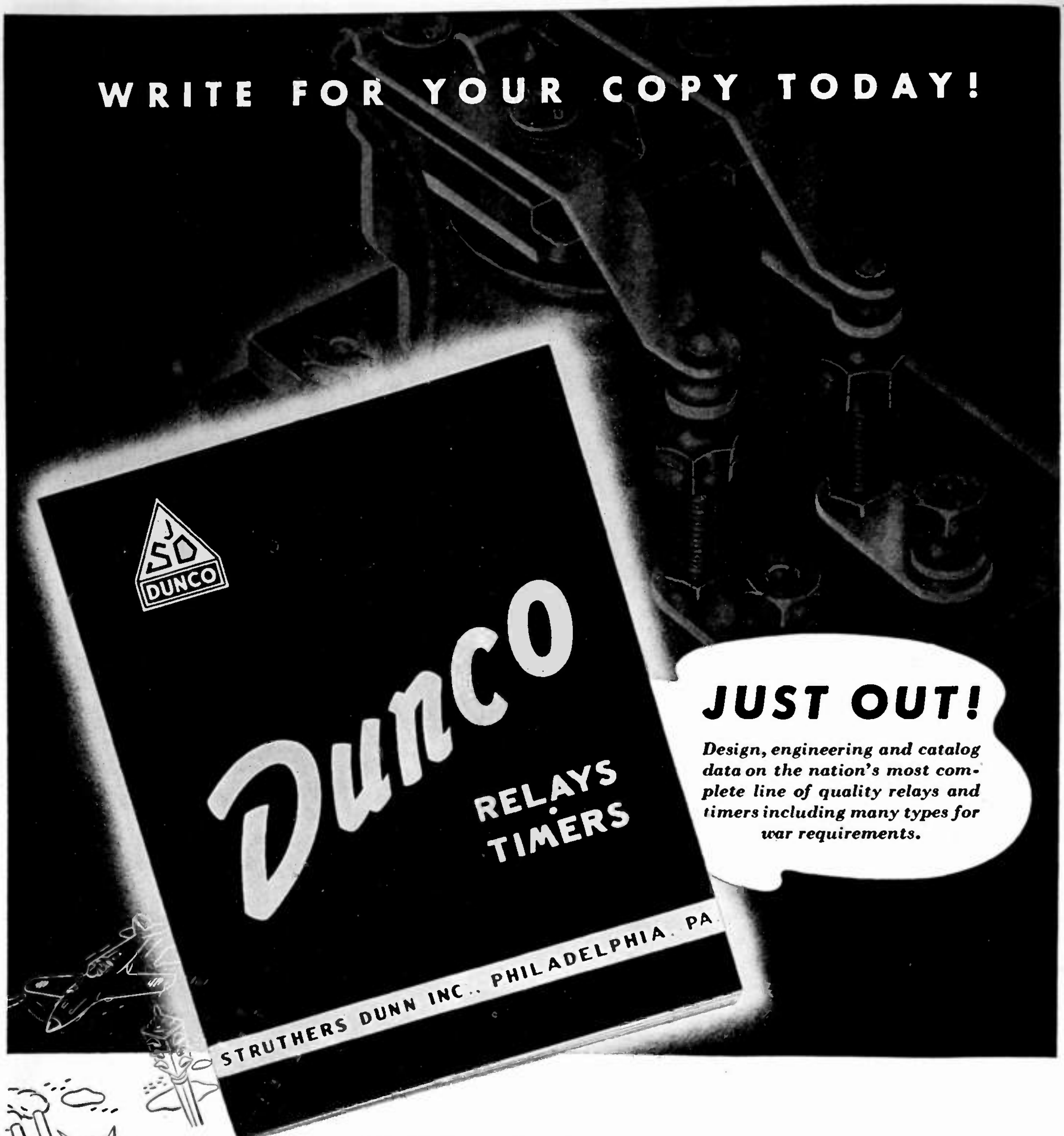
So we ask you not to put through a Long Distance telephone call to Washington unless it is absolutely necessary. Even then please be as brief as possible — to keep the lines open for the vital needs of war.

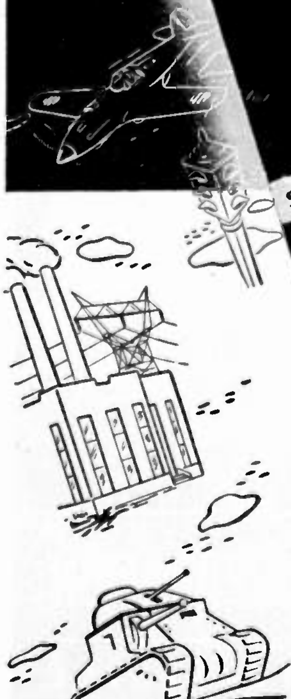
***BELL***
***TELEPHONE SYSTEM***

**WAR CALLS**

**COME FIRST**


ALSIMAG
AlSiMag
Coil Forms
The coil form is one of the most important components of every transmitter and receiver. It must be rigid, non-warping. It must have low dielectric loss and accurate dimensions.
The majority of AlSiMag Coil Forms are custom-made in compositions 35 or 196. For unusual requirements, other compositions are available. AlSiMag coil forms, meeting all needs, are favorites of designing engineers.
ALSIMAG
Trade Mark Reg. U. S. Pat. Off.
AMERICAN LAVA CORPORATION
CHATTANOOGA, TENNESSEE
CHICAGO • CLEVELAND • NEW YORK • ST. LOUIS • LOS ANGELES • SAN FRANCISCO • BOSTON • PHILADELPHIA • WASHINGTON, D. C.


WRITE FOR YOUR COPY TODAY!
JSO
DUNCO
Dunco
RELAYS
TIMERS
STRUTHERS DUNN INC., PHILADELPHIA, PA.
JUST OUT!
Design, engineering and catalog data on the nation's most complete line of quality relays and timers including many types for war requirements.



