The Bell System Technical Journal

Devoted to the Scientific Engineering Aspects of Electrical Communication

July, 1923

Transient Oscillations in Electric Wave-Filters

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I. INTRODUCTION

THE electric wave-filter has been very fully discussed with respect to its remarkable steady-state properties.[1] In the present paper it is proposed to give the results of a fairly extensive theoretical study of its behavior in the transient state. This study is of particular interest and importance in connection with the wave-filter, because, as we shall find, its remarkable selective characteristics are peculiarly properties of the steady state and become sharply defined only as the steady state is approached. To this fact, it may be remarked in passing, is to be ascribed the uniform failure of wave-filters to suppress irregular and transient interference, such as "static," in anything like the degree with which they discriminate against steady-state currents outside the transmission range. This limitation is common to all types of selective networks and restricts the amount of protection it is possible to secure from transient or irregular interference. In fact the general conclusions of the present study are applicable to all types of selective circuits.

In the present paper the discussion will be principally concerned with the following phases of the general problem.

1. *The indicial admittances of a representative set of wave-filters.* The *indicial admittance*, as explained below, is equal to the current, expressed as a time function, in response to a uniform steady e.m.f. of unit value, applied to the network at time $t=0$. It has been shown in previous papers that a knowledge of the indicial admittance of an invariable network completely determines its behavior, both in the transient and steady state; to all types of applied forces. Its determination is therefore fundamental to the whole problem.

2. *The mode in which the steady-state is built up after a sinusoidal voltage within the frequency transmission range is applied to the wave-*

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[1] Physical Theory of the Electric Wave-Filter, G. A. Campbell, *B. S. T. J.*, Nov., 1922; Theory and Design of Uniform and Composite Electric Wave-Filters, O. J. Zobel, *B. S. T. J.*, Jan., 1923.

filter. Formulas are deduced and a set of representative curves computed and plotted which show the dependence of the building-up process on the constants and number of sections of the filter and the frequency of the applied e.m.f. The outstanding deduction from this phase of the problem is that as the selectivity of the filter is increased either by narrowing the transmission band or increasing the number of sections, the time required for sinusoidal currents to build up is proportionally increased. This fact, it may be remarked, sets a theoretical limit to the amount of selectivity which can be employed in communication circuits.

3. *The character and duration of the transient current when a sinusoidal voltage outside the frequency transmission range is applied to the filter.* It will be found that in this case a transient disturbance penetrates the filter which is enormous compared to the final steady state. The magnitude of this disturbance decreases very slowly with the number of filter sections and its duration increases therewith. This phenomenon is an important special case of the general limitations of the selectivity of the filter in the transient state.

4. *The energy which penetrates through selective circuits from random interference.* The energy spectrum of random interference, that is, interference from random disturbances is discussed and a formula is deduced which defines the *figure of merit* of a selective circuit with respect to random interference. This formula leads to general deductions of practical importance regarding the relative merits of selective networks in the transient state and their inherent limitations. It also provides a method for experimentally determining the spectrum of random interference.

Unfortunately the complexity of transient phenomena is such as to absolutely require a large amount of mathematical analysis. Consequently, while the mathematics has been relegated as far as possible to Appendices, a considerable amount necessarily appears in the text. The writers, however, have endeavored to emphasize the physical significance of the mathematics and have included only that which is absolutely essential to an understanding of the problem and the appropriate methods of attack.

In order to keep the analysis within manageable limits and in a form to admit of relatively simple and instructive interpretation, the formulas will be restricted for the most part to non-dissipative filters and the effects of terminal reflections will be ignored.[2] These

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[2] The general solution for the case of arbitrary terminal impedances is given in Appendix IV and briefly discussed.

restrictions are desirable on their own account, because the selective properties, both in the transient and steady-state, are isolated and exhibited in the clearest manner when the disturbing effects of dissipation and reflections are absent. As regards dissipation, its effect is usually small for filters of ordinary length and, as regards transient phenomena, is always of such a character as to require no essential modification of the conclusions reached from a study of the ideal non-dissipative filter. In fact the conclusions reached in this paper regarding the inherent limitations of selective circuits in the transient state are conservative.

II. General Theory and Formulas

Before taking up the investigation of wave-filters it is necessary to write down the fundamental formulas of electric circuit theory, which are required in the analysis, and briefly discuss their application to the investigation of transient phenomena in networks in general. The theory and calculation of electrical networks may be approached in a number of ways, as for example, from the Fourier integral.[3] Perhaps the simplest way, however, is to base the theory on the fundamental formulas

$$I(t) = \frac{d}{dt}\int_0^t f(t-y)A(y)dy, \qquad \text{I}$$

and

$$1/pZ(p) = \int_0^\infty e^{-pt}A(t)dt. \qquad \text{II}$$

In these formulas $I(t)$ is the current (expressed as an explicit time function) in any branch or mesh of an electric network which flows in response to the electromotive force $f(t)$ which is applied to the network at time $t \geqq 0$ in the same or any other branch or mesh of the network. The function $A(t)$ is a characteristic function of the constants and connections of the network only which may be termed the *indicial admittance* or the *Heaviside Function.* Its physical significance may be inferred by setting $f(t)=1$, whence it follows that $I(t)=A(t)$. That is to say $A(t)$ *is equal to the current in response to a "unit e.m.f." (zero before, unity after time* $t=0$).

In the following we shall be principally concerned with the case when the applied electromotive force is sinusoidal. To deal with this case we set $f(t)=\sin(\omega t+\Theta)$ and equation I becomes

$$I(t) = a(\omega, t)\sin(\omega t+\Theta) + b(\omega, t)\cos(\omega t+\Theta) \qquad \text{III}$$

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[3] The Solution of Circuit Problems, T. C. Fry, *Phys. Rev.*, Aug., 1919.

where, denoting $d/dt\, A(t)$ by $A'(t)$,

$$a(\omega, t) = A(o) + \int_0^t \cos \omega y \, A'(y)\, dy$$

and

$$b(\omega, t) = -\int_0^t \sin \omega y \, A'(y) dy. \tag{IV}$$

The ultimate steady-state amplitudes are evidently the limits of the foregoing as t approaches infinity. Thus if we write the steady-state current as

$$I = \alpha(\omega) \sin (\omega t + \Theta) + \beta(\omega) \cos (\omega t + \Theta),$$

then

$$\alpha(\omega) = A(o) + \int_0^\infty \cos \omega y \, A'(y) dy$$

and

$$\beta(\omega) = -\int_0^\infty \sin \omega y \, A'(y) dy. \tag{V}$$

For the derivation and a fuller discussion of the foregoing formulas the reader is referred to "Theory of the Transient Oscillations of Electrical Networks and Transmission Systems," Proc. A. I. E. E., March, 1919.

In the majority of the more important selective networks $A(o) = 0$; that is to say the initial value of the current is zero and the current in response to the applied sinusoidal voltage of the frequency $\omega/2\pi$ is built up entirely from the progressive integrals

$$a(\omega, t) = \int_0 \cos \omega y \, A'(y) dy$$

and

$$b(\omega, t) = -\int_0^t \sin \omega y \, A'(y) dy$$

in accordance with formula IV. The derivative $A'(t) = d/dt\, A(t)$ of the indicial admittance which appears in the integrals will be termed the *impulse function* of the network to indicate its direct physical significance; it is equal to the current in response to a "pulse" of infinitesimal duration and moment (or time integral) unity, or, stated in the terminology of the radio engineer, it is equal to the response of the network to "shock-excitation." These formulas therefore establish a definite quantitative relation between the selective properties of the network and its response to "shock-excitation"; a relation which is of great importance in understanding and interpreting the behavior of selective networks to transient disturbances.[4]

The indicial admittance $A(t)$ is calculable from and may be regarded as defined by the very compact formula II.[5] In this equation $Z(p)$ is the operational impedance of the network. It is derived from the differential equations of the problem by replacing the differential operator d/dt by the symbol p, thus formally reducing the equations to an algebraic form from which the ratio $1/Z(p)$ of the current to electromotive force is gotten by ordinary algebraic processes. $Z(p)$ will involve the constants and connections of the network and will depend, of course, on the mesh or branch in which the electromotive force is inserted and that in which the required current is measured.

The procedure in formulating the transient behavior of networks is as follows. Derive the operational impedance $Z(p)$ as stated above. With $Z(p)$ formulated, the corresponding indicial admittance $A(t)$ is determined by the integral equation II. The appropriate methods of solution of the integral equation are briefly discussed in "The Heaviside Operational Calculus." Sometimes the solution can be recognized by inspection as in the case of the low pass wave-filter. Otherwise the procedure in general is to expand $1/Z(p)$ in such a form that the individual terms of the expansion are recognizable as identical with infinite integrals of the required type. Two expansions of this kind lead to the Heaviside Expansion and power series solution, respectively. The appropriate form of expansion depends on the particular problem in hand and often calls for considerable ingenuity and experience. An excellent illustration of the appropriate process is furnished by the detailed derivation [6] of the indicial admittance of the band pass filter which is rather intricate.

In connection with the problem of the energy absorbed from forces of finite duration, and from random interference, the following formulas are required, of which VIII and IX are original and hitherto unpublished. Formula X, which is a special case of VIII and IX was derived by Rayleigh (*Phil. Mag.*, Vol. 27, 1889, p. 466), in connection with an investigation of the spectrum of complete radiation.

If an applied force $f(t)$ exists only in the finite time interval $0 \leqq t \leqq T$, during which it has a finite number of discontinuities and a

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[4] It may be noted in passing that these formulas show the futility of attempting, as so many inventors have done in connection with the problem of protection from "static" disturbances, to design a circuit, which, in the language of patent specifications, shall be unresponsive to shock excitation while at the same time shall be sharply responsive to sustained forces.

[5] The Heaviside Operational Calculus, J. R. Carson, *B. S. T. J.*, Nov., 1922.

[6] See Appendix I.

finite number of maxima and minima, it is representable by the Fourier integral

$$f(t)=\frac{1}{\pi}\int_0^\infty |F(\omega)| \cos\,[\omega t+\Theta(\omega)]\,d\omega, \tag{VI}$$

where

$$|F(\omega)|^2=\left[\int_0^T f(t)\cos\omega t\,dt\right]^2+\left[\int_0^T f(t)\sin\omega t\,dt\right]^2. \tag{VII}$$

Let this force be applied to a network in branch 1 and let the resultant current $I_n(t)$ be measured in branch n. Let the steady-state transfer impedance at frequency $\omega/2\pi$ be denoted by $Z_{1n}(i\omega)$ and let $z_n\,(i\omega)$ and $\cos\Theta_n$ denote the impedance and power factor of branch n at frequency $\omega/2\pi$. It may then be shown that

$$W'=\int_0^\infty [I_n(t)]^2dt=\frac{1}{\pi}\int_0^\infty \frac{|F(\omega)|^2}{|Z_{1n}(i\omega)|^2}\,d\omega \tag{VIII}$$

and, as special cases,

$$\int_0^\infty [A'_{1n}(t)]^2dt=\frac{1}{\pi}\int_0^\infty \frac{d\omega}{|Z_{1n}\,(i\omega)|^2}, \tag{IX}$$

and

$$\int_0^T [f(t)]^2dt=\frac{1}{\pi}\int_0^\infty |F(\omega)|^2d\omega. \tag{X}$$

The total energy W, absorbed by branch n from the applied force is given by

$$W=\frac{1}{\pi}\int_0^\infty \frac{|F(\omega)|^2}{|Z_{1n}\,(i\omega)|^2}\,|z_n\,(i\omega)|\,\cos\Theta_n\cdot d\omega. \tag{VIIIa}$$

Comparison of the formulas for W′ and W shows that, if the branch n is a simple series combination of impedance elements, W′ *is the energy absorbed by a unit resistance element in branch n from the applied force f(t).*

In the subsequent discussion of the behavior of selective circuits to random interference and applied forces of finite duration, W′ of formula VIII will be taken, therefore, as a measure of the energy absorbed by the receiving branch or element. Similarly formula IX measures the energy absorbed when the applied force is impulsive. The application of formula VIII rather than VIIIa, when they differ except for a constant, is justified because we are concerned with the energy absorbed by a receiving element proper, which can be represented by a simple resistance.

The advantage of formula VIII, in addition to its simplicity, resides in the fact that the right hand side is usually quite easily computed, since the integrand is everywhere positive, and this without any explicit reference to the transient phenomena themselves. Formula IX is of particular importance, because, as will be shown in a subsequent part of this paper, it represents, except in limiting cases, the relative amount of energy absorbed from random interference.

III. THE INDICIAL ADMITTANCES OF WAVE-FILTERS

We are now in possession of the necessary formulas and mathematical processes for investigating the behavior of wave-filters in the transient state. We shall first write down the indicial admittances of the representative types investigated, their derivation being discussed in Appendix I. The formulas given for the low pass and the high pass are exact, while those of the band pass filters are approximations based on the assumption that the transmission band-width is small compared with the "mid-frequency" of the transmission band. They are therefore formally restricted in their application to "narrow band" filters. The analysis of the exact formula, given in Appendix I, shows, however, that the deductions drawn from the approximate formulas of the text are quite generally applicable without errors of any practical consequence to band pass wave-filters, even when the transmission band is relatively wide. These questions are fully discussed in the Appendix.

In the formulas given below the filters are assumed to be infinitely long and the voltage to be applied at "mid-series" position to the initial or zero-th section. $A_n(t)$ is then equal to the current in the nth section in response to a unit voltage (zero before, unity after time $t=0$.)

1. *Low Pass Wave-Filter, Type* L_1C_2, *Fig.* 1.

$$A_n(t) = \frac{1}{k}\int_0^x J_{2n}(x)dx, \tag{1a}$$

where $x=\omega_c t$,

$\omega_c = 2/\sqrt{L_1C_2} = 2\pi$ times the critical or cut-off frequency,

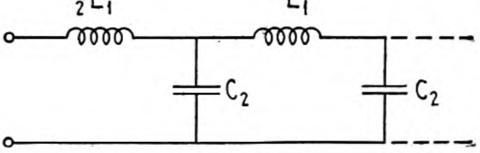

Fig. 1

$J_{2n}(x)$ = The Bessel function of the order $2n$ and argument x, and the filter elements, in terms of the parameters w_c and k, are given by

$$\omega_c = 2/\sqrt{L_1 C_2}, \qquad L_1 = 2k/\omega_c,$$
$$k = \sqrt{L_1/C_2}, \qquad C_2 = 2/\omega_c k.$$

For values of time such that $x < 2n$, $A_n(t)$ is very small and positive, while for $x > 2n$, the character of the solution is exhibited by the approximate solution

$$A_n(t) = \frac{1}{k}\left[1 + \sqrt{\frac{2}{\pi x}}\ \frac{h_{2n}}{q_{2n}} \sin\left(q_{2n} x - \Theta_{2n}\right)\right]. \tag{1b}$$

The formula is deduced from the approximate formulas given in Appendix II for Bessel functions, and h_{2n}, q_{2n} and Θ_{2n} are determined by

$$h_n = \left(\frac{1}{1 - n^2/x^2}\right)^{1/4},$$

$$q_n = \sqrt{1 - n^2/x^2},$$

and

$$\Theta_n = \frac{2n+1}{4}\pi - n \sin^{-1}(n/x).$$

For sufficiently large values of x, $A_n(t)$ is ultimately given by the asymptotic formula

$$A_n(t) \sim \frac{1}{k}\left[1 + \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{2n+1}{4}\pi\right)\right]. \tag{1c}$$

Formula (1a) was first given by one of the writers (Trans. A. I. E. E., 1919) as a special case of the solution for the dissipative low pass filter (series resistance and shunt leakage).

2. *High Pass Wave-Filter, Type C_1L_2, Fig. 2.*

$$A_n(t) = \frac{1}{k}\left\{\phi_0(x) - \frac{2n}{1!} D^{-1}\phi_1(x) + \frac{(2n)\,(2n-1)}{2!} D^{-2}\phi_2(x) - \dots \right.$$
$$\left. \dots + D^{-2n}\phi_{2n}(x)\right\} \tag{2a}$$

where

$x_c = \omega_c t$,

$\omega_c = 2\pi$ times critical frequency *below* which the filter attenuates,

$C_1 = 1/2\omega_c k$, $\quad k = \sqrt{L_2/C_1}$,

$L_2 = k/2\omega_c$, $\quad \omega_c = 1/2\sqrt{L_2 C_1}$.

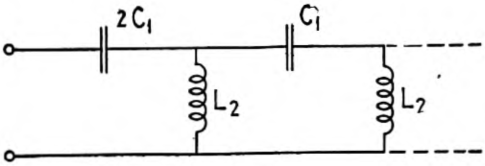

Fig. 2

The symbol D^{-m} denotes multiple integrations, repeated m times and

$$\phi_m(x) = J_o(x) - \frac{m}{1!} J_1(x) + \frac{(m)\ (m-1)}{2!} J_2(x) + \ . \ . + (-1)^m J_m(x).$$

A large amount of time and effort have been devoted to an attempt to reduce this and other forms of solution (see Appendix I) to a form in which its properties would be exhibited by direct inspection, but without success. Numerical computations and curves must, therefore, be largely relied upon in the study of the high pass filter in the transient state.

For sufficiently large values of x ($x > 4n^2$) the ultimate behavior of the filter is shown by the asymptotic formula

$$A_n(t) \backsim (-1)^n \frac{1}{k} \sqrt{\frac{2}{\pi x}} \cos\ (x - \pi/4). \tag{2b}$$

Band Pass Wave-Filters.

In all the band pass types of filters discussed below the transmission band lies in the frequency range between $\omega_1/2\pi$ and $\omega_2/2\pi$ so that the band width is $(\omega_2 - \omega_1)/2\pi$. We shall write $\sqrt{\omega_1 \omega_2} = \omega_m$ and $\omega_2 - \omega_1 = w$. For each type the filter elements are determined by the parameters ω_m, w and a third parameter[7] k which may be so chosen as to fix the magnitude of the impedance of the filter.

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[7] The parameter k is equal to the characteristic impedance, both mid-series and mid-shunt, at mid-frequency of the confluent band, "constant k" type of wave-filter. See Theory and Design of Uniform and Composite Electric Wave-Filters, this Journal, Jan., 1923.

The formulas for the indicial admittances of all the band pass filters are approximate, as stated above, and are deduced on the assumption that the band width is narrow. Practically, however, as regards the essential deductions drawn from them, they are not so restricted but are applicable to the case of relatively wide bands. (See Appendix I.)

There are, of course, an infinite variety of band pass filters; the ones investigated in the present paper are, however, representative and the conclusions drawn from a study of them are, in their general aspects, applicable to all types.

3. *Band Pass Wave-Filter, Type* $L_1C_1L_2C_2$, *Fig.* 3.

$$A_n(t)=\frac{w}{\omega_m k}J_{2n}(y)\sin x \tag{3a}$$

where $x=\omega_m t$; $y=wt/2$; and the filter elements are given by

$$L_1=2k/w, \qquad L_2=wk/2\omega_m^2,$$

$$C_1=w/2k\omega_m^2, \qquad C_2=2/wk.$$

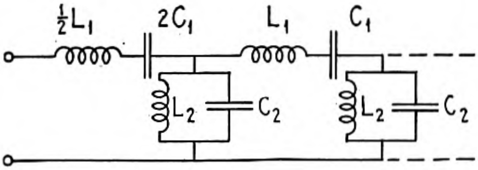

Fig. 3

This is the "constant k" type of filter and, as will be noted, the elements are so proportioned that $L_1C_1=L_2C_2=1/\omega_m^2$, and $L_1/C_2=L_2/C_1=k^2$.

From the properties of Bessel functions discussed in Appendix II, it follows that $A_n(t)$ is very small until $y\geqq 2n$. For values of $y>2n$, the character of the function is clearly exhibited by the following approximate formulas, although these are not sufficiently accurate for the purposes of precise computation.

$$A_n(t)\doteq\frac{w}{\omega_m k}h_{2n}\sqrt{\frac{2}{\pi y}}\cos\left(q_{2n}y-\Theta_{2n}\right)\sin x \tag{3b}$$

$$\doteq\frac{w}{2\omega_m k}h_{2n}\sqrt{\frac{2}{\pi y}}\left[\sin\left(x-q_{2n}y+\Theta_{2n}\right)+\sin\left(x+q_{2n}y-\Theta_{2n}\right)\right] \tag{3c}$$

and ultimately,

$$A_n(t) \sim \frac{w}{2\omega_m k}\sqrt{\frac{2}{\pi y}}\left[\sin\left(x-y+\frac{4n+1}{4}\pi\right)+\sin\left(x+y-\frac{4n+1}{4}\pi\right)\right]. \tag{3d}$$

h_{2n}, q_{2n}, Θ_{2n} are determined by the formulas given in Appendix II,—

$$h_n=\left(\frac{1}{1-n^2/y^2}\right)^{1/4},$$

$$q_n=\sqrt{1-n^2/y^2},$$

and

$$\Theta_n=\frac{2n+1}{4}\pi-n\sin^{-1}(n/y).$$

4. *Band Pass Wave-Filter, Type $L_1C_1C_2$, Fig. 4.*

$$A_n(t)=\frac{w}{\omega_m k}J_n(y)\sin(x-n\pi/2) \tag{4a}$$

where, as above, $x=\omega_m t$; $y=wt/2$, and the filter elements are given by

$$L_1=2k/w;\ C_1=w/2k\omega_1^2;\ C_2=\frac{2}{(\omega_1+\omega_2)k}.$$

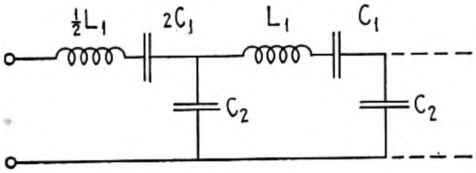

Fig. 4

The approximate formulas for $y>n$, are,

$$A_n(t)\doteq\frac{w}{\omega_m k}h_n\sqrt{\frac{2}{\pi y}}\cos(q_n y-\Theta_n)\sin(x-n\pi/2) \tag{4b}$$

$$\doteq\frac{w}{2\omega_m k}h_n\sqrt{\frac{2}{\pi y}}\left[\sin(x-q_n y+\Theta_n-n\pi/2)+\sin(x+q_n y-\Theta_n-n\pi/2)\right] \tag{4c}$$

and ultimately

$$A_n(t)\sim\frac{w}{2\omega_m k}\sqrt{\frac{2}{\pi y}}\left[\sin(x-y+\pi/4)+\sin\left(x+y-\frac{4n+1}{4}\pi\right)\right]. \tag{4d}$$

5. *Band Pass Wave-Filter, Type* $L_1C_1L_2$, Fig. 5.

$$A_n(t)=\frac{w}{\omega_m k}J_n(y)\sin(x+n\pi/2) \tag{5a}$$

where $x=\omega_m t$, $y=wt/2$ and the filter elements are determined by

$$L_1=2\omega_1 k/w\omega_2;\quad C_1=w/2k\omega_m^2;\, L_2=\frac{\omega_1+\omega_2}{2\omega_m^2}k.$$

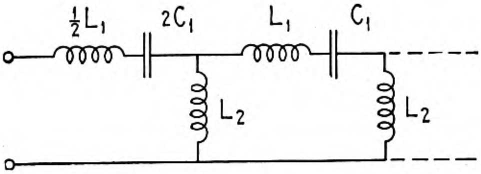

Fig. 5

The approximate formulas for $y>n$ are

$$A_n(t)\doteq\frac{w}{\omega_m k}h_n\sqrt{\frac{2}{\pi y}}\cos(q_n y-\Theta_n)\sin(x+n\pi/2) \tag{5b}$$

$$\doteq\frac{w}{2\omega_m k}\,h_n\sqrt{\frac{2}{\pi y}}[\sin(x-q_n y+\Theta_n+n\pi/2)+\sin(x+q_n y-\Theta_n+n\pi/2)] \tag{5c}$$

and ultimately

$$A_n(t)\sim\frac{w}{2\omega_m k}\sqrt{\frac{2}{\pi y}}\left[\sin\left(x-y+\frac{4n+1}{4}\pi\right)+\sin(x+y-\pi/4)\right]. \tag{5d}$$

6. *Band Pass Wave-Filter, Type* $L_1L_2C_2$, *Fig.* 6.

$$A_n(t)=\frac{2w}{\omega_m k}[J_n(y)\sin(x-n\pi/2)-J_n'(y)\cos(x-n\pi/2)] \tag{6a}$$

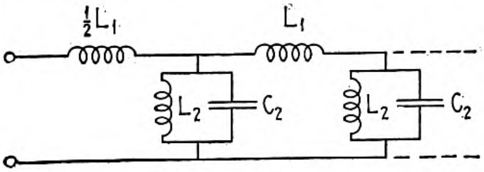

Fig. 6

where $x=\omega_m t$; $y=wt/2$; $J_n'(y)=d/dy\;J_n(y)$, and

$$L_1=\frac{2k}{\omega_1+\omega_2};\quad L_2=wk/2\omega_1{}^2;\quad C_2=2/wk.$$

The approximate formulas for $y>n$ are

$$A_n(t) \doteq \frac{2w}{\omega_m k} h_n \sqrt{\frac{2}{\pi y}} [\cos (q_n y-\Theta_n) \sin (x-n\pi/2) + q_n \sin (q_n y-\Theta_n) \cos (x-n\pi/2)] \tag{6b}$$

$$\doteq \frac{w}{\omega_m k} h_n \sqrt{\frac{2}{\pi y}} \begin{bmatrix} (1-q_n) \sin (x-q_n y+\Theta_n-n\pi/2) \\ +(1+q_n) \sin (x+q_n y-\Theta_n-n\pi/2) \end{bmatrix} \tag{6c}$$

and ultimately

$$A_n(t) \backsim \frac{2w}{\omega_m k} \sqrt{\frac{2}{\pi y}} \sin\left(x+y-\frac{4n+1}{4}\pi\right). \tag{6d}$$

7. *Band Pass Wave-Filter, Type $C_1L_2C_2$, Fig. 7.*

$$A_n(t) = \frac{2}{\omega_m k}\left(\frac{w}{2\omega_m}\right)^n P + \frac{2w}{\omega_m k} [J_n(y) \sin (x+n\pi/2) + J'_n(y) \cos (x+n\pi/2)], \tag{7a}$$

where $x=w_m t$; $y=wt/2$, and

$$C_1 = \frac{\omega_1+\omega_2}{2k\omega_m^2};\ L_2 = wk/2\omega_m^2;\ C_2 = 2\omega_1/w\omega_2 k.$$

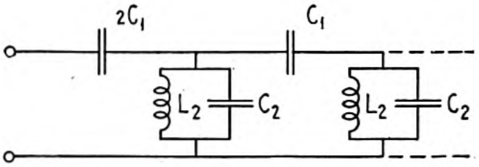

Fig. 7

The symbol P in the first term denotes a "pulse" at time $t=0$; that is

$$P = \infty \text{ at } t=0,$$
$$=0 \text{ for } t>0,$$

and

$$\int_0^\infty P dt = 1.$$

The first term in $A_n(t)$ exists in consequence of the fact that at the instant the voltage is applied the filter behaves like a pure capacity network. For narrow band filters the factor $(w/2\omega_m)^n$ is small so that this term does not contribute appreciably to the steady state. As a matter of fact in actual filters which necessarily have some series resistance, it does not exist.

The approximate formulas for $y > n$ are

$$A_n(t) \doteq \frac{2w}{\omega_m k} h_n \sqrt{\frac{2}{\pi y}} \, [\cos (q_n y - \Theta_n) \sin (x + n\pi/2) - q_n \sin (q_n y - \Theta_n) \cos (x + n\pi/2)] \qquad (7b)$$

$$\doteq \frac{w}{\omega_m k} h_n \sqrt{\frac{2}{\pi y}} \, [(1+q_n) \sin (x - q_n y + \Theta_n + n\pi/2) + (1-q_n) \sin (x + q_n y - \Theta_n + n\pi/2)] \qquad (7c)$$

and ultimately

$$A_n(t) \backsim \frac{2w}{\omega_m k} \sqrt{\frac{2}{\pi y}} \sin \left(x - y + \frac{4n+1}{4} \pi \right). \qquad (7d)$$

8. *Discussion of Indicial Admittances.*

The indicial admittances for the low pass filter, that is, the current in response to a steady unit e.m.f. applied at time $t=0$, are shown in the curves of Figs. 8, 9 and 10, for the initial or zero-th, the 3rd and the 5th sections. These curves together with the exact and approximate formulas given above are sufficient to give a reasonably comprehensive idea of the general character of these oscillations and their dependence on the number of sections and the constants of the filter.

It will be observed that the current is small until a time approximately equal to $2n/\omega_c = n\sqrt{L_1 C_2}$ has elapsed after the voltage is applied. Consequently the low pass filter behaves as though currents were transmitted with a finite velocity of propagation $\omega_c/2 = 1/\sqrt{L_1 C_2}$ sections per second. This velocity is, however, only apparent or virtual since in every section the currents are actually finite for all values of time>0.

After time $t = n\sqrt{L_1 C_2}$ has elapsed the current oscillates about the value $1/k$ with increasing frequency and diminishing amplitude. The amplitude of these oscillations is approximately

$$\frac{1/k}{\sqrt{1-(2n/\omega_c t)^2}} \sqrt{\frac{2}{\pi \omega_c t}}$$

and their instantaneous frequency (measured by intervals between zeros)

$$\frac{\omega_c}{2\pi}\sqrt{1-(2n/\omega_c t)^2}.$$

The oscillations are therefore ultimately of cut-off or critical frequency $w_c/2\pi$ in all sections, but this frequency is approached more and more slowly as the number of filter sections is increased.[8]

The indicial admittances of the band pass filter, type $L_1C_1L_2C_2$, are shown in Figs. 11, 12 and 13 for the initial, the 3rd and the 5th sections. These curves show not the actual oscillations but their *envelopes*. That is to say the curves must be multiplied by sin $\omega_m t$ to give the actual oscillations. The "mid-frequency" $\omega_m/2\pi$ may therefore be regarded as the "carrier frequency" which is modulated by the relatively low frequency oscillations shown in the curves.

Comparison of the formulas for the indicial admittances of the band filters of type $L_1C_1C_2$ and $L_1C_1L_2$ with that of type $L_1C_1L_2C_2$ shows that these curves are applicable to the two former types provided the number of sections is doubled and the phase of the oscillations of frequency $\omega_m/2\pi$ is correctly modified.

Referring to Figs. 11, 12, 13 it will be observed that the oscillations are small until time $t=4n/w$; consequently they are transmitted with an apparent velocity of propagation roughly equal to $w/4=1/2\sqrt{L_1C_2}$[9] sections per second.

After time $t=4n/w$, the low frequency oscillations shown in the curves are of increasing frequency and diminishing amplitude, their envelope being roughly equal to

$$\frac{w}{\omega_m k}\sqrt{\frac{4}{\pi w t}}.$$

The actual oscillations are analyzable into two frequencies

$$\frac{1}{2\pi}\left(\omega_m+\frac{w}{2}\sqrt{1-(4n/wt)^2}\right) \text{ and } \frac{1}{2\pi}\left(\omega_m-\frac{w}{2}\sqrt{1-(4n/wt)^2}\right)$$

so that the ultimate oscillations are of the two critical frequencies

$$\frac{1}{2\pi}(\omega_m+w/2) \text{ and } \frac{1}{2\pi}(\omega_m-w/2).$$

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[8] For curves showing the indicial admittance of the low pass filter when n is very large, the reader is referred to Transient Oscillations, Trans. A. I. E. E., 1919.

[9] For types $L_1C_1C_2$ and $L_1C_1L_2$ the velocity in sections per second is double this. This corresponds to the fact that two sections of these types are approximately equivalent, as regards their selectivity, to one section of type $L_1C_1L_2C_2$.

For both the low pass and band pass filters the oscillations of the indicial admittances are of continuously variable frequency which traverses the frequency transmission band and ultimately reaches the critical frequencies of the filter.[10]

The indicial admittances of the band pass filter, type $L_1L_2C_2$ are shown in Figs. 14, 15, 16 for the initial, the 6th and 10th sections.[11] The curves show the oscillation envelopes $\sqrt{(J_n^2+J_n'^2)}$, whereas the actual oscillations are within a constant,

$$\sqrt{[J_n^2(wt/2)+J_n'^2(wt/2)]}\ \sin(\omega_m t-n\pi/2-\Theta_n),$$

where $\Theta_n=\tan^{-1}(J_n'/J_n)$. For a narrow band filter the variation in the phase angle Θ_n is very slow.

The principal difference between these curves and the corresponding curves for type $L_1C_1L_2C_2$ is that the envelope of the oscillation does not go through zero as in the latter. In addition the oscillations are ultimately of a single frequency $\frac{1}{2\pi}\ (\omega_m+w/2)$ while for type $C_1L_2\ C_2$the ultimate frequency is $\frac{1}{2\pi}\ (\omega_m-w/2)$.

The indicial admittance of the high pass filter, shown in the curves of Figs. 17, 18, 19, 20 for the initial, the 1st, 2nd and 3rd sections, differs in important respects from those of the low pass and band pass filters. In the first place the current jumps instantaneously to its maximum value $1/k$ in all sections, so that the velocity of propagation is infinite.[12] After this initial jump the current oscillates with decreasing frequency and decreasing amplitude, the oscillation frequency becoming ultimately the critical or cut-off frequency $\omega_c/2\pi$. The initial frequency and the time required for the oscillation frequency to reduce to $\omega_c/2\pi$, increases, practically linearly with the number of sections. *The oscillation frequency varies continuously and traverses the frequency transmission range of the filter from infinite frequency (represented by the initial jump) down to the critical frequency of the filter, below which it attenuates sinusoidal currents.*

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[10] From a purely mathematical viewpoint, this fact explains the transmission, without attenuation, of a continuous band of frequencies.

[11] These curves are applicable to the $C_1L_2C_2$ type of band pass filter, due regard being had to difference in phase, and to the initial jump of current. See formulas (6a) and (7a).

[12] This is, of course, a consequence of the assumption of zero series inductance and shunt capacity. Actually, of course, the circuit must include a finite amount of both.

IV. THE BUILDING-UP OF ALTERNATING CURRENTS IN WAVE-FILTERS

If an e.m.f. $\sin(\omega t+\Theta)$ is applied to the low pass wave-filter (type L_1C_2) at time $t=0$, then by formulas I and (1a), the resultant current in the nth section builds up in accordance with the expression

$$\frac{1}{k}\left[\sin\Theta\int_0^x J_{2n}(x_1)\cos\lambda(x-x_1)dx_1+\cos\Theta\int_0^x J_{2n}(x_1)\sin\lambda(x-x_1)dx_1\right],$$

where $x=\omega_c t$ and $\lambda=\omega/\omega_c$.

For the band pass filter, type $L_1C_1L_2C_2$, the corresponding formula, based on the approximations discussed in the preceding, is by I and (3a).

$$\frac{1}{k}\left[\sin(\mu y+\Theta)\int_0^y J_{2n}(y_1)\cos(\lambda-\mu)(y-y_1)dy_1\right.$$
$$\left.+\cos(\mu y+\Theta)\int^y J_{2n}(y_1)\sin(\lambda-\mu)(y-y_1)dy_1\right]$$

where $y=wt/2$; $\lambda=2\omega/w$; and $\mu=2\omega_m/w$ so that $\mu y=\omega_m t$. Similar formulas are deducible for the other types of band pass filters considered in the preceding section.

Comparison of these formulas shows that, in both the low pass and band pass wave-filters, the genesis and growth of the current in response to an e.m.f. $\sin(\omega t+\Theta)$, applied at time $t=0$, is mathematically determined by definite integrals of the form

$$S_n(z;\nu)=\int_0^z J_n(z_1)\sin\nu(z-z_1)dz_1,$$

and

$$C_n(z;\nu)=\int_0^z J_n(z_1)\cos\nu(z-z_1)dz_1.$$

These integrals[13] have been extensively studied in the course of this investigation; their general properties and the appropriate methods of computation are discussed in Appendix III.

The subsidence of the current, when a sinusoidal e.m.f. is removed, is also determined by the above formulas for the low pass and band pass filters. To show this suppose that prior to the reference time $t=0$, that steady-state currents are flowing in the filter in response

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[13] The writers take pleasure in acknowledging their indebtedness to T. H. Gronwall, consulting mathematician, who furnished asymptotic formulas for the computation of these integrals. See Appendix III.

to an e.m.f. sin $(\omega t+\Theta)$, which is removed at time $t=0$. We can represent this condition correctly by regarding the e.m.f. sin $(\omega t+\Theta)$ as continuing, while a negative e.m.f., $-\sin(\omega t+\Theta)$, is applied at time $t=0$. The resultant current for $t\geqq 0$, is then

$$\alpha_n(\omega)\sin(\omega t+\Theta)+\beta_n(\omega)\cos(\omega t+\Theta)$$
$$-\frac{1}{k}[\sin\Theta\cdot C_{2n}(x;\lambda)+\cos\Theta\cdot S_{2n}(x;\lambda)]$$

for the low pass filter with a corresponding expression for the band pass. $\alpha_n(\omega)$ and $\beta_n(\omega)$ are the real and imaginary parts of the steady state admittances of the filter at frequency $\omega/2\pi$.

Figs. 21–32 exhibit the phenomena attending the building-up of alternating currents in the low pass filter for a sufficient number of representative cases to show the effects of the length of the filter and the applied frequency. For $\omega_c t>25$, the curves represent the *transient distortion*, that is the difference between the final steady state and actual current. For $\omega_c t<24$ the actual current is shown. An important outstanding result which follows from a study of these curves and the formulas of Appendix III may be stated as follows:

The time T required for an alternating current of frequency $\omega/2\pi$ to build up to its proximate steady state in the nth section of a low pass wave-filter is given approximately by the formula

$$T=\frac{2n}{\omega_c}\frac{1}{\sqrt{1-(\omega/\omega_c)^2}}.$$

The first factor $2n/\omega_c$ represents the delay due to the apparent finite velocity of propagation, while the second factor represents the effect of the applied frequency in its relation to the cut-off frequency of the filter.

This formula is a rather rough approximation when the number of sections n is small. Furthermore the time at which the current reaches its *proximate* steady state does not admit of precise definition.[14] Nevertheless the formula is in substantial agreement with the facts as regards the effect of a number of filter sections, cut-off frequency and applied frequency on the phenomena, and is of great practical importance.[15]

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[14] Actually the time T corresponds to a singularity in the mathematical formulas. See Appendix III.

[15] This formula has been applied in the design of periodically loaded cable circuits, which are of such length in the Bell System as to make transient phenomena a factor which must be taken into account. The formula is in close agreement with a large amount of experimental evidence.

The *transient distortion*, it is interesting to note, is, as regards frequency, independent of the applied frequency, and ultimately attains the cut-off frequency of the filter. Its envelope is ultimately

$$\frac{1}{k}\frac{\omega/\omega_c}{1-(\omega/\omega_c)^2}\sqrt{\frac{2}{\pi\omega_c t}}$$

when a voltage sin ωt is applied, and

$$\frac{1}{k}\frac{1}{1-(\omega/\omega_c)^2}\sqrt{\frac{2}{\pi\omega_c t}}$$

when a voltage cos ωt is applied.

Figs. 33 and 34 show the form of the current in the 5th section when sinusoidal voltages sin ωt and cos ωt of frequency 25 per cent above the cut-off frequency of the filter are applied. The transient current shown in the curves increases in frequency up to the critical frequency of the filter, the oscillations being ultimately given by

$$\frac{1}{k}\frac{\omega/\omega_c}{(\omega/\omega_c)^2-1}\sqrt{\frac{2}{\pi\omega_c t}}\cos\left(\omega_c t-\frac{2n+1}{4}\pi\right)$$

and

$$\frac{1}{k}\frac{1}{(\omega/\omega_c)^2-1}\sqrt{\frac{2}{\pi\omega_c t}}\sin\left(\omega_c t-\frac{2n+1}{4}\pi\right)$$

corresponding respectively to applied voltages sin ωt and cos ωt. The amplitude of these transient oscillations are enormous compared with the final steady state, and the curves furnish a clear illustration of the fact, stated in a previous part of this paper, that the selective properties of wave-filters are essentially properties of the steady state only.

Figs. 35–41 show the building-up phenomena in the band pass filter, type $L_1C_1L_2C_2$, and are applicable also to types $L_1C_1C_2$ and $L_1C_1L_2$ when proper values are assigned to the constants and parameters.[16] The curves actually show the envelopes of the oscillations which are of slowly variable frequency in the neighborhood of $\omega_m/2\pi$.

A study of these curves and the formulas of Appendix III lead to the following proposition, analogous to that stated above for the low pass filter.

The time T required for an alternating current of frequency $\omega/2\pi$ within the transmission range $w/2\pi$ of a band filter to build up to its

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[16] n sections of type $L_1C_1L_2C_2$ are approximately equivalent to $2n$ sections of type $L_1C_1C_2$ or of type $L_1C_1L_2$.

proximate steady state in the nth section is given approximately by the formula

$$T=\frac{4n}{w}\frac{1}{\left[1-4\left(\frac{\omega-\omega_m}{w}\right)^2\right]^{1/2}}$$

for type $L_1C_1L_2C_2$ and one half this amount for the other types of band pass filters discussed in this paper.

These curves show the envelope of the oscillations with fidelity but are not well adapted to exhibit the actual frequencies. These are given by the formula

$$\sqrt{C^2+S^2}\sin\left[\omega_m t+\Theta+\tan^{-1}(S/C)\right]$$

where C and S denote the definite integrals

$$C_{2n}\left(\frac{wt}{2};\ \frac{2(\omega-\omega_m)}{w}\right)\text{ and }S_{2n}\left(\frac{wt}{2};\ \frac{2(\omega-\omega_m)}{w}\right).$$

The envelope is therefore substantially independent of the phase angle Θ of the applied e.m.f. The frequency is ultimately the applied frequency $\omega/2\pi$. The *transient distortion* is analyzable into two frequencies

$$\frac{1}{2\pi}\left(\omega_m+\frac{w}{2}\sqrt{1-(4n/wt)^2}\right)\text{ and }\frac{1}{2\pi}\left(\omega_m-\frac{w}{2}\sqrt{1-(4n/wt)^2}\right),$$

and its envelope is ultimately

$$\frac{1+4\left(\frac{\omega-\omega_m}{w}\right)^2}{1-4\left(\frac{\omega-\omega_m}{w}\right)^2}\sqrt{\frac{4}{\pi wt}}.$$

The building-up of alternating currents in the high pass filter has been investigated only qualitatively owing to the extremely laborious computations required. The process is essentially different from that in the low pass and band pass filters. When an e.m.f. $\sin(\omega t+\Theta)$ is applied the current in all sections jumps instantly to the value

$$\frac{1}{k}\sin(\omega t+\Theta).$$

Therefore the process depends on the applied frequency. If the applied frequency is within the transmission band ($\omega>\omega_c$), the current builds up to its ultimate frequency, the time required being given approximately by the formula

$$T=\frac{2n\omega_c}{\omega^2}\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}},$$

(by the principle of stationary phase; see footnote 31).

It thus requires an infinite time when the applied frequency is equal to the critical frequency while infinite applied frequencies build up instantly.

When the applied frequency is outside the transmission band, the current *subsides* to its steady value, the time required being proportional to the ratio n/ω_c and decreasing as the applied frequency is decreased.

The fact that the initial value of the current is of the same order of magnitude as that of steady state currents in the transmission range is an outstanding feature of the process and reflects the failure of the selective properties of this type of filter in the transient state.

V. THE ENERGY ABSORBED FROM TRANSIENT APPLIED FORCES

In only a relatively few cases is the solution for the transient current, in response to suddenly applied forces, reducible to a manageable form, which admits of interpretation or of computation without prohibitive labor. Fortunately, however, it is usually possible to calculate the energy absorbed by a receiving element in a selective network from suddenly applied forces of finite time duration and such a calculation throws a great deal of light on the general properties of selective circuits in the transient state. The calculation is based on formulas VI to IX of Section II.

A particularly important example is the energy absorbed from the force $\sin(pt+\Theta)$, applied at time $t=0$ and removed at time $t=T$. If the energy is averaged with respect to the phase angle Θ, we get [17]

$$\int_0^\infty [I(t)]^2 dt = \frac{1}{2\pi}\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}\left\{\frac{1-\cos(\omega-p)T}{(\omega-p)^2}+\frac{1-\cos(\omega+p)T}{(\omega+p)^2}\right\}.$$

If $p/2\pi$ is in the neighborhood of the frequency which the network is designed to select, this becomes approximately

$$\int_0^\infty [I(t)]^2 dt = \frac{1}{2\pi}\int_0^\infty \frac{1-\cos(\omega-p)T}{(\omega-p)^2}\,\frac{d\omega}{|Z(i\omega)|^2}. \tag{8}$$

In the *steady state* the time integral of the square of the current in response to the e.m.f. $\sin(pt+\Theta)$ during the time interval T is simply $T/2|Z(ip)|^2$. The expression

$$\frac{|Z(ip)|^2}{\pi T}\int_0^\infty \frac{1-\cos(\omega-p)T}{(\omega-p)^2}\,\frac{d\omega}{|Z(i\omega)|^2} \tag{9}$$

is therefore *the relative amount of energy actually absorbed from the*

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[17] Here $Z(i\omega)$ is the steady-state transfer impedance and the integral measures the energy absorbed by a unit resistance in the receiving branch.

force sin $(pt+\Theta)$ *acting during the time interval* T, *to that calculated on the assumption of a steady state in this interval.*

Calculations of these formulas are of particular interest and importance in multiplex carrier telephone and telegraph systems where they furnish a measure of the interference between channels operating at different frequencies.

In order to exhibit clearly the significance of the formulas without detailed computation, consider an ideal selective circuit, for which in the range $\omega_1 \leqq \omega \leqq \omega_2, |Z(i\omega)| = Z_T$ (a constant) and everywhere else $|Z(i\omega)| = Z_s$ (a constant, very large compared with Z_T). Under these assumptions, formulas (8) and (9) become approximately, for the case when $p > \omega_2$,

$$\frac{T}{2Z_s^2} + \frac{1}{2\pi} \frac{\omega_2 - \omega_1}{(p-\omega_2)\ (p-\omega_1)} \frac{1}{Z_T^2} \tag{8a}$$

and

$$\left(1 + \frac{1}{\pi T} \frac{Z_s^2}{Z_T^2} \frac{\omega_2 - \omega_1}{(p-\omega_2)\ (p-\omega_1)}\right). \tag{9a}$$

These formulas admit of some quite interesting deductions which are applicable to band filters in general.

(1) The energy absorbed in excess of that calculated in the steady state basis is

$$\frac{1}{2\pi} \frac{\omega_2 - \omega_1}{(p-\omega_2)\ (p-\omega_1)} \frac{1}{Z_T^2}.$$

This is independent of the duration of the applied force and of the degree to which the filter discriminates against steady state currents outside the frequency range $\omega_1 \leqq \omega \leqq \omega_2$. It is proportional to the *band width* and inversely to the product $(p-\omega_2)\ (p-\omega_1)$. It follows therefore that *no amount of selectivity will appreciably reduce the energy absorbed from a sinusoidal force of finite duration outside the transmission range of the filter, below the value given above.*

(2) The fractional excess of energy absorbed is given by

$$\frac{1}{\pi T} \left(\frac{Z_s}{Z_T}\right)^2 \frac{\omega_2 - \omega_1}{(p-\omega_2)\ (p-\omega_1)}.$$

This decreases with the duration of the applied force but *increases as the square of the selectivity* (Z_s/Z_T) *of the filter.* Hence for forces of short duration the energy absorbed may be very large compared with that calculated on the steady state basis.

VI. RANDOM INTERFERENCE

We have hitherto confined attention to the transient phenomena when the form of the applied voltage was explicitly given. In the problem of the behavior of wave-filters and selective circuits in general to such disturbances as "static" in radio transmission and "noise" in wire transmission this is not the case, and the applied force is usually more or less completely *random*. By this it is meant that the interfering disturbance, which may be supposed to originate in a large number of unrelated sources, varies in an irregular, uncontrollable manner, and is characterized statistically by no predominant frequency. Consequently the wave form of the applied force at any particular instant is entirely indeterminate. This fact makes it necessary to treat the problem as a statistical one, and deal with mean values. In the following we shall derive formulas for the mean *energy* absorbed from random interference; and then define and discuss the selective figure of merit of networks with respect to random interference.

The mathematical treatment of the problem will be based on formulas VI to VIII of section II. To apply these formulas to the problem of random disturbances and their effect on selective networks, consider a long interval of time, or epoch, say from 0 to T. During this epoch we suppose that the network is subjected to a large number of individual impressed forces $f_1(t), f_2(t) \ldots f_n(t)$, which are unrelated and vary in intensity and wave form in an irregular, indeterminate manner, and thus constitute what will be called *random interference*. If we write

$$\sum(t) = f_1(t) + f_2(t) + \ldots + f_n(t),$$

then by VI, $\sum(t)$ is representable as a Fourier integral, thus:

$$\sum(t) = \frac{1}{\pi}\int_0^\infty |F(\omega)| \cos\left[\omega t + \Theta(\omega)\right] d\omega$$

while, in accordance with formula VIII, the energy absorbed by the selective network from this random interference is measured by [18]

$$W' = \frac{1}{\pi}\int_0^\infty \frac{|F(\omega)|^2}{|Z(i\omega)|^2} d\omega.$$

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[18] It should be clearly understood that $Z(i\omega)$ is the transfer impedance of the receiving with respect to the driving branch of the network, and that W' is the energy absorbed by a unit resistance located in the former.

