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Some Physical Characteristics of Speech and Music *

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Kinematic and statistical descriptions of the physical aspects of speech and music are given in this paper. As the speech or music proceeds, the kinematic description consists in giving the principal melodic stream, namely, the pitch variation and also the intensity and the quality variations. For speech and song, the quality changes are principally described by giving, besides the main melodic stream, two secondary melodic streams corresponding, respectively, to the resonant pitches of the throat and mouth cavities. To this must also be added the positions of the stops and the high pitched components of the fricative consonant sounds as functions of the time. The statistical description consists in giving the average, the peak, and the probable variations of the power involved as the various kinds of speech and music proceed. These general ideas are illustrated by numerous experimental data taken by various instrumental devices which have been evolved in the Laboratories during the past fifteen years.

A speech or musical sound is transmitted from the mouth of a speaker or from a musical instrument through the air to the ear of the listener by means of a pressure wave, a succession of condensations and rarefactions of the air. Such a wave spreads in all directions away from the source of sound and soon encounters solid objects which cause reflections. These reflected waves combine with the original one and thus modify the pressure changes taking place at any point. In this paper we shall be concerned chiefly with the pressure changes which take place before reflections occur.

Speech is composed of fundamental sounds called vowels and consonants. As a conversation proceeds there is a constant shifting from one of these sounds to another, only one of them being sounded at one time. Most of these sounds may be continued as a steady tone and hence may be designated as continuants. The others require that the sound stream be interrupted and are therefore called stops. The first class includes the long and short vowels, the diphthongs, the semi-vowels, and the fricative consonants, the sounds ā, ĭ, ou, l and s being typical, respectively, of each of these groups. The pure stops are p, t, ch, and k. In producing the corresponding voiced stops, b, d, j and g, the voiced stream is not entirely interrupted, although the tones from the vocal cord are very much subdued. A conversation,

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then, consists of a succession of continuants and stops and a physical interpretation of speech consists, therefore, of a description of these continuants and a discussion of the manner of joining the continuants together either directly or by means of stops.

Melodic Streams of Speech

As an example of how this analysis of speech may be made consider the sentence, "Joe took father's shoe bench out," an oscillogram of which is shown in Fig. 1.[1] This silly sentence was chosen because it

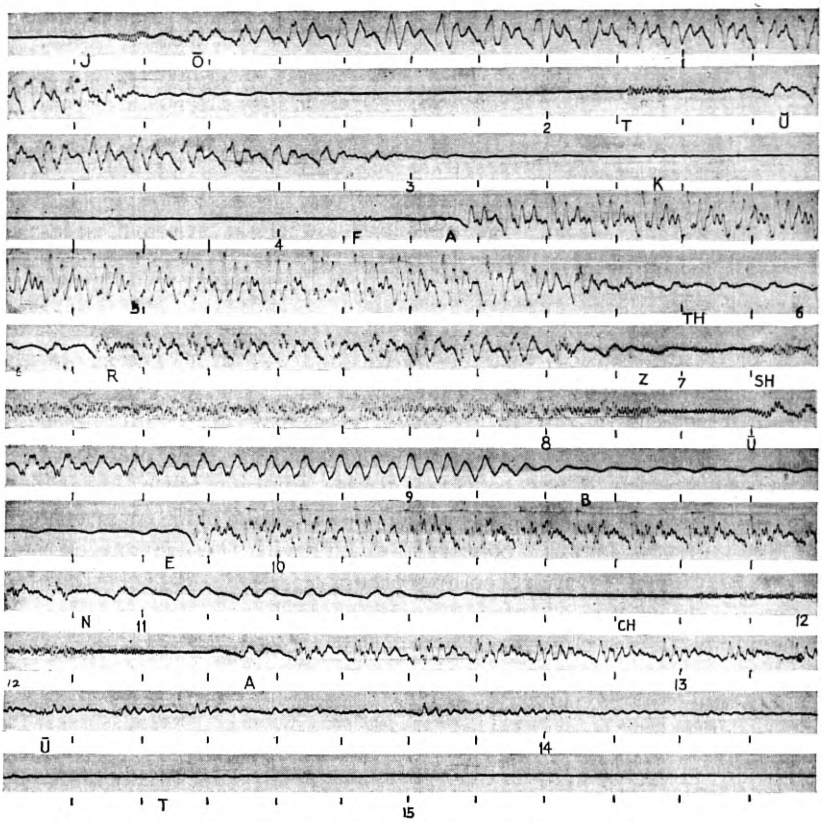


Fig. 1—Oscillogram: "Joe took Father's shoe bench out"—spoken.

is used in our laboratory for making tests on the efficiency of telephone transmitters. This sentence together with its mate "She was waiting at my lawn" contains all of the fundamental sounds in the English

[1] This oscillogram and the others following it were taken with the new high quality and high speed oscillograph which has recently been developed in our laboratory. It has an approximately uniform response for amplitude and phase from 20 to 10,000 cycles per second.

language that contribute toward the loudness of speech. In Fig. 1 the ordinates are proportional to the pressure change in bars and the abscissas are time intervals of .01 second. The eighteen fundamental sounds in this sentence are joined together without the stream of sound being interrupted except for the stops t, k and ch. The stop consonant b is voiced so that although the vocal cord sound is interrupted by the closing of the lips, it continues to sound in a subdued way until the stop is removed and the e sound begins. Pauses, that is, silent

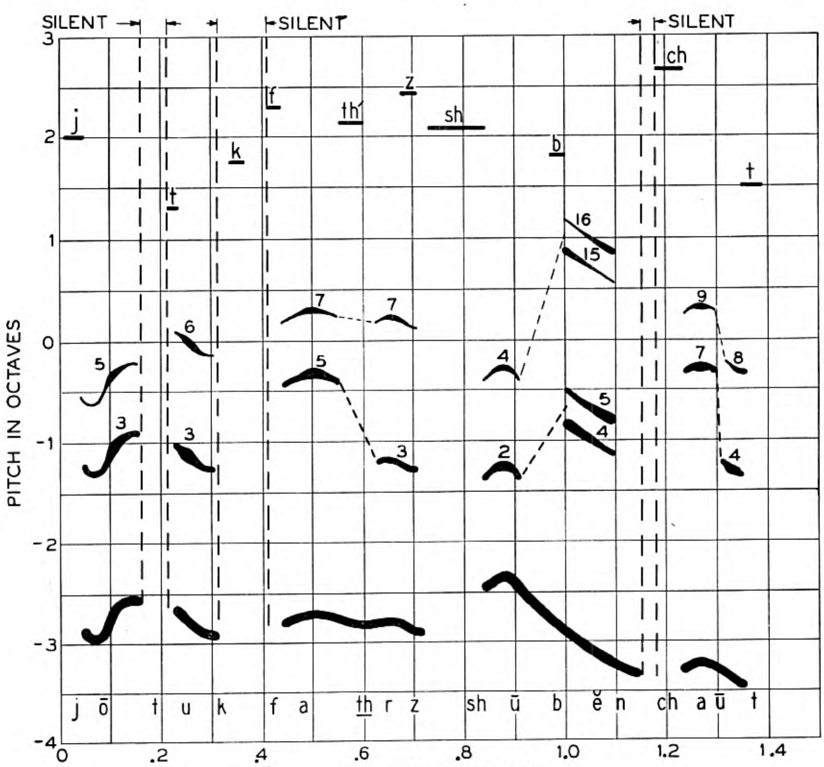


Fig. 2—Melodic curves: "Joe took Father's shoe bench out"—spoken.

intervals, are made between sentences and sometimes between words. It will be noticed that a brief pause was inserted at the intervals .17 to .21 and .32 to .335 and .34 to .41 and 1.16 to 1.18 seconds. There is no such pause between "shoe" and "bench."

Speech, then, consists of a series of comparatively steady states of vibration joined together in time, either by silences or transitions from one steady state to another. Each one of these steady states is characterized by a pitch and a tone quality, and the sequence is essentially a melody. The melody of the sentence whose wave form is shown in Fig. 1 may be illustrated graphically as indicated in Fig. 2. In this figure the ordinates represent the pitch in octaves below or above a tone having a frequency of one kilocycle per second; or if the frequency f is measured in kilocycles, then the pitch P is given by the equation

$$P = \log_2 f. \tag{1}$$

The abscissas represent the time in seconds. The lower curve gives the changes in the pitch of the fundamental and represents the melody as ordinarily understood in music. The middle two curves represent the pitch positions of the strongest harmonics. The location of these positions is determined by the resonant properties of the throat and mouth cavities. These curves may be considered as secondary melodic streams. The combination of these two secondary melodic streams is interpreted by the senses as a sequence of spoken vowels rather than as a series of pitch changes. The small number above each part of the curve gives the number of the harmonic which is augmented by the resonance of the mouth or throat. For the sound e in bench the 4th harmonic was the strongest at the beginning of the sound, but the 5th came in strongest near its end. I have tried to indicate the relative intensities of the harmonics as the sound proceeds by the relative thicknesses of the lines. An examination of the oscillogram shows that the intensity of the harmonic always increases as its pitch becomes nearer the characteristic pitch for the vowel being spoken.

As indicated by the short lines at the top of the chart, there exists at certain intervals high pitched components which are characteristic of the fricative sounds. The unvoiced sounds t, k, f, z and sh, exist only when the three melodic streams are stopped. The high pitched components of the voiced sounds, j, *th* and b, are superimposed upon the three melodic streams.

Besides these four important streams of speech (Fig. 2), there are a great many others with intensities which are in general much lower, but when combined with the main streams they determine the kind of voice, that is, whether it is smooth and musical or rough and harsh. The main melodic stream for a woman's voice is between the pitches -1 and -2 octaves while for a man's voice it is between -3 and -2 octaves. The secondary melodic streams produced while speaking the same sentence are approximately the same for man and woman and of pitches shown in Fig. 2.

In Fig. 3 is shown an oscillograph of the sentence "How are you?".

This sentence contains no stops. The sound stream is not interrupted; it is just a continuous variation from one vowel to another. In Fig. 4 the main melodic stream is given.

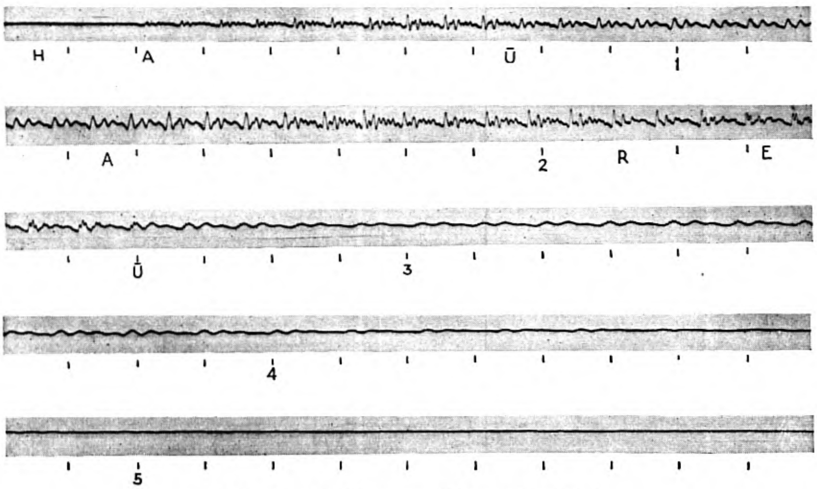


Fig. 3—Oscillogram: "How are you?"

In Fig. 5 an oscillograph of the sentence "Joe took father's shoe bench out" is shown when the vowels of this sentence are intoned on the simple melody do-re-me-fa-me-re-do, and in Fig. 6 the melodic

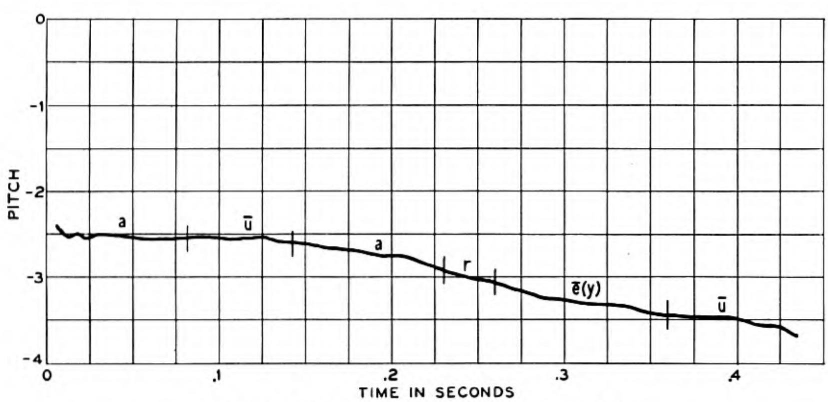


Fig. 4—Melodic curve: "How are you?"

streams are given. In this case only the characteristic resonant pitch positions for the two secondary melodic streams are given. The chief difference between this figure and that for the spoken sentence is in the main melodic stream. For purposes of comparison the curves of the spoken and sung sentence are enlarged and shown together in Fig. 7. In the case of the sung sentence the pitch changes are in definite intervals on the musical scale while for the spoken sentence

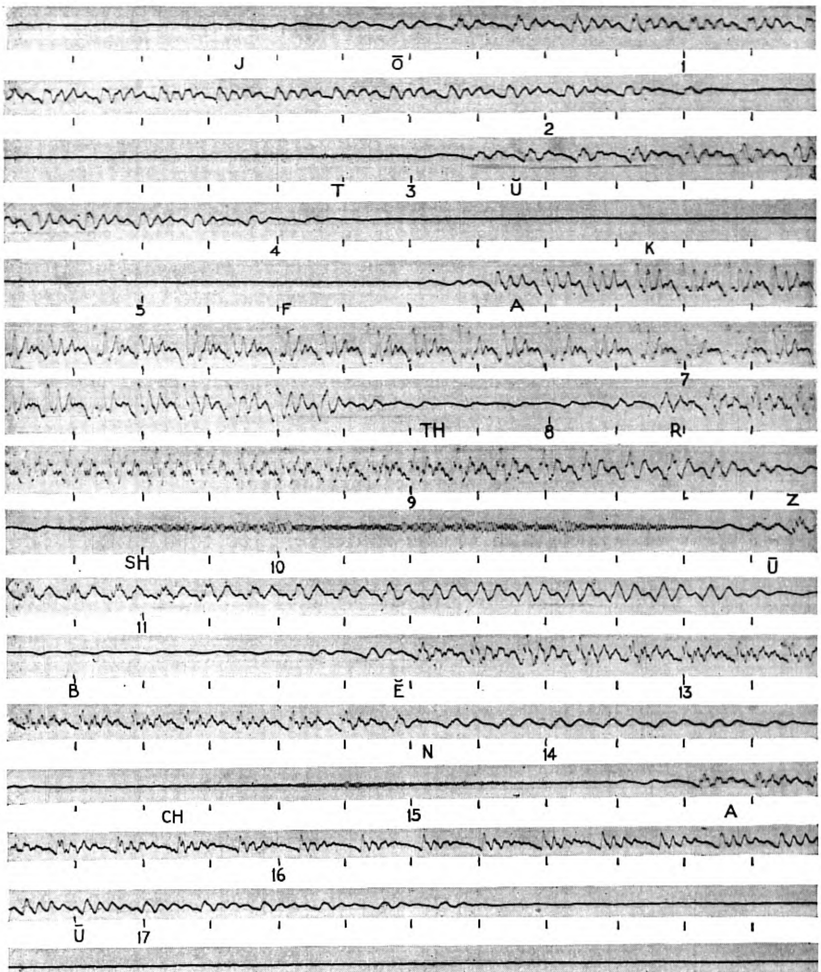


Fig. 5—Oscillogram: "Joe took Father's shoe bench out"—sung.

the pitch varies irregularly, depending upon the emphasis given. The pitch of the fricative and stop consonants is ignored in the musical score, and since these consonants form no part of the music they are generally slid over, making it difficult for a listener to understand the

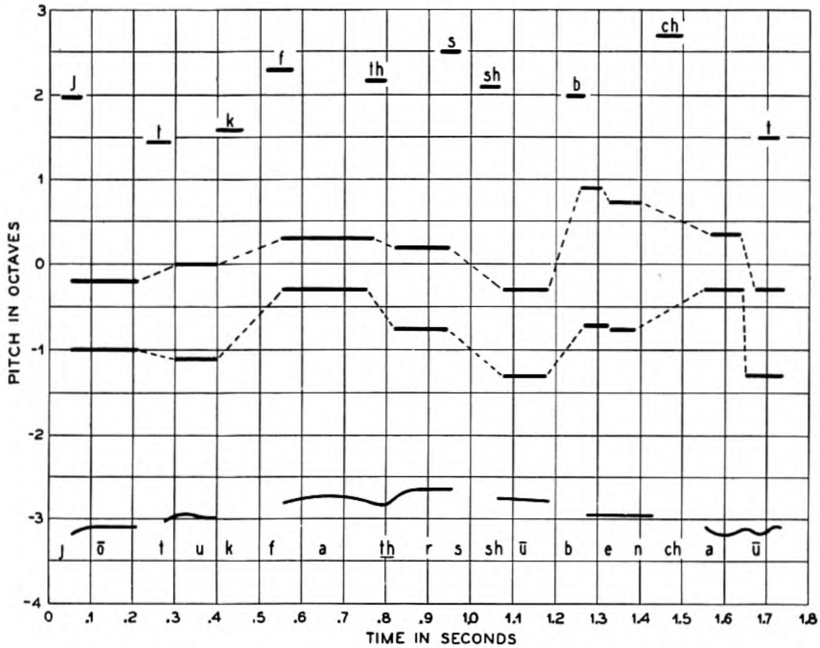


Fig. 6—Melodic curves: "Joe took Father's shoe bench out"—sung.

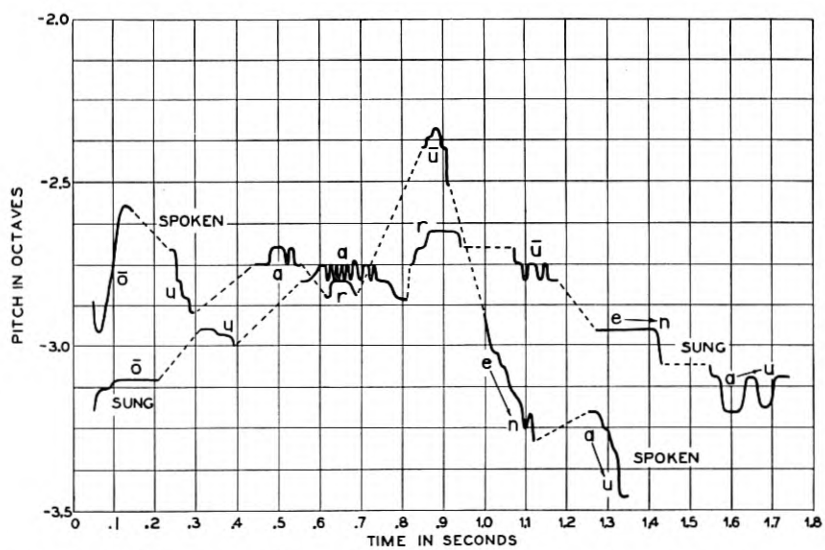


Fig. 7—Melodic curves: "Joe took Father's shoe bench out"—spoken and sung.

meaning of the words. Some of my friends in the musical profession object to this statement of the situation but I think you will agree that a singer's principal aim is to produce beautiful vowel quality and to manipulate the melodic stream so as to produce emotional effects. To do this, it is necessary in singing to lengthen the vowels and to shorten and give less emphasis to the stop and fricative consonants. It is for this reason that it is more difficult to understand song than speech.

Characteristic Pitch or Frequency Levels for the Vowels

Now let us examine part of the speech wave of Fig. 1 in more detail. Consider the vowel in the word "shoe."

The fundamental cycle was repeated 170 times per second. It is evident that the second harmonic is very much magnified until it is nearly as intense as the fundamental. In Fig. 8 is shown another

Fig. 8—Oscillogram of vowel ū.

oscillogram of ū intoned at 120 cycles per second. In this case the 3rd harmonic is magnified. An analysis of a number of ū sounds shows that components falling between 300 and 400 cycles per second are always reinforced. This reinforcement is probably due to the resonance characteristic of the mouth cavity.

Similar characteristic low pitch regions exist for the vowels in the words, put, tone, talk, ton and father. A characteristic *high* pitch region also exists for these sounds but the intensity of the components falling in it are much less. For the vowels in the words tap, ten, pert, tape, tip and team there are two characteristic regions of reinforcement which are of approximately the same intensity and which are independent of the fundamental pitch. This is illustrated in Fig. 9, which gives a spectrum analysis of the vowel "ē" pronounced at the four pitches indicated. The characteristic regions are at 375 cycles per second and 2400 cycles per second corresponding to pitches − 1.4 octaves below and + 1.3 octaves above the reference pitch.

Experimental work[2] has indicated that for American speech the characteristic pitch regions for the vowels and semi-vowels are those shown in Fig. 10. For the first six vowels the components corre-

[2] "Speech and Hearing," Harvey Fletcher, pp. 58, 59.

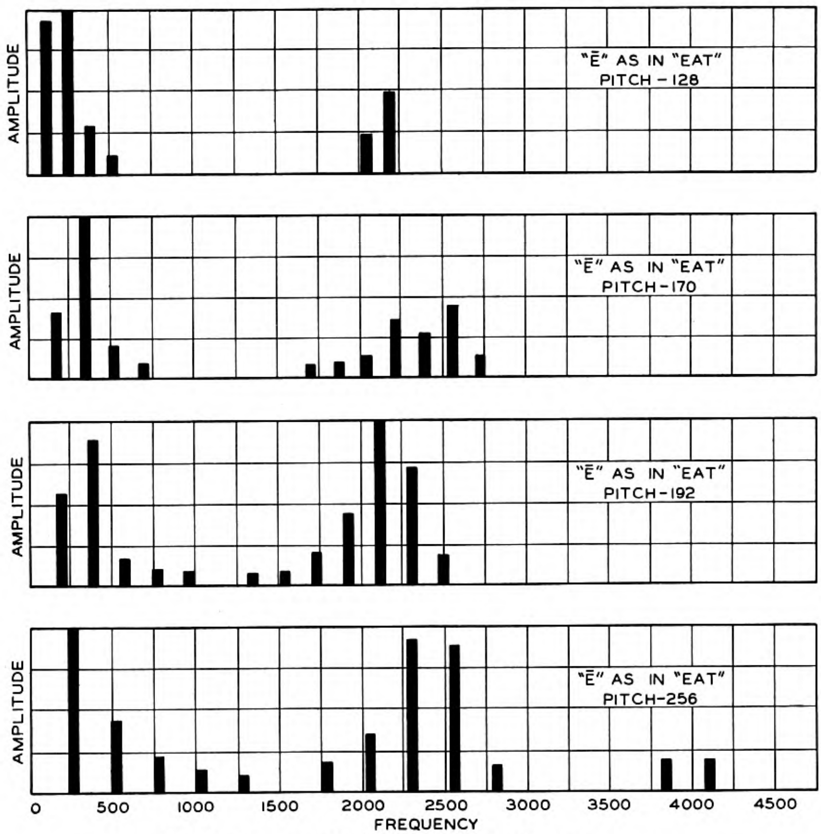


Fig. 9—Spectra of "Ē" intoned at different pitches.

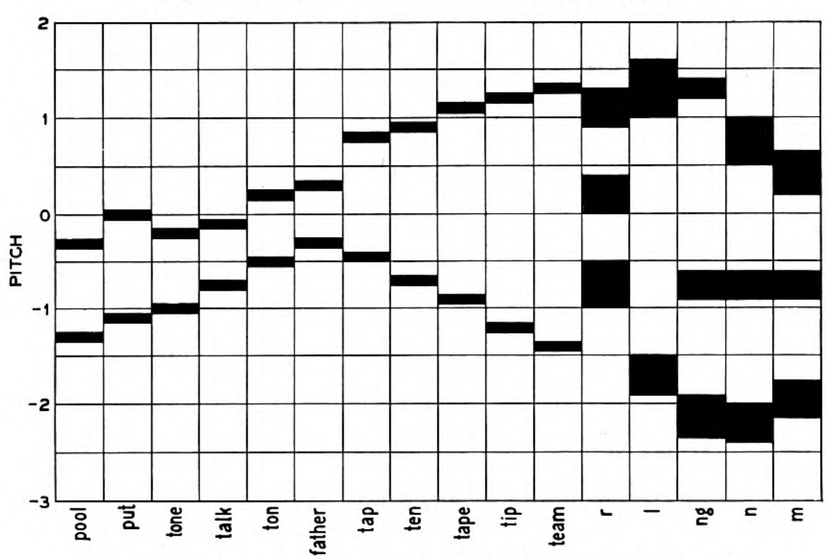


Fig. 10—Characteristic resonance positions for the spoken vowels.

sponding to the characteristic region of high pitch are much less intense than those of low pitch. For the other vowels the intensities of both regions are about alike.

Oscillograms of the Unvoiced Continuants

Now let us examine more closely the wave forms for the fricative sounds, s, sh, f, th. They are shown in Fig. 11. These show only

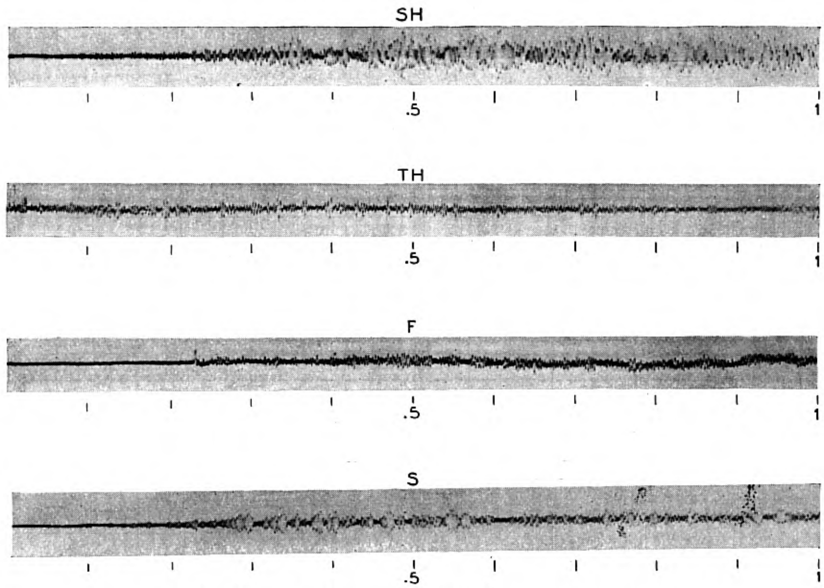


Fig. 11—Oscillograms of fricative consonants.

part of the oscillogram produced when each of these sounds was continued for about one second. It is seen that these sounds contain components having high pitches mostly above + 1. It is seen that they do not have the wave form repeated as uniformly as was the case with the vowel sounds. They seem to be composed of a series of explosions. For example, the oscillogram for "sh" looks very much like one obtained from the sound of a sky rocket.

The f and th sounds are magnified six times in amplitude compared to the sh and s sounds. Although much fainter they still show this explosive character. There are 40, 45, 37 and 55 waves per each .01 second interval, respectively, for these four sounds corresponding to 4000, 4500, 3700, 5500 cycles per second.

Acoustical Power of Speech Waves

Keeping this picture before us, as to the physical composition of speech, and its kinematic nature, let us now consider some statistical averages. If ten different persons spoke the sentence discussed above, there would be a considerable range of differences in the frequencies and intensities used to transmit it through the air. To get a typical cross-section of American speech, it would require at least 100 such sentences pronounced by at least 5 men and 5 women. This would involve the analysis of 18,000 fundamental sounds besides the transitions between them. Also, as was seen from the oscillograms given above, the wave form changes even where it is ideally supposed to be constant so that three or four sample waves from each steady state condition should be analyzed to find the components in each sound. Thus, we have the problem of recording and analyzing about 70,000 such waves. To analyze such a wave by the usual academic methods, namely, to plot the wave to a definite scale and then analyze it into its components by means of a Henrici or similar analyzer, would require at least two or three hours. So such a job for analyzing only the steady-state part of speech would require about 210,000 hours, or 100 years working seven hours a day for 300 days per year. In other words, such a method of attacking the problem is altogether too slow. To find the average intensities and frequencies involved in conversational speech, much more powerful methods for obtaining statistical averages were adopted.

There is a to and fro movement of the air particles simultaneously with the alteration of the air pressure. When the source is so far away that the disturbance can be considered as a plane wave, then the following relations exist between the pressure p, the displacement y, the velocity v, and the acceleration a of a layer of air particles, and the frequency of vibration $\frac{\omega}{2\Pi}$, namely,

$$y\omega^2 = v\omega = a, \tag{2}$$

$$p = rv, \tag{3}$$

where r is the radiation resistance of the air and is given by the product of the air density by the velocity propagation of the wave. The intensity J of the sound at any point is the power passing through a square centimeter of the wave front and is given by

$$J = \frac{p^2}{r}. \tag{4}$$

If J is expressed in microwatts and p in bars, this reduces to

$$J = \frac{p^2}{415} \cdot \tag{5}$$

The intensity level I is defined by

$$I = \log_{10} J \tag{6}$$

and is expressed in bels. These relations hold for any complex sound as well as for a pure tone if p is interpreted as the root mean square value of the pressure change.

It is seen then that all of these quantities can be determined by making experimental measurements of the pressure change. For accomplishing this the following methods were used.

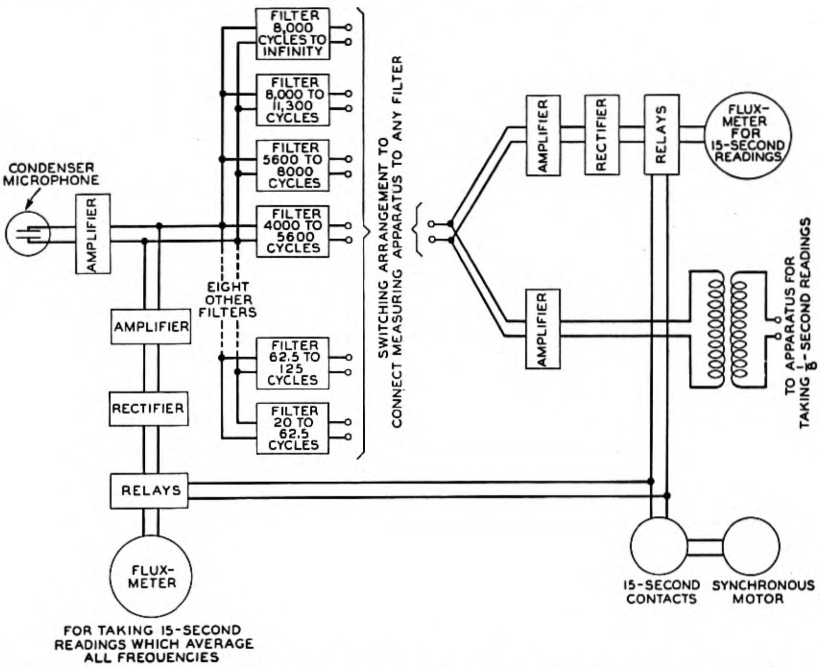


Fig. 12—Schematic of electrical circuit for measuring the average power-frequency distribution of sounds.

The speech to be analyzed is picked up by a Wente condenser microphone and sent into a vacuum tube circuit. This circuit is arranged so that any one of 14 band pass filters can be inserted. After passing through the filter the electrical speech wave is then sent through a rectifier and finally into a meter. A schematic [3] of

[3] See paper entitled "A New Analyzer of Speech and Music" by H. K. Dunn (*Bell Laboratories Record*, November, 1930) and also paper entitled "Absolute Amplitudes and Spectra of Certain Musical Instruments and Orchestras" by Sivian, Dunn & White, *Jour. Acous. Soc. of America*, Jan., 1931.

the circuit is shown in Fig. 12. Two kinds of meters are used. The first is a flux meter as shown in Fig. 12 for integrating the speech energy over any desired interval. When the rectifier is designed to give a value which is proportional to the average voltage, then the deflection of the needle of the flux meter will be proportional to the average pressure times the time. In other words, this device will read the average pressure during any desired time interval. In this

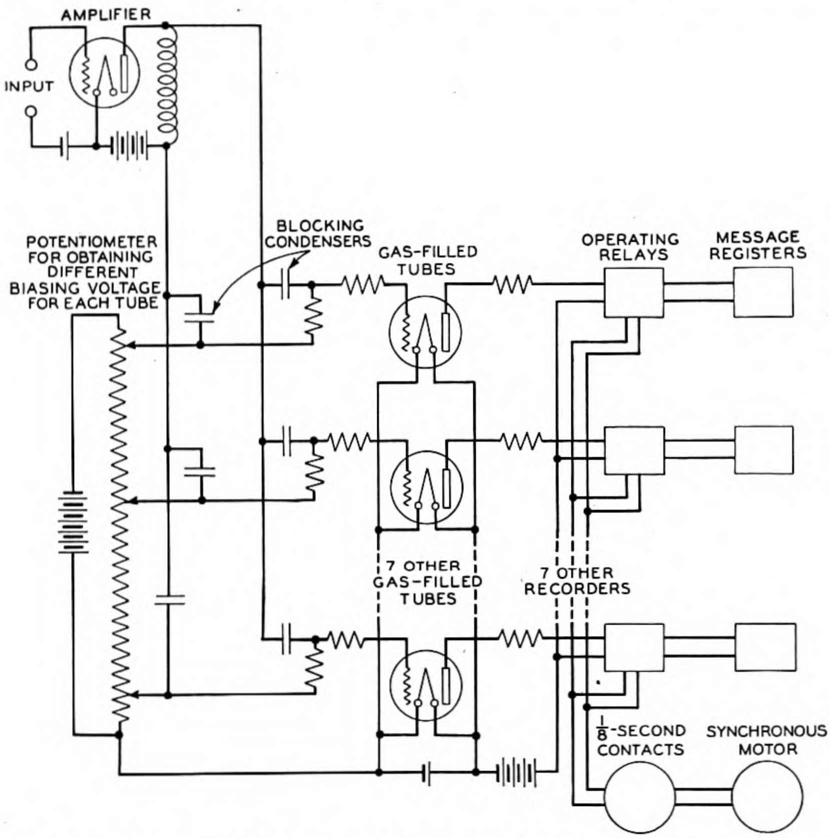


Fig. 13—Schematic of electrical circuit for measuring the peak power-frequency distribution of sounds

way it is possible to find the average pressure in any one of the 14 bands. If the rectifier is adjusted so that the reading is proportional to the square of the impressed voltage then the reading will correspond to the average power. Knowing the calibration [4] of the transmitter

[4] "Speech and Hearing," page 305, and also paper entitled "Absolute Calibration of Condenser Transmitters" by L. J. Sivian, *Bell System Tech. Jour.*, Jan., 1931.

and also its distance from the mouth of the speaker, it is possible to calculate approximately the average speech power.

The other type of meter shown in Fig. 13 consists of a series of parallel circuits, each containing an argon filled three-electrode tube connected in such a way that in adjacent circuits the tube breaks down and allows the passage of current for voltage levels which are 6 db (decibels) apart. Ten such circuits then cover a range of 54 db.

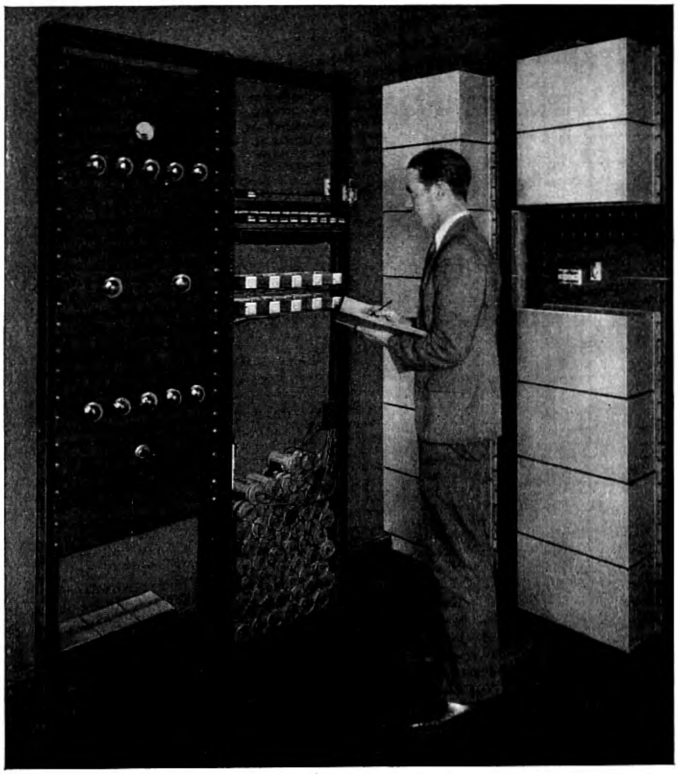


Fig. 14—Photograph of the level analyzer.

In each of these circuits a relay and counter are connected so that for each tube discharge the counter operates. In this way the number of times the tube breaks down is automatically registered. The speech wave coming from the rectifier is sent into this meter where the peak values are measured; that is, the number of times the pressure exceeds a value fixed by each of these circuits will be registered automatically by the corresponding counter. The apparatus is arranged so that every other 8th second interval is measured, the intervening interval being required for resetting the apparatus. In Fig. 14 an observer is shown reading the message registers after a test has been taken. The breakdown tubes are seen at the left and the filters at the right mounted on relay racks.

It is thus seen that with this apparatus 1000 observations may be recorded on a four minute conversation, the final results being read directly from the series of counters.

By the use of this and similar apparatus the following results have been obtained. The average conversational speech power is 10 microwatts or 100 ergs per second. About 1/3 of the time no sound is flowing due to the pauses and the stops to form consonants so that the average conversational speech power is about 50 per cent higher than this value if the silent intervals are excluded. Some of the speakers will use a greater and some a lesser speech power than this average. In Table I are shown the results with a large number of speakers. It will be seen that about 7 per cent of the speakers will use in conversation average powers less than 1/16 the average while about 4 per cent will use powers which are from 4 to 8 times as much as the average. This value of 10 microwatts per second is of course for average conversational intensity. When one shouts as loudly as possible, this average speech power is raised about 100 fold and when one whispers about as softly as possible and still produces intelligible speech, it is reduced to about 1/10,000.

TABLE I

RELATIVE SPEECH POWERS USED BY INDIVIDUALS IN CONVERSATION

Region of Average Speech Power	below 1/16	1/16 to 1/8	1/8 to 1/4	1/4 to 1/2	1/2 to 1	1 to 2	2 to 4	4 to 8	above 8
Per Cent of Speakers	7	9	14	18	22	17	9	4	0

For describing in greater detail the powers involved in speech, we will define the terms Mean Speech Power, Phonetic Speech Power and Peak Speech Power. They are defined as follows:

The Mean Speech Power is the average speech power within any one one-hundredth of a second period.

The Phonetic Speech Power is the maximum value of the mean speech power of a fundamental vowel or consonant.

The Peak Speech Power is the maximum value of instantaneous power over the interval considered.

It was seen from the oscillographs that the vowels have much greater phonetic powers than the consonants. Studies of these phonetic powers for average conversation have indicated that for a typical speaker they are as shown in Table II. The most powerful sound is

TABLE II

o′	680	ū	310	ch	42	k	13
a	600	i	260	n	36	v	12
o	510	ē	220	j	23	*th*	11
a′	490	r	210	zh	20	b	7
ō	470	l	100	z	16	d	7
u	460	sh	80	s	16	p	6
ā	370	ng	73	t	15	f	5
e	350	m	52	g	15	th	1

the vowel in the word "awl" which carries about 900 times as much power as the weakest sound which is th as in thigh. This most powerful vowel when intoned without emphasis is about 50 micro-watts. The relative position in this table depends upon the emphasis given. An emphasized syllable has about three times as much syllabic power as an average one and as will be seen from the table this is about the range of powers among the different vowels.

An analysis of a few oscillograms such as we first considered for determining the peak powers was made and showed that the peak powers are from 10–20 times the phonetic power. It is thus seen that when the vowel in the word "awl" is emphasized, the peak power is from 50 to 200 times the average speech power. To find how frequently these peak powers occur, the apparatus described above using the glow discharge tube circuits was used. The results obtained are shown in Table III.

TABLE III

Per Cent of 1/8 Second Intervals	Number of db the Peak Power in the Interval is Above the Average Level
2	above 20
3	18 to 20
6	16 to 18
8	14 to 16
10	12 to 14
11	10 to 12
11	8 to 10
10	6 to 8
8	4 to 6
6	2 to 4
4	0 to 2
21	Below the average

These values confirm earlier results obtained by oscillographs and give a much more detailed picture of the variation of the peak values as the speech proceeds. About 2 per cent of the time the peak power in 1/8th second intervals exceeds the average power level by 20 db; that is, it is more than 100 times greater. It is seen that a system designed to transmit conversational speech of the best quality should be capable of handling at least 1000 microwatts instead of 10 microwatts. It is also seen that the most frequently occurring peak is at about 10 times the average speech power. For 21 per cent of the time the peaks are below the average level. A large number of the 1/8th second intervals in this class are silent.

To find how the speech powers are distributed throughout the pitch range similar measurements were made introducing successively each one of the 14 band filters as indicated in Fig. 12. These bands

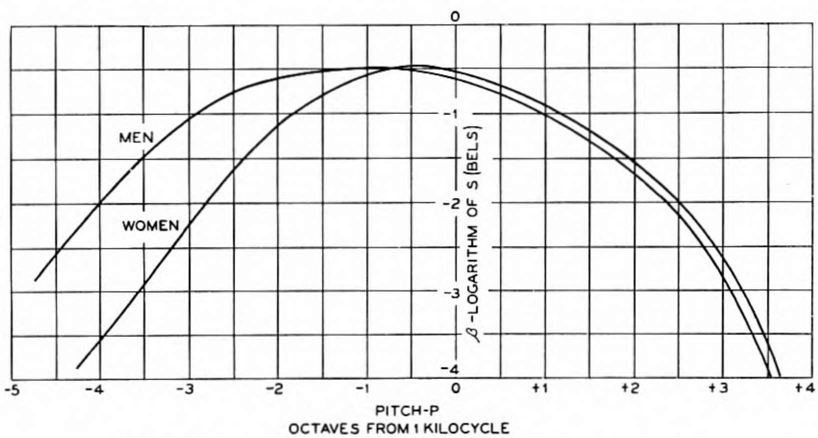


Fig. 15—Distribution function for conversational speech.

$$\text{Fractional energy} = \int_{P_1}^{P_2} S dP.$$

were arranged so as to cover about 1/2 octave pitch range except at the two lower octaves where they cover a complete octave. From the measurements on the average speech power in each band the curves in Fig. 15 were constructed. They give the results for average conversational speech for both men's and women's voices. The ordinates are such that the fraction of the total power F which is carried by any pitch interval between P_1 and P_2 is given by

$$F = \int_{P_1}^{P_2} 10^{\beta} \cdot dP. \tag{7}$$

In other words β is the intensity level per octave expressed in bels. For example, the octave containing the most energy in men's voices is -1.75 to $-.75$ and it contains about $10^{-.5}$ or 31 per cent. The octave below -3 contains about 4 per cent and the octave above $+1$ about 5 per cent. For women's voices these figures are 31 per cent for the most intense region, which is the octave from $-.85$ to $+.15$, and .2 per cent and 7 per cent, respectively, for the other two octaves.

Audible Pitch Limits

The audible pitch limits for conversational speech received at various intensities are determined in the following way. It is seen from Table III that the peak power exceeds the average power by 17 db 10 per cent of the time. The loudness of speech near the threshold is probably determined by these louder components. For convenience the term "effective intensity level" will be used when speaking of these components only. With this nomenclature the effective intensity level is 17 db above the average intensity level. Using these figures and assuming that three-fourths of the speech power is radiated through the hemisphere in front of the speaker, then one can calculate that the effective intensity at one meter's distance will be 6×10^{-3} microwatts per square centimeter or at an effective intensity level of 22 db below one microwatt.

To determine the sensation level the pitches and intensities of the components in the vowels must be considered. A study of the frequency spectra of these vowels indicates that the loudest component contains from 1/2 to 1/5 of the total power of the vowel. From this it is concluded that the components determining the threshold are from 3 to 7 db below the effective level of the speech. The threshold of hearing for pure tones in the pitch region between -1 and $+1$ octaves is from -85 to -95 db with an average value of -91 db. Consequently, it is concluded that at the threshold the effective intensity level for the speech is approximately -86 db and the average level approximately -103 db. Since the effective level of the speech at one meter's distance was shown to be -22 db, it is seen that the sensation level at one meter's distance is 64 db. If the speech wave is uninterrupted by reflections then this level decreases 6 db when the distance between the speaker and the listener is doubled. This level will be raised or lowered in accordance with the intensity of the speaking, the variation for different speakers being in accordance with the data in Table I.

For example, using these relations one finds that the most probable average speech power used by a person in conversation is 5 microwatts. The most probable sensation level of such speech at 1 meter's distance is 61 db, at 10 meters' distance it would be only 41 db and could be brought back to level of conversational speech at one meter's distance only by the speaker shouting as loudly as possible.

If we use the peak voltmeter as shown in Fig. 13 and make measurements upon the peaks in 1/8th second intervals in each of the half octave bands the results will be as represented by the curves of Fig. 16.

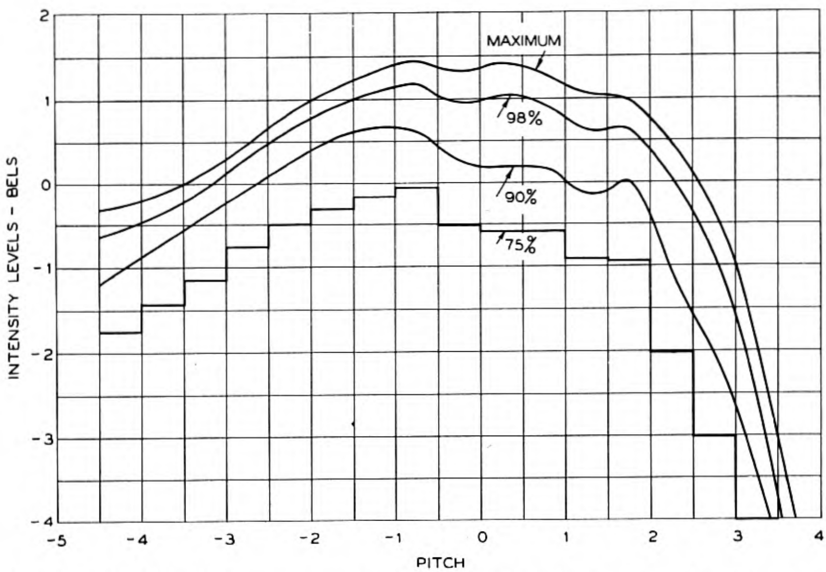


Fig. 16—Peak levels for conversational speech (3 male voices), using ½ octave average pitch intervals.

The top curves give the maximum level of the peak compared to the average intensity. The other two give levels such that the peak levels are below them 98 per cent, 90 per cent or 75 per cent of the time. It will be seen that the most intense peaks occur in the pitch range of − 1 to + 1 octaves. In this pitch range the intensity levels of the maximum peaks for the different components are approximately the same, being 13 or 14 db above the average speech level.

It is interesting to note that in the higher pitch range the curves in this figure are more widely separated than in the lower pitch range. This illustrates an important characteristic of speech, namely, that although components in the pitch range from zero to 2 octaves occur which are just as intense as those in the lower range, they occur less frequently. In other words, the spread in the intensities of the components which are successively occurring as the speech proceeds is very much greater in the higher pitch regions.

As shown above, the threshold is determined for conversational speech when the average speech level is at a − 103 db. For the same reason that only 10 per cent of the peaks having the highest levels determined the threshold for the speech as a whole, the curves labelled 90 per cent of this figure can be used as a basis for determining the sensation level in each of the bands. When the ear of the listener is 10 centimeters from the mouth of the speaker the sensation level will be 84 db and the average intensity level will be − 19 db. If α_0

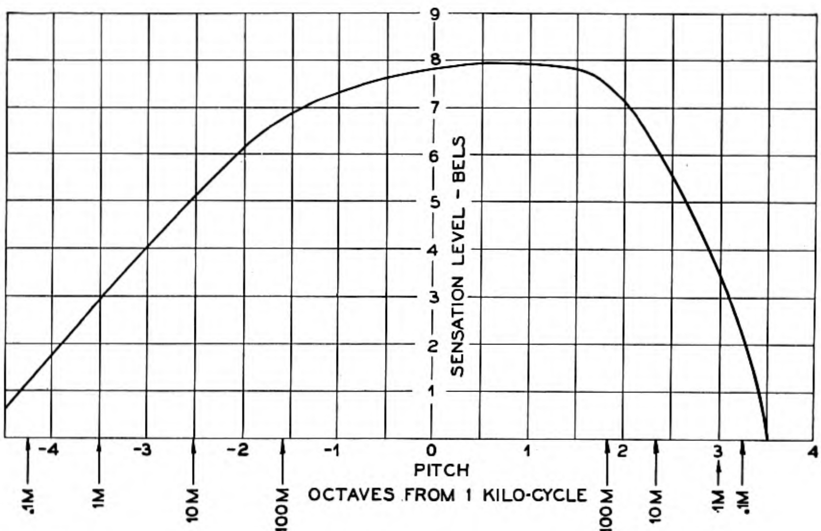


Fig. 17—Speech audibility curve (male voices).

is the average threshold level for tones in each of the half octave bands, then, if we subtract $\alpha_0 - 19$ from each ordinate of the curve in Fig. 16, we will obtain the sensation level of each half octave band. A curve constructed in this way will be called an audibility curve and is given in Fig. 17. This curve is for the case when the lips of an average male speaker are 10 centimeters from the ear of an average listener. It will be seen that the half octave bands above 3.25 octaves and below − 4.25 octaves are just audible. If the distance between speaker and listener is increased to one meter, which is the most commonly used distance, then the audibility curve would be one which is lowered 20 db from that one shown in Fig. 15 and the audible limits would be + 3 and − 3.5 octaves, corresponding to frequencies of

8000 c.p.s. and 90 c.p.s. Similarly, if the distance is increased to 100 meters, the limits will be found to be + 1.85 and − 1.55 octaves. These relations are true only when no other sounds are present. Similar limits are easily determined when the listener is in the presence of any other sound whose noise audiogram is known. In that case, the ordinates in the audibility curve are reduced by an amount equal to the corresponding ordinate in the noise audiogram.

These values are such that any half octave by itself within the pitch limits will transmit audible sounds. This does not necessarily imply that, when the undistorted speech is acting upon the ear, such a half octave will transmit sounds whose presence can be detected. To test this point several observers listened to speech reproduced by a high quality loud speaker system which would reproduce all frequencies from 40 to 15,000 uniformly and into which filters could be introduced. These filters limited at desired cut-off positions the upper and lower frequencies which were reproduced.

A large group of observers then listened to this reproduced speech and they were asked to judge which was filtered and which was unfiltered. The results of such tests are shown in Fig. 18. The

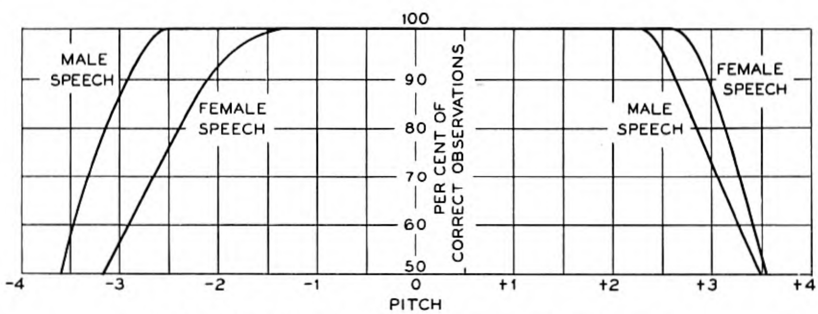


Fig. 18—Audible pitch limits for conversational speech.

ordinates give the per cent of correct observations and the abscissæ the cut-off frequency of the filter. Taking a 60 per cent correct judgment as a criterion for determining the detectable pitch limits, then it will be seen that the lower limit is − 3.5 octaves and the upper limit 3.25 octaves for male speech which agrees with the results taken from the audibility curve established directly from power measurements upon speech and the threshold of hearing as described above. For female speech the limits are − 2.9 and + 3.4 octaves. Summarizing, then, it is seen that the most powerful components carrying conversational speech, which are of any practical importance, are about 4000 or 5000 microwatts while the principal components in the weakest sound carry only about 1/20th of a microwatt. Even for an extremely loud shout or for the most intense singing the maximum power will not exceed more than about 100 times these values; that is, they will not exceed 1 watt. The pitch range necessary for faithfully transmitting men's and women's speech is from -3.5 to $+3.3$ octaves or from 90 to 10,000 cycles per second.

