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CONTENTS

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THE PIEZO-ELECTRIC RESONATOR*

By

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In the course of experiments with piezo-electric crystals, extending over a number of years, certain radio frequency phenomena were brought to light, the practical application of which appeared worthy of development. The two applications that seem most promising at present are (1) as a frequency-standard, and (2) as a frequency-stabilizer, or means of generating electric oscillations of very constant frequency. It is with these that this paper is chiefly concerned. The fundamental phenomena will first be described, followed by the mathematical theory, and finally an account of the applications will be given.[1]

I. Fundamental Phenomena

1. A plate or rod suitably prepared from a piezo-electric crystal, and provided with metallic coatings, can be brought into a state of vigorous longitudinal vibration when the coatings are connected to a source of alternating emf. of the right frequency. Under these conditions the plate reacts upon the electric circuit in a remarkable manner. Owing to the piezo-electric polarization produced by the vibrations, and to the absorption of energy in the plate, the apparent electrostatic capacity and resistance of the plate are not constant, but depend upon the frequency somewhat as does the motional impedance of a telephone receiver.[2] Over a certain very narrow range in frequency the capacity becomes negative. An analogy may also be drawn between the vibrating plate and a synchronous motor. The man-

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[1] Preliminary reports on this work have appeared in "The Physical Review," 17, page 531, 1921, and 18, page 142, 1921. The writer wishes to acknowledge the aid that he has received thru a grant from the American Association for the Advancement of Science.

[2] For an explanation of the motional impedance of a telephone receiver, see Proceedings of The Institute of Radio Engineers, volume 6, 1918, page 40.—Editor.

ner in which the reactions upon the circuit are utilized will be described below. It is necessary, however, to consider the theory of the phenomenon first.

II. Piezo-Electric Theory

2. Four decades have elapsed since the discovery of piezo-electricity by the Curie brothers, and the prediction of the converse effect by Lippmann, which the Curies promptly verified. During this time much has been accomplished, both theoretically and experimentally, in systematizing and extending our knowledge of the behavior of crystals under static mechanical or electric stress. Only in very recent years, however, has consideration been given to rapidly varying stresses in piezo-electric crystals.

Nicolson[3] has had marked success in the use of suitably treated Rochelle salt crystals at telephonic frequencies, both as transmitters (direct piezo-electric effect) and as receivers (converse effect). The writer has also experimented with crystals at audio frequencies, but has devoted his attention chiefly to radio frequency vibrations in the neighborhood of the natural frequency of the crystal plates or rods.

We now summarize briefly those features of Voigt's theory of which we shall make use hereafter.[4]

When a piezo-electric crystal is mechanically strained, there results a dielectric polarization, the magnitude of which is proportional to the strain, and the direction and magnitude of which depend upon the direction of the strain and upon the class to which the crystal belongs. Except in the case of the class of crystals of lowest symmetry (triclinic), not all of the six components of strain are effective in producing a polarization. The higher the degree of symmetry, the smaller does this number become. Of the 32 classes, ten are devoid of piezo-electric properties.

The only two crystals the piezo-electric applications of which have hitherto been considered important are quartz and Rochelle salt; the latter, because it is far more strongly piezo-electric than any other crystal thus far examined; and quartz, because of its excellent mechanical qualities, which make it for most purposes decidedly preferable to Rochelle salt, in spite of its

[3] Nicolson, "Proceedings of the American Institute of Electrical Engineers," 38, page 1315, 1919; "Electrical World," June 12, page 1358, 1920.

[4] For a more complete statement, see Voigt, "Lehrbuch der Kristallphysik," Leipzig, 1910; Graetz, "Handbuch der Elektricität und des Magnetismus," Leipzig, 1914, volume 1, page 342; or Winkelmann, "Handbuch der Physik," 1905, volume 4, part 1, page 774.

being only moderately piezo-electric. The present paper has to do only with quartz, tho obviously the theory applies to any piezo-electric crystal.

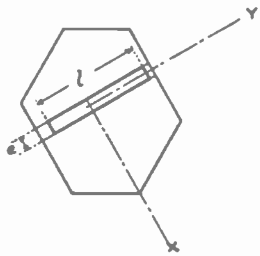

FIGURE 1—Section of a Quartz Crystal perpendicular to the Optical Axis

3. PIEZO-ELECTRIC PROPERTIES OF QUARTZ—Quartz belongs to the trigonal trapezohedral class of crystals. Figure 1 shows a cross-section of a quartz crystal, of which the Z-axis (optical axis) is perpendicular to the paper. The Y-axis is normal to two opposite prismatic faces. Owing to the threefold symmetry of quartz, the Y-axis may be drawn in any one of three directions 120° apart. The three X-axes (electric axes) are perpendicular to the Z- and Y-axes. For piezo-electric experiments, a plate is usually cut from the crystal with its length l, breadth b, and thickness e parallel respectively to the Y-, Z-, and X-axes. The two faces perpendicular to the X-axis are provided with conductive coatings, which may or may not be in actual contact with the quartz.

DIRECT EFFECT—If the plate is compressed in a direction parallel to the X-axis (*longitudinal effect*), the resulting polarization induces equal and opposite charges on the coatings, and the charges change sign with the pressure. Similarly, in the *transverse effect*, an endwise compression of the plate, parallel to the Y-axis, causes the coatings to become charged. *A compression of the plate parallel with the X-axis causes a polarization in the same direction as an extension parallel with the Y-axis.*

CONVERSE EFFECT—In terms of the *converse effect*, if the plate is polarized by an external electric field in the same direction in which it would become polarized by compression along the X-axis, it tends to contract along the X-axis and to expand along the Y-axis.

From what has been said, two important conclusions should be borne in mind: first, that, in quartz, just as the *direct* effect may be produced by compression along either one of two directions (longitudinal and transverse effects), so both of these effects manifest themselves in connection with the *converse* effect; and second, that in both the direct and converse effects, a given strain is always associated with an electric polarization *in the same direction* and *of the same algebraic sign*.

SYMBOLS

$l, b, e,$	length, breadth, and thickness of quartz plate or rod.
$\epsilon, \delta,$	piezo-electric constant and modulus respectively. From section 11 on, a special meaning is attached to δ.
$M, N, g,$	equivalent mass, resistance, and stiffness of resonator.
$x,$	displacement of end of resonator.
$F,$	equivalent mechanical force on resonator.
$E,$	voltage impressed on circuit.
$V,$	potential difference across resonator.
$D,$	piezo-electric polarization in resonator.
$I, i,$	currents in coil and resonator branches, Figure 3.
$C_1,$	normal capacity of resonator, vibrations damped.
$C_2,$	capacity of tuning condenser.
$C_1', C_1'',$	equivalent series and parallel capacity of resonator.
$R_1', R_1'',$	equivalent series and parallel resistance of resonator.
$C_a,$	"apparent" capacity of resonator.
$C_t, R_t,$	equivalent series capacitv and resistance of entire circuit, Figure 3.
$R_{12},$	equivalent series resistance of resonator and C_2, together.
	When printed without subscripts, x, F, E, V, D, I, and i denote instantaneous values. x_o and so on, denote maximum values.
$f,$	frequency.
$\omega,$	angular velocitv $=2\pi f$. ω_o and f_o denote resonance values.

4. In the case of quartz, the general polarization-strain equations reduce to the following form:

$$P_1 = \epsilon_{11}\, x_x + \epsilon_{12}\, y_y + \epsilon_{14}\, y_z \tag{1}$$

$$P_2 = \epsilon_{25}\, z_x + \epsilon_{26}\, x_y. \tag{2}$$

P_1 and P_2 are X and Y components, respectively, of polarization (electric moment per unit volume), and the ϵ's are the *piezo-electric constants*. x_x and x_y are, in Voigt's notation, the components of extension (elongation or contraction per unit length), and y_z and so on, the components of shearing strain.

If, instead of the components of strain, we have given the components of *stress*, (1) and (2) become

$$-P_1 = \delta_{11} X_x + \delta_{12} Y_y + \delta_{14} Y_z \quad (3)$$

$$-P_2 = \delta_{25} Z_x + \delta_{26} X_y. \quad (4)$$

The δ's are the *piezo-electric moduli*, which are related to the piezo-electric constants ε by equations involving also the elastic constants.

As is evident from equations (2) and (4), the polarization P_2 is produced only by shears, which may be neglected in the present paper, as may also the third term in (1) and (3). Of the two remaining terms on the right-hand side of (1) and (3) the first expresses the longitudinal effect, the second the transverse effect.

We shall need also the following expressions for the *converse effect*, in which the stresses along the X- and Y-axis are given in terms of the X-component E_1 of impressed electric intensity·

$$-X_x = \varepsilon_{11} E_1 \quad (5)$$

$$-Y_y = \varepsilon_{12} E_1. \quad (5a)$$

The other stress-components are of no concern here. The equation (5) expresses the longitudinal effect, and (5a) the transverse. In the applications described in the present paper, only the *transverse effect* is utilized.

One more fundamental equation must be added, namely the strain-equation for the transverse converse effect, which is analogous to (5a)·

$$y_y = \delta_{12} E_1 \quad (6)$$

According to Voigt's theory, in the case of the class of crystals to which quartz belongs, $\varepsilon_{26} = \varepsilon_{12} = -\varepsilon_{11}$, $\varepsilon_{25} = -\varepsilon_{14}$, $\delta_{12} = -\delta_{11}$, $\delta_{25} = -\delta_{14}$, and $\delta_{26} = -2\delta_{11}$. Hence in all only two different numerical values of δ and ε have to be known, and of these only one occurs in the present investigation. The following values of ε_{11} and δ_{11} were determined by Riecke and Voigt:[5]

$$\varepsilon_{11} = -4.77 \times 10^4, \quad \delta_{11} = -6.45 \times 10^{-8}.$$

The ε's and δ's as indeed all electric and magnetic quantities in this paper, unless otherwise stated, are in c. g. s. electrostatic units. As is evident from (1) and (2), ε has the dimensions of an electrostatic polarization, while from (3) and (4) it may be seen that δ has the dimensions of the reciprocal of an electric intensity. Hence

$$\varepsilon = [k^{\frac{1}{2}} M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}], \quad \delta = [k^{\frac{1}{2}} M^{-\frac{1}{2}} L^{\frac{1}{2}} T].$$

Other observers have obtained slightly different values for

[5]Voigt, previous citation, pages 869-870.

ε_{11} and δ_{11}. Fortunately, in the practical applications under consideration, the absolute values need not be accurately known.

III. Theory of Longitudinal Vibrations in Rods

5. The theory of electric reactions of vibrating piezo-electric plates is a structure built upon two main piers. First, there is the fundamental piezo-electric theory which has just been set forth; and second, the theory of longitudinal mechanical vibrations in rods, which will now be briefly summarized. The "plates" which the writer uses are, as far as mechanical considerations permit, in the form of thin rods. The advantage of this procedure, in addition to economy of material, is that the fundamental vibration together with harmonics of considerable purity may be secured, free from the disturbing effects of other modes of vibration. The theoretical treatment is also greatly simplified.

In a paper which is to appear in "The Physical Review," the general theory of forced longitudinal vibrations in rods is developed. The characterizing feature is the insertion in the equations of a symbol representing the *viscosity* of the material composing the rod; for that property of the rod whereby it absorbs energy and damps its own vibrations is as important here as is the resistance in an oscillating electric circuit. It is possible to measure the actual value of the viscosity by a purely electrical method, at any desired frequency; this, as well as the effect upon the resultant viscosity of air friction and of restraints imposed by the method of mounting, need not concern us here. It is only necessary to remark that a successful piezo-electric resonator must be prepared and mounted so as to reduce the damping to a minimum.

6. When an alternating emf. is applied to the metallic coatings of a rod of this sort, an alternating mechanical stress is set up in the rod in accordance with equation (5a), which is uniform throughout the mass of the rod. In this statement we neglect the "edge effect" of the condenser formed by the quartz and its coatings. Considering the thinness of the quartz and its high dielectric constant—about 4.5—this procedure is justifiable as a first approximation. In the paper referred to above, it is shown that the vibrations are the same as if the rod had impressed upon its ends two alternating forces, numerically equal to the actual internal stress, of like amplitude but opposite phases, and it is on this basis that the theory of forced vibrations is developed.

The general equation of motion is

$$\frac{\partial^2 \xi}{\partial t^2} = P \frac{\partial^2 \xi}{\partial u^2} + Q \frac{\partial^3 \xi}{\partial u^2 \partial t}. \qquad (7)$$

ξ is the displacement, at the time t, of that cross-section of the rod whose undisturbed co-ordinate is u. P is defined by the equation $P = G/\rho$, where G is Young's modulus and ρ the density; P is therefore the square of the wave-velocity in absence of damping.[6] For brevity, we call Q the "viscosity," and treat it as a constant of the material, implying thereby that it is independent of the frequency. Its possible dependence upon frequency can be tested experimentally. The dimensions of Q are $[L^2 T^{-1}]$.

7. In the paper referred to, equation (7) is solved, but its application to actual cases of forced vibration is somewhat cumbersome. It is, however, shown that, for the fundamental vibration in the neighborhood of resonance, the rod may be replaced by a *fictitious "equivalent mass" M possessing one degree of freedom.* The equation of motion then has the familiar form

$$M \frac{d^2 x}{dt^2} + N \frac{dx}{dt} + g x = F = F_o \cos \omega t. \qquad (8)$$

Here M is half the actual mass of the rod, or $M = \frac{1}{2} \rho ble$. In place of Young's modulus G in (7) we use the "equivalent stiffness," $g = M\omega_o^2$, which is related to G by the equation $g = \pi^2 beG/2l$. This follows from the equation $\omega_o = 2\pi f_o$, and $\sqrt{G/\rho} = 2 l f_o$, $2l$ being the fundamental wave-length.[7]

x is the mechanical displacement at time t of the end of the rod, so that the actual elongation (or contraction) of the entire rod at any instant is $2x$.

M and g correspond to L and $1/C$ in an electric circuit hav-

[6] In crystalline media, the elastic constants depend, of course, upon the direction with respect to the axis of the crystal. Slight differences are found between individual crystals. Moreover, in the case of our rods, the elastic modulus is modified by lateral effects, unless the rod is extremely narrow, and by any discrepancy between the axis of the rod and the true Y-axis of the crystal. The effective value of G with the rods employed by the writer ranges from 8×10^{11} to 10×10^{11}. The value for quartz as given by Voigt is 8.51×10^{11}.

[7] Strictly, ω_o is the angular velocity when the amplitude of the *velocity* of the equivalent mass M is a maximum under forced vibrations; it is also the free angular velocity in absence of damping. The maximum amplitude of equivalent *displacement* x (equation (10)) comes (under forced vibrations) at the angular velocity $\sqrt{\frac{g}{M} - \frac{N^2}{2M^2}}$, while the angular velocitv of free damped vibrations is $\sqrt{\frac{g}{M} - \frac{N^2}{4M^2}}$. The distinction between these three values may under ordinary circumstances be ignored.

ing concentrated, as contrasted with distributed, constants. N is the equivalent resistance, and bears to the viscosity Q the relation $N=\pi^2 \rho\, b\, e\, Q/2l$. For the proof of this the paper on longitudinal vibrations must be consulted. F is the equivalent impressed force. If the actual stress acts thruout the entire length of the rod, it may be proven that F is twice the actual force at any cross-section, or $F=2beX$, where X is the instantaneous stress. The expression for X in terms of the piezo-electric constant is given below, section 11.

We now write the steady-state solution of equation (8), which is of prime importance for the graphical method described in section 12:

$$x=x_o \sin(\omega t-\theta), \tag{9}$$

in which the maximum displacement is

$$x_o=\frac{F_o}{\omega\sqrt{N^2+\left(\omega M-\frac{g}{\omega}\right)^2}}, \tag{10}$$

and

$$\tan\theta=\frac{\omega M-\frac{g}{\omega}}{N}=\frac{\pi(\omega-\omega_o)}{\omega_o \Delta} \tag{11}$$

approximately, since $g/\omega=\omega_o M$ very nearly, and the logarithmic decrement per period, Δ, is, as in the electrical analogy, $N/2fM$.

The *power expended in maintaining vibrations*, as in the case of the electrical analogy, is easily proved to be

$$p=\frac{1}{2}\cdot\frac{F_o^2}{N}\ (ergs\ per\ sec.) \tag{12}$$

The *maximum stress when in resonance* may easily become so great as to break the quartz rod. On the assumption that the distribution of stress is sinusoidal, being zero at the ends, and, for the fundamental, a maximum at the center, we find that the maximum stress at the center is $\pi x_o G/l$, where x_o is half the maximum elongation of the rod of length l, and G is Young's modulus.

IV. The Resonance Circle

8. In applying the foregoing theory to investigations with piezo-electric resonators, it is advantageous to employ a graphical method, based on the properties of what may be called, for brevity, the resonance circle. In principle, this curve is similar to the "motional impedance" circle which has been used by

Kennelly and his collaborators in their studies of the telephone receiver.[8]

The equation of the curve in question is obtained by eliminating $\omega M - g/\omega$ between equations (10) and (11):

$$x_o = \frac{F_o}{\omega N} \cos \theta. \tag{13}$$

If ω were constant, this would be the polar equation of a circle passing thru the origin. In reality, as θ varies from $-90°$ thru zero to $+90°$, ω varies from zero to infinity. Nevertheless, when N is very small, as is the case with quartz, not only is the "diameter" of the "circle" in Figure 2 large, but that portion of the curve corresponding to the neighborhood of resonance comprises nearly the entire curve. For all other values of ω, θ is nearly equal either to $-90°$ or to $+90°$, so that with quartz, to the precision attainable by ordinary graphical methods, the curve cannot be distinguished from a perfect circle. The distortion of the curve owing to varying ω in Figure 2 is very greatly exaggerated in order to illustrate the principle.

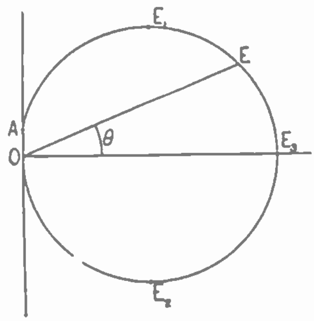

FIGURE 2—The Resonance Circle

$O\,E$ represents one value of the modulus x_o, with the corresponding argument θ. It has been found most convenient to draw the maximum modulus $O\,E_3$ horizontally to the right from the origin, and to lay off positive values of θ *below* the horizontal axis, so that *increasing frequency* is represented by a *clockwise movement* of the point E around the curve. Strictly speaking, the maximum modulus, $x_o = F_o/\omega N$, should be inclined slightly upwards, corresponding to a small negative value of θ. Here

[8] "Proc. Am. Acad. Arts and Sci.," 48, page 113, 1912; 51, page 421, 1915; "Proc. Am. Phil. Soc.," 54, page 96, 1915; 55, page 415, 1916. The circle diagram is also used by Hahnemann and Hecht, "Phys. Zeitschr.," 20, page 104, 1919, and 21, page 264, 1920, and by Wegel, "Journal of the American Institute of Electrical Engineers," 40, page 791, 1921.

again the damping in the case of quartz is so slight that the maximum is practically the line $O\,E_3$.

