# The Impedance Concept and Its Application to Problems of Reflection, Refraction, Shielding and Power Absorption

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This paper calls attention to the practical value of a more extended use of the impedance concept. It brings out a certain underlying unity in what otherwise appear diverse physical phenomena. Although an attempt has been made to trace the history of the concept of "impedance" and many interesting early suggestions have been found, reference to these lies beyond the scope of this paper. Apparently, Sir Oliver Lodge was the first to use the word "impedance," but the concept has been developed gradually as circumstances demanded through the efforts of countless workers.

The main body of the paper is divided into three parts: Part I, dealing with the exposition of the impedance idea as applied to different types of physical phenomena; Part II, in which the general formulæ are deduced for reflection and transmission coefficients; Part III, presenting some special applications illustrating the practical utility of the foregoing manner of thought.

THE term "impedance" has had an interesting history, in which one generalization has suggested another with remarkable rapidity. Introduced by Oliver Lodge,[1] it meant the ratio $V/I$ in the special circuit comprised of a resistance and an inductance, $I$ and $V$ being the *amplitudes* of an alternating current and the driving force which produced it. This was soon extended to the somewhat more general circuit consisting of a resistance, an inductance coil and a condenser.[2] The usage did not develop much further until the use of

[1] Dr. Oliver Lodge, F.R.S., "On Lightning, Lightning Conductors, and Lightning Protectors," *Electrical Review*, May 3, 1889, p. 518

[2] It is interesting to note that the first impulse was to introduce a new word rather than to extend the meaning of the old term. Thus in 1892, F. Bedell and A. Crehore write as follows: "From the analogy of this equation to Ohm's law, we see that the expression $\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}$ is of the nature of a resistance, and is the apparent resistance of a circuit containing resistance, self-inductance and capacity. This expression would quite properly be called 'impedance' but the term impedance has for several years been used as a name for the expression $\sqrt{R^2 + L^2\omega^2}$, which is the apparent resistance of a circuit containing resistance and self-inductance only. We would suggest, therefore, that the word 'impediment' be adopted as a name for the expression $\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}$ which is the apparent resistance of a circuit containing resistance, self-induction and capacity, and the term impedance be retained in the more limited meaning it has come to have, that is $\sqrt{R^2 + L^2\omega^2}$, the

complex quantities, which had begun early in the nineteenth century among mathematicians, was popularized among engineers by Kennelly and Steinmetz. Then the proportionality relation $V = ZI$, which had previously been true only if $V$ and $I$ were interpreted as amplitudes, acquired a more general significance, for it was found that this relation could express the phase relationship as well, provided $Z$ was given a suitable complex value.

An important generalization came when the close similarity of the laws connecting $V$ and $I$ in an electric circuit to those governing force and velocity in mechanical systems suggested that the ratio "force/velocity" be called a "mechanical impedance." This usage is now well nigh universal.

The next step was a short one: it amounted to extending the term to include also the ratio "force per unit area/flow per unit area"; that is, "pressure/flux." This usage is well known in such fields as acoustics, but it has not penetrated as far into the electrical field as convenience seems to warrant.

If we read these remarks with a view to appraising the direction in which future growth might be expected, we are immediately impressed by the strong trend toward interpreting the ratio "force/velocity" in an ever widening sense. It is my purpose in the present paper to indicate some further extensions which I have found to be useful. They are founded upon five basic ideas. The first is to recognize and use whenever possible analogies between dynamical fields in which the impedance concept is common and others (heat, for instance) in which it is not. The second is the idea of extending the $V/I$ relation from circuits to radiation fields, in much the same way that the "force/velocity" concept has been made to embrace "pressure/flux" in hydrodynamics. The third is, to regard the impedance as an attribute of the *field* as well as of the body or the medium which supports the field, so that the impedance to a plane wave is not the same as the impedance to a cylindrical wave, even when both are propagated in infinite "free space." The fourth basic idea is that of assigning *direction* to the impedances of fields. This does not mean, however, that the impedances are vectors; in fact, they are not, since they fail to obey the laws of addition and the laws of transformation peculiar to vectors. And finally the fifth is a generalization of the idea of a *one-dimensional transmission line* or simply a *transmission line*. While

apparent resistance of a circuit containing resistance and self-induction only." Frederick Bedell and Albert C. Crehore, "Derivation and Discussion of the General Solution of the Current Flowing in a Circuit Containing Resistance, Self-Induction and Capacity, With Any Impressed Electromotive Force," *Journal A. I. E. E.*, Vol. IX, 1892, pp. 303–374, see p. 340.

all physical phenomena are essentially three-dimensional, frequently all but one are irrelevant and can be ignored or are relatively unimportant and can be neglected. In the mathematical language, this means that only one coordinate (distance, angle, etc.) is retained explicitly in the *equations of transmission.*

The paper is divided into three parts. Part I discusses broadly the ratios to which the term "impedance" can appropriately be applied in a wide variety of physical fields, ranging from electric circuits and heat conduction to electromagnetic radiation. In this part the concept is gradually broadened until at the end it has acquired the property of direction mentioned above. Parts II and III consider the general laws governing reflection, refraction, shielding and power absorption, and rephrase them as theorems regarding the generalized impedances. To make the illustrations more effective, familiar examples are chosen.

## PART I

## THE IMPEDANCE CONCEPT

### Electric Circuits

In an electric circuit comprised of a resistance $R$ and an inductance $L$, the instantaneous voltage-current relation is described by the following differential equation

$$L\frac{dI_0}{dt} + RI_0 = V_0, \tag{1}$$

where $V_0$ is the applied electromotive force. If $V_0$ varies harmonically with frequency $f$, ultimately $I_0$ will also vary harmonically with frequency $f$. What happens is that the solution of (1) consists of two parts, the *transient* part and the *steady state* part, the former decreasing exponentially with time and the latter being periodic.

The steady state solution of (1), or indeed of the most general linear differential equation with constant coefficients, can be found by means of a simple mathematical device based upon the use of complex numbers. Thus if $V_0$ and $I_0$ vary harmonically, they may be regarded as real parts of the corresponding complex expressions $Ve^{i\omega t}$ and $Ie^{i\omega t}$, where $f = \omega/2\pi$ is the frequency. The quantities $V$ and $I$ are complex numbers whose moduli represent the amplitudes and whose phases are the initial phases (at the instant $t = 0$) of the electromotive force and the electric current. The time rate of change of $I_0$ is then the real part of the derivative of $Ie^{i\omega t}$, that is, the real part of $i\omega Ie^{i\omega t}$.

If we form another equation after the pattern of (1), replacing $I_0$ and $V_0$ by the *imaginary* part of $Ie^{i\omega t}$ and $Ve^{i\omega t}$, and add the new equation to (1), we shall have

$$L\frac{d(Ie^{i\omega t})}{dt} + R(Ie^{i\omega t}) = V_0 e^{i\omega t}.$$

Differentiating and cancelling the time factor $e^{i\omega t}$, we obtain

$$(R + i\omega L)I = V.$$

The ratio $Z = V/I = Ve^{i\omega t}/Ie^{i\omega t}$ is called the *impedance* of the electric circuit. In the present instance

$$Z = R + i\omega L.$$

In general, the impedance $Z = R + iX$ has a real and an imaginary part, the former being the *resistive* component of the impedance and the latter the *reactive.*

## Mechanical Circuits

Linear oscillations of a mass in a resisting medium are described by equations identical with (1) and (2) except for the customary difference in lettering

$$m\frac{d(ve^{i\omega t})}{dt} + r(ve^{i\omega t}) = Fe^{i\omega t}.$$

In this equation, $v$ represents the velocity and $F$ the applied force, $m$ the mass and $r$ the resistance coefficient. The mechanical impedance is then

$$Z = r + i\omega m.$$

Similarly, for torsional vibrations the impedance is defined as the ratio "torque/angular velocity."

## Electric Waves in Transmission Lines

Let $x$ be the distance coordinate specifying a typical section of an electric transmission line. Let the complex quantities $V$ and $I$ be the voltage across and the electric current in the transmission line.[3] Then the space rate of change of the voltage is proportional to the current and the space rate of change of the current is proportional to the voltage

$$\frac{dV}{dx} = -ZI, \quad \frac{dI}{dx} = -YV. \tag{2}$$

[3] The time factor $e^{i\omega t}$ is usually implicit.

The coefficients of proportionality $Z$ and $Y$ are known as the distributed *series impedance* and the distributed *shunt admittance* of the line; they depend upon the distributed series resistance $R$, shunt conductance $G$, series inductance $L$ and shunt capacity $C$ in the following manner:

$$Z = R + i\omega L, \qquad Y = G + i\omega C. \tag{3}$$

In a generalized transmission line $Z$ and $Y$ may be functions of $x$ and may depend upon $\omega$ in a more complicated manner than that suggested in (3).

If $Z$ and $Y$ are independent of $x$, (2) possesses two exponential solutions:

$$\begin{aligned} I^+ &= Ae^{-\Gamma x + i\omega t}, & V^+ &= Z_0 I^+; \\ I^- &= Be^{\Gamma x + i\omega t}, & V^- &= -Z_0 I^-; \end{aligned} \tag{4}$$

where

$$\Gamma = \alpha + i\beta = \sqrt{ZY}, \qquad Z_0 = \sqrt{\frac{Z}{Y}} = \frac{\Gamma}{Y} = \frac{Z}{\Gamma}.$$

It is customary to designate by $\Gamma$ that value of the square root which is in the first quadrant of the complex plane or on its boundaries; the other value of the square root is $-\Gamma$.

The two "secondary" constants $\Gamma$ and $Z_0$ are called, respectively, the *propagation constant* and the *characteristic impedance*. The real part $\alpha$ of the propagation constant is the *attenuation constant* and $\beta$ is the *phase constant*.

Equations (4) represent *progressive* waves because an observer moving along the line with a certain finite velocity beholds an unchanging phase of $V$ and $I$. This velocity $c$ is called the *phase velocity* of the wave. Setting $x = ct$ in the upper pair of (4), we obtain the condition for the stationary phase

$$-\beta c + \omega = 0, \qquad c = \frac{\omega}{\beta}.$$

Hence, $V^+$ and $I^+$ represent a wave traveling in the positive $x$-direction. Similarly we find that $V^-$ and $I^-$ represent a wave traveling in the opposite direction.

Consider two points in which the phases of $V$ and $I$ differ by $2\pi$ when observed at the same instant; the distance $\lambda$ between these points is called the wave-length. By definition

$$\beta\lambda = 2\pi, \qquad \lambda = \frac{2\pi}{\beta}.$$

If the transmission line is non-uniform, that is, if $Z$ and $Y$ are functions of $x$, then the solutions of (2) are usually more complicated. In any case, however, there are two linearly independent solutions $I^+(x)$ and $I^-(x)$ in terms of which the most general solution can always be expressed

$$I(x) = AI^+(x) + BI^-(x).$$

These independent solutions may represent either progressive waves in two opposite directions or certain convenient combinations of such waves.

The corresponding $V$-functions are found by differentiation from (2); thus

$$V^+(x) = -\frac{1}{Y}\frac{dI^+}{dx}, \qquad V^-(x) = -\frac{1}{Y}\frac{dI^-}{dx}.$$

The *impedance* of the $V^+$, $I^+$-wave is then

$$Z_0^+(x) = \frac{V^+(x)}{I^+(x)} = -\frac{1}{YI^+}\frac{dI^+}{dx} = -\frac{1}{Y}\frac{d}{dx}(\log I^+).$$

Similarly the impedance of the $V^-$, $I^-$-wave is

$$Z_0^-(x) = -\frac{V^-(x)}{I^-(x)} = \frac{1}{YI^-}\frac{dI^-}{dx} = \frac{1}{Y}\frac{d}{dx}(\log I^-). \tag{5}$$

The negative sign in (5) is merely a matter of convention: the "positive" and the "negative" directions of the transmission line are so defined that the real parts of $Z_0^+$ and $Z_0^-$ are positive.

In general, $Z_0^+$ and $Z_0^-$ are not equal to each other. Moreover, there is a considerable amount of arbitrariness in our choice of the basic solutions $I^+$ and $I^-$. Thus, we are brought face to face with the fact that we must regard the impedance as an attribute of the wave as well as of the transmission line. This point of view will become even more prominent when we come to deal with the wave transmission in three-dimensional media. There even progressive waves may have different characters (they may be plane, cylindrical, spherical, etc.) and the impedances of the *same* medium to these waves will be different. And naturally, it goes without saying that the impedances to like waves in *different* media may also be different. One could, perhaps, take the position that geometrically similar waves in different media are not really alike if the corresponding "force/velocity" ratios are not equal and that under all circumstances the "impedance" is the property of a wave. However, "intrinsic impedance" will be used to designate a constant of the medium without reference to any particular wave.

## Vibrating Strings

In strings under constant tension $\tau$, simply periodic waves may be described by the following two equations:

$$\frac{dF}{dx} = -(r + i\omega m)v, \qquad \frac{dv}{dx} = -\frac{i\omega}{\tau}F,$$

where $m$ is the mass and $r$ the resistance per unit length of the string. The variable $F$ represents the force on a typical point of the string at right angles to the string and $v$ is the velocity at that point.

Hence the characteristic impedance and the propagation constant are given by

$$Z_0 = \sqrt{\frac{(r + i\omega m)\tau}{i\omega}}, \qquad \Gamma = \sqrt{(r + i\omega m)\frac{i\omega}{\tau}}.$$

In the non-dissipative case we have simply

$$Z_0 = \sqrt{m\tau}, \qquad \Gamma = i\omega\sqrt{\frac{m}{\tau}}.$$

## Heat Waves

Transmission of heat waves is also a special case of the generalized transmission line theory. In the one-dimensional case we have

$$\frac{\partial T}{\partial x} = -\frac{v}{K}, \qquad \frac{\partial v}{\partial x} = -c\delta\frac{\partial T}{\partial t},$$

where: $T$ is the temperature, $v$ the rate of heat flow, $K$ the thermal conductivity, $\delta$ the density and $c$ the specific heat. For simply periodic waves, we obtain

$$\frac{dT}{dx} = -\frac{1}{K}v, \qquad \frac{dv}{dx} = -i\omega c\delta\, T.$$

Thus the characteristic impedance and the propagation constant of heat waves are

$$Z_0 = \frac{1}{\sqrt{i\omega c\delta K}}, \qquad \Gamma = \sqrt{\frac{i\omega c\delta}{K}}.$$

The ratio "the temperature of the source/the rate of heat flow from the source" is the impedance "seen" by the heat source.

## Electromagnetic Waves

The transmission equations of *uniform linearly polarized*[4] plane waves are:

$$\frac{dE}{dx} = -\, i\omega\mu H, \qquad \frac{dH}{dx} = -\,(g + i\omega\epsilon)E,$$

where: $E$ is the electric intensity, $H$ the magnetic intensity, and $g$, $\epsilon$, $\mu$ are, respectively, the conductivity, the dielectric constant and permeability of the medium. These equations are of the same form as (2). Even the physical meanings of $E$ and $H$ are closely related to those of $V$ and $I$; thus $E$ is $V$ per unit length and $H$ is $I$ per unit length.

The propagation constant and the characteristic impedance of an unbounded medium to linearly polarized plane waves are:

$$\sigma = \sqrt{i\omega\mu(g + i\omega\epsilon)}, \qquad \eta = \sqrt{\frac{i\omega\mu}{g + i\omega\epsilon}} = \frac{\sigma}{g + i\omega\epsilon} = \frac{i\omega\mu}{\sigma}.$$

These constants are so directly related to the fundamental electromagnetic constants of the medium that they themselves may be regarded as fundamental constants. On this account, we call $\sigma$ and $\eta$, respectively, the *intrinsic propagation constant* and the *intrinsic impedance of the medium*. The intrinsic impedance will frequently occur as a multiplier in the expressions for the impedances of various types of waves.

The intrinsic impedance of a non-dissipative medium is simply $\eta = \sqrt{\mu/\epsilon}$; in air, this is equal to $120\pi$ or approximately 377 ohms.* Thus in the uniform linearly polarized plane wave traveling in free space, the relation between $E$ and $H$ is

$$E = 120\pi H \quad \text{or} \quad E \doteq 377H,$$

provided the positive directions of $E$ and $H$ are properly chosen.

An electromagnetic field of general character can be described by means of three electric components $E_x$, $E_y$, $E_z$, and three magnetic components $H_x$, $H_y$, $H_z$. We can form the following matrix whose components can be regarded as impedances:

[4] In this connection the word "uniform" is used to mean that equiphase planes are also equi-amplitude planes.

* See the letter from G. A. Campbell to Dean Harold Pender reproduced at the end of this paper.

$$\left\| \begin{array}{ccc} \dfrac{E_x}{H_x}, & \dfrac{E_x}{H_y}, & -\dfrac{E_x}{H_z} \\ -\dfrac{E_y}{H_x}, & \dfrac{E_y}{H_y}, & \dfrac{E_y}{H_z} \\ \dfrac{E_z}{H_x}, & -\dfrac{E_z}{H_y}, & \dfrac{E_z}{H_z} \end{array} \right\|$$

The algebraic signs preceding the ratios of components with different subscripts are assigned as follows. If a right-hand screw is rotated through 90° from the positive axis indicated by the subscript in the numerator toward the positive axis indicated by the subscript in the denominator, it will advance either in the positive or in the negative direction of the remaining axis. In the former case the ratio is given the positive sign and in the latter the negative sign. This convention happens to be particularly convenient in expressions for the Poynting vector.

Thus two impedances are associated with any pair of perpendicular directions, the $x$-axis and the $y$-axis, let us say; these impedances are:

$$Z_{xy} = \frac{E_x}{H_y}, \qquad Z_{yx} = -\frac{E_y}{H_x}.$$

If these two impedances are equal, then we define the *impedance in the direction of the positive z-axis* as follows:

$$Z_z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}.$$

Similar definitions hold for the impedances in other directions.

While the impedances as now defined possess an attribute of direction, they are neither vectors nor tensors because they do not add in the proper fashion. However, in practical applications this lack of vectorial properties does not seem to be a drawback.

The above definitions can be extended to other systems of coordinates. Let $r$ be the distance of a point $P$ $(r, \theta, \varphi)$ from the origin of the spherical coordinate system, $\theta$ the polar angle or colatitude and $\varphi$ the meridian angle or the longitude (Fig. 1). Then the "radial" impedance in the outward direction is defined as

$$Z_r = \frac{E_\theta}{H_\varphi} = -\frac{E_\varphi}{H_\theta}, \tag{6}$$

provided the two ratios of the field components are equal. The radial impedance looking *toward* the origin is defined as the negative of (6). Similarly the "meridian" impedance in the direction of increasing $\theta$

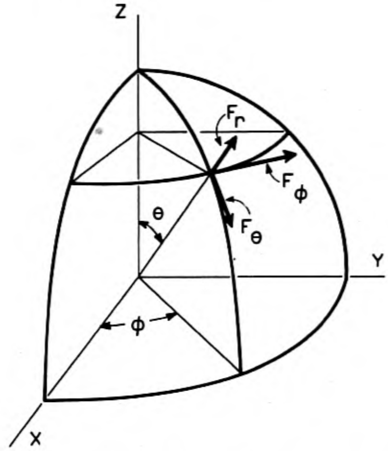


Fig. 1—Spherical coordinates. The positive directions of $r$-, $\theta$-, and $\varphi$-components of a vector are, respectively, the directions of increasing $r$, $\theta$, and $\varphi$.

and the impedance in the direction of increasing $\varphi$ are:

$$Z_\theta = -\frac{E_r}{H_\varphi} = \frac{E_\varphi}{H_r}, \qquad Z_\varphi = \frac{E_r}{H_\theta} = -\frac{E_\theta}{H_r}.$$

In cylindrical coordinates we have (Fig. 2):

$$Z_\rho = -\frac{E_z}{H_\varphi} = \frac{E_\varphi}{H_z}, \qquad Z_\varphi = \frac{E_z}{H_\rho} = -\frac{E_\rho}{H_z}, \qquad Z_z = \frac{E_\rho}{H_\varphi} = -\frac{E_\varphi}{H_\rho}.$$

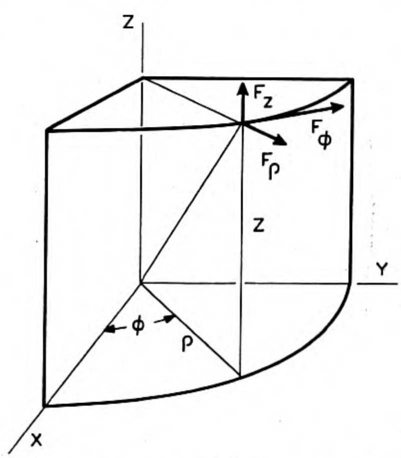


Fig. 2—Cylindrical coordinates. The positive directions of $\rho$-, $\varphi$-, and $z$-components of a vector are, respectively, the directions of increasing $\rho$, $\varphi$, and $z$.

Usually it is only one of the entire set of three-dimensional impedances that is of particular importance, the preferred direction being frequently the direction of the wave under consideration. When the ratios involved in the above definitions are unequal, it is expedient to resolve the field into component fields for which the ratios are equal. We shall now consider some special examples.

The field of the spherical electromagnetic wave emitted by a Hertzian doublet is known to be

$$
\begin{aligned}
E_\theta^+ &= \frac{i\omega\mu Il e^{-\sigma r}}{4\pi r}\left(1 + \frac{1}{\sigma r} + \frac{1}{\sigma^2 r^2}\right)\sin\theta,\\
E_r^+ &= \frac{\eta Il e^{-\sigma r}}{2\pi r^2}\left(1 + \frac{1}{\sigma r}\right)\cos\theta, \qquad\qquad (7)\\
H_\varphi^+ &= \frac{\sigma Il e^{-\sigma r}}{4\pi r}\left(1 + \frac{1}{\sigma r}\right)\sin\theta,
\end{aligned}
$$

where: $Il$ is the moment of the doublet in ampere-meters, $r$ is the distance from the doublet, $\theta$ the angle made by a typical direction in space with the axis of the doublet, and $\varphi$ is the angle between two planes containing the doublet, one of which is kept fixed for reference. The radial impedance of this wave is

$$Z_r^+ = \frac{E_\theta}{H_\varphi} = \eta\,\frac{1 + \dfrac{1}{\sigma r} + \dfrac{1}{\sigma^2 r^2}}{1 + \dfrac{1}{\sigma r}}.$$

In a non-dissipative medium this becomes

$$Z_r^+ = \eta\,\frac{1 + \dfrac{1}{i\beta r} - \dfrac{1}{\beta^2 r^2}}{1 + \dfrac{1}{i\beta r}}.$$

At a distance large compared with the wave-length, the radial impedance to the spherical wave emitted by a doublet is substantially equal to the intrinsic impedance of the medium. Very close to the doublet (compared with the wave-length) the radial impedance is substantially a capacitive reactance; in fact, we have approximately $Z_0^+ = 1/i\omega\epsilon r$.

Reversing the sign of $\sigma$ in (7), we obtain a spherical wave traveling toward the origin. At first sight, this *inward bound* wave appears to be the natural mate to the outward bound wave. Two such waves move in opposite directions in the same sense in which two plane waves spread out from a plane source in an infinite homogeneous space. However, the analogy is not complete. The inward bound spherical wave cannot exist without an appropriate receiver of energy at the origin. In absence of such a receiver, the energy condenses at the origin and spreads outward again. The result of interference between two such progressive waves will be called the "internal" spherical wave.[5] It is natural to regard a thin spherical source in an infinite *homogeneous* medium as an analogue of a thin plane source and to consider the waves *on the two sides* of such a spherical source as the mates. In accordance with this idea the (+) and the (−) signs are used to distinguish between the waves produced by a source on its two sides rather than to indicate "progressive" waves moving in opposite directions. This attitude is not only a possible and a natural attitude but almost a necessary one in view of the fact that no generally applicable criterion is known by which "progressive" waves could be identified in any particular case. As often happens, in simple situations there is no need for arguing as to which attitude is the more proper one; thus the waves on the two sides of a plane source in an infinite homogeneous medium are two progressive waves moving in opposite directions.

The field of the internal spherical wave is

$$E_\theta^- = \frac{i\omega\mu A}{2\pi r}\left(\sinh \sigma r - \frac{\cosh \sigma r}{\sigma r} + \frac{\sinh \sigma r}{\sigma^2 r^2}\right)\sin\theta,$$

$$E_r^- = \frac{\eta A}{\pi r^2}\left(\frac{\sinh \sigma r}{\sigma r} - \cosh \sigma r\right)\cos\theta,$$

$$H_\varphi^- = \frac{\sigma A}{2\pi r}\left(\frac{\sinh \sigma r}{\sigma r} - \cosh \sigma r\right)\sin\theta.$$

The corresponding impedance is then

$$Z_r^- = -\frac{E_\theta^-}{H_\varphi^-} = \eta\,\frac{\sinh \sigma r - \dfrac{\cosh \sigma r}{\sigma r} + \dfrac{\sinh \sigma r}{\sigma^2 r^2}}{\cosh \sigma r - \dfrac{\sinh \sigma r}{\sigma r}}.$$

Close to the origin we have approximately

$$Z_r^- = \frac{2}{(g + i\omega\epsilon) r}.$$

[5] If the medium is non-dissipative, this wave is a *standing* wave; but, in general, it is simply a combination of two progressive waves in such proportions that the field is finite at the origin.

If the source of electromagnetic waves is a small coil rather than a small doublet, the field is

$$
\begin{aligned}
E_\varphi^+ &= -\frac{\eta\sigma^2 SIe^{-\sigma r}}{4\pi r}\left(1+\frac{1}{\sigma r}\right)\sin\theta,\\
H_\theta^+ &= \frac{\sigma^2 SIe^{-\sigma r}}{4\pi r}\left(1+\frac{1}{\sigma r}+\frac{1}{\sigma^2 r^2}\right)\sin\theta, \qquad (8)\\
H_r^+ &= \frac{\sigma SIe^{-\sigma r}}{2\pi r^2}\left(1+\frac{1}{\sigma r}\right)\cos\theta.
\end{aligned}
$$

In this equation $I$ is the current in the loop and $S$ is the area. The corresponding radial impedance is then:

$$
Z_r^+ = -\frac{E_\varphi^+}{H_\theta^+} = \eta\,\frac{1+\dfrac{1}{\sigma r}}{1+\dfrac{1}{\sigma r}+\dfrac{1}{\sigma^2 r^2}}.
$$

This impedance approaches $\eta$ as $r$ increases indefinitely. Close to the loop we have approximately

$$
Z_r^+ = i\omega\mu r. \qquad (9)
$$

The field of the internal wave having the same type of amplitude distribution over equiphase surfaces as the diverging wave (8) is

$$
\begin{aligned}
E_\varphi^- &= \frac{\eta\sigma^2 A}{2\pi r}\left(\cosh\sigma r-\frac{\sinh\sigma r}{\sigma r}\right)\sin\theta,\\
H_\theta^- &= \frac{\sigma^2 A}{2\pi r}\left(\sinh\sigma r-\frac{\cosh\sigma r}{\sigma r}+\frac{\sinh\sigma r}{\sigma^2 r^2}\right)\sin\theta,\\
H_r^- &= \frac{\sigma A}{\pi r^2}\left(\frac{\sinh\sigma r}{\sigma r}-\cosh\sigma r\right)\cos\theta.
\end{aligned}
$$

The radial impedance to this wave is then

$$
Z_r^- = \frac{E_\varphi^-}{H_\theta^-} = \eta\,\frac{\cosh\sigma r-\dfrac{\sinh\sigma r}{\sigma r}}{\sinh\sigma r-\dfrac{\cosh\sigma r}{\sigma r}+\dfrac{\sinh\sigma r}{\sigma^2 r^2}}.
$$

Close to the origin we have approximately

$$
Z_r^- = \tfrac{1}{2}i\omega\mu r. \qquad (10)
$$

A line doublet formed by two parallel electric current filaments produces a cylindrical wave. Close to the doublet (compared with the wave-length) we have

$$H_\varphi{}^+ = \frac{Il}{2\pi\rho^2}\cos\varphi, \qquad H_\rho{}^+ = -\frac{Il}{2\pi\rho^2}\sin\varphi. \tag{11}$$

In this equation $Il$ is the moment of the doublet per unit length, $I$ being the current and $l$ the distance between the filaments. These equations are well known in the elementary theory of electromagnetism. The electric field is obtainable from (11) with the aid of Faraday's law of electromagnetic induction. This field and the corresponding radial impedance are

$$E_z{}^+ = -\frac{i\omega\mu Il}{2\pi\rho}\cos\varphi, \qquad Z_\rho{}^+ = i\omega\mu\rho. \tag{12}$$

The exact field of the line doublet and the corresponding radial impedance are:

$$E_z{}^+ = -\frac{\eta\sigma^2 Il}{2\pi}K_1(\sigma\rho)\cos\varphi, \qquad H_\varphi{}^+ = -\frac{\sigma^2 Il}{2\pi}K_1'(\sigma\rho)\cos\varphi,$$

$$H_\rho{}^+ = -\frac{\sigma Il}{2\pi\rho}K_1(\sigma\rho)\sin\varphi, \qquad Z_\rho{}^+ = -\eta\frac{K_1(\sigma\rho)}{K_1'(\sigma\rho)}.$$

The internal cylindrical wave with the same relative amplitude distribution over equiphase surfaces as in the wave originated by the line doublet is [6]

$$E_z{}^- = i\omega\mu A\, I_1(\sigma\rho)\cos\varphi, \qquad H_\varphi{}^- = \sigma A\, I_1'(\sigma\rho)\cos\varphi,$$

$$H_\rho{}^- = \frac{A}{\rho}I_1(\sigma\rho)\sin\varphi, \qquad Z_\rho{}^- = \eta\frac{I_1(\sigma\rho)}{I_1'(\sigma\rho)}.$$

Close to the doublet we have approximately

$$E_z{}^- = i\omega\mu P\rho\cos\varphi, \qquad H_\varphi{}^- = P\cos\varphi,$$
$$H_\rho{}^- = P\sin\varphi, \qquad Z_\rho{}^- = i\omega\mu\rho.$$

Another familiar field is that produced by two parallel line charges in a perfect dielectric. Close to the doublet this field is

$$E_\varphi{}^+ = \frac{ql\sin\varphi}{2\pi\epsilon\rho^2}, \qquad E_\rho{}^+ = \frac{ql\cos\varphi}{2\pi\epsilon\rho^2},$$
$$H_z{}^+ = \frac{i\omega ql\sin\varphi}{2\pi\rho}, \qquad Z_\rho{}^+ = \frac{1}{i\omega\epsilon\rho}, \tag{13}$$

[6] The symbols $I_n(x)$ and $K_n(x)$ designate the modified Bessel functions as defined in G. N. Watson's "Bessel Functions."

$ql$ being the moment of the doublet. The last equation is obtained from the first two with the aid of Ampére's law. The exact expressions for any medium are

$$E_\varphi^+ = -\frac{i\omega q l \sigma \eta}{2\pi} K_1'(\sigma\rho) \sin\varphi, \qquad E_\rho^+ = \frac{i\omega q l \eta}{2\pi\rho} K_1(\sigma\rho) \cos\varphi,$$

$$H_z^+ = \frac{i\omega q l \sigma}{2\pi} K_1(\sigma\rho) \sin\varphi, \qquad Z_\rho^+ = -\eta \frac{K_1'(\sigma\rho)}{K_1(\sigma\rho)}.$$

For an internal cylindrical wave, we have

$$E_\varphi^- = A\sigma I_1'(\sigma\rho) \sin\varphi, \qquad E_\rho^- = -\frac{A}{\rho} I_1(\sigma\rho) \cos\varphi,$$

$$H_z^- = -A(g + i\omega\epsilon) I_1(\sigma\rho) \sin\varphi, \qquad Z_\rho^- = \eta \frac{I_1'(\sigma\rho)}{I_1(\sigma\rho)}.$$

Close to the doublet this becomes substantially

$$E_\varphi^- = P \sin\varphi, \qquad E_\rho^- = -P \cos\varphi,$$

$$H_z^- = -P(g + i\omega\epsilon)\rho \sin\varphi, \qquad Z_\rho^- = \frac{1}{(g + i\omega\epsilon)\rho}.$$

In concluding this set of examples we shall emphasize the fact that the impedance to a wave depends upon the particular manner in which the applied electromotive force is distributed in space, in very much the same way as it depends upon the manner of distribution of this force in time, that is, upon the frequency of the wave. Just as the impedance has a meaning only if the applied electromotive force varies harmonically with a certain well defined frequency,[7] there are definite types of applied force distribution in space for which the impedance has a meaning and other types for which it has not. Arbitrary spatial distributions of force may be decomposed into "space harmonics" in a manner analogous to Fourier's frequency analysis of arbitrary time distributions of force. This is just another way of interpreting the well-known method of solving Maxwell's equations with the aid of characteristic wave functions.

Here is a simple example of the dependence of the impedance to a wave upon the manner of applied force distribution. Consider the wave generated by an infinite electric current filament of radius $a$

[7] Strictly speaking, the impedance concept is applicable to any impressed force which varies exponentially with time, the exponent being in general a complex number. The only exceptions are the exponents which are either zeros or infinities of the impedance function. Undamped impressed forces constitute merely an important subclass of exponential forces.

when the electromotive force driving the current is distributed uniformly along the filament. In this case we have

$$E_z = -\frac{\eta I K_0(\sigma\rho)}{2\pi a K_1(\sigma a)}, \qquad H_\varphi = \frac{I K_1(\sigma\rho)}{2\pi a K_1(\sigma a)}, \qquad Z_\rho = \eta \frac{K_0(\sigma\rho)}{K_1(\sigma\rho)},$$

where $I$ is the current in the filament. On the other hand if the electromotive force is applied to the filament with a uniform progressive phase delay so that it varies along the filament as $e^{-ikz}$ for instance, then the field and the impedance are

$$E_z = -\frac{\eta I K_0(\Gamma\rho)}{2\pi a K_1(\Gamma a)} e^{-ikz}, \qquad H_\varphi = \frac{I K_1(\Gamma\rho)}{2\pi a K_1(\Gamma a)} e^{-ikz},$$

$$Z_\rho = \eta \frac{K_0(\Gamma\rho)}{K_1(\Gamma\rho)}, \qquad \Gamma = \sqrt{\sigma^2 + k^2}.$$

PART II

# REFLECTION, REFRACTION, SHIELDING AND POWER ABSORPTION—GENERAL FORMULÆ

## Uniform Transmission Lines

While the following discussion refers specifically to an electric transmission line, the results apply to all generalized transmission lines of which the former may be considered typical. These results depend upon certain boundary conditions and are not influenced by the names of the variables.

Consider a semi-infinite transmission line terminated by a prescribed impedance $Z_t$. Suppose that an "impressed" wave is coming from infinity. If $V_i$ and $I_i$ are the voltage and the current, their ratio must equal the characteristic impedance $Z_0$ of the wave. On the other hand, the ratio of the voltage across the impedance $Z_t$ to the current through it is $Z_t$ by definition. Thus, unless $Z_t$ is equal to $Z_0$, a "reflected" wave must originate at the terminal and travel backwards. Let $V_r$ and $I_r$ be the voltage and the current of the reflected wave at the terminal. The total values of the voltage and the current will be designated by $V_t$ and $I_t$. Then at the terminal

$$I_i + I_r = I_t, \qquad V_i + V_r = V_t. \tag{14}$$

By (9) and by the definition of $Z_t$, we have

$$V_i = Z_0 I_i, \qquad V_r = -Z_0 I_r, \qquad V_t = Z_t I_t. \tag{15}$$

Designating the ratio of the characteristic impedance of the line to the terminal impedance by $k$, we use (15) to rewrite (14) in the following form:

$$I_i + I_r = I_t, \qquad I_i - I_r = \frac{I_t}{k}, \qquad k = \frac{Z_0}{Z_t}.$$

Solving, we obtain the reflection and transmission coefficients:

$$\begin{aligned} R_I &= \frac{I_r}{I_i} = \frac{k-1}{k+1}, \qquad & R_V &= \frac{V_r}{V_i} = \frac{1-k}{1+k}, \\ T_I &= \frac{I_t}{I_i} = \frac{2k}{1+k}, \qquad & T_V &= \frac{V_t}{V_i} = \frac{2}{1+k}. \end{aligned} \tag{16}$$

Thus *when* $k = 1$, *that is when the terminal impedance equals the characteristic impedance, there is no reflection. When the ratio of the impedances is zero or infinity the reflection is complete: in the first case the current vanishes and the voltage is doubled, and in the second the current is doubled and the voltage vanishes. The amount of reflection is completely determined by the ratio of the impedances.*

The terminal impedance may be another semi-infinite transmission line and its characteristic impedance will play the part of $Z_t$. It is important to note that *neither the propagation constants nor the velocities of the wave in the lines have anything to do with reflection.* No reflection will take place if the lines have equal impedances and there will be reflection in the case of unequal impedances even if the velocities are the same.

The variables $V$ and $I$ can stand for any two physical quantities satisfying equations (2). It will be observed that if we disregard the physical significance of the variables $V$ and $I$, the characteristic impedance can be defined either as the ratio $V/I$ or as $I/V$. We are perfectly free to make our choice. It is evident from (16) that if we interchange $V$ and $I$ and replace $k$ by its reciprocal, the expressions for the reflection and transmission coefficients remain unaltered.

## Non-Uniform Transmission Lines

The foregoing analysis has to do only with *uniform* lines. In the case of non-uniform lines the impedances looking in the opposite directions may be different. These two impedances will be defined by the following equations:

$$Z_0^+ = \frac{V^+}{I^+}, \qquad Z_0^- = -\frac{V^-}{I^-},$$

where $V^+$, $I^+$ and $V^-$, $I^-$ refer to the two waves.

At the terminal we have as before:

$$I_i + I_r = I_t, \qquad Z_0^+ I_i - Z_0^- I_r = Z_t I_t.$$

Hence the more general expressions for the reflection and transmission coefficients are:

$$R_I = \frac{Z_0^+ - Z_t}{Z_0^- + Z_t}, \qquad R_V = -\frac{Z_0^-}{Z_0^+} R_I,$$

$$T_I = \frac{Z_0^- + Z_0^+}{Z_0^- + Z_t}, \qquad T_V = \frac{Z_t}{Z_0^+} T_I.$$

These reflection and transmission coefficients can be expressed in terms of the ratios of the line impedances to the terminal impedance.

## Shielding

When a source of electromagnetic waves is enclosed in a metallic box, the field outside the box is substantially weaker than it would have been in the absence of the box. The box is said to act as a "shield." Under some conditions, transmission of electromagnetic waves in free space and in the metallic shield is governed by equations of the form (1). In those cases the shielding effect can evidently be regarded as due to a reflection loss at the boundaries of the shield and to an attenuation loss in the shield itself. A schematic representation of a single layer shield is shown in Fig. 3. The source of disturbance

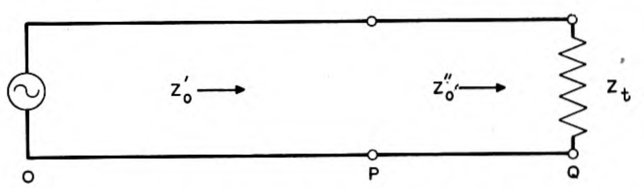


Fig. 3—Transmission line representation of a shield. The generator represents the source of the electromagnetic disturbance, the section $OP$ the space surrounding the source, the section $PQ$ the shield, and the impedance $Z_t$ the space outside the shield.

is shown as a generator, the space around this source is represented by a piece of a transmission line $OP$, the shield by a piece $PQ$ and the space outside the shield by the impedance $Z_t$.

The simplest case to consider is that of an *electrically thick* shield, in which the attenuation between $P$ and $Q$ is so great that waves reflected at $Q$ do not affect appreciably the situation at $P$. In such a case the impedance at $P$ looking toward $Q$ equals the characteristic impedance $Z_0''$ and the same is true of the impedance at $Q$ looking toward $P$. The effect of the inserted piece is comprised of two independent reflections at $P$ and $Q$ and of attenuation with concomitant phase change between $P$ and $Q$. Thus the transmission coefficients across $PQ$, that is, the ratios of the quantities at $Q$ to the impressed quantities at $P$, are

$$T_I = T_{I,P} T_{I,Q} e^{-\Gamma'' l}, \qquad T_V = T_{V,P} T_{V,Q} e^{-\Gamma'' l},$$

where $T_{I,P}$ is the transmission coefficient for $I$ at $P$ and the remaining $T$'s have similar meanings.

If $PQ$ is a piece of a uniform transmission line inserted into a uniform semi-infinite line, $Z_t = Z_0'$. In this case, we have

$$T_I = T_V = \frac{4k}{(k+1)^2} e^{-\Gamma'' l},$$

where $k$ is the ratio of the characteristic impedances. The factor $4k/(k+1)^2$ represents the reflection loss and $e^{-\alpha'' l}$ the attenuation loss.

Let us now assume that $PQ$ is electrically short and that all the transmission lines in question are uniform. By the transmission line theory, the ratios of the total currents and voltages at $P$ and $Q$ are:

$$\frac{I_Q}{I_P} = \frac{Z_0''}{Z_0'' \cosh \Gamma'' l + Z_t \sinh \Gamma'' l},$$

$$\frac{V_Q}{V_P} = \frac{Z_t}{Z_t \cosh \Gamma'' l + Z_0'' \sinh \Gamma'' l}.$$

On the other hand, we have

$$\frac{I_P}{I_i} = \frac{2Z_0'}{Z_0' + Z_P}, \qquad \frac{V_P}{V_i} = \frac{2Z_P}{Z_0' + Z_P},$$

where $Z_P$ is the impedance at $P$ looking toward $Q$

$$Z_P = Z_0'' \frac{Z_t \cosh \Gamma'' l + Z_0'' \sinh \Gamma'' l}{Z_0'' \cosh \Gamma'' l + Z_t \sinh \Gamma'' l}.$$

The transmission coefficients across $PQ$ can be represented as

$$T_I = \frac{I_Q}{I_i} = \frac{I_Q}{I_P} \cdot \frac{I_P}{I_i}, \qquad T_V = \frac{V_Q}{V_i} = \frac{V_Q}{V_P} \cdot \frac{V_P}{V_i}.$$

Making appropriate substitutions into this equation, we obtain

$$T_I = p(1 - qe^{-2\Gamma'' l})^{-1} e^{-\Gamma'' l}, \qquad T_V = \frac{Z_t}{Z_0'} T_I, \tag{17}$$

where

$$p = \frac{4Z_0'Z_0''}{(Z_0'' + Z_0')(Z_0'' + Z_t)},$$

$$q = \frac{(Z_0'' - Z_0')(Z_0'' - Z_t)}{(Z_0'' + Z_0')(Z_0'' + Z_t)}.$$

In the special case when $Z_t = Z_0'$, we have

$$p = \frac{4k}{(k+1)^2}, \qquad q = \left(\frac{k-1}{k+1}\right)^2, \qquad \text{and} \qquad T_V = T_I.$$

If $PQ$ is electrically long, (17) becomes simply

$$T_I = pe^{-\Gamma''l}, \qquad T_V = \frac{Z_t}{Z_0'} T_I.$$

An interesting physical interpretation of (17) will follow if we expand the factor in parentheses into a series

$$T_I = pe^{-\Gamma''l} + pqe^{-3\Gamma''l} + pq^2e^{-5\Gamma''l} + \cdots.$$

The first term represents what remains of the original wave on the first passage through $PQ$. A part of the original wave is reflected back at $Q$ and then partially re-reflected from $P$; the second term represents that fraction of the re-reflected wave which is transmitted beyond $Q$. The following terms represent succeeding reflections. In making this analysis, we must remember that $p = p_1p_2$ where $p_1$ and $p_2$ are respectively the transmission coefficients across the first and the second boundaries on the supposition that the inserted piece is infinitely long. Similarly, $q = q_1q_2$, the product of the two reflection coefficients.

Let us now consider a non-uniform transmission line. The propagation of a disturbance is no longer exponential and we introduce the ratios $\kappa^+ = V^+(x_2)/V^+(x_1)$ and $\kappa^- = V^-(x_1)/V^-(x_2)$ for the voltage ratios in the waves moving in opposite directions. In what follows $x_1$ and $x_2$ are the coordinates of the beginning and the end of the inserted piece. The transmission coefficient $T$ across the insertion, that is, the ratio of the total quantity at $x = x_2$ to the impressed quantity at $x = x_1$, is then

$$T = p_1\kappa^+p_2 + (p_1\kappa^+)(q_2\kappa^-q_1\kappa^+)p_2 + (p_1\kappa^+)(q_2\kappa^-q_1\kappa^+)(q_2\kappa^-q_1\kappa^+)p_2 + \cdots.$$

This can be rewritten as follows:

$$T = p[1 + q\kappa + (q\kappa)^2 + (q\kappa)^3 + \cdots]\kappa^+ = \frac{p}{1 - q\kappa}\kappa^+,$$

where

$$p = p_1 p_2, \qquad q = q_1 q_2, \qquad \kappa = \kappa^+ \kappa^-.$$

The same formula applies of course to $I$ provided we interpret the $p$'s, $q$'s and $\kappa$'s as referring to the variable $I$ rather than the variable $V$. If the inserted piece is electrically long, we have approximately

$$T = p\kappa^+.$$

In many practical applications the inserted piece is a uniform transmission line, so that

$$\kappa^+ = \kappa^- = e^{-\Gamma l}, \qquad \kappa = e^{-2\Gamma l},$$

where $\Gamma$ is the propagation constant and $l$ is the length of the piece. In this case

$$T = \frac{p}{1 - qe^{-2\Gamma l}} e^{-\Gamma l}.$$

## Power Absorption and Radiation

The power transferred from left to right across $P$ is the real part of the following function $\Psi$:

$$\Psi_P = \tfrac{1}{2} V_P I_P^* = \tfrac{1}{2} Z_P I_P I_P^*, \tag{18}$$

where the asterisk denotes the complex number conjugate to the one represented by the letter itself. The power absorbed by the impedance $Z_t$ is

$$\Psi_T = \tfrac{1}{2} Z_t I_Q I_Q^*. \tag{19}$$

The difference between (18) and (19) represents the power absorbed by the section $PQ$.

The power absorbed by a shield is calculated in a similar manner. The energy flow per unit area of the shield is given by an expression closely analogous to (18); the tangential component of $H$ appears in the place of $I$ and $Z_P$ is to be interpreted as the impedance in the direction normal to the shield. The formula is derivable from the Poynting expression for energy flow. Thus the power flow per unit area is $\frac{1}{2} Z_n H_t H_t^*$ where $H_t$ is the tangential component of $H$ and $Z_n$ is the impedance in the direction normal to the shield.

PART III

# REFLECTION, REFRACTION, SHIELDING AND POWER ABSORPTION

## Special Applications

The general formulæ derived in the preceding part are directly applicable to a variety of special cases such as reflection of plane waves at a plane boundary, shielding action of cylindrical shields upon electric waves produced by an infinite parallel pair of electric current filaments, shielding action of spherical shields upon electric waves produced by a coil or a condenser, etc. Of course, most of these results have already been obtained and published, each special problem having been treated on its own merits rather than as a particular case of a general formula. For this reason, we shall confine ourselves largely to a discussion of those aspects of reflection which are particularly illuminated by the general point of view.

## Cylindrical Waves

Consider two parallel wires carrying equal and opposite alternating currents. At a distance from the wires two or three times as large as their interaxial separation, the wave is substantially that of a line doublet and the radial impedance in free space is approximately [8] $i\omega\mu\rho$ so long as $\rho$ is much less than the wave-length. This restriction on $\rho$ is permissible in the present communication art. In metallic media this expression for the radial impedance is good only at very low frequencies. At high frequencies the radial impedance in metallic media is substantially [9] $\sqrt{i\omega\mu/g}$.

If the pair of wires is surrounded by a metal cylinder, the latter will act as a shield by virtue of reflections taking place at the boundary and attenuation through the shield.

The attenuation constant is substantially $\sqrt{\pi\mu g f}$ nepers per meter.[10] Thus the attenuation in logarithmic units through the shield is proportional to the first power of its thickness and to the square roots of the conductivity, the permeability and the frequency.

The reflection loss depends upon the impedance ratio. In the neighborhood of $f = 0$, the impedance ratio is seen to be equal to the ratio of the permeabilities. Consequently, at very low frequencies non-magnetic shields are relatively inefficient since there is no reflec-

[8] Equation (12).

[9] The approximate error is $1/2\sigma\rho$; at 10 kc. the error is about 2.5 per cent at a distance 1 cm. from the line source.

[10] This is true even at low frequencies if the shield is thin compared to its diameter. Otherwise, the cylindrical divergence of the wave must be taken into account.

tion loss. Inasmuch as the radial impedance in air is proportional to the first power of the frequency and in metal it is proportional only to the square root of the frequency, a point is reached beyond which the radial impedance in air always exceeds the radial impedance in metals. Thus the air-to-magnetic metal impedance ratio is less than unity near $f = 0$ and greater than unity for sufficiently high frequencies. Consequently, the absolute value of this impedance ratio is equal to unity at some intermediate frequency at which the reflection

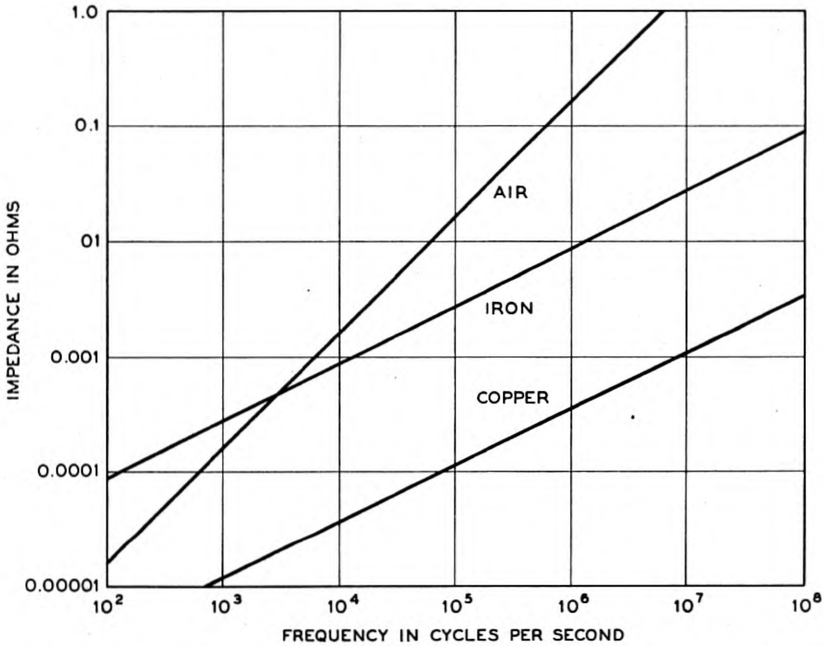


Fig. 4—The radial impedances in air, copper and iron at a distance of 2 centimeters from the axis for cylindrical waves generated by line doublets comprised of infinitely long electric current filaments. The conductivity of copper = $5.8005 \times 10^7$ mhos per meter, the conductivity of iron = $10^7$ mhos per meter, the permeability of air and copper = $1.257 \times 10^{-6}$ henries per meter, the permeability of iron = $1.257 \times 10^{-4}$ henries per meter.

loss will be quite small.[11] Some typical curves of radial impedances are shown in Fig. 4. The radial impedances in non-magnetic metals are always less than the impedance in air.

At high frequencies the reflection loss between metals is substantially independent of the frequency. At copper-iron boundaries this loss is always high and at copper-air boundaries it increases steadily with the frequency and becomes quite substantial at frequencies as

[11] A small reflection loss exists because the impedances have different phases.

high as 100,000 cycles. On the other hand, in a certain frequency range the reflection loss at iron-air boundaries may be very low. Since the attenuation loss of a complete shield made of coaxial layers of copper and iron is independent of the sequence of the layers, considerable gain in shielding may be secured by placing an iron layer between two copper layers rather than a copper layer between two iron layers so as to take advantage of the added reflection loss, assuming of course that the amounts of copper and iron are the same in both cases.

Since the high-frequency impedance ratio is proportional to the diameter of the shield, the size has a substantial influence upon the effectiveness of the shield. Each time the diameter of a *non-magnetic* shield is doubled, the shielding is increased by 6 decibels. In the case of *magnetic* shields, this is true only at frequencies considerably higher than the critical frequency at which the reflection loss is minimum. Considerably below this frequency, the effectiveness of a magnetic shield is *decreased* by 6 decibels with each doubling of the diameter of the shield. For the transition region we can say that with increasing size the effectiveness of the magnetic shield decreases below the critical frequency and increases above it.

## "Electrostatic Shielding"

If the cylindrical wave is originated by two parallel oppositely charged wires, alternating with a given frequency $f$, the radial impedance in free space is $1/i\omega\epsilon\rho$ provided $\rho$ is small compared with the wave-length.[12] As in the preceding case, in metallic media the radial impedance is $\sqrt{i\omega\mu/g}$ provided the frequency is not too low; for very low frequencies the radial impedance becomes $1/g\rho$.

It is clear at once that for these waves the reflection loss is tremendous. Thus in air $\epsilon = (1/36\pi)10^{-9}$ farads per meter; if $f = 10^6$ and $\rho = 0.01$ m., then the radial impedance is 1,800,000 ohms. The corresponding impedance in copper is only 0.000369 ohms. At lower frequencies the disparity between the radial impedances becomes even greater. The impedance ratio tends to infinity as the frequency approaches zero.

In the elementary theory a metal shield is regarded as a perfect shield against this "electrostatic field." An "electrostatic" field alternating 1,000,000 cycles per second is probably a misnomer. And the shielding is excellent but not perfect. Nevertheless the distinction between two possible types of waves is a valid one, at least in the frequency range usually employed in the communication art. In one wave the electric field is normal to the direction of propagation and

[12] Equation (13).

in the other the magnetic field is so disposed. The former wave may be called *transverse electric* and the latter *transverse magnetic*. The product of the corresponding radial impedances of these waves is equal to the square of the intrinsic impedance. Hence if one wave is a low impedance wave (as compared to the intrinsic impedance), the other is a high impedance wave. Under the usual engineering conditions these waves are unmistakably different in air, although this distinction disappears in metallic media. It must be pointed out, however, that for micro-waves the dimensions of the shield may be comparable to the wave-length, in which case the radial impedances may be of the same order of magnitude.

In the above discussion we have supposed that the line source was on the axis of the shield. If it is not, it is possible to represent the actual source by means of an equivalent system of sources along the axis and calculate the shielding effect. The latter is different for cylindrical waves of different orders. This will result in somewhat different shielding for different positions outside the shield. Ordinarily, however, the difference is not large enough to be considered in practical problems.

## Spherical Waves

A small coil carrying an alternating current will give rise to a transverse electric spherical wave and a small condenser to a transverse magnetic wave. Consider a shield concentric with the coil or the condenser. In the shield the radial impedance is $\sqrt{i\omega\mu/g}$, again excepting very low frequencies. In air the radial impedance of the outward bound electric wave is [13] $i\omega\mu r$ and that of the internal wave $\frac{1}{2}i\omega\mu r$. The corresponding impedances of transverse magnetic waves are $1/i\omega\epsilon r$ and $2/i\omega\epsilon r$. The conditions for reflection and shielding are substantially the same as in the case of cylindrical waves. Some quantitative difference results from the inequality of the radial impedances in opposite directions.

## Plane Waves

The next example of uniform linearly polarized plane waves is particularly well known.[14] When the boundary between two media coincides with an equiphase surface of the impinging wave, the formulæ

[13] Equations (9), (10).

[14] The general formulæ for the reflection and transmission coefficients have been obtained by T. C. Fry on the basis of the Maxwell theory in his paper "Plane Waves of Light II," published in the *Journal of the Optical Society of America* and *Review of Scientific Instruments*, Vol. 16, pp. 1–25 (1928). The earliest formulæ are probably due to A. Cauchy, who obtained them from the "elastic solid" theory of light waves.

of Part II are directly applicable and the reflection coefficient depends upon the ratio of the intrinsic impedances of the media.

A more interesting situation arises when the incidence is oblique. Let the $xy$-plane be the boundary between two homogeneous media and let the *electric vector be parallel* to this boundary. We may assume it to be parallel to the $x$-axis. In this case the electric field strength is given by

$$E_x = E_0 e^{-\sigma s + i\omega t}, \qquad E_y = E_z = 0, \tag{20}$$

where $E_0$ is the amplitude and $s$ is the distance from the equiphase surface passing through the origin. If the angle of incidence is $\vartheta$ (Fig. 5), this distance may be expressed as: $s = y \sin \vartheta + z \cos \vartheta$.

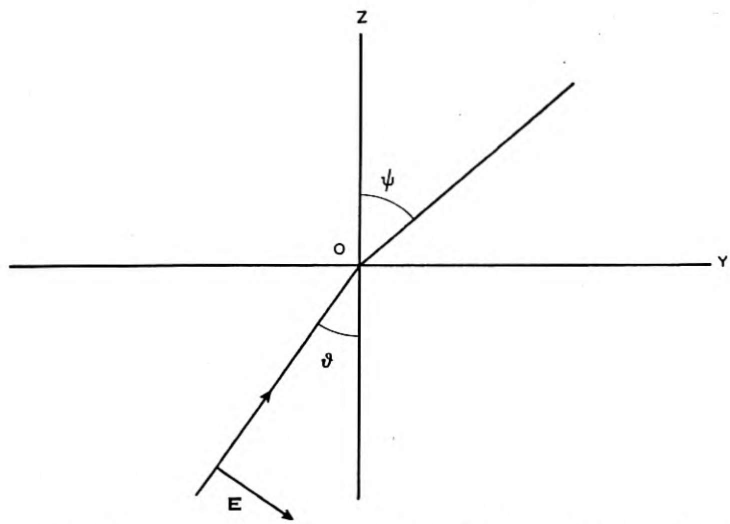


Fig. 5—Reflection of plane waves. The $x$-axis is toward the reader, the $xy$-plane is the boundary between the media, the $E$-vector is toward the reader, the angle of incidence = $\vartheta$, and the angle of reflection = $\psi$.