We now introduce the function $R(\omega)$ which will be termed the *energy spectrum of the random interference*, and which is defined by the equation

$$R(\omega)=\frac{1}{T}|F(\omega)|^2. \tag{10}$$

Dividing both sides by T and writing $W'/T=\epsilon$, formula VIII becomes

$$\epsilon=\frac{1}{\pi}\int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2}\,d\omega. \tag{11}$$

Both ϵ and $R(\omega)$ become independent of T provided the epoch is made sufficiently great, and ϵ *measures the mean energy absorbed per unit time from the random interference.* The practical significance of this formula is contained in the statement that *the required function of the selective network, as regards random interference, is to minimize the ratio of* ϵ *to the signal energy. Consequently this ratio furnishes an index of the merit of the network.*

In order to rigorously evaluate the integral of formula (11) the energy spectrum $R(\omega)$ of the interference must be completely specified over the entire interval of integration. Obviously this information cannot be deduced without imposing some restrictions on the character of the interference, or making some hypothesis regarding the mechanism in which it originates. On the other hand if the forces $f_1(t), f_2(t) \ldots f_n(t)$ are absolutely random in a strict mathematical sense, it would appear that all frequencies are equally probable in the spectrum $R(\omega)$ and that, consequently, the most probable energy distribution is that which makes $R(\omega)$ a constant, independent of ω. This inference, however, has not been theoretically established; indeed, the problem does not appear to admit of satisfactory solution by the calculus of probabilities. Furthermore, deductions based on the assumption that the interference is random in a strict mathematical sense might well be inadequate for the applications contemplated, and the "most probable" spectrum in serious disagreement with the spectrum of the actual interference [19] to which we wish to apply the results of the present study.

Fortunately, in view of these difficulties, a complete specification of $R(\omega)$ is not at all necessary for a practical solution of the problem. This is a consequence of the following facts:

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[19] For example, the spectrum of the interference presented to the terminals of the selective network will be modified by the characteristics of the "transducer," over which the disturbances are transmitted. Thus both in radio and wire systems, the greater attenuation suffered in transmission by high frequencies, will reduce the relative intensity of the high frequency part of the spectrum.

(a) In the case of efficient selective networks, the important contributions to the integral (11) are confined to a finite continuous range of ω which includes, but is not greatly in excess of, the range which the network is designed to select.[20] This fact is a consequence of the impedance characteristics of selective networks and of the following properties of the spectrum $R(\omega)$.

(b) $R(\omega)$ is a continuous, finite function of ω which converges to zero at infinity and is everywhere positive. It possesses no sharp maxima or minima,[21] and its variation with respect to ω, where it exists, is slow. These properties of $R(\omega)$ are believed to be evident from physical considerations, and will not be elaborated.

Now referring to formula (11), since the numerator and denominator of the integrand are everywhere positive, it follows that a value ω_m of ω exists, such that

$$\epsilon=\frac{1}{\pi}R(\omega_m)\int_0^\infty\frac{d\omega}{|Z(i\omega)|^2}.$$

Now suppose that the network is designed to select frequencies in the range $\omega_1 \leqq \omega \leqq \omega_2$. Then from the properties of the network and of the spectrum $R(\omega)$ discussed above, it follows that ω_m lies close to, or within, the range $\omega_1 \leqq \omega \leqq \omega_2$. In any case, if the band $\omega_2-\omega_1$ is made so narrow that the curvature of $R(\omega)$ over the interval is negligible, then with negligible error ω_m may be taken as 2π times the "mid-frequency" of the band. That is to say, with negligible error, ω_m may be defined either as $(\omega_1+\omega_2)/2$ or as $\sqrt{\omega_1\omega_2}$.

The foregoing argument may be summarized in the following proposition:

The mean energy ϵ absorbed per unit time from random interference by a selective network designed to select the band of frequencies corresponding to $\omega_1 \leqq \omega \leqq \omega_2$ is measured by the formula

$$\epsilon=\frac{1}{\pi}\rho R(\omega_m), \tag{12}$$

where ρ denotes the infinite integral

$$\rho=\int_0^\infty\frac{d\omega}{|Z(i\omega)|^2}$$

---

[20] This statement excludes from present consideration networks, which, like the high pass filter, select an infinite band of frequencies. This limitation, however, is of no practical consequence, because such networks are quite useless as regards random interference. This question will be briefly discussed later.

[21] The existence of sharp maxima and minima would indicate the presence of *systematic* interference, which should not be regarded as part of the random interference.

and $R(\omega_m)$ *is the spectral energy level of the interference at frequency* $\omega_m/2\pi$. ω_m *lies close to or within the band* $\omega_1<\omega<\omega_2$, *and when this band is sufficiently small with respect to the curvature of* $R(\omega)$, $\omega_m/2\pi$ *may be aken as the mid-frequency of the band.*

Formula (12) is of very considerable practical and theoretical importance. It furnishes a basis for the experimental determination of the energy spectrum $R(\omega)$, and this determination, for any given epoch, can be made as accurate as desired by employing a band filter which selects a sufficiently narrow band of frequencies. It also leads immediately to the following important proposition.

If a selective network is required to select the band of frequencies corresponding to $\omega_1\leqq\omega\leqq\omega_2$, *the mean energy absorbed per unit time by the network from random interference is necessarily greater than*

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2}\frac{R(\omega)}{|Z(i\omega)|^2}d\omega \doteq \frac{1}{\pi}R(\omega_m)\int_{\omega_1}^{\omega_2}\frac{d\omega}{|Z(i\omega)|^2}. \tag{13}$$

This formula, therefore, determines the theoretical limit, beyond which it is not possible to discriminate against random interference.

We are now prepared to introduce a formula which defines *the figure of merit of a selective network with respect to random interference.* This formula gives the signal-to-random-interference energy ratio of the network as compared with the corresponding ratio in an ideal reference circuit (defined below).

Let the network, as above, be designed to select frequencies in the band $\omega_1\leqq\omega\leqq\omega_2$. Then the energy absorbed per unit time from *steady-state* forces in this frequency range is proportional to

$$\sigma=\frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2}\frac{d\omega}{|Z(i\omega)|^2}.$$

The corresponding mean energy absorbed from random interference is proportional to

$$\rho=\int_0^\infty\frac{d\omega}{|Z(i\omega)|^2}$$

when the energy level of the interference is corrected to unity.

The ratio $S=\sigma/\rho$ defines the selective figure of merit of the network with respect to random interference.

Stated in words, *the selective figure of merit of a network with respect to random interference is equal to the statistical signal-to-random-interference energy ratio, divided by the corresponding ratio in an ideal band filter which transmits without loss all frequencies in a "unit" band* $(\omega_2-\omega_1=1)$, *and absolutely extinguishes all frequencies outside this band.*

In the foregoing argument, the theoretical limitations have been carefully pointed out and even emphasized. In practical applications, however, it is believed that these limitations are of small or negligible importance, and that the formula for and definition of the selective figure of merit furnish all the information, as regards the behavior of selective circuits to random interference, which we are in a position to make use of. Thus the formula is immediately applicable to the problem of determining the effect of band width, number of sections, dissipation, and terminal reflections on the selectivity of filters with respect to random interference. It furnishes likewise, a means of estimating the comparative merits of the very large number of circuits which have been invented for the purpose of eliminating "static" in radio communication, and leads to general deductions of practical value regarding the inherent limitations imposed on the solution of the "static" problem.

The utility and significance of the foregoing formulas will now be illustrated by application to some representative selective circuits. It is easily shown that, to a good approximation, in the case of the low pass filter (type L_1C_2)

$$S=\frac{1}{\omega_c(1+1/16n^2)},$$

and for the band pass filter (type $L_1C_1L_2C_2$)

$$S=\frac{1}{w(1+1/16n^2)}.$$

In these formulas n denotes the number of filter sections while ω_c is 2π times the cut-off frequency of the low pass filter and w is 2π times the transmission band width of the band filter. In both cases the filters are assumed to be terminated in their characteristic impedances and to be non-dissipative.[22] These formulas show at once the effect of band width and number of sections n on the behavior of wave-filters to random interference, and lead to the following proposition.

In filters designed to select a band of frequencies of width w, the ratio of energy transmitted through the network by the signal and by random interference is inversely proportional to the band width and increased inappreciably when the number of sections is increased beyond two.

As regards the effect of dissipation, a second proposition is deducible.

The effect of introducing dissipation into a network designed to select a single frequency or a band of frequencies is always such as to reduce the ratio of signal energy to that absorbed from random interference.

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[22] These approximate formulas are in very good agreement with actual calculations for filters terminated in resistances.

An inference drawn from the study of band filters in the preceding section may be stated as follows:

The selective figure of merit of a wave-filter designed to select a finite band of frequencies is approximately proportional to the minimum time required for sinusoidal currents within the transmission band to build up their approximate steady values, divided by the number of filter sections.

Another circuit of practical interest, which has been proposed as a solution of the "static" problem in radio-communication consists of a series of sharply tuned oscillation circuits, unilaterally coupled through amplifiers.[23] This circuit is designed to receive only a single frequency to which all the individual oscillation circuits are tuned. The figure of merit of this circuit is approximately

$$S=\frac{L}{R}\,\frac{2^{2n-2}[(n-1)!]^2}{(2n-2)!}$$

where n denotes the number of sections, or stages, and L and R are the inductance and resistance of the individual oscillation circuits. The outstanding fact in this formula is the slow rate of increase of S with the number of stages. For example, if the number of stages is increased from 1 to 5, the figure of merit increases only by the factor 3.66, while for a further increase in n the gain is very slow. This gain, furthermore, is accompanied by a serious increase in the "sluggishness" of the circuit; that is, in the particular example cited, by an increase of 5 to 1 in the time required for signals to build up to their steady-state.[24]

The outstanding deduction of practical importance to be drawn from the preceding is that, as regards disturbances which are predominantly random, irregular, or discontinuous, it is useless to employ selective circuits of extremely high selectivity. The gain in signal-to-interference ratio is very small when the selectivity is increased beyond a moderate amount, and is only gotten by making the circuit relatively sluggish and slowly responsive.

The preceding discussion is, for the reasons discussed above, not applicable to selective circuits like the high pass filter, which transmit an infinite band of frequencies. Considerable information, however, regarding the behavior of the high pass filter to random disturbances can be gotten by returning to formula (10) and comparing the energy absorbed by the high pass filter, with that absorbed by a *pure-resistance network*. Reference to formula (10) shows that the

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[23] See U. S. Patent No. 1173079 to Alexanderson.

[24] When the number of stages n is fairly large, the selective figure of merit becomes proportional to $\sqrt{n}$ and the building-up time to n.

energy absorbed from random disturbances by a pure resistance network is proportional to

$$\int_0^\infty R(\omega)d\omega.$$

The relative amount of energy absorbed by the high pass filter is *greater than*

$$\int^\infty R(\omega)d\omega.$$

The function $R(\omega)$ represents, as above, the statistical energy spectrum of the interference.

Comparison of these formulas shows at once that, unless the energy of the random interference is largely confined in the range $\omega<\omega_c$, little protection is afforded by the high pass filter.

APPENDIX I

DERIVATION OF WAVE-FILTER INDICIAL ADMITTANCES

1. *Low Pass Wave-Filter, Type* L_1C_2.

The derivation of the indicial admittance of this type of filter is given in detail by one of the writers in a previous paper.[25] The method of solution there employed, which is quite generally applicable to periodic structures, consists in writing down the Heaviside Expansion formula for the current in the nth section of a filter of s sections in length $(s>n)$, short circuited at the sth section. The expansion is converted into a definite integral by letting s become infinite and the formula becomes that of the indicial admittance of the nth section of an infinitely long filter. For the non-dissipative filter having mid-series termination, this procedure leads to the formula

$$A_n(t)=\frac{1}{k}\int_0^x dx_1\frac{2}{\pi}\int_0^{\pi/2}\cos(2n\lambda)\cdot\cos(x_1\sin\lambda)d\lambda, \qquad x=\omega_c t,$$

which is identifiable, from known formulas, as

$$A_n(t)=\frac{1}{k}\int_0 J_{2n}(x_1)dx_1. \tag{1.1}$$

A much more direct and flexible method of solution and one which avoids the necessity of setting up the Heaviside expansion formula

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[25] Transient Oscillations, Trans. A. I. E. E., 1919. This paper should be consulted for the details of this method.

and then converting into a definite integral, is to employ the integral equation II. If $Z_n(p)$ denote the transfer operational impedance of the nth section of the infinitely long low pass filter, we have

$$\frac{1}{Z_n(p)}=\frac{\omega_c}{k}\frac{1}{\sqrt{p^2+\omega_c^2}}\left(\frac{\sqrt{p^2+\omega_c^2}-p}{\omega_c}\right)^{2n} \tag{1.2}$$

and writing $x=\omega_c t$, $F_n(x)=kA_n(t)$, the integral equation II becomes

$$\int_0^\infty F_n(x)e^{-px}dx=\frac{1}{p\sqrt{p^2+1}}\left(\sqrt{p^2+1}-p\right)^{2n}. \tag{1.3}$$

The solution of this integral equation is known[26] ; it is

$$F_n(x)=\int_0^x J_{2n}(x_1)dx_1$$

which agrees with the preceding.

The "mid-series" termination is chosen not only for its importance in practical applications but because in general the indicial admittance has been found to take the simplest form when the voltage is applied at this position. This is not always the case, however. For example in the low pass filter if the e.m.f. is applied, not directly at mid-series but through a terminal inductance $L=L_1/2=k/\omega_c$, the integral equation becomes

$$\int_0^\infty F_n(x)e^{-px}dx=\left(1-p(\sqrt{p^2+1}-p)\right)\frac{1}{p\sqrt{p^2+1}}\left(\sqrt{p^2+1}-p\right)^{2n},$$

whence

$$F_n(x)=\int_0^x J_{2n}(x_1)dx_1-J_{2n+1}(x). \tag{1.4}$$

Unless, however, the terminal impedance is related in some simple manner to the constants of the filter, the resulting formula is necessarily complicated.

2. *High Pass Wave-Filter, Type* C_1L_2.

For this type of filter it can be shown, by the first method discussed above in connection with the low pass filter, that the indicial admittance is expressible as the definite integral

$$A_n(t)=\frac{2}{\pi k}\int_1^\infty\frac{\cos\left(2n\ \sin^{-1}\frac{1}{\lambda}\right)\sin x\lambda\, d\lambda}{\sqrt{\lambda^2-1}}, \tag{2.1}$$

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[26] Nielsen, Cylinderfunktionen, page 186, formula 13.

where $x=\omega_c t$. For the case $n=0$, the solution can be recognized as

$$A_0(t)=\frac{1}{k}J_0(x).$$

To attack the problem by means of the integral equation **II**, we write down the operational transfer impedance

$$\frac{1}{Z_n(p)}=\frac{1}{k}\,\frac{1}{\sqrt{1+\omega_c^2/p^2}}\left(\sqrt{1+\omega_c^2/p^2}-\omega_c/p\right)^{2n}. \tag{2.2}$$

Writing $\omega_c t=x$, and $A_n(t)=\dfrac{1}{k}F_n(x)$, and substituting in II gives, as the integral equation of the problem

$$\int_0^\infty F_n(x)e^{-px}dx=\frac{1}{p^{2n}}\,\frac{1}{\sqrt{p^2+1}}\left(\sqrt{p^2+1}-1\right)^{2n}. \tag{2.3}$$

The solution of this equation can be expressed in a number of different forms, depending on the type of expansion of the right hand side which we adopt. One form is as follows:

Expansion of the bracketted expression on the right hand side by the binomial theorem and rearrangement gives

$$\int_0^\infty F_n(x)e^{-px}dx=\frac{1}{\sqrt{p^2+1}}\left[\frac{1}{p^{2n}}+\frac{(2n)(2n-1)}{2!}\,\frac{(p^2+1)}{p^{2n}}+\frac{(2n)\,.\,.\,(2n-3)}{4!}\right.$$
$$\left.\times\frac{(p^2+1)^4}{p^{2n}}+\,.\,.\right]-\left[\frac{2n}{1!}\,\frac{1}{p^{2n}}+\frac{(2n)(2n-1)(2n-2)}{3!}\,\frac{(p^2+1)}{p^{2n}}+\,.\,.\,.\,.\,.\right].$$

Recognizing that

$$\int_0^\infty J_0(x)e^{-px}dx=\frac{1}{\sqrt{p^2+1}},$$

the solution, after rearrangement, becomes the terminating series [27]

$$F_n(x)=kA_n(t)$$
$$=\left(1+\frac{4n^2}{2!}D^{-2}+\frac{4n^2(4n^2-2^2)}{4!}D^{-4}+\frac{4n^2(4n^2-2^2)(4n^2-4^2)}{6!}D^{-6}+\,.\,.\right)J_0(x)$$
$$-2n\left(x+\frac{(4n^2-2^2)}{3!}\,\frac{x^3}{3!}+\frac{(4n^2-2^2)(4n^2-4^2)}{5!}\,\frac{x^5}{5!}+\,.\,.\,.\right), \tag{2.4}$$

where D^{-m} indicates multiple integration, repeated m times. Thus

$$D^{-1}J_0(x)=\int_0^x J_0(x_1)dx_1;\quad D^{-2}J_0(x)=\int_0^x dx_1\int_0^{x_1}J_0(x_2)dx_2;\ \text{etc.}$$

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[27] This solution has been derived from the definite integral also.

Another type of expansion, leading to the formula given in the text, is suggested by the known identity

$$\int_0^\infty J_n(x)e^{-px}dx = \frac{1}{\sqrt{p^2+1}}(\sqrt{p^2+1}-p)^n.$$

To introduce this identity, we write the integral equation in the form

$$\int_0^\infty F_n(x)e^{-px}dx = \frac{1}{p^{2n}}\,\frac{1}{\sqrt{p^2+1}}\Big((\sqrt{p^2+1}-p)+(p-1)\Big)^{2n}$$

and expand the bracketted expression by the binomial theorem. Identification of the individual terms and rearrangement gives the terminating series

$$F_n(x) = \phi_0(x) - \frac{2n}{1!}D^{-1}\phi_1(x) + \frac{(2n)(2n-1)}{2!}D^{-2}\phi_2(x) - \ldots\ldots$$
$$-\frac{2n}{1!}D^{-(2n-1)}\phi_{2n-1}(x) + D^{-2n}\phi_{2n}(x),$$

where $\phi_m(x)$ denotes the terminating series

$$\phi_m(x) = J_0(x) - \frac{m}{1!}J_1(x) + \frac{(m)(m-1)}{2!}J_2(x) + \ldots + (-1)^m J_m(x)$$

and as above D^{-m} denotes multiple integration.

It is an easy matter to derive solutions in the form of infinite series, as for example power series and Bessel series. These solutions, however, which have been carefully investigated, have not proved manageable for either computation or interpretation. The solutions given above are also unfortunately, extremely difficult to compute or interpret. For computation, numerical integration of the following difference equations, is sometimes preferable

$$\begin{aligned} &F_0(x) = J_0(x), \\ &F_1(x) - F_0(x) = 2\int_0^x dx_1\int_0^{x_1}F_0(x_2)dx_2 - 2x, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &F_{n+1}(x) - 2F_n(x) + F_{n-1}(x) = 4\int_0^x dx_1\int_0^{x_1}F_n(x_2)dx_2, \\ &\qquad\qquad n \geqq 1. \end{aligned} \tag{2.5}$$

3. *Band Pass Wave-Filter.*

The mathematical discussion of the band pass filters will be limited to the $L_1C_1L_2C_2$ type shown in Fig. 3. This type is representative and the appropriate mathematical procedure is essentially the same for all the band pass wave-filters.

The first method of solution outlined above for the low pass and high pass filters, leads, for the $L_1C_1L_2C_2$ type of band pass filter, to the definite integral formula

$$A_n(t)=\frac{w}{\omega_m k}\frac{2}{\pi}\int_0^{\pi/2}\frac{\sin gx}{g}\cos 2n\mu\cdot\cos(y\sin\mu)d\mu, \qquad (3.1)$$

where $x=\omega_m t$; $y=wt/2$; $\rho=w/2\omega_m$; and

$$g=\sqrt{1+\rho^2\sin^2\mu}.$$

In solving this definite integral, use is made of the known formulas,

$$J_{2n}(y)=\frac{2}{\pi}\int_0^{\pi/2}\cos 2n\mu\cdot\cos(y\sin\mu)d\mu \qquad (3.2)$$

and

$$(-1)^s\frac{d^{2s}}{dy^{2s}}J_{2n}(y)=\frac{2}{\pi}\int_0^{\pi/2}\sin^{2s}\mu\cdot\cos 2n\mu\cdot\cos(y\sin\mu)d\mu. \qquad (3.3)$$

If in (3.1) g is replaced by unity, it follows from (3.2) that, to this approximation

$$A_n(t)=\frac{w}{\omega_m k}J_{2n}(y)\sin x \qquad (3.4)$$

which is formula (3a) of the text [28]. Clearly this becomes an increasingly good approximation as the parameter ρ becomes smaller; that is, as the ratio of the band width $w/2\pi$ to the mid-frequency $\omega_m/2\pi$ becomes smaller. The approximate formulas of the text for the other types of band pass filters were derived by precisely similar procedure and involve approximations of the same character and order of magnitude.

To investigate the approximate solution, we proceed as follows: If we write

$$\frac{\omega_m k}{w}A_n(t)=F_n(x,y)=\frac{2}{\pi}\int_0^{\pi/2}\frac{\sin gx}{g}\cos 2n\mu\cdot\cos(y\sin\mu)d\mu, \qquad (3.5)$$

and

$$G_n(x,y)=\frac{2}{\pi}\int_0^{\pi/2}\sin gx\cdot\cos 2n\mu\cdot\cos(y\sin\mu)d\mu, \qquad (3.6)$$

and if we substitute for $1/g$ in (3.5) the expansion

$$1-\frac{1}{2}\rho^2\sin^2\mu+\frac{1\cdot 3}{2\cdot 4}\rho^4\sin^4\mu-\ldots\ldots,$$

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[28] If a series resistance R_1 and a shunt resistance $R_2=k^2/R_1$ are included in the filter sections, the formula becomes (3.4) multiplied by the factor $\exp(-R_1y/2k)$.

it follows from a formula exactly analagous to (3.3) that

$$F_n(x,y)=\left(1+\frac{1}{2}\rho^2\frac{\partial^2}{\partial y^2}+\frac{1\cdot3}{2\cdot4}\rho^4\frac{\partial^4}{\partial y^4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\rho^6\frac{\partial^6}{\partial y^6}+\dots\right)G_n(x,y) \qquad (3.7)$$

so that the problem is reduced to the solution of the definite integral $G_n(x, y)$.

In the integral (3.6), write $g=1+h$, so that

$$h=\sqrt{1+\rho^2\sin^2\mu}-1, \qquad (3.8)$$

whence

$$\begin{aligned}G_n(x,y)&=\sin x\cdot\frac{2}{\pi}\int_0^{\pi/2}\cos hx\cdot\cos 2n\mu\cdot\cos(y\sin\mu)d\mu\\&\quad+\cos x\cdot\frac{2}{\pi}\int_0^{\pi/2}\sin hx\cdot\cos 2n\mu\cdot\cos(y\sin\mu)d\mu \qquad (3.9)\\&=P_n\sin x+Q_n\cos x, \qquad (3.10)\end{aligned}$$

where P_n and Q_n denote the definite integrals of (3.9). This effects a further reduction of the problem to the solution of the definite integrals P_n and Q_n.

In the integrands of these integrals expand cos hx and sin hx in the usual power series, and in each term thereof introduce the expansion

$$h^s=\left(\frac{\rho^2}{2}\right)^s(\sin^{2s}\mu)\,(1+a_{s1}\rho^2\sin^2\mu+a_{s2}\rho^4\sin^4\mu+\dots),$$

where the coefficients are given by

$$a_{sj}=(-1)^j s\frac{(s+2j-1)!}{(s+j)!\;j!}\left(\frac{1}{4}\right)^j.$$

By aid of this procedure it is easily shown that

$$\begin{aligned}P_n=J_{2n}(y)&-\frac{(\rho^2x/2)^2}{2!}\frac{d^4}{dy^4}\left(1-a_{21}\rho^2\frac{d^2}{dy^2}+a_{22}\rho^4\frac{d^4}{dy^4}-\dots\right)J_{2n}(y)\\&+\frac{(\rho^2x/2)^4}{4!}\frac{d^8}{dy^8}\left(1-a_{41}\rho^2\frac{d^2}{dy^2}+a_{42}\rho^4\frac{d^4}{dy^4}-\dots\right)J_{2n}(y)\\&-\frac{(\rho^2x/2)^6}{6!}\frac{d^{12}}{dy^{12}}\left(1-a_{61}\rho^2\frac{d^2}{dy^2}+a_{62}\rho^4\frac{d^4}{dy^4}-\dots\right)J_{2n}(y)\\&+\dots, \qquad (3.11)\end{aligned}$$

with a corresponding expansion formula for Q_n.

It is now convenient to introduce the symbolic notation

$$P_n = \cos\left[x\left(\sqrt{1-\rho^2 d^2}-1\right)\right] J_{2n}(y) \tag{3.12}$$

and

$$Q_n = \sin\left[x\left(\sqrt{1-\rho^2 d^2}-1\right)\right] J_{2n}(y) \tag{3.13}$$

where the symbol d denotes the differential operator d/dy operating on $J_{2n}(y)$. The actual numerical significance of these formulas is gotten by expanding as in (3.11).

With the same symbolic notation we get finally,

$$A_n(t) = \frac{w}{\omega_m k}\sin\left(x\sqrt{1-\rho^2 d^2}\right)\frac{1}{\sqrt{1-\rho^2 d^2}} J_{2n}(y). \tag{3.14}$$

The exact solution (3.14) is too complicated, as it stands, to be of any practical value. Fortunately, however, it is possible to sum the expression asymptotically, and the resultant formula shows clearly the behavior of $A_n(t)$ and in particular the character and magnitude of the errors in the approximate formula of the text.

When y is large compared with $(4n)^2$,

$$J_{2n}(y) \doteq \sqrt{\frac{2}{\pi y}}\cos\left(y-\frac{4n+1}{4}\pi\right)$$

and

$$\frac{d^{2s}}{dy^{2s}} J_{2n}(y) \doteq (-1)^s\sqrt{\frac{2}{\pi y}}\left\{\begin{aligned} &\left[1-\frac{3}{2}\,\frac{2s(2s-1)}{4y^2}\right]\cos\left(y-\frac{4n+1}{4}\pi\right)\\ &\quad -\frac{s}{y}\sin\left(y-\frac{4n+1}{4}\pi\right)\end{aligned}\right. \tag{3.15}$$

to order $1/y^2$.

If this expression is substituted in the expanded form of (3.14), some rather intricate and tedious operations finally give as the asymptotic limit of $A_n(t)$

$$A_n(t) \sim \frac{w}{\omega_m k}\sqrt{\frac{2}{\pi y}}\left\{\begin{aligned} &\left(1-\frac{1}{8}\rho^2+\,..\right)\sin\left(x\sqrt{1+\rho^2}\right)\cos\left(y-\frac{4n+1}{4}\pi\right)\\ &-\left(\frac{1}{2}\rho+\,..\right)\cos\left(x\sqrt{1+\rho^2}\right)\sin\left(y-\frac{4n+1}{4}\pi\right).\end{aligned}\right. \tag{3.16}$$

The coefficients of the two terms of (3.16) are even and odd power series in ρ respectively, powers of ρ beyond the second being neglected.

Formula (3.16) is important, as showing the effect of the band width, that is of the parameter ρ, on the indicial admittance. It can be used for numerical computation, however, only when $y > (4n)^2$. A corresponding formula, valid over a much wider range, is obtainable from the expression derived in Appendix II for the Bessel function, namely

$$J_{2n}(y) = B_{2n}(y) \cos \Omega_{2n}(y).$$

If this expression is employed instead of (3.15), we get corresponding to (3.16),

$$\mathrm{A}_n(t) \doteq \frac{w}{\omega_m k} B_{2n}(y) \left\{ \begin{array}{l} \left(1 - \frac{1}{8}\sigma^2 + \ldots\right) \sin (x\sqrt{1+\sigma^2}) \cos \Omega_{2n}(y) \\ \quad - \left(\frac{1}{2}\sigma + \ldots\right) \cos (x\sqrt{1+\sigma^2}) \sin \Omega_{2n}(y) \end{array} \right. \tag{3.17}$$

$$\doteq \frac{w}{\omega_m k} \left\{ \begin{array}{l} \left(1 - \frac{1}{8}\sigma^2 + \ldots\right) \sin (x\sqrt{1+\sigma^2}) \; J_{2n}(y) \\ \quad + \left(\frac{1}{2}\rho + \ldots\right) \cos (x\sqrt{1+\sigma^2}) \; J'_{2n}(y), \end{array} \right. \tag{3.18}$$

where $\sigma = \rho q_{2n} = \rho\sqrt{1-(2n/y)^2}$.

Formula (3.17) is valid when $y > 2n$, and ultimately approaches the limit (3.16) as y becomes indefinitely large.

We are now prepared to discuss the character of the approximations of the formula of the text, which may be written as

$$\frac{w}{2\omega_m k} B_{2n}(y) \left\{ \sin [x + \Omega_{2n}(y)] + \sin [x - \Omega_{2n}(y)] \right\}. \tag{3.19}$$

Correspondingly (3.17) may be written as

$$\frac{w}{2\omega_m k} B_{2n}(y) \left\{ \begin{array}{l} \quad \left(1 - \frac{1}{2}\sigma + \ldots\right) \sin [x\sqrt{1+\sigma^2} + \Omega_{2n}(y)] \\ + \left(1 + \frac{1}{2}\sigma + \ldots\right) \sin [x\sqrt{1+\sigma^2} - \Omega_{2n}(y)]. \end{array} \right. \tag{3.20}$$

Comparison of (3.19) and (3.20) shows that the approximate formula of the text ignores slowly variable correction factors in the amplitudes of the component oscillations, and a slowly variable change in their frequencies. For band pass filters employed in practice these corrections are not only slowly variable but in most cases are quite small. In any case, it is important to observe that failure to include these corrections does not appreciably affect any essential features of the building-up phenomena discussed in the text. Consequently the deductions from the formula of the text are valid not only for narrow-band pass filters, but also for filters of quite wide bands. This statement is substantiated by the fact that the steady-state characteristics, deduced from the approximate formula in accordance with the general formula V, are in excellent agreement with the exact values.

As illustrating the appropriate methods in the solution of problems in electric circuit theory, it is of interest to derive the formula for the band pass filter directly from the integral equation II. The method is not only more generally applicable, but avoids the necessity of deriving the definite integral (3.1). We therefore start with the formulas:

$$\int_0^\infty e^{-pt} A_n(t)dt = 1/p\, Z_n(p)$$

or

$$\int_0^\infty e^{-pt} A'_n(t)dt = 1/Z_n(p), \text{ where } A'_n(t) = d/dt\, A_n(t).$$

For all wave-filters of the "ladder" type it may be shown that

$$\frac{1}{Z_n(p)} = \frac{1}{z_2} \frac{(\sqrt{1+r/4} - \sqrt{r/4})^{2n}}{\sqrt{r+r^2/4}}, \tag{3.21}$$

where z_1 and z_2 are the series and shunt impedances respectively, and $r = z_1/z_2$. This expression admits of series expansion

$$\frac{1}{Z_n(p)} = \frac{2}{z_1}\left[\frac{1}{r^n} - \frac{2n+2}{1!}\frac{1}{r^{n+1}} + \frac{(2n+3)\,(2n+4)}{2!}\frac{1}{r^{n+2}} \right.$$
$$\left. - \frac{(2n+4)\,(2n+5)\,(2n+6)}{3!}\frac{1}{r^{n+3}} + \ldots\right]. \tag{3.22}$$

For the $L_1C_1L_2C_2$ type of filter

$$1/r = \left(\frac{w}{2}\right)^2 \left(\frac{p}{p^2+\omega_m^2}\right)^2$$

and

$$1/z_1 = \frac{1}{k}\left(\frac{w}{2}\right)\left(\frac{p}{p^2+\omega_m^2}\right).$$

It follows from (3.22) and the integral identity,

$$\int_0^\infty e^{-pt} A'_n(t)dt = 1/Z_n(p)$$

that $A'_n(t)$ has an expansion solution of the form

$$A'_n(t) = \frac{w}{k}\left\{ \rho^{2n} f_{2n}(x) - \frac{2n+2}{1!}\rho^{2n+2} f_{2n+2}(x) + \right.$$
$$+ \frac{(2n+3)\,(2n+4)}{2!}\rho^{2n+4} f_{2n+4}(x) \tag{3.23}$$
$$\left. - \frac{(2n+4)\,(2n+5)\,(2n+6)}{3!}\rho^{2n+6} f_{2n+6}(x) \ldots\ldots \right\},$$

where $x=\omega_m t$; $\rho=w/2\omega_m$; and the $f_s(x)$ functions are defined and determined by the integral identities,

$$\int_0^\infty f_s(x)e^{-px}dx=\left(\frac{p}{p^2+1}\right)^{s+1} \qquad (3.24)$$

for all integral values of s.

For $s=0$, the solution of this equation is known; it is

$$f_0(x)=\cos x.$$

The solutions for $s>0$ are gotten from the recurrence formulas[29]

$$f_s(x)=\int_0^x \cos\,(x-\lambda)\,f_{s-1}(\lambda)d\lambda.$$

Repeated applications of this formula give

$$f_{2s}(x)=\frac{1}{2^{2s}}\Big(P_{2s}(x)\cos x+Q_{2s}(x)\sin x\Big)$$

where P_{2s} and Q_{2s} are polynomials in x of the $2s^{th}$ and $(2s-1)^{th}$ orders respectively. Thus:

$$P_{2s}=\alpha(s)\,\frac{x^{2s}}{2s\,!}+\beta\,(s)\,\frac{x^{2s-2}}{(2s-2)\,!}+\gamma(s)\,\frac{x^{2s-4}}{(2s-4)\,!}+\;.\;.$$

(terminating in term in $x^2/2!$),

and

$$Q_{2s}=a(s)\,\frac{x^{2s-1}}{(2s-1)\,!}+b(s)\,\frac{x^{2s-3}}{(2s-3)\,!}+c(s)\,\frac{x^{2s-5}}{(2s-5)\,!}+\;.\;.$$

(terminating in term in $x/1!$).

The α, β, γ . . . a, b, c, . . . coefficients are functions of the order s; the first few coefficients are:

$$\alpha(s)=1,\ s\geqq 0,$$

$$a(s)=\frac{2s+1}{2},\ s\geqq 1,$$

$$\beta(s)=-\frac{(2s-2)\ (2s+1)}{8},\ s\geqq 2,$$

$$b(s)=\frac{(2s-2)\ (2s+1)}{8}-\frac{(2s-3)\ (2s+1)\ (2s+2)}{3\cdot 16},\ s\geqq 2.$$

If the foregoing expressions for the f_s functions are substituted in the series solution (3.23) for $A'_n(t)$ and if the series are rearranged as explained below, we get writing $wt/2=\rho x=y$,

$$A'_n(t)=\frac{w}{k}\left\{J_{2n}(y)\cos x+\rho R_1(y)\sin x+\rho^2R_2(y)\cos x\;.\;.\right\}.$$

---

[29] See equation 10, The Heaviside Operational Calculus, *B. S. T. J.*, Nov., 1922.

The first term $J_{2n}(y)$ is gotten by picking out the leading terms in the P polynomials; the second term $R_1(y)$ by picking out the leading terms in the Q polynomials; the third term $R_2(y)$, from the second terms in the P polynomials; etc.

The work of rearranging and identifying the "remainder" functions $R_1(y), R_2(y) \ldots$ is rather intricate and tedious. The first few functions can be written as

$$R_1(y) = \left(\frac{y}{2}\right)\left(\frac{d^2}{dy^2} + \frac{2}{y}\frac{d}{dy}\right) J_{2n}(y),$$

$$R_2(y) = -\frac{1}{2!}\left(\frac{y}{2}\right)^2\left(\frac{d^4}{dy^4} + \frac{4}{y}\frac{d^3}{dy^3}\right) J_{2n}(y),$$

$$R_3(y) = -\frac{1}{3!}\left(\frac{y}{2}\right)^3\left(\frac{d^6}{dy^6} + \frac{6}{y}\frac{d^5}{dy^5} - \frac{6}{y^2}\frac{d^4}{dy^4} - \frac{24}{y^3}\frac{d^3}{dy^3}\right) J_{2n}(y), \text{ etc.}$$

If we substitute these expressions, rearrange and write $\rho y/2 = z$, we get finally

$$A_n'(t) = \frac{w}{k}\left\{\begin{aligned} &\cos x\left[1 - \frac{z^2}{2!}\frac{d^4}{dy^4} + \frac{z^4}{4!}\frac{d^8}{dy^8} \cdots\right] J_{2n}(y) \\ &+ \sin x\left[\frac{z}{1!}\frac{d^2}{dy^2} - \frac{z^3}{3!}\frac{d^6}{dy^6} + \cdot\cdot\right] J_{2n}(y) \\ &+\rho \sin x\left[\frac{d}{dy} - \frac{z^2}{2!}\frac{d^5}{dy^5} + \cdot\cdot\right] J_{2n}(y) \\ &-\rho \cos x\left[\frac{z}{1!}\frac{d^3}{dy^3} - \frac{z^3}{3!}\frac{d^7}{dy^7} + \cdot\cdot\right] J_{2n}(y) \\ &+ \text{series involving factors in } \rho^2 \text{ and higher powers.} \end{aligned}\right.$$

Neglecting factors in ρ^2, this becomes

$$A_n(t) = \frac{w}{w_m k}\left\{\begin{aligned} &\sin x\left[1 - \frac{z^2}{2!}\frac{d^4}{dy^4} + \frac{z^4}{4!}\frac{d^8}{dy^8} \cdots\cdots\right] J_{2n}(y) \\ &-\cos x\left[\frac{z}{1!}\frac{d^2}{dy^2} - \frac{z^3}{3!}\frac{d^6}{dy^6} + \frac{z^5}{5!}\frac{d^{10}}{dy^{10}} \cdots\cdots\right] J_{2n}(y). \end{aligned}\right.$$

The character of this solution in the region $y > 2n$, is shown by the asymptotic approximation

$$A_n(t) = \frac{w}{\omega_m k} J_{2n}(y) \sin\left(1 + \tfrac{1}{2}\rho^2 q_{2n}^2\right) x \tag{3.25}$$

where

$$q_{2n} = \sqrt{1 - \frac{(2n)^2}{y^2}}.$$

To the same order of approximation in $\rho = w/2\omega_m$, this agrees with the solution (3.18) given above.

APPENDIX II

PROPERTIES OF THE BESSEL FUNCTION $J_n(x)$

The Bessel functions have been studied and tabulated more exhaustively than any other functions largely owing to their great importance and frequent occurrence in mathematical physics. Qualitatively their behavior for integral orders n and real arguments x may be described as follows.

When the argument is less than the order $(0 \leqq x < n)$ the function is very small and positive, and is initially zero (except when $n=0$). In the neighborhood of $x=n$, the function begins to build up and reaches a maximum a little beyond the point $x=n$. Thereafter the function oscillates with increasing frequency and diminishing amplitude, and ultimately behaves as

$$\sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{2n+1}{4}\pi\right).$$

When $n=0$, the initial value is unity, but the subsequent behavior of the function is as described above.

In order to get a more accurate picture of this function the following approximate formula was developed in the course of the present investigation.[30]

$$J_n(x) \doteq B_n(x) \cos \Omega_n(x), \qquad \text{for } x > n$$

where

$$B_n(x) = \sqrt{\frac{2}{\pi x}} \frac{1}{\left(1 - \frac{m^2}{x^2} + \frac{3}{2}\frac{m^2}{x^4}\frac{1}{(1-m^2/x^2)^2}\right)^{1/4}},$$

$$\Omega_n(x) = x\left[\sqrt{1-\frac{m^2}{x^2}} + \frac{m}{x}\sin^{-1}\left(\frac{m}{x}\right) - \frac{m^2}{4x^4}\frac{1}{(1-m^2/x^2)^{3/2}}\right] - \frac{2n+1}{4}\pi,$$

$$\Omega_n'(x) = \frac{d}{dx}\Omega_n(x),$$

$$= \sqrt{1 - \frac{m^2}{x^2} + \frac{3}{2}\frac{m^2}{x^4}\frac{1}{(1-m^2/x^2)^2}},$$

and

$$m^2 = n^2 - 1/4.$$

---

[30] It was subsequently discovered that somewhat similar formulas had previously been developed by Graf and Gubler (Einleitung in die Theorie der Besselschen Funktionen), and by Nicholson (*Phil. Mag.*, 1910, p. 249).

This approximate formula is valid only where $x>n$, its accuracy increasing with x and with n. For all orders of n it is quite accurate beyond the first zero of the function.

The "instantaneous frequency" of oscillation is approximately

$$\frac{1}{2\pi}\,\Omega_n'(x)=\frac{1}{2\pi}\sqrt{1-\frac{m^2}{x^2}+\frac{3}{2}\frac{m^2}{x^4}\frac{1}{(1-m^2/x^2)^2}}.$$

By this it is meant that at any point x $(>n)$ the interval between successive zeros is approximately $\pi/\Omega'(x)$. Otherwise stated, in the neighborhood of any point x, the function behaves like a sinusoid of amplitude $B_n(x)$ and frequency $\omega/2\pi$ where $\omega=\Omega_n'(x)$.

The following approximate formulas, while not sufficiently precise for the purposes of accurate computation except for quite large values of x, clearly exhibit the character of the functions for values of the argument $x>n$, and of the order $n>2$.

$$J_n(x)\doteq h_n\sqrt{\frac{2}{\pi x}}\cos\,(q_n x-\Theta_n),$$

$$J_n'(x)=-q_n h_n\sqrt{\frac{2}{\pi x}}\sin\,(q_n x-\Theta_n),$$

$$\int_0^x J_n\,(x)dx=1+\frac{h_n}{q}\sqrt{\frac{2}{\pi x}}\sin\,(q_n x-\Theta_n),$$

where

$$h_n=\left(\frac{1}{1-n^2/x^2}\right)^{1/4}\doteq 1+\frac{n^2}{4x^2},$$

$$q_n=\sqrt{1-n^2/x^2},$$

and

$$\Theta_n=\frac{2n+1}{4}\pi-n\,\sin^{-1}(n/x).$$

APPENDIX III

BUILDING-UP OF ALTERNATING CURRENTS IN WAVE-FILTERS

The integrals

$$S_n(z;\nu)=\int_0^z J_n(z_1)\,\sin\,\nu(z-z_1)dz_1$$

and

$$C_n(z;\nu)=\int_0^z J_n(z_1)\,\cos\,\nu(z-z_1)dz_1,$$

on which the genesis and growth of alternating currents in the low pass and band pass filters depends, have been computed as follows.

For values of $z<24$, $n \leqq 10$ and $\nu \leqq 1$, they are accurately calculable from the following series expansions

$$C_n(z;\nu)=2(c_1J_{n+1}(z)+c_3J_{n+3}(z)+c_5J_{n+5}(z)+\ .\ .),$$

and

$$S_n(z;\nu)=4\nu(c_2J_{n+2}(z)+c_4J_{n+4}(z)+c_6J_{n+6}(z)+\ .\ .),$$

where the coefficients $c_1, c_2 \ldots$ are polynomials in 2ν, and are independent of the index n. They are

$$c_1=1,$$

$$c_3=1-(2\nu)^2,$$

$$c_5=1-\frac{3}{1!}(2\nu)^2+(2\nu)^4,$$

$$c_7=1-\frac{3\cdot4}{2!}(2\nu)^2+\frac{5}{1!}(2\nu)^4-(2\nu)^6,$$

$$c_9=1-\frac{3\cdot4\cdot5}{3!}(2\nu)^2+\frac{5\cdot6}{2!}(2\nu)^4-\frac{7}{2!}(2\nu)^6+(2\nu)^8,$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$c_2=1,$$

$$c_4=\frac{2}{1!}-(2\nu)^2,$$

$$c_6=\frac{2\cdot3}{2!}-\frac{4}{1!}(2\nu)^2+(2\nu)^4,$$

$$c_8=\frac{2\cdot3\cdot4}{3!}-\frac{4\cdot5}{2!}(2\nu)^2+\frac{6}{1!}(2\nu)^4-(2\nu)^6,$$

$$\cdots\cdots\cdots\cdots\cdots$$

The tabulation of $J_n(z)$ for values of z up to 24 and of n up to 60 given by Gray and Mathews and by Jahnke und Emde make the computation for integral values of z rapid and precise.

For large values of n the integrals can be accurately computed, except in the neighborhood of the critical point $z=n/\sqrt{1-\nu^2}$, $(\nu<1)$, from the asymptotic formulas furnished by Gronwall.

Without detailed computation, however, the general character of the integrals can be shown as follows with an accuracy usually sufficient for engineering purposes. By differentiation S_n and C_n satisfy the differential equations

$$S_n'=\nu C_n,$$

and

$$C_n'=J_n(z)-\nu S_n,$$

where the primes denote differentiation with respect to the argument z. The solution of these differential equations is based on the approximation, valid only when $z > n$,

$$\frac{d^2}{dz^2} J_n(z) \doteq -q_n^2 J_n(z), \qquad q_n = \sqrt{1 - n^2/z^2}.$$

To this approximation, which becomes more and more accurate as z and n increase, the differential equations are satisfied by solutions of the form

$$S_n = \frac{\nu}{\nu^2 - q_n^2} J_n(z) + A \sin(\nu z - \alpha),$$

and

$$C_n = \frac{1}{\nu^2 - q_n^2} J_n'(z) + A \cos(\nu z - \alpha).$$

A and α in the complementary terms are arbitrary constants, which must be determined. These complementary terms, periodic in νz, are evidently the ultimate values of the integrals when z approaches infinity, which are known. Other considerations, however, show that these terms should be omitted when $\nu < 1$ and $z < n/\sqrt{1-\nu^2}$. Consequently we arrive at the following approximations.[31]

For $\nu < 1$ *and* $n < z < n/\sqrt{1-\nu^2}$,

$$S_n(z;\nu) = \frac{\nu}{\nu^2 - q_n^2} J_n(z),$$

$$C_n(z;\nu) = \frac{1}{\nu^2 - q_n^2} J_n'(z),$$

and

$$q_n = \sqrt{1 - n^2/z^2}.$$

This approximation is not accurate at $z = n$, and breaks down at the critical point $z = n/\sqrt{1-\nu^2}$. In the interval between, however, it is a fair approximation, particularly when ν is nearly equal to unity and n is not too small.

For $\nu < 1$ *and* $z > n/\sqrt{1-\nu^2}$,

$$S_n(z;\nu) = \frac{\nu}{\nu^2 - q_n^2} J_n(z) + \frac{1}{\sqrt{1-\nu^2}} \sin(\nu z - n \sin^{-1}\nu),$$

and

$$C_n(z;\nu) = \frac{1}{\nu^2 - q_n^2} J_n'(z) + \frac{1}{\sqrt{1-\nu^2}} \cos(\nu z - n \sin^{-1}\nu).$$

---

[31] The qualitative properties of these definite integrals can be deduced from the principle of stationary phase (See Theory of Bessel Functions, G. N. Watson, p. 229).

This formula can be safely employed only when z considerably exceeds the critical value $n/\sqrt{1-\nu^2}$.

For $\nu>1$ *and* $z>n$, the ultimate periodic terms are very small, and may be omitted unless n is too small. Consequently in this region,

$$S_n(z;\nu) \doteq \frac{\nu}{\nu^2-q_n^2} J_n(z),$$

and

$$C_n(z;\nu) \doteq \frac{1}{\nu^2-q_n^2} J_n'(z).$$

In the range of values for which the foregoing approximations are valid we have also to the same approximation (see Appendix II)

$$J_n(z) \doteq \sqrt{\frac{2}{\pi z}} \cos(q_n z-\Theta_n),$$

and

$$J_n'(z) \doteq -q_n\sqrt{\frac{2}{\pi z}} \sin(q_n z-\Theta_n).$$

APPENDIX IV

THE EFFECTS OF TERMINAL IMPEDANCES

In the text of this paper, the calculation of the wave-filter indicial admittances is based on the assumption that the voltage is applied directly to the filter at "mid-series" position and that the filter is either infinitely long or else, what amounts to the same thing, is terminated in its characteristic impedance. By virtue of these assumptions, the disturbing effects of terminal reflections are eliminated, and, as shown in the text, the solution is reducible to a relatively simple form, which admits of considerable instructive interpretation by inspection, and is rather easily computed.