$O\,A$ represents the "amplitude" when $\omega=0$, that is, $O\,A$ is the equilibrium elongation under static stress at zero frequency, and must therefore have the value $X_o\, l/2\,G$. In the case of a typical quartz plate (Quartz Resonator N 2, to which further reference will be made), 3.07×0.41×0.14 cm. (1.21×0.16×0.055 inch), the fundamental frequency of which is 89,870, the equilibrium elongation at either end under a potential difference of one electrostatic unit (300 volts) is 6.5×10^{-7} cm., while the maximum amplitude of vibration at either end is (by calculation) 0.0025 cm. (0.001 inch). Thus we see that $O\,E$ is about 4,000 times as large as $O\,A$: in other words, the growth of am plitude at resonance is 4,000-fold.

For our purposes, the advantage of the resonance circle as outlined above is two-fold: the moduli $O\,E$, being proportional to elongations of the plate, are thereby also proportional to the piezo-electric polarization; and since the argument θ is a phase angle, the resonance circle can be incorporated into an ordinary alternating current vector diagram in studying the reaction of the plate upon the circuit. We now come to a consideration of the latter.

V. Reaction of Piezo-Electric Resonator Upon Circuit

9. Description of Circuit—The circuit that I have most frequently used in these experiments is that shown in Figure 3.

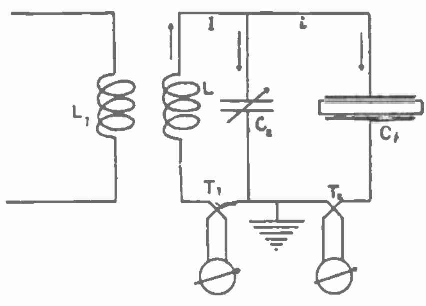

Figure 3—Resonator connected in a Secondary Circuit, with Thermo-elements for measuring Currents. The arrows indicate positive directions in the three branches

The main conclusions derived from the following paragraphs are, however, applicable to any circuit to which the resonator is likely to be connected. L_1 is a coil of about 5 millihenrys in the anode circuit of a vacuum-tube generating set kindly loaned by the Western Electric Company, so designed as to minimize the effect of load upon frequency. The latter is an extremely important requirement, and was found to be admirably fulfilled. Loosely coupled to L_1 is the litzendraht coil L of 3 or 4 millihenrys, in parallel with which are the precision variable condenser C_2 (maximum 1,500 $\mu\mu$f.) and the piezo-electric resonator C_1. Most of the quantitative work has been done with resonators about 3 cm. (1.2 inch) long, having frequencies around 90,000, corresponding to radio wave lengths of the order of 3,000 meters. References to other wave lengths will be made later, and also to the method of mounting C_1. T_1 and T_2 are thermo-elements of resistances 4.5 and 2,810 ohms, respectively, T_2 being a Western Electric vacuum thermo-element. The thermo-elements are in Figure 3 represented as connected to separate galvanometers, but in practice a single Leeds and Northrup high-sensitivity galvanometer of 10 ohms resistance was employed, provided with a change-over switch and suitable shunts. It was thus possible, at each frequency used, to measure the current I in the coil, and the current i flowing to the resonator. C_2 was provided with a worm gear for fine regulation, and readings were taken by means of a lamp and calibrated scale, a mirror being mounted on the condenser handle. By means of a specially constructed parallel-plate air condenser with micrometer control in the generating circuit an extremely fine regulation of frequency was possible. The voltage induced in L was practically constant over the small range in frequency involved.

10. The most instructive data are obtained by observing C_2, I, and i at a number of frequencies, and plotting the results as in Figure 6. These curves will be further discussed in section 15. At each frequency, C_2 is varied until either I or i is a maximum.

At a certain critical frequency, the absorption of energy by the resonator is so great that the coil-current I falls almost to zero, even when C_2 is at its most favorable setting, as represented in Figure 6. Any change in C_2 causes I to decrease still more: but a slight change in frequency causes I to increase enormously, showing that the mechanical tuning of the resonator is very much sharper than the electrical tuning of the circuit. At this point, then, *I is a maximum with respect to C_2, but a minimum with respect to f.*

Further consideration of Figure 6 will be deferred until the theory of the resonator's reactions has been given.

11. Theory of the Reactions—We assume that an emf.

$E_0 \cos \omega t$ is impressed upon coil L, Figure 3, E_0 being constant at all frequencies. Let V represent the instantaneous potential difference across C_2 and C_1, which, of course, varies with frequeney, and R the combined resistance of L and T_1. The capacity of the coil should be included in C_2. The impedance of C_1 is so great that the resistance of T_2 may, as a first approximation, be neglected. We then have

$$E = E_0 \cos \omega t = V + L\dot{I} + RI. \tag{14}$$

The link between this equation and that for the vibration of the resonator is the quantity V. For the electric field in the quartz has the value V/e, where e is the thickness of the quartz; and by equation (5b), the mechanical stress is $X = \varepsilon V/e$. From section 7, we see that the value of the equivalent force F in equation (8) must be $F = 2\,b\,e\,X = 2\,\varepsilon\,b\,V$. Equation (8) may, therefore, be written thus:

$$2\,\varepsilon\,b\,V = M\ddot{x} + N\dot{x} + Gx. \tag{15}$$

Next, we consider the components of polarization within the resonator. Let D_1 represent the component due to the potential difference between the coatings. If this p. d. is V, and if the air-gaps between coatings and crystal be neglected, then D_1 is the ordinary displacement in the dielectric, as given by the equation $D_1 = kV/4\pi e$. k, the dielectric constant of quartz, has the value 4.5, approximately.

A second component of electric polarization, that which is responsible for all the effects described in this paper, is caused by the deformation of the resonator. Using the same notation as heretofore, we assume that at any given instant the p. d. across the resonator is V, and that the total elongation (difference, + or −, between instantaneous length and normal length) is $2x$. Of this elongation, a portion, say $2x_1$ is that which would be produced under static conditions with a constant potential-difference V. This portion is the "equilibrium elongation," and exerts no reaction upon the circuit. The value of $2x_1$ may be derived from (6) thus:

$$y_y = \delta_{12} E_1 = \delta_{12} \frac{V}{e}.$$

Further, we have the fundamental relation $2x_1 = l\,y_y$, hence $2x_1 = \delta_{12} V\,l/e$. x_1 is readily seen to be the same as the $OA - X_0 l/2G$ in section 8, since by Hooke's law $y_y = X_0/G$. That portion of the total elongation $2x$ which is due to the vibrations, and therefore, effective in producing an electric reaction, is $2x - 2x_1$. Applying equation (1) (only the second term on the right side of which remains, since x_x and y_z do not appear) to this elongation, we find $P_1 = 2\,\epsilon_{12}\,(x - x_1)/l$, or, writing D in place of P_1, dropping the subscript from ϵ_{12}, replacing $2\,x_1$ by its value above, and letting δ represent the quantity $\delta_{12}\,l/e$,

$$D = \frac{\epsilon}{l}(2\,x - \delta\,V). \qquad (16)$$

At frequencies sufficiently removed from resonance, $2\,x$ may be so small as to be even less than δV; but with quartz, in the neighborhood of resonance, the static elongation δV is negligible, as stated already in section 8.

Equations (14), (15), and (16) contain four unknown quantities, namely: V, I, x, and D. The fourth equation necessary to a solution of the problem is found either in the expression for the resonator current:

$$i = b\,l\,(\dot{D}_1 + \dot{D}) = C_1\,\dot{V} + b\,l\,\dot{D}, \qquad (17)$$

or preferably that for the coil current:

$$I = C_2\,\dot{V} + i = C_2\,\dot{V} + b\,l\,(\dot{D}_1 + \dot{D}) = (C_1 + C_2)\,\dot{V} + b\,l\,\dot{D}. \qquad (18)$$

Equation (18) follows from Kirchhoff's first law.

These equations are simply expressions of the principles that current is time-rate of change of dielectric flux, and that the polarization in C_1 has the two components D_1 and D discussed above. Dielectric flux is of course polarization times area $b\,l$.

From equations (14), (15), (16), and (18) a single differential equation of the fourth order may be formed, giving x as a function of time in terms of impressed voltage E and the circuit constants. A much more convenient and satisfactory procedure, exhibiting at a glance all the characteristic phenomena, is the graphical method now to be described.

12. Graphical Method of Solution—We shall construct a vector diagram showing quantitatively not only the current and voltage components and their phases, but the effective capacity and resistance of the resonator at any frequency as well. Our starting-point is the resonance circle described in section 8. For the purpose of this paper it is amply accurate to regard the curve as a true circle, letting the distance $O\,A$, Figure 2, equal zero, and neglecting the second term in equation (16). Equation (16) then reduces to

$$D = \frac{2\,\epsilon\,x}{l}, \qquad (19)$$

or, in terms of maximum values,

$$D_o = \frac{2\,\epsilon\,x_o}{l} \qquad (19a)$$

D is thus approximately proportional to x; in other words, *the resonance circle is applicable also to the piezo-electric polarization D, and consequently to the piezo-electric flux in the resonator $D\,b\,l$.*

The first problem before us is, to represent I graphically as the vector sum of the two terms in equation (18). This involves, in the first term, the phase-angle of V, which is not yet known. We therefore write V in the form

$$V = V_o \cos(\omega t - \gamma), \qquad (20)$$

leaving γ to be determined later. The derivative is

$$\dot{V} = -\omega V_o \sin(\omega t - \gamma).$$

For the second term in (18), we have the following relations, from equations (9) and (19), together with (15) and (20):

$$b\,l\,\dot{D} = 2\,b\,\varepsilon\,\dot{x} = 2\,b\,\varepsilon\,\omega\,x_o \cos(\omega t - \gamma - \theta)$$
$$- 2\,b\,\varepsilon\,\omega\,V_o\,x_o' \cos(\omega t - \gamma - \theta). \qquad (21)$$

x_o' equals x_o/V_o, and is therefore the amplitude of vibration per unit maximum potential difference. Similarly, D' and I' will signify D/V_o and I/V_o, respectively. Hence equation (18) may be expressed thus:

$$\frac{I}{\omega V_o} = \frac{I'}{\omega} = -(C_1 + C_2) \sin(\omega t - \gamma)$$
$$+ 2\,b\,\varepsilon\,x_o' \cos(\omega t - \gamma - \theta). \qquad (22)$$

In Figure 4, the two terms on the right-hand side of equation (22) are represented by the vectors $O\,q$ and $O\,E$, respectively. $O\,X$ is the positive direction corresponding to $\sin(\omega t - \gamma)$, and $O\,Y$ that corresponding to $\cos(\omega t - \gamma)$. The moduli $O\,q$ and $O\,E$ are assumed equal to $-(C_1 + C_2)$ and $D_o'\,b\,l = 2\,b\,\varepsilon\,x_o'$, respectively.[9] As ω varies, $O\,q$ remains constant (unless C_2 is altered), while the point E describes the resonance circle. The half of the circle to the left of $O\,Y$ in Figure 4 is the low-frequency side, for which θ is negative. $O\,A$ is thus the vector representing I'/ω, and the modulus OA is equal to the maximum value I_o'/ω.

Our graphical construction so far has been based upon equations (15), (16), and (18). We will now make use of (14) in order to make the transition from I' to I. Since this involves only multiplying I' by ωV_o, the argument $\theta_2 = \angle XOA$ remains unchanged. We have

$$I' = I_o' \sin(\omega t - \gamma + \theta_2), \qquad (23)$$

---

[9] If the material and construction of the resonator are such as to make the retention of the second term δV in equation (16) desirable, it is only necessary to shift the center of the circle in Figure 4 to the left by an amount equal to $\delta\,b\,l$, leaving all else unchanged. In the problem of the telephone receiver, the analogue of the term δV is Kennelly and Affel's "damped impedance" ("Proc. Am. Acad. Arts & Sci.," 51, page 423, 1915).

and find, by substitution of (20) and (23) in (14), and division by $\omega^2 L V_o$,

$$\frac{E}{\omega^2 L V_o} = \frac{E_o}{\omega^2 L V_o} \cos \omega t = \frac{1}{\omega^2 L} \cos(\omega t - \gamma) + \frac{I_o'}{\upsilon} \cos(\omega t - \gamma + \theta_2) + \frac{R I_o'}{\omega^2 L} \sin(\omega t - \gamma + \theta_2). \qquad (24)$$

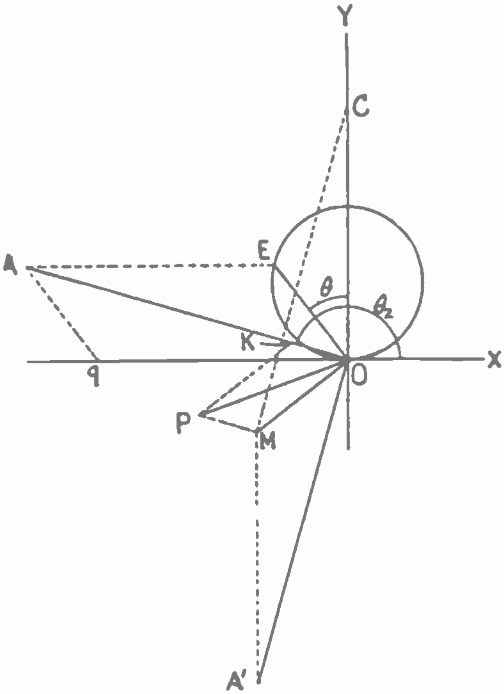

Figure 4—Vector Diagram for Determination of Current I

The advantage of this form is, that the second term on the right-hand side is, as we have seen, already represented by the vector OA in Figure 4. If now we lay off $OC = 1/\omega^2$ L along OY, $OA' = OA$, $\angle AOA' = 90°$, and $OK = R \cdot OA/\omega L$, we see that the three terms on the right of equation (24) are correctly represented by the three vectors OC, OA', and OK, respectively. The resultant of OC and OA' is OM, and the final resultant is O P. By (24), the modulus O P is therefore equal to $E_o/\omega^2 L V_o$, and $\gamma = \angle POC$. These operations might of course be performed analytically, but the graphical method is rapid, and amply accurate.

If the frequency is too low for the resonator to vibrate perceptibly, OE is practically zero, $Oq > OC$ (that is, $C_1 + C_2 > 1/\omega^2 L$), and, if R is small, O P is directed nearly vertically downward and $O\,A$ to the left, indicating the usual lead of nearly 90° if the circuit is tuned electrically to approximately the frequency of the resonator. At frequency f_o (natural resonator frequency), the angle $P\,O\,A$ (phase angle between E and I) may be positive, zero, or negative, depending upon the value assigned to the known capacity C_2.

The actual value of I is not shown directly on the diagram. It may, however, be found as follows. We have, from statements made above

$$V_o = \frac{E_o}{\omega^2 L} \cdot \frac{1}{O\,P}, \tag{25}$$

and

$$I_o = I_o' \, V_o = \frac{E_o}{\omega L} \cdot \frac{O\,A}{O\,P}. \tag{26}$$

13. The construction of the vector diagram may be simplified, and its range of application extended, by the following modification, shown in Figure 5. First we draw the resonance circle to the same scale as in Figure 4, namely with modulus $OE = 2b\,\varepsilon\,x_o'$, but with the angle θ laid off, as in Figure 2, from OX instead of from $O\,Y$. $Oq = C_1 + C_2$, where C_2 may have any value. Draw $E\,A$ and $q\,A$ parallel to $O\,q$ and $O\,E$, respectively; also $P\,C\,T$ perpendicular to $O\,A$, letting $PC = R\,.\,OA/\omega L$. Draw $A\,P$, cutting $O\,C$ at V. By comparison with Figure 4 we see at once that the phases of X, V, E, and I are given by the directions of $O\,E$, $O\,Y$, A P, and T P, respectively, and that

$$\theta_2 = \angle AOY' \tag{27}$$

$$\gamma = \angle AVO \tag{28}$$

$$V_o = \frac{E_o}{\omega^2 L} \cdot \frac{1}{AP} \tag{29}$$

$$I_o = \frac{E_o}{\omega L} \cdot \frac{OA}{AP}. \tag{30}$$

The entire diagram is determined by the electric constants and the characteristic resonance circle of the resonator. From the above equations, V_o and I_o and their corresponding phases may therefore be found for any frequency, in terms of the impressed E_o. A comparison of these theoretical values with those derived from observation can then be made, as will be seen in section 15. For the present it need only be remarked that, as the point E travels around the circle clockwise (increasing frequency), V_o and I_o pass thru a minimum, which is more pronounced the smaller the value of R, that is, the shorter the line

PC. The range in ω is usually so small that the distance OC may be given a mean value, practically independent of frequency.

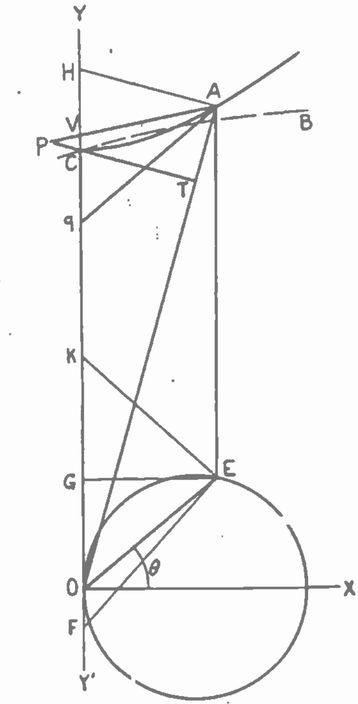

FIGURE 5

Vector diagram showing reactions of resonator upon electric circuit. Summary of relations derived from this diagram:

$$OE = D_o' b l \qquad I_o = \frac{E_o}{\omega L} \cdot \frac{OA}{PA} \qquad C_1' = FK \qquad R_1' = \frac{1}{\omega} \cdot \frac{EG}{FE^2}$$

$$OC = \frac{1}{\omega^2 \kappa_s} \qquad i_o = \frac{E_o}{\omega L} \cdot \frac{FE}{PA} \qquad C_1'' = FG \qquad R_1'' = \frac{1}{\omega . EG}$$

$$Oq = C_1 + C_2 \qquad Z_1 = \frac{1}{\omega \cdot EF} \qquad C_l = OH \qquad R_{12} = \frac{1}{\omega \cdot OC} \cdot \frac{CT}{OA}$$

$$V_o = \frac{E_o}{\omega^2 L} \cdot \frac{1}{AP} \qquad C_1 = OF \qquad C_a = H_q + OF \qquad R_t = \frac{1}{\omega \cdot OC} \cdot \frac{PT}{OA}$$

14. We will next consider the graphical solution for the case of greatest practical importance, namely that in which C_2 at each frequency is adjusted to such a value as to make the current I a maximum. We will call this the *condition of I-resonance.* In an ordinary resonance circuit this would mean simply bringing I into phase with E. But when the piezo-electric resonator is present, we must proceed as follows. From equation (30), since over the range considered ω is nearly constant, it is evident that I_o will be a maximum when C_2 is so adjusted as to make the ratio

OA/AP a maximum. That is, since $Oq=C_1+C_2$, that location of q on the line OY must be found for which OA/AP is a maximum, subject to the condition that, for any given frequency, E is fixed and AE must remain parallel to OV. The point C is fixed, since $OC=1/\omega^2 L$, and PT remains always perpendicular to OA. It is easily proven that *A must lie on one branch of a rectangular hyperbola, as shown in Figure 5, the center of which is midway between C and O.* The hyperbola, which will usually be rather flat, is readily drawn when two or three of its points have been determined by simple geometrical construction. Therefore, when the circuit is at every frequency tuned to resonance by maximum I, the construction consists simply in drawing a line from E parallel to OV, meeting the hyperbola at A; and then drawing Aq parallel to EO. Thus we find, *not only the resonant current, but the corresponding capacity $C_2=Oq-C_1$, as well.*

15. We will next examine the graphical construction for the *resonator-current* i, and from it find the effective capacity and resistance of the resonator. The starting-point is equation (17), and the method exactly similar to that by which the vector representation of I_o (see equation (30)) was derived from equation (18). We therefore pass at once to the result:

$$i_o=\frac{E_o}{\omega L}\cdot\frac{FE}{PA}. \tag{31}$$

This differs from equation (30) only in that in place of OA we have FE, the resultant of OF and OE. The point F is taken such that $OF=C_1$, while, as has been shown above, $OE=D_c' bl$. The vectors OF and OE correspond to the two terms on the right side of equation (17). In the case of a good resonator, OF is small in comparison with the diameter of the circle.

