SUPPLEMENT

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Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

For economic reasons, alternate issues of the Supplement are published in 32-page and 16-page sizes.

TELECOMMUNICATION PRINCIPLES A, 1975

Students were expected to answer 6 questions, including at least one from Q9–10

Q 1 (*a*) *What factors in a magnetic circuit correspond to voltage, current and resistance in an electric circuit?*

(*b*) *With reference to a flux-density/magnetizing-force* (B/H) *curve, show why*

(*i*) *the relative permeability of iron varies with increasing magnetic flux, and*

(*ii*) *there is a limit to the magnetic flux that iron can carry.*

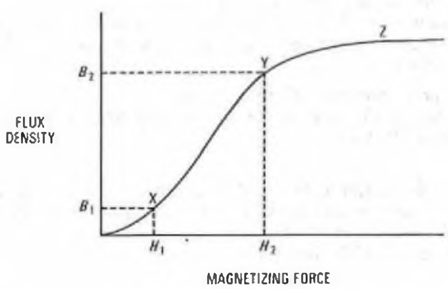
(*c*) *A closed magnetic circuit has a mean length of 0·5 m. When the circuit is wound with a coil of 100 turns, a current of 0·25 A produces a flux density of 0·1 T in the iron. Find the relative permeability of the iron at this flux density.*

A 1 (*a*) The corresponding factors for electric and magnetic circuits are shown in the table.

Electric Circuit	Voltage	Current	Resistance
Magnetic Circuit	Magnetomotive force	Magnetic flux	Reluctance

In an electric circuit, the 3 factors are related by Ohm's law, which states that the voltage across a circuit is equal to the current flowing multiplied by the resistance of the circuit. In a magnetic circuit, there is a similar relationship: the magnetomotive force is equal to the flux multiplied by the reluctance of the complete magnetic path.

The analogy is not complete because the reluctance of iron varies with flux density, whereas resistance is constant for a given temperature.



(*b*) (*i*) A typical B/H curve is shown in the sketch. The relative permeability, μ_r, is given by

$$\mu_r = \frac{B}{\mu_0 H}, \qquad \ldots\ldots (1)$$

where B is in teslas, H is in amperes/metre, and μ_0 is the constant absolute permeability of free space, equal to $4\pi \times 10^{-7}$ H/m. The graph is not linear and, therefore, B is not proportional to H. It can be seen that B/H varies as H increases; that is, μ_r varies as H increases. For example, the relative permeability at point Y, equal to $B_2/\mu_0 H_2$, can be seen to be greater than the relative permeability at point X, which is equal to $B_1/\mu_0 H_1$.

(*ii*) For a given cross-sectional area of iron, the magnetic flux increases with magnetizing force as shown by the B/H curve, due to the increasing alignment of molecular-sized magnetic domains. When all the magnetic domains are aligned (point Z), no further increase in magnetic flux is possible; the iron is magnetically saturated. Thus, at high values of H, there is no further increase in B as H increases.

(*c*) From equation (1),

$$H = \frac{B}{\mu_0 \mu_r},$$

$$= \frac{0 \cdot 1}{4\pi \times 10^{-7} \times \mu_r} \text{ A/m.} \qquad \ldots\ldots (2)$$

Also,
$$H = \frac{NI}{l},$$

where N is the number of turns on the coil, I is the current in the coil (A), and l is the length of the magnetic circuit (m),

$$= \frac{100 \times 0 \cdot 25}{0 \cdot 5} = 50 \text{ A/m.} \qquad \ldots\ldots (3)$$

Equating (2) and (3) gives

$$\mu_r = \frac{0 \cdot 1}{4\pi \times 10^{-7} \times 50} = \underline{1592.}$$

Q 2 (*a*) *Using the circuit of Fig. 1 as an example, explain the application of the superposition theorem in finding the currents in an electric circuit.*

(*b*) *Using the superposition theorem, calculate the magnitude and direction of the current flowing through each battery and the 6 Ω resistor of the circuit shown in Fig. 1.*

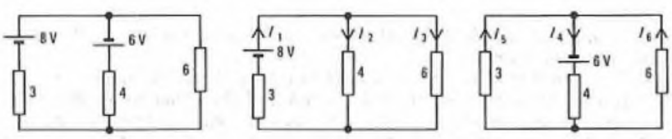


Fig. 1 (*a*) (*b*)

A 2 (*a*) In the application of the superposition theorem, a circuit containing parallel arms is reduced to 2 or more simpler circuits which can easily be solved individually. The currents in the complete circuit can then be found by superimposing the results for the individual circuits. The current in each arm is the algebraic sum of the superimposed currents.

The circuit in Fig. 1 has 2 batteries and 3 parallel arms. It can be represented by 2 separate circuits, each containing one battery, the other battery being represented only by its internal resistance. The application of Ohm's law then gives the current in each arm of each circuit due to the battery in that circuit, and these currents are then superimposed to solve the complete circuit.

(*b*) The first simplified circuit is shown in sketch (*a*). The 6 V battery is replaced by its internal resistance; in this case, it has no internal resistance and, so, is short-circuited. The directions of currents I_1, I_2, and I_3 are assumed to be those shown.

Now, the total resistance of the 4 Ω and 6 Ω resistors in parallel

$$= \frac{1}{\frac{1}{4}+\frac{1}{6}} = \frac{1}{\frac{3}{12}+\frac{2}{12}} = \frac{12}{5} = 2\cdot4\ \Omega.$$

Hence,
$$I_1 = \frac{8}{3+2\cdot4} = 1\cdot48\ \text{A}.$$

$$\therefore\ I_2 = 1\cdot48 \times \frac{6}{10} = 0\cdot89\ \text{A}.$$

$$\therefore\ I_3 = 1\cdot48 - 0\cdot89 = 0\cdot59\ \text{A}.$$

In the second simplified circuit, shown in sketch (*b*), the 8 V battery is removed and replaced by its internal resistance, which is again zero. The directions of currents I_4, I_5, and I_6 are assumed to be as shown.

Now, the total resistance of the 3 Ω and 6 Ω resistors in parallel

$$= \frac{1}{\frac{1}{3}+\frac{1}{6}} = \frac{1}{\frac{2}{6}+\frac{1}{6}} = \frac{6}{3} = 2\ \Omega.$$

Hence,
$$I_4 = \frac{6}{4+2} = 1\ \text{A}.$$

$$\therefore\ I_5 = 1 \times \frac{6}{9} = 0\cdot67\ \text{A}.$$

$$\therefore\ I_6 = 1 - 0\cdot67 = 0\cdot33\ \text{A}.$$

Superimposing the circuits in sketches (*a*) and (*b*),

(*i*) the current in the 8 V battery is

$$I_1 + I_5 = 1\cdot48 + 0\cdot67 = \underline{2\cdot15\ \text{A}},$$

in the same direction as I_1 and I_5,

(*ii*) the current in the 6 V battery is

$$I_2 + I_4 = 0\cdot89 + 1 = \underline{1\cdot89\ \text{A}},$$

in the same direction as I_2 and I_4, and

(*iii*) the current in the 6 Ω resistor is

$$I_3 - I_6 = 0\cdot59 - 0\cdot33 = \underline{0\cdot26\ \text{A}},$$

in the direction of current I_3.

Q 3 (a) With the aid of a sketch, describe the construction of a variable capacitor.

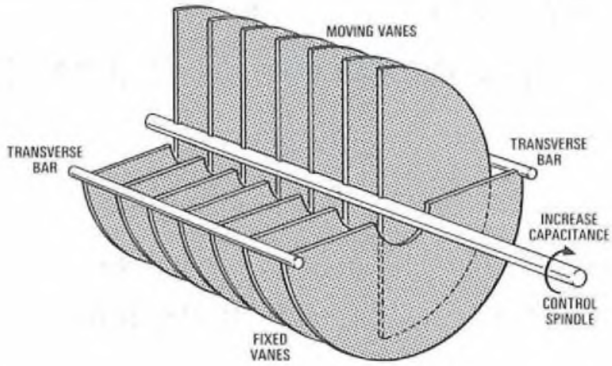
(b) A capacitor is made up of 2 parallel plates, each 100 mm × 50 mm, spaced 1 mm apart in air. A potential difference of 50 V exists between the plates. Calculate, stating the units in each case,

(i) the potential gradient between the plates,
(ii) the capacitance, and
(iii) the energy stored.

(c) If the air dielectric were replaced by a dielectric having a relative permittivity of 10, what would be the effect on the value of the capacitance?

A 3 (*a*) The sketch shows the basic construction of an air-dielectric variable capacitor.

The capacitor consists of 2 sets of interleaved parallel plates, usually referred to as *vanes*, one set being fixed and the other movable. The moving vanes are clamped to, and spaced by, the control spindle, which rotates in bearings on the capacitor's chassis (not shown). Electrical contact to the moving vanes is made through a wiper acting on the spindle. The fixed vanes are spaced and braced by 2 transverse bars, attached to the chassis by insulated supports. The capacitance is proportional to the area of overlap of the fixed and moving vanes and hence, for semi-circular vanes, proportional to the angle of rotation. Maximum capacitance is obtained when the vanes are fully meshed.



(*b*) (*i*) The potential gradient between the plates, E volts/metre, is given by

$$E = \frac{V}{d}\ \text{volts/metre},$$

where V is the voltage across the plates (V), and d is the distance between the plates (m),

$$= \frac{50}{1\times10^{-3}}\ \text{V/m},$$

$$= \underline{50\ \text{kV/m}}.$$

(*ii*) The capacitance, C farads, is given by

$$C = \frac{\epsilon_0\epsilon_r A}{d}\ \text{farads},$$

where ϵ_0 is the permittivity of free space, equal to $10^{-9}/36\pi$ F/m, ϵ_r is the relative permittivity of the dielectric, and A is the effective common area between the plates (m^2).

$$\therefore\ C = \frac{10^{-9}\times1}{36\pi} \times \frac{50\times10^{-3}\times100\times10^{-3}}{1\times10^{-3}}\ \text{F},$$

$$= \underline{44\cdot2\ \text{pF}}.$$

(*iii*) The energy stored, W joules, is given by

$$W = \frac{CV^2}{2}\ \text{joules},$$

$$= \frac{44\cdot2\times10^{-12}\times50^2}{2}\ \text{J},$$

$$= \underline{55\cdot3\ \text{nJ}}.$$

(*c*) If the air dielectric, which has a relative permittivity of unity, is replaced by a dielectric of relative permittivity 10, it can be seen from the equation in part (*b*)(*ii*) that the value of C increases by a factor of 10 to $\underline{442\ \text{pF}}$. (The stored energy also, therefore, increases by a factor of 10, but the potential gradient is unaltered.)

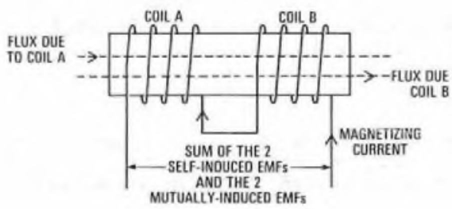
Q 4 (a) What is meant by the term flux linkage?

(b) When inductors are arranged such that mutual inductance exists between them, the term series-aiding may be used. With the aid of a diagram, explain what is meant by this.

(c) Two inductors, having self-inductances of 100 mH and 50 mH are connected in series-aiding. The total inductance of the circuit is 190 mH. Calculate

(i) the value of mutual inductance, and
(ii) the value of the total induced e.m.f. if the current changes at a constant rate of 20 A/s.

A 4 (*a*) In electromagnetism, flux linkage defines the amount of interaction between a coil and the magnetic flux embracing it. Flux linkage is given by the product of the number of turns on the coil and the flux linked with those turns.



(*b*) The term *series-aiding* is used when 2 mutually-inductive coils are connected in series so that the magnetic flux produced by each reinforces that of the other. The mutually-induced e.m.f. in each coil is in the same direction as the self-induced e.m.f. in that coil. The sketch shows such an arrangement. The total inductance of the arrangement, L_T henrys, is given by

$$L_T = L_A + L_B + 2M \text{ henrys,}$$

where L_A and L_B are the self-inductances of coils A and B respectively (H), and M is the mutual inductance between them (H).

(*c*) (*i*) From part (*b*),

$$M = \frac{L_T - L_A - L_B}{2} \text{ henrys,}$$

$$= \frac{190 - 100 - 50}{2} \text{ mH,}$$

$$= \underline{20 \text{ mH.}}$$

(*ii*) The induced e.m.f., e volts, is given by

$$e = -L_T \frac{di}{dt} \text{ volts,}$$

where di/dt is the rate of change of current (A/s), and the minus sign indicates the direction of the induced e.m.f. is opposite to that of the change of current.

$$\therefore\ |e| = 190 \times 10^{-3} \times 20 \text{ V,}$$

$$= \underline{3{\cdot}8 \text{ V.}}$$

Q 5 (a) Draw a labelled diagram showing the construction of a moving-coil milliammeter.

(b) Is it possible to use this type of instrument on its own in

(i) a d.c. circuit, and

(ii) an a.c. circuit?

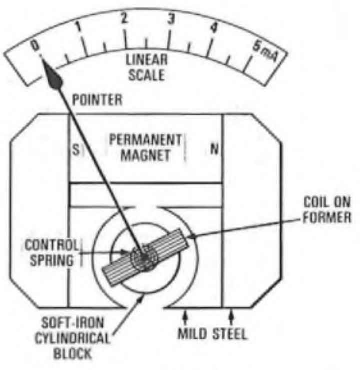
Give reasons for your answers.

(c) A moving-coil milliammeter is to be converted for use as a voltmeter. With the aid of a circuit diagram, show how this can be done.

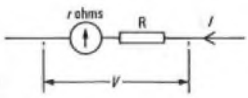
(d) A moving-coil voltmeter has a sensitivity of 2 kΩ/V and a full-scale deflexion (FSD) of 250 V. Calculate the resistance of the instrument and the current taken at FSD.

A 5 (*a*) Sketch (*a*) shows the main constructional features of a moving-coil milliammeter.

(*b*) The moving-coil milliammeter can operate only in a d.c circuit, since the deflecting torque must be unidirectional. In an a.c. circuit, the effect of an alternating current on the highly-damped coil system would be to generate an alternating torque that would give no resultant deflexion of the pointer. A rectifier would be necessary to provide a direct current to operate the meter.



(*a*)



(*b*)

(*c*) A moving-coil milliammeter operates on a direct current of only a few milliamperes. The resistance, r ohms, of its coil is of the order of 25 Ω, so that the voltage required across the coil for FSD is approximately 125 mV. Hence, to convert the instrument to a voltmeter, a voltage-dropping resistor, R ohms, must be connected in series with it, as shown in sketch (*b*). Resistor R is chosen such that the potential difference across it, for the FSD current, is the maximum voltage that needs to be measured minus the small potential difference across the milliammeter's coil.

For an FSD current of I amperes and a maximum voltage of V volts,

$$V = I(r + R) \text{ volts.}$$

$$\therefore\ R = \frac{V}{I} - r \text{ ohms.}$$

(*d*) The resistance of a voltmeter having a sensitivity of 2 kΩ/V and an FSD of 250 V

$$= 2 \times 10^3 \times 250\ \Omega,$$

$$= \underline{500 \text{ k}\Omega.}$$

Hence, the current taken at FSD

$$= \frac{250}{500 \times 10^3} \text{ A,}$$

$$= \underline{0{\cdot}5 \text{ mA.}}$$

Q 6 The voltage across a capacitor of reactance 100 Ω is given by

$$v = 340 \sin 200\pi t \text{ volts,}$$

where t is in seconds.

(a) Find the expression for the current in the capacitor.

(b) For the current in part (a), calculate

(i) the r.m.s.value,

(ii) the frequency, and

(iii) the periodic time.

(c) Sketch, on the same axes, the waveforms of voltage and current over 1 cycle.

A 6 (*a*) Ohm's law applies to alternating currents in reactances in the same way as to direct currents in resistances, but allowance must be made for phase changes. The instantaneous voltage is given by $v = 340 \sin 200\pi t$ volts; the current in the capacitor leads this voltage by 90°, which is equivalent to $+\pi/2$ rad. The peak value of current, by Ohm's law,

$$= \frac{340}{100} = 3{\cdot}4 \text{ A.}$$

Therefore, the expression for the current in the capacitor is

$$\underline{i = 3{\cdot}4 \sin\left(200\pi t + \frac{\pi}{2}\right) \text{ amperes.}}$$

(*b*) (*i*) The r.m.s. value of a sinusoidal current

$$= \frac{\text{peak value}}{\sqrt{2}} \text{ amperes,}$$

$$= \frac{3{\cdot}4}{\sqrt{2}} \text{ A,}$$

$$= \underline{2{\cdot}4 \text{ A.}}$$

(*ii*) From the expression for v, the angular velocity is 200π rad/s, and this is equal to $2\pi f$, where f is the frequency (Hz). Hence,

$$f = \frac{200\pi}{2\pi} \text{ Hz,}$$

$$= \underline{100 \text{ Hz.}}$$

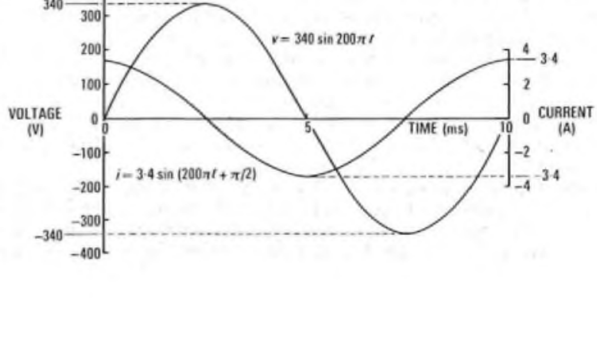
(*iii*) The periodic time (that is, the time for one complete cycle of alternation)

$$= \frac{1}{f} \text{ seconds,}$$

$$= \frac{1}{100} \text{ s,}$$

$$= \underline{10 \text{ ms.}}$$

(*c*) The waveforms are shown in the sketch.



Q 7 Describe one of the following experiments.

(a) The measurement of the e.m.f. of a cell, using a simple potentiometer.

(b) The determination of the efficiency of an electric kettle.

Set out the answer in the form of a laboratory report, which should include a circuit diagram, details of the equipment used, the procedure, typical results and conclusions. Give 2 possible sources of error.

A 7 See A2, Telecommunication Principles A, 1970, *Supplement*, Vol. 63, p. 95, Jan. 1971.

Q 8 (a) (i) What is meant by thermionic emission in a thermionic tube? State 2 factors upon which it depends.

(ii) Sketch the anode-current/anode-voltage characteristic for a vacuum diode, and explain the reasons for its shape.

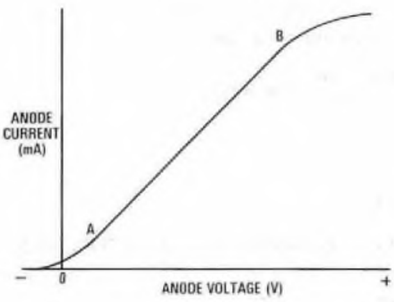
(b) Show, with the aid of a circuit diagram, how diodes can be arranged to provide full-wave rectification from a single-phase supply. Explain, by reference to the diagram, why the current through the load is always in the same direction.

A 8 (*a*) (*i*) Thermionic emission is the ejection of electrons from a thermionic tube's cathode into the vacuum surrounding it.

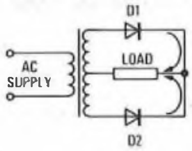
The surface of the cathode must be specially treated with an emissive material, such as barium or strontium, and the cathode temperature must be high enough to give free electrons sufficient energy to overcome the restraining forces acting on them.

Thermionic emission occurs at a usable rate only when the cathode's surface is heated to above about 500°C, heating usually being achieved by passing a current through a resistive element embedded in the cathode. The emitted electrons form a negatively-charged cloud, known as the *space charge*, above the cathode, and a stream of electrons is attracted from the space charge to the tube's anode when the latter is connected to a positive potential with respect to the cathode. Thus, a current is established between the cathode and anode.

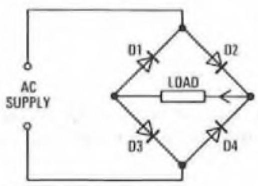
(*ii*) The anode-current/anode-voltage characteristic for a vacuum diode is shown in sketch (*a*).



(*a*)



(*b*)



(*c*)

The current established between the cathode and anode is proportional to the applied potential difference over most of the working range of the tube; that is, the tube has a constant internal resistance (region A–B on the characteristic). When a small negative potential is applied to the anode, of such a value that the anode is less negative with respect to the cathode than is the space charge, some electrons reach the anode, and a minute current thus flows. This gives the features shown on the characteristic below point A. Above point B, when the anode voltage is large, the electrons in the space charge are drawn off as fast as thermionic emission can replace them. This is known as *saturation*, and the characteristic levels out; to obtain a higher anode current, the cathode temperature must be raised.

(*b*) Full-wave rectification can be obtained either by using 2 diodes in a push-pull configuration, or 4 diodes in a bridge formation; sketches (*b*) and (*c*) respectively show the 2 arrangements.

In the push-pull circuit, the diodes are connected in the same direction relative to one side of the load and fed from opposite ends of a centre-tapped transformer winding, the other side of the load being connected to the tapping point. For half-cycles that cause the upper end of the centre-tapped winding to go positive with respect to the lower end, diode D1 conducts and supplies the load, while diode D2 is reverse biased. Alternate half-cycles are then supplied through diode D2 while diode D1 is reverse biased. Hence, a unidirectional voltage is applied to the load.

In the bridge circuit, the load is connected across one diagonal of a 4-diode bridge, and the supply across the other. For half-cycles that cause the upper point of the bridge to go positive with respect to the lower point, diodes D2 and D3 conduct in series with load. For alternate half-cycles, diodes D4 and D1 conduct. Thus, a unidirectional voltage is applied to the load.

Q 9 (a) Show, with the aid of a sketch, the basic construction and principle of operation of either a moving-coil microphone or a balanced-armature telephone receiver.

(b) State, with reasons, the factors that control the sensitivity of the instrument described.

Q 10 (a) Explain briefly the meaning of depletion layer in a transistor.

(b) Name 2 conditions that must be avoided if permanent damage to a transistor is not to occur in operation.

(c) Show, by using the output characteristics, how a transistor connected in the common-emitter configuration with a resistive load in the collector circuit can act as an amplifier. Sketch a single-stage amplifier circuit of this type and include typical component values.

