PROCEEDINGS

of

The Institute of Radio Engineers

Institute of Radio Engineers

Forthcoming Meetings

CINCINNATI SECTION
Cincinnati, Ohio, October 17, 1929

CLEVELAND SECTION
Cleveland, Ohio, October 18, 1929

CONNECTICUT VALLEY SECTION
Hartford, Conn., October 14, 1929

DETROIT SECTION
Detroit, Mich., October 16, 1929

NEW YORK MEETING
New York, N. Y., November 6, 1929

PITTSBURGH SECTION
Pittsburgh, Penna., October 15, 1929

TORONTO SECTION
Toronto, Canada, October 15, 1929

WASHINGTON SECTION
Washington, D. C., October 10, 1929

Eastern Great Lakes District Convention

November 18-19, 1929, Rochester, N. Y.

Sponsored by the
Rochester, Buffalo - Niagara, Cleveland and Toronto Sections

All members of the Institute are invited to participate in the very interesting two-day program being arranged

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The Institute of Radio Engineers

GENERAL INFORMATION

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PROCEEDINGS

Preparation of Paper

Form—Manuscripts may be submitted by member and non-member contributors from any country. To be acceptable for publication manuscripts should be in English, in final form for publication, and accompanied by a summary of from 100 to 300 words. Papers should be typed double space with consecutive numbering of pages. Footnote references should be consecutively numbered and should appear at the foot of their respective pages. Each reference should contain author's name, title of article, name of journal, volume, page, month, and year. Generally, the sequence of presentation should be as follows: statement of problem; review of the subject in which the scope, object, and conclusions of previous investigations in the same field are covered; main body describing the apparatus, experiments, theoretical work, and results used in reaching the conclusions and their relation to present theory and practice; bibliography. The above pertains to the usual type of paper. To whatever type a contribution may belong, a close conformity to the spirit of these suggestions is recommended.

Illustrations—Use only jet black ink on white paper or tracing cloth. Cross-section paper used for graphs should not have more than four lines per inch. If finer ruled paper is used, the major division lines should be drawn in with black ink, omitting the finer divisions. In the latter case, only blue-lined paper can be accepted. Photographs must be very distinct, and must be printed on glossy white paper. Blueprinted illustrations of any kind cannot be used. All lettering should be $\frac{1}{16}$ in. high for an 8 x 10 in. figure. Legends for figures should be tabulated on a separate sheet, not lettered on the illustrations.

Mathematics—Fractions should be indicated by a slanting line. Use standard symbols. Decimals not preceded by whole numbers should be preceded by zero, as 0.016. Equations may be written in ink with subscript numbers, radicals, etc., in the desired proportion.

Abbreviations—Write a.c. and d.c., (a-c and d-c as adjectives), kc, μf, $\mu\mu f$, e.m.f., mh, μh, henries abscissas, antennas. Refer to figures as Fig. 1, Figs. 3 and 4, and to equations as (5). Number equations on the right in parentheses.

Summary—The summary should contain a statement of major conclusions reached, since summaries in many cases constitute the only source of information used in compiling scientific reference indexes. Abstracts printed in other journals, especially foreign, in most cases consist of summaries from published papers. The summary should explain as adequately as possible the major conclusions to a non-specialist in the subject. The summary should contain from 100 to 300 words, depending on the length of the paper.

Publication of Paper

Disposition—All manuscripts should be addressed to the Institute of Radio Engineers, 33 West 39th Street, New York City. They will be examined by the Committee on Meetings and Papers and by the Editor. Authors are advised as promptly as possible of the action taken, usually within one month.

Proofs—Galley proof is sent to the author. Only necessary corrections in typography should be made. *No new material is to be added.* Corrected proofs should be returned *promptly* to the Institute of Radio Engineers, 33 West 39th Street, New York City.

Reprints—With the galley proof a reprint order form is sent to the author. Orders for reprints must be forwarded promptly as type is not held after publication.

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Washington, D. C.

J. H. DELLINGER
Member of Board of Direction, 1929

J. H. Dellinger was born in Cleveland, Ohio, July 3, 1886. He was educated at Western Reserve (1903–07), George Washington University (A.B. degree, 1908), and Princeton University (1912–13, Ph.D. degree in 1913). He was an instructor in the physics department of Western Reserve in 1906–07.

Dr. Dellinger joined the staff of the Bureau of Standards as physicist in 1905. From 1919 to date he has been Chief of the Bureau's Radio Section. He has been particularly active in special government service. In 1912 he was a U. S. delegate to the Interallied Technical Conference on Radio Communication at Paris and member of technical staff of the Conference on Limitation of Armament and Far East Problems, at Washington. In 1922–23 he served as secretary of the U. S. Government Inter-department Radio Advisory Committee. He was a member of all four National Radio Conferences (1922–25), and since 1924 has been chairman of the Committee on Radio Apparatus, Federal Specifications Board. Dr. Dellinger was on the technical advisory staff of the International Radio Conference at Washington in 1927 and has recently left for The Hague, where he is one of the technical advisors to the American Delegation of the International Technical Consulting Committee.

Dr. Dellinger was vice-president of the Institute in 1924 and president in 1925. In 1928 he was Chairman of the Committee on Meetings and Papers and is the present chairman of the Committee on Standardization. He was elected a member of the Board of Direction of the Institute by the membership in 1928 to serve until 1931. He is a Fellow in the Institute of Radio Engineers and the American Physical Society.

Eastern Great Lakes District Convention

An innovation in general meetings of Institute members is to be provided at the forthcoming Eastern Great Lakes District Convention which is to be held in Rochester, New York, on November 18–19, 1929.

The Convention, to which the entire membership of the Institute is cordially invited, is sponsored by the Rochester, Buffalo-Niagara, Toronto, and Cleveland sections of the Institute. A two-day program of diversified and interesting activities including four technical papers of timely importance and three inspection trips has been arranged.

The headquarters will be at the Sagamore Hotel, where all technical sessions and the banquet on the second evening will take place.

Registration cards will be mailed from the Institute office in advance of the meeting. Members contemplating attending the Convention should return these cards promptly to the Convention Headquarters.

<div align="center">Program</div>

Sunday, November 17th—3:00 P.M. to 6:00 P.M.—Registration at Convention Headquarters, Sagamore Hotel.

Monday, November 18th—8:00 P.M.—Registration at Sagamore Hotel, Convention Headquarters.

10: A.M. to 12:30 P.M.—Opening technical session. Address of welcome by E. C. Karker, chairman of the Rochester section. Technical papers: "Considerations in Screen-Grid Receiver Design," by W. A. McDonald, Hazeltine Corporation, Hoboken, N. J., and "What Executives Expect of Engineers," by I. G. Maloff, Valley Appliance Corp., Rochester.

2:00 P.M.—Inspection trip to Kodak Park and Valley Appliance Corporation. 8:00 P.M.—Technical session: "Ultra High-Frequency Transmission and Reception," by A. Hoyt Taylor, Naval Research Laboratory, Washington, D. C.

<div align="center">1671</div>

Tuesday, November 19th—9:30 to 12:30 P.M. Technical session: "A Broadcast Receiver for Special Purposes," by Paul O. Farnham, Radio Frequency Laboratories, Boonton, N.J. and "Standardization in the Radio Vacuum-Tube Field," by W. C. White, Research Laboratory, General Electric Company, Schenectady.

2:00 P.M.—Inspection trip to Stromberg-Carlson Plant.

6:30 P.M.—Banquet at Sagamore Hotel. Speaker, H. B. Richmond, President of the Radio Manufacturers' Association.

The following committees have been appointed and are arranging the entire Convention:

Executive: Virgil M. Graham, chairman; publicity, R. A. Hackbusch, chairman; transportation-accommodations, J. Eichman, chairman; fellowship, J. A. Victoreen, chairman; trips, A. L. Schoen, chairman; entertainment, E. C. Karker, chairman; registration, A. E. Soderholm, chairman; exhibits, I. G. Maloff, chairman.

September Meeting of Board of Direction

At the meeting of the Board of Direction of the Institute held at 2:00 P.M. on September 4th, the following board members were present: J. H. Dellinger (acting chairman), Melville Eastham, treasurer; John M. Clayton, secretary; Arthur Batcheller, Ralph Bown, Alfred N. Goldsmith, R. H. Marriott, and L. E. Whittemore.

The following were transferred or elected to the higher grades of membership in the Institute: transferred to the grade of Fellow: C. W. Latimer; transferred to the grade of Member: V. D. Landon, C. B. Mirick, E. R. Pfaff, Paul M. Segal, and Clinton H. Suydam; elected to the grade of Member: J. H. Barron, J. A. Beveridge, R. J. Bird, Louis G. Caldwell, A. L. Duval d'Adrian, Lionel Harvey, C. M. Johnson, R. H. Lyne, George McElrath, R. A. McLennan, L. Osiatinsky, H. A. Pidgeon, E. G. Ramirez, D. E. Replogle, Hans Roder, W. G. Russell, B. A. Schwarz, A. C. Shaw, K. H. Thow, Glen E. West, R. M. Wilmotte, W. L. Winner, E. C. Woodruff.

One hundred and eighteen Associate members and fourteen Junior members were elected.

The Board approved the tentative program of the 1930 Convention as submitted by the Convention Committee, deciding upon August 18-21 as the most suitable dates.

The Board approved the petition for the formation of a Cincinnati section of the Institute. W. W. Boes and R. H. Langley were prominent in the organization work which led to the petition. The petition bore the signatures of the following members of the Institute: C. D. Barbulesco, Carl L. Flory, C. F. Fraasa, C. C. Dixon, R. M. Blair, R. P. Glover, F. J. Franzwa, W. S. Hannon, C. W. Peterson, H. S. Meyrose, E. J. Bussard, H. E. Youngs, C. E. Kilgow, D. D. Israel, J. A. Chambers, F. Clifford Estey, E. T. Flewelling, H. J. Loftis, W. K. Aughenbaugh, R. H. Langley, W. W. Boes, L. A. Kellogg, L. A. Hendricks, John Gerstle, and R. S. Copp.

Nomination of 1930 Officers and Managers

At the September 4th meeting of the Board of Direction the following were placed in nomination for 1930 elective officers and Board members:

For President: R. A. Heising, engineer, technical staff, Bell Telephone Laboratories, New York, N. Y. Member of the Board of Direction, 1927-29; Chairman, Committee on Admissions, 1929. Fellow of the Institute.

For Vice President: A. G. Lee, staff engineer in charge of radio section, engineering department, General Post Office, London, England. Member, Committee on Admissions, 1928-29. Fellow of the Institute.

For Managers (three-year terms)

J. V. L. Hogan, consulting engineer, New York, N. Y. Vice-president of the Institute, 1916-19, president, 1920. Member, Board of Direction, 1913-15, 1921-29. Member, Committee on Broadcasting, 1928-29. Fellow of the Institute.

R. H. Marriott, consulting engineer, New York, N. Y. President of the Institute, 1912. Member, Board of Direction, 1914-16, 1920-22, 1926-29. Member, Committee on Broadcasting, 1928, vice-chairman, 1929; Chairman, Committee on Constitution and Laws, 1928-29. Fellow of the Institute.

Ballots will be in the mails on or before October 20, 1929.

Institute Representative at International Conference

The Institute has been invited by the Department of State to send a representative to the first meeting of the International Technical Con-

sulting Committee on Radio Communication held at The Hague in September, 1929. The International Technical Consulting Committee is charged with the study of technical and allied problems which relate to international radio communication. Its functions are to give advice on questions submitted by the various governments. The Dutch government organized the first meeting of the Committee and prepared the program of work.

The Institute has appointed Dr. W. Wilson as its representative at the Conference. Dr. Wilson sailed for The Hague early in September.

The American delegation is composed of the following members: Major-General Charles McK. Saltzman, Major-General George S. Gibbs, and Captain S. C. Hooper. Among the technical assistants appointed for the delegation are J. H. Dellinger, C. B. Jolliffe, Lieut. Comdr. T. A. M. Craven, G. C. Gross, and K. B. Warner. The representatives of American communication companies attending the conference include Lloyd Espenschied, W. Wilson, J. Nathansonn, L. A. Briggs, Colonel S. E. Reber, A. Y. Tuel, H. H. Buttner, W. E. Beakes, R. M. Heintz, C. J. Pannill, J. W. Swanson, L. G. Caldwell, and H. C. Leuteritz.

Announcement of Civil Service Examination

The U. S. Civil Service Commission announces an open competitive examination for the position of assistant inspector (radio enforcement) at a salary of $2,400 a year. Competitors will be rated on the following subjects: theoretical and practical questions on radio and electrical engineering, 60 per cent; education, training and experience, 40 per cent.

The duties of the assistant radio inspector will be primarily to assist the radio inspector in enforcement of the radio act, and will include the inspection of radio equipment on vessels and at land stations. Members of the Institute interested in this examination should secure Form 2600 from the U. S. Civil Service Commission, Washington, D. C., mentioning examination number 205, and the exact title of examination desired.

Convention Reprints Available

By addressing a letter to the Secretary of the Institute, members will be supplied with one copy of any of the following 1929 Convention reprints, gratis. Mention should be made of the reprint number when writing.

No. 75—Symposium on Technical Problems of Radio Regulation

No. 76—Symposium on Photoradio and Television

No. 77—Nineteen papers presented at the joint meeting of the Institute and the American Section of the International Scientific Radio Union.

Past Issues of the Proceedings

On page xxi of the advertising section of this issue will be found a statement of the back issues of the PROCEEDINGS, both in bound volumes and single copies, on hand for sale at the Institute office.

Institute members contemplating completing their files of back issues are informed that the stock of certain numbers and years is very small, and are advised to place an order with the Institute for those copies required in the near future before the present stock is exhausted.

Members desiring photostat copies of issues of the PROCEEDINGS or individual papers no longer available for sale should communicate with the United Engineering Societies Library, 33 West 39th Street. The price for photostat copies is twenty five cents per double page.

Institute Meetings

NEW YORK MEETING

On Wednesday, September 4th, the first meeting of the fall season was held in the Engineering Societies Building, 33 West 39th Street, presided over by R. H. Marriott. The following papers were presented: "Plate Voltage Supply for Naval Vacuum Tube Transmitters," by Commander E. C. Raguet, Bureau of Engineering, Navy Department, and "'Hot-Cathode Mercury-Vapor Rectifier Tubes," by H. C. Steiner and H. T. Maser of the ResearchLaboratory of the General Electric Company at Schenectady. The first paper was delivered by Lieutenant Commander R. C. Starkey, of the Brooklyn Navy Yard, New York, and the second paper by H. C. Steiner.

Following the presentation of these papers the following participated in their discussion: H. C. Steiner, H. T. Maser, H. E. Hallborg, W. C. White, Robert M. Arnold, Austin Bailey, and Frank R. Stansel.

ATLANTA SECTION

The first meeting of the fall season of the Atlanta section was held on September 4th in the Hotel Winecoff. An informal banquet preceded the technical meeting.

Henry L. Reid presented a paper on "Modern Methods of Servicing Radio Receivers." Phil Bangs presented an informal talk on benefits

of I. R. E. membership. Lieutenant S. A. Jones, U. S. N., gave a talk on "Experiences in China."

Mr. Reid's paper described the service problem from 1920 through to 1929 and demonstrated the latest model of a set analyzer, explaining the function of the various circuits. Lieutenant Jones' paper dealt with the trade conditions, customs and possibilities of the Far East as a market for American goods.

Mr. Bangs emphasized the fact that members of the sections of the Institute are interested in all phases of the radio art and that section meetings should offer an opportunity for members to discuss problems arising in their particular field.

PITTSBURGH SECTION

A joint meeting of the Pittsburgh sections of the Institute and the A. I. E. E. will be held on November 12th. Mr. Taylor, of the General Electric Company, will present a paper, "Making Sound Visible and Light Audible."

Committee Work

COMMITTEE ON ADMISSIONS

A meeting of the Committee on Admissions was held at 6:30 P.M. on August 14th in the Western Universities Club, Hotel Biltmore, New York City. The following were present: R. A. Heising, chairman; E. R. Shute, H. S. Smith, A. F. Van Dyck, R. S. Kruse, H. F. Dart, George Lewis, and F. R. Brick.

The committee took action on thirty-three applications for transfer or election to higher grades of membership in the Institute, approving twenty-five of that number.

COMMITTEE ON BROADCASTING

Raymond Guy, of the Research Department of the Radio Corporation of America, New York, N. Y., has been appointed to membership on the Committee on Broadcasting under the chairmanship of L. M. Hull.

COMMITTEE ON MEETINGS AND PAPERS

As foreign members of the Committee on Meetings and Papers to officially represent the Institute in that capacity in their individual countries, President Taylor has appointed the following:

Dr. Alexander Meissner, Germany
Professor E. V. Appleton, England
Dr. Balth. van der Pol, Holland
Prof. V. T. Bashenoff, U. S. S. R
Prof. H. Yagi, Japan

COMMITTEE ON SECTIONS

A meeting of the Committee on Sections was held at 6:30 P.M. on August 29th in the Western Universities Club, New York, N. Y., with attendance as follows: E. R. Shute, chairman; Austin Bailey and D. H. Gage.

The Committee approved the petition from members in the Cincinnati, Ohio, territory, recommending to the Board of Direction that a Cincinnati section be organized.

The Committee completed its revision of the Manual of Section Organization and Operation, and took action on a number of the detailed section operation matters.

SECTIONAL COMMITTEE ON RADIO

Reorganization of the Sectional Committee on Radio of the American Standards Association, of which the I.R.E. and the A.I.E.E. are joint sponsors, is in progress.

The Board of Direction of the Institute has designated L. E. Whittemore, L. M. Hull, and J. V. L. Hogan representatives of the Institute of Radio Engineers on the Sectional Committee on Radio.

Personal Mention

Alfred Crossley has resigned from the Steinite Radio Company to become chief engineer of the Howard Radio Company of Chicago.

M. C. Batsel, until recently in the radio receiver section of the Westinghouse Electric and Manufacturing Company at East Pittsburgh, is now chief engineer of RCA Photophone, Inc., in New York City.

D. J. H. Leitch has joined Radio Industries of Canada, Ltd., as branch manager, with headquarters at Toronto.

C. G. Lemon, for some time in the research and experimental department of Messrs. Lissen, Ltd. of Isleworth, England, is associated with Tungsram Electric Lamp Works of London as technical adviser.

Frank Leslie, formerly assistant chief engineer of Farrand Manufacturing Company, is now recording engineer at Paramount Famous Lasky Corporation in Astoria, N. Y.

I. Dale Ball has joined the inspection force of the Supervisor of Radio, 8th inspection district, Detroit, in the capacity of assistant radio inspector.

T. J. Bindner, formerly radio operator at station KJR, Seattle, is associated with the Masterphone Sound Equipment Corp., of Seattle.

Sidney Bloomenthal has joined the research department of the Radio Corporation of America in New York. He was, until recently, laboratory assistant at Ryerson Physical Laboratory, University of Chicago.

Robert E. Cain has become affiliated with RCA Photophone, Inc., at New York City. Mr. Cain left the engineering department of the Philco Radio Company at Philadelphia to assume his new connection.

E. C. Carlson, until recently laboratory assistant with Acoustic Products Manufacturing Corp., of Stamford, Conn., has become engineer with Sonora, Inc., of Buffalo, N. Y.

Joseph A. Davis is in the radio engineering department of the Temple Corporation at Clearing, Ill. He was formerly connected with the research department of the Automatic Electric Company.

J. D. Durkee, formerly officer in charge Radio Central, Navy Department, Washington, is now chief communications engineer of Universal Wireless Communication Company, Inc., at Chicago.

OBITUARY

With deep regret the Institute announces the death of

Edward Beech Craft

Mr. Craft was born in Cortland, Ohio, September 12, 1881. He received a common and high school education in Warren, Ohio. From 1900 to 1902 he was superintendent of the lamp department of the Warren Electrical and Supply Company. Shortly after that he joined the Western Electric Company at Chicago. In 1907 Mr. Craft moved to New York City to become development engineer with the Western Electric Company. By 1918 he had become assistant chief engineer in charge of all development work in the engineering department of the Western Electric Company. In 1922 he became chief engineer in charge of all research activities of Western Electric. With the incorporation of Bell Telephone Laboratories in 1925 Mr. Craft became its executive vice-president, and later a director of the corporation.

His contributions to the telephone and radio art through important inventions were numerous.

He was elected a Fellow in the Institute in April, 1920, and was a member of several other scientific and engineering societies.

His death occurred on August 20th, 1929.

GEOGRAPHICAL LOCATION OF MEMBERS ELECTED
SEPTEMBER 4, 1929

Transferred to the Fellow grade

New York	New York City, c/o RCA Communications, Inc.	Latimer, Chester W.

Transferred to the Member grade

California	Palo Alto, Federal Telegraph Company	Suydam, Clinton H.
Dist. of Columbia	Washington, 3314 Newark St.	Mirick, C. B.
	Washington, c/o Federal Radio Commission	Segal, Paul M.
Illinois	Chicago, 6658 Loleta Ave.	Pfaff, Ernest R.
Pennsylvania	Wilkinsburg, 204 Avenue F.	Landon, V. D.

Elected to the Member grade

California	San Francisco, Signal Office, Presidio of San Francisco	Winner, William Lane, Jr.
Dist. of Columbia	Washington, Dept. of Commerce, Radio Division	Barron, J. H., Jr.
Illinois	Alton, 1124 Washington St.	d'Adrian, A. L. Duval
	Chicago, 1400 Union Trust Bldg.	Caldwell, Louis G.
Massachusetts	Waban, 100 Devon Road	Replogle, Delbert E.
Montana	Bozeman, c/o Montana State College	West, Glenn Edwin
New Jersey	Boonton, Radio Frequency Laboratories, Inc.	Wilmotte, R. M.
	Maplewood, 26 Berkeley Road	Pidgeon, Howard A.
New York	New York City, USS *Bridge*, c/o Postmaster	Johnson, C. M.
	New York City, 711 Fifth Avenue, Room 1223	McElrath, George
Pennsylvania	State College, 234 West Fairmount Ave.	Woodruff, Eugene C.
Canada	Toronto, Ont., Windsor Arms Apts., St. Thomas and Sultan Sts.	Schwarz, Bertram A.
England	Leeds, Yorks, 35 Reginald Terrace, Chapeltown	Harvey, Lionel
	Leeds, 6 Blenheim Terrace, Northeastern School of Wireless	Russell, M. W. G
	Manchester, British Broadcasting Corp.	Bird, R. J.
	Norbury, SW, 119 Headcorn Road	Shaw, A. C.
Germany	Berlin, Fritschestrasse 41, I, Charlottenburg	Roder, Hans
Italy	Milan Corso Italia, 13, c/o S.I.R.A.C.	Osiatinsky, L.
New Zealand	Christchurch, Canterbury University College, Elec. Dept.	McLennan, Roderick Arthur
	Wellington, G. P. O. Box 638	Thow, Keith H.
Scotland	Dundee, 1 Lochee Road	Lyne, R. H.
	Edinburgh, "Dunelm," 8 Cluny Drive	Beveridge, John A.
Spain	Cartagena, Calle de Ignacio Garcia No. 3	Ramirez Eduardo Garcia

Elected to the Associate grade

Arkansas	Little Rock, 2324 S. Maple St.	Spierling, P. R.
California	Los Angeles, 442 N. Sweetzer	Johnston, Charles D.
	Napa, 2143 Main St.	Rawson, Clifford L.
	Mayfield, 1627 College Ave. (Box 503)	Redeker, Harry E.
Connecticut	Stamford, 757 Main St.	Sullivan, Robert J.
Indiana	Angola, 407 West South St.	Huntley, Kenneth L.
Illinois	Chicago, 3451 Beach Ave.	Blom, Roy
	Chicago, 6533 Woodlawn Ave.	Carothers, R. F.
	Chicago, Bremer Tully Mfg. Co., 656 W. Washington Blvd.	Glassman, George
	Chicago, c/o Triangle Elec. Co., 600 W. Adams St.	Nelson, R. G.
	Chicago, 4253 W. Winchester Ave.	Rech, Charles B.
	Chicago, 4540 Clifton Ave.	Ross, Frank C.
	Chicago, 1464 Larrabee St.	Sieben, Clarence M.
	Decatur, 1352 Riverview Ave.	Boggs, R. A.
	Evanston, 936 Hinman Ave.	Clarke, R. E.
	Fort Sheridan, Hq. Co. 12th Brigade	Hoover, Richard T.
	Lombard, 7 East St. Charles Road	Pukalski, Donald
	Oak Park, 610 North Grove Ave.	Willing, Wm. P.
	Peoria, c/o Nailon Corp., 108–110 Liberty St.	Snyder, Roy A.
	Quincy, 1124–6 Ave. N	Daly, F. Grey
	Rankin, Box 195	Denton, Frank O.
	Winnetka, 934 Westmoor Rd.	Manly, Harold P.
Louisiana	New Orleans, 2807 Jefferson Ave.	Breedlove, Frederick Wallace
Maryland	Baltimore, 1413 John St.	Dudley, Charles B., Jr.
	Mt. Rainier, 352 Harris St.	Flather, Bryan S.
Massachusetts	Cambridge, 70 Lake View Ave.	Spike, J. Edward, Jr.
	East Holliston, Washington St.	Peters, John L.
	East Springfield, 95 Ardmore St.	Russ, George H.
	Lawrence, 223 Hampshire St.	Theberge, Albert R.

Michigan	Ann Arbor, R.F.D. #6	Campbell, George R.
	Detroit, 13156 Ward Ave	Crary, Frederick W.
	Detroit, 5432 Livernois Ave	Goldman, Sherman
	Jackson, 1024 Lansing Ave	Williams, Billy G.
	Rives Junction	Ostrander, R. M.
Missouri	Kansas City, 403 Bellefountain	Screechfield, R. M.
	Kansas City, 3654 Campbell St	Vasen, Gustave
	Nevada, Ellis Music Co.	Kluth, Wm. J., Jr.
	St. Louis, 3330 Abner Place	Bush, Walter S.
	St. Louis, 4417 Clayton Ave.	Koehler, Elmer F.
	St. Louis, 5179 Easton Ave.	Suggs, Thomas B.
	Waldron	Barker, L. T.
New Jersey	Merchantville, 1442 Lexington Ave.	Frenz, H. J.
	Newark, 113 Huntington Terrace	Reiner, Leonard
	Newark, 165 Mt. Pleasant Ave.	Warren, Chester L.
	Ridgewood, 21 Phelps Road	Bondy, Hugo Alois, Jr.
New York	Astoria, L. I., 3411 Astoria Ave.	Berhley, Fred
	Brooklyn, 121 Sands St.	Burney, William Shelley
	Brooklyn, 1904 Albany Ave.	Easton, J. H.
	New York City, 109–123 West 64th St.	Credner, Louis L.
	New York City, 45 Vesey St.	Leopold, Aaron
	New York City, 583 Riverside Drive	Link, Fred
	Schenectady, P. O. Box 51	Petterson, Junior
North Carolina	Washington	Mitchell, Jowdy
Ohio	Cincinnati, 2861 Marshall Ave.	Kolks, Richard H.
	Cleveland, National Carbon Co., Inc., Edgewater Works	Baeckler, Walter
	Norwood, 4558 Smith Road	Bishop, Chester R.
Oklahoma	Stigler	Sigmon, L. C.
	Tulsa, 2808 E. 7th St.	Bowen, Leonard R.
	Tulsa, 114 East 24th St	Morrison, John P., Jr.
Oregon	Salem, Willamette University	Leamer, Frank D.
Pennsylvania	Erie, WRAK	Swanson, Merrill J.
	Lancaster, R.F.D. #5	Rohrer, Richard L.
	Lancaster, 644 East King St.	Snavely, B. L.
	Philadelphia, 5642 N. 7th St.	Hofberg, Alf. H.
	Philadelphia, 5416 Whitby Ave.	Reeves, E. H.
	Pittsburgh, 405 Sheridan Ave.	Woodward, J. D.
	Wilkinsburg, 520 Wallace Ave.	Gregory, S. D.
Rhode Island	Providence, 89 Pleasant St.	Leibow, Saul A.
Texas	Dallas, 912 Commerce St., Room 500	Miller, Gerald E.
Washington	Chelan	Kaul, William
	Seattle, 5212–18th Ave. N. E	Renhard, Julius A.
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Canada	Saint John, N. B., P. O. Box 1225	Thorne, Frank D.
	Toronto, Ont., 153 Beech Ave.	Purser, D. E.
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	Gillingham, Kent, 5 Gillingham Road	Potts, Edward
	Liverpool, Bootle, 64 King's Road	Jones, H. Richardson
	London EC1, Radio Section, Engineer in Chief's Office, G. P. O.	Warren, A. C.
France	Paris, 17, 7 Place Pereire	Berche, P.
Scotland	Aberdeen, 71 Braemar Place	Chadder, E. G.

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Illinois	Springfield, 508 South 6th St	Barregarye, Laurence H.
	St. Elmo, Box 43	Fortner, Obed D.
Indiana	Marion, c/o M. J. Wine, Route #2	Roman, Frank E.
Missouri	Independence, 1316 W. Kensington St	Howe, Lloyd F.
	St. Louis, 1935 Semple Ave.	Schechter, Leo
Nebraska	Lincoln, 1845 D St.	Norris, William C.
Pennsylvania	Allentown, 438 Allen St.	Hilliard, Winfield Edward
China	Shanghai, Nanyang University, c/o K. C. Chang.	Chin, Tsunyi
	Shanghai, Nanyang University, c/o K. C. Chang.	Fung, Huo-Siu
	Shanghai, Nanyang University, c/o K. C. Chang.	Hawng, Ju-tsu

APPLICATIONS FOR MEMBERSHIP

Applications for election to membership have been received from the persons listed below. Members objecting to election of any of these applicants should communicate with the Secretary on or before October 31st. These applicants will be considered by the Board of Direction at its November 6th meeting.

For Election to the Associate grade

Alabama	Birmingham, 901 S. 38th St.	Duran, Albert E.
	Montgomery, 418 Adams Ave.	Persons, S. G.
Arizona	Prescott, O. A. Heala Co., Box 658	deWitt, Harold
Arkansas	Little Rock, Y.M.C.A.	Tracy, Kermit F.
California	Los Angeles, 6220 La Mirada St.	Greger, J. G.
	Los Angeles, 405 E. Pico St.	Kruger, Bernard
	Los Angeles, 1467 Temple St.	Richards, W. S.
	Los Angeles, 5327 Monte Vista St.	Roberts, Russell B.
	Palo Alto, Federal Telegraph Co.	Lacabanne, Washington D.
	Palo Alto, 1040 Channing Ave.	Penther, Carl Joseph
	Palo Alto, 551 Lytton Ave.	Wagener, Winfield G.
	Redwood City, 523 King St.	Christensen, C. W.
	San Diego, 3000 A St.	Kinney, E. S., Jr.
Dist. of Columbia	Washington, c/o Bureau of Navigation, Navy Dept.	Pierrepont, John Jay
	Washington, Dept. Terres. Mag., Carnegie Inst. of Wash.	Seaton, Stuart L.
Illinois	Chicago, 1634 S. Springfield Ave.	Chernikoff, Leo J.
	Chicago, 6415 Stewart Ave.	Chatlos, Albert
	Chicago, 1925 Granville Ave.	Clark, Paul H.
	Chicago, 3710 Leclaire Ave.	Fitzgerald, John J.
	Chicago, c/o Shure Bros. Co., 335 W. Madison St.	Gram, Shirley L.
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	Chicago, Montgomery Ward & Co., Div. 62, Radio & Elect. Lab.	Thompson, Sidney J.
	Chicago, 4500 N. Sacramento Ave.	Waterman, S. S.
	Glenview, c/o WBBM	Fancy, Raymond C.
	Hinsdale, 206 Ayres Ave.	Doyle, E. J.
	Gary, 540 Lake St.	Gunther, W. J.
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Iowa	Cedar Rapids, 1718 C St. W.	Hajny, George F.
	Red Oak, 104 W. Reed St.	Anderson, Donald C.
Louisiana	New Orleans, 4515 Freret St.	Elliott, Harry M.
	New Orleans, 3421 Prytania St.	Dumestre, Alexis M.
	New Orleans, 1114 Pere Marquette Bldg.	Hancock, Olan W.
	New Orleans, c/o Western Union Tel. Co.	Hilgedick, W. C.
	New Orleans, 717 Greenwood St.	Scheffer, Roy J.
	New Orleans, 605 Belleville St.	Voegtlin, Elmo
Massachusetts	Chelsea, 124 Williams St.	Fox, William
	Marion, Radio Corporation of America	Brunette, Deo Z.
	Roxbury, 120 George St.	Ashenden, G. K., Jr.
	Springfield, 547 Page Blvd.	Cole, Neil D.
	Springfield, 32 Ardmore St.	Knapp, Harold D.
Michigan	Bay City, Radio WBCM	Carpenter, Ralph Harvey
	Detroit, 4322 Grand Ave. W.	Davis, Chester
	East Lansing, 348 Oakhill Ave.	Clark, Ralph L.
	Tecumseh	McConnell, Harley H.
Minnesota	Detroit Lakes	Hetland, L. C.
	Minneapolis, 3848 Harriet Ave. S.	Brooke, Robert O.
	Minneapolis, 915 Queen Ave. N.	Tynan, Thomas E.
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New York	Auburn, 10½ Pleasant St.	Lumb, Frank J.
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	Long Island City, 3986 47th St.	Gilcher, V. J.
	New Brighton, 62 Westervelt Ave.	Macken, H. I.
	New York City, 961 Tiffany St.	Cilin, Louis
	New York City, Bell Telephone Labs., Inc., 463 West St.	Cole, Burton R.

New York (*cont'd*)	New York City, 195 Broadway, Room 2014.........	Herrmann, Henry J.
	New York City, 43 Exchange Place................	Palmer, Robert T.
	New York City, 655 W. 160th St................	Stokvis, Morris, Jr.
	New York City, 1020 Walton Ave..............	Weiland, Christian Frederick
	New York City, 281 E. 7th St................	Weiss, Samuel
	New York City, 615 W. 113th St. (Apt. 42)........	Wilson, H. Warden
	New York City, c/o Technidyne Corp., 644 Broadway..................................	Yolles, Jacob
	Schenectady, Research Lab., General Electric Co.....	Nergaard, Leon S.
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Ohio	Cincinnati, 3881 Reading Road.................	Klein, Helen
	Columbus, 625 W. 5th St................	Blum, Louis M.
	Lakewood, 17545 Madison Ave.................	KaDell, Harold W.
Oklahoma	Tulsa, 1309 E. 15th St...........................	Banks, J. Vernon
	Tulsa, 2628 E. 6th St................	Carpenter, Hugh
	Tulsa, Radio Station KVOO................	Richardson, Harry K.
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	Philadelphia, 5531 Master St................	Stark, Harry W.
	Pittsburgh, 6503 Landview St................	Stayer, David
	Upper Darby, 202 Heather Road................	Cahill, James A.
	Wilkinsburg, 414 Ella St................	Ballard, Randall C.
	Wilkinsburg, 815 Rebecca Ave................	Lehman, James N.
	Wilkinsburg, R. D. #1, Box 223................	Sinnett, Chester M.
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Utah	Salt Lake City, c/o Radio KDYL................	Barbre, Tom
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	Skipton, Dyneley House................	Carr, John
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	Marton, Ngahima St................	Ruscoe, Chas. R.
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Philippine Islands	Cebu, 37 Tres de Abril St................	Lauza, Nemesio D.
South Africa	Cape Province, Klipheuvel, Beam Wireless Station..	Osborn, Eugene Wilson
	Johannesburg, 62 Persimmon Street Malvern.......	Lukat, John Frederick
South Wales	Cardiff, Llanishen, "Ciscar," Hilary Gardens, Cyn Coed Road................	Jinman, Arthur Melville
Sweden	Stockholm, Kungsholmsgatan 21................	Elmquist, Torsten

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Indiana	Valparaiso, 712 Calumet Ave................	DeHart, Delmar W.
Louisiana	New Orleans, 2434 Valence St................	Dahlstrom, Hugo Wolf
New Jersey	Camden, Audio Vision Appliance Co., Radio Eng. Dept................	Lathrope, Kenneth W.
	Weehawken, 62 Liberty St................	Pollack, Dale
Oklahoma	Alva, Radio KGFF, 709 Noble St................	Sears, Garold D.
Canada	Montreal, P. Q., 637 Craig St., Room 808........	Harvey, Fred E.

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HIDETSUGU YAGI

Part II

TECHNICAL PAPERS

STATIC AND MOTIONAL IMPEDANCE OF A MAGNETO-STRICTION RESONATOR*

By

E. H. LANGE AND J. A. MYERS

(School of Engineering, The Johns Hopkins University, Baltimore, Md.)

Summary—*The equivalent series inductance and resistance of a long solenoid with nickel-steel bar has been investigated in relation to the excitation frequency for frequencies up to 14000 cycles per second. Beyond 400 cycles per second, after which only a negligible flux penetrates to the center of the bar, the inductance varies inversely with the square root of the frequency, and the resistance directly with the square root of the frequency. The product of the resistance and inductance is substantially constant over the frequency range; also, the power factor angle of the bar is substantially constant. The field for the higher frequencies is shown to be confined largely to the circumference of the bar, at which zone the greater part of the magnetostrictive force is produced. The results are discussed in relation to the theory of flux distribution in the bar.*

The effect of motion of the bar under the action of magnetostriction has been measured in terms of the motional impedance, and a circle diagram obtained. The ratio of motional resistance to reactance, in the vicinity of mechanical resonance of the bar, is of the order of magnitude of one to six thousand. The theory of total impedance, static and motional, is given, and the nature of the angular displacement of the resonant circle indicated.

Self-excitation by means of one 201A tube was found to be possible for the particular solenoid used in the measurements, by introducing in series with each half of the solenoid an air-core inductance to improve the low reactance-resistance ratio of the static impedance.

INTRODUCTION

THE electrical impedance of a solenoid containing a bar of magnetic substance may, for convenience, be considered as divided into two types; the volts per ampere in the solenoid due to the effective resistance and reactance, and the volts per ampere in the solenoid due to longitudinal vibration of the bar under the action of a periodic mechanical force, in this instance due to magnetostriction. The former impedance is called, in this paper, the static impedance, and the latter the motional impedance.

The two types can be considered separately, for, when motional impedance is present at all, the whole resonance phenomenon occurs for so small a frequency variation, due to the extreme sharpness of resonance, that the static impedance is substantially constant over this frequency range.

The variation of the static impedance with frequency, or what is the same, the effective resistance and reactance with frequency, is primarily of interest in connection with the experimental examination of the flux distribution within the bar, and subsequent comparison with the theoretical relations for flux distribution. These relations are also important in determining the active annular zone within the bar over which the major part of the periodic mechanical force is operative, and the allied relation for the rotation of the motional impedance circle, due to the fact that the coil current and the resultant bar flux are not in phase. This latter relation requires consideration of the varying magnitude and phase of the magnetic induction within the bar for various distances from center to circumference of the bar.

Apart from the utility of the static impedance characteristic with varying frequency for the primary purpose stated, the relatively simple experimental procedure necessary for determining the effective depth of penetration of flux into a bar may be of interest as a possible method of examination of a magnetic substance, in bar form, with particular reference to changes in structure at or near the surface of the bar.

Mechanical oscillations in bars due to periodic magnetostriction induced by periodic magnetic intensities have been extensively investigated by Prof. G. W. Pierce,[1] who also examined the frequency stabilizing action of such bars when excited by means of a vacuum-tube oscillator. Further work on the motional impedance properties of various magnetic materials has been carried out by K. C. Black.[2] The change in length of bars of iron, nickel, and cobalt has been studied under static conditions, with varying amounts of magnetic intensity and tensile stress by several observers.[3] Recently, attention has been given to certain nickel-iron alloys (permalloys),[4] in which the negative magnetostriction of nickel is, to a certain extent, balanced against the positive magnetostriction which the iron possesses up to its saturation point. An explanation of the relative ease with which magnetism spreads through such alloys, for which the gross magnetostriction is practically zero, has been given by L. W. McKeehan[5] in terms of atomic magnetostriction.

[1] G. W. Pierce, "Magnetostriction Oscillators," *Proc.* American Academy of Arts and Sciences, April, 1928.

[2] K. C. Black, "A Dynamic Study of Magnetostriction," *Proc.* American Academy of Arts and Sciences, April, 1928.

[3] See bibliography.

[4] O. E. Buckley and L. W. McKeehan, *Phys. Rev.*, Second Series, **26**, No. 2; August, 1925.

[5] L. W. McKeehan, *Phys. Rev.*, Second Series, **26**, No. 2, August, 1925.

DESCRIPTION OF APPARATUS

The apparatus used for both the static and motional impedance measurements is shown in Fig. 1. A solenoid extending somewhat beyond the ends of the bar was used for excitation purposes. There are two coils, each having 812 turns consisting of two layers of No. 22 enamelled copper wire, connected in series, the whole being practically continuous except for a small separation necessary for the nodal-point support of the bar. This type of support is of course essential for the investigation of motional impedance, and consists of two V-shaped jaws of moderately sharp edges which can be screwed together to clamp the bar and support it in the axis of the solenoid. In order to establish a steady magnetic flux within the bar a U-shaped permanent magnet was attached at each end of a steel bar, mounted
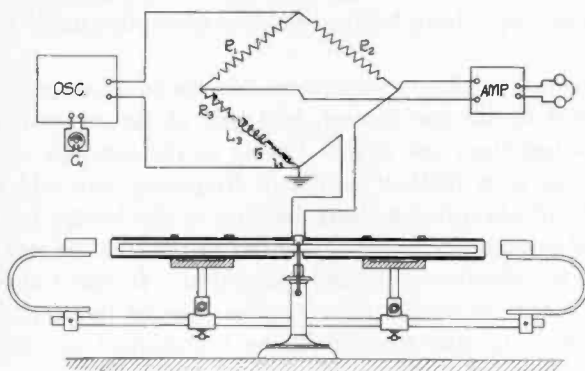
Fig. 1—Arrangement for measuring depth of flux penetration, static and motional impedance.

as shown in Fig. 1, thus forming a closed magnetic circuit through the specimen bar. This arrangement was used because of its simplicity and also to obviate any coupled circuit which would be formed by a separate d-c excitation circuit and consequent reaction in the solenoid circuit under a-c conditions. As relatively small polarizations are needed, this arrangement proved entirely satisfactory for present purposes.

The bridge arrangement used for static and motional impedance measurements is also shown in Fig. 1. A type 8A Western Electric oscillator served as a driving source for the circuit. In measurements of the static impedance, the oscillator calibration was used over the frequency range, and for the frequency control in measurements of the motional impedance a vernier condenser, giving 2.5 cycles per sec. range, was used. The arms R_1, R_2, and R_3 are decade resistance boxes

having non-inductive coils; L_3 is an 85-mh Brooks inductometer; l_3 is an inductometer for final balancing whose range is 0.6 mh; and r_3 is a 1-ohm slide wire. An amplifier circuit was connected to the arms as shown.

EXPERIMENTAL PROCEDURE

The oscillator was adjusted and allowed to warm up for at least four hours before making observations. This was found necessary in order to reduce frequency drift which, however, cannot be entirely eliminated. The slow frequency drift was of no consequence in the static impedance measurements. Observations were made to determine the resistance and reactance of the solenoid without the bar of magnetic material, for a frequency range of 100 to 14,000 cycles per second, also for the same range with the bar clamped in position in the axis of the coil. Sharp bridge balances were obtainable throughout this range.

The bar used in these experiments has its fundamental frequency at about 3834 cycles per second, and half of the resonance circle is traversed in less than one cycle. Owing to the extreme sharpness of resonance, any slow drift of oscillator frequency can add greatly to the difficulty of obtaining a sharp balance of the bridge for points on the motional impedance circle. After the oscillator drift was definitely identified this difficulty was easily obviated. It was found that by means of the vernier condenser any slow drift of frequency could be readily followed up, and sharp balances maintained anywhere on the motional impedance circle.

INDUCTANCE AND RESISTANCE CURVES

The measured inductance and resistance values give directly the series inductance and resistance of the solenoid. A typical curve of total inductance of solenoid with bar, $L_i + L_0$, for the frequency range 625 to 14,000 cycles per sec., is shown in Fig. 2. The curve L_0 represents the inductance due to flux linkages with the solenoid through the space outside the bar. This is obtained from the measured air-core inductance by computation on the basis of ratio of area of space surrounding the bar to total coil area. Due to the fact that it is not feasible for the motional impedance measurements to have a closely fitting coil, this procedure is necessary. Since the ratio of solenoid length to diameter is about 22 to 1, the computation of L_0 as indicated is sufficiently accurate. The difference between the total measured inductance and L_0 is the inductance due to flux linkages within the bar.

The curve of total inductance shows how rapidly the flux linkages

per ampere in the coil are decreasing as the frequency is increased. From theoretical considerations, given in a following paragraph, (eq. 13 and 14), it appears that the effective flux-area of the bar, for
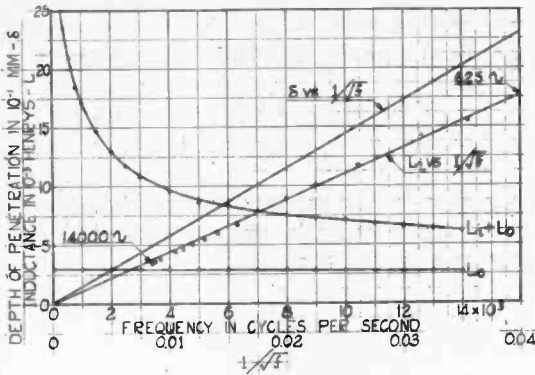

Fig. 2

frequencies sufficiently high to manifest skin-effect, is given by the relation $\pi a \delta$; in which a is the radius of the bar and δ the depth of penetration of the flux.

Specifically,

$$\delta = 5033 \sqrt{\frac{\rho}{\mu f}} \, \text{cm} \tag{1}$$

and the resultant flux vector is

$$\phi_i = \pi a \delta \cdot B_a (1-j)$$

in which B_a is the flux density at the surface of the bar. In accordance with equation (1) the inductance due to the bar was plotted against $1/\sqrt{f}$.

The linear relation between L_i and $1/\sqrt{f}$ holds very closely for the frequency range shown. Examination of this relation below 300 cycles has shown that as the frequency is decreased the straight line approaches a constant value of inductance, indicating an increasing uniformity of flux across the section of the bar. Further, the linear relation between L_i and $1/\sqrt{f}$ in the range shown indicates that μ is substantially constant in this range and for the excitation currents used (about 30 ma), so that from the measured value of ρ(90 x 10^{-6} ohms per cm cube) and experimental data on inductance variation with frequency, the depth of flux penetration δ can be computed. Thus

$$L_i = \frac{N\phi_i'}{I} \times 10^{-8} = 4\pi N^2 \left(\frac{\text{effective area}}{\text{length}}\right) \times \mu \times 10^{-9} \cdots \text{ henries.}$$

Substituting the value of the effective area in terms of the depth of penetration, and noting that ϕ_i' in the above equation is the component of the bar flux in phase with the coil current,

$$L_i = \left(\frac{5033 \times 4\pi^2 \times 10^{-9} N^2 \cdot a \cdot \sqrt{\rho\mu}}{\text{length}}\right) \cdot \frac{1}{\sqrt{f}} \tag{2}$$

The value of μ computed from the experimental curve shown, using equation (2) is $\mu = 64.9$. Thus, for the bar under test the depth of flux penetration as related to the frequency is

$$\delta = \frac{5.91}{\sqrt{f}} \text{ cm} \tag{3}$$

Length of bar $= 58.45$ cm
Radius of bar $= 0.635$ cm $= a$
Mean diameter of coil $= 2.70$ cm

Analysis of the resistance of the solenoid is somewhat more involved. The curve of resistance, R_c with frequency, for the coil without the bar is shown in Fig. 3. The total resistance of the solenoid
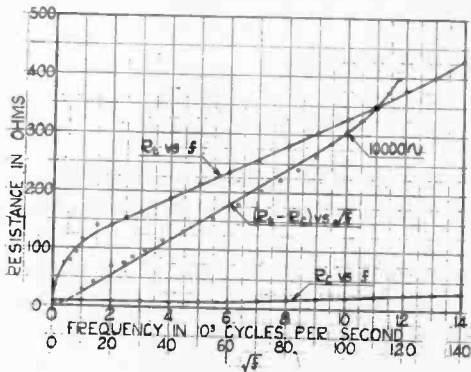
Fig. 3

and bar is indicated by R_t, and the difference $R_t - R_c$ is the series resistance due to the bar alone. The experimental values of $(R_t - R_c)$ plotted against $\sqrt{f}$ show a slight curvature up to several hundred cycles per second, until the flux has ceased to penetrate to the center of the bar, after which the resistance is substantially linear with $\sqrt{f}$

over the whole range. The deviation from linearity in the vicinity of 10,000 cycles is believed to be due to the fact that at the higher frequencies the true resistance due to the bar cannot be accurately obtained by deducting the coil resistance, for the field distribution about the coil with the bar removed is somewhat changed.

The equivalent series resistance due to the losses in the bar, i.e. $(R_t - R_c)$ is made up of eddy-current and hysteresis effects. The short circuit or circulating currents around the bar are confined largely to the outer circumference for frequencies above 1000 cycles, and equation (1) measures both the effective depth of penetration of the flux and short-circuit current. The power loss due to the bar in terms of the coil current I is

$$P = I^2(R_t - R_c) = I^2(R_e + R_h) \tag{4}$$

in which R_e is the series resistance due to eddy currents and R_h due to hysteresis. In the case of R_e, since the short-circuit currents are proportional to the current I, and the effective section of the short-circuit path directly proportional to δ, the value of R_e will vary directly with $\sqrt{f}$. In the case of R_h, the effective volume is proportional to δ, i.e., $1/\sqrt{f}$, so that as far as effective volume and number of hysteretic cycles are concerned, R_h varies directly as $\sqrt{f}$; the lack of complete linearity between I^2 and the energy loss per loop in its relation to some exponent of the flux density is, to a large extent, compensated in the total loss by the fact that the coil current I does not vary over a wide range, also by the considerable excess of eddy-current loss over hysteresis loss. The relations for the variation of R_e and R_h with frequency can be more clearly seen from the following equations. The power loss due to the eddy currents is

$$I^2 R_e = I_b^2 R_b = \frac{2\pi a \rho}{l \delta} \cdot I_b^2 \cdot$$

Thus,

$$R_e = \left(\frac{2\pi a \rho}{l} \frac{I_b^2}{I^2} \right) \cdot \frac{1}{\delta}$$

where I_b is the aggregate circulating current in the bar, l the bar length, a the bar radius, and R_b the resistance for the circulating current. Taking into account the actual distribution of current across the section of the bar, the aggregate current I_b can be shown to be equal to the effective current density at the surface of the bar $u_a/\sqrt{2}$ multiplied by $\delta \times l$, the effective cross section; also the current I_b to be directly proportional to the surface value of flux density B_a, i.e., to the coil

current, and finally, the resistance R_b to be that of a hollow tube of thickness δ, radius a and length l. The power loss due to hysteresis can be expressed in the form $I^2R_h = \text{constant} \times \text{vol.} \times f \times B^n$, and as the effective volume of material in which the hysteresis loss is taking place is proportional to δ, this becomes

$$R_h = \text{constant} \times \sqrt{f} \times \frac{B^n}{I^2}$$

THEORY OF INDUCTANCE VARIATION

The distribution of a harmonically changing flux in a solid conductor has been treated by numerous writers.[6] In the particular problem at hand what is needed is not only the flux distribution, but also the resultant flux in phase and magnitude, in order that the distribution may be determined in terms of its manifestations in the solenoid sur-
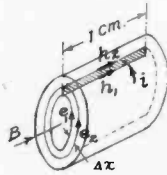
Fig. 4

rounding the bar. The bar can be considered to be divided into annular zones, and the vector sum of the fluxes obtained, also the phase of the resultant flux with reference to the coil current. For present purposes it is desirable to limit the discussion to the distribution of flux for frequencies sufficiently high to reduce the field in the central portion of the bar to negligible values. Only such frequencies are of interest here, and the equations under these conditions are very much simplified.

Referring to Fig. 4, the voltage difference between two successive cylinders due to changing magnetic induction is

$$e_2 - e_1 = -2\pi x \Delta x \frac{\delta B}{\delta t} \times 10^{-8}.$$

Thus we may write

$$\frac{\delta F}{\delta x} = -\frac{\delta B}{\delta t} \times 10^{-8} \tag{5}$$

[6] Alex. Russell, "Alternating Currents," Vol. I; J. H. Jeans, "Electricity and Magnetism."

in which F is the electric intensity along the circumference.[7] For any area of elementary width parallel to the induction, the m.m.f. of the circulating current is

$$h_2 - h_1 = -0.4\pi u \Delta x, \text{ or } \frac{\delta h}{\delta x} = \frac{1}{\mu} \frac{\delta B}{\delta x} = -0.4\pi u \qquad (6)$$

Also, the current around any ring in terms of the resistivity ρ (ohms per cm cube) and the current density u (amperes per cm²) is

$$e/2\pi x \rho / \Delta x = u \Delta x \quad \text{or} \quad F = \rho u \qquad (7)$$

Combining (5, 6, 7)

$$\frac{\delta^2 B}{\delta x^2} = \frac{4\pi\mu \times 10^{-9}}{\rho} \cdot \frac{\delta B}{\delta t} = k \frac{\delta B}{\delta t} \qquad (8)$$

and

$$\frac{\delta^2 u}{\delta x^2} = k \frac{\delta u}{\delta t}.$$

Solutions of (8) are of the form $B = B_m \epsilon^{j\omega t} \cdot \epsilon^{(\alpha + j\beta)x}$ where

$$\alpha + j\beta = \pm \sqrt{2}j \sqrt{\frac{k\omega}{2}} = \pm(1+j)\sqrt{\frac{k\omega}{2}}.$$

Put

$$\gamma = \sqrt{\frac{k\omega}{2}}; \ \omega = 2\pi f.$$

Then,

$$B = \left[B_1 \epsilon^{(1+j)\gamma x} + B_2 \epsilon^{-(1+j)\gamma x} \right] \epsilon^{j\omega t} \qquad (9)$$

Noting that the flux density must have circular symmetry about the axis of the bar, $B_1 = B_2 = B_m$, and that for the condition assumed of negligible flux density in the central portion of the bar in comparison with the flux density at the surface, the second term of (9) may be neglected, the equation becomes

$$B = B_m \epsilon^{j\omega t} \cdot \epsilon^{(1+j)\gamma x} \qquad (10)$$

Substituting $x = a$ to obtain the flux density at the circumference,

$$B_a = B_m \epsilon^{j\omega t} \cdot \epsilon^{(1+j)\gamma a} \qquad (11)$$

[7] Strictly, $\partial(Fx)/\partial x = -x(\partial B/\partial t) \times 10^{-8}$; however, under the conditions above assumed, (5) is sufficiently accurate.

The vector flux density in terms of its value at the circumference is therefore

$$B = B_a \epsilon^{(1+j)\gamma(x-a)}.$$

If, now, distances are measured from the circumference instead of from the center, $z = a - x$, and the vector flux density is

$$B = B_a \epsilon^{-(1+j)\gamma z}. \tag{12}$$

The instantaneous value is therefore $B = B_a \epsilon^{-\gamma z} \cos(\omega t - \gamma z)$. From (12) γ is a reciprocal length. Put $\delta = 1/\gamma$, then

$$\delta = \frac{10^4}{2\pi} \sqrt{\frac{10\rho}{\mu f}} = 5033 \sqrt{\frac{\rho}{\mu f}} \text{ cm}. \tag{13}$$

The phase of the flux varies continuously from one annular zone to the next, so that the resultant bar flux is the vector sum of the fluxes of the various annular zones. The flux of any zone of radius x is

$$d\phi = 2\pi x dx \cdot B = -2\pi(a-z)Bdz$$

and the total flux between the circumference and any inner zone is

$$\phi = 2\pi B_a \int_o^z (z-a)\epsilon^{-\gamma(1+j)z} \cdot dz.$$

Evaluation for the whole bar gives

$$\phi = 2\pi a \frac{\delta}{2} B_a (1-j). \tag{14}$$

Equation (14) shows that the magnitude of the flux components is obtained by considering the surface value of density B_a as spread uniformly over an effective area equal to the circumference $2\pi a$ times one half the depth of penetration δ. The phase of the resultant flux is 45 deg., lagging the surface density B_a which is in phase with the coil current.

The above result shows that, provided the frequencies are such as to confine the flux largely to the outer rim, the ratio of the reactance to eddy-current resistance is constant for all frequencies, and equal to one. The constancy of the ratio is also shown by the experimental curves, for $L\sqrt{f}$ is constant, also $R/\sqrt{f}$ is constant, so that by division fL/R is constant; i.e., the eddy-current angle is independent of frequency. The experimental curve for resistance includes the hysteresis resistance, so that in this case the reactance-resistance ratio is constant and less than one, that is, the resultant bar flux lags the current in the coil by an angle greater than 45 deg. It is of interest to note that be-

cause of the skin effect, which gives rise to the particular relations above stated between L, R, and $\sqrt{f}$, the product of the inductance L and resistance R is constant and independent of frequency. Attention appears to have been first directed to the constancy of this product for the magnetic circuit of telephone receivers, by means of an empirical relation.[8]

Taking the coil current vector for reference, and denoting by θ_i the total angle of flux lag (eddy-current and hysteresis), the flux vector is $\phi_i[\cos\theta_i - j\sin\theta_i] = \phi_i\cos\theta_i(1-jp)$ in which ϕ_i is the total flux, and $p = \tan\theta_i > 1$. The coil e.m.f. necessary to maintain the flux ϕ_i is therefore

$$j\omega N\phi_i\cos\theta_i(1-jp).$$

Substituting for $\phi_i\cos\theta_i$ its value in terms of the inductance and coil current, the e.m.f. becomes

$$j\omega LI(1-jp) = \omega LI(p+j).$$

Thus the impedance due to the bar is

$$Z_s = \omega L(p+j). \tag{15}$$

The copper coil resistance R_c and leakage inductance L_0 give rise to the impedance $Z_c = R_c + j\omega L_0$. The total impedance when the bar is not in motion is therefore the vector sum $Z = Z_c + Z_s$.

MOTIONAL IMPEDANCE CIRCLE

A typical motional impedance circle for the nickel-steel bar used in these experiments is shown in Fig. 5. The line Z_s is the static impedance line for the frequencies shown, and includes the copper coil resistance R_c and the leakage reactance due to L_0. The maximum room temperature variation during the measurements was less than one-half deg. C. Between the frequency 3750 and 3900, the resistance and reactance values follow around the circle shown, with frequency increasing in the clockwise direction. The origin of the motional reactance and resistance axes is indicated by the intersection of the static impedance line and the motional impedance circle. In accordance with (23) the ratio of motional resistance to motional reactance is 1 to 6500. The small energy loss of the bar also manifests itself in the persistence of oscillations which beat with the driving source, when the impressed frequency is changed so as to pass through the bar frequency. For the

[8] A. E. Kennelly and G. W. Pierce, "The Impedance of Telephone Receivers as Affected by the Motion of Their Diaphragms," *Electrical World*, September 14, 1912.

frequencies of the motional circle, the bar emits sound of considerable intensity, which becomes very penetrating and disagreeable when listened to for a considerable length of time. A part of the mechanical

Fig. 5—Static and motional impedance curves.

resistance is sound radiation resistance, so that the ratio of actual mechanical resistance due to internal losses of the bar, to the reactance, is even less than the above value. The reactance and resistance varia-

Fig. 6

tions with frequency, in the vicinity of resonance, are shown in Fig. 6 and Fig. 7.

MOTIONAL IMPEDANCE THEORY

The characteristic magnetostriction curves for iron under a constant tension indicate that as the magnetic intensity is increased the

deformation is positive up to a certain value of H, after which the deformation decreases and becomes negative. A family of such curves results for different constant values of tension. In the present arrangement, since a steady polarizing field is used with a small superposed alternating field, the increment of the main field is of principal interest in relation to any change of tension or deformation. Calling the tension p, and the deformation or strain q, the increment of the flux density may be stated in the form

$$dB = K_p p + K_q q$$

where $\delta B/\delta p = K_p$, and $\delta B/\delta q = K_q$. The sign of the coefficients is determined from the characteristic curves for the particular magnetic material and polarizing field. Thus,

$$dB = dB_i + dB_q \tag{16}$$

in which dB_i may be regarded as the incremental change due to the external alternating field which produces a tension dB_i/K_p, and
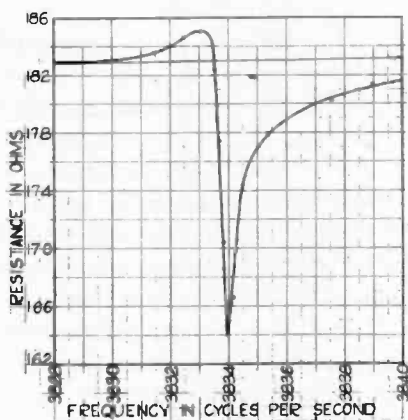
Fig. 7

$dB_q = K_q q$ the change due to increase or decrease in the length of the bar. The time rate of change of dB_i and dB_q gives rise to the ordinary voltage and the motional voltage, i.e., for any harmonic coil current, to the ordinary impedance Z_s and the motional impedance Z_m.

Under ordinary conditions, the deformation is so small as to cause a negligible change in the flux, but when the excitation frequency is such as to cause mechanical resonance of the bar the deformation is greatly increased, and the motional impedance becomes a part of the total impedance. Computation of the motional impedance caused by

the solid bar is complicated by the fact that the flux distribution is not uniform across the section of the bar and the deformation is not uniform along the bar; however, the motional impedance in terms of basic factors can be closely approximated by taking account of the simplest aspect of these effects.

The effect of flux distribution will be considered first, with reference to the ordinary flux and the deformation flux. The force at any one of the annular zones previously considered is $\Delta P = 2\pi x \Delta x \cdot B_i / K_p$, and in accordance with the development of equations (14) and (15) the total force or vector sum of all the forces at any section is

$$P = \frac{\phi_i}{K_p} \cdot \epsilon^{-j\theta_i} = \frac{\phi_i}{K_p} \cos \theta_i (1 - jp).$$

The resultant force vector therefore lags the current in the coil by the angle θ_i. Substituting $\phi_i \cos \theta_i = LI/N$.

$$P = \frac{LI}{K_p N \cos \theta_i} \epsilon^{-j\theta_i}. \tag{17}$$

The sinusoidal force P gives rise to a sinusoidal deformation q, and to a resultant deformation flux ϕ_q. The relation $B_q = K_q \cdot q$ for the deformation flux density in terms of the deformation is true for static deformations or for a very fine wire in which the deformation is rapidly alternating; for a solid bar mechanically excited at high frequencies the flux distribution is governed by (8). The deformation flux as influenced by the eddy currents is assumed in accordance with (14) to be $\phi_q = \pi a \delta \cdot B_{qa} \cdot (1-j)$, and since the deformation flux is also subject to the effects of hysteresis, this may be written $\phi_q = \pi a \delta \cdot B_{qa}(1-pj)$, where B_{qa} is the deformation flux density at the surface of the bar and $p' = \tan \theta_q > 1$, θ_q being the angle by which the resultant deformation flux lags the surface flux density B_{qa}. The surface zone of infinitesimal thickness is free from the counter m.m.f. of the interior circulating currents, and it may be assumed that for this zone $B_{qa} = K_q \cdot q$, thus

$$\phi_q = \pi a \delta K_q q (1 - jp') = \frac{K_q A \epsilon^{-j\theta_q}}{\cos \theta_q} \cdot q \tag{18}$$

where A is the effective flux area.

The effect of variation of the deformation q along the bar will now be considered. When the bar is excited at its fundamental resonant frequency, a stationary wave of deformation exists along the bar, the deformation being a maximum in the center and zero at the ends.

The extreme sharpness of resonance indicates that the energy loss per cycle per unit length of bar is very small compared with the stored energy per unit length, so that the system closely approximates a purely reactive resonator, and the deformation may be considered as sinusoidal in distribution along the bar, all points being in the same time phase. In accordance with (18) the deformation flux is therefore distributed sinusoidally along the bar, and on the assumption that all of the deformation flux links with the turns of the solenoid, the total flux linkages are $(2/\pi)N\phi_q$. Denoting by N_q the effective turns of the solenoid for the deformation flux linkages, the total linkages will in general be $N_q\phi_q$. The e.m.f. which the line must supply to overcome the motional e.m.f. of the bar in the coil is therefore

$$E_m = N_q\dot{\phi}_q = \frac{N_q K_q A \epsilon^{-i\theta_q}}{\cos\theta_q} \cdot \dot{q}_0, \tag{19}$$

in which $\dot{q}_0$ refers to the time rate of change of the deformation half-way between the ends of the bar.

The magnitude and phase of $\dot{q}_0$ in relation to the total exciting force P are determined by the mechanical resonance properties of the bar. The mechanical properties of the resonator for the frequencies at or near resonance, can in the present case of a highly reactive system be most simply expressed in terms of an equivalent lumped resonator. Denoting by m the mass of the resonator, r the resistance, g the elastic constant, l the length of the resonator, A_b the cross-section, q the deformation, and y the displacement of the end of the resonator, then for a lumped resonator

$$\frac{P}{l} = m\ddot{q} + r\dot{q} + gq.$$

When the total exciting force P is a harmonic function of the time, the rate of change of deformation is

$$\dot{q} = \frac{P}{[r + j(\omega m - g/\omega)]l} \tag{20}$$

in which $r + j(\omega m - g/\omega)$ is the mechanical impedance of the resonator. The phase angle of $\dot{q}$ with reference to the force P is

$$\psi = \tan^{-1}\frac{\omega m - g/\omega}{r}. \tag{21}$$

and mechanical resonance occurs for the frequency given by

$$\omega_0 = \sqrt{\frac{g}{m}}. \tag{22}$$

In the actual resonator, the quantities, $\ddot{q}$, $\dot{q}$, and q, are sinusoidally distributed along the bar, and for any one of these quantities the values along the bar are in time phase. The actual resonator thus corresponds to numerous lumped resonators running in synchronism but having different amplitudes. The total energy content of the actual resonator, i.e., the kinetic and potential energy, can be readily determined in terms of the maximum velocity V and displacement Y at the ends of the bar, and the mass m and elastic constant g determined in relation to the actual resonator. Since the velocity distribution is sinusoidal, the mean square velocity is $V^2/2$, thus if M is the mass of the actual resonator the equality of kinetic energies gives $(m/2)V^2 = M/2 \cdot V^2/2$, or $m = M/2$. The equivalent lumped mass is therefore one-half the actual mass. Equality of the potential energies of the lumped and actual resonators gives (average $q^2e/2)A_bl = gY^2/2$, in which e is Young's modulus and $q^2e/2$ is the potential energy per unit volume. Noting that the maximum displacement of the ends of actual resonator is $Y = q_{av} \cdot l/2 = (2/\pi)q_0 \cdot l/2$, and that the mean square deformation is $q_0^2/2$, the equivalent elastic constant reduces to $g = \pi^2/2 \cdot A_b e/l$. Thus the equivalent elastic constant is $\pi^2/2$ times the value for a uniformly stressed bar.

