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EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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PROCEEDINGS OF THE SECTIONS

WASHINGTON SECTION

On the evening of Wednesday, June 21, 1916, a meeting of the Washington Section of The Institute of Radio Engineers was held at the University Club, Washington, D. C. A dinner was tendered Captain William H. G. Bullard, Superintendent of the Naval Radio Service, and Member of the Board of Direction of the Institute, on the occasion of his departure for duty at sea.

BOSTON SECTION

A meeting of the Boston Section of the Institute was held on the evening of Thursday, May 25, 1916, at the Cruft High Tension Laboratory, Harvard University, Cambridge, Massachusetts. Two papers were presented. These were: "On the Energy Losses in Radio Telegraph Transmitters" by Messrs. Bowden Washington and P. H. Royster, and "Notes on Tone Circuits" by Mr. Fulton Cutting. The papers were followed by discussion. Professor G. W. Pierce of Harvard University presided, and the attendance was fifty-nine.

SEATTLE SECTION

On the evening of April 8, 1916, a meeting of the Seattle Section of the Institute was held at Denny Hall, University of Washington. A paper on "Safety Thru Radio" was presented by Mr. V. Ford Greaves, Radio Engineer, United States Department of Commerce. A general discussion followed. Mr. Robert H. Marriott, Expert Radio Aide of the United States Navy, presided; and the attendance was thirty-five. Mr. T. M. Libby was elected Secretary-Treasurer of the Section, and a future place of meeting chosen.

A meeting of the Seattle Section of the Institute took place on the evening of May 6, 1916, at the Chamber of Commerce, Central Building, Seattle. A paper on "The Mechanism of Radiation and Propagation in Radio Communication" by Mr. Fritz Lowenstein was presented. A discussion followed. Mr. Robert H. Marriott presided, and the attendance was fifteen.

On the evening of June 10, 1916, a meeting of the Seattle Section was held in the Chamber of Commerce, Seattle. A paper on "Sustained Wave Range Chart" was presented by Mr. Tyng M. Libby of the Bremerton Navy Yard. This was followed by a discussion. Mr. Robert H. Marriott presided, and the attendance was twenty.

The Institute of Radio Engineers announces
with regret the death of

Mr. Frank B. McSoley

(Engineer of the Narragansett Electric Lighting Company, of Providence, R. I., and Associate-Member of the Institute since June 25, 1913).

ON A DETERMINATION OF THE ENERGY LOSSES IN A RADIO TELEGRAPH TRANSMITTER*

By

Bowden Washington

(Radio Engineer, Cutting & Washington)

and

P. H. Royster

(Research Student, Harvard University)

The research herein described was originally taken up with the idea of determining the efficiency of the spark gap of a radio telegraph transmitter. Calorimetry immediately suggests itself as the simplest, if not the only available, method. This method was found so exceedingly satisfactory and convenient at the desired accuracy that it was decided to extend the research to cover the entire transmitter circuit. The radio apparatus used comprised a later modification of the Chaffee system, consisting of a symmetrical copper gap in alcohol vapor, operating on 250 volts, 500 cycle A. C. The action of the gap may be considered as identical with that of the Chaffee gap when used with a tone circuit, the feed current being in that case unidirectional pulsating current; i. e., A. C. with a displaced zero axis. Pure impact excitation still exists with the later type of gap, the condenser discharging thru the gap in a large number of discrete loops or half cycles during each cycle of the A. C. feed.

This has been proven in several ways:

(1) By examining the gap current and antenna current with a special high-speed Duddell oscillograph, using circuits of high audio frequencies;

(2) By the peculiar period relations existing between the closed and radiating circuits;

(3) When the circuit is adjusted for maximum radiation, a careful search with an exceptionally good wave meter discloses but one wave, whose period is the natural period of the radiating circuit, and whose decrement is apparently the decrement of that circuit alone.

---

* Presented before the Boston Section of The Institute of Radio Engineers, May 25, 1916.

This operation was performed with great care; and determination having been made of where the "coupling waves" should occur if the gap behaved as a "quenched gap," particular care was taken to search at these points.

Under the best conditions, when the closed circuit consists of an exceedingly large condenser and a very small inductance, and the least obtainable uncoupled or free inductance, maximum radiation is obtained when the natural period of the primary is from 0.5 to 0.8 that of the secondary. Adjustment between circuits is far from critical, as shown by the following table taken on a ¼ kilowatt set with a 10.7 ohm phantom antenna. The secondary current and secondary wave length are given. Nothing whatever was changed but the capacity of the secondary condenser. The natural period of the primary was 440 meters.

λ_s	I_s	λ_s	I_s
460	3.8	815	4.0
575	4.0	1000	3.9
640	4.0	...	...

It will be noticed that the amplitude of the secondary current decreases as "resonance" is approached.

The most widely used forms of calorimeter consist essentially of a liquid bath carefully heat-insulated and properly stirred; and some device to measure the rise in temperature from which is determined the quantity of heat liberated. This determination can be made in two ways: viz., by knowing the heat capacity of the calorimeter and its contents, or by comparison with a known quantity of electrical energy introduced by means of a suitable heating coil. Both these methods, tho accurate, have not appealed to the engineer, on account of the laboriousness and attention to detail required. The most serious obstacles in the way of applying this method of calorimetry to commercial apparatus are the facts that it is impracticable to submerge certain parts of the apparatus in an oil bath, and that a calorimeter of sufficient size to contain some of the larger parts of the apparatus, in which the loss is small, would show an almost inappreciable temperature rise occasioned by this small loss. We were faced, therefore, by the engineering problem of devising some form of calorimeter which, tho giving the desired accuracy, would be quick, convenient in operation, and of sufficient size to contain the largest single unit in the apparatus, and at the same time be appropriate for the measurement of heat losses of a widely divergent magnitude. The method which we have evolved has proved so satisfactory that we feel that a short note on this method may be of use, if not of interest, to engineers engaged in a similar line of work.

If the interior of a closed vessel is at a uniform temperature, T_1, the temperature of the surrounding medium being T_o, then h, the heat lost per second by conduction, radiation, convection and the leakage of air, is some function $F\ (T_1,\ T_o)$, which for small values of T_o may be written as an explicit function of the temperature difference, $\phi\ (T_1 - T_o)$. The value of h, expressed in watts, may be ascertained for a given vessel by liberating suitable variable quantities of electrical energy, and observing the temperature when thermal equilibrium is attained. This thermal equilibrium will obtain when the heat generated is equal to the heat lost to the surrounding medium. This temperature will be a measure of heat evolved in the apparatus, and can be read off immediately from the previously computed calibration curve.

This method does not require the heat capacity of the calorimeter and its contents to be known, nor does it require knowledge of how the heat is dissipated into the surrounding medium; and the result is not affected by the leakage of air into or out of the calorimeter, or by changing the contents during the experiment. If the container is made of a substance of good thermal conductivity, the changes in room temperature will affect the result. But if the walls of the vessel are made of a material of low heat conductivity and high specific heat, the effect of any change in room temperature will take quite a long while to affect the interior; so where results of greater precision are required, calibration points should be taken within an hour or so of the experiment; but for ordinary engineering purposes one calibration curve will be adequate. There is an objection to using a calorimeter with a low time period on account of the length of time before equilibrium to be attained. It is found, however, that the temperature within the calorimeter when a constant amount of heat is being generated rises exponentially with the time; i. e., $T = a\ (1 - \varepsilon^{bt})$. The value of the constants a and b will vary with the contents of the calorimeter and its previous thermal state. If, however, observations are made of the temperature at successive intervals of time, and the temperature computed against the anti-logarithm of time, its intersection with the axis of $log^{-1}(-b\,t)=0$ will give the final temperature.

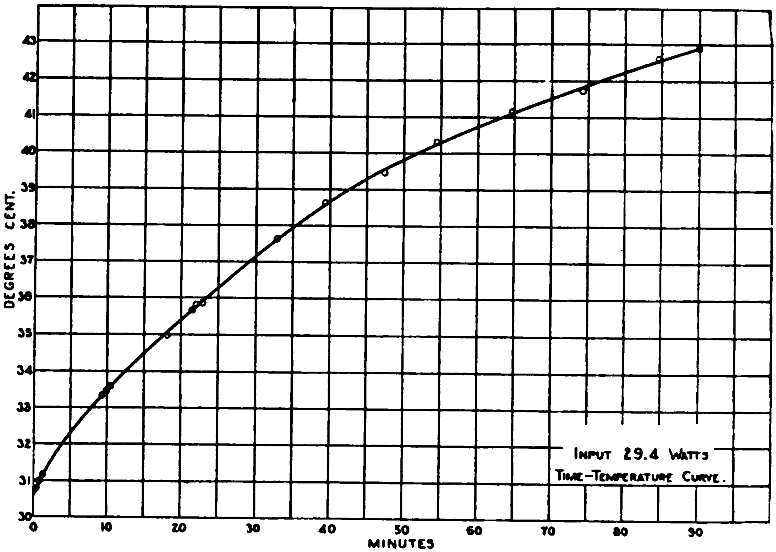

FIGURE 1

Figure 2 shows the points computed in a run which was carried out until the final temperature was maintained, in this case after 145 minutes; but it will be seen that observations on the first 10 minutes would have given this final temperature within 4 per cent. Making use of this device greatly shortens the time without any marked loss of accuracy; also where a calorimeter of too high sensitivity is used, the final temperature might be so great as to prove injurious to the apparatus. A certain amount of judgment is necessary in using this method of extrapolation if the calorimeter has been recently used for another experiment, as the presence of heat waves and loops sometimes leads one astray. Since, however, the cost of a calorimeter is negligible, it is no great matter always to have a cold calorimeter at hand.

It will be seen that the use of air as a calorimetric fluid obviates the necessity of stirring. If the apparatus is not too near the radiating walls, the temperature within will be sensibly constant. We have found it possible with a 30 degree rise of temperature to maintain the final temperature constant within

0.01 of a degree during a period of fifteen or twenty minutes. This temperature being constant to one part in 3,000 requires the heat evolved to remain constant to the same order of magnitude. The result in accuracy is not only greatly beyond engineering needs, but extremely difficult to attain with the highest form of precision instruments. It is impossible to conceive of the temperature remaining constant thru any such period of time, were any temperature gradients to exist in the large and unobstructed air spaces.

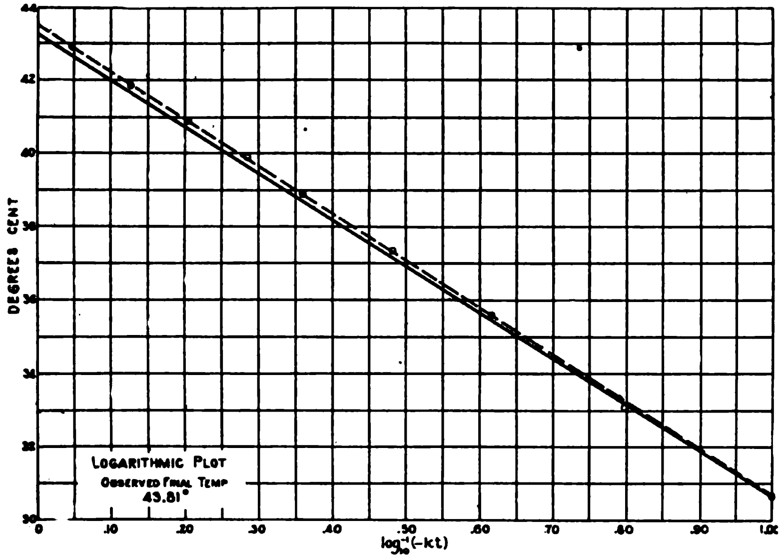

FIGURE 2

For the purpose of measuring energies up to 100 watts, we have found a satisfactory container in a corrugated double-walled cardboard packing case or carton, resting upon the top of an inverted and similar packing case, the junction between the two being rendered reasonably air-tight by strips of felt permanently attached to the lower box. The apparatus to be tested is placed on the lower case, and the upper case placed over it. A thermometer is introduced thru a hole in the top. It will be seen that since the calibration and the measurements are taken with the same thermometer, it is not necessary for the thermometer to be accurate. We have used a Fritz Köhler thermometer, graduated in tenths, which can be read with great ease to 0.01 of a degree. This thermometer happened to have been carefully calibrated against the Bureau of Standards platinum thermometer, but there seems to be no reason why the worst thermometer would not serve equally well. The calibration curve of this calorimeter is shown in Figure 3.

This curve approaches the equation

$$h = 0.60\,(T-25) + 0.124\,(T-25)^2$$

hence if T_2 is the final temperature in an experiment, and T_1 is the final temperature in a calibration test, then

$$h_2 = h_1 + D_T\,h\,(T_2 - T_1).$$

If $(T_2 - T_1)$ is small, this obviates the necessity of adjusting the calibration energy by successive approximations for absolute coincidence of temperature. Where the apparatus under test operates at a perceptibly higher temperature than the calorimeter, such as the gap shown in Figure 4, two sources of error may prove troublesome: loss of heat directly thru the walls of the calorimeter, and radiation to the bulb of the thermometer. In the arrangement shown in Figure 4, the gap rests on a piece of heavy felt, thus greatly minimizing the heat transmitted to the lower wall. Further, the heat transferred thru this lower

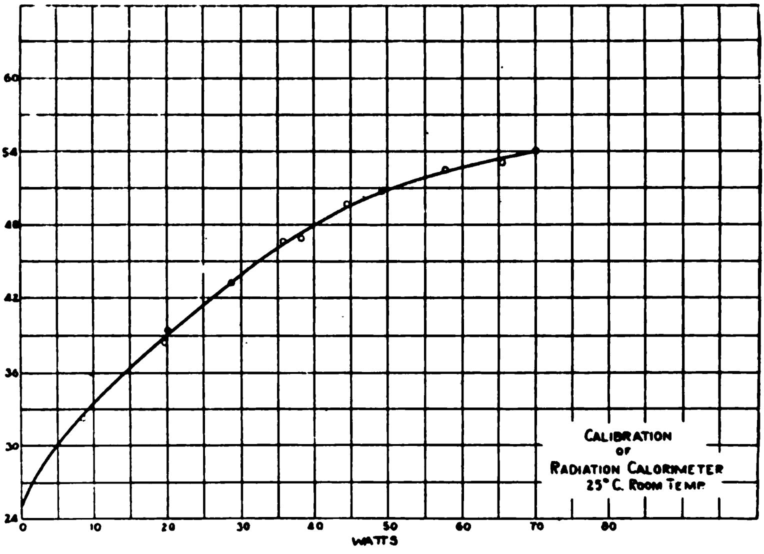

FIGURE 3

wall is a very small percentage of the total heat lost, on account of the insulating effect of the air space in the lower carton. Radiation to the bulb of the thermometer is prevented by the shield shown in the figure, a piece of cardboard secured to a wooden base.

The transmitter circuit under test consisted of a 500-cycle transformer charging a mica dielectric primary condenser of 0.07 μ f. capacity; and a symmetrical copper gap, coupling coil and a phantom antenna composed of a standard copper-coated jar of 0.002 μ f. and a 10-ohm resistance. This very excellent instrument, made for us by Mr. Melville Eastham, consisted of a 0.125 inch by 0.005 inch (0.32 cm. x 0.013 cm.) manganin strip wound on a flat asbestos holder. This was compared by Mr. Eastham with a number 38 manganin wire* at 3,000,000 cycles, and no discrepancy of resistance was noted.

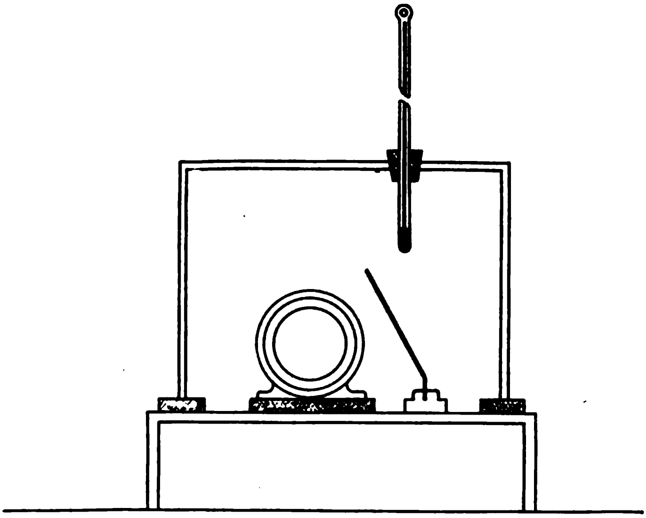

FIGURE 4—CALORIMETER

Table 1 gives the resultant loss when the set was operated at its best adjustment.

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* Diameter of number 38 wire = 0.004 inch = 0.0101 cm.

TABLE I

	Final Temperature	Watts Loss
Transformer	42.85	27.35
Primary Condenser	37.82	17.1
Gap	52.21	55.8
Coupling Coil	31.73	7.1
Secondary Condenser	33.25	8.3

The input in this case was 225 volt-amperes. The total energy in this set, as we can show later, was 207.2 watts, indicating a power factor of 0.92, and from the above table a transformer efficiency of 86.9 per cent. The gap loss given in the above table has been shown by repeated trials to be that yielded when the gap remains relatively cold. Under such conditions the closed circuit current is 14.7 amperes. However, during a test of some hours' continuous operation, due either to the increase of temperature of the primary condenser or of the gap, or both, the radiated energy decreases and the gap and primary condenser losses increase, resulting in a lowering of set efficiency.

Table 2 gives the result of an attempt to follow out to some extent the mutual dependence of these phenomena.

TABLE II

Test	Final Temperature	Watts Loss	Closed Circuit Current	Apparent Gap Resistance	Antenna Watts
1	52.21	55.78	14.7	0.258	100.2
2	53.45	62.55	15.2	0.211	93.1
3	54.15	68.18	15.9	0.260	87.6
4	54.75	74.05	16.5	0.272	82.0

We have carried out in the above table the gap loss to the second place of decimals; for altho the probable error in absolute magnitude of the energy measurements is greater than 0.01 of a watt, yet for changes of energy the figures carried are significant. The increase of gap loss appears to account for the loss of radiated energy between tests 1 and 2. The radiation decreases 7.1 watts; the gap loss increases 6.73 watts. For tests 2 and 3 these figures are 5.5 watts for the radiation, and 5.68 watts for the gap; while for 3 and 4 the energy changes are 5.6 against 5.83.

The value of the gap resistance given in the fifth column of the above table seems to indicate that the average gap resistance is somewhat independent of the current, certainly for such small variations, and within the limit of experimental error of the ammeter measuring the closed circuit current. The primary condenser during these tests we attempted to keep as cool as possible with a fan. Changing the impressed frequency on this condenser from 60 to 500 cycles, and varying the closed circuit current from the range as given in Table II, caused a maximum variation of condenser loss of 2 watts. We have as yet found no satisfactory explanation of this change of energy distribution from radiation to gap loss, and the accompanying falling off in set efficiency.

Since the value of the radiated energy was a matter of extreme importance, our measurements upon this quantity were made with greater care than in any other single test. The manganin-strip phantom antenna placed in the calorimeter was connected to a double-throw switch so arranged that the antenna current and a suitably chosen direct current could be applied alternately, the switch being thrown over very rapidly, and the direct current being adjusted until thermal conditions within the calorimeter were unaffected by the change.

Three interpretations of our results are possible. Since $W = I^2 R$, we may assume; first, that the phantom resistance R had the same resistance at 300,000 cycles that it showed on direct current, giving the true value of the current $I = \frac{\sqrt{W}}{R}$; second, we may select any one of the eight hot band ammeters which were placed in series as indicating the correct radio frequency current I, giving a measure then of the high-frequency resistance of the phantom $R = \frac{W}{I^2}$; finally, we may assume that both the observed R and I are correct, and that "e" is the measure of our calorimetric error where $e = W - I^2 R$. We think we may safely accept the value of the antenna resistance as the closest approach to the truth. The readings of the eight ammeters, as shown in the following table, being so discordant, it is impossible to select any one as a standard.