The magnetic vector is perpendicular to $E$ and to the ray and its value is

$$H = H_0 e^{-\sigma s + i\omega t}, \qquad E_0 = \eta H_0. \tag{21}$$

The cartesian components are then

$$H_y = H_0 \cos \vartheta \, e^{-\sigma s + i\omega t}, \qquad H_z = - H_0 \sin \vartheta \, e^{-\sigma s + i\omega t}, \qquad H_x = 0.$$

Equations (20) and (21) represent the motion of equiphase planes in the direction specified by the angle $\vartheta$. It is equally possible to regard them as representing the motion of *phase-amplitude patterns* in the direction *normal* to the $xy$-plane. We need only to rewrite these equations as follows:

$$\begin{aligned} E_x &= (E_0 e^{-\sigma y \sin \vartheta}) e^{-\sigma z \cos \vartheta + i\omega t}, \\ H_y &= (H_0 \cos \vartheta \, e^{-\sigma y \sin \vartheta}) e^{-\sigma z \cos \vartheta + i\omega t}. \end{aligned} \tag{22}$$

The relative distribution of the amplitude and the phase of the wave are governed by the factor $e^{-\sigma y \sin \vartheta}$ and this phase-amplitude pattern is propagated in the direction of the $z$-axis, the propagation constant being $\sigma \cos \vartheta$.

The advantages of this point of view are clear. In attempting to find the reaction of the second medium upon the incident wave, it is necessary to satisfy certain boundary conditions at *every point* of the interface. This can be insured by requiring the reflected and the refracted waves to have the same phase-amplitude patterns at the interface and by adjusting their relative amplitude and phases to secure the fulfilment of the boundary conditions at some one point. In other words, the problem is reduced to that for which the general solution was given in Part II.

The impedance to the incident wave in the $z$-direction is found from (22):

$$Z_z = \frac{E_x}{H_y} = \frac{E_0}{H_0 \cos \vartheta} = \eta \sec \vartheta.$$

This impedance is seen to be a function of the intrinsic impedance of the medium and of the angle of incidence.

For the refracted wave in the second medium the transmission equations are similar to (22):

$$\begin{aligned} E_x' &= (E_0' e^{-\sigma' y \sin \psi}) e^{-\sigma' z \cos \psi + i\omega t}, \\ H_y' &= (H_0' \cos \psi \, e^{-\sigma' y \sin \psi}) e^{-\sigma' z \cos \psi + i\omega t}, \qquad E_0' = \eta' H_0. \end{aligned} \tag{23}$$

The "angle of refraction" $\psi$ is, in general, different from $\vartheta$. In our equations we may regard $\psi$ merely as a parameter. Its value is obtained from the condition that at the $xy$-plane the phase-amplitude pattern of the incident and the refracted waves must be the same, and consequently

$$\sigma \sin \vartheta = \sigma' \sin \psi. \tag{24}$$

In dielectrics this relation is known as Snell's law of refraction.

By (23), the impedance to the refracted wave in the $z$-direction is

$$Z_z' = \frac{E_x'}{H_y'} = \eta' \sec \psi.$$

The reflection and the transmission coefficients are then obtained from (22) in terms of the impedance ratio

$$k = \frac{\eta \sec \vartheta}{\eta' \sec \psi} = \frac{\eta \cos \psi}{\eta' \cos \vartheta}. \tag{25}$$

Thus, we have

$$R_H = \frac{k-1}{k+1}, \qquad R_E = \frac{1-k}{1+k},$$

$$T_H = \frac{2k}{k+1}, \qquad T_E = \frac{2}{1+k}.$$

These coefficients refer to the *tangential* components of the field.

In a similar way we can deal with the case in which the magnetic vector of the incident wave is parallel to the boundary. The parts played by $E$ and $H$ are interchanged and the impedance ratio becomes

$$k = \frac{\eta \cos \vartheta}{\eta' \cos \psi}. \tag{26}$$

The cosine factors have changed their places.

The general case, in which neither $E$ nor $H$ is parallel to the boundary, cannot be treated in the above manner. In this case the components of $E$ and $H$ which are parallel to the boundary are not perpendicular to each other, the impedances $Z_{xy}$ and $Z_{yx}$ are not equal to each other and the unique impedance $Z_z = Z_{xy} = Z_{yx}$, upon which the results of Part II are based, does not exist. In accordance with a suggestion made in Part I, the incident wave must be resolved into components possessing unique impedances in the direction normal to the boundary. It is well known that such a decomposition is possible for ordinary plane waves; the latter can always be decomposed into two components, in one of which $E$ is parallel to the boundary and in the other $H$ is so disposed.

It is not surprising that reflection of arbitrarily oriented waves cannot be treated directly. The impedance ratios (25) and (26) for two basic orientations are in general different and the polarization of the reflected wave will be changed. An exceptional case arises *when the intrinsic propagation constants of the media are equal.* In this case $\psi = \vartheta$, as seen from (24), and *the impedance ratio is independent of the angle of incidence and of the particular orientation of the wave.* Consequently, *the reflection and the transmission coefficients depend solely upon the ratio of the intrinsic impedances of the media.*

Frequently the permeabilities of the media are assumed to be the same, in which case the ratio of the intrinsic impedances is equal to the inverse ratio of the "indices of refraction" of the media. Much could be said, however, in favor of not making such an assumption when formulating the general results since in many applications the permeabilities may be unequal.

### Images

A few additional interesting results can be obtained for the special case of two semi-infinite homogeneous media having equal propagation constants. If the media are separated by a plane boundary, problems of reflection and refraction can be solved by the *method of images.* This method is frequently used in electrostatics and one or

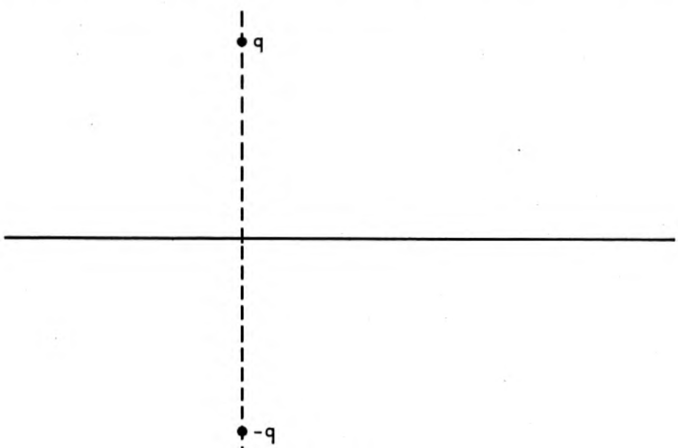


Fig. 6

two simple examples from that science will serve as an introduction to the later generalizations.

The field of a point charge $q$ above a conducting plane can be found by assuming another point charge $(-q)$. This "image" charge (Fig. 6) is the same distance below the plane as the actual charge is above the plane, both charges lying on the same perpendicular. The field due to the original charge and to the image charge satisfies the boundary conditions at the conducting plane since it makes the latter an equipotential. This combined field gives the correct resultant field on the *same* side of the plane as the original charge; on the opposite side the field is zero.

If the boundary is the interface between two perfect dielectrics (Fig. 7) with dielectric constants respectively equal to $\epsilon_1$ and $\epsilon_2$, the results are almost equally simple. Above the boundary we have a reflected field in addition to the original field. This reflected field is produced by an image charge $q' = (\epsilon_1 - \epsilon_2)q/(\epsilon_1 + \epsilon_2)$ on the supposition that the dielectric constant is everywhere equal to $\epsilon_1$. Below the plane the field is such as would be produced by a charge $q'' = 2\epsilon_1 q/(\epsilon_1 + \epsilon_2)$ if placed where the original charge is, also on the assumption that the dielectric constant is everywhere $\epsilon_1$. The charge producing the correct field below the boundary would be $q''' = 2\epsilon_2 q/(\epsilon_1 + \epsilon_2)$ if we were to assume $\epsilon_2$ as the dielectric constant of the whole space.

Inspecting equations (7) for an electric current element, which we assume to be perpendicular to the plane interface of two homogeneous

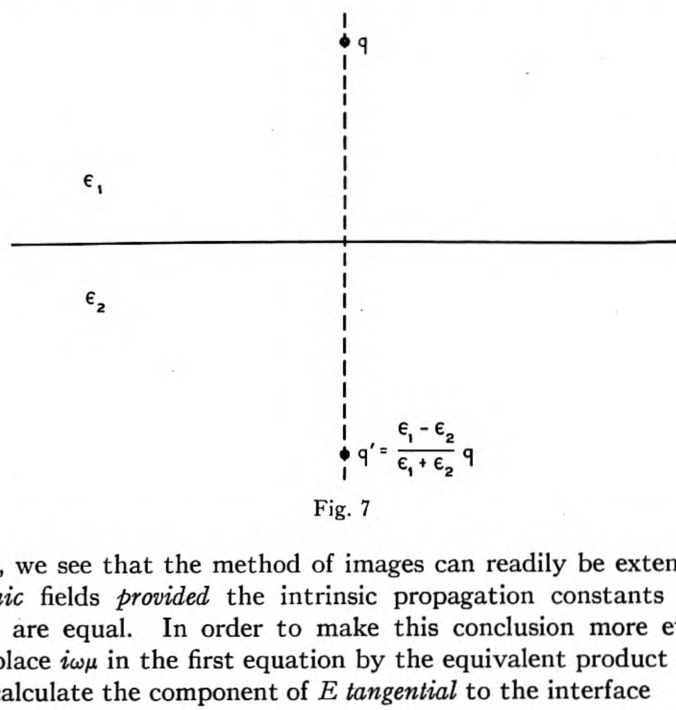


Fig. 7

media, we see that the method of images can readily be extended to *dynamic* fields *provided* the intrinsic propagation constants of the media are equal. In order to make this conclusion more evident, we replace $i\omega\mu$ in the first equation by the equivalent product $\eta\sigma$ and then calculate the component of $E$ *tangential* to the interface

$$E_\rho^+ = E_\theta^+ \cos\theta + E_r^+ \sin\theta = \eta Il \frac{\sigma e^{-\sigma r}}{4\pi r}\left(1 + \frac{3}{\sigma r} + \frac{3}{\sigma^2 r^2}\right) \sin\theta\cos\theta.$$

It is easy to see that the continuity of the tangential field components will be preserved if we assume a reflected field on the same side of the boundary and a refracted field on the opposite side in accordance with the following specifications. *The reflected field is such as would be produced by an image current element of moment* $(\eta_1 - \eta_2)Il/(\eta_1 + \eta_2)$

*and the refracted field is such as could be produced by a current element of moment* $2\eta_1 Il/(\eta_1 + \eta_2)$, *occupying the same position as the source. In calculating these fields a uniform intrinsic impedance* $\eta_1$ *is assumed throughout the whole space.*

Since the current $I$ in the element implies two point charges, $-I/i\omega$ and $I/i\omega$, at its terminals, we can interpret the above rule of images in terms of the charges. *The image of a point charge q for calculating the reflected field is* $(\eta_2 - \eta_1)q/(\eta_2 + \eta_1)$. *For calculating the refracted field a charge* $2\eta_1 q/(\eta_1 + \eta_2)$ *must be assumed in the same position as the original charge.* For perfect dielectrics the expressions of the image charges reduce to those given by electrostatics.

## ACKNOWLEDGMENT

I wish to express my appreciation to Dr. T. C. Fry for his valuable criticism in the preparation of this paper.

## AN HISTORICAL NOTE

The following memorandum written in 1932 by Dr. G. A. Campbell, formerly of the American Telephone and Telegraph Company, represents an interesting historical comment and it is reprinted with Dr. Campbell's permission.

A letter discussing the characteristic impedance of free space, written to me seven years ago by Dr. H. W. Nichols, is of possible interest in connection with both this impedance and the question of superfluous units. He derives the impedance from the Poynting vector by simple substitutions. Specific use is made, however, of five systems of units. The letter also supplies an illustration of confusion arising from the multiplicity of units in use. Apparently, Heaviside's 30 ohms ("Electrical Papers," ii, p. 377, 1888) was in ordinary ohms and not in Heaviside's own units, as Nichols quite naturally assumed. The correct explanation of the 30 ohms seems to be that Heaviside's "resistance-operator of an infinitely long tube of unit area" was not intended to be the characteristic impedance, as I define it.

In definitive units the characteristic impedance of free space equals the square of the effective volts per meter, in a plane electromagnetic wave, divided by the transmitted watts per square meter. For a numerical example, take the figures for strong sunlight (Maxwell, ii, footnote p. 441) which correspond to 666.1 effective volts per meter and 1176 watts per square meter. The characteristic impedance of free space implicitly assumed was thus 377.3 ohms, which checks well with my 376.54 international ohms.

If free space could be bounded in one direction by a thin, plane film having surface resistivity equal to the characteristic impedance of space, a normally incident plane wave would be completely absorbed by the film; there would be neither reflected wave nor transmitted wave beyond the film. This picture is suggested by the analogy of a transmission line terminated, at the receiving end, in its characteristic impedance, so that there is no reflected wave. The difficulty with the analogy is that free space exists beyond the film and cannot be cut off. This idealized picture may serve, however, to indicate the simplification made possible by the introduction of characteristic impedances in practical problems involving reflection, refraction and absorption.

The characteristic impedance of free space may be usefully introduced into formulas for the characteristic impedances of transmission lines. Thus, assuming perfect conductors, we have:

For flat strips, width $w$, separation $d$, if $w/d$ is large or the guard-ring method is employed in measurements,

$$K_f = 376.54\frac{d}{w};$$

For concentric cylinders, with radii $b$ and $a$,

$$K_c = \frac{376.54}{2\pi}\log\frac{b}{a}.$$

These characteristic impedances will each agree with the characteristic impedance of free space if $w = d$ and $b = 535.49\,a$. Since these strips are not wide compared with the separation, it would be necessary to employ the guard-ring method to maintain the plane wave assumed in the square shaft between the two strips. These two characteristic impedances would each become one ohm if $w = 376.54\,d$, and $b = 1.0168\,a$.

Practically, the finite conductivity of copper would add a reactance component and change the resistance component. It would be interesting to investigate simple cases numerically and include mutual characteristic impedances between two metallic circuits.

My own interest in the applications of the impedance concept to the electromagnetic field theory dates back to the last quarter of 1931.

# On the Theory of Space Charge Between Parallel Plane Electrodes

By C. E. FAY, A. L. SAMUEL and W. SHOCKLEY

The problem of the potential distribution, current, and electron transit time resulting from the perpendicular injection of electrons into the space between parallel planes is considered. The electrons are assumed to be injected uniformly with velocities corresponding to the potential of the plane through which they are injected. Consideration of all possible solutions of the basic equation shows that four general types of potential distribution are possible. Curves are given which enable the easy calculation of transmitted current and transit time and show the complete potential distribution for any concrete example. The case for current injected through both planes is also considered.

The complete mathematical treatment is given in the appendix.

SPACE charge has been studied extensively since the publication of the first papers on the subject by Child [1] in 1911 and Langmuir [2] in 1913. These papers in common with many which followed [3,4,5,6,7] dealt for the most part with potential distributions which occur when electrons are injected into a region with relatively small initial velocities.

The problem of space charge between parallel planes when the electrons possess arbitrary initial velocities, although contained implicitly in some of this early work, was first considered in detail by Gill [8] in 1925. Gill appears to have clearly understood the phenomenon but did not publish a complete analysis. Other workers, beginning with Tonks [9] in 1927, have considered various aspects of the problem,[10,11] and recently Plato, Kleen, and Rothe [12,13] published an extensive analysis. They did not include transit time calculations and their published curves are not easily adaptable for numerical

[1] C. D. Child, *Phys Rev.*, **32**, 492 (1911).
[2] I. Langmuir, *Phys. Rev.*, **2**, 450–486 (1913); also *Phys. Zeits*, **15**, 348 (1914).
[3] W. Schottky, *Phys. Zeits*, **15**, 526 and 624 (1914).
[4] P. S. Epstein, *Vehr. d.D. Phys. Ges.*, **21**, 85 (1919).
[5] T. C. Fry, *Phys. Rev.*, **17**, 441 (1921); *Phys. Rev.*, **22**, 445 (1923).
[6] I. Langmuir, *Phys. Rev.*, **21**, 419 (1923).
[7] I. Langmuir and K. B. Blodgett, *Phys. Rev.*, **22**, 347 (1923); *Phys. Rev.*, **24**, 49 (1924).
[8] E. W. B. Gill, *Phil. Mag.*, **49**, 993 (1925); *Phil. Mag.*, **10**, 134 (1930).
[9] L. Tonks, *Phys. Rev.*, **30**, 501 (1927).
[10] H. C. Calpine, *Wireless Engineer*, **13**, 473 (1936).
[11] Myers, Hartree and Porter, *Proc. Roy. Soc.* **158**, 23 (Jan. 1937).
[12] G. Plato, W. Keen, and H. Rothe, *Zeit. f. Phys.*, **101**, 509 (July 1936).
[13] H. Rothe and W. Keen, *Telefunken Rohre* No. 9 (April 1937).

calculations. In this country Salzberg and Haeff [14] have presented a solution, apparently quite complete, which has not as yet been published. The present independently derived solution differs from other published work in the manner of presentation and in an attempt at completeness. The results are presented in the form of curves which give potential distributions, electron transit times, and current-voltage relations, directly in terms of units which have a simple physical significance.

## Statement of the Problem

In common with some of the earlier treatments, the present discussion will be restricted to a consideration of the steady state potential distributions which can exist in an evacuated region between two parallel planes at known potentials when electrons having normal velocities corresponding to these potentials are injected into the region through one or both planes. For definiteness the case in which all of the current is injected through one plane will be considered first. It will then be shown that the results may be applied to the more general problem. It is supposed that the electrostatic potential is a function of one rectilinear coordinate only and that all the electrons move parallel to this coordinate and have the same total energy. The assumption of equal energy electrons leads to indeterminancy at planes where the potential and its gradient are both zero. At these planes the existence of non-uniform electron velocities will be recognized in so far as it provides a selective mechanism to resolve the mathematical indeterminancy.[15]

## Units

It has been found convenient to express voltages, currents, distances and transit times in terms of some derived units which are related to these quantities in certain ways by the simple Child's law equation for space charge limited current and by the corresponding transit time equation. The use of such derived units makes it possible to present a limited number of curves which are then applicable to a wide variety of conditions and may be used with a minimum amount of computations. The physical significance of the units also simplifies the interpretation of the results.

In order to understand how these units are obtained consider the hypothetical situation shown in Fig. 1 in which an electron current from a space charge limited cathode is injected into the region to the

[14] B. Salzberg and A. V. Haeff, *Proc. I.R.E.*, **25**, 546 (May 1937).—Abstract only. This paper appears in full in the January 1938 issue of the *R.C.A. Review*.

[15] This procedure has been justified by Langmuir. See *Reviews of Modern Physics*, vol. 3, pp. 237–244 (1931).

right of a plane at a potential $V_1$.[16] From Child's equation the current in amperes per square centimeter is given by

$$I = 2.33 \times 10^{-6} \frac{V_1^{3/2}}{S_0^2} = \frac{a^2 V_1^{3/2}}{S_0^2} \text{ (amps. per sq. cm.)} \tag{1}$$

or solving for $S_0$

$$S_0 = 1.527 \times 10^{-3} \frac{V_1^{3/4}}{I^{1/2}} = \frac{a V_1^{3/4}}{I^{1/2}} \text{ (centimeters).} \tag{2}$$

Accordingly, whenever the conditions at the first plane are represented

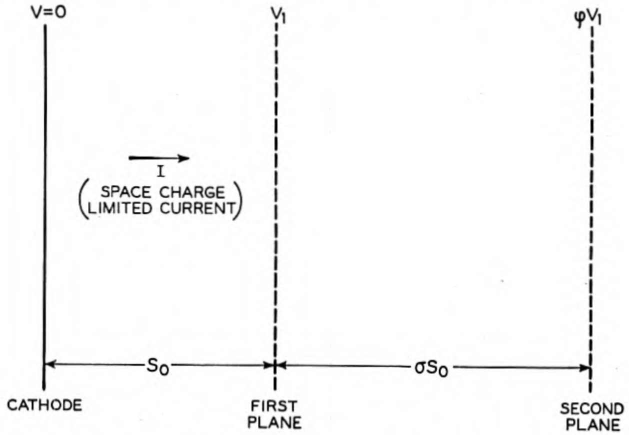


Fig. 1—Hypothetical conditions to assist in visualizing the unit of distance $S_0$.

by $I$ and $V_1$, distance from the first plane may be measured in units of $S_0$. The distance in centimeters is then

$$S = \sigma S_0. \tag{3}$$

Similarly a natural unit of potential is $V_1$ and the potential of any plane at a distance $S$ is then given by

$$V = \varphi V_1. \tag{4}$$

## Potential Distributions

All possible potential distributions to the right of the first plane can now be expressed in terms of $\varphi$ as a function of $\sigma$. The mathematical derivation is straightforward and is given in the appendix.

[16] This analogy is useful in getting a physical picture of the units but it must not be carried too far as will be evident when the problem of reflected currents is considered.

The results are shown in Figs. 2, 3 and 4. Figures 6 to 11 refer to current voltage relationships discussed in a later section.

To find the potential distribution when a second plane at a distance $\sigma$ away from the first plane is held at a potential $\varphi$, one enters the figures with the values of $\varphi$ and $\sigma$. Any member of the family of curves drawn in heavy solid lines passing through this point represents a possible potential distribution. The regions occupied by the heavy curves are bounded by certain limiting curves identified by lower case letters. These curves correspond to certain space charge conditions between the planes and are indicated by the same letter whenever met. The dashed curves refer to transit time and are explained below. Situations corresponding to low potentials and large spacings occur, for the most part, on Fig. 2, while those for high potentials and small spacings occur on Fig. 3. An overlap region exists for which potential distributions may be found on both families of curves. The interpretation of this is found by inquiring more closely into the nature of the curves of Figs. 2, 3 and 4.

There are in all four distinct types of potential distributions. These are:

Type *A*—The second plane is at a negative potential. All the injected current is reflected. Potential distributions are the same as for the case of temperature limited emission with the current $2I$ from the zero potential plane.

Type B—Both planes positive with potential zero between them. Injected current partially transmitted and partially reflected. Potential distributions corresponding to space charge limited emission on both sides of a virtual cathode at the zero of potential.

Type C—Both planes positive with a potential minimum (at a positive potential) between them. Complete transmission of injected current.

Type D—Both planes positive with no minimum between them. Complete transmission of injected current.

The curves on Fig. 2 are seen to fall into two groups, those which extend to negative values of $\varphi$ (marked with values of $\beta$) and those which contain the value $\varphi = 0$ but which remain positive. The first group (Type A) obviously corresponds to conditions under which all of the injected current returns to the first plane. The slopes of these potential curves at the reflection planes are not zero but are related to the parameter $\beta$ (shown on the curves) by the equation

$$\frac{d\varphi}{d\sigma} = -\frac{4}{3}\sqrt{2}\beta^{1/4}. \tag{5}$$

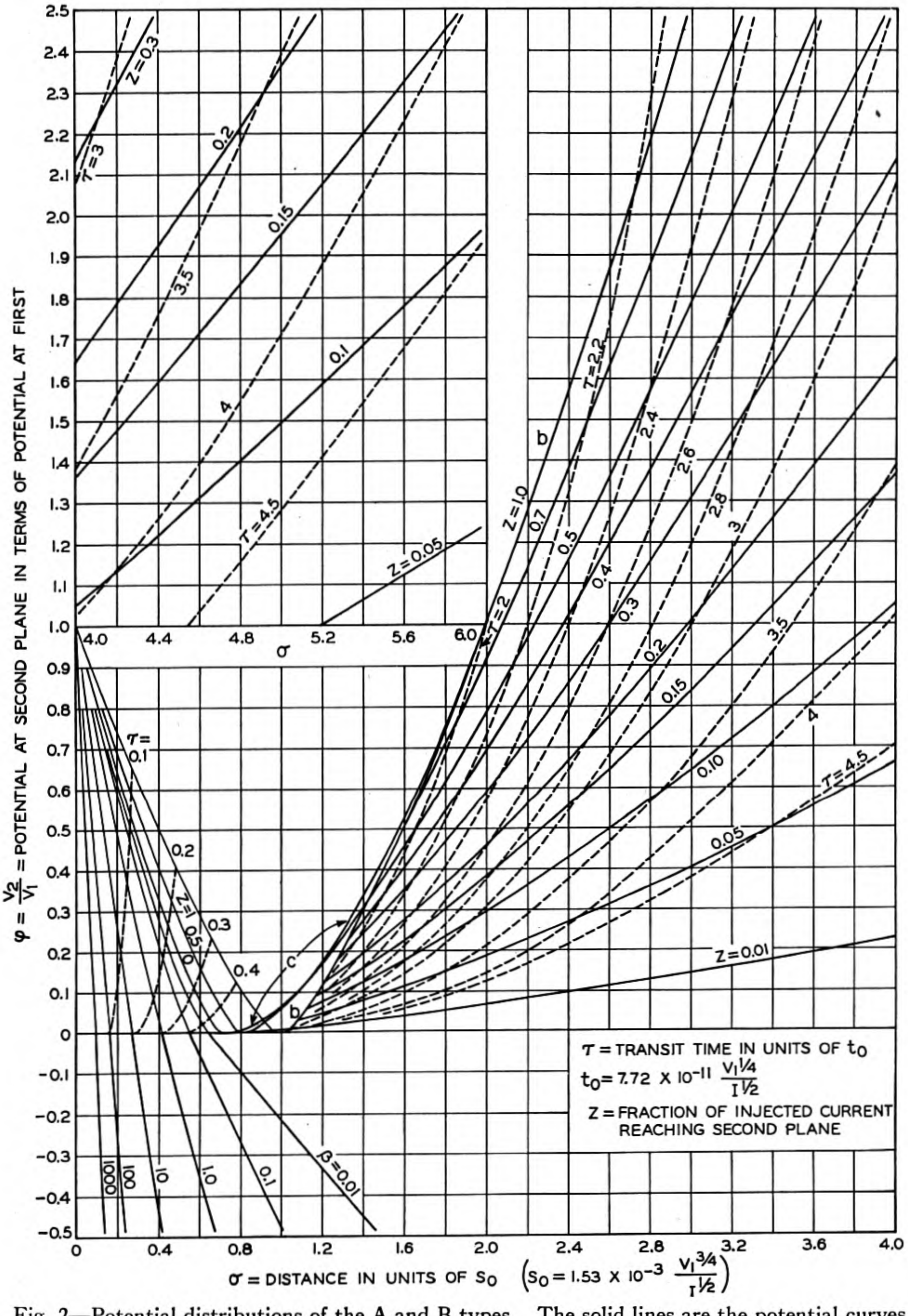


Fig. 2—Potential distributions of the A and B types. The solid lines are the potential curves; the broken lines indicate the transit times.

For the second group of curves (Type B) a certain fraction of the current denoted by $Z$ (and so indicated on the curves) will be transmitted while the fraction $1 - Z$ will be reflected toward the first plane. These potential curves have zero slopes at the reflection planes so that they correspond to solutions of Child's equation on both sides of a so-called virtual cathode existing at the reflection plane. For each value of $Z$ from 0 to 1 a possible potential distribution is obtained.

It will be observed that the portions of the solid curves to the left of the zero points are drawn lighter than the rest. These portions correspond to potential distributions resulting from conditions with reflected current and so while applying as extensions of the heavy curves to the right they cannot be entered directly with values of $\varphi$ and $\sigma$. Should values used to enter the figure fall in this region the absence of a potential zero is indicated and the correct distribution is obtained by entering Fig. 3. Before leaving Fig. 2 the existence should be noted of a small domain lying between the curves marked $b$ and $c$ for which two different $B$ solutions are possible for the same injected current, spacing, and potential, one corresponding to larger values of $Z$ than the other. The significance of this region will be apparent when the current voltage relationships are considered. All solutions of the A and B types are represented on Fig. 2.

Solutions of the C and D types are to be found on Fig. 3 except for a small overlap region which for clearness is shown on Fig. 4. Values of $\varphi$ and $\sigma$ which cannot be entered on Fig. 2 as well as values common to both B and C types are to be found on these figures. The additional overlap solutions of the C type are shown in Fig. 4. These are extensions of the potential distribution curves of Fig. 3 after they reach the right-hand boundary curve where they turn inward as shown and overlap the other solutions.

For convenience each curve of the C type is labeled by the value of its minimum potential. The parameters of the D curves do not have this simple physical significance. However, for all cases the value of the curve parameter is simply related to the electric field at the first plane by the equations:

type C
$$\frac{d\varphi}{d\sigma} = -\frac{4}{3}\sqrt{1 - \varphi_{min.}{}^{1/2}}. \tag{6}$$

type D
$$\begin{cases} \dfrac{d\varphi}{d\sigma} = \dfrac{4}{3}\sqrt{1 - \alpha^{1/2}}, & (7) \\[2ex] \dfrac{d\varphi}{d\sigma} = \pm\dfrac{4}{3}\sqrt{1 + \beta^{1/2}}. & (8) \end{cases}$$

type A
$$\frac{d\varphi}{d\sigma} = -\frac{4\sqrt{2}}{3}\sqrt{1 + \beta^{1/2}}. \tag{9}$$

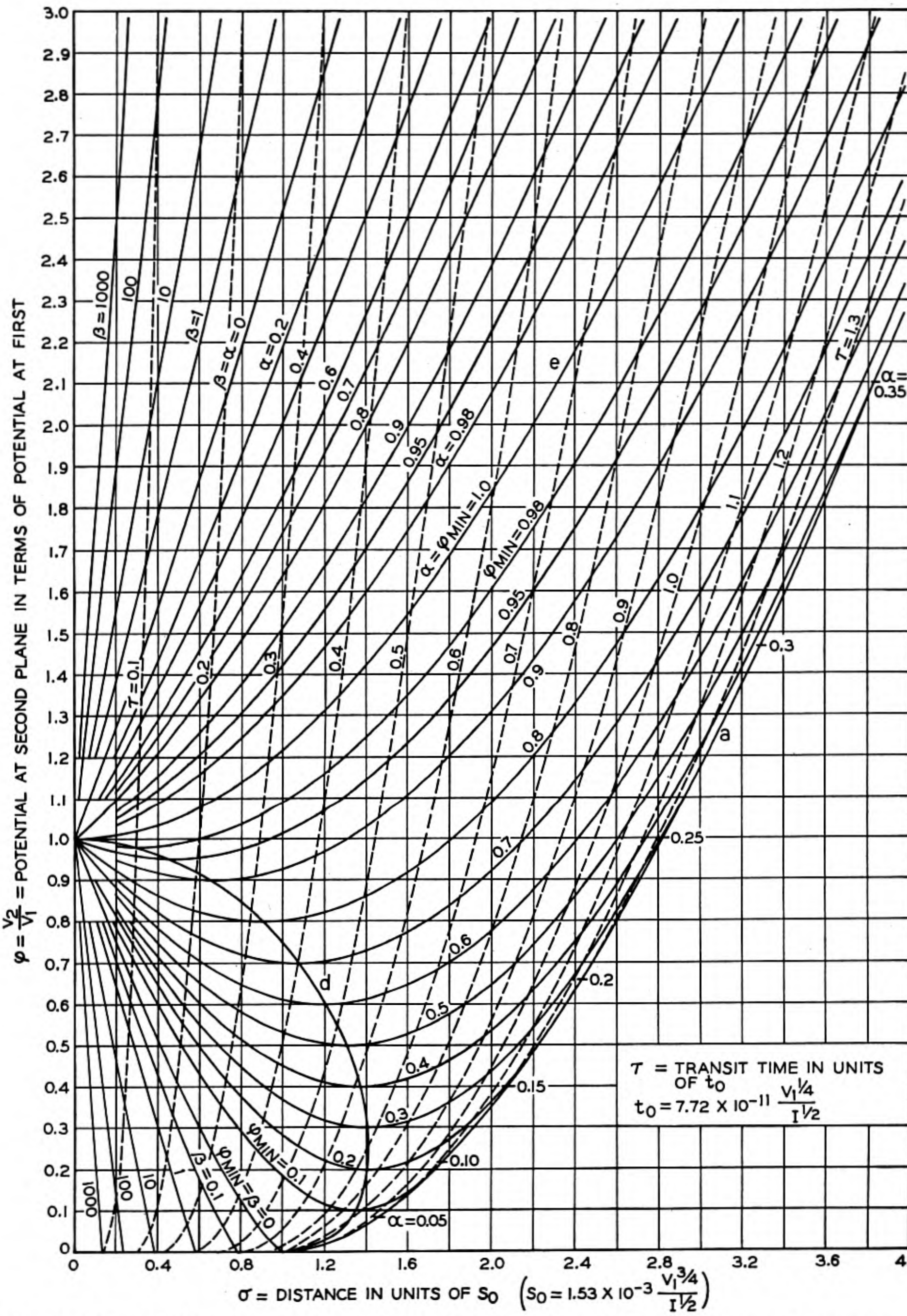


Fig. 3—Potential distributions of the C and D types. The solid lines are potential curves, the broken lines indicate the transit times.

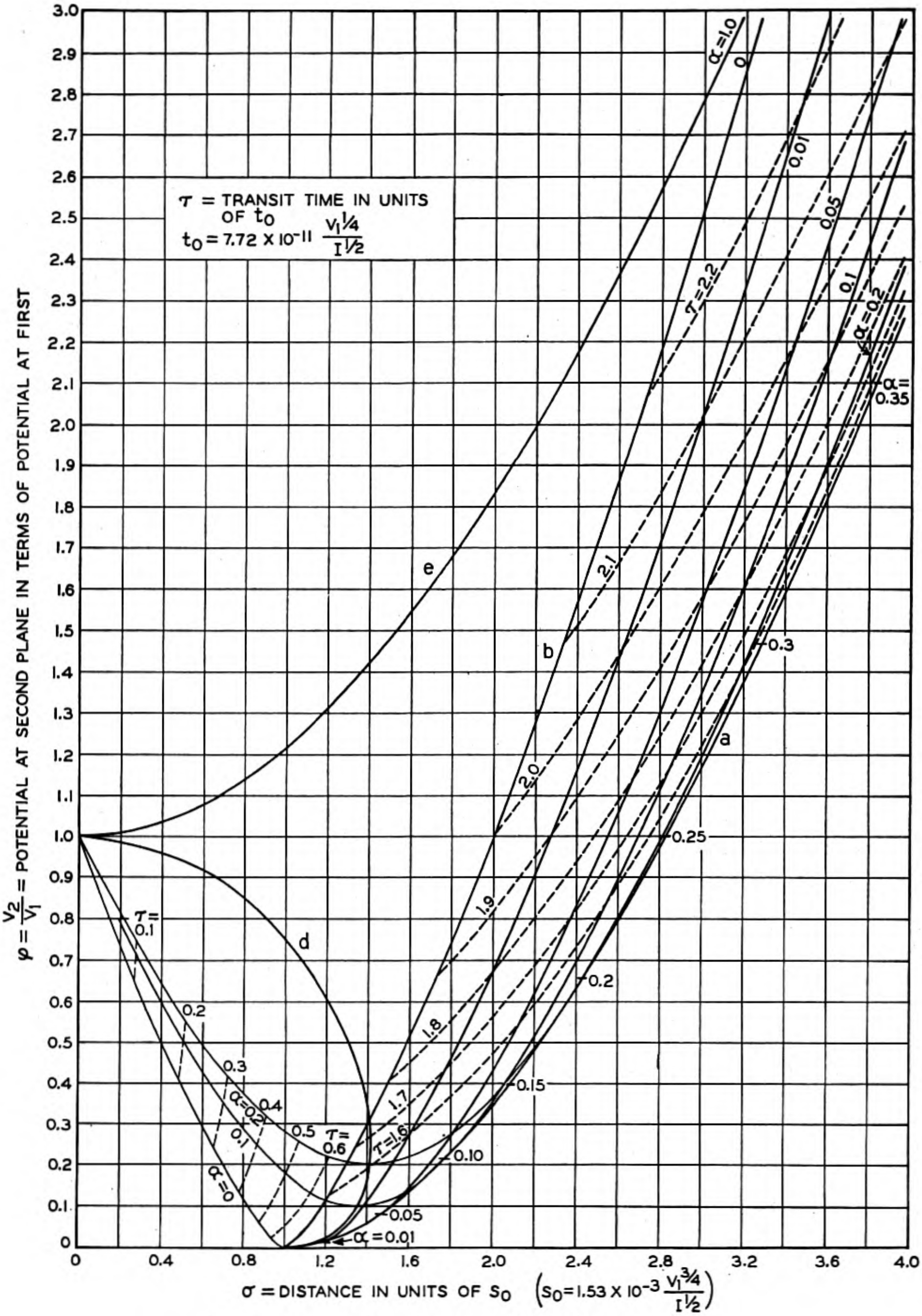


Fig. 4—Potential distributions of the C overlap type.

It should be noted that while some of the curves in the D region shown in Fig. 3 resemble the portions of the curves drawn in light lines on Fig. 2, the curves in Fig. 3 have essentially different physical content, representing solutions in which all of the injected current is transmitted. Values of $\sigma$ and $\varphi$ may, therefore, be entered in all regions on Fig. 3 while entering values on Fig. 2 are restricted to the regions covered by the heavy lines.

The occurrence of overlap regions indicates that for certain boundary conditions more than one type of potential distribution may exist. In the practical case the choice between these various possible solutions depends upon the manner in which the boundary conditions are established.[17]

## Current Injection from Both Sides

The present analysis although derived on the assumption of current injection from one side only is equally applicable to the situation shown schematically in Fig. 5. The potential distribution curves

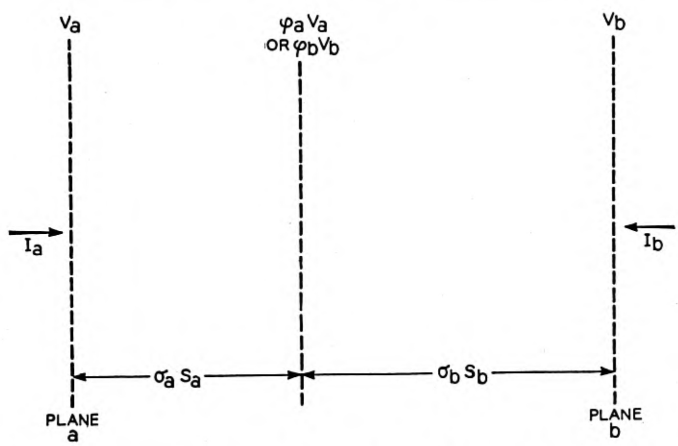


Fig. 5—Schematic representation of the general conditions under which the potential distribution analysis may be applied.

which occur when the currents $I_a$ and $I_b$ are injected from opposite sides will be identical with those obtained for an assumed injection from one side only of a current $I$ equal to the numerical sum of $I_a$ and $I_b$. Values of $V_1$, $\varphi$ and $\sigma$ are chosen to correspond with an assumed direction of injection and these values are entered on the figures. For solutions of the B type, different distributions will result depending

[17] This matter will be treated in more detail in the section on circuit characteristics.

upon the assumed direction of $I$. In some instances all solutions may correspond to physically realizable distributions for the double injection case. The solutions will, in general, require that the potential zero be located at different places and may require that the net flow of current across the zero plane be in opposite directions. The physical reality of each of these distributions must be checked by noting the value of $Z$ for the indicated solution and solving for the derived current $(2 - Z)I$ on the injected side of the zero potential plane and the derived transmitted current $ZI$. If these are respectively greater than the numerical values of $I_a$ and $I_b$ (paired consistent with the assumed direction of injection) the indicated solution is possible. In addition there may exist a solution in which the currents $I_a$ and $I_b$ are each totally reflected at two different zero potential planes, separated by a region of zero potential. The possible existence of this solution must be tested for separately.

## Electron Transit Time

So far no use has been made of the dashed curves shown in Figs. 2, 3 and 4. These give the electron transit time from the first plane to any desired plane. The significance of the unit of time is to be found by again referring to Fig. I. The time an electron takes to travel from the cathode to the first plane is given by

$$t_0 = c\,\frac{V_1^{1/4}}{I^{1/2}} = 7.72 \times 10^{-11}\,\frac{V_1^{1/4}}{I^{1/2}}\ \text{(seconds)}. \tag{10}$$

This value of $t_0$ is a natural unit of time to be used whenever the conditions at the first plane are expressed by $V_1$ and $I$ just as $S_0$ is a natural unit of distance. Accordingly, electron transit times from this plane to any other plane are expressed in units of $t_0$. The time in seconds is then

$$T = \tau t_0. \tag{11}$$

Transit times for reflected electrons are not given but may be computed by taking the time to the reflection plane and adding to it the returning time. This latter is obtained by taking the difference between the time to the reflection plane and the direct time to the plane under consideration.

## Circuit Characteristics

In many practical cases the first plane will coincide with a mesh or grid electrode in a vacuum tube and the second plane will correspond with a plate electrode. Under these conditions the current-voltage relations which exist at the second plane are of considerable interest. Since the distance between the planes is now fixed, the parameter $\sigma$ is not appropriate. However, the spacing $d$ between planes and the voltage $V_1$ serve to define a natural unit of current density $i_0$, which is indicated by the hypothetical situation of Fig. 6. The value of $i_0$ is

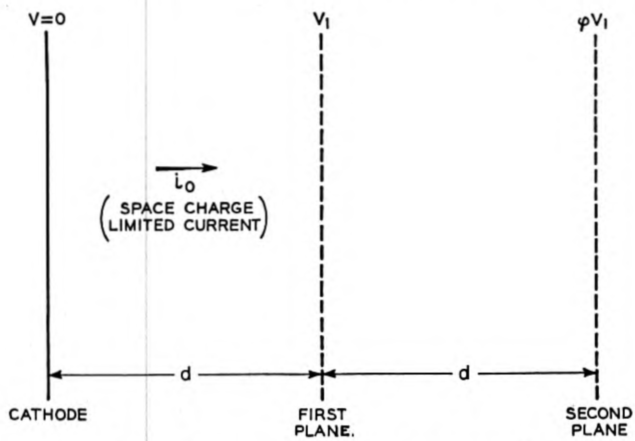


Fig. 6—Hypothetical conditions to illustrate the significance of the unit of current $i_0$.

$$i_0 = a^2 \frac{V_0^{3/2}}{d^2} = 2.33 \times 10^{-6} \frac{V_1^{3/2}}{d^2} \text{ (amperes per sq. cm.)} \qquad (12)$$

The current in units of $i_0$ is denoted by $\gamma$ so that the injected current $I$ in amperes per sq. cm. is given by

$$I = \gamma i_0. \qquad (13)$$

Similarly, the transmitted current under conditions corresponding to type B potential distributions will be

$$ZI = Z\gamma i_0. \qquad (14)$$

Specifying $\gamma$ and $\varphi$ is equivalent to specifying $\sigma$ and $\varphi$.[18] For this reason many of the limiting curves of the circuit characteristic plots are simply related to those shown in the potential distribution plots and are designated by the same letters. The regions for which the different types of potential distributions may exist are shown in Fig. 7 now in terms of $\gamma$ and $\varphi$. In Fig. 8 the regions are defined in terms of $\varphi$ and the transmitted current $Z\gamma$. The boundaries of the *C* and *D*

[18] This is a consequence of the relationship $\gamma = \sigma^2$, which is derived in the appendix.

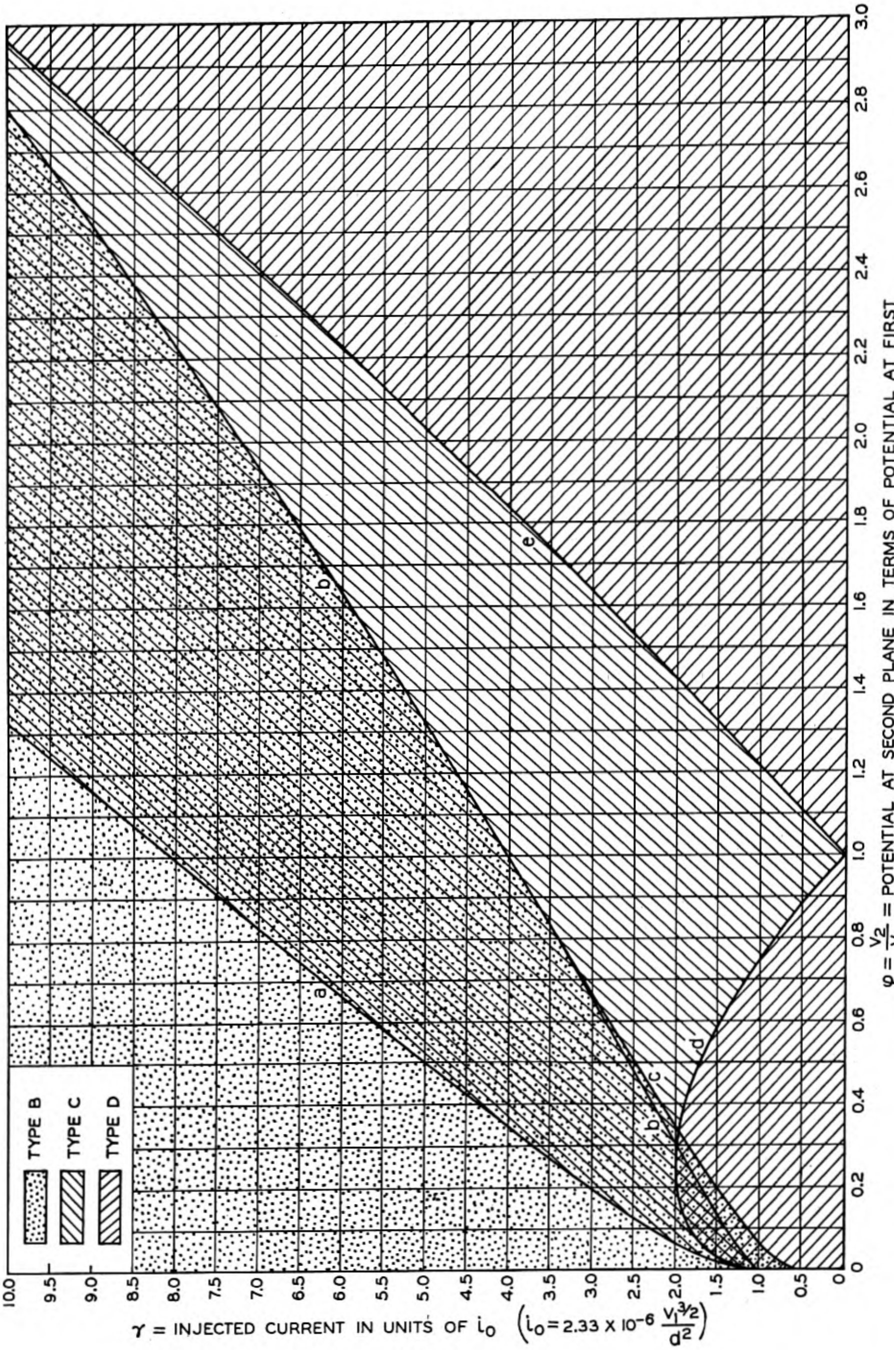


Fig. 7—Key chart giving boundary conditions for the different distributions in terms of the potential and the injected current.

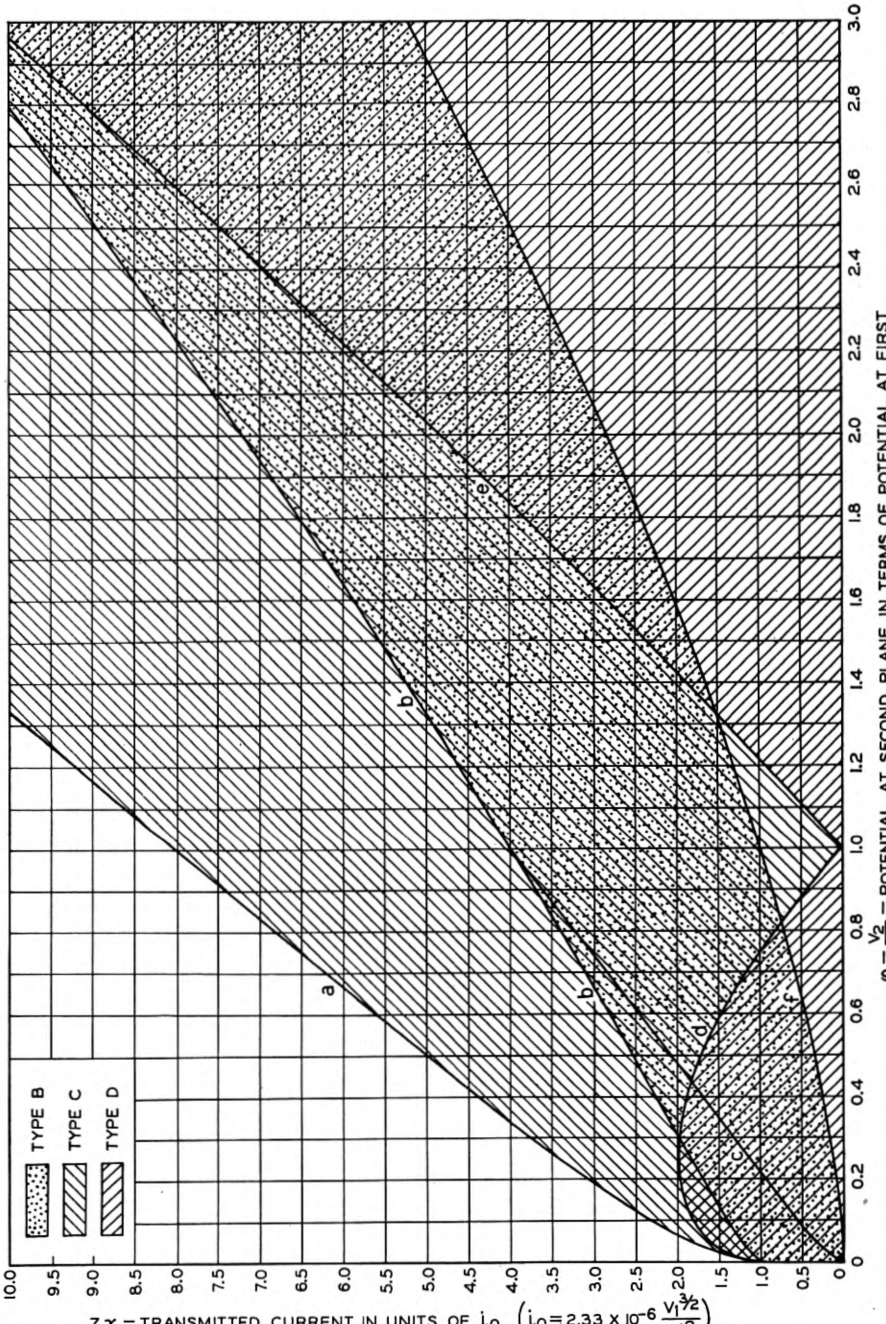


Fig. 8—Key chart giving boundary conditions for the different distributions in terms of the potential and the transmitted current.

regions are of course the same in the two plots (i.e., $Z = 1$) while the boundaries of the $B$ region are changed, since only the portion $Z\gamma$ of the injected current is transmitted. The $A$ region, which corresponds to negative values of $\varphi$ for all values of $\gamma$ and gives $Z\gamma = 0$, is not indicated. It may be of interest to note that for any given value of $\varphi$ it is impossible to transmit more than a certain maximum current, shown by curve $a$, and that this maximum occurs for a C type distribution. For injected currents greater than this amount only type B distributions are possible.

An enlargment of the $B$ region is shown in Fig. 9$a$ and a portion of the region to a still larger scale in Fig. 9$b$. Lines for constant injected current $\gamma$ are plotted with the potential $\varphi$ and the transmitted current $Z\gamma$ as the coordinates. These plots correspond to plots frequently used to describe vacuum tube characteristics and may be used accordingly. For values of $\gamma$ less than four and values of $\varphi$ less than unity, double values of $Z\gamma$ appear. In the region lying between the lines $b$ and $c$ on Fig. 9, the slopes of the constant $\gamma$ lines are negative corresponding to conditions which are unstable unless sufficient resistance is included in the external circuit. Conditions outside of this region are always stable and need no further comment.

Solutions of the C type are shown in Fig. 10$a$. For these solutions the injected and transmitted currents are equal. The depth of the potential minimum between the two planes is, however, of interest and is shown in the figure in terms of the lesser of the two boundary potentials. For values of $\varphi$ greater than unity the value of $\varphi$ min. (i.e., the minimum potential in units of $V_1$) is indicated while for values of $\varphi$ less than unity the value of $\varphi_{min.}$ (i.e., the minimum potential in units of $V_2$) is shown. In the enlargement, Fig. 10$b$, both values are given. For values of $\varphi_{min.} = 1$, curve $e$, and $\varphi_{min.} = 1$, curve $d$, the potential minimum is equal to the potential at one of the planes and is located at this plane. For currents greater than the value indicated by these limiting curves the potential minimum becomes deeper and moves away from the plane while for lesser currents it disappears entirely. The minimum value to which the potential may be forced by increasing the injected current before the distribution changes precipitously to one of the B type is indicated by the values on curve $a$.

Again to avoid confusion the overlap type C region is shown separately in Fig. 11. Conditions are somewhat different for this region in that with any assumed value of $\varphi$ a minimum potential of 0 (but with complete transmission of injected current) occurs for a certain value of $\gamma$. With increasing injected current, the potential minimum

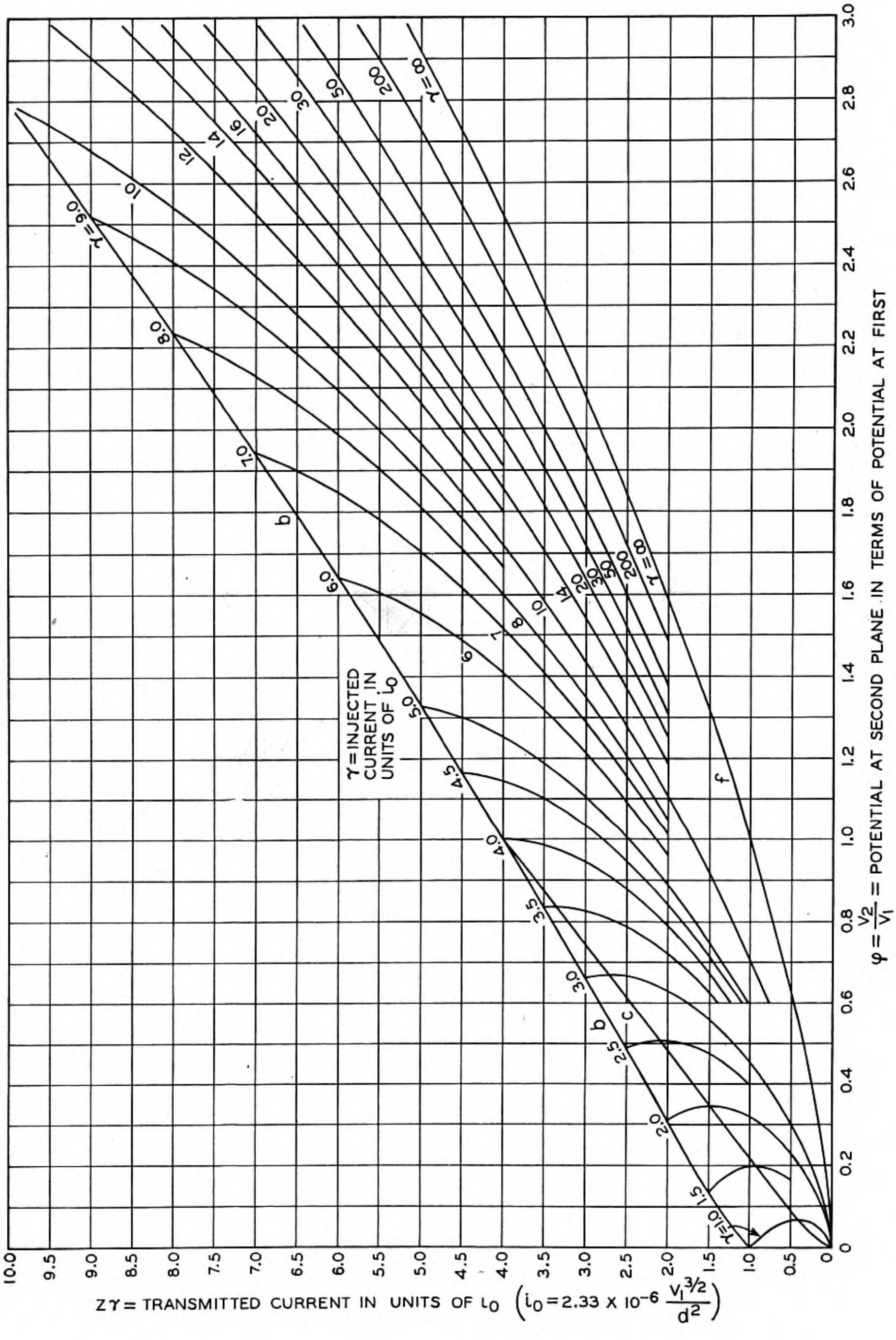


Fig. 9*a*—Electrical characteristics for type B distributions.