In the following the general solution will be given for the indicial admittance $A_n(t)$ in the nth section of a wave-filter of s sections or length, with the e.m.f. applied to the initial or zero-th section through an impedance $Z_1(p)=Z_1$ and the last or sth section closed by an impedance $Z_2(p)=Z_2$.

For any type of periodic structure, including as a limiting case, the smooth line, it can readily be shown that

$$\frac{1}{Z_n(p)}=\sigma\frac{1}{K_1}\frac{e^{-n\Gamma}+\rho_2 e^{-(2s-n)\Gamma}}{1-\rho_1\rho_2 e^{-2s\Gamma}} \tag{1}$$

where

K_1 = characteristic impedance, as seen from terminals of initial or zero-th section,

K_2 = characteristic impedance, as seen from terminals of last or sth section,

Γ = propagation constant per section,

Z_1, Z_2 = terminal impedances,

$$\sigma=\frac{K_1}{K_1+Z_1},$$

$$\rho_1=\frac{K_1-Z_1}{K_1+Z_1},$$

and

$$\rho_2=\frac{K_2-Z_2}{K_2+Z_2}.$$

K_1, K_2, Z_1, Z_2, and consequently σ, ρ_1, ρ_2 are, of course, functions of the operator p.

The corresponding indicial admittance $A_n(t)$ is given by the integral equation

$$\int_9^\infty e^{-pt}A_n(t)dt=\frac{1}{pZ_n(p)}. \tag{2}$$

By aid of (1) the right hand side of (2) can be expanded as

$$\begin{aligned}\sigma\frac{e^{-n\Gamma}}{pK_1}+\sigma\rho_2\frac{e^{-(2s-n)\Gamma}}{pK_1}+\sigma\rho_1\rho_2\frac{e^{-(2s+n)\Gamma}}{pK_1}+\sigma\rho_1\rho_2^2\frac{e^{-(4s-n)\Gamma}}{pK_1}\\+\sigma\rho_1^2\rho_2^2\frac{e^{-(4s+n)\Gamma}}{pK_1}+\ldots\ldots.\end{aligned} \tag{3}$$

Now if $a_m(t)$ denotes the indicial admittance in the mth section of an infinitely long periodic structure, when the e.m.f. is applied directly to the sending end terminals, it follows from (2) and (3) that

$$\int_0^\infty e^{-pt}a_m(t)dt=\frac{e^{-m\Gamma}}{pK_1}. \tag{4}$$

From (2), (3) and (4) it follows at once that

$$A_n(t) = \frac{d}{dt}\int_0^t dy \left\{ \begin{array}{l} r_0(t-y)a_n(y) + r_1(t-y)a_{2s-n}(y) + r_2(t-y)a_{2s+n}(y) \\ + r_3(t-y)a_{4s-n}(y) + r_4(t-y)a_{4s+n}(y) + \ldots \end{array} \right. \tag{5}$$

provided the functions $r_0(t)$, $r_1(t)$, $r_2(t)$ . . . satisfy, and are defined by, the equations

$$\begin{aligned} \int_0^\infty e^{-pt} r_0(t)dt &= \frac{\sigma}{p} = \frac{1}{p}\frac{K_1}{K_1+Z_1}, \\ \int_0^\infty e^{-pt} r_1(t)dt &= \frac{\sigma\rho_2}{p} = \frac{1}{p}\frac{K_1}{K_1+Z_1}\cdot\frac{K_2-Z_2}{K_2+Z_2}, \\ \int_0^\infty e^{-pt} r_2(t)dt &= \frac{\sigma\rho_1\rho_2}{p} = \frac{1}{p}\frac{K_1}{K_1+Z_1}\cdot\frac{K_1-Z_1}{K_1+Z_1}\cdot\frac{K_2-Z_2}{K_2+Z_2}, \text{ etc.} \end{aligned} \tag{6}$$

If the indicial admittance in any section of an infinitely long periodic structure is determined, and equations (6) solved for $r_0(t)$, $r_1(t)$, $r_2(t)$ . . . (by aid of any of the methods discussed in the present paper), then $A_n(t)$ is given by (5) by a single quadrature. The solution may appear quite involved; as a matter of fact it is the simplest and most easily interpreted and computed form of solution possible and its complexity merely reflects the complicated character of reflection effects due to terminal impedances. This considered statement is made in the light of an extensive study of the whole problem and the literature bearing on it and has been tested in many specific cases.

When the terminal impedances Z_1 and Z_2 are complicated and entirely unrelated to the corresponding characteristic impedances K_1 and K_2, the solution of equations (6) and the numerical computations of (5) are laborious but entirely possible, the only questions being as to whether the importance of the problem justifies the necessary expenditure of time and effort. In many cases, also, approximate solutions are obtainable. Without any computations, however, the solution (5) admits of considerable instructive interpretation by inspection. The first term represents the current in the nth section of an infinitely long structure when a unit e.m.f. is impressed through a terminal impedance Z_1. $r_0(t)$ is the corresponding voltage which exists across the terminals proper. The second term is a reflected wave from the other terminals due to the terminal impedance irregularity which exists there. The third term is a reflected wave from the sending end terminals due to the corresponding terminal impedance irregularity, etc. The solution, consequently, is expanded in a form which corresponds exactly with the actual sequence of phenomena which occur.

The solution takes a particularly simple and instructive form when $Z_1=k_1K_1$ and $Z_2=k_2K_2$ where k_1 and k_2 are numerics. In this case the solutions of (6) give

$$r_0(t)=r_0=\frac{1}{1+k_1},$$

$$r_1=\frac{1}{1+k_1}\cdot\frac{1-k_2}{1+k_2},$$

$$r_2=\frac{1}{1+k_1}\cdot\frac{1-k_1}{1+k_1}\cdot\frac{1-k_2}{1+k_2},\quad \text{etc. and}$$

$$A_n(t)=\frac{1}{1+k_1}\left\{a_n(t)+\frac{1-k_2}{1+k_2}a_{2s-n}(t)+\frac{1-k_1}{1+k_1}\cdot\frac{1-k_2}{1+k_2}a_{2s+n}(t)+\ldots\right\}.$$

The solution for the special cases of open and short circuit terminations follow at once by assigning the values of zero or infinity, as the case may be, to k_1 and k_2. If $k_1=o$; $k_2=1$, $A_n(t)$ reduces to $a_n(t)$ as, of course, it should.

Low Pass Wave-Filter, Type L_1C_2

Divide ordinates by k and abscissae by ω_c to read current in amperes and time in seconds.

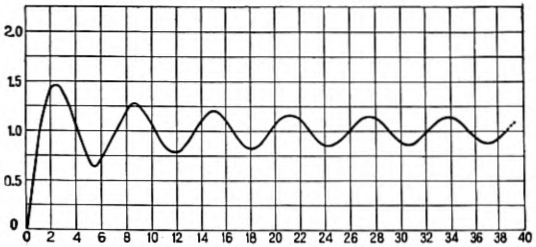

Fig. 8
Indicial Admittance of Initial Section.

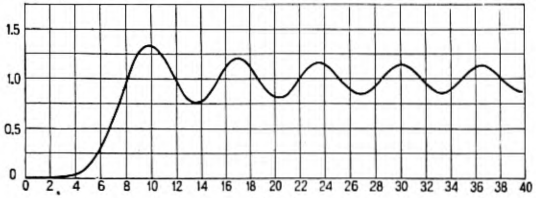

Fig. 9
Indicial Admittance of Third Section.

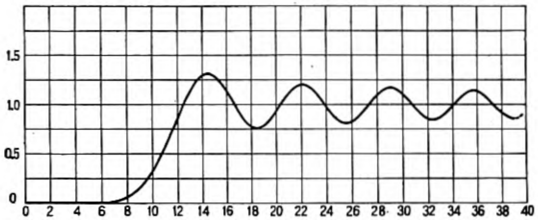

Fig. 10
Indicial Admittance of Fifth Section.

Band Pass Wave-Filter, Type $L_1C_1L_2C_2$

Divide ordinates by $\omega_m k/w$ and abscissae by $w/2$ to read current in amperes and time in seconds.

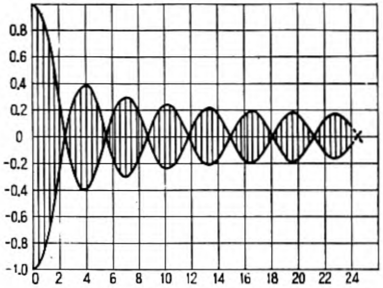

Fig. 11
Indicial Admittance of Initial Section.

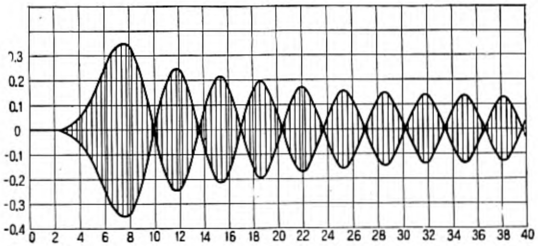

Fig. 12
Indicial Admittance of Third Section.

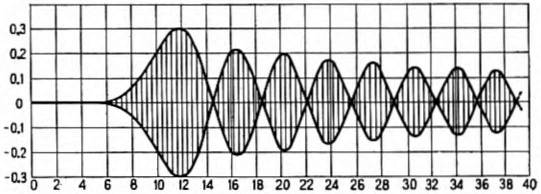

Fig. 13
Indicial Admittance of Fifth Section.

Band Pass Wave-Filter, Type $L_1L_2C_2$

Divide ordinates by $\omega_m k/2w$ and abscissae by $w/2$ to read current in amperes and time in seconds.

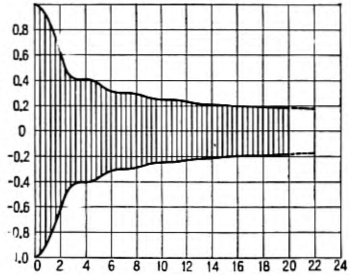

Fig. 14
Indicial Admittance of Initial Section.

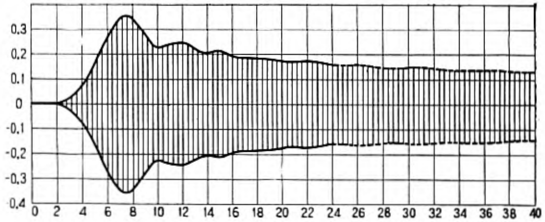

Fig. 15
Indicial Admittance of Sixth Section.

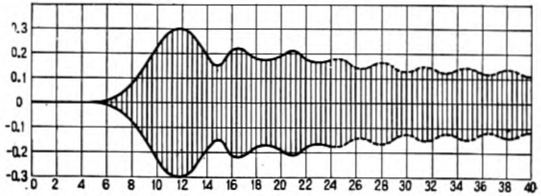

Fig. 16
Indicial Admittance of Tenth Section.

High Pass Wave-Filter, Type C_1L_2

Divide ordinates by k and abscissae by ω_c to read current in amperes and time in seconds.

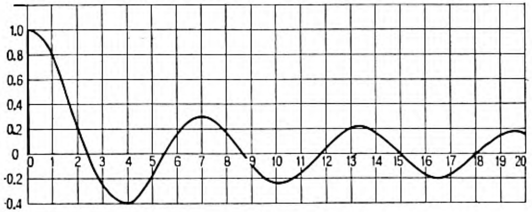

Fig. 17

Indicial Admittance of Initial Section.

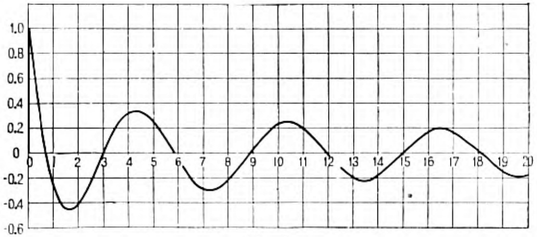

Fig. 18

Indicial Admittance of First Section.

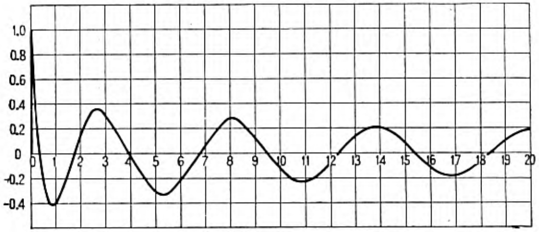

Fig. 19

Indicial Admittance of Second Section.

High Pass Wave-Filter, Type C_1L_2—*Contd.*

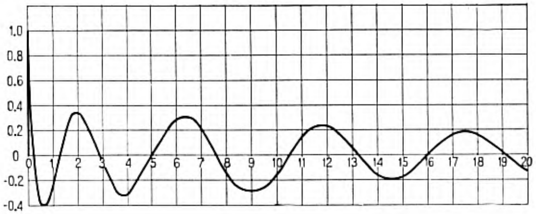

Fig. 20

Indicial Admittance of Third Section.

Low Pass Wave-Filter, Type L_1C_2

Divide ordinates by k and abscissae by ω_c to read current in amperes and time in seconds.

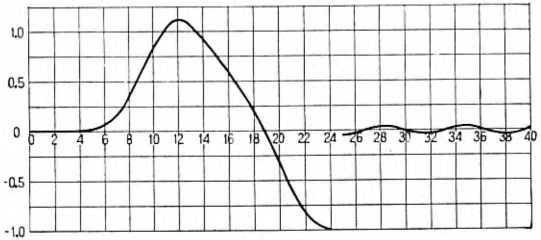

Fig. 21

Current in Third Section in Response to E.M.F. sin $\frac{1}{4}\omega_c t$.

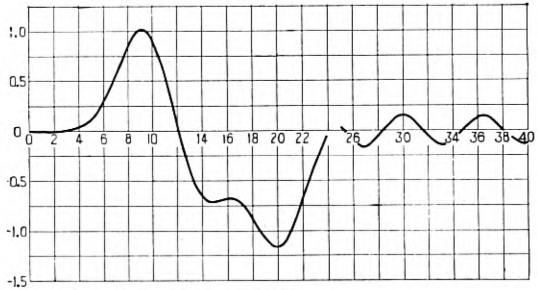

Fig. 22

Current in Third Section in Response to E.M.F. cos $\frac{1}{4}\omega_c t$.

Low Pass Wave-Filter, Type L_1C_2—*Contd.*

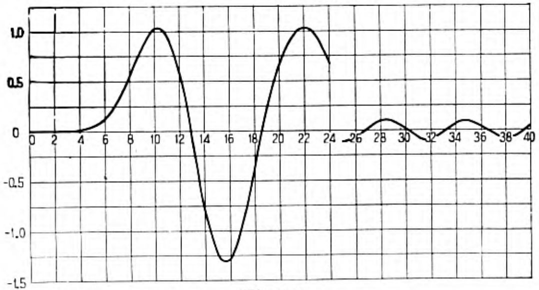

Fig. 23

Current in Third Section in Response to E.M.F. sin $\frac{1}{2}\omega_c t$.

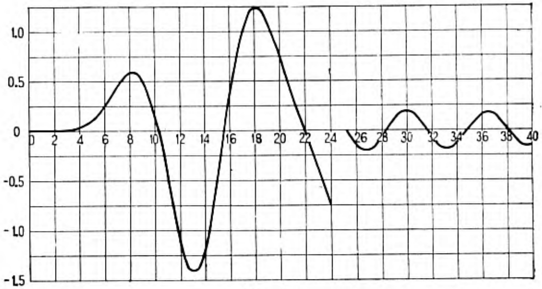

Fig. 24

Current in Third Section in Response to E.M.F. cos $\frac{1}{2}\omega_c t$.

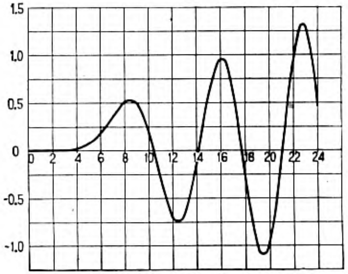

Fig. 25

Current in Third Section in Response to E.M.F. sin $\omega_c t$.

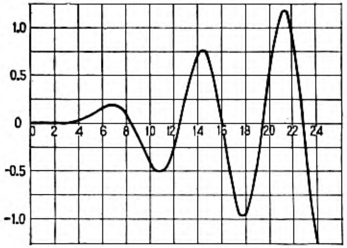

Fig. 26
Current in Third Section in Response to E.M.F. cos $\omega_c t$.

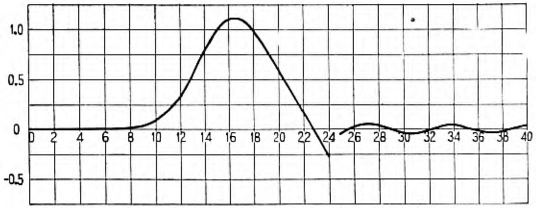

Fig. 27
Current in Fifth Section in Response to E.M.F. sin $\frac{1}{4}\omega_c t$.

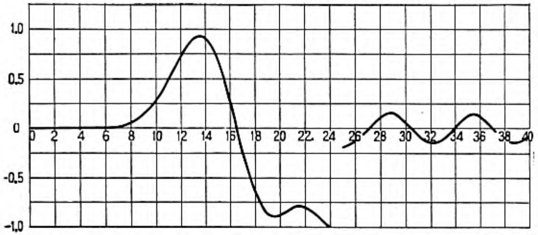

Fig. 28
Current in Fifth Section in Response to E.M.F. cos $\frac{1}{4}\omega_c t$.

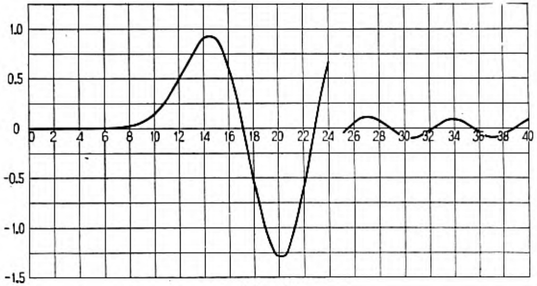

Fig. 29
Current in Fifth Section in Response to E.M.F. sin $\frac{1}{2}\omega_c t$.

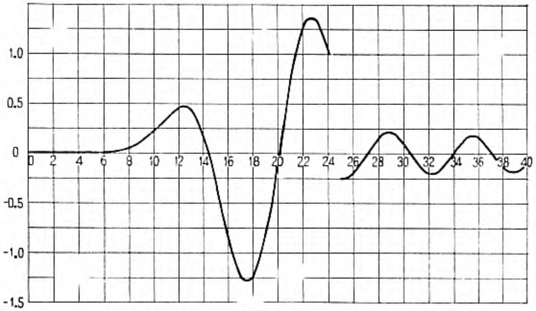

Fig. 30
Current in Fifth Section in Response to E.M.F. cos $\frac{1}{2}\omega_c t$.

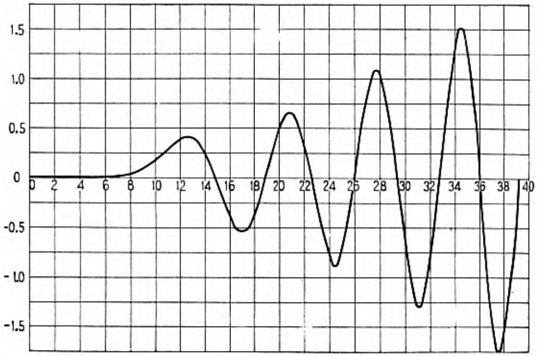

Fig. 31
Current in Fifth Section in Response to E.M.F. sin $\omega_c t$.

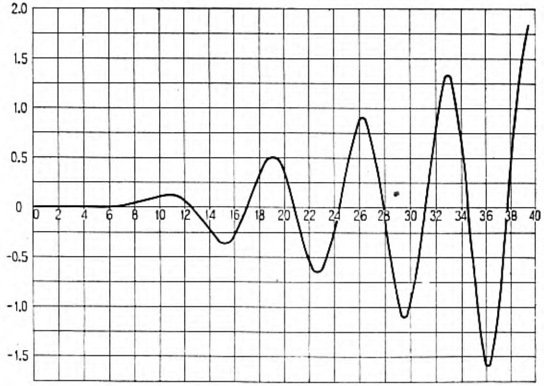

Fig. 32
Current in Fifth Section in Response to E.M.F. cos $\omega_c t$.

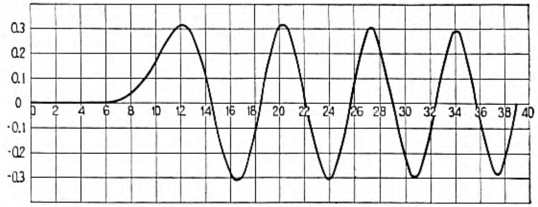

Fig. 33
Current in Fifth Section in Response to E.M.F. sin 1.25 $\omega_c t$.
Steady-state Amplitude=0.0013.

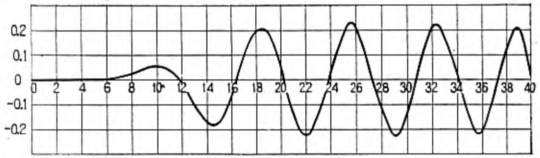

Fig. 34
Current in Fifth Section in Response to E.M.F. cos 1.25 $\omega_c t$.
Steady-state Amplitude=0.0013.

Band Pass Wave-Filter, Type $L_1C_1L_2C_2$

Divide ordinates by ω_{mk}/w and abscissae by $w/2$ to read current in amperes and time in seconds.

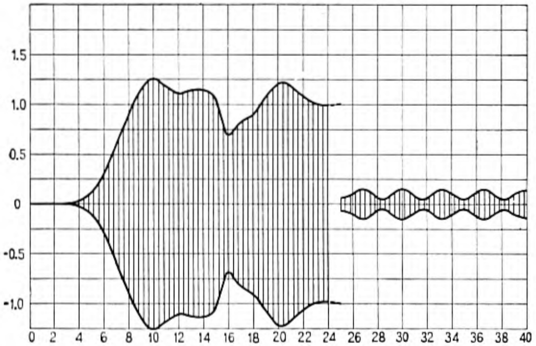

Fig. 35

Envelope of Current in Third Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/8)$.

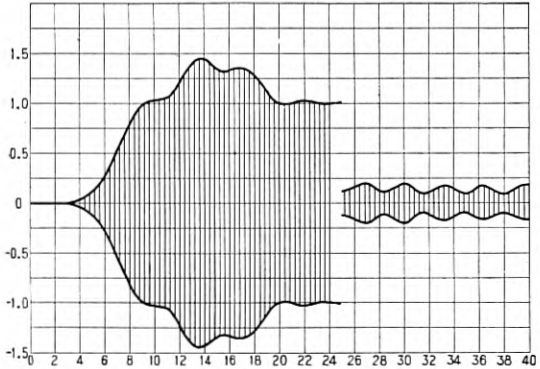

Fig. 36

Envelope of Current in Third Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/4)$.

Band Pass Wave-Filter, Type $L_1C_1L_2C_2$, *Contd.*

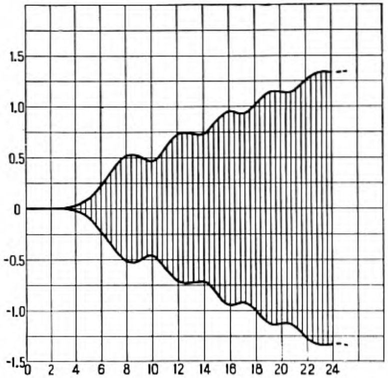

Fig. 37

Envelope of Current in Third Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/2)$.

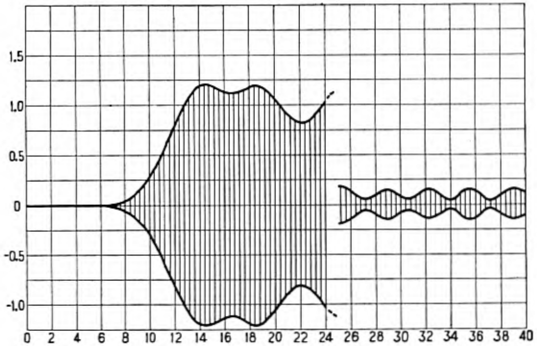

Fig. 38

Envelope of Current in Fifth Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/8)$.

Band Pass Wave-Filter, Type $L_1C_1L_2C_2$, *Contd.*

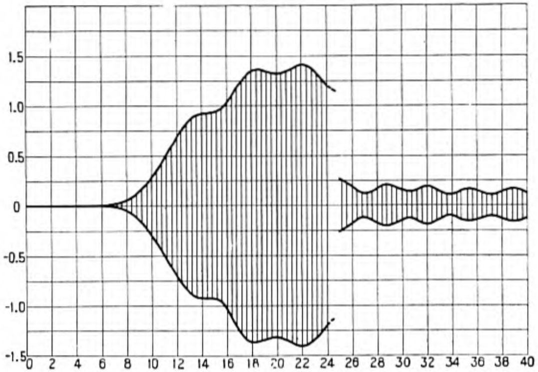

Fig. 39

Envelope of Current in Fifth Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/4)$.

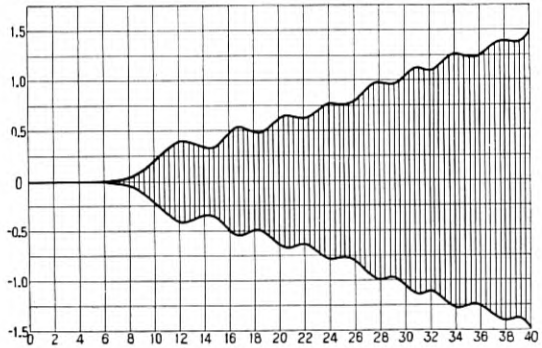

Fig. 40

Envelope of Current in Fifth Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}$ $(\omega_m \pm w/2)$.

Band Pass Wave-Filter, Type $L_1C_1L_2C_2$, *Contd.*

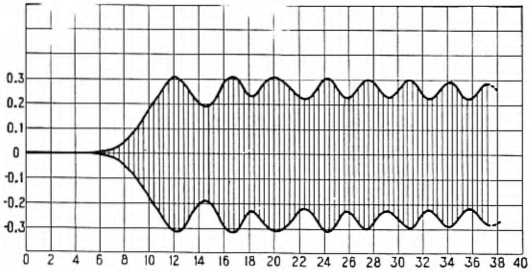

Fig. 41

Envelope of Current in Fifth Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}\left(\omega_m \pm \frac{5}{4} w/2\right)$.

Band Pass Wave-Filter, Type $L_1L_2C_2$

Divide ordinates by ω_{mk}/w and abscissae by $w/2$ to read current in amperes and time in seconds.

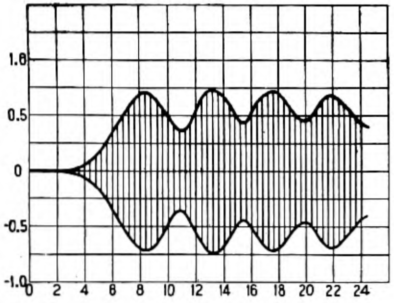

Fig. 42

Envelope of Current in Sixth Section in Response to E.M.F. of Frequency $\frac{1}{2\pi}(\omega_m \pm w/4)$.

High Pass Wave-Filter, Type C_1L_2

Divide ordinates by k and abscissae by ω_c to read current in amperes and time in seconds.

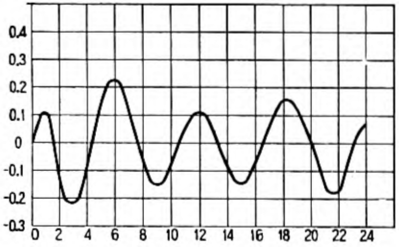

Fig. 43

[Current in First Section in Response to E.M.F. sin $\frac{1}{2}\omega_c t$. Steady-state Amplitude = 0.0415.

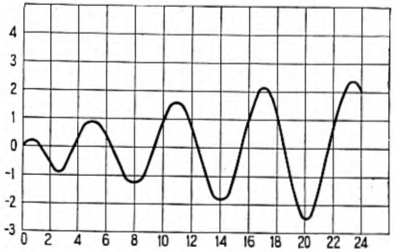

Fig. 44

Current in First Section in Response to E.M.F. sin $\omega_c t$.

Use of Labor-Saving Apparatus in Outside Plant Construction Work

By J. N. KIRK

INTRODUCTION

IN the January issue of this Journal was discussed the adaptation of transportation equipment to telephone construction and maintenance work. Closely associated with the operation of such equipment is the problem of utilizing various labor-saving machinery which in many cases has been so designed as to form an integral part of the transportation unit.

It is the purpose of this article to describe some of the more important developments along this line such, for example, as the application of different types of derricks, trailers for various kinds of work, earth boring machines, numerous applications of air compressors and compressed air tools, etc., and in some instances to contrast the latest types of equipment with former manual methods of carrying out similar operations.

POLE DERRICKS

There are erected in the Bell System each year in the neighborhood of 600,000 new poles. In addition, the maintenance of the existing plant of over 14,000,000 poles involves the moving, removing, resetting and straightening of large numbers of poles annually. This immense task emphasizes the importance of devising means for offsetting, in so far as is practicable, the old manual methods of handling these poles on the job and from point to point in the field as occasion demands.

In 1914 there was developed and put into use a pole derrick of the tripod type which was mounted upon a 5-ton truck from which the derrick received the necessary power for operation. As the use of this derrick, which weighed something over $\frac{1}{2}$ a ton, was extended it became apparent that while the fundamentals of the design and operation were reasonably well adapted to the average construction job, the weight and bulk of the apparatus introduced a very real factor with regard to the available truck capacity. The derrick members, being large and heavy, were difficult for the men to handle and there was not in all cases the desired amount of flexibility to meet the varied and often difficult requirements. This derrick, however, clearly demonstrated the inestimable value of apparatus capable of doing in a few minutes the work ordinarily requiring a large gang of men, many times as long to complete.

An active period of development and experimental field work soon followed the advent of this labor-saving device which has resulted in making available a light type of high grade steel tube derrick.

Figs. 1 and 2 show a pole derrick of the latest type mounted on a 2½ ton truck. Fig. 1 illustrates the method of erecting a pole where the truck can be maneuvered into a position in close proximity to the proposed location of the pole. Fig. 2, on the other hand, shows the possibility of handling a pole at a considerable distance from the location of the truck, which for any reason may be more practicable or desirable.

These illustrations show the derrick in each of the two possible operating positions; in the first instance supported entirely upon the truck, and in the second, supported from the ground by one of the three pipe members. The derricks of this type are constructed of high grade steel tubing having a strength at the yield point of approximately 70,000 pounds per square inch.

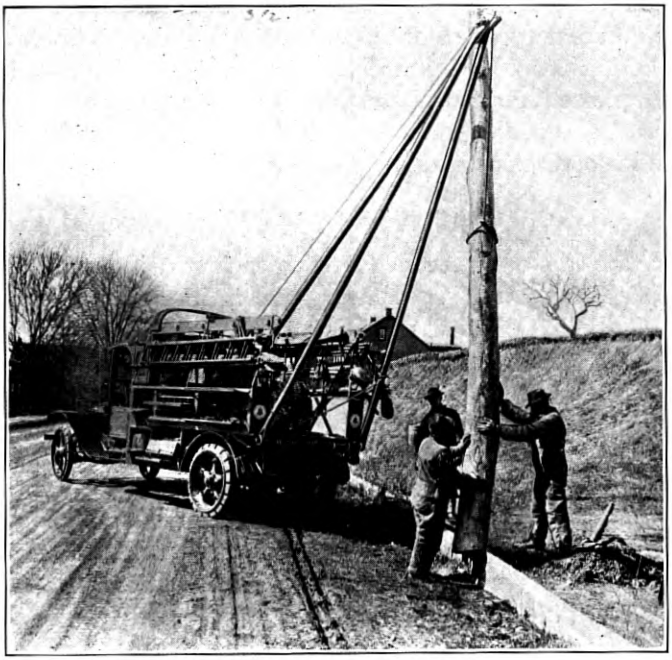

Fig. 1—Erecting Pole, all Derrick Members Mounted on Truck

In order that country-wide conditions may be satisfactorily met, the present type of derrick has been made available in two general types which are known as the "middle" and "corner" types for use, as the names imply, from the rear middle or corner of the truck. Each of these types are further available in light and heavy weights, depending upon the lengths and the kinds of poles, cedar or chestnut or other kinds of similar weights, that are generally used in any particular part of the country.

As contrasted with the early type of derrick, the present types weigh from 370 to 520 pounds, depending upon the size used, and are capable of readily and safely handling any load within the limits of the winch rope capacity, which leaves a satisfactory margin when doing practically any work for which the derrick has a place in telephone construction. Each of the four classes of derricks above mentioned is designed with a view to making its operation as rapid as is consistent with safety. The chauffeur and one man can remove the derrick members from the carrying racks provided on the truck, assemble them and erect the derrick ready for work in from three to four minutes. The disassembling of the derrick requires about the same length of time.

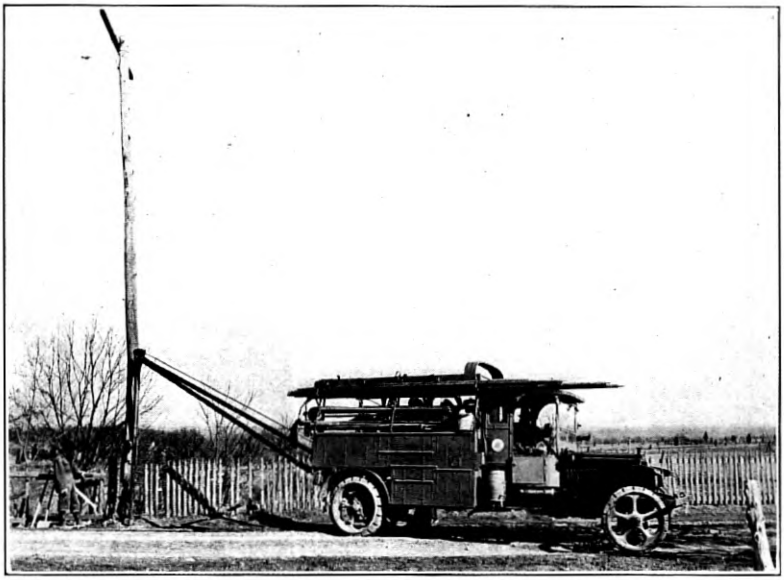

Fig. 2—Erecting Pole at Distance from Truck, One Derrick Member on Ground

Naturally, the greatest economies may be made in the application of this apparatus where the poles to be handled constitute a consecutive line, the holes for which have been dug in advance. However, because of the short time required for assembling and taking down the derrick, it is generally economical to use it for placing only one or two poles at a location. As indicative of the possibilities with regard to rapidity of operation, it may be of interest to note that in erecting a number of 30 to 35 foot poles under average conditions in a line for which the holes had previously been prepared, a gang of three men have averaged approximately two minutes per pole erected but not tamped.

The use of the derrick has thus far been described as applied to the economical erection of poles. There are, as a matter of fact, many other important uses for which the winch-operated, derrick equipped truck is well adapted, a few of which are enumerated below.

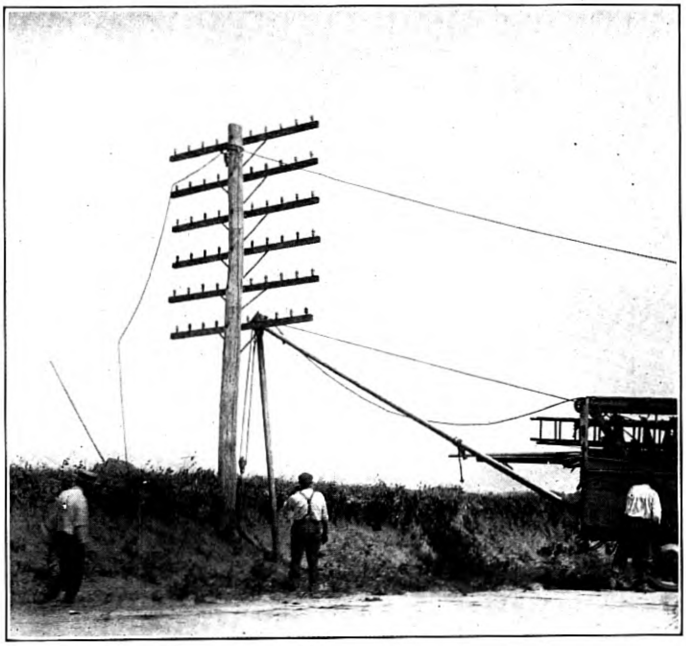

Fig. 3—Derrick in Position to Pull Pole Out of Ground

Road and highway changes and improvements throughout the country make it necessary for the telephone companies to annually move thousands of poles to the new highway limits or curb lines. In many instances these pole lines carry heavy loads of wire or cable or both. With the pole derrick many of these moves can readily be accomplished without in any way disturbing the wire or cable loads. The derrick pulls the pole out of the ground and with the aid of the truck, the pole with its load intact is moved to the new location where it is lowered into the hole prepared without even untying a wire or loosening a cable clamp. It will also be readily appreciated that the rehandling of cable and particularly the untying of open wires is not only an expensive operation in point of first cost, but that each such operation is distinctly detrimental to the plant, shortening its life and greatly increasing maintenance expenses. It will be seen, therefore, that the use of the derrick where practicable in connection with the moving of existing lines will largely eliminate the undesirable and costly procedure which is involved in the manual handling of the poles.

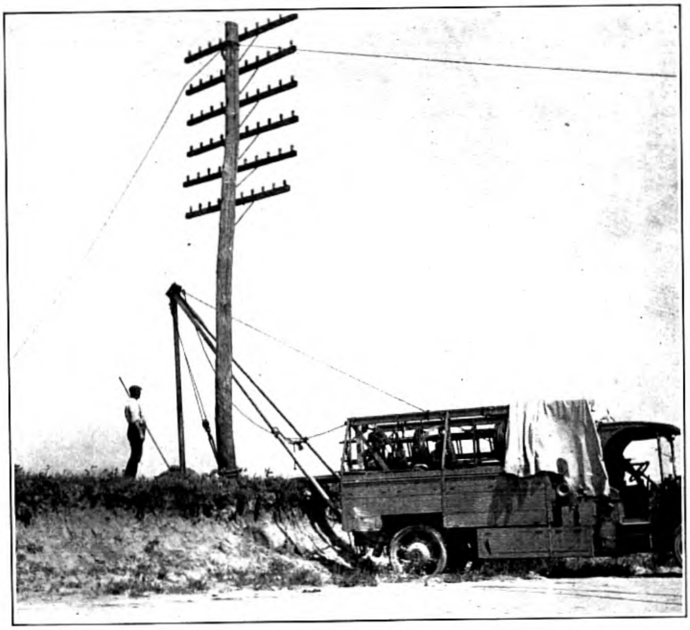

Fig. 4—In Position to Shift Pole to New Location. Pole Has Been Moved Over Bank with Wires Intact

As an example of one of the many uses to which the pole derrick can very satisfactorily be put, Figs. 3 and 4 illustrate the initial and final steps in moving back a pole in a 6-arm lead of wires and lifting it up an embankment to its new location in connection with highway widening. This particular line is about 60 miles long and the distance the poles were moved varied between 6 and 125 feet. It is reported that the move of this entire lead which averaged about 4 arms was completed without untying a single wire, without cutting any slack and with practically no trouble on the circuits. It is needless to say that the saving involved by being able to move this line rather than rebuild at the new location was an item of considerable importance.

The above illustration shows the derrick in position to pull a pole out of the ground, the top of the pole being temporarily side-guyed.

In Fig. 4 the pole is shown after having been pulled out of the ground and placed on top of the embankment. The derrick is ready to shift and slide the pole back to the new hole. Two men and the chauffeur pulled and completed the moving of this pole with its load of six arms of wires in twenty-five minutes.

Fig. 5—Derrick Operating Under Difficult Conditions

As a further example of the usefulness of the derrick in pole work, Fig. 5 shows a job where the pole derrick was operated under rather unusual conditions to erect a pole at the side of the road where the pole hole was dug under water and the pole erected in barrels. It would be difficult to pike a pole into such a hole because there is nothing against which to rest the butt while raising it.

Another important function of the derrick is that in connection with the resetting of poles or the removal of abandoned poles when it is necessary to remove the butts. The slow and laborious process of pulling the pole out of the ground with a jack or other equipment is practically eliminated as the derrick, properly handled, is capable of doing the greater part of this work in much less time, more economically and with greater safety to the men.

In addition, it might be pointed out that the derrick equipped truck is also becoming more and more indispensable in connection with the handling or moving of any heavy loads in the storage yards, in unloading or in moving stock supplies of poles under adverse conditions and many other uses.

Fig. 6—Erecting Pole by Manual Methods. Contrast with Previous Operations

In contrast with the mechanical methods of erecting and handling poles as previously shown, Fig. 6 shows the old manual method of erecting a large pole. Not only is the number of men required large, but the observance of most rigid precautions does not entirely preclude the possibility of hazard to the men when handling the heavier poles. Further, the pole locations are not always such that a considerable number of men with pikes can properly distribute themselves about the pole so as to complete the raising and lowering operations in a reasonably safe and efficient manner.

Earth Boring Machines

One of the slowest and most difficult physical tasks connected with outside construction work is that of digging pole holes. It is estimated that upwards of 1,000,000 holes must be dug annually to accommodate the poles erected in new locations, and those replaced, moved and reset in the Bell System. Under soil conditions reasonably free from obstructions a man can generally average about three holes per day with perhaps five to six as a maximum under ideal soil conditions, while in more difficult digging one or possibly two holes may represent a good average day's work. It probably requires somewhere in the neighborhood of 3,500,000 man-hours per year simply to dig pole holes.

For a number of years the availability of a practical pole hole digger has been the objective of telephone linemen. Development work has progressed rapidly during recent years and the high point of perfection which has been reached in automobile truck design and performance has greatly simplified the adaptation and increased the practicability of the boring apparatus. It is of interest to note in this connection that the solution of the problem comes at a time when there is a pronounced shortage of common labor.

The construction in 1914 of that portion of the transcontinental line extending across Nevada, marks the first really economical application of a machine to bore pole holes. In about 1917 the need for labor relief led to renewed activity in connection with adapting the fundamental principles of the original boring apparatus to machines sufficiently flexible to meet the general and rather exacting requirements of telephone work.

Fig. 7 shows one of the latest developments in earth boring machines, which is cleancut and rugged. This machine is mounted upon a 4-wheel drive truck and is otherwise specially equipped which enables it to reach practically any location where it is necessary to bore holes for the erection of poles. As a matter of fact it has been demonstrated that these machines are able to reach approximately 95% of the pole locations. Further, the machine being equipped with a pole raising derrick makes possible the digging of the hole and the erecting of the pole with but one setting of the truck.

With the boring machine from 30 to 80 poles per day can be set in their holes by a force of three men. This, of course, does not include straightening the poles and backfilling the holes. To do this amount of work with manual labor only would ordinarily require from 15 to 50 men. It is of particular interest to note that the more difficult the digging, exclusive of rock, of course, the greater the saving by using the machine. It might be mentioned that one of the most important features of the boring machine is its ability to bore holes through frost thus enabling a more uniform apportionment of pole work over the entire year. This feature is also of particular value in connection with the restoration of service subsequent to sleet storm breaks in winter at which time hand digging is in many cases a practical impossibility.

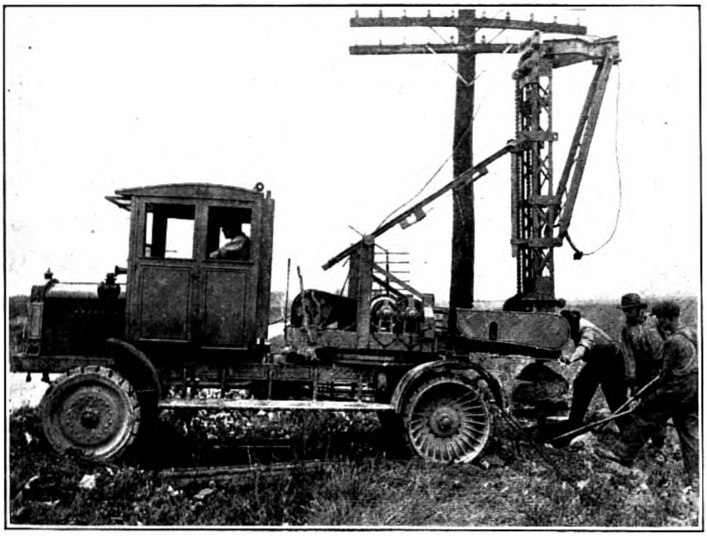

Fig. 7—Boring Hole for "H" Fixture

Fig. 8 illustrates the ability of this 4-wheel drive outfit to negotiate difficult ground conditions. In this instance one rear wheel has dropped into a hole while traveling over a plowed field covered with snow. It required only a few minutes to lift the wheel by moving the turn table so that the auger was just behind the buried wheel, then raising that corner of the truck by forcing down the auger with power from the engine, sliding a skid board under the wheel thus raised, lowering the wheel to this board and driving away.

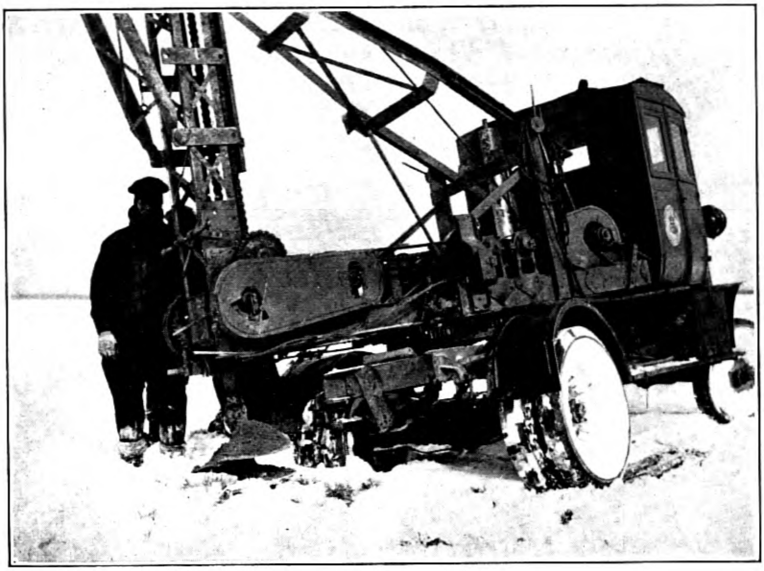

Fig. 8—Machine Extricating Itself from Hole

Cable Reel Trailers

To meet the need for a device suitable for trailing a single reel of cable and also for use as a reel "set-up" preparatory to a "pull" of either underground or aerial cable, a type of cable reel trailer has been developed as illustrated in Figs. 9 and 10.

A number of trailers of this type have been in service for some length of time and their use has brought out many advantages, some of the more important of which are:

> A reel of cable can be loaded on and unloaded from the trailer in less time and with less effort than when a reel is carried in the body of the truck. In this connection, it might be pointed out that an important safety feature is involved in that the hazards to the men in loading and unloading heavy reels of cable by the old method are practically eliminated. Of course, even where reels of cable are carried in the truck the use of the winch and spindle as previously discussed in the January issue eliminates the hazard that was present in the old method of loading and unloading, involving the use of skids.

Fig. 9—Truck Being Used to Load Reel of Cable on Trailer

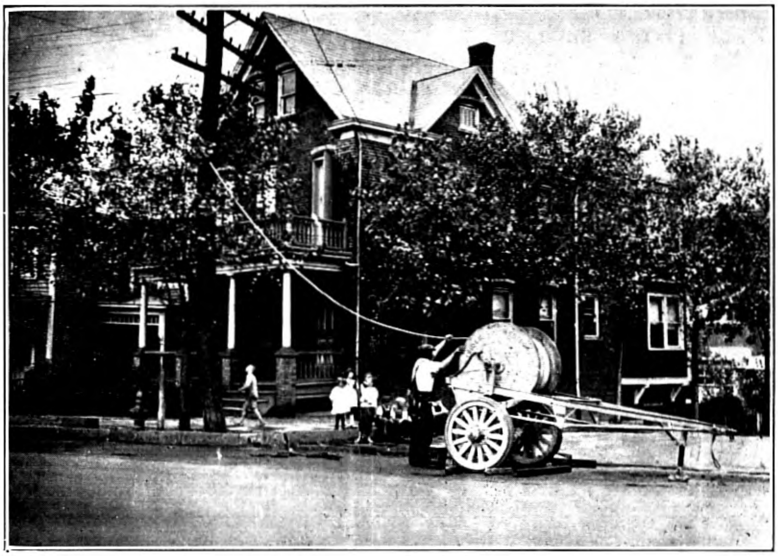

Fig. 10—Cable Being Pulled into Rings from Reel "Set-up" on Trailer

Fewer men are required for loading, unloading and "setting-up." For example, two men with a chauffeur and truck (not necessarily equipped with a winch) can satisfactorily handle a 3-ton reel of cable with the trailer, where ground conditions are such that they can maneuver the reel on the ground.

