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LONG DISTANCE RADIO RECEIVING MEASUREMENTS AT THE BUREAU OF STANDARDS IN 1923*

By
L. W. Austin

(Chief, Radio Physical Laboratory, Bureau of Standards, Washington District of Columbia)

(Communication from the International Union for Scientific Radio Telegraphy)

The results of the signal intensity and atmospheric disturbance measurements for 1923 are shown in the tables of monthly averages. During the past year changes have been made in the wave frequency of Lafayette, and at the same time the antenna at Nauen has been rebuilt so that its antenna current and radiation height have been at times uncertain. These two facts make any comparison with the results of the receiving measurements of the previous year somewhat unsatisfactory. The radiation height of Nauen is now given as 170 meters and the current as 270 amperes, as compared with 150 meters and 380 amperes in 1922. This represents a net decrease of about 25 percent in meter-amperes. Notwithstanding the irregularities in transmitting conditions, the average strength of the Nauen signals for the year is only slightly less than in 1922.

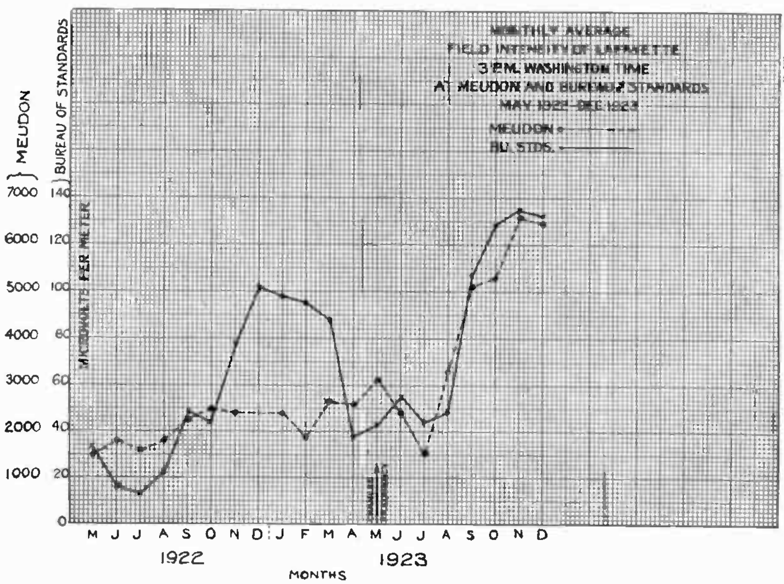
The most remarkable thing to be noticed in connection with the measurements of 1923 is the great increase in signal intensity which has accompanied Lafayette's change in frequency about May 1st, from 12.8 kc. (23,400 m.) to 16.2 kc. (18,500 m.). Thus far no explanation of the increase in the intensity has been offered. Its truth, however, cannot be doubted as the high readings have been noted not only at the Bureau of Standards but also at other American laboratories and at the French army experimental laboratory at Meudon[1] near Paris. The daily records at Lafayette show that there has been practically no change in average antenna current due to the change in frequency, the extremes for 1923 being approximately 400 amperes and 540 amperes with

*Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce. Received by the Editor March 13, 1924.

[1] "Onde Electrique," November 25, page 46. 1924

an average of 475 amperes. Such a large increase in strength cannot be due to the suppression of harmonics, nor has there been any change in the antenna system. If the effect were supposed to be due to a change in antenna radiation height, assuming that the height measurement of 170 meters made in 1921 was correct, the present radiation height, as estimated from the increase in intensity observed at the Bureau of Standards and at Meudon, would have to be more than 60 meters above the tops of the towers.

The following figure shows the comparative trend of the monthly averages of the measurements made on Lafayette's URSI signals at 3 P. M., Washington time, at Meudon near Paris, and at the Bureau of Standards. The noticeable thing in the curves aside from the difference in summer and winter values in Washington due to the afternoon fading is the lack of agreement between the Paris and Washington observations before Lafayette's change of wave length and the remarkable agreement shown since that change.



Another fact to be noted in connection with Lafayette's change of frequency is the decrease in its summer afternoon fading at Washington. The unusual weakness of Nauen in the forenoon in December is also worthy of mention. This is probably connected with the time of the European sunset and was noticeable, tho in a lesser degree, in other European stations measured. It has been noticed before in the case of Nauen, but in former years the very weak period has more commonly come in November than in December. The European sunset in summer comes in the middle of the afternoon, Washington time, gradually shifting back until during the shortest days of winter the sun sets in Nauen before 10 A. M., Washington time. Observations taken at various times of day thru the different seasons clearly indicate the shift of the weak periods of the European stations, with the changing sunset hour.

In addition to the measurements of Nauen and Lafayette, daily observations were begun in May, 1923, on the Radio Corporation station at Bolinas, California, and on the new French high-power station at Ste. Assise, near Paris. The latter station has a double Marconi type antenna, each half of which is of the same height and length as the antenna at Lafayette, and is so arranged that the two parts can be used simultaneously, one with the call letters UFT employing a frequency of 20.8 kc. ($\lambda = 14{,}400$ m.), and the other UFU with a frequency of 15.0 kc. ($\lambda = 19{,}900$ m.). The observations on Bolinas were discontinued at the end of October on account of extensive repairs and alterations which were being made there, but have now been taken up again.

The frequencies and the approximate antenna currents of the different stations are given in Table III.

As the difficulties of reception in America are almost entirely confined to the summer months, the averages for this period have been collected in Table IV. The degree of fading of the stations, that is, the ratio of the 3 P. M. signal to the 9 A. M. signal is shown in column 5 of the table. Here Lafayette shows the least fading, the ratio of afternoon to morning signal being 0.48. This is an improvement over the value for 1922 in the proportion of 0.48 to 0.35. The Nauen fading is slightly worse than in the year before, being 0.184 against 0.198. It is interesting to note that the fading on the two Ste. Assise frequencies is nearly the same, notwithstanding the considerable difference in frequency, which may indicate that the very marked difference between the fading of Lafayette and Nauen may depend not on the difference of wave length as was surmised last year, but rather on situation or other causes not yet known. The fading on all stations in 1923 continued considerably later than in 1922.

It is worthy of note that Bolinas, tho of nearly the same frequency as Ste. Assise (UFT) has a much more favorable afternoon fading ratio in Washington, which may be due partly to the difference in conditions along the signal path, and partly to the fact that the sunset effect in California does not coincide in time with the local afternoon absorption around Washington, while the two effects do coincide in the case of Ste. Assise.

The European sunset fading as observed in America is not by any means a daily phenomenon, but occurs irregularly tho often on a number of successive days. It seems to be most noticeable near the time of the longest and the shortest days.

A comparison of the measurements made at the Bureau of Standards with observations made on European stations close to the seacoast indicate that a considerable part of the afternoon fading in Washington is due to absorption taking place within two hundred miles(320 km.).

As has been said before, all conclusions drawn from observations on radio transmission muct be considered strictly tentative until the phenomena have been verified by observations extending over a number of years.

The atmospheric disturbances in the summer of 1923 were less violent than in 1922, but began considerably earlier and continued much later than in that year.

Radio Physical Laboratory,
Bureau of Standards,
March 4, 1924.

TABLE I

MONTHLY AVERAGES OF SIGNAL INTENSITIES FOR NAUEN AND LAFAYETTE AND ATMOSPHERIC DISTURBANCES IN MICROVOLTS PER METER

1923	Nauen POZ				Lafayette LY			
	A. M.		P. M.		A. M.		P. M.	
	Sig.	Dist.	Sig.	Dist.	Sig.	Dist.	Sig.	Dist.
January	22.3	9	29.6	12	....	19	98.5	25
February	21.2	6	26.3	10	82.9	12	95.8	20
March	34.3	15	30.5	26	81.7	27	89.0	48
April	35.5	63	15.0	152	....	122	38.5	244
May	27.4	97	3.8	306	69.3	129	41.4	370
June	25.3	57	6.3	219	113	77	56.5	383
July	35.2	41	5.5	197	132	56	41.9	232
August	27.8	41	5.0	217	108	49	48.9	255
September	46.2	39	7.9	197	159	46	105	211
October	37.4	23	20.8	78	166	29	129	95
November	24.1	52	20.8	88	146	55	136	95
December	7.7	23	20.2	40	118	29	132	44
Average	28.7	38.8	16.0	129	107	54.2	84.4	168

TABLE II

MONTHLY AVERAGES OF SIGNAL INTENSITIES FOR BOLINAS AND STE. ASSISE IN MICROVOLTS PER METER

1923	Bolinas		Ste. Assise			
	KET		UFT		UFU	
	A. M.	P. M.	A. M.	P. M.	A. M.	P. M.
May	49.8	28.8	43.2	14.2	....	....
June	53.6	21.9	52.3	18.4	101.4	27.0
July	69.4	19.0	56.9	13.4	92.0	24.7
August	55.7	22.0	49.9	10.7	91.4	17.4
September	78.0	32.0	61.9	13.0	112.0	43.4
October	68.8	53.4	52.5	39.0	84.0	66.1
November	....	....	49.4	43.7	109.0	73.2
December	....	....	24.9	33.8	42.0	68.4
Averages	62.5	29.5	48.9	23.2	90.2	45.8

TABLE III

FREQUENCIES AND APPROXIMATE ANTENNA CURRENTS—1923

	Bolinas	Nauen	Ste. Assise UFT	Ste. Assise UFU	Lafayette
Frequency kc.	22.9	23.4	20.8	15.0	15.9
Wave length, m.	13,100	12,800	14,400	19,000	18,900
Antenna current, amp.	420	270	380	475	475

TABLE IV

SIGNAL AVERAGES AND DISTURBANCES DURING AFTERNOON FADING SEASON, JUNE TO SEPTEMBER, 1923

	A. M. Sig.	A. M. Dist.	P. M. Sig.	P. M. Dist.	Sig. P.M. / Sig. A.M	Sig./Dist. A.M.	Sig./Dist. P.M.
Nauen	33.6	44.5	6.2	207	0.18	0.92	0.030
Bolinas	64.2	45.0	23.7	208	.38	1.42	.114
Ste. Assise UFT	55.3	44.6	13.8	206	.25	1.24	.067
Ste. Assise UFU	100.5	56.9	28.1	271	.27	1.85	.103
Lafayette	129	56.9	62.3	271	.48	2.27	.230

Department of Commerce,
Washington, D. C.

SUMMARY: A summary is given of the signal intensities and strength of atmospheric disturbances at Washington from Nauen, Lafayette, Bolinas, and Ste. Assise stations during 1923.

NOTE: It has been decided to discontinue the bi-monthly publication of the daily observations at the Bureau of Standards on European signals and in atmospheric disturbances, but these will be furnished in mimeographed form to any one making application for them.

DISTRIBUTION OF RADIO WAVES FROM BROADCASTING STATIONS OVER CITY DISTRICTS*

By

Ralph Bown

and

G. D. Gillett

(Department of Development and Research, American Telephone and Telegraph Company, New York)

With the advent of popular interest in radio broadcasting many of the peculiarities of radio transmission, previously known only to the radio engineer and the radio amateur, have become matters of common experience and broadened interest. The broadcast listener who goes in for receiving distant stations has found that the difference in transmission by day and by night, the enormous fluctuations in night transmission usually termed fading, together with widely varying conditions of natural and man-made interference, lend to his pursuit the sporting aspects of a game of chance. The listener interested in receiving from local stations has learned that two equally powerful stations equally distant from him but in different directions do not necessarily give equal signal strength in his receiver, and again that his friend who lives in a different part of the city gets quite different results. Many of these variations are chargeable to the use of different receiving sets the sensitivity of which varies with frequency in different ways. But after these factors have been cancelled out the fact remains that at short distances from the broadcasting station such that time variations of transmission are inappreciable, or within what might be called the service area of the station, there are space variations or inequalities of distribution which seem to depend on the physical character of the landscape.

There are thus distinguished two kinds of transmission variations; time variations or fading, which are the outstanding characteristics at long distances, and space variations or inequalities of distribution, which are most easily distinguished at short dis-

*Received by the Editor, January 8, 1924. Presented before The Institute of Radio Engineers, New York, January 16, 1924.

tances where they are not complicated by the presence of time variations. This paper is concerned only with space variations. These are a permanent characteristic of the transmission and lend themselves readily to quantitative studies. To the engineer, a study of irregularities of distribution is valuable not only in disclosing scientific information on radio transmission, but also in providing an empirical basis for the design of radio broascasting distribution systems. We have endeavored to set forth below the results of just such a study of the conditions in the urban and suburban districts of New York City and the City of Washington, District of Columbia.

Theoretical Considerations

Before considering specific cases, it is worth while to discuss very briefly just what the natural obstacles are which lie in the way of theoretically perfect broadcasting distribution.

Looking upon radio broadcasting as a means for distributing intelligence-bearing energy over a given territory, the most desirable result would be to produce the same field strength of radio waves at all points in the territory and to produce little or no disturbance outside that area. This would enable a given receiving set to be used with equal satisfaction anywhere in the territory of the station. Unfortunately radio transmission from a single station is ill adapted to afford such a result. It is by nature a means for throwing out a large amount of energy at one point and allowing it to spread out, diminishing rapidly at first and then more gradually, until finally at considerable distance it becomes unstable or too attenuated to be discernible.

Not only is the field strength reduced with increasing distance on account of the spreading of the energy over a larger and larger circle, but it is also reduced in addition by the fact that some of the radiated energy is transformed into heat by electrical losses in the transmission media and therefore lost to radio uses.

If the surface of the earth were flat and of uniform electrical characteristics, the falling off in field strength of the waves due to spreading of the energy and losses in the earth might be expressed as a law in mathematical form. Several such expressions have been developed from experimental data and are useful in many cases. In the usual case, however, the earth's surface is irregular in form or material and corresponding transmission irregularities are introduced. The characteristics of the earth's surface which affect radio transmission may be roughly classified under three headings as follows:

1. Areas of different electrical constants—fresh water, salt water, dry land, wet land, rock, snow, and so on.
2. Differences of elevation—hills, valleys, mountains, and the like.
3. Absorbing structures—man-made buildings, towers, or other structures many of which have resonance characteristics producing selective absorption.

It is almost hopeless to expect that we shall ever be able properly to take account of such varied factors as these in any transmission formula. We may, however, thru the collection of experimental data hope to obtain a fairly clear picture of how they affect transmission and be able to make allowances for them in engineering problems and calculations.

Experimental Methods

Fortunately the means for taking experimental data of this kind are already at hand in the form of radio field-strength measuring sets of the kind described in a recent paper before The Institute of Radio Engineers on "Radio Transmission Measurements," by Bown, Englund, and Friis.[1] One of the small portable type of short wave measuring sets described in that paper was used in obtaining the data herewith presented. In order to be able to cover ground rapidly with this outfit, it was transported and used in an automobile as shown in Figure 1.

In our measurements at New York and Washington, schedules were arranged with certain stations so that a carrier frequency of constant amplitude would be sent out while observations were being taken. During some of the periods this carrier was modulated or, more strictly speaking, two intelligence-bearing side bands were transmitted in addition to the constant carrier. This was immaterial from the measuring standpoint, because the measuring set measures substantially only the carrier.

It is obvious that taking any considerable number of points to the square mile would be very laborious. The data contained in this paper represent the results of about two months of field work during which the automobile carrying the measuring set traveled 3,000 miles (4,800 kilometers). An endeavor was made, therefore, to reduce the amount of measuring work by taking the observations at points selected in advance as most likely to show

[1] "Proceedings of The Institute of Radio Engineers," volume 11, 1923, number 2, page 115.

up the effects being investigated. In many places the data are very sketchy, but they serve as a guide in picturing phenomena which are more accurately delineated in other sections where the data are more complete.

The observations in each city group were made during daylight hours, during the same season of the year, and in most instances under uniform weather conditions. Relatively, the measured values of field strength are believed to be correct within a few percent. The absolute accuracy of the values does not enter into this discussion, so the possible 10 or 15 percent constant error in the calibration of the measuring set used is of no direct

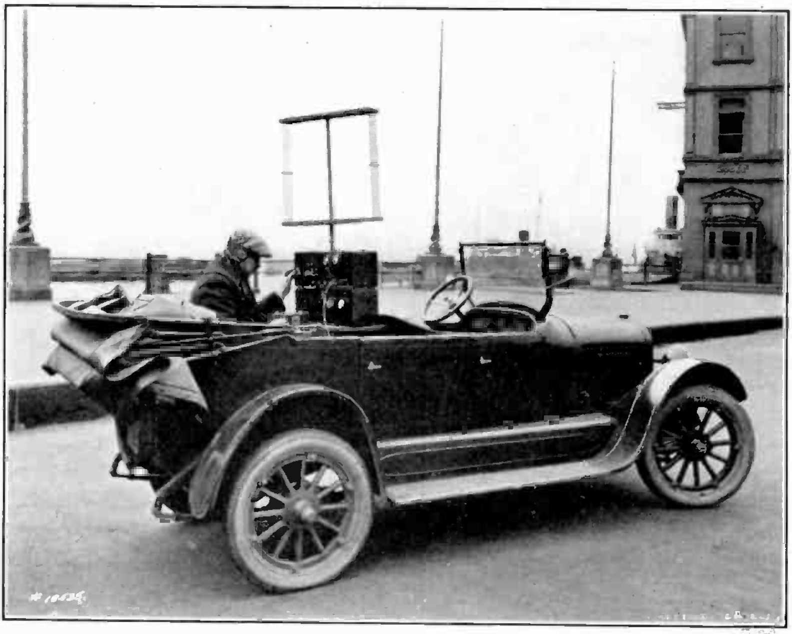


FIGURE 1

importance. Where observations were made in settled areas, as open locations as possible were chosen, for instance, in parks, or squares and in the centers of wide streets. Such points tend to represent general conditions as distinct from the very local conditions which may obtain on the roofs of or inside of particular buildings. No doubt if the study were carried to sufficient refinement, each building or group of buildings would be found to have its own little distribution pattern.

In order to facilitate an orderly consideration of the data, we have presented them below in such sequence as to bring out, first, the separate effects of the three classes of transmission factors previously mentioned, and, second, the complications into which combinations of the factors lead.

Open Country Transmission

The first measurements to be considered were made around Washington, District of Columbia, in the summer of 1923. The broadcasting station WCAP of the Chesapeake and Potomac Telephone Company was used as a transmitting station for the tests. The transmitting frequency was 640 kilocycles (wave length 469 meters). As shown on the map, Figure 2, observations were taken on two lines leading out of the city, one to the southeast and the other to the northwest. The points at which

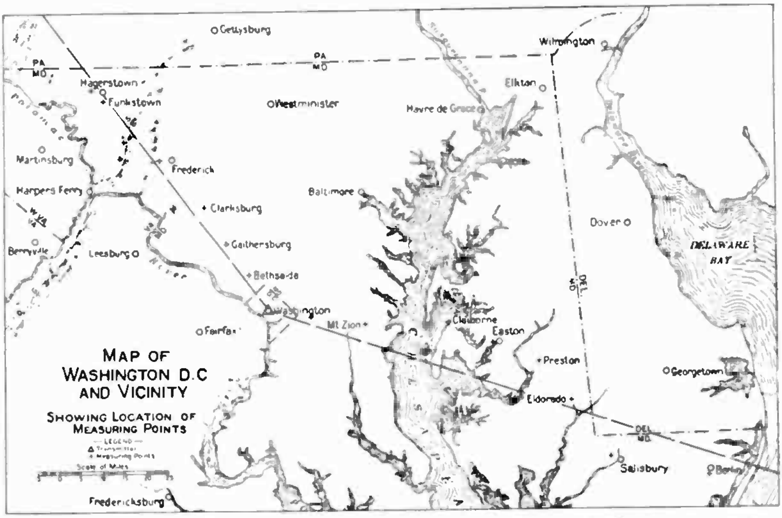


FIGURE 2—Map of Washington, District of Columbia, and Vicinity showing Location of Measuring Points

measurements were made are marked. In Figure 3 are given curves showing plots of field strength against the distance from the transmitting station. The smooth upper curve *A* shows the falling off which would be obtained under the ideal condition of simple spreading, the field strength being inversely proportional to the distance. The calculation of the ideal curve requires an assumption or measurement of the effective height of the transmitting antenna. In the present case, the effective height was calculated from field strength measurements made at a distance of about eight wave lengths from the transmitting station.[2]

Curve *B* is for the southeast line. A profile of the land and water corresponding to this line is given just below at *B'*. It starts out over rolling country and the field drops rapidly. About 25 miles (40 km.) out it strikes Chesapeake Bay, and the next 30 miles (48 km.) over water is characterized by a flattening of the field strength curve, due to lowered attenuation. Then, going over land again, the curve drops more rapidly. The alternating steep and flat portions bring out clearly the effect of the different electrical characteristics of the land and brackish water over which the radio waves pass.

Curve *C*, with its corresponding profile of the landscape *C'*, is for the northwest line, which is entirely over land. The land increases in elevation and irregularity as the distance from the city increases. The field strength falls off at first much the same as in Curve *B*. Farther out, as the foothills of the Blue Ridge Mountains are reached, the rate of falling off increases until a low point is registered in the shadow of a mountain range.

A simple theory suffices to explain this shadow effect. If the ground is considered to be a fairly good conductor, the penetration of the waves will not be great, and there will be but little transmission directly thru the mountain. As the waves pass the crest and the ground falls away, the bottom of the wave front is stretched and the unit field strength reduced. Simultaneously the wave front is given a forward tilt. Since the flow of energy is perpendicular to the wave front, this means that energy is being fed down from above. The effect of the feeding down of energy is gradually to erase the shadow, as is brought out by the rising of Curve *C* after the mountain has been well passed. What occurs is probably very similar to the diffraction of light, altho it should be noted that the dimensions of the obstructions are small relative to the wave length. It is probable that if a hundred or more points had been taken, closely spaced along this northwest line, it would be continually rising up and down in minor undulations reflecting the shadows of the smaller hills and mountains. The seven points shown serve to give a general indication of the average rate of attenuation and to make more prominent the one large shadow caused by a natural land elevation.

[2] For a brief discussion of this method of determining the effective height, see paper on "Radio Transmission Measurements," by Bown, Englund, and Friis, "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," volume 11, number 2, page 117

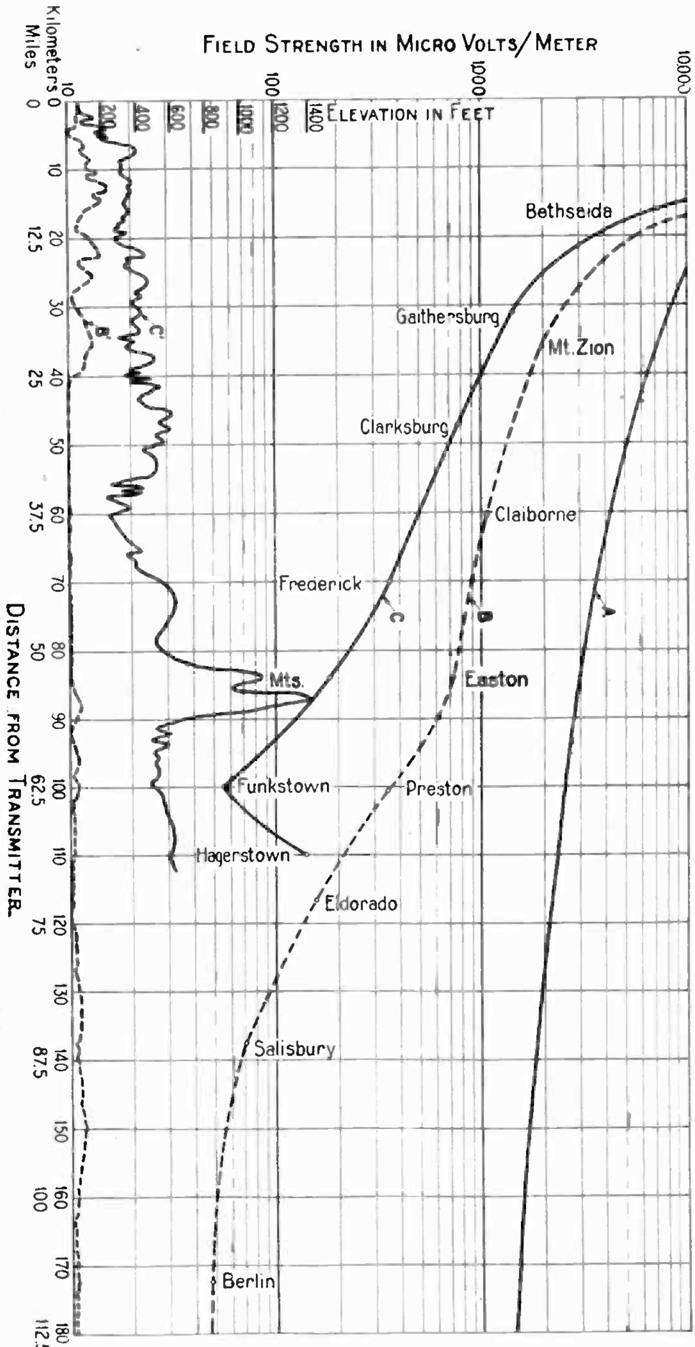


FIGURE 3—Plot showing Effect of Topography on Radio Transmission

An effort was made to classify the data taken in the country districts outside the city of Washington along the two radial lines just discussed and along other radial lines in other directions, with the object of obtaining an attenuation factor or formula by means of which the transmission over the soil of Maryland and Virginia could be calculated. The wide scattering of the values caused this to be abandoned as impractical. During the analysis there was calculated for each individual point the attenuation factor α which, inserted in the familiar transmission formula, caused it to give the correct field strength for that particular point. The formula is

$$\text{Field strength} = 377 \frac{h_s I_s}{\lambda d} \varepsilon^{-\frac{\alpha d}{\sqrt{\lambda}}} \text{ micro-volts per meter,}$$

where $h_s I_s$ is the antenna radiation constant in meter-amperes, and λ and d are the wave length and distance, respectively, both in kilometers. The values of α obtained by this process ran all the way from 0.012 to 0.035, with a rough average at about 0.022.

One method of analysis applied to the curves of Figure 3 consisted in fitting attenuation factors to sections of the curves which are uniform and correspond to transmission over uniform territory. For instance, the attenuation factor for the section of Curve *B* between Preston and Salisbury is 0.028. Thoe lan is low but dry and sandy. The rolling upland of more mist soild from Gaithersburg out to Frederick (Curve *C*) has an attenuation factor of 0.009. The over-water transmission of Curve *B* between Mt. Zion and Easton shows a factor of 0.0025. This is relatively quite close to the 0.0015 factor of the Austin-Cohen formula which seems to hold well for daylight transmission over sea water thru a fairly wide frequency range. A simple calculation discloses that at a distance of 100 kilometers (62 miles) from a 750-kilocycle (400-meter) transmitting station, the sea water factor (0.0015) gives a received field strength about seventy times as great as that given by the dry sand factor (0.028). This strikingly illustrates the wide variation in the attenuations caused by different surface conditions.

City Transmission

In cities, where large steel buildings and similar structures enter as factors in transmission, the situation is much the same as over open country except that the effects are more intense and of smaller dimensions. For instance, the fact that heavy shadows can be cast is well illustrated by the curves in Figure 5.

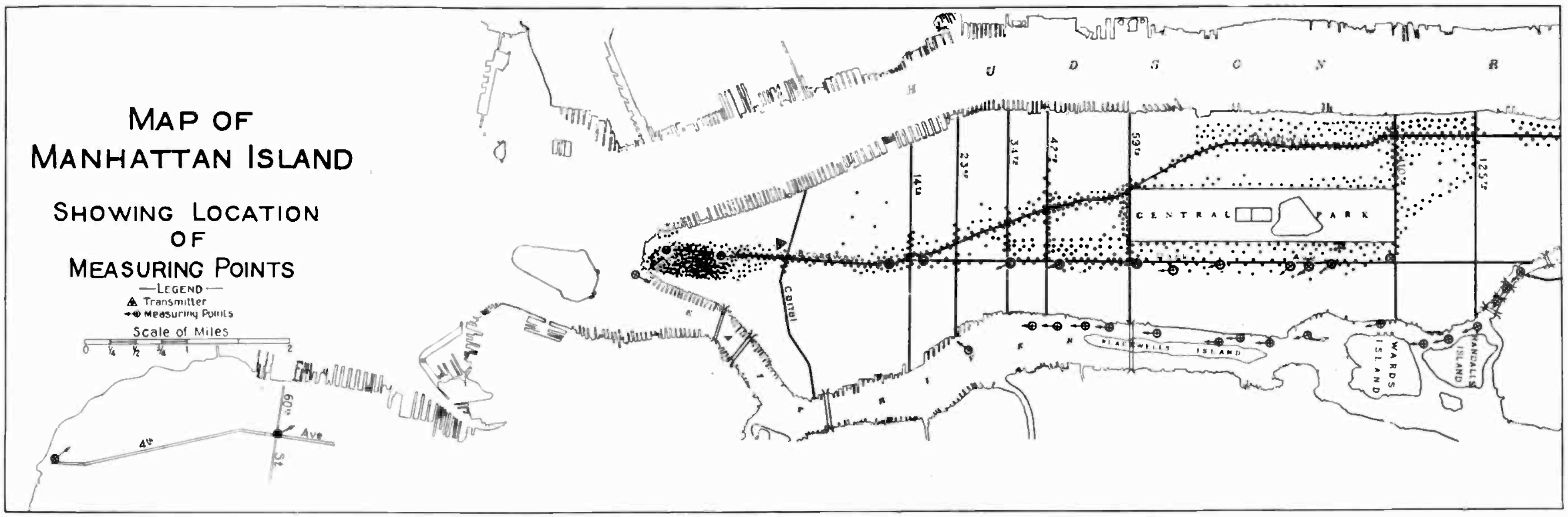


Figure 4—Map of Manhattan showing Location of Measuring Points

This figure, together with the map in Figure 4, gives the results of some measurements taken in New York City. The map shows by circled arrows the locations of points at which measurements were taken and by stipple shading indicates congested regions of high buildings. The transmitting station was the American Telephone and Telegraph Company's experimental station 2XY, located at 24 Walker Street, about one mile (1.6 km.) north of the heavy congestion of steel skyscrapers near the tip of Manhattan Island. The transmitting frequency was 610 kilocycles (wave length 492 meters). The measurements were taken during the fall of 1923. At that time the station was also carrying a radio broadcasting schedule as WEAF. In Figure 5, Curve *A*, as before, shows the simple spreading law. The effective height used in drawing Curve *A* was calculated from two field strength measurements taken at a distance of about seven wave lengths in the Hudson River where a clear line to the station was obtainable. Curve *B* shows the attenuation on a line running southerly thru Battery Park and down toward Fort Hamilton. The first point, taken in City Hall Park, starts an abysmal drop which ends in the third point taken on a boat just off the Battery. That this heavy shadow is soon erased by the feeding in of energy from above and the sides is indicated by the dotted extension of the curve running out thru two points on Long Island near the water's edge. At the farthest point, the transmission has so far recovered itself as to be almost up to the ideal curve. While this heavy shadow can be explained by the same reasoning as was used to explain that behind the Blue Ridge Mountains, it should be noted that the skyscraper area is not only a great hill of steel lattices, but that this hill is divided by the deep criss-crossing cuts of the streets into a group of lattice towers or pillars. Many of these pillars have natural electrical oscillation frequencies near the frequency of transmission and so are excellent absorbers of the radio waves. This doubtless contributes materially to the sharpness and depth of the shadows.

These shadows, drops, and flat spots in the transmission curves, while sufficiently outstanding to be rather startling and to constitute definite limitations on the effectiveness of radio transmission, looked at from another standpoint, merely serve to expose the puny size of landscape features in comparison with the vast overhead space thru which the radio waves are propagated. The wave fronts, rising to great heights, proceed forward, for hundreds of wave lengths at least, with an irresistible

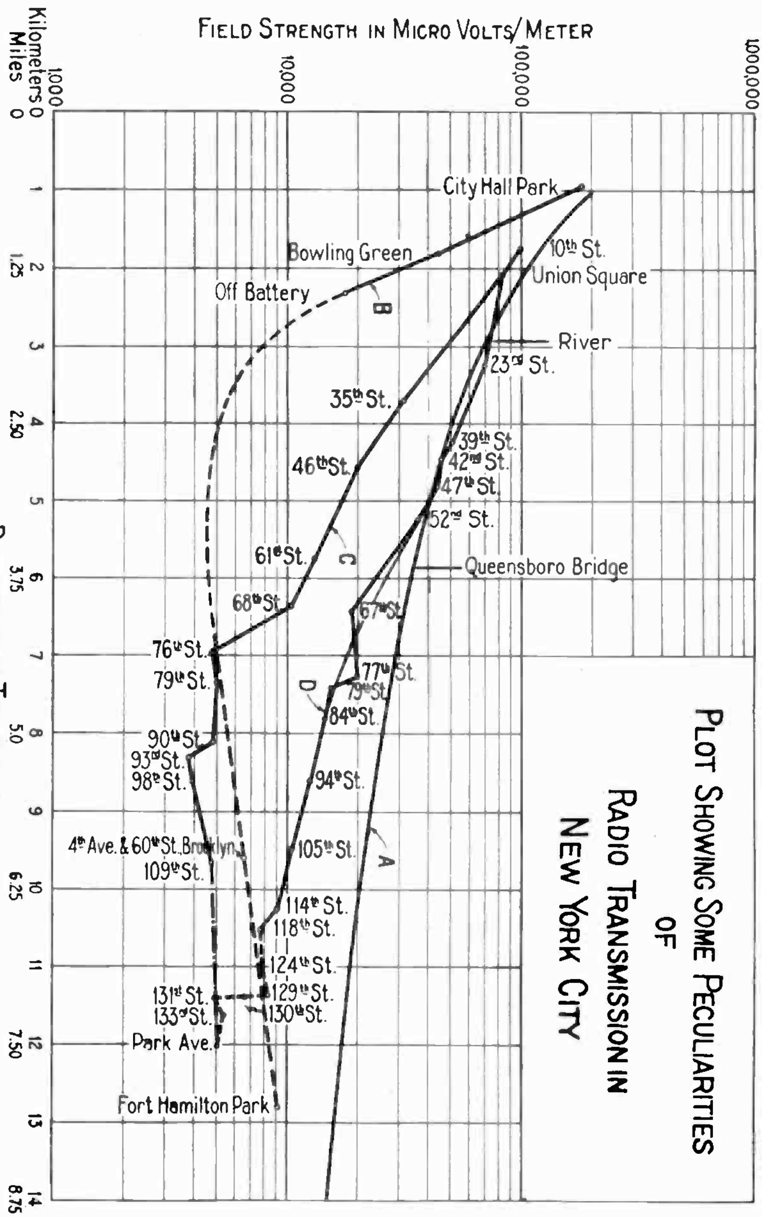


Figure 5—Plot showing Some Peculiarities of Radio Transmission in New York

sweep. At their feet the earth sometimes tends to drag them down, and sometimes casts obstacles in their way, but the gaps are quickly healed as the greater store of energy above is partially directed downward

The way in which this self-healing action occurs is brought forward strikingly in the case of transmission northward along Manhattan Island. In Figure 5, Curve *C* gives measurements taken on a line running northward along Park Avenue. The first point at Union Square is just in front of a group of fairly large buildings, the effect of which is registered in a sharp dropping of the field strength. Beyond this, running up the center of Park Avenue, the drop is more uniform and less rapid. The high attenuation of the buildings along the avenue effectively prevents any appreciable healing up of the initial drop. At about 68th Street, a group of big apartment buildings and the crest of a slight hill cause another irregularity. The rest of this curve can best be considered in conjunction with Curve *D*, which is for a line of points up the East River. The river points were taken on the upper deck of a small steamer. The field strength holds up well until the Queensboro Bridge is approached. The low point, and the subsequent recovery to a smooth curve, account for the shadow caused by the bridge. The fact that the field strength over the river is much larger than that over the adjacent city causes a continual feeding in of energy from the river to the land. This is manifested in three ways:—(1) the recovery and flattening out of Curve *C* as the action becomes pronounced, (2) the fact that Curve *D* shows a higher attenuation than for clear over-water transmission, and (3) the direction of wave arrival on land, as indicated by the arrows on the map, is deflected away from the transmitting station and toward the river. The definite swinging round of the waves at the boundary line of two areas of different characterictics is closely analogous to the refraction of light. At its outer end Curve *D* is shown dotted because here the points depart radically from a radial line to the station and follow up the Harlem River to the place where the two lines cross and the curves come together.