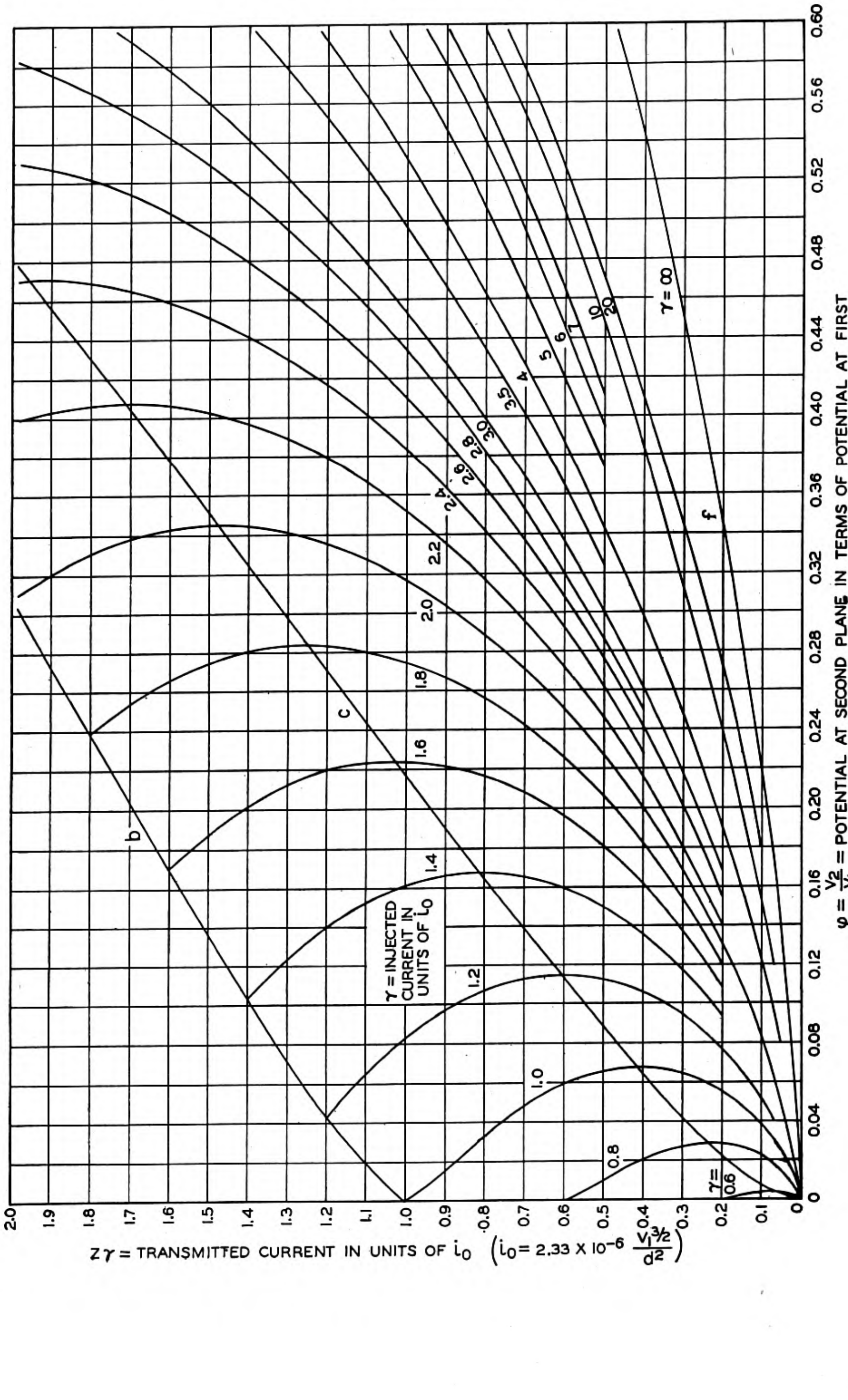


Fig. 9*b*—Enlargement of a portion of Fig. 9*a*.

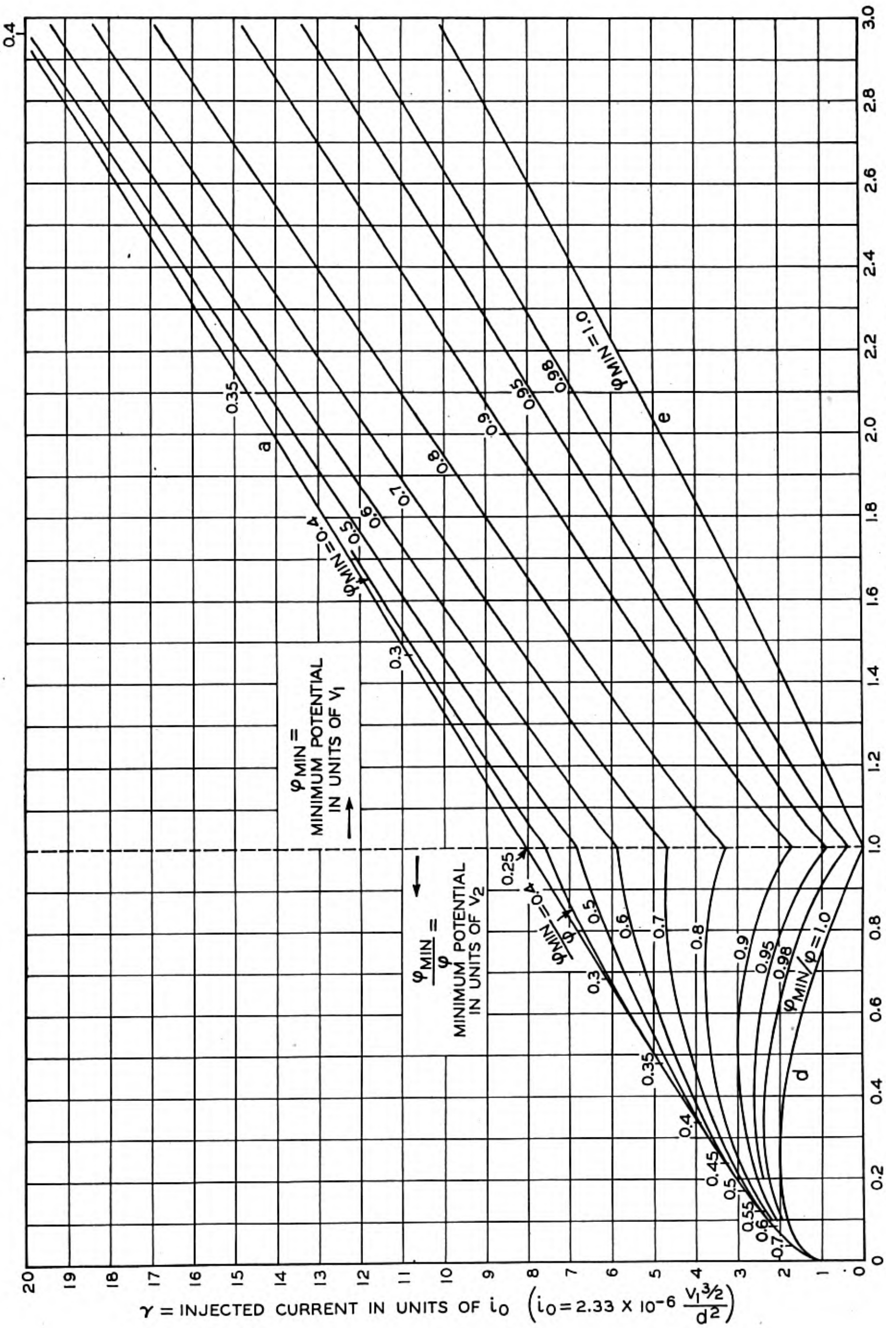


Fig. 10*a*—Electrical characteristics for type C distributions (overlap solutions on Fig. 11).

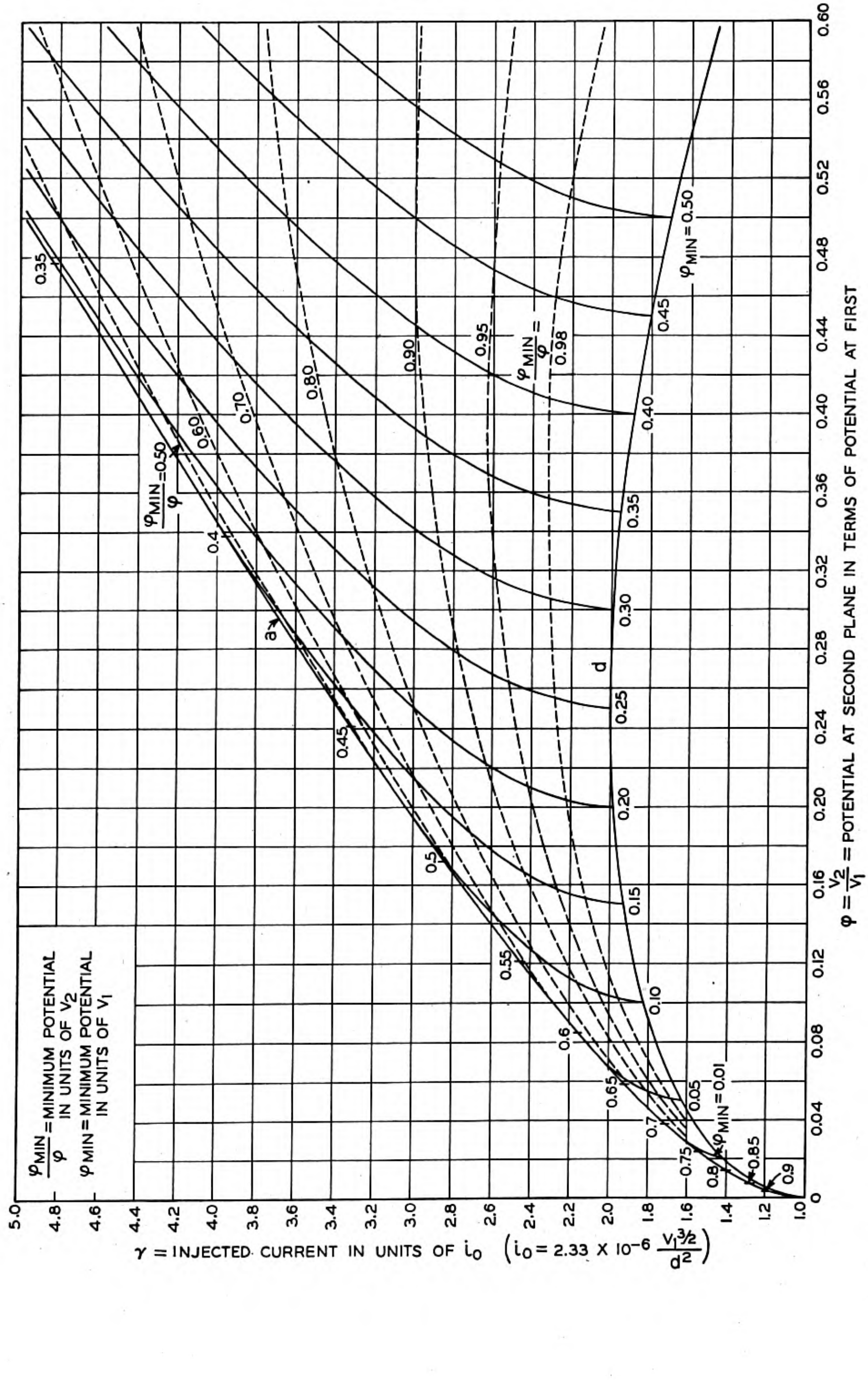


Fig. 10b—Enlargement of a portion of Fig. 10a.

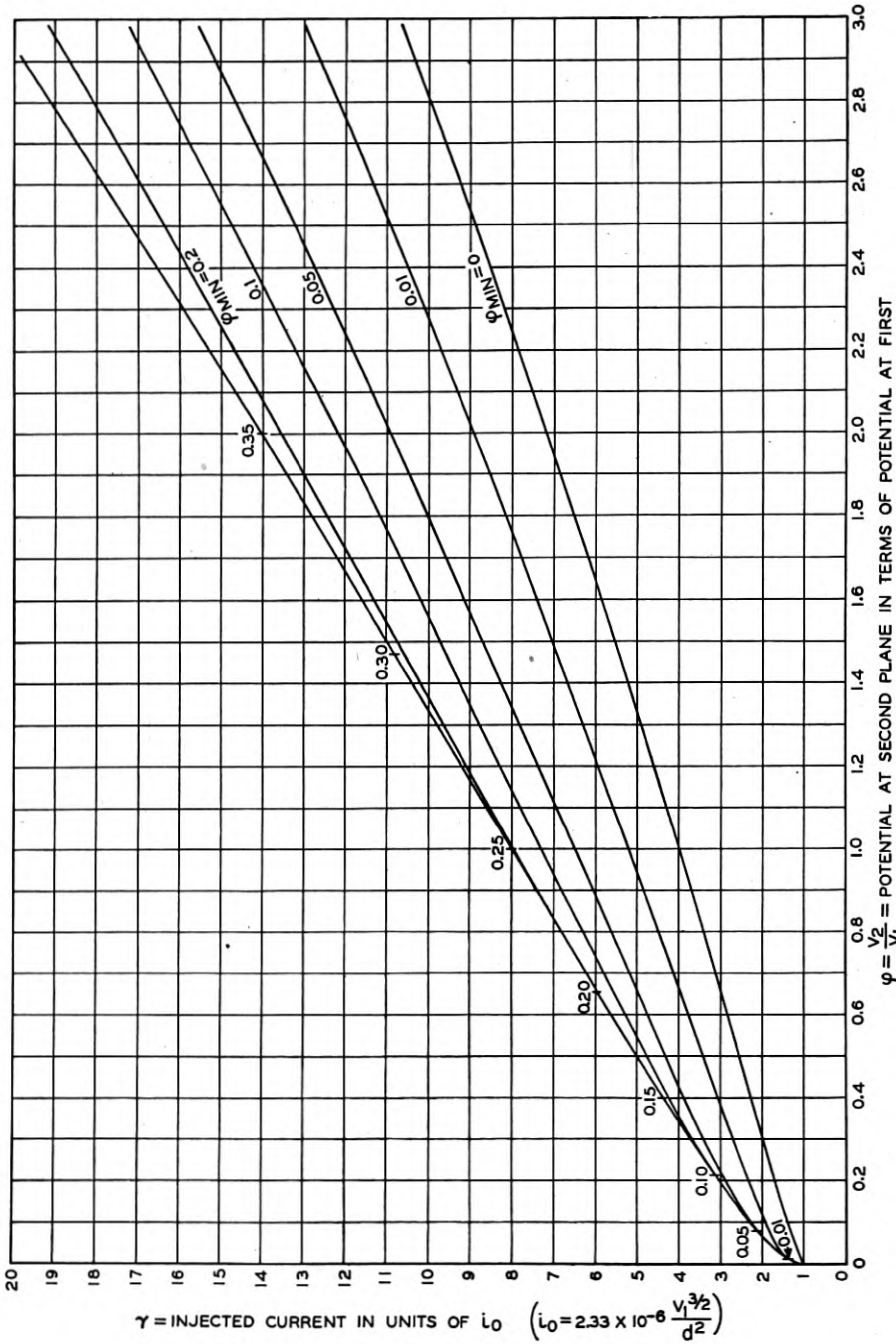


Fig. 11—Electrical characteristics for type C overlap distributions.

rises, finally reaching the limiting values indicated on the upper curve of Fig. 11; the limiting conditions there are precisely the same as for curve *a* of Fig. 10. Further increase of injected current produces a transition to type *B* solutions.

It seems questionable that this overlap type C condition can exist in a practical case. An investigation of this question involving external circuit considerations is beyond the scope of this paper.

## Transition Between Distribution Types

The physical choice between the different possible potential distributions which may exist with a given set of boundary conditions is determined by the sequence in which the boundary conditions are established. Extreme values of any parameter are seen from Figs. 7 and 8 to lie in regions for which only one solution is possible. If the boundary conditions are varied slowly and continuously from these values, the indicated type of distribution will persist until the limit of this region is reached at which time a sudden transition must occur to another indicated type of distribution. Inspection of Figs. 7 and 8 will show that at such transitions only one other type of distribution is ever possible. The determination of the correct physical distribution can thus be made without ambiguity.

Certain peculiarities are, however, to be noted. A survey of all possible transitions in which $\gamma$ and $\varphi$ are treated as independent variables will indicate that, starting from extreme conditions and changing conditions continuously in the same direction, distributions of the overlap C type shown in Figs. 4 and 11 never occur. A second peculiarity has to do with the unstable region of type B solutions shown on Fig. 9. When this region is entered with insufficient resistance in the external circuit, instability results with a sudden transition to a corresponding stable type of distribution.

The space model shown in Fig. 12 has been found to be of value in visualizing problems involving transitions. The three coordinates used in its construction are the second electrode potential $\varphi$ (to the right) the injected current $\gamma$ (to the left) and the transmitted current $Z\gamma$ (vertical). Solutions corresponding to potential distributions of the C and D type, for which $Z\gamma = \gamma$, appear as a celluloid plane inclined at 45 degrees, on which the values of the potential minima are indicated. Solutions of the C overlap type have been omitted. Solutions of the B type are represented by the concave surface of the model. Viewing the model from a different angle as shown in Fig. 13 (where the celluloid sheet is removed) the unstable B region appears as an over-hanging cliff extending between $\gamma$ values of 0.5 and 4. Bounding curve *c* separating the unstable region from the stable region is drawn on the model and lies along the vertical surface of the cliff. The small region lying between $\gamma$ values of 0 and 0.5, for which direct transitions between $A$ solutions and $D$ solutions occur, can be seen in both views as a small vertical cliff.

Solutions corresponding to varying conditions will be given by the position of a movable particle confined to the surface of the model.

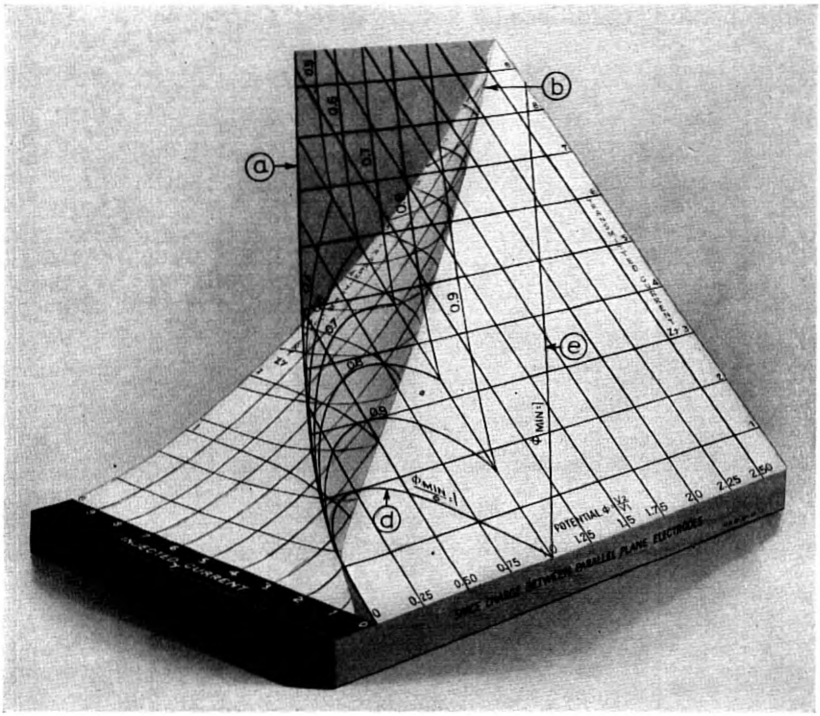


Fig. 12—A three-dimensional model illustrating the current-voltage relationships. Solutions of types C and D appear on the celluloid surface and solutions of the B type appear on the concave surface.

When this particle lies on the celluloid surface it may be moved around at will and will not "fall off" unless an attempt is made to go to larger values of the injected current $\gamma$ than is permitted by the bound-curve $a$ (the curved edge of the celluloid). The particle may, however, "fall off" at this edge and land on the concave surface from which it can escape only by climbing up the surface to the bounding curve $b$ which is common to both surfaces. The overhanging cliff is, however, particularly treacherous, for unless the particle is supported by the existence of sufficient resistance in the circuit connected to the second electrode, it will go through the model coming to rest on the celluloid surface above. With no resistance in the external circuit this curious behavior will occur at the vertical part of the cliff indicated by the bounding curve *c*. The presence of some resistance will cause this to occur for larger values of $Z\gamma$. The exact place can of course be determined by noting the point of tangency between a resistance line drawn on Fig. 9 and one of the characteristic curves—just as the stable operating conditions for any negative resistance device is found.

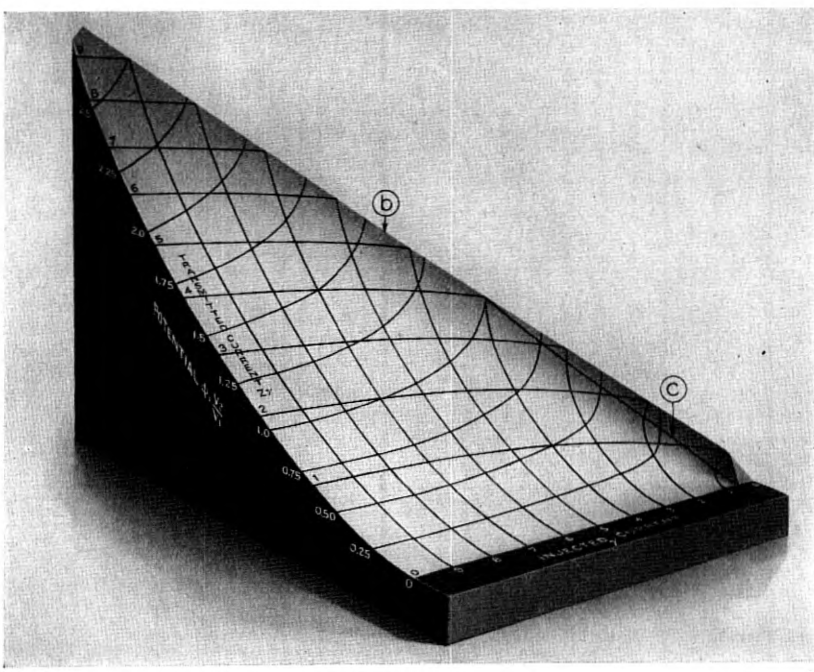


Fig. 13—A different view of the concave surface of the model shown in Fig. 12. (The celluloid surface has been removed.)

The entire surface is stable if the external resistance is greater than approximately 0.25 in units of $\varphi/Z\gamma$.

## Illustrative Examples

### *I. Class B Operation of a "Critical Distance" Tetrode*

Since the purpose of "critical distance" operation is to prevent the passage of secondary electrons from the plate (our second plane) to the screen grid (our first plane), the existence of a potential minimum is desired; however, type B solutions with some reflected electrons would be objectionable. The permissible operating range must therefore lie in the $C$ region on Fig. 7. If, for example, we operate the tube with $V_2 = V_1$ as the quiescent point and with a resistance load, the operating curve is a straight line through the point $\gamma = 0$, $\varphi = 1$. The upper excursion of this line is set by the upper limit, curve $a$, of the $C$ region. For characteristics which cross curve $d$ and enter the $D$ region a loss of suppression would result. Maximum efficiency occurs with the largest possible excursion of $V_2$, that is for a line tangent to the $d$ curve at the quiescent point. The minimum instantaneous value of $V_2$ for the conditions specified is seen to be 25 per cent of its quiescent value as contrasted with a usual value of about 10 per cent for a pentode when similarly operated. For the instantaneous value of $V_2$ to fall to 10 per cent of its initial value (without loss of suppression) the quiescent point must be at $\gamma = 0$, $\varphi = 1.3$ with the operating line tangent to curve $d$ at about $\varphi = 0.5$. Operation in the $D$ region above $\varphi = 1$ does not result in a secondary current from plate to grid.

### *II. Triode Operation in the Positive Grid Region*

Figure 1 may be taken as representing an idealized triode. The second plane represents the plate so that $V_2 = V_p$ and the first plane represents the grid plane with effective potential $V_1$. We let

$$V_1 = V_g + V_p/\mu,$$

where $\mu$ will be a variable if space charge is formed in the plate-grid region and may differ considerably from its "cut-off" value.

If the fraction of the electrons stopped by the grid is negligible and if operation does not enter the $B$ region, with disturbing effects produced by reflected electrons, then Fig. 1 becomes an accurate representation and in view of the choice of units $\sigma$ is a geometrical constant of the tube. Hence the operating characteristic is a constant $\gamma$ line ($\gamma = \sigma^2$) on Fig. 7.[19]

For normal operation the characteristic must not extend into the $B$ region. If the grid is to be driven far positive it is desirable to stay in the $C$ region for all values of $\varphi < 1$ to prevent secondary electrons from leaving the plate. The minimum value of $\varphi$ as a function of $\gamma$ may be read from the intersections of the constant $\gamma$ lines with limiting curve $a$ if $\gamma > 2$ and limiting curve $d$ if $\gamma < 2$. Now since

$$\varphi = \frac{V_P}{V_1} = \frac{V_P}{V_g + \dfrac{V_P}{\mu}}$$

[19] Proof of the relationship $\gamma = \sigma^2$ is contained in the appendix.

and

$$V_g/V_P = \frac{1 - \varphi/\mu}{\varphi} = \frac{1}{\varphi} - \frac{1}{\mu},$$

if $\mu$ and $\sigma$ are known, the maximum value of $V_g/V_P$ may be calculated. Fig. 7 shows that a value of $\gamma$ just greater than 2 will give the lowest minimum value for $\varphi$ ($\varphi = 0.07$) by intersection with curve *a*. In this region $\varphi/\mu \ll 1$ so that

$$\left(\frac{V_g}{V_P}\right) \text{max.} = \frac{1}{\varphi} = 14.$$

For values of $\gamma$ only slightly less than 2 the minimum value of $\varphi$ rises abruptly, being given by the intersection with curve *d*, while for values greater than 2 the rise in $\varphi$ is not so abrupt.

It should be noted that for the usual cylindrical triode the departure from the parallel plane case is not very marked in the grid-plate region while it can be taken into account by the usual cylindrical formulæ for the cathode grid region. It therefore becomes possible to extend this analysis in an approximate sort of way to cover the cylindrical case.

## APPENDIX

### Definition of Symbols

$V_1$ = Potential of the first plane in volts,
$V$ = Potential of a second plane in volts,
$\varphi$ = Ratio of the potential at the second plane to the potential of the first plane,
$I$ = Injected current in amperes per sq. cm.,
$x$ = Distance in centimeters,
$d$ = Spacing between the two planes in centimeters (used when this distance is regarded as a fixed quantity),
$S$ = Distance between the two planes in centimeters (used when this distance is regarded as a variable),
$\sigma$ = Distance between the planes expressed in units $S_0$,
$S_0 = 1.527 \times 10^{-3} \dfrac{V_1^{3/4}}{I^{1/2}}$ (centimeters),
$a = 1.527 \times 10^{-3} = [10^{13}(2e/m\mathbf{c})^{1/2}/9\pi\mathbf{c}^2]^{1/2}$,[20]
$a^2 = 2.33 \times 10^{-6}$,
$Z$ = Fraction of the injected current $I$ which reaches the second plane,
$t$ = Time in seconds,
$T$ = Electron transit time between the planes in seconds,

$\tau$ = Electron transit time expressed in units of $t_0$,

$t_0 = 7.72 \times 10^{-11} \dfrac{V_1^{1/4}}{I^{1/2}}$ (seconds),

$c = 7.72 \times 10^{-11} = 3a(m\mathbf{c}/2e)^{1/2}10^{-4}$,[20]

$\gamma$ = Injected current expressed in units of $i_0$,

$i_0 = 2.33 \times 10^{-6} \dfrac{V_1^{3/2}}{d^2} = \dfrac{a^2 V_1^{3/2}}{d^2}$ (amperes per sq. cm.),

$\alpha$ = Ratio of the potential at a potential minimum when one exists to the potential of the first plane, otherwise merely a convenient constant of integration. (In the text the symbol $\varphi_{min.}$ is used in the cases where the physical significance can be attached.)

$\beta$ = An integration constant similar to $\alpha$ but associated with an opposite sign.

### Relationship between $\gamma$ and $\sigma$

By definition

$$S_0 = \frac{a V_1^{3/4}}{I^{1/2}} = \frac{S}{\sigma} \tag{1}$$

and

$$i_0 = \frac{a^2 V_1^{3/2}}{d^2} = \frac{I}{\gamma}. \tag{2}$$

Solving (1) for $\sigma$

$$\sigma = \frac{S I^{1/2}}{a V_1^{3/4}}. \tag{3}$$

Solving (3) for $\gamma$

$$\gamma = \frac{d^2 I}{a^2 V_1^{3/2}}. \tag{4}$$

But since $S = d$ for the conditions under which $\gamma$ is used

$$\gamma = \sigma^2 \tag{5}$$

### Derivations

The space charge equation based upon Poisson's equation and the energy equation for an electron is[21]

$$\frac{d^2 V}{dx^2} = \frac{4}{9a^2} I V^{-1/2}. \tag{6}$$

[20] In this work $e$ is the charge on the electron in e.s.u., $m$ its mass in grams, **c** is the ratio of electrostatic and electromagnetic units, 1 volt = ($10^8$/**c**) e.s.u., 1 ampere = (**c**/10) e.s.u.

[21] See, for example, Dow, "Fundamentals of Engineering Electronics," John Wiley, 1937, pg. 100.

This implies that all of the electrons at any plane have the same speed, a condition which must be borne in mind when the present analysis is applied.

Integration of equation (6) yields

$$\left(\frac{dV}{dx}\right)^2 = \frac{16}{9}\frac{I}{a^2}(V^{1/2} + \text{const.}) \tag{7}$$

The choice of zero for this constant leads to type B distributions. A negative constant gives type C distributions, and some of the type D distributions while a positive value gives type A and type D distributions. These will be considered in detail in the sections which follow.

Transit time solutions are obtained by writing the energy equation for an electron in a conservative field which in practical units is

$$\frac{dx}{dt} = \left(\frac{2eV10^8}{m\mathbf{c}}\right)^{1/2} = \frac{3a}{c}V^{1/2}. \tag{8}$$

Solving for $t$ and introducing numerical values

$$t = \int \frac{c}{3a} V^{-1/2}\,dx = 1.68 \times 10^{-8} \int V^{-1/2} dx \tag{9}$$

$$= 1.68 \times 10^{-8} \int \left(\frac{dV}{dx}\right)^{-1} V^{-1/2} dV.$$

Specialization of this equation for the various types is carried out below.

*Integration Constant Zero—Type B*

If the constant in equation (7) is set equal to zero, the next integration gives Child's equation, applicable to the type B distribution when the correct values of current are used, corresponding to conditions before and after the potential zero. Before the potential zero the total current, i.e., the arithmetic sum of injected and reflected currents, is $(2 - Z)I$ so that

$$(2 - Z)I = \frac{a^2 V^{3/2}}{x^2}, \tag{10}$$

where $V$ is the potential $x$ centimeters before the zero. Hence measuring distance from the first plane in units of $S_0$ and expressing potential in units of $V_1$,

$$\sigma_- = \frac{1}{(2 - Z)^{1/2}} - \frac{\varphi^{3/4}}{(2 - Z)^{1/2}}. \tag{11}$$

Similar analysis for the region beyond the potential zero where

$$ZI = \frac{a^2 V^{3/2}}{x^2}, \tag{12}$$

yields

$$\sigma_+ = \frac{1}{(2 - Z)^{1/2}} + \frac{\varphi^{3/4}}{Z^{1/2}}. \tag{13}$$

Introducing the identity $\gamma = \sigma^2$ in equation (13) gives the relationship

$$Z\gamma = \left[ \left( \frac{Z}{2 - Z} \right)^{1/2} + \varphi^{3/4} \right]^2. \tag{14}$$

Some of the limiting curves associated with the $B$ solutions are closely related, as is implied by the use of a common letter. Curve $c$ in Fig. 2 corresponds to maximum values of $\varphi$ for fixed values of $\sigma$ and hence of $\gamma = \sigma^2$. By inspection this is seen to be the same condiion as is implied by curve $c$ in Fig. 9. To find these curves we make $\varphi$ a maximum in (13) with respect to $Z$ holding $\sigma$ constant and obtain:

(c)
$$\begin{cases} Z = 2\varphi^{1/2}/(1 + \varphi^{1/2}), & (15) \\ \sigma = (1 + \varphi^{1/2})^{3/2} 2^{-1/2}. & (16) \end{cases}$$

From these the various other forms of curve $c$ are readily found by the relationships $\gamma = \sigma^2$ and $Z\gamma = Z\sigma^2$. The curve $b$ of Fig. 2 corresponds to $Z = 1$ and gives in equation (13)

(b)
$$\sigma = 1 + \varphi^{3/4}. \tag{17}$$

The minimum transmitted current for fixed $\varphi$, curve $f$ in Figs. 8 and 9, is seen to correspond to $\gamma \rightarrow \infty$; for this the virtual cathode recedes to the first plane and

(f)
$$Z\gamma = \varphi^{3/2}. \tag{18}$$

Introducing $dV/dx$ obtained from equation (7) with the correct current and a zero constant into equation (9) and integrating gives

$$T = \frac{c V^{1/4}}{(2 - Z)^{1/2} I^{1/2}} + \text{const.}, \tag{19}$$

which in units of $t_0$ measured from the first plane with potentials in units of $V_1$ gives

$$\tau_- = \frac{1 - \varphi^{1/4}}{(2 - Z)^{1/2}}. \tag{20}$$

Similar analysis to points beyond the potential zero yields

$$\tau_+ = \frac{1}{(2 - Z)^{1/2}} + \frac{\varphi^{1/4}}{Z^{1/2}}. \tag{21}$$

*Integration Constant Negative—Types C and D*

Introducing for the constant in equation (7) a negative value, say $-(\alpha V_1)^{1/2}$, will give a positive value of $V$ (equal to $\alpha V_1$) for $dV/dx = 0$ with $d^2V/dx^2 > 0$ and must therefore lead to solutions of the C type.

Integrating once more and introducing the unit $S_0$ gives

$$x = \pm S_0(\varphi^{1/2} + 2\alpha^{1/2})\sqrt{\varphi^{1/2} - \alpha^{1/2}} + \text{const.} \tag{22}$$

Expressing distance from the first plane in units of $S_0$, we find two possibilities:

$$\sigma_D = +(\varphi^{1/2} + 2\alpha^{1/2})\sqrt{\varphi^{1/2} - \alpha^{1/2}} - (1 + 2\alpha^{1/2})\sqrt{1 - \alpha^{1/2}} \tag{23}$$

and

$$\sigma_- = -(\varphi^{1/2} + 2\alpha^{1/2})\sqrt{\varphi^{1/2} - \alpha^{1/2}} + (1 + 2\alpha^{1/2})\sqrt{1 - \alpha^{1/2}}. \tag{24}$$

The first of these solutions gives a potential distribution rising continuously as $\sigma$ increases from zero, hence of type D as was anticipated by the subscript. The second solution decreases to a minimum at

$$\sigma_{min.} = (1 + 2\alpha^{1/2})\sqrt{1 - \alpha^{1/2}} \tag{25}$$

and then increases, the equation to the right of the minimum being

$$\sigma_+ = (\varphi^{1/2} + 2\alpha^{1/2})\sqrt{\varphi^{1/2} - \alpha^{1/2}} + (1 + 2\alpha^{1/2})\sqrt{1 - \alpha^{1/2}}. \tag{26}$$

Curves given by equations 23, 24 and 26 are drawn in Fig. 3. If values of $\sigma$ and $\varphi$ corresponding to conditions on the boundary planes are entered in the figure, a $C$ solution is indicated only if the point falls upon a curve of the $\sigma_+$ type. This curve then gives the potential distribution to the right of the minimum; to the left of the minimum the distribution is given by the $\sigma_-$ curve with the same value of $\alpha$, which has the interpretation $\alpha = \varphi_{min.}$ for this case. Points entered on the $\sigma_-$ or $\sigma_D$ curves will clearly give D type solutions.

Three equations for limits of the $C$ region can easily be written down on the basis of the above equations:

(b) $$\sigma = 1 + \varphi^{3/4}. \tag{27}$$

(d) $$\sigma = (1 + 2\varphi^{1/2})\sqrt{1 - \varphi^{1/2}}. \tag{28}$$

(e) $$\sigma = (\varphi^{1/2} + 2)\sqrt{\varphi^{1/2} - 1}: \tag{29}$$

The curve *a* is obtained by making $\sigma_+$ a maximum with respect to $\alpha$ while holding $\varphi$ constant. This gives

$$\text{(a)} \qquad \begin{cases} \alpha = \varphi_{min.} = \varphi(1 + \varphi^{1/2})^{-2}, & (30) \\ \sigma = (1 + \varphi^{1/2})^{3/2}. & (31) \end{cases}$$

Here $\varphi$ and $\sigma$ are coordinates of a point on the *a* curve and $\alpha = \varphi_{\min.}$ is the parameter value for the potential distribution curve tangent to curve *a* at that point. The $\sigma_+$ curves give type C solutions for values before the tangent point and give *C* overlap solutions beyond this point.

All the curves described in this section are readily transformed to current voltage plots by the relationships $\gamma = Z\gamma = \sigma^2$.

The transit times for the various curves are found from equations (7) and (9) using the value $-(\alpha V_1)^{1/2}$ for the constant. Integrating and measuring time from the first plane, they are

$$\tau_D = +(\varphi^{1/2} - \alpha^{1/2})^{1/2} - (1 - \alpha^{1/2})^{1/2}. \qquad (32)$$

$$\tau_{c-} = -(\varphi^{1/2} - \alpha^{1/2})^{1/2} + (1 - \alpha^{1/2})^{1/2}. \qquad (33)$$

$$\tau_{c+} = +(\varphi^{1/2} - \alpha^{1/2})^{1/2} + (1 - \alpha^{1/2})^{1/2}. \qquad (34)$$

*Integration Constant is Positive—Type D*

Type D solutions include those given by equations (23) and (24).

Other solutions are obtained by giving the integration constant of equation (7) a positive value, say $+(\beta V_1)^{1/2}$. Integrating the equation and measuring distances from the first plane in units of $S_0$ we obtain the two possibilities:

$$\sigma = +(\varphi^{1/2} - 2\beta^{1/2})\sqrt{\varphi^{1/2} + \beta^{1/2}} - (1 - 2\beta^{1/2})\sqrt{1 + \beta^{1/2}}, \qquad (35)$$

wh:ch applies for $\varphi > 1$, and

$$\sigma = -(\varphi^{1/2} - 2\beta^{1/2})\sqrt{\varphi^{1/2} + \beta^{1/2}} + (1 - 2\beta^{1/2})\sqrt{1 + \beta^{1/2}}, \qquad (36)$$

which applies for $\varphi < 1$. Corresponding transit times are:

$$\tau = +(\varphi^{1/2} + \beta^{1/2})^{1/2} - (1 + \beta^{1/2})^{1/2}. \qquad (37)$$

$$\tau = -(\varphi^{1/2} + \beta^{1/2})^{1/2} + (1 + \beta^{1/2})^{1/2}. \qquad (38)$$

*Integration Constant is Positive—Type A*

The potential distribution curves of the A type are identical in form with those of the D type, where $\varphi < 1$, which result from the positive value of the constant in equation (7). They differ in numerical values in that the current $I$ must be replaced by the value $2I$ to allow for the reflected current. The correct equation is then

$$\sigma = [(1 - 2\beta^{1/2})(1 + \beta^{1/2})^{1/2} - (\varphi^{1/2} - 2\beta^{1/2})(\varphi^{1/2} + \beta^{1/2})^{1/2}]2^{-1/2}. \quad (39)$$

The corresponding transit times are given by

$$\tau = [(1 + \beta^{1/2})^{1/2} - (\varphi^{1/2} + \beta^{1/2})^{1/2}]2^{-1/2}. \quad (40)$$

To the right of $\varphi = 0$ the space is free of charge so that the potential gradient is constant and equal to the value at $\varphi = 0$ obtained by taking the derivative of equation (39). This value is

$$\frac{d\varphi}{d\sigma} = -\frac{4\sqrt{2}\beta^{1/4}}{3}. \quad (41)$$

## Concerning Completeness

We may now review our work and see that no possible space charge distributions can have been omitted. Starting from the fundamental equation (6) we obtain equation (7) with an undetermined integration constant. Setting this constant equal to zero we could integrate once more, obtaining a solution formally identical with Child's equation. If we supposed that the cathode plane defined by the Child's solution lay to the right of the initial plane, then the only freedom left in the solution was represented by $Z$, the fraction of current passing through the plane. All physically sensible values of $Z$, i.e., 0 to 1, are included in the solutions. If the cathode is assumed to lie to the left of the plane, then $Z$ must equal 1 and the solution which arises is given by $\alpha = 0$ in equations (23) or (26), or $\beta = 0$ in equation (35)—that is, a $D$ solution. For a negative value of the constant, a further integration gave only two possibilities. Each of these was investigated for all possible values of the constant. A similar statement is true for positive values of the constant.

As was stated in the text, space charge distributions corresponding to injection from both bounding planes can be handled in formally the same way as injection from one plane; therefore we may conclude that all solutions to the problem given by specifying the boundary conditions on two planes, subject to the assumptions represented by equation (6), have been determined.

## Equations for Boundary Curves

For convenience we append a table of equations for the limiting curves occurring in the figures.

| Symbol | Equation numbers | $\sigma$ | $\gamma$ | $Z$ | $Z\gamma$ |
|---|---|---|---|---|---|
| $a$ | 31 | $(1+\varphi^{1/2})^{3/2}$ | $(1+\varphi^{1/2})^3$ | 1 | $(1+\varphi^{1/2})^3$ |
| $b$ | 17, 27 | $1+\varphi^{3/4}$ | $(1+\varphi^{3/4})^2$ | 1 | $(1+\varphi^{3/4})^2$ |
| $c$ | 15, 16 | $(1+\varphi^{1/2})^{3/2}2^{-1/2}$ | $(1+\varphi^{1/2})^3/2$ | $\dfrac{2\varphi^{1/2}}{(1+\varphi^{1/2})}$ | $\varphi^{1/2}/(1+\varphi^{1/2})^2$ |
| $d$ | 28 | $(1+2\varphi^{1/2})\sqrt{1-\varphi^{1/2}}$ | $(1+2\varphi^{1/2})^2(1-\varphi^{1/2})$ | 1 | $(1+2\varphi^{1/2})^2(1-\varphi^{1/2})$ |
| $e$ | 29 | $(\varphi^{1/2}+2)\sqrt{\varphi^{1/2}-1}$ | $(\varphi^{1/2}+2)^2(\varphi^{1/2}-1)$ | 1 | $(\varphi^{1/2}+2)^2(\varphi^{1/2}-1)$ |
| $f$ | 18 | $\infty$ | $\infty$ | 0 | $\varphi^{3/2}$ |

The further relationship

$$\alpha = \varphi_{\mathrm{min.}} = \varphi(1+\varphi^{1/2})^{-2} \tag{30}$$

holds for the $a$ curve.

# A Carrier Telephone System for Toll Cables *

By C. W. GREEN and E. I. GREEN

A new 12-channel carrier telephone system for existing cables is described. This system, which incorporates a number of interesting departures from the previous carrier art, is now being manufactured in considerable quantities to meet increased traffic requirements.

AN important advance in the art of carrier telephony has been made by the development of a new 12-channel system, known as the type K, for toll telephone cables of existing type. It is applicable both to cables installed underground, and also to aerial cables, for which the wide range of temperature variation introduces quite difficult transmission problems. Field trials on cables previously installed between Toledo and South Bend have been successful, and the system is now being manufactured to meet field demands.

This new development is an outgrowth of the experiments at Morristown, New Jersey, described by Messrs. Clark and Kendall before the American Institute of Electrical Engineers in 1933,[1] and the essential principles of the new system were included in those experiments. The earlier work dealt, however, with cable specially designed for carrier operation, and only underground cable was experimented with. As that work drew to a close, it became clear that because of general economic conditions several years would elapse before the Bell System would require any substantial increase in toll facilities. Hence this early system was not put into commercial form, but work was continued to determine the extent to which carrier could be applied to existing cables, of which more than 15,000 miles were available for such use. Serious problems of cross-talk at high frequencies had to be reckoned with. A more serious problem, however, was that of maintaining stability of transmission, since with aerial cable, which comprises about two-thirds of the existing cable mileage, the total variation in attenuation, due to temperature variation, is about three times that for underground cable, and the rate of variation not infrequently is several hundred times as great.

In spite of these and other difficulties, the capabilities of the present system go far beyond those of previous systems. As a develop-

* Presented at Winter Convention of A.I.E.E., New York, N. Y., Jan. 24–28, 1938.
[1] For references see end of paper.

ment objective the maximum length was taken as 4000 miles, with as many as five separate systems linked together. On the basis of results thus far obtained it is expected that for these exacting conditions the performance with respect to crosstalk, noise, transmission stability, width of voice band and other characteristics will equal or exceed that of previous facilities for much shorter distances.

Superior performance has been achieved without material effect on the cost of the system. For distances of a few hundred miles, on moderately heavy traffic routes, it will provide telephone circuits at a much lower cost than previous facilities. The minimum distance for which the system will be useful may be less than 100 miles.

Interesting features of the new system are:

(1) A line of very high attenuation, requiring high-gain repeaters spaced at approximately 17-mile intervals. This would mean, for the maximum distance for which the system is designed, more than 200 repeaters in tandem.
(2) The use in the repeaters of the negative feedback principle of amplification to obtain the requisite stability and freedom from modulation.
(3) Small auxiliary repeater stations, established between existing voice-frequency repeater stations, housing equipment which can be left for considerable periods of time without attention.
(4) A system of transmission regulation whereby huge variations of attenuation, differing at each frequency, are automatically equalized to a high degree of accuracy.
(5) New methods of crosstalk and noise reduction. Small adjustable mutual inductance coils are connected between carrier pairs to balance out the crosstalk. The noise is kept at an extremely low level to permit the high gains.
(6) Channel terminal equipment designed so that it may be used in other types of carrier systems, thus simplifying development and manufacture, and facilitating the interconnection of different types of systems.
(7) Speech bands considerably wider than those of existing facilities. The increase is obtained by spacing the channels at uniform 4000-cycle intervals, and employing channel band filters containing quartz crystal elements.
(8) High-speed transmission, which is of considerable value from the standpoint of minimizing delays and echoes.

A general description of the system is presented herein, and the different parts are taken up in greater detail in other papers.

## General Considerations

The type K system, the elements of which are illustrated schematically in Fig. 1, operates on a "four-wire" basis, using the same frequency range, but different electrical paths, for opposite directions of transmission. Thus it differs from open-wire carrier systems, for which the line is not suitable for four-wire operation, and which therefore require complicated and expensive filters to separate the different frequency bands used for transmission in opposite directions. A high degree of shielding between the two cable paths is necessary to avoid the effects of near-end crosstalk, which would be serious because of the large level differences existing at the repeaters and the terminals. On routes where two or more cables exist, such shielding is obtained by employing two separate cables, with transmission in one direction only, in each section of cable. On single-cable routes, a similar arrangement is obtained by adding a small cable. Where there is no cable, two small cables may be provided. Also, satisfactory shielding between the carrier pairs used for opposite directions of transmission has been obtained in short experimental lengths of cable by the use of a layer shield.

### *Frequency Allocation*

In contrast to the original Morristown system, which gave nine one-way channels per pair in the range from 4 to 40 kilocycles, the type K system has twelve channels in the range from 12 to 60 kilocycles. As shown in Fig. 2, the frequency range of the type K system is roughly double that of preceding open-wire carrier systems.[2] The choice of 12 and 60 kilocycles as the lower and upper frequency limits was governed by economic considerations, and there is nothing technically insurmountable either in going to considerably higher frequencies or in utilizing the lower frequency range, which is now idle except for the use of the d-c. path for purposes of transmission regulation and fault location. Important factors influencing the selection of the upper frequency are the crosstalk, which depends on the number of pairs utilized for carrier in one cable and the extent to which special crosstalk balancing means are used, and the attenuation, which largely controls the spacing between repeaters. Factors affecting the lower limit include the difficulty of maintaining accurate transmission regulation over the whole frequency range, and the design of the repeater, which becomes harder as the ratio of maximum to minimum transmitted frequency is increased.

The frequency range between 12 and 60 kilocycles accommodates 12 speech channels, each occupying a gross band of 4 kilocycles. The

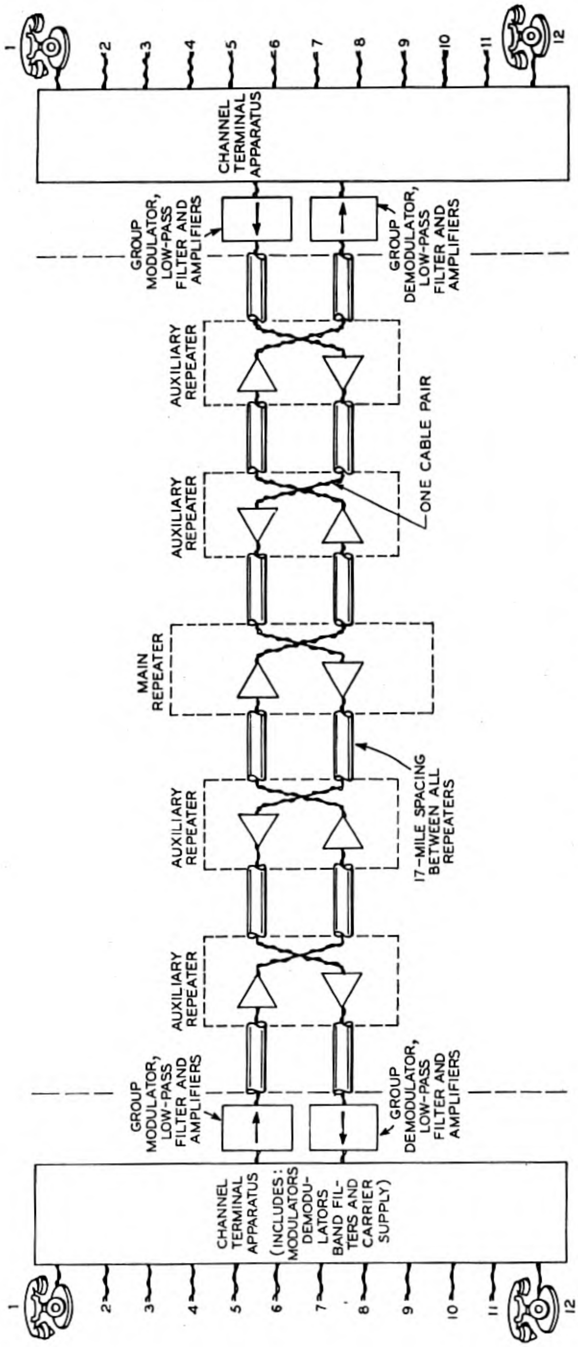


Fig. 1—Schematic of Type K system.

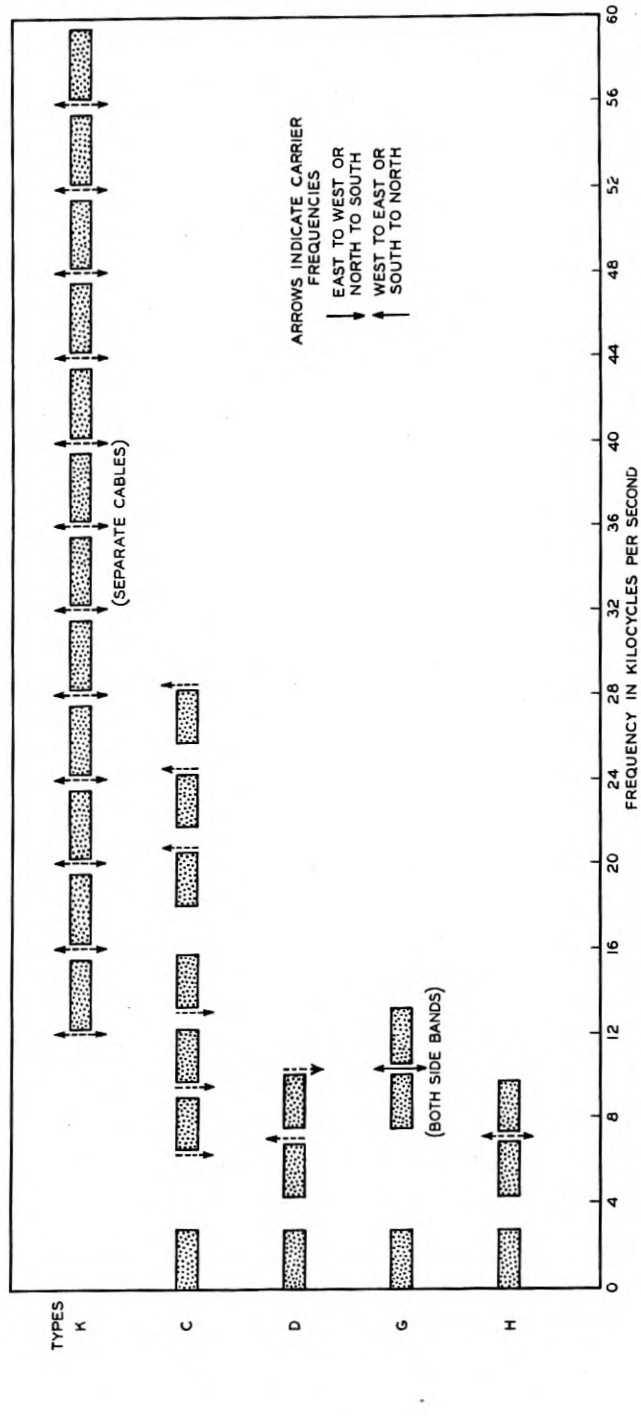


Fig. 2—Frequency allocation of carrier telephone systems.

single sideband method of transmission is employed, with carrier frequencies suppressed. The choice of a group comprising 12 channels was influenced not alone by the requirements of the type K system itself but also by those of other broad-band systems. From the earliest stages of the broad-band development it was recognized that there would be considerable advantage from the standpoints of flexibility of interconnection, of minimum development effort, and of large scale production of equipment units, if the designs of different broad-band systems could be so coordinated as to enable the same design of channel terminal equipment to be employed for each. A common 12-channel terminal unit developed for this purpose is used in the type K system.

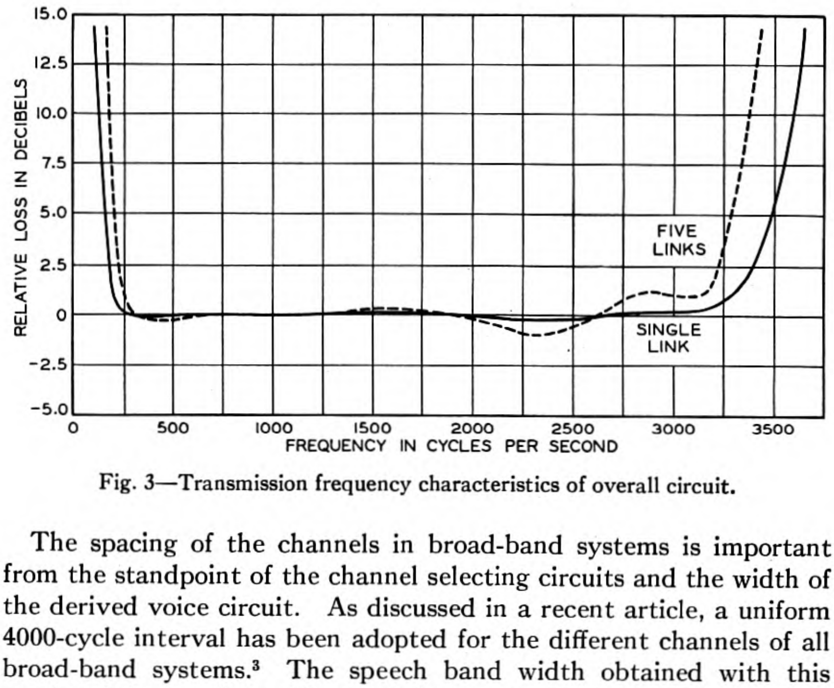


Fig. 3—Transmission frequency characteristics of overall circuit.

The spacing of the channels in broad-band systems is important from the standpoint of the channel selecting circuits and the width of the derived voice circuit. As discussed in a recent article, a uniform 4000-cycle interval has been adopted for the different channels of all broad-band systems.[3] The speech band width obtained with this spacing is in keeping with recent improvements in telephone instruments and other parts of the telephone plant. Overall transmission-frequency characteristics for a single link and a five-link connection are shown in Fig. 3.

*Cable Attenuation*

The type K system is designed to be applied to the No. 19 AWG (0.9 mm.) pairs commonly found in existing cables. (The Morristown system used 16-gauge pairs.) Because the conductors are small and closely spaced, with paper and air dielectric, the attenuation of a non-loaded 19-gauge pair at the frequencies involved is inherently high, as will be seen from Fig. 4. Because of the high attenuation, the repeaters must be placed much closer together than is necessary for voice-frequency cable circuits. Fortunately this effect is partly offset by the fact that it is possible, as discussed later, to use higher gains in the carrier repeaters.

The cable pairs exhibit the rise in attenuation with frequency which is familiar in most transmission circuits. This effect is brought about largely by the increase in conductor resistance, due to skin effect, and

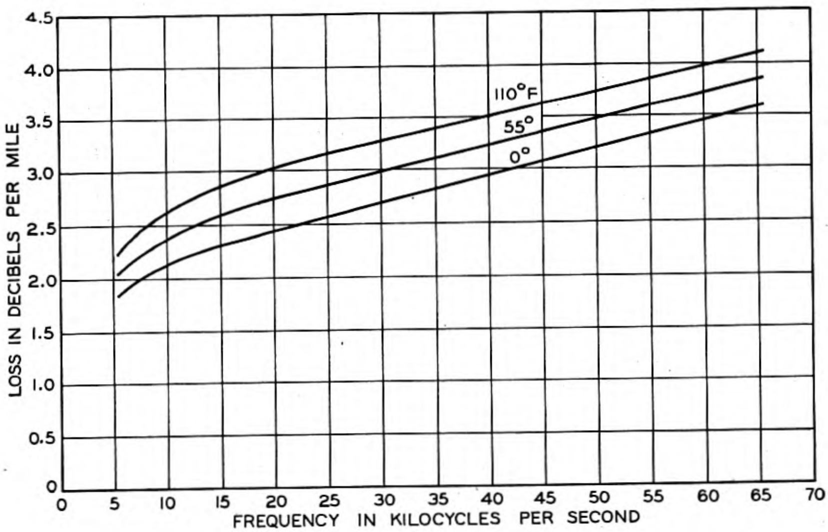


Fig. 4—Attenuation of 19-gauge non-loaded cable pair.

the increasing dielectric losses. More important than this, however, is the fact that the resistance of the wires and the other "constants" of the cable pair undergo variations with temperature, which in turn affect the attenuation. The magnitude of the result for a representative non-loaded 19-gauge cable pair is illustrated by the curves of Fig. 4, which show, respectively, the attenuation for an average temperature, assumed to be 55° F., and for 0° F. and 110° F. The latter values, often taken as the extremes of annual variation for an aerial cable, are in fact frequently exceeded. One reason for this is that when the sun is shining directly on an aerial cable, it may assume a temperature from 15° to 25° above that of the ambient air. The range of temperature variation (and attenuation variation) for an aerial cable may be half as much in one day as in an entire year. For an underground cable, changes of temperature occur quite gradually and the total annual variation is about one-third of that for an aerial cable.