A 10 (*a*) The term *depletion layer* applies to a p n junction in a junction diode or transistor. Such a junction may consist of a region of n-type germanium in juxtaposition with one of p-type germanium. The n-type region has an excess of electrons, which are negative-charge carriers, and the p-type an excess of holes, which are positive-charge carriers. At the junction, electrons from the n-type germanium tend to diffuse into the p-type, and holes from the p-type germanium tend to diffuse into the n-type. This movement of charges across the junction sets up a small potential barrier, consisting of positively-charged n-type germanium on one side and negatively-charged p-type germanium on the other. The resulting interface region is known as a *depletion layer*, and it behaves as if a battery were connected internally across the interface region. The electric field from the depletion layer prevents the further transfer of charge carriers across the junction.

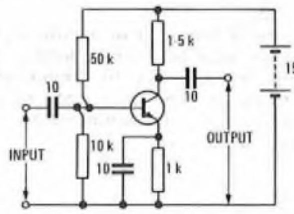
(*b*) Two conditions that must be avoided if a transistor is not to be destroyed are

(*i*) a temperature rise beyond the specified limits for the transistor, and

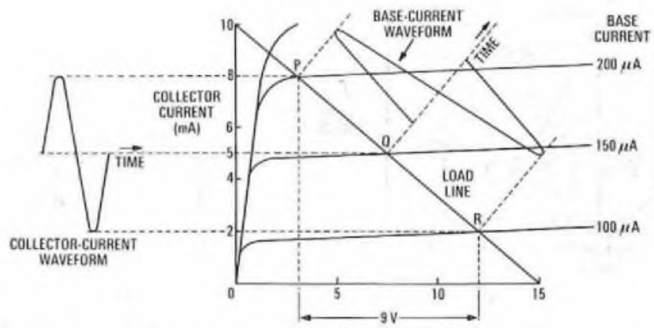
(*ii*) the application of peak voltages in excess of those specified by the manufacturer.

High temperatures increase the energy content of atoms in the transistor and, if the thermal agitation becomes excessive, the depletion layer can be broken down beyond recovery. High-voltage surges produce a similar destructive effect by forcing electrons through the interface region, thereby breaking down its structure.

(*c*) A typical single-stage common-emitter transistor amplifier, with component values marked, is shown in sketch (*a*).



(*a*)



(*b*)

A family of output characteristics, that is, curves of collector current against collector voltage for various constant values of base current, is shown in sketch (*b*). A load line is shown for a resistive collector load of 1·5 kΩ. The amplifying action can be readily demonstrated. An input signal to the stage consists of a base-current waveform, as illustrated, at right angles to the load line and centred on the quiescent operating point, Q. The variation of base current between 100–200 μA causes the actual working point to move up and down the load line between points P and R. The corresponding variation in collector current is about 6 mA, giving a collector-voltage swing of about 9 V in a 1·5 kΩ load. Thus, the transistor acts as an amplifier.

Students were expected to answer any 6 questions

Q 1 (*a*) *Express each of the following as 2 factors:*

(*i*) $5x^2 + 3x - 2$, *and*

(*ii*) $10xy - 5x - 4y + 2$.

State the common factor.

(*iii*) *Hence, express the following as one fraction in its simplest form:*

$$\frac{1}{10xy - 5x - 4y + 2} + \frac{1}{5x^2 + 3x - 2}.$$

(*b*) *Rearrange the following formula to make t the subject:*

$$\frac{x}{a} = \frac{1 + t^2}{1 - t^2}.$$

(*c*) *Simplify, writing your answer with positive indices only,*

$$\frac{6a^{-3/2} \times c^{-2} \times \sqrt{b}}{2b^{-5/2} \times a^{1/2} \times c}.$$

A 1 (*a*) (*i*) $5x^2 + 3x - 2 = \underline{(5x - 2)(x + 1)}.$

(*ii*) $10xy - 5x - 4y + 2 = 5x(2y - 1) - 2(2y - 1),$

$= \underline{(5x - 2)(2y - 1)}.$

The factor common to equations (*a*) (*i*) and (*a*) (*ii*) is $\underline{5x - 2}$.

(*iii*)
$$\frac{1}{10xy - 5x - 4y + 2} + \frac{1}{5x^2 + 3x - 2}$$
$$= \frac{1}{(5x - 2)(2y - 1)} + \frac{1}{(5x - 2)(x + 1)},$$
$$= \frac{x + 1 + 2y - 1}{(5x - 2)(2y - 1)(x + 1)},$$
$$= \underline{\frac{x + 2y}{(5x - 2)(2y - 1)(x + 1)}}.$$

(*b*)
$$\frac{x}{a} = \frac{1 + t^2}{1 - t^2}.$$
$$\therefore x(1 - t^2) = a(1 + t^2).$$
$$\therefore x - xt^2 = a + at^2.$$
$$\therefore t^2(x + a) = x - a.$$
$$\therefore t^2 = \frac{x - a}{x + a},$$
$$\therefore \underline{t = \sqrt{\left(\frac{x - a}{x + a}\right)}}.$$

(*c*)
$$\frac{6a^{-3/2} \times c^{-2} \times \sqrt{b}}{2b^{-5/2} \times a^{1/2} \times c} = \frac{3b^{(1/2)+(5/2)}}{a^{(1/2)+(3/2)}c^{1+2}},$$
$$= \underline{\frac{3b^3}{a^2c^3}}.$$

Q 2 (*a*) *Form the quadratic equation whose roots are* $(4/5, -1/2)$.

(*b*) *A sinusoidal e.m.f. is given by* $e = E \sin(\omega t + \phi)$, *where* E, ω *and* ϕ *are constants, and* ϕ *is in radians. Find the values of* E, ω *and* ϕ *if*

(*i*) *the maximum value of e is 10 V,*

(*ii*) $e = 10$ *V at* $t = 0$, *and*

(*iii*) *the frequency is 5 Hz.*

(*c*) *Three voltages,* V_1, V_2 *and* V_3, *with respective magnitudes 8 V, 11 V and 14 V, are such that* V_2 *leads* V_1 *by 40° and* V_3 *lags* V_1 *by 50°. Calculate their resultant, giving its phase with reference to* V_1.

A 2 (*a*) The required equation is given by

$$\left\{x - \frac{4}{5}\right\}\left\{x - \left(-\frac{1}{2}\right)\right\} = 0.$$
$$\therefore x^2 - x\left(\frac{4}{5} - \frac{1}{2}\right) - \frac{4}{5} \times \frac{1}{2} = 0.$$
$$\therefore x^2 - \frac{x}{10}(8 - 5) - \frac{2}{5} = 0.$$

Multiplying through by 10 gives the required quadratic equation as

$$\underline{10x^2 - 3x - 4 = 0.}$$

(*b*) The maximum value of the sine function is unity, and occurs when $(\omega t + \phi) = \pi/2$ rad. Hence, from part (*i*) of the question,

$$\underline{E = 10 \text{ V}.}$$

Thus, $e = 10 \sin(\omega t + \phi)$.

From part (*ii*) of the question, $e = 10$ V at $t = 0$.

$$\therefore 10 = 10 \sin \phi.$$
$$\therefore \sin \phi = 1.$$
$$\therefore \underline{\phi = \pi/2 \text{ rad}.}$$

Now, $\omega = 2\pi f$ radians/second, where f is the frequency (Hz). Therefore, from part (*iii*) of the question,

$$\omega = 2\pi \times 5 = \underline{10\pi \text{ rad/s}.}$$

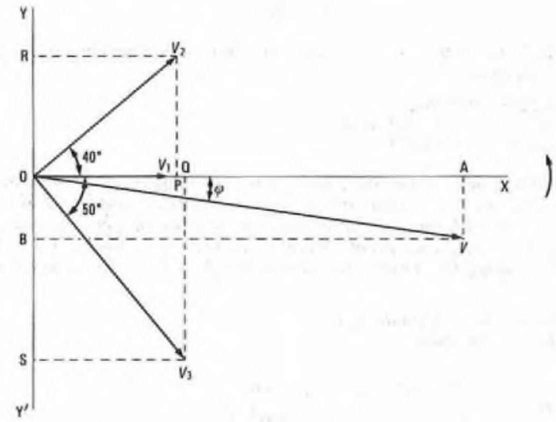
(*c*) Assuming the voltages to be sinusoidal, let $v_1 = V_1 \sin \theta$, where v_1 is the instantaneous value of V_1, and θ is the phase angle in degrees of arc.

$$\therefore v_1 = 8 \sin \theta.$$

Hence, $v_2 = 11 \sin(\theta + 40°)$,

and $v_3 = 14 \sin(\theta - 50°)$.

The 3 voltages are represented by the phasor diagram in the sketch, with axes OX and Y'OY, and using the conventional counter-clockwise rotation of phasors. Phasor V_1 is shown on the reference axis, OX. Phasors V_2 and V_3 can be resolved into components along the axes.



Resolving for horizontal components,

$$\text{OP} = V_2 \cos 40° = 11 \times 0{\cdot}7660 = 8{\cdot}427 \text{ V},$$

$$\text{and OQ} = V_3 \cos 50° = 14 \times 0{\cdot}6428 = 8{\cdot}999 \text{ V}.$$

Hence, the net component, OA, along the horizontal axis

$= V_1 + 8{\cdot}427 + 8{\cdot}999$ volts,

$= 8 + 8{\cdot}427 + 8{\cdot}999$ V,

$= 25{\cdot}426$ V.

Resolving for vertical components,

$\text{OR} = V_2 \sin 40°$,

$= 11 \times 0{\cdot}6428 = 7{\cdot}071$ V,

and $\text{OS} = V_3 \sin 50° = 14 \times 0{\cdot}7660 = 10{\cdot}725$ V.

Hence, the net component, OB, along the vertical axis

$= 10{\cdot}725 - 7{\cdot}071$ V,

$= 3{\cdot}654$ V in direction OY'.

The net components, OA and OB, are shown in the sketch, and their resultant, V, is represented by the diagonal of the rectangle formed by the components.

$$\therefore\ V^2 = 25{\cdot}426^2 + 3{\cdot}654^2,$$
$$= 646{\cdot}5 + 13{\cdot}35,$$
$$= 659{\cdot}85.$$
$$\therefore\ V = 25{\cdot}69\text{ V}.$$

Also, $\tan\phi = \dfrac{3{\cdot}654}{25{\cdot}426},$

where ϕ is the phase angle between V and V_1,

$$= 0{\cdot}1437.$$
$$\therefore\ \phi = 8^\circ\ 11'.$$

Hence, the resultant of voltages V_1, V_2 and V_3 has a magnitude of $\underline{25{\cdot}69\text{ V}}$ and lags V_1 by $\underline{8^\circ\ 11'}$.

Q 3 (*a*) *Evaluate, with the aid of tables,*

(*i*) *cos 142° 19′,*
(*ii*) *tan (−45° 14′), and*
(*iii*) *sin 262° 25′.*

(*b*) *Construct a table of values, at intervals of 10° between 0–90°, for* $y_1 = \sin 2\theta$, $y_2 = \sin(\theta + 30^\circ)$ *and* $y_3 = y_1 + y_2$.
Draw the graph of $y_3 = \sin 2\theta + \sin(\theta + 30^\circ)$, *and use your graph to solve the equation*

$$\sin 2\theta + \sin(\theta + 30^\circ) = 1{\cdot}3.$$

A 3 (*a*) (*i*) $\underline{-0{\cdot}7914}$; (*ii*) $\underline{-1{\cdot}0082}$; (*iii*) $\underline{-0{\cdot}9913}$.

(*b*) The solution of the equation is $\theta = \underline{17^\circ\ 15' \text{ or } 79^\circ\ 30'}$.

Q 4 (*a*) *By writing cos A for c, and sin A for s, find the numerical value of*

$$\frac{c^2 - s^2}{1 - 2s^2}.$$

(*b*) *Find the values of θ between 0–360° that satisfy each of the following equations:*

(*i*) $\sin\theta = 0{\cdot}9063$,
(*ii*) $\cos\theta = -0{\cdot}8290$, *and*
(*iii*) $\tan\theta = -2{\cdot}9042$.

(*c*) *The distance, d metres, travelled in t seconds by a car moving in a straight line with constant acceleration a metres/second², is given by* $d = ut + \frac{1}{2}at^2$, *where u is the initial velocity (m/s). The car passes an initial point, A, and then points B and C in succession, where AB = 12 m and BC = 36 m. It takes 2 s to travel from A to B, and 3 s from B to C. Find*

(*i*) *the velocity at point A, and*
(*ii*) *the acceleration.*

A 4 (*a*)
$$\frac{c^2 - s^2}{1 - 2s^2} = \frac{\cos^2 A - \sin^2 A}{1 - 2\sin^2 A},$$
$$= \frac{(1 - \sin^2 A) - \sin^2 A}{1 - 2\sin^2 A},$$

since $\sin^2 A + \cos^2 A = 1$.

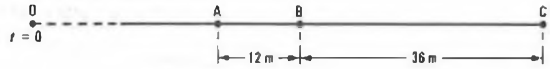
$$\therefore\ \frac{c^2 - s^2}{1 - 2s^2} = \underline{1}.$$

(*b*) (*i*)
$$\sin\theta = 0{\cdot}9063.$$
$$\therefore\ \theta = 65^\circ \text{ or } (180^\circ - 65^\circ),$$
$$= \underline{65^\circ \text{ or } 115^\circ}.$$

(*ii*)
$$\cos\theta = -0{\cdot}8290.$$
$$\therefore\ \theta = 180^\circ \pm 34^\circ,$$
$$= \underline{146^\circ \text{ or } 214^\circ}.$$

(*iii*)
$$\tan\theta = -2{\cdot}9042.$$
$$\therefore\ \theta = (180^\circ - 71^\circ) \text{ or } (360^\circ - 71^\circ),$$
$$= \underline{109^\circ \text{ or } 289^\circ}.$$

(*c*) Considering the formula $d = ut + \frac{1}{2}at^2$, when $t = 0$, $d = 0$. Thus, the car starts from rest at some point O, as shown in the sketch, and reaches point A with velocity u metres/second.



Hence, at point B,

$$12 = u \times 2 + \frac{a \times 2^2}{2}.$$
$$\therefore\ u + a = 6. \qquad \ldots\ldots\ (1)$$

Also, at point C,

$$48 = u \times 5 + \frac{a \times 5^2}{2}.$$
$$\therefore\ 5u + \frac{25a}{2} = 48. \qquad \ldots\ldots\ (2)$$

(*i*) From equation (1),

$$a = 6 - u. \qquad \ldots\ldots\ (3)$$

Substituting for a in equation (2) gives

$$5u + \frac{25}{2}(6 - u) = 48.$$
$$\therefore\ 10u + 150 - 25u = 96.$$
$$\therefore\ 15u = 54.$$
$$\therefore\ \underline{u = 3\tfrac{3}{5}\text{ m/s}}.$$

(*ii*) Substituting for u in equation (3) gives the constant acceleration as

$$a = 6 - 3\tfrac{3}{5} = \underline{2\tfrac{2}{5}\text{ m/s}^2}.$$

Q 5 (*a*) *Solve the following quadratic equations:*

(*i*) $12x^2 + 7x - 10 = 0$ *by the method of factors, and*
(*ii*) $2y^2 + 5y - 4 = 0$ *by the formula method.*

(*b*) *The resistance, R, of a length of wire varies as the length, l, and inversely as the square of the cross-sectional diameter, d.*

(*i*) *Write a formula for R in terms of l, d and a constant.*
(*ii*) *If R = 50 when l = 750 and d = 0·1, find the value of the constant. (Leave your answer as a fraction.)*
(*iii*) *Hence, calculate R when l = 1000 and d = 0·15.*
(*iv*) *Find the percentage change in R if l is increased by 50% and d is decreased by 20%.*

A 5 (*a*) (*i*)
$$12x^2 + 7x - 10 = 0.$$
$$\therefore\ (4x + 5)(3x - 2) = 0.$$
$$\therefore\ 4x + 5 = 0 \text{ or } 3x - 2 = 0.$$
$$\therefore\ \underline{x = -\tfrac{5}{4} \text{ or } \tfrac{2}{3}}.$$

(*ii*)
$$2y^2 + 5y - 4 = 0.$$
$$\therefore\ y = \frac{-5 \pm \sqrt{\{5^2 - 4 \times 2 \times (-4)\}}}{2 \times 2},$$
$$= \frac{-5 \pm \sqrt{57}}{4},$$
$$= \frac{-5 \pm 7{\cdot}55}{4},$$
$$= \underline{0{\cdot}6375 \text{ or } -3{\cdot}1375}.$$

(*b*) (*i*) $\underline{R = \dfrac{\rho l}{d^2}}$, where ρ is a constant.

(*ii*) From part (*b*) (*i*),

$$\rho = \frac{Rd^2}{l},$$
$$= \frac{50 \times 0{\cdot}1^2}{750},$$
$$= \frac{1}{15} \times \frac{1}{100} = \underline{\frac{1}{1500}}.$$

(*iii*) Substituting values gives

$$R = \frac{1}{1500} \times \frac{1000}{0{\cdot}15^2},$$
$$= \underline{29{\cdot}63}.$$

(*iv*) If l is increased by 50%, let $l_1 = 1{\cdot}5l$ be the increased value.

Similarly, let $d_1 = 0{\cdot}8d$ be the decreased value of d. Then, the new value of resistance, R_1, is given by

$$\begin{aligned} R_1 &= \frac{\rho l_1}{d_1^2}, \\ &= \rho \times \frac{1{\cdot}5l}{(0{\cdot}8d)^2}, \\ &= \frac{1{\cdot}5}{0{\cdot}64} \times \frac{\rho l}{d^2}, \\ &= 2{\cdot}344R, \end{aligned}$$

since $R = \rho l/d^2$.

Hence, the percentage change in R

$$\begin{aligned} &= \frac{2{\cdot}344R - R}{R} \times 100\%, \\ &= \underline{134{\cdot}4\%}. \end{aligned}$$

Q 6 *(a) Plot accurately the graph of $y_1 = x^2 + 2x - 2$ between $x = -4$ and $x = 2$.*

(b) Using the graph in part (a), solve the equations

(i) $x^2 + 2x - 2 = 0$, and
(ii) $x^2 + 2x - 5 = 0$.

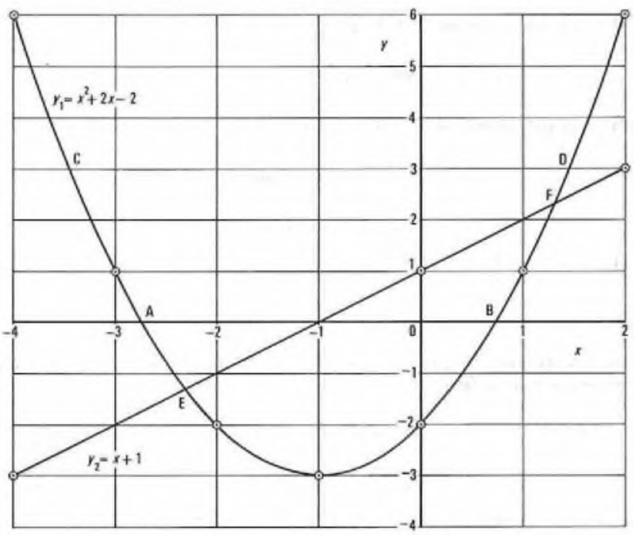
(c) On the axes used in part (a), draw the graph of $y_2 = x + 1$ for the same range of x. Determine the equation whose solutions are given by the x co-ordinates of the points of intersection of

$$y_1 = x^2 + 2x - 2 \text{ and } y_2 = x + 1.$$

Write the equation in the form $x^2 + ax + b = 0$, where a and b are constants.

A 6 (*a*) The graph of $y_1 = x^2 + 2x - 2$ is plotted from the following table of values, and is shown in the sketch.

x	-4	-3	-2	-1	0	1	2
x^2	16	9	4	1	0	1	4
$2x$	-8	-6	-4	-2	0	2	4
-2	-2	-2	-2	-2	-2	-2	-2
y_1	6	1	-2	-3	-2	1	6



(*b*) (*i*) The solution of $x^2 + 2x - 2 = 0$ is given by the graph when $y_1 = 0$; that is, where the graph crosses the x-axis (points A and B). From the graph, the solution is

$$\underline{x = -2{\cdot}73 \text{ or } 0{\cdot}73.}$$

(*ii*) $\qquad x^2 + 2x - 5 = 0,$

or $\qquad x^2 + 2x - 2 = 3.$

Therefore, the solution of $x^2 + 2x - 5 = 0$ is given by the intersection of $y_1 = x^2 + 2x - 2$ with the line $y = 3$ (points C and D on the graph). From the graph, the solution is

$$\underline{x = -3{\cdot}45 \text{ or } 1{\cdot}45.}$$

(*c*) The graph of $y_2 = x + 1$ is a straight line, which can, therefore, be plotted using 2 points (although it is advisable to use 3 points to obtain a check).

When $x = -4$, $y_2 = -3$; when $x = 0$, $y_2 = 1$; and when $x = 2$, $y_2 = 3$.

The graph of $y_2 = x + 1$ is also shown in the sketch, and intersects the curve $y_1 = x^2 + 2x - 2$ at points E and F where $x \approx -2{\cdot}3$ or $1{\cdot}3$.

The equation that has the roots $-2{\cdot}3$ and $1{\cdot}3$ is given by

$$(x + 2{\cdot}3)(x - 1{\cdot}3) = 0.$$

$$\therefore\ \underline{x^2 + x - 2{\cdot}99 = 0.}$$

Note: By calculation, it can be shown that the equation should be $x^2 + x - 3 = 0$, which has the solution $x = -2{\cdot}303$ or $1{\cdot}303$. Graphically, this degree of accuracy could be obtained only from a very carefully drawn large-scale graph.

Q 7 *Values of current, I amperes, and voltage, V volts, observed in an experiment, are related by the law $I = kV^n$, where k and n are constants. If $I = 8{\cdot}7$ A when $V = 2{\cdot}0$ V, and $I = 45{\cdot}5$ A when $V = 5{\cdot}0$ V,*

(a) calculate the values of k and n, and
(b) hence, determine the value of V when $I = 20$ A.

A 7 (*a*) $\underline{n \approx 1{\cdot}8}$, and $\underline{k \approx 2{\cdot}49}$.

(*b*) $\underline{V = 3{\cdot}182 \text{ V}}$.

Q 8 *(a) A triangle has angles in the ratio 5 : 6 : 7. Calculate the angles of the triangle and the ratio of the longest side to the shortest.*

(b) A radar station is to be built on a triangular site ABC, where AB = 450 m, AC = 520 m and angle BAC = 120°. Calculate

(i) the length of BC,
(ii) the angle ACB, and
(iii) the area of the site.

