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Some Ceramic Manufacturing Developments of the Western Electric Company

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A general picture is given of the development work involved in the introduction of manufacturing processes for vitreous enameled resistances, vitreous enameled iron and copper base number plates, pressed glass lenses, extruded and pressed porcelain parts, and close tolerance ceramic insulators for use in telephone apparatus. The reasons for undertaking the manufacture of these products, some of the major problems encountered in developing suitable processes, and the work done in overcoming these difficulties including several major contributions to commercial methods of manufacturing similar parts are described.

ORIGINALLY, the ceramic parts used in the telephone and associated equipment were not manufactured by the Western Electric Company because the technical requirements and volume of consumption of such parts did not warrant the development or establishment of processes or the facilities for manufacture. The later development of such manufacturing processes for some of the ceramic parts has been necessitated largely by inability to secure an adequate supply of parts meeting the close limits required for satisfactory functioning of the apparatus, although there have usually been other influencing factors. Such developments have, in most instances, been advantageous from an economic standpoint. The experimental work has been confined to that required for the above ends and only a very limited amount of research work has been done. Some of the major projects for which it was necessary to develop new methods of processing to obtain the desired quality at a satisfactory cost are outlined.

Switchboard Lamp Cap Lenses

The first major project undertaken was the development for manufacture of switchboard lamp cap lenses of the types shown in Fig. 1. One factor necessitating this undertaking was the difficulty experienced change. The resultant compositions are illustrated by the following batch which was developed and used for clear amber glass; and the functions of the various raw materials in this composition are given below:

Glass Sand	45
Red Lead	30
Sodium Nitrate	10
Sodium Carbonate	10
Manganese Dioxide	3
Ferric Oxide	2
	100

Scrap Glass—50 parts approximately.

As is common practice, glass sand was used as the most economical means of obtaining the desired silica content. The sodium content was introduced by the use of sodium nitrate and sodium carbonate. The oxidizing action of the nitrate and manganese dioxide assisted in (1) the prevention of lead reduction; (2) the oxidation of any organic materials present; and (3) the maintenance of the iron in ferric form. The liberation of gas during the decomposition of the sodium nitrate and carbonate tended to stir the glass during melting and in addition the escape of large gas bubbles during this decomposition assisted in the removal of small bubbles of occluded gas. Some of the sodium was introduced as sodium carbonate because it was cheaper than the nitrate. Red lead was used as an economical means of obtaining the desired lead oxide content and to lessen the possibility of any difficulties from unoxidized lead particles. A percentage of glass scrap from the punching and drawing operations was used in each batch as a means of reclaiming the scrap, facilitating melting and improving the working characteristics of the glass when drawn into rods. The amber color obtained in this glass was of course dependent on the predominance of the brown color of ferric iron. If sufficiently oxidizing conditions were not maintained during melting and working, the iron would be reduced to the ferrous state resulting in a greenish color. The color intensity obtained was very sensitive to changes in the amount of heating and to atmospheric conditions in the furnace. This complicated the problem of maintaining the glass within close limits for color and translucency.

After satisfactory glasses with twice the impact strength of the previously imported glasses were developed, open pot manufacture of clear glasses was started on a limited basis.[1] It was then found desirable in order to obtain better signaling characteristics to obtain

[1] H. T. Bellamy *Patent* 1,271,652, "Method of Making Colored Glass," July 9, 1918.

the required light dispersion in certain colors of lenses without the use of sandblasted surfaces. Several methods of dispersing the light were tried including the application of a translucent layer of glass on the back of a clear lens, but as it was difficult to control economically the amount of light dispersion by these methods, it was decided to use opalescent glasses. Calcium phosphate and cryolite were found suitable as opacifiers and satisfactory compositions were developed by means of further progressive changes to suit the particular working conditions in the shop.

Several serious objections were found to the open pot method of manufacture, the most important of which were the long heating period required for new pots and their relatively short life. The manufacture of opalescent glasses increased these difficulties because of the more corrosive nature of these glasses as a result of which the maximum life of the pots was approximately twelve days. In view of this, a small 500-pound capacity gas fired melting furnace known as a day tank was designed and constructed. This tank consisted of a rectangular box shaped furnace lined with refractory blocks about twelve inches thick. With this equipment, a complete batch was melted each night and the resultant glass formed into rods during the next day. Under continuous operation, furnace life of about three months was obtained which was considered very satisfactory in view of the corrosive nature of these glasses.

Satisfactory compositions and methods of manufacture were finally developed for the production of glasses in the required colors. This development resulted in the elimination of an unsatisfactory supply situation, reduced the cost of lenses appreciably, and greatly improved the quality of lenses.

Spirally Grooved Resistance Cores

At the same time that development work on glasses was being carried on, a preliminary survey was made of the advantages of manufacturing instead of purchasing the ceramic cores used in filament resistances. As the preliminary survey indicated that definite advantage would be realized, development for manufacture was undertaken. The part, shown in Fig. 2, consists of a thick-walled tube with a spiral groove on the outer surface in which a resistance filament is placed. Tests first were made on pressing blanks from sodium silicate and powdered slate mixtures. These parts adhered to the die, were difficult to dry, and were very weak in the fired state. Further work was done with talc and sodium silicate mixtures which were stronger but still had the objectionable feature of adhering to the dies. Additional compositions were then made of talc and clay with and without sodium silicate and feldspar. It was found that most satisfactory results were obtained with a ball clay, kaolin, and talc body in the proportions of forty per cent, ten per cent and fifty per cent, and this body was therefore adopted.

Originally, attempts were made to cut the groove in the fired core with a diamond tool but this resulted in excessive chipping of the

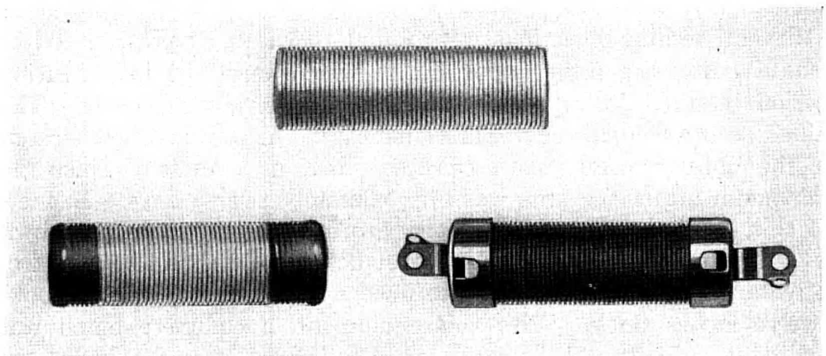

Fig. 2—Ceramic core and completed filament resistance.

groove. A chaser with alternate teeth was then tried out, with the thought that the gradual cutting action would prevent chipping. This also proved unsuccessful. The use of a circular saw, emery wheel, and a phosphor bronze disc charged with diamond dust were also considered. These methods were not completely satisfactory although better results were obtained. Rolling the thread in the core while in the leather hard state after extrusion was then tried with good results and a suitable machine was developed for performing this operation.[2]

With this machine, an extruded blank of slightly oversize diameter was placed on a revolving mandrel. An arm was provided to hold a shaving tool ahead of a disc which formed the thread. This arm was attached to a segment of a nut and the movement of the arm when the nut segment was engaged with a thread integral with the mandrel, shaved the core to exact diameter and carried the disc longitudinally across the core forming the spiral groove. An auxiliary arm carrying two knives was then engaged which cut the core to exact length and

[2] H. T. Bellamy *Patent* 1,384,587, "Manufacture of Composition Cores," July 12, 1921.

chamfered the ends. The finished core was then removed from the mandrel, dried and fired. This method of manufacture produced cores of superior quality at a greatly reduced cost.

Porcelain Protector Blocks

The next major development was that of the manufacture of protector blocks. These small porcelain blocks, used in open space cut outs and shown in Fig. 3, are illustrative of parts where it was necessary

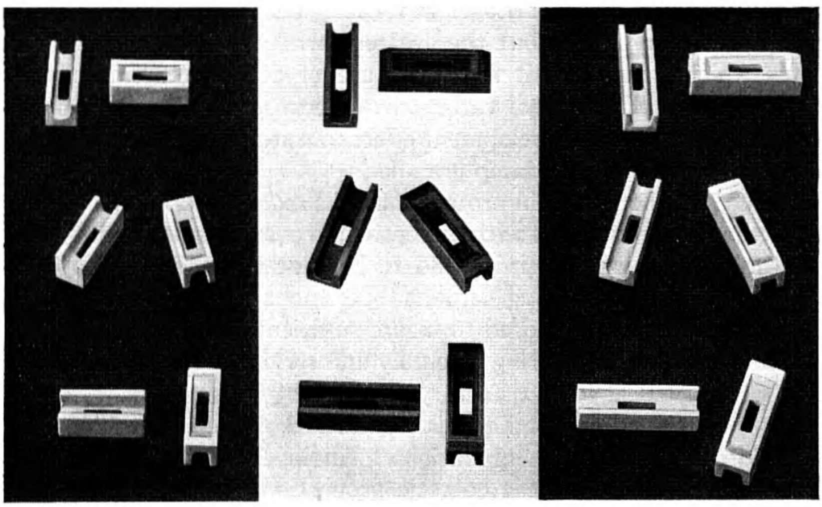

Fig. 3—Porcelain protector blocks.

to undertake manufacture for quality considerations and where such manufacture resulted in cost reduction. They were originally purchased from domestic sources which were unable to meet consistently, required limits as close as +.020-inch and −.015-inch on a .390-inch dimension. The dimensional deviations encountered necessitated sorting to insure proper functioning in the field, and it was necessary to scrap a large percentage of the purchased parts. Difficulties were also experienced in the assembly of the blocks because of manufacturing defects such as small fins and low strength.

Common commercial practice on most porcelain parts of this type at that time was to form the parts on a hand screw press, remove fins by hand, and fire the blocks in refractory containers in intermittent furnaces. The amount of hand labor involved was large and early studies of the economics of manufacture showed it would be necessary to develop new methods of manufacture before the development and plant expenditures associated with the installation of manufacturing facilities would be warranted. A survey of commercial practices indicated that mechanization of the forming operation and simplification of the finning, firing, and material preparation procedures were possible.

The two general methods of processing first considered were: (1) extrusion of a plastic column having the end cross section of the block, cutting this column to block lengths and forming complete in the plastic state; and (2) automatic pressing of damp granules. The first method offered some advantages in forming because of the thin walls of the protector blocks, but the greater shrinkage from raw to fired states which would result from the use of plastic material would involve greater dimensional variations. Because of this factor it was decided to confine the development effort to a study of the possibilities of automatic pressing of damp granules.

The uses of the porcelain protector blocks required a body as highly vitrified as was consistent with dimensional requirements, to minimize moisture absorption in service and to prevent the adherence of carborundum particles during lapping operations in assembly. High vitrification was also required to insure sufficient mechanical strength to withstand handling during assembly and service. Accurate dimensions were essential for satisfactory functioning in service.

Two general types of bodies were considered, talc-clay combinations and feldspar-clay-silica combinations. An investigation of talc-clay mixtures indicated that the eutectic proportion of the two minerals was approximately sixty-five per cent talc and thirty-five per cent clay with small variations dependent upon various clay compositions. The fusion temperature of this eutectic was approximately cone 12 or 2390° F. This combination, however, was not satisfactory since it softened over an extremely small temperature range and formed a very fluid glass in the melted state. A longer temperature range for softening and greater melted viscosity was obtained by the addition of feldspar. A eutectic composition of twelve and one-half per cent talc and eighty-seven and one-half per cent spar was found which fused at cone 6 or 2174° F. Using ten per cent to twenty per cent of this flux, a well vitrified body was obtained at cone 8 or 2237° F The firing range of this body was still much narrower than desired and any excess firing resulted in blistering. Although it was evident that commercial use of this body would require extremely close regulation of temperature, it was decided to investigate its pressing characteristics in view of the small amount of abrasive material it contained and the importance of abrasion on dies and equipment with automatic molding.

A study of the pressing behavior of the body under automatic molding speeds and conditions indicated that development work would be necessary to prevent the molded parts adhering to die surfaces. An investigation of this factor indicated that the sticking to dies was caused primarily not by adhesion between the metal and the molded clay surface but rather by the vacuum effect of a dense air-tight layer of material against the metal. This was shown by the facts that the tendency for sticking decreased with (1) a decrease in the plastic content of the body or a decrease in moisture content; (2) a decrease in the viscosity of the die lubricant which thereby tended to clog the pores of the molded surface to a less extent; and (3) an increase in the volatility of the die lubricant. Two methods of overcoming the sticking difficulties with molding compositions were therefore suggested: (1) opening up the structure of the molded part by the use of coarser material to provide capillaries for the escape of entrapped air, and (2) the use of an improved lubricating compound. Since it was not feasible to improve the lubricant sufficiently, an attempt was made to obtain much coarser talc. The talc normally available at that time was such that on sieve tests approximately five per cent to ten per cent remained on the 300-mesh screen. The availability of coarser talc was investigated and it was found that material coarser than eighteen per cent on 300-mesh was not available at an economical price. In view of the fact that the talc was very fine grained and non-plastic, it gave a very dense molded structure without contributing materially to the strength required to hold the molded part together. It therefore seemed advisable to use a clay, feldspar, and silica body and to minimize abrasion by the selection of suitable tool steels and the proper design of equipment.

In arriving at a suitable body composition of the feldspar type it was decided to use a composition which would mature at about cone 12 or 2390° F. Sufficient feldspar was used to obtain a low porosity when fired over a reasonably wide temperature range. The amount of clay used was governed by the raw strength required. Enough silica was used to obtain sharp definite outlines and to avoid warpage. The following composition was arrived at:

Flint	22.5
Feldspar	37.5
Ball Clay	20.0
Kaolin	15.0
China Clay	5.0
	100.0

Further development work was then confined to methods of processing this body to obtain satisfactory results on an automatic machine. A survey of available commercial pressing equipment indicated that machines of the type used in the manufacture of various pharmaceutical tablets or pellets offered the most promise for adaptation to molding protector blocks. The development of suitable equipment was complicated by the extremely thin walls of the parts and the necessity for rapidity of operation. In the hand molding method commonly used in the industry, a slow application of the molding force was possible at the end of the stroke and likewise a gradual withdrawal of the top die was possible after completion of the forming operation.

After some preliminary work with various types of tableting machines, we concentrated our efforts on single-plunger-type machines with double dies. One of the major problems was a satisfactory method of die lubrication since with the machine operating at twenty-eight strokes a minute, the die surfaces were exposed for oiling only an instant during each cycle. The use of an atomizer-type device with a mixture of lard oil and kerosene was finally adopted with the amount of lubricant closely controlled by oil sight cups. Exact timing of the application of the spray to dies was obtained by automatically operating air check valves. This method proved more satisfactory than wiping with saturated felt or incorporating a lubricant in the body particles before molding.

The various stages of the molding cycle are shown in Fig. 4. The cycle of operation at twenty-eight strokes per minute was as follows: as the bottom die reached the lowest position, a feed hopper was vibrated over the cavity. The withdrawal of this hopper removed excess material, after which the top punch descended forming the part. The bottom and top punches then moved upward until the bottom of the part was flush with the top of the die. A projection on the hopper then pushed the part free. Before the hopper reached a position over the cavity, any particles of clay adhering to the dies were blown off and the lubricant was sprayed over each die. The lower and upper dies were then returned to the original positions.

A large amount of work was also necessary to adjust the size and moisture characteristics of the pressing material not only to secure well formed parts and prevent sticking but also to secure fired parts meeting the desired requirements. A mixture of colored and uncolored particles was used in the study of these characteristics in order that the flow movements in the die during compression could be studied. As a result of this work, it was found that most satis-

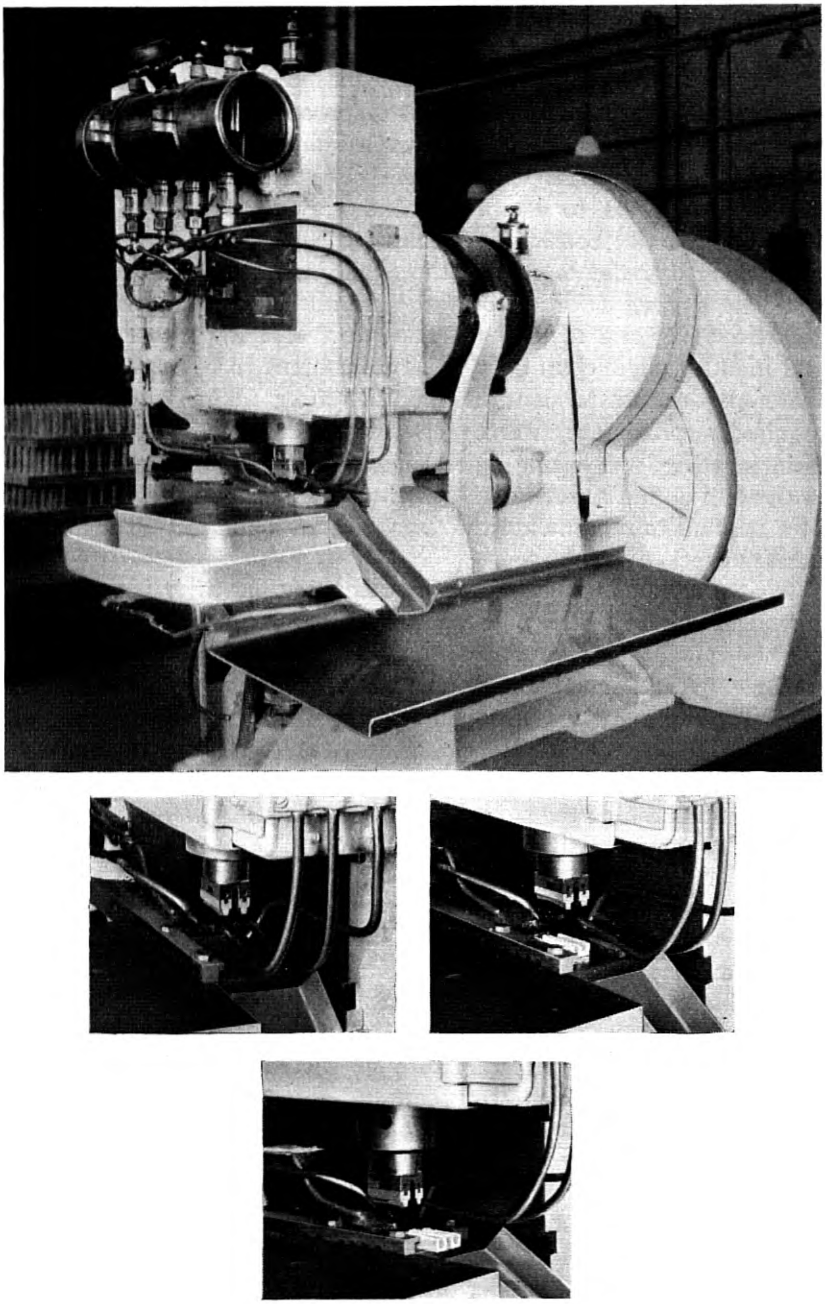

Fig. 4—Block forming press.

factory results could be obtained with 10–24-mesh material and a 12.75 per cent $\pm$ .75 per cent moisture content. This moisture content was sufficient to give a compact part and body mix still could be fed satisfactorily to the dies. Methods of processing the body mix were then worked out to hold it within these limits.

The use of material within these close size limitations involved considerable effort to establish economical methods of production. Common practice consisted of slaking the clay in water, adding the other body ingredients, mixing thoroughly with water, filter pressing, complete drying, addition of water to obtain the desired moisture content, aging and screening. The effect of aging was investigated and found negligible with the moisture content to be used and it was therefore decided to dry the material after filter pressing to the required approximate twelve per cent moisture content before the disintegrating and sizing operations. Methods of handling were evolved to obtain a maximum percentage of material between 10 and 24-mesh and to regranulate the fines without again mixing them with excess water.

Various methods of economically removing fins after forming were investigated and initially the fired parts were tumbled with small porcelain balls. This method removed fins and produced smooth surfaces. Another advantage of the method was the automatic elimination of any weak or flawed parts by breakage during the tumbling. Later, further developments in methods of firing described hereafter made it more economical to remove the fins in the raw state by vacuum brushing the parts in multiple after they were arranged on trays at the pressing machines.

Initially the parts were fired using the practice then commonly followed in the industry. With this method, the parts were placed in saggers and fired in an intermittent kiln. This method involved costly handling, heat losses due to heating and cooling the furnace at each firing, and considerable expense from sagger replacements. A small continuous kiln was therefore installed in which the parts were carried in layers on top of cars through successive preheating, firing, and cooling zones which were continuously maintained at definite temperatures, the heat from the cooling fired ware being used to heat the incoming ware.

Summarizing, the method of manufacture finally developed for porcelain blocks consisted in mixing feldspar, clay and flint with water to get an intimate mixture, filter pressing, drying to proper moisture content, sizing, automatically molding the parts, removing fins in multiple, and firing in a continuous kiln. This method resulted in a marked improvement in quality and reduced the cost of parts. The method of automatic pressing developed constituted a major contribution to existing commercial methods of manufacturing small porcelain parts.

VITREOUS ENAMELED COPPER BASE NUMBER PLATES

Manufacture of parts similar to the vitreous enameled number plates used on calling dials and shown in Fig. 5 was limited to producers

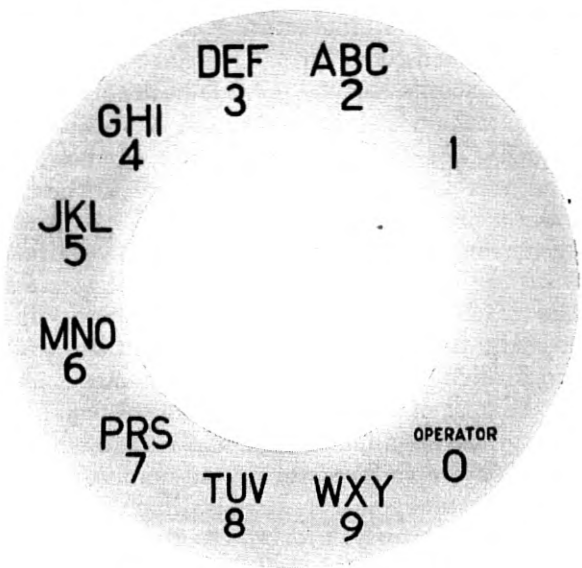

Fig. 5—Copper base number plate.

of enameled parts of the watch dial type. Because of our unusual requirements for dimensions and character location together with the need for a collar and locating pins, only one source of supply could be developed. With rapidly increasing schedules for calling dials and because of the possibility of conditions deyond the control of the supplier interfering with the continuity of supply, this situation was unsatisfactory and made it advisable to undertake manufacture to remove possibilities of any embarrassment from a supply standpoint. As sources of supply for copper enamels were also limited, various enamel compositions were investigated. It was found that the following composition would give an enamel satisfactory for color, texture, gloss, fusibility, and durability when fired on copper blanks:

Red Lead	40
Pearl Ash	6
Sodium Nitrate	9
White Arsenic Oxide	6
Flint	31
Borax	8
	100

In practice constituents of this enamel were first thoroughly melted to a homogeneous glass, giving on cooling a glass magma having high opalescence. The dead white opacity of this enamel could only be developed by slow cooling through the range necessary to precipitate the arsenic compounds. Manufacturing considerations, such as the necessity of an enclosed room for commercial smelting to avoid contamination as well as to avoid the possible health hazards involved in the smelting of arsenic-lead combinations, led us to purchase the required enamel. The fact that a suitable composition had been developed and was available for manufacture if necessary was an advantage from a supply standpoint.

Various enameling procedures were considered. In order to cover the vertical surface of the collar satisfactorily, it was essential that this surface be coated either by dipping or by spraying. It was equally important to apply the enamel coating to the flat surface of the plate by dusting on a thick coat of dry powdered enamel. This dust coat was necessary because of the thickness of enamel required on the flat portion to strengthen the number plate and also to obtain the desired quality of finish on the surface bearing the numerals and characters. From an economic standpoint, it was also imperative that only one enamel fire be used. Initially, efforts were made to dust enamel on a blank already completely coated with a thin coat of enamel slip consisting of finely divided enamel frit suspended in water by means of clay or bentonite. It was found on firing that the added refractoriness of the enamel slip containing the clay or bentonite resulted in a roughened fired surface over the dusted area. This was caused by the formation of gases in the decomposition of the clay or bentonite while the enamel was in a viscous state. It was therefore necessary to protect the flat portion of the plates by templates during spraying. As this was costly, a study was made of other means of floating the enamel frit for collar application.

In order to overcome these process difficulties, it was desirable to find a material which would (1) satisfactorily hold the heavy lead enamel particles in suspension and prevent packing, (2) not attack the enamel or impair its durability, (3) decompose before the enamel started to fuse, and (4) be inexpensive. Soluble alginates appeared to possess these properties and excellent results were obtained from their use.[3] These substances were made from kelp. Their most interesting property as a suspending medium was the ability of the alginates when added to water even in small percentages to make solutions of high viscosity. For example, water solutions of ten per cent ammonium alginate would stand stiff. Some of the advantages in our use of alginates for suspending number plate enamel were: (1) uniformity of composition resulting from the alginates being a manufactured product rather than a natural mineral; (2) the fact that dried sprayed coats of alginate suspended enamel were less subject to damage from handling; (3) a low decomposition temperature which resulted in the material being driven off before fusion of enamel, thus avoiding bubbles in the enamel; and (4) increased resistance of the finished enamel surfaces to chemical attack and their ability to withstand greater mechanical shock and distortion without damage, since any refractory materials present when the enamel was fired would not be completely fused or incorporated into the glass, leaving points more readily attacked chemically as well as lines of mechanical weakness.

Using alginate suspended enamels, suitable manufacturing processes were developed for the application and firing of enamel and the application of characters to the fired plates. A machine was devised for the application of the sifted coating, and rotary continuous furnaces were installed for the firing operations.

Originally the decalcomania method was used for character application. In this process, the enameled parts were first coated with a thin coat of sizing and, after partial drying, they were placed in a locating fixture mounted on a small arbor press and pressure was applied to a properly located transfer by means of a soft rubber pad. The paper backing of the transfer was then removed by soaking in water and, to insure contact, the characters were repressed with a silk covered pad. The sizing was then baked off before firing to remove organic materials and eliminate shadows around the characters. This method was costly and even well trained, careful operators did not produce satisfactory plates.

To eliminate these defects, an offset printing and dusting method was developed in which an electrotype printing plate was covered with printer's ink and an impression was transferred to the number plate by means of a rubber transfer pad. Powdered vitrifiable colors were then dusted over the entire surface of the plate and the unprinted areas of the part brushed clean with a camel's hair brush. In printing two color plates, the black letters were printed and dusted first, after

[3] L. I. Shaw *Patent* 1,806,183, "Suspension," May 19, 1931.

which the red numerals were printed and dusted. By using a special black powder which would give an intense black in combination with a thin film of red powder, one firing for both colors was possible. This method required close control of temperature and humidity of the air in the room which was therefore air conditioned.

Even under good conditions considerable difficulty was experienced at times with the adherence of the powder to unprinted areas. In addition, the application of the powder and the brushing operation required the installation of a special well exhausted unit and involved some problems in the recovery of the ceramic dust which were quite expensive. Efforts were therefore made to incorporate the glass powder directly in the printing vehicle. The development of a

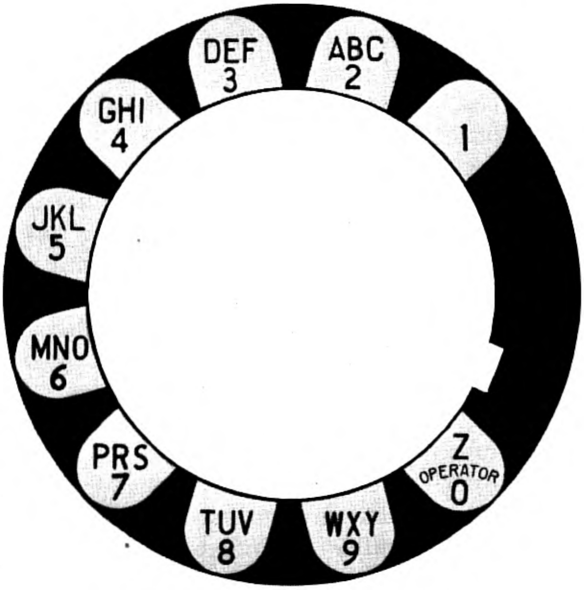

Fig. 6—Iron base number plate.

ceramic printing ink covered tests on various printing vehicles to determine what vehicle or mixture of vehicles would be most suitable. Difficulty was encountered in incorporating a sufficient amount of inert, finely pulverized, intensely colored glasses into a vehicle and still retaining the properties essential for offset printing by transferring an impression from an electrotype plate to a vitreous enamel. This problem was finally solved by the use of a relatively large percentage of uncalcined ceramic material in combination with light and heavy ink varnishes.[4] Motor driven presses using this ink were also developed to facilitate the printing operations.[5]

The manufacture of these parts was undertaken primarily to eliminate an undesirable supply situation but Western Electric manufacture resulted in improvements in quality of enameling, quality of printing, and in mechanical strength. This latter characteristic was important since it reduced assembly losses from cracked plates.

While vitreous enameled copper base number plates have been replaced by other types, the developments outlined were the basis of subsequent enameling developments.

Vitreous Enameled Iron Base Number Plates

The low level of illumination at some pay stations led to the design by the Bell Telephone Laboratories of a large iron base number plate, shown in Fig. 6, to be mounted flush with the finger wheel of the dial. Since the demand for these plates was relatively small, they were originally made by the usual process followed in the industry in enameling similar articles. This process consisted of applying and firing one ground coat for adherence and then applying and firing two sprayed cover coats to obtain the whiteness and opacity desired; all being felspar enamels. The whiteness was not as good as that obtained on the copper base plates with lead enamels and in addition considerable difficulty was experienced in the field due to the fading of the characters as a result of chemical action on plates exposed to corrosive gases such as sulphurous fumes in certain locations. The process was also costly.

Since maximum whiteness and opacity was obtainable in the lead-arsenic type of enamels previously described when applied by dusting on dry, it was desirable that the coating be applied in this manner. In order to avoid several enamel applications and firings, it was also desirable that other portions of the plate be protected by some corrosion resistant coating other than vitreous enamel which would necessarily have to retain such corrosion resisting properties after exposure to a temperature of 1500° F. for six minutes and also be capable of being enameled with satisfactory results. Numerous coatings were tried and it was found that a Western Electric black oxide finish on iron would satisfactorily meet all requirements.[6] Using this finish, it was possible to fuse the enamel directly on the upper surface of plates, to retain corrosion resistant qualities on all other exposed surfaces, and to reduce the number of process operations. A number plate of greatly improved appearance and durability also resulted. In addition, the curved

[4] L. McLaughlin *Patent* 2,030,999, "Ink," February 18, 1936.
[5] L. McLaughlin *Patent* 1,951,430, "Printing Apparatus," March 20, 1934.
[6] W. J. Scott *Patent* 1,962,751, "Ceramic Coated Articles," June 12, 1934.

surface obtained in dusting a base plate having a groove around the edge prevented the entrapment of air between the plate and the printing pad during the printing operation, thereby resulting in a simplification of that process.[7]

As a result of our development of enameling over the black oxide finish it would have been possible to replace the previously described copper-base number plate by one employing a sifted coat of enamel over such finish on a steel blank. However, the application of the black oxide finish on enameling iron was so costly that manufacture of number plates by this process was not competitive. We therefore continued our developments and found that it was possible to enamel directly over an electroplated copper-nickel finish consisting of a minimum of 25 m.s.i. each of copper and nickel on a mild steel blank and get a smooth enamel coat having very good adherence.[8] As this finish had the necessary rust resistance and the blank was relatively flat, the enamel could be applied in a single sifted coat on the face only to produce a satisfactory number plate. Also with the steel base it was not necessary to have a thick coating of enamel for strength as was the case with the copper base number plate. In fact, due to the good adherence of the enamel coat, if the thickness of enamel after firing was less than 0.010 inch the plate could be flexed considerably without chipping the finish. On the other hand, it was necessary to have a minimum of 0.007 inch of enamel to hide sufficiently the gray color of the nickel surface. Additional refinements of the enameling process were effected by improvements in the uniformity of enamel distribution and in the printing of characters; and the process was generally automatized. These developments produced a number plate of superior quality and appearance at a reduced cost. As all final details for commercial manufacture have not been completed further details of this process will not be given here.

Vitreous Enameled Resistances

With the increased use of panel-type machine switching, the demand for vitreous enameled resistances for controlling the current for operating relays and switches increased materially and manufacture of these parts was undertaken. These resistances were required to dissipate a considerable amount of heat in service and to reach a high operating temperature without being damaged. The units therefore consisted of a suitable resistance wire wound on a ceramic core and covered with a vitreous enamel. Some of the types now being manufactured at Hawthorne are shown in Fig. 7.

[7] W. J. Scott *Patent* 2,020,476, "Ceramic Articles," November 12, 1935.
[8] S. R. Mason and W. J. Scott *Patent* 2,020,477, "Ceramic Article," November 12, 1935.

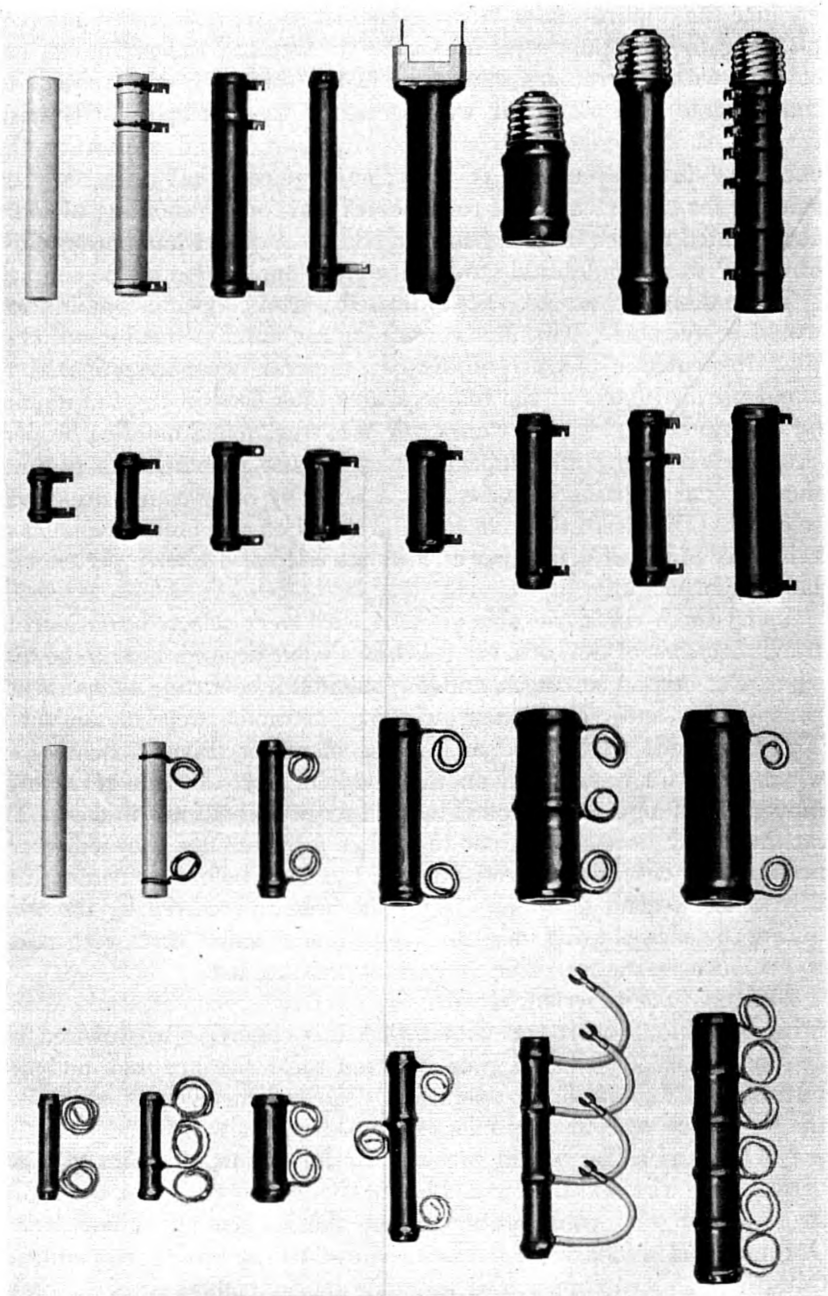

Fig. 7—Ceramic cores and vitreous enameled resistances.

Since the application of vitreous enamel to the resistances involved plunging the porcelain cores into a hot furnace and their removal into cold air without cracking, and since in use they were also subject to considerable heat shock, it was necessary to develop a body with thermal shock resistance characteristics that would still have the necessary fired strength. It was also required that this body be suitable for the extrusion of round cores and for the molding of more complicated shapes from a granular body. A somewhat porous fired structure was also desirable to facilitate the application of the enamel.

These desired characteristics indicated that a clay-talc combination would be suitable. Mixtures containing a greater percentage of clay than the eutectic mixture of the two minerals were investigated to avoid vitrified parts at the temperatures then used in the tunnel kiln for other products. Tests were made of extrusion and molding properties, fired, breaking and impact strengths, and resistance to thermal shocks. On the basis of these tests a talc-clay body composition was selected. Dies were then constructed based on the fired shrinkage of this body of about fifteen per cent in extruded and twelve per cent in molded forms.

Using these cores, suitable sizes of wire were selected considering the dimensions of the core, the resistance value desired, heat to be dissipated at certain wattages, and the maximum operating temperature permissible, and satisfactory winding methods were established. These methods were based on the use of motor driven machines in which the wire was spaced on the revolving cores by the transverse movement of a guide controlled by lead screws of various pitches. To facilitate any necessary minor resistance adjustments, methods were provided for checking the resistance of the units before connecting the wire to the second terminal. Since the heating received by the wire during the enamel firing increased its resistance value, tests were made to establish resistance value factors for winding use.