These relations between the attenuation of a cable pair and the frequency and temperature are of fundamental importance in the design of the type K system. First of all, since the attenuation at 60 kilocycles is about 4 db per mile, the total attenuation for a cable circuit of the length used in designing the type K system, i.e., 4000 miles, would be approximately 16,000 db. This must be offset by a corresponding gain.

In the next place, differences in the attenuation at the different frequencies would, if uncorrected, become so great that signals of the less attenuated channels would overload the repeaters, while those of the more attenuated channels would drop down into the noise region. Hence, each repeater must be given a gain-frequency slope which is complementary to the attenuation slope of the line.

Finally, the changes of transmission due to temperature variations and other causes must be compensated so precisely that the net variation in each channel is held within very narrow limits. The method of doing this is explained later. Here it is interesting merely to consider the magnitude of the problem. For the top channel, assuming a 4000-mile circuit, the annual variation in attenuation of an aerial cable pair might be approximately 2000 db. The systems thus far installed have, of course, been limited to much shorter distances than this.

Even if the change of attenuation with temperatures were related to frequency by a simple law, correct compensation over the frequency range would be far from easy. To a casual inspection the differential between any two curves of Fig. 4, for example those for 55° F., and 110° F., will not appear serious. This differential, which becomes very large for a long circuit, is a complicated function of the frequency.

The attenuation differential with temperature can be considered as made up of two components, one which is independent of frequency and another which varies with frequency. The former component, which is much the larger, requires a gain adjustment which is uniform or flat over the frequency range of the system. The latter component is frequently referred to as the "twist." For the range from 12 to 60 kilocycles, the maximum change of attenuation with temperature occurs near 28 kilocycles. Hence this frequency has been used as a datum point in determining the twist. The shape of the twist component is apparent from Fig. 5, which shows the net loss per mile at temperatures of 0° F., and 110° F., assuming that the attenuation has been equalized so as to obtain a flat characteristic at 55° F., and that the gain is then adjusted so as to hold the transmission constant at 28 kilocycles as the temperature varies. Although the twist is small enough so that it need not be corrected at each repeater, it is too large to be allowed to accumulate over a very long distance.

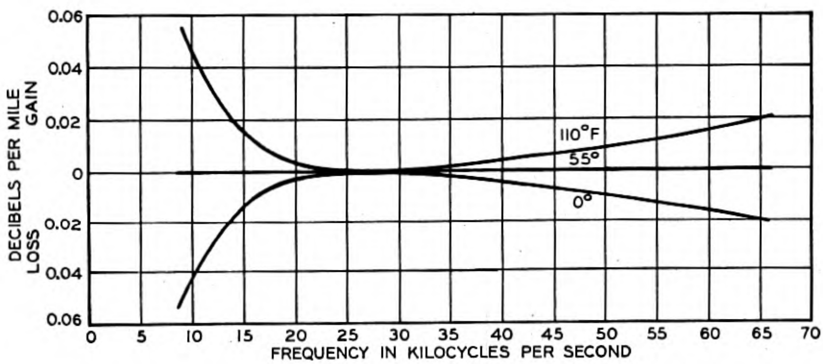


Fig. 5—Twist characteristics of 19-gauge non-loaded cable pair.

*Crosstalk*

As noted above, crosstalk between opposite directions of transmission is avoided by using two separate cables (or shielded compartments in the same cable). To prevent crosstalk in offices, special measures are employed. There remains the problem of "far-end" crosstalk between pairs in the same cable which transmit in the same direction. The pairs are packed closely together, and substantial crosstalk occurs between them because of small departures from symmetry and slight imperfections of twisting. However, by abandoning the use of phantoms and by connecting small adjustable mutual inductance coils between each carrier pair and every other carrier pair, sufficient crosstalk reduction is obtained to permit transmission up to 60 kilocycles on a substantial number of pairs.[4] The scheme is illustrated in Fig. 6.

In the original Morristown cable, the crosstalk was reduced in part by separating the 16-gauge carrier pairs from one another by 19-gauge quads which served as spacers. With existing cables, however, the use of spacers would be impracticable since this would require resplicing the cable at every joint, and therefore reliance must be placed largely on balancing. Since the number of combinations to be balanced increases approximately as the square of the number of pairs employed for carrier, the number of balancing coils required for even a moderate complement of carrier pairs becomes quite large. With 40 carrier pairs, for example, the number of coils required is 780. The balancing coils are mounted on panels as shown in Fig. 7 and are connected together in a crisscross arrangement. Each repeater section is balanced separately, the balancing panels being located in the repeater station.

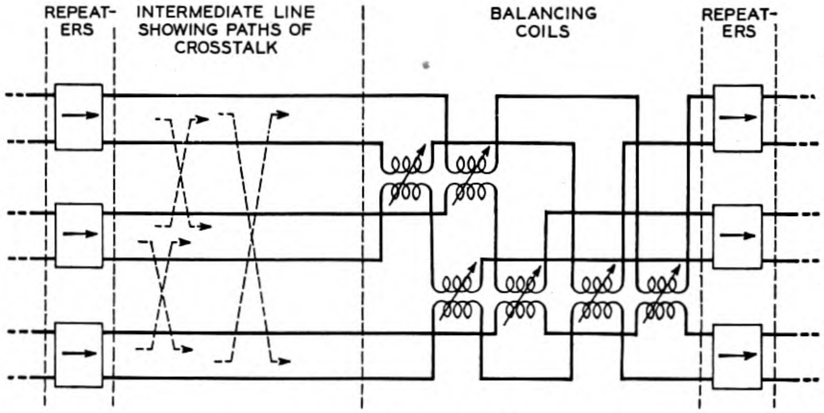


Fig. 6—Method of balancing out crosstalk.

Other measures are necessary to supplement this balancing technique. To reduce the crosstalk coupling, and also to average the transmission characteristics of different pairs, the carrier pairs in different quads of an existing cable are respliced about every mile so as to approach random splicing. The crosstalk coupling between the two sides of a quad is reduced by test splicing at the middle of a repeater section and the quads are split at repeater points. In a new cable, of course, the desired splicing arrangements are introduced at the time of installation. The carrier pairs are also transposed from one cable to the other as indicated in Fig. 1. This avoids interaction crosstalk that would take place, through the medium of the voice-frequency pairs, between the high-level carrier outputs on one side of a repeater station and the low-level inputs on the other side. There is of course, a similar effect between carrier pairs at any point in a repeater section. This is much less serious since no level difference is involved, but it does tend to limit the effectiveness of the balancing over a range of frequencies.

Reflections resulting from impedance irregularities reverse the direction of propagation and therefore produce far-end crosstalk from near-end crosstalk. This crosstalk cannot readily be balanced out over a range of frequencies. To avoid it, it is necessary that the impedances of successive lengths of cable pair be substantially uniform and also that the impedance of the equipment be closely matched to the characteristic impedance of the cable pair.

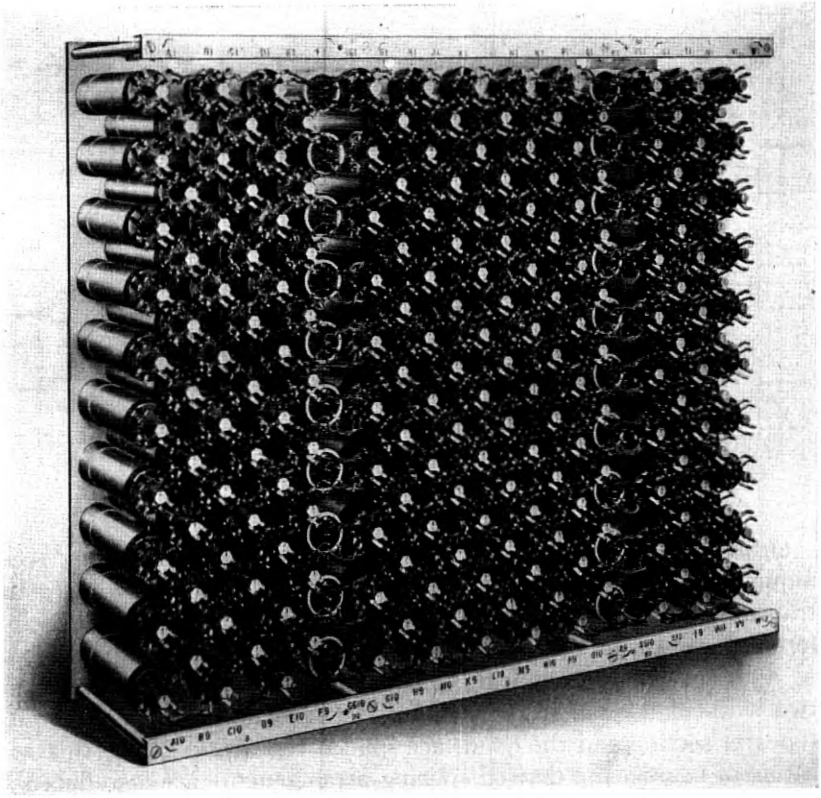

Fig. 7—Crosstalk balancing panel.

*Noise*

The cable pairs are fairly well protected by the lead sheath from external electrical disturbances. However, high-frequency noise originating in the voice-frequency repeater stations due to relay operation, etc., would, unless prevented, enter the cable over the voice-frequency pairs and thence would be induced in the carrier pairs. Such noise would be excessive at the low-level carrier inputs. It is avoided by connecting, in each voice-frequency pair in the "low level" cable on each side of a voice-frequency repeater station, a coil which suppresses longitudinal noise currents. Similarly, it is necessary to keep high-frequency noise from entering the cable where open-wire pairs tap into it and frequently also where branch cables are connected. For this purpose simple noise suppression filters are employed. With these and other measures the noise on the carrier pairs can, at the highest frequencies where the amplification is greatest, be brought within a few db of the basic noise due to thermal agitation of electricity in the conductors themselves.

*Velocity of Transmission*

Voice waves travel through loaded cable circuits at from 10,000 to 20,000 miles per second, the higher speed being used for the longer circuits. On very long connections, even if echo suppressors are employed this velocity results in transmission delays which introduce difficulties in conversation.[5] The use of non-loaded conductors for the type K systems results in an overall velocity of transmission of about 100,000 miles per second, a speed so high that such difficulties are greatly reduced and satisfactory telephone conversations are possible over the longest distances for which connections may be required.

## Repeaters

Since the noise level in the cable circuits can be made quite low, the carrier currents may be permitted to drop to levels below those used on voice-frequency circuits or on open-wire carrier circuits, and the repeaters may have higher gains. In the cable carrier system, the noise has been so reduced that the level of the top channel at the repeater input may on the average be dropped about 60 db below the voice level at the transmitting switchboard. The amplifier gains at the top frequency range from about 50 to 75 db, and the output level of each of the 12 channels is about 10 db above that at the switchboard. The average repeater spacing is about 17 miles.

The tube which was developed for the gain stage of the amplifier is a pentode with indirect heater. The heater requires a potential of 10 volts and a current of 0.32 ampere and the plate 150 volts. The tube in the power stage is similar in type but requires a heater current of 0.64 ampere at 10 volts. With this power tube a feedback of about 40 db has been found to provide a satisfactory reduction of interchannel modulation.[6] Both tubes were designed to have long life with very reliable performance.

*Description of Amplifier*

Each repeater comprises two amplifiers of the type illustrated in Fig. 8. A schematic diagram of the amplifier circuit is shown in

Fig. 9. Three stages with impedance coupling are used and the feedback circuit is connected between the plate circuit of the last tube and the grid circuit of the first. The amount of gain and the slope of the gain-frequency characteristic are controlled by the condensers and line equalizer in the feedback circuit.

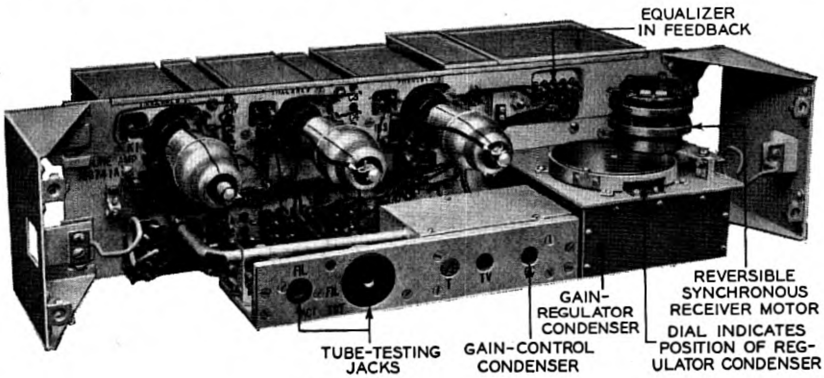


Fig. 8—Line amplifier.

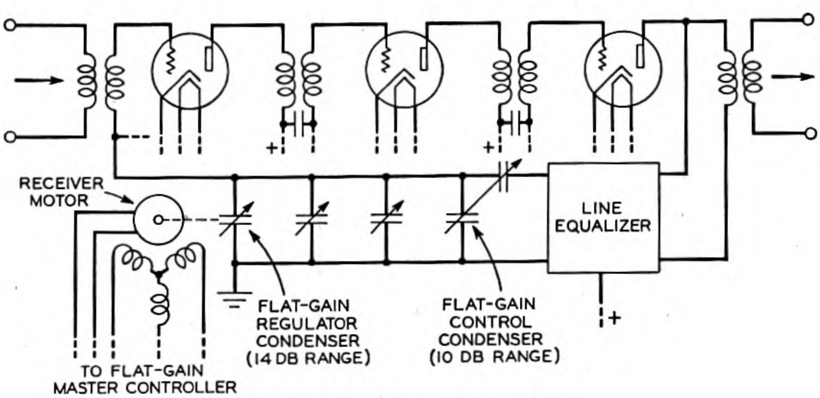


Fig. 9—Schematic of line amplifier.

The line equalizer has the same attenuation slope as the line. In the introduction of this equalizer in the feedback circuit careful attention to phase shift requirements was required. Four types of equalizers are available, for different repeater spacings, to compensate for the cable distortion which occurs at a temperature of 55° F., additional means being provided to compensate for variations which occur as the cable temperature swings away from this value. The solid curve of Fig. 10 shows a repeater gain characteristic with one of these equalizers in the feedback circuit.

The correction introduced by the line equalizers is subject to errors which, although small at each repeater point, become important for a moderate length of system. Supplementary equalizers have been designed to correct for these. Two types of deviation are considered,

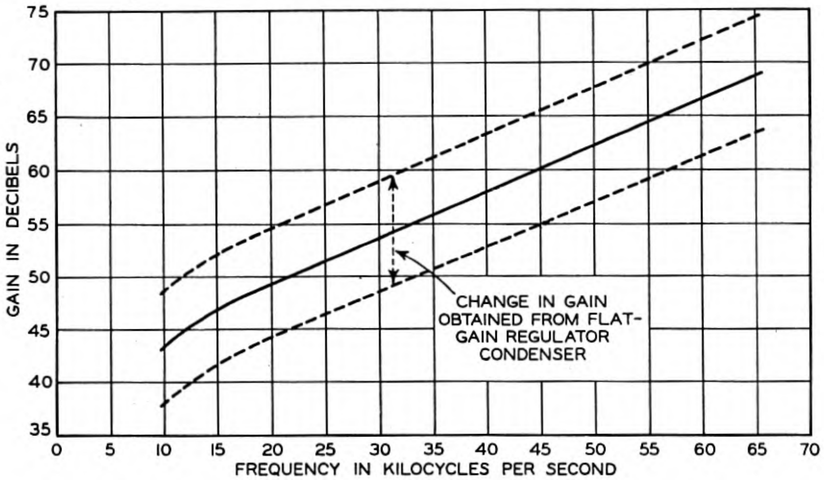


Fig. 10—Amplifier gain characteristics.

that of the cable and that of the amplifier. As the characteristics of cables manufactured at different times show slight departures from one another, two shapes are required to correct their deviations. There is one correction for concave deviations and another for convex.

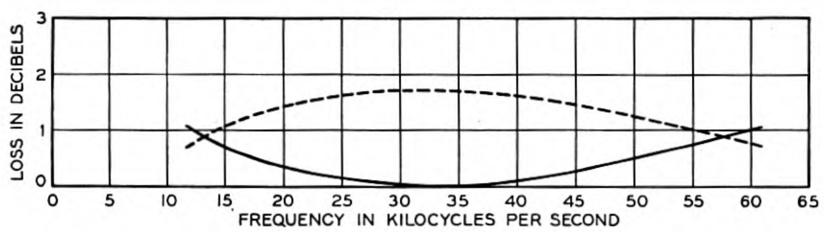


Fig. 11—Characteristics of cable deviation equalizers.

The amplifier requires but one type of correction. The characteristics of these equalizers are shown in Figs. 11 and 12. The equalizers for amplifier deviations are used about every 10th repeater and those for the cable deviations, at distances of 300 to 400 miles. At normal temperature (55° F.), the correction applied by these networks will, for a 500-mile system, result in a frequency characteristic which is flat to within less than 2 db over the range from 12 to 60 kilocycles. For a longer circuit further corrective measures will be provided.

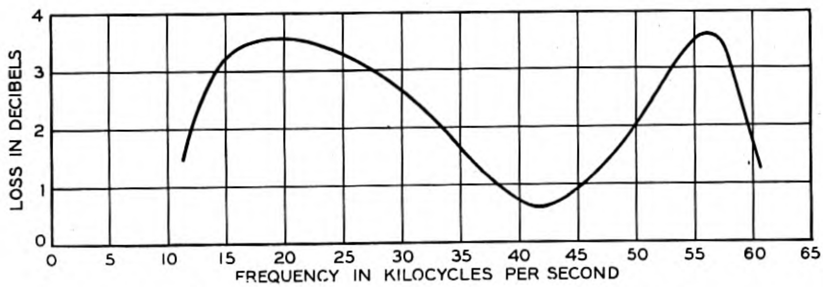


Fig. 12—Characteristics of amplifier deviation equalizer.

*Regulation for Temperature Effects.*

The method adopted for controlling the repeater gain to compensate for temperature changes is similar to that which has been found satisfactory for voice-frequency cable circuits. This is the pilot wire method in which a pair of cable conductors extending over the section

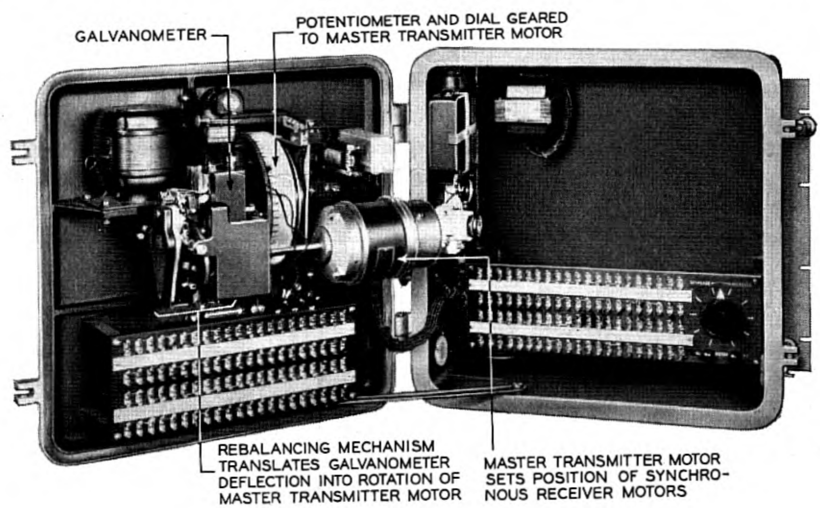


Fig. 13—Flat gain master controller.

to be regulated forms one arm of a Wheatstone bridge. This bridge is designed for automatic self-balancing and the mechanical motion required for establishing the balance has been made to adjust the gain of the amplifiers. The d-c. resistance of the pilot wire gives an accurate indication of the temperature of the carrier pairs which determines their attenuation to a close approximation. The motion of the bridge mechanism is communicated to the repeater amplifiers by means of self-synchronizing motors, a master motor being associated with the bridge and an individual motor with each amplifier.

With aerial cable a flat gain correction must be made at every repeater. With underground cable the flat gain correction may be

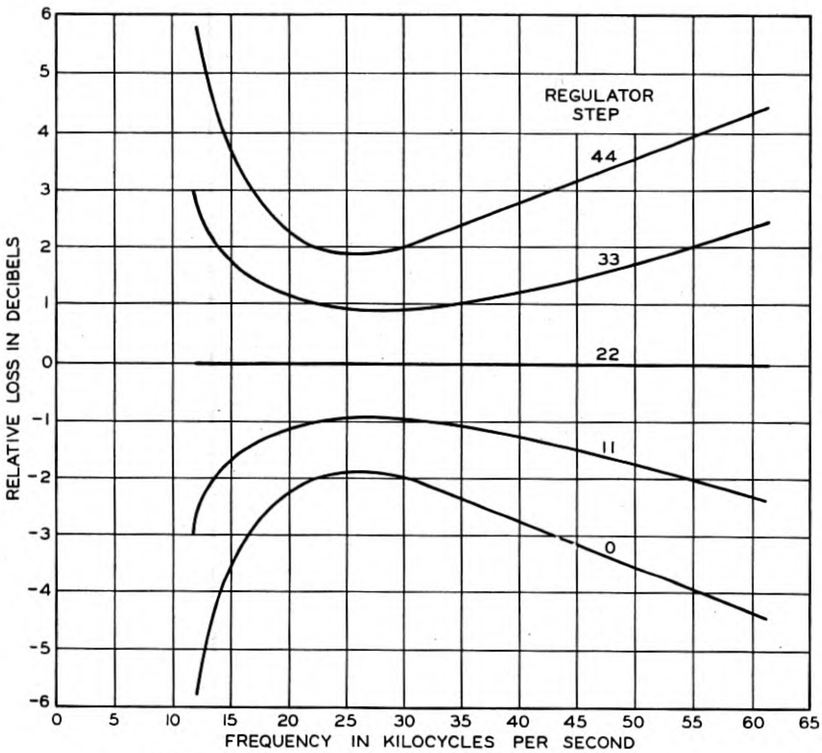


Fig. 14—Characteristics of twist regulator networks.

omitted at some repeater points. Figure 13 shows a master controller with its galvanometer, driving motor, and self-synchronizing motor. The air condenser in the feedback circuit of Fig. 8 makes this correction. The small self-synchronizing motor which may be seen in this figure is geared to the condenser and it moves the air condenser corresponding to the motion of the master motor. The resulting change in repeater gain is virtually the same for all frequencies in the transmitted band. In Fig. 10 the repeater gain is plotted against frequency for three angular positions of the condenser.

As was mentioned earlier, an additional correction for the residual effect or "twist" is required about every six repeaters to supplement the flat gain adjustment. This distortion is a function of frequency and has been found to vary from cable to cable. A network the characteristics of which are shown in Fig. 14 has been developed to meet this condition. Certain fixed resistances in the network are selected to correspond to the length and twist characteristic of the cable section considered. A variable resistance in the network is adjusted automatically using a control similar to the flat gain regulator. Figure 15 gives the transmission characteristics of a 150-mile regulator-controlled circuit under two temperature conditions.

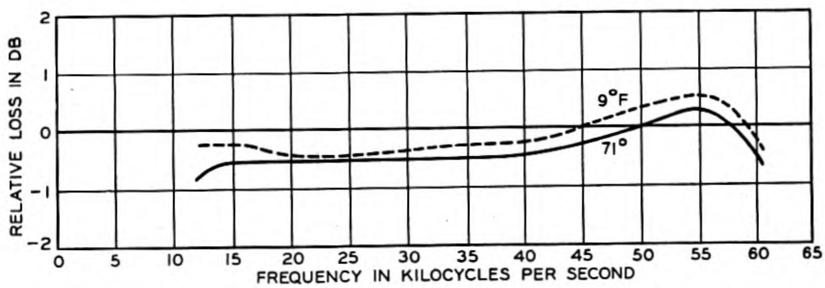


Fig. 15—Overall transmission-frequency characteristic of 150-mile line.

*Auxiliary Repeater Stations*

Cable carrier systems are expected to be used largely on existing toll cable routes which now carry voice-frequency circuits. The average spacing of the stations housing the voice-frequency repeaters on these routes is about 50 miles. The same buildings with their power plants will also care for the cable carrier repeaters. Since the maximum spacing for the carrier repeaters is about 19 miles, additional carrier repeaters must be provided at intermediate stations (two is the usual number). The various design features of the equipment to be located in these stations have been made the subject of extensive development work and field tests. These stations are designed to function with a minimum of attention and are visited at intervals for routine testing work or as required by some emergency, but resident maintenance forces are not planned for them. The present equipment is expected to be suitable not only for auxiliary stations on existing cable routes but also for cases where a greater spacing than 50 miles between the attended stations may be desired on new routes.

A voice-frequency repeater station for a single cable and a cable carrier auxiliary station are shown to approximately the same scale in Figs. 16a and 16b, respectively. Many of the existing voice-frequency stations are even larger than that shown in Fig. 16a. The auxiliary building shown in Fig. 16b has about 600 feet of floor space

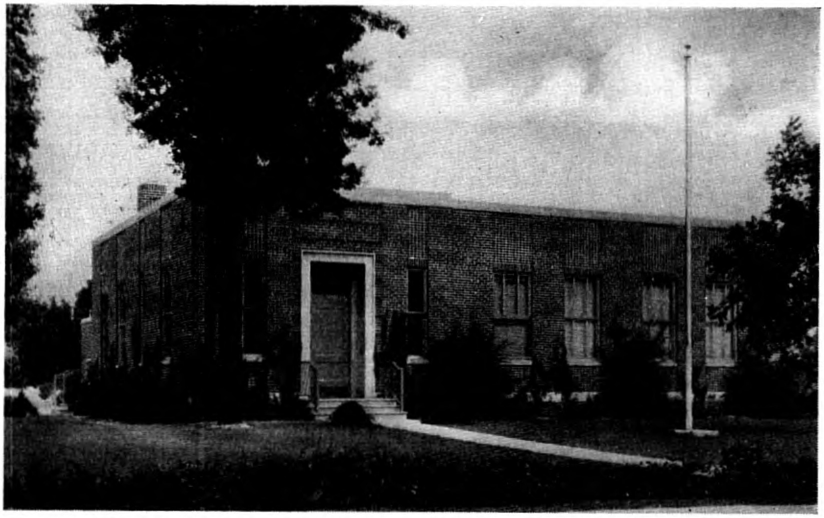

Fig. 16*a*—Voice frequency repeater station on single cable route.

with a ceiling height sufficient to take care of 11′6″ relay racks. This building will house 100 repeaters with necessary auxiliary equipment, thus providing ultimately for a total of 1200 carrier circuits. The interior of a typical auxiliary station is shown in Fig. 17.

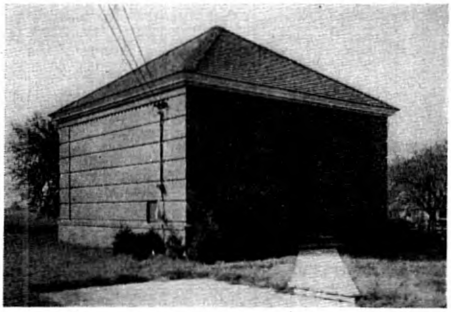

Fig. 16*b*—Auxiliary cable carrier repeater station.

The main power plant for the repeaters consists of a 152-volt storage battery, which is continuously floated across a grid-controlled rectifier fed from the 60-cycle power mains. The voltage of the entire battery supplies the plate voltage for the tubes. Each amplifier requires about 22 volts for the tube heaters and this is obtained by dividing the battery into seven sections, each section supplying several amplifiers in parallel. Additional power supplies of 55-volt alternating

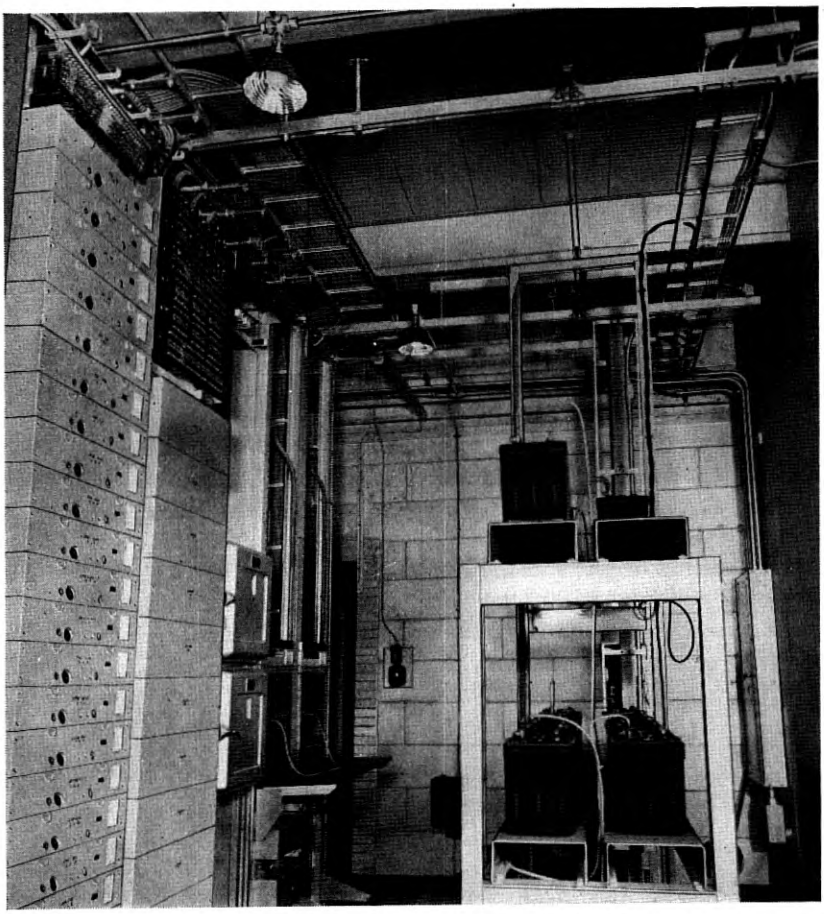

Fig. 17—Interior of auxiliary repeater station.

current and 140-volt direct current are required for the regulator system.

In the station, there are alarm circuits, which signal the nearest attended office if trouble develops. There are alarms for blown fuses, high or low battery voltages, power failures, etc. A telephone order wire to the nearest attended station is provided for the maintenance force.

In addition to the line amplifiers with their regulating equipment, there are racks mounting the crosstalk balancing coils. There are also sealed terminal units between the outside cable or the balancing units and the office cable. These furnish access to the line or equipment through jacks.

## Terminals

The minimum distance over which a cable carrier system can be operated economically is determined in large measure by the cost of the terminal apparatus. Hence, the field of usefulness of the system is greatly increased by keeping the terminal cost as low as is consistent with satisfactory performance. Numerous developments during the past few years in connection with modulation, filtering and methods of carrier supply have all contributed materially toward this end. At the same time, the standards of performance have not only been maintained, but in many respects substantially improved.

### *Channel and Group Modulation*

In the design of the terminals for the type K system, a number of circuit arrangements were considered, the final choice being influenced to a considerable extent by the conditions imposed upon the filters. As noted above, the desirability of using the channel terminal equipment in other broad-band systems, such as those for open-wire or coaxial cable, was also an important factor. The circuit arrangements selected have a first stage of modulation which raises the voice frequencies of the 12 channels up to a range of 60 to 108 kilocycles. This range is favorable to the use of crystal type band filters,[7] which have transmission characteristics superior to the coil and condenser type and seem to be no more costly. For the type K system, a single stage of group modulation shifts the frequencies to the range required on the line, 12 to 60 kilocycles, and a similar stage at the receiving end returns them to the 60 to 108-kc. range. Other carrier systems will also use the 60 to 108-kc. channels and by group modulation shift them to the desired position in the frequency spectrum.

The band filter occupies a space on the relay rack equal to 1/8 of that required by the coil and condenser type which was used in the earlier model of this system. Its attenuation characteristic in the transmitting region is flat to within 1 db over a range of about 3100 cycles. Immediately outside of this range the attenuation rises very rapidly, thus permitting very efficient use of the frequency spectrum.

Another new device on the terminal is the copper-oxide unit used in the modulating process. These units are expected to show a stability of the same order as that of coils and condensers, and require practically no maintenance as compared to vacuum tubes.

The translation of the channels from the 60 to 108-kc. range to the position required for cable carrier, 12 to 60 kilocycles, is made by a stage of group modulation. A copper-oxide group modulator is used and a carrier frequency of 120 kilocycles. The reverse of this process in a similar group demodulator at the receiving end steps the frequency back to its original range, 60 to 108 kilocycles. These processes of modulation take place at points of low-energy level in the circuit with a comparatively high level of carrier, so that the inter-channel crosstalk which results from unwanted products of modulation is unobjectionable. Low-pass filters are inserted after the group modulator and demodulator, and amplifiers with flat gain characteristics are supplied to raise the levels of the output currents of the group modulators or demodulators.

### *Carrier Supply*

The carrier frequencies which are required at a terminal are obtained from the harmonics of a base frequency. The carrier supply system is common to as many as 10 systems in one office. This simplification was made possible by the selection of the channel frequencies as multiples of a base frequency, 4 kilocycles being chosen for this system. This base frequency is produced by an oscillator in which the control element is a tuning fork, the whole unit being designed to have the necessary output and frequency stabilities. The output of the oscillator is amplified and fed to a circuit which produces the desired harmonics. All of the carrier frequencies which are required for the different channels as well as for group modulation and demodulation are obtained from these harmonics. A small coil with a permalloy core is the important agent in this process.[8]

Failure of the 4-kc. supply, or failure of the 120-kc. supply used for group modulation, would cause failure in the channels of all systems operating from this supply. Provision is made for such a contingency by an emergency carrier supply which is automatically switched into service when the regular supply fails. This reserve source duplicates all of the parts of the regular supply, 4-kc. fork, amplifier, harmonic producer, and amplifier for the 120-kc. carrier.

### *Assembly*

The different panel units which make up the terminal of a type K system are assembled on a functional basis with similar panels of other K systems, the channel modulator-demodulator panels in one bay, the carrier supply in a second, the group modulator and demodulator in a third, etc. The compactness of the equipment makes it possible to mount the modulators and demodulators for 18 channels on one 11 ft. 6 in. bay 19 inches wide.

*Signaling*

The same type of ringdown signaling equipment is used with the channels of this system as with the voice-frequency toll circuits. A 1000-cycle tone, interrupted 20 times per second, is impressed on a channel terminal, modulated, and transmitted over the carrier system in the allotted channel band. At the far end, it is demodulated to operate the receiving end of the standard voice-frequency signaling circuit, or to be transmitted along an extended voice-frequency circuit to its terminal.

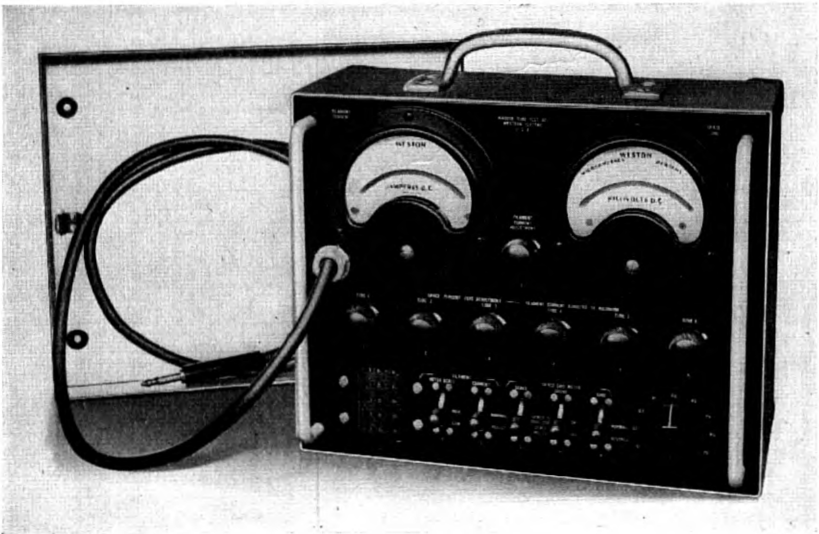

Fig. 18—Vacuum tube test set.

*Telegraph and Program Applications*

Voice-frequency telegraph can be superimposed on any of the carrier channels as is now done on the three-channel open-wire systems. Equipment is being developed to include a program channel on the cable carrier system. This will be done by devoting to the program circuit the frequency space occupied by two of the 4-kc. speech bands.

## System Maintenance

Arrangements are provided whereby the tubes may be tested on a routine basis as has been done in voice-frequency practice. The amplifier panels, however, are provided with test jacks which are con-

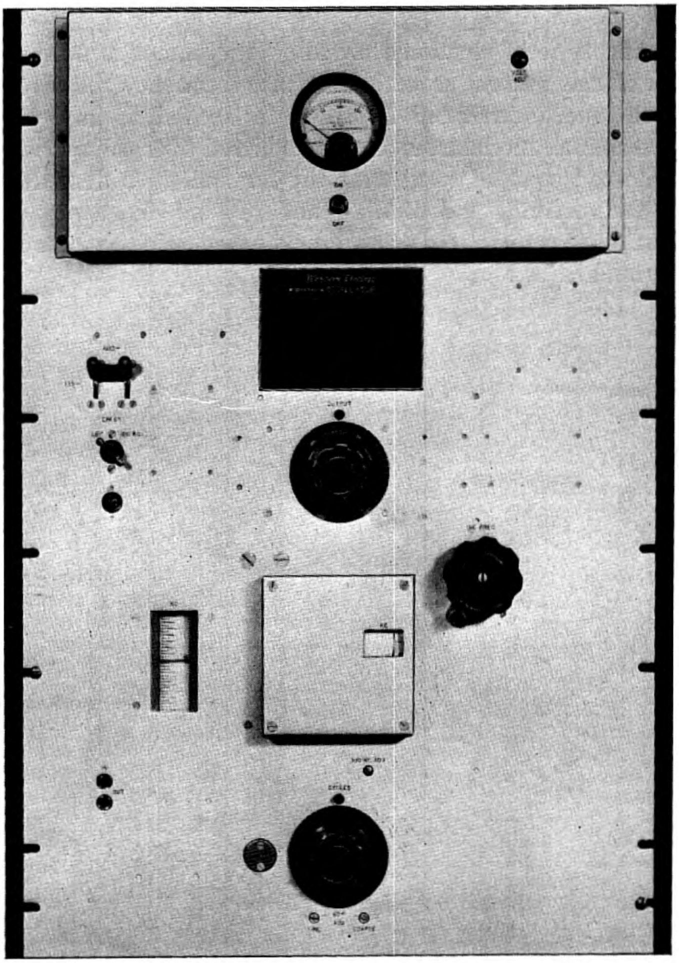

Fig. 19—Testing oscillator.

nected to resistances in the plate circuits. A reading of the voltage across the resistance gives a measure of the plate current for the associated tube without disturbing the amplifier performance while the amplifier is in service. A portable tube testing set, Fig. 18, has been designed for this measurement.

Provision is being made for the removal of an amplifier from an active circuit without interruption of service. A spare amplifier at each repeater station can be substituted for the active one by connecting it to jacks at the sealed terminal and operating associated relays to make a quick transfer.

Apparatus is also furnished which permits the substitution of a new link between attended points for one which develops trouble. A complete high-frequency circuit for each direction of transmission will generally be reserved as a spare. It can be substituted for any working

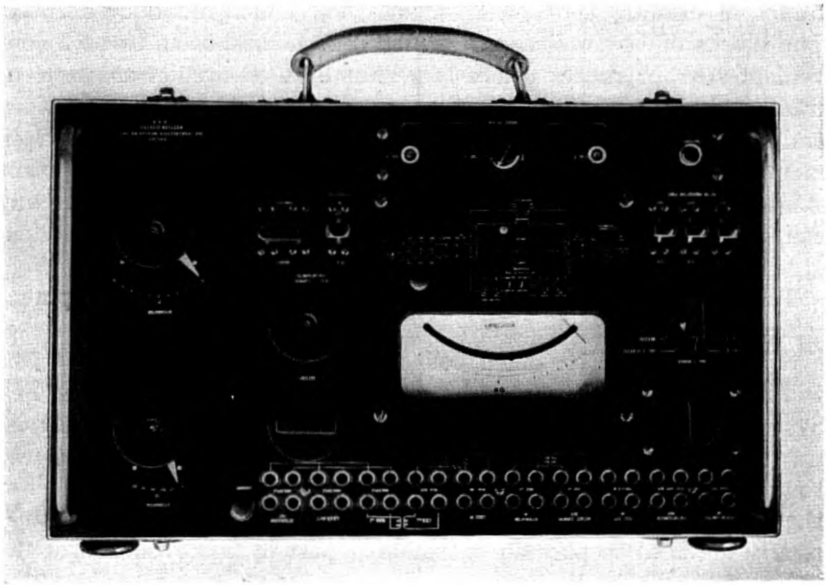

Fig. 20—Transmission measuring set.

high-frequency circuit without interfering with service by paralleling the transmitting ends of the spare and working circuits and patching the receiving ends through relays. The operation of a key controlling these relays substitutes the spare circuit for the working one with a transient disturbance of but 1 or 2 milliseconds.

Three pilot frequencies, 15.9, 27.9 and 55.9 kilocycles, which are produced at the transmitting terminals, may be used to check the levels at the main repeater points and the receiving terminals. This is done by means of a special testing circuit which can be bridged across a pair to detect the level of the pilots without interference to service.

A heterodyne oscillator having a frequency range from 60 cycles to 150 kilocycles has been developed for use in testing this and other carrier systems. Its frequency is calibrated at 60 cycles against the power mains and at 100 kilocycles against a quartz crystal. This oscillator is shown in Fig. 19. A portable test set, developed for measuring transmission gains and losses with high precision, is shown with the cover removed in Fig. 20.

## Conclusion

The type K system makes possible the application of carrier to toll cables of existing type, whether installed underground or aerially. The blocks of 12 circuits each, which it furnishes, seem to be a convenient size for routes where large numbers of circuits are concentrated. It is to be expected, of course, that substantial modifications and improvements will be made in this system through further development effort. In its present form, however, it constitutes an important stage in the history of carrier development. Plans already under way call for the application of large numbers of such systems to meet rapid growth in long distance traffic.

This new system forms merely one phase of a concerted development effort on broad-band carrier transmission systems.[9, 10] There is every indication that, taken collectively, these broad-band systems will have far reaching effects upon the toll telephone plant of the Bell System. A transition is already under way from the time when carrier was used only on open wire, and comprised only a small part of the toll plant, to a time when carrier systems will furnish a major part of the toll circuit mileage of the Bell System. The type K system is clearly destined to play an outstanding part in this evolution of the toll plant along carrier lines.

## References

1. "Carrier in Cable" by A. B. Clark and B. W. Kendall, *Electrical Engineering*, Vol. 52, page 477, July 1933; also *Bell Sys. Tech. Jour.*, Vol. XII, p. 251, July 1933.
2. "Carrier Systems on Long Distance Telephone Lines" by H. A. Affel, C. S. Demarest and C. W. Green, *Bell Sys. Tech. Jour.*, Vol. VII, pages 564–629, July 1928; also *Electrical Engineering*, Vol. 47, pp. 1360–1367, October 1928.
3. "Transmitted Frequency Range for Circuits in Broad Band Systems" by H. A. Affel, *Bell Sys. Tech. Jour.*, Vol. XVI, p. 487, October 1937.
4. A. G. Chapman, U. S. Pat. No. 1863651; M. A. Weaver and O. H. Coolidge, U. S. Pat. No. 2008061; M. A. Weaver, U. S. Pat. No. 2080217.
5. "The Time Factor in Telephone Transmission" by O. B. Blackwell, *A.I.E.E. Transactions*, Vol. 51, pages 141–147, March, 1932; also *Bell Sys. Tech. Jour.*, Vol. XI, pp. 53–66, January 1932.
6. "Stabilized Feed-Back Amplifiers" by H. S. Black, *Electrical Engineering*, Vol. 53, p. 114, January 1934.

7. "Electrical Wave Filters Employing Quartz Crystals as Elements" by W. P. Mason, *Bell Sys. Tech. Jour.*, Vol. XIII, p. 405, July 1934.
8. "Magnetic Generation of a Group of Harmonics" by E. Peterson, J. M. Manley and L. R. Wrathall, *Electrical Engineering*, Vol. 56, No. 8, p. 995, August 1937; also *Bell Sys. Tech. Jour.*, October 1937.
9. "Wide-Band Transmission in Sheathed Conductors" by O. B. Blackwell, *Bell Telephone Quarterly*, Vol. XIV, p. 145, July 1935.
10. "Systems for Wide-Band Transmission Over Coaxial Lines" by L. Espenschied and M. E. Strieby, *Electrical Engineering*, Vol. 53, pp. 1371–1380, October 1934; also *Bell Sys. Tech. Jour.*, Vol. XIII, pp. 654–679, October 1934.
11. "Modern Systems of Multi-Channel Telephony on Cables" by A. S. Angwin and R. A. Mack, *Journal of the Inst. of Electrical Engineers*, Vol. 81, No. 941, p. 573, November 1937.

# Cable Carrier Telephone Terminals *

By R. W. CHESNUT, L. M. ILGENFRITZ and A. KENNER

This paper describes the circuits, performance and equipment features of the terminals of a new 12-channel carrier system for application to existing toll cables. The 12-channel group of terminal apparatus has been designed also to form a basic part of the terminals of other carrier systems now under development, such as the type J system for open wire and the coaxial system.

## Introduction

ABOUT twenty years ago the first commercial carrier telephone system was installed between Baltimore and Pittsburgh. Until recently, telephone circuits were obtained by carrier methods largely on open-wire lines. The notable exceptions were on short deep sea submarine cables.[1, 2] Ten years ago, experiments were initiated which have now resulted in the design of a carrier system which can be applied with substantial economy to existing long distance toll cables on land. Its general features are described in another paper.[3] The present paper describes in detail the circuits and performance of the carrier terminals of this system.

## General Features

The carrier system for existing cables, designated type "K," is designed to provide twelve telephone channels in the frequency range between 12 and 60 kilocycles, using one non-loaded 19-gauge paper insulated cable pair in each direction. Previous carrier systems employed for open-wire lines used vacuum tubes for the modulating or translating circuits and electrical filters composed of coil and condenser networks for separating the frequency bands associated with the respective channels. The terminals of the new type "K" system are simpler and yet provide improved performance by using copper oxide bridges for the modulation function and quartz crystal filters [4] for the separation of the individual channel bands.

The quartz crystal filter is economical only in a comparatively high-frequency range, necessitating the use of high intermediate frequencies. The high intermediate frequencies are reduced by a second stage of modulation to the desired range of frequencies for transmission over the

* Presented at Winter Convention of A. I. E. E., Jan. 24–28, 1938.

line. Copper oxide bridge circuits again are used for this group modulation stage. In all cases they are connected to suppress the carrier. To provide the various carriers required for modulation and demodulation, a carrier supply system has been designed somewhat along the lines of an office power distribution system using bus bars and protective arrangements for the various carriers. Each carrier supply system is capable of supplying as many as ten carrier terminals, or a total of one hundred and twenty two-way channels.

Because of the large number of circuits involved, every effort has been made to provide reliable operation of the carrier supply and common terminal equipment. The terminal and carrier supply equipment is designed to permit maintenance tests for checking the performance of amplifier tubes and to permit switching between regular and spare equipment without interruption of the large number of circuits involved.

The emphasis placed upon ease of maintenance and the necessity for more careful handling of higher-frequency circuits have resulted in new equipment design features. These include new cable terminals, new shielded office cabling, and panels arranged for front wiring and maintenance which are mounted on racks having wiring ducts at both edges of the bays. In the following sections a more detailed description is given of the circuits, their performance, equipment and maintenance features.

## Circuits

The frequency allocation for one direction of transmission and a block schematic of one terminal are shown in Figs. 1 and 2, which supplement each other and need little explanation. The twelve voice bands shown at the left in Fig. 1 are modulated individually in the channel modems.* This forms a 12-channel block lying between 60 and 108 kc. which is then modulated in the group modulator by a 120 kc. carrier to move the block down in the range from 12 to 60 kc. for transmission to the distant terminal. On the receiving side the processes are reversed. One of the channels, as well as the group modem of Fig. 2, is presented in more circuit detail in Fig. 3. This shows the circuit from the point where the voice comes into the carrier system to the point where the twelve carrier sidebands go out onto the cable and vice versa.

At the left the four-wire terminating circuit serves, not only as a device to transform from a two-wire to a four-wire circuit, but also as a

* The term "modem" has been coined to mean a panel or equipment unit in which there is both a modulator and a demodulator to take care of both the outgoing and the incoming signal.

high-pass filter to eliminate, from the input to the carrier system, noises below about 200 cycles, such as telegraph harmonics, 20-cycle ringing, 60-cycle power, etc., which may be present on connected voice-frequency circuits. Otherwise these noise frequencies, which are below the voice range, would modulate and pass through the terminal to load unnecessarily the carrier repeaters along the line, as well as to interfere with the level indications of the pilot channels.

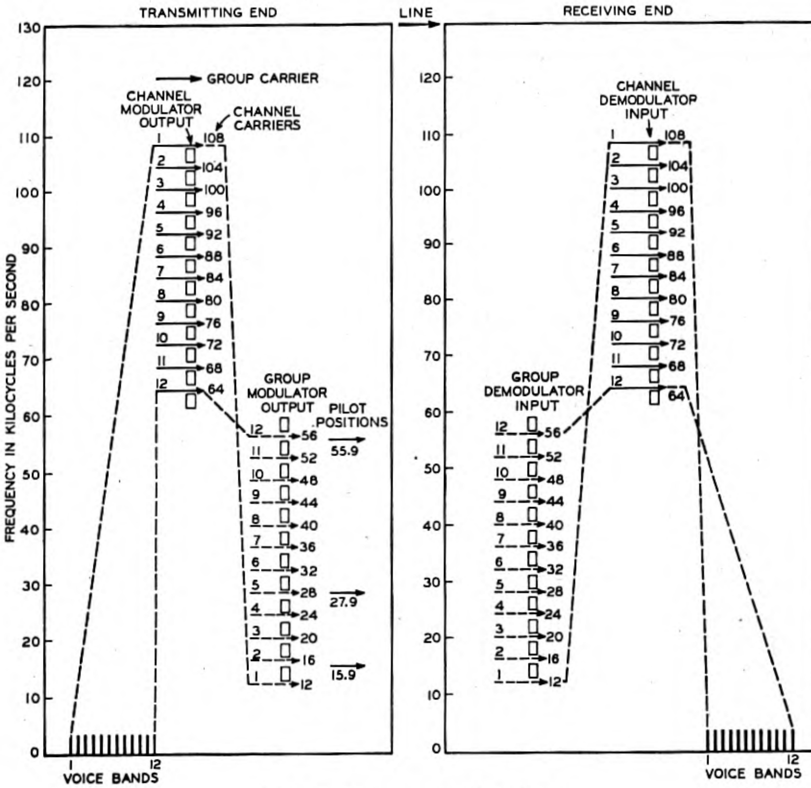


Fig. 1—Frequency allocation.

From the terminating equipment the circuit loops through jacks which have paralleled contacts for reliability. The level at this point is − 13 db compared with the transmitting toll switchboard, which level is expected to be generally used in the Bell System for all multi-channel carrier telephone systems. Then comes the channel modulator which consists of four copper-oxide discs, each three-sixteenths of an inch in diameter, potted in a small can. This makes a very simple and inexpensive modulator which is much more satisfactory than tubes.

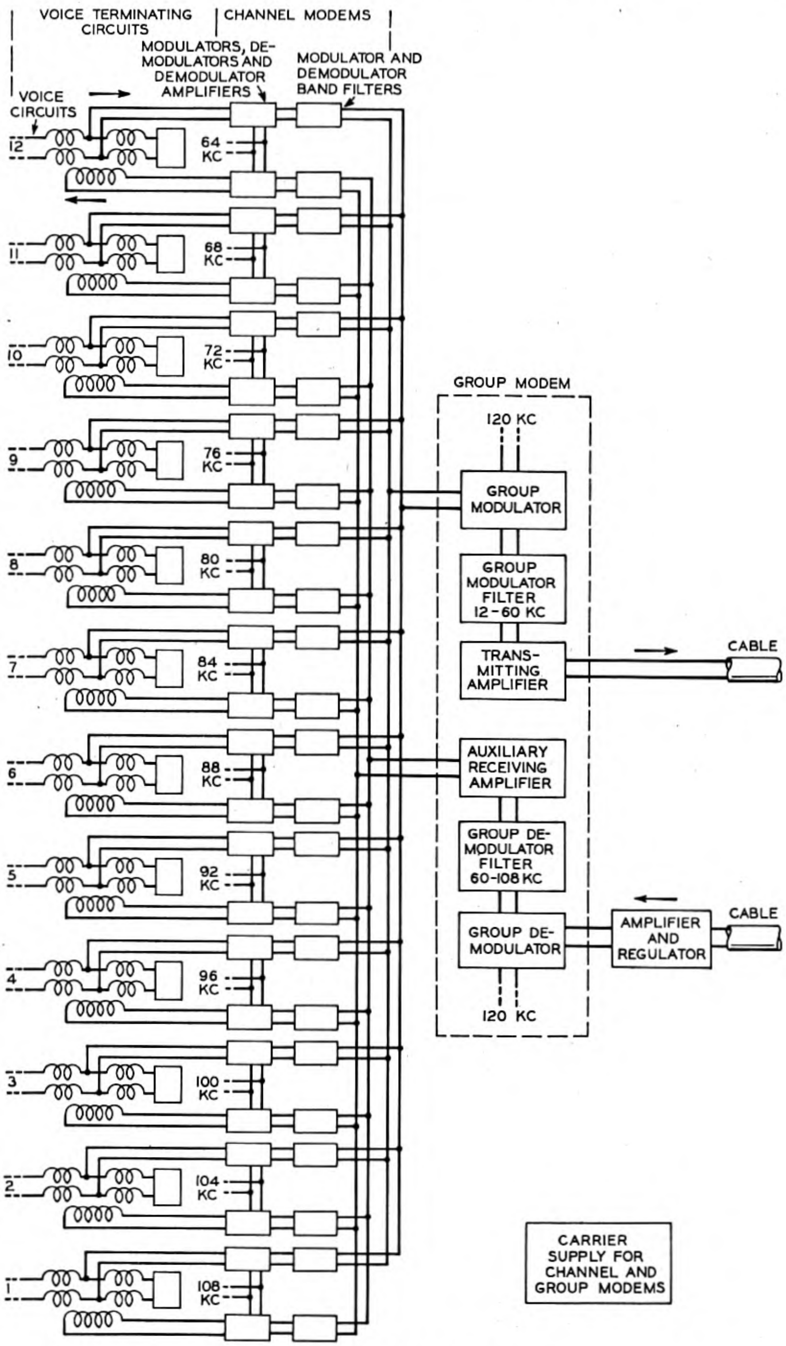


Fig. 2—Block schematic.

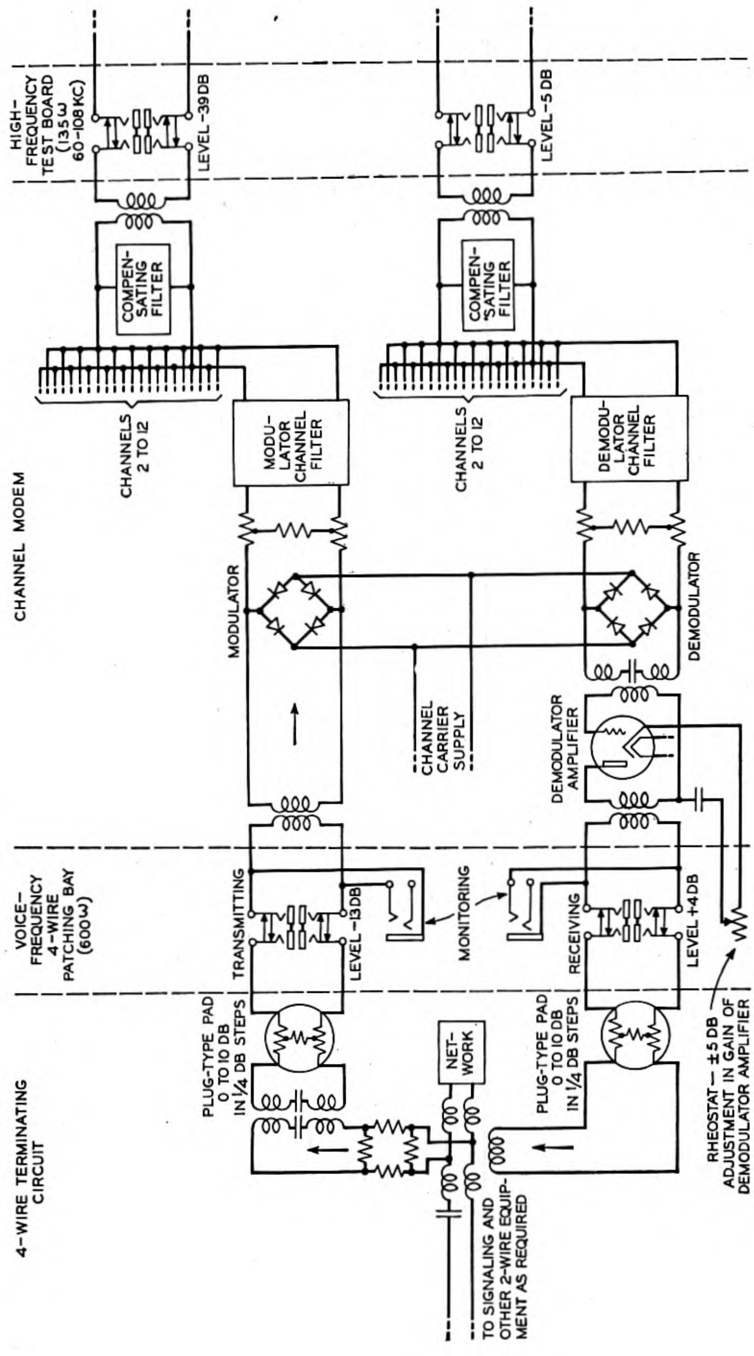


Fig. 3—Circuit schematic.