A 8 (*a*) The sum of the angles of any triangle is 180°. If the angles are in the ratio 5 : 6 : 7, the 3 angles are given by

$$\frac{5}{5+6+7} \times 180^\circ = \frac{5 \times 180^\circ}{18} = \underline{50^\circ},$$

$$\frac{6}{5+6+7} \times 180^\circ = \frac{6 \times 180^\circ}{18} = \underline{60^\circ},$$

and

$$\frac{7}{5+6+7} \times 180^\circ = \frac{7 \times 180^\circ}{18} = \underline{70^\circ}.$$

Sketch (*a*) shows a triangle, ABC, having angles of 50°, 60° and 70°. The longest side, BC, is opposite the largest angle, A = 70°, and the shortest side, AC, is opposite the smallest angle, B = 50°.

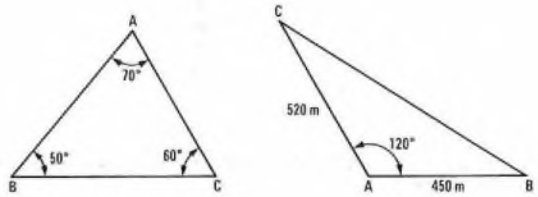
From the sine rule,

$$\frac{\sin A}{\sin B} = \frac{\text{BC}}{\text{AC}}.$$

$$\begin{aligned} \therefore\ \frac{\text{BC}}{\text{AC}} &= \frac{\sin 70^\circ}{\sin 50^\circ}, \\ &= \frac{0{\cdot}9397}{0{\cdot}7660}, \\ &= 1{\cdot}227. \end{aligned}$$

Hence, the ratio of the longest side to the shortest is

$$\underline{1{\cdot}227 : 1.}$$



(*a*) (*b*)

(*b*) The radar-station site is illustrated in sketch (*b*).

(*i*) The length of BC can be determined using the cosine rule, which gives

$$BC^2 = AB^2 + CA^2 - 2 \times AB \times AC \times \cos \angle BAC,$$

whence $\underline{BC = 840 \cdot 8 \text{ m.}}$

(*ii*) From the sine rule,

$$\frac{\sin \angle ACB}{AB} = \frac{\sin \angle BAC}{BC},$$

whence $\underline{\angle ACB = 27° \, 37'.}$

(*iii*) The area of the site is the area of triangle ABC, and is given by

$$\text{area} = \tfrac{1}{2} \times AB \times AC \times \sin \angle BAC \text{ m}^2,$$

$$= \frac{450 \times 520}{2} \times \sin 120° \text{ m}^2,$$

$$= 117\,000 \times 0 \cdot 8660 \text{ m}^2,$$

$$= \underline{101\,325 \text{ m}^2.}$$

Q 9 (*a*) *If $10^{2x} = 437 \cdot 6$, find the value of x.*

(*b*) *If $\log (y + 2) = \log (2y - 1) + 3$, and the logarithms are to the base 2, determine the value of y.*

(*c*) *Express the denary number 175 in binary form.*

(*d*) *Working throughout in binary notation, calculate*

(*i*) *110 111 + 11 011, and*

(*ii*) *110 101 − 10 111.*

(*e*) *The calculation*

$$\frac{2 \cdot 86 \times 11 \cdot 36^2 \times \pi}{\sqrt{(15 \cdot 63)} \times 0 \cdot 562}$$

gives a result having the significant figures 5219. By suitable approximation and cancelling, obtain an approximate value for the expression and, hence, insert the decimal point in its correct place in the result.

A 9 (*a*) $10^{2x} = 437 \cdot 6.$

Taking logarithms,

$$2x = \log_{10} 437 \cdot 6,$$

$$= 2 \cdot 6411.$$

$$\therefore \underline{x = 1 \cdot 3206.}$$

(*b*) $\log_2 (y + 2) = \log_2 (2y - 1) + 3.$

$$\therefore \log_2 (y + 2) - \log_2 (2y - 1) = 3.$$

$$\therefore \log_2 \left(\frac{y + 2}{2y - 1}\right) = 3.$$

$$\therefore \frac{y + 2}{2y - 1} = 2^3 = 8.$$

$$\therefore y + 2 = 16y - 8.$$

$$\therefore 15y = 10.$$

$$\therefore \underline{y = \tfrac{2}{3}.}$$

(*c*) To express, in binary form, any number on the denary scale, it is simplest to divide the number successively by the radix, 2, the remainder being noted at each division.

	Remainder
2) 175	
87	1
43	1
21	1
10	1
5	0
2	1
1	0
0	1

The remainders are written down in reverse order to obtain the binary number. Thus,

$$\underline{175_{10} = 10\,101\,111_2.}$$

Note: The remainders are written in reverse order so that the least-significant digit is on the right and the most-significant digit on the left. This can more clearly be appreciated when the power associated with the respective digits is considered; that is,

$$175 = (1 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0).$$

(*d*) (*i*)

```
  110 111
   11 011 +
1 010 010
```

Hence, the binary sum is $\underline{1\,010\,010}$.

(*ii*)

```
110 101
 10 111 −
 11 110
```

Hence, the binary difference is $\underline{11\,110}$.

(*e*)

$$\frac{2 \cdot 86 \times 11 \cdot 36^2 \times \pi}{\sqrt{(15 \cdot 63)} \times 0 \cdot 562} \approx \frac{3 \times 120 \times 3}{4 \times 0 \cdot 5},$$

$$= \frac{9 \times 120}{2},$$

$$= 9 \times 60,$$

$$= 540.$$

Therefore, the correct result is $\underline{521 \cdot 9}$.

Q 10 (*a*) *A solid metal sphere, of diameter 240 mm, is to be recast to form a number of conical spacing units of base diameter 60 mm and perpendicular height 20 mm. Calculate the number of conical units that can be produced.*

(*b*) *The instantaneous value of a current, i amperes, at stated times, t milliseconds, is given in the table.*

t	0	5	10	15	20	25	30	35	40	45	50
i	5·0	8·5	4·5	8·2	11·6	5·0	−6·0	−7·0	1·5	1·0	5·0

Apply the mid-ordinate rule, using 5 intervals of 10 ms, to determine, for the period given,

(*i*) *the average value of the current, and*

(*ii*) *the r.m.s. value of the current.*

A 10 (*a*) The volume of the sphere

$$= \frac{4}{3} \pi r^3 \text{ metres}^3,$$

where r is the radius of the sphere (m),

$$= \frac{4}{3} \times \pi \times 120^3 \text{ mm}^3,$$

$$= 2 \cdot 304\pi \times 10^6 \text{ mm}^3.$$

The volume of one cone

$$= \frac{1}{3} \pi r^2 h \text{ metres}^3,$$

where r is the radius of the base of the cone (m), and h is the vertical height of the cone (m),

$$= \frac{1}{3} \times \pi \times 30^2 \times 20 \text{ mm}^3,$$

$$= 6\pi \times 10^3 \text{ mm}^3.$$

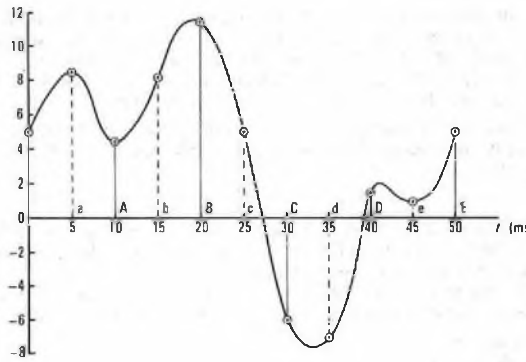
The number of conical units that can be produced

$$= \frac{\text{volume of the sphere}}{\text{volume of one cone}},$$

$$= \frac{2 \cdot 304\pi \times 10^6}{6\pi \times 10^3},$$

$$= \underline{384.}$$

(*b*) The graph of i/t is shown in the sketch.



The time axis is divided into 5 equal intervals of 10 ms at points A, B, C, D and E, and the mid-ordinates are erected at points a, b, c, d and e.

(*i*) By the mid-ordinate rule, the area under the graph between $t = 0$ and $t = 50$ ms is given by the length of one interval multiplied by the sum of the mid-ordinates. Hence,

$$\text{area} = 10 \times (8{\cdot}5 + 8{\cdot}2 + 5{\cdot}0 - 7{\cdot}0 + 1{\cdot}0)\text{ ms A},$$
$$= 10 \times 15{\cdot}7 \text{ ms A},$$
$$= 157 \text{ ms A}.$$

The average value of current between $t = 0$ and $t = 50$ ms

$$= \frac{\text{area under graph between } t = 0 \text{ and } t = 50 \text{ ms}}{50 \text{ ms}}\text{ A},$$
$$= \frac{157}{50}\text{ A},$$
$$= \underline{3{\cdot}14 \text{ A}}.$$

(*ii*) The r.m.s. value of current is defined as the square root of the mean value of the square of the current; that is,

$$i_{\text{r.m.s.}} = \sqrt{\{(i^2)_{\text{mean}}\}}\text{ A}.$$

If the mid-ordinate rule is to be used, it is necessary to determine the mean, or average, value of i^2. This can be done, without the need for a second graph, by squaring the values of i at the mid-ordinate points, a, b, c, d and e. The values are shown in the table.

t (ms)	5	15	25	35	45
i (A)	8·5	8·2	5·0	−7·0	1·0
i^2 (A^2)	72·25	67·24	25·00	49·00	1·00

By the mid-ordinate rule, the area under the graph of i^2 between $t = 0$ and $t = 50$ ms

$$= 10 \times (72{\cdot}25 + 67{\cdot}24 + 25{\cdot}00 + 49{\cdot}00 + 1{\cdot}00)\text{ ms A}^2,$$
$$= 10 \times 214{\cdot}49 \text{ ms A}^2,$$
$$= 2144{\cdot}9 \text{ ms A}^2.$$

The average value of i^2 between $t = 0$ and $t = 50$ ms

$$= \frac{\text{area under graph of } i^2 \text{ between } t = 0 \text{ and } t = 50 \text{ ms}}{50 \text{ ms}}\text{ A}^2,$$
$$= \frac{2144{\cdot}9}{50}\text{ A}^2,$$
$$= 42{\cdot}898 \text{ A}^2.$$
$$\therefore\ i_{\text{r.m.s.}} = \sqrt{42{\cdot}898}\text{ A},$$
$$= \underline{6{\cdot}55 \text{ A}}.$$

RADIO AND LINE TRANSMISSION A, 1975

Students were expected to answer any 6 questions

Q 1 (*a*) *The envelope of an amplitude-modulated waveform varies sinusoidally between maximum values of ±8 V and minimum values of ±2 V. Sketch the waveform and determine*

(*i*) *the amplitude of the unmodulated carrier,*
(*ii*) *the amplitude of the modulating signal, and*
(*iii*) *the modulation factor (expressed as a percentage).*

(*b*) *This waveform is applied to the input of the detector circuit shown in Fig. 1. Explain, with reference to further waveforms, the functions of the various components in the demodulating and filtering processes.*

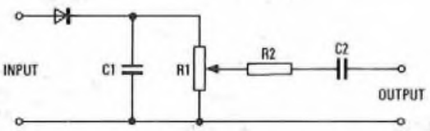


Fig. 1

A 1 See A1 and A2, Radio and Line Transmission A, 1974, *Supplement*, Vol. 68, p. 1, Apr. 1975.

Q 2 (*a*) *Give typical capacitance values and working voltages for each of the following types of capacitor:*

(*i*) *air-dielectric,*
(*ii*) *mica-dielectric,*
(*iii*) *paper-dielectric, and*
(*iv*) *electrolytic.*

(*b*) *What other factors influence the choice of each of the above for use in communication equipment?*
(*c*) *State one application for each type.*

A 2 See A4, Radio and Line Transmission A, 1974, *Supplement*, Vol. 68, p. 2, Apr. 1975.

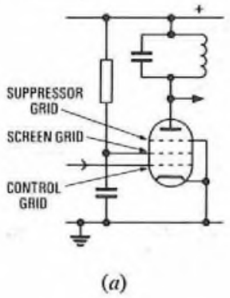
Q 3 (*a*) *State the limitations of a thermionic triode when used in a high-frequency-amplifier stage.*
(*b*) *Briefly explain the reason for these limitations.*
(*c*) *Explain how the 2 additional grids in a pentode help to reduce these limitations.*
(*d*) *Draw typical anode-current/anode-voltage characteristics for a pentode.*

A 3 (*a*) A thermionic triode, used in a high-frequency-amplifier stage, is limited by instability, and restricted gain and selectivity.

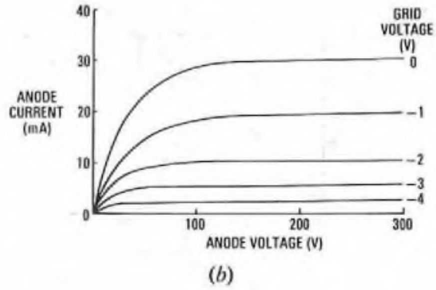
(*b*) These limitations arise basically from the inter-electrode capacitances of the triode, principally the grid–anode capacitance. Since capacitive reactance is inversely proportional to frequency, the grid–anode capacitance provides a low-impedance feedback path between the output and the input of the valve at high frequencies. Should any of this feedback be in phase with the input, oscillation occurs, and the valve therefore becomes unstable. If the feedback opposes the input signal, the latter tends to be suppressed, and amplification is reduced.

Also, at high frequencies, the anode–cathode capacitance and other stray capacitances that appear in parallel with the anode load, tend to shunt the output signal, thus reducing the amplification of the stage.

(*c*) The additional grids used in a pentode, the screen grid and the suppressor grid, are connected as shown in sketch (*a*). The screen grid is effectively at earth potential to the alternating signal, hence acting as an earthed screen between the control grid and the anode, reducing the grid–anode capacitance. Thus, the basic cause of instability that exists in a triode is eliminated. The suppressor grid reduces the effects of secondary emission resulting from the introduction of the screen grid.



(*a*)



(*b*)

A further effect of the 2 additional grids in a pentode is to increase considerably the amplification factor and anode a.c. resistance compared with those of a triode. Thus, the stage gain of a pentode can be made much greater than that of a triode. The high anode a.c. resistance of the pentode, in parallel with the anode load, has much less effect in reducing the selectivity of the tuned circuit, compared with the same effect in a triode high-frequency amplifier.

(*d*) Sketch (*b*) shows typical anode-current/anode-voltage characteristics for a pentode at a particular value of screen-grid voltage.

Q 4 (*a*) *Define the decibel.*

(*b*) *Use the definition to derive an expression for voltage ratios in decibel notation. State the conditions for which the expression is valid.*

(*c*) *The measured response of a parallel tuned circuit is shown in the table.*

Frequency (kHz)	244	246	248	250	252	254	256
Voltage (V)	60	100	160	200	160	106	70

Plot the response curve in decibels relative to 1 V (dBV) against the frequency in kilohertz. Use the curve to determine the bandwidths at

(*i*) *the half-power points, and*
(*ii*) *the −6 dB points*

A 4 (*a*) If the ratio of 2 powers, P_1 and P_2, is to be expressed in decibels, the number of decibels, N, is given by

$$N = 10 \log_{10} \frac{P_1}{P_2} \text{ dB.} \qquad \ldots\ldots (1)$$

(*b*) The power, P watts, dissipated in a resistance, R ohms, is given by

$$P = \frac{V^2}{R} \text{ watts,}$$

where V is the voltage (V).

Hence, from equation (1), for 2 resistances, R1 and R2, having voltages V_1 and V_2 respectively,

$$N = 10 \log_{10} \frac{V_1^2/R_1}{V_2^2/R_2} \text{ dB.}$$

If $R_1 = R_2$,

$$N = 10 \log_{10} \frac{V_1^2}{V_2^2} = 10 \log_{10} \left(\frac{V_1}{V_2}\right)^2 \text{ dB,}$$

$$= 20 \log_{10} \frac{V_1}{V_2} \text{ dB.} \qquad \ldots\ldots (2)$$

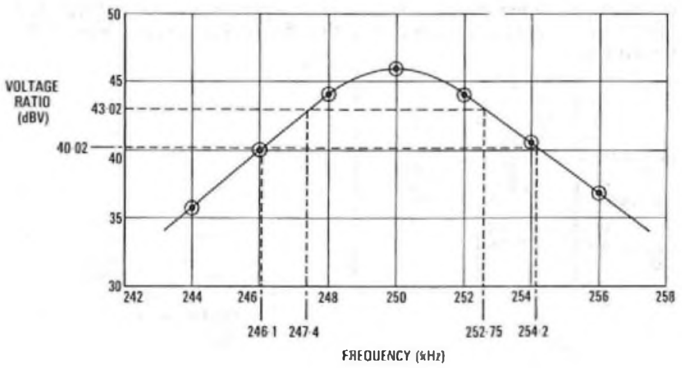
This expression is valid only when the 2 resistances, across which voltages V_1 and V_2 are developed, are equal.

(*c*) The table shows the voltage ratios in decibel notation against frequency. The ratios are calculated from equation (2), as shown in the following example for a frequency of 244 kHz.

$$N = 20 \log_{10} \frac{60}{1} = 20 \times 1{\cdot}7782 = \underline{35{\cdot}56 \text{ dBV.}}$$

Frequency (kHz)	244	246	248	250	252	254	256
Voltage Ratio (dBV)	35·56	40	44·08	46·02	44·08	40·51	36·9

The response curve, plotted from the table, is shown in the sketch.



(*i*) The maximum value of the response curve is 46·02 dBV. The half-power points occur at −3 dB relative to the maximum value; that is, at 43·02 dBV. From the graph, voltage ratios of 43·02 dBV occur at 247·4 kHz and 252·75 kHz. Hence, the bandwidth at the half-power points is 252·75 − 247·4 = 5·35 kHz.

(*ii*) The −6 dB points occur at a voltage ratio of 40·02 dBV. From the graph, the bandwidth at the −6 dB points is 254·2 − 246·1 = 8·1 kHz.

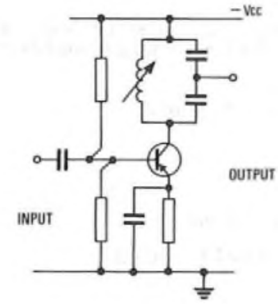
Q 5 (*a*) *Sketch the circuit of a tuned class-A transistor amplifier.*

(*b*) *Explain why a parallel tuned circuit is used in the radio-frequency-amplifier section of a radio receiver.*

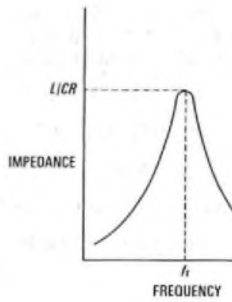
(*c*) *Sketch suitably labelled input-waveform/output-waveform diagrams to illustrate what is meant by the following conditions of amplification:*

(*i*) *class A,*
(*ii*) *class B, and*
(*iii*) *class C.*

A 5 (*a*) One arrangement of a tuned class-A transistor amplifier is shown in sketch (*a*).

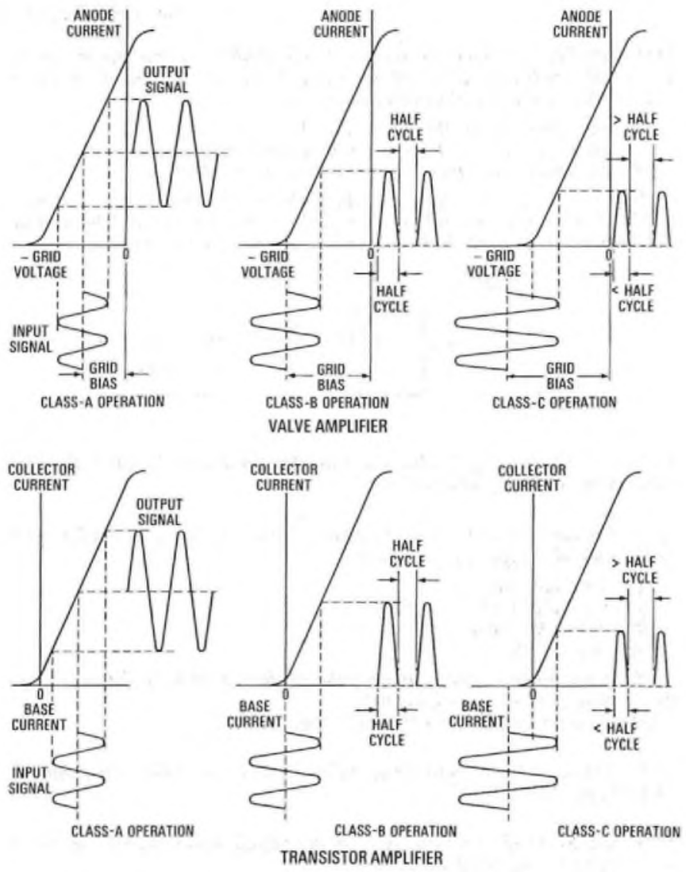


(*a*)



(*b*)

(*b*) The impedance of a parallel tuned circuit at resonance is high, and can be shown to have a value of L/CR ohms, where L is the inductance in the circuit (H), C is the capacitance (F), and R is the series resistance (Ω). (The resistance is usually the series resistance of the inductor.) A typical impedance/frequency characteristic for a parallel tuned circuit



(*c*)

is shown in sketch (*b*), which illustrates the impedance reaching a maximum value at the resonant frequency, f_r, and falling off rapidly each side of f_r.

The voltage amplification factor of a radio-frequency amplifier is approximately proportional to the load impedance. Thus, by connecting a parallel tuned circuit, resonant at the desired signal frequency, as the load of such an amplifier, the characteristic ensures that the desired signal frequency is amplified considerably more than undesired frequencies. This property is known as *selectivity*, and is dependent upon the slope of the impedance/frequency characteristic each side of resonance; the greater the slope, the better the selectivity. Further, by changing the values of inductance and capacitance in the tuned circuit, it is possible to vary the resonant frequency over a wide frequency range.

(*c*) (*i*) In class-A valve operation, the operating point and amplitude of the input signal are so adjusted that anode current flows at all times during a cycle of the input signal. Similarly, for class-A transistor operation, the operating point and input-signal amplitude are adjusted so that collector current flows at all times during a cycle of the input signal. Both types of amplifier are operated within the straight portion of their characteristics so that minimum distortion occurs.

(*ii*) In class-B operation, the valve or transistor is biased so that the output just ceases in the absence of an input signal, and an output is obtained only during alternate half-cycles of the input signal.

(*iii*) In class-C operation, the valve or transistor is biased beyond cut-off, so that anode current or collector current flows for less than half of each input cycle.

Input-waveform/output-waveform diagrams for each case are shown in sketch (*c*).