Where a single reel of cable is to be used for one "pull" or for a number of short "pulls," the trailer is used to haul the reel to the job and to "set up" the reel for each "pull." The reel may be trailed, in addition to carrying materials, tools, etc., in the body of the truck, thus making it unnecessary to unload or disarrange the equipment regularly carried on the truck.

When delivering a number of reels, one reel may be trailed in addition to carrying one or more on the body of the truck, thus materially increasing the hauling capacity of the truck, with a proportionate reduction in delivery costs.

As the photographs indicate, these trailers are equipped with springs and rubber tires which afford material protection to the cable while in transit.

POLE TRAILERS

For the transportation of poles under ordinary conditions, the use of a two-wheel trailer with the poles balanced on the trailer and towed behind the truck is ordinarily the most satisfactory method. Fig. 11 shows such a trailer loaded and ready for action. This method has the advantage that the trailer loaded with poles can be readily detached from the truck and left at any desired location, thus releasing the truck for other work. Also, in case of the load being stuck on a hill or in the mud, the trailer can be readily detached while the truck runs forward and from the top of the hill or from firm ground, pulls the trailer load of poles by means of the winch line.

Limiting the weight to conform with requirements of state laws materially limits the size of the load in hauling chestnut and creosoted pine poles. However, in the case of cedar poles, the bulk of the load rather than its weight is ordinarily the limiting factor.

To meet these different conditions, three sizes of pole trailers have been designed, a heavy duty trailer rated at about 8 tons with ample overload capacity, a medium duty trailer rated at 5 tons, and a light duty trailer of 2½ ton capacity for use in districts where it is desirable to maintain a standard tread between the wheels rather than to use the narrow tread dinkeys for the lighter pole loads.

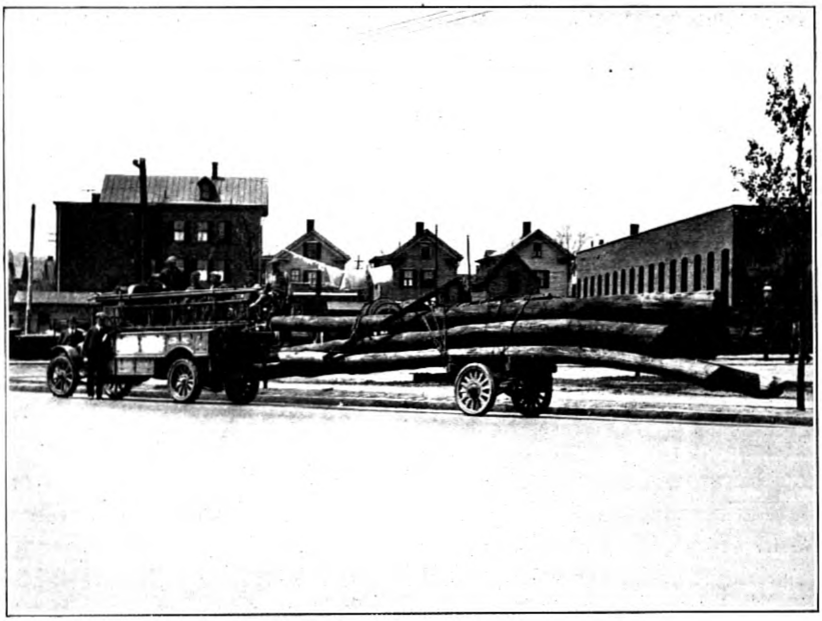

Fig. 11—Balanced Load of Chestnut Poles on Trailer

BLOCK GANG TRAILER

Fig. 12 illustrates a type of trailer which has been developed recently for the use of gangs doing interior block construction work. In a

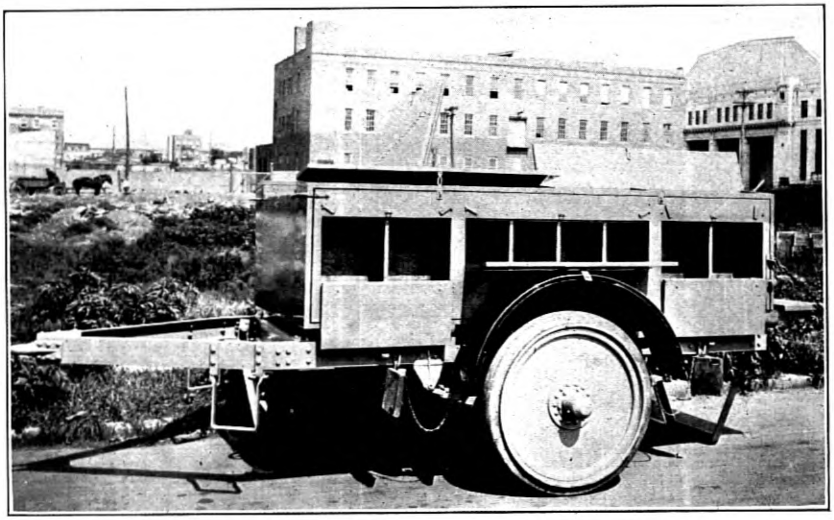

Fig. 12—Trailer Equipped with Special Body for Interior Block Construction Work

case of this kind, the gang is ordinarily located on a job from one-half day to three or four days, and since the power equipment on a truck would be of no value in connection with placing a cable on the rear walls of buildings, for instance, it is more economical to serve this gang by means of a trailer.

This light type of trailer contains sufficient space for carrying all the necessary miscellaneous tools and materials required in connection with block work and the compartments into which it is divided are such that these articles can be arranged in an orderly and readily accessible manner, thus making for increased efficiency in executing the work.

CONCRETE MIXERS

In connection with the construction of underground conduit and particularly in the work of building concrete manholes, which are now being employed to a rather large extent, it is essential that concrete mixers be available which will be especially adapted to telephone work. Some of the requirements of this service are that the outfit be of light weight, compact, embody maximum portability, and be reliable in operation. The failure of a mixer on a telephone job may seriously handicap the operations of a large gang of men.

Fig. 13 shows a commercial type of mixer which has been modified

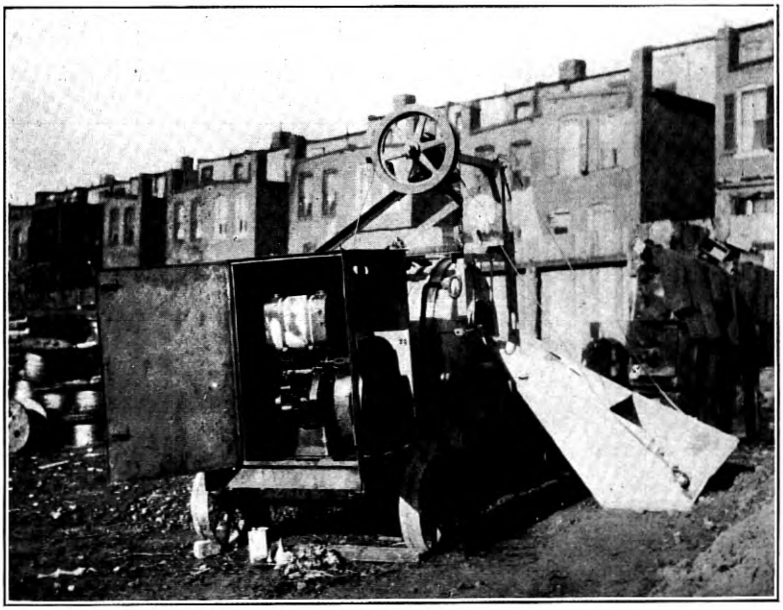

Fig. 13—Concrete Mixer Adapted to Meet Telephone Construction Requirement

in several respects to meet the particular requirements of telephone construction work.

Units of this type which are now in service are operating very satisfactorily, both from the viewpoint of reliability and adaptability to the work. This outfit will mix as much concerte as ten men and will do it much better.

Fig. 14—Pouring Concrete Manhole. Note 4-way Chute for Distribution

Fig. 14 shows one of the batch mixers in service pouring a concrete manhole, the concrete being uniformly distributed to all sides of the structure by means of a four-way chute. In connection with the broadening use of concrete manholes it might be mentioned that the availability of improved compressed air tools has greatly simplified and cheapened the making of any changes that may be required subsequent to the initial construction of the manholes.

In order to provide a concrete mixer unit having maximum portability and having proper capacity and operating features for telephone work, we have cooperated with the manufacturer in the development of such a unit which is shown in Fig. 15. This consists of a batch mixer permanently mounted upon a Ford 1-ton truck chassis and operated through a suitable power take-off from the Ford engine. This unit loads from the ground by means of a power loader and distributes the concrete from the opposite side of the drum through a long swinging adjustable chute (not shown). A small trailer if desired

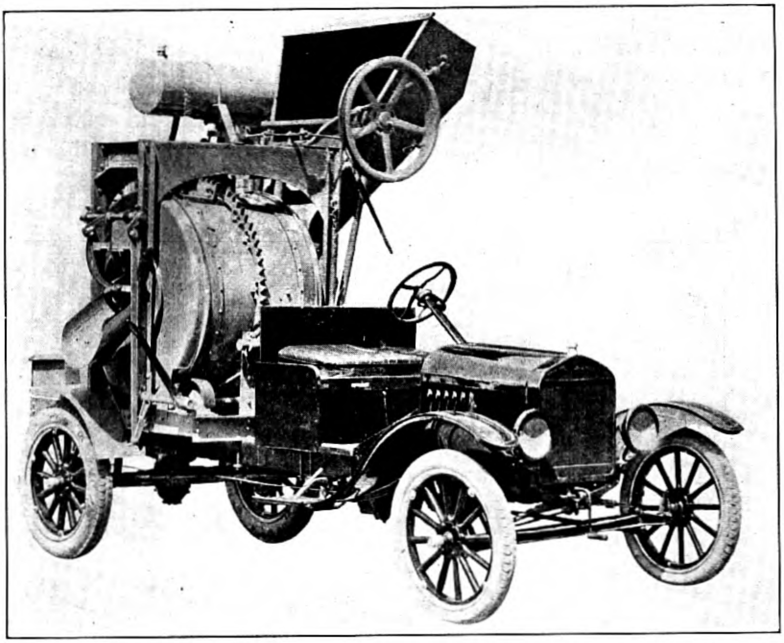

Fig. 15—Concrete Mixer on Ford One-Ton Truck. Maximum Portability for Small Jobs

Fig. 16—Light Weight Trenching Machine.

can be used behind the Ford truck to transport the supplies and tools necessary in connection with isolated jobs.

TRENCHING MACHINES

Where it is necessary to do a considerable amount of trenching for underground conduit in outlying districts, it is sometimes possible to utilize a trenching machine with marked economy. In fact under normal conditions a machine of this kind will dig trench about as fast as a gang of 50 men.

The machine shown in Fig. 16 is a recent development which has advantages over the older type units in that the size and weight are such as to admit of its being transported from point to point on a heavy truck or trailer.

PUMPS

In handling the water from excavations and also from manholes where splicers are working, the diaphragm pump illustrated in Fig. 17 is giving a good account of itself, particularly because of certain features incorporated in the design which especially adapt it to telephone conditions.

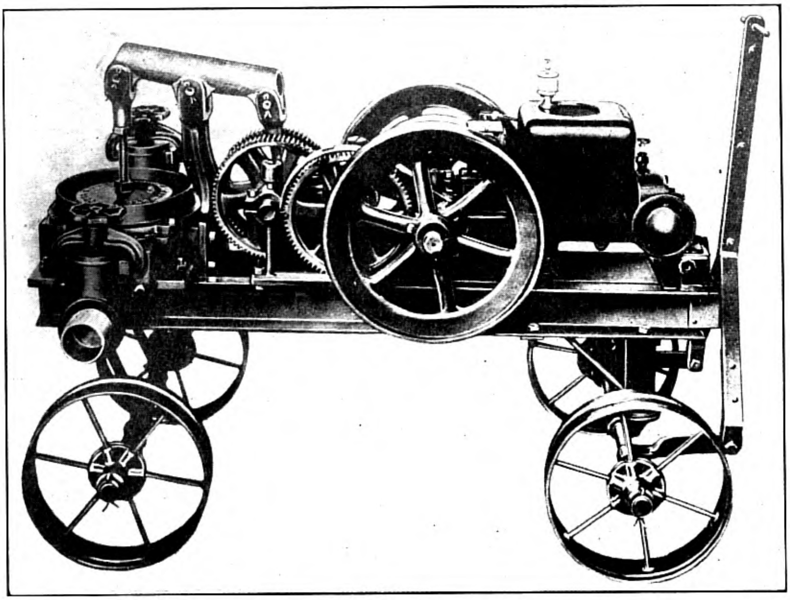

Fig. 17—Enclosed Discharge Diaphragm Pump. Capacity One Barrel per Minute

This little unit will pump water at the rate of over one barrel per minute and discharge it through a hose away from the job to any location desired. It will operate all day with practically no attention, upon a gallon or two of gasoline. When pumping under ordinary conditions it will handle water faster than 12 men with hand pumps.

One very desirable feature of the diaphragm pump is that it is self-priming. For instance, if splicers are working in a manhole the pump can be started and the initial volume of water removed, then as seepage water enters the manhole it will be immediately taken up and discharged without any attention from the splicers or helpers.

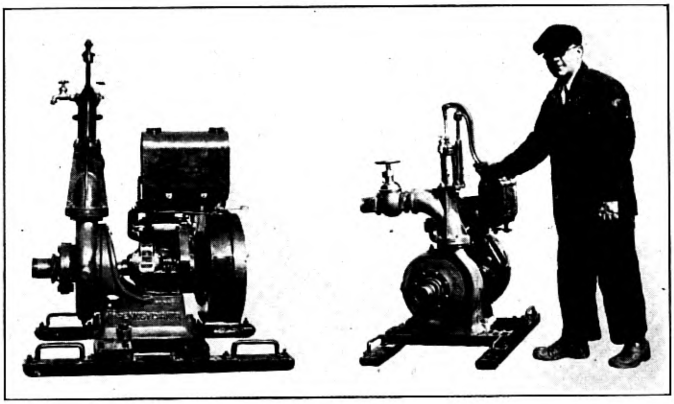

Fig. 18—Light Weight Centrifugal Pump. Capacity Seven Barrels per Minute

For handling larger volumes of water there has just been developed, as the result of careful study and cooperation with the manufacturer, a new type of centrifugal pump shown in Fig. 18. This unit consists of an air cooled engine similar to that used in the concrete mixers. On the end of the engine shaft is mounted the centrifugal pump impeller. The pump casting also forms a base for the engine.

As an indication of the capacity of this pump it might be of interest to note that it would not be possible to get enough men with hand pumps around a manhole to handle water as fast as this unit. It will pump seven barrels of water per minute and mounted on skids as shown it weighs only about 500 pounds.

In the case of trucks which do a considerable amount of underground cable placing in districts where water must be removed from manholes in advance of the cable placing operation, centrifugal pump equipment mounted on the truck is desirable. As soon as the gang arrives at a wet manhole, the pump if promptly applied will remove the water in the few minutes during which preparations are being made for placing the cable, so that ordinarily the gang is not delayed in the least by the water.

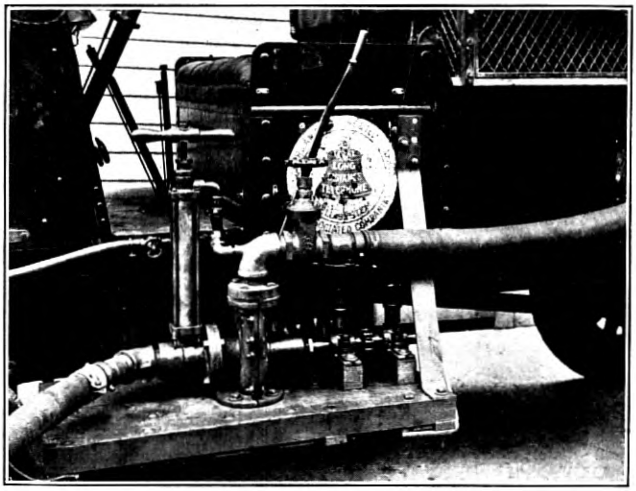

Fig. 19—Centrifugal Pump Mounted on Underground Cable Placing Truck

There are several points in favor of locating the pump on the running board as shown in Fig. 19 rather than in the body at the rear of the cab as has been the usual practice in the past. With the running board installation the water is not carried up into the truck body where it has a tendency to get into the tool and material boxes and equipment and also to cause deterioration of the body. In addition space is economized and the pump is located considerably lower than would otherwise be the case, thus resulting in a reduction of the suction lift for the water between its level in the manhole and the pump impeller.

AIR COMPRESSORS AND COMPRESSED AIR TOOLS

Of the many applications for mechanical equipment to offset the scarcity and high cost of certain types of labor such as for excavating, etc., the use of air compressors and compressed air tools is of prime importance in the outside plant construction work. Through special adaptations to meet each peculiar condition, this class of labor saving equipment has been made available for use on such jobs as the opening of all kinds of street pavements preparatory to laying underground conduit, cutting frozen ground, loosening the earth in excavating instead of using picks, drilling rock preparatory to blasting for underground conduit or for pole holes, tamping back filled earth, cutting iron pipe covering from cable, etc.

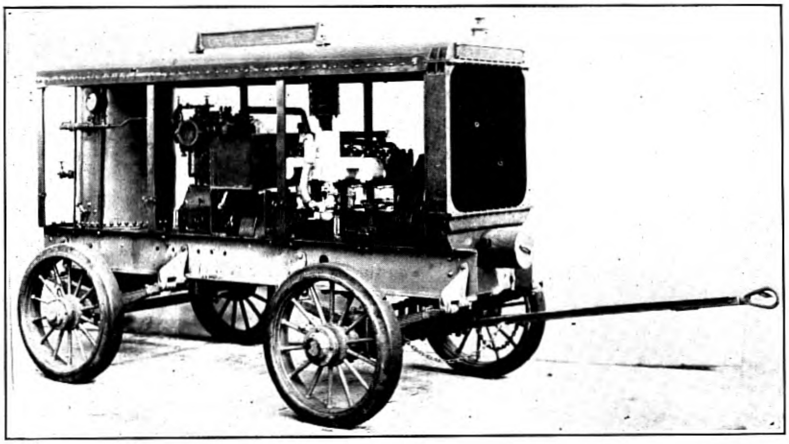

Fig. 20—Air Compressor Mounted on Trailer for Maximum Portability

Fig. 20 shows a new type of portable gasoline engine driven compressor unit which is being satisfactorily used for the larger jobs of opening street pavements, for rock drilling, etc.

Fig. 21—Removing Granite Blocks and Breaking Concrete Base

Where trenching work involves cutting through paved streets one compressor unit with three men will ordinarily accomplish as much in a given period of time as 27 men employing former methods.

In Fig. 21 two operators are shown opening pavement which consists of granite blocks set in cement, on a concrete base. One man goes ahead and wedges the blocks loose, while the man following breaks the concrete base. Some pavements of this type are very difficult to open. When the cement filling is of good quality the granite blocks often break before the cement loosens.

Fig. 22—Air Gun Cutting Asphalt

Fig. 22 shows an operator cutting asphalt pavement. With the wedge-shaped blade cutting at intervals as shown, small cracks are opened between the holes so that when cross cuts are made square blocks of asphalt can be readily lifted out.

The above illustrations contrast rather strikingly with the old methods of cutting pavements by means of sledges and bars as shown in Figs. 23 and 24.

In the use of the old manual method of cutting with sledges and bars there is always present a certain degree of hazard to the men. There is the possibility of the striker missing the steel and striking the holder's wrist, also the danger to the men's eyes from flying steel chips. These safety points, of course, are outside the labor saving considerations.

Fig. 23—Manual Method of Breaking Concrete. Contrast with Fig. 21

Fig. 24—Manual Method of Cutting Asphalt. Contrast with Fig. 22

While the labor saving is large in connection with opening street pavements, it is even greater in the work of drilling rock for blasting, where two men and a compressor can ordinarily do as much work in a given length of time as 35 to 40 men using hand methods.

In Fig. 25 is shown another interesting and efficient application of compressed air tools. Compressed air spades are being used to an increasing extent for loosening hard earth instead of doing this work by the usual pick method. A tool of this kind requires very little air and while this particular application is rather new, it is felt that further study may result in an appreciable saving over hand pick methods.

Fig. 25—Pneumatic Spade Replacing Hand Pick Method of Loosening Hard Soil

Compressed air can also be used to advantage in tamping back filled earth. Under certain conditions, however, it now seems that a suitable mechanically operated tamper will probably show greater economy on all except jobs in congested areas where the underground pipe interference is serious or where the trench or opening extends in a diagonal direction, thus often precluding the use of a rigid mechanical device.

The utilization of the portable air compressor is a comparatively recent development undertaken by the telephone companies in cooperation with one of the large air compressor manufacturers.

While the large capacity units have reached the stage where they give satisfactory operation, there is a field in the telephone business for a much more compact, lighter weight unit of lower capacity and cost, for such work as the opening of trench for subsidiaries, cutting frost, drilling rock for pole hole blasting, etc. With this in mind there has recently been developed in cooperation with an air compressor manufacturer, a type of compressor which is suitable for operating either one jack hammer for rock drilling or one tool for street opening with a corresponding capacity for other types of compressed air work. It is expected that the weight of this unit can through further study be reduced to such an extend that it will be practicable to mount it upon a Ford one-ton truck and still leave sufficient carrying capacity to handle the necessary guns, steels and hose for operating. Where there will be practically constant use for this lighter unit it may be desirable to mount it permanently upon the truck, while, in cases where the use will be intermittent, a very economical and convenient mounting can be made upon one of the Army type trailers.

CONCLUSION

In this article an endeavor has been made to cover in a very brief way some of the more important items of mechanical application which have a place in telephone construction work. The adaptation of mechanically operated tools and other devices to assist in the necessary manual operations will undoubtedly continue to occupy an important place in the work. Further study and development should result in many improvements in the present-day way of doing things which will make not only for marked economies of operation, but for greatly increased features of safety to the men engaged in constructing and maintaining the telephone plant.

A Method of Graphical Analysis

By HELENE C. BATEMAN

INTRODUCTION

IN connection with many telephone problems of an economic character, it is necessary to develop methods for making estimates and forecasts of the effects of changes in conditions. When the changes in conditions are such that direct experimentation is impracticable the development of logical methods and bases for estimates involves analyses of past experience in specific situations and, in so far as is feasible, the generalization of such experience. It is the purpose of this paper to describe briefly a graphical method by which complex economic data may be generalized for use in forecasting probable future conditions.

In some problems, it is necessary to determine the effects of changes in a specific situation, the results being applicable particularly to the given situation, and only very generally to other situations. The effect of a change in population upon station growth in a given exchange is an example of such a problem. In other problems, it is practicable to generalize experience so that the results of analyses may be applied, under proper conditions and limitations, to various specific situations. Moreover, it is often necessary to apply a general conclusion to a specific situation because no specific experience is available. An example of this type of analysis is the generalization of results of various rate treatments in different exchanges. In meeting this type of problem graphical methods are utilized to compare experience of a similar nature in various situations. The factors which may be indices of differences in conditions among various situations are studied to determine their relation to the differences encountered. Finally an attempt is made to derive quantitative relationships from the experience analyzed.

The assumption made in utilizing such methods is that the experience in different situations, from which generalizations are to be made, is *essentially similar* in certain respects, and that the variation in the quantitative unit to be estimated is due to varying conditions, as between the different situations, which may be measured in part by quantitative factors. There are, of course, certain types of problems where essential similarity between different situations does not exist or where it is difficult, if not impossible, to isolate quantitative factors sufficiently reliable to form a basis for estimates. On the other hand, there are many problems to which these methods may properly be applied and in which it is practically impossible to derive a reliable and satisfactory basis for making estimates without some such methods of analysis. Certain economic problems, in particular, because of the impracticability of experimentation and because the complex reactions of a group of individuals are involved, are not adapted to solution by the statistical methods which have proved useful in biometric sciences, but may be dealt with by graphical methods. This has been found particularly true in problems involving local telephone message use, and throughout the following discussion, illustrations are drawn from analyses of this type.

DATA

Since the ultimate aim of a graphical analysis of this type is to provide a basis for making estimates, the first step is to determine the estimates which will be required and the type of cases and conditions under which they will be used. In this way the aim and scope of the analysis is clearly defined. The unknown factor (the dependent variable) is to be estimated from certain known factors (independent variables). Various factors, quantitative and qualitative, which might logically appear to be indices of conditions controlling the dependent variable are, therefore, considered.[1] Only factors as to which data are available at the time and place where estimates are to be made are useful as independent variables. It is usually advisable to test a suggested factor by means of any data, even in small amounts, which may be available before a complete body of data is collected. Such preliminary investigations are useful in indicating the scope and detail in which data should be secured. In general the data should:

1. Be adequately representative of the type of cases for which estimates must be made,
2. Be adequate from a sampling standpoint for each situation,
3. Be as nearly homogeneous as practicable, i.e., cases having any outstanding peculiarities should be excluded,[2]
4. Include what appear to be the important factors or indices for each case.

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[1] It should be noted that such relationships need not be those of cause and effect. If two factors vary together (as do, for instance, different effects of a common cause) the values of the one which are hard to determine can be estimated from the more easily measured values of the other.

[2] For instance, if estimates are to be made for small exchanges, it would not be advisable to include data from large exchanges in the analysis.

PRELIMINARY ANALYSIS

After the data have been collected and summarized in accordance with the general plan of the study, the graphical phase of the analysis begins with trial setups in which the dependent variable is plotted against each of the independent variables in turn. Such charts are intended only to give a general idea of the types of relationships and to determine which of the factors tested are most closely related to the dependent variable. Factors which do not vary with the dependent variable are not necessarily to be discarded permanently since the effect of one factor may obscure that of another. It is not to be expected that the data plotted on any of these charts will fall along smooth curves. They will probably be widely scattered but in the case of the more important factors a general trend is usually evident.

On the next series of trial charts, several of the more important factors are considered simultaneously. If a qualitative factor is under consideration, separate charts are plotted for the different classes. If these charts are essentially similar, the qualitative factor may be disregarded for the time being and the data considered as a whole. If, however, the qualitative factor appears to influence the relationships in a logical manner the data must be sub-divided and a number of practically independent studies carried on. In fact, the analysis of the effect of a qualitative factor is intended to determine whether or not the data forms an essentially homogeneous whole. If there is a discontinous variable, it is often convenient to hold it constant, i.e., a separate chart may be plotted for each value or group of values of this factor. The factor, which from the preliminary charts, seems most important is usually plotted against the dependent variable. One or two other factors are coded. The codes may be either in colors or symbols or both. The color codes are usually the more easily distinguished and are, therefore, the better for working charts. For final charts, however, color codes are not usually practicable because of the difficulties of reproduction. Both colors and symbols may be used when two coded factors are to be tested simultaneously.

In these preliminary sets of charts, it is well to test as many different factors and combinations of factors as appear logically to vary with the dependent variable. It is usually best, however, to consider not more than three or four independent variables at a time, one plotted against the dependent variable with one or possibly two coded and one held constant on each chart. An attempt to hold constant a greater number will often sub-classify the number of data points so far as to obscure the real trends. Furthermore, the complexity of charts increases rapidly with the inclusion of more variables and makes the analysis and estimating complex and cumbersome.

Fig. 1 is a typical preliminary trial chart from a study of average telephone message use under message rate service. Each data point represents one class of service in a particular exchange. The independent factors taken into account are:

1. Major Service Classifications [3]—held constant since this chart is for one class only.
2. Rank of Service [4]—plotted.
3. Message Allowance—coded.

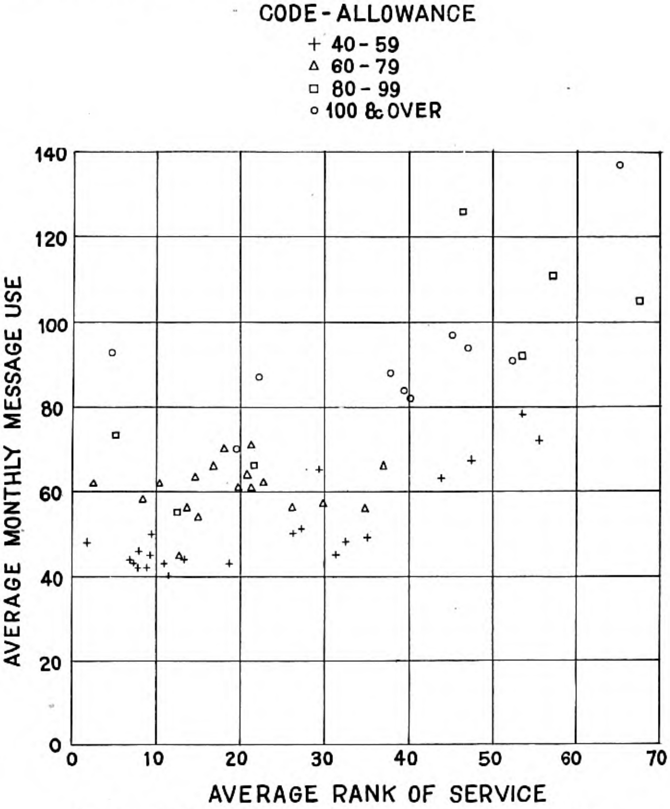

Fig. 1—Preliminary Chart Involving Three Variables

The trend of the relationships between message allowance, rank of service and average message use is fairly well defined on this chart.

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[3] Business Main Station, Residence Main Station, P. B. X., etc.

[4] A statistical index indicating the relative ranking of subscribers in accordance with their demands for service.

When several different sets of charts such as are described above have been scrutinized, definite trends will usually be fairly clearly established. It will often be found that while these trends are well defined, nevertheless a number of points may scatter widely. Such points are studied carefully. If, after the original data are checked, the points are found to be correctly plotted, each case is investigated in detail to account for the observed divergence. Sometimes it will be found due to a factor which has not been taken into account, the inclusion of which will often improve the results of the study as a whole. On the other hand, peculiar local conditions or history may give rise to such divergence. These cases are not really a part of the similar group under consideration. If they are sufficient in number and similar with respect to each other they may be studied independently. If not, they are either excluded entirely or given slight weight in the general study. Because of wide differences in problems and material, it is not practicable to describe in detail the process of analyzing such preliminary charts in arriving at decisions as to data and process.

CURVE DRAWING

The next step is the construction of curves through these data which will truly represent the relationships involved. This can be facilitated by plotting the average values of the dependent variable for all cases having the same values (within certain limits) for all the independent variables.

> On Fig. 2 the data points are the same as those plotted on Fig. 1. The closed symbols which have been added are average points representing the data points of the same symbol. The abscissa of each average point is the mid-point of an interval of rank of service (0–20, 10–30, etc.) and the ordinate is the average of the message uses of all the points falling within that interval.

The average most often used on such charts is the median not only because it is most easily located but because it is usually the most representative, giving little weight to extreme cases. Whatever average is used, it is well to make it a moving average, i.e., covering overlapping intervals such as 20–30, 25–35, 30–40, etc., rather than 20–30, 30–40, 40–50, etc.

These averages serve as a guide for drawing the preliminary curves through the data but the actual data points are considered at the same time. In constructing the curves, the significance to be attached to any data point depends chiefly on:

1. The number of individual cases on which it is based.
2. The probable degree of accuracy of the data.

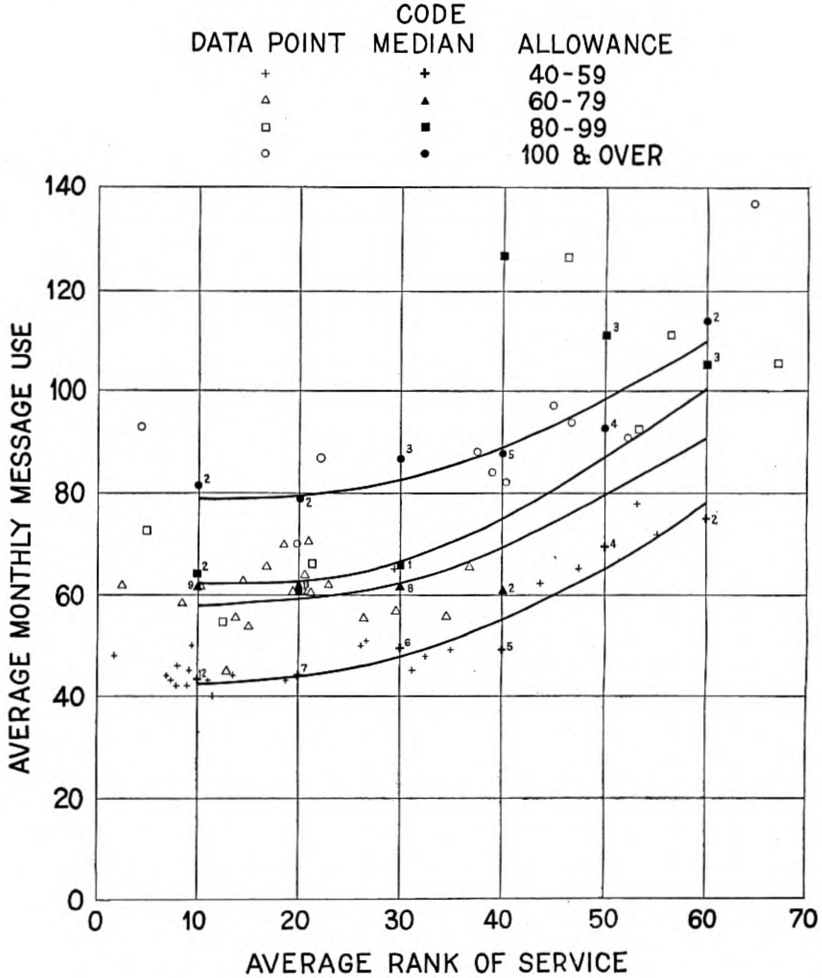

Fig. 2—Preliminary Chart With Averages and Curves

Since these characteristics are considered simultaneously it is usually advisable to depend on judgment using the averages as a general guide, rather than to rely on any formal mathematical system. The first set of curves is drawn to fit the data as closely as practicable and still be *logical* and *consistent.*

> On Fig. 2, the number of data points on which each average is based has been noted as an aid to judgment and a set of rough preliminary curves has been drawn. These, of course, are not necessarily the most accurate curves which could be constructed from these data. A method of progressing to final curves is described below.

CURVE SMOOTHING

The first set of curves constructed from the data may not be an entirely consistent and reasonable family. The relation between different curves on the same chart or between different charts indicates the influence of factors other than the one plotted and must, therefore, be made consistent and logical. The process of transforming the preliminary curves into the final normals is known as smoothing.

The original curves are first studied for reasonableness. Their general shape (whether straight line, convex or concave, having maximum or minimum points, being asymptotic to a certain line, etc.) is, in so far as practicable, determined on logical grounds. If a large majority of the curves, or the curves based on the greatest amount of data, have a certain clearly defined trend, the remainder of the curves are made to conform to this trend, if it is reasonable, at the same time keeping as closely in line with the data as possible.

Each chart will usually have one independent variable plotted against the dependent variable and another independent variable coded. Each curve, therefore, indicates the relationship between the dependent variable and one independent variable for a certain constant value or range of values of a second independent variable. If the relative positions of the curves of a family on one chart are adjusted, the relationship of the coded variable to the other two is altered. The effect of this alteration may be seen by plotting the coded variable against the dependent variable and coding the one which previously was plotted, all values being read from the preliminary curves. This is sometimes called cross-sectioning. The families of curves formed by cross-sectioning are then smoothed until they are reasonable and consistent. In doing this, the original curves are automatically departed from, and when the original curves are replotted from the cross-sections, it may be found that the resulting family of curves is not smooth, consistent or reasonable.

The smoothing process must, therefore, be repeated back and forth a number of times until both sets of curves appear to be smooth, reasonable and consistent families. During this process, it is important that the various families of curves be tested against the original data. If this is not done, it may happen that a series of small changes will accumulate in such a way as to bring portions of the curves outside the limits of the original data. Furthermore, the factor or factors held constant on each chart must not be lost sight of. These factors should be plotted against the dependent variable holding constant

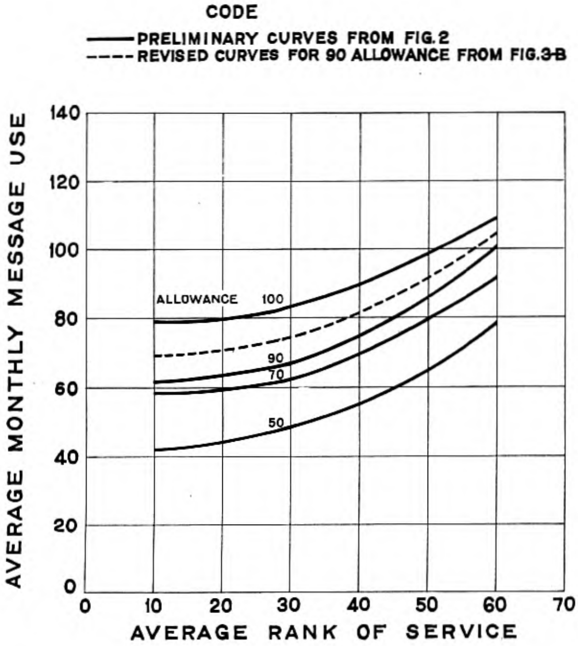

Fig. 3A

all other factors (values being read from the smoothed curves) to see that these relationships also are being made reasonable, consistent and smooth.

The process of smoothing described above is a long and laborious one involving at every step the making of special decisions based upon knowledge of the data and the logic of the situation with regard to the particular problem. Various methods of facilitating the work have, however, been devised some of which are described below. Figs. 3A and 3B illustrate the advantage of having both sets of curves on the same chart with the same scale for the dependent variable so that when the smoothing process is applied to one family, the effects on the other may be more readily ascertained.

The curves on Fig. 3A are the rough curves which were drawn through the data on Fig. 2. Fig. 3B shows the cross sections of these curves, message allowance being plotted and rank of service coded. It is evident that neither set of curves is a consistent family. Most of the curves of Fig. 3B are irregular instead of being smooth. They might be smoothed considerably either by lowering the points corresponding to a 70 message allowance or by raising those for an allowance of 90 messages. A study of Fig. 3B shows that the curves at 90 allowance are further apart than at any other point. This might be used as an argument either for raising the lower points or lowering the higher. In scrutinizing the data, however, it is found that of the classes of service having from 80 to 99 allowance all have allowances of either 80 or 83. Therefore, the midpoint (90 allowance) is too high to represent the group, or, conversely, the message use plotted is too low for an allowance of 90 messages. In order to have a guide in the amount of shifting necessary, data points for the actual message allowances of the 80–99 group have been plotted

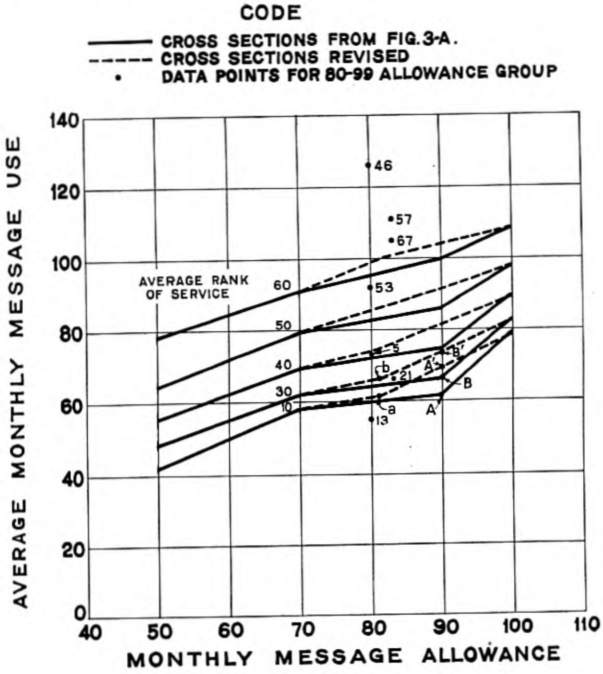

Fig. 3B

on Fig. 3B, and the values formerly entered at 90 (points A, B, etc.) entered at 81½ (points a, b, etc.). With these points and those at 100 as a guide, new values for 90 allowance have been estimated (points A′, B′, etc.). The shifting of a point up or down on Fig. 3B results in shifting the corresponding points of the other family (Fig. 3A) the same distance in the same direction resulting in the dotted curve. There is much more smoothing necessary to make Figs. 3A and 3B satisfactory and reasonable, but by proceeding in the manner just described, taking into account the appearance of the curves, the logic of the situation and the original data, a smooth and consistent family of curves can finally be evolved.

Another excellent method of smoothing involves the use of a three dimensional figure. Just as a plane surface gives a complete representation of two variables and a partial representation (by coding) of a third, so a three dimensional system can be used to give a complete representation of three variables, and a partial representation of a fourth. It also aids greatly in smoothing simultaneously. A device for three dimensional representations consists of a plane surface marked off with rectilinear coordinates and having at frequent intervals holes into which pegs can be set. The pegs also have coordinate markings. The values of two variables, then determine the point at which the peg is set and the value of the third determines a distance along the peg. The point is marked by a small rubber ring which fits around the peg. The values of a fourth variable may be coded by using rings of different colors. When the device is used for smoothing curves involving only three variables, the data points may be indicated in one color and the smoothed values in another. The data points, remaining constant as the smoothed curves are shifted, form a continuous check on the divergence of the smoothed curves from the data. This is an automatic process of cross-sectioning. When the position of a point is changed, the effects of the change on the various relationships are seen by studying different aspects of the setup. This device gives the best results with discontinuous variables (such as message allowances, rates, etc.) as the pegs can then be set in at regular intervals without resorting and regrouping the data. It is also especially valuable when one of the variables is a complex factor (such as distribution of development among more than two classes of service) which cannot easily be represented by one curve.

Fig. 3C illustrates such a setup with the revised curves of Figs. 3A and 3B. The independent variables, message allowance and rank of service are represented by the rectilinear coordinates of the plane surface. Message use is indicated by the rings on the upright pegs.

In general, the various steps in the analysis leading up to the final[5] or normal[6] relationships require continuous exercise of judgment. The problem is never one of securing simply curves of "best fit" to the data. It is broader, more fundamental and much more involved than this. It requires a combination of logic with the data that results in normal relationships which fit the data and at the same time are reasonable. It is necessary to consider such questions as the following: Why do the data indicate this relationship? As a generalization, is such a relationship reasonable? What should be the character of this relationship? Should it be a straight line, concave up or concave down? Particular attention is given to the reasonableness of maxima or minima points and to points of inflexion when indicated by the data. It is only by considering such fundamental questions that a sound basis can be established for building up normal rela-

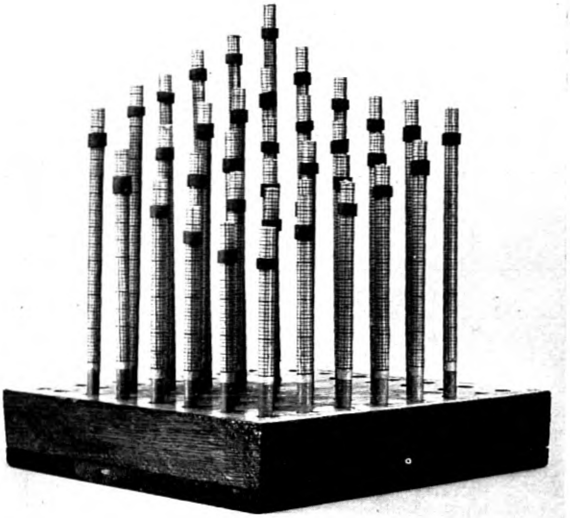

Fig. 3C

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[5] Final in a relative sense. In economic studies of this type involving human reactions and relationships normal relationships are never final in an absolute sense.

[6] The term "normal curve" is used throughout this paper to designate a final curve from which estimates are to be made. A normal distribution curve in this sense may or may not be "normal" in the statistical sense of an evenly balanced bell-shaped curve.

tionships which will be a true generalization of experience. The importance of dealing with economic problems in this way can hardly be over emphasized. It is a recognition of the complexities which are inherent in problems involving human reactions and the dangers of untrue generalization if rigorous and more or less inflexible methods of analysis are utilized.

FINAL RESULTS

The result of the smoothing process is the development of a consistent series of charts and curves by means of which the value of the dependent variable may be estimated from the values of the various independent variables.

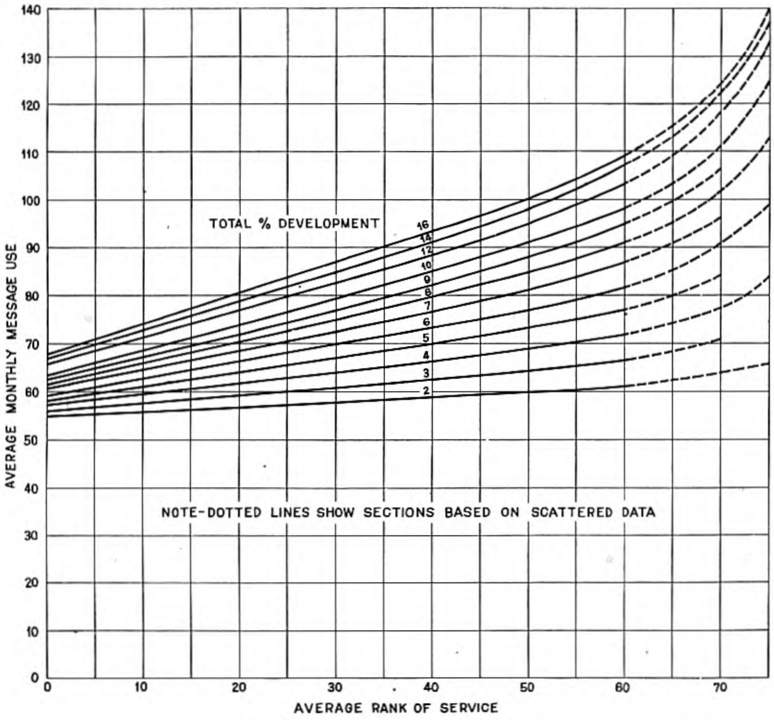

Fig. 4—Final Family of Curves

Fig. 4 is an example of such a chart for estimating average message use. Charts of this type may be used under proper conditions for estimating either an actual value which is unknown (such as average message use under an existing rate schedule) or the value which may be expected to result from some change (such as average message use under a proposed rate schedule).

After deriving a series of final charts estimates are made of the value of the dependent variable for all the cases on which the study was based. Consideration of the differences between the estimated and the actual values is an excellent general criterion of the accuracy of the normals. In general, the positive deviations should be approximately equal to the negative both in number and in the sum of their numerical values. If either positive or negative deviations are decidedly predominant, it is probable that the general level of the normal curves is too low or too high.

When the deviations (without regard to sign) are plotted as a frequency curve, the curve should be fairly smooth. It need not be and usually is not a bell shaped curve, but if there are sudden and decided breaks, it is probable that either certain portions of the data have not been given proper consideration or that the data were not originally essentially homogeneous. The cumulative frequency curve based on the deviations makes possible the easy reading of the median or probable error of estimate. The probable error may be used as a general criterion of the value of future estimates made from these normals and the ratio of the probable error to the median value of the dependent variable forms a basis for comparison of the relative accuracy of different sets of normals.

The deviations (sign being taken into account) when plotted against the various factors included in the study should be fairly evenly scattered and show no trend or relationship. If a consistently occurring variation is discovered between the deviations and any of the independent variables it indicates that the relationship of that variable to the dependent variable has not been properly taken into account in deriving the normals. If this variation appears in connection with the dependent variable, it indcates that some of the curves are not of proper shape. For instance, if a straight line is fitted to data having a decided non-linear trend, the errors plotted against the dependent variable will fall along a well defined U-shaped (or inverted U-shaped) curve.

Additional information may also be obtained by plotting the deviations against factors not included in the study. Relationships will sometimes become apparent which previously were obscured by the effect of the more important factors. The influence of such factors may account for seemingly abnormal cases and their inclusion would tend to reduce the mean and to a lesser extent the median deviation.

FREQUENCY DISTRIBUTIONS

Normal curves, such as those described above, form a basis for estimates of an *average value* for a group of items comprising a unit such as has been utilized in developing the study. In many instances, however, it is necessary to know not only the average value but also the distribution of items about that average.

> Thus, the normal curves of the type of those shown in Fig. 4 serve as a basis for estimating the average message use of all subscribers to a given class of measured service in a given city. Additional curves are, however, required for estimating the distribution of subscribers by message use.

