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INDEX FOR VOLUME 5, 1917

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ALFRED N. GOLDSMITH, Ph.D.

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SOME EXPERIMENTS WITH LONG ELECTRICAL CONDUCTORS*

By

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The object of this paper is to give an elementary non-mathematical discussion of long electrical conductors, showing how the apparent inductance, capacity and resistance of such conductors depend upon the frequency, loading, etc.; also to show the distribution of current and potential along such conductors, with particular reference to the case of quarter wave length oscillations such as are used in radio antennas. The subject is treated altogether from the experimental standpoint; from the experiments described certain relations are obtained which may prove useful in calculating the oscillation period of an antenna.

By the term "long electrical conductor" is meant one on which an appreciable portion of a wave length is developed. In this paper the conductors considered particularly have electrical lengths equal to from one quarter to one twentieth of the wave length of the impressed frequency. It is to be remembered that the electrical length of a conductor depends entirely upon the frequency used in exciting it; thus a 100 mile (160 km.) 25 cycle transmission line is a short electrical conductor, while a 1,000 foot (300 m.) long antenna excited by a frequency of 100,000 cycles per second is a long electrical conductor.

Another way of distinguishing short conductors from long ones is this:—in a short conductor (loaded only at its end) the current is practically the same thruout the whole length of the line but in a long electrical conductor the value of current varies widely thruout the length of the conductor. Thus the transmission line 100 miles (160 km.) long might have a current of 100 amperes at the generator end and 98 amperes at the load end while the 1,000 foot (300 m.) antenna might have 100 amperes at the beginning and would have no current at all at the farther end.

* Received by the Editor, March 19, 1917.

Wave propagation over long conductors has been presented critically by such writers as Heaviside, Pupin, Campbell, Kennelly, Cohen, and others; study of these discussions gives one a complete knowledge of wave phenomena but there are many who have not the time for such study and who would doubtless appreciate an elementary discussion, especially of the standing wave phenomena encountered in radio work. It is hoped that the experiments outlined in the following pages may prove interesting to such readers.

An electrical conductor, such as an antenna, differs from the ordinary alternating current circuit, in that its inductance and capacity are *distributed*; the formulas which solve circuits having concentrated inductance and capacity do not, in general, hold good just because of the distributed character of the inductance and capacity. To illustrate this point let us consider two circuits as illustrated in Figure 1. In sketch *a* are shown a certain inductance L and a capacity C; if the voltage of the exciting alternator is held constant and the frequency is varied it will be found that at a certain frequency the current is a maximum and power factor is unity. Moreover there will be found only *one frequency* where these conditions hold. This frequency is given by the well known formula

$$f=\frac{1}{2\pi}\sqrt{LC}.$$

Now in sketch *b* is shown the same amount of inductance and capacity but it is divided into small parts and distributed uniformly. If now we excite this circuit as before, with constant voltage and varying frequency, there will be found *many frequencies* at which the power factor is unity and at which the current has a maximum value and none of these frequencies satisfy the above formula for resonance. Of course there can be only one frequency which gives a real maximum of current but the other resonant frequencies give current values nearly equal to the maximum value and much larger than the values of current for frequencies slightly higher or lower than the one in question.

The first experiments were carried out to illustrate this feature of multiple resonance of the distributed L and C conductor. Instead of actually using a long line an artificial line was used in the laboratory. This line was made up of nine coils and condenser sections. The coils were wound of stranded conductor and each had an inductance of 0.0415 henry and resistance of 0.702 ohm. They were air core coils and weighed about 75 lbs. (34 kg.) each. The capacity units were some specially built telephone condensers, having rather small losses (at the frequencies used) and capable of withstanding 750 volts. They were rated at 1 μf each but on measurements showed an average capacity of 0.915 μf. About four hundred of these condensers were used in some of the tests.

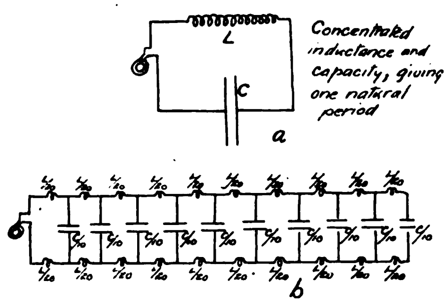


FIGURE 1

These coils and condensers made possible an electrical conductor of large enough current capacity that ordinary ammeters and voltmeters could be used for most of the measurements. One would not be justified in procuring such expensive capacity and inductance for carrying out the tests to be described, but the equipment had been purchased for some other experimental work and proved well suited for the work I wanted to do. Altho low frequencies were used the damping was so low that the results obtained probably compare favorably with those obtaining on an actual antenna.

This line made up of coils and condensers is styled a "lumpy line"; but it is shown in standard works that when the number of lumps is greater than six per wave length the lumpy line acts nearly enough like a uniform line. In most of my tests there were more than twenty-five lumps per wave length.

The results of the first experiment are shown in Figures 2 and 3. A voltage of 20 was maintained constant as the frequency impressed varied from 12 cycles to 152 cycles per second. It will be seen from Figure 1 that three resonant points (meaning by resonance, maximum current) were obtained and a fourth

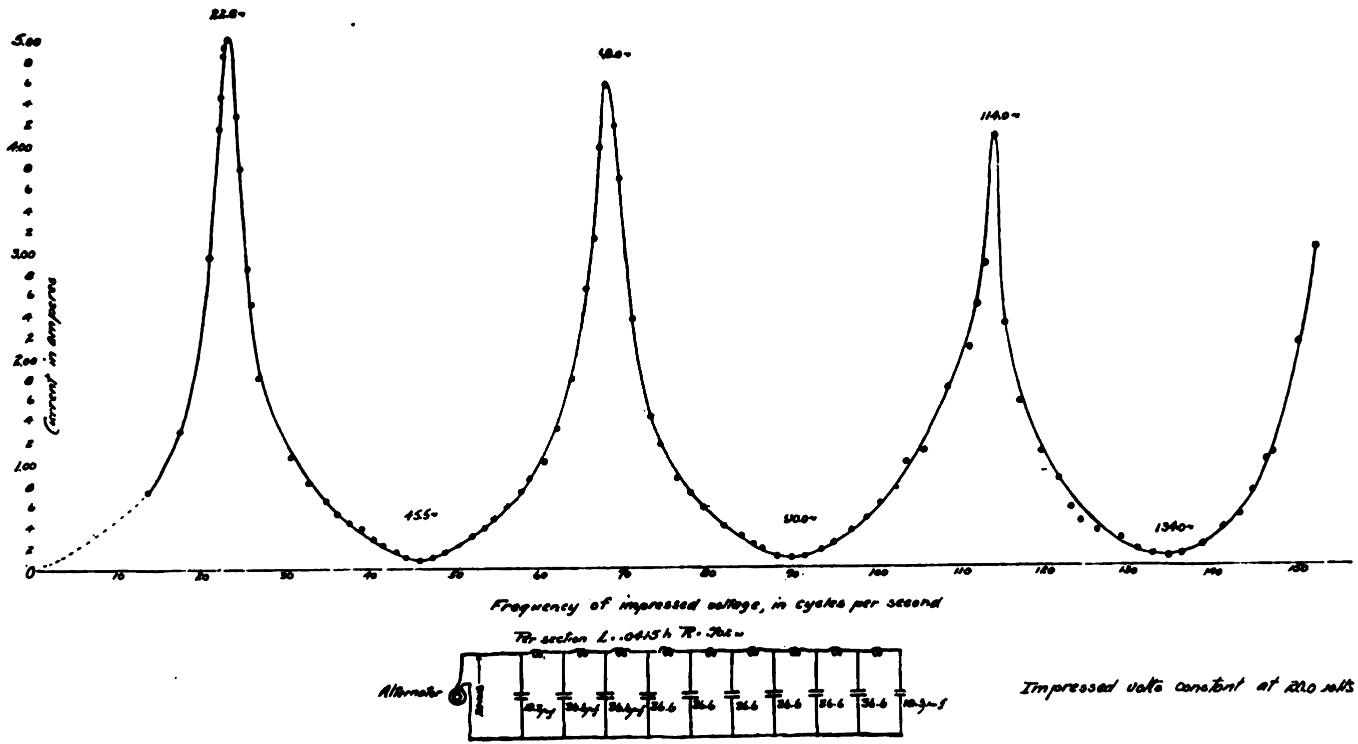

22.8~
45.5~
114.0~
134.0~
Current in amperes
Frequency of impressed voltage, in cycles per second
Alternator
Impressed volts constant at 220 volts

FIGURE 2

one was nearly defined. The upper frequency was limited by safe speed of the alternator. Other alternators were available which could extend the frequency range to 600 cycles; they could not be used however, as their wave forms were not pure and purity of wave form is absolutely necessary in performing tests of this character.

Besides the ammeter and voltmeter, a wattmeter was used to measure the power supply to the line. Then from the readings of the three instruments, voltmeter, ammeter, and wattmeter, it was possible to obtain the apparent resistance of the line, the reactance and the angle of phase difference between E and I. These results are shown in Figure 3.

It at once appears that such a line cannot be said to have any definite resistance or reactance unless the frequency is specified at which these constants are given. The resistance, for example, varies between 3 ohms and 350 ohms. The reactance varies all the way between 165 positive ohms (inductance) and 165 negative ohms (capacitance). And the angle between the voltage and current periodically changes from 75° lead to 75° lag.

When the resistance of a long line is small compared to the reactance the frequencies at which resonance occurs are all odd multiples of the lowest frequency. Thus 68 cycles is three times 22.8 cycles (the lowest frequency) and 114 cycles is five times the lowest frequency. But if we define resonance in the more general sense as being that frequency which makes the reactance of the line zero, then we notice that both even and odd multiples of the fundamental frequency give resonance. Thus we have as the resonant frequencies of this line 22.8, 45.5, 68.0, 90.0, 114.0. Within experimental error* these are all multiples of the fundamental frequency, 22.8 cycles per second.

The results of above tests could have been predicted by the use of suitable formulas, such as are given in Kennelly's "Application of Hyperbolic Functions to Engineering Problems."

Thus if

r = resistance per unit length
L = inductance per unit length
g = conductance per unit length
C = capacity per unit length
f = impressed frequency
$\omega = 2\pi f$

* The accuracy of the tests outlined in this paper is probably between 0.5 and 1 per cent.

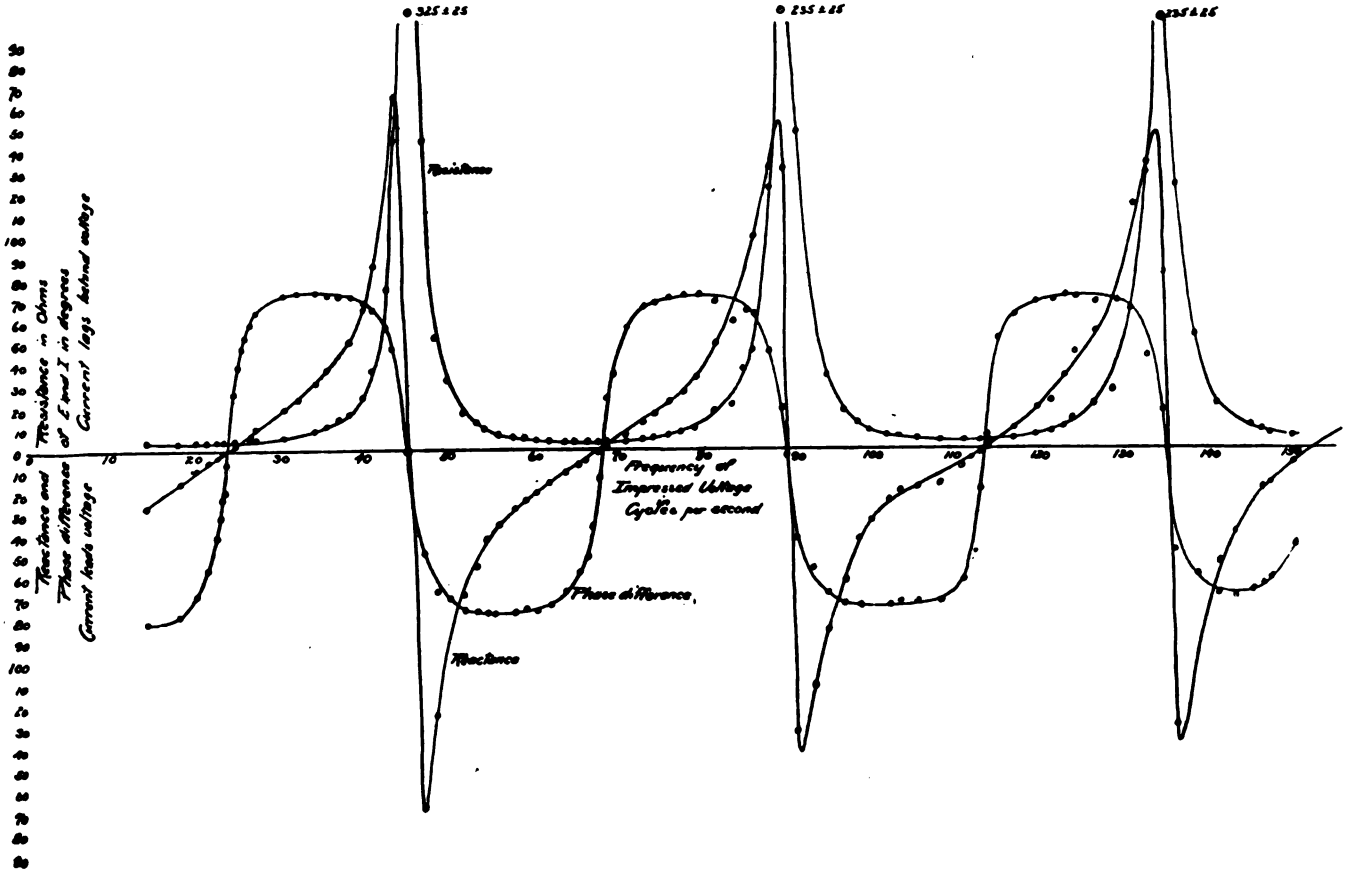

Reactance and Resistance in Ohms
Phase difference of E and I in degrees
Current leads voltage
Current lags behind voltage
Frequency of Impressed Voltage Cycles per second
Resistance
Reactance
Phase difference
325 ± 25
235 ± 25
235 ± 25

FIGURE 3

$$z = r + j\,\omega L$$
$$y = g + j\,\omega C$$

Then the propagation constants of this line is

$$a = \sqrt{zy} = \sqrt{(r + j\,\omega L)\,(g + j\,\omega C)} = a + j\,\beta \qquad (1)$$

and the surge impedance of the line is given by

$$z_o = \sqrt{\frac{r + j\,\omega L}{g + j\,\omega C}} \qquad (2)$$

The electrical length of the line is given by al where l is the actual length, in whatever units the other constants are defined.

Now in an open circuited line the current and impressed voltage are related by the formula

$$I_A = \frac{E_A}{z_o}\tanh al \qquad (3)$$

where I_A = entering current
E_A = impressed voltage and

tanh al is the hyperbolic tangent of the complex angle *al*.