The resonator current i leads the potential difference V by the angle FEG; G being the foot of the perpendicular from E upon OV. This angle, when out of resonance, becomes 90° corresponding to the fact that the resonator is then a simple condenser, and absorbs no energy. The *maximum* of i would, if V were constant, come practically at that frequency for which FE is a maximum. But, as has been seen, V varies in a manner determined in part by the arbitrary variation of the condenser C_2. Hence the variation of i with frequency depends upon C_2, that is, upon the distance Oq. If C_2 is varied in such a manner as to keep I a maximum at each frequency ("*I-resonance*," section 14), then A, the operating point for the current I, lies on the hyperbola CA, and the value of PA to substitute in (31) in determining i_o is that shown in Figure 5.

In Figure 6 are shown the results of a series of observations made with quartz resonator N 2, already mentioned in section 8.[10] The arrangement shown in Figure 3 was used, the circuit being adjusted for the condition of I-resonance at each frequency by means of C_2. Values of C_2, I, and i are plotted as functions of

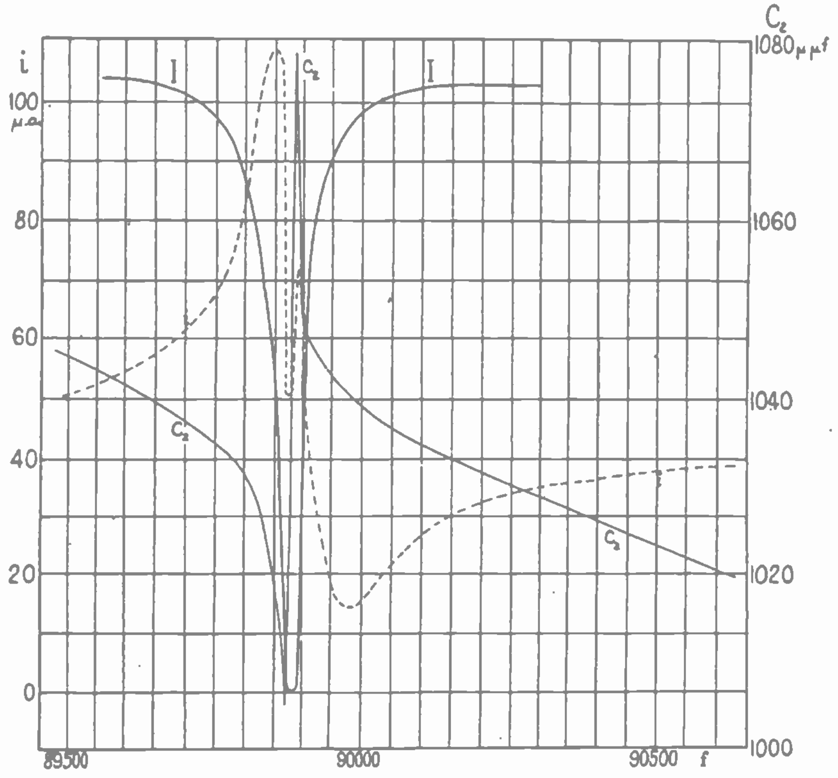

FIGURE 6—Characteristic curves of piezo-electric Resonator N2. The values of current I in micro-amperes are obtained by multiplying by fifty the ordinates of the I-curve, expressed in terms of the i-scale

frequency. The minimum and maximum of C_2 correspond to the lowest and highest positions of the point q in Figure 5. The minimum of I comes at a frequency between the frequencies for maximum and minimum C_2, as it should according to theory (see equation (30). Attention is directed especially to the sharpness of resonance, and to the practical extinction of the current I at resonance. This suggests a possible application of the resonator as a wave-filter.

[10] These observations, which required no little patience and skill, were made by Mr. T. Fujimoto.

The form of the i-curve is more complicated, since the resonator current, as is seen by inspection of equation (31), depends essentially upon both $F E$ and $P A$. As the point E travels around the resonance circle, the variations in $F E$ and $P A$ are such that i passes thru two maxima and two minima. The i-curve in Figure 6, which is based on direct observation, is in full agreement with the theory.

The values of C_2 shown in Figure 6 are for the variable condenser alone, uncorrected for the capacity of the coil, which was about 20 $\mu\mu$f.

By constructing the resonance circle as indicated in section 19, the equivalent parallel capacity of the resonator may be found according to equation (36). For resonator N2 the maximum value is 41.5, minimum -32.5 $\mu\mu$f. The normal value is 4.5 $\mu\mu$f. The logarithmic decrement is 0.00066.

Figure 5 is essentially a diagram of admittances, each vector having the dimensions of a *capacity*. In this sense the resonance circle represents what may be termed the *motional admittance* of the resonator.

One interesting deduction from Figure 5 is that when C_2 is adjusted for I-resonance or for i-resonance, as long as the resonator is vibrating, the current *I lags behind E* by the angle $A P T$ (see section 12). In other words, the resonator load is not capacitive, but *inductive*. This is also made evident by the large value of the total equivalent capacity C_t (equation (38)) of the circuit, since by Figure 5, $O H$ is always larger than $O C$, which latter represents $1/\omega^2 L$.

If one were to take the difference between the value of C_2 with resonator vibrating and that with the vibrations damped, setting for I-resonance in each case, this difference would not be a measure of the capacity of the resonator, since the phase of the current would be different in the two cases. The difference between the two C_2 values would be Cq in Figure 5; it passes thru positive and negative values in a manner similar to C_a and C_1'' (sections 17 and 18).

16. The Condition of i-Resonance—In section 10 it was stated that C_2 might be varied in such a way as to keep the *resonator-current* i a maximum at each frequency. Altho the settings of C_2 in this case differ only slightly from those required for I-resonance, the distinction must not be overlooked. It will be remembered that the hyberbolic locus $C A$ of the operating point A was derived from the condition that $O A/A P$ in equation (30) should be a maximum. In the present case, for i-reson-

ance, we must find a locus for A such that $F\,E/P\,A$ in (31) shall be a maximum (ω may be regarded as practically constant over the narrow range involved). Since $F\,E$ depends onlv on ω, it follows that i_o is a maximum when P A is a minimum, that is, when P A is perpendicular to $A\,E$. It is not difficult to prove that under these conditions the locus of A is not a hyberbola, but a *parabola*, with its axis horizontal and midway between O and C, and its vertex to the left of $O\,C$ by a distance equal to $O\,C^2/4$ P T. The dotted line $C\,B$ in Figure 5 represents that portion of the parabola which comes into play here. Its closeness to the hyperbola $C\,A$ shows how nearly identical the two types of resonance are, especially when $O\,C$ is relatively large (L small, C_2 large). When the parabolic arc $C\,B$ has been drawn, the value of i_o is easily computed for any frequency.

By using a variometer for L, it would of course be possible to leave C_2 fixed and tune for electrical resonance by varying L. This would lead to a different graphical construction, offering no special advantage.

17. EQUIVALENT RESISTANCE AND CAPACITY OF RESONATOR—These are found by forming the expressions for impedance and admittance, which are then resolved into their R and X components. Since we are dealing with ratios, it is unnecessary to transform maximum into effective values of current and voltage.

Considering first the resonator alone, we have, for any frequency $\omega/2\pi$, the maximum current given by equation (31), and maximum potential difference by (29). Hence the impedanee Z_1 of the resonator is evidently

$$Z_1 = \frac{V_o}{i_o} = \frac{1}{\omega \cdot FE}. \tag{32}$$

The angular relations are those of the triangle $F\,E\,G$, Figure 5. Hence we have (still using electrostatic units) for the *equivalent series resistance R' and capacity C' of the resonator*

$$R_1' = \frac{1}{\omega} \cdot \frac{EG}{FE^2} \tag{33}$$

$$C_1' = \frac{EF^2}{FG} = FK, \tag{34}$$

the line $E\,K$ being drawn perpendicular to $E\,F$. C_1' becomes infinitely great at resonance.

For some purposes the *equivalent parallel resistance R_1'', and capacity C_1''*, are more useful. They are found by taking the reciprocal of Z_1 (admittance), still using the triangle $F\,E\,G$. By writing the expressions for conductance and susceptance we have

$$R_1'' = \frac{1}{\omega . EG} \tag{35}$$

$$C_1'' = FG. \tag{36}$$

The last equation is noteworthy, as it makes the variation of equivalent parallel capacity with frequency evident from a glance at Figure 5. These four expressions for resistance and capacity depend only on the resonator, and not at all upon the rest of the circuit.

18. Effect of Resonator Upon Constants of Circuit— We seek expressions, in terms of vectors in Figure 5, for the following quantities: equivalent series resistance R_t and capacity C_t of entire circuit; equivalent series resistance R_{12} of the resonator and C_2 together; and the *apparent* capacity C_a of the resonator, this being defined as the difference between C_t and C_2.

The method is similar to that followed in section 17. From equation (30) we find the impedance of the entire circuit to be $Z_t = E_o/I_o = \omega\, L\, .\, AP/OA = A\, P/\omega . O\, C . OA$, since $OC = 1/\omega^2 L$. From the statement in section 13 respecting phase angles it is evident that our impedance triangle is similar to the triangle $A\ T\ P$ in Figure 5. By resolving Z_t into components we readily find that

$$R_t = \omega\, L \cdot \frac{PT}{OA} \tag{37}$$

$$C_t = \frac{OC\, .\, OA}{OT} = OH. \tag{38}$$

H is the intersection of $O\,C$ with a perpendicular erected on OA at A. Now remembering that $O\,q = C_1 + C_2$, and $C_1 = O\,F$, we see at once that the *apparent capacity* C_a of the resonator is

$$C_a = C_t - C_2 = O\,H - O\,q + O\,F = H\,q + O\,F \tag{39}$$

The apparent capacity of the resonator thus differs from both the equivalent series and the equivalent parallel capacity of the resonator alone, being a function also of C_2. The excess of C_a over C_1'' is greater the smaller the value of C_2.

The equivalent series resistance R_{12} of C_2 and the resonator is of importance, because it represents the increase in effective resistance of the circuit brought about by the absorption of energy in the resonator. It may be computed by the familiar method from C_2, C_1'' and R_1'', but more simply by writing $R_{12} = R_t - R$. Using the graphical representation of R given in section 13 we find

$$R_{12} = \frac{1}{\omega . OC} \cdot \frac{CT}{OA}. \tag{40}$$

Since $O\,A$ does not vary greatlv over the range of resonance,

R_{12} is evidently greatest when $C\,T$ is greatest, that is, when $O\,E$ in Figure 5 is a maximum (mechanical resonance). By substituting the appropriate values in (40), the value of R_{12} may be computed and compared with that derived from observation.

All the results in sections 17 and 18 are entirely independent of the question of electrical resonance, and hold for all values of C_2 and L.

Thus we see that the performance of the resonator is completely described in terms of the resonance circle and the accompanying vectors in Figure 5.

19. APPLICATIONS OF GRAPHICAL METHOD—Besides the pur pose that it serves in connection with the experimental test of the theory, the graphical method is useful in predicting the performance of the resonator in any circuit to which it may be connected, and also in finding the coefficient of viscosity Q of the material of the resonator.

In testing the theory, it must be stated at the outset that, owing to our ignorance of the actual amplitude of vibration x_o of the ends of the resonator, and owing also to the fact that we are not certain of the precise value of the piezo-electric constant ε, especially at radio frequencies, the diameter of the resonance circle must be found indirectly from the electrical data. A check on this could be obtained if x_o could be measured, and on the other hand observation of x_o could be employed for finding ε at high frequency. I hope soon to undertake the measurement of x_o with the aid of an interferometer. There are several possible ways of determining the resonance circle, based on observations of I, i, and C_2, in some cases inserting also a known resistance in series with L. Only one such method need be described here.

In this method we adjust C_2 to satisfy the condition of I-resonance at each frequency. Let it be recalled that the point A in Figure 5 then lies on the hyperbola $C\,A$; that $q\,A$ is equal and parallel to $O\,E$; and that $O\,q = C_1 + C_2$. As the impressed frequency passes thru resonance, E travels around the circle, and $O\,q$ passes thru a minimum, followed by a maximum. The points on the resonance circle corresponding to these two values of $O\,q$ are those points where the hyperbola $C\,A$, if moved vertically downward parallel to itself, would be tangent to the circle. Conversely, if the two values of $O\,q$ are known from observations of C_2, the circle fulfilling the above requirements is easily determined.

The same method may also be applied by adjusting C_2 at each frequency so as to satisfy the condition of i-resonance.

Exactly the same procedure is followed, save that the parabola $C\,B$, Figure 5, is used in place of the hyperbola $C\,A$. This has its advantages, inasmuch as, when R is made as small as possible, the parabola becomes practically a horizontal straight line. The points on the circle corresponding to the maximum and minimum values of C_2 are then the "quadrantal points" E_1 and E_2, Figure 2, for which $\tan\theta = \pm 1$.

By observing the frequency corresponding to any point on the circle for which θ is known, for example the point of tangency with a hyperbola mentioned above, or one of the quadrantal points, we may find the decrement Δ in equation (11). Having found Δ, the value of ω for any θ is easily computed. The theoretical value of C_2, i, or I for any frequency may then be found by construction.[11]

VI. Construction of Resonators

20. It was found in the early stages that strongest and most constant results were obtained when the crystal plate was placed between and not quite touching the metallic coatings. For the latter small flat plates of brass are now used, of the same size as the crystal or somewhat shorter.[12] Light contact between quartz and brass at one or two points is, for most purposes, of no consequence. The quartz rod may be supported by a thread tied about its center, or balanced on edge upon a small block. Successful tests have also been made with a quartz rod or plate silvered on both sides. It is easy to deposit chemically a coating of silver sufficiently thick for the electrical effects, without decreasing the frequency more than one or two tenths of a per cent. The advantages of this method are not sufficient, however, to offset the difficulty of making an electrical contact with the silver that is both permanent and delicate.

For a portable unit, the best method of mounting is to let the crystal plate lie in a small pocket in which it is just free to vibrate. The sides of the pocket are formed by the brass "coatings," the

[11]In the paper on longitudinal vibrations already mentioned, it is proved that $\Delta = \pi\,\omega\,Q/c^2$, where c, the wave velocity, equals $4\,l^2 f^2$ for the fundamental vibration. Thus by finding Δ we can compute the coefficient of viscosity Q. It is planned to deal with this more fully in a later paper. Due regard must of course be paid to air-friction.

[12]When the metallic coatings are shorter than the crystal, the behavior of the resonator is qualitatively unchanged. The consequent small modifications to the theory are easily made, if needed. Strictly, a correction should also be made in the theory on account of the air-space between crystal and the coatings. This is not so simple a matter. Fortunately the essential performance of the crystal is not altered, except in intensity, by the air space, and for quantitative tests of the theory, it is possible so to mount the rod as to make the air-space negligible.

bottom is of glass, and the ends of bakelite or hard rubber. By its own vibrations the plate keeps itself sufficiently free from contact with sides and ends, while any particles of dust on which it may rest serve as roller bearings to reduce friction with the bottom of the pocket.

Figure 7 shows a partially completed unit containing four plates, of which the longest is about 3 cm. (1.2 inch) in length. To the left is seen a small resonator containing one 3 cm. plate.

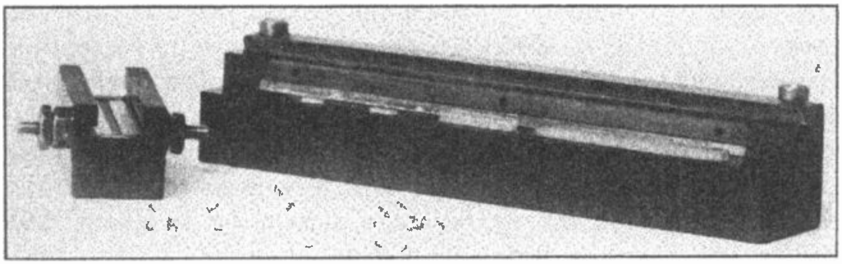

FIGURE 7—Quartz Resonators. Left: a single unit, mounted, for 3,040 m. Right: partially completed mounting for four quartz plates, which are seen lying on the front edge of the bakelite base. The wave-lengths are approximately 3,000, 1,200, 500, and 400 meters. These plates will stand on edge on the glass strip immediately behind them, having as one common "coating" the long brass bar which rests on the glass. The four individual "coatings," and the spacers to keep the quartz plates in place, are not shown.

21. THE CONSTANTS OF QUARTZ are such that the radio wave length in meters is roughly 100 times the length of the resonator in millimeters (2,500 times the length in inches). Concerning the relative dimensions of the crystal plates, the following are determining factors, which follow directly from the theory. Since all the essential phenomena are proportional to the diameter of the resonance circle as drawn in Figure 5, and since this diameter (compare section 13) contains the width b of the plate as a factor, it follows that b should be large. The limit to the width of the plate is set by the fact that disturbing modes of vibration enter in when the plate is not relatively narrow. In practice, the best value of the ratio l/b has been found to vary from 2 for the smallest plates (length about 2 mm. or 0.08 inch) to 10 or 20 for the largest. Thin plates are always an advantage, since for same voltage the electric field in the quartz is then more intense.