Acoustical Power Produced by Musical Instruments

Now we will look briefly at some of the same results obtained for music by the use of some of these same measuring tools. In Fig. 19

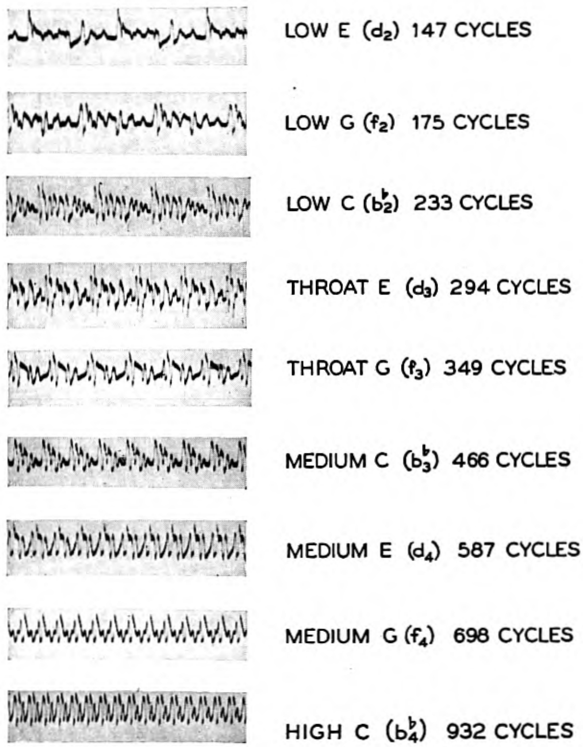


Fig. 19—Major triads of B-flat clarinet.

are shown typical waves produced by the clarinet. A complete oscillogram of the waves produced when the instrument played its full range of three octaves on the chromatic scale was taken. The simple waves shown in the figure are those corresponding to the major triad in each of these octaves. The entire record was about 250 feet long. Such musical tones have a much more uniform wave form than those from the voice.

The measurement of the peak power from typical musical instruments used in an orchestra gave the following results.[6]

TABLE IV

PEAK POWER OF MUSICAL INSTRUMENTS (Fortissimo Playing)

Instrument	Peak Power in Watts
Heavy Orchestra	70
Large Bass Drum	25
Pipe Organ	13
Snare Drum	12
Cymbals	10
Trombone	6
Piano	0.4
Trumpet	0.3
Bass Saxophone	0.3
Bass Tuba	0.2
Bass Viol	0.16
Piccolo	0.08
Flute	0.06
Clarinet	0.05
French Horn	0.05
Triangle	0.05

The most powerful single instrument is the bass drum which gives powers which exceed 25 watts in successive 1/8th second intervals about 6 per cent of the time it is being played. A 75-piece orchestra

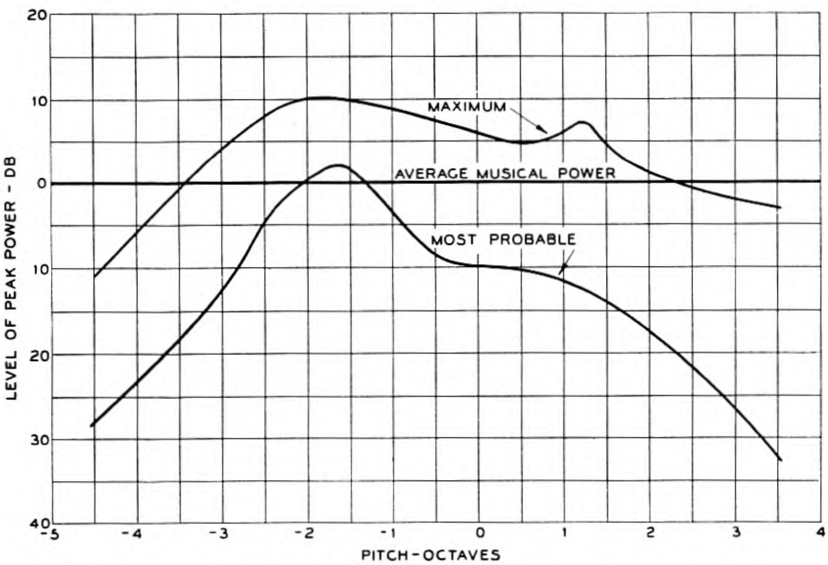


Fig. 20—Maximum and most probable peak levels for a 75-piece orchestra.

[6] These results and those in Fig. 19 were taken from a paper by Sivian, Dunn and White entitled "Absolute Amplitudes and Spectra of Certain Musical Instruments and Orchestras," *Jour. Acous. Soc. of America*, Jan., 1931.

playing with full volume will produce peak acoustic powers as great as 70 watts.

When such an orchestra played the four different selections, the maximum peak powers varied from 8 to 66 watts, but the average powers were .08, .07, .07 and .13 watts, respectively. Hence the variation of the average power from selection to selection was much less than that of the peak power. Both the peak powers and also the average powers for the orchestra are about 10,000 times the corresponding powers for conversational speech. In Fig. 20 the curves show how the peak power was distributed among the different pitch bands for this 75-piece orchestra. The curves give the average values for the four selections. The zero line corresponds to a power of approximately 1/10th of a watt. The levels correspond to that which was obtained in the half octave band acting alone. Although the maximum peak was 70 watts for the unfiltered music when the heaviest piece was being played, the most probable peak value in any half octave band is less than 1/10 of a watt except for the octave between -2 and -1 octaves, where it is slightly higher than this value. The distance between the two curves increases as you go to either side of this octave which is approximately that between middle "C" and the "C" above it. This indicates that the components in this region are more nearly alike in intensity and occur more frequently than in the other regions. The top curve indicates that from the standpoint of maximum peak values the half octaves from $-2\frac{1}{2}$ to $+1\frac{1}{2}$ octaves are all about equally important. As the pitch of a component goes below $2\frac{1}{2}$ octaves, its intensity decreases rapidly as indicated in the figure. Very intense peaks occur occasionally with frequencies as high as 10,000 or 12,000 cycles.

To find the lowest level used in orchestral music a violin player was asked to play as softly as is ever customary while playing before the public. Its average power was found to be about 4 microwatts. It is thus seen that the peak power from a large orchestra is about 20,000,000 times the average power produced by soft violin playing.

Audible Pitch Limits for Musical Sounds

Measurement of the detectable pitch limits was determined in a way similar to that described for conversational speech. The results [7] for typical musical instruments are shown in Fig. 21. For comparison the results for speech and some common noises are also included. It will be seen that the lower limit for music is determined by the bass

[7] A more comprehensive report of this work will soon be given in a paper by W. B. Snow.

tuba, the bass viol, and the kettle drum, and its value is about 40 c.p.s. The upper limit is determined by the snare drum, the violin, and the cymbals, and is shown to be about 15,000 c.p.s. Summarizing, then, for music the range of pitches covered by the components is

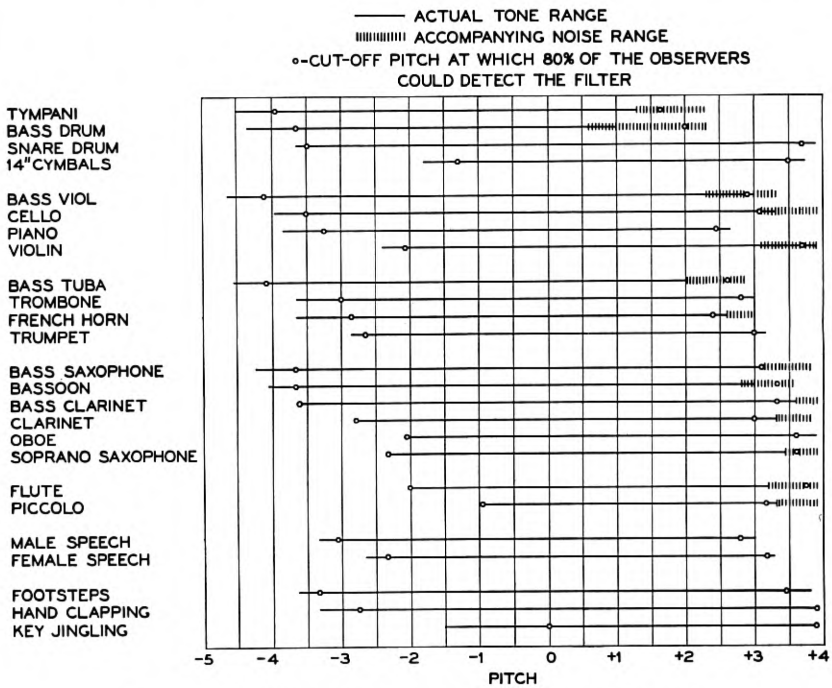


Fig. 21—Audible pitch range for speech, music and noise.

from -4.7 to $+3.9$ octaves, corresponding to the frequency range from 40 to 15,000 cycles per second. The intensity ranges from about 70 watts to 4 microwatts, corresponding to an intensity level range of 73 db going from the average level of the softest violin playing to the peaks in the heaviest playing of a full 75-piece orchestra.

The Statistical Energy-Frequency Spectrum of Random Disturbances

By JOHN R. CARSON

A mathematical discussion of the statistical characteristics of Random Disturbances in terms of their "energy-frequency spectra" with applications to such typical disturbances as telegraph signals and "static".

IN a paper entitled "Selective Circuits and Static Interference" (*B. S. T. J.*, April, 1925) the writer discussed the "energy-frequency spectrum" (hereinafter precisely defined) of irregular random disturbances extending over a long interval of time. In view of our lack of even statistical information regarding static or atmospheric disturbances the specification of the energy-frequency spectrum, denoted by $R(\omega)$, was necessarily qualitative, and it was merely postulated that

"$R(\omega)$ is a continuous finite function of ω which converges to zero at infinity and is everywhere positive. It possesses no sharp maxima or minima and its variation with respect to $\omega(\omega = 2\pi f)$, where it exists, is relatively slow."

In a paper entitled "The Theory of the Schroteffekt,"[1] T. C. Fry deals with a similar problem, namely, the energy or "noise" absorbed in a vacuum tube from a stream of electrons with random time distribution. His method of attack is widely different from that of the present paper. In a more recent paper on "The Analysis of Irregular Motions with Applications to the Energy-Frequency Spectrum of Static and of Telegraph Signals" (Phil. Mag., Jan., 1929), G. W. Kenrick, by making certain hypotheses regarding the wave-form of the elementary disturbances whose aggregate is supposed to represent static interference, and by applying probability analysis, arrives at explicit formulas for the "statistical" or "expected" value of $R(\omega)$ for a number of different cases.

I

In the present paper the statistical or "expected" energy-frequency spectrum $R(\omega)$ of random disturbances is investigated by a method which is believed to be somewhat more general and direct than that of Kenrick.[2] The results are applicable to the Schroteffekt, telegraph

[1] *Jour. Franklin Inst.*, Feb., 1925.

[2] Kenrick's analysis is based on a formula derived originally by N. Wiener instead of proceeding directly from the Fourier integral.

signals and similar disturbances. The writer, however, concludes that their application to "static" or "atmospheric" disturbances is of questionable value owing partly to our lack of the necessary statistical information regarding such disturbances and also to the fact that they cannot be expected to have the "quasi-systematic" characteristics necessary to the application of probability theory.

The energy-frequency spectrum of a disturbance, as the concept is here employed, will now be defined. Let a disturbance $\Phi(t)$ exist in the epoch $0 \leq t \leq T$ and let

$$\begin{aligned} F(i\omega) &= C(\omega) + iS(\omega) \\ &= \int_0^T \Phi(t) e^{i\omega t} dt. \end{aligned} \tag{1}$$

Then, as shown in my paper referred to above,

$$\frac{1}{\pi}\int_0^\infty |F(i\omega)|^2 d\omega = \int_0^T \Phi^2 dt.$$

The energy-frequency spectrum is defined by the equation

$$G(\omega) = \operatorname*{Lim}_{T\to\infty} \frac{1}{\pi T} |F(i\omega)|^2, \tag{2}$$

so that

$$\int_0^\infty G(\omega) d\omega = \operatorname*{Lim}_{T\to\infty} \frac{1}{T}\int_0^T \Phi^2 dt. \tag{3}$$

It is on this last equation that the physical application of the concept of the energy-frequency spectrum rests; namely, that it determines the mean square value of $\Phi(t)$, as the epoch T is made indefinitely great. Its principal application in electrotechnics depends upon the further fact that, if $\Phi(t)$ represents an electromotive force applied to a network of impedance $Z(i\omega)$, the mean square current $\overline{I^2}$ absorbed by the network is given by [3]

$$\overline{I^2} = \operatorname*{Lim}_{T\to\infty} \frac{1}{T}\int_0^T I^2 dt = \int_0^\infty \frac{G(\omega)}{|Z(i\omega)|^2} d\omega. \tag{3a}$$

We now suppose that the function or disturbance $\Phi(t)$ is composed of a number N of elementary disturbances; thus

$$\Phi(t) = \sum_1^N a_m \phi_m(t - t_m), \tag{4}$$

[3] A somewhat more involved formula gives the mean power absorbed. See my paper referred to in the first paragraph.

the mth elementary disturbance being supposed zero until $t = t_m$. If we now write

$$f_m(i\omega) = c_m + is_m = \int_0^T \phi_m(t)e^{i\omega t}dt, \tag{5}$$

it is easy to show by the methods employed in my previous paper that

$$\begin{aligned} |F(i\omega)|^2 = \sum_1^N a_m^2 |f_m(i\omega)|^2 & \\ + 2\sum_{m=1}^{N-1} \sum_{n=m+1}^{N} & a_m a_n (c_m c_n + s_m s_n) \cos \omega(t_n - t_m) \\ + 2\sum_{m=1}^{N-1} \sum_{n=m+1}^{N} & a_m a_n (c_m s_n - s_m c_n) \sin \omega(t_n - t_m). \end{aligned} \tag{6}$$

This is more compactly expressible as

$$\begin{aligned} |F(i\omega)|^2 = \sum_1^N a_m^2 |f_m(i\omega)|^2 & \\ + 2\sum_{m=1}^{N-1} \sum_{n=m+1}^{N} & \{a_m a_n \cdot f_m(i\omega) \cdot f_n(-i\omega)e^{i\omega(t_n - t_m)}\}_{\text{Real Part}}. \end{aligned} \tag{6a}$$

Now, obviously, if the amplitudes $a_1, \cdots, a_N$ and the wave form of the elementary functions $\phi_1, \cdots, \phi_N$ are specified, $G(\omega)$ is uniquely defined and determined by the preceding formula. This, however, is not the case in the problem under consideration, where at best the functions are specified only statistically by probability considerations. Under such circumstances, when the problem is correctly set and sufficient statistical information is furnished for its solution, we introduce the idea of the *statistical* energy-frequency spectrum $R(\omega)$ defined as follows:

The statistical energy-frequency spectrum $R(\omega)$ is equal to the weighted average of $G(\omega)$ for all possible values of $G(\omega)$, the weighting being in accordance with the probability of the occurrence of each particular possible value.

For example, the statistical value of a function $f(x_1, x_2, \cdots, x_n)$, where the variables $x_1, \cdots, x_n$ are defined only by probability considerations, is, in accordance with the foregoing definition,

$$\int_{-\infty}^{\infty} dx_1 p_1(x_1) \cdot \int_{-\infty}^{\infty} dx_2 p_2(x_2) \cdots \int_{-\infty}^{\infty} dx_n p_n(x_n) \cdot f(x_1, x_2, \cdots, x_n),$$

where $p_m(x_m)dx_m$ is the probability that x_m lies between x_m and $x_m + dx_m$.

To apply the foregoing concept and definition of the statistical value of a function to the problem at hand it is necessary to suppose that the typical impulse $f_m(i\omega)$ is a function of ω and certain parameters $\lambda_1, \lambda_2, \cdots, \lambda_n$, and that these parameters are statistically specified by probability considerations. Thus we suppose that $p_m(\lambda_m)d\lambda_m$ is the probability that λ_m lies between λ_m and $\lambda_m + d\lambda_m$. $G(\omega)$ will then be a function of ω and $\lambda_1, \lambda_2, \cdots, \lambda_n$, the amplitudes $a_1, \cdots, a_N$ being regarded as parameters, when defined by probability functions. We then have, in accordance with the foregoing,

$$R(\omega) = \int_{-\infty}^{\infty} d\lambda_1 p_1(\lambda) \cdot \int_{-\infty}^{\infty} d\lambda_2 p_2(\lambda_1) \times \cdots \int_{-\infty}^{\infty} d\lambda_n p_n(\lambda_n) G(\omega, \lambda_1, \lambda_2, \cdots, \lambda_n). \quad (7)$$

II

To apply the foregoing to the simplest possible case let us suppose that the elementary impulses are all identical; $a_1 = a_2 = \cdots a_N = 1$, and that their distribution in time is purely random. With these assumptions it follows at once from (6) that

$$R(\omega) = \frac{\nu}{\pi} |f(i\omega)|^2 + 2 \cdot \frac{\nu^2}{\pi} |f(i\omega)|^2 \frac{1 - \cos \omega T}{\omega^2 T}, \qquad T \to \infty. \quad (8)$$

If $f(i0) \neq 0$, this has a singularity at $\omega = 0$; however

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{T} \int_0^T \Phi^2 dt &= \int_0^{\infty} R(\omega) d\omega \\ &= \nu \int \phi^2 dt + \nu^2 \left[\int \phi dt \right]^2. \end{aligned} \quad (9)$$

Here $\nu = N/T$ = mean frequency of occurrence of the elementary impulses. This formula is in entire agreement with Fry's results for the Schroteffekt (l.c.).

To consider a somewhat more involved problem, we shall suppose that the *durations* of the individual impulses and their *amplitudes* are distributed at random. We further denote the probability that the duration of any impulse, selected at random, lies between λ and $\lambda + d\lambda$ by $p(\lambda)d\lambda$. Correspondingly, $q(a)da$ denotes the probability that its amplitude lies between a and $a + da$. The durations and the amplitudes are then the statistically specified parameters.

We now postulate that $\Phi(t)$ is an *alternating series* of impulses of the *same wave form;* i.e.

$$\Phi(t) = \sum_1^N (-1)^m a_m \phi_m(t - t_m),$$

$$\phi_m(t) = \phi(t), \qquad 0 \leq t \leq \lambda_m$$

$$= 0 \qquad t > \lambda_m,$$

$$t_m = \lambda_1 + \lambda_2 + \cdots + \lambda_{m-1},$$

and we denote the mean frequency of occurrence, N/T, by ν.

Substitution in the preceding formulas and straight-forward operations give

$$R(\omega) = \frac{\nu}{\pi} \int_0^\infty a^2 q(a) da \cdot \int_0^\infty |f(i\omega, \lambda)|^2 p(\lambda) d\lambda$$

plus the real part

$$\frac{2\nu}{\pi} \left[\int_0^\infty aq(a)da \right]^2 \cdot \int_0^\infty f(i\omega, \lambda) p(\lambda) e^{i\omega\lambda} d\lambda \cdot \int_0^\infty f(-i\omega, \lambda) p(\lambda) d\lambda$$
$$\times \operatorname*{Lim}_{N \to \infty} \frac{1}{N} \sum_{m=1}^N \sum_{n=m+1}^N (-1)^{n-m} \left[\int_0^\infty p(\lambda) e^{i\omega\lambda} d\lambda \right]^{n-m-1}. \quad (10)$$

If we write

$$\int_0^\infty p(\lambda) e^{i\omega\lambda} d\lambda = \rho(i\omega) = \rho, \quad (11)$$

we have by straightforward procedure

$$\operatorname*{Lim}_{N \to \infty} \frac{1}{N} \sum_{m=1}^N \sum_{n=m+1}^N (-1)^{n-m} \left[\int_0^\infty p(\lambda) e^{i\omega\lambda} d\lambda \right]^{n-m-1} = \frac{-1}{1 + \rho(i\omega)}, \quad (12)$$

whence

$$R(\omega) = \frac{\nu}{\pi} \int_0^\infty a^2 q(a) da \int_0^\infty |f(i\omega, \lambda)|^2 p(\lambda) d\lambda$$
$$- \frac{2\nu}{\pi} \left[\int_0^\infty aq(a)da \right]^2 \cdot \left\{ \frac{U(\omega) \cdot V(\omega)}{1 + \rho(i\omega)} \right\}_{\text{Real Part}}, \quad (13)$$

where

$$f(i\omega, \lambda) = \int_0^\lambda \phi(t) e^{i\omega t} dt = c(\omega, \lambda) + is(\omega, \lambda),$$

$$U(\omega) = \int_0^\infty f(i\omega, \lambda) p(\lambda) e^{i\omega\lambda} d\lambda, \quad (14)$$

$$V(\omega) = \int_0^\infty f(-i\omega, \lambda) p(\lambda) d\lambda.$$

If, on the other hand, we suppose that the impulses, instead of systematically alternating in sign, are equally likely to be positive or negative, the double summation term of (9) vanishes and

$$R(\omega) = \frac{\nu}{\pi}\int_{-\infty}^{\infty} a^2 q(a)da \cdot \int_0^{\infty} |f(i\omega, \lambda)|^2 p(\lambda)d\lambda. \tag{15}$$

This follows from the fact that the amplitude a is equally likely to be positive or negative. Consequently the integration with respect to da must be extended from $-\infty$ to $+\infty$ and, since by hypothesis $q(-a) = q(a)$, it follows that

$$\int_{-\infty}^{\infty} aq(a)da = 0.$$

To apply the preceding formulas to actual calculations, it is necessary to know the function $f(i\omega, \lambda)$ and in addition the probability functions involved. These latter may be supposed known from statistical data or calculable on theoretical assumptions. For example, if we assume that the times of incidence of the elementary disturbances are distributed entirely at random, the application of well-known probability theory gives $p(\lambda) = \nu e^{-\nu\lambda}$.

A third case is of interest. Here, instead of postulating that the termination of one impulse coincides with the start of the next (i.e. $t_{m+1} = t_m + \lambda_m$), we suppose that the times of incidence are entirely unrelated, and that the amplitudes are equally likely to be positive or negative. For this case the formula for $R(\omega)$ is formally identical with (15).

III

The foregoing analysis will now be applied to deriving what represents more or less accurately the statistical energy-frequency spectrum of telegraph signals. To this end we shall suppose that the elementary disturbance may have any one of three possible values (all equally probable), characterized by durations λ_1, λ_2, λ_3 and amplitudes a_1, a_2, a_3. The corresponding spectra of the elementary disturbances are then determined by the equations,

$$\begin{aligned} f_1(i\omega) &= \int_0^{\lambda_1} \phi(t)e^{i\omega t}dt, \\ f_2(i\omega) &= \int_0^{\lambda_2} \phi(t)e^{i\omega t}dt, \\ f_3(i\omega) &= \int_0^{\lambda_3} \phi(t)e^{i\omega t}dt. \end{aligned} \tag{16}$$

The application of the preceding analysis to this case gives

$$R(\omega) = \frac{\nu}{3\pi}(a_1^2|f_1(i\omega)|^2 + a_2^2|f_2(i\omega)|^2 + a_3^2|f_3(i\omega)|^2)$$

plus the *real part* of

$$\begin{aligned} &\frac{2\nu}{9\pi}(a_1f_1(i\omega)e^{i\omega\lambda_1} + a_2f_2(i\omega)e^{i\omega\lambda_2} + a_3f_3(i\omega)e^{i\omega\lambda_3}) \\ &\qquad \times (a_1f_1(-i\omega) + a_2f_2(-i\omega) + a_3f_3(-i\omega)) \\ &\qquad \times \operatorname*{Lim}_{N\to\infty}\frac{1}{N}\sum_{m=1}^{N-1}\sum_{n=m+1}^{N}[\tfrac{1}{3}(e^{i\omega\lambda_1} + e^{i\omega\lambda_2} + e^{i\omega\lambda_3})]^{n-m-1}. \end{aligned} \tag{17}$$

It is to be understood that the *real part* of the second term is alone to be retained.

If we write

$$x = \tfrac{1}{3}(e^{i\omega\lambda_1} + e^{i\omega\lambda_2} + e^{i\omega\lambda_3}),$$

$$\frac{1}{N}\sum\sum[\tfrac{1}{3}(e^{i\omega\lambda_1} + e^{i\omega\lambda_2} + e^{i\omega\lambda_3})]^{n-m-1} = \frac{1}{1-x}\left(\frac{N-1}{N} - \frac{x}{N}\frac{1-x^{N-1}}{1-x}\right)$$

and

$$\begin{aligned} \operatorname*{Lim}_{N\to\infty}\frac{1}{N}\sum\sum &= \frac{1}{1-x} & x &< 0 \\ &= \frac{N}{2} & x &= 1. \end{aligned}$$

There is therefore an infinity at $\omega = 0$, as we should expect. Its measure, however, is finite.

The preceding is merely an example which admits of extension to more complicated types of signals, as will be obvious to the reader. For example, the probabilities of the elementary signals need not be the same and their number need not be restricted to three.

IV

In all the cases discussed above it will be observed that the disturbance is "quasi-systematic" in the sense that the elementary disturbances are all of the same wave-form differing only in duration and amplitude. Indeed, some such assumptions as these are essential to the application of the mathematical theory. In the case of atmospheric disturbances we have no reason to suppose any such quasi-systematic character exists. Furthermore, even if for the sake of argument, we suppose that the elementary disturbances, which make up static, have a common wave form at the point at which they originate, they would vary widely in this respect after arriving at a common receiver. The writer is therefore of the opinion that the quotation from his previous paper appearing at the start of this article, represents all that can safely be said regarding the spectrum of static and that our present knowledge is insufficient to justify the application of probability analysis to the problem. All that we can say is that the part of $R(\omega)$ which contributes to "static interference" is simply

$$\operatorname*{Lim}_{N\to\infty} \frac{\nu}{\pi} \cdot \frac{1}{N} \sum_{1}^{N} a_m^2 |f_m(i\omega)|^2,$$

a result deducible from (6) and in agreement with the conclusion of my original paper (l.c.). It is here supposed that the times of incidence are distributed at random. This formula, however, supplies no useful information in the absence of data regarding the wave forms and amplitudes of the individual disturbances.

Bridge Methods for Locating Resistance Faults on Cable Wires

By T. C. HENNEBERGER and P. G. EDWARDS

In this paper are discussed bridge methods for locating resistance faults on cable wires, with special reference to the theory of methods for (1) locating insulation faults which cause complete cable failure, (2) locating insulation faults of high resistance, and (3) locating series resistance unbalances.

The methods described are better adapted to the toll than to the exchange telephone cable plant, since they require that the conductor resistances of the wires used for measurements be equal and, in general, that measurements be made from each end of the faulty cable.

IN the toll telephone plant, insulation faults such as "grounds" and "crosses" are usually located by the "Varley loop" method, which involves essentially the measurement of the d.-c. resistance of the faulty wire between the point of fault and one end of the cable, and the comparison of this resistance with the total conductor resistance of the wire to obtain the "percentage location" of the fault on a resistance basis. Corrections are then applied to account for such factors as the resistance of the leads between the cable and the bridge, the resistances of loading coils, and non-uniformity of conductor resistance caused by temperature differences between underground and aerial sections of the cable. After all corrections are applied the corrected percentage location is converted into distance from one cable end to the fault.

In general, the most troublesome insulation fault to locate is a "wet spot" due to absorption of moisture by the insulation through a defect in the lead covering of the cable, which results in low insulation resistance between wires and to ground. Standard apparatus now available for locating grounds and crosses is sufficiently sensitive to permit accurate locations of wet spots up to about five megohms in resistance. The Varley loop methods ordinarily employed in connection with the apparatus will give accurate results provided a wire of very much higher insulation resistance than the faulty wire is used as the "good" wire for measurements. These are the conditions which usually are found when wet spots occur. Cases occur occasionally, however, in which a "good" wire having sufficiently high insulation resistance as compared to the faulty wire cannot be obtained, either because all of the wires available for measurements are affected by the fault or because the fault resistance is high. The methods for locating insulation faults discussed in this paper are especially applicable to such cases.

Resistance unbalances on cable wires are of relatively infrequent occurrence and are usually difficult to locate. A method frequently employed for locating such faults is to measure the impedance unbalance at various frequencies of a circuit containing the faulty wire and to analyze periodic impedance-frequency curves plotted from the measurements.[1] The methods for locating series resistance unbalances discussed in this paper involve the use of ordinary Wheatstone bridges, are simple to apply, and give results which are believed to be comparable to those obtained by the impedance-frequency method.

Normal Insulation Resistance of Cable Wires

The values of insulation resistance obtained by measurements on cable wires which are not faulty are dependent on the circumstances in which the measurements are made. In the case of paper-insulated telephone cable the most important factors affecting insulation resistance are electrification period and temperature.

The following discussion of normal insulation resistance refers particularly to measurements between wires of pairs in a typical repeater section of aerial toll cable approximately 50 miles long, the wires being at ground potential at the time of application of the testing potential. Insulation resistance to ground is also of interest, but is difficult to measure accurately in long lengths of cable because of interfering potentials. As a rough approximation, normal insulation resistance between a wire and ground can be considered to be about two thirds as great as normal insulation resistance between wires.

A curve illustrating the variation of insulation resistance between wires of a typical cable pair over a 30-minute electrification period is shown in Fig. 1. In general, the electrification periods necessary for obtaining reasonably constant values of insulation resistance differ appreciably for different pairs, and for the same pair at different times. The usual period ranges from 15 minutes to an hour for a pair 50 miles long. Routine measurements are generally made, however, using electrification periods of one minute.

The paper used for insulating the wires of telephone cable has an appreciable negative temperature coefficient of insulation resistance. This is indicated by the curve of Fig. 2 which shows variations of average insulation resistance with temperature. The points for the

[1] "Telephone Circuit Unbalances," by L. P. Ferris and R. G. McCurdy, A. I. E. E. Transactions, 1924, Volume XLIII, page 1331.

curve were obtained by averaging, for each five-degree range of temperature, the insulation resistances obtained by measurements made

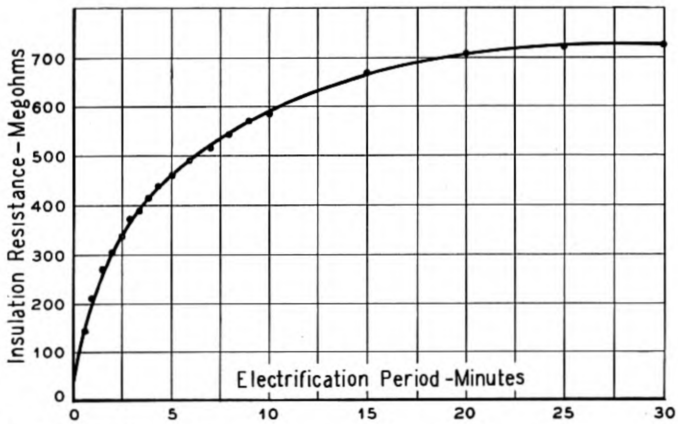


Fig. 1—Variation of insulation resistance with electrification period—typical 50 mile aerial cable pair.

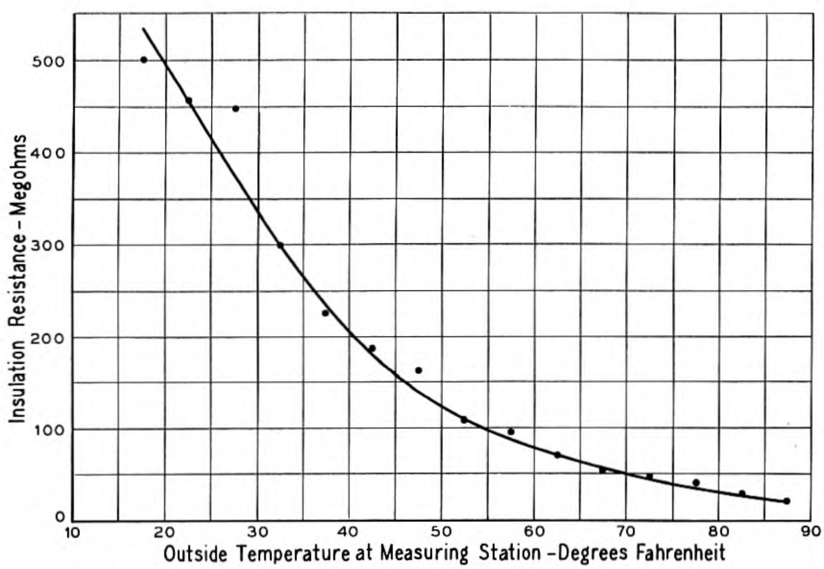


Fig. 2—Variation of average insulation resistance with temperature—typical repeater section of aerial cable.

daily over the course of a year on representative pairs in a repeater section, using electrification periods one minute long. It has been found that the percentage change in insulation resistance per degree change in temperature differs widely for different cable sections and even for the individual pairs in a particular section. The average change per degree Fahrenheit is probably about four per cent, for the temperature range encountered in the plant.

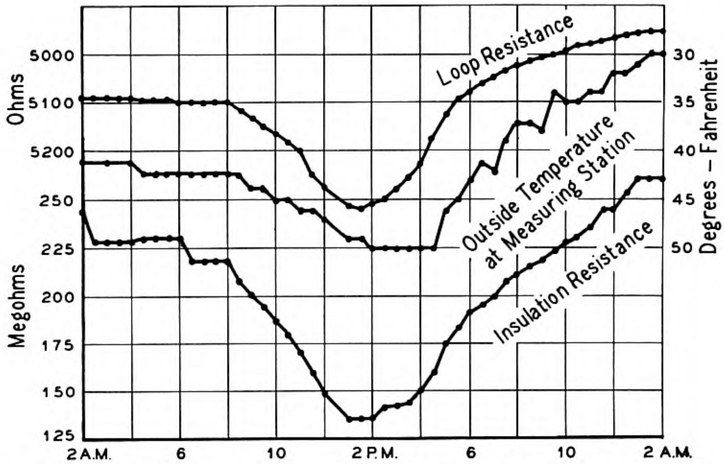


Fig. 3—Variation of insulation resistance, loop resistance and temperature over 24 hour period—typical 50 mile aerial cable pair.

The curves of Fig. 3 illustrate comparative variations of insulation resistance between wires of a representative cable pair, conductor resistance of the pair, and outside temperature, during a 24-hour period which included a sunny summer day. The curves were plotted from measurements made every half hour, one-minute electrification periods being used when measuring insulation resistance. It is not uncommon to find that the insulation resistances of particular pairs vary by factors of three to one during the course of a day.

Comparative variations of average insulation resistance between wires of pairs and of mean outside temperature over the course of a year are illustrated by the curves of Fig. 4. The points for the insulation resistance curve were obtained by measuring the insulation resistances of a number of pairs each working day during the year, using one-minute electrification periods, and averaging the measured values for each day.

In general, average insulation resistance is likely to vary by a factor of 15 to one during the course of a year. Individual pairs are, of course, subject to much wider seasonal variations in insulation resistance. During winter it is not uncommon to find particular pairs in a 50-mile repeater section with insulation resistances between wires

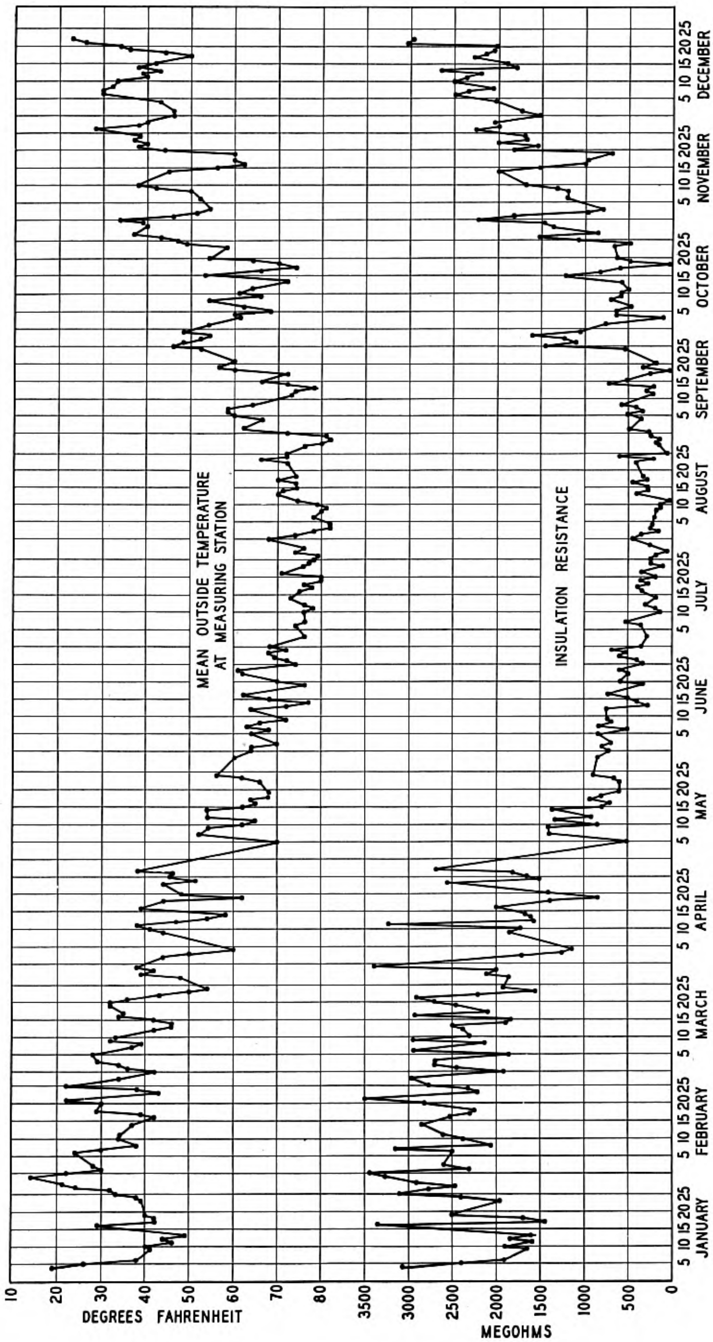


Fig. 4—Seasonal variation of average insulation resistance—typical repeater section of aerial cable.

of several thousand megohms, while during summer, especially in cables which have been in service for a number of years, the insulation resistances between wires of some pairs in a 50-mile repeater section may be as low as 25 megohms (1250 megohm-miles).

VARLEY LOOP METHOD

The Varley loop circuits which are used ordinarily for locating grounds and crosses on wires of toll cable are illustrated in Figs. 5 and 6. The Wheatstone bridge has equal ratio arms, A. The "good"

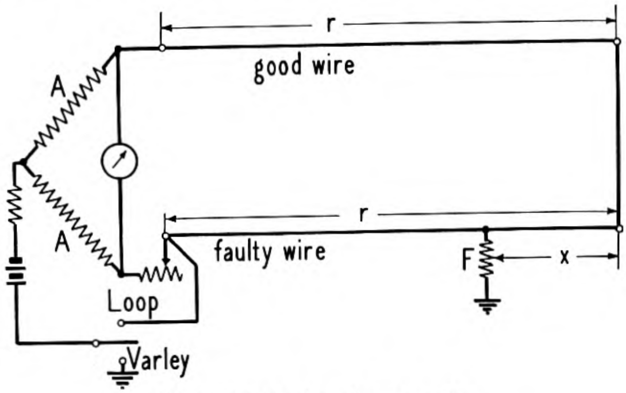


Fig. 5—Varley loop for grounds.

and faulty cable wires have equal conductor resistances, r, and are connected together at the distant end of the cable. F is the resistance of the fault, and x is the conductor resistance of the faulty wire between the fault and the distant end of the cable.

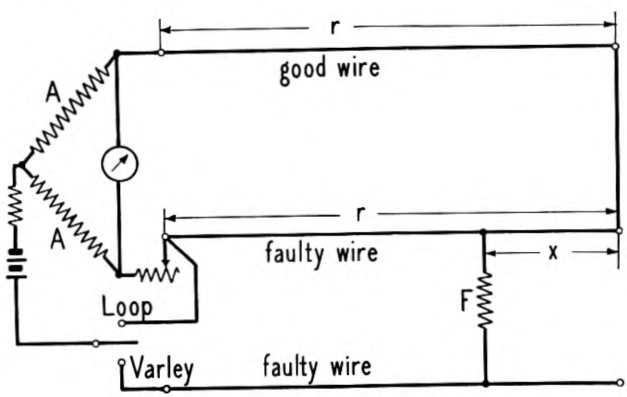


Fig. 6—Varley loop for crosses.

With the battery switch in "Varley" position, a Varley measurement is made by balancing the bridge to a rheostat value, V, at which there is no galvanometer current. Then:

$$\frac{A}{A} = \frac{r + x}{r - x + V},$$

$$x = \frac{V}{2}. \tag{1}$$

It will be noted that the fault resistance, F, is in series with the battery and has no effect on the measurement except to limit the sensitivity of the bridge.

With the battery switch in "loop" position, a loop resistance measurement is made by balancing the bridge to a rheostat value, L. Then:

$$r = \frac{L}{2}.$$

From these Varley and loop measurements the percentage location of the fault, on a resistance basis, can be calculated as follows:

From the distant end: $\frac{V}{L}$(100 per cent).

From the measuring end: $\frac{L - V}{L}$(100 per cent).

Corrections for resistances of bridge leads, loading coils, etc., are then made, the corrected percentage location is converted into feet, and the location of the fault is determined by reference to cable records.

These Varley circuits and formulas are well adapted to the toll cable plant where wires are usually well balanced in conductor resistance, and the resistance of the leads between the bridge and the cable is small compared to the conductor resistance of the cable wires. In exchange cable work, modified forms of the Varley loop, which do not require that the "good" and faulty wires be of equal conductor resistance and which correct automatically for the resistance of bridge leads, are frequently used.

Total Cable Failures

In the case of total cable failure, due, for instance, to a wet spot, there are no wires in the cable which are unaffected by the fault, and the fault resistances of a large number of the wires are low, i.e., of the same order of magnitude as the conductor resistances of the wires.

Two methods by which such faults can be located are discussed below: A "Corrected Varley" method which may be used provided two wires having fault resistances to ground differing by at least 25 per cent are available for measurements; and a "Straight Resistance" method which does not require that the two wires have faults of unequal resistances.

Corrected Varley Method

Consider a cable in which all wires have low insulation resistance to ground because of a wet spot, and assume that from among the faulty wires two wires are selected for a Varley measurement. Assuming a bridge having equal ratio arms, A, the Varley network can be represented as shown in Fig. 7, where M and F are the effective resistances of the faults on the two wires, r is the conductor resistance of either wire, and x is the resistance of that portion of either wire which is between the distant end of the cable and the faults.[2]

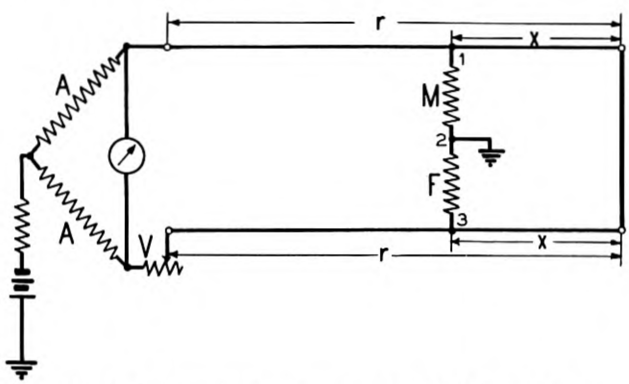


Fig. 7—Schematic circuit—corrected Vaıley method.

The Varley circuit of Fig. 7 is equivalent to the Varley circuit of Fig. 8, where the "π" type network formed by the three resistances, M, F and $2x$, has been replaced by a "T" type network having resistance values as indicated. When the bridge is balanced by adjustment of the rheostat to a resistance, V, at which there is no galvanometer

[2] The actual faults form a "π" type network consisting of a resistance between wires and a resistance between each wire and ground. The "π" type network has been replaced by a "T" type network having resistances, M and F, between the two wires and the branch point of the network, and a third resistance connecting the branch point to ground. This third resistance is in series with the bridge battery and its only effect is to limit the sensitivity of the bridge. To simplify discussion the resistances, M and F, are shown connected directly to ground, and the third resistance is considered to form a part of the resistance shown connected between the battery and the junction point of the ratio arms of the bridge.

current:

$$r - x + \frac{2Mx}{M + F + 2x} = r - x + \frac{2Fx}{M + F + 2x} + V.$$

Solving for x:

$$x = \frac{V}{2}\frac{(M + F)}{(M - F - V)}. \tag{2}$$

Comparison of this formula with Formula (1) indicates that the factor $V/2$, as determined by Varley measurement, represents the

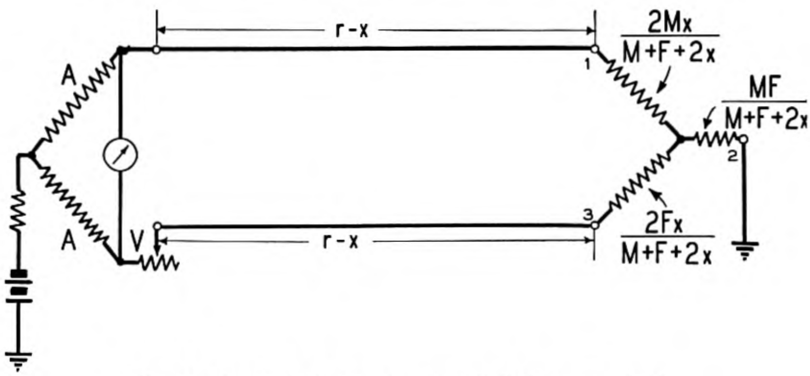


Fig. 8—Equivalent circuit—corrected Varley method.

apparent rather than the true resistance between the distant end of either wire and the location of the faults. The factor $\dfrac{(M + F)}{(M - F - V)}$ is a correction factor and expresses the relation between $V/2$ and the true resistance, x. If the fault, M, is very much higher in resistance than either the fault, F, or the balancing resistance, V, the correction factor becomes practically equal to one and $V/2$ becomes practically equal to x. In these circumstances the wire having the fault, M, can properly be called a "good" wire and Formula (1) will give accurate results.

Since the apparent resistance, $V/2$, can be determined by Varley measurement the faults can be located if the value of the correction factor can be determined. The correction factor can be evaluated by additional measurements made on the two faulty wires from the end of the cable opposite to that used for the Varley measurement, as described below.

Referring to Fig. 7, the resistance of either wire between the faults and the end of the cable opposite to that used in making the Varley measurement is x. If a loop resistance measurement is made from this opposite end, using a bridge having equal ratio arms, *with the distant ends of the wires open*, and the resistance in the bridge rheostat at balance is designated L_0:

$$M + F = L_0 - 2x.$$

If a Varley measurement is made from the same end, using a bridge having equal ratio arms, *with the distant ends of the wires open*, and the resistance in the bridge rheostat at balance is designated V_0:

$$M - F = V_0.$$

Substituting these values of $(M + F)$ and $(M - F)$ in (2):

$$x = \frac{V}{2} \frac{L_0}{V_0}. \tag{3}$$

Application: To apply the Corrected Varley method, an ordinary Varley measurement is made from one end of the cable, and additional loop resistance and Varley measurements, as described above, are made from the opposite end. The values of balancing resistance thus obtained are substituted in Formula (3). The location of the trouble on a resistance basis, x/r, can then be calculated, and the location can be converted into feet in the usual manner.

Usually it will be necessary to determine the loop conductor resistance, $2r$, of the faulty wires from cable records rather than by measurement at the time the location is being made. A measurement of loop conductor resistance would be in error because of the low resistance shunt $(M + F)$ on the portion of the loop between the faults and the short-circuited ends of the wires. The accuracy of location is dependent, therefore, on the accuracy to which conductor resistance can be estimated.

In cases where it is desirable to use the Corrected Varley method the fault resistances will be low, so that usually the balancing resistance, L_0, will not exceed 10,000 ohms. If L_0 is too high to measure using a bridge with equal ratio arms, unequal ratio arms, A and B, may be used and the quantity $\frac{A}{B} L_0$ substituted for L_0 in Formula (3). Measurement of V_0, however, should be made using a bridge with equal ratio arms.

The Corrected Varley method will give accurate results only under the following conditions:

(1) Both faults must be at the same point along the cable.
(2) The fault resistances must remain constant throughout the test.

(3) The resistance of the fault on one wire must be higher than the resistance of the fault on the other wire.

(4) The conductor resistances of the faulty wires must be equal.

In the practical application of the method, care must be exercised in selecting the wires to be used for measurements. The resistance, M, of the fault on the wire which is connected to the ratio arm of the bridge when measuring V should be appreciably higher (at least 25 per cent higher) than the resistance, F, of the fault on the wire connected to the rheostat arm of the bridge. This can be understood by considering that as M and F approach each other in value the correction factor becomes larger and the Varley balancing resistance, V, approaches zero, i.e., the apparent location of the trouble approaches the distant end of the cable. Errors in measurement become increasingly important as V and V_0 become smaller.

Accurate results will not be secured if the resistances of the faults vary while a set of measurements to determine V and the correction factor is being made. It is advisable, therefore, to make a number of separate sets of measurements, and to base the location on those sets which appear to be consistent.

Straight Resistance Method

In many cases of complete cable failure the faults on all of the wires are of practically equal resistance, and the Corrected Varley method cannot be used successfully. The Straight Resistance method described below has the advantage that the wires used for measurement need not be unequal in fault resistance.

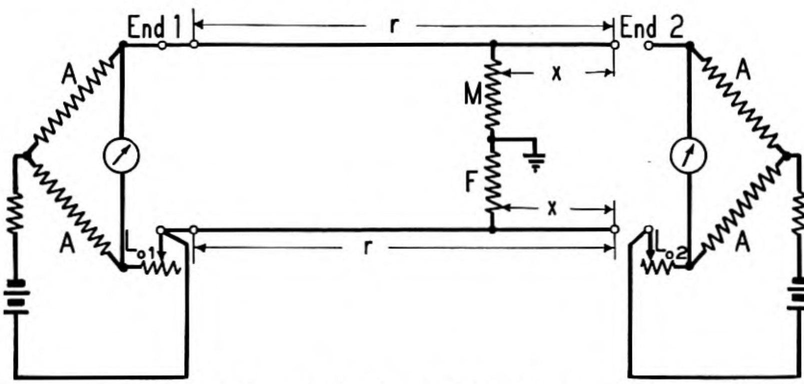


Fig. 9—Schematic circuit—straight resistance method.

The Straight Resistance method is based on the assumptions that the wires on which the tests are made are of equal conductor resistance, that the fault resistances are comparable in magnitude to the conductor resistances, and that the fault resistances remain constant while a set of measurements is being made.

Referring to Fig. 9, assume that, from among the faulty wires, two wires are selected having a fault of low effective resistance, $(M + F)$, between wires. Let r be the conductor resistance of either wire between cable Ends 1 and 2; and let $(r - x)$ and x be the conductor resistances of either wire from Ends 1 and 2, respectively, to the fault.

With the wires open at End 2, the resistance between wires is measured at End 1 by means of a bridge having equal ratio arms and arranged for an ordinary loop resistance measurement. Calling the rheostat resistance at balance, L_{01}:

$$L_{01} = 2(r - x) + (M + F).$$

Similarly, with the wires open at End 1, the resistance between wires is measured at End 2. Calling the rheostat resistance at balance, L_{02}:

$$L_{02} = 2x + (M + F).$$

Combining the equations for L_{01} and L_{02}:

$$L_{02} - L_{01} = 4x - 2r.$$

and therefore:

$$x = \frac{2r + (L_{02} - L_{01})}{4}, \tag{4}$$

$$(r - x) = \frac{2r - (L_{02} - L_{01})}{4}. \tag{5}$$

Application: The Straight Resistance method involves only simple resistance measurements, L_{01} and L_{02}, from the two ends of the cable. The loop conductor resistance of the faulty wires is obtained from cable records. The values thus secured are substituted in Formula (4) or (5), and the location, x or $(r - x)$, is converted into feet in the usual manner.

Since the conductor resistances of the faulty wires must be equal, measurements should be made on the two wires comprising a pair when practicable. The effective fault resistance between wires should be low; otherwise slight errors in measurement might cause large errors in calculated location. However, in cases where the fault resistances are too high to be measured using bridges with equal ratio arms, unequal arms, A and B, may be used and the quantity $\frac{A}{B}(L_{02} - L_{01})$ substituted for $(L_{02} - L_{01})$ in the formulas.

In connection with both the Corrected Varley method and the Straight Resistance method, it is possible to modify the measuring schemes and obtain somewhat more complicated formulas for the location of the faults. The specific measuring schemes which have been described are those which it is felt are most practicable for fault locating work on toll cable.

Insulation Faults of High Resistance

In order to locate faults of high resistance, sensitive galvanometers and highly insulated bridges must be employed, and the fault locating methods must correct for factors peculiar to the locating of such faults. If the resistance of the fault is high enough to be comparable in magnitude to the normal insulation resistance of the faulty wire, the effect of normal insulation resistance must be taken into account. In the case of a high resistance wet spot, it may happen that all wires in the cable are affected to some extent by the fault so that no wire of high insulation resistance compared to the selected faulty wire is available for measurements.

The solutions of the Varley networks for high resistance faults are more readily obtained by approximate than by exact mathematical reasoning, and will be worked out by the process of combining all of the "effective faults" on the wires into a single resultant fault and then solving the bridge network for this fault. The approximate solution is based on a principle which for the purposes of the present discussion can be stated as follows:

> Any two shunt faults of high resistance along a wire can be replaced by a single resultant shunt resistance located between the two faults at a point the distance of which from either fault is directly proportional to the fault resistances.

Thus, if M and F are the resistances of two faults at separated points along a wire, and m and f are their respective distances from the resultant, then:

$$\frac{M}{F} = \frac{m}{f}.$$

The application of this "Rule of Resultant Faults" to Varley measurements can be shown as follows: Let M and F be the effective resistances of the faults on two cable wires at the same point along the cable; let r be the conductor resistance of either wire between the cable ends, and x the resistance of that portion of either wire which is between End 2 of the cable and the faults. Let V be the value of balancing resistance for a Varley measurement made from End 1, using a bridge with equal ratio arms, as indicated in Fig. 10.