Q 6 (*a*) *Briefly explain the principles of operation of the following types of microphone:*

(*i*) *carbon-granule, and*
(*ii*) *moving-coil.*

(*b*) *List the relative advantages and disadvantages of each type.*
(*c*) *State one use for each type.*

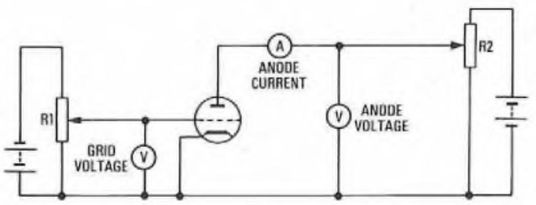
A 6 See A3, Radio and Line Transmission A, 1970, *Supplement*, Vol. 63, p. 87, Jan. 1971; and A6, Radio and Line Transmission A, 1972, *Supplement*, Vol. 65, p. 93, Jan. 1973.

Q 7 (*a*) *Sketch a circuit that can be used to determine the static characteristics of a thermionic triode.*
(*b*) *The data shown in the table were obtained for a triode.*

Anode Voltage (V)	*Anode Current (mA) for Grid Voltage =* −1 V	−3 V	−5 V	−7 V
40	4·00	2·00	—	—
80	8·75	6·25	4·00	2·00
120	13·50	10·75	8·25	5·25
160	18·50	15·25	12·50	9·00
200	23·25	19·50	16·50	13·00

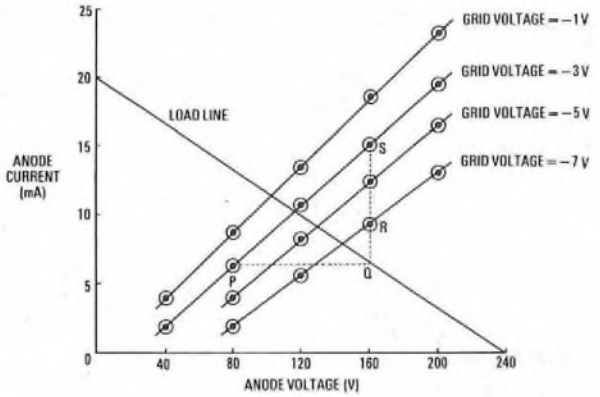
Plot the anode characteristics for each value of grid voltage.
(*c*) *Use these characteristics to determine the anode (a.c.) resistance, mutual conductance and amplification factor of the triode.*
(*d*) *Draw the load line for an anode load resistance of 12 kΩ at a supply voltage of 240 V.*

A 7 (*a*) A circuit suitable for use in determining the static characteristics of a triode valve is shown in sketch (*a*). Potential dividers R1 and R2 enable the supply voltages to the grid and anode to be set and varied, while the anode current is measured at each step.



(*a*)

(*b*) The anode characteristics are shown in sketch (*b*).



(*b*)

(*c*) The anode (a.c.) resistance, r_a, is defined as the change in anode voltage divided by the corresponding change in anode current, at a constant value of grid voltage.

From the anode characteristics, at a grid voltage of −3 V,

$$r_a = \frac{PQ}{QS} = \frac{160 - 80}{(15\cdot25 - 6\cdot25) \times 10^{-3}}\ \Omega,$$
$$= \underline{8\cdot9\ k\Omega.}$$

The mutual conductance, g_m, is the change in anode current divided by the corresponding change in grid voltage, at a constant value of anode voltage.

From the anode characteristics, at an anode voltage of 160 V,

$$g_m = \frac{SR}{-3-(-7)} = \frac{(15\cdot25 - 9) \times 10^{-3}}{4}\ S,$$
$$= \underline{1\cdot56\ mS.}$$

The amplification factor, μ, is given by the mutual conductance multiplied by the anode (a.c.) resistance.

$$\therefore\ \mu = 1\cdot56 \times 10^{-3} \times 8\cdot9 \times 10^3,$$
$$= \underline{13\cdot9.}$$

(*d*) When the anode voltage is zero, the supply voltage appears across the load resistance. Therefore, the anode current

$$= \frac{240}{12 \times 10^3}\ A = 20\ mA.$$

When the anode current is zero, the anode voltage is equal to the supply voltage, 240 V.

The load line is constructed from these 2 points, and is shown on the anode characteristics.

Q 8 (*a*) *State whether the transistor is basically a current-amplifying or a voltage-amplifying device.*
(*b*) *Explain, with the aid of circuit diagrams, the operation of a simple resistance-loaded transistor amplifier in either the common-emitter or the common-base configuration. Show clearly the directions of the electrode currents.*
(*c*) *A transistor is connected in the common-base configuration with a collector-load resistance of 20 kΩ. The current gain of the transistor is 0·98 and its input resistance is 80 Ω. Calculate the voltage and power gain of the amplifier.*
(*d*) *State the factors that affect the frequency response of a 2-stage audio-frequency amplifier using resistance-capacitance interstage coupling.*

A 8 (*a*) The transistor is basically a current-amplifying device.

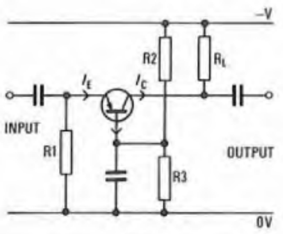
(*b*) Sketch (*a*) shows a simple resistance-loaded small-signal p n p transistor amplifier in the common-base configuration, with the directions of the electrode currents marked. The emitter–base junction is forward biased by resistors R1, R2 and R3. During positive half-cycles of an alternating input signal, the forward bias of this junction is increased, causing an increase in both the emitter and collector currents, I_E and I_C respectively. During negative half-cycles, I_E and I_C decrease. Thus, the collector current varies in accordance with the input waveform, producing an alternating output-signal voltage across load resistor R_L.

For a small-signal resistance-loaded common-base amplifier, the current gain approximates to the short-circuit current gain, α, of the transistor, which is given by the change in collector current, δI_C, divided by the corresponding change in emitter current, δI_E, and is approximately 0·98. Thus, if the input resistance is R_{in}, the voltage gain

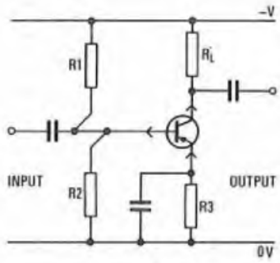
$$= \frac{\delta I_C R_L}{\delta I_E R_{in}} = \alpha \frac{R_L}{R_{in}}, \qquad \ldots\ldots (1)$$

and the power gain

$$= \frac{(\delta I_C)^2 R_L}{(\delta I_E)^2 R_{in}} = \alpha^2 \frac{R_L}{R_{in}}. \qquad \ldots\ldots (2)$$



(*a*)



(*b*)

Sketch (*b*) shows a simple resistance-loaded small-signal p n p transistor amplifier in the common-emitter configuration, with the directions of the electrode currents marked. The emitter–base junction is forward biased by resistors R1, R2 and R3. During positive half-cycles of an alternating input signal, the forward bias of this junction is decreased, causing a reduction in both the emitter and collector currents. During negative half-cycles, the emitter and collector currents are increased. Thus, the collector current varies in accordance with the input waveform, but in anti-phase to it, producing an alternating output-signal voltage across the load resistor. The voltage and power gain of a common-emitter amplifier are higher than those of a common-base amplifier, since the current gain is much greater (generally of the order of 50–200).

(*c*) From equation (1), the voltage gain

$$= 0{\cdot}98 \times \frac{20 \times 10^3}{80} = \underline{245.}$$

From equation (2), the power gain

$$= 0{\cdot}98^2 \times \frac{20 \times 10^3}{80} = \underline{240{\cdot}1.}$$

(*d*) The factors that affect the frequency response of a 2-stage resistance-capacitance-coupled audio-frequency amplifier are

(*i*) the potential-dividing effect of the resistance-capacitance coupling which, at low frequencies when the reactance of the capacitor is relatively large, causes a significant portion of the output signal to be developed across the capacitance, allowing only a small portion to be developed across the resistance, and

(*ii*) the output shunt capacitance of the first stage and the input shunt capacitance of the second stage; at high frequencies, these capacitances have a low reactance and tend to reduce the gain by shunting the effective load of the first stage.

Q 9 (*a*) *Draw the circuit diagram of a simple tuned-collector transistor oscillator.*

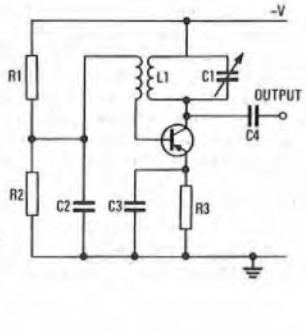
(*b*) *Briefly explain*

(*i*) *the biasing and stabilization arrangements,*
(*ii*) *what governs the frequency of oscillation, and*
(*iii*) *how the oscillations are maintained.*

(*c*) *Indicate the point from which the output signal may be obtained.*

(*d*) *A tuned circuit is resonant at 600 kHz. By what factor would the capacitance be changed if the circuit were retuned to 300 kHz?*

A 9 (*a*) The sketch shows the circuit diagram of a simple tuned-collector transistor oscillator.



(*b*) (*i*) The circuit is basically that of a common-emitter amplifier, with resistors R1, R2 and R3 providing the conventional bias and stabilization arrangements for such an amplifier. The potential divider formed by resistors R1 and R2 fixes the base bias voltage. Resistor R3 stabilizes the circuit because, if the collector current increases due to temperature instability, the emitter current increases by an almost equal amount, thereby increasing the voltage drop across resistor R3 and reducing the forward bias of the emitter–base junction. Hence, the base current is reduced to compensate for the original change in collector current.

(*ii*) The tuned circuit, consisting of capacitor C1 and inductor L1, determines the frequency of oscillation, which is approximately equal to $1/2\pi\sqrt{(L_1C_1)}$ Hz.

(*iii*) The oscillations are maintained by feeding back energy from the collector circuit, via the transformer, to the base circuit. The transistor, acting as an amplifier, is arranged just to overcome the circuit losses.

(*c*) The point from which an output signal can be obtained is shown in the sketch.

(*d*) The resonant frequency, f_r hertz, of a tuned circuit is given by

$$f_r = \frac{1}{2\pi\sqrt{(LC)}} \text{ hertz,}$$

where L is the inductance (H), and C is the capacitance (F).

Let the circuit be resonant at a frequency f_{r1} hertz when the value of the capacitor is C_1 farads, and at f_{r2} hertz when the value of the capacitor is C_2 farads. Then,

$$f_{r1} = \frac{1}{2\pi\sqrt{(LC_1)}} \text{ hertz,}$$

and

$$f_{r2} = \frac{1}{2\pi\sqrt{(LC_2)}} \text{ hertz.}$$

$$\therefore \frac{f_{r1}}{f_{r2}} = \frac{1/2\pi\sqrt{(LC_1)}}{1/2\pi\sqrt{(LC_2)}} = \sqrt{\left(\frac{C_2}{C_1}\right)}.$$

$$\therefore \frac{600 \times 10^3}{300 \times 10^3} = \sqrt{\left(\frac{C_2}{C_1}\right)}.$$

$$\therefore \frac{C_2}{C_1} = 4,$$

$$\text{or } C_2 = \underline{4C_1.}$$

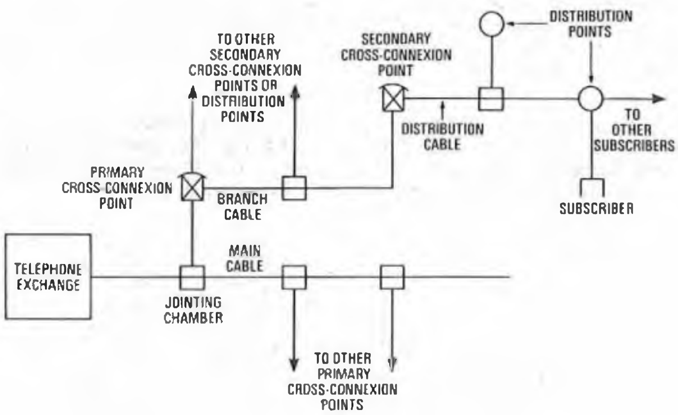
Thus, to retune the circuit to 300 kHz, the capacitance has to be increased by a factor of 4.

Q 10 (*a*) *Draw suitably-labelled block diagrams to show the equipment and links required for 3 of the following connexions:*

(*i*) *a telephone subscriber to the local telephone exchange,*
(*ii*) *a repeatered audio junction circuit between 2 telephone exchanges,*
(*iii*) *a teleprinter circuit between a shipping-company's office and one of the company's ships, and*
(*iv*) *an outside-broadcast event to the national broadcasting station.*

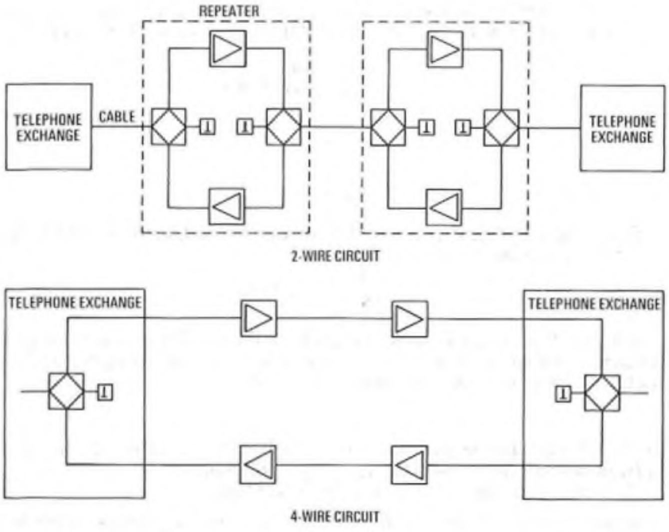
(*b*) *Briefly explain the purpose of the terminating sets used in either part (ii) or (iii) above.*

A 10 (*a*) Sketches (*a*)–(*d*) respectively show the required connexions.



(*a*)

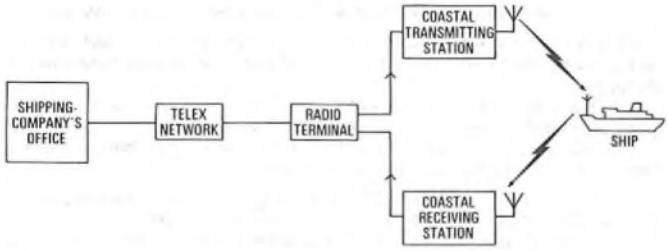
(*b*) Amplifiers are unidirectional devices, while telephone circuits are bidirectional. Hence, for the junctions shown in sketch (*b*), it is



(*b*)

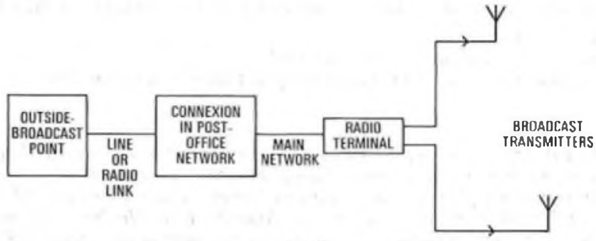
necessary to separate the GO and RETURN paths by means of 2-wire–4-wire terminating sets and amplify each direction of transmission separately. The terminating sets are fitted with balancing networks to prevent interaction between the GO and RETURN paths, which leads to howling.

In sketch (*c*), a terminating set is required at the radio terminal to



(*c*)

separate the GO and RETURN paths to the ship. To prevent interaction between the transmitted and received radio signals, the coastal transmitting and receiving stations are generally geographically separated. Thus, outgoing signals to the ship are routed via a different station from incoming signals from the ship; hence the need to separate the signals.



(*d*)

LINE PLANT PRACTICE A, 1975

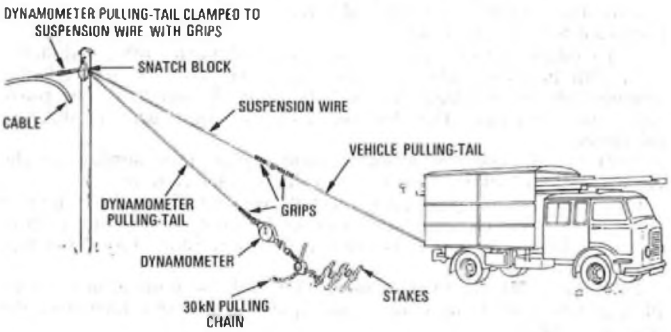
Students were expected to answer any 6 questions

Q 1 A combined self-supporting aerial cable is to be erected manually.

(a) Describe, with the aid of sketches, how it is tensioned and terminated at a supporting pole.

(b) Why are aerial cables terminated to supporting poles at intervals along the route?

A 1 (*a*) Sketch (*a*) shows how a manually-erected combined self-supporting aerial cable is tensioned. The procedure is as follows.



(*a*)

(*i*) The cable's suspension wire is connected to a vehicle pulling-tail fixed to the back of a suitable vehicle. The connexion is made by folding the suspension and pulling-tail wires back on themselves to form linked eyes; the wires are then secured with grips.

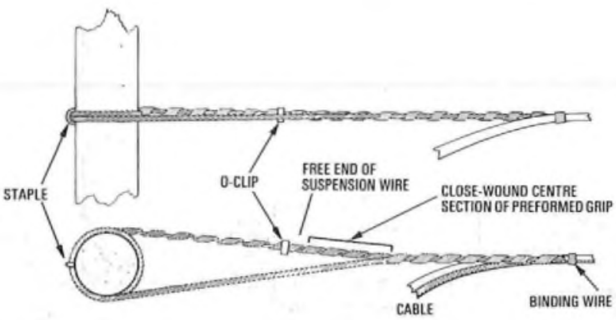
(*ii*) The aerial cable is pulled by the vehicle to remove any slack and place the cable near to its final position.

(*iii*) Another pulling tail, attached to a 30 kN dynamometer and pulling chain anchored to stakes, is connected to the suspension wire with grips, as shown.

(*iv*) The suspension wire is tensioned to 0·9 kN above the required tension to ensure an equal tension throughout the aerial-cable section.

(*v*) The tension is then reduced to its correct value, and the termination made.

Sketch (*b*) shows how the cable is terminated at a supporting pole. The procedure is as follows.



(*b*)

(*i*) Several turns of binding wire are applied to the cable where the suspension wire leaves the sheath.

(*ii*) The suspension wire is wrapped 1½ times round the pole and stapled to it.

(*iii*) One half of a preformed grip is attached to the free end of the suspension wire and clamped in place with an O-clip.

(*iv*) The other half of the grip is wrapped around the suspension wire, towards the cable, to complete the termination.

(*b*) At the ends of cable sections, where cables must be jointed, it is necessary to terminate the cables so that joints can be made and attached to the pole.

It is also necessary to terminate cables at poles where the "pull-on-pole" exceeds 9 m, the pull-on-pole being defined as the minimum distance between the pole and an imaginary line joining points 33 m from the pole on the 2 adjacent spans.

Q 2 (a) State 3 basic requirements for ensuring a good joint between 2 cables.

(b) What are the 2 basic methods used for making a joint between conductors in an audio cable?

(c) What type of joint would be used for jointing aluminium conductors? Why is this method preferred?

(d) State 2 instances when twist-jointed copper conductors must be soldered.

(e) What are the 3 main advantages of using a jointing machine?

A 2 (*a*) Three basic requirements for ensuring good joints are that

(*i*) when random-pair jointing is not being used, correct pair-to-pair jointing according to the code markings, or correct balancing, is required,

(*ii*) the joint must be physically good, having low resistance, adequate mechanical strength and good insulation, and

(*iii*) the sheath closure must be strong and watertight, and the joint enclosed must be dry before closure.

(*b*) The 2 basic methods used for making joints between conductors in an audio cable are twist jointing and crimp jointing.

(*c*) Crimped joints are used for jointing aluminium conductors, because aluminium is relatively brittle, and breakages tend to occur if twist jointing is used. Also, aluminium tends to corrode, and this can produce high-resistance joints when twist jointing is used.

(*d*) Twist-jointed copper conductors must be soldered

(*i*) when one or both conductors have a diameter of 1·27 mm, or when a 0·9 mm or 0·5 mm diameter conductor is jointed to one of 0·9 mm diameter, and

(*ii*) on junction and main-network circuits, and on balanced cables.

(*e*) The 3 main advantages of using a jointing machine are that it is

(*i*) faster,

(*ii*) easier, causing less fatigue, and

(*iii*) more consistent in producing good electrical connexions.

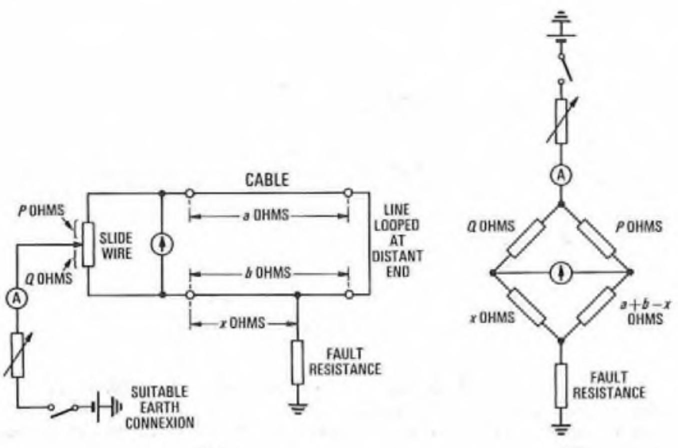
Q 3 (a) Draw the circuit used to measure the distance to a full-earth fault on a conductor in a cable when using the Murray test.

(b) Derive the formula used to determine the resistance of a conductor to a full-earth fault when using the Murray test. Having determined the resistance of a conductor to an earth fault, how is this converted into a distance?

(c) Using the derived formula, determine the distance to an earth fault, given that the loop resistance is 100 Ω, the ratio arms have a ratio of 1 : 4, and the single-wire resistance of the conductor is 15 Ω/km.

(d) When is a Varley test preferred to a Murray test?

A 3 (*a*) Sketch (*a*) shows the circuit used to measure the distance to a full-earth fault on a conductor in a cable.



(*a*) (*b*)

(*b*) The 2 sections of the slide-wire resistance, *P* ohms and *Q* ohms, and the 2 sides of the line loop to the fault, $a + b - x$ ohms and x ohms, form the 4 resistance components of a Wheatstone bridge, as illustrated in sketch (*b*). It can be seen that the fault resistance does not form part of the bridge circuit and, therefore, does not affect the balance conditions. The variable resistor allows control of the testing current.

When the bridge is balanced, indicated by a null reading on the galvanometer, then

$$\frac{P}{Q} = \frac{a + b - x}{x}.$$

$$\therefore\ Px = Qa + Qb - Qx.$$

$$\therefore\ x(P + Q) = Q(a + b).$$

$$\therefore\ x = \frac{Q(a + b)}{P + Q}\ \text{ohms}.$$

If the resistance per unit length of the conductor is known, the resistance, x ohms, can be converted into a distance measurement.