The mechanical resistance can be compared with the reactance by determining the frequency deviation from resonance necessary to make $\psi = 45$ deg. Denoting by ω_1 the deviated angular velocity, and noting that due to extreme sharpness of resonance $\omega_0 + \omega_1$ may be replaced by $2\omega_0$,

$$\frac{\omega_1 m - g/\omega_1}{r} = 1 = \frac{(\omega_1^2 - \omega_0^2)m}{\omega_1 r}$$

or

$$\frac{r}{\omega_0 m} = \frac{2(f_1 - f_0)}{f_0} = \frac{r}{\sqrt{gm}}. \tag{23}$$

The motional e.m.f. given by (19) in terms of the maximum rate of change of deformation can now be expressed in terms of the force by means of (17) and (20), and the motional impedance obtained. Thus

$$E_m = \frac{\pi N_q K_q A P \epsilon^{-i\theta_q}}{\cos \theta_q \cdot l[r + j(\omega m - g/\omega)]} = \frac{\pi N_q K_q A L I \epsilon^{-i(\theta_i + \theta_q)}}{K_p N l \cos \theta_i \cos \theta_q [r + j(\omega m - g/\omega)]}$$

or

$$Z_m = \frac{E_m}{I} = \frac{K \epsilon^{-i(\theta_i + \theta_q)}}{r + j(\omega m - g/\omega)}. \tag{24}$$

The coefficient K is used to designate the product of the coefficients in the previous equation. The reciprocal of the mechanical impedance gives a circle locus with maximum electrical motional impedance along the resistance axis; however, due to eddy currents and hysteresis this maximum has an angular displacement which causes the motional
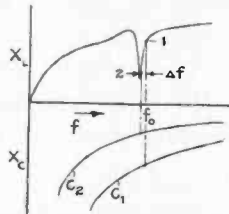
Fig. 8—Frequency stabilizing action of motional inductance.

e.m.f. to lag behind the coil current. The area A in (24) is the effective flux area as defined in (14) and is small compared with the area of the bar. In the case of a tube in which the thickness of the walls is less than δ, the area A approaches in value the actual cross-section of the tube, and the angles θ_i and θ_q are decreased, thereby decreasing the angle of lag between the coil current and the motional e.m.f.

VECTOR DIAGRAM OF RESONATOR

Referring to Fig. 9, the results of the foregoing equations are shown in the form of a vector diagram. The ordinary flux ϕ_i lags the coil current I by the angle θ_i which is substantially independent of fre-
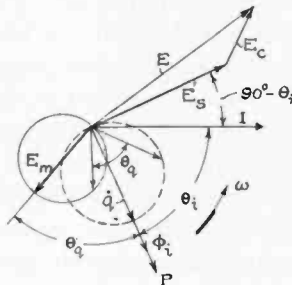
Fig. 9

quency. The part of the impressed e.m.f. necessary to maintain this flux is shown by E_s, 90 deg. in advance of the flux, and E_c is the part of the impressed e.m.f. necessary to overcome the leakage reactance and resistance of the solenoid. When the motion of the bar is negligible, the vector sum of E_s and E_c or E gives the total impressed e.m.f. necessary to maintain the current I. Since the resonance of the bar is

very sharp, the frequency may be considered variable over a sufficient range to include resonance without appreciably changing the phase or magnitude of the vectors E_s or E_c. The force P is in phase with the flux ϕ_i and at exact resonance the rate of change of deformation $\dot{q}$ is in phase with P. The dotted circle shows the locus of $\dot{q}$ as the frequency is changed. The motional e.m.f. E_m lags the rate of change of deformation by the angle θ_q; the resonance circle for the motional e.m.f. is shown by the solid line. The total impressed e.m.f. is the vector sum of E_s, E_c, and E_m, or the total impedance the vector sum of Z_s, Z_c, and Z_m, the impedances being given by (15) and (24).

Use as a Frequency Stabilizer

The action of the bar in stabilizing frequency, that is, in rendering the generated frequency to a large extent independent of the capacity and static inductance of a vacuum-tube oscillator circuit, may be seen by reference to Fig. 8, which shows the reactance diagrams for variable frequency of a circuit having static and motional inductance, and capacitance. The upper curve shows the static and motional inductive reactance, and the lower curves are the capacitive reactance for two different constant values of capacitance. For frequencies in the vicinity of mechanical resonance of the bar, for a variation of Δf, there are a group of inductance values between the points 1 and 2 available for balancing any capacitive reactance in the range C_1 to C_2. Thus, a relatively large variation of capacitance C produces a small change of frequency between zero and Δf. The motional impedance vector simply shifts to a new position on the resonance circle, the whole of which is executed in an extremely small frequency variation. The extent of frequency stabilization depends therefore upon the sharpness of mechanical resonance $\Delta f/f_0$ and upon the magnetostriction activity of the bar, that is, upon the extent to which the kinetic and potential energies of the oscillatory system can be concentrated in the mechanical rather than the electrical system. The larger the motional impedance circle in relation to the static impedance value at the resonant frequency, the greater may the variation of capacitance be without appreciable frequency variation.

The arrangement of coil and bar shown in Fig. 1 was found to operate satisfactorily in a Hartley circuit, using a 201A tube, and additional inductance to increase the ratio of reactance to resistance, in both the grid and plate branch of the solenoid. This of course further decreases the motional inductance in relation to the total inductance; however, even under these conditions, the stabilizing

action of the bar is evident for a variation of the capacitance connected between the grid and the plate.

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CALCULATION OF CHARACTERISTICS AND THE DESIGN OF TRIODES*

By

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Summary —*The primary object of the paper is to present a simple method of designing triode vacuum tubes. It comprises three parts; the first part deals with the calculation of characteristics and constants of the triode from the electrode structures, special considerations being taken for applying the formulas already established for tubes of complicated structures. In the second part the derivation of various working conditions of a triode from its static characteristic is treated, in which the writer worked out a graphical representation of d-c and a-c components of the anode working current at various working voltages. The resulting dynamic characteristic diagram is applicable to any type of triodes in evaluating the working voltages, currents and powers, whether the tube be used as an amplifier, oscillator, or modulator. The third part presents the designing procedure for a typical case in which the use of the triode is indicated and its power output is given. First the working points are determined on the dynamic characteristic diagram so as to give the required working condition; then all the quantities arising in the operation are known in their relative amounts. The type of the tube is selected between the high-impedance type and the low-impedance one, which necessitates technical and economical considerations. Then all the quantities are known in numerical values and characteristics of the tube are determined. The electrode structures are calculated to give the required characteristics.*

The present paper, being an abridgment of a paper published in Japan,[1] is primarily intended for presenting a simple method of designing triodes.

Part 1 : Computation of Characteristics from Electrode Structures

THE well-known space-charge equation[2]

$$i_a = Ge_a{}^{3/2} \tag{1}$$

applies to diodes in actual cases, when it has a cathode of uniform potential all over the surface or when the anode voltage is very high compared with the filament voltage. This equation may also be applied, in ordinary cases, if the anode voltage is referred to the neutral point of the cathode, or when the filament is lighted by alternating current.

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* Dewey decimal classification: R 130. Original manuscript received by the Institute, April 9, 1929.

[1] Y. Kusunose, "Calculation on Vacuum Tubes and the Design of Triodes," Researches of the Electrotechnical Laboratory, No. 237; September, 1928, (written in English). Published by Kōseikai, No. 1/1 Yūrakucho, Tokyo, Japan. (Price about $1.70).

[2] I. Langmuir, *Phys. Rev.*, 2, 450, 1913.

When the cathode consists of a filament heated by direct current at a terminal voltage e_f, the above equation must be corrected for the non-uniformity of anode-to-cathode potential difference along the length of the filament, and the modified forms[3] are as follows:
for $e_a < e_f$:

$$i_a = \tfrac{2}{5}G\frac{1}{e_f}e_a^{5/2} = \tfrac{2}{5}Ge_f^{3/2}\left(\frac{e_a}{e_f}\right)^{5/2} \tag{2}$$

and for $e_a > e_f$:

$$i_a = \tfrac{2}{5}G\frac{1}{e_f}\left[e_a^{5/2} - (e_a - e_f)^{5/2}\right] = \tfrac{2}{5}Ge_f^{3/2}\left[\left(\frac{e_a}{e_f}\right)^{5/2} - \left(\frac{e_a}{e_f} - 1\right)^{5/2}\right] \tag{3}$$

The two equations may be reduced to a single one, as

$$i_a = \tfrac{2}{5}Ge_f^{3/2}\cdot f\left(\frac{e_a}{e_f}\right). \tag{4}$$

The constant G which is to be called "perveance" is determined from the electrode configurations, and in case of cylindrical anode,

$$G = 2.33 \times 10^{-6}\frac{A}{x_a^2} \tag{5}$$

in which A = effective anode area; x_a = distance of the anode surface from the axis of the cathode, or radius of the anode in this case.
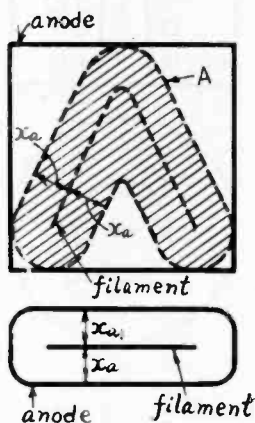
Fig. 1—Effective area of plane anode.

All the quantities relating to dimensions are to be expressed in centimeter units, and those relating to electricity in volts, amperes, ohms, watts, etc., unless otherwise specified.

[3] Van der Bijl, "Thermionic Vacuum Tube," p. 64, 1920.

The writer has found that the above expression of G is equally applicable to anodes of plane forms, if the effective anode area is taken as that area comprised in the breadth $2x_a$ along the filament length projected on the anode, as illustrated in Fig. 1.*

TRIODE CHARACTERISTICS

In a triode, an anode voltage e_a and a grid voltage e_g produce as their combined effect a certain strength of electric field around the cathode, and the same electric field can be produced if a single anode exists at the grid suface and is at the potential[4]

$$e_g' = \frac{e_a + ke_g}{1+k} \tag{6}$$

where k is the amplification constant.

The electron current is mainly governed by the electrostatic action
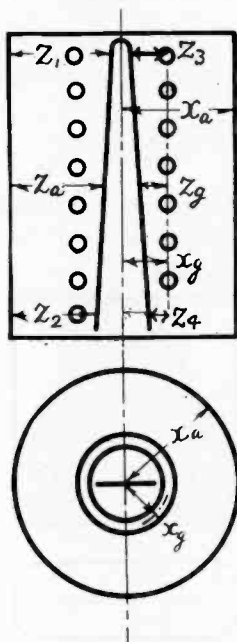
Fig. 2—$Z_a = (Z_1 + Z_2)/2$
$Z_g = (Z_3 + Z_4)/2$

of the system of electrodes, and if it be assumed that the two cases above cited give equal electron current, the characteristic of the triode

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* This has lately been proved theoretically by N. Kato and S. Koizumi.
[4] W. H. Eccles, "Continuous Wave Wireless Telegraphy," Part I, p. 338; 1921.

will be expressed by the following equations which are obtained by replacing e_a in (1) and (4) by e_g':

$$i = Ge_g'^{3/2} \tag{7}$$

$$i = \tfrac{2}{5}Ge_f^{3/2} \cdot f\left(\frac{e_g'}{e_f}\right) \tag{8}$$

in which $f(e_g'/e_f)$ may be obtained from Table I.

TABLE I

e_g'/e_f	$f(e_g'/e_f)$
0	0
0.25	0.031
0.50	0.177
0.75	0.414
1.00	1.00
1.5	2.57
2.0	4.65
2.5	7.13
3.0	9.94
4.0	16.51
5.0	24.1
6.0	32.5
8.0	53.0
10	74.0
15	138
20	211
40	622

In this case

$$G = 2.33 \times 10^{-6} \frac{\text{grid surface area}}{x_g{}^2}$$

and for cylindrical electrodes, grid surface area $= 2\pi x_g l$ and anode area $= A = 2\pi x_a l$, l being the effective length of the electrodes.

Hence

$$G = 2.33 \times 10^{-6} \frac{A}{x_a x_g}$$

in which x_a and x_g are the distances of the anode and grid surfaces respectively from the cathode axis. But the writer found by experiment that in case of triodes having cylindrical anode and grid, and a cathode of other than a single axially-spanned filament, such as V-shaped, the values of x_a and x_g in the expression of G should be replaced by the mean shortest distances from the respective electrodes to the cathode z_a and z_g which should be taken in such a way as illustrated in Fig. 2.†

Then generally

$$G = 2.33 \times 10^{-6} \frac{A}{z_a z_g} \cdot \tag{9}$$

† This can be verified by conformal transformation and integration as worked out by S. Koizumi.

UV-206 (R.C.A)	228-A (Western. Water-cooled)	S-53 (Nihon Musen)
$X_a=2.0$ $Z_a=1.71$ $A=103$	$X_a=1.73$ $Z_a=1.20$ $A=148$	$X_a=2.77$ $Z_a=2.46$ $A=146$
$X_g=0.98$ $Z_g=0.69$ $e_f=11$	$X_g=1.05$ $Z_g=0.50$ $e_f=21.5$	$X_g=0.98$ $Z_g=0.67$ $e_f=15$
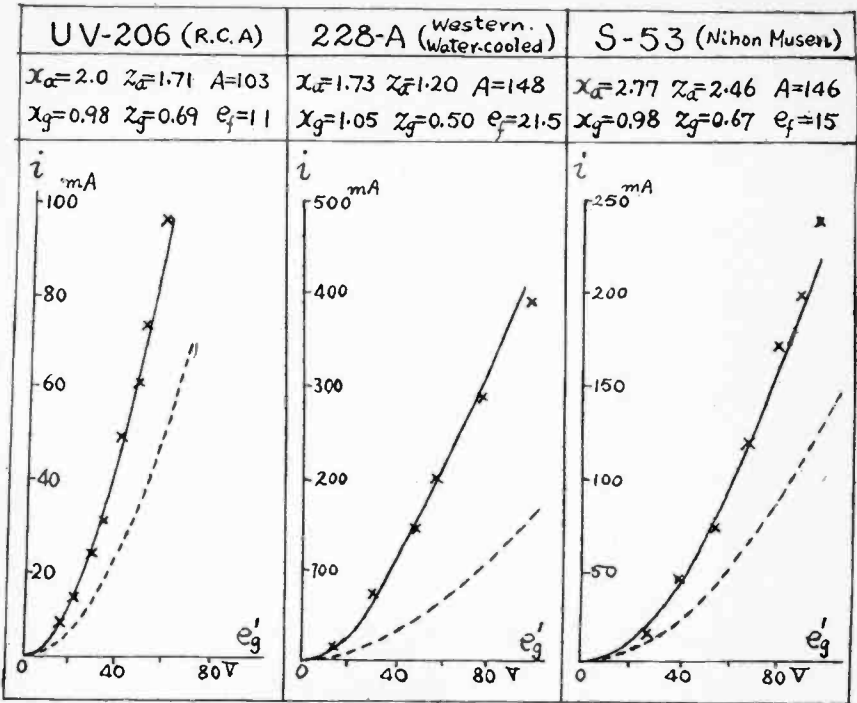
Fig. 3—Calculated characteristics of tubes with cylindrical anode and V-shaped filament.

Calculated with $G = 2.33 \times 10^{-6}$ $\left\{ x \dfrac{A}{Z_a Z_g} \rule{2cm}{0.4pt} \right.$

Observed points x. $\qquad x \dfrac{A}{X_a X_g}$ ‑ ‑ ‑ ‑ ‑ ‑
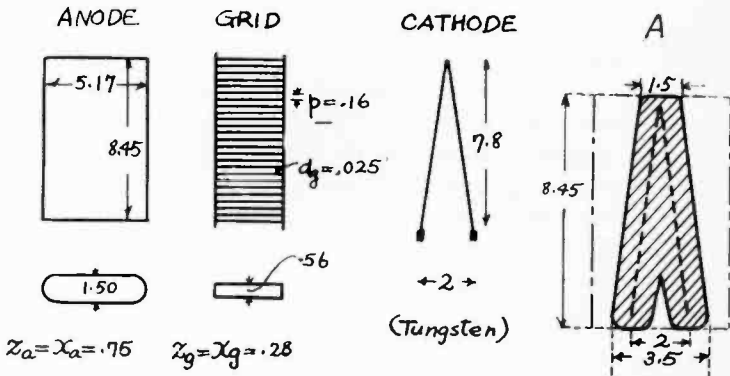
Fig. 4—Electrode dimensions and effective anode area of type UV204 tube.

This expression may also be applied to tubes with plane forms of anode and grid and in this case evaluation of the effective anode area. *A* should be made as described before (Fig. 1).
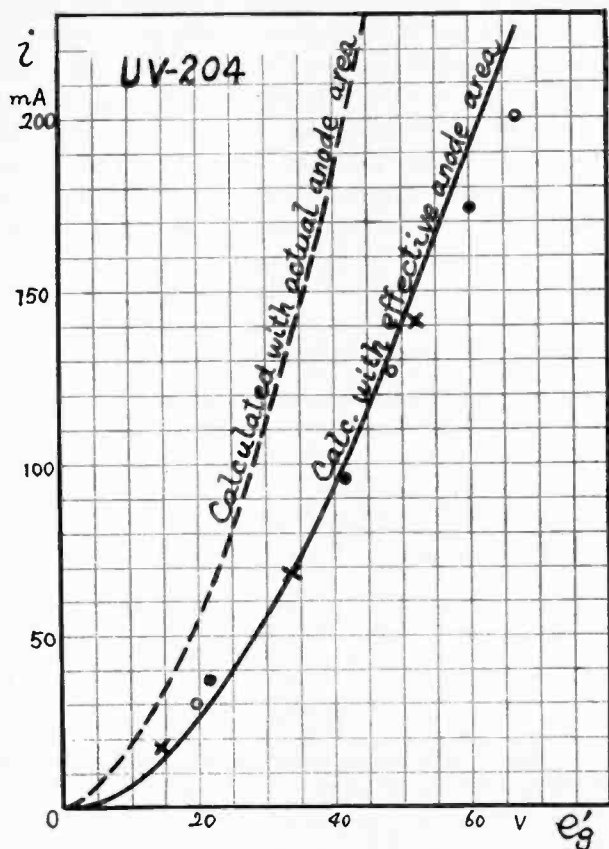
Fig. 5—Calculated characteristics of UV204.

Observed points $\begin{cases} x \text{ at } e_a = 1500 \text{ v} \\ \cdot \text{ " " } = 2000 \text{ "} \\ 0 \text{ " " } = 2500 \text{ "} \end{cases}$

Example 1: Calculation of characteristics of triodes by equation (8), one with the value of *G* obtained from expression (9) (full lines in Fig. 3), and the other with $G = 2.33 \times 10^{-6} A/x_a x_g$ (dotted lines in Fig. 3). Observed characteristics (points in Fig. 3) show the validity of taking the former expression of *G*.

Example 2: Evaluation of the effective anode area of a plane anode. A triode type UV204 (manufacturer: Tokyo Denki Co.) has dimensions as shown in Fig. 4. The effective anode area obtained after the principle described above is $A = 2 \times (8.45 \times (1.5 + 3.5)/2) = 42.3$ and for this area $G = 0.446 \times 10^{-3}$. If, on the other hand, actual area of the anode surface is

taken $A' = 2 \times (8.45 \times 5.17) = 87.3$ and for this area $G' = 0.965 \times 10^{-3}$. The characteristics are calculated by equation (8) for both and are plotted in Fig. 5. Observed data shows the validity of taking $A = 42.3$.

The depiction of the anode-current grid-voltage characteristic curves may be simplified in the following manner. First obtain the numerical relation between i and (e_g'/e_f), and from it the relation of i to e_g' for a given value of e_f. Then assume $e_a = 0$, or $e_g' = \{k/(1+k)\}e_g$, and i versus e_g curve can be drawn and this corresponds to the characteristic for the anode voltage zero. In order to obtain a characteristic for any anode voltage e_a, this curve for $e_a = 0$ should be shifted horizontally toward the negative direction of the grid voltage by an amount of e_a/k in the scale of grid voltage.

It should be remembered that a slightest error in the calculated value of k causes a considerable amount of error on the resulting values of i, especially at high anode voltages.

Example 3: Computation of characteristics of a triode type 211A (Western Electric Co.). Dimensions are shown in Fig. 6.

Calculations:

$$A = 2 \times \left(5.7 \times \frac{2.9+3.1}{2}\right) = 34.2$$

$$Z_a = x_a = 0.68 \qquad Z_g = x_g = 0.25$$

$$G = 2.33 \times \frac{34.2}{0.25 \times 0.68} \times 10^{-6} = 0.469 \times 10^{-}$$

$$e_f = 9.0 \qquad 2/5Ge_f^{3/2} = 0.0051$$

$$i = 0.0051 f\left(\frac{e_g'}{e_f}\right).$$

Characteristic curve representing the relation between i and e_g' is shown in Fig. 7, together with some observed points.

TABLE II

(e_g'/e_f)	$f(e_g'/e_f)$	e_g'	i
0	0	0	0
2	4.65	18	0.024
4	16.51	36	0.084
6	32.5	54	0.166
8	53.0	72	0.270

Amplification constant calculated by equation (13) given later is $k = 12.7$

$$\left(L_g = 8.3 \qquad \frac{x_a}{x_g} = 2.7 \qquad c = 1.8\right)$$

while observed value is $k = 12.8$ at $e_a = 1,000$ and $e_g = -45$.

To draw i versus e_g characteristic curves, first assume $e_a = 0$; then $e_g' = \{k/(1+k)\}e_g$ or $e_g = 1.08\,e_g'$ and Table III shows the result of calculation of the characteristic for $e_a = 0$, which is depicted in Fig. 8.

TABLE III

	$e_a = 0$	
i	e_g'	e_g
0	0	0
0.024	18	19.5
0.084	36	39
0.166	54	58
0.270	72	78

For other anode voltages, this curve is to be shifted to the negative direction by amounts as shown in Table IV.

TABLE IV

e_a	e_a/k
500	39
1,000	79
1,500	118

The resulting characteristic curves are drawn in Fig. 8 and are consistent with the observed data.

The electron current i is the sum of anode and grid currents,

$$i = i_a + i_g. \tag{10}$$

As long as the grid is negative $i_g = 0$ and $i_a = i$, but when grid becomes positive, grid current begins to flow. In ordinary tubes, and at
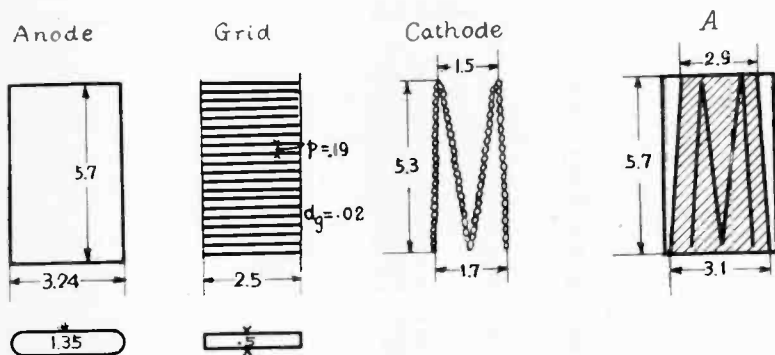
Fig. 6—Dimensions and effective anode area of type 211A tube.

low grid voltages where no remarkable effect of secondary emission appears, the following formula[5] is applicable for estimation of the grid current:

$$i_g = \gamma a \sqrt{e_g/e_a} \qquad i_a = \frac{\gamma a \sqrt{e_g/e_a}}{1 + \gamma a \sqrt{e_g/e_a}} i \tag{11}$$

[5] H. Lange, *Z. f. Hochfreq.*, **31**, 105; April, 1928.

and hence

$$i_a = \frac{i}{1 + \gamma a \sqrt{e_g/e_a}} \tag{12}$$

in which

$$\gamma = \sqrt{\log \frac{x_a}{r_c} \Big/ \log \frac{x_g}{r_c}}$$

(γ_c = radius of the cathode) are usually of the order: $\gamma = 1 \sim 2 \doteqdot 1.5$ and a = ratio of the grid conductor projected area to the grid surface area.

At $e_a = 0$, the electron current is totally absorbed in the grid if the grid voltage is positive.

When the grid voltage becomes so high as to approach the anode voltage, the characteristics are severely affected by secondary emissions from the electrodes, and grid current usually increases very rapidly while anode current falls off as shown in Fig. 9.

Example 4: Calculation of grid current.

TABLE V

Tube	Observed data				Calculated grid current
	e_a	e_g	i_a	i_g	$i_g = 1.5\, a \sqrt{e_g/e_a} \cdot i_a$
102-D $\left(\begin{matrix}a = 0.156\\ k = \quad 30\end{matrix}\right)$	100 100 148 148	5.2 8.3 5.2 8.3	0.0046 0.0074 0.0065 0.0100	0.0003 0.0004 0.0003 0.0004	0.00025 0.00050 0.00029 0.00055
UV-199 $\left(\begin{matrix}a = 0.108\\ k = 6.6\end{matrix}\right)$	20 20 60 100	6 16 10 16	0.0023 0.0040 0.0038 0.0042	0.0002 0.0005 0.0003 0.0003	0.00020 0.00058 0.00025 0.00027
UV-201A $\left(\begin{matrix}a = 0.105\\ k = 7\end{matrix}\right)$	20 20 60 60	5 15 5 10	0.0025 0.0095 0.0065 0.0113	0.0001 0.0013 0.0001 0.0004	0.0002 0.0013 0.0003 0.0007
UV-204A $\left(\begin{matrix}a = 0.16\\ k = 25\end{matrix}\right)$	1,000 1,000 5,000	30 60 21	0.172 0.274 0.220	0.011 0.019 0.009	0.006 0.009 0.006
MT-9 $\left(\begin{matrix}a = 0.245\\ k = \quad 90\end{matrix}\right)$	1,000 3,000 5,000	80 40 20	0.305 0.223 0.230	0.027 0.012 0.003	0.031 0.0094 0.0033

AMPLIFICATION CONSTANT

Notations regarding the electrode dimensions are illustrated in Fig. 10.

For $r_g \ll p \ll x_g$, and for plane-electrode tubes,[6]

[6] W. Schottkey, *Archiv f. Elektrot.*, 8, 21, 1919.

$$k = \frac{2\pi x_g}{p} \cdot \frac{\dfrac{x_a}{x_g} - 1}{\log e \dfrac{p}{2\pi r_g}} \tag{$k-1$}$$

and for cylindrical-electrode tubes,[7]

$$k = \frac{2\pi x_g}{p} \cdot \frac{\log \dfrac{x_a}{x_g}}{\log \dfrac{p}{2\pi r_g}}. \tag{$k-2$}$$
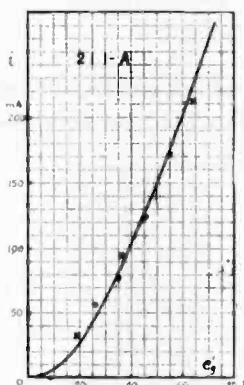
Fig. 7—Calculated characteristic of 211A.
Observed points $\begin{cases} x \text{ at } e_a = 500 \text{ v} \\ \bullet \ \ " \ \ " \ = 1000 \ " \\ 0 \ \ " \ \ " \ = 1500 \ " \end{cases}$

If the condition $r_g \ll p \ll x_g$ is not fulfilled, the more general formula for cylindrical-electrode tubes is to be applied,[8]

$$k = \frac{\dfrac{2\pi x_g}{p} \log \dfrac{x_a}{x_g} - \log \cosh \dfrac{2\pi r_g}{p}}{\log \cosh \dfrac{2\pi r_g}{p} - \log \sinh \dfrac{2\pi r_g}{p}}. \tag{$k-3$}$$

The formulas for cylindrical electrodes were originally derived for grid wires parallel to the filament, but they are equally applicable to grids of a spiral or a system of parallel wires spanned crossways to the length of the filament.

[7] M. Abraham, *Archiv f. Elektrot.*, **8**, 42, 1919. R. W. King, *Phys. Rev.*, **15**, 256, 1920.
[8] F. B. Vogdes. *Phys. Rev.*, **24**, 683, 1924; **25**, 255, 1925.

Example 5: Calculation of amplification constant of a triode type
ABT (made at the Laboratory), which has dimensions as shown in Fig. 11.

$$x_a = 0.20 \qquad x_g = 0.14 \qquad p = 0.12 \qquad r_g = 0.0085$$

$$\frac{2\pi x_g}{p} \; 7.3 \qquad \frac{2\pi r_g}{p} = 0.44 \qquad\qquad \log\frac{x_a}{x_g} = 0.155$$

By $(k-2)$:

$$k = 7.3 \times \frac{0.155}{0.357} = 3.2$$

By $(k-3)$:

$$k = \frac{7.3 \times 0.155 - 0.040}{0.040 + 0.345} = 2.8 \ .$$

Observed value of $k = 2.9$.

In an actual triode in which anode and grid are plane, the cathode
is not actually plane but is a filament arranged on a plane surface and
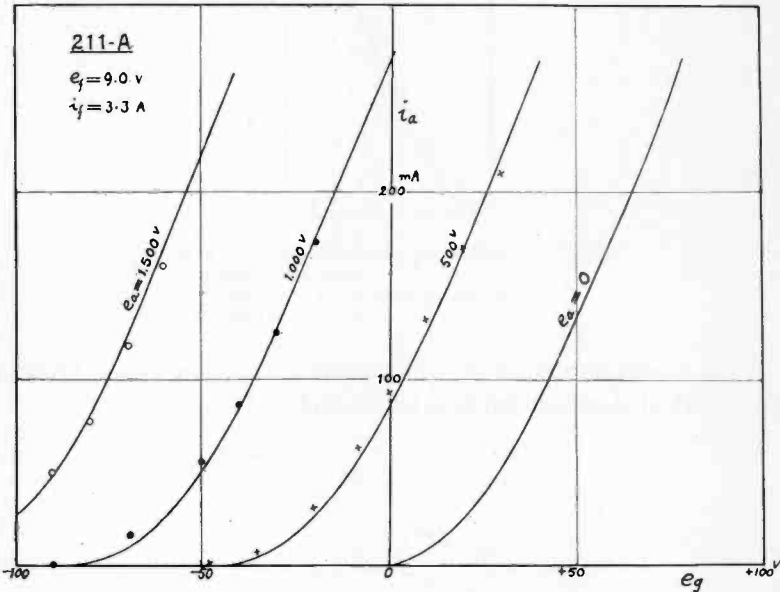
Fig. 8—Characteristics of 211A.
Calculated——— ⎧ 1500 0
Observed at $e_a =$ ⎨ 1000 •
                   ⎩ 500 x

accordingly does not strictly conform to the condition from which the
formula $(k-1)$ has been derived. But by trying the formulas on actual
cases, the writer found that the formula for plane $(k-1)$ is applicable

for a tube with W-shaped filaments, while it gives somewhat larger value for tubes with V-shaped filaments. The formulas for cylinder $(k-2)$, if applied to these tubes, give lower values than observed. Comparing the two formulas, we find that they are different only in
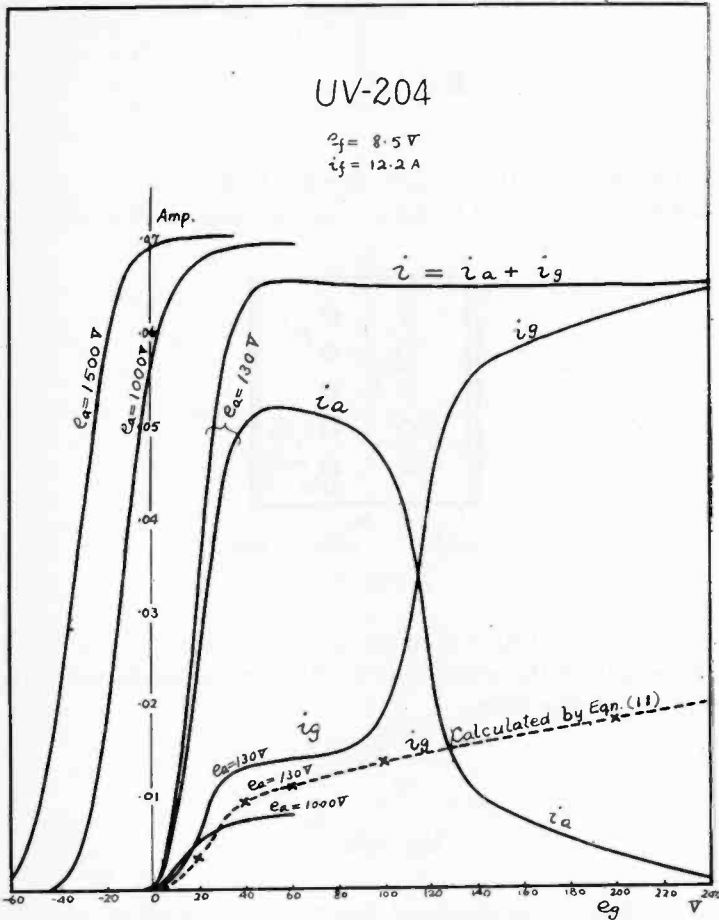
Fig. 9—Observed characteristics of UV204 showing the effect of secondary emission.

terms of x_a/x_g, and the writer suggests that for approximate calculation of k for plane-electrode tubes, the formula for cylindrical electrodes may be applied by multiplying a factor c which depends on x_a/x_g and the shape of the filament as given in Table VI.

TABLE VI

VALUES OF C

Cylindrical-electrode tubes;	$C=1$
Plane-electrode tubes;	
with grid of a mesh:	$C=1$
with grid of parallel wires:	

| | C | |
x_a/x_g	W-shaped filament	V-shaped filament
7	1.0	1.0
2	1.5	1.3
3	1.9	1.6
4	2.4	1.9
5	2.8	2.1

The formula $(k-3)$ is a more general form of $(k-2)$, and this is to be taken as a standard formula. In this formula $(\log \cosh 2\pi r_g/p)$
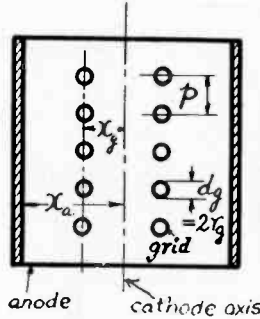
Fig. 10

is of the order of 0.1 and is small compared with $(2\pi x_g/p \cdot \log x_a/x_g)$ which is usually of the order of 10 to 100, and so the formula may be reduced to a simpler form:

$$k = \frac{c \dfrac{2\pi x_g}{p} \log \dfrac{x_a}{x_g}}{\log \coth \dfrac{2\pi r_g}{p}} \quad (k-4)$$

Example 6: Calculation of amplification constant of a triode type UV203 (Tokyo Denki Co.) which is a plane-electrode tube with a cathode of W-shaped filament as shown in Fig. 12.

$$x_a = 0.49 \qquad x_g = 0.19 \qquad p = 0.12 \qquad r_g = 0.0085$$

$$\frac{2\pi x_g}{p} = 9.95 \quad \frac{2\pi r_g}{p} = 0.445 \quad \frac{p}{2\pi r_g} = 2.25 \quad \log \frac{x_a}{x_g} = 0.412.$$

By equation $(k-1)$,

$$k = 9.95 \times \frac{1.60}{2.34 \times 0.353} = 19.6$$

and by equation $(k-2)$,

$$k = 9.95 \times \frac{0.412}{0.353} = 11.6 .$$

Equation $(k-4)$ gives

$$k = \frac{9.95 \times 0.412 \times 1.74}{0.467} = 15.3 \quad (c = 1.74)$$

and the observed value $k = 15.9$.

Example 7: Calculation of amplification constant of a triode type UV201A (R. C. A.), dimensions of which are given in Fig. 13.

$$x_a = 0.34 \qquad x_g = 0.17 \qquad p = 0.102 \qquad r_g = 0.0053$$

$$\frac{2\pi x_g}{p} = 10.5 \quad \frac{2\pi r_g}{p} = 0.33 \quad \frac{p}{2\pi r_g} = 3.03 \quad \log \frac{x_a}{x_g} = 0.301$$

By equation $(k-1)$,

$$k = 10.5 \times \frac{1.0}{2.30 \times 0.482} = 9.4$$

by $(k-2)$,

$$k = 10.5 \times \frac{0.301}{1.022} = 3.1$$

and by $(k-4)$, as the filament is V-shaped $c = 1.3$ for $x_a/x_g = 2.0$ and

$$k = \frac{10.5 \times 0.301 \times 1.30}{0.505} = 8.1 .$$

Observed value, $k = 8.2$.

In the above formulas grid is considered to be formed of parallel wires only. But in actual cases supports are usually added and in some cases grid is constructed of wires weaved into a mesh.

The writer brought out the following deductions in order to make the formula equally applicable to these cases.

In formula $(k-4)$, the quantity $2\pi x_g/p$ means the "total active length of grid wires per unit axial length of the grid" in a cylindrical-electrode tube, because $2\pi x_g$ is the length of one turn of grid wire in circular form and $1/p$ is the number of turns per unit length. This quantity is to be denoted by L_g, thus for a grid of closely-wound spiral wire

$$L_g = \frac{2\pi x_g}{p}.$$

Example 8: Amplification constant of a triode type RE-84 (Telefunken) which has a grid of spiral wire of wide pitch of winding. (Fig. 14.) If the value of $L_g = 2\pi x_g/p$ is taken, i.e. if the grid is assumed to be formed of circular rings spaced at the pitch p,

$$x_a = 0.263 \quad x_g = 0.125 \quad p = 0.25 \quad r_g = 0.025 \quad \frac{2\pi x_g}{p} = 3.14$$

$$k = \frac{3.14 \times 0.323}{0.25} = 3.96 .$$

If, on the other hand, actual length of the spiral grid wire per unit axial length of the grid as defined above is taken,

$$L_g = \sqrt{1 + \left(\frac{2\pi x_g}{p}\right)^2} = 3.30$$

$$k = \frac{3.30 \times 0.323}{0.25} = 4.4 .$$

The observed value $k = 4.5$ proves that the latter is more consistent.

For a grid consisting of a square mesh of very thin wires

$$L_g = \frac{4\pi x_g}{p}$$

but when grid wire is not of small diameter, effect of overlapping of the wires should be considered. At each crossing point of two wires, active
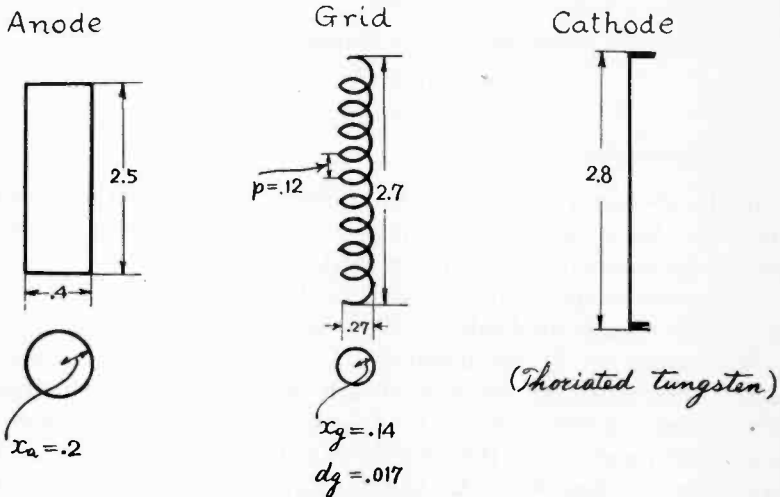
Fig. 11—Dimensions of type *ABT* tube.

length of one wire is reduced by an amount equal to the diameter of the wire, and as the number of crossing points per unit axial length of the grid is $2\pi x_g/p^2$ (Fig. 15),

$$L_g = \frac{4\pi x_g}{p} - \frac{2\pi x_g}{p^2} \cdot d_g = \frac{4\pi x_g}{p}\left(1 - \frac{r_g}{p}\right).$$

Effect of grid supports should be considered in the same way and for a grid of parallel wires,

$$L_g = \frac{2\pi x_g}{p} + s - \frac{st}{p}$$

in which s is the number of grid supporting wires which are active in influencing on the space charge, and st/p is the amount of reduction of
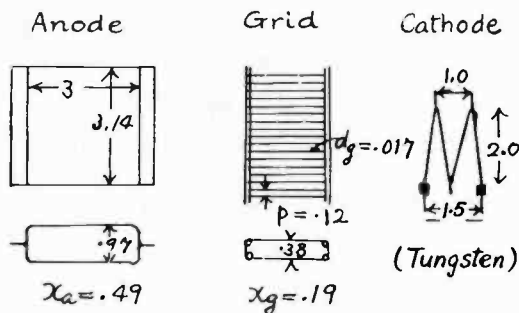
Fig. 12—Dimensions of type UV203 tube.

effective length of the grid wires due to the grid support covering a portion of each grid wire (Fig. 16).

For a grid made of a mesh, the effect of supports may not be considered, as the total length of the wires is very large, and moreover the
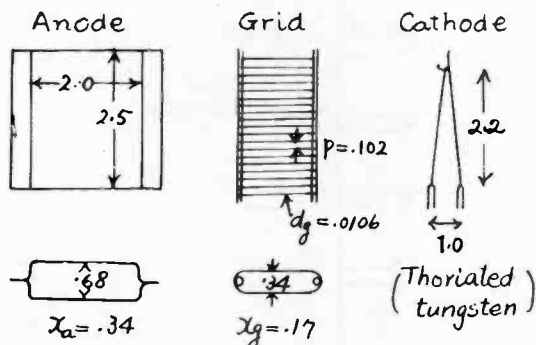
Fig. 13—Dimensions of type UV201A tube.

increase of L_g due to the supports is usually canceled by the reduction of it due to the support covering a portion of the mesh.

The quantity $2\pi r_g/p$ in formula $(k-4)$ signifies π-times d_g/p or "π-times the ratio of the grid conductor projected area to the grid surface area," and this ratio of areas being denoted by a,

$$\pi a = \pi \frac{\text{grid wire projected area per unit length of the grid}}{2\pi x_g}.$$
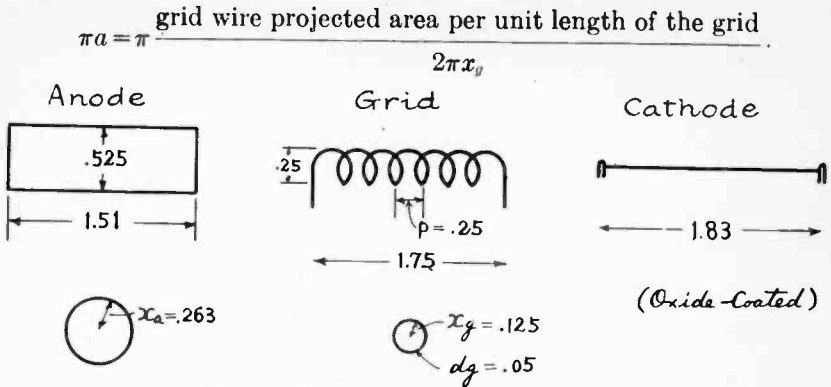
Fig. 14—Dimensions of type RE84 tube.

For a grid of a mesh or of a system of parallel wires,

$$\pi a = \frac{L_g d_g}{2x_g} = \frac{L_g r_g}{x_g}.$$

If the parallel wire grid has supports,

$$\pi a = \frac{(L_g - s)d_g + st}{2x_g}.$$
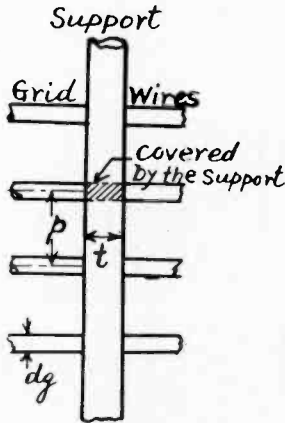
Fig. 15

Example 9: A triode type TA08/10 (Philips' Lamp Works) has a grid supported by a thick wire. (Fig. 17.)

$$x_a = 0.64 \quad x_g = 0.24 \quad p = 0.112 \quad r_g = 0.014 \quad \log \frac{x_a}{x_g} = 0.436.$$

Without considering the support:

$$L_g = \frac{2\pi x_g}{p} = 13.2 \qquad \pi a = \frac{2\pi r_g}{p} = 0.79 \qquad k = 31.6.$$

Considering the support: $s = 1, t = 0.092$

$$L_{,g} = \frac{2\pi x_g}{p} + s\left(1 - \frac{t}{p}\right) = 13.4 \qquad \pi a = \frac{(L_g - s)d_g + st}{2x_g} = 0.93 \qquad k = 43.$$

Observed value: $k = 46$.

Example 10: A triode type MT-4 (Marconi Co.) has a grid of a mesh as shown in Fig. 18.
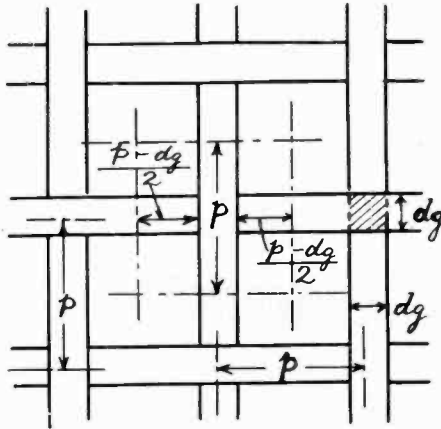
Fig. 16

$$x_a = 2.17 \quad x_g = 0.93 \quad p = 0.13 \quad r_g = 0.011 \qquad \text{og}\,\frac{x_a}{x_g} = 0.368.$$

Without considering the overlapping of the grid wires,

$$L_g = \frac{4\pi x_g}{p} = 90 \qquad \pi a = \frac{L_g r_g}{x_g} = 1.06 \qquad k = \frac{90 \times 0.368}{0.105} = 315$$

and considering the overlapping;

$$L_g = \frac{4\pi x_g}{p}\left(1 - \frac{r_g}{p}\right) = 90 \times 0.91 = 81 \quad \pi a = \frac{L_g r_g}{x_g} = 0.96 \quad k = \frac{81 \times 0.368}{0.129} = 230.$$

Observed value: $k = 210$.

The formula for the amplification constant finally attains the form;

$$k = \frac{c\, L_g \log \dfrac{x_a}{x_g}}{\log \coth \pi a}. \tag{13}$$

The estimation of L_g and πa is summarized in Table VII.

TABLE VII

Grid construction	L_g	πa
Wires parallel to the filament	n	$\dfrac{nd_g}{2x_g}$ or $\dfrac{L_g r_g}{x_g}$
Parallel wires or spiral, no support	$\dfrac{2\pi x_g}{p}$	$\dfrac{\pi d_g}{p}$ or $\dfrac{L_g r_g}{x_g}$
Ditto, with supports in the path of electrons	$\dfrac{2\pi x_g}{p}+s\left(1-\dfrac{t}{p}\right)$	$\dfrac{(L_g-s)d_g+st}{2x_g}$
Square mesh, with or without support	$\dfrac{4\pi x_g}{p}\left(1-\dfrac{r_g}{p}\right)$	$\dfrac{\pi d_g}{p}\left(2-\dfrac{d_g}{p}\right)$ or $\dfrac{L_g r_g}{x_g}$

These are equally applicable to tubes of either cylindrical or plane electrodes.

The amplification constant is mainly governed by grid construction and depends little on anode area and filament configuration. Main factors determining k are x_a/x_g, p/x_g, and r_g/p, and similar tubes of
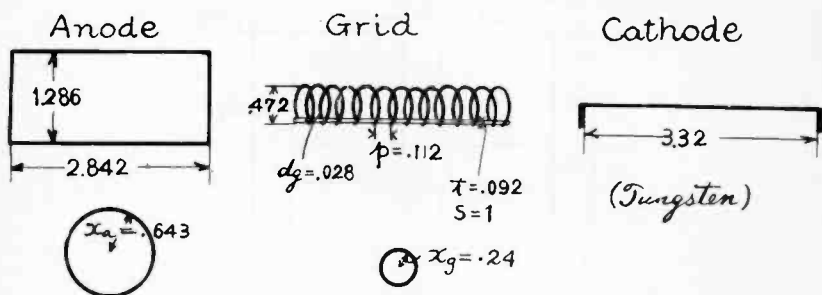
Fig. 17—Dimensions of type TA18/10 tube.

equal relative dimensions will give equal values of k whatever the size may be. k is affected by grid current at positive grid voltages and by the non-uniformity of grid action along the length of the cathode at very low anode currents, observed value of k being slightly reduced in both the cases.

ELECTROLYTIC METHOD OF DETERMINING k

By the amplification constant k is meant that the electrostatic influence of the grid upon the cathode is k-times that of the anode. Electrostatic field in a space assumes similar distribution as stream lines of electric current that would take place if the space were filled with conducting material. The amplification constant may thus be determined by means of a model as shown in Fig. 19, in which only a single section of grid wire is taken, as this is sufficient for the purpose. By taking the ratio of electric conductance of grid-to-cathode to that

of anode-to-cathode, after the bridge is balanced, the amplification constant is known as

$$k = \frac{R_a}{R_g}.$$

Table VIII shows an example obtained by the simplest structure as shown in Fig. 19, and calculated values are given for comparison.
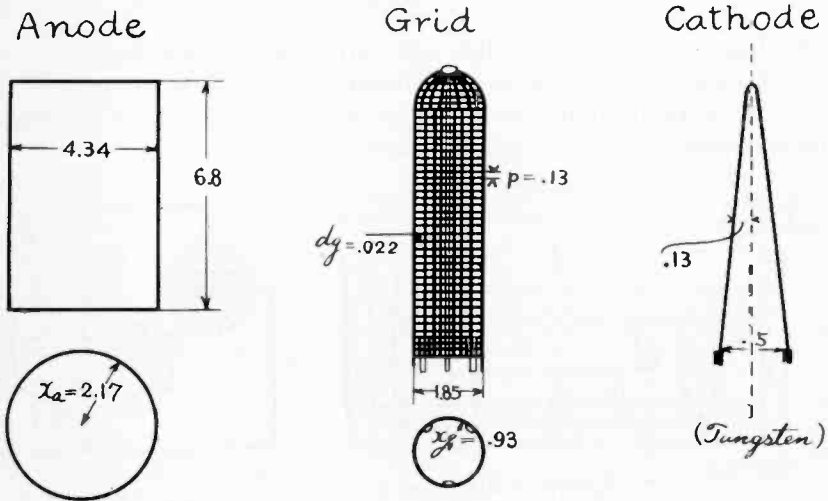
Fig. 18—Dimensions of type MT4 tube.

TABLE VIII

x_a	x_g	p	r_g	k obs.	k calc.
4.0	3.0	3.0	0.125	1.4	1.27
10.0	3.0	4.0	0.125	3.2	3.45
10.0	2.0	4.0	0.250	5.4	5.09
10.0	4.0	3.0	0.250	10.0	10.3
7.0	3.0	2.0	0.250	21.0	20.4

That the cathode configuration gives little effect on k could be evidenced by trying various forms and arrangements of the cathode conductor.

In designing a triode with very complicated structures that do not permit calculation, this method of predetermining the value of k will save much the effort of evacuation and readjustment of completed tubes.

MUTUAL CONDUCTANCE AND ANODE RESISTANCE

The mutual conductance g for given anode and grid voltages e_a and e_g can be obtained by differentiation of the characteristic equation, thus

$$g = \frac{\partial i_a}{\partial e_g} = \tfrac{2}{5} G \frac{1}{e_f} \cdot \frac{\partial}{\partial e_g} \left[e_g'^{5/2} - (e_g' - e_f)^{5/2} \right]$$

$$\doteqdot 1.5 G \frac{k}{1+k} \left(\sqrt{e_g'} \frac{1}{4} \cdot \frac{e_f}{\sqrt{e_g'}} \right) \tag{14}$$

in which

$$e_g' = \frac{e_a + k e_g}{1+k}.$$

This is strictly applicable when grid current is not appreciable.

The mutual conductance is principally governed by the perveance G which is proportional to the anode effective area and is inversely proportional to the electrode distances.
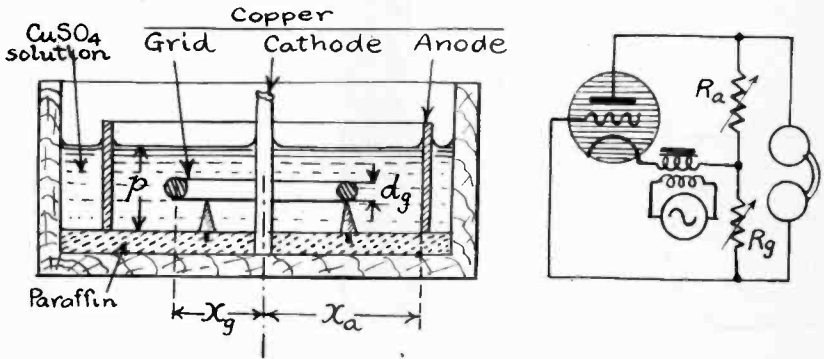
Fig. 19—Determination of the amplification constant by means of models.

The anode resistance r may also be obtained as $\partial e_a / \partial i_a$, but it is more convenient to calculate it from the known values of k and g as

$$r = \frac{k}{g} \tag{15}$$

Example 11: Calculation of mutual conductance and anode resistance of a triode type T-100 (Marconi Co.).

From the dimensions shown in Fig. 20.

$x_g = 1.15 \quad x_g = 0.44 \quad p = 0.095 \quad r_g = 0.0075 \quad A = 2\pi x a l = 23.9$

$k_{\text{calc.}} = 52.7 \qquad Z_a = 1.15 - 0.08 = 1.07 \qquad Z_g = 0.44 - 0.08 = 0.36$

$e_f = 10$ (rating) $\quad G = 0.127 \times 10^{-3} \qquad 1.5 G \dfrac{k}{1+k} = 0.187 \times 10^{-3}$.

(1) To obtain g and r for $e_a = 1,500$ and $e_g = 0$,

$e_g' = 27.9 \qquad \sqrt{e_g'} = 5.28$

$g = 0.187 \times 10^{-3} \times 4.81 = 0.00090 \qquad r = \dfrac{52.7}{0.00090} = 585,000$.

observed values: $g = 0.00089$, $r = 540,000$.

(2) For $e_a = 2,000$ and $e_v = -20$,

$$e_g' = 17.7 \qquad \sqrt{e_g'} = 4.20$$
$$g = 0.000675 \qquad r = 78,100$$

observed values: $g = 0.000675$, $r = 71,000$.

SATURATING PART OF THE CHARACTERISTIC

Saturation current I_s depends on the material of which the cathode is made, and for a given material it depends on the dimensions and working temperature of the cathode. General characteristics of the
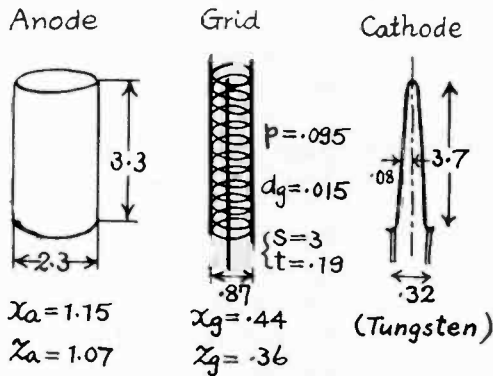
Fig. 20—Dimensions of type T100 tube.

three types of cathode, i.e. tungsten, thoriated tungsten, and oxide-coated cathode, were already published by many authorities and no particular mention is made here.

The point at which the anode current begins to saturate occurs at an equivalent voltage e_g' such as

$$e_s = \left(\frac{I_s}{G}\right)^{2/3}.$$

If the cathode consists of a uniformly emissive surface of equal potential all over it, the characteristic would sharply bend to saturate as soon as the voltage reaches the saturating voltage e_s. But in actual cases, slow bending of the characteristic curve is invariably observed, which is caused by the non-uniformity of filament temperature as well as the voltage drop along the filament. The expression of electron current considering the latter cause only is as follows;[3] in a range of the voltage tending to saturation: $e_s < e_g' < e_s + e_f$

$$i = \frac{1}{5} G \frac{1}{e_f} \left[e_s{}^{3/2} (5 e_g{}' - 3 e_s) - 2 (e_g{}' - e_f)^{5/2} \right],$$

This expression does not fully represent the saturating part in actual cases, and the non-uniformity of emissive power of the cathode, especially due to the cooling of the ends, is by far the important factor. Even in an indirectly-heated tube the bending takes place very slowly.

Calculation of this part of the characteristic is very difficult as the temperature distribution must be known, and moreover the emissive power of the cathode is affected by various unexpected causes. The writer finds no need of calculating this part as it is of little practical importance for the designing.

In the original paper numerical examples of computation of characteristics on about fifty types of diodes and triodes were given, being compared with their observed characteristics.

Part II: Determination of Operations of a Triode from its Static Characteristics

Working Anode Current

The operations of a triode as an amplifier, oscillator, modulator, etc., can be determined from its static characteristics for a given circuit arrangement, and strict and elaborated theories in this respect have been published by many authors. The following discussion is intended only for a rough estimation of the operating conditions, simplicity being aimed at with an accuracy sufficient for the designing purpose.

The static characteristic of a triode comprises two parts:

1st Part: the anode current is determined from the approximate relation,

$$i_a = G e_g{}'^{3/2} = \frac{G}{(1+k)^{3/2}} (e_a + k e_g)^{3/2} = G'(e_g + k e_g)^{3/2}. \tag{15}$$

2nd Part: anode current reaches saturation, and forms the upper limit of the characteristic.

The characteristic curve is assumed to have a shape as depicted in Fig. 21 with its sharp bending point at the saturating voltage E_s. When the characteristics are simplified in such a manner, all the tubes will have the similar curves, and if we express the anode current in the unit of the saturation current I_s, and all voltages in the unit of saturating voltage E_s, a simple and universal representation of the triode characteristic is obtained.

Let E_g = grid steady voltage,

E_a = anode steady voltage,

$\mathcal{E}_g$ = grid alternating voltage in maximum value,

$\mathcal{E}_a$ = anode alternating voltage in maximum value,
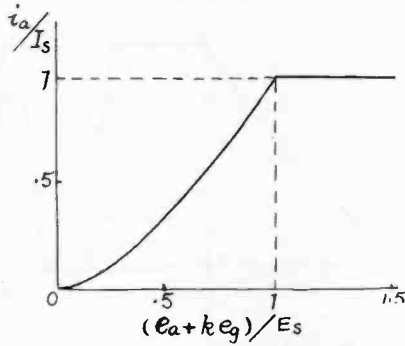
Fig. 21

positive directions being taken in the direction of direct current (Fig. 22). The load connected in the anode circuit is assumed to act as a resistance R at the operating frequency and to produce no potential drop by the d-c component of the anode current.
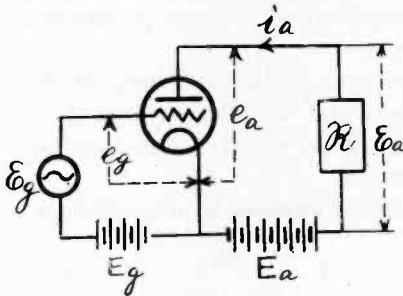
Fig. 22

In ordinary working conditions $\mathcal{E}_a$ and $\mathcal{E}_g$ are generally in opposite phase, and if the instantaneous value of the grid voltage is represented by

$$e_g = E_g + \mathcal{E}_g \cos \omega t$$

anode voltage will be

$$e_a = E_a - \mathcal{E}_a \cos \omega t$$

and for a given tube, instantaneous value of anode current will be determined by

$$e_a + ke_g = (E_a + kE_g) + (k\mathcal{E}_g - \mathcal{E}_a) \cos \omega t.$$

Thus in Fig. 23, a steady voltage $E_a + kE_g$ determines the working point p on the characteristic and the alternating component $k\mathcal{E}_g - \mathcal{E}_a$ gives the amplitude of oscillation, the working point being the center.
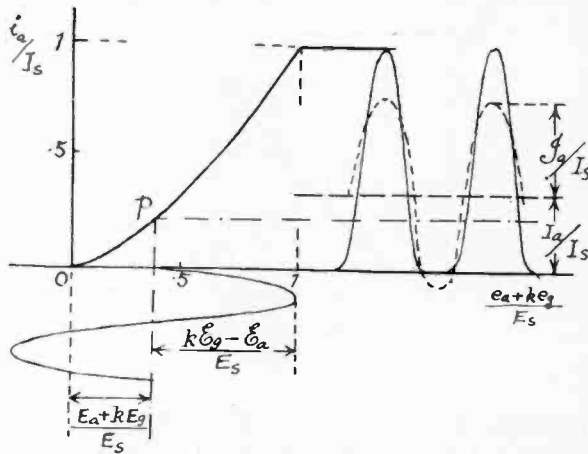
Fig. 23*

The variation of anode current due to this oscillating voltage is sinusoidal when the amplitude of voltage is very small, but becomes distorted when it is large.

Let I_a = mean value or d-c component of the anode current,

I_a = fundamental a-c component of the anode current, in maximum value, positive direction being taken as that of direct current.

For an infinitesimal amplitude of oscillation,

$$\mathbf{I}_a = \frac{di_a}{d(e_a + ke_g)}(k\mathcal{E}_g - \mathcal{E}_a)$$

and for a constant anode voltage,

$$\frac{di_a}{d(e_a + ke_g)} = \frac{1}{k} \cdot \frac{di_a}{de_g} = \frac{g}{k} = \frac{1}{r}$$

and

$$\mathbf{I}_a = \frac{k\mathcal{E}_g - \mathcal{E}_a}{r}$$

this gives the well-known fact that the triode circuit can be represented by an equivalent circuit as shown in Fig. 24, regarding the a-c components of working quantities.

* The symbol Ia in the illustrations is designated by Ia in the text.

For general cases, the writer obtained the wave forms of the anode current, and calculated the d-c and a-c components of the anode current by the Fourier analysis. (Fig. 23)

DYNAMIC CHARACTERISTIC DIAGRAM

The result of calculation of the d-c and a-c components of the anode current at various working voltages is summarized in a diagram shown in Fig. 25. This diagram is universally applicable to any particular
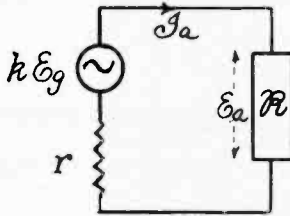
Fig. 24

triode as long as voltages are expressed in the unit of its saturating voltage E_s, and currents in the unit of the saturation current I_s. These quantities taken as units can be obtained from the static characteristics and E_s can also be calculated from the following relations:

$$E_s = \left(\frac{I_s}{G'} \right)^{2/3} . \qquad G' = \frac{G}{(1+k)^{2/3}} .$$

The dynamic characteristic diagram represents the surfaces of I_a/I_s and I_a/I_s as determined by the voltages $(E_a + kE_g)/E_s$ and $(kE_g - E_a)/E_s$, and comprises the following regions of practical importance, as designated in Fig. 26.

TABLE IX

Regions	Working point	Working range of anode current	Practical value
A	On the slope of the static characteristic	Entirely in the slope, not limited	Distortionless amplification
B	Ditto.	Limited by zero line.	Self-oscillation practicable and effectively subjected to modulation.
C	Ditto.	Limited by saturation.	Ditto, but with lower efficiency.
D	Ditto.	Limited by both zero-line and saturation.	Self-oscillation or amplification with large power.
E	At zero anode current with highly negative grid	Lower half-cycle entirely suppressed, upper peak not limited.	Power amplification of modulated waves.
F	Ditto.	Ditto, upper peak limited by saturation.	Self-oscillation or amplification of large power with high efficiency.

It should be remembered that this diagram has been derived under the following assumptions:

(a) static characteristic has sharp bending toward saturation,

(b) anode and grid voltages vary in sine wave form and just in opposite phase,

(c) grid current is not appreciable,

(d) load connected in the anode circuit does not produce appreciable voltage drop by direct current as well as by harmonic waves.
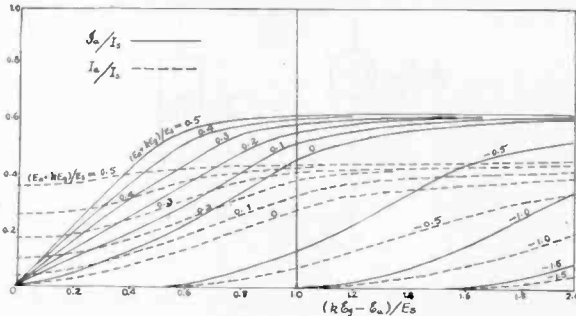
Fig. 25—Dynamic characteristics.

Thus the diagram is not strictly applicable when there is considerable grid current flowing, which takes place when the maximum grid voltage $E_g + \mathcal{E}_g$ becomes comparable with the minimum anode voltage $E_a - \mathcal{E}_a$. In such a case the anode alternating current attains a drooping characteristic at high excitation as shown in Fig. 27.