If the air-space is increased, or if the ends of the quartz extend far out beyond the brass coatings, the frequency is slightly raised. In order to understand this, one should compare the polarized plate with a short, wide bar magnet. When the coatings are close to the quartz, they and the associated circuit are analogous to a massive yoke of highly permeable material, allowing the induction to attain the greatest possible value. As the air gap increases, the plate partially depolarizes itself, owing to the turning back of some of the lines of electrostatic induction that are produced by its state of strain. The piezo-electric action of this depolarizing component is always to tend to make the plate regain its normal form; that is, the effective value of Young's modulus is larger, the greater the air gaps. In the most extreme case hitherto observed, the increase in frequency when the coatings, originally touching the quartz, were entirely removed, amounted to about 0.6 per cent. Hence no perceptible inaccuracy need be feared from this direction. Data at present available indicate that the frequency of the larger quartz resonators, when permanently mounted, is constant at least to within one part in 10,000. The quartz-steel resonators are subject to a small correction for temperature, amounting to about 0.01 percent decrease per degree centigrade.

The decrement of the resonator is easily proven to be independent of its cross-section, but directly proportional to the frequeney. Resonators for large wave lengths are therefore more efficient and more sharply tuned than those for short waves. This statement assumes that the viscosity Q is constant. The larger resonators are for this reason much more in danger of fracture from excessive voltage. It is doubtful whether a good resonator for 3,000 meters will stand safely as much as 50 volts at the resonant frequency.

The *electrostatic capacity* of the resonators varies from a few micro-micro-farads down to a small fraction of a $\mu\mu$f.

22. Quartz-Steel Resonators—For the longer wave lengths used in radio it will hardly be possible to secure sufficiently long quartz rods. The writer has used quartz up to about 4,000 m. (length of rod about 4 cm. or 1.6 inch), and beyond this has had good results with flat rods of tool steel or invar, excited to longitudinal vibration by means of small quartz plates cemented to the sides with solid shellac. The wave velocity in steel is not very different from that in quartz. For a 10,000 m. resonator, a steel rod about $95 \times 9 \times 3$ mm. ($3.71 \times 0.35 \times 0.12$ inch) is used, quartz plates about $9 \times 10 \times 1$ mm. ($0.35 \times 0.39 \times 0.04$ inch) being cemented to each side at the center, as shown in Figure 8. The steel itself forms one "coating," the quartz plates being so placed that the same polarity of each faces the steel. The other coatings are of tinfoil, to which fine wires are soldered and connected in parallel. The decrement of this combination is not very differ-

ent from that of the larger quartz resonators. A small hook is screwed into the steel at the exact center, between the quartz plates, to serve as a suspension and as one terminal of the resonator, leaving the rod free to vibrate at its fundamental frequency. Thru the action of the transverse effect the quartz rods expand and contract when connection is made to an alternating current supply, causing an alternating condensation and rarefaction at the center of the steel rod, whereby the longitudinal vibrations are excited. The frequency is essentiallv that of the steel rod, and the electric reactions take place exactly as with the quartz resonators, with an intensity sufficient to produce a strong response.

FIGURE 8—Steel resonator, the exciting quartz plates having tinfoil coatings electrically connected. The hook by which the steel rod is suspended serves as the other terminal

In the earlier experiments, the steel rods had plates of quartz or Rochelle salt at their ends, but this construction proved less reliable than that with the quartz side-plates.

A photograph of a mounted quartz steel resonator for 11,000 m. is shown in Figure 9.

VII. Applications

23. The Resonator as a Wave Length Standard—

1*st Method*—Resonator in parallel with tuning condenser of a low-power regenerative electron tube circuit containing a telephone receiver. If the tuning condenser is varied back and forth thru the setting for resonance with the resonator, short musical clicks are heard, the ringing quality of which is an indication of the low damping of the resonator. The note is due to the fact that the resonator, when once set into vibration, generates a radio frequency current and produces beats with the oscillating current proper. If the resonator has previously been calibrated, the setting of the condenser at which the click occurs may be noted, thus establishing one point in the calibration of the circuit.

2*nd Method*—The above method is made more precise if in parallel with the tuning condenser there are connected a key and small capacity in series. The tuning condenser is slowly varied the key being tapped continually. When a setting is reached such that, with the key open, the resonator is set into vibration, then on closing the key a heterodyne note is heard if the auxiliary capacity is of the right value. The note possesses a very sharp maximum at the resonant frequency.

3rd Method—Exactly like the first two methods, save that the resonator is not in the generating circuit, but in parallel with the tuning condenser of a non-regenerative receiving circuit which is tuned to the frequency of the resonator. If the two circuits

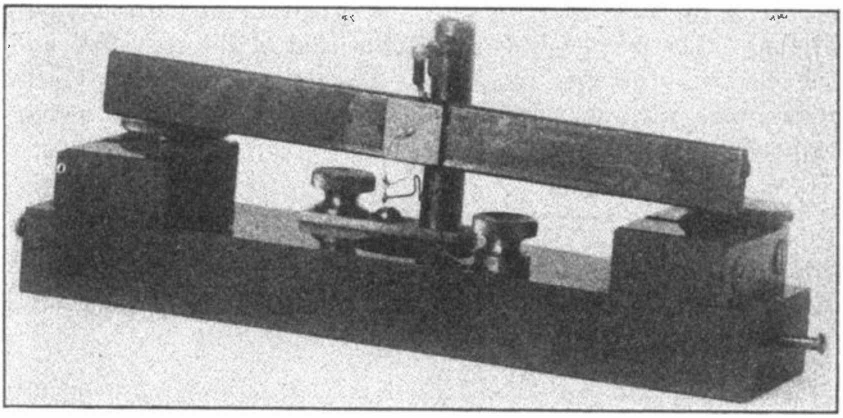

FIGURE 9—Steel resonator, excited by quartz plates at its center. Length about 10 cm. (3.9 inch), for 11,000 m. wave-length. When not in use, the resonator is unhooked and clamped to the bakelite block

are loosely coupled, a heterodyne note is heard, formed by beats between the current from the vibrating resonator and that impressed by the generating circuit. This is the most precise telephone method. For example, with a small 300-meter resonator the frequency can be determined to one part in about 5,000, while a quartz-steel resonator for 10,000 meters allows a precision of about 1 in 20,000.

Various modifications of the above methods are possible. For example, the resonator may be connected across the grid condenser or in still other parts of the generating or receiving circuit.

4th Method—Resonance indicated by ammeter or galvanometer. If the resonator is in the generating circuit, an ammeter in series with either anode or grid will give a sharp indication of resonance. Or the meter and resonator may be in a tuned circuit coupled to the generating circuit, in which case we have practically the arrangement shown in Figure 3.

5th Method—It is possible to use a resonator for the calibration of a buzzer-driven wave-meter, the receiving circuit containing a crystal detector. Owing to the impurity of the wave the method is difficult and not generally to be recommended.

24. The Resonator as a Frequency Stabilizer

There are several methods whereby the frequency of an electron tube generating circuit can be rendered practically free from disturbing capacity effects, variations in battery voltage, and so on. All make use of one or other of the properties of the piezo-electric resonator that have already been described.

The *first method* is virtually a mechanically-tuned feed-back. A three-stage resistance-coupled amplifier is used, as shown in Figure 10. The resonator 12 has two independent pairs of metallic "coatings," one pair, 13 and 14, being connected between output anode and ground, and the other pair, 15 and 16, between input grid and ground. Any slight increase in potential of the anode 5, by altering the electric field between 13 and 14, sets the resonator into vibration. The charges thereby excited in 15 and 16 vary the potential of grid 4 with respect to the ground. This varying potential, by virtue of the amplification taking place in the system, will, if the proper coating is connected to the grid, still further increase the variations in potential of 5 and maintain the resonator in vibration. The power output is of course very small—in my tests it was only about 0.05 watt—but by the insertion of a coil at 21, or by connecting the plate 5 to the grid of another amplifier, this small power may be further amplified indefinitely. With coatings connected as shown in Figure 10, the resonator vibrates at its second harmonic, with twice the fundamental frequency, there being a loop of compression midway between each end and the center. If either pair of coatings is reversed the fundamental vibration is excited. If a quartz-steel resonator is used for this purpose, a higher degree of amplification is necessary.

Three methods will now be described, in which only a single tube is needed. In each case we require a generating circuit with the usual coils, condensers, and feed-back.

In the *second method* we use a resonator with two pairs of coatings as before, connecting one pair between anode and ground, the other across the grid condenser. If, with the resonator disconnected, the regenerative coupling is made so loose that the circuit just fails to oscillate, then when the resonator is connected it furnishes the necessary additional feed-back, and oscillations ensue, of a frequency determined solely by the resonator The circuit must be tuned approximately to the resonator, but small variations in the electric constants are practically without effect upon the frequency.

The *third method* makes use of the fact that the relation between resonator current i (Figure 6) and frequency is such that a resonator with one pair of coatings, connected across the grid condenser of a generating set, will stabilize the frequency to a large degree. Regenerative coupling and electrical tuning are as in the second method.

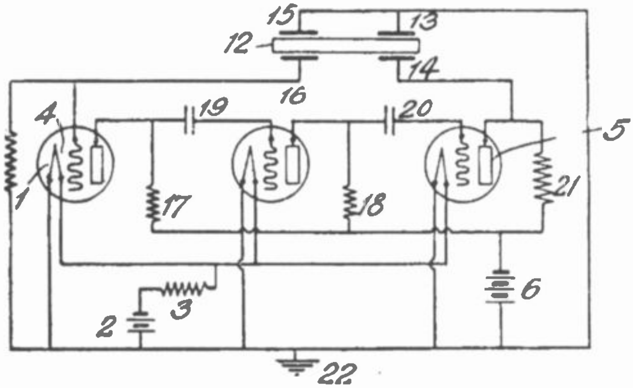

FIGURE 10—Resonator connected to three-stage amplifier, generating oscillations with mechanically tuned feed-back

The *fourth method* is of particular interest, as it makes use of the variation of apparent capacity of the resonator with frequency. *A* resonator with one pair of coatings is simply connected across the tuning condenser of any generating circuit the voltage of which is not high enough to endanger the crystal. This is, in fact, precisely the connection already described in Methods 1 and 2, section 22.

In order to make the operation of this method clear, we shall suppose the circuit to be oscillating initially with a frequency below the natural frequency of the resonator. As the tuning capacity is decreased, the frequency rises, and were it not for the resonator, the curve connecting capacity with frequency would be as indicated by the line 1, 9, 2, 4, 6, 8, 7 in Figure 11. If now the resonator is present and the resonant frequency is represented by the vertical line thru 4, then shortly before this frequency is reached the resonator begins to vibrate, but since the frequency is still relatively low, the capacity C_a of the resonator is abnormally high, to compensate for which the capacity of the tuning condenser C must be made abnormally small. The curve bends down along 2, 3, becoming practically vertical for a considerable range of C. When this decrease in C has become greater than the maximum value of C_a, the curve springs abruptly from 3 to 8, the frequency increases by a large amount, and any further decrease in C gives rise simply to the undisturbed portion 8, 7. By similar reasoning, when C is increased the curve 7, 8, 6, 5, 9, 1, is described. The operating part of the curve for the stabilizing effect is thus either 2, 3, or 5, 6.

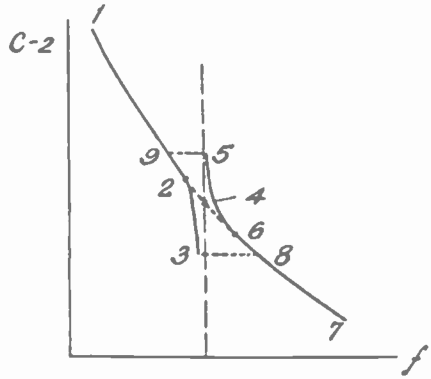

FIGURE 11—Stabilizing effect of resonator in a generating circuit. Over the portions 2, 3 and 5, 6 of the curve a large variation of C_2 produces a relatively small change in frequency

For example, a quartz plate 3.9 cm. (1.52 inch) long vibrated at a frequency of about 69,700. A change in C which, when the crystal was removed, altered the frequency by 3 percent., varied it by less than one part in 20,000 when the crystal was replaced. This resonator had coatings so small that its normal capacity was only 0.67 $\mu\mu$f.; yet when vibrating, its equivalent parallel capacity varied from +10 to −9 $\mu\mu$f.

25. The first method of stabilization described in section 24 suggests another possible application, namely as a *tuned mechanical coupling* between two circuits. For if the resonator is provided with two pairs of coatings, one pair being connected across a reactance in each of the circuits to be coupled, then the secondary circuit will receive energy only at the particular wave length (and, usually, certain harmonics) to which the resonator responds.

26. The nodes for the fundamental vibration or any harmonic may be detected directly by touching the crystal, while it is vibrating, with the point of a pencil. When the crystal is touched at a loop of displacement, the vibrations are easily stopped, while they are hardly affected when the contact is at a node.

The effect of energetic vibrations may be rendered visible by viewing through a magnifying glass particles of lycopodium powder sprinkled over the edge of the resonator near its ends.

Under certain conditions a sustained musical note is heard when a resonator is connected to a generating circuit provided with a telephone. The pitch depends upon the condenser setting, as if the circuit, while oscillating at its own electrical frequeney, nevertheless maintained the resonator in vibration at its mechanical frequency. The effect is especiallv pronounced with steel rods excited by plates of Rochelle salt.

Wesleyan University,
October 8, 1921.

SUMMARY: The general theory of piezo-electricity is summarized, and the equations applicable to quartz are stated. The "concentrated mass theory" of longitudinal vibrations in rods is then developed, and the properties of the "resonance circle" discussed. On this theoretical basis the equations for the reaction of a piezo-electric resonator upon a circuit are derived, and solved by a graphical method. From the vector diagram, important parameters, including the capacity and resistance of the resonator at any frequency, may readily be found.

The construction and operation of quartz and of quartz-steel resonators is described, and an account is given of their application as wave length standards and as frequency-stabilizing devices. Other practical applications are also suggested.

A STUDY OF THE OSCILLATIONS OCCURRING IN THE CIRCUITS OF THE PLIOTRON*

By

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1. Introduction

During the months of March and April, 1918, the authors determined the nature of the currents in the circuits of a pliotron tube, for low frequencies (100 to 500 oscillations per second), by means of a Duddell oscillograph. Altho the work was interrupted by the war, and it has not been possible to complete it in the form originally intended, it is thought best to publish the results obtained.

The circuits used are shown in Figure 1, where *PGF* is the pliotron; B_1, the battery in the plate circuit; B_2, the filament battery; L_1 and L_3, inductances in the plate circuit; L_2 and L_4, inductances in the grid circuit, and C, a condenser. L_3 and L_4 were closely coupled. The condenser C and the coil, L_3, therefore acted as "tickling" condenser and coil, respectively. k_1 and k_2 are keys in the plate and grid circuits, respectively. Sometimes the condenser, C, was used without the coils L_3 and L_4, and sometimes they were used without the condenser.

With sueh an arrangement we have three principal circuits, viz.: $P\ L_3\ k_1\ L_1\ B_1\ F$, the plate circuit; $G\ L_4\ k_2\ L_2\ F$, the grid circuit; and $C\ L_3\ k_1\ L_1\ B_1\ L_2\ k_2\ L_4$, the external circuit. The oscillations were studied by inserting the oscillograph successively in each of the circuits, and obtaining oscillograms showing the current flowing for a steady oscillation and also for its rise and fall.

The Duddell oscillograph was made in the shop of the Physical Laboratory of Clark University. The pliotron tube was kindly given by the General Electric Company to Professor A. G. Webster for experimental purposes. The diameter of its bulb

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was about 12.6 cm. (5 inches); the dimensions of the plates about 6.0 by 4.5 cm. (2.34 by 1.76 inch), and their distance apart about one cm. (0.39 inch). The filament, midway between the plates, was at a distance of about 2 millimeters (0.078 inch) from the symmetrical grid.

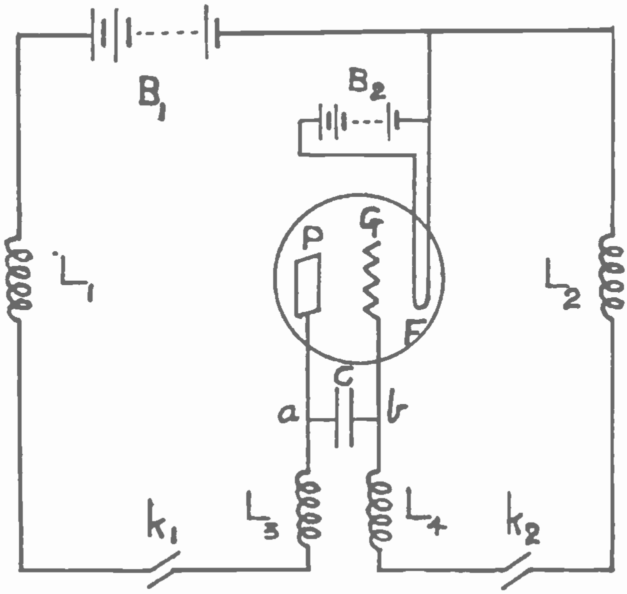


FIGURE 1—Diagram of Pliotron Circuit

The static characteristic for the tube, showing the current from the plate to the filament, for different voltages between the plate and the filament, is given in Figure 2. The current thru the filament for these values was 3.56 amperes.

The variation of the steady current from the plate to the filament as the current thru the filament is varied, when there is a difference of 71 volts between the plate and the filament, is shown in Figure 3.

On the oscillograms, the vibrations of a tuning fork, having a frequency of 101.3 vibrations a second, are shown either above or below the oscillogram. The two curves can be distinguished from each other since that of the pliotron usually has a straight line running thru it, the line of zero current.

The battery B_1, in the plate circuit, was of 80 volts and produced a direct current, when the circuits were not oscillating, of from 26 to 62 milliamperes, and a mean oscillating current, of from 73 to 100 milliamperes. The mean oscillating current in the grid circuit varied from 60 to 70 milliamperes. In a typical case, the direct and oscillating currents in the plate circuit were 40 and 73 milliamperes, respectively, and the oscillating current in the grid circuit was 60 milliamperes. The current thru the filament was about 3.6 amperes.