Difficulty was experienced with the resistance becoming open in the firing operation and it was shown that this condition was caused by the formation of a film of glass between the resistance wire and the terminal during the firing operation. Various methods of attaching the resistance wire to terminals were tried and it was found that the use of lead as solder would prevent the formation of a film of glass between the wire and the terminal even though the fusion temperature of the lead was considerably below the enameling temperature. Ordinary soft solder could not be used due to the tin content embrittling the standard copper lead wires during the enamel firing. While silver solder could be used it was costly both for material and in application.

In developing an enamel to be used for resistances, it was desirable that the melting temperature be as low as possible consistent with good durability in order to maintain at a minimum the thermal shocks received by the porcelain cores and any changes in the resistance of the wire during firing. A very high viscosity during fusion was also desirable in order to avoid running of the enamel during firing, an undesirable feature which would result in exposed wires and unsightly lumps unless the enamel was applied in numerous thin coats. Conversely to these requirements, it was necessary that the enamel coating be glassy in appearance, smooth, free from blisters and pin holes, and capable of being fired in a relatively short time.

These factors indicated the desirability of investigating lead-boron-silica mixtures and the elimination of any raw clay or similar refractory substance in the enamel slip. The enamel finally developed was as follows:

Red Lead	48.0
Boric Acid	24.0
Flint	10.0
Soda Ash	3.7
Cryolite	6.0
Tin Oxide	1.6
Manganese Dioxide	0.5
Cobalt Oxide	0.6
Iron Oxide	4.0
Zinc Oxide	1.6
	100.0
White Lead	10.0
Light Calcined Magnesia	1.3

All of the materials other than the white lead and light calcined magnesia of the above composition were fritted or melted to a glass and then quenched in water. The fritting of these materials was done to insure complete formation of stable compounds and to permit more rapid firing of the enamel coating on resistances to a smooth homogenous glass. The proportions of sodium, lead, boron and silica were selected to obtain a stable coating with the desired viscosity characteristics at as low temperature as possible. Sufficient opacity of the coating was obtained through the use of cryolite and tin oxide. The cryolite also functioned as a flux. A pleasing dark color was obtained economically with the iron, cobalt and manganese contents. The zinc oxide functioned as an additional flux and also aided considerably in the formation of a smooth coating. Slight variations in the sodium content of this enamel affected its viscosity markedly and also affected its expansion characteristics.

After fritting, the resultant glass was sized and suspended in a water suspension of white lead and light calcined magnesia in a tank provided with mechanical agitation. This method of suspension aided in increasing the fired viscosity without the formation of blisters and pin holes. Smooth glassy resistances were obtained with this enamel in a ten minute firing at 1150° F. without appreciable bubbling or flowing of the coating. This eliminated the necessity of three or four thin fired coats and resultant greater variations in resistance values after firing.

Close Tolerance Ceramic Barriers and Insulators

In the design of the handset type of telephone transmitter, it was found desirable to use a thin washer type insulator, shown in the lower portion of Fig. 8, as a barrier to control the path of the current between

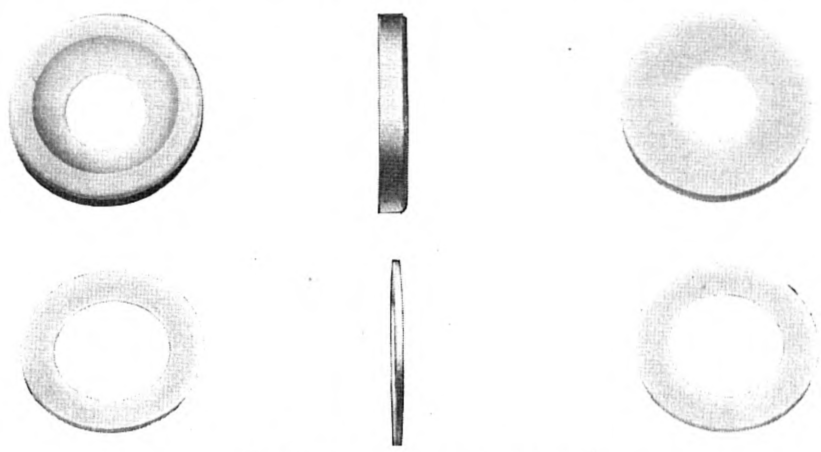

Fig. 8—Close tolerance insulators.

electrodes. This necessitated very close dimensional tolerances, unusual freedom from surface and edge defects, and reasonable strength to withstand the clamping force used in assembly.

Various materials such as fiber, lava and metal coated with vitreous enamel, were tried and lava gave the most promising results. In view of the cost of lava parts, experiments were made using the usual process of dry pressing a porcelain body in which the clay content furnished the raw strength. The difficulties inherent with this process were the fragility of the raw part and the variable dimensions resulting from uneven drying and firing shrinkages. Because of the fragility of the parts, it was necessary to mold them 0.050 inch thick and then lap the fired parts to the desired thickness of 0.030 inch. This operation was costly and losses from breakage were high. The narrow dimensional limits of $\pm .002$ inch on thickness were also hard to maintain because of the difficulty of keeping the lapping surfaces parallel. In view of this, it was decided to machine the parts from natural talc rod or lava.

The mineral talc or lava, being soft, was easy to machine and the firing shrinkage was only one per cent as compared to about ten per cent with dry pressed porcelain. While less difficulty with warpage and dimensional variations was experienced, the machined surfaces, while reasonably smooth and accurate, were not equal in quality to surfaces obtainable with molded parts. The chief difficulty with the process was in obtaining a satisfactory raw material free from flaws and fissures. The first work was done with domestic lava which was somewhat granular in structure but large rejections resulted from pitted surfaces and chipped edges. A survey of domestic lavas showed that only a small percentage was sufficiently dense. Chinese white lava was found to be homogeneous and fine grained but of uneven shrinkage. Best results were obtained with Italian green lava and this material was used in commercial production. Due to breakage because of fissures, the number of good insulators per foot of rod was very low and the manufacturing cost was therefore excessive.

In view of this, various domestic manufacturers of glass, porcelain, lava and other types of ceramic parts were canvassed but no source of supply that could meet the required quality limits could be located. It was therefore decided to make a thorough investigation of new molding compositions for the job. As a first step in this study, it was necessary to do away with drying shrinkage which required a binder which would give sufficient strength in the raw state to withstand the various finning and handling operations prior to firing. It was also desirable that such a binder should not affect the fired structure of the parts. Various organic substances such as pitches, phenolic resins, asphalts, paraffins, and waxes were tried in both hot and cold molded bodies. It was found that a large percentage of these binders could be incorporated into a body without deformation during firing.[9] As a mixture of paraffin and carnauba wax was found satisfactory for cold molding and in addition possessed sufficient hardness to furnish the necessary molded strength, this combination of materials was chosen for the binder.[10]

[9] W. J. Scott *Patent* 1,847,102, "Ceramic Material," March 1, 1932. W. J. Scott *Patent* 1,977,698, "Ceramic Material and Method of Making the Same," October 23, 1934.

[10] L. I. Shaw and W. J. Scott *Patent* 1,847,197, "Ceramic Material and Method of Making the Same," March 1, 1932.

Another major factor in the development of a suitable molding compound was the abrasive effect of the molding body on the die parts. Because of the close tolerances required, this factor was important in order to avoid excessive tool expense. Talc was therefore chosen as the chief body constitutent to obtain a long die life. The balance of the body was made up of twenty-five per cent clay which gave the desired density in both molded and fired states. With the talc-wax compound, a long die life was obtained even with the close tolerances required. As a result of the use of a combination of waxes as a binder this composition had a low uniform shrinkage of approximately four per cent as compared to about ten per cent with most dry pressed porcelains. In addition, variable shrinkage and warpage resulting from drying strains were eliminated.

In molding this body, the lubrication of die surfaces was found to be critical because of the extreme thinness of the part. It was impracticable to apply a sufficiently exact amount of a liquid lubricant to prevent the parts from either adhering to the dies or being weakened from the absorption of the liquid. This problem was overcome by tumbling the granulated molding material with a fraction of a per cent of zinc stearate.[11] The stearate coated grains of material were then molded without any additional die lubricant.

Using the above composition, the process developed was as follows: The talc and clay were thoroughly milled in a carbon tetrachloride solution of the waxes. After drying, this mixture was disintegrated and sized, after which the particles of compound were coated with zinc stearate. The parts were then molded four at a time in a commercial self-contained hydraulic press within an accuracy of $\pm$ 3 per cent of the total thickness and $\pm$ 1 per cent of the inside diameter. After molding, any fins were removed and the parts trimmed within $\pm$ 1.5 per cent of the total thickness in a finning machine which was an adaptation of a commercial automatic indexing head drill press. In this machine, the parts were fed to a rotating end cutter by a revolving indexing head and were held under this cutter by a vacuum applied to the underside of the parts. Tungsten carbide cutters were used to obtain long tool life. After finning, the parts were fired in small trays in a continuous kiln. The parts were then individually gauged for thickness, roundness and inside diameter and individually inspected for cracks, flaws, and burrs. They were then examined under a 10 to 1 glass for smoothness and regularity of inner edge before being used in the assembly of the transmitter.

[11] W. J. Scott *Patent* 1,847,196, "Ceramic Article and Method of Making the Same," March 1, 1932.

This development permitted the manufacture of ceramic insulators within limits not feasible with other methods of manufacture at that time except by machining from mineral talc and to closer dimensional tolerances than ever before attained in molded ceramic parts. The cost of the parts was reduced to a fraction of that of machined parts and their quality was greatly improved. Since that time the process has been used in the manufacture of other close tolerance ceramic parts for telephone use such as the insulator shown in the upper part of Fig. 8.

Although the outline of miscellaneous manufacturing developments given herein does not include all of the engineering development effort on glass, porcelain and vitreous enamel problems it gives a general picture of the type and scope of past engineering work in the production of ceramic articles for telephone apparatus. The miscellaneous ceramic parts used in telephone apparatus were described in an earlier publication.[12]

[12] A. G. Johnson and L. I. Shaw, "Ceramics in the Telephone," *Industrial and Engineering Chemistry*, Vol. 27, pp. 1326–1332, November, 1935.

The Production of Ultra-High-Frequency Oscillations by Means of Diodes

By F. B. LLEWELLYN and A. E. BOWEN

The general problem of obtaining oscillations by the use of diodes with critical electron transit time is outlined. Some of the properties of a 10 cm. oscillator tested experimentally are included. Extraneous losses were reduced when the oscillator was enclosed within a wave guide.

THE theory of the production of negative impedance by means of an electron discharge between two parallel planes has been known for some years.[1] The negative resistance appears whenever the electron transit time is approximately 1¼, 2¼, 3¼, etc. cycles of a given high-frequency current. Using this property, Müller was able to construct tubes giving 100 cm. oscillations.[2] The operating efficiencies were quite low, and in the frequency range covered by these tubes it seems fairly conclusive that other methods of producing oscillations are more effective than the critical transit time diode. However, there is promise in the application of diode operation to much higher frequencies than those of Müller.

In a diode where the electron discharge occurs between two parallel planes where one performs the function of electron emitting cathode and the other constitutes an anode biased at a positive potential, the effective impedance presented to an external source is inherently low in magnitude. This is because of the capacitance between the two planes which causes the decrease in impedance at high frequencies. For the production of oscillations, the capacitance must be combined with a resonant structure having the proper inductance to resonate at the desired frequency and having a resistance which effectively is less in magnitude than that of the electron stream. Because of the low losses thus required of the coupling or tuning circuit the properties of concentric lines and of tuned cavities offer a favorable method of attack. These structures also have the property that the impedance presented to the diode proper may be made low to match its capacitive reactance at the high frequencies desired.

The two most important sources of circuit resistance are ordinary ohmic loss modified in the usual way by skin effect in the conducting

[1] For numbered references see end of paper.

material forming the resonant system and secondly the losses caused by radiation of energy. These latter are extremely important where the negative resistance is only a few ohms as in the present instance and necessitate the use of nearly closed structures. This again directs attention to the properties of cavities and concentric lines tuned by internal capacitive resonance, the low capacitance being formed by the electrodes between which the electron discharge flows. It was on the basis of these principles that the actual diode models were constructed.

The general aspect of these tubes is shown in Fig. 1 which presents a section through the axis of revolution. The cylinders of radii r_1 and r_2 respectively constitute the outer and inner conductors of a concentric line. At one end of the inner conductor a flange partly closes the system thus confining most of the energy within the cavity. At the other end of the inner conductor the flat surface of the inner conductor constitutes an emitting cathode while the opposing surface of the outer conductor constitutes the positively biased anode which also completely closes the end of the cylinder. The system is tuned by the capacitance between cathode and anode and the effective inductance of the coaxial line of length h. The emitter was coated in the experiments with an oxide of the uncombined type and was heated by a filament located within the inner cylinder. The spacers for separating the inner cylinder from the main body of the outer conductor were composed of fused quartz in order to obtain low losses and good mechanical rigidity. A water jacket was supplied to assist in cooling the anode.

In reference to Fig. 1, the tuning relation between the cathode-anode capacitance and the inductance of the resonant circuit connected to it requires the following relation to be satisfied,

$$\frac{1}{\lambda} \tan \frac{2\pi h}{\lambda} = \frac{x}{\pi r_2^2 \log_e \dfrac{r_1}{r_2}}. \tag{1}$$

Here λ stands for the free space wave-length. The other quantities in the formula are illustrated in Fig. 1 and all dimensions are in centimeters. The radii r_1 and r_2 refer respectively to the inner surface of the outer cylinder and the outer surface of the inner cylinder. Improved formulas for the resonant frequencies of cavities of this type have recently been published by Hansen.[3]

The formula (1) is based on the approximation that the presence of electrons between cathode and anode does not affect the dielectric constant of the resulting capacitance. This approximation is a good one when the electron transit time is greater than a cycle, as is the case here.

The next design formula required is the resistance of the electron

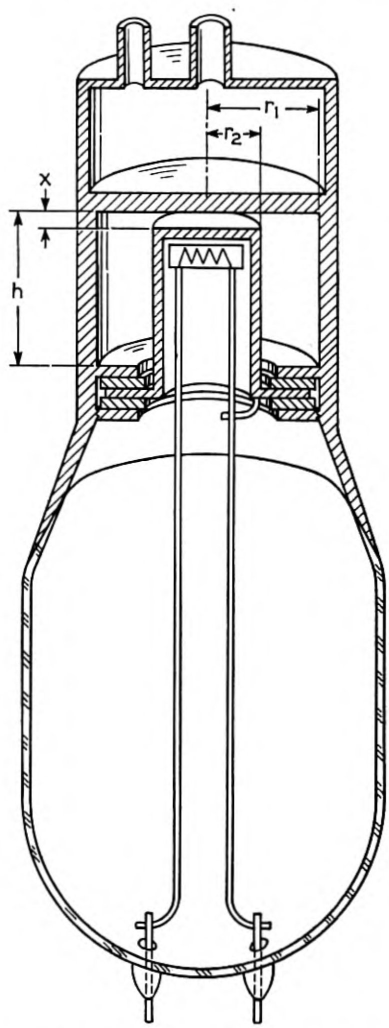

Fig. 1—Ten-centimeter diode used in tests.

Diode	r_1	r_2	x	h
No. 24	1.270	0.635	0.203	1.870
No. 37	1.220	0.635	0.105	1.870

Centimeters

stream. Reckoned per square centimeter of area this may be written,[4*]

$$r_p = \frac{1.78\ \lambda^4\ I_0}{10^4}[2(1 - \cos\theta) - \theta\sin\theta] \text{ ohms for cm.}^2, \tag{2}$$

where I_0 is the direct current density in amperes per square centimeter flowing to the anode and θ is the electron transit angle, given by

$$\theta = \frac{Ax}{\lambda\sqrt{V_0}} \text{ radians.} \tag{3}$$

Here V_0 is the constant potential difference in volts between the cathode and anode and A is a numerical factor which depends upon the amount of space charge within the electron discharge, being equal to 6300 for negligible space charge and to 9500 for complete space charge with intermediate values for intermediate space charge. As an alternative the resistance (2) may be written

$$r_p = \frac{12\ r_0}{\theta^4}[2(1 - \cos\theta) - \theta\sin\theta] \text{ ohms for cm.}^2, \tag{4}$$

where r_0 is the low-frequency series resistance of the device. With space charge, r_0 is the slope of the static characteristic derived from Child's equation

$$I_0 = \frac{2.33}{10^6}\frac{V_0^{3/2}}{x^2} \text{ amperes/cm.}^2. \tag{5}$$

More generally r_0 is given by the expression

$$r_0 = \frac{1.48}{10^5} I_0 \frac{x^4}{V_0^2} A^4 \text{ ohms for cm.}^2, \tag{6}$$

where A is the same as was defined under (3).

Figure 2 shows a graph of the electron stream resistance as a function of transit angle and is repeated from previous papers.[1] However, it may not have been emphasized in the literature that the graph as well as equations (2) and (4) apply not only with complete space charge but with intermediate values when interpreted correctly, namely in terms of the d-c. current density I_0 rather than in terms of the applied potentials.

Whenever the transit angle is equal to $2\pi n + \frac{\pi}{2}$ where n is 1, 2, 3,

* Equation 41 in this reference applies where the initial velocities are very small. With complete space charge $q = J$ and $a_a = 0$ whereas without space charge $q = 0$. Either condition gives the same series resistance in terms of I_0.

etc. then the electron stream exhibits a negative resistance. From this it may be inferred that oscillations are possible not only for values of n equal to unity but also for larger values, thus yielding the possibility of higher order oscillations when the circuit coupled to the electron stream is properly proportioned. If we start with a given cathode temperature with space charge and attempt to obtain the longer transit times by a decrease in applied potential, the smaller currents obtained will decrease the negative resistance. Hence, with space charge it is advisable to employ the smallest value of n possible in actual circuit design.

For computation work the general formula (2) may be greatly simplified because we need to know only the maximum values which

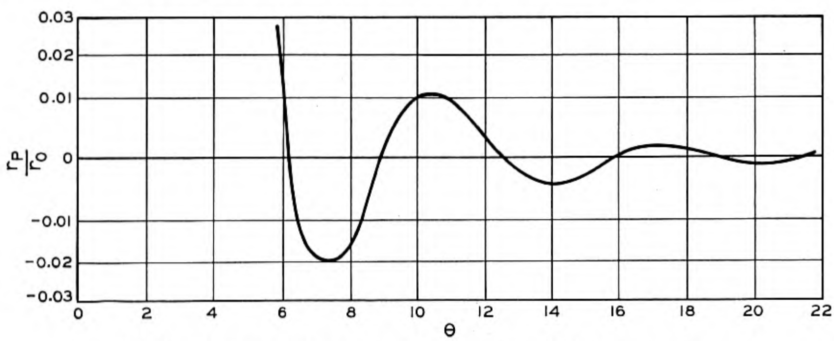

Fig. 2—Relation between transit angle θ and diode resistance.

the negative resistance attains. These occur in the neighborhood of transit angles given by

$$\theta = 2\pi n + \frac{\pi}{2}, \qquad n = 1, 2, 3, \cdots \tag{7}$$

and under these conditions the effective negative resistance is

$$R_p = \frac{r_p}{\text{area}} = -\frac{1.4\,\lambda^4\,I_0}{10^3\,\pi r_2^2}\left(\frac{4n+1}{5}\right)\text{ohms.} \tag{8}$$

The detailed steps in circuit design are these: First the allowable value of current density I_0 must be determined. This depends upon the ability of the cathode to emit electrons and as a practical limit something in the neighborhood of 300 mils per square centimeter cannot be exceeded. When this current has been decided upon then the value x of the separation between cathode and anode may be found from (3) and (7) with the lowest value of n which will give practical figures. The space charge condition (4) also gives the lowest allowable potential for which the required current can flow and hence the best efficiency in the simple diode of Fig. 1. From these values the negative resistance may be computed from (8).

As a next step the remainder of the circuit must be proportioned. The tuning relation (1) yields the values of height h for a given diameter. The next consideration is to insure that the sum of all positive resistances is less than the negative resistance of the diode. For a circuit with dimensions small compared with the wave-length, approximate formulas for the resistances associated with the losses in the circuit conductors can be readily derived from classical circuit analysis.

A most important resistance, not so readily computed, is caused by radiation of energy through the gap between the insulating flanges which separate cathode and anode. In most uses of the device, this radiated energy constitutes the useful load on the oscillator but care must be taken that the load is not so heavy as to stop the oscillations altogether. An important distinction must be made as to whether the tube is to radiate into free space or into some enclosure such as a hollow wave guide, for example. In the latter case the radiation may be regulated to a large extent by the geometry of the enclosure. For values of radiation resistance when energy is directed into free space an article by S. A. Schelkunoff [5] may be referred to.

For oscillation, as pointed out, the sum of all these positive resistances must be less than the negative resistance of the electron discharge and for high efficiency the radiation resistance should be much greater than the sum of all of the other positive resistances. This is usually found to be the case, and in fact the radiation resistance is likely to be so great as to stop oscillations unless the gap is made sufficiently small.

In designing a hollow wave guide mounting for diodes of the sort pictured in Fig. 1 it was recognized that since the high-frequency wave energy issues from the coaxial resonator as a wave guided along the heater leads, the natural and probably most effective thing to do was to dispose these leads so that the field associated with them would conform as nearly as possible to one of the wave types which can be supported in a hollow wave guide. Of these wave types, the so-called H_1 type[6] is readily generated by high-frequency current in a wire extending across a diameter of the guide, and the wave guide mounting shown in cross section on Fig. 3 is such as to give rise to this type of wave. For mechanical reasons a brass pipe of circular cross sections was chosen for the guide, and its diameter ($3\frac{7}{8}$ inches) was chosen large enough so that it would freely transmit an H_1 wave of the expected frequency. In the mounting, the high frequency circuit is completed from the anode to the wall of the guide through a stopping condenser.

Preliminary experiments with diode no. 24 had shown that when wave power issuing from the tube was allowed to radiate into free space, a space current of 500 milliamperes with anode voltage of about 300 volts was required to maintain oscillations. An interesting and instructive experiment is then to determine by how much this current

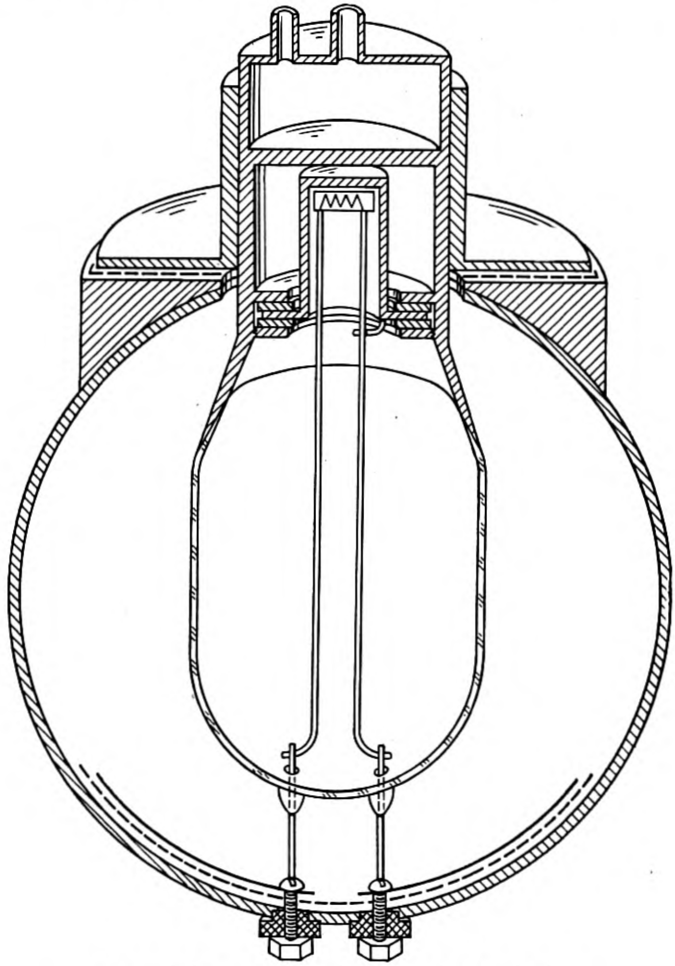

Fig. 3—Hollow wave guide mounting for diodes.

is reduced when radiation is held to as low a value as possible. For this purpose an assemblage as in Fig. 4 was used. Here a wave guide mounting of form comparable to Fig. 3 is clamped into sections of wave guide closed at the two ends by closely fitting but longitudinally adjustable reflecting pistons. By thus completely enclosing the tube, escape of energy into free space is avoided, and the losses external to the tube are reduced to the ohmic losses (including dielectric losses) incident to the existence of the wave within the guide. The presence of wave power within the guide is indicated by a crystal detector-microammeter combination connected to an antenna extending a short distance within the guide, as shown in Fig. 4. Adjustment of the pistons closing the

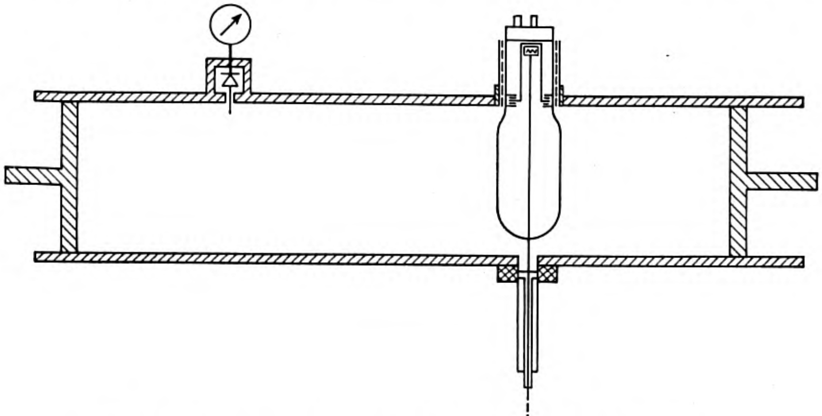

Fig. 4—Apparatus arrangement for "No Load" test of diode.

ends of the wave guide system allowed the attainment, for each value of anode current and voltage, of the most favorable impedance conditions for abstraction of energy from the diode.

The results of this experiment are shown on Fig. 5, which gives the boundaries of the domain of oscillation of the two tubes whose dimensions are given on Fig. 1. The large gain in extent of the oscillation region of tube no. 24 is immediately apparent; the free space oscillation limit of $E_p = 300$ volts, $I_p = 500$ ma. has been lowered to $E_p = 210$ volts, $I_p = 110$ ma. For tube no. 37 oscillations occur at much lower voltages, as is to be expected from the smaller anode-cathode distance, and the minimum plate current required to maintain oscillations is also somewhat smaller.

In the arrangement of Fig. 4 no useful power is extracted from the diode. To examine the oscillation domain of the diodes when delivering useful power, the arrangement of Fig. 6 was used. Here in a section of wave guide closed at both ends by tightly fitting but longitudinally adjustable pistons there are placed the diode mounting shown on Fig. 3 and a power absorbing and measuring element. The assemblage constitutes in effect a wave guide transformer, for by suitably adjusting the positions of the pistons with respect to the source and the power absorber, and by a proper choice of the distance between the source and the power absorber, the impedance of the latter can be matched to that of the source, so as to ensure the maximum delivery of power.

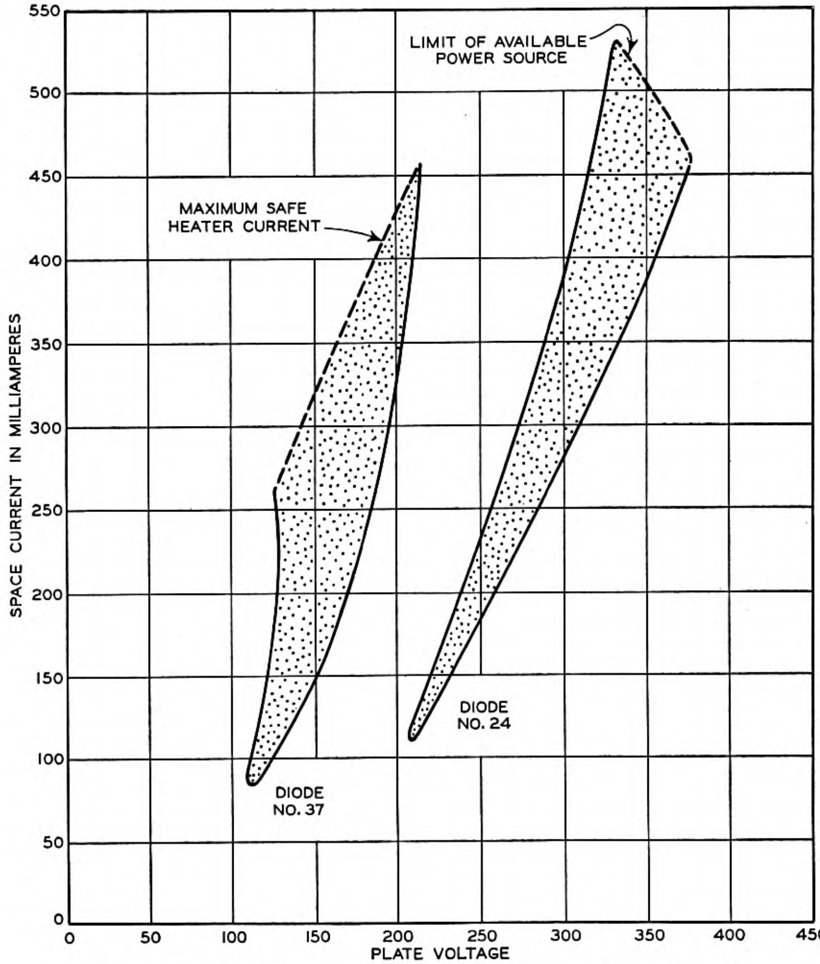

Fig. 5—"No Load" oscillation domain of diodes no. 24 and no. 37.

The power absorbing and measuring element shown on Fig. 6 represents a new and useful way of measuring power at the very high frequencies involved in this investigation. It makes use of the high negative temperature coefficient of resistance of boron. In the middle of a wire extending across a diameter of the wave guide, parallel to the lines of electric force in an H_1 wave, there is placed a small crystal of boron. Connection to the crystal is made by fine platinum wires, melted into two small globules on opposite sides of the crystal.* By virtue of its small size and the fine leads connected to it, small amounts of power dissipated in the resistance of the crystal will raise its temperature materially, with a consequent large change in its resistance. With a stopping condenser, an ohmmeter connected as shown in Fig. 6 serves to indicate the resistance of the crystal when absorbing high-frequency power, and calibration curves showing resistance as a function of power absorption can be obtained with direct current.

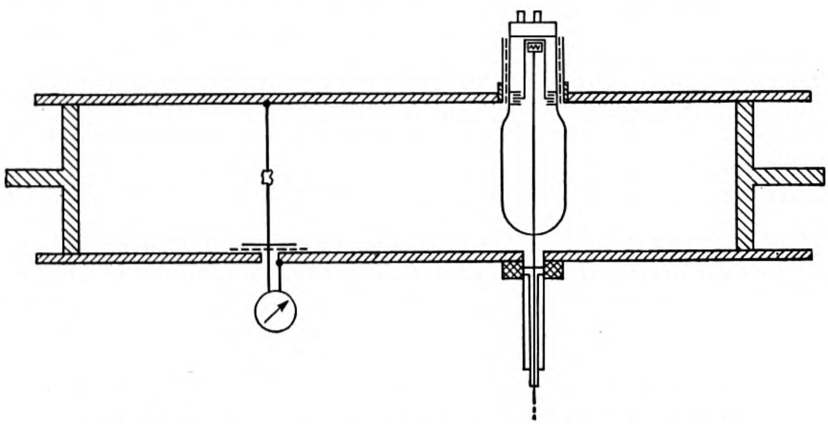

Fig. 6—Apparatus arrangement for "Loaded" test of diode no. 24.

Power output and efficiency data obtained during these measurements are shown on Fig. 7. Power outputs of a few tenths of a watt at efficiencies ranging from one to two tenths of a per cent are obtainable.

In consideration of the wave-length of the oscillations generated by these two diodes, it will be recalled that they were designed nominally for a wave-length of about 10 centimeters. For diode No. 24 the wave-length was close to 10.6 cm. (2830 mc.) and for diode no. 37 it was somewhat higher, about 11.55 cm. (2600 mc.). This difference is of the order to be expected from the difference in the dimensions of the two tubes.

While the wave-length should be fixed largely by the dimensions of the coaxial resonant circuit built into the diode, it is to be expected that it will be affected to a small extent by the applied voltage and by

* These were developed by Mr. G. L. Pearson of the Bell Telephone Laboratories.

the position of the piston closing one or both of the ends of the wave guide. In the case of diode No. 37 the wave-length was found to vary over a range between 11.50 and 11.65 cm. with plate voltage and over a range between 11.52 and 11.56 cm. with piston position.

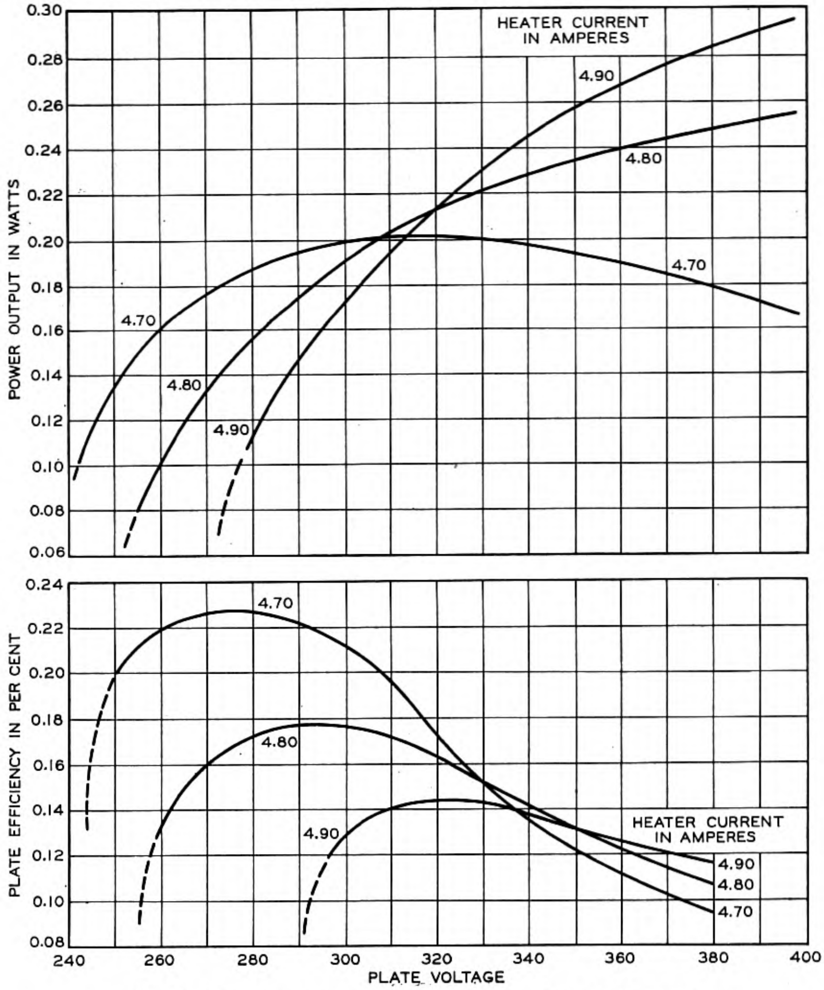

Fig. 7—Power output and plate efficiency for diode no. 24.

In conclusion, the writers wish to mention the work done by Mr. C. A. Bieling of the Bell Telephone Laboratories in working out suitable mechanical design features and in the actual assembly and processing of tubes which were built and tested.

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A Representation of the Sunspot Cycle *

By C. N. ANDERSON

ALTHOUGH sunspots had been observed occasionally back to ancient times, their study may be said to date from their rediscovery by Galileo in the spring of 1610 with the then newly invented telescope. Since then much has been written about their nature, their periodicities and possible influence on human affairs.

The purpose of the study reported on in this paper was to analyze the components of the sunspot data and thereby to reconstruct a curve which would not only represent the variation in sunspot numbers from 1749 to date but would also be consistent with times of maxima and minima from 1610 to 1749. A number of attempts along this line have been made in the past,[3, 4, 5, 6, 9, 11] all of which have neglected the data previous to 1749 and all have used a slightly different method of analysis. It is believed that the agreement in the present study is somewhat better than in those of the past; nevertheless, no claim is made for any great accuracy in predicting future sunspot activity. Harmonic analysis based on a fraction of a period is always a source of danger and, furthermore, we have no assurance that all the components of the sunspot curve are periodic functions.[8] In fact, some papers have appeared in which each cycle was treated as a more or less independent outburst.[2, 10, 12, 13] Nevertheless, because of the long base line over which agreement is obtained in this present study, it is hoped that the results may not be too much in error for at least a few cycles.

The data used are the series of relative sunspot numbers begun by Rudolph Wolf,[1] Professor of Astronomy at Zurich and continued by his successors Wolfer and Brunner. Wolf began his systematic observations of sunspot numbers in 1849. He endeavored to make some allowance for the area of the spots and to avoid having a small spot of short duration count as much as a large group. With this in mind, he applied the following formula to his observation:

$$\text{Relative sunspot number} = k(10g + f),$$

where g and f are the group and total spot numbers respectively,

* Presented before the Astronomy Section of A.A.A.S. at Richmond, Va., December 28, 1938.

and k is a constant depending on the type of telescope and other factors affecting the observation. The figure 10 is an arbitrary one arrived at by Wolf from investigation of a number of individual cases and which seemed to give him the proper relationship.

A careful study was then made by Wolf of all existing records of prior data. Hofrat Schwabe of Dessau supplied data for the period 1826 to 1855 to which a correction was applied determined from a study of the overlapping data and from the percentage of spotless days. Johann Casper Staudacher of Nürnberg had made a total of 1131 observations (from one to ten every month) by means of a helioscope during the period 1749 to 1799. He often gave detailed descriptions and included many sketches. Imagine a man making observations for fifty years without any attempt at analysis and then have the data resurrected fifty years after his death to form an important contribution to the record. Flaugergues (1794–1830), Tevel (1816–1836), Adams (1819–1823) and Arago (1822–1830) supplied most of the data for the intervening period between the observations of Staudacher and Schwabe. Wolf lists about a hundred references (225 up to 1866) to sunspots prior to 1850. In most cases, the observations were incidental to other solar observations such as culminations, solar diameter, eclipses, transits, or on the nature of sunspots rather than their number. Each record was carefully studied, and from them all Wolf obtained a representation of sunspot numbers for as far back as 1749 and established the times of maxima and minima with an accuracy of $\pm$ 2 years or better back as far as 1610 A.D. Although the data prior to 1849 include a certain element of unreliability and all the data represent *relative* numbers which have been obtained from the observations by applying a weighting factor, it is, however, not only the best record but the only one for such a long period of time. In the aggregate it is probably a good indication of the variation of solar activity.