EXPERIMENTAL VERIFICATION

The dynamic characteristic diagram was obtained by experiment in the following way. A triode was used as a radio-frequency power amplifier in a circuit shown in Fig. 28. The static characteristics of the tube are shown in Fig. 29, from which the necessary quantities for the units were calculated as follows:

$$I_s = 0.071 \qquad E_s = 1.220 \qquad k = 25.$$

During the experiment, under constant filament current, E_a and E_g were varied and at each step of variation, I_0 and I_a were observed with variation of I' and C_g. Calculations were then made as follows,

$$\mathcal{E}_a = \frac{I_0}{\omega C} \qquad \mathcal{R} = \frac{L}{CR} = \frac{1}{\omega^2 C^2 R} \qquad I_a = \frac{\mathcal{E}_a}{\mathcal{R}} \qquad \mathcal{E}_a = \frac{I'}{\omega C_g}.$$

In the actual case, $\omega = 3 \times 10^6 (\lambda = 628 \text{ m})$ and high-frequency resistance of the oscillatory circuit was 3 ohms so that $R = 3 + R'$. Hence

$(k\mathcal{E}_g - \mathcal{E}_a)/E_s$ could be calculated for each value of $(E_a + kE_g)/E_s$, and $\mathbf{I}_a/I_s$ and I_a/I_s were plotted against $(k\mathcal{E}_g - \mathcal{E}_a)/E_s$ to obtain the dynamic characteristic curves.

The resulting curves are shown in Fig. 30, which are in conformity with the theoretical diagram in Fig. 25 in the main.

APPLICATION OF THE DIAGRAM

In Fig. 31, the dynamic characteristic diagram is shown with its left side extended to another diagram in which the ordinate gives
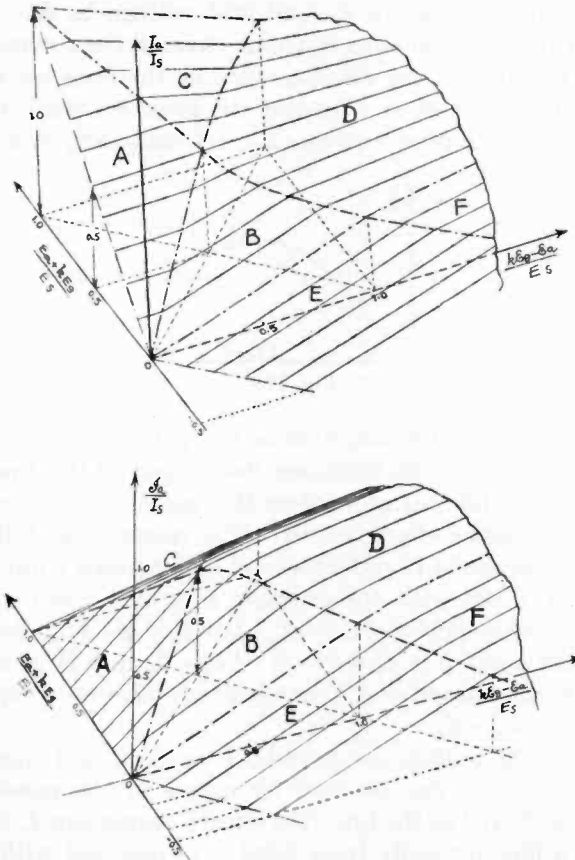
Fig. 26—Dynamic characteristic surfaces.

currents in the unit of I_s, and the abscissa gives voltages in the unit of E_s. In this part of the diagram a group of straight lines starting at the origin represent the external resistance R in the unit of the mean

anode internal resistance $\bar{r}$ which is derived as $\bar{r} = E_s/I_s$, and hyperbolas give the contour lines of power, unit of which is to be taken as $E_s I_s$.

To explain the use of the diagram, assume that a triode having saturation current I_s, amplification constant k, and saturating voltage E_s, is to be used as an amplifier with an anode circuit impedance R, and its operating conditions are to be studied. First calculate the unit quantities $\bar{r} = E_s/I_s$ and $E_s I_s$ from the given E_s and I_s. All the currents, voltages, resistances and powers are to be expressed in the units of I_s, E_s, $\bar{r}$ and $E_s I_s$, respectively. In the following explanations quantities expressed in this manner are taken instead of actual values.

Let the anode voltage be E_a and grid voltage be E_g. Calculate $E_a + k E_g$ and the corresponding dynamic characteristic curves of I_a/I_s and I_a/I_s are found on the diagram such as the lines *oeg* and *def* in Fig. 32. Draw a line *oh* to represent the given external impedance. For a given grid exciting voltage $\mathcal{E}_g$, calculate $k\mathcal{E}_g$ and take this
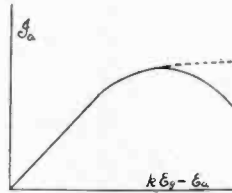
Fig. 27

quantity on the right abscissa such as the point i; from i a straight line is drawn parallel to the resistance line *oh* and let this line intersect with the dynamic line *oeg* at p; then this point p gives the working point on the dynamic characteristic. The reason is as follows: the ordinate of p represents I_a and let a level line through p intersect with the *oh* line at j and with the ordinate axis at k, then $\bar{ok} = I_a$ and $\tan < jok = R$. Therefore $\bar{jk} = I_a R = \mathcal{E}_a$, but $\bar{pj} = \bar{oi} = k\mathcal{E}_g$ and thus the abscissa of the point p is $\bar{pk} = \bar{pj} - \bar{jk} = k\mathcal{E}_g - \mathcal{E}_a$ and is in conformity with the dynamic characteristic curves which have been expressed by the abscissa of $k\mathcal{E}_g - \mathcal{E}_a$.

The area of the triangle *ojk* gives $\frac{1}{2}\mathcal{E}_a I_a$ or $\frac{1}{2}I_a^2 R$ and represents the power output, which can be read by means of the power contour hyperbolas numbered at the top. The steady component I_a is obtained by drawing a line vertically from point p to intersect with the I_a/I_s dynamic line at a point q. Take $\bar{o\bar{a}}$ equal to E_a. A level line through q intersects with the vertical line through a at l, and with the ordinate axis at m. Then $\bar{om} = I_a$, and the rectangular area *omla* is $E_a I_a$ and represents the anode power input to the tube which can be read from the position of the point l on the contour hyperbolas numbered at the

left end. The efficiency of power conversion can be calculated from the two power values as $\eta = \frac{1}{2}I_a^2 R / E_a I_a$.

When the triode is used as a self-excited oscillator,[9] the grid voltage $\mathcal{E}_g$ is in definite relation to the anode voltage $\mathcal{E}_a$ as determined from the circuit conditions. In an example shown in Fig. 33, I_a is determined from $\mathcal{E}_g$ by a dynamic line *ops*, while I_a and external impedance $R = L/CR$ determine $\mathcal{E}_a = I_a(L/CR)$ and this induces an e.m.f. $\mathcal{E}_g = \mathcal{E}_a(M/L)$ in the grid circuit. Hence $I_a/\mathcal{E}_g = CR/M$ is a straight line, the inclination of which is $\tan^{-1} CR/M$ as shown by a line *ot* in Fig. 33. Thus the dynamic characteristic curve *ops* gives the relation in which I_a is determined by $\mathcal{E}_g$ from the tube characteristics, while the excitation line *ot* gives the relation in which $\mathcal{E}_g$ is determined by
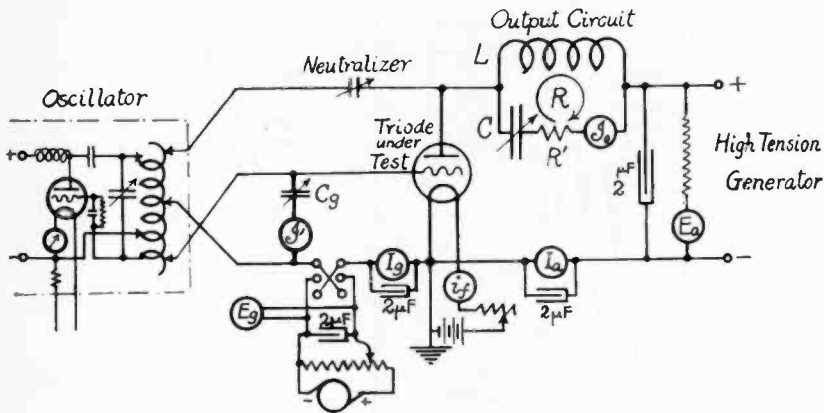
Fig. 28

I_a from circuit conditions. Then the point of intersection p shall be the working point of the self-oscillator. If the coupling becomes loose, the excitation line is shifted to left and finally will not intersect with the dynamic line, and in this case oscillation cannot be produced.

The regeneration can be explained in the same diagram.[9,10] When regeneration is applied, anode circuit is coupled to grid circuit but not so close as to produce oscillation, such as a line *ot′* in Fig. 33. If the regenerative connection is not made, for a certain grid voltage such as $\overline{ou}$, the anode current is $\overline{uv}$. But on regenerative condition, a line is to be drawn from the point u parallel to the line *ot′* and the intersecting point w on the characteristic gives the working point. The reason is that the level line from w intersecting with *ot′* at r and with the ordi-

[9] H. G. Möller, "Elektronenröhren und ihre technischen Anwendungen," 1920.

[10] E. H. Lange, *Phil. Mag.*, 750; October, 1925.

nate axis at z represents the grid excitation which is the sum of the originally impressed voltage $\overline{ou} = \overline{rw}$ and the voltage induced by the anode current $\overline{rz}$. The ratio of regenerative amplification to non-regenerative one may be expressed by $\overline{oz}/\overline{uv}$.

Now these ideas can be applied to the dynamic characteristic diagram as shown in Fig. 34. Comparing the figure with Fig. 33, resistance line oh corresponds to the ordinate axis of Fig. 33, and the abscissa representing $k\mathcal{E}_g$ corresponds to that of Fig. 33. The excitation line ot in Fig. 33 is obtained in Fig. 34 by taking the quantity $k\mathcal{E}_g/\mathbf{I}_a$ from the point h to the right on the level line as a point t, and by connecting this point to the origin; in other words $\{(k\mathcal{E}_g/\mathbf{I}_a)-R\}/\bar{r}$ shall be taken on right abscissa as $\overline{zt}$ in order to obtain the excitation line ot.
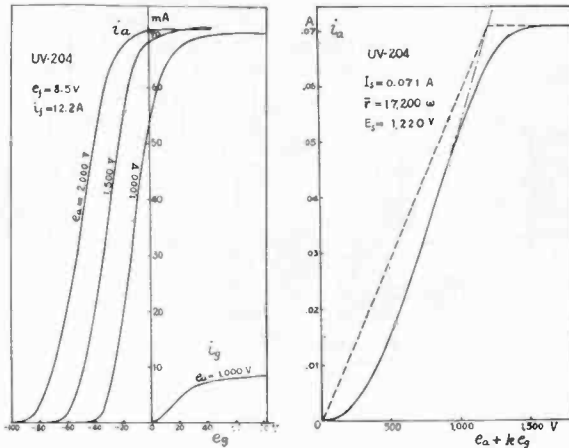
Fig. 29—Characteristics of type UV204 tube.

The point of intersection of the excitation line with the dynamic characteristic curve gives the working point p and all the other quantities can be derived in the same way as explained before. In Fig. 31 $(k\mathcal{E}_g/\mathbf{I}_a)_c$ means that the quantity is to be derived from the circuit conditions.

The modulation of a self-oscillating tube consists in altering the anode or grid steady voltage with audio frequency, hence producing the corresponding variations in the amplitude of oscillation. This means the variation of E_a+kE_g, other things being unvaried. Thus, for example, if anode voltage E_a is varied in sinusoidal wave form as shown in Fig. 35, E_a+kE_g will vary accordingly and the dynamic characteristic curve is shifted up and down. The excitation line ot remaining unvaried, the working point p must travel on the ot line and the oscil-

lating current varies its amplitude in accordance with the variation of the anode voltage.

Applications of the diagram have been further illustrated by numerical examples in the original paper. The diagram gives a bird's-eye view of various working conditions.

Part III: The Design of Triodes

Working Points on the Dynamic Characteristics

The design of a triode to satisfy specified conditions can be accomplished in the following way, which shows a typical case where the use of the triode is indicated and its power output is given.

Optimum working points can always be found on the dynamic characteristic diagram for any particular use of a triode. These optimum conditions which the writer conceives to be appropriate are given below.

TABLE X

Use of Triode	$(E_a+kE_g)/E_s$	I_a/I_s	R/r
Low power amplifier	0.3—0.5	0.2—0.3	1.0
Distortionless power amplifier or modulator	0.3—0.5	0.2—0.4	2.0
Oscillator or power amplifier for radio-telegraphy	—1.0	0.55	1.0
Self-oscillator for radio-telephony	0.3	0.35	1.2
Power amplifier for radio-telephony, amplifying modulated wave	0—0.2	0.25	1.0
Ditto, amplifying continuous wave and subjected to modulation	0	0.3	1.5

If the above three quantities are decided, the following may be directly obtained from the diagram:

$$\frac{P}{E_s I_s}, \qquad \frac{k\mathcal{E}_g}{E_s}, \qquad \frac{\mathcal{E}_a}{E_s}, \qquad \frac{I_a}{I_s}.$$

Moreover E_a/E_s can be roughly estimated according to the use of the tube and in relation to the value of $\mathcal{E}_a/E_s$, and from this the approximate values of $E_a I_a/E_s I_s$ and η can be found.

High-Impedance or Low-Impedance Tube

For a given power output P, $\alpha = P/E_s I_s$ having been known, $E_s I_s$ can be obtained and next comes an essential problem of splitting $E_s I_s$ into the saturating voltage E_s and the saturation current I_s.

If one of any other operating conditions, such as anode voltage E_a

or anode circuit impedance R, is given at the same time, the problem is directly solved.

If this is not the case, especially in the design of standard types of tubes, most precise considerations must be taken for this from technical a nd economical points of view. This is the problem of selecting
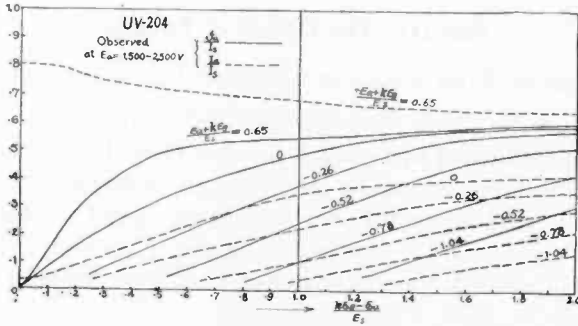
Fig. 30—Dynamic characteristics of UV204 tube obtained by experiment at radio frequency.

the type of the tube between a high-voltage low-emission type, and a low-voltage high-emission type. The general considerations on the two alternatives, high-impedance tube or low-impedance tube in other words, are as follows for a given power output:

TABLE XI

	High impedance tube	Low impedance tube	Remarks
Anode voltage	high	low	← Anode voltage must be within a practicable limit, from standpoint of insulations of the tube and other apparatus.
Saturation current	low	high	
Cathode power consumption	little	large	← Lower filament power is preferable as overall operating efficiency becomes higher and anode radiating area can also be reduced.
Circuit impedance	high	low	
Amplification constant	high	low	

Anode power conversion efficiency is not very different when it is considered on the diagram as it is determined from the working points, but in a higher voltage tube, anode voltage is more completely utilized and net efficiency will be a little higher than lower voltage tube.

This problem involves not only the economy of the tube itself but also that of apparatus with which the tube is to be used, such as output circuits, anode source of power, amplifiers preceding the tube under consideration, etc. The practicable limits in the values of E_a, R, cathode heating power, and relative amount of cathode heating power to anode loss, should also be taken into consideration.

Regarding the oscillatory power output of a triode, the following consideration is sometimes necessary. Output P, somewhat larger than
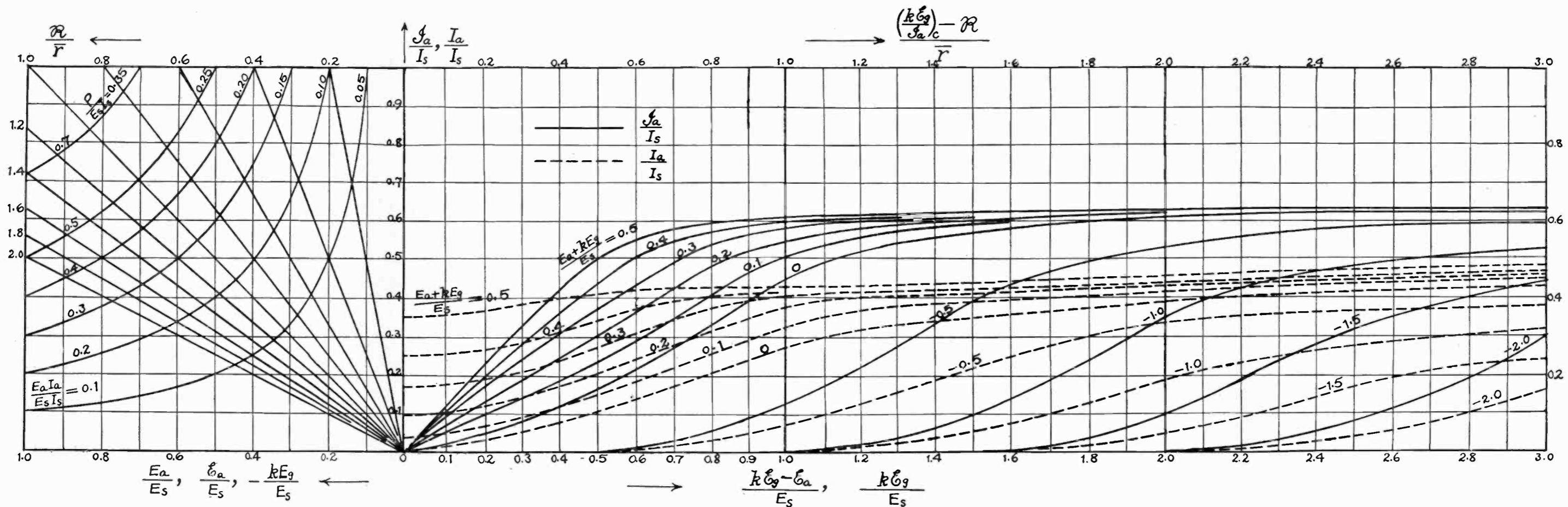
Fig. 31—Dynamic characteristic diagram.

the required value, has to be taken as the designing basis in order to take into account effect of the grid current, which was not done in the derivation of the dynamic characteristic diagram. The grid current affects the power output in two ways; first it reduces the anode current and hence the output, and secondly it absorbs some portion of the output in case of self-oscillators. An increase of 10 or 20 per cent of the required value will generally be sufficient, according as the tube is used as a power amplifier or a self-oscillator. More rigorous estimation of grid excitation loss will be given later.

AMPLIFICATION CONSTANT

From equation (15),

$$I_s = \frac{G}{(1+k)^{3/2}} E_s^{3/2} \text{ or } (1+k)^3 = G^2 E_s^3/I_s^2 = G^2 \bar{r}^3 I_s \qquad (16)$$

and

$$P = \alpha E_s I_s$$

hence

$$(1+k)^3 = \left(\frac{G\alpha}{P}\right)^2 E_s^5. \qquad (17)$$

G is not very much different with the type of tubes and is of the order as shown in the following table. This is because G is determined by the ratio of anode area to distances between electrodes, and in practical construction of the tube there are limits in the attainable anode area for a given length of the filament and for a given span of the filament the distances between electrodes must not be brought too close together in order to prevent an accidental short-circuit of the electrodes during operation.

TABLE XII

Tubes		No. of tubes examined	G	
Anode voltage	Electrodes		Range	Average
6,000–10,000	cylinder	14	$0.08-0.4\times10^{-3}$	0.15×10^{-3}
200– 5,000	cylinder	8	$0.08-0.4$	0.17
"	plane	15	$0.15-0.9$	0.38
below 100	cylinder	10	$0.12-0.5$	0.18
"	plane	5	$0.28-0.5$	0.33

Total average: G for plane electrodes, 0.35×10^{-3}
G for cylindrical electrodes, 0.16×10^{-3}.

α and P having been known, and G being assumed, k is in definite relation to E_s as given by equation (17), and once E_s is determined k is directly obtainable.

In case of oscillators, self or separately excited, power lost in the excitation depends on the value of k. In a self-excited oscillator, this power is converted from the output of the tube and too much exciting power results in the lowering of the net output and overloading of the grid. In a separately-excited oscillator or power amplifier, the grid loss
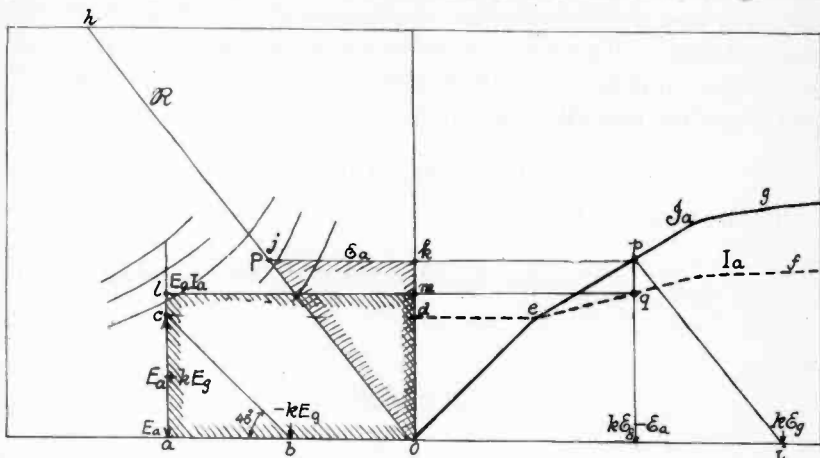
Fig. 32

is to be supplied from the preceding tube. In these cases the value of k may be estimated as follows; for the most favorable operation delivering maximum power at a given anode voltage, the minimum anode voltage $E_a - \mathcal{E}_a$ must be nearly equal to the maximum grid voltage $E_g + \mathcal{E}_g$,[11] E_g being negative, and this requires that

$$E_a - \mathcal{E}_a = E_g + \mathcal{E}_g$$

or

$$\frac{E_a}{E_s} - \frac{\mathcal{E}_a}{E_s} = \left(\frac{kE_g}{E_s} + \frac{k\mathcal{E}_g}{E_s} \right) \frac{1}{k}$$

hence

$$k = \frac{\left(\dfrac{k\mathcal{E}_g}{E_s} \right) + \left(\dfrac{E_a + kE_g}{E_s} \right) - \left(\dfrac{E_a}{E_s} \right)}{\left(\dfrac{E_a}{E_s} \right) - \left(\dfrac{\mathcal{E}_a}{E_s} \right)} \tag{18}$$

in which all quantities except E_a/E_s have accurately been known. Thus if k is given, E_a/E_s can be accurately determined and $E_a I_a/E_s I_s$ and η are accordingly known.

[11] D. C. Prince, Proc. I. R. E., **11**, 280; June, 1923.

For a rough estimation of the grid excitation loss in the above working condition, the grid current may be assumed to have a tri-angular wave form as shown in Fig. 36, and its peak value to reach
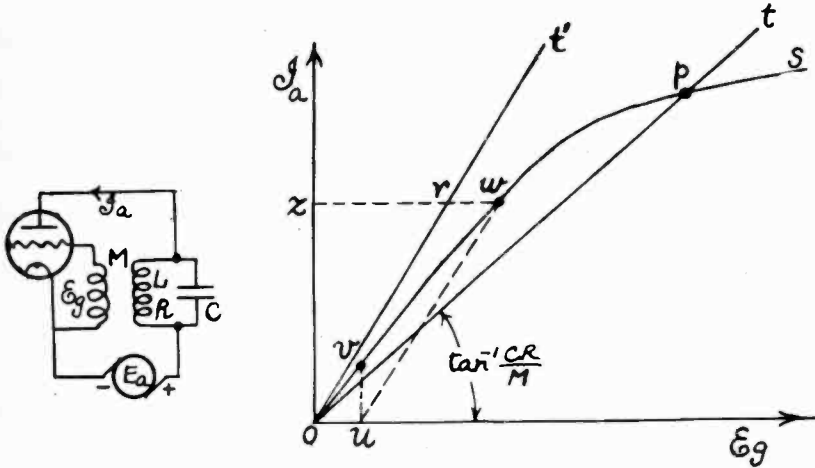
Fig. 33

$\frac{1}{2}I_s$. Of course this condition may not take place in actual case, but this will not be far from it. The results of calculations are as follows:

Grid current d-c component, $\qquad I_g = \mu I_s.$

Grid leak loss (or loss in charging the grid
biasing battery), $\qquad W_l = -\mu E_g I_s.$

Grid excitation loss, $\qquad W_e = \nu \mathcal{E}_g I_s.$

Grid loss inside the tube or grid heating
power, $\qquad W_g = (\nu \mathcal{E}_g + \mu E_g) I_s.$

Grid leak resistance required, $\qquad R_g = -E_g/I_g = -E_g/\mu I_s.$

in these relations E_g is a negative quantity, and μ and ν are the func-tions of $-E_g/\mathcal{E}_g$ or $-(kE_g/E_s)/(k\mathcal{E}_g/E_s)$ as given in Fig. 37.

If the grid loss inside the tube be allowed up to $1/b$ of the anode loss, $1/b\,(E_aI_a-P) \geqq (\nu \mathcal{E}_g + \mu E_g)I_s$ or

$$\frac{1}{b}\left(\frac{E_aI_a}{E_sI_s} - \frac{P}{E_sI_s}\right) \geqq \left(\nu\frac{k\mathcal{E}_g}{E_s} + \mu\frac{kE_g}{E_s}\right)\frac{1}{k}$$

hence

$$k \geqq b\frac{\nu\left(\dfrac{k\mathcal{E}_g}{E_s}\right) + \mu\left(\dfrac{kE_g}{E_s}\right)}{\left(\dfrac{E_aI_a}{E_sI_s}\right) - \left(\dfrac{P}{E_sI_s}\right)}. \qquad (19)$$

If, on the other hand, the grid excitation loss is to be limited within $1/d$ of the power output,

$$\frac{P}{d} \geqq \nu \mathcal{E}_g I_s \quad \text{or} \quad k \geqq d\nu \left(\frac{k\mathcal{E}_g}{E_s}\right) / \left(\frac{P}{E_s I_s}\right). \tag{20}$$

All the quantities in the parentheses have been known and the lower limit of k can be determined from these relations. The exact value of E_a/E_s must be derived by equation (18), after the value of k is determined.

It is advisable to calculate the required quantities for several values of k or E_s, and to select among them a most preferable one.

DIMENSIONS OF THE ELECTRODES

The main factor determining the anode construction is the anode loss W_a which may be taken as $(E_a I_a - P)$. But in the case of amplifiers or modulators for radiotelephony, the dead anode loss corre-
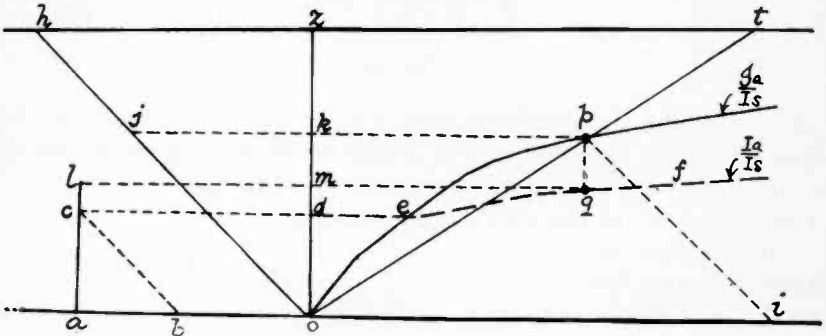
Fig. 34

sponding to the working point on the static characteristics, i.e., $GE_a\{(E_a + kE_g)/(1+k)\}^{3/2}$ must be taken as the anode loss, as this amount of loss actually takes place when speech is not being transmitted.

The allowable limit of anode loss is dependent on the material of the anode and its cooling condition. The following data giving the maximum limit of the loss continuously applicable to the anode may be taken in the ordinary construction.

TABLE XIII

Materials	Allowable loss (watt/cm²)
Nickel	$w = 3$
Molybdenum	5
Tungsten	8
Copper, water-cooled	20

The filament heating power W_f must also be considered in estimating the anode cooling area, as some portion of it will have to dissipate through the anode. Suppose q part of the filament power is radiated through the anode, then

$$A \geqq \frac{W_a + qW_f}{w} \qquad (21)$$

q depends on the degree at which the filament is hidden in the anode, and in ordinary cases in which the anode is open on both ends, q may be taken as about 0.5. The writer deems it reasonable to take the effective area A explained in Part I, instead of the actual area, in the above expression.
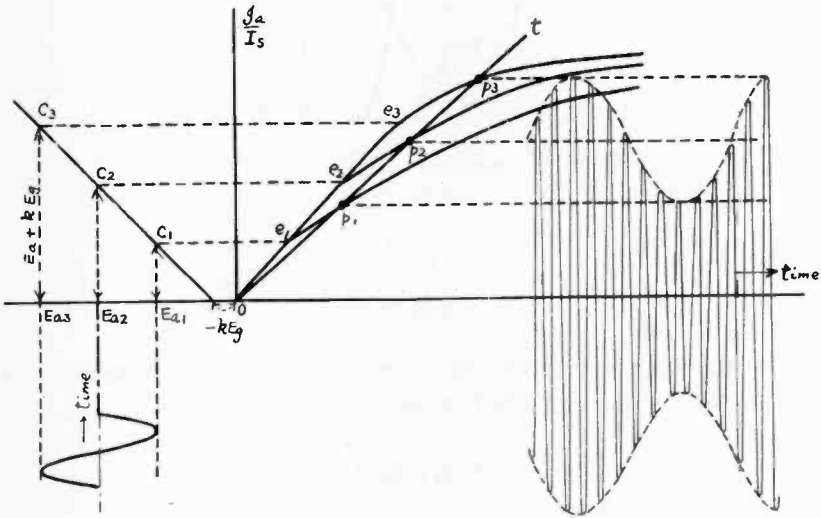
Fig. 35

In high-voltage tubes grid voltage also becomes high with respect to the cathode, and appreciable distances must be kept between grid and cathode in order to avoid severe electrostatic attraction. On the other hand, in low-voltage tubes, x_g can be reduced, and if the cathode is made of W-shaped filament arranged in a plane, A may be greatly increased without difficulty in construction. Thus for anode voltages less than say 4,000 v, plane anode is preferable, while for higher voltages cylindrical anode is advantageously applied.

The order of x_a in the existing tubes is as follows:

TABLE XIV

E_a below 100 V	$X_a = 0.2–0.5$
E_a 100–1,000	0.3–1.0
E_a 1,000–10,000	0.6–3.5

The actual anode surface area should be made as effective as possible in order to reduce the mass of the anode material and hence to minimize the absorbed gas evolving in the evacuation process, as well as from an economical point of view. The plate should be as thin as mechanically allowable, but not so thin as to avoid free conduction heat, otherwise local overheating or melting may occur at accidental overload of the anode.
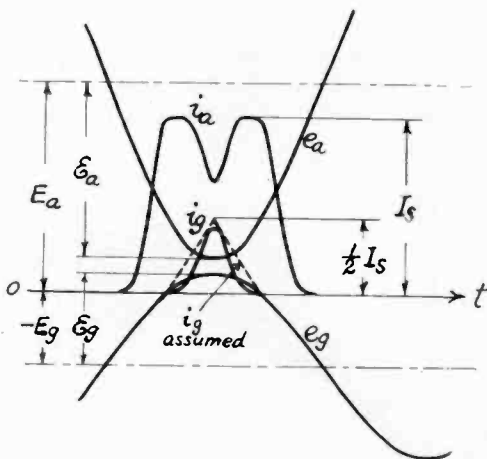
Fig. 36

The anode area A and anode-to-cathode distance x_a or z_a must be in conformity with each other to satisfy the expression,

$$G = 2.33 = 10^{-6} \frac{A}{z_a z_g}.$$

The grid construction is determined from the amplification constant. When x_a is given and other dimensions p and d_g remain unvaried, k is maximum at $x_g \doteqdot 0.4\, x_a$, and it is advisable to take this in order to reduce the mass of the grid as well as to minimize the effect of non-uniformity of x_g on the value of k. Thus

$$\frac{k}{\log \dfrac{x_a}{x_g}} = \frac{k}{0.4} = \frac{cL_g}{\log \coth \pi a}.$$

The table in Part I is to be referred to for the calculation, and ranges of actual values of L_g and πa in existing tubes are shown in Fig. 39.

L_g and πa having been determined, the next problem is to determine

p and r_g. This means a decision as to whether to use thick wires at broad pitch or to use thin wires closely arranged.

It is preferable to take πa small in order to reduce the mass of the grid and to facilitate evacuation. This is effected by thin grid wires at comparatively narrow pitch, but in extreme cases mechanical rigidity of the grid will be lost and deformation or break down will occur during
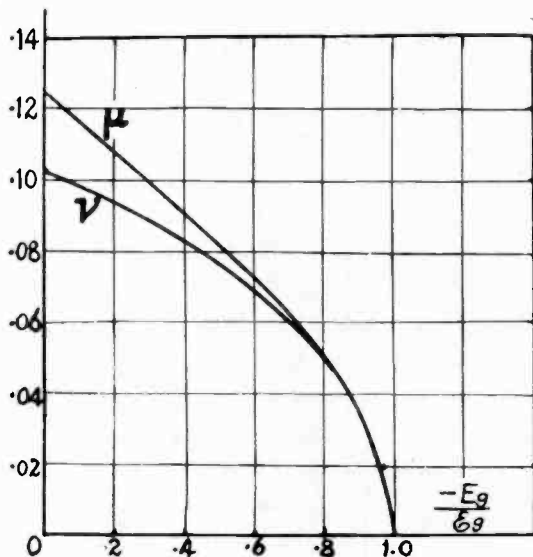
Fig. 37

operation by the heating of the grid. Fig. 38 shows actual values of p and r_g in existing tubes, from which the general tendency can be observed. Materials best suited for grid are molybdenum and nickel, but the latter has a remarkable effect of secondary emission and often causes the "blocking" of oscillators.

DESIGN OF THE CATHODE

Design of a tungsten cathode to give the required emission is fully described in the original paper, applying a chart worked out by N. Kato[12] based on the data given by Forsythe and Worthing.[13] There have been presented various other methods on the design of cathode, and no explanation is made here.

For a close approximation of the reasonable life of a tube, which in

[12] N. Kato, "Graphs for the Design of Bright-emitting Tungsten Filament," Circulars of the Electrotechnical Laboratory, No. 50, February, 1928. (Written in Japanese.)
[13] W. E. Forsythe and A. G. Worthing, *Astrophysical J.*, **61**, 146, 1925.

ideal case is the life of the cathode, an economical condition must be dealt with in such a way that the total running cost of a tube is a
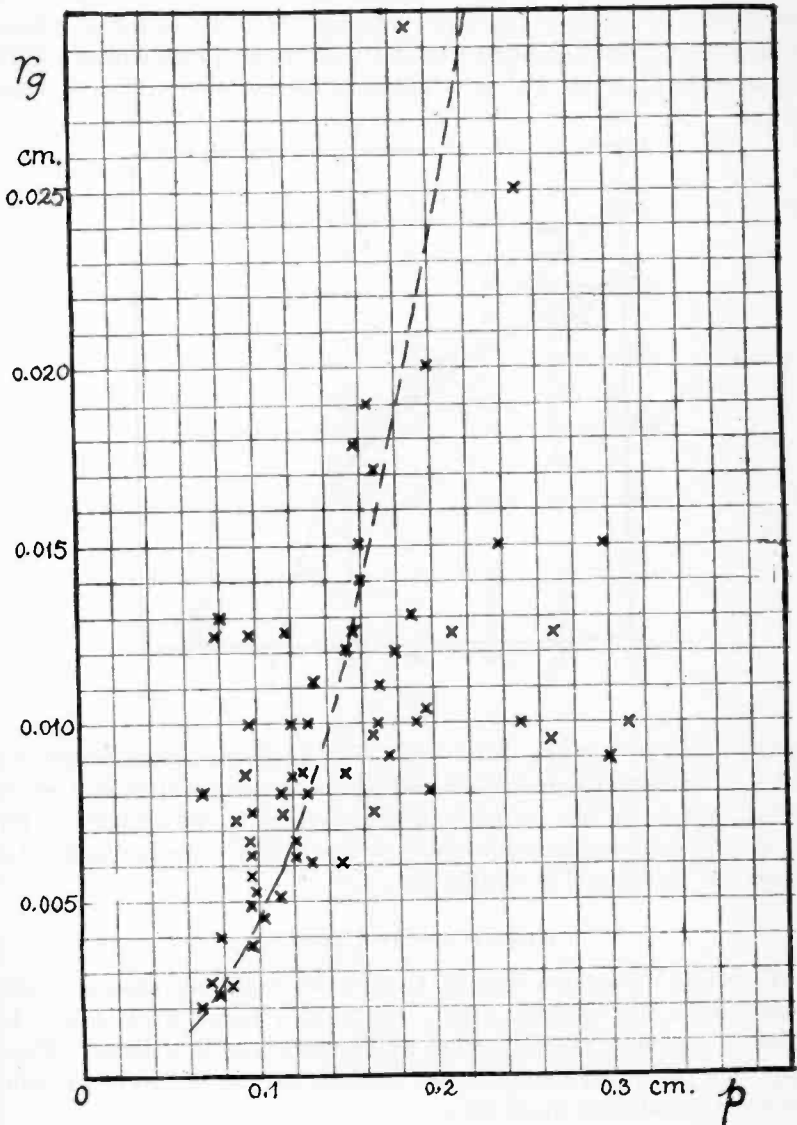
Fig. 38

minimum for a given power output. This condition is given by the following relations: total cost per watt output per hour,

$$T = \frac{C_t}{PL} + \left(\frac{1}{\eta} = \frac{W_f}{p}\right)C_g$$

in which C_t = cost of the tube, and C_p = cost of electric power per watt-hour. On the other hand, for the filament of a certain diameter the life L depends on its working temperature, which in turn is determined by the filament efficiency I_s/W_f at which it is operated, thus
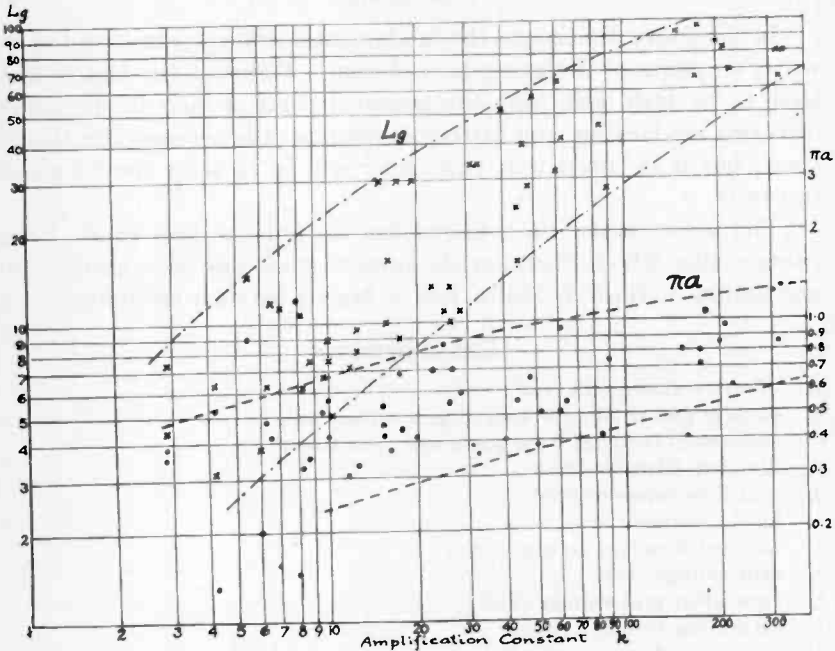
Fig. 39—Ranges of $Lg(x)$ and $\pi a(\cdot)$ in existing tubes.

$$L = \phi\left(\frac{W_f}{I_s}\right)^{\sigma}.$$

Minimum of the operating cost T occurs at a life

$$L_{opt.} = \phi^{1/1+\sigma}\left(\frac{\sigma C_t}{C_g I_s}\right)^{\sigma/1+\sigma}.$$

For tungsten filament, the two constants σ and ϕ are of the following orders,[14]

$$\sigma \doteqdot 2.6$$

d_f	0.005	0.020	0.050
ϕ	0.00034	0.0026	0.033

[14] H. Simon, *Telefunken Z.*, Fig. 44 on p. 39; October, 1927.

It is preferable to design the filament to give a slightly higher value of emission than desired in order to give allowance for the discrepancies which might take place, as the emission is quite a variable quantity.

For tungsten filaments it is sometimes necessary to take a slightly longer life than calculated, as a portion of the designed life will be lost during the evacuating process by evaporation.

CONCLUSION

In the above discussions the fundamental principles involved in the writer's system of designing is explained. Many of the factors that have to be dealt with from the practical point of view in the manufacturing are lacking, and further development is necessary in this respect, but it is hoped that this paper will be of some use to manufacturers.

The writer wishes to acknowledge his indebtedness to E. Yokoyama, under whose direction the present work has been carried out, and also to N. Kato, I. Miura, and S. Maeda for their assistance.

List of Symbols

A effective anode area (cm^2)

a ratio of grid conductor area to grid surface area

c conversion factor of k for plane-electrode tubes

d_f filament diameter (cm)

d_g grid wire diameter (cm)

e_a anode voltage (volt)

e_f filament terminal voltage (volt)

e_g grid voltage (volt)

e_g' equivalent grid voltage (volt)

E_S saturating voltage (volt)

E_a steady anode voltage (volt)

$\mathcal{E}_a$ alternating anode voltage, max. value (volt)

$\mathcal{E}_v$ alternating grid voltage, max. value (volt)

g mutual conductance (mho)

G perveance

i electron current (ampere)

i_a anode current, instantaneous value (ampere)

i_g grid current (ampere)

i_f filament current (ampere)

I_a anode mean or direct current (ampere)

I_g grid mean or direct current (ampere)

I_S saturation current (ampere)

$\mathbf{I}_a$ anode alternating current, fundamental component max. value (ampere)

k amplification constant

l life of a cathode (hour)

L_g ratio of total length of effective grid wires to axial length of the grid.

m number of points of support of the filament

n number of grid wires parallel to filament

p pitch of adjacent grid wires between centers (cm)

P power output of a triode (watt)

q portion of the filament power radiated through the anode

r anode resistance (ohm)

r mean anode resistance (ohm)

R a-c impedance (resistive) of the anode circuit (ohm)

r_f filament radius (cm)

r_g grid wire radius (cm)

s number of supports of grid wires which lie in the stream of electrons

t thickness of grid support (cm)

w permissible anode loss per unit area (watt/cm^2)

W_a anode loss (watt)

W_f filament heating power (watt)

W_g grid internal loss (watt)

W_e grid excitation loss (watt)

W_l grid leak loss (watt)

x_a distance of anode to cathode axis (cm)

x_g distance of grid to cathode axis (cm)

z_a mean shortest distance of anode to filament (cm)

z_g mean shortest distance of grid to filament (cm)

η anode power conversion efficiency

WIRELESS ECHOES OF LONG DELAY*

BY

P. O. PEDERSEN

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Summary—*The paper discusses the various possible ways in which long delayed radio echoes may be propagated. It is proved that the echoes cannot be due to waves which are propagated in any of the following manners:*

(1) Along very long paths in the highly ionized part of the earth's atmosphere.

(2) In any part of the earth's atmosphere so highly ionized that the group or signal velocity is very small.

(3) In any part of space outside the earth's atmosphere so highly ionized that the group velocity is very small.

(4) In the non-conductive layer of air between the earth's surface and the lower surface of a highly conductive layer in the higher atmosphere.

In all of these four cases the attenuation will be altogether prohibitive for all echoes with more than 10 seconds retardation.

In case (3) a pure electron atmosphere would not give any attenuation, but it is also proved that such an atmosphere having the necessary density will be so rapidly dispersed by the electrostatic forces that it cannot persist for any appreciable length of time. In the ionization bands which may cause these echoes the total space charge must consequently be very nearly equal to zero. And in this case the attenuation is great.

The long delayed echoes must therefore be due to signal waves having travelled very long distances (about $3.10^5 t$ km, t being the retardation in seconds, either along bounding surfaces between ionized and empty parts of space outside the earth's atmosphere but "within the earth's magnetic field," or in free space, having been subjected to one or more reflections at such boundary surfaces.

The geometry of those ionized layers which causes delays up to 30 (possibly 60) sec. will mainly be controlled by the earth's magnetic field as shown by Störmer.

Beside these echoes originating within the earth's magnetic field the writer has predicted that echoes might occur with delays of several minutes by reflections from ionization-bands outside the influence of the earth's magnetic field. Such long delayed echoes—3 min. 15 sec. and 4 min. 20 sec.—have recently been reported by Jörgen Hals.

INTRODUCTION†

WITH regard to the origin of the long delayed echoes of short radio waves—λ about 31 m—observed by Jörgen Hals, C. Störmer,[1] Balth van der Pol,[2] and others, rather great dif-

* Dewey decimal classification: R113.6. Original manuscript received by the Institute, April 17, 1929. Read before Royal Danish Society of Science, February 8, 1929.

† Footnotes in square brackets are added after the date when this paper was read before the Royal Danish Society of Science.

[1] C. Störmer, *Nature*, **122**, 681, 1928; **123**, 16, 1929. *C. R.* **187**, 811, 1928; *E. N. T.* 5, 483, 1928. ["*L'Onde Electrique*," 7, 531–532, 1928.]

[2] Balth. van der Pol, *Nature* **122**, 878, 1928. ["*L'Onde Electrique*," 7, 534–537, 1928.]

ference of opinion prevails between the various authors who have treated the question. Some, as for instance C. Störmer,[1] P. O. Pedersen,[3] and K. W. Wagner,[4,7] assume the echoes to be caused by the radio waves being reflected from or propagated along swarms or bands of electrons out in space while others, as for instance Balth. van der Pol,[2] E. V. Appleton[5] and M. von Ardenne,[6,8] assume the long delay of the echoes to be due to particular conditions existing along the path of the waves in—or bounded by—the ionized part of the earth's atmosphere. As to the manner in which the propagation of the waves should then occur in order to show such great delays—up to about 30 seconds as were at the time observed—these authors have, however, very different opinions. M. von Ardenne[6] thus assumes that the waves simply travel round the earth a sufficient number of times—some hundreds—in the Kennelly layer the attenuation of which he assumes to be so small for short waves that in spite of the great distance travelled they will arrive at the receiver with sufficient strength. Appleton[5] points out that the length of the time of delay depends upon the group velocity of the waves in the medium in question and if this velocity is small then the length of path will also be correspondingly smaller. Appleton[9] further points out, however, that the waves will be too strongly attenuated in the ionized part of the atmosphere—at least at heights less than about 600 km. But he suggests another possibility, namely that the lower boundary of the ionized layer of the upper atmosphere acts as a sharply defined reflecting "shell."[10] Balth. van der Pol[11] also suggests the long echo times to be due to very small group velocities at places where the electrons are so densely crowded that the refractive index for the waves in question approaches zero. But he hardly pays the necessary attention to the attenuation of the waves under such circumstances.

We shall later discuss these various possibilities, but since a detailed treatment of the very complicated conditions of the path is extremely difficult, and since it is hardly possible to give an account of all the geometrically possible tracks of the waves, and, finally, since we know so very little about the atmosphere above 150 km, we shall

[3] P. O. Pedersen, "Radiofoniens Aarbog 1929," 9–25 (Copenhagen, October, 1928).
[4] K. W. Wagner, *E. N. T.* 5, 483, 1928.
[5] E. V. Appleton, *Nature* 122, 879, 1928.
[6] M. v. Ardenne, *Populär Radio*, 345, Copenhagen, December, 1928.
[7] [Deslandres, *L'Onde Electrique*, 7, 532–3, 1928.]
[8] [H. S. Jelstrup, *L'Onde Electrique*, 7, 538–540, 1928.]
[9] loc. cit.
[10] [Jelstrup, loc. cit., holds a similar view of the phenomenon.]
[11] loc. cit.

try to find some criteria of general validity to throw light upon the matter.

1. The Attenuation of Electromagnetic Waves in a Homogeneous Ionized Atmosphere.

Plane electromagnetic waves propagated in a homogeneous medium are attenuated at the rate

$$e^{-\gamma_0 x + j\omega(t-x/v)} = e^{-\gamma_0 x + j\omega(t-n(x/c))} \tag{1}$$

where γ_0 is the attenuation constant and

$$v = \frac{c}{n} \tag{2}$$

is the phase velocity of the waves; $c = 3.10^{10}$ cm sec^{-1} being the velocity of the waves in empty space and n the refractive index of the medium for waves of the frequency $f = \omega/2\pi$.

The parameters γ_0 and n are determined by [12]

$$\gamma_0 = \frac{\omega}{c} \sqrt{ \sqrt{ \frac{\epsilon^2}{4} + \left(2\pi c^2 \frac{\sigma}{\omega}\right)^2 } - \frac{\epsilon}{2} } \tag{3}$$

and

$$n = \sqrt{ \sqrt{ \frac{\epsilon^2}{4} + \left(2\pi c^2 \frac{\sigma}{\omega}\right)^2 } + \frac{\epsilon}{2} } \tag{4}$$

where ϵ is the dielectric constant in *esu* of the medium and σ its conductibility in *emu*

From (3) and (4) it follows that [13]

$$n\gamma_0 = 2\pi c\sigma. \tag{5}$$

A signal "carried" by a wave of frequency $\omega = 2\pi f$ travels with the velocity u determined by [14]

$$u = \frac{c}{n + \omega \dfrac{dn}{d\omega}} = \frac{nc}{n^2 + \dfrac{1}{2}\omega \cdot \dfrac{d(n^2)}{d\omega}} = \frac{nc}{g}; \; g = n^2 + \frac{1}{2}\omega \cdot \frac{d(n^2)}{d\omega}. \tag{6}$$

The attenuation of a wave which has travelled the distance x is according to (1) determined by

$$e^{-\gamma_0 x} \tag{1'}$$

[12] P. O. Pedersen, "Propagation of Radio Waves," p. 117. (G. E. C. Gad. Copenhagen, 1927.) Cited in the following as *P. R. W.*
    [13] *P. R. W.*, p. 118 (6a).
    [14] *P. R. W.*, p. 174 (68).

and the attenuation of a signal having travelled t seconds is consequently

$$e^{-\gamma_0 \mu t} \tag{7}$$

since $x = u \cdot t$.

According to (5) and (6) we have

$$\gamma_0 x = \gamma_0 u t = \frac{\gamma_0 n c}{n^2 + \dfrac{1}{2}\omega \cdot \dfrac{d(n^2)}{d\omega}} \cdot t = \frac{2\pi c^2 \sigma}{g} \cdot t = \frac{t}{T_0} \tag{8}$$

where

$$\tau_0 = \frac{g}{2\pi c^2 \sigma} \tag{9}$$

is the time the signal wave must travel to have its amplitude decreased in the ratio e^{-1}.

The above formulas are valid generally for homogeneous media in which the attenuation of the waves is not excessively high. If the medium is an ionized atmosphere with n ions (electrons) per cm³ and these are on the average exposed to ν collisions per second, then the dielectric constant of the medium and its conductibility are determined by [15]

$$\epsilon = 1 - N \cdot 4\pi \frac{e^2}{m} \cdot \frac{1}{\omega^2 + \nu^2} \;[\text{esu}] \text{ and } \sigma = N \cdot \frac{e^2}{mc^2} \cdot \frac{\nu}{\omega^2 + \nu^2} \;[\text{emu}] \tag{10}$$

and these formulas are valid in the cases in which we may neglect the influence of the magnetic field of the earth.

By inserting

$$k = N \cdot 2\pi \frac{e^2}{m} \cdot \frac{1}{2} = \kappa \omega^2 \quad \text{or} \quad \kappa = N \cdot 4\pi \cdot \frac{e^2}{m\omega^2} \tag{11}$$

the equations (10) will be

$$\epsilon = 1 - \frac{2k}{\omega^2 + \nu^2} = 1 - \kappa \frac{\omega^2}{\omega^2 + \nu^2} \text{ and } 2\pi c^2 \frac{\sigma}{\omega} = \frac{\nu}{\omega} \frac{k}{\omega^2 + \nu^2} = \frac{\frac{1}{2}\kappa\omega\nu}{\omega^2 + \nu^2}. \tag{10'}$$

From (4) and 10) we then find [16]

[15] *P. R. W.*, p. 121, (11).
[16] *P. R. W.*, p. 172, (55).

$$g = n^2 + \frac{\omega}{2} \cdot \frac{d(n^2)}{d\omega} = \frac{1}{2}\left\{1 - \frac{\kappa\omega^2\nu^2}{(\omega^2+\nu^2)^2}\right.$$

$$\left. + \frac{\left[(1-\kappa)\omega^2+\nu^2\right]^2 - \left[1-\frac{1}{2}\kappa\right]\frac{\kappa\omega^2\nu^2}{(\omega^2+\nu^2)^2}}{\sqrt{\left[(1-\kappa)\omega^2+\nu^2\right]^2+\kappa^2\omega^2\nu^2}}\right\} \quad (12)$$

For $(1-\kappa)\omega^2+\nu^2 \gg \kappa\omega\nu$ or $\kappa \ll \omega^2+\nu^2/\omega^2+\omega\nu$ we get

$$g \cong 1 - \frac{1}{2} \cdot \frac{\kappa\omega^2\nu^2}{(\omega^2+\nu^2)^2} \cdot \frac{(2-\frac{3}{2}\kappa)\omega^2+(2-\frac{1}{2}\kappa)\nu^2}{(1-\kappa)\omega^2+\nu^2} . \quad (13)$$

If further $\nu \ll \omega$, then we have

$$g \cong 1 - \kappa\frac{\nu^2}{\omega^2} \cdot \left(1 + \frac{\kappa}{4(1-\kappa)}\right) \quad (13')$$

which for $\kappa \ll 1$ reduces to

$$g \cong 1 - \kappa\frac{\nu^2}{\omega^2} . \quad (13'')$$

In all cases where $\kappa \ll \omega^2+\nu^2/\omega^2+\omega\nu$, $g=1$ will consequently give a good approximation.

For $\kappa = 2$ we have

$$g = \frac{\nu^4}{(\omega^2+\nu^2)^2} \underset{\longrightarrow}{\cong} \nu^4 _{-4\omega} \quad (13''')$$
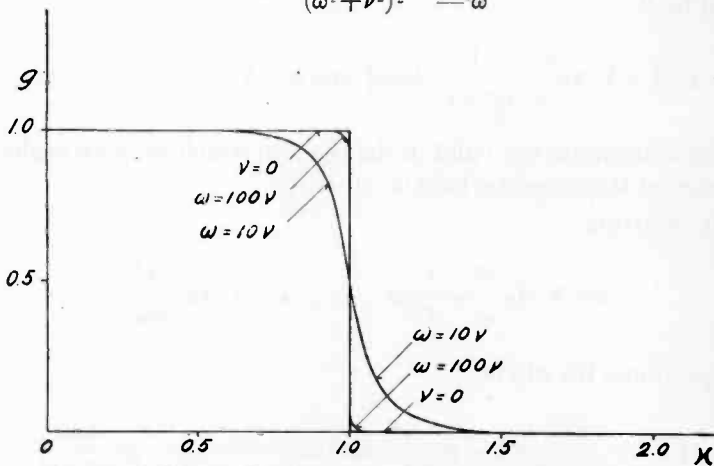
Fig. 1—The factor g as a function of κ.

Fig. 1 shows g as a function of κ and we see that for $\nu \leqq 10^{-2} \cdot \omega$ we get with good approximation $g=1$ for $0 < \kappa < 1$ while g is very nearly zero for $\kappa > 1$. For $\kappa=1$ we get $g \cong 1/2$.

From (6) it appears that for $g = 1$ the group velocity is proportional to n. Therefore it will be of some interest to determine the smallest value which n can assume. By means of the equations (4), (10), and (11) we easily deduce that n will have minimum value for $\kappa = 2$ and that

$$n_{\min} = \frac{\nu}{\sqrt{\omega^2 + \nu^2}} \cong \frac{\nu}{\omega} \tag{14}$$

while the corresponding value of the dielectric constant will be

$$\epsilon = \frac{\nu^2 - \omega^2}{\nu^2 + \omega^2} \cong -1 . \tag{15}[17]$$

Another point of particular interest is the one corresponding to $\kappa = (\omega^2 + \nu^2)/\omega^2 \cong 1$, here we get $n \cong \sqrt{\nu/2\omega}$ and $\epsilon = 0$.

The density of electrons [18] corresponding to the various values of κ is according to (11) determined by

$$N = \frac{\kappa \omega^2}{4\pi \dfrac{e^2}{m}} = 3.14 \cdot 10^{-10} \cdot \kappa \cdot \omega^2 \text{ electrons per cm}^3 . \tag{16}$$

The refractive index is thus independently of the air pressure minimum for

$$N = 6.29 \cdot 10^{-10} \cdot \omega^2 = 2N_0 . \tag{17}$$

Assuming that $\nu \ll \omega$ we further get $\epsilon \cong 0$ for $N = N_0 = 3.14 \cdot 10^{-10} \cdot \omega^2$.

In Table I is given a view of the constants which are of interest for the propagation problem here considered. Besides ϵ, n, and g the table contains the formulas for the group velocity u, for the constant T_0; for the attenuation constant γ_0 and for the attenuation-factor $e^{-\gamma_0 \lambda}$ which latter indicates the attenuation of the amplitude by the

[17] In the deduction of (14) and (15) no account is taken of the influence of the earth's magnetic field on the conductibility and on the dielectric constant. If we do so then we get (see *P. R. W.*, p. 122) that for

$$\kappa = 2\frac{\omega - h}{\omega} \text{ we get } n_{\text{I min}} \quad \frac{\nu}{\omega - h}$$

and for

$$\kappa = 2\frac{\omega + h}{\omega} \text{ we get } n_{\text{II min}} \cong \frac{\nu}{\omega + h}$$

[18] Generally we speak only of electrons because one electron will influence the propagation of the waves about as much as 50,000 ions.

wave travelling a distance equal to one wavelength in the medium in question.

TABLE I

The values given are valid for $\nu < < \omega$; $N_0 = 3.14 \cdot 10^{-10} \cdot \omega^2$.

$\kappa = \dfrac{N}{N_0}$	ϵ	n	g	u	T_0	γ_0	$e^{-\gamma_0\lambda}$
$0 \leq \kappa < 1$	$1-\kappa$	$\sqrt{1-\kappa}$	1	$c\sqrt{1-\kappa}$	$\dfrac{2}{\kappa\nu}$	$\dfrac{\nu}{2c}\cdot\sqrt{\dfrac{\kappa^2}{1-\kappa}}$	$e^{-\pi\,\kappa\nu/(1-\kappa)\,\omega}$
$\kappa = 1$	0	$\sqrt{\dfrac{\nu}{2\omega}}$	$\dfrac{1}{2}$	$c\sqrt{2\dfrac{\nu}{\omega}}$	$\dfrac{1}{\nu}$	$\dfrac{1}{c}\cdot\sqrt{\dfrac{\omega\nu}{2}}$	$e^{-2\pi}$
$\kappa = 2$	-1	$n_{min} = \dfrac{\nu}{\omega}$	$\dfrac{\nu^4}{\omega^4}$	$\left(c\cdot\dfrac{\omega^3}{\nu^3}\right)^*$	$\left(\dfrac{\nu^3}{\omega^4}\right)^*$	$\dfrac{\omega}{c}$	$e^{-2\pi(\omega/\nu)}$

* These expressions are formally in agreement with formulas (6) and (9), but have no physical significance. See the text.

From the table it appears that for $\kappa = 2$ we find $u > c$. Since, however, we must always necessarily have $u \leq c$ the method of determining the group velocity applied has led to wrong results, and the corresponding value of T_0 is consequently also of no significance. To emphasize these circumstances both of these expressions are placed in brackets.

The reason we come to such unreasonable results at great densities of electrons—or more correctly, at great values of κ—is quite evident. In the deduction of the formula (6) for the group velocity we assumed the "signal" to be produced by the superposition of a number of continuous waves having a frequency only very slightly different from the carrier frequency $f = \omega/2\pi$. From the last column in the table it appears, however, that for $\kappa = 2$ the attenuation will be so great that the waves will lose entirely their periodical character. If for example we put $\omega = 6.10^7$ and $\nu = 6.10^3$, then by travelling one wavelength the amplitude will have decreased at the rate $e^{-2\pi.10^4}$, and in this case the formula (6) must consequently lead to unreasonable results. There is, however, no reason to enter further into this question since it is quite evident that the attenuation here is so great that waves entering a medium of the character considered within a period will be reduced practically to zero.

Even for $\kappa = 1$ where we get $u < c$, it is doubtful whether the values found for u and T_0 will be correct, since in this case the attenuation over a wavelength will be $e^{-2\pi}$. The long delayed echoes observed would consequently be unable to pass through a medium corresponding to $\kappa = 1$, and in the following we shall therefore confine ourselves to treat the transmission of signals through such media for which $0 \leq \kappa < 1$. For these we get

$$T_0 = \frac{2}{\kappa \nu} \tag{18}$$

and after t seconds the attenuation of plane waves in such a medium will be

$$e^{-t/T_0} = e^{-\kappa \nu / 2 \cdot t}. \tag{19}[19]$$

In order to get in t sec. a track length travelled by the wave of ξct instead of ct as would be the case in empty space we must have

$$\xi = \sqrt{1 - \kappa} \quad \text{or} \quad \kappa = 1 - \xi^2. \tag{20}$$

During t sec. the amplitude will decrease at the rate

$$e^{-t/T_0} = e^{-\kappa \nu t / 2} = e^{-\frac{1}{2}(1 - \xi^2)\nu t}. \tag{21}$$

If the track traversed by the wave is to be essentially shorter than what it would be during an equal time in empty space then must $\xi^2 \ll 1$. In that case the attenuation S during t sec. will be approximately

$$S = e^{-1/2\nu t} \tag{22}$$

which is exactly the expression found by Appleton.

To $\xi \to 0$ corresponds $\kappa \to 1$ and the density of electrons $N \to N_0 = 3.14 \cdot 10^{-10} \cdot \omega^2$.

Appleton assumes that for a height of 250 km above the surface of the earth we may put $\nu = 1000$. I should be inclined for this height to apply $\nu = 360$.* For $t = 10$ sec. the attenuation factors would then be respectively

$$S = e^{-5000} \quad \text{and} \quad S = e^{-1800}$$

and accordingly we would get an attenuation much too great even for delay of 10 sec. by applying even the value for ν assumed by me. For a height of 400 km Appleton assumes $S = e^{-50}$ for $t = 10$ sec. which he, however, considers as too great an attenuation; nevertheless he adds: "But if there were sufficient ionization at heights of 600 km or more, it is certain that retardation without much absorption could take place, although our inadequate knowledge of the values of ν for such regions precludes a more quantitative statement."

[19] For $\kappa \to 1$ the attenuation deduced here is in agreement with the one given by E. V. Appleton (loc. cit.).

* *P. R. W.* Appendix, p. 6, Fig. IX, 1; compare also Table III below.

In estimating the value of ν, only the collisions of the electrons with neutral molecules have, however, so far been considered, and not their collisions with other electrons or ions. The error thus introduced is small as long as the number of electrons and ions are exceedingly small in comparison to the number of neutral molecules, which will always be the case— at least for the circumstances here considered —as long as the air pressure is not extremely small. The error will, however, be very great if the air pressure approaches zero, or if the number of electrons plus ions approaches—or even exceeds—the number of neutral molecules.

We shall, therefore, now consider the free path l and the number of collisions ν of an atmosphere having solely N electrons per cm³. According to W. Schottky and H. Rothe[20] the mean free path is determined by

$$l = \frac{1}{\pi\sqrt{2}\cdot Nd^2} \tag{23}$$

where d is the "diameter" of the electron.   The latter is determined by the potential electric energy of the two colliding electrons—or of an electron and a positive (or negative) ion—being equal to $1/\eta$ of the kinetic energy of a single electron, when they are at a distance d from each other.

The mean value of the kinetic energy of a single electron will, at a temperature T^{0abs}, be determined by

$$\frac{1}{2}mU^2 = \frac{3}{2}kT = 2.058\cdot10^{-16}T, \tag{24}$$

where $k = 1.372\cdot10^{-16}$ is the Boltzmann constant.

The number of collisions ν is determined by

$$\nu = \frac{U}{l} = \pi\sqrt{2}Nd^2\sqrt{\frac{3\,k\,T}{m}} \tag{25}$$

the mean velocity of the electron—according to (24)— being

$$U = \sqrt{\frac{3kT}{m}} = 6.76\cdot10^5\sqrt{T}. \tag{25'}$$

[20] *Handb. d. Experimentalphysik*, **XIII**, 2, 41, 1928.

The diameter d will then be determined by

$$\frac{e^2}{d} = \frac{1}{\eta} \cdot \frac{1}{2} m U^2 = \frac{1}{\eta} \cdot \frac{3}{2} kT = \frac{1}{\eta} \cdot 2.058 \cdot 10^{-16} T \qquad (26)$$

or

$$d = \frac{\eta e^2}{\frac{3}{2} kT} = 1.107 \cdot 10^{-3} \frac{\eta}{T}. \qquad (26')$$

Schottky and Rothe assume as an upper limit $\eta = 100$ to which corresponds

$$d = 0.1107 \cdot \frac{1}{T} \cdot [\text{cm}]. \qquad (27)$$

By (23) and (26') the free path will generally be

$$l = 1.84 \cdot 10^5 \frac{T^2}{\eta^2 N} \qquad (28)$$

and the number of collisions

$$\nu = 3.68 \cdot \frac{\eta^2}{T^{3/2}} \cdot N. \qquad (29)$$

At for instance $T = 300$ deg. abs. we then get $\nu = 7.08 \cdot 10^{-4} \eta^2 N$.