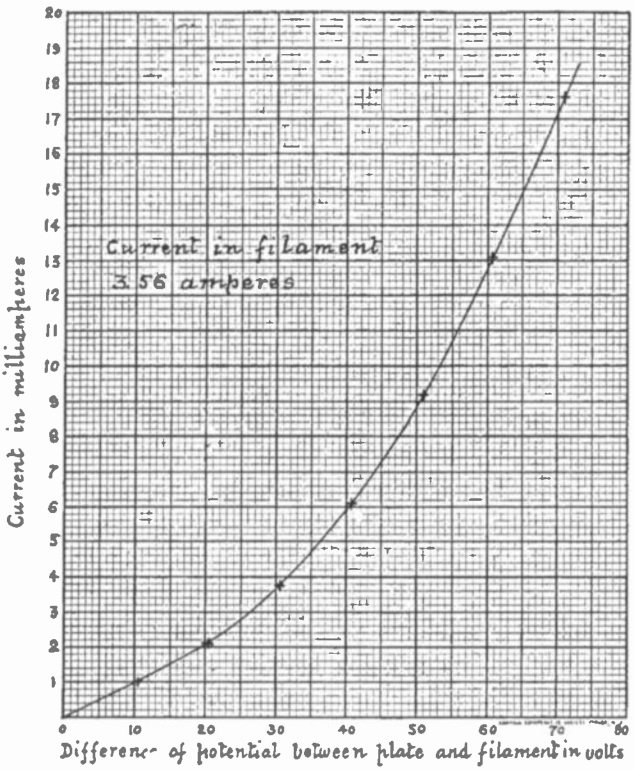


FIGURE 2—Static Characteristic for the Pliotron

2. RISE AND FALL OF THE CURRENT

To study the manner in which the oscillations begin and end in the plate and grid circuits, respectively, oscillograms 1-6 (Figure 4) were taken. For oscillograms 1-4 there were both magnetic and electrostatic coupling, and for 5 and 6, only electrostatic coupling. For 1-4, L_1 was equal to 0.143 henry; L_3, 0.147 henry; L_4, 1.95 henry; L_2, 0.203 henry, and C 0.95 microfarad. The mutual inductance of L_3 and L_4 was 0.402 henry. For 5, these values were the same except that L_3 and L_4 were taken out of their circuits and C was equal to 1.00 microfarad.

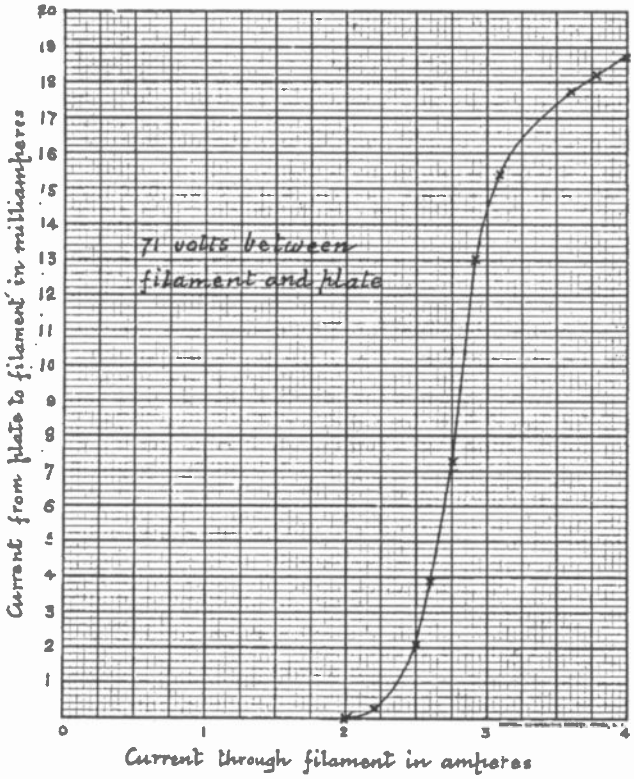


FIGURE 3—Showing Relation of Plate and Filament Currents

For 6, L_3 and L_4 were also absent; L_1 was equal to 0.210 henry; L_3, 3.25 henrys, and C, 0.50 microfarad.

Oscillograms 1 and 2, for both magnetic and electrostatic coupling, show the current in the plate circuit, the oscillograph being placed between L_3 and k_1. For 1, k_1 was closed before k_2. For, 2, k_2, was closed before k_1. If k_1 is closed before k_2, a steady direct current will be flowing in the plate circuit when k_2 is closed. If k_2 is closed before k_1, there will be no current in the grid circuit until k_1 is closed, and when this happens, both circuits will begin to oscillate immediately. Also for 1, the oscillation was *stopped* by opening k_2, and for 2, by opening k_1. It will be noticed that in these oscillograms the current is mainly positive and starts off with a positive flow.

Oscillograms 3 and 4, also for both magnetic and electrostatic coupling, show the current in the grid circuit, the oscillograph being placed between L_2 and k_2. For 3, k_1 was closed before k_2, and for 4, k_2 before k_1.

For oscillograms 5 and 6, with electrostatic coupling only,

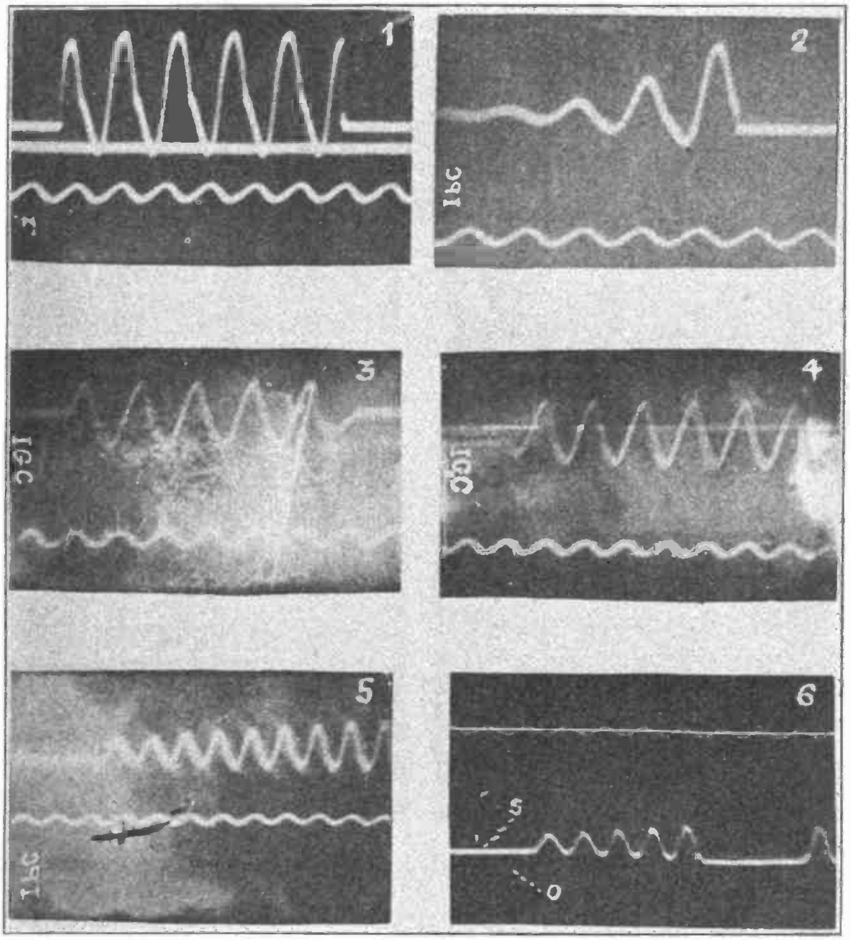


Figure 4—Oscillograms 1-6, Showing Rise and Fall of Current in Plate and Grid Circuits

the current builds up gradually and is not as strong as when both magnetic and electrostatic couplings are used. For 5, the oscillograph was in the plate circuit between L_3 and k_1, and k_1 was closed before k_2. For 6, the oscillograph was next to the plate. It shows the current flowing into the plate, which is seen to be all positive. The line marked *s* on the oscillogram is that for steady current, and that marked *o* is that for no current. The oscillogram shows that, in this case, the oscillating current into the plate does not return to zero for every oscillation.

3. Steady State; Electrostatic Coupling Only

Oscillograms 7-11 (Figure 5) show the current in the steady state when inductances L_3 and L_4 were taken out and there was therefore electrostatic coupling only. For 7-10, L_1 was equal to 0.210 henry; L_2, 3.25 henrys, and C, 0.95 microfarad.

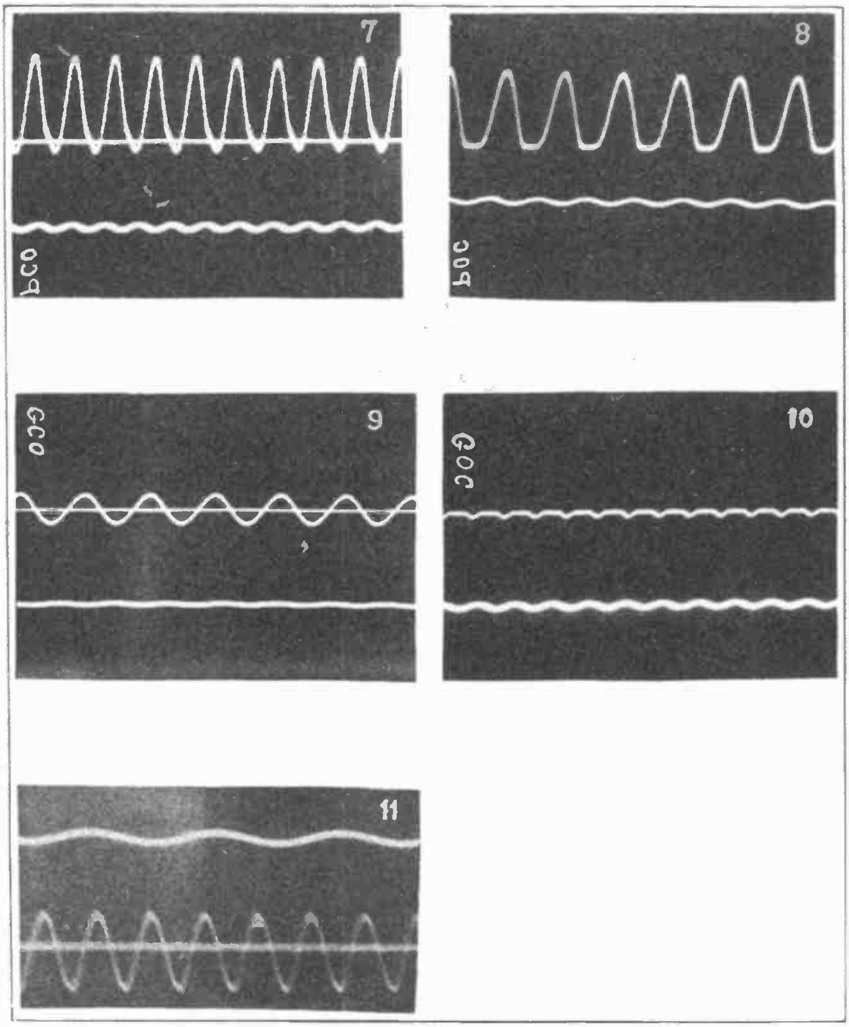


Figure 5—Oscillograms 7-11, Showing Current in Steady State for Electrostatic Coupling Only

For 7, the oscillograph was in the plate circuit between k_1 and L_1. It shows a large oscillating current in this circuit, mostly positive and deformed from a harmonic oscillation in a curious way. For 8, the oscillograph was next to the plate. It shows an interrupted positive current into the plate.

For 9, the oscillograph was in the grid circuit between k_2

and L_2. It shows that the current in the main part of the grid circuit is simple harmonic. For 10, the oscillograph was placed next to the grid. It shows a small negative current into the grid.

For 11, the oscillograph was next to the condenser, C, between a and C, and shows the current flowing thru the condenser. It is seen to be simple harmonic, slightly more negative than positive. This oscillogram was taken at a different time from 7-10. C was equal to 1 microfarad, and L_1, to 0.143 henry, but no record was kept of the value of L_2.

These five curves show that, for the case of electrostatic coupling only, there is a pulsating positive current from the plate to the filament, shown in 8, which produces a harmonic current, 9, in the grid circuit. This current divides at b, a very small part of it going into the grid and the rest of it, 11, harmonic or nearly so, flowing thru the condenser. This condenser current flows back into the grid circuit by way of $a\, L_3\, k_1\, L_1\, B_1$ (see Figure 1), and, combined with 8, produces the main current, 7, of the plate circuit.

4. Steady State; Magnetic Coupling Only

Oscillograms 12-17 (Figure 6) were taken to show the current when there was magnetic coupling only; that is, when the condenser C was omitted.

For 12-15, L_1 was equal to 0.210 henry; L_3, 0.325 henry; L_4, 1.71 henry; and L_2, 0.203 henry. The mutual inductane of L_3 and L_4 was 0.505 henry.

For 12 and 13 the oscillograph was in the plate circuit; for 12 between L_3 and L_1; and for 13 next to the plate. For 14 and 15 the oscillograph was in the grid circuit; for 14 between L_4 and L_2, and for 15 next to the grid. Curve 13 shows that the pulsating positive current from the plate to the filament is nearly all positive. The small negative component added to it to form the main current, 12, of the plate circuit, may, we think, be due to the electrostatic capacity between the coupling coils L_3 and L_4. The same effect is to be noted for the grid circuit in 14 and 15. The action of the electrostatic capacity between the coupling coils in this case is of somewhat the same nature as that of the coupling condenser described in the preceding section.

For oscillograms 16 and 17, L_1 was equal to 0.143 henry; L_2, 0.203 henry; L_3, 0.147 henry, and L_4, 1.99 henry. The mutual inductance between L_3 and L_4 was 0.402 henry. 16 and 17 were taken under the same conditions except that the drum was turning faster for 17 than for 16. The conditions were much the same as for 12, but 16 and 17 show the presence of an oscillation of higher frequency imposed on that of the plate circuit, which the authors suppose to be due to the mutual electrostatic capacity of the coupling coils, L_3 and L_4.

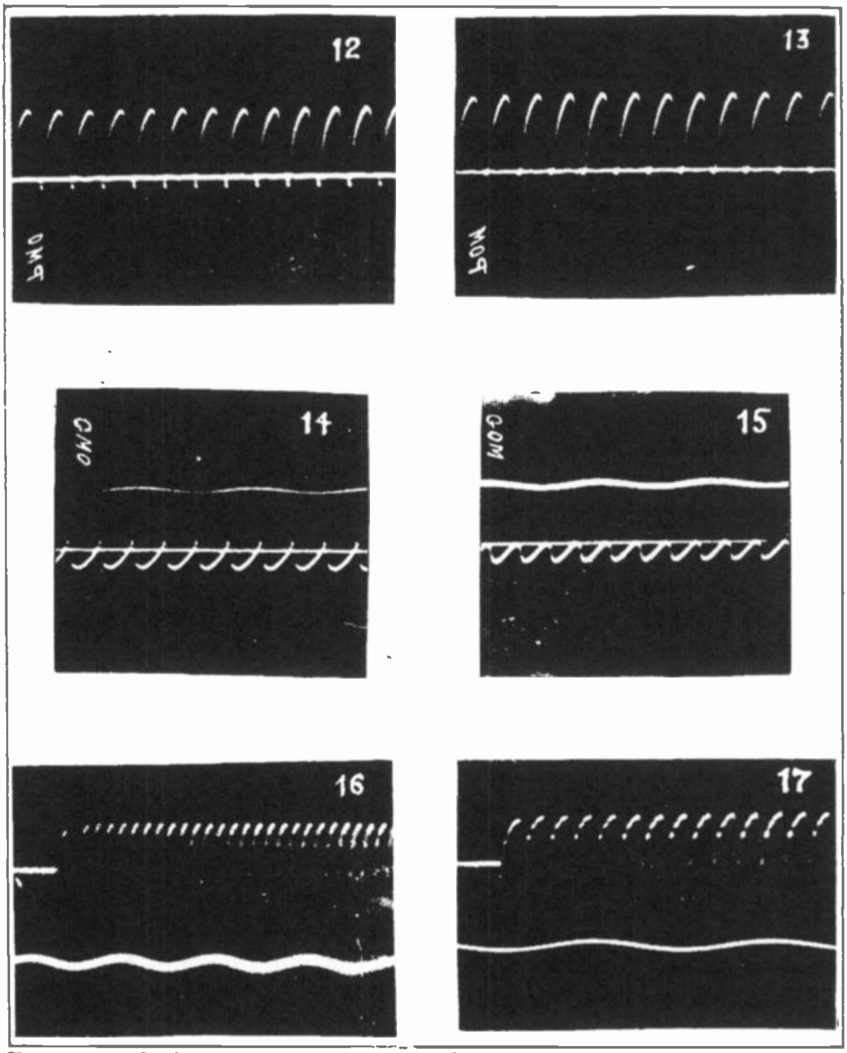


FIGURE 6—Oscillograms 12-17, Showing Current in Steady State for Magnetic Coupling Only

5. Steady State; Both Magnetic and Electrostatic Coupling

Oscillograms 18-23 (Figure 7) give the steady current in different parts of the plate and grid circuits when there is both magnetic and condenser coupling between them. In this case L_1 was equal to 0.210 henry; L_3, 0.325 henry; L_4, 1.71 henry; L_2, 0.203 henry, and C, 0.95 microfarad. The mutual inductance between L_3 and L_4 was 0.505 henry.

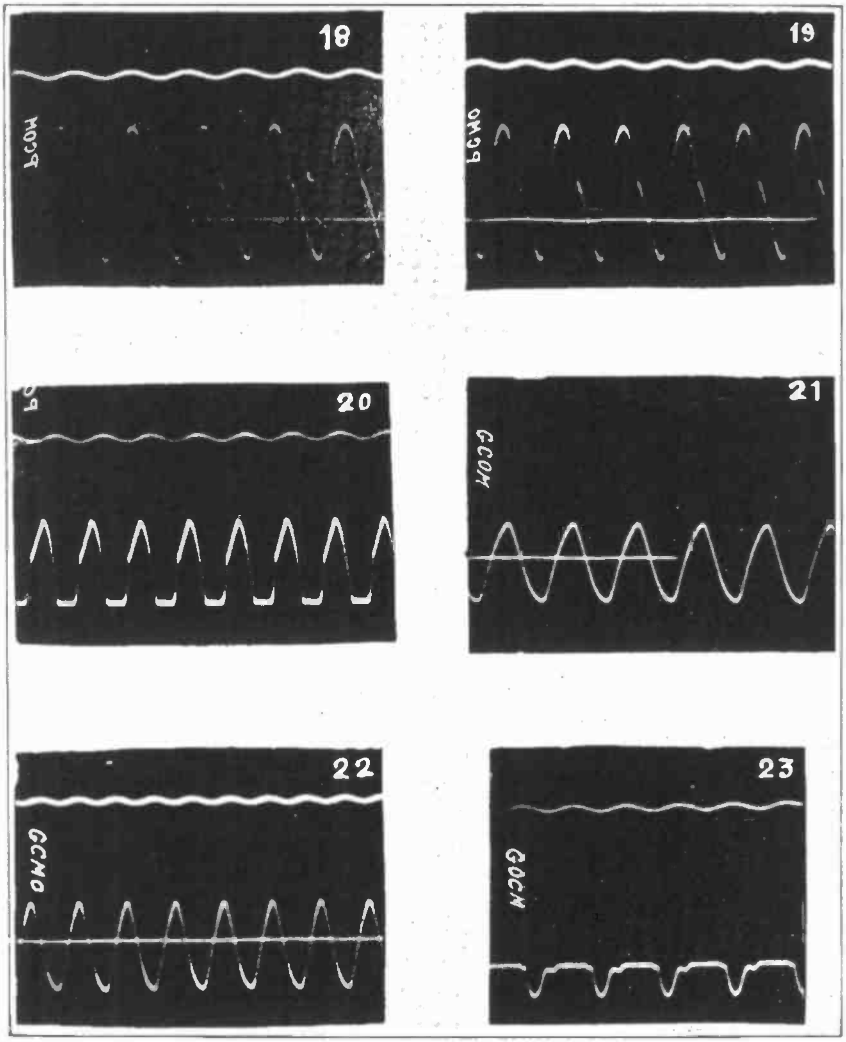


FIGURE 7—Oscillograms 18-23, Showing Current in Steady State for Both Magnetic and Electrostatic Coupling

For 18, 19, and 20 the oscillograph was in the plate circuit, and for 21, 22, and 23 in the grid circuit.