TABLE III

Ammeter	$W=I^2R$		
	Test I	Test II	Difference I–II
1	92.3	92.3	0.0
2	95.7	90.4	−5.3
3	89.2	92.0	+2.8
4	91.6	92.9	+1.3
5	91.0	91.0	0.0
6	89.2	88.2	−1.0
7	94.5	92.5	−2.0
8	95.7	96.6	+0.9
Mean	92.4	92.0	

The heat generated in the calorimeter was in Test I 91.6 watts, in Test II 92.2 watts, indicating an increase of 0.6 in the second test, whereas the mean of the ammeter readings shows a decrease of 0.4—an outstanding discrepancy of 1.0 watts between the two methods. But the variation noted between the individual meters is so much greater that the actual calorimetric error unfortunately cannot be determined.

Accepting then our antenna resistance as 10 ohms, and the heat generated corresponding to the ammeter reading in test 1 of Table II as 91.9, we arrived at the value of the radio frequency current, of 3.03 amperes. The energy loss of the secondary condenser being 8.3 watts, the resistance is 0.90 ohms. Our total antenna resistance is then 10.90 ohms, and the antenna energy 100.1 watts. This gives a set efficiency of 48.4 per cent.

The conclusions drawn from these experiments led us to construct a small set of about the same capacity but of the panel type, care being taken to shorten the closed circuit leads as much as possible, and to provide sufficient radio frequency conductance. The primary inductance in this case consisted of two turns of a cable composed of seven strands of 5-ampere "Litzendraht," each strand containing twenty-four number 30 wires.* The condenser was mounted directly on the back of the gap, the coupling coil immediately above; the primary wiring then consisting of practically nothing but these two coupled turns. It was found that the lead foil used in the primary condenser

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* Diameter of number 30 wire = 0.010 inch = 0.025 cm.

was carrying approximately 0.25 ampere per sheet, and had not sufficient conductance. Heavier copper foil was substituted. The secondary of the coupling coil was wound with a single strand of 5-ampere "Litzendraht."

The generator was wound to a suitable voltage, namely, 500 volts no load, 250 volts full load, thus doing away with the transformer losses. The results obtained with this set thoroly justified the small amount of labor expended in calorimetry.

With the same 10.7 ohm phantom antenna, an antenna current of 4.5 amperes was obtained (216 watts) with an input of 308 watts—an efficiency of 70 per cent.

SUMMARY: A simple method of calorimetry is devised to measure the total losses of a modified Chaffee gap 0.25 K. W. transmitter. The equilibrium temperature of the air in a slightly heat-conductive box containing the transmitter under test enables the direct determination of the heat evolved. A supplementary calibration with known heat evolution renders the method simple and free from certain previously common calorimetric complications. The thermometer used must be shielded from direct radiation from the transmitter.

Guided by results thus obtained, the authors increased the over-all efficiency of the set under test from 48 to 70 per cent.

THE HEAVISIDE LAYER*

BY

E. W. MARCHANT, D. SC., M. I. E. E.

(DAVID JARDINE PROFESSOR OF ELECTRICAL ENGINEERING, UNIVERSITY OF LIVERPOOL)

The formulas which have been obtained for the strength of signals received at a station when a measured amount of high frequency power is sent into a distant transmitting antenna (as the result of investigations and measurements by Austin and others) are well known, and have been dealt with fully in Mr. Fuller's paper before the American Institute of Electrical Engineers.[1]

It is within the knowledge of all radio operators that signals vary widely in strength, often in the course of a few minutes; and such variations can most easily be explained by reflection and refraction from moving masses of "cloud" or ionic fog. The surface wave theory developed by Sommerfeld, while explaining transmission over long distances, round the curvature of the earth, does not explain these sudden changes. The fact that these changes occur more by night than by day provides further evidence that the reflection and refraction theory of which Dr. Eccles has been, in this country, the chief exponent, is the most likely one to explain observed phenomena.

The experiments described by Balsillie[2] in which he found that dust storms occurring along the line of transmission affect signal strength, when the transmission is in the direction in which the wind is blowing, are of interest, as they indicate that the *atmosphere* immediately adjacent to the earth is a factor in the absorption of waves. The chief phenomena, however, which require further explanation are (*a*) the sudden variations in signal strength at night, and (*b*) that comparatively small changes in wave length may make relatively enormous changes in the strength of received signals. The experiments recently described by Mr. Fuller[3] have added much exact information

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*Received by the Editor, April 18, 1916.

[1] "Proceedings of the A. I. E. E." Volume 34, p. 567.

[2] "Proceedings of British Association (Australia)," p. 514. See also the "Electrician."

[3] "Proc. A. I. E. E.," *loc. cit.*

to that already available for the discussion of this subject, and it may be useful, therefore, to consider them in their bearing on the existence and probable nature of what, in this country, is generally called the "Heaviside layer."

Tho it is usually called by this name, Professor Fleming[4] observed recently that it was Sir James Dewar, who was one of the first to draw attention to its existence. In a lecture to the Royal Institution in 1902[5], when discussing the constitution of the atmosphere Dewar pointed out that there were really two parts to it: the lower part in which atmospheric currents circulated, and in which the constituents were similar to those of the atmosphere at lower levels, and the upper part in which the distribution of gases was governed by their density. In two lectures delivered recently to the Royal Institution[6], Professor Fleming has discussed the formation of this upper ionized layer and the causes which produce it. He points out that, in order to produce ionization in such a gas as oxygen, by light radiation, it is necessary to have a wave length of the order of 1,500 to 1,800 ångstrom units (10^{-7} mm.), that is, light which is far beyond the ultra violet end of the spectrum. If such light really produces any ionization, then it is to be expected that the ionization would be reduced at night; and, therefore, that signals might be expected to vary in strength at night, if these ionized gases are the cause of signal variation. Professor Fleming suggests, however, that at heights of the order of 60 miles (100 km.), where the ordinary constituents of the atmosphere disappear and are replaced by hydrogen and helium and possibly other lighter gases, the most likely agency in producing ionization is the solar dust projected from the sun and transmitted to the earth thru the agency of light pressure. This explanation of the production of an upper ionized layer of gas is verified by the fact that the time interval elapsing between the passage of a sun spot across the solar meridian and the corresponding magnetic storm, as shown by Arrhenius, is about 45 hours, a figure which agrees fairly closely with the time Professor Fleming calculates that a particle of 1,200 ångstrom units diameter would take to pass from the sun to the earth. Whatever the cause which produces this layer, there is little doubt that such a layer exists, not necessarily in the form of a shell concentric with the earth, with fairly flat surfaces, but more likely in the form of large masses of gases in the upper regions

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[4] "Electrician," Volume 75, p. 348.

[5] "Proc. Royal Institution," Volume 17, p. 223.

[6] *Loc. cit.*

of the atmosphere which act as reflectors and refractors for the waves that are used for the transmission of radio signals.

Other facts bearing on the presence of this layer have been dealt with by Dr. Eccles in a paper published by the Royal Society.[7] It will not be necessary to reproduce the argument he uses to prove its existence, that may be assumed. It is the object of this paper to discuss to what extent the Heaviside layer can explain the phenomena described by Mr. Fuller.

THEORY OF INTERFERENCE BANDS ON A SPHERICAL REFLECTOR

In the first place, the conditions governing the production of interference bands on a spherical surface with a surrounding envelope, also of spherical form, whose internal surface acts as a reflector, may be discussed. It is usually assumed that the height at which the Heaviside layer becomes sufficiently conducting to act as a reflector is about 50 miles (80 km.). This is shown to scale in Figure 1, the points *A* and *B* corresponding respectively with San Francisco and Honolulu. It is clear that the passage between this layer and the earth is very much in the nature of a narrow crevasse between two parallel surfaces or reflecting mirrors.

FIGURE 1

It is sufficient, as a first approximation, to consider, therefore the formation of interference bands by reflection from a pair of flat parallel surfaces. The distance between the bands of electromagnetic lightness and darkness is determined by the fact that the difference between the paths of the rays reaching these points by two alternative routes is half a wave length. If the difference in distance along the two paths is a multiple of a wave length the point is one of brightness; if it is an odd number of half wave lengths the point is dark.

It may be assumed in the first instance that one ray travels along the surface of the earth, as supposed in Sommerfeld's

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[7] Eccles, "Proc. Roy. Soc.," A, Volume 87, pp. 79–99.

theory, and that the other is reflected by the Heaviside layer and by the earth's surface.

Altho it is hardly to be expected that reflection will be regular, in the first instance it may be advisable to consider the conditions which govern the width of interference bands formed by regular reflections between plane surfaces. Let the two surfaces be represented by $O\,P$ and $Q\,R$ distant h kilometers from each other.

Let n be total number of successively reflected rays, and θ the angle at which the reflected ray strikes the Heaviside layer. It is easily seen that

$$\tan\theta = \frac{n\,h}{D} \qquad \text{(See Figure 2)}$$

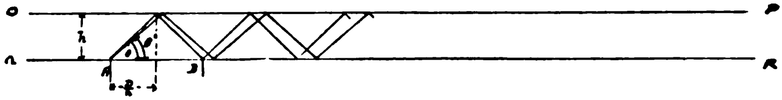

FIGURE 2

The difference in length of path for two rays going from A to B, one directly along the surface of the earth and the other reflected by the surface $O\,P$ is given by

$$2\left(\frac{D}{n}\sec\theta - \frac{D}{n}\right)$$

For the whole series of n reflections, this equals $D\,(\sec\theta - 1)$, and it is seen that this is dependent only on the value of θ and D. If one assumes a ray impinging on the reflecting layer at a slightly different angle, θ', the difference in length of path to a point distant $D+a$ from the sending station will be $(D+a)\,(\sec\theta' - 1)$. To get the distance between interference bands, differentiate $D\,(\sec\theta - 1)$ with respect to D, making the necessary substitution for $\sec\theta$,

$$\frac{d\,[D\,(\sec\theta - 1)]}{d\,(D)} = \frac{d\left(D\left\{\sqrt{1+\left(\frac{n\,h}{D}\right)^2} - 1\right\}\right)}{d\,(D)}$$

$$= \left(\sqrt{1+\left(\frac{n\,h}{D}\right)^2} - 1\right) - \left(\frac{n^2\,h^2}{D^2\sqrt{1+\left(\frac{n\,h}{D}\right)^2}}\right)$$

If a be half the width of an interference band, i. e., the distance from a dark to a light patch, the difference between the differences in length of path for the two rays at angles θ and θ' which interfere at the points distant D and $(D+a)$ from the sending station respectively must equal $\pm\frac{\lambda}{2}$.

Hence
$$a\left(\frac{d[D(\sec\theta-1)]}{d(D)}\right)=\pm\frac{\lambda}{2}.$$

Reducing this quantity we arrive at the condition
$$\left(1-\sqrt{1+\left(\frac{nh}{D}\right)^2}\right)=\pm\frac{\lambda}{2a}\sqrt{1+\left(\frac{nh}{D}\right)^2}$$

But
$$\sec\theta=\sqrt{1+\left(\frac{nh}{D}\right)^2},$$

then
$$\sec\theta=\frac{1}{1\pm\frac{\lambda}{2a}}.$$

Putting now $\lambda=6$ km., $a=14.4$ km. (the values given in Mr. Fuller's paper), $\sec\theta=1.26$ and $\theta=37.6°$, since it is evident that the $-$ sign must be taken.

Substituting now for $\sec\theta$, its value found above,
$$\frac{nh}{D}=0.76.$$

If $D=3{,}700$ km. and $h=80$ km., D being the distance between San Francisco and Honolulu, and h the height usually assumed for the Heaviside layer, it follows that
$$n=\frac{0.76\times3{,}700}{80}=35 \text{ times},$$
or the rays reflected by the Heaviside layer must go up and down 35 times between it and the earth to give an interference band of this width. This is certainly not likely to happen without considerable loss of energy of the reflected ray, which would prevent sharpness of the interference band. Moreover, such reflections as these would give a difference in path for the two rays of
$$D(\sec\theta-1)=3{,}700\times0.26=960 \text{ km}.$$

THE HEAVISIDE CLOUD THEORY

Now the very interesting curves given by Mr. Fuller in which he shows that weakening and strengthening of the signal occurs as the wave length at the sending station is altered, indicate that the difference in length of path of the two interfering rays is much less than that calculated above.

In Figure 13 of his paper he obtains the following results—

Wave lengths for which minimum values of signal strength are found	Wave lengths for which maximum values of signal strength are found
5 km.	6 km.
7 "	8 "
10 "	—

If the two interfering rays travel along their two different paths in the same direction independently of wave length, it is evident that the difference in length of path must be an odd number of half wave lengths for 5 kilometers, 7 kilometers, and 10 kilometers waves, and an even number of half wave lengths for the 6 kilometers and 8 kilometers waves.

Let δ be the difference in length of path,

then $$\delta=\frac{5}{2}m=\frac{7}{2}(m-2)=\frac{10}{2}(m-4)$$

where m is any odd number.

In the same way $\delta=\frac{6}{2}(m-1)=\frac{8}{2}(m-3)$.

This gives a series of equations from which m and δ can be found, which give fairly consistent results. The average value obtained by solving them gives $m=7$ or 9 (the values of m found are 7.5, 8 and 9) and $\delta=17.5$ or 22.5 kilometers, a result very different from that found by the simple reflection theory.

From the other curves given in Mr. Fuller's paper, in which maximum and minimum points are shown with varying wave length, the difference in length of path of the two sets of interfering waves may be readily calculated. The results of these calculations are given on the following page. (All records are for transmission from San Francisco to Honolulu.)

There is no relation observable between the difference in length of path of the interfering rays and the state of the sky, i. e., whether it is all light or all dark, but the variation in the difference of path is evidence that the observed phenomena are due to reflections from irregularly placed surfaces and lends support to the "Heaviside Cloud" theory.

Curve, Figure, Number	Date, 1914	San Francisco Time	Difference in length of path of interfering waves.	Condition of intervening space
5	Mar. 8	3 p.m.— 3.54 p.m.	28 km.	all light
6	" 15	10.30 a.m.— 12 p.m.	21 "	" "
7	" 15	3 p.m.— 4 p.m.	35 "	" "
8	Apr. 1	3 a.m.— 4 a.m.	28 "	all dark
10	May 17	11.30 a.m.— 12 a.m.	15 "	all light
11	" 24	10.25 a.m.—11.25 a.m.	36 "	" "
12	" 24	11.25 a.m.— 12 a.m.	14 "	" "
13	June 13	2.45 a.m.— 3.30 a.m.	20 "	all dark
14	" 22	10.30 a.m.—11.15 a.m.	average 15 km.	all light
15	" 28	10.45 a.m.—11.30 a.m.	36 "	" "

There is, of course, a difference between the day and night records in that the variations at night, due to altering wave length, are usually greater than they are in the day time, thus showing that the interfering waves are more nearly equal in intensity at night than they are by day. This is clearly shown in Figure 13 of Mr. Fuller's paper. The morning records give large variations also, and it would appear that altho the intervening space is fully lit, night conditions of semi-transparency still supervene. One of the most interesting results mentioned at the end of Mr. Fuller's paper is that he finds that the character of the land or water between the stations appears to make very little difference in long distance transmission, i. e., that the signals over land are as strong as they are over water. This seems almost conclusive proof that the observed signals are due to rays which are almost entirely refracted and reflected. The result, of course, is at variance with observations with shorter wave lengths over smaller distances. In dealing with interference phenomena it may be assumed therefore that the two rays travel round the earth thru the narrow passage formed by the earth and the Heaviside layer, being refracted as they pass, as was explained by Dr. Eccles, and being also reflected from the lower surface of the Heaviside layer.

If one assumes a ray refracted so as to follow nearly the curvature of the earth, which strikes a reflecting surface as shown in Figure 3, interference might take place between the waves reflected from two regions *C E* and *D F*. If one assumes a reflecting surface of such a nature[8] that the angle of incidence of a ray on it changes considerably for slightly differing altitudes as indicated in the figure, i. e., if the reflecting surface is irregular, a series of rays may be reflected at different angles by such a cloud at such points as *C E* (closely adjacent) in directions *C A*

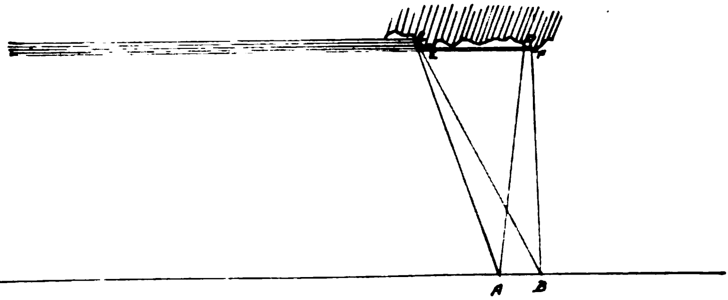

FIGURE 3

and *E B*, and it is easy to see that the difference in length of the path from *C* to *A* direct, and by the path *C D A* may be an odd number of half wave lengths, while the difference in length of path from *E* to *B* direct, and by the path *E F B* may be an even number of half wave lengths.[9] If this is the case, the distance between the two positions *A* and *B* at which the two waves would reinforce and neutralize each other respectively, may vary within wide limits. The diagram has been drawn to correspond, as nearly as possible, with the conditions given in Mr. Fuller's paper in which he has two stations 9 miles apart. The difference in length of the two paths to *A* is about 30 kilometers and to *B* about 26 kilometers. For a wave length of 7,500 meters, the signals at *A* would be strong, since the difference in path is 8 half wave lengths, at *B* the signals would be weak, since the difference in path is nearly 7 half wave lengths. For a wave length of about 8,500 meters the difference in path to *A* would be just under 7 half wave lengths so that signals

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[8] It is perhaps misleading to speak of angle of incidence when the dimensions of the reflecting surface are of the same order of magnitude as the wave length, the incident radiation is really scattered.

[9] Altho the diagram has been drawn with the two interfering rays in one plane, it is clear that the rays may be reflected from other directions not coplanar.

would be weak, whereas at *B* the difference in path would be just over 6 half wave lengths and the signals would therefore be strong. These phenomena can hardly be described as due to the production of an interference band in the sense in which it is usually understood, when speaking of interference phenomena in light. The interference between the rays traveling along the different paths is entirely tortuitous, and the regions of electromagnetic lightness and darkness are probably scattered in a most irregular way. The explanation here given, tho only one of an infinite number of possible explanations, is consistent with all the observations made by Mr. Fuller, and it is one which corresponds very closely with what may be anticipated from our knowledge of the upper atmosphere. The interesting calculation made recently by Mr. Cohen[10] that Austin's results could be represented by a formula of the form:—

$$I_R=\frac{K}{D}(1+N\,D)\,\varepsilon^{-0.0019\frac{D}{\sqrt{\lambda}}}$$

also has a bearing on the subject, as it would seem to point to the strength of scattered waves, reflected from the lower face of the Heaviside layer. Mr. Fuller's results therefore, lend weight to the theory that the reflecting surface formed by the Heaviside layer is quite irregular, or rather they point to the existence of an irregular mass of reflecting clouds which form the lower surface of the layer. Combined with this, there must be a certain amount of refraction to enable the rays to get round the arc of nearly 30° of the earth's surface which they have to cover in going from San Francisco to Honolulu, as already explained by Eccles.[11]

Altho one can do no more than speculate on the causes of phenomena occurring in media of whose properties one can have no direct experimental knowledge, the facts seem to be adequately explained by some such conception. The Heaviside cloud theory may therefore be considered as further established. The author's own experiments which, unfortunately, have been completely interrupted by the war, pointed in the same direction, tho the distance over which he was working, London to Paris (650 km.) was comparatively short. In the account of his tests published by the Institution of Electrical Engineers[12] he emphasized the "cloud" theory first suggested by Fessenden as the most likely explanation of the phenomena observed, and Mr.