The value $\eta = 100$ assumed by Schottky and Rothe[21] for other purposes is no doubt too great for the object here considered. $\eta = 1$ is, on the other hand, no doubt too small. The formula (29) shows, however, the necessity of fixing somewhat narrower limits for η or for the diameter d which depends on η.

For reasons which will be given in the following we shall, however, not consider collisions between two electrons but between an electron and a positive—or negative—ion, and we assume the latter to be at rest as well before as after the collision.

The angle of deflection, 2θ, of the electron (see Fig. 2) is determined by [22]

$$\operatorname{tg} \theta = \frac{e^2}{mU^2 p} \qquad (30)$$

[21] loc. cit.
[22] See, for example, *Handb. d. Physik*, **XXIV**, p. 4, 1927.

where U is the velocity of the electron and p the distance of the ion from the straight path of the electron.

By means of (25′) and (26) equation (30) is reduced to

$$\text{tg } \theta = \frac{1}{2\eta} \cdot \frac{d}{p} = \frac{\alpha}{p} \tag{31}$$

where

$$\alpha = \frac{d}{2\eta} = \frac{e^2}{3kT} = \frac{5.5 \cdot 10^{-4}}{T} . \tag{31'}$$

The velocity component of the electron after the collision in the original direction will be equal to the original velocity multiplied by

$$\xi' = 1 - 2 \sin^2 \theta = \frac{p^2 - \alpha^2}{p^2 + \alpha^2} . \tag{32}$$

That fraction of the original velocity which is deflected from the original direction is consequently

$$\xi = 1 - \xi' = \frac{2\alpha^2}{p^2 + \alpha^2} . \tag{32'}$$

By integrating this quantity over a circular area at right angles to the original direction of the electron, and having the radius p_0, we get

$$A = \int_0^{p_0} \xi \pi \cdot 2\pi p dp = 2\pi \alpha^2 \cdot \lg \frac{p_0^2 + \alpha^2}{\alpha^2} . \tag{33}$$

If we put

$$A = \pi d_0^2 \tag{34}$$

then d_0 may be taken as a kind of equivalent diameter and we get

$$d_0 = \alpha \sqrt{2 \lg \frac{p_0^2 + \alpha^2}{\alpha^2}} . \tag{35}$$

Table II contains some values of d_0/α for various values of p_0/α. d_0 increases steadily but slowly with increasing values of p_0.

TABLE II

Values of $\sqrt{2 \lg \dfrac{p_0^2 + \alpha^2}{\alpha^2}}$ for various values of $\dfrac{p_0}{\alpha}$.

$\dfrac{p_0}{\alpha}$	1	2	3	4	5	10	20	100	1000
$\sqrt{2 \lg \dfrac{p_0^2 + \alpha^2}{\alpha^2}} = \dfrac{d_0}{\alpha}$	1.18	1.79	2.15	2.38	2.55	3.04	3.46	4.29	5.26

These calculations assume, however, that along the entire curved part of the path the colliding particles are so far away from other charged particles that their movements are practically unaffected by these. Since the density of electrons, according to Table I shall be $N = N_0 = 3.14 \cdot 10^{-10} (2\pi)^2 \cdot 10^{14} = 1.24 \cdot 10^6$ electrons per cm^3 in order to reduce to a considerable extent the group velocity of the 30-m wave, and since according to (31') $a = 10^{-6}$ cm for $T = 550°$ abs. it will hardly be justifiable to assign to p_0/α a value much higher than 100 to which corresponds $d_0 = 4.29\alpha$. On the other hand, the value p_0/α should no doubt be higher than 3 to which corresponds $d = 2.15\alpha$. Since for the following it is not essential to know the exact value of d_0 we simply put

$$d_0 = 3\alpha = \frac{1.65 \cdot 10^{-3}}{T} \ [\text{cm}] \tag{35'}$$

which is 67 times smaller than the one assumed by Schottky and Rothe.
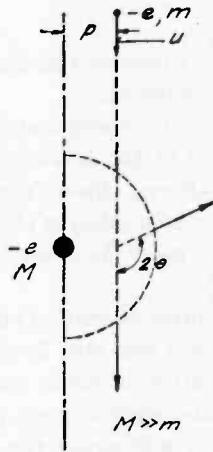
Fig. 2—Collision between an electron and a fixed negative ion.

To (35') corresponds a length of free path

$$l = 8.3 \cdot 10^4 \frac{T^2}{N} \tag{36}$$

and a number of collisions

$$\nu = \frac{U}{l} = \frac{6.7 \cdot 10^5 T^{1/2}}{8.3 \cdot 10^4 T^2} \cdot N = 8.1 \cdot \frac{N}{T^{3/2}}. \tag{37}$$

Table III shows for some values of T the corresponding values of the ratio v/N.

TABLE III
The ratio v/N calculated by means of (37). Values of v for $N = 1.2 \cdot 10^6$

$T =$	$100°$	$400°$	$900°$	$1600°$	$2500°$	$3600°$	$10000°$	$250.000°$ abs.
$\dfrac{v}{N} =$	$8.1 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$3.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$6.5 \cdot 10^{-5}$	$3.8 \cdot 10^{-4}$	$8.1 \cdot 10^{-5}$	$6.5 \cdot 10^{-6}$
$N = 1.2 \cdot 10^6; v =$	$1 \cdot 10^4$	$1.2 \cdot 10^3$	$3.6 \cdot 10^2$	$1.6 \cdot 10^2$	78	46	9.7	$7.8 \cdot 10^{-2}$

We cannot, however, insert the values of v thus obtained in the formula (22) and by this means calculate the attenuation of the waves in an atmosphere consisting only of electrons. In this case where the colliding particles are perfectly identical a collision will exert no influence at all on the propagation of the waves since we have

$$v_{1x} + v_{2x} = v_{1x}' + v_{2x}' ; \quad v_{1y} + v_{2y} = v_{1y}' + v_{2y}' ; \quad v_{1z} + v_{2z} = v_{1z}' + v_{2z}' ;$$
$$\text{and} \qquad\qquad v_1{}^2 + v_2{}^2 = v_1'{}^2 + v_2'{}^2 , \tag{38}$$

where v_1 and v_2 are the velocities of the electrons before and v_1' and v_2' their velocities after the collision.

The sum of the "current" components, viz. $-e \cdot (v_{1x} + v_{2x})$, $-e \cdot (v_{1y} + v_{2y})$, $-e \cdot (v_{1z} + v_{2z})$, and of the kinetic energy, viz. $\frac{1}{2} m \cdot (v^2{}_1 + v_2{}^2)$, of two electrons is thus not altered. *Radio waves are consequently propagated in an atmosphere consisting solely of electrons without suffering any loss and in the same manner as if there were no collisions at all between the electrons.*[23]

On account of their unstable character such pure electron "atmospheres" ("layers" or "bands") can not, however, play any important rôle in the cases of propagation of radio waves here considered. To illustrate this we will consider such a large, plane band of electrons or ions. The mutual repulsion will cause the thickness of the band to increase and consequently the density of electrons to decrease.

It is easily proved that the density q of the charge within a plane layer, see Fig. 3, part I, decreases with increasing time according to the following formula

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[23] [In a letter in *Nature* (February 2, 1929, p. 166) L. H. Thomas has also called attention to the influence which the other electrons and ions, which are present in the neighbourhood of an electron or ion, exert on the length of the effective mean free path of this electron or ion. He does not, however, mention the fundamental difference, with regard to the attenuation of radio waves, between collisions taking place between charged particles of equal charges and equal masses and collisions in which the two particles have unequal masses or charges (or both).]

$$q_t = \frac{q_0}{1 + 2\pi q_0 \cdot \dfrac{e}{m} \cdot t^2}, \quad [\text{esu}] \tag{39}$$

q_0 being the density at the time $t = 0$ and q_t the density t seconds later.

Fig. 3, part II, sections, 1, 2, and 3 show the dispersion of a plane homogeneous layer having at the time $t = 0$ a density of $N = 1 \cdot 10^6$ electrons per cm³. Section 2 shows the distribution $1 \cdot 10^{-7}$ sec. and section 3 shows $2 \cdot 10^{-7}$ sec. later.

Fig. 3 $ABCD$ represent parts of large, plane bands of electrons or ions. In part I q_0 is the density of the charge at the time $t = 0$.

From this it appears that a band of electrons having a density of 10^6 can exist only if inside the band there is a positive space charge of practically equal value, and evidently it must be mainly the less mobile positive particles which determine the geometric relations of the band.

If the positive space charge consists of mono-valent positive ions

Fig. 3—$ABCD$ represent parts of large plane bands of electrons or ions. In part Ig_0 is the density of the charge at the time $t = 0$.

then the number of "effective" collisions ν is determined simply by means of (37), N being the density of the positive and negative atomic or molecular ions, and the attenuation of the wave is found by inserting in (22) the value of ν thus obtained. If the positive and negative ions are poly-valent, then ν and accordingly also the attenuation will be even greater.

In order that a 30-m wave shall travel at a slow group velocity the electron density of the medium must, as is shown in the preceding, be about $1.2 \cdot 10^6$ electrons per cm³. For a temperature of 900 deg. abs.

according to Table III we then get $\nu = 360$. A wave travelling in such a medium for 10 seconds will, according to (22), suffer an attenuation $S = e^{-1/2\nu t} = e^{-1800}$. For $T = 1600$ deg. abs. we get $S = e^{-800}$.

From the preceding it appears that even if we pay no attention to the attenuation suffered by the waves in consequence of collisions between electrons and neutral air molecules, the radio waves will, nevertheless, in consequence of the collisions between the electrons and the necessarily existing positive ions be so greatly attenuated that the long delayed wireless echoes cannot have travelled a considerable part of the time within such an electron-ion-atmosphere.[24]

2. The Propagation of Electromagnetic Waves Along the Boundary Surface between an Ionized and a Non-Ionized Part of the Earth's Atmosphere.

Having now shown that the long delayed echoes cannot be due to the propagation of the waves *in* a space containing so many electrons that the group velocity is many times smaller than the velocity of the waves in free space, because in such a medium the attenuation will be too great, it may be appropriate to investigate whether the long delays may be due to the fact that the waves are propagated along the boundary between such a medium and either empty space or an non-ionized air space. A smaller attenuation would be possible in either case, since the loss of energy in the ionized part could be compensated for by the field energy in the non-ionized part. Something similar occurs in a very pronounced manner in the propagation of the waves along the surface of the earth.[25] We shall therefore consider now a little more closely the propagation of the waves along such a plane boundary surface.

The boundary surfaces will, of course, generally be curved, but the radii of curvature must at all events be very great if propagation along such surfaces shall be of any significance at all for the cases here considered. Further the concavity of the boundary surface must face toward the earth and toward the non-ionized part of space, and will

[24] Even if the long delayed echoes cannot be due to propagation in media having a very small group velocity, such propagation may play an important rôle in the various methods of determining the altitude of the highest point of the path of radio rays, often called the height of the ionized layer, and may have caused some of the discrepancies between the values which various experimenters have found for this altitude, insofar as the discrepancies are not due to "reflections" from various "layers." This question is treated of in *P. R. W.*, pp. 171–178, 209–212, and the theory is further developed by E. V. Appleton (*Proc.* Phys. Soc., 41, 43–56, 1928) and J. C. Schelleng (Proc. I. R. E., 16, 1471–76; November, 1928), whose papers will be a great help to the investigation of this problem in the future.

[25] *P. R. W.*, chap. III, p. 22 and 29–32.

thus serve to collect the radiation-energy. The border between the conductive and the non-conductive part of space will never be perfectly sharp because some electrons and ions will penetrate into the otherwise nonconductive part. Under such conditions the waves will be refracted and show a tendency to follow the curved boundary surface.

The conditions existing at a plane boundary surface will, no doubt, give an approximately correct representation of the actual conditions. Considerable mathematical difficulties would also be met with in the treatment of a propagation along a curved surface, while the problem of propagation along a plane boundary surface, as shown by J. Zenneck[26], may be solved in a simple manner.

By considering the boundary surface to be an XY-plane and then assuming the plane waves to travel along the plane in the positive
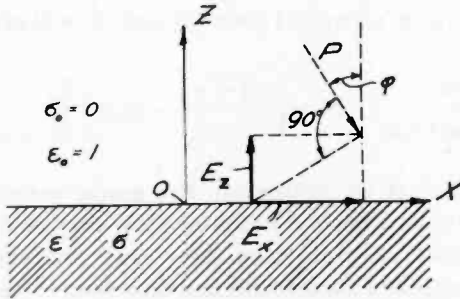
Fig. 4—Propagation of waves along the boundary surface XY. In the upper medium the energy is propagated in the direction indicated by the arrow P.

direction of the x-axis (see Fig. 4) attenuation along the boundary surface will be determined by

$$e^{-\gamma_0 x} \tag{1}$$

If we call the phase velocity along the surface v then γ_0 and v can be determined by the following equation[27]

$$\gamma_0 + j\frac{\omega}{v} = j\frac{\omega}{c}\sqrt{\frac{1+j\dfrac{\omega}{\sigma}\cdot\dfrac{\epsilon}{4\pi c^2}}{1+j\dfrac{\omega}{\sigma}\cdot\dfrac{1+\epsilon}{4\pi c^2}}} \tag{2}$$

[26] J. Zenneck, *Ann. d. Phys.* (4) **23**, 846, 1907. *P. R. W.*, chap. III.
[27] *P. R. W.*, p. 17 (the symbols used are different).

provided the medium (0) has the dielectric constant $\epsilon_0 = 1$ and the conductivity $\sigma_0 = 0$, while the corresponding constants for the ionized medium are ϵ and σ [emu.].

Let the X and Z components of the electric field at the boundary surface be E_x and E_z; then we have

$$\left|\frac{E_x}{E_z}\right|_{z=+0} = \sqrt{\frac{j\dfrac{\omega}{\sigma}\cdot\dfrac{1}{4\pi c^2}}{1+j\dfrac{\omega}{\sigma}\cdot\dfrac{\epsilon}{4\pi c^2}}} \tag{3}$$

where the index $z = +0$ indicates that the field components shall be taken in the insulated medium but immediately at boundary surface $z = 0$.

If $0 < \epsilon < 1$ and $\omega/\sigma \cdot \epsilon/4\pi c^2 \gg 1$ then (2) and (3) will give

$$\gamma_0 = \frac{2\pi c\sigma}{(1+\epsilon)\sqrt{\epsilon(1+\epsilon)}}, \quad v = c\sqrt{\frac{1+\epsilon}{\epsilon}} = u \text{ and } \left|\frac{E_x}{E_z}\right|_{z=+0} = \sqrt{\frac{1}{\epsilon}} \tag{4}$$

Since here there is no dispersion, the group velocity is equal to the phase velocity, and both are greater than the velocity c in empty space. This is quite natural, since the energy propagation does not occur along the boundary surface but in a direction P (see Fig. 4) which is perpendicular to the resultant of E_x and E_z. If ϵ is extremely small, then the energy propagation will take place practically perpendicular to the boundary surface.

For the attenuation exponent we get (compare equation (10′) in sect. 1)

$$\gamma_0 x = \gamma_0 u t = \frac{2\pi c^2\sigma}{\epsilon(1+\epsilon)}\cdot t = \frac{\dfrac{1}{2}\kappa\dfrac{\omega^2\nu}{\omega^2+\nu^2}}{\left(1-\kappa\dfrac{\omega^2}{\omega^2+\nu^2}\right)\left(\cdot 2 - \kappa\dfrac{\omega^2}{\omega^2+\nu^2}\right)}\cdot t \cong \frac{\dfrac{1}{2}\kappa\nu t}{(1-\kappa)(2-\kappa)} \tag{5}$$

where the last term is correct only for $\nu \ll \omega$.

In order to obtain a small attenuation exponent the smallest possible value of κ should be chosen. If $\kappa \ll 1$ then for (5) we get

$$\gamma_0 x = \gamma_0 u t = \tfrac{1}{4}\kappa\nu t. \tag{5′}$$

We will later come back to the application of this result.

Next we will consider the case where the density of electrons is so great that $\kappa > 2$, and we therefore have

$$\epsilon = 1 - \kappa \qquad \frac{\omega^2}{\omega^2 + \nu^2} < 0.$$

We then get

$$\frac{\omega}{\sigma} \cdot \frac{\epsilon}{4\pi c^2} = \frac{\omega^2 + \nu^2}{\kappa \omega \nu} - \frac{\omega}{\nu} \quad \text{and} \quad \frac{4\pi c^2 \sigma}{\omega} = \kappa \frac{\omega \nu}{\omega^2 + \nu^2}.$$

For $\nu \ll \omega$ by means of (2) we get

$$\gamma_0 = \frac{\nu}{2c} \cdot \frac{\kappa}{(\kappa-2)^{3/2} \cdot (\kappa-1)^{1/2}} \quad \text{and} \quad v = c\sqrt{\frac{\kappa-2}{\kappa-1}} = u. \tag{6}$$

The attenuation exponent will then be

$$\gamma_0 x = \gamma_0 u t = \frac{\kappa \nu}{2(\kappa-1)(\kappa-2)} \cdot t \cong \frac{\nu}{2\kappa} \cdot t \tag{6'}$$

where the last term holds good for $\kappa \gg 1$.

Finally, if $\nu \gg \omega$ then

$$\gamma_0 = \frac{\kappa \omega^2}{4\sqrt{2} \cdot c\nu}, \quad \nu = c\sqrt{2} = u \tag{7}$$

to which corresponds

$$\gamma_0 x = \gamma_0 u t = \frac{\kappa}{4} \cdot \frac{\omega^2}{\nu} \cdot t. \tag{7'}$$

For a judgment of the conditions we must know how rapidly the amplitude of the waves decrease in the non-conductive medium and in a direction perpendicular to the boundary surface.

Using the symbols from Fig 4 this decrease is determined by

$$e^{-\nu_0 z}, \tag{8}$$

where r_0 and the phase velocity of the waves w in the direction of the Z-axis is determined by [28]

$$r_0 + j\frac{\omega}{w} = j\frac{\omega}{c} \sqrt{\frac{j\frac{\omega}{\sigma} \cdot \frac{1}{4\pi c^2}}{1 + j\frac{\omega}{\sigma} \cdot \frac{1+\epsilon}{4\pi c^2}}}. \tag{9}$$

[28] *P. R. W.*, p. 17.

TABLE IV.

The attenuation-exponents for the Propagation of Plane Waves along a Boundary Surface between an Ionized and a Non-ionized Part of Space.

Assumptions $\epsilon_0 = 1;\ \sigma_0 = 0$			Attenuation exponent $\gamma_0 z = \gamma_0 ut$		Group-velocity u	Attenuation-constant r_0	$\left\|\dfrac{E_z}{E_x}\right\|_{z \to +0}$	$\phi = \text{arctg}\left\|\dfrac{E_z}{E_x}\right\| = \tan^{-1}\left\|\dfrac{E_z}{E_x}\right\|$
			ν	$\nu = \kappa \nu t$				
$0 < \epsilon < 1$	$\nu \ll 3$	$0 < \kappa < 1$	$\dfrac{\kappa \nu}{2(1-\kappa)(2-\kappa)} \cdot t$	$\dfrac{\kappa^2 \nu_1}{2(1-\kappa)(2-\kappa)} \cdot t$	$c\sqrt{\dfrac{1+\epsilon}{\epsilon}}$	$\dfrac{\kappa \nu}{2c(2-\kappa)^{3/2}}$	$\sqrt{\dfrac{1}{1-\kappa}}$	$\text{arctg}\sqrt{1-\kappa}$ $= \tan^{-1}\sqrt{1-\kappa} < 45°$
	$(\epsilon \lesssim 1-\kappa)$	$0 < \kappa \ll 1$	$\dfrac{1}{4}\kappa \nu t$	$\dfrac{1}{4}\kappa^2 \nu_1 t$	$c\sqrt{\dfrac{1+\epsilon}{\epsilon}} \leqq c\sqrt{2}$	$\dfrac{\kappa \nu}{4\sqrt{2} \cdot c}$	1	$45°$
$\epsilon < -1$	$\nu \ll 3$	$\kappa \gg 2$	$\dfrac{\nu}{2\kappa} \cdot t$	$\dfrac{\nu_1}{2} \cdot t$	$\sqrt{\dfrac{\kappa-2}{\kappa-1}}$			
	$\nu \gg 3$	$\kappa > 2$	$\dfrac{\kappa \omega^2}{4\nu} \cdot t$	$\dfrac{\omega^2}{4\nu_1} \cdot t$	$c\sqrt{2}$			

For $\kappa < 1$ and $\nu \ll \omega$ we consequently get

$$r_0 = \frac{\kappa\nu}{2c(2-\kappa)^{3/2}}. \tag{10}$$

The values of r_0 and of the ratio $\left|\frac{E_x}{E_z}\right|_{(z=+0)}$ and further of $\phi = \text{arctg}$ $\left|\frac{E_x}{E_z}\right|$ as well as the above obtained results are shown in Table IV.

From this table it appears that the attenuation becomes very great for $\epsilon < -1$ ($\kappa > 2$) both for $\nu \ll \omega$ and still more for $\nu \gg \omega$. In both cases it is so great that it is out of question that the corresponding wave propagation can form the basis of the long delayed echoes. A closer consideration further shows that the attenuation exponent becomes very great within the interval $0 > \epsilon > -1$ so that this assumption also is out of question.

We then only have the interval $0 < \epsilon < 1 (1 > \kappa > 0)$. For $\kappa \to 1$ and consequently $\epsilon \to 0$ we have $\gamma_0 u t \to \infty$ while for $\kappa = 1/2$ and $\epsilon = 1/2$ the attenuation-exponent $\gamma_0 u t = \nu_1/6 \cdot t$. Taking $\nu_1 = 360$ and $\iota = 10$ sec. we get an attenuation-factor $S = e^{-600}$ while $\kappa = 1/4$ and $\epsilon = 3/4$ gives $S = e^{-85.7}$ for $t = 10$ sec. In order to obtain by the waves propagated in this manner the long delayed echoes we must consequently have $0 < \kappa < 1/4$ and accordingly $3/4 < \epsilon < 1$.

From Table IV we learn that the attenuation-exponent $\gamma_0 u t$ along the boundary surface will be the smaller, the smaller we have the value of κ. We cannot, on the other hand, have $\kappa = 0$, as in that case we should have no boundary surface at all and therefore the entire energy of the waves would disappear out into space. We are unable to fix any definite limit for the value of κ. However, if the waves must be able to travel several hundred times round the earth, which is necessary in order to obtain echoes up to 10 sec. or even more, then the refractive index $n = 1 - \Delta n$ of the ionized layer must assume such small values that waves leaving the transmitter in a horizontal direction are totally reflected at the boundary surface. At a smaller value of the reduction Δn of the refractive index practically all of the energy radiated would proceed out into space and get lost. We therefore must have

$$\Delta n \geqq \frac{h}{R} \tag{11}$$

where h is the height of the boundary surface above the earth and R the radius of the earth.

Since

$$n = 1 - \Delta n \leqq \sqrt{\epsilon} \leqq \sqrt{1-\kappa} \leqq 1 - \tfrac{1}{2}\kappa$$

we get

$$\kappa = 2\Delta n \geqq 2\frac{h}{R} . \tag{12}$$

Taking $h = 160$ km and $R = 6400$ km we find by means of (11) and (12) that $\kappa \geqq 1/20$ and the corresponding value of the attenuation exponent for $t = 10$ sec. is, according to Table IV

$$\gamma_0 ut = \tfrac{1}{4}\kappa^2\nu_1 t = 2.25 \tag{12'}$$

for $\nu_1 = 360$. The attenuation factor will then be $S = e^{-2.25}$.

This attenuation is so small that it will be of no importance in the case here considered; and therefore such a manner of propagation would appear to be a solution of the question and may be so for ionization bands in free space, see sect. 4 below. We shall, however, immediately show that this is not the case for a wave travelling some hundreds of times round the earth. To realize this we must, however, consider the field energy relations in the non-conductive medium during the wave propagation in question. According to Table IV the attenuation constant r_0 (compare (8) and (10)) for $\kappa = 1/20$ and for $\nu_1 = 360$ is

$$r_0 = \frac{\kappa^2\nu_1{}^2}{4\sqrt{2}\cdot c} = 5.3 \cdot 10^{-12}\left[\text{cm}^{-1}\right] = 5.3 \cdot 10^{-7}\left[\text{km}^{-1}\right]$$

and for $\kappa = 1/4$

$$r_0 = 1.3 \cdot 10^{-5}\left[\text{km}^{-1}\right].$$

In the derivation of Zenneck's formulas for wave propagation along a plane surface, the non-conductive space is assumed to be of infinite height perpendicular to the boundary surface. The total electromagnetic field energy in this space is equal to the field energy of a layer of the height H_0 with a uniformly distributed field having all over the same intensity as that found at the boundary-surface, where H_0 is determined by

$$H_0 = \int_0^\infty e^{-2r_0z}\cdot dz = \frac{1}{2r_0} . \tag{13}$$

For $\kappa = 1/20$ we have $H_0 = 9.4 \cdot 10^5$ km, and for $\kappa = 1/4$ we have $H_0 = 3.8 \cdot 10^4$ km. If we assume the boundary surface to be at a height of 160 km above the earth the total energy will be respectively $940 \cdot 10^3/160 = 5900$ and $38 \cdot 10^3/160 = 240$ times smaller than assumed. The effective attenuation will, therefore, be very much greater than the value 2.25 according to (12').

The conditions are rather complicated, partly owing to the fact that the boundary surface is not plane but spherical, and partly owing to the presence of the earth's surface. We cannot therefore directly infer that the above found values of the attenuation exponents, namely $\gamma_0 u t_0 = 85.7$ and 2.25 shall be multiplied respectively by $1/2 \cdot 240 = 120$ and $1/2 \cdot 5900 = 2950$ which would raise their values to respectively 10280 and 6700. The correct figures are, however, not essential in this connection but only the fact, which we can easily derive, that the attenuation with this manner of propagation would be so great that a wave propagation subject to such conditions cannot form the basis of the long delayed echoes.
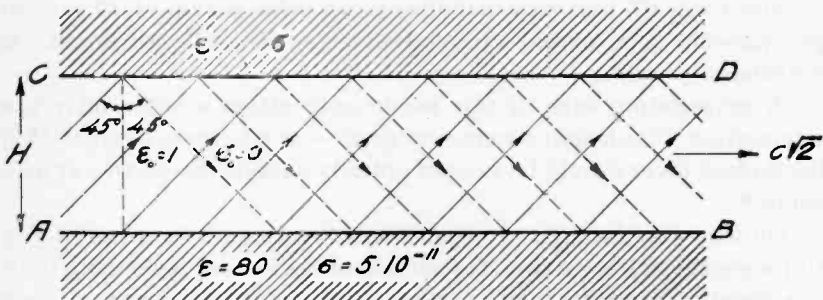
Fig. 5—Schematical representation of the propagation of radio waves between the surface A-B of the earth and an ionized layer C-D. Assuming the height H to be infinitely great, the waves would travel in the direction shown in full drawn lines forming an angle of incidence $\kappa \lesseqgtr 45$ deg. at a very small value of κ.

If the waves were propagated in that manner then the longer waves as for instance 60 and 90 m would at all events be propagated with a smaller attenuation and consequently give longer delays. Further, echo should generally be obtained after each passage round the earth. Both of these consequences are contradictory to experience and we consequently consider it as impossible that the long delayed echoes are produced in that manner.

This result is also verified by a consideration of Fig. 5 where the full drawn lines show the direction of the energy propagation under the assumption that within the limited layer between the earth's surface $A - B$ and the boundary surface $C - D$ the propagation occurs in the same manner as in unlimited space. The actual propagation will evidently suffer very great losses partly by reflection (at an angle of incidence of about 45 deg.[29] from the ionized layer, partly by reflec-

[29] Compare *P. R. W.*, p. 140.

tion of the wave from, and its propagation along, the surface of the earth.

The most favorable case would be if the propagation occurred between a perfectly even ocean-surface and a perfectly conductive outer shell at a height $h(\cong 160$ km). If we further assume the wave energy to be uniformly distributed between the earth's surface and the outer shell then at $\omega = 6 \cdot 10^7 (\lambda \cong 30m)$ we should get an attenuation constant γ_0 with a value of [30]

$$\gamma_0 \cong \frac{5 \cdot 10^{-3}}{h} \cong 3.1 \cdot 10^{-5} [\text{km}^{-1}].$$

For $x = 3 \cdot 10^6$ km corresponding to an echo period of 10 sec. we get $\gamma_0 x = 93$. The actual attenuation exponent will, no doubt, be considerably greater.

A propagation such as this would only attain a sufficiently low attenuation if the height h were very great —at all events so great that the ionized layer should be located entirely outside the earth's atmosphere.[31]

On the other hand, a propagation in the manner here considered, and repeated reflection from the surface of the ionized layer contribute in a great measure to the remarkably efficient transmission of short waves in the earth's atmosphere which we know from experience. We have in fact that while the direction of the wave-ray in the non-conductive medium forms the angle $\psi = 90° - \phi$ with the boundary-surface, where

$$\text{tg } \psi_0 = \left| \frac{E_x}{E_z} \right|_{z=+0} \cong \sqrt{\frac{1}{\epsilon}} \tag{14}$$

the ray will in the ionized medium form an angle ψ with the boundary surface, where

$$\text{tg } \psi \cong \sqrt{\epsilon}, \tag{15}[32]$$

so that $\psi \ll \psi_0$ when $\epsilon \ll 1$.

For refraction, the direction of the wave ray in the two media will be determined by

$$\sin (90° - \psi_0) = \sqrt{\epsilon} \cdot \sin (90° - \psi). \tag{16}$$

[30] *P. R. W.*, p. 32, Fig. III, 9.
[31] With regard to the limit of the earth's atmosphere we may refer to A. Veronnet, "Constitution et Evolution de l'Univers," p. 153 (Paris, 1927).
[32] *P. R. W.*, p. 18, Table I.

In both cases there is thus a pronounced tendency to transfer the electromagnetic energy from the non-conductive medium to the ionized medium, and further, for the direction of the wave ray in the latter to become parallel to the boundary surface. And this tendency must exist even if the transition from the one medium to the other may not be quite sharp but changes gradually.

The well known far-reaching ability of the short waves may thus be satisfactorily explained by means of the propagation here discussed, but this manner of propagation does not afford an explanation of the long delayed echo signals.

3. The Propagation of Radio Waves in a Space between an Upper Layer, having a Sharply Defined Lower Boundary—and the Surface of the Earth.[33]

This problem has been partly treated in the preceding, but must be considered a little more closely. A ray SP (see Fig. 6) leaving the transmitter S strikes at the point P a part of the upper boundary surface which is not quite concentric with the earth's surface but having such an acclivity that the reflected ray PQ does not come quite down to the earth. If the upper boundary surface taken as a whole is otherwise a spherical shell concentric with the earth's surface, then such a ray may travel on in the manner referred to without ever returning again to the earth. If the intervening space is perfectly non-conductive then the rays may travel round the earth several times since comparatively little attenuation will in general be suffered at each reflection from the upper boundary surface. The number of reflections a during a complete circumscription is

$$a \geqq \pi \sqrt{\frac{R}{2h}} . \tag{1}$$

If the boundary surface is 100 km above the earth and if the radius of the earth is $R = 6400$ km then $a \geqq 17.8 \leqq 18$. The height can hardly be estimated as more than 100 km, since otherwise we should have to reckon with a comparatively strong ionization in the intervening space.

To simplify the case we will consider the losses by reflection at the upper boundary to be negligible. In that case the ray in Fig. 6 will be able to travel very great distances with very small losses; *we are, however, also not receiving any signals.*

If signals are to be received at the earth's surface we may assume as the simplest case that after each reflection from the upper layer the

[33] *P. R. W.*, p. 197–200.

ray returns to the earth, which will be at least 18 times during each circumscription of the earth. The ray may then be either reflected from the earth's surface at an angle of inflection very little less than 90 deg., or it may just graze it. In the first case the amplitude will decrease in the ratio of about 0.8.* In the last case the ray will travel at least about 16 km along the surface of the earth corresponding to that the earth "dips" 5 m inside of the ray. In this manner we get for each reflection an attenuation factor [34] of about $e^{-0.3}$. One circumscription of the earth gives thus an attenuation of respectively $0.8^{18} = 10^{-1.75}$ and $e^{-5.4} = 10^{-2.35}$. After 100 circumscriptions the attenuation would be respectively 10^{-175} and 10^{-235}.
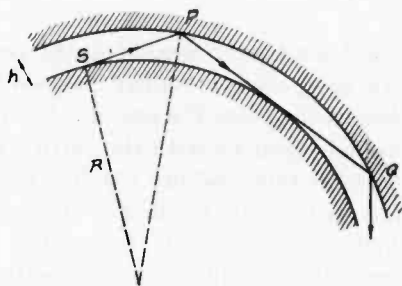
Fig. 6—Schematical representation of a possible path of a ray in the non-conductive space between an upper ionized layer and the surface of the earth.

If we assume that reception is possible within a zone of 40 km at each reflection from, or grazing along, the earth's surface—according to the assumption made above, this width may hardly be estimated at a higher value —then the probability that a signal is received at a given place during one circumscription is equal to $40 \cdot 18/40000 \cong 1/55$.

This probability is of the right order of magnitude since, if a certain signal is giving a long delayed echo at all, it very often gives two or more;[35] and for 110 circumscriptions we shall then actually have a probability of getting two signals.

On the other hand, the above calculated attenuation is unreasonably great. The attenuation exponent should be reduced to at least

* *P. R. W.*, p. 132–135, Fig. VIII, 11–18.
[34] *P. R. W.* p. 19, Fig. III, 2.
[35] *Balth. van der Pol,* loc. cit. Of a series of 11 long delayed echoes observed in Oslo the three gave only one echo, six gave two echoes and two gave three echoes. This gives an average of $(21/11) \cong 2$ echoes for the cases where echoes are obtained at all. The present experimental material is, however, too small to justify a definite conclusion with regard to the average number. In the observations here considered the average of delay was 16 sec.; the longest being 30 sec., the shortest about 3 sec.

1/50 of its value, and consequently also the number of reflections or grazings at the surface of the earth should be reduced to 1/50 of that assumed above in order to get a reasonable attenuation. In that case, however, the probability that a signal will give two successive echoes is reduced also to about 1/50, which is in contradiction to the available experimental material.

Even though the above considerations may not prove the impossibility of obtaining long delayed echoes in the manner last referred to, they show, at all events, that such a probability is extremely small. And there is further another circumstance which is decidedly against such a probability.

The losses due to the reflections at both the upper boundary surface and at the surface of the earth, or by grazing along the earth's surface, will decrease with increasing wavelength. Somewhat longer waves, as for instance 60 or 90 m, should consequently show extraordinarily long ranges under such conditions as are considered here. Two circumstances may, however, possibly counterwork the longer range of the longer waves: Due to the lower frequency the rectilinear radiation would be less pronounced. This, however, is again compensated by a more regular reflection of these longer waves at the uneven surface of the earth. The second circumstance is the attenuation caused by the presence of ions or electrons in the intervening space hitherto considered perfectly non-conductive. No doubt, such ions are present and they will attenuate the longer waves most. The attenuation constant may namely in this case be determined by[36]

$$\gamma_0 = N \cdot \frac{2\pi e^2}{mc} \cdot \frac{\nu}{\omega^2 + \nu^2}, \tag{2}$$

which for the 60-m wave will be at most 4 times, and for the 90-m wave at most 9 times greater than for a 30-m wave. Since all of the other attenuations, under the conditions postulated, are smaller for the 60 and for the 90 wave than for a 30-m wave, then the 60-m wave must have a range of at least 1/4, and the 90-m wave at least 1/9 that of the 30-m wave. If the latter is able to travel round the earth for instance 100 times, then the longer waves should be able to do so at least 25 and 11 times respectively. Any such feature has, as far as the writer is aware, never been observed. In all of the cases where complete circumscription of the earth has been observed the wavelength was between 15 and 25 m.

Presumably therefore we may consider it as an established fact

[36] *P. R. W.*, p. 121 (8a).

that the long delayed echoes are not obtained in the manner last referred to.

Also, presumably, we have now tried every conceivable possibility of explaining the long delayed echoes by means of the propagation of radio waves within the earth's atmosphere.

4. The Reflection of Radio Waves From—or Propagation Along—Bands of Ions out in Space.

Since, according to the preceding, the long delayed echoes cannot arise either by the propagation of radio waves within the earth's atmosphere, or by the waves travelling outside the latter in a medium so strongly ionized that the group velocity approaches zero, they must
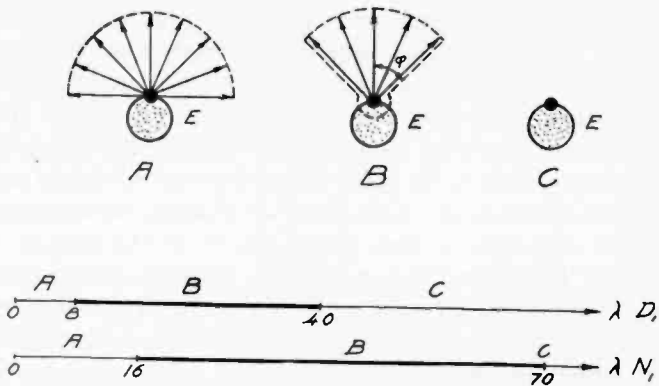
Fig. 7—All wavelengths within the interval C (D_1 represents the conditions at noon, N_1 the conditions at midnight; about 40 deg. northern latitude) cannot penetrate out to space but are refracted or reflected back to the earth's surface. All waves within the interval A penetrate the atmosphere and go out into space. Within the interval B it depends upon the angle of ascent of the wave whether the waves leave the atmosphere or are reflected back again to the earth's surface.

be due to the fact that the waves have travelled very great distances outside the earth's atmosphere. I shall not enter too deeply into the astrophysical problems connected with the present case, but shall only consider some of the relevant wave propagation problems.

The first problem in that respect is: which radio waves are able to penetrate the earth's atmosphere and proceed out into space? This question is thoroughly treated elsewhere,[37] and according to what is set forth there we assume (see Fig. 7) that all waves shorter than about 8 m will always penetrate out into space with comparatively small attenuation. At midnight this will be the case for all waves up

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[37] *P. R. W.*, chap. XI, especially section (i), p. 214–218. See also "Radiofoniens Aarbog 1929," p. 16–20 (Copenhagen, October, 1928.)

to about 16 m. All waves longer than 40 m are completely refracted or reflected back to the earth at noon, and at midnight all waves longer than about 70 m. The given limits are, however, not fixed values but vary according to the varying state of ionization of the upper atmosphere. All experimental data indicate, however, that the figures given are reasonable mean values.
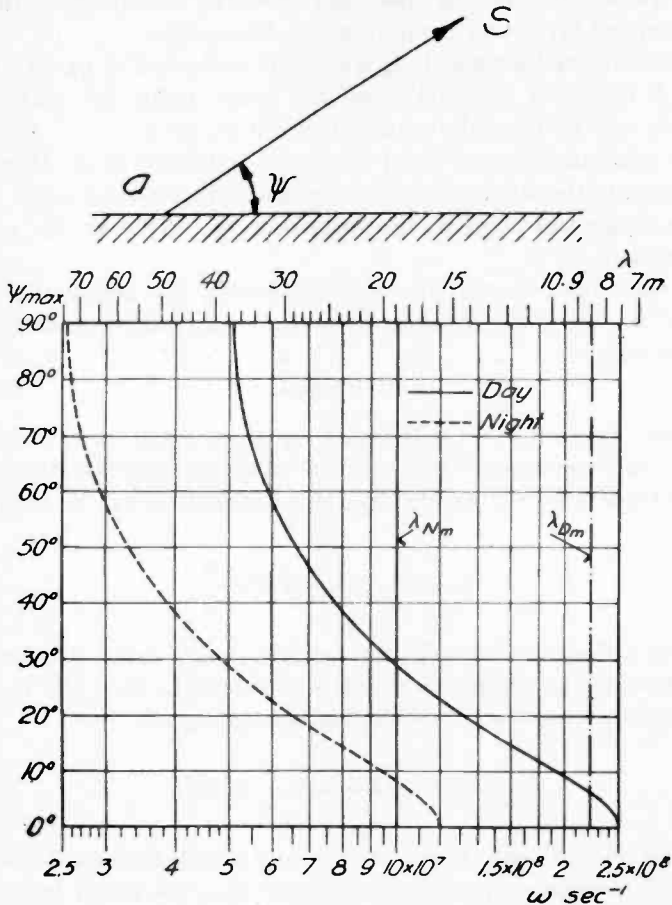
Fig. 8—The earth angle Ψ_{max} as function of the wavelength λ. The full drawn curve represents the conditions at noon, the dotted curve at midnight.

For waves between 8 and 40 m at noon it will consequently depend upon their angle of ascent ψ (see Fig. 8) whether they penetrate out into space or return to the earth. The greatest angle of ascent at which a wave does not penetrate out into space is called ψ_{max}, and the dependency of this angle upon the wavelength at noon is shown in the

full drawn curve in Fig. 8, while the dotted curve shows the same relation at midnight. These curves also depend of course upon the state of ionization in the upper atmosphere, but represent reasonable mean values.

According to the above, the long delayed echoes can arise only with waves shorter than 70 m. Waves longer than 70 m can generally neither penetrate out into space nor from the outside penetrate the upper ionized layer and come down to the earth.

The relations for these long waves are indicated in part C, Fig. 7, where E indicates the earth and the black point the transmitter. No wave rays go from the transmitter out to space.

The relations for very short waves are indicated at A. Here emission occurs to the whole of the hemisphere which has for lower boundary the tangential plane to the surface of the earth at the point of transmission.

Of most interest are the relations within the active short-wave interval B. Here the emission to space occurs within a cone having the apex angle

$$\phi = 90° - \psi_{max}. \tag{1}$$

If the transmitter radiates with equal strength in all directions, then the ratio between that fraction η which penetrates out into space and the total radiation within the whole hemisphere will be determined by

$$\eta_1 = 1 - \cos \phi = 2 \sin^2 \frac{\phi}{2}. \tag{2}$$

If the radiation occurs from a vertical, linear aerial, the length of which is small in comparison to the wavelength, then the radiation intensity is proportional to $\sin^2\phi$ and we then get[38]

$$\eta_2 = 1 + \frac{1}{8} \cos 3\phi - \frac{9}{8} \cos \phi. \tag{3}$$

In Fig. 9 are shown the values of η_1 and η_2 as functions of the wavelength λ. Corresponding values of λ and ψ_{max} are taken from Fig. 8 and then ϕ is found by means of equation (1).

[38] Beside the direct radiation to space calculated by means of (2) and (3), where the wave rays out in space proceed in nearly the same direction as the one at which they left the transmitter, a more or less regular radiation will take place from—and in a direction nearly parallel to—the upper surface of the ionized layer. This radiation is mainly due to those rays which leave the transmitter at an earth angle in the vicinity of ψ_{max}. Rays may be emitted from any point of the upper surface of the ionized layer on wavelengths within the interval B, Fig. 7, but the intensity of such rays is generally rather small.

From Fig. 9 it appears that at wavelengths appreciably more than 30 m η_1 and η_2 decrease to very small values. Above 30 m the radiation out to space will consequently be comparatively very small, and from the figure it appears at all events that the probability that waves may penetrate out into space in a favorable direction decreases rapidly at increasing wavelength for waves from about 30 m and upwards.

On the other hand, we have shown above that for total reflection at normal incidence of a 30-m wave a density of electrons of about
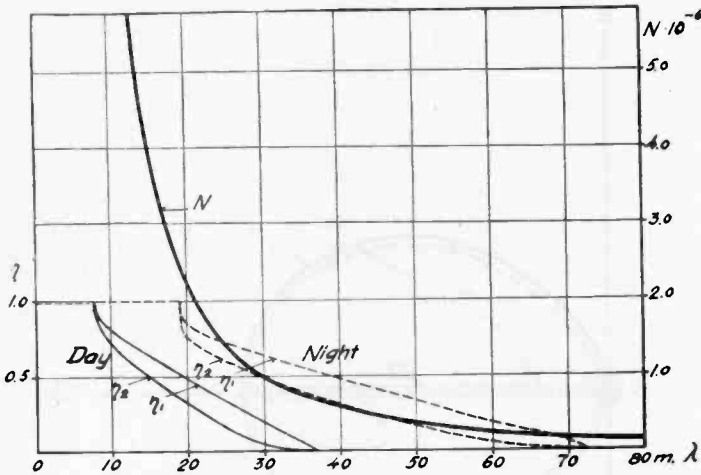
Fig. 9—The values of the constants η_1 and η_2 at day and at night, calculated on the basis of Fig. 8. The N-curve shows the values of N calculated by means of equation (4).

$N_0 = 1.2 \cdot 10^6$ per cm³ is necessary. At other wavelengths the necessary density of electrons is determined with an accuracy sufficient in this connection by

$$N = N_0\left(\frac{30}{\lambda}\right)^2 \cong 1.2 \cdot 10^6 \left(\frac{30}{\lambda}\right)^2. \tag{4}$$

The N curve in Fig. 9 shows the values of the density of electrons determined in this manner, and we see that these values increase rapidly with decreasing wavelengths. The probability of finding a band of ions having a density of electrons equal to or greater than N is of course smaller the greater the value of N.

From the preceding it appears that there must be a certain wavelength for which the probability of getting long delayed echoes is the greatest, and from Fig. 9 we may draw the conclusion that this most favorable wavelength must be about 30 m; but the possibility exists of course that echoes may be observed on somewhat shorter

as well as on somewhat longer waves; but to the best of our knowledge, long delayed echoes so far have been observed only on waves of about 30 m.

If we were to judge solely from Fig. 9 we would be apt to assume that a somewhat longer wave, for instance about 40 m, would be the most favorable for the observation of echoes. By calculating the η_1 and η_2 curves we have, however, not considered the loss suffered by the waves by their passage through the ionized part of the upper atmos-
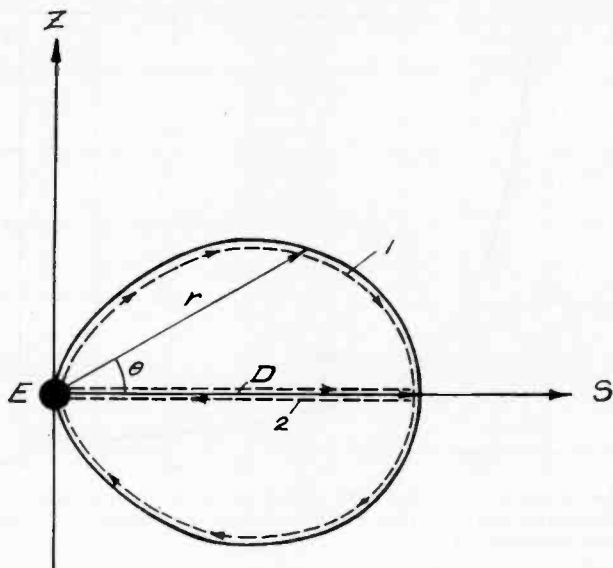
Fig. 10—The full drawn curve indicates the inner limits of the paths of charged particles according to equation (5). *E* represents the earth with its magnetical axis along the *Z*-axis. The dotted curves 1 and 2 indicate the two echo tracks possible, the direction *S* pointing toward the sun.

phere, and since this loss increases with increasing wavelength the said curves for the effective radiation out to space should decrease somewhat more rapidly than shown in Fig. 9.

Another circumstance of some importance in this connection may be mentioned. All of the preceding considerations are based upon a state of ionization of the atmosphere which is assumed by the writer in consequence of the experimental material at hand with regard to the propagation of radio waves in general. Considering this very comprehensive material there is some reason to believe that the ionization distribution assumed as a whole is approximately correct from the earth's surface up to the altitude of maximum ionization, i.e., up to about 130–150 km. As to the density of electrons and the con-

ductivity above that height where the ionization is maximum, our ordinary terrestrial experience with propagation of radio waves is unable to give any information. The density of electrons at greater heights may consequently very well exceed somewhat the values indicated in *P.R.W.* Hitherto it has been possible only to judge about the state of ionization in these regions above the altitude of maximum ionization by means of multiple echoes of short delays[39] and by means of the terrestrial magnetical conditions.[40] The investigation of the ionization of these very high layers may possibly in the future profit by the experiences gained from the long delayed echoes.

With regard to the character and the location of the bands of ionization which cause the echoes, we may just mention that presumably they may be divided into two groups, namely, those along which the waves are propagated, and those from which the waves are reflected.

Of the first kind are presumably those bands of electrons which according to Störmer are due to the invasion of electrons into the magnetic field of the earth; see for instance Fig. 10. As long as it is a question of relatively few electrons only, the classical work of Störmer offers the necessary information with regard to the form of the tracks possible, and to the bounding of those spaces to which such tracks are limited. Above we have shown, however, that in those bands or layers which may be of importance for the phenomena here considered of the propagation of radio waves, the density of electrons is so great that the electrostatic forces will prevent the development of pure electron bands or of dense bands consisting exclusively of ions of one sign. In the ionized bands which are effective echo reflectors the positive and the negative space charges must be of nearly equal value, *i.e.*, the resulting space charge must be comparatively small. Such a band of ions will also have a tendency to follow the tracks calculated

[39] *P. R. W.*, p. 209–212.

[40] The bearing of the magnetic evidence on the value of the total ionization of the atmosphere is discussed in a valuable paper by S. Chapman just published ("On the Theory of the Solar Diurnal Variation of the Earth's Magnetism," *Proc. Roy. Soc.* (A) 122, 369–386, 1929.) This author comes to the conclusion that the total ionization required from a magnetical point of view is of the same order of magnitude as that assumed by the writer in *P. R. W.*, the magnetic evidence indicating a somewhat higher value. Chapman adheres to the "dynamo" theory eventually combined with the "drift current theory" which he suggests in the above mentioned paper. Upon numerical considerations he concludes that the "diamagnetic" theory of Ross Gunn (*Phys. Rev.*, 32, 133–141,1928) cannot be right. It may be added that N. Bohr ("Studier over Metallernes Elektronteori," Chap. IV, Copenhagen, 1911) has proved that such a diamagnetic effect does not exist. The remarks of R. Gunn (loc. cit., p. 136) do not meet the main point of Bohr's arguments and do not invalidate his general conclusions.]

by Störmer, and the shape of the track and the velocity will in the main be determined by the heavy ions.

In Fig. 10 we have as an example shown the boundaries of the tracks of the corpuscular rays in a particular case for which the equation of the bounding curve is [41]

$$r = \frac{\cos^2 \theta}{1 + \sqrt{1 + \cos^3 \theta}} \cdot \sqrt{\frac{M}{\dfrac{cm}{e} \cdot U}} \tag{5}$$

where r is the radius vector, $(\pi/2 - \theta)$ its angle with the magnetic axis Z of the earth. $M = 8.6 \cdot 10^{25} =$ the magnetical moment of the earth, $c = 3 \cdot 10^{10}$ cm sec.$^{-1} =$ the velocity of light in empty space, m and e the mass and the charge of the ion [esu].

According to Störmer the bounding surfaces which may give long delayed echoes constitute the boundary surfaces between those parts of space into which the ions in question may penetrate at a given velocity, and those parts of space into which these ions cannot come. Such boundary surfaces may be efficient as guiding planes for a propagation of radio waves, and such a propagation is indicated by the dotted curve 1 in Fig. 10.

For such a propagation along a band of electrons or ions, which connects the northern and southern polar areas, and which has very great radii of curvature, only a comparatively small density of electrons is demanded and, according to Table IV, the propagation may occur with comparatively small losses.

The other manner of obtaining echoes is indicated by track 2 in Fig. 10. Here the wave ray is reflected from the bounding surface and returns to the surface of the earth. This manner of propagation may occur with only very little attenuation, but demands in the bounding surface the density of electrons determined by (4).

These bands of ions, formed by the magnetic field of the earth, presumably play an important rôle in the production of wireless echoes up to 30 sec. and possibly even up to 60 sec. as was originally suggested by Störmer[42, 43].

But there is also the possibility that outside that space in which the magnetic field of the earth exerts its influence in this manner, bands

[41] C. Störmer, loc. cit., p. 129.
[42] C. Störmer, *Nature*, 122, 681, 1928; *C. R.* 187, 811, 1928.
[43] For this kind of echo the probability of obtaining good echo signals will, as shown by Störmer (*Nature*, 123, 16, 1929), depend upon the angle between the magnetic axis of the earth and the direction to the sun, being greatest when these two directions are at right angles to each other. [The latest evidence seems to be in agreement with Störmer's prediction.]

of ions having sufficient density of electrons may attain such forms that they act as reflectors which after one or more reflections return the radio waves to the earth. If a wave ray is to return to the earth with sufficient intensity after a single reflection from a very distant ionization band the center of the curvature of that band must necessarily be located at or near the earth. Such curvatures may partly be due directly to the influence of the earth's magnetic field and, outside that space where the latter appreciably influences the shape of the ionization bands, partly to the electric field from charges directly
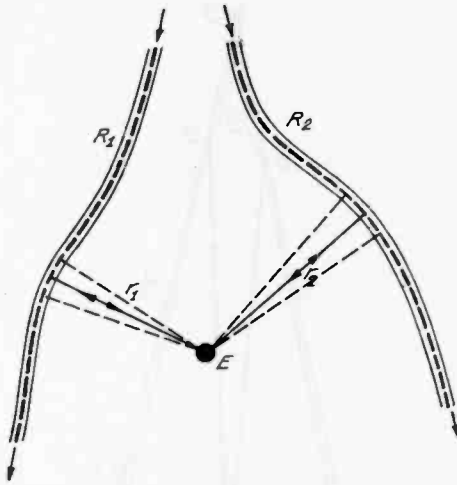
Fig. 11—Two curved ionization bands acting as concave mirrors at the places indicated. *E* represents the earth.

on the earth and particularly from such charges which may be "arrested" by the magnetic field of the earth. Such bent ionization tracks are outlined in Fig. 11.

Long delayed echoes may further be obtained in the manner indicated in Fig. 12, where R_1 and R_2 are two bands of ions extending from the sun and which at *a* and *c* reflect a wave ray coming from the earth *E*. Between *a* and *c* the wave ray traverses the curved path *abc*, the curved form of which is due to increasing density of electrons in the direction toward the sun.

Since, generally, the bands of ions are not perfectly electrically neutral they will mutually act upon one another, thus forming more or less curved bands. In this case the center of curvature will, however, in general not be located near the earth and consequently the bent part of the band will not be efficient in producing echoes. If, however, the radiation from the earth is emitted at a comparatively great space

angle an exceedingly large space within our solar system will be "searched" and consequently the probability of finding a favorable constellation may be not altogether vanishingly small, although such echo phenomena presumably are rather seldom.

In consequence of the above considerations the writer therefore anticipates that in the future, as the study of the echoes is carried on more systematically and on a more extensive scale there will occasionally be reported echoes of very long delays, possibly up to 10 to 15 min. or even more.[44]
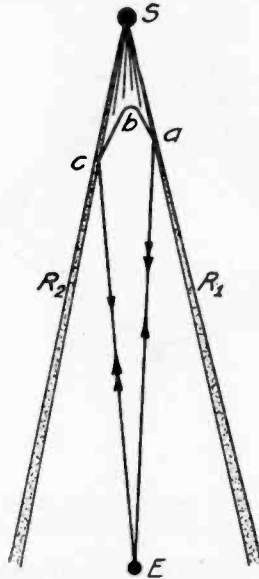
Fig. 12—Schematical representation of a possible manner of obtaining echoes after two reflections (at *a* and *c*), and of a curved path *abc* of a wave ray between the two points of reflection. *E* represents the earth.

From the considerations set forth in this paper, it presumably appears that:

(1) Echoes delayed more than 10 sec. cannot be due to the propa-

[44] This prediction, set forth some time ago by the writer ("Radiofoniens Aarbog 1929," p. 22–24, October 1928—the above Figs. 8, 11, and 12 are taken from this paper) has received an unexpectedly quick confirmation, since in a letter dated February 2, 1929, Jörgen Hals of Oslo communicates that he has observed echoes up to 4 min. 20 sec. corresponding to a path-length of 78,000,000 km. This observation, if correct, confirms the above considerations, namely, that the long delayed echoes are not due to propagation entirely within the earth's atmosphere, and further that echoes may occur with so long delay that they must be due to ionization bands located outside that space in which the magnetic field of the earth directly exerts its influence. [Another echo, having a retardation of 3 min. 15 sec. is reported by Mr. Hals, February 14, 1929.]

gation of radio waves entirely within the atmosphere of the earth, nor to a propagation of the waves outside this in a medium being so densely crowded with electrons that the group velocity decreases to such small values that the distance travelled will be comparatively short.

(2) Echoes delayed up to 30(possibly to 60) sec. are probably due to propagation along—or reflections from—"Störmer bands" of electrons within the magnetic field of the earth.

(3) Occasionally echoes may be obtained with such great delay that those bands of ions to which the echoes are due must be located at such great distances from the earth (more, for example, than 40.000.000 km) that they are outside the space in which the magnetic field of the earth exerts any appreciable direct influence.

AN ECHO INTERFERENCE METHOD FOR THE
STUDY OF RADIO WAVE PATHS*

By

L. R. HAFSTAD AND M. A. TUVE

(Dept. of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.)

Summary—*An experimental determination of the rate of change of radio-frequency phase of the separate downcoming echoes has been carried out. The small power crystal-controlled oscillator circuit of the 4,435-kc transmitter operates continuously, the high-power pulse transmission being produced by modulating the power-amplifier circuits. A receiver was operated very near the transmitter, having rapid recovery from the paralyzing effect of the pulse ground wave and sufficient sensitivity to receive the echoes with good amplitude, and having a very slight coupling to the crystal-oscillator circuit such that the crystal "pick-up" was comparable to the echoes received. The echoes showed their changing radio-frequency phase by alternately adding to and subtracting from the constant crystal oscillator pick-up. This "interferometer" is naturally sensitive to small changes in the optical path of the waves. The phase changes are regular, but the time of one 360 deg. phase change on 4,435 kc varies from 1 sec. to 60 sec., or possibly longer during the day and evening, and at times changes between these limits in as short a time as 15 min. When multiple echoes are present, the second and third echoes phase in and out more rapidly than the first echo, but not by an even factor.*

I T has been obvious from the time of its inception that the echo-method possesses singular advantages for the study of radio wave propagation, and particularly for the study of the ionized Kennelly-Heaviside layer, by reason of the fact that the waves reaching the receiver by different paths are separated, instead of being received simultaneously, provided, of course, that the transmitted pulses are of sufficiently short duration. The complex echo pattern received on 4,435 kc, especially during the evening and night hours,[1] has emphasized the importance, at least in this region of frequencies, of such an analysis of the energy received over the various paths. By suitable arrangements the polarization, phase, amplitude fading, and other characteristics of the waves received over each separate path can be studied.

We have recently carried out an experiment which determines the rate of change of radio-frequency phase of the e.m.f.'s in the receiver due to the separate downcoming echoes. This has been done by allowing the separate echoes to combine at the receiver with a small (radio-frequency) "pick-up" from the transmitter crystal. At present the

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[1] L. R. Hafstad and M. A. Tuve, PROC. I. R. E., 17, 1513; September, 1929.

experiment has been carried only far enough to verify the possibility of studying radio wave paths by such a method, but the results obtained seem interesting enough to justify a brief report. The small power, crystal-controlled oscillator circuit of the 4,435-kc transmitter operates continuously, the high power pulse transmission being produced by modulating the power-amplifier circuits with an unbalanced multivibrator.[2]   A receiver was operated very near to the transmitter, the receiver having rapid recovery from the paralyzing effect of the pulse ground wave and sufficient sensitivity to receive the echoes with good amplitude and having a coupling to the crystal-oscillator circuit such that the crystal pick-up was comparable to the echoes received.   The echoes showed their changing phase by alternately adding to and subtracting from the constant crystal-oscillator pick-up.

Before the experiment was performed it could not be predicted with certainty that the transmitter crystal would provide a suitably constant phase reference for comparison with the echoes, particularly because of the fact that its phase and even its frequency might be shifted slightly by the pulse modulation of the power amplifiers excited by the crystal circuit. We have observed, however, that with great regularity the echoes phase in and out with respect to the crystal pick-up. A constant phase shift of the crystal with each pulse, of course, would not be detected, nor would it affect the interpretation of the results of the experiment as measuring the changes in optical path for each echo. It is possible that the phase of the crystal between pulses may be variable with respect to the phase during emission. This point remains to be studied, but if so the observations here reported do not correctly measure phase changes due to changes in optical path. Nevertheless, since the separate multiple echoes from a given set of pulses phase in and out at different rates, and as it is very unlikely that the phase of the crystal is disturbed in the intervals between multiple echoes, at least part of the observed phasing must be due to changes in optical path. It is difficult to imagine that the crystal phase or frequency could have been changing *progressively* in such a way as to cause the regular phasing which was observed. In addition to possible sudden phase shifts due to the modulation, another source of error to be considered is that involved in the change of the crystal frequency with temperature. One may question whether the electrical phase of the crystal alters appreciably from its correct value as a reference phase, due to changing temperature, in the interval between a pulse and its echo. The crystal frequency changes with temperature about one part in 40,000 per deg. C. To change the electri-

[2] M. A. Tuve and O. Dahl, Proc. I. R. E., **16**, 794–798; June, 1928.

cal phase by even 1/1000 radian from its correct value, in the 0.002-sec. interval between a pulse and its echo, would require the temperature to change at the rate of about 1 deg. C per sec. Actually the temperature of the crystal does not alter, at most, by more than 3 deg. to 5 deg. C during several hours. A second point to be considered is whether a slowly changing frequency, due to changing temperature of the crystal, might produce phasing of the echo with respect to the crystal, for reasons similar to those underlying the wavelength change method of Appleton and Barnett,[3] i.e., phasing of echo with respect to crystal due to the varying frequency causing changes in optical path and changes in the exact number of crystal cycles taking place between pulse and echo. However, a phase change of 360 deg. observed to occur on the average in perhaps 10 to 15 sec., means a change in optical path or number of cycles between pulse and echo of approximately one part in 3,500. Since a temperature change as large as 1 deg. C causes a frequency change of only one part in 40,000, it is clear that only a negligibly small rate of phasing of the echoes can be caused by temperature (frequency) change of the crystal, unless the frequency happened to pass through a possible critical value at which very large path changes might be caused by very slight changes in frequency.

The receiver was located about 300 feet from the transmitter, with the two antennas approximately at right angles. The apparatus used was entirely similar to that previously used here for echo observations,[4] with the exception that grid bias detection was substituted for grid leaks and condensers, and moving-picture film was used for recording, giving a record of any desired time length. Continuous records of 15 sec. or longer were found most suitable. Visual counts giving the 360-deg. phasing time were found to be a valuable supplement to the records, especially for the phasing time of the first echo. In fact, with our present arrangements suitable photographic records of phasing times over a period of several hours would require large quantities of film. Low time constants throughout the receiver, giving very rapid recovery, were of obvious importance so near the transmitter, but as careful elimination of condensers in the receiver and the use of grid bias detection instead of grid leaks had been resorted to when first using multivibrator modulation, no trouble was experienced in this respect. Using super-speed motion-picture film an ordinary 6-volt tungsten lamp gave a satisfactory trace with an optical lever of about

[3] See *Proc*. Phys. Soc., London, **41**, 43; December, 1928, and references there quoted.
[4] G. Breit and M. A. Tuve, *Phys. Rev.*, **28**, 554–575; 1926.

50 cm. Timing marks were obtained using an auxiliary spot of light interrupted by a two-sectored disk mounted on a 25-cycle synchronous motor. Film speeds of about 25 and 50 cm per second were used, the latter giving much the better records.

A typical section of the record obtained is shown in Fig. 1. The oscillograph is deflected by the radio-frequency pick-up from the transmitter crystal-oscillator circuit to about half the saturation value for the power amplifier of the receiver. Following the ground peak, which
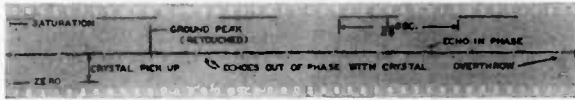
Fig. 1—Typical record showing echoes in phase and out of phase with crystal at 17 h. 55 m., March 3, 1929 (effective heights 250–500–780 km).

shows a moderate overthrow due to underdamping of the oscillograph element, three echoes are present in this specimen of the record, with retardations equivalent to multiples of about 250 km layer height, as usual. The first is "in phase" with the crystal, deflecting toward saturation, and the second and third are "out of phase" with the crystal, deflecting toward zero. The echo is strictly in phase with the crystal only for an instant during each 360-deg. phase change, of course. The terms "in phase" and "out of phase" are used here somewhat inexactly to refer to the periods during which the resultant of the crystal vector and the echo vector is greater or less than the crystal vector alone. When the ray path is increasing, for example, the echo may be represented by a vector which is rotating, and possibly also changing in amplitude, with respect to the constant crystal vector. During our observations the echo amplitude was always less than that from the crystal. For the simple case of a constant amplitude echo, the deflection due to the echo should be zero (resultant vector equals crystal vector) twice during each 360-deg. phase change of the echo, and due to the difference in amplitudes these points both should lie between 180 deg. and 270 deg. from the point of exact phase, that is, the echo should be "in phase" a longer time than it is "out of phase." Amplitude fading of the echo, possible polarization of the downcoming wave, which may also be changing, antenna directivity, non-linearity of the receiver, and other variables prevent an exact analysis of our present records. However, it seems very probable that the *average* 360-deg. phasing time obtained from the records should closely approximate the actual average time of increase (or decrease) of the optical path of the echo by one wavelength.