(*c*) From the given data, $a + b = 100\ \Omega$, and $P/Q = 4/1$; that is, $P = 4Q$. Note that P is always greater than Q since $(a + b - x) > x$.

$$\therefore\ x = \frac{Q \times 100}{4Q + Q}\ \text{ohms},$$

$$= \frac{100}{5}\ \Omega,$$

$$= 20\ \Omega.$$

Since the resistance per unit length of the conductor is 15 Ω/km, the distance to the fault

$$= \frac{20}{15} = 1{\cdot}33\ \text{km}.$$

(*d*) The Varley test is preferred on long lines to locate the fault to a particular cable section. The Murray test is more accurate, and is used to locate the exact position of the fault.

Q 4 (a) List the steps required to seal a polyethylene sleeve to a polyethylene-sheathed cable using epoxy-resin putty.

(b) What types of cable are currently used for

(i) the local main network from exchange to primary cross-connexion points, and

(ii) the distribution network between primary cross-connexion points and distribution points?

A 4 (*a*) The steps required to seal a polyethylene sleeve to a polyethylene-sheathed cable using epoxy-resin putty are listed below.

(*i*) The stripped cable ends are set up for jointing such that the cable-sheath butts will overlap the sleeve at each end by 45 mm.

(*ii*) The butts are thoroughly degreased for 300 mm using methylated spirit.

(*iii*) The places on the sheaths where the putty wipes are to be made are roughened for about 100 mm with a scratch brush applied longitudinally to the cable.

(*iv*) A special polymer-backed aluminium foil is lapped around the sheath, with the polymer backing adjacent to the sheath.

(*v*) The foil is bound with copper wire to within 3 mm of its edge.

(*vi*) The binding wire is gently heated with a blow torch until molten polymer begins to ooze out of the edges of the foil.

(*vii*) When cool, the binding wire is removed.

(*viii*) The aluminium foil, which is now firmly bonded to the sheath, is scratch brushed.

(*ix*) Foil is then bonded to the ends of the sleeve in a similar manner.

(*x*) An air valve is fitted to the sleeve.

(*xi*) The sleeve is placed over one of the cable ends.

(*xii*) After the conductors have been jointed, an electrical-continuity wire is connected between the aluminium moisture barriers of each cable by soldering at the sheath butts.

(*xiii*) Desiccative packs are inserted into the joint.

(*xiv*) The sleeve is positioned over the joint and spaced with expanded-polystyrene spacers.

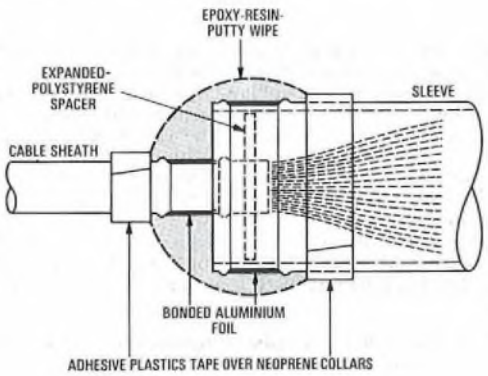
(*xv*) Collars, 5 mm thick, are made around both cables and sleeve ends with neoprene adhesive tape on the sides of the bonded aluminium foil remote from the seal, to limit the length of the putty wipes and to ensure that the edges of the wipes are of adequate thickness.

(*xvi*) A thin coat of synthetic-rubber primer is applied to the cleaned foil and allowed to dry naturally to a tacky state.

(*xvii*) With hands properly protected, the putty is made by mixing the resin and hardener, kneaded around the joint and roughly shaped.

(*xviii*) The final shaping is done using a moleskin cloth lubricated with water.

(*xix*) The putty is allowed to harden and the joint is finished by placing bands of 12 mm wide black plastics adhesive tape over the neoprene collars.



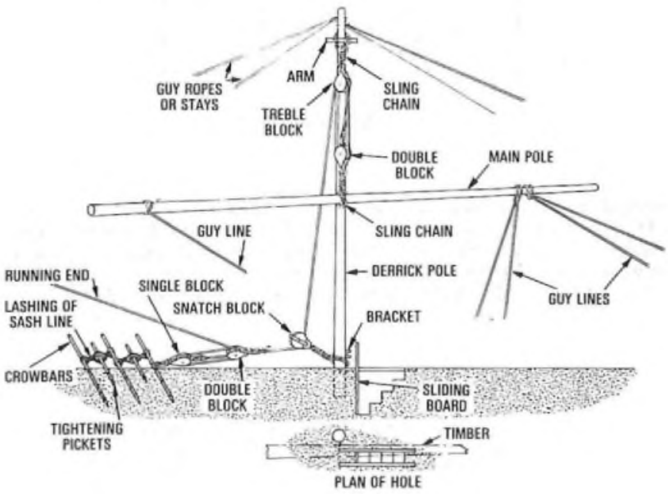
(*xx*) After 2–3 h, the joint is pressure tested. The sketch shows the completed seal at one end of the sleeve.

(*b*) (*i*) Paper-insulated polyethylene-sheathed unit-twin pressurized cable is currently used in the local main network.

(*ii*) Polyethylene-insulated polyethylene-sheathed pair-type jelly-filled cable is currently used in the local distribution network.

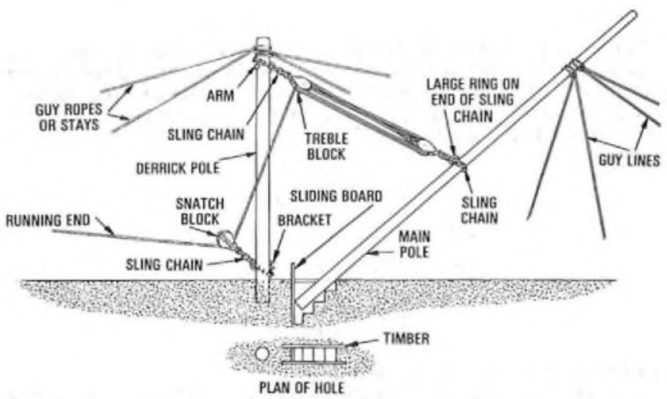
Q 5 (*a*) *Using sketches, show 2 methods whereby a tall telephone pole can be erected manually with the aid of a derrick pole.*
(*b*) *State the circumstances in which these methods would be used.*

A 5 (*a*) Sketch (*a*) illustrates the method of erecting a tall pole known as the *middling method*, whereby the main pole is suspended from the derrick pole just to the butt side of its centre of gravity. The main pole is raised until its butt can be guided over the hole, using the guy lines, and is then allowed to drop into the hole.



(*a*)

Sketch (*b*) shows the method known as the *end-on method*, whereby the main pole is gradually raised to the vertical position by a tackle attached above its centre of gravity. The butt is guided into the hole by a sliding board.



(*b*)

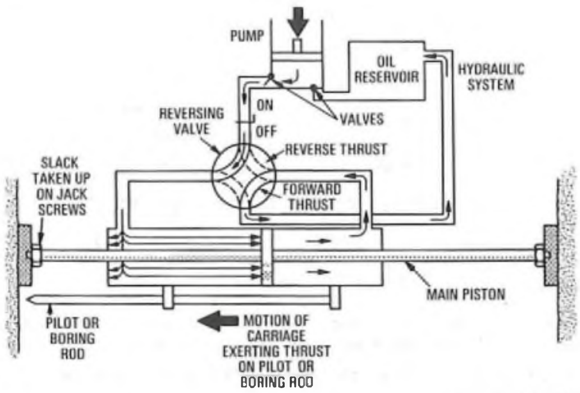
(*b*) The middling method is the preferred method of manually erecting a tall telephone pole using a derrick pole, since the stresses on the derrick pole are smaller. Sometimes, however, space does not permit the use of the middling method, but may allow the end-on method to be used. Manual methods are used where mechanized pole-erection units cannot be accommodated.

Q 6 (*a*) *State 2 situations where thrust-boring can conveniently be used for providing a duct.*
(*b*) *What types of soil are unsuitable for the thrust-boring?*
(*c*) *Describe briefly the operation of thrust-boring.*
(*d*) *What is the planned minimum clearance required of a thrust-bore from obstructions?*
(*e*) *What is the minimum depth of a thrust-bore beneath a concrete road?*

A 6 (*a*) For completeness, 4 situations where thrust-boring can conveniently be used are given. They are

(*i*) under roads,
(*ii*) under level-crossings,
(*iii*) under pavements that are expensive to reinstate, and
(*iv*) under embankments.

(*b*) Soil that does not readily compact, or that contains obstructions such as boulders and tree roots, is unsuitable for thrust-boring.



(*c*) The sketch illustrates the principle of the thrust-borer. The sequence of operations is as follows.

(*i*) A thrust pit is dug on one side of the obstacle under which the bore is to be made, to accommodate the thrust-boring machine.
(*ii*) A receiving pit is dug on the other side.
(*iii*) The machine is lowered into the thrust pit, correctly positioned, and lined up with the receiving pit using sighting rods.
(*iv*) The pilot rod is placed on the carriage and thrust into the soil using the hydraulic ram.
(*v*) The carriage is returned using the reversing valve, and a boring rod is coupled in and thrust forward. This process is repeated as boring proceeds.
(*vi*) A check is maintained on the hydraulic pressure, as an increase in pressure is an indication that an obstruction has been met.
(*vii*) When the pilot rod arrives at the receiving pit, it is removed and replaced by a larger-diameter boring head, which is then drawn back through the bore.
(*viii*) The bore can be further increased in size by using successively larger boring heads.
(*ix*) Finally, duct or cable is secured directly to an enlarging head and pulled through. (Wrought-iron or mild-steel duct can be pushed through.)

(*d*) The minimum clearance of the bore from any obstruction, such as gas, water or electricity services, is 450 mm.
(*e*) The minimum depth of a bore beneath a concrete road is 600 mm.

Q 7 (*a*) *State 2 reasons why cable ducts are necessary.*
(*b*) *Name 6 types of material used for ducts.*
(*c*) *Sketch the plastics joint used on unidiameter earthenware ducts.*
(*d*) *State 3 advantages of unidiameter earthenware ducts.*
(*e*) *Briefly describe the laying of a straight single-way track of unidiameter earthenware ducts in an open trench.*
(*f*) *What test is made on a newly constructed duct route?*

A 7 (*a*) Cable ducts are necessary to

(*i*) provide protection for cables, and
(*ii*) enable tracks to be laid down in advance of cabling.

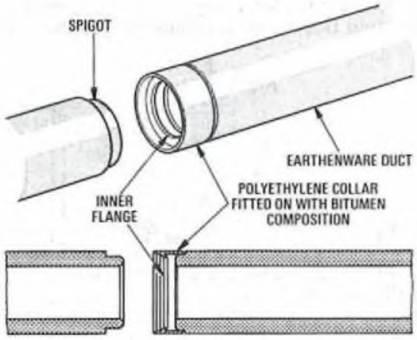
(*b*) For completeness, 9 materials used for ducts are given. They are

(*i*) earthenware,
(*ii*) PVC,
(*iii*) polyethylene,
(*iv*) steel,
(*v*) wrought iron,
(*vi*) asbestos cement,
(*vii*) pitch fibre,
(*viii*) glass-reinforced plastics, and
(*ix*) concrete.

(*c*) The sketch shows the plastics joint used on unidiameter earthenware ducts.
(*d*) Six advantages of unidiameter earthenware ducts are given. They are that

(*i*) they have a low coefficient of friction,
(*ii*) they are easy to lay,

(*iii*) small obstructions can be negotiated by setting the duct at small angles,
(*iv*) the length of trench open at any time can be minimized,
(*v*) they are durable (except under impact loads), and
(*vi*) they are inexpensive.



(*e*) The procedure for laying a straight single-way track of uni-diameter earthenware ducts in an open trench is as follows.

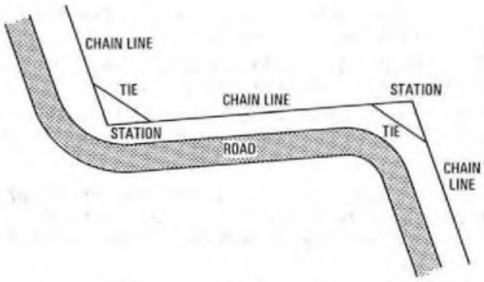
(*i*) Each duct is laid in the trench with the collar towards the direction of laying.
(*ii*) The spigot is cleaned and pushed into the collar of the preceding duct, simultaneously being given half a turn.
(*iii*) A 1·5 m long by 52 mm diameter wooden mandrel is drawn through each duct as it is laid.

(*f*) When a duct route has been laid, a 240 mm long cast-iron mandrel, 79 mm in diameter, with a cylindrical brush attached to its following end, is drawn by hand through each duct track to test the bore.

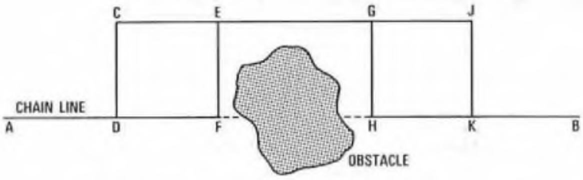
Q 8 (*a*) *What is the prime purpose of the preliminary survey made prior to the provision of an underground duct scheme?*
(*b*) *What is the purpose of the paving schedule?*
(*c*) *List 10 items of information that should be contained in the paving schedule.*
(*d*) *Draw a sketch showing the ideal relative positions of public utility services under the footway.*
(*e*) *What minimum clearance is permitted between underground telecommunications plant and underground high-voltage (over 650 V) single-core power cables?*

Q 9 (*a*) *With the aid of a sketch, illustrate the chain-surveying technique used for surveying the route of a winding road.*
(*b*) *What is this technique called?*
(*c*) *Explain, with the aid of sketches, the techniques used in surveying to overcome*
(*i*) *an obstacle in the path of a chain line, and*
(*ii*) *an obstacle that cannot be crossed, such as a river, but where a station is required on the other side.*
(*d*) *When laying down a chain line, how is it ensured that the line is straight?*

A 9 (*a*) The chain-surveying method is based on the fact that, if a triangle is set out on the ground and the lengths of its sides measured, that triangle can be accurately reproduced to any scale on paper. The angles of the triangle are automatically reproduced since they depend on the relative side lengths. Sketch (*a*) shows how this technique is applied to surveying a winding road.
Straight chain lines are laid along each straight section of the route and, at each bend, a tie chain is laid to form a triangle with the 2 adjacent chain lines. By measuring the sides of the triangles and the lengths of the chain lines, the route is exactly defined, and can later be reproduced on a scale map. Each salient point in the survey is called a *station*.
(*b*) The technique described in part (*a*) is called the *open-traverse method*.
(*c*) (*i*) Sketch (*b*) shows a solid obstacle in the path of a chain line, AB.
To overcome the obstacle, 2 stations, D and F, are marked on the chain line approaching the obstacle, and perpendicular offsets are set out to stations C and E with DC = FE. A chain line is projected beyond the obstacle from stations C and E. Stations G and J are then

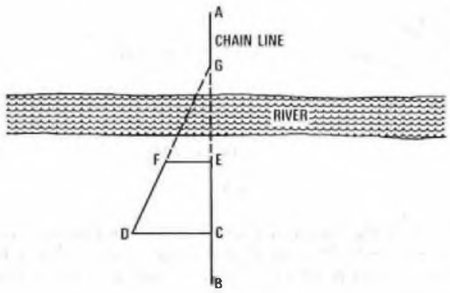


(*a*)



(*b*)

marked, and perpendicular offsets are taken to stations H and K with GH = JK = DC = FE. Stations H and K therefore lie on the main chain line, which can be projected towards station B, and the distance between stations F and H is equal to the distance EG.
(*ii*) Sketch (*c*) shows a river in the path of a chain line, AB.



(*c*)

Station G is marked on the chain line approaching the river, and stations C and E are marked on the opposite bank, sighted in line with stations A and G. Perpendicular offsets are set out from stations C and E to stations D and F, such that DFG is a straight line. Triangles GEF and GCD are similar

$$\therefore \frac{GE}{EF} = \frac{GE + EC}{CD}.$$

$$\therefore GE \times CD = GE \times EF + EC \times EF.$$

$$\therefore GE \times (CD - EF) = EC \times EF.$$

$$\therefore GE = \frac{EC \times EF}{CD - EF},$$

and EC, EF and CD can be measured.
Thus, stations E and C lie on the chain line, which can be projected towards station B, and the distance between stations G and E can be calculated.
(*d*) A chain line is checked for straightness by erecting 3 ranging poles at points along its length and sighting them to ensure that they are in line.

Q 10 (*a*) *What is the basis of electrolytic corrosion?*
(*b*) *Where lead-covered cables are laid in the proximity of d.c. traction or supply systems, electrolytic corrosion of the sheaths may sometimes take place. Explain how this happens.*
(*c*) *With a sketch, describe an insulating gap in a lead-sheathed cable.*
(*d*) *If the lead sheath is used for electrical screening, how is continuity maintained for high-frequency currents at an insulating gap?*

Students were expected to answer any 6 questions

Q 1 (a) A digital computer contains arithmetic, control, storage, input and output units. Draw a block diagram showing how these units are connected together in a typical digital computer. Explain the function of each of the blocks and illustrate clearly the direction of flow of data and control information.
(b) List 3 typical input devices and 3 typical output devices.

A 1 See A1, Computers A, 1969, *Supplement*, Vol. 63, p. 7, Apr. 1970.

Q 2 (a) Explain what is meant by the term radix notation.
(b) Using the denary number 29 as an example, compare the following notations and explain why each is used in digital computers:

(i) binary,
(ii) octal, and
(iii) hexadecimal.

A 2 (*a*) The radix is the base of a number system. The radix notation is a method of expressing a number by positional representation. The number is represented by a set of digits in such a way that both the position of each digit and its value are of significance.
Any number, N, can be represented by radix r and digits D_n, D_{n-1}, D_{n-2} etc., thus:

$$N = D_n r^n + D_{n-1} r^{n-1} \ldots . + D_0 r^0 + D_{-1} r^{-1} \ldots . + D_{-m} r^{-m},$$

where terms up to and including $D_0 r^0$ represent the integral part, terms after $D_0 r^0$ represent the fractional part, and D can take any integral value from 0 to $r - 1$.
(*b*) (*i*) A binary number has a radix of 2.

$$\therefore\ 29_{10} = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0,$$
$$= \underline{11\ 101_2}.$$

Binary notation, in which only the 2 digit values 0 and 1 are permissible, is used in computers because most electrical and electronic components have only 2 stable states; for example, a relay can be operated or released, or a transistor can be in an ON or OFF state.
(*ii*) An octal number has a radix of 8.

$$\therefore\ 29_{10} = 3 \times 8^1 + 5 \times 8^0,$$
$$= \underline{35_8}.$$

Octal notation is often used in modern computing because conversion from binary notation to octal can easily be performed. The primary use of octal notation is to record values stored in binary registers.
(*iii*) A hexadecimal number has a radix of 16. The 10 decimal digits 0–9 are used, together with 6 more digits, usually A, B, C, D, E and F, which represent 10, 11, 12, 13, 14 and 15 as single characters. Thus,

$$29_{10} = 1 \times 16^1 + 13 \times 16^0,$$
$$= 1 \times 16^1 + \mathrm{D} \times 16^0,$$
$$= \underline{1\mathrm{D}_{16}}.$$

Hexadecimal notation is used because it is easy and convenient to convert the contents of 8 bit bytes into hexadecimal digits. The number of digits representing, for example, an address in a main store, is reduced by a factor of 4 when converted from binary to hexadecimal notation, thus making it easier for the programmer to manipulate the code. Only 2 hexadecimal digits are necessary to represent each 8 bit byte.

Q 3 (a) Multiply 11 101₂ by 1 011₂ using binary arithmetic. Convert the result to denary form, showing all working.
(b) Divide 10 000 110 001₂ by 11 101₂ using binary arithmetic. Convert the result to denary form, showing all working.
(c) What is a quick method of calculating the denary value of a binary number that has been operated on to shift it to the left or right, if the original value is known in denary form?

A 3 (*a*) Binary multiplication can be carried out by copying the multiplicand for each binary 1 in the multiplier, the least significant digit of the multiplicand being aligned with the operating digit in the multiplier for each case. The results are then summed.

Multiplicand:	11 101
Multiplier:	1 011 ×
Copy multiplicand:	11 101
Copy multiplicand, shifted 1 place:	111 010 +
Copy multiplicand, shifted 3 places:	11 101 000 +
Sum:	100 111 111

Thus,

$$\underline{11\ 101_2 \times 1\ 011_2 = 100\ 111\ 111_2}.$$

To convert the result to denary form, each digit is multiplied by its weight and the results are summed.

Binary Digit	Weight	Result
1	$2^8 = 256$	256
0	$2^7 = 128$	0
0	$2^6 = 64$	0
1	$2^5 = 32$	32
1	$2^4 = 16$	16
1	$2^3 = 8$	8
1	$2^2 = 4$	4
1	$2^1 = 2$	2
1	$2^0 = 1$	1
		319

Hence,

$$\underline{11\ 101_2 \times 1\ 011_2 = 319_{10}}.$$

Note: To check the result, the multiplicand and multiplier are converted to denary form and their product is recalculated, thus:

$$11\ 101_2 = 2^4 + 2^3 + 2^2 + 0 + 2^0 = 29_{10},$$
$$1\ 011_2 = 2^3 + 0 + 2^1 + 2^0 = 11_{10},$$

and

$$29 \times 11 = \underline{319}.$$

(*b*) Binary division can be carried out in long-division form, thus:

```
              100 101
      11 101)10 000 110 001
              1 110 1
                 10 010 0
                  1 110 1
                     11 101
                     11 101
```

Thus,

$$\underline{10\ 000\ 110\ 001_2 \div 11\ 101_2 = 100\ 101_2}.$$

Converting the result to denary form:

Binary Digit	Weight	Result
1	$2^5 = 32$	32
0	$2^4 = 16$	0
0	$2^3 = 8$	0
1	$2^2 = 4$	4
0	$2^1 = 2$	0
1	$2^0 = 1$	1
		37

Hence,

$$\underline{10\ 000\ 110\ 001_2 \div 11\ 101_2 = 37_{10}}.$$

Check:

$$10\ 000\ 110\ 001_2 = 2^{10} + 2^5 + 2^4 + 2^0 = 1073_{10},$$
$$11\ 101_2 = 2^4 + 2^3 + 2^2 + 2^0 = 29_{10},$$

and

$$1073 \div 29 = \underline{37}.$$

(*c*) A quick method of calculating the denary value of a binary number that is shifted to the left or right, if the original denary value is known, is to multiply the original value by 2 for each place shifted to the left, or to divide the original value by 2 for each place shifted to the right.
For example, if the binary number 11 101 (denary 29) is shifted one place to the left, it becomes 111 010. Evaluating this gives

$$111\,010_2 = 2^5 + 2^4 + 2^3 + 0 + 2^1 + 0 = 58,$$

but a shorter method is to multiply 29 by 2, also giving 58.