The method employed in the present analysis is briefly as follows:

(*a*) The yearly averages of sunspot numbers from 1749 to the end of 1937 were first plotted in the conventional way as shown in Fig. 1; it was noted that in certain sections of the curve, notably after 1840, the maximum amplitudes of alternate eleven-year periods were higher than the intervening ones.

(*b*) The data were redrawn with alternate eleven-year periods above and below the time axis; this not only smoothed out the envelope of the maxima but also simplified the analysis by eliminating a computed mean value base line which has been employed in previous analyses; the maximum-amplitude component becomes approximately

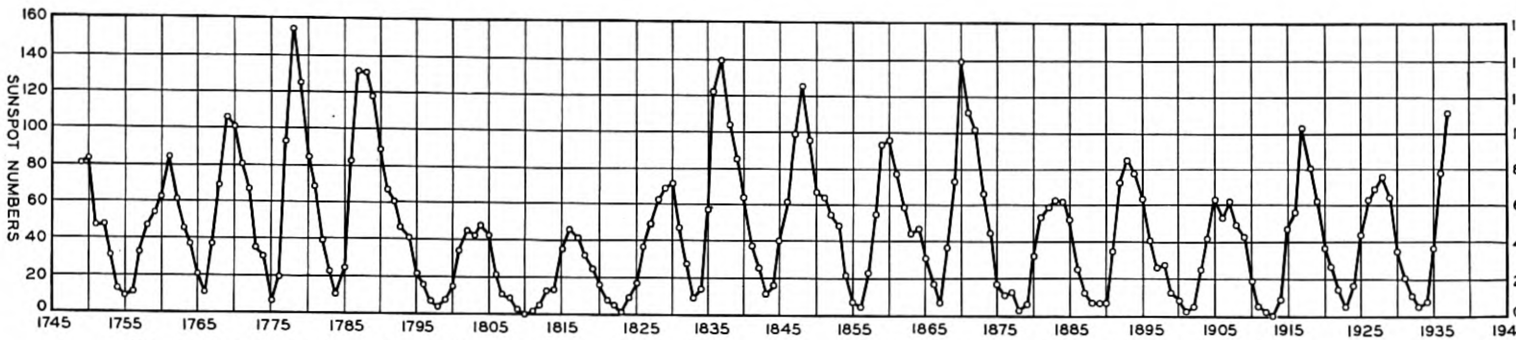

Fig. 1—Yearly averages of monthly sunspot numbers. This series of relative sunspot numbers was begun by Wolf of the Zurich Observatory and continued by his successors, Wolfer and Brunner.

twenty-two years instead of eleven years and the physical justification is the similarity in the polarity of the leading spots in alternate eleven-year periods.

(*c*) Quarterly values were obtained from the curve in order to obtain a smaller unit and to approximate better such periods as 22.25 years, 17.33 years and other periods not an integral number of years; the values were assigned + or − signs in accordance with whether they were above or below the time axis.

(*d*) A periodogram was computed and the component of maximum amplitude appeared to be 22.75 years; this component was eliminated, a new periodogram computed and so on; after many trials it was finally decided that no solution could be found which would reduce the residue satisfactorily with 22.75 years as the main component.

(*e*) Inspection of the analysis indicated an improvement would be obtained by a decrease in the period and accordingly a change was made to 22.5 years and the computations repeated; a solution was finally obtained which fit the 1749–1937 curve of sunspots fairly well but, when extrapolated back to 1600 A.D., did not fit particularly well the observed times of maxima and minima for the period 1610 to 1749; a still further reduction in the chief component was necessary.

(*f*) The computations were repeated with 22.25 years as the chief component; after the computation of the series was completed, it was discovered that the components were either harmonics or nearly integral harmonics of 312 years; many of these components had been in use since the original periodogram.

(*g*) A search was made for the 312-year period, since it should be possible to check in the cases of two minima and two maxima, with the following results:

Minima	1610.8 ± 0.4	1923.1	Diff. 312.3 ± 0.4
Maxima	1615.5 ± 1.5	1928.6	313.1 ± 1.5
Minima	1619 ± 2	1933.6	314.6 ± 2
Maxima	1626 ± 0.5	1937.7	311.7 ± 0.5

giving a weighted average of 312.5 years or an average of 312.0 years using the two most reliable values.

(*h*) Computations were again repeated using harmonic components of 312 years. This gave a very good representation of the data from 1749 to date and also agreed with the times of maxima and minima for the period 1610 to 1749. The resultant curve is shown in Figs. 2 and 3.

The components of the reconstructed curve are shown in Fig. 4. It is of interest to note that the 22.25-year component which is largely

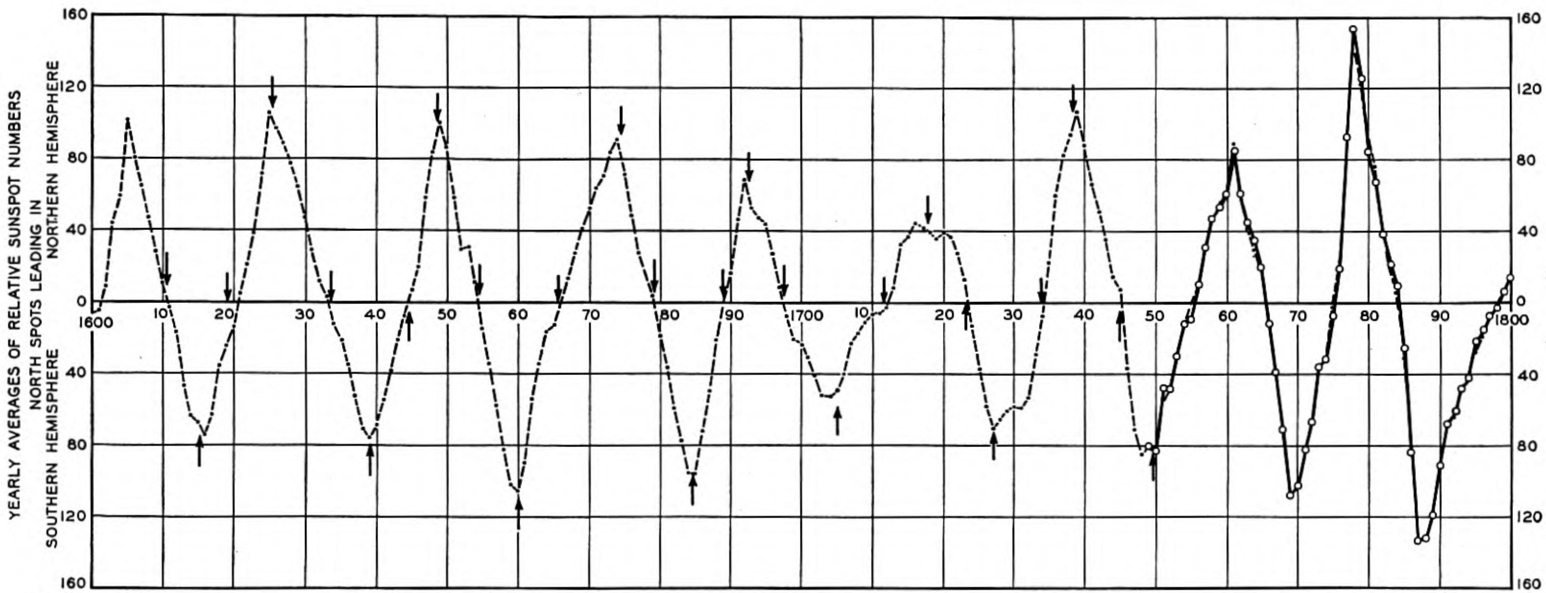

Fig. 2—Measured and computed sunspot numbers, 1600–1800 A.D. Solid line indicates measured values; light dashed line indicates values obtained by adding up the components computed from the measured values; components are harmonics of a 312-year period; arrows indicate the years when maxima and minima were observed.

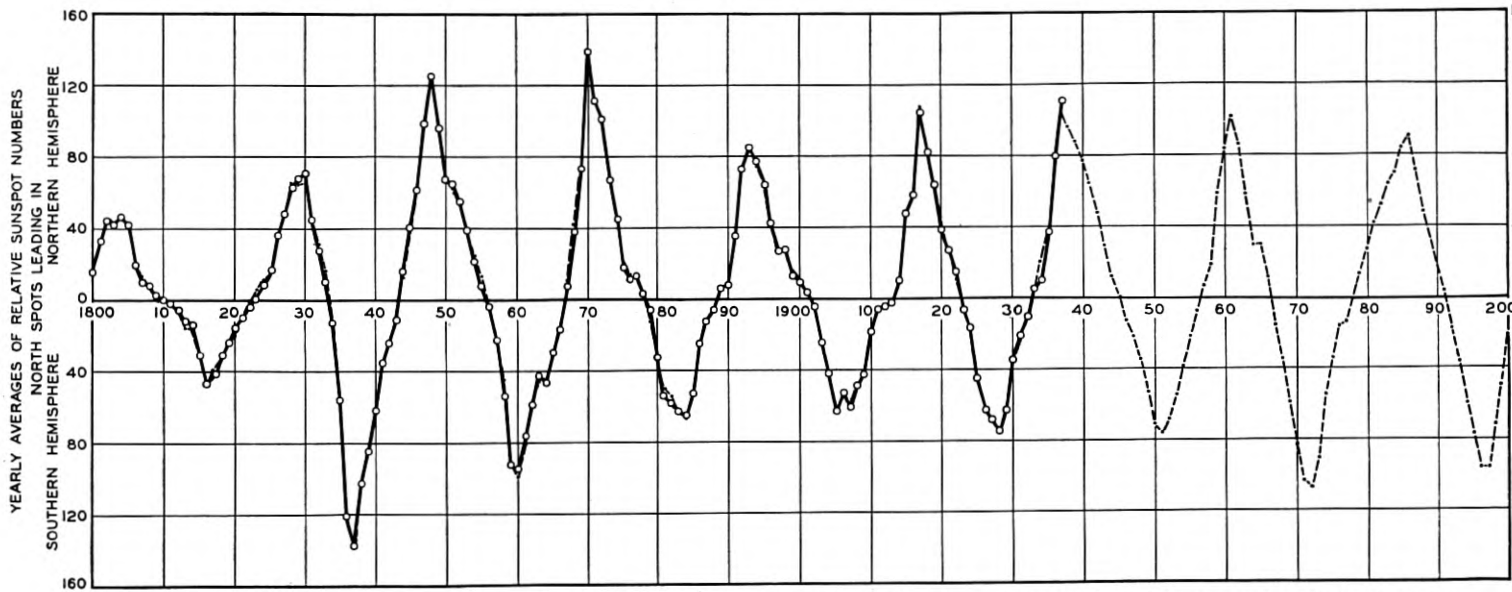

Fig. 3—Measured and computed sunspot numbers, 1800–2000 A.D. Solid line indicates measured values; light dashed line indicates values obtained by adding up the components computed from the measured values; components are harmonics of a 312-year period.

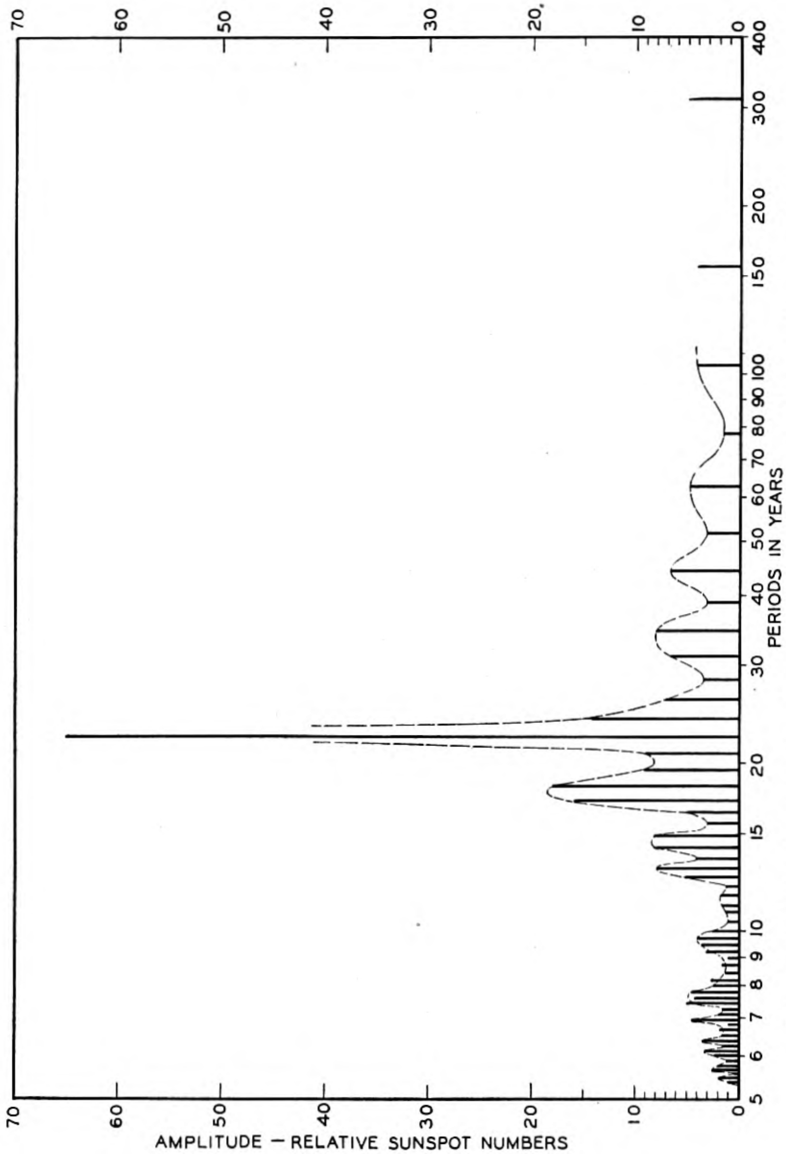

Fig. 4—Analysis of components of the sunspot cycle. Vertical lines indicate amplitudes of harmonic components of an apparent 312-year cycle.

responsible for the eleven-year periodicity of sunspots has an amplitude of only about 2/5 of the greatest amplitudes of the resultant maxima. Next in importance are the two adjacent periods at 17.3 and 18.4 years, respectively.

In conclusion, the study has resulted in a representation of yearly sunspot averages which agrees as well as could be desired with the data and which is also consistent with the reported times of maxima and minima back as far as 1610 A.D. The chief component has been treated as a 22-year (22.25 years) component instead of eleven years. In the course of computation the components appeared to be harmonics, or nearly so, of a 312-year period. A substantiation of this 312-year cycle was found in a check of the overlapping data (maxima and minima from 1923 to date).

It had been hoped that the resultant distribution of amplitudes versus frequency of the components might be capable of simple interpretation and that a rather simple explanation of the phenomena of sunspot periodicity might result, such as one or two forces acting upon a nonlinear system with a given fundamental period. Further studies may still indicate this to be the case.

The author wishes to express his appreciation of the assistance of Miss Helen Grant with the rather tedious computations.

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The Number of Impedances of an n Terminal Network

By JOHN RIORDAN

This paper gives the enumeration of impedances measurable at the n terminals of a linear passive network. The enumeration supplies background for the study of network representations and the numerical results which are given up to ten terminals are perhaps surprising in the rapidity of the rise of the number of impedances with the number of terminals; almost 126,000,000 impedances, e.g., are measurable for ten terminals.

A LINEAR passive network having n accessible terminals may be completely represented by an equivalent direct impedance network,[1] consisting of branches, devoid of mutual impedance, connecting the terminals in pairs. The number of elements (branches) in this representation is equal to the number of combinations of n things taken two at a time, i.e., $\frac{1}{2}n(n - 1)$. Each of the elements is defined by an impedance measured by energizing between one of the terminals it connects and the remaining terminals connected together and taking the ratio of the driving voltage to the current into the other terminal it connects. The network then is represented by a particular set, of $\frac{1}{2}n(n - 1)$ members, of impedances measurable at its terminals; as will appear later, the set is of short-circuit transfer impedances.

The direct impedance network is one among many network representations; it is taken as illustrative of two aspects, (i) the necessity of a certain number of elements $\frac{1}{2}n(n - 1)$ and (ii) the expression of these elements in terms of measurable impedances. It is well known that any linearly independent set, of $\frac{1}{2}n(n - 1)$ members, of the measurable impedances of an n-terminal network will serve as a network representation; hence the enumeration of representations may be taken in two steps, the first of which, the enumeration of measurable impedances, is dealt with in the present paper.

The number of measurable impedances for two to ten terminal linear passive networks is given in Table I, which lists the driving-point impedances, D_n, transfer impedances (open or short circuit), T_n, certain additional transfer impedances to be described later, U_n, and the total N_n. As mentioned below, this total counts once only

[1] Item (*b*) in the list of equivalent networks given by G. A. Campbell "Cisoidal Oscillations," *Trans. A.I.E.E. 30*, pp. 873–909 (1911), p. 889; or p. 81, "Collected Papers of George Ashley Campbell," Amer. Tel. & Tel. Co., New York, 1937.

impedances which are equal by the reciprocity theorem; the doubling of T_n in forming the total is due to the equality in number of open-circuit and short-circuit transfer impedances. The numbers increase rapidly with n, reaching almost 126,000,000 for ten terminals. The number of representations, which is the number of combinations of the measurable impedances $\frac{1}{2}n(n-1)$ at a time less the number of non-independent sets, at a guess increases even more rapidly, indicating a variety of equivalents, few of which seem to have been investigated.

TABLE I

MEASURABLE IMPEDANCES OF AN n-TERMINAL NETWORK

n	D_n	T_n	U_n	$N_n = D_n + 2T_n + U_n$
2	1	0	0	1
3	6	3	0	12
4	31	33	60	157
5	160	270	1,050	1,750
6	856	2,025	12,540	17,446
7	4,802	14,868	129,570	164,108
8	28,337	109,851	1,257,060	1,505,099
9	175,896	827,508	11,889,990	13,720,902
10	1,146,931	6,397,665	111,840,180	125,782,441

Because the field of the work is somewhat unusual, considerable space is given to details in the formulation of the problem before proceeding to the enumeration proper. The enumerating expressions obtained are found susceptible of some mathematical development which, though subsidiary to the main object of the paper, seems of sufficient interest to justify the relatively brief exposition given. The arrangement is such that readers not interested in this mathematical half may obtain the substance of the paper without it.

FORMULATION OF THE PROBLEM

The enumerating problem is essentially one of combinations, as indicated schematically in Fig. 1, which shows the n terminals of a linear passive network together with the apparatus required for impedance measurement, that is, a source, a voltmeter and an ammeter, each supplied with two terminals (shown solid to distinguish them from the network terminals). Each of these latter may be connected across any pair of the n terminals except that the ammeter, which constitutes a short circuit, may not be connected to terminals to which either the source or voltmeter is connected; in the former case no current will be supplied to the network and in the latter the voltmeter will read zero. The ammeter may be connected in series with the source to read the source current, of course.

Although but one source, voltmeter, and ammeter are shown, as many of each as will produce distinct impedances should of course be included. Multiple sources are not required because if the source voltages are in defined proportions, as is necessary to determine impedances independent of source voltage, the corresponding measurable admittances are linear combinations of single-source admittances, by the principle of superposition; a similar requirement on source currents produces impedances which are linear combinations of single-source impedances. A single voltmeter is sufficient because it has no effect on network currents or voltages and it is immaterial whether

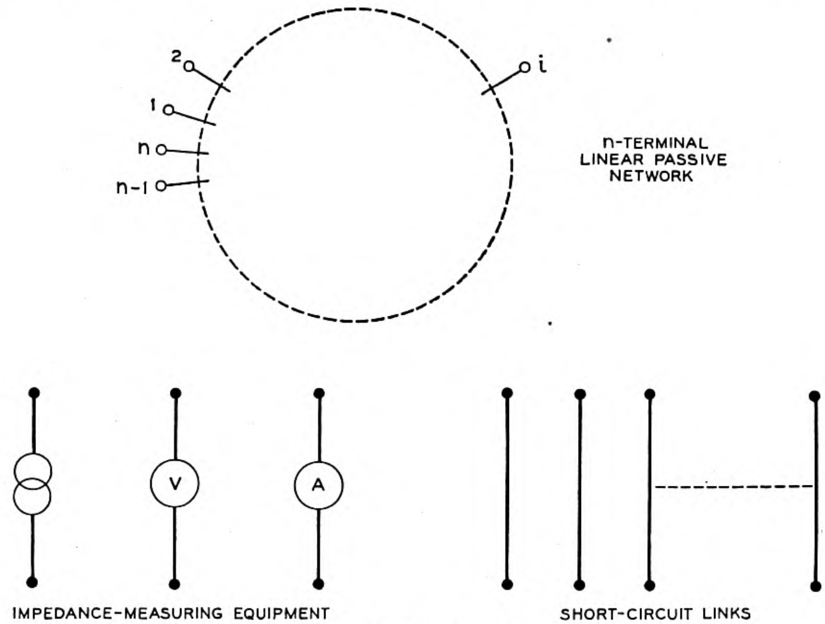

Fig. 1.—Elements involved in impedance enumeration.

impedances are supposed measured by successive positions of a single voltmeter or by many voltmeters. The connection of an ammeter is equivalent to a short circuit (except of course when in series with a source) across the terminals the ammeter connects; this alters network voltages and currents and the impedances measured without the ammeter differ from those with it. Hence a plurality of ammeters or its equivalent is required; for convenience, all ammeters except that one determining a specific impedance under consideration are supposed replaced by the short-circuiting links on the right of Fig. 1, thus focussing attention on the single items of the enumeration.

The classification under which the enumeration is conducted is illustrated by Fig. 2, which shows typical positions of source, voltmeter and ammeter for measuring impedances of three classes. In the first of these, the ammeter reads the source current, the voltmeter source voltage (across some pair of the network terminals) and the class is that of driving-point impedances, D_n. In the second class, that of transfer impedances T_n, there are two types of connection: in the first the ammeter reads the source current, the voltmeter a non-source voltage, the voltage-current ratios being open-circuit transfer impedances; in the second the voltmeter reads the source voltage and

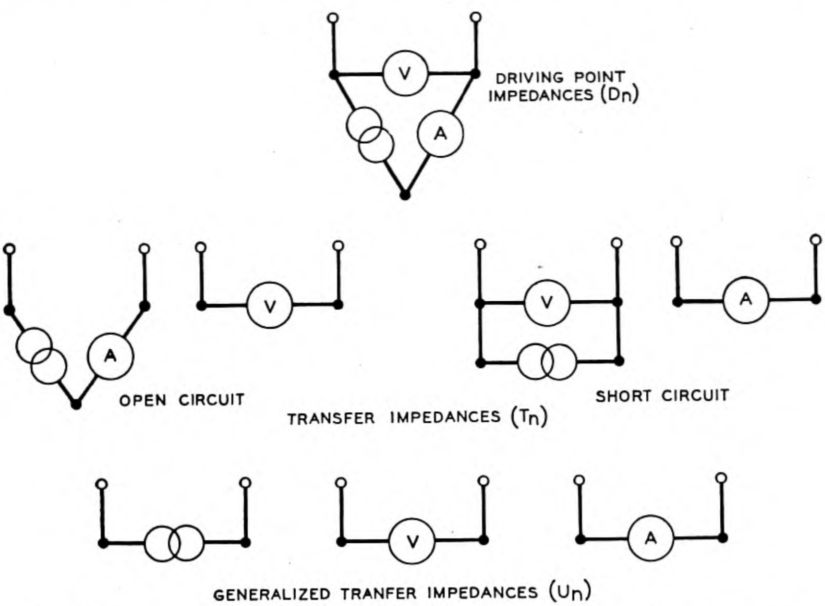

Fig. 2—Arrangement of apparatus for measuring impedances of three classes.

the ammeter a non-source current, the voltage-current ratios being short-circuit transfer impedances. It will be noted that the two connections differ only in that the ammeter and voltmeter are interchanged. The third class is that of generalized transfer impedances U_n, in which both voltmeter and ammeter are across non-source terminals.

The last class, of course, might be supposed to include the two preceding ones but the separation proves convenient not only for numerical work, as will appear, but also for keeping distinct both well-recognized and formally different classes.

The most important of these differences in classes is that arising from the reciprocity theorem. Of the three classes, only the T (open-circuit and short-circuit transfer impedances) includes members which are equal by the reciprocity theorem; this follows because the reciprocity theorem requires interchange of voltmeter (or ammeter) and source with associated ammeter (or voltmeter) and the T class alone permits this. It is a matter of taste whether such duplicates should be counted separately or as one; in the interest of keeping large figures as low as possible they are here counted as one, since the classification is such that the other alternative may be taken merely by doubling the T_n.

Another reason for keeping the T class distinct is that total open-circuit and short-circuit transfer impedances for a given number of terminals are equal in number. This is proved immediately by observing that the two connections shown in Fig. 2 for this class are in one-one correspondence: each may be obtained from the other by interchanging voltmeter and ammeter. Moreover, if $T^o{}_{x,\,n}$ and $T^s{}_{x,\,n}$ are the numbers of open-circuit and short-circuit transfer impedances measurable when short circuits have been placed across the n terminals in all possible ways so as to realize x terminals, each merged group of terminals counting as a single terminal, the correspondence leads to the relation

$$T^o{}_{x+1,\,n} = T^s{}_{x,\,n}, \tag{1}$$

since the interchange of voltmeter and ammeter in the measuring arrangement for open-circuit transfer impedances results in one less available terminal, two terminals being merged by the ammeter short circuit. Note that $T^o{}_{2,\,n} = T^s{}_{n,\,n} = 0$, since with just two terminals, no non-source voltages and with n terminals no non-source currents are measurable.

Equation (1) is important in determining enumerating expressions in the section following.

Enumerating Expressions

The laws of enumeration appear most simply exposed by examining the simplest cases first.

For two terminals, there is but one measurable impedance, the driving-point impedance between the terminals.

For three terminals, with the terminals distinct, there are three driving-point and three open-circuit transfer impedances, for there are three ways of selecting driving pairs of the three terminals and for each selection two ways of selecting pairs for open-circuit voltage measurement, the total of six transfer impedances being halved to eliminate reciprocity theorem duplicates. With two of the terminals connected by an ammeter, there are again three driving-point and three transfer impedances, the latter being short-circuit transfer impedances, for there are three ways of connecting pairs of terminals and one driving-point and two transfer impedances for each, the total of transfer impedances again being halved to eliminate duplicates.

There are no generalized transfer impedances because with an ammeter connected, there is only one measurable voltage, the driving-circuit voltage.

With terminals designated by t_1, t_2 and t_3, the conditions arising from connection of terminals may be exhibited as follows:

Terminals distinct	$t_1 \vert t_2 \vert t_3$		
Pairs connected	$t_1t_2 \vert t_3$	$t_1t_3 \vert t_2$	$t_1 \vert t_2t_3$

the lines of separation dividing the terminals into groups such that the terminals in any group are merged into a single terminal. Paying attention only to the number of terminals in each group, the groups illustrated may be designated by the partition notation (111) or (1^3) and (21), the numbers in the designation being partitions[2] of the number 3.

The enumeration for three terminals may then be exhibited as follows:

Measurable Impedances

Group	Driving Point	Open-Circuit Transfer	Short-Circuit Transfer	Total
(1^3)	3	3	0	6
(21)	3	0	3	6
	6	3	3	12

It will be noted that the open-circuit and short-circuit transfer impedances satisfy equation (1), that is, $T^o{}_{3,\,3} = T^s{}_{2,\,3}$.

This table and its correspondents for larger values of n show that the impedances may be expressed as sums with respect to x, where x is the number of terminals defined as in equation (1), from 2 to n; thus e.g., $D_n = \sum D_{x,\,n}$ where $D_{x,\,n}$ is the number of driving-point impedances measurable for all conditions of merging of n terminals such that the resulting number of terminals is x. Moreover, con-

[2] A partition of a number n is any collection of positive integers whose sum is equal to n. It may be noted that the number of parts of a partition is the number x of equation (1); the partition (1^3) has three parts corresponding to the three distinct terminals; (21) has two parts corresponding to two terminals, each merged pair of terminals counting singly.

sidering for the moment only the driving-point and open-circuit transfer impedances, the numbers $D_{x,\,n}$ and $T^o_{x,\,n}$ are the products of two factors: (i) the number of such impedances measurable for x terminals, which is independent of n and (ii) the number of ways the n terminals may be merged so as to result in x terminals, which is independent of the impedance classes. By equation (1) this result applies also to $T^s_{x,\,n}$ and, as $U_{x,\,n}$ is related to $T^s_{x,\,n}$ by a factor independent of n, as will be shown, it applies generally.

This leads to the following equation:

$$\begin{Bmatrix} D_n \\ T_n \\ U_n \end{Bmatrix} = \sum_{x=2}^{n} \begin{Bmatrix} d_x \\ t_x \\ u_x \end{Bmatrix} S_{x,\,n}. \tag{2}$$

The small letters are the several factors of the first kind and $S_{x,\,n}$ is the common second factor.

The small letters are determined as follows: A driving point impedance may be measured between every pair of terminals; hence d_x is the number of combinations of x things taken two at a time, that is:

$$d_x = \binom{x}{2} = \tfrac{1}{2}x(x-1) = \tfrac{1}{2}(x)_2, \tag{3}$$

where $(x)_i$ is the factorial symbol $x(x-1)\cdots(x-i+1)$.

For a given pair of driving terminals, there are $\binom{x}{2} - 1$ measurable open-circuit transfer impedances since a voltmeter can be connected to every pair of the x terminals except the driving pair; hence, multiplying by the number of driving terminals and by the factor one-half to eliminate reciprocity theorem duplicates:

$$\begin{aligned} t_x &= \tfrac{1}{2}\binom{x}{2}\left[\binom{x}{2} - 1\right], \\ &= \tfrac{1}{8}[4(x)_3 + (x)_4]. \end{aligned} \tag{4}$$

The second, factorial, form is given for convenience of later development.

By equation (1) this serves for enumeration of both open-circuit and short-circuit transfer impedances; the direct enumeration of the latter appears more difficult.

Considering, for the generalized transfer impedances, a fixed source and an ammeter in a fixed (non-source) position, the voltmeter may be connected across $\binom{x}{2}$ pairs of terminals when x terminals are available; one of these pairs is the source pair measuring a short-circuit transfer impedance which must be excluded; hence, remembering that reciprocity theorem duplicates are eliminated in the latter:

$$U_{x,\,n} = 2\left[\binom{x}{2} - 1\right] T^{s}{}_{x,\,n}$$
$$= 2\left[\binom{x}{2} - 1\right] T^{o}{}_{x+1,\,n} = 2\left[\binom{x}{2} - 1\right] t_{x+1} S_{x+1,\,n}.$$

Degrading x by unity to obtain the form of equations (2), the third of the lower case factors is reached as follows:

$$u_x = \binom{x}{2}\left[\binom{x}{2} - 1\right]\left[\binom{x-1}{2} - 1\right] \qquad (5)$$
$$= \tfrac{1}{8}[20(x)_4 + 10(x)_5 + (x)_6].$$

The common factor $S_{x,\,n}$ remains for determination.

Returning to the connection conditions illustrated for three terminals, this number is the number of ways separators may be placed between letters of the collection $t_1,\ t_2 \cdots t_n$ symbolizing the terminals so as to produce x compartments, symbolizing merged terminals. The terminal symbols $t_1 \cdots t_n$ may be thought of as the prime distinct factors (excluding unity) of some number and the number $S_{x,\,n}$ is then identically the number of ways a number having n distinct prime factors may be expressed as a product of x factors. The enumeration for this latter problem is given by Netto,[3] who gives the recurrence relation

$$S_{x,\,n+1} = xS_{x,\,n} + S_{x-1,\,n}$$

with

$$S_{n,\,n} = 1, \qquad S_{x,\,n} = 0,\ x > n, \qquad S_{0,\,n} = 0,\ n \neq 0.$$

This is the recurrence relation for the Stirling numbers of the second kind,[4] the notation for which has been adopted in anticipation of the result. These numbers are perhaps better known as the "divided differences of nothing," that is, as defined by the equation:

$$S_{x,\,n} = \lim_{z=0} \frac{1}{x!} \Delta^x z^n = \frac{1}{x!} \Delta^x 0^n,$$

where Δ^x denotes x iterations of the difference operator with unit

[3] "Lehrbuch der Combinatorik," Leipzig, 1901, pp. 169–170; Whitworth, "Choice and Chance," Cambridge, 1901, Prop. XXIII, p. 88, gives a generating function for the solution of this problem which, it is not difficult to show, leads to the same answer.

[4] Ch. Jordan, "Statistique Mathematique," Paris, 1927, p. 14.

interval, that is, of the operator defined by

$$\Delta f(z) = f(z+1) - f(z).$$

For convenience of reference, a short table of the numbers follows:

$n \backslash x$	0	1	2	3	4	5
	$S_{x,\,n}$					
0	1					
1	0	1				
2	0	1	1			
3	0	1	3	1		
4	0	1	7	6	1	
5	0	1	15	25	10	1

The table may be verified and extended readily by the recurrence relation.

With this table (extended to $n = 10$) and corresponding tables of d_x, t_x and u_x running to $x = 10$, the values given in Table I may be calculated by equations (2) and in this sense this paper is completed at this point. The sections below contain an algebraic and arithmetical examination of the numbers.

Generating Identities

The generating identity for the function

$$\sum_{x=0}^{n} a^x S_{x,\,n}$$

is [5]

$$\exp\left[a(e^t - 1)\right] = \sum_{n=0}^{\infty} \frac{t^n}{n!} \sum_{x=0}^{n} a^x S_{x,\,n}.$$

This leads, by differentiating s times with respect to a and setting a equal to unity, to the generating identity:

$$(e^t - 1)^s \exp\,(e^t - 1) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \sum_{x=0}^{n} (x)_s S_{x,\,n}.$$

This relation may be rendered more summarily by introducing the notation of the symbolic or umbral calculus [6] of Blissard; the expression on the right is written $\exp t\delta$ where δ is an umbral symbol standing for the sequence $(\delta_0,\ \delta_1,\ \cdots \delta_n \cdots)$ in this case infinite, through the relation $\delta^n = \delta_n$ and:

[5] E. T. Bell, "Exponential Polynomials," *Annals of Math. 35*, 2 (April, 1934) p. 265; or J. Riordan, "Moment Recurrence Relations . . . ," *Annals of Math. Statistics 8*, 2, pp. 103–111 (June, 1937), eq. 3.4.

[6] Cf. Bell, l.c. p. 260 where further references are given.

$$\delta_n = \sum_{x=0}^{n} (x)_s S_{x,\,n}.$$

All algebraic operations on umbral symbols are carried out as in ordinary algebra except that the degrading of subscripts must not be performed until operations are completed. It must be noted that $\delta^0 = \delta_0$, hence is unity only when $\delta_0 = 1$, as in the present case and not always as in ordinary algebra.

The umbrae for the impedance numbers are written D, T and U, and by use of the generating identity above have the following generating identities:

$$\begin{aligned}
\exp tD &= \tfrac{1}{2}(e^t - 1)^2 \exp (e^t - 1),\\
\exp tT &= \tfrac{1}{8}[4(e^t - 1)^3 + (e^t - 1)^4] \exp (e^t - 1), \qquad (6)\\
\exp tU &= \tfrac{1}{8}[20(e^t - 1)^4 + 10(e^t - 1)^5 + (e^t - 1)^6] \exp (e^t - 1).
\end{aligned}$$

These follow immediately from the base generating identity and the factorial expressions for d_x, t_x and u_x.

Expanding these expressions in powers of e^t gives alternate expressions as follows:

$$\begin{aligned}
\exp tD &= \tfrac{1}{2}(e^{2t} - 2e^t + 1) \exp (e^t - 1),\\
\exp tT &= \tfrac{1}{8}(e^{4t} - 6e^{2t} + 8e^t - 3) \exp (e^t - 1), \qquad (6.1)\\
\exp tU &= \tfrac{1}{8}(e^{6t} + 4e^{5t} - 15e^{4t} + 35e^{2t} - 36e^t + 11) \exp (e^t - 1).
\end{aligned}$$

To recapitulate, these expressions mean that D_n, T_n and U_n are the coefficients of $t^n/n!$ in the expansions of the right-hand sides; taking D_n, for example, the first equation of (6) is equivalent to the equation:

$$D_n = \lim_{t=0} \frac{d^n}{dt^n} [\tfrac{1}{2}(e^t - 1)^2 \exp (e^t - 1)],$$

which may be shown to be equivalent to the first of equations (2).

The generating identities lead immediately to recurrence relations, as will now appear.

Recurrence Relations

Recurrence relations to be derived are all obtained by differentiation with respect to t. Under this operation umbrae behave like ordinary variables; thus

$$\begin{aligned}
\frac{d}{dt} \exp tD &= D \exp tD\\
&= D_1 + D_2 t + D_3 \frac{t^2}{2!} + \cdots D_{n+1} \frac{t^n}{n!} + \cdots,
\end{aligned}$$

as may be verified readily.