Fig. 2 shows a series of sections cut from a single record of about 15 sec. duration, to illustrate the phasing in and out of the first echo. The ground peak has been retouched to make reproduction possible. The time for each section is given in sec. Due allowance must be made for the overthrow of the oscillograph with large amplitudes, due to underdamping. From 0.0 to 0.94 sec. the first echo is out of phase
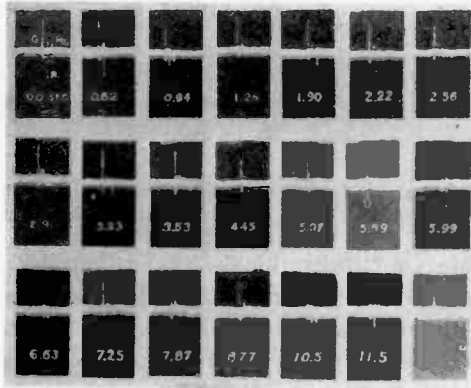
Fig. 2—Successive sections of typical record showing phasing of first echo with more rapid phasing of second echo also evident; time of each section given in seconds. Recorded at 20 h. 19 m., March 3, 1929 (effective heights 250–480 km).

with the crystal; it is in phase from 1.26 to 4.45 sec., out of phase from 5.07 to 6.63 sec. in phase from 7.25 to 8.77 sec., and out of phase from 10.5 to 12.4 sec.

The phasing of the echo was always quite regular, alternately adding to and subtracting from the deflection due to crystal pick-up, but the time for one complete 360-deg. phase change (change of one wavelength in optical path) varied greatly during any extended period, as illustrated by Table I for Sunday, March 3, 1929.

TABLE I
Phasing Times for First Echo

Time P. M.		Average time for 360-deg. phase change, first echo	Time P. M.		Average time for 360-deg. phase change, first echo
h.	m.	sec.	h.	m.	sec.
3	52	15	7	38	5.4
4	18	16	7	51	4.0
4	24	4	8	15	3.0
4	37	7.5	8	23	20
4	52	11.5	8	29	5.0
5	16	3.3	8	44	22
5	21	3.5	8	51	30
6	37	18	9	02	4.3
6	47	50	9	15	5.4
6	56	7.5	9	24	20
7	09	4.8	9	35	4.
7	19	5.4	9	53	30

During the hours covered by Table I the layer height, measured by the retardation of the first echo, changed from 240 km to 300 km approximately. Due to low film speed the accuracy of these figures is not better than about ±12 km. A change of 60 km in effective height would mean an average path increase of about one wavelength (360-deg. phase change) in 12 sec. Averaging the figures given in Table I is obviously impossible, but at least the phasing time would seem to be of the right order of magnitude.

The phasing time for the first echo has been observed to be variable and usually of the order of 10 sec. during midday hours as well. It is perhaps not to be expected that the wave path should be constant at any time, and of course, with the present method of recording, slow fluctuations in the optical path would show phase changes similar to those due to a steady increase or decrease.

We have only a comparatively small number of records, made during the evening when marked multiple echoes are present, which give suitable measurements of the phasing times of the second and higher order echoes. Those we have, however, fail to show any obvious relation between the phasing times of the various echoes. The second and third echoes phase faster than the first echo, but not by even factors of 2 or 3 as might be expected if they were due to multiple reflection over the same ray path. The retardation times for these records were usually multiples to within the errors of measurement, however.

The times for successive 360-deg. phase changes, measured on the film by the distance between successive passages of the echo amplitude through the "shifted zero" (resultant vector equals crystal vector, that is, no echo peak visible) necessarily cannot be accurately determined, but seem to vary much more erratically for the second and third echo than for the first. Amplitude fading should affect the individual measurements, but the average measured phasing time should not be affected by fading unless fortuitously the phasing time is related to the fading time in a particular way. We have observed that at different times for the various echoes the fading time may be much longer or considerably shorter than the phasing time, with no apparent relation between the two.

A fundamental source of trouble in measuring phasing times for multiple echoes accurately was the fact that at times when multiplicity appeared the reflections were often doublets which were too close to be completely resolved. The two echoes comprising a doublet phased independently, giving rise to indefinite echo amplitude "zeros" (resultant vector equals crystal vector). A fairly well resolved doublet,

with the components phasing separately through "zero," is shown in Fig. 3. At 0.0 sec. both components are in phase with the crystal, at 0.49 sec. the first component is in phase and the second component is out of phase, at 2.07 sec. the reverse is true, and at 5.41 sec. both components are out of phase. Long records, made with a higher film-speed than 50 cm per second, will be necessary before quantitative statements can be made concerning the relative phasing times of multiple echoes.
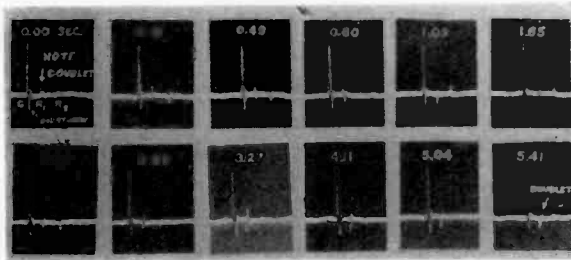
Fig. 3—Successive sections of record showing phasing of doublet echo through the "shifted zero." Recorded at 22 h. 40 m., March 3, 1929.

A brief attempt was made to determine the direction or sense of the phase changes of the first echo, in order to distinguish between increase and decrease of optical path (ionized layer rising or falling)by obtaining records alternately on two antennas, using a relay switch. The results obtained were too indefinite to be of significance, especially as the antenna constants were undefined, but it may be of interest to point out that quite definite information on this point could be obtained by introducing a known periodic phase change in the pick-up from the crystal circuit.

This experiment was performed at the suggestion of our colleague, G. Breit, to whom we are also greatly indebted for discussion and other assistance. The necessary special schedules of signals were very kindly transmitted by the members of the staff of the U.S. Naval Research Laboratory, who have provided the special transmissions for all observations by the Department of Terrestrial Magnetism using the echo method. For this experiment a room was also provided at the Naval Research Laboratory in which our recording apparatus was operated. We are grateful to Captain Oberlin, Dr. Taylor, Dr. Hulburt, and others there for their very cordial cooperation, and to J. A. Fleming of the Department of Terrestrial Magnetism, whose interested support has made the work possible.

ON THE RELATION BETWEEN LONG-WAVE RECEPTION AND CERTAIN TERRESTRIAL AND SOLAR PHENOMENA*

By

K. Sreenivasan

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Summary—It is shown in the paper that there appears to be an inverse relation between long-wave reception on 75 kc per sec. and the temperatures at the transmitting and receiving stations, when the distance between them is not great. The explanation is offered that the variations ι the temperatures and in the signal strength are due to a common source, the changes in the medium between the two places.

If at all, only a direct relationship has been noticed between field intensity and the barometer reading at the receiving station.

There is found to be good correlation between atmospheric ozone in northwestern Europe and Bangalore reception, and between the two and sunspots. Little correlation is found between these and terrestrial magnetism as represented by the international magnetic character of day numbers. Analysis of the data on the basis of the 27-day period has yielded very satisfactory agreement between reception and sunspots, with a six-day period in reception for which no explanation is found.

The lag of one to two days between reception and sunspots occasionally occurring has been explained on the basis of the emission of high speed corpuscles from the sun. Regarding some of the differences between long- and short-wave transmission, the transmission of short waves is attributed to electron movement in the higher regions of the upper atmosphere and that of long waves to less mobile ions below. It is on this basis that the correlation between reception and ozone may be explained.

The paper is in connection with the signal intensity measurements of Madras (Fort) Radio working on 75 kc per sec., made at the Radio Laboratory of the Indian Institute of Science, Bangalore, between March 1926 and August 1927, a period of eighteen months.

1. Introduction

DURING the last few years, attention has often been drawn to the possible connection between the strength of received radio signals and some of the terrestrial and solar phenomena. In the undoubted influence of the sun on radio wave transmission, interest has been centered on the understanding of the relation of the spots on the sun to the field intensity of distant radio stations. Of the terrestrial phenomena are (a) local temperature and pressure, (b) atmospheric ozone, and (c) the earth's magnetism, with which relation is sought. The success that has attended long distance communication by the use of very high frequencies and some of the opposite effects observed in long and short waves by the same cause have stimulated interest in the study.

In a general way, the problem can be considered to be a statistical one, depending on data accumulated for a considerable period over as much as possible of the radio spectrum, parallel to those on phenomena with which relation is sought. Such complete data do not exist for a short period of even a year. The statistical study has, for some time, receded into the background, due to the fruitful results obtained by the application of optical methods to the study of wave propagation sometimes under special conditions, as in the experiments of Breit and Tuve, Appleton and Hollingworth.

Of the statistical investigations, those of G. W. Pickard[1] and of L. W. Austin[2] are important both for their reliable observations and the fairly long period of time which they cover. Of special interest are the observations of Austin which date back to 1914.

Daily measurements of the received field intensity of Madras (Fort) Radio (VWO), on a test signal transmitted at 0800 I.S.T., have been going on since March 1926 at the Radio Laboratory of the Indian Institute of Science, the frequency of the transmission being 75 kc per sec. Bangalore is about 295 km from Madras, almost directly to the west of it. The method of measurement and some preliminary observations have been described elsewhere.[3]

Although the results in the paper relate to a period of eighteen months, the conclusions drawn can only be tentative in character and cannot partake of the nature of established facts. There have been occasional breaks in the field intensity measurements; in August 1926, the monsoon was responsible for the breakdown of the apparatus for some period, due to the high humidity in the air; traffic pressure

[1] G. W. Pickard:—
  (a) "Correlation of Radio Reception with Solar Activity and Terrestrial Magnetism," PROC. I. R. E., 15, 83; February, 1927.
  (b) "The Relation of Radio Reception to Sunspot Position and Area," PROC. I. R. E., 15, 1004; December, 1927.
  (c) "The Correlation of Radio Reception with Solar Activity and Terrestrial Magnetism, II," PROC. I. R. E., 15, 749; September, 1927.
  (d) "Some Correlations of Radio Reception with Atmospheric Temperature and Pressure," PROC. I. R. E., 16, 765; June, 1928.
  (e) "Discussion on Long Distance Receiving Measurements at the Bureau of Standards," PROC. I. R. E., 15, 539; June, 1927.
[2] L. W. Austin:—
  (a) "Long Wave Radio Measurements at the Bureau of Standards in 1926 with Some Comparisons of Solar Activity and Radio Phenomena," PROC. I. R. E., 15, 825; October, 1927.
  (b) "Radio Atmospheric Disturbances and Solar Activity," PROC. I. R. E., 15, 837; October, 1927.
  (c) "On the Influence of Solar Activity on Radio Transmission," PROC. I. R. E., 16, 166; February, 1928.
[3] K. Sreenivasan:—
  (a) "Madras (Fort) Radio Field Intensity Measurements at Bangalore," *Jour. Ind. Inst. Sc.,* 9B, 37, 1926.
  (b) "Intensity Variations of Madras (Fort) Radio Station," *Jour. Ind. Inst. Sc.,* 10B, 35, 1927.

during the latter half of December and the early part of January rendered the transmission of the test signals very irregular.

2. METHODS OF ANALYSIS

The methods of analysis followed in arranging and correlating the data of the various elements are mainly two. (a) Periodic averages of a weekly or monthly type or of both, are compared with respect to the variations therein. (b) With any one of the elements as a reference quantity, the variations of that quantity and of reception on either side of the maximum or minimum days of the former are studied. This method has been extensively used, a very interesting instance being the paper by Chree and Stagg in a study of the 27-day period in terrestrial magnetic phenomena. Further, to get rid of comparatively transient changes and bring the main similarity or dissimilarity between the two quantities under examination, considerable use has been made of the smoothing formula $(a+2b+c)/4$.

3. TEMPERATURE AND SIGNAL STRENGTH

The daily local temperature at Bangalore has been obtained from the Bangalore Meteorological Observatory and relates to 0819 I.S.T., 19 minutes after the signal has been transmitted. Those for Madras for the same time were obtained from the Madras Observatory. That the temperature of the room in which the signal is measured is not necessary will be evident at a later stage.[4]

Practically every experimenter in field intensity measurements has come to the general conclusion that the months October to January form a period of very pronounced changes in the daily intensities of radio stations, especially of long waves, in the band 100 kc per sec. to 12 kc per sec. At moderate distances, as Hollingworth[5] showed, the change may be either a decrease or an increase depending upon the position of the receiving station in the interference pattern produced by the ground and the downcoming rays. In general, at fairly big distances, the variations are an increase, including in this statement the Bangalore observations on Madras, though with only 295 km between them.

It is exactly this period of the year which is characterized by low temperatures, the hotter seasons being characterized by low field intensity. Austin's curves (loc. cit.) for Tuckerton and New Brunswick are very good examples of this.

[4] C. Chree and Stagg, "Recurrent Phenomena in Terrestrial Magnetism," *Roy. Soc., Phil. Trans.*, **227**, 21–62, Aug. 3, 1927.
[5] J. Hollingworth, "Propagation of Radio Waves," *Jour.* I. E. E., **64**, 1926.

To determine if the interrelation between the two is purely restricted
to the receiving station temperature or if there is any relation with
the temperature at the transmitting station as well, the two tempera-
tures were plotted together with the rather unexpected result that
the curves march almost parallel to each other. Evidence of this can
be seen in the detailed curves *a* to *j* of Fig. 1, where the daily deviations
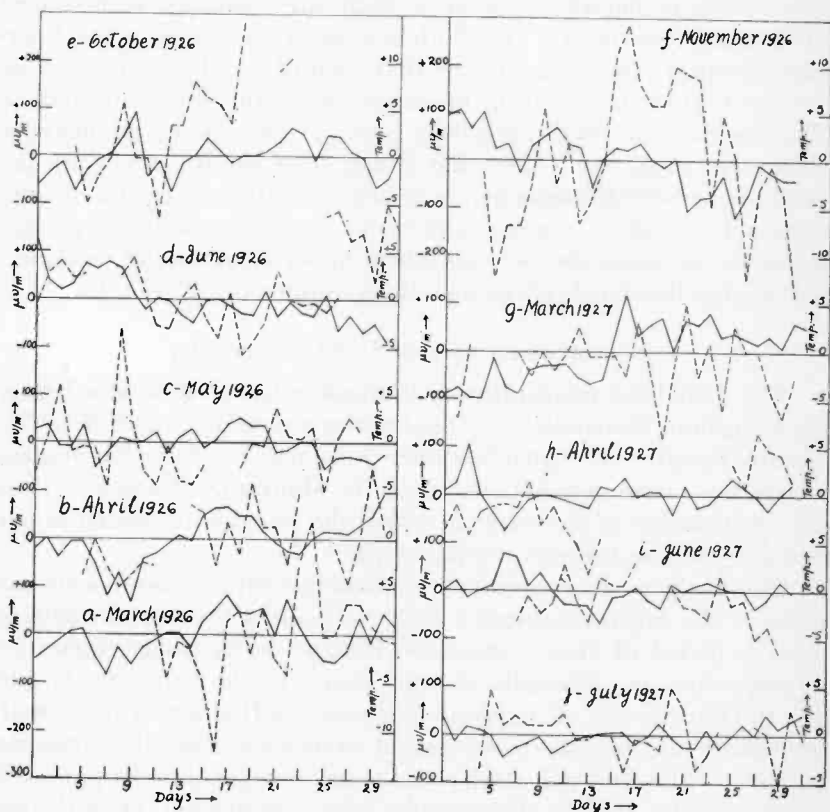
Fig. 1

from monthly averages are plotted. Even in the matter of details, the
agreement between the weekly averages plotted in Fig. 2 as the ratio
$100/T$ is very close. With monthly averages, the parallelism between
the two temperature curves is hardly broken. (Fig. 3)

These curves show a broad inverse relation between temperature
(at either station) and field intensity. Of closer similarity there is
none, due obviously to the masking influence of other factors of both
terrestrial and solar origin. That the inverse relation is not a purely

local matter, but is due to changes that are common to the temperature throughout the distance between the stations is the chief point of interest. In view of this, it is legitimate to expect that in the results of Austin too, the transmitting station temperature will have parallel changes with those at the receiving station.
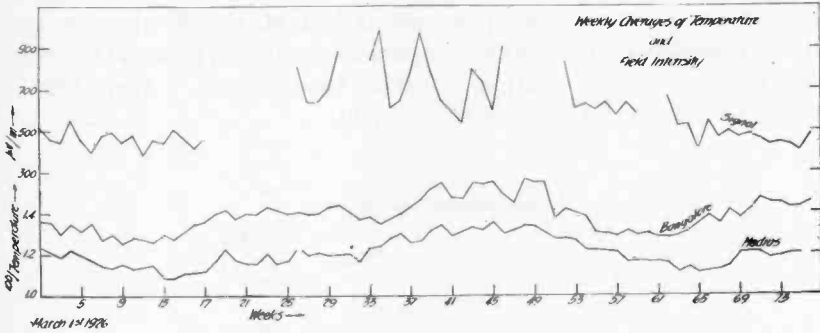
Fig. 2

The intervening medium between the two places undergoes changes which are reflected in those of the temperatures at the two stations, so that the inverse relation between field intensity and temperature is not strictly of a local character but is due to the non-local type of changes occurring between the two stations.
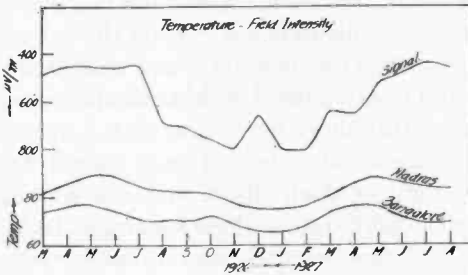
Fig. 3

Of interest are the curves of daily departures from monthly means, although they sometimes give contradictory results (Fig. 1). During some months, such as November 1926, the daily variations of intensity are very big; but they are not accompanied by as pronounced changes in temperature. In this month, Bangalore temperature shows a gradually diminishing tendency; the field strength decreases in the first week, then increases rapidly and as rapidly gets low with a suggestion of a six-day period. During March 1927, however, the temperatures

at both places are generally on the increase while signal strength varia-
tions, though big, show an increasing and then a decreasing character
just opposite to what we had for November, 1926.

Although it may appear contradictory to the inverse relationship,
the curves for the months April, June, and October 1926 indicate that
the broad variations in both temperature and field intensity are similar;
but during the "steady" months, especially in May 1926, when the day-
to-day intensity variations are comparatively smaller than in the winter
months, the inverse relation is better brought out.   April 1927 is
another instance of the "steady" months.
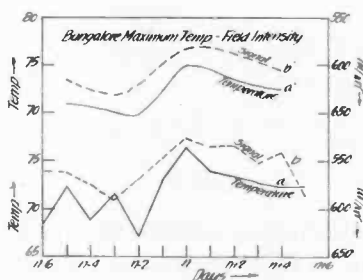
Fig. 4

The second method of study is to select the day in the month on
which the temperature has been highest for the whole month. Call
this day n; the day previous to it is $n-1$ and the one after it $n+1$ and
so on. The temperatures on these days are written under the corres-
ponding number and the average of each is calculated to give the general
variation of temperature on either side of day n, for the whole period
of 18 months. For these same days of each month, the field strength
is also correspondingly written down and the average variation of
signal intensity is calculated from these readings. In Fig. 4, the curves
a and b refer to temperature and field intensity, respectively. The
scale for b increases from top to bottom, in order to indicate better
the inverse effect. The curves a' and b' in the same figure obtained by
using the smoothing formula $(a+2b+c)/4$ show in a much more pro-
nounced manner that as temperature increases, field intensity decreases
and vice versa.

Fig. 5 relates to a similar analysis with minimum temperature day.
Here the similarity between the two quantities is very slight, with only
a trace of the inverse.

Whether taken over long periods or judged by the second method of
average daily variations, there is evidence to conclude that in-

crease or decrease of temperature is generally attended by the opposite variation in signal intensity.

The question naturally arises whether such an inverse effect is evident with long distance observations, say, over more than 1500 km. Signal intensity variations are primarily connected with the state of ionization in the upper regions; the duration of sunlight and the angle of sun's rays at any two places far from each other vary so much all along the path that any similarity in temperature variations, due even to this cause alone, is non existent. So far, in long distance measurements, no results in this matter have been obtained, due certainly to numerous other influences that prevent any useful comparison.
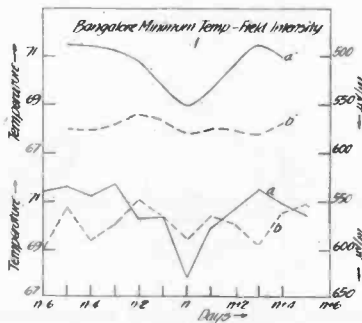
Fig. 5

Temperature and Atmospherics. On the other hand, the close correlation between local temperature and intensity of atmospheric disturbances found by Austin (loc. cit.) is perhaps due to the fact that the latter are bound up with the electric potential gradient of the air in the locality. Proof of this is forthcoming in the observations made at the Central College, Bangalore, by Prof. A. Venkata Row Telang. Low humidity and high temperatures produce high potential gradients of as much as 300 volts per meter or more, even at comparatively low heights from the ground; and atmospheric disturbances are likely to be violent under such conditions. Disturbances due to high potential gradients of a strictly local character are more numerous than those due to others and are distinct from the ones caused by agencies like near or distant thunderstorms.

4. BAROMETRIC PRESSURE AND FIELD INTENSITY

In examining the dependence or, more correctly, the relation between field intensity of Madras and the changes in the barometer readings (supplied by the Bangalore Observatory) at 0819 I.S.T., we

notice in the curves for monthly averages (Fig. 6) what appears to be
a direct relationship between the two. Between March and July, 1926,
the pressure steadily decreases while signal intensity is practically
constant. July is also characterized as being the month of lowest barom-
eter reading for the whole year. From July onward, however, there
is good measure of similarity between the two curves. The smoothed
curves a' and b' of Fig. 6 give the main variations for the whole period
under queston.
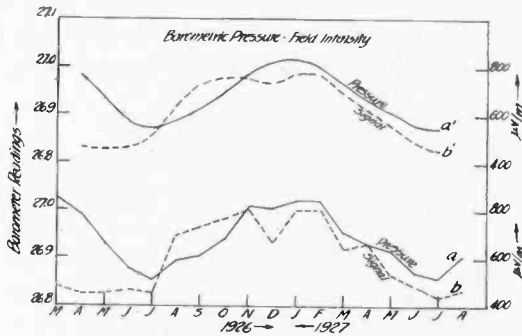
Fig. 6

As with temperature, Figs. 7 and 8 respectively give the average
daily variations of pressure and field intensity on either side of the
maximum and minimum pressure days of each month for the eighteen
months from March, 1926, to August, 1927.

The curves a and b in the two figures may apparently lead one to
think that there is an inverse relation between pressure and field
strength. But the curves of monthly averages have distinctly shown
a direct connection. That this is more probable will be realized if a for-
ward shift of two days is given to the field intensity curve. This has
been done in Fig. 7, giving a' and b', and they show the appreciable
similarity between pressure and signal intensity. The two-day shift
has been incorporated in the smoothed curves a'' and b''.

With the variations of the two quantities on either side of the day
of minimum pressure as shown in Fig. 8, there appears to be little
agreement. The forward shift too appears to be one day instead of
two. However, the interesting fact is that pressure always lags behind
reception, indicating that pressure variations are in no way the cause
of signal strength changes.

The statement occasionally made that there is no connection be-
tween pressure and reception is perhaps due to this uncertain lag, which
undoubtedly causes indefiniteness except in cases of prolonged obser-

vations. Even then, the relationship is none too marked, nor absolutely definite.

Unlike pressure, there is no lag between temperature and field intensity as we have already seen. The day of lowest temperature is also the day of highest signal strength and vice versa. It is therefore difficult to say whether or not reception changes are caused by temperature variations.
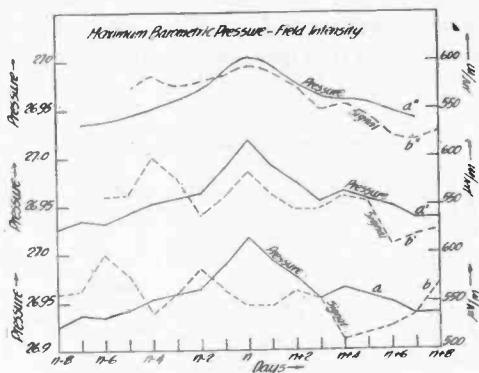
Fig. 7

5. ATMOSPHERIC OZONE AND FIELD STRENGTH

The interest, evident of late, in regard to the presence of ozone in the atmosphere of the earth is shown by the attempts that have been made to understand its relation to a variety of things, such as weather, solar phenomena, and earth's magnetism. The formation of ozone by light in the band spectrum of oxygen from 1300 to 1850 A.U. and the breaking up of ozone into oxygen between 2300 and 2900 A.U.,—the latter figure corresponding to the limit of the sun's ultra violet spectrum[6]—are properties which are likely to have definite relation to the state of ionization of the upper atmosphere.

Apart from the ozone values determined at about half a dozen places in northwestern Europe by Dobson[7] and his associates, there is little information to be obtained. Of the condition of ionization of the gas, the hourly changes in its ionization, the velocity of the ionized molecules in the field of the earth, any changes in the average height at which the ozone layer exists, and other information, nothing is known. Further, there seems to be no method yet devised to determine

[6] E. O. Hulburt, "Ionization in the Upper Atmosphere of the Earth," *Phys. Rev.*, 31, 1018, 1928.

[7] The author understands that ozone measurements have recently been started at Kodai Kanal and a few other places in the lower latitudes.

these. It seems possible that radio wave propagation experiments might be of some help in this direction.

Clayton[8] pointed out the possible existence of some relation between atmospheric ozone and solar phenomena. The properties of the gas in relation to ultra violet radiation from the sun immediately suggests an examination of its possible relation to radio reception. With the figures for ozone values for northwestern Europe kindly supplied by Dr. Dobson, an analysis of the variations of Madras signal strength in relation to the ozone content of the atmosphere was undertaken and the results communicated in a letter to *Nature*.[9]
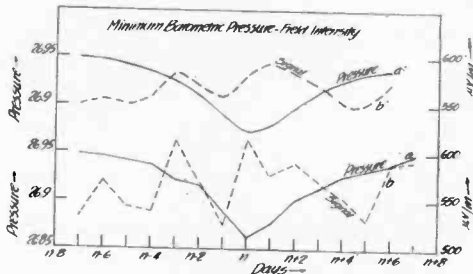
Fig. 8

The period of comparison was limited to the six months from March to August 1927, which was the only interval common to both the measurements. The conclusions arrived at in the paper regarding this matter would have been more reliable if comparison over the whole period were possible, especially with local values of ozone.

The curves a and b, Fig. 9, represent the variations in the weekly averages of field intensity and ozone value. Except for the ninth and tenth weeks, the curves march almost parallel to each other, particularly from the eleventh week. The reason for the sudden shooting up of signal strength in the ninth week and its equally rapid decrease in the tenth is not understood, as no corresponding pronounced change occurs in any other terrestrial or solar elements. The parallel changes in the two quantities show up much better in the curves a' and b' (Fig. 10) of smoothed values. If the values for the two abnormal weeks are omitted, the correlation, in the smoothed values, between field intensity and ozone is 0.88 ± 0.023, representing a very satisfactory measure of agreement, in view of the nature of the phenomena.

[8] H. H. Clayton, "Atmospheric Ozone and Solar Variability," *Nature*, 120, 153, 1927.

[9] K. Sreenivasan, "Long Wave Radio Reception and Atmospheric Ozone," *Nature*, 122, 613, 725, 1928.

The inference naturally arises that there is a direct relation between the ozone present in the air and long-wave intensity even when averages over short periods like a week are taken. From the ionization theory of propagation, as far as it is understood, it appears that an inverse relation might be anticipated between ozone and short-wave reception. This is likely, as very high frequency waves in traveling into much higher heights than that at which the ozone layer is estimated to exist get attenuated in penetrating through it.

Long-wave intensity is proportional to the conductivity of the conducting layer. The curves therefore indicate that the variation of the ozone content is proportional to that of conductivity. From this, it appears that the ionization of the ozone in the atmosphere is proportional to the ozone content therein.
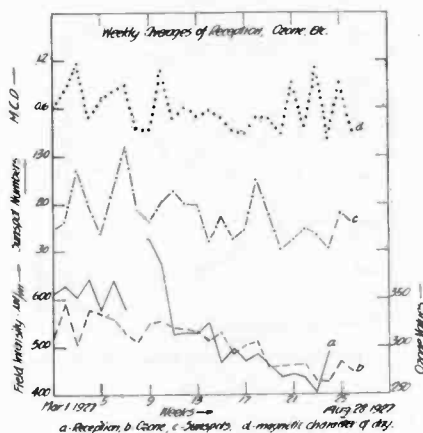
Fig. 9

It is appropriate that, during the period of similarity between field intensity and ozone, a comparison should be made with the variations of the magnetic elements. Curve *c*, Fig. 9, gives the weekly averages of Wolfer's sunspot numbers, while *d* shows those of the international mean magnetic character of day. But for the peak of the eighteenth week, the sunspot curve shows the same general tendency to decrease as the ozone and intensity curves do. After the eleventh, the peaks and troughs in the ozone and sunspot curves occur together, though the ratio of amplitudes in the variations is not constant. The corresponding smoothed curve is *c'*. A further examination into this question cannot fail to yield interesting results.

It is explained that variations in sunspots generally mean changes in the high velocity (charged or neutral) corpuscles and the electro-

magnetic radiations streaming out of the sun. If a correspondence between ozone and sunspots is quite definitely established, it will facilitate a better understanding of the relation between reception and sunspots. On the other hand, there seems to be no tangible relation between field intensity or ozone to terrestrial magnetism, as represented by the international mean magnetic character of day numbers, as curves d in Figs. 9 and 10 show. Neither in general variation nor in details is there any direct or inverse relation that can be definitely traced. This is the more remarkable as one would think that terrestrial magnetic changes would be expected to be accompanied by fluctuations in reception.
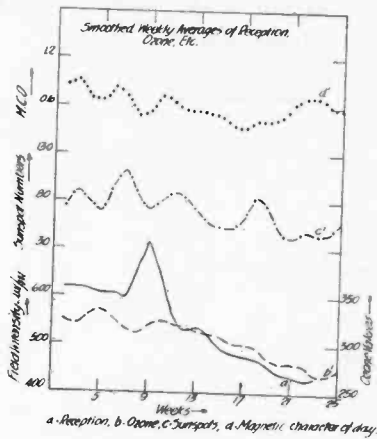
Fig. 10

As pointed out in the letter in *Nature*, already referred to, the close correlation between ozone and reception on 4 km is striking in view of the great distance of about 8000 km, from Bangalore and Madras to the stations in northwestern Europe, where the ozone values have been observed. The conclusion seems to be warranted that changes in atmospheric ozone partake of the nature of a world phenomenon. That this is not so has been pointed out by Dobson. The reasons given are (a) that ozone values have an annual variation in higher latitudes, (b) that the corresponding variations in lower latitudes are correspondingly small, and (c) that no evidence has been found of any world wide variations in the amount of ozone.

While great care has to be exercised in coming to any definite conclusion on this point, the similarity between the curves is so marked over a comparatively considerable period of time, that the correlation may not be found to be spurious after an examination of further data.

Fig. 11

Weekly Averages of Field Intensity, Sunspots and Magnetic Character of Day.
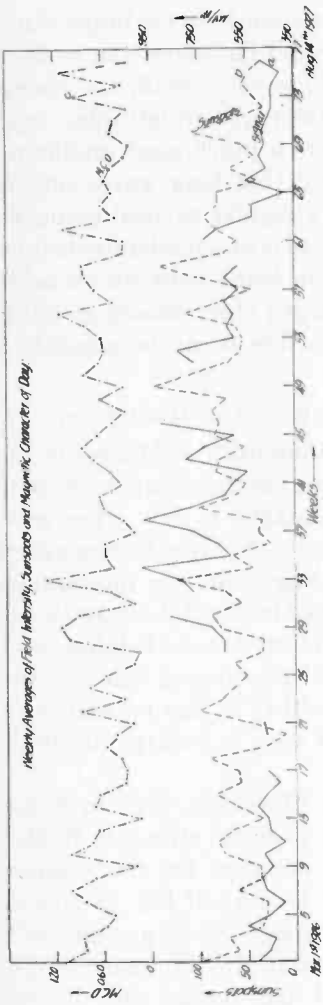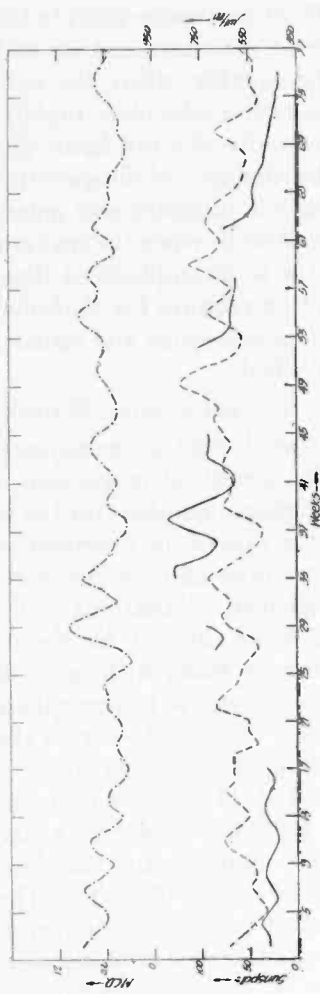
Fig. 12

Dobson estimates the height of the ozone layer at about 30 to 40 km. Eckersley's estimate[10] of the height up to which long waves travel is between 40 and 50 km. Hollingworth (loc. cit.) has given the limits as 75 and 90 km. If Eckersley's figure is taken, the existence of the ozone layer at about that height is of very great interest.

What part ozone plays in the ionization changes in the upper atmosphere at periods of sunrise and sunset would be interesting to know; at the equator, where the twilight period is very small, the changes in ionization take place rapidly while in the northern latitudes, where the period is of a few hours duration, there is much more gradualness in the change. Hollingworth[11] has shown that long waves undergo changes in intensity and polarization in a regular cyclical manner at these periods, when the transmitting station is at a moderate distance.

It is to be emphasized that the relation found between ozone and reception requires for confirmation prolonged observations at Bangalore on both ozone and signal intensity before it can be accepted as established.

6. Terrestrial Magnetism, Sunspots and Reception

In studying the variations in the field intensity of Madras in relation to terrestrial magnetism, considerable use has been made of the international numbers for the magnetic character of day. They represent the sum of the character of day figures of the forty-three magnetic observatories all over the world, cooperating under the international arrangement. Variations in declination and horizontal intensity were found more difficult for computation and were not therefore used; and for the study with sunspots, Dr. Wolfer's sunspot numbers have been used. It is to be remembered that the M.C.D. has no real physical basis by itself and from that point of view is perhaps unsuitable for comparisons of this kind.

In Fig. 11, *a*, *b*, and *c* represent from March 1st, 1926 to August 14th, 1927, the respective weekly averages of signal strength, Wolfer's sunspot numbers and the international numbers for the magnetic character of day (M.C.D.) The curves *a*, *b*, and *c* of Fig. 12 give the smoothed values obtained as described earlier. There appears to be no relation of any kind between reception and M.C.D., either of a direct or of an inverse nature. Analysis of daily values and monthly averages too have shown no connection whatever. A day of magnetic storm represented by a high character number is generally unaccompanied by any abnormality of reception even with some reasonable

[10] Round, Eckersley, Tremellen, and Lunnon, "Report on Measurements Made on Signal Strength at Great Distances," *Jour.* I. E. E., **63**, 933, 1925.
[11] J. Hollingworth, "The Polarisation of Radio Waves," *Proc.* Roy. Soc., **119 A**, 444, 1928.

difference in time. The curves of smoothed values too (Fig. 12) show no relation between reception and M.C.D.

Further examination of this point was made by analyzing the readings on the basis of the five selected D days of the month. Following the method of Chree and Stagg (loc. cit.), each of these five D days of the month is given the number n; the succeeding days are $n+1$, $n+2$, etc.; while the days preceding it are $n-1$, $n-2$, etc. Under each number the values of the M.C.D. and signal strength for the corresponding dates are written side by side. The process is repeated for each of the eighteen months, thus giving ninety n days and an equal number of $n-1$, $n-2$ etc., and $n+1$, $n+2$ etc., days. The average of the two quantities for each of the days on either side of, and including day n, is taken. Thus we get their average daily variation on both sides of the D days. Table 1 for April 1927 is given to make the process clear. The D days for this month are 9th, 11th, 12th, 14th, and 24th. The readings for the rest of the months go one after another in the respective columns.
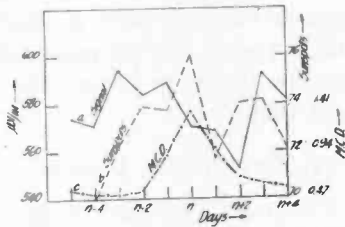
Fig. 13

The averages for the five days before and four days after day n—ten days in all—are given as curves in Fig. 13. No evidence of similarity or of any inverse relation is present. While the M.C.D. varies between the wide limits of 0.5 to 1.5, i.e., from a comparatively mild to a highly disturbed day, the maximum variation in sunspots is from 70 to 76, about 8 per cent, and that in field intensity from 550 to 590 μv per m, about 7 per cent. Earlier in the paper, it was seen that the ozone value too exhibited little relation with terrestrial magnetism as represented by the M.C.D. Reference to this will be made again a little later.

$n-4$			$n-3$			$n-2$			$n-1$		
Date	M.C.D.	F.S.	Date	M.C.D.	F.S.	Date	M.C.D.	F.S.	Date	M.C.D.	F.S.
Apr.			Apr.			Apr.			Apr.		
5	29	638	6	10	640	7	37	635	8	37	691
7	37	635	8	37	690	9	57	600	10	31	638
8	37	690	9	57	600	10	31	638	11	58	590
10	31	638	11	58	590	12	40	560	13	37	495
20	4	—	21	2	—	22	1	—	23	35	—

n			n+1			n+2			n+3		
Date	M.C.D.	F.S.	Date	M.C.D.	F.S.	Date	M.C.D.	F.S.	Date	M.C.D.	F.S.
Apr. 9	57	600	Apr. 10	31	638	Apr. 11	58	590	Apr. 12	40	560
11	58	590	12	40	560	13	37	495	14	76	578
12	40	560	13	37	495	14	76	573	15	34	580
14	76	573	15	34	580	16	5	580	17	1	790
24	49	—	25	34	—	26	9	675	27	4	—

Reception and Sunspots. In their main variation as shown by curves *a* and *b* of Fig. 11, there appears to be a direct relationship between reception and sunspots, an increase or decrease of sunspots being usually accompanied by a corresponding change in reception. It is to be strongly emphasized that there is little similarity between the curves in the matter of details, due perhaps to other influences on reception. There is also a shift of phase between the two sets of curves of no constant character; and even in the cases of similarity, the proportional changes of amplitude in the two quantities are not constant.

From the curves of smoothed averages (Fig. 12), the general characteristics are seen to be that from March to July 1926, the values are fairly constant varying only a little from the average. From July onward, they are on the increase till March 1927, when there is a steeper increase back to about the same value for the corresponding month of 1926.

7. The 27-Day Period

In their analysis (loc. cit.) Chree and Stagg have found a 27-day interval in magnetic disturbances of any type and concluded that there is no definite departure from this period, whatever be the sunspot conditions prevailing. Further, the period of solar rotation in the region of the greatest number of spots is approximately 27 days. This coincidence is of help in the present study of the relation between solar phenomena and radio reception.

The whole period of 18 months has been divided up into twenty groups of 27 days each and the observational data arranged accordingly. The field intensities of the first days in each group are added and the average calculated. The readings for the second days in each group are similarly added and the average taken. The process is repeated for each of the 27 days. Thus a series of average daily values for the period is obtained, giving the mean day to day variation for the 27 days, removing almost all the undesired variations. Similarly we get the figures for sunspot numbers and the M.C.D. These give us Fig. 14 with the curves *a*, *b*, and *c* while *a'*, *b'*, and *c'* give the smoothed values. The coefficient of correlation between sunspots and reception works out to 0.31 ± 0.117, from curves *a* and *b*, Fig. 14.

The agreement between the curves of sunspot and reception even in the matter of peaks and troughs is pronounced. On four of the days there is a phase shift of a day, with sunspots as the earlier occurrence. On the nineteenth day, however, the reverse is true. The ratio of field strength to sunspots for the averages over the whole interval is 8.86. The maximum ratio is 10.6 and the minimum 7.2, giving a maximum variation range of about 20 per cent. In view of the long period considered and the nature of the phenomena, the above is certainly satisfactory.
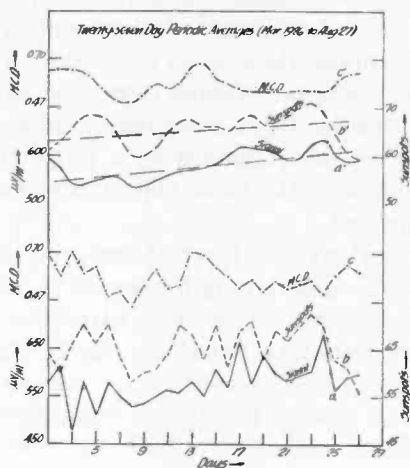
Fig. 14

In addition to the daily variations, a main steady increase in both the quantities is evident at a glance as Fig. 14 will show. This is better brought out in the smoothed curves *a'* and *b'* of Fig. 14. In this connection, it is necessary to point out that while the main general increase is a common feature and there is agreement in the details of peaks and troughs, the proportionate changes of amplitude are not the same right through.

Examining the curves of the M.C.D., there are a few days when the peaks and troughs correspond with those in the sunspot curve, otherwise there is no other feature of any interest.

The 6-Day Period. Curve *a'* (Fig. 14) exhibits what appears to be a six-day period. The peaks occur on the days 1, 6, 12, 17, and 24; and the troughs on the days 4, 9, 14, 21, and 27. No such shorter period is seen in the sunspot curve. This feature is more prominent when the 27-day periodic averages are made out for the 12 months from March 1926 to March 1927, as given in Fig. 15; the coefficient of correlation in this case is much higher, 0.73 ± 0.06.

While the appearance of this period seems definite from the curves, it is interesting to note that Austin has discovered a 9-day periodic variation of intensity with some stations. With the reasonable assumption of the absence of periods smaller than 27 days in solar and terrestrial magnetic disturbances, it follows that the shorter periodic variations of reception, if their existence is world wide, should be due to peculiarities of the locality itself. The Bangalore and Washington observations lead to this inference. What this local influence can be, it is difficult to see, unless it be temperature.

Variations in Amplitudes. With qualitative similarity in the changes in sunspots and reception, there is no constancy in the ratios of the amplitudes of such variations. A small increase in sunspots may result in an increase in reception. But a much bigger change in sunspots may not, and usually is not, accompanied by a proportionate increase in reception. This along with the phase shift of about two days suggests the following explanation.

On the Kennelly-Heaviside layer theory, the main cause of the changes in long-wave signal strength consists in the changes in the conductivity of the upper layers; the variations in ionization at these heights, apart from any radiations due to the earth's surface, are due to the ionizing agents emitted by the sun. These consist mainly of charged or neutral high speed corpuscles and the portion of the electromagnetic radiations which have ionizing properties, such as the ultra violet portion of the spectrum. Of the total emission, the earth gets its tiny share. The only pertinent point about the spots on the surface of the sun is that when some of these come in the direction of the earth, there is a change in the quantity of the ionizing agents reaching the earth. So, if every sunspot was equally effective in regard to the changes in atmospheric conductivity, then Wolfer's numbers would truly correspond with reception in direct proportion. In the absence of it, either all the spots are not equally active or the emission of the high speed particles or of changes in the electromagnetic radiations, due to some of the spots, are directed away from reaching the earth's surface. In such a case, Wolfer's numbers are accompanied by only a little or no change at all in field intensity.

In Figs. 14 and 15, there appears on a few days a lag of about two days between sunspots and signal strength, while for the most part of the 27 days there is full agreement. This can be accounted for on the assumption that the sunspots change only the number of the high velocity corpuscles shot out into the earth's atmosphere.

When the spots are few, if solar activity is violent the velocity of the particles shot out is very high, possibly as much as $1/10$ of that of

light. Neglecting any retarding influences on the way, these would enter the earth's atmosphere within a few hours and cause changes in the conductivity of the atmosphere, resulting in an almost immediate response in radio field strength variations. But when the velocity of the particles is comparatively low, a much longer time would be necessary to reach the earth's atmosphere. Even if their numbers were high, the velocity being low, the effective change in ionization of the
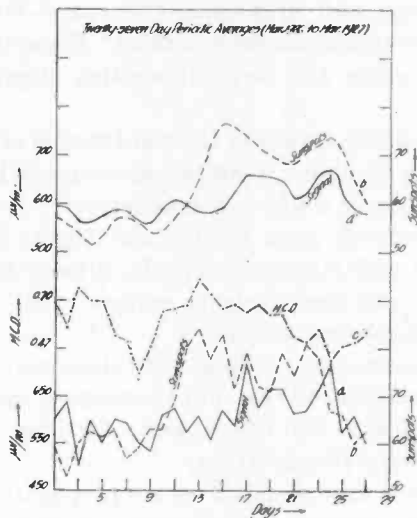
Fig. 15

earth's atmosphere in the region where long waves are supposed to travel is not great. The lag of two days would mean an average velocity for the particles of about 1/360 of the velocity of light. Further, the diffusion process from the heights at which these high velocity particles enter into the earth's atmosphere to the heights above the surface of the earth where long waves are believed to travel, is likely to take considerable time.

8. Certain Differences Between Long- and Short-Wave Transmission

It has often been pointed out that there are many differences in the transmission of long and short waves. Of these the present paper attempts an explanation of one or two only, mainly concerned with the subject matter of the paper.

We have, first of all, the phenomenon that during a time when short-wave propagation shows great abnormality, lower frequencies may ex-

hibit none. A striking instance of this was in October 1926, when short-wave communication the world over broke down while long-wave reception was hardly affected. A milder instance was on July 8, 1928, when for over 24 hours, short-wave communication was very difficult.

Secondly, there is the occasional lag of about two days, at the most, between long-wave reception and sunspots, whereas short-wave propagation indicates almost instantaneous response.

Thirdly, we have the possibility of a true physical relation between long-wave reception and atmospheric ozone. With short waves, however, no such relation has been noticed. These three are not independent of each other and any explanation should consider them together.

Now, apart from the details of the constitution of the upper atmosphere about which complete definite knowledge is lacking, there are two points which can be safely taken as correct.

(a) At comparatively great heights, the density is so low that free electrons can exist with a long mean path; at lower heights, say 50 km, density is higher, and free electrons cannot exist, so that there are only ions of very much less mobility.

(b) All the measurements agree that short waves penetrate into greater heights than long waves. For the former, varying figures have been given, from 100 to 250 or 300 km. For long waves the highest figure given is between 75 and 90 km.

The suggestion is that short waves are propagated in an absorbing refractive medium consisting of free electrons, while for the transmission of lower frequencies the conductivity of the lower layers of less mobile ions is responsible.

The fundamental equation for the passage of a radio ray of velocity $\omega = 2\pi f$ in a cloud of electrons of density N and collisional frequency n is

$$I_z = NeV_z = \frac{Ne^2 E(kn - i\omega)}{m(kn^2 + \omega^2)}$$

where e and m represent the charge and mass of the electron, while E the electric vector of the plane polarized ray is parallel to z. The displacement or condenser portion of the current is represented by the second term, while the first one represents the collisional momentum of the electrons, the two having a 90-deg. phase difference. The coefficient of collision differs little from unity. While the second term helps in the propagation of the wave, the first one robs the energy in the wave and thus attenuates it giving the familiar exponential term $\epsilon^{-\alpha d/\phi(u)}$ in its most general form. When $\omega \gg n$, the attenuation is

small and the electrons execute a number of oscillations before collid-
ing, a case of refraction with negligible absorption. But when the
frequency of collision is high, that is, when $n \gg \omega$, the ray is attenuated
rapidly.

Short-wave observations seem to agree with the above rough pic-
ture of propagation; for the increase in the density of the electron cloud
due to increased sunspots or to any other agency is always attended
with poorer short-wave reception, as demanded by theory. Larmor[12]
suggested that the magnetic storm of October, 1926, might be attributed
to the incursion of electrons—not necessarily a large number—into
our atmosphere, tending to twist out all the usual ray paths. This
seems hardly possible in view of the world wide blocking of short-wave
communication. A more satisfactory explanation is that of absorption
due to the additional electrons, as represented by the first term of the
above equation. The number of them, while sufficient to have a pro-
nounced effect on short-wave reception may not in any way affect
long-wave transmission even if the velocities of the electrons were
high; the smallness in their numbers would effectively prevent the
ionization due to them from penetrating into the lower regions of long
waves.

As remarked earlier, the effect of the spots on the sun coming in the
direction of the earth is to cause a change in the ionizing agents of
the earth's atmosphere, that is, in the high speed particles and in the
electromagnetic radiations. The latter make their presence felt by
the corresponding changes of ionization occurring almost immediately
by variations in short-wave reception. The effect on long-wave trans-
mission depends upon the magnitude of the change in the radiation.
If of sufficient intensity to penetrate into the lower heights, there will
be variations on long-wave propagation too.

In the case of high speed corpuscles, the number and the velocity
of the particles determine the change. A large number with small ve-
locities, besides taking a long time to travel to the earth's atmosphere,
may not cause any serious changes in reception.

These considerations and the correlation between the Bangalore
measurements on 4 km, with the ozone content of the atmosphere,
give an indication that electrons are responsible for short-wave prop-
agation while the less mobile and heavier ions take part in long-wave
transmission. As though to support this view, we have the tantalizing
estimate of the ozone layer at about 30 or 40 km. High sunspot num-
bers, poor short-wave reception, improved long-wave transmission

[12] J. J. Larmor, "Magnetic Storms and Wireless Communication," *Nature*,
118, 662, 1926.

seem to go together with increase in the ozone value of the atmosphere
and reduced temperature. The suggestion made above is qualitative.
A quantitative treatment with experimental data for an extended
period will no doubt take into consideration the action of the earth's
field. Considerable data and analysis are necessary before some of the
results of the paper can be accepted as established and beyond doubt.

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Royal and D. K. R. Ramanathan. To Dr. G. M. B. Dobson I owe the
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to Dr. M. O. Forster for kind permission to publish the paper in the
PROCEEDINGS. It is with pleasure that I acknowledge the interest that
Prof. J. K. Catterson-Smith has always taken in these measurements
which were made with his encouragement at Bangalore.

Addendum

Information so far obtained since January, 1929, from measurements
of ozone at lower latitudes shows that its seasonal variations are quite
small. Also the only figure, that due to Eckersley, for the height of the
Heaviside layer, which at all approximated to that of the ozone layer,
has been revised to lie between 80 and 100 km. This, of course, is the
usually accepted view for long waves.

THE SIGNIFICANCE OF OBSERVATIONS OF THE PHASE OF RADIO ECHOES*

By

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Summary—A method of observing the phase of radio echoes has been developed by Messrs. Tuve and Hafstad. The present note is intended to show to what extent the phase can be expected to be constant throughout the echo if the frequency-dispersion of the Kennelly-Heaviside layer is taken into account. It is shown that for reflections of 4000-kc waves with a small retardation (effective height of 200 km) the phase can be expected to be the same for the whole echo and that in this case the observed phase is a measure of the optical path. For echoes which spend a longer time in the layer (effective height of 1800 km) the phase should not be constant and the average phase is not expected to be a measure of the optical path.

It is shown that by measurements on reflections with low retardation the ratio between the changes of (1) the equivalent height for interference and (2) the effective height for echo retardation is a measure of how much of the change is due to the layer moving as a whole and how much is due to a redistribution of electron-densities through the layer. A compression or expansion of a layer having an electron-density increasing in proportion to the height above the lower boundary should give a result by the interference-method of approximately one-third the value obtained by the echo-retardations.

It is shown that the broad echoes observed at night by Hafstad and Tuve are in all probability due to multiple echoes and not to frequency-distortion in the sidebands of the emitted pulses.

THE information received by means of radio about the height of the Kennelly-Heaviside layer has heretofore been principally from the following sources: (a) skip-distance measurements according to Taylor and Hulburt;[1] (b) wavelength variation measurement;[2] (c) measurement of the angle of the downcoming wave;[3] (d) observation of the echoes of short pulses.[4] It has been shown by Appleton[5] and Kenrick[6] that (b) and (d) are equivalent in the sense that they both give the time which a group of waves spends in travelling from the transmitter to the receiver. A combination of the two is therefore incapable of giving more information than either by itself, although of course the multiplicity of paths and fading of components is seen more clearly by means of (d). Method (c) is for short waves equivalent to

¹ A. H. Taylor and E. O. Hulburt, *Phys. Rev.*, 27, 189, 1926.
² E. V. Appleton and M. A. F. Barnett, *Proc. Roy. Soc.*, A, 113, 450, 1927.
³ E. V. Appleton and M. A. F. Barnett, *Proc. Roy. Soc.*, A, 109, 621, 1925.
⁴ G. Breit and M. A. Tuve, *Phys. Rev.*, 28, 554, 1926.
⁵ E. V. Appleton, *Proc. Phys. Soc.*, 41, 43, 1928.
⁶ G. W. Kenrick and C. K. Jen, PROC. I. R. E., 17, 711; April, 1929.

(b) and (d) since the tangents to the ray at the earth determine the equivalent path for echo-retardation. For longer waves it may be possible to obtain additional information from (c), particularly for the circularly polarized components. Short waves, however, penetrate the layer better than the long, and it is especially desirable to obtain additional data by means of them. Method (a) serves as a useful check but on account of the complexity of the phenomena involved gives average values only and is also concerned primarily with the angles which the rays make with the vertical.

For short waves these methods give us essentially $\int(1/\mu)ds$ where μ is the refractive index and the $\int$ is taken from end to end of the signal path. The echo-interference method suggested by the writer and carried out by Messrs. Tuve and Hafstad[7] is intended to give information about $\int\mu ds$, the optical path of the ray. The method consists in introducing an e.m.f. in the receiver due to the continuously going crystal oscillator used for frequency control in the transmitter. As the echoes arrive their effect depends on their phase with respect to the phase of the crystal. If, for instance, the echo and the crystal are in phase the effect on the receiver is large; if they are out of phase the combined effect is small. It is shown below that to a good approximation the relative phase depends on $\int\mu ds$. If the echo-interference and the echo-retardation are measured simultaneously a comparison of changes in the equivalent path for interference and the effective path for echo-retardation gives at least an idea of the manner in which the change in the layer takes place, for instance, of whether the layer moved as a whole or whether the lower boundary remained fixed, but in the layer the regions of larger electron densities moved up. By means of this method it is possible to detect smaller changes in height than have been observed otherwise. On account of the complexity of the echo-pattern it is important to resolve the pattern rather than to be analyzing fading curves which are the result of the superposition of a large number of interfering echoes. In order to be sure of the interpretation of the results it is of course necessary also to study the polarization of the echoes. In the calculations made below nothing is said about polarization. This omission is not important since polarization can be taken into account later and would needlessly complicate the present discussion, which is concerned with the frequency-distortion due to the sidebands of the pulses used.

Let us consider a set of rays connecting the transmitter T to the receiver R. For every frequency f there is a definite $\int(v(f))^{-1}ds$ where $v(f)$ is the phase-velocity. The disturbance at R due to a pulse at T is

[7] L. R. Hafstad and M. A. Tuve, Proc. I. R. E., **17**, 1786; October, 1929.

$$E = \int_0^\infty A(f) \exp 2\pi i (ft - \int (f/v(f))ds)fd \tag{1}$$

while the disturbance sent out by T is

$$F(t) = \int_0^\infty A(f) \exp 2\pi i ft \ df = g(t) \exp 2\pi i f_0 t. \tag{2}$$

We let

$$\int (f/v(f))ds = P(f) \tag{3}$$

and expand by Taylor's theorem

$$P(f) = P(f_0) + (f - f_0)P'(f_0) + (1/2)(f - f_0)^2 P(f_0) + \cdots \tag{4}$$

For the present we confine ourselves to the first two terms. Substituting (4) in (1) we have

$$E = \int_0^\infty A(f) \exp 2\pi i (f(t - P_0') - P_0 + P_0'f_0)df$$

$$= F(t - P_0') \exp 2\pi i (f_0 P_0' - P_0) = g(t - P_0') \exp 2\pi i (f_0 t - P_0) \tag{5}$$

As long as the first two terms of the expansion (4) are sufficiently accurate we see that the envelope g of the disturbance E is retarded by the amount P_0' while the phase of the vibrations modulated by g is retarded by $2\pi P_0$. According to (3) the phase depends simply on the optical path since

$$P(f) = \int (f/c)\mu(f)ds \tag{6}$$

where $\mu(f)$ is the refractive index at frequency f. The retardation of the envelope, or in other words the arrival of the group, depends on P_0'. By Fermat's theorem the differentiation can be performed inside the integral (4) so that

$$P' = \int (v_g(f))^{-1}ds \ ; \qquad (v_g(f))^{-1} = \frac{d}{df}(f/v(f) \tag{7}$$

where v_g is the group velocity. We conclude that as long as terms beyond the second are negligible in (4) the time of arrival of the group of waves represented by (2) is given by the group-velocity while the phase of the vibrations when they arrive is given by the phase velocity. It is not clear without calculation to what extent the terms disregarded in (4) are negligible. The third term when inserted in the integral of (5) gives on expanding the exponential a corrected value of E

$$E = \sum_{n=0}^{\infty} (-\pi i P_0'')^n (1/n!)$$

$$\int_0^\infty (f-f_0)^{2n} A(f) \exp 2\pi i f t' df \exp 2\pi i (f_0 P_0' - P_0) \qquad (8)$$

where

$$t' = t - P_0'.$$

In order to evaluate the integrals involving $(f-f_0)^{2n}$ we differentiate (2) with respect to t twice and obtain

$$\int_0^\infty f A(f) \exp 2\pi i f t \, df = ((g'/2\pi i) + f_0 g) \exp 2\pi i f_0 t \ ;$$

$$\int_0^\infty f^2 A(f) \exp 2\pi i f t \, df = (-(g''/4\pi^2) + (f_0 g'/\pi i) + f_0^2 g) \exp 2\pi i f_0 t$$

whence combining with (2)

$$\int_0^\infty (f-f_0)^2 A(f) \exp 2\pi i f t \, df = -(g''/4\pi^2) \exp 2\pi i f_0 t$$

and repeating the process n times

$$\int_0^\infty (f-f_0)^{2n} A(f) \exp 2\pi i f t \, df = (-1/4\pi^2)^n \frac{d^{2n} g}{dt^{2n}} \exp 2\pi i f_0 t$$

substituting this in (8)

$$E = \sum_{n=0}^{\infty} (i P_0''/4\pi)^n (1/n!) \left(\frac{d^{2n} g}{dt^{2n} \, t'} \right) \exp 2\pi i (f_0 t - P_0). \qquad (9)$$

The factor in front of the exponential is complex, and therefore the phase of the vibrations is affected. The effect is in general different throughout the received hump. In the special case of derivatives of g higher than the second vanishing we have a simplified form

$$E = (g + (i P_0'' g''/4\pi)) \exp 2\pi i (f_0 t - P_0). \qquad (9')$$

The phase correction to be applied to (5) is

$$\theta = \tan^{-1} (P_0'' g''/4\pi g) \cong P_0'' g''/4\pi g \qquad (10)$$

measured in radians. A special case of an envelope g is a sine-curve of period $2T$, the "on time" being T. For this case $(g''/g) = -\pi^2/T^2$ and the phase correction is

$$\theta = -\pi P_0''/4T^2. \qquad (10')$$

Let us examine this formula for the case of a uniformly increasing density of electrons and a somewhat simplified dependence of the refractive index on the frequency: $\mu = 1 - 2by/f^2$ where y is the height above the lower boundary of the layer. For normal incidence we find $P(f) = 2f^3/3bc = 4hf/3c$, where $h = f^2/2b$ gives the height to which the signal rises above lower boundary. Letting h' be the effective height to which the signal rises above lower boundary as measured by the echo-method, we have $P' = 2f^2/bc = 2h'/c$ so that $h' = f^2/b$. The effective height h' is in this case just twice the actual height h. The equivalent height for interference is a still third and different quantity $h_1 = 2h/3 = h'/3$. The error in the phase due to variation in group velocity is by $(10')$ $\theta = -\pi h'/cfT^2$. If $h' = 100$ km, $T = 10^{-4}$ sec., $f = 4 \times 10^6$ cycles, corresponding to the conditions of the experiment we have $\theta = -1.5$ deg. This is a small and at present hardly measurable error. In this estimate f has been underestimated on purpose and T is probably also underestimated. Taking h' to be 100 km amounts to supposing that the first reflection of approximate effective height 200 km is due to a 100-km path (one way) in the lower un-ionized atmosphere and an equal path in the Kennelly-Heaviside layer. For the first reflection h' is surely less than 200 km so that the maximum error in the phase is for this case 3 deg. For reflections with higher retardations, say for effective heights of 1800 km, we expect proportionately higher errors which in this case have to be obtained by means of (9).

Since we have disregarded terms higher than the third in (4) we have still an error in (9), but for practical purposes this error is small. The fourth term introduces in (5) a correction factor $\exp (\pi i/3)(f - f_0)^3 P_0'''$. For the special case just considered this means $\exp (4\pi i/3c)h'(f - f_0)^3 f^{-2}$. If $f - f_0 = 10^4$ the angle is 0.005 deg. At the most we are concerned with $f - f_0 = 3 \times 10^4$ in which case the angle is 0.13 deg. The effect of this term on the resultant vibrations is presumably much less than that, because the first odd term in (4) involving $(f - f_0)^{-1}$ has no effect on the phase. We can draw another conclusion from (9). It shows that if g and its derivatives vanish there is no signal. This means that to within the first three terms of (4) the arrival of the humps is accurately given by the group-velocity and that their width is the original width of the pulse sent out by the transmitter. It is therefore very unlikely that the broadened reflections observed at night by Hafstad and Tuve[8] are due to distortion of the signals by the dispersing action of the ionized atmosphere. They are rather to be explained as the result of the superposition of several

[8] L. R. Hafstad and M. A. Tuve, PROC. I. R. E., **17**, 1513; September, 1929.

echoes. This conclusion is especially supported by an experiment per-
formed during one of the night tests. The carrier-frequency was
changed by 20 kc. No observable change in the retardation of the
broad echoes was found. If the variations of group-velocity in the side-
bands were responsible for the width, this change of all the frequencies
concerned would have produced a measurable change in the effective
height.

The law $\mu^2 = 1 - 2by/f^2$ is of course only approximate. The conclu-
sions we have reached by means of it are essentially correct. Instead
of taking the electron-density to be proportional to the height y above
the lower boundary of the layer, we shall suppose that it is proportional
to y^n where n is a positive constant, and we also attempt to take into
account the magnetic properties of the medium by letting

$$\mu^2 = 1 - a^n y^n/(f^2 - ff_1) \tag{11}$$

where f_1 may be the magnetic absorption frequency or its negative.
This gives

$$P(f) = 2f(f^2 - ff_1)^{1/n} q_n/ac = 2fhq_n/c \tag{12}$$

where

$$q_n = \int_0^1 (1 - x^n)^{1/2} dx, \quad h = (f^2 - ff_1)^{1/n}/a$$

As before h is the height reached by the signal. The equivalent height
for interference is

$$h_1 = hq_n. \tag{13}$$

The effective height for echo measurements is

$$h' = h_1 [(1 + 2/n)f - (1 + 1/n)f_1](f - f_1)^{-1} \tag{14}$$

while the phase-error is

$$\theta = -(\pi h'/2T^2c) \{(1/nf) + (1 - n)/(n(f - f_1))$$
$$+ (1 + 2/n)[(1 + 2/n)f - (1 + 1/n)f_1]^{-1}\}$$

Relation (14) shows how the effective height for echo-retardation h' is
connected with the equivalent height for interference h_1. If n is large
$h' = h_1$, while if n is small h' is larger than h_1. This of course is to be
expected because for large n the condition of specular reflection is
approached, while for small n the rays are gradually refracted. The
phase error may become unusually large if $f = f_1$ or if $(n + 2)f = (n + 1)f_1$.
In the region of 4000 kc neither of these conditions takes place and the
estimate made with the simplifying assumptions is essentially correct.

As a working hypothesis it is probably satisfactory to take $n = 1$ in
the layer and to suppose that the lower boundary is at some height h_0

above the ground. If changes in height as observed by the two methods are equal the interpretation is that the layer moves as a whole. If the interference method gives about one-third of the echo retardation all of the change is likely to have taken place as a redistribution of electrons in the layer, resulting in a compression or expansion of the layer as a whole with a fixed lower boundary.

On account of the sensitivity of phase to the magnetic field of the earth it may be possible to detect a correlation between the observed phase-changes and changes in the field.

INDUCTANCE AS AFFECTED BY THE INITIAL
MAGNETIC STATE, AIR-GAP, AND
SUPERPOSED CURRENTS*

By

H. M. TURNER

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THE object of this paper is to present information regarding the manner in which the inductance of iron-cored coils depends upon the initial magnetic state, the length of the air-gap, and the amount of direct and alternating current flowing in the winding. Important contributions have been made by Ball, Chubb, Spooner, Prince, Hanna, Morecroft, Howe, and others, but considering the importance of the subject it is surprising that there are so few data or curves in the literature showing the performance of inductance coils under widely varying conditions. It is hoped that the results here reported will be of interest and value to those who use or occasionally design choke coils or audio-frequency transformers, although no attempt was made to simulate closely the constants of either.