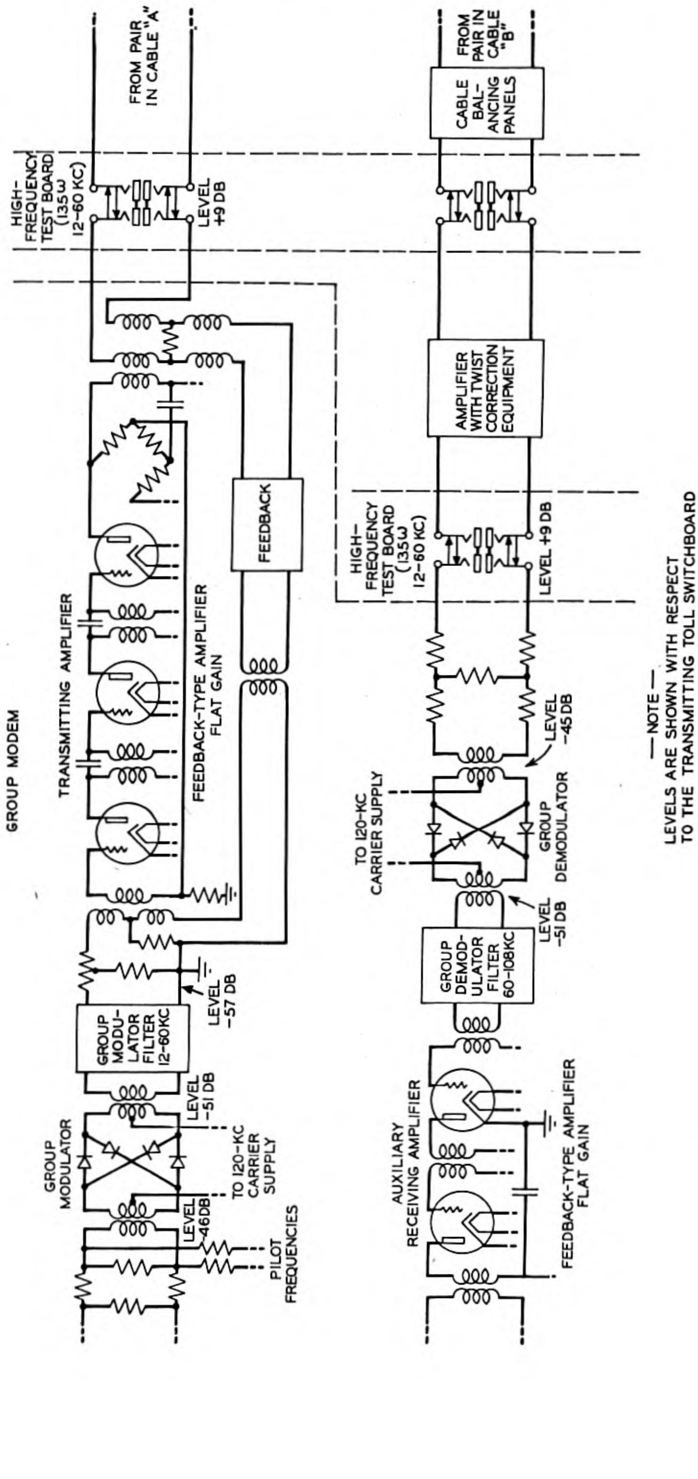


Fig. 3—*Continued from page 110.*

It seems to have an indefinite life. (Some have been on life tests as modulators for about five years.) The carrier power required is about 1/2 milliwatt to modulate satisfactorily a single telephone circuit level of − 13 db.

The modulator produces the usual two sidebands and the lower one is selected by the quartz crystal channel filter described in another paper.[4] This sideband, joined by eleven others, is stepped down to about the iterative impedance of the shielded office cabling. In the office cabling the twelve channels pass through the high-frequency patching bay to the double balanced group modulator of copper oxide where they are joined by three pilot channel frequencies.

The group modulator uses the same copper oxide as that in the channel modulator described above, but the carrier power is about 50 times greater (about 25 milliwatts) in order to keep down unwanted modulation produced between the twelve sidebands. To that same end the level of each sideband is made low (− 46 db), and the double balanced type of circuit is used to balance out some of the undesired products. It also balances out the twelve incoming bands in the range 60 to 108 kc. from the output and so simplifies the following group modulator filter.

From a level of − 57 db the twelve channels, now in the range from 12 to 60 kc., are amplified to + 9 db for delivery to the 19-gauge pair in the lead covered toll cable. The amplifier is a three-tube negative feedback type, using pentodes and operating with 154 volts plate battery which is composed of the usual 24-volt filament battery and 130-volt plate battery in series. The last tube is a power tube and does not overload until a single-frequency output of about one watt is reached.

On the receiving side in Fig. 3, the twelve incoming channels, in the range from 12 to 60 kc., pass from the amplifying and regulating equipment,[3] to the group demodulator. This is identical with the group modulator described above and transfers the twelve channels to the range 60 to 108 kc. The channels are then amplified to a − 5 db level by an amplifier of the negative feedback type using two low-power pentodes with 154-volt plate battery as described above for the transmitting amplifier.

From there the twelve channels are separated by the filters which are identical with those on the transmitting side, and are then demodulated and amplified to a + 4 db level as shown for one channel in Fig. 3. The demodulator is identical with the modulator but it is poled oppositely on the carrier supply so that the d-c. components of modulation in the modulator and demodulator neutralize each other and thereby avoid developing an undesirable voltage bias. The poling also reduces somewhat the amount by which stray frequencies have to be suppressed in the carrier supply. The demodulator amplifier has a slide wire gain control rheostat to equalize channel levels, which functions by changing both the grid bias on the tube and the amount of negative feedback which is introduced by the rheostat. The sliding contact in the slide wire is made practically free from contact trouble by the space current of the tube flowing through it. As the rheostats are only about 1000 ohms and small in size, they can easily be mounted at a distance from the amplifier in the voice-frequency jack field.

The carrier supply for the twelve channels from 64 to 108 kc., and for the group modems of 120 kc., is derived in the circuit shown in Fig. 4. A regular generator is shown at the top in solid lines and an emergency generator at the bottom is shown in dotted lines. Between the two is an automatic transfer circuit (in dotted lines) which transfers to the emergency whenever the regular generator fails to supply the proper amount of 120 kc. to the 120 kc. bus.

At the upper left-hand corner is shown a 4-kc. tuning fork, of an alloy having a low temperature coefficient driven by the tube to its right to operate as an oscillator of very stable frequency. The next, or control tube, amplifies the 4 kc. to drive the push-pull power stage where a power of about 4 watts is developed. This passes through the 4 kc. filter to the non-linear coil where odd harmonics of 4 kc. are produced. The underlying principles of operation of this coil have been published.[5] To derive even harmonics of 4 kc., the copper-oxide bridge is used which rectifies about half the energy of the complex wave of odd harmonics but, by balance, greatly reduces the amount of the odd harmonics present in its output. Odd harmonics are obtained at one point and even at the other. This separation into odd and even harmonics by the balance of the copper-oxide bridge provides effective loss of about 30 to 40 db and reduces the requirements on the carrier supply filters which follow.

The two branches pass through hybrid coils to the banks of channel carrier supply filters. These separate the frequencies and feed them to twelve carrier supply bus bars, one for each channel frequency. From these the individual modems are fed through protective resistances so that an accidental short circuit on one of the modems will not cut off the carrier supply to the others.

The hybrid coils permit the two generators to be connected so that either can feed into the same bank of channel carrier supply filters without being reacted upon by the other. No switching is required when changing from regular to emergency supply.

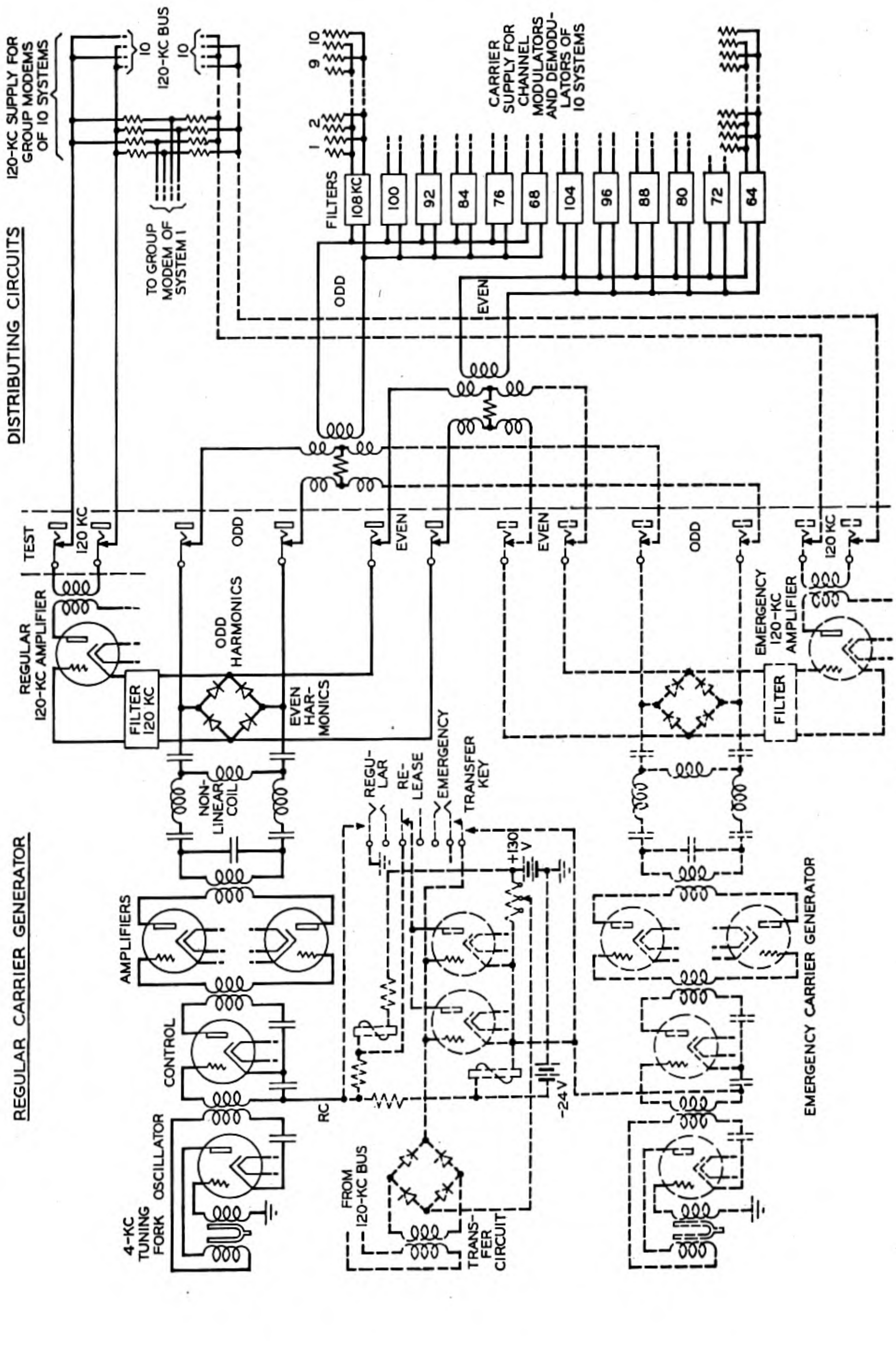


Fig. 4—Carrier supply circuit.

The 120-kc. carrier which feeds the group modulators of ten systems or a total of one hundred and twenty talking channels must be very dependable. Therefore separate filters are used for the regular and emergency supply and separate amplifiers for the large power required by group modulators. Regular and emergency distributing buses are provided. Each group modulator and each group demodulator is wired through protective resistances to the regular bus and through another set of protective resistances to the emergency bus. With this arrangement an accidental short circuit even across one of the busses or across one of the output coils of one of the 120-kc. amplifiers will not stop the whole supply of 120 kilocycles.

The 4-kc. oscillator of the emergency generator is in constant operation so that when it is needed no time is required to start it, but the grid bias on the second tube is held above its cutoff value by the automatic transfer circuit. This prevents the 4 kc. from going further until called for in an emergency.

An emergency is indicated when there is no 120-kc. supply on either the regular or emergency bus. When this happens, the copper-oxide rectifier in the transfer circuit gets no 120 kc. and so loses its rectified voltage. This triggers off one or both of the two gas-filled tubes (multipled for safety) which increases the grid bias on the control tube of the regular generator to stop its 4 kc. supply and at the same instant restores the bias to normal on the control tube of the emergency to let its 4 kc. pass through and put the whole emergency circuit into operation. The keys in the transfer circuit are provided for maintenance purposes, and to return from emergency to regular operation, since the gas tube circuit is arranged to transfer automatically in only one direction.

The pilot supply circuit is shown in Fig. 5. The 3.9-kc. tuning fork oscillator at the left supplies that frequency, through the three transformers, to the three copper-oxide modulators the carriers of which are obtained from the regular channel carrier supply bus-bars as shown. The three filters, which are identical with channel carrier supply filters, select the lower sidebands to be used for pilot frequencies at 64.1, 92.1 and 104.1 kc. The three pilot frequencies are distributed to the different systems through protective resistances from a bus-bar as shown. They are set 100 cycles off the carrier frequencies to obtain locations of minimum interference from carrier leak and other sources.

Signaling circuits do not form an integral part of the carrier terminal equipment. Signaling equipment of a type already widely used in the Bell System for toll circuits, is connected between the toll switchboard and the four-wire terminating set of the individual channel.

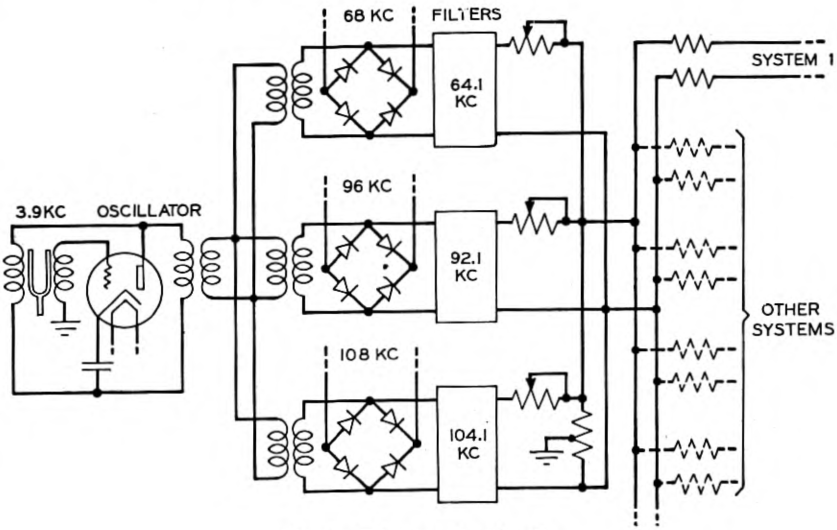


Fig. 5—Pilot supply circuit.

## Transmission Performance

In general, the performance requirements set down as objectives in the development of this system were based on the assumption that five carrier links operating in tandem and over a 4000-mile circuit should give satisfactory, high-grade service.

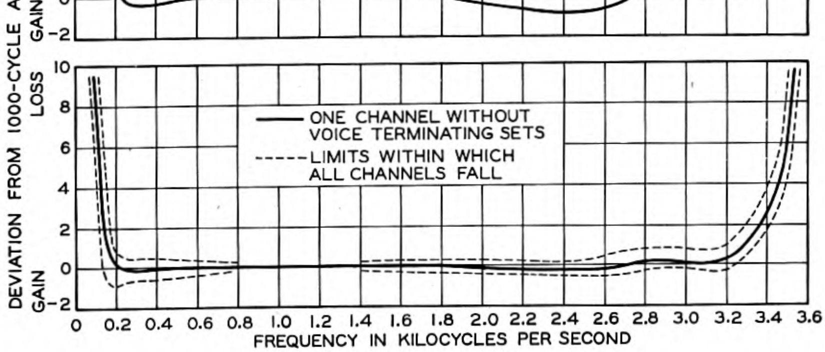


Fig. 6—Channel frequency characteristics.

The channel frequency characteristic which has been attained in the terminals is shown in Fig. 6. The solid curve below shows the frequency characteristic of a representative channel, while the dotted curves near it show the limits within which the characteristics of all single channels, so far measured, would fall. Above in the figure is shown the characteristic of five representative channels in tandem, each channel having its two voice terminating circuits included.

The delay distortion and time of transmission, contributed by all terminal apparatus at both ends of a system except voice terminating sets, are shown in Fig. 7 for a single channel.

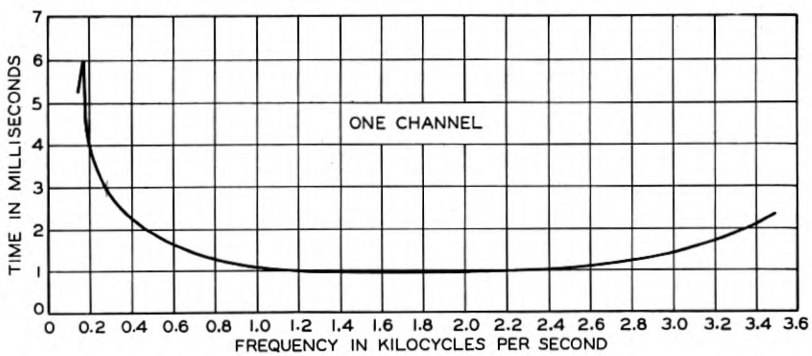


Fig. 7—Delay characteristic.

The channel modulators have been adjusted so that they will cut off the peaks of excessively loud talk to prevent overloading the carrier repeaters or other parts of the circuit, but this cutting is not enough to degrade the quality of speech. The single-frequency load curve of one complete channel is plotted in two ways in Fig. 8.

The frequency stability of the oscillating tuning forks is expected to be within $\pm 1 \times 10^{-6}$ parts per degree Fahrenheit on all systems, with negligible variations due to other causes. The amplitude stability of each frequency at its distributing bus is expected to be within $\pm$ 1/4 db over a period of months. The impedances of the bus-bars are sufficiently low so that crosstalk from one system into another through this path is unimportant. The effectiveness of the protective resistances at the carrier supply bus-bars is such that a short circuit on one modulator or demodulator will increase the loss in the remaining modulators and demodulators less than 1/2 db. The speed of switchover to emergency carrier supply is such that the disturbance to transmission will be less than 10 milliseconds. The effect on speech is not detectable.

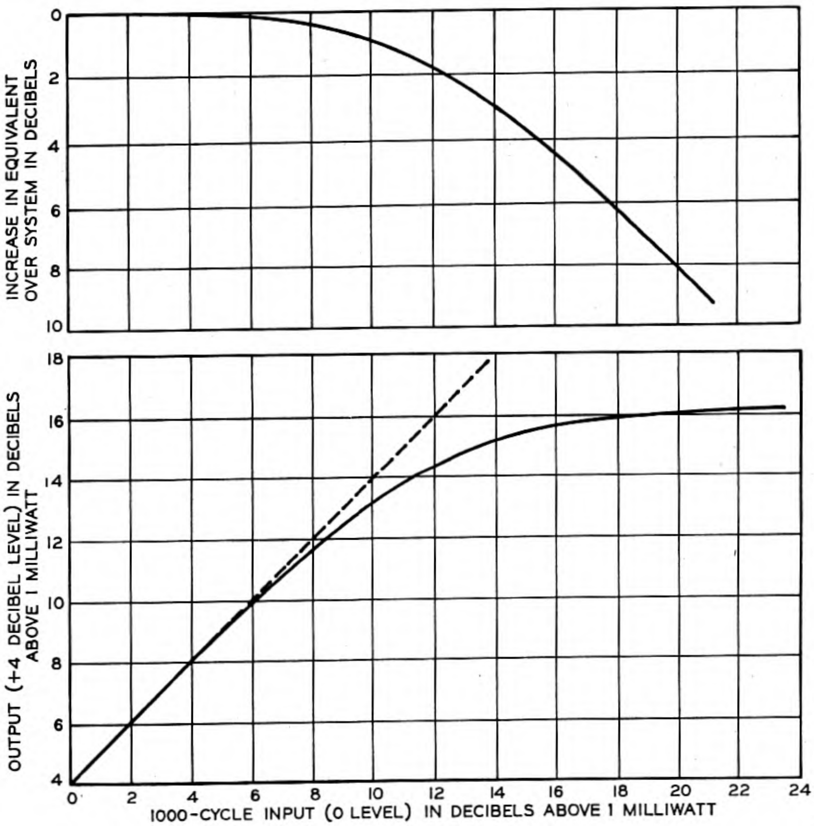


Fig. 8—Channel load curve at 1000 cycles.

## Maintenance Features

Since the type "K" system provides more circuits in a group than ever before, it is essential that appropriately better maintenance facilities be furnished. Wherever vacuum tubes are used, jacks have been included to permit testing the condition of the tube by plugging in a new type of test set. The testing of a working tube with this set will not produce an appreciable reaction on performance of the circuits involved. When it has been determined that a tube in the common equipment is nearing the end of its useful life, a special transfer cord circuit is used to remove the circuit involving the tube from service and to substitute a spare circuit temporarily while the defective tube is replaced. This transfer from a regular to a spare and vice versa can be made without effect upon service.

In a type K terminal office transmission tests are made at the four-wire test board shown in Fig. 9, *A*, where the incoming and outgoing voice-frequency circuits appear, and at the high-frequency test board shown in Fig. 9, *B*. At the former, four-wire talking, monitoring and testing circuits have been provided for voice-frequency maintenance.

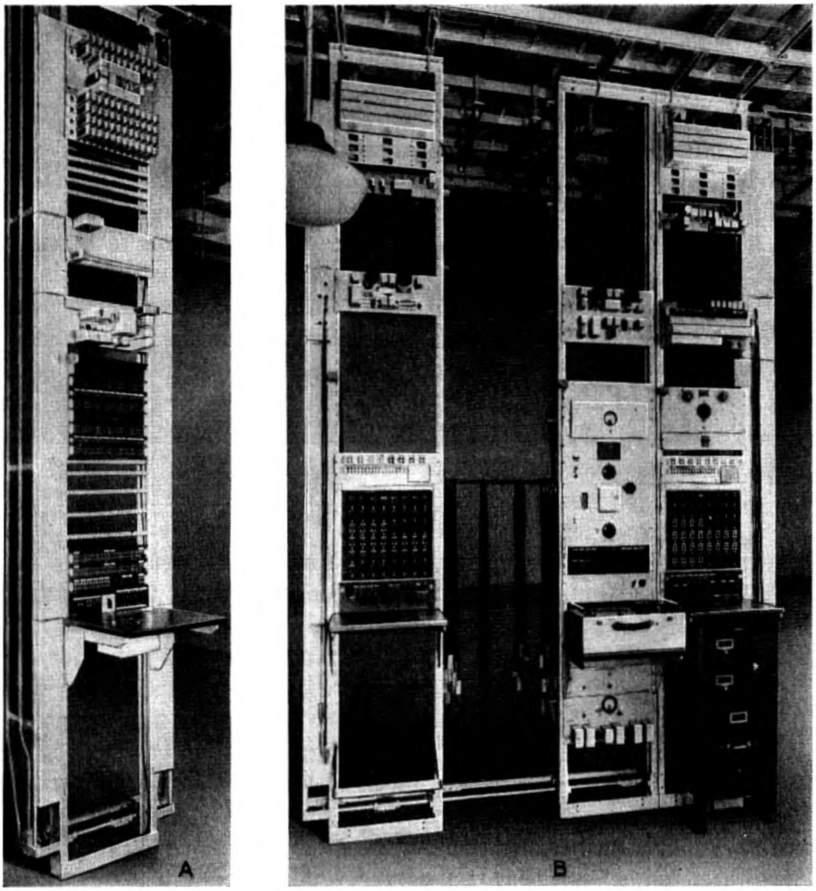


Fig. 9—Test positions: "A"—voice frequency; "B"—high frequency.

Adjustment of the equivalents of the individual channels can be made from this point as previously described. Much can be done from this position by means of monitoring and noise measuring to diagnose troubles.

At the high-frequency testboard the circuits are brought through jacks and high-frequency measuring apparatus is provided. Measurements can be made on operating systems to determine the performance of the intermediate repeaters and regulators with respect to level and equalization over the frequency range from 12 to 60 kilocycles. Loss and gain measurements also can be made either between this point and the voice-frequency four-wire test board, through the carrier terminal equipment or through the next adjacent repeater or terminal office at high frequencies. It is possible to test the high-frequency portion of the terminal and to substitute a spare, by patching or rapid transfer, for a defective or potentially defective group modulator, transmitting amplifier, group demodulator or receiving amplifier.

Some of the high-frequency testing equipment is shown mounted on the middle bay of Fig. 9, *B*, of which one of the most important units is the 1 to 150-kc. test oscillator located at the center of this bay. It is a heterodyne type of oscillator which covers the frequency range with a continuous film strip scale about 300 inches long. Its maximum output is about one watt and this varies less than 1 db over the entire range. It is provided with built-in calibrating features and can be set to any frequency with an absolute accuracy of about 25 cycles. It is used as the tuning control of the pilot level measuring circuit. An auxiliary scale on the oscillator permits tuning the measuring circuit directly in terms of frequency.

The pilot level measuring circuit is of the double heterodyne type and includes a copper-oxide modulator which is supplied with carrier from the heterodyne oscillator, an intermediate frequency 130-kc. crystal filter of 10 cycles band width, a high-frequency amplifier for this frequency, a copper-oxide demodulator supplied with carrier from a 129-kc. fixed frequency oscillator, a voice-frequency amplifier and calibrating circuit. The input impedance of the measuring circuit is high so that when it is bridged across a line pair at the high-frequency testboard jack fields it does not produce appreciable loss to the line. The circuit permits measuring each of the three pilot frequencies to check levels and equalization of operating systems. The panels comprising the circuit are shown below the oscillator in Fig. 9, *B*.

Mounted on a shelf just below the oscillator is the transmission measuring set, which contains a highly accurate thermocouple and meter combination with calibrating circuits, wide range repeating coils, a test key circuit, and attenuators, one of which can be set in steps of 1 db up to a total of 90 db.

## Equipment Features

Because of the large number of systems likely to be terminated in an office, the jacks are concentrated in a group of bays located together for ease in patching and testing. There are in general five major divisions of the terminal equipment consisting of channel modem bays, group modem bays, carrier supply bays, high-frequency testboard, and four-wire voice-frequency patching board and associated voice termin-

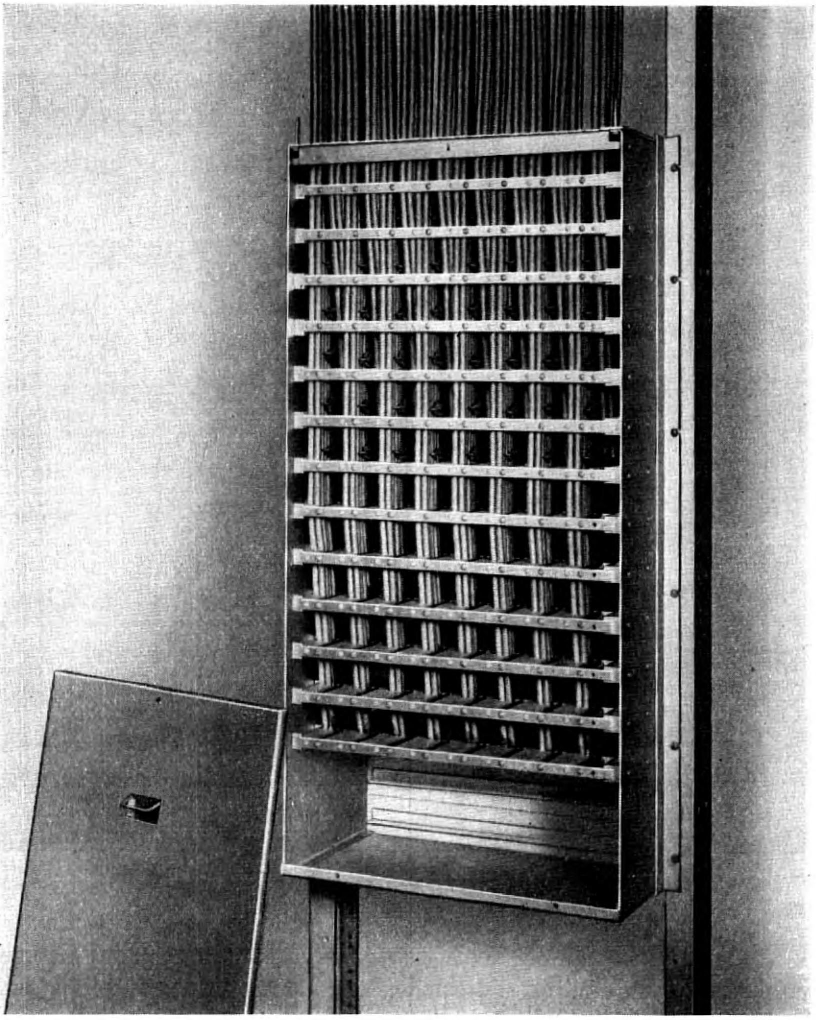

Fig. 10—Cabling of high-frequency jack bay.

ating equipment. The general arrangement in an office is such as to simplify the cabling between various groups of equipment

The cabling of a high-frequency jack bay, shown in Fig. 10, illustrates the congested wiring condition occurring when a large number of heavy shielded wires is run to one location. Because of this congestion the jacks in this bay are mounted or removed from the front.

The concentration of equipment in the modem unit is made possible by the small size of the copper-oxide bridges and the filters. Fig. 11, *A*, shows twelve modem units for two systems on adjacent bays with space left at the bottom for the six modems of a third system.

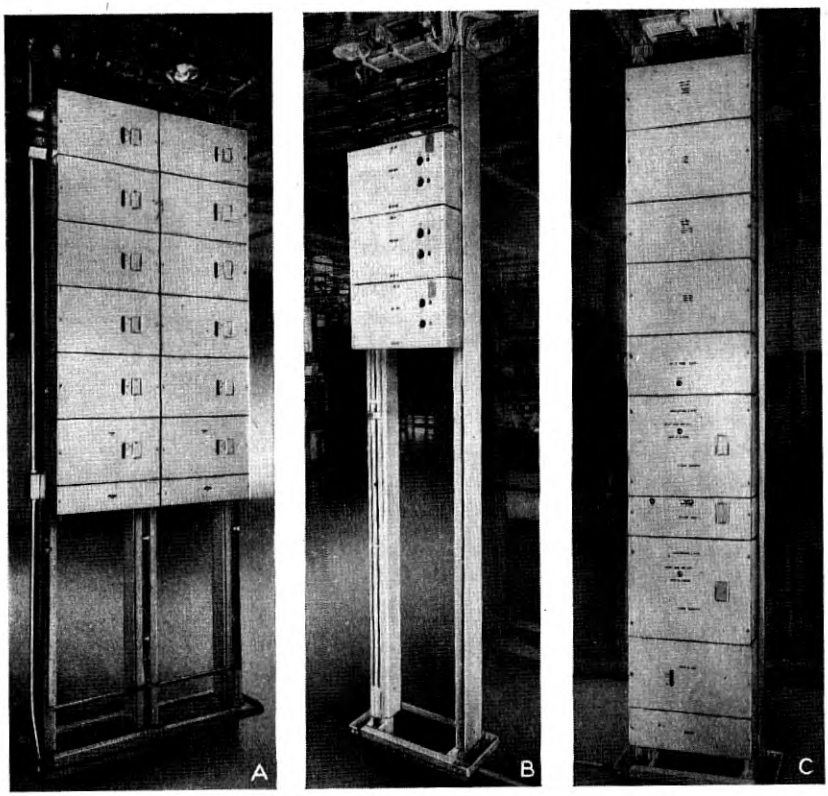


Fig. 11—Carrier equipment bays: "A"—channel; "B"—group; "C"—carrier and pilot supply.

The group modems are about the same size as the channel modems and include a modulator, a demodulator, a transmitting amplifier, and auxiliary receiving amplifier with associated filters. Fig. 11, *B*, shows three units for three systems with space for six additional units at the bottom of the bay.

The carrier supply equipment for ten systems is mounted in one bay as shown in Fig. 11, *C*, which includes the regular and emergency generators, transfer unit, distributing equipment and pilot channel supply panel. The bays of this type are located near their associated channel equipment because the supply is chiefly for channel modems. One carrier distributing unit provides for the even and another for the odd harmonics. All terminals and bus bars of these units which are common to the ten systems are protected by insulating covers.

The four-wire voice-frequency jacks for all the systems in an office will ordinarily be grouped in associated bays, one of which is shown on Fig. 9, *A*. A bay will accommodate five systems as an average, that is, 60 voice circuits including the necessary pads and telephone set.

The high-frequency testboard is an arrangement of sealed test terminals, high-frequency patching jacks and high-frequency testing equipment mounted on bays as shown in Fig. 9, *B*. Only a few high-frequency patching jacks were required initially and these were therefore mounted above the sealed terminals. This arrangement of bays with the addition of a high-frequency patching bay at the right of each sealed terminal bay will accommodate 100 systems.

The carrier pairs are split off from the main toll cables at splices in the cable vault. The input circuits are carried thence in lead covered cable to the cable crosstalk balancing bays and thence to the input sealed terminal. The output pairs run directly from the output sealed test terminal to the splice in the cable vault. The remaining high-frequency wiring from rack to rack is shielded wire.

## Conclusion

The carrier telephone terminals for the type "K" system which have been described are simpler, occupy less space and provide better transmission performance than multi-channel carrier terminals used previously in the Bell System. As part of a general development of broad-band transmission systems, it is very desirable to employ equipment which can be used in common with several systems. The 12-channel bay, much of the carrier supply and all of the voice-frequency terminating equipment of this type "K" system terminal will be used to form corresponding parts of the terminals for the 12-channel open-wire system and the coaxial system, both of which are under development. This not only has simplified the development work, but also will result in greater mass production of these common parts and provide desired uniformity of voice-frequency terminating levels and maintenance arrangements.

## References

1. "Carrier Current Communication on Submarine Cables," H. W. Hitchcock, *Jour. of A. I. E. E.*, October 1926; *Bell Sys. Tech. Jour.*, October 1926.

2. "New Key West-Havana Carrier Telephone Cable," H. A. Affel, W. S. Gorton and R. W. Chesnut, *Bell Sys. Tech. Jour.*, April 1932.
3. "Carrier Telephone Systems for Toll Cables," C. W. Green and E. I. Green. Presented at the 1938 Winter Convention of the A. I. E. E. Published in this issue of the *Bell System Technical Journal.*
4. "Crystal Channel Filters for the Cable Carrier System," C. E. Lane. Presented at the 1938 Winter Convention of the A. I. E. E. Published in this issue of the *Bell System Technical Journal.*
5. "Magnetic Generation of a Group of Harmonics," E. Peterson, J. M. Manley and L. R. Wrathall, *Electrical Engineering*, August 1937.

# Crystal Channel Filters for the Cable Carrier System * †

By C. E. LANE

SINCE the channel selecting filters used at the terminals of the twelve-channel cable carrier system are the principal filters in the system this paper is concerned primarily with these. Their importance is evident from the fact that they represent over one-third of the cost of the system terminals.

Many new features appear in these channel filters. The most outstanding is the use as filter elements, along with inductance coils and condensers, of plates cut from crystalline quartz. It is for this reason they are called "crystal filters." In addition, however, the inductance coils, some of the condensers, and also the filter assemblies have in them new features. Only after a number of years of laboratory experimentation with filters using crystal elements, studying their advantages and limitations, was the cable carrier system planned to use such filters.

There are twelve channel filters which transmit the lower side bands derived from the modulation of the speech signals with carrier frequencies spaced 4 kilocycles apart from 64 to 108 kilocycles. An insertion loss frequency characteristic which applies for each of the twelve filters is shown in Fig. 1. Regarding a 10 db loss increase as the cut-off as compared with transmission at 1000 cycles, the voice-frequency band for a single-carrier link, largely determined by the characteristics of the channel filters, extends from approximately 150 to 3600 cycles. For five links the band extends from about 200 to 3300 cycles. This is a 600 or 700 cycles wider frequency band than the present three-channel open-wire carrier system. The maximum delay distortion in the transmission band of each of the filters is about 0.4 millisecond. As many as ten of these filters may appear in tandem in the longest talking circuits. The total delay distortion in such cases would then not exceed 4 milliseconds. This is not objectionable since the average listener can not observe the effect of delay distortion unless it exceeds about 10 milliseconds. A representative filter schematic is shown in Fig. 2. The condenser shown by the dotted line at the left is used only in the two lowest frequency filters to obtain

* This is a companion paper to other papers covering different parts of the twelve-channel cable carrier system.

† Presented at Winter Convention of A.I.E.E., New York, N. Y., Jan. 24–28, 1938.

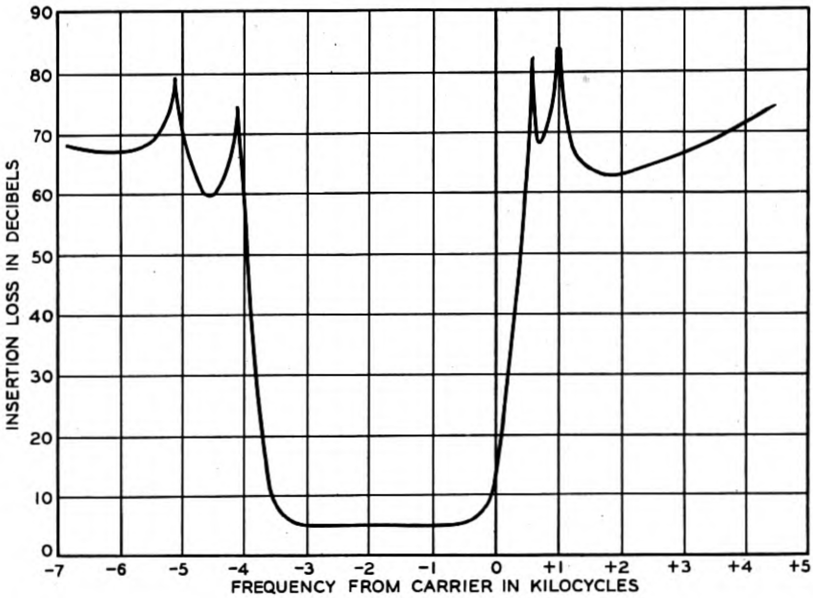


Fig. 1—When plotted in cycles removed from the carrier frequency, the insertion loss frequency characteristics of each of the twelve crystal channel filters are for all practical purposes identical.

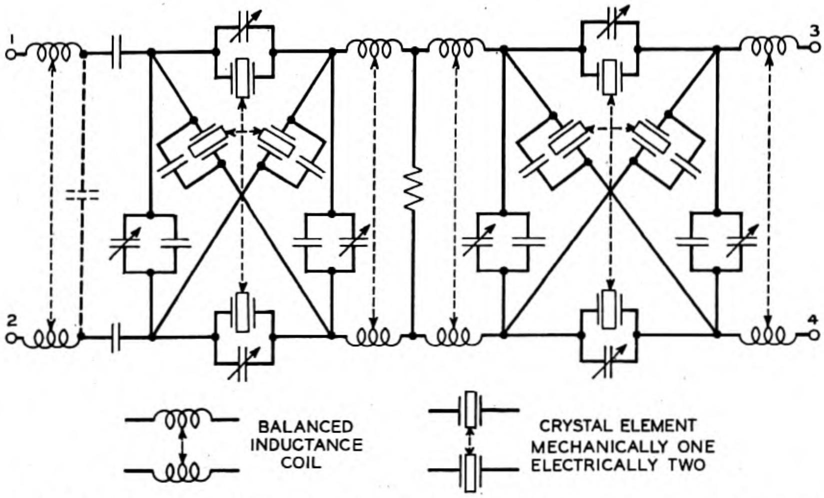


Fig. 2—The schematic circuits of each of the twelve filters are the same except for the addition of the condenser shown by the dotted lines which appears only in the two lowest frequency filters.

an impedance transformation internal to the filters and thereby permit the use of crystals of practical thicknesses for these filters. However, the equivalent circuit for each of the twelve filters is the same. In the system the filters work in parallel at one end and between terminating impedances of 600 ohms. The two condensers appearing in the series arms at the left end of the filter schematic are used in obtaining satisfactory operation of the filters in parallel and otherwise might be omitted provided the inductance at this end was made smaller at the same time.

Figure 2 indicates the separate physical elements and the manner in which these are connected in the filters. In considering the performance of the filters the crystal elements are replaced by their equivalents, an inductance and capacitance in series, shunted by a second capacitance as shown in Fig. 3. Also, the condensers in

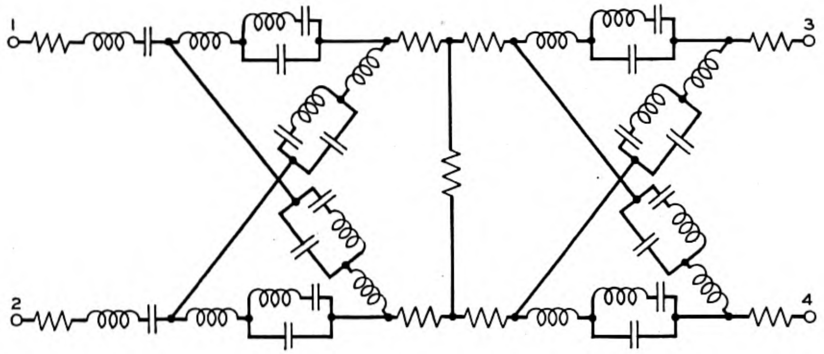


Fig. 3—The schematic circuits of the filters are each equivalent to two lattice sections with a resistance pad between them, resistances at each end and at the paralleling end a coil and condenser which resonate at the mid-band frequency.

shunt across the filter are shown inside the lattice combined with the direct capacitance of the crystals and the inductances are relocated in series in each lattice arm. In making this conversion, however, the effective resistance of the inductance coils are, for reasons which will appear later, shown remaining outside the lattice. Also the capacitances and the portion of the inductance which are used solely for purpose of paralleling are left outside the lattice. The basis for the conversion from Fig. 2 to Fig. 3 is shown in Fig. 4.

Before considering further the filter as a whole, the nature of the crystal elements and the reason for using them will be considered. It is common knowledge to those familiar with the performance of electrical wave filters that the energy loss unavoidably associated with inductances imposes limitations upon the filter characteristics obtainable. Capacitances may be designed so that the energy dissipation is small and negligible as compared to that in the inductances. With ideal reactance elements entirely free from dissipation, filters might be designed for any band width with as little loss in the band as wanted and at the same time frequencies might be rejected outside the band by any amount desired, no matter how near such frequencies

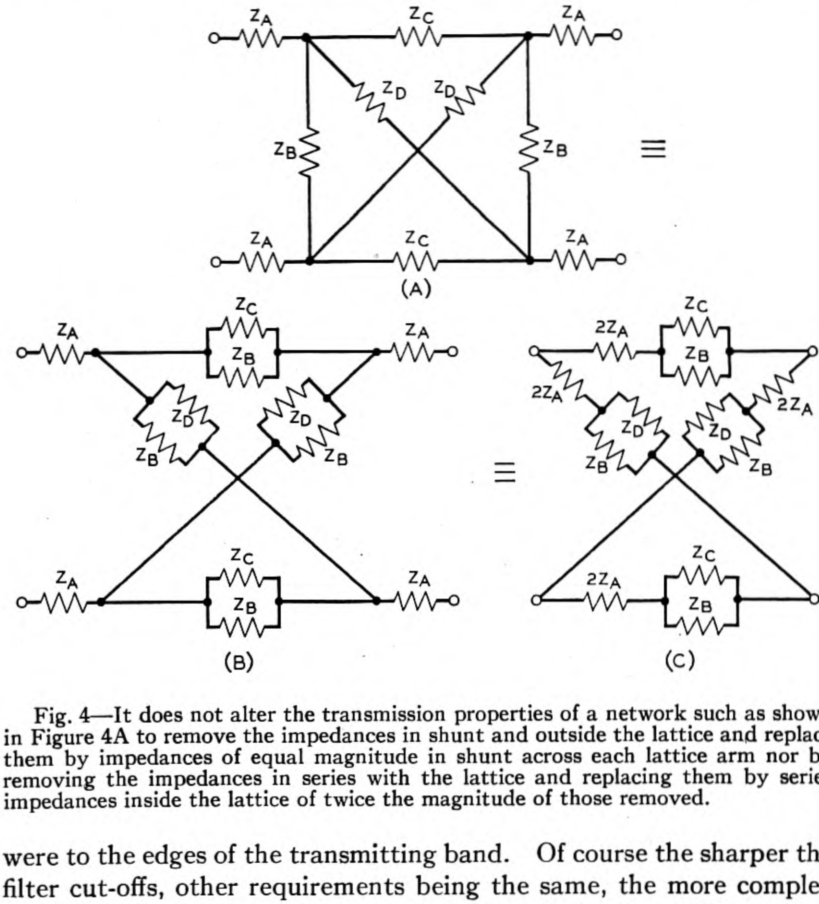


Fig. 4—It does not alter the transmission properties of a network such as shown in Figure 4A to remove the impedances in shunt and outside the lattice and replace them by impedances of equal magnitude in shunt across each lattice arm nor by removing the impedances in series with the lattice and replacing them by series impedances inside the lattice of twice the magnitude of those removed.

were to the edges of the transmitting band. Of course the sharper the filter cut-offs, other requirements being the same, the more complex the filter structure would be even neglecting dissipation. The greater the dissipation in the filter elements, the greater the loss in the transmitting range of the filters and the greater the number of cycles required for this loss to rise from the relatively low and uniform loss in the transmitting band to the high loss wanted outside the band. In the design of channel filters for carrier systems, the presence of dissipation in the filter elements is costly in that the channels must be spaced farther apart than would otherwise be necessary, thereby wasting frequency space. At the same time the loss to transmitted frequencies must be made up for by amplification. The amount of dissipation in a reactance element is measured by the ratio of the effective resistance component of its impedance to the reactance component at any frequency. The reciprocal of this ratio is called the $Q$ of the reactance and hence is a measure of efficiency or freedom from dissipation. In the design of inductances in the form of wire wound coils, it is generally not practical to obtain $Q$'s much in excess of 200 or 300 at any frequency. The quartz crystal element used in the filters as previously stated is equivalent electrically to a two-terminal reactance consisting of an inductance and capacitance in series shunted by a second capacitance. For the $Q$ of the inductance in the equivalent circuit of the crystal element a value of 15,000 or more can readily be obtained. It is for the purpose of utilizing this high $Q$ inductance and obtaining the benefits therefrom that crystal elements are used in these filters.

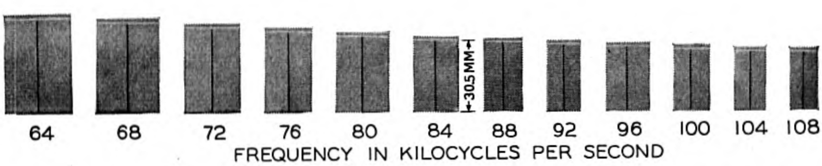


Fig. 5—The length of the crystal elements used in the different filters varies about inversely as the frequency of the filter band location.

The filter schematic in Fig. 2 shows crystal elements in each filter section; the two in the lattice arms and the two in the series arms in each case are identical. Electrically there are four crystals in each section but for reasons of economy and for convenience in handling and adjusting the crystals those in corresponding arms are physically one. This is possible since the filter is a balanced structure and the two like crystals vibrate in unison. Figure 5 is a photograph which shows a representative double crystal element taken from each of the twelve filters. The four crystals in the lowest frequency filter range from 40.2 millimeters to 41.8 millimeters in length and those in the highest frequency filter from 23.8 to 24.3 millimeters. The thickness of the crystals in all four of the lowest frequency filters are 0.63 milimeters, in the next four filters 0.82 millimeters, and in the highest frequency filters 1.1 millimeters. Uniformity in thickness is maintained as far as practicable since it contributes to economy in manufacture of the crystal. Within the range using the same crystal thickness the impedance and frequency differences, called for by the design of the different filters, can be provided by variations in width and length of the crystals. The ratio of width of the crystals to their length ranges from about 1/2 to 4/5.

The major surfaces of the crystals are plated with a thin layer of aluminum deposited by an evaporation process. This plating is divided along the center line lengthwise of the crystals to form the two

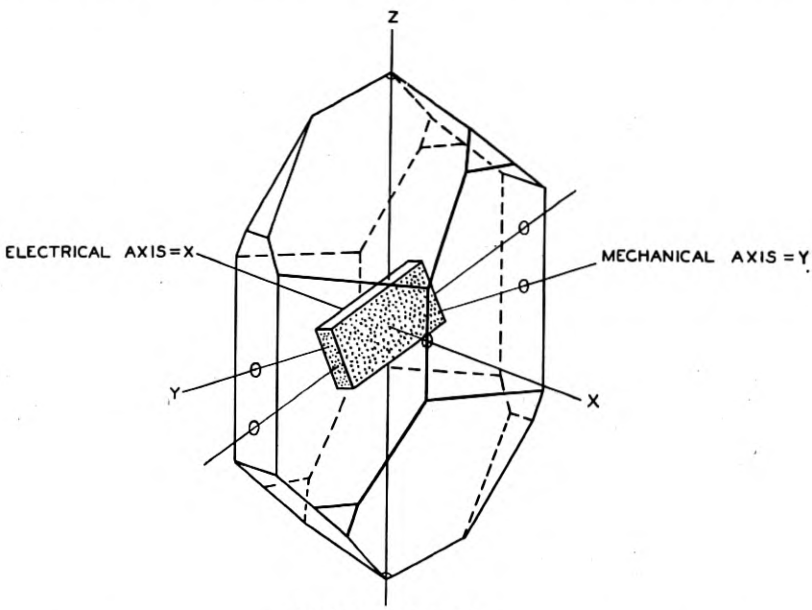


Fig. 6—The crystal elements are out with their major surfaces perpendicular to the electrical axis of the natural quartz, with their side surfaces making a small angle with the mechanical axis, and with their end surfaces making a small angle with the optical axis.

electrically independent crystals. Since the crystals vibrate longitudinally with a node across the middle, they are clamped at this node in mounting. Figure 6 shows the orientation of the crystal plates with respect to the natural axes of the quartz from which they are cut. The plates are cut as accurately as is practicable to the dimensions computed making a small allowance in length and then the crystal is finally adjusted in an electric circuit by grinding the end of the crystal until the resonant frequency falls within five or ten cycles of that desired.

Considering again the filter schematic as a whole (Fig. 3) and neglecting the dissipation in the crystals and condensers, the filter may be regarded as made of two lattice filter sections having ideal reactance elements, that is, elements free from dissipation. The location of the effective resistance of the coils outside the lattice, for purpose of performance analysis, shows how at the end of the filter these resistances may be regarded as part of the terminating impedance between which

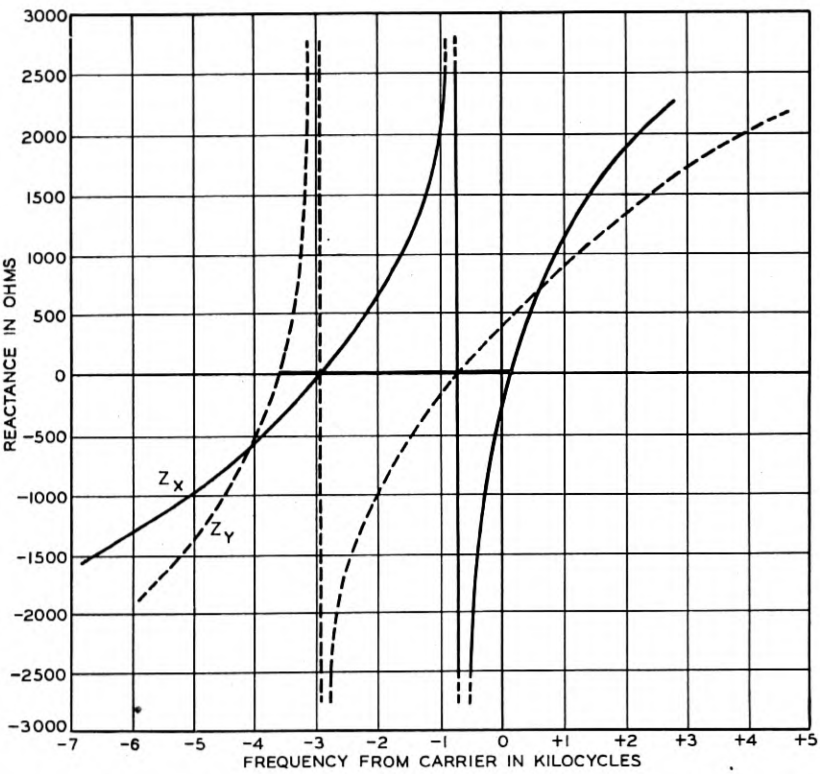


Fig. 7—In a lattice filter section transmission occurs for frequencies where the impedances for the two arms of the lattice are of opposite sign and attenuation peaks of very high loss occur where the impedances cross.

the filter works, and how between the filter sections the resistances may be combined with a shunt resistance to form a resistance pad which matches the image impedance of the two filter sections. The effect, then, of the coil resistances is primarily to provide a flat loss over the entire frequency range and does not affect appreciably the shape of the loss characteristic furnished by the reactance inside the lattice sections.

In considering the performance of lattice type filter sections, it is common practice to sketch together the frequency reactance curve of the two lattice arms $Z_x$ and $Z_y$. This is done for one of the filter sections and is shown in Fig. 7. In the frequency range where the two curves are of opposite sign the filter transmits, and where they are of

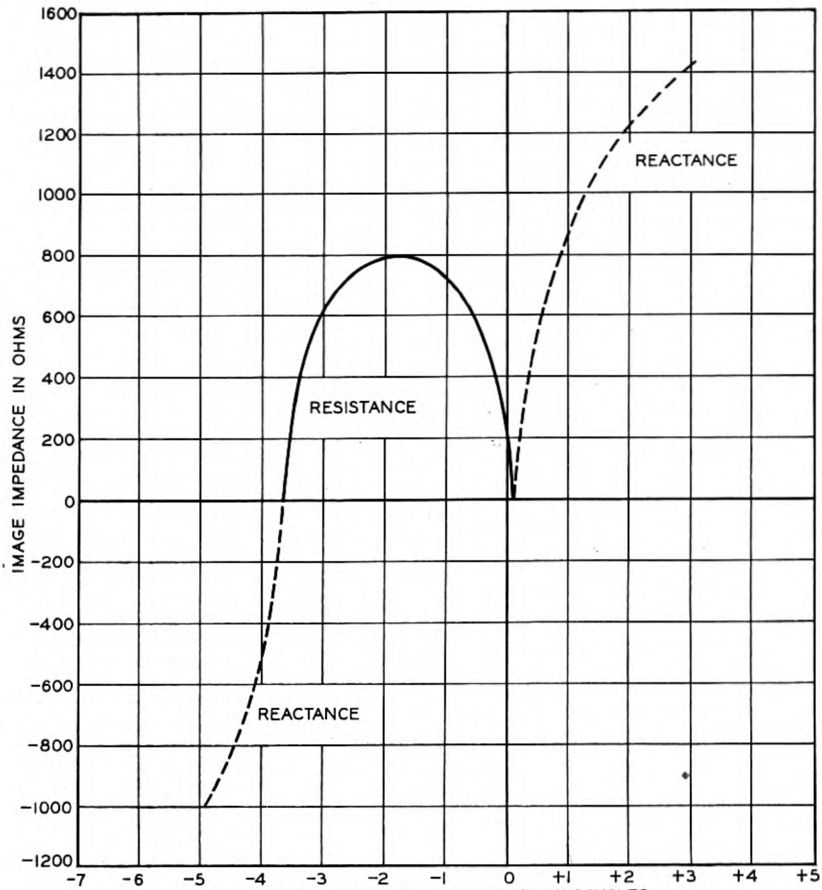


Fig. 8—The large reflection losses occurring within the transmission band of the filters and near the edges of this band has the effect of narrowing the width of the transmission band.

the same sign there is attenuation. At the point where the two curves intersect there are attenuation peaks of very high loss. The reactance curves of Fig. 7 are for the filter section accountable for the pair of attenuation peaks shown in the filter characteristic which are the closer to the edges of the transmitted band. For the other section the cross-over points of the two reactance curves are farther away from the band, since this section is responsible for the outer pair of attenuation peaks. The design of the filters consisted in determining values for the inductance coils, condensers, and crystals, such that the reactance curves of the lattice arms of the filter passed through infinity and intersected with each other at the desired frequencies and, at the same time, in determining the impedance level for all of the elements such that the filters would have the right image impedances.

The curves of Fig. 7 would seem to indicate a somewhat greater band width for these filters than shown by the insertion loss characteristic of Fig. 1. The reason for this can best be explained by referring to the image impedance of one of the filter sections as shown in Fig. 8. Within the band the image impedance is, of course, a pure resistance which varies with frequency. It is about 800 ohms at mid-band frequency and falls rapidly to zero near the edges of the band. Assuming the effective resistance of the coils, which is about 100 ohms, as belonging to the terminating impedances, the filter sections actually work between impedances of about 700 ohms. This means that large reflection losses occur at each end of each filter section near the edges of the transmission band where the image impedance of the filter is very small. It is these reflection losses that are responsible for the actual transmission band being much narrower than it would be with the filter sections terminated in their actual image impedances. The filter sections are designed with 800 ohms image impedance at mid-band frequency instead of 700 ohms to make the band flatter and somewhat wider than it would be otherwise.

When a number of band filters are operated in parallel it is generally necessary to connect across the paralleled end a two-terminal network to correct for the distortion that would otherwise be present in the highest- and lowest-frequency filters in the group. A circuit of the network used for this purpose with the channel filters is shown in Fig. 9.

The filters employ crystal elements in order to obtain abrupt discrimination between wanted and unwanted frequencies and at the same time to secure low and uniform loss in their transmitting bands. This characteristic must not only be obtained at the time the filters are assembled and adjusted but must be maintained throughout the service life of the filters and not appreciably affected by temperature variations. This imposes severe stability requirements upon the elements used in the filters. The crystal elements themselves are very stable when properly designed and once adjusted will retain at a given temperature their frequencies of resonance within one or two cycles seemingly indefinitely. Their temperature coefficient is only about twenty-five parts per million per degree centigrade, which is not objectionable.