**THE INSTITUTE** serves those interested in radio and allied electrical communication fields through the presentation and publication of technical material.

**THE PROCEEDINGS** has been published without interruption since 1913 when the first issue appeared. The contents of each paper published in the PROCEEDINGS are the responsibility of the author and are not binding on the Institute or its members. Material appearing in it may be reprinted or abstracted in other publications on the express condition that specific reference shall be made to its original appearance in the PROCEEDINGS. Illustrations of any variety, however, may not be reproduced without specific permission from the Institute.

**STANDARDS** reports are published from time to time and are sent to each member without charge. The four current reports are on Electroacoustics, Electronics Radio Receivers, and Radio Transmitters and were published in 1938.

**MEMBERSHIP** has grown from a few dozen in 1912 to about seven thousand. Practically every country in the world in which radio engineers may be found is represented in our roster. Dues for the five grades of membership range from $3.00 to $10.00 per year. The PROCEEDINGS is sent to each member without further payment.

**SECTIONS** in twenty-six cities in the United States, Canada, and Argentina hold regular meetings. The chairmen and secretaries of these sections are listed on the page opposite the first article in this issue.

**SUBSCRIPTIONS** are accepted for the PROCEEDINGS at $10.00 per year in the United States of America, its possessions, and Canada; when college and public libraries order direct from the Institute, the subscription price is $5.00. For other countries there is an additional charge of $1.00.

**The Institute of Radio Engineers, Inc.**

**Harold P. Westman, Secretary**
**330 West 42nd Street**
**New York, N.Y.**

## The Institute of Radio Engineers

**Incorporated**

**330 West 42nd Street, New York, N.Y.**

To the Board of Directors

Gentlemen:

I hereby make application for ASSOCIATE membership in the Institute of Radio Engineers on the basis of my training and professional experience given herewith, and refer to the sponsors named below who are personally familiar with my work.

I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I shall be governed by the Constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power.

..................................................................
(Sign with pen)

..................................................................
(Address for mail)

..................................................................
(City and State)

..................................................................
(Date)

### SPONSORS

(Signatures not required here)

Mr. ..................................................................

Address ..................................................................

City and State ..................................................................

Mr. ..................................................................

Address ..................................................................

City and State ..................................................................

Mr. ..................................................................

Address ..................................................................

City and State ..................................................................

REMITTANCE SCHEDULE

| Month During Which Application Reaches I.R.E. Headquarters | Amount of Remittance (=Entrance Fee+Dues) Which Should Accompany Application | |
|---|---|---|
| Grade → | Associate | Period Covered by Dues Payment |
| Jan., Feb. | $7.50 (=$3+$4.50*) | Apr.-Dec. (9 mo. of current year)† |
| Mar., Apr., May | 6.00 (= 3+ 3.00*) | July-Dec. (6 mo. of current year)† |
| June, July, Aug. | 4.50 (= 3+ 1.50*) | Oct.-Dec. (3 mo. of current year† |
| Sept., Oct., Nov. | 9.00 (= 3+ 6.00*) | Jan.-Dec. (entire next year) |
| Dec. | 7.50 (= 3+ 4.50*) | Apr.-Dec. (9 mo. of next year)† |

† You can obtain the PROCEEDINGS for the entire year by including with your application a request to that effect and a remittance of $9.00.

* Associate dues include the price of the PROCEEDINGS, as follows: 1 year, $5.00; 9 months, $3.75; 6 months, $2.50; 3 months, $1.25. This may not be deducted from the dues payment.

## TO APPLY FOR ASSOCIATE MEMBERSHIP

*To Qualify* for Associate membership, an applicant must be at least 21 years of age, of good character, and be interested in or connected with the study or application of radio science or the radio arts.

*An Application* should be filed, preferably on blanks obtainable on request from I.R.E. Headquarters or from the secretary of your local Section. If more convenient, however, the accompanying abbreviated form may be submitted. Additional information will be requested later on.

*Sponsors* who are familiar with the work of the applicant must be named. There must be three, preferably Associates, Members, or Fellows of the Institute. Where the applicant is so located as not to be known to the required number of member sponsors, the names of responsible nonmembers may be given.

*Entrance Fee and Dues:* The Associate entrance fee is $3.00. Annual dues are $6.00 per year, which include the price of the PROCEEDINGS as explained in the accompanying remittance schedule.

*Remittance:* Even though the I.R.E. Constitution does not require it, you will benefit by enclosing a remittance with your application. We can then avoid delaying the start of your PROCEEDINGS.

Your PROCEEDINGS will start with the next issue after your election, if you enclose your entrance fee and dues as shown by the totals in the accompanying remittance schedule. Any extra copies sent in advance of the period for which you pay dues (see last column) are covered by your entrance fee.

Should you fail to be elected, your entire remittance will be returned.

(Typewriting preferred in filling in this form) No. ..........

## RECORD OF TRAINING AND PROFESSIONAL EXPERIENCE

Name ..........................................................................
(Give full name, last name first)

Present Occupation ..........................................................
(Title and name of concern)

Business Address ............................................................

Home Address ................................................................

Place of Birth ..................... Date of Birth .......... Age ......

Education ...................................................................

Degree ......................................................................
(College) (Date received)

### TRAINING AND PROFESSIONAL EXPERIENCE

(Give dates and type of work, including details of present activities)

Record may be continued on other sheets this size if space is insufficient.

Receipt Acknowledged .......... Elected .......... Notified ..........

4111

**OTHER GRADES** are available to qualified applicants. Those who are between the ages of eighteen and twenty-one may apply for Junior grade. Student membership is for full-time students in engineering or science courses in colleges granting degrees as a result of a four-year course. A special application blank is provided and requires the signature of a faculty member as the sole sponsor. Member grade is open to older engineers with several years of experience. Fellow grade is by invitation only. Information and application blanks for these grades may be obtained from the Institute.