Applying the Rule of Resultant Faults, the apparent location of the faults as determined by the Varley measurement will be at a point between the two faults, and at a distance from either fault which is directly proportional to the fault resistances. Let c be the resistance

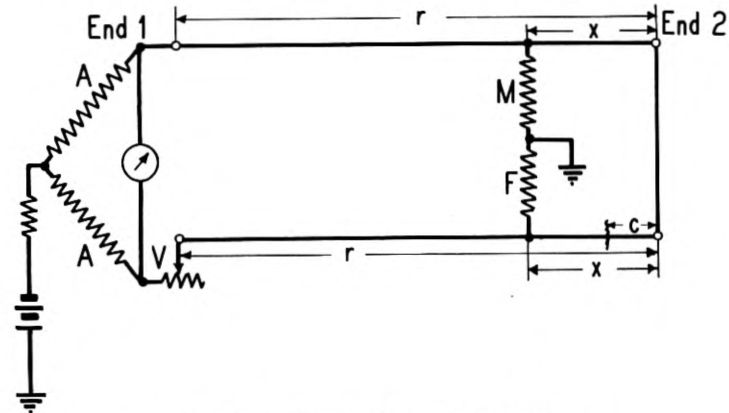


Fig. 10—Location of a resultant fault.

of the portion of the wire between the distant end of the cable and the apparent location. Then:

$$\frac{M}{F} = \frac{x + c}{x - c},$$

$$c = x\frac{M - F}{M + F}.$$

When the bridge is balanced for the Varley measurement:

$$c = \frac{V}{2}.$$

Equating the two values of c and solving for x:

$$x = \frac{V}{2}\frac{M + F}{M - F}. \qquad (6)$$

Comparison of Formula (6) with the more exact Formula (2) for the same case indicates that the Rule of Resultant Faults will give accurate results only if the fault resistances are high compared to the conductor resistances, and if M is of appreciably higher resistance than F.

If M equals F, the location will be indeterminate: The two faults will have no effect on the balance point of the bridge and V will be zero.

Double Varley Method [3]

The distributed normal insulation resistances of cable wires can be considered, in so far as fault locating measurements are concerned, as though they were single resistances concentrated at some point along the wires (Rule of Resultant Faults). Consider two wires having equal and correspondingly distributed normal insulation resistances, N, which appear to be concentrated at some point b ohms from End 2 of the wires, and assume faults of effective resistances, M and F, on the wires at a point x ohms from End 2. Let r be the conductor resistance of either wire, and V_1 and V_2 the balancing resistances for Varley measurements from Ends 1 and 2 of the wires, respectively, using bridges with equal ratio arms as indicated in Fig. 11.

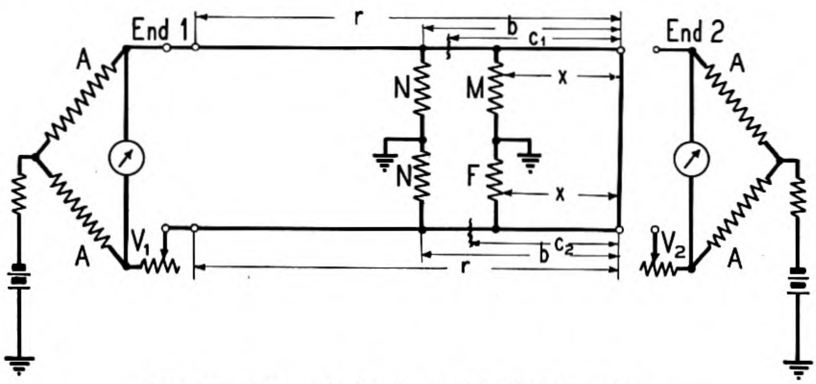


Fig. 11—Schematic circuit—double Varley method.

Applying the Rule of Resultant Faults, let c_1 be the apparent location, in ohms from End 2, of the resultant of M and N, and let c_2 be the corresponding location of the resultant of F and N. Then:

$$\frac{M}{N} = \frac{c_1 - x}{b - c_1},$$

$$c_1 = \frac{Mb + Nx}{M + N},$$

and correspondingly:

$$c_2 = \frac{Fb + Nx}{F + N}.$$

The equivalent resistance of the resultant of M and N is $MN/M + N$, and of the resultant of F and N is $FN/F + N$. Let c_3 be the apparent

[3] The Double Varley method has been described in "Cable Testing," a paper read by E. S. Ritter before the Nottingham Centre of the Institute of Post Office Electrical Engineers (British), May 25, 1922. In that paper it is stated that the method is due to Mr. H. T. Werren.

location, in ohms from End 2, of the resultant of these two resultants, as indicated in Fig. 12.

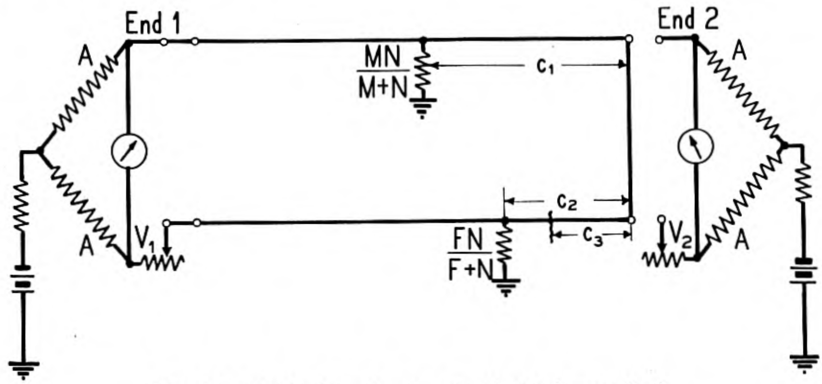


Fig. 12—Equivalent circuit—double Varley method.

Again applying the Rule of Resultant Faults:

$$c_3 = \frac{Nx(M - F)}{M(F + N) + F(M + N)}.$$

For the Varley measurement from End 1 of the cable:

$$c_3 = \frac{V_1}{2}.$$

Equating these two values of c_3 and solving for x:

$$x = \frac{V_1}{2}\left[\frac{M + F}{M - F} + \frac{2MF}{N(M - F)}\right]. \tag{7}$$

Likewise, for the Varley measurement from End 2 of the cable:

$$x = r - \left\{\frac{V_2}{2}\left[\frac{M + F}{M - F} + \frac{2MF}{N(M - F)}\right]\right\}. \tag{8}$$

By equating the two values of x found in (7) and (8), the value of the "correction factor" for the Varley measurements can be determined:

$$\frac{M + F}{M - F} + \frac{2MF}{N(M - F)} = \frac{2r}{V_1 + V_2}.$$

Substituting this value of the correction factor in Formula (7):

$$x = \frac{rV_1}{V_1 + V_2}. \tag{9}$$

Likewise, the resistance of one wire between End 1 of the cable and the faults is:

$$(r - x) = \frac{rV_2}{V_1 + V_2}. \tag{10}$$

Application: To apply the Double Varley method, ordinary Varley measurements, V_1 and V_2, are made from the two ends of the cable, using bridges with equal ratio arms, and the loop resistance, $2r$, of the wires is measured. The location, x or $(r - x)$, can be calculated from Formula (9) or (10), and then converted into feet in the usual manner.

Similarly, using the Rule of Resultant Faults, it can be shown that Formulas (9) and (10) also apply when only one of the wires used for Varley measurements is faulty. In this case the resistance, x, of the portion of the faulty wire between the distant end of the cable and the fault is:

$$x = \frac{V}{2} + V\frac{F}{N},$$

where V is the balancing resistance for a Varley measurement made from one end of the cable. This formula indicates that, where the ordinary Varley method (Figs. 5 and 6) is used, the insulation resistance of the "good" wire should be at least several hundred times as high as the fault resistance of the faulty wire. If this condition does not obtain the Double Varley method should be used. It will be clear, however, that the Double Varley method may be used, if desired, instead of the ordinary Varley method in cases where a wire of sufficiently high insulation resistance to be a "good" wire is available. In such cases the sum of the Varley balancing resistances obtained by measurements from the two ends of the cable will be equal to the loop resistance and Formula (9) will reduce to Formula (1).

The Double Varley method is workable only if the conductor resistances of the two wires used for measurements are equal. It can be shown that, if the conductor resistance of the wire having the fault, M, is r_m and that of the wire having the fault, F, is r_f, and if the normal insulation resistances of the wires are equal and uniformly distributed so that they may be regarded as concentrated at the middle of each wire, Formula (9) becomes:

$$x = r_f\left\{\frac{\frac{V_1}{2}[2MF + N(M + F)] + \frac{r_f - r_m}{2}[MF + N(M + F)]}{\frac{V_1 + V_2}{2}[2MF + N(M + F)] + (r_f - r_m)[MF + N(M + F)]}\right\}.$$

As indicated by the above discussion, the limitations of the Double Varley method are as follows:

1. There must be only one actual fault on any one cable wire.
2. The fault resistances must remain constant throughout a set of measurements to determine V_1 and V_2.
3. If both of the wires used for the Varley measurements are faulty, the faults must be at the same point on each wire, the resistances of the faults must be unequal, and the resistance of the fault on at least one of the wires must be high compared to the conductor resistance of the wire.
4. If the fault resistances are high enough to be comparable in magnitude to the normal insulation resistances of the faulty wires, the normal insulation resistances must be equal, and correspondingly distributed along the wires.
5. The conductor resistances of the wires must be equal.

It will be understood that since the Double Varley method is applicable only when the resistance of the fault, M, is high compared to the conductor resistances of the wires, the Corrected Varley method or the Straight Resistance method should be used in cases where M is comparable in magnitude to the conductor resistances.

Series Resistance Unbalances

The methods for locating series resistance unbalances discussed in this paper involve essentially the balancing of the faulty wire against a "good" wire of equal capacitance by adding resistance to the "good" wire at the testing end until the effective impedances of the two wires are equal. A simple relationship then exists between the balancing resistance required, the resistance of the fault, the length of the faulty wire between the distant end of the cable and the fault, and the total length of the faulty wire. The circuit arrangement used depends on whether the cable under test is long or short.

The circuit arrangement for applying the test to short cables is shown in Fig. 13.

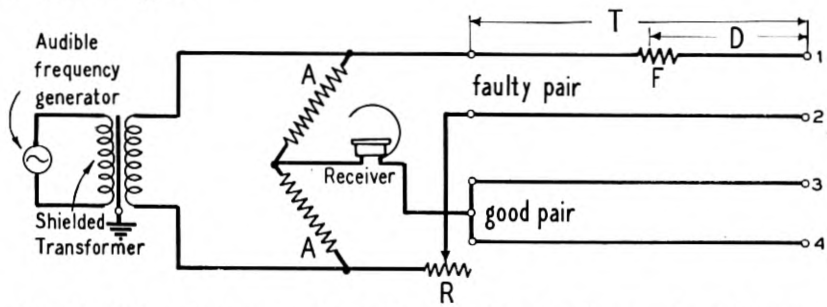


Fig. 13—Schematic circuit—short cable method for locating a series resistance unbalance.

The wires 1–2 and 3–4 form the pairs of a quad containing a series unbalance of resistance, F. The total length of the faulty wire is T, and the length of the portion of the faulty wire between the distant end of the cable and the fault is D. The bridge has equal ratio arms, A, and a balancing resistance, R. The audible frequency generator is a buzzer or other source of relatively low frequency current.

The bridge is balanced first with the distant ends of wires 1, 2, 3 and 4 open, and then with the distant ends of wires 1, 2, 3 and 4 connected together. The location of the unbalance from the distant end can be calculated from the formula:

$$D = T\sqrt{\frac{R_0}{R_c}},$$

where R_0 and R_c are the balancing resistances for the measurements with the distant end open and the distant end short-circuited, respectively. This test is suitable for use only on non-loaded cable, up to a few miles in length.

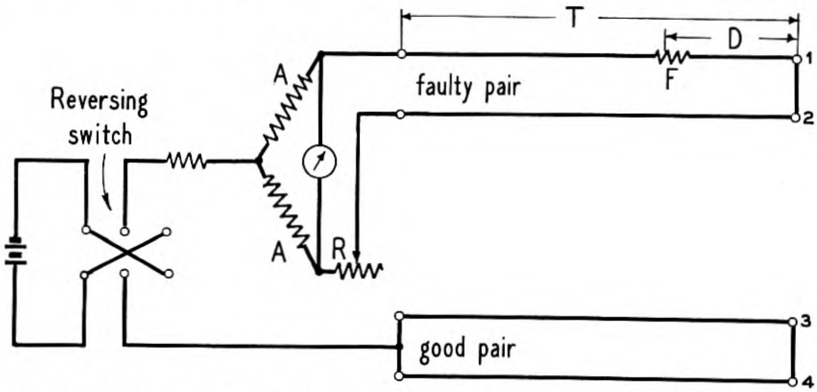


Fig. 14—Schematic circuit—long cable method for locating a series resistance unbalance.

The bridge arrangement for applying the test to long (either loaded or non-loaded) cables differs from that for short cables in that the wires of each pair, 1–2 and 3–4, are connected together at the distant end when measuring R_0, and a testing current of very low frequency is used. A battery, reversed either manually or by means of a motor-driven commutator, provides a satisfactory source of current, as indicated in Fig. 14.

With the wires of each pair, 1–2 and 3–4, connected together at the distant end as shown, the balancing resistance is adjusted to a value

R_0 at which no deflection of the galvanometer occurs when the battery is reversed. The two short-circuited pairs are then connected together at the distant end, the reversing switch is left in one position, and the rheostat is adjusted to a value R_c to balance the bridge. The location of the unbalance from the distant end is:

$$D = T\frac{R_0}{R_c}.$$

As will be clear from the following discussion, both the formula for the short cable method and that for the long cable method are based on the assumption that the wires under test are of short electrical length. Theoretically, either method could be used with cables of any physical length provided the testing frequency were chosen properly. The specific measuring schemes described here are well adapted to practical application, however.

Short Cable Method [4]

When the bridge measurement is made with the distant ends of wires 1, 2, 3 and 4 open, as shown in Fig. 13, the impedance of wire 1 to 3–4 is compared to the impedance of wire 2 to 3–4. Assume a

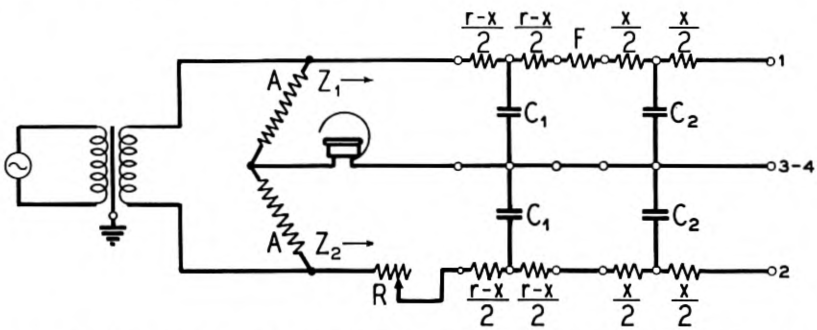


Fig. 15—Equivalent circuit—short cable method for locating a series resistance unbalance.

testing current of sufficiently low frequency that the wires are electrically short. Calling the capacitance and the conductor resistance of the length $(T - D)$ of each wire, C_1 and $(r - x)$, respectively, and of the length D of each wire, C_2 and x, respectively, the bridge circuit of Fig. 13 is practically equivalent to that of Fig. 15.

The impedance presented to the bridge terminals by the network

[4] The short cable method is described briefly in the paper, "Cable Testing," by E. S. Ritter, loc. cit.

containing F can be determined by inspection to be:

$$Z_1 = \frac{r-x}{2} + \frac{\dfrac{1}{j\omega C_1}\left[\dfrac{r}{2} + F + \dfrac{1}{j\omega C_2}\right]}{\dfrac{r}{2} + F + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}},$$

where j is the operator $\sqrt{-1}$ and ω is 2π times the testing frequency.

Likewise, the impedance presented to the bridge terminals by the network containing R is:

$$Z_2 = R + \frac{r-x}{2} + \frac{\dfrac{1}{j\omega C_1}\left[\dfrac{r}{2} + \dfrac{1}{j\omega C_2}\right]}{\dfrac{r}{2} + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}}.$$

When the bridge is balanced, these two impedances are equal, so that:

$$\frac{r-x}{2} + \frac{\dfrac{1}{j\omega C_1}\left[\dfrac{r}{2} + F + \dfrac{1}{j\omega C_2}\right]}{\dfrac{r}{2} + F + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}} = R_0 + \frac{r-x}{2} + \frac{\dfrac{1}{j\omega C_1}\left[\dfrac{r}{2} + \dfrac{1}{j\omega C_2}\right]}{\dfrac{r}{2} + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}}.$$

This equation reduces to:

$$\left[\frac{r}{2} + F + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right]\left[\frac{r}{2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right] = \left[\frac{1}{j\omega C_1}\right]^2 \frac{F}{R_0}.$$

For a testing current of relatively low frequency the capacitive reactances, $1/j\omega C_1$ and $1/j\omega C_2$, are much larger than the resistances, r and F, and the above equation can be written as follows, the symbol $\doteqdot$ being used to denote "is practically equal to":

$$\left[\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right]^2 \doteqdot \left[\frac{1}{j\omega C_1}\right]^2 \frac{F}{R_0},$$

$$\sqrt{\frac{R_0}{F}} \doteqdot \frac{C_2}{C_1 + C_2}.$$

Since C_2 is proportional to the length D and $(C_1 + C_2)$ to the total length, T:

$$\sqrt{\frac{R_0}{F}} \doteqdot \frac{D}{T}.$$

When the bridge is balanced to the value R_c, with the distant ends of wires 1, 2, 3 and 4 connected together, the amount of unbalance between wires 1 and 2 is measured. Assuming that F is the only unbalance present, and that the conductor resistances of wires 1 and 2 are equal:

$$R_c = F$$

and therefore:

$$D \doteq T\sqrt{\frac{R_0}{R_c}}. \tag{11}$$

Application: It will be clear from the above theory that Formula (11) will give accurate results only if the following requirements are met:

1. The resistance, F, must be the only unbalance on the wires.
2. The resistance of the unbalance must remain constant throughout a set of measurements to determine R_0 and R_c.
3. The conductor resistances of wires 1 and 2 must be equal.
4. The capacitive reactances of wires 1 and 2 to 3–4 must be large as compared to the conductor resistances of the wires and the fault resistance.
5. Capacitance unbalances of wires 1 and 2 to 3–4 must be negligible.

In general, the short cable method is suitable for locating, with a fair degree of accuracy, series resistance unbalances ranging from a few ohms to several hundred ohms on non-loaded cable not exceeding three or four miles in length. In cases of unbalances of only a few ohms resistance, however, it is essential that the wires of the faulty quad be very well balanced in conductor resistance; and the bridge rheostat should be variable in steps of 0.1 ohm. Usually, best results are secured when measurements are made from the cable end nearer the fault.

The bridge voltage used should be as small as practicable in order to minimize changes in fault resistance. A sufficient number of separate determinations of the location should be made to insure that consistent results are being secured.

The measurement with the distant ends of wires 1, 2, 3 and 4 connected together is made merely to obtain the actual value of fault resistance. The value of fault resistance can be obtained instead by a d.-c. Varley measurement, if desired. If this is done, however, arrangements should be made so that the bridge connections can be changed rapidly, as it is desirable to make measurements of R_0 and R_c in quick succession to avoid errors due to changing fault resistance.

The short cable method is applicable to paired cable as well as to quadded cable. In the case of paired cable, ground may be substituted for wires 3–4, and measurements made of impedance to ground rather than of impedance between wires. Usually in these circumstances, however, the bridge cannot be balanced very sharply.

Long Cable Method [5]

Referring to Fig. 14, assume that the wires under test are non-loaded and that a testing current of very low frequency is used so that the wires are electrically short. Calling the capacitance and the

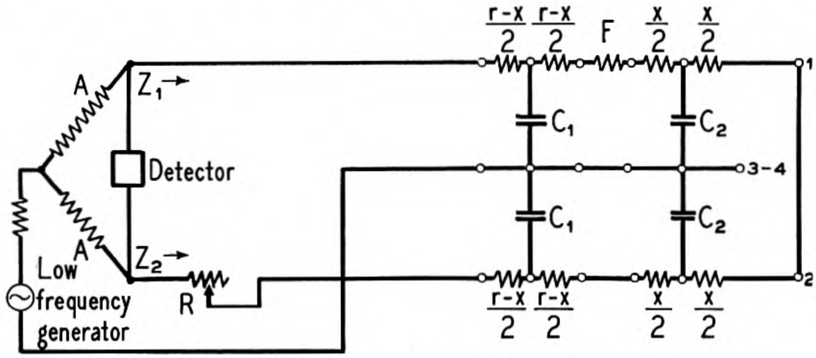


Fig. 16—First equivalent circuit—long cable method for locating a series resistance unbalance.

conductor resistance of the length $(T - D)$ of each wire, C_1 and $(r - x)$, respectively, and of the length D of each wire, C_2 and x, respectively, the bridge circuit of Fig. 14 is practically equivalent to that of Fig. 16.

When the bridge is balanced so that there is no current through the detector, the impedance Z_1 looking into the upper branch of the net-

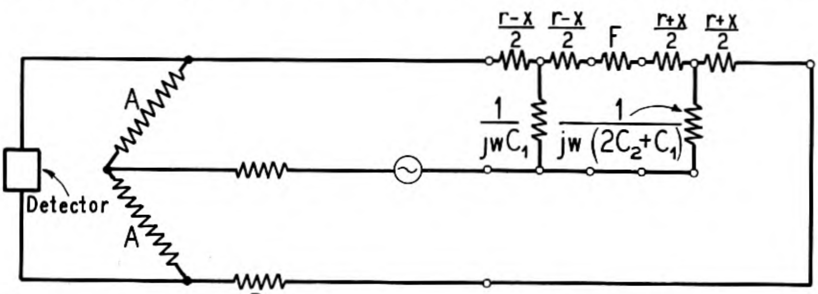


Fig. 17—Second equivalent circuit—long cable method for locating a series resistance unbalance.

work must be equal to the impedance Z_2 looking into the lower branch. At the balance point the bridge circuit is practically equivalent to that shown in Fig. 17, in which the network up to the point of fault, as seen from the bridge terminals of the lower branch, is replaced by a single resistance-capitance network.

[5] Credit for the long cable method is given to Capt. F. Reid in the paper, "Cable Testing," by E. S. Ritter, loc. cit.

The network of Fig. 17 can be replaced by the equivalent network of Fig. 18. The values of the impedances h, k and p of Fig. 18 are:

$$h = \frac{r-x}{2} + \frac{\frac{1}{j\omega C_1}(r+F)}{\frac{1}{j\omega C_1} + \frac{1}{j\omega(2C_2+C_1)} + r + F},$$

$$k = \frac{\frac{1}{j\omega C_1}\left[\frac{1}{j\omega(2C_2+C_1)}\right]}{\frac{1}{j\omega C_1} + \frac{1}{j\omega(2C_2+C_1)} + r + F},$$

$$p = \frac{r+x}{2} + R_0 + \frac{\frac{1}{j\omega(2C_2+C_1)}(r+F)}{\frac{1}{j\omega C_1} + \frac{1}{j\omega(2C_2+C_1)} + r + F}.$$

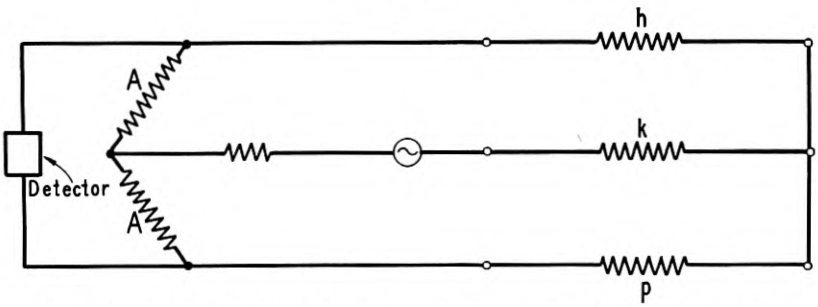


Fig. 18—Third equivalent circuit—long cable method for locating a series resistance unbalance.

It is evident from inspection of Fig. 18 that if h equals p the network is balanced so that there is no current through the detector. Equating the values of h and p, and solving gives:

$$\frac{R_0}{F} = \frac{\left[\frac{1}{j\omega C_1} - \frac{1}{j\omega(2C_2+C_1)}\right] + \frac{r}{F}\left[\frac{1}{j\omega C_1} - \frac{1}{j\omega(2C_2+C_1)}\right]}{\frac{1}{j\omega C_1} + \frac{1}{j\omega(2C_2+C_1)} + r + F} - \frac{x}{F}.$$

If the capacitive reactances of the wires are very high compared to the conductor resistances and the fault resistance, this last equation can be reduced to:

$$\frac{R_0}{F} \doteqdot \frac{C_2}{C_1+C_2} + \frac{r}{F}\left[\frac{C_2}{C_1+C_2}\right] - \frac{x}{F},$$

and since, for a testing current of very low frequency, C_2 and x are proportional to D, while $(C_1 + C_2)$ and r are proportional to T:

$$r\left[\frac{C_2}{C_1 + C_2}\right] = x,$$

and we may write:

$$\frac{R_0}{F} \doteq \frac{D}{T}.$$

When the bridge is balanced to the value R_c with wires 1, 2, 3 and 4 connected together at the distant end, the amount of unbalance between wires 1 and 2 is measured. Assuming that F is the only unbalance present, and that the conductor resistances of wires 1 and 2 are equal:

$$R_c = F$$

and therefore:

$$D \doteq \frac{R_0}{R_c} T. \tag{12}$$

Application: The same general requirements set down for the short cable method must be met to secure accurate results with the long cable method. While Formula (12) has been developed specifically for non-loaded cable, it is clear that it applies also to loaded cable, provided the effective series impedances of the wires, including the loading coils, are very low compared to the effective shunt impedances of the wires. A testing frequency of three or four cycles per second is sufficiently low to satisfy this requirement on telephone cables up to a repeater section in length. If, however, the cable is only a few miles in length, the effective sensitivity of the bridge may be too low for satisfactory results.

In general, the long cable method is suitable for locating, with reasonable accuracy, series resistance unbalances ranging from about 10 ohms to several thousand ohms. A well insulated bridge and a fairly sensitive galvanometer are desirable, especially when working with faults of low resistance.

An essential requisite for accurate results is that the resistance of the fault remain constant while a set of measurements to determine R_0 and R_c is being made. In the application of the method, therefore, the bridge voltage used should be as low as practicable. Bridge voltages of, say, 100 volts for measuring R_0 and six volts or less for measuring R_c are usually satisfactory. In this connection it can be pointed out that if measurements R_{01} and R_{02} are made from the two ends of the cable it is unnecessary to measure R_c since $(R_{01} + R_{02})$ will equal F

and Formula (12) can then be written:

$$D \doteqdot T \frac{R_{01}}{R_{01} + R_{02}}.$$

In cases where the fault resistance appears to be affected appreciably by the testing current this scheme of measuring may be found desirable.

It has been found that, when a battery and manually operated battery reversing switch are used and the balance point of the bridge is determined by observing the galvanometer kicks as the battery is reversed, the action of the galvanometer is somewhat as follows: For settings appreciably below the balance point the galvanometer kicks are definitely in one direction while for settings which are too high the kicks are definitely in the opposite direction (assuming, of course, that the polarity of the battery is taken into account). When the rheostat setting is very close to the point of balance but slightly too low, the galvanometer gives a quick double kick, i.e., the needle moves away from galvanometer zero, then returns toward zero a short distance and again moves away from zero. When the rheostat setting is slightly too high, the galvanometer gives a single kick and then coasts toward the end of the scale. The balance point of the bridge is where the transition from double to single kick occurs.

When the value of R_0 is low a rheostat variable in steps of 0.1 ohm may be necessary if the transition point is to be accurately obtained.

Seasoned judgment is an essential adjunct to a knowledge of theory in the practical application of fault locating methods. This is especially true in the case of methods such as those discussed here, with which accurate results cannot be secured unless the fault resistances remain constant in value while a set of measurements to determine location is being made. Experience has indicated that cable faults of the types discussed are apt to be inconstant in resistance. Great care must be exercised, therefore, in interpreting the results of measurements. It is very important to make a sufficient number of separate sets of measurements to insure that consistent data are being obtained.

Mutual Impedance of Grounded Wires Lying on the Surface of the Earth *

By RONALD M. FOSTER

This paper presents a formula for the mutual impedance between two insulated wires of negligible diameter lying on the surface of the earth and grounded at their end-points. The formula holds for frequencies which are not too high to allow all displacement currents to be neglected. For any two elements dS, ds of the two wires the mutual impedance is obtained from their direct-current mutual impedance by introducing the complex factor $2(\gamma r)^{-2}[1 - (1 + \gamma r)e^{-\gamma r}]$ in the reactance term, γ being the propagation constant in the earth, and r the distance between the elements dS and ds.

THE mutual impedance of grounded circuits may be derived from certain results obtained by A. Sommerfeld,[1] who has developed formulæ for the electric and magnetic fields in the earth and in the air due to horizontal and vertical electric and magnetic antennæ situated at the surface of the earth. For our present problem we use his formulæ for the electric field in the earth due to a horizontal electric doublet, since this doublet may be regarded as a short element dS of a wire of negligible diameter carrying a finite current. At the end of this present paper we shall show how the same formula for the mutual impedance may be obtained directly from first principles.

Sommerfeld uses rectangular coordinates (x, y, z) and the corresponding cylindrical coordinates (r, ϕ, z), the surface of the earth, assumed flat, being the xy plane, and the z axis extending upward into the air. The doublet is at the origin, and its axis along the x axis. Then the components of the Hertzian vector [2] in the earth ($z < 0$) from which the electric field is determined are [3]

$$\Pi_x = C\frac{k_0^2}{k^2}\int_0^\infty \frac{J_0(\rho r)}{N'}e^{z\sqrt{\rho^2-k^2}}\rho d\rho, \tag{1}$$

$$\Pi_y = 0, \tag{2}$$

* Presented by title at the Eugene, Oregon meeting of the American Mathematical Society, June 20, 1930, as "Mutual Impedances of Grounded Circuits."

[1] A. Sommerfeld, "Über die Ausbreitung der Wellen in der drahtlosen Telegraphie," *Annalen der Physik*, (4), **81**, 1135–1153 (December 1926). This paper is a summary and an extension of earlier work by Sommerfeld and von Hoerschelmann, references to which will be found in the paper.

[2] H. Abraham and A. Föppl, "Theorie der Elektrizität," 5th ed., Leipzig and Berlin, 1918; Vol. I, § 79, page 331.

[3] A. Sommerfeld, *loc. cit.*, pages 1145 and 1146, introducing the constant factor defined on page 1152.

$$\Pi_z = C(k^2 - k_0^2)\frac{k_0^2}{k^2}\cos\phi\int_0^\infty \frac{J_0'(\rho r)}{NN'}e^{z\sqrt{\rho^2-k^2}}\rho^2 d\rho, \tag{3}$$

where the time factor $e^{-i\omega t}$ is omitted throughout. J_0 is the Bessel function of order zero, and the constants k and k_0 are the propagation constants in the earth and in the air for plane waves varying with the time as $e^{-i\omega t}$. Their values in Heaviside units are given by Sommerfeld as

$$k^2 = \frac{1}{c^2}(\epsilon\omega^2 + i\sigma\omega), \qquad k_0^2 = \frac{1}{c^2}\epsilon_0\omega^2, \tag{4}$$

where ϵ and ϵ_0 are the dielectric constants of the earth and of the air, respectively, σ is the conductivity of the earth, assumed uniform, and c is the velocity of light. In both media the permeability is taken as unity. Also

$$N = k^2\sqrt{\rho^2 - k_0^2} + k_0^2\sqrt{\rho^2 - k^2}, \tag{5}$$

$$N' = \sqrt{\rho^2 - k_0^2} + \sqrt{\rho^2 - k^2}, \tag{6}$$

and C is a constant measuring the electric moment of the doublet.

We now replace the doublet by a short element of wire dS carrying a current $Ie^{i\omega t}$, and at the same time we assume that ϵ and ϵ_0 are both negligible, so that all displacement currents are neglected. This is a simplification which is ordinarily made as a first approximation at power frequencies for the shorter transmission lines. Then, introducing c.g.s. electromagnetic units, in which the conductivity of the earth is λ, and noting that we have changed the sign of ω, formulæ (4)–(6) become

$$k^2 = -i4\pi\lambda\omega = -\gamma^2, \tag{7}$$

$$k_0^2 = 0, \tag{8}$$

$$N = -\gamma^2\rho, \tag{9}$$

$$N' = \rho + \sqrt{\rho^2 + \gamma^2}, \tag{10}$$

and the constant C is such that

$$\frac{Ck_0^2}{k^2} = \frac{1}{\sigma}\times \text{current} \times \text{effective length of doublet} \tag{11}$$

$$= \frac{IdS}{2\pi\lambda}.$$

Substituting from (7)–(11) in (1)–(3) we have, therefore,

$$\text{(12)} \qquad \Pi_x = \frac{IdS}{2\pi\lambda}\int_0^\infty \frac{J_0(r\rho)}{\rho + \sqrt{\rho^2+\gamma^2}}\epsilon^{z\sqrt{\rho^2+\gamma^2}}\rho d\rho = \frac{IdS}{2\pi\lambda\gamma^2}\left(\frac{\partial^2 P}{\partial z^2} + \frac{\partial^3 Q}{\partial x^2 \partial z} + \frac{\partial^3 Q}{\partial y^2 \partial z}\right),$$

$$\text{(13)} \qquad \Pi_y = 0,$$

$$\text{(14)} \qquad \Pi_z = \frac{IdS}{2\pi\lambda}\frac{\partial}{\partial x}\int_0^\infty \frac{J_0(r\rho)}{\rho + \sqrt{\rho^2+\gamma^2}}\epsilon^{z\sqrt{\rho^2+\gamma^2}}d\rho = -\frac{IdS}{2\pi\lambda\gamma^2}\left(\frac{\partial^2 P}{\partial x \partial z} - \frac{\partial^3 Q}{\partial x \partial z^2}\right),$$

where

$$\text{(15)} \qquad P = \int_0^\infty J_0(r\rho)\epsilon^{z\sqrt{\rho^2+\gamma^2}}\frac{\rho d\rho}{\sqrt{\rho^2+\gamma^2}} = \frac{1}{R}e^{-\gamma R},$$

and

$$\text{(16)} \qquad Q = \int_0^\infty J_0(r\rho)\epsilon^{z\sqrt{\rho^2+\gamma^2}}\frac{d\rho}{\sqrt{\rho^2+\gamma^2}} = I_0[\tfrac{1}{2}\gamma(R+z)]K_0[\tfrac{1}{2}\gamma(R-z)],$$

with $R^2 = r^2 + z^2$.

The integral P is well known,[4] while Q is evaluated by a suitable transformation of a Fourier integral.[5] $I_0(z) = J_0(iz)$ and $K_0(z) = \frac{1}{2}\pi i H_0^{(1)}(iz)$ are the Bessel functions of the first and second kinds for imaginary arguments as defined by G. N. Watson.[6] In reducing Π_x to this form we use the differential equation [7] for J_0 to obtain the relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)J_0(r\rho) + \rho^2 J_0(r\rho) = 0.$$

The components of the electric force in the earth are obtained from Π by the formula

$$\text{(17)} \qquad E = \text{grad div } \Pi - \gamma^2 \Pi,$$

[4] See *e.g.* H. Bateman, "Electrical and Optical Wave-Motion," Cambridge, 1915, page 72; or G. N. Watson, "Theory of Bessel Functions," Cambridge, 1922, page 416, formula (2) of § 13.47, with $\mu = 0$ and $\nu = \frac{1}{2}$.

[5] G. A. Campbell, "The Practical Application of the Fourier Integral," *Bell System Technical Journal*, **7**, 639–707; using pair 936 of Table I, with $\alpha = \frac{1}{2}$, substituting x^2 for $(g^2 - 4)$ in the integral of G, and generalizing the resulting integral to include complex quantities.

[6] G. N. Watson, *op. cit.*, pages 77, 78.

[7] G. N. Watson, *op. cit.*, page 19, formula (1) of § 2.13.

and we thus obtain E_x, E_y, E_z in the compact form

$$(18) \qquad (E_x, E_y, E_z) = \frac{IdS}{2\pi\lambda}\left(-\frac{\partial^3 Q}{\partial y^2 \partial z} - \frac{\partial^2 P}{\partial z^2},\ \frac{\partial^3 Q}{\partial x \partial y \partial z},\ \frac{\partial^2 P}{\partial x \partial z}\right),$$

where P and Q are given by (15) and (16). In deriving this form we use the fact that Q satisfies the wave equation

$$\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2} - \gamma^2 Q = 0.$$

At the surface of the earth ($z = 0$) the electric force takes the simple form

$$(19) \qquad (E_x, E_y) = \frac{IdS}{2\pi\lambda}\left[-\frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) + \frac{1 + \gamma r}{r^3} e^{-\gamma r},\quad \frac{\partial^2}{\partial x \partial y}\left(\frac{1}{r}\right)\right],$$

where we have used the expressions for the derivatives [8] of the Bessel functions, $I_0'(z) = I_1(z)$, $K_0'(z) = -K_1(z)$, and also the identity [9] $I_0(z)K_1(z) + I_1(z)K_0(z) = 1/z$.

The mutual impedance dZ_{12} between two infinitesimal elements dS and ds is now written down as the ratio of the resulting electric force in one element to the current in the other, with sign reversed:

$$\begin{aligned}(20) \quad dZ_{12} &= \frac{dSds}{2\pi\lambda}\left[\cos\epsilon \frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) - \cos\epsilon \frac{1+\gamma r}{r^3} e^{-\gamma r} \right. \\ &\qquad\qquad \left. - \sin\epsilon \frac{\partial^2}{\partial x \partial y}\left(\frac{1}{r}\right)\right] \\ &= \frac{dSds}{2\pi\lambda}\left[\frac{3\sin\Phi\sin\phi - \cos\epsilon}{r^3} - \frac{\cos\epsilon}{r^3}(1+\gamma r)e^{-\gamma r}\right] \\ &= \frac{dSds}{2\pi\lambda}\left\{\frac{d^2}{dSds}\left(\frac{1}{r}\right) + \frac{\cos\epsilon}{r^3}[1 - (1+\gamma r)e^{-\gamma r}]\right\},\end{aligned}$$

where Φ and ϕ are the angles which the elements dS and ds make with r, and $\epsilon = \Phi - \phi$ is the angle they make with each other.

Integration over the two wires S and s gives a general formula for the mutual impedance of grounded wires lying on the surface of the earth:

$$\begin{aligned}(21) \quad Z_{12} &= \frac{1}{2\pi\lambda}\int\int\left\{\frac{d^2}{dSds}\left(\frac{1}{r}\right) + \frac{\cos\epsilon}{r^3}[1 - (1+\gamma r)e^{-\gamma r}]\right\} dSds \\ &= \int\int\left[\frac{1}{2\pi\lambda}\cdot\frac{d^2}{dSds}\left(\frac{1}{r}\right)\right. \\ &\qquad\qquad \left. + i\omega\frac{\cos\epsilon}{r}\left\{\frac{2}{(\gamma r)^2}[1 - (1+\gamma r)e^{-\gamma r}]\right\}\right] dSds.\end{aligned}$$

[8] G. N. Watson, *op. cit.*, page 79, formula (7) of § 3.71.

[9] G. N. Watson, *op. cit.*, page 80, formula (20) of § 3.71, with $\nu = 0$.

The factor

$$\frac{2}{(\gamma r)^2}[1 - (1 + \gamma r)e^{-\gamma r}] \tag{22}$$

approaches unity as ω approaches zero, and Z_{12} then agrees with the

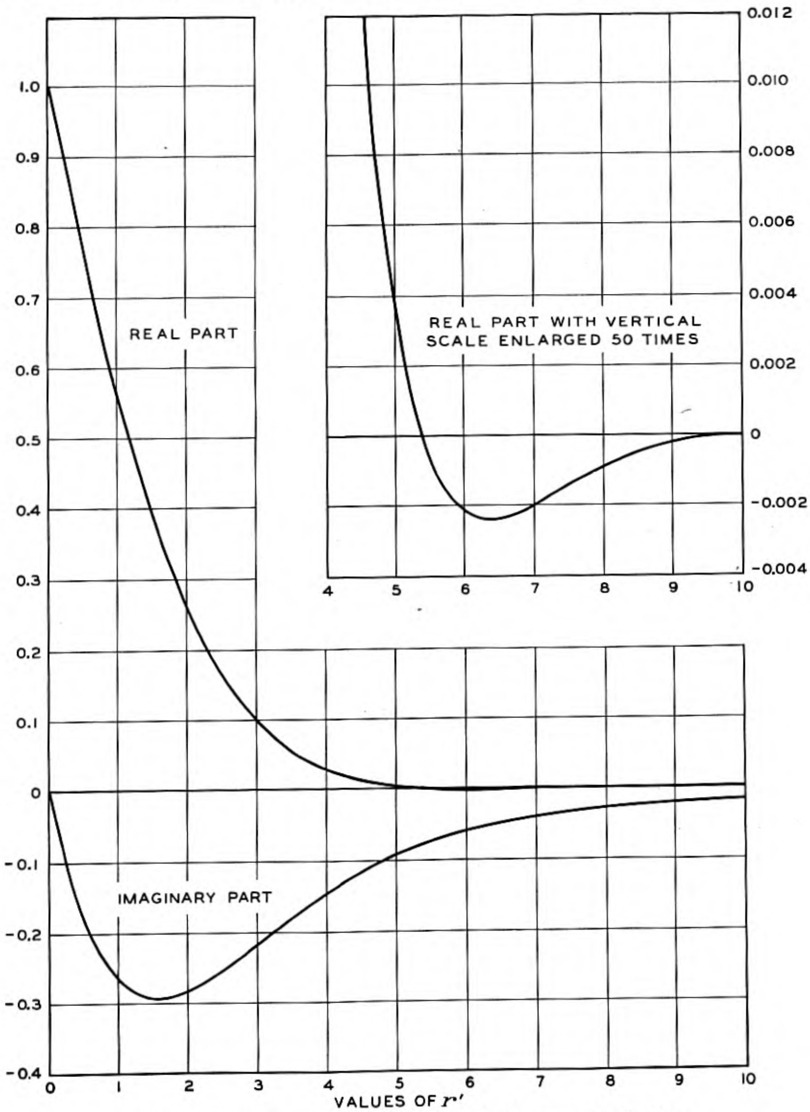


Fig. 1—Real and imaginary parts of the complex factor,

$$\frac{2}{(\gamma r)^2}[1 - (1 + \gamma r)e^{-\gamma r}],$$

plotted as functions of $r' = |\gamma r| = (4\pi\lambda\omega)^{1/2}r$.

direct-current mutual impedance as given by G. A. Campbell.[10] Introducing this factor, which is a function of γr only, into the reactance term for the direct-current mutual impedance between two elements dS and ds gives the general expression for their mutual impedance corresponding to the propagation constant γ. It is interesting also to determine, for any given value of γ, the variation of the factor (22) for increasing values of r. This is shown very clearly in Fig. 1, where the real and imaginary parts of (22) are plotted for increasing values of $r' = |\gamma r| = (4\pi\lambda\omega)^{1/2}r$. The real part, we note, decreases rapidly from the initial value unity as r' increases, while the imaginary part is always negative, decreasing from zero to a minimum value (approximately -0.3 for $r' = 1.5$) and then increasing towards zero, although it does not approach zero so rapidly as the real part does.

The first three terms in the expansion of Z_{12} for low frequencies are given by

$$(23)\quad Z_{12} = \frac{1}{2\pi\lambda}\left(\frac{1}{Aa} - \frac{1}{Ab} - \frac{1}{Ba} + \frac{1}{Bb}\right) + i\omega N_{Ss} + (1 - i)\tfrac{1}{3}(8\pi\lambda\omega^3)^{1/2}AB\,ab\cos\theta + \cdots,$$

where N_{Ss} is the mutual Neumann integral between the two wires S and s of arbitrary form but with end-points A, B and a, b respectively; θ is the angle between the straight lines AB and ab. The first two terms in this expansion are precisely the direct-current mutual impedance as given by G. A. Campbell.

The first term in the expansion of Z_{12} for a long straight wire S and any wire s located near the midpoint of S is

$$(24)\qquad \int \left[\frac{1}{\pi\lambda x^2} - \frac{\gamma}{\pi\lambda x}K_1(\gamma x)\right]\cos\epsilon\,ds,$$

x being the positive distance from ds to S, and ϵ the angle between ds and S. $K_1(z) = -\frac{1}{2}\pi H_1^{(1)}(iz)$ is the Bessel function of the second kind for imaginary argument as defined by G. N. Watson.[11] In obtaining (24) from (21) we use the derivative with respect to x of the integral

$$\int_0^\infty \frac{e^{-\gamma r}}{r}\,dz = K_0(\gamma x),$$

which is a special case of the integral used above in evaluating Q, with x assumed positive.

[10] G. A. Campbell, "Mutual Impedances of Grounded Circuits," *Bell System Technical Journal*, **2**, 1–30 (October 1923).

[11] G. N. Watson, *op. cit.*, page 78.

The expression in square brackets in (24) is the mutual impedance gradient parallel to an infinite wire at a positive distance x from the wire. It agrees with the results published independently by F. Pollaczek,[12] J. R. Carson,[13] and G. Haberland,[14] and has been employed by us to obtain numerical results since 1917. Pollaczek has also investigated the case of two gounded circuits of finite length.[15]

The mutual impedance dZ_{12} between a short grounded circuit dS and a counterclockwise small loop of area da, on the surface of the earth, is given by the formula

$$dZ_{12} = \frac{dSda}{2\pi\lambda} \cdot \frac{\sin\phi}{r^4}[3 - (3 + 3\gamma r + \gamma^2 r^2)e^{-\gamma r}], \tag{25}$$

where ϕ is the angle which dS makes with r, the line from da to dS. This may be obtained from Sommerfeld's formulæ for the horizontal electric force due to a vertical magnetic antenna, or it may be obtained by an application of Stokes's theorem to formula (20) above.

By a further application of Stokes's theorem we may obtain the mutual impedance between two counterclockwise small loops dA and da, namely,

$$dZ_{12} = \frac{dAda}{2\pi\lambda} \cdot \frac{1}{r^5}[(9 + 9\gamma r + 4\gamma^2 r^2 + \gamma^3 r^3)e^{-\gamma r} - 9]. \tag{26}$$

This result might also be derived from Sommerfeld's formula for the vertical magnetic force due to a vertical magnetic antenna.

We shall now indicate briefly how the same value of E as given in (18) above may be obtained directly, though more laboriously, from first principles. In this method we start from the fundamental solution [16]

$$u = e^{lx+my+nz}e^{i\omega t} \tag{27}$$

of the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \gamma^2 u = 0, \tag{28}$$

[12] F. Pollaczek, "Über das Feld einer unendlich langen wechselstromdurchflossenen Einfachleitung," *Elektrische Nachrichten-technik*, **3**, 339–359 (September 1926).

[13] J. R. Carson, "Wave Propagation in Overhead Wires with Ground Return," *Bell System Technical Journal*, **5**, 539–554 (October 1926).

[14] G. Haberland, "Theorie der Leitung von Wechselstrom durch die Erde," *Zeitschrift für angewandte Mathematik und Mechanik*, **6**, 366–379 (October 1926).

[15] F. Pollaczek, "Gegenseitige Induktion zwischen Wechselstromfreileitungen von endlicher Länge," *Annalen der Physik*, (4), **87**, 965–999 (December 1928). His assumptions regarding conditions at the ground connections seem to depart considerably from the conditions assumed in the present paper, and moreover his results are not expressed in convenient form for direct comparison with the formula given above for Z_{12}.

[16] H. Bateman, *op. cit.*, § 4, pages 6, 7; § 11, page 26.

which is satisfied by the electric force in the earth; $\gamma = (i4\pi\lambda\omega)^{1/2}$ is the propagation constant for plane waves which vary with the time as $e^{i\omega t}$. The parameters l, m, n satisfy the relation

$$l^2 + m^2 + n^2 - \gamma^2 = 0. \tag{29}$$

In the air, the same equations hold, but with the propagation constant γ equal to zero, and we note that the solution in the air must be chosen to vanish at an infinite height, while in the earth the solution must vanish at an infinite depth.

For convenience in this method we start with a short straight wire of length $2a$ lying along the x axis, later allowing a to approach zero. Thus we suppose that the current $Ie^{i\omega t}$ enters the earth at the point $(a, 0, 0)$ and leaves it at the point $(-a, 0, 0)$. The factor $e^{i\omega t}$ will be omitted, however, throughout the following work. The current flow in this system is symmetrical with respect to the vertical plane through the wire, the xz plane, and is also symmetrical, but with sign reversed, with respect to the vertical plane normal to the wire at its midpoint, the yz plane. Then if we replace the three parameters l, m, n of (27) by two independent parameters μ, ν, such that

$$l = \pm\, i\mu, \qquad m = \pm\, i\nu, \qquad n = \pm\sqrt{\mu^2 + \nu^2 + \gamma^2}, \tag{30}$$

formula (29) is identically satisfied, and we can then replace the four solutions $e^{\pm i\mu x \pm i\nu y}$ by their corresponding expressions in terms of sines and cosines, namely,

$$\sin x\mu \sin y\nu, \qquad \sin x\mu \cos y\nu, \qquad \cos x\mu \sin y\nu, \qquad \cos x\mu \cos y\nu.$$

The above considerations of symmetry will eliminate, for each component of the electric force, all but one of these forms. With the remaining solution as a basis we build up, by means of the Fourier integral, a general expression for any possible steady harmonic oscillation. Hence we may write down the general solutions for the total electric force in the earth ($z < 0$), as follows.

$$E_x = \int_0^\infty \int_0^\infty F_x(\mu, \nu) e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \cos x\mu \cos y\nu \, d\mu d\nu, \tag{31}$$

$$E_y = \int_0^\infty \int_0^\infty F_y(\mu, \nu) e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \sin x\mu \sin y\nu \, d\mu d\nu, \tag{32}$$

$$E_z = \int_0^\infty \int_0^\infty F_z(\mu, \nu) e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \sin x\mu \cos y\nu \, d\mu d\nu, \tag{33}$$

where the positive sign is chosen in the exponential term containing z since the solution must vanish at an infinite depth, z being negative in the earth; and that value of the radical is taken which has a positive real part. F_x, F_y, F_z are arbitrary functions of their arguments, to be determined by the physical conditions of the problem.

In the air $(0 < z)$ we may formulate the corresponding solutions for the total electric force as

$$E_x = \int_0^\infty \int_0^\infty P_x(\mu, \nu) e^{-z\sqrt{\mu^2+\nu^2}} \cos x\mu \cos y\nu \, d\mu d\nu, \tag{34}$$

$$E_y = \int_0^\infty \int_0^\infty P_y(\mu, \nu) e^{-z\sqrt{\mu^2+\nu^2}} \sin x\mu \sin y\nu \, d\mu d\nu, \tag{35}$$

$$E_z = \int_0^\infty \int_0^\infty P_z(\mu, \nu) e^{-z\sqrt{\mu^2+\nu^2}} \sin x\mu \cos y\nu \, d\mu d\nu, \tag{36}$$

where the propagation constant is zero in the air; the negative sign is chosen in the exponential term containing z since the solution must vanish at an infinite height, z being positive in the air; and P_x, P_y, P_z are arbitrary functions of their arguments.