In the first type of recurrence only successive values of the numbers themselves appear. The derivation is illustrated for the D_n, the simplest case. Differentiating the first of equations (6) leads to the relation:

$$D \exp tD = \tfrac{1}{2}(e^{3t} - e^t) \exp (e^t - 1),$$

or

$$\begin{aligned}(e^t - 1)D \exp tD &= e^t(e^t + 1)\tfrac{1}{2}(e^t - 1)^2 \exp (e^t - 1)\\ &= (e^{2t} + e^t) \exp tD.\end{aligned}$$

Equating coefficients of $t^n/n!$ in this relation gives the umbral recurrence:

$$D(D + 1)^n - D_{n+1} = (D + 2)^n + (D + 1)^n,$$

which in ordinary form is:

$$(n - 2)D_n = \sum_{i=1}^{n} \left[\binom{n}{i} (2^i + 1) - \binom{n}{i + 1} \right] D_{n-i}.$$

The process is common to the three classes of numbers and produces similar results which may be put in general form as follows:

$$a_n A_n = \sum_{i=1}^{n} \left[\binom{n}{i} b_i - \binom{n}{i + 1} c_{i+1} \right] A_{n-i}, \tag{7}$$

where A_n, a_n, b_i and c_i are defined for the three cases by the following table:

A_n	a_n	b_i	c_i
D_n	$n-2$	2^i+1	1
T_n	$4n-12$	$3^i+6\cdot 2^i+5$	2^i+2
U_{n-3}	$480\left[\binom{n}{4} - \binom{n}{3}\right]$	$7^i+10\cdot 6^i+5\cdot 5^i-60\cdot 4^i$ $+35\cdot 3^i+34\cdot 2^i-25$	$6^i+4\cdot 5^i-15\cdot 4^i$ $+35\cdot 2^i-36$

Somewhat more convenient recurrences may be obtained by allowing the presence of numbers other than those for which the recurrence is sought. For this purpose it is expedient to introduce the exponential numbers ϵ_n of E. T. Bell.

These are defined by the generating identity:

$$\exp t\epsilon = \exp (e^t - 1)$$

or by the equivalent formula:

$$\epsilon_n = \lim_{t=0} \frac{d^n}{dt^n} \exp (e^t - 1) = \sum_{x=1}^{n} S_{x,\,n},$$

which shows their close relation with the impedance numbers. They have the recurrence relation:

$$\epsilon_{n+1} = (\epsilon + 1)^n$$

and

$$\epsilon_0 = \epsilon_1 = 1.$$

Now, returning to the first of equations (6) and again differentiating:

$$\begin{aligned} D \exp tD &= \tfrac{1}{2}[2(e^t - 1)e^t + (e^t - 1)^2 e^t] \exp (e^t - 1), \\ &= (e^t - 1)e^t \exp (e^t - 1) + e^t \exp tD, \\ &= 2 \exp tD + (e^t - 1) \exp (e^t - 1) + \exp t(D + 1), \\ &= 2 \exp tD + \exp t(\epsilon + 1) - \exp t\epsilon + \exp t(D + 1), \end{aligned}$$

from which, passing to the coefficient relation, comes the umbral recurrence:

$$D_{n+1} = 2D_n + (D + 1)^n + \epsilon_{n+1} - \epsilon_n.$$

Similar recurrences for the T and U numbers are derived in the same way; writing $\Delta\epsilon_n = \epsilon_{n+1} - \epsilon_n$, the results may be summarized as follows:

$$\begin{aligned} D_{n+1} &= 2D_n + (D + 1)^n + \Delta\epsilon_n, \\ T_{n+1} &= 4T_n + (T + 1)^n + 3D_n, \\ U_{n+1} &= 6U_n + (U + 1)^n + 46T_n - 4T_{n+1} \\ &\qquad + 30D_n - 6D_{n+1} + 6\Delta\epsilon_n. \end{aligned} \tag{8}$$

The expressions in parentheses, it will be remembered, are shorthand binomial expansions; thus:

$$(D + 1)^n = \sum_{i=0}^{n} \binom{n}{i} D_i.$$

Relations with the Exponential Integers

The generating identities in equations 6.1 furnish immediate relations with the exponential integers, ϵ_n. Writing $\exp (e^t - 1)$ as $\exp t\epsilon$, as above, and passing from generating relations to coefficient relations, these results are as follows:

$$\begin{aligned} D_n &= \tfrac{1}{2}[(\epsilon + 2)^n - 2(\epsilon + 1)^n + \epsilon_n], \\ T_n &= \tfrac{1}{8}[(\epsilon + 4)^n - 6(\epsilon + 2)^n + 8(\epsilon + 1)^n - 3\epsilon_n], \\ U_n &= \tfrac{1}{8}[(\epsilon + 6)^n + 4(\epsilon + 5)^n - 15(\epsilon + 4)^n \\ &\qquad + 35(\epsilon + 2)^n - 36(\epsilon + 1)^n + 11\epsilon_n]. \end{aligned} \tag{9}$$

Expanding internal parentheses by the binomial theorem, the general result is as follows:

$$A_n = \sum_{i=1}^{n} \binom{n}{i} \alpha_i \epsilon_{n-i},$$

where the coefficients α_i for the three cases are as follows:

A_n	α_i
D_n	$2^{i-1} - 1$
T_n	$(2^{i-1} - 1)(2^{i-2} - 1)$
U_n	$\frac{1}{8}[6^i + 4\cdot 5^i - 15\cdot 4^i + 35\cdot 2^i - 36]$

Note that in the first case $(D_n)\alpha_1 = 0$, in the second $(T_n)\alpha_1 = \alpha_2 = 0$, in the third $(U_n)\alpha_1 = \alpha_2 = \alpha_3 = 0$. Thus a given table of values of ϵ_n up to $n = k$ determines D_n up to $k + 2$, T_n up to $k + 3$, and U_n up to $k + 4$.

Somewhat simpler relations may be derived as follows. Repeated differentiation of the generating identity of the ϵ_n with respect to t, and passage from the generating relations to coefficient relations leads to the following:

$$\begin{aligned}
\epsilon_{n+1} &= (\epsilon + 1)^n, \\
\epsilon_{n+2} &= (\epsilon + 1)^n + (\epsilon + 2)^n, \\
\epsilon_{n+3} &= (\epsilon + 1)^n + 3(\epsilon + 2)^n + (\epsilon + 3)^n,
\end{aligned}$$

or, in general:

$$\epsilon_{n+m} = \sum_{x=1}^{m} (\epsilon + x)^n S_{x,\,m}.$$

This formula may be inverted by the reciprocal relations for the Stirling numbers of the first and second kinds [7] which run as follows: If

$$a_m = \sum_{x=1}^{m} b_x S_{x,\,m}$$

then

$$b_m = \sum_{x=1}^{m} a_x s_{x,\,m}$$

where $s_{x,\,m}$ is the Stirling number of the first kind defined by the recurrence relation

$$s_{x,\,m+1} = s_{x-1,\,m} - m s_{x,\,m}$$

and the boundary conditions $s_{m,\,m} = 1$, $s_{x,\,m} = 0$ $x > m$, $s_{0,\,m} = 0$, $m > 0$.

[7] Nielsen: "Handbuch der Gamma Funktion," Leipzig, 1906, p. 69.

The inverted formula[8] is:

$$(\epsilon + m)^n = \sum_{x=1}^{m} \epsilon_{n+x} s_{x,\, m}.$$

A short table of the Stirling numbers of the first kind follows:

$m \diagdown x$	$s_{x,\,m}$ 0	1	2	3	4	5	6
0	1						
1	0	1					
2	0	−1	1				
3	0	2	−3	1			
4	0	−6	11	−6	1		
5	0	24	−50	35	−10	1	
6	0	−120	274	−225	85	−15	1

The three equations resulting from applying this transformation to equations (9) are as follows:

$$\begin{aligned} D_n &= \tfrac{1}{2}[\epsilon_{n+2} - 3\epsilon_{n+1} + \epsilon_n], \\ T_n &= \tfrac{1}{8}[\epsilon_{n+4} - 6\epsilon_{n+3} + 5\epsilon_{n+2} + 8\epsilon_{n+1} - 3\epsilon_n], \\ U_n &= \tfrac{1}{8}[\epsilon_{n+6} - 11\epsilon_{n+5} + 30\epsilon_{n+4} + 5\epsilon_{n+3} \\ &\qquad - 56\epsilon_{n+2} - 5\epsilon_{n+1} + 11\epsilon_n]. \end{aligned} \tag{10}$$

For computing purposes, values of ϵ_n and $\Delta\epsilon_n$ up to $n = 10$ are given in Table II.

TABLE II

EXPONENTIAL NUMBERS

n	ϵ_n	$\Delta\epsilon_n$
0	1	0
1	1	1
2	2	3
3	5	10
4	15	37
5	52	151
6	203	674
7	877	3,263
8	4,140	17,007
9	21,147	94,828
10	115,975	562,595

[8] Noting that $\sum_{x=1}^{m} a^x s_{x,\, m} = (a)_m$, where $(a)_m$ is the factorial symbol used throughout, the inverse relation may also be written:

$$(\epsilon + m)^n = \epsilon^n(\epsilon)_m.$$

In this notation, the inverses to equations (2) for the impedance numbers have the following simple forms which are worth noting:

$$\begin{aligned} (D)_n &= d_n \\ (T)_n &= t_n \\ (U)_n &= u_n \end{aligned}$$

Congruences

For numerical checks, it is convenient to note the simplest congruences[9] for the three numbers. These follow from the Touchard congruence for the ϵ numbers[10] which runs as follows:

$$\epsilon_{p+n} \equiv \epsilon_{n+1} + \epsilon_n \mod p,$$

where p is a rational prime greater than 2.

Since by equations (10) each of the impedance numbers is a linear function of the ϵ numbers, each has a similar congruence as follows:

$$\begin{aligned} D_{p+n} &\equiv D_{n+1} + D_n \mod p, \\ T_{p+n} &\equiv T_{n+1} + T_n \mod p, \\ U_{p+n} &\equiv U_{n+1} + U_n \mod p. \end{aligned} \tag{11}$$

Special values for the first few congruences are as follows:

	Remainder, mod p		
n	D_{p+n}	T_{p+n}	U_{p+n}
0	0	0	0
1	1	0	0
2	7	3	0
3	37	36	60

These are sufficient for checking every value in Table I at least once and the values for $n = 5, 6, 7, 8$ are checked twice.

Acknowledgment

This paper arose as a result of a suggestion made by R. M. Foster on a former paper[11] and thanks are also due him for continuous counsel and critical scrutiny which have enlarged the boundary and sharpened the outline of the problem.

[9] The congruence $D_n = r \bmod p$ is equivalent to the equation $D_n = mp + r$, where m is an integer; that is, r is the remainder after division by p (or the remainder plus some multiple of p).

[10] See E. T. Bell, "Iterated Exponential Integers," *Annals of Math.*, *39*, 3 (July, 1938), eq. 1.101, p. 541.

[11] "A Ladder Network Theorem," *Bell System Technical Journal 16*, pp. 303–318 (July, 1937); see especially footnote 3; I take this opportunity to draw attention to an error in that footnote: for four terminals (see Table I) there are 157, not 64, measurable impedances; hence the upper bound to the number of representations is 18,883,356,492, not 74,974,368.

Copper Oxide Modulators in Carrier Telephone Systems *

By R. S. CARUTHERS

Copper oxide modulators are widely used in telephone systems for translating either single speech channels or groups of speech channels to carrier frequency locations on the lines. A number of simple circuit arrangements have been developed that enable suppression of certain undesired frequencies to a degree that is impractical in tube modulators. These modulators transmit equally well in either direction and the modulating elements are more non-linear than in tube modulators. As a result numerous effects are found that ordinarily are not important in the tube arrangements. Analytical studies have been considerably simplified by the use of a small signal, and a large carrier controlling the impedance variation of the copper oxide. It is found in this case that the superposition and reciprocity theorems hold for all the circuits that it has been possible to analyze even though the modulator is made up of non-linear elements. Open and short-circuit impedance measurements can be made use of as in four-terminal linear networks, and a generalized reflection theory developed. Performance data are given for an idealized modulator under a variety of operating conditions.

Introduction

At least as early as 1927, copper oxide rectifiers were being tried as modulators for the speech channels of carrier telephone systems in this country. At this time only a rather large type of rectifier was available, better adapted for power use rather than in modulating the few milliwatts of a speech signal. Largely because of instabilities these early units were found to be unsatisfactory for modulator use. Further developments in copper oxide rectifiers made in various laboratories extended the variety and improved the quality of the product available, so that by about 1931 they began to be promising as serious competitors for vacuum tubes in modulators. Since 1931 continued improvements in copper oxide rectifiers have rapidly increased their field of application until now they are employed in practically all modulators of the latest types of carrier telephone systems.

In the new systems a copper oxide modulator is used instead of the previous push-pull arrangement of two vacuum tubes. In cable,[1]

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[1] The general features of carrier telephone terminals have been described in a paper "Cable Carrier Telephone Terminals" by R. W. Chesnut, L. M. Ilgenfritz and A. Kenner, *Electrical Engineering*, January 1938.

open wire and coaxial carrier systems, from twenty-six to twenty-eight of these modulators are needed in each direction for translating each twelve-channel group of speech bands from voice to carrier and back again. These copper oxide modulators have no power costs, tube replacements or possibilities of power failures. In Fig. 1 the four 3/16 inch diameter copper oxide discs generally used in a carrier telephone modulator are shown individually, assembled with connections, and potted in a can.

The carrier terminals have tended to become increasingly complex as it became their function to place more and more channels on a single pair of wires. The extreme simplicity and reliability of copper oxide modulators have been of great value in helping to overcome this tendency. Copper oxide modulators have been used from zero frequency to nearly four million cycles. Certain modulators for coaxial carrier systems have been designed to modulate simultaneously as many as sixty speech channels spaced over a 240,000-cycle band of frequencies.

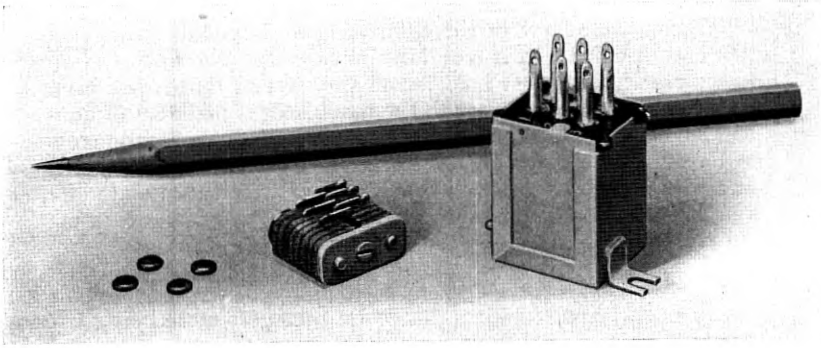


Fig. 1—Four disc copper oxide modulator.

Copper oxide modulators probably differ most from tube modulators because the simplicity of the rectifier elements allows a much greater variety of circuit arrangements to be used. Although the underlying principles of operation are not new, it has become necessary to investigate numerous transmission effects that could be neglected in tube modulators. This has resulted not only from the newer circuit arrangements with their smaller losses, but also from higher transmission standards for the overall system along with the greatly increased numbers of modulators in long circuits. Copper oxide modulators, unlike tube modulators, transmit signals equally well in either direction. While this is a simplification in allowing a modulator also to be used as a demodulator, the modulator becomes complicated by the effects of reflections back and forth into the signal bands of numerous frequency bands of modulation products.

Circuit Arrangement

The circuit arrangements used in copper oxide modulators generally are concerned either with carrier suppression, with carrier transmission along with the signal, or with balancing action to suppress certain unwanted bands of signal frequencies. In most carrier telephone systems economy of frequency space and amplifier load capacity demands the use of single-sideband, carrier-suppressed transmission.

In Figs. 2(a), 2(b), and 2(c) three types of copper oxide modulators are shown, each arranged to suppress the carrier in both the signal input and the signal output circuits. In Figs. 2(d) and 2(e) the carrier is balanced out in only one signal branch. In the usual arrangements a signal band selective filter must be used in each signal branch to restrict transmission to that of the wanted frequency band. Largely in this way interferences are guarded against, not only into other channels or systems to which the modulator output circuit is connected on the line or at the distant end, but also back into the complex array of facilities to which the input circuit may be connected.

In any of the circuits shown, modulation results from either the reduction or reversal of the current flow between the input and output signal circuits at periodic intervals as the carrier varies the copper oxide resistance back and forth between high and low values. In Fig. 2(a) where the connections of the input and output signal circuits are periodically short-circuited by the carrier-actuated copper oxide, transmission of the modulated signal into the input circuit or the unmodulated signal into the output circuit is prevented by filters, each of high impedance at the frequency of the other signal. In Fig. 2(b) the connections between the signal and modulated signal circuits are open-circuited periodically by the carrier. In this case each filter should have a low impedance at the other signal frequency. In Figs. 2(c), 2(d) and 2(e) the copper oxide rectifiers are made to become alternately low and high resistance in pairs as the polarity of the carrier is either in the same direction as the arrows or in the opposite direction. As a result, current flow from the input signal circuit into the output is periodically reversed by provision of a periodically reversing low impedance path. In effect each signal is balanced from the other's circuit.

Although an indefinite number of other circuit configurations can be used, no novel transmission feature would be found which was not

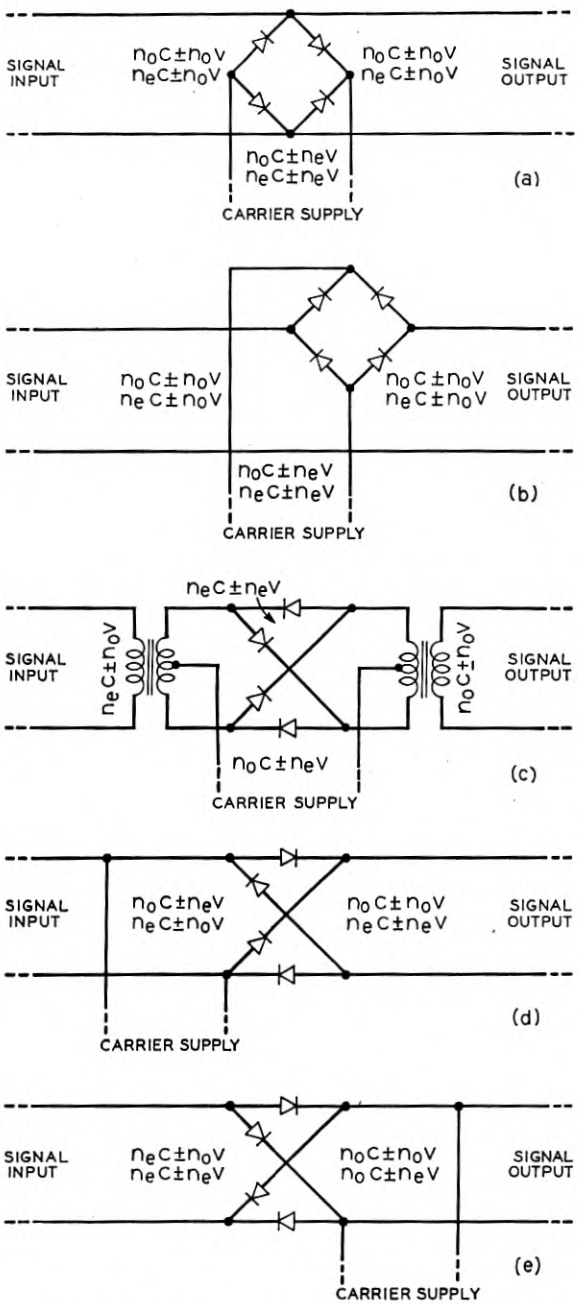


Fig. 2—Types of copper oxide modulator circuits.

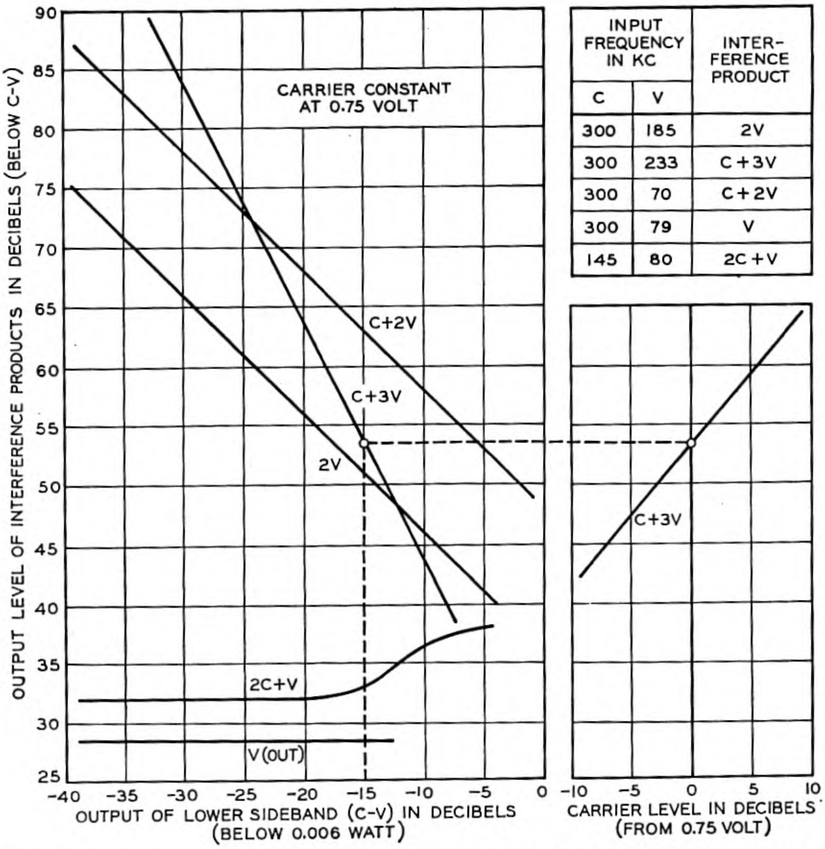
present in the five circuits already shown. Third order modulators in which the copper oxide is arranged to give equal interruptions to the signal in both positive and negative half cycles of the carrier are exceptions not considered here. In addition, circuits like Hartley's, in which phase discriminations have been obtained in the sideband outputs from two modulators by altering both the carrier and signal input phase of one, can be viewed as composed of two modulators of any of the types illustrated.

In these copper oxide modulators all modulation product frequencies can be grouped into four classes:

$$n_0c \pm n_0v,$$
$$n_0c \pm n_ev,$$
$$n_ec \pm n_0v,$$
$$n_ec \pm n_ev,$$

in which c and v are the carrier and input signal frequencies and n_0 is any odd number 1, 3, 5 etc., while n_e is any even number 0, 2, 4, 6, etc. If c and v contain more than one frequency each, n_0 and n_e are respectively the odd and even combinations of all multiples of the c and v frequencies. All frequencies of one of these four types appear together in a specific branch of the modulator circuit; and they will not appear in another branch unless from a dissymmetry among the copper oxide units or unless inherent in the circuit configuration. The branches in which the modulation products appear are shown in the circuits illustrated. It is apparent that only in the case of the double-balanced circuit of Fig. 2c, are all of these types of products completely separated in different parts of the circuit. In the other circuits shown the classes of products appear together in combinations of two types. In any balanced circuit that can be drawn the above relationships will be found to hold.

Modulation products will be of a type to which the circuit offers some degree of balance, of a type that can be made to vary in importance relative to the signal by adjustment of either the carrier or signal voltage, or of a type to which neither balance nor level adjustment is of any benefit. Satisfactory operation of such modulators requires large carrier voltages relative to those of the signal, so that products like $c \pm v$, $2c \pm v$, $3c \pm v$, etc. tend to be of large magnitude while products like $c \pm 2v$, $c \pm 3v$, etc. tend to be small. Furthermore, the former types can be made to predominate even more over the latter types either by increasing the carrier amplitude or by decreasing the signal levels. A 6 db reduction in signal results in 12 db reduction of $c \pm 2v$ and 18 db reduction in $c \pm 3v$. In any circuit application interferences of this type lend themselves to reduction in so far as carrier power is available for high signal level operation, or in so far as noise does not limit for low signal levels. Laboratory measurements of some of these modulation products made during the development of a group modulator for a twelve-channel open-wire carrier system are shown in Fig. 3 for a double-balanced modulator



INPUT FREQUENCY IN KC		INTERFERENCE PRODUCT
C	V	
300	185	2V
300	233	C+3V
300	70	C+2V
300	79	V
145	80	2C+V

Fig. 3—Intensity of modulation products in a representative double-balanced copper oxide modulator.

like that of Fig. 2(c). Single 3/16 inch diameter discs as shown in Fig. 1 were used in each bridge arm. About 20 to 30 db reduction in interference by balance alone is quite readily obtained in the normal run of manufactured copper oxide rectifier elements, for those products to which the circuit arrangement offers a balance. Any further improvement must be obtained either by closer control of manufacture or disc selection, by artificial balancing with some means such as condenser-resistance potentiometers, or by statistical averaging through use of numbers of discs in each bridge arm.

In single-channel modulators interferences caused by the signal into its own signal band will occur only in the presence of the signal. In such cases they need be only 20 to 30 db below the signal, except in special cases, as for example, modulators for broad-band program channels. In multi-channel systems interferences may be produced in the silent channels by the active channels. This kind of interference or crosstalk is ordinarily made to be 70 db or more below the wanted signals for commercial telephone service. In such cases overlapping bands of frequencies not improved by level adjustment are avoided by judicious choice of the carrier frequencies.

Circuit Impedance and Loss

In all modulators the carrier serves merely as a means for obtaining a simple periodic variation of the impedance presented to the signals. It is not only immaterial to the signals how this variation is obtained, but the signals also are totally unaware of whether electrical, mechanical or other means are used, just so long as the signals themselves are unable to affect the time variation of this impedance. In a copper oxide modulator, only by making the carrier amplitude large compared to the signal amplitudes across the rectifier elements, can the impedance of the rectifiers be made to vary at carrier rather than signal rates. Too large a signal amplitude not only results in the production of undesired frequencies, but also the impedance and loss characteristics of the modulator vary with the signal amplitude. With small signals the carrier energy is used up in maintaining the copper oxide at prescribed impedance values at each instant of time, and none of the modulation products involving the signal receive more than a negligible amount of energy from the carrier. As a result the output signal energy will always be less than that of the input signal, partly because of i^2r losses within the copper oxide, and partly from the diversion of the input signal energy into the energies of the many modulation product frequencies.

The signal impedance of a copper oxide modulator is a combination of a characteristic impedance of the rectifier elements and the impedance of the connected circuits at all the modulation product frequencies. The characteristic impedance of the rectifier can be viewed crudely as an average of the impedance encountered by a small signal over a cycle of the carrier, treating each instantaneous value of the carrier voltage as a d-c. bias. If the impedance for small superimposed a-c. voltages is measured on a single copper oxide disc at various d-c. bias voltages, this impedance generally changes with both bias and frequency. Measurements to 200 kilocycles made on a 3/16 inch diameter disc are shown in Fig. 4. At all negative bias

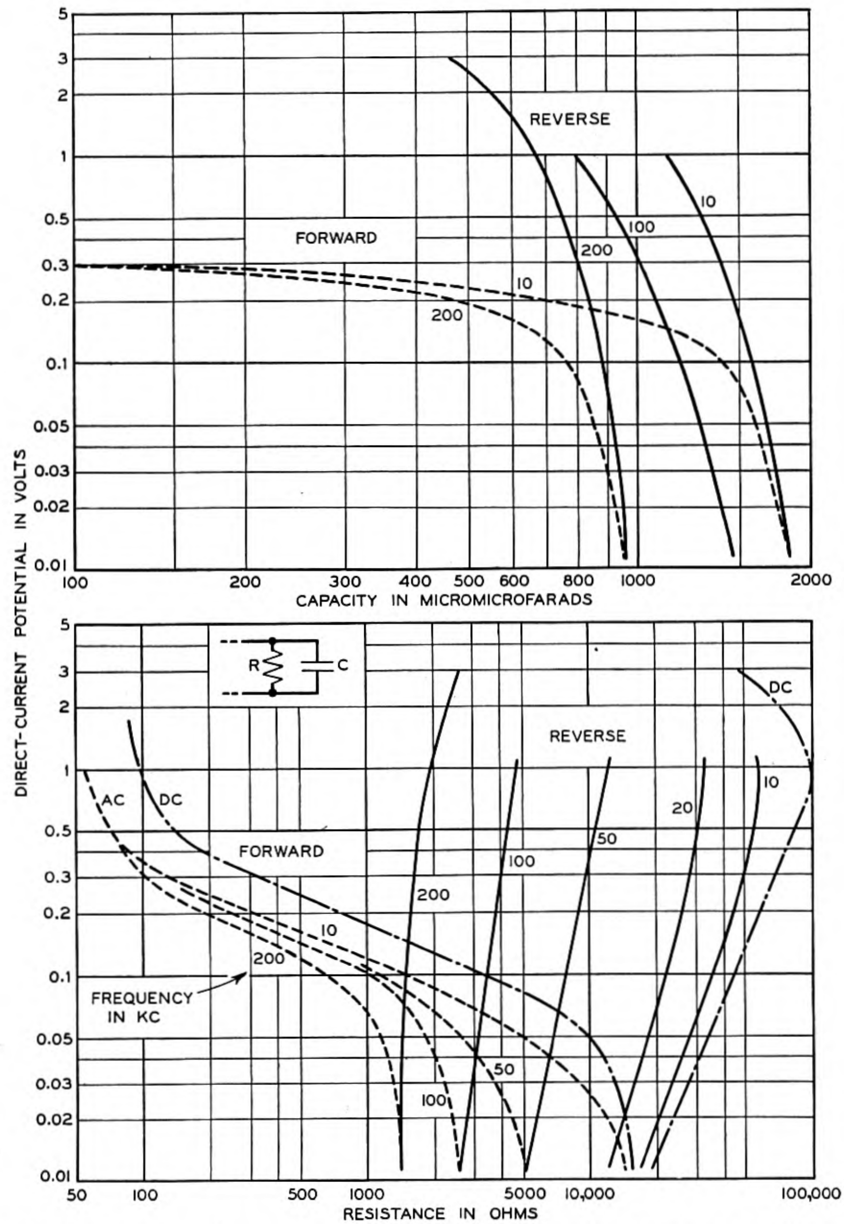


Fig. 4—Impedance of a copper oxide disc at various forward and reverse d-c. voltages for small superposed a-c. voltage.

voltages the impedance is a resistance in parallel with a condenser. This resistance decreases rapidly with increasing frequency while the shunt capacity decreases only a moderate amount. At moderate positive voltages (about 1/2 volt) the impedance becomes resistive and does not change appreciably with frequency. Experimentally it is found that the signal impedances in resistance terminated modulators can be made largely free of reactance at high frequencies by using either large carrier amplitudes, inductive tuning of the copper oxide capacities, or lower impedance connected signal circuits to accentuate the importance of the low-resistance part of the copper oxide characteristic. Very much lower circuit impedances must be used at the higher frequencies. Where 600 to 1000 ohms is a satisfactory impedance at speech frequencies, less than 50 ohms may be the best impedance to use, at three megacycles.

Impedance measurements on a double-balanced modulator designed to translate a twelve-channel group of frequencies for cable carrier systems from a band at 60 to 108 kilocycles to a 12- to 60-kilocycle band are shown in Fig. 5 for several resistance terminations. Absence of any impedance irregularities with frequency is apparent. Also, the tendency is shown for the modulator impedance to become less reactive with lower resistance terminations.

Inasmuch as copper oxide discs are available in sizes from 1/16 inch to more than an inch in diameter, a wide range of circuit impedances are possible varying from only a few ohms to thousands of ohms. Large area discs roughly are equivalent to small area discs in parallel. Thus by using a disc of n times the area of a small one or n of the small ones in parallel, the best circuit impedance becomes $1/n$th at the same carrier voltage. Either discs in series or ones of smaller diameter enable the impedance to be raised in a corresponding manner. The lower impedance and greater energy dissipations of larger discs, or paralleled smaller ones at the same carrier voltage, obviously allow greater input signal energies before the signal impedance and loss begin to vary with the signal, and overload distortion appears. Similarly series stacks of discs, or series-parallel combinations, offer wide choice in both the signal levels that can be satisfactorily modulated and in the impedance levels. Usually r.m.s. carrier voltages across individual discs in the conducting direction will best be made somewhere between 3/10 and 3/4 volts.

The impedances of the connected circuits at the modulation product frequencies react back on the signal impedances in a way similar to the way that the two terminating impedances of a four-terminal linear network react on each other. In the case of the copper oxide modulator a reaction from some modulation product back into the signal impedance is less and less as the product becomes of higher order, or as the circuit loss to it becomes greater. Where the impedances of the connected circuits have bothersome interactions with the copper oxide impedance either at the signal frequencies or the lower loss modulation products, resistance pad separation is usually the simplest solution if the increased loss can be tolerated.

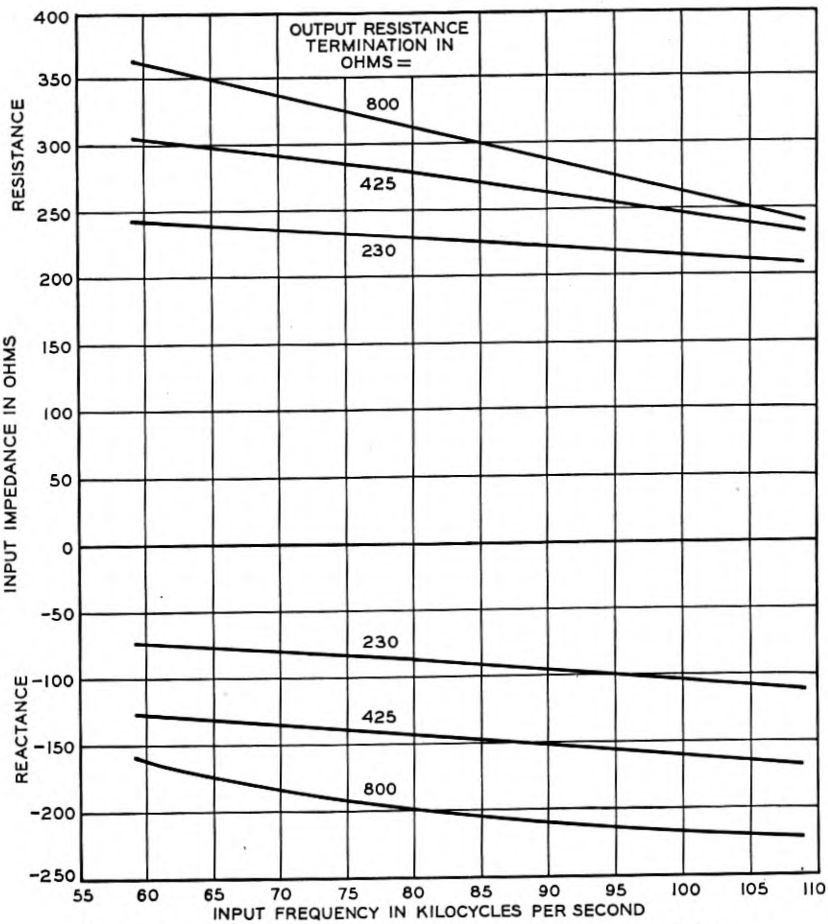


Fig. 5—Impedance of a representative double-balanced modulator.

Energy losses in copper oxide modulators between signal input and single sideband signal output have been found to be no greater than 8 or 9 db even at frequencies of 3 or 4 megacycles. At lower frequencies 5 or 6 db losses are normal, but losses as low as 2 db have been obtained under less practical operating conditions. Experimental loss measurements are shown in Fig. 6 for a double-balanced modulator using single 3/16 inch diameter discs in each bridge arm. This modulator was designed to simultaneously modulate sixty speech channels occupying a 240,000-cycle band width. The modulator loss, like the impedance, depends on the impedance terminations of the modulator at all the modulation product frequencies as well as on the internal losses of the modulator. Short circuit, open circuit, or reactive terminations at the unwanted frequencies, permit energy losses only through reflections at the signal circuit junctions to the modulator or within the modulator. With proper terminations and loss-free copper oxide, 100 per cent efficiency frequency translations are theoretically possible. In a practical case, a larger carrier amplitude results in a smaller percentage of the time in which the rectifier

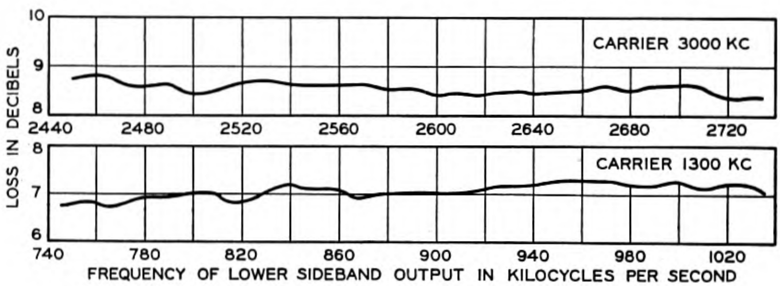


Fig. 6—Loss in a double-balanced group modulator for coaxial systems.

elements have impedances that are comparable to the connected circuits and that are neither blocking nor conducting. Signal energies are lost in this time interval, so a higher efficiency modulator results. The time spent on the intermediate resistance parts of the rectifier characteristic can be further reduced by introducing harmonics into the carrier wave, so that a square type of wave results. The resistance of the rectifier is abruptly switched back and forth between blocking and conducting values in this manner. When the connected circuit impedances at the unwanted frequencies are very high or low, best efficiencies result when transmission between the signal circuits is blocked most of the time. Thus in a circuit like that of Fig. 2(a) when the filters are high impedance at the unwanted products, highest efficiency results when the copper oxide is a low resistance short circuit for the major portion of the carrier cycle. In Fig. 2(b) an open circuit is desirable most of the time.

Linear Modulator Theory

The analytical studies that have been of most benefit in the development of copper oxide modulators have made use of a variable resistance characteristic controlled by the carrier. This assumption has made it possible to investigate modulator performance [2] for a wide variety of characteristics under a great many operating conditions. Copper oxide modulator performance in particular cases as well as the effects of the circuit elements on this performance can readily be inferred from the data at hand about these idealized modulators.

In limited space it is not possible to discuss the varieties of resistance modulators that have been analyzed. However, certain viewpoints will be discussed that have been very useful not only for obtaining solutions for some of the hypothetical cases, but also in supplementing laboratory experiments on actual modulators.

All of these analytical studies have assumed a signal sufficiently small compared to the carrier, that it can be varied in magnitude without noticeable changes in the signal impedance or in the linearity between input and output signal amplitudes. This is in agreement with design procedure, as the circuit impedances and losses are determined on such a linear basis.

Superposition Principle

All of the modulator circuits with which we have been dealing, though composed of non-linear elements, have been resolved into the equivalent of linear systems by virtue of using a large carrier and small signal amplitude. We may simultaneously apply any number of signal frequencies, but all have negligible effect on the periodic changing of the non-linear element resistance by the carrier. These frequencies may be modulation product voltages, some applied at the output terminals and some at the input terminals, but in all cases, even though frequencies may coincide, it can be shown that the principle of superposition will hold without interaction between the applied forces and the responses. This permits a great simplification in the mathematical approach to modulator analysis, because the modulation product or signal voltages can be applied one at a time and the current responses summed. The voltage-current ratios at each frequency can then be replaced by equivalent impedances.