## SUPPLIES

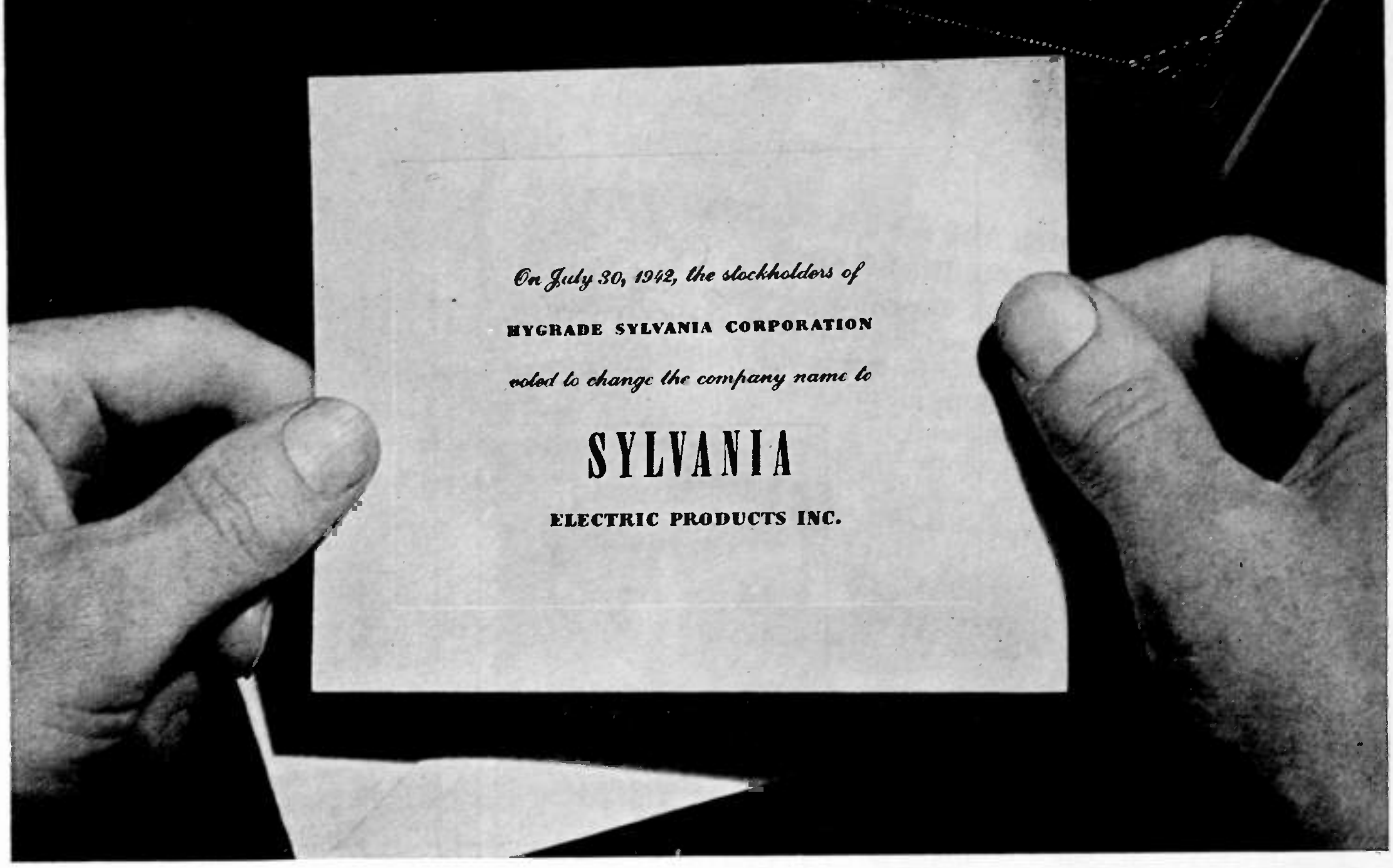
**BACK COPIES** of the PROCEEDINGS may be purchased at $1.00 per copy where available. Members of the Institute in good standing are entitled to a twenty-five per cent discount.

**VOLUMES**, bound in blue buckram, may be purchased for $14.25; $11.25 to members.

**BINDERS** are $1.50 each. The volume number or the member's name will be stamped in gold for fifty cents additional.

**INSTITUTE EMBLEMS** of fourteen-carat gold with gold lettering on an enameled background are available. The lapel button is $2.75; the lapel pin with safety catch is $3.00; and the watch charm is $5.00. All of these are mailed postpaid.

# The name is new...the Purpose is not Changed




● There's little in a name beyond what men put into it by their sincere effort and their determination to give it significance.

There has been a great deal, though, in the name Hygrade Sylvania because of the high standards that have marked the manufacture of its products — Hygrade Incandescent Lamps, Hygrade Fluorescent Lamps and Fixtures, and Sylvania Radio Tubes.

But there are advantages in simplicity, and in a single title that stands for single-minded insistence on the highest quality possible in any field with which it is linked.

So as of July 30, the name of the Hygrade Sylvania Corporation, adopted in 1931, became Sylvania Electric Products Inc.

Largely as a matter of conserving stocks of cartons and scarce packaging materials, changes in the trade names of Hygrade products will be made gradually.

Eventually, however, all products of the company will take the name *Sylvania,* which will become the single emblem of our one and unchanged purpose to produce the best in our field and to make our products widely known.

You who have known the quality of Hygrade Lamps and Sylvania Tubes will continue to find that same superlative quality in those products. The only change will be in the name, to which we intend to add new luster at every opportunity that is given us to improve on the quality and performance of our products.