The basic principles governing the derivation of normal curves are the same whether these normals be concerned with averages or with distributions. The detailed methods involved are, however, quite different because of the inherent differences in the material. An average can be expressed in one arithmetic term which can be plotted against other factors. A distribution, on the other hand, is a complex entity which may itself be expressed as a curve but which obviously cannot be measured by an index to be plotted against other variables without losing sight of certain detailed characteristics of the distributions. The procedure and methods described above for deriving normal curves are modified somewhat in the derivation of normal distribution curves. Some of the methods which have been found advantageous for these analyses are described below.

The first step in the analysis is usually to plot the actual detail and cumulative distributions for each group of items and to compare the various distributions in order to determine points of similarity or difference. For purposes of comparison, percentage distributions are used, i.e., the per cent. of total items rather than the actual number occurring in each interval is plotted. With homogeneous material it will usually be found that when plotted to the same scales the detail frequency curves are all of the same general shape but differ in three primary characteristics.

1. The spread or extent of variation.
2. The location of the peak or point of maximum frequency.
3. The concentration of items in the peak interval.

These characteristics are, however, interrelated and to a certain extent related to the average.[7] Other things being equal, it might be expected that:

1. The greater the average the greater the spread.
2. The greater the spread the less the concentration at the peak.
3. The higher the peak the nearer it will fall to the average.

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[7] Throughout this section the term "average" is used in the sense of arithmetic mean.

Since the average is of much importance in determining frequency curves, it will usually be found that differences will be reduced if the curves are plotted with each interval of the horizontal scale as a per cent. of the average instead of an actual value. For example:

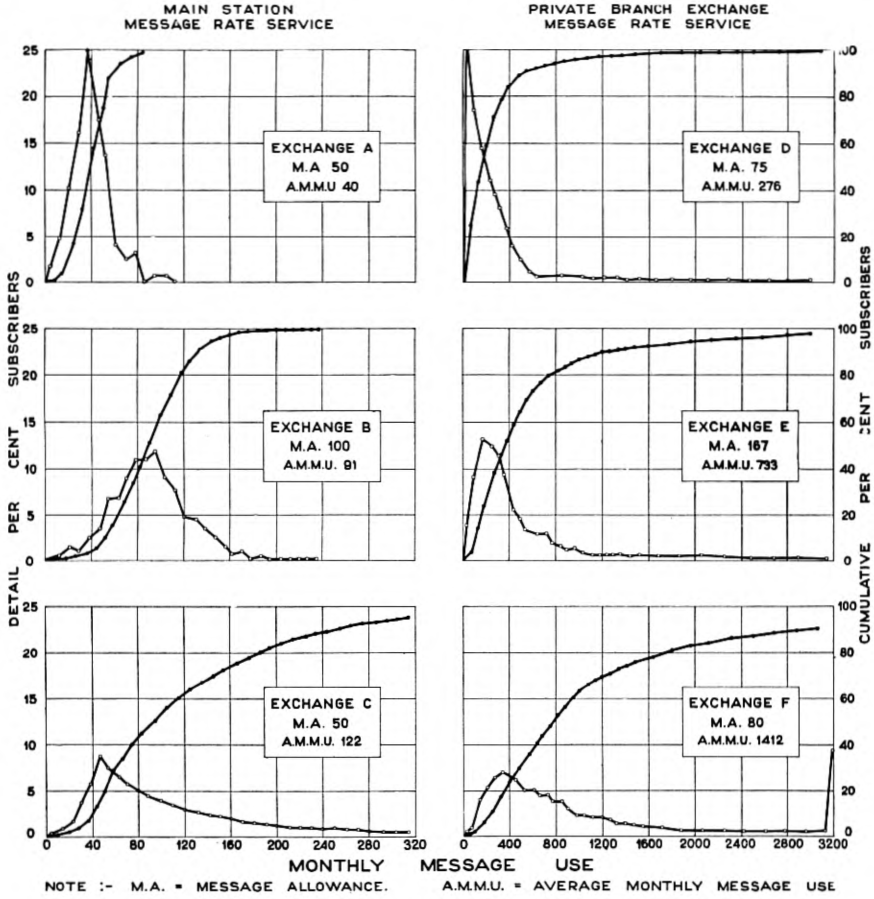
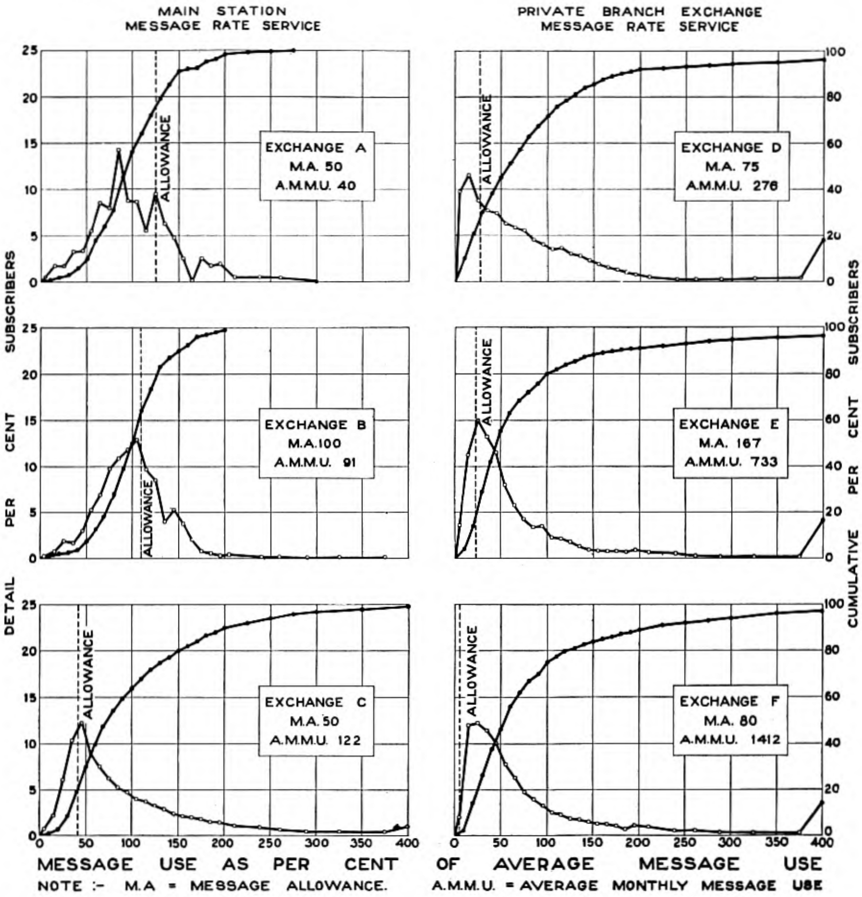


Fig. 5—Sample Distributions of Subscribers by Absolute Message Uses

Fig. 5 illustrates distributions of subscribers by message use plotted in terms of actual values for different classes of message rate service. On each chart the average message use is indicated. It will be noted that, in general, the greater the average message use, the greater the spread of the curves, the less the concentration at the peak interval and the less marked the correspondence of the peak with the average value. On Fig. 6 the frequency curves have been replotted using instead of each actual message use interval the per cent. that the message use is to average message use. The result of this statistical process is to make the spread of the curves and the height of the peaks much more nearly uniform.


MAIN STATION MESSAGE RATE SERVICE

PRIVATE BRANCH EXCHANGE MESSAGE RATE SERVICE

NOTE :- M.A = MESSAGE ALLOWANCE. A.M.M.U. = AVERAGE MONTHLY MESSAGE USE

Fig. 6—Sample Distributions of Subscribers by Message Uses in Per Cent. of Average

In some instances, the frequency curves plotted with each interval expressed as per cent. of the average may be so similar for the different groups that satisfactory normals may be derived from this setup alone without including any other factor. This appears to be the case for the distribution of P. B. X. subscribers by message use as illustrated on Fig. 6. It is necessary, however, to test whether or not the full effect of the average on the distribution has been eliminated. This may be done on a detail basis by plotting a series of charts showing the relation between the absolute amount of the average and the per cent. of cases falling within a given message use interval (expressed as per cent. of the average). On a cumulative basis the per cent. of cases falling below a given per cent. of the average is plotted against the average. For example:

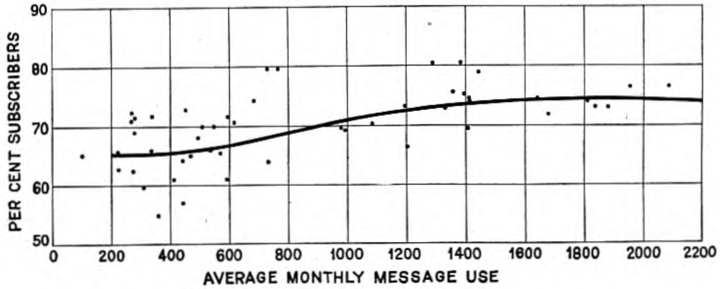


Fig. 7—Preliminary Chart of Cumulative Series

Fig. 7 shows the relationship between average message use and the per cent. of subscribers using less than 100 per cent. of the average message use for a given service classification. Each plotted point represents the reading from the cumulative curve for a different exchange. It is evident that the two factors vary together.

Curves similar in type to that shown in Fig. 7 are constructed on each of the charts of the detail and cumulative series. The curves of each series are smoothed by cross-sectioning and developed into consistent and reasonable families.

In connection with the smoothing of the cumulative series a method described below has been found useful. This method can be used with any setup of three variables but is simplest in setups of a cumulative type which have no maxima or minima within the limits of the curves. To simplify the explanation of the method the cumulative distribution of certain subscribers by message use is referred to as an example.

Fig. 8 shows preliminary curves representing the relationship between average message use and the per cent. of subscribers using less than the various per cents. of the average message use from 10 per cent. to 500 per cent. of the average. These curves are derived from a series of charts similar to Fig. 7 for different message uses. Cross sections of the family of curves of Fig. 8 give a series of cumulative curves. The successive curves of this cumulative series have been plotted in Fig. 9 at regular intervals apart, the intervals being the same distance as the average monthly message use scale of Fig. 8. The horizontal scale of Fig. 9 used in plotting these cumulative distributions must therefore be movable so as to apply in turn to each of the cumulative curves. With the cumulative curves plotted, the family of curves on Fig. 8 have been drawn in on Fig. 9, the curves representing the various message uses being exactly the same as those of Fig. 8 except that the method of drawing the cumulative curves has automatically shifted successive curves of Fig. 8 further and further to the right. It follows from the methods which have been used in constructing Fig. 9 that any given cumulative curve must intersect each curve of the other family somewhere on the vertical line corresponding to the message use (expressed as per cent. of the average message use) represented by that curve. For instance, the cumulative curve for 1000 average message use (indicated by A) must intersect the curve representing 30 per cent. of the average message use on the vertical line corresponding to a co-

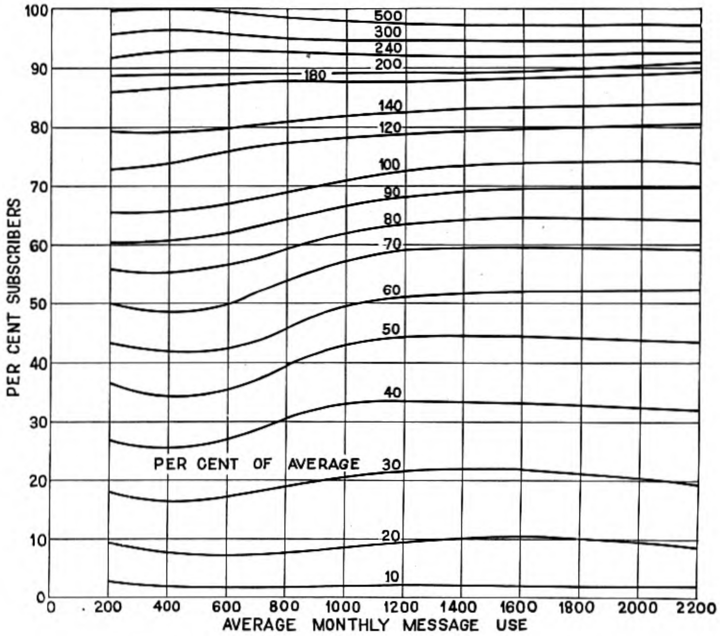


Fig. 8—Preliminary Family of Cumulative Curves

ordinate of 30 on the horizontal movable scale when the zero point of the horizontal movable scale falls at 1000 average message use on the fixed horizontal scale. The point of intersection described in this illustration is indicated on Fig. 9 by P. This characteristic (intersections falling on given vertical lines) forms the basis for smoothing the two families of curves simultaneously. A point of intersection may be shifted vertically but cannot be shifted horizontally since it must fall somewhere on a definite vertical line. Dashed lines (- - - -) on Fig. 9 indicate the manner in which a few of the points have been shifted in smoothing.

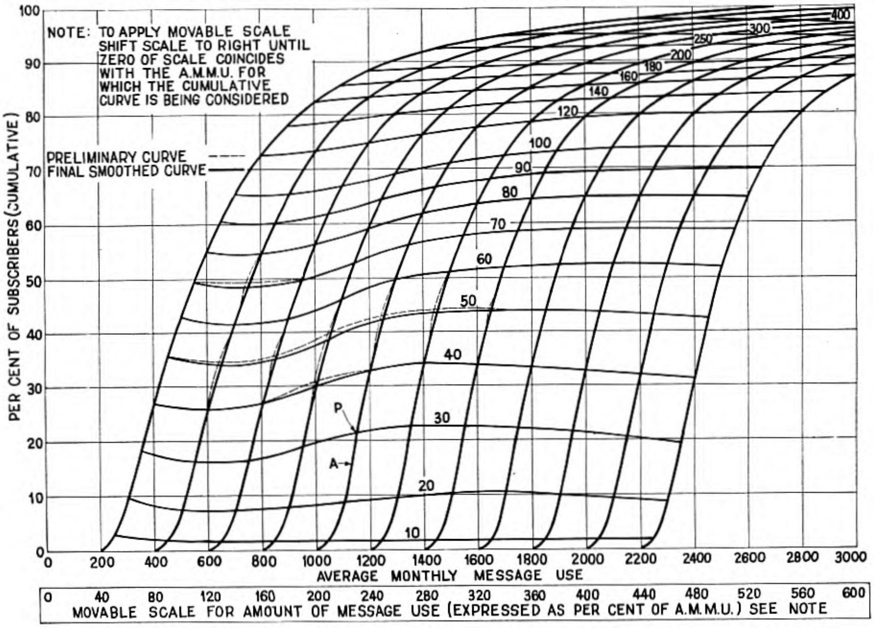


Fig. 9—Simultaneous Smoothing

A family of cumulative curves may appear easier to smooth than the corresponding family of detail curves. On the other hand, the detail curves give, in some respects, a more vivid picture of the outstanding characteristics of the distributions than do the cumulative, and certain important characteristics of the distributions may be more easily studied on a detail basis.

It is important, therefore, that both series be taken into account in deriving final normal distribution curves. For the detail series, charts are plotted showing the relationship between the amount of the average and the per cent. of cases falling in a particular interval (expressed as per cent. of the average). Fig. 10 is such a chart for the interval 80–90 per cent. of the average message use.

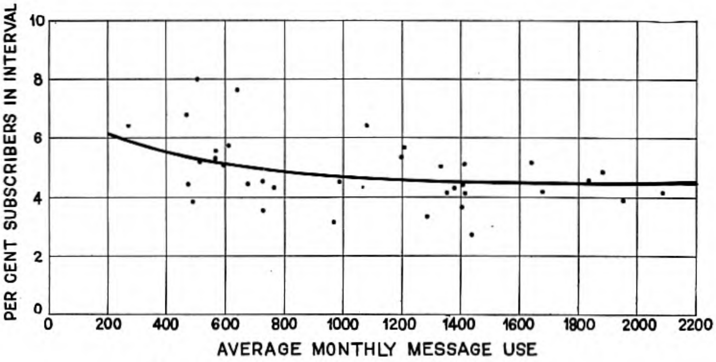


Fig. 10—Preliminary Chart of Detail Series

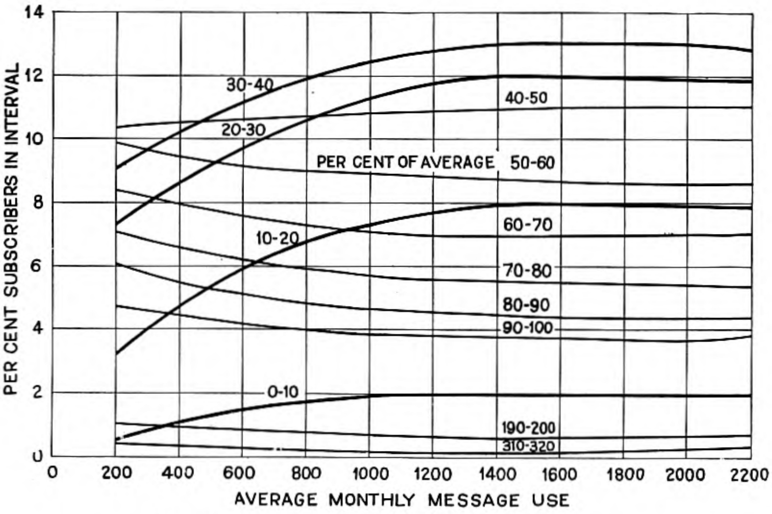


Fig. 11—Preliminary Family of Detail Curves

Cross-sections of a family of curves such as those on Fig. 11 give a series of detail frequency curves. Further smoothing may be facilitated by a study of these curves. As an aid in this process of smoothing it is desirable to determine the normal location of the peaks and the normal proportion of cases occurring in the peak interval, as these are important characteristics of such curves. These normal values may be determined by plotting these factors against the average as shown on Figs. 12 and 13. For this data, it is noted that, on an absolute message use interval basis, the greater the average the greater the abscissa of the peak value. However, with an increase in the average, the abscissa of the peak interval on an absolute basis does not increase as rapidly as does the average, because large users increase their usage relatively more than small users. Therefore, when the intervals are plotted in terms of per cent. of the average, it will be found that the greater the average the less the abscissa of the peak. For the same reasons, the peak interval expressed as per cent. of the average becomes relatively wider as the average increases, and the height of the peak increases as illustrated on Fig. 13. With

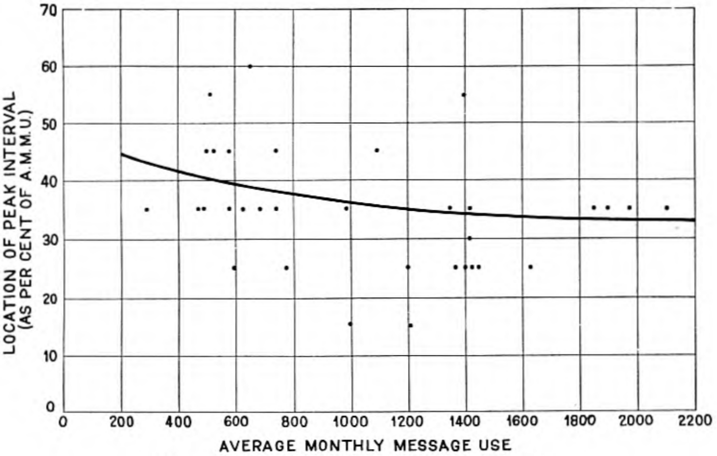


Fig. 12—Determination of Normal Peaks

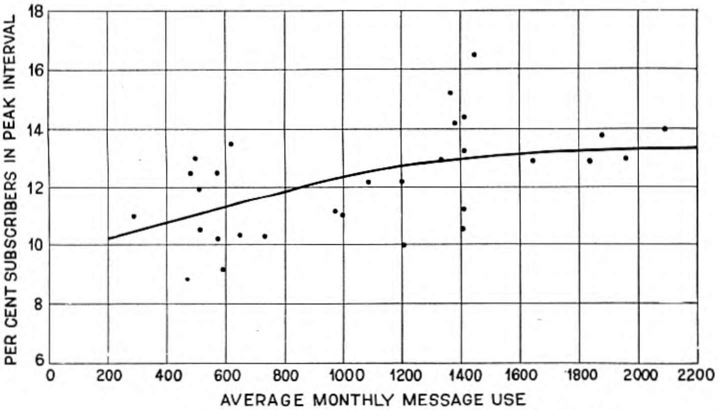


Fig. 13—Determination of Normal Heights of Peaks

the location and height of the peaks normally determined, the process of constructing preliminary curves for the various intervals of the detail series is, in many cases, considerably simplified. These preliminary curves are then cross-sectioned and smoothed into a consistent family.

Finally the smooth curves of the cumulative and detail series are checked with each other and averages are computed from these curves as a check against the assumed average. When the minor discrepancies disclosed by these checks have been corrected, normal curves are plotted and comparisons are made with the actual distributions. Further adjustments may then be necessary.

ADDITIONAL FACTORS IN DERIVING NORMAL DISTRIBUTION CURVES

The smoothing processes described above give a series of normal[8] distribution curves taking into account completely the effect of the amount of the average upon the distribution. In some cases, however, it will be found that some outside factor has also a decided effect upon the distribution.

When the effect of an outside factor is apparent it may be necessary to derive a series of normal distribution curves, each curve corresponding to a constant value of the factor under consideration. If this is done, the curves are smoothed by cross-sectioning and the various other methods described above so as to form a consistent and reasonable family. The type of the final family derived will, however, depend largely upon the character of the relationships developed during the smoothing process. For instance, in the case of main station message rate service, a series of distribution curves was plotted, one for each message allowance. In the course of smoothing these curves it seemed reasonable that there might be a relationship between the type of distribution and the proportional relationship of average message use to message allowance. That is, with an annual message allowance of 600 and an average annual message use of 400 the distribution of subscribers by amount of message use might be similar, on a proportional basis, to the distribution of subscribers under an annual message allowance of 900 with an average annual message use of 600; or under an annual message allowance of 1,200 with an average annual message use of 800. This idea was tested by use of the various sets of normals which had been derived for the different message allowances and was found to hold so closely that this pro-

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[8] See Note 6.

portion factor (ratio of the average message use to the message allowance) might be used in deriving a revised setup of normal distribution curves.

In certain cases it may be found that some expedient such as that described above may be used to eliminate or take account of the effect of an outside factor. Whether this is done or a separate set of curves is derived for different values of that factor, the process of deriving the detail and cumulative distribution curves would in general be the same as that described above.

Some of the processes involved in studying averages and distributions of subscribers by message use have been described because they are typical and illustrate what have been found to be satisfactory methods of analysis for problems of this type. It is clearly impossible, however, to set up any rigid methods for such studies. Any economic problem which permits of analysis by these methods must be treated in the manner best suited to the data available, the purposes of the study, the degrees of accuracy necessary, etc. Where these methods can logically be employed the results obtained, an important part of which are the background and sidelights, on the problem, disclosed during the process of building up the normal relationships, will generally be found superior to those obtained through the use of more rigorous methods.

APPLICATION

Before final results are obtained, there will naturally be developed by those concerned in the study a very definite conception of the field of their usefulness and their limitations. It is important that a knowledge of these limitations be extended to those who may have occasion to use the results. Given a set of smooth curves from which quantitative estimates can be made, there is a great temptation to make estimates under any and all circumstances, and often to give such estimates an undue appearance of accuracy. The final results are merely the general expression of the information contained in the original data logically developed according to the knowledge and judgment of the investigator. It is always necessary in applying such results to consider the effect of special and local conditions. Where it is known that actual conditions in a specific case are far from normal, it is often possible to estimate the effect of a proposed change by applying differences based on the normal experience.

Care must also be taken in extrapolation estimates, i.e., estimates where the value of one or more factors lies beyond the limits of the original data. Such estimates, of course, are always subject to considerable error. In other cases it may happen that some part of the data necessary for making a complete estimate is not available. It may be practicable, however, to approximate the required information and make a rough estimate which may be more accurate than the alternative of basing the estimate upon less complete analysis.

In applying the results of such analyses, satisfactory conclusions can be reached only if due consideration is given to the following points:

1. The quantitative readings from the normal curves.
2. All the qualitative relationships developed in the course of the analysis.
3. Any additional data available for the particular case or cases in question.
4. Any peculiar special conditions known to exist for that case or which probably exist because of comparison with similar cases.
5. Changes affecting general levels since the date of the study.

It follows that the making of such estimates cannot be left in inexperienced hands any more than can the progress of the original study. Good judgment and a complete knowledge of the problem are of paramount importance both in making the general analysis and in the application of results to specific problems.

To those accustomed to working in the more exact fields of physics, chemistry, etc., it will undoubtedly appear that the methods described above may be inexact and unsatisfactory. Undoubtedly, the average errors of estimate are considerably greater than would be allowed in fields where more exact data are obtainable. Yet the reason for this lies rather in the material itself than in the methods of dealing with it. An economic quantity is extremely complex and difficult to estimate because it is usually dependent upon the action of hundreds or thousands of individuals each one of whom is influenced by individual needs and desires which at best can only be partially measured by such quantitative factors as reflect variations in these needs or desires. Estimates of such quantities are necessarily subject to a relatively high degree of error if comparisons are made with the fields of physical science. Yet such estimates must be made and the problem is to make them as accurately as practicable. Judged from this standpoint, experience indicates that such analyses are an important aid in connection with certain phases of many economic problems.

Permalloy, A New Magnetic Material of Very High Permeability

By H. D. ARNOLD and G. W. ELMEN

SYNOPSIS: The magnetic alloy described in this paper is a composition of about 78.5% nickel and 21.5% iron and at magnetizing fields in the neighborhood of .04 gauss and with proper treatment has a permeability running as high as 90,000. This is about 200 times as great as the permeability of the best iron for these low magnetizing fields. This high permeability is attendant upon proper heat treatment and also upon other factors among which is freedom from elastic strain. The presence of other elements than iron or nickel and specially carbon, reduces the permeability, but slight variations in heat treatment produce large changes compared with those due to small quantities of impurities.

So far as discovered, other physical properties show no peculiarities at the composition which brings out the remarkable magnetic properties of permalloy. The equilibrium diagram, electric conductivity, crystal structure, mean spacing between adjacent atom centers and density are among the physical properties which have been studied.

To the engineer in electrical communication the development of permalloy is very significant. It assures a revolutionary change in submarine cable construction and operation and promises equally important advances in other fields.—*Editor.*

SOME time ago it was discovered in the Bell System laboratory[1] that certain nickel-iron alloys, when properly heat-treated, possess remarkable magnetic properties. These properties are developed in alloys which contain more than 30 per cent of nickel and which have the face-centered cubic arrangement characteristic of nickel crystals, rather than the body-centered structure characteristic of iron. The entire range above 30 per cent nickel exhibits these properties to some degree and offers new possibilities to those interested in magnetic materials. The most startling results, however, are obtained with alloys of approximately 80 per cent nickel and 20 per cent iron, whose permeabilities at small field strengths are many times greater than any hitherto known. To alloys of this approximate composition we have given the name "permalloy". The development of permalloy has assured us a revolutionary change in submarine cable construction and operation, and promises equally important advances in other fields of usefulness. It also presents questions of great interest to the scientist, and emphasizes again the meagreness of our fundamental information about ferromagnetism. The present paper is intended to give a general discussion of the preparation and testing of permalloy, with sufficient detail to indicate its unusual characteristics. Detailed statements of numerical results are reserved for publication in separate articles dealing with specific properties.[2]

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[1] British Patent No. 188,688.

[2] L. W. McKeehan, The Crystal Structure of Iron-Nickel Alloys, Phys. Rev. (2), 21, (1923).

In making permalloy we use the purest commercial nickel and Armco iron. Our samples for laboratory study are prepared by melting these metals in a silica crucible, using a Northrup high-frequency induction furnace. The particular furnace which we use will conveniently melt a charge of about six pounds. An analysis typical of the resulting billets is as follows:

Ni	78.23
Fe	21.35
C	.04
Si	.03
P	trace
S	.035
Mu	.22
Co	.37
Cu	.10

The presence of other elements than nickel and iron is of course to be expected after any practical method of preparation. To determine their effects, samples were prepared in which the usual impurities were present in various proportions. It was found that their presence does affect the permeability of the alloys and that carbon is especially harmful. Since, however, the variations produced by slight changes in heat-treatment are very large compared with those due to small quantities of impurities we have found it unnecessary for most purposes to require higher purity than that indicated in the analysis above given.

In our laboratory studies we have made it a practice to reduce the billets through the forms of rod and wire to tape 3.2 mm. wide and 0.15 mm. thick. Accordingly test samples are available in a variety of forms and conditions. Thin narrow tape is particularly adapted to use in experiments involving heat-treatment, since it possesses a high ratio of area to volume and is easy to manipulate. Fortunately the entire nickel-iron series can be mechanically worked if sufficient care is exercised and we have thus been able to use samples of the same size, shape, and mechanical condition in all measurements upon which we have based comparisons between alloys. This practice has also made possible strictly comparable micrographic studies throughout the series.

Permeability is the magnetic characteristic of permalloy in which we first became interested and we have used its numerical value as an index in establishing the effects of mechanical and thermal treatments. Most of the measurements have been made in a ring permeameter of special design. The ring sample is prepared by winding twenty or more turns of tape around a disk about three inches in diameter. The disk is then removed leaving the material in the form of a spirally laminated ring with a rectangular cross-section approximately 3.2 mm. by 6 mm. A single massive copper conductor is linked with this ring, and constitutes also the secondary of a transformer whose primary winding forms one arm of an inductance bridge. From the bridge measurements, and the dimensions of the ring the permeability of the latter may readily be computed. For most of the measurements 112-cycle alternating current has been employed, permitting the use of telephone receivers in adjusting the balance of the bridge. The ring is sufficiently well laminated so that no serious troubles are introduced at this frequency by eddy currents. This fact was verified by making a number of permeability determinations at alternating current frequencies both above and below that chosen for routine use, and also by comparing the results of ring permeameter tests with those of ballistic tests on specially wound ring samples. The bridge method is particularly well adapted to the measurement of permeability in very weak magnetic fields since amplifiers may readily be used to increase the delicacy of the bridge adjustment to almost any degree desired. As a matter of convenience we have usually included in our test program measurements with fields of 0.002, 0.003, and 0.010 gauss, and on the graph of permeability against magnetizing field strength the straight line through these points has been extended to field strength zero. We have called the permeability read from the graph at this point the "initial permeability" of the sample.

The form of permeameter used is especially adapted to making measurements quickly and with minimum handling of the sample, since it makes use of but a single magnetizing turn. The ring is laid on suitable insulating supports in an annular copper trough, and placing the copper cover on this trough completes the electrical circuit. In a modified instrument, the "hot permeameter", provided with a heating device, permeabilities may be measured from liquid air temperatures up to about 1000°C. without altering the position of the sample.

The heat-treatment of permalloy is of the utmost importance. To develop its maximum initial permeability it must be cooled not only through the proper temperature ranges, but also at the proper rates.

It is obvious that only a small part of any sample can be given the most favorable treatment, since the interior portions of the sample cool at rates which are dependent upon the geometrical configuration and thermal properties of the material and are only indirectly under the control of the experimenter. For these reasons each shape and size of sample will have its own best heat-treatment and it is obviously difficult to establish the correct heat-treatment for a small element of volume, characteristic of permalloy as a material. By the use of thin tape, however, we secure fairly uniform treatment of the whole volume so long as the cooling is not too rapid, and fortunately the best cooling rate is not much different from the normal cooling rate of the tape in the open air. It has been found that temperature changes below 300°C. have very little effect upon the resultant properties of permalloy, but the rate of cooling from just above the magnetic transformation temperature down to about 300°C. is a controlling factor. By a long series of experiments a heat-treatment has been established which is especially well adapted to the permalloy test rings already described. They are first heated at about 900°C. for an hour and allowed to cool slowly, being protected from oxidation throughout these processes. They are then reheated to 600°C., quickly removed from the furnace and laid upon a copper plate which is at room temperature.

Not only does each size and shape of sample require its own special heat-treatment, but samples differing only in composition also differ in their most suitable heat-treatments. In our investigation of the nickel-iron series we have not, however, attempted to determine the best heat-treatment for ring samples of each of the many alloys studied. By careful exploration we located the region about 80 per cent nickel, 20 per cent iron as the one promising the highest initial permeability and established the best heat-treatment for this composition. Keeping this treatment unchanged we then relocated the best composition, finding it to be at about 78.5 per cent nickel, 21.5 per cent iron. There is a maximum temperature in the equilibrium diagram for this binary at about 70 per cent nickel,[3] and it was natural to suspect that the maximum in initial permeability which we had found at 78.5 nickel might be displaced to 70 nickel by proper treatment. The 70 per cent nickel alloy was accordingly subjected to a great variety of heat-treatments, but no method was found capable of producing in it an initial permeability as high as that readily obtainable in the 78.5 per cent nickel alloy.

Fig. 1 shows the general way in which initial permeability has been found to vary throughout the nickel-iron series when the heat-treat-

[3] Bureau of Standards Circular No. 58, April 4, 1916.

ment determined as best for the 80 per cent nickel alloy is used. It is obvious from what has been said above that too much weight must not be given to the actual values recorded at any composition. Had the best heat-treatment been determined for each sample the curve might have been altered considerably in detail, particularly outside the permalloy range. We believe, however, that its general form is approximately correct. Alloys were made at 5 per cent steps throughout the range except in the vicinity of 80 per cent nickel where a great number of slightly different compositions were investigated. The chemical analysis, rather than the intended composition, was used in every case, although the difference was never considerable.

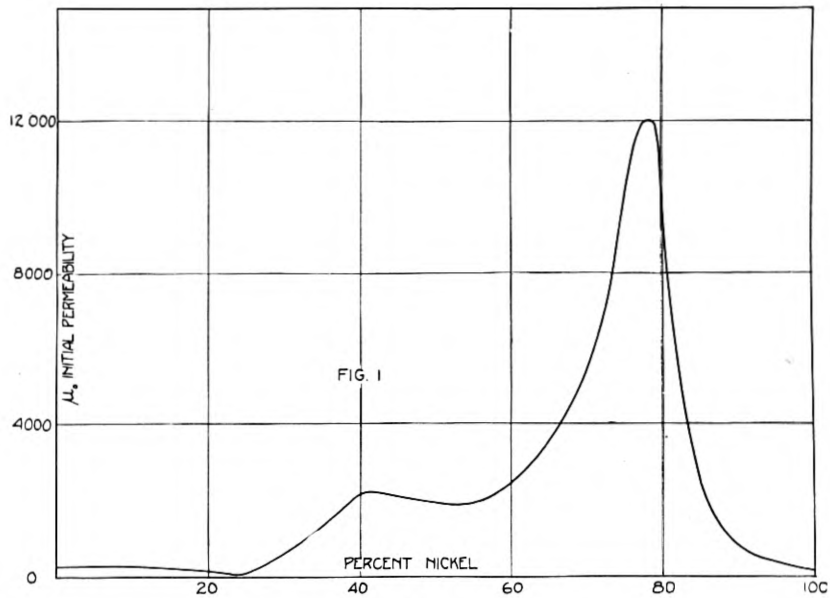

Fig. 1

The largest value of initial permeability for permalloy at room temperature which we have so far found in the ring permeameter is about 13000, more than 30 times the corresponding value for the best soft iron. How extraordinary this is may be appreciated by considering that this material, although it has a saturation value of magnetic intensity comparable with that of iron, approaches magnetic saturation in the earth's field. Unusual caution must therefore be exercised in measuring the properties of permalloy to protect the sample from the influence of stray magnetic fields. Fig. 2 shows, to different scales, the values of initial permeability in similar ring samples of permalloy and of annealed armco iron, and small portions of the corresponding μ-H curves from which these were obtained.

We have measured the magnetization of permalloy at saturation and find that it is not sensitive to heat-treatment. The saturation values of magnetization per gramatom are known to vary almost

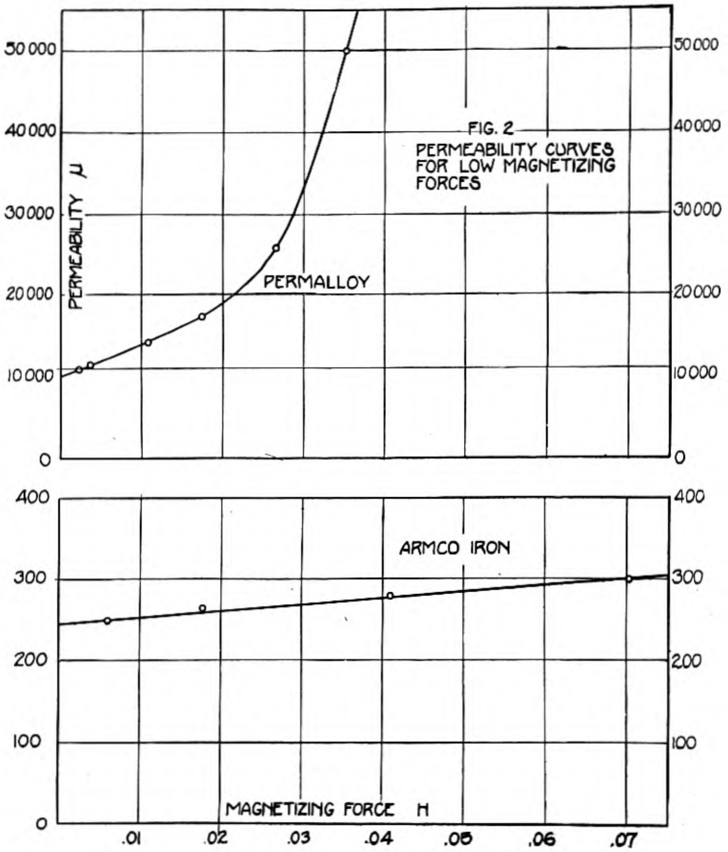

Fig. 2

linearly with composition throughout the nickel-iron series, from 222 for iron to 59 for nickel.[4] The value 84 which we have found for the 78.5 per cent nickel alloy is therefore not abnormal.

The magnetic characteristics of heat-treated ring samples of the same alloy have also been determined through a wider range of field

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[4] P. Weiss, Faraday Society Trans. 8, 149-156 (1912).

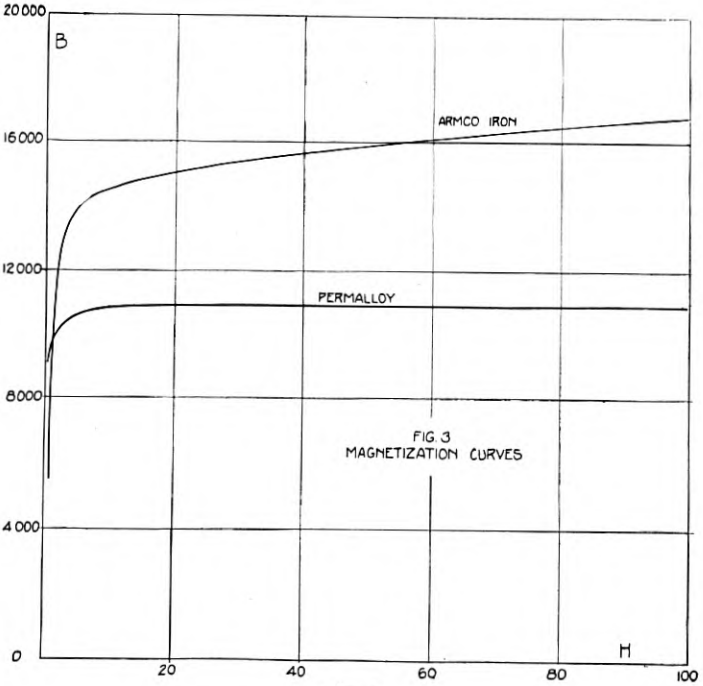
20000
16000
12000
8000
4000
0
B
ARMCO IRON
PERMALLOY
FIG. 3
MAGNETIZATION CURVES
H
20
40
60
80
100

Fig. 3

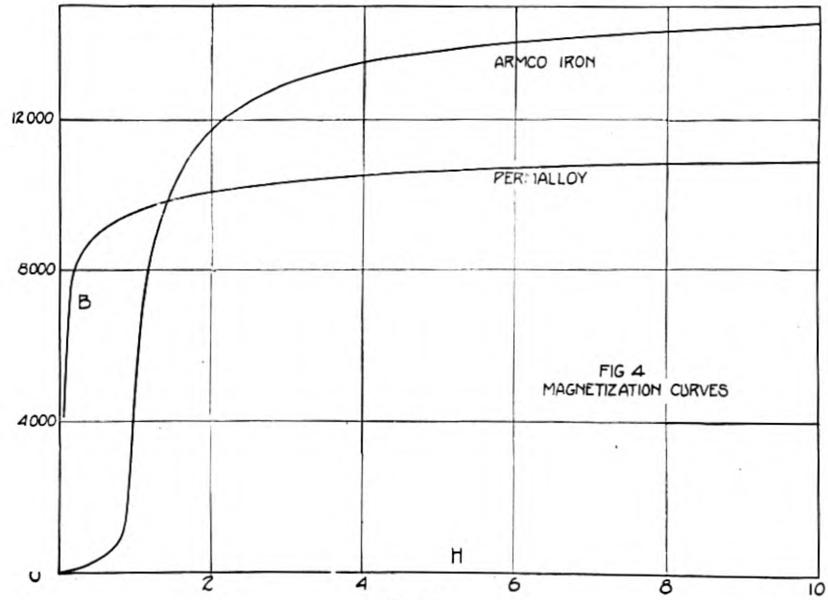
12000
8000
4000
0
B
ARMCO IRON
PERMALLOY
FIG 4
MAGNETIZATION CURVES
H
2
4
6
8
10

Fig. 4

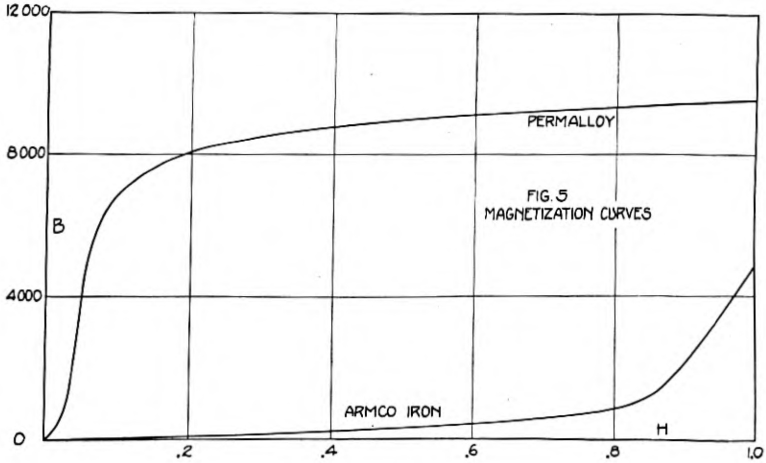
12 000
8000
4000
0
B
PERMALLOY
FIG. 5
MAGNETIZATION CURVES
ARMCO IRON
H
.2
.4
.6
.8
1.0

Fig. 5

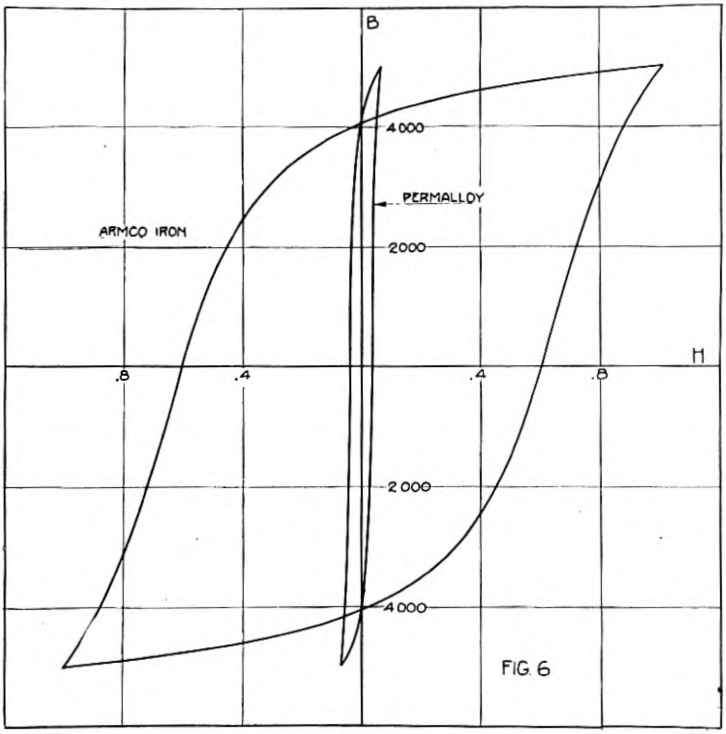
B
4000
PERMALLOY
ARMCO IRON
2000
H
.8
.4
.4
.8
2 000
4 000
FIG. 6

Fig. 6

strengths by ballistic methods. Figs. 3, 4, and 5 show B-H curves for such a sample of permalloy and for a sample of annealed armco iron. From Fig. 5 is apparent the enormous susceptibility of the former material in the weak fields so important in communication engineering. Fig. 6 shows for the same two materials hysteresis loops carried to a maximum induction of 5000 maxwells. The area of the permalloy loop is only one sixteenth that of the loop for soft iron. Fig. 7 shows the μ-B curves for these materials. The maximum permeability here shown, $\mu = 87000$, which is not exceptionally high for permalloy largely exceeds the highest values obtainable in silicon steel[5] and of course occurs at a much lower flux density.

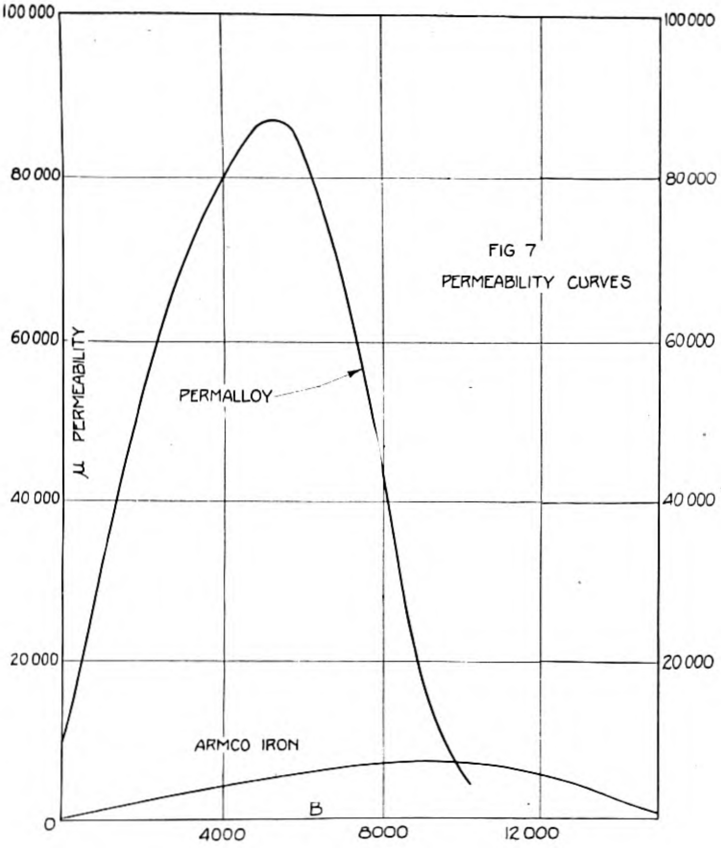

Fig. 7

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[5] T. D. Yensen, U. S. Patent 1,358,810.

Early in the investigations it was found that heat-treated permalloy is sensitive to strain, and the routine measurements were so conducted as to avoid this disturbing effect. Separate investigations of the effects of strain upon permeability and electrical conductivity in straight samples, and of the converse effects of magnetization upon dimensions and conductivity were also undertaken. While these studies are not yet complete it can be stated that all these effects are large in comparison with the corresponding effects in hitherto available magnetic materials. So long as the elastic limit of the material is not exceeded the effects due to strain are reproducible and disappear when the strain is relieved. The effects of magnetization, however, show the expected hysteretic properties. As an example of the magnitude of the effects producible it may be stated that between its value in the unstrained condition and about one-tenth that value the initial permeability of a heat-treated strip of certain of these materials can, by the mere variation of strain, be adjusted to any value we may for the moment desire. The range through which the conductivity can similarly be adjusted by strain is much narrower, the maximum reduction being about 2 per cent, which, however, is a large effect compared with that found in other metals.

The effect of magnetization in reducing conductivity is as much as 2 per cent for fields of the order of one gauss. This makes it easy, for example, to measure the earth's magnetic field to within about 1 per cent by finding the strength of the opposing field necessary to give a permalloy strip its maximum conductivity. It will be noted that the conductivity change which we have mentioned as attainable by magnetization is the same as that attainable by elastic strain. This is no mere coincidence, for we find that the maximum change due to either cause alone is not further increased by superposition of the other, although the effects of small tensions and magnetizing fields are additive. This suggests, of course, that both causes ultimately produce the same change in the mechanism responsible for conduction.