In the line used there was no perceptible leakage in the condensers. The condensers did have some loss due to dielectric hysteresis but it was very small at the frequencies used. Moreover this loss did not vary strictly with the square of the impressed voltage and hence could not be accurately accounted for by selecting the proper value for g, the conductance. It was decided therefore, to neglect the condenser losses and call the conductance negligible. Then the surge impedance z_o was calculated for the various frequencies used in the test and was found to vary in magnitude but slightly. It was found to be $33.8 \diagdown\overline{3.3^\circ}$ ohms for the low frequency and $33.6 \diagdown\overline{0.6^\circ}$ ohms for the high frequencies. Its magnitude was taken as 33.7 ohms for all frequencies.

The current I_A should therefore vary in accordance with the variations of *tanh al* as the frequency varied. Now by inspection of charts or tables it may be seen that *tanh al* goes from maximum to minimum values as *al* goes thru the multiples of $\frac{\pi}{2}$. With the values of *al* occurring in my test *tanh al* goes from maximum values of about 11 to minimum values of about 0.08.

The calculated range of current variation is then

$$\text{maximum } I_A = \frac{20}{33.7} \times 11 = 6.4 \text{ amperes}$$

$$\text{minimum } I_A = \frac{20}{33.7} \times .08 = 0.05 \text{ ampere}$$

Actually the variation was from 5.00 amperes to 0.06 ampere, a very fair agreement when it is considered that the condenser losses were neglected. The effect of such losses is to decrease the theoretically predicted variation of current, which decrease the test actually showed.

Such an open circuited line will give various voltages at the far end as the frequency is varied, the impressed voltage remaining constant.

Theoretically, if

$$E_A = \text{impressed voltage}$$
$$E_B = \text{voltage at open end}$$
$$al = \text{electrical length of line}$$

then
$$E_B = E_A / \cosh al \qquad (4)$$

where *cosh al* is the hyperbolic cosine of the complex angle *al*.

Now as *al* continually increases with increase of frequency, *cosh al* varies cyclically just as does *tanh al*. For the line tested *cosh al* varies from 0.09 to a value slightly greater than unity. The minimum values of *cosh al* occur when *al* is some odd multiple of $\frac{\pi}{2}$, that is when the line is some odd number of quarter-wave-lengths long. Formula (4) predicts that with an impressed voltage of 20 volts at 22.8 cycles the voltage at the open end should be about 220 volts. The measured value was only 157 volts or about eight times the impressed voltage. The condenser losses undoubtedly account for this discrepancy as a very slight condenser loss increases the minimum value of *cosh al* very much.

It is interesting to note here a difficulty in carrying out such tests with the facilities of the ordinary engineering laboratory. In getting the results of Figure 2, e. g., as the current varied from 5 amperes to 0.06 amperes, it was necessary to use several different range meters to read these currents. Now if a 1 ampere meter was substituted for a 5 ampere meter, everything else being the same, the input current would vary as much as 10 per cent., *due to the inductance of the meters themselves*. In getting voltages across the line for different conditions an ordinary closed circuit voltmeter could not be used at all. On some of the lines tested the voltage across the open end would drop as much as 20 per cent. as soon as the meter was connected. An electrostatic voltmeter was used for all voltage measurements; its capacity was so small compared to the line capacity that its connection did not appreciably disturb the line potential.

When an open circuited line is one quarter wave length long $\left(\text{or when its electrical length is } \frac{\pi}{2}\right)$ standing waves are set up on the conductor. These standing waves are due to successive reflections from the open end and generator end combining in the right phase to change travelling waves into stationary waves. The standing wave for quarter wave length conductor has a minimum potential at the generator end and maximum at the open end; this form of wave is exactly expressible only in hyperbolic functions of complex angles but is nearly represented by an ordinary sine or cosine wave. The current curve is also a nearly sinusoidal curve, its maximum value occurring at the generator end and zero value at the open end.

These curves of current and potential distribution are shown in all texts on radio telegraphy but, in so far as the writer knows, experimental results are rather meagre. I therefore obtained curves of voltage distribution for the nine section conductor previously described, not only for that impressed frequency which gave quarter wave length but for several others, each being such as to make the line some multiple of $\frac{\pi}{2}$ hyperbolic angles long. These frequencies are the resonant frequencies shown in Figure 3, each frequency making the reactance of the line zero. The form of voltage curve for each frequency is shown in Figures 4, 5, and 6, the notation on each curve sheet making them self explanatory. The voltage at any point on the line is obtained by scaling the distance from the zero line to the curve for the frequency in question.

In Figure 4, curve 2–2′, the form of voltage distribution curve is such as would apparently go thru zero at the center of the line. But actually the voltage at the apparent node *does not go thru zero*; actually there is no voltage node in the sense that the voltage is zero. And there never can be a node of potential on such a conductor unless energy is being supplied everywhere along the line at the same rate as it is being dissipated at that point in the line. As this is practically never the case the so-called nodes are only pseudo nodes, the minimum amount of voltage being fixed by the amount of energy which must flow past the nodal point to maintain the line in a state of oscillation. In Figure 4 this minimum voltage is about 10 volts.

The point might be raised as to how the voltage vector passes thru this nodal point. On one side of the node the voltage has one phase and on the other side of the node the phase is just opposite, i. e., nearly 180° shift in phase occurs in passing thru the nodal point. How can this be so if the vector nowhere

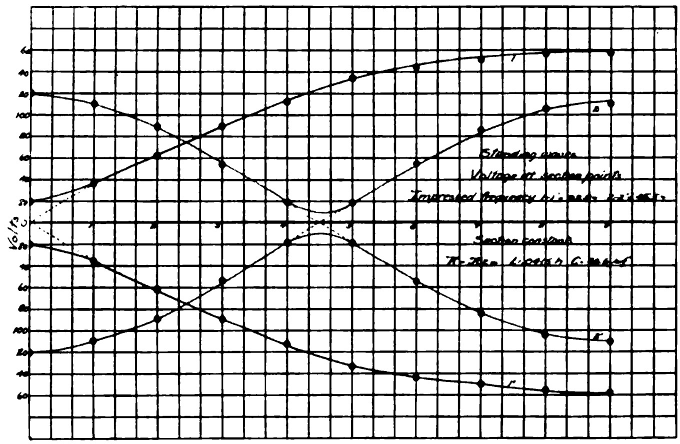


FIGURE 4

goes thru zero value? The apparent ambiguity is due to the attempt to use a two dimensional figure where a three dimensional figure is required. The voltage vector winds around the zero line in a kind of spiral as we consider consecutive parts of

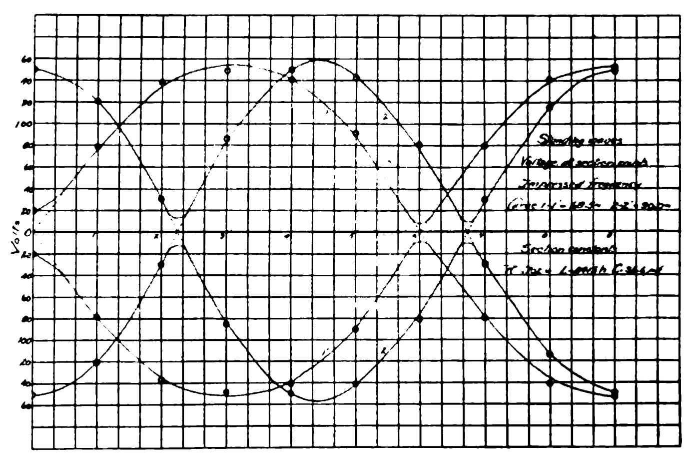


FIGURE 5

the line; this spiral comes closest to the zero line at the nodal point but does not go thru the zero line.

The foregoing material apparently has no direct application to radio telegraphy. Such is not the case, however, because there are certain problems where the distributed capacity and inductance may play a very important part in determining the node of oscillation, etc. Thus one of the long Marconi aerials,

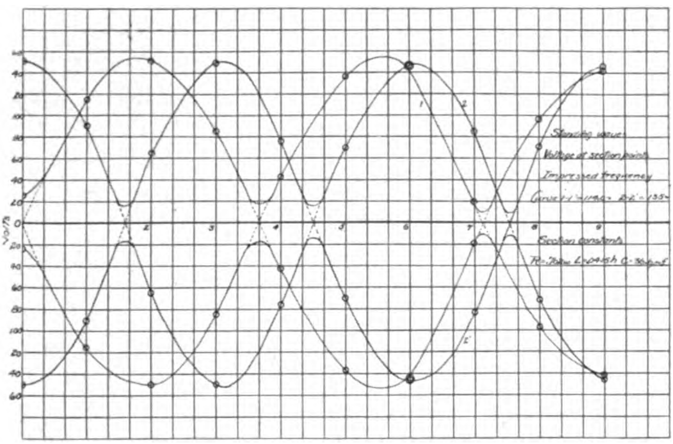


FIGURE 6

having a natural wave length of perhaps 7,500 meters, if adjusted to emit a wave length of 10,000 meters or even more, will by no means obey the ordinary equations for oscillations, connecting frequency, inductance and capacity,

Also in determining the distributed capacity of coils the ordinary formulas give entirely erroneous results. For an accurate solution of such problems it is necessary to consider the fact that the current and potential distribution largely affect the value of the constants to be determined. For attacking such problems the material already presented furnishes the right view point.

OSCILLATIONS IN AN ANTENNA

An antenna is always excited to oscillate in the quarter wave length conditions as shown in Figure 4, curve 1–1′. It could not be made to oscillate strongly at the half wave length condition of curve 2–2′ but might oscillate very well at the three quarter wave length condition as shown by curve 1–1′ of Figure 5; also as shown by curve 1–1′ of Figure 6 which is the one and a quarter wave length oscillation.

A long uniform antenna could be made to have any of these three modes of oscillation (or others) but the normal operation for radiation efficiency requires oscillation at the quarter wave length condition. Hence further experimental data was obtained to show current and voltage distribution in a line oscillating at quarter wave length.

If a series inductance or series condenser be added in the beginning of the laboratory model the distribution of current and potential will be just the same as occur on an actual antenna to which such adjustments have been made for the purpose of lengthening or shortening the radiated wave.

Experimental data on such oscillations is given in Figures 7 thru 11. The natural frequency of the line used was 45.2 cycles. By adding 0.142 henries in series, the quarter wave length frequency dropped to 32.6 cycles per second; and by adding 0.617 henries this was further reduced to 20.5 cycles. The current and potential distribution shown in Figure 9 give the conditions as they occur in a long uniform aerial emitting a wave about 2.5 times the natural wave length. It is to be noticed that as inductance is added in the base of an antenna the potential tends to become uniform over the whole length of the antenna and the current decreases from the ground end toward zero at the open end on a nearly straight line instead of a sine curve as it does for the unloaded antenna.

With a series condenser of 165 microfarads in the base Figure 10 was obtained, and with 82.5 microfarads in the base Figure 11 was obtained. The latter corresponds to an antenna oscillating at a frequency about 30 per cent. greater than its natural frequency. The current is now a maximum not at the beginning of the line (base of the antenna) but somewhat along the line. Where the current is a maximum the voltage is a minimum. The voltage curve crosses the zero line in both Figures 10 and 11 but the voltage does not go thru zero and so the curve is shown dotted in this portion. This anomaly comes about in trying to represent the voltage curve in two dimensions, as previously noted.

The voltage impressed on the line is the same in Figures 7 thru 11. But it is seen that whereas the entering current is nearly 5 amperes for the unloaded line and the maximum voltage is 345 volts none of the other curves show values as large as these. Thus when by loading the quarter wave length frequency has been reduced to about 40 per cent. of the natural frequency,

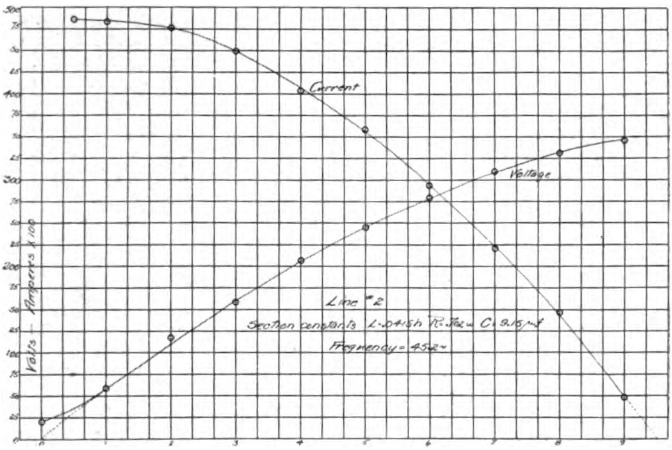


Figure 7

the entering current is only 1.44 amperes and maximum voltage on the antenna is 148 volts. To get the same input current as for the unloaded line would require that the impressed voltage

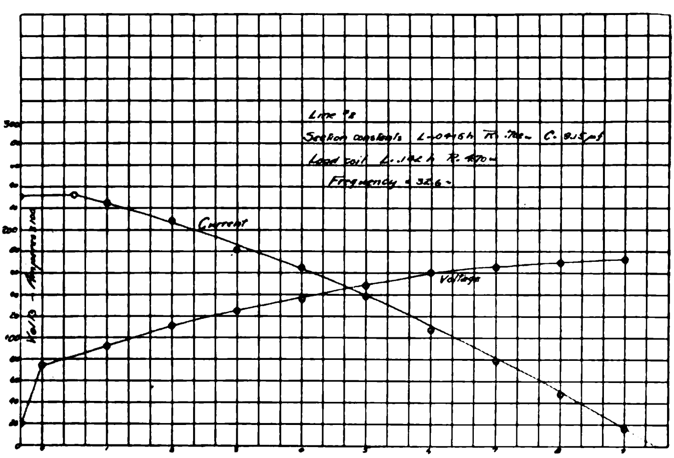


Figure 8

be increased more than three times. The same effect occurs when the natural frequency is increased by series condenser. Hence we see that when the wave length of an antenna is in-

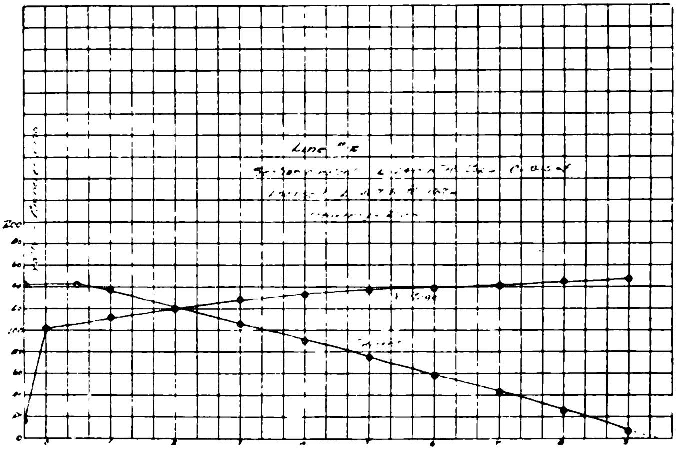


FIGURE 9

creased by loading or decreased by series condenser, it is necessary to change the adjustment of the coupling transformer if the same current is to be maintained in the antenna. The

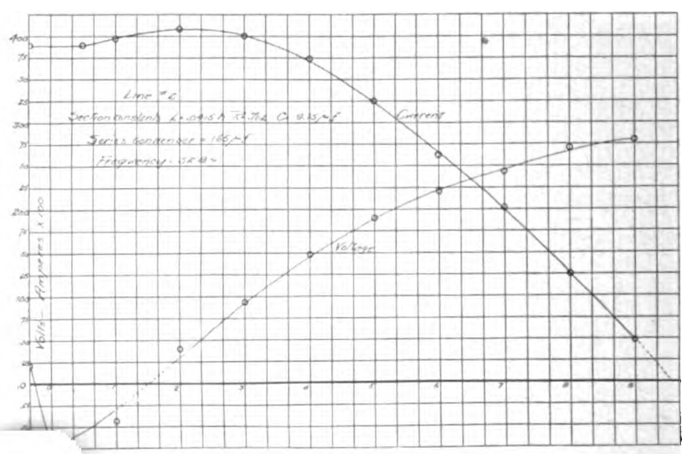


FIGURE 10

results given above apply directly, of course, only to continuous wave operation but hold good also for damped wave oscillation.