To determine these six arbitrary functions we need six independent relations among them. Two of these relations are obtained by utilizing the fact that the divergence of the electric force either in the earth or in the air is equal to zero, that is,

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0.$$

By means of this we obtain from (31)–(33),

$$- \mu F_x + \nu F_y + \sqrt{\mu^2 + \nu^2 + \gamma^2} F_z = 0, \tag{37}$$

and from (34)–(36),

$$- \mu P_x + \nu P_y - \sqrt{\mu^2 + \nu^2} P_z = 0. \tag{38}$$

Since the horizontal components of the electric force are continuous at the surface of the earth $(z = 0)$ we see that we must also have, from (31) and (34),

$$F_x = P_x, \tag{39}$$

and from (32) and (35),

$$F_y = P_y. \tag{40}$$

We may obtain a fifth relation from the fact that the current I flows through the earth from one grounding point to the other. To utilize this fact let us compute the total current flowing out through five faces of a rectangular prism in the earth, the sixth face being a rectangle in the surface of the earth surrounding the grounding point $(a, 0, 0)$, the prism extending from $x = a - \xi$ to $x = a + \xi$, from $y = -\eta$ to $y = \eta$, and from $z = -\zeta$ to $z = 0$. The components of the electric force being given by (31)-(33), and λ being the conductivity of the earth, we obtain for this current the expression

$$-4\lambda \int_0^\infty \int_0^\infty F_z \frac{\sin a\mu \sin \xi\mu \sin \eta\nu}{\mu\nu} d\mu d\nu, \tag{41}$$

after simplifying by means of the divergence condition (37). This current flowing out through the prism is I if the face in the surface of the earth includes only the one grounding point $(a, 0, 0)$, but is zero if it includes both grounding points; that is, the above integral (41) equals I if $\xi < 2a$, but equals zero if $2a < \xi$, for any positive value of η. It is readily verified that the Fourier integral

$$\frac{8I}{\pi^2} \int_0^\infty \int_0^\infty \frac{\sin^2 a\mu \sin \xi\mu \sin \eta\nu}{\mu\nu} d\mu d\nu \tag{42}$$

has the desired properties. Accordingly, we must have

$$F_z = -\frac{2I}{\pi^2\lambda} \sin a\mu. \tag{43}$$

To obtain the one additional relation which is needed, we make use of the fact that the current I flows through the straight wire from one grounding point to the other. Let us integrate the magnetic force around a rectangle in a plane perpendicular to the wire, that is, perpendicular to the x axis, the rectangle extending from $y = -\eta$ to $y = \eta$ and from $z = -\zeta$ to $z = \zeta$, the path of integration being taken in the clockwise direction looking along the positive direction of the x axis, and then equate this integral to 4π times the total current threading the rectangle. The components of the magnetic force which we need, H_y and H_z, are found from the fact that curl $E = -i\omega H$, that is,

$$i\omega H_y = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}, \tag{44}$$

$$i\omega H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}, \tag{45}$$

where the E's are given by (31)–(33) for $z < 0$ and by (34)–(36) for $0 < z$. We now subtract from this integral 4π times the current *in the earth* which threads the rectangle, this quantity being found by the appropriate integration of E_x, as given by (31), over that portion of the area of the rectangle which lies below the surface of the earth. As a final result we obtain the expression

$$\frac{2}{i\omega}\int_0^\infty \int_0^\infty \frac{1}{\nu}\left(-\sqrt{\mu^2+\nu^2+\gamma^2}F_x+\mu F_z-\sqrt{\mu^2+\nu^2}P_x-\mu P_z\right) \times \cos x\mu \sin \eta\nu \, d\mu d\nu, \tag{46}$$

after simplifying by means of the divergence conditions (37) and (38). The net current threading the rectangle, after subtracting the current in the earth, is I if the rectangle is situated between the two grounding points, but is zero if it is outside them; that is, the above integral (46) equals $4\pi I$ if $|x| < a$, but equals zero if $a < |x|$, for any positive value of η. It is readily verified that the Fourier integral

$$\frac{16I}{\pi}\int_0^\infty \int_0^\infty \frac{\sin a\mu \cos x\mu \sin \eta\nu}{\mu\nu} d\mu d\nu$$

has the desired properties. Accordingly we must have

$$-\sqrt{\mu^2+\nu^2+\gamma^2}F_x+\mu F_z-\sqrt{\mu^2+\nu^2}P_x-\mu P_z = \frac{8i\omega I}{\pi}\cdot\frac{\sin a\mu}{\mu}. \tag{47}$$

We can now solve equations (37)–(40), (43), and (47) for the six arbitrary functions, obtaining

$$F_x = P_x = \frac{2I}{\pi^2\lambda}\left[\frac{\nu^2}{\mu\sqrt{\mu^2+\nu^2}} - \frac{\sqrt{\mu^2+\nu^2+\gamma^2}}{\mu}\right]\sin a\mu, \tag{48}$$

$$F_y = P_y = \frac{2I}{\pi^2\lambda}\cdot\frac{\nu}{\sqrt{\mu^2+\nu^2}}\sin a\mu, \tag{49}$$

$$F_z = -\frac{2I}{\pi^2\lambda}\sin a\mu, \tag{43}$$

$$P_z = \frac{2I}{\pi^2\lambda}\cdot\frac{\sqrt{\mu^2+\nu^2+\gamma^2}}{\sqrt{\mu^2+\nu^2}}\sin a\mu. \tag{50}$$

Substituting these values in equations (31)–(33) and letting a approach zero such that $2a = dS$, we find, for the electric force in the earth,

$$(51)\quad E_x = \frac{IdS}{\pi^2\lambda}\int_0^\infty\int_0^\infty \left[\frac{\nu^2}{\sqrt{\mu^2+\nu^2}} - \sqrt{\mu^2+\nu^2+\gamma^2}\right] e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \times \cos x\mu \cos y\nu \, d\mu d\nu,$$

$$(52)\quad E_y = \frac{IdS}{\pi^2\lambda}\int_0^\infty\int_0^\infty \frac{\mu\nu}{\sqrt{\mu^2+\nu^2}} e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \sin x\mu \sin y\nu \, d\mu d\nu,$$

$$(53)\quad E_z = -\frac{IdS}{\pi^2\lambda}\int_0^\infty\int_0^\infty \mu e^{z\sqrt{\mu^2+\nu^2+\gamma^2}} \sin x\mu \cos y\nu \, d\mu d\nu.$$

These are precisely the values found by the former method, for the integrals P and Q may be expressed as double integrals by substituting for $J_0(r\rho)$ the integral expression given by the formula [17]

$$(54)\qquad J_0(r\sqrt{\mu^2+\nu^2}) = \frac{2}{\pi}\int_0^{\frac{1}{2}\pi} \cos(r\mu\cos\theta)\cos(r\nu\sin\theta)d\theta,$$

and introducing rectangular coordinates in place of r, θ. These integrals may, therefore, be written in the equivalent forms,

$$(55)\quad P = \frac{2}{\pi}\int_0^\infty\int_0^\infty \frac{e^{z\sqrt{\mu^2+\nu^2+\gamma^2}}}{\sqrt{\mu^2+\nu^2+\gamma^2}} \cos x\mu \cos y\nu \, d\mu d\nu,$$

$$(56)\quad Q = \frac{2}{\pi}\int_0^\infty\int_0^\infty \frac{e^{z\sqrt{\mu^2+\nu^2+\gamma^2}}}{\sqrt{\mu^2+\nu^2}\sqrt{\mu^2+\nu^2+\gamma^2}} \cos x\mu \cos y\nu \, d\mu d\nu,$$

and comparison with (51)–(53) again leads to the values

$$(18)\quad (E_x, E_y, E_z) = \frac{IdS}{2\pi\lambda}\left(-\frac{\partial^3 Q}{\partial y^2\partial z} - \frac{\partial^2 P}{\partial z^2},\ \frac{\partial^3 Q}{\partial x\partial y\partial z},\ \frac{\partial^2 P}{\partial x\partial z}\right),$$

where P and Q are evaluated in (15) and (16). Thus the mutual impedance formula presented in this paper may be derived directly from first principles, without reference to the work of Sommerfeld.

I am greatly indebted to my colleague, Dr. Marion C. Gray, for putting into its present form the derivation of my formula from Sommerfeld's results.

[17] G. N. Watson, *op. cit.*, page 21, formula (1) of § 2.21.

Transients in Grounded Wires Lying on the Earth's Surface *

By JOHN RIORDAN

Voltages during transient conditions in a grounded wire lying on the earth's surface due to current in a second grounded wire also on the earth's surface are formulated for types of transient currents ordinarily obtained in a.-c. and d.-c. circuits. The fundamental formula is for voltage due to a unit step current, that is, a current zero for time less than zero, and unity for time greater than zero; curves are given for the function determining this voltage for a wide range of values of its two parameters. The formulas for other types of currents are not well adapted for numerical computation, which should be more conveniently carried out by numerical integration using the above curves.

I

A FORMULA for the mutual impedance of grounded wires lying on the earth's surface has recently been published by R. M. Foster.[1] The object of the present paper is to derive formulas for the voltages during transient conditions in one such grounded wire due to current in a second for types of transient currents ordinarily obtained in a.-c. and d.-c. circuits, and particularly for the voltage due to unit step current, zero for time less than zero, unity for time greater than zero.

The voltage due to unit step current is expressed in closed form for straight parallel wires; closed form expressions have not been obtained for straight parallel wires for the exponential forms of current for a.-c. and d.-c. transients. While the integrals might be evaluated numerically, or transformed to asymptotic expressions, it appears more desirable in practical calculation to use the curves given for the unit step voltage directly; a single integration is necessary to find the voltage for current of arbitrary wave form, from the unit step result.

The fundamental physical assumptions upon which the steady-state formula is based are as follows: The surface of the earth is assumed flat, the earth semi-infinite in extent, of uniform conductivity λ, unit

* A brief report of the results in this paper was given at the Summer Convention of the American Institute of Electrical Engineers, Toronto, Ontario, Canada, June 23–27, 1930, in Discussion of "Mutual Impedances of Ground Return Circuits—Some Experimental Studies," by A. E. Bowen and C. L. Gilkeson; *A. I. E. E. Trans.*, Oct. 1930.

[1] R. M. Foster: "Mutual Impedances of Grounded Circuits" (Abstract), *Bulletin of the American Mathematical Society*, May, 1930, pp. 367–368; "Mutual Impedance of Grounded Wires Lying on the Surface of the Earth," *Bell System Technical Journal*, July, 1931.

permeability and negligible dielectric constant. The air above the earth is of zero conductivity, unit permeability, and negligible dielectric constant. Because of the assumption of negligible dielectric constant, the formulas for voltages during transient conditions do not hold strictly for small values of the time, that is, during the initial stages of the transient. The wires are of negligible diameter, lying on the surface of the earth, and insulated from it except at the ends, where there is point contact.

In using the steady-state solution as the basis of transient solutions, the Heaviside operational calculus is employed after replacing $i\omega$, where $\omega = 2\pi f$ is the radian frequency and $i = \sqrt{-1}$, by $p = d/dt$, the time differentiator, since $(d^n/dt^n)(\exp i\omega t) = (i\omega)^n \exp i\omega t$, where n is integral.

II

The mutual impedance of grounded wires lying on the surface of the earth and insulated from it except at the ends is given by the following formula:[2]

$$Z_{12} = \frac{1}{2\pi\lambda}\int\int\left\{\frac{d^2}{dSds}\left(\frac{1}{r}\right) + \frac{\cos\epsilon}{r^3}\left[1 - (1+\gamma r)e^{\gamma r}\right]\right\}dSds.$$

The integration is extended over the two wires S and s, having arbitrary paths, r and ϵ are the distance and angle, respectively, between differential elements dS and ds, and $\gamma = (4\pi\lambda i\omega)^{1/2}$; λ is the ground conductivity and $\omega = 2\pi f$ is the radian frequency.

Replacing $i\omega$ by $p = d/dt$ in γ, the resulting forms to be evaluated are $\exp(-\alpha\sqrt{p})$ and $\sqrt{p}\exp(-\alpha\sqrt{p})$ where $\alpha = r\sqrt{4\pi\lambda}$. The first of these is known and, following Heaviside,[3] may be developed as follows.

Expressing the exponential in series form:

$$\exp(-\alpha\sqrt{p}) = 1 - \alpha\sqrt{p} + \frac{\alpha^2 p}{2!} - \frac{\alpha^3 p\sqrt{p}}{3!} + \cdots.$$

Integral powers of p are neglected, since (omitting the discontinuity at $t = 0$) the operand is unity and the derivative of a constant is zero. Then:

$$\exp(-\alpha\sqrt{p}) = 1 - \alpha\sqrt{p}\left[1 + \frac{\alpha^2 p}{3!} + \frac{\alpha^4 p^2}{5!} + \cdots\right].$$

The bracketed terms may now be assumed to operate on $\sqrt{p} = (\pi t)^{-1/2}$

[2] Foster, loc. cit.

[3] Heaviside: "Electromagnetic Theory," Vol. II, pp. 49–51, equations (4) and (12).

and, if p^n is replaced by d^n/dt^n,

$$\exp(-\alpha\sqrt{p}) = 1 - \left[1 + \frac{\alpha^2}{3!}\frac{d}{dt} + \frac{\alpha^4}{5!}\frac{d^2}{dt^2} + \cdots\right]\frac{\alpha}{\sqrt{\pi t}}$$
$$= 1 - \left[1 - \frac{1}{3x1!}\left(\frac{\alpha^2}{4t}\right) + \frac{1}{5x2!}\left(\frac{\alpha^2}{4t}\right)^2 - \cdots\right]\frac{\alpha}{\sqrt{\pi t}}$$
$$= 1 - \operatorname{erf}\frac{\alpha}{2\sqrt{t}},$$

since the term in brackets with its accompanying multiplier is the absolutely convergent expansion of the error function (erf);

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-z^2)dz.$$

The result may also be established either by use of an integral equation [4] or the Fourier integral; it is given as pair 803, Table I, in tables published by G. A. Campbell.[5] In the present use of the tables, for unit step current, the mate of $F(p)/p$, where $F(p)$ is a function of p to be evaluated, is taken since the unit step function is expressed by p^{-1} (pair 415).

The second operational form required may be derived from the first by differentiating with respect to α, since $(d/d\alpha)F(p) = (d/d\alpha)f(t)$ where $F(p)$ and $f(t)$ are corresponding functions of p and t. Thus,

$$\alpha\sqrt{p}\exp(-\alpha\sqrt{p}) = \frac{\alpha}{\sqrt{\pi t}}\exp\left(-\frac{\alpha^2}{4t}\right),$$

since

$$\frac{d}{dt}\operatorname{erf}[\psi(t)] = \frac{2}{\sqrt{\pi}}\psi'(t)\exp\left\{-[\psi(t)]^2\right\}.$$

The unit step voltage may now be expressed, by substitution of these results, by the following formula:

$$V_{12}(t) = \frac{1}{2\pi\lambda}\int\int\left\{\frac{d^2}{dSds}\left(\frac{1}{r}\right) + \frac{\cos\epsilon}{r^3}\left[\operatorname{erf}\left(r\sqrt{\frac{\pi\lambda}{t}}\right) - 2r\sqrt{\frac{\lambda}{t}}\exp\left(-\frac{\pi\lambda r^2}{t}\right)\right]\right\}dSds. \quad (1)$$

In equation (1), as in the steady-state formula from which it is derived, the wires are unrestricted in path or length on the surface of

[4] J. R. Carson: "Electric Circuit Theory and The Operational Calculus," McGraw Hill Co., 1926, p. 19, eq. 29.

[5] "The Practical Application of the Fourier Integral," *Bell System Technical Journal*, October, 1928.

the earth. The formula for straight parallel wires, wire S extending along the z axis from $-a$ to $+a$, and wire s from z_1 to z_2 at distance x from it, is obtained by double integration between these limits with $r^2 = x^2 + (S-s)^2$, $\cos\epsilon = 1$.

The result of integrating once, with respect to S, is:

$$V_{12}(t) = \frac{1}{2\pi\lambda}\int\left\{\frac{d}{ds}\left[\frac{1}{\sqrt{x^2+(a-s)^2}} - \frac{1}{\sqrt{x^2+(\alpha+s)^2}}\right] + \phi(s+a) - \phi(s-a)\right\}ds, \quad (2)$$

where

$$\phi(u) = \frac{u}{x^2\sqrt{(x^2+u^2)}}\operatorname{erf}\left(\sqrt{x^2+u^2}\sqrt{\frac{\pi\lambda}{t}}\right) - \frac{1}{x^2}\exp\left(-\frac{\pi\lambda x^2}{t}\right)\operatorname{erf}\left(u\sqrt{\frac{\pi\lambda}{t}}\right),$$

where u is to be replaced by $s+a$ and $s-a$ in equation (2).

Equation (2) is checked as follows. In the first term substitute limits after removing differentiation and integration with respect to S, which cancel each other. In the second term integrate by parts:

$$\int\frac{1}{[x^2+(S-s)^2]^{3/2}}\operatorname{erf}\sqrt{\frac{\pi\lambda}{t}[x^2+(S-s)^2]}\,dS$$
$$= \frac{S-s}{x^2\sqrt{x^2+(S-s)^2}}\operatorname{erf}\sqrt{\frac{\pi\lambda}{t}[x^2+(S-s)^2]}$$
$$- 2\sqrt{\frac{\lambda}{t}}\int\frac{(S-s)^2}{x^2[x^2+(S-s)^2]}\exp\left\{-\frac{\pi\lambda}{t}[x^2+(S-s)^2]\right\}dS.$$

The integral coming from this operation combines with the remaining term to give:

$$-2\sqrt{\frac{\lambda}{t}}\int\frac{1}{x^2}\exp\left\{-\frac{\pi\lambda}{t}[x^2+(S-s)^2]\right\}dS,$$

which can be simplified in terms of the error function to the form in equation (2).

Integration from z_1 to z_2 gives the result:

$$V_{12}(t) = \frac{1}{2\pi\lambda x}[\psi(z_2+a) - \psi(z_2-a) - \psi(z_1+a) + \psi(z_1-a)], \quad (3)$$

where

$$\psi(u) = -\frac{x}{\sqrt{x^2+u^2}} + \frac{\sqrt{x^2+u^2}}{x}\operatorname{erf}\left(\sqrt{x^2+u^2}\sqrt{\frac{\pi\lambda}{t}}\right) - \frac{u}{x}\exp\left(-\frac{\pi\lambda x^2}{t}\right)\operatorname{erf}\left(u\sqrt{\frac{\pi\lambda}{t}}\right).$$

As before, u is to be replaced in the equation by the functional arguments, which are the four sums of the z-coordinates of position. The factor x in $\psi(u)$ is introduced to make it a function of two parameters, ux^{-1} and $\pi\lambda x^2 t^{-1}$; the result of integration is $x^{-1}\psi(u)$. The result has the dimensions of abohms when all quantities are in electromagnetic c.g.s. units.

To check equation (3) notice that the integration of the first term of equation (2) is effected by removal of differentiation and integration signs, and substitution of limits; its contribution is identical with the d.-c. mutual resistance.[6] The integration of $\phi(u)$ may be effected by integrating the first term by parts and employing the indefinite integral:

$$\int \operatorname{erf}(ax)dx = x \operatorname{erf}(ax) + \frac{1}{a\sqrt{\pi}} \exp(-a^2x^2) + \text{const.}$$

The result is checked by differentiating, that is, by the relation:

$$\frac{d}{du}\left[x^{-1}\psi(u) + \frac{1}{\sqrt{x^2+u^2}}\right] = \phi(u).$$

For large values of u,

$$\psi(u) \sim \frac{|u|}{x}\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right],$$

since

$$\operatorname{erf}(\pm\infty) = \pm 1,$$

so that for $a = \infty$ the unit step voltage approaches the limit:

$$V_{12}(t) = \frac{z_2 - z_1}{\pi\lambda x^2}\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right]$$
$$= \frac{l}{\pi\lambda x^2}\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right],$$

where $l = z_2 - z_1$ is the length of the second wire.

This result is in agreement with a result published by F. Ollendorff, *Elektrische Nachrichten—Technik*, October, 1930, eq. (26), and by L. C. Peterson, *Bell System Technical Journal*, October, 1930, equation (5).

The case of collinear straight wires is obtained by taking the limit $x = 0$, which gives

$$\lim_{x=0} x^{-1}\psi(u) = \frac{1}{u}\left[-1 + \left(\frac{1}{2} + \frac{\pi\lambda u^2}{t}\right)\operatorname{erf}\left(u\sqrt{\frac{\pi\lambda}{t}}\right) + u\sqrt{\frac{\pi\lambda}{t}}\exp\left(-\frac{\pi\lambda u^2}{t}\right)\right]$$
$$= u^{-1}\zeta(u).$$

This result involves the evaluation of an indeterminate form.

[6] G. A. Campbell: "Mutual Impedances of Grounded Circuits," *Bell System Technical Journal*, October, 1923, eq. (3), p. 5.

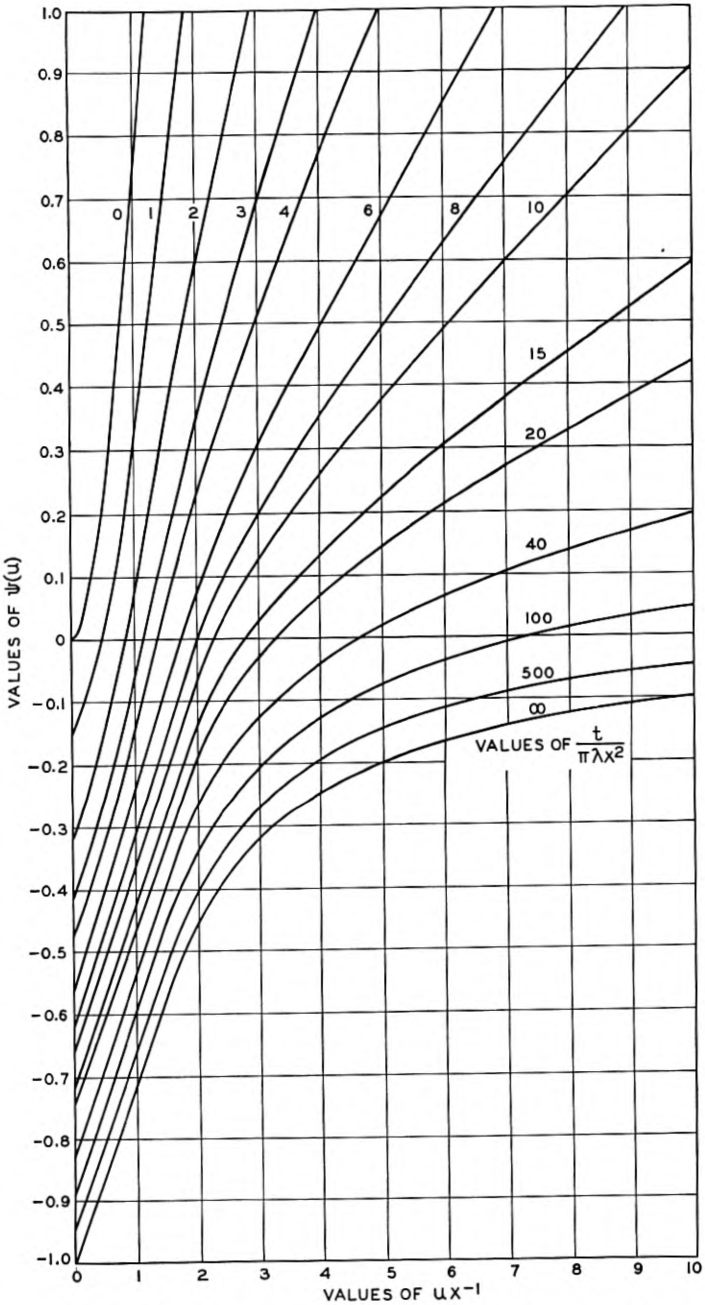


Fig. 1—$\psi(u)$ for the range in which $\psi(u) \leq 1$, $0 \leq ux^{-1} \leq 10$.

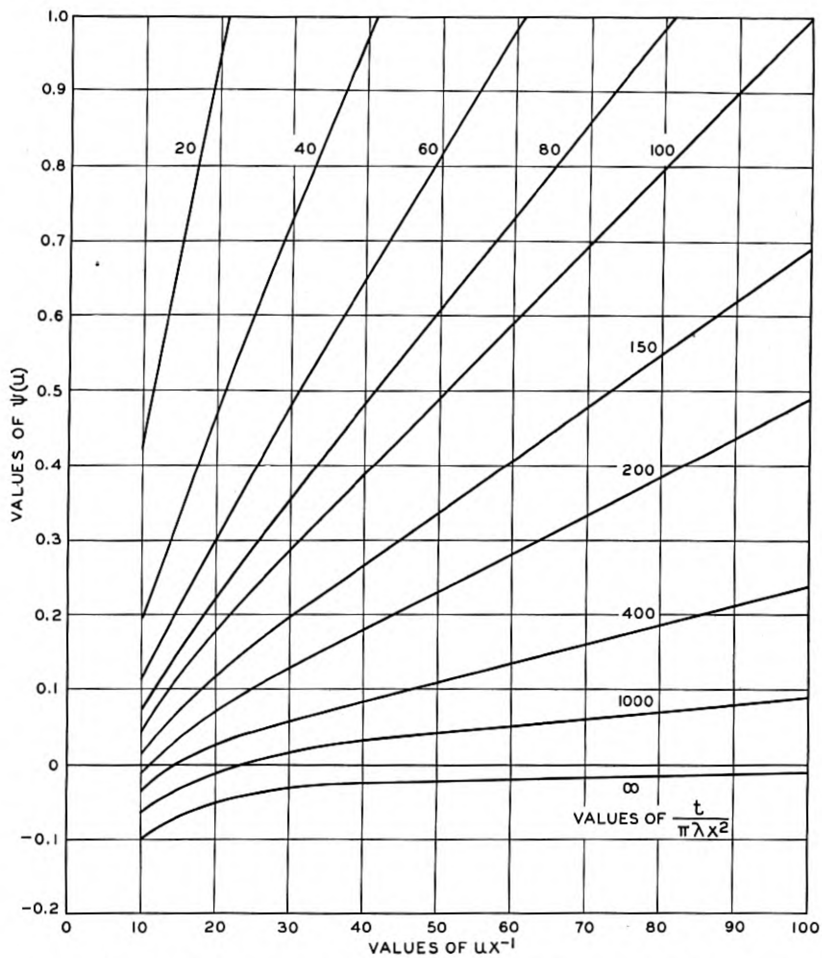


Fig. 2—$\psi(u)$ for the range in which $\psi(u) \leq 1$, $10 \leq ux^{-1} \leq 100$.

Curves for $\psi(u)$ as a function of ux^{-1} with $t/(\pi\lambda x^2)$ as parameter of the curve families are shown on Figures 1, 2, and 3. The range $\psi(u) \leq 1$, is shown on Figures 1 and 2 for $ux^{-1} \leq 10$ and 100, respectively; both figures cover the entire range of $t/(\pi\lambda x^2)$ in the intervals. The remaining range $\psi(u) > 1$ is shown on Figure 3. For the greater part of the range on Figure 3 the function is determined by its limiting form for ux^{-1} large, that is, by the equation

$$\psi(u) = ux^{-1}\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right]$$

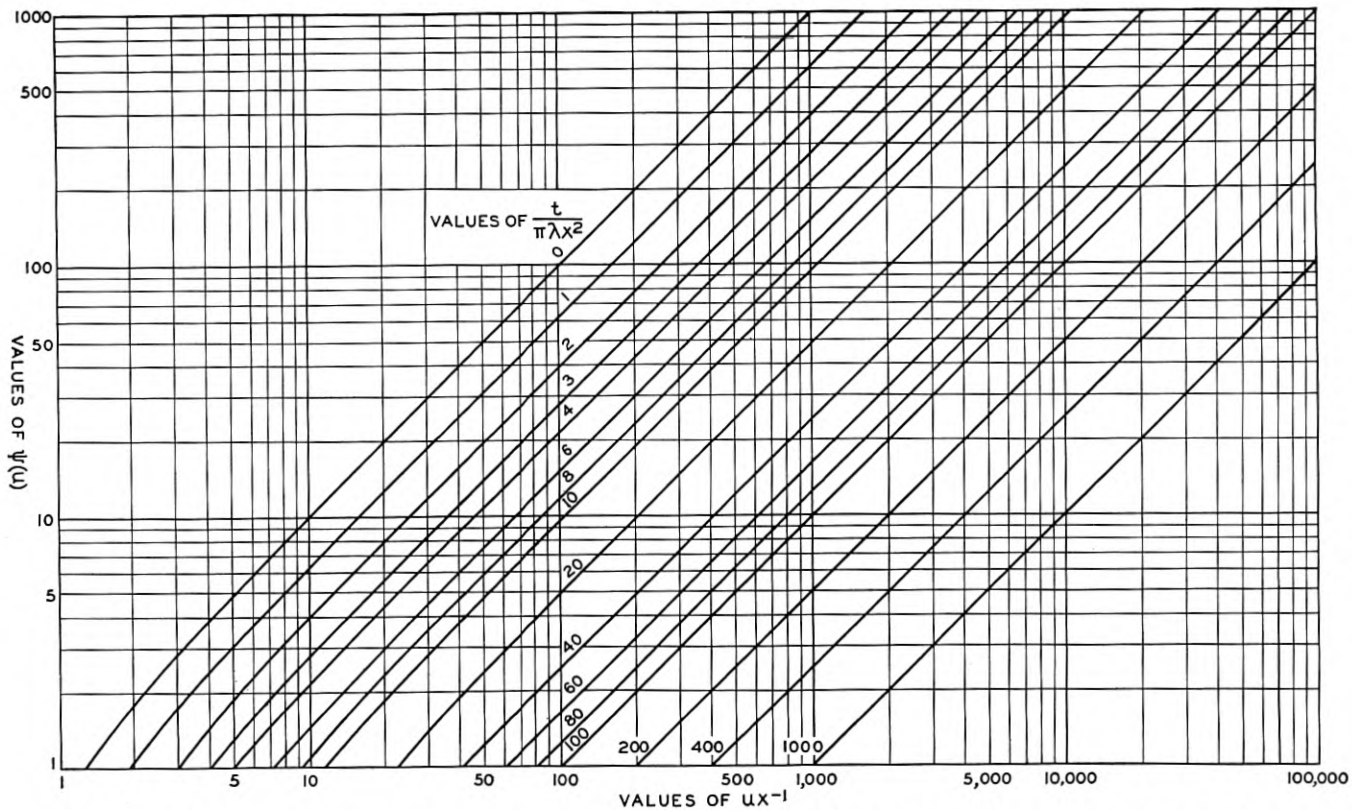


Fig. 3—$\psi(u)$ for a range in which $\psi(u) \geqq 1$, $1 \leqq ux^{-1} \leqq 100{,}000$. The straight line portion of the curve plots the equation

$$\psi(u) = ux^{-1}\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right]$$

which is the limiting form for large values of u.

or

$$\log \psi(u) = \log ux^{-1} + \log\left[1 - \exp\left(-\frac{\pi\lambda x^2}{t}\right)\right].$$

Thus Figure 3 may be used to indicate the range of applicability of the limiting form, which is quite large; in this range the unit step voltage is simplified as shown above.

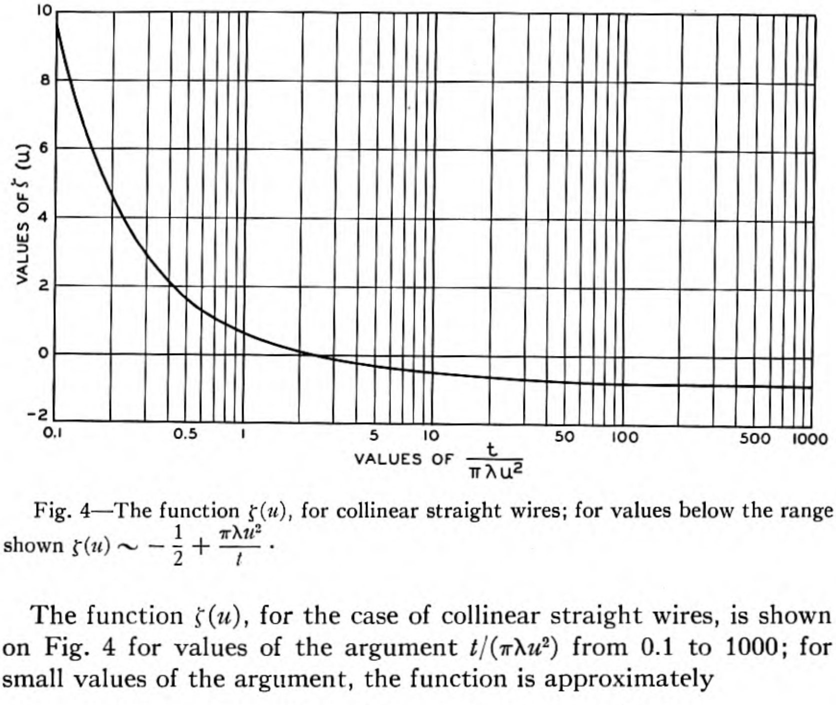


Fig. 4—The function $\zeta(u)$, for collinear straight wires; for values below the range shown $\zeta(u) \sim -\frac{1}{2} + \frac{\pi\lambda u^2}{t}$.

The function $\zeta(u)$, for the case of collinear straight wires, is shown on Fig. 4 for values of the argument $t/(\pi\lambda u^2)$ from 0.1 to 1000; for small values of the argument, the function is approximately

$$\zeta(u) \sim -\frac{1}{2} + \frac{\pi\lambda u^2}{t} \qquad \left(\frac{t}{\pi\lambda u^2} < 0.4\right).$$

These curves may be employed to obtain voltages due to other forms of disturbing currents by numerical or mechanical integration of the following integral:[7]

$$\begin{aligned} E_{12}(t) &= \frac{d}{dt}\int_0^t I(\tau)V_{12}(t-\tau)d\tau \\ &= \frac{d}{dt}\int_0^t I(t-\tau)V_{12}(\tau)d\tau, \end{aligned}$$

where $I(t)$ is the disturbing current as a function of time.

[7] J. R. Carson: loc. cit., p. 16, eq. (20) and (20a).

III

The equation above may be used to obtain a formula for voltage due to suddenly applied current $\exp i\omega t$; or the operational product, of which it is an expression in terms of t, may be carried out directly in terms of p. The current is expressed in terms of p by:

$$\exp i\omega t = \frac{p}{p - i\omega}.$$

The second term in $\psi(u)$ is transformed by the operational equivalent already developed:

$$\operatorname{erf} \frac{\alpha}{2\sqrt{t}} = 1 - \exp(-\alpha\sqrt{p}).$$

The last term in $\psi(u)$ is not known in closed form in p.

The operational product of exp $i\omega t$ and the second term is evaluated by

$$\begin{aligned}\frac{p[1 - \exp(-\alpha\sqrt{p})]}{p - i\omega} &= \frac{p}{p - i\omega} - \frac{p \exp(-\alpha\sqrt{p})}{p - i\omega} \\ &= \exp i\omega t - \frac{1}{2}\left[\exp(i\omega t - \alpha\sqrt{i\omega}) \operatorname{erfc}\left(\frac{\alpha}{2\sqrt{t}} - \sqrt{i\omega t}\right)\right. \\ &\qquad \left. + \exp(i\omega t + \alpha\sqrt{i\omega}) \operatorname{erfc}\left(\frac{\alpha}{2\sqrt{t}} + \sqrt{i\omega t}\right)\right],\end{aligned}$$

the last term of which is given by pair 819 (with $\beta = 0$) in the tables referred to. Erfc is the error function complement;

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z).$$

The operational product of exp $i\omega t$ and the last term in $\psi(u)$ may be expressed in integral form by the formula:

$$\begin{aligned}\frac{p}{p - i\omega} f(t) &= \left[1 + \frac{i\omega}{p - i\omega}\right] f(t) \\ &= f(t) + i\omega \exp i\omega t \int_0^t \exp(-i\omega t) f(t)\, dt.\end{aligned}$$

The complete expression for the voltage due to cisoidal current is as follows:

$$E_{12}(t) = \frac{1}{2\pi\lambda x}[\Phi(z_2 + a) - \Phi(z_2 - a) - \Phi(z_1 + a) + \Phi(z_1 - a)], \quad (4)$$

where

$$\Phi(u) = \frac{u^2}{x\sqrt{x^2+u^2}} - \frac{u}{x}\exp\left(-\frac{\pi\lambda x^2}{t}\right)\operatorname{erf}\left(u\sqrt{\frac{\pi\lambda}{t}}\right)$$
$$- \frac{\sqrt{x^2+u^2}}{2x}\left[\exp\left(i\omega t - \gamma\sqrt{x^2+u^2}\right)\operatorname{erfc}\left(\sqrt{\frac{\pi\lambda}{t}(x^2+u^2)} - \sqrt{i\omega t}\right)\right.$$
$$\left. + \exp\left(i\omega t + \gamma\sqrt{x^2+u^2}\right)\operatorname{erfc}\left(\sqrt{\frac{\pi\lambda}{t}(x^2+u^2)} + \sqrt{i\omega t}\right)\right]$$
$$- \frac{u}{x} i\omega \exp i\omega t \int_0^t \exp\left(-i\omega t - \frac{\pi\lambda x^2}{t}\right)\operatorname{erf}\left(u\sqrt{\frac{\pi\lambda}{t}}\right)dt.$$

The integral appearing in $\Phi(u)$ apparently cannot be expressed in closed form in terms of known functions; for numerical results series or asymptotic expressions may be derived but it appears more desirable to employ numerical or mechanical integration using the unit step voltage since tables or charts of the error function of complex variable which also appears in $\Phi(u)$ are not available.

A useful check on the above formula is obtained by taking the limit for $t = \infty$, which gives the steady-state mutual impedance between straight parallel wires; the result is as follows:

$$Z_{12} = E_{12}(t)\exp(-i\omega t)$$
$$= \frac{1}{2\pi\lambda x}\left[\Psi(z_2+a) - \Psi(z_2-a) - \Psi(z_1+a) + \Psi(z_1-a)\right], \quad (5)$$

where

$$\Psi(u) = \frac{u^2}{x\sqrt{x^2+u^2}} - \frac{\sqrt{x^2+u^2}}{x}\exp\left(-\gamma\sqrt{x^2+u^2}\right)$$
$$- \frac{u}{x}\int_0^\infty \exp\left(-w - \frac{\gamma^2 x^2}{4w}\right)\operatorname{erf}\frac{\gamma u}{2\sqrt{w}}\,dw$$
$$= -\frac{x}{\sqrt{x^2+u^2}} + \frac{\sqrt{x^2+u^2}}{x}\left[1 - \exp\left(-\gamma\sqrt{x^2+u^2}\right)\right]$$
$$- \frac{\gamma u}{x}\int_0^u \exp\left(-\gamma\sqrt{x^2+w^2}\right)dw,$$

where as before $\gamma^2 = 4\pi\lambda i\omega$.

The third term in $\Phi(u)$ approaches the limit given because $\operatorname{erfc}(-\sqrt{i\,\infty}) = 2$, $\operatorname{erfc}(\sqrt{i\,\infty}) = 0$; the integral term as given in the first form of $\Psi(u)$ has been transformed by the substitution $w = i\omega t$.

The first form of $\Psi(u)$ may be checked directly from equation (3) by introducing $i\omega = p$ in the operationally equivalent function of p; the third term of (3) being expressed by the infinite integral:

$$F(p) = p \int_0^\infty e^{-pt} f(t)dt.$$

The second form of $\Psi(u)$ is obtained by separating the d.-c. mutual resistance term, and transforming the infinite integral as follows: express the error function in integral form, put $y = \gamma v/(2\sqrt{w})$ where y is the variable of integration for the error function, and invert the order of integration; thus

$$\begin{aligned}\int_0^\infty \exp\left(-w - \frac{\gamma^2 x^2}{4w}\right) \operatorname{erf} \frac{\gamma u}{2\sqrt{w}} dw &= \frac{\gamma}{\sqrt{\pi}} \int_0^u dv \int_0^\infty \exp\left(-w - \frac{\gamma^2(x^2+v^2)}{4w}\right)\frac{dw}{\sqrt{w}} \\ &= \frac{2\gamma}{\sqrt{\pi}} \int_0^u dv \int_0^\infty \exp\left(-z^2 - \frac{\gamma^2(x^2+v^2)}{4z^2}\right) dz \qquad (z = \sqrt{w}) \\ &= \gamma \int_0^u \exp\left(-\gamma\sqrt{x^2+v^2}\right)dv.\end{aligned}$$

The infinite integral evaluated in the third line is No. 495 in Peirce's "Short Table of Integrals," third edition.

The second form of $\Psi(u)$ may be verified by direct double integration of the mutual impedance; it agrees with the known result in the limit for one wire infinite, and, when expanded in powers of γ, with the terms given in the second form for the mutual impedance by R. M. Foster, loc. cit.

Expressions for voltages due to suddenly applied currents $\exp(-kt)\sin\omega t$ or $1 - \exp(-kt)$, which are important forms for a.-c. and d.-c. networks, may be readily obtained from equation (4), the first by use of the expression:

$$\exp(-kt)\sin\omega t = \frac{1}{2i}[\exp(-kt+i\omega t) + \exp(-kt-i\omega t)]$$

and the second by the substitution $-k = i\omega$ and subtraction from the unit step voltage.

The results attained in this paper depend in appreciable measure on advice and suggestions received from Mr. R. M. Foster of the American Telephone and Telegraph Company; I am also appreciative of the interest and advice of Messrs. K. L. Maurer and H. M. Trueblood of this company.

Developments in the Manufacture of Lead-Covered Paper-Insulated Telephone Cable *

By JOHN R. SHEA

This paper describes developments in the manufacture of lead covered paper insulated telephone cable completed during the past three years. The introduction describes the manner in which cable is used in the telephone system and briefly outlines the manufacturing processes and equipment as they existed about three years ago. The new developments are then treated in considerable detail, the most outstanding of which are the application of wood pulp insulation direct on the wire instead of spirally wrapping manila rope ribbon paper; new equipment for vacuum drying and storing cable in which a large storage room of unique construction is provided with conditioned air at a relative humidity of .5 per cent at 100° F.; the central melting of large quantities of lead alloy and its distribution through piping systems to a number of lead presses; improved and larger sheathing presses; and precision electrical testing of the finished cable. Most of these improvements are incorporated in the new Baltimore Cable Plant of the Western Electric Company.

PAPER-INSULATED lead-covered telephone cable constitutes approximately 25 per cent of the Bell System telephone plant. The cost of new telephone cable each year, including installation, averages $100,000,000. Developments in the process and equipment for its manufacture are numerous and have been a large contributing factor in the establishment of a high standard of service in the long-distance communication field. The problems involved in manufacturing engineering are extremely interesting both from an economic and technical standpoint to the mechanical and the electrical engineer, the physicist, and the chemist, and the illustrations which follow contain fundamental engineering principles of use in many lines of industry.

Before proceeding directly with these problems, a brief outline of how cable and its associated apparatus function in the long distance communication field will be of value. After presenting this broad picture, the bulk of the paper will be devoted to an engineering discussion of developments in the process and equipment for manufacturing cable as illustrated by recent improvements introduced in the new cable plant of the Western Electric Company at Baltimore and at the Kearny, New Jersey, and Chicago plants.

* Presented at A. S. M. E. meeting, Cleveland, Ohio, April 13–17, 1931. Published in abridged form in Mech. Engg., April, 1931.

General Information on Use of Cable

The rapid increase with which cable is being added to the toll plant is illustrated quite strikingly by Fig. 1, which shows the present and proposed increases in cable in comparison with open wire and carrier

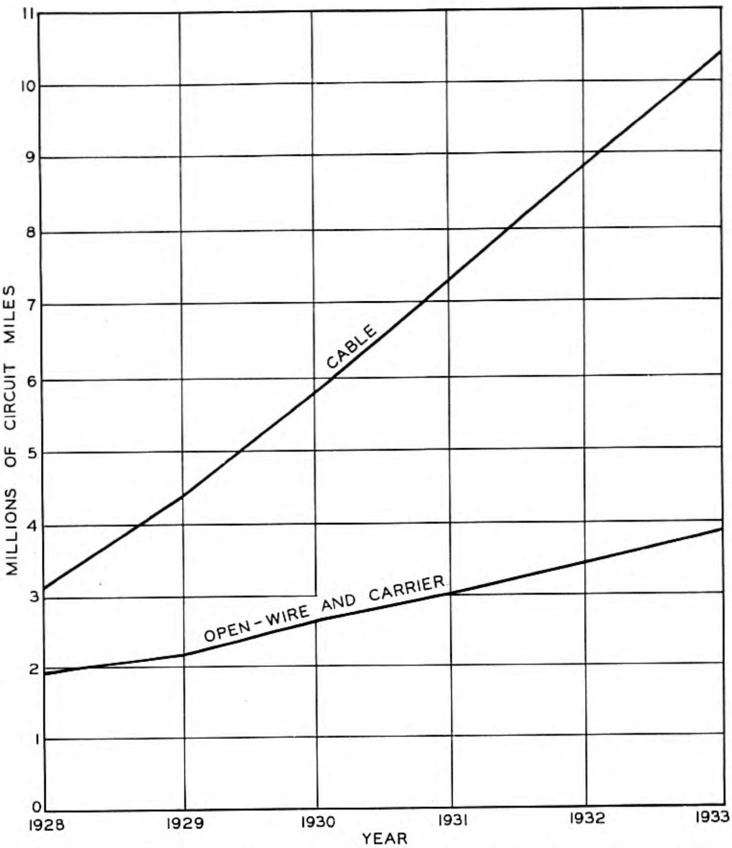


Fig. 1—Present and proposed increase in cable in comparison with open wire and carrier circuits.

circuits.[1] The future scope of this expansion is shown by Fig. 2, which indicates the present and proposed main toll cable routes in the United States. The exact program on which these cables will be extended will depend upon how rapidly the business develops; however, definite future plans have been outlined to extend the cable to Omaha, Nebraska,** and across the continent to San Francisco, thus replacing and increasing the capacity of existing open wire lines.

[1] "Recent Developments in Toll Telephone Service" by W. H. Harrison, *Jour. A. I. E. E.*, March, 1930; *Bell Telephone Quarterly*, April, 1930.

** This cable was completed in May, 1931.

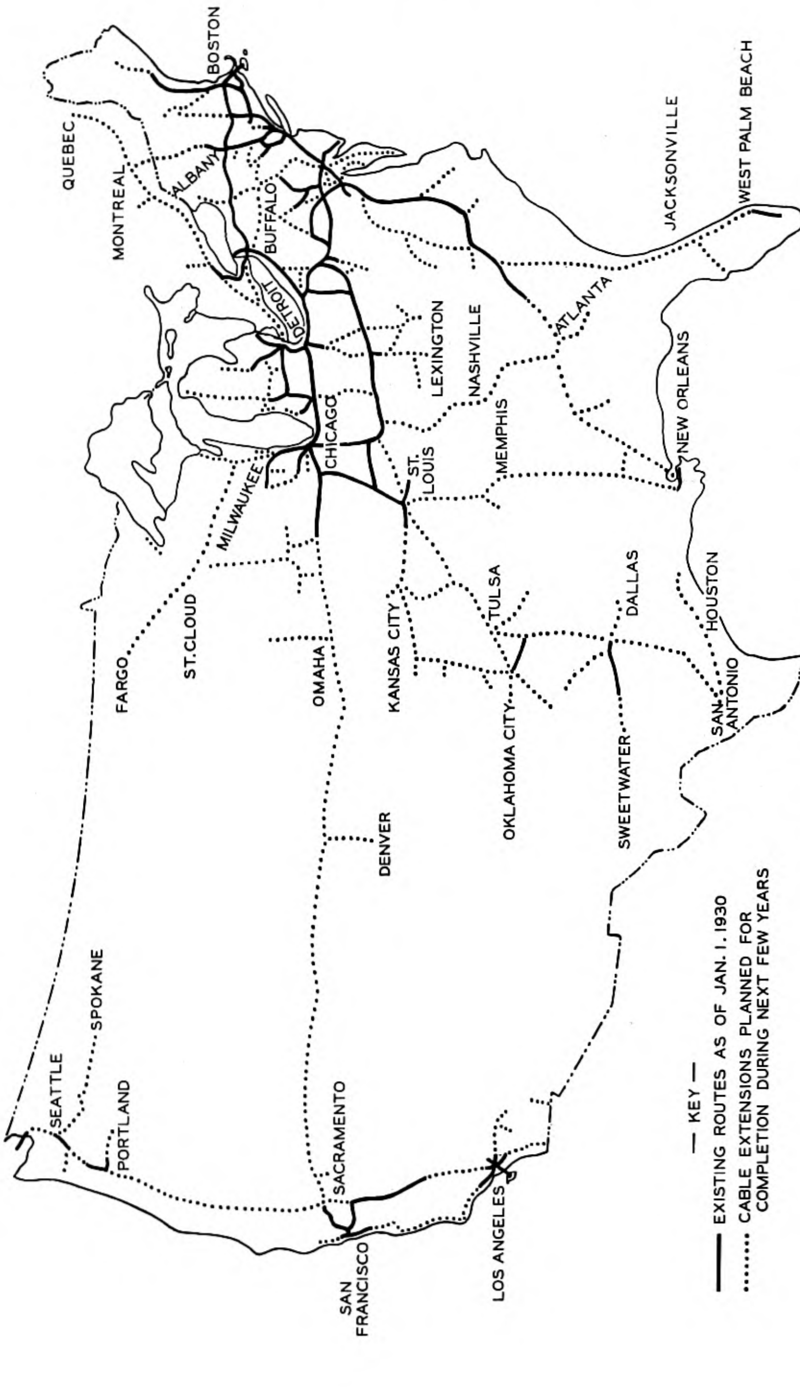


Fig. 2—Main toll cable routes of United States and Canada.

The elements of a typical cable route are illustrated in the New York to Pittsburgh cable chart shown in Fig. 3. A Pittsburgh call originating at a subscriber's station, for example, in Yonkers, New York, passes through the toll board of the local telephone exchange to the toll center located at Walker Street, New York City. At this point the connections are completed for the call to Pittsburgh through the toll cable circuits and repeater stations between the two cities.

The speech currents as they travel along this circuit diminish in intensity. Loading coils placed along the cable circuit at regular intervals reduce these losses to a considerable degree but even with

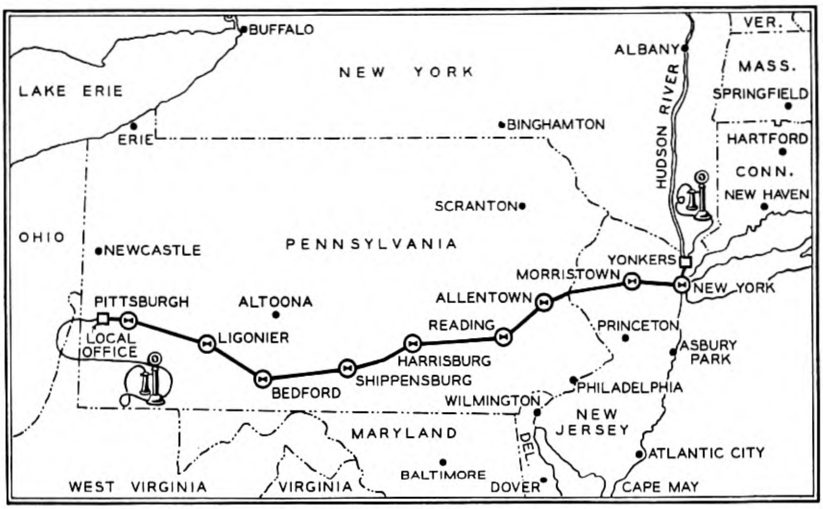


Fig. 3—Typical cable route.

these it is necessary to supply amplifiers (repeaters) [2, 3] at intervals of approximately fifty miles to boost the energy level.

The amount of amplification required for intelligible speech varies with the resistance of the cable conductors which changes with the temperature. In order to regulate the amount of the amplification to compensate for these variations, what is known as a pilot wire regulator is installed at certain repeater points which automatically adjusts the gain of the repeaters to correct for the changing line losses.

Difficulty is also experienced on long toll lines due to the voice currents being reflected back to the speaker. To prevent this, a device is provided which automatically short circuits one side of the

[2] *A. I. E. E. Transactions* (1919), Vol. XXXVIII, Part 2, "Telephone Repeaters," by Bancroft Gherardi and Frank B. Jewett.

[3] *A. I. E. E. Transactions* (1923), Vol. XLII, "Telephone Transmission over Long Cable Circuits," by A. B. Clark. *Bell. Sys. Tech. Jour.*, Jan., 1923.

line while speech is being transmitted in the opposite direction on the other side. This device is known as an "echo suppressor." [4]

The enormous increases in long distance telephone traffic together with the necessity of providing better transmission quality in connection with radio broadcasting [5] and trans-oceanic messages, have led to continuous design changes in telephone plant with more exacting requirements for manufacture. To permit adequate and predetermined spacing of loading coils and repeater stations, the cable design must be such as to insure definite capacitances per mile. There must be a minimum of unbalance between circuits to insure that interference or "crosstalk" is held to a low value. To handle the ever increasing load of messages promptly and to secure further overall economies, cables are being designed with a greatly increased number of wire pairs, but of approximately the usual outside diameters to permit the use of existing cable ducts. All of these design problems are reflected in the machinery and methods of manufacture.

Manufacture of Cable [6]

A typical long-distance telephone cable (toll cable) consists of "quads" (double pairs) of paper-insulated electrolytic copper wire (No. 16 to No. 22 B. & S. gauge) built up in layer construction and covered with a lead-antimony alloy sheath $2\frac{5}{8}$ in. in diameter and $\frac{1}{8}$ in. thick. (Fig. 4.)

The raw materials for such cable consist of high-grade lead in pig form, annealed electrolytic copper wire, and large jumbo rolls of manila-rope wood-pulp paper. The first operation consists of slitting the large rolls of paper into disk-shaped pads (Fig 5). A sufficient number of these pads are placed in an insulating machine which applies the paper to the copper wire in spiral form at a head speed of from 1,470 to 2,400 r.p.m. (Fig. 6). The insulated wires are paired very carefully and then placed in a machine which first twists the pairs and then forms them into twisted quads (Fig. 7).

The quads of wire thus built up are placed into a strander. One quad serves as a center about which other quads are laid in alternate layers as the material progresses through the machine (Fig. 8). Step

[4] *A. I. E. E. Proceedings*, Vol. XLIV, "Echo Suppressors for Long Telephone Circuits," by A. B. Clark and R. C. Mathes.

[5] *Bell Sys. Tech. Jour.*, July 1930, "Long Distance Cable Circuit for Program Transmission," By A. B. Clark and C. W. Green.

[6] See paper "Recent Developments in the Process of Manufacturing Lead-Covered Telephone Cable," by C. D. Hart, for historical treatment and developments prior to 1927—presented at the Regional Meeting of District No. 5 of the A. I. E. E., Chicago, Illinois, November 28 to 30, 1927. Published in *Bell Sys. Tech. Jour.*, April, 1928.

by step it is thus built up, one layer being applied by each drum until the full amount is obtained, after which an outer wrapping of paper is applied to retain the insulated wires in shape and also serve as an additional insulation from the lead sheath.