Having thus seen with a considerable degree of clearness how the impediments to transmission operate in certain outstanding cases, and having a considerable number of measured points in and about New York and Washington, we have had the temerity to draw what may be called radio field-strength contour maps of the two cities. It must be admitted frankly that these maps are at best only an approximation, and that in many areas the data is too meagre to do more than vaguely support the contour lines. But a careful inspection will usually disclose a surprising agreement between the local territorial features and the values of the measured points. as to where the contour lines should lie.

On the maps the measuring locations are indicated by crosses. Where loop direction was taken the cross is elongated into an arrow denoting the apparent direction of wave arrival. The value of field strength in milli-volts per meter is written beside the cross or arrow. The contour interval is not uniform thruout, but the value of each contour in milli-volts per meter is indicated in the usual way by a number inserted in a short gap in the line.

Figure 6 shows one of these maps embracing only the island of Manhattan. An outstanding characteristic is the relation of the contour lines to the Park Avenue and East River measurements discussed above. In the river, the contours bend rapidly inward toward the island until at some places on land in the northern part of the island the contours are actually parallel to the radial line from the transmitting station.

This does not mean that the wave fronts are necessarily traveling perpendicular to their starting direction; in fact, there is some doubt as to the meaning of the term "wave front" in such a case where there must be a conglomeration of reflected, refracted, diffracted, and re-radiated elements. The direction from which the energy is apparently arriving is, however, given by the loop position for maximum received current. The arrival directions, as was said above, favor the river. They also tend toward perpendicularity to the contour lines. In places where the wave travel is smooth and unobstructed, this becomes more strictly true—the wave fronts and the contours coincide, and the loop direction is perpendicular to both.

On the west side of the city, the same heavy attenuation on land and the same feeding in from the river is present as on the east side; in fact, it is even more pronounced at some points. At a few points, such for instance as at Columbus Circle, the loop direction does not follow the general trend. In most of these cases there was evidence of heavy local distortion of the waves due to the presence of a nearby building which was resonant at the transmitting frequency used.

As might be anticipated from the foregoing discussion, Central Park, surrounded by the city on all sides, has a low area or dead spot. Several contours close on themselves within this area and the field strength at one point drops to a value as low as it has 30 miles (48 km.) or more out in the country.

Another low point is indicated near Spuyten Duyvil. This one lies down in a valley behind a hill, and is probably due to the shadow of the hill.

In order to bring out more clearly how the various types and

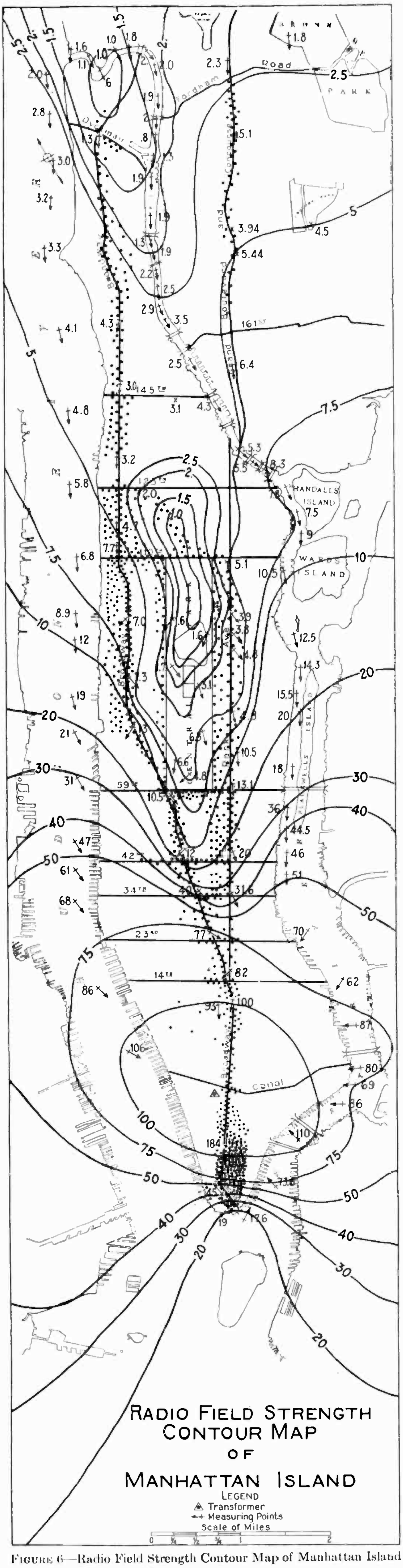


FIGURE 6—Radio Field Strength Contour Map of Manhattan Island
(Field Strengths in Millivolts per Meter)

heights of buildings affect the transmission, we have plotted the contours on an aerial photograph of New York City. This is reproduced as Figure 7. Because of the location of the airplane when the photograph was taken, Manhattan Island is very much fore-shortened. In the original plot, which is made on a large photographic print, individual buildings may easily be seen, so that the reasons for many of the peculiarities of the contour lines are evident.

Figure 8 gives a radio field strength contour map of the whole Metropolitan area. From this map, the high radio attenuation of the thickly settled city is unmistakable. It gives the contours a roughly dumb-bell shape. Several other features are notable. To the west, a marked indentation of the two outside contours shows the effect of the city of Newark. This is substantiated by only a few points, but the points were carefully checked. To the northeast, the contours loop out over Long Island Sound indicating low attenuation over the water. Perhaps the most interesting feature of this map is the way in which the 10- and 20-milli-volt contours approach and recede from each other. Starting at the north on Manhattan, where they are closest, and proceeding clockwise, they spread gradually until in the residential district of lower Brooklyn they are well apart. Then, the abrupt indentation of the inner one and the slight indentation of the outer one show the extent of the shadow of lower Manhattan. For the rest of the circuit, except for the effect of the city of Newark, they spread widely apart in response to good transmission over the New Jersey meadows.

New York City, from the transmission and distribution standpoint, presents a "horrible example" of the difficulties which may be encountered. On the other hand, the situation at Washington, District of Columbia, is a fair example of an excellent distribution. The complete contour map of the Washington area is given in Figure 9. While the effect of land and water is well marked on this map, the deviation of the contours from the circular form which they would have in the ideal case is not sufficient to constitute serious irregularity. At only one point, to the west of Washington, is there an indication of a real dead spot. This is substantiated by only a single point. The measurement at this point was carefully made, but it was not checked and no other measurements were taken nearby. We were unwilling, therefore, to give the contours a radical shift to account for the single unsupported point and have left them undrawn thru this locality.

From the information which this study has disclosed many interesting and useful conclusions may be drawn. Some of them are as follows:

1. The radio attenuation over different kinds of earth surface varies widely. It is low for sea water and for flat moist ground. For dry ground, the attenuation is relatively much greater. In the case of closely built cities filled with steel buildings the local attenuation may be enormous.

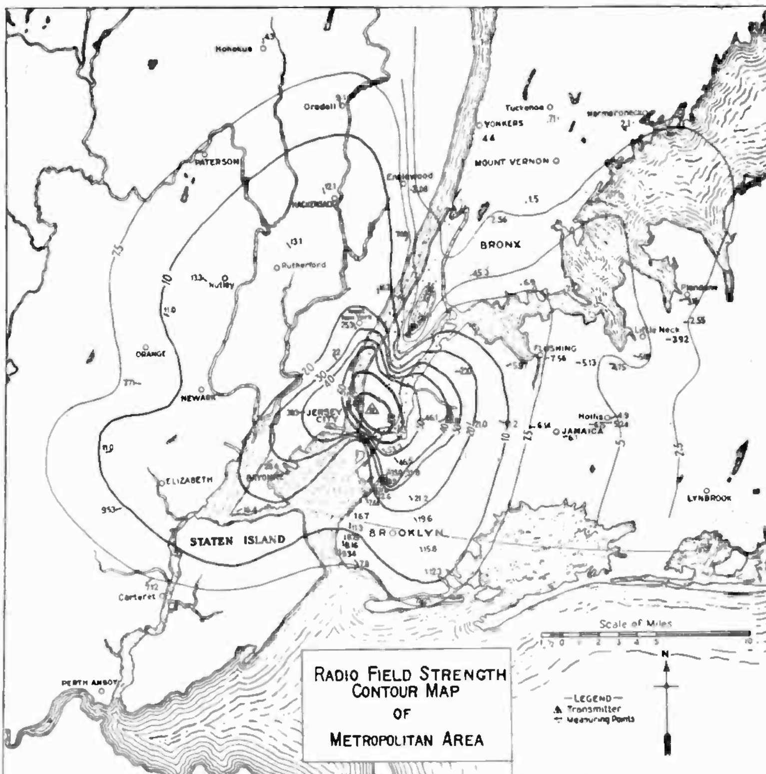


FIGURE 8—Radio Field Strength Contour Map of Metropolitan Area
(Field Strengths in Millivolts per Meter)

2. Sudden changes in land elevation and large masses of conducting material cast radio shadows which may be very heavy in extreme cases.

3. Shadows cause local dead spots; but usually within a relatively short distance beyond, the shadow is wiped out by refraction or diffraction.

All of these effects can be predicted by purely theoretical considerations, so it is not astonishing to find them. It is impos-

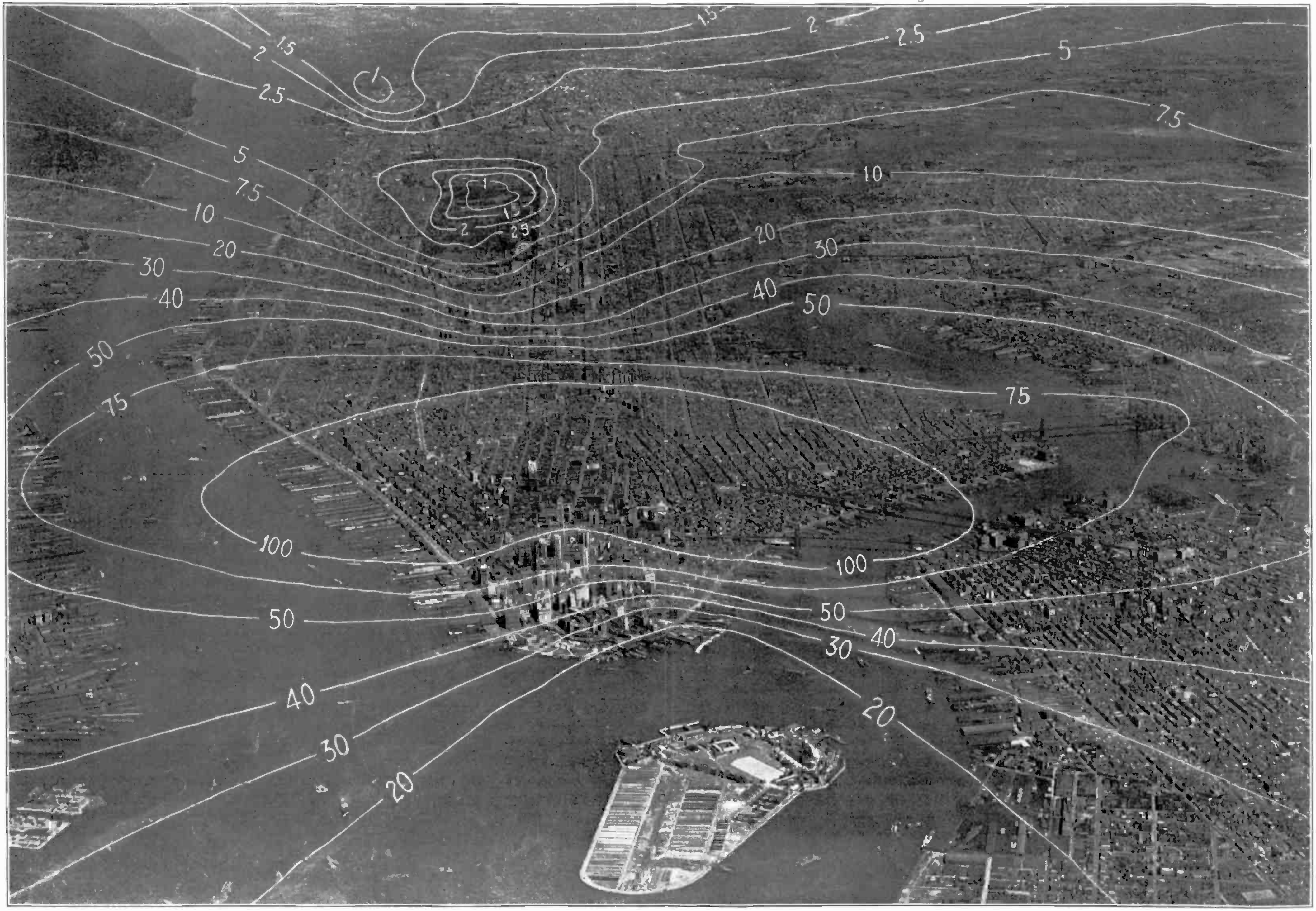


FIGURE 7—Radio Field Strength Contours Superimposed on Aerial Photograph of New York City

(Field Strength in Millivolts per Meter)

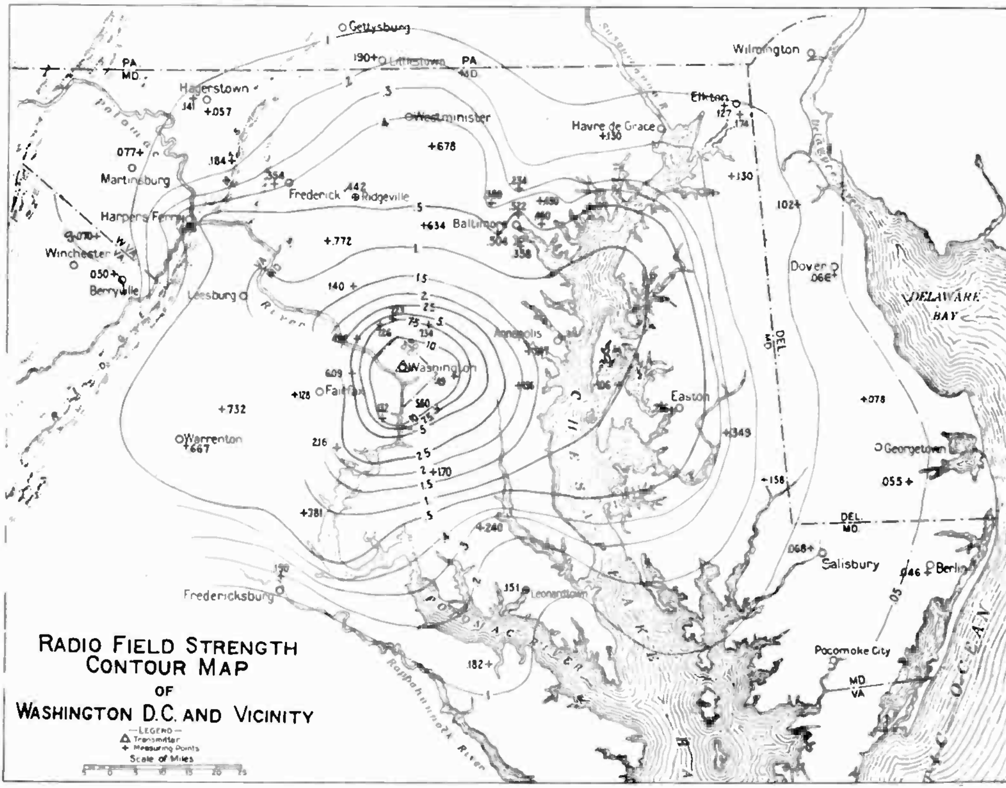


Figure 9—Radio Field Strength Contour Map of Washington, District of Columbia and vicinity

sible to predict the magnitude of the effects, however, on account of the irregular and complex nature of the transmission media and the lack of quantitative data on their electrical characteristics. The experimental study not only shows the magnitudes for specific cases but, as a whole, throws the picture into focus and provides the mind with a sort of scale or measuring stick which is useful in estimating the probable effects in other similar situations.

SUMMARY: The paper presents the results of a quantitative study of radio distribution in and about the cities of New York and Washington, District of Columbia. The wide variations of attenuation over different kinds of territory, the causation of radio shadows or dead spots, and the phenomena of refraction and diffraction are all illustrated by the data and curves. Radio field strength contour maps of the two territories are presented.

THE MARCONI FOUR-ELECTRODE TUBE AND ITS CIRCUIT*

By

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Before I commence the subject of my paper this evening, I should first like to express my pleasure at having the honor of reading a paper before this Institute of world-wide reputation, the journal of which is read with such avidity by all the leading men in the science of radio.

I think that I am correct in saying that up to the present time little or nothing has been done commercially on this side of the Atlantic with the four-electrode tube. However, I understand they are being experimented with at the present moment by the General Electric Company. In Great Britain these tubes have found considerable favor amongst the radio engineers, and this type of tube is now a current store issue to all the large vessels of the British mercantile marine where the radio installations are under the care of the Marconi Company.

In order that this paper may be as complete as possible, it will perhaps be of interest to trace the evolution of the modern quadrode and to describe some of the early types of this form of tube which are interesting if only from a historic point of view.

The classification of the different types of quadrodes now in existence is not an easy matter, as each individual valve may be said to have been evolved for some particular purpose; that is to say, some are utilized for the purpose of rectification only, and others for magnification of radio signals, while others are designed for the reception of continuous waves.

Roughly these tubes may be classified into three separate groups, this classification being only useful from a constructional point of view. In every case, however, two of the electrodes function as controlling elements.

This construction classification can be detailed as follows:

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Type	Types of Electrodes
Group 1	Tubes with Two Grids in addition to the usual filament and anode
Group 2	Tubes with Two Anodes in addition to the usual filament and grid
Group 3	Tubes with Three Anodes in addition to the usual filament

Due to the presence of this additional electrode in the various types of quadrodes, the function of the electronic stream varies considerably from that of the well-known thermionic current produced between the hot cathode or filament and the anode of the popular triode. It is for this reason that the quadrodes present an interesting study. In some types of four-electrode tubes there is a diversion of electrons from one main electrode to another; the electrons during this period do the useful work required to produce the resulting effects, whilst in another instance the electrons are projected by virtue of their high velocity to a destination more distant than the usual anode of the triode.

The origin of the quadrode can, I think, be safely attributed to the genius of Majorana, who took out a patent in the United Kingdom for his tube in the year 1912. The circuit in which this tube was employed and the general arrangement of the four electrodes within the tube are shown in Figure 1. The tube in this instance functions purely as a rectifier, the quality of which is rather poor.

It will be seen from the figure that the grid itself consists of two separated and insulated portions, A and B, which form two separate grids in addition to the usual filament and anode and, therefore, comes under Group 1 of the above table. These two grids, it will be noticed, are interposed between the remaining two electrodes. The circuit in which the oscillations to be rectified are taking place is shunted across these two elements A and B.

A potential difference is then established between A and B, due to the oscillations taking place in the circuit, the result of which is to bring about an attraction and repulsion alternatively of electrons to and from A and B. The electrons are, therefore, diverted from their true path, namely, towards the anode with a resulting diminution of the filament anode current. This brings about variations of current in the external telephone circuit which is directly connected with the anode and a partially rectified current of the induced radio oscillations is effected.

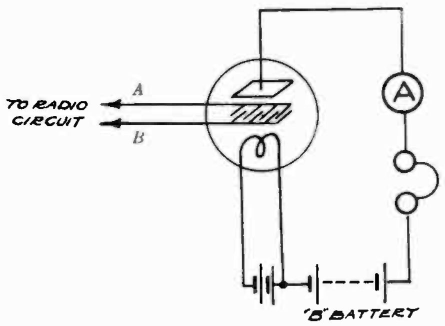

FIGURE 1—Majorana's Quadrode and Circuit

In view of the fact that this is a potential-operated device it is necessary to employ a large inductance across A and B in order to produce maximum efficiency, and consequently this particular type of tube is most sensitive when employed for the reception of long waves.

Another tube which functions in a very similar manner to that of Majorana but of a somewhat later date is that due to Professor Fleming. This particular quadrode comes under Group 3, and consists of three anodes in addition to the usual hot cathode or filament.

The mechanical construction and attendant circuit is shown in Figure 2. The tube, it will be seen, consists of four metal plates which are curved along their axes so that their convex faces are presented towards the filament, the filament being a short, straight vertical wire made of tungsten situated in the middle of the four plates.

The one pair of diametrically opposite plates A and B are joined together externally and constitute one single electrode and are known as the collecting plates. The second pair of plates C and D are connected directly across the receiving circuit and are the controlling elements functioning similarly to the two grids in the Majorana tube.

The function of the tube can be briefly described as follows:

The incoming radio oscillations in the receiver produce potential differences across C and D and thus diminish the established filament-anode current towards A and B, bringing about the

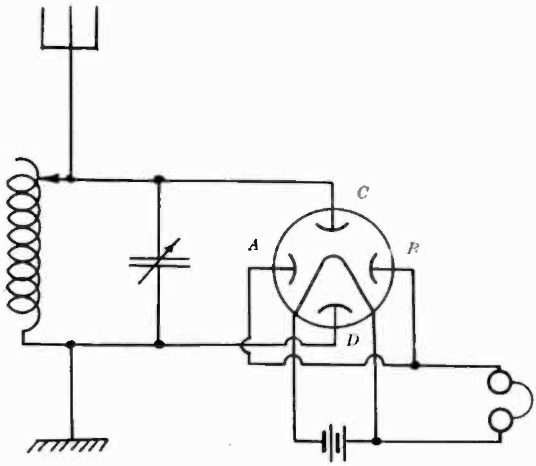

FIGURE 2—The Fleming Quadrode and Circuit

same results as those achieved within the Majorana tube. The characteristic curve illustrating this section is shown in Figure 3, which shows clearly that the decrease in filament-anode current

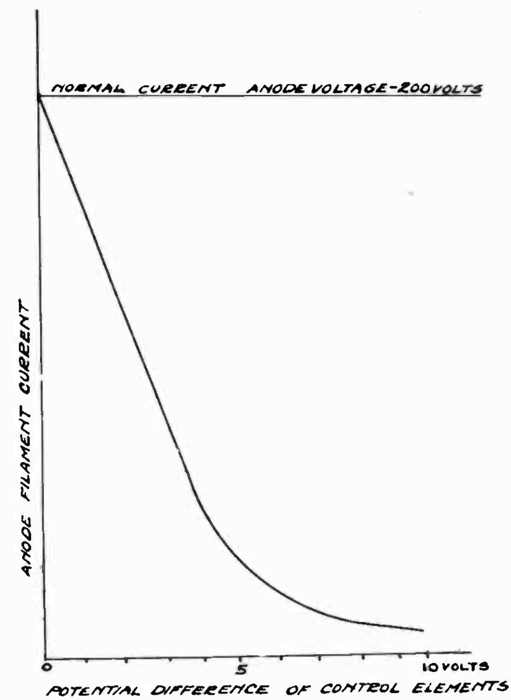

FIGURE 3—Curve for Fleming Quadrode

is determined by the potential difference across plates *C* and *D*, so that the results obtained are proportional to the amplitude of the radio oscillations being received.

Practically speaking the only difference between the Fleming and Majorana tubes is in their construction and position of electronic disturbance.

I now come to the main feature of my paper in the description of the Marconi quadrode or FE I type of tube as it is commercially described. This instrument is of particular interest since it is now a commercial issue to radio installations for shipboard use.

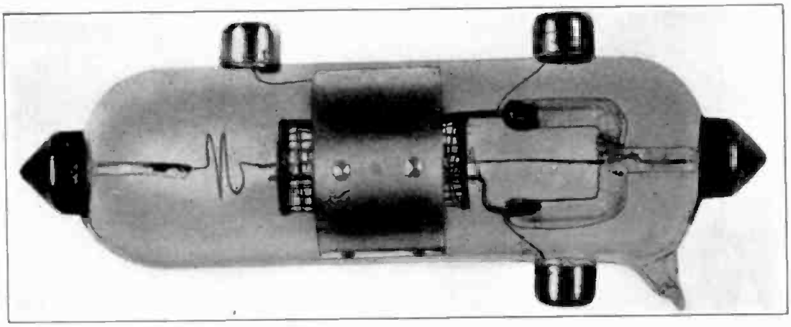

FIGURE 4—The Marconi FE I Quadrode

This tube is used in conjunction with a special amplifier known as Amplifier type number 91, Figure 5, and effects a one-stage radio frequency amplification, rectification, and a one-stage audio-frequency amplification, using the single tube, and consequently this quadrode may be said to do the work of three triodes.

Figure 4 illustrates the construction of the tube, the dimensions of which are about 3¾ inches (9.5 cm.) in length and ⅞ inches (2.2 cm.) in diameter. The following table gives the electrical constants of this tube.

Voltage Across Filament	Working Filament Current	Maximum 2nd Grid Current	Anode Voltage
4.5	1.2 amp.	0.75 milliamps.	45 volts

The tube, it will be seen, contains in addition to the usual grid (which is of a spiral nature) an additional grid which is of a wide open mesh form and, therefore, comes under Group 1 of the classification table. The electrodes in this tube are known as the "first grid," "second grid," and "outer electrode," and external connections are made to these electrodes by means of metal pips secured to the outside of the glass by means of plaster of Paris. Excluding the outer electrode this tube produces the usual characteristic curve of a fairly straight line, the second grid operating as the anode of a triode.

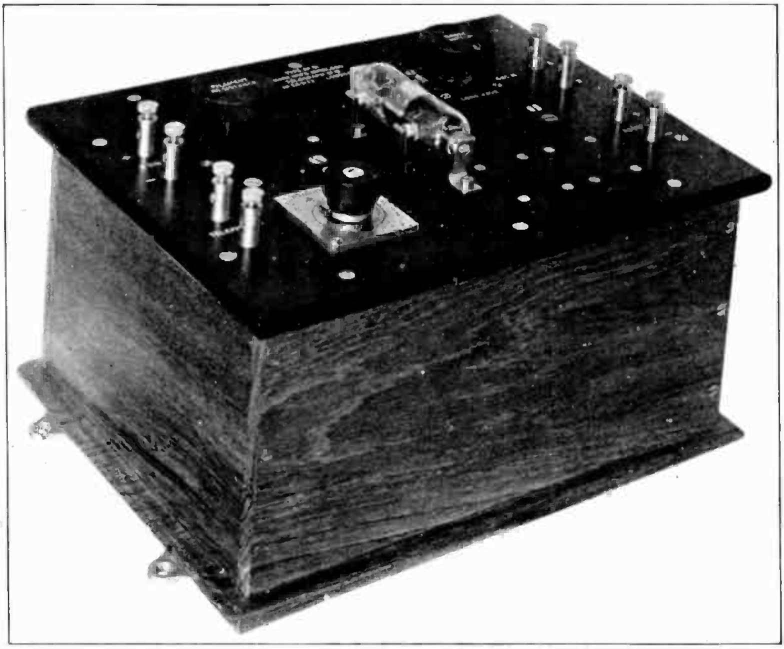

FIGURE 5—The Marconi Amplifier Type 91 for use with Four-Electrode Tubes

Having described the tube, it is next essential to trace the circuits of the amplifier in order to illustrate clearly the functioning of this particular type of tube. This type of amplifier will be of particular interest at the present moment when the interference to broadcasting is being so freely discussed by the press and Government Departments dealing with radio, for the simple reason that this amplifier, whilst producing a high degree of magnification, is not associated with any irradiation effect.

Figure 6 illustrates the circuit when the tube is employed purely for rectification purposes. It will be noticed that the second grid circuit corresponds to the usual anode circuit of a triode arrangement and includes the usual high tension or "B" battery for the purpose of establishing a steady filament-anode current. Now, if the first grid be given a positive charge with regard to the negative limb of the filament, the electrons will travel towards the second grid with a high velocity, and having due regard to the constructional design of this electrode, the electrons, because of their inertia, will pass thru it.

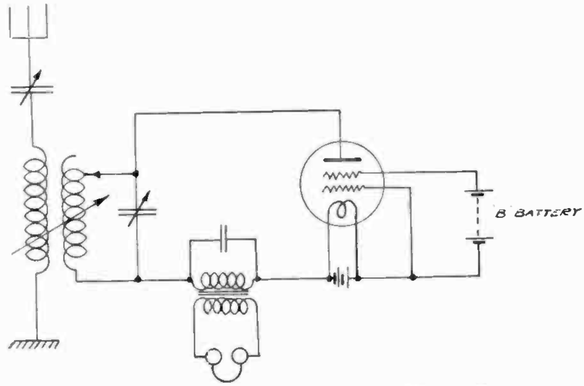

FIGURE 6—Rectification Circuit for FE 1 Quadrode

Now, the outer electrode, it will be seen, is connected to the negative limb of the filament and is, therefore, of the same potential, so that immediately an electron passes thru the second grid, it comes under the retarding electrostatic charge of this outer electrode.

Since there is a drop in voltage between the outer electrode and the second grid equal in value to the rise in voltage between the filament and first grid, the speed of the electron on its arrival at the outer electrode will be equal to the speed of the electron at the filament.

Assuming the initial speed of the electron at the moment of emission from the filament to be zero, the velocity of the electron on arrival at the outer electrode will also be zero. Now, if the resulting alternating emf., due to the induced radio oscillations, be impressed between the outer electrode and negative limb of the filament, a current will flow in that circuit whenever the outer electrode is made positive by those halves of the oscillations giving that potential, as when this condition is effected, the electrons are naturally attracted towards the outer electrode.

This function, it will be seen, is nothing more or less than a Fleming two-electrode tube rectification and the results are comparable to those obtained with a crystal. The rectification current arriving at the outer electrode has a saturation value of about 80 micro-amperes. However, with the use of the 91 type of amplifier, the oscillations are first submitted to a one-stage radio frequency amplification before the above form of rectification is effected.

The circuit embodying these two functions utilizing the one tube is shown in Figure 7. It has already been pointed out that this second grid of this type of quadrode functions according to the principle of the anode of a triode. It is, therefore, possible

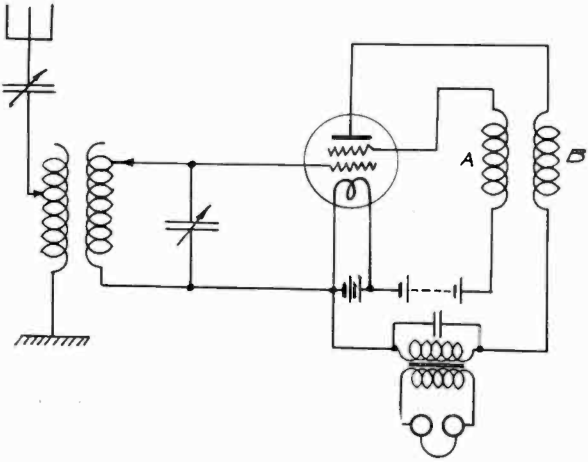

FIGURE 7—Radio Frequency and Rectification Circuits for FE 1 Quadrode

to produce large variations of filament anode current in the second grid circuit for small variations in potential of the first grid. This condition naturally occurs when the potential on the first grid is adjusted so that the tube is working on the straight part of the characteristic curve, and the magnified current produced in the second grid circuit will have the same shape and will be of the same frequency as that of the induced radio oscillations. In this circuit there is included the primary *A* of a radio frequency transformer, the secondary *B* of which is in the outer electrode circuit. It follows, therefore, that the magnified radio oscillations taking place in the second grid circuit will produce an alternating emf. across the secondary of the radio frequency transformer, or, in other words, across the outer electrode and negative limb of the filament; and consequently rectification is satisfactorily effected after the manner described above.

It is possible to submit these rectified and magnified signals to a one-stage audio-frequency amplification. This is obtained by impressing the rectified signals back on the first grid by means of an iron core transformer as is shown in Figure 8. The windings of this iron core transformer have the ratio of 1-to-1, and it will readily be seen that a simple low frequency triode arrangement follows in the first and second grid circuits. The audio-frequency oscillations are produced in the second grid circuit in the usual manner, in which circuit is located a telephone transformer. The windings, which are located in the circuits where radio oscillations are taking place, are naturally shunted with by-pass condensers.

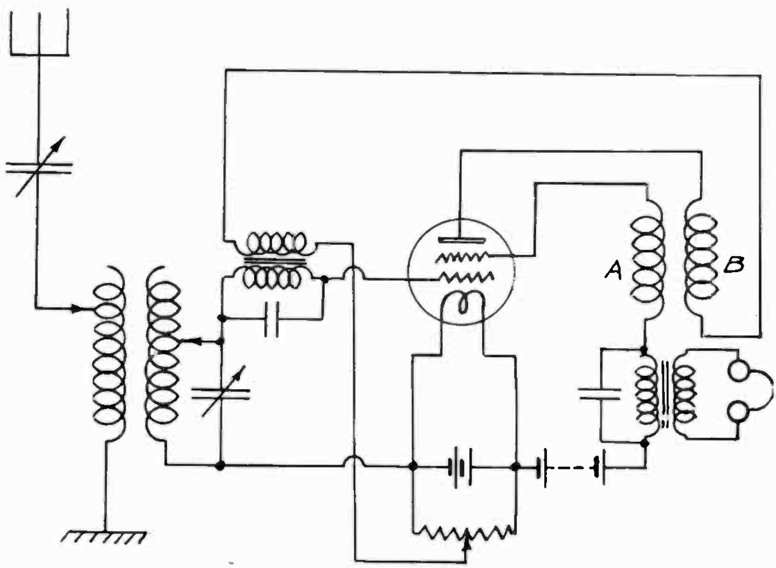

FIGURE 8—Radio Frequency, Rectification and Audio Frequency Circuits for FE 1 Quadrode

A feature of this type of amplifier is an adjustment, within limits, for the elimination of "jamming" or interference. For this particular purpose a potentiometer is employed, the high resistance winding of which is connected directly across the filament or "A" battery and the slider to the outer electrode thru the various elements of that circuit. This allows of alterations in the potential of the outer electrode being effected relative to the filament, and the well-known "limiting" effect can be satisfactorily produced.

The actual instrument is designed to cover a range of wave lengths from 200 meters to 2,800 meters on spark signals and up to about 12,000 meters on continuous waves. Above this wave length, the radio frequency amplification falls away slowly to about 2-to-1 at 20,000 meters.

Figure 9 shows the actual connections of the amplifier. The high frequency transformer *T* 1 has triple windings, all in series, each set having an equal number of turns in the primary and secondary. The first set has 220 turns each, the second 330, and the third 1,625. In the "short wave" position the switch short-circuits the second and third sets, in the "600" position the third set is short-circuited, and in the "long wave" position all are in circuit.