Any non-linear resistance like copper oxide will have a current-voltage characteristic that can be expressed as accurately as de-

[2] A physical picture of modulator performance in terms of linear networks is developed in a paper published in the January 1939 *Bell System Technical Journal*, "Equivalent Modulator Circuits" by E. Peterson and L. W. Hussey.

sired by

$$i = a_1 e + a_2 e^2 + \cdots + a_n e^n. \tag{1}$$

If a carrier and signal voltage are applied to this non-linear element, each term beyond the first will independently produce currents of new frequencies composed of the intermodulation products of these two voltages. If in turn the external impedances at these new frequencies are not zero, new voltages will appear across the non-linear element to produce still more new frequencies. In this case the simplest consideration is to minimize the number of voltages by presenting zero impedance to the modulation products. If the carrier voltage is $C \cos ct$ and the input signal voltage $S \cos st$, the current flow in the nth term is

$$i = a_n(C \cos ct + S \cos st)^n. \tag{2}$$

In the binomial expansion of this expression it is obvious that linear response to the signal and freedom from distortion result when the ratio of carrier to signal is made sufficiently large so that only the first two terms are important.

$$i \approx a_n[(C \cos ct)^n + n(C \cos ct)^{n-1} \cdot S \cos st]. \tag{3}$$

The first term in equation (3) is the current flow at d-c. and harmonics of the carrier; it has no effect on the input signal and output signal except in so far as impedance termination presented across the non-linear element at the carrier harmonic frequencies may alter the carrier voltage harmonic content. The signal input current and the signal output current, as well as the unwanted modulation products of the signal, result from the second term. The even values of n produce the even order sidebands, second order being the output signal, while the odd values of n produce the input signal current and the odd order sidebands. These currents can be evaluated from

$$\cos^n b = \frac{K^n_{\frac{n}{2}}}{2^n} + \sum_{m=1}^{m=n} \frac{K^n_{\frac{n-m}{2}}}{2^{n-1}} \cos mb,$$

in which $K^n_{\frac{n-m}{2}}$ is equal to the combination of n things taken $\frac{n-m}{2}$ at a time for $\frac{n-m}{2}$ integral and is equal to zero for $\frac{n-m}{2}$ non-integral.

The signal input current is

$$i_s = \frac{a_n n K^{n-1}_{\frac{n-1}{2}}}{2^{n-1}} C^{n-1} S \cos st, \tag{4}$$

while the second order output signal sideband is

$$i_{c\pm s} = \frac{a_n n K^{n-1}_{\frac{n-2}{2}}}{2^{n-1}} C^{n-1} S \cos (c \pm s)t. \tag{5}$$

Similarly, if the output signal voltage at second order sideband frequency $(c \pm s)$ had been applied along with the carrier in place of the input signal, and of an equal amplitude, then the following currents of the output and input signal frequencies would result:

$$i_{c\pm s} = \frac{a_n n K^{n-1}_{\frac{n-1}{2}}}{2^{n-1}} C^{n-1} S \cos (c \pm s)t, \tag{6}$$

$$i_s = \frac{a_n n K^{n-1}_{\frac{n-2}{2}}}{2^{n-1}} C^{n-1} S \cos st. \tag{7}$$

If both signal input and output frequency voltages are applied simultaneously, equation (3) then becomes

$$i \approx a_n[(C \cos ct)^n + n(C \cos ct)^{n-1} \cdot S \cos st + n(C \cos ct)^{n-1} \cdot S \cos (c \pm s)t]. \tag{8}$$

The current responses obviously are the sum of the separate responses from independent application of the two frequencies. Even if a complex array of terminating impedances are supplied so that voltages appear across the non-linear element at all the modulation product frequencies, each new voltage will individually produce its own current response, quite independently of the responses that are being produced by the other voltages. It can readily be seen then that superposition does not depend on any assumptions about what the terminating impedances may be.

Reciprocal Theorem

Equations (5) and (7) show that the sideband response to an input signal voltage is exactly equal in magnitude to the input signal response to the same amplitude sideband voltage. It can readily be seen that any two modulation products also bear such a reciprocal relationship between their voltages and currents, as a result of using the same amplitude and frequency of carrier harmonic multiplier of their respective voltages to modulate between the two frequency positions. Although reciprocity has been proved valid here only for short-circuit terminations at the modulation product frequencies, it can also be proved under numerous other conditions of circuit operation. It seems that, regardless of modulator complexity of impedance terminations or frequency loss effects, *the reciprocal theorem is a necessary attribute of such a linear and bilateral system in which there are no internal energy sources.* Two-way systems in which an amplifier for example, is included as an internal energy source in one or both directions will, of course, violate the reciprocal theorem if the gains in the two directions are different. This arrangement is, however, both bilateral and linear.

Complete Performance Criteria

The laws for transmission between a signal input frequency and a signal output frequency can be completely specified from open and short circuit impedance measurements at the signal input and output frequencies, regardless of the complexity of the modulator. (From such measurements optimum impedance terminations can even be determined for linear-bilateral systems with internal energy sources.)

The four-terminal network of Fig. 7 is assumed to represent a modulator with a large carrier amplitude and having small signal voltage P of frequency p applied at the input terminals 1–2 at the left. Current of the output signal frequency q flows out of the terminals 3–4 into the impedance R_q. The generator P is assumed to have zero internal impedance at its own frequency. Impedance terminations at the 1–2 and 3–4 terminals at all other modulation product frequencies are perfectly general; whatever they are in a particular case, it is assumed that they are undisturbed as the terminations of the input and output at the signal frequencies are varied between open circuit and short circuit. The following symbols are used for the impedances looking into the modulator at the input terminals at input signal frequency p and at the output terminals at signal output frequency q.

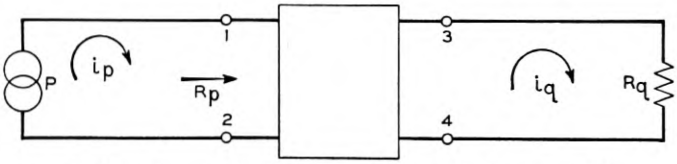


Fig. 7—Four terminal network equivalent of a linear modulator.

Z_{p0} = impedance at p for open circuit at q;
Z_{q0} = " " q " " " " p;
Z_{ps} = " " p " short " " q;
Z_{qs} = " " q " " " " p;
R_q = impedance termination at frequency q;
R_p = impedance of modulator at frequency p with R_q at 3–4 terminals for frequency q;
K_a = transfer admittance between voltage at frequency p applied at the 1–2 terminals and short-circuit current at frequency q flowing from 3–4 terminals;
K_b = transfer admittance between voltage applied at 3–4 terminals at frequency q and short-circuit current at frequency p flowing from 1–2 terminals.

For R_q first short-circuited

$$i_{q_1} = PK_a, \tag{9}$$

$$i_{p_1} = \frac{P}{Z_{ps}} \cdot \tag{10}$$

If the short circuit on R_q is removed, the following two additional currents will flow due to superposition of a new voltage $-i_qR_q$

$$i_{q_2} = \frac{-i_qR_q}{Z_{qs}} \tag{11}$$

and

$$i_{p_2} = -i_qR_qK_b, \tag{12}$$

$$i_q = i_{q_1} + i_{q_2} = \frac{PK_a}{1 + \dfrac{R_q}{Z_{qs}}}, \tag{13}$$

$$i_p = i_{p_1} + i_{p_2} = \frac{P}{Z_{ps}} - \frac{PK_aK_bR_q}{1 + \dfrac{R_q}{Z_{qs}}} \cdot \tag{14}$$

The efficiency of the frequency translation, measured by the ratio of the power delivered to R_q to the power into terminals 1–2, is

$$\eta = \frac{i_q^2R_q}{i_pP} = \frac{\left(\dfrac{K_a}{1 + \dfrac{R_q}{Z_{qs}}}\right)^2 R_q}{\dfrac{1}{Z_{ps}} - \dfrac{K_aK_bR_q}{1 + \dfrac{R_q}{Z_{qs}}}}, \tag{15}$$

which is maximum for

$$R_q = \frac{Z_{qs}}{\sqrt{1 - K_a K_b Z_{ps} Z_{qs}}}. \tag{16}$$

The maximum efficiency is then

$$\eta_{\text{max.}} = \frac{K_a^2 Z_{ps} Z_{qs}}{(1 + \sqrt{1 - K_a K_b Z_{ps} Z_{qs}})^2} \tag{17}$$

When equation (16) is substituted in (14) it is found that

$$R_p = \frac{P}{i_p} = \frac{Z_{ps}}{\sqrt{1 - K_a K_b Z_{ps} Z_{qs}}}. \tag{18}$$

In order to evaluate $K_a K_b$, open circuit impedance measurements must also be made. By superposition methods like those used in obtaining equations (11) and (12) it can readily be shown that

$$\frac{1}{Z_{p0}} = \frac{1}{Z_{ps}} - K_a K_b Z_{qs}, \tag{19}$$

$$\frac{1}{Z_{q0}} = \frac{1}{Z_{qs}} - K_a K_b Z_{ps}. \tag{20}$$

From these two equations, it follows that

$$\frac{Z_{ps}}{Z_{p0}} = \frac{Z_{qs}}{Z_{q0}}, \tag{21}$$

$$K_a K_b = \frac{\dfrac{1}{Z_{ps}} - \dfrac{1}{Z_{p0}}}{Z_{qs}}, \tag{22}$$

and $K_a K_b$ can be determined from any three of the open-short measurements by using (21) and (22). It follows that

$$K_a K_b Z_{ps} Z_{qs} = 1 - \frac{Z_{ps}}{Z_{p0}}. \tag{23}$$

Upon substitution in (16) and (18) it is found that

$$R_q = \sqrt{Z_{q0} Z_{qs}} \tag{24}$$

and

$$R_p = \sqrt{Z_{p0} Z_{ps}} \tag{25}$$

when 3–4 is terminated in R_q for maximum efficiency.

Open and short-circuit measurements enable us to compute the optimum efficiency from equation (17) only if the transfer admittance

K_a is known. If the reciprocal theorem holds, $K_a = K_b$ and K_a can be determined from (23). The optimum efficiency is then

$$\eta_{\text{max.}} = \frac{1 - \sqrt{\dfrac{Z_{ps}}{Z_{p0}}}}{1 + \sqrt{\dfrac{Z_{ps}}{Z_{p0}}}}. \tag{26}$$

If the input signal generator has an internal impedance, most efficient energy delivery to the modulator will, of course, result if this impedance is made equal to R_p.

Equivalent T, π and bridge networks can obviously be drawn from the open and short-circuit measurements as in four-terminal linear networks.

It appears that even in a plate or grid circuit modulator the formulae of equations (24), (25) and (26) can be applied to the plate or grid circuit, respectively, where the signals are small compared to the carrier, inasmuch as the modulating parts of these circuits are linear and bilateral with no internal energy sources.

Double-Balanced or Reversing-Switch Modulator [3]

A number of interesting conclusions can be reached about copper oxide modulators by assuming that the copper oxide acts like a switch having a low-resistance value when the positive half-cycle of the carrier voltage is across the disc and a high-resistance value during the negative half-cycle. The circuits of Fig. 2(c), 2(d) or 2(e) can then be represented by the equivalent circuit of Fig. 8.

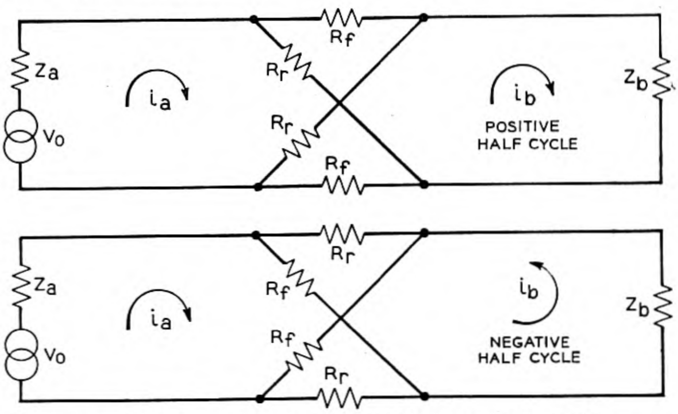


Fig. 8—Equivalent circuit of a double balanced modulator.

[3] Referred to in the German literature as the "ring modulator."

The input signal generator V_0 is applied out of an input circuit impedance Z_a, and the output signal is delivered to the output circuit impedance Z_b. Both Z_a and Z_b in general will be functions of frequency.

If $Z_a = Z_b = R$ at all frequencies it can easily be shown that the circuit will operate most efficiently for

$$R = \sqrt{R_r R_f}. \tag{27}$$

The signal input current will then be the only frequency in i_a:

$$i_a = \frac{V_0}{2R}. \tag{28}$$

From the properties of lattice networks it follows that

$$i_b = \frac{V_0}{2R} \cdot \frac{\sqrt{\dfrac{R_r}{R_f}} - 1}{\sqrt{\dfrac{R_r}{R_f}} + 1} \cdot f(t), \tag{29}$$

where

$$f(t) = \begin{cases} +1 \text{ for positive half-cycle of carrier} \\ -1 \text{ " negative " " " " } \end{cases}.$$

Letting

$$f(t) = \frac{4}{\pi}\left(\cos ct - \frac{1}{3}\cos 3ct + \frac{1}{5}\cos 5ct \cdots\right)$$

and

$$k = \frac{\sqrt{\dfrac{R_r}{R_f}} - 1}{\sqrt{\dfrac{R_r}{R_f}} + 1}, \tag{30}$$

$$i_b = \frac{V_0 k}{2R} \cdot \frac{4}{\pi}\left[\cos ct - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5ct \cdots\right]. \tag{31}$$

The magnitude of the single-sideband output current is

$$i_{1+} = i_{1-} = \frac{2k}{\pi} \cdot \frac{V_0}{2R} \tag{32}$$

and the efficiency

$$\eta = \frac{4}{\pi^2} k^2. \tag{33}$$

In addition currents flow in the output circuit at both sidebands of all odd harmonics of the carrier frequency. These can be represented by i_{3+}, i_{3-}, i_{5+}, etc. No currents flow at the sideband frequencies of the even harmonics of the carrier, i_{2+}, i_{2-}, $\cdots$.

If V_0 should be replaced by an equal amplitude generator at any of the sideband frequencies, V_{1+}, V_{2-}, etc., then the input current would in any case be

$$I_{1+} = \frac{V_{1+}}{2R}, \qquad I_{2-} = \frac{V_{2-}}{2R}, \qquad \text{etc.}$$

The correspondence between the magnitudes of these entering currents and the magnitudes of the output currents at the modulation product frequencies are shown in Table I. Reciprocal relations between the

TABLE I

CORRESPONDENCE BETWEEN CURRENTS ENTERING AND LEAVING A REVERSING-SWITCH MODULATOR

Component on One Side of Modulator	Corresponding Components on Other Side of the Modulator				
	$\frac{2k}{\pi}$*	$-\frac{2k}{3\pi}$	$\frac{2k}{5\pi}$	$-\frac{2k}{7\pi}$	
I_0	I_{1-} I_{1+}	I_{3-} I_{3+}	I_{5-} I_{5+}	I_{7-} I_{7+}	$\cdots$
I_{1-}	I_0 I_{2-}	I_{2+} I_{4-}	I_{4+} I_{6-}	I_{6+} I_{8-}	$\cdots$
I_{1+}	I_0 I_{2+}	I_{2-} I_{4+}	I_{4-} I_{6+}	I_{6-} I_{8+}	$\cdots$
I_{2-}	I_{1-} I_{3-}	I_{1+} I_{5-}	I_{3+} I_{7-}	I_{5+} I_{9-}	$\cdots$
I_{2+}	I_{1+} I_{3+}	I_{1-} I_{5+}	I_{3-} I_{7+}	I_{5-} I_{9+}	$\cdots$
I_{3-}	I_{2-} I_{4-}	I_0 I_{6-}	I_{2+} I_{8-}	I_{4+} I_{10-}	$\cdots$
I_{3+}	I_{2+} I_{4+}	I_0 I_{6+}	I_{2-} I_{8+}	I_{4-} I_{10+}	$\cdots$
I_{4-}	I_{3-} I_{5-}	I_{1-} I_{7-}	I_{1+} I_{9-}	I_{3+} I_{11-}	$\cdots$
I_{4+}	I_{3+} I_{5+}	I_{1+} I_{7+}	I_{1-} I_{9+}	I_{3-} I_{11+}	$\cdots$
I_{5-}	I_{4-} I_{6-}	I_{2-} I_{8-}	I_0 I_{10-}	I_{2+} I_{12-}	$\cdots$
I_{5+}	I_{4+} I_{6+}	I_{2+} I_{8+}	I_0 I_{10+}	I_{2-} I_{12+}	$\cdots$
I_{6-}	I_{5-} I_{7-}	I_{3-} I_{9-}	I_{1-} I_{11-}	I_{1+} I_{13-}	$\cdots$
I_{6+}	I_{5+} I_{7+}	I_{3+} I_{9+}	I_{1+} I_{11+}	I_{1-} I_{13+}	$\cdots$
I_{7-}	I_{6-} I_{8-}	I_{4-} I_{10-}	I_{2-} I_{12-}	I_0 I_{14-}	$\cdots$
I_{7+}	I_{6+} I_{8+}	I_{4+} I_{10+}	I_{2+} I_{12+}	I_0 I_{14+}	$\cdots$
I_{8-}	I_{7-} I_{9-}	I_{5-} I_{11-}	I_{3-} I_{13-}	I_{1-} I_{15-}	$\cdots$

NOTE: A current of the frequency indicated in the first column will be modulated to produce the components written on the same line, the magnitudes of which are the magnitude of the generating current multipled by the factors at the top of the columns.

$$*k = \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1}.$$

driving voltage at one frequency and the output current at another frequency are obvious.

TABLE II

PERFORMANCE OF DOUBLE-BALANCED MODULATOR (IDEAL REVERSING SWITCH) FOR VARIOUS INPUT AND OUTPUT TERMINATIONS

Modulator Terminations				Modulator Impedance		Modulator Loss or Efficiency	
Input Circuit		Output Circuit		Input Signal	Output Signal	Voltage Ratio	db
Signal	Others	Signal	Others				
R	R	R	R	R	R	$\frac{2}{\pi}$	3.9
R	any value	R	R	R		$\frac{2}{\pi}$	3.9
R	R	R	0	$\frac{2R}{\pi^2 - 2}$	R	$\frac{2}{\pi}$	3.9
R	R	R	∞	$\frac{(\pi^2 - 2)R}{2}$	R	$\frac{2}{\pi}$	3.9
R	$\frac{(\pi^2 - 2)R}{2}$	$\frac{(\pi^2 - 2)R}{2}$	0	R	$\frac{\pi^2(\pi^2 - 2)R}{6\pi^2 - 16}$	$\frac{\pi}{\sqrt{2(\pi^2 - 2)}}$	2
R	$\frac{2R}{\pi^2 - 2}$	$\frac{2R}{\pi^2 - 2}$	∞	R	$\frac{(6\pi^2 - 16)R}{\pi^2(\pi^2 - 2)}$	$\frac{\pi}{\sqrt{2(\pi^2 - 2)}}$	2
R	0	R	0	0	0		∞
R	∞	R	∞	∞	∞		∞
R	0	R	∞	$\frac{\pi^2 R}{4}$	$\frac{4}{\pi^2} R$	$\frac{4\pi}{4 + \pi^2}$	.85
R	∞	R	0	$\frac{4}{\pi^2} R$	$\frac{\pi^2}{4} R$	$\frac{4\pi}{4 + \pi^2}$	.85
R	0	$\frac{4}{\pi^2} R$	∞	R	$\frac{4}{\pi^2} R$	1	0
R	∞	$\frac{\pi^2}{4} R$	0	R	$\frac{\pi^2}{4} R$	1	0
R_s	$R_{s'}$	R_r	$R_{r'}$	Z_i*	Z_0*	η*	

$$*Z_i = \frac{\pi^2 R_r'(R_r + R_s') + 4(R_r - R_r')R_s'}{\pi^2(R_r + R_s') - 4(R_r - R_r')},$$

$$Z_0 = \frac{\pi^2 R_s'(R_s + R_r') + 4(R_s - R_s')R_r'}{\pi^2(R_s + R_r') - 4(R_s - R_s')}.$$

$$\eta = \frac{4\pi(R_r' + R_s')\sqrt{R_s R_r}}{\pi^2(R_s + R_r')(R_r + R_s') - 4(R_r - R_r')(R_s - R_r')}.$$

Generalized Reflection Theory

Superposition permits us to apply simultaneously driving forces of the frequencies tabulated above in any relative phases and amplitudes that we care to choose on either side of the modulator. If simultaneously I_0 is applied on one side of the modulator and $(I_{1+})\frac{2k}{\pi}\cdot\frac{Z_{1+}-R}{Z_{1+}+R}$ is applied to the other set of modulator terminals, then the total current at the output terminals at the sideband frequency (1^+) will be

$$(I_{1+})\frac{2k}{\pi}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]. \tag{34}$$

This is equivalent to saying that a resistance R at the sideband frequency (1+) has been connected to the output terminals of the modulator and in this resistance is an internal zero impedance generator of voltage

$$2R(I_{1+})\frac{2k}{\pi}\cdot\frac{Z_{1+}-R}{Z_{1+}+R}. \tag{35}$$

This resistance R at sideband frequency (1 +) must be infinite at all other frequencies, if in parallel we assume another resistance of R at all frequencies except (1 +) at which it is infinite.

The equivalent impedance at frequency (1 +) at the modulator terminals connected to the (1 +) resistance with its internal generator, is the ratio of (1 +) voltage to (1 +) current.

$$Z=\frac{\frac{2k}{\pi}I_{1+}\cdot 2R-\frac{2k}{\pi}\cdot I_{1+}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]R}{\frac{2k}{\pi}I_{1+}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]}, \tag{36}$$

which reduces to

$$Z=Z_{1+}. \tag{37}$$

Z_{1+} may be real or complex as it involves only the amplitude and phase of the superimposed voltage of upper sideband frequency. It can readily be seen then that the solution for current flow at this frequency of equation (34) is identical with the case of linear networks in which the current is expressed as that flowing in a matched circuit modified by a reflection factor. Reflection from any modulation product frequency can be similarly treated.

A number of cases have been worked out of efficiencies and impedances in such modulators for transmission between an input signal and a single-sideband output signal. The modulating element has been assumed perfect ($k = 1$) and the terminations pure resistances. The results are shown in Table II.

Acknowledgments

The writer wishes to acknowledge his appreciation of the assistance of numerous associates in the Bell Telephone Laboratories in arriving at the views on copper oxide modulator performance recorded in this paper. In particular, acknowledgment is due to Mr. R. W. Chesnut, Dr. E. Peterson and Dr. G. R. Stibitz.

Some Applications of the Type "J" Carrier System *

By L. C. STARBIRD and J. D. MATHIS

Previous papers before the American Institute of Electrical Engineers describe the development of a twelve-channel type J Carrier System. This paper discusses some of the practical problems encountered in extending the circuit capacity of existing open-wire lines by the use of this carrier system.

The first systems of this type were placed in commercial operation late in 1938. One of these systems is discussed in detail from the standpoint of obtaining satisfactory operation with the most economical arrangement of new and existing facilities.

A TWELVE-CHANNEL carrier telephone system for open-wire lines was described before the American Institute of Electrical Engineers early this year,[1] and a discussion of the requirements of line facilities for its operation is being presented.[2] Since the first three systems to be placed in commercial operation are located in Texas, it seems appropriate to present to the Southwest District Convention the major problems arising from the practical application of this type system on existing open-wire plant.

In 1935 it became apparent that existing open-wire facilities on some of the major toll lines in Texas would soon be exhausted. In the case of the Dallas-Houston, Dallas-San Antonio, and Dallas-Longview lines, current growth and requirements for the future indicated that while a toll cable would probably have to be provided ultimately, the development of the open-wire twelve-channel J carrier system makes available an arrangement for obtaining a large number of additional circuits over the existing lines to provide for the immediate requirements and also permit postponement of more costly relief measures for a number of years.

The type J system operates in a frequency range above that of the three-channel type C carrier system and can be superposed on the same conductors with the type C, thereby providing a total of sixteen circuits from one pair of conductors. However, conductors suitable

* Presented April 18, 1939 before the A.I.E.E. in Houston, Texas.

[1] "A Twelve-Channel Carrier Telephone System for Open Wire Lines," by B. W. Kendall and H. A. Affel, Winter Convention, A.I.E.E., 1939. *Bell System Technical Journal*, January 1939.

[2] "Line Problems in the Development of the Twelve-Channel Open-Wire Carrier System," by L. M. Ilgenfritz, R. N. Hunter, and A. L. Whitman, District Convention, A.I.E.E., Houston, 1939. This issue of the *Bell System Technical Journal*.

for type C carrier operation are not necessarily satisfactory for the operation of the new system.

The three lines under consideration were practically of the same construction, being twelve-inch phantomed lines originally built for voice frequency circuits only and later modified for the application of type C carrier systems. Over lines of this type, it is practicable to operate a single type J system without any material change in the line wire because no crosstalk considerations are involved, although it is necessary to select by transmission measurement pairs which are free from absorption effects. Where more than one system is required a transposition arrangement has been designed for use with line conductors of a non-phantomed pair spaced six inches apart and thirty inches between conductors of horizontally adjacent pairs. This design can be used either for new wire or for existing wire retransposed, and can be applied without regard to the existing phantomed transposition design, thereby permitting respacing and retransposing any portion of the existing wire, a phantom group at a time if desired.

Advance Engineering

With these operating limitations a review of the circuit requirements established a plan to place a J carrier system on one of the phantom groups of the Dallas-Longview line during 1938. This system would not only provide sufficient circuits to meet the additional requirements but would furnish sufficient spare circuits to release one phantom group of twelve-inch wire for respacing and retransposing. This plan was not applicable to the Dallas-Houston and Dallas-San Antonio lines since circuit relief was required for the 1937 business, and the J carrier system would not be available until 1938. These lines each consisted of five crossarms of 104 mil wire over the greater portion of their length. An inspection showed that, although the poles were of sufficient strength to support additional crossarms, it would be difficult to maintain the necessary wire clearance with an additional crossarm below the existing wire and also that new wire so placed would be susceptible to interference from possible breaks in the wire above.

The solution of this problem was the addition of a crossarm two feet above the others on a simple extension fixture. This fixture shown in Fig. 1 consists of a four-inch steel "H" beam fastened to the pole by the through bolts which also support the two upper crossarms. By placing four pairs of six-inch spaced conductors on the new crossarm and by using four type C carrier systems, sixteen additional circuits were obtained to furnish the circuit relief for 1937 and, in addition to

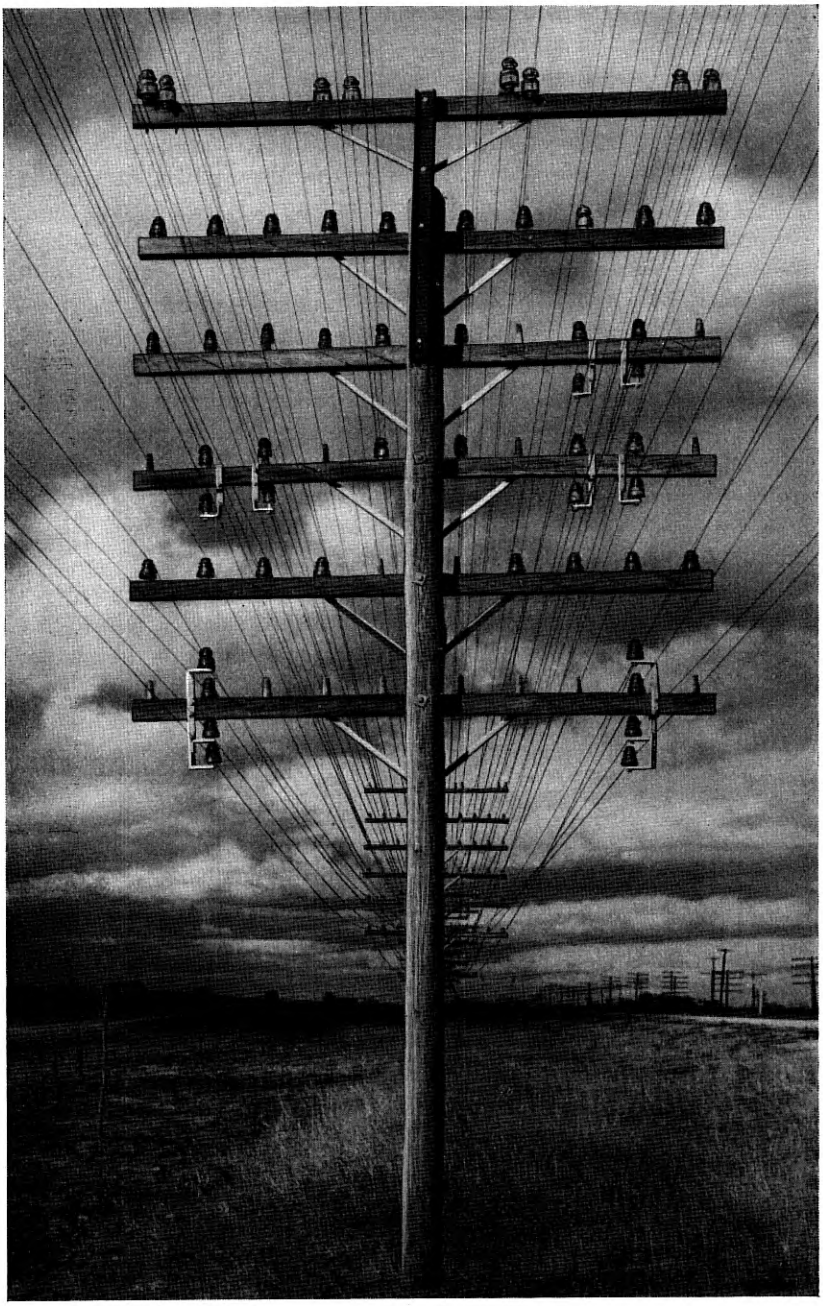


Fig. 1—Typical pole with extension fixture.

the immediate relief, four suitable J carrier paths were provided of which one on each line was needed in 1938. Figure 2 is a typical pole head and shows how the ultimate circuit capacity of this open-wire plant has been expanded from 69 to 133 circuits by the addition of one crossarm and eight conductors. The use of 128-mil wire instead of 104-mil wire provides greater strength and, considering the particular location, reduces the probability of interrupting sixteen circuits by a single wire break or other physical interference.

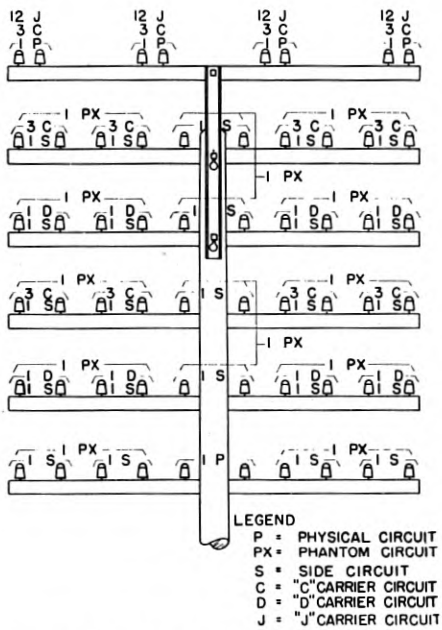


Fig. 2—Pole head diagram showing circuit capacity of the Dallas-Houston and Dallas-San Antonio lines.

The program of placing three type J carrier systems in service in Texas during 1938 was established. Figure 3 is a map of a portion of the state showing the routes of the lines and the principal cities along the routes. Since the length and attenuation of each of these lines are such that the carrier systems can not operate without intermediate amplification, it was necessary that the number and locations of repeater stations be determined.

Typical System

The layout of a particular system is largely controlled by available repeater gain, existing entrance cables, line attenuation under normal

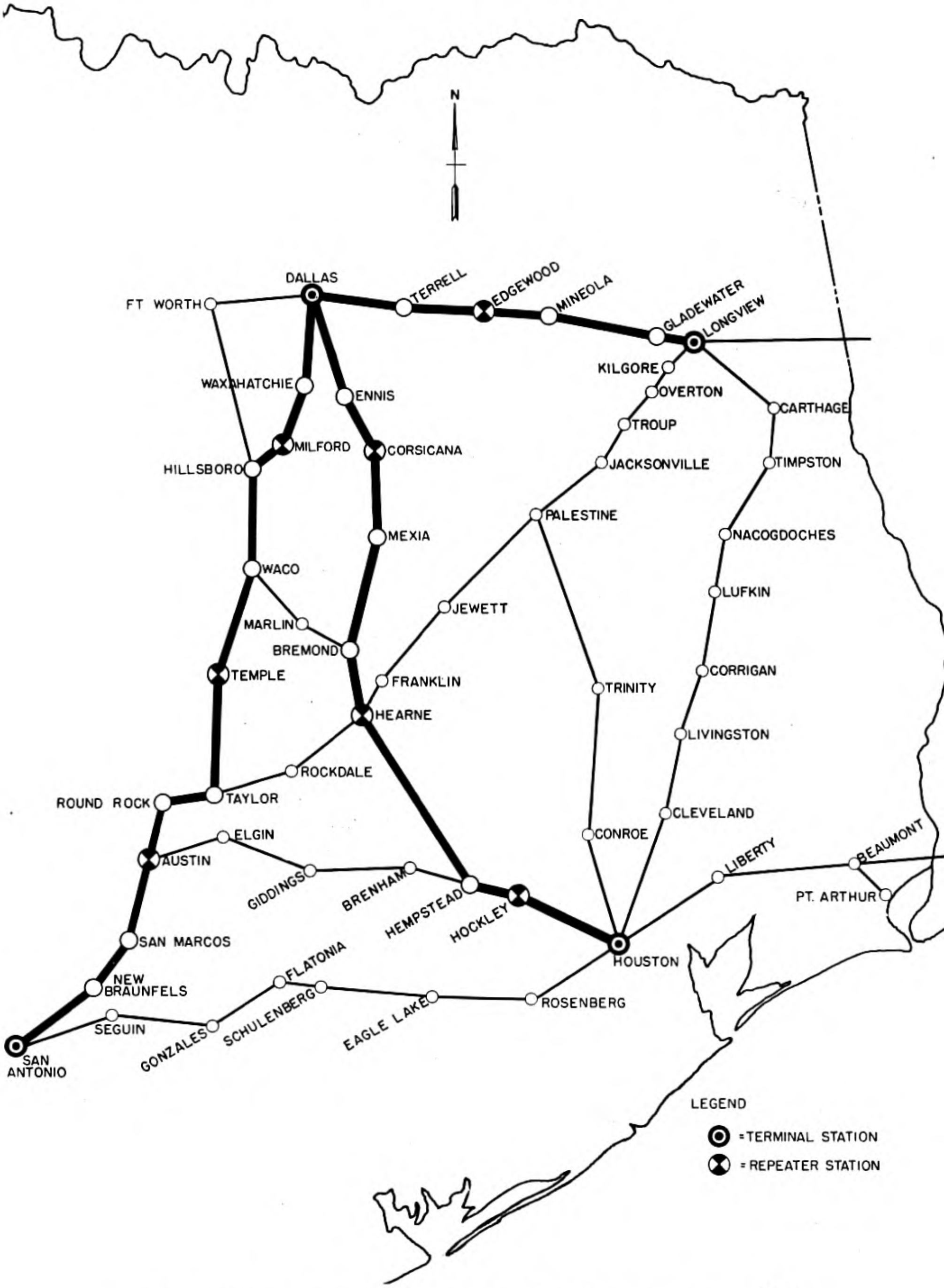


Fig. 3—Routes of toll lines on which the J carrier systems are applied.

and adverse weather conditions, location and availability of existing telephone buildings, and availability of commercial power for new buildings. Line attenuation is increased greatly by deposits of ice on the wire during sleet conditions. Although data are available regarding the frequency of large deposits of ice, there is very little information as to the amounts or frequency of occurrence of small deposits. Under normal wet weather conditions the maximum attenuation of six-inch spaced 128-mil facilities at 140 kilocycles is 0.35 db per mile.

On the Dallas-San Antonio line the facilities available consisted of 286 miles of six-inch spaced 128-mil copper wire and 42,000 feet of 16-gauge non-loaded paper insulated cable. Using repeaters having a maximum amplification of 45 db in each direction of transmission, the provision of two intermediate repeaters would provide sufficient gain to take care of the wet weather conditions with no extra margin; three repeaters would provide 45 db margin and four would provide 90 db margin for the overall system. Considering the location of this line and the small probability of obtaining large deposits of ice in accordance with past experience, it was decided to select tentatively three intermediate repeater stations which would provide sufficient gain to take care of attenuation up to about 0.5 db per mile as compared to the wet weather value of 0.35 db per mile.

For type C operation over this line, only one type C carrier repeater point is required, and it is at Austin. Considering the availability of power equipment and operating personnel and the possibility of future J carrier terminals being located at Austin, it is desirable that this be one of the repeater points on the J system. A division of the attenuation of the facilities north of Austin indicated that the other stations should be in the vicinity of Temple and Milford.

At these repeater stations amplification is needed only on the type J system and the other circuits on the line pass through these stations without amplification. Under these conditions energy may be transferred from the output of one type J repeater to the input of the same repeater or to the input of a repeater on another J system via crosstalk paths involving the wires which are not used for type J systems. The effect of this transfer of energy is accentuated by the fact that there is a large difference in transmission level between the output of one type J repeater and the input of the same or another repeater. In order to minimize these effects it is necessary that all wires on the line be given special treatment, including a gap in the toll line, longitudinal choke coils in all wires at terminal poles and crosstalk suppression filters in the non-J pairs in the repeater station itself. In selecting locations for repeater stations, consideration must also be given to the possible coupling between type J systems by interaction paths involving other conductors adjacent to the toll line.