The obtaining of inductance coils and condensers that were adequate in stability for use in conjunction with the crystals required considerable development effort. The inductance coils were required to have not only a high degree of stability with respect to temperature and time but also a high ratio of reactance to effective resistance, low modulation, and at the same time be small in size. Air core coils might have been designed for the purpose but they would have been quite large. The coils used are of the toroidal type wound on about one and three-fourths inch rings of molybdenum-permalloy. To reduce eddy current losses the cores are made of very fine powder and then annealed to reduce hysteresis losses. The particles are mixed with insulating

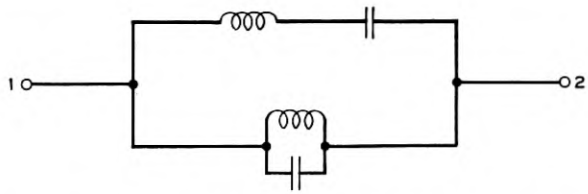


Fig. 9—A two-terminal reactance network is connected in shunt across the filters at their paralleling end to improve the characteristic of the highest and lowest frequency filters.

material and formed into rings by extremely high pressure. The inductance of the coils has a temperature coefficient of less than 40 parts per million per degree centigrade. The cores of the coils for the higher-frequency filters are wound with finely stranded wire to help secure good $Q$'s (about 225). Because of the high impedance of the coils called for by the filter design, care is taken to make the capacitance between the windings and the core and between the windings and the case as low as practicable and also to make stable all such small capacitances as must be present.

The two extra condensers used at one end of each filter for paralleling purposes are of a high grade mica type. The other condensers are all quite special. The fixed ones, ranging in magnitude from about 7 mmf to 100 mmf, are made by plating short lengths of high grade glass tubing inside and outside with silver. Because of the intimate association of electrodes with the surfaces of the tubes and the low expansion coefficient of the glass used, a condenser is obtained that has a temperature coefficient comparable with that of the coils and crystals. No aging effect has been observed. It will be noticed that four small adjustable air condensers appear in each filter section. These are used to secure precise initial adjustment of the filter capacitances.

To protect the filter elements against moisture, the filters are hermetically sealed in a container made from a rectangular section of seamless brass tubing with closely fitting plates soldered in each end. One end plate carries four metal-glass seal terminals and a nozzle through which dry air is blown after the filter is assembled. The other end plate is provided with a small hole for the escape of the drying air. After the drying operation the hole in the nozzle and the hole in the opposite end of the filter are closed by soldering. The elements that make up the filter are assembled on a chassis which is completely wired and then slid into the container in assembly. Figure 10 is a photograph showing this chassis and the arrangement of the

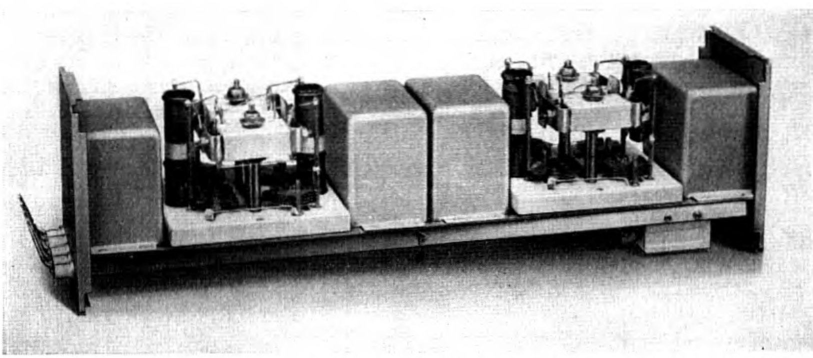

Fig. 10—The filter parts are assembled and wired on a chassis which is slid as a unit inside the filter container.

elements. The elements are located in such a way as to use very short wiring connections which reduce the magnitude of any stray admittances. The wired filter chassis is carefully adjusted by setting the values of the air condenser such that for each filter section the resonance frequencies looking into each end of each section occur where they theoretically should. This compensates for the effect of small capacitances between the filter parts. In the design of the filter parts care is taken to use no material which absorbs moisture readily since such moisture would later be released and raise the relative humidity of the air inside the filter.

If a potential much in excess of about twenty volts is applied across crystal elements at frequencies near resonance, the crystals will break from the mechanical strain of their vibration. The maximum safe voltage across the channel filters at the resonant frequencies of the crystals is considerably greater, however, since at resonance the full voltage does not appear across the crystals. In normal use the voltages across the filters will be very much less than twenty volts.

Other filters forming part of the terminal apparatus are the group modulator and group demodulator low-pass filters, the channel and group carrier supply filters, and the pilot supply filters. The group modulator and demodulator filters are of the low-pass type employing only coils and condensers as elements. The group carrier supply filter is the same in schematic and mechanical design as the crystal channel filters described. The pilot supply filters and the channel carrier supply filters are equivalent in schematic to one section of the channel filters; but of course they are only about half the size and are hermetically sealed in the same manner.

### References

For a further discussion of crystal filters the reader is referred to "The Evolution of the Crystal Wave Filter" by O. E. Buckley, *Journal of Applied Physics*, October 1936, and "Electrical Wave Filters Employing Quartz Crystals as Elements" by W. P. Mason, *Bell System Technical Journal*, July 1934.

# Crosstalk and Noise Features of Cable Carrier Telephone System *

By M. A. WEAVER, R. S. TUCKER and P. S. DARNELL

CROSSTALK and noise are important factors in cable carrier transmission as outlined in the paper "A Carrier Telephone System for Toll Cables" by Messrs. C. W. Green and E. I. Green. Crosstalk and noise limit the number of carrier channels which can be utilized in any one cable, not only by limiting the number of channels which can be placed on a single pair, but by limiting the number of pairs which can be used. Noise also controls the transmission loss which can be permitted between repeaters. Without the crosstalk and noise reduction measures described in this paper, the number of carrier channels per cable would be so few and the spacing between repeaters so short, that the type K carrier system would be impracticable.

## Crosstalk

To utilize existing toll cables in the Bell System for frequencies up to 60 kilycycles required the solution of many new crosstalk problems because: (1) Crosstalk increases rapidly with the frequency, (2) Non-loaded carrier pairs due to their high speed of propagation are especially suitable for very long distances and hence the crosstalk requirements per unit length are relatively severe, (3) The large gains of the carrier repeaters amplify certain crosstalk currents much more than in the case of voice frequency circuits.

Two general effects need to be considered: intelligible crosstalk must be prevented; and, a large number of circuits crosstalking into a particular circuit must not contribute an undue amount of noise. The second effect is called babble, since it consists of a multiplicity of unrelated voice sounds which, in the aggregate, are unintelligible.

An important feature is the use of different cables for opposite directions of transmission. This makes the major crosstalk problem the reduction of crosstalk between pairs in the same cable used for transmission in the same direction. The crosstalk currents due to transmission at one end of a disturbing circuit through the distributed couplings with a disturbed circuit tend to arrive at the distant end at the same time since the currents via any of the couplings travel sub-

* Presented at Winter Convention of A. I. E. E., Jan. 24–28, 1938.

stantially the same distance. This makes it possible to greatly reduce the total effect of these distributed couplings by the use of small adjustable mutual inductance coils connected between pairs at one point in each repeater section.

If nothing more were done, there would still be objectionable crosstalk since currents from the outputs of carrier repeaters could crosstalk into voice frequency circuits and these circuits could then again crosstalk into other carrier frequency circuits at points near their repeater inputs. This effect is minimized by transposing the carrier pairs from one cable to the other at carrier repeater points.

At common voice frequency and carrier frequency repeater points there would be an unsatisfactory crosstalk path from a carrier repeater output into all the wires not used for carrier frequencies and from them through coupling between office wiring into similar wires in the other cable and finally into carrier repeater inputs in the second cable. This crosstalk is minimized by the use of carrier frequency suppression coils in the voice frequency circuits. These coils also serve the purpose of preventing carrier frequency noise originating in voice frequency circuits from being transmitted into the cables and inducing noise at points near carrier repeater inputs.

### *Near-End Crosstalk*

Near-end crosstalk is the result of coupling between circuits transmitting in opposite directions, while far-end crosstalk is the result of coupling between circuits transmitting in the same direction. Near-end crosstalk coupling between different carrier circuits of the same frequency must be kept very small, particularly near a repeater point, since crosstalk from the output of a repeater into an opposite directional pair near the input of its repeater will be greatly amplified by this repeater.

Crosstalk between carrier circuits within the offices is kept low by careful shielding, segregation, suppression of spurious paths through battery supply, common grounding arrangements, etc.

Since the type K system operates on a "four-wire" basis, different electrical paths are used for opposite directions of transmission. Satisfactory near-end coupling in the outside plant is obtained, therefore, by placing east bound pairs in one cable and west bound pairs in another. When two cables have relatively heavy sheaths as in the larger Bell System cables, their coupling is sufficiently small even with the two cables in close proximity.

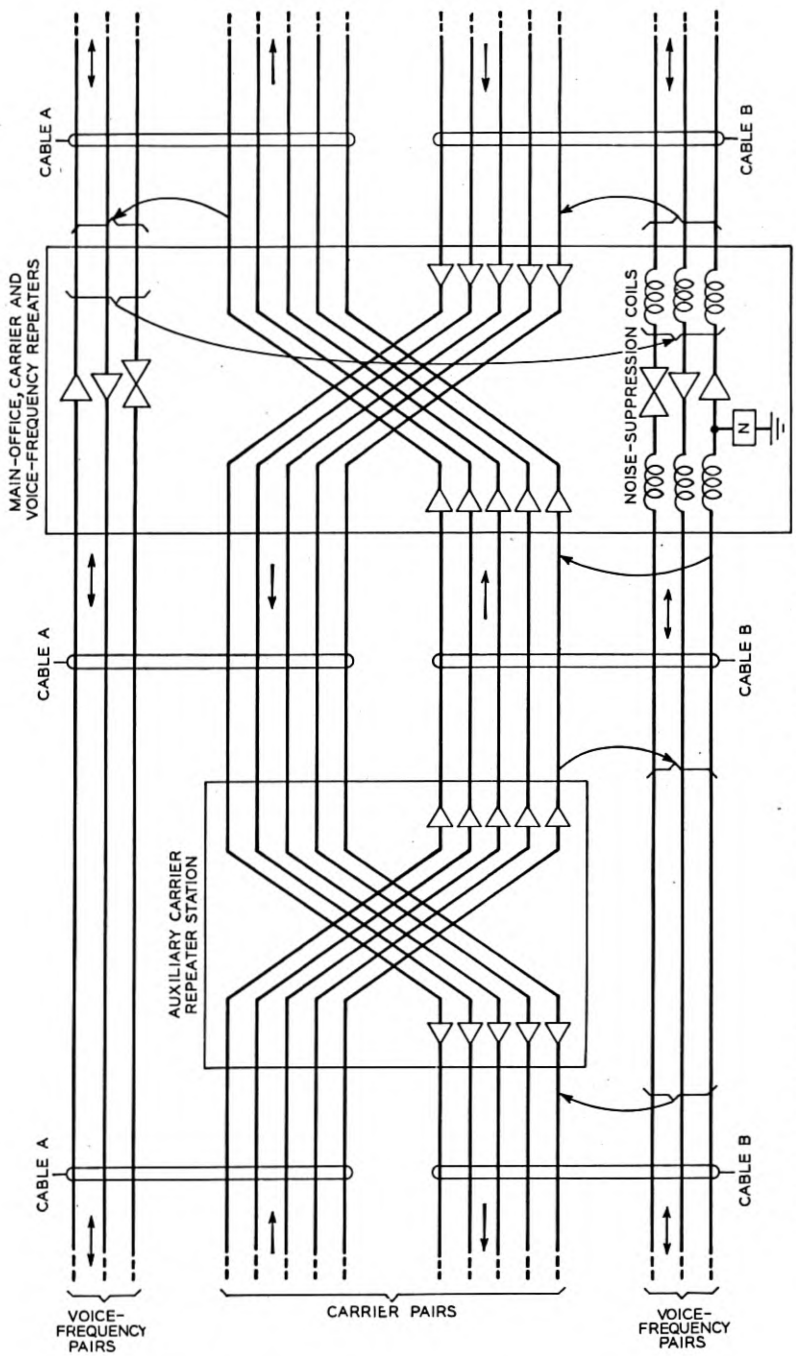


Fig. 1—Schematic showing carrier pairs transposed between cables, coupling paths and noise suppression coils.

*Interaction Crosstalk*

The crosstalk currents from a carrier repeater output into voice frequency circuits in the same cable must be limited, since they crosstalk again into carrier circuits near repeater inputs and, consequently, are amplified by the high gain repeaters. Intermediate circuits most responsible for crosstalk of this type are made up of combinations of pairs and phantoms and the sheath, i.e., longitudinal paths.

One case of crosstalk of this kind would occur if the same cable were used for carrier pairs transmitting in the same direction on both sides of a repeater. This is prevented by transposing carrier pairs from one cable to the other at each repeater point, as shown on Fig. 1.

A second interaction crosstalk problem is encountered at the common voice and carrier repeater points and involves coupling between cables as well as in the same cable. Here the coupling path is from carrier repeater outputs to intermediate circuits in the same outside cable, back into the common office over these intermediate circuits and then via office coupling to intermediate circuits in a second outside cable and from there to carrier repeater inputs connected to pairs in the second cable. Referring to Fig. 1, a set of noise (and crosstalk) suppression coils is encountered in this path. The high longitudinal circuit impedance of these coils minimizes this interaction crosstalk.

*Far-End Crosstalk*

Far-end crosstalk currents are subjected to line attenuation and amplification similarly to the main transmission currents, and do not have extra amplification as in the case of near-end crosstalk. Furthermore, far-end crosstalk currents due to couplings at different points along the line tend to arrive at the distant end of the disturbed circuit at the same time. Hence a considerable portion of the far-end crosstalk over the type K frequency range, which occurs between circuits transmitting in the same direction in the same cable, can be neutralized by introducing compensating unbalances at only a comparatively few points, such as one per repeater section. The far-end crosstalk reduction problem is greatly simplified because phantom circuits are not used for carrier operation.

Theoretically, for the same precision of match between the impedances in the two directions at the balancing point, the crosstalk reduction would be about the same whether the balancing is done at an intermediate point or at either end of a repeater section. Balancing will be done at repeater inputs rather than at an intermediate point, such as the middle, because it is practicable to obtain repeater impedances matching the average line impedance sufficiently well so that the effectiveness of balancing is reduced only slightly.

*Nature of Far-End Crosstalk Coupling*

The coupling between two cable pairs in a short length may be represented by a mutual admittance and a mutual impedance. The former is due almost entirely to capacitance unbalance, which varies but little with frequency, so that its effect could be practically balanced out by means of a simple condenser. The latter, however, involves a complex mutual inductance of the form $M_a + jM_b$, because of the proximity effect of the wires of a pair and of other cable conductors.[1] As shown on Fig. 2, both components vary considerably with fre-

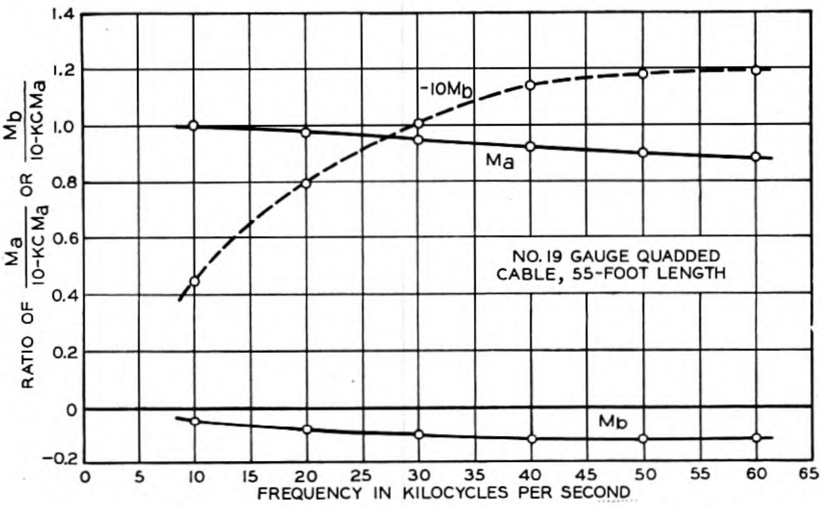


Fig. 2—Mutual inductance between cable pairs in terms of value for $M_a$ at 10 kilocycles.

quency; $M_a$ on the average decreasing as the frequency increases while $M_b$ in the general case is of negative sign and reaches a maximum value at 56 kilocycles.

*Type of Balancing*

To obtain maximum reduction in crosstalk it would be necessary to use a condenser for balancing the mutual admittance and an inductance coil for balancing the mutual impedance or to use some equivalent complex network. Experimental balancing in a particular cable using the coil-condenser method reduced the mean crosstalk over the type K

[1] "Cable Crosstalk—Effect of Non-Uniform Current Distribution in the Wires," R. N. Hunter and R. P. Booth, *Bell System Technical Journal*, April 1935.

range about 20 db, which is close to the maximum reduction possible with a universal type of balancing unit. The reduction is limited by the fact that two pairs having identical crosstalk couplings in each of two short lengths at different points in the cable will not produce two identical elements of crosstalk current at a circuit terminal because: (1) Cable circuits are not perfectly smooth. Reflections, as at junctions of reel lengths or at terminals, alter the two crosstalk currents differently, (2) The propagation constants of each circuit vary slightly from reel to reel in a random fashion and therefore the two crosstalk currents are of slightly different phase and magnitude, (3) In any short length the disturbing circuit produces crosstalk currents in intermediate circuits, which are propagated along these circuits and crosstalk again into the disturbed circuit at various points, producing an additional crosstalk current at the circuit terminal. At any frequency, this interaction crosstalk current has a random phase and magnitude relation to the crosstalk current for the short length considered by itself, and depends also upon the position in a repeater section of the short length.

A 20 db crosstalk reduction is not required, considering the number of K systems anticipated in any one cable. Studies were made, therefore, to determine whether satisfactory results could be obtained with a less expensive type of balancing, as outlined below.

The effects of frequency and circuit impedance on crosstalk coupling are as follows: (1) Crosstalk in a short length due to capacitance and to inductance coupling increases about directly as the frequency increases for circuits whose impedance is independent of frequency. (2) Crosstalk due to capacitance coupling varies directly as the impedance of the circuits while that due to inductance coupling varies inversely as the impedance. Changing the impedance from about 800 ohms for loaded voice circuits to about 135 ohms for non-loaded carrier circuits and changing the frequency from about 4 kc. to 60 kc. increases the crosstalk due to capacitance coupling by a factor of about 2.5 and that due to inductance coupling by a factor of about 90.

Capacitive coupling in existing cables was reduced by design to as great a degree as practicable, particularly for the most closely associated circuits, because it is of most importance in the loaded voice frequency case. These same design measures also reduce inductive coupling but not to the same extent. Capacitive coupling decreases rapidly with separation due to the shielding effect of copper in intervening circuits while inductive coupling is not much affected by intervening copper wires. To minimize magnetic coupling it is necessary to use different lengths of twist for the pairs. Existing cables have relatively few lengths of pair twists.

As the net result, capacitance coupling is no longer all important, inductance coupling at 60 kc. actually predominating by a factor of about 3 to 1 in existing cables. Capacitance balancing should, therefore, be less effective than balancing designed to reduce the inductance coupling. Tests have shown that capacitance balancing alone gives a crosstalk reduction of about 11 db while inductance balancing alone gives a reduction of about 16 db. Since the latter reduction is sufficient, except possibly for small cables or special cases, the type K balancing has been designed on this basis. Far-end crosstalk currents due to the two kinds of coupling have phase relations not differing from zero or 180 degrees by more than about 15 to 40 degrees, depending on whether the upper or lower type K frequencies are considered. There is, therefore, a tendency for either type of balancing unit to annul both kinds of coupling.

To obtain as much as 16 db reduction it is necessary that the frequency characteristic of the balancing coil simulate that of the cable (Figure 2). This was found practicable, as discussed later, by shunting the primary (or secondary) of the coil by a properly designed impedance.

### *Size of Balancing Coil*

To meet the crosstalk requirement it is necessary to balance each carrier pair against every other carrier pair. If 50 carrier pairs were used, there would be 49 balancing coils connected to each pair for balancing to all the other pairs, a total of 1225 coils. For convenience, adjustable coils having the same mutual inductance range and the same self-inductance are used. Hence, the insertion loss per coil, resulting from the self-inductance and resistance of the coils, must be kept small. In addition, the self-inductance of the coil presents a problem from the impedance standpoint. To keep the impedance at any point in the balancing panel as nearly like the average cable impedance as practicable, the self-inductance of a series of coils must be neutralized by capacitances shunted at suitable intervals. It is very desirable, therefore, to use coils whose self and mutual inductances are no larger than actually essential. Consequently, an attempt has been made to keep the maximum crosstalk before balancing low.

Due to special measures, described below, it appeared that the maximum inductance unbalance per repeater section could be kept below about 1.3 to 1.5 microhenries, with the possible exception of side-to-side unbalances, and trial balancing coils were designed accordingly.

### *Crosstalk Reduction Before Balancing*

Changes in the original splicing are made at approximately 6000-foot intervals, i.e., at points where voice frequency loading coils must be removed from the carrier pairs. In most existing voice frequency toll cables the 19-gauge quads were spliced as three groups, one a two-wire circuit group, one an east bound four-wire circuit group and the third a west bound four-wire circuit group. Ordinarily, the carrier pairs will be selected from the four-wire groups because these groups are usually larger than the two-wire group and since the quads within a group are spliced at random there is less chance of a large value of coupling between pairs of different quads, i.e., two pairs are less apt to be recurrently in a relation of high coupling. The carrier pairs are divided equally between the two four-wire groups, in order that the least number of four-wire voice circuits will be lost.

In cables with large four-wire groups it is satisfactory to maintain the grouping arrangement on the pairs converted to carrier. In such cables, however, one four-wire group is in the center or core of the cable and the other group in the outer periphery. In order that all circuits will have about the same velocity and attenuation and be subjected to about the same temperature conditions for both transmission and crosstalk reasons, one (four-wire) carrier group in these cables will be spliced to the other (four-wire) carrier group and vice versa at the 6000-foot intervals.

In cables with relatively small four-wire groups, there is more chance of two pairs being recurrently in a relation of high coupling. To reduce this chance, a special splicing arrangement has been devised for use at the 6000-foot intervals. With existing splicing the maximum coupling in cables with small groups is about 2.5 times that for cables with large groups. This ratio is appreciably reduced by the special splicing, likewise reducing the maximum mutual inductance that must be supplied by the balancing unit.

The foregoing was with particular reference to crosstalk between pairs in different quads. Crosstalk between pairs in the same quad (side-to-side crosstalk) is an additional problem. A quad consists of two twisted pairs of wires which are twisted together to permit the use of voice frequency phantom circuits. Since the two sides of a quad are so closely associated, side-to-side crosstalk is generally much greater that than between pairs of different quads. The electrical size of the balancing unit, therefore, is determined by the side-to-side crosstalk, which is reduced by "poling."

To apply poling, the quads are carried through as quads for an entire carrier repeater section. From measurements of side-to-side crosstalk in phase and magnitude, quads in one half repeater section are chosen and spliced to quads in the other half in such manner as to partially neutralize the side-to-side crosstalk. In effect, quads in one half-section serve as balancing units for the other half.

In most existing toll cables the side-to-side capacitance coupling was reduced when the cables were installed, by means of test-splicing within the 6000-foot sections. Obviously, for poling to be effective it is necessary to operate mainly on the inductance component. The poling measurements, therefore, are made at about 1 kc. where an approximate measure of the inductance component can be obtained directly since the capacitance and inductance components of the crosstalk are at an angle of almost 90° at this frequency. Figure 3 shows the crosstalk results obtained by means of 1-kc. poling on 14 repeater sections. It has been shown that this 9 db reduction is within 2 to 3 db of the

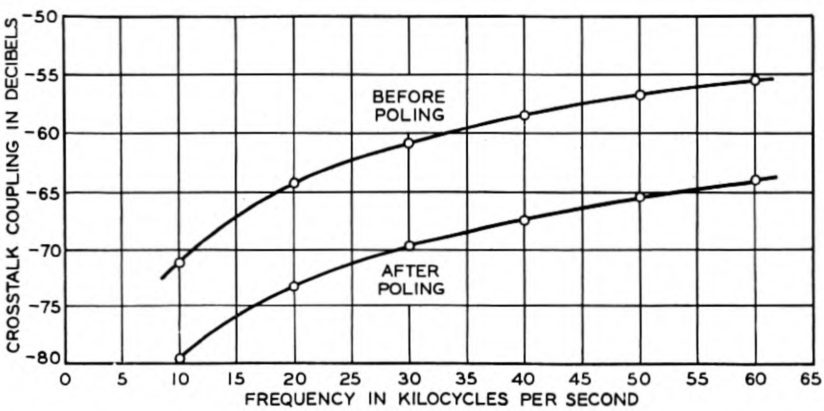


Fig. 3—R.M.S. side-to-side far-end crosstalk per repeater section from measurements on 14 repeater sections.

maximum reduction possible with much more complicated poling involving measurement and consideration of both components at carrier frequencies.

After side-to-side poling, coil balancing cannot be expected to give as much as 16 db reduction in crosstalk. This is unimportant, however, as long as the required reduction can be obtained more economically by the combined methods rather than by balancing alone.

### *Crosstalk Balancing Coil*

Since the voltage which causes the crosstalk current in the disturbed circuit may be in either a clockwise or counter-clockwise direction, the balancing device, for flexibility reasons, should be capable of establishing voltages in either direction. A balancing coil was developed, therefore, which in operation may be likened to that of two separate transformers with simultaneously movable cores. The primary windings are in series, as are the secondary windings, and are connected as shown in Fig. 4, for example. The relative direction of each secondary winding is the same, whereas the relative directions of the primary windings are reversed. With the cores in mid-position, the voltages induced in the two secondaries are equal in magnitude but opposite in phase, and the net induced voltage in the disturbed circuit is zero. As the cores are moved toward the left the respective components of the voltage induced in circuit 3–7–8–4 increase in a counter-clockwise direction and decrease in a clockwise direction, the net result being a voltage in a counter-clockwise direction. Such a setting of the balanc-

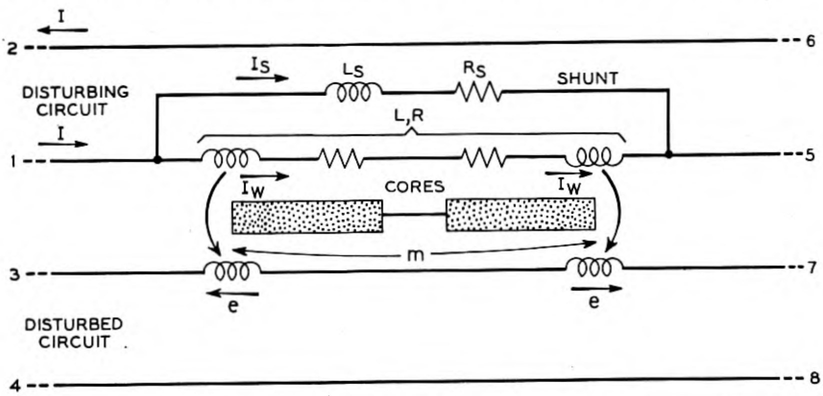


Fig. 4—Schematic of a simple balancing coil designed to produce a complex mutual impedance.

ing coil would be used to counteract a clockwise crosstalk voltage, the amount of departure of the cores from mid-position being dependent on the magnitude of the crosstalk voltage being counteracted. Movement of the cores toward the right produces the opposite effect.

This device, disregarding any proximity effects therein and the effects of the shunt, acts to set up a net voltage $e$ which is in phase quadrature with the disturbing current $I$. Hence,

$$e = -j\omega m I, \tag{1}$$

in which $m$ is the net mutual inductance of the device. To obtain the required mutual impedance characteristic, the primary (or secondary) windings of the coil are shunted by an inductive resistance. Let the effective self-inductance and resistance of the line windings (primaries) be denoted by $L$ and $R$, respectively, and the current through these windings by $I_w$. Let the effective self-inductance and resistance of the shunt be denoted by $L_s$ and $R_s$, respectively. At balance, that is, when no crosstalk current flows in 3–7–8–4 due to $I$ (the disturbing current), the current $I_w$ is

$$I_w = \left[ \frac{R_s(R + R_s) + \omega^2 L_s(L + L_s)}{(R + R_s)^2 + \omega^2(L + L_s)^2} - j\frac{\omega(R_s L - RL_s)}{(R + R_s)^2 + \omega^2(L + L_s)^2} \right] I \quad (2)$$

$$= [a - jb]I, \quad (3)$$

where $a$ and $b$ are, respectively, the coefficients of the real and imaginary parts of the expression. Hence, with a shunted coil the voltage induced in the disturbed circuit is:

$$e = -j\omega m I_w = -j\omega(ma - jmb)I. \quad (4)$$

The mutual impedance, $Z_m$, equals $j\omega(ma - jmb)$, or the effective mutual inductance $M$ of the balancing coil may be written

$$M = M_a + jM_b, \quad (5)$$

wherein $M_a = ma$ and $M_b = -mb$. Assuming $R$, $L$, $L_s$ and $R_s$ to be constant with respect to frequency of current and position of the cores, it is seen from (2) and (5) that for any core setting, $M_a$ and $M_b$ are functions of frequency only and their ratio at a given frequency is theoretically constant throughout the operating range.

To keep inductance $L$ constant irrespective of the mutual inductance settings, the length of the coil windings, the length of the magnetic cores and their spacing with respect to the winding spacing are so related that the change in inductance of one primary (or secondary)

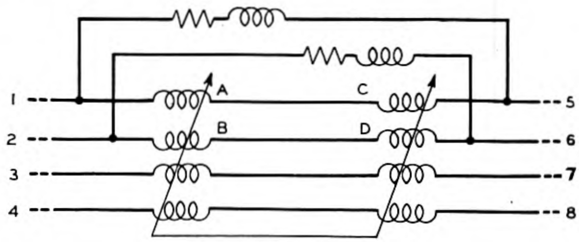


Fig. 5—Schematic of winding arrangement of trial balancing coil.

winding caused by motion of its associated core is equal and opposite to the change caused by the movement of the core associated with the other primary (or secondary) winding. To keep $R$ low over the type K frequency range, cores of finely powdered molybdenum permalloy pressed into a cylindrical form are used.

Because of other requirements which a balancing coil must satisfy, the winding arrangement actually employed is shown in Fig. 5. The simple device of Fig. 4 is not balanced from the standpoint of longitudinal crosstalk for any coil setting except that of zero mutual induc-

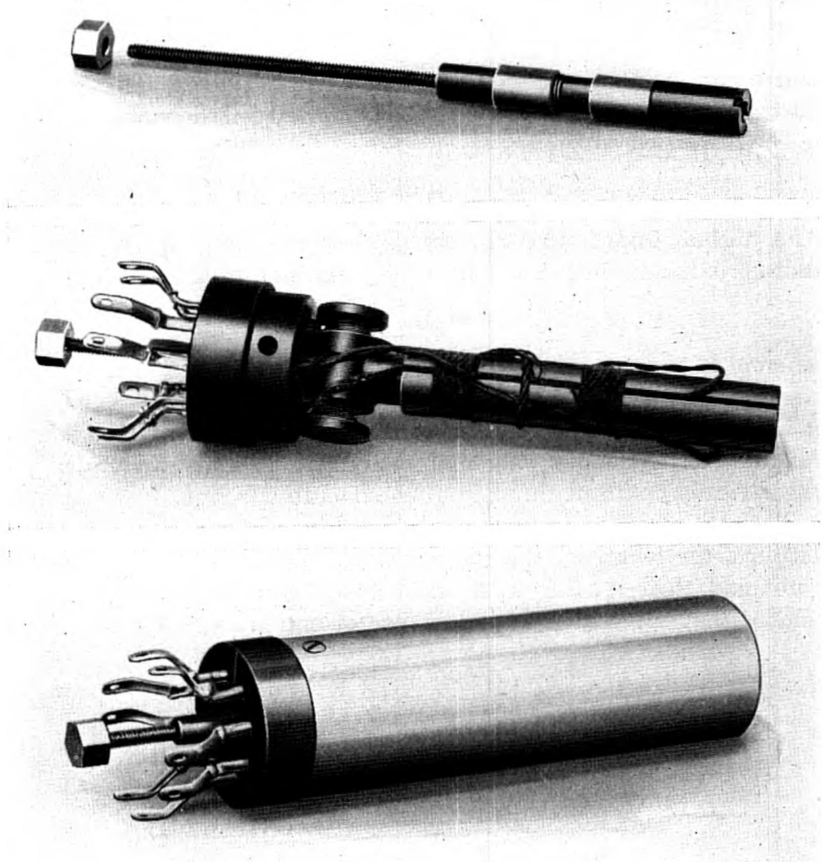

Fig. 6—Trial balancing coil construction. Container is 4½″ in length and 1⅜″ in diameter.

tance. The Fig. 5 arrangement is such that theoretically there is no magnetic coupling between the two circuits for longitudinal currents in either one, regardless of the coil setting. Unless the capacitance between primary and secondary windings can be kept very small, the resultant admittance unbalance produces crosstalk which is not completely balanced out when the coil is adjusted. The turns of conductor in the Fig. 5 coil are so located that this side-to-side capacitance unbalance is less than 5 micro-microfarads. The capacitances between wires of either the primary or secondary winding do not affect the unbalance but contribute a part of the capacitance loading which compensates for the line inductance of the coils.

In the actual balancing coil, shown in Fig. 6, the windings are located in channels cut in a fibre tube which is secured to a head carrying the winding terminals and a bushing through which passes the threaded brass rod supporting the two cores. Below the head are small spool

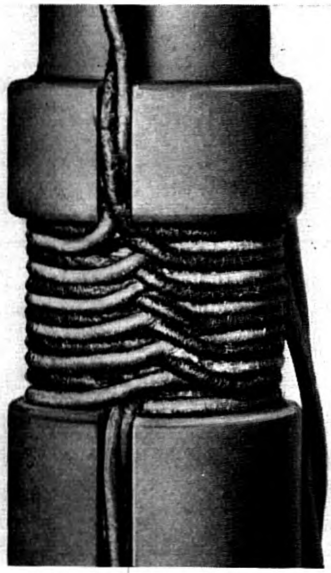

Fig. 7—Arrangement of inner winding of trial balancing coil.

forms on which the shunts are wound. Insulating material such as bakelite is used to obtain proper spacing of the two cores.

The rather unusual manner in which the turns are applied is illustrated in Fig. 7, which is a closeup view of the two wires forming the inner winding. These two wires alternately cross over each other, progressing along the axis in opposite directions of rotation. The outer winding is similarly applied. This type of winding eliminates all splices within the coil, removing hazards incident to interior splices.

The complete coil assembly is enclosed by an aluminum container which serves the dual purpose of a shield and a convenient means of holding the coil for mounting purposes as this container fits snugly into an aluminum cup riveted to the assembly panel. The windings are dried and impregnated and the space between the coil assembly and container is filled with insulating compound.

The mutual inductance of a typical coil varies as shown in Fig. 8 as the cores are moved. The range, with the shunts disconnected, is approximately + 1.6 to − 1.6 microhenries, which is covered in about 16 turns of the screw (a total core travel of 0.5 inch). With the shunts connected, the effective mutual inductance at a given setting becomes less as the frequency rises, the two components, $M_a$ and $M_b$, varying with frequency as shown in Fig. 9. To determine the proper values of $L_s$ and $R_s$ for the shunt, allowance must be made for the complex mutual inductance inherent in the coil due to proximity effect within the windings.

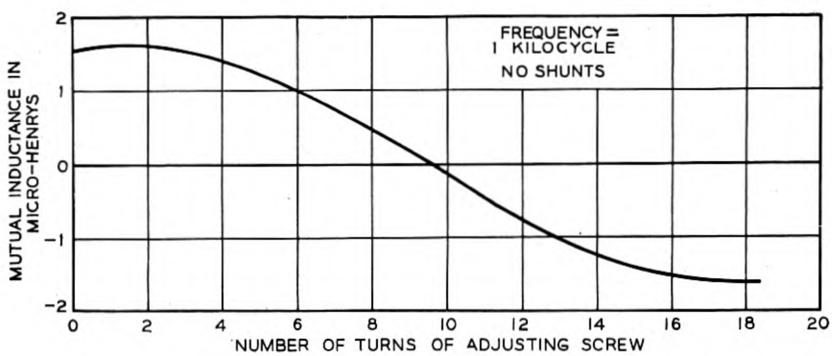


Fig. 8—Mutual inductance of trial balancing coil.

The series inductance of the balancing coil without shunts varies as shown in Fig. 10. As the cores are moved, the inductances of windings 1–*A*–*B*–2 and 5–*C*–*D*–6 (Fig. 5) behave as shown by their respective curves, one increasing as the other decreases. The sum of these two curves is shown by the dotted line, and the measured value of 1–5–6–2 is shown by the solid line. It is seen that the overall self-inductance of 1–5–6–2 is constant to within ± 0.1 microhenry. The difference between the curves (about 0.1 microhenry) is caused by the slight mutual inductance existing between winding 1–*A*–*B*–2 and 5–*C*–*D*–6, which is negative owing to reversed winding direction in this side of the balancing coil. The measured inductance around 3–7–8–4 would slightly exceed that obtained by adding the inductances of the two sections owing to positive mutual inductance between the two ends. These end effects could be reduced by greater separation of the two sets of windings, but this refinement is not necessary.

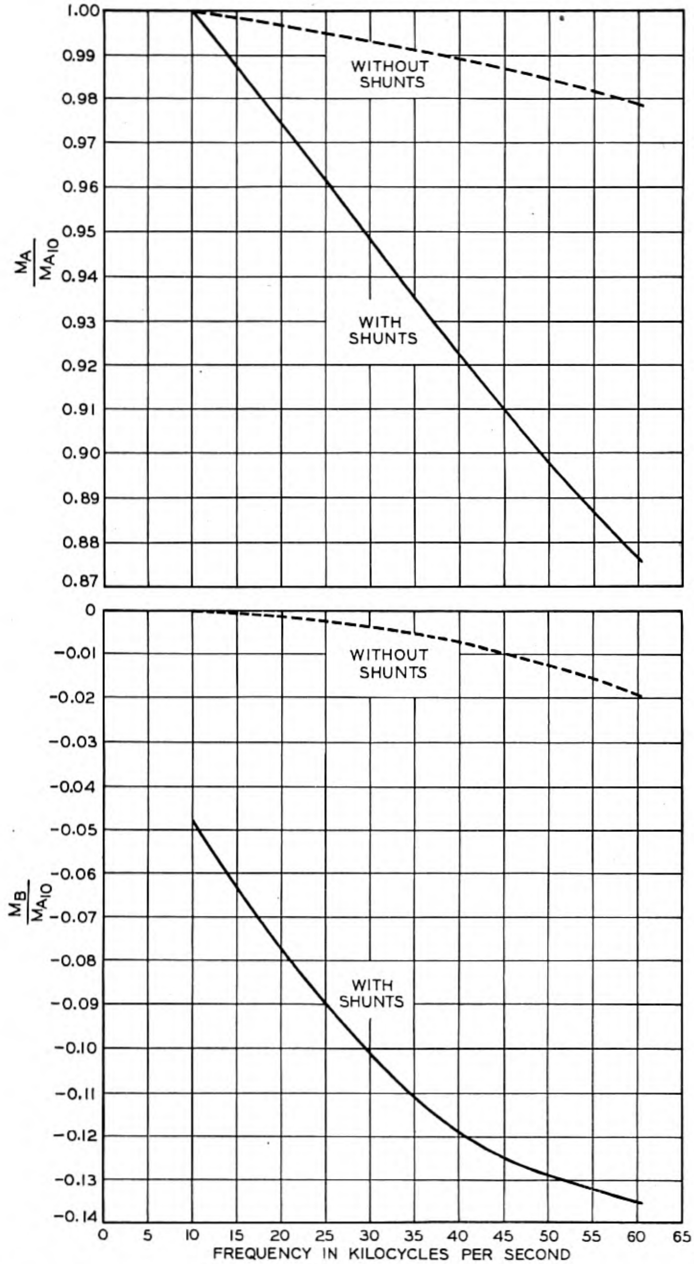


Fig. 9—Variation of $M_a$ and $M_b$ components of trial balancing coil with frequency.

When the shunts are connected, the inductance around 1–5–6–2 is lowered slightly, and the effective resistance is increased. To simplify the capacitance loading and in order not to introduce more resistance in one cable pair than another, the balancing coil assembly is so arranged that shunted and non-shunted windings are alternately introduced into a pair.

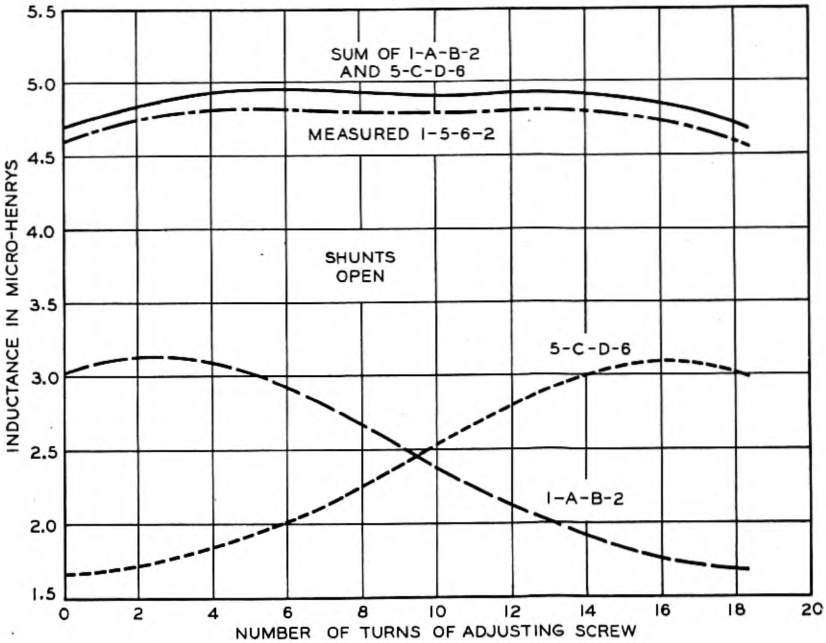


Fig. 10—Series inductance of non-shunted trial balancing coil.

*Balancing Panels*

In assembling the balancing coils on panels, the same number of coils should be traversed on each of two pairs before reaching the coil that balances these two pairs, in order that the phase shift up to this balancing coil on one pair will be essentially the same as that on the other pair. If these phase shifts differed materially, the coil setting for minimum crosstalk when one pair is the disturbing circuit might be quite different from the best setting when the other pair is the disturbing circuit. To obtain this equality objective a "criss-cross" arrangement, as shown schematically on Fig. 11, was devised, whereby the number of coils on one pair up to a particular balancing coil never differs by more than one from the number of coils on the other pair up to this same balancing coil.

For economic reasons it is undesirable to install a complete panel for the ultimate number of pairs, possibly 100 in some cases, but rather to install sections conforming more closely to the circuit growth. The placing at different times and properly connecting of sections obtained from the 100-pair criss-cross panel and at the same time maintaining service on operating circuits appeared rather formidable. This problem was solved by the use of two types of criss-cross panels; an

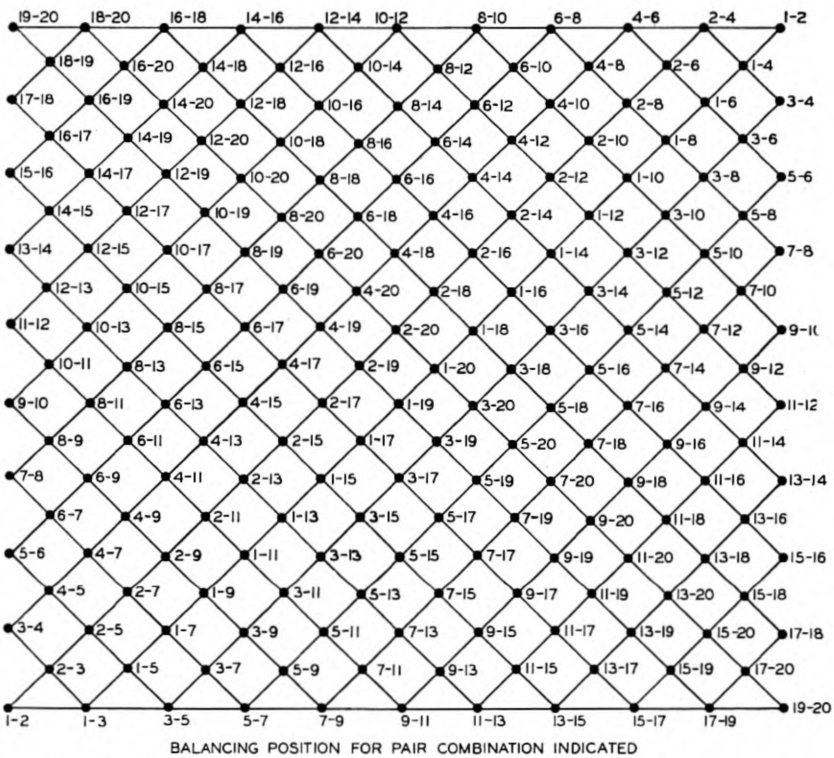


Fig. 11—Schematic of criss-cross wiring for 20-pair balancing panel, designed to maintain phase equality of coils.

intra-group panel for balancing within one group of carrier pairs and an inter-group panel for balancing pairs in one group against pairs in a second group of equal size. In the present design, an intra-group panel takes care of 20 pairs (190 combinations) and an inter-group panel of the 400 combinations between two 20-pair groups. To maintain phase equality through a number of panels, it is necessary to install them following a definite pattern. Figure 12 shows a suitable pattern for the 15 panels required for 100 pairs.

In the criss-cross scheme (Fig. 11) the side-to-side combinations, which are those marked 1/2, 3/4, 5/6, etc., appear twice, i.e., along the left and right edges of the panel. Advantage of this is taken by installing balancing coils at both locations. This is done because one side-to-side coil of about 1.3 microhenries may not be large enough in all cases in spite of the fact that the mean side-to-side crosstalk has been reduced 9 db by poling.

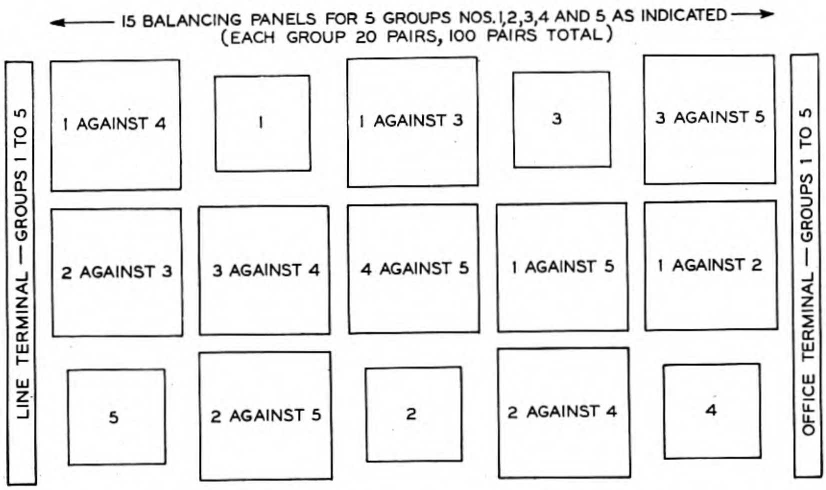


Fig. 12—Allocation of balancing panels designed to maintain phase equalization of coils at all stages. Panels with suitable cross-connections between them are installed in following order.

For first group—Install 1
Add second group—Add 2, and 1 against 2
Add third group—Add 3, 1 against 3, and 2 against 3
Etc.

### *Balancing Procedure*

As stated above, the far-end crosstalk in a repeater section can not be balanced out completely over the frequency range with a single balancing unit. To determine the balanceable as distinct from the non-balanceable crosstalk, involves crosstalk measurements in phase and magnitude at a number of frequencies, using each pair of a two-pair combination as a disturbing circuit in turn. The balanceable crosstalk may then be separated from the non-balanceable crosstalk by computation. Balancing by this method would be impracticable because of the time required. As a practical scheme, it has been shown that balancing at a frequency of about 40 kc. will produce satisfactory results over the type K range even though part of the non-balanceable crosstalk may be neutralized at this frequency. This is theoretically undesirable since the crosstalk reduction at other frequencies is impaired.

To prevent undue interference into operating carrier circuits when balancing, a frequency falling between the transmitted bands must be used. For this reason, the balancing coils are adjusted at a test frequency of 39.85 kc. and a measurement to check the suitability of the adjustment is made at 28.15 kc. Figure 13 shows the crosstalk vs. frequency before and after coil balancing by this method on three repeater sections.

### *Additional Crosstalk Remedial Measures*

Although poling as well as balancing is done to reduce side-to-side crosstalk, this crosstalk is still considerably greater than the pair-to-pair crosstalk. For this reason, side-to-side crosstalk is diluted among

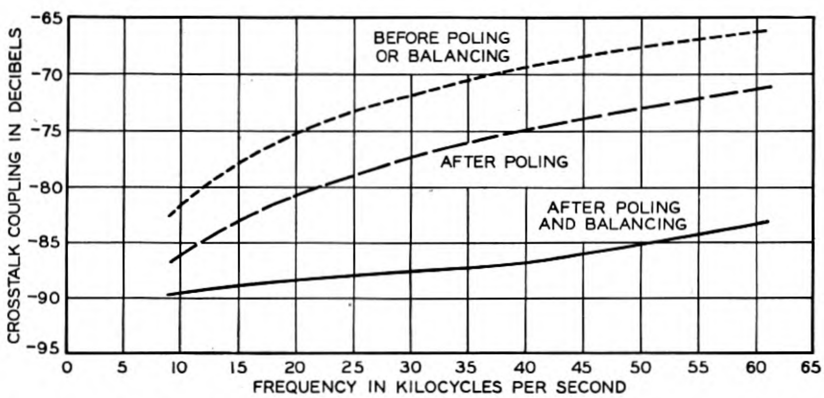


Fig. 13—R.M.S. far-end crosstalk per repeater section from measurements on 3 repeater sections.

the pair-to-pair combinations by a system of quad-splitting at repeater points.

The crosstalk after balancing (Fig. 13) is considerably higher at the upper end of the frequency band than at the lower end. Consequently, if circuits were set up to use the same channel throughout, the crosstalk in the upper-frequency channels would be materially greater than that in the lower-frequency channels. In order that all circuits may be equally satisfactory from the crosstalk standpoint, a system of special channel assignments in successive intervals, say 500 to 1000 miles, can be used. This will tend to equalize both the crosstalk and the noise on all circuits, thus permitting a somewhat cheaper design then if each channel had to meet the crosstalk and noise limits by itself.

## Noise

Besides babble, many other sources of noise need to be considered in cable carrier design. Figure 14, which shows the approximate magnitude of several of these if no means are taken to suppress them, indicates the noise at the end of a single 17-mile repeater section when

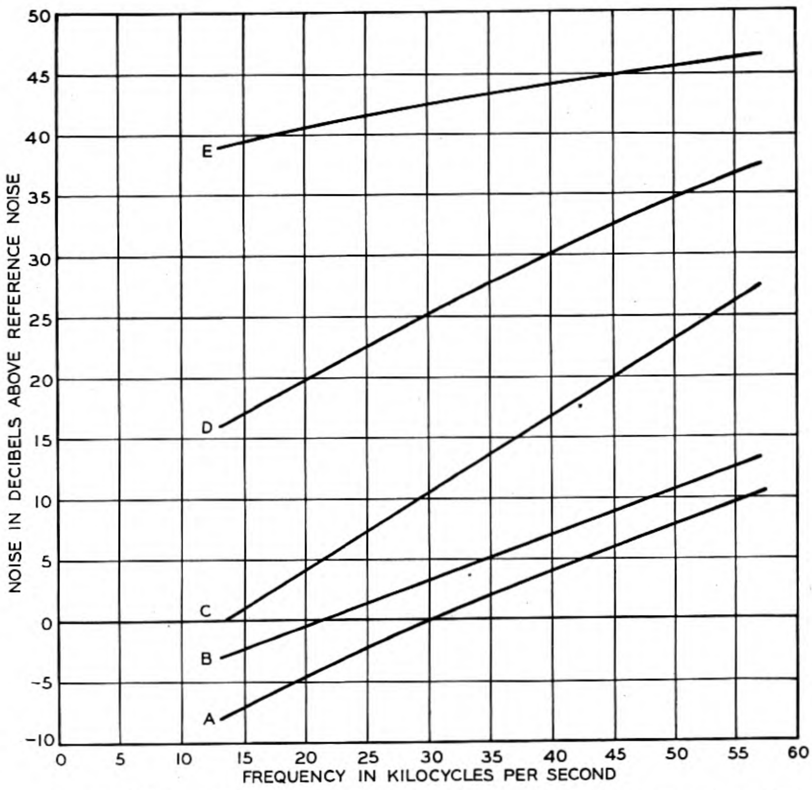


Fig. 14—Noise, prior to suppression measures, per repeater section at output of repeater whose gain equals line loss.

*A*—Noise from thermal agitation.
*B*—Thermal agitation plus tube noise.
*C*—Noise from voice frequency telephone repeater office.
*D*—Noise from telephone and telegraph repeater office.
*E*—Noise from heavy static on open-wire tap close to carrier repeater input.

amplified by a repeater whose gain equals the hot-weather line loss. Curve *A* shows the unavoidable lower limit of noise, that produced by thermal agitation of the electrons in the cable conductors and the repeater.[2] This amounts to about $2 \times 10^{-17}$ watts per telephone

[2] J. B. Johnson, *Phys. Rev.*, **32**, 97 (1928); H. Nyquist, *Phys. Rev.*, **32**, 110 (1928).

channel per repeater section, at the repeater input. If there were no other noise sources, the repeater section length would necessarily be limited by this effect. Curve $B$ shows the sum of thermal noise and noise due to the vacuum tubes in the repeaters, which is little in excess of thermal noise alone. The other three curves show noises of considerably higher magnitude which require suppression in order to arrive at an economical carrier system. Curve $E$ shows the order of magnitude of noise on carrier circuits due to connecting open-wire pairs directly to non-carrier pairs in the outside cable near the carrier repeater input. The source of the noise is heavy atmospheric static of a magnitude experienced several times during the summer.

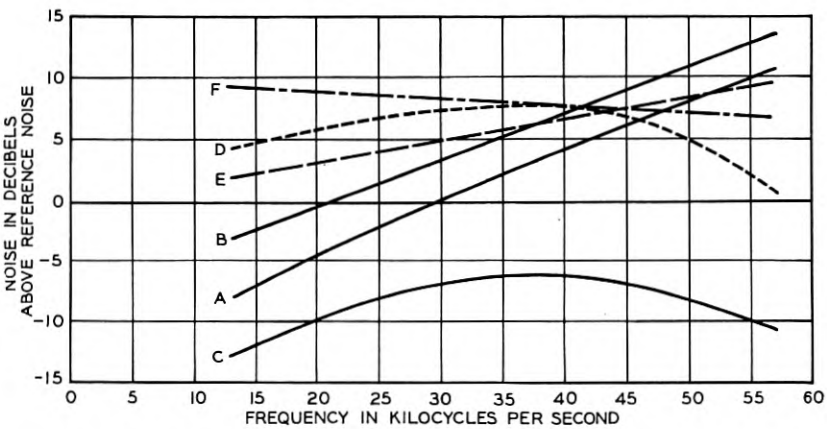


Fig. 15—Noise, subsequent to suppression measures, per repeater section at output of repeater whose gain equals line loss.

$A$ to $E$—Same sources as in Fig. 14.

$F$—Noise from heavy static induced directly into outside cable.

The other curves show typical magnitudes of noise originating in the existing telegraph and voice frequency telephone plant; this is generated in existing repeater stations and transmitted by the non-carrier pairs to the outside cable where it is induced into the carrier pairs. Curve $D$ represents the situation at a combined telephone and telegraph repeater station, and Curve $C$, the situation at a station where there are no telegraph repeaters.

Figure 15 indicates the results after suppression measures have been applied. As shown, at the top frequency, which controls the carrier repeater section length, these sources of noise have been reduced to be well below thermal plus tube noise. It is also shown that the noise due to heavy atmospheric static induced directly into a carrier pair in the outside cable is below thermal plus tube noise at the top frequency.

There are additional types of noise, not shown, whose sources lie within the carrier system: e.g., modulation in amplifiers, inter-system cross-induction, battery noise. While control of such noise is an integral part of the fundamental carrier system design, it is not the purpose of this paper to cover this class of noise.

### *Conductors Tapping the Carrier Cable*

Carrier noise may come from open-wire pairs which connect to conductors in the cable. Its sources may be static; corona on power lines; power line carrier or other carrier frequency voltages on power lines paralleling the open wire; induction from radio telegraph stations; or carrier frequency voltages arising in the office to which the open

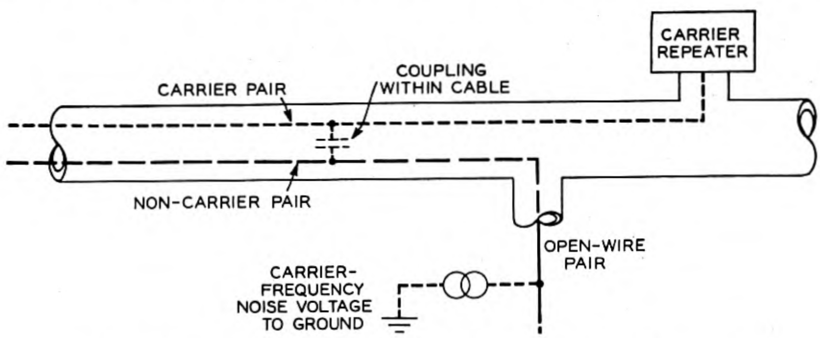


Fig. 16—Schematic of path followed by induction from open-wire taps.

wire is connected, such as voltages generated by d-c. telegraph or telephone signaling systems. The limited experience to date indicates that, in a long cable carrier system, the effect of heavy static will be larger than that of the other sources if telephone and power supply plants are coordinated so as to be satisfactory from voice frequency and low frequency standpoints. Branch cables connected to the carrier cable have a similar but generally smaller effect than that of open-wire taps.