All telephone cable for local service (exchange cable) until recently was made in much the same manner. Recently two new processes have completely revolutionized its manufacture.

Fig. 4—Typical construction of long distance telephone cable.

Direct Application of Wood-Pulp Insulation

The process and machine recently developed to apply wood pulp direct on wire combines the steps of paper making, slitting, (Fig. 5) and insulating (Fig. 6) into one operation, and gives a continuous sleeve of pulp paper around the wire.

Essentially, the process consists in forming simultaneously on a modified cylinder paper machine, 50 narrow continuous sheets of paper, with a single strand of wire enclosed in each sheet, pressing the excess moisture from the sheets, turning them down by means of a rapidly rotating polishing device, so as to form a uniform cylindrical coating of wet pulp around the wire, and then driving the water from this coating by drying.

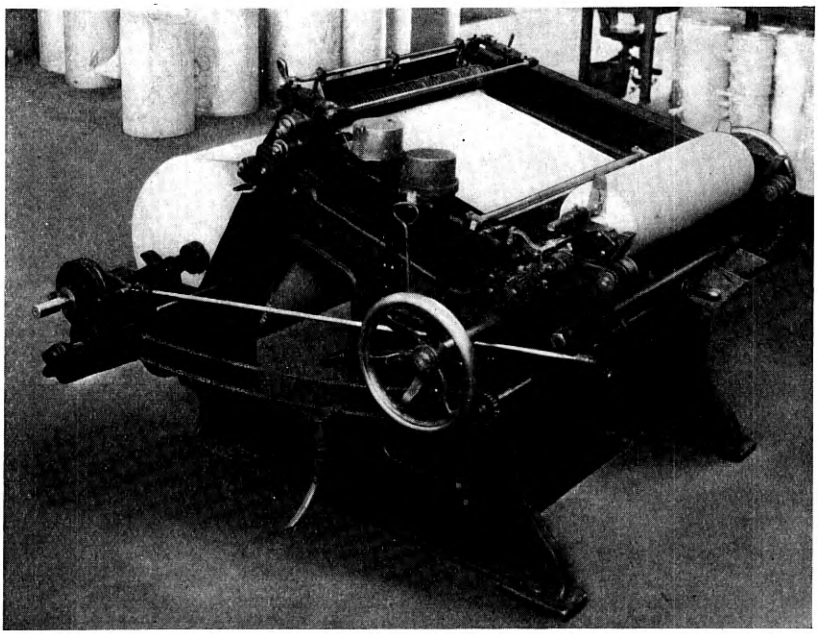


Fig. 5—Slitting of paper.

The material used in making this insulation is Kraft pulp, which is prepared for use on the machine by beating as in the ordinary paper-making operation (Fig. 9) and fed to the machine in a somewhat more diluted form than in standard paper making practice.

In theory, the whole process is simple, but from a practical stand-point, many interesting problems had to be solved before satisfactory operation was possible. A continuous supply of wire must be furnished, as it is not feasible to shut the machine down to change supply spools. This was taken care of by removing the wire from the supply spool by means of a flier without rotating the spool. This allows time to braze the end of the wire from one spool to the next. Ordinary annealed copper wire has a non-uniform surface due in part to the residual drawing compound. A satisfactory surface is obtained by passing the wire through an alternating-current electrolytic cleaning bath before it enters the paper forming machine. A narrow sheet is formed on each conductor in an ordinary single-cylinder paper machine, the mold of which has been divided into 50 parts by means of celluloid strips and so arranged that a part of the sheet of paper is formed before the wire comes in contact with it. The remainder of the sheet is then laid down on top of the wire without any break in the formation,

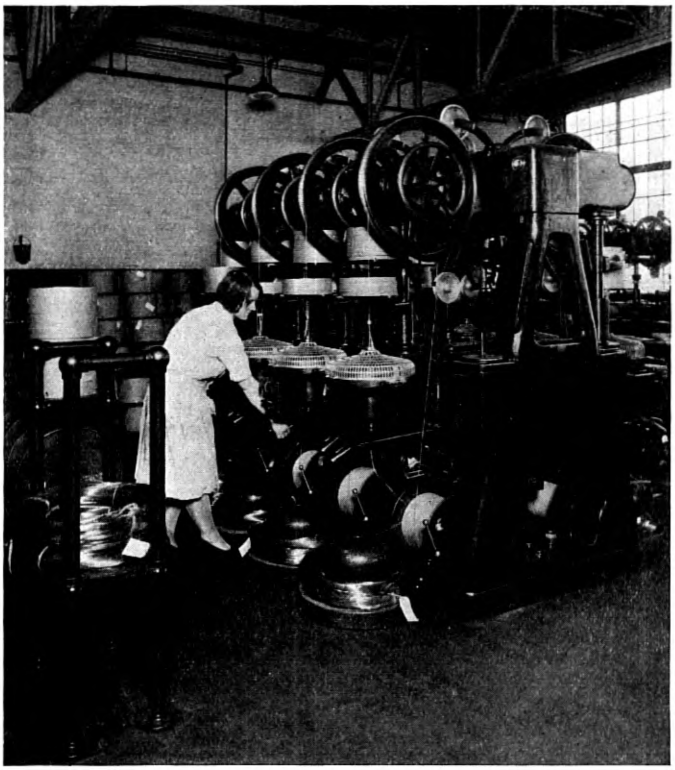


Fig. 6—Paper insulating machine.

and the resulting narrow ribbon of paper carries the wire imbedded in it. Thus fifty conductors are being insulated simultaneously. Two sets of press rolls take the excess moisture from the sheet, and leave it ready for the polishing operation. Various types of polishers have been developed and the one now in use consists of two short, specially shaped blocks, with a third block located about centrally to the other two. These polishers are rotated very rapidly around the wire

(Fig. 10). Their construction is such that if an occasional lump or break occurs in the sheet it does not cause clogging of the polisher. Polished wet insulation carries about 70 per cent water by weight, which has to be driven off by heat. The drier consists of a 25-ft.-long electric box-type furnace, with heating elements extending the full length of the top, and additional heating elements in the first 8-ft.

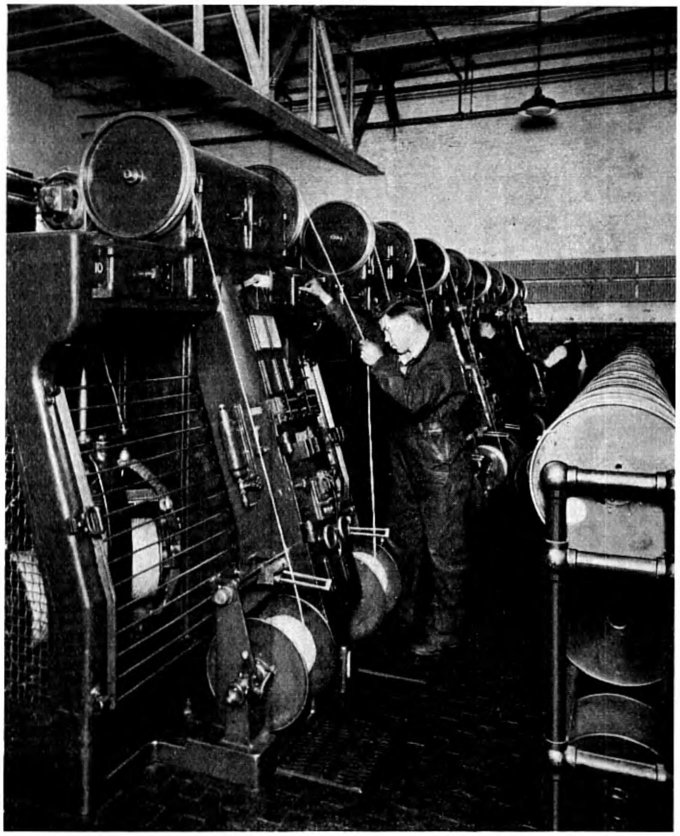


Fig. 7—Twisting and quadding machine.

section of the bottom. These elements are thermostatically controlled so that the temperature of the furnace can be set so as not to cause charring of the insulation as it passes through the drier (Fig. 11). Two spooling positions are furnished at the take-up for each wire, so that as soon as one spool is full, the wire can be shifted to an empty spool, and the full spool removed (Fig. 11). In this way, no shutdowns for

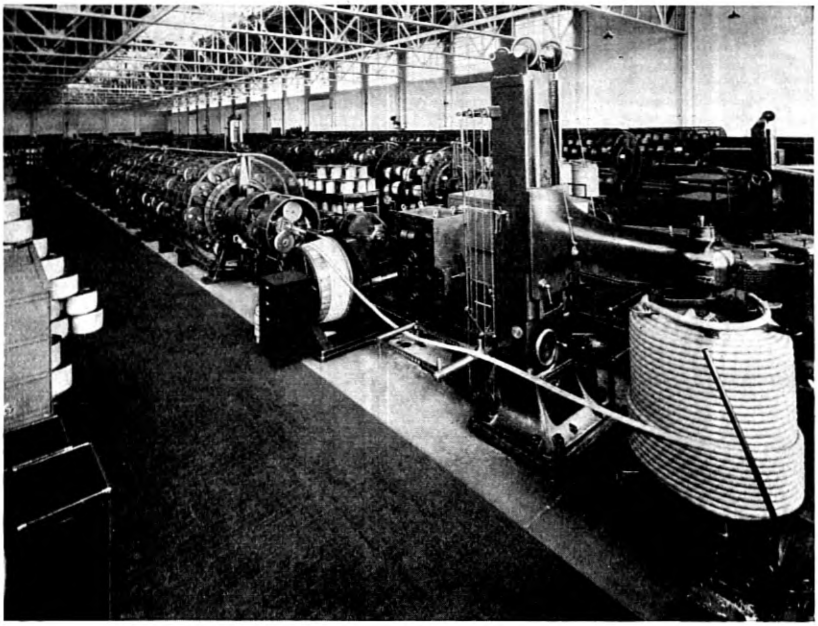


Fig. 8—Stranding machine.

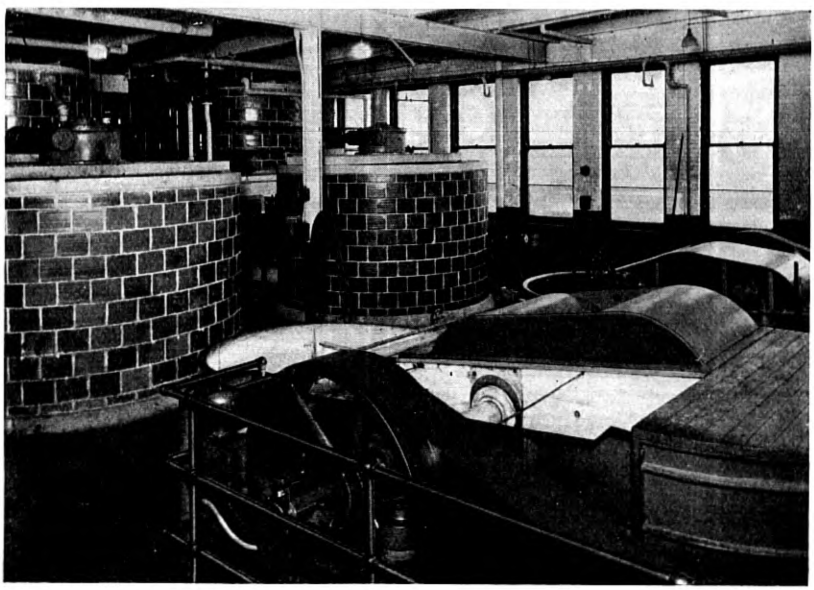


Fig. 9—Beating equipment and pulp storage tanks.

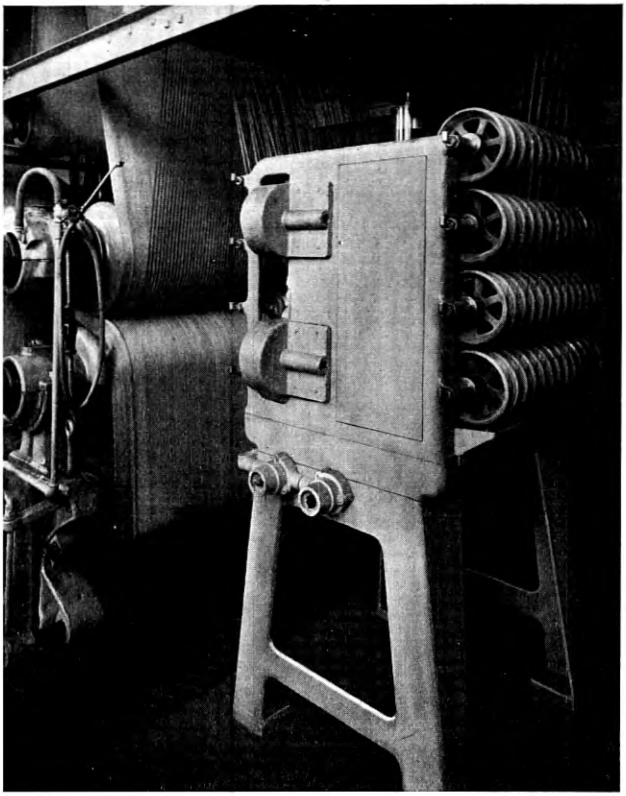


Fig. 10—Machine for polishing pulp insulation after its application to the wire.

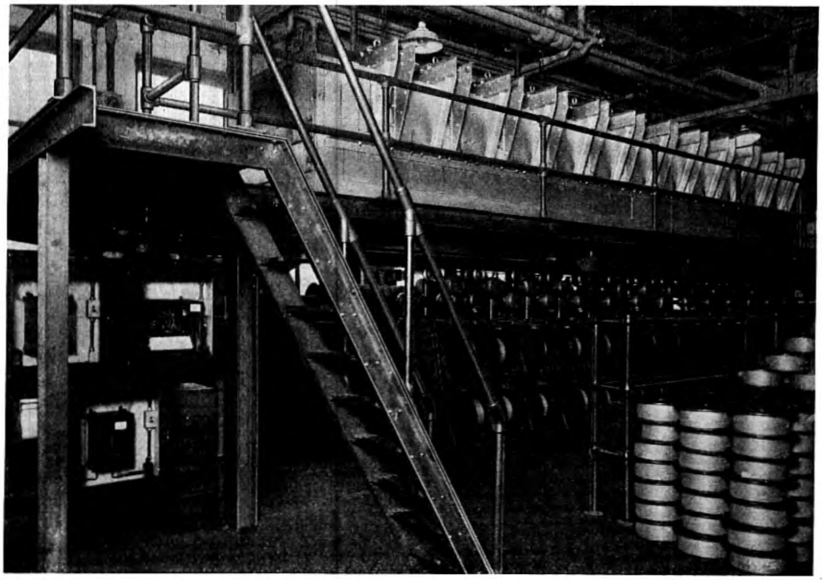


Fig. 11—Drying and take-up units.

changing take-up spools are necessary. Individual wires are strung in without shutting down. Tension devices are incorporated in the take-up so as to avoid the possibility of any undue tension being put on the finished wire. The normal speed of the machine is approximately 110 ft. per min., and the output per week is about 45 million conductor feet.

The electrical properties of telephone exchange cables made from this material compare favorably with those made from ribbon insulation, and the annual saving per machine is an appreciable factor due largely to the lower cost of raw material.

Improved Cable Stranding

Until recently all local cable (exchange cable) was built up or stranded by the concentric layer method at a speed of 50 to 100 ft. per min. (Fig. 8). This construction is being rapidly superseded by a unit

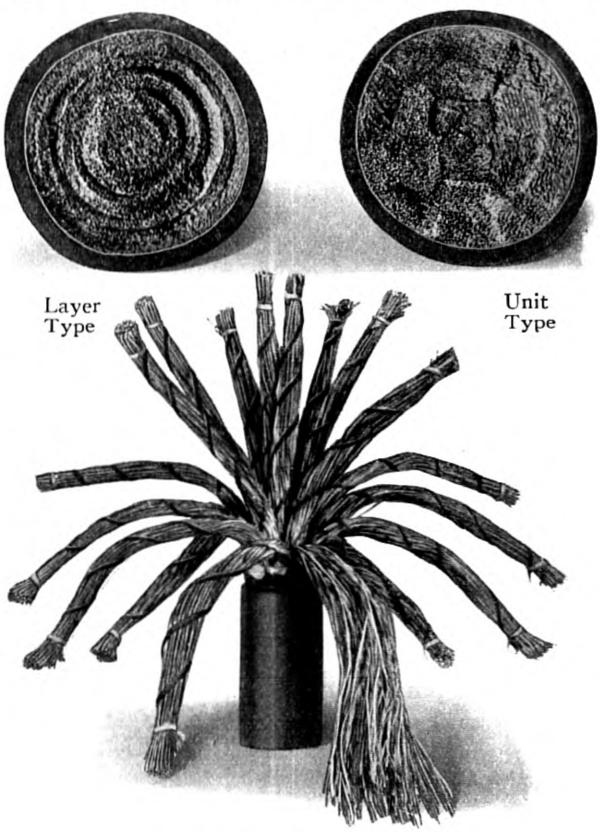


Fig. 12—1818-pair unit cable.

method, the first application of which was made on the 1818-pair 26 B. & S. gauge cable.[7]

The unit method consists of two distinct steps. A flier strander is used to strand pairs into individual color groups known as units, which usually consist of 50, 51, or 101 pairs. A cabling machine then assembles a definite number of these units into a round core form. Thus the final cable size is some multiple of 50, 51, or 101 pairs. An 1818-pair cable built in this manner is shown in Fig. 12.

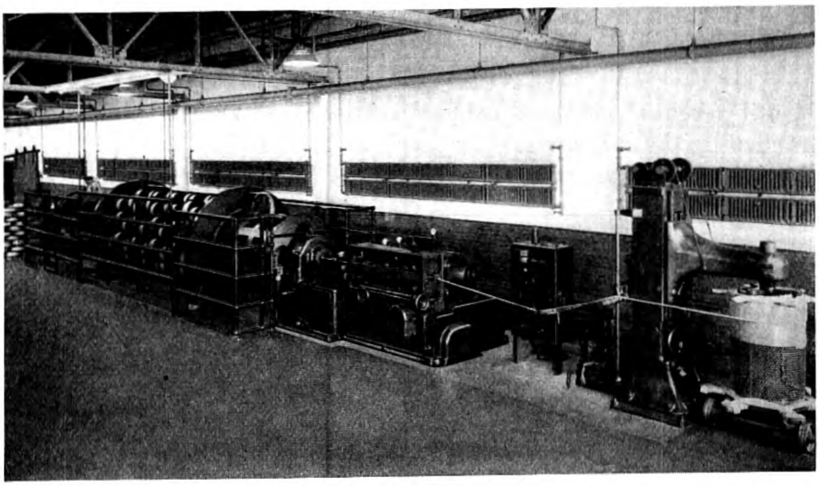


Fig. 13—Flier strander.

The flier strander shown in Fig. 13 consists of a reel carriage or drum for holding 101 supply reels of paired wire; a cotton serving head for winding a cotton thread about the unit; a flier for stranding the unit; a pulling mechanism or capstan for advancing the unit through the machine; and a take-up for reeling the finished unit on a core truck.

By revolving the flier about the normally stationary supply it is possible to obtain two twists in the unit per flier revolution. This combined with the low inertia of the flier permits units to be stranded at the rate of 300 ft. per min.

The cabling machine shown in Fig. 14 consists of 18 supply stands equipped with suitable pneumatic brakes for holding and maintaining tensions on the trucks of units, and a rotating capstan take-up. The units are pulled through a distributer plate and covered with a protective wrap of paper. A twist is put in the cable between the dis-

[7] *Bell Telephone Quarterly*, January 1929, "1800-pair Cable Becomes a Bell System Standard," by F. L. Rhodes.

tributer plate and the entrance point of the cable to the capstan. The finished cable is taken up on reels capable of carrying three times as much cable as the core trucks used with the concentric stranding machine. These reels of cable are then handled through subsequent manufacturing processes by electric trucks.

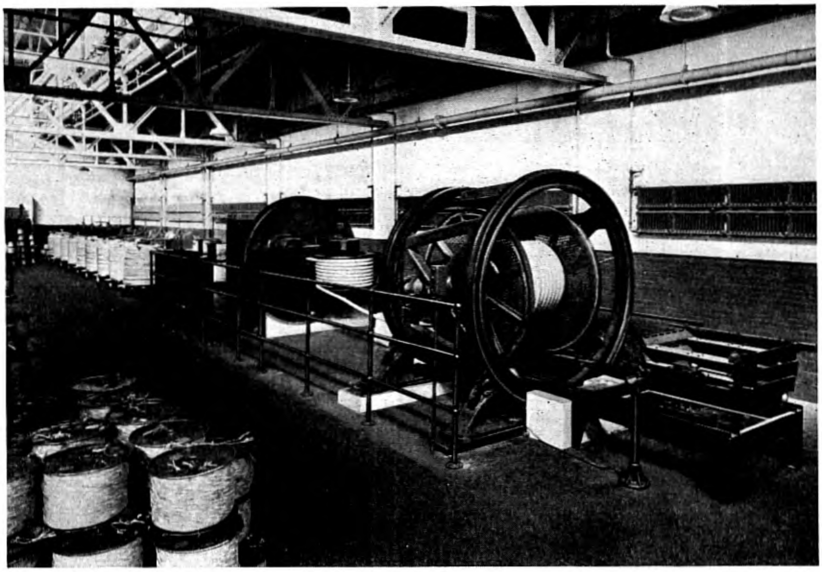


Fig. 14—Cabling machine.

The principal advantages of this construction are that slightly less copper and paper are required in large sizes of cable due to the shorter lay in the outer strands. With the same investment in machinery and building, a much larger production may be obtained. Much finer gauges of wire may be stranded without danger of stretching beyond its elastic limit.

Vacuum Drying

Dry paper is an excellent insulation for the conductors of a telephone cable, but it must be bone dry. Dry paper takes up moisture rapidly and 1000 lbs. loosely packed in a few hours will absorb 90 lbs. of moisture in a room at summer temperature and 60 per cent relative humidity.

A vacuum drying operation is applied to stranded cable prior to the lead sheathing operation at a temperature of 270° F. for a period of from 12 to 42 hours, depending on the size of cable. The vacuum maintained toward the end of the drying cycle is less than 2 in. Hg.

The vacuum drying system installed at the Point Breeze plant has incorporated in its design many improvements in order to improve cable quality, and also to reduce that part of the manufacturing cost.[8] It consists of fifteen horizontal driers, each 40 ft. in length and $7\frac{1}{2}$ ft. in diameter, and one horizontal drier 40 ft. in length and 10 ft. 4 in. in diameter (Fig. 15). The former driers are used for the ordinary toll

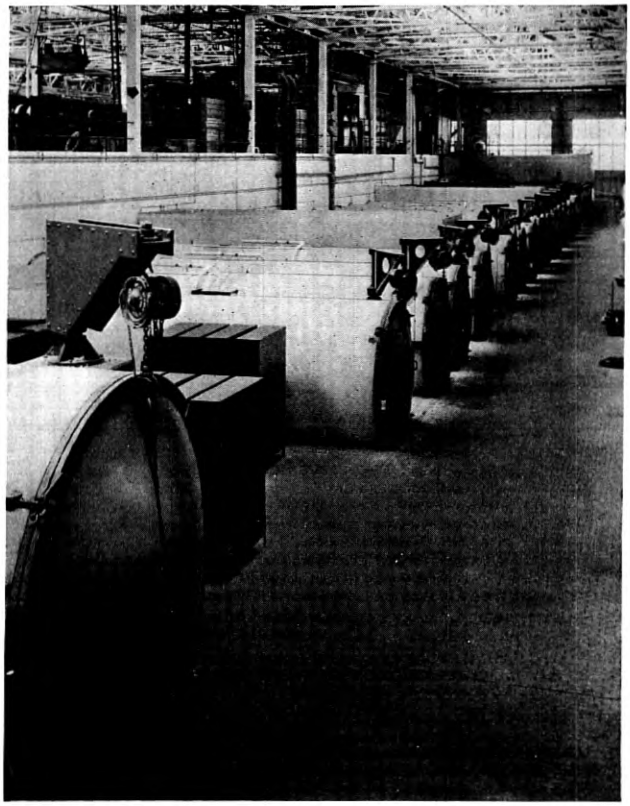


Fig. 15—Vacuum driers.

cable, while the latter single tank is used for drying submarine cables of long lengths.

The drying ovens are arranged so that the loading end is located in the cable room proper, and the unloading end in the dehumidified cable storage room (Fig. 17). To prevent the exfiltration of dry air from the storage room through joints between oven and brick wall a novel type of seal is used. This consists of a flexible sheet of copper, to allow for tank expansion, fastened and gasketed on the inner cir-

[8] For further discussion and detailed factory layout of this system, see paper by J. C. Hanley, *Mech. Engg.*, March, 1931.

cumference to the tank, and on the outer circumference to the brick wall of the storage oven.

Auxiliary equipment used with the vacuum drying ovens consists of two welded vacuum lines (twelve inches in diameter) vacuum pumps, condensers and receiver tanks. A general view can be obtained from Fig. 16.

Fig. 16—Auxiliary equipment for vacuum driers.

One vacuum line is used to establish vacuum in a new tank load of cables, and the second is used for maintaining vacuum in the tanks once they have reached the proper point. The pump equipment consists of four reciprocating feather valve vacuum pumps. The pistons on these pumps have a diameter of twenty-nine inches and a stroke of eighteen inches or a displacement of ten hundred and twenty five C.F.M. The pumping capacity has been based on maintaining absolute pressures of one-half to one inch in the vacuum tanks. These values are based on a vacuum tank activity of eighty-five per cent and on maximum leakage of approximately twenty pounds of air into each tank through the door gaskets.

Two two hundred and twenty five C.F.M. surface condensers are incorporated in the layout ahead of the pumps to condense moisture given off by the insulated paper. Three thousand pounds of water may be extracted in twenty-four hours.

New features incorporated in the oven are design changes of the heater coil and tank. This coil, of which there are four in each oven, consists of steam header inlet and outlet, instead of a continuous length of eleven hundred and twenty feet of pipe. This type of coil not only makes a much neater appearance in the heating system due to its rigidity, but also insures positive draining, with the elimination of steam hammer, and also more uniform heating in all portions of the tank. The tanks are completely welded instead of riveted. This method of assembly insures a better average vacuum as well as eliminating considerable maintenance work in caulking rivets, which become loosened by the repeated expansion and contractions of the drier.

Cable Storage Prior to Lead Covering

The air conditioned room (Fig. 17) is provided for the storage of cable prior to lead sheathing in order to facilitate the covering of varying

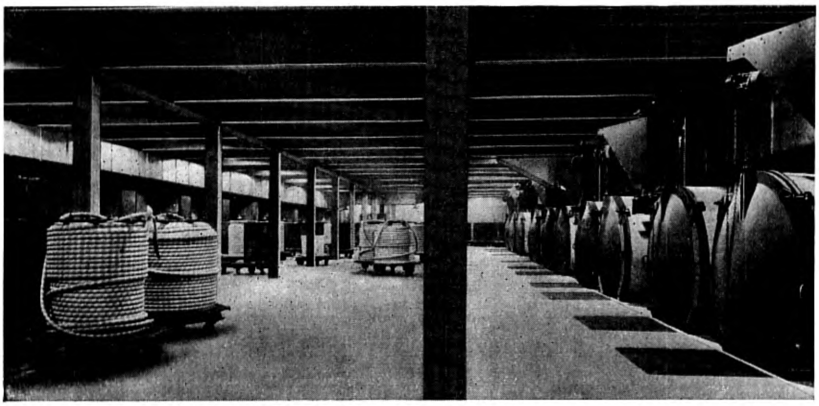


Fig. 17—Air conditioned cable storage room.

diameters of cable with a minimum of lead-press die-block changes, and also to act as a reservoir for the fluctuating delivery of large quantities of vacuum-treated cable. An alternative, that of storing cable in the vacuum driers until ready for lead covering would require an excessive investment in vacuum drying tanks and their operation.

The storage room, from which the cable is paid out directly to the presses, is approximately 270 ft. long, 50 ft. wide and 12 ft. high, and has been designed to prevent infiltration of moisture. Without moisture proofing, the outside wet air would penetrate a concrete or brick wall since the vapor pressure in the storage room is only approximately .007 in. Hg as compared to 1.02 inch outside the room on a hot humid day. The moisture proofing was accomplished as follows: An aluminum foil was placed over the inner surface of the outer portion of the brick wall. This foil was suitably protected by a layer of saturated rag felt and roofers asphalt. The remainder of the brick wall was placed in position over the moisture proofing membrane. The floor was prepared in a similar manner.

The concrete ceiling of the room was covered with a layer of aluminum foil suitably overlapped and held in place by varnish.

As an added protection, all entrances are vestibuled and all cable ports are equipped with air tight cable tubes leading to the presses. When the press is not in use an air tight door is closed over the inner end of the cable port.

Air Conditioning Equipment

The primary object in drying toll cable is to obtain as low conductance and capacitance values and as high insulation resistance as possible. This has a very important effect on the transmission quality of the cable, and consequently justifies considerable expense.

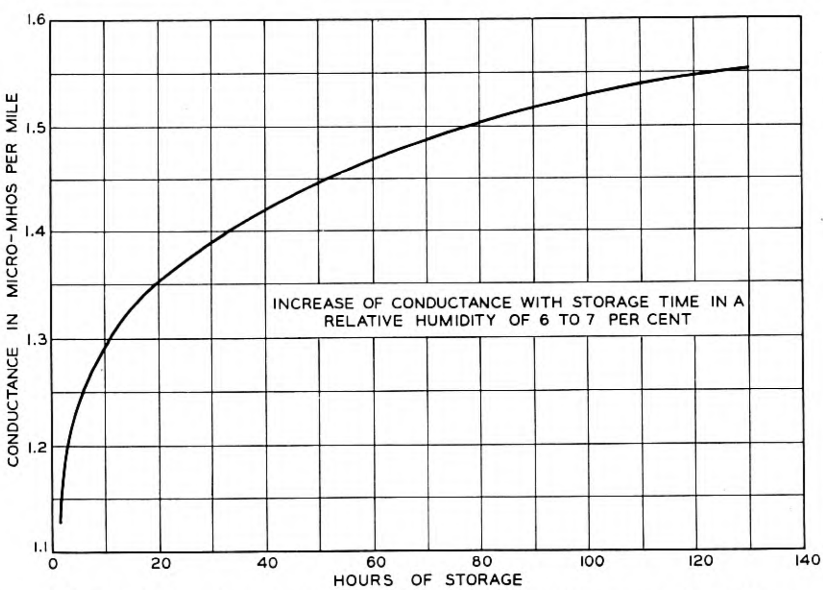


Fig. 18—Effect of moisture regain on conductance of vacuum dried cable.

A large amount of experimental work has been done to determine the best methods of obtaining and retaining dry cables. At the end of the vacuum drying cycle the cable paper is in such a dry condition that its moisture regain when exposed to higher humidities is exceedingly rapid. This is indicated by Fig. 18, showing the increase in conductance over a period of hours when dry cable is exposed to approximately 6–7 per cent relative humidity.

Working from these data and an estimate of the manufacturing advantages from storage due to the elimination of lead press changes, it was decided that a minimum moisture condition of .5 of one per cent with storage periods not greater than 24 hours would result in minimum conductance and capacitance values consistent with manufacturing costs. The limit of .5 of one per cent was decided upon since to maintain humidities lower than that, costs would increase very rapidly and entirely out of proportion to the change in relative humidity conditions and the final result.

The air conditioning equipment installed at the Baltimore plant is unique, in that a relative humidity of .5–.8 per cent is maintained at a temperature of 100° F. without resorting to refrigeration. Silica gel, highly porous form of silicon dioxide, or sand, is used as the water absorptive medium. Before deciding upon this method of dehydration other existing types of equipment were investigated. To obtain such low humidities with the usual types of dehumidification systems would require more than one stage of cooling and result in more expensive operation costs in comparison with silica gel units.

The design requirements of this equipment were based on data established for the following:

(1) Heat losses in the walls and infiltration of moisture.
(2) The movement into the storage room of core trucks filled with dry cable at temperatures of approximately 260° F., and the incident rush of storage room air into the vacuum driers when the vacuum was broken.
(3) The loss of conditioned air when cables are being pulled through the bell mouth openings to the press and also when the storage room doors are opened.
(4) The actual moisture content of outside air, which must be dried to replace losses in the storage room.

Based on a summary of the B.T.U. losses and gains which could be expected in the manufacturing process, a study of the Baltimore temperature conditions over a period of years, and an analysis of the humidity conditions which would be encountered, equipment was designed which will handle a volume of 13,000 cu. ft. of air per minute amounting to a complete change of room air five times per hour. Of this total amount approximately 10,300 cu. ft. is re-circulated, cooled, dehydrated and brought back to the storage room requiring adsorber capacity for only .6 pound of water per minute. Twenty-six hundred cu. ft. of air is drawn from the outside to compensate for air losses at various points in the room and to maintain an overall room pressure of about $\frac{1}{2}$ ounce in excess of outside air pressures, requiring additional adsorber capacity of approximately 4 pounds of water per minute.

To maintain a normal operating temperature, it is necessary to remove 17,500 B.T.U.'s per minute. This is accomplished by cooling the air which is re-circulated plus the fresh air taken into the system to 72° F.

The method of air distribution within the storage oven was carefully designed since the rate of regain of moisture by paper insulated cable is dependent not only on the difference in vapor pressure of the cable paper itself and that of the air passing over it but also on the velocity of the air. The dry air is supplied through grill openings along the side of the room at approximately 3–4 ft. from the floor, and at low velocities consistent with positive circulation. Thus the driest air is supplied at the point where it is most needed and, since the return

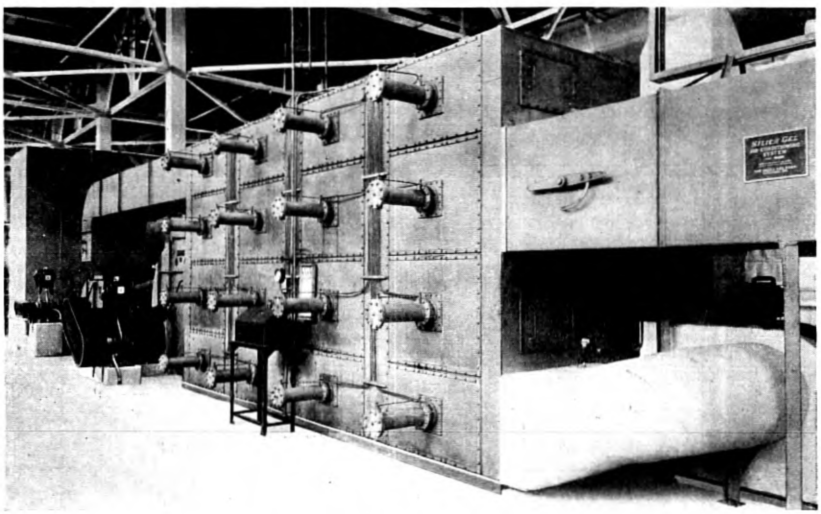
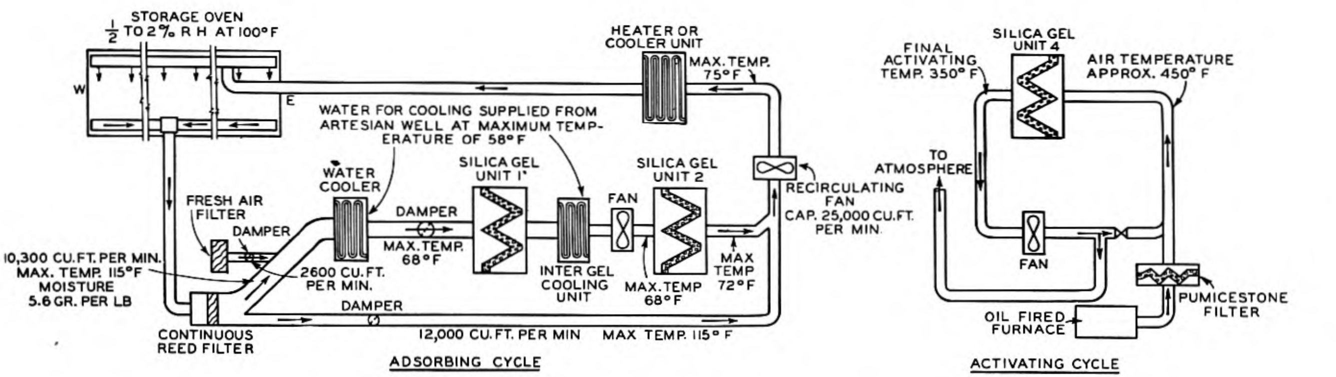


Fig. 19—Silica gel drying unit.

ducts are located at the ceiling opposite to the grill openings, any regain of moisture in the room itself is largely concentrated in air strata above the cables.

Operation of the Baltimore conditioning system (Fig. 19) may be described briefly as follows: Approximately 10,300 cu. ft. of air per minute from the storage room is mixed with 2600 cu. ft. per minute of outside fresh air. The temperature of this air mixture which may be as high as 100° F. is lowered to a maximum of 68° F. by passing it over and around copper tubes through which water at 58–60° F. is circulating. The cool air then passes through the first silica gel adsorber where it is partially dehydrated; then it is again cooled and is passed into the second adsorber where the drying is completed and from which



COMPLETE AUTOMATICALLY OPERATED CYCLE				
CYCLE	UNIT 1	UNIT 2	UNIT 3	UNIT 4
1	ADSORBING 1	ADSORBING 2	COOLING	ACTIVATING
2	ACTIVATING	ADSORBING 1	ADSORBING 2	COOLING
3	COOLING	ACTIVATING	ADSORBING 1	ADSORBING 2
4	ADSORBING 2	COOLING	ACTIVATING	ADSORBING 1

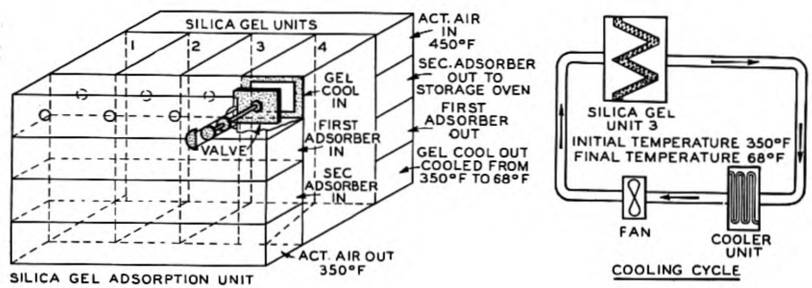


Fig. 20—Operating cycles on silica gel unit.

it passes to the supply ducts in the storage room. The system is so constructed that there are three simultaneous cycles (Fig. 20): one in which the silica gel is used as an absorbent, the second where it is reactivated, and the third where a freshly reactivated bed is cooled to 68° F. Automatic controls switch the air currents into their respective channels at established intervals. The condition of the vital parts of the system is indicated continuously on a control board where temperatures, air volumes, and relative humidities [9] are shown.

Lead Sheathing

The thoroughly dried cable core passes from the storage oven through a tube, designed to minimize any exposure to outside air, into the press where it receives its protective cover of lead. The

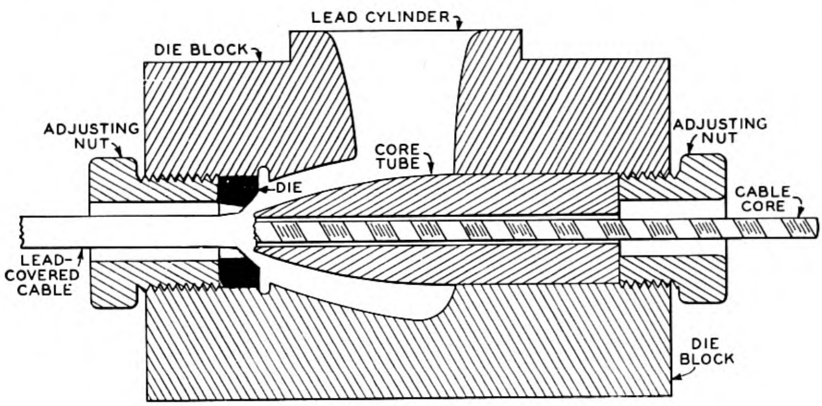


Fig. 21—Cross section of typical die block.

basic principle of applying lead sheath to cable is illustrated by Fig. 21 which shows a cross-section of a typical die block. This die block consists of a core tube and a die, ring shaped, mounted in a hollowed-out block. This arrangement provides an opening adjacent to the cable core which aids in definitely controlling the thickness and diameter of the sheath. This die block is placed underneath a large cylinder for receiving molten lead, and both are placed in a hydraulic press.

In covering large cable, more than half of the total time is taken up in filling the cylinder with lead and cooling it under pressure to a point where it can be extruded. The tendency, therefore, has been to build presses with larger lead containers, and in turn of larger capacity,

[9] Page 134, Vol. 2, *Industrial and Engineering Chemistry*, April 15, 1930—article by A. C. Walker and E. J. Ernst, Jr.

in order to make the productive time of extrusion a larger percentage of the complete cycle of operation. Until recently presses were used having a 30 in. diameter ram and a 42 in. stroke. Such a press has a capacity of 1100 lbs. of lead per charge and extrudes a maximum of 4500 lbs. per hour. This type of press has the water ram located below the floor line. The die block and lead cylinder therefore rise slowly as the lead is forced out around the cable core. This varying height of the cable as it is extruded in relation to the floor introduced some difficulties in the operation.

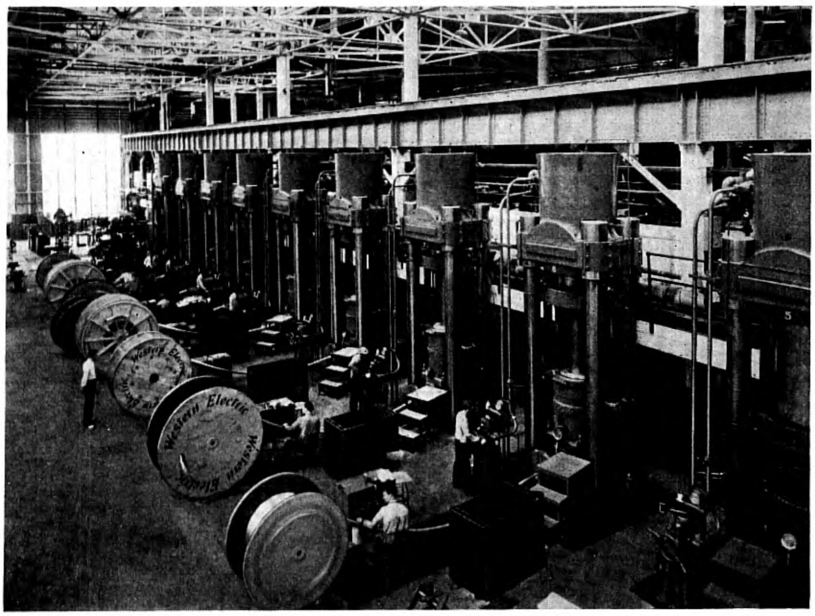


Fig. 22—34-in. inverted press.

The latest type of press used at Baltimore is illustrated by Fig. 22 and is known as the 34 in. inverted press. It was designed and built by one of our outstanding American engineering firms. Its stroke is 56 in.; the diameter of the ram is $10\frac{1}{2}$ in., with a lead capacity of 1800 lb. per charge and a maximum extrusion rate of 5680 lb. per hour. This press is approximately 21 ft. in height above the floor line, and has the water cylinder mounted between the four columns at the top of the press. The 34 in. diameter water ram has the steel lead ram bolted to it. Connection is made from the water cylinder to a hydraulic pump, Fig. 23, supplying water at a maximum pressure of

5500 lb. per sq. in. The four steel columns supporting these top castings are $12\frac{1}{2}$ in. in diameter. The steel ram exerts during extrusion a pressure of approximately 59,000 lb. per sq. in. on the lead. At the floor level of the press there is a cast-steel plate which carries a steel spacing block upon which the die block rests. Above the die block is a water-jacketed lead cylinder which is exactly centered over the feed orifice of the die block. The die block and lead cylinder are held in place on the cast-steel plate by four $2\frac{1}{2}$ in. bolts. All these parts are stationary on this press, facilitating handling and inspection, and insuring that the cable core always enters and leaves the die block at the same angle.

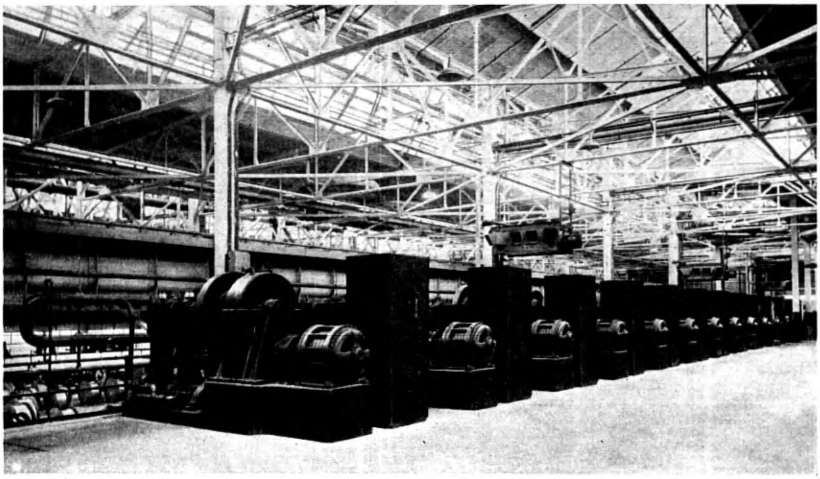


Fig. 23—Lead press hydraulic pumps.

The concentricity of the sheath is affected not only by the contour of the extrusion chamber, including core tube and die, but also by the manner in which heat is applied; and the thickness is affected by temperature and speed of extrusion so that the human element is an important factor, and it is necessary to have thoroughly trained and reliable operators on this kind of work. Temperature indicators are used to show die-block temperatures, and the temperature of the molten lead is automatically controlled and recorded.

Aside from increasing output, many studies have been made to determine the exact mechanism of lead extrusion, the relative flow of lead in different parts of the extrusion block, the effect of application of heat at different points, etc.

As the lead-covered cable leaves the press, it is wound upon either wood or steel reels, depending upon its type. A full reel may weigh as much as 10,000 lbs. These reels are rotated by means of power-driven floor rolls which are controlled by the press operator's helper. After the reel was filled with cable, it was formerly the practice to push the reels off the rolls manually. The latest type of floor rolls are equipped with automatic ejector devices which lift one roll and cause the loaded reel to roll off on the floor. This is done by means of a small hydraulic cylinder connected to a pump which is operated by a valve mounted adjacent to the floor rolls.

The Central Lead-Melting System

In order to supply the presses just described, large quantities of lead-antimony alloy must be delivered frequently. The old and new arrangements are shown in Fig. 24. With the old arrangement lead

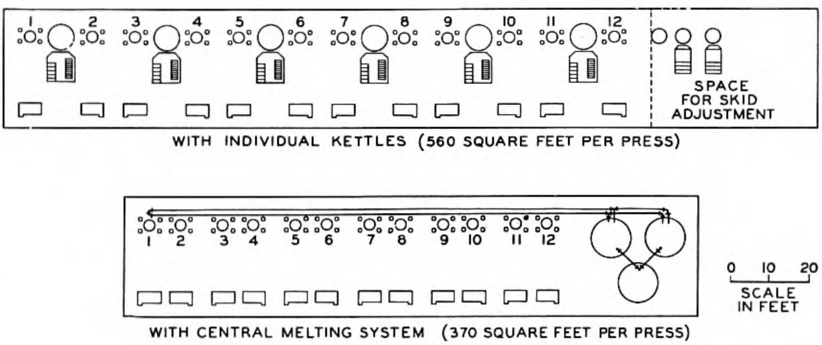


Fig. 24—Space required for 34-inch inverted presses.

was delivered in skids by an overhead traveling crane to small melting kettles adjacent to each pair of presses. This arrangement also involved considerable manual handling, and introduced some variation in the finished alloy sheath.

The new arrangement consists of melting all of the lead alloy in a large furnace at a central location and distributing this molten lead through a long-loop pipe line running back of the presses. Near each press a loop branch from this line is made and equipped with the proper kind of control valve. This line is heated electrically and the lead is in constant circulation. Such a system was built on a small scale and tested under continuous operation for over a period of six months, at the conclusion of which it was considered entirely feasible to incorporate it as a part of our new Baltimore plant. In order to take full advantage of such a system, the presses were placed close together, thus saving the space formerly occupied by the small individual melting kettles, and the large central-supply kettles were placed adjacent to the lead storage pit in order to minimize handling. Views of this system now in use at Baltimore are shown in Figs. 25–28.

The details of this central lead-melting and distributing system will be of interest to manufacturers using large quantities of lead or lead alloy. Three oil-heated kettles are used (Fig. 25), and pipe and valve arrangements have been set up so that the middle kettle is used for melting and preparing the alloy to the exact composition. The second kettle

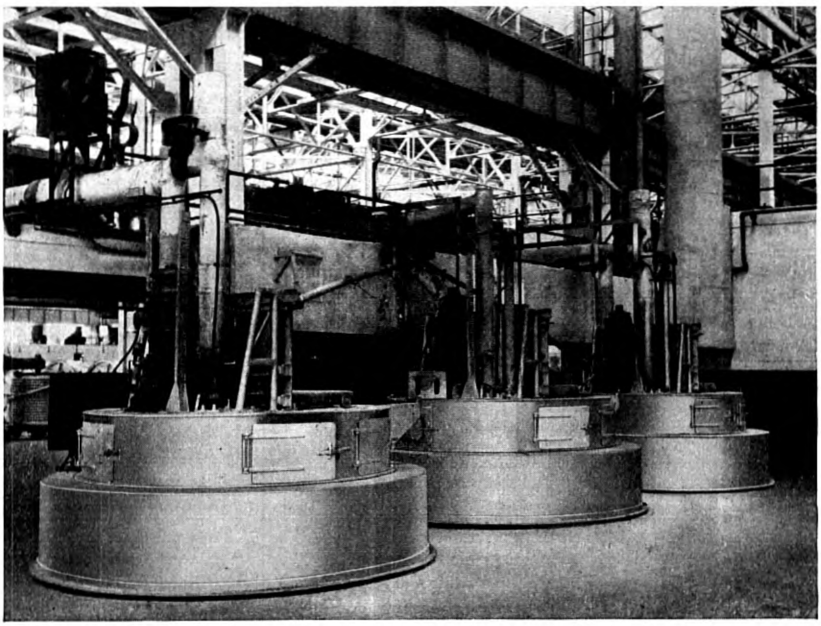


Fig. 25—General view of melting and supply kettles.

is used as a main supply and connected up to the distributing system. The third kettle is a spare, and the piping is so arranged that it can be used either as a melting or supply kettle. Each kettle has a capacity of 120,000 lbs. of lead, and the melting capacity of the system is 80,000 lbs. per hour. Space is provided for a fourth kettle to take care of the ultimate expansion of the cable plant.

Each kettle has two sets of low-pressure oil burners installed diagonally across from each other. An impeller type of vertical pump having its intake about 12 in. above the bottom of the kettle, and driven by a 20 hp. vertical motor, creates sufficient agitation by the circulation of the metal to assure a uniform composition.

The charging of the melting kettle with virgin lead is accomplished by means of a specially designed lead-handling grapple (Fig. 26) which has a capacity for 100 billets of the standard size or a total weight of about 8500 lbs. Five to six of these charges or about 40,000 to 50,000 lbs. constitutes one melting cycle. The corresponding amount of

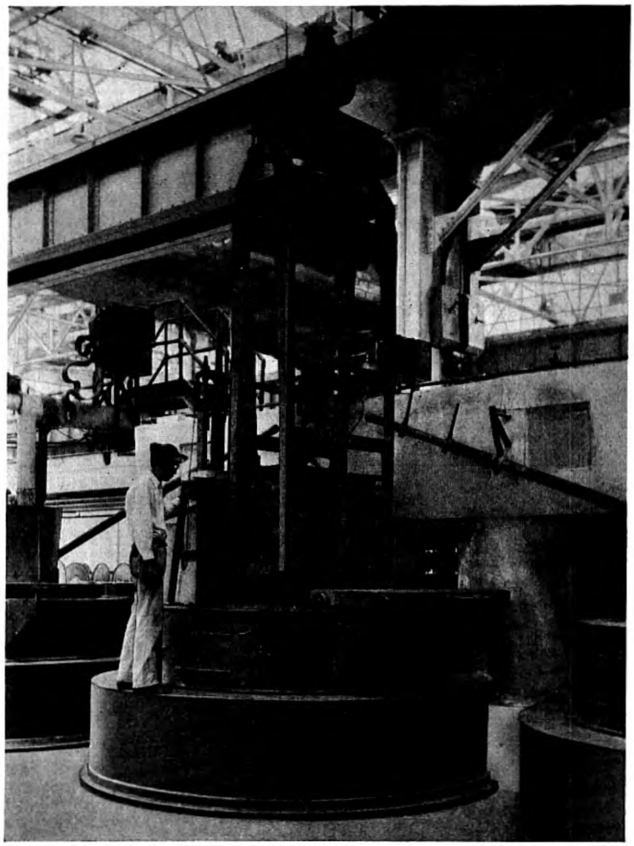


Fig. 26—Charging of lead-melting kettle.

antimony is loaded into a special cradle which moves in a separate chamber and is lowered below the surface of the lead, where the antimony is dissolved by the washing action of the stream of lead from the return line of the pump (Fig. 27).