The curves of Figures 7 thru 11 show that no matter what the frequency may be at which the line is oscillating, the current and voltage distribution are nearly sine waves, just as for the unloaded line. But when the frequency is less than the natural, less than one quarter of a complete sine wave occurs ard when

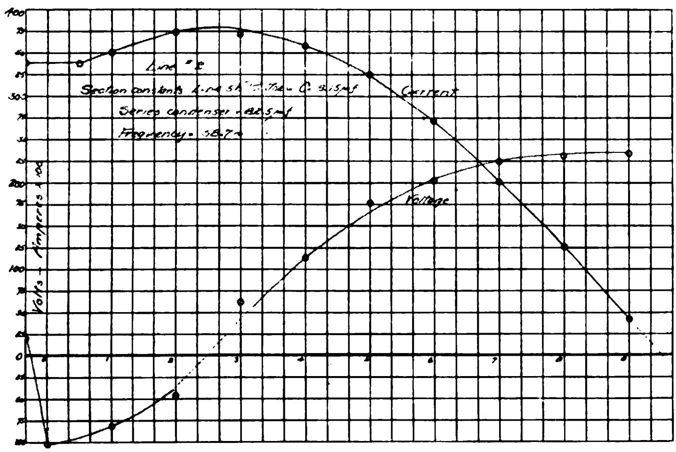


FIGURE 11

the frequency is greater, more than one quarter of a whole sine curve occurs. Thus in Figure 8, the voltage curve is 72.2 per cent. of a quarter of a sine curve when calculated from the frequency ratio, 32.6 to 45.2. This gives 64.8° of a curve and so the voltage at the beginning should be equal to voltage at the end of line multiplied by *sin* $(90° - 64.8°) = 0.426$. But the end voltage $= 172$ volts. Hence beginning voltage should be $172 \times 0.426 = 73$ volts. It measures from the curve 75 volts.

Many antennas have an irregular distribution of capacity as for example a *T* antenna which has more capacity per unit length at the top of the antenna than it has at the beginning. In Figures 12, 13, and 14 are shown the distribution of current and potential in such an antenna. The total capacity and inductance of the line is the same as was used for the previous uniform line but it is seen that the natural frequency is now only 40.1 cycles as compared to 45.2 cycles for the uniform line.

A given series condenser raised the natural frequency of the uniform line by 30 per cent. The same series condenser in the assymetrical line raised the frequency by 32 per cent. A

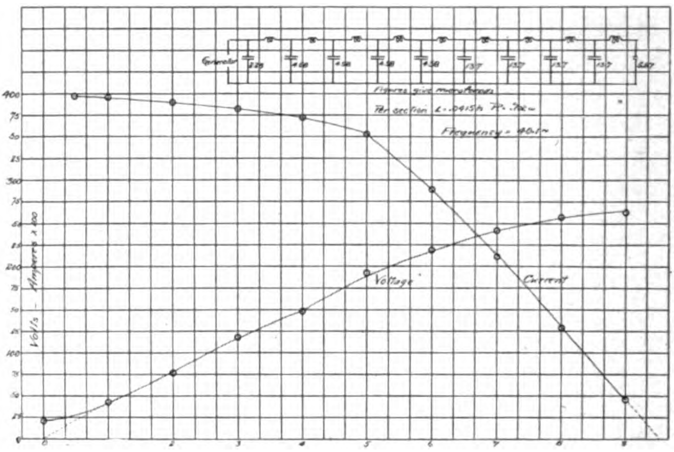


FIGURE 12

given loading inductance lowered the frequency of the uniform line to 72.3 per cent. of the natural frequency. The same inductance lowered the frequency of the assymetrical line to 77.2 per cent. of natural frequency. So that the series con-

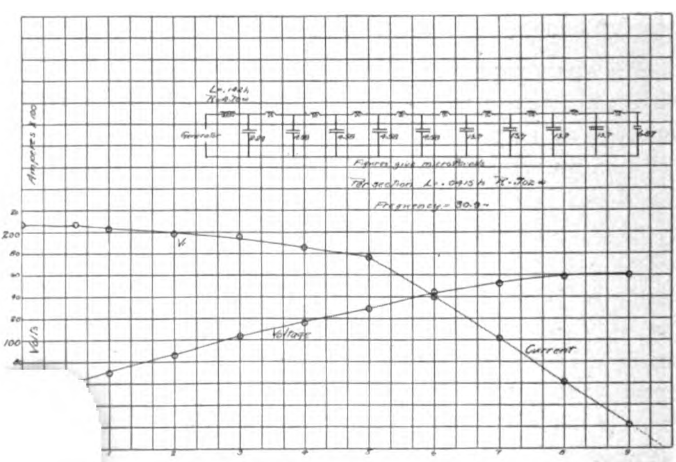


FIGURE 13

denser has more effect and the series inductance has less effect in the non-uniform line than in the uniform line.

Now as the capacity of an antenna and inductance of an antenna are continually being measured and the values so obtained used in calculations it is worth while to investigate what these quantities really are.

Three uniform conductors were first tested for quarter wave length frequency. The inductance and resistance per section in each case were 0.0415 henry and 0.702 ohm. In one case the capacity was 36.6, in the second 9.15, and in the third it was 1.83 microfarads per section.

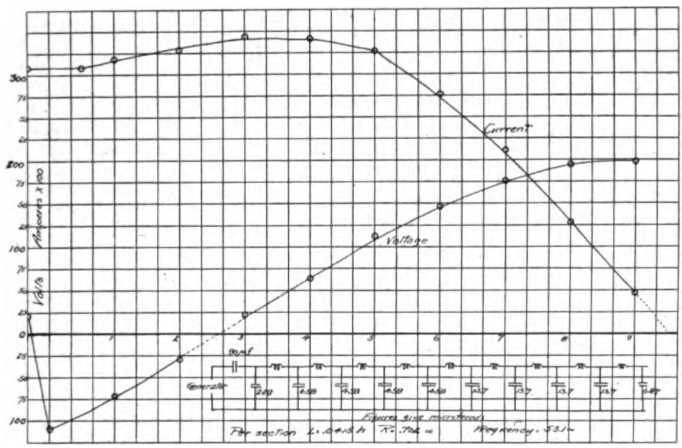


FIGURE 14

In the following table are shown the total inductances and capacities for each line, the frequencies calculated from these quantities, the frequency which actually gave quarter wave length vibration and the ratio of these two frequencies.

Total L		Total C	$\frac{1}{2\pi\sqrt{LC}}$	Measured Frequency	$\frac{\text{Meas. Freq.}}{\text{Cal. Freq.}}$
Line 1	0.374	329.	14.3	22.5	63.6 per cent
Line 2	0.374	82.3	28.7	45.2	63.5 per cent
Line 3	0.374	16.45	64.2	100.5	63.8 per cent

Now as in each case above the lines have no added series inductance or capacity the line was quarter wave length long and the voltage and current curves were sine waves very nearly.

But the average value of a sine wave from 0° to 90° is 63.6 per cent. of the maximum; it appears therefore, that the natural quarter wave length period of such a uniform line is a function of the form of current and potential curve of the line. As the electrostatic energy is a function of the potential curve and the electromagnetic energy is a function of the current curve and both curves have the same shape, it is logical to say that the effective self induction of such a line is equal to 63.6 per cent. of the total self induction and the effective capacity of such a line is 63.6 per cent. of the total capacity of the line.

Now even when inductance or capacity is added in series with such a line the current and potential curves are both sine curves when the frequency of impressed e. m. f. is such as to give quarter wave length vibration; it seems, therefore, that the effective capacity and self induction of such a line can be predicted for any amount of loading.

With this idea in mind a series of results was obtained to show how line number 3 acted as the series inductance loading was continually increased. For each frequency the average value of the curve of potential and current was calculated, calling the maximum value of either curve equal to unity.

Thus the quarter wave length frequency of this line unloaded was 100.5 cycles; curves of this line are shown in Figure 15. With a series loading of 0.413 henries the frequency dropped to 52.4 cycles. At this frequency the line has not an electrical length of 90°, but of $\frac{52.4}{100.5} \times 90° = 47.°$. Hence the current distribution curve will be a sine wave from 47° to 0°, and the potential curve will be a sine curve from 43° to 90°. The average value of the sine curve from 47° to 0° is 0.531 of the value of the sine of 47°. The average value of the sine curve from 43° to 90° is 0.894 of the sine of 90°.

As the total L of the line was 0.374 henries, its effective L (if previous reasoning is correct) will be $0.374 \times 0.531 = 0.199$ henries. The effective value of capacity will be $16.45 \times 0.894 = 14.73$ μf. As the added inductance is 0.413 henries we would expect the line to oscillate like a circuit having $(0.413 + 0.199) = 0.612$ henries and 14.73 μf.

The natural frequency of such a system

$$= \frac{1000}{2\pi\sqrt{0.612 \times 14.73}} = 53.1 \text{ cycles.}$$

actual measured value of resonant frequency was 52.4 The other values are tabulated herewith. It will be noticed that the agreement between predicted and measured frequencies is all that could be expected from the precision with which the test was carried out.

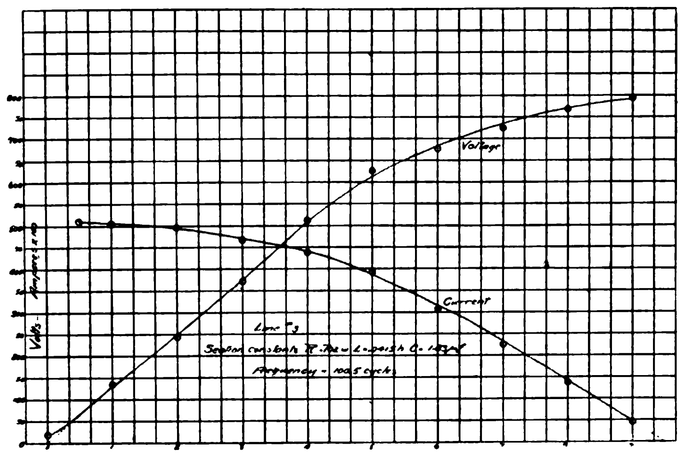


FIGURE 15

Added L	Capacity		Inductance		Calculated f	Measured f
	Av. Sine	Effect. Cap.	Av. Sine	Effect. L		
0.000	0.636	10.52	0.636	0.238	100.5	100.5
0.073	0.736	12.12	0.586	0.220	84.7	83.8
0.142	0.791	13.05	0.563	0.211	74.2	73.6
0.207	0.828	13.65	0.552	0.207	67.0	66.8
0.292	0.855	14.08	0.540	0.203	60.5	60.2
0.348	0.878	14.48	0.534	0.200	56.6	55.6
0.413	0.894	14.73	0.530	0.198	53.1	52.4
0.617	0.920	15.15	0.521	0.195	45.4	45.1
0.823	0.936	15.40	0.516	0.193	40.3	39.6
1.245	0.952	15.70	0.513	0.192	33.6	33.8

The true value of line capacity is 16.45 microfarads.

The true value of line inductance 0.374 henry.

The above results were plotted in the forms of curves and given in Figure 16. They seem important as they have an immediate application to radio calculations. In the form given above they apply directly only to a uniform antenna such as some of the Marconi stations use, but with suitable modifications can probably be suited to fit other fairly simple forms of antennas.

It will be seen at once that the common measurement to get antenna capacity is more or less meaningless because the capacity of the antenna is a function of the frequency with which it is vibrating. The ordinary method of getting antenna capacity

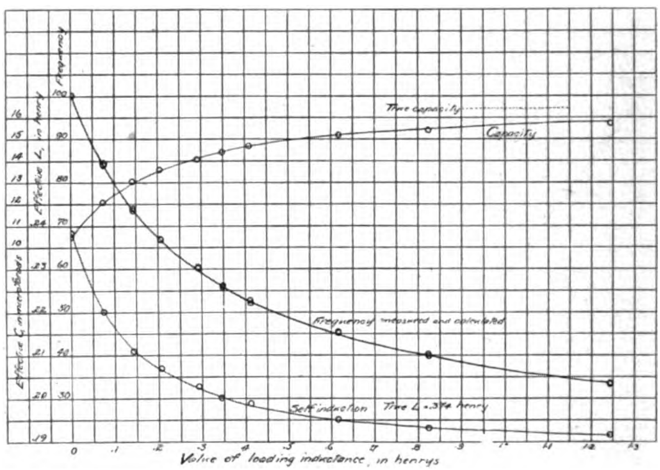


FIGURE 16

is to measure the oscillation constant when a large loading coil of known value has been inserted in the base of the antenna. From the LC thus measured, and known L, the C is calculated. Then the LC of the unloaded antenna is measured and from the C just obtained the antenna L is obtained.

Trying this method on the line used in above test would give C = 16.40 μf (nearly). The LC of the line unloaded gives $LC = 2.56$.

From this $L = 0.156$ henries. But the actual effective L when line is unloaded is 0.238 henries, and under no conditions does it become lower than about 0.190 henries.

Summing up the above results shows that as the series loading of an antenna is increased the effective capacity of the antenna varies from 63.6 per cent. of its true capacity up to 100 per cent. true capacity while the inductance decreases from 63.6 per cent. of its true value to 50 per cent. of its true value.

However, this analysis, which assumes change in both capacity and inductance with frequency, yields no more accurate a solution of the problem than the present method of treating both L and C constant. Thus, if we assume $C = 16.40$ μf and $L = 0.156$ henry, and calculate the natural periods of the loaded line with these values, the frequency will come out just the same as it does with the method outlined. Thus if the loading is 0.5 henry, $L = 0.656$ and $C = 16.40$. The natural frequency calculated from these values is 48.7 cycles, which agrees with the experimental result.

Resistance

As the current distribution in the conductor changes from a sine curve to a straight line with increase in loading (assuming quarter wave length mode of oscillation) the heat generated in the conductor by the $I^2 R$ loss must, of course, vary also. Hence the conductor resistance, which is obtained by dividing the total $I^2 R$ loss by the square of the value of current at the beginning of the line must vary with increased loading in some manner similar to that in which the capacity and inductance vary.