For 18 and 19, the oscillograph was placed between L_3 and a, and L_3 and k_1, respectively. The curves are alike and show the current flowing in the main branch of the plate circuit. It is seen that, altho it is mainly positive, it has a negative component. Its form departs strongly from a harmonic curve. For 20 the oscillograph was placed next to the plate and therefore shows the current going into it. This is seen to be all positive. The current in the main branch of the plate circuit, shown in 18 and 19, is made up of the current flowing into the plate and the current flowing into the condenser. Therefore, if 20 is subtracted from 18, we get the current flowing thru the condenser. This being oscillating will give the negative component of 18.

For 21 and 22, the oscillograph was placed between L_4 and b, and L_4 and k_2, respectively. 21 and 22 are alike and show the current in the main branch of the grid circuit. The current is nearly harmonic and is made up of that flowing into the condenser and into the grid. For 23, the oscillograph was placed next to the grid and shows the current flowing into it. It is seen to be all negative. If 23 is taken away from 21, the resultant will be the current flowing into the condenser.

6. Effect of the Mutual Capacity of the Coupling Coils

Attention has been drawn to the presence of a higher frequency imposed on the fundamental frequency of the plate circuit in oscillograms 16 and 17, and the suggestion has been made that this is due to the mutual electrostatic capacity of L_3 and L_4.

This effect would be much the same as that of a capacity shunted across L_3. To determine such an effect, a capacity of 8.3 microfarads was shunted across L_3 and oscillograms 24-29 (Figure 8) were obtained. The circuit, with its inductances and capacity, was the same as that described in the preceding section, except that for 28 and 29, the capacity C was reduced to 0.05 microfarad. For 24 the oscillograph was placed between L_3 and k_1; for 25, in the closed circuit formed by the shunting condenser and L_3; for 26, between b and L_4, and for 27, next to the grid. For 28, the oscillograph was placed in the closed circuit formed by the shunting condenser and L_3, and for 29, next to the plate. These curves show that a shunting capacity of 8.3 microfarads produces an oscillation of higher frequency than that of the fundamental of the external circuit and that the former is superimposed on the latter. Comparing these oscillograms with those given in the preceding section, we see that 24 has the same general form as 19, 26 as 21, and 29 as 20, except that the oscillation of greater frequency is superimposed on

24, 26, and 29. Oscillograms 27 and 23, for the current going into the grid, are very much alike.

These oscillograms, therefore, show how the mutual capacity of the coupling coils L_3 and L_4 might produce the effect shown in 16 and 17.

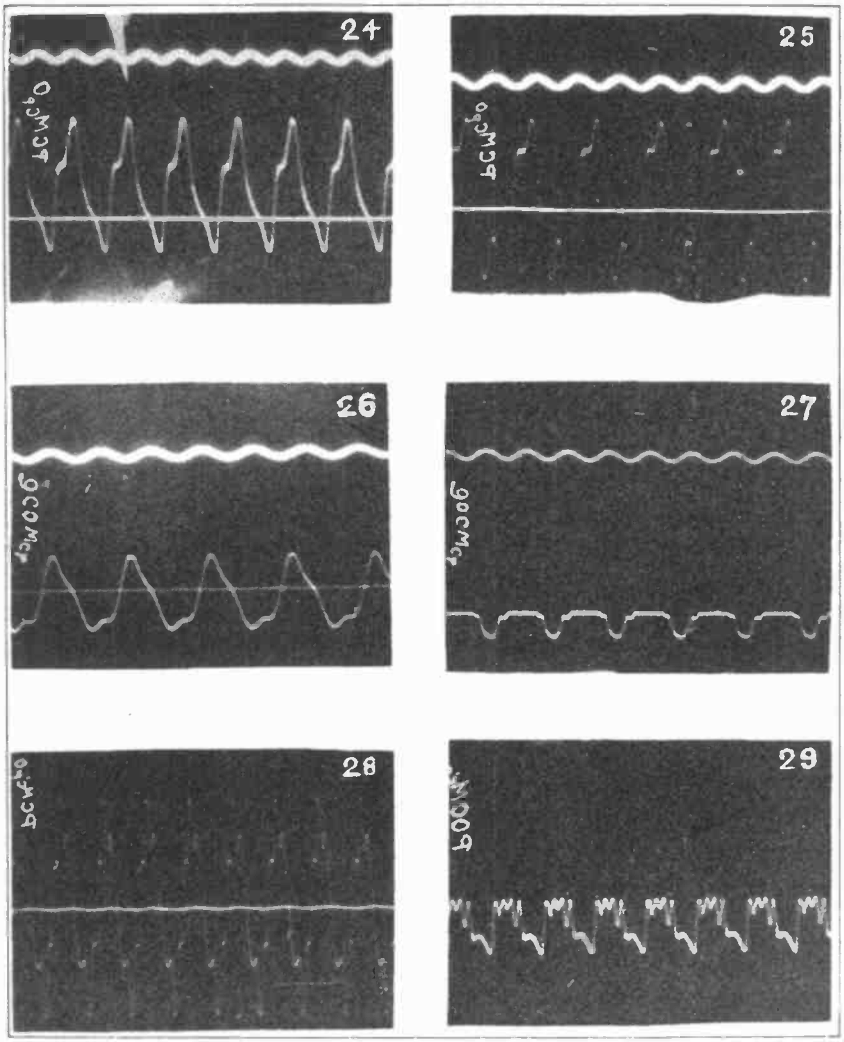


FIGURE 8—Oscillograms 24-29, Showing Effect of Condenser Shunted Across L_3

7. Discussion of Results

The oscillations in the pliotron circuit shown in Figure 1 can be compared with those of the pendulum and escapement of a clock. The plate circuit contains a battery which supplies the energy of the oscillations just as the energy of the clock is supplied by the wound up weight or spring. The external circuit has an oscillation of a definite period, just as the pendulum has in a clock, and is kept in oscillation by the tickling coil, L_3, or the tickling condenser, C, just as the pendulum is kept in oscillation by the escapement.

It is to be noticed that in all cases the current into the grid is negative. This current into the grid produces a negative charge on it which periodically checks, or stops, the flow of electrons from the filament to the plate.

Consider the case represented by curves 12-15. Here the coupling is entirely by the coils L_3 and L_4 and there is no condenser present. Suppose that in the first place k_2 is closed and k_1 is open. Then there will be no current in either circuit. Suppose now that k_1 is closed. Then the current will grow in the plate circuit according to the law

$$i=\frac{E}{R}\left(1-\varepsilon^{-\frac{R}{L}t}\right) \quad (1)$$

If the grid circuit is open, the plate current will grow until it reaches a steady value. If, however, the grid circuit is closed, the growing current in L_3 will create a counter electromotive force in L_4 which will produce a negative current into the grid and consequently a negative charge on it. This negative charge will reduce, or stop entirely, the flow of electrons from the filament to the plate and the current in the plate circuit will be decreased, or fall to zero, as shown in oscillogram 13. The process will then be repeated. The resistance of the plate circuit lies principally in the passage of the electrons from the hot filament to the plate, but also contains the resistances of the inductances L_1 and L_3 and of the battery B_1. It can be found approximately if we know the steady current corresponding to a given voltage of the battery B_1. For we then have

$$C=\frac{E}{R}$$

and

$$R=\frac{E}{C}$$

For our case the battery B_1 had a potential of 80 volts and produced a steady current in the plate circuit of 0.062 ampere. The equivalent resistance was, therefore, about 1,290 ohms. Since we know the inductance in the plate circuit we can calculate its time constant.

When the flow of electrons from the filament to the plate ceases, the current in the plate circuit will decay according to the law

$$i = I \varepsilon^{\frac{R}{L}t}$$

It will be seen from oscillogram 13 that the current in the plate circuit grows and decays as would be expected in a circuit containing inductance and resistance.

If both electrostatic and magnetic coupling are used, then the periodic charging and discharging of the condenser will set up an oscillating current around the external circuit $C\ L_3\ L_1\ B_1\ L_2\ L_4$. (See Figure 1.) The oscillograms obtained, 18-23, show that this current is the resultant of a harmonic current thru the condenser and the current flowing into the plate. In the same way the current in the main branch of the grid circuit is the resultant of the harmonic oscillating current thru the ocndenser and the current flowing into the grid. The currents in the main branches, both of the plate and grid circuits, are modified slightly by the mutual electrostatic capacity of the coupling coils L_3 and L_4.

When the coupling is made only by a condenser the harmonic oscillation due to the condenser is well marked, as is shown in oscillogram 9.

It is evident that there are three cases to be considered

1. Magnetic coupling only.
2. Electrostatic coupling only.
3. Combined magnetic and electrostatic coupling.

For the first case the period of the oscillation will be determined by the values of the time constants, $\frac{L'}{R'}$, and $\frac{L''}{R''}$, of the plate and grid circuits, respectively, where L′ and L″, and R' and R'' are the total inductances and resistances of these two circuits.

For the second case, and also approximately for the third case, the period will be determined by the value of the capacity, C, and inductances, L_1, L_3, L_4, and L_2, in the external circuit.

If L is the total inductance and C the capacity in the external circuit, the period, T, of the oscillation in this circuit can be calculated from the formula

$$T = 2\pi\sqrt{LC} \qquad (3)$$

The mean period of oscillograms 7-10 was determined from the curves and the known frequency of the tuning-fork and found to be equal to 0.0118 sec. The period calculated from the constants of the circuit was 0.0114 sec. The calculated values are somewhat lower than the observed values, but they show that, at least for oscillations of about this frequency, and for the case of electrostatic coupling, the fundamental frequency is approximately that of the external circuit and can be calculated from the constants of this circuit.

For oscillograms 18-23, where we have both electromagnetic and electrostatic coupling, the period determined from the curves was 0.0123 sec. The calculated value was 0.0114. For oscillograms 1, 3, and 4, also cases of combined electromagnetic and electrostatic coupling, the period determined from the curves was 0.0121 sec., whereas that calculated was 0.0113 sec. In these cases, since the inductance L_3 and L_4 were very closely coupled, being in fact parts of a single large coil, the total inductance of the coil, 3.05 henrys, was taken instead of the sum of L_3 and L_4. Also, on account of their very close coupling, L_3 and L_4 probably had a large mutual electrostatic capacity, which would help to make the experimental value greater than the calculated.

The oscillograms obtained would therefore, indicate that for the cases of electrostatic, and combined magnetic and electrostatic coupling, for the oscillating circuits and frequencies used, the fundamental frequency produced by a pliotron tube can be determined, at least approximately, from formula (3) where L is the total inductance and C the total capacity of the external circuit.

Clark University,
Worcester, Massachusetts.
March 28, 1921.

SUMMARY: The currents in the grid and plate circuits of a pliotron are studied oscillographically, both for the transient conditions at the beginning of oscillations, and for the steady oscillating state. Magnetic, electric, and combined magnetic and electric couplings between the grid and plate circuits are tested. The effects obtained are explained, and the experimentally determined periods of oscillation checked against the suitably calculated values.

A SIMPLE METHOD OF CALCULATING RADIATION RESISTANCE*

By

FULTON CUTTING

INDEPENDENT WIRELESS TELEGRAPH COMPANY, NEW YORK)

The output of radio transmitting stations is the product of the input and the antenna efficiency. In designing new stations it is therefore essential to have some idea of the latter quantity in order to be able to choose the size of the generator required for a given result. Expressing this mathematically, we have the relation:

$$W_o = \frac{R_r}{R_g + R_c + R_r} W_i \qquad (1)$$

where:

W_o = output

W_i = input

R_g = ground resistance

R_c = conductor resistance

R_r = radiation resistance

For a given output the cost of a station may be given as:

$$\text{COST} = f_1(R_r) + f_2\left(\frac{1}{R_g}\right) + f_3\left(\frac{1}{R_c}\right) + f_4(W_i) \qquad (2)$$

In this equation F_1 must be determined by calculation, f_2 by experiment, f_3 by experiment (or by calculation if you know how), f_4 by ascertaining the current prices of suitable generators. Knowing these functions a radiating system and generator can be chosen to give a certain output at a minimum cost. Whether this solution will be desirable from a practical standpoint in which operating and maintenance expenses are considered, is a matter that will not be discussed here. All that is desired to be brought out is the essential importance of knowing the radiation resistance

If, for instance, the ground resistance and conductor resistance are low, and the radiation resistance at the wave length to

* Presented before THE INSTITUTE OF RADIO ENGINEERS, New York January 23, 1922. Received by the Editor, February 1, 1922.

be used high, so that the antenna efficiency is over 50 per cent, any increase of radiation resistance effected by using higher towers will result in a relatively unimportant gain in antenna efficiency compared to the expense involved. If, however, the antenna efficiency is around 5 per cent, due, let us say, to unavoidably high ground resistance together with the necessity of using a very long wave, it is then well worth while increasing the radiation resistance, as the antenna efficiency and the output will be practically proportional to the radiation resistance.

The radiation resistance of an L antenna has been very ably computed by Professor Pierce. The method he uses is applicable to any shape of antenna, but it is very laborious. The present method is a modification of the doublet formula given by Hertz. It has little theoretical justification, but it is extremely simple, and checks fairly well with Pierce's results.

The doublet formula assumes that the current is uniform over the entire vertical length of the antenna. No account, moreover, is taken of the radiation from the flat top. Neglecting the radiation from the flat top is not a serious error; first, because its magnitude is small, and secondly, because its direction is such that it contributes nothing towards the electric or magnetic field in the horizontal plane. The assumption of uniform current is, of course, very far off. To correct this, it is assumed that the current decreases linearly along the antenna from maximum at the base to zero at the open end. The radiation resistance is then taken as the value given by the doublet formula multiplied by the ratio of the mean squared vertical current to the square of the current at the base.

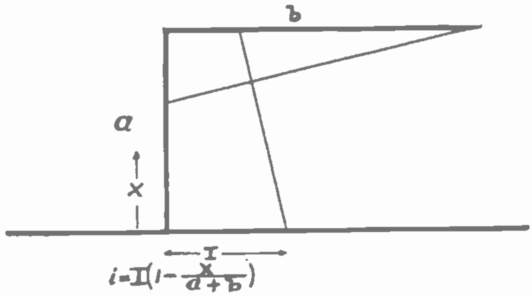
That is:

$$R_r = 1578 \left(\frac{h}{\lambda}\right)^2 \frac{I_{eff}^2}{I^2} \qquad (3)$$

The linear current distribution is assumed, first, because it is easy to handle, and secondly, because most antennas are worked sufficiently far from their natural period to approximate this condition. A strictly linear current distribution would be obtained if the antenna were worked very far from its natural period and provided also that the capacity per unit length of the antenna remains constant along its entire length. Neither of these conditions are exactly fulfilled, but the errors introduced by the usual departure from these conditions are such as to tend to offset each other.

Let us apply (3) to an L antenna of heighth a and length b: The current distribution is shown in Figure 1:

$$I_{eff}^2 = \frac{1}{a}\int_0^a i^2\,dx$$
$$= \frac{I^2}{a}\int_0^a \left(1 - \frac{2x}{a+b} + \frac{x^2}{(a+b)^2}\right)dx$$
$$= I^2\left[1 - \frac{a}{a+b} + \frac{1}{3}\,\frac{a^2}{(a+b)^2}\right]$$



EIGURE 1

Therefore the radiation resistance is:

$$R_r = 1578\left(\frac{a}{\lambda}\right)^2\left[1 - \frac{a}{a+b} + \frac{1}{3}\,\frac{a^2}{(a+b)^2}\right] \qquad (4)$$

In cases where the flat top is as long as, or longer than the vertical portion, the last term in the brackets can be omitted without serious error. This gives the following simple formula for rough calculations:

$$R_r = 1600\left(\frac{a}{\lambda}\right)^2\left[\frac{b}{a+b}\right]$$

Using Pierce's notation:

$$\gamma = \frac{b}{a+b} \qquad \lambda^o = 4\,(a+b) = \textit{natural wave length}$$

$$R_r = 32.9\,\frac{(1-\gamma^2(}{\left(\frac{\lambda}{\lambda_o}\right)}\,[1+\gamma+\gamma^2]$$

Figures 2 and 3 show this expression compared with Pierce's values. The latter are shown dotted. The agreement is good for $\gamma = 0.4$, $\gamma = 0.6$, and $\gamma = 0.8$, except near the natural period. This is to be the expected, since at this point the current distribution is more nearly sinusoidal than linear, and therefore the linear assumption would give too small a value for the radiation resistance.

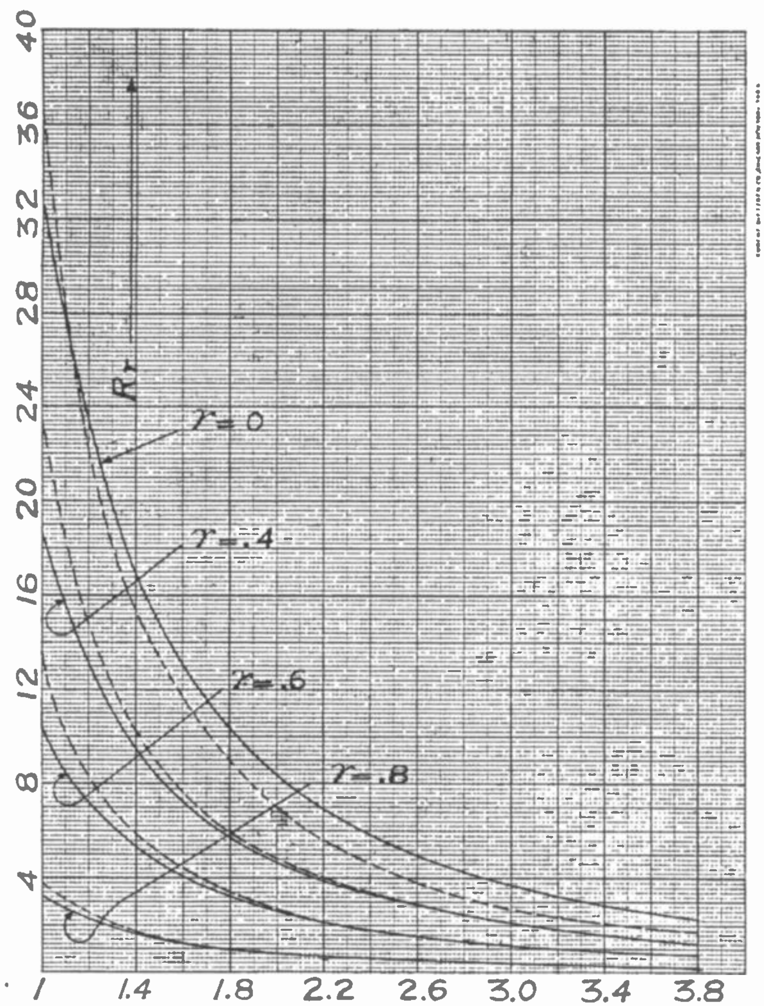


FIGURE 3

The dotted curve crosses and falls slightly below the solid curve for $\gamma = 0.4$, and markedly so for $\gamma = 0$.