The supply kettle is charged with the desired amount of molten lead of the correct composition and temperature from the melting kettle by means of the pump on the transfer line. Each kettle has one recording controller for regulating the temperature and one controller as a check instrument and to actuate an alarm if the temperature goes above or below a predetermined limit. Each instrument has its own thermocouple.

To reduce to a minimum the possibility of a prolonged shutdown due to a breakdown in the lead conveying line, a duplicate pipe system

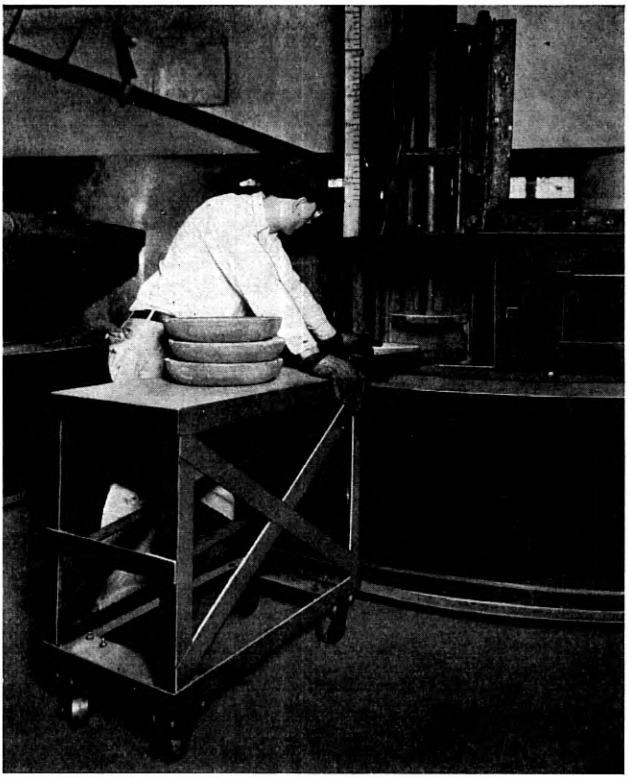


Fig. 27—Antimony charging mechanism.

is provided which can be put into service in a short time in case of failure of the line in use. The line ordinarily used is the one nearest to the presses and is the service line while the line one foot to the rear but at the same height, is called an emergency line.

The main-line piping system is made of seamless steel tubing supported on a roller-conveyor system to take care of the expansion and contraction which amounts to $6\frac{1}{2}$ in. per 100 linear feet at 750° F. or a total of approximately 20 in. under normal working conditions for the system. The down spouts are of seamless steel tubing and have a steel valve at each joint with the main line and a service valve at one corner of the "U" bend. All joints are oxyacetylene welded, and no fittings are used throughout the system. The lines are insulated with pipe covering protected by a layer of fireproofed canvas (Fig. 28).

The lines are heated initially by a series of transformers which supply a low-tension, high-amperage current directly into the pipe by forming a loop of the supply and return line. Once circulation of the lead has been established in the piping system, the main line requires little additional heat from the transformers, as the flow of the

Fig. 28—Main lead supply lines.

lead will ordinarily keep the line up to temperature. Approximately 4 KVA are required on each down spout while in use. The connections leading from the transformer to the pipe are flexible, to allow for expansion and contraction of the system.

Switches are provided on each building column opposite the presses to enable an operator to shut down the pumping system in case of a serious leak or failure of a valve.

This system has been in operation for about nine months and has resulted in a higher quality of lead sheath due to more uniform composition maintained. In addition there are considerable savings in fuel, reduction in dross, and elimination of a large amount of heavy manual effort. The press room is now clean and cool, resulting in much better working conditions and in turn an indirect improvement in the quality of the product.

Testing Lead Covered Cable

After the cable is stranded each conductor is tested from end to end for continuity and against every other conductor for crosses. Defects are repaired and after the cable core has been dried the lead sheath is applied. After the application of the sheath the cable is allowed to stand until it cools to room temperature. Fig. 29 shows the cooling floor and test mezzanine in the Point Breeze cable plant. The reels of cable issue from the lead presses at the right; are cooled in the central area and tested beneath the mezzanine at the left.

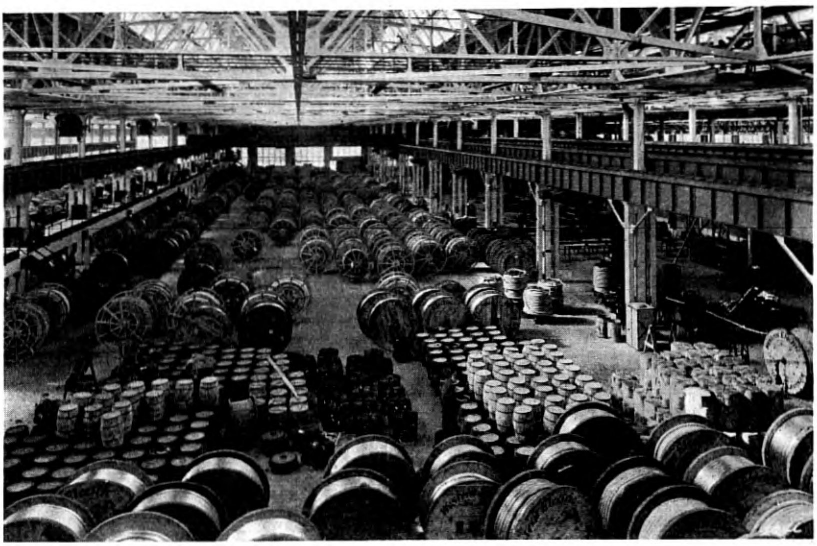


Fig. 29—Cooling floor and test mezzanine.

When the cables are cooled the conductors are given a final test for opens and crosses which may have developed due to strains imposed during the sheathing process. Most toll cables have a number of spare wires and if fewer than the allowable number of above defects are found the cable is tested for dielectric strength, insulation resistance, mutual capacitance, capacitance unbalance and defects in the sheath. Dielectric strength tests are made between each conductor and every other adjacent conductor to which failure may occur and between all conductors and the lead sheath. The potential used for these tests ranges between 350 volts, A.C., the lowest value used for certain conductor to conductor tests and as high as 5,000 volts, A.C. for some conductor to sheath tests. In making the conductor to conductor tests a large number of circuits are involved so that interesting problems arise in designing switching devices to apply the test potential between all conductors.

Defects found by continuity, cross or dielectric strength tests must be located within the cable in order that repairs may be made. The point of break in open conductors is located by comparing the capacitance between the defective conductor and the adjacent conductors with the capacitance between a conductor known to be good and its adjacent conductors. Preliminary locations of crosses between conductors and between conductors and the sheath are made by means of the modified Murray Loop test. Final locations are made by means of a search coil and telephone receiver which responds to currents of audible frequency circulated through the crossed conductors.

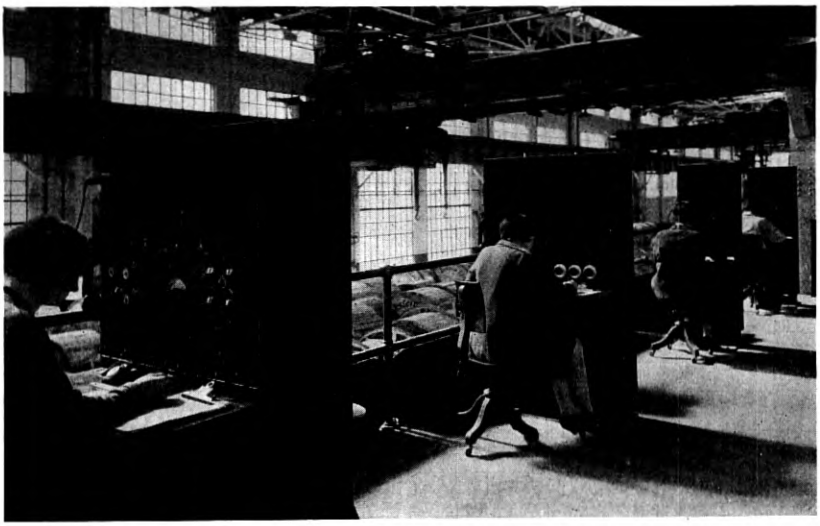


Fig. 30—Typical test set installation at Baltimore plant.

Fig. 30 shows a closer view of a section of the test mezzanine at the left of the cooling area. The test desk in the foreground is designed for making insulation resistance, D.C. capacitance, A.C. capacitance, and conductor resistance tests. The test desk in the center is a shielded precision bridge for making capacitance and conductance measurements at audio frequencies. Two test desks in the background are capacitance unbalance bridges. All desks on the mezzanine floor are provided with test leads which terminate in outlet boxes on the test floor below.

Figs. 31 and 32 show a front and rear view respectively of the insulation resistance, D.C. capacitance, A. C. capacitance meter, and conductor resistance test desk which appears in the foreground of Fig. 30. Insulation resistance measurements are made between conductors and between all conductors and the sheath by observing the deflection obtained with a high sensitivity reflecting type D'Arsonval galvanometer through which a potential of 500 volts D.C. is impressed on the insulation of the conductors under test. Due to the

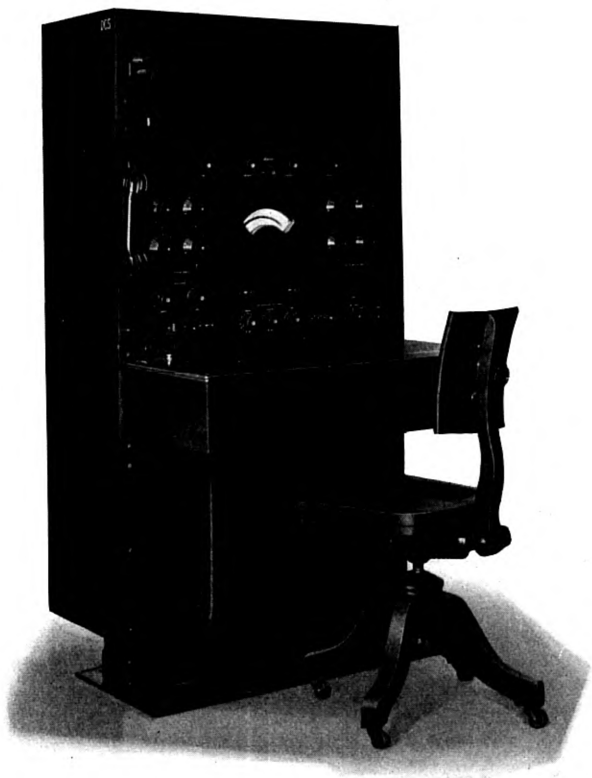


Fig. 31—D-C. insulation resistance test desk—front view.

high insulation resistances involved and the extreme sensitivity of the measuring circuit, considerable difficulty is likely to be encountered with leakage in the test apparatus itself, especially during times of high relative humidity. To overcome this source of error special test circuits have been designed which employ a shield (Fig. 33) to eliminate from the measurement all extraneous leakage other than that of the cable. The direct reading capacitance meter is used extensively for mutual capacitance measurements where the highest accuracy is not essential and where conductance readings are not desired. D.C. capacitance tests are made by the charge and discharge method, employing a ballistic galvanometer. In general, D.C. capacitance tests are not fully indicative of the characteristics of the cable at telephonic frequencies and for this reason are not extensively employed.

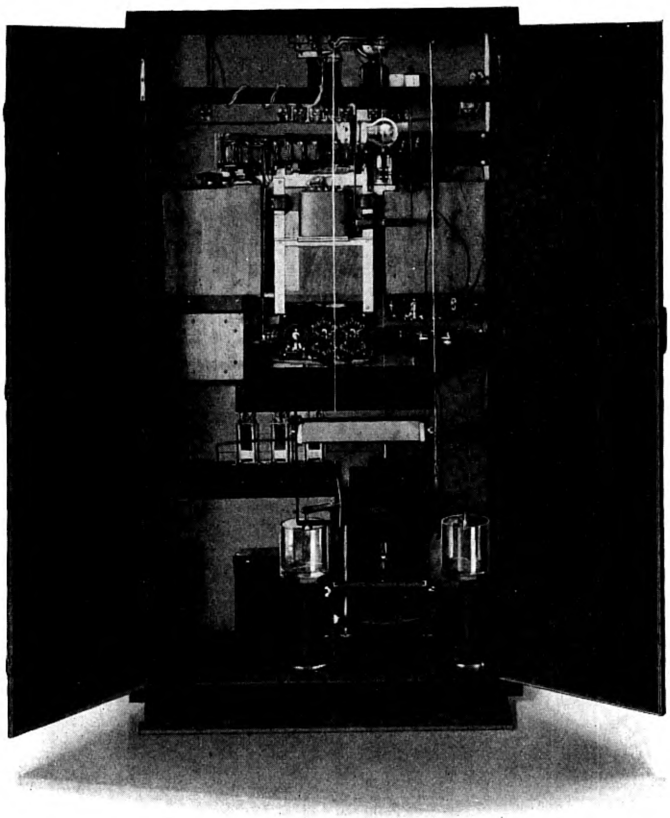


Fig. 32—D-C. insulation resistance test desk—back view.

Conductor resistance tests, Fig. 34, are made by means of a Wheatstone bridge circuit specially arranged to read directly the conductor resistance per mile at 68° F.

Although the majority of mutual capacitance measurements are made by means of the direct reading capacitance meter, the capacitance and conductance of a percentage of all cables are measured at a frequency of 900 cycles per second by means of the shielded capacitance bridge.[10] Due to the fact that these bridges are frequently employed in shop areas where some noise exists it has been necessary to develop a

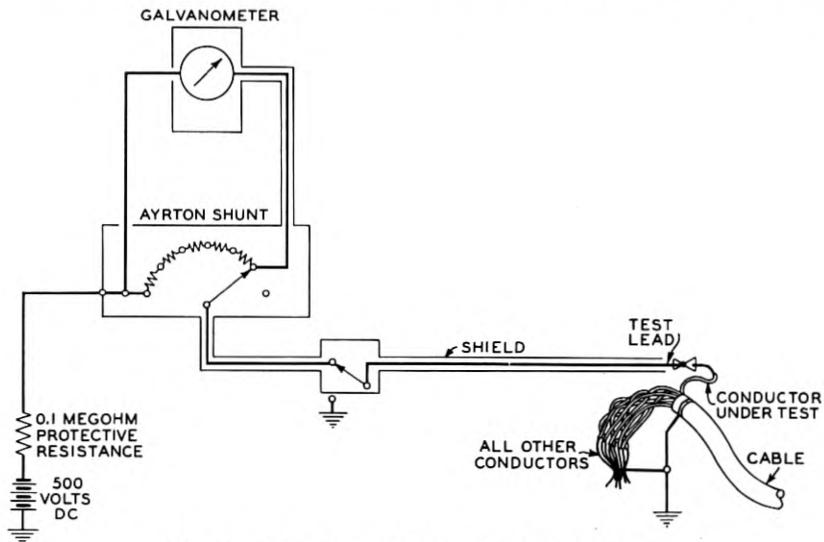


Fig. 33—Shielded insulation resistance test circuit.

device to replace the telephone receiver as a means of indicating bridge balances. The visual bridge balance indicator used consists essentially of a vacuum tube circuit in which the alternating current

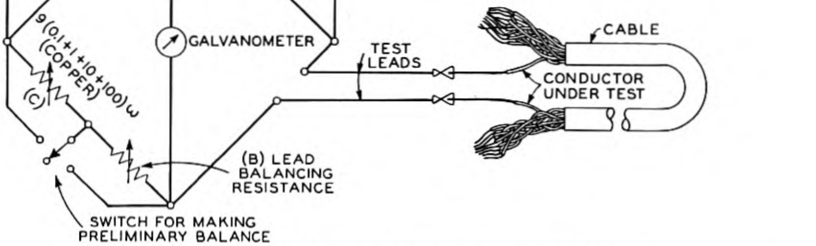


Fig. 34—Conductor resistance measuring circuit.

[10] *Bell Sys. Tech. Jour.*, July, 1922: "Measurement of Direct Capacities," G. A. Campbell. *Transactions A. I. E. E.*, Vol. XLVI, May, 1927: "High Frequency Measurement of Communication Apparatus," W. J. Shackelton and J. G. Ferguson.

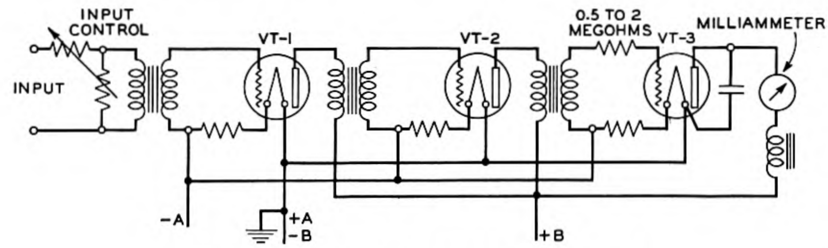


Fig. 35—Visual bridge balance indicator circuit.

input to the indicator is amplified and the rectified output is indicated by the reading of a D.C. milliammeter. When the bridge is balanced there is no input to the indicator so that the milliammeter pointer

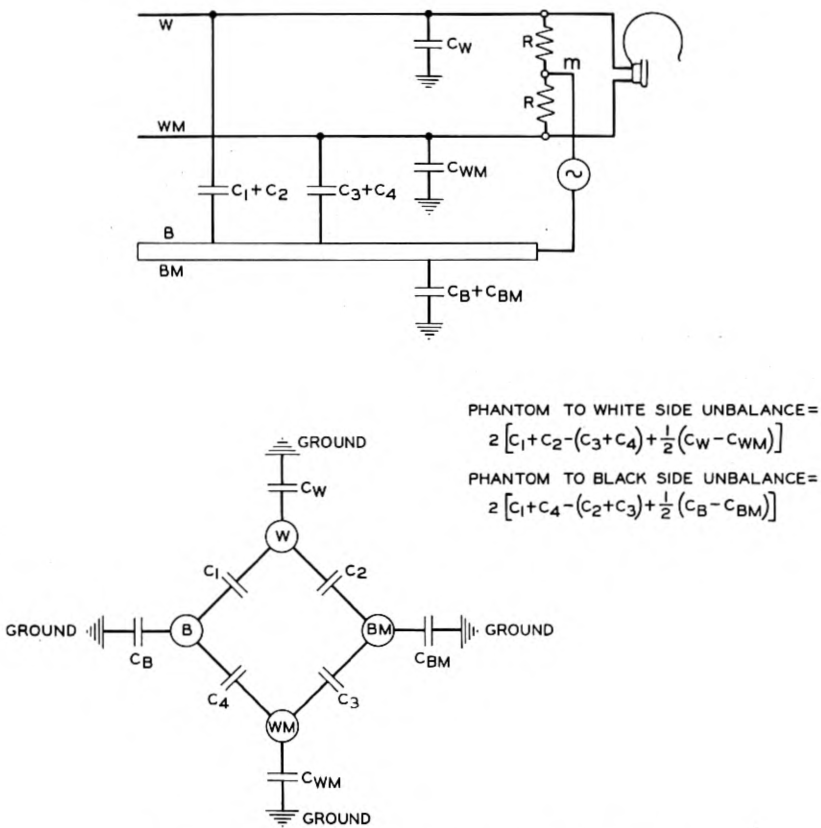


Fig. 36—Phantom-to-side capacitance unbalances.

returns to the lower end of the scale, See Fig. 35. The high resistance in series with the grid of the third or rectifier tube prevents the overloading of the milliammeter when a large input voltage is impressed on the indicator circuit.

Toll cable in addition to the above receives a capacitance unbalance

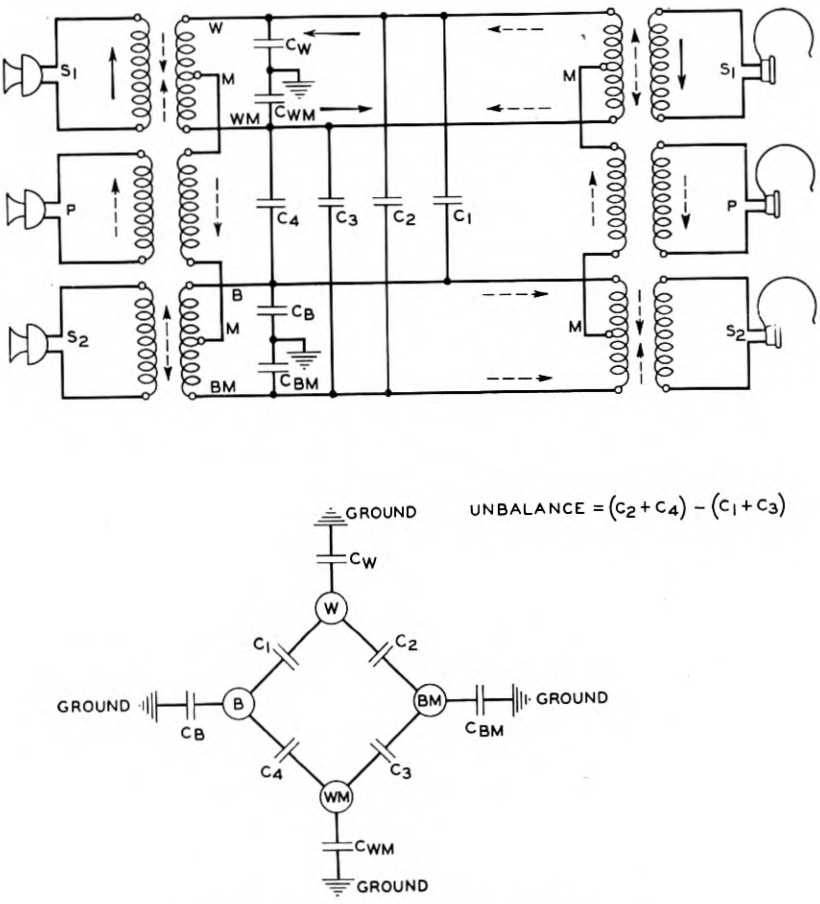


Fig. 37—Side-to-side capacitance unbalances.

test which is indicative of the cross-talk existing between circuits. These tests are made with a special shielded bridge mentioned above, which measures the capacitance unbalance between side and phantom and side circuits of "quads" (Figs. 36 and 37). These bridges are also provided with visual balance indicators as described above.

After the cable has successfully met all electrical requirements both ends are sealed and the cable is prepared for shipment. Certain types of cable receive an additional gas pressure test to detect minor defects in the lead sheath which otherwise may have escaped attention. Dry nitrogen is forced into the cable to a predetermined pressure and the cable is allowed to stand for a specified period. Loss of pressure during the test period indicates that the sheath or seals contain one or more defects.

Armoring of Telephone Cable

Two types of armored telephone cable are in use, Fig. 38. Submarine telephone cables for rivers and harbors are usually protected by

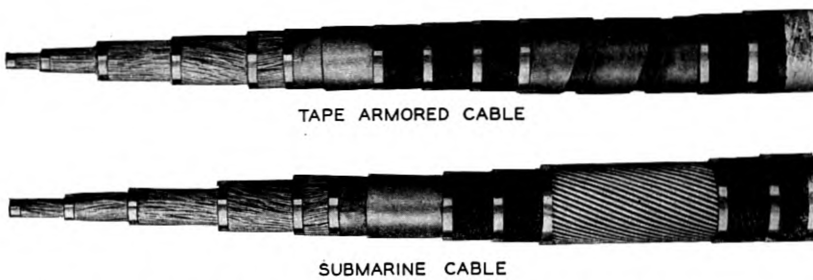


Fig. 38—Typical construction of tape and wire armored cables.

layers of jute and wire placed on the outside of the lead sheath. This type of armor is quite familiar and is called wire armoring. Cable buried in a dirt trench is armored in a similar way except the wire is replaced by two layers of steel tape. This is called tape armoring. It is adapted to certain localities where there are long stretches of open country and the conditions indicate one or two cables will handle the requirements for a considerable number of years.

A typical wire armor is made up of a bedding of 100 or 150 pound jute roving, impregnated with suitable preservative after serving, by passage through immersion troughs, over which a layer of armor wires is applied. In some cases, a covering of outer jute flooded with coal tar is used. When an unusual degree of protection is desired, a second layer of armor wire is applied. In such cases a bedding of jute is used between the layers.

Recent trends in the design of wire armored cables are leading toward cables of much larger diameter. At the Point Breeze plant there is an unusually large wire armoring machine (Fig. 39). It is designed to handle cable up to $5\frac{1}{2}$ inches in diameter over the armor.

Tape armored cable differs somewhat in construction depending upon the kind and diameter of cable armored. A typical design is made up as follows: A coating of asphalt is first applied to the cable and over this a layer of impregnated kraft paper. Another layer of asphalt compound is put on and then two servings of impregnated jute roving

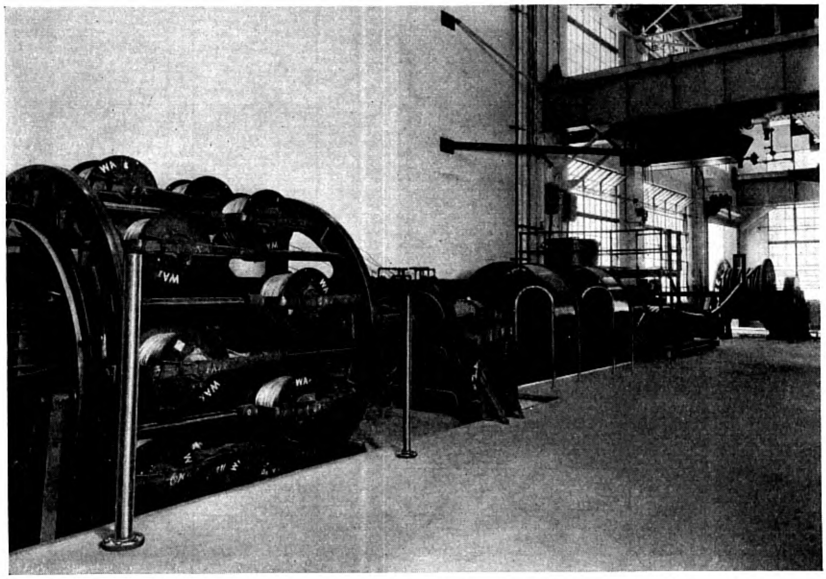


Fig. 39—Wire armoring machine.

with opposite directions of lay. Asphalt coatings are used between the two servings and on the outside of the second. Next two steel tapes are served with the same direction of lay and with the second tape overlapping the gap between the edges of the first. Again the cable is given a coat of asphalt. One serving of impregnated jute roving, a coating of asphalt and a layer of impregnated jute yarn with opposite direction of lay are next applied. An application of non-adhesive compound composed of whiting, glue, and water completes the armor coating. The machines used for tape armoring are shown by Fig. 40. They consist of a supply position for the lead sheathed cable, asphalt tanks, paper heads, jute heads, two steel tape heads, a capstan and take-up. Tanks for melting the asphalt compounds before their use in the machine are also provided. This type of cable is protected from mechanical injury and soil corrosion, and can be laid very quickly and cheaply. One interesting advantage gained through the use of this type of armor is that a magnetic shield is thus placed around the cable greatly reducing the effects of induction.

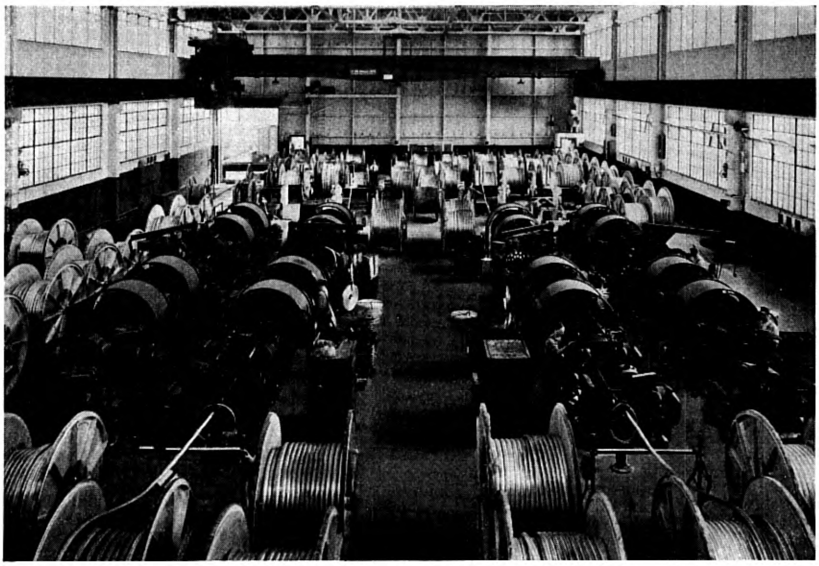


Fig. 40—Tape armoring machine.

Conclusion

The application of scientific and engineering effort to improvements in the processes and machine equipment for manufacture of telephone cable is fully justified by the results which have been obtained from both an economic and quality standpoint. New raw materials and alloys together with new designs of cable will be forthcoming in the future in the effort to improve and extend the long distance telephone service. New communication devices will be invented and perfected for use in connection with such cable and these in turn will have a radical effect upon the cable design, the process and the equipment for its manufacture. The engineers and scientists engaged in such manufacturing activities are indeed rendering a broad service not only to the men and women employed in the immediate industry but also to the people at large who use these facilities.

In concluding, the writer wishes to acknowledge the efforts of the men who have carried these developments to a conclusion, in particular Mr. H. G. Walker on the pulp wire process; Mr. L. O. Reichelt and Mr. H. J. Boe on the unit cable machinery; Mr. H. F. Carter on the central lead melting system; and Mr. J. Wells on the air conditioning system.

Effect of Ground Permeability on Ground Return Circuits

By W. HOWARD WISE

The formulas for the self and mutual impedances of ground return circuits are derived without restricting the ground permeability. Curves are given to show the effect of a ground permeability 1.7 on the mutual impedance between two parallel ground return circuits with the wires lying on the ground.

ON account of the irregular and heterogeneous character of the major portion of the earth's surface and the consequent difficulty in choosing a conductivity to be used in a computation of ground return circuit impedance it has heretofore been considered useless to take into consideration the possibility of an earth permeability greater than unity. However, since the permeability may sometimes be known to be appreciably different from unity and it is always desirable to reduce the probable error in a computation and since the inclusion of the permeability in the formulas may sometimes lead to a better agreement between the theory and experiments it seems worth while to provide formulas which include the permeability.

The self impedance of a ground return circuit is

$$Z = z + i2\omega \log \rho''/a + 4\omega(P + iQ),$$

where $z + i2\omega \log \rho''/a$ is the self impedance with a perfectly conducting ground and $4\omega(P + iQ)$ contains the effect of the finite conductivity and permeability of the ground. Carson [1] has derived an infinite integral and series expansions for $P + iQ$ on the basis of unit permeability. The infinite integral derived here is arrived at merely by going through Carson's paper and writing in the permeability wherever Carson has replaced it by unity. The reader will be expected to have a copy of Carson's paper at hand as not all of the steps in his paper will be here reproduced.

Equations (23) and (24) respectively are the new infinite integral formulas for self and mutual impedance. Equations (A) and (C) respectively are the new asymptotic and convergent series formulas for P and Q. The functions m and l occurring in equations (C) are functions of the permeability. Since some of them are defined by series and their computation is consequently rather laborious, enough

[1] John R. Carson, *Bell System Technical Journal*, Oct., 1926.

of them are tabulated for values of μ from 1 to 1.7 to provide for the computation of P and Q for values of r_1 up to 2.

Equation (1)[1] is unchanged but there is a new definition for α

$$\alpha = 4\pi\lambda\mu\omega.$$

Since curl $E = -(\partial/\partial t)\mu H$ equations (2) and (3) have the factor μ added to their left hand sides.

The next change is in the application of the boundary conditions. At the surface of the ground H_x and μH_y must be continuous. The equations to be solved for $F(\tau)$ and $\phi(\tau)$ now become

$$\frac{1}{\mu i\omega}\sqrt{\tau^2 + i\alpha}\,F(\tau) = 2Ie^{-h\tau} + \phi(\tau),$$

$$\frac{1}{i\omega}\tau F(\tau) = 2Ie^{-h\tau} - \phi(\tau),$$

whence

$$F(\tau) = \frac{\mu i\omega e^{-h\tau}}{\sqrt{\tau^2 + i\alpha} + \mu\tau}4I, \tag{11}$$

$$\phi(\tau) = \frac{\sqrt{\tau^2 + i\alpha} - \mu\tau}{\sqrt{\tau^2 + i\alpha} + \mu\tau}e^{-h\tau}2I. \tag{12}$$

The new equations (13), (14), (18), (19), (20), (23) and (24) are

$$E_z = -i4\omega I\mu\int_0^\infty \frac{\cos x\tau}{\sqrt{\tau^2 + i\alpha} + \mu\tau}e^{-\tau h + y\sqrt{\tau^2 + i\alpha}}d\tau, \tag{13}$$

$$E_z = -i4\omega I\mu\int_0^\infty \frac{\cos x'\tau}{\sqrt{\tau^2 + i} + \mu\tau}e^{-\tau h' + y'\sqrt{\tau^2 + i}}d\tau, \tag{14}$$

$$E_z = -i4\omega I\mu\int_0^\infty \frac{e^{-(h'+y')\tau}}{\sqrt{\tau^2 + i} + \mu\tau}\cos x'\tau d\tau - i2\omega I\log\frac{\rho''}{\rho'} - \frac{\partial}{\partial z}V, \tag{18}$$

$$zI = -i4\omega I\mu\int_0^\infty \frac{e^{-(h'+y')\tau}}{\sqrt{\tau^2 + i} + \mu\tau}d\tau - i2\omega I\log\frac{\rho''}{a} + \Gamma V, \tag{19}$$

$$\Gamma^2 = (G + i\omega C)\left[z + i2\omega\log\frac{\rho''}{a} + i4\omega\mu\int_0^\infty \frac{e^{-2h'\tau}}{\sqrt{\tau^2 + i} + \mu\tau}d\tau\right], \tag{20}$$

$$R + iX = Z = z + i2\omega \log \frac{\rho''}{a} + i4\omega\mu \int_0^\infty \frac{e^{-2h'\tau}}{\sqrt{\tau^2 + i} + \mu\tau} d\tau \tag{23}$$
$$= z + i2\omega \log \frac{\rho''}{a} + 4\omega(P + iQ),$$

$$Z_{12} = i2\omega \log \frac{\rho''}{\rho'} + i4\omega\mu \int_0^\infty \frac{e^{-(h_1'+h_2')\tau}}{\sqrt{\tau^2 + i} + \mu\tau} \cos x'\tau d\tau \tag{24}$$
$$= i2\omega \log \frac{\rho''}{\rho'} + 4\omega(P + iQ).$$

The principal steps in the derivation of equation (18) are given in Appendix I.

The new definition of $P + iQ$ is

$$P + iQ = i\mu \int_0^\infty \frac{e^{-(h'+y')\tau}}{\sqrt{\tau^2 + i} + \mu\tau} \cos x'\tau d\tau.$$

Replacing i by v^2 and assuming that v is a real quantity this is

$$P + iQ = \mu v^2 R \int_0^\infty \frac{e^{-v(h'+y'+ix')\tau}}{\sqrt{\tau^2 + 1} + \mu\tau} d\tau,$$

where R is used to indicate that the real part is to be taken.

The asymptotic expansion is easiest derived by expanding $1/(\sqrt{\tau^2 + 1} + \mu\tau)$ into an ascending power series in τ and integrating termwise.

$$1/(\sqrt{\tau^2 + 1} + \mu\tau) = 1 - \mu\tau + (\mu^2 - \tfrac{1}{2})\tau^2 - (\mu^3 - \mu)\tau^3$$
$$+ (\mu^4 - \tfrac{3}{2}\mu^2 + \tfrac{3}{8})\tau^4 - (\mu^5 - 2\mu^3 + \mu)\tau^5 + - \cdots,$$

whence, writing $h' + y' + ix' = re^{i\theta}$,

$$P + iQ = \mu v^2 \Bigg[\frac{\cos\theta}{vr} - \mu \frac{\cos 2\theta}{v^2 r^2} + (\mu^2 - \tfrac{1}{2}) \frac{\cos 3\theta}{v^3 r^3} 2!$$
$$- (\mu^3 - \mu) \frac{\cos 4\theta}{v^4 r^4} 3! + (\mu^4 - \tfrac{3}{2}\mu^2 + \tfrac{3}{8}) \frac{\cos 5\theta}{v^5 r^5} 4!$$
$$- \mu(\mu^2 - 1)^2 \frac{\cos 6\theta}{v^6 r^6} 5! + \cdots \Bigg],$$

whence, separating the real and imaginary parts,

$$P = \frac{\mu}{\sqrt{2}} \Bigg[\frac{\cos\theta}{r} + (2\mu^2 - 1) \frac{\cos 3\theta}{r^3}$$
$$+ 3(12\mu^2 - 8\mu^4 - 3) \frac{\cos 5\theta}{r^5} + \cdots \Bigg]$$

$$
-\frac{\mu^2}{\mu^2-1}\left[\left(\frac{\mu^2-1}{r^2}\right)1!\cos 2\theta-\left(\frac{\mu^2-1}{r^2}\right)^3 5!\cos 6\theta\right.
$$

$$
\left.+\left(\frac{\mu^2-1}{r^2}\right)^5 9!\cos 10\theta-+\cdots\right], \tag{A}
$$

$$
Q=\frac{\mu}{\sqrt{2}}\left[\frac{\cos\theta}{r}-(2\mu^2-1)\frac{\cos 3\theta}{r^3}\right.
$$

$$
\left.+3(12\mu^2-8\mu^4-3)\frac{\cos 5\theta}{r^5}-\cdots\right]
$$

$$
+\frac{\mu^2}{\mu^2-1}\left[\left(\frac{\mu^2-1}{r^2}\right)^2 3!\cos 4\theta-\left(\frac{\mu^2-1}{r^2}\right)^4 7!\cos 8\theta\right.
$$

$$
\left.+\left(\frac{\mu^2-1}{r^2}\right)^6 11!\cos 12\theta-+\cdots\right].
$$

It is worth noticing that when r is so large that only the leading terms in P are of importance

$$
P=[\mu+(h_1+h_2)\sqrt{2\pi\lambda\omega\mu}]/4\pi\lambda\omega[x^2+(h_1+h_2)^2].
$$

At power frequencies $(h_1+h_2)\sqrt{2\pi\lambda\omega\mu}$ is small in comparison with μ.

When $\mu-1$ is small a series in powers of $\mu-1$ is a convenient form of solution. This is readily arrived at by writing

$$
\frac{1}{\sqrt{\tau^2+1}+\mu\tau}=\frac{1}{\sqrt{\tau^2+1}+\tau}\left[1-\left(\frac{\epsilon\tau}{\sqrt{\tau^2+1}+\tau}\right)\right.
$$

$$
\left.+\left(\frac{\epsilon\tau}{\sqrt{\tau^2+1}+\tau}\right)^2-+\cdots\right].
$$

The expansion is absolutely convergent for all values of τ if $\epsilon=\mu-1<2$.

$$
\begin{aligned}
1/(\sqrt{\tau^2+1}+\mu\tau) &= (\sqrt{\tau^2+1}-\tau)-\epsilon\tau(\sqrt{\tau^2+1}-\tau)^2 \\
&\quad +\epsilon^2\tau^2(\sqrt{\tau^2+1}-\tau)^3-+\cdots \\
&= \sqrt{\tau^2+1}[1+\epsilon 2\tau^2+\epsilon^2(4\tau^4+\tau^2)+\epsilon^3(8\tau^6+4\tau^4) \\
&\quad +\epsilon^4(16\tau^8+12\tau^6+\tau^4)+\epsilon^5(32\tau^{10}+32\tau^8+6\tau^6) \\
&\quad +\epsilon^6(64\tau^{12}+80\tau^{10}+24\tau^8+\tau^6) \\
&\quad +\epsilon^7(128\tau^{14}+192\tau^{12}+80\tau^{10}+8\tau^8)+\cdots] \\
&\quad -[\tau+\epsilon(2\tau^3+\tau)+\epsilon^2(4\tau^5+3\tau^3) \\
&\quad +\epsilon^3(8\tau^7+8\tau^5+\tau^3)+\epsilon^4(16\tau^9+20\tau^7+5\tau^5) \\
&\quad +\epsilon^5(32\tau^{11}+48\tau^9+18\tau^7+\tau^5)+\cdots].
\end{aligned}
$$

Writing $c = v(h' + y' + ix') = vre^{i\theta}$ we have then

$$\begin{aligned} P + iQ &= \mu v^2 R \int_0^\infty \frac{e^{-c\tau}}{\sqrt{\tau^2+1} + \mu\tau} d\tau \\ &= \mu v^2 R \left[1 + \epsilon 2 \frac{d^2}{dc^2} + \epsilon^2 \left(4 \frac{d^4}{dc^4} + \frac{d^2}{dc^2} \right) + \cdots \right] f(c) \qquad (B) \\ &- \mu v^2 R \left[\frac{1}{c^2} + \epsilon \left(2 \frac{3!}{c^4} + \frac{1}{c^2} \right) + \epsilon^2 \left(4 \frac{5!}{c^6} + 3 \frac{3!}{c^4} \right) + \cdots \right], \end{aligned}$$

$$\begin{aligned} \text{where } f(c) &= \int_0^\infty \sqrt{\tau^2+1}\, e^{-c\tau} d\tau = \int_0^\infty \cosh^2\phi\, e^{-c \sinh \phi} d\phi \\ &= \frac{K_1(c)}{c} + \frac{c}{3} - \frac{c^3}{3^2 5} + \frac{c^5}{3^2 5^2 7} - + \cdots .^2 \end{aligned}$$

$\mu v^2 R[f(c) - (1/c^2)] = \mu$ times Carson's $P + iQ$ with $\alpha = \mu 4\pi\lambda\omega$.

The problem is now reduced to the tedious procedure of differentiating $f(c)$ and separating real and imaginary parts twice for each power of ϵ. The chief steps are given in Appendix II. The result is best written in the form

$$\begin{aligned} P = \frac{\pi}{8} \Bigg[& m_0 - \frac{m_4}{2!3!} \left(\frac{r_1}{2}\right)^4 \cos 4\theta + \frac{m_8}{4!5!} \left(\frac{r_1}{2}\right)^8 \cos 8\theta \\ & - \frac{m_{12}}{6!7!} \left(\frac{r_1}{2}\right)^{12} \cos 12\theta + - \cdots \Bigg] \\ + \frac{\theta}{2} \Bigg[& \frac{m_2}{1!2!} \left(\frac{r_1}{2}\right)^2 \sin 2\theta - \frac{m_6}{3!4!} \left(\frac{r_1}{2}\right)^6 \sin 6\theta \\ & + \frac{m_{10}}{5!6!} \left(\frac{r_1}{2}\right)^{10} \sin 10\theta - + \cdots \Bigg] \\ - \frac{1}{\sqrt{2}} \Bigg[& m_1 \frac{r_1 \cos\theta}{3} - m_3 \frac{r_1^3 \cos 3\theta}{3^2 5} - m_5 \frac{r_1^5 \cos 5\theta}{3^2 5^2 7} \\ & + m_7 \frac{r_1^7 \cos 7\theta}{3^2 5^2 7^2 9} + - \cdots \Bigg] \\ + & \frac{r_1^2 \cos 2\theta}{2^3 1!2!} \left(l_2 + m_2 \log \frac{2}{r_1} \right) - \frac{r_1^6 \cos 6\theta}{2^7 3!4!} \left(l_6 + m_6 \log \frac{2}{r_1} \right) \\ & + \frac{r_1^{10} \cos 10\theta}{2^{11} 5!6!} \left(l_{10} + m_{10} \log \frac{2}{r_1} \right) - + \cdots, \end{aligned} \qquad (C)$$

$$\begin{aligned} Q = -\frac{\pi}{8} \Bigg[& \frac{m_2}{1!2!} \left(\frac{r_1}{2}\right)^2 \cos 2\theta - \frac{m_6}{3!4!} \left(\frac{r_1}{2}\right)^6 \cos 6\theta \\ & + \frac{m_{10}}{5!6!} \left(\frac{r_1}{2}\right)^{10} \cos 10\theta - + \cdots \Bigg] \end{aligned}$$

[2] See Jahnke & Emde, "Funktionentafeln," pages 171 and 93.

$$-\frac{\theta}{2}\left[\frac{m_4}{2!3!}\left(\frac{r_1}{2}\right)^4 \sin 4\theta - \frac{m_8}{4!5!}\left(\frac{r_1}{2}\right)^8 \sin 8\theta + \frac{m_{12}}{6!7!}\left(\frac{r_1}{2}\right)^{12} \sin 12\theta - + \cdots\right]$$

$$+\frac{1}{\sqrt{2}}\left[m_1\frac{r_1 \cos\theta}{3} + m_3\frac{r_1^3 \cos 3\theta}{3^2 5} - m_5\frac{r_1^5 \cos 5\theta}{3^2 5^2 7} - m_7\frac{r_1^7 \cos 7\theta}{3^2 5^2 7^2 9} + + \cdots\right]$$

$$+\frac{1}{2\cdot 0!1!}\left(l_0 + m_0 \log\frac{2}{r_1}\right) - \frac{r_1^4 \cos 4\theta}{2^5 2!3!}\left(l_4 + m_4 \log\frac{2}{r_1}\right) + \frac{r_1^8 \cos 8\theta}{2^9 4!5!}\left(l_8 + m_8 \log\frac{2}{r_1}\right) - + \cdots,$$

where $r_1 = r/\sqrt{\mu} = \sqrt{4\pi\lambda\omega[x^2 + (h+y)^2]}$ = Carson's r and the permeability is contained in the functions m_x and l_x.

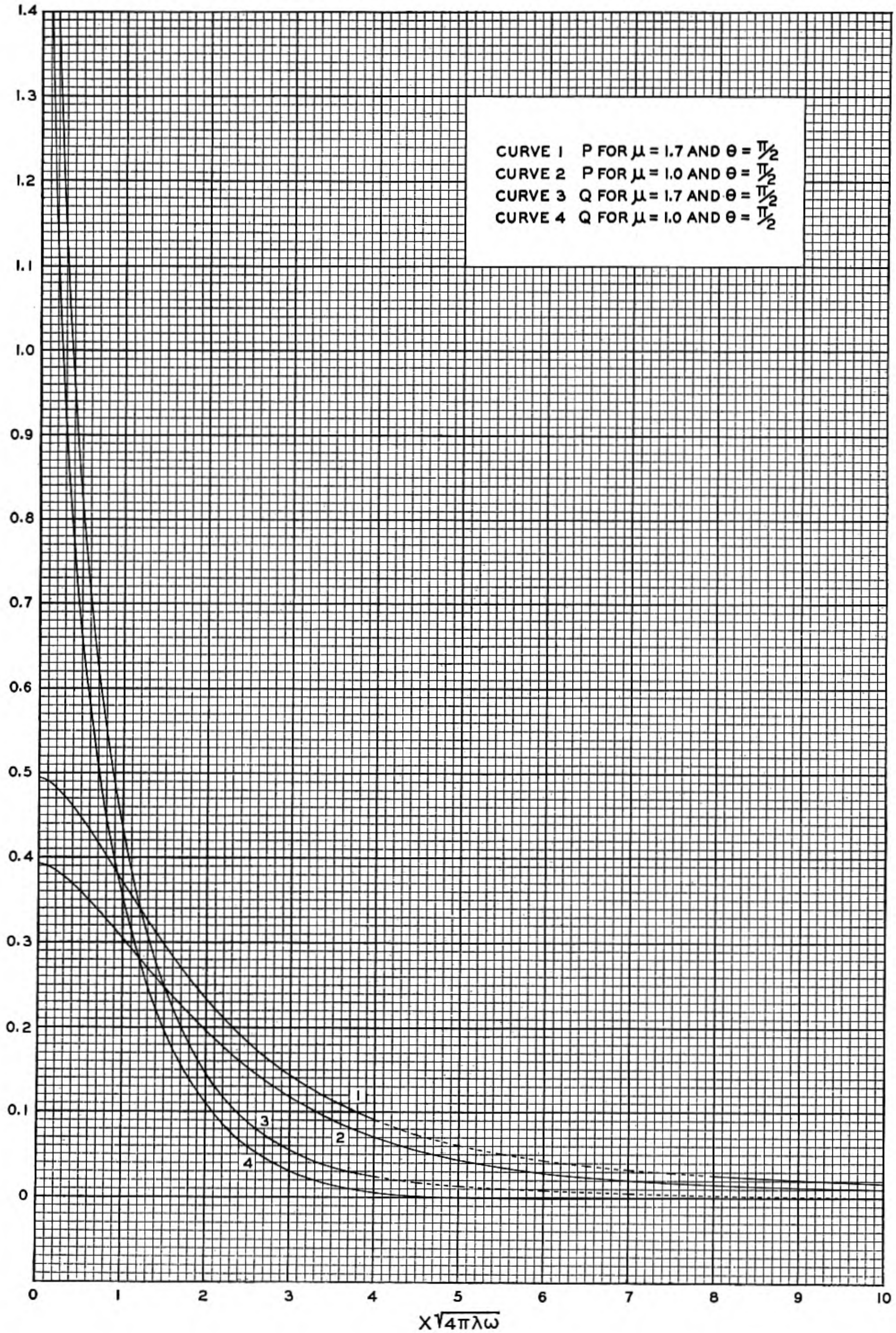
The definitions of the m_x and l_x will be found in Appendix II. The table of numerical values should suffice for most needs.

TABLE 1

μ	$-\frac{1}{2}l_0$	l_2	l_4	l_6	l_8	l_{10}
1	0.03861	0.67278	1.08945	1.38112	1.60612	1.78945
1.1	0.04619	0.70382	1.23834	1.71141	2.1758	2.6549
1.2	0.05264	0.72954	1.38429	2.07062	2.8568	3.7890
1.3	0.05808	0.75059	1.52655	2.45663	3.6558	5.2371
1.4	0.06261	0.76756	1.66456	2.86745	4.5785	7.0466
1.5	0.06631	0.78095	1.79799	3.30122	5.6305	9.2669
1.6	0.06923	0.79121	1.92660	3.75623	6.8167	11.9492
1.7	0.07159	0.79871	2.05026	4.23089	8.1417	15.1467

μ	m_0	m_1	m_2	m_3	m_4	m_5
1	1	1	1	1	1	1
1.1	1.04762	1.06700	1.09751	1.13529	1.17851	1.22625
1.2	1.09091	1.12837	1.19008	1.26928	1.36318	1.47078
1.3	1.13043	1.18469	1.27788	1.40147	1.55291	1.73236
1.4	1.16667	1.23643	1.36111	1.53153	1.74676	2.00996
1.5	1.20000	1.28403	1.44000	1.65922	1.94400	2.30254
1.6	1.23077	1.32793	1.51479	1.78442	2.14402	2.60929
1.7	1.25926	1.36845	1.58573	1.90706	2.34630	2.92942

μ	m_6	m_7	m_8	m_9	m_{10}	m_{11}
1	1	1	1	1	1	1
1.1	1.27799	1.3334	1.3926	1.4554	1.5217	1.5918
1.2	1.59192	1.7270	1.8767	2.0420	2.2241	2.4244
1.3	1.94162	2.1834	2.4613	2.7798	3.1441	3.5602
1.4	2.32690	2.7055	3.1556	3.6895	4.3217	5.0696
1.5	2.74752	3.2957	3.9685	4.7925	5.8003	7.0324
1.6	3.20326	3.9566	4.9089	6.1108	7.6264	9.5370
1.7	3.69388	4.6903	5.9856	7.6674	9.8497	12.6816


CURVE 1 P FOR μ = 1.7 AND θ = π/2
CURVE 2 P FOR μ = 1.0 AND θ = π/2
CURVE 3 Q FOR μ = 1.7 AND θ = π/2
CURVE 4 Q FOR μ = 1.0 AND θ = π/2
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
0
1
2
3
4
5
6
7
8
9
10
X√4πλω

The curves show the effect of $\mu = 1.7$ on the mutual impedance between two parrallel ground return circuits with the wires lying on the ground. The dashed portions of the curves were not computed.