Since the effect of tension upon permeability is in some of these cases so marked it seemed surprising that the only reported study[6] of the converse effect, that is of magnetostriction, indicated a zero value within the permalloy range. It appeared advisable therefore to study the magnetostriction of the series of alloys here available. Preliminary results indicate that under usual conditions of experiment, heat-treated 78.5 per cent nickel alloy exhibits larger magnetostriction than does iron.

[6] K. Honda and K. Kido, Tohoku Univ. Sci. Rep., 9, 221-232, (1920). It should be noted, however, that their alloys had received different treatments than ours.

With the remarkable ferromagnetic behavior of permalloy in mind one naturally looks for analogous peculiarities in its other properties. As has been shown, however, the equilibrium diagram does not point accurately to the composition exhibiting highest initial permeability. The conductivity curve is even less indicative of a peculiarity at this point, its minimum lying at about 35 per cent nickel. The crystal structure is that of nickel and its type does not change until the nickel content is made less than 35 per cent. Even the mean spacing between adjacent atom-centers, and with it the density, varies continuously throughout the entire range. Our experience in working these alloys also indicates that the series has no mechanical peculiarities at or near 80 per cent nickel. Not only do these characteristics indicate no abnormality as the nickel content is increased beyond 70 per cent, but, what is more surprising they are little affected by the heat-treatments which so profoundly change the magnetic properties. So far as has been determined, therefore, it is only in connection with its magnetic properties that permalloy is unusual.

To the engineer the discovery of permalloy means the realization of plans long impossible of accomplishment for lack of a suitable material. For the scientist the principal interest in these materials may well lie in the large response of their magnetic properties to simple external controls. Without alteration of composition these properties may be adjusted through extraordinary ranges by strain, by magnetization, or by heat-treatment. This allows a more definite study of the way in which these factors are related to magnetic properties than has been possible with materials hitherto available in which their effects are comparatively small and may be associated with complicated and irreversible changes in other properties. The behavior of permalloy demonstrates that ferromagnetism is associated with material structure in a different way than are the ordinary physical and chemical properties and its extreme sensitiveness to control gives us a powerful method for use in magnetic investigations.

Telephone Equipment for Long Cable Circuits[1]

By CHARLES S. DEMAREST

SYNOPSIS: Some of the important developments contemplated in the apparatus and equipment for long toll cable circuits are described. The large number of equipment units per station in the cable plant and the greater number of stations in a given length of cable than in an open-wire system have made the economic importance of the equipment design such that a comprehensive program of development, affecting many types of equipment, has been undertaken. The outstanding features of some of the more important of these, including the telephone repeater equipment, test board equipment and signaling equipment, are described. The necessity for compactness in the dimensions of equipment units, uniformity in assembly arrangements, and simplicity in design, together with the need of careful correlation of the electrical and mechanical requirements, are emphasized. The methods proposed for meeting these requirements generally, are described.

INTRODUCTION

THE use of lead covered cables in place of bare copper wires for long distance telephone lines has been an important development and much interesting information on this subject has already been presented to the Institute. The engineering and construction features involved in a cable system of this sort were described by Mr. Pilliod[2] in his article on the Philadelphia-Pittsburgh Section of the New York-Chicago cable, while the transmission characteristics of such a system were brought out in the recent paper by Mr. Clark.[3] It is the purpose of the present paper to deal with some of the important developments in apparatus and equipment which are contemplated for the cable plant.

A cable system requires repeater stations at more frequent intervals throughout its length than an open-wire line, because of the much smaller gauge conductors which it employs and the increased electrical capacity due to closer proximity of the wires. Consequently, in such a system, a greater proportion of the plant investment is represented by the equipment within the offices than is the case with open-wire construction. Furthermore, the number of equipment units per station in a cable system is ordinarily much larger than in an open-wire office, due to the fact that the chief advantages in the use of long cable circuits, in place of open-wire construction, have occurred on routes carrying heavy traffic where many circuits are

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[1] Presented before A. I. E. E., June 27, 1923.

[2] Journal of the A. I. E. E. for August, 1922; and *Bell System Technical Journal*, July, 1922.

[3] Transactions of A. I. E. E., Vol. 38, part 2, p. 1287; and *Bell System Technical Journal*, January, 1923.

needed. Thus, the requirements of the cable plant have been such as to emphasize the economic importance of the equipment design.

To meet these requirements it has been necessary to undertake a comprehensive plan of development affecting many types of equipment. This has involved careful consideration of both the electrical arrangements and the mechanical design, which are being closely coordinated with the purpose that both should contribute to the highest degree of efficiency in the functioning of the system.

It is not possible in this paper to give many details concerning these developments, but it is desired to present some of the principal features as applied to typical cases. Among the more important of these are the telephone repeater equipment, the testboard equipment and the signaling equipment. These are closely associated with each other in their operation, as well as in their physical location, and it has been necessary, in the design of all units to have due regard to the system as a whole.

TELEPHONE REPEATER EQUIPMENT

The function of the telephone repeater as an amplifier in long distance lines is well known. The telephone repeater in its present form has been the chief factor in making long distance cable telephony practicable, and it is probable that the developments in connection with telephone repeaters have been among the most rapid and comprehensive of any in the toll equipment. It will, therefore, be very interesting to note, in the illustrations which follow, the principal

Fig. 1—Box Type Repeater Installation

steps which have been taken in working out the form of the equipment to the degree of efficiency now required for the cable plant.

In Fig. 1 is shown one of the original forms of repeaters, a number of which were installed as early as 1914. In this case the repeater apparatus was assembled in boxes designed to mount on the wall, each box containing a one-way amplifier. Two such one-way units would be required to form what is now known as a two-way, two element repeater. Although the particular amplifiers shown in the illustration were actually used for one-way operation only, they are typical in their general form, of the units employed for two-way operation. The balancing networks, associated coils, etc., were

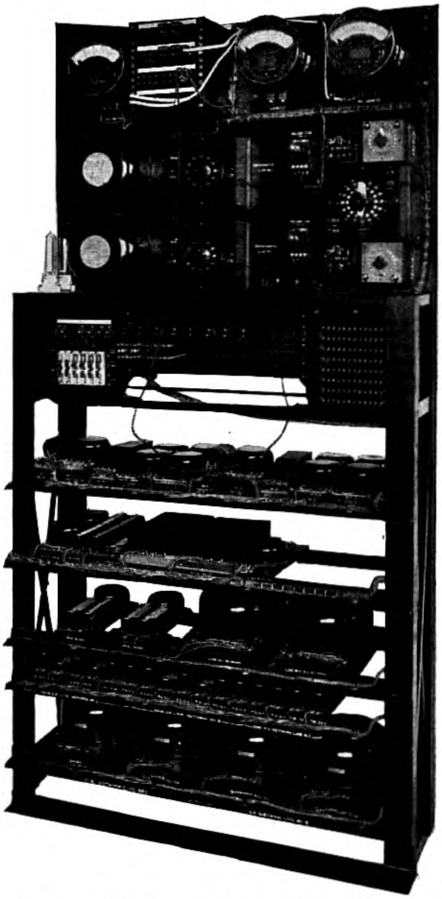


Fig. 2—Wide Rack Through Line Repeater

mounted on separate racks while the plate batteries for the vacuum tubes were mounted in an adjacent enclosed cabinet. The floor space required for a two-way repeater employing this type of apparatus was in the neighborhood of 15 square feet per unit, including the usual allowances for aisle space and associated apparatus.

Fig. 2 shows the first type of vacuum tube telephone repeater designed for commercial manufacture. The particular unit shown is a single "through line" set, that is, one which remains connected to a through circuit at all times instead of being used at a switching point to establish built-up connections at the will of the operator, as a "cord circuit" repeater is used. The "cord circuit" repeater of the same type was similar in arrangement and dimensions to the "through line" repeater shown. In this type of repeater, all of the

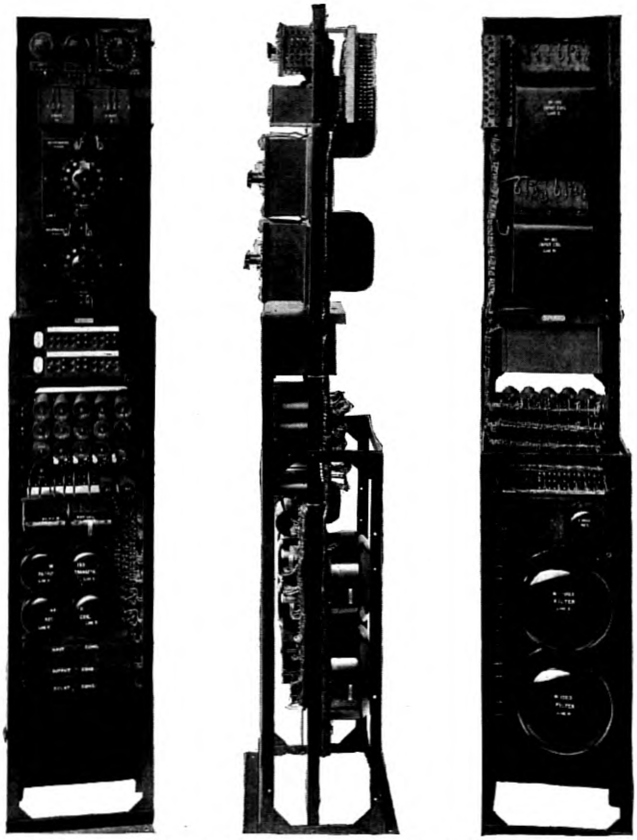


Fig. 3—Standard Through Line Repeater for Open-Wire Use

apparatus for a two-way circuit was mounted together on one rack. The testing equipment and signaling apparatus were duplicated in each repeater set. This apparatus, as well as the balancing networks and other miscellaneous apparatus were mounted on the same rack as the repeater. A unit of this type required about 10 square feet of floor space.

Fig. 3 shows the type of "through line" set which was standardized in 1917 for use on open-wire lines. Fig. 4 shows a group of cord circuit sets of the same type of design, together with the testing unit. This form of set was a great improvement over the earlier types. It employed, however, many of the same large types of individual pieces of apparatus as were used in the former sets and the testing equipment, which was mounted on the middle rack of the group, was required to be duplicated for each group of 6 repeaters. This type of set has been used in many of the smaller installations, but it has not met the requirements for large cable installations. The average floor space area required was about 6 square feet per set.

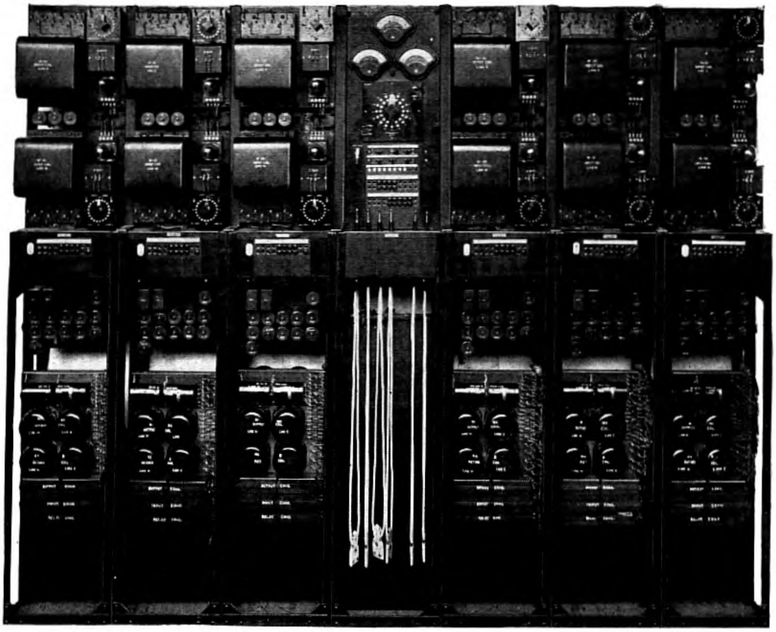


Fig. 4—Group of Six Cord Circuit Repeaters and Associated Testing Unit

Fig. 5 shows the proposed assembly of the type of telephone repeater which is now being developed for all classes of installations employing the two-way, two-element circuit. Fig. 6 shows the general arrangement proposed for a group of sets of this type in a large installation, as in a cable office. This set is expected to have many advantages adapting it particularly to cable installations. When mounted as shown in Fig. 6 it will occupy but 1.5 square feet of floor space per unit. Fig. 7 shows how this set may be arranged

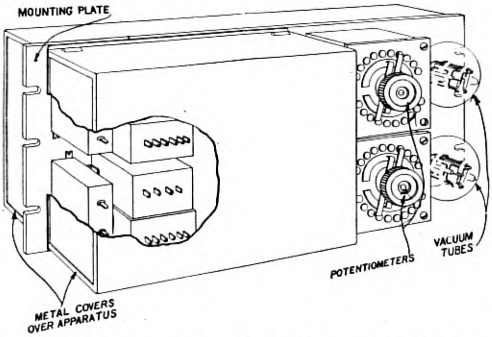


Fig. 5—Typical Assembly of Panel Mounted Repeater Set

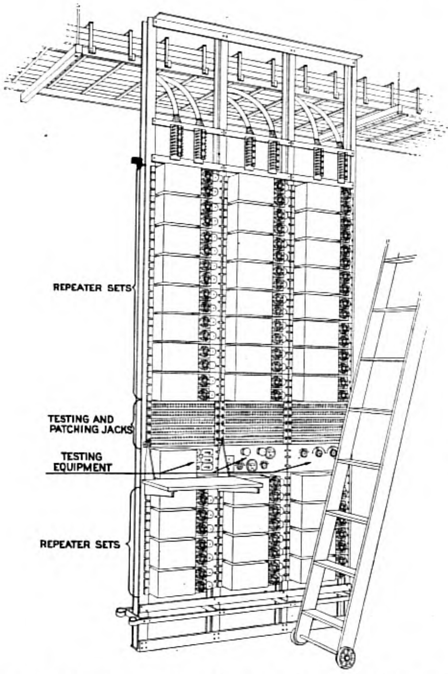


Fig. 6—Group of Panel Mounted Repeaters as Arranged in a Large Installation

in small installations where it may be desired to be mounted on a low rack.

By the uniform use of these general mounting arrangements for all of the new repeater equipment, including the accessory apparatus as well as the repeater sets themselves, it will be possible, where desired, to serve a large number of repeaters with a small amount of testing equipment. For example, in the case of the voltmeters and ammeters required, it will be possible to employ but one meter panel for as many as 120 repeaters. Thus, an economy in equipment as well as a saving in space will be effected.

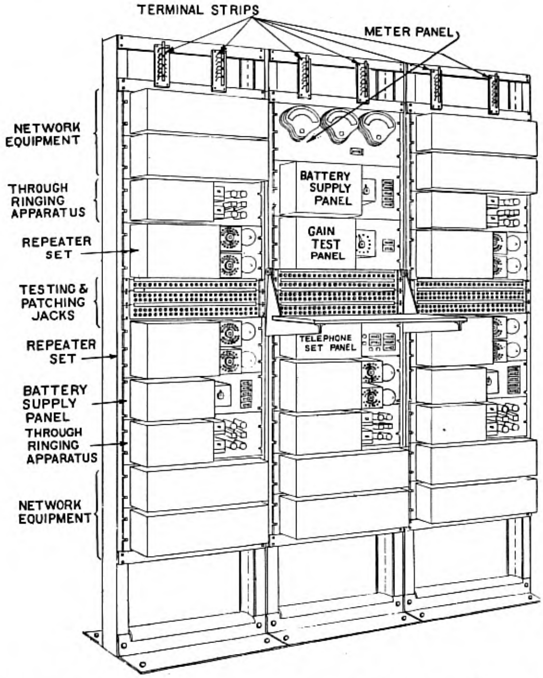


Fig. 7—Panel Mounted Repeaters as Arranged in a Small Installation

Fig. 8 shows how some of the principal features of this proposed type of set, which distinguish it from the earlier types of repeaters, are related to the circuit arrangement, as well as to the mechanical design. Previously, the apparatus which it is now proposed to mount in distinct groups on separate panels, as indicated in this diagram, was assembled together in one repeater unit. Several types of sets were accordingly necessary to meet the various field conditions. This is to be avoided in the new design by separating from the basic repeater unit such apparatus as may be required to be different under different conditions of use. For example, the basic repeater unit in the new repeater is to be the same for both "through line" and "cord circuit" use, and for large installations as well as for small ones. The signaling apparatus which may have different features in different

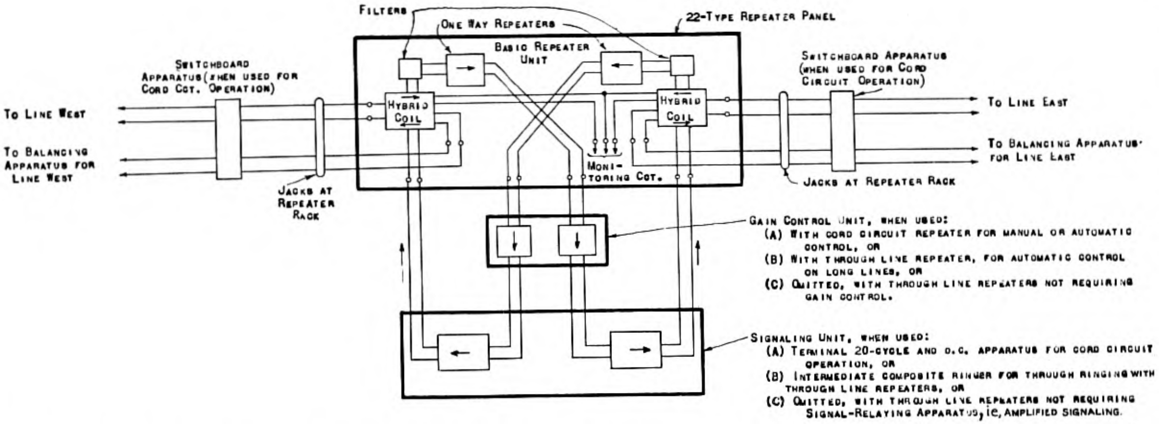


Fig. 8—Schematic Circuit Diagram Showing Two-Way Two-Element Repeater Arranged to Employ One Type of Basic Repeater Unit for all Classes of Service

types of offices, and which may not be needed with "through line" repeaters in some cases, will be furnished as a separate panel from the basic repeater unit. The apparatus which will permit the repeater to be used for cord circuit operation is also to be furnished as a separate unit and may be used in place of the through signaling unit, without changing the basic repeater. The filter which will determine the cut-off frequency of the repeater will also be furnished as a separate piece of apparatus mounted on the repeater set, thus the repeater may be suited to any desired type of line by providing the proper filter.

Another interesting phase of repeater development has been that in connection with the power supply for the vacuum tubes. This includes (1) a source of filament current (2) a source of plate potential and (3) a source of grid potential. In meeting the requirements of cable installations the principal improvements desired have included the use of batteries common to as many repeaters as possible, in place of individual batteries, closer regulation of potentials and the elimination of dry cells where practicable.

In some of the earliest installations a separate six-volt storage battery was used to supply the filament current for the vacuum tubes of each repeater set. Later, the filament current supply was taken from an 11-cell central office storage battery through a rheostat. As the potential of the 11-cell central office battery, normally 24 volts, varied from 20 to 28 volts during the operation of a charge and discharge routine, it was necessary to adjust the rheostat at frequent intervals to maintain constant current in the vacuum tube filaments. With the greatly increased number of repeaters per station which has occurred in cable systems, the maintenance involved in readjusting the filament currents would have become prohibitive on this basis. Accordingly, for the larger installations, duplicate 11-cell batteries normally floated from generators and provided with an emergency cell to maintain voltage during an emergency discharge are proposed. By this improved arrangement it is expected to be possible to maintain the filament voltage within one volt up or down from its normal value, even during an emergency discharge, until the batteries are almost completely discharged. This improved regulation will entirely eliminate adjustments of the individual repeaters during operation to secure proper values of filament current.

For the plate voltage supply dry cells have sometimes been used in small installations. These are now being displaced to a large extent, by small storage cells, two groups being used so that one group may be charged while the other is in service. In the large cable installations, the current drain on the 130-volt plate batteries has sometimes reached values as great as four or five amperes, so that it is now planned to float these batteries also, instead of operating on the charge and discharge basis. It is expected by this means to obtain regulation of the plate voltage within plus or minus five volts from the normal value of 130, at all times.

Consideration is being given to another possible improvement in the power arrangements for large repeater installations. This involves the proposal to use a storage battery common to all of the repeaters for supplying the grid potential. If it is found that this arrangement is practical, it will permit the elimination of the individual dry cell batteries which have been employed, with a consequent saving in maintenance that might be appreciable. It seems likely that a very small storage battery would serve for this purpose since the current drain is negligible.

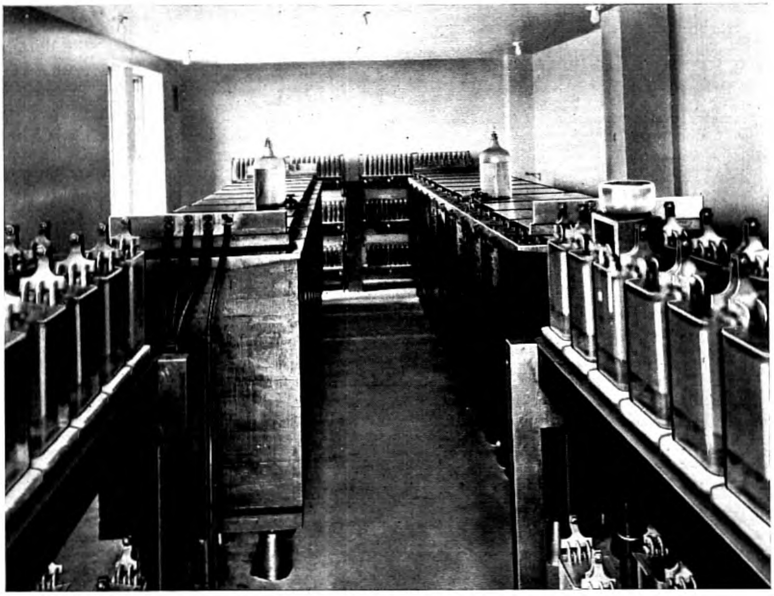


Fig. 9—Typical Storage Battery Room for Cable Repeater Station

The amount of power required to operate the vacuum tubes in a large telephone repeater installation is considerable. Each filament requires a current in the neighborhood of one ampere and, while the filaments are connected in series in such a way as to utilize as efficiently as practicable the full potential of the central office battery, the load on this battery sometimes amounts to several hundred amperes.

Fig. 9 gives some idea as to the size of the storage batteries for a typical office of this kind. The large cells in wooden tanks are those making up the filament batteries, each of which is an 11-cell battery of 24-volt nominal rating. These two batteries in parallel are large enough to carry the office load for at least 12 hours in the event of a complete failure of charging equipment.

Fig. 10 shows the charging generators and power switchboard for a typical cable repeater office. The gas engine drives emergency generators to float the filament and plate batteries in the event that the regular electric power supply service fails. Alarms are provided to indicate any abnormal condition such as a voltage higher or lower than normal, a blown fuse, etc. These are grouped in an annunicator cabinet on the wall.

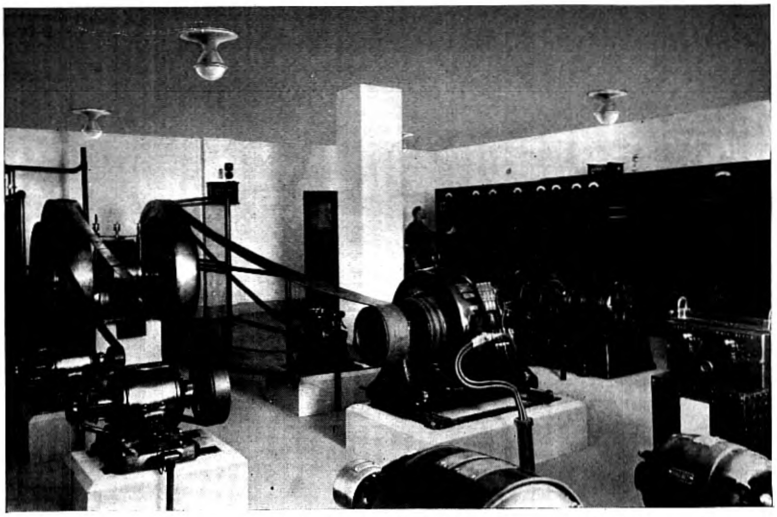


Fig. 10—Typical Power Room for Cable Repeater Station

TEST BOARD EQUIPMENT

The test board forms an important part of the cable plant equipment, since it is the one point in the office where all of the lines and the equipment as a whole may be reached readily for purposes of testing or re-routing, in cases of line trouble or changes in layout. The form and arrangement of the test board equipment consequently have an important bearing on the effectiveness with which the facilities are handled by the maintenance forces. This is particularly true in the cable plant where the number of circuits involved is large.

In general, all toll line conductors are brought into a central office, from the outside, through a cable and first appear at a rack called a "distributing frame" where they are soldered to exposed terminal lugs so that they may be reached for the purpose of connecting them to apparatus within the office. This arrangement is well suited to the permanent connections but is not intended to permit frequent changes or the ready removal of the apparatus for line testing.

The necessity for rearranging the connections between the apparatus and the lines in cases of line trouble makes it desirable at times to be able to make such changes quickly, and the means which have been provided for this purpose are located at the "test board." Here both the line conductors and the apparatus units are wired to "jacks" which are arranged to permit the transfer of the normal connections by the insertion of plugs wired to flexible cords. A temporary connection made in this manner through a conducting cord wired to two plugs is called a "patch." The apparatus for determining the location of line trouble is also located at the test board and is wired to cords and plugs so that it may be connected to any line in the office readily, upon the occurrence of line trouble, without necessitating changes in soldered connections.

The test boards used for the open-wire plant have been designed to take care of 40 to 80 line conductors in one position, that is, in a board three feet long. The amount of testing and patching work required on open-wire lines has been such as to make this a convenient number of circuits to handle within this space. In cable installations, however, the amount of testing and patching per line conductor is less, while the number of circuits in such an office is much greater. Consequently, in cable offices it is possible to concentrate a larger number of wires within a given test board space. This is desirable from the standpoint of economy in space as well as from that of convenience in operation.

One of the first steps considered in the development of efficient test board equipment for cable installations has been the reduction of the number of jacks per circuit. In open-wire installations it has been the practice to equip each line circuit and each equipment unit, such as a composite set or phantom coil, with a full complement of jacks suited to provide the maximum degree of flexibility in "patching," thus permitting the ready interchanging of individual equipment units, lines and drop circuits. In cable installations, where the circuits are more likely to be uniformly equipped with the same types of associated apparatus and where the line troubles are less frequent, it is expected to be possible to eliminate certain of the jacks,

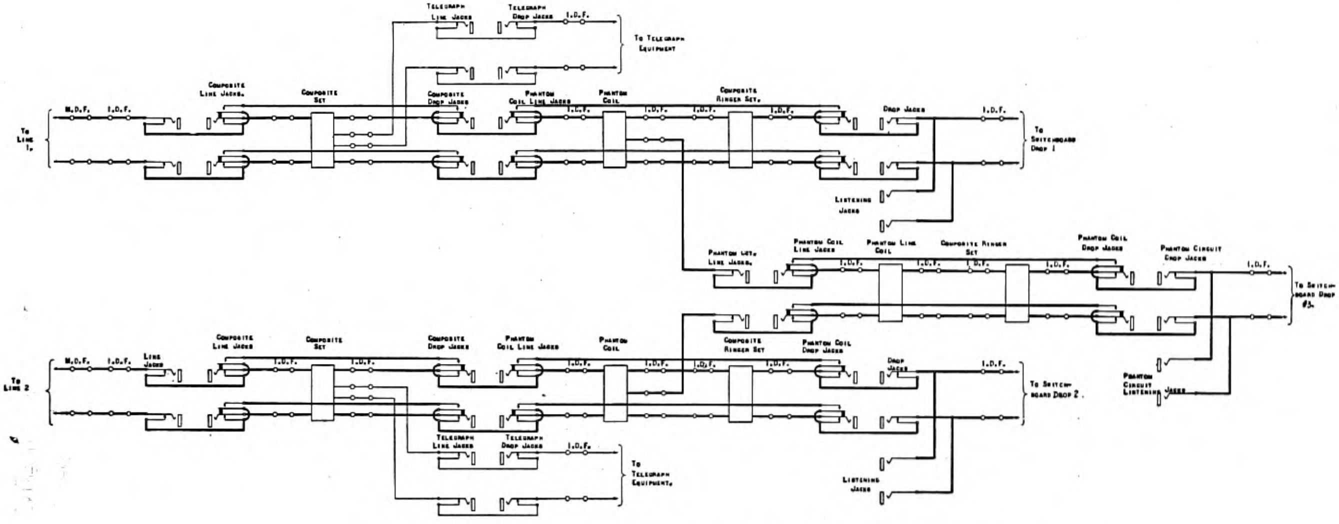


Fig. 11—Typical Phantom Group Circuit for Open-Wire Installations

such as those associated with the composite sets and phantom sets, thus simplifying the equipment for terminating the toll lines. Fig. 11 shows the typical open-wire arrangement for a terminating phantom group circuit in which the maximum number of jacks is furnished. This requires a total of 46 jacks. Fig. 12 shows the arrangement of a terminating phantom group circuit as planned for a cable installation. In this case a total of but 30 jacks is required.

Another important development expected in the test board arrangements to suit them to cable use is the grouping together of the jacks serving similar functions. Considerable improvement in operation is thought to be possible with the jacks having different functions located at different test board positions. In this way all of the line conductor jacks, for example, may be assembled together in consecutive order, and since several hundred of these may be involved in a single installation, this should greatly facilitate the identification of the desired circuits by the attendant in the process of patching and testing. This grouping of the jacks will also effect a saving in testing equipment, since it will eliminate the need of the line testing apparatus, such as the Wheatstone bridge, at positions where the line conductors will not appear.

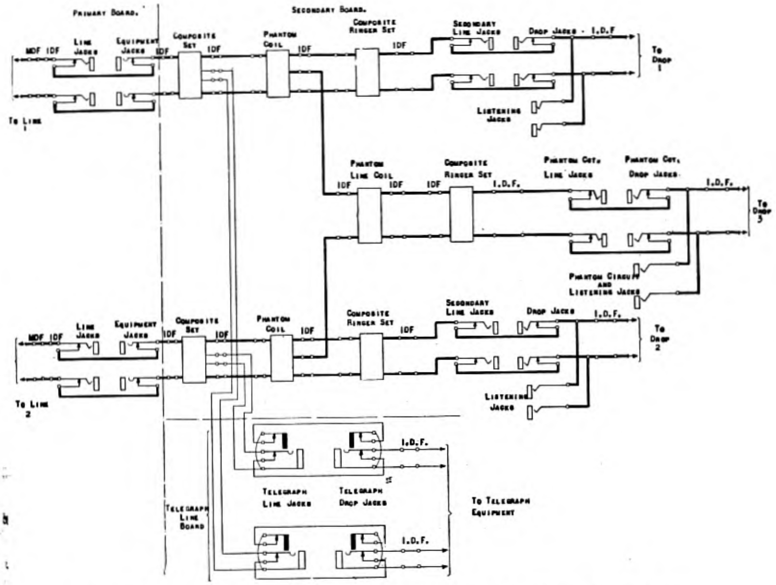
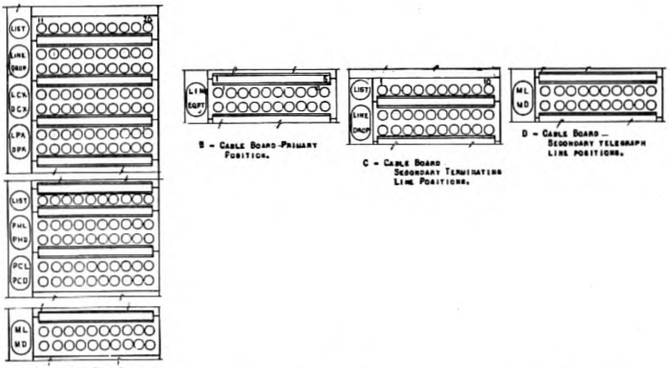


Fig. 12—Typical Phantom Group Circuit for Cable Installations

Fig. 13 illustrates both the open-wire and the proposed cable methods of grouping the jacks. In the arrangement for open-wire circuits the jacks associated with both the lines and equipment are located adjacent to each other in the same test board panel. In cable installations the jacks having similar functions are to be grouped together, the groups having different functions being mounted in different panels which may be located in different test board positions. These various groups, in the latter case, are planned as follows:



A - Open-Wire Test Board

B - Cable Board-Primary Position.

C - Cable Board Secondary Terminating Line Positions.

D - Cable Board Secondary Telegraph Line Positions.

Fig. 13—Typical Jack Assembly Arrangements at Test Board

1. Primary line testing position for testing and "patching" toll lines only. This position to be used for locating faults in the cable circuits and equipped with a Wheatstone bridge and voltmeter, to permit the necessary electrical measurements for this purpose. This position is also to be used for making temporary changes in the assignments between the lines, and the equipment as a whole, but is not to be arranged to permit changes in individual equipment units, such as composite sets, phantom sets, etc. The jacks to be located at this board are to include those designated as "line jacks" and "equipment jacks," in Fig. 12.
2. Secondary terminating line positions for testing and "patching" the lines between the "drop" side of the equipment and the toll switchboard circuit. This position is to be used for determining the general nature of a trouble and its general location, i.e., whether it is in the direction of the line or in the direction of the "drop," and for clearing troubles not requiring line tests. This position is not to be equipped with Wheatstone bridge testing

apparatus as at the primary board. The jacks to be located at this position are to include those designated as "secondary line jacks," "drop jacks" and "listening jacks," in Fig. 12.

3. Secondary telegraph line positions for testing and "patching" the telegraph line circuits. This position is to be used solely for interchanging telegraph lines and telegraph equipment, in cases of temporary changes in assignment and is not to be equipped with the line testing apparatus. This position will permit changes to be made in the telegraph assignments without interfering with the telephone circuits. The jacks to be located at this position are to include those designated in Fig. 12 as "telegraph line jacks" and "telegraph drop jacks."

A further and more extensive improvement in test board design is anticipated as a result of development work whereby it will be possible to employ panel mounted keyshelf equipment units, jacks and testing apparatus, which in the standard board are now housed in a large wooden section. This will have the advantage of uniformity with the other toll equipment, as well as requiring less space. It will also permit flexibility in the use and installation of the various combinations of keyshelf equipment, jack equipment and other testing

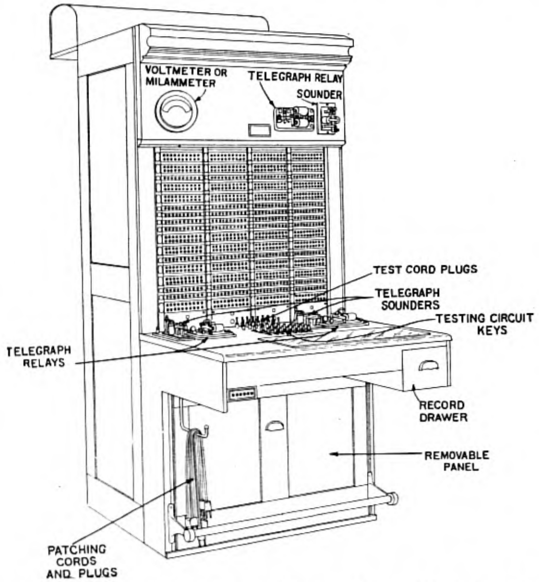


Fig. 14—Typical Assembly of Floor-Mounted Test Board—One Position

apparatus which may be required to suit each case. While this arrangement will have its chief advantages when applied to large cable installations, it will also be well suited to open-wire use and small installations, since its design will permit the highest degree of flexibility with respect to both the amount and type of equipment.

These points may be illustrated by comparing the general features of the two types of boards. Fig. 14 shows the assembly of a one-position section of the present type of board employing a wooden framework. This board, with the necessary allowances for aisle space, requires a floor area amounting to about 24 square feet, while it houses a maximum of about 1000 jacks corresponding roughly to about 40 jacks per square foot. Fig. 15 shows a typical position em-

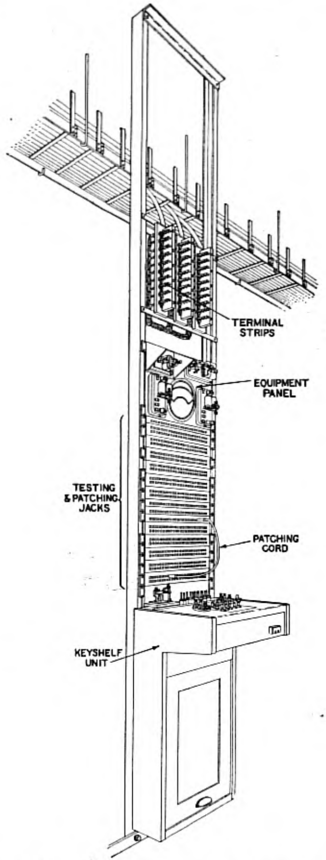


Fig. 15—Typical General Assembly of Panel Mounted Test Board

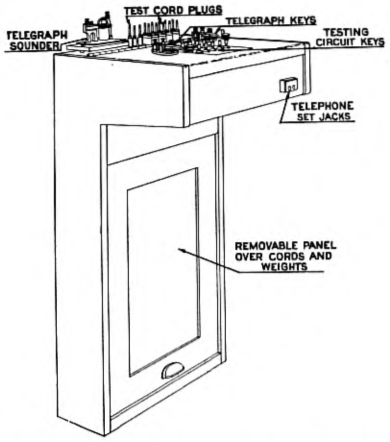


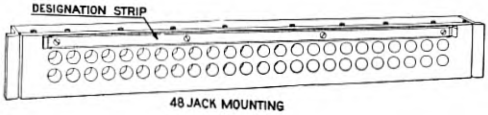
Fig. 16—Typical Assembly for Keyshelf Equipment Unit for Panel Mounted Test Board

ploying the proposed panel mounting method. Such a board will occupy a floor space of about 10 square feet and will take care of about 600 jacks, corresponding roughly to a capacity of 60 jacks per square foot.

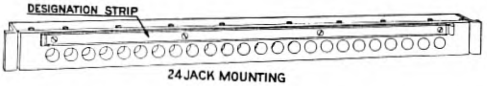
This latter type of board is to be made up of a number of panel units which are to be assembled on two vertical supports. The principal types of units to be provided for the purpose are the keyshelf units, the jack mountings and the equipment panels which may be combined together as desired to give the necessary facilities.

Fig. 16 shows a typical keyshelf unit designed for the panel type board. By constructing the keyshelf unit as a separate piece of apparatus, it is expected to be possible to standardize the necessary types of keyshelves to fit all ordinary field conditions and to specify the desired type of keyshelf to go with any particular arrangement or number of jacks. The number of keyshelf units of any given type may be as desired for each installation, thus the proportion between the jacks and the keyshelf equipment may be suited to each type of office.

Fig. 17 shows a typical arrangement of the jack equipment and the mountings which are to be employed for the jacks. This type of jack mounting will make it possible to mount the jacks on the same supports as the testing equipment. The mountings are to be attached to the supports by fasteners, each occupying a vertical space of 1¾ inches and drilled to fit the usual drillings in the supports. This will permit the close association of the jacks with the desired testing apparatus. It will also be possible, by this means, to use for the jacks only such of the available vertical space as may be desired, the remainder being used for other equipment, thereby effecting economy in the use of the space. The arrangement is thus expected to be advantageous both in large installations, where the various groups of



48 JACK MOUNTING



24 JACK MOUNTING

Fig. 17—Typical Assembly of Jack Mounts for Panel Mounted Test Board

jacks are desired to be arranged at separate primary and secondary positions, and in small installations in which but a few jacks may be required for all purposes.

Fig. 18 shows the proposed assembly of a typical panel equipped with voltmeter, telegraph relay and sounder, such as are usually mounted above the jack field. The advantage of using the panel mounting for this equipment is expected to be the same from the standpoint of convenience and economy as that obtained from this method of mounting the keyshelves in relation to the various jack fields. It will permit the location of this equipment at any desired position and eliminate duplication at positions where this apparatus is not necessary. It will also permit flexibility in regard to the type of panel associated with a given jack field.

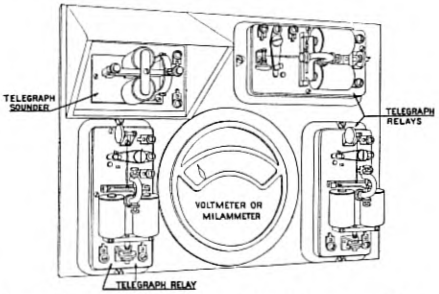


Fig. 18—Typical Assembly of Apparatus Panel for Panel Mounted Test Board

SIGNALING EQUIPMENT

The principal purposes of the signaling equipment in telephone lines are (1) to permit a subscriber to signal the central office operator, as he does automatically in removing the receiver from the switchhook, (2) to permit the central office operator to ring the subscriber's bell and (3) to permit the operators at different central offices to signal each other. At repeater stations this equipment serves to pass the signals around the telephone repeaters which might otherwise interfere with their transmission.

The switchboards in both the local and toll central offices have been provided with a source of 20-cycle current for signaling, as current of this frequency is suited to operate the subscriber's bell directly. This frequency has also been satisfactory for operating the signaling apparatus in the various local trunk circuits and in short toll lines without superposed telegraph. The introduction of superposed telegraph and telephone repeaters, however, has prevented

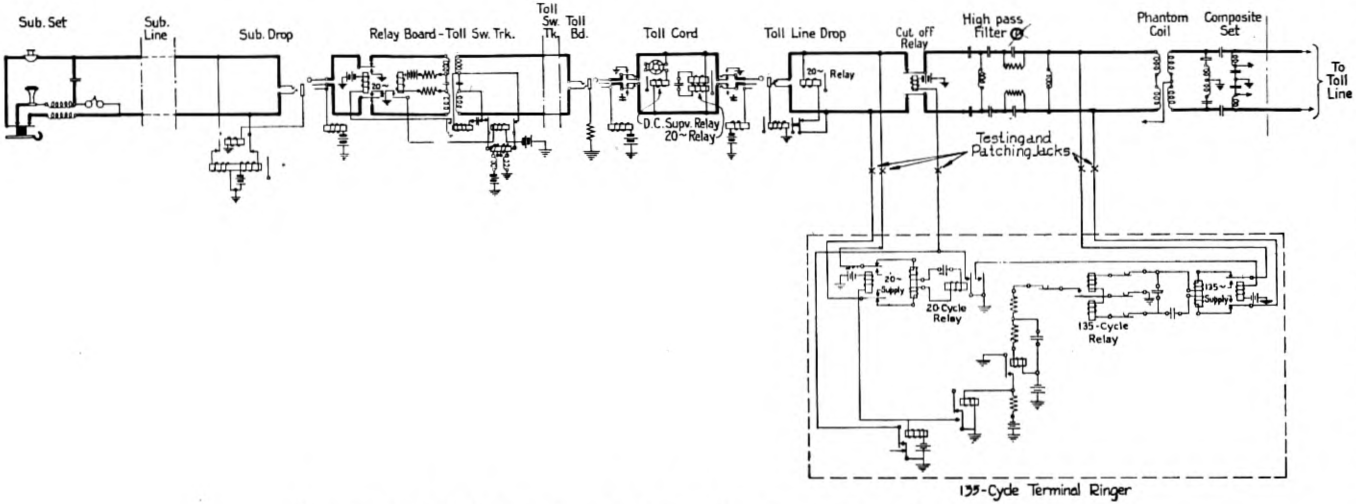


Fig. 19—135-Cycle Ringdown Toll Signaling System—Simplified Diagram

the use of 20-cycle signaling on many toll circuits. It has accordingly been expedient to use a higher frequency for the majority of these circuits, and the one which has been most generally used for this purpose has been 135 cycles. It is desired, therefore, to bring out here some of the more important features of the 135-cycle signaling arrangements which are being developed for cable systems.

The signaling equipment for cable systems has been required to meet more severe conditions than those ordinarily encountered on open-wire lines. In order not to interfere with the direct current telegraph system used on the cables, it has been necessary to limit the signaling current to a few milliamperes. Furthermore, the characteristics of the cable apparatus have been such as to attenuate the signaling currents to a greater extent than in open-wire systems. It has accordingly been necessary to undertake the design of signaling equipment of greater sensitivity, as well as to provide a source of supply of signaling current possessing a high degree of freedom from harmonics and capable of being closely regulated.

The desired increase in sensitivity of the signaling system is expected to be obtained through the use of a highly selective circuit in conjunction with a very sensitive 135-cycle relay. Fig. 19 shows the general scheme of the circuit which is being developed for the purpose.

It is seen that this includes a filter which may be inserted in the line circuit between the signal-receiving apparatus and the switchboard "drop" to give the desired terminal impedance at the signaling frequency, thus preventing the low impedance shunt on the signaling relay which would otherwise be caused by the "drop" circuit at a frequency as low as 135 cycles. The filter is arranged so that it need be inserted in the circuit only in such cases as may require the increased signaling range obtained in this manner, thus, it may be omitted on the shorter circuits where it is usually unnecessary.

The general circuit arrangement shown in Fig. 19 will permit the ready interchange of different types of signaling apparatus. With this arrangement of the apparatus, any "ringer" of any desired frequency combination such as 20–135 cycles, 20–20 cycles, 135–135 cycles, etc., may be connected temporarily to the system when desired. by means of "patching" cords, without requiring any changes in the permanent wiring.

Much of the sensitivity and selectivity which may be obtained with this signaling system are due to the design of the 135-cycle relay, As shown in Fig. 20, the relay is designed to make it capable of close and accurate adjustment. Both the magnetic air gap and the contact spacing may be adjusted, about 0.0015 inch or 0.038 millimeter being used ordinarily for the latter. The relay is mechanically tuned, the natural period of the reed with its adjustable weight corresponding closely to the frequency of the signaling current. A stop pin is provided which prevents undue vibration of the reed due to transient impulses or excessive currents. Also, the relay circuit is electrically tuned by a shunt capacity and a series inductance and capacity. It is thus very selective and is relatively free from the ordinary sources of interference such as those caused by mechanical vibration, telegraph signals, switchhook impulses, voice currents, etc. The sensitivity of the relay is such that it will operate on as little power as 30 microwatts, corresponding to a current in the neighborhood of 0.25 milliampere.

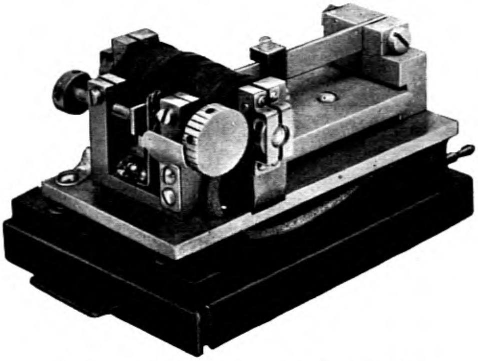


Fig. 20—Assembly of 135-Cycle Relay

This relay is to be mounted so that it may be inserted in the circuit or removed from it in the manner of a plug and jack, without requiring changes in the permanent wiring and without affecting the circuit operation, except to interrupt the signal-receiving system, while the relay is removed. Thus it will be very convenient to make the necessary adjustments of the relay separate from the circuit with which it may be associated in service. The relay will be well protected from mechanical interference, such as building vibration, by padding in the mounting which prevents rigid mechanical connection between the relay and its external support.

The assembly arrangements proposed for the new signaling apparatus are such as will fit in closely with the panel mounting methods designed for the remainder of the toll equipment in cable installations. Fig. 21 shows the assembly proposed for the new composite ringer set. This method of mounting the composite ringer set is a very desirable one since it permits a complete set to be manufactured as a unit and installed as such.

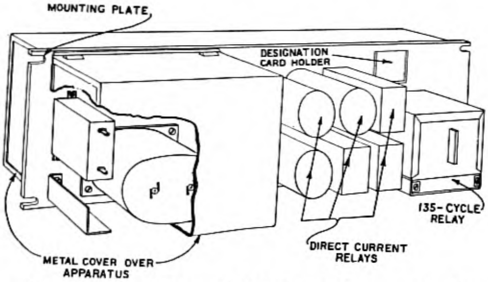


Fig. 21—Typical Assembly of 135-Cycle Composite Ringer Set

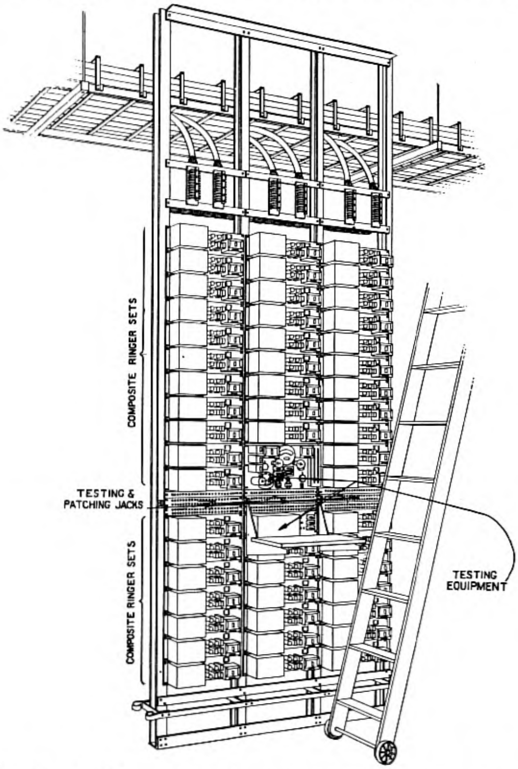


Fig. 22—Typical Assembly of Group of Composite Ringer Sets

Fig. 22 shows the manner in which a number of these composite ringer panels are planned to be mounted together with the associated testing equipment in a large installation. This close association of the composite ringers with the jacks and testing equipment is expected to be of considerable advantage in facilitating the maintenance of the apparatus.