Radio tubes of many types and for many purposes have made the name Sylvania synonymous with utmost dependability and high quality. Such products will continue to be branded with this name, but will now be joined by similarly labeled incandescent lamps, fluorescent lamps and fluorescent lighting fixtures which were formerly brand-marked Hygrade.

Ben G. Erskine
PRESIDENT
SYLVANIA ELECTRIC PRODUCTS INC.
Emporium, Pa.

*Hygrade Incandescent Lamps, Fluorescent Lamps, Fixtures and Accessories, Sylvania Radio Tubes*

# Booklets, and

**The following commercial received by**

REFERENCE GUIDE TO ULTRA HIGH FREQUENCIES··· *Zenith Radio Corp., 620 N. Michigan Ave., Chicago, Ill.* (Booklet, 56 pages, 6×9 inches.) A useful reference list of published information on ultra high frequency, compiled by E. Kelsey, associate member of I.R.E. Requests for copies will be granted to I.R.E. members if addressed to: E. Kelsey at Zenith Radio Corp.

RELAY TIMERS···(*Struthers Dunn, Inc., 1321 Arch St., Philadelphia, Pa.* (Catalog F, 48 pages, $8\frac{1}{2}\times11$ inches.) This "Dunco" catalog is designed to serve as a guide for relay selection and use. Classification of every type of relay has been worked out and simplified. Illustrations are excellent and specifications complete.

GRAPHIC INSTRUMENTS··· *The Esterline-Angus Company, Inc., Indianapolis, Ind.* (Bulletin Number 1241 "The Graphic," 4 pages, $8\frac{1}{2}\times11$ inches.) Treats new uses and applications for graphic instruments. Illustrated.

RESISTORS···*Lectrohm, Inc., Cicero, Illinois.* (Bulletin Number 98, 8 pages, $8\frac{1}{2}\times11$ inches.) Describes Lectrohm vitreous enamel resistors, fixed and variable; "Rib-on-edge" and Ferrule Terminal types; also chokes and brackets. Illustrated and full specifications given.

PILOTRON SPECIFICATIONS ···*General Electric Company, Schenectady, N. Y.* (Pamphlet, 10 pages, $8\times10\frac{1}{2}$ inches, ring punched.) Complete descriptions and ratings for three-electrode tubes, water cooled, vacuum, high frequency and power. Gives characteristic charts.

# Catalogs
# Pamphlets

literature has been
the Institute.

INDUSTRIAL ELECTRONIC TUBES · · · *General Electric Company, Schenectady, N. Y.* (Pamphlet, 4 pages, 9×12 inches.) This folder spreads open to make a quick selection chart listing G. E. electronic tubes for industrial use. Gives tables of specifications and prices.

HEAT TRANSFER THROUGH METALLIC WALLS · · · *The International Nickel Co., 67 Wall St., New York, N. Y.* (Booklet, 24 pages, 8½×11 inches.) This booklet charts and gives the formulas for heat conductivity of various metals including thickness, temperature and time factors.

RECEIVING TUBES · · · *RCA Manufacturing Co., Inc., Harrison, N. J.* (Catalog, 16 pages, 8½×11 inches.) The RCA Receiving Tube booklet contains three charts. Chart 1 classifies RCA Receiving Tubes according to their cathode voltages and function. Chart II gives characteristics of each of 329 receiving types arranged in numerical-alphabetical sequence. Chart III includes information on certain tubes closely allied to receiving tubes but customarily tabulated separately and identified as "special purpose." These tubes are particularly of interest for applications involving special performance requirements.

*RCA* GUIDE FOR TRANSMITTING TUBES · · · *RCA Manufacturing Co., Inc., Camden, N. J.* (Book, 76 pages, 8½×11 inches.) This is the 1942 edition of the 1941 Guide with a new special reference chart on air cooled and water cooled transmitting tubes and many other revisions. Three parts cover: I—Transmitting Tube



Data, 37 pages; II—Transmitting Circuit Facts, 6 pages; and III—Transmitter Construction, 20 pages. Price \$0.35. Really a remarkably fine guide.

CURRENT FLOW TEST SETS · · · *Shallcross Manufacturing Co., Collingdale, Pa.* (Bulletin "E," 4 pages, $8\frac{1}{2} \times 11$ inches.) Describes two Shallcross instruments that greatly facilitate the testing and adjustment of relays. One is quite comprehensive in its scope of operation while the other takes care of a small scope of requirements. Illustrated, gives prices and specifications.

OIL-FILLED PAPER CAPACITORS · · · *Solar Manufacturing Corp. Bayonne, N. J.* (Part of catalog 12, section C. 6 pages, $8\frac{1}{2} \times 11$ inches.) These are special data pages on oil-impregnated and oil-filled paper capacitors in metal tubes, of interest for their compactness and stamina.

AUTOMATIC FREQUENCY RESPONSE RECORDER · · · *Sound Apparatus Co., 150 West 46th Street, New York, N. Y.* (Pamphlet, "Sound Advances," 4 pages, $8\frac{1}{2} \times 11$ inches.) Concise and useful description of this equipment.

RADIO INTERFERENCE ELIMINATION MANUAL · · · *Sprague Products Co., North Adams, Mass.* This is the 1942 edition of the manual for radio engineers and service men on locating and filtering out interference noise. Well illustrated and specifications given for proper filters to correct noise conditions from such sources as fluorescent fixtures, motors, D.C. generators, switches, thermostats, etc.

## Manufacturers' House Publications

THE AEROVOX RESEARCH WORKER · · · *Aerovox Corp., New Bedford, Mass.* (4 page issues,

(*Continued on page xxii*)

*(Continued from page xx)*

$8\frac{1}{2}\times 11$ inches.) "Transformerless Power Supplies." Part I—February, 1942; Part II—March, 1942. "Transmitter Bias Supplies"—April, 1942.