Similarly, if the binary number 11 101 (denary 29) is shifted 3 places to the right, it becomes 11·101. Evaluating this gives

$$11\cdot101 = 2^1 + 2^0 + 2^{-1} + 0 + 2^{-3},$$

$$= 2 + 1 + 0\cdot5 + 0 + 0\cdot125 = 3\cdot625,$$

but it is quicker to divide 29 by 2 three times, thus:

$$\frac{29}{8} = 3\cdot625.$$

Q 4 (a) Write down the truth table for each of the following logic functions: AND, OR, NAND and NOR.
(b) Illustrate each of the logic functions in part (a) using
(i) a simple Boolean expression,
(ii) a Venn diagram, and
(iii) a Karnaugh, or Veitch, map.

A 4 (*a*) The truth tables are shown below. Two-variable functions have been assumed, A and B being the inputs and F the output.

AND

A	B	F
0	0	0
1	0	0
0	1	0
1	1	1

OR

A	B	F
0	0	0
1	0	1
0	1	1
1	1	1

NAND

A	B	F
0	0	1
1	0	1
0	1	1
1	1	0

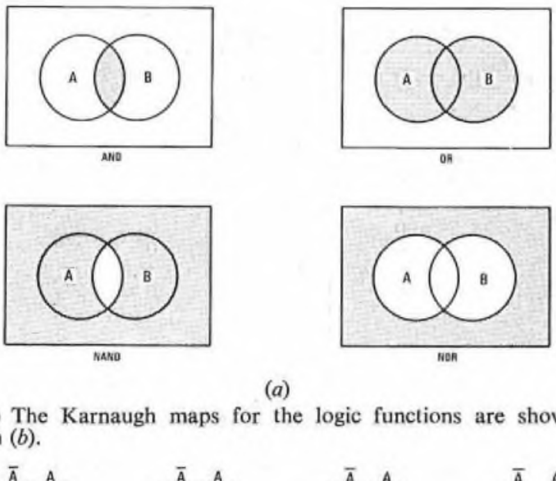
NOR

A	B	F
0	0	1
1	0	0
0	1	0
1	1	0

(*b*) (*i*) The Boolean expressions for the logic functions are given in the table.

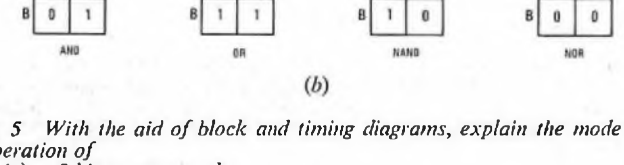
Logic Function	Boolean Expression
AND	$F = A\,.\,B$
OR	$F = A + B$
NAND	$F = \overline{A\,.\,B}$
NOR	$F = \overline{A + B}$

(*ii*) The Venn diagrams for the logic functions are shown in sketch (*a*). The shaded areas represent $F = 1$ and the unshaded areas represent $F = 0$.



(*a*)

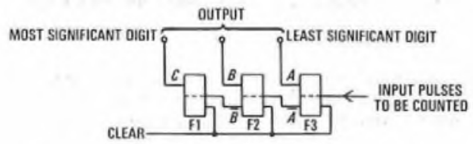
(*iii*) The Karnaugh maps for the logic functions are shown in sketch (*b*).



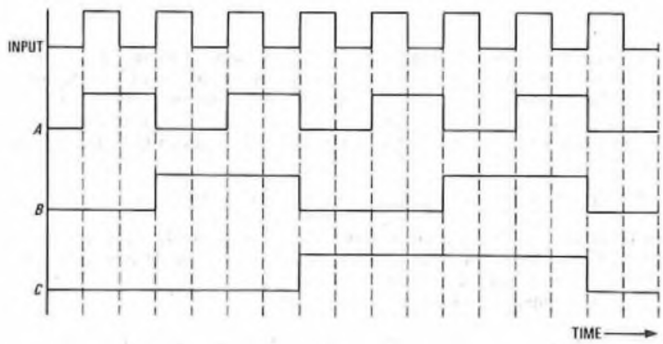
(*b*)

Q 5 With the aid of block and timing diagrams, explain the mode of operation of
(a) a 3 bit counter, and
(b) a 3 bit shift register.

A 5 (*a*) A counter is a device that records the number of times an event occurs. The input to an electronic counter usually consists of a series of pulses, which are counted over a given period. The basic memory device used in an electronic counter is the bistable circuit with a triggered input. The 3 bistable circuits required for a 3 bit counter are arranged as shown in sketch (*a*). The output represents the digits of the 3 bit number. The largest number that can be held in such a counter is binary 111 (i.e., denary 7). The least significant digit is that given by bistable circuit F3.



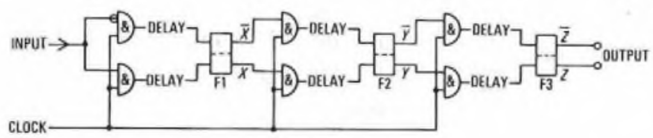
(*a*)



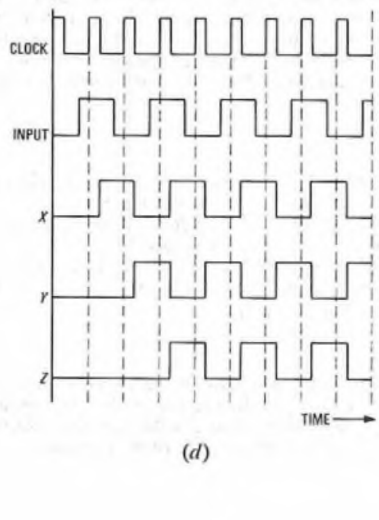
(*b*)

The counter operates to positive pulses in the following manner. It is cleared by a positive pulse on the CLEAR lead. The first positive input pulse triggers bistable circuit F3, so that A becomes logic 1 and $\bar{A}$ becomes logic 0. The effect of the latter is a negative pulse applied to the trigger of bistable circuit F2, which does not change its state. The counter now contains binary 001. On receipt of the next positive pulse at bistable circuit F3, A reverts to logic 0 and $\bar{A}$ to logic 1. This results in a positive pulse being applied to bistable circuit F2 which causes it to change state. A negative pulse from $\bar{B}$ is therefore applied to the trigger of bistable circuit F1. The counter now contains binary 010. The next positive pulse increments the counter to binary 011. Subsequent pulses increment the counter to its maximum value of binary 111, and the next pulse resets the counter to zero.

The timing diagram of sketch (*b*) shows the outputs of bistable circuits F1, F2 and F3 for the full cycle of 8 input pulses. The higher level of each waveform represents logic 1, and the lower, logic 0.



(*c*)



(*d*)

(*b*) A shift register is a device capable of storing information, and is used extensively within computers. The shift register shown in sketch (*c*) uses bistable circuits as the storage elements. The information to be stored is applied serially to the left-hand bistable circuit. Each time a binary digit is to be read into the register, a clock pulse is applied. This causes all the digits in the register to be shifted right, and the new digit is stored in bistable circuit F1.

The input signal is applied to the SET input of bistable circuit F1, while its inverse is applied to the RESET input. The AND gates ensure that the bistable circuits remain in their previous states until a clock pulse arrives, by holding the SET and RESET inputs at zero.

The output from each stage is extended to the input of the next stage, so that, when a clock pulse occurs, each bistable circuit adopts the state of its predecessor. Bistable circuit F1 adopts the state of the input signal.

The delay units associated with each bistable circuit allow time for the AND gates to operate on receipt of a clock pulse before the bistable circuits change state.

The timing diagram of sketch (*d*) shows the outputs of bistable circuits F1, F2 and F3 as the input alternates between logic 1 and logic 0. The higher level of each waveform represents logic 1, and the lower, logic 0.

Q 6 (*a*) *At what stage in the preparation of a programme for a digital computer are flowcharts used?*

(*b*) *Give 3 reasons why flowcharts are considered to be so important.*

(*c*) *Draw the flowchart of a programme that will compute the value of y for values of x between 1–3 inclusive, where* $y = x^2 + bx + c$.

A 6 (*a*) Flowcharts are used at an early stage in the preparation of a programme as an aid to the conversion of the programme specification into a statement of sequential operations. The flowchart then guides further development of the programme, and is a diagrammatic representation of the required logical process. The flowchart is used as the basis for coding the programme, either in a high-level language or in machine code and, thus, the level of detail must be sufficient for unambiguous coding. The detail varies with the language used and the complexity of the programme, but it is usually sufficient to represent a sequence of instructions forming a logical unit of the programme by a single symbol.

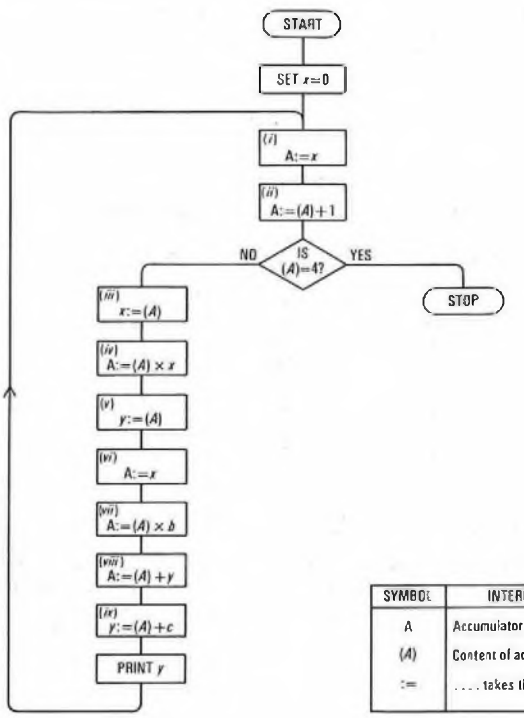
(*b*) The use of flowcharts is important because

(*i*) they help to ensure that no input or output record is overlooked, and that all the requirements of the programme specification are met,

(*ii*) they act as an aid to removing defects from the programme and checking for logical errors, and

(*iii*) they make the coded programme more intelligible and, therefore, aid documentation of the programme.

(*c*) A flowchart of a programme that will compute the value of y for values of x between 1–3 inclusive, where $y = x^2 + bx + c$, is shown in the sketch.



SYMBOL	INTERPRETATION
A	Accumulator
(A)	Content of accumulator
:=	.... takes the value of ....

Operations (*i*) and (*ii*) indicate that x is placed in the accumulator and incremented, to establish its value. Operation (*iii*) indicates that the content of the accumulator is stored in x. In operations (*iv*) and (*v*), x^2 is calculated and stored in y. In operations (*vi*) and (*vii*), bx is calculated and held in the accumulator. In operations (*viii*) and (*ix*), bx and c are added to y, which contains the value of x^2, giving the final value of y for the output.

Q 7 (*a*) *With the aid of sketches, describe how data is held on the following input/output media, and discuss the advantages and disadvantages of each method:*

(*i*) *8-hole punched paper tape, and*

(*ii*) *80-column punched cards.*

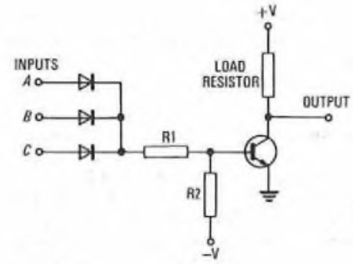
(*b*) *What is meant by the term parity bit? Illustrate your answer by deriving a binary code that uses a parity bit.*

A 7 See A7, Computers A, 1971, *Supplement*, Vol. 65, p. 19, Apr. 1972.

Q 8 (*a*) *Draw a circuit diagram of a positive-logic 3-input resistor-transistor* NOR *logic element, and explain its electrical and logical operation.*

(*b*) *What logic function will the circuit in part (a) perform if negative logic were applied? Verify your answer with voltage and truth tables.*

A 8 (*a*) The circuit diagram shows a positive-logic 3-input resistor-transistor NOR logic element.



The transistor is normally held in the OFF state by a negative potential applied via biasing resistor R2. The output voltage is, therefore, positive, and is virtually equal to +V.

When a positive signal, defined as *high-level*, is applied to any of the inputs, the corresponding diode conducts and the signal is applied to the base of the transistor via resistor R1. This causes the transistor to switch on, and the output voltage falls to almost zero; that is, to a *low level*. Thus, any high-level input signal is converted to a low-level output signal; only when all the inputs are simultaneously low is the output high.

In positive logic, the more positive voltage is defined as logic state 1, and the less positive voltage as logic state 0. The positive-logic truth table for the gate is, therefore, as shown below.

Inputs			Output
A	*B*	*C*	
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

This is the truth table for a 3-input NOR gate.

(*b*) Let the positive voltage applied to any input be V_i volts, and the positive output voltage be V_o volts. From the description and truth table given in part (*a*), a voltage table can be constructed, as shown below.

Inputs			Output
A	B	C	
0	0	0	V_o
0	0	V_i	0
0	V_i	0	0
0	V_i	V_i	0
V_i	0	0	0
V_i	0	V_i	0
V_i	V_i	0	0
V_i	V_i	V_i	0

If the less positive voltage is defined as logic state 1, and the more positive voltage as logic state 0, the system is said to have *negative logic*. Using this system, the voltage table can be converted to a negative-logic truth table as shown below.

Inputs			Output
A	B	C	
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

This is the truth table for a 3-input NAND gate. Hence, a logical NAND operation is performed when negative logic is applied to the circuit described in part (*a*).

Q 9 (*a*) *Explain how variables are represented on an analogue computer.*
(*b*) *What is meant by the term scale factor?*
(*c*) *Three variables, +12, +7, and −3, are applied to a passive analogue summing network. With the aid of a sketch, calculate the output of the network using resistor values of your own choice. If the input scale factor is 1, what is the scale factor of the result?*

A 9 See A10, Computers A, 1974, *Supplement*, Vol. 68, p. 32, Apr. 1975.

The formula derived in part (*a*) of that answer shows that the resistor values are irrelevant, provided they are equal. The formula also shows that, if the input scale factor is unity, the result is one quarter of the sum of the inputs.

Q 10 Devise a machine code and use it to write a programme to calculate the area of 3 circles whose radii can be entered directly from a peripheral device. Output the results to a peripheral device of your choice. Give a key to the machine code devised.

A 10 A suitable set of machine-code instructions for use in the required programme is given in Table 1.

Table 1

Instruction	Function
STOP	Stops the programme
LD X	Loads the accumulator with the content of store location X. Information already in the accumulator is overwritten
ST X	Stores the content of the accumulator in store location X. The content of the accumulator is unaffected

Table 1 (*continued*)

Instruction	Function
SUB X	Subtracts the content of store location X from the accumulator
MULT X	Multiplies the content of the accumulator by the content of store location X. The result remains in the accumulator
JNZ L	Jumps to label L if the content of the accumulator is not zero
READ N	Inputs data from peripheral device N to the accumulator
PRINT N	Outputs content of the accumulator to peripheral device N

The area of a circle is the square of its radius multiplied by the constant π. The required programme is shown in Table 2, and Tables 3 and 4 give details of the constants and devices used.

Table 2

Label	Programme	Comments
	LD THREE ST COUNT	Sets the count to 3
L1	READ C	Reads radius from peripheral input device
	ST R	Stores value of radius
	MULT R	Computes square of radius
	MULT PI	Computes area of circle
	PRINT B	Prints area on peripheral output device
	LD COUNT SUB ONE ST COUNT	Subtracts 1 from the count
	JNZ L1	Jumps to L1 if count is not zero
	STOP	

Table 3

Constant Store	Value
THREE	3
PI	3·142
ONE	1

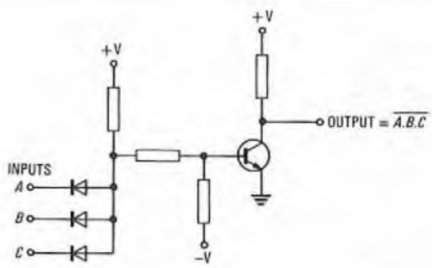
Table 4

Device	Description
C	Card reader
B	Line printer

CORRECTION

COMPUTERS A, 1973 (*Supplement*, Vol. 67, July 1974)

A 7 (*b*) Sketch (*b*) should have been as shown below.



When any of the inputs is low, negative potential is applied to the base of the transistor. Hence, the transistor is in the OFF state and the output is high. Only when all 3 inputs are high is the transistor switched on and the output low. Therefore, the device functions as a positive-logic NAND gate.

Students were expected to answer any 6 questions

Q 1 (a) Define the term Q-factor for a resonant circuit, and discuss its significance.

(b) Explain the meaning of the 3 dB points, and show how they are related to the Q-factor at resonance.

(c) Describe an experiment to show the relationship between the impedance of a parallel tuned circuit and the frequency of the supply when the latter is varied through resonance.

Q 2 (a) What is meant by the term power factor of an a.c. circuit?

(b) An inductor of 0·3 H and 20 Ω resistance is connected in series with 40 Ω across a 10 V supply at 50 Hz. Draw a phasor diagram showing the relationship between current and the voltages in the circuit.

(c) Calculate

(i) the power factor, and

(ii) the energy dissipated in the circuit.

(d) What components, added in series with the circuit under test, would make the power factor 0·6?

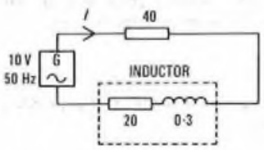
A 2 (*a*) The power factor of an a.c. circuit is the ratio of the true power to the apparent power in the circuit. It is given by the cosine of the phase angle between the current in the circuit and the voltage across it. Thus,

$$\text{power factor} = \frac{\text{true power}}{\text{apparent power}} = \cos\phi,$$

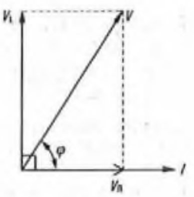
$$\text{or } P = VI\cos\phi \text{ watts},$$

where P is the true power (W), V is the voltage (V), I is the current (A), and ϕ is the phase angle.

(*b*) The circuit is shown in sketch (*a*), and the phasor diagram in sketch (*b*).



(*a*)



(*b*)

The current is taken as the reference phasor since the circuit is connected in series. The voltage, V_R volts, appearing across the total resistance in the circuit is in phase with the current, and is given by $V_R = (40 + 20)I$ volts. The voltage across the inductance, V_L volts, leads the current by 90°, and is given by $V_L = (2\pi \times 50 \times 0{\cdot}3)I$ volts. The resultant, V volts, is the supply voltage, equal to 10 V.

(*c*) (*i*) From sketch (*b*),

$$\phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\frac{(2\pi \times 50 \times 0{\cdot}3)I}{(40+20)I},$$

$$= \tan^{-1}\frac{94{\cdot}25}{60} = 57°\ 31'.$$

$\therefore$ power factor $= \cos\phi = 0{\cdot}5370.$

(*ii*) The power in the circuit is given by

$$P = \frac{V_R{}^2}{R} \text{ watts},$$

where R is the total resistance in the circuit (Ω).

Now, $\quad V_R = V\cos\phi = 10 \times 0{\cdot}537 = 5{\cdot}37$ volts.

$$\therefore\ P = \frac{5{\cdot}37^2}{60}\text{ W} = 480\text{ mW}.$$

Hence, the energy dissipated per second is 480 mJ.

(*d*) To make the power factor 0·6, ϕ must equal 53° 08′. This can be achieved by increasing the total resistance in the circuit to

$$\frac{94{\cdot}25}{\tan 53°\,08'} = \frac{94{\cdot}25}{1{\cdot}33} = 70{\cdot}7\ \Omega.$$

Hence, an additional resistor is required, of value

$$70{\cdot}7 - 60 = 10{\cdot}7\ \Omega.$$

Note: An important alternative method is to add a capacitor of such a value as to reduce the total reactance to 60 tan 53° 08′ = 80 Ω (inductive). This requires a capacitor of reactance 94·25 − 80 = 14·25 Ω; that is, a capacitance of $1/(2\pi \times 50 \times 14{\cdot}25)$ F = 223 μF. This method has the advantage that the power dissipated in the circuit is not increased, and is the method by which power-factor correction is achieved in practice.

Q 3 (a) Explain the principles of

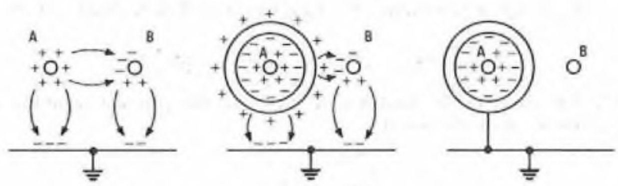
(i) electrostatic shielding, and

(ii) magnetic shielding.

(b) A battery-operated amplifier is found to be subject to 2 types of interference. One is due to a radio station having a frequency in the passband of the amplifier. The other is a hum from adjacent power equipment. Describe how these interfering signals can be eliminated by screening.

(c) Are 2 separate screens necessary to deal with both interfering signals?

A 3 (*a*) (*i*) Consider 2 conductors, A and B, situated adjacent to an earthed conducting surface. If conductor A is at a positive potential, an equal and opposite charge is induced on conductor B and on the conducting surface, as shown in sketch (*a*). The induced charge on conductor B also gives rise to a charge on the conducting surface. However, as the conducting surface is earthed, the charges on it leak away.



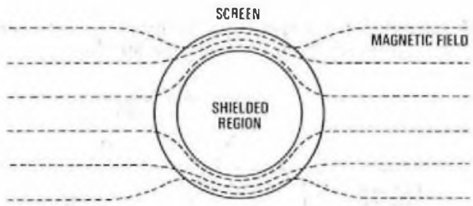
(*a*) (*b*) (*c*)

If conductor A is enclosed in a hollow conducting screen, as shown in sketch (*b*), a negative charge is induced on the inside of the screen, giving rise to a positive charge on its outside. The latter then induces charges on conductor B and the conducting surface as before.

If the screen is now earthed, the positive charge on its outside leaks directly to earth, as shown in sketch (*c*), and no charge is induced on conductor B.

Thus, an earthed screen can be used to prevent an electrostatic field setting up potential differences between neighbouring surfaces. The process is known as *electrostatic shielding*, and the screen, or shield, usually consists of thin metal sheeting.

(*ii*) Shielding from magnetic fields is obtained by enclosing the component to be protected in a screen made from a magnetic material of very low reluctance, thus providing a magnetic shunt. The magnetic lines of force tend to follow the low-reluctance path of the screen rather than enter the screened region, which has a relatively high reluctance. Sketch (*d*) illustrates this principle.



(*d*)

Iron of very high permeability is a suitable material for magnetic shielding where the magnetic field is steady or alternates at a low frequency.

(*b*) The hum from the adjacent power equipment requires magnetic shielding as described in part (*a*) (*ii*), as the interference is almost certainly caused by an unscreened power transformer. A soft-iron container will be effective in reducing the interference.