The need in cable systems of a particularly well-regulated source of 135-cycle signaling current, with sufficient capacity for a large number of lines, has made it desirable to undertake the development of both a special type of interrupter and a motor-generator for this purpose. Close frequency regulation is also very desirable in signaling over cable circuits, in view of the increased sensitivity in receiving which may be obtained by the use of very selective receiving apparatus.

Fig. 23 shows the circuit arrangement of the interrupter which is expected to be provided for this purpose. This includes a vibrating reed actuated by an electromagnet when direct current is applied. The contacts on the reed being in series with the battery circuit, intermittent operation is secured in the manner of an ordinary buzzer, the speed of operation for a given applied voltage being determined by the natural period of the reed. The actuating circuit of the vibrator also includes the primary side of a transformer, the secondary side of which is connected to a filter for suppressing harmonics in the output. The maximum output capacity of the interrupter is about three-fourths of a watt.

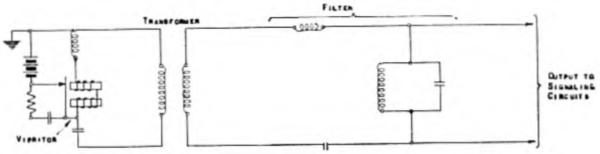


Fig. 23—135-Cycle Interrupter Circuit—Simplified Diagram

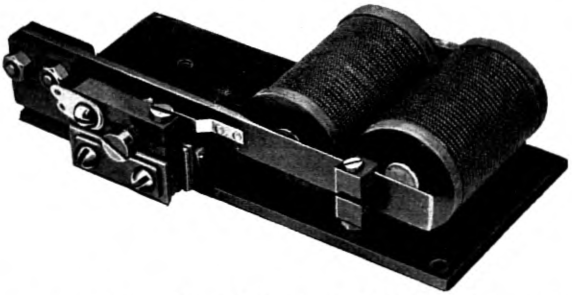


Fig. 24—Assembly of Vibrator for 135-Cycle Interrupter

Fig. 24 shows the vibrating element to be employed in this interrupter. This is equipped with an adjustable weight so that the frequency of the output may be regulated. For an input voltage variation of 20 to 28 volts, the output frequency will vary about 5 cycles.

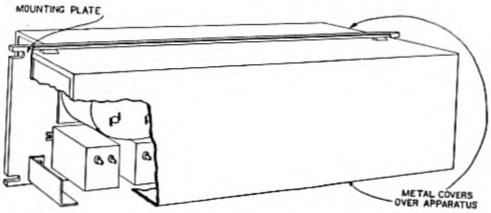


Fig. 25—Assembly of 135-Cycle Interrupter

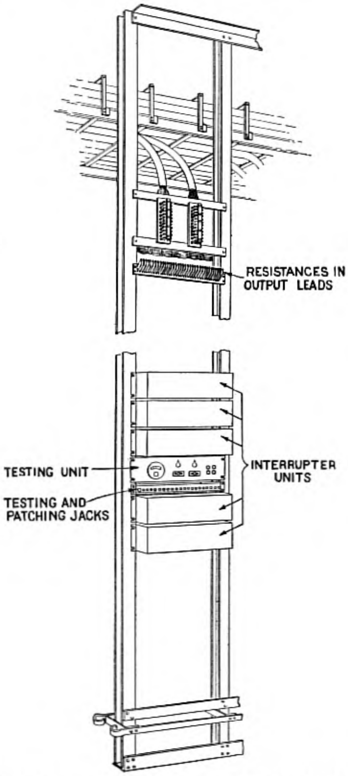


Fig. 26—Typical Assembly of Group of 135-Cycle Interrupters

Fig. 25 shows the assembly proposed for the complete interrupter unit. This includes the vibrator, filter, transformer, etc., mounted on a panel under metal covers. Fig. 26 shows a typical arrangement of a group of interrupters with the associated testing equipment. These assembly arrangements are uniform with those previously described for other types of equipment.

The 135-cycle motor-generator developed for signaling purposes is shown in Fig. 27. This outfit includes two motor-generators, one for regular service and one for reserve, on one panel, the control equipment being mounted on a second panel. The output of the motor-generator, which is provided with a filter, is practically free from harmonics. Its voltage range is from 30 to 40, while its frequency range is from 133 to 137 cycles, including full load to no load conditions. The output capacity of this machine is approximately 10 watts.

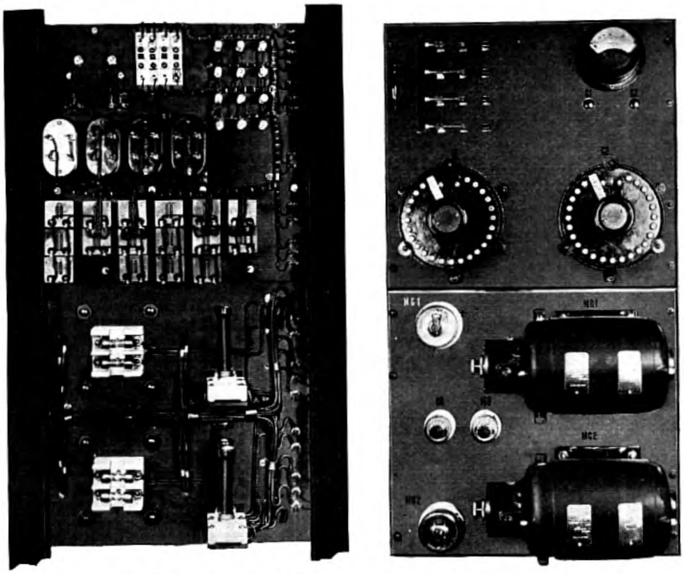


Fig. 27—Assembly of 10-Watt, 135-Cycle Motor-Generator Set and Associated Apparatus

GENERAL ASSEMBLY ARRANGEMENTS

In the preceding descriptions some mention has been made of the panel assembly method which is expected to be used with much of the cable equipment. It might be well to speak briefly here of some of the features of this mounting method which are expected to have general application.

The large number of equipment units per station in the cable plant has been one of the principal factors in determining the requirements for efficiency in the design of the equipment. These requirements have been mainly (1) compactness in dimensions (2) uniformity in assembly arrangements and (3) simplicity in design.

To make possible the housing of the equipment for a cable installation within a building of reasonable dimensions, it has been necessary to economize carefully in space. Fig. 28 shows in a general way how the panel assembly method may be employed to accomplish this. As seen from this view, the equipment panels are assembled on vertical supports consisting of I-beams or channels which extend from the floor to the ceiling, rolling ladders being employed to reach the apparatus above when necessary. As has been shown in the preceding views of individual equipment units, all of the adjustable apparatus is mounted on the front of each equipment panel and the

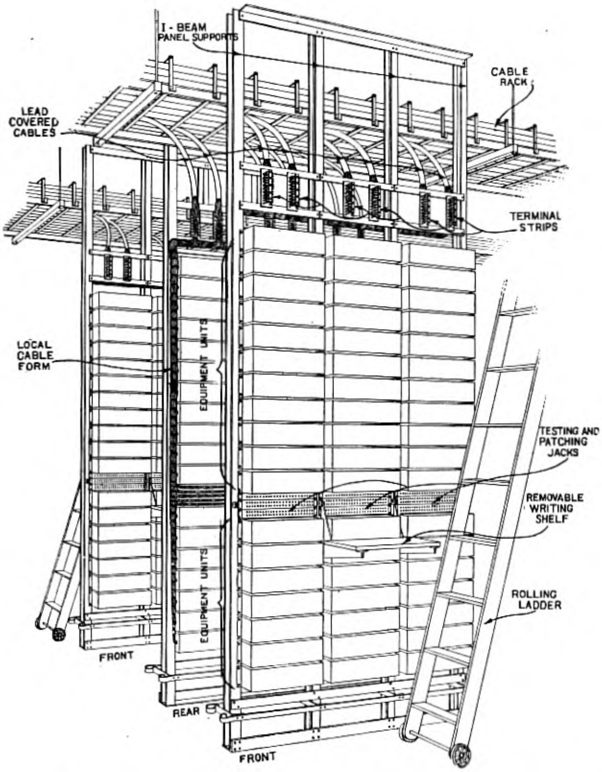


Fig. 28—General View Showing Typical Group of Equipment Units Employing Panel Mounting Method

material not requiring adjustment when in operation is mounted on the rear, since by this means the rear aisles can be made narrower than the front aisles. Thus, all available space is utilized to the extent that the floor area required per equipment unit in a cable installation will be as small as the maintenance, manufacturing and installing requirements for the apparatus will permit.

In view of the many different types of equipment units having a variety of special functions which are required to make up a complete cable installation, a high degree of uniformity in design is necessary to permit efficiency in their use. It is expected that this will be accomplished effectively by applying the panel assembly method in a uniform manner to practically all types of equipment units. All panels are to be of a uniform length, designed to mount on vertical supports spaced 19½ inches between centers. The height of the different panels will vary, according to the amount of apparatus in each unit, but this vertical dimension is in all cases to be a whole multiple of 1¾ inches. By applying these specifications widely, it will be possible to secure interchangeability between panels and to employ uniform methods in grouping the different units, thus facilitating their installation and use.

The simplicity of the design of the equipment units comprising the various panels is also of great importance. This has necessitated careful attention to the forms of the individual pieces of apparatus, in order that these might fulfill their specific functions efficiently while at the same time fitting in well with the general equipment arrangements. To this end, new types of apparatus, such as repeating coils, retardation coils, etc., are being developed especially for cable use. Much will also be accomplished toward the simplification of the panels by carefully avoiding duplication in the accessories to the different types of equipment and by dissociating from the individual units of all types any pieces of apparatus capable of being made common to a number of units, or subject to different methods of application in different types of offices.

Other important advantages are anticipated in the panel assembly method. One of these is that it will permit the assembling together on one panel of all of the pieces of apparatus of different types which may be desired to form a distinct equipment unit. In large installations, completely equipped racks including a number of equipment units with the associated testing apparatus may be assembled in the factory on the supporting uprights and wired to the terminals at the top of the rack, thus simplifying the installation work. The location of the testing apparatus on the same rack with the equipment panels has the further advantage that it will place the apparatus to be adjusted within easy reach of the testing facilities. Furthermore, the testing apparatus will serve for the maintenance of a greater number of equipment units when all of the space between the floor and ceiling is utilized, thus reducing the amount of testing equipment required.

Radio Extension of the Telephone System to Ships at Sea[1]

By H. W. NICHOLS and LLOYD ESPENSCHIED

SYNOPSIS: The paper describes the development of a two-way radio-telephone system and its use in extending the Bell Telephone System to connect with ships at sea. The electrical considerations and the experimental work involved in determining the system-design of the radio link are discussed. Two land stations were established, one of them a permanent three-channel station on the New Jersey coast. Two coastal vessels and finally one trans-Atlantic liner were equipped. These installations are briefly described in the paper.

The operation of the combined radio and wire system is explained, particularly in respect to the transmission characteristics of the over-all system and the effect thereupon of the movement of the vessel and of variations in atmospheric conditions. Measurements of the variations in the field strength received from field vessels at sea show why it is possible to receive over very long distances at favorable times at night and not during the day. The method of establishing combined radio-telephone-wire circuits to ships is described and representative results are given of the considerable telephone traffic which was handled over the system experimentally during a period of trial operation. Tests of multi-channel telephone operation to several ships through the Deal Beach shore station, and also tests of simultaneous telegraph and telephone operation from the same vessel are described. Connection of a vessel thru the transcontinental telephone line to the Catalina Island radio-telephone system, whereby the vessel in the Atlantic talked with an island in the Pacific, is briefly described, and finally the outstanding conclusions of the entire development work are given.

IN 1919, the American Telephone and Telegraph Company and the Western Electric Company initiated a development program which had for its object the development of a radio telephone system capable of enabling the service of the Bell Telephone System to be extended to include vessels at sea. The program involved extensive development work in the laboratory and field, the establishment of shore and ship stations, and the putting of the system into practical operation, altho on a limited and experimental scale.

It is the purpose of this paper to describe the results of this development work from the standpoint of the complete system, with emphasis upon the general transmission and operating features rather than upon the details of the apparatus developed to perform the necessary functions. The development divides itself, naturally, into two parts: first, the determination of the system-design and the establishment of the necessary stations, and, second, the study of the transmission and operating characteristics of the system.

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[1] Presented before The Institute of Radio Engineers, New York, January 3, 1923, Reprinted with minor changes from the Procd. of Institute of Radio Engineers, June, 1923.

PART I

RADIO SYSTEM-DESIGN AND ESTABLISHMENT OF STATIONS

GENERAL PLANS

The fundamental condition laid down at the beginning of this work was the very general one that there should be developed a system by which any telephone subscriber of the Bell System could carry on a conversation with a telephone station located on a ship, and that, from the point of view of the speakers, the operation should be similar to the carrying on of an ordinary toll call between land wire subscribers. This, of course, involves the development of a satisfactory two-way radio telephone system for ship use. Furthermore, it was desired to be able to carry on three simultaneous and independent conversations between three ships and one land station, since a final commercial system will involve the establishment of several circuits simultaneously. These 2-way transmissions were to be obtained without employing an excessively large frequency band.

A rough study of the problem resulted in a decision to locate the experimental land stations about 200 or 250 miles (320 or 400 km.) apart and to try for reliable commercial transmission to ships at a distance of approximately 200 miles (320 km.).

The transmission problems involved in this work, which were different from those in wire telephone engineering, were:

(a) A much greater variability in the transmission equivalent to be expected in the radio link;
(b) A much greater and more variable interference, both natural and artificial;
(c) A lack of secrecy in the sense of a wire system;
(d) Greater possibilities of cross-talk between channels because of the use of a single medium;
(e) More complication in the matter of signaling and in the setting up of the telephone circuit.

The apparatus problems were, of course, entirely different from those of wire transmission and will not be considered in detail in this paper.

An engineering project of this kind divides itself naturally into two phases; that of the development in the laboratory of systems and apparatus which are technically suitable for the work and, second, the providing in the field of a model system, incorporating the knowledge obtained in the laboratory as a means for enabling the system to be tried out. A preliminary survey of the purely technical problems convinced us that the more important ones were the development of two-way radio telephone apparatus and of multi-channel systems which would operate from a single transmitting station without interference between channels; the design of transmitting apparatus which would satisfy the requirements and which could be built with the vacuum tubes available and the development of a type of receiving system which would provide sufficient selectivity to allow an economical use of the frequency range and at the same time fit in with the two-way system most likely to be adopted. It was decided that during the laboratory development work preparations should be made in the field for providing the necessary experimental stations. This field work as it developed included the location of station sites, the actual construction of the station buildings and the antennas, the equipping of the stations with the apparatus as developed in the laboratory and as further developed in the station, the equipment of the ships, and, finally, the operation and tests of the overall system.

SYSTEM DESIGN CONSIDERATIONS

In the beginning it was thought that to cover the required 200 miles (320 km.) range about one or one and one-half kilowatts in the antenna would be necessary. It was not known that wave lengths would be made available for this work by the Department of Commerce. To produce this amount of power in the antenna there were available Western Electric 250 watt tubes which it was decided to employ. The question then arose as to the particular type of transmission systems most suitable for the work. The points of importance in solving this problem are as follows:

The greatest economy both in power and in wave length range may be secured by transmitting only one side band of the modulated wave. Moreover, this method has the great advantage that variations in the transmission characteristics of the medium do not cause as great fluctuations in the received signal. This is because the received signal is proportional to the product of the carrier and side bands and if the carrier is supplied locally instead of being transmitted, it is not affected by transmission factors. The use of such a system, however, or of one in which only the carrier is suppressed, throw upon the receiving set the burden of maintaining a constant oscillator frequency not only complicating it but also making reception impossible for the great majority of ships which are equipped with only straight detectors. This would defeat general inter-communication in emergency. Further, it practically restricts the transmitting set to one in which the power tubes are used as amplifiers, and it was known that some difficulty might be experienced in operating a number of 250 watt tubes in parallel if it should be necessary to transmit at wave lengths as low as 300 meters. For these reasons and after some development work it was decided that the proper system to use in the first experiment was one in which modulation is carried on by the constant current method which requires about an equal number of modulator tubes and power tubes and sends out all components of the modulated wave.

The simultaneous transmission of three channels from the land station may be accomplished in several ways. It is possible, for example, to carry on such multi-channel operation from one antenna, which is multi-tuned, or from three separate antennas. The antenna power may be supplied by one system of tubes carrying all three conversations or the power tube system may be split into three parts. Also, using a single antenna simply tuned it would be possible to transmit the three channels from one system of tubes by a system of double modulation which had been installed by the Western Electric Company on United States battleships two or three years earlier. The difficulties which are likely to arise in these various schemes are as follows: The use of multi-tuned antennas involves loss of power in the circuits used to give the antenna three degress of freedom. The use of a single system of power tunes for three channels requires that the tube system be capable of handling a large overload at times without impairment of quality, since it is possible that the peaks of three channels may occur simultaneously. It was expected that under conditions of this kind there would be inter-modulation of the channels due to the modulating action in the plate circuits of the power tubes. The use of three separate antennas located very close to one another and tuned to frequencies differing by three or four per cent. might lead to such close coupling of the three channels that cross-talk and modulation of one channel by another would result, the latter by plate modulation of one set of tubes by the currents induced in its antenna. The use of the double modulation system—altho requiring but one radiated carrier—is open to the objection of overloading and cross modulating of channels and also to the objection that the receiving apparatus aboard ship must be more complicated. An analysis of these and other proposed methods of operation resulted in the decision to employ at that time three separate but closely adjacent antennas and three separate transmitting sets using the constant current modulation system. This choice was made because of conditions peculiar to this particular problem and to the vacuum tubes then available. Of course, improvements can be made in the system at the present time as a result of the information obtained in the development using the very much larger vacuum tubes now available.

These decisions, therefore, determined the general type of system to be used, namely, one in which many of the known advantages of single side band transmission were sacrificed in order to secure simple apparatus, to make use of then existing power tubes and to enable the transmission to be received generally.

The problem of securing the two-way operation necessary aboard ship and for combined radio and wire operation may be attacked in several ways. In general, there are three methods available:

(1) In which the east and west channels are established alternately and not simultaneously, by switching. The push-button scheme is a familiar example, although unsuitable for tying in with the wire telephone system. Another arrangement is the use of voice-operated relays to throw the terminal apparatus into the sending or reciving condition, depending upon the direction of transmission.

(2) The use of the principle of balance to separate the outgoing from the received transmission. The radio receiving antenna circuit is balanced with respect to the transmitting antenna circuit.

(3) Employment of different frequencies for the two directions of the two-way transmission, relying upon frequency-selecting circuits for affecting separation. The first two methods allow of operation on the same or on different carrier frequencies.

All of these fundamental methods were considered in their several possible embodiments, and compared from the standpoint of the conditions to be met in the radio system itself and in linking it with a public service telephone system. The system finally adopted employed different frequencies for sending and receiving and secured discrimination by frequency selection supplemented at the land station by a moderate degree of special separation and balance. By using sharply selective receiving circuits, a moderate frequency difference between east and west channels sufficed to give the necessary degree of separation.

PRELIMINARY TESTS

By the time these decisions had been made there was available for experimental purposes a plot of land near Cliffwood, New Jersey. It was decided to construct a model of the proposed antenna system on this plot and to operate small transmitting sets to determine the cross-talk and other important conditions. The antenna system decided upon consisted of three poles arranged in the form of an equilateral triangle supporting three antennas—one from the middle of each span to the transmitting shack at the center of the triangle. The dimensions of this model system were 50 meters (164 ft.) by 10 meters (33 ft.) high. Three experimental transmitting sets of small power were set up under the antenna system and studies were made of the interference produced between channels when all three channels were in use. Three receiving sets of the general form proposed were built and taken to a location near Elberon, New Jersey, about 16 miles (26 km.) from Cliffwood, at which place it was decided to locate the three-channel receiving station to co-operate with the New Jersey transmitting station a mile (1.6 km.) away.

In November, 1919, the first test of a three-channel system was held between Cliffwood and Elberon with the result that the receiving sets resolved conversations on carriers of frequencies of 725, 750, and 775 kilocycles without any cross-talk altho the received volume was so large as to be audible all over the room. This is a frequency difference between channels of approximately three per cent. A change in frequency to 747, 759, and 777 kilocycles resulted in a barely perceptible cross-talk on the middle channel, with no cross-talk on the others. These results indicated that the loop receivers which had been developed were sufficiently sensitive and selective to carry out the proposed three-channel work; and, altho a great deal of development work was done later on the receiving sets, the general principles were retained. It was found that some reliance must be placed upon the directional properties of the loop antennas, and considerable care was used to secure very sharp directional selectivity. This was done by compensating for the vertical antenna effect of the loop by a balanced connection to ground.

During the whole course of this ship-to-shore work very little trouble was experienced thru interference by continuous wave stations, even when their frequencies came within two or three per cent. of those to be received. We did, however, have much difficulty due to interference from spark stations, since they inherently occupy a wide frequency range.

PROVISION OF STATIONS AND DEVELOPMENT OF APPARATUS

During the time the model system was being constructed at Cliffwood, land had been purchased at West Deal, Monmouth County, New Jersey, for the permanent transmitting station. This station as it now appears is shown in Fig. 1. The permanent building was preceded by a temporary structure to house an experimental transmitting station which could be used as a model for the design of the four final sets to be located in the permanent building.

Preliminary studies were also made to determine the proper form to give to antennas suitable for three channel operation without excessive cross-talk, and this study indicated that by the use of series inductance and capacity the antennas could be stiffened enough to prevent excessive coupling effects and still pass the required frequency band.

While this work was going on, a two-way telephone set for use aboard ships was developed, and in the spring of 1920 one of these sets was installed aboard the steamship *Ontario* of the Merchants and Miners Transportation Company. Experimental communication with this ship, by means of the model transmitters at both Deal Beach and Green Harbor stations, showed that commercial operation, at least for one channel, could be maintained.

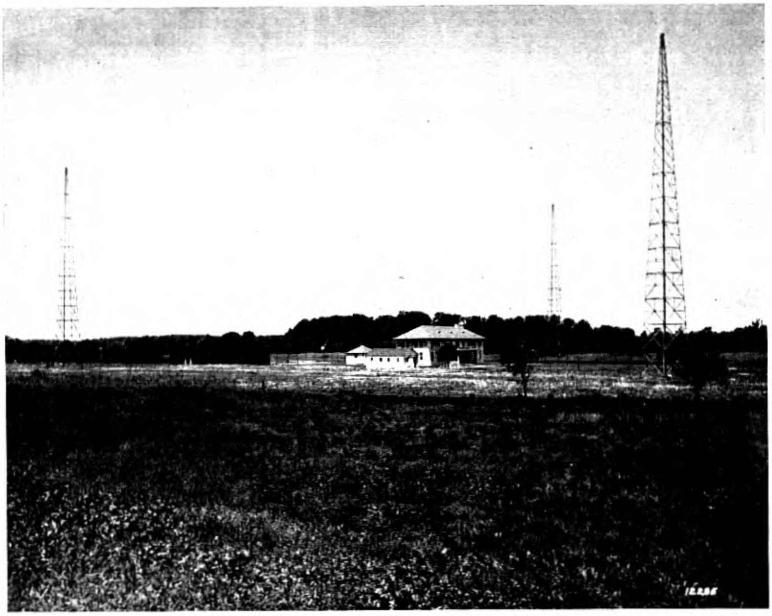

Fig. 1

By the fall of 1920, the construction work on the four transmitting and receiving channels was completed and early in December a demonstration of simultaneous three-channel operation from this station to ships was carried out with satisfactory results.

Fig. 1 shows a general view of the outside of the transmitting station at Deal Beach. The three steel towers form an equilateral triangle of sides five hundred feet (150 m.) and each is one hundred and sixty-five feet (50 m.) high. Steel cables to support the antennas are strung between these towers and also three cables extending inward support a fourth antenna which rises directly from the building in the middle of the triangle. One antenna goes to the middle of each of the first mentioned steel cables, so that there are a total of four transmitting antennas. One of these is intended for use at six hundred meters. The building is thirty by ninety feet (9.1 by 27.3 m.) and two stories high. The southern half comprises the operating room which rises two full stories. The other part of the building is taken up by an office, shop, power room, living and dining room and kitchen, and by six bedrooms.

DESCRIPTION OF THE EXPERIMENTAL RADIO STATIONS

The system as developed at Deal Beach consists of four transmitting sets, operating into four separate, altho naturally coupled, antennas, one set and antenna being intended primarily for 600 meter calling and for emergency. The receiving station co-operating with Deal Beach is located about a mile (1.6 km.) north of that station and contains four receiving sets receiving energy from four loop antennas. The transmitting sets are capable of putting about one kilowatt of modulated radio frequency power into each antenna and are controlled from a telephone switchboard into which run trunk lines from New York City. A ten-pair telephone cable connects Deal Beach and Elberon and another telephone switchboard at Elberon permits the transfer of received signals back to the wire line. The radio station operates, therefore, generally as a telephone repeater arranged for two-way operation with two repeaters. At the ship stations, because of the small amount of space involved, transmitting and receiving was accomplished on the same antenna at different frequencies in the two directions. Because of the better receiving conditions on the shore the proper transmission balance was obtained by making the output of the ship transmitting set about one-quarter that of the land station.

The general principle of operation of one channel of the wire-to-radio repeater will be described from the schematic circuit diagram of Fig. 2, which shows, in the dotted blocks, one channel of the transmitting station, a ship station, and one receiving set. At the transmitter station the master oscillator, very carefully shielded to maintain constant frequency, operates into a two-stage amplifier, the last stage being fifty watt tubes, and from there into a bank of six radio frequency power tubes, each with a rating of 250 watts plate dissipation. Speech to modulate this radio frequency output enters from a telephone line and is applied to a speech amplifier the output of which operates into a bank of 250 watt modulator tubes in parallel. Thus both the radio frequency and the speech frequency currents are brought up to the high power level before modulation takes place. The six radio frequency and six speech frequency tubes have their plate circuits connected together and operate as a constant current modulation system. Thus current of the frequency F_1, generated by the master oscillator and amplified and modulated, is radiated from the antenna. The notation $F_1 \pm S_1$ indicates the radiation of the carrier and two side bands from this antenna. The incoming speech S_1, as it comes from the telephone line, passes through the hybrid coil and to the balancing network shown. This balancing network has an impedance characteristic similar to that of the incoming line, and the combination of hybrid coil and network is similar to that used in telephone repeater practice to secure two-way operation. The object of this arrangement is, of course, to prevent signals, coming in from the receiving station, operating upon the transmitter of the outgoing channel. If the balancing network is an exact picture of the incoming line and if the hybrid coil is properly made, incoming

TWO-WAY RADIO-WIRE SYSTEM

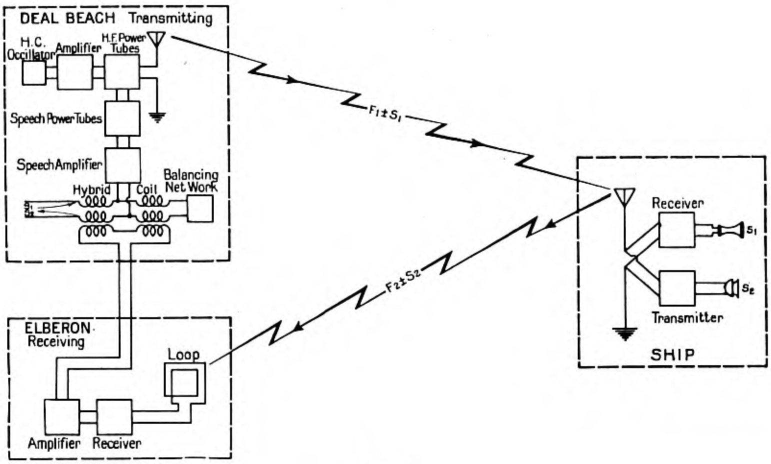

Fig. 2

signals for transmission west on the telephone line will produce no voltage at the terminals of the transmitting amplifier.

At the receiving station the incoming wave is impressed upon a loop antenna and the receiving set. The resulting detected output is then amplified as indicated and returns to the hybrid coil, passing out on the telephone line without producing a voltage on the speech amplifier of the transmitting set if the hybrid coil balance is perfect.

On the ship, this physical separation of transmitting and receiving set is, of course, not practical, and, as indicated before, transmission and reception take place upon one antenna so arranged that the receiving circuit offers a high impedance to currents of the outgoing frequency and low impedance to the incoming signal. Actually, the outgoing signal is not entirely excluded from the ship's receiver and there is present a side tone of about the magnitude of the incoming signal. This is by no means an undesirable condition and is the one which holds approximately in an ordinary telephone subscriber's instrument. The presence of side tone assures the speaker that his system is functioning properly.

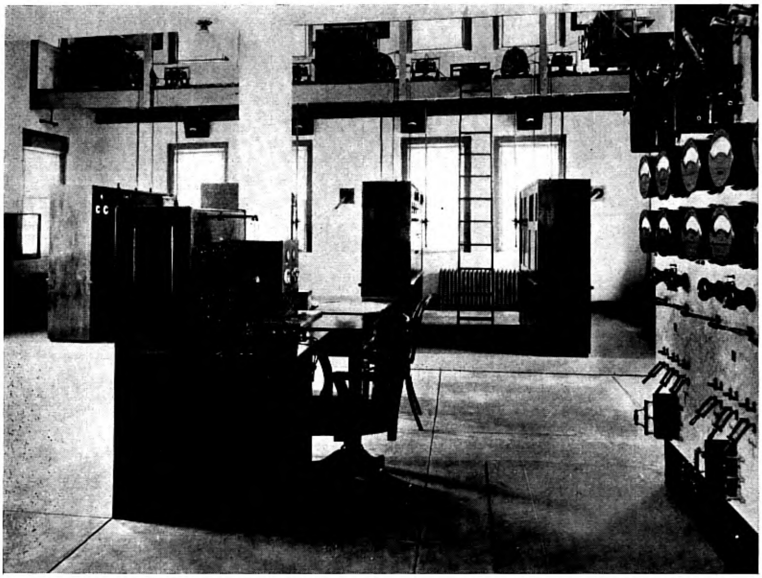

Fig. 3

Fig. 3 is a view of the operating room showing the four transmitting units at the back; the power switchboard for supplying the plate circuits of the tube at the right; the telephone switchboard for the four speech or telegraph channels in the center; and on the gallery above the transmitting units, the coupling coils, loading inductances, and so on, between the sets and the antennas. The motor generator sets capable of supplying as much as five kilowatts at eighteen hundred volts to each of the transmitting sets are located in an adjoining room and controlled from the operating room.

Fig. 4 shows the interior of one of the transmitting units. In the shielded box at the upper right hand corner is the master oscillator which sets the frequency to be used for that particular channel.

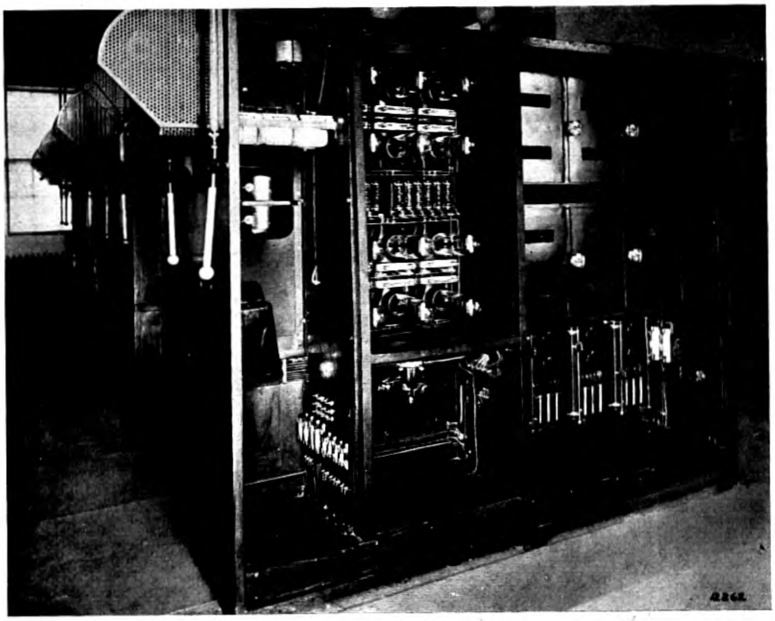

Fig. 4

Next to the left are two more shielded compartments, each of which contains an amplifier. The last stage of this amplifier employs a fifty watt tube. In the larger unshielded compartment are located above, the six radio frequency power amplifier tubes. The reason for introducing two amplifiers between the master oscillator and the power tubes is to prevent any reaction from the attenna circuit back to the master oscillator. By taking this precaution the frequency of the master oscillator never varies more than fifty cycles in eight hundred thousand. The lower set of shielded compartments, at the right, contains the audio frequency telephone amplifiers which supply currents to the six modulator power tubes shown in the lower par of the open compartment. These two sets of six tubes each are connected together to secure constant current modulation. The output of these twelve tubes is led to terminals on the output of the transmitter unit at the left. To secure cooling in hot weather, a fan is installed below the power tube compartment. In the extreme left compartment are shown choke coils in the power circuits, and at the extreme left on the outside are circuit breakers and two handles for operating the tuning and coupling apparatus in the gallery above.

Fig. 5 shows one set of radio frequency apparatus in this gallery. The two inductometers at the left are for coupling and tuning, and the large condenser at the right, the plates of which consist of brass frames covered with copper window screen, is inserted in series with the antenna. This capacity together with the inductance immediately above it stiffens the antenna circuit and increases the frequency selectivity to prevent radio frequency interaction between the several antennas.

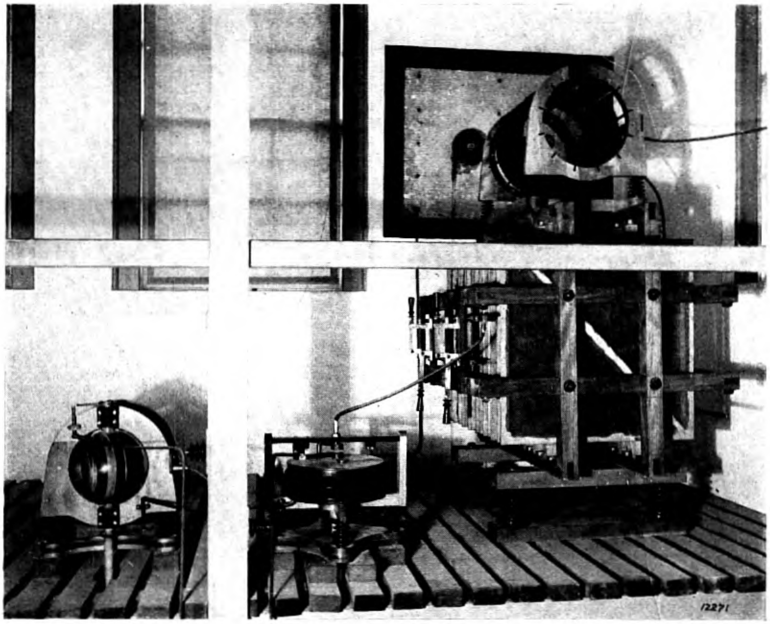

Fig. 5

The telephone switchboard shown in Fig. 6 is a special type of P. B. X. (private branch exchange), constructed to provide the necessary shielding and to include telegraph oscillator, phantom coils, and other special apparatus. This switchboard provides for four channel telephone or telegraph operation and for the control and monitoring of all channels. In the operation of the system one operator, located at this switchboard, has complete control of the entire transmitting plant. The operating board was especially built for the experiments, and altho not the final form contains features which are of interest in that they illustrate well the technique involved in combined wire and radio operation.

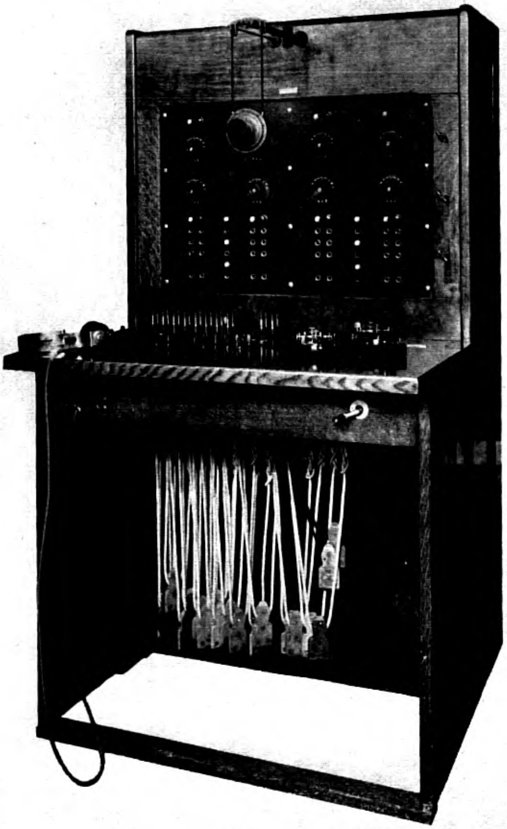

Fig. 6

The four vertical rows of jacks correspond to the four two-way radio channels. At the top of each row will be seen the dials for controlling amplification. On the apron are telegraph keys, telephone keys, and operating cords. The cord circuits, by being plugged into the jacks, interconnect any one of the New York toll circuits with

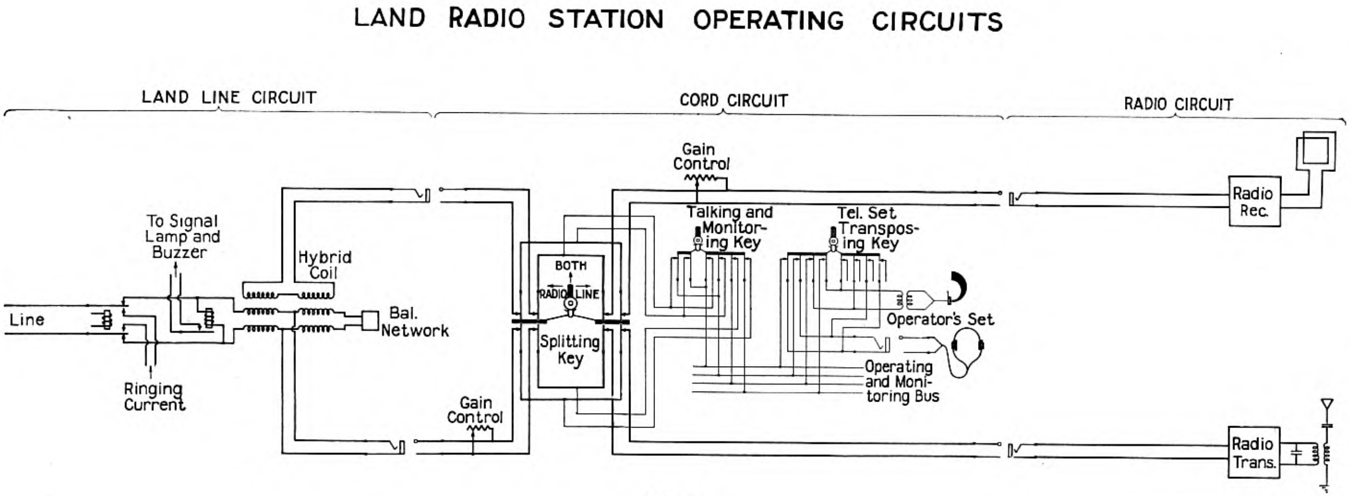
LAND RADIO STATION OPERATING CIRCUITS
LAND LINE CIRCUIT
CORD CIRCUIT
RADIO CIRCUIT
Line
To Signal Lamp and Buzzer
Ringing Current
Hybrid Coil
Bal. Network
Gain Control
BOTH
RADIO
LINE
Splitting Key
Gain Control
Talking and Monitor-ing Key
Tel. Set Transpos-ing Key
Operator's Set
Operating and Moni-toring Bus
Radio Rec.
Radio Trans.

Fig. 7

any one of the four radio circuits. The cord circuits contain the switching keys seen in front, by means of which the radio station operator is enabled to split the circuit and talk either way, connect the circuit thru and bridge on it and talk or monitor. This cord

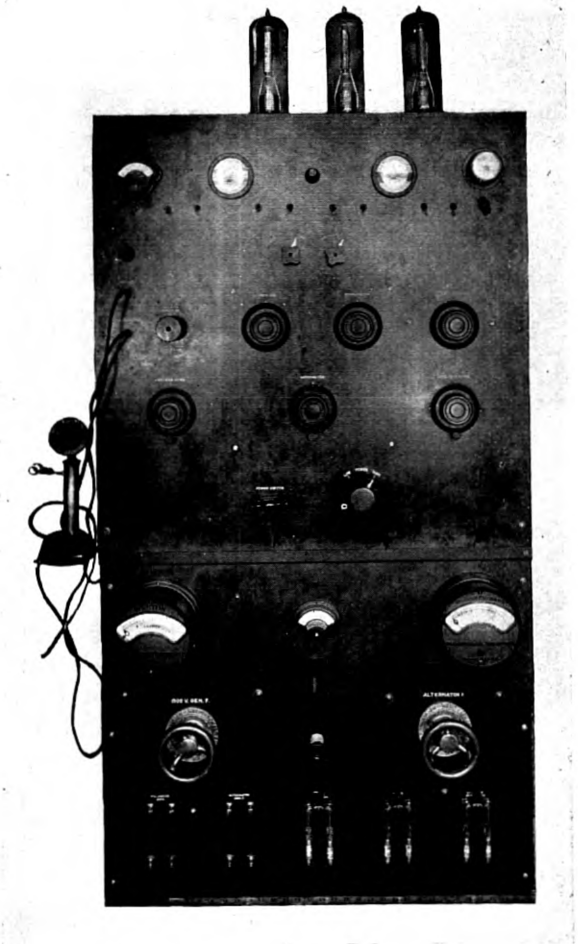

Fig. 8

circuit is shown in Fig. 7 in relation to the rest of the wire-radio junction circuit. It will be seen to be of a four-wire instead of the more usual two-wire type and to comprise in reality two circuits, one for east-bound and the other for west-bound transmission. This arrangement was used in order to obtain flexibility in the experiments. It enables the circuits to be continued inland as four-wire circuits and permits of the switching operations being carried out with a minimum effect upon the 2-way balance of the transmission system.

The receiving station at Elberon is located on a rented plot of ground and was not built in permanent form, since we did not regard this location as entirely suitable for receiving from the Atlantic. Reception is carried on the four channels by means of four loop antennas operating into four receiving sets. A telephone switchboard similar to that at Deal Beach provides for the connection to the wire system. Of course, two telephone switchboards are not necessary but one was installed at each station in order that we might determine by operating tests whether the control of the system should be from the transmitting or the receiving station.

The receiving sets as finally developed were extremely selective and pass only a band of speech width with a large attenuation outside this band. They will be described in another paper.

Fig. 8 shows a front view of one of the experimental transmitters used aboard ship. The lower half consists of power control apparatus. Three 250 watt tubes are used of which one is a master oscillator, one a power amplifier and one a modulator. The large capacity tube was used as a master oscillator and only a very small part of its output applied to the second power tube. This was done in order to prevent reaction of the antenna system upon the oscillator.

Apparatus of this type was installed on the *Ontario* and *Gloucester* of the Merchants and Miners Line, and operated in conjunction with Deal Beach and Green Harbor. Later another electrically similar set was built by the General Electric Company and was installed and operated by the Radio Corporation on the steamship *America*. This installation is illustrated in Fig. 9.

PART II

OPERATION OF THE COMBINED RADIO AND WIRE SYSTEM

The development work as described in Part I had resulted in establishing an experimental ship-to-shore radio telephone plant of some proportions. This will be seen by reference to the accompanying map of Fig. 10 which gives a picture of the field setting, as it were, of the experimental operations. The experimental plant included:

Two operating shore stations—Deal Beach, New Jersey, and Green Harbor, Massachusetts.

A field experimental station at Cliffwood, New Jersey.

Two ship installations, on the S. S. *Gloucester* and the S. S. *Ontario.* The vessels operated between Boston and Philadelphia or Baltimore

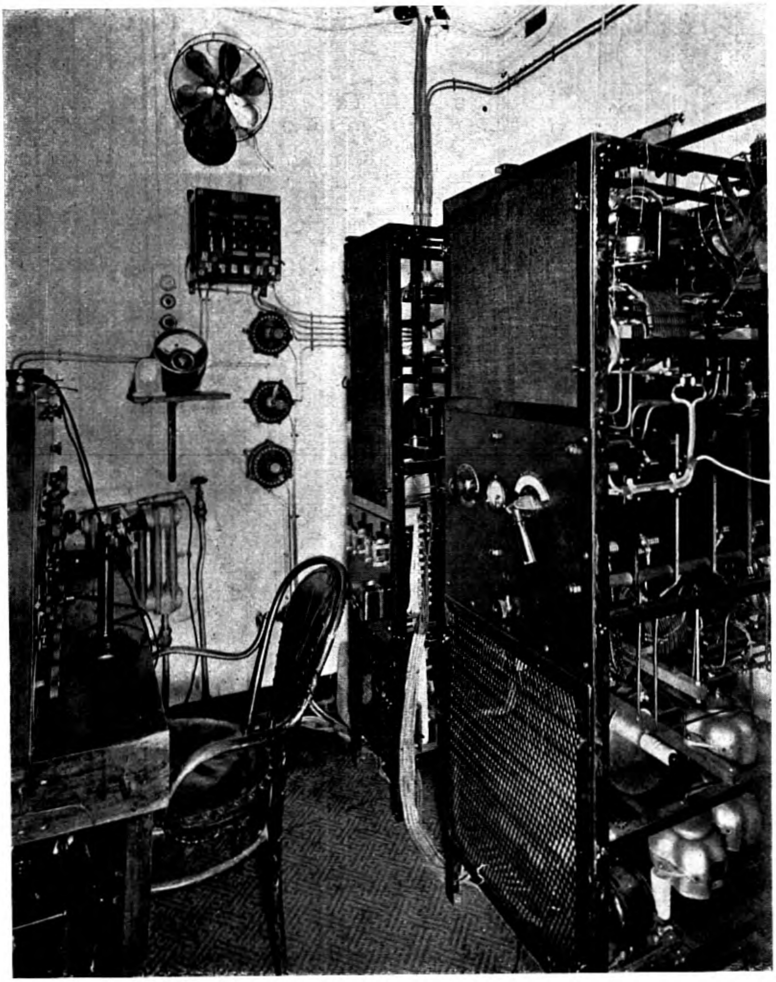

Fig. 9

and took any of several courses, two representative ones of which are as plotted in the figure.

Let us now consider this plant from the communication standpoint and look into its characteristics, first as an electric transmission system, and then as a message handling facility.

Each of the two land stations was tied into its nearest center by wire circuits, Deal Beach to New York and Green Harbor to Boston. We will take for our example the New York-Deal Beach-Ship circuit pictured in Fig. 11 and shown diagrammatically in Fig. 12. This is a combination of wire-radio toll circuit, one end of which terminates on a vessel of variable position and the other end of which is capable of being extended either over a local circuit to a New York subscriber or over a long distance circuit to reach subscribers at more distant inland points.