Figure 16 illustrates the path followed by this induction. A voltage to ground impressed on the open-wire pairs passes by secondary induction over to the carrier pairs in the cable, and, on account of the unbalance to ground of these pairs, produces a metallic voltage on these pairs at the repeater input. The effect may be greatly reduced by interposing a filter at the junction of the open wire and the cable.

It is necessary to filter only the longitudinal circuit at an open-wire tap, because: (1) the voltage to ground on the open wire is larger than the metallic circuit voltage, and (2) the coupling between the longitudinal circuit and the disturbed carrier pair is greater than the coupling between metallic circuits.

Figure 17 is a schematic diagram of the longitudinal filter developed for a phantom group. It consists of two longitudinal retardation coils and a set of condensers connected between the line wires and the cable sheath. This filter has relatively high carrier frequency longitudinal impedance to minimize effects of impedance in the ground con-

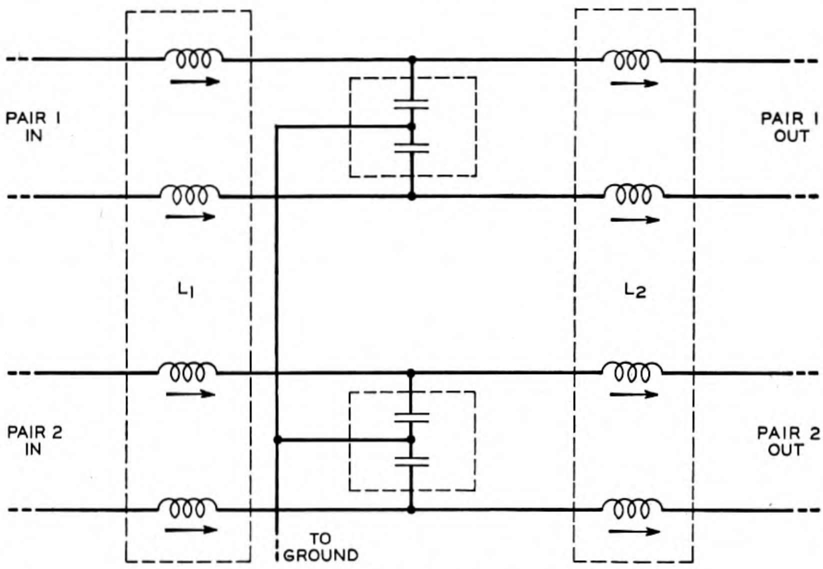


Fig. 17—Schematic of longitudinal filter.

nection. The major portion of the carrier frequency impedance of the coils is obtained by designing them to have high core loss at these frequencies. The filter has little effect on voice frequency transmission, precaution having been taken to hold the transmission loss, crosstalk and unbalance to ground to low values.

### *Noise Arising in Existing Repeater Offices*

The noise caused by carrier frequency voltages generated in existing repeater offices is due to d-c. telegraph, telephone speech and signaling voltages, power supply, etc. Figure 1 shows the path by which they reach the carrier plant and the means used to suppress them. In this figure, $N$ represents a source of carrier frequency voltage in a repeater office, connected to a voice frequency pair which transmits this voltage into the outside cable where it is induced on the carrier pairs. These voltages are reduced by inserting suppression coils in the longitudinal voice frequency paths at the junction between the office and the outside cable connected to carrier inputs.

The design of coils giving the requisite carrier frequency suppression without appreciably affecting voice frequency transmission on the circuits in which they are connected was difficult. One coil is used for each phantom group. Each coil has sixteen windings, four for each line wire. These windings are so paired and disposed about the core

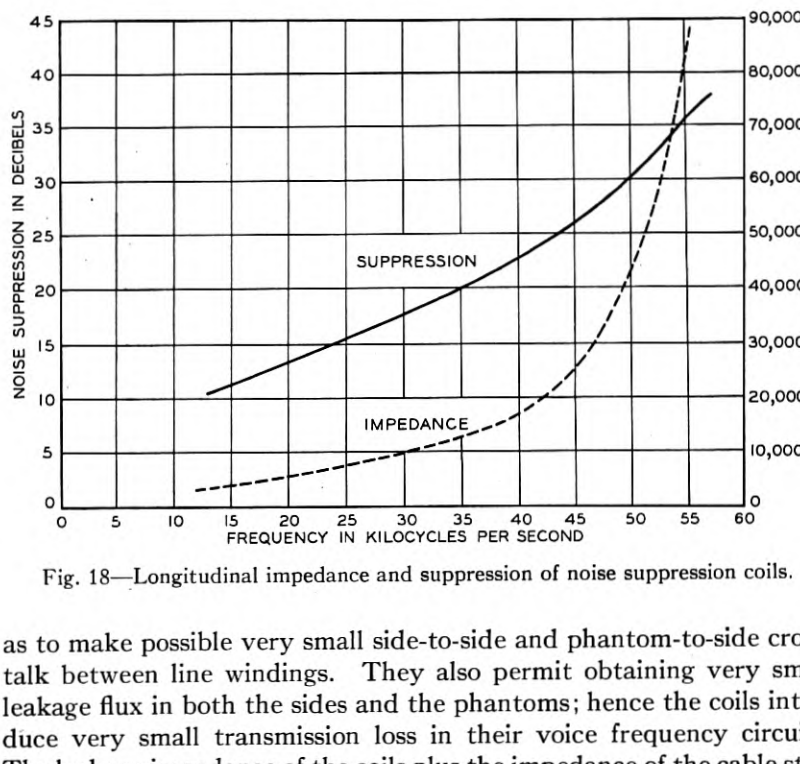


Fig. 18—Longitudinal impedance and suppression of noise suppression coils.

as to make possible very small side-to-side and phantom-to-side crosstalk between line windings. They also permit obtaining very small leakage flux in both the sides and the phantoms; hence the coils introduce very small transmission loss in their voice frequency circuits. The leakage impedance of the coils plus the impedance of the cable stub used to connect them into the circuit is held down so that the effect on repeater singing and echoes in the voice circuits is very small. The coils are so wound that their longitudinal inductance is in antiresonance with their distributed longitudinal capacitance at approximately the top cable carrier frequency, resulting in a large increase in their suppression in this critical frequency range. The longitudinal impedance of one of these coils, and the approximate suppression which a set of them provides, are shown in Fig. 18.

In addition, the carrier circuits are carefully separated, electrically and physically, from existing voice frequency circuits in common repeater stations. To this end the carrier pairs in the outside cable are brought out on the line side of the noise suppression coils into a separate cable connected directly to a sealed terminal. From this terminal they are carried in shielded wire to the units in the carrier office and then to a similar sealed terminal leading to the outside cable in the opposite direction. Filters for filament and plate battery supply are included in the carrier amplifiers and additional filament battery supply filters are provided at the carrier fuse panels.

# A New Single Channel Carrier Telephone System *

By H. J. FISHER, M. L. ALMQUIST and R. H. MILLS

The single channel carrier telephone system described in the following paper is designated the Type H. It is characterized by several new features, making it applicable not only to the needs of telephone companies but also to those of railroads, power systems and oil companies. It replaces the Type D single channel carrier system, more than 500 of which are now in operation in the Bell System, and, in addition because of its lower cost is applicable to shorter distances. It therefore marks another step in extending the use of carrier. Reduction in size and provision for operating on a-c supply simplify its installation, and its portability makes it well suited to provide emergency circuits.

ON open-wire lines where the growth is not rapid, there is frequently need for adding telephone circuits one at a time. When the Type D single-channel carrier telephone system was developed a few years ago it became possible to meet this need without stringing additional wires.[1] More than 500 of these systems have been placed in service in the Bell System plant. A new single-channel carrier telephone system, known as the Type H, has recently been developed and is now being applied. This new system offers improved performance, and also, because of its lower cost, is applicable to providing service over shorter distances than were economical with the earlier system.

The Type H system, which is characterized by a number of new features and special developments, is applicable not only to the needs of telephone companies but also to those of railroads, power systems, and oil companies.[2] In the first place it is designed to operate either on alternating current or on direct-current plate and filament supply. A repeater is available to extend the range of operation. Through the use of specially designed but simple filters the system can be employed on circuits which are equipped with bridged telephone

* Presented at Winter Convention of A. I. E. E., New York, N. Y., January 24–28, 1938. Published in *Electrical Engineering*, January 1938.

[1] "Carrier Telephone System for Short Toll Circuits," H. S. Black, M. L. Almquist and L. M. Ilgenfritz, *A. I. E. E. Transactions*, Vol. 48, January 1929, pp. 117–140.

[2] "Carrier Telephone Systems—Application to Railroad Circuits," H. A. Affel, *Proceedings of the Association of American Railroads, Telephone and Telegraph Section*, October 1936, pp. 654–672.

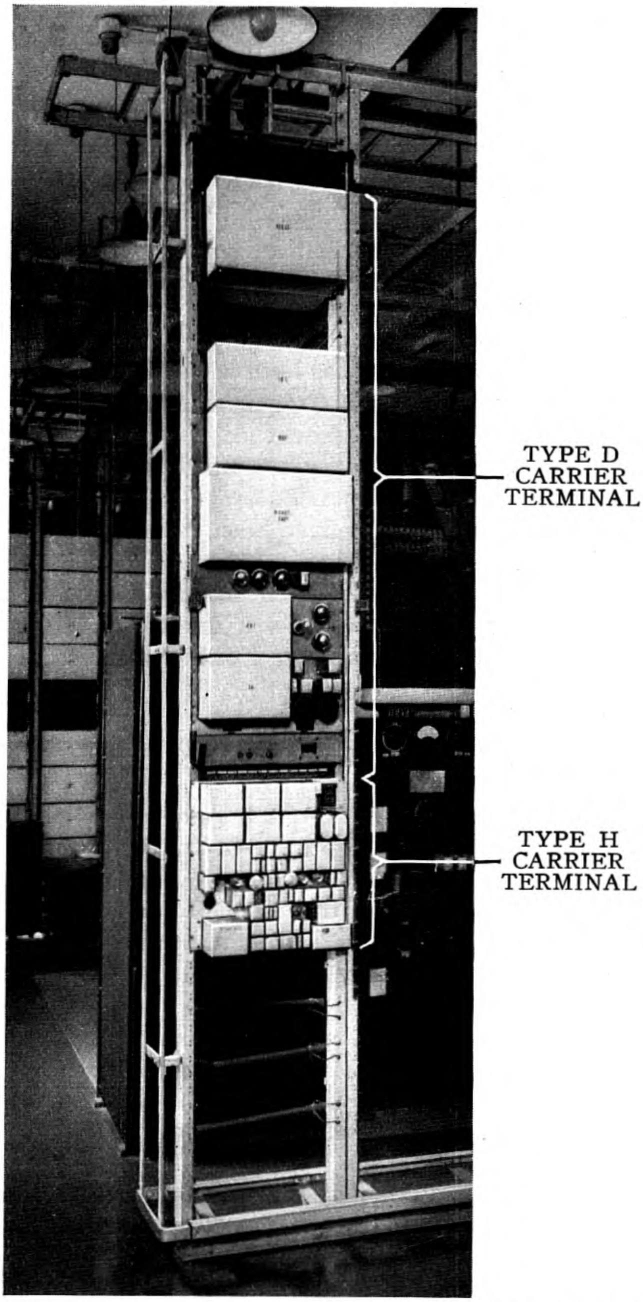


Fig. 1—Installation of Type H carrier telephone system at Charlotte, North Carolina.

stations at intermediate points as is frequently the case in railroad operation.

A unique feature is the use of opposite sidebands of the same carrier frequency for opposite directions of transmission. The upper sideband is used in one direction and the lower sideband in the other, the carrier being suppressed. For the modulators and demodulators copper oxide "varistors" are employed in place of vacuum tubes. The amplifiers are single stage, employ pentode tubes and are stabilized in performance by feedback. The filters have been simplified in construction by the use of coils with a new type of core material and by improved designs of paper condensers.

The size of the new terminal has been so reduced that it occupies less than 40 per cent of the space required for a Type D terminal, as indicated in Fig. 1. The equipment may be mounted on racks as is customary in telephone offices, or a complete terminal or repeater may be mounted in a small cabinet.

Single-channel carrier systems have been used in the Bell System principally for short open-wire toll circuits. Thus, the Type D systems are for the most part between 50 and 200 miles in length. The Type H system, since it includes a repeater, can be used for greater distances, and due to its lower cost is economical for shorter distances.

## General Description of System

### *Basic System*

The basic system consists of two terminals one of which is referred to as an "east" terminal and the other as a "west" terminal, as indicated in Fig. 2. The two terminals differ only in minor respects, the differences being due to the fact that at one terminal the upper sideband is transmitted and the lower sideband is received, while at the other terminal the reverse takes place. In order to simplify coordination between various types of carrier systems operating on the same pole line, the frequencies between 7400 cycles and 10,150 cycles are transmitted in the east to west (or north to south) direction, and the frequencies between 4150 cycles and 6900 cycles in the west to east (or south to north) direction. The frequency allocation of the Type H system and those for the Type D and the three-channel Type CS system are shown in Fig. 3. All three types may be operated on the same pole line.

The circuit arrangement is given in greater detail in Fig. 4, which shows a schematic diagram of one terminal, with the exception of the power supply circuit. Each terminal is made up of a transmitting

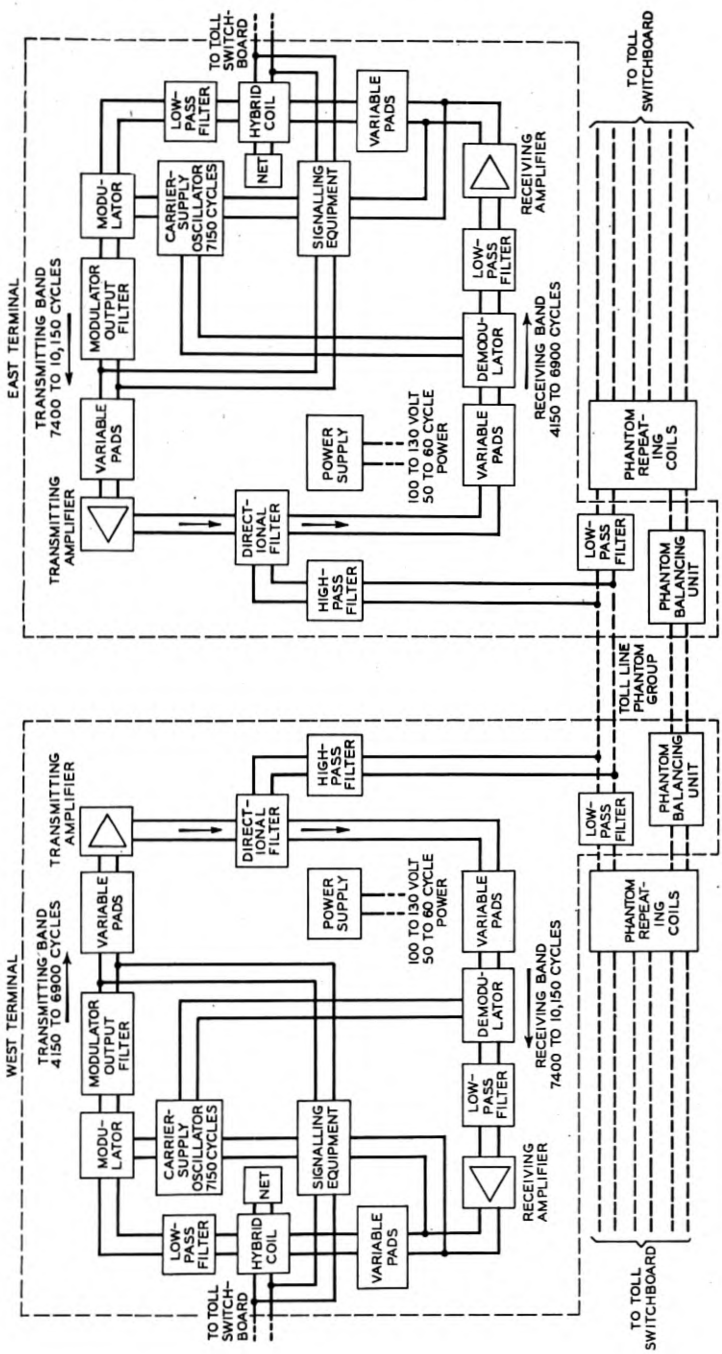


Fig. 2—General schematic of Type H system applied to side circuit of phantom group.

branch which includes a modulator, a receiving branch which includes a demodulator, and a hybrid coil to combine the two branches into a two-wire voice-frequency circuit. The carrier is generated by a vacuum tube oscillator which supplies both the modulator and demodulator. The output of the modulator, which is of the copper oxide type, consists principally of the two sidebands. The desired sideband is selected by the modulator output filter and an amplifier raises the level to that desired for transmission over the line. The demodulator is also of the copper oxide type; its output consists principally of the two sidebands, one of which is a reproduction of the original voice-frequency input. This is selected by means of a low-pass filter and applied to a voice-frequency amplifier which provides the necessary receiving gain. Adjustable pads serve as a means for adjusting the transmitting and receiving gains. The characteristics and functions of the various filters are described later.

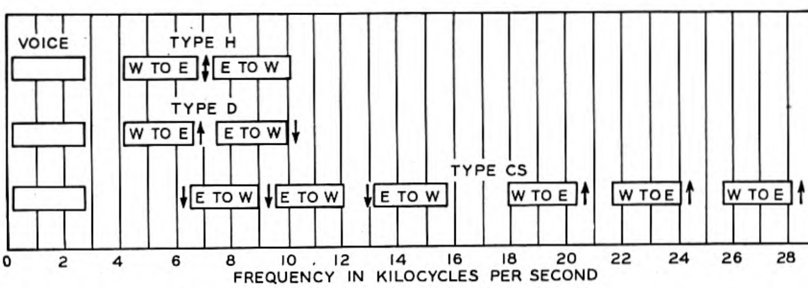


Fig. 3—Frequency allocation.

For signaling over the carrier circuit, a 1000-cycle signaling system is employed. The method is similar to that used on other types of circuits, but includes a number of simplifications. The number of tubes has been reduced, and the vibrating relays used in the older type circuits have been replaced by copper oxide rectifiers and simple d-c. relays requiring little maintenance. When 20-cycle ringing current is received from the switchboard, the frequency of the carrier supply oscillator is shifted by 1000 cycles and its output is interrupted at a 20-cycle rate, and applied to the input of the transmitting amplifier. At the receiving end this appears at the input of the signal receiving circuit as a 1000-cycle current interrupted at a 20-cycle rate. It is then demodulated in a copper oxide rectifier and the resulting 20-cycle current is amplified and applied to a second rectifier the output of which is connected to a d-c. relay. Operation of this relay causes 20-cycle current from a local source to be sent toward the switchboard.

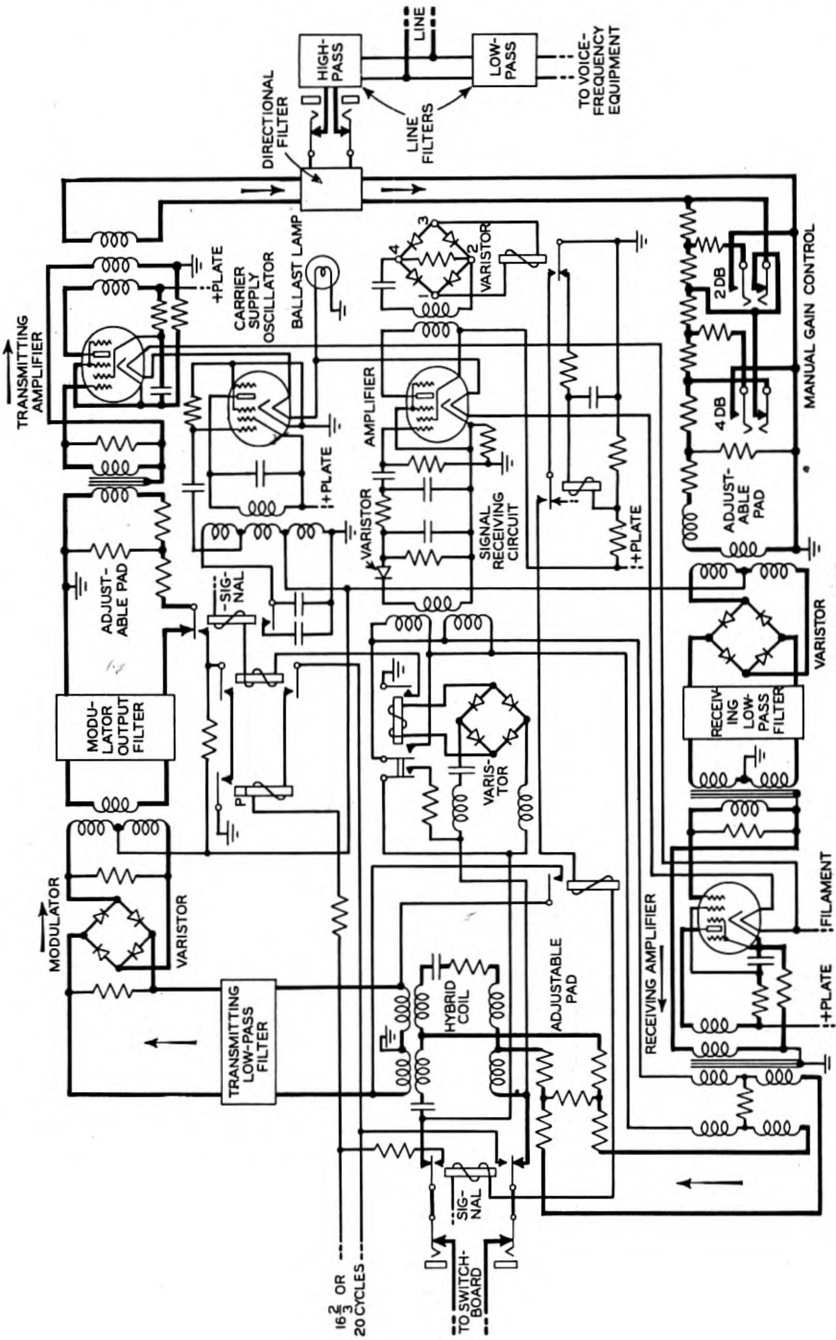


Fig. 4—Schematic of Type H terminal.

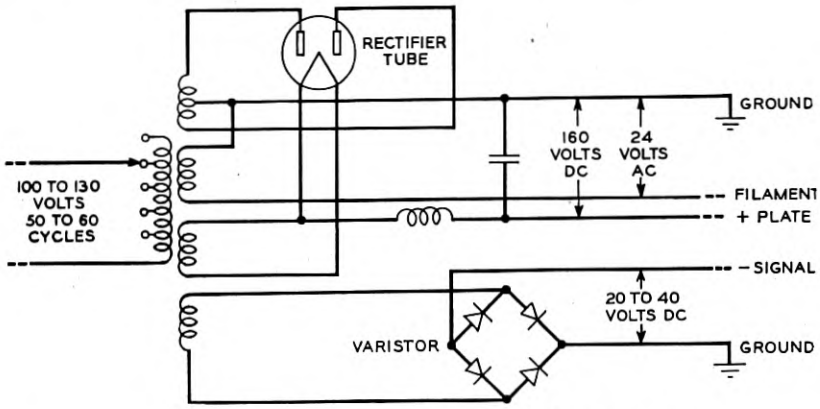


Fig. 5—Schematic of a-c. power supply for terminal.

Freedom from false operation on speech is obtained by tuning the receiving circuit to the rate of interruption and by providing a slow operating relay such that the signal must persist for several tenths of a second before 20-cycle current is passed to the switchboard.

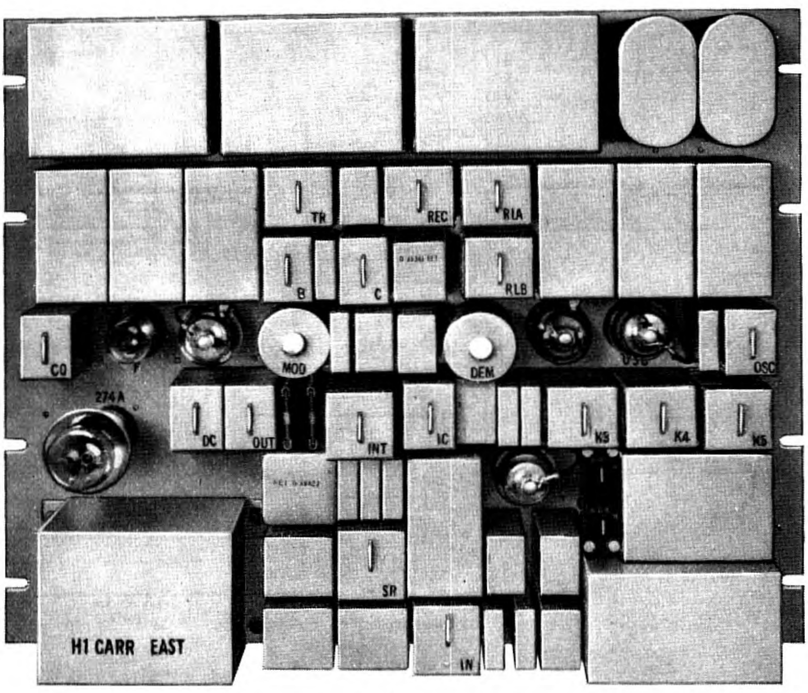


Fig. 6—Terminal panel—front view.

The power supply circuit, which is shown in Fig. 5, operates from an alternating-current source of 100 to 130 volts, 50 to 60 cycles, and requires about 50 watts. It supplies alternating current for the filament supply, 160 volts direct current for plate supply and 24 volts direct current for relay operation. Provision is also made for direct operation from 24-volt and 130-volt central office batteries.

The equipment for a terminal is mounted on two panels. One of these, shown in Fig. 6, is $15\frac{1}{4}$ inches by 19 inches in size and contains all the apparatus except the line filter circuit. The second panel, shown in Fig. 7, is $3\frac{1}{2}$ inches by 19 inches and contains the line filters, and other equipment which is required for balancing purposes.

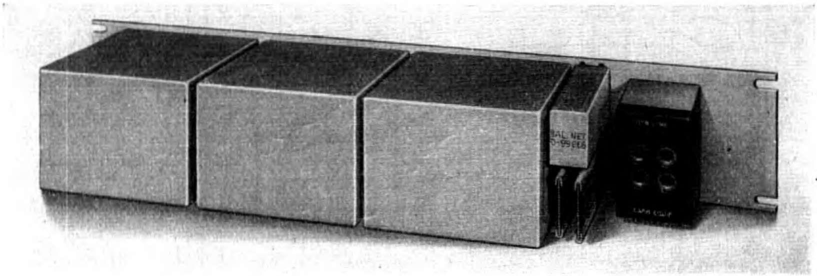

Fig. 7—Line filter and balancing panel—front view.

*Repeater*

A schematic of the repeater is shown in Fig. 8. The amplifiers are the same in design as the transmitting amplifier at the terminals. Two sets of line filters are required and are identical with those used at the terminals. The a-c. power supply circuit is substantially the same as the one used at the terminals except that the 24-volt supply for relay operation is omitted. The power required for operation is about 35 watts.

The complete repeater consists of three panels—the repeater panel, which is $10\frac{1}{2}$ inches by 19 inches, and two line filter and balancing panels. The repeater panel is shown in Fig. 9.

## Transmission Performance

*Line Considerations*

In outlining the transmission performance of the system, it is necessary to consider the characteristics of the lines as well as those of the equipment. At carrier frequencies the line losses and the variations in these losses with weather are considerably greater than at voice frequencies. This is illustrated in Fig. 10, which shows the

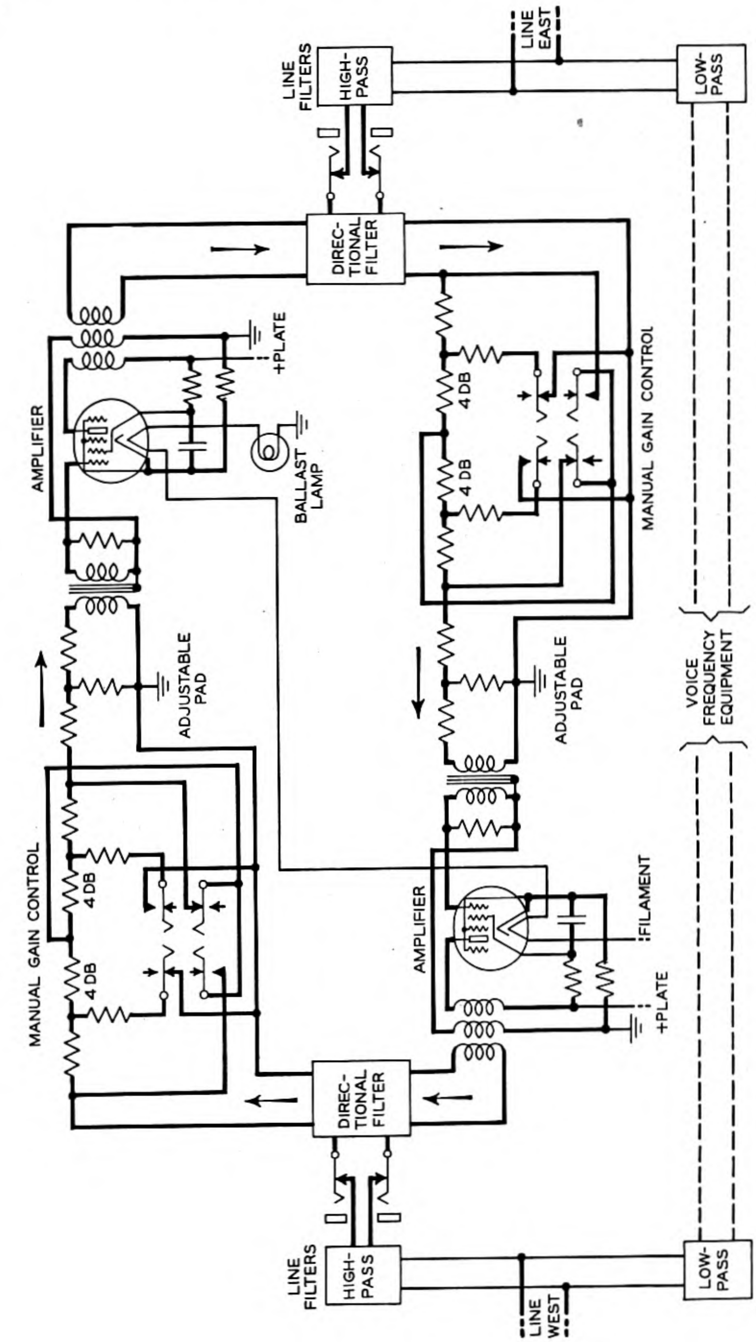


Fig. 8—Schematic of repeater.

attenuation-frequency characteristics for four commonly employed gauges of open-wire line under dry and wet weather conditions, respectively, at a temperature of 68° F. These curves are for 12-inch spaced copper wires equipped with the type of insulators ordinarily

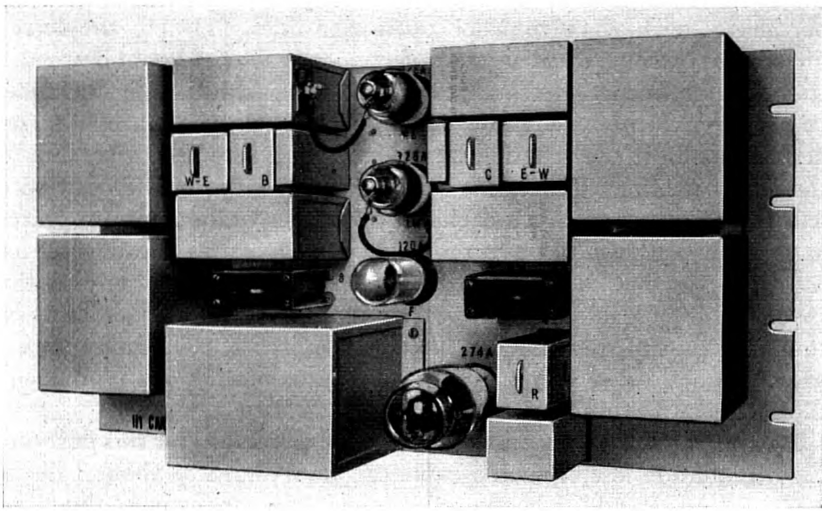


Fig. 9—Repeater panel—front view.

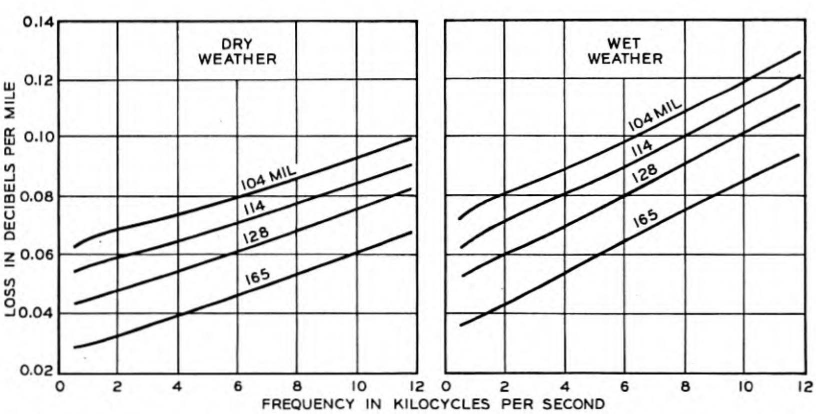


Fig. 10—Attenuation of open-wire side circuits—12-inch spaced copper wire with double petticoat glass insulators.

employed for toll circuits. Changes in temperature also affect the attenuation, the loss increasing as the temperature rises. The variations due to changes in temperature are, however, smaller than those due to changes from dry to wet weather and are more likely to occur gradually. It is apparent that the variations in attenuation with weather increase with the length of the system, and that more frequent adjustments will be required on the longer systems to maintain the overall circuit net loss within given limits.

The carrier frequency attenuation of cable circuits is much greater per unit length than that for open-wire lines. Hence, the losses introduced by comparatively short sections of cable, either at entrances to offices or at intermediate points, are of considerable importance. In addition to the attenuation of the cable itself, there are large reflection losses at the junction of the cable and the open wire, due to the difference between the characteristic impedance of the open-wire and that of the non-loaded cable. The carrier terminal and repeater have been designed to have the same impedance as the average of the open-wire line facilities, so the same considerations apply at the junction of the cable and the carrier equipment. As an example of the magnitude of these effects, the insertion loss at 8000 cycles of two miles of non-loaded 16-gauge cable is approximately 6 db, or about the same as that of 60 miles of 104-mil open-wire.

By applying carrier loading that has been developed for this purpose, the attenuation loss of such a cable can be reduced to about 1 db at 8000 cycles. In addition, the reflections at the terminals of the cable will be reduced to satisfactory low values by virtue of the impedance matching properties of the loading. This method of treatment has the important advantage that it improves the transmission characteristics in both the voice and carrier ranges.

In cases where substantial transmission margins exist, it is sometimes practicable to use impedance matching transformer networks at the cable terminals, as a substitute for carrier loading, with economies that are proportional to the length of the cables. At the present stage of development, this so-called transformer treatment is much less satisfactory in the voice-frequency range than in the carrier-frequency range. Certain inherent limitations in simple transformer treatment result from the fact that the ratio of the (non-loaded) cable impedance to the open-wire impedance varies widely over the frequency band to be transmitted, and the transformer impedance ratio that is optimum at carrier frequencies is distinctly disadvantageous in the voice-frequency range. The choice of optimum transformer ratio for the complete transmission band may thus involve different compromises for different sets of conditions and service requirements.

Where several carrier systems are to be placed on a pole line, crosstalk between systems becomes an important consideration. Where only a few single-channel systems are involved, it is sometimes possible, by separating the systems widely on the pole line, to operate with only the regular voice-frequency transpositions, but, in general, additional transpositions are required. A comparatively inexpensive transposition system for this purpose was designed at the time the Type D system was developed. It permits operation of Type H systems on all pairs of a four-crossarm line with the exception of the pole pairs, and Type C three-channel systems on the top crossarm. In addition to transposing it is important that reflections at junctions between open wire and cable be reduced as described above, in order that near-end crosstalk will not through reflection appear as crosstalk at the distant terminal of the system.

*Range of Operation*

The terminals and repeaters have sufficient load carrying capacity so that they may be operated at an output level 16 db above that at the transmitting toll switchboard. About 19 db transmitting gain and 14 db receiving gain are available at each terminal, of which a total of 20 db may be used at the east terminal and 22 db at the west terminal. The lower permissible loop gain (sum of transmitting and receiving gains) at the east terminal is not controlling, since the line loss is greater for the frequencies used in the east to west direction than for those used in the west to east direction. Thus, the terminals are capable of providing a 9 db circuit over a line the attenuation of which does not exceed 31 db at 8150 cycles and 29 db at 6150 cycles. These figures correspond roughly to the wet weather attenuation of about 280 miles of 104-mil open wire where no intermediate cable or equipment is involved. The presence of even a small amount of cable will considerably increase the attenuation so that in most cases the distance which can be spanned is not greater than 150 to 200 miles, and may be even less.

Where greater distances are to be covered, an intermediate repeater may be added. The repeater has a useful gain of about 23 db in each direction, with some flexibility as to allocation of gains between the two directions of transmission. More than one intermediate repeater can, of course, be employed, although as the system is lengthened maintenance effort will be increased, as more frequent adjustments of the overall net loss will be required to compensate for the variations in line attenuation with weather. No provision is made for a pilot channel such as is generally provided on the long multi-channel systems, and adjustments of overall net loss must be made manually. Also, no provision has been made for equalizing the variation in line attenuation with frequency. These factors are not important on the shorter circuits for which the system has primarily been designed.

*Overall Transmission Characteristics*

The circuit provided by a Type H system without a repeater has a band width of about 2750 cycles, extending from about 250 to 3000 cycles. This is somewhat wider than that for the Type D system. The introduction of repeaters will tend to narrow the band somewhat. Representative frequency characteristics are shown in Fig. 11. One

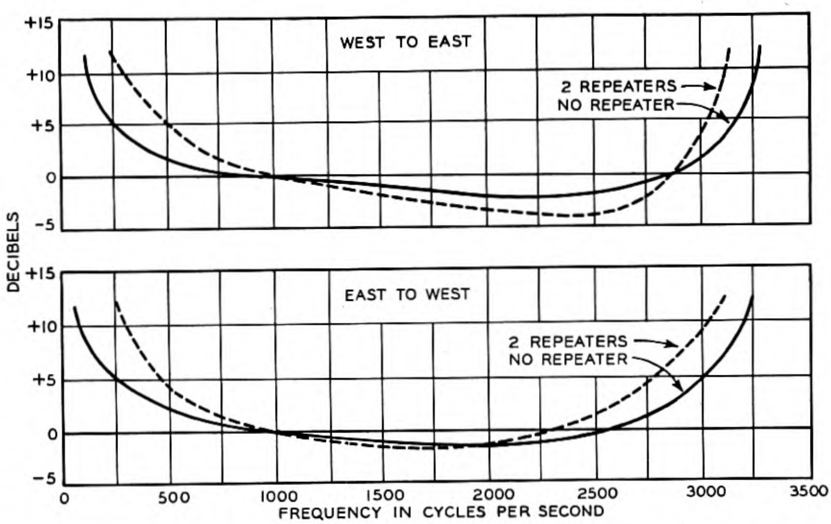


Fig. 11—Representative overall circuit net loss characteristic of system.

set of characteristics is for a typical circuit without a repeater, and the other for a circuit including two repeaters. It is assumed that the line conditions are such that no large reflection effects are present.

The band width is limited principally by the characteristics of the band filters. The small differences in the characteristics for the two directions of transmission are due partly to differences in the filters and partly to the fact that the attenuation of the line increases with frequency and is greater at the 3000-cycle point in the east-to-west channel than for the 3000-cycle point of the west-to-east channel. As the circuit is increased in length this difference tends to increase.

Variations in overall circuit net loss are due largely to variations in the loss of the high-frequency line. For a circuit 200 miles long these may amount to $\pm$ 3 db. The key controlled pads which are included at each terminal and repeater are provided for making adjustments to compensate for these variations. Variations due to the equipment are small in comparison with the line variations. The transmitting gain at a terminal may vary $\pm$ 0.5 db and the receiving gain $\pm$ 0.3 db for variations of $\pm$ 10 volts in the a-c. supply. With a more stable a-c. supply or when operated from regulated plate and filament batteries such as are employed in the larger telephone offices, these variations will be less than half the figures given above. With suitable maintenance it should be possible to maintain the overall circuit net loss within $\pm$ 2 db of its normal value.

A representative load characteristic, as measured with 1000-cycle current for a system without a repeater, is shown in Fig. 12. On a

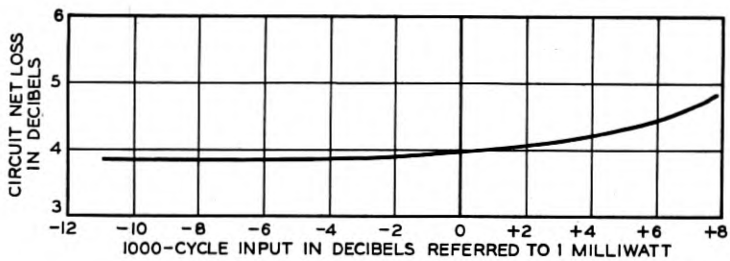


Fig. 12—Representative system load characteristic.

repeatered system some additional limiting of high inputs may occur. However, even on repeatered systems, there should be no noticeable distortion for input volumes such as are obtained directly from a switchboard.

*Reactions on Voice Frequency Circuit*

In superimposing a carrier system on a voice-frequency circuit, line filters are added to provide separate paths for the voice and carrier circuits. The introduction of a filter in one side of a phantom group requires the addition of a network on the other side to maintain the balance of the phantom circuit from a noise and crosstalk standpoint. It is also necessary for return loss reasons to balance these units in the network circuits of voice-frequency repeaters that may be located at the same point as a carrier terminal or repeater. The networks required to take care of these conditions are included with the carrier system.

In some cases it is desired to apply the Type H system to circuits equipped with bridged telephone stations at intermediate points. Such arrangements are common on railroad communication systems, and occur to a small extent in the Bell System. In such cases, excessive interference into the carrier system due to talking at the way station, and into the way stations due to talking on the carrier system, is likely to occur unless suitable filters are provided at each way station. A simple filter for this purpose has been developed for use with the 501 type subscribers set, and work is proceeding on a similar filter for use with other types of subscriber sets.

The line filters and the filters for use at way telephone stations each introduce a loss of about 0.15 db to the through voice-frequency transmission. Where the voice-frequency circuit is equipped with repeaters and return loss conditions permit, these additional losses may be taken care of by readjusting the voice-frequency repeater gains. In extreme cases, particularly where a considerable number of filters are to be added, it may be necessary to resort to other means of improving the transmission on the voice-frequency circuit, such as loading of incidental cables or the addition of a voice-frequency repeater.

On circuits equipped with way stations, selective signaling by means of selectors is sometimes used. Such signaling systems are generally arranged to apply an "answer back" tone to the line when a station has been called to indicate to the calling party that the selector has operated. This tone contains a considerable amount of high-frequency currents so that it is necessary to modify the selector circuit to filter out the high frequencies. The modification is a simple one and makes the answer back tone inaudible on the carrier circuit.

## Design Features

In the development of the Type H system advantage was taken of many new devices which have been perfected in recent years, adapting them to the particular conditions of this application. A discussion of the more interesting features relating to the design of the various parts of the system is given below.

### *Modulators*

The modulator and demodulator used in the Type H system are of the double-balanced copper-oxide type. Each modulator or demodulator consists of an input transformer, an output transformer, a copper-oxide "varistor" and a carrier supply. Although the modulators are bilateral, in the present application they are used in one direction only. The varistor consists of 48 copper-oxide discs assembled on a single bolt and connected as shown in Fig. 13.

The principal advantage of this type of modulator or demodulator is that in the ideal case (and to a lesser degree in the practical case) each modulation product appears only in one of the four branches of the circuit. For example in the case of the modulator, if a voltage of frequency "$V$" is applied to the input and a voltage of frequency "$C$" is applied by the carrier supply circuit, resulting products of modulation will appear in the ideal case as shown in Fig. 13. It is obvious that the only unwanted products in the output which cannot be suppressed by filters or balance are those which are of the frequency $(C \pm A V)$ which for some values of $A$ and $V$ fall in the frequency range of the desired sideband $(C + V)$ or $(C - V)$.

These components, however, are normally more than 50 db weaker than the sideband and are not noticeable. Of course, the term $A V$ represents not only odd harmonics of $V$ but odd order intermodulation

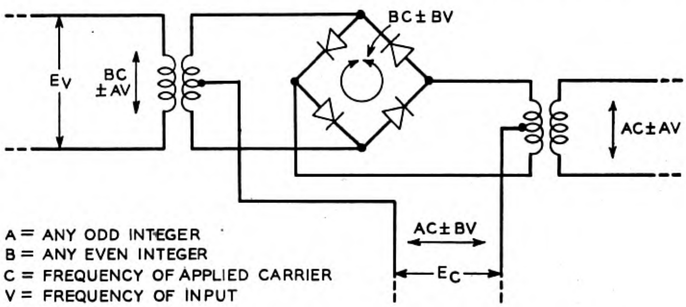


Fig. 13—Simplified schematic of copper oxide modulator.

products such as $(2 V_1 \pm V_2)$. The relative amplitudes of the $(C \pm A V)$ terms increase with load in a similar manner to that for the distortion products of an amplifier as the overload point is approached, and the effect on articulation is the same.

For the actual case the modulator balance is not perfect and all products and the original frequencies do appear in all branches of the circuit including the output. However, the balance in most cases is greater than 30 db and the filter requirements are helped to that extent over some portions of the frequency range. This is particularly helpful in connection with suppressing the carrier from the output and input since it lies only about 200 cycles from the pass band and it would be costly to obtain all of the suppression required by means of filters.

With a single disc in each arm, taking the factory run of discs and making no attempt to select units, the balance for many assemblies would be less than 15 db. By selecting units, this balance could be improved to any desired amount. In the present design, however, to save the cost of selection, twelve discs were used per arm to obtain the better balance resulting from the averaging of the characteristics of a large number of discs. There is some sacrifice in efficiency due to using the large number of discs but in this application it was of minor importance.

The averaging obtained by using twelve discs in each arm is helpful in several other respects. First, the normal impedance, transmission and balance do not vary greatly from unit to unit. Secondly, although each disc has a negative coefficient of resistance *vs.* temperature and there is a variation in the coefficient among discs, the average coefficient of twelve discs chosen at random will be very nearly equal to the average of any other twelve discs chosen at random and the balance between arms will, therefore, remain practically constant with temperature, even though the impedance and efficiency vary. A similar advantage is obtained in the case of aging and a good balance is obtained throughout the life of the equipment.

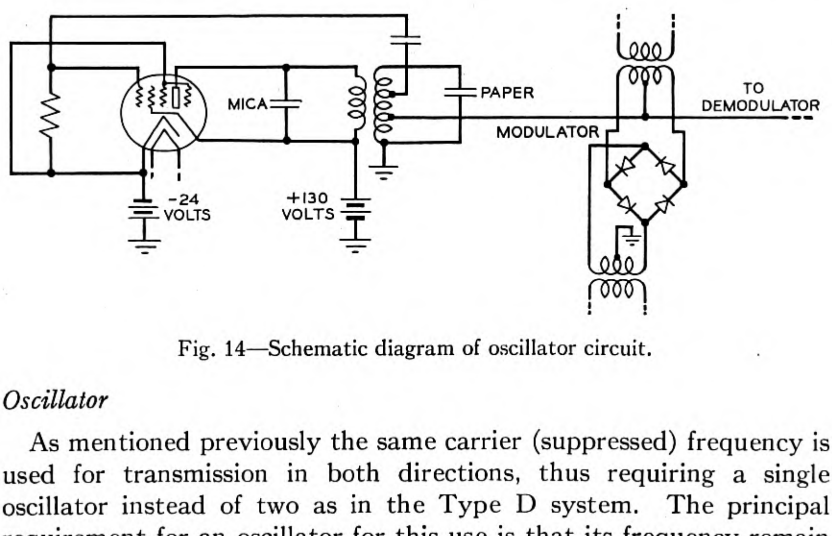


Fig. 14—Schematic diagram of oscillator circuit.

*Oscillator*

As mentioned previously the same carrier (suppressed) frequency is used for transmission in both directions, thus requiring a single oscillator instead of two as in the Type D system. The principal requirement for an oscillator for this use is that its frequency remain stable under the variations in power supply voltage and temperature which will occur. The new design, which is shown schematically in Fig. 14, provides a degree of stability such that no operating adjustments will be required due to these factors. Relatively high stability with changes in temperature is obtained by balancing the positive capacitance-temperature coefficient of the copper-oxide load and the mica tuning condenser against the negative coefficient of the paper tuning condenser.

The stability of frequency with plate voltage variations is about $5/10^6$ parts per volt. This is adequate and was obtained without the use of an expensive tuning inductance. The coil used, which also serves as output and feedback transformer, has a ratio of reactance to resistance of about 20 and is an air-core solenoid potted in a copper can.

*Amplifiers*

Both the receiving and transmitting amplifiers employ a single pentode with about 9 db feedback. For this amount of feedback, the variations of gain and impedance due to power supply variations are reduced to at least one-third of the amount of the variation obtained without feedback, and the load-carrying capacity is increased about 1 db.

The two amplifiers differ in that the frequency range transmitted is different and in that the output transformer of the receiving amplifier also acts as an inequality ratio hybrid coil to separate the receiving

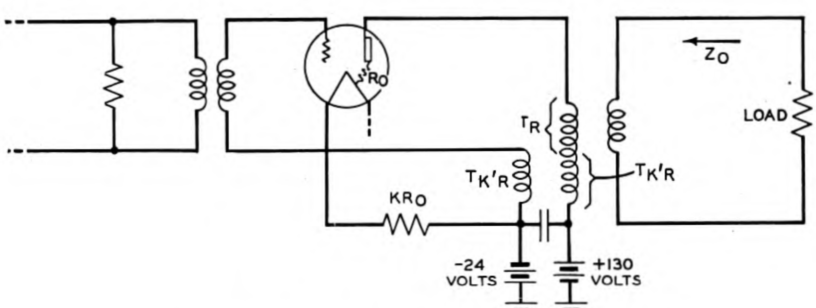


Fig. 15—Simplified schematic of amplifier.

signaling circuit from the two-wire voice circuit. The two circuits are shown in Fig. 4. In each case, the feedback is accomplished by means of a bridge circuit in the output and a series connection in the input. This can be more readily seen from Fig. 15, which is a simplified circuit representing both amplifiers. There is a considerable saving in circuit elements as compared to the familiar resistance bridge feedback connection. The output power loss due to shunt arms of the resistance bridge is eliminated. Furthermore, the impedance of the feedback circuit is relatively low, and consequently some wiring difficulties were avoided. In this application, the bridge is unbalanced, and the impedance $Z_0$ is a function of $KR_0$. As a result, it was convenient to adjust $Z_0$ to the optimum value by choosing the proper value of $KR_0$.

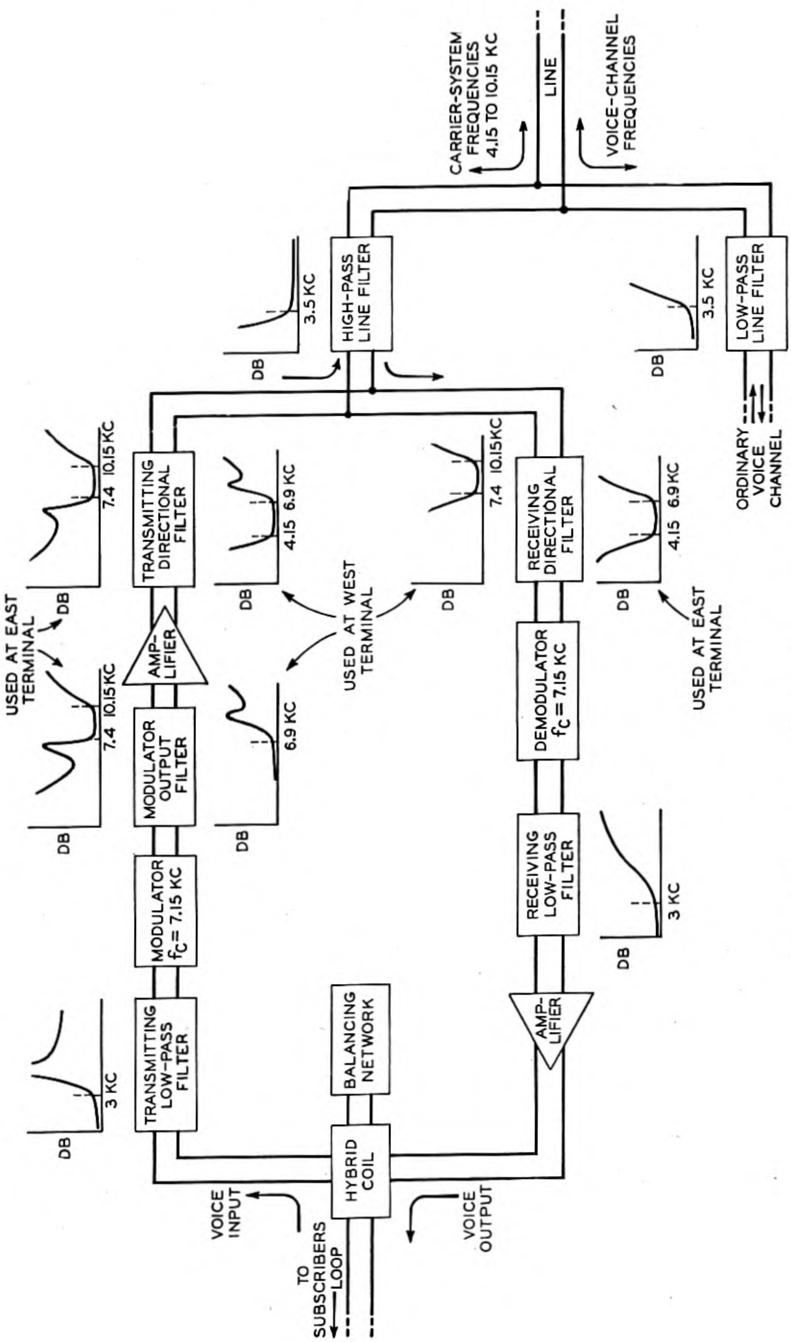


Fig. 16—Attenuation characteristics of filters at terminal.

*Filters*

Filters constitute an important part of the Type H carrier system. They represent about 30 per cent of the cost of the terminal and occupy about 25 per cent of the total space.

The various filters required at a terminal are indicated in Fig. 16. The transmission characteristic of each filter is given in miniature above or below the block representing the filter.

The high-pass and low-pass line filters separate the ordinary voice telephone channel and the added carrier telephone channel made available by this system. The low-pass line filter passes voice frequencies and suppresses all other frequencies. The high-pass line filter passes the carrier frequencies and suppresses the voice frequencies. Each filter offers a high impedance to the frequencies passed by the other, and bridges off only a very small part of the energy of these frequencies.

The remaining filters are associated with the carrier terminal proper, where they serve to separate the transmitting and receiving paths and suppress unwanted frequencies. The voice frequencies pass through the hybrid coil and the transmitting low-pass filter to the modulator. This filter limits the path between the hybrid coil and the modulator to voice frequencies only. Modulation of the voice with the carrier frequency of 7.15 kc. produces two sidebands extending from 4.15 to 6.90 kc. and from 7.40 to 10.15 kc. At an east terminal, the upper sideband is transmitted, and the modulator output filter passes this sideband and suppresses the lower sideband, together with other unwanted modulation products. In this manner it limits the load on the amplifier to the desired sideband. The transmitting directional filter offers further suppression to frequencies lying outside this band. The receiving directional filter will not pass this band but has a high impedance to these frequencies. The high-pass line filter passes all frequencies above roughly 3.5 kc. and, therefore, this band passes through it readily and out onto the line for transmission to the distant terminal. Transmission from a west terminal is identical in principle but here the lower sideband is passed by the modulator output filter and transmitting directional filter while the upper sideband is suppressed.

It is apparent from Fig. 16 that the received sideband coming in on the line from the distant repeater or terminal is operated upon by the filters in a reverse manner from that described above for the transmitted sideband. The incoming frequencies are directed through the receiving directional filter to the demodulator, where modulation with the original carrier reproduces the voice frequencies together with other modulation products. The desired voice-frequency band is then separated from these products by the receiving low-pass filter.

In addition to performing the function of selecting desired and rejecting undesired currents, a filter, if operating in parallel with another as in the case of the directional filters or the line filters, should offer a high impedance to the transmitted currents of the other and thus prevent an excessive drain of these currents. Since these filters are

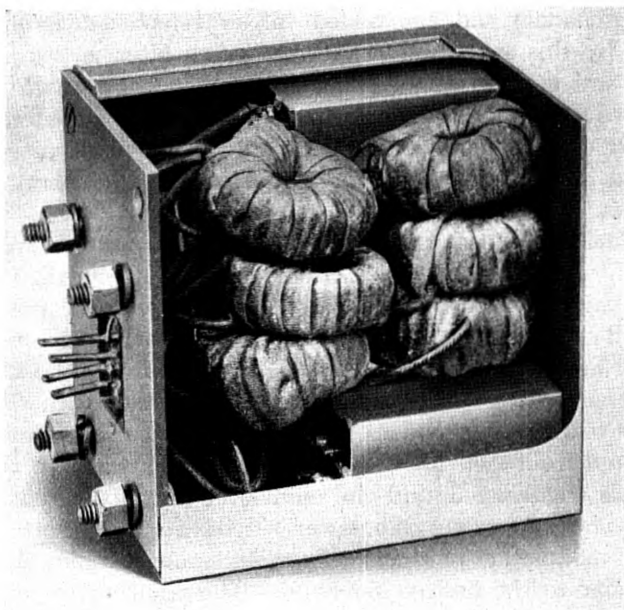

Fig. 17—Filter (with cover cut away) showing general method of assembly.

designed for operation in parallel, either filter operated without the other would have somewhat different electrical characteristics.