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[10] Cohen. "Electrician," Volume 76, p. 743.
[11] Eccles, "Proc. Roy. Soc.", *loc. cit.*
[12] "Proc. I. E. E.", Volume 53, p. 329.

Fuller's results now confirm this explanation, tho it is necessary in addition, in the San Francisco-Honolulu tests, to assume refraction. It is to be hoped that experiments may be continued, since, besides giving a great deal of information on radio telegraphic technology, such tests may enable us to gain further knowledge of the nature of the upper atmosphere, at altitudes higher than those at which balloon observations are possible. The author wishes to express his indebtedness to Professor Wilberforce for his co-operation.

SUMMARY: Variations in received signal strength are ascribed partly to the existence of a "Heaviside cloud" layer consisting of masses of ionized gas at considerable heights. A partial bibliography of the subject is given.

The theory of interference caused by a spherical reflector is given, and results of receiving experiments due to Mr. Leonard Fuller, are studied in the light of the derived theory. It is shown that the Heaviside layer is probably quite irregular, and that refraction of the traveling waves is probably existent

DISCUSSION

Leonard F. Fuller (by letter): Operators using continuous waves have frequently observed that altho the received night signals may be stronger on the shorter waves, fading is so much more frequent on these that they are often commercially inferior to longer waves even for night work.

This may be explained by the following:

If the mean of the observations on curve Figure 13 of the American Institute of Electrical Engineers' paper mentioned by Professor Marchant are plotted, a curve of received watts is obtained following approximately the curve of energy radiated from the transmitting antenna.

This is as it should be and is the reason for the first of the practical observations of operators. The second may be explained by the following consideration of the theory of Fgure 3 of Professor Marchant's paper.

As he points out, interference bands, in the sense in which they are usually understood in light phenomena, do not exist if we consider the path of transmission parallel to the earth surface as a whole or even locally at *A* and *B*. Inasmuch as the possible number of Heaviside cloud arrangements are infinite, there are possibilities of an infinite number of points similar to *A* and *B*. Thus regions of weak or strong signals may be entirely irregular as to dimensions and spacing.

These regions are produced by the superposition of two waves of the same frequency, *out of phase*, rather than by the combination of two waves of different frequency. The relative amount of signal fading or amplification at a given point is therefore dependent upon the angular phase displacement of the two interfering waves. This displacement is dependent upon the actual mechanical dimensions of the convolutions on the under side of the Heaviside layer. Assuming the contours of this layer constantly changing, it is obvious that the longer the wave length the less the phase displacement of the interfering waves for a given change in the Heaviside layer. Thus fading may be less troublesome on the longer waves and their commercial value enhanced thereby.

SKIN-EFFECT RESISTANCE MEASUREMENTS OF CONDUCTORS

AT RADIO-FREQUENCIES UP TO 100,000 CYCLES PER SECOND*

By

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The research here reported was conducted in the Research Division, of the Electrical Engineering Department of the Massachusetts Institute of Technology, during 1915-16, under an appropriation from the American Telephone and Telegraph Company. The object of the research was to obtain experimental data on the alternating-current resistances of various electrical conductors at frequencies up to 100,000 ∾.

OUTLINE OF PRECEDING HIGH-FREQUENCY ALTERNATING-CURRENT RESISTANCE MEASUREMENTS

Publications of skin-effect measurements at high frequencies are very rare. The Bibliography of the Kennelly-Laws-Pierce A. I. E. E. paper of 1915, and that appended to this paper, contain references to the work of Fleming, Lindemann, Dolezalek and others, in this direction. Fleming's measurements were published in 1910, and he was able to obtain confirmation of Kelvin's formula for the skin-effect resistance ratio of certain straight wires using oscillatory currents. The measurements here reported seem to differ from those previously published, in that they have been obtained with steadily sustained alternating currents, furnished by an a. c. generator, and using a null method, with a somewhat sensitive induction bridge. The research

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constitutes a continuation, at higher frequencies, of that reported in the A. I. E. E. 1915 paper above mentioned.

In that paper it was reported that the standard skin-effect formulas for round wires had been observed to hold, within the limits of experimental error, up to 5,000~, the highest frequency used. It was also noted that the skin-effect resistance ratio of conductors formed of 7 round bare strands, with normal spirality, was slightly greater than that of the equisectional solid wire. Straight flat-strip conductors were found to have a skin-effect resistance ratio far in excess of that predicated from existing available formulas.

The ratio Z/R of the linear a. c. internal impedance of a conductor (Z ohms ∠ per centimeter), to the linear d. c. resistance (R ohms per centimeter), is a complex numeric called the *skin-effect impedance ratio.*[1] Its real component R'/R, is called the *skin-effect resistance ratio*, and is the usual factor which expresses the magnitude of skin-effect. The imaginary component X/R, is similarly called the *skin-effect reactance ratio.* This paper deals almost exclusively with the skin-effect resistance ratio R'/R. When subdivided wires are cabled, they are spiralled for mechanical reasons. This usually increases the resistance ratio, owing to what is called the *spirality effect.* When wires carrying alternating currents are placed in such a degree of proximity as to affect appreciably the distribution of alternating-current density over their cross section, owing to their mutually interacting magnetic fields, the resistance ratio R'/R is ordinarily increased, owing to *proximity effect.* The ordinary skin-effect resistance ratio of a long straight conductor is therefore to be understood as occasioned by the distortion of alternating-current density over its cross section, due entirely to the magnetic field of that conductor. This research has been directed to the measurement of the resistance ratio in straight conductors only, in view of the fundamental character of such measurements, and the sparsity of published observations in this field.

GENERATING APPARATUS

The source of the alternating currents used in all the tests was an Alexanderson radio-frequency[2] alternator made by the General Electric Co., and kindly loaned to our Department, by the National Electric Signaling Co. and Professor R. A. Fes-

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The angle sign ∠ indicates a complex quantity or "plane vector."

Radio frequencies are, for convenience, arbitrarily chosen as those over cycles per second.

senden. A picture of the machine appears in Figure 1. The direct-current driving motor M, on the right-hand side, is a 4-pole 125-volt 82-ampere machine, operating, when at full speed, at 2,000 r. p. m., and driving the alternator on the left, thru a 10:1 step-up gearing, at a speed of 20,000 r. p. m. The alternator, of the inductor type, has a steel disk rotor, represented in Figure 2, with 300 radial slots. The stationary armature windings are supported on polar projections, on each side of the rotor, so that there is one complete cycle of induced e. m. f. in the armature winding at the passage of each tooth on the rotor. The full-speed frequency is therefore 20,000×300 = 6,000,000 cycles per minute, or 100,000 cycles per second. The

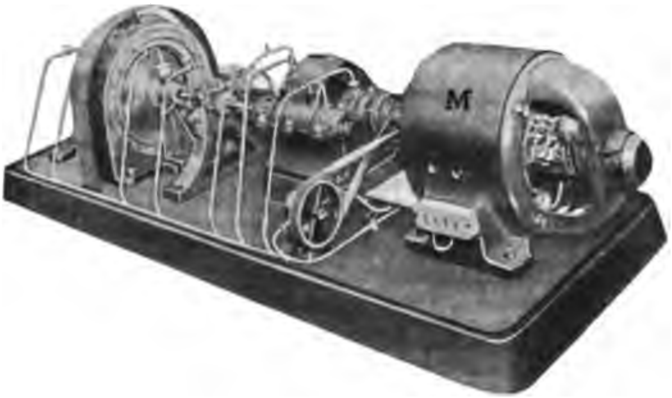

FIGURE 1—Motor-Driven High-Frequency Generator

FIGURE 2—Rotor and Shaft of Standard 100,000-Cycle Alternator

alternator is so designed that the clearance between the rotor and the stator pole faces can be closely adjusted. With a small air gap, power up to 2 kilowatts, at about 15 amperes and 130 volts, can be obtained. In the tests described, the current taken from the machine rarely exceeded 5 amperes, thus enabling a less troublesome mechanical clearance adjustment to be used. The machine, which is self-oiling by forced lubrication, could be run at constant full speed for an hour or more without particular attention.

The wave form of the generated e. m. f. was not investigated, but no evidence has presented itself of any appreciable irregularities, to which moreover the additional reactance automatically developed would be enormous.

The generator set was placed in a room about 20 meters away from the testing laboratory, and controlled from the latter, thru a set of armature and field resistances. A calibrated little magneto generator, geared to the motor shaft, but not shown in the picture, actuates a calibrated d. c. voltmeter on the testing table, so as to indicate continuously both speed and frequency.

METHODS OF MEASUREMENT

The first means for measurement tried, consisted of a simple Ohm's Law circuit, illustrated in Figure 3, using a hot-wire voltmeter V, and ammeter A. A series circuit was made up, comprising the alternator G, a hot-wire ammeter A, the test conductor X, and the adjustable condenser C. This condenser was needed in measuring the resistance of the test-wire X, in order to compensate for the relatively high reactance, that even a simple grid distribution of a few meters of test conductor will offer at so high a frequency. The operation consists in causing the radio-frequency current to flow in the circuit, and so adjusting the variable condenser C, that the minimum e. m. f. is indicated by the hot-wire voltmeter V, connected as shown. Under these circumstances, the ratio E/I of the branch circuit $a\,b\,c$, will be the effective a. c. resistance of the test conductor and condenser, the latter being relatively small.

The method has the advantages of simplicity and directness of measurement. It is swift and convenient. It has the disadvantages that it is not a null method, and that the final resonance adjustment is very troublesome; also, that at least two ranges of voltmeter reading are required, in order to complete the adjustment. At resonance; i. e., when the inductive reactance of the branch circuit $a\,b\,c$, is equal and opposite to the reactance of the condenser C, the current thru the ammeter A is relatively large; while the e. m. f. indicated on the voltmeter V is relatively small. The capacitance C is such that for low-resistance test wires, at frequencies not exceeding $100{,}000^{\sim}$, an air condenser cannot be used, and hence the apparent resistance of the test wire is increased, owing to the presence of power loss in condensers with solid dielectrics. For these reasons, the method was later discarded, in favor of an inductive-bridge method; but it is nevertheless recommended as a suitable method in commencing the study of high-frequency resistances; besides serving as a convenient check upon such null methods as may later be employed.

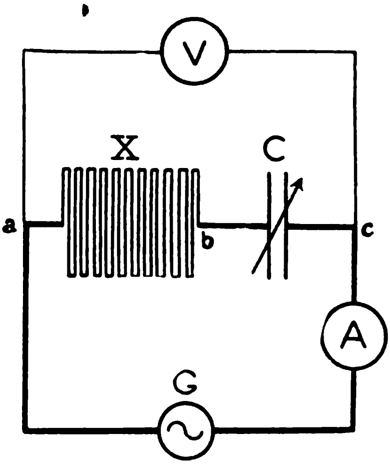

FIGURE 3—Radio-Frequency Resistance Measurement by Ohm's Law Circuit

The test wires employed had apparent radio-frequency resistances of less than half an ohm. The condenser C was adjustable up to a maximum of 26 μf. (microfarads), by steps of 0.001 μf., and a final adjustment, with the aid of a little rotary oil-condenser, continuously variable to a total of about 4 mμf. (millimicrofarads).

There were two multiple-scale Hartmann and Braun voltmeters available, covering a range from 0 to 200 volts, and a multiple-scale ammeter, covering a total range from 0 to 5 amperes without shunt. Altho calibrated with continuous currents, the design and construction of these instruments warrants the belief that their calibration is substantially correct for alternating currents over the range of frequency employed.

INDUCTIVE BRIDGE METHOD

The bridge method later employed corresponds, in essential features, to that described by A. Hund* in 1915. The diagrammatic connections are indicated in Figure 4. *A* is the radio-frequency alternator, and *X* is the conductor under test. It is balanced, as to resistance and inductance, by the adjustable resistor *R*, and the adjustable inductor *L*. The equal arms of the bridge are the two opposing primary coils T_1 and T_2 of a small special testing transformer, the secondary winding T_0 of which is connected to the detector *D*. When zero balance is obtained in the detector *D*, for any measured impressed frequency within the range of the alternator *A*, the values of *R* and *L* are respectively the apparent resistance and inductance of the test conductor *X*, corresponding to that frequency.

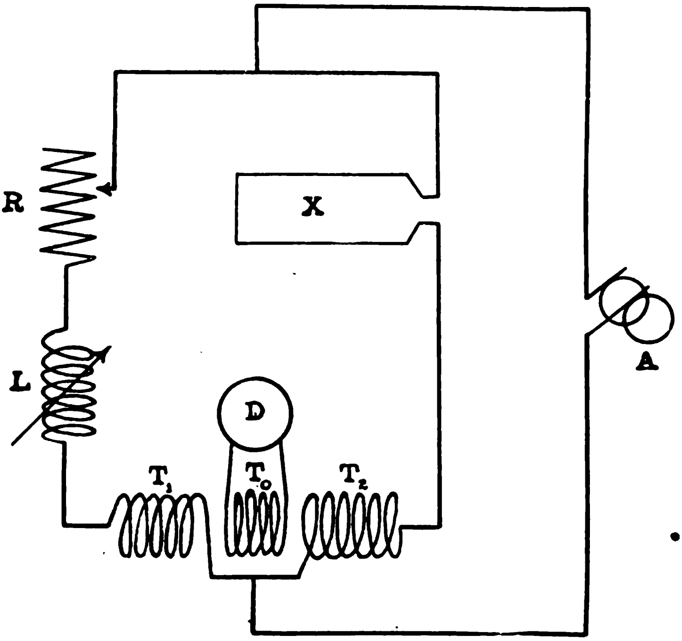

FIGURE 4—Differential Transformer Bridge System

The details of the apparatus, as used in practice, are shown in Figures 27 and 28, the description of which is given in Appendix I, in such detail as should enable the apparatus to be duplicated if desired.

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*Bibliography, Number 14.

Straight Solid Wires.—A round solid annealed copper wire number 12 A. W. G. (diameter 0.205 cm. = 0.0808 inch) was tested by both of the above methods. The results are given in Figure 5. The curve is the locus of the skin-effect resistance

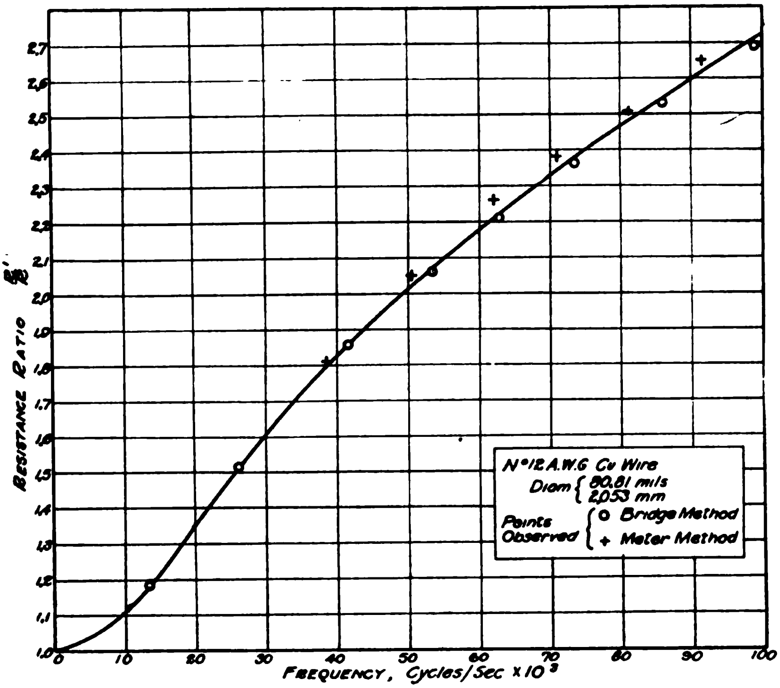

FIGURE 5—Skin Effect Ratio for Number 12 A.W.G. Copper Wire as Observed and Computed

ratio R'/R, as computed from the real part of the Bessel-function formula:

$$\frac{Z}{R} = \frac{a_0 X}{2} \cdot \frac{J_0(a_0 X)}{J_1(a_0 X)} \qquad \text{numeric} \angle \quad (1)$$

where Z is the internal linear impedance of the wire, of which R' is the linear resistance component, X is the radius of the wire in centimeters, and $a_0 = \sqrt{-j\,4\,\pi\gamma\mu\,\omega}$ (see Appendix II). The crosses represent observations by the Ohm's law method, and the circles by differential method. It will be seen that the latter observations are in substantial accordance with the computed curve. The wire was mounted over insulators, on a flat frame illustrated in Figure 6, which was made standard in these tests. The advantage of this particular mounting is that it holds the wires in one plane at a uniform spacing (about 5 cm. or 2 inches), sufficiently great to make the proximity effect negligibly small.

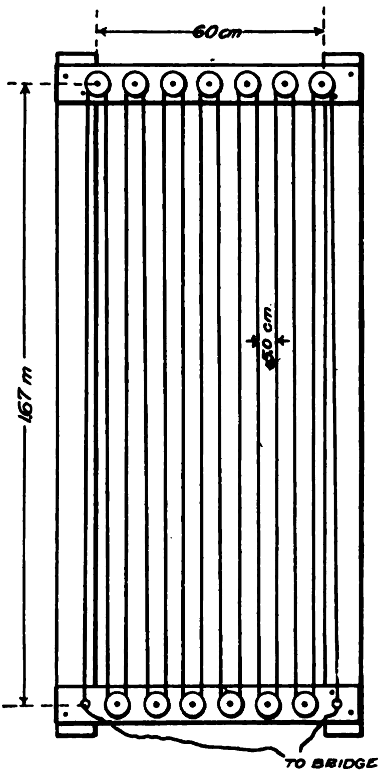

FIGURE 6—Test Wire on Frame

The external inductance of the wire, thus mounted, is also relatively small. It was ascertained that altering the spacing of the conductor on the frame; even to the extent of placing the 25 meters of wire in a single horizontal loop, 20 centimeters wide, on the wall, did not appreciably affect the skin-effect resistance ratio R'/R, to the degree of precision used in these tests. The mounting of the test wire on such a frame has the additional advantage that it brings the wire into a small compass, and minimizes its external electromagnetic disturbing fields on the measuring apparatus. Moreover, it enables a framed test wire to be kept indefinitely for reference, when desired.