Figure 4 shows the radiation resistance of the vertical portion of the antenna, calculated by Pierce's method, as compared with the total radiation resistance. The error involved in neglecting the flat top is seen to be small. Only one curve is plotted ($\gamma = 0.6$) since the maximum radiation from the flat top comes close to this point. The maximum radiation from the flat top comes when the length b is twice the height a (that is $\gamma = 1.666$).

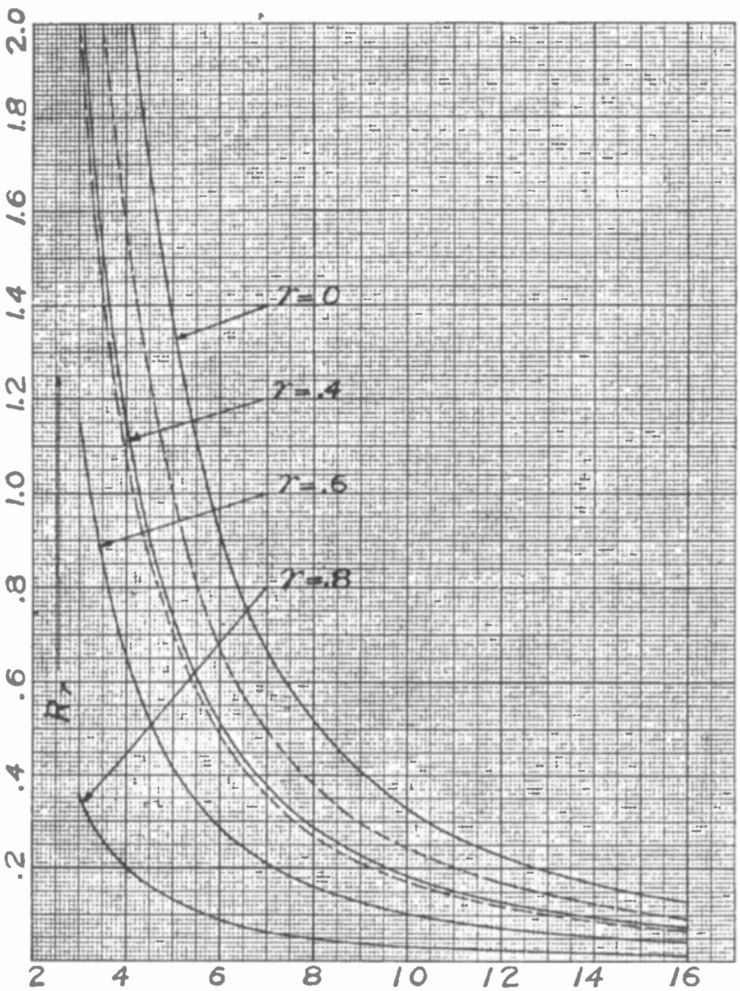


FIGURE 3

We shall now consider types of antenna different from the L. Let us take an antenna such as is shown in Figure 5.

The length to be used in the doublet formula is taken as $a+b \sin \theta$ and the effective current squared as:

$$I^2_{eff} = \frac{I^2}{a+b\sin\theta}\left[\int_0^a \left(1-\frac{x}{a+b}\right)^2 dx + \left(1-\frac{a}{a+b}\right)^2 \int_0^{b\sin\theta}\left(1-\frac{x}{b\sin\theta}\right)^2 dx\right]$$

$$= \frac{I^2}{a+b\sin\theta}\left[a-\frac{a^2}{a+b}+\frac{a^3}{3\,(a+b)^2}+\frac{b^3\sin\theta}{3(a+b)^2}\right]$$

Therefore:

$$R_r = 1578\ \frac{a+b\sin\theta}{\lambda^2}\left[a\left(1-\frac{a}{a+b}+\frac{1}{3}\frac{a^2}{(a+b)^2}\right.\right.$$
$$\left.\left. + \frac{b\sin\theta}{3}\left(\frac{b}{a+b}\right)^2\right] \qquad (5)$$

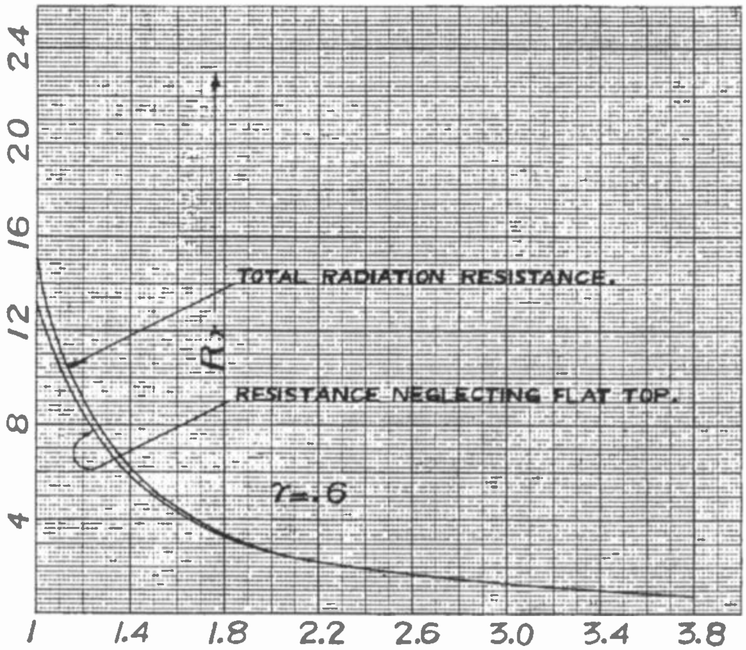


FIGURE 4

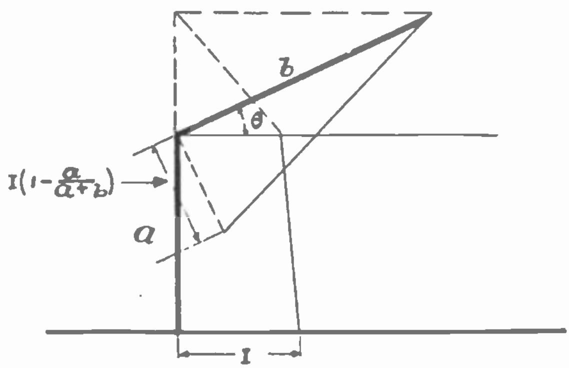


FIGURE 5

In this case the current in the flat top partly neutralizes the current in the vertical portion so that the resultant current is represented by the line $A\,B\,C$. The effective current squared is then:

$$I_{eff}^2 = \frac{I^2}{a}\left[\int_0^{a-b\sin\theta}\left(1-\frac{x}{a+b}\right)^2 dx + \left(1-\frac{a-b\sin\theta}{a+b}\right)^2 \int_0^{b\sin\theta}\left(1-\frac{x}{b\sin\theta}\right)^2 dx\right]$$

$$R_r = 1578\frac{a}{\lambda^2}\left[(a-b\sin\theta)\left(1-\frac{a-b\sin\theta}{a+b}+\frac{1}{3}\left(\frac{a-b\sin\theta}{a+b}\right)^2\right) + \frac{b\sin\theta}{3}\left(\frac{b+b\sin\theta}{a+b}\right)^2\right] \qquad (6)$$

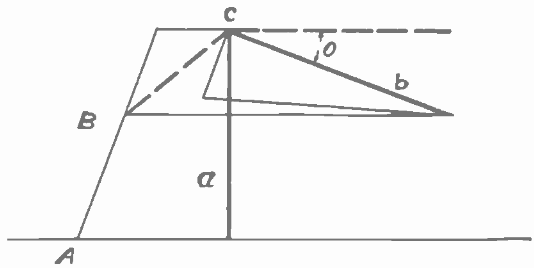


FIGURE 6

As an example of the use of these expressions, let us consider the case of an antenna which must be erected on a single mast. Three types of such an antenna are shown in Figure 7.

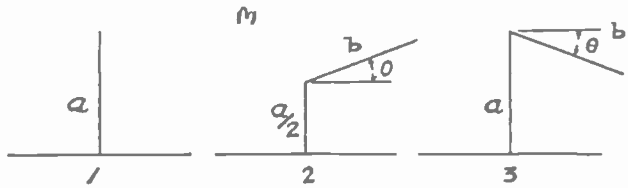


FIGURE 7

For definiteness let us take $a=b$, $\theta=\pi/6$. From (4) we have

$$R_{\mathrm{I}} = 1578\left(\frac{a}{\lambda}\right)^2(0.33)$$

From (5) we have:

$$R_{\mathrm{II}} = 1578\left(\frac{a}{\lambda}\right)^2(0.43)$$

And from (6) we have:

$$R_{\mathrm{III}} = 1578\left(\frac{a}{\lambda}\right)^2(0.48)$$

We see, therefore, that Type II has a radiation resistance 28 per cent greater than Type I, and Type III a radiation resistance 44 per cent greater. Besides this, Types II and III are more advantageous than Type I owing to their larger capacity and the consequent need of less loading inductance.

Any type of antenna is subject to this simple analysis. A T antenna, for instance, can be correctly taken as an L antenna of the same length and height. The vertical current distribution is identical in both cases, the only difference being that the vertical radiation is smaller in the case of the T; this, however, is not considered in this treatment. An umbrella antenna, or in fact anv antenna with regularly or irregularly arranged elevated capacity, may be treated simply by replacing the elevated capacity by an equivalent capacity arranged either as in Figure 5, or as in Figure 6.

In general, the method described here should be useful until such time as there becomes available complete radiation resistance data calculated on a firm theoretical basis. Comparative calculations of various types of antenna can be easily made, and in fact, absolute values of resistance can be obtained with a resaonable degree of approximation.

Caution, however, should be exercised in placing too great reliance in the accuracy of calculations of radiation resistance by any methods used so far. The effect of conducting masts and stays has never been taken into account, and until the effect of these can be accurately determined, radiation resistance caleulations are open to a certain measure of doubt.

SUMMARY: Proceeding on the basis of the formulas for the radiation resistance of a flat-top antenna and on the assumption that the current distribution along an antenna worked at a wave length far from its fundamental is linear, there are calculated the approximate radiation resistances for various forms of antennas (flat top, upward inclined top, downward inclined top). The general application of the formulas and curves to other forms of antennas is given, together with a discussion of possible sources of error in such calculations.

DIGEST OF UNITED STATES PATENTS RELATING TO RADIO TELEGRAPHY AND TELEPHONY*

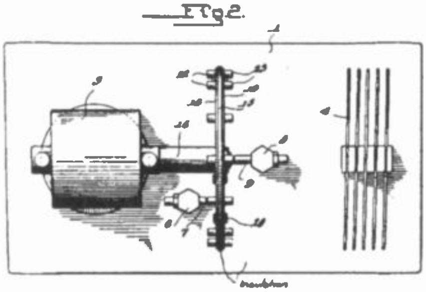
ISSUED JANUARY 3, 1922–FEBRUARY 21, 1922

BY

JOHN B. BRADY

(PATENT LAWYER, OURAY BUILDING, WASHINGTON, D. C.)

1,402,235—H. Jorgenson, filed January 20, 1920, issued January 3, 1922.



NUMBER 1,402,235—Oscillation Generator

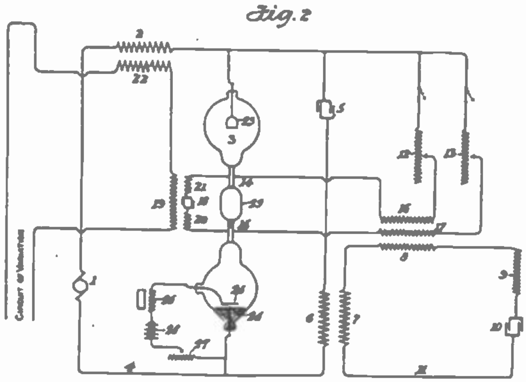
OSCILLATION GENERATOR. This patent shows a form of transmitter unit including a transmitter inductance, a rotary spark gap and condenser arranged with minimum length of primary circuit leads to secure transmission at relatively short wave lengths.

1,402,931—P. C. Hewitt, filed March 18, 1915, issued January 10, 1922.

METHOD AND APPARATUS FOR THE PRODUCTION AND UTILIZATION OF ELECTRIC CURRENTS. This patent relates to generation of alternating currents of a definite time period from the production of periodic electric pulses of a definite time period by means of a gas, vapor or vacuum tube. The frequency of the impulses thru the pulsator is controlled by all the electrical factors of the associated circuits and oscillations generated, modulated, and transferred to an antenna system for transmission.

* Received by the Editor, March 15, 1922.

1,402,932—P. C. Hewitt, filed November 12, 1921, issued January 10, 1922.



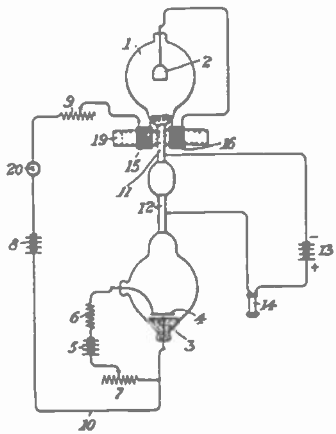
Number 1,402,932—Method of Reproducing Variations in an Electric Circuit

Method of Reproducing Variations in an Electric Circuit. This patent relates to a method of reproducing amplified electrical variations in a circuit employing a gas or vapor tube pulsator. Figure 2 shows the apparatus employed in a line telephone system wherein the effects of the variations produced by the operation of the telephone are reproduced in the inductances 19 and 22. Thru the action of inductances in combination with the other devices in the system and their relation to each other, the variations are reproduced or modified in the inductance 2, and being so reproduced or modified exercise an effect upon the inductance 22 and indirectly upon the inductance 19, whereby the entire telephone line is affected. At any desired point in said line, as at a local station, the variation in the line may be heard by means of a telephone and accordingly the telephone line becomes the world circuit.

1,402,933—P. C. Hewitt, filed July 6, 1916, issued January 10, 1922.

Method of and Apparatus for Controlling Electric Current. This invention relates to a vacuum, gas or vapor tube and a circuit arrangement whereby amplification of current may be secured. Figure 3 of the patent shows a circuit arrangement wherein sound variation introduced at 20 are amplified and reproduced at 14. A magnetic field is employed to control the gas or vapor current carrying path within the tube.

Fig. 3



Number 1,402,933—Method of and Apparatus for Controlling Electric Current

1,402,991—H. A. Affel, filed October 21, 1920, issued January 10, 1922. Assigned to American Telephone and Telegraph Company.

Carrier Telegraph Circuit. This invention relates to an arrangement in carrier wave telegraphy, whereby carrier signals may be translated into carrier frequencies for transmission and whereby received carrier frequencies may be translated into low frequency telegraph signals.

1,403,172—F. H. Kroger, filed July 3, 1916, issued January 10, 1922. Assigned to Westinghouse Electric and Manufacturing Company.

Control of Electric Machines. This patent shows a mechanical method for regulating the position of the high speed rotor with respect to the stator of a high frequency alternator.

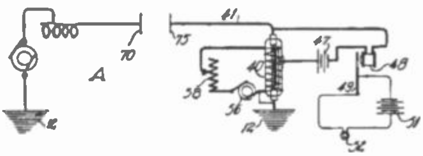
1,403,475—H. D. Arnold, filed November 11, 1920, issued January 17, 1922. Assigned to Western Electric Company.

Vacuum-Tube Circuits. A circuit for the connection of a plurity of vacuum tubes in tandem is shown in this patent. The output circuit of one vacuum tube is provided with a path for direct current, comprising an impedance, and the input circuit of the succeeding tube is connected across a portion of the direct current path containing the impedance. A condenser is placed between the direct current circuit and the second tube to prevent the flow of direct current, and a direct connection is also made between the input electrodes of the second tube so that a normal potential difference may be applied there-between.

1,403,640—H. J. Round and G. M. Wright, filed February 20, 1920, issued January 17, 1922. Assigned to Radio Corporation of America.

"WIRELESS" TELEGRAPHY. This patent shows a circuit arrangement for a direction finder system, employing two closed loop aperiodic antennas with an inductance included in each loop circuit and a connection to ground at the midpoint of each inductance. A third inductance is coupled to the two inductances and a variable condenser shunted across the leads to the receiver. The mutual inductance between the field coils and the exploring coil is as great as possible whereby the variable condenser across the exploring coil affords all necessary tuning over a wide range.

1,403,700—F. S. McCullough, filed October 30, 1916, issued January 17. 1922. Assigned to Glenn L. Martin.



NUMBER 1,403,700 — Apparatus for Directive Transmission of Electromagnetic Waves

APPARATUS FOR DIRECTIVE TRANSMISSION OF ELECTROMAGNETIC WAVES. This invention has reference to an electro magnetic wave transmission and reception system in which directive capacities each consisting of a conductive flat ribbon bent upon itself in a succession of loops and arranged in the form of a parabolic reflector, are employed as antennas. The system is described for operation along a trackway or guideway where a plurality of receivers are located at intervals along the trackway or guideway, while a co-acting transmitting device may communicate selectively with the receiving devices, as it is moved along the trackway or guideway past the several receivers.

1,403,835—O. B. Blackwell, filed September 30, 1919, issued January 17, 1922. Assigned to American Telephone and Telegraph Company.