Let R = the actual conductor resistance, i. e., resistance per unit length × length,

I = current at beginning of line,

r = effective resistance of conductor.

Then for no loading it is seen that the effective value of the current is equal to the root mean square of a sine wave from 90° to 0°. This root mean square value is $0.707 \times I$, as proved in elementary alternating current theory.

Hence the actual heat loss is given by

$$\text{Loss} = (I \times 0.707)^2 R = 0.5\, I^2 R$$

But this gives for effective resistance of antenna

$$I^2 r = 0.5\, I^2 R \text{ or}$$

$$r = 0.5\, R$$

So the effective resistance of such a uniform conductor is only one half the actual resistance.

Now as the loading is increased the current distribution tends to become a straight line. But the effective value (root mean square) of such a straight line is $\frac{1}{\sqrt{3}} \times$ maximum value. Hence for straight line current curve (very heavy loading)

$$I^2 r = \left(\frac{I}{\sqrt{3}}\right)^2 R \text{ or}$$

$$r = \frac{R}{3}.$$

Hence we should expect the effective resistance of the line to vary from 50 per cent. of its actual resistance to 33 per cent. of its actual value as loading was increased.

Readings were taken to see whether such a change actually did occur in the line used in previous tests. After adjusting the line for quarter wave length oscillation with various values of loading the power input was read by wattmeter. Then another reading of wattmeter was taken with current the same but the potential coil of the wattmeter was connected to read the power used in meters, connections, loading coils, etc. The difference of these two readings gave the power lost in heating the line.

This loss divided by the square of the current supplied to the line gave the effective resistance of the line. The total actual resistance of the line was 6.60 ohms; the effective resistance with no added inductance should be 3.30 ohms. Actually it came out 3.56 ohms. Other values of resistance for different values of loading were obtained but the results were erratic and apparently unreliable. It involved the determination of a small difference between two comparatively large quantities and moreover the condenser losses (which were neglected) apparently affected the results considerably.

Of course the results I have obtained experimentally could have been at once predicted from the laws of wave propagation.

Thus using same symbols as before

$$a = \sqrt{(r + j\,\omega L)\,(g + j\,\omega C)} = a + j\,\beta \text{ where}$$

β is the so called wave length constant of the line.
If r and g are small compared to ωL and ωC (this condition was fulfilled in my experiments) then we may write approximately

$$\beta = \omega\sqrt{LC},$$

or if we are considering a certain length of the line l

$$\beta\, l = \omega l\,\sqrt{LC} = \omega\,\sqrt{(l L)\,(l C)} = \omega\,\sqrt{L' C'},$$

where L' and C' are the actual total self induction and capacity of the line.

Now λ, the wave length developed on a line, is given by

$$\lambda = \frac{2\pi}{\beta},$$

and if we choose such a length of line that quarter wave length is developed we have

$$\text{if } l = \frac{\lambda}{4} \qquad \beta\, l = \frac{\pi}{2}.$$

Then
$$\beta l = \omega \sqrt{L' C'} = \frac{\pi}{2}, \text{ or, as } \omega = 2\pi f,$$

$$f = \frac{1}{4\sqrt{L' C'}},$$

so that
$$f^2 = \frac{1}{16 L' C'}.$$

As the ordinary equation for resonance (lumped inductance and capacity) is

$$f^2 = \frac{1}{4\pi^2 L C},$$

we evidently must put

$$L = \frac{2}{\pi} L' \text{ and } C = \frac{2}{\pi} C'$$

if the two frequencies are to be the same:—that is, in so far as determining quarter wave length resonance frequency is concerned, the total inductance L' acts as tho it had only 63.6 per cent. of its actual value, and similarly for the capacity.

SUMMARY: Working at frequencies from 12 to 152 cycles per second, an artificial antenna made up of numerous sections each having lumped capacity and inductance (and closely simulating an actual antenna), is carefully studied. The effective capacity, inductance, and resistance are measured and found to agree with theory; and the theoretical effects of capacity and inductance loading are similarly experimentally verified.

DISCUSSION

Dr. A. E. Kennelly (communicated): Professor Morecroft's paper is particularly interesting, as furnishing a new cross-connection between the radio laboratory and the power-transmission laboratory; or between oscillatory phenomena, at perhaps 1,000,000∽, and steady-state phenomena at perhaps 100∽. It shows an ingenious means of deducing what happens in the oscillations of an antenna during a cycle of say a few microseconds, from observations of what happens, in the steady state of standing waves, on a low-frequency artificial line. We have had artificial electric antennas, consisting of a fixed condenser associated with a reactor; but here we have an artificial antenna consisting of a suitably modified artificial power-transmission line. This opens up the prospect of developing a low-frequency multi-section antenna, in which the inductance and capacitance are arranged to correspond, point for point, with that of an imitated actual antenna; so that a study of the electric behavior of the model may reveal the behavior of the high-frequency tower system. The prospect of experimental investigation thus opened up is very fascinating.

There is one difficulty in the design of an artificial multi-sectional antenna, and that is the insertion of radiation resistance. An actual antenna develops a virtual resistance of radiation R' in addition to its conductor resistance R; so that $I^2 R'$ becomes the radiated power dissipated externally, in addition to the $I^2 R$ of power dissipated within the conductor as heat. It would be very desirable to insert the correct amount and distribution of R' in the artificial antenna, as deduced from the observed behavior with L, C, and R.

DISTRIBUTED INDUCTANCE OF VERTICAL GROUNDED ANTENNAS*

By

A. Press

(Assistant Professor of Electrical Engineering, University of Oklahoma)

Formulas have been suggested for estimating the self induction of vertical antennas. Such formulas have moreover been employed in estimating the effect of inductance and capacity on the distribution of stationary waves on wires. The whole theory rests on the tacit assumption that the inductance and capacity are uniformly distributed along the wires in question, which in fact is not the case. In the subjoined paper the inductance distribution of an earthed antenna will be considered and based on the fact that the ether displacement currents, or electrostatic stresses in the medium will be properly indicated by taking two imaged antennas oppositely disposed with respect to an infinitely conducting lamina providing a return for both earth currents.

It will be necessary to postulate such a system of voltages set up at the base O of the antenna that a uniform current distribution is set up in the antenna, flowing in opposite directions with respect to the infinitely conducting lamina or image plane representing the earth's surface. In this way the displacement currents will take paths substantially the same as those usually assumed for a Hertzian oscillator except that the flux lines of displacement on the lower half are imaged with respect to the upper half. That this is nearer approximation to actual conditions should follow from the fact, first, that the image field restricts the actual field to its own domain, and, secondly, that in this way there is not the usual difficulty in calculating inductances of finite open circuited wires where the so-called return wire is assumed to be infinitely removed from the actual vertical antenna under consideration.

* Received by the Editor April 10, 1917.

The antenna will therefore be assumed to have impressed upon it an alternating current uniformly distributed thruout the length of the antenna. The magnetic intensity H will be

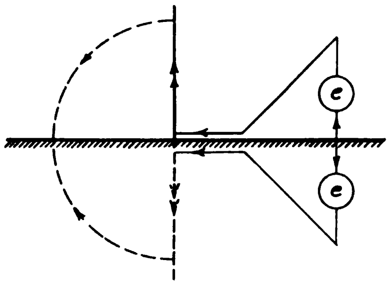


FIGURE 1

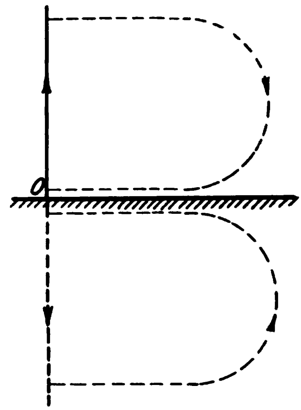


FIGURE 2

considered first for a point P distant r from the antenna axis for a height $\lambda = l + x$ from the image end S of the aerial.

The intensity due to the current element $i\,dx$ at R is

$$H_{dx} = \frac{i}{10}\,dx\,\frac{\sin PRM}{(MP)^2+(MR)^2} = \frac{i\,r\,dx}{10\{(x-\lambda)^2+r^2\}^{\frac{3}{2}}}$$

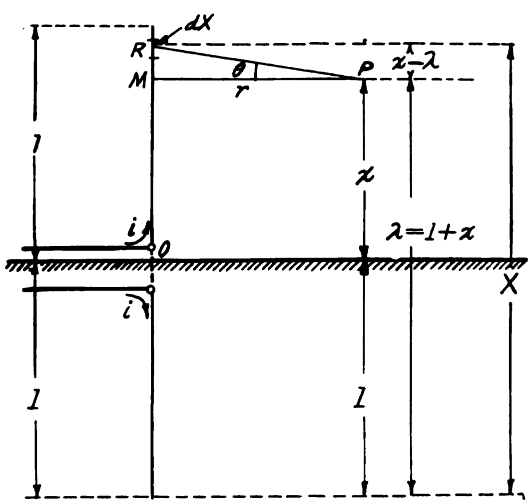


FIGURE 3

Thus the intensity H_x at P due to the entire antenna system, which latter includes the image antenna, is

$$H_x = \int_{x=l}^{x=2l} \frac{i\,r\,d\,x}{10\,\{(x-\lambda)^2+r^2\}^{\frac{3}{2}}} + \int_{x=0}^{x=l} \frac{-i\,r\,d\,x}{10\,\{(x-\lambda)^2+r^2\}^{\frac{3}{2}}}.$$

To integrate the above let

$$x-\lambda = r\tan\theta$$

and then

$$H_x = \frac{i}{10\,r}\left[\frac{2l-\lambda}{\sqrt{(2l-\lambda)^2+r^2}} + \frac{(-\lambda)}{\sqrt{(-\lambda)^2+r^2}} - 2\,\frac{l-\lambda}{\sqrt{(l-\lambda)^2+r^2}}\right].$$

To find the total number of external flux linkages per centimeter at the distance x above the plane of the earth's surface, it will be necessary to find

$$\int_{r=a}^{r=\infty} H_x\,d\,r = \phi_x$$

where a is the radius of the antenna wire. Performing the necessary integrations

$$\phi_x = \frac{1}{10}\,log\,\frac{\{(2l-\lambda)+\sqrt{(2l-\lambda)^2+a^2}\}\,\{\sqrt{\lambda^2+a^2}-\lambda\}}{\{(l-\lambda)^2+\sqrt{(l-\lambda)^2+a^2}\}^2}$$

Removing the origin of coordinates by referring the points along the antenna to distances x removed from the earth's surface we have $\lambda = l+x$, and therefore

$$\phi_x = \frac{i}{5}\,log_\varepsilon\left[\frac{\sqrt{x^2+a^2}+x}{a}\right].$$

The inductance in henrys being defined as the flux per ampere divided by 10^8, we have

$$L_x = 2\,(10)^{-9}\,log_\varepsilon\left[\frac{\sqrt{x^2+a^2}+x}{a}\right]$$

or practically

$$L_x = 2\,(10)^{-9}\,log\,e\,\left(\frac{4\,x}{d}\right).$$

where d is the diameter of the antenna wire. It is easily seen that the inductance at the earthed end of the antenna is zero whereas at the top of the antenna it is the greatest.

SUMMARY: Assuming a uniform current distribution along the antenna, the variation of distributed inductance along the antenna is studied.

Appendix

The following may be of value in considering the validity of the foregoing development:

In considering the self induction coefficient of a long transmission line in which the overhead conductor is run parallel to the earth's surface, the self induction coefficient L is considered constant. This amounts to regarding the end effect of the line at the source of e. m. f. as negligible.

In the above consideration the distributed leakage conductance from the line to earth is taken to have no influence on the self induction coefficient as such, tho necessarily the current density in the circuit must always be far from uniform from the source outward. Evidently the self induction coefficient is calculated on the basis that the leakage conduction currents from the source outward along the line as well as the variable condensance current is assumed to have no influence on the self induction coefficient which is calculated on the basis that a uniform or unit ampere of current is flowing along the line.

Whereas the self induction coefficient, therefore, is assumed as independent of leakance or condensance, the *inductance drop* is dependent on the current density from point to point of the line.

MUNICIPAL REGULATIONS COVERING RADIO STATIONS*

(A Discussion on "ENGINEERING PRECAUTIONS IN RADIO INSTALLATIONS" by ROBERT H. MARRIOTT)

BY

LIEUTENANT ELLERY W. STONE, U.S.N.R.F.

(DISTRICT COMMUNICATION SUPERINTENDENT, SAN DIEGO, CALIFORNIA)

Mr. Marriott's paper very aptly covers a subject which has been of interest to the writer for the past eight years. While the various phenomena encountered in low voltage circuits caused by the operation of radio transmitters in their vicinity are often capable of explanation, it seems somewhat difficult to predict what will happen in a given case, so that Mr. Marriott's plan of stating the results of individual cases appears to be the best method of approaching the problem of interaction between radio frequency circuits and audio frequency or direct current circuits.

In connection with Mr. Marriott's suggestion that the "Underwriters' Code" be enlarged so as to include within its scope the installation of radio apparatus, the following may be of interest. During the month of December, 1916, the writer was asked by the Electrical Department of the City of Oakland, California, to collaborate in the drawing up of an ordinance to regulate the installation of radio apparatus in that city. It is the custom of most municipalities, upon the recommendation of the civic electrical department, to pass an ordinance adopting the "National Underwriters' Code" as the criterion in the regulation of electrical installations, with such additional provisions as may be considered expedient. In Oakland, for instance, it is required that an electrical installation be made by a registered electrician who is required to deposit a cash bond with the city, and the customary provisions are made for "rough-in"

* Presented before the San Francisco Section of The Institute of Radio Engineers, February 20, 1917. Received by the Editor, March 2, 1917. Mr. Marriott's paper appeared in the "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," volume 5, number 1 (February, 1917), page 9.

and "final" inspections by city electrical inspectors who hold their positions by local civil service.

After several conferences, the following provisions were decided upon and incorporated in legal form in the proposed ordinance, which, by the time this paper appears in print, will probably be passed.

1. No radio apparatus, whether for receiving or transmitting purposes, connected to an elevated antenna, may be installed within the City of Oakland without a permit from the City Electrical Department.

2. All radio apparatus must be installed subject to the provisions of the "National Underwriters' Code" in force and subject to the following additional provisions:

a. No antenna wires shall be strung over electrical wires whose potential to ground exceeds 250 volts.

b. Every radio station shall be equipped with a lightning switch which shall serve to ground the antenna at all times when the station is not in use and during electrical storms.

c. The wire from the antenna to the lightning switch, commonly known as the lead-in, shall be of such size that its cross-sectional area in circular mils shall not be less than the cross-sectional area of the wire used in the antenna times the number of antenna wires. The current carrying capacity of the lightning switch shall not be less than the accepted current carrying capacity of the "lead-in." The cross-sectional area of the wire from the lightning switch to the earth connection shall not be less than that of the "lead-in."

d. The "lead-in" must be brought thru an approved non-inflammable, non-combustible insulator.

e. The earth connection shall conform to that specified in the "Underwriters' Code."

f. Approved surge protectors, i. e., mica condensers with capacitances not less than 0.5 microfarad, lamps or carbon rods, shall be connected between earth and all electrical wires whose potential to earth exceeds 50 volts.

g. In installations where it is impossible to eliminate surges or antenna induction, the service to the radio station shall be run from a power transformer, the secondary of which is connected to no other lighting or power circuit.