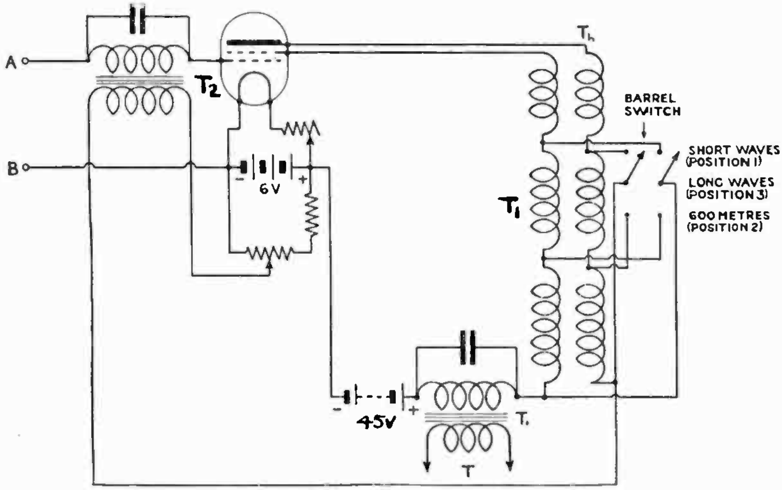

FIGURE 9—Connections of Type Number 1 Amplifier (Marconi)

Each winding of the transformer *T*, which is the closed iron core type, consists of 3,000 turns.

It will be noticed that the slider or contact of the potentiometer used for the "anti-jamming" adjustments operates over only half the total resistance, which is 220 ohms in all. This is to prevent this particular adjustment from being too critical. It will be found necessary, when using this adjustment, to vary at the same time the filament current within small limits.

The actual life of the tube is found to be in practice about 1,300 hours, altho, as a general rule, it is in excess of this, and one of these tubes has a record of 10,000 hours.

The amplifier itself has been criticized by some as not giving a high rate of magnification, but in this connection it is necessary to point out that the instrument gives a maximum increase of signals for feeble radio oscillations and does not respond so readily to strong signals, an ideal condition for shipboard reception. It is also useful from an economic point of view in that it renders maximum amplification with a minimum number of tubes, thereby reducing maintenance charges.

This instrument, which is fitted on a fleet of vessels running between England and Australia, regularly receives signals from the Leafield Station whilst these vessels are in Australian waters.

I think that this example alone will conclusively prove the efficiency of this type of tube as a commercial asset in the radio world.

SUMMARY: The paper deals firstly with the early forms of quadrodes or four-electrode tubes and leads up to a complete description of the Marconi type of quadrode or F. E. I. type and the attendant amplifier circuit.

The amplifier described utilizes one F. E. I. tube and effects a one-stage radio frequency, a rectification, and a one-stage audio frequency.

Information is detailed as to the practical experience obtained with this amplifier and quadrode, this particular combination now being a commercial issue to ships the radio installations of which are controlled by the Marconi International Marine Communication Company.

THE PERFORMANCE AND THEORY OF LOUD SPEAKER HORNS*

By

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and

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The loud speaker is an essential part of a radio receiving set. The poor performance of either will result in unsatisfactory reception. The theory and operation of the latter have been dealt with in numerous places but the acoustic problems of reception, even tho of great importance, have received comparatively little attention. We believe, therefore, the present article dealing with the loud speaker is one which will prove of considerable interest to radio engineers.

A few years ago, Professor G. W. Stewart[1] published data on the performance of conical horns used as acoustic receiving apparatus. It will also be of interest to publish similar data on conical horns used for transmitting or producing sounds rather than for receiving. The present paper contains data which are of practical importance since they cover the range of horn lengths and solid angles which are apt to be considered in practice.

The curves show sound pressure produced at various frequencies. The ear is primarily a pressure device, and for this reason the pressure units of sound can be associated directly with the loudness as perceived by the brain.

Method of Test

The method of testing consisted in placing the horns in a well-padded sound proof booth and measuring the sound pressure at a point on the axis of the horns about 15 cm. (6 inches) from the mouth of the horn. A measuring system consisting of a con-

*Received by the Editor, April 14, 1924.

[1] "Physical Review," pages 313-326, 1920

denser transmitter, a five-stage resistance-coupled amplifier feeding into a thermocouple and galvanometer was calibrated to enable sound pressure to be determined from the galvanometer readings. Tests were made at several different frequencies to ascertain the manner in which the pressure along the axis varied with the distance from the opening of the horns. These test-curves showed that the sound pressure varied inversely as this distance except within a few centimeters of the opening and within a few centimeters of the walls of the booth. From about 10 to 75 cm. (4 to 30 inches) away from the mouth of the horns the end effect and the reflection from the walls were of no consequence. The pressure then decreased with the distance just as it would have done in the free, unconfined space. The curves shown in Figure 1 are given to illustrate these results. The curves show that a point 15 cm. (6 inches) from the opening is satisfactory to avoid room effects. The dotted lines are calculated values.

All the tests were made with the same receiver—namely, one of high impedance—operated at a constant voltage of about 15 volts thru 20,000 ohms. The results, therefore, are strictly comparable. A three-electrode tube oscillator was used to supply the current to the receiver attached to the horns. The oscillograms are included to show that the voltage and current waves are practically pure sine waves. The oscillograms also show the sound output from the horns as picked up by a condenser transmitter and amplified by a six-stage resistance-coupled amplifier. Some of the curves were taken at resonant frequencies of the horns and others were taken at anti-resonant frequencies. Hence, in the former case, any impurities are masked by the intense output at resonance whereas in the second case the harmonics show up prominently. No trouble from impurities occurred above, say, 400 cycles. Our curves, then, at low frequencies are sufficiently correct at the resonant frequencies, but are not so at the low anti-resonant frequencies. At low frequencies, then, we must take these factors into account. Reference to these oscillograms will be made later.

Horns

Sixteen conical horns were made of heavy galvanized iron. Sufficient rigidity of the walls was secured to avoid material vibrations. The results can be taken as closely representative of horns with rigid walls and, therefore, the data are characteristic of the air columns themselves.

The following table summarizes the horn-dimensions in cm.:

Axial length	31	34	34	34	66.3	65.3	65.0	63.0	124	123	122	120	183	182	180	180
Diam., small end	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Diam., large end	5.0	10.3	15.3	20.3	7.6	15.7	30.5	45.5	15.5	31.0	45	60	15	30	60	90

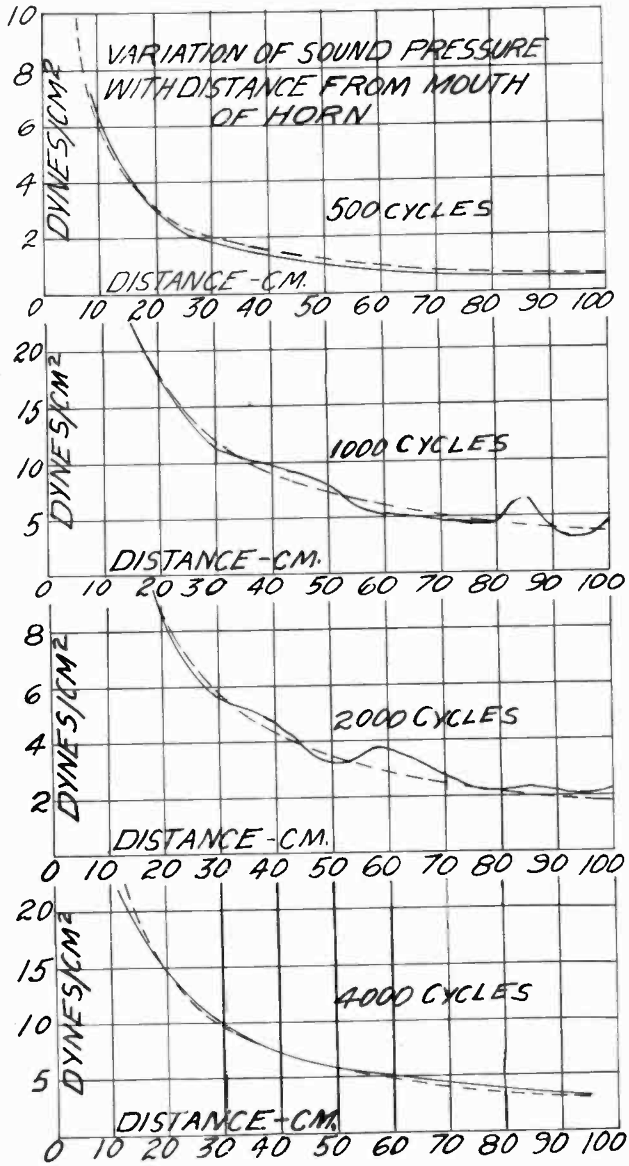


FIGURE 1

Experimental Results

The curves given in Figures 3, 4, 5, and 6 for the sixteen horns are shown in four groups; each group contains the curves for horns of constant length with varying opening. A comparison can also be made with constant opening and varying length. Still a third comparison can be made with constant solid angle at various lengths. The comparisons can be made at constant frequencies or at the fundamentals and also at the various overtones of the horns.

In Figure 2 is given the sound output of the receiver unit without a horn attached to it. The condenser transmitter was placed 15 cm. (6 inches) from the opening of the unit to the surrounding space. For this reason we can gain some information

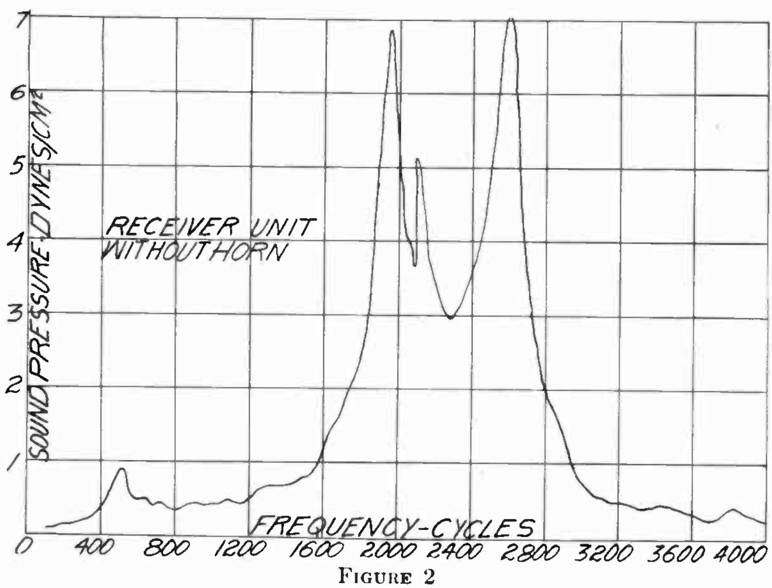
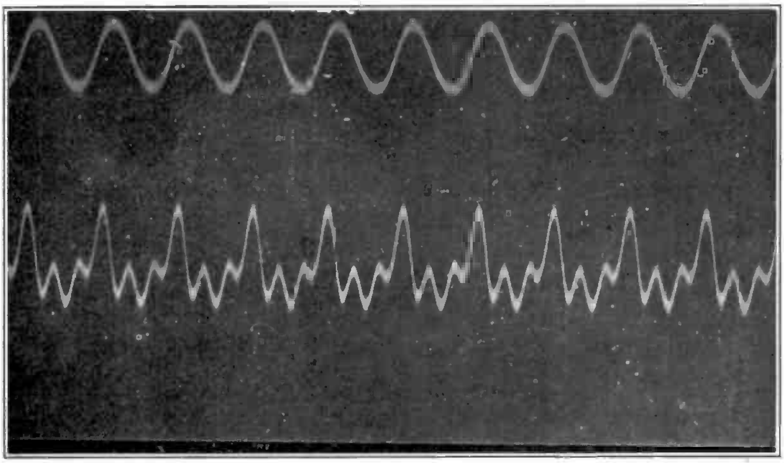


Figure 2

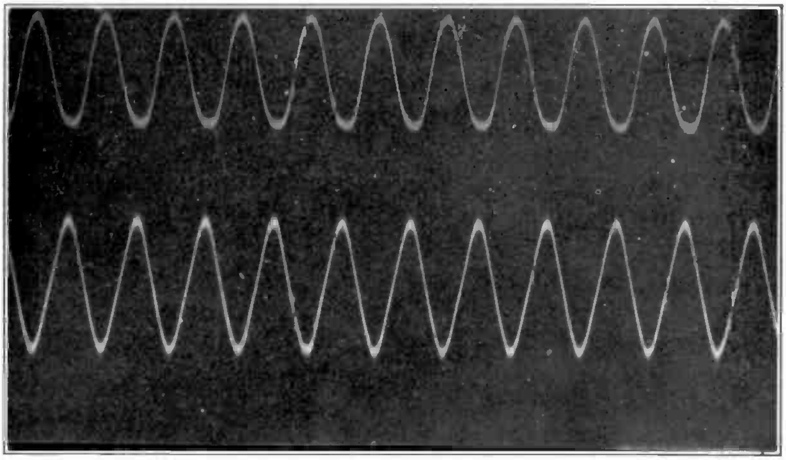
as to the amplification produced by the horn. We can also use this receiver unit curve to assist in the interpretation of the curves. The fundamental of the receiver diaphragm as shown by the curve is 500 cycles. The peaks at the higher frequencies are produced by the diaphragm vibrating in some of its higher modes of vibration. These effects will be modified some by the various horns and our curves give a comparative basis for the horns with the unit used.

The first group of curves gives the pressure-frequency curves for the horns of the shortest length, namely 34 cm. approximately.

These are shown in Figure 3. Several characteristics are distinctly noticeable. There are pronounced peaks in all the horns. The two large ones in the neighborhood of 500 and 2,300 cycles are due to the receiver diaphragm; the others (except some of the "ripples" below 300 cycles) are due to the resonating properties of the horns. The fundamental on the basis of its wave length equals four times the length of the horn and should occur at about 270 cycles. The overtones—the odd ones only being present in a horn closed at one end—should occur at 810, 1,350, 1,890, 2,430, 2,970, 3,510, 4,050 cycles, and so on. The first horn peak, however, is not 270, but 320 cycles. The two oscillo-



Current Thru Unit—Sound Output—Plate I (1), 269 Cycles



Current Thru Unit—Sound Output—Plate I (2), 320 Cycles grams, shown in Plate I, show that the 269-cycle peak is composed almost entirely of the double frequency term. This is due to the second harmonic introduced by the unit itself, which corresponds to the natural frequency of the diaphragm. The oscillogram for the 320-cycle peak indicates that this peak is composed almost entirely of the fundamental.

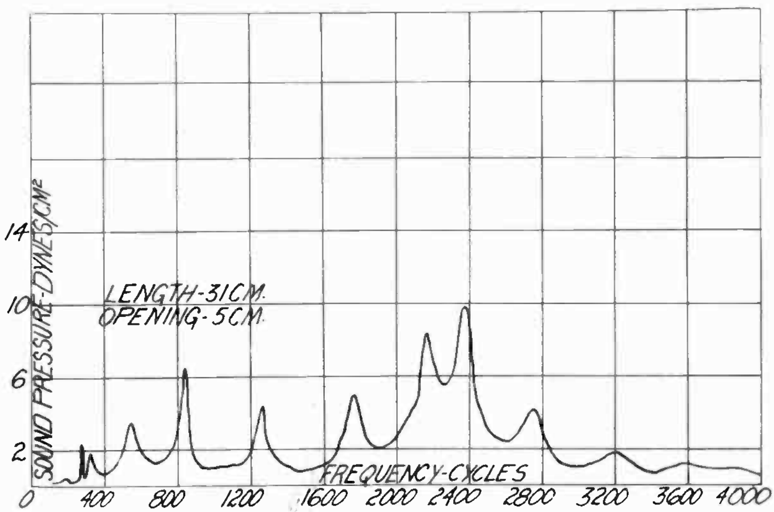


FIGURE 3—GROUP I

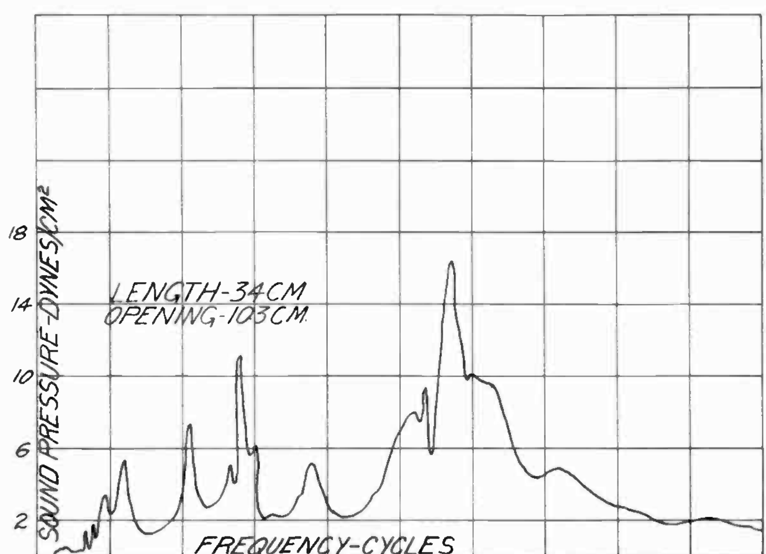


FIGURE 3—GROUP I

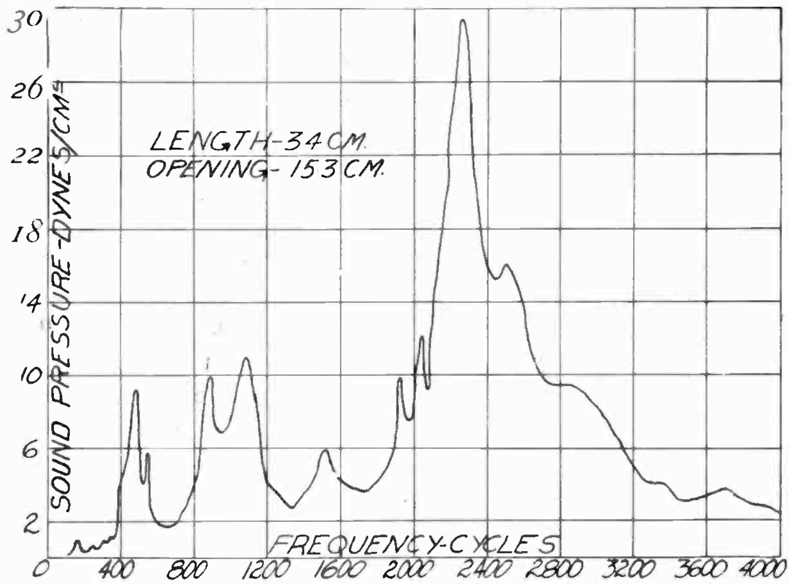


Figure 3—Group I

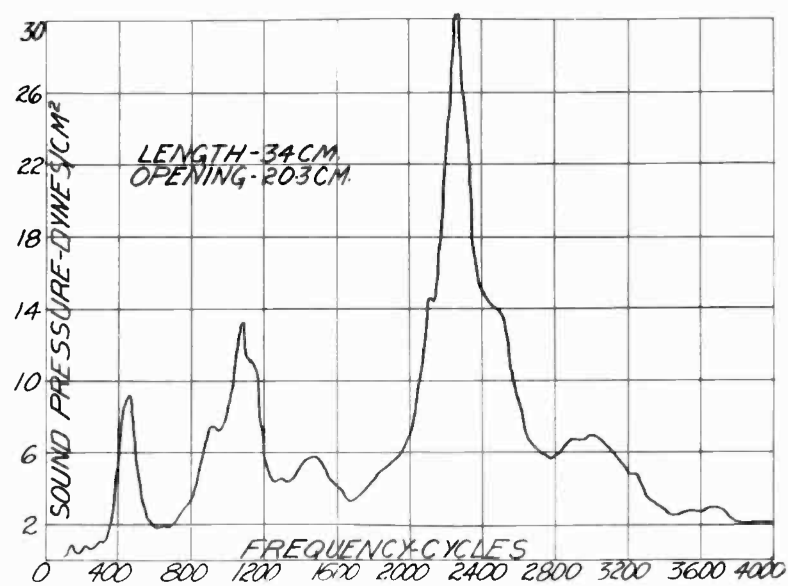


Figure 3—Group I

Reflection of sound at the smaller openings is considerable, and marked resonance will occur with these openings. Reflection is much less for the larger openings, and, hence resonance will be less marked in these cases. Furthermore, as pointed out by Professor Stewart,[2] reflection takes place along the entire inner surface of the horn and therefore "resonance" will occur at all frequencies above the fundamental.

This phenomenon is most pronounced for horns with large openings, the length being kept constant. Two effects may therefore be anticipated for horns of large openings. Firstly, the peaks at definite resonance frequencies in the curves should be less prominent because there is 'ess end reflection in the horns with large openings. And secondly, the large "valleys" in the curves should be relatively filled in or elevated because there is increased reflection along the walls of the horns, and therefore, more reinforcement of *all* frequencies above the fundamental.

With this viewpoint as a guide, it is interesting to study the peaks. They are sharp and pronounced with the horn of the smallest opening and less sharp and pronounced with the large opening. Using an end correction of 0.7 times the radius of the opening as recommended by Stewart, we find that for the smallest horn there should be 8 horn peaks from 253 cycles, the fundamental, to 3,800 cycles, the 15th overtone (if we call the fundamental 1). Assuming a simple closed pipe the peaks should occur at the following frequencies: 253, 760, 1,265, 1,770, 2,270, 2,790, 3,290, and 3,800. Of course, the resonant peaks will follow a closed pipe only roughly. The agreement is not exact, but it is satisfactory. What has been said about the smallest horn may be repeated in substance for the next larger size. Here, however, the end correction is larger, and the peaks should occur at 221, 665, 1,110, 1,,545, 1,990, 2,430, 2,870, 3,310, 3,760 cycles, and so on. Since the end reflection vanishes for the higher frequencies with the larger openings, there should be no marked resonant frequencies in this region. About all that is actually observed is a "wave," so to speak. Such waves are, however, for example, present at the last four frequencies, mentioned above. The 1,990-cycle peak is interfered with by its proximity to the receiver peak. The 1,100 and 1,500-cycle peaks are properly located, but the lower ones are not. The fundamental and first odd overtone are too high to check with the simple theory for pipes which we are here using. Apparently the end correction of 0.7 radius is too large for the low resonant frequencies. The 270-cycle peak is a second harmonic one—the first horn peak being 320 cycles.

Horn resonance is less marked for the 15 cm. opening and still less marked for the 20 cm. opening. As for the average sound

[2] Previous citation, page 324.

output, however, the smallest horn gives the least, the next large size still more, and the two larger ones about equal outputs with perhaps a preference for the horn with the 20 cm. opening. One is not to conclude, however, that the horn with the least amount of resonance and largest opening will give the largest output. The data for the longer horns indicate an optimum opening for best results, in accordance with Professor Stewart's results.[3]

The basis for consideration of the longer horns is the same as that given for the shortest ones tested. It should be remembered in studying these curves that the sound outputs at the low frequencies are small. Overtones, therefore, which are always generated in the receiver are particularly disturbing for the low frequency measurements. A peak may exist at a low frequency reading but the sound pressure may be due to a considerable extent to the overtone. This effect is not important, however, above 300 cycles.

The data for the 65 cm. horns are plotted in Figure 4 in Group II. As we would expect, resonance is pronounced, the peaks are sharp, and the ratios of maxima to minima are large for the horns with the 7.6 cm. and 15.7 cm. openings. The horn resonance is much less marked for the 30.5 cm. opening and is practically absent for the 45.5 cm. opening. The average sound output from the largest aperture horn of this length is about the same as that from one having the smallest opening. The two intermediate horns give superior results, the larger of the two intermediate sizes have less resonance and a more uniform output. As in the case of the shortest horns, the ratio of maxima to minima approaches unity for the higher frequencies and for the largest openings.

The peaks for this group of horns should be about 200 to 250 cycles apart. The fundamental for these 65 cm. horns should be in the region of 120 or 130 cycles. The curves show small peaks in this region. However, the vibrational energy of the diaphragm is so small at low frequencies that a small sound output must nevertheless be the result. This is still true for the long horns tested. The horn will be of little value for low frequencies, even if it is a long one, unless the receiver unit itself has at least a reasonable amount of vibrational energy at those frequencies. This shows the importance of having a low natural frequency unit to give low frequencies. On the other hand, the low frequency unit will not vibrate with any intensity, at the high frequencies, which would rather indicate the desirability of a high frequency

[3] Previous citation.

unit. Since, for high frequencies, reflection will occur at the end of the horn if the opening is small, it means that the opening will need be rather small to take advantage of any resonance effects, when high frequency response is needed. In the theoretical part

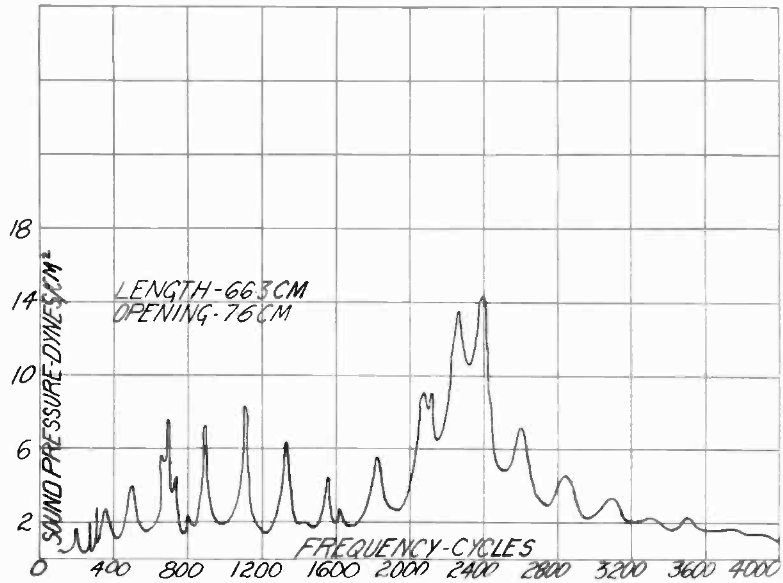


FIGURE 4—GROUP II

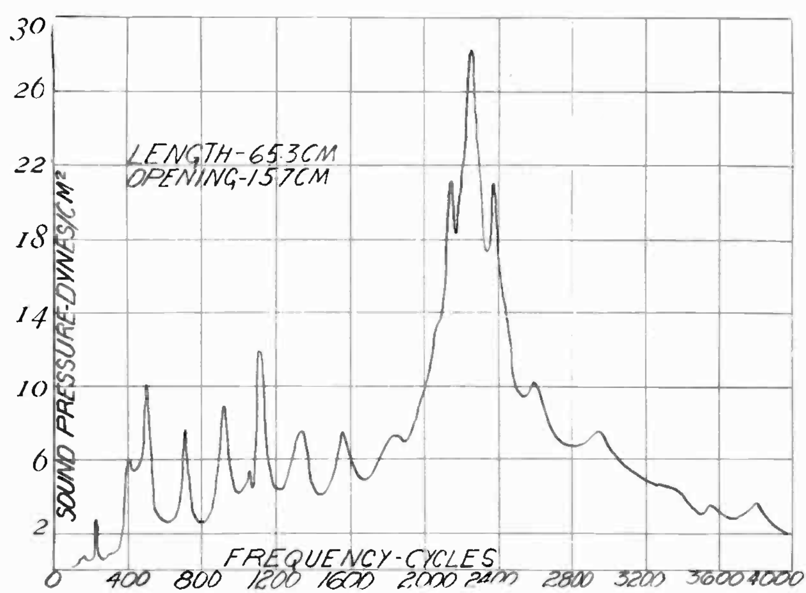


FIGURE 4—GROUP II

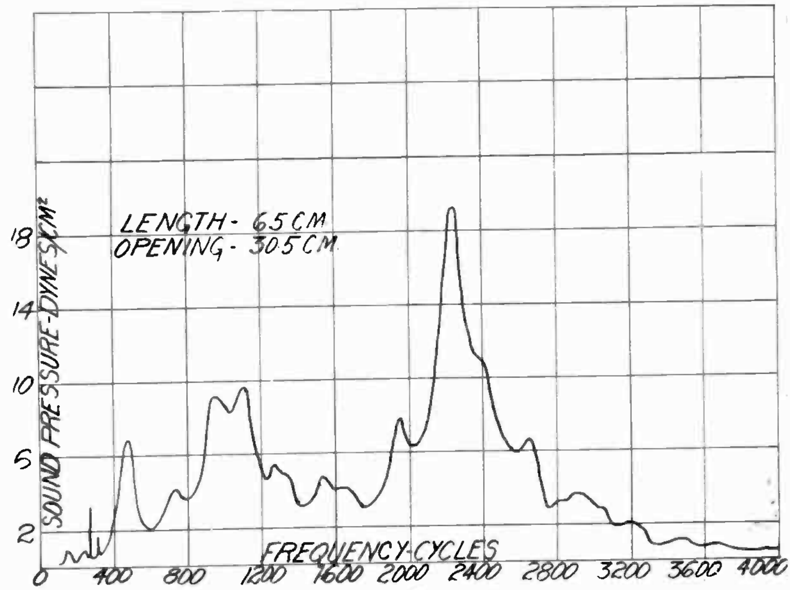


FIGURE 4—GROUP II

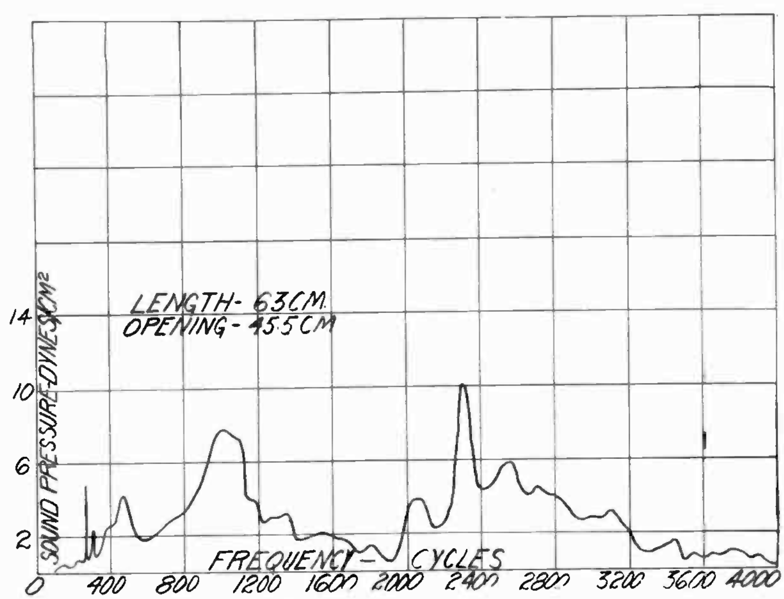
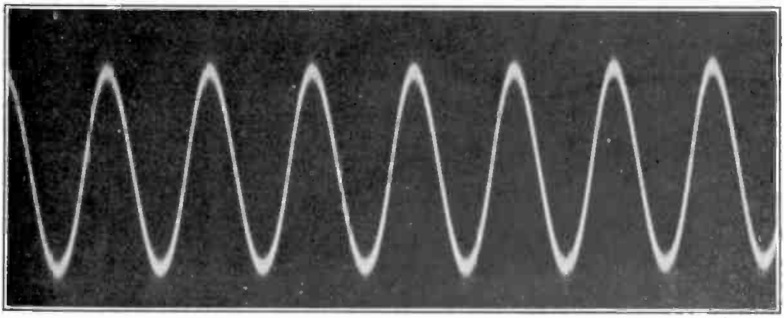
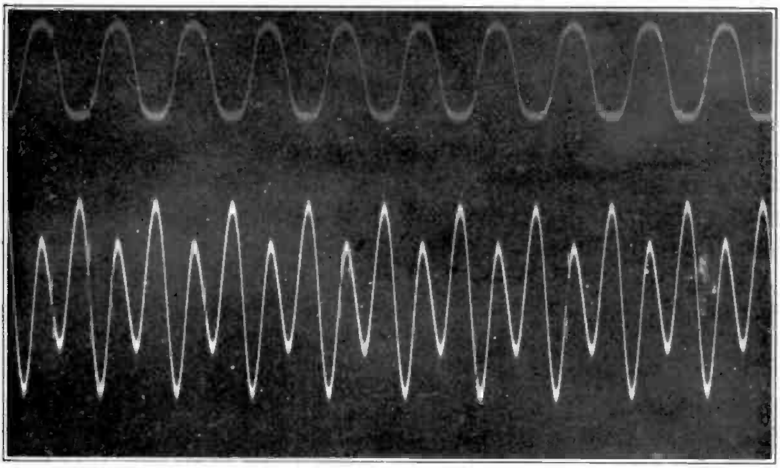
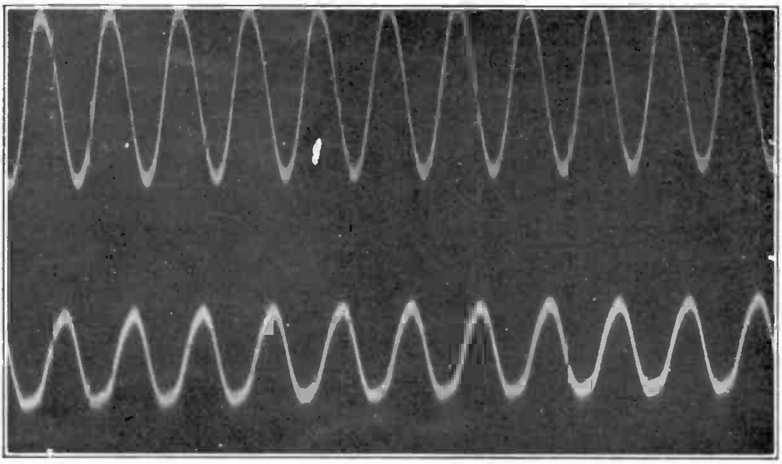


FIGURE 4—GROUP II


Sound Output—Plate II (1), 200 Cycles


Current Thru Unit—Sound Output—Plate II (2) 270 Cycles


Current Thru Unit—Sound Output—Plate II (3) 320 Cycles

of the paper it is pointed out that the air velocity and pressure are largely out of phase at the low frequencies. This prevents large sound radiation at such frequencies.

In these 65 cm. horns we may call attention to the sharp peaks at 270 cycles. The oscillograms in Plate II show that these peaks are chiefly second harmonic. Those peaks at 200 cycles and 320 cycles, however, are chiefly fundamentals as shown by the reproduction.

An analysis for the 270-cycle oscillograms was made by Mr. E. W. Smith of this laboratory. The first of the two following equations is an analysis of the oscillogram itself, and the second equation is the analysis of the sound itself. The latter was obtained from the former by correcting for the frequency characteristics of the measuring system.

OSCILLOGRAM EQUATION

$$y=1.00\ sin\,(\omega t+301^\circ)+5.45\ sin\ (2\ \omega t+338^\circ)+0.562\ sin\ (3\ \omega t+77^\circ)+0.993\ sin\ (4\ \omega t+325^\circ)+0.342\ sin\ (5\ \omega t+222^\circ)+0.03\ sin\ (6\ \omega t+45^\circ)+0.127\ sin\ (7\ \omega t+105^\circ)+0.333\ sin\ (8\ \omega t+320^\circ)+0.475\ sin\ (9\ \omega t+153^\circ)+0.206\ sin\ (10\ \omega t+248^\circ)+0.255\ sin\ (11\ \omega t+65^\circ)$$

and for the corrected sound output:

$$y=1.00\ sin\ (\omega t+301^\circ)+6.52\ sin\ (2\ \omega t+338^\circ)+0.787\ sin\ (3\ \omega t+77^\circ)+1.438\ sin\ (4\ \omega t+325^\circ)+0.513\ sin\ (5\ \omega t+222^\circ)+0.046\ sin\ (6\ \omega t+45^\circ)+0.196\ sin\ (7\ \omega t+105^\circ)+0.516\ sin\ (8\ \omega t+320^\circ)+0.74\ sin\ (9\ \omega t+153^\circ)+0.323\ sin\ (10\ \omega t+248^\circ)+0.4\ sin\ (11\ \omega t+65^\circ).$$

As stated above, the 270-cycle peak is due chiefly to the second harmonic, nevertheless the analysis shows that other harmonics, even up to the eleventh, are present to a noticeable degree. These equations, then, serve to illustrate the significance of the distortion introduced by a receiver unit and horn and how the original sounds may be modified greatly at the receiving end.