Before definite selection of repeater station locations may be made, it is necessary that each repeater section be checked in detail and in this check the entrance cable arrangement may be controlling. The newly developed spacer insulated spiral-four cable, either loaded or non-loaded, or non-loaded pairs of the conventional paper insulated cable may be used between the open wire and equipment. Generally the existing voice and C carrier circuits use loaded entrance cable pairs and in most cases a change to non-loaded facilities would require extensive rearrangements in these circuits. In order to use non-loaded pairs for the J carrier and leave the C carrier and voice on loaded facilities, filters are placed at the terminal pole to separate the J carrier frequencies from the C and voice frequencies at that point. A limitation on the use of existing cable is that suitable pairs must be selected by crosstalk measurements and balanced at 140 kilocycles to meet the requirements of the system. The paper insulated conductors have the largest attenuation of any of these facilities, and the loaded spiral-four the least. The various entrance arrangements from the open wire to the office equipment are described in more detail elsewhere.[2] The choice of the facility used in any particular case will depend upon the resultant overall economy.

The large number of non-loaded pairs in the existing 1.6 mile entrance cable at San Antonio indicated that sufficient pairs could be selected which would be satisfactory from the crosstalk standpoint for J carrier operation. Six pairs were subsequently selected and balanced.

At Austin a single toll entrance cable, one mile in length, with two complements, terminates the line from the two directions. Although the two complements are separated by a layer shield, this cable is not suitable from a crosstalk standpoint for operation of the J carrier in and out of the office; therefore, at least one additional cable is required from the central office to the toll line. For this purpose a new non-loaded spiral-four entrance cable was indicated for the type J system with the type C and voice circuits continuing to use the existing cable. The separation of the type J circuits from the non-J circuits on the same pairs is accomplished by filters which are located in a small building at the junction of the toll line and the entrance cables. The use of a single entrance cable for the non-J wire in both directions on the telephone line indicated that it might be necessary in the future to use crosstalk suppression filters at this point. Accordingly, the filter hut was made large enough to include future crosstalk suppression

[2] Loc. cit.

filters if required as well as the line filters which separate the type J from the non-J circuits.

A repeater station at Temple could have been located in the existing central office or could be located in a separate building in or near the city. In either case a new power plant was needed since the existing plant could not be economically modified to serve the J carrier repeaters. The telephone line is continuous through the city, only those wires used for Temple circuits being terminated in the office through one entrance cable 0.6 mile in length. This cable is not suitable for J operation in both directions, which would require one additional cable if the repeaters were located in the central office. Numerous signal and supply lines in proximity with the telephone line within the city offered interaction crosstalk complications. A separate repeater station near the toll line in or near the city avoids the placing of a long entrance cable, reduces the overall system attenuation, and eliminates the problem of interaction crosstalk from paralleling lines. Other factors including cost showed very little difference between a separate station and placing the repeaters in the central office. An unattended station near the toll line was indicated.

A common entrance cable at Dallas terminates the wire on both the Houston and San Antonio lines, the terminal of the Houston line being 2.9 miles from the central office, and of the San Antonio line one mile further. This cable previously had been placed in three different sections, each section having a different make-up, and there was considerable doubt as to the number of suitable pairs for J operation that could be obtained. The use of either a loaded or non-loaded spiral-four cable would not improve attenuation sufficiently to change the number or materially alter the locations of the repeater stations from those tentatively selected, but would provide some additional margin for sleet conditions. The expense of loading the spiral-four cable, if placed, could not be justified by the improvement in overall attenuation. Using either non-loaded spiral-four or existing non-loaded paper insulated conductors requires filters at the open-wire terminus. With these considerations, it was decided that suitable pairs would be used in the existing cable until exhausted. Subsequent crosstalk selection tests have indicated that twelve pairs, six for each line, are available.

Since there was no suitable central office building at Milford, the repeater station in that vicinity must of necessity be in a new building preferably near the toll line. Commercial power is available only near the town, forcing a tentative location to be selected at the edge of the city.

TABLE I

Distribution of Gain and Loss by Repeater Sections

Repeater Section	Cable		Open Wire			
	Length Miles	Loss db	Length Miles	Wet Weather Loss db	Maximum Tolerable Attenuation in db per Mile at 140 KC Using	
					45 db Repeaters	75 db Repeaters
Dallas-San Antonio System						
Dallas-Milford	3.9 mi., 16 ga.	17.50	49.2	16.4	0.560	1.165
Milford-Temple	Nominal	Nominal	84.7	27.3	0.532	0.885
Temple-Austin	1.1 mi., Spiral-4	2.20	74.4	24.8	0.575	0.980
Austin-San Antonio	1.1 mi., Spiral-4 1.8 mi., 16 ga.	10.30	78.5	26.2	0.635	0.825
Dallas-Houston System						
Dallas-Corsicana	2.9 mi., 16 ga.	13.20	52.5	17.5	0.610	1.175
Corsicana-Hearne	0.5 mi., 16 ga.	2.25	90.5	30.2	0.484	0.805
Hearne-Hockley	Nominal	Nominal	85.6	28.6	0.545	0.875
Hockley-Houston	5.6 mi., 16 ga.	25.20	29.1	9.7	1.190	1.700
Dallas-Longview System						
Dallas-Edgewood	1.4 mi., 16 ga.	6.30	55.3	21.0	0.984	1.240
Edgewood-Longview	0.5 mi., 16 ga.	2.30	69.7	26.5	0.614	1.040

With these selections of entrance cable facilities and tentative repeater station locations, the distribution of gain and line loss by repeater sections is shown in Table I. A satisfactory distribution of line loss has been obtained and an analysis of these data shows that further improvement is impracticable. Therefore, the tentative repeater station locations were adopted.

Figure 4 is a diagram of the major line and equipment parts of the Dallas-San Antonio lead. The J carrier path is shown by heavy solid lines, the C and voice on the same wire with the J by light solid lines, and all other circuits, classed as non-J, by dotted lines. Figures 5 to 8, inclusive, show in more detail the arrangements at the huts and repeater stations. The figures for the Dallas Hut and Temple Repeater Station are typical, and huts and unattended repeater stations not shown differ from these only in minor details. It will be noted that all wire on the toll line is brought through the repeater stations while only that wire on which J carrier is superposed is routed through the huts except at Austin where all wire to the north is brought through the hut to allow the future application of crosstalk suppression filters if required. For both huts and unattended repeater stations, short lengths of loaded spiral-four conductors are used from the six-inch spaced wire at the terminal poles to the equipment in the buildings. A single continuously adjustable load unit is used for each pair and is located with the equipment. Paper insulated pairs under the same cable sheath as the spiral-four conductors are used for the non-J wire.

As previously mentioned, the conditions at Austin were complicated by a single cable for existing circuits and a new cable for J carrier in both directions. Figure 9 is a diagram of the existing and new cables to the filter hut and terminal poles, and Fig. 10 shows the interconnection of circuits and equipment used at the filter hut, terminal poles, and central office.

Terminal and repeater equipment in existing offices is located in space adjacent to other equipment terminating toll circuits, and makes use of the common office equipment and power plant. The relation of the J carrier terminals to the other equipment in the Dallas Toll Office is shown in Fig. 11.

The new repeater stations and the filter huts are arranged for unattended operation. The equipment in the filter huts is such that no adjustment or attention is required other than periodic inspections. In the unattended repeater stations the power supply equipment is automatic in its operation. Although periodic maintenance attention is necessary, it is desirable that any abnormal condition be recognized as soon as practicable and a system of alarms has been provided from each unattended station to an adjacent main repeater or terminal

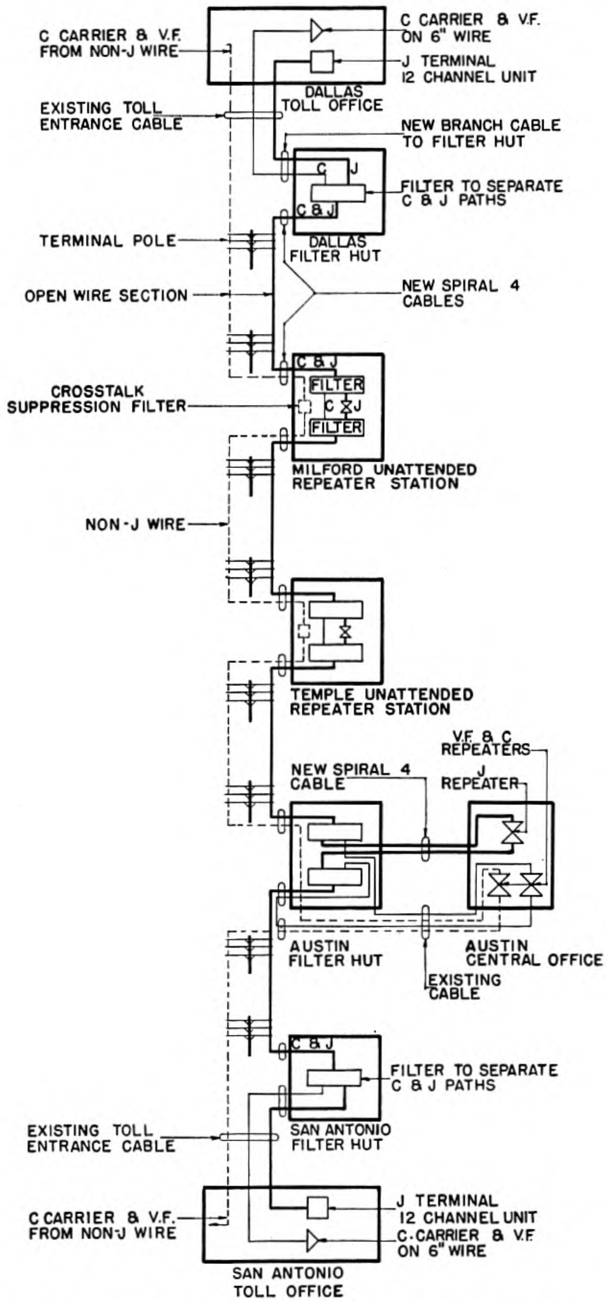


Fig. 4—Arrangement of facilities for a typical J carrier system.

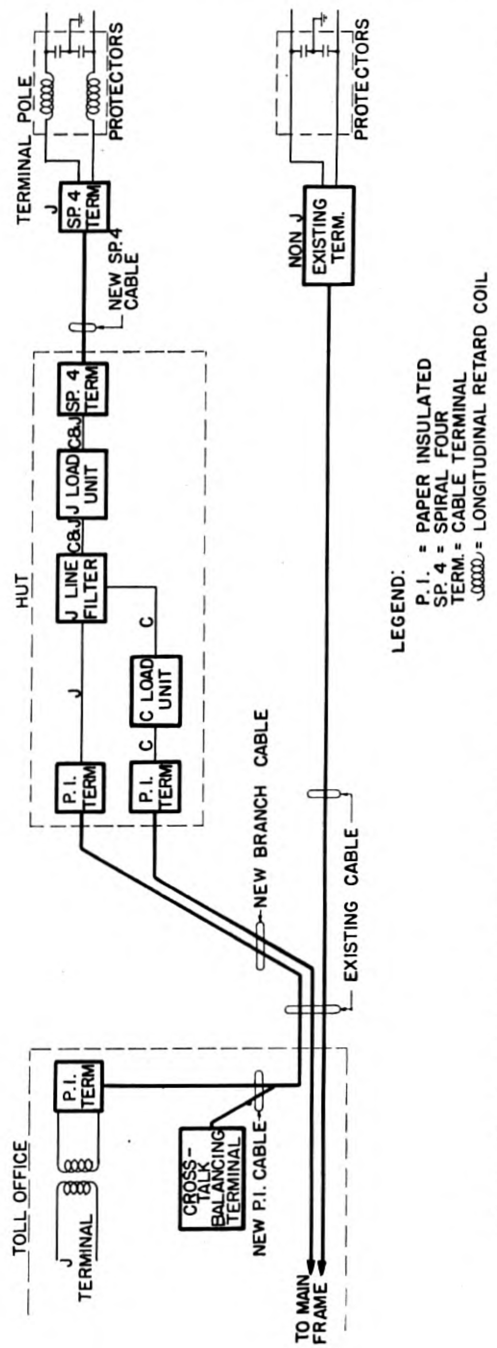


Fig. 5—Circuit connections through Dallas filter hut from open wire to terminal equipment at central office.

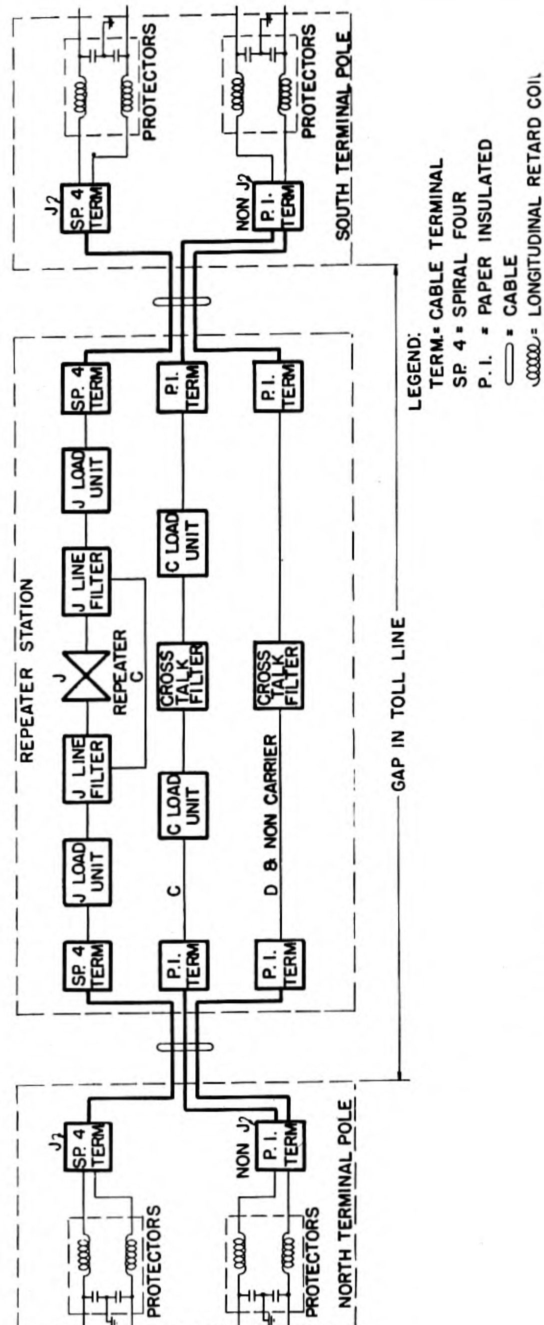


Fig. 6—Circuit connections through Temple unattended repeater station.

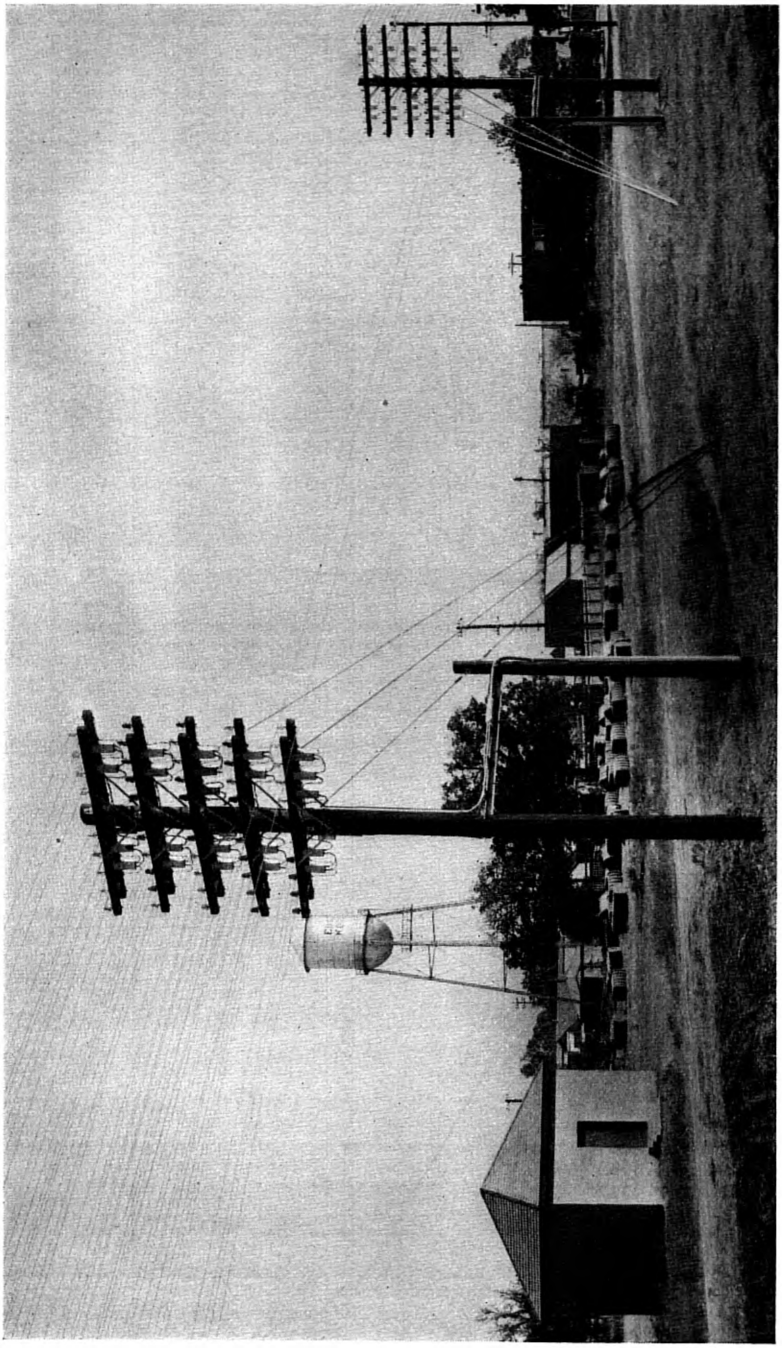


Fig. 7—Arrangement of gap in toll line at unattended repeater station.

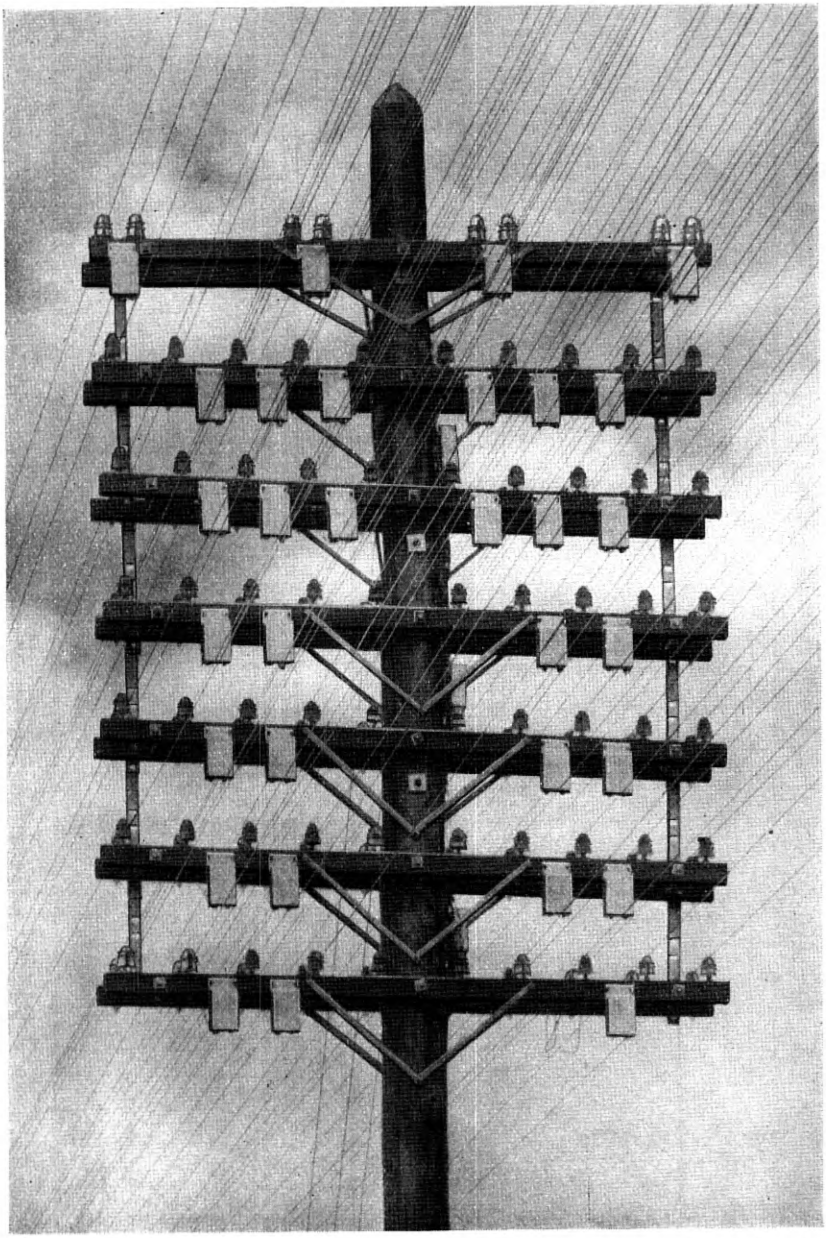


Fig. 8—Longitudinal choke coils and protectors on terminal pole at unattended repeater station.

office. This alarm has been arranged to operate by direct current over one conductor between offices without interfering with existing telephone circuits but at the expense of one DC telegraph path. For fuse failure, rectifier failure, power off, power restored, high-low voltage, high-low temperature, fire, burglary, pilot channel failure, and end of pilot channel control, alarms are sent and identified. A questionable alarm may be rechecked from the attended office.

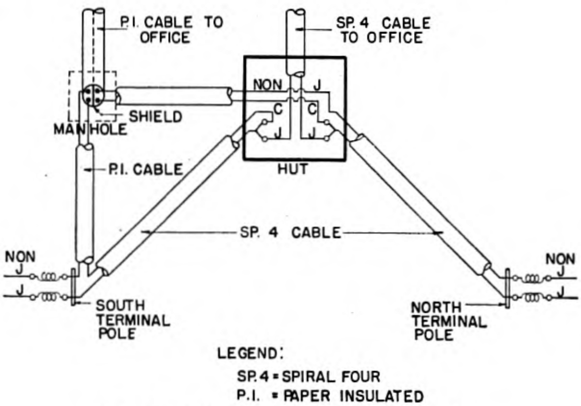


Fig. 9—Cable arrangement at Austin.

Special Problems

Some of the problems encountered in connection with the other two systems may be of interest. At Corsicana, a repeater point on the Dallas-Houston system, a filter hut was used on only one side of the repeater station. The situation which led to this arrangement is that an intermediate cable in the Dallas-Houston line extends 0.2 mile north and 0.5 mile south from the local central office. As it is necessary that the J system operate through this entrance cable and since space was available in the local central office building, repeater equipment similar to that installed in unattended buildings was placed in one room in the office.

The section of cable north of the central office terminates on a corner in a business district with all adjacent property occupied by buildings, making it more economical to use loaded spiral-four cable to this location than to extend the existing cable to an available site and provide the necessary filter hut and equipment. For the longer cable, it was more economical to provide the filter equipment in a hut in order to use existing facilities. Although this cable terminates in a fully developed residential area, a site for a filter hut was obtained adjacent to an alley in the rear of one of the residences facing the street on which the terminal pole is located.

The use of non-loaded paper insulated pairs in existing entrance cables has been mentioned. However, it is in general not practicable for crosstalk reasons to use all the non-loaded pairs which are available in one cable, and the selection of pairs suitable for type J operation is illustrated by a discussion of the methods used on the Dallas cable.

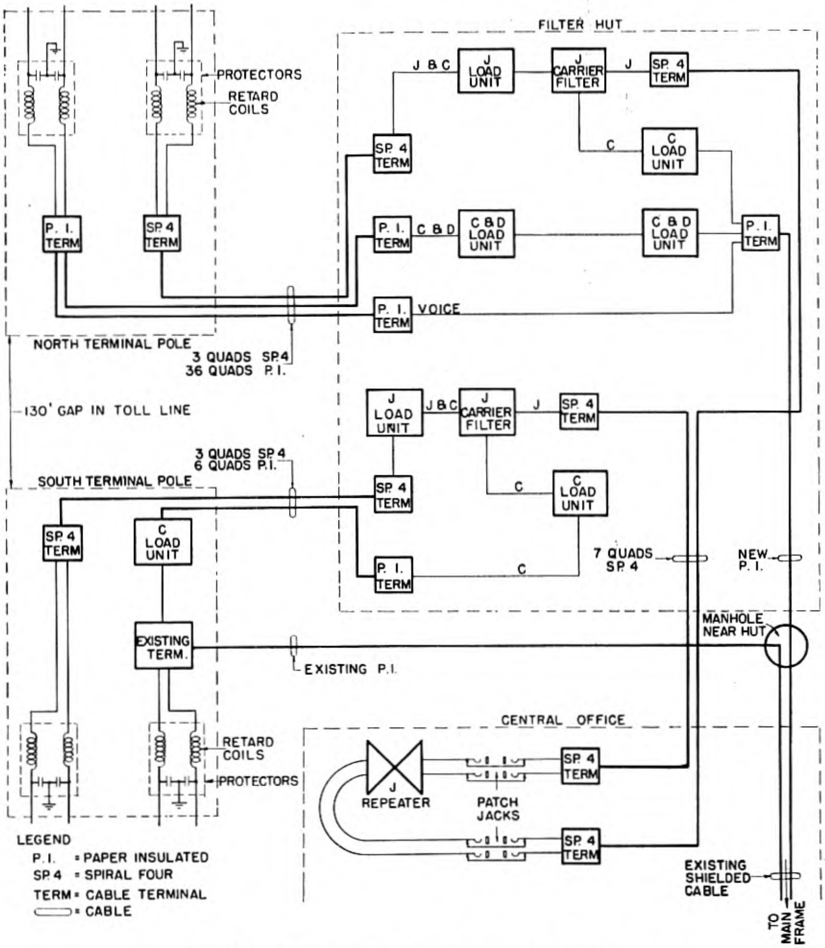


Fig. 10—Circuit connections through Austin.

The Dallas cable is composed of three sections of different make-up. The section nearest the central office, 1.3 miles long, and the intermediate section, 1.6 miles long, each contained 22 idle non-loaded pairs, and the third section, one mile long, had only six. The Houston line

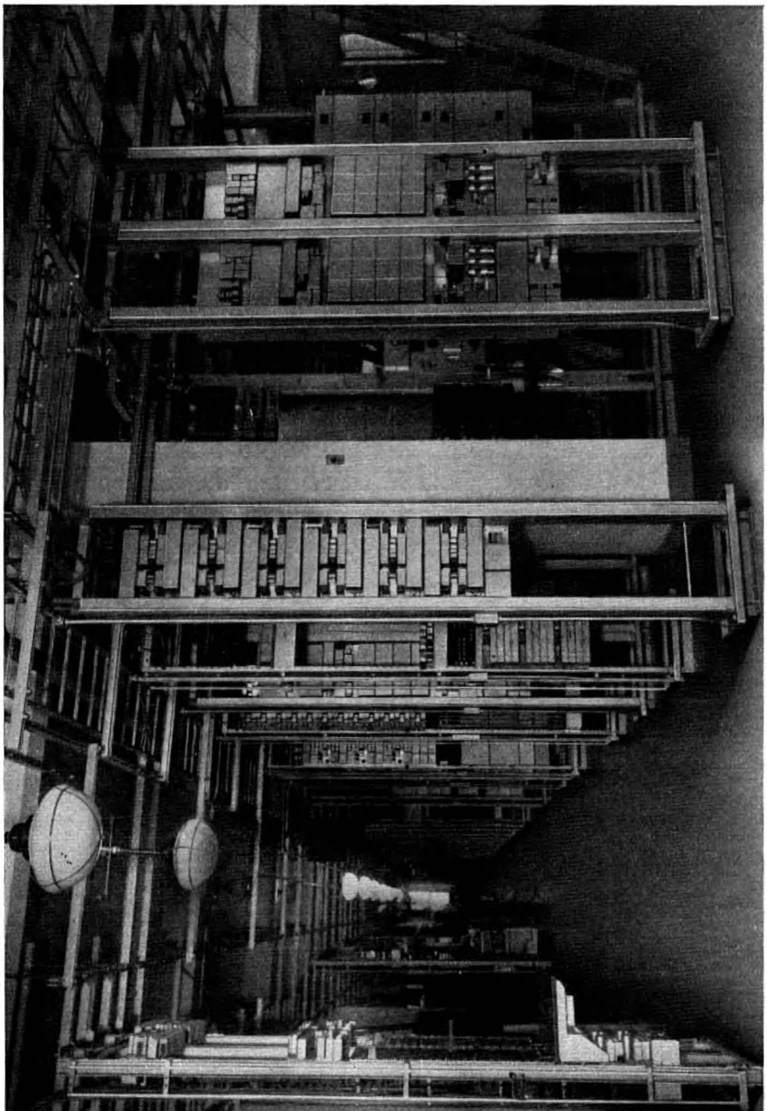


Fig. 11—Toll line terminating and testing equipment at Dallas with J carrier terminals in foreground.

terminates at the end of the second section, the third section extending the cable to the San Antonio line.

Since the number of cable pairs to the San Antonio line was limited to a maximum of six and since the rate of circuit growth over the two lines was expected to be approximately the same, requiring cable relief over the entire distance when the branch to the San Antonio line was exhausted, an objective of six pairs to each line was set up.

Measurements of crosstalk coupling at 140 kilocycles in terms of inductance and capacitance unbalance were made between each pair and all other pairs in each section and the pairs rated in their order of desirability. It is of interest that this required a total of 854 measurements. Those pair combinations whose coupling of the mutual inductance type was high were rated as the least desirable. This was done because capacitance balancing was to be used to obtain crosstalk reduction. The more desirable pairs in the first two sections were connected through to the six pairs in the last section by cut and try method until the overall condition was such that all six pairs were acceptable. By a similar procedure, using the remaining pairs in the first two sections, six pairs to the Houston line were made acceptable. No record is available as to the number of tests made in the cut and try process.

A cable terminal on which balancing condensers were mounted was installed in the central office building and connected to the selected pairs. This terminal contained sixty-six small adjustable wire wound condensers which were connected between each pair and every other pair. The condensers were adjusted to reduce to a minimum the capacitance component of the crosstalk coupling.

Buildings

For the three J carrier systems, four new repeater stations and eight filter huts were needed. The same type of construction was used for all: Concrete foundation with floor slab above grade, double four-inch brick walls with rock wool insulation between but with solid brick at corners and openings, pitched roof with wood framing, fire resistant wall board ceiling, fire resistant composition shingles, and heat insulation above ceiling and below floor slab.

All of the racks for equipment in the unattended repeater stations are arranged in three rows with power, repeater, and line equipment in separate rows within a floor space of 17 feet by 16 feet which will allow the ultimate installation of six repeaters in each building. The entrance cables from the terminal poles enter from iron conduit through the floor and are racked and spliced on the side wall adjacent to the line bays. The stubs from the cable terminals at the top of the line bays are carried overhead to splices on the wall. A ceiling height of 13 feet is maintained above the equipment but reduced along the pitch of the roof to 11 feet 8 inches at the side walls.

For all huts except that at Austin, three adjacent bays of racks are needed. With these along one side wall of the hut, the opposite side is available for splicing the entrance cable. At Austin an ultimate of

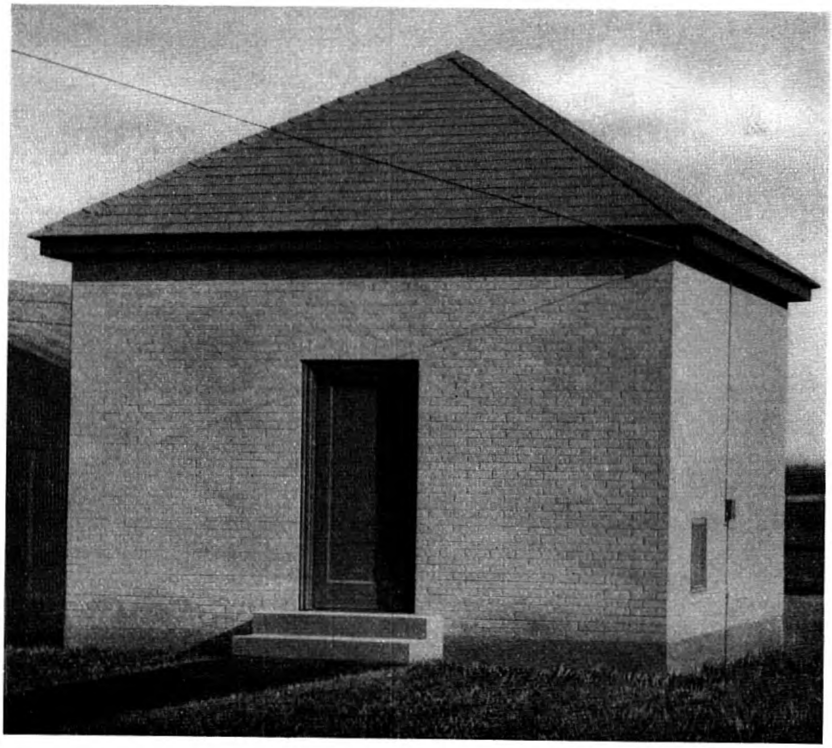


Fig. 12—Unattended repeater station.

nine racks, for filters in both directions of transmission, led to the use of racks along opposite sides of the hut with a splicing pit under the floor made accessible by trap doors in the floor between the lines of racks. In this case, the cable terminals are installed at the bottom of the racks with their stubs dropped directly through the floor slab into the splicing pit. The racks in the hut are seven feet high and a ceiling height of eight feet is used. Figures 12, 13, 14, and 15 are pictures of a typical repeater station, typical filter hut, and the special hut at Austin.

For correct operation of the equipment, temperature limits of 32 to 110 degrees Fahrenheit are desirable. Also, it is necessary that there be no precipitation of moisture on wiring or equipment. To maintain the desired conditions, each of the huts is equipped with a 2 kw. blower type electric heater arranged to operate at low temperature or high relative humidity, but with operation blocked when the temperature

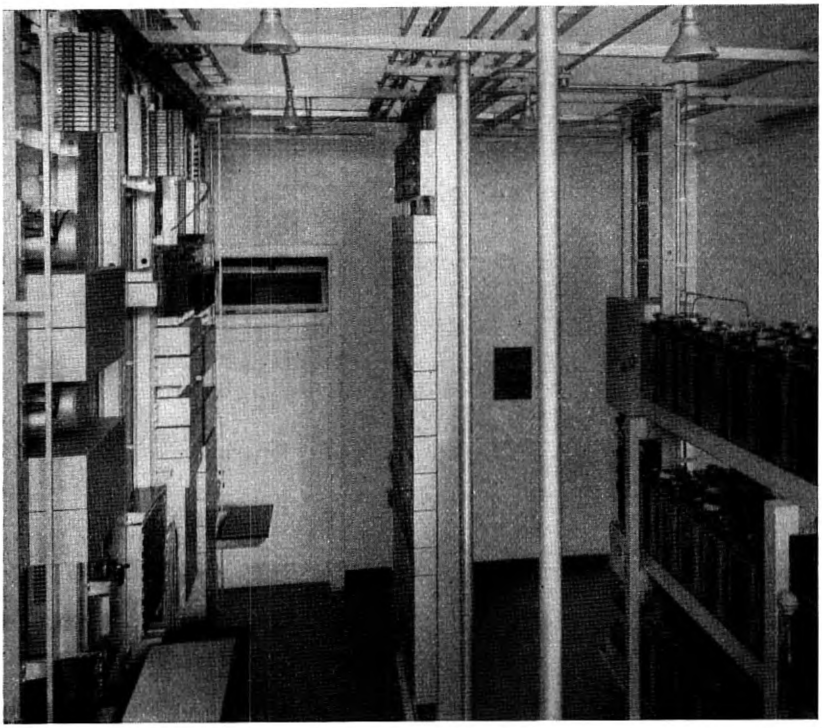


Fig. 13—Equipment in unattended repeater station.

reaches 95 degrees. Each new unattended repeater station is equipped with a 4 kw. heater similarly controlled, and, on account of the heat dissipation of power plant and vacuum tubes, also has forced ventilation which is operative under conditions of high temperature. The system of forced ventilation consists of spun glass intake filter, exhaust fan, electric solenoid controlled shutters at intake and exhaust, and thermostat, and is interconnected with the office alarms to prevent fan operation in case of fire.

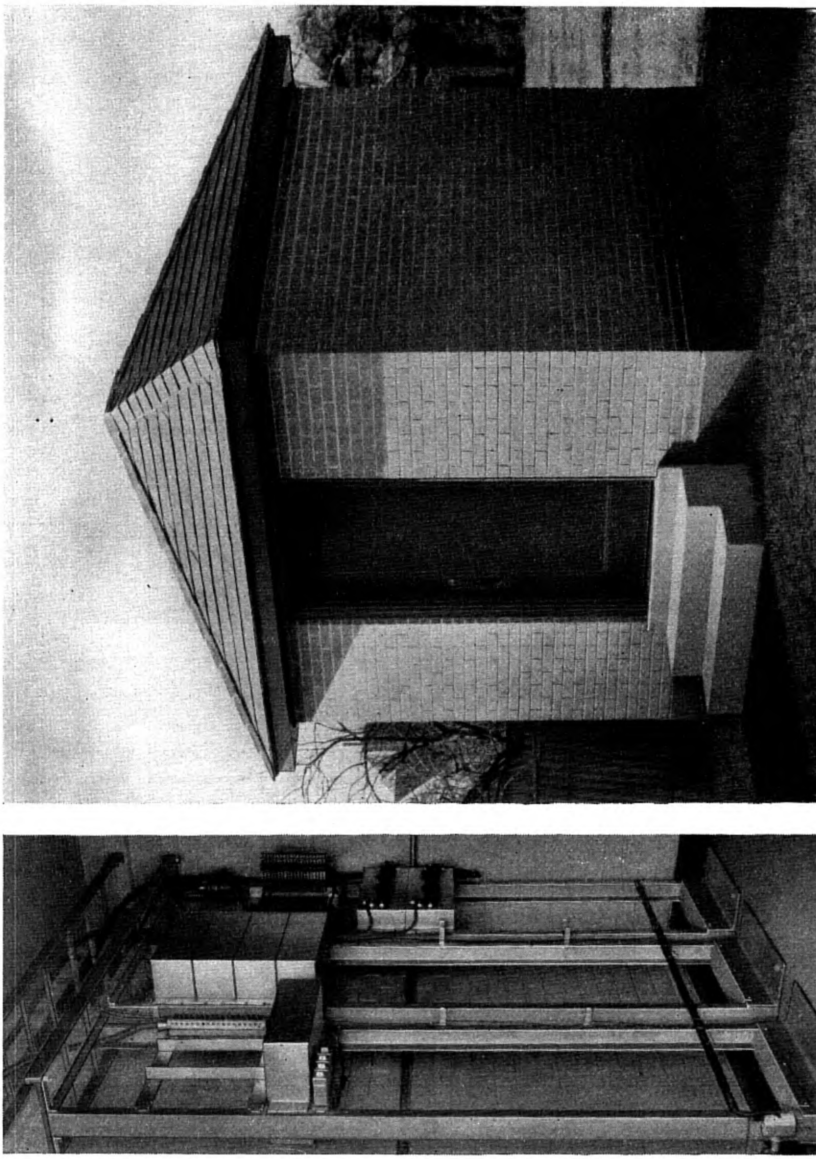


Fig. 14—Filter equipment and typical hut.