3. The radio installation shall not be supplied with power for operation until same has been approved by the City Electrical Department.

4. All high voltage circuits shall be so insulated as to reduce the fire hazard to a minimum.

The reason for the adoption of some of these provisions is obvious, the others will be discussed.

Number 1 was intended to bring all radio experimenters under the jurisdiction of the Electrical Department in order that their activities might be watched. It was considered that the fire hazard from an improperly erected antenna, due to its proximity to high voltage wires or danger from lightning was sufficient justification for requiring a receiving station to be brought within the scope of the ordinance.

While natural electrical phenomena of any violence on the Pacific coast were extremely uncommon a few years ago, several electrical storms have occurred within recent months and the frequency of their occurrence seems to be increasing. This fact is responsible for the adoption of 2 (b). It is doubtful whether the use of a lightning switch, no matter how installed, will completely safeguard a wooden building in the event that an antenna connected thereto be struck by lightning. However, an ungrounded antenna may often be charged to enormous potentials owing to a heavily charged "thunder" cloud or to a lightning discharge occurring in the vicinity, and such charges may be readily dissipated to earth by proper precaution. The writer has witnessed examples of the former phenomenon at his experimental station.

2 (c) was intended to obviate the absurdity of grounding an antenna having a number 18 "lead-in" by a number 4 ground wire and a 100-ampere lightning switch as the 1915 "Underwriters' Code" at present requires.*

The adoption of such an ordinance to regulate radio installations was considered imperative by the Oakland Electrical Department because of the fact that irresponsible experimenters have for many years been raising havoc with lighting circuits, because of surges and antenna induction burning out lamps and appliances connected to such circuits, as set forth in Mr. Marriott's paper. The fire hazard from such practice was actually considered to be greater than that from any other type of electrical installation. That such a state of affairs is reaching a vexing magnitude is shown by the fact that the California Railroad Commission (a state committee for the regulation of public utilities), acting on the petitions of various power companies,

* Diameter of number 18 wire = 0.040 inch = 0.102 cm.
Diameter of number 4 wire = 0.204 inch = 0.520 cm.

has recently ruled to the effect that power companies, at their option, may refuse to supply consumers with electricity in the event that radio transmitters are connected to their circuits

The writer has witnessed cases where high potentials have been completely eliminated on the power lines at the radio station by the usual methods, only to lead to breaking down insulation on the same lines several hundred feet away. It would appear that the surge takes the form of a stationary electric wave, that by grounding the line thru protective devices at the radio apparatus, we secure a node of potential, the loop occurring at some point removed from the station.

The subject of stationary electric waves on wires is an interesting one and has been investigated by many physicists. Fleming has developed a set of equations showing that if a simple harmonic E.M.F. is applied at one end of a wire of finite length, the potential at any point on this wire may be obtained by taking the algebraic sum of two potentials, one due to the source at the origin, the other due to an electrical image of this source. The distance from the open end of the wire to the position of the image is equal to the length of the wire. That is to say, there is a wave traveling on the wire from the end at which the periodic E.M.F. is applied, and a wave reflected from the open end of the wire. When the length of the wire bears the proper relation to the frequency of the applied E.M.F., the interference between these two waves causes a resultant series of stationary nodes and loops of potential to be set up, corresponding to the stationary air waves in stopped organ pipes.

If the length of the wire is one fourth of the wave length of the applied E.M.F., the combination of the initial and reflected waves is such as to cause a steady increase of potential following the ordinates of a sine curve, from the origin to the open end of the wire. It may be shown that if the length of the wire is any multiple of a quarter wave length, loops of potential will occur at each quarter wave length along the wire.

A more complete theoretical investigation of this subject has been made by Professor H. M. Macdonald, who finds that the length of the fundamental wave on a wire is more nearly five times the length of the wire. However, for the purpose of this paper, it will be considered sufficiently accurate to note that if any of the various low voltage circuits in the vicinity of the radio station have lengths approximating one-fourth of the transmitting wave length, or multiples thereof, stationary electric waves with potentials of some magnitude will be generated therein. With modern power distribution in cities and customary house wiring, this is a condition not difficult of fulfilment, so that while the induced current set up by a radio transmitter may have a node of potential at the station, where every care has been taken to secure such a node by proper surge protection, the loop of potential may occur at some point removed from the radio installation where no provision has been made to protect the line and the appliances connected thereto. Hence, it is advisable in cities, where the secondary distributing leads of the power systems are often several blocks in length, and from which many consumers are supplied, to place each radio installation on a separate power transformer. The secondary lead may then be a hundred feet or less in length and there will not only be less opportunity for excessive potentials to be built up, but there will be no place where such potentials can cause damage.

There are three ways by which stationary waves may be set up on wires, i. e., by direct, inductive, or static (capacitive) coupling.

Applied to the subject under consideration, direct discharge from the transmitting apparatus is illustrative of the first method of coupling. If the transformer of the radio transmitter be poorly constructed so as to permit actual leakage, high potentials of audio frequency may find their way back on to the line, and if resonance effects in the windings are present, radio frequency potentials may also be present on the line. Such cases as this, however, are rare.

Inductive coupling between radio frequency circuits and low potential circuits is probably the chief cause for most of the surges present on the latter circuits. The primary requisite for this type of coupling is that the circuits, or portions of them, be parallel.

Static coupling is brought about by having the two circuits under discussion in close proximity to each other but not necessarily parallel. The writer had occasion to witness a case where potentials were set up in a lighting circuit by an antenna lead run exactly at right angles to the lighting circuit a few inches away, clearly a case of static coupling.

It would appear that most of the induction of high potentials takes place by virtue of the inductive or static coupling of the antenna circuit with other circuits in its proximity. The writer has never seen a case where induced currents were set up in low potential circuits with the antenna circuit disconnected.

Besides the induction of high potentials of radio frequency on low voltage circuits, currents of audio frequency equal to the train frequency of the antenna may be set up as well. The method of their generation is obscure, it may be due to some form of excitation by which an impulse or surge occurs for each train in the antenna circuit, but whatever their source, their presence may be easily detected with a telephone receiver, even without the use of a loose contact or rectifying detector. It is the presence of these audio frequency currents in telephone lines which is responsible for the annoyance caused subscribers by radio transmitters in their vicinity, and since their frequency is of the same order as voice frequency, which is assumed to be 796 in telephone practice, they cannot be drained from the line by condensers having a high, low frequency impedance as in the case of radio frequency currents.

Some years ago, the writer witnessed a curious case of induced surges on a power line. An experimenter had a special power service supplied for his station, being the only consumer on a three kilowatt power transformer, which was wound in the usual 10:1 step-down ratio. Full protection was made for the elimination of induced surges by the usual methods, nevertheless, when the station was in operation, brush and corona discharge appeared on the primary leads of the power company's transformer which finally resulted in setting fire to a tree thru which the primary leads were strung. The theory of resonance effects in an audio frequency transformer, due to the distributed capacitance of multiple layer winding, is probably the correct explanation of the phenomenon, altho it is barely possible that audio frequency potentials or surges, having the same frequency as that of the wave trains in the antenna, were induced in the secondary leads of the power distribution system by the antenna which ran parallel to them, and were stepped up in the power transformer. That the potential was stepped up in the transformer by some method was demonstrated by the fact that no corona existed on the secondary leads from the transformer.

In addition, the writer has observed a case of high potential generation in a lighting circuit such that when the radio transmitter was being operated, lights connected to this circuit would flash up to an alarming brilliancy. Grounded condensers, whose radio frequency impedance was very low, but with high audio frequency impedance, connected to the line in the usual fashion did not serve to ameliorate conditions, whereas a few tungsten lamps connected to earth eliminated the trouble entirely.

This would appear to be another proof of the presence of audio frequency impulses or surges.

In this connection, the writer wishes to endorse the use of the metallic filament lamp bank as a surge protector. Its impedance being independent of frequency, if enough lamps are used, it will serve equally well in handling audio or radio frequency surges and is indestructible. As Mr. Marriott points out in his paper, mica or paper condensers are liable to puncture and, when protected by fuses, as they should be, are removed from the circuit without any warning. A small bank of lamps for this purpose may be controlled thru a proper switch or the transmitting key so as not to consume current except when the transmitter is being operated.

In cities where telephone lines are run underground or in aerial lead cable suspension, inductive interference from radio stations is not particularly annoying. Cases arise frequently where trouble is caused by an experimenter grounding his transmitter on the same ground as that used by the telephone, but the remedy for this is obvious. In the event that sparking occurs in the carbon block lightning arrester which will result in fusing the arrester, grounding the line and rendering it inoperative, the telephone company upon request can furnish copper blocks such as are used on toll lines to replace the carbons. However, in localities where the telephone wires are not run in a grounded sheath, very severe interference may take place.

The writer was called upon recently by the Pacific Telephone and Telegraph Company to assist in the elimination of a troublesome case of radio interference caused by a three kilowatt station located on the outskirts of a small town, in which locality none of the telephone wires were run in grounded cable.

Figure 1 is a diagram of the situation. Extremely loud signals of the same frequency as the train frequency or spark frequency of the radio station were received at the exchange board some four miles (6 km.) from the station when the transmitter was operated. A series of experiments were accordingly conducted to see if the interference could be reduced.

Telephone line number 1 served the radio station. Telephone line 2 served a subscriber living two miles past the radio station. Line 3 connected with a small town four miles (6.5 km.) away in a direction at right angles to line 2.

The loudest interference at the exchange was experienced on these three lines altho signals could be heard on other lines as well, due to the fact that all lines into the exchange were brought thru a common cable.

In the belief that the interference was caused by induction from the antenna to that part of line 1 marked AB, the line was opened at A. No effect was noticed.

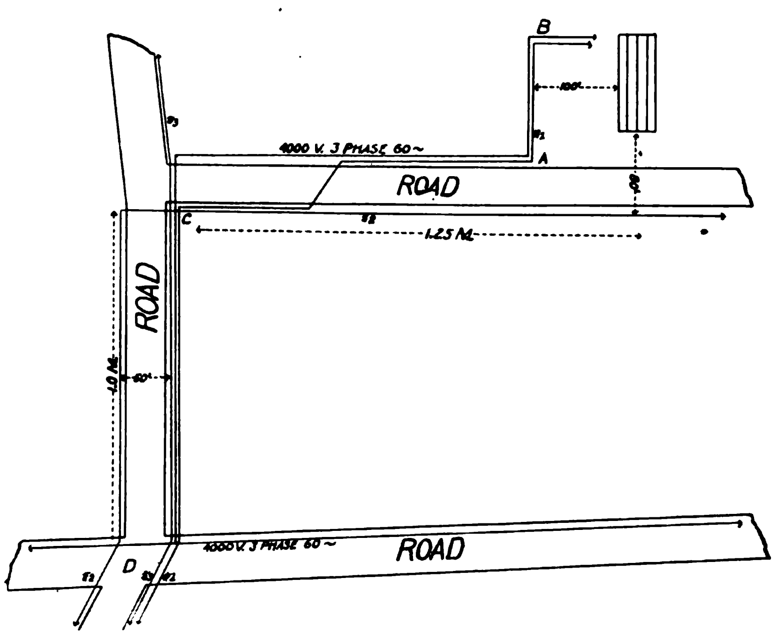


FIGURE 1—Chart Showing Antenna, Power, and Telephone Lines with Induced Interference

Line 1 was next opened at C. This reduced strength of signals on Line 1 at the exchange, all other lines remained the same.

Line 2 was cut at C. Interference was reduced on this line, all other lines remained the same.

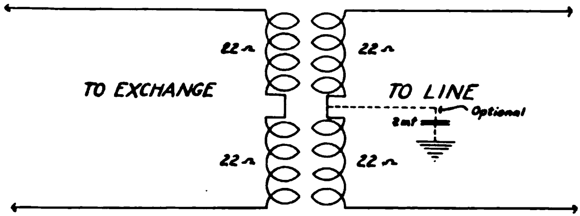
Lines 1 and 2 were cut at D. Interference was eliminated on these lines, interference remained the same on Line 3. Line 3 was cut at D, all interference being eliminated.

This had but served to locate the source of trouble: the elimination of the interference with lines 1, 2, and 3 restored to service was another problem.

In telephone practice, it is customary to isolate a "noisy" line, that is to say, a line experiencing moderate interference from power lines or a line with a partial ground, by a repeating coil, as shown in Figure 2. A repeating coil, as it is termed in telephone parlance, is a 1:1 transformer, designed to operate on 796 cycles with a D. C. resistance of 22 ohms in each of its windings.

These were interposed in lines 1, 2, and 3 at *D* and reduced the interference considerably but not entirely.

It is significant that the interference was not eliminated until all lines were opened at *D*, which was the last point at which the telephone lines ran parallel with the power line.



REPEATING COIL - 25 A

FIGURE 2—Use of Repeating Coil to Isolate "Noisy" Telephone Line

This power line was 4,000 volt, 3 phase, 60 cycle service supplying the ranch on which the radio station was located. Power for the station was obtained from a 20:1 step-down transformer, one leg of the primary of which was connected to earth, the other to one wire of the 3 wire, 3 phase system.

It is obvious that line 3 could receive no induction from the radio antenna directly, nor could lines 1 and 2 when cut at *C*. With line 2 at right angles to the antenna and 80 feet (25 m.) from it at its nearest point, it does not appear that it could receive much interference from the antenna at any time.

With lines 1 and 2 open at *C*, the only source from which 1, 2, and 3 could receive induction was the power line, which parallels all lines for the mile (1.6 km.) from *C* to *D*. Lines 1 and 3 were but 6 feet (2 m.) from the power line over this distance.

Surges in the power line were probably induced by the antenna between the points *A* and *B* and in turn induced in the telephone lines along their entire length. These surges may have been of radio or audio frequency, but whatever their frequency, the frequency of the current in the telephone lines was of audio frequency.

For a complete elimination of the interference, such as was obtained by opening all lines at *D*, it was recommended that the telephone wires in lines 1, 2, and 3 be run in a grounded lead sheath or lumped together and surrounded with a frequently grounded "messenger."

In connection with the induction of audio frequency currents in circuits near radio transmitters, it has been the writer's experience that these audio currents are of moderate potential, are only set up within a wire when in extremely close proximity to the exciting circuit, and have the same frequency as the train frequency in that circuit.

Besides the annoyance caused by radio circuits on power circuits the latter may similarly react on the former. Most experimenters have observed the inductive interference caused in their receiving circuits due to alternating current lines in their vicinity. This may be even of such magnitude as to render the reading of weak signals impossible as the writer found it to be in his station. The situation here was as follows. Power for the transmitter was furnished by a 2 wire, 220 volt, 60 cycle service, run in conduit from a special power transformer to the transmitting room, adjoining the receiving room. The lighting service in the building was of the usual 3 wire, 110 volt, 60 cycle, grounded neutral system. By a series of tests, it was found that the interference was caused by the 220 volt circuit, one leg of which was dead-ended in the transmitting apparatus, the other being open thru the transmitting key and controlling switch. By connecting the dead-ended leg to earth, eliminating the possibility of a static charge accumulating on the network of apparatus and connecting wires, the inductive hum was entirely eliminated.