A second test wire, formed of 25.3 meters of number 14 A. W. G. solid round copper wire (diameter 1.625 mm. = 0.064 inch) was tested by differential bridge at 22° C., at a later period, and with greater precision. Its linear resistance was found to be 0.00825 ohm per meter. The skin-effect resistance ratio of this wire was found to agree with the value computed from the standard formula, within the limits of observational error, the following results being obtained:

Frequency Cycles/second	Skin-effect Resistance Ratio R'/R		Ratio $\frac{\text{Observed}}{\text{Computed}}$
	Observed	Computed	
96,100	2.192	2.189	1.002
82,200	2.051	2.049	1.001
59,300	1.775	1.774	1.001
38,000	1.453	1.458	0.997
20,000	1.162	1.170	0.993

The entire series of observations are presented by the circles in Figure 7, the curve showing the Bessel-function computed values, according to formula (1). It will be seen that there is a satisfactory agreement between the observations and this curve. Altho no reason can perhaps be assigned why the skin-effect resistance ratio of a round wire should depart, at greater frequencies, from the theoretical formula, already found to hold up to 5000∾; yet it is satisfying to find the agreement maintained. It should be noted that when attempts are made to measure with precision the skin-effect resistance ratio of such a straight round wire, the temperature of the wire must be carefully noted.[*]

STRANDED WIRES

Stranded Copper Wires.—After ascertaining that round solid copper wires, at radio frequencies, conformed to standard formulas, measurements were extended to subdivided or stranded copper wires with insulated strands. A test was made of stranded wire, of 20 strands, each number 26 A. W. G. (0.405 mm.) and enamel insulated to a diameter of 0.42 millimeter. The strands had a usual amount of spiralling or lay (4 cm. pitch). The unbraided diameter, i. e., the diameter of the bundle of strands,

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[*] The result is also in conformity with that obtained by Fleming, thru another method, to frequencies up to 900,000∾. See Bibliography, Number 3.

under the braiding, was about 25 per cent. greater than that of an equisectional solid or unstranded wire. The skin-effect resistance ratio R'/R, for this stranded wire, between 50,000 ∿ and 100,000 ∿, was about 10 per cent. less than that computed for the equisectional solid wire.

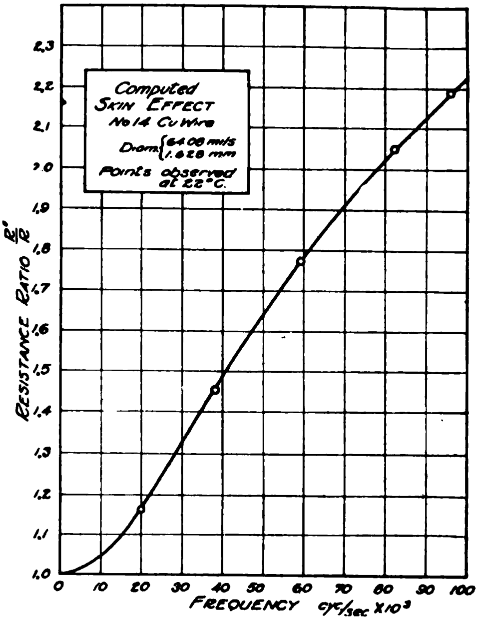


Figure 7—Agreement between Measured and Computed Skin-Effect Ratios for Round Wire

Another stranded wire had 50 strands of number 28 A. W. G. copper wire (0.32 mm., silk covered to 0.45 mm.) the strands being made up with a very long lay (about 1 turn per meter) and covered with braiding. The unbraided diameter was about 80 per cent. greater than that of the equisectional solid wire. This conductor had a skin-effect resistance ratio, at 100,000 ∿, about 33 per cent. less than that computed for the equisectional solid wire. Since measurements of the skin-effect resistance ratio of seven-strand copper cables, last year, showed a slight increase over the equisectional solid conductor, at frequencies up to 5,000 ∿, as stated in the above-mentioned A. I. E. E. paper of August, 1915, it seemed desirable to separate the effects of subdivision thru insulation, from those of spirality. By the courtesy of the Simplex Wire and Cable Co. of Boston, we were able to secure a sample of 26 meters of a stranded and braided wire, containing 48 strands, each of number 30 A. W. G. copper wire, diameter 0.255 millimeter, silk insulated to about 0.33 millimeter. These strands were braided up without twist, by being carefully drawn thru a die, whipped with silk thread by hand, and then laid on the supporting wooden frame for testing. By this treatment, it was hoped and believed that each individual strand would preserve its proper position substantially unchanged in the cross-section thruout the sample. The over-all unbraided diameter was about 50 per cent. greater than that of the equisectional solid wire. Another sample of stranded wire was made up at the same time, except that it had the usual amount of spirality, i. e., 1 turn in 4 centimeters about the central axis, and was machine braided.

Figure 8 shows the results of the skin-effect measurements on these two stranded conductors, and gives the resistance ratio R'/R against impressed frequency, up to 100,000∾. The curve A refers to the stranded untwisted sample, B to the stranded twisted sample, and C to the computed solid equisectional conductor. It will be seen that the subdivision of the wire into separate parallel untwisted strands, with this amount o separation, reduced the resistance ratio, at 100,000∾, from 2.38 to 1.86; but that the spirality effect of the lay in sample B, brought the ratio nearly half way back, or to 2.11. The B curve lies roughly midway between the A and C curves thruout. The reasons for the greater skin effect in the B sample over that of the A sample are probably twofold; namely (1), the additional copper losses in B due to the spirality effect; i. e., the longitudinal component of alternating magnetic field in the interior of the wire B, due to the action of the lay as a solenoid, and (2) the greater tightness under the braiding of the spiralled B sample, than that which could be safely given to the unspiralled A sample by hand, with a corresponding diameter enlargement ratio of only 1.4 for B, as against 1.5 for A.

If there were no insulation separating the individual parallel strands in the A sample, then it is believed that the skin-effect resistance ratio would rise into coincidence with C, or would become the same as in the equisectional solid wire. In other words, the theory and tests, to date, of skin effect in round wires, seem to show that the mere subdivision of the wire into a round bundle of parallel round filaments, which make contact with each other, but entrain channeled air spaces, has no appreciable effect on the skin-effect ratio; because the magnetic flux maintains a cylindrical distribution. When, however, even a thin layer of insulation separates the individual strands, some magnetic flux circulates radially to the mass, around each strand, and diminishes the skin-effect resistance ratio of the whole, while raising somewhat the skin-effect reactance ratio of the whole.

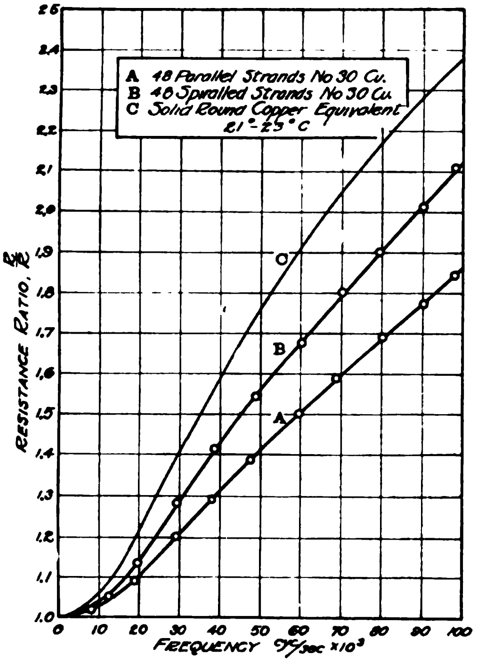


FIGURE 8—Effects of Stranding with Insulated Strands

Measurements with Definitely Spaced Strands.—In order to study, in greater detail, the effect of separating the strands of an unspiralled stranded conductor, a test loop was constructed, about 10 meters long, of a copper conductor comprising 7 strands of number 22 A. W. G. wire (diameter 0.644 mm.). These strands were supported parallel to each other in air, at the positions they would occupy in any 7-strand cable; i. e., one wire at the center, and the six others forming a hexagon about this, as indicated in Figure 9. The spacings were, however, changed in the different tests, by means of specially prepared insulating frames, strung on the wires at suitable distances. In this way, the interaxial spacing distance could be varied from

1 strand diameter, to nearly 10 strand diameters, in 6 successive steps.

The results of the skin-effect resistance-ratio measurements in these successive geometrical arrangements are presented in the six curves of Figure 10. Curve *A* is for the 7 wires bound

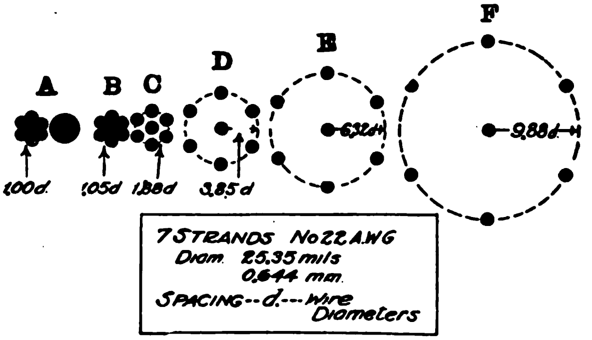


FIGURE 9—Wire Spacing

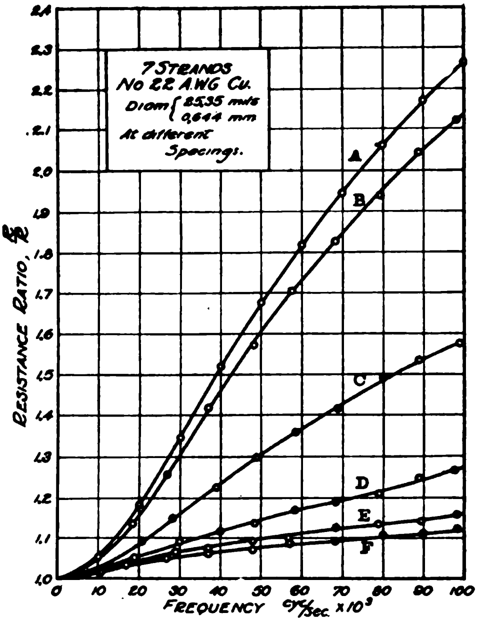


FIGURE 10—Effects of Strand Spacing on the Skin Effect of a Stranded Conductor

together in close parallel contact. It agrees with the computed resistance ratios of the equisectional solid conductor, within the limits of observational error. Curve *B* gives the corresponding ratios when all of the 7 wires were insulated with an enamel thickness of approximately 0.015 millimeter, thus increasing the interaxial spacing distance to 1.05 times the strand diameter. It will be seen that the ratio R'/R has thus been lowered about 10 per cent., with respect to number 1, or the equisectional solid wire. The other curves, which will be self explanatory, show how rapidly the spacing distance diminishes the skin-effect resistance ratio. At 100,000$^{\sim}$, this ratio is only 1.12, for a spacing distance of 9.88 diameters, as against 2.26 for either the solid wire, or a spacing distance of 1.00 diameter. At great separating distances, it appears that the resistance ratio of the 7-strand conductor would substantially coincide with that of any one of its components.

First-approximation formulas have been worked out (see Appendix IV) for dealing with the geometrical relations of such variable "strands," as are indicated in Figure 9. It thus far appears that these formulas represent the observed facts fairly well for the larger spacings, but give lower ratios than are measured at the smaller spacings; perhaps because there is extra skin effect in the component strands at close spacings, due to proximity effect, or current-density distortion due to the magnetic fields of neighboring strands.

The approximate proportionality maintained between the curves in Figure 10 permits of using suitable approximate reduction factors for all frequencies between 40,000 and 100,000$^{\sim}$. This relation is presented more clearly in Figures 11 and 12. The upper curves of Figure 11 here approach simple hyperbolas; so that doubling the spacing distance approximately halves the extra resistance of skin effect.

Very Finely Stranded Wire.—Two 10-meter samples of highly subdivided wire, with enamel-insulated strands, were obtained thru the courtesy of Dr. A. N. Goldsmith of New York. The first (Figure 13) contained 169 wires, arranged in 13 twisted bundles, each of 13 twisted strands, of number 40 A. W. G. diameter 0.080 millimeter, enamelled to about 0.085 millimeter. The total diameter over all wires, and under the braiding, was approximately 1.47 millimeters, representing a diameter enlargement ratio of about 1.4 with respect to the equisectional solid copper wire. It is obviously difficult to remove the enamel successfully from all of these little wires, and to solder them together into a common terminal with the certainty that none have failed to make good electrical contact therewith. Moreover, it would be very difficult to make sure that each of the individual insulated strands is intact, without discontinuity, thruout the test piece. In fact, the linear resistance of the sample was observed to be 0.0221 ohm per meter at 20° C.; whereas, without allowance for spirals, the computed linear resistance was 0.0204 ohm per meter, if all the wires were active, indicating a conductance defect from all causes of 8.4 per cent. However, treating the sample as a stranded wire of, say, 162 strands, having this observed linear resistance, the measured skin-effect resistance-ratio curve is given in Figure 13. It will be observed that, at 100,000 ~, the ratio R'/R is 1.17, and that the extra resistance of skin effect is roughly 60 per cent. less than that computed for the round solid copper wire of equal d. c. linear resistance. The reduction may be attributed partly to the insulation separation of the strands, and partly to transposition effects; since the twisting of the component strands in each

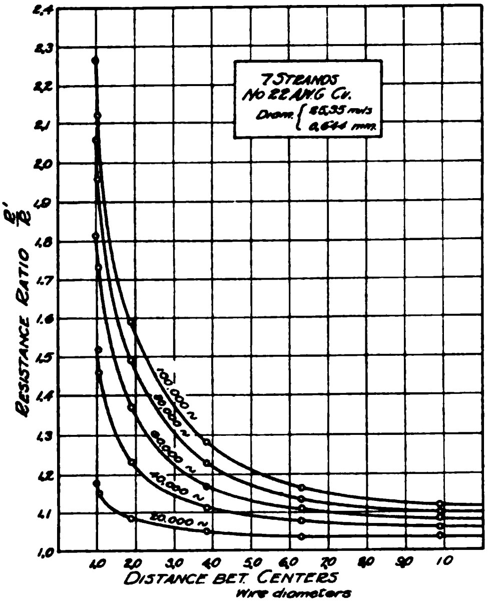


FIGURE 11—Curves Showing Drop in Skin Effect with Increased Spacing Distance between Strands of Conductor

bundle tends to interchange their relative positions in the cross-section of the whole.

The second sample was made up, in a similar manner, of 49 strands, each number 38 A. W. G. (0.1007 mm. enamelled), arranged in 7 bundles of 7 strands each. The observed linear resistance indicated that all of the 49 strands were probably intact and connected in parallel between the soldered extremities. The total cross-section of conductor is approximately 0.39 square millimeter. The unbraided diameter was 0.83 millimeter, representing a diameter enlargement ratio of about 1.5. The skin effect of this strand at various frequencies is shown in Figure 14, and is similar in regard to equivalent round wire to that shown in Figure 13 for the other and more finely subdivided stranded wire.

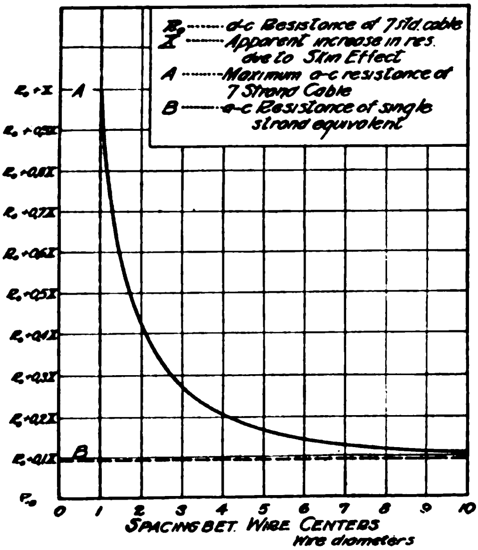
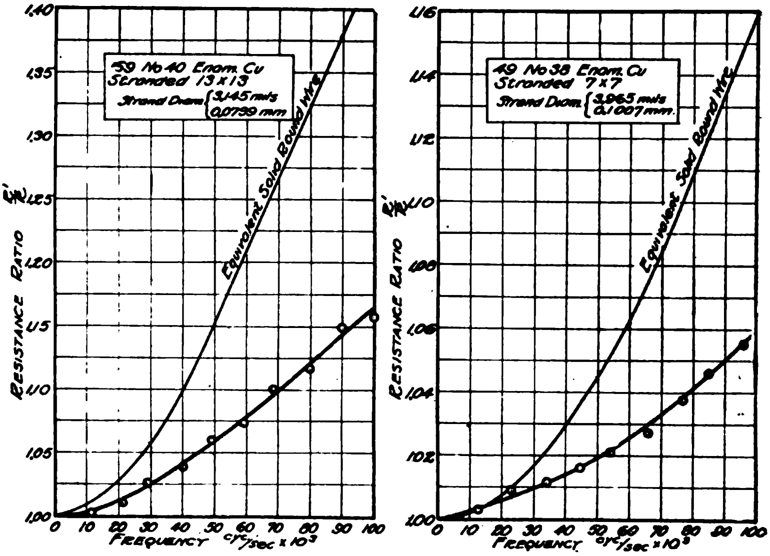


FIGURE 12—Average Decrease in Skin Effect of 7-Strand Cable with Increased Wire Spacing

Litzendraht Wire.—Thru the courtesy of Dr. Goldsmith, a sample of "Litzendraht" wire, 13 millimeters long, was secured for test. This wire is constructed by braiding enamelled copper wires, stocking fashion, in the form of a continuous long cylinder. This sample contained 48 number 38 copper strands, in 16 groups of 3 each. Each strand had a diameter of approximately 0.1

millimeter, enamelled to about 0.12 millimeter. Such a basket braiding necessarily provides a certain central air space, of diameter depending on the braiding and tension. The external diameter, under a heavy braiding of silk, was approximately 1.15 millimeter (0.045 in.), representing a diameter enlargement ratio of nearly 1.7. Such a construction involves a transposition of strands at regular intervals, between the inner and outer layers, also a hollow cylindrical cross-section. The results of the test of this sample are given in Figure 15. The total cross-section of conductor is approximately 0.38 square millimeter, or nearly the same as in the previous case, but the skin-effect resistance ratio is only 1.016 at 100,000∾.



FIGURES 13 and 14—Skin-Effect Resistance Ratios of Finely Divided and Interwoven Wires

Spirality Effect in Stranded Wires.—Thru the courtesy of the Simplex Wire and Cable Co., six samples of stranded bare copper wires were obtained, and of the same particulars and construction, except in regard to the degree of spiralling or lay. It was considered desirable to ascertain, with the aid of these conductors, the influence of spiralling upon their skin-effect resistance ratio. It had already been ascertained that the resistance of a 7-strand conductor, with the strands uninsulated, and in contact without spiralling, is the same as that of the solid equisectional round wire of the same metal and temperature. Consequently, any increase in the resistance ratio of 7-strand uninsulated and spiralled conductors could be attributed to spirality effect, assuming that the extra length of the external strands in the spiralled conductors was taken into account.

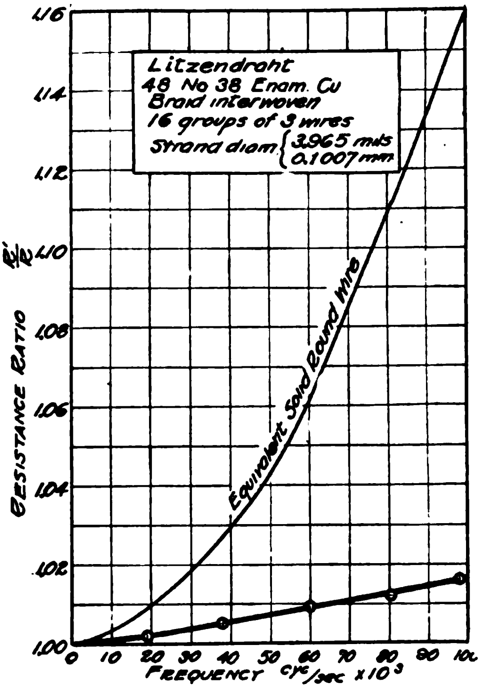


Figure 15—Skin-Effect Resistance Ratio for Litzendraht

Each of the six special test conductors was 25 meters long, and consisted of seven strands of bare tinned copper wires, each number 22 A. W. G., of diameter 0.668 millimeter (0.0263 in.), made up in the usual manner by a stranding machine, except in regard to the lay. The lay varied between 1 turn in 7.62 centimeters, to 1 turn in 1.52 centimeter; i. e., a spirality of 0.131 to 0.656 turns per centimeter. To assist in maintaining insulation between these bare copper strands, they were passed thru oil before entering the stranding machine in each case. They were then braided and thus held closely in contact.