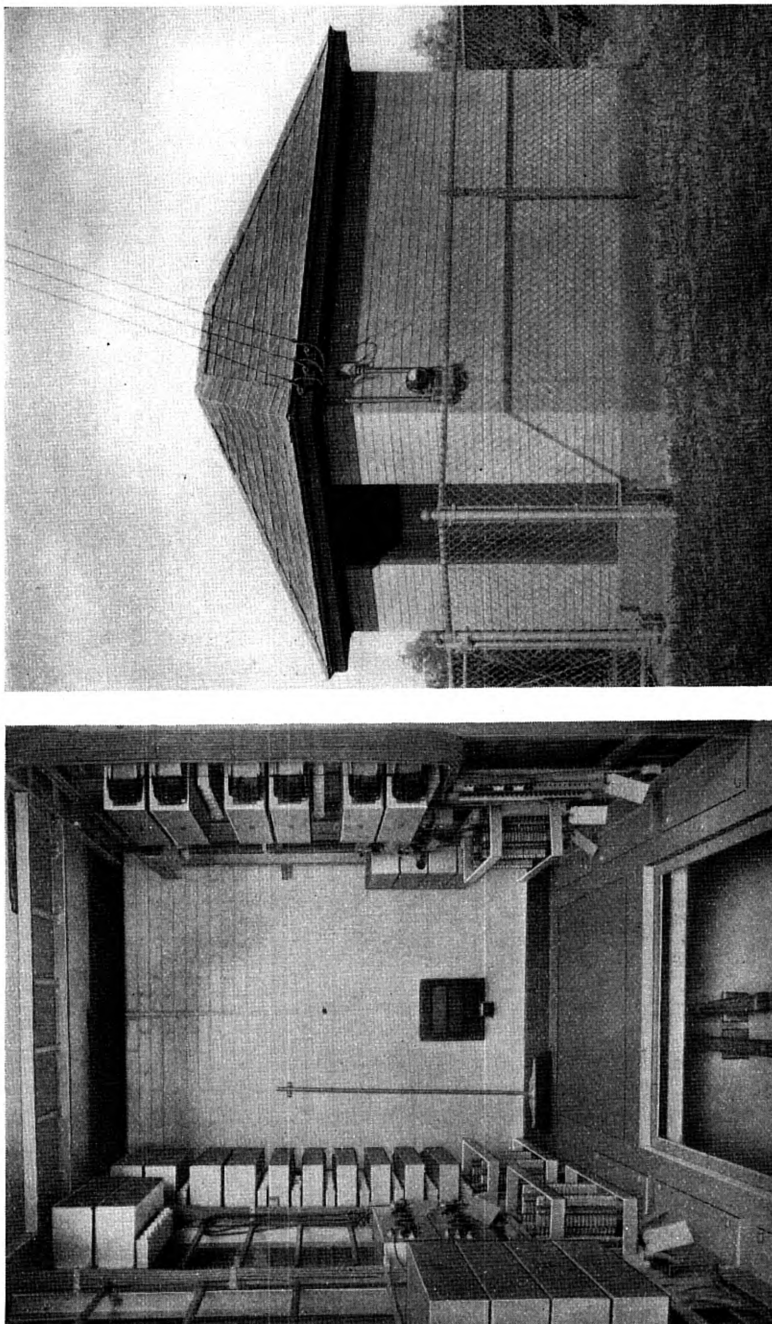


Fig. 15—Filter equipment and hut in Austin.

Conclusion

Upon completion of the buildings, equipment installation, and line facility rearrangements, adjustments in the equipment were made to match the lines used. Networks associated with the terminal and intermediate amplifiers were adjusted so that the amplification for any particular frequency would be equal to the attenuation at that frequency in the preceding repeater section; the automatic pilot channel equipment[1] compensates for attenuation changes. In repeater sections containing long toll entrance cables, it was necessary to sacrifice range of automatic pilot channel control to obtain the best equalization. However, satisfactory equalization and range of pilot channel control were obtained in every case.

As mentioned previously, the Dallas-Longview system operates on twelve-inch spaced phantomed wire. In Fig. 16 the attenuation

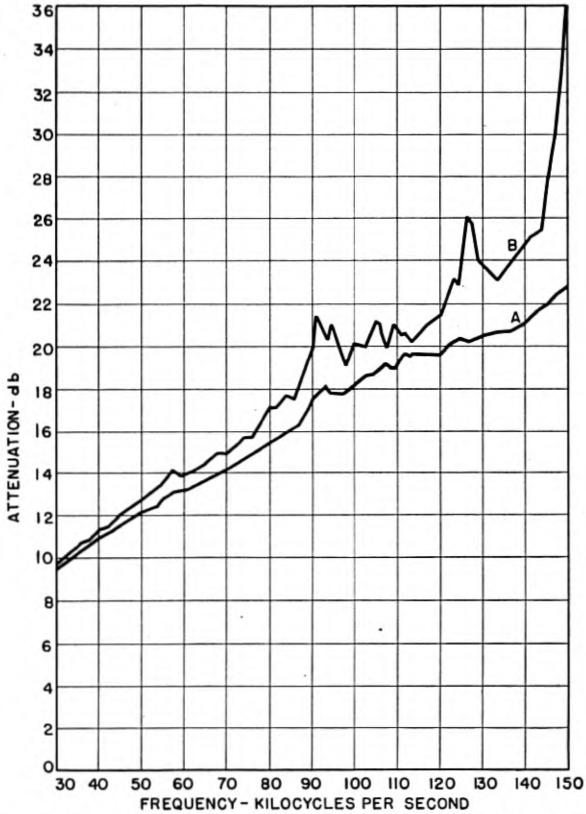


Fig. 16—Attenuation of two twelve-inch spaced phantomed pairs of the Edgewood-Longview repeater section.

[1] Loc. cit.

characteristics of two possible pairs are shown. The absorption peaks of pair "A" at 92 and 127 kilocycles are within the frequency range of channels the fourth and twelfth of the J system and would impair the quality of those channels if pair "A" were used. Therefore, pair "B" is used as the regular path for the system. The quality of the channels obtained from these systems is shown by Fig. 17. Curve

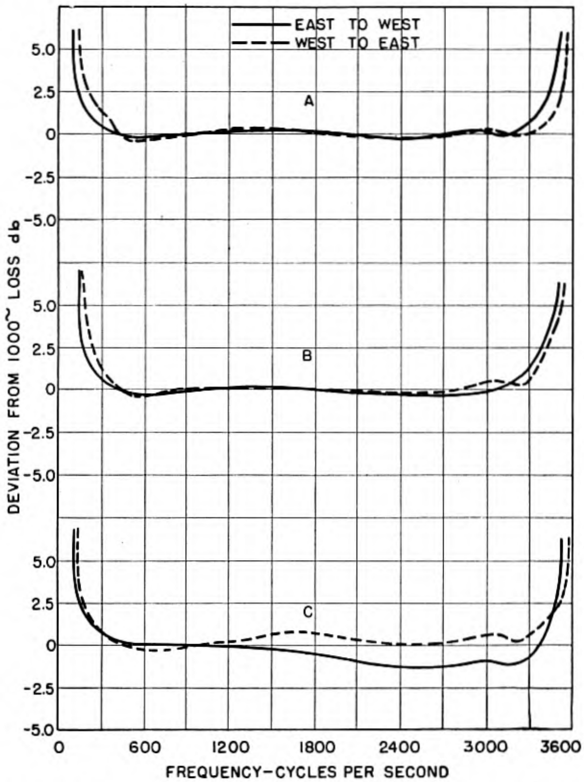


Fig. 17—Quality of derived circuits, "A" for typical channel of systems on six-inch spaced wire, "B" and "C" for the best and poorest channels of system on twelve-inch spaced wire.

"A" is representative of that obtained from a system operating over six-inch spaced wires; "B" and "C" are the best and poorest obtained from the Dallas-Longview system.

The Dallas-San Antonio, Dallas-Longview, and Dallas-Houston systems were placed in commercial service in September, October, and November, 1938, respectively. Experience with these systems is that the circuits obtained compare favorably with those obtained from any other facilities in quality and continuity of service, and that a definite need has been fulfilled in providing an economical method of increasing the capacity of the existing plant.

Line Problems in the Development of the Twelve-Channel Open-Wire Carrier System *

By L. M. ILGENFRITZ, R. N. HUNTER, and A. L. WHITMAN

The development of the type J twelve-channel carrier telephone system for open-wire lines required an increase of nearly 5 to 1 in the transmission frequency range of the lines. In the provision of suitable line facilities a number of new problems were encountered with respect to attenuation, noise and crosstalk. Methods for meeting these problems and the results obtained are described.

Introduction

A NEW carrier telephone system for open-wire telephone lines has been described recently.[1] This system increases the number of two-way telephone circuits which can be obtained on a single pair of wires from the previous maximum of 4 to a total of 16. This has been achieved by extending the frequency range from a maximum of about 30 kilocycles to more than 140 kilocycles. The exploitation of this new range of frequencies on open wire has involved the solution of a number of interesting problems, among which are these:

(1) Not only does the attenuation of an open-wire line under ordinary weather conditions rise substantially with frequency but extremely large increases in attenuation occur at the higher frequencies when ice forms on the wires.[2, 3] In spite of these effects a high degree of stability of transmission has been secured on all channels by the provision of automatic control of repeater gain and equalization.

(2) New crosstalk problems created by the extension of the frequency range have been solved by the development of transposition designs with numbers of transpositions not greatly in excess of those employed for the lower frequency systems. Problems have also arisen in controlling the crosstalk around the repeaters and in reducing the effect of impedance departures between the line circuits and the equipment.

Frequency Allocations

The type J system operates on circuits on which type C carrier systems were already operating in the frequency range up to about 30 kilocycles. To provide enough frequency separation between the two

* Presented April 18, 1939 before the A. I. E. E., in Houston, Texas.

[1] Reference numbers refer to the list of references appearing at the end of the article.

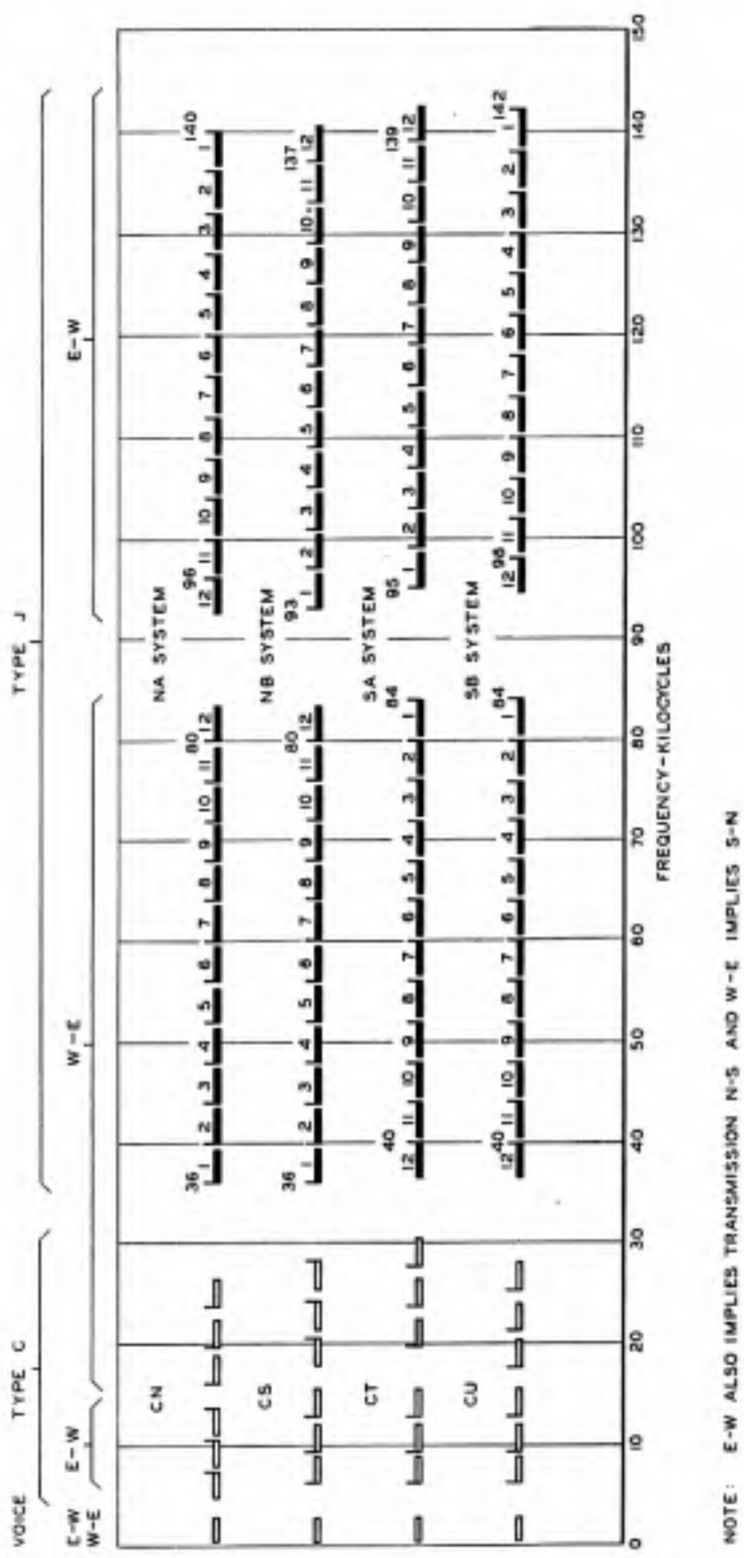


Fig. 1—Frequency allocation.

systems the lower frequency limit of the J system was set at 36 kilocycles; the necessary frequency space for 12 channels in each direction set the upper limit at about 140 kilocycles. This range is split into two parts, one used for transmission in one direction and the other for the opposite direction. Figure 1 illustrates the relation of the frequency bands occupied by the type J and type C systems and the voice-frequency channel. Different "staggered" locations of the frequency bands are to be employed in order to simplify crosstalk problems.

Filters are used for separation of the type J from the type C and lower frequency facilities on the same pair of wires. This separation is done by means of a combination of high and low pass filters which split apart the frequency ranges above and below the band between 30 and 36 kilocycles. To simplify the design of these filters, the low frequency group of the type J system is transmitted in the same direction as the high frequency group of the type C system. This arrangement of transmitting certain frequencies in a particular direction is generally used throughout the telephone plant in order to avoid serious crosstalk difficulties. Accordingly, with few exceptions, west to east transmission or south to north transmission takes place in the same frequency bands throughout the country and similarly, east to west or north to south transmission employs the same frequency bands. These are indicated in Fig. 1.

Line Attenuation

An open-wire pair affords the lowest loss transmission medium of any conductor employed in the telephone plant. It is, however, peculiarly subject to the effect of weather, which may cause large and often rapid changes in the attenuation. In consequence, some form of gain regulation is required.

Even for carrier systems operating up to 30 kilocycles, manual regulation is inadequate for the longer systems and automatic devices have been provided for most systems over 500 miles in length. The attenuation changes caused by changes in resistance of the wire with temperature or by changes in the shunt losses when insulators become wet are much larger at the higher frequencies of the J system, and therefore, an automatic regulating scheme is required. Tests were made on open-wire circuits to determine more precisely the characteristics needed for such a regulator. During sleet storms, when wires are covered with ice, the increases in attenuation are far beyond any caused by rain. Figure 2 shows increases which may be caused by ice as compared with the normal dry and wet weather values.

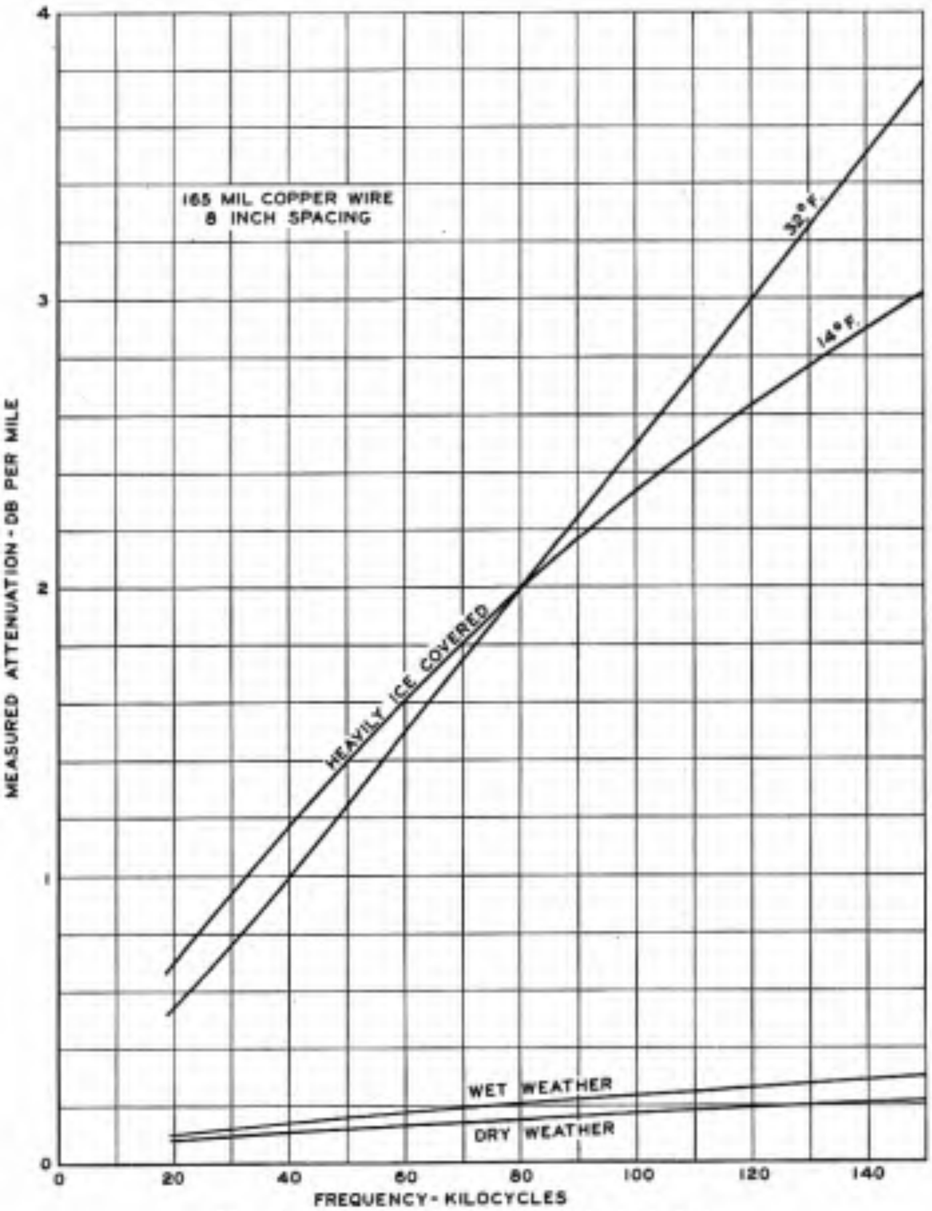


Fig. 2—Attenuation variation with weather.

The deposits on the wire may be actual ice, or in some cases wet snow or frost adhering to the wire. Figure 3 shows an example of such deposits. Theory shows that the increase in attenuation is caused by energy losses in the ice itself and that leakage across the insulators is usually a negligible factor.

An extensive survey of the effects of ice has been carried on at various points throughout the country during the past four years and a large amount of information has been accumulated. These tests have shown that the shape of the attenuation-frequency characteristic differs considerably for different ice formations and even if the ice deposit remains the same for a time, the attenuation-frequency characteristic may vary with temperature as in Fig. 2. The two upper curves of the figure were measured at different times during the same storm. There

Fig. 3—Ice on wires and insulators near Amarillo, Texas.

was no apparent change in deposit between the two measurements. This change in shape of the characteristic, of course, makes the regulation problem more difficult. In spite of the extreme severity of ice effects in certain regions, it is expected that satisfactory reliability will be obtained on type J systems by placing the repeaters sufficiently close together.

Regulation Problem

In the first type J systems the regulator, actuated by a single pilot frequency in each direction, compensates for the attenuation changes caused by temperature and wet weather.

The required varieties of attenuation slopes with ice on the wires could not be provided by a simple regulator. Hence provision is to be made in later designs for a regulator with variable slope controlled by two pilot frequencies which is expected to be satisfactory in areas subjected to sleet conditions. The regulating range will also be increased so that a completely automatic control of gain up to about 75 db will be available.

It was found that during periods when ice coated the wires the circuit noise measured at the end of a repeater section usually decreased as the attenuation increased. This is important because otherwise the extra increase in the repeater gain to take care of the higher attenuation at such times would make the noise excessive. The study of ice conditions throughout the country which has been carried on and is still continuing will be useful in laying out repeater stations along some of the routes which eventually will be candidates for the application of type J systems.

Open-Wire Crosstalk [4]

The crosstalk problem on open-wire lines is one of the most important. Crosstalk is controlled by transpositions which are introduced into the various pairs in accordance with a predetermined design. The creation of the necessary designs requires consideration both of the complex theory of transpositions and measurements on lines constructed by practical methods.

However, the design of transposition systems is considerably simplified by the use of different frequencies for the two directions of transmission. The only crosstalk between systems which is directly important is that known as far-end crosstalk, which is that between a talker at one end of one circuit and a listener at the opposite or far end of another. Near-end crosstalk, which is that between a talker and a listener at the same or near ends of two circuits, becomes a source of interference between circuits only when portions of it appear as far-end crosstalk because of reflections at points of impedance irregularity in the circuits.

Because of the high cost of a transposition design to keep both near-end and far-end crosstalk down to small values, only small reflections are permitted where open-wire and cable meet, or where circuits are terminated in equipment. A number of the difficulties which had to be overcome to attain small reflections are discussed later in the paper. With this control the transposition designer can concentrate most of his attention on far-end crosstalk, the near-end crosstalk requirements are relaxed, and a cheaper transposition arrangement can be used.

What can happen when reflection occurs may be seen by comparison of the near-end and far-end crosstalk curves in Fig. 4. The similarity in the shapes of the two curves, and particularly the fact that the peaks occur at the same frequencies, show that what appears to be far-end crosstalk is in this case mostly reflected near-end crosstalk. It is for pair combinations such as this one, where the near-end crosstalk is much larger than the far-end, that the closest control of reflection effects is required. With the values of reflection realized in the J system, reflected crosstalk will ordinarily be unimportant.

To obtain satisfactory crosstalk conditions at the higher frequencies some changes in line construction are necessary. To use type J carrier systems on existing open-wire routes, methods were devised for modifying the line construction in as economical a manner as possible.

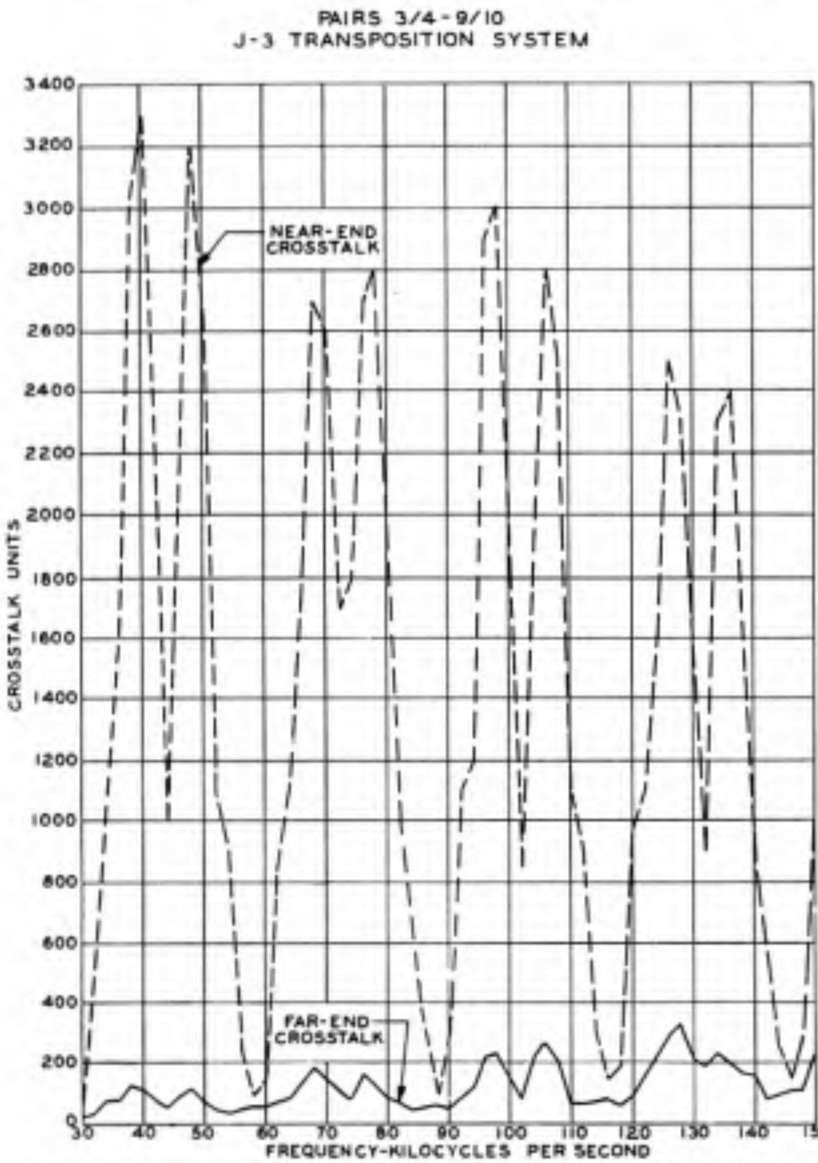


Fig. 4—Near-end and far-end crosstalk—J-3 transposition system.

For new lines, such as the new part of the Fourth Transcontinental line,[5] advantage was taken of the greater degree of freedom in structural design which was possible.

Figure 5 shows three types of open-wire pole head configuration

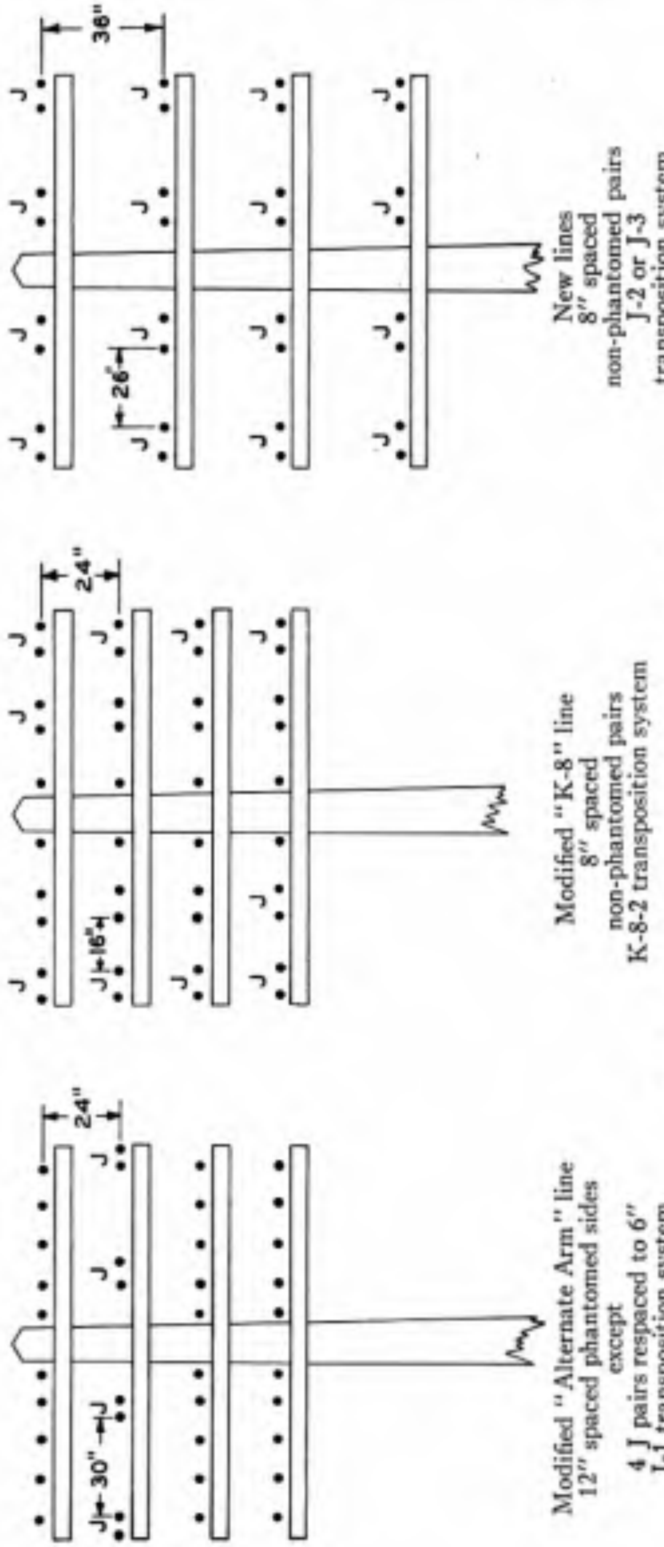


Fig. 5—Three types of open-wire pole head configuration.

suitable for J system operation. The left-hand diagram shows a method of reconstructing part of one of the older types of open-wire lines built with 12-inch spacing between wires of the pairs and with the "Alternate Arm" transposition system which was developed for the use of type C systems on the side circuits of the horizontal phantom groups on alternate arms. This method is a flexible one in that one or more phantom groups may be converted at a time, as on the second crossarm shown. For such an application not only was removal of the phantoms and retransposition necessary, but the spacing of the two wires of each pair was reduced to 6 inches. This general method of construction was used for the Dallas-Houston and Dallas-San Antonio lines,[6] except that the 6-inch pairs were constructed with new wire on a new crossarm rather than by respacing 12-inch pairs.

Another common type of open-wire pole head configuration, the middle diagram of Fig. 5, is that made up of 8-inch spaced non-phantomed pairs transposed in accordance with the K-8 transposition system on an eight-span base. Through design studies supplemented with field experiments it was found that such a line could be converted for J systems much more cheaply than an Alternate Arm line. If J systems are restricted to the pairs on the outer ends of the crossarms, with two inner pairs, about one or two transposition changes in each pair per mile are enough. This scheme was followed in reconstructing the line between Charlotte, North Carolina, and West Palm Beach, Florida.

For new lines yet to be built, a greater degree of latitude in structural design is naturally possible. The right-hand diagram of Fig. 5 shows an open-wire pole head configuration designed to allow J systems to be operated on all of the pairs. The unique feature of this configuration is that, while 8-inch spacing is preserved between the wires of the various pairs, the adjacent non-pole pairs on a crossarm are separated by twenty-six inches and the crossarms by thirty-six inches. The reduction in coupling made possible by this increased spacing keeps the crosstalk for any combination of pairs down to a suitable value with transposition arrangements not necessarily more complicated than those employed for the other configurations. This type of construction was used for the new parts of the Fourth Transcontinental line.

Fig. 6 shows a comparison of the number of transpositions used in a typical section of open-wire line for various types of circuits from voice frequency phantomed circuits to non-phantomed circuits intended for J system operation. From the original arrangement where there was one transposition point in every ten spans, about

¼ mile, the number of transpositions for J carrier operation has been increased so that for the J-3 design, which was used for the new wires on the Fourth Transcontinental line, there are four transpositions in each eight-span interval and every pole is a potential transposition point.

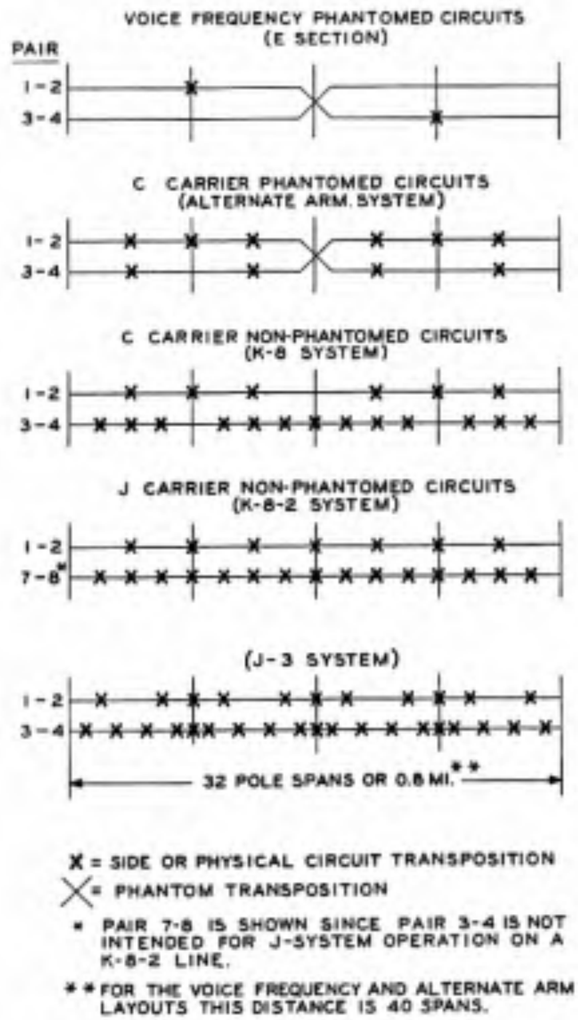


Fig. 6—Illustrative transposition arrangements.

It may be seen from Fig. 6, however, that the number of transpositions required in pairs for J carrier operation is not necessarily larger than the number employed in systems intended for C carrier operation with a top frequency of 30 kilocycles. The superiority of the J system transposition arrangements as compared with those designed for C system operation results from the choice of specific arrangements which best limit the systematic effects for frequencies in the J system range.

Typical far-end crosstalk measured between 8-inch spaced pairs 11/12 and 19/20 on a new J-3 line and on a reconstructed K-8-2 line is shown by Fig. 7. The superiority of the new line with its fewer wires, greater

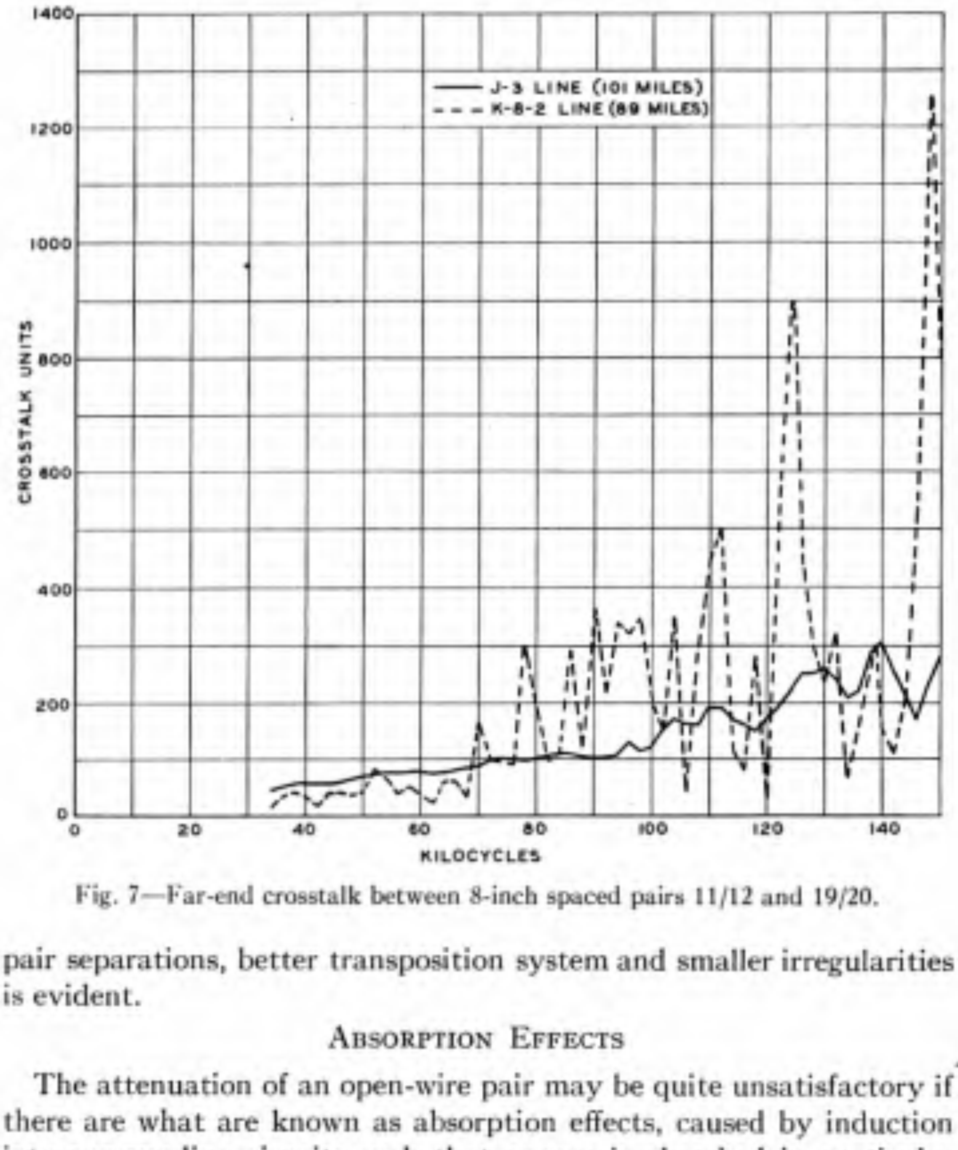
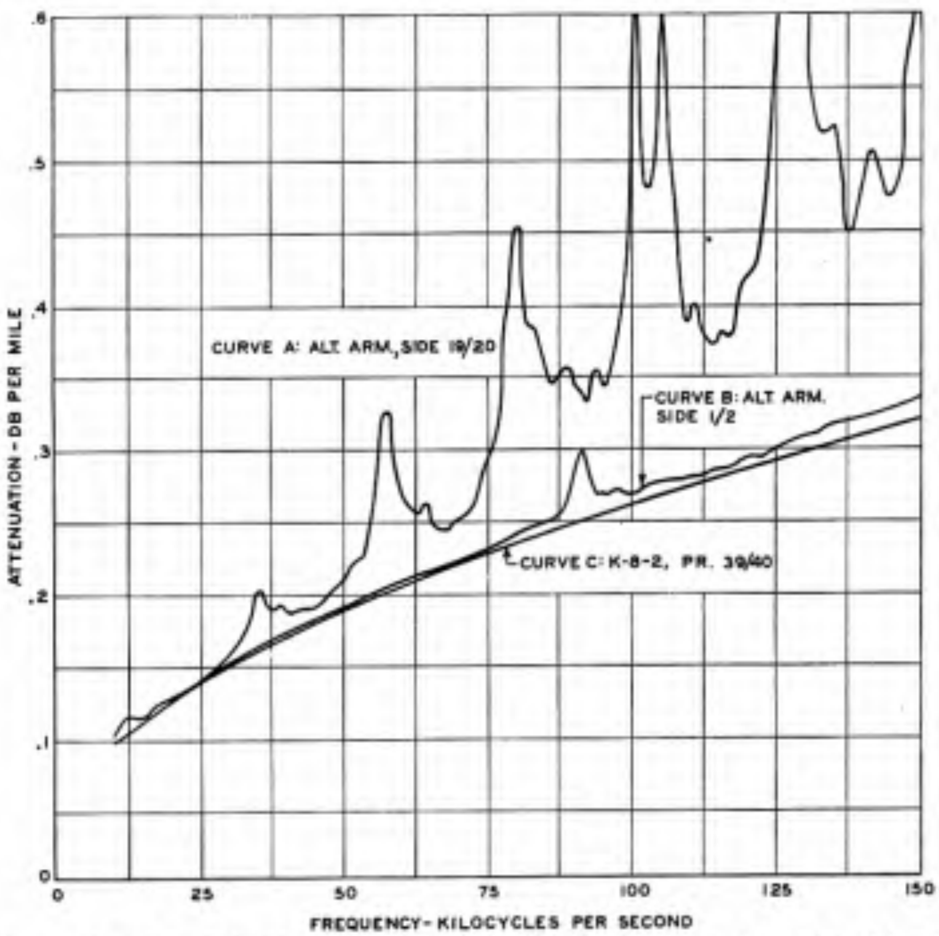


Fig. 7—Far-end crosstalk between 8-inch spaced pairs 11/12 and 19/20.

pair separations, better transposition system and smaller irregularities is evident.