THE CAPACITOR··· *Cornell-Dubilier Corp., Hamilton Boulevard, South Plainfield, N. J.* (16 pages, $5\frac{1}{4}\times 7\frac{3}{4}$ inches.) Articles on "Solving Shortage Problems in Oscillator Circuits" and on "Servicing AC-DC Compacts."

THE GENERAL RADIO EXPERIMENTER···*General Radio Co., 30 State St., Cambridge, Mass.* (8 pages, $6\times 9$ inches.) July, 1942 has articles on "Bringing the Beat Oscillator Up-to-Date" and "Recent Priority Orders of Interest to Buyers of G. R. Equipment."

SYLVANIA NEWS···*Sylvania Electric Products, Inc., Emporia, Pa.* (8 pages, $9\frac{1}{2}\times 12\frac{1}{2}$ inches.) July, 1942 reports change of Hygrade Sylvania Corporation name to Sylvania Electric Products, Inc. Technical Section describes operation and service of "The Solovox," made by Hammond Organ Company.

# Current Literature

**New books of interest to engineers in radio and allied fields—from the publishers' announcements. A copy of each book marked with an asterisk (*) has been submitted to the Editors for possible review in a future issue of the Proceedings of the I.R.E.**

---

DEFINITIONS OF ELECTRICAL TERMS···*American Institute of Electrical Engineers.* (300 pages, $8\times 11$ inches, bound in dark blue fabricoid.) Approved American Standard, August 12, 1941. Approved Canadian Standard, March 2, 1942. An invaluable aid for engineers, scientists and technicians. \$1.00 in U.S.A. \$1.25 (U.S.A. Currency) elsewhere.

NEW RADIO STANDARDS··· *Institute of Radio Engineers, 330*

*West 42nd St., New York, N. Y.* (Distributed July, 1942 to I.R.E. members.) Standards on Radio Wave Propagation: Definition of Terms, 1942. Price $0.20. Standards on Radio Wave Propagation: Measuring Methods, 1942. Price $0.50. Standards on Facsimile: Definition of Terms, 1942. Price $0.20.

TABLE OF SINE AND COSINE INTEGRALS FOR ARGUMENTS FROM 10 TO 100 · · · *National Bureau of Standards, Washington, D. C.* (189 pages, bound in buckram.) Prepared by the Federal Works Agency, W.P.A. under sponsorship of the bureau and is a sequel to "Table of Sine, Cosine and Exponent Integrals," Vol. II. Price $2.00.

RADIO COMMUNICATION SERIES. Edited by Beverly Dudley, Acting Manager "Electronics" · · · *McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, N. Y.* Designed to provide a well coordinated program of texts covering various specialized branches of communication developed in the last two decades. First book, ready in September—"Frequency Modulation" by August Hund. Not yet priced.

*RADIO CODE MANUAL. By Arthur R. Nilson, Chief Instructor, Nilson Radio School, New York City · · · *McGraw-Hill Book Company, Inc., 330 West 42nd St., New York, N. Y.* xii+174 pages, illustrated, $5 \times 7\frac{1}{2}$ inches. This is a spiral bound text of 20 lessons on the radio code and selected projects on code-learning equipment. It gives a complete course for self-study and classroom use, particularly designed for the training of military radio operators. It has a chapter devoted to "Setting Up and Running a Radio Class" and a question and answer chapter on obtaining a restricted radiotelephone operator's permit. Cloth. $2.00.

*ACOUSTICS OF MUSIC. By Wilmer T. Bartholomew, Instructor in Peabody Conservatory of Music. New York · · · *Prentice-*

*(Continued on page xxvi)*

# POSITIONS OPEN

The following positions of interest to I.R.E. members have been reported as open. Apply in writing, addressing reply to company mentioned or to Box No. ..........

**PROCEEDINGS of the I.R.E.**
330 West 42nd Street, New York, N.Y.

### NOTICE TO APPLICANTS

All men having employment records on file at Institute headquarters are requested to notify us immediately of their continued availability and at least every three months thereafter. Failure to receive notification will void application, as we are endeavoring to keep accurate files of only those men still available for placement.

### I.R.E. ASSISTANT SECRETARY

The Institute has immediate need for the services of a man with some business experience and a good general knowledge of communications engineering for the position of Assistant Secretary. Duties include carrying on correspondence, assisting the Secretary in conducting the Institute's activities, acting as secretary to technical committees, etc. Writing ability and publication experience desirable. Apply in writing to the Secretary, Institute of Radio Engineers, 330 West 42nd Street, New York, N.Y.

### TRANSMITTER ENGINEER

Radio engineer for low-powered transmitter development and design; experienced in high- and ultra-high-frequency amplitude-modulated and frequency-modulated applications. Permanent position. Write stating full qualifications to Transmitters Engineering Department, The Hammarlund Manufacturing Company, Inc., 460 West 34 Street, New York, N.Y.

### CRYSTAL ENGINEERS

A New England organization requires the services of a highly experienced engineer on finishing and calibrating quartz crystals. Also need several assistants. Box 272.

### YOUNG ELECTRICAL ENGINEER

Wanted by a Chicago manufacturer now engaged wholly in engineering and manufacturing of military material. Work at start will be experimental and consist of electrical, electronic, and magnetic measurements. A good knowledge of fundamentals is essential. Address a report of your training and experience to Box 273.

### COMMUNICATIONS TEACHER

A prominent eastern University wishes to employ a man to head up the Communications Division in the Electrical Engineering Department. This would include all the work given in the junior and senior years in wire and wireless communications. He would have charge of the laboratory work. The position is a permanent one. Additional information can be obtained and all communication should be sent to Box 274.