Appendix I

Equations (4), (7) and (17) substituted into (16) give

$$E_z(x, y) = E_z(x, 0) - i\omega \int_0^y \left[\frac{2I(h - y)}{x^2 + (h - y)^2} + \int_0^\infty \phi(\tau) \cos x\tau \cdot e^{-y\tau} d\tau \right] dy - \frac{\partial V}{\partial z}$$

$$= E_z(x, 0) + i\omega \int_0^\infty \phi(\tau) \cos x\tau (e^{-y\tau} - 1) \frac{d\tau}{\tau} + i\omega I \log \frac{x^2 + (h - y)^2}{x^2 + h^2} - \frac{\partial V}{\partial z}$$

$$= - i4\omega I \int_0^\infty \frac{\mu e^{-h\tau} \cos x\tau}{\sqrt{\tau^2 + i\alpha} + \mu\tau} d\tau + i2\omega I \int_0^\infty \frac{\sqrt{\tau^2 + i\alpha} - \mu\tau}{\sqrt{\tau^2 + i\alpha} + \mu\tau} (e^{-(h+y)\tau} - e^{-h\tau}) \frac{\cos x\tau}{\tau} d\tau + i\omega I \log \frac{x^2 + (h - y)^2}{x^2 + h^2} - \frac{\partial V}{\partial z}$$

$$= - i4\omega I \int_0^\infty \frac{\mu e^{-(h+y)\tau}}{\sqrt{\tau^2 + i\alpha} + \mu\tau} \cos x\tau d\tau + i2\omega I \int_0^\infty (e^{-(h+y)\tau} - e^{-h\tau}) \frac{\cos x\tau}{\tau} d\tau + i\omega I \log \frac{x^2 + (h - y)^2}{x^2 + h^2} - \frac{\partial V}{\partial z}$$

$$= - i4\omega I \int_0^\infty \frac{\mu e^{-(h+y)\tau}}{\sqrt{\tau^2 + i\alpha} + \mu\tau} \cos x\tau d\tau + i\omega I \log \frac{x^2 + (h - y)^2}{x^2 + (h + y)^2} - \frac{\partial V}{\partial z}.$$

Appendix II

The succeeding analysis has been considerably shortened by writing

$$\zeta_{nm} = \omega_n - \omega_{2n} + \frac{1}{2n - 1} + \omega_{2(n-1-m)},$$

where

$$\omega_n = \sum_1^n \frac{1}{s}, \qquad \omega_0 = 0.$$

$$f(c) = \frac{1}{c^2} + \left[\frac{c}{3} - \frac{c^3}{3^2 5} + \frac{c^5}{3^2 5^2 7} - \frac{c^7}{3^2 5^2 7^2 9} + - \cdots \right]$$
$$+ \frac{1}{2}\left[\zeta_{10} - \zeta_{20}\frac{1}{1!2!}\left(\frac{c}{2}\right)^2 + \zeta_{30}\frac{1}{2!3!}\left(\frac{c}{2}\right)^4 - \zeta_{40}\frac{1}{3!4!}\left(\frac{c}{2}\right)^6 + - \cdots \right]$$
$$+ \frac{1}{2}\left[1 - \frac{1}{1!2!}\left(\frac{c}{2}\right)^2 + \frac{1}{2!3!}\left(\frac{c}{2}\right)^4 - \frac{1}{3!4!}\left(\frac{c}{2}\right)^6 + - \cdots \right] \log \frac{2}{\gamma c}$$

$$f^{(n)}(c) = \frac{(n+1)!}{c^{n+2}} + \frac{1}{2}\frac{(n-1)!}{c^n} - \frac{1}{2\cdot 4}\frac{(n-3)!}{c^{n-2}} + \frac{1\cdot 3}{2\cdot 4\cdot 6}\frac{(n-5)!}{c^{n-4}} - + \cdots \frac{1\cdot 3\cdot 5 \cdots (n-3)}{2\cdot 4\cdot 6 \cdots n}\frac{1!}{c^2}$$
$$+ (-2)^{n/2}\left[\frac{(n/2)!\,c}{3\cdot 5\cdot 7 \cdots (n+3)} - \frac{(1+n/2)!\,c^3}{1!3^2 5\cdot 7 \cdots (n+5)} + \frac{(2+n/2)!\,c^5}{2!3^2 5^2 7\cdot 9 \cdots (n+7)} - + \cdots \right]$$
$$- \left(-\frac{1}{2}\right)^{(n/2)+1}\left[\zeta_{[(n/2)+1]n/2}\frac{1\cdot 3\cdot 5 \cdots (n-1)}{0!\left(\frac{n}{2}+1\right)!} - \zeta_{[(n/2)+2]n/2}\frac{3\cdot 5\cdot 7 \cdots (n+1)}{1!\left(\frac{n}{2}+2\right)!}\left(\frac{c}{2}\right)^2 + - \cdots \right]$$
$$- \left(-\frac{1}{2}\right)^{(n/2)+1}\left[\frac{1\cdot 3\cdot 5 \cdots (n-1)}{0!\left(\frac{n}{2}+1\right)!} - \frac{3\cdot 5\cdot 7 \cdots (n+1)}{1!\left(\frac{n}{2}+2\right)!} \times \left(\frac{c}{2}\right)^2 + \frac{5\cdot 7\cdot 9 \cdots (n+3)}{2!\left(\frac{n}{2}+3\right)!}\left(\frac{c}{2}\right)^4 - + \cdots \right] \log \frac{2}{\gamma c},$$

the n being an even integer.

The inverse powers of c all cancel out, in equation (B), and there remains

$$P + iQ = \mu v^2 R \left\{ \frac{c}{3} q_1 - \frac{c^3}{3^2 5} q_3 + \frac{c^5}{3^2 5^2 7} q_5 - \frac{c^7}{3^2 5^2 7^2 9} q_7 + - \cdots \right.$$

$$+ \frac{1}{2} \left[\zeta_{10} \frac{1}{0!1!} p_0 - \zeta_{20} \frac{(c/2)^2}{1!2!} p_2 + \zeta_{30} \frac{(c/2)^4}{2!3!} p_4 \right.$$

$$\left. - \zeta_{40} \frac{(c/2)^6}{3!4!} p_6 + - \cdots \right] \quad (D)$$

$$+ \frac{1}{2} \log \frac{2}{\gamma c} \left[q_0 - \frac{(c/2)^2}{1!2!} q_2 + \frac{(c/2)^4}{2!3!} q_4 \right.$$

$$\left. \left. - \frac{(c/2)^6}{3!4!} q_6 + - \cdots \right] \right\}.$$

where

$$\zeta_{10} p_0 = \zeta_{10} - \epsilon 2 \zeta_{21} \frac{1}{2 \cdot 2!} + \epsilon^2 \left(4 \zeta_{32} \frac{1 \cdot 3}{2^2 \cdot 3!} - \zeta_{21} \frac{1}{2 \cdot 2!} \right)$$

$$- \epsilon^3 \left(8 \zeta_{43} \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 4!} - 4 \zeta_{32} \frac{1 \cdot 3}{2^2 \cdot 3!} \right)$$

$$+ \epsilon^4 \left(16 \zeta_{54} \frac{1 \cdot 3 \cdot 5 \cdot 7}{2^4 \cdot 5!} - 12 \zeta_{43} \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 4!} + \zeta_{32} \frac{1 \cdot 3}{2^2 \cdot 3!} \right) - + \cdots,$$

$$\zeta_{20} \frac{1}{2!} p_2 = \zeta_{20} \frac{1}{2!} - \epsilon 2 \zeta_{31} \frac{3}{2 \cdot 3!} + \epsilon^2 \left(4 \zeta_{42} \frac{3 \cdot 5}{2^2 \cdot 4!} - \zeta_{31} \frac{3}{2 \cdot 3!} \right)$$

$$- \epsilon^3 \left(8 \zeta_{53} \frac{3 \cdot 5 \cdot 7}{2^3 5!} - 4 \zeta_{42} \frac{3 \cdot 5}{2^2 \cdot 4!} \right) + \cdots,$$

$$\zeta_{30} \frac{1}{3!} p_4 = \zeta_{30} \frac{1}{3!} - \epsilon 2 \zeta_{41} \frac{5}{2 \cdot 4!} + \epsilon^2 \left(4 \zeta_{52} \frac{5 \cdot 7}{2^2 \cdot 5!} - \zeta_{41} \frac{5}{2 \cdot 4!} \right)$$

$$- \epsilon^3 \left(8 \zeta_{63} \frac{5 \cdot 7 \cdot 9}{2^3 \cdot 6!} - 4 \zeta_{52} \frac{5 \cdot 7}{2^2 \cdot 5!} \right) + \cdots,$$

. . . .

$$q_0 = 1 - \epsilon 2 \frac{1}{2 \cdot 2!} + \epsilon^2 \left(4 \frac{1 \cdot 3}{2^2 \cdot 3!} - \frac{1}{2 \cdot 2!} \right)$$

$$- \epsilon^3 \left(8 \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 4!} - 4 \frac{1 \cdot 3}{2^2 \cdot 3!} \right) + - \cdots,$$

$$\frac{1}{2!} q_2 = \frac{1}{2!} - \epsilon 2 \frac{3}{2 \cdot 3!} + \epsilon^2 \left(4 \frac{3 \cdot 5}{2^2 \cdot 4!} - \frac{3}{2 \cdot 3!} \right)$$

$$- \epsilon^3 \left(8 \frac{3 \cdot 5 \cdot 7}{2^3 \cdot 5!} - 4 \frac{3 \cdot 5}{2^2 \cdot 4!} \right) + - \cdots,$$

$$\frac{1}{3!}q_4 = \frac{1}{3!} - \epsilon 2\frac{5}{2\cdot 4!} + \epsilon^2\left(4\frac{5\cdot 7}{2^2\cdot 5!} - \frac{5}{2\cdot 4!}\right) - \epsilon^3\left(8\frac{5\cdot 7\cdot 9}{2^3\cdot 6!} - 4\frac{5\cdot 7}{2^2\cdot 5!}\right) + - \cdots,$$

. . . .

$$q_1 = 1 - \epsilon 2\frac{2^1\cdot 1!}{5} + \epsilon^2\left(4\frac{2^2\cdot 2!}{5\cdot 7} - \frac{2^1\cdot 1!}{5}\right) - \epsilon^3\left(8\frac{2^3\cdot 3!}{5\cdot 7\cdot 9} - 4\frac{2^2\cdot 2!}{5\cdot 7}\right) + - \cdots,$$

$$1!q_3 = 1! - \epsilon 2\frac{2^1\cdot 2!}{7} + \epsilon^2\left(4\frac{2^2\cdot 3!}{7\cdot 9} - \frac{2^1\cdot 2!}{7}\right) - \epsilon^3\left(8\frac{2^3\cdot 4!}{7\cdot 9\cdot 11} - 4\frac{2^2\cdot 3!}{7\cdot 9}\right) + - \cdots,$$

$$2!q_5 = 2! - \epsilon 2\frac{2^1\cdot 3!}{9} + \epsilon^2\left(4\frac{2^2\cdot 4!}{9\cdot 11} - \frac{2^1\cdot 3!}{9}\right) - \epsilon^3\left(8\frac{2^3\cdot 5!}{9\cdot 11\cdot 13} - 4\frac{2^2\cdot 4!}{9\cdot 11}\right) + - \cdots.$$

. . . .

The way in which the succeeding terms of each series are to be formed will be made clear by comparing the numbers just preceding the ζs in the p series with the numbers in the expansion of $1/(\sqrt{\tau^2+1} + \mu\tau)$ into a power series in ϵ.

The series converge if $\epsilon = \mu - 1 < 2$.

The q series are all represented by the single formula

$$\begin{aligned} q_x &= \left(\frac{2}{1+\mu}\right)^{1+(x/2)} F\left(\frac{x}{2}, 1 - \frac{x}{2}, 2 + \frac{x}{2}, -\frac{\epsilon}{2}\right) \\ &= \left(\frac{2}{1+\mu}\right)^{1+(x/2)}\left[1 + \frac{x}{x+4}\left(\frac{\epsilon}{2}\right)\frac{x-2}{2}\right. \\ &\quad + \frac{x(x+2)}{(x+4)(x+6)}\left(\frac{\epsilon}{2}\right)^2\frac{(x-2)(x-4)}{2\cdot 4} \\ &\quad + \frac{x(x+2)}{(x+6)(x+8)}\left(\frac{\epsilon}{2}\right)^3\frac{(x-2)(x-4)(x-6)}{2\cdot 4\cdot 6} \\ &\quad \left. + \frac{x(x+2)}{(x+8)(x+10)}\left(\frac{\epsilon}{2}\right)^4\frac{(x-2)(x-4)(x-6)(x-8)}{2\cdot 4\cdot 6\cdot 8} + \cdots\right], \end{aligned}$$

good for all values of ϵ if x is even; good for $\epsilon \leq 2$ if x is odd. Other series are available for odd x and $2 < \epsilon$ but there is little likelihood of their being needed.

The p series are all comprised in the single formula

$$\zeta_{(1+x/2)0}p_x = \left(1+\frac{x}{2}\right)!\left\{\zeta_{(1+x/2)0}\frac{1}{\left(1+\frac{x}{2}\right)!} - \epsilon 2\zeta_{(2+x/2)1}\frac{x+1}{2\left(2+\frac{x}{2}\right)!}\right.$$
$$\left. + \epsilon^2\left(4\zeta_{(3+x/2)2}\frac{(x+1)(x+3)}{2^2\left(3+\frac{x}{2}\right)!} - \zeta_{(2+x/2)1}\frac{x+1}{2\left(2+\frac{x}{2}\right)!}\right) - \cdots\right\}.$$

Since $\zeta_{nm} = \zeta_{n(n-1)} + \omega_{2(n-1-m)}$ we can write this

$$\zeta_{(1+x/2)0}p_x = \omega_x q_x + \delta_x,$$

where

$$\delta_x = \left(1+\frac{x}{2}\right)!\left\{\zeta_{(1+x/2)x/2}\frac{1}{\left(1+\frac{x}{2}\right)!} - \epsilon 2\zeta_{(2+x/2)(1+x/2)}\frac{x+1}{2\left(2+\frac{x}{2}\right)!}\right.$$
$$+ \epsilon^2\left(4\zeta_{(3+x/2)(2+x/2)}\frac{(x+1)(x+3)}{2^2\left(3+\frac{x}{2}\right)!}\right.$$
$$\left.\left. - \zeta_{(2+x/2)(1+x/2)}\frac{x+1}{2\left(2+\frac{x}{2}\right)!}\right) - \cdots\right\}.$$

$$\delta_0 = \frac{2\mu}{\mu^2-1}\log\frac{1+\mu}{2},$$
$$\delta_2 = \left(\frac{2}{1+\mu}\right)^2\left(\frac{1}{2}+\frac{1}{\mu-1}-\frac{2\mu}{(\mu-1)^2}\log\frac{1+\mu}{2}\right),$$
$$\delta_x = \frac{1}{\mu^2-1}\cdot\frac{x+2}{x-1}(\zeta_{x/2(x/2-1)} - \delta_{x-2}) \text{ for } 4 \le x.$$

By separating the real and imaginary parts one gets from equation (D)

$$P = \frac{\pi}{8}\mu\left[q_0 - \frac{1}{2!3!}\left(\frac{r}{2}\right)^4\cos 4\theta\cdot q_4 + \frac{1}{4!5!}\left(\frac{r}{2}\right)^8\cos 8\theta\cdot q_8 - + \cdots\right]$$
$$+ \frac{\theta}{2}\mu\left[\frac{1}{1!2!}\left(\frac{r}{2}\right)^2\sin 2\theta\cdot q_2 - \frac{1}{3!4!}\left(\frac{r}{2}\right)^6\sin 6\theta\cdot q_6 + - \cdots\right]$$
$$- \frac{\mu}{\sqrt{2}}\left[\frac{r\cos\theta}{3}q_1 - \frac{r^3\cos 3\theta}{3^25}q_3 - \frac{r^5\cos 5\theta}{3^25^27}q_5 + + - - \cdots\right]$$
$$+ \frac{1}{2}\frac{\mu}{1!2!}\left(\frac{r}{2}\right)^2\cos 2\theta\left(\zeta_{20}p_2 + q_2\log\frac{2}{\gamma r}\right)$$
$$- \frac{1}{2}\frac{\mu}{3!4!}\left(\frac{r}{2}\right)^6\cos 6\theta\left(\zeta_{40}p_6 + q_6\log\frac{2}{\gamma r}\right)$$
$$+ \frac{1}{2}\frac{\mu}{5!6!}\left(\frac{r}{2}\right)^{10}\cos 10\theta\left(\zeta_{60}p_{10} + q_{10}\log\frac{2}{\gamma r}\right) - + \cdots,$$

$$
\begin{aligned}
Q = &-\frac{\pi}{8}\mu\left[\frac{1}{1!2!}\left(\frac{r}{2}\right)^2\cos 2\theta\cdot q_2 - \frac{1}{3!4!}\left(\frac{r}{2}\right)^6\cos 6\theta\cdot q_6 + - \cdots\right] \\
&-\frac{\theta}{2}\mu\left[\frac{1}{2!3!}\left(\frac{r}{2}\right)^4\sin 4\theta\cdot q_4 - \frac{1}{4!5!}\left(\frac{r}{2}\right)^8\sin 8\theta\cdot q_8 + - \cdots\right] \\
&+\frac{\mu}{\sqrt{2}}\left[\frac{r\cos\theta}{3}q_1 + \frac{r^3\cos 3\theta}{3^2 5}q_3 - \frac{r^5\cos 5\theta}{3^2 5^2 7}q_5 - + + \cdots\right] \\
&+\frac{1}{2}\,\frac{\mu}{0!1!}\left(\zeta_{10}p_0 + q_0\log\frac{2}{\gamma r}\right) \\
&-\frac{1}{2}\,\frac{\mu}{2!3!}\left(\frac{r}{2}\right)^4\cos 4\theta\left(\zeta_{30}p_4 + q_4\log\frac{2}{\gamma r}\right) \\
&+\frac{1}{2}\,\frac{\mu}{4!5!}\left(\frac{r}{2}\right)^8\cos 8\theta\left(\zeta_{50}p_8 + q_8\log\frac{2}{\gamma r}\right) - + \cdots.
\end{aligned}
$$

Equations (C) are now got by writing $r = r_1\sqrt{\mu}$, $m_x = \mu^{1+(x/2)}q_x$ and

$$
\begin{aligned}
l_x &= \mu^{1+(x/2)}(\zeta_{[1+(x/2)]0}p_x - q_x\log\gamma\sqrt{\mu}). \\
l_x &= m_x(\omega_x - \log_e\gamma\sqrt{\mu}) + \mu^{1+(x/2)}\delta_x \\
\log_e\gamma &= 0.5772157.
\end{aligned}
$$

Negative Impedances and the Twin 21-Type Repeater

By GEORGE CRISSON

This paper discusses negative resistances and impedances. It describes their properties and some devices by which they may be produced physically. Certain properties of negative impedances when used as series and shunt boosters for amplifying speech waves in telephone circuits are discussed. The paper concludes with a description of the circuit and properties of the twin 21-type repeater.

WHEN an e.m.f. is applied to the terminals of an ordinary positive resistance a current flows in at the terminal connected to the positive pole of the source and out at the other terminal. This direction of current flow is considered positive and the value of the resistance R, in ohms is given by Ohm's law as $R = E/I$ where E is the applied voltage and I is the current in amperes. Similarly a definite current I may be passed through the resistance and a potential difference or drop $E = RI$ will appear across its terminals. With positive resistances it makes no difference whether we "apply an e.m.f." or "pass a current". The resistance may be a very simple device such as a coil of wire which absorbs energy from the circuit at a rate $W = EI = I^2R$ watts.

It is possible, however, to construct assemblages of apparatus which have the property of keeping the ratio of the voltage across a pair of terminals to the current at the terminals constant, but with the relative direction of the voltage and current opposite to that which a positive resistance would give. In such devices the resistance is negative and the apparatus contributes power to the circuit with which it is connected. Each such device necessarily includes a source of energy such as a battery and some means such as a vacuum tube for controlling the delivery of this energy to the circuit. There are two varieties of such devices. In one case, the internal arrangement of the mechanism is such that, if a definite voltage is applied to the terminals, a current flows in a direction opposite to the applied e.m.f. In the other, if a definite current is passed through the system, the drop across the terminals will be opposite in direction to that caused by a positive resistance. These two arrangements are essentially different and cannot be used interchangeably in a given circuit, though either one can give any desired value of negative resistance. If the wrong arrangement is used instability or singing will occur. To know whether a given negative resistance will work satisfactorily in a given circuit it is not sufficient to know its value in ohms. Something must be known about its internal arrangement and about the impedance of the circuit in which it is to work.

Regenerative Negative Resistances

One of the simplest ways to produce a negative resistance is to interconnect the input and output terminals of a one-way amplifier. This gives a regenerative arrangement because part of the output energy of the amplifier is fed back into the input circuit. The type of negative resistance obtained depends upon the way in which the interconnection is made.

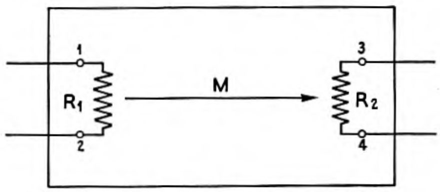


Fig. 1—Ideal one-way amplifier.

Fig. 1 shows schematically an ideal one-way amplifier for this purpose. It has a pair of input terminals 1, 2, and a pair of output terminals 3, 4. The impedances between the input and output terminals are pure resistances R_1 and R_2, respectively. Some mechanism, indicated symbolically by the arrow, is provided, which produces an e.m.f. in the output circuit which is proportional to the input current. The nature of this mechanism is not of importance to this discussion except that it is a one-way device. The mutual impedance M is the ratio of the e.m.f. generated in the output circuit to the current in the input circuit. This ratio may be adjusted by suitable means such as a potentiometer but is otherwise constant and includes no phase shift. The internal connections are assumed to be such that when the input terminal 1 is positive to 2 the e.m.f. in the output circuit tends to make terminal 3 positive with respect to 4.

Series Negative Resistance

In Fig. 2 the input and output circuits of the ideal amplifier are connected in series with each other to a source of e.m.f. E and a re-

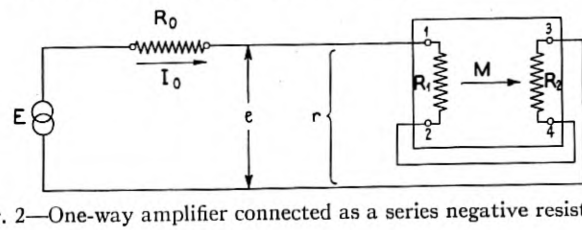


Fig. 2—One-way amplifier connected as a series negative resistance.

sistance R_0 in such fashion that the e.m.f. in the output circuit of the amplifier tends to increase the current. Assume now that the e.m.f. E is applied and a current I_0 flows in the series circuit.

$$E + (M - R_0 - R_1 - R_2)I_0 = 0. \tag{1}$$

The drop across the amplifier is:

$$e = (R_1 + R_2 - M)I_0, \tag{2}$$

and the net resistance of the whole amplifier is:

$$r = \frac{e}{I_0} = R_1 + R_2 - M. \tag{3}$$

It may aid in understanding the behavior of this system to assume, first, that M is zero so that the circuit consists simply of the three positive resistances R_0, R_1 and R_2 in series and then consider what happens as M is gradually increased. The e.m.f. appearing in the output circuit of the amplifier acts to reduce the drop e across the terminals 1, 3 and to increase the current I_0. The e.m.f. E must be reduced if the current is to be kept constant. The curves of Fig. 3 show how the resistances and current vary as M changes, E being constant.

When $M = R_1 + R_2$ the drop e and the resistance r become zero. The amplifier then ceases to take power from the circuit and supplies its own losses. If this condition could be exactly obtained the terminals 1, 3 might be short-circuited and the e.m.f. E removed, without changing the current which would continue to flow in the amplifier. If, however, the e.m.f. were removed or the circuit opened without short-circuiting the terminals of the amplifier the current in the input circuit, and, consequently, the e.m.f. in the output circuit of the amplifier would disappear and the system would become inactive.

If, now, M is further increased so that it approaches $R_0 + R_1 + R_2$ the current increases indefinitely, or the e.m.f. E required to sustain the current at a given value approaches zero. Under these conditions the drop e and the resistance r become negative and the amplifier supplies not only its own losses but also part of the energy dissipated by the resistance R_0. It does so under the control of the e.m.f. E, however, and if this e.m.f. is removed the system becomes inactive as before. At the limit when $M = R_0 + R_1 + R_2$, the amplifier supplies all the losses in the system and any current I_0, once started, continues indefinitely.

This ideal condition is not realized in practice. Either M is slightly too small, in which case the current decreases when E is removed, or it is too large so that any value of E however small starts a current which thereafter increases because the amplifier supplies more than enough energy to sustain the current. This increase continues until checked by the inability of the amplifier to deal with larger currents. In effect M is reduced to the point where r is again equal to $-R_0$, after which the current continues at a constant value.

The arrangement shown in Fig. 2 can therefore be made to provide any negative resistance between $r = 0$ and $r = -R_0$ without causing instability or a tendency to sing. *Such a system is stable when the algebraic sum of all the resistances in series in the circuit is positive.* This behavior is typical of a large number of arrangements that are able to furnish negative resistances. All such arrangements will be referred to as *series negative resistances* to distinguish them from another type which will be described below.

It should be noted that if the sign of M is reversed, for example, by interchanging the two wires connected with the output terminals 3, 4, no negative resistance results. As M increases, the current I_0 decreases, or the e.m.f. E must be increased to maintain the current, but no matter how large M is made, the direction of the drop e and sign of the resistance r do not change though the latter approaches ∞.

The Unstable Condition

So far nothing has been said as to the nature of the e.m.f. E. In the ideal case, when the system is stable, the current wave is a copy of the voltage wave as in any circuit having a pure resistance. What happens when the circuit is unstable depends upon the nature of the amplifier or other device used to produce the negative resistance and not upon the e.m.f. E. This may be of any kind and of minute size, such as that resulting from thermal agitation in the resistances forming part of the apparatus. If the amplifier is able to amplify direct currents, the resulting disturbance may be a direct current limited only by the ability of the apparatus to supply energy to the circuit. Where transformers, condensers, etc., are involved the disturbance settles down to an alternating current which may contain many harmonics or may be almost a pure sine wave. These effects are called "singing." The final frequency, amplitude and wave shape depend upon the makeup of the apparatus in a way which is beyond the scope of this paper.

Shunt Negative Resistance

By connecting the terminals of the ideal one-way amplifier in parallel as shown in Fig. 4, a negative resistance will be obtained which is typical of the second type or *shunt negative resistance.*

Referring to Fig. 4, the current in the input circuit of the amplifier is:

$$I_1 = \frac{e}{R_1}. \tag{4}$$

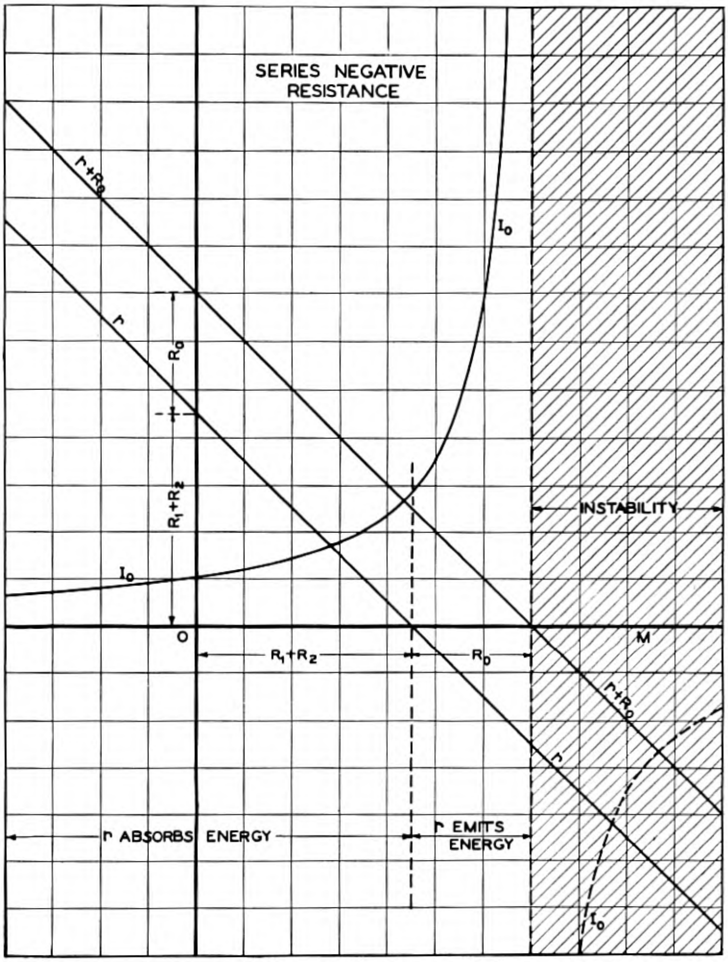


Fig. 3—Curves illustrating properties of series negative resistances.

The current in the output circuit is:

$$I_2 = \frac{e - MI_1}{R_2} = \frac{R_1 - M}{R_1 R_2} e, \tag{5}$$

and the current in the main circuit is:

$$I_0 = \frac{E - e}{R_0} = I_1 + I_2 = \frac{R_1 + R_2 - M}{R_1 R_2} e, \tag{6}$$

from which

$$r = \frac{e}{I_0} = \frac{R_1 R_2}{R_1 + R_2 - M}, \tag{7}$$

and the applied voltage E is:

$$E = I_0(R_0 + r) = \left[1 + \frac{R_0(R_1 + R_2 - M)}{R_1 R_2} \right] e. \tag{8}$$

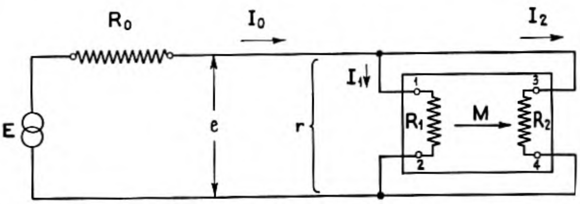


Fig. 4—One-way amplifier connected as a shunt negative resistance.

With this arrangement, the e.m.f. generated in the output circuit of the amplifier opposes the current I_2 due to the e.m.f. E, and as M increases, the current I_0 in the main circuit decreases and the resistance of the amplifier increases. The curves of Fig. 5 show how the resistances and current vary as M changes, E being constant. To keep I_0 constant, it would now be necessary to increase E.

When $M = R_1$ the current I_2 becomes zero.

When $M = R_1 + R_2$ the current I_0 falls to zero, the potential $e = E$, the current I_2 has reversed in direction, the resistance $r = \infty$ and the amplifier just supplies its own losses. If the circuit outside the amplifier is now opened, the condition of the amplifier is the same as when the short circuit was applied to Fig. 2 and the current circulating in the amplifier will continue. If E is removed without opening the circuit, R_0 will draw energy from the amplifier, thus reducing I_1 and causing all currents and voltages to disappear. The amplifier is still under the control of the e.m.f. E.

For the arrangement of Fig. 4 to become unstable it is necessary for the amplifier to maintain or increase the voltage e after the controlling e.m.f. E is removed. For the amplifier to maintain the voltage e it is necessary that:

$$e = \frac{e}{R_1} M \frac{\dfrac{R_0 R_1}{R_0 + R_1}}{\dfrac{R_0 R_1}{R_0 + R_1} + R_2}, \tag{9}$$

from which

$$M = R_1 + R_2 + \frac{R_1 R_2}{R_0}, \tag{10}$$

and from (7),

$$r = -R_0. \tag{11}$$

Hence if

$$R_1 + R_2 < M < R_1 + R_2 + \frac{R_1 R_2}{R_0}, \tag{12}$$

the impedance r is a negative resistance greater in magnitude than R_0 but the system cannot sing because the amplifier cannot maintain or increase

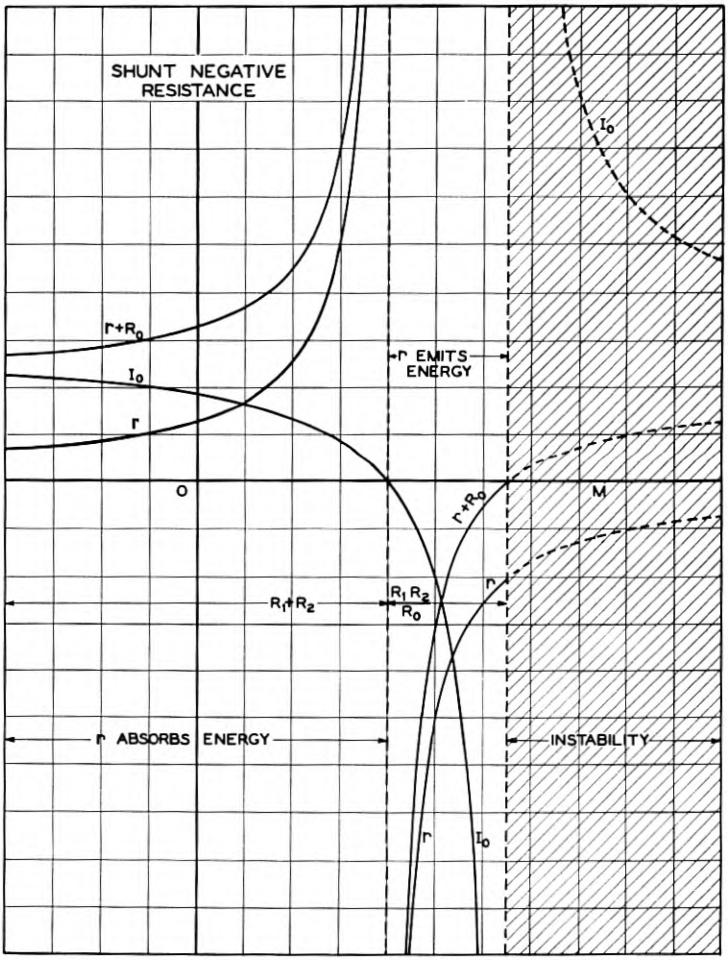


Fig. 5—Curves illustrating properties of shunt negative resistances.

the voltage e after E is removed, even though the current I_0 flows against E and the source is receiving energy from the amplifier.

If M becomes greater than the upper limit given by equation (10) the system passes out of control by the e.m.f. E and becomes unstable or sings. By short-circuiting the terminals 1, 2, it would be possible to increase M until it is greater than the value given by equation (10) which would make r numerically smaller than R_0. On removing the short circuit, however, a disturbance would begin and grow until checked by the limitations of the amplifier so that, in effect, M would be reduced and r again made equal to $-R_0$.

If M is reversed in sign, for example, by interchanging the two wires connected to the output terminals 3, 4, no negative resistance results. As M increases, the current I_0 increases. The resistance r decreases, approaching zero as M becomes indefinitely great.

From these facts it is seen that a negative resistance of any desired value may be inserted in a circuit having any positive resistance R_0 provided that the inserted resistance has the characteristics of the series type when the inserted negative resistance is numerically smaller than the positive resistance or the characteristics of the shunt type when the negative resistance is numerically larger than the positive resistance.

Other Forms of Negative Resistance

All known devices for producing negative resistance fall into one or the other of the two classes described above.

Arrangements are known which exhibit one type of negative resistance at one pair of terminals and the other type at a different pair but not both types at the same pair of terminals at the same time.

Certain apparatus involving gaseous conduction or electronic discharge exhibit negative resistance effects. Fig. 6, for example, shows an arc burning between two electrodes which are connected in series with a resistance and inductance serving as ballast to a source of d-c. power. The ballast serves to stabilize the arc and hold the current drawn from the source constant and also to prevent the passage of alternating current through the source from the arc. The arc has a positive resistance with respect to the d-c. circuit, since it consumes d-c. power, but this resistance varies with the current in such a way that an increase of current is accompanied by a reduction of the potential drop across the arc.

If an alternating current is superimposed upon the direct current through the arc by means of the taps a and b it encounters a negative resistance. If a circuit consisting of a resistance R, inductance L and condenser C is bridged across the arc as shown and the resistance is made large, nothing occurs, but if the resistance is reduced to a certain critical value a state of oscillation is established. This oscillation causes an alternating current to flow through the resonant circuit and the arc. If the oscillation is of audible frequency the arc will emit a

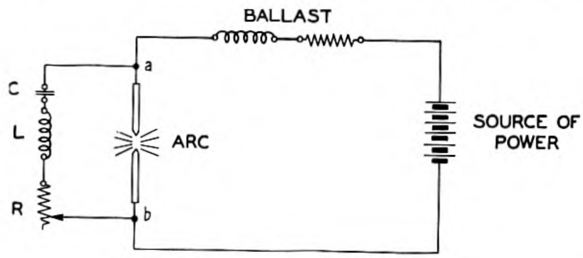


Fig. 6—Arc as a negative resistance.

singing or whistling sound. This property of the arc has found useful application as a generator of high-frequency oscillations in the Poulsen arc used in radiotelegraphy. The negative resistance of the arc has series characteristics as oscillations will not occur if there is an excess of positive resistance in the oscillating circuit.

The dynatron,[1] on the other hand, has shunt characteristics as it is unstable when the external resistance is made large.

Negative Resistances of the Ideal 21-Type Circuit

Fig. 7 shows the ideal one-way amplifier of Fig. 1 connected with an ideal hybrid coil to form a 21-type repeater circuit. The ideal hybrid

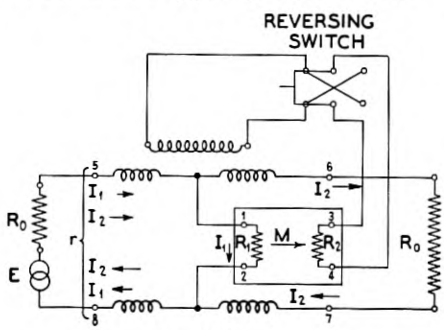


Fig. 7—Ideal 21-type circuit.

coil is assumed to have windings of zero resistance, no leakage reactance, no capacitance in or between the windings, no core loss and negligible exciting current.

[1] See "The Dynatron," by A. W. Hull, *Proc. I. R. E.*, February, 1918.

This 21-type circuit is connected between two equal resistances, R_0. $R_1 = \frac{1}{2}R_0$ and $R_2 = R_0$, assuming that the hybrid coil is designed for equal impedances at the two pairs of line terminals and the drop terminals. If an e.m.f. E acts in series with the resistance R_0 at the left side of the repeater and the mutual impedance M of the amplifier is zero, a current,

$$I_1 = \frac{E}{2R_0}, \tag{13}$$

flows at the left hand terminals 5, 8 of the hybrid coil and in the input circuit of the amplifier. One-half of the power entering the repeater is absorbed in the input resistance R_1 of the amplifier and the other half is absorbed in the output impedance R_2. In accordance with a well known property of the hybrid coil, no current will flow in the right-hand resistance R_0. At a given instant this input current may be assumed to have the direction indicated by the short arrows I_1. By increasing M the amplifier can be made active, causing an amplified current I_2 to flow in series through the line windings of the hybrid coil and the connected resistances R_0. By throwing the reversing switch to one side I_2 may be made to flow in the same direction as I_1 at the terminals 5, 8 as indicated by the long arrows marked I_2. For convenience, this will be referred to as the "direct connection." Changing the reversing switch changes the direction of I_2 with respect to I_1, giving the "reverse connection." As the hybrid coil is balanced, the output power of the amplifier does not react upon the input circuit. Putting A for the amplifying ratio of the 21-type circuit,

$$A = \frac{I_2}{I_1}. \tag{14}$$

The total current flowing at the terminals 5, 8 is:

$$I_0 = I_1 + I_2 = \frac{E}{2R_0}(1 + A), \tag{15}$$

and the active resistance of the 21-type circuit is:

$$r = \frac{E}{I_0} - R_0 = \frac{1 - A}{1 + A} R_0. \tag{16}$$

As the amplification is increased, the current I_0 increases while r falls to zero and becomes negative, thus exhibiting series characteristics. If A is increased without limit, r approaches $-R_0$ in magnitude but cannot reach it, while A remains finite. That is, the system shown in Fig. 7 cannot sing. This is also obvious from the fact that the hybrid coil is balanced. However, the resistance r does not depend upon holding R_0 at the terminals 5, 8 constant. If the resistance at the terminals 5, 8 is reduced to a lower value R_0', while that at the terminals 6, 7 is held constant at R_0, the output energy of the amplifier is permitted to reach the input terminals 1, 2 and when $-R_0' = r$ instability or singing can occur.

Throwing the reversing switch to give the reversed connection has the effect of reversing the sign of the amplification A. The total current I_0 at the terminals 5, 8 decreases to zero, reverses and increases as A increases, while r increases, passes discontinuously from $+\infty$ to $-\infty$ and decreases in magnitude. Again r approaches $-R_0$ as A increases indefinitely, but cannot reach it. However, by increasing the resistance connected to the terminals 5, 8 to a higher value R_0' such that $-R_0' = r$, instability will occur. The reversed connection thus gives a negative resistance of shunt characteristics.

Referring to Fig. 7 and assuming that the switch is thrown to give the directions of current flow indicated by the arrows, transfer the e.m.f. E to the right-hand end of the diagram. This change will not change the direction of I_1 in the input circuit of the amplifier or the direction I_2 at any point. The current I_1 will now be found at terminals 6, 7 instead of 5, 8 and will be flowing in the direction opposite to I_2. From this it will be seen that a 21-type circuit which is direct-connected with reference to terminals 5, 8, giving a series type negative resistance, will be reverse-connected, and give a shunt type negative resistance at the opposite terminals 6, 7. Changing the reversing switch reverses the conditions at both pairs of terminals.

Non-Ideal Devices

The discussion has so far been confined principally to certain ideal conditions which can only be approximated in practice, but consideration of these simple cases will serve to illustrate the important fundamental properties of negative resistances and the requirements that must be met to insure stable operation.

To obtain a pure negative resistance from a one-way amplifier or from a 21-type repeater circuit requires that there shall be no phase shift in the process of amplification. This can only be approximated in practice because even a resistance coupled amplifier system involves small inductances and capacitances in the tubes and wiring which produce phase shifts at high frequencies. Commercially practicable transformers, choke coils, and condensers which are so useful in assemblages of apparatus which include vacuum tubes further limit the range of frequency over which an approximately pure negative resistance may be obtained. In some cases, this may not be a serious disadvantage. Suppose, for example, it is desired to reduce the effective resistance of a series resonant circuit in order to obtain more nearly ideal performance at the resonant frequency. It would be sufficient to arrange a negative resistance in series with the resonant circuit which would produce the desired result at and near the resonant frequency and which would produce no harmful effect at other frequencies even though it departed widely from the value at the resonant frequency. In other cases the variation of the negative resistance with frequency and the introduction of reactive components do no serious harm and may even be quite useful as in the case of the twin 21-type repeater to be described below. In still other cases the difficulties of producing a negative resistance of satisfactory characteristics may be very great.

General Negative Impedance

The arrangements described above produce under ideal conditions pure negative resistances.

It has been shown by R. C. Mathes and H. W. Dudley that it is possible to produce any desired negative impedance provided that the positive of this impedance can be constructed in the form of a network.

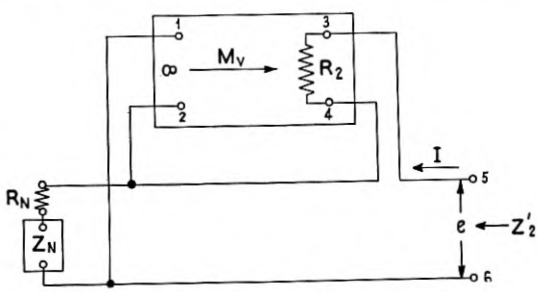


Fig. 8—Series type negative impedance.

Fig. 8 shows in simplified form the arrangement invented by Mathes, and Fig. 9 shows the arrangement due to Dudley. Each of these arrangements requires a distortionless one-way amplifier whose input impedance (terminals 1, 2) is substantially infinite. This condition is easily approximated by using vacuum tubes. In discussing the behavior of such arrangements, it is necessary to use the ratio, M_v, of the e.m.f. generated in the output circuit of the amplifier to the voltage impressed on its input terminals, instead of the mutual impedance of the amplifier, because the input current is negligibly small. This ratio may be adjusted by some suitable means such as a potentiometer.

Referring to Fig. 8, let Z be the positive of any desired negative impedance such that a network having the impedance, $Z_N = Z/M_v - 1$, may be constructed of physically available parts, M_v being a real

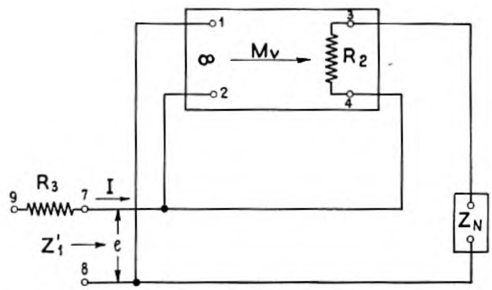


Fig. 9—Shunt type negative impedance.

number greater than 1. $R_N = R_2/M_v - 1$ is a pure positive resistance. Next assume that a current, I, is flowing through the circuit between terminals 5 and 6. The e.m.f. generated in the output circuit of the amplifier is $(R_N + Z_N)IM_v$. It acts in the direction which tends to increase the current. The voltage e required at the terminals 5, 6 to produce this current is, then,

$$e = (R_N + Z_N + R_2)I - (R_N + Z_N)IM_v, \tag{17}$$

from which the impedance Z_2' is:

$$Z_2' = \frac{e}{I} = -Z, \tag{18}$$

which is the desired negative impedance. Due to the arrangement of the circuit this impedance has series characteristics.

Referring to Fig. 9, Z_N is a positive network. Assuming that an e.m.f. e is applied to the terminals 7, 8, the e.m.f. generated in the output circuit of the amplifier is eM_v which acts in opposition to e to reduce or reverse the current. The current at the terminals 7, 8 is, then,

$$I = \frac{e - eM_v}{R_2 + Z_N}, \tag{19}$$

and the impedance Z_1' at the terminals 7, 8 is:

$$Z_1' = \frac{e}{I} = \frac{R_2}{1 - M_v} - Z, \tag{20}$$

which consists of the desired negative impedance $-Z$ and a negative resistance if $M_v > 1$. By connecting the positive resistance, $R_3 = R_2/M_v - 1$, in series with Z_1' this negative resistance is neutralized and the desired negative impedance is found between the terminals 8 and 9. This impedance has shunt characteristics.

In both of these arrangements it is possible, without changing the constants of the network Z_N, to give the negative impedance any desired magnitude by adjusting the value of M_v and making the corresponding change in the resistance R_N or R_3.

Boosters

The name "booster" has been applied to a negative impedance of suitable characteristics connected in series with or bridged across a telephone circuit in order to introduce energy when a wave passes and so produce a transmission gain. Such devices have certain interesting theoretical properties.

Series Booster

Fig. 10 shows an impedance Z_s connected in series between the two parts of a telephone line having the characteristic impedance Z_0.

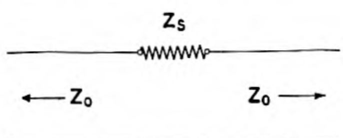


Fig. 10—The series booster.

Assume first that Z_s is a positive impedance having the same angle as Z_0 and that a wave is traveling over the line, for example, from left to right. The effect of the inserted impedance is to reduce the current in the line wires at the point of insertion, weakening the wave that passes on to the receiver and causing a reflected wave to return to the source. The transmission loss[2] caused by the inserted impedance is:

$$L = 20 \log_{10}\left(1 + \frac{Z_s}{2Z_0}\right), \tag{21}$$

[2] The values of losses, return losses and gains will be expressed in decibels (db) throughout this paper.

and the return loss [3] due to the irregularity is:

$$S = 20 \log_{10}\left(1 + \frac{2Z_0}{Z_s}\right). \tag{22}$$

If, now, Z_s is made a negative impedance of the series type smaller in magnitude than $2Z_0$, the potential difference between its terminals reverses in sign, the current at the point of insertion increases, the loss becomes a gain and the reflected wave reverses in sign. As Z_s approaches $-2Z_0$, the transmitted and reflected waves increase until singing occurs; but the reflected wave is always smaller than the transmitted though they approach each other as the gain increases. Such a booster, therefore, causes a smaller returned wave or echo than an ideal 21-type repeater circuit working between ideal line impedances which always returns a wave toward the source which is equal to that transmitted toward the receiver.

The series booster would also operate if Z_s were made a shunt type negative impedance greater in magnitude than $2Z_0$, but in this case the current at the booster and the wave traveling toward the receiver would be reversed in phase and the reflected wave or echo would be greater than the wave traveling toward the receiver. This arrangement would, therefore, give greater echoes for a given gain than a 21-type repeater. The curves of Fig. 12 show the relation between the return loss and transmission gain for these boosters in comparison with a 21-type repeater.

The echoes referred to above are, of course, those inherent in the operation of the devices described and would not occur if a 22-type repeater were used with perfect lines. Echoes due to line irregularities would be amplified to the same extent by boosters as by any other type of two-way repeater giving the same gain.

Shunt Booster

Fig. 11 shows an impedance Z_b bridged across the line. The effect of this impedance is to reduce the wave traveling toward the receiver, causing a transmission loss,

$$L = 20 \log_{10}\left(1 + \frac{Z_0}{2Z_b}\right), \tag{23}$$

and causing a reflected wave to return to the source with a return loss,

$$S = 20 \log_{10}\left(1 + \frac{2Z_b}{Z_0}\right). \tag{24}$$

[3] When a wave is partially reflected at an irregularity the relation between the reflected part and the original wave, expressed in decibels, is called the return loss.

In this case the current in the line leading toward the source is increased; that is, the reflected wave is of opposite phase to that reflected by an impedance in series with the line.

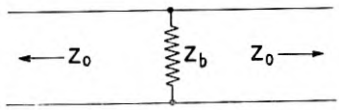


Fig. 11—The shunt booster.

If Z_b is made a negative impedance with shunt characteristics and greater in magnitude than $Z_0/2$, the current through Z_b reverses in sign, the wave transmitted toward the receiver increases, the transmis-

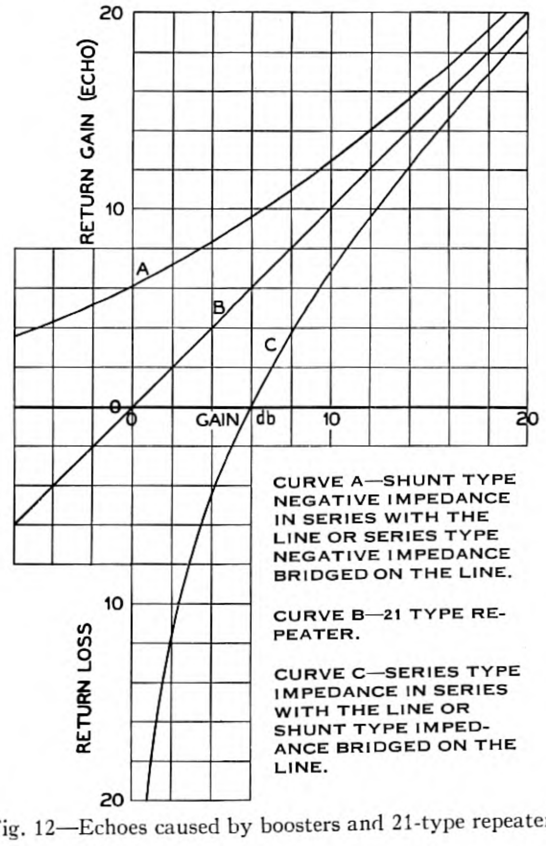


Fig. 12—Echoes caused by boosters and 21-type repeaters.

sion loss becoming a gain and the reflected wave reverses in sign, thus reducing or reversing the current in the line leading toward the source. The relation between the magnitude of the echo and the gain is the same as for the series type booster described above except that the reflected waves are opposite in phase. This makes it possible to eliminate the echo by combining two boosters in one repeating device as described below.

The shunt booster would also operate if Z_b were made a series type negative impedance smaller in magnitude than $Z_0/2$, but in this case the wave traveling toward the receiver would be reversed in phase and the echo wave would be greater than the transmitted wave.

Singing Points of Various Forms of Repeaters

When a line of characteristic impedance Z_0 has a certain return loss S_l, its impedance will lie between a maximum value of mZ_0 and a minimum of Z_0/m where

$$S_l = 20 \log_{10} \frac{m+1}{m-1} \cdot \tag{25}$$

If two pieces of such a line are joined through a repeating device the high and low impedances may combine in three different ways which give the greatest tendency to sing with different types of apparatus.

The series type negative impedance, whether connected in series with or across the line, has the greatest tendency to sing when the minimum impedances of both lines occur at the same frequency and the shunt type negative impedance has the greatest tendency to sing when the maximum impedances occur at the same frequency. The 21-type repeater has the greatest tendency to sing when the maximum impedance of one line and the minimum of the other occur at the same frequency, the internal connections of the repeater determining which impedance must be high. In the 22-type repeater any of these combinations may be the worst, depending upon the internal arrangement of the repeater circuit.

The series booster (with series type negative impedance) will sing when

$$Z_s + \frac{2Z_0}{m} = 0. \tag{26}$$

Substituting Z_s obtained from this relation in equation (21) and remembering that the loss L becomes a gain G_s when Z_s is negative, the gain which will produce singing is:

$$G_s = 20 \log_{10} \left(1 - \frac{1}{m} \right) \cdot \tag{27}$$

This gain is, of course, the gain which a booster having the impedance Z_s obtained from equation (26) would produce when connected between two impedances Z_0. The actual gain of the booster, like that of any other type of repeater approaches infinity as the singing condition is approached.

The shunt booster (with shunt type negative impedance) will sing when

$$Z_b + \frac{mZ_0}{2} = 0. \tag{28}$$

Substituting the value of Z_b from this equation in equation (23) shows that the relation given in equation (27) also holds for the shunt type booster.

It is well known that when a 22-type repeater giving the gain G_{22} in each direction is connected between two lines having the return loss S_l singing will occur when

$$G_{22} = S_l, \tag{29}$$

if the worst combination of unbalances occurs.

It is also well known that under similar conditions the gain of a 21-type repeater is:

$$G_{21} = S_l - 6\text{db}, \tag{30}$$

because of the fact that waves reflected from the irregularities in both lines combine in the input circuit of the amplifier.