In concluding, the writer wishes to acknowledge his indebtedness to Mr. H. U. Linkins, Division Toll Inspector of the Pacific Telephone and Telegraph Company, for his helpful assistance in the preparation of this discussion.

SUMMARY: The chief features of a proposed municipal code (for the city of Oakland, California), covering all radio stations, are given, together with the basis of each regulation.

The induction of radio and audio frequency surges in power and telephone lines is then considered; and a particularly complex case of both sorts is described in detail. The methods of investigation and elimination of trouble are discussed.

Finally, some methods for the elimination of the hum induced in radio receivers from power lines are described.

THE MANUFACTURE OF VACUUM DETECTORS*

By

O. B. Moorhead

(Chief Engineer, Moorhead Laboratories, San Francisco, California)

Altho the majority of radio engineers are familiar with the use and operation of vacuum tube detectors, a brief description of their manufacture may be interesting.

In the early experimental work on this type of device, we strove to produce a detector which would combine maximum operating efficiency with inexpensive manufacture. The next point considered was the production of desirable conditions, i.e.; tubes that possessed oscillating characteristics, tubes that were exceptional detectors, and tubes that displayed both qualities. The third consideration was the production of a device easily handled and shipped without disturbing the adjustment of the elements and damaging the filaments.

Tubes and bulbs of various shapes and sizes were tried using a gaseous medium ranging from one millimeter to 0.025 millimeters of vacuum, many materials being employed as elements. Various exhausts were applied but it was soon found that the employment of a gaseous medium introduced considerable difficulty in the matter of accurate reproduction of a desired result. Gases at pressures ranging from one millimeter to 0.0013 millimeters were next experimented with.

I found that a tube containing a platinum filament in an atmosphere of hydrogen, at pressures comparable with one millimeter, gave fair results. Tungsten filaments were then tried in higher vacua as well as at the so-called "gaseous medium" pressure. It was immediately noticed that conditions could be duplicated as soon as vacua above that which allowed a "gaseous medium" to exist, were obtained. Moreover, tungsten was ideal as a filament not only because of its refractory qualities and low volatility but also because it acts as a purifying agent by attacking any traces of residual gases that may remain in the tube and forming compounds which are then volatilized on the walls of the tubes.

* Received by the Editor, February 15, 1917.

As the parts are small and complicated, the glass is worked before the blowpipe, after it has been brought into the form of tubes by the glass works. This tubing is obtained by first blowing a bulb, then fusing an iron rod to a point diametrically opposite the blowpipe and rapidly separating the two points of attachment from each other.

Various grades of glass were experimented with, and a mixture containing a high percentage of lead and a small quantity of silicic acid was found to be the easiest to work and produced a detector of maximum sensitiveness when used in conjunction with the aluminum plate and copper grid. In the selection of the glass to be used, the devitrification of the glass had to be considered, as during exhaustion of the tubes it is necessary to subject them to a temperature near the point of softening and nearly all glasses, when maintained at this temperature for any length of time, have a tendency to separate out into the crystalline state.

There has been considerable discussion regarding the elements in this type of device and I may say that aluminum plates and copper grids were first selected on account of their electrochemical relation to the tungsten filament. Later, numerous other metals were tried under the same and other conditions of exhaustion and showed widely different operating characteristics.

The selection of metals for the elements is very difficult, as a slight difference in either the copper or aluminum changes the whole system of exhaust. For instance, copper and aluminum purchased from one factory will require a certain degree of applied temperature during the evacuation, while another factory lot of the same weight and size will require an entirely different exhaust.

I have eliminated this variation to some extent by subjecting the aluminum plates to a temperature of approximately 600 degrees Fahrenheit (315° C.), immersing them in a saturated solution of cyanide of potassium, and finally rinsing in alcohol. The copper is subjected to heat until it glows, when it combines with the oxygen of the air to form a black, brittle oxid which breaks off in scales and exposes the underlying metal which is of rose red color. It is then placed in a current of moist air and becomes covered with a layer of oxygen compounds, which remains very thin but closes the pores of the metal.

The exhaustion of the tubes is the most important operation because of the fact that the low vacuum of the round bulb nickel element audion which permits of gas conduction is not used in the tubular "electron relay," wherein all gas phenomena must be eliminated.

To produce the high vacuum necessary, I have found that a Gaede mercury pump capable of producing a vacuum of 0.00001 millimeter, backed by a piston pump, such as the Geryck type, is the most satisfactory method of evacuation.

The manifold to which the tubes to be exhausted are attached and the vacuum line connecting the manifold to the pumps are preferably made of large diameter tubing. A container filled with pentoxid of phosphorus is connected in the vacuum line between the pump and the manifold. The manifold is contained in an oven heated by gas and arranged so that the tubes during exhaustion may be heated to high temperatures.

The lead glass tubing, used as the container for the elements in the tubular type detector, is obtained from the glass works in lengths of 6 feet (2 m.) with an inside diameter of 0.875 inch (2.2 cm.) and a wall of 0.032 inch (0.7 mm.) thickness. This tube is cut in lengths of about 6 inches (15 cm.) and one end is drawn down to a point. Two stems are made of glass tubing similar to those used in an incandescent lamp, one stem contains the grid and two filament leads and the other contains the plate connection and one filament lead. After the wire is sealed into these stems, they must be annealed very carefully. The annealing consists in allowing the temperature to drop very slowly, since quickly cooled glass is subject to internal strains which arise in the following manner: In rapid cooling, a low temperature is soon established at the surface and the outermost layer solidifies while the interior tends to contract, thereby exerting a pressure on the outer layer which is directed inwards. This may cause the stem to crack.

After the stems are annealed, the grid is wound to the proper diameter and the filament is clamped onto the two leads. The plate is mounted on the other stem and the two stems are then connected together by means of the filament. Final adjustment of the plate and grid is then made. The spacing between the elements is not very critical in this type of device, but it is best to wind the grid to a large enough diameter so that it will strike the plate rather than the filament when the tube is jarred.

After adjustment on the plate and grid has been made, the assembly is inserted into the prepared tubes and the end seals made. A short length of small diameter tubing is attached to the seal at one end of the tube, this being for connection to the pump manifold. The tube is then carefully annealed and is ready for exhaustion.

A number of tubes are sealed on the manifold in the oven and the temperature is gradually increased to 900 degrees Fahrenheit (480° C.) at which point the pumps are started. The tubes are heated in this manner before the pumps are started so that the air contained in the tubes may conduct the heat to the central elements and drive off the occluded gases. When the pumps have produced a vacuum of one micron, the temperature of the tubes is very gradually increased to 1000 degrees (540° C.). At this point they must be watched very closely as the melting point of this glass varies greatly and should the walls of the tubes become soft, the vacuum would cause collapse. From one micron, the vacuum slowly increases, and after about five hours of continuous pumping the tubes are sealed off at the manifold and allowed to cool in the oven.

McLeod gauges are used in the measurement of vacua but I have found that a much more accurate vacuum comparison can be made using a large induction coil. For this purpose an electrode is sealed to the manifold or at some point in the vacuum line. One terminal of the coil is connected to this electrode and the other coil terminal is connected to the low vacuum pump. A calibrated spark gap is used on the coil and when the vacuum is high enough and the residual gases are properly pumped from the tube a spark will jump the gap without a glow in the vacuum line or tubes. The vacuum used in the tubular detector will permit a five inch (12.5 cm.) spark between needle points in air.

Prof. Richardson has shown that when new metals are heated to incandescence they emit positive ions, probably because of the impurities or gases in the metal. I have found that this positive discharge must be eliminated to obtain maximum sensitiveness of the tubular detector, and this is accomplished during the manufacturing stage by burning the filament on alternating current for about two hours. Tubes that have not been treated in this manner are found to be less sensitive than those in which the positive ionization has been destroyed.

SUMMARY: Experiments with three-electrode vacuum detectors are described. Various filament, grid and plate metals were tested. Different degrees of exhaustion were used.

The paper then describes in detail the manufacture of a tubular detector and the testing thereof.

DISCUSSION

H. R. Sprado: Referring to your last paragraph regarding the emission of positive ions, I gather that this phenomena is rather transient. If a tube is left idle for quite a period of time, would it recover the positive ionization?

O. B. Moorhead: The phenomena referred to I have noticed to be an emission from fresh wires only, and when these wires are heated in a vacuum the positive ionization decays rapidly at first and then more slowly until it finally disappears. This rapid disappearance can be facilitated by applying a positive potential to the hot metals. I do not know if it will re-appear when left absolutely idle but it can be revived by burning a fresh wire near it, the old wire being cold. This must be due to a substance which is distilled from one metal to another.

H. R. Sprado: In some research work that I have recently done with the "gaseous medium" type of device, I have noticed that this power of emitting positive ions can be restored if the plate or filament end of the audion is held to one terminal of a high tension coil and a luminous discharge be caused to fill the bulb.

Do you believe that this rapid decaying of the positive ionic emission bears any relation to that phenomenon commonly called "photo-electric fatigue"?

O. B. Moorhead: It probably does, as photo-electric sensitiveness is not recovered after the surface has rested. Have you ever tried your luminous glow experiment on a plate that shows photo-electric fatigue and ascertained if it regains its sensitiveness?

H. R. Sprado: No, I have not. I note that you have experimented with platinum filaments in hydrogen. Did you make any experiments with tungsten in the same or other gases?

O. B. Moorhead: I have found that tungsten in hydrogen operates very poorly and that exceedingly small amounts of gas cause very great changes in the values of the constants. This applies to all the gases with which I have experimented.

H. R. Sprado: While making the above experiments I had occasion to use different pressures of argon in your tube. I found that the saturation currents in this gas have the same values as in the higher vacua. I noted that, when small quantities of argon were used the attainment of saturation was greatly facilitated because of the action of positive ions formed by impact ionization, in reducing the effect of the mutual repulsion of the electrons. When argon was present in greater quantities, the saturation current was considerably higher. Have you any theory regarding this increase in current?

O. B. Moorhead: I presume this was due to ionization by collision of the electrons with the argon gas molecules.

ON THE PHENOMENA IN RESONANCE TRANSFORMER CIRCUITS*

By

HIDETSUGU YAGI

(PROFESSOR IN THE COLLEGE OF ENGINERING, TOHOKU IMPERIAL UNIVERSITY, SENDAI, JAPAN)

About three years ago, while staying in Germany, I was engaged in the study of the phenomena in resonance transformer circuits. Prof. H. Barkhausen of the Technische Hochschule (Institute of Technology) in Dresden was so kind as to give me valuable suggestions and to permit me to experiment in the Institut für Schwachstrom Technik (Division of Feeble Current Engineering) of the above named college.

In June, 1914, I submitted to him a manuscript with dozens of oscillograms verifying the results of an analytical solution, which was nearly ready for publication. Owing to the sudden outbreak of war, the publication was unfortunately suspended, and much of the material rendered unavailable.

There had previously been an excellent paper by M. Blondel,[1] in which he pointed out the necessity for taking the spark discharge of the condenser into consideration. Many papers have since been published treating the same problem under the same conditions. Among recent publications are, as far as I know, those of Mr. Weinberger[2], Mr. Bouchardon[3] and Mr. Cutting[4]. The chief omission in these studies seems to me to be that they do not properly treat the superposition of the transients produced by the successive discharges.

Since it is not possible for me to secure the experimental data, I shall communicate here, as a further discussion of this problem, only the theoretical part of the investigation on the basis of my recollections. As the purpose is not to give any exact mathematical expressions but to make clear the complex phenomena by plain representations, and since cumbersome

* Received by the Editor, January 13, 1917.

[1] A. Blondel, "L'Eclairage Electrique," **18**, 1907, May 25, and June 8, or "Journal de Physique," **7**, (Serie 4), 1908, p. 89.

[2] J. Weinberger, "Proc. I. R. E." Dec., 1915.

[3] V. Bouchardon, "Lumière Electrique," **33**, 1916, April 29, and May 6.

[4] F. Cutting, "Proc. I. R. E.," April, 1916.

expressions seem rather likely to obscure mental images, the treatment is made as easy as possible and every permissible approximation is introduced. Nevertheless, I hope the deduction will prove interesting in throwing more light on the method of solving the problem.

The Transient Phenomenon

Let us neglect R_1 the effect of which can as well be studied by considering R_2. This obviously makes the solution much simpler.

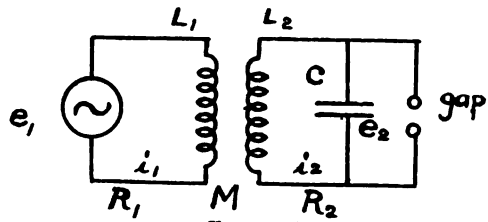


FIGURE 1

It is well known that the secondary current i_2 and the condenser potential e_2 can be expressed, for the non-sparking condition, by:

$$i_2 = I_2 \sin(\omega t + \phi_2) \tag{1}$$

$$e_2 = -\frac{I_2}{\omega C} \cos(\omega t + \phi_2) \tag{2}$$

Suppose that the spark gap is so adjusted that the condenser discharges at the potential E_o. An instantaneous discharge of the condenser across the gap is equivalent to the superposition of a transient equivalent to that which would occur if the condenser C, charged to an equal and opposite potential $-E_o$, discharged back from the secondary thru the transformer to the primary, assuming no source of E. M. F. in the primary. (Figure 2.)

This transient can be obtained neglecting R_1, from the equations:

$$\left.\begin{aligned} L_1 \frac{d i_1}{d t} + M \frac{d i_2}{d t} &= 0 \\ R_2 i_2 + L_2 \frac{d i_2}{d t} + M \frac{d i_1}{d t} + \int \frac{i_2}{C} d t &= 0 \end{aligned}\right\} \tag{3}$$

and the solution is, as well known,

$$i_2 = A\, \varepsilon^{-\alpha t} \sin(\beta t + \gamma) \tag{4}$$

and $$e_2 = \int \frac{i_2}{C} dt \qquad (5)$$

where $$\alpha = \frac{L_1 R_2}{2(L_1 L_2 - M^2)} \qquad (6)$$

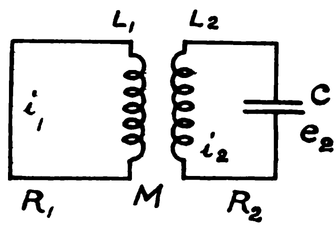


FIGURE 2

$$\beta = \sqrt{\frac{L_1}{C(L_1 L_2 - M^2)} - \frac{L_1^2 R_2^2}{4(L_1 L_2 - M^2)^2}} \qquad (7)$$

A and γ are integration constants.