The radio station causes high-frequency electromagnetic interference, and this is most effectively prevented using a conducting shield. Conducting shields have low resistivity, and an electromagnetic flux incident on such a shield induces voltages which give rise to eddy currents in the shield. These currents set up fields which oppose the incident flux and, thus, prevent its penetration of the shield.

(*c*) At high frequencies, high-permeability magnetic shields are also effective as conducting shields. Therefore, separate screens for the 2 sources of interference are not necessary.

Q 4 (a) Explain the principle of operation of a transformer, and show why an iron core is often desirable.

(b) An ideal transformer is supplying 400 W to a load of 250 Ω. The primary winding is connected to a 240 V a.c. mains supply. Calculate

(i) the voltage across the load,
(ii) the turns ratio of the transformer, and
(iii) the current taken from the supply.

A 4 (*a*) A transformer consists essentially of 2 coils, known as the *primary* and *secondary* windings, wound on an iron core so as to be as closely magnetically coupled as possible. An alternating current in the primary winding causes an alternating magnetic flux to be set up in the iron core. This flux links with the secondary winding, and induces in it an e.m.f. proportional to the relative number of turns on both windings. The voltage ratio of a transformer is, therefore, equal to the ratio of the number of turns on the 2 windings.

For maximum efficiency, all the flux must cut all the turns of both windings, and an iron core helps to concentrate the flux through the windings by providing a low-reluctance path. The high relative permeability of the iron also allows the primary-winding current to generate a relatively high magnetic flux. An iron core ceases to be an advantage at high frequencies because the eddy-current and hysteresis losses in the iron become very high. A granulated-iron, or dust, core is used at high frequencies to reduce these losses.

(*b*) (*i*) The load power, P watts, is given by

$$P = \frac{V_L^2}{R} \text{ watts,}$$

where V_L is the voltage across the load (V), and R is the load resistance (Ω).

$$\therefore\ V_L = \sqrt{(400 \times 250)} = \underline{316 \cdot 2 \text{ V.}}$$

(*ii*) The ratio of the number of turns on the primary winding to the number on the secondary

$$= \frac{240}{316 \cdot 2} = \frac{1}{1 \cdot 32}.$$

Hence, the turns ratio is $\underline{1 : 1 \cdot 32.}$

(*iii*) An ideal transformer is a theoretical concept in which the internal losses of the transformer are ignored. Hence, the supply current, I amperes, is given by

$$I = \frac{P}{V} \text{ amperes,}$$

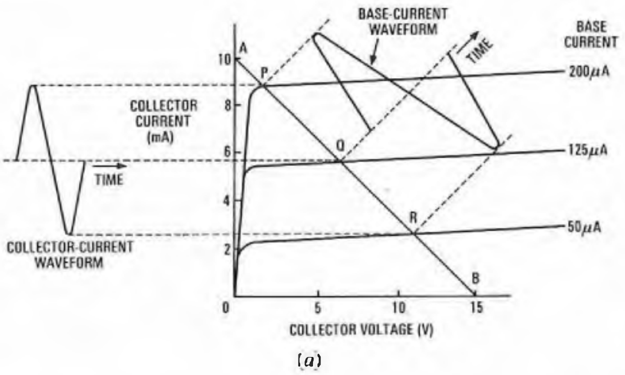
where V is the supply voltage,

$$= \frac{400}{240} = \underline{1 \cdot 67 \text{ A.}}$$

Q 5 (a) Explain how the amplification of a single-stage common-emitter transistor amplifier with a resistance load can be calculated from a load-line drawn on the output characteristics.

(b) Describe briefly an experiment from which the output characteristics used in part (a) can be found. Include a circuit diagram and give typical results.

A 5 (*a*) Sketch (*a*) shows a specimen family of output characteristics for a single-stage common-emitter transistor amplifier.



(*a*)

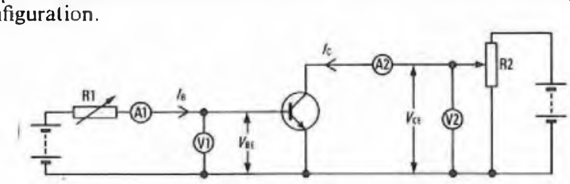
The load line, AB, corresponds to a collector load of 1·5 kΩ working to a supply voltage of 15 V. Hence, the load line intercepts the vertical axis at a collector current of 10 mA, and the horizontal axis at a collector voltage of 15 V. The quiescent operating point, Q, is fixed by the biasing current of 125 μA, which is itself set by a suitable choice of biasing resistance; to avoid distortion, point Q is chosen such that the base-current intercepts on each side of it are equal.

A peak-to-peak input current of 150 μA is shown, so that the base current varies about Q between points P and R. It can be seen that the corresponding variation in collector current is between about 8·9 mA and 2·6 mA, giving a peak-to-peak output current of 6·3 mA.

The current amplification is given by the output current divided by the input current, and can therefore be determined from a load line drawn on the output characteristics. In the above example, the current amplification

$$= \frac{\text{peak-to-peak output current}}{\text{peak-to-peak input current}} = \frac{6 \cdot 3 \times 10^{-3}}{150 \times 10^{-6}} = \underline{42.}$$

(*b*) Sketch (*b*) shows a circuit that can be used to determine the output characteristics of a transistor used in the common-emitter configuration.



(*b*)

The output characteristics, or collector-current/collector-voltage (I_C/V_{CB}) characteristics, are a family of curves drawn for various constant values of base current (I_B). Therefore, the procedure is to set the base current to a suitable value (say, 50 μA), and maintain it at that value using variable resistor R1, and vary the collector voltage using potentiometer R2. The values of V_{CB} and I_C are noted at each step. This process is repeated for other values of I_B to give a family of curves as illustrated by the output characteristics shown in sketch (*a*). The values shown in sketch (*a*) are typical.

Q 6 (a) Find the series-connected circuit components which, at a frequency of 1 kHz, are represented by the following impedances:

(i) 10 + j8 Ω, and
(ii) 10 − j4 Ω.

(b) Draw phasor diagrams to illustrate each of the above impedances. Explain each line on your phasor diagrams.

A 6 (*a*) (*i*) An impedance of 10 + j8 Ω is a resistance of 10 Ω in series with a positive reactance, which must be due to an inductance, of 8 Ω. The value of the inductance is deduced from the formula

$$X_L = 2\pi f L \text{ ohms,}$$

where X_L is the inductive reactance (Ω), f is the frequency (Hz), and L is the inductance (H).

$$\therefore\ L = \frac{8}{2\pi \times 1000} \text{ H} = 1 \cdot 273 \text{ mH.}$$

Hence, the circuit consists of a resistance of $\underline{10\ \Omega}$ in series with an inductance of $\underline{1 \cdot 273 \text{ mH.}}$

(*ii*) An impedance of 10 − j4 Ω is a resistance of 10 Ω in series with a negative reactance, which must be due to a capacitance, of 4 Ω. The value of the capacitance is deduced from the formula

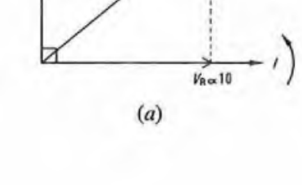
$$X_C = \frac{1}{2\pi f C} \text{ ohms,}$$

where X_C is the capacitive reactance (Ω), and C is the capacitance (F).

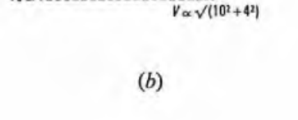
$$\therefore\ C = \frac{1}{2\pi \times 1000 \times 4} \text{ F} = 39 \cdot 8\ \mu\text{F.}$$

Hence, the circuit consists of a resistance of $\underline{10\ \Omega}$ in series with a capacitance of approximately $\underline{40\ \mu\text{F.}}$

(*b*) Sketch (*a*) shows the phasor diagram for an impedance of 10 + j8 Ω. Phasor V_R represents the voltage across the resistance and is in phase with the circuit current, I. The voltage across the inductance, V_L, leads the current by 90°. The resultant, V, is the voltage across the total impedance; that is, the voltage across the complete circuit.



(*a*)



(*b*)

Sketch (*b*) shows the phasor diagram for the impedance of $10 - j4\ \Omega$. Phasor V_R is again in phase with I, but the voltage across the capacitance, V_C, lags the current by 90°. The resultant is the voltage across the total impedance.

Q 7 Describe the principle of operation of either an oscilloscope or an electronic voltmeter.

(b) Give experimental details of how to set-up and operate the instrument. Give the voltage and frequency limits within which the instrument will operate satisfactorily.

A 7 See A4, Telecommunication Principles B, 1972, *Supplement*, Vol. 66, p. 23, Apr. 1973.

Q 8 (a) The circuit given in Fig. 1 shows a 2-way switch that can connect a 100 V battery and a 10 kΩ resistor in series with an inductor or a capacitor. Write down expressions for the instantaneous readings of voltage and current for each of the switch positions in terms of the time from the operation of the switch.

Sketch the shape of the charging curves.

(Assume that the capacitor is discharged before the switch is operated to connect it into the circuit.)

(b) Find the initial and final currents and the time constant in each case.

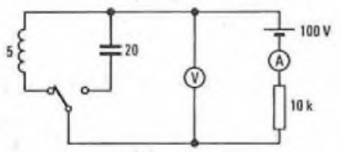


Fig. 1

A 8 (*a*) The instantaneous value of current in the inductance, i_L amperes, t seconds after the operation of the switch, is given by

$$i_L = I(1 - e^{-Rt/L}) \text{ amperes,}$$

where I is the final value of current (A), R is the resistance (Ω), and L is the inductance (H).

But $$I = \frac{V}{R} \text{ amperes,}$$

where V is the battery voltage (which is also the final voltage across the inductance).

$$\therefore\ I = \frac{100}{10 \times 10^3} = 10^{-2} \text{ A.}$$

$$\therefore\ i_L = 10^{-2} \times (1 - e^{-10\,000t/5}) \text{ A,}$$

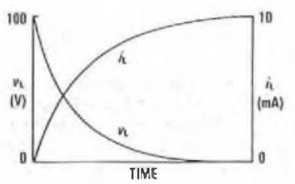
$$= \underline{10 \times (1 - e^{-2000t}) \text{ mA.}}$$

The instantaneous value of voltage across the inductance, v_L volts, is given by

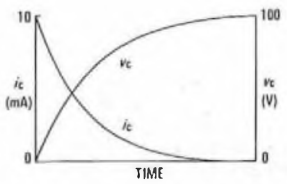
$$v_L = Ve^{-Rt/L} \text{ volts,}$$

$$= \underline{100e^{-2000t} \text{ V.}}$$

The charging curves for the inductance are illustrated in sketch (*a*).



(*a*)



(*b*)

The instantaneous voltage across the capacitance, v_C volts, is given by

$$v_C = V(1 - e^{-t/CR}) \text{ volts,}$$

where C is the capacitance (F),

$$= 100(1 - e^{-t/(0\cdot00002 \times 10\,000)}) \text{ V,}$$

$$= \underline{100(1 - e^{-5t}) \text{ V.}}$$

The instantaneous value of current through the capacitor, i_C amperes, is given by

$$i_C = Ie^{-t/CR} \text{ amperes,}$$

$$= \underline{10 \times e^{-5t} \text{ mA.}}$$

The charging curves for the capacitance are illustrated in sketch (*b*).

(*b*) For the inductance, the time constant

$$= \frac{L}{R} \text{ seconds} = \frac{5}{10 \times 10^3} \text{ s} = \underline{0\cdot5 \text{ ms.}}$$

When $t = 0$, the initial current

$$= 10^{-2} \times (1 - e^0) = \underline{0 \text{ A.}}$$

When $t = \infty$, the final current

$$= 10^{-2} \times (1 - e^{-\infty}) \text{ A} = \underline{10 \text{ mA,}}$$

since $e^{-\infty} = 1/e^{\infty} = 1/\infty = 0$.

For the capacitance, the time constant

$$= CR \text{ seconds} = 20 \times 10^{-6} \times 10 \times 10^3 \text{ s} = \underline{200 \text{ ms.}}$$

When $t = 0$, the initial current

$$= 10^{-2} \times e^0 \text{ A} = \underline{10 \text{ mA.}}$$

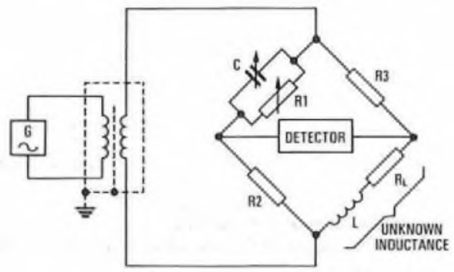
When $t = \infty$, the final current

$$= 10^2 \times e^{-\infty} = \underline{0 \text{ A.}}$$

Q 9 (a) Explain the principle of operation of a bridge method of measuring inductance. Include a description of the bridge circuit, the method of connexion of the source to the bridge, and the type of detector used to indicate a balance.

(b) Deduce the conditions for balance in the circuit, and state the range of frequencies for which the bridge is suitable.

A 9 (*a*) An unknown inductance can be measured by balancing its impedance against a known reactance and resistance in a bridge circuit energized by an alternating current. With a suitable bridge circuit, the balance conditions can be made independent of frequency.



The sketch shows a bridge in which the unknown inductance, L henrys, with resistance R_L ohms, is balanced by known capacitance C farads, and known resistance R1 ohms. A practical inductor always contains resistance and, hence, has a phase angle of less than 90°. The combination of capacitor C and resistance R1 allows both the phase and amplitude of the unknown impedance to be balanced, so that a null can be obtained on the detector. The energizing source for the bridge is an oscillator, connected via a screened transformer. There are 2 screens: the first is an iron container, to prevent interference from surrounding fields, and the second is an earthed interwinding electrostatic screen, to prevent interfering longitudinal currents flowing. Screened leads are used throughout the bridge circuit. A telephone receiver, used in conjunction with a frequency changer if necessary, is a suitable detector. Alternatively, an electronic voltmeter can be used.

(*b*) At balance,

$$\frac{1}{\dfrac{1}{R_1} + j\omega C} \times \frac{1}{R_2} = \frac{R_3}{R_L + j\omega L},$$

where $\omega = 2\pi f$ radians/second, and f is the frequency (Hz).

$$\therefore\ \frac{R_1}{R_2 + j\omega CR_1R_2} = \frac{R_3}{R_L + j\omega L}.$$

$$\therefore\ R_L + j\omega L = \frac{R_2R_3}{R_1} + \frac{j\omega CR_1R_2R_3}{R_1}.$$

Equating real and imaginary parts gives

$$R_L = \frac{R_2R_3}{R_1},$$

and $$\underline{L = R_2R_3C.}$$

It can be seen that the balance equations are independent of frequency, but it is good practice to make measurements at the normal operating frequency of the unknown inductance to allow, for example, for skin effects.

Q 10 *(a) Define the decibel.*
(b) What is the meaning of the units
(i) dBm, and
(ii) dBW?
(c) Why is a negative sign sometimes prefixed to a decibel value? Calculate the power implied by −10 dBm.
(d) (i) An amplifier has a voltage gain of 60 when terminated in a 50 Ω load resistor. Calculate the power in the load for an input of 1 mV into the amplifier.
(ii) If the 50 Ω resistor is replaced by the input to a network that has a loss of 10 dB when connected to a 50 Ω load, calculate the voltage across this load for an input of 1 mV into the amplifier.

A 10 (*a*) If the ratio of 2 powers, P_1 and P_2, is to be expressed in decibels, the number of decibels, N, is given by

$$N = 10\log_{10}\frac{P_1}{P_2}\text{ decibels.} \qquad \ldots\ldots (1)$$

Since the power dissipated in a resistor is given by the square of the voltage divided by the resistance, then, for equal resistances having voltages V_1 and V_2 respectively across them,

$$N = 20\log_{10}\frac{V_1}{V_2}\text{ decibels.} \qquad \ldots\ldots (2)$$

(*b*) (*i*) The unit dBm denotes that the decibel ratio expressed, N, is that of some power, P_1 watts, relative to a power of 1 mW; that is, P_2 in equation (1) is 1 mW.

(*ii*) The unit dBW denotes that the decibel ratio expressed, N, is that of some power, P_1 watts, relative to a power of 1 W; that is, P_2 in equation (1) is 1 W.

(*c*) A negative decibel value denotes that $P_2 > P_1$; that is, if P_1 is the output power of a circuit and P_2 is the input power, then an attenuation of power is incurred.

A decibel value of −10 dBm represents a power level 10 dB lower than 1 mW. From equation (1),

$$-10 = 10\log_{10}\frac{P_1}{1\times10^{-3}}.$$

$$\therefore\ -1 = \log_{10}\frac{P_1}{1\times10^{-3}}.$$

$$\therefore\ \frac{P_1}{1\times10^{-3}} = 10^{-1}.$$

$$\therefore\ P_1 = 1\times10^{-4}\text{ W} = \underline{0\cdot1\text{ mW}}.$$

(*d*) (*i*) Since the gain is 60, the voltage across the load is 60 mV. Therefore, the power in the load

$$= \frac{(60\times10^{-3})^2}{50}\text{ W} = \underline{72\ \mu\text{W}}.$$

(*ii*) From equation (2),

$$-10 = 20\log_{10}\frac{V_1}{60\times10^{-3}},$$

where V_1 is the voltage across the network's load (V).

$$\therefore\ 0\cdot5 = -\left(\log_{10}\frac{V_1}{60\times10^{-3}}\right) = \log_{10}\frac{60\times10^{-3}}{V_1}.$$

$$\therefore\ 3\cdot162 = \frac{60\times10^{-3}}{V_1}.$$

$$\therefore\ \underline{V_1 = 18\cdot97\text{ mV}}.$$

TELECOMMUNICATION PRINCIPLES B, 1975

Students were expected to answer any 6 questions

Q 1 *(a) Explain the meaning of reactance.*
Write down expressions, including the units, for the reactances of
(i) a capacitor, and
(ii) an inductor.
(b) A 100 Ω resistor, a capacitor and an inductor are connected in series across a 120 V 500 Hz supply. The r.m.s. voltage across the capacitor is equal to that across the resistor and is half the voltage across the inductor.
(i) Draw a phasor diagram for the circuit, relating the voltages with the current.
(ii) Hence, or otherwise, calculate the values of the capacitor and the inductor.

A 1 (*a*) Reactance is the ability of an inductive or capacitive circuit to resist the flow of alternating current without dissipating energy. It is the ratio of the quadrature component of voltage across the reactive circuit to the current in the circuit.

(*i*) Capacitive reactance, X_C ohms, is given by

$$\underline{X_C = \frac{1}{2\pi fC}\text{ ohms}},$$

where f is the frequency (Hz), and C is the capacitance (F).

(*ii*) Inductive reactance, X_L ohms, is given by

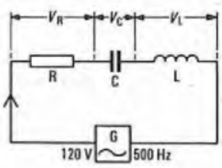
$$\underline{X_L = 2\pi fL\text{ ohms}},$$

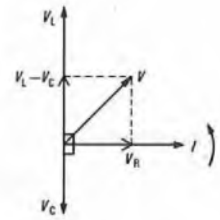
where L is the inductance (H).

Note: Using the operator j, capacitive reactance can be expressed as $1/j\omega C$ or $-j/\omega C$, and inductive reactance as $j\omega L$, where $\omega = 2\pi f$ radians/second.

(*b*) (*i*) The circuit and phasor diagram are shown in the sketch.

Now, $$|V_R| = |V_C| = \left|\frac{V_L}{2}\right|,$$





where V_R, V_C and V_L are the voltages across the resistance, capacitance and inductance respectively.

$$\therefore\ |100I| = \left|\frac{I}{2\pi fC}\right| = \left|\frac{2\pi fLI}{2}\right|. \qquad \ldots\ldots (1)$$

For a series circuit, the current, I, is taken as the reference phasor. Phasor V_R is drawn in phase with the current. Phasor V_C, of the same magnitude as V_R, lags the current by 90°. Phasor V_L, of twice the magnitude of V_R, leads the current by 90°. The resultant reactance voltage, $V_L - V_C$, is therefore a phasor of the same magnitude as V_R, leading the current by 90°. Phasor V is the resultant of the reactance and resistance voltages, and is the supply voltage of 120 V.

(*ii*) From equation (1),

$$100 = \frac{1}{2\pi fC}\text{ ohms}.$$

$$\therefore\ C = \frac{1}{2\pi\times500\times100}\text{ F} = \underline{3\cdot18\ \mu\text{F}}.$$

Again, from equation (1),

$$100 = \frac{2\pi fL}{2}\text{ ohms}.$$

$$\therefore\ L = \frac{100}{\pi\times500}\text{ H} = \underline{63\cdot7\text{ mH}}.$$

Q 2 (a) Define the decibel. Why is the decibel used in calculations in telecommunications engineering?

(b) Explain the meanings of

(i) a gain of 3 dB,
(ii) a gain of −5 dB, and
(iii) a level of 3 dB relative to 1 mW.

(c) An amplifier has a voltage gain of 30 when terminated by a 75 Ω resistor. The resistor is replaced by a 10 dB attenuator of 75 Ω resistive impedance, terminated in 75 Ω. Calculate

(i) the input voltage to the amplifier to give 5 mV across the output of the attenuator, and
(ii) the power delivered by the amplifier to the attenuator.

A 2 (*a*) If the ratio of 2 powers, P_1 and P_2 watts, is to be expressed in decibels, the number of decibels, N, is given by

$$N = 10 \log_{10} \frac{P_1}{P_2} \text{ dB.} \qquad \ldots\ldots (1)$$

The advantages of using logarithmic units for expressing power ratios in telecommunications engineering are

(*i*) the enormous ranges of power values used in communication work can be expressed in simpler numbers, and

(*ii*) the overall gains or losses of complicated tandem-connected circuits can be simply calculated by taking the algebraic sum of the logarithmic ratios for their various components.

(*b*) (*i*) From equation (1),

$$10 \log_{10} \frac{P_1}{P_2} = 3 \text{ dB.}$$

$$\therefore \frac{P_1}{P_2} = \text{antilog}_{10}\, 0{\cdot}3 \approx 2.$$

Hence, a gain of 3 dB implies that P_1 is twice P_2. In a telecommunications system, it is usual to regard P_2 as the input power and P_1 as the output power, so that a gain of 3 dB implies a power increase of 2: 1.

(*ii*)
$$10 \log_{10} \frac{P_1}{P_2} = -5 \text{ dB.}$$

$$\therefore 10 \log_{10} \frac{P_2}{P_1} = 5 \text{ dB.}$$

$$\therefore \frac{P_2}{P_1} = \text{antilog}_{10}\, 0{\cdot}5 \approx 3.$$

Hence, a gain of −5 dB implies that the input power is 3 times the output power; that is, a power reduction of 3: 1 has occurred.