The economies and reduction in size of these filters as compared to those of the Type D system are due to several factors. Improved paper condensers having the required stability are used. Compared to mica condensers, these condensers are less costly and smaller for a given capacity. Moreover, at a very slight increase in cost paper condensers are used which will withstand 1000-volt line surges. Most of the condensers of the Type D system were designed for only 500-volt protection.

New types of coils with molybdenum permalloy cores having the necessary stability and low hysteresis losses are employed instead of the air core solenoidal coils used in the Type D system. Since these new coils have less dissipation, the filters have flatter and wider transmission bands which contribute to improve telephone quality in the system. These coils are toroidal in shape and have very small stray fields and therefore small coupling with any nearby coils. This permits them to be packed closely together, which results in a large decrease in the size of the filters. A further reduction in size and cost of the filters was effected by dispensing with individual containers for each coil. The elements of the filter are wired together and held in place in the filter can by a potting compound. This is poured around them, hermetically sealing the whole assembly.

The small size of the elements and the very low coupling between them permit the assembly of more than one filter in the same can. For example, by a careful placing of the elements it was possible to place the transmitting and receiving low-pass filters and the modulator output filter in one can approximately $3\frac{1}{4}$ inches by $4\frac{1}{4}$ inches by $4\frac{1}{4}$ inches in size. A photograph of a high and low-pass line filter with the can cut away is shown in Fig. 17. The filters for a terminal of this system require 70 square inches of mounting space or about 1/5 of that required for those of a Type D system.

## Conclusions

The development of the Type H system is another step in extending the use of carrier systems. Improvements in performance and simplifications which are effective in reducing its cost as compared with the Type D system which it supersedes have been obtained. Reduction in size and provision for operation on a-c. supply simplify its installation, particularly in outlying offices where suitable d-c. power supply is not ordinarily available. Its portability makes it well suited to provide additional circuits required in cases of emergency. The Type H system is expected to have a large application in the Bell System telephone plant, and in addition to provide carrier circuits for the communication systems of other companies.

## Abstracts of Technical Articles from Bell System Sources

*Directional Ferromagnetic Properties of Metals.*[1] R. M. Bozorth. This is a review of the magnetic properties of single crystals of iron, cobalt and nickel and their alloys with each other. After a general introduction and a description of recently developed technique some new results are described which show for the first time that such crystals are anisotropic in low as well as in high fields. The amount of anisotropy is expressed usually by one "anisotropy constant" but sometimes a second constant is necessary. These constants vary with composition and temperature and their values are shown in tables and curves. When the constants are small the material is especially sensitive to strain and heat-treatment; this condition applies to some of the iron-nickel alloys (permalloys). Small amounts of impurities affect the magnetic properties in low fields. By careful purification and heat-treatment several crystals have been made which have permeabilities of over 1,000,000.

*Use of Negative Regeneration in Radio Receivers.*[2] C. B. Fisher. This paper gives a brief and simple explanation of gain stabilization which can be secured by employment of the feedback property of an amplifier.

*Electron Diffraction Studies of Cuprous Oxide.*[3] L. H. Germer. Etched surfaces of single crystals of cuprous oxide produce electron diffraction patterns consisting of spots and Kikuchi lines. In some cases lines are observed corresponding to all of the most widely spaced planes of copper atoms which are so situated as to be able to give reflections on the photographic plate. An example is given, however, of a diffraction pattern from which lines due to the most widely spaced plane of atoms (111), are missing when this plane lies parallel to the primary beam direction; the lines appear when the crystal is rotated to destroy this parallelism. The existence of Kikuchi line patterns indicates that the surfaces of etched cuprous oxide single crystals are relatively smooth. When spot patterns also appear they show anomalies which seem attributable to refraction, thus confirming the conclusion regarding smoothness. An example is given of an array of

[1] *Jour. of Applied Physics*, September 1937.
[2] *R. M. A. Engineer*, November 1937.
[3] *Phys. Rev.*, November 1, 1937.

spots made up of three distinct patterns which are displaced with respect to each other and with respect to the primary beam position. Polycrystalline surfaces made up of rather large crystals produce patterns of spots which are often drawn out into streaks lying approximately parallel to the plane of incidence. This effect, which is attributable to refraction and is a proof of smoothness of the surface, is most pronounced after etching in nitric acid. It is rarely, or never, found after etching in a mixture of hydrogen peroxide and ammonium hydroxide. Nitric acid etch produces upon the surface of a single crystal of cuprous oxide, or upon a polycrystalline surface made up of rather large crystals, crystals of some new compound which are cubic with cell edge 8.35A. These are sharply oriented with one or another of the most important planes parallel to the gross surface plane of the oxide. The sharpness of this orientation is proof of the flatness of the underlying oxide surface upon which the crystals are formed. This new 8.35A structure is produced by potassium cyanide etch as well as nitric acid, but never by sulphuric acid nor by ammonium hydroxide.

*On the Ionization of the $F_2$ Region.*[4] W. M. GOODALL. In this paper the available data on $F_2$ region ionization for Peru, Australia, and this country are analyzed in a way that permits the separation of effects due to variations in solar ionizing force from effects due to seasonal and annual changes. It is shown that for constant solar activity the expected curves of critical frequency for Australia and this country appear to indicate both seasonal and annual tendencies. It is suggested as a possibility that the apparent "annual" effect may in fact be due to meteorological conditions which cannot be eliminated without data from more locations.

*Coupling Between Parallel Earth-Return Circuits Under D-C Transient Conditions.*[5] K. E. GOULD. In tests conducted in connection with several d-c railway electrifications, the induced voltages recorded in paralleling communication circuits at times of short circuit on the railway have shown marked divergences from values computed on the basis of uniform earth resistivity and a rate of change of earth current determined from measurements in trolley and rail circuits. Due to the numerous factors which might contribute to these divergences, such as non-uniform division of transient current along the tracks and associated return conductors, the presence of shielding conductors along or near the right-of-way, etc., it was felt that a better understanding of the problem of induction under d-c transient conditions

[4] *Proc. I. R. E.*, November 1937.
[5] *Elec. Engg.*, September 1937.

could be obtained by experimental studies of the transient coupling between parallel earth-return circuits, free from the effects of shielding conductors, and with concentrated, rather than distributed, grounds. The study described in this paper was undertaken for this purpose.

*Energy of Lattice Distortion in Cold Worked Permalloy.*[6] F. E. HAWORTH. The lattice distortion produced by severe cold working of permalloy of 70 per cent Ni content has been studied by measuring the broadening of the reflection of the Fe $K\alpha$ doublet by the (311) planes, with a focusing camera. The broadening decreases upon annealing and recovery is complete at 650° C, when the breadth of the x-ray intensity curve at half-maximum is as small as that obtained by use of the two-crystal spectrometer. The mean square distortion in the lattice spacing due to cold work is derived from the X-ray measurements after photometering the x-ray film, converting the curve into an x-ray intensity curve, fitting the latter with an empirical equation and using an analysis worked out by S. O. Rice. The energy of the distortion is then calculated by using an equation derived by G. R. Stibitz. The root-mean-square distortion was found to be 0.31 per cent of the lattice spacing after the material had been reduced 96 per cent in cross-sectional area by cold working. The energy of distortion in the hard worked condition is thus found to be $23 \times 10^6$ ergs/cm$^3$ or 0.065 calorie/gram.

*Optical Constants of Rubidium and Cesium.*[7] HERBERT E. IVES AND H. B. BRIGGS. In previous papers the authors have presented the results of measurements of the optical constants of potassium and sodium in the ultra-violet and visible ranges of the spectrum, with a description of the apparatus used and the method of making measurements. The present paper deals with the results of measurements of the optical constants of rubidium and cesium by the same methods and for the same wave-length range. As with sodium and potassium, previous investigations of the optical constants of these metals were confined to the visible region of the spectrum.

The author's original interest in these constants lay in their application to the theory of photoelectric emission from thin films. In order to test this theory a knowledge of both the refractive index and the extinction coefficient of the metals concerned is needed for the spectral range embracing the characteristic photoelectric emission of the thin films. The work described in this paper is intended to supply this need.

[6] *Phys. Rev.*, September 15, 1937.
[7] *Jour. Opt. Soc. Amer.*, November 1937.

*Minimum Noise Levels Obtained on Short-Wave Radio Receiving Systems.*[8] KARL G. JANSKY. The theoretical minimum noise level of receivers in the absence of any interference, the source of which is external to the receiver, is discussed and compared with the limit actually measured on various antennas over a limited frequency range in the short-wave spectrum. It is pointed out that, on the shorter wave lengths and in the absence of man-made interference, the usable signal strength is generally limited by noise of interstellar origin. The powers obtained from this noise with the various antennas and for different times of the day are given.

Recently, man-made interference, of which that caused by diathermy machines constitutes the greatest part, has become so extensive that it is now the limiting noise during most of the daylight hours. Data are given on the intensity and extent of this form of interference.

*Superstructures in Alloy Systems.*[9] FOSTER C. NIX. A review of the literature treating superstructures in alloys. The presence of a superstructure produces new X-ray diffraction lines,—commonly called superstructure lines. The elements of superstructure theories are presented including the Bragg-Williams and Bethe-Peierls treatments. The author discusses the effect of a superstructure or an ordered phase on the thermal, mechanical, electrical and magnetic properties of alloys. A comparison is made between the theoretical predictions and the experimental results for both the specific heat and the energy of transformation. A $Cu_3Au$ alloy becomes disordered more rapidly near the critical temperature of order, Tc, than the theories predict. A large anomalous specific heat is observed above Tc,—as predicted by the Bethe-Peierls theory.

*Moment Recurrence Relations for Binomial, Poisson and Hypergeometric Frequency Distributions.*[10] JOHN RIORDAN. This paper gives a uniform development of recurrence relations for moments about the origin and mean of binomial, Poisson, and hypergeometric frequency distributions. Uniformity is obtained through the use of the moment arrays of H. E. Soper. Both types of moments are expressed in terms of coefficients which are alike for the three distributions; for the moments about the origin these coefficients are the Stirling numbers of the second kind. Moment recurrence relations follow from recurrence relations for these coefficients. The recurrences for the hypergeometric moments appear to be new. For working purposes,

[8] *Proc. I. R. E.*, December 1937.
[9] *Jour. Applied Physics*, December 1937.
[10] *Annals of Math. Statistics*, June, 1937.

moments about the mean of binomial and Poisson distributions are expressed in terms of auxiliary moment coefficients with recurrence relations which also appear to be new.

*Extending the Frequency Range of the Negative Grid Tube.*[11] A. L. SAMUEL. The conventional vacuum tube when adapted for use at ultra-high frequencies carries with it many of the desirable attributes which it possesses at lower frequencies, such for example as its ability to amplify and the ease with which satisfactory frequency stability can be obtained. However, difficulties are encountered because of the effects of the finite electron transit time and because of certain circuit limitations. Methods of circumventing these restrictions are discussed and illustrated by reference to specific tubes. The paper closes with a brief review of recent work directed toward extending the frequency range of the negative grid tube both as an oscillator and as an amplifier.

*Transmission Theory of Plane Electromagnetic Waves.*[12] S. A. SCHELKUNOFF. This paper deals with transmission theory of plane electromagnetic waves in free space and in cylindrical regions of arbitrary cross section. Transmission properties of such waves can be expressed very simply in the same terms as the properties of electric waves guided by a pair of parallel wires. The earlier parts of the paper are concerned with general theorems and the latter parts with their application to plane waves in metal tubes of circular and rectangular cross sections.

*The Empty Lattice Test of the Cellular Method in Solids.*[13] W. SHOCKLEY. The cellular method of constructing wave functions for electrons in crystals developed principally by Wigner and Seitz and Slater is tested by applying it to an artificial crystal in which the potential is constant. Knowledge of the exact solutions for this case, plane waves, shows that the cellular method is quite accurate in the first Brillouin zone but may be in error by a factor of two in the second. Hence calculations of occupied levels in Li and Na are probably quite good; for Cu, Ca, diamond, LiF, and NaCl the errors will be larger. Calculations of excited states are likely to be very much in error. The accuracy of the cellular method is shown to improve very slowly with increasing number of continuity conditions.

*Sound Propagation in Ducts Lined with Absorbing Materials.*[14] L. J. SIVIAN. In ventilator and exhaust systems it is desirable to provide

[11] *Jour. of Applied Physics*, October 1937.
[12] *Proc. I. R. E.*, November 1937.
[13] *Phys. Rev.*, October 15, 1937.
[14] *Jour. Acous. Soc. Amer.*, October 1937.

a high degree of attenuation for audio-frequency waves while offering low resistance to continuous or slowly pulsating air flow. For that purpose ducts lined with absorbing materials are sometimes used. This paper deals with sound propagation, particularly with its attenuation constant, in such ducts. Two types of lining are considered: (a) non-vibratile, i.e., a lining in which there is no wave motion propagated in the direction of the duct axis; (b) vibratile, i.e., a lining admitting of such motion. Methods for computing the propagation constants in terms of the acoustic constants of the lining, are given. Some experimental data are presented. Comparison of observed and computed values indicates that the computational procedure is substantially valid up to a frequency at which the sound wave-length is about twice the internal duct diameter.

*New Experimental Methods Applicable to Ultra-Short Waves.*[15] G. C. SOUTHWORTH. This paper presents a new approach to the problem of electrical measurements at extremely high frequencies. It makes use of a relatively new principle whereby electromagnetic waves may be propagated through the interiors of hollow metal tubes. These tubes have for convenience been called wave guides. Their diameters must be at least 0.585 wave-length in order that power may be propagated. Some of the difficulties of previous methods have been incidental to radiation and spurious coupling effects. These have been largely eliminated by this method since the power resides almost entirely within the pipe. Short sections of wave guide may be made to resonate electrically much as organ pipes and air columns resonate acoustically. A high degree of sharpness may be obtained. Such a pipe may, therefore, play the role of a resonant circuit and become effectively a wave meter, a frequency determining unit for an oscillating vacuum tube or an impedance matching device. Specimens under study may be placed inside a resonant chamber and be subjected to electric fields of controlled intensities.

*Preparation of Large Single Crystals of Sodium Chloride.*[16] H. WALTHER. An apparatus and a method are described, whereby single crystals of sodium chloride are produced in the form of bars 2 cm in diameter and 30 cm long. The crystal is drawn from the melt by means of a platinum rod which is dipped into it and which is raised and rotated simultaneously by a clock mechanism. An air stream from a circular nozzle surrounding the growing crystal immediately above the surface of the molten salt provides the steep

[15] *Jour. Applied Phys.* October 1937.
[16] *Rev. Sci. Instruments,* November 1937.

temperature gradient necessary for the continuous growth of the crystal. The orientation of the crystal with respect to the axis of the bar may be chosen at will.

*Magnetic Properties of Single Crystals of Silicon Iron.*[17] H. J. WILLIAMS. The magnetization curves for the [100], [110] and [111] directions of single crystals of iron containing 3.85 per cent silicon were obtained from single crystal specimens cut in the form of hollow parallelograms so that the sides of each specimen were parallel to the tetragonal, digonal or trigonal axes, respectively. This method avoids the errors due to demagnetizing fields, inherent in previous measurements on single crystals. In addition to the well-known anisotropy at magnetizations above half of saturation, the data show for the first time considerable anisotropy at low magnetizations. A maximum permeability of 1,380,000, by far the highest ever reported for silicon iron, was observed in the [100] specimen after careful annealing. The magnetic anisotropy constants $K_1$ and $K_2$ were obtained from the magnetization curves and from torque measurements on a disk cut parallel to a (110) plane.

*The Quest of Vitamin $B_1$.*[18] R. R. WILLIAMS. An account is given in semi-popular form of the events which led to a recognition of a dietary deficiency as the cause of Oriental beriberi. The steps are indicated which led to the isolation and synthesis of the lacking substance which is now known as vitamin $B_1$. The author began his researches on this subject in Manila in 1910 and has prosecuted them continuously since that time. In 1936 he had the gratification of effecting with Dr. J. K. Cline of Merck and Company a practical synthesis of this vital compound so that it is now produced artificially much more cheaply than it can be obtained in pure form from nature.

In presenting his work on the architecture of the molecule and its artificial reconstruction from synthetic sources, the author endeavors to make clear to the layman by analogy and otherwise the methods which the organic chemist employs in such a study. The highly inferential deductions concerning complex interatomic relationships which result from tearing the natural molecule apart reach a dramatic verification only when one finds he has actually reproduced in the laboratory the precise product of nature's art.

[17] *Phys. Rev.*, October 1, 1937.
[18] *Jour. Franklin Institute*, November 1937.

## Contributors to this Issue

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F. H. Best, M.E., Cornell University, 1911. American Telephone and Telegraph Company, Engineering Department, and Department of Development and Research, 1911–34. Bell Telephone Laboratories, 1934–. Mr. Best has been engaged principally in the development of methods and arrangements used in maintaining the transmission efficiency of telephone circuits.

R. W. Chesnut, A.B., Harvard University, 1917; War Department of French Government, 1917; U. S. Army, 1917–19. Western Electric Company, Engineering Department, 1920–25; Bell Telephone Laboratories, 1925–. Mr. Chesnut has been engaged in the development of carrier telephone and long-wave radio systems.

Paul S. Darnell, B.S. in Electrical Engineering, University of Pennsylvania, 1922; M.A., Columbia University, 1925. Western Electric Company, Engineering Department, 1922–1925; Bell Telephone Laboratories, 1925–. Mr. Darnell's work has been primarily in connection with the development of retardation coils for operation at voice and carrier frequencies.

Clifford E. Fay, B.S. in Electrical Engineering, Washington University, 1925; M.S., 1927. Bell Telephone Laboratories, 1927–. Mr. Fay has been engaged principally in the development of power vacuum tubes for radio purposes.

H. J. Fisher, E.E., Cornell University, 1920; U. S. Signal Corps, 1917–19. Western Electric Company, Engineering Department, 1920–25; Bell Telephone Laboratories, 1925–. Mr. Fisher has been engaged in the development of toll switching and transmission systems.

C. W. Green, B.S. in Electrical Engineering, University of Wisconsin, 1907; Instructor and Assistant Professor, Massachusetts Institute of Technology, 1907–17; Captain 1917, Major 1918, U. S. Army. Bell Telephone Laboratories, 1919. Mr. Green's work has had to do with the development of carrier telephone systems, voice frequency repeaters, and radio terminating circuits.

E. I. Green, A.B., Westminster College, Fulton, Missouri, 1915; University of Chicago, 1915–16; Member of Faculty, Westminster College, 1916–17; U. S. Army, 1917–19 (Captain, Infantry); B.S. in Electrical Engineering, Harvard University, 1921. American Telephone and Telegraph Company, Department of Development and Research, 1921–34; Bell Telephone Laboratories, 1934–. Mr. Green's work has been concerned principally with multiplex transmission systems.

L. M. Ilgenfritz, B.S. in Electrical Engineering, University of Michigan, 1920. American Telephone and Telegraph Company, Department of Development and Research, 1920–34; Bell Telephone Laboratories, 1934–. Mr. Ilgenfritz has been engaged in the development of carrier systems.

A. Kenner, B.S. in Electrical Engineering, Purdue University, 1913. Western Electric Company, Engineering Department, 1913–25; Bell Telephone Laboratories, 1925–. Mr. Kenner has been engaged in equipment development work for toll systems, particularly those items such as telephone repeaters and carrier associated with toll cables.

C. E. Lane, A.B., University of Iowa, 1920; M.S., University of Iowa, 1921. Western Electric Company, Engineering Department, 1921–25; Bell Telephone Laboratories, 1925–. Mr. Lane is engaged in the development of special filters such as mechanical filters, filters in which quartz crystals are used as elements, and high-frequency filters of all types.

R. H. Mills, S.B. in Electrical Engineering, Massachusetts Institute of Technology, 1916. Western Union Telegraph Company, 1916–18. Western Electric Company, Transmission Development Branch, 1918–25; Bell Telephone Laboratories, Apparatus Development Department, 1925–. Mr. Mills has been engaged in the development of carrier frequency filters for use in commercial communication systems.

A. L. Samuel, A.B., College of Emporia (Kansas), 1923; S.B. and S.M. in Electrical Engineering, Massachusetts Institute of Technology,

1926. Instructor in Electrical Engineering, Massachusetts Institute of Technology, 1926–28. Bell Telephone Laboratories, 1928–. Mr. Samuel has been engaged in research and development work on vacuum tubes.

S. A. SCHELKUNOFF, B.A., M.A. in Mathematics, The State College of Washington, 1923; Ph.D. in Mathematics, Columbia University, 1928. Engineering Department, Western Electric Company, 1923–25; Bell Telephone Laboratories, 1925–26. Department of Mathematics, State College of Washington, 1926–29. Bell Telephone Laboratories, 1929–. Dr. Schelkunoff has been engaged in mathematical research, especially in the field of electromagnetic theory.

W. SHOCKLEY, B.Sc., California Institute of Technology, 1932; Ph.D., Massachusetts Institute of Technology, 1936. Bell Telephone Laboratories, 1936–. Dr. Shockley's work in the Laboratories has been concerned with problems in electronics.

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The Bell System Technical Journal

*Vol. XVII* *January, 1938* *No. 1*

# New Transmission Measuring Systems for Telephone Circuit Maintenance

By F. H. BEST

Transmission measurements on telephone circuits have long been recognized as essential aids in the furnishing of good service. Recent development work has produced new testing methods and apparatus which greatly simplify and expedite transmission measuring. This paper gives a brief description of the existing and the new arrangements.

ABOUT two decades ago pioneering work was being done in the development and introduction of methods and apparatus for the measurement of the transmission characteristics of local and toll telephone circuits, it having been realized that with the increasing complexity of the telephone plant, these measurements were necessary to insure satisfactory transmission performance. Transmission measurements now have a well established place in maintenance activity and development work is constantly in progress to improve the testing apparatus so that it can be operated more rapidly and conveniently, will cost less or do more things, and also to improve testing methods.

The principle employed in making transmission measurements has not changed. A standard testing power is supplied to one end of a telephone circuit and the power received at the other end is measured. The ratio between these powers, expressed in decibels, is a measure of the transmission loss or gain in the circuit. The magnitude of the testing powers is small, the sending power for measuring being one-thousandth of one watt and the received power ranging from one ten-thousandth to one-millionth of a watt. Sensitive meters must be used, or the power amplified before measuring it. Direct-current meters are more sensitive than alternating-current meters and it is the practice to convert the weak received alternating current to direct current by means of vacuum tube or copper-oxide rectifiers and employ direct-current meters for measuring it.

Until recently the developments have been along the line of improvements in the apparatus itself, there being practically no change in the general methods and arrangements employed. For the large toll offices where much testing is done the measuring apparatus has been installed at one or more points in an office, the installations being known as transmission test boards. From these points trunks radiate to repeaters, test boards, switchboards, etc., and the circuits and equipment to be tested are connected to them by patching cords or switchboard cords. The testing has been done by a trained force of transmission testers, this force often being separate from that of the test board attendants whose work has consisted chiefly in correcting line faults, signaling troubles, etc., which do not require transmission measurements for their location.

In the local plant, testing has been done almost entirely with portable transmission measuring apparatus in the hands of a transmission testing force who travel from office to office, testing the cord circuits, switching trunks, etc., at intervals of one or two years. The measuring apparatus has been costly and its permanent installation in each office could not be justified. The apparatus used in both local and toll testing has been described in a number of articles.[1, 2, 3.]

During the last few years new methods and apparatus have been developed and radical changes made that greatly improve the situation both from an economic and operating standpoint. The new measuring instruments are much less costly than the types heretofore available and are as stable in operation and as simple to use as ammeters and voltmeters. In the local plant the decrease in cost and complexity and improvement in operating methods has justified permanent installations in many of the larger offices while the newer forms of portable apparatus are being much more extensively distributed than earlier types. For the toll plant the improvement consists in doing away largely with the centralized transmission testing point. The regular test board force now can make transmission measurements at their test boards and the repeater attendants can measure the performance of the repeaters while working on them. The work is so simple and the maintenance forces are so well trained that a special transmission testing force is not required.

This change has been brought about by the development of new instrumentalities and improved circuits, chief among which are more sensitive meters, the copper-oxide rectifier and the negative feedback amplifier.[5] Until recently, the most sensitive meters which were suitable for general use would not measure the weak power used in transmission measuring so that amplification of this power was required. Using the alloy steels now available, meters of much greater sensitivity and equal ruggedness have been manufactured and transmission losses of 20 db can be measured without an amplifier and without using abnormal testing power. Vacuum tubes formerly used to rectify the received alternating current testing power have been largely superseded by the copper-oxide rectifier which is small, inert and requires no added power for its operation. The combination of this rectifier with sensitive meters greatly simplifies and reduces the cost of the measuring apparatus. Figure 1 shows the simplicity of the circuit of a transmission measuring set which will measure losses up to 20 db.

While indicating meters have been used in transmission measuring for many years, it is only recently that they have been calibrated directly in db. This is the preferable way of measuring and its adoption has been delayed only because of the limitations of available apparatus and circuits which prevented a stable device from being developed. The earlier copper-oxide rectifiers were too unstable for

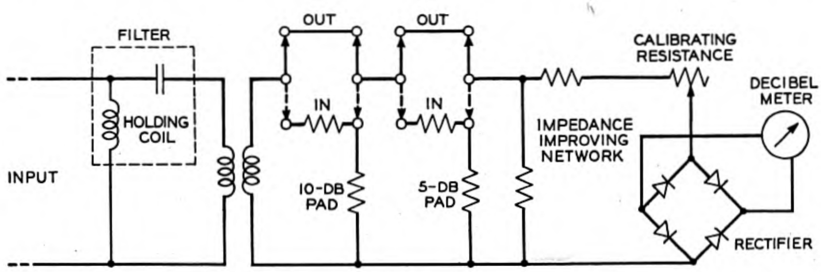


Fig. 1—Simplified circuit of transmission measuring set having a range of 20 db without an amplifier.

measuring and until the development of the negative feedback amplifier, all vacuum tube amplifiers varied in amplification with changes in filament and plate voltage and the aging of tubes. The voltage of available power plants, while sufficiently stable for commercial use, caused the pointer of the meter in a transmission measuring set to fluctuate and over intervals of a few hours the change might be as much as one db. Smaller changes occurred frequently. These changes were slow enough so that measurements could be made, but they required frequent adjustment of amplifier gain to compensate. With such instability, the most rapid and accurate measurements can be made by using the comparison method of testing where a known "standard" power is attenuated by calibrated potentiometers or networks until it equals the unknown received testing power. A meter in this case serves merely as a means for telling when the two are equal and the result is read from calibrated dials on the attenuator. A majority of the measuring sets now in the plant operate on this "flip flop" principle. The use of these dials generally requires that the measuring instrument be built as a unit and that measurements be made at its location. The latter requirement often prevents its installation at the most desirable point because space is not available.

For toll circuit testing in the larger offices where a considerable number of routine and trouble locating tests are made, the required frequency range and loss range are beyond the scope of a simple copper-oxide rectifier and meter. The ability to make level measurements is also a desirable feature. To meet these requirements, an amplifier is provided in the receiving circuit. By applying the negative feedback principle to the amplifier and rectifier of the latest toll transmission measuring system a remarkably stable circuit has been obtained so that the meter can be calibrated directly in db and oper-

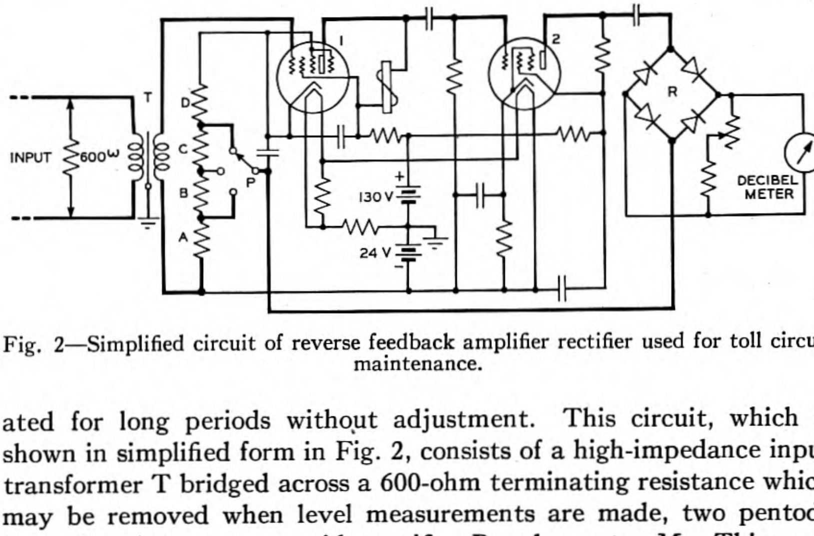


Fig. 2—Simplified circuit of reverse feedback amplifier rectifier used for toll circuit maintenance.

ated for long periods without adjustment. This circuit, which is shown in simplified form in Fig. 2, consists of a high-impedance input transformer T bridged across a 600-ohm terminating resistance which may be removed when level measurements are made, two pentode tubes 1 and 2, a copper-oxide rectifier $R$ and a meter $M$. This combination has much more amplification than is required, so the excess is used to improve stability by introducing part of the output voltage into the input circuit of the first tube in such phase relation with respect to the applied input voltage that the net input voltage is reduced. This reverse or negative feedback voltage is introduced into the grid circuit of the first tube through resistances $A$, $B$ and $C$ by connecting one of the rectifier terminals to the movable arm of the potentiometer $P$. These resistances, and resistance $D$ form the cathode drop resistance of the grid circuit and any potential applied across them affects the potential on the grid of the tube. In the position shown, the reverse feedback is a maximum and the net amplification of the amplifier is a minimum. Moving the potentiometer arm to the lowest step gives less feedback and the amplifier net gain is greater.

The principle of stabilization is as follows: If a constant potential is applied to the input terminals of the amplifier a constant potential will also be applied to the grid of the first tube. As long as the tubes or the rectifier do not change in characteristics the output of the rectifier will likewise be constant. Now if through any cause, such as a change in tube characteristics or in the rectifier, the gain of the amplifier-rectifier should increase, the output will increase and the neutralizing voltage fed back into the input circuit will also increase. This will reduce the voltage on the grid so that the net output voltage of the amplifier-rectifier will not be changed noticeably. Conversely, if a change in the tubes or the rectifier should cause a reduction in gain, the voltage fed back into the input circuit will decrease, there will be less feedback voltage and the output voltage will be substantially the same as before. The reverse feedback amplifier-rectifier is so stable that once it is adjusted to have the proper characteristics it will remain constant for long periods.

The meters used with this amplifier-rectifier have a range of 15 db. This is less than the required measuring range so that it is necessary to increase it by changing the amplifier gain in steps of 10 db. The reverse feedback amplifier lends itself readily to this as the variation of a single resistance in the feedback circuit is sufficient and no expensive balanced attenuators are required. In Fig. 2 this is done by potentiometer *P*. In practice this resistance is controlled by relays which form a part of the amplifier, these relays in turn being remotely controlled by keys, jacks or dials at various points in the office.

The db meters can be located where desired without reference to the location of the amplifier-rectifier. They may also be placed in lantern slide projectors which throw a greatly enlarged meter scale on a screen so that it can be read from distances up to 50 feet or more. The new arrangements are extremely flexible, one set of equipment supplying measuring facilities for several different points in an office where previously several sets would be necessary. Where the use of the equipment is intermittent more than one meter may be used with a single amplifier-rectifier. The meters from which the results are read may be of the conventional indicating type which can be mounted on keyshelfs or on vertical panels, they may be of the projector type or they may be of the recording type which records on paper the characteristics which are being measured. All meters are interchangeable, being similar electrically.

These new instrumentalities have removed many of the limitations to the design of measuring systems and the recently developed arrangements are therefore better coordinated with other facilities than those which they replace. A few examples will be given to illustrate the recent advances in transmission measuring work.

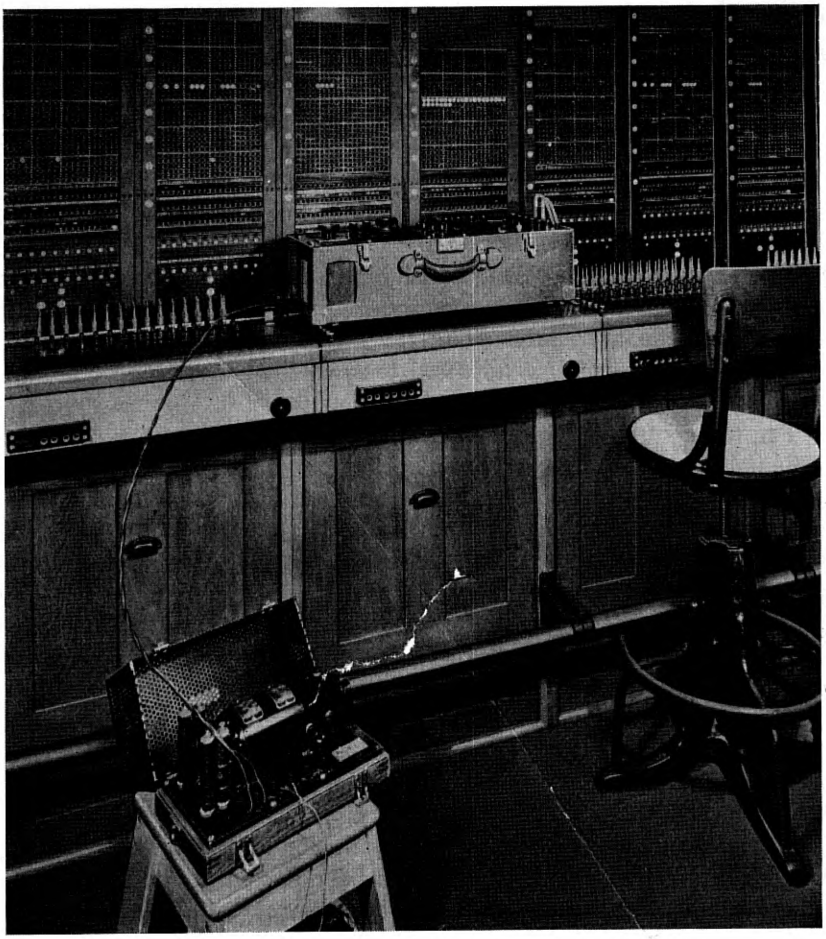

Fig. 3—Transmission measuring set developed about 1920 and associated generator as used at a switchboard in testing cord circuits or interoffice trunks. Both units are required for either sending or receiving.

Figure 3 shows the testing power generator and measuring set developed about 1920 for use in the local plant to measure central office equipment and interoffice trunks with testing power of a single frequency, the frequency employed usually being 1000 cycles. When measuring circuits between offices, it is necessary either to have duplicate sets of equipment at the two ends of the circuit or to connect two circuits together at one end, testing them at the other end as one circuit. This latter arrangement, while not wholly satisfactory, has been the one generally employed because of the greater expense of the

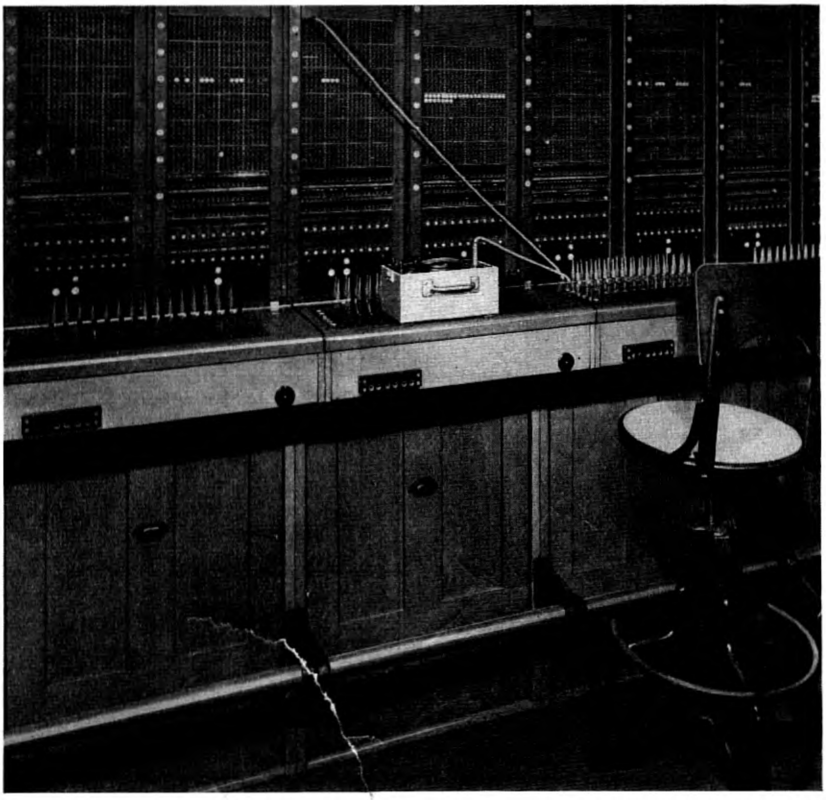

Fig. 4—Latest type of transmission measuring set as used at a switchboard. The associated generator shown in Fig. 5 is usually permanently mounted in the office and testing power is obtained through the multiple as shown. For one-way measurements on circuits between offices, the generator is connected to one end of the circuit and the transmission measuring set at the other, both units not being required at each end.

other method which has involved not only two sets of expensive testing apparatus but two testers, these being necessary because the testing apparatus required frequent adjustment. If circuits between a group of offices in a city are to be tested between circuit terminals rather than by the looping back method, considerable time is consumed in traveling between offices when using this apparatus.

The new apparatus shown in Figs. 4 and 5 not only costs but one-tenth as much and weighs one-quarter as much as the older apparatus but also has electrical advantages which enable a new and improved measuring technique to be employed. The generator is a magneto inductor alternator driven by a 50-cycle or 60-cycle induction motor and gives a constant output without attention. The output is adjusted at the factory or on installation. Some of these machines have

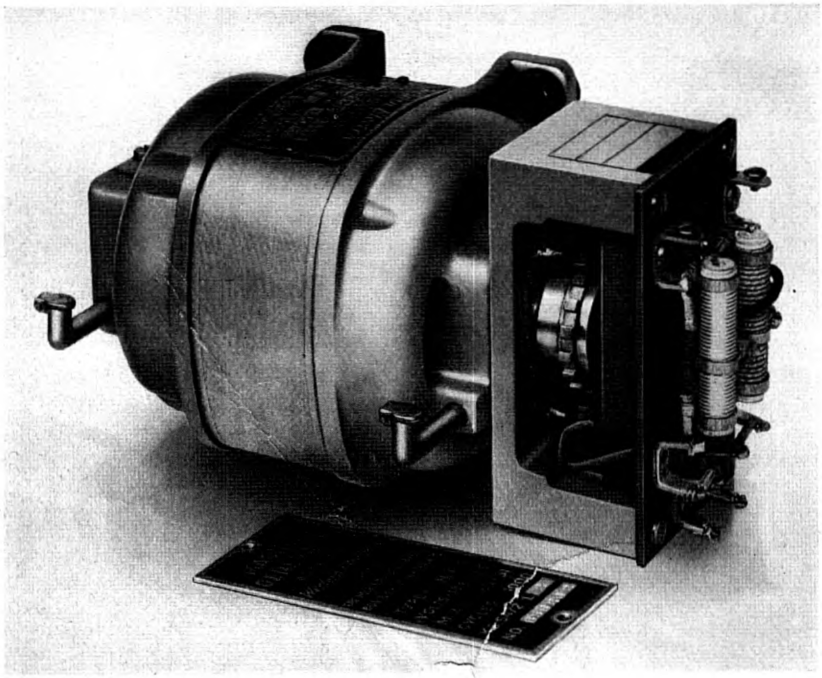

Fig. 5—One thousand-cycle magneto generator used generally for transmission testing. Cover plate removed to show the generator construction. Overall length 7 inches.

been running continuously for about a year without showing any appreciable change in output. They can, therefore, be mounted permanently in an office and the output terminals wired to convenient testing points so that the generator need not be carried around. Because of this output stability, it is not only unnecessary to have an experienced tester at the distant end of the circuit but in the larger offices auxiliary switching equipment is arranged so that testing power can be supplied automatically to one end of the circuit by direction of the tester at the other end, who simply calls or dials a designated number over the circuit to be tested.[7] The testing power is cut off at the sending end when the connection is broken at the receiving end. Tests can be made in the same manner from private branch exchanges and subscribers' stations without a charge being registered against the subscriber.

Fig. 6—Small receiving set as used to measure the transmission loss of a subscriber's line. The 1000-cycle generator has been connected to the central office end of the line by calling a designated number. When the telephone instrument has been replaced on the mounting, the meter in the measuring set will read the line loss.

The receiving set shown in Fig. 4 is based on the electrical circuit of Fig. 1. It has a transmission range of 20 db, and is provided with all the jacks and facilities required for testing cord circuits, trunks and central office equipment. A still smaller and less expensive receiving set of a similar type having a 10 db range is shown in Fig. 6. It is extremely portable and light and is useful for work where the limited range is sufficient. Jacks and other testing conveniences have been omitted to save bulk and cost.

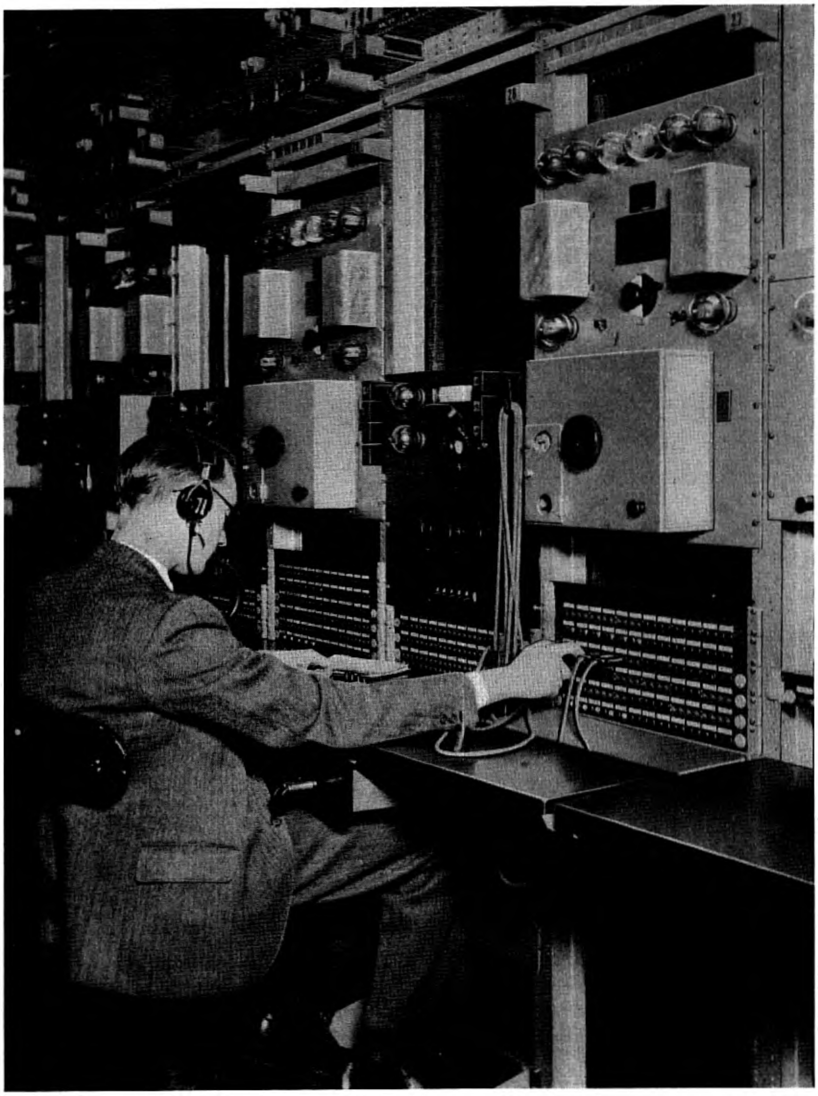

Fig. 7—Transmission testboard for maintaining long distance circuits.

The new instruments for the local plant are not limited to local testing but also have extensive application in the toll plant especially in the smaller offices which do not have many toll circuits. Because of its excellent performance the 1000-cycle generator shown in Fig. 5 will be used generally in the toll plant for 1000-cycle testing.

Figure 7 shows several positions of the transmission test board now widely used for measuring toll telephone circuits. Introduced in 1926, it preceded the development of the reverse feedback amplifier-rectifier and the transmission measuring set used in this board is therefore of the comparison type in which the results are read fron calibrated dials. Testing power is provided by a variable frequency oscillator from which frequencies in the voice range can be obtained. The size of this equipment precluded its being installed in test boards which contain the circuit terminals so that trunks between the measuring

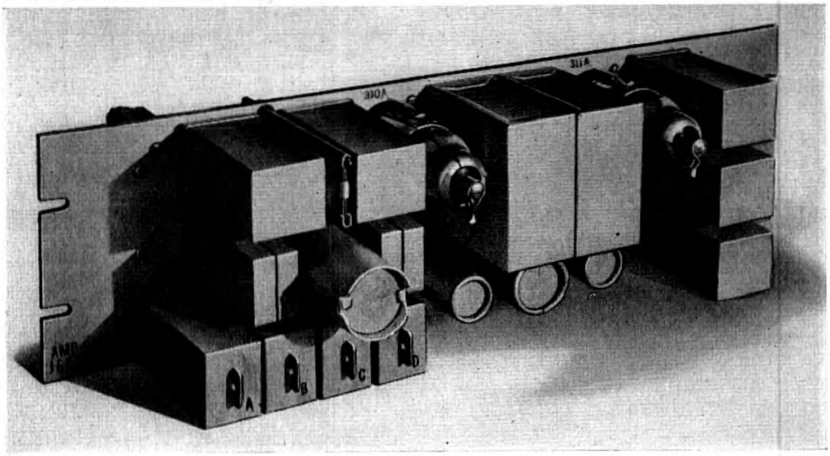

Fig. 8—New amplifier rectifier for general use in toll transmission maintenance.

equipment and these terminals are necessary. In large offices a number of these transmission test boards have been provided.

Figure 8 shows the new reverse feedback amplifier-rectifier used with the latest toll transmission measuring system. This simple panel, which is about one-fourth the size of the measuring set employed in the test board shown in Fig. 7, contains everything required at the receiving end of a circuit excepting the meter and the keys for changing the measuring range. With its associated meter and keys, it is less than one-half as expensive as corresponding elements in the transmission testboards. It can be used with the variable frequency oscillator shown in Fig. 7 or with the 1000-cycle machine shown in Fig. 5.

The meters used with this amplifier-rectifier have a specially designed magnetic circuit which gives an evenly spaced scale on the meter.[8] A conventional meter would have large db divisions at one end of the scale and small ones at the other.

When a direct reading transmission measuring set is used to make measurements over a wide frequency range, it is essential that the response be the same at all frequencies. Variation of response of the new feedback amplifier-rectifier with frequency is so small that it can be calibrated with 1000-cycle current and measurements made over a wide frequency range without recalibration.

From the standpoint of efficiency, the best place for making trans-

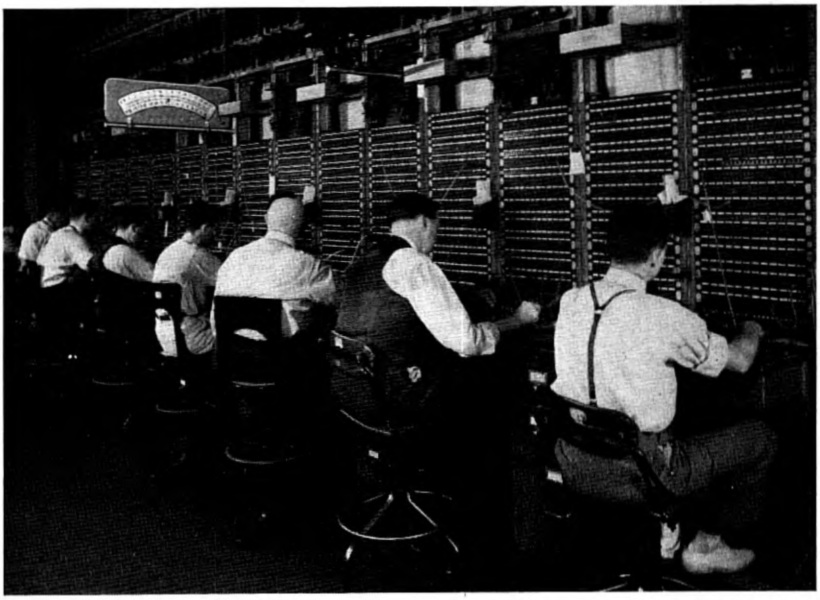

Fig. 9—Projection meter as used for measuring transmission at a secondary testboard. The meter is being read by the third man from the right.

mission measurements on complete toll circuits is at the test board to which all troubles are reported by the operator and where overall signaling and other operating tests are made. Practically all of the space in this board is taken up by jacks on which the circuits terminate so that the older types of measuring set could not be provided at this point. The new arrangement is ideal for application to this type of board since all that is required in the board are the meter and a few jacks or keys for controlling its range. This type of measuring device can be applied equally well to new and existing boards of various types. This feature is of particular value as the maximum efficiency of high-speed measuring systems can be obtained only when all test boards are equipped with them. Two arrangements are available for test board use. One employs a conventional type of meter mounted in the keyshelf or on a panel and the other employs the projection type of meter which is illustrated in Fig. 9.[6] The method of operation is simple. When a transmission test is to be made the tester listens until he hears the tone caused by testing power coming over the circuit from the distant generator, then connects the circuit to a jack in which the measuring set input terminates. The meter indicates immediately the net loss of the circuit at the testing frequency. With the projection meter arrangement the lamp in the projector is turned on automatically when a connection is made to the test jack.

The new amplifier-rectifier will measure transmission losses and

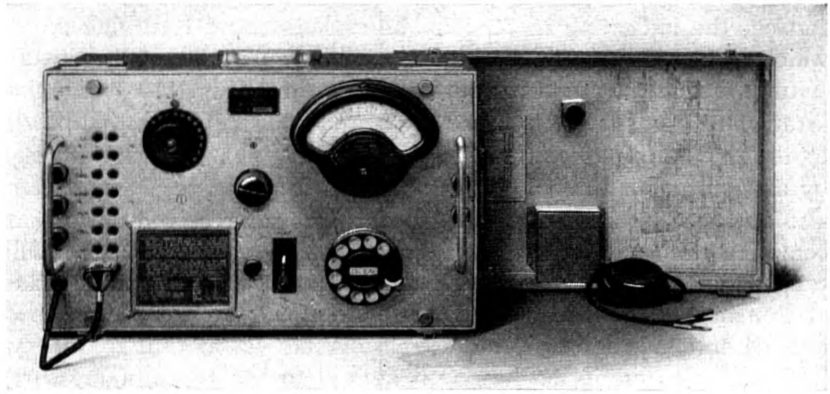

Fig. 10—Noise measuring set.

gains and also transmission level. For the latter type of measurement the input impedance of the amplifier is raised to a high value so that it is, in effect, a voltmeter. This change in impedance can be made from a remote point, a relay for making the change being a part of the amplifier.

In addition to the measurement of transmission losses, gains and levels on telephone circuits, it is also necessary at times to make measurements of noise on the circuit. This noise may be caused by currents induced by power systems or from power plants in the telephone offices or it may be in the form of crosstalk, sometimes unintelligible, from other telephone circuits. Noise measurements are now made with meter indicating devices, the latest type of self-contained portable noise measuring set being shown in Fig. 10.[9] Where enough noise measurements are made to justify a permanent installation, an arrangement similar to that described for transmission measurements can be used. While a different amplifier-rectifier is required for noise measurements the same meters and methods of control can be employed as for transmission measurements.

All of the methods so far described are manually operated in so far as the recording of the results is concerned. There are occasions when a fully automatic recording device is desirable.[4] Such cases are the making of transmission versus frequency runs on repeaters or circuits where measurements are desired over a wide range of frequencies. Another class of measurements are those in which the single-frequency transmission loss of a circuit is to be determined over a long period of time to obtain a measure of the stability of the circuit. For this purpose the new method of measuring transmission is well adapted. With a stable amplifier-rectifier having practically no frequency distortion, the indicating meter may be replaced by a recording meter which will record continuously the received power expressed in db. A fully automatic recording system is shown in Fig. 11. With this arrangement an oscillator at one end of the line supplies testing power to the line, the frequency of the power being changed continuously by a synchronous motor. At the receiving end the recording meter, also operated by a synchronous motor, plots the received power. A complete transmission frequency run on a message telephone circuit can be made in less than one minute. When records are to be made of transmission vs. time, the oscillator frequency at the sending end is fixed and the meter plots the transmission loss at that frequency.

The above description has been limited to the types of measurements commonly made on complete circuits or parts of circuits. In addition to these there are a number of types of measuring apparatus which are used in connection with the installation of new equipment, changes in installations and the detailed running down of trouble.

Transmission measurements have proved to be of great value in maintaining satisfactory transmission performance of telephone circuits. Periodic routine tests avoid service impairment by detecting troubles. Troubles which are thus detected or which cause service complaint are located readily. Another large field of use is in the adjustment of repeaters and complete circuits to prescribed transmission characteristics. The importance of this work to the Bell System is indicated by the fact that there are nearly 1000 of the portable transmission measuring equipments and 1500 transmission test boards now in use, with which several million measurements are made annually. The new measuring systems enable this work to be done more rapidly than in the past, and the reduced cost of the equipment is resulting in its greater distribution.

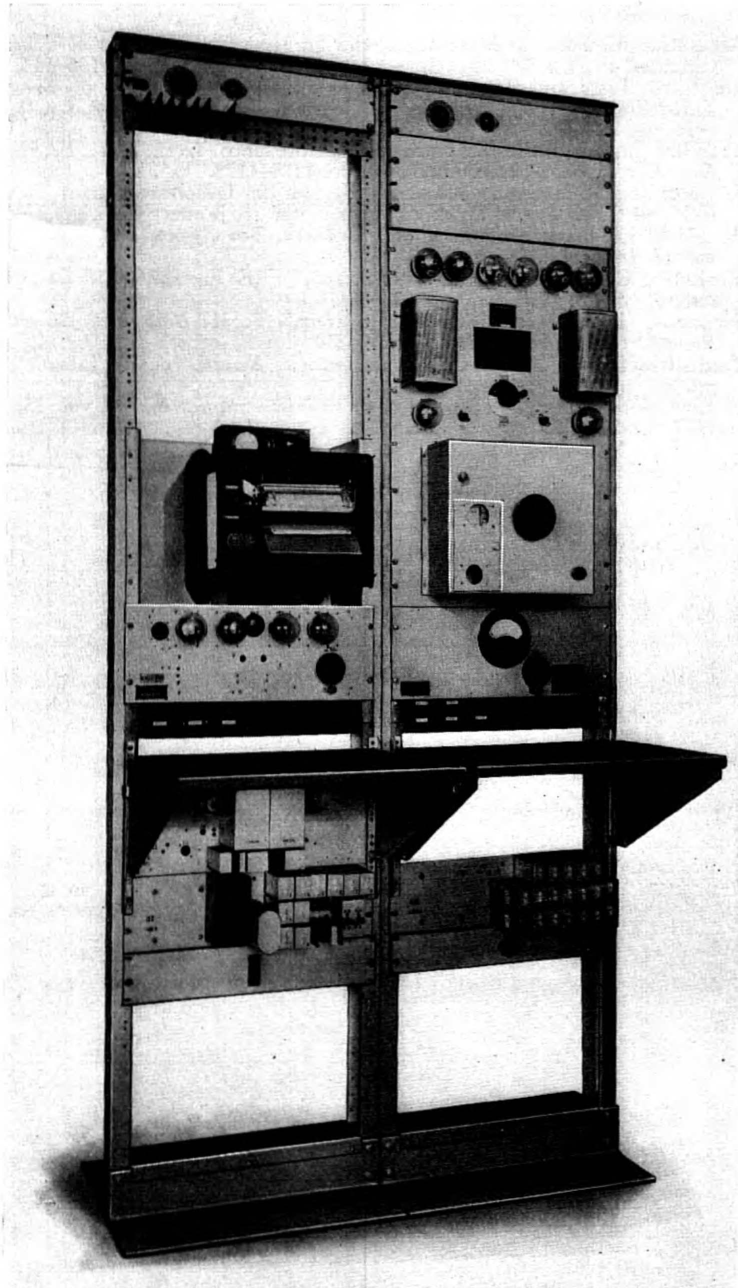

Fig. 11—Automatic recording transmission measuring system.

## References

1. "Measuring Methods for Maintaining the Transmission Efficiency of Telephone Circuits," F. H. Best, *Elec. Engg.*, vol. 43, February 1924, pp. 136–144.
2. "Electrical Tests and Their Applications in the Maintenance of Telephone Transmission," W. H. Harden, *Bell System Technical Journal*, vol. 3, July 1924, pp. 353–392.
3. "Practices in Telephone Transmission Maintenance Work," W. H. Harden, *Elec. Engg.*, vol. 43, December 1924, pp. 1124–1128.
4. "A Recording Transmission Measuring System for Telephone Circuit Testing," F. H. Best, *Bell System Technical Journal*, vol. 12, January 1933, pp. 22–34.
5. "Stabilized Feedback Amplifiers," H. S. Black, *Bell System Technical Journal*, vol. 13, January 1934, pp. 1–18.
6. "Projecting Circuit Performance on a Screen," F. E. Fairchild, *Bell Laboratories Record*, vol. 13, July 1935, pp. 328–331.
7. "Improved Transmission Measuring System," F. H. Best, *Bell Laboratories Record*, vol. 14, March 1936, pp. 237–239.
8. "Decibel Meters," F. H. Best, *Bell Laboratories Record*, vol. 15, January 1937, pp. 167–169.
9. "A New Noise Meter," J. M. Barstow, *Bell Laboratories Record*, vol. 15, April 1937, pp. 252–256.