When $t = 0$, $\quad i_2 = 0$

and $\quad e_2 = E_o$

therefore $$i_2 = -\frac{\alpha^2 + \beta^2}{\beta} C E_o \varepsilon^{-\alpha t} \sin \beta t \qquad (8)$$

and the condenser potential dies away according to the equation:

$$e_2 = \frac{\sqrt{\alpha^2 + \beta^2}}{\beta} E_o \varepsilon^{-\alpha t} \cos \left(\beta t - \tan^{-1} \frac{\alpha}{\beta}\right) \qquad (9)$$

Neglecting α in comparison with β, we have approximately

$$i_2 = -\beta C E_o \varepsilon^{-\alpha t} \sin \beta t \qquad (8')$$

and $$e_2 = E_o \varepsilon^{-\alpha t} \cos \beta t \qquad (9')$$

We will consider the two most important forms of regular sparking, which we will call:

(I) Alternate discharge—one spark per half cycle.

(II) Unidirectional discharge—one spark per cycle.

(I) Alternate Discharge

Under steady sparking condition, the condenser potential between two consecutive discharges will be made up of a sinusoidal sustained oscillation and an infinite number of transient oscillations, i. e.,

$$e_2 = E_c \sin(\omega t + \phi) - E_o \varepsilon^{-at} \cos \beta t + E_o \varepsilon^{-a\left(t+\frac{\pi}{\omega}\right)} \cos \beta\left(t + \frac{\pi}{\omega}\right)$$

$$- E_o \varepsilon^{-a\left(t+\frac{2\pi}{\omega}\right)} \cos \beta\left(t + \frac{2\pi}{\omega}\right) + E_o \varepsilon^{-a\left(t+\frac{3\pi}{\omega}\right)} \cos \beta\left(t + \frac{3\pi}{\omega}\right) - \cdots$$

$$= E_c \sin(\omega t + \phi) - \sum_{n=0}^{n=\infty} (-1)^n E_o \varepsilon^{-a\left(t+\frac{n\pi}{\omega}\right)} \cos \beta\left(t + \frac{n\pi}{\omega}\right) \quad (10)$$

The first term is what was already given in (2) as

$$-\frac{I_2}{\omega C} \cos(\omega t + \phi_2)$$

and the second term, the infinite series, is absolutely convergent and becomes

$$\sum_{n=0}^{n=\infty} (-1)^n E_o \varepsilon^{-a\left(t+\frac{n\pi}{\omega}\right)} \cos \beta\left(t + \frac{n\pi}{\omega}\right) = K E_o \varepsilon^{-at} \cos(\beta t - \theta) \quad (11)$$

where

$$K = \frac{1}{\sqrt{1 + 2\left(\varepsilon^{-a\frac{\pi}{\omega}}\right) \cos \beta \frac{\pi}{\omega} + \left(\varepsilon^{-a\frac{\pi}{\omega}}\right)^2}}$$

$$= P_o(x) - r P_1(x) + r^2 P_2(x) - \cdots \quad (12)$$

and

$$\tan \theta = \frac{\varepsilon^{-a\frac{\pi}{\omega}} \sin \beta \frac{\pi}{\omega}}{1 + \varepsilon^{-a\frac{\pi}{\omega}} \cos \beta \frac{\pi}{\omega}} \quad (13)$$

in which

$$x = \cos \beta \frac{\pi}{\omega}$$

$$r = \varepsilon^{-a\frac{\pi}{\omega}}$$

and $P_o(x), P_1(x), \cdots$ zonal harmonics.

Thus $$e_2 = E_c \sin(\omega t + \phi) - K E_o \varepsilon^{-at} \cos(\beta t - \theta) \quad (14)$$

(II) Unidirectional Discharge

Similarly, for the unidirectional discharge,

$$e_2 = E_c \sin(\omega t + \phi) - \sum_{n=0}^{n=\infty} E_o \varepsilon^{-a\left(t+\frac{2n\pi}{\omega}\right)} \cos \beta\left(t + \frac{2n\pi}{\omega}\right) \quad (15)$$

or $$e_2 = E_c \sin(\omega t + \phi) - K E_o \varepsilon^{-at} \cos(\beta t + \theta) \tag{16}$$
where
$$K = \frac{1}{\sqrt{1 - 2\left(\varepsilon^{-a\frac{2\pi}{\omega}}\right)\cos\beta\frac{2\pi}{\omega} + \left(\varepsilon^{-a\frac{2\pi}{\omega}}\right)^2}}$$
$$= P_o(x) + r P_1(x) + r^2 P_2(x) + \cdots \tag{17}$$

and $$\tan\theta = \frac{\varepsilon^{-a\frac{2\pi}{\omega}} \sin\beta\frac{2\pi}{\omega}}{1 - \varepsilon^{-a\frac{2\pi}{\omega}} \cos\beta\frac{2\pi}{\omega}} \tag{18}$$

in which
$$x = \cos\beta\frac{2\pi}{\omega}$$
$$r = \varepsilon^{-a\frac{2\pi}{\omega}}$$
and $P_o(x)$, $P_1(x)$, $\cdots$ zonal harmonics.

The Factor K

Thus in both alternate and unidirectional discharges, the variations of e_2 can be represented by the superposition of a sustained oscillation of the forced frequency and a damped occillation of the natural frequency of the circuit. K is an important factor which comes in whenever one treats of the superposition of periodic transient phenomena. For its calculation, the series of spherical harmonics (12) or (17) is convenient when $r \left(= \varepsilon^{-a\frac{\pi}{\omega}} \text{ or } \varepsilon^{-a\frac{2\pi}{\omega}}\right)$ is comparatively small. In our present case r is not much smaller than unity, as $\frac{\pi}{\omega}$ is extremely small, and the series form of K is not convenient. Figure 3 and Figure 4 show the values of K and $\frac{1}{K}$ when r is assumed equal to unity.

In Figure 4 the broken lines represent the curves of K and $\frac{1}{K}$ for $\varepsilon^{-a\frac{2\pi}{\omega}} = \frac{1}{\varepsilon}$, or $a\frac{2\pi}{\omega} = 1$; i.e., when the damping factor is extremely large.

Terminal Conditions

There are two conditions which we desire to have fulfilled from the practical point of view; namely: the rise of e_2 before a discharge should be steep, or $\left(\frac{d e_2}{d t}\right)_{t=\frac{\pi}{\omega}}$ be large in order that the discharge takes place sharply at a definite phase, and the rise of e_2 after a discharge should be slow, or $\left(\frac{d e_2}{d t}\right)_{t=0}$ be

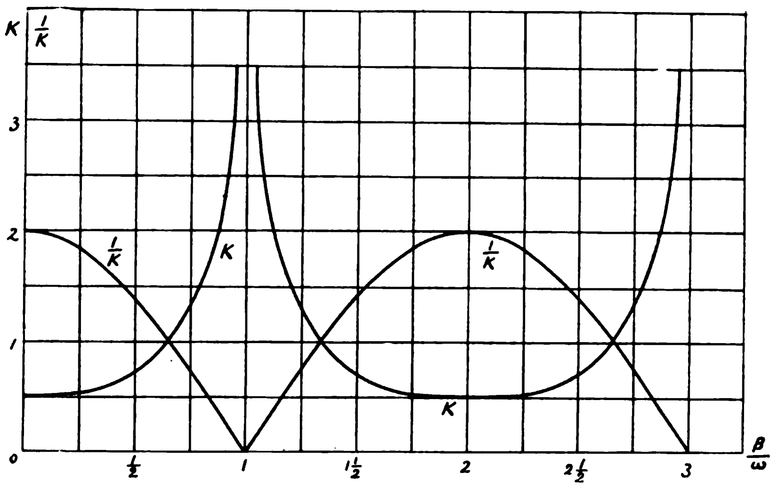


FIGURE 3—Alternate Discharge

small in order that the ionization of the gap is sufficiently reduced before e_2 begins to rise from zero, so that no extra spark may take place.

These two conditions are theoretically inconsistent, because, since what is effected in the circuit by a discharge is nothing but the superposition of a transient of (8′) or (9′), $\frac{d e_2}{d t}$ will be equal just before and after the discharge, i. e.,

$$\left(\frac{d e_2}{d t}\right)_{t=0} = -\left(\frac{d e_2}{d t}\right)_{t=\frac{\pi}{\omega}} \quad \text{for alternate discharge}$$

and

$$\left(\frac{d e_2}{d t}\right)_{t=0} = \left(\frac{d e_2}{d t}\right)_{t=\frac{2\pi}{\omega}} \quad \text{for unidirectional discharge.}$$

The equations (8′) and (9′) are approximate results and the phase angle $tan^{-1}\frac{\alpha}{\beta}$ was also neglected. It may seem that this term might cause some change of $\frac{d e_2}{d t}$ before and after the discharge; however, its effect can never be very large, and it is almost out of the question to attempt to make the two abovementioned conditions consistent simply by means of the proper selection of damping. On the other hand, these conditions can and must evidently be adjusted by the proper choice of the ratio $\frac{\beta}{\omega}$.

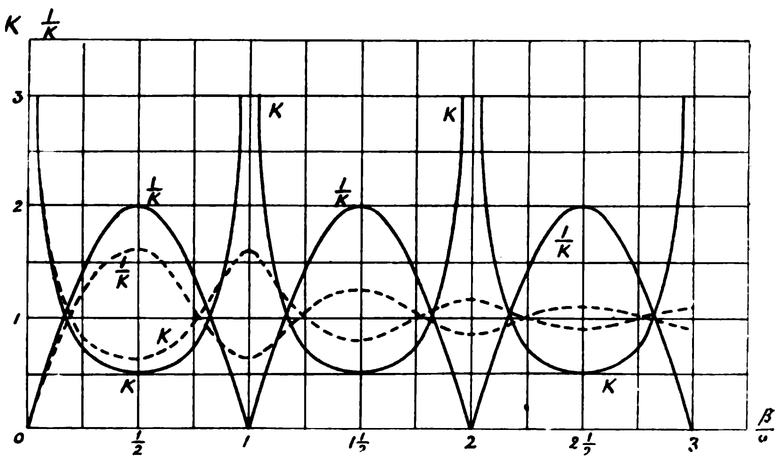


FIGURE 4—Unidirectional Discharge

Now at

$$\left.\begin{array}{ll} & t=0,\ e_2=0 \\ \text{and} & \dfrac{d\,e_2}{d\,t} \geqq 0 \end{array}\right\} \qquad (19)$$

Another terminal condition is:

$$\left.\begin{array}{ll} \text{at} & t=\dfrac{\pi}{\omega},\quad e_2=-E_o \\ \text{and} & \dfrac{d\,e_2}{d\,t} \leqq 0 \end{array}\right\} \text{for alternate discharge,}$$

$$\left.\begin{array}{ll} \text{and at} & t=\dfrac{2\pi}{\omega},\quad e_2=E_o \\ \text{and} & \dfrac{d\,e_2}{d\,t} \geqq 0 \end{array}\right\} \text{for unidirectional discharge.}$$

Possible Range of Steady Sparking

So far, we have assumed the discharge potential E_o to be arbitrary and independent of E_c; but, in order that the assumed steady sparking state may be possible, E_o ought to have a certain relation to E_c.

One of the limiting conditions is, as shown,

$$\left(\frac{d e_2}{d t}\right)_{t=0} \geqq 0.$$

The other limiting condition is that the absolute value of e_2 must never reach E_o before the assumed moment of discharge. In other words, the maximum values $e_{2,max}$ of Figure 8 must always be smaller than E_o, or,

$$|e_{2,max}| < E_o$$

(I) For alternate discharge, the first limit of $\frac{E_c}{E_o}$ is determined as follows:

From (14)

$$\left(\frac{d e_2}{d t}\right)_{t=0} = \omega E_c \cos\phi + \sqrt{\alpha^2+\beta^2}\, K E_o \sin\left(\tan^{-1}\frac{\alpha}{\beta} - \theta\right) \qquad (20)$$

hence the limiting condition $\left(\frac{de_2}{dt}\right)_{t=0} = 0$ becomes approximately

$$\omega E_c \cos\phi - \beta K E_o \sin\theta = 0. \qquad (21)$$

By (19) and (14)

$$(e_2)_{t=0} = E_c \sin\phi - K E_o \cos\theta = 0 \qquad (22)$$

Eliminating ϕ from (21) and (22)

$$\frac{E_c}{E_o} = K\sqrt{\cos^2\theta + \left(\frac{\beta}{\omega}\right)^2 \sin^2\theta}$$

Substituting the values of K, $\sin\theta$, and $\cos\theta$,

$$\frac{E_c}{E_o} = \frac{\sqrt{\left(1+\varepsilon^{-\alpha\frac{\pi}{\omega}}\cos\frac{\beta}{\omega}\pi\right)^2 + \left(\frac{\beta}{\omega}\right)^2\left(\varepsilon^{-\alpha\frac{\pi}{\omega}}\sin\frac{\beta}{\omega}\pi\right)^2}}{\left(1+\varepsilon^{-\alpha\frac{\pi}{\omega}}\cos\frac{\beta}{\omega}\pi\right)^2 + \left(\varepsilon^{-\alpha\frac{\pi}{\omega}}\sin\frac{\beta}{\omega}\pi\right)^2} \qquad (23)$$

To determine the other limit of $\frac{E_c}{E_o}$, put $\frac{d e_2}{d t} = 0$, or

$$\omega E_c \cos(\omega t+\phi) + \sqrt{\alpha^2+\beta^2}\, K E_o \varepsilon^{-\alpha t} \sin\left(\beta t - \theta + \tan^{-1}\frac{\alpha}{\beta}\right) = 0,$$

solve for t which gives the time t_o corresponding to $e_{2,max}$ and substituting t_o for t of the expression (14) and putting $|e_{2,max}| < E_o$ the required ratio $\frac{E_c}{E_o}$ can be obtained.

As this process of solution is difficult, several curves with various $\frac{E_c}{E_o}$ have been plotted for different $\frac{\beta}{\omega}$ and the critical values of $\frac{E_c}{E_o}$ determined, beyond which the assumed regular sparking becomes impossible.

(II) For unidirectional discharge, the first limit is similarly determined by

$$\frac{E_c}{E_o}=\frac{\sqrt{\left(1-\varepsilon^{-a\frac{2\pi}{\omega}}\cos\beta\frac{2\pi}{\omega}\right)^2+\left(\frac{\beta}{\omega}\right)^2\left(\varepsilon^{-a\frac{2\pi}{\omega}}\sin\beta\frac{2\pi}{\omega}\right)^2}}{\left(1-\varepsilon^{-a\frac{2\pi}{\omega}}\cos\beta\frac{2\pi}{\omega}\right)^2+\left(\varepsilon^{-a\frac{2\pi}{\omega}}\sin\beta\frac{2\pi}{\omega}\right)^2}\tag{24}$$

and the other can be deduced by the process similar to that for the alternate discharge.

Figure 5 gives the possible range of the regular alternate discharge and Figure 6 the same for the regular unidirectional discharge.