## NAVY LABORATORY

An activity of the United States Navy is in need of civil Junior, Assistant and Associate Radio Engineers; Assistant and Associate Physicists and Physicists for Laboratory research and development work in conjunction with the war effort program. Salaries range from $2000.00 to $3800.00 per annum. For further information and application for employment form write the Director, U. S. Navy Radio and Sound Laboratory, San Diego, California.

## RADAR LABORATORY

The Signal Corps Radar Laboratory has urgent need for Physicists and Engineers with Mechanical, Electrical, and Radio training. Inexperienced engineering graduates can also qualify.

Salaries range from $2000 to $3200 and up.

Draftsmen, Engineering Aides, Electricians, and Radio Mechanics also are wanted.

Apply in writing stating full qualifications to: Civil Service Representative, Signal Corps Radar Laboratory, Camp Evans, Belmar, New Jersey.

## CIVILIAN ORDNANCE ENGINEERS

The Naval Ordnance Laboratory of the Washington Navy Yard is seeking research and engineering men holding Ph.D. degrees or like calibre to head up new research problems—Salaries are very attractive.

Also—men with proven ability as Mechanical Engineers, Electrical Engineers, Physicists and Draftsmen are urgently needed.

An unusual opportunity as the work is among men proven leaders in their respective fields who have been loaned to the Navy by their civilian employers, and excellence of war-time work will certainly receive recognition in peacetime industry.

Only American citizens can be considered. Apply by letter—stating full experience, education and qualifications to Robert F. Moore, Naval Ordnance Laboratory, Washington, D.C.

## Attention Employers . . .

Announcements for "Positions Open" are accepted without charge from employers offering salaried employment of engineering grade to I.R.E. members. Please supply complete information and indicate which details should be treated as confidential. Address: "POSITIONS OPEN," Institute of Radio Engineers, 330 West 42nd Street, New York, N.Y.

**The Institute reserves the right to refuse any announcement without giving a reason for the refusal.**



## RADIO TECHNICIANS
## WIREMEN
## COIL WINDERS
## ASSEMBLERS

An outstanding opportunity for Radio Technicians, Wiremen, Coil Winders and Assemblers with experience on aircraft receivers, transmitters, and electronic equipment in an organization with large war orders as well as post war plans. Plant located in the east.

Write giving full personal history, experience and present salary; personal interview will be arranged later. We do not desire applications from men in key defense positions.

Please reply to:

Box 275
Institute of Radio Engineers
330 West 42nd St.
New York


(Continued from page xxiii)

*Hall, Inc., 1942.* xvi+238+4 index pages, illustrated, 6×9 inches. Cloth. $3.00.

*PRINCIPLES OF RADIO (Fourth Edition). By Keith Henney, Editor, "Electronics," Member, Institue of Radio Engineers · · · *John Wiley & Sons, Inc., 440 Fourth Avenue, New York, N. Y.* xii+549 pages, illustrated, $5\frac{1}{2}$×8 inches. This is a "standard text" on radio, revised to date for use in U. S. Signal Corps courses as chosen by the U. S. Office of Education. Cloth. $3.50.

*RADIO AND ENGLISH TEACHING. Edited by Max J. Herzberg. Sponsored by the National Council of Teachers of English and entitled "English Monograph No. 14" · · · *D. Appleton-Century Company, 35 W. 32nd St., New York, N. Y.* viii+246 pages, $6\frac{1}{4}$×$9\frac{1}{2}$ inches. Discusses the experiences, problems and procedures in teaching English by radio. Cloth. $2.00.

*A GRAPHIC TABLE COMBINING LOGARITHMS AND ANTILOGARITHMS. By Adrien Lacroix and Charles L. Ragot · · · *MacMillan Company, 60 Fifth Avenue, New York, N. Y.* Originally published in September, 1925, reprinted May, 1942. vi+40 pages, 7×10 inches. 40 pages devoted to a five-place graphic table. Cloth. $1.60.

*FUNDAMENTALS OF RADIO. By Edward C. Jordan, Instructor in Electrical Engineering, Ohio State University; Paul H. Nelson, Assistant Professor of Electrical Engineering, University of Connecticut; William Carl Osterbrock, Professor of Electrical Engineering, University of Cincinnati; Fred H. Pumphrey, Professor of Electrical Engineering, Rutgers University; Lynne C. Smeby, Director of Engineering for the National Association of Broadcasters · · · *Prentice-Hall Inc., 70 Fifth Avenue, New York, N. Y.* xiii+400 pages, $6\frac{1}{4}$×$9\frac{1}{4}$ inches, illustrated, includes three unfolding circuit diagrams. Cloth. $5.00.

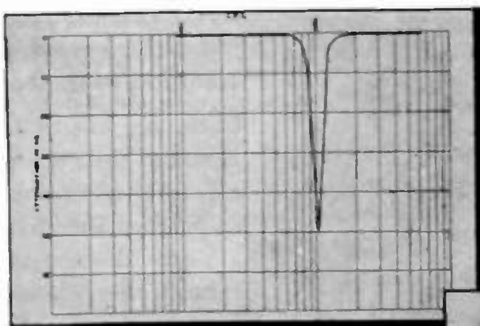
# INDEX



## DISPLAY ADVERTISERS

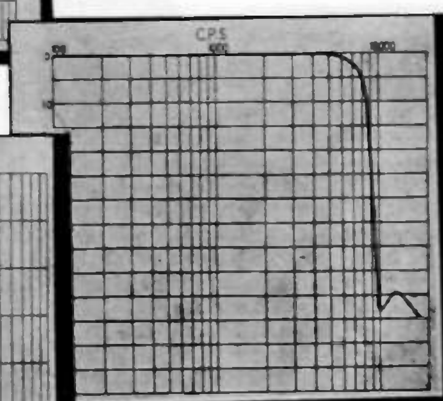


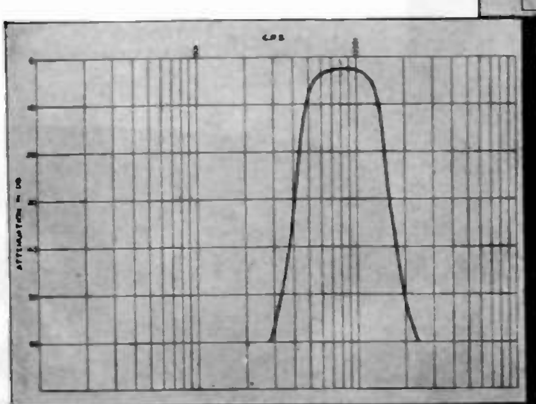

Low pass, high pass, band pass or band rejection types of the size, weight and characteristics to serve your purpose. Discuss your filter problem with experienced Thordarson engineers.