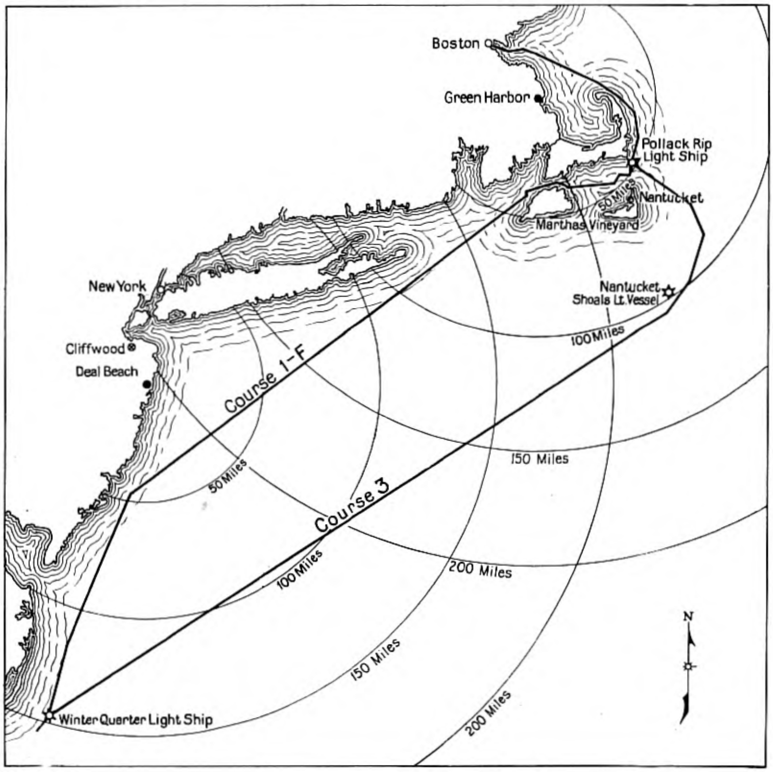

Fig. 10

This communication circuit must fulfil two general requirements. In the first place it must be so constituted electrically as to preserve the feeble voice currents launched upon it by one subscriber so that they be rendered to another person with sufficient volume and fidelity of wave shape as to be readily intelligible. This requires that the circuit be properly engineered as an electrical transmission network.

Secondly, given a circuit capable of talking, it is necessary that this circuit be flexible in use so that it can be put at the disposal of any land line subscriber for connection to a ship at sea, at any time the ship is within range. This requires that the proper switching facilities be provided and brings in operating and traffic problems.

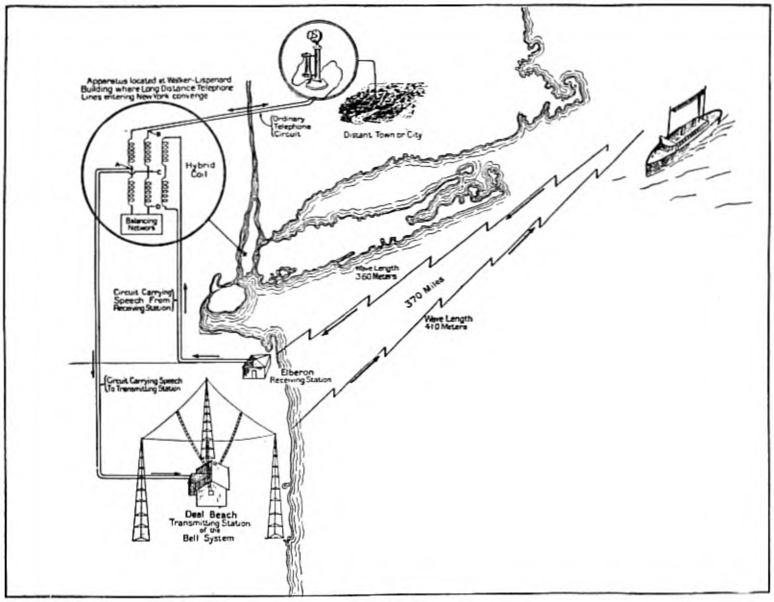

Fig. 11

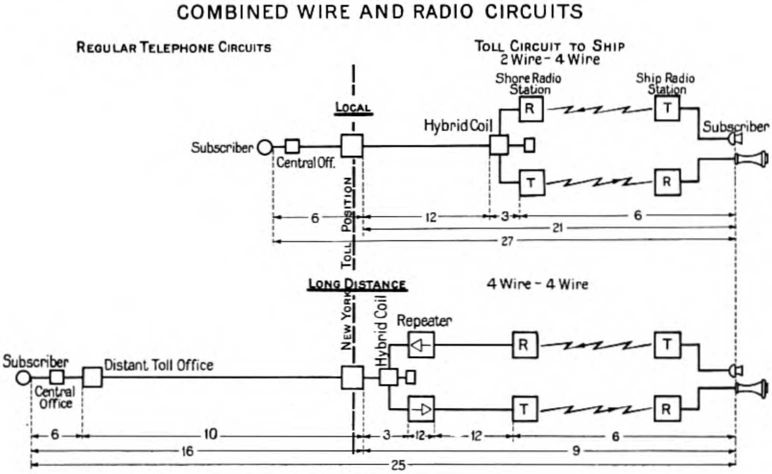

Fig. 12

THE RADIO-WIRE TRANSMISSION CIRCUIT

Two types of circuits were used in the experiments in operating between New York and the vessel, as shown schematically in Fig. 12. The radio link is the same for both. Different frequencies are used for transmitting in the two directions so that in the radio link we have the equivalent of two circuits, one for transmitting east and the other for transmitting west. This is the same as the four-wire telephone circuit. In the thru circuit first shown the radio four-wire circuit is brought into a two-wire circuit at the land radio station, making a regular two-wire telephone circuit from the radio station to the New York toll terminal. This circuit was used in the tests for calls local in New York. In the arrangement of the second circuit of Fig. 12, each of the one-way radio channels is extended back to the New York toll office by its own wire line and there joined into a two-wire circuit, making the wire-radio toll line from New York to the ship a four-wire system. This arrangement forms a high grade circuit and is the one used in the experiments for connecting with long distance lines.

Taking the four-wire circuit, the path of the voice currents may be traced thru from one end to the other as follows: The currents which are initiated by the land subscriber, for example, upon arrival at the New York toll office divide in the hybrid coil between the east-bound and the west-bound circuits. The currents are prevented from being propagated over the west-bound branch because of the unilateral nature of the repeater. The currents of the east branch are amplified in the repeater of this circuit in order to make up for the attenuation suffered in the cable circuit, and upon arrival at Deal Beach are amplified to power proportions, modulated upon the radio carrier and radiated into space. Upon being received at the ship end of the radio circuit they are sharply selected in respect to frequency, are again amplified, and delivered to the listener. When the ship subscriber talks, the voice currents are amplified, pass directly into the radio transmitter, are transmitted over the radio link in the usual way, received at the shore station, amplified and sent out over the wire circuit to New York. Here they pass in the reverse direction thru the hybrid coil and divided between the two-wire circuit on the one hand and the balancing network on the other, thus getting back into a regular two-wire telephone circuit.

INTERCONNECTION BETWEEN RADIO AND WIRE CIRCUITS

It will be well to recall at this point just what it is that makes possible automatic repetition or thru transmission between the wire and radio circuits.

There is both an outgoing and an incoming radio channel. The automatic repetition from the wire to the outgoing radio channel is made possible thru ability to control the transmitter wave power by the voice currents set up at the distant end of the telephone line. It will be recalled that in the early radio telephone art, before the vacuum tube, modulation was effected by the microphone transmitter which required that the talker be present at the radio station. It is, therefore, the electric-control type of modulator such as the vacuum tube, as distinguished from the air-wave control modulator, which permits of the talker being at the far end of a wire circuit. Conversely in the receiving channel, it is the fact that the detecting action yields telephone currents directly, ready for propagation over a wire circuit, that enables the radio channel to be extended to a distant listener.

Thus it is the thermionic tube modulator and detector which have made possible the radio-wire transfer. It is the thermionic tube as a reliable high-quality amplifier, however, that makes the transfer practical; for it is the amplifier which enables the weak voice currents received at the radio station from a land line subscriber to be boosted to power proportions and thus control the considerable radio frequency power required for transmission; and, again, it is the amplifier which enables the extremely weak currents received from the radio link to be so augmented that upon being placed upon a wire circuit, and perhaps being further amplified en route, they may be heard in the regular telephone at the other end.

The other important feature of the radio-wire inter-connection is the junction of the four-wire and the two-wire circuits by means of the hybrid coil and balancing network as shown in Fig. 11. The windings of such a coil are so designed as to establish a sort of Wheatstone bridge circuit. This bridge circuit accomplishes the joining of the regular two-wire telephone circuit with the sending radio channel on the one hand and the receiving radio channel on the other, while still maintaining an electrical separation between the two radio channels. It is really, therefore, the connecting link between the two-wire type of circuit of the telephone plant and the four-wire circuit of the radio link. The hybrid coil type of circuit is taken from the telephone repeater and carrier current art.[2] The radio receiving circuit corresponds to the generator branch of the Wheatstone bridge, and radio transmitting circuit to the detector branch. The

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[2] "Telephonic Repeaters," B. Gherardi and F. B. Jewett, *Journal of American Institute of Electrical Engineers*, pages 1255–1395, November, 1919.

"Carrier Current Telephony and Telegraphy," E. H. Colpitts and O. B. Blackwell, *Journal of American Institute of Electrical Engineers*, pages 205–300, February, 1921.

two-wire telephone line corresponds to the "X" arm of the bridge and the balancing artificial line to the "Y" arm. The ratio arms are in effect formed by the windings of the hybrid coil.

Speech Received from Ship Re-Transmitted from Shore Station

Now this junction circuit always has some unbalance because it is obviously impossible to maintain a perfect symmetry between the telephone line and the balancing network. Especially is this true where the telephone line is a type not designed for repeater operation and is switched at its terminal to any of a number of lines of different impedances, as was the case with the circuit used in the tests.

This unbalance between the line and its balancing network will be seen to permit some of the speech-current received over the radio link to get across into the transmitting circuit, to modulate the shore station carrier and to get out into the ether again on the transmitting wave length. As a matter of fact during the experiments the unbalance was sometimes such as to permit of fairly strong transmission around back thru the shore transmitter, so that incoming speech was repeated out thru the shore station transmitter in amplified form. This enabled listeners in the vicinity of New York to hear the conversation originating on the ship almost as well as that originating on land, and they naturally thought that they were picking up the ship's radio transmission directly, whereas they were actually overhearing the re-transmission of the shore-station's reception.

The Thru Circuit as a Repeated Telephone Circuit

This re-transmission makes all the more evident the true role of the shore station, namely, that of a large telephone repeater between two sections of line, the one a land line and the other a "space" line, and functioning also to convert between the voice frequencies of one section and the radio frequencies of the other. As such, we can consider the over-all circuit from a transmission standpoint much as we do long distance repeated telephone circuits.

Now one of the most important transmission considerations in such a long distance circuit is that of how the amplification is applied in relation to the losses in the circuit. This question of amplification is particularly important in the case of combination radio-wire systems, because the radio circuit possesses inherently large transmission losses and requires correspondingly large amplification. The necessary large amplification is supplied at both ends of the radio link, partly in the transmitting station where the voice currents are amplified up to power proportions and partly at the receiving end where the amplification is likewise large altho at small power. In the radio telephone circuits which were operated in the experimental work the power in the sending antenna to that in the receiving antenna is in the ratio of roughly 10^{10}. This requires amplification which was distributed somewhere near equally between the sending and the receiving ends. It has been found convenient to express such transmission losses in terms of a power ratio using $10^{0.1}$ as a unit.[3] Thus the above antenna to antenna power ratio would correspond to 100 of such units.

CIRCUIT TRANSMISSION EQUIVALENTS

It is necessary that the amplification of such a circuit be sufficient to offset very closely the loss, in order that the net loss be small. Actually, in the tests, the radio portion of the circuit was worked with a net transmission loss of about six units, meaning that at least 95 per cent. of the radio over-all circuit losses were wiped out. This means that if a change of say 10 per cent. occurs in the amplification, or in the ether loss as by fading or movement of the vessel, the circuit equivalent will be greatly affected—changed by about 200 per cent. The difficulty of maintaining the ship circuit stable will therefore be appreciated.

Fig. 12 shows the transmission loss (of six units) obtained for the radio link during the tests and also the other losses which are in the wire portion of the combination system. The distribution of losses in the first circuit will be noted to be approximately as follows:

6 units in the radio link.
3 units in the hybrid coil—balancing network.
12 units in the wire circuit to New York.
6 units from the New York central office to the subscriber.

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[3] The unit used in this paper is one which has been found convenient in which expressing the transmission loss or gain of a circuit. One unit is taken as that power ratio which is equal to $10^{0.1}$. Thus, if the attenuation or amplification of a circuit is one unit the power at the two ends are in ratio of $10^{0.1}$; if ten units, in the ratio of $10^{10\times 0.1}$ or 10; twenty units would therefore have a power ratio of 100, and so on. The advantage of using a power ratio instead of a current ratio is that it is undependent of the impedances of the two portions of the circuit considered. The advantage of expressing the power ratio as an exponent is that on account of the exponential nature of attenuation it enables the net transmission efficiency of a system to be readily derived by algebraically summing up the individual losses and gains. This unit has been selected as more suitable for general use in expressing transmission efficiencies than the 800 cycle "mile of standard cable" which has sometimes been made. The ratio between these two units is as follows: 1 mile of standard cable equals 0.95 units as used in this paper. $\frac{P_1}{P_2} = 10^{0.1} = \frac{1}{0.95} \epsilon^{(0.109)}$ units.

This makes a total loss between subscribers of 27 units which is satisfactory for a good talk under fairly quiet conditions. This equivalent was usually realizable under the conditions of the test and the majority of the calls put thru from local stations in New York with the ship 100–200 miles (160–320 km.) out were successful despite occasional spark interference in the radio circuit.

The transmission loss is, however, too high in such a circuit to enable it to be extended inland over long distance wire circuits. If this is attempted, two limitations come into play. In the first place, the volume of the talk becomes too weak. If the call were extended over a toll circuit having a 10-unit equivalent, for example, the overall equivalent would become something like 37 units, which is excessive. This could be overcome to some extent by a cord circuit repeater at New York. A second limitation which existed in the experimental set-up resided in the unbalance between the line and the balancing network at the radio station. This unbalance permitted currents received over the radio link to be fed back thru the radio transmitter of the land station, as described above. These fed-back currents overload the radio transmitter if they are large compared with the currents being supplied to the radio transmitter from the shore subscriber. In other words, if there is sufficient amplification in the shore transmitter to enable very weak voice currents arriving over a line of high equivalent to load the transmitter fully, then the transmitter is likely to be overloaded by currents which get through the hybrid coil from the associated radio receiver. For these reasons, the two-wire-four-wire circuit of Fig. 12 is not good enough for extension over long distance circuits.

The four-wire type of circuit which is suitable for long distance land line connections is shown in the second diagram with representative transmission equivalents. A brief comparison of the two-wire and the four-wire circuits will make it evident why the four-wire circuit gives the better equivalent. It enables the land line loss between the radio station and the toll center to be more or less wiped out, thus in effect placing the radio station electrically at the toll center. Another way to express the situation is this: regard the hybrid coil unbalance as the limiting factor, then assume that, while holding to a given unbalance, the four-wire circuit (the loss in which can be largely wiped out by one-way amplifiers) is extended inland. The length of the remaining two-wire line back to the land subscriber is thereby decreased and the ratio of the current received at the radio station over the line as compared with that transmitted across the hybrid coil thru unbalance is increased. It will be observed that with the circuit conditions as illustrated, the over-all equivalent between, say, a Chicago subscriber and a ship, including a 10-unit toll circuit loss, is approximately 25 units, which should give a good talk.

POWER LEVELS AND INTERFERENCE

It is necessary that the magnitude of stray currents be so kept down in comparison to the transmission currents thruout the system as to obviate noise interference with telephone conversation. This requirement is particularly difficult of realization in the radio link because of static and, especially in the vicinity of New York, interference from spark telegraph stations. It is, of course, this interference, caused by the presence in the ether, on the wave length band being used, of extraneous wave components, which sets the actual range limit of the radio link. Actually it was found that in transmitting on about 400 meters in the vicinity of New York the receiving field strength could not be permitted to go on the average below about 200 micro-volts per meter, and even then the spark situation is so bad in the present art as to give periods of prohibitive interference. In less congested zones along the coast to the north, probably lower field strengths could be permitted.

TRANSMISSION VARIATIONS IN RADIO CIRCUIT

One of the outstanding transmission characteristics of a ship-to-shore radio telephone system is the variation which the attenuation of the radio link undergoes as a result of the movement of the vessel. In order to determine the magnitude of these variations, a series of measurements were made of the telephone transmission over the radio circuit as the vessel proceeded on her course.

The method of making these measurements is shown schematically in Fig. 13. Take for example the case of measuring a one-way circuit as distinguished from a circuit looped back. A 1,000-cycle current of predetermined power of the order of one miliwatt is impressed upon the input circuit of the radio transmitter. This tone is received in the output of the distant radio receiver. There it is passed to the measuring apparatus where it is amplified, rectified, and made to operate an indicating instrument. The receiving end measuring apparatus is then switched to a local source of 1,000-cycle current giving the same power as was applied to the transmitter at the sending end. The proportion of this power which enters the measuring apparatus is then varied by a variable network calibrated in power ratio loss (it was actually in miles of standard cable) until the indicator reading is the same as that obtained from the radio receiver. The setting of the variable network then indicates the transmission loss of the circuit. The method is similar to that developed for measuring

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mile. This was one of the cases where the vessel was well south and close in shore and, of course, represents very poor transmission. It means that with the vessel traveling as slowly as ten miles (16 km.) per hour, a transmission change of about five units per hour will occur. In telephone practice it is desirable to keep the transmission equivalent constant to within two or three units, so that this condition would require re-adjusting the amplification as often as every half hour.

These curves show the necessity for so designing

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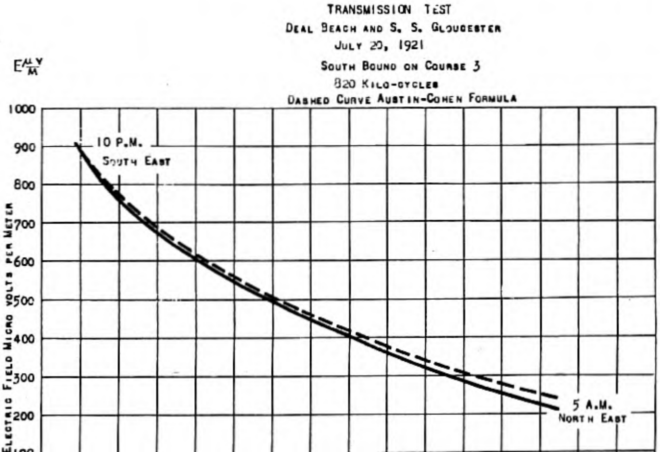
portion of this curve is for straight-out-to-sea transmission where the attenuation law is seen to be normal.

In the field strength measurements made during the ship-to-shore development, those of the S. S. *America* en route across the Atlantic are especially illuminating. The results are given in Fig. 20. The vessel was in-bound so that the curve develops from the right to the left, altho the effect is just the same as if it developed in the reverse direction with the vessel out-bound as was shown by another set of measurements which gave generally similar results. The actual measurement results are indicated by the points and by the con-



as during the best times at night. These enormous day to night fluctuations are now familiar to broadcast listeners. This curve shows the impossibility of giving continuous ship-to-shore telephone service at these relatively short wave lengths for distances as great as 1,000 miles (1,600 km.). For such distances much longer wave lengths will be required, as well as more sending power.

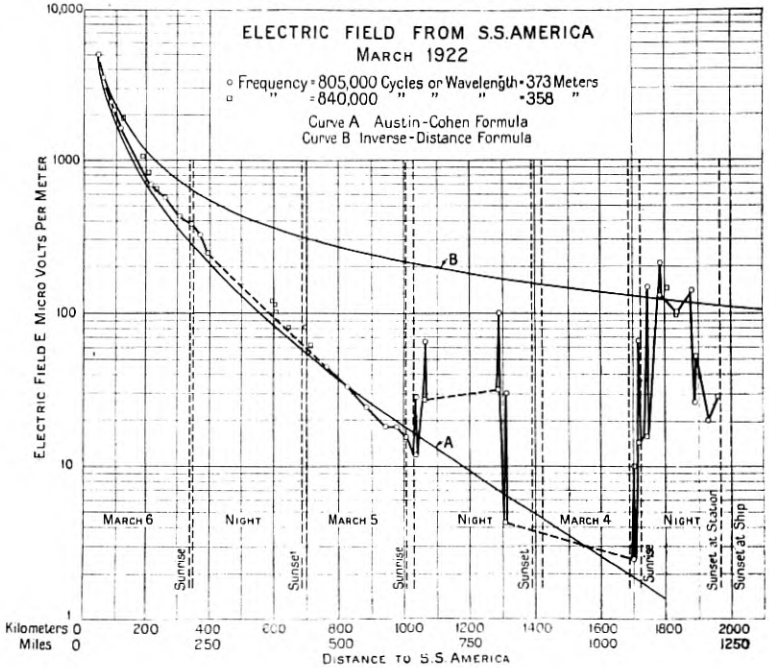


Fig. 20

(2) The wide fluctuations which occur thruout the night period. Altho smaller than the day to night fluctuations, their effect upon transmission is still very large. The fluctuations during the third night out, for example, are as much as 10:1 in field strength or 100:1 in power, or about 20 of the power ratio units we have used above. In view of the rapidity with which these fluctuations occur—within a very few minutes—it is practically impossible to maintain a circuit under these conditions satisfactory for regular telephone service.

(3) The most interesting thing to observe is that the fluctuations tend to fall within the two curves A and B. The day trans-

mission is a pretty definite proposition, following closely the Austin-Cohen formula. The night transmission appears in the nature of a "bob-up" from the day condition but seems to be limited in the extent of its "come-back" by the loss imposed by the simple inverse-with-distance law. The fact that the difference between curves A and B is entirely one of absorption suggests that the very large and rapid night fluctuations, which are now so well known to broadcast listeners, may be explained in large part if not in whole, by variations in atmospheric absorption.

Occasional Long Distance Transmissions

Many of the long distance records which have been made on short waves and low power can be accounted for simply on this basis—that the absorption which ordinarily obtains during daylight has been temporarily wiped out. The way in which it is possible for the range to "open up" tremendously under exceptionally favorable conditions will be seen from this: Referring to Fig. 20, assume that the normal daylight range between S. S. *America* and New York was 250 miles (400 km.) as fixed by a limit taken as 200 micro-volts per meter. Then, at night, this same field strength may be delivered over a distance of about 700 miles (1,100 km.) if the absorption is wiped out in accordance with curve B.

Furthermore, so favorable is the simple spreading-out law at such distances, that the field strength is only halved in going another 700 miles to 1,400 miles (2,200 km.) and only halved again in doubling this distance to 2,800 miles (4,500 km.), and so on. In other words under no-absorption conditions, by increasing the receiving radio frequency amplification by a current ratio of only $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$, or about 12 power radio units, the range of transmission may be increased from the reliable daylight range of 250 miles (400 km.) to a possible night range of ten times this distance. It is therefore seen that many if not all of the long distance transmissions which have been realized for short periods of time probably can be explained simply on the basis of there having occurred an exceptional clearing up of absorption at a time of unusually favorable interference conditions.

Setting Up and Operating Combined Radio-Wire Circuits

The operating problems presented by the combination wire-radio telephone system are more difficult than those involved in the operation of either a straight telephone toll line on the one hand or the ordinary radio-telegraph circuit on the other. In regular long distance telephone circuits we have a fixed type of system which is maintained continually in good talking condition and the operators turn the terminals over to the use of the subscribers themselves. On the other hand in a radio telegraph circuit operating between land and vessel the circuit is kept entirely in the hands of skilled operators who have access to the apparatus and who handle the traffic directly between themselves. In no case before have we had the requirement of taking a radio link of varying length, building it up as occasion requires with wire circuits and, upon call, putting the combined system at the disposal of people experienced only in the use of the regular telephone. The technical difficulties of the combination system, together with the necessity of coming as close as possible to meeting telephone standards in the quality of talk given, greatly reduce the length of the radio link which can be used for a service as compared with those distances which can be spanned for short periods of time under the most favorable conditions. The effect which the requirement of reliability has in reducing the range of transmission will be appreciated from the discussion of field strength measurements.

SETTING UP A TELEPHONE CONNECTION

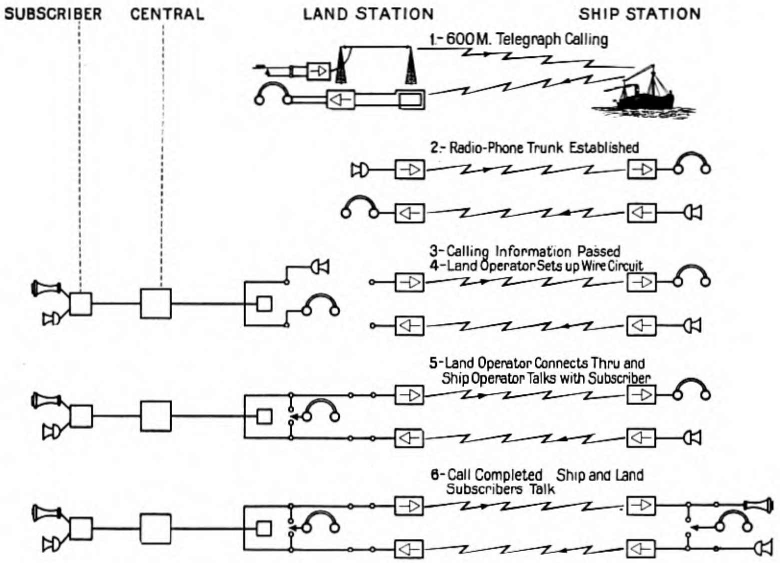
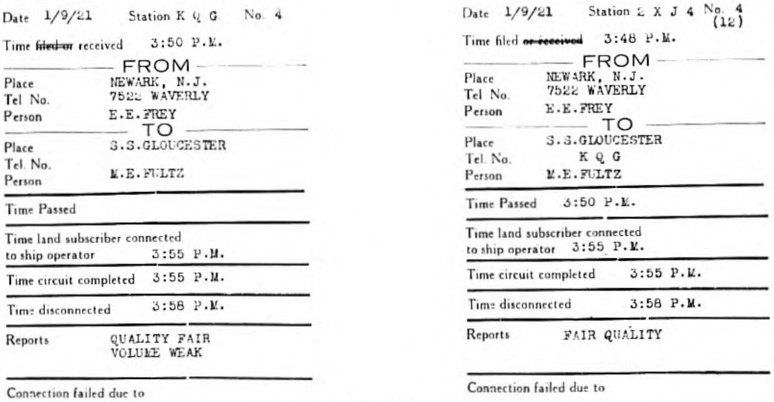


Fig. 21

There are various ways in which the combination circuit can be set up and operated and it will take further experience before the most satisfactory arrangement is determined upon. In order to explain the operation generally, however, we will describe how the circuit was actually set up during the tests. Take the case of a call originating on the vessel; then the procedure is as illustrated in Fig. 21, namely:

1. When the ship comes within range she calls the land station by telegraph on 600 meters, and informs the land station of her message business.
2. The land station then assigns a pair of telephone channels to which both stations switch over and the circuit tested out for talking. In case of important long distance land line connection, this test may involve circuit transmission measurements.
3. The ship operator then passes to the land operator by voice (or by tone modulation telegraph) the information as to the connection desired.
4. The land operator then tells the ship operator to stand by while he switches to the wire circuit and passes the call to the telephone central, who in the case of a local call is a local operator (actually, for this case, she was the operator on the Cortlandt Official Board of the American Telephone and Telegraph Company at New York; or in the case of a long distance call is a toll operator.
5. The land line connection is made in the usual way and the shore station radio operator greets the land line subscribers.
6. The shore station operator then joins together the land line and the radio link thus connecting the land subscriber with the ship operator, who proceeds to tell the subscriber that this is the steamship so and so and that Mr. Blank wishes to talk with him. While this is going on, the land operator is monitoring on the circuit and makes such final adjustments of the amplification as may be necessary.
7. The ship operator then summons the ship subscriber and the latter takes up the conversation.

The handling of calls originated by the land line subscriber presents a more difficult operating problem because of the uncertainty as to the radio link—it not being known whether it can be established and, if so, as to how long a wait will be involved in getting the connection. During the tests, most of the calls originated in New York area. For these cases the land subscriber was connected to the Deal Beach station and there the call was put thru directly to the ship in case the radio telephone circuit was available. When not available, information as to the call was recorded by the radio operator and the telephone circuit released for the time being. The call was then completed by first setting up the radio link and then calling back the initiating subscriber. It is obvious that the giving of commercial service will involve: first, the ascertaining of whether or not the vessel is within range; second, the "lining up" of the radio link preparatory to the thru connection; and third, the building up of the land line connection back to the calling subscriber and the making of the thru connection. Many detailed variations are possible in the procedure and the determination of the best operating methods will have to await upon experience obtained in actually giving service.



Date 1/9/21 Station K Q G No. 4
Time ~~filed or~~ received 3:50 P.M.
FROM
Place NEWARK, N.J.
Tel No. 7522 WAVERLY
Person E.E.FREY
TO
Place S.S.GLOUCESTER
Tel. No.
Person M.E.FULTZ
Time Passed
Time land subscriber connected to ship operator 3:55 P.M.
Time circuit completed 3:55 P.M.
Time disconnected 3:58 P.M.
Reports QUALITY FAIR
VOLUME WEAK
Connection failed due to

Date 1/9/21 Station 2 X J 4 No. 4 (12)
Time filed ~~or received~~ 3:48 P.M.
FROM
Place NEWARK, N.J.
Tel No. 7522 WAVERLY
Person E.E.FREY
TO
Place S.S.GLOUCESTER
Tel. No. K Q G
Person M.E.FULTZ
Time Passed 3:50 P.M.
Time land subscriber connected to ship operator 3:55 P.M.
Time circuit completed 3:55 P.M.
Time disconnected 3:58 P.M.
Reports FAIR QUALITY
Connection failed due to

Fig. 22

In order to become familiar with the problems involved in maintaining the ship-to-shore system in operation, a series of operating tests were carried out for a period of about three months, starting in January, 1921, and operating between Deal Beach station and the S. S. *Gloucester*. In accordance with a pre-arranged schedule (unknown to the engineer-operators), calls were entered by a considerable number of Bell System engineers in the vicinity of New York, and calls were initiated from the vessels also by the opening of sealed envelopes carrying instructions to call one or more parties on shore. Fig. 22 is a facsimile of the message form or "ticket" used in the operating tests. The table below gives a representative record sheet recording the calls which were made on a partciular day and also the time which elapsed in putting each one thru. These data, of course, are not especially representative of what can be done with a system after it has begun smoothly in commercial service, but are interesting in giving a general idea of the way the system worked and in showing that calls were successfully put thru in a reasonably short time. In the aggregate a large number of calls were made, and as a result the system was put to a fairly severe operating test. It was found, as was to be expected, that the time required to put thru the radio connections to the vessels is large in comparison with the connecting time on the wire lines, and will require that precaution to be taken in the operating routine to minimize the time during which wire circuits are held up pending connection with the radio link. (The wire circuits used in the tests appeared in New York on a busy P.B.X. (private branch exchange) and did not receive the operating attention that they would in regular service.) But even tho the radio holding time was larger than the usual wire time, it is in itself rather surprisingly good considering the difficulties which attended this maiden

THREE CHANNEL OPERATION

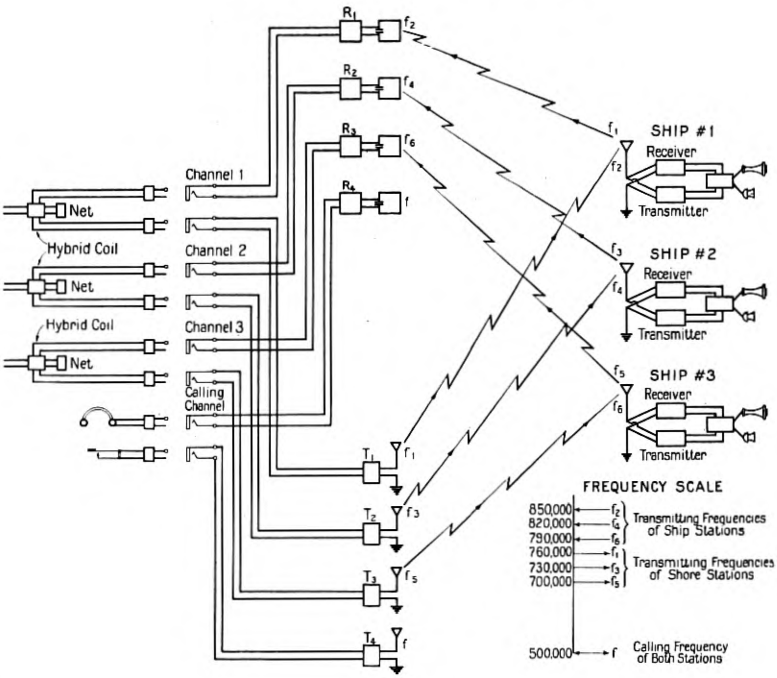


Fig. 23

operation of telephone circuits to ships and must be regarded as full of promise for the extension of telephone service to the highways of the sea.

TESTS OF THREE-CHANNEL OPERATION

Of course, any comprehensive ship-to-shore radio telephone system must be capable of establishing a number of telephone connections from a common land station to a number of ships. The Deal Beach experiments, therefore, had as one of their objectives the trying out of multi-channel operation. These tests were conducted during the fall of 1920 and thru January, 1921, with the S. S. *Gloucester* and the S. S. *Ontario*. A third boat was simulated by a small-power experimental set installed at the Cliffwood, New Jersey, experimental station. The three channel operation is illustrated diagrammatically in Fig. 23, which also shows the scheme of frequencies. The three channels transmitting from Deal Beach were grouped in one frequency range, spaced 30,000 cycles apart. The frequencies transmitted from the ships and received at Deal Beach were grouped in another frequency range removed 30,000 cycles from the first and having frequency intervals likewise of 30,000 cycles. Transmitting and receiving channels differing by 90,000 cycles were paired in the manner indicated to form two-way circuits. While it is possible to squeeze channels together more closely than this, it was not desired on the experiments to go to the limit of frequency squeezing, particularly because of the severe selectivity requirements imposed upon vessel equipment. These frequencies represented a fair balance between technical perfection on the one hand and practically realizable conditions on the other. It will be seen that the set-up was really a four-channel system, with the fourth channel used on 600 meters for calling purposes. Under these conditions three conversations were carried on successfully from the single land station, two to actual ships and one to a "dummy" ship at the Cliffwood experimental station.

EQUIPPING OF S. S. "AMERICA"

The primary development work of the ship-to-shore system was carried out, as described above, in conjunction with coastal vessels. Such vessels were chosen because the rapidity of their turn-round gave much more frequent test periods than could be obtained by means of vessels pursuing a longer route. It remained, however, to equip a trans-oceanic vessel and connect her into the telephone system.

In 1921, the development tests of ship-to-shore telephony were extended to include the General Electric Company and the Radio

Corporation of America. The engineers of these companies built a ship set similar to that developed in the work described above, but of a more commercial design, and installed in on the S. S. *America* in January. 1922.[4] During the succeeding few months, tests were made between the S. S. *America* and the shore, and on a number of these trips connections were put up to various interested parties around New York when the ship was within about 300 miles (480 km.) of the Deal Beach station. Of course, the *America* was carried out much farther than this at night, but the circuits were not sufficiently reliable to be used in connection with the land lines as will be appreciated from the field strength measurements given above.

A photograph of a portion of the installation on the S. S. *America* is reproduced in Fig. 9 above. The talking tests made with the *America* were the occasion of much interest on the part of the listeners-in, and several of the demonstrations which were given the subject of newspaper accounts and need not be described. The more technical phases of the tests with the S. S. *America* are (a) the field strength measurements, and (b) the simultaneous telegraph tests discussed below.

SIMULTANEOUS TELEPHONE AND TELEGRAPH OPERATION BETWEEN SHIP AND SHORE

During the experiments with the Steamships *Gloucester* and *Ontario*, the radio telephone transmissions were carried on alternately with the conduct of the regular radio telegraph service of the vessels. Simultaneous operation was impossible because the vessels were equipped with spark transmitters. While this arrangement of having to switch between either telephone and telegraph operation is permissible for small vessels where the communication load is light, it is, of course, not satisfactory for large trans-oceanic vessels where the message business may be such as to require practically continuous operation on the part of both services.

Recognizing, therefore, that one of the problems attending the successful application of radio telephony to large vessels is that of simultaneous telephone and telegraph operation, tests of such transmission were conducted in co-operation with the Radio Corporation of America from the S. S. *America*. These were made during February and March of 1922. On the land ends, the two radio circuits terminated at different stations, the telegraph at the Bush Ter-

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[4] See article "Duplex Radio Telephone Transmitter," by Baker and Byrnes. *General Electrical Review*, August, 1922.

minal, New York City station of the Radio Corporation, and the telephone at our Deal Beach, New Jersey, station. The telegraph transmitter was of the continuous wave, vacuum tube type manufactured by the General Electric Company. The telephone and telegraph sets used individual antennas on the ship.

Altho certain apparatus difficulties were experienced aboard the vessel because of the short notice at which the tests were made, nevertheless the tests were successful and demonstrated that a telephone set can be made to operate simultaneously with a suitable C. W. (continuous wave) telegraph transmitter. The final solution of this problem of simultaneous operation, however, will undoubtedly require further work in co-ordinating the two types of systems, in order to permit them to be operated on wave lengths relatively close together. During the tests, the wave lengths were widely different, the telegraph operating on about 2,100 meters and the telephone on about 375 meters. The work done at Deal Beach in the development of multiplex telephone operation, where three telephone channels were operated in the vicinity of 400 meters and a fourth channel was operated for telegraphy at 600 meters, demonstrates that it should be feasible to operate telephone and telegraph channels simultaneously on closely adjacent wave length bands. However, in determining wave length allocations, these limiting factors will have to be considered: first, the greater susceptibility of the telephone to interfering noises, such as beat tones, and second the fact that the telephone requires two bands one for each direction of transmission and that these bands are required to be spaced a little apart in frequency. It is obvious that by controlling both types of channels from the same station they can be better co-ordinated in respect to frequency and general service use than if operated from separate stations, so that combined telephone-telegraph shore stations present interesting possibilities for the future.

Another method of operation, and one which requires fewer wave lengths, is that of superimposing the telephone and telegraph channel on the same carrier wave after the general manner of compositing long distance telephone lines with telegraph. This can be done by combining the two channels on one circuit as is done in wire practice and then modulating the combined channels upon the radio carrier. At the receiving end both channels can be detected simultaneously and then the channels separated by composite sets or filters. This method is mentioned to show the ultimate possibilities of combined operation and is not put forward as one which is sufficiently practical, all things considered, for use in the art of the immediate future.

RADIO LINKED WITH TRANSCONTINENTAL LINE AND CATALINA ISLAND

The ship-to-shore radio link was on several occasions connected with very long distance circuits in order to demonstrate the extreme conditions under which combined radio and wire operation are possible.

Perhaps the most interesting case is that in which the ship was linked up with the transcontinental telephone line and connected thru to Catalina Island in the Pacific thus bringing together the two oceans. The circuit arrangements for one of these demonstrations are given schematically in Fig. 24. Both the Deal Beach and Green Harbor shore stations were used, since it was desired to reach the ship anywhere on her course from Boston to the Delaware capes. The demonstration was, therefore, also an example of connecting the ship into the land telephone system thru either of two shore stations. As a matter of fact, at one time the vessel could be reached thru both stations. It happened that the vessel was coming up the coast. The night before the demonstration the ship was communicated with thru the Deal Beach station and connected thru to Catalina Island for a rehearsal. For the demonstration of the following morning, connection was made thru Green Harbor. During both the rehearsal and the demonstration the operator on the vessel talked successfully, altho with some difficulty, with the Catalina Island operator, while New York listened in. This demonstration was

TRANSCONTINENTAL LINE WITH RADIO EXTENSIONS

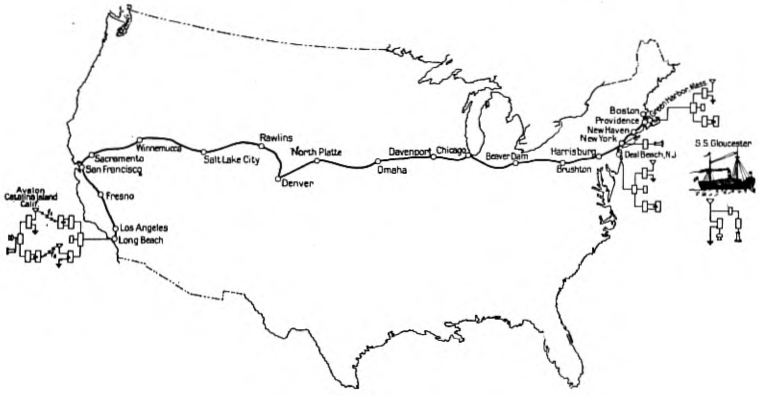

Fig. 24

made for General J. J. Carty on February 14, 1921. An earlier demonstration of a similar nature, although not involving Green Harbor, was made for the delegates of the Preliminary International Communications Conference on October 21, 1920.

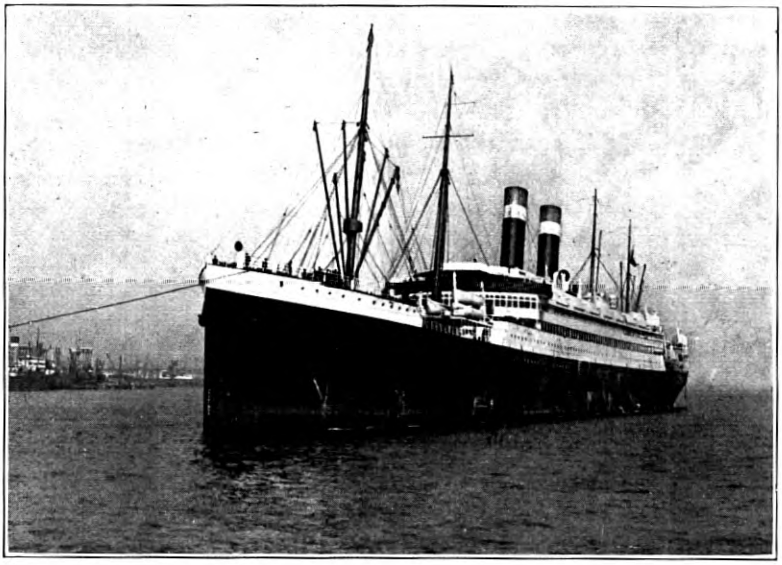

Fig. 25

CONCLUSIONS

The results of this development may be summed up as follows:

(1) It has realized a radio-telephone system capable of giving two-way transmission and meeting the requirements imposed by joint radio-wire operation.

(2) It has demonstrated the actual use of this radio-telephone system in a wire-radio toll circuit as a means for extending the telephone service of the country to include vessels at sea.

(3) The experiments have demonstrated also the practicability of multi-channel operation from a common land station whereby a number of land subscribers may be connected simultaneously to a number of different vessels.

(4) The transmission and operating tests show the difficulties attending the establishment and maintenance of the radio-telephone link to a moving vessel and the necessity for careful

adjustment of the transmission conditions of the circuit and for a diligent maintenance of these adjustments during operation.

(5) In the experiments in multi-channel operation and in simultaneous telephone and telegraph transmission from the same vessel, a beginning has been made in one of the most important problems concerned with the early application of radio telephony to the marine service, namely, that of the co-ordination between radio-telegraph and radio-telephone transmission. It is obvious that the general development of the art of selective transmission, as well as the entrance of radio telephony, calls for the use of purer carrier waves and of a minimum transmission band in radio telegraphy.

(6) As regards the important question of wave lengths, the development has shown that the relatively short waves employed in the experiments are satisfactory up to several hundred miles but that for longer distances longer wave lengths will be required. The difficulty of obtaining for the marine service a wave length sufficiently wide for permitting the handling of any considerable traffic is obvious. The band which can be allocated to this service will naturally be limited by the requirements of other services; and the intensiveness with which this band can be worked by closing up the frequency spacing between channels is limited by the consideration of intercommunication between different types of systems and by apparatus expense.

In general it may be said that the present development has contributed to the communication art the means whereby the universal land line telephone system may be extended to ships at sea. The actual giving of such service must await the working out of the economic problems involved and the necessary business and organization arrangements between the communication companies and the steamship companies.

DAILY RECORD SHEET

SHIP TO SHORE RADIO SERVICE

System Test

April 6, 1921.

			DESTINATION	TOTAL TIME TO CONNECT		DELAYED	INTERRUPTIONS	CANCELLED	COMPLETED		REMARKS
									UNSATIS-FACTORY	SATIS-FACTORY	
TELEPHONE CALLS	From ship			hr.	min.						
		1	N.Y.		41					✓	Good.
		2	N.Y.		37					✓	Good.
		3	N.Y.		16					✓	Good. Weak arc QRM.
		4	N.Y.		4					✓	Fair, (weak) bad arc QRM.
		5	Newark	3 -	53	P. (11.55 A.M. Out till P.M. (2.14 P.M. Still out.				✓	Good, (weak QRM, spk.)
		6	Long Branch	3 -	01	P. (11.53 A.M. Don't. ans. (2.13 P.M. Answ'd. Party out.				✓	Good, but weak.
		7	Asbury Park	2	40	P. 2.19 P.M. Out.				✓	Fair. QRM from [illegible].
		8	N.Y.		22					✓	Good.
		9	Newark		36	P. 3.00 P.M. Out of Bldg.				✓	Good, but weak.
		10	Asbury Park		-			3.47 P.M. Out till 5 P.M.			
		11	Newark		15					✓	Good. Quite noisy.
		12	N.Y.		50					✓	Good. Spk. QRM.
		13	Allenhurst, NJ		57					✓	Good. Weak QRM, 3 QM.
		14	Asbury Park		29					✓	Good. (Bad spark QRM)
	To ship	1	N.Y.C.		21					✓	Good.
		2	N.Y.C.		45					✓	Good. Ship has spark QRM.
		3	N.Y.C.		53					✓	Fair.
		4	N.Y.C.		47					✓	Fair.
		5	N.Y.C.		-			10.44 A.M; 3.43 P.M; don't ans.			
		6	N.Y.C.		19					✓	Good.
		7	N.Y.C.		11					✓	Good.
		8	N.Y.C.		12					✓	Good, Mr. Maxfield tries out transmitters.
		9	Spring Lake		18					✓	Good.
		10	N.Y.C.		20					✓	Good.
		11	N.Y.C.		03	P. (2.15 P.M. Not in. (2.30 P.M. Will Call.				✓	Good.
		12	N.Y.C.		3					✓	Fair.
		13	N.Y.C.		-			3.02 P.M. Not there today.			
		14	N.Y.C.		26	P. 3.30 P.M. Line busy.				✓	Fair.
		15	N.Y.C.		18		E. 3.50-3.51 P.M. Cut off.			✓	Fair.
		16	N.Y.C.		9					✓	Fair.
		17	N.Y.C.		25					✓	Fair.

			NO. OF WORDS	TOTAL TIME TO TRANSMIT	ACTUAL TIME TO TRANSMIT	DELAY TO TELEPHONE SERVICE
TELEGRAMS	From ship	1				
	To ship	1	114	13	8	0
		2	17	4	1	0

SUMMARY

Total calls	31
Completed	28
Cancelled	3
Long Distance	0
Toll	1
Local	30

Calls completed and satisfactory	28	90.3%
Calls completed and unsatisfactory	0	
Calls cancelled due to system failure	0	
Calls cancelled not due to system failure	3	9.7%
		100%

Average time

Radio A	13.6	mins.
" B	.16	"
Wire A	8.0	"
" B	0	

$$\frac{\text{Total minutes operation}}{\text{Total calls}} = \frac{551}{31} = 17.7$$

The Contributors to this Issue

John R. Carson, B.S., Princeton, 1907; E.E., 1909; M.S., 1912; Research Department, Westinghouse Electric and Manufacturing Company; 1910–12; instructor of physics and electrical engineering, Princeton, 1912–14; American Telephone and Telegraph Company, Engineering Department, 1914–15; Patent Department, 1916–17; Engineering Department, 1918; Department of Development and Research, 1919—. Mr. Carson's work has been along theoretical lines and he has published several papers on theory of electric circuits and electric wave propagation.

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Engineering Department of the Western Electric Company, 1906—. One of Mr. Elmen's principal lines of work has been magnetic investigations.

Charles S. Demarest, E.E., University of Minnesota, 1911; American Telephone and Telegraph Company, Engineering Department, 1911–19; Department of Development and Research, 1919—. Mr. Demarest's work has been connected with the development of equipment for toll telephone systems, including that for long distance cables, carrier and radio systems.

H. W. Nichols, B.S., Armour Institute, 1908; E.E., 1911; M.S., Chicago, 1909; Ph.D., 1918; assistant professor electrical engineering, Armour, 1909–14; Engineering Department of Western Electric Company, 1914—. Since joining the research staff of the Western Electric Company, Dr. Nichols has been taking a leading part in the development of radio apparatus.

Lloyd Espenschied, Pratt Institute, 1909; United Wireless Telegraph Company as radio operator, summers, 1907–08; Telefunken Wireless Telegraph Company of America assistant engineer, 1909–10; American Telephone and Telegraph Company, Engineering Department and Department of Development and Research, 1910—. Took part in long distance radio telephone experiments from Washington to Hawaii and Paris, 1915; since then his work has been connected with the development of radio and carrier systems.