The data for the 125 cm. horns are plotted in Figure 5 in Group III. The resonant peaks should be about 130 cycles apart, the fundamental being about 65 cycles on the basis of a closed end at the receiver. In the case of the horns of this length the peaks should occur roughly at 65, 190, 320, 450, 580, 710, 840, 970, 1,100, etc., where the horn is considered closed at the receiver end. The agreement of the actual positions of the peaks with the predicted positions is fair.

For the Group III horns, the sound output was greatest for the 31 cm. opening just as was the case for the 65 cm. horns. The horn with a 15 cm. opening gave about the same average

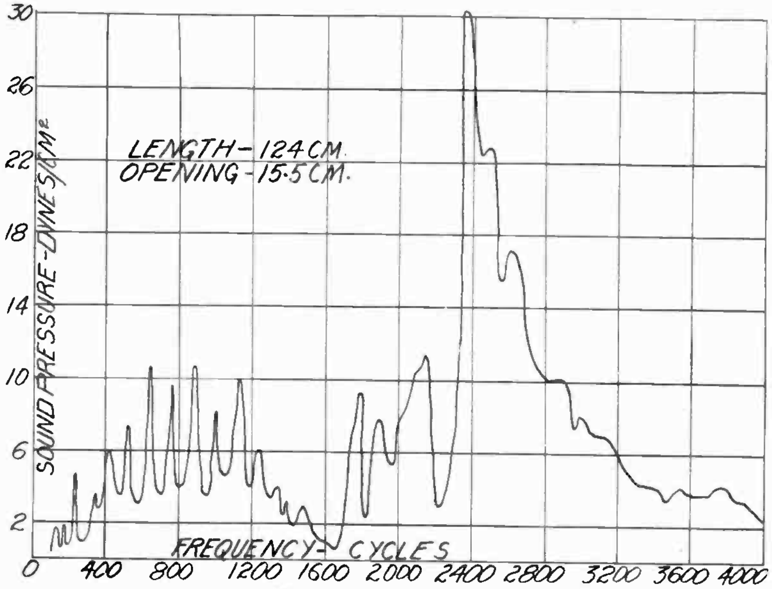


FIGURE 5—GROUP III

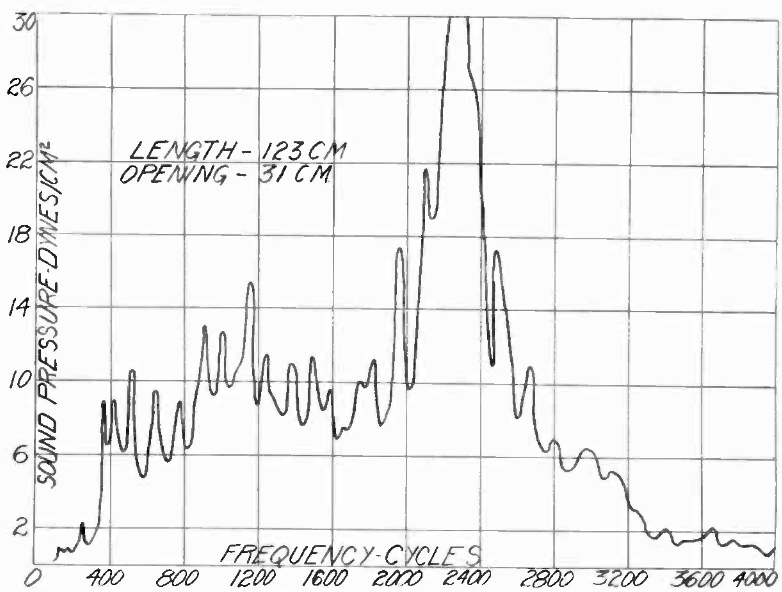


FIGURE 5—GROUP III

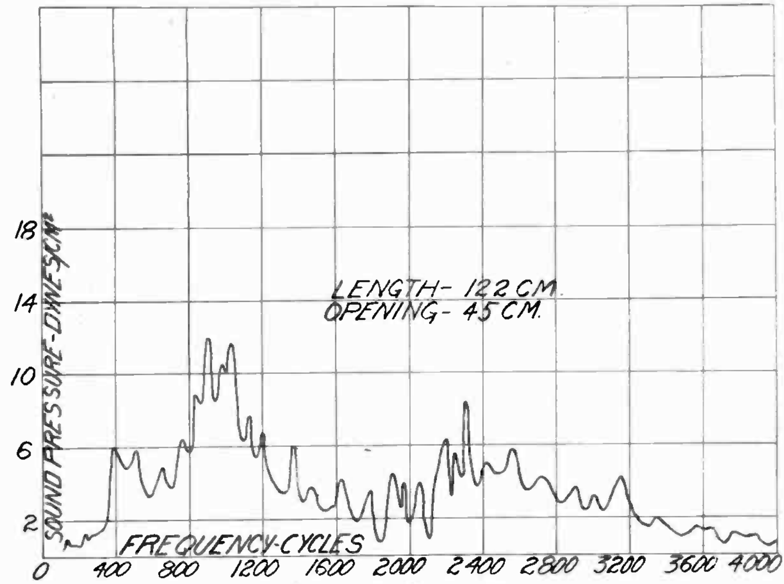


FIGURE 5—GROUP III

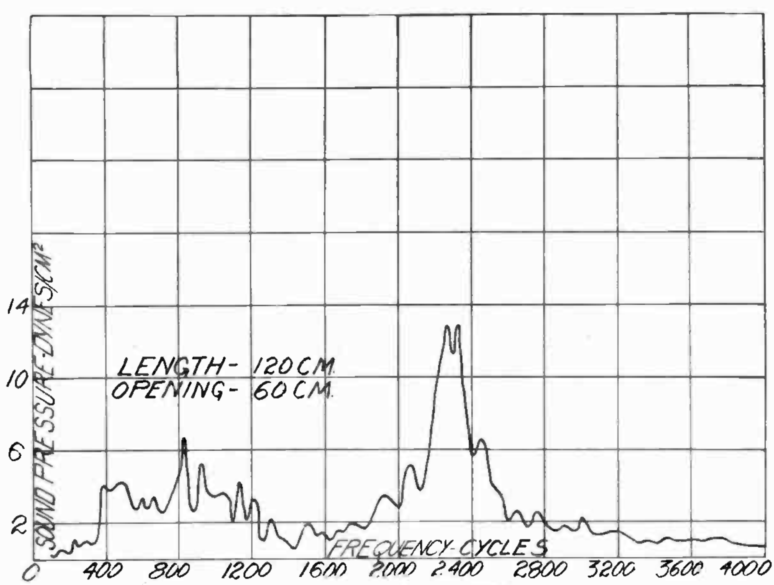


FIGURE 5—GROUP III

response as was obtained with the 45 cm. opening. The horn with the 60 cm. opening is quite inefficient compared with these of the smaller openings.

The data for the 180 cm. horns are plotted in the curves shown in Figure 6 in Group IV. The sound output from the horns with the 60 cm. and 90 cm. openings is small, the latter

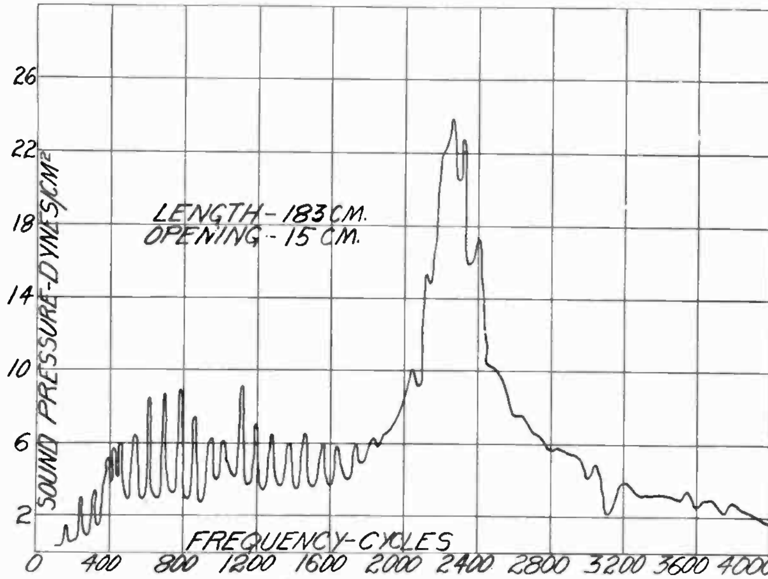


FIGURE 6—GROUP IV

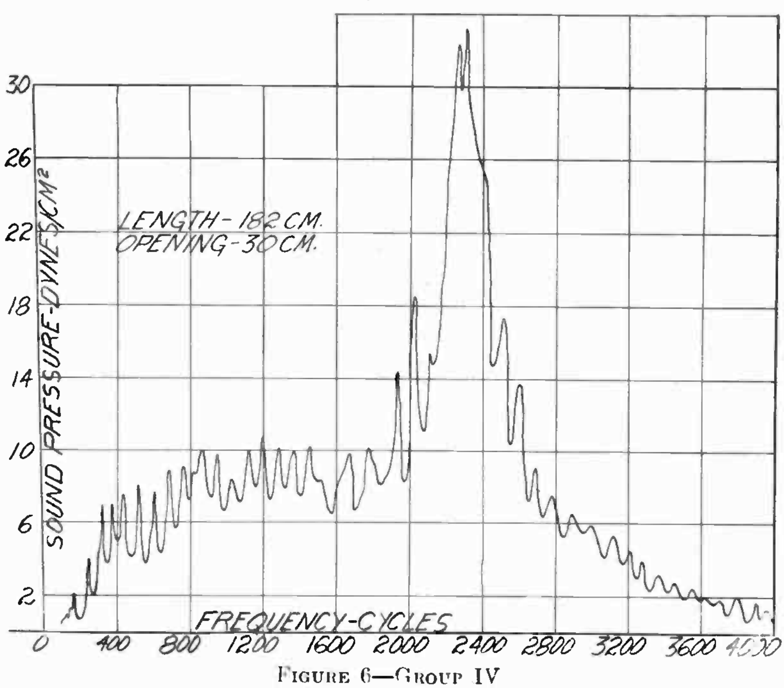


FIGURE 6—GROUP IV

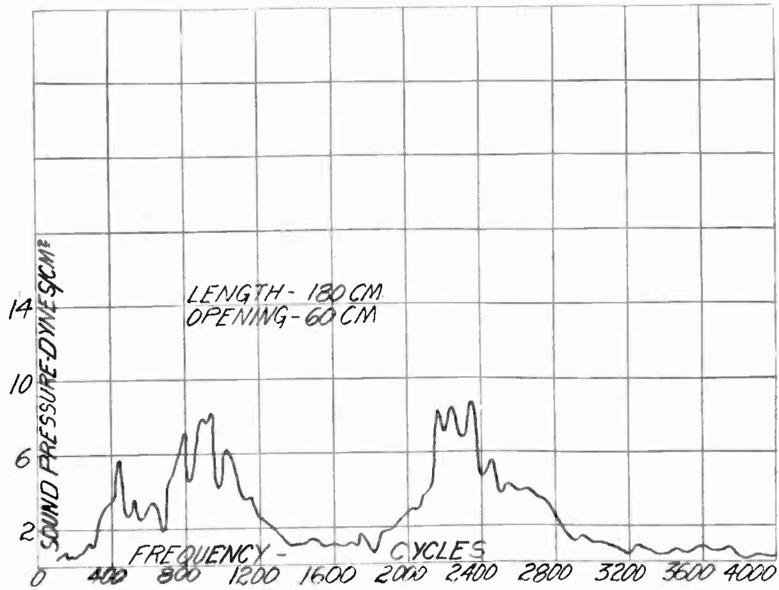


FIGURE 6—GROUP IV

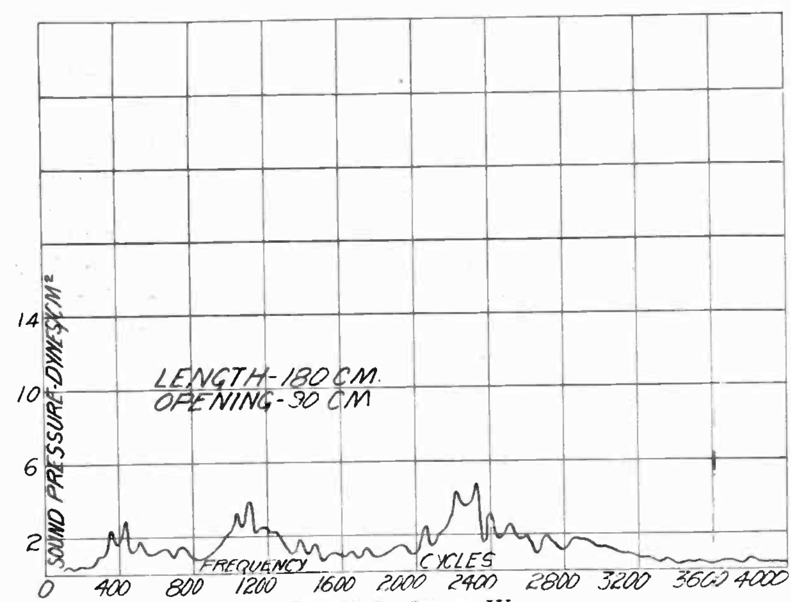


FIGURE 6—GROUP IV

horn giving the smaller output of the two. The horn with the 30 cm. opening gives much larger outputs than do those with the larger openings. The horn also gives larger outputs than does the one with the 15 cm. opening.

There are pronounced resonant peaks for the two smaller size horns, but not for the horns with the 60 cm. and 90 cm. openings. On the basis of simple, straight pipe theory, the peaks should be about 80 or 90 cycles apart. This is in agreement with the separation of the peaks as shown in the curves. If the end is considered closed the peaks should occur at approximately 45, 135, 225, 315, 405, 495, 585, 675, 765, 855, 945, 1,035, 1,125, 1,215, 1,305, 1,395, 1,485, 1,575, 1,665, 1,755, 1,845, 1,935, etc. The actual location of the peaks is in quite satisfactory agreement with positions predicted on the basis of a pipe with one end closed. In discussing the peaks it is interesting and important, theoretically, to observe that the peaks for the various horns of constant length occur at the same frequencies. It is also important to observe that resonance plays a sufficiently prominent part to cause as much as 3-to-1 or 4-to-1 pressure variation in the output over a very small frequency range; and for the horns with the smaller openings the variations are sometimes as large as 8 or 10-to-1. Of course, such horns with sharp resonant peaks, due to large end reflections, are of perhaps quite limited value as far as a loud speaker horn is concerned.

We will next consider a comparison for constant opening and varying length of horn. Taking first the frequencies above 1,800 cycles, the curves for the horns with 15 cm. openings indicate that there is little preference among the four horns. The sound output is essentially the same for these conical horns having a 1.6 cm. initial opening and a 15 cm. final opening. The phenomenon of resonance plays no important, or at least no controlling part, and all of them have little or no damping effect on the receiver diaphragm for this frequency range. The output over this region for these particular horns is almost entirely a property, apparently, of the receiver itself.

Below 1,800 cycles the horn length plays an important part. The average response from about 400 cycles to 1,800 is about the same for the four horns with the 15 cm. opening, but the variation from this average is quite different in the four cases. The peaks are close together for the 180 cm. horn and far apart for the 35 cm. horn. The results over this region for the 180 cm. horn can perhaps be considered the most satisfactory of the four horns. The 125 cm. and 180 cm. horns are certainly more satisfactory than the two shorter ones over this low frequency region. This, of course, should be the case for the longer horns, simply because, by means of resonance, the low frequencies are amplified more with these longer horns than with the shorter ones.

Also, as pointed out in the theory, for a given low frequency, the longer horn will have a better power factor at the large end than the shorter one. and for this reason, too, the longer horn will radiate more sound. The question of radiation and the effect of the matching of acoustical impedance will be considered in detail in this connection in the theoretical discussion.

A comparison of the three longest horns with a 30 cm. opening can be made. Even a casual comparison will exclude the 65 cm. horn as far as superiority of the average sound output is concerned. The superior performance of the 180 cm. horn is not only noticeable below 400 cycles, but is also noticeable at all other frequencies where the fluctuation from the average response, due to resonance for small frequency ranges (plotted as a function of the frequency) are relatively small. The cause of this is that the resonant peaks are close together. A horn of about this same opening but about 300 cm. long was constructed, and its behavior is given in Figure 7.

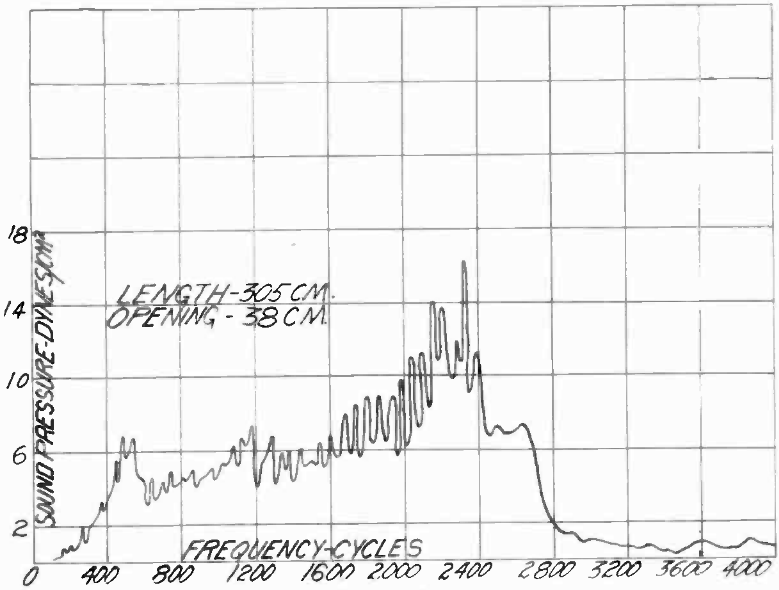


FIGURE 7

The curve for this long horn, which had a 38 cm. opening at the large end and a 1.6 cm. opening at the small end, shows resonant effects which are not very marked. The peaks are close together, being about 50 or 60 cycles apart as they should be for a horn of this length. Since the peaks are close together, there cannot be large depressions between the peaks. The range of response or width of each peak is sufficient to eliminate large fluctuations within small frequencies ranges. At high frequencies the output from this long horn does not compare favorably with that from some of the short horns. The output at the very low frequencies is no greater than that of some of the shorter horns either. As previously pointed out, a long horn cannot be expected to give a large response at low frequencies if the receiver unit itself is not designed particularly for low frequencies, even tho the power factor may be more favorable.

A comparison can be made between the two horns with 45 cm. openings and with lengths 65 and 125 cms. The latter is the more satisfactory, but neither of them gives a performance equal to that of either of the 125 cm. or 180 cm. horns with the 30 cm. openings. The effect of resonance, radiation, and acoustical impedance on these performances will also be considered in the theoretical discussion.

There is little to choose between the two longer horns with the 60 cm. openings. Neither of them is comparable in performance with the two longer horns with the 30 cm. openings.

As is to be expected the short horns with the small openings give unsatisfactory performance. We conclude, therefore, the optimum opening and length for the sixteen straight conical horns are 30 cm. and 180 cm., respectively. The practical importance of this conclusion is at once apparent.

Reaction of Horn on Diaphragm

We do not desire to give in this paper an experimental and theoretical consideration of the reaction of the horn on the receiver diaphragm, nevertheless, it seems appropriate to give a set of curves for a typical impedance analysis. For this purpose, we include impedance tests for the receiver unit alone and for the unit when attached to the 125 cm. conical horn with a 31 cm. opening. These results are shown plotted in Group V. The tests cover the same frequency range as was covered in the tests previously described. In presenting these curves, perhaps we should call attention to some of the outstanding points. The curves of reactance and resistance are given—the impedance analysis of them are omitted. The curves show that the natural period of the diaphragm without the horn is about 500 cycles. When the horn is used the natural period decreases to 440 cycles. This change can be due either to an increase in the effective mass or a decrease in the effective elasticity of the diaphragm due to the horn effect. Perhaps there is no decrease in elasticity at all, but most probably an increase. Hence, the decrease in natural period is due entirely to an increase in the effective mass brought about by the added inertia due to the air confined in the horn neck.

We note that the horn peaks at 360 and 520 cycles affect the reactance and resistance of the receiver unit to an extent equivalent to about one-half the effect produced by the diaphragm resonance. That is, near the resonance of the diaphragm, where its motion is controlled largely by its mechanical resistance, the reaction of the horn produces reactance and resistance changes

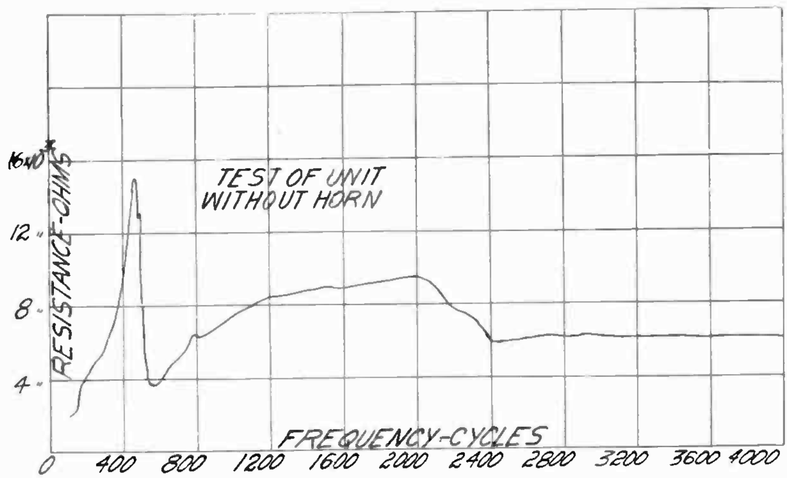


FIGURE 8—GROUP V

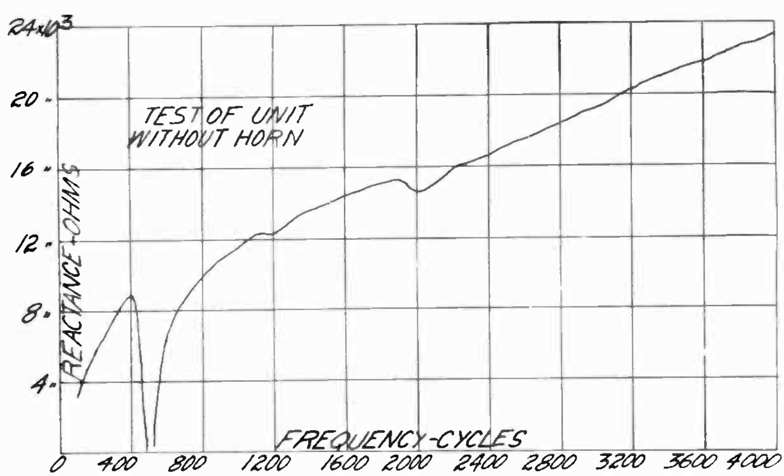


FIGURE 8—GROUP V

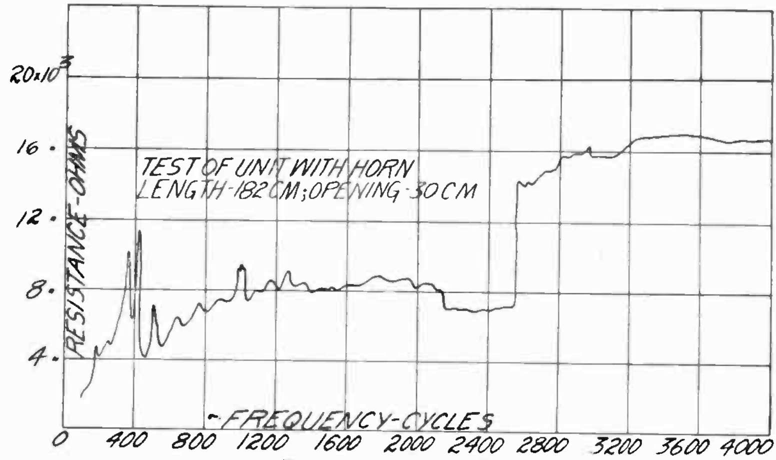


FIGURE 8—GROUP V

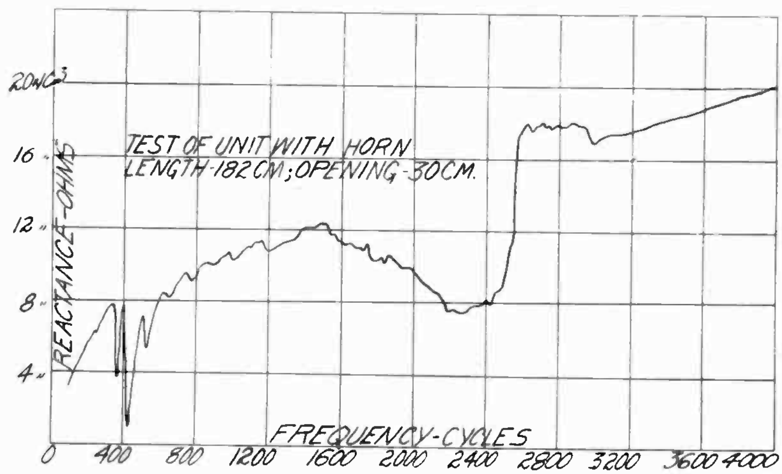


FIGURE 8—GROUP V

about half those produced by the diaphragm. The sound energy radiated, therefore, produces dissipation of energy about equal to that produced by the mechanical resistance of the diaphragm itself. Consequently we can study the horn effects by means of impedance analysis.

Away from the resonant frequency of the diaphragm, the motion is controlled largely by either the elastic or the inertia force and the resistance force plays a smaller part than near resonance. In the same way the horn reaction will be comparatively small away from the region of the diaphragm resonance. The group of curves shown in V brings out this point quite clearly. Nevertheless, the horn resonance has its effect, which is distinctly noticeable up to about 3,000 cycles. The marked change in the curves in the region of 2,500 cycles corresponds to the changes in the mode of vibration of the diaphragm. Perhaps these remarks are sufficient to illustrate the method of study employed and the magnitude of the effects brought about by the use of a horn. We shall hope to have more to say on this phase of the subject at a later date.

EXPERIMENTAL CONCLUSIONS

The following experimental conclusions are arrived at based on the results of the above data.

(1) Resonance of the air column plays a marked effect for all the horns tested.

(2) The frequencies at which resonant peaks occur are close to integral multiples of the fundamental frequency of the column.

(3) Most of the curves indicate that the fundamental frequency of a horn closed by a receiver corresponds to a horn length of one-quarter wave length. Peaks always occurred at the predicted frequency differences, as is to be expected, even up to high frequencies.

(4) The horns give considerable amplification for any frequency nearby equal to or greater than the fundamental. That is, the horns resonate for all frequencies equal to or greater than the fundamental due to reflections occurring in proper phase relations along the walls of the horns. The resonating effect is grestest for the end reflection.

(5) The greater the frequency, the more nearly does the ratio between the maximum responses and adjacent minima approach unity. That is, at the higher frequencies, there is little amplification due to the horn resonance and it is important to observe that the horns have little effect on the sound output in this region provided not too small an orifice and too long a neck are used at the receiver end.

(6) Desirable results were obtained with those horns having a 30 cm. opening. Among the four horns having this size opening the best results were obtained with a horn of 180 cm. length.

(7) Low frequencies can be increased first, by resonance, provided the horn is long enough to bring its fundamental resonant frequency into this region; second, provided the receiver diaphragm will respond to the low frequency currents, and third, by obtaining a better power factor between pressure and current with a long horn.

(8) The horn cannot of itself introduce the sound of the various frequencies. These must be present in the receiver unit in order that the horn may produce an appreciable re-enforcement of them.

(9) A long horn of appropriate shape, orifice, and end opening promises marked amplification of sound without highly marked resonant peaks thruout a considerable frequency range.

(10) Horn reaction on receiver diaphragm due to confined air and to sound radiation is marked, particularly near diaphragm resonance. For the present, however, conclusions based on impedance analysis are deferred for future publication.

Theoretical Considerations

The theoretical basis for the performance of horns of finite lengths and various shapes has been given by Professor A. G. Webster.[4] Prof. G. W. Stewart[5] extended Webster's theory to include additional results on conical horns. The present writers present in this paper a further extension of Webster's work to include the exponential shape for finite lengths. A review of the straight pipe and cone of unlimited lengths is given, and some original work on unlimited exponential and parabolic shapes is included. This is followed by a discussion of horns of finite lengths.

We are interested in the amount of sound energy that can be radiated into the space surrounding the horn. If a pipe or horn is of infinite length so as to avoid any end-reflection, and, if we assume no reflection along the walls of the horn, it is not difficult to derive the expressions for the radiant acoustical energy. This has been done by Lord Rayleigh.[6] Dr. Slepian and Mr. Hanna[7] have given an engineering interpretation to this work and extended it to cover horns of exponential shapes of infinite length, that is, those in which no end reflections occur.

1. Theory of Conical Horn of Unlimited Length

If we follow Rayleigh, the waves may be considered spherical and originating from a point and progressing outwardly. Their velocity potential is represented, therefore, by:

$$\phi = -\frac{A}{4\pi r} \cos K(ct - r) \qquad (1)$$

[4] "Journal of the National Academy Sciences," pages 275-282, 1919.

[5] "Physical Review," pages 313-326, 1920

[6] Previous citation.

[7] "Journal of the American Institute of Electrical Engineers"; mid-winter Convention, Philadelphia, February, 1924.

where A determines the strength of the source, r is the distance out and $K=\frac{2\pi}{\lambda}$; λ being the wave length.

If we consider the case of a cone, with the sound source at the vertex, (1) is replaced by

$$\phi=-\frac{A}{\omega r}\cos K(ct-r) \qquad (2)$$

where ω is the solid angle of the cone. The sound radiation is denoted by the work done per second by the sound pressure at the source forcing a "current" of air out thru the horn. The sound pressure is given by $-\rho\frac{d\phi}{dt}$, ρ being the air density. The total "current" per second is given by $\omega r^2\frac{d\phi}{dr}$. Using these relations with (2) we get:

$$\rho\frac{d\phi}{dt}=\frac{\rho A K c}{\omega r}\sin K(ct-r) \qquad (3)$$

and $$\omega r^2\frac{d\phi}{dr}=A\{\cos K(ct-r)-Kr\sin K(ct-r)\} \qquad (4)$$

A comparison of these two equations will show that one component of the current is in quadrature with the pressure and the product of this component and the current, therefore, does not represent sound radiation or "loading" as used by Dr. Slepian and Mr. Hanna. The other component of the current is in phase with the pressure and their product represents power output in the form of sound radiation. This represents the "loading" as defined and used by the authors to whom we have just referred.

The phase angle between the pressure and the current is seen to be lagging and represented in Figure 9. The power factor is given by the cosine of the phase angle θ. It is:

$$\cos\theta=\frac{Kr}{\sqrt{1+K^2r^2}} \qquad (5)$$

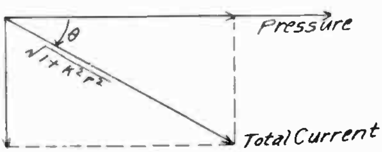


FIGURE 9

At frequencies sufficiently high or at values of r sufficiently large to make K^2r^2 large compared to unity, $\cos\theta$ becomes unity as in the case of the tube. At low frequencies or at small values or r where $K^2 r^2$ can be neglected with respect to 1, $\cos \theta = K r$. It does not become zero therefore, until zero frequency is reached. We also note that when r is small, that is, near the source, $\cos \theta$ approaches zero. This does not mean, however, that the radiation approaches zero. On the contrary, it remains finite (see equation (6) below), because the pressure becomes increasingly large near the source to compensate for the change in power factor toward zero. This indicates, then, why it is difficult to obtain low frequencies with a horn.

The power output along the horn from the source of sound situated at the vertex of the cone, therefore, is given by the product of (3) and (4). The average value of sine-square being ½, the power output is:

$$P = \frac{\rho K^2 c A^2}{2 \omega} \tag{6}$$

In the open space, of course, $\omega = 4\pi$. The power output, therefore, varies directly as the frequency squared and inversely as the solid angle.

2. Theory of Cylindrical Tube of Unlimited Length

We place the source of sound within the tube of cross sectional area σ. The energy is transmitted in both directions continually (that is, with no reflection) away from the source which we assume to be the same as that represented in equation (1). A short distance away from the source the sound waves are plane and are of the type:

$$\phi = A \cos K(c t - x) \tag{7}$$

Just as in the case of spherical radiation outward, near the source where r is small we have:

$$4 \pi r^2 \frac{d \phi}{d r} = A \cos K(c t - r) \tag{8}$$

and also just as in the conical case where r is small (see equation 4)

$$\omega r^2 \frac{d \phi}{d r} = A \cos K(c t - r) \tag{9}$$

So, near the source situated in the pipe we have in the case of the unlimited pipe where all the energy is radiated through the total area 2σ

$$2 \sigma \frac{d \phi}{d x} = A \cos K(c t - x) \tag{10}$$

which represents the current passing along the pipe.

All three of the cases are similar, the area factor entering in some way for the sphere, the cone, and the pipe.

Whence,

$$d\phi = \frac{A}{2\sigma} \cos K(ct - x)dx \tag{11}$$

or

$$\phi = \frac{A}{2\sigma K} \sin K(ct - x) \tag{12}$$

Therefore

$$\rho \frac{d\phi}{dt} = \frac{\rho A c}{2\sigma} \cos K(ct - x) \tag{13}$$

and the pressure and current are in phase.

The power radiated in both directions from the source at $x=0$ is:

$$P = \left[\left(2\sigma \frac{d\phi}{dx} \right) \left(-\rho \frac{d\phi}{dt} \right) \right]_{x=0} = [\text{current} \times \text{pressure}] \tag{14}$$

This is similar to the relation in mechanics whereby the product of velocity, force, and time gives the amount of work.

Substituting (10) and (13) into (14) we get:

$$P = \frac{\rho A^2 c}{4\sigma} \tag{15}$$

where ½ has been substituted for $\cos^2 Kct$. As pointed out by Rayleigh, if a rigid barrier is placed in the tube near the source so that the energy is radiated in one direction only, the total radiation will be doubled, because by deflection the pressure itself is double while the total current remains unchanged. In this case

$$P = \frac{\rho A^2 c}{2\sigma} \tag{16}$$

The radiation, therefore, varies inversely as the cross section and is similar to the cone where we saw it varied inversely as the solid angle. That is, σ in the straight pipe case is analogous to the solid angle in the case of the cone. In the pipe, however, the radiation is independent of frequency and the current and pressure are in phase at all frequencies. Radiation, therefore, along an infinite pipe, is constant at all frequencies.