As might be expected, it was found that the skin-effect resistance ratio, at any one frequency, such as 100,000~, increased with the spirality. Figure 16 shows the results obtained with the conductor of greatest spirality, (0.656 turn per centimeter). At 100,000~, the observed resistance ratio was 2.51; whereas the ratio computed for an unspiralled conductor of the same linear resistance was 2.34. This is the maximum deviation observed among the six samples tested. The corresponding curves for the other samples are omitted, partly because they would lie very close together, and partly because they are not strictly comparable on the same curve sheet, owing to small differences in d. c. linear resistance. In the case of the sample with least spirality, the resistance ratios were 2.41 observed, and 2.38 computed for the solid wire.

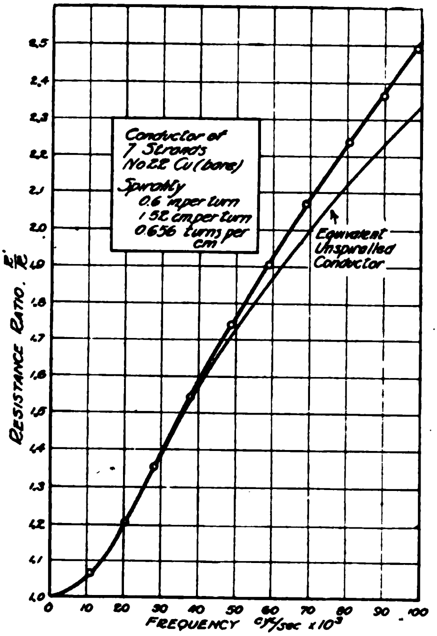


FIGURE 16—Increase in Resistance Ratio of 7-Strand Conductor Due to Spirality

The extra linear resistance of spirality may perhaps, as already mentioned, be attributed to loss of power in the substance of the conductor accompanying the longitudinal magnetic field within the spiral. It may therefore be regarded as an extra percentage of the d. c. linear resistance at normal temperature. On this basis, Figures 17 and 18 have been drawn. Figure 17 shows the extra percentage resistance of spirality in four different samples plotted against different frequencies; while Figure 18 shows the same percentage resistance at six different frequencies, plotted against spirality. The Figures magnify the spirality

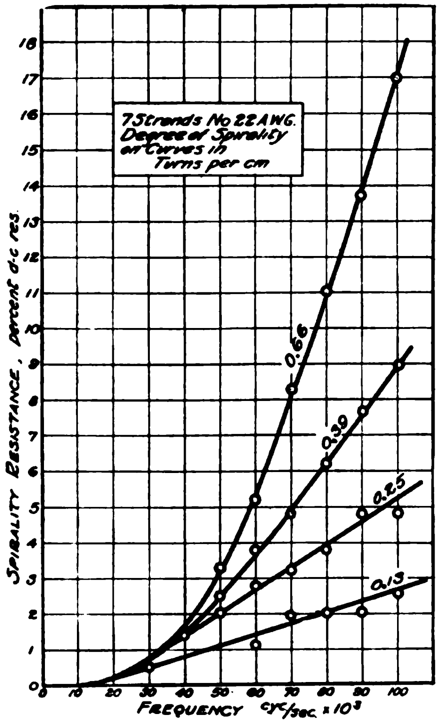


FIGURE 17—Loss Due to Strand Spiralling for Different Degrees of Spirality

effect at the expense of some apparent loss in precision. It will be seen that the spirality effect is almost negligible up to 30,000 ∾. Above 50,000 ∾, the spirality extra resistance is nearly proportional to the frequency. Moreover, the spirality resistance is roughly proportional to the spirality, especially in the neighborhood of 70,000 ∾.

Summing up the results for stranded wires, it appears that the mere subdivision of a solid round wire into seven uninsulated parallel round strands, has no appreciable effect upon the skin-effect resistance ratio. The separation of the strands, as by their insulation, rapidly diminishes the resistance ratio, but increases the reactance ratio of the conductor. Spiralling the strands, to the extent ordinarily employed, slightly increases the resistance ratio.

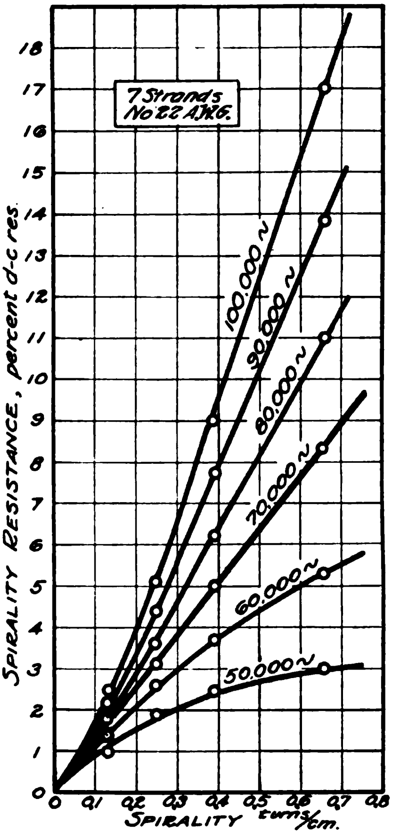


FIGURE 18—Variation in Loss with Degree of Spiralling

Copper Strips—Several samples of copper strip 0.003 inch (0.0076 cm.) were supplied by the courtesy of the Western Electric Co. These were tested in various widths from 0.356 centimeter to 3.8 centimeters (0.14 inch to 1.5 inch). The observed skin-effect resistance ratios are indicated in Figure 19, for five different widths of strip, the abscissas being impressed frequencies, and the ordinates R'/R. Each tested strip was mounted in a horizontal loop, or loops, on the testing-room wall. The length of each tested loop varied from about 3 meters, for the narrowest strip, to 25 meters for the widest strip. The spacing between the sides of the loop or loops, varied from 10 to 20 centimeters, and the proximity effect was found to be negligible under these conditions. It will be observed that at 100,000 ∾, the resistance ratio varies from 1.08 for the 0.356-centimeter strip width, to 1.44 for the 3.8-centimeter width. Owing to slight differences of thickness and of test temperatures the curves are not precisely comparable; but they resemble the curves of resistance ratio for wires of increasing diameter, and the extra resistance is, to a first approximation, proportional to the width of the strip. This relation is presented more clearly in the series of curves of Figure 20, which are taken from the last Figure, for selected frequencies. The abscissas are strip widths, in inches and cm., and the ordinates are resistance ratios. The curves are seen to be, within the limits of frequency presented, rough approximations to straight lines, particularly in the neigh-

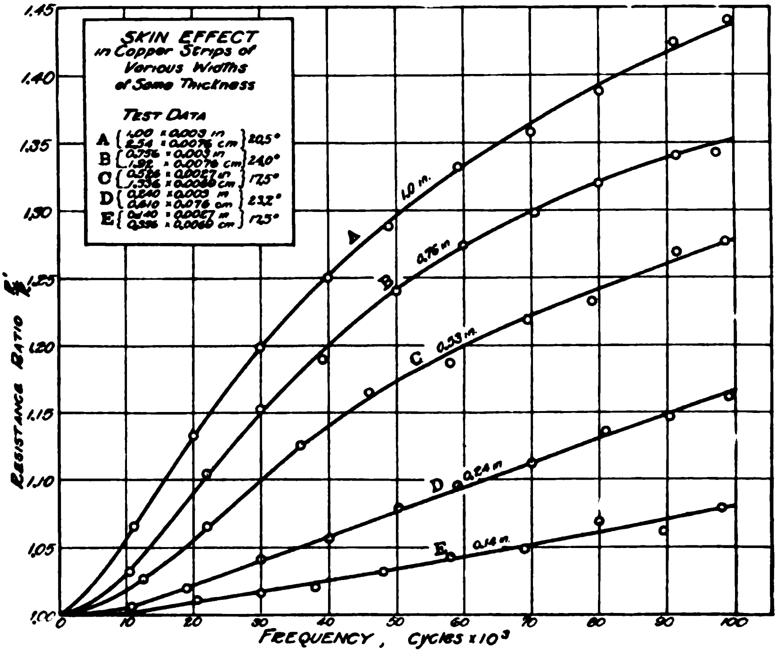


FIGURE 19—Skin Effect for Strips of Different Width but the same Thickness

borhood of 30,000 ∾. It is evident that for 0.76-millimeter strip, the effect of increasing width is to increase the resistance ratio in somewhat the same manner as increasing diameter in a round wire, so that a 4-centimeter strip at 100,000 ∾, is by no means twice as good a conductor as a 2-centimeter strip. A strip at radio frequencies is, however, always a better conductor than

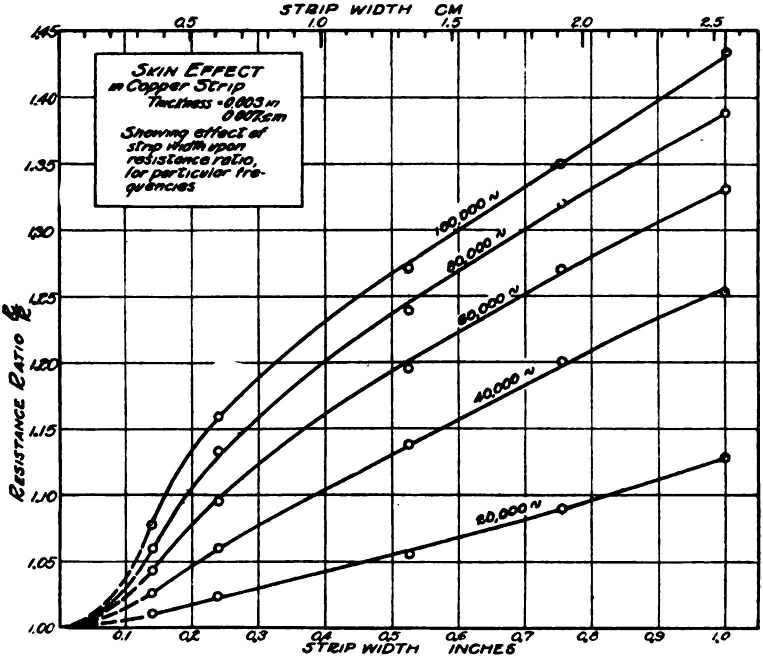


FIGURE 20—Skin Effect Variations with Strip Width, for Particular Frequencies

an equisectional round wire of the same material. Thus, Figure 21 shows the resistance ratio of the 3.8-centimeter strip, by comparison with that of an equisectional round wire 0.192 centimeter in diameter. Up to the frequency of 16,000 ∾, the round wire is the better conductor; but for all higher frequencies, the strip shows distinctly increasing advantages.

Similar tests were made upon samples of strip 0.16 millimeter thick (0.0063 in.) and of widths from 0.64 centimeter to 3.81 centimeters. The comparison between these strips and the solid round copper wire, equisectional to the widest, is shown in Figure 22. It will be observed that the curves are of the same general appearance as those for the 0.76 millimeter strips, yet they rise somewhat higher, owing to greater skin-effect in thickness.

Formulas for Skin-Effect in Strips—It has been already pointed out* that the Rayleigh skin-effect formula for indefinitely wide strip is quite inapplicable to single strips of ordinary widths. For particular ranges of dimensions and frequency, empirical formulas have recently been suggested.** It is evident experimentally that a long flat strip, carrying an alternating current, sets up a more or less cylindrical magnetic flux distribution about the center, which flux intersects the strip itself. This must give rise to eddy-current e. m. fs. in the strip. If these e. m. fs. produce local eddy-currents in the strip, they would tend to increase the power loss and the effective a. c. resistance. If, however, they produce along the entire length of the strip, an organized e. m. f. distribution, the effect will be to crowd the current toward the edges of the strip, and away from the center; thus producing what may be called an *edge-effect*, perpendicular to that plane.

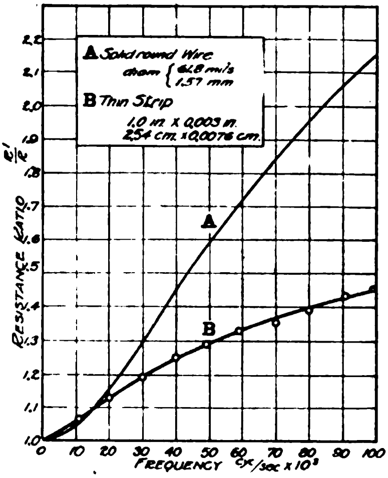


FIGURE 21—Comparative Skin Effects of Thin Copper Strip and Equisectional Round Copper Wire

In order to ascertain which eddy-current action predominated, the local eddy-current effect or the edge-effect, a sample of copper

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* Bibliography, Kennelly, Laws & Pierce, number 16.

** Bibliography, Dwight, number 17.

strip 2.22 centimeters wide (⅞ inch) and 13 meters long, was slit longitudinally near the edges with a sharp knife, in the manner indicated in Figure 23. These slits were from 6 to 8 centimeters

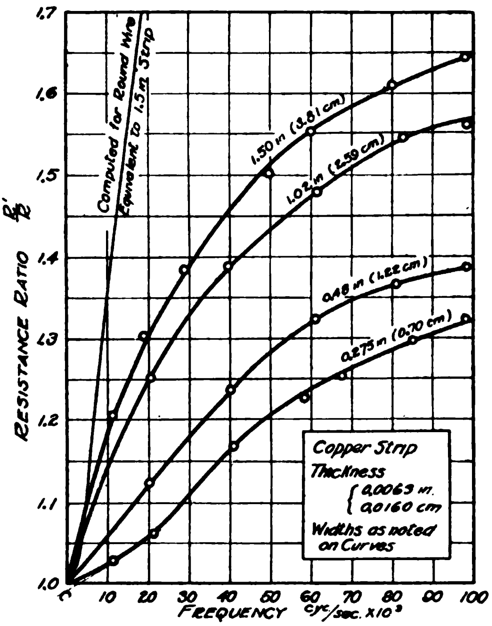

FIGURE 22—Skin-Effect for Strips of Different Width but the Same Thickness.

long, and then ceased for a distance of about 0.5 centimeter, the gaps in the slits overlapping one another; so as to maintain the structural rigidity of the conductor, while at the same time greatly hampering the circulation of eddy-currents across the strip. A strip of similar dimensions was also mounted without any slits. Up to 100,000 ∾, no appreciable difference could be detected in the skin-effect resistance ratios of the slitted and unslitted strips. The inference may, therefore, be drawn that the large increase in skin-effect resistance ratio, observed in copper strips, with reference to what should be found in infinitely wide strip, is attributable, at least in large measure, to edge-effect.

We may, therefore, consider, for purposes of discussion, that any long straight conductor of rectangular cross-section, such as a bar, prism, strap or strip, has a skin-effect in two directions, namely across its thickness and along its width. The former is commonly regarded as a skin-effect, and the latter may be called, in contradistinction thereto, an edge-effect; but actually, the two effects are essentially manifestations of one and the same phenomenon; that is, a tendency of alternating current to leave the center, and to crowd into outlying positions of the cross-section. This distortion of current density may be regarded in either of two ways; namely (1) as due to imperfect penetration of the current from the outlying positions towards the center, (2) as due to the opposing mutual inductive influence, (see Appendix IV) of parallel filaments, whereby the retarding and diminishing effects of mutual inductance are greatest at the center, and least at most remote points. According to either of these ways of viewing the matter, it must appear that the tubular form of conductor can be made to offer the minimum skin-effect at any given frequency, by reducing the wall thickness to a minimum. In the case of a rectangular section, however, the advantage of thinness may be overbalanced by the disadvantage of width, the edge-effect increasing as the skin-effect is reduced. There will thus be a best form of cross-section for each particular cross-sectional area, and frequency.

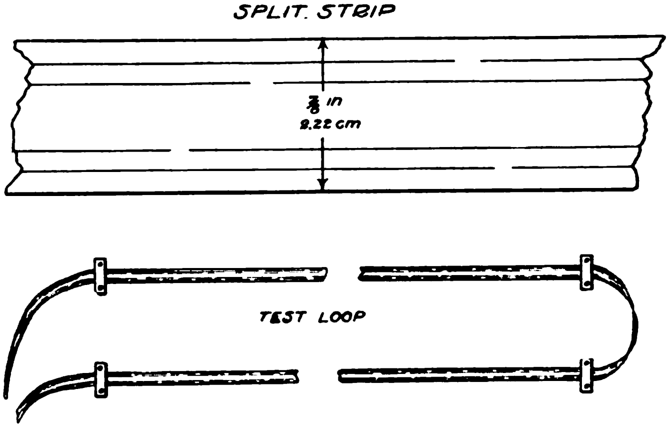

FIGURE 23—Strip Split to Reduce Eddy Currents

Parallel Opposed Strips in Close Proximity—In view of the interesting fact that in the case of a pair of parallel going and returning flat strip conductors, in close juxtaposition, one above the other, and separated only by a strip of thin insulator, the skin-effect resistance ratio at frequencies below 5,000$\sim$ is much lower than when the strips are widely separated, it seemed important to check this result at frequencies up to 100,000$\sim$. It has been pointed out by Mr. H. B. Dwight, that the data for such surface-juxtaposed parallel strips, published in the A. I. E. E. paper of 1915,* follow a certain formula which he developed, and which, as pointed out in Appendix IV, coincides with the formula for infinite strips given in that paper, when X is taken as the full thickness, instead of the half thickness of the strip, as is necessary for infinite widths:

$$\frac{Z}{R} = \frac{aX}{\tanh\,(aX)} \qquad \text{numeric} \angle \qquad (2)$$

This case constitutes the first reliable skin-effect formula for flat strips of finite width known to the writers.† Available formulas for finite strips seem to be all based on Rayleigh's infinite-strip formulas, and are therefore open to very large correction for edge-effect.

The test was made with a loop 6.1 meters long, of two flat strips, each 2.54×0.0076 centimeters (1.0×0.003 inch) laid on a flat board, the lower strip being entirely enveloped in tissue paper, 0.0038 centimeter (0.0015 inch) thick, and the upper strip laid carefully over this, so as to keep the edges parallel. Little bindings of paper were then applied to the two insulated strips, after which weights were applied uniformly to the system. Great care had to be taken to secure good alignment of the edges in the opposing strips, since it was found, in preliminary trials, that a relatively small amount of overprojecting at the edges, added materially to the losses and apparent resistance of the loop.

The d. c. resistance of the loop at 20° C. was 0.102 ohm, and the a. c. resistance at 93,000$\sim$, was the same, within the limits of observational error; so that the skin-effect resistance ratio up to this frequency was sensibly 1.00. In such a disposition of conductors, virtually forming a paper condenser, the susceptance of the dielectric may have an appreciable effect on the measurement. In this case, however, the loop was so short that the error due to capacity susceptance was negligible.

Copper tubes are of especial interest in relation to skin-effect; since, as already mentioned, their resistance ratio should be capable of being made lower than that of any other form of

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* Bibliography, number 16.

† A somewhat similar case is, however, discussed by Lord Rayleigh, Bibliography, number 1.

conductor. A tube of large diameter and very thin wall should presumably have a skin-effect resistance-ratio calculable on the basis of infinitely wide strip with X, see formula (2), taken as equal to the wall thickness.