METHOD OF MEASUREMENT

The measurements, with the exception of a few with the ballistic galvanometer, were made with the constant impedance method[1] on account of its simplicity, precision, ease of operation, and the fact that it is particularly adapted to the measurement of inductances from five henries up to large values and requires only a small standard condenser which is usually at hand. Since the details of the method are fully covered in the paper referred to, only a brief explanation will be given here.
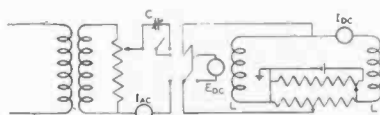
CONSTANT IMPEDANCE MEASURING CIRCUIT
Fig. 1

Two similar coils are connected in parallel to provide a local path for the direct-current magnetizing component so as to keep it out of the

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    [1] H. M. Turner, "The Constant Impedance Method for Measuring Inductance of Choke Coils," PROC. I.R.E., 16, 1559; November, 1928.

measuring circuit. With the double-pole switch thrown to the right, adjust the lower sliding contact, Fig. 1, until there is no battery potential between the lines to the parallel inductances as indicated by a low reading voltmeter. This is particularly important when using large values of direct current, for a relatively small unbalanced voltage from the battery might burn out the line instrument or at least introduce slight errors. Throw the double-pole switch to the left or
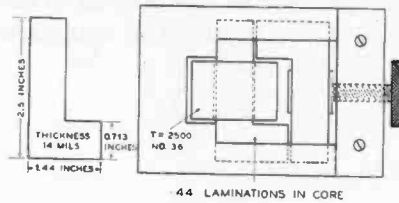
Fig. 2

operating position and proceed as follows: with the single-pole switch to the left measure the alternating current in the parallel inductive circuit by means of a thermocouple instrument, electron-tube voltmeter or otherwise, then throw the switch to the right, connecting the standard condenser in series with the line, and starting with a low value of capacity increase it until the line current is the same as before. The impedance is the same for the two positions of the switch, and should be carefully verified by throwing it first to one and then the other, adjusting the capacity until the permanent current is the same.
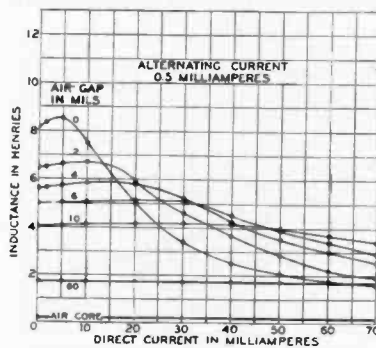
Fig. 3

Since the impedance is the same and no change has been made in the resistance then

$$X_L = X_C - X_L, \quad \text{or} \quad X_C = 2X_L \tag{1}$$

that is, the capacity is half that required for resonance at the funda-

mental frequency and twice that for resonance to the second harmonic, so harmonic components will have negligible effect on accuracy. The adjustment is made on the steep portion of the resonance curve where the critical value of capacity to satisfy equation (1) may be accurately determined. If the line current is too small with the condenser in the circuit the capacity should be increased or vice versa. For critical capacity the permanent line current is the same for the two positions of the single-pole switch, but will lag in one case by θ degrees and lead by θ degrees in the other. The inductance of the two coils in parallel is found from the equation

$$L = \frac{1}{2w^2C} \tag{2}$$

and that for one of the coils by

$$L = \frac{1}{w^2C} . \tag{3}$$

The transient currents due to switching will be limited by the inductance and are of no importance, however, if preferred balance may be obtained without opening the circuit. This is done by adjusting the capacity until the voltage across the inductive circuit is the same as that of the line as determined by an electron-tube voltmeter.

MAGNETIC CIRCUIT

The magnetic circuit is shown in Fig. 2 and consists of two L-sections each having 44 laminations 14 mils in thickness, the material
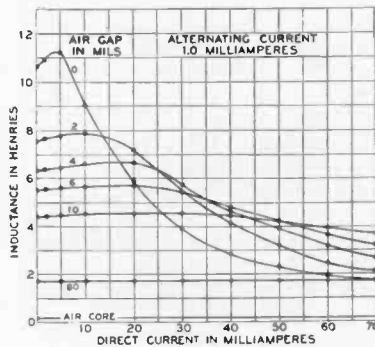
Fig. 4

being 3.5 per cent silicon steel. The dimensions are: length 16.3 cm, cross-section 2.82 sq. cm and volume 46.2 cu. cm or 6.42 in., 0.437 sq. in. and 2.8 cu. in., respectively. Where the length of the air-gap is

given it will be the measured value of the two in series. The coil has 2500 turns of No. 36 wire.

CONDITIONS AND MEASUREMENTS

The first series of measurements, the results of which are plotted in Figs. 3 to 13 inclusive, were made under the conditions of practical use, that is, without first demagnetizing the core. As a matter of convenience and to confine the study to the variation of inductance
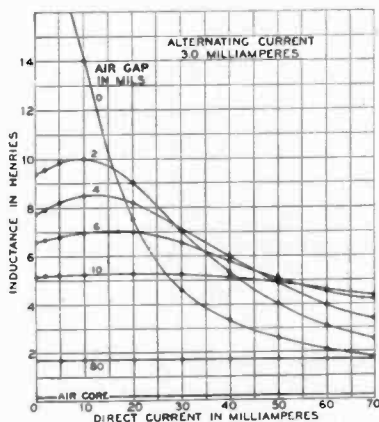
Fig. 5

as determined by the magnetic circuit, all measurements in this series were made at 60 cycles. Data were taken over a wide range of conditions: the air-gap being varied from zero to 80 mils and then the core was entirely removed; the direct current was varied from zero to 80 ma and the alternating from 0.5 to 10 ma effective. In order to see more clearly the effect of these various factors families of curves were plotted with each of these quantities as the independent variable. In some respects it would have been better had ampere-turns per unit length of magnetic path or gilberts been used, but with varying air-gap and saturation much of the advantage that would result in the case of complete magnetic circuits would have been lost due to the uncertainty of the portion to be assigned to the air-gap.

The second series of measurements, made with the ballistic galvanometer, includes the determination of the inductance for butt and lap joint magnetic circuits where the core was first demagnetized before making measurements and where the core was not demagnetized; normal hysteresis loops for butt and lap joint and the slope of the minor hysteresis loops as determined by the way a given magnetic state is approached, that is, by the past magnetic history.

The third series of measurements was made of the primary inductance of an audio-frequency transformer over a wide range of values of direct and superposed alternating current. These measure-
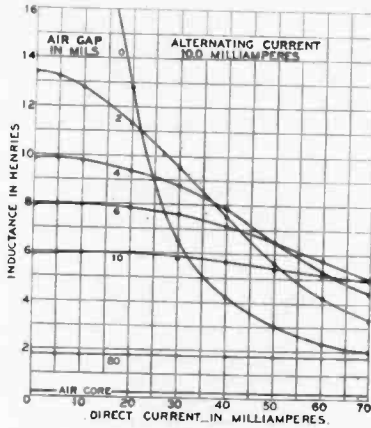
Fig. 6

ments were made with the constant impedance method at 60 cycles to avoid effects that would be present at higher frequencies, such as distributed capacity; however, these effects are small up to a few hundred cycles.

DISCUSSION OF CURVES

In analyzing these curves the information regarding the number of turns and core dimensions should be kept in mind. For direct
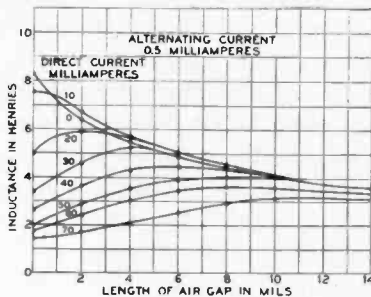
Fig. 7

currents up to approximately 12 ma the inductance is greatest for zero air-gap and decreases as the length of the gap increases, as shown in Figs. 3 to 6 inclusive. However, as the direct current increases this order is gradually changed until it is finally completely reversed, except

for the 80-mil gap which falls below the other curves throughout the entire range, being less than half that for the 10-mil gap.

Inductance is usually expressed in terms of the incremental permeability, length of path in iron, length of path in air, and other factors. It is thought that the use of the term incremental reluctance will simplify the discussion and give a clearer understanding of the action taking place. The inductance is given by

$$L = n\frac{d\phi}{di} = \frac{0.4\pi n^2}{R}10^{-8} \text{ henries} \tag{4}$$

$$= \frac{k}{R} = \frac{k}{R_i + R_a} \tag{5}$$

where R_i, the incremental reluctance of the iron, varies with the constant magnetizing component due to saturation of the iron and R_a is that of the air path.
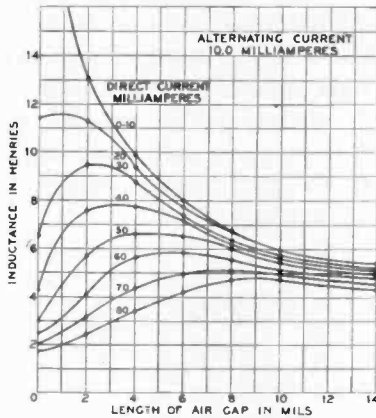
Fig. 8

Overlooking for the moment some secondary effects, the inductance for zero air-gap and low values of direct current is determined largely by R_a, which is appreciable for butt joints even though the two parts of the core are clamped together. R_i is small due to the law saturation of the core.

If the reciprocal of the inductance obtained from Fig. 3 for zero direct current be plotted against the length of the air-gap the curve is practically linear from zero up to 10 mils; however, above this value the reluctance increases less rapidly due to the increase in cross-section of the air path and finally becomes constant. The fact that the

curve is linear down to zero indicates that R_a still constitutes the major portion of the reluctance even though there is no apparent air-gap. In order to obtain a measure of the remaining effective air-gap, the core was next built up with a lap joint and the inductance found from which the reluctance was calculated, its value being the ordinate to the
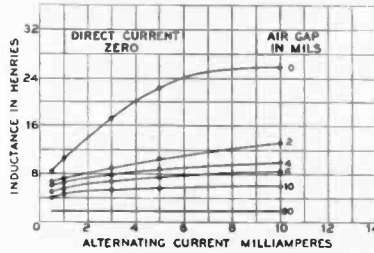
Fig. 9

dotted horizontal line. The corresponding abscissa is not known; however, it may be approximated by extending the solid line until it intersects the dotted line as shown in Fig. 13, which indicates an eqiuvalent air-gap for the butt joint of approximately 4 mils over that for the lap joint. In the vicinity of the intersection the incremental reluctance of the iron becomes of increasing importance. This was checked by using data obtained from a demagnetized core, and the results were in essential agreement.
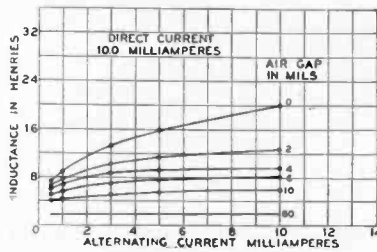
Fig. 10

With a large direct-current component the saturation is such that the incremental reluctance of the iron path exceeds that of the air path, in which case an increase in the length of the gap reduces the constant flux and thereby greatly decreases the incremental reluctance of the iron, that is, R_i decreases faster than R_a increases, and the inductance is larger. This is clearly shown in Figs. 3 to 6, where the inductance for a 10-mil gap and 70 ma direct current is more than

twice that for zero gap; however, a point is finally reached where the reverse is true, for example, at the same point the inductance for an
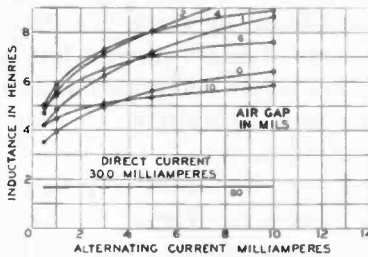
Fig. 11

80-mil gap is less than half that for the 10-mil gap. It should be noted that for 0 and 80 mils the inductance is the same, while if the iron core is entirely removed it is reduced to 0.1 this value.
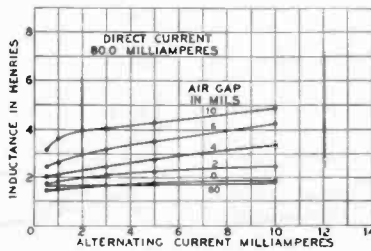
Fig. 12

The inductance increases with the alternating current, indicating the importance of knowing the alternating current as well as the direct current at which the measurements are made if they are to be sig-
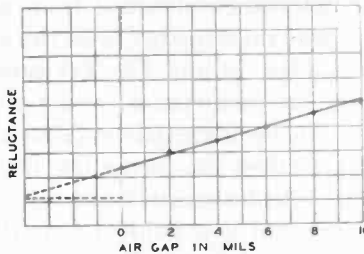
Fig. 13

nificant. Where measurements are made without first demagnetizing the core the inductance increases or tends to increase with direct current up to a maximum and then falls off. The greatest increase is for zero gap, and is of the order of a few per cent, say 6 or 8. As the

air-gap is increased the maximum occurs at higher values of direct current, but the percentage becomes less and less.

From a theoretical point of view it might be argued that the core should first be demagnetized, in which case the inductance always decreases with an increase of direct current, but in practice the cores are seldom demagnetized before being used and the inductances may differ widely in some cases. However, for some purposes this procedure is to be recommended.

Figs. 7 and 8 show the effect of air-gap upon the inductance for different values of direct current. Figs. 9 to 12 show the variation as a function of the alternating current. Fig. 13 has already been referred to.
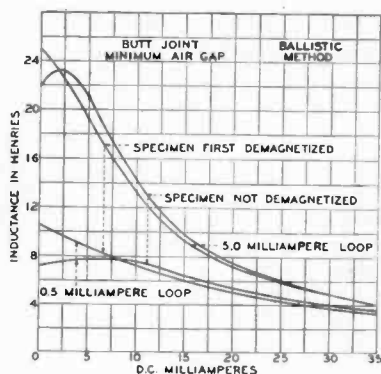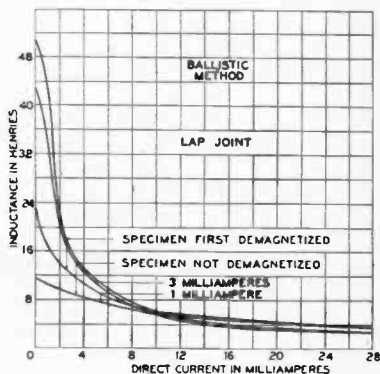
Fig. 14   Fig. 15

SECOND SERIES—BALLISTIC METHOD

The difference in inductance obtained from magnetized and demagnetized specimens was investigated, and the results for butt and lap joints are shown in Figs. 14 and 15. For zero direct current the magnetized core gives less inductance, and for small minor loops very much less than for the demagnetized specimen. In some cases this general relation is maintained for increasing direct current, but in others the curves cross. Unfortunately the minor loops in Figs. 14 and 15 are not the same, but it is evident that the lap joint gives a much higher inductance, or lower incremental reluctance than the butt joint, showing definitely that there is a relatively large equivalent air-gap in the best butt joint. This is further borne out by the hysteresis loops in Fig. 16. Attention is called to the low residual for the butt joint, only about 12 per cent, indicating a large equivalent air-gap, while that for the lap joint is about 37 per cent, giving further sup-

port to the statement in connection with Fig. 13 that R_a was the major part of the total incremental reluctance when there was no direct current even though the apparent air-gap was zero.
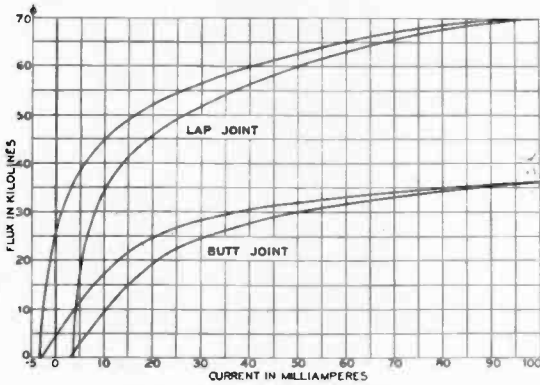
Fig. 16

Had the maximum current been 50 ma in the lower curve, Fig. 16, the upper branch would have joined the one plotted at 18 ma so the maximum value could be considerably reduced below this value and still leave practically the same residual flux.
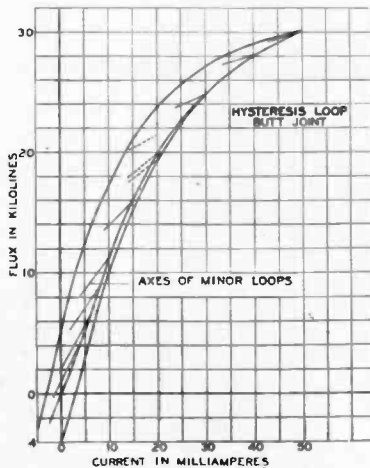
Fig. 17

In Fig. 17 the slope of the minor loops, which is a measure of the incremental permeability, for a demagnetized core decreases with increasing magnetizing current as shown by the solid lines. While they show the correct slope they are not in the proper vertical position

and should not have their tips on the initial magnetization curve, but were placed there as a matter of convenience. The dotted lines show the slope and correct vertical position of the minor loops depend-

A. C. MICROAMPERES (EFFECTIVE)
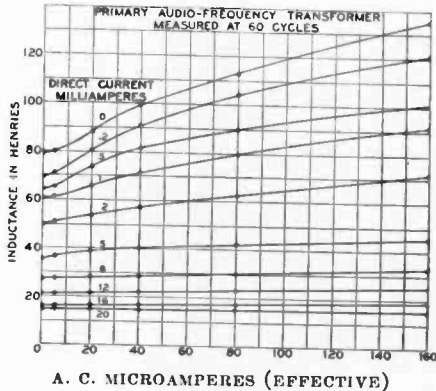
Fig. 18

ing on the approach or past magnetic history of the core. The upper one, the slope of which is taken as 100 per cent, was approached from the positive tip of the loop. The middle one was approached from zero on the initial magnetization curve and its slope is 106.8 per cent;
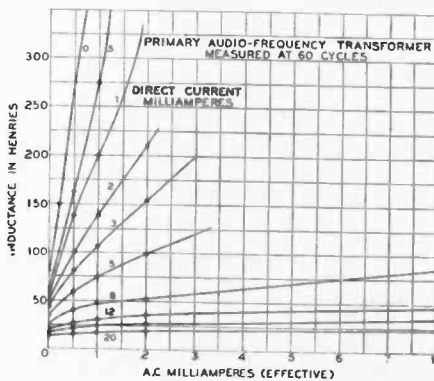
Fig. 19

the lower one is approached from the negative tip of the normal loop and its slope is 111 per cent.

THIRD SERIES—CONSTANT IMPEDANCE METHOD

The inductance of the primary of an audio-frequency transformer measured over a wide range of direct and alternating currents is

given in Figs. 18, 19, and 20. No change was made in the magnetic circuit. The first shows the inductance as a function of the alternating current from extremely low values up to several times the maximum that would ever be encountered. The inductance as a function of the direct current is shown in Fig. 20.
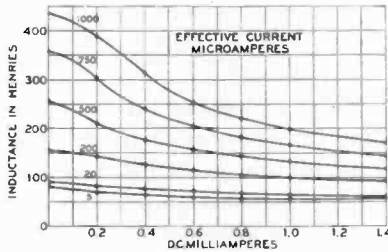
Fig. 20

Thanks are due Lieut. J. W. Green and Mr. H. T. Lyman, Jr., graduate students in Communication Engineering, and Mr. William Sherwood, instructor, for the care exercised in obtaining the data.

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NOTES ON THE DETECTION OF LARGE SIGNALS*

By

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Summary—The effect of large signals applied to the grid of a detector is discussed. It is shown that signals even as small as fifty mv appreciably affect the tube parameters and influence the frequency distortion. The nature of detector overloading is discussed and overload curves of the plate rectifier are presented.

MUCH admirable work has been done in developing the theory of rectification in detectors of both types—grid and plate—but almost all of this work has been confined to the case where the radio-frequency signal is quite small—of the order of 50 mv or less. The conditions were assumed to be such that the signal produced no appreciable variation of the tube parameters.
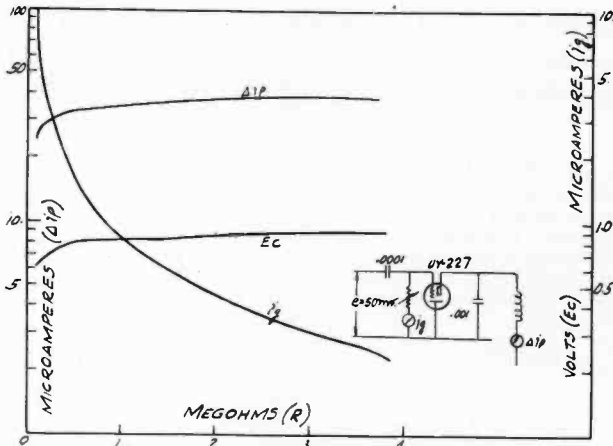
Fig. 1

Actually, it is found, in the case of receivers designed for broadcast reception, where sufficient voltage must be developed to operate a loud speaker at what might be termed "good room volume," that the radio-frequency voltage applied to the detector nearly always exceeds 50 mv, and generally is of the order of 200 or 300 mv, depending of course, on the gain of the audio amplifiers. As the voltage at the detector input is increased, changes in the fidelity of reproduction will be noticed; in addition to this a loss of volume occurs beyond a certain voltage, depending upon the tube parameters.

* Dewey decimal classification: R134. Original manuscript received by the Institute, April 13, 1929.

In the grid rectifier the audio voltage at the grid is given by[1]

$$e_g = \left(\frac{1}{2}\,\frac{d^2 i_g}{de_g{}^2}|\,Z\,|\right) m E_0{}^2 \sin\,(at+\phi)\,. \tag{1}$$

The quantity in brackets is the detection factor; the term Z is given by

$$Z = \frac{1}{g_g + A_c(d) + A_i(a)}\,. \tag{2}$$

In these formulas

m = the modulation coefficient;

E_0 = the r-f voltage applied to the grid;

g_g = the grid electron conductance = $1/R_g$;

A_c = the conductance of the grid-leak, grid-condenser combination.

A_i = the input conductance of the tube due to electrode capacities and plate circuit load.
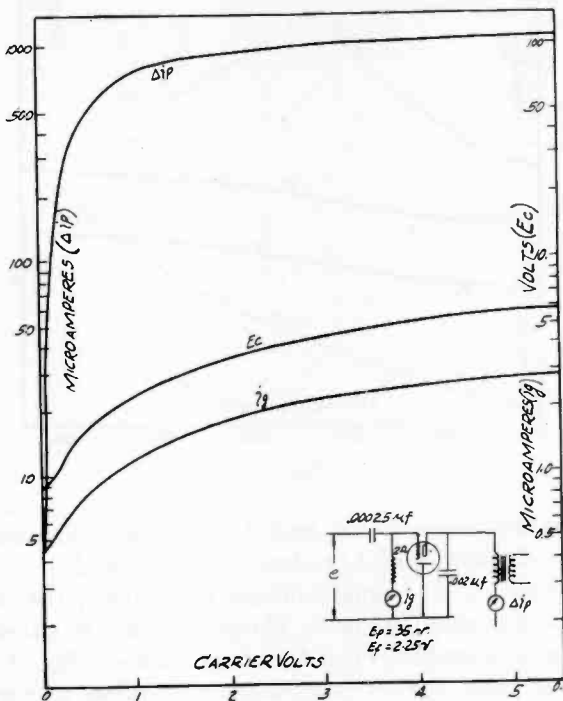
Fig. 2

[1] S. Ballantine, Proc. I. R. E., **16**, 593; May, 1928.

The values of d^2i_g/de_g^2 are fixed by the constant grid voltage applied through the grid leak. In the case of the UY227 tube, which was used in this work, this initial bias is practically independent of the value of the grid-leak resistance where the latter is returned directly to the cathode. This fact is illustrated in the curves of Fig. 1; a constant signal voltage of 50 mv was applied to the detector; i_g is the grid
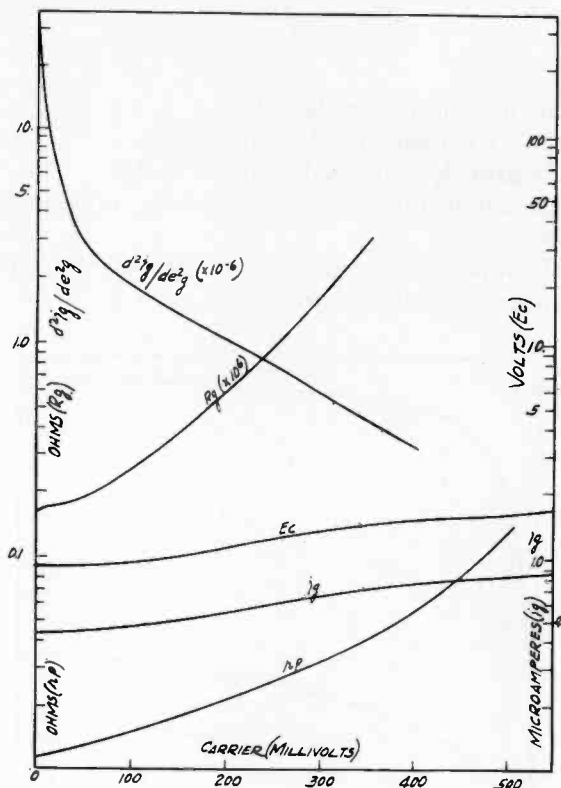
Fig. 3

current; E_c is the product of i_g and the grid-leak resistance, and Δi_p is the change of plate current produced by the signal.

When, however, the signal voltage was varied, the effects produced were as illustrated in Fig. 2. The grid voltage E_c varies considerably with signal strength, even for small values. Fig. 3 shows the lower parts of these curves magnified, in addition to curves which show the variation of the tube parameters with the signal voltage. These variations are quite large; a 400-mv carrier produces an r_p of 55,000 ohms and an R_g of 2.7 megohms; as compared with initial values

of 11,500 and 0.165, respectively. The $d^2i_g/de_g{}^2$ changes from 31.5×10^6 to 0.33×10^6.

The effects of these variations are therefore felt at both the input and output of the detector. The effect at the input can be found by applying the values given by the curves of Fig. 3 to formulas (1) and (2), and is indicated by the curves of $|Z|$ and of the detection factor D_g in Fig. 4, where $D_g = \frac{1}{2}(d^2i_g/de_g{}^2)|Z|$.

The ratio of rectified voltage at a modulation frequency of 5000 cycles to the rectified voltage at very low frequencies was calculated and plotted in Fig. 5A. On referring this curve to the R_g curve of Fig. 3, the curve of Fig. 5B was obtained, which indicates the effect of signal strength on the attenuation of the higher audio frequencies.
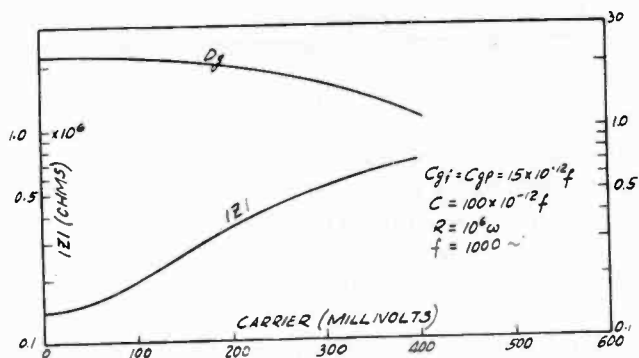
Fig. 4

A much more serious effect is found in the great increase of the r_p with the carrier voltage. The effect of this on the low audio frequencies is well known. In addition, it is found that for carrier voltages in the neighborhood of one volt and over, the great increase of r_p may actually cause a diminution of audible response. In connection with the selective properties of the radio-frequency amplifier this produces an interesting effect.

On adjusting the tuning condenser, approaching the setting for resonance with a strong modulated carrier, the loud speaker response (or the audio-frequency output of the detector) increases in accordance with the combined selective properties of the r-f amplifier and the rectifying property of the detector. The frequency distortion changes considerably all the while; at first the low frequencies are missing due to the selectivity of the amplifier. On approaching nearer the resonance setting the low frequencies begin to be heard and the output increases rapidly. At the same time, however, the r_p of the tube is increasing,

and soon a setting is reached where the r_p is so great that the output of
the detector diminishes, reaching a minimum at the setting of reso-
nance. On passing over the resonance point the signal voltage on the
detector decreases, the output increases and reaches a maximum, and
then falls off again. A "double hump" has occurred, which has often
unwittingly been attributed to causes in the radio-frequency amplifier.
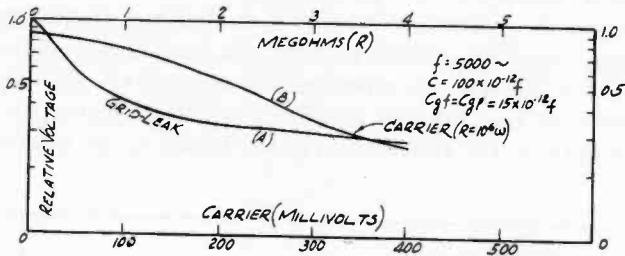
Fig. 5

A similar effect is found in the plate rectifier, although in this case
it is due to the grid swing exceeding the bias voltage, so that the grid
takes current. Up to the overload point, however, an increase of signal
voltage causes a decrease of r_p improving the reproduction of the low
audio frequencies. This is the opposite of what happens in the grid
rectifier.

Fig. 6 shows curves obtained experimentally on a plate rectifier,
for a weak signal, a signal just beginning to overload, and a signal
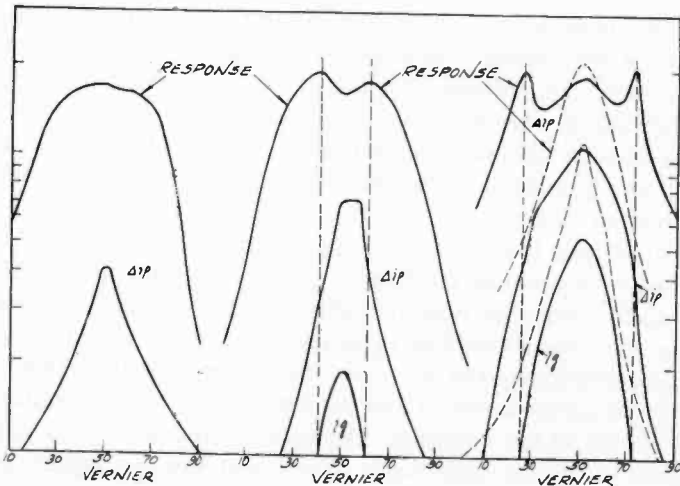
Fig. 6

which considerably overloads the detector. In the two latter cases grid current begins to flow exactly where the peaks occur. In the third case an additional peak occurs at the resonance setting, due to the fact that the increase of signal voltage exceeds the loss due to the flow of grid current. The effect of such overloading is to "chop off" the top of the response curve. Thus, in the case of the strongest signal, when the grid bias was increased to the point where the grid current just ceased to flow, and the plate voltage readjusted to furnish the same constant plate current, the broken curves were obtained. The apparent "broadness" of tuning due to detector overloading is evident .

SINGLE-WIRE TRANSMISSION LINES FOR SHORT-WAVE ANTENNAS*

BY

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Summary—*The versatility of the possible arrangements of short-wave antennas can only be made available through the use of transmission lines. The phenomena of high-frequency transmission lines are discussed with respect to the effect of the low attenuation per wavelength and the influence of the termination on standing waves and radiation, for both an antenna and a transmission line.*

The single-wire transmission line is effective when properly terminated. It is the easiest method of feeding the Hertz antenna. The adjustment may be divided into two parts. The frequency which makes the Hertz antenna a pure resistance termination must first be determined. The proper point of connection between the line and antenna must then be found to make the terminating resistance equal to the characteristic impedance of the line.

When so terminated, experiment and theory show that radiation from the feeder will be small and that the feeder will act efficiently.

Theoretical and experimental curves to show the nature of a horizontal Hertz antenna are shown, since they influence the behavior of the line under changing conditions.

THE transmitting antenna is one of the most important and interesting of the component parts of a radio system. With the increased use of the shorter waves a greater flexibility is possible in the design of an antenna, and there is therefore a greater need for knowledge of its possibilities. When long waves are used, the shape of the antenna is generally determined by factors of cost, the largest item of which is the expenditure for the supporting towers. For instance, a vertical antenna to operate at a fundamental wavelength of 1000 meters would need to be about 750 ft. high, which would be very expensive. Therefore, long-wave antennas are usually of one general design, consisting of a flat top as high as can be built for the money available, with a vertical section connecting it to the transmitter at the bottom, the latter being coupled directly to the antenna circuit. At the shorter waves, however, the antenna is correspondingly smaller, so that it is possible to make it entirely vertical or horizontal, to operate it on harmonics, and in general to use a much greater latitude in design. One of the important advantages of this is that it makes it possible to construct the antenna so that the maximum radiation may occur at angles with the horizon such that the greatest effectiveness is obtained when this energy is reflected

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from the Heaviside layer. A great deal of work has been done on such arrangements, and this field of investigation appears very fruitful.

Since the limitations placed upon the antenna dimensions at longer wavelengths are removed in short-wave work, it often becomes desirable to separate the radiating antenna from the oscillator supplying the energy, for then all restrictions upon the placing of the antenna are removed. This necessitates the use of a transmission line to convey the energy from one to the other, without itself being a part of the radiating system. It is possible to design such a transmission line, but it is necessary to operate it properly if the desired result is to be obtained. It is the purpose of this discussion to outline what these proper conditions are.

It should not be thought that the advantageous use of transmission lines is confined to short waves, as there are many fields of application in the medium and long-wave field. One of the most important is the possibility of removing the transmitter with its associated amplifiers from the strong field under the antenna, and by this means reduce the trouble due to feedback either in the radio-frequency amplifiers or in the audio amplifiers if there is some modulating action in the latter. This application becomes increasingly important as the power of the station is increased with a corresponding higher gain in the amplifiers.

GENERAL THEORY OF TRANSMISSION LINES

The action of transmission lines is a most absorbing study. It has been investigated thoroughly in its application to telephone and power. While the theory is quite general, some effects are more important in one field than the other. For instance, it is common to speak of the electrical length of the line, which is the length measured in terms of wavelength. In power use, the longest lines are about one-tenth of a wavelength long, the per cent power loss being comparatively small, while telephone lines may be several wavelengths long and the power received may be less than one per cent of that put into the sending end. In the radio transmission lines which we will discuss, their electrical length may also be several wavelengths, but the power loss will be small. It is apparent that it is desirable to give particular attention to a study of each application.

The fundamental treatment of a transmission line is usually mathematical in nature, but its physical concepts are quite easy to grasp and it is these concepts which are of importance to the engineer. Since they are often lost sight of in the formulas, consider physically what happens on an electrically long line, paying particular attention to the case where the energy dissipated along the line is small, as it is

this case which has the most bearing upon lines operating at radio frequency.

The outstanding physical fact about the transmission line is that its capacitance, inductance, leakance, and resistance are distributed evenly along the line instead of being in lumps, as is the case in the ordinary tuned circuit. The simplest sort of a line to consider is one which is infinitely long.
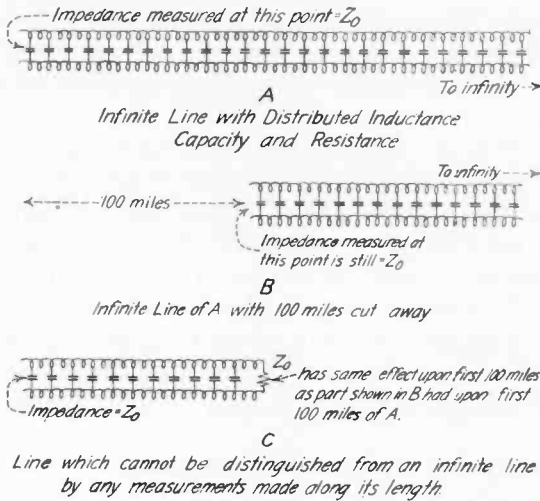
Fig. 1

If a line were infinitely long, and a length of say 100 miles is cut off, the line would still be infinitely long. Therefore, if the impedance of the line is measured before and after the piece is cut off, the same value will be obtained. This impedance is called the characteristic, surge, or iterative impedance of the line. Now if a study is made on the first hundred miles of the line, *there would be no way to distinguish between a line infinitely long and another one which was a hundred miles long and terminated in an impedance equal to that of an infinite line of the same characteristics.* This is illustrated in Fig. 1. We can therefore actually have a line which shows all the characteristics of an infinite line.

If an a-c generator were placed at the sending end of an infinite line, its effect would not be felt immediately everywhere along the line, but there would be a definite rate at which the disturbance would be propagated. This speed is not always the speed of light, but is generally less than that. On a loaded telephone cable, it is

sometimes as low as 10,000 miles per second, but in the open wire line it is nearly that of light, and as the frequency increases and the magnetic flux within the wire decreases it approaches the speed of light as a limit, so that at radio frequencies the velocity is generally taken equal to approximately 300,000,000 meters a second.
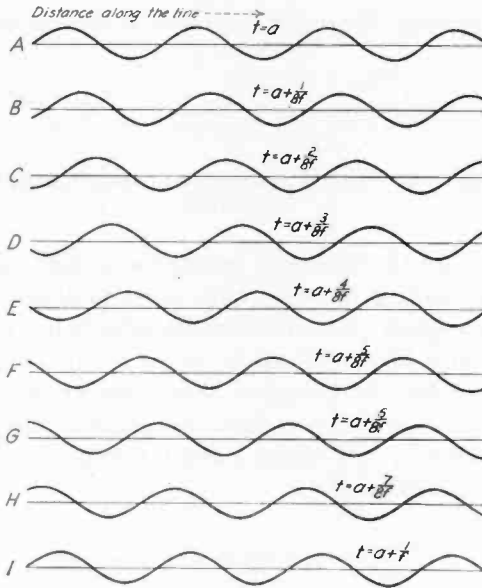
Fig. 2—Voltage distribution at different instants on line with negligible resistance.

If a group of oscillographs, or some other device which would indicate the instantaneous value of the voltage were connected at different points in the line, it would show that the voltage along the line varies sinusoidally with distance at any given instant. At some instant the distribution would be that shown in Fig. 2A. The distance between two maximum points in one direction is a wavelength and is equal to the velocity divided by the frequency. At a slightly later time the disturbance would have passed along the line and the voltage from point to point would be illustrated by Fig. 2B. And so as time passed the voltage would successively vary as shown in Figs. 2C to 2I. This cycle would be repeated as long as the voltage was maintained at the sending end. It is thus seen that there is a wave of voltage traveling down the line. In this figure it is assumed that the voltage is not attenuated as it passes along, or, in other words, that the losses due to resistance are negligible. In a telephone line the reduction in

voltage due to resistance losses over a wavelength of line is generally quite large and the voltage at some one instant would be illustrated by Fig. 3, where the ratio by which the effective voltage is reduced by any given length of line is constant. This attenuation may be reduced by reducing the resistance or by increasing the inductive reactance of the line. When this is done by adding inductance to the line it is the process called loading. Since the inductive reactance is equal to $2\pi fL$,

Fig. 3—Voltage distribution at some instant along line with appreciable resistance.

at high frequencies the inductive reactance is high, while the wavelength is short, making the resistance per wavelength small, so that the decrease in voltage over a few wavelengths is not very great. The current would also follow the same curves as the voltage and be in phase with it, for the characteristic impedance of an open wire line is practically a pure resistance at high frequencies, independent of frequency and equal to $\sqrt{L/C}$ where L and C are the values of inductance and capacity per unit length.
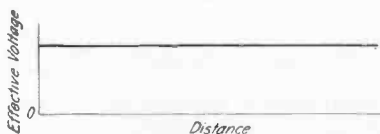
Fig. 4—Effective voltage as measured with a voltmeter along a line terminated in its characteristic impedance, showing that wave motion cannot be detected in this way.

If an ammeter and a voltmeter (which read effective rather than instantaneous values) were connected at various points along the line, since they read average effects, they would all have practically the same reading, and a plot of these values would be a straight line as shown in Fig. 4. It would not be possible with these instruments to detect the presence of the traveling waves.

The traveling waves have a counterpart in hydraulics, in the case of a canal of water very long and straight. If a disturbance is made at one end of the canal, waves will travel along, and a set of bobs on the surface would give the instantaneous values and draw curves similar to the instantaneous values of voltage and current shown in Fig. 2. If, however, the set of bobs is arranged so as to be moved up on rods in the water, but due to friction remain at the highest point of the wave

after it has passed, so as to record maximum values, then a straight line would be obtained, similar to the values read by a voltmeter on the line as shown in Fig. 4.

Suppose instead of having the canal very long a board dam is created so that the waves will strike it. There will then be a piling up of water at the dam which will start a wave back in the opposite direction.
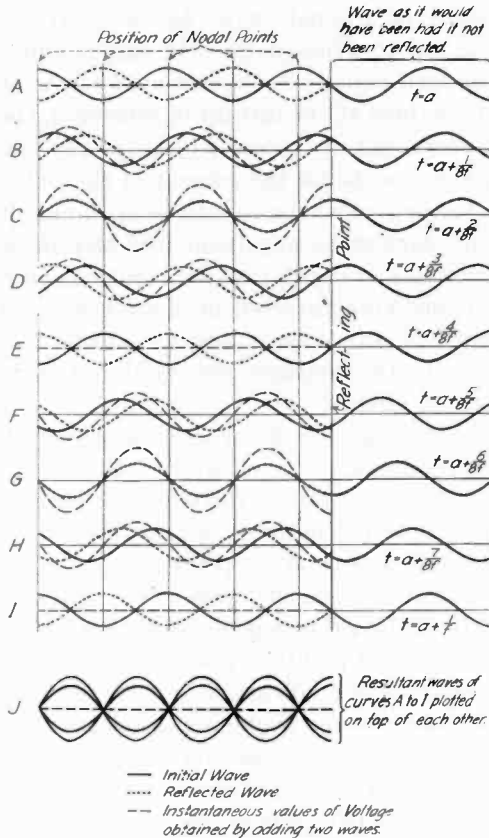
Fig. 5—Distribution of voltage on open circuited line.

In the same way, if the electric line is not infinitely long or terminated in its characteristic impedance, but instead is open circuited, then when the wave strikes the end, it also will be reflected. The electric wave in traveling has half of its energy stored in the magnetic field due to the current, and half of the energy is stored in the electric field due to the voltage. When it strikes the open circuit, the magnetic field will collapse, for the current must become zero. But a changing

magnetic field produces an electric field. It is upon this principle that all generators and transformers operate. The energy stored in the magnetic field will therefore be turned into energy in an electric field and added on to the existing field, so that the voltage at the end circuit will be increased. This increased voltage will start a wave traveling back in the opposite direction, and since there has been nothing to absorb any energy at the open circuit, the returning wave will be of the same magnitude as the original wave. As the electric field starts moving back, it will set up a magnetic field again, and the energy will once more be divided equally between the two. As the electric field has simply been doubled at the instant of reflection, the voltage of the returning wave starts out in the same phase as the original wave, but the magnetic wave, and hence the current of the reflected wave, is in opposite phase to the incident wave at the open circuited point. The total voltage and current at any point and any instant is therefore the sum of the voltages or currents of the incident and reflected waves. That the current has been reversed in phase is evident from the fact that the two current waves must add to zero at the point of open circuit, and since the two voltages are equal and do not add to zero, they must remain in phase.

In Fig. 5A is shown a plot of the two voltage waves which are present on the open circuited line at some instant. The total voltage at any point on the line at that instant is the algebraic sum of the two waves, and is also shown. It will be noticed that the reflected wave is obtained by folding back on the line from the point of reflection, the initial wave as it would have been beyond the open circuit had the line continued on. In Figs. 5B to 5I this process is continued for later instants. When the two waves are added point by point it will be seen that at some points the resultant wave is always zero. These points occur at odd quarter wavelengths from the open circuit, because at such a position the reflected wave has traveled an odd number of half wavelengths since it passed this point as part of the initial wave. At even quarter wavelengths from the open circuit, the resultant voltage is always twice the value of either wave, since the two waves at these points are equal and in the same direction. In Fig. 5J all the resultant waves obtained for the different instants are plotted on top of each other to permit a better comparison. It will be noticed that voltage varies with time and space. At every point along the line the voltage reaches a maximum at the same instant, but this maximum voltage varies sinusoidally along the line. It should be noticed in contrast, that on the line terminated in its characteristic impedance the maximum voltage occurs at different instants for different points on the line but is

the same value for all points. Since the effective voltage is the maximum divided by the $\sqrt{2}$, if a voltmeter is moved along the open circuited line, a curve similar to Fig. 6 would be obtained, the readings being always positive since the meter cannot take account of phase. Waves which can be detected by a meter which records average results are called standing waves, and are different from the traveling waves which were first discussed. This is apparent after comparing Figs. 4 and 6. The points of minimum voltage are called the nodes of voltage, while the points of maximum voltage are called the antinodes. The voltage at the nodes is not actually zero since there is always some loss due to resistance as the wave travels along the line, and the returned wave being slightly smaller will not entirely neutralize the initial wave at these points, but at high frequencies there is a close approximation to this ideal condition.
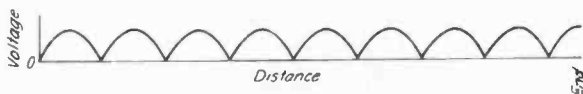
Fig. 6—Voltage readings along an open circuited line with negligible resistance showing standing waves. (Voltmeter cannot record phase so all readings are positive.)

Figs. 7A to 7I show the two waves of current on the line for different instants. It should be remembered that while a voltage wave and a current wave are discussed as though they were distinct, the voltage and current together are necessary for a physical wave, since one cannot exist without the other, but each can be considered separately in plotting results. In the figure it will be noticed that the current has been reversed at the open circuit, so that the two waves add up to zero at that point. As the instantaneous values are added point by point it will be observed that there is also a standing wave of current. This wave is the same as would be obtained for voltage if the reflection had occurred a quarter wavelength down the line, and the nodes of current occur at the antinodes of voltage and vice versa.

In any line the numerical ratio and phase between voltage and current at any point in the line is a measure of the impedance beyond that point. In the line terminated in its characteristic impedance the ratio is the same at every point and the voltage and current are in phase, and any length of line therefore has no natural period. In an open circuited line this ratio varies along the line, being small at the nodes of voltage and large at the nodes of current. By examining this ratio, the way the impedance of an open circuited line varies can be determined. In Fig. 8 the effective values of voltage and current are plotted on top of each other. Immediately below is plotted their

ratio. This shows how the impedance varies as the distance from the open circuited end is varied, and is therefore a plot of impedance vs. length of line. This is also the way the reactance of an antenna varies since it is a line with distributed constants. It will be noticed that the line is resonant at odd quarter wavelengths, and antiresonant at even
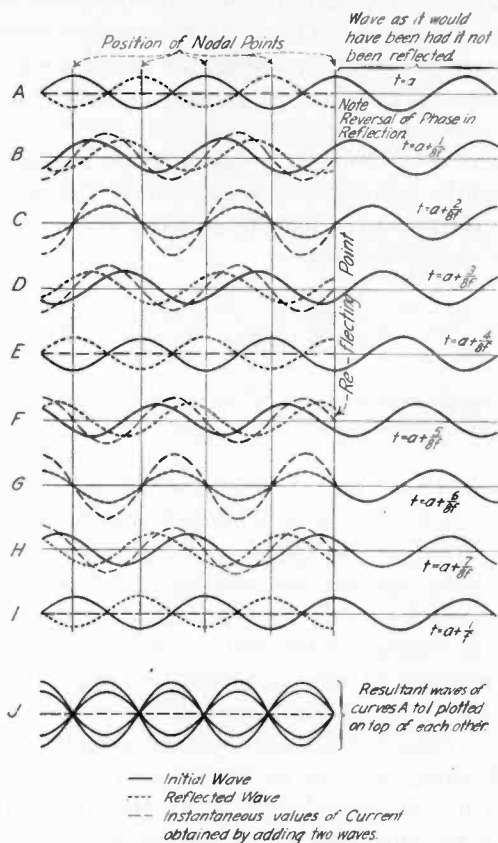
Fig. 7— Distribution of current on open circuited line.

quarter wavelengths. Since the effective voltage at any point on the line is represented by the equation $E_x = E_m \cos (2\pi x/\lambda)$ and the current is represented by a similar equation $I_x = I_m \sin (2\pi x/\lambda)$ their ratio is represented by the equation

$$Z_x = \frac{E_x}{I_x} = \frac{E_m}{I_m}\ \frac{\cos \dfrac{2\pi x}{\lambda}}{\sin \dfrac{2\pi x}{\lambda}} = Z_0 \cot \frac{2\pi x}{\lambda}$$

In these equations the distance x is measured from the open end of the line, the subscript x is used to denote the values at the distance x from the end, and the subscript m is used to denote maximum values (those that occur at the antinodes).

It is not enough ordinarily to know how the absolute value of an impedance varies, but it is also desirable to know whether it is a resistance, capacitive reactance, inductive reactance, or an impedance with both resistance and reactance. This can best be determined by means of vector diagrams.
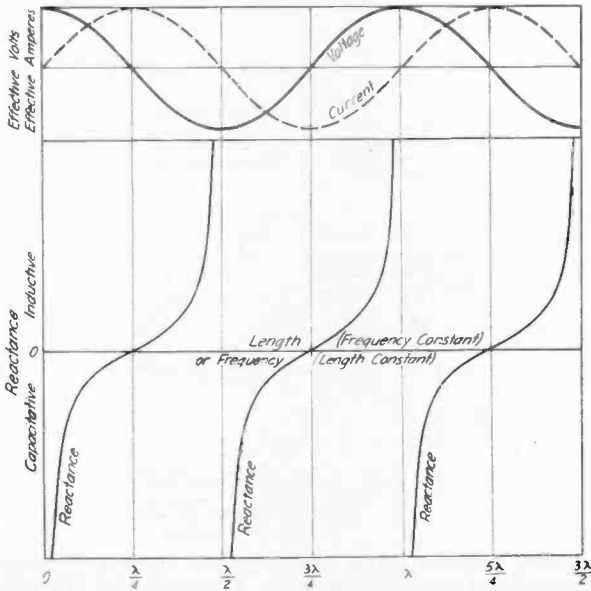
Fig. 8—Variation of impedance, open circuited line.

Fig. 9 shows a progressive vector diagram for a point less than a quarter wavelength from the end. Fig. 9A shows a vector representing the voltage and current of the initial wave at the point in question. The voltage and current of the initial wave at the open circuit are shown in Fig. 9B and lag the vectors in Fig. 9A by an angle $\phi = 2\pi x/\lambda$. Upon reflection the reflected wave voltage and current will be represented by Fig. 9C. The fact that these are 180 deg. out of phase indicates that energy is now being propagated in the opposite direction. After traveling back along the line, the vectors at the point x are shown in Fig. 9D and lag the vectors in Fig. 9C by a similar angle ϕ. In Fig. 9E the two sets of vectors representing the two waves are added together and give the resultant voltage and current E_T and I_T. It

will be noticed that if the attenuation is negligible, so that the currents
and voltages in the two waves are equal, the resultant voltage and
current will be at right angles to each other, and in this case when ϕ
is less than 90 deg. the current leads the voltage and the impedance of
such a length of line is a pure capacitive reactance.

In a similar manner Fig. 10 shows the vector diagram at a point
between one quarter and a half wavelength from the open end. In
this case the current and voltages are also 90 deg. apart, but the current
lags the voltage and the impedance is therefore a pure inductive re-
actance.
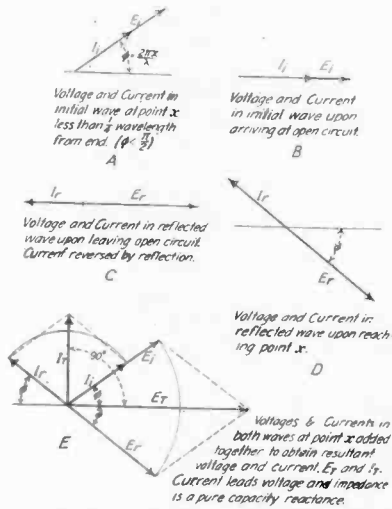
Fig. 9—Progressive vector diagram of voltage and current at point on line less
than one-fourth wavelength from end. Attenuation of line assumed negli-
gible.

Fig. 11 shows the result of attenuation upon the impedance of the
open line. It is drawn for a length less than one quarter wavelength,
but a similar diagram can readily be constructed for any length. Due
to attenuation from resistance losses the voltage and current of the
returned wave are less than those of the initial wave. The resultant
sums of voltage and current are now less than 90 deg. apart, showing
that the impedance has a resistive component.

These results are to be expected from the fact that if the line has
no loss, all the energy which goes into any length is later returned, a
condition similar to that produced by a pure reactance. On the other
hand, if the line does dissipate energy, all the energy will not be re-
turned and the impedance of any length will have a resistive com-
ponent.

If instead of varying the physical length of the line, it is kept constant and the frequency is varied, the *electrical* length of the line will be varied just as effectively as when the frequency was constant and the physical length of the line changed. The impedance curve of Fig. 8 might therefore have frequency for its abscissa instead of distance. It is possible, therefore, to obtain any reactance desired by varying either the length or frequency.
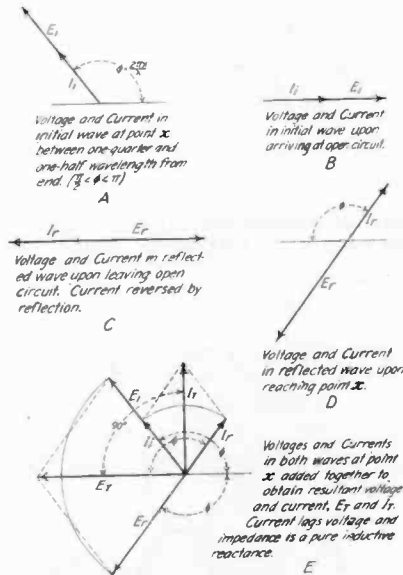
Fig. 10—Progressive vector diagram of voltage and current at a point on line between one-fourth and one-half wavelength from end. Attenuation of line assumed negligible.

So far only two terminations for a transmission line have been discussed, viz., termination in the characteristic impedance or in an open circuit. From the relations deduced for the open circuit it is possible to see what the result would be when the line is terminated in a pure reactance. Since there would not be any absorption of energy in the termination, the returned wave would be as strong as the initial wave, and therefore the same sort of standing wave would result. This may also be seen because, in passing back from the open end of a line, the short length out to the end may be made a pure reactance of any value desired including zero or a short circuit. Such a short length could be replaced by a pure reactance of the same value, and would give the same results as a termination for the rest of the line. It can therefore be seen that terminating the line in a pure reactance of any

value whatever will merely shift the standing wave along the line, but will not change its character.

If, in the hydraulic model mentioned earlier in the article, instead of having a dam so as to reflect the wave completely, a submerged board had been placed in the canal, then some of the energy would have been reflected and some would have gone on. The returning wave would therefore not be as strong as the original wave.
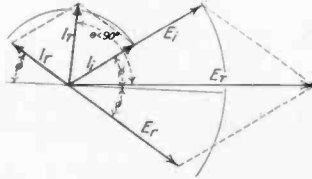
Fig. 11—Vector diagram of voltage and current at point on line less than one quarter wavelength from end. Attenuation makes impedance partly resistive.
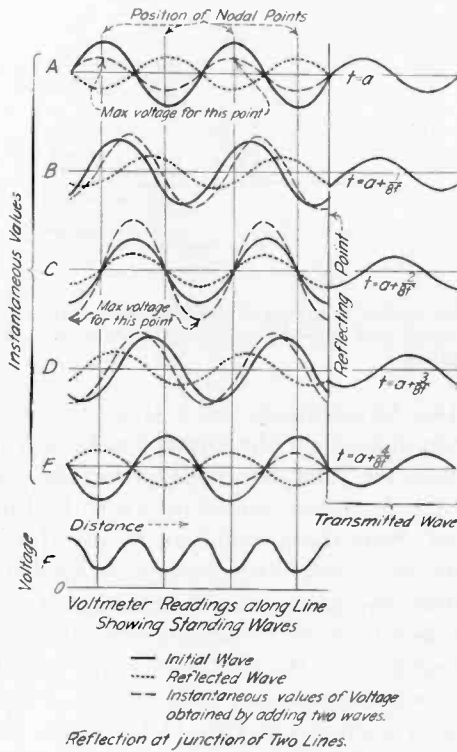
Fig. 12—Distribution of voltage on line when terminated in an impedance greater than the characteristic impedance.

If two transmission lines of different characteristics are joined together, or if the single line is terminated in something other than the characteristic impedance, reflection will also occur, with a weaker returning than incident wave. Standing waves will again be produced, but since the two waves can never cancel each other at any point, the current and voltage will never be reduced to zero, but only to a minimum value. Figs. 12A to 12E show a plot at different instants of instantaneous values of voltage for the case of partial reflection, where the return wave is half the magnitude of the original wave. Fig. 12F shows the plot of effective values of the voltage along the line and is obtained by examining curves A to E and determining the maximum voltage which ever occurs at each point of the line. If the impedance of the termination is greater than the characteristic impedance, the minimum points of current and voltage will occur at the same points as the nodes of the open circuited line, while if the terminating im-
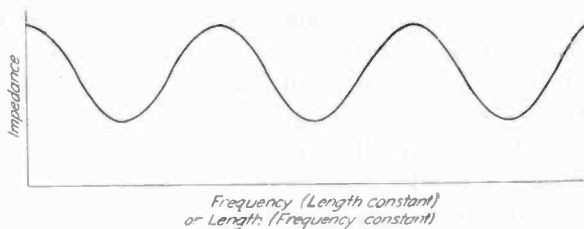
Fig. 13—Variation of impedance of line terminated in impedance greater than characteristic impedance.

pedance is less than the characteristic impedance the minimum points will correspond to the nodes of a short circuited line. In both cases the minimum points of voltage will be maximum points of current and vice versa.

In the same way the impedance will not be a pure reactance but will vary in magnitude between maximum and minimum values in the manner shown in Fig. 13. Here again the abscissa may be length for a fixed frequency, or frequency for a fixed length. This variation in impedance with frequency is used to locate faults which develop in long telephone lines, and is a measurement of the time it takes the waves to pass down to the fault and return after being reflected. It therefore corresponds in principle to recently developed methods of sounding the sea by measuring the time taken for a sound to pass to the bottom and return after reflection. A similar principle is used to detect pockets in earth oil fields before undergoing the expense of sinking a well.

That this case of reflection is a definite and not a theoretical phenomena is illustrated in a very real trouble which is encountered due to echoes from impedance irregularities on loaded telephone lines where the velocity of propagation is low. Since telephone repeaters amplify in both directions, the reflected wave is amplified along its return path, and may differ sufficiently in time and strength from the original wave to interfere with speech.

Usually reflection of a wave decreases the efficiency of a line, as the wave in passing back along the line causes additional losses. There are some exceptions in telephone work where the characteristic impedance of a cable may have a considerable capacitive component, when the best termination from the standpoint of efficiency is an impedance equal in magnitude to the characteristic impedance of the cable but having an inductive instead of a capacitive reactance. Also if the generator is not subject to adjustment and the maximum power is to be obtained from the generator, another termination may at times be desirable on shorter lines. However, for high-frequency work, the characteristic impedance of the transmission line, as has been mentioned before, is practically always a pure resistance, and since adjustments may be made at the generator, the termination desired is that which will produce no reflection.

To summarize, it can be seen that:

(a) A line terminated in its characteristic impedance has the same characteristic as an infinite line.

(b) A line terminated in its characteristic impedance will have no standing waves, the traveling waves producing the same effect on a voltmeter or ammeter anywhere along the line.

(c) A line terminated in its characteristic impedance will have a constant impedance at high frequencies which is equal to a pure resistance, and therefore has no natural period.

(d) A line terminated in an open or short circuit will have a standing wave set up on it, and the maximum points of voltage will be the minimum points of current and vice versa.

(e) A line terminated in an open circuit will be a pure reactance if the resistance is negligible, or if there is resistance, will be largely reactive except at the fundamental and odd harmonics when it will be a low resistance. The reactance will be alternately inductive and capacitive if either frequency or length is varied.

(f) A line terminated in anything but the characteristic impedance will have standing waves set up on it, but the variation between the maximum and minimum points of current and voltage will decrease

as the value of the terminating impedance is made to approach that of the characteristic impedance.

(g) A line terminated in anything but the characteristic impedance will vary in impedance with frequency or length, and will therefore have a natural period, but the magnitude of this variation in impedance with frequency will decrease as the terminating impedance is made to approach the characteristic impedance.

(h) A line for high-frequency work is most efficient when terminated in its characteristic impedance.

Thus the fundamentals are set forth upon which the proper design of an antenna and transmission line feeder can be determined.

TRANSMISSION LINES TO FEED ANTENNAS

One of the most versatile types of antenna is the so-called Hertz antenna, consisting of a single wire, without a counterpoise, in which the voltages at the two ends of the wire are opposite in phase. Its versatility lies in the fact that by operating it horizontally, vertically, or at some other angle to the earth, by varying its height above the earth, and by operating it either at its fundamental, or one of the harmonics, a wide variety of radiation patterns may be obtained. To take advantage of this versatility, and to predetermine its action, some type of transmission line feed is necessary to convey the energy from the driver to the antenna. Two types of transmission lines are in use, the two wire and the single wire feed. These have been called respectively "current" and "voltage" feeds, but it is not believed that this is a very good term, as it conveys an erroneous impression of their action. In each case, as in any case where power is transmitted, both current and voltage are necessary, and contrary to the opinion entertained when this investigation was started, it has been found that the characteristic impedances and hence the voltages and currents for a given amount of power do not differ very much between the two types of lines.

In order to minimize radiation from the feeder, it must be terminated in its characteristic impedance and thus prevent the formation of standing waves. The radiation resistance of an efficient antenna is lower than the characteristic impedance of any feasible transmission line and so means must be used to match the one against the other. In the two-wire line this may be done by means of a network equivalent to a transformer, designed for the particular frequency to be used and inserted between the line and the antenna. It is believed, however, that the single-wire transmission line has a number of advantages, which usually make it more desirable. One of the most important of

these is that it is readily possible to terminate it properly by the antenna without the use of a transformer. It is more convenient to construct, especially if the antenna is suspended at some height above the ground, where the proper spacing of the two-wire feed would be awkward to secure. It is very easy to adjust when the wavelength is changed or when the radiation of the antenna is changed by changing its height. It is possible to change the antenna system readily from operation on the fundamental to operation on some harmonic. It should be noticed that while a grounded antenna, or a Hertz with the driver connected permanently in the center, can only be operated on odd harmonics of the fundamental, the use of a transmission line makes it possible to operate a Hertz antenna on both odd and even harmonics. It has even been found with the single-wire feed that by selecting the proper height of antenna to adjust the radiation resistance, a single physical construction could be secured in which the single-wire feeder was properly terminated for both the fundamental and the second harmonic, affording a ready means of shifting from one wave band to another. These advantages are particularly important in making investigations where it is desirable to change the construction of the antenna, or the operating wavelength frequently.

One factor which has brought the single-wire line into disrepute is that, when improperly adjusted, it will radiate much more than the two-wire line, since the latter has an action similar to a loop, in which the radiation is a function of the separation of the two sides, measured in wavelengths. By making this separation small, the radiated power of an improperly terminated two-wire line may be quite small. When the proper adjustment of the single-wire line is made, its operation will be satisfactory, and it is the purpose of this discussion to indicate how those adjustments should be made.

Carson[1] has shown in a digest of the literature that radiation is negligible on a two-wire line when terminated in its characteristic impedance, and the same point was brought out by Mannebach.[2] In the discussion of the latter's paper, Slepian[3] gives a good description of the mechanism of radiation due to a principle of relativity which applies with equal force to the single-wire line. In this description the radiation occurs only when there is a snapping of the lines of force due to reflection at the end of the line. This occurs in all antenna systems.

[1] J. R. Carson, "Radiation from Transmission Lines," *Jour.* A. I. E. E., pp. 789–790, 1921.
[2] Charles Mannebach, "Radiation from Transmission Lines," *Jour.* A.I.E.E., pp. 95–105, 1923.
[3] Slepian, Discussion, *Jour.* A. I. E. E., pp. 981–982, 1923.

When a single electron is moving at a constant velocity, no radiation occurs. If the electron is accelerated or decelerated, radiation will take place. However, the radiation is due, not to the acceleration or deceleration of the electron itself but to the acceleration of a portion of the field, although it is realized that the two cannot be separated in the case of the single charge.

Similarly if a series of doublets along a transmission line are so arranged in phase that the field is propagated along the line with a constant velocity, no radiation will take place. This constant velocity occurs when there is a continuous shift in phase of the doublets, so that even though the electrons flow first in one direction and then the other, the resultant field moves with constant speed. This condition is produced on a transmission line when there is no reflection and therefore no returning wave.
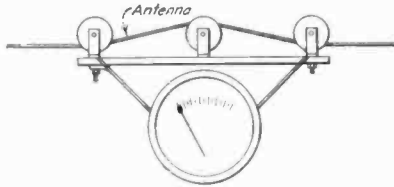
Fig. 14—Trolley for obtaining current distribution along antenna.

By application of the reciprocity theorem and a study of wave antennas it will be concluded that there will be a small radiation in the direction of the transmission line.

Hence this theory is postulated: Radiation is largely due to reflection.

In the previous discussion it has been shown that reflection always produces standing waves, and consequently the test for the absence of radiation on a transmission line is the absence of standing waves.

Actual measurements on the line show a high efficiency for the transmission line and therefore experimentally verify the absence of appreciable radiation.

A single-wire feeder was connected to a Hertz antenna at some point away from the center of the antenna, and current distributions over the system for three different frequencies were measured by the trolley arrangement shown in Fig. 14 in order to verify the theory. This trolley was used to shunt a meter across a portion of the antenna about eight inches long. The current in the meter is therefore proportional to the total current and the combination may be calibrated and the actual current known. By means of a system of cords and pulleys the trolley was drawn along the antenna or transmission line

and the distribution of current determined. The meter readings were made by means of a transit. A tube working below normal plate voltage was used in order that fluctuations due to overheating of the tube during steady operation might not be encountered.

Curve A, Fig. 15, shows the distribution for a wavelength greater than the fundamental. Curve B shows the distribution for a wavelength less than the fundamental. Curve C shows the distribution when the wavelength is equal to the fundamental of the antenna. There are two interesting features here, first that at the fundamental wavelength the currents on the two sides of the feeder are equal, and secondly, that the current at the center of the antenna may be greater when the operation is at a higher wave than the fundamental. However, while in this case the current at the center is greater, the current at the short end is smaller, and the total radiation is less than when the antenna is operated at its fundamental. This definitely shows that such an antenna system cannot be adjusted to its fundamental by placing a meter in the center and adjusting for maximum
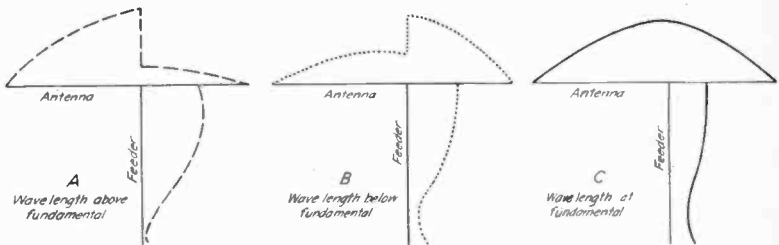
Fig. 15—Distribution of current on antenna and feeder for three different wavelengths (feeder tap not yet adjusted).

current. The criterion which was adopted was to place a meter on either side of the feeder and adjust until the two currents are equal. The intersection of the two curves of current will be the fundamental wavelength of the antenna.