3. Theory of Exponential Horn of Unlimited Length*

In Appendix I we show how Professor Webster derived the equations for the exponential horn from his general equations

*The parabolic horn is an interesting case and is treated in Appendix III.

for horns of any profile whatever. In this appendix we show that, if ϕ is the velocity potential, $g=\frac{1}{2}\sqrt{4K^2-m^2}$, $K=2\pi$ divided by λ, the wave length, m is the exponential coefficient in the horn equation, area (σ) = initial area (σ_0) times ε^{mx}, then

$$\phi=\varepsilon^{-\frac{mx}{2}}\left[A\,cas(nt-gx)+B\cos(nt+gx)\right] \tag{17}$$

for the waves in both directions along the exponential horn. It is of considerable interest to note that the general solution for plane waves of the type

$$\frac{d^2\phi}{dt^2}=c\frac{d^2\phi}{dx^2} \qquad 18)$$

along straight tube in both directions as shown by Rayleigh and others is:

$$\phi=A\cos K(ct-x)+B\cos K(ct+x) \tag{19}$$

Equations (17) and (19) are similar. $Kct=nt$, but Kx does not exactly equal gx for $g=\frac{1}{2}\sqrt{4K^2-m^2}$. If m^2 is negligible, or the frequency is high, then equations (17) and (19) are identical, except for the exponential factor in the solution of the exponential horn. This similarity between the exponential horn and the straight pipe was pointed out by Webster in his 1919 paper in the "Proceedings of the National Academy."

The solution of the exponential horn given in equation (17) is similar to the propagation of a wave motion in both directions along a string where a friction term is taken into account. This case is treated in Rayleigh's, "Theory of Sound," Volume I, page 232. The solution is also similar to the propagation of electricity in both directions along wires where the exponential factor in the present solution corresponds to the attenuation factor in the electrical case. Consequently, the exponential solution in the case of sound is due to the transverse vibrations which are not considered in the present approximate treatment.

If we assume that the total current flow into the small end of the exponential horn is the same, namely: $A\cos Kct$, as in the case of the straight tube and conical horn as already treated, then we may write for the total current at any place in the exponential horn (compare with (8), (9), (10), and (17)):

$$2\sigma_o\varepsilon^{\frac{mx}{2}}\frac{d\phi}{dx}=A\cos(Kct-gx) \tag{20}$$

corresponding to:

$$\phi = -\frac{A\,\varepsilon^{-\frac{mx}{2}}g}{2\sigma_0 K^2}\left[-sin(Kct-gx)+\frac{m}{2g}cos(Kct-gx)\right] \quad (21)$$

Since the pressure is given by $-\frac{\rho\, d\phi}{dt}$ where ρ is the air density, we get from (21)

$$-\rho\frac{d\phi}{dt} = \frac{\rho A\,\varepsilon^{-\frac{mx}{2}}cg}{2\sigma_0 K}\left[cos(Kct-gx)-\frac{m}{2g}sin(Kct-gx)\right] \quad (22)$$

In passing we desire to point out that these equations correspond to equations (78) and (79), published by Slepian and Hanna in their recent paper. The power radiation is given by the product of (20) and (22) and is

$$P = \frac{\rho A^2 c\sqrt{K^2-\frac{m^2}{4}}}{2\sigma_0 K\,\varepsilon^{\frac{mx}{2}}} \quad (23)$$

in which we have substituted the value of $g=\frac{1}{2}\sqrt{4K^2-m^2}$

In this product ½ has been substituted for the square of the cosine and on the average the product of the sine and cosine terms does not represent power radiation.

A comparison of equations (20) and (22) shows that there is a leading current as shown in Figure 10 and by equation (24):

$$cos\,\theta = \frac{1}{\sqrt{1+\frac{m^2}{4g^2}}} = \sqrt{1-\frac{m^2}{4K^2}} = \sqrt{1-\frac{m^2c^2}{4n^2}} \quad (24)$$

which is identical with equation (82) published by Slepian and Hanna. It is to be noted that the only characteristic of the exponential horn which is associated with the phase relation between velocity and pressure is that which determines the rate at which the horn opens up. The initial opening does not enter the phase relation. On the other hand, the initial opening of the exponential horn enters the pressure and power equation in exactly the same way the solid angle of the cone enters when the source of sound is situated at the vertex, and in exactly the same way the cross sectional area of the straight pipe enters Much care needs to be exercised, therefore, in making comparisons and drawing conclusions as to relative merits. Furthermore, our experimental data given in section I shows a distinct optimum opening and solid angle, and comparisons should properly be made between best results obtained with each of the various types. In the last analysis this simple theory does not check experimental data for the cone and we are quite certain that it will not check data on exponential horns. The theory has not gone far enough yet and we should be cautious about drawing conclusions from it. Nevertheless our theoretical deductions, therefore, are in agreement. We may emphasize some of the deductions that are to be drawn from this result. When $\sqrt{1-\frac{m^2}{4K^2}}$ is zero, the pressure and current are ninety degrees out of phase and, theoretically, no radiation results.

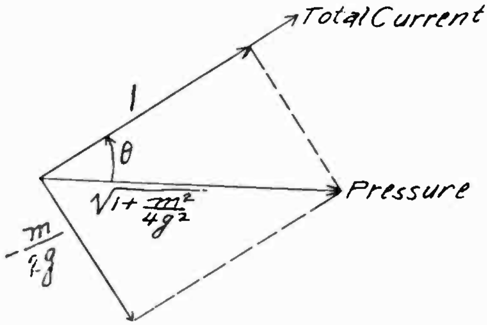


FIGURE 10

This occurs when $m^2=4K^2$ or at a frequency the wave length of which is: $\lambda=\frac{4}{m}$. If we take $m=0.07$ (a value used by Slepian and Hanna), then $\lambda=\frac{4\pi}{0.07}=180$ cm. This corresponds to a frequency of about 185 cycles. At lower frequencies the equations will not hold, because the power factor and power becomes imaginary. We can, however, increase the power output of these low frequencies by making m smaller. Such a procedure means that a horn must have a long neck. Also since σ_o the initial opening, should be small to increase the output as shown by equation (23), a small m means a long neck of smaller diameter. Such a procedure, if carried to extremes, would reduce the output to a large extent, particularly at high frequencies. As is well known, frequencies above 500 cycles are far more important for clearness of reproduction than are those frequencies below 500 cycles. In our effort to secure a greater low frequency output in this way, we should be cautious in order not to go too far.

We can make a comparison by means of our equations among the three types of radiators considered thus far. We saw that the conical horn did not cease radiating until zero frequency was reached. At the low frequencies, then, the conical horn should exceed the exponential one, as far as the simple theory we have used would cover the facts. In this respect, too, the straight pipe also should be superior to the exponential shape at these low frequencies, because it continues to radiate down to zero frequency.

If we refer to equation (23), the sound radiation at the initial end of the exponential horn is:

$$P_{x=0} = \frac{\rho A^2 c \sqrt{K^2 - \frac{m^2}{4}}}{2 \sigma_o K} \tag{25}$$

Since the velocity and pressure along the horn both vary as $\epsilon^{-\frac{mx}{2}}$ and the area varies as ϵ^{-mx}, this equation represents the total radiation along the horn and is seen to be independent of x. We may make use of equation (25) then, to study what happens under various conditions.

At frequencies sufficiently high where K^2 is large enough to make $\frac{m^2}{4}$ a negligible factor in comparison, equation (25) becomes

$$P_{x=0} = \frac{\rho A^2 c}{2 \sigma_o} \tag{26}$$

This result is identical with that for the straight pipe as given by equation (16). Under this condition we may note also that *cos* θ becomes unity (see equation (24)) and the current and pressure are in phase as was shown to be the case for the straight pipe. Under the same conditions, we saw that the same conclusions were reached for the conical horn except that the radiation increased as the square of the frequency, whereas with the pipe and exponential horn the radiation is independent of frequency at these higher frequencies. As an illustration let us use the same numerical figures for m ($=0.07$) as used above for the exponential horn. Suppose, further, that $\sigma_o = 1$ sq. cm. (perhaps too small a value for practical purposes, but a value which tends to favor theoretically the exponential horn) and that $\omega = 0.01$ radian. Such a conical horn would have a cross sectional area of 400 sq. cm. at a distance of 200 cm. from the vertex. A comparison of equations (6) and (26) shows that the ratio between the conical and exponential horns at relatively high frequencies is given by:

$$\frac{P_c}{P_e} = \frac{K^2 \sigma_o}{\omega} \tag{27}$$

Substitution of the chosen values in this equation will show that at all frequencies above 500 cycles, approximately, the conical horn is superior to the exponential horn. It is allowable to use equation (25) in making this comparison because at 550 cycles m^2 is negligible compared with K^2. If, therefore, $m=0.07$, $\sigma_o=1$, $\omega=0.01$ and the same source of sound is used on both horns, the conical would be superior to the exponential not only above 550 cycles, but also below 185 cycles as previously calculated. Between these two frequency limits the exponential horn will be superior according to our theory and the amount of superiority can be calculated from the ratio:

$$\frac{P_c}{P_{E_{x=0}}} = \frac{K^3 \sigma_o}{\omega\sqrt{K^2 - \frac{m^2}{4}}} \qquad (28)$$

which is obtained from equations (6) and (25). These results are not in disagreement with those of Slepian and Hanna—they merely show that by slightly altering conditions, the superiority or inferiority of one type of horn compared with another can be greatly changed and the whole complexion of the problem is altered. We desire to point out these differences so that a better understanding of the problem will result and thus lead to improved types of horns.

The results and comparison given above apply only to the case where the source of sound is placed at the vertex of the cone. Hanna and Slepian's results apply particularly to the case where the initial and final openings of the cone are equal to those of the exponential horn. In both these cases our theoretical deductions are in entire agreement. Since, however, the simple theory does not take all factors into consideration, and since the cones with finite openings, other than zero, have optimum angles for maximum output, it seems that exponential data will throw much additional light on the subject.

For this purpose, therefore, two straight conical and exponential horns were constructed of heavy gauge galvanized iron so that the walls would not vibrate. The initial and final areas and the horn lengths were the same, the latter being about 120 cm. An ordinary standard loud speaker unit was used for the middle and high frequency ranges. Over the middle register, the exponential horn was distinctly superior in accordance with predicted results. Over the high frequency region, the two horns performed very much alike, which is also in accordance with the theoretical results of Hanna and Slepian.

Since superior low frequencies can be obtained from a unit with a low natural period, one was made with a natural period of about 300 cycles to test the horns at these frequencies. In the region of 80 to 200 cycles, the sound output from the conical horn was many times that from the exponential horn. The cut-off frequency of the exponential horn was calculated to be 135 cycles. With this same low frequency unit, the exponential horn became superior at about 200 cycles. These results, then, are in quite satisfactory qualitative agreement with the theoretical results which have been published by Hanna and Slepian and ourselves.

As stated in the beginning of this section, the above results include a review of existing literature, and some of our own results on horns of infinite lengths. We now proceed to consider some problems of horns of finite lengths.

4. Theory of Exponential Horns of Finite Length

The above discussion and the work which Dr. Slepian and Mr. Hanna have presented apply to horns of and tubes of infinite length. In these cases the sound energy indicated can be readily calculated provided certain simplifying assumptions are made. We have given these calculations for the straight tube, the straight cone, and for the exponential horn. The equations for the air-current, pressure, phase displacement, and energy radiated from the straight pipes and cones of infinite length are in accordance with the published work of Lord Rayleigh. To this material there has been added the case of the infinite exponential horn as a result of Webster's work and the extensions discussed above.

However, since we are called upon in practice to deal with horns and tubes of finite length, the extension of the theory for horns of unlimited lengths needs to be carried out to cover the cases of practical importance. As has been pointed out above, the foundation for this extension was laid by Professor A. G. Webster and published in general form with an application in condensed form to the special case of the straight pipe, the cone, the exponential horn, and the hyperbolic horn. These results were published in the "Proceedings for the National Academy of Sciences," pages 275-282, 1919. Professor G. W. Stewart[8] extended somewhat Webster's theory for the cone of limited length to include spherical waves and secured an equation which gave the ratio between the pressures at the vertex of the cone and those just outside the large opening of the horn in terms of its length and the pitch of the sound.

[8] Previous citation.

We have extended Webster's theory for the exponential horn of finite length and have secured among others an equation for such a horn giving the same pressure ratio in terms of length, rate of opening up, and frequency as obtained by Professor Stewart for the conical horn. Since the ear is a pressure-responding device and not a power-responding device, the results in pressure changes are perhaps the more useful of the two possibilities. The following presentation of the theoretical results obtained and a comparison of the exponential and the conical horns will be of interest.

For this purpose we start with Webster's equations on page 282 of his article. These equations correspond to equation (11) (or the more general equation (13) with the time factor) in Appendix I. The time factor may be omitted for our purpose for the present. If we correct Webster's equation for typographical errors and use the quantities which we have already defined, then the pressure equation at the two ends and at any intermediate place may be written as follows:

$$\left.\begin{aligned} P_1 &= \epsilon^{-\frac{m x_1}{2}} [A \cos g x_1 + B \sin g x_1] \\ P_2 &= \epsilon^{-\frac{m x_2}{2}} [A \cos g x_2 + B \sin g x_2] \\ \text{and} \qquad P &= \epsilon^{-\frac{m x}{2}} [A \cos g x + B \sin g x] \end{aligned}\right\} \qquad (29)$$

These equations, when combined with the time factor, include propagation in both directions. As shown in Appendix I, $A = A_1 + B_1$ and $B = (A_1 - B_1)\, i$ where A_1 and B_1 represent the maximum pressures in the direct and reflected wave, respectively. The method of analysis in working with these exponential equations is the same as that used by Webster in his work on conical horns. Since our results on exponential horns have not been published and since they are of considerable interest, we believe it is of value to present a summary of them at this time. As shown in Appendix II, we start with the above equations and derive equations for pressures, displacements, impedances, the coefficients A and B, A_1 and B_1, etc. These equations are given here rather in summary form. They are worked out in detail in the Appendix II.

Professor Webster defined acoustic impedance by:

$$Z = \frac{P}{X} \qquad (30)$$

rather than by P divided by $\frac{dx}{dt}$ according to the electrical analogy. X corresponds to the total displacement of fluid. For the general case Webster had for the acoustic impedance of the two ends of a horn:

$$Z_1=\frac{Z_2\,d-b}{-Z_2\,c+a}\text{ and }Z_2=\frac{Z_1\,a+b}{Z_1\,c+d}\tag{31}$$

We make use of equations (29) to determine the constants a, b, c, and d for the exponential horn and thus arrive at the expression for the acoustic impedances at the two ends of the horn. The proper adjustment of the impedances, according to Webster's viewpoint, at the two ends of the horn determine its efficiency in performing its functions of helping the radiation of sound from vibrating body, such as a diaphragm.

The mathematical work involved in determining the value of the constants a, b, c, and d is given in Appendix II. If the values given there are substituted in equations (31) we arrive at the following values for the acoustic impedances of the ends:

$$Z_1=\frac{Z_2\frac{\sigma_2}{\sigma_1}\left(\cos gL-\frac{m}{2g}\sin gL\right)-\frac{\beta K}{\sigma_1 g}\sin gL}{Z_2\frac{K\sigma_2}{g\beta}\sin gL+\cos gL+\frac{m}{2g}\sin gL}\tag{32}$$

$$Z_2=\frac{Z_1\left(\cos gL+\frac{m}{2g}\sin gL\right)+\frac{\beta K}{\sigma_1 g}\sin gL}{-Z_1\frac{K\sigma_2}{g\beta}\sin gL+\frac{\sigma_2}{\sigma_1}\left(\cos gL-\frac{m}{2g}\sin gL\right)}\tag{33}$$

All the quantities have been defined in the previous work. These equations are similar to those for the impedance of conical horns given by Professor Webster. A discussion of these equations and a comparison of them with the equations for the conical horns is deferred until the additional equations are derived.

If X_1 and X_2 represent the total air displacement at the small and large end of the horn, respectively, then, as shown by Webster:

$$P_2=a\,P_1+b\,X_1\text{ and }X_2=c\,P_1+d\,X_1\tag{34}$$

If we substitute the values for a, b, c, and d determined in Appendix II in these equations and place $X_1=0$ and $X_2=L$, then the pressure at the large end is given by*:

*In this and the following equations we may place

$$\varepsilon^{\frac{mL}{2}}=\sqrt{\frac{\sigma_2}{\sigma_1}}\text{ and }\varepsilon^{-\frac{mL}{2}}=\sqrt{\frac{\sigma_1}{\sigma_2}}$$

if we so desire.

$$P_2 = \epsilon^{-\frac{mL}{2}} \left[\cos gL + \frac{m}{2g} \sin gL \right] P_1 + \frac{K\beta \epsilon^{-\frac{mL}{2}}}{g\sigma_1} X_1 \sin gL \quad (35)$$

And the displacement at the large end is given by:

$$X_2 = -\frac{P_1 K \sigma_2 \epsilon^{-\frac{mL}{2}}}{g\beta} \sin gL + \frac{\sigma_2}{\sigma_1} \epsilon^{-\frac{mL}{2}} \left[\cos gL - \frac{m}{2g} \sin gL \right] X_1 \quad (36)$$

A discussion of these two equations is deferred also until additional relations are obtained for the exponential horn.

In Appendix II there has been derived the following equation which shows the relation between the pressure, P_1, at the small end of the exponential horn and the pressure, P_3, just outside the large end of the horn:

$$P_1 = \frac{P_3 \varepsilon^{\frac{mL}{2}}}{\cos gL + \frac{m}{2g} \sin gL + \frac{\sigma_2 K Z_0}{g\beta} \sin gL} \quad (37)$$

In this equation L is the axial length of the exponential horn, σ_2 is the area of the large opening, Z_0 is the acoustic impedance of the opening treated as a fictitious cylinder of definite length and an opening equal to the diameter of the large end. The equation for Z_0 is derived in the Appendiz II, equation (19). $\beta = \rho C K^2$ where ρ is the air density, C is the velocity of sound and $K = 2\pi$ divided by the wave length. g is defined by $\frac{1}{2}\sqrt{4K^2 - m^2}$. m is defined by the exponential area, σ, thus, $\sigma = \sigma_1 \varepsilon^{mx}$. Our treatment of the opening is the same as developed by Rayleigh. Both Webster and Stewart used the same scheme in their theoretical work on horns.

To study the direct and reflected waves separately and in combination we make use of the equations for A and B, and A_1 and B_1 derived in Appendix II. The following equations are derived for these four constants:

$$A = -P_1 \text{ and } B = \frac{P_1 \varepsilon^{-\frac{mL}{2}} \cos gL - P_2}{\varepsilon^{\frac{mL}{2}} \sin gL}$$

$$A_1 = \frac{1}{2}\left(-P_1 + \frac{P_1 i \varepsilon^{-\frac{mL}{2}} \cos gL - P_2 i}{\varepsilon^{-\frac{mL}{2}} \sin gL} \right) \quad (38)$$

and

$$B_1 = \frac{1}{2}\left(-P_1 - \frac{P_1 i \varepsilon^{-\frac{mL}{2}} \cos gL - P_2 i}{\varepsilon^{-\frac{mL}{2}} \sin gL} \right)$$

In our study with the direct and reflected waves we are to work with A_1 and B_1 and not with A and B. The two former quantities represent the maximum pressures in the direct and reflected pressure waves of the type A

$$A_1 \varepsilon^{-\frac{mL}{2}} \cos(nt - gx) \text{ and } B_1 \varepsilon^{-\frac{mL}{2}} \cos(nt + gx)$$

respectively. These equations hold only for the case where $x = 0$ and $x_2 = L$, that is, the zero of the abscissas begin at $x = 0$.

Since the pressure, P_1 equals $\rho a^2 s$ where s is the compression $\left(\text{which is equal to } \frac{dq}{dx}\right)$, we can make use of the pressure equations (29) as shown in Appendix II, to obtain the following expression for the total air displacement X, at any point in the horn:

$$X = -\frac{\sigma g \varepsilon^{-\frac{mx}{2}}}{\rho C^2 K^2}\left[A\left(\sin gx + \frac{m}{2g}\cos gx\right) - B\left(\sin gx - \frac{m}{2g}\cos gx\right)\right] \quad (39)$$

We are now in a position to make a comprehensive study of the exponential horn of finite length and compare the performance of this horn with that of the conical horn.

Discussion of Theory of Exponential Horn of Finite Length and Comparison to a Similar Conical Horn and Straight Pipe.

First, let us collect the equations we have derived for the exponential horn. We consider $X = 0$ and $X_2 = L$ for this summary.

Acoustic impedances:

Small end:

$$Z_1 = \frac{Z_2\left[\frac{\sigma_2}{\sigma_1}\left(\cos gL - \frac{m}{2g}\sin gL\right)\right] - \frac{\beta k}{\sigma_1 g}\sin gL}{Z_2\frac{\sigma_2 K}{g\beta}\sin gL + \cos gL + \frac{m}{2g}\sin gL} \quad (32)$$

and for the large end:

$$Z_2 = \frac{Z_1\left(\cos gL + \frac{m}{2g}\sin gL\right) + \frac{\beta k}{\sigma_1 g}\sin gL}{-Z_1\frac{K\sigma_2}{g\beta}\sin gL + \frac{\sigma_2}{\sigma_1}\left(\cos gL - \frac{m}{2g}\sin gL\right)} \quad (33)$$

Pressure and displacement at the large end in terms of corresponding quantities at small end:

$$P_2=\varepsilon^{-\frac{mL}{2}}\left[P_1\left(\cos gL+\frac{m}{2g}\sin gL\right)+\frac{K\beta X_1}{g\sigma_1}\sin gL\right] \quad (35)$$

and

$$X_2=\epsilon^{-\frac{mL}{2}}\left[\frac{\sigma_2 P_1 K}{\beta g}\sin gL+\frac{\sigma_2 X_1}{\sigma_1}\left(\cos gL-\frac{m}{2g}\sin gL\right)\right] \quad (36)$$

Pressure at small end in terms of pressure just outside large end:

$$P_1=\frac{P_3\varepsilon^{\frac{mL}{2}}}{\cos gL+\frac{m}{2g}\sin gL+\frac{K\sigma_2 Z_0}{g\beta}\sin gL} \quad (37)$$

For the pressure as a function of x we had:

$$P=\varepsilon^{-\frac{mx}{2}}(A\cos gx+B\sin gx) \quad (29)$$

where, as shown in Appendix II,

$$\left.\begin{aligned} A&=-P_1 \text{ and } B=-\frac{P_1\varepsilon^{-\frac{mL}{2}}\cos gL-P_2}{\varepsilon^{-\frac{mL}{2}}\sin gL}\\ A_1&=\frac{1}{2}\left(-P_1+\frac{P_1 i\varepsilon^{-\frac{mL}{2}}\cos gL-{}_2Pi}{\varepsilon^{-\frac{mL}{2}}\sin gL}\right)\\ &\text{for the direct waves and for the reflected waves:}\\ B_1&=\frac{1}{2}\left(-P_1-\frac{P_1 i\varepsilon^{-\frac{mL}{2}}\cos gL-P_2 i}{\varepsilon^{-\frac{mL}{2}}\sin gL}\right)\end{aligned}\right\} \quad (38)$$

Finally the displacement as a function of x is shown by equation (5′) in Appendix II:

$$X=-\frac{\sigma g}{\rho C^2K^2}\varepsilon^{-\frac{mx}{2}}\left[A\left(\sin gx+\frac{m}{2g}\cos gx\right)+B\left(\sin gx-\frac{m}{2g}\cos gL\right)\right] \quad (39)$$

The general equations (32) and (33) for the impedances may be considered from the practical importance of loud speakers. For this case the small end of the horn is closed. No energy, therefore, will be transmitted beyond the diaphragm closing the horn. A good method to employ in order to demonstrate that the receiver and the end of the horn is closed is to excite a long horn with a low frequency current thru the unit. The sound output will be quite marked if the frequency corresponds to one of the natural frequencies of the horn. If a small hole or crack is introduced at the connection between the horn and the receiver case, the low frequency sound output practically vanishes. The horn is now acting like one with both ends opened. It is astonishing to find that so small an opening will cause so marked an effect. The practical importance of exercising care in this direction is apparent.

We may follow Webster and Stewart, therefore, by placing the impedance, Z_1, at the small end of the horn equal to infinity. The impedance at the large end, then, can be written from equation (33) as follows:

$$Z_2 = -\frac{\cos g L + \frac{m}{2g} \sin g L}{\frac{K \sigma_2}{\beta g} \sin g L} \qquad (40)$$

Put $g = \frac{1}{2}\sqrt{4K^2 - m^2}$ and $\beta = \rho C^2 K$. We get, then,

$$Z_2 = \frac{\rho C^2}{2\sigma_2}\left[\sqrt{4K^2 - m^2} \cot \frac{L}{2}\sqrt{4K^2 - m^2} + m\right] \qquad (41)$$

We observe that the impedance of the large end varies inversely as its area. Since the acoustic impedance of the large end should be as small as possible in order to have a minimum reflection loss, it is necessary to have a large final opening. The above equation is a general one to cover the whole frequency range. We may simplify it if we desire to study special cases. For example, if we consider m^2 small in comparison with $4K^2$ as it will be for frequencies above about 1,000 cycles for practical values of m, then we may write equation (41) as follows:

$$Z_{2 \text{ high freq.}} = -\frac{\rho C^2}{2\sigma_2}\left[2K \cot K L + m\right] \qquad (42)$$

When $K L = \frac{n_2 \pi}{2}$ where $n_2 = 2, 4, 6$, etc., we have $\cot KL = \infty$;

and when $K L = \frac{n_1 \pi}{2}$ where $n_1 = 1, 3, 5, 7$, etc., we have $\cot K L = 0$.

The former case corresponds to high frequency anti-resonance where Z_2 is very large. The latter corresponds to:

$$Z_2 = \frac{\rho C^2 m}{2\sigma_2} \qquad (43)$$

which corresponds to high frequency resonance where Z_2 has its smallest value. The smallest value, then, is limited by the value of m and that of the final area. There is, then, even at these high frequencies a wide variation in the acoustic impedance of the exponential horn.

The special case where $\frac{1}{2}\sqrt{4K^2-m^2}$ approaches zero, which occurs at some definite low frequency, is of interest.

$cot\frac{L}{2}\sqrt{4K^2-m^2}$ can be replaced by $\frac{2}{L\sqrt{4K^2-m^2}}$. Substitute this in (41) and we get:

$$Z_2=\frac{\rho C^2}{2\sigma_2}\left[\frac{2}{L}+m\right] \tag{44}$$

Consequently, at these low frequencies near the critical frequency the acoustical impedance is a constant finite value and is determined by the final opening, length, and exponential coefficient, m, of the horn.

A comparison between the impedances for a straight pipe, the exponential, and the conical horns is of interest. Webster[9] showed that the impedance for a straight pipe is

$$Z_2=\frac{\beta}{\sigma}\left(\frac{Z_1 \cos Kl+\frac{\beta}{\sigma}\sin Kl}{-Z_1 \sin Kl+\frac{\beta}{\sigma}\cos Kl}\right) \tag{45}$$

If Z_1 is infinity, as in the case of the pipe closed at the end, then, when β is replaced by $\rho C^2 K$

$$Z_2=-\frac{\rho C^2 K}{\sigma}\cot Kl \tag{46}$$

This is the same as equation (42), provided m is zero and $\sigma=\sigma_2$. When m is zero, the exponential horn reduces to a straight pipe and the reduction of equation (42) for the exponential horn to the form for a pipe as given by Webster, speaks for the correctness of our equations. Z_2 for both the pipe and the exponential with a small m varies between wide limits corresponding to resonance and anti-resonance. In both types of apparatus, at the anti-resonance frequencies, the value of Z_2 becomes larger with increasing frequencies because of the factor K. This would tend to decrease the sound radiation more and more the higher the frequency.

A comparison of the acoustic impedance of the large end of the conical horn can be made with that for the exponential horn also. The acoustic impedance for the large end of the conical horn when the opposite end is closed by a receiver diaphragm is:[10]

$$Z_2=-\frac{\rho C^2 K}{\sigma_2}\left[\frac{\sin K(L+\varepsilon_1)\sin K\varepsilon_2}{\sin K(L+\varepsilon_1-\varepsilon_2)}\right] \tag{47}$$

[9] Previous citation 4.
[10] Previous citation 4.

This equation is derived from the first equation for Z_2 on page 280 of Webster's article by placing $Z_1 = \infty$. In this equation ϵ_1 and ϵ_2 are defined by $\tan K\epsilon_1 = k x_1$ and $\tan K\epsilon_2 = K x_2$. If $x_1 = 0$ then $x_2 = L$, the length of the horn. Also, if $x_1 = 0$, then $\tan K\epsilon_1 = 0$, and therefore $\epsilon_1 = 0$. When $x_2 = L$, $\tan K\epsilon_2 = KL$ or $K\epsilon_2 = \tan^{-1} KL$. Using these relations, (47) reduces to:

$$Z_2 = -\frac{\rho C^2 K}{\sigma_2}\left[\frac{\sin KL \sin(\tan^{-1} KL)}{\sin(KL + \tan^{-1} KL)}\right] \qquad (48)$$

The corresponding equation for the exponential horn is given by equation (41). The final opening of the two horns enter the equations the same way. The impedance of the conical horn and the straight pipe increases directly with the frequency; that of the exponential varies in practically the same way. Hence, at high frequencies the reflection losses will be large, due to too high an acoustic impedance for the three types of devices considered here,—each performing in essentially the same manner. Because of this, the horns are more efficient the lower the frequency.

We shall next consider equation (37). It may be written:

$$P_3 = P_1 \varepsilon^{-\frac{mL}{2}}\left[\cos\frac{1}{2}\sqrt{4K^2 - m^2}\,L + \frac{m}{\sqrt{4K^2 - m^2}}\sin\frac{1}{2}\sqrt{4K^2 - m^2}\,L + \frac{2\sigma_2 K Z_o}{\beta\sqrt{4K^2 - m^2}}\sin\frac{1}{2}\sqrt{4K^2 - m^2}\,L\right. \qquad (49)$$

The impedance, Z_o, of the large end treated as a tube of finite length radiating into the outside space as given by equation (19) in Appendix II is:

$$Z_o = \rho C^2 K^2\left(\frac{Ki}{2\pi} - \frac{1}{C_o}\right) \qquad (50)$$

In this equation C_o is the acoustic conductance of the opening as treated and defined in connection with the development of this equation in Appendix II. i is the square root of (minus one), and merely means that the vector sum of the two terms of equation (50) should be taken. The absolute value of Z_o would need to be used in equation (49). To use this equation we would need to know the value of C_o, the acoustic conductance of the open end treated in the manner stated. If R is the radius of the open end, then Rayleigh on page 181, volume 2, "Theory of Sound," gives

$$C_o = \frac{\pi R^2}{\frac{1}{4}\pi R}.$$

Experimental data (Rayleigh, page 202) indicated that

$$C_o = \frac{\pi R}{0.6}$$

which we can use in equation (50) to determine Z_o.

At the higher frequencies, m^2 can be neglected compared with $4K^2$. If $m^2 = 0.005$, then when $4K^2 = 0.05$ or $K = 0.11$, corresponding to about 600 cycles. Equation (49) can be written:

$$P_{3\,\text{high freq.}} = P_1 \epsilon^{-\frac{mL}{2}} \left[\cos KL + \left(\frac{m}{2K} + K\sigma_2 \left(\frac{Ki}{2\pi} - \frac{1}{C_o} \right) \right) \sin KL \right] \quad (51)$$

To get this equation from (49) we made the following substitution

$$\frac{2K\sigma_2 Z_o}{\beta\sqrt{4K^2 - m^2}} = K\sigma_2 \left(\frac{Ki}{2\pi} - \frac{1}{C_o} \right)$$

which is legitimate for the high frequencies which we are considering. In a typical example of practical value, $\frac{1}{C_o}$ will be negligible compared with $\frac{K}{2\pi}$, σ_2 will be perhaps 2,000 sq. cm., in which case $\frac{m}{2K}$ will be negligible compared with $\frac{\sigma_2 K^2}{\pi}$.

Introducing these restrictions in (51) we get:

$$P_3 = P_1 \epsilon^{-\frac{mL}{2}} \left[\cos KL + \frac{\sigma_2 K^2}{\pi} \sin KL \right] \quad (52)$$

The coefficient of the second term is large compared with that of the cosine term and the pressure just outside the horn shows large increase over that at the small end due to the phenomenon of resonance. The minimum value of P_3 will be determined by $\cos KL$, and will occur at frequencies corresponding to 2, 4, 6, etc., times the fundamental and the maximum value of P_3 will occur at frequencies 1, 3, 5, 7, etc., times the fundamental. In this respect the pressure outside the large end of the exponential horn follows the changes in acoustic impedance of the large end but in the reverse direction. That is, increased pressure occurs with decrease acoustic impedance and vice versa.

At the lower frequencies we must use the more general relation given in equation (49). This equation shows the same resonant and anti-resonant frequencies as did the restricted equations (51) and (52). In the latter case, however, there was a harmonic relation of the resonant frequencies. In the former case, however, the maxima and minima of the sine and cosine terms occur when $\frac{1}{2}\sqrt{4K^2-m^2}\,L=\frac{n\pi}{2}$, where n is 0, 1, 2, 3, 4, etc. This relation gives $\lambda=\frac{4\pi L}{\sqrt{n^2\pi^2+m^2L^2}}$ for the maxima and minima of the sine and cosine terms of equation (49).

The special case of $n=0$, that is, $\sqrt{4K^2-m^2}=0$ is of interest and has been discussed in connection with the impedance of the open end. We also encountered this case for the exponential horn of infinite length where the phase angle between the current and pressure became $\frac{\pi}{2}$ and the power radiation became zero. In the case of the pressures under discussion here, when $\sqrt{4K^2-m^2}$ approaches and finally becomes zero, all the terms in (49) remain finite and the pressures are therefore finite. When, however, $\sqrt{4K^2-m^2}$ becomes imaginary, as it does at low frequencies, then equation (49) becomes

$$P_3=P_1\varepsilon^{-\frac{mL}{2}}\left[\cosh\frac{1}{2}\sqrt{m^2-4K^2}L+\frac{m}{\sqrt{m^2-4K^2}}\sinh\frac{1}{2}\sqrt{m^2-4K^2}L+\frac{2\sigma_2 Z_o K}{\beta\sqrt{m^2-4K^2}}\sinh\frac{1}{2}\sqrt{m^2-4K^2}\,L\right] \qquad (53)$$

To arrive at this relation we substituted $\sin ix=i\sinh x$, $\cos ix=\cosh x$ and $\sqrt{4K^2-m^2}=i\sqrt{m^2-4K^2}$. We see, therefore, that the pressure, P_3 remains finite at these low frequencies, but varies continuously. That is, there are no resonant effects for frequencies below the critical frequencies. At the very low frequencies $4K^2$ can be neglected compared with m^2. In this case for a moderately long exponential horn with an m equal to about 0.07 the hyperbolic sines and cosines are practically equal. The coefficient of the second term reduces to unity, in which case the first two terms combine, but their coefficient is negligible compared with that of the third term. This leaves:

$$P_3=P_1\,\varepsilon^{-\frac{m2}{2}}\frac{2\sigma_2K^2}{m}\left(\frac{Ki}{2\pi}-\frac{1}{C_o}\right)\sinh\frac{mL}{2} \qquad (54)$$

where, for

$$\frac{KZ_o}{\beta\sqrt{m^2-4K^2}}$$

we are permitted for this case to substitute

$$\frac{K^2}{m}\left(\frac{Ki}{2\pi}-\frac{1}{C_o}\right)$$

This same equation (54) also is sufficiently accurate at the critical frequency because the first two terms are small in comparison with the third one. Consequently the variation in P_3 with frequency at these low values of frequency is determined by the change introduced because of K in equation (54). The lowest limit of P_3 is obtained when K is placed equal to zero, when the pressure P_3 becomes equal to zero.

As stated earlier in the paper, we are interested mainly in pressure changes because the ear itself responds to pressure changes in the air and also because we are able to make experimental tests on such pressure changes. For these reasons in the discussion of our theoretical work we have given much space to studying these effects. Furthermore, the amount of sound energy radiated into the surrounding space is directly associated with the acoustic impedances of the large end of the horn. It is for this reason that much consideration has been given to a study of acoustic impedances.