☆ ABOVE *Band rejection filter*

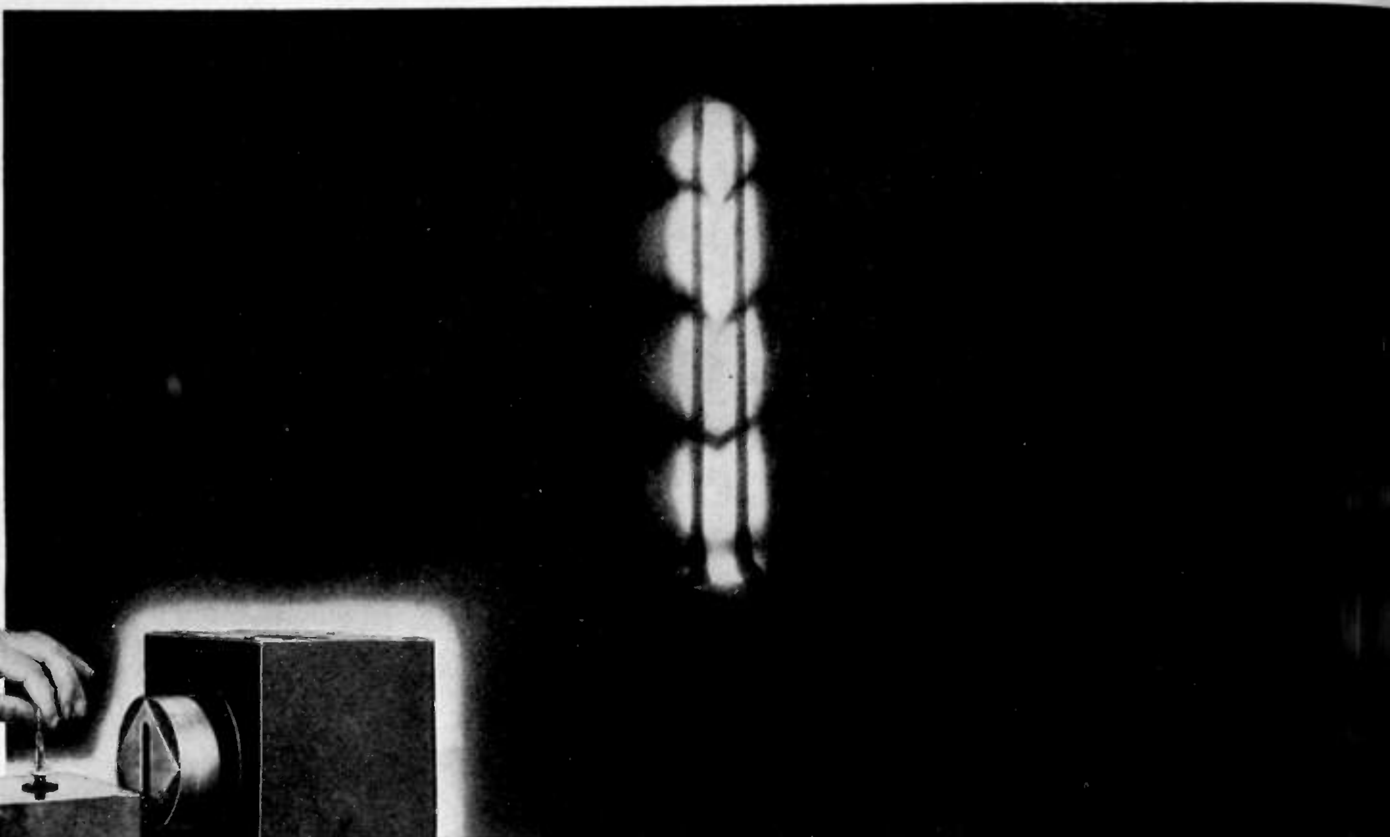
**THORDARSON**
ELECTRIC MFG. COMPANY
500 WEST HURON STREET CHICAGO ILL.
*Transformer Specialists Since 1895*