Two copper tubes were tested. One was a slit tube of 2.22 centimeter (⅞ inch) perimeter, made by rolling a length of the thin flat strip previously tested (2.22 cm. wide × 0.076 mm. thick) over a cylindrical wooden form. This produced a slit copper tube 12 meters long, and of an external diameter approximately 7.2 millimeters. The calculated skin-effect resistance ratio of 0.076 millimeter copper strip of indefinitely great width is, by (2) 1.005 at 100,000∾. When the tube conductor made of this strip was tested at 100,000∾, its skin-effect resistance ratio was found to agree with this value, within the limits of observational error. The same strip, when tested flat, before being rolled into tube, was found to give 1.38 at 100,000∾. This experiment gave a striking demonstration of the influence of edge-effect in one and the same single loop of copper conductor. With the edge-effect present in the flat strip, the a. c. resistance at 100,000∾ was nearly 40 per cent. greater than the d. c. resistance. With the edge-effect removed, by bending the same strip into a simple slit tube, the a. c. resistance, at the same frequency, was hardly greater than the d. c. resistance.

The second tube was a soft drawn continuous copper tube of external diameter 3.18 millimeters and of wall thickness 0.65 millimeter (number 22 A. W. G.). Its length was approximately 25 meters, in two horizontal loops, or four horizontal passages, along the wall, with an interaxial distance of about 20 centimeters between adjacent conductors.

Figure 24 shows the observed skin-effect resistance ratios of this tube at various frequencies up to 100,000∾. It will be observed, that at 100,000∾, this ratio is 2.46. The round wire of equivalent cross-section, shown in dotted lines, would have, at the same frequency, a ratio of 3.17. The curve C shows the computed ratios for infinitely wide strip, of the same thickness (0.65 mm.), that is, a tube of infinite diameter and this wall thickness; while curve D shows the corresponding ratios for a tube of zero internal diameter; that is, a solid wire of radius 0.65 millimeter. It will be seen that the curve OA of observations lies nearly midway between the curve of infinite internal diameter, and of zero internal diameter, having the same wall thickness, on the assumption that the magnetic flux density vanishes within a uniform straight hollow cylinder carrying a current. The curve OA corresponds very closely to formula (2) when X is taken as 82 per cent. of the actual wall thickness.

Comparisons of Skin-Effect Resistance Ratios for Different Types of Conductor—When an alternating current of radio-frequency has to be carried with the least loss for a given amount of copper, it is evident that the skin-effect resistance ratio must be made as near unity as possible. It is therefore important to consider what form a cross-section of given number of square millimeters of copper should have, in order to make the conductor most efficient. The best form of the conductor will depend to some extent upon the frequency.

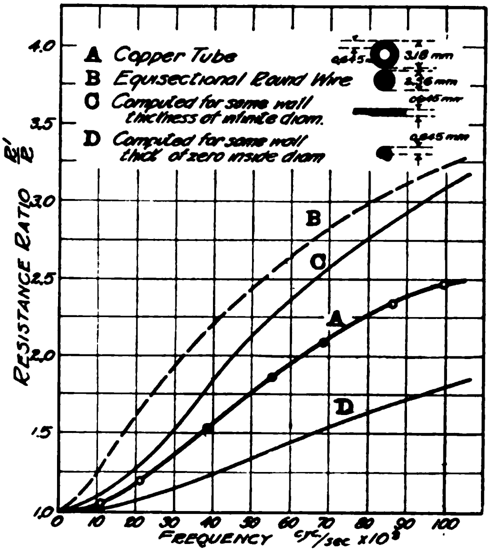

FIGURE 24—Skin-Effect in Copper Tube

For the particular frequency of 100,000~, Figure 25 shows the skin-effect resistance ratios of various kinds of geometrical section. The ordinates are in R'/R. The abscissas are in square millimeters, and also in square mils (10^{-6} sq. inch) of copper cross-section. It will be seen that round solid wire has the greatest resistance ratio thruout almost the entire range of cross-section shown, altho spiralled 7-strand conductor, devoid of internal insulation, would have a resistance ratio in excess of this by an amount depending on the degree of spirality. All other forms of conductor, for a given cross-section, have less resistance ratio.

The few tests thus far made of stranded conductors are plotted on the diagram. The diminution in resistance ratio depends upon the number of strands, their internal transposition and separation. Below most of these come three-mil (0.076 mm.) copper strip, which is not so low as six-mil (0.15 mm.) strip. As already explained, the edge-effect of 3-mil strip, which must be twice as wide as 6-mil strip for the same cross-section, predominates over skin-effect at 100,000∾, so that there is a disadvantage, within the limits of cross-section shown, in rolling 0.15-millimeter strip into 0.075-millimeter strip for 100,000∾. The limit of thickness to which flat strip should be rolled, for a given cross-section, is evidently a function of the frequency, which determines the equivalent penetration skin-depth δ. The tests have not yet been carried sufficiently far to assign the economical thickness limit; except that, at 100,000∾, it is above 3 mils (0.076 mm.) for cross-sections less than 6 square millimeters.

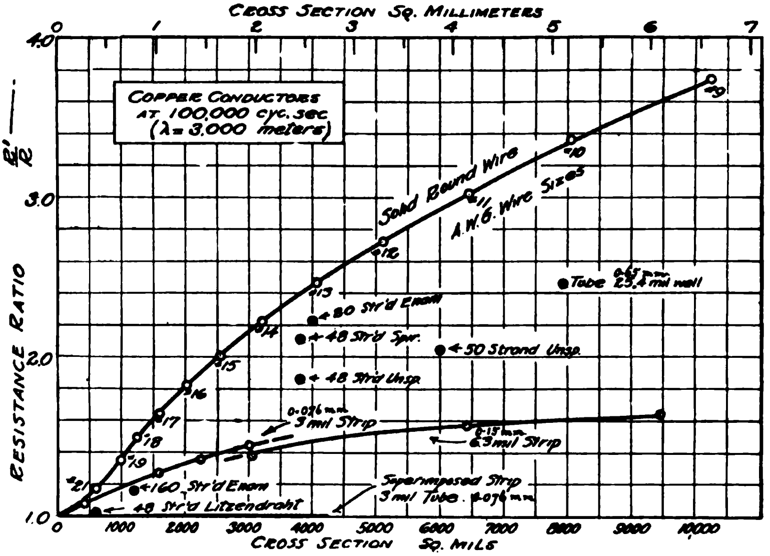

FIGURE 25—Comparative Skin-Effects for Different Types of Conductors

The only apparent means of avoiding the edge-effect, so as to permit of using very thin sheets of conductor, is either to use two opposing parallel strips in close proximity, as described in

Appendix III; or to roll a single strip up into circular tube form. In either case, as Figure 25 shows, the resistance skin-effect ratio of 0.075-millimeter strip fell to almost 1.0. The use of opposing juxtaposed parallel strips, as going and returning conductors, has limitations, owing to the large linear capacitance involved. No matter how high the frequency, there seems to be no electrical limit to the reduction of extra skin-effect resistance, by using tubular conductors of sufficiently thin wall. Mechanical considerations suggest that this might best be accomplished by using a flexible insulating core, with fine copper wires braided over the surface. In such a flexible conductor, by taking thin enough wire, and avoiding spirality effects, the skin-effect should be reduced indefinitely, since there could be no edge-effect.

The data contained in Figure 25 are presented in a somewhat different form in Figure 26, which shows the total linear resistance of copper conductors of different types of cross-section at 100,000 ~ and 20° C, for different cross-sectional areas as abscissas. By reference to this diagram, the most suitable conductor can be chosen for this particular frequency, so far as the experimental data, thus far obtained, will permit.

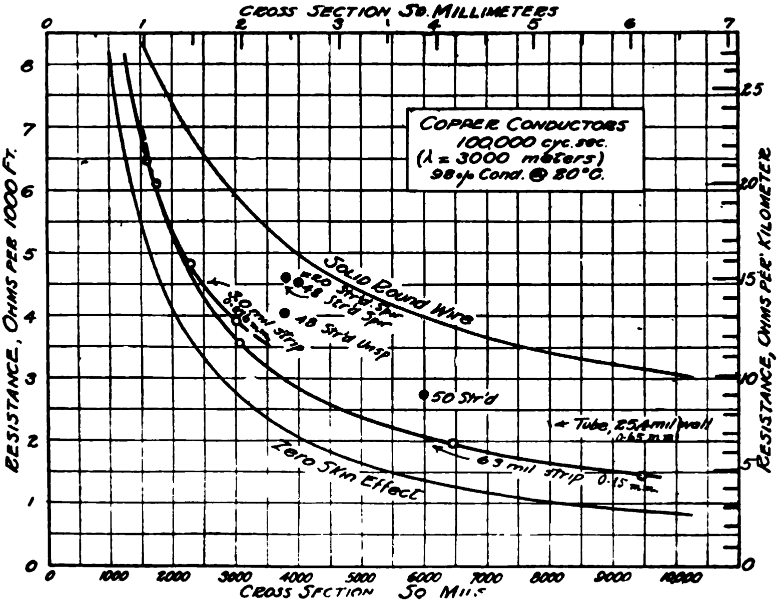

FIGURE 26—Comparative Linear Resistances of Conductors of Different Types at 100,000 Cycles

SUMMARY

The following conclusions have been reached from tests at frequencies up to 100,000∿, on straight conductors, shielded from proximity effects, and of cross-sections up to 6 square millimeters.

(1) The skin-effect resistance ratio of round copper wires has been found to conform to the standard Heaviside-Kelvin Bessel-function formulas, within the limits of experimental error.

(2) The skin-effect resistance-ratio of a copper conductor formed of seven equal and parallel round bare strands, (six surrounding a central seventh), was found to be the same as that of the equisectional solid round wire up to 100,000∿, within the limits of observational error. It is therefore inferred that the subdivision of a round wire into a round cable of uninsulated contacting strands does not alter the skin-effect.

(3) The effect of simple spiralling of strands in the same direction, increased the resistance ratio.

(4) The resistance ratio of a subdivided wire, with parallel insulated strands, fell below that of the equisectional solid wire, and diminished rapidly with the spacing, or degree of strand separation.

(5) The braiding of strands, so as to effect their transposition in the cross-section at frequent intervals, was found to diminish the skin-effect.

(6) The skin-effect in copper strips was found to be usually, but not invariably, less than that of equisectional round wires.

(7) Increasing the width of a copper strip within the limits reported, was found to increase the skin-effect resistance ratio; owing to what may be called "edge-effect."

(8) A pair of parallel going and returning flat strips, separated by a thin insulating layer, was found to have a skin-effect depending only on the strip thickness; that is, without perceptible edge-effect.

(9) Rolling a flat strip into the form of a slit tube destroyed the edge-effect, and reduced the resistance ratio to a minimum, dependent only on the strip thickness.

(10) Cutting longitudinal slits with a sharp knife in a thin flat copper strip, was found not appreciably to affect its skin-effect resistance ratio.

(11) In order to employ a stranded wire of minimum skin-effect at radio-frequency, it seems desirable to employ thick insulation on the strands, such as double cotton, to transpose

the strands by braiding, and to avoid spirality in one and the same direction.

(12) In order to employ a flat strip conductor most effectively, it is necessary to stop rolling it out laterally when the increasing edge-effect more than offsets the reduction in skin-effect.

(13) The hollow tubular form seems to be the most efficient type of radio-frequency metallic conductor, since, with proper mechanical internal support, the skin-effect can be indefinitely diminished by diminishing the wall thickness, and the edge-effect is absent.

APPENDIX I

DETAILS OF APPARATUS AND PROCEDURE FOR MEASURING SKIN-EFFECT RESISTANCE RATIOS, AT FREQUENCIES BETWEEN 10,000 ∾ AND 100,000 ∾

The detailed electrical connections of the induction bridge referred to above, under the title "Inductive Bridge Method," are presented in Figure 27. The alternator *A* supplies a current of from 4 to 5 amperes r. m. s. thru the hot-wire ammeter *M* to the bridge, with the aid of the adjustable condenser C_3 of about 1 μf. (microfarad), which partially compensates the inductive reactance of the bridge circuit. The adjustable bridge resistance is a single slide-wire meter of Ia-Ia high-resistivity wire, number 18 A. W. G. (diameter 1.02 mm.) offering 0.0058 ohm per linear centimeter, or 0.58 ohm in all. The resistance measurements are all taken in terms of lengths of this slide wire, which possesses negligible skin-effect.

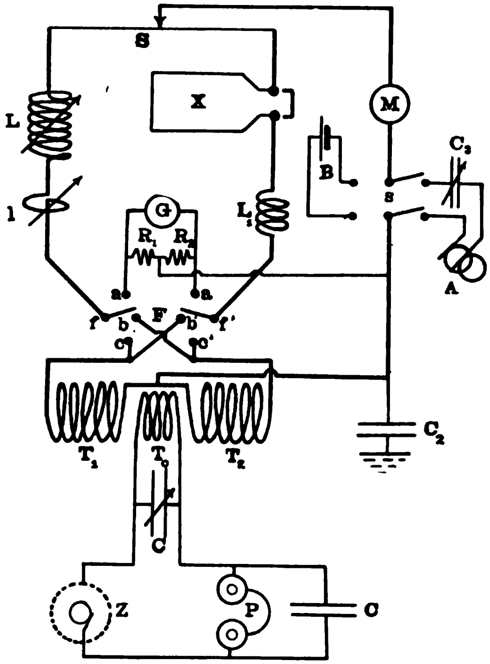

Figure 27—Detailed Diagram of Differential Bridge Circuit as Employed in Tests.

The test conductor *X* is connected between two stout terminals containing mercury, that can be shorted by a heavy copper link. A bridge balance is obtained by adjusting L and S, first with X shorted, and then with X unshorted. The difference between the readings of S in these two tests, measures the resistance of X at the impressed frequency.

The switch s changes from the test dry battery B, of about 2 volts, for d. c. measurements, to the alternator A, for a. c. measurements. The switches ff' are thrown to aa', for d. c. measurements, and to bb' for a. c. measurements. The contacts cc' provide for the reversal of bb', that is, for the exchange of the two similar and equal inductive arms T_1 and T_2. Each of these is wound in two layers of 8 turns, or 16 turns all told, in a groove 1.5 centimeter wide cut in a wooden cylinder, the mean winding diameter being 15 centimeters. The wire in T_1 and T_2 consists of 9 strands each of 9 enamel-insulated number 30 A. W. G. (0.25 mm.) wires, or 81 insulated number 30 wires in all. The mid-point between these two coils T_1 T_2 is connected to ground thru an adjustable condenser C_2 of from 0.5 to 4 μf., which helps to minimize electrostatic disturbances in the detector P. The secondary winding T_0 consists of 180 turns of plain double cotton-covered number 18 A. W. G. copper wire (1.02 mm. diam.) equally distributed in two grooves in the wooden cylinder, one on each side of the primary winding.

The fixed inductance L_1 has 20 turns in 2 layers of 90 insulated strands number 30 A. W. G. wire. The mean winding diameter is 8 centimeters and winding breadth 3.8 centimeters. The object of this inductor is to assist the test wire X to make up an inductance within the range of the inductance variometer L. This variometer has an outer fixed coil and an inner frictionally movable coil, the angle between the axes of the two being adjustable by hand. Each of the two coils has 12 turns in 2 layers. The mean diameter of the pair is 15 centimeters. The wire has 90 enamelled strands of number 30 A. W. G., and the total maximum inductance is about 0.1 mh. After an approximate inductance balance has been reached in L, a yet finer adjustment to balance can be made with the single turn l, suspended beneath the testing table, and capable of being altered in its area, and therefore in its self-inductance, by a winding spindle controlled from a distance, so that the observer's hand does not have to reach over the table when making the final adjustment.

The tuning adjustable condenser C has about 4 m.μf. (millimicrofarads) and C_1, the condenser shunting the detector telephones, about 3 m. μ f. C is adjusted for best hearing with the impressed frequency. The frequency being ordinarily above the limit of audibility, the commutator interruptor Z, motor-driven at approximately 1,000 makes and breaks per second, produces an audible effect in the head telephones P, which have each a d. c. resistance of approximately 1,000 ohms.

For the d. c. measurements, the equal-resistance arms R_1 R_2 have each 0.25 ohm, and the portable galvanometer G has 250 ohms, with a sensitivity of 1 scale division for 1 microampere.

To make a test of a conductor X, it is connected to the shorted binding posts. The switch s is thrown to the left, f to a and f' to a'. A d. c. Wheatstone-bridge balance is then obtained on G, by adjusting the position of the slider S. The short is then removed from X, and the balance repeated, at a new position of S. The change in the slide-wire reading measures R, the d. c. resistance of X. Arrangements, not shown in Figure 27, also permit of reversing the arms R_1 and R_2, so as to reveal any inequality they may possess.

The switch s is then thrown to the right, the alternator A having first been brought to the desired speed and frequency. Switch f is thrown to b and f' to b'. The alternating current in the bridge circuit is observed at M, and adjusted to between 4 and 5 amperes r. m. s., with the aid of the condenser C_3. The conductor X being shorted, a balance for both resistance and inductance is obtained telephonically at P, by successive adjustments of S and L. It is important that the observer should take up a stationary position with respect to the apparatus, and that the final adjustments should be made at S and l, without changing his position. The short is now removed from the conductor X, and a new balance obtained at S and $L + l$. The change in the reading of S measures R', the a. c. resistance of X, including the skin-effect. Reversals of f and f' enable average values to be secured.

With careful manipulation, the skin-effect resistance ratio in specimens of wire X, whose total d. c. resistance lies preferably between 0.2 and 0.4 ohm, may be measured to within a deviation probable error, for a single observation, of about ⅓ of 1 per cent.

A photograph of the testing table is shown in Figure 28, with the elements marked to correspond with Figure 27.

It has been found advisable to select a pair of head telephones with cases of insulating material instead of metal, since the capacitance disturbance thru the body of the observer on the bridge balance is thereby greatly reduced.

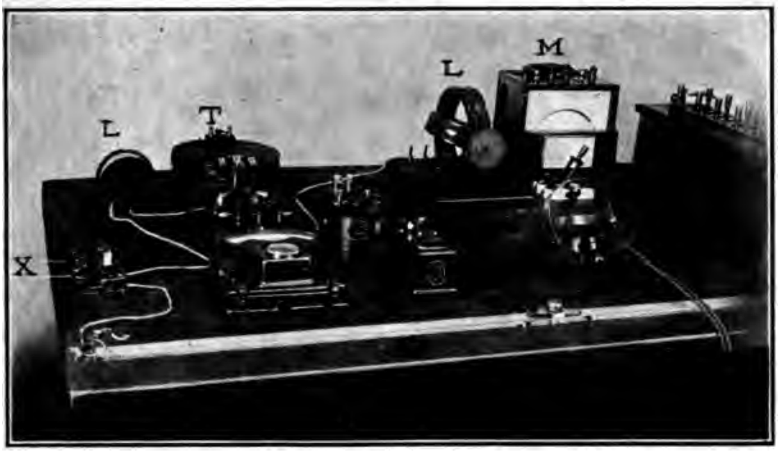

FIGURE 28—Photograph of Testing-Table Apparatus

APPENDIX II

COMPUTATION OF SKIN-EFFECT RESISTANCE RATIO IN ROUND WIRES

The theory of skin-effect in round wires has been developed and published by a number of investigators, commencing with the work of Clerk Maxwell in 1873. A bibliography of the subject appears in the paper by Kennelly, Laws and Pierce in the "Transactions of the American Institute of Electrical Engineers," September, 1915. The formula, as developed in that paper, following Jahnke and Emde is

$$\frac{Z}{R} = \frac{a_0 X}{2} \cdot \frac{J_0(a_0 X)}{J_1(a_0 X)} \qquad \text{numeric} \angle \qquad (3)$$

where Z is the linear impedance of the wire at the impressed single frequency $f^{\sim}$, R is its linear resistance to steady currents. X is the radius of the wire in centimeters, $J_0(a_0 X)$ and $J_1(a_0 X)$ are respectively Bessel functions of zero and first order, a_0 is the semi-imaginary quantity

$$a_0 = \sqrt{-j\,4\pi\gamma\mu\omega} = \sqrt{2\pi\gamma\mu\omega} - j\sqrt{2\pi\gamma\mu\omega} = a_2 - j\,a_2 \qquad \text{cm.}^{-1} \angle \qquad (4)$$

where $j = \sqrt{-1}$
$\pi = 3.14159 \ldots$
γ = conductivity of wire at actual temperature $\left(\frac{\text{abmhos}}{\text{cm.}}\right)$

μ = permeability of wire at actual temperature

$$\left(\frac{\text{gausses}}{\text{gilberts per cm.}}\right)$$

$\omega = 2\pi f$ the impressed angular velocity

$$\left(\frac{\text{radians}}{\text{sec.}}\right)$$

This formula gives a vector skin-effect impedance ratio

$$\frac{Z}{R} = M \angle \beta^\circ \qquad \text{numeric} \angle \qquad (5)$$

the real component of which is the skin-effect resistance-ratio

$$\frac{R'}{R} = M \cos \beta \qquad \text{numeric} \qquad (6)$$

or

$$\frac{R'}{R} = \left| \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \right| \cdot \cos \beta \qquad \text{numeric} \qquad (7)$$

In the above mentioned paper, the values of a_o (at $\rho = 1724$ absohm-cm.), $J_o(Z \angle = 45^\circ)$ and $J_1(Z \angle = 45^\circ)$, were tabulated and graphed over a certain range. Altho those tables assist computation, yet since for ordinary purposes the resistance ratio $\frac{R'}{R}$ only is required, a simpler basis of computation is obtained by tabulating and plotting the values of (3) assuming a certain representative value of γ.