It is of interest to compare Professor Stewart's pressure equation for the conical horn with our equation for the exponential horn. He had (equation 2, page 322, "Physical Review," 1920):

$$P_1 = \frac{P_3}{\dfrac{\sin Kr}{Kr} + \dfrac{\sigma_2 \sin K(r-\varepsilon)}{r \sin K\varepsilon}\left(\dfrac{Ki}{2\pi} - \dfrac{1}{C_o}\right)} \tag{55}$$

where $\tan \epsilon = Kr$, r being the length of the horn. The other quantities involved in this equation are the same as used by us to derive equation (37). Reference may be made to equation (19) in Appendix II to see that we use the same expression for Z_o as used by Professor Stewart. Equation (55) shows the same type of resonant phenomenon as does our equation (37). In the conical horn equation, however, there is no "critical" frequency as found for the exponential case; neither is there an exponential factor as obtained in the exponential pressure equation. At very low frequencies (much below the fundamental of the horn) equation (55) can be replaced by:

$$P_3 = P_1\left[1 + \frac{\sigma_2 (r-\varepsilon)}{r\varepsilon}\left(\frac{Ki}{2\pi} - \frac{1}{C_o}\right)\right] \tag{56}$$

which corresponds to equation (54).

But, for these small values of Kr, $K\varepsilon = \tan^{-1} Kr$ can be replaced by $K\varepsilon = Kr$ or $\varepsilon = r$. Whence at all the very low frequency region where this condition is satisfied, equation (56) states that $P_3 = P_1$ and is independent of the horn. This is not the case for the exponential horn as shown above where $P_3 = 0$ at zero frequency.

At the high frequencies where Kr is large (say for all frequencies above 1,000 cycles), where r is about 200 cm., equation (55) can be modified. The term $\frac{\sin Kr}{Kr}$ can be neglected. Since Kr is large, $\tan K\varepsilon$ is large, and, therefore, $K\varepsilon$ can be placed equal to $\frac{\pi}{2}$. In this case $\sin K\varepsilon$ is essentially unity. Also, since $K\varepsilon = \frac{\pi}{2}$ it means that ε is about 7 or 8 at 1,000 cycles and becomes less with increasing frequencies. Consequently, if r is 200 cm. $(r-\epsilon)$ can be placed equal to r. With these simplifications equation (55) can be written

$$\frac{P_3}{P_1} = \frac{\sigma_2 \sin Kr}{r}\left(\frac{Ki}{2\pi} - \frac{1}{C_o}\right) \tag{57}$$

The higher the frequency, the more accurate this equation becomes, finally taking on the form,

$$\frac{P_3}{P_1} = \frac{\sigma_2 K \sin Kr}{2\pi r} \tag{58}$$

because of the relative smallness of $\frac{1}{C_o}$ at these high frequencies. Consequently, the resonant frequencies are harmonic and continue up to these higher frequencies. The performance, then, of the exponential and conical horns are quite similar at these high frequencies. A comparison of equation (52) with (58) shows both the similarities and differences at these high frequencies between the two types of horns.

A word as to the reflection effects and to the energy in the direct and reflected waves will also be of interest. A detail report of these effects is reserved for a future publication. For the present, however, we may call attention to the equations for the coefficient of the direct and reflected waves given in (38). From these equations the ratio of the pressure coefficients for the reflected and direct waves is:

$$\frac{B_1}{A_1} = \frac{P_1 \varepsilon^{-\frac{mL}{2}}[\sin gL + i\cos gL] - P_2 i}{P_1 \varepsilon^{-\frac{mL}{2}}[\sin gL - i\cos gL] + P_2 i} \tag{59}$$

But the absolute value of

$$\sin gL + i\cos gL = \sqrt{\sin^2 gL + \cos^2 gL} = 1$$

and the absolute value of

$$\sin gL - i\cos gL = \sqrt{\sin^2 gL + \cos^2 gL} = 1$$

so that:

$$\frac{B_1}{A_1} = \frac{P_1 \varepsilon^{-\frac{mL}{2}} - P_2 i}{P_1 \varepsilon^{-\frac{mL}{2}} + P_2 i} \tag{60}$$

If equation (35) is substituted for P_2 in terms of P_1 it will be possible to make a comparative study of the ratio of reflected to direct pressures. Other equations may be developed, but these are sufficient to indicate the direction the investigation takes.

A study of energy radiations at the large end can be made by means of equations (35 and (36). To do this, it is first necessary to secure the total "current" from the total displacement X_2. This is done merely by using the $\frac{d x_2}{d t}$ instead of X_2. It is necessary first to put in the time factor in both equations (35) and (36) corresponding to equation (13) in Appendix I. The results of this study, however, will be presented with experimental data in a future publication to which we have just referred.

APPENDIX I

Webster's Derivation of Equations for Exponential Horns

The general equation for wave motion in three directions is given by the familiar equation:

$$\frac{d^2 \phi}{d t^2} = C^2 \nabla^2 \phi = C^2 \left[\frac{d^2 \phi}{d x^2} + \frac{d^2 \phi}{d y^2} + \frac{d^2 \phi}{d z^2} \right] \tag{1}$$

ϕ is the velocity potential for sound waves. For plane waves we use

$$\frac{d^2 \phi}{d t^2} = C^2 \frac{d^2 \phi}{d x^2} \tag{2}$$

To derive his general equations for horns of various shapes, Professor Webster introduced a variable cross section, σ, varying as a function of x. Introducing σ in (2) we get:

$$\frac{d^2 \phi}{d t^2} = C^2 \left[\frac{1}{\sigma} \frac{d}{d x} \left(\sigma \frac{d \phi}{d x} \right) \right] \tag{3}$$

or

$$\frac{d^2 \phi}{d t^2} = C^2 \left[\frac{d^2 \phi}{d x^2} + \frac{1}{d x} \frac{d \sigma}{\sigma} \frac{d \phi}{d x} \right] \tag{4}$$

whence

$$\frac{d^2 \phi}{d t^2} = C^2 \left[\frac{d^2 \phi}{d x^2} + \frac{d (\log \sigma)}{d x} \frac{d \phi}{d x} \right] \tag{5}$$

For the exponential horn:

$$\sigma = \sigma_1 \varepsilon^{m x} \tag{6}$$

so that

$$\frac{d}{d\,x}(log\,\sigma)=m \tag{7}$$

Introduce this in (5) and we get:

$$\frac{d^2\,\phi}{d\,t^2}=C^2\left[\frac{d^2\,\phi}{d\,x^2}+m\frac{d\,\phi}{d\,x}\right] \tag{8}$$

For potentials of the harmonic type, such as $\phi \propto \varepsilon^{i\,n\,t}$ we have $\frac{d^2\,\phi}{d\,t^2}=-n^2\phi$. With this change equation (8) becomes

$$\frac{d^2\,\phi}{d\,x^2}+m\frac{d\,\phi}{d\,x}+K^2\phi=0 \tag{9}$$

where $K^2=\frac{n^2}{C_2}=\left(\frac{2\,\pi}{\lambda}\right)^2$ where λ equals the wave length.

To solve this equation we assume the ϕ is proportional to $\varepsilon^{a\,x}$ and substitute in (9). We get two roots—determined by $a^2+m\,a+K^2=0$—for a; namely

$$a_1=-\frac{m}{2}+\frac{i}{2}\sqrt{4\,K^2-m^2} \text{ and } a_2=-\frac{m}{2}-\frac{i}{2}\sqrt{4\,K^2-m^2}.$$

For propagation in both directions, therefore, the solution of (9) is:

$$\phi=A_1\,\varepsilon^{a\,x}\ +B_1\,\varepsilon^{a_2\,x} \tag{10}$$

or, in trigonometric form:

$$\phi=\varepsilon^{-\frac{m\,x}{2}}\left[A\,cos\,g\,x+B\,sin\,g\,x\right] \tag{11}$$

where $A=A_1+B_1$, $B=(A_1-B_1)i$ and $g=\frac{1}{2}\sqrt{4\,K^2-m^2}$.

We should observe that A_1 and B_1 take care of the direct and reflected waves, respectively, but A and B do not. However, we may introduce the time factor in (10) and thus obtain the general solution of equation (8). Thus

$$\phi=\varepsilon^{-\frac{m\,x}{2}}\left[A_1\varepsilon^{i\,g\,x}+B_1\,\varepsilon^{-i\,g\,x}\right]\varepsilon^{i\,n\,t} \tag{12}$$

Remembering that:

$$cos\,a\,x+i\,sin\,a\,x=\varepsilon^{i\,a\,x} \text{ and } cos\,a\,x-i\,sin\,a\,x=\varepsilon^{-i\,a\,x},$$

w get for (12) where only the real portion is retained:

$$\phi=\varepsilon^{-\frac{m\,x}{2}}\left[A_1\,cos(n\,t-g\,x)+B_1\,cos(n\,t+g\,x)\right] \tag{13}$$

The equation (11) was published by Webster in his article referred to. It is only a step to equation (13). The first factor takes care of the wave motion in the outward direction, while the second term represents the reflected wave. A comparison of A_1 and B_1, therefore, enables us to study the reflected and direct wave. When, therefore, we considered the propagation in one direction we studied a special case of Professor Webster's more general equations. This, of course, is what we (and Slepian and Hanna, independently) have done in section 2 of the paper.

APPENDIX II

Analysis of the Exponential Horn Based on Webster's General Method

The equations given in Appendix I are entirely due to Webster. The equations giuen in this section, altho based on Webster's method of analysis, are new and of considerable importance in the study of horns. For this reason we believe it is worth while to present here a summary of this work.

If P, P_1, and P_2 are the sound pressures at any point along the horn, at the initial end and at the final or large end, then according to equation (11) of Appendix I, we may write:

$$P=A\,u+B\,v,\; P_1=A\,u_1+B\,v_1, \text{ and } P_2=A\,u_2+b\,v_2 \qquad (1)$$

where $u\ (g\,x)$ and $v\ (g\,x)$ are independent solutions of equation (9) in Appendix I. The pressure, P, is equal to $-\rho\,C^2\frac{d\,q}{d\,x}$ where q is the displacement of air, ρ the density and C the velocity of sound. We can make use of this relation to determine equations for q. A comparison of (1) with equation (11) shows that

$$u=\varepsilon^{-\frac{m\,x}{2}}\cos g\,x \text{ and } v=\varepsilon^{-\frac{m\,x}{2}}\sin g\,x \qquad (2)$$

If we write, therefore,

$$p=-\rho\,C^2\frac{d\,q}{d\,x}=A\,\varepsilon^{-\frac{m\,x}{2}}\cos g\,x+B\,\varepsilon^{-\frac{m\,x}{2}}\sin g\,x \qquad (3)$$

we obtain

$$-\rho\,C^2\,q=A\int\varepsilon^{-\frac{m\,x}{2}}\cos g\,x\,d\,x+B\int\varepsilon^{-\frac{m\,x}{2}}\sin g\,x\,d\,x \qquad (4)$$

whence

$$\rho\,C^2\,q=-\frac{A\,g}{K^2}\varepsilon^{-\frac{m\,x}{2}}\left[\sin g\,x+\frac{m}{2\,g}\cos g\,x\right] -\frac{B\,g}{K^2}\varepsilon^{-\frac{m\,x}{2}}\left[\sin g\,x-\frac{m}{2\,g}\cos g\,x\right] \qquad (5)$$

Parenthetically, we may include the total displacement

$X\sigma = q\,g$) by means of the following equation obtained by multiplying the two sides of (5) by σ and dividing by $\rho\,C^2$. Thus,

$$X = -\frac{g\,\sigma}{\rho\,C^2 K^2}\varepsilon^{-\frac{m x}{2}}\left[A\left(\sin g\,x + \frac{m}{2g}\cos g\,x\right) + B\left(\sin g\,x - \frac{m}{2g}\cos g\,x\right)\right] \tag{5'}$$

On the other hand if we differentiate (2) with respect to $(g\,x)$, (not x) we find that equation (5) can be written:

$$\beta\,q = A\,u^1 + B\,v^1$$

and therefore $\beta\,q_1 = A\,u_1{}^1 + B\,v_1{}^1$ and $\beta\,q_2 = A\,u_2{}^1 + B\,v_2{}^1$ (6)

where $\beta = \rho\,C^2\,K$. Equations given in (1), (2), and (6) are used in developing the results which are to follow.

For convenience of manipulation the following determinants are defined and used:

$$\left.\begin{aligned} D_1 &= \begin{vmatrix} u_1 & v_1 \\ u_1{}^1 & v_1{}^1 \end{vmatrix}, \quad D_2 = \begin{vmatrix} u_2 & v_2 \\ u_2{}^1 & v_2{}^1 \end{vmatrix}, \quad D_3 = \begin{vmatrix} u_1 & v_1 \\ u_2{}^1 & v_2{}^1 \end{vmatrix} \\ D_4 &= \begin{vmatrix} u_2 & v_2 \\ u_1{}^1 & v_1{}^1 \end{vmatrix}, \quad D_5 = \begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}, \quad D_6 = \begin{vmatrix} u_1{}^1 & v_1{}^1 \\ u_2{}^1 & v_2{}^1 \end{vmatrix} \end{aligned}\right\} \tag{7}$$

With these definitions, the following four constants are defined:

$$a = \frac{D_4}{D_1}, \quad b = \frac{\beta\,D_5}{\sigma_1 D_1}, \quad c = -\frac{\sigma_2\,D_6}{\beta\,D_1}, \quad d = \frac{\sigma_2\,D_3}{\sigma_1\,D_1} \tag{8}$$

where σ_1 represents the small opening and σ_2 the large one of the exponential horn defined by $\sigma = \sigma_1\,\varepsilon^{m x}$. Using the equations given by (2) we find from (7) that:

$$\left.\begin{aligned} D_1 &= \frac{g}{K}\varepsilon^{-m x_1}, \quad D_2 = \frac{g}{K}\varepsilon^{-m x_2} \\ D_3 &= \frac{g}{K}\varepsilon^{-\frac{m}{2}(x_2+x_1)}\left[\cos g(x_2-x_1) - \frac{m}{2g}\sin g(x_2-x_1)\right] \\ D_4 &= \frac{g}{K}\varepsilon^{-\frac{m}{2}(x_2+x_1)}\left[\cos g(x_2-x_1) + \frac{m}{2g}\sin g(x_2-x_1)\right] \\ D_5 &= \varepsilon^{-\frac{m}{2}(x_2+x_1)}\sin g(x_2-x_1), \quad D_6 = \varepsilon^{-\frac{m}{2}(x_2+x_1)}\sin g(x_2-x_1) \end{aligned}\right\} \tag{9}$$

Substitute these in (8) and we find:

$$\left.\begin{aligned} a &= \varepsilon^{-\frac{m}{2}(x_2-x_1)}\left[\cos g(x_2-x_1) + \frac{m}{2g}\sin g(x_2-x_1)\right] \\ b &= \frac{K\beta}{g\sigma_1}\varepsilon^{-\frac{m}{2}(x^2-x_1)}\sin g(x_2-x_1) \\ c &= -\frac{K\sigma_2}{g\beta}\varepsilon^{-\frac{m}{2}(x_2-x_1)}\sin g(x_2-x_1) \\ d &= \frac{\sigma_2}{\sigma_1}e^{-\frac{m}{2}(x_2-x_1)}\left[\cos g(x_2-x_1) - \frac{m}{2g}\sin g(x_2-x_1)\right] \end{aligned}\right\} \tag{10}$$

In the general case Professor Webster showed that the acoustic impedances of the two ends are given by:

$$\left.\begin{aligned} Z_1 &= \frac{Z_2 d - b}{-Z_2 c + a} \text{ for the small end} \\ \text{and} \qquad Z_2 &= \frac{Z_1 a + b}{Z_1 c + d} \text{ for the large end} \end{aligned}\right\} \tag{11}$$

He derived the expressions for Z_1 and Z_2 for the conical horns. If we substitute our values of a, b, c, d, in equation (11) we arrive at the corresponding equations for the exponential horns. If we place $x_2 - x_1 = L$, the length of the horn, then we get for the impedances:

$$\left.\begin{aligned} Z_1 &= \frac{Z_2 \frac{\sigma}{\sigma_1}\left(\cos g L - \frac{m}{2g} \sin g L\right) - \frac{\beta K}{\sigma_1 g} \sin g L}{Z_2 \frac{K\sigma_2}{g\beta} \sin g L + \cos g L + \frac{m}{2g} \sin g L} \\ Z_2 &= \frac{Z_1\left(\cos g L + \frac{m}{2g} \sin g L\right) + \frac{\beta K}{\sigma_1 g} \sin g L}{-Z_1 \frac{K\sigma_2}{g\beta} \sin g L + \frac{\sigma_2}{\sigma_1}\left(\cos g L - \frac{m}{2g} \sin g L\right)} \end{aligned}\right\} \tag{12}$$

We now extend the analysis to secure a relation between the pressure, P_1, at the small end of the horn and that, P_3, just outside the large end. This analysis is similar to one used by Professor Stewart for the conical horn and published by him in the "Physical Review" for 1920, pages 313-326.

As the air passes from the end, X_2, to the surrounding space, it does so by effectively passing thru a short fictitious tube the length of which is ΔL, known as the end correction, and the diameter of which is that of the large end of the horn. At the initial end of this tube the pressure is P_2, and that at the other end is P_3. There is thus a change in pressure, $P_2 - P_3$, thru the length of this fictitious tube. If we call Z_o the acoustic impedance of this "tube," then by definition

$$X_2 Z_o = P_2 - P_3 = X_2 Z_2 - P_3 \tag{13}$$

where X_2 is the total displacement ($\sigma_2 q_2$) in the "tube," being the same as that at the end of the horn. From (13) we get

$$X_2 = \frac{P_3}{Z_2 - Z_o} \tag{14}$$

or

$$P_2 = \frac{P_3 Z_2}{Z_2 - Z_o} \tag{15}$$

which is a general relation for all horns. In Webster's paper he showed that for the general case:

$$P_2 = a\,P_1 + b\,X_1 \tag{16}$$

which we apply to the exponential horn by using the values determined above for the exponental case. From (15) and (16) we eliminate P_2, place $X_1 = \frac{P_1}{Z_1}$ and substitute the second equation of (11). This gives

$$P_1 = \frac{P_3}{a - c Z_o} \tag{17}$$

corresponding to one of Professor Stewart's equations. We substitute the values of a and c given in (10) and arrive at the following equation for the exponential horn; (in this equation we have placed $x_1 = 0$ and $x_2 = L$):

$$P_1 = \frac{P_3 \epsilon^{\frac{mL}{2}}}{\cos g\,L + \frac{m}{2g} \sin g\,L + \frac{\sigma_2 K Z_o}{g\beta} \sin g\,L} \tag{18}$$

Professor Stewart derived a similar equation for the conical horn which, in connection with our equation, is discussed in the paper. All the quantities involved in this equation have been defined. The equation cannot be made use of, however, until we can calculate Z_o.

This has been done by Lord Rayleigh in his second volume of "Theory of Sound." He showed that if air is passing thru a short tube the acoustic conductivity of which is C_o, the inertia of the air gives an apparent mass factor of $\frac{\rho}{C_o}$. He also showed that if air escapes (that is, sound is radiated) from a circular opening into the outide space, the energy radiated gives a dissipational pressure coefficient of $\frac{\rho\, n^2}{2\pi C}$ where n is 2π times the frequency, ρ is the air density, and C is the velocity of sound. The acoustic impedance, therefore, in the sense we are using the term, of such a tube radiating sound energy into the surrounding space is:

$$Z_o = -\frac{\rho\, n^3}{C_o} + i\,\frac{\rho\, n^3}{2\pi C} = \rho\, C^2 K^2 \left(\frac{K\,i}{2\pi} - \frac{1}{C_o}\right) \tag{19}$$

where $K = \frac{n}{C}$.

This reason for this equation will be clear if we consider the mechanical case for vibratory displacement, corresponding to acoustic displacement. In mechanics we have the familiar equation

$$m\frac{d^2\xi}{dt^2}+r\frac{d\xi}{dt}+s\xi=f \tag{20}$$

If $f=Fe^{int}$ and $\xi=\xi_o e^{int}$, then

$$\xi=\frac{F}{s-mn^2+irn} \tag{21}$$

The denominator of this equation represents the mechanical impedance and if s is zero, this impedance is given by:

$$Z_m=-mn^2+irn \tag{22}$$

In the acoustic analogy, m is replaced with $\frac{\rho}{C_o}$ and the resistance coefficient, r, is replaced with $\frac{\rho n^2}{2\pi C}$. In the acoustic definition the pressure is equal to the product of the air displacement and the acoustic impedance. Hence, $\frac{\rho}{C_o}$ is the pressure per unit acceleration and $\frac{\rho n^2}{2\pi C}$ is the pressure per unit velocity. Further reference to this work may be made to Rayleigh, "Theory of Sound," volume II, pages 193-194, equation 3 (where X is unity) and page 172, equation 1. Equation (19), then, can be used to determine Z_0 which occurs in equation (18).

To study the direct and reflected waves we use equation (1) from which we get:

$$A=\frac{\begin{vmatrix}P_1 & v_1\\ P_2 & v_2\end{vmatrix}}{\begin{vmatrix}v_1 & u_1\\ v_2 & u_2\end{vmatrix}} \quad\text{and}\quad B=\frac{\begin{vmatrix}P_1 & u_1\\ P_2 & u_2\end{vmatrix}}{\begin{vmatrix}v_1 & u_1\\ v_2 & u_2\end{vmatrix}} \tag{23}$$

Whence, if $x_1=0$ and $x_2=L$, and if we substitute equations (2) in (23), we get:

$$A=-P_1 \quad\text{and}\quad B=-\frac{P_1\varepsilon^{-\frac{mL}{2}}\cos gL-P_2}{\varepsilon^{-\frac{mL}{2}}\sin gL} \tag{24}$$

In Appendix I we saw that:

$$A=A_1+B_1 \quad\text{and}\quad B=(A_1-B_1)\,i \tag{25}$$

where A_1 is the maximum velocity potential of the direct wave and B_1 the maximum of the reflected wave. In a similar manner we can apply this relation to the maximum pressures produced by the direct and reflected waves in the pressure equation

$$P = A_1 \epsilon^{-\frac{mL}{2}} \cos(nt - gx) + B_1 \epsilon^{-\frac{mL}{2}} \cos(nt + gx) \qquad (26)$$

which is obtained in the same manner as equation (13) in Appendix I.

From equation (25) we get

$$\frac{B_1}{A_1} = \frac{Ai - B}{Ai + B} \qquad (27)$$

If we substitute the results of equation (24) in (27), we get, after reduction:

$$\frac{B_1}{A_1} = \frac{-iP_1 \epsilon^{-\frac{mL}{2}} \sin gL + P_1 \epsilon^{-\frac{mL}{2}} \cos gL + P_2}{-iP_1 \varepsilon^{-\frac{mL}{2}} \sin gL - P_1 \varepsilon^{-\frac{mL}{2}} \cos gL + P_2} \qquad (28)$$

In this equation the square root of (-1) occurs, so that in getting absolute values of the ratio of the reflected to the direct wave it is necessary to take the vector sum of (28). By means of the above ratio, we can study the reflection effects for the exponential horn.

From equation (25) we also get:

$$A_1 = \frac{1}{2}(A - Bi) \text{ and } B_1 = \frac{1}{2}(A + Bi) \qquad (29$$

In these we substitute equation (24) and get:

$$\left.\begin{aligned} A_1 &= \frac{1}{2}\left(-P_1 + \frac{P_1 i \varepsilon^{-\frac{mL}{2}} \cos gL - P_2 i}{\varepsilon^{-\frac{mL}{2}} \sin gL}\right) \\ &\text{and} \\ B_1 &= \frac{1}{2}\left(-P_1 - \frac{P_1 \varepsilon^{-\frac{mL}{2}} \cos gL - P_2 i}{\varepsilon^{-\frac{mL}{2}} \sin gL}\right) \end{aligned}\right\} \qquad (30)$$

Using the absolute values of these equations we are able to make a study of the maximum values for the pressures in the direct and reflected waves separately.

Appendix III

The parabolic horn offers an interesting contrast with the straight pipe, the conical, the exponential, and the hyperbolic horns. In the case of the parabolic horn the area varies directly as the distance x from the small end of the horn. The velocity potential can be written as a function of x and t, according to Webster.

$$\frac{d^2\phi}{dx^2} + \frac{d \log \sigma}{dx} \frac{d\phi}{dx} = \frac{d^2\phi}{dt^2} \qquad (1)$$

Since, in the parabola, $\sigma = \sigma_1 x$, then $\frac{d}{dx}(log\, \sigma) = \frac{1}{x}$.

Substitution in (1) gives for the parabola

$$\frac{d^2 \phi}{d x^2} + \frac{1}{x}\frac{d \phi}{d x} = \frac{d^2 \phi}{d t^2} \qquad (2)$$

This is a familiar equation met with in vibrating diaphragms, drum heads, and the like, and has been treated in a number of places.[1] It is solved by placing $\phi = X\, T$, X being a function of x only and T being a function of t only. Substitute in (2) and we get

$$\frac{d^2 T}{d t^2} + C^2 T = 0 \qquad (3)$$

and

$$\frac{d^2 X}{d x^2} + \frac{1}{x}\frac{d X}{d x} + C^2 X = 0 \qquad (4)$$

The general solution of the former is

$$T = A\, cos\, K\, at + B\, sin\, K\, at \qquad (5)$$

Equation (4) is the Bessel equation of zero order and its general solution is

$$X = C\, J_o(x) + D\, Y_o(x) \qquad (6)$$

Consequently the general solution of (2) is

$$\phi = [C\, J_o(x) + D\, Y_o(x)]\, [A\, cos\, Kat + B\, sin\, K\, at] \qquad (7)$$

The pressure is, $-\rho \frac{d \phi}{d t}$, or

$$P = -[C\, J_o(x) + B\, Y_o(x)]\, [-A\, Ka\, sin\, K\, at + B\, Ka\, cos\, K\, at] \qquad (8)$$

and the velocity is $\frac{d \phi}{d x}$, or

$$-V = -[C\, J_1(x) + D\, Y_1(x)]\, [A\, cos\, K\, at + B\, sin\, K\, at] \qquad (9)$$

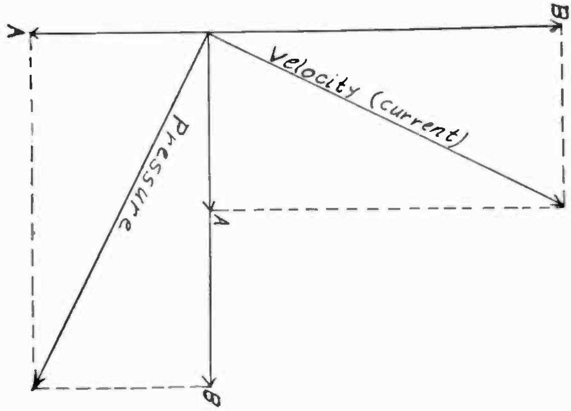
On the average, the energy radiation is proportional to the product of (8) and (9), which is zero because the pressure and velocity are always 90° out of phase. The accompanying figure shows the result diagramatically. Such a case as this is unusually interesting and is highly suggestive.

It is interesting to observe that equation (8) is identical to the solution of the general equation for the vibrating drumhead or diaphragm. There are, therefore, "modes" of vibration in the infinite parabolic horn corresponding to the roots of the Bessel functions just as there are modes of vibrations for a vibrating diaphragm with which we are familiar.

In case we have a parabolic horn of finite length, we can proceed as in Appendix II and derive a number of interesting results.

[1] Rayleigh, "Theory of Sound," volume I, page 318, article 200, equation (1), where W is independent of θ. Byerly, "Fourier's Series and Spherical Harmonics," page 12, article 11. Gray and Matthews, "Treatise on Bessel Functions," chapter X.

If we do this, we observe that $u = J_o(Kx)$, $u_1 = J_o(Kx_1)$, $u_2 = J_o(Kx_2)$, $v = Y_o(Kx)$, $v_1 = Y_o(Kx_1)$, $v_2 = Y_o(Kx_2)$, $u^1 = -J_1(Kx)$, $u_1^1 = -J_1(Kx_1)$, $u_2^1 = -J_1(Kx_2)$, $v^1 = -Y_1(Kx)$, $v_1^1 = -Y_1(Kx_1)$, and $v_2^1 = -Y_1(Kx_2)$.



With these results we can derive the expression for a, b, c, and d in equation (8) Appendix II. We can then derive all the equations for the parabolic horn of finite length. The resulting equations will all be of the Bessel function types and consequently the variations with frequency and distance will not be determined by the roots of the simple sine and cosine functions, but by the roots of the Bessel functions. The resonant effects, therefore, will be of a distinct type and unlike the usual ones which occur for horns of the usual curvature. Since the parabolic type of horn offers results the practical application of which require considerable imagination, it is felt that the detailed results need not be here given.

SUMMARY: (1) Experimental: By means of a vacuum tube oscillator to operate a loud speaker unit and a calibrated condenser transmitter, resistance-coupled amplifier, and a thermocouple and galvanometer, tests have been made at various frequencies to obtain the sound output from straight conical horns. The horns were made of heavy galvanized sheet iron and include lengths from 30 cm. to 305 cm. and final openings from 5 cm. to 90 cm. The results are plotted to show the sound pressure in dynes at the various frequencies up to 4,000 cycles.

Marked resonance was obtained at all the lower frequencies and the effect was still observable up to 4,000 cycles. The resonant peaks due to the horns at the lower frequencies are frequently as high as three or four times the response at the adjacent anti-resonant frequencies. The ratio approaches unity for all horns at the higher frequency range and for all the horns with the large openings there are not large resonant peaks. At the peaks the resonant is most marked for the horns with small openings, and the "valleys" are not "filled in." The larger solid angle, however, causes sufficient resonance at all frequencies above the fundamental to fill in the valleys and to cut off the sharpness of the peaks. The result is a more uniform sound output. For the

horns of each length and varying solid angle there is a particular solid angle and opening which give the optimum sound output. The best results were obtained with the longer horns of not too small or too large an opening.

Impedance analysis indicated marked effects of the horns on the diaphragm near the fundamental frequency of the diaphragm.

Dissipation due to sound radiation from the horns near the natural frequency of the diaphragm is about equivalent to the dissipation of the natural resistance of the diaphragm itself. This is not the case if the horn is not a good radiator of sound.

(2) Theoretical: Based on the assumption of plane waves, equations for pressure, velocity, sound radiation, and phase of the unlimited straight pipe and exponential and parabolic horns are reviewed or developed. The corresponding equations for the straight conical horns with the source at the vertex are reviewed and a comparison made among them. In the case of the parabolic horn the pressure and velocity are always out of phase and no sound energy is radiated. The velocity and pressure equations of a parabolic horn have solutions similar to a vibrating membrane and therefore the series of resonant and anti-resonant frequencies is determined by the maxima and minima of Bessel functions rather than by the maxima and minima of sine or cosine functions as in the pipe and exponential horn.

The equations for pressure and velocity show these equations to be in phase at all frequencies for the pipe; and sound radiation along it is independent of frequency. In the case of the cone with the source at the vertex and the exponential horn, these quantities are in phase for all the high frequencies and for this frequency region the sound radiation along the cone exceeds that along the exponential horn. Over the intermediate or middle frequency region the sound radiation along the exponential horn exceeds that along the cone. At the low frequency region the radiation along the cone again exceeds that along the exponential horn. Experimentally, these statements have been shown to hold at the low and intermediate frequencies for horns of finite initial openings. In this case, however, the horns are much alike at the high frequencies.

There is a finite frequency where the power radiation becomes zero and the velocity and pressure become 90 degrees out of phase for the exponential horn whereas this does not occur in the case of the cone until zero frequency is reached.

In the case of horns of finite lengths, Webster's method of treatment is extended to cover in detail the exponential horn and a comparison is made between it and the straight pipe and cone. The acoustic impedance of the large end is studied in detail and an equation is derived for the exponential horn to show the sound pressure just outside the large end in terms of the pressure at the receiver diaphragm. This equation shows all the phenomenon of resonance of exponential horns and can be compared with a similar equation for the conical horn derived by Stewart.

The solution of the parabolic horn of finite length has been worked out and briefly indicated.

Equations for the direct and reflected sound energy from the open end have been derived and stated in brief form. By means of these we are able to study end reflection in conjunction with the resonance phenomenon which is of great importance in our use of loud speaker horns as amplifiers of sound. In addition to resonance, the horn at the small end has a high acoustic impedance which gradually diminishes to a small value at the large end. In this way the horn greatly increases the sound radiation from a receiver diaphragm because it adjusts the acoustic impedance of the small end to match the unit and in the same way it adjusts the impedance at the large end to fit the surrounding air.

This study has led to a comprehensive understanding of the performance of loud speaker horns and will undoubtedly lead to new and improved types of loud speakers.

THE LIMIT OF REGENERATION*

By

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It is a well-known fact that the regenerative effect of a vacuum tube on an oscillatory circuit may be considered as the reduction of the effective resistance of that circuit. Further, if the regeneration is pushed too far, that is, if the coupling between grid and plate circuits is increased beyond a certain critical value, the effective resistance becomes negative and the system gives rise to self-sustained oscillations of a frequency corresponding to zero reactance. In the use of a regenerative system for radio reception, the borderland region between the purely regenerative state and that of spontaneous oscillation is of paramount importance. At the suggestion of Professor E. L. Chaffee, research was undertaken at Cruft Laboratory, Harvard University, to investigate the limit of pure regeneration and the conditions of stability at that limit.

The system studied consisted of a simple oscillatory circuit with the regeneration produced by the inductive coupling of a reaction coil in the plate circuit of the tube. The adjustment of this reaction coupling was made extremely fine. In fact, by employing two reaction coils, one of a comparatively large number of turns and one of a single turn, it was possible to vary the coupling by steps of one part in a million. By this means, it was possible to arrive at that critical adjustment just beyond which the system would give rise to self-sustained oscillations. This adjustment, however, was obtained only by extremely patient manipulation, first by increasing the reaction coupling in minute steps and then, after each increase, ascertaining that the system was not oscillating of itself. However, when this critical adjustment is obtained, the system has zero reactance and, as will be shown later, practically zero resistance to the frequency which it is desired to receive. The state of regeneration, corresponding to this critical adjustment, is the practical limit to which pure

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regeneration may be pushed in the laboratory and is called *critical regeneration.* In its other details the system studied was not unique. The grid and filament of the tube were connected, as usual, across the condenser of the oscillatory circuit. The load in the plate circuit was made as small as possible.

The preliminary work was done at a frequency of 1,000 cycles. By balancing, thru a series of mutual inductances, the electromotive force impressed upon the oscillatory circuit with that due to the alternating component of the plate current, a measure of the relative magnitudes of the impressed and received signals was obtained. Measurements were made at that limit of regeneration just beyond which the system gives rise to self-sustained oscillations, namely, at critical regeneration. Results obtained with VT-1 and a UV-201, both used with varying amounts of grid bias, show that *the relative magnitude of received signal in the plate circuit to that impressed on the grid is inversely proportional to the latter.* This is called the inverse signal strength law. It states that the response of a system adjusted to critical regeneration is independent of the strength of the impressed signal, that no matter how weak the oscillating field surrounding an antenna may be, if the regeneration is pushed to its limit, a finite signal may always be obtained.