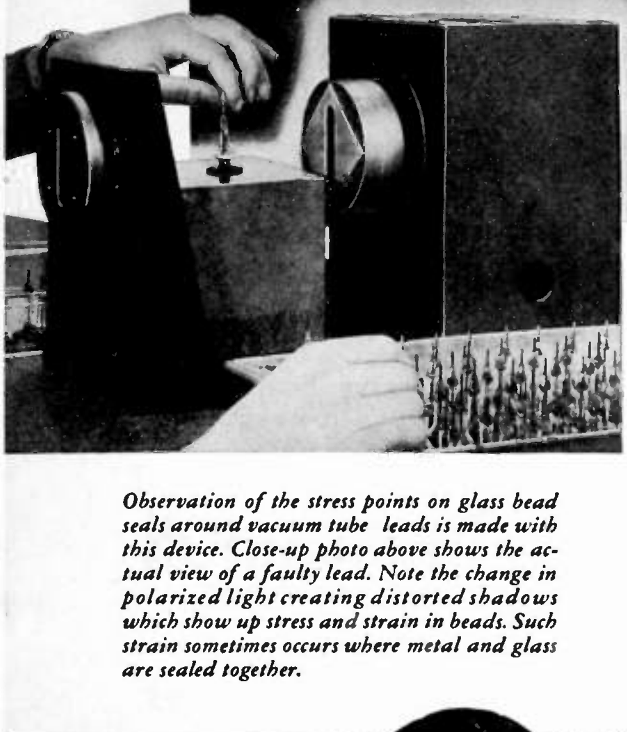


☆ ABOVE *Low pass filter*
☆ LEFT *Band pass filter*

# DEATH *before* DISHONOR

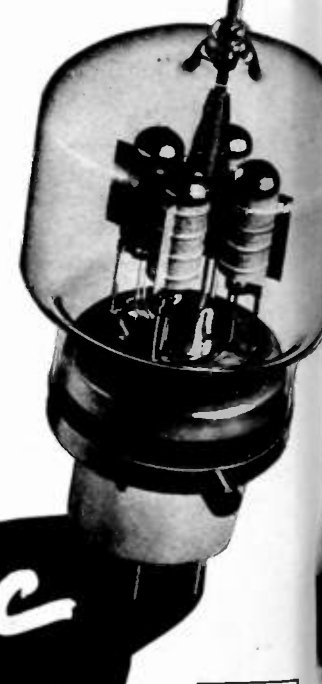




*Observation of the stress points on glass bead seals around vacuum tube leads is made with this device. Close-up photo above shows the actual view of a faulty lead. Note the change in polarized light creating distorted shadows which show up stress and strain in beads. Such strain sometimes occurs where metal and glass are sealed together.*

Casual observation of a vacuum tube does not reveal its flaws. That's why Eimac engineers have developed many devices for the purpose of exposing even slight weaknesses in construction. The above is not a dungeon window, but a close-up photo of a faulty bead on a filament stem as viewed through a special bead testing device. Needless to say, this stem will never reach final assembly . . . better "death before dishonor" to the Eimac tradition of dependability.

Such care in production plus constant research into the phenomenon of the electron tube assures you of the utmost in performance from every Eimac tube . . . provides the answer to why Eimac tubes are first choice by most of the leading engineers throughout the world.



(*Export Agents:*
**FRAZAR & CO., LTD., 301 Clay St.**
San Francisco, California, U. S. A.)

*Inspecting the entire glass bulb with the help of a polarized light. This device shows up stress and strain on the glass which might be created during the shaping operations.*

*Follow the leaders to*

**Eimac TUBES**

FOR VICTORY BUY UNITED STATES SAVINGS BONDS AND STAMPS

*Mfg. by Eitel-McCullough, Inc., San Bruno, California, U. S. A.*

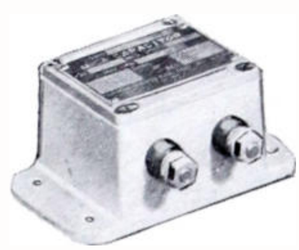

**TODAY'S** ***C-D Capacitors Speed Victory . . .***

**TOMORROW'S** ***C-D Capacitors assure more hours of use per dollar for American industry . . .***

Day in, day out, Navy blimps patrol our coastal sea lanes. When a sub is sighted, a flash brings a tough "sub-buster" to track the enemy down.

Teamwork of blimps and boats, of men and equipment, is winning the battle of the Eastern Seaboard. Here, as with a hundred other electrical and electronic devices on vital war duty, C-D capacitors are meeting the enemy challenge with the *finer* performance assured by 32 years' concentration on the building of superior capacitors.

Today, war-spurred "impossible" improvements and applications promise measureless peacetime benefits; giving a new meaning to C-D's well-known pledge to industry of "more hours of capacitor use per dollar". Cornell-Dubilier Electric Corporation, South Plainfield, New Jersey; New England Div.: New Bedford, Mass.



**Special Mica Transmitter Capacitors**

***copied • imitated • but never duplicated***

Type 21 dual-section mica capacitors are designed for filament by-pass applications in broadcast transmitters. They are encased in aluminum castings and permit mounting close to the filament terminals of power tubes. Compound filled, the type 21 mica capacitors have low inductance characteristics — thus affording excellent by-pass of r.f. current from filament leads to grounds.

Described in Catalog No. 160T free on request.

**Cornell Dubilier Capacitors**

MICA • PAPER • DYKANOL • WET & DRY ELECTROLYTIC CAPACITORS

**MORE IN USE TODAY THAN ANY OTHER MAKE**


# Capacitance Standards

General Radio air and mica condensers are widely used as standards of capacitance. For this reason, the General Radio Company maintains a careful supervision of the calibration standards in its own standardizing laboratory. These fixed standards cover a range of capacitance from 1000 μμf to 1 μf in values of 1, 2 and 5 for each decade.

These standards are kept in a temperature-controlled room. A complete intercomparison of units to detect drift of one unit relative to the others is made at least twice a year; additional measurements of selected groups are made more frequently. To detect drifts of the system as a whole, a complete set of mica standards is checked once in two years by the National Bureau of Standards, and any necessary corrections in the recorded capacitance values are made. Discrepancies are seldom as great as 0.02%. This capacitance standardization system is, therefore, internally consistent at all times to 0.02% or better.

The guaranteed accuracy of commercial standards calibrated in the General Radio laboratories is limited to 0.1% for capacitance, which is the accuracy to which the National Bureau of Standards certifies capacitance calibrations.

*The technique of comparing capacitances in our laboratories has been developed to the point where the inherent errors in the comparison process are negligible in terms of the accuracy of the standards. The overall precision of measurement is 0.02% for capacitance and 0.000050 for dissipation factor.*

**GENERAL RADIO COMPANY · Cambridge, Massachusetts**