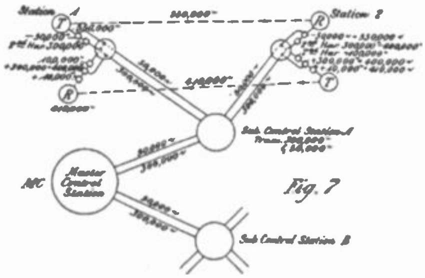


Fig. 7

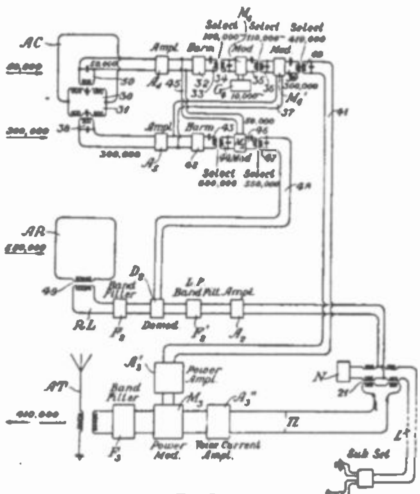


Fig. 8

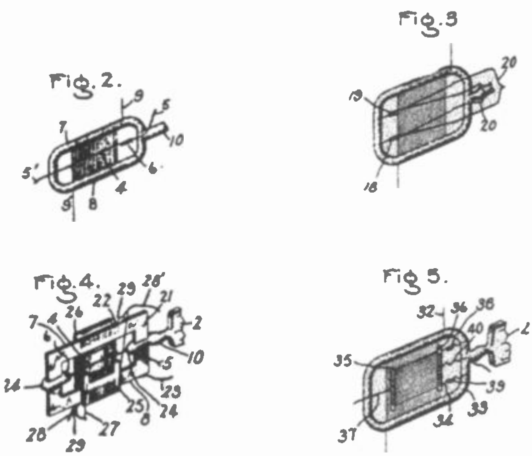
NUMBER 1,403,835 — Frequency-Control System

FREQUENCY-CONTROL SYSTEM. This patent is directed to the organization of radio signaling systems to provide non-interfering wave lengths for stations owned and operated by different business corporations. A master station is employed to generate one or more fundamental frequencies which may be radiated to the stations of the various groups or systems, the various signaling frequencies to be used by individual stations of the groups being derived from the received fundamental frequency or frequencies. The stations are divided into groups, each group having assigned to it a definite frequency range which shall not interfere with the frequency range assigned to any other group. The fundamental frequency transmitted from the master station to the stations of the various groups is translated at the stations into the frequencies within the range assigned to the group.

1,403,841—J. R. Carson, filed September 30, 1919, issued January 17, 1922. Assigned to American Telephone and Telegraph Company.

FREQUENCY-CONTROL SYSTEM. This patent relates to a system of operating a large number of signaling stations simultaneously without interference. For this subject matter see also Patent number 1,403,835. The system comprises the organization of transmission and receiving stations whereby various signaling stations may be divided up into groups depending upon the controlling corporate business owner thereof to provide the maximum number of channels of non-interfering communication. The stations are divided into groups of inter-communicating stations, each group having assigned to it a definite frequency range which shall not interfere with the frequency ranges assigned to any other group. A master station generates a radio frequency and a relatively low frequency, too low to radiate, which modulates the radio frequency. The resultant frequencies are radiated to the stations of the groups. The small frequency difference is detected at these stations, thereby producing a low frequency. A radio frequency is selected and frequencies within the range assigned to each group produced by variously combining the high and low frequencies and harmonies of said frequencies.

REISSUE 15,278—I. Langmuir, filed April 23, 1919, issued January 31, 1922. Assigned to General Electric Company.



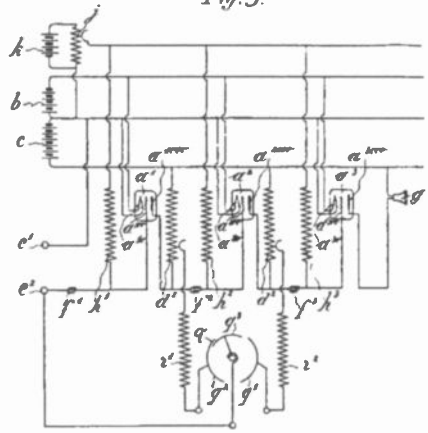
REISSUE 15,278—Electron-Discharge Apparatus

ELECTRON-DISCHARGE APPARATUS. This patent relates to a construction and arrangement of elements within a thermionic vacuum tube, wherein the cathode and grid are insulatingly supported by the same mounting.

1,404,573—L. N. Brillouin and G. A. Beauvais, filed June 13, 1917, issued January 24, 1922.

TELEPHONE INSTALLATION IN "WIRELESS" TELEGRAPHY. This patent relates to an amplifier circuit involving a plurality of vacuum tubes. The filaments of each of the tubes are heated from a common source. The plate of the first tube is connected thru a resistance to a source of plate potential. The grid of the first tube connects thru a resistance to a source of grid potential, and thru a condenser to one of the receiver terminals. The plate of the tubes of the series connect thru condensers to the grids of the succeeding tubes and the telephone receiver is included in the plate circuit of the last tube.

Fig. 3.

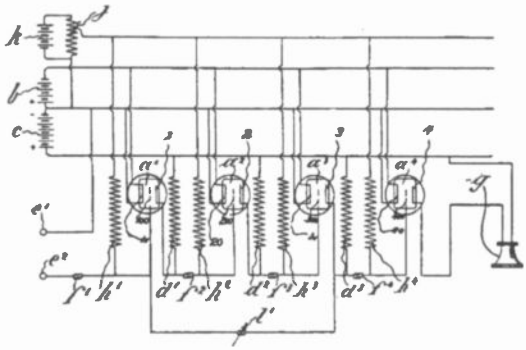


NUMBER 1,404,573—Telephone Installation in "Wireless" Telegraphy"

1,404,574—L. N. Brillouin, filed April 25, 1918, issued January 24, 1922.

TELEPHONE AND "WIRELESS" TELEGRAPHY INSTALLATION. This patent shows a circuit connection for a plurality of vacuum tubes. An amplifier circuit is shown, wherein the grid of the first vacuum tube is connected to one of the receiver terminals and the grids of the succeeding tube are connected to the plates of the preceding tubes in series, and a variable condenser is connected between the grid of the first tube and the plate of the last tube of the series for adjusting the circuit to heterodyne or damped wave reception.

Fig 1



NUMBER 1,404,574—Telephone and "Wireless" Telegraphy Installation

1,404,726—E. F. W. Alexanderson, filed January 14, 1919, issued January 31, 1922. Assigned to General Electric Company.

REMOVING SLEET FROM ANTENNAS. The invention here is directed to a circuit arrangement for passing heating current thru an antenna of the multiple tuned type whereby to melt ice from the wires. The heating current is supplied from a transformer with its secondary connected to the antenna wires. The disconnecting switches in the individual ground leads at the intermediate tuned points are opened and switches in the individual antenna wires manipulated by the operator to pass heating current thru each pair of antenna conductors successively The antenna system has an arrangement of cross connections which connect with only a portion of the antenna conductors and then connect with the tuned sections to ground.

1,404,756—Alfred N. Goldsmith, filed May 9, 1916, issued January 31, 1922. Assigned to General Electric Company.

SIGNALING SYSTEM. This patent shows a form of modulating circuit for a radio transmitter. The circuit shows a pliotron oscillator producing radio frequency oscillations in the primary circuit including a primary inductance. The output is modulated by varying the resistance of another pliotron from a control microphone. The second pliotron has its anode connected to the midpoint of the primary inductance and its cathode connected to ground.

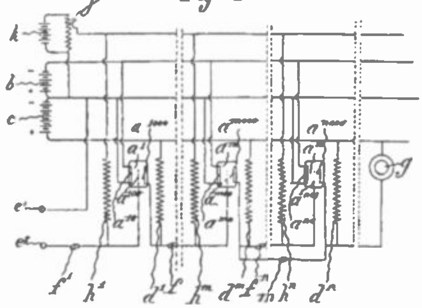
1,404,799—S. E. Shackleton, filed May 31, 1919, issued January

31, 1922. Assigned to American Telephone and Telegraph Company.

CURRENT-SUPPLY CIRCUIT FOR VACUUM BULBS. This patent relates to a constant current filament lighting supply system for vacuum tubes employed in telephone repeater installations with means for automatically supplying a constant filament heating current to the vacuum tubes in the event of tubes being switched out or otherwise removed from the circuit. An automatically operated polarized differentially wound relay is employed to introduce resistance into the circuit to compensate for the removal of individual vacuum tubes.

1,405,267—L. N. Brillouin and G. A. Beauvais, filed June 10, 1918, issued January 31, 1922.

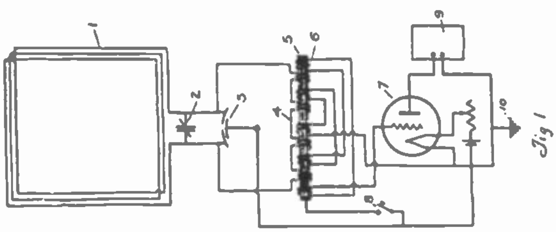
Fig. 1



NUMBER 1,405,267 — Telephone and "Wireless" Telegraphy Installation

TELEPHONE AND "WIRELESS" TELEGRAPHY INSTALLATION. This patent relates to a circuit arrangement for a vacuum tube amplifier. A plurality of vacuum tubes are employed having filaments heated from a common source of current. The plates of the respective tubes are connected thru a resistance to one pole of a source of plate circuit potential; the other pole of which is connected to one of the receiver terminals. The grid of the first tube is connected to the other receiver terminal. The grid of each of the other tubes is connected thru a condenser to the plate of the respective preceding tubes. Means are provided for modifying the amplifying effect of the equipment comprising a connection containing an impedance between the plates of two of the tubes.

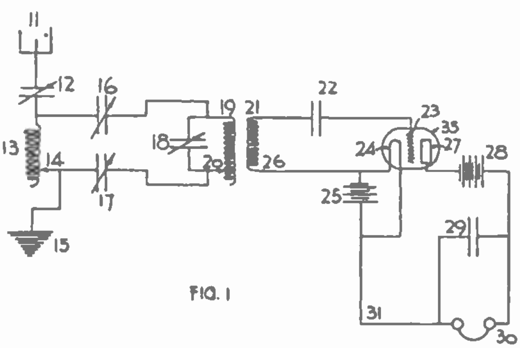
1,405,905—F. W. Dunmore, filed February 4, 1921, issued February 7, 1922.



NUMBER 1,405,905—Radio Receiving Apparatus

RADIO RECEIVING APPARATUS. A loop receiving system is shown in this patent utilizing a circuit for reducing the effects of electrical dissymmetry or so-called antenna or capacity effect to ground of the loop. The object is to secure a quieter, sharper and less distorted minimum, improving the operation of the coil as an interference preventer and its accuracy as a direction finder. An iron core transformer having primary and secondary windings is employed, one connected to the terminals of the coil antenna and the other to the input circuit of a vacuum tube detector. The windings are formed in a plurality of individual sections connected in series with the primary and secondary sections alternately arranged. The iron core and filament circuit are maintained at ground potential. 3 is a balancing condenser or "Mesny compensator," which may be used in balancing what little dissymmetry in capacity to earth is not taken care of by the coupling coil. The core 5 serves a two-fold purpose. First, to couple the coil aerial to the detector and distribute the capacity effect more evenly to earth; and second, partially to bipass to earth thru the core disturbing influences such as motor and ignition noises.

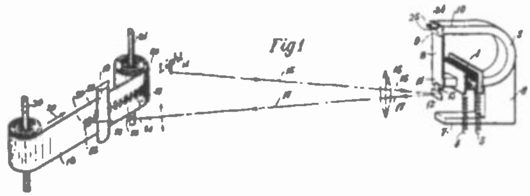
1,406,442—L. Cohen, filed December 28, 1918, issued February 14, 1922.



NUMBER 1,406,442—Radio Receiving System

Radio Receiving System. This patent relates to a receiving circuit having high selectivity for waves of definite frequency. Static coupling is employed between the antenna ground circuit and the vacuum tube detector circuit.

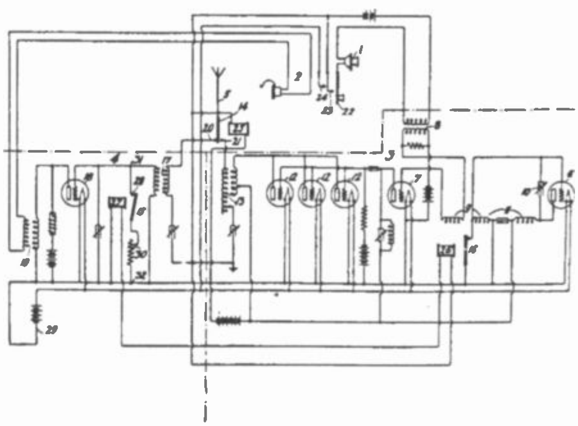
1,406,445—C. A. Culver, filed June 19, 1919, issued February 14, 1922.



Number 1,406,445—Photographic Receiving Apparatus

Photographic Receiving Apparatus. This patent shows a recording apparatus for radio signals wherein the received signals cause vibration of an adjustably loaded spring element carrying a mirror which directs light through a lense upon a photographic strip. The vibratory element includes a spring having a natural stiffness sufficiently low to give substantially simple harmonic vibration for small displacements when loaded.

1,406,857—R. A. Heising, filed October 26, 1916, issued February 14, 1922.



Number 1,406,857—"Wireless" Signaling

"Wireless" Signaling. This patent relates to a transmitter and receiver and a protective circuit in the receiver for regulating the intensity of the side tone in the head receivers when the transmitter at the same station is being operated. Three relays, 25, 26 and 27 are employed to transfer connections from transmitting to receiving and vice versa. To disconnect the receiving system and connect the antenna to the transmitting system, it will be necessary to start the oscillator 6 and shunt the receiving system to eliminate the side-tone. These objects are attained by the use of key 22 and the three relays, for when the key 22 is pressed down, so as to make an electrical connection between the contacts 23 and 24, the circuit containing the relays and the battery 29 is closed so that the relays become energized. Switch 14 is then pulled over to contact 21, thereby connecting the transmitting system to the antenna; the switch 16 is opened, thereby breaking the shunt around part of inductance 11, thus allowing the oscillator to oscillate and to impress its oscillations on the input circuit of the modulator by means of the inductive relation between the primary and secondary of coil 9; and the switch 15 is pulled over to contact 28, thereby shunting the detector 18 so that no energy will go thru to the receiver 2. In actual practice it is sometimes desirable not to eliminate the side-tone completely, but to allow a portion of the energy ot the transmitted signals to pass thru the detector 18 to the receiver, so that the operator will know when the transmitting apparatus is working satisfactorily. If there is no resistance between points 31 and 32, there will be silence in the receiver. But there will be a desired small side-tone effect if a resistance 30 which may be variable is added to the circuit, as is shown in the drawing. By varying resistance 30, the side-tone may be regulated to any desired intensity, even to the point of complete elimination.

1,406,996—J. B. Morrill, filed September 30, 1920, issued February 21, 1922.

Electric Wave Ranging System. This patent relates to a system for determining the position of a body with respect to three fixed points which comprises ascertaining the differences in length with respect to a given electrical wave length of the direct paths from said body to said points and calculating therefrom the desired position. The system may employ other than long ether waves, that is light or radiant heat waves. It is particularly applicable to permit the pilot of an aircraft or ship to determine his location with respect to shore stations. According to this invention, electric waves are simultaneously transmitted over three courses between three fixed observing stations and the aircraft or point to be located. The velocity of propagation (approximately 3×10^8 meters per second) is known. The only data necessary is the difference in arrival time. Obviously, of two waves of the same phase starting at the same instant, the wave travelling the longer distance will arrive behind the wave travelling the shorter distance, and the phase difference of the two waves is a measure of the difference of their arrival time. According to the present invention the energy of the earlier arriving wave is caused to traverse a phase retarding path so as to make the phases of the two waves the same, as indicated by a null indication obtained when their energies are opposed to each other in a differential indicator. A feature of the invention consists in transmitting the energies over the different courses, as carrier waves of correspondingly different frequencies, to enable them to be readily separated at the receiver. In order to secure an audible note, so as to determine when a phase balance has been secured, each of these carrier waves may be modulated in accordance with the same audible frequency wave.

Fig. 2

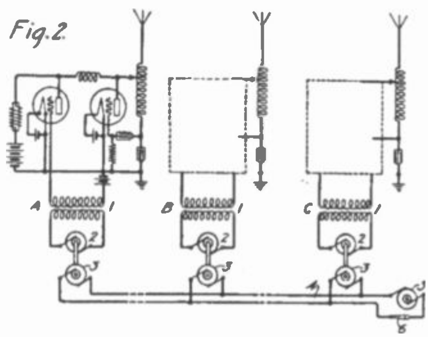
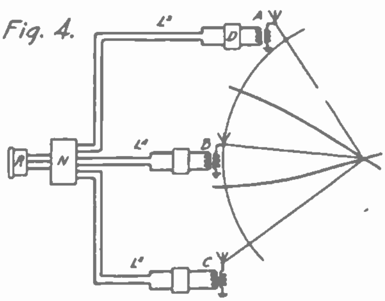


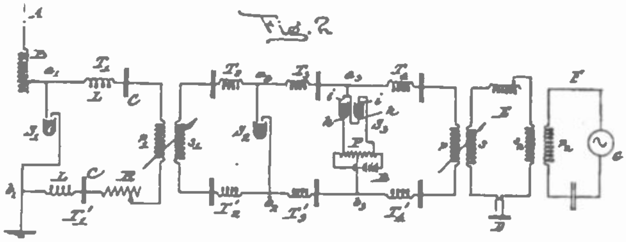
Fig. 4.



NUMBER 1,406,996 — Electric Wave Ranging System

1,407,103—F. K. Vreeland, filed February 17, 1917, issued February 21, 1922.

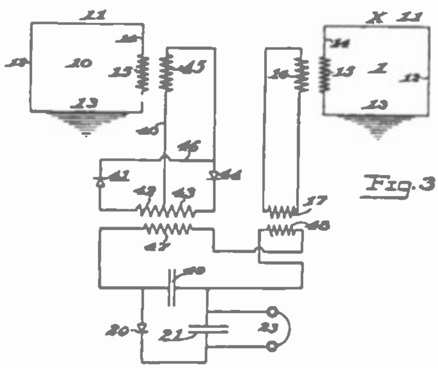
RECEIVING SYSTEM FOR RADIO TELEGRAPHY AND TELEPHONY. This patent is directed to a receiving circuit for the elimination



NUMBER 1,407,103—Receiving System for Radio Telegraphy and Telephony

of interference by strays or static disturbances. The circuit employs electrolytic cells as "intensity selectors" which discriminate between the strong static impulses and feebler signal impulses, permitting the passage of the feebler impulses but diminishing or diverting the stronger disturbances.

1,407,205—R. H. Marriott, filed December 26, 1916, issued February 21, 1922.



NUMBER 1 407,205—Radio Receiving System

RADIO RECEIVING SYSTEM. This invention relates to a circuit for a radio receiver arranged to neutralize or dissipate the effects of undesirable currents. These undesirable currents cause the production of two currents of definite but different periods. One of these currents is transformed into a current of the same period as the other and then, by combining these two currents into a current of the same period as the other, both are neutralized, leaving only the effects produced by the periodic signal current which it is desired to translate at the receiver.