It should be remembered, however, that this law applies only to the state of critical regeneration, a condition rarely obtained in engineering practice and only in the laboratory with extremely cautious adjustment. The effective resistance of the regenerative system under these circumstances, however, may be made so low that the temperature variations of the resistance in the inductances are appreciable in comparison. Transient phenomena, which under ordinary circumstances may be neglected, now may readily be observed, as the sound in the telephone receivers is heard to die away gradually upon the removal of the impressed signal. In fact, assuming the original sound in the telephone receivers to be a million times that of the sound just audible and also proportional to the square of the current in the oscillatory circuit, it is possible from the exponential law of current decay, to estimate the effective resistance of the system. At the lower frequency (1,000 cycles), where the inductance was of the order of 300 millihenrys with an initial resistance of 25 ohms, the effective resistance at critical regeneration was found to be 0.014 ohms. At the higher frequency (750 kilocycles), where the inductance was of the order of 0.2 millihenrys and the oscillatory circuit had an initial high frequency resistance from 1 to 10 ohms, this critical effective resistance was found to be only 0.00014 ohms.

The effective resistance measured by the above method is that which obtains when the potential impressed on the grid is very small. In order to measure the effective resistance when that potential is large, it is necessary to study the form of the resonance curve. For this purpose a radio-frequency carrier-wave completely modulated by a single audio frequency is used. It may be shown, both graphically and analytically, that for the resolution of a single band, that is, for the resonance curve of the regenerative system to show in addition to a central maximum two adjacent side maxima, it is necessary that the damping constant, $R/2L$, be less than 0.06 of the modulating frequency. In the resonance curves obtained, the side bands were easily detected. The corresponding effective resistance, however, was of the order of an ohm. Thus, it is seen that the effective resistance of a system using extreme values of regeneration is not a constant but a function of the potential applied to the grid.

That such must be the case is apparent at once when one considers that the effective resistance depends on the tube constants. Consider, for example, the internal plate-filament resistance. Characteristics of this, plotted against grid potential, consist of a series of curves concave upward with their maximum points moving toward the values of negative grid potential as the filament current or plate voltage is increased. It is at once apparent that the average or effective values of this tube constant, which obtains when the operating point moves over a finite portion of the arc of this characteristic, is greater than the static value corresponding to the mid-point of this arc. On the other hand, for the amplification factor with a characteristic concave downward, the effective operating value for large oscillations of the grid potential is less than that for small. The effect, then, of an increase in the potential variation applied to the grid is to increase the plate-filament resistance and to decrease the amplification factor of the tube of the regenerative system. These two changes result in an increase of the effective resistance of the system. Thus the discrepancy between the two values of the effective resistance is due to the non-linear characteristics of the tube.

Considering as a first approximation the characteristic curves to be of the second degree, it may be shown that the relation between current amplitudes and effective resistance is such as to give the inverse signal strength law mentioned above, and that the condition of critical regeneration is essentially stable, that is, an impressed signal will not set the system into a state of self-sustained oscillations.

In conclusion, the results of this study of regeneration give the following suggestions for the use of the regenerative method in practical cases: First, the reaction coupling as employed on the majority of commercial receiving sets does not permit of sufficiently fine adjustment. Such sets should be equipped with a vernier coupling coil of a single turn. By this means, the apparent instability due to overstepping the state of critical regeneration on account of the coarseness of the coupling adjustment, will be eliminated.

Second, the inverse signal strength law shows that the weaker signals will be as effective as the stronger ones in producing a response in the telephones. It is also well-known that a circuit with little resistance has a very sharp resonance curve. Thus, the regenerative method is ideal for selective radio-telegraph reception. For radio-telephony, however, the variation with signal strength will produce distortion and the peak of the resonance curve may be too narrow to include the complete band of frequencies it is desired to receive. For this reason, probably the extreme regeneration used in this investigation is not suited to commercial radio telephony. However, this research does suggest that better results will be obtained if the tube used for regenerating is not also used for rectifying. The rectifying properties of a tube depend upon the curve of its characteristic, and it is this very curvature which introduces the variation with signal strength. Thus, using a crystal rectifier in conjunction with a regenerating tube, operated on the linear portion of its characteristic and with the adjustment slightly below that of the state of critical regeneration, one should have a simple efficient method for radio-telephonic reception.

Finally, in regard to the sensitiveness of the method. A system with zero resistance should give an infinitely large response to its resonant frequency. Or, stating the fact in terms of radio-reception, a circuit adjusted for critical regeneration will give a finite sound in the telephones for an infinitely weak signal. It makes no difference to the regeneration whether the resistance for which it is compensating is due to the losses in the wire of the inductances and in the dielectric of the capacities, or to the energy which is re-radiated. In any case, at critical regeneration, the system is in a stable state with a net effective resistance, to an incoming signal of the proper frequency, of practically zero. Thus, other than that necessitated by the difficulty in arriving at the adjustment for critical regeneration, there is no limit to the sensitiveness of the simple regenerative method.

SUMMARY: It is shown that, at the limit of regeneration, the relative magnitude of received signal in the plate circuit to that impressed on the grid is inversely proportional to the latter. This inverse signal strength law is explained in terms of the characteristics of the tube. Two methods of determining the effective resistance of a regenerative system are outlined. Practical suggestions are offered for the design of regenerative systems.

ON THE CALCULATION OF THE INDUCTANCES AND CAPACITIES FOR A MULTI-RANGE OR OTHER CONSECUTIVE SERIES OF TUNED TRANSMITTING OR RECEIVING CIRCUITS, THE TOTAL RANGE AND ACCURACY REQUIRED BEING GIVEN*

By

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1. When a large range of natural frequencies (or wave lengths) has to be covered by one instrument, it is usual to use a number of capacities or inductances of fixed values in combination with a single variable capacity or inductance of comparatively small magnitude. The total range is thus divided into a series of consecutive scales and at the commencement of each scale a new fixed inductance or capacity, or both, is switched in and the scale is then covered by use of the variable. The values of these quantities have in general been arrived at by a process of "trial and error" and the possible accuracy of reading has varied from scale to scale.

In order to place the design of such an instrument on an engineering basis, the following method of calculation has been worked out. By its aid the design of a multi-range wavemeter, transmitter or receiver, to fulfil given requirements of range and accuracy, becomes a simple and straightforward matter.

2. Symbols used in this paper:

λ = wave length in meters.

$\lambda_1 \ldots \lambda_n$ = wave lengths at commencements of intermediate ranges (or scales).

f = frequency in cycles per second.

$f_1 \ldots f_n$ = frequencies at commencements of intermediate ranges.

L = inductance in microhenrys (mics.).

$L_1 \ldots L_n$ = inductances, fixed, at commencements of scales.

L_v = maximum value of variable inductance.

L_{vo} = minimum value of variable inductance.

* Received by the Editor, December 7, 1923.

L_u = portion of L_e required to cover scale without overlapping next scale.

L_F = actual fixed inductance $(L_F)_n = L_n - L_{vo}$.

C = capacity in milli-microfarads (mifs.).

C_n and so on = C with subscripts similar to L_n and so on.

$d°$ = smallest angular difference readable on scale of variable.

c = percentage accuracy, of λ or f.

b = minimum frequency difference, in cycles, to be readable.

a = a percentage of variable; $a/100$ = fraction of scale of variable used to cover a range without overlapping the next.

$(L\,C)_n$ = oscillation constant for commencement of nth scale.

3. Total Range—Let the total range of wave lengths to be covered be from λ_1 to λ_m, the corresponding frequencies being f_1 to f_m and $L\,C$ values $(L\,C)_1$ to $(L\,C)_m$. $(L\,C)_m$ is thus the maximum value on the scale commencing at $(L\,C)_{m-1}$, and there are only $(m-1)$ scales required.

4. Accuracy—Let the desired accuracy of wave length reading thruout the nth intermediate range be c_n percent of the shortest wave length λ_n of this range. This will ensure that the accuracy thruout the range λ_n to λ_{n+1}/λ is not less than c percent of the wave length read. The cases where accuracy is defined as a percentage of frequency, or as a difference of frequency, are considered in (6) and (7) below.

5. The possible accuracy depends on the ratio, λ_{n+1}/λ_n, of the wave lengths at the beginning and end of the scale, and on $d°$, the minimum angular difference readable with certainty on the scale of the variable condenser or inductance. In the case of a receiving circuit or wavemeter, $d°$ depends not only on the radius and graduation of the scale, but also on the damping of the circuit and type of resonance current indicator used.

6. Thus, in order to obtain the desired accuracy, we find that for a movement of $d°$ on the scale of the variable, λ_n may increase to $\lambda_n\left((1+\frac{c}{100}\right)$. In the whole scale of the variable, say 0° to 180°, there are $180/d$ such steps, assuming that the calibration curve of the variable is substantially a straight line. Each step represents an equal increase of wave length (see (4)); the accuracy therefore increases towards the upper ends of the scale. Hence, putting in the whole variable increases λ_n to

$$\lambda_n\left(1+\frac{180}{d}\frac{c}{100}\right).$$

Thus if the whole variable were used, we should have the end of the scale and commencement of the next at

$$\lambda_{n+1}=\lambda_n\left(1+\frac{1.8\,c}{d}\right).$$

However, for practical reasons it is necessary that successive scales should overlap. Let us therefore take only a percentage a (say 80 percent) of the variable scale as giving the whole intermediate range. Hence to provide for the overlap we find

$$\lambda_{n+1}=\lambda_n\left(1+\frac{180}{d}\frac{c\,a}{10{,}000}\right) \qquad \text{(A)}$$

and when the variable is being moved from $\frac{a}{100}\times 180°$ to $180°$ the scale will be overlapping the next in order.

We have, therefore, found that to obtain the desired accuracy of reading, the intermediate range commencing at λ_n must finish at the value of λ_{n+1} given in (A); and since (LC) values vary as the square of the wave length

$$(L\,C)_{n+1}=(L\,C)_n\left(1+\frac{180\,c\,a}{10{,}000\,d}\right)^2 \qquad \text{(B)}$$

7. If frequencies be reckoned instead of wave lengths

$$f_{n+1}=f_n\left(1-\frac{180\,c\,a}{10{,}000\,d}\right) \qquad \text{(C)}$$

Since the frequency decreases for increasing capacity, and

$$\frac{(L\,C)_{n+1}}{(L\,C)_n}=\frac{(1/f_{n+1})^2}{(1/f_n)^2}=\frac{f_n^2}{f_{n+1}^2}$$

Hence

$$\frac{(L\,C)_{n+1}}{(L\,C)_n}=\frac{1}{\left(1-\frac{180\,c\,a}{10{,}000\,d}\right)^2} \qquad \text{(D)}$$

and the limits of successive scales may be determined as for wave lengths.

8. If the accuracy be defined as an actual difference of frequency, b cycles (for instance 100 cycles), instead of a percentage difference, a system which is convenient when a heterodyne receiver is used, we have

$$f_{n+d°}=f_n-b$$

therefore

$$f_{n+1}=\left(f_n-b\frac{180\,a}{100\,d}\right) \qquad \text{(E)}$$

and $$\frac{(LC)_{n+1}}{(LC)_n}=\frac{f_n^2}{f_{n+1}^2}=\frac{f_n^2}{\left(f_n-b\frac{180\,a}{100\,d}\right)^2} \qquad \text{(F)}$$

So that in this case also the limit of each intermediate range can be found when the total range and accuracy have been given.

9. Since a variable condenser cannot be got to ge down to zero capacity, but has some lower limit C_{vo}, its rango is actually C_{vo} to C_v, if C_v be its maximum. C_{vo} is thus a fixed capacity which forms part of the calculated fixed capacity for the range. Thus the magnitude of the fixed condenser to be switched in at the beginning of the nth range is $C_F=C_n-C_{vo}$, where C_n is the calculated fixed capacity. In general, if C_v be chosen of any convenient dimensions, for instance 1 milli-microfarad, the fixed capacity and inductance to fulfil the desired conditions are then determinable as is shown in sections (10), (11), (12), and (13).

10. Determination of Fixed Capacities and Inductances, the Variable Being a Capacity—Let L_n and C_n be the fixed inductance and capacity for the range beginning at λ_n or $(LC)_n$. This may, for instance, be the first scale of the total range required, in which case λ_n is known. In other cases λ_n is determinable by working out the previous ranges starting with λ_1, or by the method of (23).

Then for the short-wave end of this scale

$$(LC)_n=L_n\,C_n$$

and at the other limit where the variable capacity

$$=C_w=(C_v-C_{vo})\,\frac{a}{100}$$

$$(LC)_{n+1}=L_n\,(C_n+C_w)$$

See (6).

Hence $$\frac{(LC)_{n+1}}{(LC)_n}=\frac{C_n+C_w}{C_n}$$

in which C_w and the (LC) values are known, gives C_n. The cases for different conditions of accuracy are given below.

11. If the accuracy be a percentage of wave length, as in (6)

$$\frac{C_n+C_w}{C_n}=\frac{(LC)_{n+1}}{(LC)_n}=\frac{\lambda_{n+1}^2}{\lambda_n^2}=\left(1+\frac{180\,c\,a}{10{,}000\,d}\right)^2$$

Hence $$C_n=\frac{C_w}{\left(1+\frac{180\,c\,a}{10{,}000\,d}\right)^2-1}=\frac{C_w}{\frac{1.8\,c\,a}{100\,d}\left(2+\frac{1.8\,c\,a}{100\,d}\right)} \qquad \text{(G)}$$

and since c, a, and d are known, this gives C_n and the fixed condenser required is $C_n - C_{vo}$. Also from $(LC)_n = L_n C_n$, we get L_n the fixed inductance.

12. If the accuracy be defined by percentage of frequency we get from (7) by a process similar to that of (11)

$$C_n = C_w \frac{\left(1 - \frac{180\,c\,a}{10{,}000\,d}\right)^2}{\frac{1.8\,c\,a}{100\,d}\left(2 - \frac{1.8\,c\,a}{100\,d}\right)} \qquad \text{(H)}$$

and L_n is calculable from $(LC)_n$ as in (11).

(13) If the accuracy be defined as a given change in number of cycles, $b \backsim$, we find from (8)

$$\frac{(LC)_{n+1}}{(LC)_n} = \frac{f_n^2}{\left(f_n - b\,\frac{180\,a}{100\,d}\right)^2}$$

Hence

$$C_n = C_w \frac{\left(f_n - b\,\frac{180\,a}{100\,d}\right)^2}{b\,\frac{180\,a}{100\,d}\left(2\,f_n - b\,\frac{180\,a}{100\,d}\right)} \qquad \text{(I)}$$

and for this case also the fixed capacity and inductance for each intermediate range are easily determinable. Examples of the calculation of several multi-range circuits are given in (16) and following.

14. Instead of using the equations (*G*), (*H*), or (*I*) to determine the fixed capacities and inductances for the various intermediate ranges, we may employ a graphical method. Thus a series of hyperbolas may be drawn in which L and C are coordinates, one for each value of $(LC)_n$ as determined in (6), (7), and (8), starting with the required initial (LC) of the shortest wave to be received or produced in the instrument. By inspection of each successive pair of curves (see Figure 1), a point P may be found on the curve of lesser (LC), such that C_w, measured parallel to the axis of C, will just reach the next curve at some point Q. The inductance L_1 and capacity C_1 of the point P are the fixed values required in order that the variable C_w may cover the first section of the total range with the desired accuracy. By repeating this process from curve to curve the fixed capacity and inductance for each range is obtained.

15. In certain circuits it is necessary that the ratio L/C should be greater or less than a certain limiting value, k, where $k = \frac{L}{C} = \frac{R^2}{4}$; for instance, in order to control the damping of the

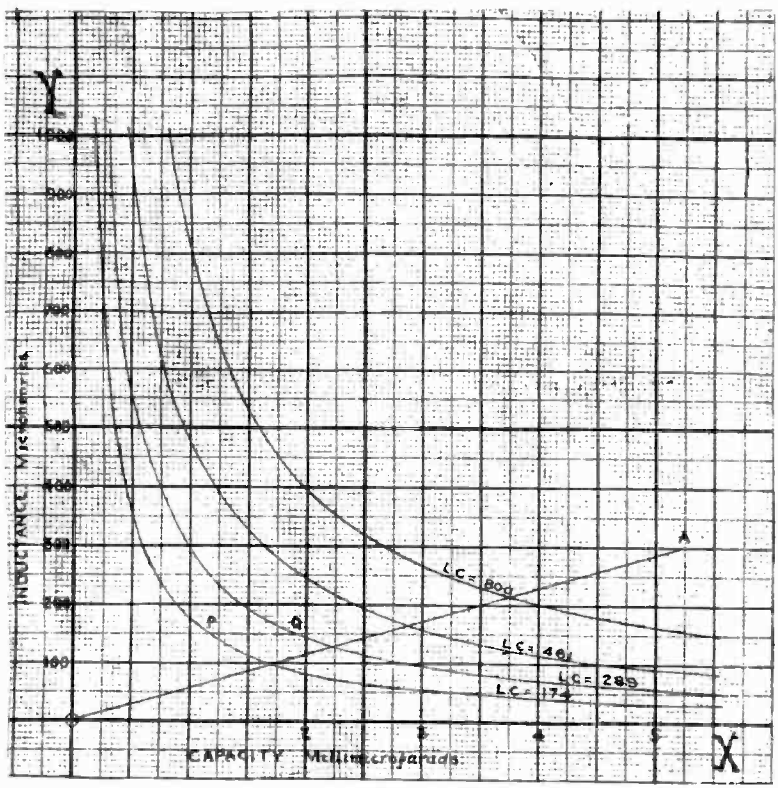

FIGURE 1

circuit and thus facilitate or hinder the production of oscillations. The graphical method of (14) lends itself readily to the introduction of such a condition. Thus if a line $O\,A$ be drawn from the origin of co-ordinates making an angle the tangent of which is k with the axis of capacity, each hyperbola will be cut in two by $O\,A$. If L/C is to be greater than k, only that part of the diagram between $O\,A$ and $O\,Y$ may be used, since for any point in this area $L/C>k$. If L/C is to be less than k the portion between $O\,A$ and $O\,X$ must likewise be used. In cases like these it will generally be convenient to defer the choice of the variable condenser or inductance until the curves have been examined.

16. THE VARIABLE AN INDUCTANCE—If a variable inductance be used instead of a capacity, the process of finding the values of fixed capacities and inductances can be carried out from the same curves by working on the other co-ordinate. The values of $(L\,C)_n$ are the same as in (14), since they depend only on c, a, and d. (See (6)). The algebraic method is also similar, and

if L_v, L_{vo} and L_w be the maximum, minimum and reduced maximum (see (6)), of the variable inductance, we obtain in place of the equations (G), (H), and (I), the values of the fixed inductance in terms of the variable as

$$L_n = \frac{L_w}{\frac{1.8\,c\,a}{100\,d}\left(2+\frac{1.8\,c\,a}{100\,d}\right)} \tag{G'}$$

for percentage wave length accuracy, and

$$L_n = \frac{L_w\left(1-\frac{1.8\,c\,a}{100\,d}\right)^2}{\frac{1.8\,c\,a}{100\,d}\left(2-\frac{1.8\,c\,a}{100\,d}\right)} \tag{H'}$$

for percentage frequency accuracy, and

$$L_n = L_w \frac{\left(f_n - b\,\frac{180\,a}{100\,d}\right)^2}{b\,\frac{180\,a}{100\,d}\left(2f_n - b\,\frac{180\,a}{100\,d}\right)} \tag{I'}$$

for a fixed frequency difference as accuracy.

17. EXAMPLE—As an example of the method let us assume that a series of circuits is required in accordance with the following data:

(I) Total range of wave lengths 800 m. to 2,000 m.
(II) Accuracy to be to 0.1 percent of λ, that is, 1 m. in 1,000 m.
(III) Smallest readable angle in scale $d^\circ = 0^\circ.5$.
(IV) Variable condenser $C_{vo} = 0.04$ mif. to $C_v = 1.06$ mif.
(V) $C_w = (C_v - C_{vo})\,\frac{80}{100} = 0.816$ mif.

From the condition (II) the limits of the intermediate ranges are determinable thus:

$$(L\,C)_2 = (L\,C)_1\left(1+\frac{1.8\,c\,a}{100d}\right)^2 = (L\,C)_1\,(1+0.288)^2 = (L\,C)_1\,(1.66)$$

To allow a margin let us start at 785 m. instead of 800 m., then

$(L\,C)_1 = 173.6$ mics.×mifs.
$(L\,C)_2 = 289.1$ mics.×mifs.
$(L\,C)_3 = 480.0$ mics.×mifs.
$(L\,C)_4 = 796.8$ mics.×mifs.

Now

$$C_1 = \frac{C_w}{0.288\,(2.288)} = \frac{0.816}{0.660} = 1.236 \text{ mifs from } (G),$$

therefore $C_F = C_1 - C_{vo} = 1.236 - 0.04 = 1.196$ mifs. and also $L_1 = 173.6/1.236 = 140.4$ mics. The others are as shown in the table. It should be noted that, as the accuracy is to be the same thruout, C_F has the same value for all ranges, as may be seen from equation (G). Table I shows the results of calculation for the example. The overlap of each pair of scales is found by taking $L_n\ (C_n + C_v - C_{vo})$ as the largest $(L\ C)$ of the scale.

Thus there will be four scales required to cover the range under the conditions stated. One fixed capacity will serve for all ranges and the inductance for each range is shown in the table.

TABLE I

Scale Number	$(L\ C)_n$ mic.×mif.	λ meters	C_n mifs.	L_n mics.	Other Numerics
1	173.6	785–1061	1.236	140.4	$C_F = 1.236 - 0.04 = 1.196$ mif.
2	289.1	1013–1360	1.236	234.0	$C_v = 1.06$ $C_F + C_v = 2.256$
3	480.0	1310–1763	1.236	388.5	$C_{vo} = 0.04$ $C_w = 0.816$
4	796.8	1686–2282	1.236	645.0	$d^\circ = 0.5$ $a = 80\%$ $c = 0.1\%$

18. If a variable inductance, instead of a variable condenser, be used in the instrument calculated in Table I, we find the same $(L\ C)$ values for the limits of the intermediate ranges of the calibration curve of the variable inductance is approximately straight. For the values of L_n we have the equation (G') of (15)

$$L_n = \frac{L_w}{\frac{1.8\,c\,a}{100\,d}\left(2 + \frac{1.8\,c\,a}{100\,d}\right)}$$

where, as before $c = 0.1$ percent, $a = 80$ percent, $d = 0^\circ.5$. Hence, if, for instance, $L_v = 140$ mics., and $L_{vo} = 15$ mics., then

$$L_w = \frac{80}{100}\left(L_v - L_{vo}\right) = 100 \text{ mics.},$$

from G',

$$L_1 = \frac{100}{0{,}288\ (2.288)} = 156 \text{ mics.}$$

and since the accuracy is to be the same thruout the whole range of the instrument $L_2 = L_1 = \quad . \quad . \quad . \quad ,$ and from the values of $(L\ C)$ calculated as for Table I, we find C_1, C_2, and so on.

TABLE II

Scale Number	(LC) mic.×mif.	λ meters	L_n mics.	C_n mifs.	Other Numerics
1	173.6	785–1054	156.0	1.112	$L_p = L_n - L_{ro}$ $= 156 - 15$ $= 141$ mics.
2	289.1	1013–1362	156.0	1.852	$L_w = 100$ mics. $L_p + L_s = 281$ mics.
3	480.0	1310–1750	156.0	3.078	$d° = 0°.5$ $c = 0.1\%$
4	796.8	1686–2267	156.0	5.110	$a = 80\%$

19. Similar calculations are as easily made for accuracy based on percentage of frequency by use of the equations (C), (D), (H), and (H').

20. The calculation for an accuracy based on fixed frequency dfference gives interesting results and illustrates how the dif-iculty of heterodyning at high frequencies may be overcome, at least theoretically, by the proper choice of ranges.

Let us assume the same total wave length range as before, that is, 800 m. to 2,000 m.—hence

(I) Total range of frequencies 375,000 – 150,000 cycles.

(II) Accuracy to be defined as a change of frequency of 100 cycles per 0°.5 thruout.

(III) Variable condenser 0.04 to 1.06 mifs. and $C_w = 0.816$ mifs.

Using equation (E) the scales come out as shown in Table III, the first scale commencing at 382,500 cycles in order to give a margin for constructional imperfections of the instrument. The overlap between successive scales is about 7,000 cycles in each case. The calculations are as follows, from (8), equation (E),

$$f_{n+1} = \left(f_n - 100\,\frac{180}{0.5}\,\frac{80}{100}\right) = (f_n - 28.800)$$

Commencing with scale (1), $f_1 = 382{,}500$ corresponding to 784.3 m.

$$f_2 = 382{,}500 - 28{,}800 = 353{,}700$$

and as the accuracy is to be the same thruout the total range, the difference is common and each f_n is found by subtracting 28,800 cycles from the one before. The Table III shows the results. From equation (I)

$$C_n = C_w \frac{(f_n - 28{,}800)^2}{28{,}800\,(2f_n - 28{,}800)}$$

Thus

$$C_1 = 0.816 \frac{(353{,}700)^2}{28{,}800\,(2\times 382{,}500 - 28{,}800)}$$

$$= 4.91 \text{ mifs.}$$

But $(L\ C)_1 = 174.2$, therefore $L_1 = 35.2$ mics., and the other *L's* and *C's* are similarly calculated. The overlaps of the scales are, as before, found by taking C_v instead of C_w to get the largest $(L\ C)$ value of each range.

TABLE III

Scale Numbers	Frequency Cycles per second	$(L\ C)_n$ mic. × mif.	C_n mifs.	L_n mics.	C_F mifs.	Other Numerics
1	382,500 −347,000	173.2	4.91	35.2	4.87	$C_v = 1.06$
2	353,700 −318,000	202.6	4.43	45.7	4.39	$C_w = 0.816$
3	324,900 −288,500	240.1	4.01	59.9	3.97	$C_{vo} = 0.04$ $d° = 0°.5$
4	296,100 −255,900	289.1	3.58	80.8	3.54	$b = 100\ \sim$ $a = 80\%$
5	267,300 −231,500	354.7	3.17	111.9	3.13	
6	238,500 −203,200	445.6	2.78	100.4	2.74	
7	209,700 −173,300	576.4	2.36	244.0	2.32	
8	180,900 −145,000	774.5	1.95	397.0	1.91	

It should be noted that in order to obtain the same heterodyne accuracy thruout it is necessary to change both fixed capacity and fixed inductance as one advances from one scale to the next.

21. If a variable inductance be used, the total range being the same, the calculations are similar to those of (20), but are based on equations (E) and (I') and give the results shown in Table IV. The intermediate ranges are the same as in Table III since they depend on the accuracy only. With the numerics given in the table the calculation of L_n takes the form

$$L_n = 100 \frac{(f_{n+1})^2}{28{,}800\,(2\times f_n - 28{,}800)}$$

and it is advisable to get these values correct to four figures, by using seven figure logarithms or a Fuller slide rule.

TABLE IV

Total Range, 375,000 ∾ to 150,000 ∾ = 800 m. to 2,000 m.

Scale Numbers	Frequency ∾	$(LC)_n$ mic.× mifs.	C_n mifs.	L_n mics.	L_F $L-L_{v_0}$	Other Numerics
1	382,500 −346,500	173.2	0.319	543	528	$L_w = 100$ mics.
2	353,700 −317,000	202.6	0.408	497	482	$L_{v_0} = 15$ mics.
3	324,900 −287,500	240.1	0.532	451	436	$L_v = 140$ mics.
4	296,100 −260,500	289.1	0.715	405	390	$a = 80\%$
5	267,300 −230,000	354.7	0.988	359	344	$b = 100$ ∾
6	238,500 −202,000	445.6	1.420	314	299	$d = 0°.5$
7	209,700 −173,000	576.4	2.148	268	253	
8	180,900 −144,500	774.5	3.485	222	207	

With these values a difference of 0°.5 on the scale of the variable gives 100 ∾ difference of frequency.

22. It may be noted that a useful check on the figures for C_n in Table III and L_n in Table IV may be obtained by plotting them as a curve with frequency as abscissa. In both cases the curve should be a straight line and any arithmetical error is shown up at once.

23. It is possible to find the number of scales required and the initial wave length of each, directly from the original data without successive calculation of each range. Thus, if the accuracy be uniform thruout, let

$$\left(1+\frac{180\,a\,c}{10{,}000\,d}\right)=k \qquad \text{as in (6) } (A),$$

therefore

$$\lambda_n = \lambda_1 (k)^{n-1} \qquad \text{(K)}$$

$$(LC)_n = (LC)_1 (k)^{2(n-1)} \qquad \text{(L)}$$

Hence from (K)

$$\log \lambda_n = \log \lambda_1 + (n-1) \log k$$

thus if λ_n and λ_1 be the given limits of the total range and if k be the same for all ranges, $(n-1)$, the number of scales, is determinable. More generally, if k vary from scale to scale

$$\lambda_n = \lambda_1 (k_1 \times k_2 \times k_3 \cdot \cdot \cdot)$$

and $$log\,\lambda_n = log\,\lambda_1 + log\,k_1 + log\,k_2 + \text{ and so on,}$$

hence λ_n may be determined directly from the conditions of accuracy. The same is true for $(L\,C)_n$, since

$$(L\,C)_n = (L\,C)_1 (k_1 \times k_2 \times \cdot \cdot \cdot)^2$$

$$log\,(L\,C)_n = log\,(L\,C)_1 + 2\,(log\,k_1 + log\,k_2 \cdot \cdot \cdot)$$

If the accuracy be defined by a difference of frequency as in (8) we have

$$(L\,C)_n = (L\,C)_1 \left[\frac{f_1^2}{f_n^2}\right] = (L\,C)_1 \left[\frac{f_1^2}{(f_1 - (n-1)\,\Delta)^2}\right]$$

where $\Delta = b\,\dfrac{180\,a}{100\,d}$ = difference in frequency between the initial frequencies of two successive ranges.

Thus the formulas of this section form an alternative method for the calculation of the limits of the intermediate ranges.

SUMMARY: It is often necessary to use a series of consecutive tunablr circuits to cover a large range of wave lengths or frequencies. The papee describes a method of determining the constants of each circuit when the total wave length range and the accuracy of setting required in each intermediate range are given. Overlap between successive scales is provided for, and a method is indicated by which further conditions, such for instance as a limiting value of the damping of each circuit, may be introduced. The paper thus reduces the calculations of such circuits from an empirical problem to a straightforward engineering proposition.

DIGESTS OF UNITED STATES PATENTS RELATING TO RADIO TELEGRAPHY AND TELEPHONY*

ISSUED MAY 6, 1924–JUNE 17, 1924

BY

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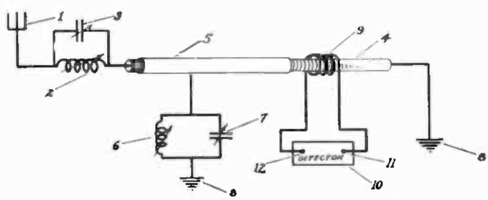
1,493,024—Louis Cohen and Joseph O. Mauborgne, filed August 6, 1920, issued May 6, 1924.

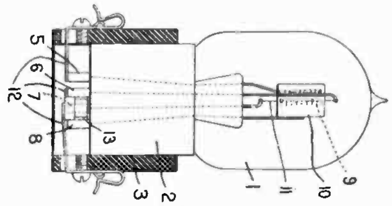


NUMBER 1,493,024—Radio Signaling

RADIO SIGNALING, utilizing a wave coil for the reduction of static disturbances and other interferences in the reception of radio signals. A long helix uniformly wound with wire is connected thru a tuning circuit with the antenna and to ground. The radio receiving apparatus is connected with the wave coil and an adjustable grounded metallic tube is provided in relation to the wave coil, which tube is electrically connected in a tuned circuit with ground. The adjustment of the metallic tube and the grounded loop circuit operates to change the electrical characteristics of the coil. Persistent waves set up in the coil will be received while foreign electrical disturbances which may act on the antenna, such as electrostatic effects causing a shock excitation being of a steep wave front are rapidly damped out in their travel thru the wave coil.

1,493,148—Montgomery B. Cohen, filed July 23, 1920, issued May 6, 1924.

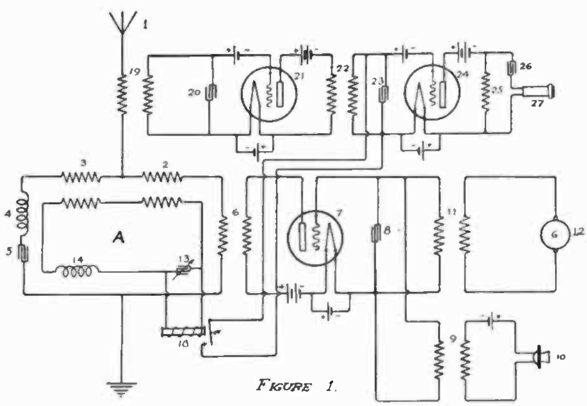
*Received by the Editor, July 18, 1924.



NUMBER 1,493,148—Fuse for Vacuum Bulbs

FUSE FOR VACUUM BULBS for protecting the filament from destructive effects of excessive current. The fuse consists of a cylindrical member which is applied to one of the bulb terminals and constructed to engage the corresponding socket contact.

1,493,151—Mihran M. Dolmage, filed September 17, 1920, issued May 6, 1924.



NUMBER 1,493,151—Radio Duplex Signaling System

RADIO DUPLEX SIGNALING SYSTEM which does not require the manual operation of switches to transfer the system from transmitting or receiving or vice versa. An auxiliary circuit tuned to the currents of the transmitting frequency is provided in association with the transmitting antenna. A control relay is connected in the auxiliary circuit in such manner that it is operated when currents exist in the auxiliary circuit. The operation of the relay causes the receiving apparatus to be protected against the transmitting currents during the periods of transmission.

1,493,696—Stanley R. Mullard, filed November 9, 1920, issued May 13, 1924. Assigned to the Mullard Radio Valve Company, London, England.

Thermionic or Oscillation Valve, wherein a cylindrical plate electrode is supported within the tube by a collar member which surrounds a re-entrant portion of the tube which extends into one end of the tube.

1,494,347—Lloyd Espenschied, filed April 30, 1920, issued May 20, 1924. Assigned to American Telephone and Telegraph Company, Incorporated, New York.

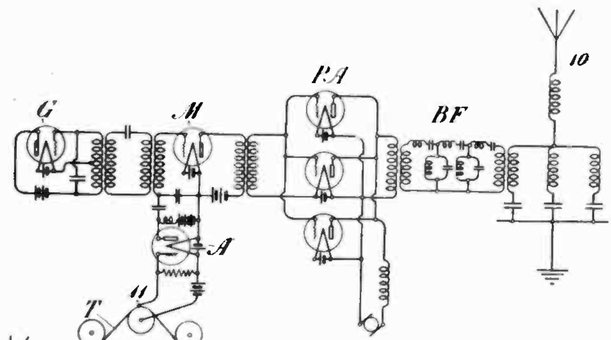


Fig. 1

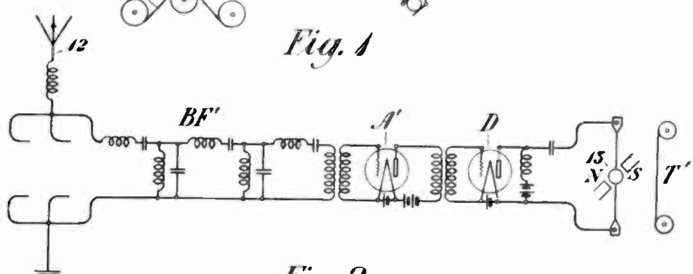
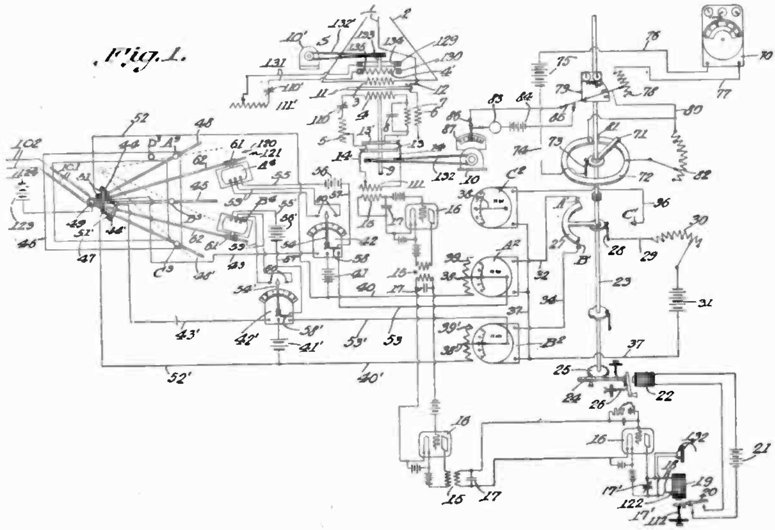


Fig. 2

Number 1,494,347—High Speed Radio System

High Speed Radio System for accommodating a large volume of traffic over wave bands in close proximity to each other. The system consists on modulating a carrier frequency in accordance with signaling current produced at a speed sufficiently high so that the band of frequencies resulting from modulation will be too wide for transmission thru a sharply tuned selective circuit. The modulated band is selected with substantially negligible and uniform attenuation while adjacent frequencies lying outside of the bands will be sharply discriminated against by the use of band filters. At the receiving station a light stream is modulated in accordance with detected current and then the modulations recorded upon a medium sensitive to light.