The curves of Fig. 13 show the singing gain as a function of line return loss for boosters, 21-type and 22-type repeaters. These curves together with the curves of Fig. 10 indicate that ideal boosters consisting of series type negative impedances in series with the line or shunt type negative impedances bridged across the line have properties intermediate between those of 21 and 22-type repeaters with respect to the amount of echo and margin against singing for a given transmission gain. These properties are particularly favorable at low gains.

In practice, however, it is usually necessary to limit the amplification to a definite band of frequencies in order to avoid the effect of impedance unbalances and interfering disturbances at frequencies outside these limits. This must be accomplished by the use of inductance and capacitance in the form of filters, transformers, choke coils or condensers. It is also desirable to couple the series booster to the line by means of a transformer having two equal windings, one in each line conductor, to enable one booster mechanism to operate without unbalancing the line and to permit the passage of low frequency signaling waves from one part of the line to the other without interference from the booster. For similar reasons, condensers must be connected in series with the shunt booster when it is bridged on the line. These devices, particularly the filters, shift the phase of the amplified waves, and modify the negative impedances so that the gain varies with frequency in the useful range to a greater extent than is the case with the 21 and 22-type repeaters and the echoes are increased. This variation of gain is due to the fact that the booster, in effect, superimposes an amplified wave upon the wave that would exist if the

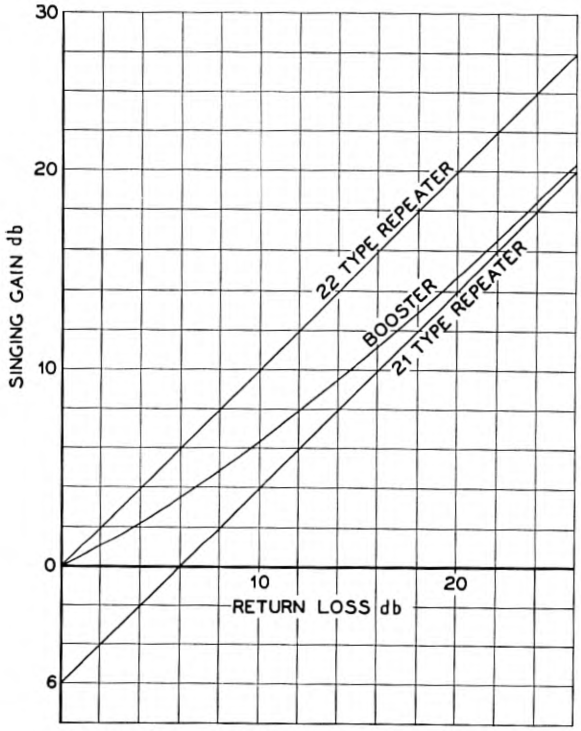


Fig. 13—Singing gains of boosters and repeaters.

booster were removed. The received wave, being the resultant of these two waves, varies with the phase angle between them.

It should also be noted that boosters do not avoid the problem of matching line impedances or the difficulties due to impedance irregularities in the line. To obtain a gain that is constant over a wide range of frequencies, the negative impedance must be fitted to the line impedance over this range and there must be no large irregularities. It will be shown below that most of the difficulties described above may be avoided by using a series and a shunt booster in combination.

Negative Impedances Arranged in T or Π Networks

It has been pointed out by G. A. Campbell, H. Mouradian,[4] and possibly by others, that three negative impedances can be grouped into a T or a π network which may be inserted in a telephone line. Such a network is able to amplify waves traversing the line without causing echoes if the values of the impedances are suitably chosen. In order to avoid singing, the impedances in series with the line must be of the series type, and those bridged across the line, of the shunt type.

A Double Booster

Fig. 14 shows a network of impedances connected between two pieces of telephone line having the characteristic impedance Z_0. These lines are assumed at first to be free from irregularities. The branches *ac* and *bc* are fixed networks, each having the impedance Z_0. Branches *ab* and *cd* are networks whose impedances can be varied reciprocally from the value Z_0, that is, if one impedance is multiplied by a factor

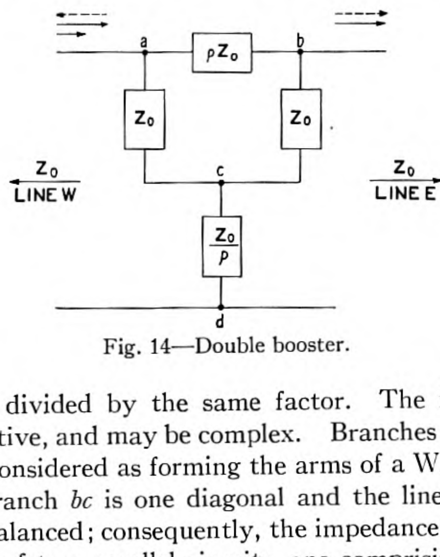


Fig. 14—Double booster.

ρ, the other is divided by the same factor. The factor ρ may be positive or negative, and may be complex. Branches *ab*, *ac*, *cd* and the line E may be considered as forming the arms of a Wheatstone bridge, of which the branch *bc* is one diagonal and the line W is the other. This bridge is balanced; consequently, the impedance connected to the line W consists of two parallel circuits, one comprising the branch *ab* in series with the line E and the other comprising the branches *ac* and *cd* in series. This impedance is independent of ρ, being equal to Z_0. By symmetry, the impedance connected to the line E is also equal to Z_0, so no reflection occurs at the terminals of the network.

Assuming that a wave arrives, for example, over the line W and is

[4] "Long Distance Transmission Problems," by H. Mouradian, *Journal of the Franklin Institute*, Vol. 207, No. 2, February, 1929.

transmitted to the line E, the ratio of the voltage across the terminals a, d to that impressed on the line E is $(1 + \rho)/1$ and the transmission loss through the network is:

$$L = 20 \log_{10} (1 + \rho). \tag{31}$$

This loss becomes a gain when ρ becomes negative and the network acts as an amplifier.

Examination of Fig. 14 shows that the branches ab and cd are each connected to a constant impedance Z_0, hence, if ρ lies between 0 and -1, the branch ab must be a series type negative impedance and cd of the shunt type. If ρ lies between -1 and $-\infty$, these types must be interchanged.

The physical behavior of this network may be readily understood if the properties of negative impedances are kept in mind. At first let ρ be infinite and assume that a wave arrives over the line W. This wave tends to produce a current in the upper conductor which at a given instant flows in the direction indicated by the short solid arrow. This wave will be absorbed by the impedance Z_0 of the branch ac. If now ρ is made negative the series type negative impedance in the branch ab will cause additional currents to flow in the lines E and W the directions and relative magnitudes of which are indicated by the longer solid arrows. The shunt type negative impedance in the branch cd tends to produce currents having the directions indicated by the dotted arrows, thus further increasing the wave in the line E but annulling the effect of the series type impedance in the line W. The network of Fig. 12, therefore, amplifies waves traveling in either direction without causing echoes to return to the source. It resembles, in this respect, a 22-type repeater, but it cannot give different gains in the two directions.

Putting a shunt type impedance in the branch ab and a series type in the branch cd would reverse the sign of the amplified wave.

For such a network to function as described above, it is not necessary for the ratio ρ to be independent of frequency. Phase shifts in the negative impedances are permissible provided they are kept equal so that the echoes will be eliminated. It is, therefore, possible to use filters and other apparatus to cause the gain to vary with frequency in a desired manner without encountering the troubles which occur in the single booster.

It is further possible to couple the series branch ab of the network of Fig. 12 to the line by means of a transformer and the bridged branch cd by means of a condenser without seriously altering the reciprocal relation of these impedances. This provides a method for permitting low frequency signals to pass over the line without serious interference from the rest of the network.

The Twin 21-Type Repeater Circuit

A simplified diagram of a twin 21-type circuit is shown in Fig. 15. This consists of a line hybrid coil whose line windings are connected in series with the line conductors and two 21-type circuits. One pair of terminals of one of the 21-type circuits is connected with the drop winding of the hybrid coil which couples it effectively in series between the two parts of the line. A network N_S is connected to the remaining terminals which balances the impedance of the two parts of the line as seen from the 21-type circuit. One pair of terminals of the second 21-type circuit is connected to the bridge terminals of the hybrid coil which bridges it across the line. A network N_B is connected to the remaining terminals which balances the impedance of the two parts of the line as seen from the bridge. The internal connection of the series 21-type circuit is direct with respect to the line hybrid coil and the bridged circuit is reversed.

At first, assume that the potentiometers of the two 21-type circuits are turned down and that a wave arrives at the W line terminals of the twin 21-type circuit. At the peak of the positive half-cycle, currents will flow in the line hybrid coil in the directions indicated by the arrows marked I. The passive impedances are chosen to fit the normal impedances at the drop and bridge terminals of the line hybrid coil; hence, none of this wave will reach the E end of the line.

Next, turn the potentiometer of the series 21-type circuit up until this circuit gives a gain. Due to the internal arrangement of this circuit, an amplified current will flow in the line conductors in the directions indicated by the large arrows marked I_{OS}. Little or none of this current will reach the bridged circuit because of the balance between the two parts of the line.

Finally, turn the potentiometer of the bridged 21-type circuit up until this circuit gives the same gain as the series circuit. Due to the internal arrangement of the bridged circuit, amplified currents will flow in the line conductors in the directions indicated by the arrows marked I_{OB}. These currents are equal in magnitude to those caused by the series 21-type circuit. In the line W the output currents annul each other so that echoes returning toward the speaker are suppressed while the currents in the line E co-operate and an amplified wave travels over the line E to the listener.

If the incoming wave arrives over the line E, the action is the same except that the direction of the current I_{OS} due to the series 21-type circuit is reversed with respect to the current I_{OB} which causes the amplified wave to travel toward the W end of the line.

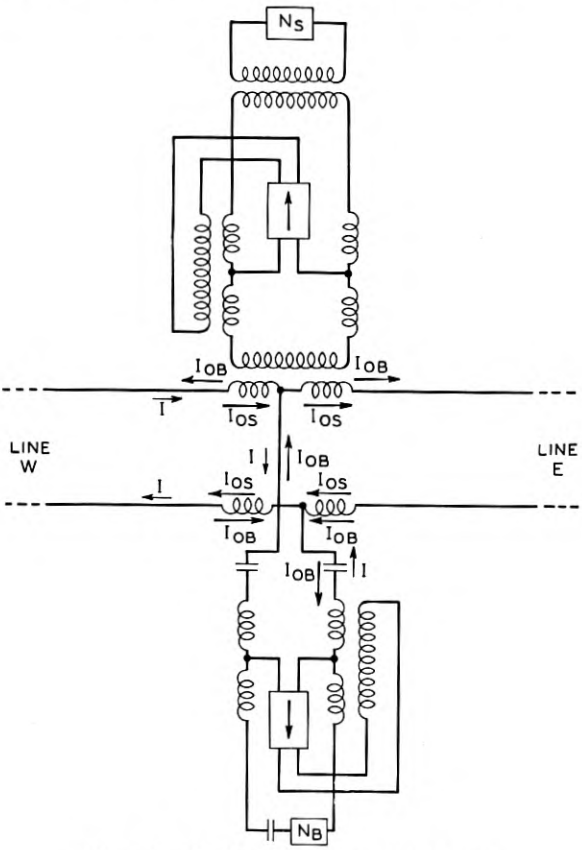


Fig. 15—The ideal twin 21-type repeater.

If the internal connection of both 21-type circuits is changed the direction of both output currents will be reversed. The action of the twin 21-type circuit will not be altered except that the phase of the amplified wave will be reversed. Changing only one of the 21-type circuits, however, will cause the amplified wave to travel toward the speaker as a powerful echo and will prevent transmission toward the listener.

Effect of Phase Shift

In the foregoing description the discussion has been simplified by assuming ideal transformers and 21-type circuits. It is sufficient, however, that at any frequency where amplification occurs, the gains and phase shifts of the two 21-type circuits should be equal. This insures that the echoes will balance out and that the maximum output power will be directed toward the listener. To accomplish this it is merely necessary to make the corresponding parts of the two circuits alike within the allowable tolerance. Filters, condensers and other devices may be used as required provided that the corresponding parts in the two 21-type circuits are nearly enough alike.

Referring to Fig. 15, the series 21-type circuit is coupled to the line inductively by the line hybrid coil and the shunt circuit is connected through condensers. This arrangement makes it possible for the line conductors to be joined through the windings of the twin 21-type circuit without a conductive bridge across the line, and so provides the desired path for d-c. impulses or low frequency alternating current.

The inductance of the line hybrid coil cannot, of course, be infinite. Practically, it must be a compromise between the opposing requirements that it shall be low enough not to interfere seriously with the transmission of low frequency signaling impulses or transfer too much of their energy to the series 21-type circuit and that it shall be high enough to prevent too great a transmission loss at the lower frequencies of the voice range. Due to this finite inductance, the amplified voice currents from the series 21-type circuit will be shifted in phase at the lower frequencies.

The capacitance of the condensers in series with the bridged 21-type circuit is similarly limited, and shifts the phase of the amplified voice currents from that circuit. These two shifts are in the same direction which makes it possible to keep the amplified currents from the two 21-type circuits in phase and prevent the production of echoes.

The transmission loss and phase shift due to the finite inductance of the line hybrid coil will be approximately equal to the loss and phase shift due to the condensers when

$$\frac{L_1}{C_1} = \frac{L}{C} = R^2, \tag{32}$$

in which

L_1 = Inductance of the whole line winding of the line hybrid coil, with the drop open.

C_1 = Capacitance in series with the bridged circuit.

L = Inductance per unit length of the line.

C = Capacitance per unit length of the line.

R = Nominal impedance of the line.

When two condensers are used in series, as shown, to keep the circuit balanced, each one must, of course, have a capacitance $2C_1$.

It would be possible to carry this principle still further, if necessary, so that anything introduced between the series 21-type circuit and the line which results in adding impedance in series or shunt with the series 21-type circuit can be matched by adding suitable impedance in shunt or series, respectively, with the bridged 21-type circuit. Similarly, anything which affects the impedance of the bridged circuit can be matched by a corresponding addition to the series circuit. In order for these additional impedances to match, the following relation must be established at all frequencies in the useful range:

$$z_S \times z_B = R^2, \tag{33}$$

in which z_S is an impedance effectively in series, or parallel, with the series 21-type circuit and z_B an impedance effectively in parallel, or series, respectively, with the bridged 21-type circuit. The value z_S is referred to the line windings of the line hybrid coil, that is, if the element contributing this impedance is connected to the drop winding of the line hybrid coil its actual impedance must be multiplied by the square of the turn ratio of the entire line winding to the drop winding to obtain z_S.

If more than one part of the series 21-type circuit must be compensated by corresponding parts of the shunt circuit, it is necessary that the corresponding parts be arranged in the same order between the line and the 21-type circuits.

Special Properties of the Twin 21-Type Circuit

The twin 21-type repeater differs in a number of important respects from the 22-type repeater and others that have been used or proposed in the past. It is essentially a network of impedances two of which include negative resistance components. These are the two 21-type circuits. Each 21-type circuit is connected to the line by only one pair of terminals through which the input wave enters and the amplified wave leaves it; hence, it may be treated as a single impedance which has a negative resistance component. It follows from this that the twin 21-type circuit follows the *reciprocal law,* and that the gain at any frequency is the same for both directions of transmission. This is true even if the two 21-type circuits are not set for the same gain. If the gains of the two circuits are different the amplified current wave will be the sum of the current waves from the two 21-type circuits (as measured in milliamperes or other current units) and an echo equal to the difference will travel toward the speaker.

Another difference lies in the fact that both amplifiers work at the same time. Even though the echo waves are cancelled out their energy is not lost, but is added to the amplified wave. The output current is twice, and the output power is four times what either 21-type circuit acting alone would send toward the listener. In the 21 or 22-type circuit, only half the output power of one amplifier reaches the line, the other half being absorbed in the opposite line or in the network. The output power is, therefore, 3 db less than that which the amplifier actually produces. In the twin 21-type circuit one-half the output power of each amplifier is also absorbed in a network, but the remaining halves are combined in the output wave. Consequently, the total output is equal to that of one amplifier. For this reason, with a given size of vacuum tube, the twin 21-type circuit can deliver twice as much useful power, or 3 db larger volume to the line, than either the 21 or 22-type repeater.

Push-Pull Effect

If the connection between the line hybrid coil and either of the 21-type circuits is transposed, the directions of current flow in the 21-type circuit are all reversed, but the directions of the input and output currents in the line conductors are not affected. If the amplifiers are perfect, such a transposition will have no effect upon the operation of the twin 21-type circuit. When vacuum tubes are used as amplifiers, however, there is a certain amount of distortion due to the curvature of the operating characteristics of the tubes.

If the connections are so arranged that the grids of the tubes in both of the 21-type circuits receive positive potentials from the incoming wave during the same half-cycle, this distortion will appear in the output wave of the twin 21-type circuit. If, for example, the input wave is a pure sinusoid, the output wave will contain a series of harmonics. Some of these harmonics will be of even number, principally the second harmonic, and correspond to a difference of the shapes of the positive and negative half-cycles.

Transposing the connection of one of the 21-type circuits as described above causes one of the grids to receive positive potential from the input wave at the same time that the other grid receives negative potential. This reverses the phase of the even numbered harmonics from one of the 21-type circuits with respect to those from the other, and so eliminates the even numbered harmonics from the output wave of the twin 21-type circuit. This result is similar to that obtained by means of the familiar push-pull arrangement of vacuum tubes used in an amplifier to reduce distortion, but no increase of the number of tubes is required.

The even numbered harmonics from the two 21-type circuits are not annihilated, however, but combine to form an echo which travels toward the speaker, and this echo must not be permitted to become too strong. It would be possible to eliminate such echoes by using the push-pull connection in the amplifier of each 21-type circuit, but this, of course, would double the number of tubes required.

The Twin 21-Type Phantom Group of Repeaters

Three twin 21-type circuits may be connected with the wires of a phantom group so that voice-frequency waves traveling over either side circuit or the phantom may be amplified and low-frequency signals may be passed through the apparatus. This arrangement does not break up the phantom group and requires no phantom repeating coils or compositing apparatus. It cannot be used, of course, at points where it is necessary to separate the side and phantom circuits. A simple diagram of the phantom group of repeaters is shown in Fig. 16. Each 21-type circuit with its own hybrid coil, amplifier, network, etc., is indicated by a small square. A twin 21-type circuit is connected in tandem with each side circuit. The repeater in the side S_1 comprises a line hybrid coil S_1Hy, a series 21-type circuit S_1S and a bridged 21-type circuit S_1B as indicated. The side circuit S_2 is similarly equipped. Repeating coils R_1 and R_2 are shown between the bridged 21-type circuits and the bridge terminals of the line hybrid coils in the side circuits. Taps are provided at the mid-points of the line windings of these coils by which the 21-type circuit PB is bridged across the phantom circuit. While separate transformers are shown in the diagram to provide the connections for the phantom bridged 21-type circuit, they might be omitted if the side circuit bridged 21-type circuits S_1B and S_2B are each so arranged as to provide a tap which is symmetrical with respect to the line wires. This arrangement, however, introduces additional possibilities of unbalance with the resulting noise and crosstalk which are avoided by the use of the coils R_1 and R_2.

The phantom series 21-type circuit PS is coupled effectively in series with the phantom circuit by means of the phantom line hybrid coil PHy. This is a special transformer having eight carefully balanced sections in the line winding and a drop winding to which the 21-type circuit is connected. Two of the line winding sections are connected in series aiding with each line conductor, one on each side of the side circuit line hybrid coil. The sections in series with the several line wires are so poled that they are non-inductive to waves traversing the side circuit, but they are inductive to waves traversing the phantom, thus producing the desired coupling.

The series and bridged phantom 21-type circuits co-act in the phantom circuit to amplify without echoes, the waves traversing the phantom in the same way as the corresponding parts of the side circuits.

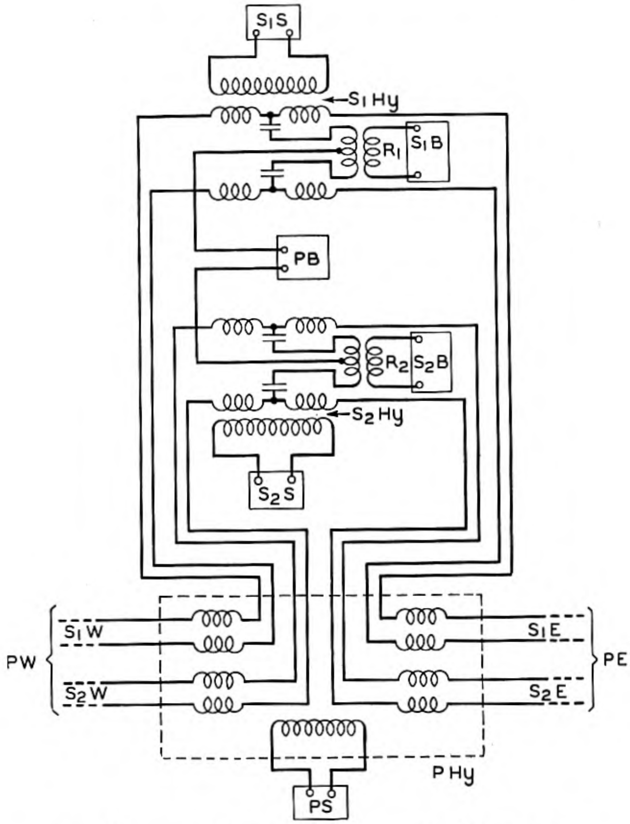


Fig. 16—Twin 21-type phantom group of repeaters.

Field Trials

In order to demonstrate the operativeness of the twin 21-type of repeater and to gain some experience with it, a complete phantom group of repeaters was built, installed at Princeton, N. J., and connected into a phantom group of cable conductors extending from New York to Philadelphia.

This apparatus functioned in a satisfactory manner despite the fact that certain transformers and other parts specially designed for this work were not available, and it was necessary to make use of some equipment designed for other purposes.

Field of Use

Although the results of the field trial were satisfactory, it is not planned to introduce the twin 21-type of repeater into the plant at the present time. It is not yet known whether the features of this type of repeater will prove of advantage as compared with the 21 or 22-type repeaters. However, with the continued increase in the use of repeatered circuits, over which it is desired to transmit d-c. signaling or dial impulses, it is possible that this may be the case and further studies are planned to determine its economic field of use.

New Standard Specifications for Wood Poles

By R. L. JONES*

This paper summarizes the work of the Sectional Committee on Wood Poles of the American Standards Association covering the preparation of specifications for northern white cedar, western red cedar, chestnut and southern pine poles. The major problems underlying the development of standard ultimate fiber stresses, standard dimension tables and practical knot limitations are discussed and illustrated by supporting tables or figures. Graphical charts comparing the old and the new dimensional classifications are described. The main points relating to the material requirements for the four pole species are outlined briefly.

REPRESENTATIVES of communication, power and light, and transportation utilities, of producers, and of public and general interests have cooperated in the preparation of the new uniform standard specifications for wood poles that were recently approved by the American Standards Association.[1] The new specifications cover dimensions and material requirements for northern white cedar, western red cedar, chestnut and southern pine poles, but rules for preservative treatment are not included. Specifications for lodgepole pine and Douglas fir poles are in preparation.

Pole specifications deal with natural rather than fabricated products. Heretofore, the larger utilities have purchased poles of the various species under specifications that have grown up more or less independently. Confusing differences in material requirements and in the dimensional tables have resulted. Economic production and utilization require the arrangement of the natural cut of pole timbers into groups defined either by top diameters and lengths, or by classes in which circumferences at the top and butt are specified in addition to length. The letter designations, such as A, B, and C, that have been applied to these classes, have had no common meaning. A pole of a given length and class of one species has not generally been equivalent in strength rating to one of the same length and class of another species; and in most cases, the longer poles of a given class have not had the same strength rating as the shorter poles of the same class.

It is perhaps quite obvious that before rational improvement could be made in the system of dimensional classification, it was necessary to create a foundation for comparison of the strength of the different

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[1] These specifications were approved on June 20, 1931.

species. For illustrative purposes a summary of part of the test results used in arriving at fiber stress values is shown in Table 1. A detailed study of the results of these tests and of other tests made on full length poles and on small clear specimens of wood of the species

TABLE 1

SUMMARY OF STATISTICAL ANALYSIS OF MODULUS OF RUPTURE VALUES OBTAINED FROM TESTS ON FULL SIZE POLES

Modulus of Rupture Pounds per Square Inch	Northern White Cedar		Western Red Cedar		Chestnut		Southern Pine (Creosoted)	
	No.	Per cent	No.	Per cent	No.	Per cent	No.	Per cent
2000–2499	2	3.57						
2500–2999	13	23.21	1	0.66				
3000–3499	11	19.64	4	2.65				
3500–3999	14	25.00	5	3.31	1	1.02		
4000–4499	8	14.29	10	6.62	4	4.08		
4500–4999	1	1.79	21	13.91	7	7.14	1	0.83
5000–5499	5	8.93	21	13.91	8	8.16	1	0.83
5500–5999	2	3.57	18	11.92	14	14.29	6	4.96
6000–6499			21	13.91	15	15.31	4	3.31
6500–6999			25	16.55	11	11.22	12	9.92
7000–7499			16	10.60	14	14.29	28	23.12
7500–7999			7	4.64	13	13.27	10	8.26
8000–8499			1	0.66	7	7.14	15	12.40
8500–8999			1	0.66	3	3.06	15	12.40
9000–9499							12	9.92
9500–9999					1	1.02	5	4.13
10000–10499							5	4.13
10500–10999							6	4.96
11000–11499							1	0.83
Total No.	56		151		98		121	
Average		3670		5813		6536		8039
Standard deviation		860[2]		1184		1223		1348
Coefficient of variation (per cent)		23.43		20.39		18.71		16.77

[2] Uncorrected for sample size.

under investigation led to the recommendation of the following figures as standard ultimate fiber stresses:

Northern white cedar	3600 lbs. per sq. in.
Western red cedar	5600 " " " "
Chestnut	6000 " " " "
Southern yellow pine (creosoted)	7400 " " " "

The fiber stress for a given species finds application in pole line engineering through the conversion of the stress value into terms of moment of resistance, usually at the ground line. The poles act as a series of supports for the wires. With this in mind one of the studies conducted in connection with the application of the new fiber stresses, which is cited here by way of illustration, was directed toward an analysis of the variation in size and variation in modulus of rupture that might be expected to affect the average ground line moment of resistance of random 3 pole groups. Approximately 400 creosoted southern pine and 500 western red cedar, class 3, thirty foot (see Table 2) poles were used in this particular study. It was found that in more than 95 per cent. of the cases the average moment of resistance of such 3-pole groups was higher than the minimum calculated for the given class and length. The result is considered reasonably representative of what would be found in a similar study of other sizes. It may be concluded that with the new standard fiber stress values as a basis practically all parts of a line when new should be equal to or better than the strength rating for the specified minimum of the class of poles used; and that when the reduced loads under the conditions usually obtaining in the higher grades of construction are considered, the bending moment developed at the ground line should rarely, if ever, approach the actual moment of resistance.

Since the standard ultimate fiber stresses are based upon tests of representative poles, they are believed to be satisfactory for all ordinary purposes. They are directly applicable in the engineering of pole lines without further adjustment or compensation for knots, variation in moisture content, or density of wood. In any case, the question of density classification may be limited for practical purposes to southern pine poles; and studies of current production show that approximately 75 per cent of such poles passing through the producers' yards could be classified as dense. The creosoting process seems to reduce the variation found in the modulus of rupture values of untreated poles. The comparatively low coefficient of variation of creosoted southern pine shown in Table 1 indicates that for general purposes an attempt to classify pine poles according to density is an unnecessary refinement.

With the standard fiber stresses as bases, dimension tables for the four species were developed in accordance with the following principles:

(*a*) The tables should specify dimensions in terms of circumference in inches at the top, and circumference in inches at six feet from the butt for poles of the respective lengths and classes except for three classes with "no butt requirement."

(*b*) All poles of the same length and class should have, when new, approximately equal strength, or in more precise terms, equal moments of resistance at the ground line.

(*c*) All poles of different lengths within the same class should be of suitable sizes to withstand approximately the same breaking load assuming that the load is applied two feet from the top and that the break would occur at the ground line.

(*d*) The classes from the lowest to the highest should be arranged in approximate geometric progression, the increments in breaking load between classes being about 25 per cent.

Item "*d*" is in accord with the preferred number principle, and the increments chosen provide the lowest number of classes that are required in service.

Tables of ten classes for each species, as shown in Table 2, have been made a part of the standard specifications. Classes 8, 9, and 10, defined simply by minimum top circumferences, have been provided to cover poles purchased on a top size basis or for rural or other lightly-loaded lines. Classes 1 to 7, defined primarily by their circumferences at six feet from the butt, have been designed to meet the following breaking loads in pounds, assuming the conditions of item (*c*):

Class 1—4500	Class 5—1900
Class 2—3700	Class 6—1500
Class 3—3000	Class 7—1200
Class 4—2400	

The required circumferences at the ground line for the respective species were calculated by means of the formula $Mr = .000264f\ C^3$, which is the well-known flexure formula applied to a cantilever beam of circular cross section, and reduced to foot pound units. The ground line circumferences thus obtained were converted into circumferences at six feet from the butt by means of approximate average taper values for the respective species.

The breaking loads are ratings for the minimum size pole for the given length and class based on the standard ultimate fiber stress for the species. The average pole of a given class will usually be considerably stronger than the class rating. The choice of sizes provided in the tables is sufficiently extensive to enable the engineer to make an economical selection of poles to meet specific requirements after the load conditions of the line have been determined.

Graphical charts have been prepared which show the relation between the dimension tables of some current specifications and the new standards. These charts should be of material assistance to suppliers and consumers who wish to compare the old with the new for inventory or record purposes. Representative blocks from the charts appear in Fig. 1. Comparisons for all lengths and classes may be found in the complete charts that are obtainable from the American Standards Association.

TABLE 2

STANDARD DIMENSIONS OF CREOSOTED SOUTHERN PINE, WESTERN RED CEDAR, NORTHERN WHITE CEDAR AND CHESTNUT POLES

Creosoted Southern Yellow Pine Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Dist. from Butt (Feet)	MINIMUM CIRCUMFERENCE AT SIX FEET FROM BUTT (Inches)							NO BUTT REQUIREMENT	NO BUTT REQUIREMENT	NO BUTT REQUIREMENT
16	3½					21.5	19.5	18.0			
18	3½			26.5	24.5	22.5	21.0	19.0			
20	4	31.5	29.5	27.5	25.5	23.5	22.0	20.0			
22	4	33.0	31.0	29.0	26.5	24.5	23.0	21.0			
25	5	34.5	32.5	30.0	28.0	26.0	24.0	22.0			
30	5½	37.5	35.0	32.5	30.0	28.0	26.0	24.0			
35	6	40.0	37.5	35.0	32.0	30.0	27.5	25.5			
40	6	42.0	39.5	37.0	34.0	31.5	29.0	27.0			
45	6½	44.0	41.5	38.5	36.0	33.0	30.5	28.5			
50	7	46.0	43.0	40.0	37.5	34.5	32.0	29.5			
55	7½	47.5	44.5	41.5	39.0	36.0	33.5				
60	8	49.5	46.0	43.0	40.0	37.0	34.5				
65	8½	51.0	47.5	44.5	41.5	38.5					
70	9	52.5	49.0	46.0	42.5	39.5					
75	9½	54.0	50.5	47.0	44.0						
80	10	55.0	51.5	48.5	45.0						
85	10½	56.5	53.0	49.5							
90	11	57.5	54.0	50.5							

Western Red Cedar Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Dist. from Butt (Feet)	MINIMUM CIRCUMFERENCE AT SIX FEET FROM BUTT (Inches)							NO BUTT REQUIREMENT	NO BUTT REQUIREMENT	NO BUTT REQUIREMENT
16	3½					23.0	21.5	19.5			
18	3½			28.5	26.5	24.5	22.5	21.0			
20	4	34.5	32.0	30.0	28.0	25.5	23.5	22.0			
22	4	36.0	33.5	31.5	29.0	27.0	25.0	23.0			
25	5	38.0	35.5	33.0	30.5	28.5	26.0	24.5			
30	5½	41.0	38.5	35.5	33.0	30.5	28.5	26.5			
35	6	43.5	41.0	38.0	35.5	32.5	30.5	28.0			
40	6	46.0	43.5	40.5	37.5	34.5	32.0				
45	6½	48.5	45.5	42.5	39.5	36.5					
50	7	50.5	47.5	44.5	41.0	38.0					
55	7½	52.5	49.5	46.0	42.5	39.5					
60	8	54.5	51.0	47.5	44.0						
65	8½	56.0	52.5	49.0	45.5						
70	9	57.5	54.0	50.5	47.0						
75	9½	59.5	55.5	52.0	48.5						
80	10	61.0	57.0	53.5	49.5						
85	10½	62.5	58.5	54.5							
90	11	63.5	60.0	56.0							

TABLE 2—*Continued*

Chestnut Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Dist. from Butt (Feet)	MINIMUM CIRCUMFERENCE AT SIX FEET FROM BUTT (Inches)							NO BUTT REQUIREMENT	NO BUTT REQUIREMENT	NO BUTT REQUIREMENT
16	3½					22.5	21.0	19.5			
18	3½			28.0	26.0	24.0	22.0	20.5			
20	4	33.5	31.5	29.5	27.0	25.0	23.0	21.5			
22	4	35.0	33.0	30.5	28.5	26.5	24.5	22.5			
25	5	37.0	34.5	32.5	30.0	28.0	25.5	24.0			
30	5½	40.0	37.5	35.0	32.5	30.0	28.0	26.0			
35	6	42.5	40.0	37.5	34.5	32.0	30.0	27.5			
40	6	45.0	42.5	39.5	36.5	34.0	31.5	29.5			
45	6½	47.5	44.5	41.5	38.5	36.0	33.0	31.0			
50	7	49.5	46.5	43.5	40.0	37.5	34.5	32.0			
55	7½	51.5	48.5	45.0	42.0	39.0	36.0				
60	8	53.5	50.0	46.5	43.5						
65	8½	55.0	51.5	48.0	45.0						
70	9	56.5	53.0								

Northern White Cedar Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Dist. from Butt (Feet)	MINIMUM CIRCUMFERENCE AT SIX FEET FROM BUTT (Inches)							NO BUTT REQUIREMENT	NO BUTT REQUIREMENT	NO BUTT REQUIREMENT
16	3½					26.0	24.0	22.0			
18	3½			32.5	30.0	28.0	25.5	23.5			
20	4	39.5	37.0	34.0	31.5	29.0	27.0	25.0			
22	4	41.0	38.5	36.0	33.0	30.5	28.0	26.0			
25	5	43.5	41.0	38.0	35.5	32.5	30.0	28.0			
30	5½	47.5	44.5	41.5	38.5	35.5	33.0	30.5			
35	6	50.5	47.5	44.0	41.0	38.0	35.0	32.5			
40	6	53.5	50.0	46.5	43.5	40.0	37.0				
45	6½	56.0	52.5	49.0	45.5	42.0					
50	7	58.5	55.0	51.5	47.5	44.0					
55	7½	61.0	57.5	53.5	49.5	46.0					
60	8	63.5	59.5	55.5	51.5						

Employment of the new standard ultimate fiber stresses of wood poles is provided for under rule 261-4-c of the National Safety Code. With the revisions necessitated by their adoption, Table 20 of the Code will appear as indicated in Table 3.

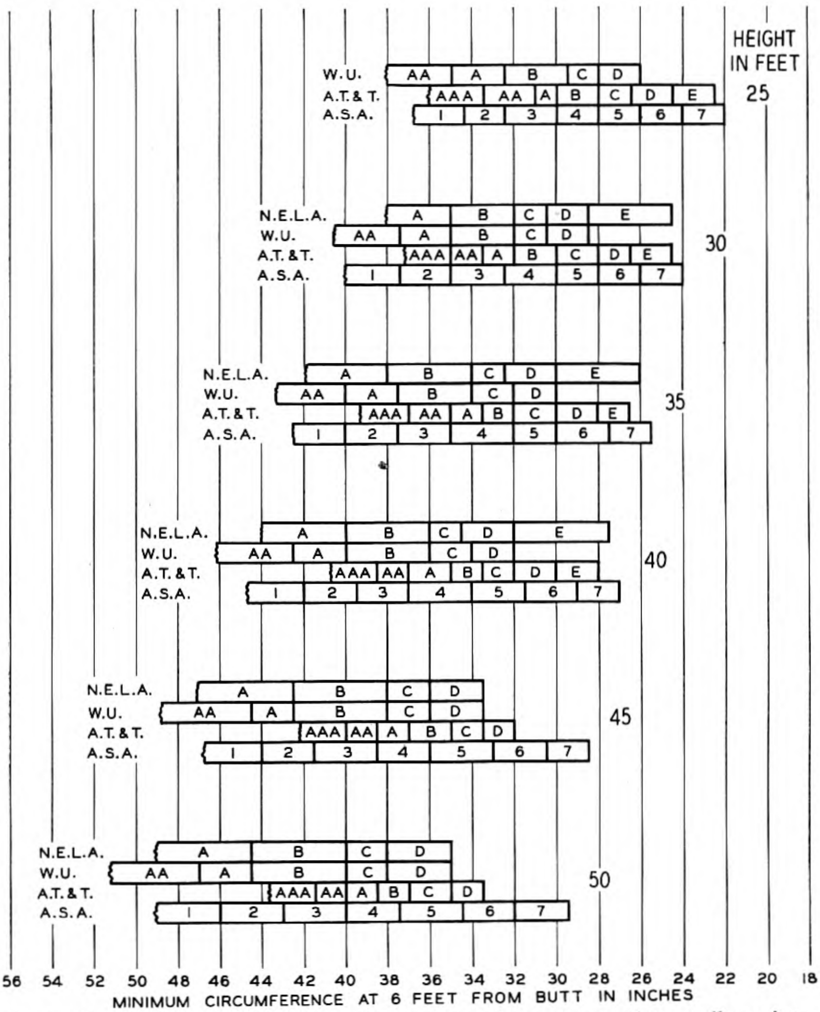


Fig. 1—Representative block from the graphical charts for southern yellow pine—current dimensions compared with the new standard tables.

The material requirements of the several specifications cover shape, and straightness of grain, and limit or prohibit such defects as knots, checks, insect damage and decay. Without detailed reference to what might be called the appearance requirements, it may be said that the

TABLE 3

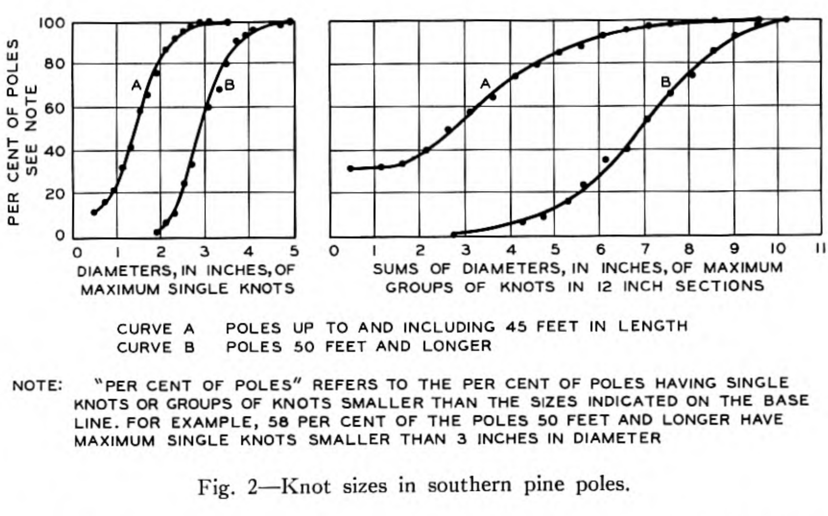
ALLOWABLE FIBER STRESSES. REVISED FORM OF TABLE 20, PAGE 154, HANDBOOK NO. 3, FOURTH EDITION, NATIONAL ELECTRICAL SAFETY CODE

	When Installed									At Replacement				
	Treated Poles					Untreated Poles				Treated or Untreated Poles				
	For Ultimate Fiber Stress of—					For Ultimate Fiber Stress of—				For Ultimate Fiber Stress of—				
	7,400	6,000	5,600	5,000	3,600	6,000	5,600	5,000	3,600	7,400	6,000	5,600	5,000	3,600
At crossings:														
Poles in lines of one grade of construction throughout—														
Grade A	2,470	2,000	1,870	1,670	1,200	2,000	1,870	1,670	1,200	3,700	3,000	2,800	2,500	1,800
Grade B	3,700	3,000	2,800	2,500	1,800	3,000	2,800	2,500	1,800	5,550	4,500	4,200	3,750	2,700
Grade C	5,550	4,500	4,200	3,750	2,700	4,500	4,200	3,750	2,700	11,100	9,000	8,400	7,500	5,400
Poles in isolated sections of higher grade of construction in lines of a lower grade of construction—														
Grade A	2,470	2,000	1,870	1,670	1,200	1,500	1,400	1,250	900	3,700	3,000	2,800	2,500	1,800
Grade B	3,700	3,000	2,800	2,500	1,800	2,000	1,870	1,670	1,200	5,550	4,500	4,200	3,750	2,700
Grade C	5,550	4,500	4,200	3,750	2,700	3,600	3,360	3,000	2,160	11,100	9,000	8,400	7,500	5,400
Elsewhere than at crossings:														
Grade A	2,960	2,400	2,240	2,000	1,440	2,000	1,870	1,670	1,200	4,440	3,600	3,360	3,000	2,160
Grade B	4,440	3,600	3,360	3,000	2,160	3,000	2,800	2,500	1,800	7,400	6,000	5,600	5,000	3,600
Grade C	7,400	6,000	5,600	5,000	3,600	4,500	4,200	3,750	2,700	11,100	9,000	8,400	7,500	5,400

specifications define poles of a quality that the major utilities have found to be satisfactory. Departures from straightness are held within practical limits for ordinary use.

Decay and the presence of wood-rotting fungi are generally prohibited. Minor exceptions are made with respect to the butts of the cedars, which are usually treated with creosote. The question of including poles cut from sound dead trees received careful consideration. Blighted chestnut is acceptable with certain restrictions, but in the case of the other three species poles from live timber are specified. While it might appear economical to salvage and use all sound dead trees standing in the woods, practical opinion at present strongly favors eliminating dead timber as a source of pole material because of the extra costs involved in handling and inspection.

It has proved impracticable to limit checks in a precise manner. Checks or lengthwise separations of the wood fibers vary so much with the age, seasoning, and moisture content of the pole that although definite limitation seemed desirable the compromise finally adopted is one which simply prohibits injurious checks. Practically the matter is left to the judgment of the supplier and consumer concerned.



CURVE A POLES UP TO AND INCLUDING 45 FEET IN LENGTH
CURVE B POLES 50 FEET AND LONGER

NOTE: "PER CENT OF POLES" REFERS TO THE PER CENT OF POLES HAVING SINGLE KNOTS OR GROUPS OF KNOTS SMALLER THAN THE SIZES INDICATED ON THE BASE LINE. FOR EXAMPLE, 58 PER CENT OF THE POLES 50 FEET AND LONGER HAVE MAXIMUM SINGLE KNOTS SMALLER THAN 3 INCHES IN DIAMETER

Fig. 2—Knot sizes in southern pine poles.

The limitation of knots was a matter of special study. Previous specifications were at variance and data were lacking to establish acceptable limits. Measurements of knots larger than one half inch were therefore made on representative poles of the four species. The size and location of about twenty-three thousand knots in some 567 poles were tabulated, and as might have been anticipated, the occurrences of large knots or large groups of knots were found to increase with the length of pole. This led to a division of the data into a group for short poles and one for long poles of each species. Figure 2, for southern pine, is a typical illustration of the curves drawn from the data. It shows, first, the per cent of poles that have single knots of the given diameters, (*A*) for poles up to 45 feet long, and (*B*) for poles 50 feet and longer; and second, the per cent of poles having groups of knots with the indicated sums of diameters in any 12 inch section, separately plotted for the same two cases. The limits set by this study for single knots and for groups of knots in a twelve inch section are shown in Table 4.

TABLE 4

SPECIFICATION LIMITS FOR KNOTS

	Southern Pine	Chestnut	Western Red Cedar	Northern White Cedar
	(Diameter—Inches)			
Single Knots				
Poles 45 ft. and under *.....	3 and 4†	4	3	2.5
Poles 50 ft. and over *......	5	5.5	3	4.5
Group of Knots				
(12 in. Sections)	(Sum of Diameters—Inches)			
Poles 45 ft. and under.......	8	7	10	9
Poles 50 ft. and over........	10	9	10	11

* Except for Northern White Cedar where the length division points are 35 ft. and 40 ft.
† 3 inches for Classes 4 to 10; 4 inches for Classes 1 to 3.

The standards referred to above which have been prepared and approved under the procedure of the American Standards Association are nine in number. One prescribes the ultimate fiber stresses for poles of northern white cedar, western red cedar, chestnut and southern pine, and four prescribe the dimensional classifications for each of the above species according to lengths and circumferences as shown in Table 2. These five are American Standards. The situation with respect to checks and dead timber led to recommending the remaining four specifications covering material requirements as American Tentative Standards. They are the first American standards for wood poles and their adoption on the sound basis outlined marks an important step toward simplified practice in an essential public utility commodity.

The application of the results of the work, as is true of other well-conceived standardization projects, should yield many engineering and economic advantages. The specifications will facilitate good engineering and help to clarify questions bearing on the joint use of poles. No attempt has been made to evaluate the economic savings, but, in the long run, bringing substantially all production and utilization together upon the basis of rational uniform sizes and specifications may be expected to produce economies and benefits in which all concerned should share.

Abstracts of Technical Articles from Bell System Sources

A Loud Speaker Good to Twelve Thousand Cycles.[1] L. G. Bostwick. A loud speaker, designed for use as an adjunct to existing types of speakers to permit efficient sound radiation at the higher audible frequencies, is described. The structural and performance characteristics are indicated, and some of the advantages and limitations of such a loud speaker are discussed.

Indicating Meter for Measurement and Analysis of Noise.[2] T. G. Castner, E. Dietze, G. T. Stanton, and R. S. Tucker. This paper describes a visual indicating meter for the measurement of noise and other sounds. Its design is based on the known characteristics of sound and hearing, which are summarized. Particular attention has been paid to the response of the meter to sounds of short duration. The aim has been to make the meter both simple in operation and portable. An attachment for the frequency analysis of noise is under development. Several fields of use of the meter and analyzer are indicated.

Some Applications of Bell System Instrumentalities and Practice to Railroad Communication Problems.[3] F. A. Cowan. Railroad communication problems are fundamentally similar to those encountered in the Bell System. As a result, the instrumentalities and practices developed for telephone company use are, to a large extent, applicable to railroad company use. Suitable Bell System circuits and equipment have, therefore, been made available to the railroad companies. Likewise, by means of representation on the American Railway Association committees, and by participation in conventions and joint discussions wherever practicable, information regarding many of the more general telephone company practices has been incorporated in the Railway Association codes.

There are, of course, some conditions which are peculiar to railroad operating procedure or plant. In these cases existing Bell System instrumentalities have been adapted for use, or new equipment suited to the particular cases involved has been developed. Catalogues and papers listing and describing this special equipment, together with

[1] *Jour. S. M. P. E.*, May, 1931.
[2] Published in abridged form in *Elec. Engg.*, May, 1931.
[3] *Proc. Amer. Railway Assoc.*, Sept., 1930.

instructions regarding its use and maintenance, have been published.

Certain classes of equipment cannot be readily treated on a general basis and specific studies of individual cases are required to insure effective application to the railroad use. Where such equipment is requested by the railroad companies the telephone company undertakes the necessary studies and furnishes the apparatus to the railroad companies on a rental basis.

Extensive use has been made of the various Bell System services by the railroad companies. Examples of the more general applications are: dispatcher and way station telephone sets, selector signaling apparatus, private branch exchanges, loud speaking equipment, cable, telephone repeaters and loading coils.

The Call Announcer: A Telephone Application of Sound Picture Ideas.[4] O. M. GLUNT. Fundamental research and development work carried on with a particular objective in one field contributes in many cases to the solution of problems in other fields. A typical example is the application of the sound reproducing elements, developed for use primarily in sound picture theater reproducing systems, in the solution of an intricate problem in telephone system operation. This article outlines the communicating problem which was presented and describes the apparatus which was developed, employing adaptations of sound picture principles to meet the need.

Design and Installation of Toll Cable in the Bell System.[5] GLEN IRELAND. This paper discusses the present status of the toll cable network of the Bell System, indicates plans for its extension and describes recent improvements in toll cable, including tape armored cable, loading coils and telephone repeaters. Present maintenance methods for toll cable circuits are also dealt with.

A Rapid Method of Estimating the Signal-to-Noise Ratio of a High Gain Receiver.[6] F. B. LLEWELLYN. It is shown that a figure of merit for the signal-to-noise ratio in a receiving system is obtained directly by noting how much the total noise output increases when the input circuit is tuned through resonance, in the absence of signal. The effect of mismatching the antenna and input circuit impedances is discussed, and it is concluded that although a small improvement may be obtained in certain ideal cases by making the circuit impedance much higher than the antenna impedance, other considerations

[4] *Jour. S. M. P. E.*, March, 1931.
[5] *Proc. Amer. Railway Assoc.*, Telegraph and Telephone Section, Sept., 1930.
[6] *Proc. I. R. E.*, March, 1931.

indicate that the matched impedance condition gives the best results in practice.

The World's Most Powerful Microscope.[7] F. F. LUCAS. In the last ten years there has been developed at Bell Telephone Laboratories a new technic of high-power micrography, which has greatly extended the limits of useful magnification possible with a microscope. Since any extension of the limits of magnification of the microscope which is accompanied by a decrease in definition is useless, it was found necessary to increase the resolving power or definition of the microscope. One way in which this can be done is by decreasing the wave length of the light used.

A microscope using ultra-violet light was developed about thirty years ago by Koehler of the Zeiss works. Due to various difficulties in operating it, this microscope soon became a scientific curiosity and was almost forgotten. About five years ago, a microscope of this type was obtained from the Zeiss works by Bell Laboratories, and the difficulties involved in the use of this instrument were largely solved by the development of a mechanical method of focusing. With this microscope, it is possible to obtain crisp, brilliant images of metallurgical specimens magnified 5000 to 6000 diameters. In studying the advantages and limitations of this microscope, it was found to be particularly applicable to the study of biological and medical specimens. Such specimens can be examined at high magnification under the ultra-violet microscope without the necessity of cutting, staining, or injuring them in any way.

A Direct Reading Audio-Frequency Phase Meter.[8] W. R. MACLEAN and L. J. SIVIAN. In connection with certain acoustic studies it was desired to measure sound pressures as vectors, i.e. to determine both the amplitudes and the phase angles. An example, more fully described at the end of the paper, is the measurement of the amplitude and phase variations in the pressure at various points in a room excited by a tone from a loudspeaker. If a microphone traverses a path in the room the amplitude and phase changes in its output voltage are equal to the corresponding changes in the sound pressure. Thus, the measurement is reduced to an electrical one, except for the absolute calibration of the microphone and associated electrical circuit. At any one frequency, relative changes of amplitude and phase with position usually are all that is of interest, in which case no calibration is necessary.

[7] *Jour. S. M. P. E.*, April, 1931.
[8] *Jour. Acous. Soc. of America*, April, 1931.

Formation of Photographic Images on Cathodes of Alkali Metal Photoelectric Cells.[9] A. R. OLPIN and G. R. STILWELL. A method of forming both negative and positive photographic images on the cathodes of potassium and sodium photoelectric cells in vacuum is described. These images are sharp and clear in every detail and can be permanently "fixed" by proper treatment. Among the materials which have been successfully used in treating the exposed surfaces to bring out these images are sulphur vapor, air, oxygen and hydrogen in the ratio of 9 to 1, hydrofluoric acid and bromine. During the time the image is forming, the photoelectric sensitivity of the illuminated portions decreases approximately 30 per cent. After the image is fixed as a permanent record there is little difference between the sensitivity of the cathode area bearing the image and neighboring areas. Photographs of photoelectric cells are shown in which such photographic images are plainly visible.

Ausgleichsstrome bei parallelen Einzelleitungen, von denen die eine in der Erde liegt und unendlich lang ist.[10] JOHN RIORDAN. This paper gives the formula for the electric force in a homogeneous semi-infinite flat earth due to unit step current (zero for time less than zero, unity for time greater than zero) in an infinite wire above the earth. The corresponding formula for the electric force in the air, due to F. H. Murray, has been published in the *Bell System Technical Journal* for October, 1930, equation (4) of L. C. Peterson's paper; the two formulas agree at the surface of the earth. The present formula is given in finite form in terms of the exponential function and the error function complement.

A Modern Laboratory for the Study of Sound Picture Problems.[11] T. E. SHEA. Recently there has been provided among the research facilities of Bell Telephone Laboratories, Inc., a separate building which is intended solely for sound picture research and development work. The prime objects of the laboratory are to find out the best methods and technic for employing sound picture recording and reproducing apparatus now in use, and of making improvements in recording and reproduction. The building contains a recording studio, film processing plant, and review room, together with testing laboratories.

[9] *Jour. Opt. Soc. Amer.*, March, 1931.
[10] *E. N. T.*, Band 8, Heft 3, March, 1931.
[11] *Jour. S. M. P. E.*, March, 1931.

Contributors to this Issue

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