Absorption Effects

The attenuation of an open-wire pair may be quite unsatisfactory if there are what are known as absorption effects, caused by induction into surrounding circuits such that energy is absorbed in particular frequency bands and the attenuation of the pair increased. These effects, which depend on the transposition arrangements in the circuits, may cause objectionable transmission distortion at critical frequencies unless the transpositions are planned to avoid them. The same arrangements necessary to control crosstalk between J systems will automatically eliminate absorption effects with one exception. If only part of the pairs on a line are designated and transposed for J systems and the remaining pairs are not so transposed, absorption in a J pair can be caused by a nearby non-J pair. Consequently, consideration of the crosstalk relations at J frequencies between all of the pairs



Curves A and B: Side 19/20 and 1/2 respectively, on Alternate Arm line, 104 mil, 12-inch, 56.7 miles, 90° F. at Mascoutah, Ill.
Curve A is transposed for voice frequencies.
Curve B is transposed for carrier operation up to 30 kc.
Curve C is pair 39/40 on K-8-2 line, 104 mil, 8-inch, 68 miles, 50° F. and CS insulation between Denmark, S. C. and Rincon, Ga. Transposed for carrier operation up to 140 kc.

Fig. 8—Attenuation of open-wire pairs of different types.

on the line cannot be avoided even though some of them will not be used for J systems.

Figure 8 illustrates the effect of absorption on three different pairs. Curves A and B show the absorption measured over the type J frequency range on a line of the Alternate Arm type. Curve A was obtained on a side circuit transposed for operation at frequencies only up to about 10 kilocycles. The absorption at frequencies above this becomes very large. Curve *B* shows the absorption present on one of the C carrier side circuits on the same line transposed for operation up to 30 kilocycles. Curve *C* shows how absorption disappears on a non-phantomed pair specially transposed for type J operation. If this pair were measured at much higher frequencies, similar absorption "bumps" would be found, perhaps at frequencies of 200–300 kilocycles or higher.

Since absorption effects depend on the systematic addition of crosstalk currents along a line, a continuous succession of identical transposition sections tends toward greater absorption while a random succession of different kinds of transposition sections of different lengths will reduce it. The Dallas-Longview J system is operating on an Alternate Arm side circuit, transposed for C carrier operation and without any modifications to adapt it for the higher frequencies. Because of the fortunately irregular succession of different transposition sections found here, it was possible to select, after tests, a pair with no serious absorption.

Construction Irregularities

With the new transposition designs, the systematic crosstalk resulting from the transposition arrangements has been reduced in nearly every case so far that the remaining crosstalk is controlled principally by construction irregularities. An important source of irregularity is the difference in sags of the various wires in each span of the line, particularly sag differences between the two wires of each pair. Another potentially important source of irregularity is the variation in the spacings between successive transposition poles. It is relatively easy to make this factor unimportant as compared with sag differences.

The large amount by which the crosstalk can be reduced by careful methods of construction coupled with the highly developed systematic transposition patterns is illustrated by the fact that between certain pairs the crosstalk in a 75-mile repeater section is reduced to a value which would be produced by a capacitance unbalance between them of less than 2 mmf, which is about the same in magnitude as the capacitance between wires of a foot of the open-wire pair. This large crosstalk reduction is in spite of the fact that at 140 kilocycles the phase change along an open-wire circuit is about 7° in a single span, the shortest distance between any two transpositions, and about 28° for the more common four-span interval.

Interaction Crosstalk at Repeater Points

Another type of problem was introduced by what is known as interaction crosstalk. This is the crosstalk which occurs from one side to the other of a J repeater station. Figure 9 illustrates two paths

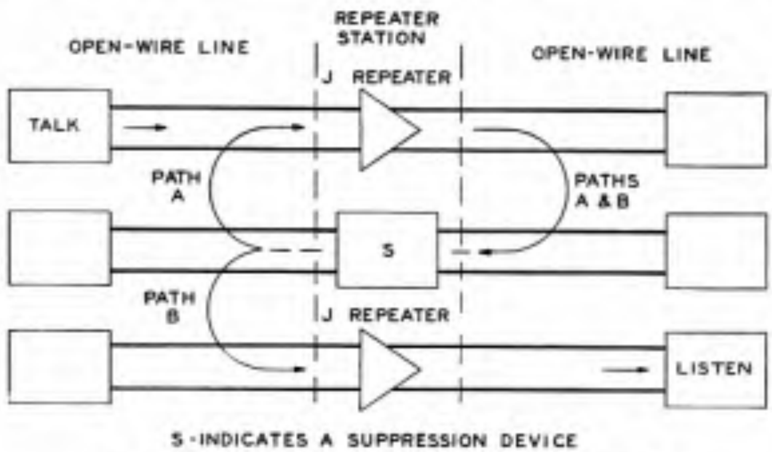


Fig. 9—Interaction crosstalk at a J repeater station.

which it may take. Path A shows the crosstalk from a system to itself which may cause transmission distortion or repeater singing while Path B is the path of crosstalk between different circuits. The essential feature of this interaction crosstalk is that, as Fig. 9 shows, the crosstalk path at a repeater station passes through the J repeater and hence the crosstalk is amplified by the repeater gain.

The new problems of controlling this crosstalk were the result of larger magnitudes of crosstalk at the higher frequencies, the larger repeater gains and the fact that with more repeaters there were more points on a system where it could occur. Magnitudes of interaction crosstalk which had previously been thought of as inconsequential assumed a new importance. For instance, with the gain of about 75 db proposed for the repeater for use in sleet areas, an initial value of unamplified interaction crosstalk as low as 0.25 crosstalk unit would be magnified to 1400 units, which might considerably exceed the far-end crosstalk existing at the same time in one repeater section.

Several new methods for reducing this interaction crosstalk were devised. In the first place, in order to prevent direct coupling between the wires of the open-wire line on the two sides of the station, it was found necessary to cut a gap in the line. With the wires entirely removed for a distance usually of about eighty feet, the line is brought into the station from the two terminal poles by means of the lead-in cables.

It was also seen to be necessary to block the paths provided by the wires of the telephone line itself. For this purpose, crosstalk suppression filters were designed and built to be installed in all of the non-J circuits on the line. These give losses of the order of 70 db at 140 kilocycles not only in the metallic transmission circuits but also in other circuits, made up of various combinations of the line wires, which may conduct crosstalk currents through the stations.

In addition to the crosstalk suppression filters and in order to provide an extra margin of safety against interaction crosstalk currents which might find their way through the repeater station by stray paths, longitudinal choke coils have been connected at the pole heads between the open wires and the lead-in cables. These coils do not disturb ordinary transmission but add high impedance in the longitudinal circuits.

These measures for controlling interaction crosstalk have been found to be adequate so far as the telephone line is concerned. At an occasional J repeater station, however, located on a right-of-way occupied by several pole lines, there is found another pole line paralleling the telephone line with a separation sometimes as little as 2 to 5 feet between the nearest wires of the two lines. Such wires provide other interaction crosstalk paths past the repeater station and impair the effectiveness of suppression measures installed in the line on which the J system is operated. The by-passing effects of such a foreign line can be controlled by crosstalk suppression devices similar to those used in the telephone line wires.

Figure 10 shows a comparison of the interaction crosstalk measured at a J repeater station before any suppression measures were installed, the other wires of the line being continuous at the station location, with the corresponding interaction crosstalk when the line was run through the suppression devices in the station. The values shown would be amplified by the gain of the J repeater on the disturbed circuit before they reached the listener. The effect of the by-passing foreign line is illustrated by the difference between the middle and bottom curves, the bottom curve showing the measured crosstalk when the by-passing line was cut to simulate the effect of suppression measures in it.

Staggered Systems

It would not be possible with the open-wire line configurations now in use to design transposition arrangements that would permit the operation of identical J systems on all pairs. For this reason four types of J systems with different channel carrier frequency allocations will be provided in the future. The frequency assignments for these systems are shown in Fig. 1.

The "staggering" advantage, or effective crosstalk reduction between systems, is effected because (1) the inversion or displacement of channels in the different systems with respect to each other makes the

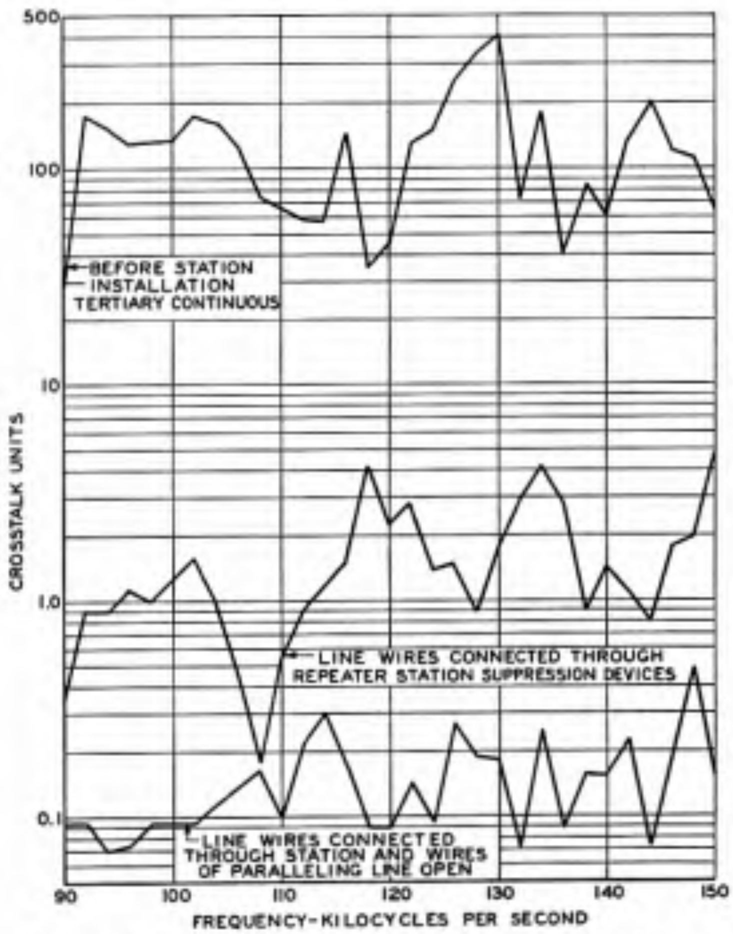


Fig. 10—Unamplified interaction crosstalk between two J circuits at an auxiliary repeater station.

crosstalk unintelligible and (2) the reduction of the overlap between channels results in less energy being transferred between them by crosstalk. The net benefits of "staggering" obtained by the allocations shown in Fig. 1 range from about 6 to 16 db.

The most effective pair assignments for the four types of J systems can best be obtained from actual crosstalk data on the particular sections of line involved. The "staggering" advantages obtained are sufficient so that the highest remaining crosstalk will usually occur between the like J systems operating on non-adjacent pairs.

Noise

Observed external sources of noise in J systems are atmospheric static, dust storms, radio stations, power line carrier and power supply systems.

Of these possible sources the more important will usually be atmospheric static which will be greatest during the summer months. In regions where dust storms occur, their effects are expected to exceed that of atmospheric static but will be more likely to occur during the winter and early spring.

The following table shows values of noise at 140 kilocycles, caused by atmospheric static, found at the open-wire line terminals of one repeater section; the values are those which it is expected will be exceeded during one per cent of the summer season extending from May to September. If the repeater spacings shown were used, the total static noise in the top channel at the end of a circuit with 20 repeaters would be 20 db above reference noise at the − 9 db level. However, other factors such as ice may require the use of shorter spacings.

Transposition System	Wire Spacing (Inches)	Noise (db) *	Repeater Spacing in Miles—128-Mil Wire
Alternate Arm	12	+ 10	67
K-8-2	8	+ 5	82
J-1	6	− 2	103

* Above reference noise, 10^{-12} watt at 1000 cycles.

Line Impedance

As mentioned previously in the discussion of crosstalk, it is important that the line impedances be matched closely and large irregularities be avoided. Because of the different wire sizes and pair spacings, a wide range of open-wire line impedances may be encountered. Novel construction arrangements and the development of new lead-in circuits have made it possible to secure a reflection coefficient of about five per cent at the junction between the open-wire pair and the toll entrance and office equipment at the highest transmitted frequency.

The transposition arrangement and wire spacing of a pair affect the smoothness of its impedance because they affect the reactions between circuits which cause absorption effects. The marked improvement which can be obtained by proper design is illustrated by comparison of Curves A and B of Fig. 11. Curve A shows the impedance of a

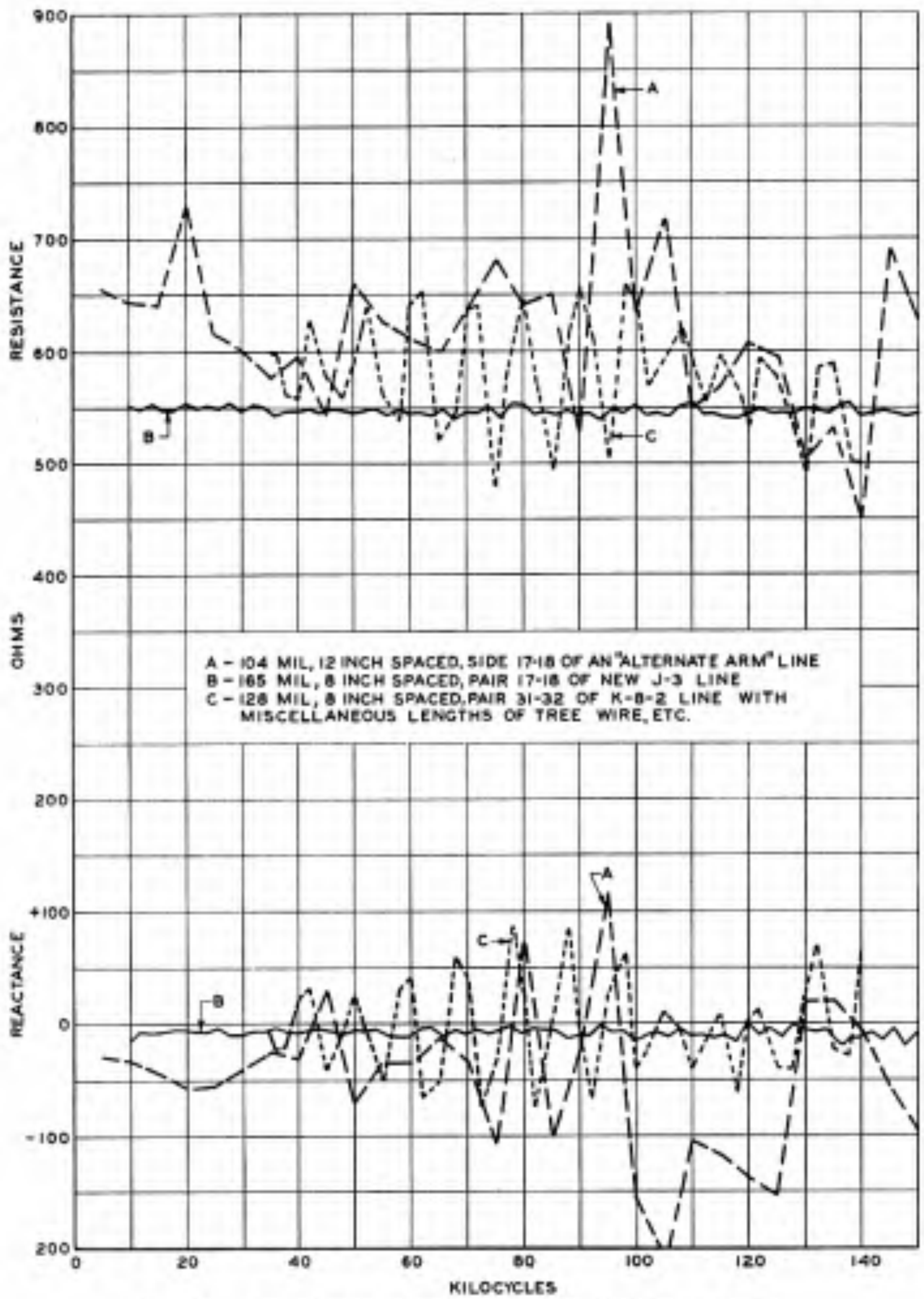


Fig. 11—Impedances of open-wire pairs of different types.

12-inch spaced side circuit on an Alternate Arm line. This particular circuit was one intended for use at frequencies not above 10 kilocycles. In striking contrast Curve B shows the comparatively smooth impedance of an 8-inch spaced non-phantomed pair on a new line transposed in accordance with the J-3 system.

"Tree" wire, a special line wire with abrasion-resistant insulation, has been used on open-wire lines for many years in places where the lines were exposed to tree branches. During line tests in Florida, another use for tree wire was found where the open-wire line, along a causeway or bridge, is subject to fouling by fishing tackle. Curve C of Fig. 11 shows what a half-mile or so of this tree wire, supplemented by

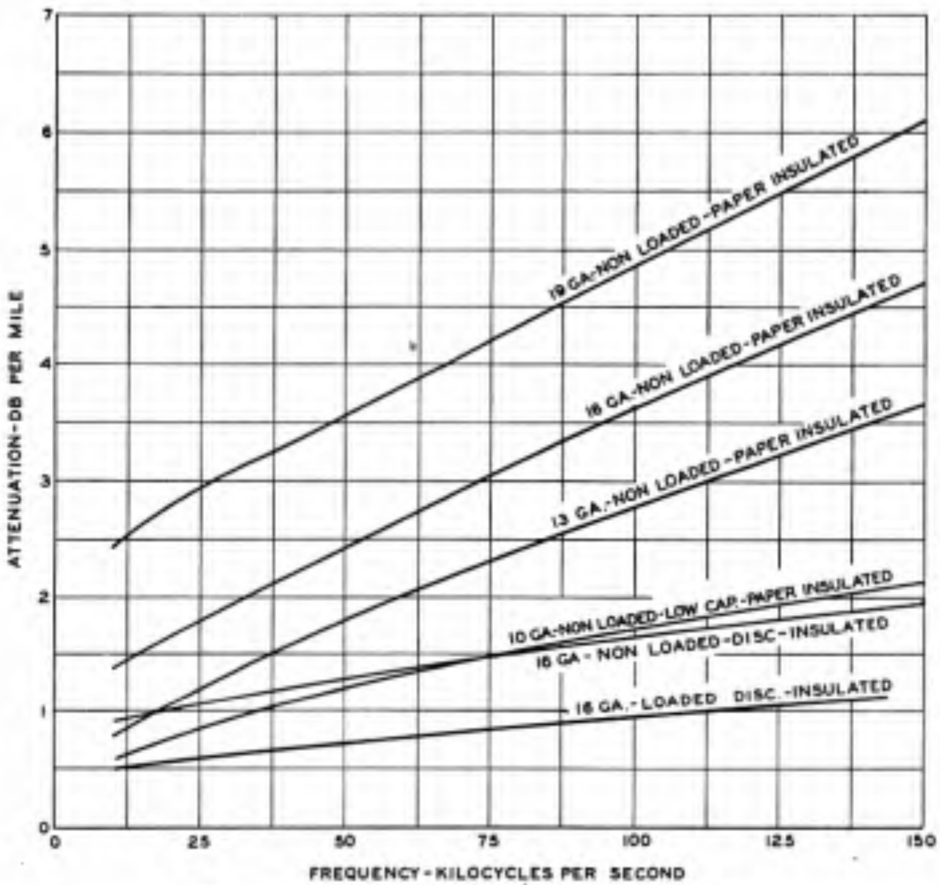


Fig. 12—Attenuation of toll entrance cable pairs.

several sections of 165-mil wire at railroad and power-line crossings, can do to the impedance of a 128-mil pair. To reduce the irregularities a new type of insulated line wire of smaller diameter and with thinner insulation was developed. This wire has about the same impedance characteristic as the line wire.

Intermediate Cable Treatment

When open-wire lines have to be placed underground to pass through towns or to cross natural barriers such as rivers which cannot be spanned economically with open wire, cable is used. In the past, the circuits in such cables were frequently loaded to reduce their attenuation and to match the impedance of the open-wire circuits in order to avoid reflection effects and degradation of voice-frequency repeater balance. To load paper-insulated cable pairs for frequencies up to 150 kilocycles would require exceedingly short loading spacing, of the order of 200 feet, which would be expensive and in many cases impractical with existing manhole locations. An alternative, the use of a transformer to match the open wire and cable impedances, was rejected as it was found impractical to design a transformer which would be adequate over the entire frequency range.

To overcome these difficulties, a new low-capacitance type of cable was developed which could be loaded to match the open-wire impedance with coil spacings about the same as those previously used. Loading coils of different sizes were developed to provide for loading to the different impedances of the open-wire circuits.

The new cable employs 16-gauge conductors in a spiral-four arrangement, supported by hard rubber disc spacers about 0.6 inch in diameter. These are surrounded by copper and iron tapes for shielding and strengthening purposes. The units so formed may be assembled either in single units in a lead sheath as for lead-in purposes, or in multiple units, up to a maximum of seven for full-sized cable, within the same lead sheath. For duct runs or submarine cables, the multiple assembly is usually employed, and, in the latter case, with outside armoring and jute protection. If the submarine span is more than about 600 feet, intermediate submarine loading is employed.

As an alternative, it sometimes happens that where a long intermediate cable is involved, an auxiliary type J repeater station can be placed conveniently at one end of this cable. In this case, the filter hut described in the discussion of toll entrance arrangements in the next section may be used at the end of the cable opposite the repeater station and the cable treated as a toll entrance cable for the auxiliary office. A further alternative is to provide filter huts at the two ends of a non-loaded intermediate cable. However, if the cable is short, the new disc-insulated cable with loading is to be preferred.

Previous practice at the ends of open-wire lines has been to use paired bridle wire with weather-proof insulation and usually of smaller gauge than the line wire to connect the open-wire pairs to cable terminals mounted on the pole. Other pairs of bridle wire were connected between the open wires and protectors. Because of the much more severe reflection requirements at the higher frequencies of the type J system, these arrangements were no longer satisfactory. The characteristic impedance of bridle wire is roughly one-fifth of that of the open-wire circuit and it has been necessary to avoid the use of even several feet of it between the open-wire and the cable terminal or protectors. To accomplish this, separate terminals for each disc-insulated unit are mounted on the crossarm near the open-wire pairs to which they connect. Four insulated wires from each terminal go by the shortest feasible route to the longitudinal choke coils and protectors and thence to the open-wire pairs.

Toll Entrance Arrangements

The new disc-insulated cable used for intermediate cables was also suited for lead-in or toll entrance cables.

When an auxiliary station is established at a point along an open-wire line where there has not previously been an office, it is usually located close to the line so that the lengths of lead-in cable required are comparatively short. Lengths of this cable up to about 175 feet can be loaded to open-wire impedances with adjustable loading units in the repeater station. For longer lead-in cables up to 300 feet, supplementary loading may be mounted directly on the pole at the cable terminals.

When an auxiliary repeater station is not close to the open-wire line, or at main repeater stations which are frequently in towns and separated from the open-wire line by greater lengths of toll entrance cable, it is still possible to use the loaded disc-insulated cable. Because of the cost of this cable and its loading, however, it has sometimes been found more economical to build a hut near the open-wire terminal pole and to separate the type J from the type C and lower frequency facilities at that point by means of filters. The connection from the open-wire line to the hut is provided by what is usually a short length of loaded disc-insulated cable. From that point, the type J frequencies are led into the toll office over non-loaded paper-insulated pairs while the C and lower frequency facilities are brought in over the existing pairs, usually loaded. By thus limiting the frequencies transmitted over the non-loaded cable pairs to the J range, it becomes practical to design transformers for suitable impedance matching.

The line filter sets located in the hut are designed for a nominal impedance of 560 ohms which is a compromise for the range of impedances normally found with different wire sizes and spacings. An accurate match with the line is obtained with a building-out network which is adjusted at the time of installation to fit the particular open-wire pair involved. On the office side of this line filter set a transformer provides for stepping down the impedance from 560 ohms to the impedance of the toll entrance cable, which is usually about 125 ohms. Adjustment of this impedance over the necessary range to match impedances of particular cable pairs is provided by means of taps on the transformer. At the office another transformer similarly tapped is employed to match the toll entrance cable pair impedance to that of the office wiring.

Fig. 12 shows the losses of the commonly used 19-, 16- and 13-gauge paper-insulated toll entrance cable, a new 10-gauge low capacity cable, and the new disc-insulated cable. Because of the high losses of the smaller gauge pairs, it is sometimes economical to place new 10-gauge cable to save repeater costs.

For the office wiring of the J system a rubber-covered shielded pair is used to provide the desired flexibility and freedom from capacitance variation due to humidity changes. Its impedance at 140 kilocycles is approximately 125 ohms. The repeater and terminal high frequency impedances are designed to match this impedance very closely.

Fig. 13 illustrates the arrangement of the toll entrance equipment involved in matching the line impedance to that of the equipment with a minimum of reflection. The terminal is illustrated to the left. The high frequency line passes to the line filter set which is here shown as located in a filter hut. There it is joined by the type C and lower frequency circuits and passes through the lead-in cable and protective arrangements on the terminal pole.

Proceeding toward the right in the figure, the arrangement at an auxiliary repeater station is shown. In this case the type J frequencies are amplified in the repeater, but the type C and lower frequencies are by-passed through filters which suppress longitudinal and metallic transmission above 30 kilocycles. At the right is shown a combined type J and type C main repeater office.

Satisfactory crosstalk between pairs in entrance and intermediate cables carrying J systems is effected through special selection methods and the application of balancing condensers.

Reflection Coefficients

The success of the various measures taken to insure good impedance matching is shown by the curves of Fig. 14, which are of reflection coefficients measured at an auxiliary repeater station. Curve A, the solid line, gives the coefficient between the open-wire pair and the lead-in cable at the terminal pole. The smaller variations are due partly to irregularities of the open-wire line and, at the lower frequencies, partly to the test terminations at the distant end. The contribution of the cable loading and office equipment is indicated by the dash-line curve

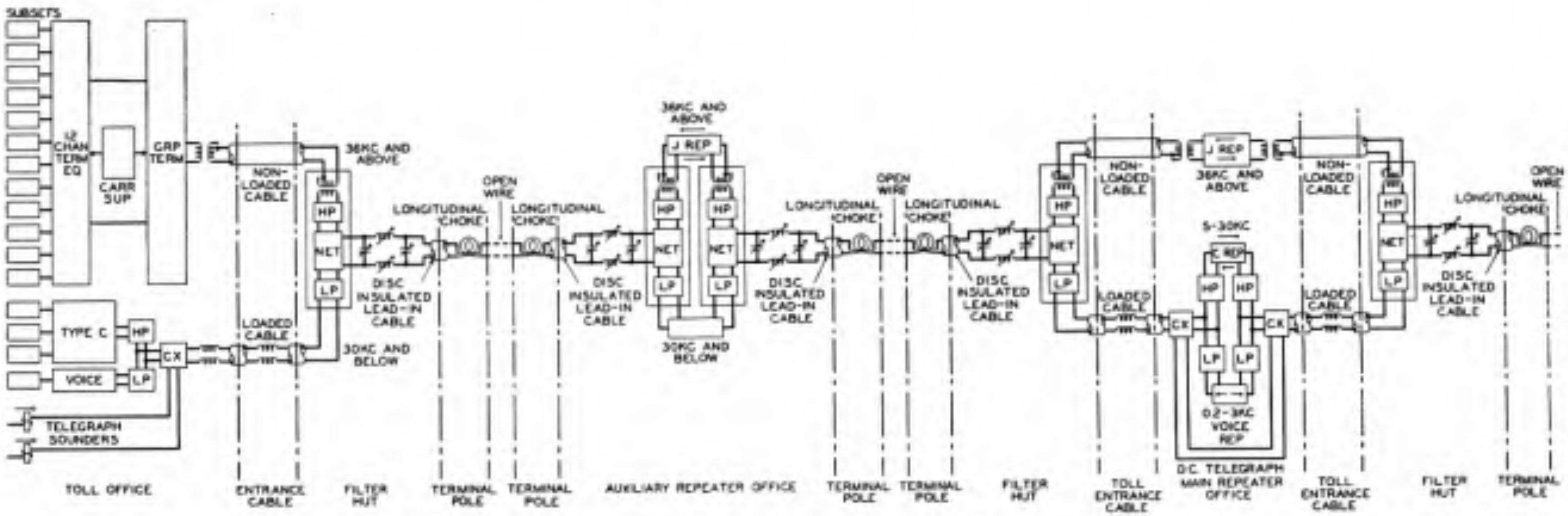


Fig. 13—Toll entrance arrangements at J terminal and repeater offices.

Abstracts of Technical Articles from Bell System Sources

Exploration of Pressure Field Around the Human Head During Speech.[1] H. K. DUNN and D. W. FARNSWORTH. A single speaker in a seated position repeated a fifteen-second sample of connected speech, while r.m.s. pressure measurements were made in thirteen frequency bands, and at seventy-six positions, in different directions and distances. The results are applicable to intelligibility and microphone placement problems. They show, in general, the greater variation with direction at higher frequencies. Directivity due to the size of the mouth opening appeared to enter above 5600 cycles per second, the axis at these frequencies being about 45° below the horizontal, in front.

Frequencies below 1000 cycles per second were found strongest directly downward from the lips, or nearly so. The power radiated in different directions has been calculated, and a summation gives a spectrum of the total speech power emitted by the mouth. It is proposed that similar spectra for other speakers may be obtained from pressure measurements at a single point, using the relations discovered for this speaker. The necessity for protecting a microphone used close to the mouth, from the puffs of air accompanying the speech, is demonstrated and explained.

A Tubular Directional Microphone.[2] W. P. MASON and R. N. MARSHALL. A tubular directional microphone is described which consists of a pressure type microphone coupled to an acoustic impedance element composed of a large number of tubes whose lengths vary by equal increments. The function of this variation in length is two-fold. First, the multiple resonances of the individual tubes occur at intervals so close together that the net effect of the bundle is that of an acoustic resistance over a fairly wide frequency range and so does not impair the high quality of the attached microphone. Second, high directivity is secured, because for sound incidence other than normal each tube introduces a different path length with phase cancellation resulting in a composition chamber between the microphone and the ends of the tubes. The theory of operation is summar-

[1] *Jour. Acous. Soc. Amer.*, January 1939.
[2] *Jour. Acous. Soc. Amer.*, January 1939.

ized and data are presented to show the performance of the instrument which is in fair agreement with the theory.

Peak Field Strength of Atmospherics Due to Local Thunderstorms at 150 Megacycles.[3] J. P. SCHAFER and W. M. GOODALL. Atmospherics in the 150-megacycle frequency range were investigated with a broad-band receiver and cathode-ray-tube scanning technique. The results are of general interest in connection with the problems of atmospheric noise interference on various types of ultra-short-wave radio-communication channels. Some of the conclusions are:

(1) The peak intensity of disturbances varies 20 decibels between different storms at the same distance. (2) The inverse distance relation is a good approximation for the calculation of the variation of peak disturbance with distance, for any distance and height of receiving antenna likely to be used in a commercial system. (3) The use of high instead of low receiving antennas increases the signal-to-disturbance ratio almost directly with height for storms within 10 miles. (4) The durations of some of the narrower peaks in any particular lightning discharge are at least as short as a few microseconds. (5) The maximum peak field strength of disturbances for a storm one mile distant is 85 decibels and for a storm ten miles distant is 65 decibels above 1 microvolt per meter at a frequency of 150 megacycles with a band width of 1.5 megacycles.

The technique of observations provided a visual indication of the noise interference which might be expected with television signals. It appears that with signal field strengths, such as might reasonably be expected, atmospherics due to thunderstorms will be noticeable for ultra-short-wave television transmission at times when storms are in progress near the point of reception.

Metal Horns as Directive Receivers of Ultra-Short Waves.[4] G. C. SOUTHWORTH and A. P. KING. The paper describes some experiments made to determine the directive properties of metal pipes and horns when used as receivers of electromagnetic waves. The experiments were of two kinds. One consisted of measurements of received power, with and without the horn in place, and the other of the determination of the directional patterns of the horns in two perpendicular planes. The results indicate that electromagnetic horns of this kind provide a simple and convenient way of obtaining effective power ratios of a hundred or more (20 decibels). The effects of varying the several horn parameters are investigated. It is shown that there is an

[3] *Proc. I. R. E.*, March 1939.
[4] *Proc. I. R. E.*, February 1939.

optimum angle of flare. The possibility of forming arrays of pipes or horns is mentioned.

Hindered Molecular Rotation and the Dielectric Behavior of Condensed Phases.[5] ADDISON H. WHITE. The polarizability of a liquid or a collection of randomly oriented single crystals in which polar molecules are unable to move except to rotate from one to the other of two equilibrium orientations separated by an angle β and of potential energies whose difference is E, is

$$\alpha = (\mu^2/6kT)(1 - \cos\beta)/\cosh^2 E/2kT,$$

where μ is dipole moment. This model accounts for the reduction of α in solids and liquids from the value $\mu^2/3kT$ observed in gases, and at the same time provides for anomalous dispersion in terms of discontinuous molecular processes.

[5] *Jour. Chemical Physics*, January 1939.

Contributors to this Issue

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A. E. Bowen, Ph.B., Yale University, 1921. Graduate School, Yale University, 1921–24. American Telephone and Telegraph Company, Department of Development and Research, 1924–34. Bell Telephone Laboratories, 1934–. With the American Telephone and Telegraph Company, Mr. Bowen's work was concerned principally with the inductive coordination of power and communication systems. Since 1934 he has been engaged in work in the ultra-high-frequency field, particularly on hollow wave guides.

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Frederick B. Llewellyn, M.E., Stevens Institute of Technology, 1922; Ph.D., Columbia University, 1928. Western Electric Company, 1923–25; Bell Telephone Laboratories, 1925–. Dr. Llewellyn has been engaged in the investigation of special problems connected with high-frequency circuits and vacuum tubes. In his present capacity as Circuit Research Engineer he directs a group in the study of amplifying problems.

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John Riordan, B.S., Sheffield Scientific School, Yale University, 1923. American Telephone and Telegraph Company, Department of Development and Research, 1926–34; Bell Telephone Laboratories, 1934–. Mr. Riordan's work has been mainly on problems associated with inductive effects of electrified railways.

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L. C. Starbird, B.E.E., University of Arkansas, 1921; Instructor, University of Arkansas, 1921–25. Southwestern Bell Telephone Company, 1925–; Equipment Engineer, 1926–32, Transmission and Protection Engineer, Texas Area, 1932–. Mr. Starbird's work has been largely in the application of carrier and repeater equipment.

A. L. Whitman, Harvard University, A.B., 1918; B.S. in Electrical Engineering, 1920. Harvard University Sheldon Fellowship for traveling study in Europe, 1920–21. American Telephone and Telegraph Company, Department of Development and Research, Inductive Interference and Noise Prevention Group, 1921–34. Member of Technical Staff, Transmission Development Department of Bell Telephone Laboratories, 1934–. Mr. Whitman is now engaged in field studies of noise and crosstalk as related to carrier-telephone transmission systems utilizing a broad band of frequencies.