Figure 29 gives the graph of $\frac{R'}{R}$ with reference to $\sqrt{\frac{f}{R'_o}}$, and also with reference to $|a_o X|$, as abscissas. It will be noticed that for values of $|a_o X|$ greater than 4.0, the curve approximates to the straight line

$$\frac{R'}{R} = \frac{|a_o X|}{2\sqrt{2}} + \frac{1}{4} = \frac{a_2 X}{2} + \frac{1}{4}.$$

Russell's formula* for the ratio $\frac{R'}{R}$ in a round wire, and using the notation of this paper, can be put in the form

$$\frac{R'}{R} = \frac{a_2 X}{2} + \frac{1}{4} + \frac{3}{32 a_2 X} - \frac{1}{16 (a_2 X)^3} + \cdots \qquad \text{numeric} \qquad (8)$$

It is evident that for large values of $a_2 X$, we may, for practical purposes, ignore all terms after the second, as is shown by the curve in Figure 29. We may also arrive at the same result by considering the depth of the equivalent skin δ centimeters in a plate conductor carrying rapid alternating currents; i. e., such a depth as, without skin effect, or with the full conductivity of continuous currents, would carry the same alternating-current

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* Proc. Phys. Soc., London, Jan., 1909.

strength as the actual plate does in the presence of skin effect. It is shown in the Kennelly-Laws-Pierce paper of 1915 that, for a flat plate, this equivalent skin depth has the value

$$\delta = \left| \frac{\tanh aX}{aX} \right| \frac{X}{\cos\beta} \qquad \text{cm.} \quad (9)$$

where X is the half thickness of the plate, a is the semi-imaginary

$$a = \sqrt{j4\pi\gamma\mu\omega} = \sqrt{2\pi\gamma\mu\omega} + j\sqrt{2\pi\gamma\mu\omega} = a_2 + ja_2 \quad \text{cm.}^{-1}\angle \quad (10)$$

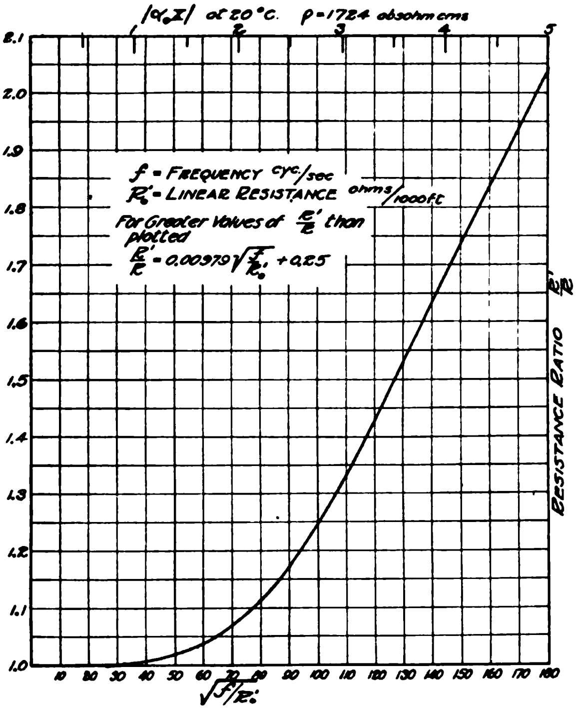

FIGURE 29—Chart for Skin-Effect Computations on Solid Round Wires

When $|aX|$ exceeds 6.0, this approximates closely to

$$\delta = \frac{1}{a_2} = \frac{1}{\sqrt{2\pi\gamma\mu\omega}} \qquad \text{cm.} \quad (11)$$

Applying this plate theory to a round wire of such dimensions that we may ignore the curvature of the surface, the linear resistance of the wire with continuous currents will be

$$R = \frac{\rho}{\pi X^2} \qquad \frac{\text{absohms}}{\text{cm.}} \quad (12)$$

where $\rho = \frac{1}{\gamma}$ is the resistivity. The linear resistance of the same wire to radio-frequency alternating currents will be

$$R' = \frac{\rho}{\delta \,.\, 2\pi\left(X - \frac{\delta}{2}\right)} \qquad \frac{\text{absohms}}{\text{cm.}} \qquad (13)$$

since the current is now supposed to be carried entirely by a superficial cylinder of depth δ centimeter and mean radius $\left(X - \frac{\delta}{2}\right)$ centimeter. The skin-effect resistance ratio of the wire will therefore be

$$\frac{R'}{R} = \frac{X^2}{2\delta\left(X - \frac{\delta}{2}\right)} = \frac{X}{2\delta\left(1 - \frac{\delta}{2X}\right)} \qquad \text{numeric} \quad (14)$$

and since $\delta/(2X)$ is a small quantity with respect to unity, we may take

$$\frac{R'}{R} = \frac{X}{2\delta}\left(1 + \frac{\delta}{2X}\right) = \frac{X}{2\delta} + \frac{1}{4}$$

$$= \frac{a_2 X}{2} + \frac{1}{4} \qquad \text{numeric} \quad (15)$$

which is Russell's formula as far as two terms.*

For practical work with round non-magnetic wires, for which $\mu = 1$, it may be noted that if R_o be the resistance in ohms of a wire of length l centimeters; then for all frequencies

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5} \sqrt{l\pi} \,.\, \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (16)$$

for a wire of R_o, expressed in ohms per meter ($l = 100$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-4} \sqrt{\pi} \,.\, \sqrt{\frac{f}{R_o}}$$

$$= 5.6 \times 10^{-4} \,.\, \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (17)$$

If, however, we use R'_o as the resistance of 1,000 feet of wire as found in the ordinary wire tables ($l = 3.048 \times 10^4$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5} \sqrt{3.048\pi \times 10^4} \,.\, \sqrt{\frac{f}{R'_o}}$$

$$= 0.00979 \sqrt{\frac{f}{R'_o}} \qquad \text{numeric} \quad (18)$$

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*A. Russell, *Phil. Mag.*, April, 1909; *Proc. Phys. Soc. London*, volume 21, 1909.

and $|a_o X| = 0.02768\sqrt{\frac{f}{R'_o}} = \frac{1}{36.1} \cdot \sqrt{\frac{f}{R'_o}}$ numeric (18a)

Consequently, in terms of wire-table resistances of a given size of wire at a certain temperature, when $a_2 X$ is greater than 3 say, formula (15) reduces to

$$\frac{R'}{R} = 5.6 \times 10^{-4}\sqrt{\frac{f}{R_o}} + 0.25 \qquad \text{numeric} \quad (19)$$

or

$$= 9.79 \times 10^{-3}\sqrt{\frac{f}{R'_o}} + 0.25 \qquad \text{numeric} \quad (20)$$

We may therefore plot a curve in terms of $\sqrt{\frac{f}{R_o}}$, or $\sqrt{\frac{f}{R'_o}}$, as abscissas, and read off the skin-effect resistance ratios as ordinates. For values of $\sqrt{\frac{f}{R'_o}}$ above say 200, the values of $\frac{R'}{R}$ fall near to the straight line of formula (15); see Figure 29.

APPENDIX III

ELEMENTARY THEORY OF A FLAT LOOP FORMED BY A PAIR OF FLAT PARALLEL COPPER STRIPS, ONE IMMEDIATELY OVER THE OTHER AND SEPARATED BY A UNIFORM THIN INSULATING LAYER

Let the width of each strip be *b* centimeters and *d* the thickness in centimeters of the thin insulating layer. The skin-effect resistance and inductance ratios of two such copper-strip loops of different sizes was recorded in Figures 14 and 15 of the paper by Kennelly, Laws and Pierce, in the A. I. E. E. Proceedings for August, 1915. It was discovered by Mr. H. B. Dwight,* that these curves correspond almost exactly (in the symbols here employed) with the formula:

$$\frac{Z}{R} = \frac{1 + \frac{(aX)^2}{2!} + \frac{(aX)^4}{4!} + \frac{(aX)^6}{6!} + \ldots}{1 + \frac{(aX)^2}{3!} + \frac{(aX)^4}{5!} + \frac{(aX)^6}{7!} + \ldots} \qquad \text{numeric}\angle \quad (21)$$

where $\frac{Z}{R}$ is the skin-effect impedance ratio of the loop, X is the thickness in centimeters of each strip, and a is a semi-imaginary quantity (affected by $\angle 45°$) defined below.

But this formula reduces to

$$\frac{Z}{R} = \frac{aX}{\tanh aX} \qquad \text{numeric}\angle \quad (22)$$

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* Bibliography 20.

which is the skin-effect impedance ratio of an infinitely wide strip as given in formula (100) of that paper, differing only in form from Rayleigh's formula, except that, whereas for an infinitely wide single strip, X should be taken as the half-strip thickness, when used for Dwight's reduction it should be taken as the whole thickness. In other words, the formula for infinite strip applies to each of a pair of parallel finite strips, when their surfaces are juxtaposed, and X is taken as the full thickness instead of the half thickness of each strip.

Referring to Figure 30, if $A\,B\,C\,D$ and $E\,F\,G\,H$ are the cross-sections of a pair of similar and parallel copper strips forming respectively the going and returning conductors of a long flat loop, the strips being separated by an insulator of permeability unity and uniform thickness d centimeters; the symmetry of the geometrical relations permits of either strip being selected for obtaining the results applying to both. Then the magnetic flux inside this loop will be parallel to the strips except near the edges. Moreover, if $+I$ be the electric current strength (abs-amperes) flowing in the lower strip, and $-I$ that in the upper; then the total magnetomotive force in the magnetic circuit will be $4\pi I$ gilberts. The reluctance of the magnetic circuit will reside almost entirely in the narrow channel of insulator between the strips, and the residual reluctance in the external widely extending return path will be relatively negligible. This means that the $4\pi I$ gilberts will be expended almost wholly in the channel between the strips; where the magnetic field intensity will be very nearly

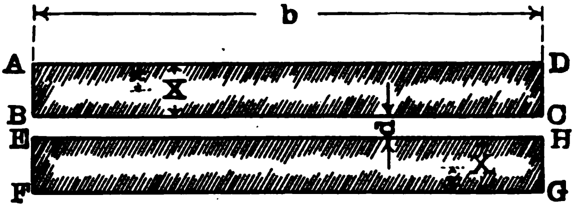

FIGURE 30—Superimposed Strips

$$\mathcal{H} = \frac{4 \pi I}{b} \qquad \text{gilberts per cm.} \qquad (23)$$

producing correspondingly a uniform flux density in the channel

$$\mathcal{B} = \frac{4 \pi I}{b} \qquad \text{gausses} \qquad (24)$$

At the outer surfaces of the strips, the flux density will be substantially zero. The differential equation of magnetic instantaneous alternating flux density at any distance x centimeters inwards from the surfaces $F\,G$ or $A\;D$ is then

$$\frac{d^2 \mathfrak{B}_x}{d\,x^2} = a^2 \mathfrak{B}_x \qquad \frac{\text{gausses}}{\text{cm.}^2}\angle \qquad (25)$$

where $a^2 = j\,4\,\pi\,\gamma\,\mu\,\omega$ cm.$^2\angle$

$j = \sqrt{-1}$

γ = conductivity of the strip in $\frac{\text{abmhos}}{\text{cm.}}$

μ = permeability of the strip taken as unity $\frac{\text{gausses}}{\text{gilberts per cm.}}$

$\omega = 2\,\pi\,f$ impressed angular velocity $\frac{\text{radians}}{\text{sec.}}$

f = impressed frequency $\frac{\text{cycles}}{\text{sec.}}$

Similarly, the differential equation for current density $\dot{i}_x$ absamperes per square centimeter at elevation x centimeters is

$$\frac{d^2 \dot{i}_x}{d\,x^2} = a^2 \,.\, \dot{i}_x \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (26)$$

The solution of both (25) and (26) is of the same type:

$$\dot{i}_x = A_1 \cosh ax + B_1 \sinh ax \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (27)$$

At $x=0$ or at the outer surface FG of the strip, $\cosh a\,x = 1$, $\sinh a\,x = 0$,

$$\dot{i}_o = A_1 \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (27a)$$

and between (27) and (27a) B_1 can be shown to be zero; so that

$$\dot{i}_x = \dot{i}_o \cosh a\,x \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (28)$$

$$\frac{\dot{i}_{xm}}{i_{Xm}} = \frac{\dot{i}_{xr}}{i_{Xr}} = \frac{\dot{i}_x}{i_X} = \frac{\cosh ax}{\cosh a X} \qquad \text{numeric}\angle \qquad (29)$$

where the subscripts m and r represent respectively maximum cyclic and r. m. s. values of the current density. If $a\,x$, a semi-imaginary quantity, (of equal real and imaginary components) is defined as the *hyperbolic position angle* corresponding to the layer of distance x from the external surface, the current density varies directly as the hyperbolic cosine of the position angle. Such cosines have all been tabulated and charted. (Bibliography 18.)

The average r. m. s. current density over the cross-section is

$$i_{qr}=\frac{1}{X}\int_0^X i_{xr}\cdot dx=\frac{1}{X}\cdot\frac{i_{Xr}}{\cosh\alpha X}\cdot\int_0^X\cosh\alpha x\ .\ dx$$
$$=\frac{1}{\alpha X}\cdot\frac{i_{Xr}}{\cosh\alpha X}\cdot\sinh\alpha X=i_{Xr}\frac{\tanh\alpha X}{\alpha X}$$
$$\frac{\text{absamperes}}{\text{sq. cm.}}\angle \qquad (30)$$

Hence the skin-effect impedance ratio $\frac{Z}{R}$ of the whole strip is

$$\frac{Z}{R}=\frac{i_{Xr}}{i_{qr}}=\frac{\alpha X}{\tanh\alpha X} \qquad \text{numeric}\angle \qquad (31)$$

The real component of this expression is the resistance ratio R'/R, the imaginary component is the reactance ratio $\frac{L'\omega}{R}$

The quantity $\frac{\tanh\alpha X}{\alpha X}$ where αX is a semi-imaginary, has been tabulated and charted as far as the modulus of $\alpha X=3.0$ and $\tanh\alpha X$ of the same semi-imaginary to a modulus of 20.

The solution of (25) is also

$$\mathfrak{B}_x=A_2\cosh\alpha x+B_2\sinh\alpha x \qquad \text{gausses}\angle \qquad (32)$$

and at $\alpha x=0$ $$\mathfrak{B}_0=A_2=0 \qquad \text{gausses}\angle \qquad (33)$$

Consequently $$\mathfrak{B}_x=B_2\sinh\alpha x \qquad \text{gausses}\angle \qquad (34)$$

and at X $$\mathfrak{B}_X=\frac{4\pi I}{b} \qquad \text{gausses}\angle \qquad (35)$$

whence $$B_2=\frac{4\pi I}{b\sinh\alpha X} \qquad \text{gausses}\angle \qquad (36)$$

so that $$\mathfrak{B}_x=\frac{4\pi I}{b}\cdot\frac{\sinh\alpha x}{\sinh\alpha X} \qquad \text{gausses}\angle \qquad (37)$$

The magnetic flux density in the strip, substantially in horizontal-layer distribution, is thus directly proportional, at any distance x from the outside surface, to the hyperbolic sine of the position angle αx. Such sines have all been tabulated and charted.

When the position angle αX at the inner surface of the strip exceeds in modulus 6.0, tanh αX is very nearly unity, and the skin-effect impedance ratio $\frac{Z}{R}$ becomes αX simply, a semi-imaginary. The real component, or skin-effect resistance-ratio, then becomes $R'/R=|\alpha X|/\sqrt{2}=\alpha_2 X$, and is equal to the imaginary component or skin-effect reactance ratio $L'\omega/R$, while the equivalent skin depth δ is $1/\alpha_2$ centimeter.

APPENDIX IV

ELEMENTARY THEORY OF THE LINEAR IMPEDANCE OF A LONG STRAIGHT CONDUCTOR FORMED OF SEVEN EQUALLY SPACED STRANDS UNSPIRALLED

Let a central round wire O Figure 31, of radius ρ centimeters be surrounded by six similar and parallel wires 1 to 6, forming a hexagon all at equal interaxial distances d centimeters. The linear resistance of each wire, with its own extra skin-effect resistance is r absohms per linear centimeter. The return conductor is a similar parallel open strand, at a distance D centimeters, which is large with respect to d. A simple alternating e. m. f. of angular velocity $\omega = 2\pi f$ radians per second, is impressed upon the loop. Required the linear impedance of the conductor.

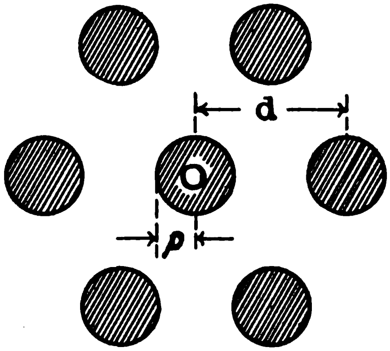

FIGURE 31—Spacing of 7-Strand Conductor.

Let i_0 be the r. m. s. complex current strength in the middle wire (absamperes ∠)

i_1 be the r. m. s. complex current strength in any outside wire (absamperes ∠)

l_0 be the linear inductance of the middle wire (abhenrys/cm.)

l_1 be the linear inductance of any outside wire (abhenrys/cm.)

z_0 be the linear impedance of the middle wire (absohms/cm. ∠)

z_1 be the linear impedance of any outside wire (absohms/cm. ∠)

m be the linear mutual inductance between middle and an outside wire (abhenrys/cm. ∠)

m' be the linear mutual inductance of any five outer wires on the sixth (abhenrys/cm. ∠)

M be the linear mutual inductance of return strand on any wire (abhenrys/cm. ∠)

e be the linear r. m. s. e. m. f. impressed on any wire $\left(\frac{\text{abvolts}}{\text{cm.}} \angle\right)$

$\varepsilon = 2.71828 \cdots, \quad j = \sqrt{-1}.$

Then on any outside wire

$$e = i_1 r + j i_1 l_1 \omega + j i_o m \omega + j 5 i_1 m' \omega - j 6 i_1 M \omega - j i_o M \omega \qquad \frac{\text{abvolts}}{\text{cm.}} \angle \qquad (38)$$

and on the inside wire

$$e = i_o r + j i_o l_o \omega + j 6 i_1 m \omega - j 6 i_1 M \omega - j i_o M \omega \qquad \frac{\text{abvolts}}{\text{cm.}} \angle \qquad (39)$$

The values of the self-inductances are

$$l_o = l_1 = 2\left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4}\right) \qquad \frac{\text{abhenrys}}{\text{cm.}} \angle \qquad (40)$$

where x is the length of the conductor, *logh.* represents hyperbolic or Napierian logarithms, and μ is the permeability of the wire which for copper may be taken as unity.