To test this criterion and to show the independence of the fundamental wavelength from the position of the feeder, curves were taken for different positions of the latter. These are shown in Fig. 16. It will be noticed that the intersections of these curves all occur at the same wavelength.

It can readily be understood why the currents will vary in the manner shown in Fig. 16. Refer to Fig. 8 which shows how the reactance of a given length of line varies with frequency. At the junction of the feeder and antenna there are two branches of the antenna. The short end will have a capacitive reactance and the long end will have an inductive reactance, but at the fundamental wavelength,

these two reactances will be equal and the currents into the two branches will be equal. If now the length of the longer one is increased and the length of the shorter branch is reduced by the same amount (which corresponds to moving the feeder tap) the reactance of both branches will increase by equal amounts as is apparent from the symmetry of Fig. 8. Therefore, the two currents will remain equal to each other as the feeder is moved outward.

If now the feeder is in a given position and the frequency is increased, the reactance of the short end will drop, while the reactance
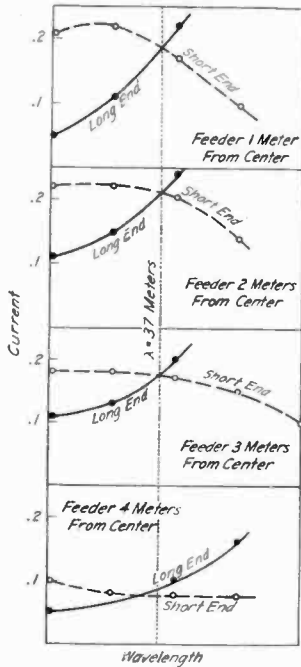
Fig. 16—Variation of current in two branches of antenna as wavelength is varied

of the long end will rise and the currents in the two branches will be unequal. Since the distribution of current along the antenna is sinusoidal in space and the wavelength has been decreased by the rise in frequency, the two sine waves will not have equal magnitudes at the junction of the feeder, and the current distribution will be as shown in Fig. 15B.

In a similar manner if the frequency is lowered, the reactance of the two branches would change in opposite directions, and the two currents will be unequal. As the wavelength has now been increased the current distribution in Fig. 15A will be obtained.

The two branches of the antenna correspond to two branches of a parallel circuit, one of capacitive reactance and the other of inductive reactance. Each branch has resistance due to the ohmic and radiation resistance of the antenna. If in such a parallel circuit the frequency is adjusted until the currents in the two branches are equal, the sum of the two currents will be less than either, and will be in phase with the voltage. This is shown by the vector diagram of Fig. 17A. The impedance of this combination is therefore a pure resistance, or very nearly so. If, however, the frequency is such that the currents are not equal, then the sum of the two currents will be out of phase with the voltage and the impedance will be largely
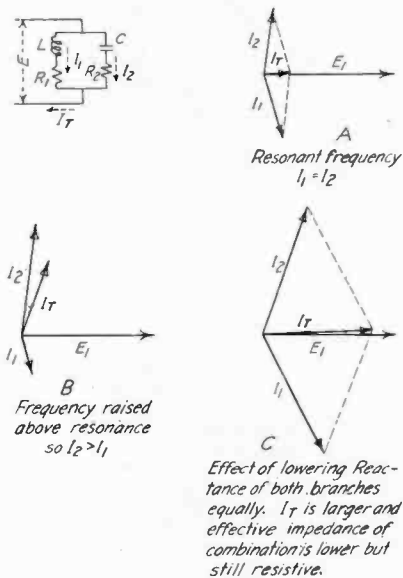
Fig. 17

reactive, provided the reactance of the two branches is large in comparison with the resistance. This is shown in the vector diagram of Fig. 17B. If in the parallel circuit, with the frequency adjusted to make the two currents equal, the reactances of both branches are decreased, but still are kept equal to each other, and the total resistance is not changed, then the two currents will increase for the same voltage, and the total current will increase. The effective impedance of the combination will be nearly a pure resistance, but of a lower value. This is shown by the vector diagram of Fig. 15C. The resistance of a parallel circuit tuned to resonance is approximately X^2/R_1+R_2,

where X is the reactance of either branch, R_1 and R_2 are the resistances of the two branches, respectively. As the point of connection of the feeder is moved along the antenna, the reactance of the two branches is simultaneously varied, being reduced as the center of the antenna is approached. The approximate formula for the variation of input resistance was derived and is

$$R_i = \frac{Z_0^2}{R_r} \cos^2 \frac{\pi x}{l}$$

where

R_i = effective input resistance

R_r = effective resistance of antenna measured at center

x = distance of connection from the end

l = length of antenna.
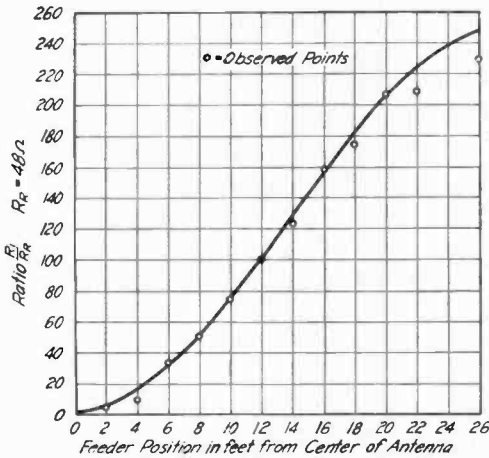
Fig. 18—Theoretical input resistance for various feeder positions.

A more accurate formula is the following:

$$R_i = \frac{R_r x(l-x)}{2 \sin^2 \pi l} + \frac{Z_0^2}{R_r} \cos^2 \frac{\pi x}{l}.$$

The first term of this equation is of importance only near the center of the antenna. To test the formula the input power was measured. This divided by the square of the current in the feeder at the junction gave the input resistance of the antenna and its effective resistance as a termination for the transmission line. Fig. 18 shows a plot of the results obtained, which is nearly a cosine squared curve, with a

plot of the theoretical curve, which deviates appreciably only at the outer end of the antenna. This deviation is to be expected since at the outer end the reflection on the feeder is pronounced, and it is therefore radiating a considerable amount of power, which will also produce an effect upon the radiation resistance of the antenna.

It is possible to obtain a proper termination for the transmission line. By adjusting the frequency to the natural period of the antenna, the termination of the line can be made   pure resistance. This is a first essential, as it has been pointed out that the characteristic impedance of the transmission line is a pure resistance. Any other frequency will not work (except harmonics of the fundamental), for then the termination would be largely reactive, and no matter what
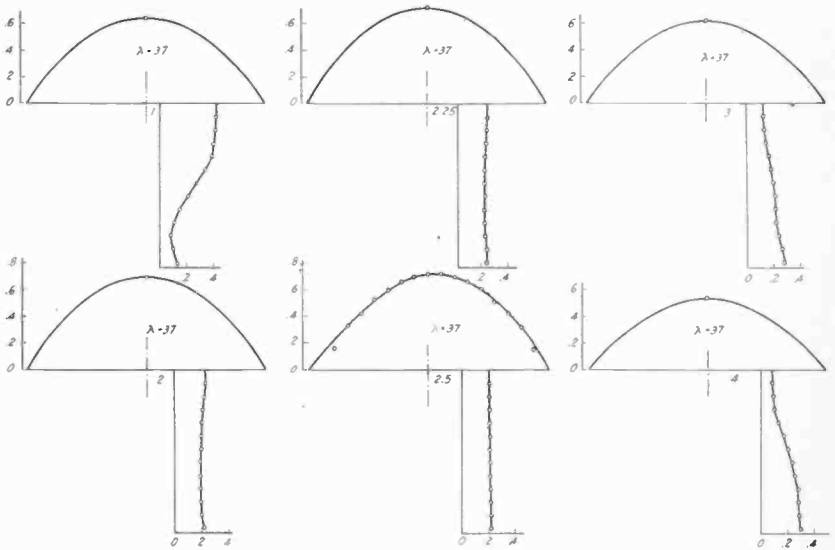
Fig. 19—Distribution of current on feeder and antenna for different positions of feeder.

the value of this reactance, reflection will occur and standing waves will be set up on the line.

The job is not quite finished when the proper frequency has been selected to make the termination a pure resistance, for the resistance must be adjusted to the proper value, which can be done by moving the tap out along the antenna until standing waves are eliminated upon the feeder. Fig. 19 shows the distribution of current for taps at different distances from the center. In this case the antenna and feeder were No. 12 wire, and the antenna was fifty feet high and eighteen meters long.

If the frequency is adjusted to give maximum current in the center of the antenna, a wavelength longer than the fundamental will always be obtained. This gives a very violent reflection at the junction, and sets up large standing waves on the feeder, and the system becomes nothing but a T antenna.

The standing waves obtained when the right frequency is used but the tap is not properly adjusted were never quite so violent. That this adjustment is important, however, is shown by the curve of Fig. 20, where with a constant adjustment at the generator the current in the center of the antenna varied with the position of the feeder tap and the maximum current was obtained at the point where standing waves were eliminated. It should be noticed that while it is not a good criterion to obtain the maximum current at the center when the frequency is varied, since the distribution will be altered, yet maximum current is a good criterion when anything else is varied, since the relative current distribution on the antenna will remain
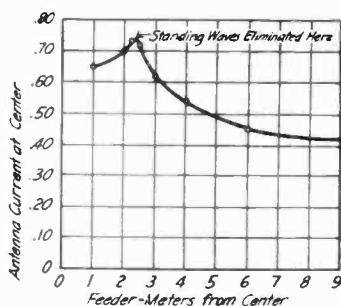
Fig. 20—Curve showing maximum antenna current at point where standing waves in feeder are eliminated. Antenna at its fundamental.

constant if the frequency is fixed. The same antenna was operated at its second harmonic and the distribution of current is shown in Fig. 21.

While the method described for finding the fundamental frequency is easy to apply, since it involves only connecting a meter to either side of the junction of antenna and feeder, the method given for determining the proper terminal point is more tedious, requiring as it does, a means of measuring the current along the line. Since reflection will tend to make the input impedance of the line reactive, any device which will determine when the line is at unity power factor may be used if the feeder is not an integral number of quarter wavelengths long.

This determination will be most accurate if the transmission line is an odd number of eighth wavelengths long, as then the power factor

will change to the greatest extent when reflection is present.   One
method of determining when the power factor is unity, which has been
tested and found to be fairly accurate, is to connect between the feeder
and ground a meter through a condenser, and connecting on either
side of the first meter another meter. The power factor may then be
determined by the three-meter method. Another method which gives
approximate results with a Hartley oscillator is to clip the feeder
on the driver circuit. If the feeder and system is inductive it will
tend to reduce the wavelength of the oscillator; if it is capacitive,
it will tend to raise it, while if it is resistive, it will not ordinarily
have much effect upon the frequency.

It might appear that the proper position of the junction of the
feeder and an antenna could be specified in fractions of a wavelength.
This cannot readily be done because of the oscillating character of
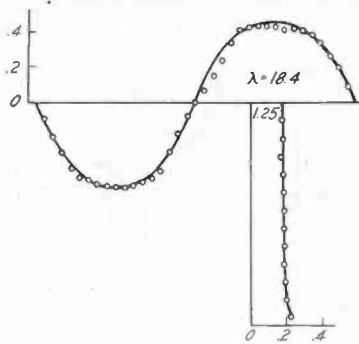
Fig. 21—Antenna working at second harmonic with correct feeder termination

the antenna constants as the height is increased. By the method of
Abraham[4] and Pierce[5] the expression for the radiation resistance of
a horizontal Hertz antenna operating at its fundamental was deter-
mined to be

$$R_R = \frac{480}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin^2 (\pi h/l \sin \theta \sin \phi) \cos^2 (\pi/2 \cos \theta)}{\sin \theta} d\theta d\phi.$$

By graphical integration this expression was evaluated and the results
obtained are shown in Fig. 22, where the abscissa is the ratio of
height to length. Pierce and Abraham obtained a radiation resistance
of a vertical quarter wave-antenna of 37 ohms, so that a half wave-
antenna removed from the earth should have a resistance of 74 ohms.
It will be seen that the computed values approach this in an oscillating

[4] M. Abraham, *Phys. Zeits.*, **2**, 329–334, 1901.
[5] G. W. Pierce, "Electric Oscillations and Electric Waves," 1920.

manner. In this respect they are similar to the curves obtained by Levin and Young,[6] in their analysis of the vertical antenna at different heights above the ground.

The curve was checked by measurement, and the results shown in Fig. 22 were obtained. In the theoretical formula the earth is considered as a perfect reflector. In Fig. 22, it is apparent that if the ground plane had been assumed as two meters below the ground, the curves would nearly coincide.

A test with a very long feeder, mentioned later, shows experimentally that the radiation of a properly terminated feeder is small. The effective resistance of the antenna may therefore be conveniently measured by determining the input to the transmission line, and the current at the center of the antenna. Neglecting losses on the transmission line other than ohmic ones, the radiation resistance is the power delivered to the antenna divided by the square of the current.
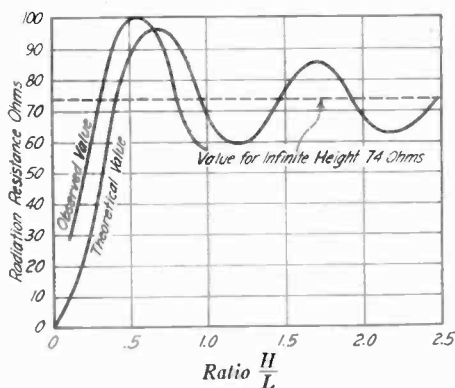
Fig. 22—Variation of radiation resistance with height—horizontal Hertz.

This gives a convenient method for determining the effective resistance of a Hertzian antenna, a method justified by the correlation of the curves in Fig. 22.

When the radiation resistance is known, a measurement can be made of the per cent of power radiated by the feeder. The power delivered to the antenna is obtained by multiplying the square of the current in the center by the radiation resistance. When this is compared with the power delivered to the transmission line by the driver, the curve of Fig. 23 is obtained. This shows conclusively that if the antenna is to be the radiating part of the system, a proper termination is essential.

[6] Levin and Young, "Radiation Resistance of Vertical Antennas," Proc. I. R. E., 14, 675–688; October, 1926.

As a final check upon the correctness of the relations deduced, a single-wire transmission line 1200 ft. long was used to feed a half-wave oscillator 50 ft. long. The input power was determined by a calorimeter test. The transmitter was immersed in oil, and a calibration curve of power dissipated vs. rise in temperature was obtained. Knowing the total input to the oscillator and the power lost, the high-frequency output may be determined.

The transmission line was No. 14 copper wire. Such a line has a high I^2R loss and such lengths would not ordinarily be practical. At the frequency used, the ohmic resistance of the 1200-ft. length as computed by the standard formula for skin effect is 48.7 ohms.

The input power used was small but definitely measurable with an accuracy of about 4 per cent. This power was 31.1 watts at a current of 0.320 amperes. The I^2R loss in the feeder was therefore 4.9 watts. The current in the antenna was 0.46 ampere and the an-
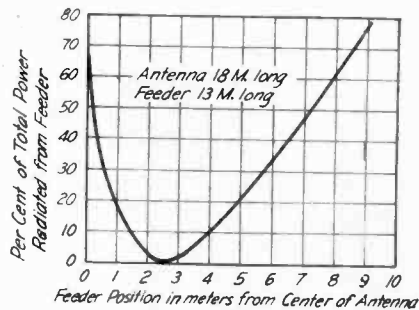
Fig. 23—Per cent of total power radiated from feeder.

tenna resistance was 128 ohms. This high resistance is probably due to losses in the supporting structure which was the University Stadium. This gave the power radiated as 27.2 watts. The power unaccounted for is therefore 1 watt, or about 3 per cent of the total, a result within the precision of measurement. This error is on the side of low rather than high loss in the line. This shows that the power radiated by the feeder, if not zero, is at least very small. With a transmission line of such length, twenty times that of the antenna, radiation, if present, might be expected to be quite prominent. This feeder was tested by J. D. Ryder and E. D. Shipley of the Ohio State University.

It should be noticed that this transmission line is about 12 wavelengths long, so that, as far as radiation problems are concerned, it would correspond at 300 meters to a line 3600 meters in length, or more than two miles.

In running a single-wire line, sharp angles should be avoided, as they cause an irregularity in the distribution of capacity and inductance. On the long feeder, such a sharp bend was at first introduced and the standing waves could not be eliminated until a more gradual bend was made. A similar phenomenon has been noticed on low-frequency power lines when trouble is experienced due to lightning surges piling up at sharp bends. This subject will be studied further.

Since the radiation resistance of the antenna varies markedly with the effective depth of the ground plane, the termination may at times vary with the weather. This would also be true if a two-wire feeder were used.

It is interesting to note that the characteristic impedance of the single-wire line ranges between 600 and 800 ohms for lines of the order of one wavelength long, and therefore is of the same order as a two-wire line. The term "voltage feed" often used to distinguish a single-wire line from a two-wire line or so-called "current feed" is not justified, as the voltage and current ratios of properly terminated lines are about the same on both types. The single-wire line has the advantage of lower I^2R losses. Higher efficiency can be obtained on long lines by loading to increase the impedance, but this is not justifiable on the lengths ordinarily needed.

The results of these and other tests and the mathematical treatment will be set forth more completely in a bulletin by the authors soon to be issued by the Engineering Experiment Station of the Ohio State University.

CIRCUIT TUNING BY WAVE RESONANCE AND APPLICATIONS TO RADIO RECEPTION*

By

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Summary—Theoretical consideration of the wave resonance system of tuning with distributed values of inductance and capacity are given, together with circuit arrangements embodying this method of tuning. It is shown that a high degree of selectivity is obtainable and that it offers an effective method for the elimination of interference. Multiplexing both in the transmission and reception of radio signals can be readily realized.

IN the present practice of the radio art, circuit tuning is accomplished by a proper combination of localized inductances and capacities; the values of the inductance and capacity being properly chosen to meet the conditions of any particular problem of design. The phenomena connected with this form of tuning have been studied quite exhaustively by many investigators; every conceivable modification of circuit structure has been subjected to careful analysis, and the results made available in many publications.

There is, however, another form of tuning which may be called "Wave Resonance" of which comparatively little (if any) advantage has been taken in the radio art, and which will on investigation offer possibilities in the accomplishment of results which cannot very well be attained by the older method of tuning. The basic principles of this form of tuning were more or less known to theorists, but the application to engineering and particularly to the radio art has been negligible.

It may be well to begin by defining what is meant by "Wave Resonance." It is the resonance condition that obtains when a conductor of distributed inductance and capacity is subjected to electrical oscillations, and either the length of the conductor or the inductance and capacity per unit length of conductor are properly adjusted in relation to the frequency of the oscillations. If an electric impulse is impressed on a conductor of distributed inductance and capacity which is open at the far end, the impulse travels along the conductor and on reaching the open end it is reflected back, returning at the same velocity. If at the time of reaching the starting point an impulse of opposite direction

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is sent into the conductor, the second impulse adds to the first. If, therefore, alternate impulses are impressed on a conductor and so timed that the intervals between them is the time required for the impulse to travel to and fro on the conductor, a high voltage or current may be produced by small impulses. This, I believe, may be properly called "Wave Resonance," that is, resonance resulting from a wave traveling on a conductor, the length of the conductor being properly adjusted in relation to the frequency or the wavelength of the oscillations.

This method of tuning offers possibilities for many novel circuit arrangements suitable for various uses in the reception and transmission of radio signals. In the discussion which follows, circuit
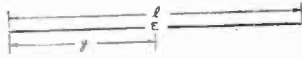
Fig. 1

systems embodying this method of tuning are analyzed. It is shown that a high degree of selectivity is obtainable, and that it offers an effective method for the elimination of interference. Also multiplexing, both in the transmission and reception of radio signals, can be readily realized by a circuit system utilizing this method of tuning.

Antenna Tuning

We shall consider first the case of a simple free antenna open at both ends, and examine the conditions for tuning under the action of a signal e.m.f. impressed uniformly at every point of the antenna. For a conductor of distributed inductance and capacity open at both ends and an e.m.f. E impressed at some intermediate point distance y from the left end of the conductor, the voltages V_1 and V_2 at points on the conductor to the right side of E and left side of E are given by the following equations[1]

$$V_1 = \frac{E \sinh ky}{\sinh kl} \cosh k(l-x),$$
$$V_2 = \frac{-E \sinh k(l-y)}{\sinh kl} \cosh kx. \tag{1}$$

The total voltage, that due to the e.m.f. acting at every point on the antenna, is readily obtainable by integrating the above expressions with respect to y. Since for every point voltage we get voltage waves

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[1] Louis Cohen. "Heaviside's Electrical Circuit Theory," p. 76.

on either side of the point of application, the total voltage is the integrated value of the sum of the voltages given by (1). That is

$$V_t = E \int_0^l \frac{\sinh ky \cosh k(l-x) - \sinh k(l-y) \cosh kx}{\sinh kl} dy$$

$$= \frac{\frac{El}{Kl}\left\{(\cosh kl - 1)\left[\cosh k(l-x) - \cosh kx\right]\right\}}{\sinh kl}. \tag{2}$$

For $x=0$,

For $x=l$,
$$\left. \begin{array}{l} V_t = \dfrac{\dfrac{El}{kl}(\cosh kl - 1)^2}{\sinh kl}. \\[3mm] V_t = \dfrac{-\dfrac{El}{kl}(\cosh kl - 1)^2}{\sinh kl}. \end{array} \right\} \tag{3}$$

At $x=l/2$, $V_t = 0$, a voltage node which we would expect.
In the above expressions

$$\left. \begin{array}{l} k = \sqrt{(L\omega + R)cj\omega} = \alpha + j\beta, \\[2mm] \alpha = \sqrt{\tfrac{1}{2}c\omega\left\{\sqrt{L^2\omega^2 + R^2} - L\omega\right\}}, \\[2mm] \beta = \sqrt{\tfrac{1}{2}c\omega\left\{\sqrt{L^2\omega^2 + R^2} + L\omega\right\}} \end{array} \right\} \tag{4}$$

For high frequencies $L^2\omega^2$ is large in comparison with R^2, and for this case to a high degree of approximation

$$\left. \begin{array}{l} \alpha = \tfrac{1}{2}R\sqrt{\dfrac{C}{L}}, \\[3mm] \beta = \omega\sqrt{LC}. \end{array} \right\} \tag{5}$$

For $kl = \alpha l + j\beta l$, we have

$$\left. \begin{array}{l} \sinh kl = \tfrac{1}{2}(e^{kl} - e^{-kl}) = \tfrac{1}{2}\left\{\cos \beta l(e^{\alpha l} - e^{-\alpha l}) + j \sin \beta l(e^{\alpha l} + e^{-\alpha l})\right\}, \\[2mm] \cosh kl = \tfrac{1}{2}(e^{kl} + e^{-kl}) = \tfrac{1}{2}\left\{\cos \beta l(e^{\alpha l} + e^{-\alpha l}) + j \sin \beta l(e^{\alpha l} - e^{-\alpha l})\right\}. \end{array} \right\} \tag{6}$$

For small values of αl which would be the case for a short conductor as that of an antenna, the above reduce to

$$\left. \begin{array}{l} \sinh kl = \alpha l \cos \beta l + j \sin \beta l, \\[2mm] \cosh kl = \cos \beta l + j\alpha l \sin \beta l. \end{array} \right\} \tag{7}$$

Introducing these values in either of equations (3) we get the following expressions for the voltage at either end of the conductor

$$V_t = \frac{\dfrac{El}{j\beta l}(\cos \beta l + j\alpha l \sin \beta l - 1)^2}{\alpha l \cos \beta l + j \sin \beta l} \cdot \tag{8}$$

A voltage resonance condition obtains for this case when $\beta l = n\pi$, $n = 1, 3, 5, \cdots$, that is $\cos \beta l = -1$, and $\sin \beta l = 0$. This gives:

$$V_{\text{res.}} = \frac{4El}{jn\pi\alpha l} = \frac{8El\sqrt{\dfrac{L}{C}}}{jn\pi Rl} \cdot \tag{9}$$

If Rl is of the order of magnitude of 3 ohms and $\sqrt{\dfrac{C}{L}} = 300$, reasonable values, we get for the resonance voltage $V_{res} = \dfrac{800El}{\pi}$.

Formula (9) for the resonance voltage on a line is of the same form as that of the voltage across the condenser of an oscillating circuit, and the voltage rise is also of the same order of magnitude that obtains in a closed oscillatory circuit. The sharpness of tuning, however, is more marked than would be the case for a closed circuit. This can be readily seen by a simple example. Suppose the frequency is slightly different from the resonance frequency; that is $\beta l = \pi + x$ where x is a small quantity, obviously the value of $\cos \beta l$ will be changed but little and $\sin \beta l$ will change to $-\sin x = -x$. The denominator in (8) will be changed from $-\alpha l$ to $-\alpha l - jx$. If $x = 2$ deg. representing a change in frequency of about one per cent, the denominator in (8) will be changed from $-\alpha l = \dfrac{1}{200}$, at resonance, to $-\dfrac{1}{200} - j\dfrac{4\pi}{360} = -0.005$ $-j\,0.035$. The change indicated is from 0.005 to 0.035. The denominator is increased seven fold giving a voltage reduction to one seventh of the resonance value for a one per cent change in frequency; a much greater change than would be the case for a closed circuit.

For the condition $\beta l = n\pi$, we have on substituting $\omega\sqrt{LC}$ for β,

$$f = \frac{n}{2l\sqrt{LC}} \cdot \tag{10}$$

In all cases of wave motion the wave velocity V is connected with the wavelength λ and the frequency f in the relation given by the formula

$$V = \lambda f,$$

but

$$V = \frac{1}{\sqrt{LC}},$$

therefore

$$f = \frac{1}{\lambda\sqrt{LC}}.$$

Substituting this in (10), we get

$$\lambda = \frac{2l}{n},$$

or

$$l = \frac{n\lambda}{2}.$$

For a conductor open at both ends, the resonance condition obtains when the length of the conductor is an odd multiple of half wavelengths.

It can be readily shown that for a conductor grounded at one end the resonance condition obtains when the length of the conductor is adjusted for an odd multiple of quarter wavelengths of the waves developed on it.

It is clear from what preceded that effective selective tuning may be accomplished by the use of open oscillators. To use straight conductors, however, would be impractical on account of the great lengths that would be required, and also because of the difficulties in the matter of adjustment and of the means for transferring the energy to some suitable receiving instruments. In what follows, methods are discussed by which this principle may be utilized in a practical system for the reception and transmission of radio signals. It is also shown that multiplexing, transmitting, and receiving can be accomplished by this method of tuning.

WAVE CONDUCTORS

To make use of wave resonance tuning in a practical way, a wave conductor must be made available that will meet the conditions of practical dimensions, and embodying convenient means of adjustment for different frequencies. This is realized in the design of a wave conductor which consists of a solenoidal coil mounted on a metal plate. A photograph of a typical wave conductor used in the experiments is shown in Fig. 2. The dimensions of the coils used for the broadcast frequency range are 8 in. long and 2 in. in diameter, wound with a suitable number of turns of wire giving the required length of conduc-

tor, and at the same time keeping the dimensions within practical limits. The dimensions of the coil as well as the number of turns may be varied to meet particular requirements of design in the matter of frequency range. By varying the distance separation of the movable plate from the coil, the distributed capacity and, to some extent, the distributed inductance are varied, affording a convenient means for adjustment. In some circuit systems the metal plate is grounded and in others ungrounded, depending upon the particular circuit arrangement in which it is used.
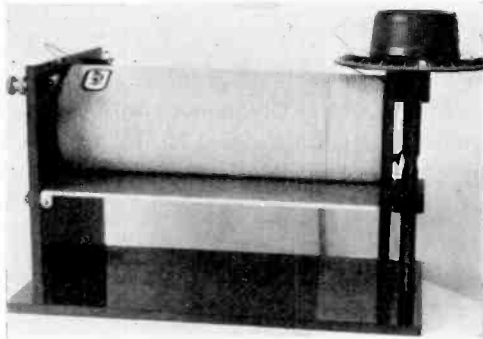
Fig. 2

The assumption that a conductor of this kind possesses the characteristics of uniform distributed inductance and capacity may not be strictly true, but sufficiently accurate for the formulation of a working theory. A more comprehensive theory which is to take cognizance of the variation of the inductance and capacity along the length of the conductor would introduce mathematical difficulties and lead to complicated formulas which may obscure the physical aspects of the problem. As a first formulation of the theory, therefore, it is considered advisable to make use of the assumption of uniform inductance and capacity which simplifies the discussion of the problem.

ANTENNA AND WAVE CONDUCTOR IN SERIES

In discussions of antenna receiving circuits it is generally assumed that the antenna may be considered as equivalent to a localized capacity in series with an emf which is that produced by the received signals acting on the entire length of the antenna. This assumption is permissible when the antenna is short in comparison with the signal wavelength. For high frequencies, where the antenna length may be comparable to the wavelength of the signals, a serious error may be introduced in considering the antenna the equivalent of a localized

capacity. It is preferable, therefore, to consider the problem in the more general form taking into accunt the distributed antenna characteristic, and where permissible the expressions arrived at will reduce to the simpler forms which would be obtained by considering the antenna as a localized capacity.
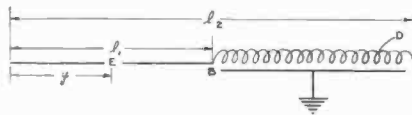
Fig. 3

We shall consider first the voltage and current distribution in a wave conductor which is connected in series with an antenna as shown in Fig. 3. Designate by L_1, R_1, C_1, and L_2, R_2, C_2 the inductance, resistance, and capacity per unit length of antenna and wave conductor respectively, and let

$$\left. \begin{aligned} k_1 &= \sqrt{(L_1 j\omega + R_1)C_1 j\omega_1} = \alpha_1 + j\beta_1, \\ k_2 &= (L_2 j\omega + R_2)C_2 j\omega = \alpha_2 + j\beta_2. \end{aligned} \right\} \tag{12}$$

For a point emf E acting at an intermediate point of the antenna distance y from the open end of the antenna we have the following expressions for the currents and voltages.

For the antenna to the left side of E

$$\left. \begin{aligned} I_1 &= A_1 e^{k_1 x} + B_1 e^{-k_1 x}, \\ V_1 &= \frac{k_1}{C_1 j\omega} \left\{ A_1 e^{k_1 x} - B_1 e^{-k_1 x} \right\}. \end{aligned} \right\} \tag{13}$$

For the antenna to the right side of E

$$\left. \begin{aligned} I_2 &= A_2 e^{k_1(x-y)} + B_2 e^{-k_1(x-y)}, \\ V_2 &= \frac{k_1}{C_1 j\omega} \left\{ A_2 e^{k_1(x-y)} - B_2 e^{-k_1(x-y)} \right\}. \end{aligned} \right\} \tag{14}$$

For the wave conductor

$$\left. \begin{aligned} I_3 &= A_3 e^{k_2(x-l_1)} + B_3 e^{-k_2(x-l_1)}. \\ V_3 &= -\frac{k_2}{C_2 j\omega} \left\{ A_3 e^{k_2(x-l_1)} - B_3 e^{-k_2(x-l_1)} \right\}. \end{aligned} \right\} \tag{15}$$

The terminal conditions from which the six constants of the above three sets of equations are determined are as follows

$$x = 0; \qquad I_1 = 0.$$
$$x = l_2; \qquad I_3 = 0.$$
$$x = l_1. \qquad I_2 = I_3 \text{ and } V_2 = V_3.$$
$$x = y; \qquad I_1 = -I_2 \text{ and } V_1 - V_2 = E. \tag{16}$$

These give the following six equations

$$A_1 + B_1 = 0,$$

$$A_3 e^{k_2(l_2 - l_1)} + B_3 e^{-k_2(l_2 - l_1)} = 0,$$

$$A_2 e^{k_1(l_1 - y)} + B_2 e^{-k_1(l_1 - y)} = A_3 + B_3,$$

$$\frac{k_1}{C_1 j\omega} \left\{ A_2 e^{k_1(l_1 - y)} - B_2 e^{-k_1(l_1 - y)} \right\} = \frac{k_2}{C_2 j\omega}(A_3 - B_3), \tag{17}$$

$$A_1 e^{k_1 y} + B_1 e^{-k_1 y} = -A_2 - B_2,$$

$$\frac{k_1}{C_1 j\omega} \left\{ A_1 e^{k_1 y} - B_1 e^{-k_1 y} + A_2 - B_2 \right\} = E.$$

From these equations all the constants are readily determined, which on substitution in (13), (14), and (15) give the complete solutions for the current and voltage distribution in the antenna and wave conductor. The main interest in this study, however, is the current and voltage distribution in the wave conductor, and we write therefore only the expressions for A_3 and B_3 which are as follows

$$A_3 = \frac{E\left\{e^{k_1 y} - e^{-k_1 y}\right\} e^{-k_2(l_2 - l_1)}}{\frac{k_1}{C_1 \omega}(e^{k_1 l_1} + e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} - e^{-k_2(l_2 - l_1)}) + \frac{k_2}{C_2 \omega}(e^{k_1 l_1} - e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} + e^{-k_2(l_2 - l_1)})} \tag{18}$$

$$B_3 = \frac{-E\left\{e^{k_1 y} - e^{-k_1 y}\right\} e^{k_2(l_2 - l_1)}}{\frac{k_1}{C_1 \omega}(e^{k_1 l_1} + e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} - e^{-k_2(l_2 - l_1)}) + \frac{k_2}{C_2 \omega}(e^{k_1 l_1} - e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} + e^{-k_2(l_2 - l_1)})}$$

$$I_3 = \frac{E(e^{k_1 y} - e^{-k_1 y})(e^{k_2(x - l_2)} - e^{-k_2(x - l_2)})}{\frac{k_1}{C_1 \omega}(e^{k_1 l_1} + e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} - e^{-k_2(l_2 - l_1)}) + \frac{k_2}{C_2 \omega}(e^{k_1 l_1} - e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} + e^{-k_2(l_2 - l_1)})},$$

$$V_3 = \frac{-\frac{k_2}{C_2 \omega} E(e^{k_1 y} - e^{-k_1 y})(e^{k_2(x - l_2)} + e^{-k_2(x - l_2)})}{\frac{k_1}{C_1 \omega}(e^{k_1 l_1} + e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} - e^{-k_2(l_2 - l_1)}) + \frac{k_2}{C_2 \omega}(e^{k_1 l_1} - e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} + e^{-k_2(l_2 - l_1)})} \tag{19}$$

Substituting these values in (15), we have the complete expressions for the current and voltage at any point on the wave conductor due to an emf acting at one point on the antenna.

For the total signal e.m.f. acting at every point on the antenna, we have

$$I_3' = \int_0^{l_1} I_3 dy,$$

$$V_3' = \int^{l_1} V_3 dy.$$

Introducting the values of I_3 and V_3 from (19) and integrating, we get

$$I_3' = \frac{\dfrac{El_1}{k_1 l_1}(e^{k_1 l_1}+e^{-k_1 l_1}-2)(e^{k_2(l_2-x)}-e^{-k_2(l_2-x)})}{\dfrac{k_1}{C_1\omega}(e^{k_1 l_1}+e^{-k_1 l_1})(e^{k_2(l_2-l_1)}-e^{-k_2(l_2-l_1)})+\dfrac{k_2}{C_2 \omega}(e^{k_1 l_1}-e^{-k_1 l_1})(e^{k_2(l_2-l_1)}+e^{-k_2(l_2-l_1)})},$$

$$V_3' = \frac{\dfrac{k_2}{C_2\omega}\dfrac{El_1}{k_1 l_1}(e^{k_1 l_1}+e^{-k_1 l_1}-2)(e^{k_2(l_2-x)}+e^{-k_2(l_2-x)})}{\dfrac{k_1}{C_1\omega}(e^{k_1 l_1}+e^{-k_1 l_1})(e^{k_2(l_2-l_1)}-e^{-k_2(l_2-l_1)})+\dfrac{k_2}{C_2 \omega}(e^{k_1 l_1}-e^{-k_1 l_1})(e^{k_2(l_2-l_1)}+e^{-k_2(l_2-l_1)})}$$

(20)

These are the most general expressions applicable for any antenna and any signal frequency. For the case, however, where the length of the antenna is only a small fraction of the signal wavelength, approximations may be introduced which simplify considerably the formulas. The antenna attenuation may be always neglected, and in that case we may put

$$e^{k_1 l_1}+e^{-k_1 l_1}=2 \cos \beta_1 l_1,$$

$$e^{k_1 l_1}-e^{-k_1 l_1}=2j \sin \beta_1 l_1.$$

Now $\beta_1 l_1 = 2\pi f\sqrt{L_1 C_1}\, l_1 = 2\pi l_1/\lambda$, and for small values of l/λ, we have approximately

$$\cos \beta_1 l_1 = 1-\frac{\beta_1 l_1^2}{2},$$

$$\sin \beta_1 l_1 = \beta_1 l_1.$$

Using these approximations, equations (20) reduce to

$$I_3' = \frac{\dfrac{E_0}{2}(e^{k_2(l_2-x)}-e^{-k_2(l_2-x)})}{\dfrac{1}{j\omega C_0}(e^{k_2(l_2-l_1)}-e^{-k_2(l_2-l_1)})+\dfrac{k_2}{C_2\omega}(e^{k_2(l_2-l_1)}+e^{-k_2(l_2-l_1)})},$$

(21)

$$V_3' = \frac{\dfrac{E_0}{2}\dfrac{k_2}{C_2\omega}(e^{k_2(l_2-x)}+e^{-k_2(l_2-x)})}{\dfrac{1}{j\omega C_0}(e^{k_2(l_2-l_1)}-e^{-k_2(l_2-l_1)})+\dfrac{k_2}{C_2\omega}(e^{k_2(l_2-l_1)}+e^{-k_2(l_2-l_1)})}.$$

Where $E_0 = El_1$, the total emf acting on the antenna, and $C_0 = Cl_1$, the total capacity of the antenna.

These are precisely the expressions that would result by considering the antenna as a localized capacity to begin with, but it must be remembered that they are only valid for the case l_1/λ small. For short waves the complete formulas (20) must be used.

If the wave conductor is of short length, which would be generally the case, $\alpha_2(l_2 - l_1)$ is very small, and to a high degree of approximation,

$$e^{k_2(l_2-l_1)} + e^{-k_2(l_2-l_1)} = 2 \cos \beta_2(l_2 - l_1) + 2j\alpha_2(l_2 - l_1) \sin \beta_2(l_2 - l_1),$$

$$e^{k_2(l_2-l_1)} - e^{-k_2(l_2-l_1)} = 2\alpha_2(l_2 - l_1) \cos \beta_2(l_2 - l_1) + 2j \sin \beta_2(l_2 - l_1). \tag{22}$$

Introducing these approximations in (21), the expression for the voltage on the wave conductor reduces to

$$V_3' = \frac{\dfrac{E_0}{2}\sqrt{\dfrac{L_2}{C_2}} \{\cos \beta_2(l_2 - x) + j\alpha_2(l_2 - x) \sin \beta_2(l_2 - x)\}}{\dfrac{1}{j\omega C_0} \{\alpha_2(l_2 - l_1)\cos\beta_2(l_2 - l_1) + j\sin\beta_2(l_2 - l_1)\} + \sqrt{\dfrac{L_2}{C_2}} \{\cos\beta_2(l_2 - l_1) + j\alpha_2(l_2 - l_1)\sin\beta_2(l_2 - l_1)\}} \tag{23}$$

For $\beta_2 (l_2 - l_1) = \pi/2 + z$, the above reduces to

$$V_3' = \frac{\dfrac{E_0}{2}\sqrt{\dfrac{L_2}{C_2}} \{\cos \beta_2(l_2 - x) + j\alpha_2(l_2 - x) \sin \beta_2(l_2 - x)\}}{\dfrac{1}{j\omega C_0} \{-\alpha_2(l_2 - l_1) \sin z + j \cos z\} + \sqrt{\dfrac{L_2}{C_2}} \{-\sin z + j\alpha_2(l_2 - l_1) \cos z\}}. \tag{24}$$

Assume now that an adjustment is made in $\beta_2(l_2 - l_1)$, which is accomplished in practice by changing the distance between the metal plate and the coil of the wave conductor, so that

$$\frac{1}{\omega C_0} \cos z = \sqrt{\frac{L_2}{C_2}} \sin z, \tag{25}$$

then (24) reduces to

$$V_3' = \frac{\dfrac{E_0}{2}\sqrt{\dfrac{L_2}{C_2}} \{\cos \beta_2(l_2 - x) + j\alpha_2(l_2 - x) \sin \beta_2(l_2 - x)\}}{j\alpha_2(l_2 - l_1) \left\{\dfrac{1}{\omega C_0} \sin z + \sqrt{\dfrac{L_2}{C_2}} \cos z\right\}}. \tag{26}$$

which is the resonance condition. At $x = l_2$, the end of the wave conductor,

$$V_3 = \frac{\dfrac{E_0}{2}\sqrt{\dfrac{L_2}{C_2}}}{j\alpha_2(l_2 - l_1)\left\{\dfrac{1}{C_0\omega}\sin z + \sqrt{\dfrac{L_2}{C_2}}\cos z\right\}} . \tag{27}$$

In practice $\sqrt{L_2/C_2}$ is large compared with $1/C_0\omega$. We may assume as reasonable values $\sqrt{L_2/C_2} = 10^4$, and $1/C_0\omega = 200$, and for these values we have by (25),

$$\tan z = 0.02,$$
$$z = 1° \, 9'.$$

Sin z is very small and cos z is practically unity, and in that case

$$V_3 = \frac{E_0\sqrt{\dfrac{L_2}{C_2}}}{R_2(l_2 - l_1)} . \tag{28}$$

At a point on the wave conductor distance x determined by the relation $\beta_2(l_2 - x) = \dfrac{\pi}{2}$, the voltage is given by, in accordance with (26)

$$V_3 = \frac{\dfrac{E_0}{2}\sqrt{\dfrac{L_2}{C_2}}(l_2 - x)}{(l_2 - l_1)\left\{\dfrac{1}{C_0\omega}\sin z + \sqrt{\dfrac{L_2}{C_2}}\cos z\right\}} ,$$

$$= \frac{E_0}{2} \, \frac{l_2 - x}{l_2 - l_1} \text{ approximately}. \tag{29}$$

The voltage at this point is very small compared with the voltage at the end of the wave conductor. If we use the value $\sqrt{L_2/C_2} = 10^4$, and assume the total resistance of the wave conductor to be 40 ohms, the ratio of the voltages, that at the end of the wave conductor and at

$$x = l_2 - \frac{\pi}{2\beta_2} = l_2 - \frac{\lambda}{4} ,$$

$$\frac{V_{3l_2}}{V_3} = \frac{2 \times 10^4}{40} = 500 .$$

At the point $x = l_2 - \lambda/4$, the voltage wave has a nodal point. If it were not for the resistance of the wave conductor, the voltage at that point would be of zero value. The voltage wave form is shown graphically in Fig. 4.
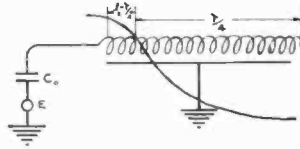
Fig. 4

It is quite evident that a double tuning is accomplished in the process of adjusting the wave conductor; the small part of the wave conductor, $l_2 - \lambda/4$, tunes out the antenna capacity reactance, and on the balance of the wave conductor a quarter wavelength is established. It must be remembered that all of this is accomplished by one adjustment, it is merely a matter of varying β_2, which is effected by changing the distance separation of the metal plate and the wave conductor until the condition $\beta l = \pi/2 + z$ is realized, which is the resonance condition.

That a high degree of selectivity is obtainable by this system of tuning will appear from the consideration of a numerical example. For the values chosen, $\sqrt{L_2/C_2} = 10^4$ and $1/C_0\omega = 200$, the resonance condition obtains when $\beta_2(l_2 - l_1) = 91$ deg. 9 min., and for that value the resonance voltage at the end of the wave conductor is by (28), assuming $R_2(l_2 - l_1) = 40$ ohms.

$$V_{3, \text{res.}} = \frac{E \times 10^4}{40} = 250E.$$

For $\beta_2(l_2 - l_1) = 90$ deg., about one per cent reduction in frequency.

$$\cos \beta_2(l_2 - l_1) = 0; \qquad \sin \beta_2(l_2 - l_1) = 1,$$

and for these values

$$V_3 = \frac{E}{2} \times \frac{10^2}{200 + j20} = 25E.$$

For $\beta_2(l_2 - l_1) = 92$ deg., about one per cent increase in frequency, we have

$$\cos \beta_2(l_2 - l_1) = -0.035,$$
$$\sin \beta_2(l_2 - l_1) = 0.9994.$$

For these values,

$$V_3 = \frac{\dfrac{E_0}{2} \times 10^4}{-j200\{-20 \times 0.035 \times 10^{-4} + j0.9994\} + 10^4\{-0.035 + j40 \times 10^{-4}\}}$$

$$= \frac{E_0}{2} \times \frac{10^4}{(200 - 350)} = 33E.$$

A change in frequency of one per cent on either side of the resonance frequency produces a reduction in voltage to about ten per cent of the resonance voltage, which shows clearly the tuning effectiveness of the system.

This form of tuning may be utilized in various ways in circuit systems for the reception and transmission of radio signals.

Special consideration, however, is required in using this system for very short waves. In that case the more complete formula (20) should be used, which gives for the voltage at the end of the wave conductor, $x = l_2$,

$$V_3 = \frac{\dfrac{El_1}{k_1 l_1} \dfrac{k_2}{C_{2j}\omega}\{e^{k_1 l_1} + e^{-k_1 l_1} - 2\}}{\dfrac{k_1}{C_{1j}\omega}(e^{k_1 l_1} + e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} - e^{-k_2(l_2 - l_1)}) + \dfrac{k_2}{C_{2j}\omega}(e^{k_1 l_1} - e^{-k_1 l_1})(e^{k_2(l_2 - l_1)} + e^{-k_2(l_2 - l_1)})} \tag{30}$$

Using the approximations given by (22) the above transforms to

$V_3 = $ a fraction, the numerator of which is $\dfrac{1}{2} \dfrac{El_1}{k_1 l_1} \dfrac{k_2}{C_{2j}\omega}\{\cos \beta_1 l_1$

$+ j\alpha_1 l_1 \sin\beta_1 l_1 - 1\}$ and the denominator of which is $\dfrac{k_1}{C_{1j}\omega}(\cos \beta_1 l_1$ (31)

$+ j\alpha_1 l_1 \sin \beta_1 l_1)\{\alpha_2(l_2 - l_1)\cos \beta_2(l_2 - l_1) + j \sin \beta_2(l_2 - l_1)\}$

$+ \dfrac{k_2}{C_{2j}\omega}(\alpha_1 l_1 \cos\beta_1 l_1 + j\sin\beta_1 l_1)\{\cos\beta_2(l_2 - l_1) + j\alpha_2(l_2 - l_1)\sin\beta_2(l_2 - l_1)\}.$

Tuning is accomplished by adjusting the wave conductor to satisfy the condition

$$\frac{k_1}{C_{1j}\omega}\cos \beta_1 l_1 \sin \beta_2(l_2 - l_1) = -\frac{k_2}{C_{2j}\omega}\sin \beta_1 l_1 \cos \beta_2(l_2 - l_1),$$

or

$$\tan \beta_1 l_1 = -\sqrt{\frac{L_1 C_2}{L_2 C_1}}\tan \beta_2(l_2 l_1). \tag{32}$$

For this condition, neglecting small quantities of the second order, that is terms in $\alpha^2 l^2$, equation (31) reduces to

$$V_3 = \frac{\dfrac{1}{2}\dfrac{El_1}{k_1 l_1}\dfrac{k_2}{C_{2,\omega}}\{\cos\beta_1 l_1 + j\alpha_1 l_1 \sin\beta_1 l_1 - 1\}}{\cos\beta_1 l_1 \cos\beta_2(l_2-l_1)\left\{\dfrac{k_1}{C_{1,\omega}}\alpha_2(l_2-l_1)+\dfrac{k_2}{C_{2,\omega}}\alpha_1 l_1\right\} - \sin\beta_1 l_1 \sin\beta_2(l_2-l_1)\left\{\dfrac{k_1}{C_{1,\omega}}\alpha_1 l_1 + \dfrac{k_2}{C_{2,\omega}}\alpha_2(l_2-l_1)\right\}} \tag{33}$$

Each of the terms in the denominator contains an αl factor which is a very small quantity, and hence the resonance voltage rise at the end of the wave conductor is great.

It is to be observed that in accordance with (32), the tuning of the wave conductor for the resonance condition is not only a matter of frequency but also of antenna length. For the same frequency the wave conductor will tune differently for antennas of different lengths.

Another point to be noticed in connection with this system for short wave tuning is the special case when the length of the antenna is a whole wavelength of the signals, which makes $\beta_1 l_1 = 2\pi$, and in that case the numerator of (33) is practically zero, and there is no voltage transfer at all to the wave conductor. That this should be so is evident from a consideration of equations (3) which show that for a distributed emf over a conductor a wavelength long the voltages are zero at the ends of the conductor, and it would follow that by connecting a wave conductor to the end of an antenna when this condition exists (zero voltage at the end), there would be no energy transfer to the wave conductor. It is obvious from this that in using this system of tuning care must be taken to avoid the condition of having the length of the antenna approximate a whole wavelength or a multiple of wavelengths of the signals.

RECEPTION OF SIGNALS

The possibility of the adoption of wave resonance tuning for the reception of radio signals is quite obvious from the preceding discussion, provided suitable means are introduced for transferring to a receiving device the voltage or current energy from the wave conductor. This can be accomplished in various ways; it is largely a matter of coupling. The simplest way is to couple a closed circuit, preferably tuned, to the wave conductor, and the voltage developed across the condenser of the closed circuit applied to the grid filament of a vacuum tube in the usual way. If desired, additional tuning may be had by the use of a tuned radio-frequency amplifier, and in that case the wave conductor tuning serves to superimpose another degree of tuning, giving a remarkably high degree of selectivity and this with-

out any loss in efficiency. In fact it may be used in connection with any of the well known types of commercial receivers. In practice it is found that a sufficient degree of coupling is obtained by placing the wave conductor close to the receiving set, the coupling being largely electrostatic. A better arrangement, however, is to connect the metal plate of the wave conductor to the receiving set as shown in Fig. 5.
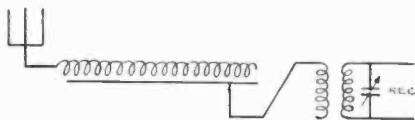
Fig. 5

In this case the metal plate serves a double purpose; that of adjusting the wave conductor, and as a coupling means. I had the use of a Stromberg-Carlson set in which the primary is untuned, the metal plate of the wave conductor connected to one terminal of the primary coil, and the other terminal open. The coupling from the primary to the tuned secondary circuit is obviously electrostatic. Very good results in the matter of volume and selectivity were obtained with this arrangement.

A somewhat startling phenomenon was observed in connection with reception tests by the arrangement of Fig. 5, that, even when
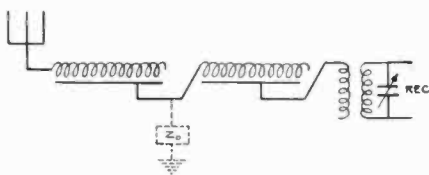
Fig. 6

grounding the plate of the wave conductor it was possible to receive signals from distant stations though with loss in volume. It would seem that since grounding the metal plate grounds the receiver, no signal reception at all could be expected. The fact is, however, that signals were received with the grounded plate arrangement, and it was also noted that the tuning of the wave conductor was sharpened to a remarkable degree, requiring an extremely delicate adjustment. The only explanation that can be offered is that the ground is not a zero potential surface, as it is customary to assume but perhaps more satisfactory explanations will be forthcoming in the future.

A modification of Fig. 5 by which the selectivity is considerably enhanced is shown in Fig. 6, in which two wave conductors connected

in tandem are used, each separately tuned. It is clear that by this arrangement a progressive tuning is effected. An additional improvement is obtained by grounding either directly, or through an impedance the junction point of the two wave conductors as indicated by the dotted lines in the figure.

ELIMINATION OF INTERFERENCES

Tuning by wave resonance is well adapted for the elimination of interferences. It was shown in the preceding discussion that a wave conductor connected in series with an antenna can be made to respond resonantly by proper adjustment to signals of a particular frequency, and it should follow from this that it should also function efficiently as an energy absorber of an interfering signal of that particular frequency. That this is actually so was demonstrated experimentally, and accomplished in the following manner. The receiver is connected to the antenna in the usual manner, and a wave conductor connected to the junction point of antenna and receiver as shown in Fig. 7. By the
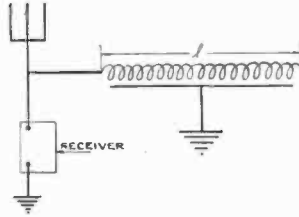
Fig. 7

proper adjustment of the wave conductor to the frequency of any particular interfering signal, a voltage node is established for that frequency at the junction point of antenna and receiver, and consequently very little if any signal current of that particular frequency will pass into the receiver. The following brief mathematical discussion will make this evident.

Let Z_1 denote the antenna reactance, and Z_2 the receiver reactance; I_1 and I_2 the currents in the antenna and receiver, respectively. The expressions for the current and voltage of wave conductors are as follows:

$$
\left.
\begin{aligned}
I &= Ae^{kx} + Be^{-kx}, \\
V &= -\frac{k}{Cj\omega}\left\{Ae^{kx} - Be^{-kx}\right\}.
\end{aligned}
\right\}
\tag{34}
$$

The constants A and B are determined from the following terminal conditions:

For $x = l;$ $I = 0,$

For $x = 0;$ $V_0 = E - Z_1 I_1.$

also,

$$I_1 = I_2 + I_{x=0},$$

and

$$I_2 = \frac{V_{x=0}}{Z_2}.$$

These conditions give,

$$
\left.
\begin{aligned}
&A e^{kl} + B e^{-kl} = 0, \\
&-\frac{k}{Cj\omega}(A - B) = E + \frac{Z_1}{Z_2}\frac{k_1}{Cj\omega}(A - B) - Z_1(A + B).
\end{aligned}
\right\} \quad (35)
$$

from which the values of A and B are readily determined. Introducing these in (34), we get the complete expression for the voltage on the wave conductor as follows:

$$
V = \frac{-E\dfrac{k}{Cj\omega}\left\{ e^{k(l-x)} + e^{-k(l-x)} \right\}}{Z_1(e^{kl} - e^{-kl}) + \dfrac{k}{Cj\omega}\left(1 + \dfrac{Z_1}{Z_2}\right)(e^{kl} + e^{-kl})}. \quad (36)
$$

At $x = 0$,

$$
V_{x=0} = \frac{-E\dfrac{k}{Cj\omega}(e^{kl} + e^{-kl})}{Z_1(e^{kl} - e^{-kl}) + \dfrac{k}{Cj\omega}\left(1 + \dfrac{Z_1}{Z_2}\right)(e^{kl} + e^{-kl})}. \quad (37)
$$

Using the approximations given by (22), the above reduces to

$$
V_{x=0} = \frac{-E\dfrac{k}{Cj\omega}\left\{ \cos \beta l + j\alpha l \sin \beta l \right\}}{Z_1(\alpha l \cos \beta l + j \sin \beta l) + \dfrac{k}{Cj\omega}\left(1 + \dfrac{Z_1}{Z_2}\right)(\cos \beta l + j\alpha l \sin \beta l)}. \quad (38)
$$

When the wave conductor is adjusted for the condition $\beta l = \pi/2$, the bracket term in the numerator of (38) reduces to αl, which is a very small quantity of the order of magnitude of 0.001 or 0.002 depending on the resistance and the ratio of C/L of the wave conductor, and therefore the interfering signal of the particular frequency for which the wave conductor is in adjustment to the condition $\beta l = \pi/2$ can have very little effect on the receiver. For signals of other frequencies,

for example differing five per cent in frequency, $\beta l = 1.05\pi/2 = 94.5$ deg., and $\cos \beta l = -0.078$, the voltage would be any where from 39 to 78 times greater than the voltage of the interfering signal for the same emf acting on the antenna. It is obvious that by connecting several wave conductors, each separately adjusted for the frequency of a different interfering signal, any number of interfering signals may be eliminated simultaneously, and this without any appreciable reduction in the efficiency of reception of signals of other frequencies. This is amply borne out by experimental tests in connection with various types of commercial receivers.

This immediately suggests a method for further increasing the selectivity of reception by the wave resonance tuning method. It consists in replacing the receiver of Fig. 7 by a leak to ground, and coupling the receiver to the wave conductor in the manner indicated in the preceding section. It is necessary, however, to introduce an inductance in the antenna to tune the antenna separately. For the resonance condition then we shall have

$$V = \frac{- E\dfrac{k}{Cj\omega}}{R_1 j + \dfrac{k}{Cj\omega}\left(1 + \dfrac{R_1}{Z_2}\right)j\alpha l} .$$

If Z_2 is a resistance of a value R_1, the above reduces to

$$V = \frac{-E\sqrt{\dfrac{L}{C}}}{j(R_1 + Rl)} ,$$

where Rl is the total resistance of wave conductor. For $\sqrt{L/C} = 10^4$, $R_1 = 10$ and $Rl = 40$,

$$V = 200E .$$

For of resonance, say 5 per cent $\beta l = 85.5$ deg., $\cos \beta l = 0.0785$, $\sin \beta l = 0.997$, and Z_1 may change from 10 to 100. For this case

$$V = \frac{- E \times 10^4}{j100(j0.997) + 10^4(1 + j10)(0.0785)} ,$$

$$= -\frac{E \times 10^4}{10^4 \times 0.785} = 1.3E .$$

A reduction in voltage to $1.3/200$ of the resonance value, which would indicate a very high degree of selectivity. It is obvious that this

method may be extended to the use of a three-quarter wavelength
conductor establishing another voltage nodal point at the half wave-
length distance, and inserting at that point another leak to ground.
This would still further increase the selectivity of the system.

Multiplex Reception

The possibility of utilizing wave resonance tuning for multiplex
reception naturally suggests itself. If, as has been indicated, it is
possible to eliminate several interfering signals of different frequencies
by the use of several wave conductors each adjusted to a frequency of
one of the interfering signals, then it should be possible to reverse the
process by coupling receivers to the wave conductors through each
of which a signal of one frequency only (for which it is in adjustment)
is transmitted to the corresponding receiver. This has been accom-
plished in a highly satisfactory manner. An outline of the theory is
given in what follows.

It was already shown that for short antennas, that is, short in com-
parison with the wavelength of the signals, it is fairly accurate to con-
sider the antenna as a localized capacity, and we shall make use of
this approximation in the following discussion which simplifies con-
siderably the mathematical analysis of the problem.

A schematic representation of the circuit system for multiplex re-
ception is shown in Fig. 8.
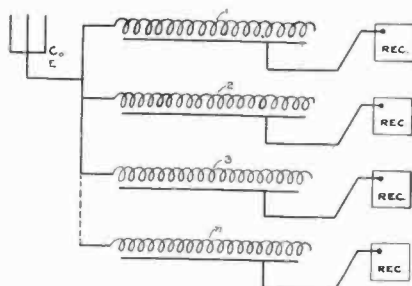
Fig. 8

Several wave conductors, n in number, are connected to the an-
tenna which is represented by the capacity C_0 and signal e.m.f. E. The
metal plates of the wave conductors are each connected to a separate
receiver, the metal plate serving in this case a double purpose: a
means for adjusting the wave conductors, and also as a means for
coupling the wave conductors to the receivers. Other means of
coupling the receivers to the wave conductors may be employed, and
in that case the metal plates may be grounded if desired.

We shall designate by subscripts 1, 2, 3, $\cdots$, n the currents and voltages on the corresponding wave conductors, and also designate the electrical constants of the wave conductor by corresponding subscripts.

We have then:

For wave conductor 1

$$\left.\begin{aligned}
I_1 &= A_1 e^{k_1 x} + B_1 e^{-k_1 x}, \\
V_1 &= -\frac{k_1}{C_1 j\omega}(A_1 e^{k_1 x} - B_1 e^{-k_1 x}), \\
k_1 &= \sqrt{(L_1 j\omega + R_1) C_1 j\omega}.
\end{aligned}\right\} \tag{39}$$

For wave conductor 2

$$\left.\begin{aligned}
I_2 &= A_2 e^{k_2 x} + B_2 e^{-k_2 x}, \\
V_2 &= -\frac{k_2}{C_2 j\omega}(A_2 e^{k_2 x} - B_2 e^{-k_2 x}), \\
k_2 &= \sqrt{(L_2 j\omega + R_2) C_2 j\omega}
\end{aligned}\right\} \tag{40}$$

Similar equations for all the other wave conductors. For the nth wave conductor, we have

$$\left.\begin{aligned}
I_n &= A_n e^{k_n x} + B_n e^{-k_n x}, \\
V_n &= -\frac{k_n}{C_n j\omega}(A_n e^{k_n x} - B_n e^{-k_n x}).
\end{aligned}\right\} \tag{41}$$

Each of the above equations must satisfy the condition that the current is zero at the end of the conductor, $x = l$, which gives the following:

$$\left.\begin{aligned}
A_1 e^{k_1 l} + B_1 e^{-k_1 l} &= 0, \\
A_2 e^{k_2 l} + B_2 e^{-k_2 l} &= 0, \\
\cdots\cdots\cdots\cdots\cdots \\
A_n e^{k_n l} + B_n e^{-k_n l} &= 0.
\end{aligned}\right\} \tag{42}$$

From these we get the relations,

$$\left.\begin{aligned}
B_1 &= -A_1 e^{2k_1 l_1}, \\
B_2 &= -A_2 e^{2k_2 l}, \\
B_n &= -A_n e^{2k_n l}.
\end{aligned}\right\} \tag{43}$$

At $x = 0$, we have the conditions that the voltages at each of the wave conductors are the same, and equal to the applied voltage less the an-

tenna capacity reactance voltage drop. The current in C_0 is obviously equal to the sum of the currents in all the wave conductors at $x=0$. Hence the following conditions:

$$\left.\begin{aligned}
\frac{k_1}{C\,j\omega}(A_1 - B_1) &= \frac{k_2}{C_2 j\omega}(A_2 - B_2) = \cdots = \frac{k_n}{C_n j\omega}(A_n - B_n), \\
\text{and} & \\
\frac{k_1}{C_1 j\omega}(A_1 - B_1) &= E - \frac{1}{C_0 j\omega}(A_1 + B_1 + A_2 + B_2 + \cdots + A_n + B_n).
\end{aligned}\right\} \quad (44)$$

Making use of the relation given by (43) we get

$$\frac{k_1}{C_1 j\omega}A_1 e^{k_1 l_1}(e^{k_1 l} + e^{-k_1 l}) = \frac{k_2}{C_2 j\omega}A_2 e^{k_2 l}(e^{k_2 l} + e^{-k_2 l}) = \cdots$$

$$= \frac{k_n}{C_n j\omega}A_n e^{k_n l}(e^{k_n l} + e^{-k_n l}), \qquad (45)$$

that is,

$$\left.\begin{aligned}
A_2 &= \frac{k_1 c_2}{k_2 c_1}\,\frac{e^{k_1 l}}{e^{k_2 l}}\,\frac{e^{k_1 l} + e^{-k_1 l}}{e^{k_2 l} + e^{-k_2 l}}A_1, \\
A_3 &= \frac{k_1 c_3}{k_3 c_1}\,\frac{e^{k_1 l}}{e^{k_3 l}}\,\frac{e^{k_1 l} + e^{-k_1 l}}{e^{k_3 l} + e^{-k_3 l}}A_1, \\
& \quad\cdots\cdots\cdots\cdots\cdots\cdots \\
A_n &= \frac{k_1 c_n}{k_n c_1}\,\frac{e^{k_1 l}}{e^{k_n l}}\,\frac{e^{k_1 l} + e^{-k_1 l}}{e^{k_n l} + e^{-k_n l}}A_1.
\end{aligned}\right\} \quad (46)$$

Introducing these values in the second equation (44), we obtain the following:

$$\frac{k_1}{C_1 j\omega}A_1 e^{k_1 l}(e^{k_1 l} + e^{-k_1 l}) = E - \frac{A_1}{C_0 j\omega}\left\{ \{(1 - e^{2k_1 l})\right. \qquad (47)$$

$$\left. + \frac{k_1 c_2}{k_2 c_1}e^{k_1 l}\frac{(e^{k_1 l} + e^{-k_1 l})}{e^{k_2 l} + e^{-k_2 l}}(1 - e^{2k_2 l}) + \cdots + \frac{k_1 c_n}{k_n c_1}e^{k_1 l}\frac{(e^{k_1 l} + e^{-k_1 l})}{e^{k_n l} + e^{-k_n l}}(1 - e^{2k_n l}) \right\}.$$

This simplifies to

$$\frac{k_1}{C_1 j\omega}A_1 = \frac{E e^{-k_1 l}}{e^{k_1 l} + e^{-k_1 l}} + \frac{1}{C_0 j\omega}A_1\left\{\frac{e^{k_1 l} - e^{-k_1 l}}{e^{k_1 l} + e^{-k_1 l}} + \frac{k_1 c_2}{k_2 c_1}\,\frac{e^{k_2 l} - e^{-k_2 l}}{e^{k_2 l} + e^{-k_2 l}} + \cdots\right.$$

$$\left. + \frac{k_1 c_n}{k_n c_1}\,\frac{e^{k_n l} - e^{-k_n l}}{e^{k_n l} + e^{-k_n l}}\right\}$$

which gives

$$A_1 = \cfrac{Ee^{-k_1 l}}{(e^{k_1 l}+e^{-k_1 l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}\left(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots+\cfrac{k_1 c_n}{k_n c_1}\tanh k_n l\right)\right\}}$$

and

$$B_1 = \cfrac{-Ee^{k_1 l}}{(e^{k_1 l}+e^{-k_1 l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}\left(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots+\cfrac{k_1 c_n}{k_n c_1}\tanh k_n l\right)\right\}} \qquad (48)$$

By (46) expressions for the other constants are readily obtained. Thus

$$A_2 = \cfrac{E\cfrac{k_1 c_2}{k_2 c_1}e^{-k_2 l}}{(e^{k_2 l}+e^{-k_2 l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}\left(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots+\cfrac{k_1 c_n}{k_n c_1}\tanh k_n l\right)\right\}}$$

$$B_2 = \cfrac{-E\cfrac{k_1 c_2}{k_2 c_1}e^{k_2 l}}{(e^{k_2 l}+e^{-k_2 l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}\left(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots+\cfrac{k_1 c_n}{k_n c_1}\tanh k_n l\right)\right\}}$$

and similarly for the other constants, that is

$$A_n = \cfrac{E\cfrac{k_1 c_n}{k_n c_1}e^{-k_n l}}{(e^{k_n l}+e^{-k_n l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots)\right\}},$$

$$B_n = \cfrac{-E\cfrac{k_1 c_n}{k_n c_1}e^{k_n l}}{(e^{k_n l}+e^{-k_n l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots)\right\}}. \qquad (49)$$

The voltage on any one of the wave conductors, say the *m*th conductor, is given by the following:

$$V_m = \cfrac{E\cfrac{k_1}{C_1 j\omega}\left\{e^{k_m(l-x)}+e^{-k_m(l-x)}\right\}}{(e^{k_m l}+e^{-k_m l})\left\{\cfrac{k_1}{C_1 j\omega}-\cfrac{1}{C_0 j\omega}\left(\tanh k_1 l+\cfrac{k_1 c_2}{k_2 c_1}\tanh k_2 l+\cdots+\cfrac{k_1 c_n}{k_n c_1}\tanh k_n l\right)\right\}}, \qquad (50)$$

which may be written in this form

$$V_m = \frac{E\left\{e^{k_m(l-x)} + e^{-k_m(l-x)}\right\}}{(e^{k_m l} + e^{-k_m l})\left\{1 - \frac{1}{C_{0j}\omega}\left(\frac{C_{1j}\omega}{k_1}\tanh k_1 l + \frac{C_{2j}\omega}{k_2}\tanh k_2 l + \cdots \frac{C_{nj}\omega}{k_n}\tanh k_n l\right)\right\}} \tag{51}$$

For $x = l$

$$V_m = \frac{2E}{(e^{k_m l} + e^{-k_m l})\left\{1 - \frac{1}{C_{0j}\omega}\left(\frac{C_{1j}\omega}{k_1}\tanh k_1 l + \frac{C_{2j}\omega}{k_2}\tanh k_2 l + \cdots \frac{C_{nj}\omega}{k_n}\tanh k_n l\right)\right\}} \tag{52}$$

To see more clearly what little effect additional wave conductors have on the efficiency of reception on any one wave conductor, it is best to express (52) trigonometrically using the approximations of (22). For wave conductor (1), we have V_1 equal to a fraction the numerator of which is E and the denominator of which is

$$(\cos \beta_1 l + j\alpha_1 l \sin \beta_1 l) - \frac{1}{C_{0j}\omega}\sqrt{\frac{C_1}{L_1}}(\alpha_1 l_1 \cos \beta_1 l_1 + j \sin \beta_1 l_1)$$

$$-\frac{1}{C_{0j}\omega}\sqrt{\frac{C_2}{L_2}}(\cos \beta_1 l_1 + j\alpha_1 l_1 \sin \beta_1 l_1)\tanh k_2 l - \cdots$$

$$-\frac{1}{C_{0j}\omega}\sqrt{\frac{C_n}{L_n}}(\cos \beta_1 l_1 + j\alpha_1 l_1 \sin \beta_1 l_1)\tanh k_n l. \tag{53}$$

If the wave conductor is adjusted for a particular signal frequency so that $\beta_1 l = \pi/2$, the above reduces to

$$V_1 = \frac{E}{j\alpha_1 l - \frac{1}{C_0\omega}\sqrt{\frac{C_1}{L_1}} - \frac{1}{C_0\omega}\sqrt{\frac{C_2}{L_2}}\alpha_1 l \tanh k_2 l - \cdots \frac{1}{C_0\omega}\sqrt{\frac{C_n}{L_n}}\alpha_1 l \tanh k_n l.} \tag{54}$$

All of the terms in the denominator which have the factor αl are very small in comparison with the first two terms unless $\tanh kl$ for any of the wave conductors is very large, which would be the case only when any of the other wave conductors should be in adjustment for a frequency extremely close, say $1/2$ of one per cent, to the frequency of wave conductor (1).