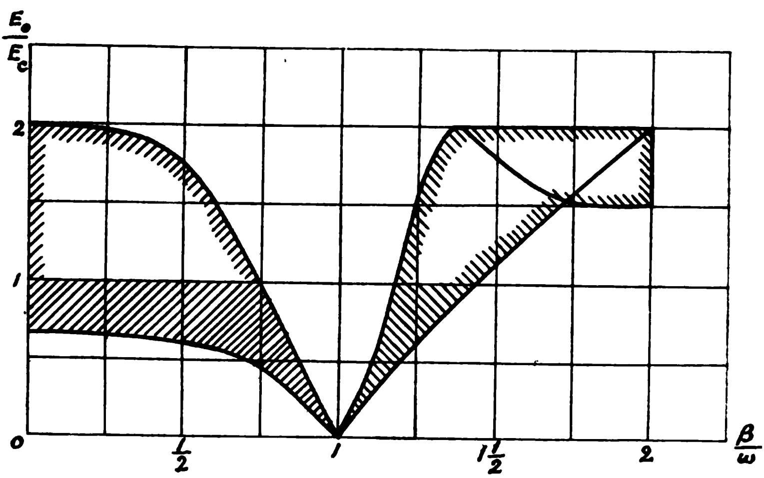


FIGURE 5—Alternate Discharge

In calculating these diagrams $\varepsilon^{-a\frac{\pi}{\omega}}$ and $\varepsilon^{-a\frac{2\pi}{\omega}}$ have been assumed equal to unity for the sake of simplicity.

Unless the value of $\frac{E_o}{E_c}$ corresponding to a certain value of $\frac{\beta}{\omega}$ lies within the shaded area of the figures, the presupposed sorts of regular sparking are impossible. It is especially noticeable that there is no possible range at the absolute resonance $\frac{\beta}{\omega} = 1$. How wide the possible range at resonance would actually be if $\varepsilon^{-\alpha\frac{\pi}{\omega}}$ or $\varepsilon^{-\alpha\frac{2\pi}{\omega}}$ were not equal to unity and $tan^{-1}\frac{\alpha}{\beta}$ of the equation (9) were taken into account, is another problem which seems worthy of a further study.

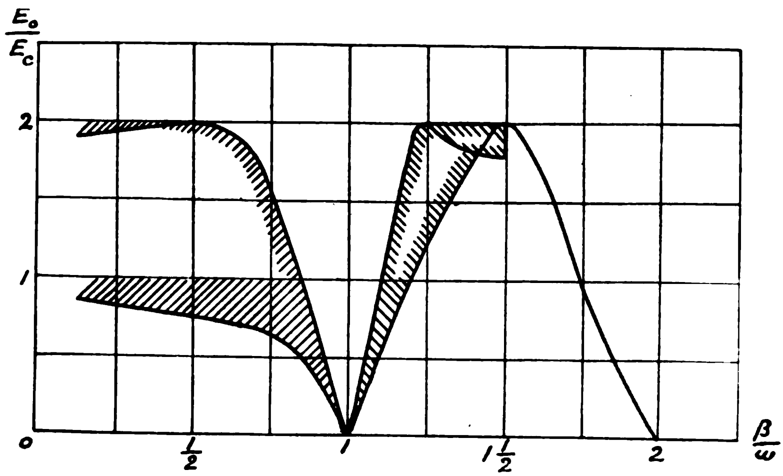


FIGURE 6—Unidirectional Discharge

Whatever be the theoretical conclusion as above deduced, there is no doubt that $\frac{\beta}{\omega}$ must not be too far from the resonance value, because otherwise the power factor of the supply circuit will become very small and the net output of the system will be much reduced. By this consideration, there should be a certain optimum condition off resonance, but not very far removed therefrom.

Unless the initial spark starts spontaneously or is effected by some special means, the condenser will never discharge when E_o is adjusted above E_c; that is to say, for $\frac{E_o}{E_c}$ larger than unity, certain devices must be provided for starting the initial spark. Or else the possible range becomes restricted to that portion of the shaded area in Figure 5 and Figure 6 that lies below the line $\frac{E_o}{E_c} = 1$. Then it may be said that there is a little wider range of possible operation for $\frac{\beta}{\omega} < 1$ than for $\frac{\beta}{\omega} > 1$. Moreover when β is larger than ω there is a greater tendency torward partial discharges. Experiment also shows that the operation is usually much steadier when the natural frequency of the circuit β is smaller than the forced frequency ω.

E_c is not a constant but varies as indicated by an ordinary resonance curve, so that the discharge potential E_o must be within the shaded area of Figure 7.

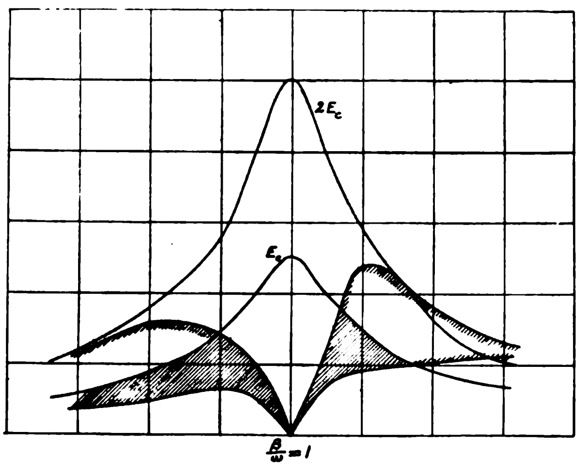


FIGURE 7

CURRENT CURVES

So, as long as the number of discharge is not more than one per half cycle, the discharge occurs after the maximum crest of the current curve. The charge corresponding to the area $B-A$ or $G-F+D$ raises the condenser potential to E_o. As e_2 must never reach E_o before the moment of discharge,

$$B > 2A$$

$$F > 2D$$

and

$$G > 2(F-D)$$

which can be easily checked by the actual oscillograms.

Mr. Weinberger[1] and also Mr. Bouchardon[2] have assumed that the discharge takes place at the maximum of the potential wave, or at the moment of zero charging current. This is

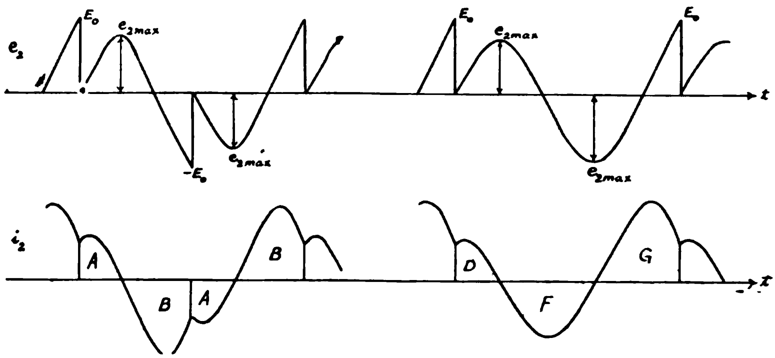
[1] *Loc. cit.*
[2] *Loc. cit.*

actually not the case and it seems rather impossible to have a discharge exactly at the maximum of the voltage wave.

In the case of partial discharges, Figure 9, (two or more discharges per half cycle), we have the relation

$$P-S=Q=R$$

which determines the time intervals between consecutive discharges.



Alternate Discharge Unidirectional Discharge

FIGURE 8

It is obvious that there is a direct current component of i_2 in the case of unidirectional discharges. This D. C. component is likely to magnetize the iron core asymmetrically. If, consequently, the potential wave becomes asymmetrical, then the tendency for unidirectional discharge will possibly be augmented. It was noticed in experiments that the unidirectional discharge was the one that could persist with the utmost steadiness.

CONCLUDING REMARKS

The above calculations have been carried out only for two particular states of regular sparking and it must be remembered that there are an indefinite number of steady sparking states, ranging between rare spark operation with each discharge at an interval of several cycles and partial discharges of many sparks per half cycle, and the two cases treated in this paper are only the particular cases best suited for tone production.

There are other states of sparking which very much resemble those treated in this paper. One of them is that in which the sparking occurs nearly once per cycle or once per half cycle, but the period of sparking is very slightly smaller than $\frac{\pi}{\omega}$ or $\frac{2\pi}{\omega}$, and there is one extra spark in several cycles. In the other case the period is very slightly larger than $\frac{\pi}{\omega}$ or $\frac{2\pi}{\omega}$ and the discharge will miss once per several cycles. The matter depends upon whether the energy supply is slightly more or less than the loss of energy by the regular discharge of the condenser at E_o.

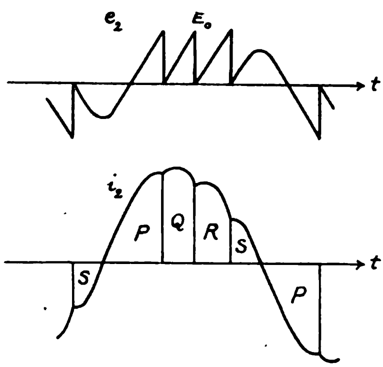


FIGURE 9—Fractional Discharge

It is not easy to distinguish these states of discharges from truly regular sparking, especially when the discrepancy is inconsiderable, altho Mr. Cutting says in answer to Prof. Morecroft's discussion that he could hear the missing of one spark in a regular sparking state.

The author once observed the change of the oscillographic images of regular sparking at 50 cycles per second, when the discharge gap was kept constant (E_o) and the resonant condition of the circuit was varied gradually from one side to the other of resonance by varying the series inductance in the primary.

It was hardly possible to observe any change in the regular sparking in passing thru absolute resonance. The conclusion that there is no range for regular sparking at resonance is based on the assumption of the ideal regular sparking and of $\varepsilon^{-a\frac{\pi}{\omega}} = 1$. Therefore it seems to be premature to say that absolute resonance is impracticable or that it is a point of minimum output. Mr. Cutting's calculation is based also on the assumption of the ideally regular sparking state and it is a different question whether the watt output would actually show a minimum when the condition is altered thru resonance with "practically" regular sparking.

It is another question whether the apparently regular sparking, tho not truly regular, produces less musical sounds in the receiving telephone than the ideally regular sparking on both sides of resonance.

The above gives but one theoretical reason, from the point of view of tone production, against operation at absolute resonance, but it must not be looked upon as conclusive.

SUMMARY: After reviewing some of the previous work in connection with the superposition of the recurrent transients of regular spark discharge of a condenser, the author develops the solutions for the cases of one and two sparks per cycle. The solutions are then studied in detail with particular reference to the possibility of operating such transformer systems at the absolute resonance point, a possibility hitherto denied. The conditions under which steady sparking may occur are also considered.

FURTHER DISCUSSION ON "THE COUPLED CIRCUIT BY THE METHOD OF GENERALIZED ANGULAR VELOCITIES" BY V. BUSH

By
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Professor Bush's interesting paper appearing in the October issue of the "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS" is accompanied by an appendix giving a "Summary of Wagner's Proof of Heaviside's Formula." This proof contains several fallacies which, curiously enough, are mutually destructive, so that the final formula is correct. The fallacies are, however, serious from a mathematical standpoint, and I therefore take this opportunity to point them out. It is a curious coincidence that Malcolm's proof, referred to in a footnote on page 377, is likewise vitiated by balancing fallacies of much the same character. I might add that, inasmuch as I have not consulted Wagner's original paper, my criticisms are directed against the proof, as given by Professor Bush, only.

Professor Bush states that the infinite integral

$$f(t)=\frac{E}{2\pi j}\int_{-j\infty}^{j\infty}\frac{\varepsilon^{nt}}{n}\,dn \qquad (1)$$

defines a function which is zero for negative values of t and E for positive values of t. That this is incorrect is easily shown as follows: Change t to $-t$ in (1), and we get

$$\begin{aligned}f(t)&=\frac{E}{2\pi j}\int_{-j\infty}^{j\infty}\frac{\varepsilon^{nt}}{n}\,dn\\&=\frac{E}{2\pi j}\int_{+j\infty}^{-j\infty}\frac{\varepsilon^{nt}}{n}\,dn\end{aligned}$$

and finally

$$\begin{aligned}f(-t)&=-\frac{E}{2\pi j}\int_{-j\infty}^{j\infty}\frac{\varepsilon^{nt}}{n}\,dn\\&=-f(t)\end{aligned}$$

This shows that the function defined by (1) has the same absolute value for positive and negative values of t but suffers a reversal of sign at $t=0$.

As a matter of fact the function defined by (1) is equal to $-\frac{E}{2}$ for $t<0$ and $+\frac{E}{2}$ for $t>0$. This may be readily shown in a number of ways; perhaps the easiest is to deduce it from the known value of the function

$$\frac{2}{\pi}\int_0^\infty \frac{sin\,(n\,t)}{n}\,d\,n$$

which is equal to -1 for $t<0$ and $+1$ for $t>0$.

Professor Bush's discussion of the contour integral by which he arrives at the conclusion that the function defined by (1) is 0 for $t<0$ and E for $t>0$, is defective in that it ignores the fact that the pole (0) cuts the path of integration. When this happens no general rule can be laid down as regards the evaluation of the residue since the pole is symmetrical with respect to the two contours of integration. It may be shown, however, that for the function under consideration, ½ the residue is to be included in each contour so that $f\,(t)=\frac{1}{2}E$ for $t>0$

$$f\,(t)=-\tfrac{1}{2}E \text{ for } t<0.$$

This same failure to evaluate properly the residue corresponding to the pole (0) accounts for the final formula (11) which is correct when the applied voltage is 0 for $t<0$ and E for $t>0$, but is incorrect when $f(t)$ is defined by (1). The correct formula in this case is, corresponding to (11),

$$i=\frac{1}{2}\frac{E}{z\,(0)}+\sum_{n_r}\frac{E}{\left(\frac{d\,z}{d\,n}\right)_{n_r}}\,\varepsilon^{n_r t},\ t>0$$

$$i=-\frac{1}{2}\frac{E}{z\,(0)}\ \text{ for } t<0.$$

Clearly this is the correct solution when we remember that

$$f\,(t)=\tfrac{1}{2}\,E,\ t>0$$

$$f\,(t)=-\tfrac{1}{2}\,E,\ t<0$$

The errors into which this proof falls seems to be due in part to the ambiguity which arises when the path of integration cuts one or more poles of the function. When this happens the evaluation of the residues is almost always a matter of doubt and should be justified by other methods and other considerations if possible. I would suggest in this connection that there is less chance of error if we start with the *current* expressed as a Fourier's integral; thus, corresponding to an impressed force $f(t)$, the resultant current, is

$$i = \int_{-\infty}^{\infty} f(\lambda) \,.\, d\lambda \int_{-j\infty}^{j\infty} \frac{\varepsilon^{n(t-\lambda)}}{z(n)} \, dn$$

I might mention here that in the September issue of the "Physical Review," I developed and proved from dynamical considerations a general expansion theorem which holds explicity when the impressed force is an exponential function of time, and implicity for functions of arbitrary form. It is there shown that if the impressed force is $E\,\varepsilon^{pt}$, the resultant current (adopting Professor Bush's notation) is given by:—

$$i = \frac{E\,\varepsilon^{pt}}{z(p)} - E \sum_r \frac{\varepsilon^{n_r t}}{(p-n_r)\left(\frac{dz}{dn}\right)_{n_r}}\,.$$

This expression degenerates into the Heaviside formula when p is put equal to zero. It enables us also to evaluate directly the transients when the impressed forces are damped or undamped sinusoidal time functions.

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All references to any individual, company, or radio station are fully listed in the index. Answers to points brought out in the Discussions will be listed under the name of the questioner. With the exception of the names of companies, all topics will be listed in general under the *noun* referred to. The numbers correspond to pages in the text. The following abbreviations are used: r. f.—radio frequency; a. f.—audio frequency; l. v.—low voltage; h. v.—high voltage.