1,494,770—Walter W. Conners, filed June 12, 1919, issued May 20, 1924.

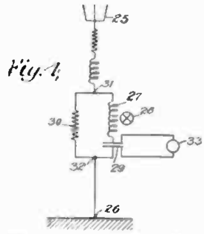


NUMBER 1,494,770—Method and Apparatus for Indicating the Geographical Location or Movement of Bodies

METHOD AND APPARATUS FOR INDICATING THE GEOGRAPHICAL LOCATION OR MOVEMENT OF BODIES by indicating in similitude or in intelligible signals the location of a moving body. The moving body carries a radio receiving station for the reception of signals from fixed wave-transmitting stations. The receiving station on the moving body is equipped with an indicating member which is controlled by received signals for indicating in similitude the signals received from the fixed stations. In this manner the course of the moving body is automatically plotted in accordance with the movement of the body with relation to the fixed stations.

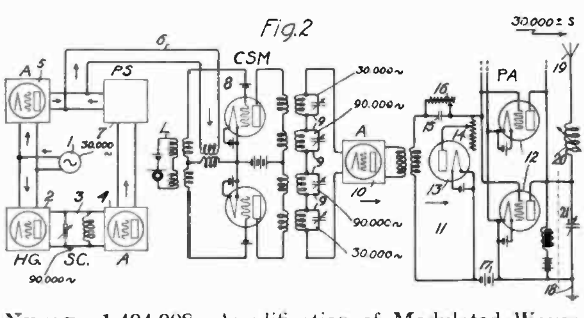
1,494,803—Michael I. Pupin, filed September 17, 1915, issued May 20, 1924. Assigned to Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pennsylvania.

ELECTRICAL TUNING apparatus by which the electrical resistance reaction which a conductor opposes to a simple harmonic electromotive force is rendered selective, that is to say, the conductor opposes a resistance reaction as small as desirable to an electromotive force of a given frequency, while at the same time this reaction may be made as large as desirable to an electromotive force of a different frequency or to electrical impulses. The invention is called a resistance compensator. It is shown as applied to a radio receiver for increasing the selectivity of the receiver by utilizing an antenna system of a resistance sufficient to dissipate the greater part of the incoming signaling energy. The incoming signaling energy of desired frequency is utilized to excite a source of energy by electromagnetic reaction and to transfer to the receiving antenna system a negative resistance reaction of high value as compared with the original resistance reaction of the conductor and of a frequency corresponding to the selected radio frequency to be received.



Number 1,494,803—
Electrical Tuning

1,494,908—Raymond A. Heising, filed February 23, 1923, issued May 20, 1924. Assigned to Western Electric Company, Incorporated, New York, N. Y.



Number 1,494,908—Amplification of Modulated Waves

Amplification of Modulated Waves for securing economy in operation of high power systems. The amplification system may be applied to a system which includes a source of carrier waves, a device for deriving from the source a triple frequency component, amplifying this component and combining it with waves from the original source to produce a peaked wave; a carrier suppression modulating device for modulating the peaked wave thus produced in accordance with a low frequency wave; a voltage step-up or low power amplifier for the modulated wave; and a vacuum tube power amplifier having a grid so negatively polarized that the amplifier is actuated only by the peak portion of each cycle of the modulated peaked wave. The repeater derives from the modulated wave a triple frequency component of large energy where the energy is chiefly in side frequencies based upon the fundamental frequency all of which energy is efficiently increased in amplitude thru the amplification system of this invention.

1,494,935—Edward O. Scriven, filed July 16, 1918, issued May 20, 1924. Assigned to Western Electric Company, New York, N. Y.

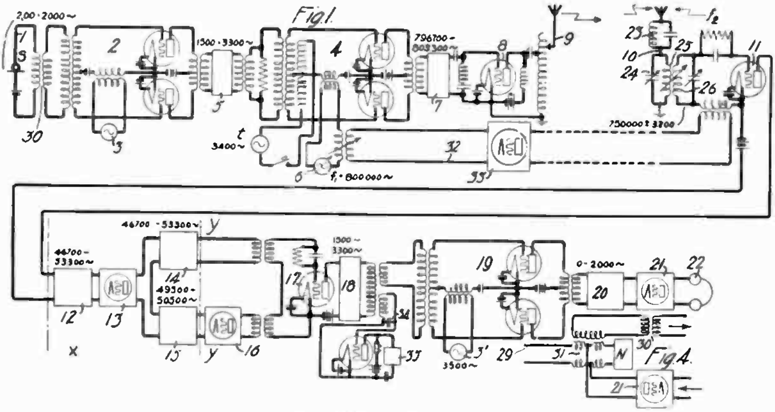
ALTERNATING CURRENT SOURCE where the frequency of oscillations generated is governed by a tuned oscillation circuit whereby the oscillator will generate substantially pure sine waves. An electron tube circuit is provided with the input and output circuit associated for feeding back energy from the output circuit to the input circuit. An additional circuit is interposed in the electron tube circuits for substantially preventing the feeding back of energy oscillations of any frequency other than that which is desired to generate by means of the electron tube oscillator.

1,495,537—Stephen F. Stafford, filed August 21, 1923, issued May 27, 1924. Assigned to Stafford Radio Company, Incorporated, of Massachusetts.

DOUBLE HELIX CAGE ANTENNA for indoor use in radio reception. The antenna consists of two helices wound in opposing relation with respect to each other with one nested within the other providing a strong cage antenna.

1,495,470—John F. Farrington, filed February 1, 1922, issued May 27, 1924. Assigned to Western Electric Company, Incorporated, of New York, N. Y.

HIGH FREQUENCY TRANSMISSION for multiplex and duplex operation where mechanical switching for two-way conversation is unnecessary. The system utilizes partial carrier wave suppression wherein the incoming and outgoing waves combine to produce an auxiliary carrier wave at each station. The transmitting system operates to transmit only feeble energy of the unmodulated component. The unmodulated component is separated from its accompanying side frequencies at a point in the system where the total energy is small relative to the transmitted energy. The energy of the selected unmodulated component is increased at the receiver and combined at the detector with the accompanying side frequencies to operate the receiving telephone.



NUMBER 1 495,470—High Frequency Transmission

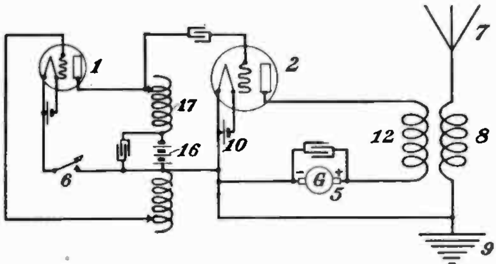
1,495,511—Robert J. Fitzgerald, filed May 12, 1920, issued May 27, 1924. Assigned one-half to J. Arthur Fischer, New York, N. Y.

CONDENSER of the rotary variable construction, wherein the movable and stationary metallic plates have an outside insulating coating thereover. The insulating coating of each of the plates is adapted to contact one with the other preventing the matallic plates from shunting each other.

1,495,577—Francis W. Dane, filed April 3, 1920, issued May 27, 1924. Assigned to Wireless Specialty Apparatus Company, Incorporated, Boston, Massachusetts.

ELECTRICAL CONDENSER of the stacked type where alternate sheets of mica and metal foil are placed within a casing with a resilient spring member compressing the stack under a pressure of the order of thousands of pounds while the stack is submerged in insulating material which is solid at ordinary temperatures, but fluid at a temperature of the order of about 140 degrees centigrade. The construction described avoids the presence of air bubbles and forces the mica and foil in intimate contact with each other, securing high capacity but with high insulation properties.

1,495,593—Lewis M. Hull, filed October 16, 1920, issued May 27, 1924.



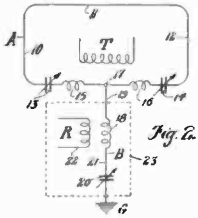
NUMBER 1,495,593—Radio Telegraph Transmitting System

RADIO TELEGRAPH TRANSMITTING SYSTEM having an electron tube amplifier whose plate-cathode circuit is operatively coupled to the transmission circuit. A self-excited electron tube generating system is so connected to the grid-cathode circuit of the amplifier that the inherent intra-electrode capacity of the said grid-cathode circuit furnishes capacity reactance for the operation of the self-excited electron tube generating system. By this system the unavoidable intra-electrode capacity of the amplification system is transformed from a wasteful absorber of electrical power into an essential tuning element, and its use as a tuning element allows a reduction in the minimum capacitive reactance attainable in the oscillatory circuits, thus increasing the maximum frequency to which these circuits are resonant.

1,495,616—Frederick G. Simpson, filed May 23, 1923, issued May 27, 1924.

ART OF MEASURING THE VELOCITY OF TERRESTRIAL BODIES RELATIVE TO EACH OTHER by applying the Doppler-Fitzeau principle, force or effect, as manifested thru the medium of electromagnetic waves. A method is described for measuring the velocity of a terrestrial body with respect to another terrestrial body which consists in comparing the frequency of generation of a generator of electromagnetic waves, at rest with respect to one of the bodies, with the frequency of the electromagnetic waves radiated by such generator as measured at a point at rest with respect to the other body.

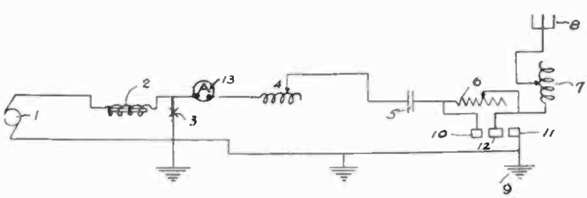
1,496,155—F. J. Fransson, filed November 8, 1919, issued June 3, 1924. Assigned to American Gas Accumulator Company, Elizabeth, New Jersey.



Number 1,496,155—Art of Radio Communication

Art of Radio Communication, in which a combination loop antenna and open antenna is employed for duplex operation. A transmitter is coupled to one of the antennas while the receiver is coupled to the other antenna with condensers arranged for balancing out interference between the transmitter and receiver during duplex operation.

1,496,214—L. F. Fuller and H. F. Elliott, filed May 22, 1919, issued June 3, 1924. Assigned to Mesne Assignments to Federal Telegraph Company, San Francisco, California.

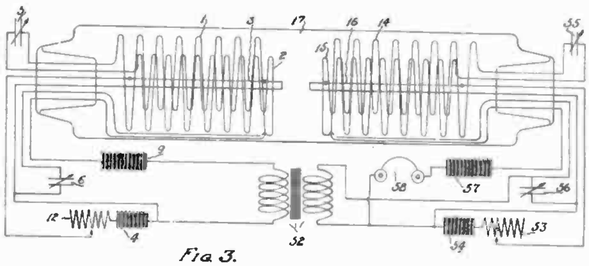


Number 1,496,214—Radiotelegraphy Transmission System

Radiotelegraphy Transmission System, employing an arc with a signaling circuit for transmitting with a single wave. A circuit is employed to replace the antenna system during periods of no radiation. The circuit consists entirely of inductive and capacitive reactance neutralizing each other or tuned to the frequency of the system, and connected in series with the antenna. Thus, the total effect of this part of the circuit on the antenna circuit is negligible, and across the terminals of the inductance and capacity a minimum potential difference is obtained for currents of the correct frequency. By proper arrangement of a resistance or other consumer of energy and a switching means, the substitute circuit can then be made to have substantially the same characteristics as the antenna circuit, so that the load on the source remains the same, independent of choice of paths.

In this manner the telegraph signals are produced without radiation of energy between signals.

1,496,243—F. S. McCullough, filed July 7, 1919, issued June 3, 1924. Assigned to G. L. Martin of Cleveland, Ohio.



NUMBER 1,496,243—Radio Telegraphy

RADIO TELEGRAPHY in which a compact form of apparatus is employed for the reception of radio signals particularly desirable for use on aircraft or in places where space is an extremely important factor. The collector coils which form the means for intercepting the radio frequency energy are wound within the evacuated tube which contains the electronic emission elements connected in the receiving circuit.

1,496,275—P. Laut, filed October 18, 1920, issued June 3, 1924.

RADIO TRANSMITTING APPARATUS employing an arc generator and a signaling circuit for producing uni-wave signals. The field coils of the arc are excited from an independent source. The antenna circuit is completely broken between signals enabling the arc to operate upon an auxiliary shunt circuit eliminating the compensation wave.

1,496,276—H. P. Rees, filed August 9, 1921, issued June 3, 1924.

RADIO TELEGRAPHY AND TELEPHONE RECEIVING SET containing constructural features for the reduction of size of the apparatus. The antenna tuning coil comprises a plurality of layers of wire, each layer consisting of a plurality of substantially plane turns. The axial length of the coils is small in relation to its mean diameter. The anode circuit of the set is reactively coupled with the grid circuit of the receiver.

1,496,311—J. H. Hammond, filed October 17, 1917, issued June 3, 1924.

Receiving System for Radiant Energy, particularly adapted for radio control operation where positive operation of the control circuit is assured without the smothering or saturating effect which may be produced by powerful signals from a hostile station endeavoring to prevent weaker signals from the distant control station from having any appreciable effect upon the receiving system. To insure the positive operation of the control a plurality of electron tube detectors connected in parallel are arranged at the receiving circuit with the control connected in the common output circuit of each of the detectors.

1,496,371—C. B. Graves, filed August 23, 1922, issued June 3, 1924.

Condenser of variable capacity where the plates are formed in parallel relationship with dielectric sheets interposed between the conductive plates with means for exerting pressure against the plates for varying the capacity of the condenser.

1,496,671—H. Gernsback, filed February 24, 1923, issued June 3, 1924.

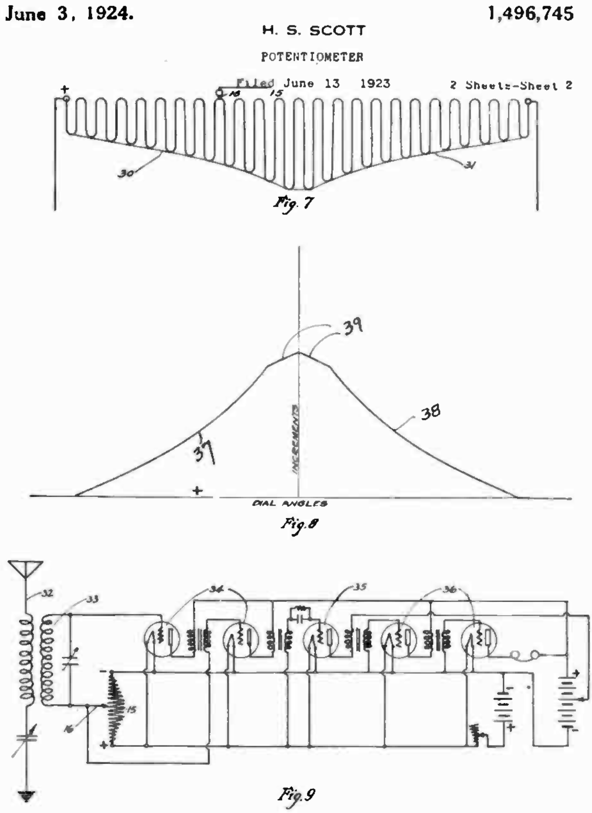
Detector of the crystal type where a sensitive crystal is enclosed by a casing which includes a feeler member in contact with the crystal. The feeler member comprises a central plate having plane opposite faces and side plates having a plurality of contact feeler points thereon which contact with a plurality of sensitive points on the crystal. By this construction a crystal detector may be placed in permanent adjustment.

1,496,745—H. S. Scott, filed June 13, 1923, issued June 3, 1924.

Potentiometer in which the variation in resistance takes place in accordance with a logarithmic law. An insulating band which carried the resistance element is relatively wide at a central section but decreases in width toward each end thereof on opposite sides of the central section. The resistance wire is wound upon this band whereby relatively large increments of adjustment are secured within the central section while relatively small increments of adjustment are obtained within the tapered portions on each side of the central section.

1,496,768—W. T. Booth and Walter A. Boyd, filed March 27, 1918, issued June 10, 1924. Assigned to Western Electric Company.

Vibration Reducing Mounting Device for Signaling Systems, where the electron tubes are supported on a horizontal panel, each end of which is mounted upon sponge rubber. The sponge rubber is carried by an intermediate supporting member and the intermediate support in turn carried by a second resilient mounting so that vibrations are prevented from reaching the tube support thru two separate damping means.

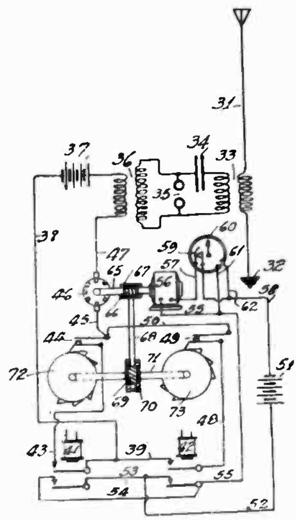


Number 1,496,745—Potentiometer

1,497,095—W. Dubilier, filed May 23, 1922, issued June 10, 1924. Assigned to Dubilier Condenser and Radio Corporation of Delaware.

Electrical Condenser of the "Micadon" variety where the alternate conductive and dielectric sheets are secured together under compression by means of a clip engaging each of the opposite ends of the condenser with a rivet passing thru the clips to secure the clips to the condenser and cause the clips to press tightly against the plates.

1,497,194—J. Nordern and A. Fetting, filed April 13, 1922, issued June 10, 1924.



NUMBER 1,497,194—Radio Alarm System

RADIO ALARM SYSTEM where a plurality of radio transmitting systems may be located in different houses to be automatically placed in operation by the happening of a condition precedent. The invention contemplates the protection of homes against burglary or fire by the installation of a radio transmitting station in the home. Upon the fusing of a fusable link in case of fire a radio transmitter is set into operation and controlled by means of a particular set of code wheels which will advise a central station that the particular home is in danger. In the case of a burglary a circuit might be opened by an intruder which places the radio transmitter in operation to advise the central police station. Each house will be identified by a particular code combination set out on the code wheels of the transmitter.

1,497,384—H. G. Saal, filed May 11, 1922. Issued June 10, 1924

RADIANT ENERGY RECEIVER, consisting of a crystal detector construction where the cat whisker is supported by a spherical member seated upon a spherically curved standard. A coil spring interior of the standard is secured to the surface of the spherical member enabling the member to be freely adjusted to different angular positions.

1,497,415—Phillips Thomas, filed April 21, 1921, issued June 10, 1924. Assigned to Westinghouse Electric and Manufacturing Company.

MOLDED OIL-FILLED CONDENSER, wherein alternate sheets of foil and solid dielectric are supported in a molded casing in position where a liquid dielectric may be filled in over the plates for increasing the insulating properties.

1,497,449—W. H. Kempton, filed October 16, 1917, issued June 10, 1924. Assigned to Westinghouse Electric and Manufacturing Company.

MOLDED CONDENSER comprising a series of stacked plates of alternate sheets of conducting material and an insulating material. The stack is wrapped with a compressible covering and the entire unit forced into a molded cover and held in compression.

1,497,927—A. Meissner, filed September 3, 1921. Assigned to Gesellschaft fur Drahtlose Telegraphie m.b.H., Hallesches, of Berlin, Germany. Issued June 17, 1924.

SENDING ARRANGEMENT, where the harmonics are suppressed by means of a grounded circuit arrangement in parallel with the grounded portion of the antenna. The grounded circuit is tuned to prevent the radiation from the antenna of harmonics of the fundamental frequency.

1,497,948—I. Shoenberg, filed April 9, 1918. Assigned to Radio Corporation of America, of Delaware. Issued June 17, 1924.

THERMIONIC DEVICE for securing an intermittent discharge of electrons from the filament. Alternating current is employed for heating the filament of a thermionic device. The temperature of the filament, provided its diameter is properly chosen, will then be proportional to the square of the current. Due to the thermal inertia, that is insufficient radiation from the filament, the temperature of the filament will lag behind the current. This property is utilized in various ways such as the reception of continuous wave radio signals. A choke coil saturated by a direct current is employed in series with the alternating current source for controlling the heating current to the filament.

1,498,129—A. E. Spicer, filed June 5, 1923. Assigned to Thomas J. F. Coady, of New York. Issued June 17, 1924.

RADIO TELEPHONE RECEIVING APPARATUS, constructed in compact form and including a miniature pick-up loop. The tuning unit includes a primary coil having a tap substantially central of the length of the coil. The ends of the coil are connected by a shunt circuit through condensers. A rotary coil is positioned adjacent each end of the primary coil. One of the rotary coils is connected in series with the primary coil and the other rotary coil is connected between the central tap and a ground connection. The receiving apparatus is connected across the tuning system with suitable amplification for bringing in signals.

LIST OF RADIO TRADE MARKS PUBLISHED BY PATENT OFFICE PRIOR TO REGISTRATION

(The numbers given are serial numbers of pending applications)

184,263—"ALL-AMERICAN" for radio apparatus, in ornamental design. Rauland Mfg. Co., Chicago, Illinois. Claims use since October 15, 1919. Published May 6, 1924.

190,373—"CROWN" for radio apparatus, in ornamental design Crown Radio Mfg., Inc., New York, N. Y. Claims use since about November 1, 1922. Published May 6, 1924.

190,773—"SELECTOFORMER" for radio apparatus, Electrical Research Laboratories, Chicago, Illinois. Claims use since December 3, 1923. Published May 6, 1924.

191,386—"HETEROFORMER" tuning coil, Elvin Radio Co., Philadelphia, Pennsylvania. Claims use since on or about December 15, 1924.

191,387—"HETEROPLEX" for radio receiving sets. Elven Radio Co., Philadelphia, Pennsylvania. Claims use since on or about December 15, 1923. Published May 6, 1924.

191,423—"LISTEN-TO-THE-WORLD" Radio for radio receiving sets. B. & C. Radio Company, Inc., Boston, Massachusetts. Claims use since about December 1, 1923. Published May 6, 1924.

192,287—"SELECTOMETER" for wave traps and tuning instruments. Vesco Radio Shop, Oakland, California. Claims use since January 1, 1923. Published May 6, 1924.

192,420—"Serenada" for loud speakers. Racon Electrical Co., Inc., New York, N. Y. Claims use since December 21, 1923. Published May 6, 1924.

192,742—"Aeolus" for radio receiving sets. H. M. Tower, Inc., West Haven, Connecticut. Claims use since November 1, 1923. Published May 6, 1924.

193,248—"Radio" for radio apparatus, in ornamental design. Radio Sales Studio, Inc., Washington, D. C. Claims use since June 1, 1923. Published May 6, 1924.

183,901—"Midget" for radio sets. Midget Radio Company, Philadelphia, Pennsylvania. Claims use since May 4, 1922. Published May 6, 1924.

185,353—"Four-Phone-Plug" for radio phone plugs. The Barkelew Electric Mfg. Co., Middletown, Ohio. Claims use since September, 1922. Published June 10, 1924. Not subject to opposition.

170,132—"Frost Radio" for radio receiving apparatus. Herbert H. Frost, Chicago, Illinois. Claims use since September 30, 1922. Published June 17, 1924.

170,133—"Like Postage Stamps, Used Everywhere" for radio receiving apparatus. Herbert H. Frost, Chicago, Illinois. Claims use since August, 1921. Published June 17, 1924.

189,924—"Radio United" for radio receiving sets, in ornamental design. United Cigar Stores of America, New York, N. Y. Claims use since November 7, 1923. Published June 17, 1924.

191,499—"The Mocking Bird" for radio receiving sets. Morrison Laboratories, Inc. Detroit, Michigan. Claims use since December 1, 1923. Published June 17, 1924.

191,584—"—Ramo—" for radio and wireless merchandise. Ramo Radium Products Co., New York, N. Y. Claims use since September 8, 1923. Published June 17, 1924.

193,844—"Strand" for radio receiving apparatus, in ornamental design. Manufacturers Phonograph Co., Inc., New York N. Y. Claims use since January, 1921. Published June 17, 1924.

194,908—"Radioscope" for variable condensers. Chelten Electric Co., Philadelphia, Pennsylvania. Claims use since November 27, 1924. Published June 17. 1924.

194,936—"NATIONAL" for radio transformers, in ornamental design. National Transformer Mfg. Co., Chicago, Illinois. Claims use since September 15, 1921. Published June 17, 1924.

195,143—"PHONODYNE" for radio receiving sets, in ornamental design. George A. Turner, Stockton, California. Claims use since March 1, 1924. Published June 17, 1924.

195,302—"WHAT'S IN THE AIR," a monthly magazine published by the Reincke-Ellis Co., Chicago, Illinois. Claims use since August 1, 1923. Published June 17, 1924.

195,317—"THURMAN" for vario couplers. Charles L. Thurman, Philadelphia, Pennsylvania. Claims use since February, 1924. Published June 17, 1924.

195,830—"RADIO PROGRESS," a magazine. Oxford Press, Providence, Rhode Island. Claims use since March 15, 1924. Published June 17, 1924.

193,794—"APCO" for battery chargers. Apco Mfg. Co., Providence, Rhode Island. Claims used since 1910. Published May 27, 1924.

193,846—"MOON" C-2A, in ornamental design, for radio receiving sets. Moon Radio, Inc., Long Island, New York. Claims use since May, 1922. Published May 27, 1924.

193,908—"EIRACO" for radio receiving sets, in ornamental design. Elgin Radio, Inc., Elgin, Illinois. Claims use since October 17, 1922. Published May 27, 1924.

194,119—"NICKELETTE" for radio receiving sets. Novelty Radio Mfg. Co., St. Louis, Missouri. Claims use since February 23, 1924. Published May 27, 1924.

183,318—"NIGHTINGALE" for radio receiving sets. The Cleveland Apparatus Co., Grafton, Ohio. Clains use since May 1, 1923. Published May 27, 1924.

187,490—"SHACTON RADIO PRODUCTS" in ornamental design, for radio apparatus. Harold M. Schwab, Inc., New York, N. Y. Claims use since August 6, 1923. Published May 27, 1924.

189,384—"MELCO" for radio receiving sets and parts thereof. Amsco Products, Inc., New York, N. Y. Claims use since July 12, 1922. Published May 27, 1924.

189,473—"OPERADIO" for radio receiving sets. The Operadio, Inc., Chicago, Illinois. Claims use since April 15, 1922. Published May 27, 1924.

191,711—"MACKLINE" in ornamental design, for radio antennas and tube-projecting apparatus. Franklin H. Mackenzie, doing business as Mack Co., Philadelphia, Pennsylvania Claims use since July 16, 1923. Published May 27, 1924.

193,080—"SUR-RAD" for radio receiving sets and apparatus therefor. Hugh Stevenson, Rochester, New York. Claims use since October 7, 1924. Published May 27, 1924.

194,630—"LAFAYETTE" for radio apparatus. R. E. Thompson Mfg. Co., Jersey City, New Jersey. Claims use since February 7, 1924. Published May 27, 1924.

194,713—"AIR SCOUT" in ornamental design for radio receiving sets. Kilbourne and Clark Mfg. Co., Seattle, Washington. Claims use since about December 1, 1923. Published May 27. 1924.

195,254—"DERESNADYNE" for radio receiving sets and parts. Andrews Radio Co., Chicago, Illinois. Claims use since March 1, 1924. Published June 17, 1924.

COUPLED CIRCUITS," PUBLISHED IN THE "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," JUNE, 1924

By

E. Leon Chaffee

All letters L, C, R, and η occurring in Part I should have the subscript unity.

In some equations the subscript is outside the exponent as $\theta^2{}_1$. These typographical errors do not change the meaning, and the above symbol should read in the usual form as $\theta_1{}^2$.

Page

300 In equation (1-1) there should be no minus sign before C in the second bracket of the denominator.

304 Equation (4-1), R^2 should read R_2.

305 In equation (5-a) the fraction in the first parenthesis of the denominator should be $\frac{2}{\theta_1{}^2}$.

The third line below equation (5-a) should read

$$1-\frac{\theta^2}{\theta_1{}^2}=\beta_1 \text{ and } 1-\frac{\theta^2}{\theta_2{}^2}=\beta_2$$

306 In equation (7-1) R_2 should be $R_1{}^2$.

Equation (7-a) should read

$$\beta_2=\frac{\tau^2\,\beta_1}{\eta_1{}^2\,\theta^2+\beta_1{}^2}$$

Italics beside equations (7-1), (7-a), (7-b), should read *Adjustment of secondary reactance to give max.* I_2. *Equation of Max.* 1-2 *line.*

307 In equation (8-a) M_1 should be η_1.

In the second line below (8-a) parenthesis should be after I_2.

309 In equation (14-a) I^2 should be I_2.

312 Omit parenthesis in third line above Figure 6.

313 In the second line below equation (17-a) η_1 should be $\frac{\eta_1}{\eta_2}$.

In the third line below equation (17-a) $\tau=\frac{\eta_1}{\eta_2}$ should be $\tau=1$.

314 In the fifth line below Figure 7 $y=y_o$ should read $\lambda=\lambda_o$.

In the fifth line from the bottom of the page change y to λ.

315 In the fourth, fifth, and seventh lines above Figure 8 change y to λ. Insert "and" before "moving" in the fifth line above Figure 8.

317 On this page every y should be changed to λ except in the eleventh line from the bottom where $y=1$.

320 In the second line below equation (20) "effects" should be "affects."

321 In equation (25-a) and in the line below and also in the third line of the second paragraph k should be K.

Equation (29) should read $\beta_2 = 1 - \dfrac{1}{\left(\dfrac{\bar{\lambda}_2}{\bar{\lambda}}\right)^2} = 1 - \dfrac{\theta^2}{\bar{\theta}_2^2}$

322 Equation (33) should read $\bar{\gamma}_2 = \dfrac{\bar{R}_2}{\bar{L}_2\,\omega_o} = \gamma_2 \dfrac{L_2}{\bar{L}_2} - H \dfrac{L_p}{\bar{L}_2} \gamma_p$

In the fifth line from the bottom of the page the last R_2 should be $\bar{R}_2$, and in the second line from the bottom R_2 should be $\bar{R}_2$.

323 Case 1 should read "Special Case of," etc.

In equation (36-a) $\gamma_2\theta$ should be $\bar{\gamma}_2\theta$.

The X_2 in the third and fourth lines from the bottom, in equation (37-1), and in the notation to the right of (37-1) should not be in Claredon Type.

The italicized notation to the right of equations (37-1) and (37-a) should read

"$\bar{X}_2$ to give max. I_2. Equation of max. 1-2 line.
β_2 to give max. I_2. Equation of max. 1-2 line."

324 The parenthesis in the denominator should be closed at the end of equation (38-1), also the parenthesis should be closed after 6 on the last line.

329 Equation (47) should read

$$[\gamma_1 \bar{\gamma}_2(\gamma_1+\bar{\gamma}_2)\bar{\theta}_2^2+\bar{\gamma}_2+\bar{\tau}^2\gamma_1]\theta_1^4+[\bar{\gamma}_2^2(\gamma_1+\bar{\gamma}_2)\bar{\theta}_2^2 - 2\bar{\gamma}_2(1-\bar{\tau}^2)]\theta_1^2\bar{\theta}_2^2+\left[\frac{\bar{\gamma}_2^2}{\gamma_1}(\gamma_1+\bar{\tau}^2\bar{\gamma}_2)\right]\bar{\theta}_2^4=0$$

332 Equation (48) should read $\theta^2 = \dfrac{\gamma_1+\bar{\gamma}_2}{\dfrac{\gamma_1}{\bar{\theta}_2^2}+\dfrac{\bar{\gamma}_2}{\theta_1^2}}$

333 In equation (49-a) in the first set of brackets $\bar{\tau}^1$ should be $\bar{\tau}^2$. $\dfrac{1}{\theta^2}$ is the coefficient before the second set of brackets.

334 Insert θ before $L_1 \omega_o$ in the second parenthesis of the denominator of equation (50-a).

In both forms of equation (50-a) E in the numerator should be E_1. In the last line of the second form of equation (50-a) the coefficient before the last set of brackets is $\frac{1}{\theta^2}$.

In the second and fourth lines below (50-a) $\frac{\lambda_2}{\lambda_o}$ should be $\frac{\bar{\lambda}_2}{\lambda_o}$.

335 The second line should read "system at point P_1," etc.

338 In equations (45-2) and (46-2) R_{21} and X_{21} should read $\bar{R}_{21}$ and $\bar{X}_{21}$.

339 In the first line S should be s.

R_{21} and X_{21} in equations (55-1) and (56-1) should be $\bar{R}_{21}$ and $\bar{X}_{21}$. In the second line of equation (58), η should be η_1.

340 In equation (60) the first brace under the second square root sign of the denominator should read

$$\left\{\eta_2 - \frac{s\,\eta_p}{\theta_2^2} + \frac{\tau^2 \eta_1}{\eta_1^2\,\theta^2 + \left(1 - \frac{\theta^2}{\theta_1^2}\right)^2}\right\}^2$$

341 In the first line above Figure 22 and the last line on the page, the number 23 should be inserted between the numbers 22 and 24.

342 In the second line from the top of the page in place of "below" insert "by equations (59) and (60)."

346 In the eleventh line from the bottom of the page insert "in" after "drop."

349 The italics after equation (61) should read "*Intersection of θ line. Tip of point.*

The fifteenth line from the bottom of the page should read "so that the settings for the tip of the point are," etc.

350 In the third line above Figure 31 $\eta_1 = -\eta$ should be $\eta_1 = -\bar{\eta}_2$. In the first line above the same figure insert a semicolon after the first inequality. The second inequality should read $|\eta_3| < \eta_1$.

In the first line above equation (63) substitute $\bar{\eta}_2$.

354 In the fifth line from the top of the page k should be K. In the seventh line fraction should read $\frac{1-\theta^2}{\theta}$.

The second line of the second paragraph should contain the fraction $\frac{I_2 L_2}{I L_1}$ in place of the one given.

In the seventh line from the bottom of the page change γ_2 to $\gamma_1'' = .05$.

357
358 In the "Nomeclature" all *k's* should be *K's*.
359

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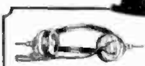

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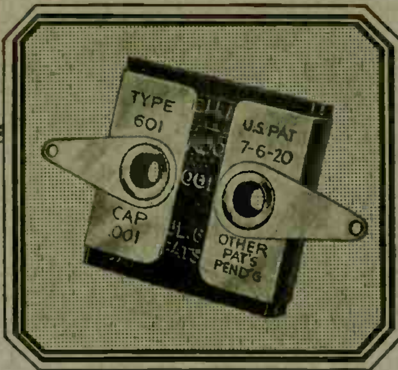

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