Assuming that the signals to be received on the different wave conductors are of appreciably different frequencies, then the ex-

pressions for the voltage resonance on any one of the wave conductors reduces to the following:

$$V_m = \frac{E}{j\alpha_m l - \dfrac{1}{C_0\omega}\sqrt{\dfrac{C_m}{L_m}}},$$

$$= \frac{E\sqrt{\dfrac{L_m}{C_m}}}{j\dfrac{R_m l}{2} - \dfrac{1}{C_0\omega}} = -EC_0\omega\sqrt{\dfrac{L_m}{C_m}} \quad \text{approx.} \tag{55}$$

This system offers also the possibility of operating a number of receivers on the same frequency from a single antenna.

If all the wave conductors are adjusted for resonance for the same frequency,

$$k_1 l = k_2 l = k_3 l = \cdots k_n l.$$

and (52) reduces to

$$V = \frac{2E}{(e^{kl}+e^{-kl})\left\{1 - \dfrac{n}{C_0 j\omega}\dfrac{Cj\omega}{k}\tanh kl\right\}}. \tag{56}$$

Expressed in trigonometric terms using the approximations of (22), the above transforms to

$$V = \frac{E}{(\cos\beta l + j\alpha l \sin\beta l) - \dfrac{n}{C_c j\omega}\sqrt{\dfrac{C}{L}}(\alpha l \cos\beta l + j \sin\beta l)}. \tag{57}$$

If each of the wave conductors is adjusted for the condition

$$\cos\beta l = \frac{n}{C_0\omega}\sqrt{\frac{C}{L}}\sin\beta l,$$

which is the resonance condition for the system, the voltage on each wave conductor is given by

$$V = \frac{E}{\alpha l\left\{j\sin\beta l + j\dfrac{n}{C_0\omega}\sqrt{\dfrac{C}{L}}\cos\beta l\right\}}$$

$$= \frac{E}{j\alpha l \sin\beta l\left\{1 + \dfrac{n^2}{C_0^2\omega^2}\dfrac{C}{L}\right\}}. \tag{59}$$

The term $n^2/C_0^2\omega^2\ C/L$ is small compared with the unity term, unless n is fairly large, and for a small number of receivers the resonance voltage is effected but very little by additional receivers. We may assume the following values: $1/C_0\omega = 200$, and $L/C = 10^8$, and for these values we have

$$n = 10; \qquad 1 + \frac{n^2}{C_0^2\omega^2}\ \frac{C}{L} = 1.04$$

$$n = 20; \qquad " \qquad = 1.16$$

$$n = 30; \qquad " \qquad = 1.36$$

$$n = 40; \qquad " \qquad = 1.64$$

$$n = 50; \qquad " \qquad = 2.00.$$

It is seen that even for fifty receivers the resonance voltage is reduced to only one half the voltage that would develop where only one receiver is connected to the antenna. If the antenna were separately tuned to that particular frequency, then $1/C_0\omega$ would be replaced by R_0 the resistance of the antenna tuning coil which would be of course very much less than $1/C_0\omega$, and in that case adding on receivers would have no appreciable effect on either amplitude of the voltages developed on the wave conductors, or on the tuning of the wave conductors.

This multiplex method should prove useful for broadcast reception in apartment houses, one large antenna serving as a supply source of the signal energy for all the receivers in the various apartments. It would be simply a matter of extending leads from the antenna to the different apartments each connecting to a wave conductor which is coupled to a receiver. In accordance with the discussion given here a small reduction in signal strength occurs only when all the receivers are operated on the same frequency, and the probability of any large numbers of receivers operating simultaneously on the same frequency is likely to occur only when receiving from a local station in which case the energy is more than sufficient, and the reduction in intensity would not be even noticed. For distant stations, if the receiving sets are operated on different frequencies, no appreciable effect is produced on either the tuning or strength of signals by the addition of other receivers connected through wave conductors.

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WAVE RESONANCE TUNING AND APPLICATION
TO RADIO TRANSMISSION*

By

WILLIAM R. BLAIR AND LOUIS COHEN
(Signal Corps, War Department, Washington, D. C)

IN a previous paper by one of the authors, the theory of wave resonance tuning was discussed, and methods described for the use of this principle of tuning in the reception of radio signals.

This paper has reference to methods for the utilization of wave resonance tuning in the transmission of radio signals. Several advantages are realized by this method of tuning, the most important of which are the elimination of harmonics, extreme sharpness of tuning, and a convenient and relatively simple method for multiplex transmission.

In the use of vacuum-tube transmitters, it is seldom that the transmission is effected on a single frequency, the radiated energy almost always having harmonic frequencies superimposed on the basic frequency. The listener in on broadcasting has ample evidence of it all the time. Also in communication transmitting stations there is no lack of harmonics. Theoretically it should be possible by proper care in design and operation to produce oscillations of a sine wave form of a single frequency, but practically this is seldom realized.

There are various causes which may contribute to the distortion of the wave form of the oscillations; some are inherent in the operating characteristics of the tube itself. The fact is that even with the use of master oscillators the oscillations are not altogether free from distortion. The method proposed here for the elimination of the harmonics is very simple and effective. This has been demonstrated repeatedly in the laboratory. One of the circuit arrangements used consists in interposing a wave conductor between the source of the oscillations and the antenna in the manner shown in Fig. 1. The wave conductor
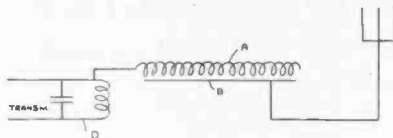
Fig. 1

A, which consists of a solenoidal coil mounted on a movable plate B, is connected to the oscillating circuit D, and the antenna is connected

to the metal plate B. The oscillations generated in D are transmitted through the wave conductor to the antenna. The wave conductor, however, in its transmission efficiency discriminates in favor of one frequency for which it is in adjustment. Assume that the oscillations produced in circuit D are of a distorted wave form comprised of fundamental frequency and harmonics, and let the wave conductor be put in adjustment for a quarter wavelength of the fundamental frequency. Then it can be shown, the theory of which was given in the previous paper referred to, that for that frequency the wave conductor will respond resonantly and serve as an efficient agency for the transfer of the fundamental oscillations to the antenna. For other frequencies if they are even harmonics of the fundamental frequency, the wave conductor will not respond at all, and even for odd harmonics only a small fraction of the energy is transmitted to the antenna. This can be readily seen from the following considerations: for a conductor of distributed inductance and capacity, a voltage applied at one end and the other end open, the voltage distribution is given by the following equation:

$$V = \frac{E\left\{e^{k(l-x)} + e^{-k(l-x)}\right\}}{e^{kl} + e^{-kl}} \tag{1}$$

where $k = \alpha + j\beta$ in the usual notation.

Since the energy transfer from wave conductor to the antenna is effected through electrostatic coupling, it is altogether a matter of voltage, and (1) is to be integrated to obtain an expression for the total voltage.

$$V_t = \int_0^l V dx = \frac{E}{k} \frac{\left[-e^{k(l-x)} + e^{-k(l-x)}\right]_0^l}{e^{kl} + e^{-kl}}, \tag{2}$$

$$= \frac{El}{kl} \frac{e^{kl} - e^{-kl}}{e^{kl} + e^{-kl}}.$$

For short conductors, we may put

$$e^{kl} + e^{-kl} = 2\cos\beta l + 2j\alpha l \sin\beta l,$$

$$e^{kl} - e^{-kl} = 2\alpha l \cos\beta l + 2j \sin\beta l.$$

Hence

$$V_t = \frac{El}{j\beta l} \frac{\left\{\alpha l \cos\beta l + j \sin\beta l\right\}}{\cos\beta l + j\alpha l \sin\beta l}. \tag{3}$$

For a frequency corresponding to a quarter wavelength, $\beta l = \pi/2$, and the above reduces

$$V_t = \frac{El}{j\pi/2\alpha l}. \tag{4}$$

Since αl is very small the voltage is great, and as a consequence efficient energy transfer to the antenna. For even harmonics $\alpha l = \pi$, 2π, 3π, etc.

$$V_t = \frac{El}{j\pi}\alpha l. \tag{5}$$

The voltage is extremely small. αl may be of the order of 0.005, and for applied voltages of the same magnitude the ratio between fundamental and even harmonic will be of the order of 25×10^{-6}, which is negligible. For odd harmonics, say $\beta l = 3\pi/2$,

$$\beta l = \frac{3\pi}{2},$$

$$V_t = \frac{El}{j3\pi/2\alpha l}.$$

It would appear that the amplitude is one-third of the fundamental, but in reality it is a great deal less because $\alpha l = Rl/2\sqrt{C/L}$, and for a frequency three times greater the resistance may be several times greater. We may therefore safely assume that for the same applied voltages the response to the harmonic would be no more than about ten per cent of that to the fundamental frequency. If the amplitude of the harmonic in the oscillation source is ten per cent that of the fundamental, the effect on the antenna would be for that harmonic only one per cent that of the fundamental, also a negligible quantity. Harmonics of higher order are even more negligible in their effects on the antenna.

It follows from the above consideration that this system offers an effective means for the elimination of harmonics, no matter what the character of the wave form generated, since oscillations of only one frequency are transmitted to the antenna. This has been fully substantiated by experimental tests in the laboratory.

The sharpness of tuning, and the consequent narrowing of the transmission band emitted by the antenna when this system of tuning is employed, is evident from the discussion given in another paper to which reference has been made above.

It is possible, on the other hand, by this method of tuning to utilize the harmonics of the oscillator as separate communication channels. All that it is necessary to do is to connect additional wave conductors to the oscillator, adjusting separately each of these additional wave conductors for one of the harmonic frequencies. Each wave conductor would be provided with a separate keying device

and suitable amplification, if desired. These wave conductors may operate either on separate antennae or on the same antenna.

It is observed that in the circuit arrangement shown in Fig. 1, the antenna proper is untuned. An improvement in the efficiency would result by tuning the antenna, but it is not altogether necessary.

A system in which the antenna is untuned immediately suggests the possibility of multiplex transmission, and this has actually been accomplished by the arrangement shown in Fig. 2. Each transmitter is separately associated with a wave conductor which is in adjustment
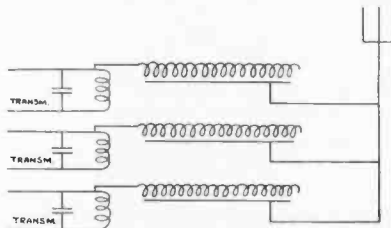
Fig. 2

for the fundamental frequency of that transmitter, and all the metal plates of the different wave conductors are connected to the antenna. It was demonstrated experimentally that there is no interaction between the different transmitters, each merely feeding energy into the antenna at its own frequency, and the current in the antenna is the sum of the currents due to the several transmitters when acting sinlgy.

The method of embodying the principle of "wave resonance" tuning described above for the elimination of harmonics and multiplex transmission is typical of many other arrangements we have used in which this principle of tuning was utilized. A more detailed discussion of the many modifications developed in the application of this method of tuning will be given at a later date.

Discussion* on
RADIO DIRECTION-FINDING BY TRANSMISSION
AND RECEPTION †

R. L. SMITH-ROSE

(Communicated in reply to previous discussion)

R. L. Smith-Rose:[1] I should like to take this opportunity of thanking the members of the Institute for the cordial reception given to my paper at the New York meeting on February 6th last. In particular, I wish to express my indebtedness to Dr. L. M. Hull for kindly presenting the paper on my behalf and for replying to various points which arose in the discussion.

Mr. Ballantine is undoubtedly correct in drawing my attention to the limitations of the reciprocal theorem, and I have noted the publication of his own and Mr. Carson's papers on this subject in the June number of the PROCEEDINGS. With regard to compensation of the rotating loop, some experiments were carried out on this at Gosport with little success due, I consider, to the unfavorable conditions at that station. It is expected that future beacons in Great Britain can be erected on much better sites.

Commander Taylor reviews the cause of errors in direction-finding due to downcoming waves and draws attention to the effect of the angle of incidence of these waves. The minimum distance at which "night" or "atmospheric" errors are experienced will undoubtedly depend upon the wave-frequency, since it is the relative strengths of the ground and atmospheric waves which is the deciding factor. The short ground waves will be attenuated more than the longer waves and so we should expect the errors to begin at shorter distances from the transmitter. I might also emphasize the fact that since, as pointed out by Mr. Dean, the available experimental evidence indicates that the ground conductivity in the United States is lower than in Great Britain, the ground waves will be attenuated more in the former country, and it is therefore to be expected that "night" errors in America will occur at shorter ranges than in Great Britain at the same frequencies. This probably accounts for Commander Taylor's experience of a 30-deg. error at six miles from a transmitter operating in the broadcast band of wavelengths. This is quite contrary to all our experience in England. The reception sensitivity of the Adcock aerial system will, of course, increase as the wavelength is reduced; but in connection with our work we have generally been able to obtain sufficient sensitivity with the aid of modern receivers. I am pleased to notice that Mr. Putnam has supported my contention as to the equality of errors experienced on CW, ICW, and spark transmissions. I am further gratified to note that the last section of my paper dealing with the very controversial subject of the relative advantages of the direction-finder and the rotating beacon in marine navigation has produced exactly the response I desired. Mr. Putnam has given me much useful information from his greater experience of the use of directional wireless in navigation. While being in close agreement with most of the points raised I think that there will still be room for the rotating beacon in providing an additional navigational service, particularly

from the smaller type of craft which would not instal a direction-finder. At the time of writing the first rotating beacon for the use of the mercantile marine has just been put into commission at Orfordness on the East coast of England.

I am indebted to Dr. Schelleng for his discussion of the behavior of directional errors with reversal of the direction of propagation in the presence of the earth's magnetic field. I trust that I did not give the impression in my paper that Fig. 14 and the last paragraph of section 4 on page 461 provided a proof of the instantaneous reversibility of night errors with a direction-finder and a rotating beacon. The remarks at the bottom of page 438 provide the necessary reservation to the application of the reciprocal theorem.

Discussion* on

MICROPHONIC IMPROVEMENT IN VACUUM TUBES†

ALAN C. ROCKWOOD AND WARREN R. FERRIS
(Research Laboratory, General Electric Company, Schenectady, New York)

M. J. Kelly:[1] I should like to ask if the filament of the UX864 is mounted with any attempt at a fixed tension.

Alan C. Rockwood:[2] Only insofar as seeing that the filament is symmetrical.

M. J. Kelly: Is it correct then that you obtain the low microphonic response described without holding to narrow limits the tension exerted on the filament by the spring at the top of the *V* of filament?

Alan C. Rockwood: Yes. The adjustment is left to the operator who adjusts the filament spring as she would for any coated filament tube with particular attention to seeing that the hook is centered on the filament. The highest disturbance occurs if an uneven filament is secured so one leg is slack. If you have a cylindrical tube high filament tension will give trouble, while with a *V* or *W* filament tube an excessively loose filament leg will cause trouble.

M. J. Kelly: Is there any evidence of a sputtering noise, somewhat similar to static and different from the microphonic noise due to variable contact between the hook of the spring and the filament?

Alan C. Rockwood: We have had no evidence of that. Such a noise might be either the result of a charge accumulating on the filament hook or of the filament sliding along the hook, but we have never found it when looking for it in this tube.

A. E. Rodd:[3] In reference to microphonic noises caused with the 227 type of tube, how about the new type with filament suspended in the center of the cathode?

Alan C. Rockwood: The microphonic characteristics of that type of 227 are no different from the old.

A. E. Rodd: The new type of cathode sleeve with the filment suspended in the center?

Alan C. Rockwood: The chief microphonic response in the 227 has been always the overall response of the whole mount assembly. Changing the interior construction of the cathode doesn't affect it. The oscillograms show the 227 has practically the lowest microphonic response of any of the small tubes with the exception of the UX864 and the $7\frac{1}{2}$-watt power tubes.

A. E. Rodd: I mean the one without it.

Alan C. Rockwood: We have tried considerable quantities of both types in broadcast receivers and find no differences between the two types.

J. E. Pristas:[4] Is the 112A type much less microphonic than the 201A type? Is there any difference?

Alan C. Rockwood: Any difference in microphonics is probably a matter of coincidence. There is a certain amount of variation for both of them. On an actual percentage basis there is little difference.

J. E. Pristas: Excuse me for differing on that. I have made a pretty accurate test in the laboratory; in practically all cases I found the 112A was less microphonic than the 201A.

Alan C. Rockwood: In testing several thousand tubes as detectors we haven't found much difference, although any variation is in favor of the coated filament tube. However, if you are referring to high gain laboratory amplifiers it is hard to predict what will happen with a certain type of tube. This will depend upon the layout and whether or not you have abnormally good tubes. It should be possible to pick out tubes of either type which are equally good if you have enough to choose from. The oscillograms [Figs. 6 and 7] show the extremes possible in the quarter ampere tube—with the poorer one being about the worst we have ever found in this structure.
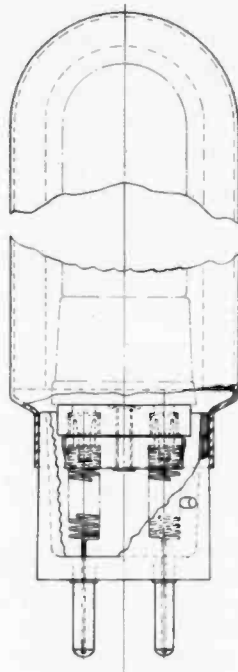
Fig. 20

Probably the best answer to your question is to say that in deciding between the UX201A and UX112A we would base a decision upon the desired electrical characteristics and not upon any differences in microphonics.

K. H. Wood:[5] I should like to ask about the 199 type tube. That is the worst offender, I believe. I wonder if any improvements were made in that type of tube.

Alan C. Rockwood: The limitation in that type of tube is the fact that economy of filament current was the primary consideration in its design and this required a small, light filament. We have to balance the microphonic re-

[5] Non-member.

sponse against the filament characteristics so the tube will be both usable and economical. Of course, improvements of detail have been made since the tube has first been introduced so as to better fit it to the service for which it was intended. However, we can't make the great improvement of the UX864 in this type as the use of a V filament implies extra end loss and loss of emission.

A. Hoyt Taylor:[6] I should like to ask Mr. Rockwood if there is any gain in any of the schemes for shielding the tube.

Alan C. Rockwood: An adapter was shown by an additional slide [Fig. 20] which is one of the best of the special shields which have been useful where abnormal demands have been placed on a tube. This was used for a UX199 detector placed directly under a dynamic speaker and represents about the extreme possible in a spring suspended socket with a surrounding acoustic shield. When you go to aircraft receivers, however, it has been found that with such an adapter you have so much motion of the tube as flexibility of the mounting is increased that you have trouble from change of tuning and of feedback before acoustic isolation is reached. These troubles limit the amount you can go in acoustic shielding.

A. Hoyt Taylor: I should like to point out that while discussion by the authors of the paper has confined itself largely to the interest of broadcast receivers and airplane receivers, there are two other fields in which the suppression of microphonic disturbance of tubes is of very great importance.

One of these is the high-frequency receiver, particularly the receiver for extremely high frequency. It is well known by all those who have had experience with such receivers that the microphonic troubles in tubes increase very rapidly with the frequency as far as their disastrous effects upon reception is concerned. In other words, tubes that will be perfectly all right, say at 1000 kc, are no good at all at 20,000 kc because of the extreme microphonic response and the disturbance of the circuit condition since we are primarily interested in CW reception where one of the tubes at least is oscillating.

The development of suitable tubes for very high frequency with a far higher degree of improvement of the microphonic properties than perhaps required even by aircraft work is therefore of great practical interest for those engaged in the development of such receivers.

There is one other field where the reduction of microphonic noises is also of very great interest and importance. That is in the power tube field. I have recently had some experience with a transmitter or several transmitters which used a master circuit followed by a balanced power amplifier circuit in working at frequency between 14,000 and 30,000 kc.

The circuit did not use crystal control. The small disturbances which in this case reflect on the frequency rather than volume or output would occur. But with the self oscillating master circuit of a special type (in fact it was the oscillator known as the Gunn oscillator) we found that while the master oscillator alone gave an excellent quality signal of very small frequency ripple, yet when we put a 750-watt balanced power amplifier outfit on top of this, we got a tremendous ripple in frequency.

Tracing this down we found this was entirely due to a microphonic disturbance originating (from the plate principally) in the power amplifier tubes. It was due to small variations in the effective capacity of the circuit which was loaded on to the power amplifier reacting back and influencing the frequency of the master circuit.

[6] Naval Research Laboratory, Bellevue, Anacostia, D. C.

We do not overlook any opportunities to learn more about the frequency control, and the crystal is not the only way of getting pretty constant frequencies. I have recently had the opportunity of examining a rather new type of transmitter tube in which an attempt was made to suppress as far as possible the microphonic disturbance in the shielded transmitter tube for the purpose of using it in such special circuits which did not use piezo-electric or equivalent control. There appear to be many possibilities of great improvement in transmitter tubes as well as receiver tubes in this matter of microphonic disturbances.

I think possibly one of the most important fields for it is in the aviation field. I spent four years in that work developing aviation radio, and ten years ago we were experimenting with a sort of hectocomb unit, a unit of six tubes consisting of three stages of radio frequency with a detector and two stages of audio. We copied this outfit more or less completely from a British wartime airplane receiver equipment, and we had to find a tube that had the proper constants to go with these circuits. We had to use a tube that is no longer produced in this country. I won't name it, but I think many of you can guess who made it at the time. They were very microphonic, but very non-uniform and I can remember on many an airplane flight we would test as many as twenty-four tubes before we got a set of six tubes we could wear with the earphones with the helmets on, without the microphonic disturbance pretty nearly driving us crazy. That shows the importance of reducing microphonic disturbance from the standpoint of aviation.

That is particularly true in dealing with receivers of high amplification as we were then, because this particular work centered about radio compass work that was done preparatory to the flight of the NC-4 to Europe across the Atlantic.

I feel that if anyone can improve very materially the microphonic properties of tubes for aviation service, they will have conferred a great benefit upon humanity.

J. E. Pristas: In reference to the screen-grid tube, how does this type compare with the three-element tubes?

Alan C. Rockwood: The microphonic response of a screen-grid tube is about the same as any three-element tube of the same general type. The three-electrode detector gives the chief trouble because of the amplification following it. Similarly, the microphonic disturbance of the screen-grid tube is largely a question of how much gain you are going to have between the detector and the output tube.

Lee Sutherlin[7]: I should like to make one statement with respect to the microphonic study which it seems to me may bear on two or three questions asked here. In making the study of a lot of tubes, I found in general that the microphonic noise of a tube of a given current and voltage with a ribbon type filament is less. Of course, that would be a coated filament.

That may be due to the fact that it is a ribbon and vibrates only in one plane, and, of course, it may be due to the difference in materials. In many cases I have found that difference. It seems to me this bears on some of the questions that were asked.

Alan C. Rockwood: That is one of the things we have never run into, although it is logical to expect this in the heavier current filaments. For the lower current filaments it is hard to say whether any difference, as compared to tungsten filaments, is due to the cross-section or the greater mass in a single leg of a coated filament having the same rating.

[7] Westinghouse Electric and Manufacturing Co., East Pittsburgh, Penna.

BOOK REVIEW

Radio Encyclopedia †—Encyclopedic dictionary of radioelectricity, by Michel Adam, engineer, E. S. E. One volume of VI—355 pages,[1] in quarto, with 1480 illustrations, 150 abbreviations and symbols, two plates of schematic symbols, 19 tables and five maps, published by Etienne Chiron, 40 rue de Seine, Paris, 1928. Price 61 French francs.

Although there are already several dictionaries and vocabularies of radioelectricity, in French and other languages, Mr. Adam's work is opportune. He condenses, in a practical and easily accessible form, technical, technological, and philological knowledge. The encyclopedia is in two parts:

I. Alphabetical index of abbreviations and symbols:

II. Encyclopedia of technical terms.

The encyclopedia, as it has been rightly termed, contains the names, definition, explanation, and an analytical description of all terms used in radioelectricity. It gives related terms of electricity and elementary mechanics and groups the different units of French, English and international measure used frequently in this subject. Moreover, the English and German translation of each term is given, this being absolutely necessary for all those who wish to follow the constant progress of the French and English scientific literature of radioelectricity. Numerous drawings and illustrations complete and make more interesting the literary and technical documentation, while facilitating the investigations of the novice as well as the works of the professional. The special alphabetical index contains all the French, English and German, and sometimes even Italian and Spanish abbreviations for radioelectricity with the exception of purely telegraph abbreviations, which interest only radiotelegraph operators and are treated in special works.

Thus, the "Encyclopedia of Radio," which answers now a real need of French, scientific literature, will certainly be welcomed favorably by Administrations, as well as by the public, both technicians and laymen.

† Translated from *Journal Télégraphique*, 53, No. 5, 113–114; May, 1929,
[1] Translator's Note: VI—355 refers to six introductory and 355 text pages.

BOOKS RECEIVED

The Physical Principles of Wireless, by J. A. Ratcliffe, Fellow and lecturer of Sidney Sussex College, Cambridge University Demonstrator in the Cavendish Laboratory. Published by E. P. Dutton and Company, Inc., 1929. 104 pages, illustrated, $4 \times 6\frac{1}{2}$ inches, cloth binding. Price, $1.15. Contents: oscillatory circuits; valves; wireless transmitters; reception of wireless signals; wireless telephony; amplifiers; miscellaneous.

Spectra, by R. C. Johnson, lecturer in physics in the University of London, Kings College, and O. W. Richardson, Yarrow research professor of the Royal Society. Published by E. P. Dutton and Company, Inc., 104 pages, illustrated, $4 \times 6\frac{1}{2}$ inches. Contents: the quantum theory; line spectra; band spectra; spectroscopy; appendix.

ABC of Television, by Raymond Francis Yates, formerly editor of *Popular Radio*. Published by The Norman W. Henley Publishing Company, 1929. 210 pages, illustrated, $5\frac{1}{2} \times 8\frac{1}{2}$ inches, cloth binding. Price, $3.00. Contents: television—the new conquest of space; television systems; telegraphing pictures;

photoelectric cells—the eyes of television; amplifying pictures; the agile neon lamp; selenium cells; the problem of scanning; synchronizing television; transmitting television at home; how to make a television receiver.

Mechanical Characteristics of Transmission Lines, by Seiichiro Noda of the Ryojun College of Engineering, Port Arthur, South Manchuria. Published as Vol. II, No. 2 of the Memoirs of that institution. 155 pages, illustrated, charts and drawings, $7\frac{1}{2}$ x $10\frac{1}{2}$ inches, paper cover. Contents: general solution, law of geometric progression, breaking of a wire in a span of continuous wire-spans, partial change in load on continuous wire-spans, failure of a suspension insulator in continuous wire-spans.

MONTHLY LIST OF
REFERENCES TO CURRENT RADIO LITERATURE

THIS is a monthly list of references prepared by the Bureau of Standards and is intended to cover the more important papers of interest to professional radio engineers which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the scheme presented in "A Decimal Classification of Radio Subjects—An Extension of the Dewey System," Bureau of Standards Circular No. 138, a copy of which may be obtained for 10 cents from the Superintendent of Documents, Government Printing Office, Washington, D. C. The various articles listed below are not obtainable from the Government. The periodicals can be secured from their publishers and can be consulted at large public libraries.

R100. RADIO PRINCIPLES

R100   Morecroft, J. H. Elements of radio communication (book). John Wiley and Sons, Inc., 1929. New York, N. Y.

(A simple text book of radio introductory to the more comprehensive text "Principles of Radio Communication" by the same author.)

R112.1   Strutt, M. J. O. Strahlung von Antennen unter dem Einfluss der Erdbodeneigenschaften; (a) elektrische Antennen, (b) magnetische Antennen. (Radiation from antennas and effects of the earth's influence). *Annalen der Physik*, pp. 721–772; 1929.

(A thorough discussion of the radiation characteristics of both horizontal and vertical antennas especially of the doublet type.)

R113   Yokoyama, E. and Nakai, T. East-West and North-South attenuations of long radio waves on the Pacific. PROC. I. R. E., **17**, pp. 1240–1247; July, 1929.

(A comparative study of the observed values of field intensity of low-frequency high-powered stations in the Pacific region seems to show a much greater east-west than north-south attenuation during daylight hours in fairly high latitudes. A comparison of observed values with values calculated by various transmission formulas indicates the need for the inclusion in the formulas of a factor depending on direction and latitude.)

R113.4   Ponte, M. and Rocard, Y. Sur la couche ionisee de la haute atmosphere. (On the ionized layer of the upper atmosphere.) *L'Onde Electrique*, **8**, pp. 179–191; May, 1929.

(In problems concerning the conductivity of the Heaviside layer the mean free path of the electrons is a relevant quantity. An analysis based on the kinetic theory in which the forces of interaction between the molecules and the electrons are considered shows value of the mean free path to be from 40 to 160 times as small as value obtained by the elementary theory.)

R113.5   Hulburt, E. O. On the ultra-violet light theory of aurorae and magnetic storms. *Phys. Review*, **34**, pp. 344–351; July 15, 1929.

(Certain details of the ultra-violet light theory of auroras and magnetic storms are developed and experimental facts which are in accord with the theory are cited.)

R113.5   Wymore, I. J. The relation of radio propagation to disturbances in terrestrial magnetism. PROC. I. R. E. **17**, pp. 1206–1213; July, 1929.

(The results of a study of the relationship between radio reception and the changes in the earth's magnetism show that for daylight reception over great distances there is an increase in the intensity of received signal which reaches its maximum in from one to two days and disappears in from four to five days after a magnetic storm.)

R113.6   Pedersen, P. O. Long delayed radio echoes. *Nature* (London), **124**, p. 164; July 27, 1929.

(Article taken from paper "Wireless Echoes of Long Delay" communicated to the Physical Section of Danish Royal Society. Shows mathematically that radio echoes occurring after 10 seconds cannot be due to propagation of waves within the earth's atmosphere, that echoes occurring after intervals up to 30 seconds are due to propagation along or reflection from Störmer bands as explained in *Nature* (122, p. 681, 1928), that echoes after several minutes must be from outside the space in which the earth's magnetic field exerts appreciable effect. Transmissions at various wavelengths are also treated.)

R113.6   Discussion on "The attenuation of wireless waves over land." *Jour. I. E. E.*, **67**, p. 931; July, 1929.

(Discussion between C. R. Englund and R. H. Barfield of certain applications of optical theory in Barfield's paper, "The attenuation of wireless waves over land.")

R113.6   The effect of the earth on short wave radiation from vertical and horizontal aerials (editorial). *Experimental Wireless and W. Engr.* (London), **6**, pp. 351–352; July, 1929.

(Results obtained by M. J. O. Strutt and given in a paper in the *Annalen der Physik*, p. 721, 1929 are discussed.)

R113.7   Burstyn, W. Die unmittelbare Messungen von Entfernungen durch elektrische Wellen. (Direct measurements of distances by means of electric waves). *Zeits. fur Hochfrequenztechnik*, **33**, pp. 181–183; May, 1929.

(Formulas and curves showing the relationship between intensity of reception and distance from the transmitting set are given with a discussion of their accuracy. This method of measuring distances is believed invaluable for ships in foggy weather.)

R114   Schindelhauer, F. Elektromagnetische Störungen. (Electromagnetic disturbances). *Elektrische-Nachrichten Technik*, **6**, pp. 231–236; June, 1929.

(A study of electromagnetic disturbances in the atmosphere and results of experiments carried on at the Meteorological-Magnetic Observatory at Potsdam, Germany.)

R114   Paoloni, D. B. and Ilardi, G. P. Compte rendu des observations radioatmospheriques faites pendant l'annee 1927. (An account of radio atmospheric observations made in 1927). *L'Onde Electrique*, **8**, pp. 222–226; May, 1929.

(Atmospherics are classed according to the Paoloni radio atmospheric scale. Observations for each class were taken daily during 1927 by au aural system. Observations are summarized in curves and general statements.)

R114   Dean, S. W. Correlation of directional observations of atmospherics with weather phenomena. PROC. I. R. E., **17**, pp. 1185–91; July, 1929.

(Description of the results obtained with a cathode-ray direction finder used by the A. T. and T. Co. at Houlton, Me. Direction of storms was determined with a considerable degree of accuracy.)

R114   Harper, A. E. Some measurements on the directional distribution of static. PROC. I. R. E., **17**, pp. 1214–1224; July, 1929.

(The utility of directional data on static is shown, and two types of apparatus for directional investigation are compared. A method which gives direction of individual crashes is found superior to the integrating methods. Distribution of thunderstorms over the world is discussed. Probable geographic locations are assigned to disturbances.)

R125.6   Stenzel, H. Uber die Richtcharakteristik von einer Ebene angeordneten Strahlern. (The directional characteristic of a polarized radiator). *Elektrische-Nachrichten-Technik*, **6**, pp. 165–181; May, 1929.

(Complete technical discussion of directional characteristics of polarized radio waves.)

R132   von Ardenne, M. Die aperiodische Verstärkung von Rundfunkwellen.
(Aperiodic amplification at broadcast frequencies.) *Zeits. fur Hoch-
frequenztechnik*, **33**, pp. 166–175; May, 1929.

(The problem of aperiodic cascade amplification is thoroughly discussed. Several
methods of a-f and r-f amplification are given, and are discussed theoretically.)

R133   Kirschstein, **F.** Zur Theorie der rückgekoppelten Röhrensenders.
(Theory of back-coupled tube transmitters). *Zeits. f. Hochfrequenz-
technik*, **33**, pp. 201–211; June, 1929.

(Graphical and integral calculus methods are used to explain the functioning of feed-
back tube generators).

R134   Ballantine, S. Detection at high signal voltages. Part 1. Plate
rectification with the high-vacuum triode. PROC. I. R. E., **17**, pp.
1153–1177; July, 1929.

(Article contains a descriptive theory, a mathematical theory for small modulation and
large carrier voltages, an apparatus setup and results, and practical applications of
reception at high voltages.)

R138   MacNabb, V. C. The production of emission from oxide-coated
filaments. *Jour.* Optical Soc. of America and Rev. of Scientific
Instruments, **19**, pp. 33–41; July, 1929.

(Results of investigation of various methods for producing oxide filaments of barium
and strontium in an emitting condition.)

R140   Watanabe, Y. Theorie des gekoppelten Schwingungskreises mit
Selbsterregung. (Theory of self-excited inductively coupled oscilla-
tory circuits). *Elektrische-Nachrichten Technik*, **6**, pp. 194–210; May,
1929.

(Theory of oscillatory circuits from a mathematical viewpoint.)

R145.3 Grover, F. W. A comparison of the formulas for the calculation of the
inductance of coils and spirals wound with wire of large cross section.
Bureau of Standards Journal of Research, July, 1929. Research Paper
No. 90. Obtainable from the Superintendent of Documents, Govern-
ment Printing Office, Washington, D. C.

(Two methods for calculating the inductance of coils of wire having a relatively large
cross section are given.)

R170   Dieckmann, M. Beitrag zur Beschreibung des Interferenzgebietes
in der Nahe von Empfangs Antennen. (Discussion on interference
regions in the neighborhood of receiving antennas). *Zeits. fur
Hochfrequenztechnik*, **33**, pp. 161–166; May, 1929.

(By making simplified assumptions the interference regions in the vicinity of re-
ceiving antennas are discussed, and pictured by models and drawings. Instantaneous as
well as average values of field intensity are given.)

R200. RADIO MEASUREMENTS AND STANDARDIZATION

R201.5 Ferguson, J. G. Shielding in high frequency measurements. *Bell
Sys. Tech. Jour.*, **8**, pp. 560–590; July, 1929.

(The purpose and usefulness of shielding for high-frequency measurements are out-
lined. General principles of electrostatic shielding are developed as applied to simple
impedances and to networks of impedances, particularly to bridge networks. Partial
applications of these principles to the shielding of adjustable impedances and in the
construction of actual bridge circuits are described.)

R201.6   Oatley, B. A. A low power audio-frequency current supply for general laboratory use. *Jour. Scientific Instruments* (London), 6, p. 217; July, 1929.

(A description is given of apparatus for providing alternating current of constant frequency and good wave form for use in bridge measurements. The apparatus derives its power from the 100-volt d-c line. Arrangements are provided whereby several observers can use the apparatus simultaneously without mutual interference.)

R202   Kallmann, H. E. Rechteckige Verformung von Resonanzkurven nach einem neuen Prinzip und ihre Anwendung beim Empfang sehr kurzer Wellen. (A new principle in the interpretation of resonance curves and its application to short-wave reception). *Zeits. fur Hochfrequenztechnik*, 33, pp. 212–223; June, 1929.

(The sensitivity and selectivity of short-wave (of the order of .30 cm) receiving sets is thoroughly discussed; also a new method for improving both.)

R210   Giebe, E. and Scheibe, A. Internationale Vergleichung von Frequenznormalen fur elektrische Schwingungen. (International comparisons of frequency standards). *Zeits. fur Hochfrequenztechnik*, 33, pp. 176–180; May, 1929.

(Piezo oscillator from the Bureau of Standards and piezo resonators from Germany were tested by the laboratories in Germany, Italy, France, England, and United States and found to vary within ± 1 part in 25,000.)

R210   Bogardus, H. L. and Manning, C. T. The routine measurement of the operating frequencies of broadcast stations. Proc. I. R. E., 17, pp. 1225–39; July, 1929.

(The method of making "zero beat" measurements of the operating frequencies of broadcasting stations in the Second Radio District is described, showing the method of comparing received signal from a broadcasting station with a signal of known frequency obtained from a 10-kc multivibrator controlled by a 90-kc quartz crystal.)

R210   Marrison, W. A. A high precision standard of frequency. Proc. I. R. E., 17, pp. 1103–1122; July, 1929. Also in *Bell Sys. Tech. Jour.*, 8, pp. 493–514; July, 1929.

(A new standard of frequency is described in which 100,000-cycle quartz crystal-controlled oscillators of very high constancy are employed. These are inter-checked automatically and continuously with a precision of about one part in one hundred million. They are checked daily in terms of radio time signals by the usual method employing a clock controlled by current maintained at a submultiple of the crystal frequency. Specially shaped crystals are used which have been adjusted to have temperature coefficients less than 0.0001 per cent per degree C.)

R214   Lack, F. R. Observations on modes of vibration and temperature coefficients of quartz crystal plates. Proc. I. R. E., 17, pp. 1123–1141; July, 1929. Also in *Bell Sys. Tech. Jour.*, 8, pp. 515–535; July, 1929.

(The characteristics of piezo-electric quartz crystal plates of the perpendicular or Curie cut are compared with 30-degree or parallel-cut plates with reference to the type of vibration of the most active modes, the frequency of these modes as a function of the dimensions, and the magnitude and sign of the temperature coefficients of these frequencies. The relation of various dimensional cuts to the temperature coefficient is discussed. The analysis offers an explanation of the low temperature coefficients which can be produced by a proper choice of the dimensional ratios.)

R270   Austin, L. W. Experiments in recording radio signal intensity. Proc. I. R. E., 17, pp. 1192–1205; July, 1929.

(The method of recording intensity of long-wave radio signals used at the Bureau of Standards and some of the results obtained are given. Variability of wave propagation in regard to field intensity and angle of incidence is shown in curves. An apparent connection between night signal variation and magnetic storms is shown. The downcoming waves seem to be reflected from rapidly changing masses of ionized gas.)

R275   Lamb, J. J. The modulometer. *QST*, 13, pp. 8–15; Aug., 1929.

(A simple device for measuring the percentage of modulation and generally checking the performance of the phone transmitter is described. Modulometer is essentially the adaptation of the electron-tube peak voltmeter for modulation measurements as previously outlined by C. B. Jolliffe.)

R300. RADIO APPARATUS AND EQUIPMENT

R320   Meissner, A. Transmitting antennas for broadcasting. PROC. I. R. E.,
17, pp. 1178–1184; July, 1929.

(Fundamental requirements for best broadcast aerials are stated. Aerials having a height of λ/2 are found to produce less upward and greater horizontal radiation than aerials having a height of λ/4, and consequently fading due to interference of indirect with ground waves is greatly reduced. Comparative efficiency of vertical and horizontal transmitting antennae at different frequencies, is presented on basis of limited tests.)

R330.4   Bedeau, F. and DeMare, J. Etude de la methode de Beatty pour la
mesure de l'amplification d'un etage a resonance. (Study on the
Beatty method of measuring the amplification of a resonant stage).
L'Onde Electrique, 8, pp. 210–211; May, 1929.

(The method of Beatty (*Experimental Wireless and W. Engr.*, 5, January, 1928) for calculating the overall amplification of a resonant stage from the tube design. Circuit constants are discussed. The method involving vectorial analysis is shown useful in design.)

R331   Bartels, H. Uber die Maximalleistungen von Schutznetzleistungroh-
ren. (On the maximum output of screen-grid tubes). *Elektrische-
Nachrichten Technik*, 6, pp. 188–193; May, 1929.

(A comparison of screen-grid with ordinary tubes, bringing out the advantages of the former.)

R342.15   Dijksterhuis, P. R. and Groeneveld, Y. B. F. J. Low-frequency
amplification with transformers. *Experimental Wireless and W. Engr.*
(London), 6, pp. 374–379; July, 1929.

(It is claimed that the problems of low-frequency amplification by transformers are met in the design of the Philip's transformer (Dutch). A nickel-iron alloy core is employed to give a high primary self-inductance independent of amplitude of the voltages to be amplified. A high resistance nickel-alloy wire in the secondary coil is used to prevent a leakage resonance peak in the characteristic curve. A special silver alloy is used in the primary winding to give low resistance and high mechanical strength.)

R344.3   Watanabe, Y. Uber den Zwischenkreisrohrensender mit stark ge-
dampften Sekundarkreis. (On the filter circuit of tube transmitting
sets with strongly damped secondary circuit). *Elektrische-Nach-
richten Technik*, 6, pp. 244–248; June, 1929.

(Mathematical analysis of this type of transmitting set giving equations and results of experimental tests.)

R360   Von Ardenne, M. Receiver with aperiodic high-frequency amplifica-
tion. *Experimental Wireless and Wireless Engr.*, 6, pp. 369–373; July,
1929.

(Loewe double triodes designed to give a high aperiodic radio-frequency amplification and a very low audio-frequency amplification are used in cascade in a long range receiver. The triodes have a high input resistance and permit great selectivity in the tuned stages. Intercoupling is prevented through shielding and the use of chokes and by-pass condensers.)

R376.3   Greaves, F. F., Kranz, F. W., Crozier, W. D. The Kyle condenser
loud speaker. PROC. I. R. E., 17, pp. 1142–1152; July, 1929.

(Describes a loud speaker developed on the Kyle principle of construction with a view to providing comparatively large actuating force and also sufficient amplitude of vibration to permit efficient and faithful reproduction.)

R376.3   Clarke, H. M. The moving coil loudspeaker. *Experimental Wireless
and Wireless Engr.* (London), 6, pp. 380–384; July, 1929.

(A compensation winding, reducing hysteresis, eddy currents and self induction is offered as the solution of the problem of response control of the moving coil loud speaker. By this winding the impedance of the moving coil is made practically independent of frequency, and by the supplementary use of shunting filters the natural resonance peaks of the speaker may be smoothed to give a desirable response curve.)

R386   Farrar, C. L. Band pass tuning. *Radio Engineering*, 9, pp. 27–30;
July, 1929.

(Reduction of sideband cutting in r-f amplifiers by use of band pass filters.)

R500. APPLICATIONS OF RADIO

R512   Ludenia, W. Reichweitenversuche mit Zentimeterwellen. (Distance
            range tests with centimeter waves). *Elektrische-Nachrichten Technik,*
            6, pp. 248–249; June, 1929.

> (A short discussion of the use of wavelengths from 10 to 100 cm. Their application is
> believed most practical where sharp narrow beams are necessary, e.g., for fog signals
> and lighthouse stations.)

R526.1   Diamond, H. and Dunmore, F. W. A course-shift indicator for the
            double-modulation type radiobeacon. Bureau of Standards Journal
            of Research, July, 1929. Research Paper No. 77. Obtainable from
            Superintendent of Documents, Government Printing Office, Washing-
            ton, D. C.

> (To further increase reliability of the visual direction beacon system developed by the
> Bureau of Standards, a course-shift indicating instrument is described which is primarily
> for station use and indicates to the operator whether the course remains unvarying during
> operation and also aids in checking of beacon calibration.)

R800. NON-RADIO SUBJECTS

621.313.7   Demontvignier, M. Les redresseurs a oxyde de cuivre. (Copper-
            oxide rectifiers). *L'Onde Electrique*, 8, pp. 192–209; May, 1929.

> (A contact of copper and copper-oxide possesses rectifying properties. The use of
> these properties in commercial a-c rectifiers is described. An analysis with theoretical
> curves and actual oscillograms showing the characteristics of the input and output is
> given. The application of the rectifiers to radio circuits is summarized.)

621.385   Fuhrer, R. Energieverhaltnisse in Ortsnetzen mit Selbstanschluss-
            betrieb. (Energy relations on local automatic telephone networks).
            Elektrische-Nachrichten Technik, 6, pp. 213–230; June, 1929.

> (Extensive discussion of the energy relations in automatic telephone networks with
> eference to the new Munich installation. Bibliography is given.)

CONTRIBUTORS TO THIS ISSUE

Blair, William R.: Born November 7, 1874 in County Derry, Ireland. Came to America in 1884. Graduated Kansas State Normal School, 1895. Received B. S. degree, University of Chicago, 1904; Ph D. degree, 1906. Entered govt. service, 1906; research director in charge upper air research, U.S. Weather Bureau, 1907; Physical Laboratory, 1910; executive office charge Mt. Weather Obs., Va., 1912—1914; professor meteorology, U. S. Weather Bureau since 1915; com. maj. N. A., 1917, and assigned to Aviation Corps with A. E. F. in France; transferred to signal corps, 1918; lt. col., 1919; maj. regular army, 1921; chief of research and engineering division, office of chief signal officer, Washington, since 1927; member technical subcommittee, Inter. Commn. for Aerial Navigation, Inter. Wireless Commn., Paris, 1919; meteorologist for world flight, 1924; member Inter. Radio Conf., Washington, 1927. Non-member of the Institute of Radio Engineers.

Breit, G.: See PROCEEDINGS for September, 1929.

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Harris Sylvan: See PROCEEDINGS for March, 1929.

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Lange, Edward H.: See PROCEEDINGS for June, 1929.

Myers, John A.: Born March 29, 1905 at Baltimore, Maryland. Received B. S. degree, Johns Hopkins University, 1927. Graduate student in electrical engineering, Johns Hopkins University, 1927 to date. With Westinghouse Electric and Manufacturing Co. during several past summers.

Pedersen, P. O.: Born June 19, 1874 in Denmark. Graduated in civil engineering, Royal Technical College, Copenhagen, 1897. Awarded the gold medal of the Royal Danish Society of Sciences. Assistant professor, Royal Technical College, 1909; professor in telegraphy, telephony, and radio, Royal Technical College, 1912 to date; principal of the college, 1922 to date. Member, Royal Danish Society of Sciences, 1917; member, State Control Board for Licensed Telephone Companies of Denmark, 1917, and president since 1920; president, Danish section of the International Electric Committee, 1926 to date; awarded the H. C. Orsted medal, 1927. Received Ph. D. degree, University of Copenhagen, 1929. Fellow, Institute of Radio Engineers, 1915, and member of several other national and international societies.

Sreenivasan, K.: Born 1899. Graduated in 1920. Apprentice in the hydro-electric department of the Mysore government, 1920—1922. Studied electrical engineering at the Indian Institute of Science, Bangalore, 1922—1924. Radio-frequency and field strength measurements, 1924 to date. Awarded the Associate-ship of the Indian Institute of Science, 1927. Non-member of the Institute of Radio Engineers.

Turner, H. M.: Born July 20, 1882 at Hillsboro, Illinois. Received B. S. degree, University of Illinois, 1910; M. S. degree, 1915; assistant instructor in electrical engineering, 1910—1912; instructor, University of Minnesota, 1912—1918; assistant professor, Yale University, 1918—1926; associate professor, 1926 to date. Frequent contributor to leading engineering journals. Associate member, Institute of Radio Engineers, 1914; Member, 1920; member of several other societies.

Tuve, M. A.: See PROCEEDINGS for September, 1929.

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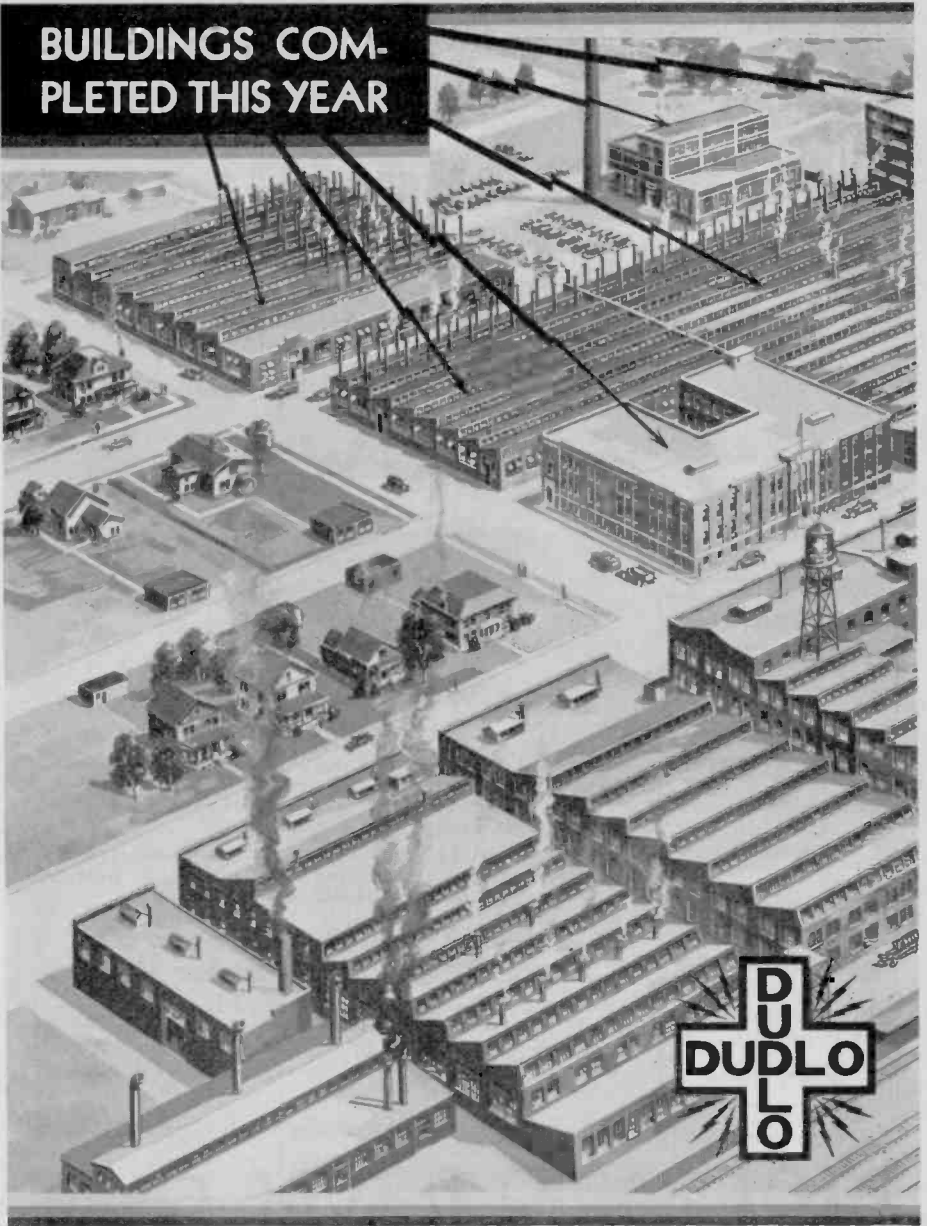
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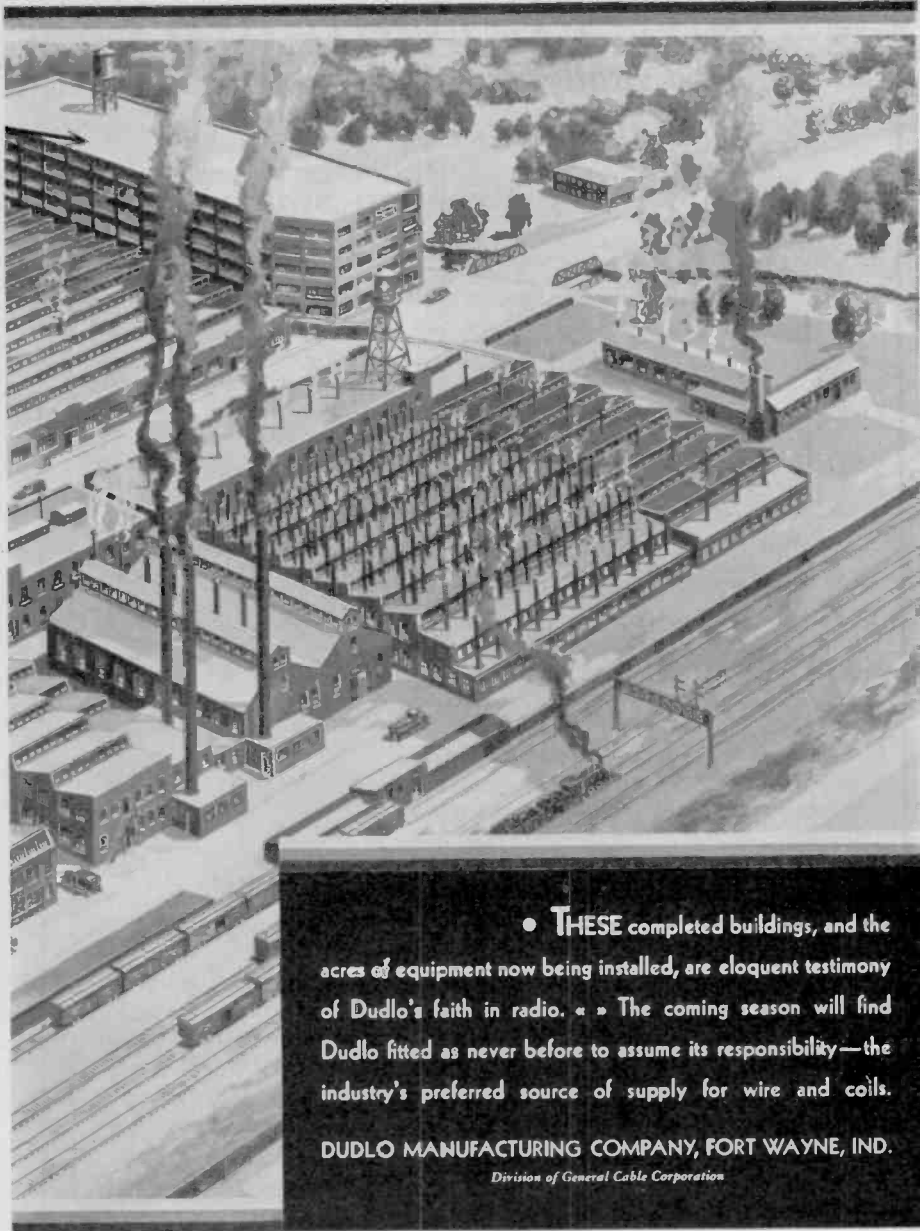
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3. Outlet Box Lock Washer.

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The following extracts from the Constitution govern applications for admission in the Institute in the Associate grade:

ARTICLE II—MEMBERSHIP

Sec. 1: The membership of the Institute shall consist of: * * * (d) Associates, who shall be entitled to all the rights and privileges of the Institute except the right to hold the office of President, Vice-president and Editor. * * *

Sec. 5: An Associate shall be not less than twenty-one years of age and shall be: (a) A radio engineer by profession; (b) A teacher of radio subjects; (c) A person who is interested in and connected with the study or application of radio science or the radio arts.

ARTICLE III—ADMISSION

Sec. 2: * * * Applicants shall give references to members of the Institute as follows: * * * for the grade of Associate, to five Fellows, Members, or Associates; * * * Each application for admission * * * shall embody a concise statement, with dates, of the candidate's training and experience.

The requirements of the foregoing paragraph may be waived in whole or in part where the application is for Associate grade. An applicant who is so situated as not to be personally known to the required number of members may supply the names of non-members who are personally familiar with his radio interest.

RECORD OF TRAINING AND PROFESSIONAL EXPERIENCE

1 Name...

<div style="text-align:center">(Give full name, last name first)</div>

2 Present Occupation..

<div style="text-align:center">(Title and Name of concern)</div>

3 Permanent Home Address...

4 Business Address...

5 Place of Birth.................. Date of Birth.............. Age.......

6 Education..

7 Degree...

<div style="text-align:center">(college)   (date received)</div>

8 Training and Professional experience to date...........................
NOTE: 1. Give location and dates. 2. In applying for admission to the grade
of Associate, give briefly record of radio experience and present employment.

DATES HERE |

9 Specialty, if any..

Receipt Acknowledged............ Elected............ Deferred...........
Grade.......... Advised of Election........ This Record Filed...........

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by GEORGE LEWIS
Vice-President
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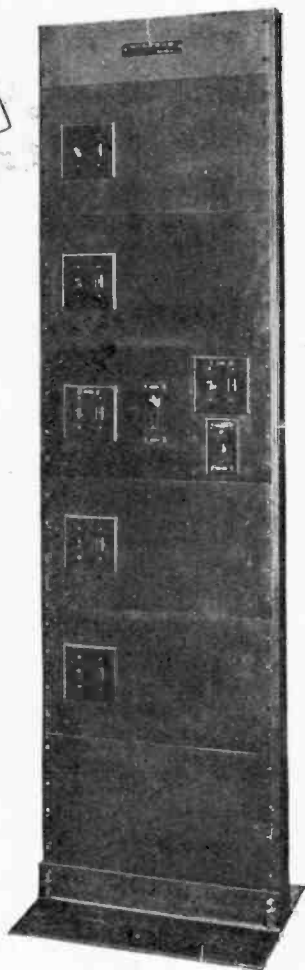
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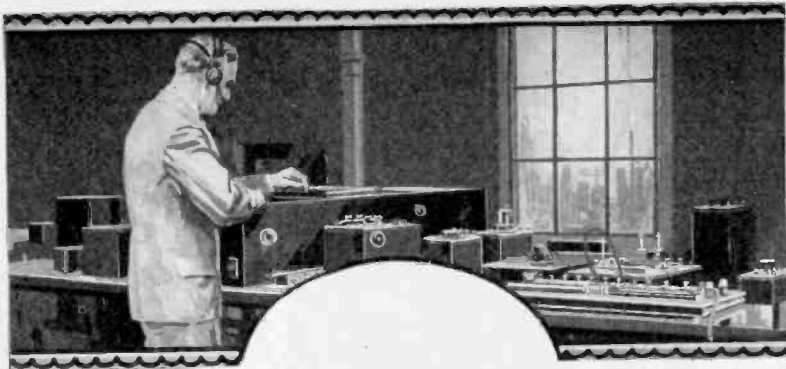
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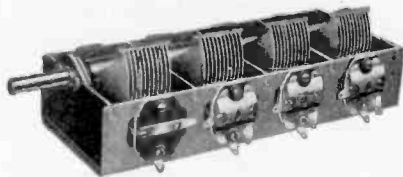
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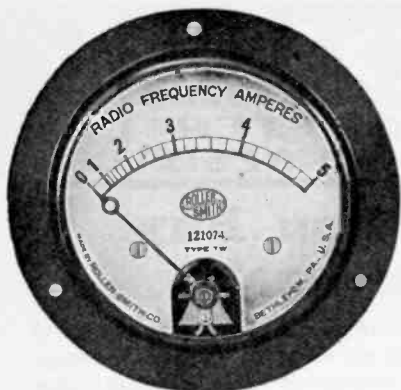
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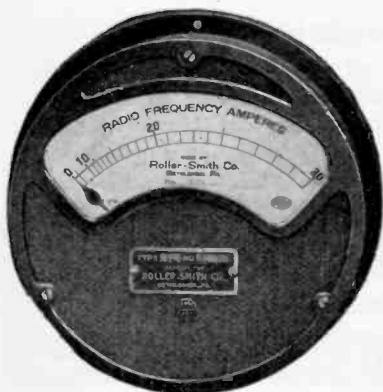
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Further, the plate current in the 227 runs anywhere from 50% to 100% more than it does with the 224. The amount of plate current drawn, of course, has a direct bearing on the life of the tube and it would therefore appear that the 224 should in reality last between 25% and 50% longer than the 227.

It is unfortunate that some man-ufacturers attempted to get into a heavy production of alternating current screen grid tubes overnight without having the background of long experience and research in this direction. The inevitable result followed, namely that the market was well covered with a motley assortment of screen grid tubes having a wide variation of characteristics and very little excuse for existence. Some of these tubes bore the name of manufacturers who had previously been favorably known and this condition undoubtedly gave rise to the belief that a screen grid tube was not practical and had a very short useful life.

As we see it, there is absolutely no logical reason why the alternating current screen grid tube should not last fully as long, if not longer, than the 227, provided the type 224 tube is made by manufacturers who know what they are doing.

—Engineering Department

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