The values of the mutual inductances are

$$m = 2 logh \left(\frac{2x}{\varepsilon d}\right) \qquad \frac{\text{abhenrys}}{\text{cm.}} \qquad (41)$$

$$M = 2 logh \left(\frac{2x}{\varepsilon D}\right) \qquad \frac{\text{abhenrys}}{\text{cm.}} \qquad (42)$$

$$m' = 2 logh \left(\frac{2x}{\varepsilon d\, 6^{\frac{1}{5}}}\right) \qquad \frac{\text{abhenrys}}{\text{cm.}} \qquad (43)$$

Since the geometrical mean distance of all five outer wires from the sixth is

$$\sqrt[5]{d \cdot \sqrt{3}\,d \cdot 2d \cdot \sqrt{3}\,d \cdot d} = d\sqrt[5]{6} \qquad (44)$$

Equating (38) and (39) we have

$$i_1 \left\{r + j\omega\,(l_1 + 5m' - 6M)\right\} + i_o \left\{j\omega\,(m - M)\right\} =$$
$$i_1 \left\{j\omega\,(6m - 6M)\right\} + i_o \left\{r + j\omega\,(l_o - M)\right\} \quad \text{abvolts}\,\angle \qquad (45)$$

or

$$i_1 \left\{r + 2j\omega \left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4} + 5\, logh \frac{2r}{\varepsilon d\, 6^{\frac{1}{5}}} - 6\, logh \frac{2x}{\varepsilon d}\right)\right\} =$$
$$i_o \left\{r + 2j\omega \left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4} - logh \frac{2x}{\varepsilon d}\right)\right\} \qquad \text{abvolts}\,\angle \qquad (46)$$

whence

$$i_o = k\, i_1 \quad \text{where } k = \frac{r + 2j\omega\left\{\operatorname{logh}\left(\frac{d}{6\rho}\right) + \frac{\mu}{4}\right\}}{r + 2j\omega\left\{\operatorname{logh}\left(\frac{d}{\rho}\right) + \frac{\mu}{4}\right\}} \quad \text{numeric} \angle \tag{47}$$

$$z_1 = r + 2j\omega\left\{\operatorname{logh}\left(\frac{D^6}{6\,d^5\rho}\right) + \frac{\mu}{4} + k\operatorname{logh}\left(\frac{D}{d}\right)\right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \tag{48}$$

$$z_o = r + 2j\omega\left\{\operatorname{logh}\left(\frac{D}{\rho}\right) + \frac{\mu}{4} + \frac{6}{k}\operatorname{logh}\left(\frac{D}{d}\right)\right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \tag{49}$$

and

$$Z = \frac{1}{\frac{6}{z_1} + \frac{1}{z_o}} \quad \frac{\text{absohms}}{\text{cm.}} \angle \tag{50}$$

Taking the case of the test represented in Figure 10, with $f = 50{,}000^{\sim}$; or $\omega = 314{,}159$, $\rho = 0.03213$ centimeter, $d = 20\rho$, or 10 diameters $= 0.643$ centimeter, $D = 30\,d = 300\rho = 9.64$ centimeters, $r = 5.711 \times 10^5$ absohms per linear centimeter, which includes a skin-effect resistance ratio of 1.030 for this wire remote from other conductors, we find $k = 0.50872 \angle -(16^\circ\, 22')$. That is, the current strength in the middle wire is only a little more than half that on any one of the six outer wires, and lags in phase 16° 22′ relatively thereto. Substituting in (48), we find $z_1 = 11.981 \times 10^6 \;\underline{\angle 86^\circ\, 05'\, 58''}$ absohms per centimeter and $z_o = 23.551 \times 10^6 \;\underline{\angle 102^\circ\, 27'\, 58''}$. The joint linear impedance of the 7 strands is then $Z = 1.8461 \times 10^6 \;\underline{\angle 87^\circ\, 21'\, 55''}$ absohms per centimeter $= (0.084865 + j\,1.8461)\,10^6 = R' + jX'$. Since the d. c. linear resistance of the same strands would be $R = 0.07920 \times 10^6$ absohms per centimeter, the skin-effect resistance ratio $R'/R = 1.0715$. The observed resistance-ratio of this 7-strand conductor at 10 diameter spacing and $50{,}000^{\sim}$ was observed to be 1.072, which is in satisfactory agreement. At closer spacings, however, the agreement does not appear to be so good, the observed resistance-ratios being greater than the computed. Thus, at $d = 10\rho$, the computed ratio is 1.08, but the observed was 1.11. It may be that at the closer spacings the current over the cross-sections of the wires is further distorted by proximity effect, thereby increasing the power loss and resistance ratio.

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LIST OF SYMBOLS EMPLOYED

A_1, B_1	Arbitrary constants of current density	(absamperes/sq. cm. ∠)
A_2, B_2	Arbitrary constants of flux density	(gausses ∠)
$a_0 = \sqrt{-j4\pi\gamma\mu\omega} = a_2 - j\,a_2$,	semi-imaginary propagation constant in round wires	(cm.$^{-1}$ ∠)
$a = \sqrt{j4\pi\gamma\mu\omega} = a_2 + j\,a_2$,	semi-imaginary propagation constant in sheets	(cm.$^{-1}$ ∠)
a_2	Components of a_0 or a.	(cm.$^{-1}$)
$\mathfrak{B}$	Magnetic flux density	(gausses)
$\mathfrak{B}_0$	Magnetic flux density at external surface of strip $x=0$	(gausses ∠)
$\mathfrak{B}_x$	Magnetic flux density at distance x	(gausses ∠)
$\mathfrak{B}_X$	Magnetic flux density at distance X	(gausses ∠)
b	Width of a flat strip	(cm.)
β°	Argument of a complex quantity	(degrees)
$\gamma = 1/\rho$	Electric conductivity of a metal	(abmhos/cm.)
D	Distance of return conductor from going conductor	(cm.)
d	Thickness of an insulating layer between two strips	(cm.)
	Also interaxial distance between strands in a 7-strand conductor	(cm.)
δ	Depth of equivalent skin	(cm.)
e	Linear alternating e. m. f. impressed upon one wire of a strand	(r. m. s. abvolts per cm. ∠)
$\epsilon = 2.71828\ldots$	the Naperian base	
f	Impressed frequency	(cycles per second)
$\mathfrak{H}$	Magnetic intensity	(gilberts/cm.)
i_{qr}	Root-mean-square mean vector current density in a conducting strip	(absamperes/sq. cm. ∠)
i_x	Instantaneous current strength at distance x from origin	$\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$
i_{xm}	Maximum cyclic current strength at distance x from origin	$\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$
i_{xr}	Root-mean-square current strength at distance x from origin	$\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$
i_0	Root-mean-square current strength in central wire of a strand	$\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$

i_1	Root-mean-square current strength in external wire of a strand	$\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$
$j=\sqrt{-1}$		
$J_0(\quad)$	Bessel function of zero order	(numeric ∠)
$J_1(\quad)$	Bessel function of first order	(numeric ∠)
k	Ratio of current strength in central strand to current strength in an external strand	(numeric ∠)
l	Length of a wire of resistance R_0 or R'_0 ohms	(cm.)
l_0	Linear self inductance of central wire in a strand	(abhenrys/cm.)
l_1	Linear self inductance of external wire in a strand	(abhenrys/cm.)
M	The modulus of the impedance ratio Z/R Also the linear mutual inductance of return conductor on going conductor	(numeric) (abhenrys/linear cm. ∠)
m_0	Linear mutual inductance between middle and any outside wire	(abhenrys/linear cm. ∠)
m'	Linear mutual inductance of any five outer wires of 7-strand conductor on the sixth	(abhenrys/linear cm.)
μ	Permeability	(gausses/gilberts per cm.)
$\pi = 3.14159\ldots$		
R	Linear resistance of a conductor to continuous currents	(ohms or absohms/cm.)
R'	Linear resistance of a conductor to alternating currents	(ohms or absohms/cm.)
R_0	Linear resistance of a conductor to continuous currents	(ohms per meter)
R'_0	Linear resistance of a conductor to continuous currents	(ohms per 1,000 feet)
$\rho=1/\gamma$	Resistivity of conductor Also radius of central wire in a 7-strand conductor	(absohm-cm.) (cm.)
X'	Linear reactance of a conductor in the presence of skin-effect	$\left(\frac{\text{absohms}}{\text{cm.}}\angle\right)$
X	Radius of a round wire. Also thickness or half-thickness of a strip	(cm.)
x	Radius of a point in the cross-section of a wire	(cm.)
z_0	Linear impedance of central strand in a 7-strand conductor	(absohms/cm. ∠)
z_1	Linear impedance of any external strand in a 7-strand conductor	(absohms/cm. ∠

Z	Linear impedance of a straight conductor in the presence of skin-effect	$\left(\frac{\text{absohms}}{\text{cm.}} \angle\right)$
	Also linear impedance of a 7-strand conductor	(absohms/cm. $\angle$)
$\omega = 2\pi f$	Impressed angular velocity	(radians/sec.)
$\sim$	Sign for cycles per second	
μ f.	Sign for microfarad	
m μ f.	Sign for millimicrofarad	
$\angle$	Sign of a complex quantity or plane vector	

SUMMARY: After defining the skin-effect resistance and reactance ratios, the "spirality" and "proximity effects" for conductors at radio-frequencies, a differential bridge circuit supplied with current from 10,000 to 100,000 cycles by an Alexanderson alternator is described.

There are tested at various frequencies straight solid wires, stranded wires (of various spiralities, and also braided), wires with definitely spaced strands, very finely stranded wire (with insulated strands), braided "litzendraht" wire, stranded twisted wire (of various spiralities), strips (both singly and in opposed proximity), and tubes.

Important practical design data, quantitative as well as comparative, are given for the range of frequencies investigated.

In a number of appendices to the paper, the details of the apparatus and certain theoretical calculations of radio-frequency resistance are given.

DISCUSSION

J. Zenneck: I was especially interested in the way the authors obtained some information on what they call the spirality effect. As far as I know, this part of their investigation is quite new.

I was, however, somewhat astonished that the authors used a mechanically driven radio frequency alternator. Of course the ideal generator for measurements with radio frequency alternating currents is an oscillating electron relay such as the audion. The constancy of that can hardly be obtained by any mechanically driven alternator and the power is quite sufficient for most laboratory measurements.

From a practical point of view the resistance of a wire, stranded or not, is especially interesting when the wire is wound in the form of a coil. The coil is the practical case of the radio engineer, not the straight wire. I fully realize why the authors have first restricted their investigation to straight conductors. But I hope, that after having in this way acquired a good deal of experience, they will extend their measurements to coils of different forms.

E. F. Northrup: I have listened to the presentation of this paper with the very greatest interest. I have known for a number of years of the very important work which Dr. Kennelly and his associates have been doing on the subject of power transmission and I am fully aware, having given considerable thought and study to the matter myself, of the very great importance which attaches to this whole subject and, in particular, in reducing the losses in cables. Every one must feel, after listening to a paper like the one presented this evening, that an exceedingly complex problem has been handled in a most thoro and masterly way. The investigators necessarily had to define and limit the extent of the problems which they would attack, for the problems connected with this subject rapidly open up in number as one attempts to investigate them; and this is probably the reason why I have been able to look at the matter of power losses in cables and how they may be investigated from a somewhat different angle.

We have ordinarily been accustomed to think of the transmission of power by radio methods as involving more complexity than when power is transmitted from a place *A* where it is generated to a place *B* where it is utilized; and is guided, in going from *A* to *B*, by means of a conductor. The radio transmission of power, however, can be shown, I think, to involve fewer complexities than the transmission of alternating-current power (especially if the frequency be high), by means of guiding conductors. When power is transmitted by means of a belt, or pneumatically, or along a tube by water in pipes, or by any mechanical system of transmission, our thoughts are concentrated upon the material links between the place *A* where the power is developed and the place *B* where it is utilized. When, however, the transmission of power is accomplished by electrical methods, our thoughts should be concentrated, not on the conductor or cable which joins the places *A* and *B*, but on the medium, the dielectric, which surrounds the conductor. In every case it may be readily shown by analysis that the transmitter of electrical power is the medium or dielectric and not the conductor at all. The conductor is not a power-transmitter but acts as a *guide* which directs power thru the medium along particular channels. When power lets go all guidance of conductors it propagates itself thru space in rectilinear lines (ordinarily) with a velocity which is the velocity of light and its course can be predicted and followed with the aid of differential equations.

We must not think of the conductor as a transmitter of electric power but only a means of guiding its passage. On the contrary, a conductor fritters power away as it goes from the place *A* to the place *B*. When it is found necessary in meeting requirements to guide power by means of a conductor, we add enormously to the number and complexity of the phenomena which are manifested over those manifested in the direct passage of power thru the ether of space.

Not the least important consideration is how we may guide high-frequency power by means of a conductor from one place to another with the least loss of power, en route, in the conductor itself. This has ceased to be an academic question. It is a question of large engineering importance; if one realizes the large investment of capital in transmission of power of every kind.

A study of the loss of power in cables at frequencies varying from 167,000 cycles to 250,000 cycles was taken up last year by two graduate students in Princeton, Mr. Thompson and Mr. Monteiro, under my guidance. Our object was to investigate the losses in cables using frequencies comparable to those used on a radio telegraph antenna. As a description of our apparatus, methods and our results will shortly appear in a forthcoming issue of the "Journal of the Franklin Institute," I shall only briefly indicate the line along which the problem was attacked.

We considered that at the frequencies employed practically all the losses in a cable would manifest themselves in the form of heat; the loss of power by radiation, especially with the arrangement of apparatus employed being quite negligible. Our method consisted in comparing directly the heat-losses in a solid-wire copper coil with the heat losses in the cable to be tested. Arrangements were, however, effected so that the heat losses in the cable might be obtained in absolute measure. The standard copper coil and the cable coil to be tested were each inclosed in a separate Dewar bulb which was unsilvered. The bulbs were filled each with the same quantity of oil and arrangements were made whereby a rise in temperature of the oil in each flask could be determined with an electrical resistance-thermometer to better than 0.01° C. High-frequency current was passed in series thru the standard and test coil and the rise in temperature in each of the bulbs at the end of a given time gave, within 1 per cent. or better, the heat-loss in the solid-wire standard and in the cable. Furthermore, a high-frequency ammeter of special construction permitted the effective value of the high-frequency (radio frequency) current to be measured within one per cent. Both the standard and cable coils were wound in some of the tests inductively and in others non-inductively and the heat-losses were compared using these two types of windings and found not to be the same. Six different types of cable were studied.

It is not practical with the space at my disposal and without diagrams and not having the data before me to give the results of these tests or to describe any of the details of their execution. As nearly as I recollect, however, the results obtained were, except in one particular, in good agreement with those which Mr. Affel has described this evening. Our results showed that with the frequencies we employed, which were considerably higher than those used by Dr. Kennelly and Mr. Affel, a cable made up of a number of round wires lying parallel and untwisted showed a greater heat loss than a solid round conductor of identical cross-section. This fact may be accounted for, possibly, as follows: The circumferencial surface of a cable, made up as described, is not continuous, the surface of the cable having a washboard surface. The very high—or radio—frequency current, seeking as it does the extreme surface, finds, in the case of the cable, less cross-section of metal thru which to flow than it does in the case of the solid round conductor.

The quite different results which we obtained when using the inductively and non-inductively wound coils I think can be explained by the difference which exists in the distributed capacity in the first and second cases. I should like to go into this matter further as it brings out some very interesting questions on the influence of distributed capacity in the production of heat losses, but space will not permit.

In addition to the many other problems connected with heat-losses at radio frequency and its dependence upon skin-effect, there is the very extensive question of the losses due to skin-effect which occur in iron wires. Mr. Carson and myself went quite extensively into this phase of the problem both theoretically and experimentally and our results are published in an article of over 40 pages in the February, 1914 number of the "Journal of the Franklin Institute." The most curious result and a quite important one from an engineering standpoint, is that the ratio of the alternating current resistance to the direct current resistance of an iron wire is, for any given frequency, a function of the current carried by the wire. This ratio is small with a very small current, increases to a maximum for a particular value of current and then continually decreases, with increase in current, to an asymptotic value which would be the ratio of the alternating current resistance to the direct current resistance of the iron if it were non-magnetic.

I wish to congratulate the speaker of the evening for the splendid attack that has been made on this very complex but very important problem—skin-effect resistance of conductors at radio frequencies. It is indeed a vast field and, with full appreciation of what has already been done, I believe there is ten times as much still to do.

H. A. Affel: It seems to me that the Ohm's law and bridge methods of measuring resistance are at least as good as the calorimetric method which Professor Northrup has described. The Ohm's law method is simplicity itself and when applied to coils can be made quite accurate, in view of the fact that coils can be balanced by air condensers. Hot wire meters as a means of measuring current and potential are usually available. For small currents calibrated thermo-couples might be employed.

The bridge method as described previously will also give results up to at least 100,000 cycles, which are even better with coils than with straight wires. One has to stretch the isolated straight wires in spans across the room for testing. This introduces certain capacity unbalancing, and radiation effects, to parts of the bridge circuit, which tend to affect the accuracy of balance. The compactness of coils decreases this source of trouble.

The matter of coils of seven-strand wire having a higher resistance than those of solid round wire was astonishing, but the explanation offered to explain the fact seems quite reasonable.

The subject of iron wires is also very interesting. Professor Kennelly has, for some time past, been investigating the skin effect of steel contact rails, such as used for railway work at commercial frequencies of 25 and 60 cycles. It is quite evident that these are but a somewhat more complex instance of skin effect in iron wires. In attempting to determine formulas for the predetermination of skin effects in these conductors, solid round iron rods were tested. These showed the usual humped-curve skin-effect characteristics as Professor Northrup describes, in which the resistance ratio $\frac{R'}{R}$ is largely determined by the permeability of the conducting medium. This is covered in the usual theory of skin effect in wires; the skin effect increasing practically as the square root of the permeability. Having determined the permeability by the usual methods, Professor Kennelly was able to check by theory the measured skin effects of these rods to a degree which justifies the mathematical predictions in the case.

Professor Zenneck suggests the use of oscillating audions as a source of current. We all realize that the oscillating audion can be made, and is a most suitable generator of radio frequency currents. The main reason for using a rotary generator in this case, however, was the fact that it was immediately available, and delivered without difficulty the 4 to 5 amperes necessary for the successful operation of the bridge. With ordinary audions, this is hardly feasible. No trouble was experienced in the matter of keeping the frequency constant within the necessary limits. It is quite conceivable that with more sensitive methods of detection, involving perhaps other vacuum tubes, the bridge might be successfully operated with the audion as a generator, and the frequency limit extended considerably.

The matter of the measurement of losses in coils is being pursued at present, and I can assure Professor Zenneck that this will be covered as thoroly as possible. The preliminary intentions are to investigate the losses in solid wire coils as a function of wire spacing, and then determine the efficiency of stranding, both as affected by the number of strands and the thickness of insulation between strands.

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