(*iii*) A level of 3 dB relative to 1 mW (normally written as 3 dBm) implies that the ratio of the actual power level to a reference level of 1 mW is 2: 1. Therefore, 3 dBm is a power level of 2 mW.

(*c*) (*i*) The power dissipated in a resistance is the square of the voltage across the resistance divided by the resistance. Therefore, for 2 equal resistances having voltages V_1 and V_2 volts,

$$N = 20 \log_{10} \frac{V_1}{V_2} \text{ dB.}$$

Thus, the input voltage to the attenuator, V_2 volts, to give an output voltage of 5 mV is given by the equation

$$-10 = 20 \log_{10} \frac{5 \times 10^{-3}}{V_2}.$$

$$\therefore V_2 = 5 \times 10^{-3} \times \text{antilog}_{10}\, 0{\cdot}5 \text{ V} = 15{\cdot}81 \text{ mV.}$$

Since the amplifier has a voltage gain of 30, the required input

$$= \frac{15{\cdot}81 \times 10^{-3}}{30} \text{ V} = \underline{527\ \mu\text{V.}}$$

(*ii*) The power delivered by the amplifier to the attenuator

$$= \frac{V_2^2}{R} \text{ watts,}$$

where R is the resistance (Ω),

$$= \frac{(15{\cdot}81 \times 10^{-3})^2}{75} \text{ W} = \underline{3{\cdot}33\ \mu\text{W.}}$$

Q 3 Describe an experiment to determine, at a frequency of 100 kHz,

(a) the alternating voltage across each component in a circuit consisting of a capacitor, inductor and resistor in series, and

(b) the phase angle between the current in the circuit and the supply voltage.

Q 4 (a) What is hysteresis loss in iron?

(b) Sketch the shape of the curve relating magnetic flux density to magnetizing force in a specimen of iron subjected to repeated cycles of alternating magnetizing force. Explain the reason for the shape of this curve.

(c) How does the curve differ between samples of iron used in

(i) a transformer core, and
(ii) a permanent magnet?

Why is it desirable that the area of the loop should be as small as possible in the transformer core?

A 4 See A10, Telecommunication Principles B, 1973, *Supplement*, Vol. 68, p. 8, Apr. 1975.

Q 5 (a) Explain the meaning of the h-parameters for a transistor in the common-emitter configuration.

(b) How would you determine the h-parameters for a transistor connected as a common-emitter amplifier? Include typical results in your answer.

A 5 (*a*) The *h*-parameters (hybrid parameters), for a transistor connected in the common-emitter configuration, are related to the gradients of the 4 main d.c. static characteristic curves for that transistor. They describe the low-frequency, small-signal performance of the transistor in terms of the input and output alternating signal voltages (V_{be} and V_{ce} respectively) and currents (I_b and I_c respectively).

The 4 *h*-parameters are the input parameter (h_i), the output parameter (h_o), the forward current gain (h_f), and the reverse voltage ratio (h_r). A second suffix, e, to each parameter denotes that the transistor is connected in the common-emitter configuration.

The *h*-parameters are defined as follows.

(*i*) $h_{ie} = V_{be}/I_b$ with the output short-circuited; i.e., $V_{ce} = 0$.
(*ii*) $h_{oe} = I_c/V_{ce}$ with the input open-circuited; i.e. $I_b = 0$.
(*iii*) $h_{fe} = I_c/I_b$ with the output short-circuited; i.e., $V_{ce} = 0$.
(*iv*) $h_{re} = V_{be}/V_{ce}$ with the input open-circuited; i.e., $I_b = 0$.

(*b*) The static characteristics, for a transistor in the common-emitter configuration, are plotted from measurements of the direct input and output voltages (V_{BE} and V_{CE} respectively) and currents (I_B and I_C respectively). The 4 characteristics, with their related *h*-parameters, are listed below.

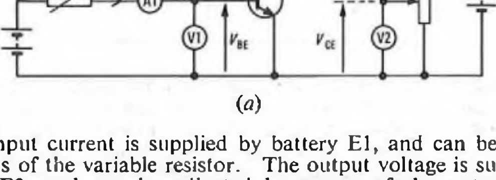
(*i*) The input characteristic shows the graph of I_B/V_{BE} for various fixed values of V_{CE}. The gradient of the curve gives the input admittance of the transistor, the reciprocal of which is the input parameter, h_{ie}.

(*ii*) The output characteristic shows graphs of I_C/V_{CE} for a series of fixed values of I_B. The gradient of the curves gives the output admittance of the transistor, or the output parameter, h_{oe}.

(*iii*) The transfer characteristic shows the graph of I_C/I_B for various fixed values of V_{CE}. The gradient of the curve gives the forward current gain, h_{fe}.

(*iv*) The feedback characteristic shows graphs of V_{BE}/V_{CE} for a series of fixed values of I_B. The gradient of the curves gives the reverse voltage ratio, h_{re}.

Sketch (*a*) shows a circuit diagram suitable for obtaining the static characteristics of a transistor connected in the common-emitter configuration.



(*a*)

The input current is supplied by battery E1, and can be adjusted by means of the variable resistor. The output voltage is supplied by battery E2, and can be adjusted by means of the potentiometer. Voltmeters V1 and V2 record V_{BE} and V_{CE} respectively, and must be high-impedance instruments, so that the circuit conditions are not disturbed. Ammeters A1 and A2 record I_B and I_C respectively.

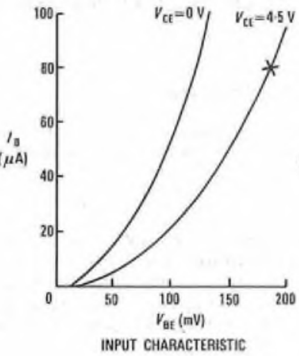
To obtain the input characteristic, V_{CE} is set to a convenient value and maintained at that value by adjusting the potentiometer as necessary. Values of V_{BE} are recorded for a range of values of I_B. The value of V_{CE} is changed and the measurements repeated.

To obtain the output characteristic, I_B is set to a convenient value and maintained at that value by adjusting the variable resistor as necessary. Values of I_C are recorded for a range of values of V_{CE}. The measurements are repeated for a range of fixed values of I_B.

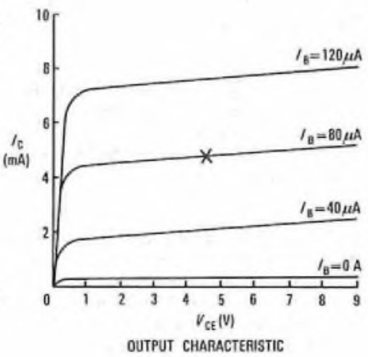
To obtain the transfer characteristic, V_{CE} is maintained at a constant value, while values of I_C are recorded for a range of values of I_B. The value of V_{CE} is changed and the measurements repeated.

To obtain the feedback characteristic, I_B is maintained at a constant value, while values of V_{BE} are recorded for a range of values of V_{CE}. The measurements are repeated for a range of fixed values of I_B.

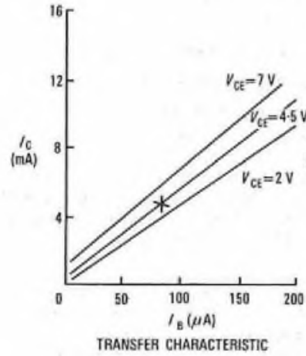
Typical input, output, transfer and feedback characteristics are shown in sketches (*b*), (*c*) (*d*) and (*e*) respectively. (Note that, by convention, current flowing into a transistor is considered to be positive, and current flowing out of a transistor is considered to be negative.)



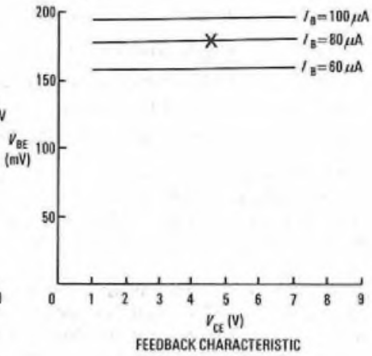
(*b*)



(*c*)



(*d*)



(*e*)

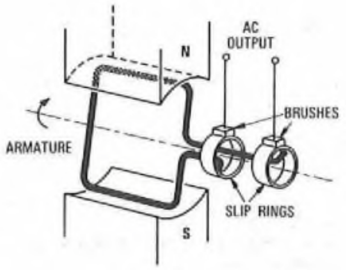
By examination of the output characteristic, a suitable operating point is given when $I_B = 80\ \mu A$ and $V_{CE} = 4{\cdot}5$ V. This point is marked by a cross on each of the characteristics. The *h*-parameters are then derived by taking the slope or reciprocal slope, as appropriate, of each characteristic about the operating point. Typical values for a transistor in the common-emitter configuration are given in the table

h-Parameter	Value
h_{ie}	1·5 kΩ
h_{oe}	80 μS
h_{fe}	50
h_{re}	1×10^{-4}

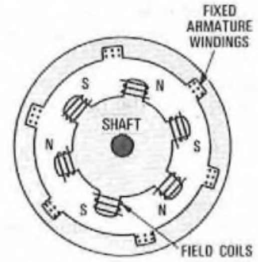
Note: Values for *h*-parameters, derived from the static characteristics, are valid provided that the characteristics are within the limits of small-signal operation, even though the open-circuit input and short-circuit output conditions are replaced by fixed-value conditions. For example, considering the output characteristic, small changes in I_C and V_{CE} behave in the same way as alternating components I_c and V_{ce} over the linear region of the characteristic. As these changes occur for a fixed value of I_B, then $I_b = 0$, and the open-circuit input condition is satisfied.

Q 6 (a) Explain the principle of a simple a.c. generator.
(b) What factors determine an a.c. generator's
(i) output voltage, and
(ii) frequency.
(c) Why is it usual to construct the armature core of iron laminations?

A 6 (*a*) An a.c. generator is an application of Faraday's law of electromagnetic induction, which states that, when the magnetic flux linking a circuit changes, an e.m.f. is induced in the circuit with a magnitude proportional to the rate of change of flux linkage. A coil of wire, called an *armature*, is made to rotate such that its conductors cut the magnetic field between the poles of a strong field magnet, thus inducing an e.m.f. in the coil. The ends of the coil are connected to slip rings. Fixed spring-mounted carbon brushes maintain sliding contact with the slip rings as they rotate, thus providing the output terminals of the generator. The principle is illustrated in sketch (*a*).



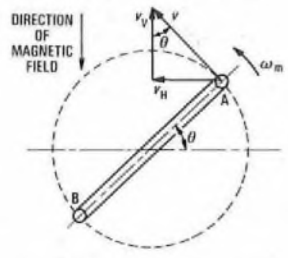
(*a*)



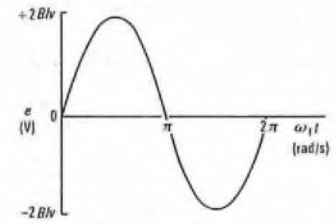
(*b*)

In a small generator, the magnetic field can be sustained by a powerful permanent magnet. In a large generator, the armature windings are normally stationary, while the field-magnet poles rotate; in such a machine, illustrated in sketch (*b*), the field-magnet poles are usually produced electromagnetically. A practical a.c. generator has a number of armature coils and pairs of field poles.

(*b*) (*i*) Sketch (*c*) shows an end-on view of a single-turn coil rotating in a magnetic field produced by one pair of poles.



(*c*)



(*d*)

Considering side A of the coil, the linear velocity, v metres/second, of the conductor acts tangentially to the circle through which it rotates. This velocity can be resolved into a horizontal component, $v_H = v \sin\theta$ metres/second, and a vertical component, $v_V = v\cos\theta$ metres/second, where θ is the angle of the coil to the horizontal axis. The vertical component cuts no flux and therefore produces no e.m.f. The horizontal component gives rise to an instantaneous e.m.f., e volts, given by

$$e = Blv \sin\theta \text{ volts,}$$

where B is the magnetic flux density (T), and l is the length of the conductor (m). Since an equal and aiding e.m.f. is induced simultaneously in conductor B, the expression becomes

$$e = 2Blv \sin\theta \text{ volts.}$$

Hence, e varies sinusoidally between values of $\pm 2Blv$ volts, as illustrated in sketch (*d*).

For an armature of N turns, the e.m.f.s induced in each turn are additive,

$$\therefore\ e = 2NBlv \sin\theta \text{ volts.}$$

If the angular velocity of the armature is ω_m radians/second, and the radius of the armature is r metres, the expression becomes

$$\underline{e = 2NBl\omega_m r \sin \omega_m t \text{ volts,}} \qquad \ldots\ldots (1)$$

since $v = \omega_m r$ metres/second, and the electrical angular velocity, ω_f, is equal to the mechanical angular velocity, ω_m.

(*ii*) The waveform of the generated e.m.f. alternates through 1 cycle each time the conductors move past a north and south pole pair. Hence, the electrical frequency, f hertz, depends on the number of pairs of poles, p, and the mechanical frequency of the armature, n revolutions/second, and is given by

$$\underline{f = np \text{ hertz.}}$$

For a machine with more than one pole pair, such as that illustrated in sketch (*b*), there are as many armature windings as there are pole pairs. As these windings are connected in series, the magnitude of equation (1) is modified by a factor of p. Also, the electrical angular velocity is no longer equal to the mechanical angular velocity, and the equation becomes

$$e = 2pNBl\omega_m r \sin \omega_m pt \text{ volts.}$$

(*c*) An iron armature core, rotating in a magnetic field, has eddy currents induced in it. These currents represent wasted energy and cause the core to heat up. To reduce them, the core is laminated, with each lamination insulated from the next. This form of armature offers a high resistance to eddy currents, which tend to flow parallel to the shaft, but maintains a low reluctance to the magnetic field, which is transverse to the shaft.

Q 7 (*a*) (*i*) *An inductor is being measured on an a.c. bridge using headphones as the detector. Two bridge components must be adjusted to obtain silence in the headphones. Explain why this is so.*

(*ii*) *Sketch a bridge circuit suitable for use at a frequency of 1 kHz.*

(*b*) *By using the operator j, or otherwise, obtain expressions for the balance conditions from which the unknown component values can be calculated. Use algebraic values for the bridge components.*

(*c*) *Why is it advantageous to use a bridge circuit in which the balance condition is not a function of the frequency of the bridge current?*

A 7 See A9, Telecommunication Principles B, 1974, *Supplement*, Vol. 69, p. 23, Apr. 1976.

Q 8 (*a*) *Explain why an electromagnetic relay, operating on a d.c. supply, can be made slow to operate by connecting a resistor in series with it.*

(*b*) *The inductance of a relay coil is 20 H, and the total resistance in the circuit is 100 Ω. The circuit is switched to a 50 V d.c. supply.*

(*i*) *Give an expression for the current, related to the inductance, resistance, supply voltage and time at the instant of switching on.*

(*ii*) *Find the current in the coil immediately after switching on.*

(*iii*) *Find the time constant of the circuit.*

(*c*) *Explain the phenomenon that occurs at the switch contacts as the switch opens. What steps are taken to minimize the phenomenon?*

A 8 (*a*) When the current in an inductor is changing, by Lenz's law, the back e.m.f. induced opposes the change in the current. Hence, when an inductor is switched to a d.c. supply, the rise of current in the circuit is delayed.

If the current at any instant is i amperes, the supply voltage, V volts, is given by

$$V = iR + (\text{back e.m.f.}) \text{ volts,}$$

where R is the resistance in the circuit (Ω).

Now, the back e.m.f. is given by the inductance, L henrys, multiplied by the rate of change of current.

$$\therefore\ V = iR + L \times (\text{rate of change of current}) \text{ volts.}$$

Hence, the rate of change of current

$$= \frac{V - iR}{L} \text{ amperes/second.}$$

Therefore, if the resistance in the circuit is increased, the rate of change of current decreases.

Thus, an electromagnetic relay, which does not operate until a specific value of current has been reached, can be made slow to operate by connecting a resistor in series with it.

(*b*) (*i*) The instantaneous value of current is related to the supply voltage, resistance, inductance, and time from the instant of switching on, t seconds, by the expression

$$i = \frac{V}{R}(1 - e^{-Rt/L}) \text{ amperes.}$$

(*ii*) At the instant of switching on, t can be considered to be zero.

$$\therefore\ i = \frac{50}{100}(1 - e^{-0}) \text{ amperes.}$$

Now, $e^{-0} = 1/e^0 = 1$.

$$\therefore\ i = 0{\cdot}5 \times 0 = 0 \text{ A.}$$

(*iii*) The time constant

$$= \frac{L}{R} \text{ seconds} = \frac{20}{100} \text{ s} = 200 \text{ ms.}$$

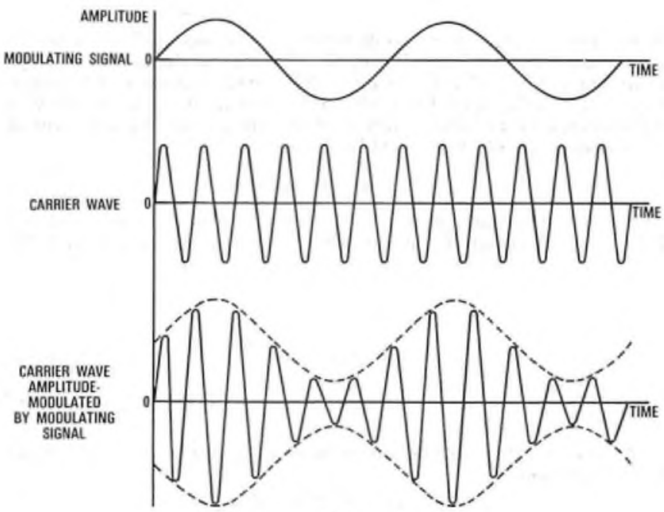
(*c*) When the current in an inductive circuit is suddenly interrupted, the magnetic flux associated with the inductor collapses. Hence, the flux linking the coil of the inductor changes rapidly, so that a large e.m.f. is induced in it. The e.m.f. appears across the contacts of the opening switch, and is of a sufficiently high value to cause an arc, or spark, across them. The energy stored in the magnetic field is dissipated in the resistance of the circuit and in the arc, and the latter causes erosion of the switch contacts. The phenomenon is reduced by preventing the induced e.m.f. appearing across the contacts. This can be achieved by connecting a resistor and capacitor in series across them, in which the energy is dissipated slowly. Such an arrangement is known as a *spark-quench circuit*. The induced e.m.f. charges the capacitor, which then slowly discharges through the resistor and the inductive circuit.

Q 9 (*a*) *Explain the meaning of the term amplitude modulation.*

(*b*) *A carrier frequency of 100 kHz is amplitude modulated by a speech-frequency band of 300–2500 Hz and, in addition, a single frequency of 3·5 kHz. Draw a diagram to illustrate the output after modulation.*

(*c*) *If the carrier frequency in part* (*b*) *is modulated by a single tone, describe briefly any practical method by which the modulation factor can be measured.*

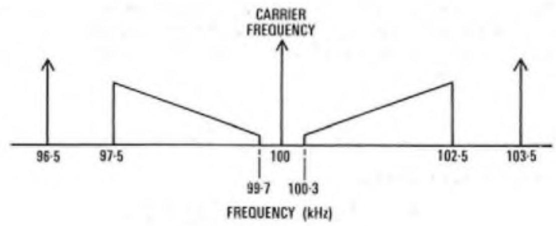
A 9 (*a*) Amplitude modulation is a method of transmitting a low-frequency signal on a high-frequency carrier wave. The amplitude of the carrier wave is made to vary in proportion to the amplitude of the signal to be transmitted, and at a rate equal to the frequency of the modulating signal. Sketch (*a*) shows the modulating signal, the carrier wave, and the resulting amplitude-modulated carrier wave.



(*a*)

(*b*) When a carrier wave of frequency f_C hertz is modulated by a signal of frequency f_M hertz, 2 side frequencies, $(f_C + f_M)$ hertz and $(f_C - f_M)$ hertz, centred about the carrier frequency, are produced. If the modulating signal is a band of frequencies, f_1–f_2 hertz, 2 sidebands are produced, one above and one below the carrier frequency, occupying the frequency ranges $(f_C + f_1)$ hertz to $(f_C + f_2)$ hertz and $(f_C - f_1)$ hertz to $(f_C - f_2)$ hertz.

Thus, the speech-frequency band of 300–2500 Hz produces sideband ranges of $(100 + 0{\cdot}3)$ kHz to $(100 + 2{\cdot}5)$ kHz and $(100 - 0{\cdot}3)$ kHz to $(100 - 2{\cdot}5)$ kHz. The single frequency of 3·5 kHz produces side frequencies of $(100 + 3{\cdot}5)$ kHz and $(100 - 3{\cdot}5)$ kHz. Sketch (*b*) shows the resulting frequency diagram.



(*b*)

(*c*) An oscilloscope can be used to measure the modulation factor of an amplitude-modulated carrier. If the modulated carrier is connected to the Y-plates, and the modulating signal to the X-plates, then the horizontal position of the display depends on the value of the modulating signal at any instant. Consideration of sketch (*a*) shows that, when the modulating signal is zero, the modulated carrier has an amplitude equal to that of the unmodulated carrier, and is displayed in the centre of the screen. When the modulating signal is at its maximum positive value, the modulated carrier is at its maximum amplitude and is displayed, say, to the left of the screen, depending on the polarity of the connexions to the X-plates. When the modulating signal is at its maximum negative value, the modulated carrier is at its minimum amplitude and is displayed to the right of the screen. As the modulating signal takes its intermediate values, the corresponding amplitudes of the modulated signal are displayed at their appropriate intermediate positions on the screen. Hence, the resulting display is trapezoidal, having its maximum amplitude on the left and tapering to its minimum amplitude on the right. By measuring the maximum and minimum amplitudes thus obtained, the modulation factor can be calculated from the expression

$$m = \frac{A_{MAX} - A_{MIN}}{A_{MAX} + A_{MIN}},$$

where m is the modulation factor, A_{MAX} is the maximum amplitude, and A_{MIN} is the minimum amplitude.

Q 10 (a) Explain the meaning of impedance.

(b) Impedances of $4 + j\,\Omega$ and $2 - j3\,\Omega$ are connected in series. Calculate the equivalent impedance and state its phase angle.

(c) (i) If the circuit is energized by a sinusoidal current of frequency 50 kHz, find the components that will produce this equivalent impedance.

(ii) What single component could be added in parallel with this circuit to make the equivalent impedance non-reactive at a frequency of 50 kHz?

A 10 (*a*) When a circuit consisting of resistance and reactance is connected across a sinusoidal supply, the ratio of the applied voltage to the current taken by the circuit is called the *impedance* of the circuit. Impedance is the opposition offered by such a circuit to the flow of alternating current, and is measured in ohms. The general form of an impedance, Z ohms, is given by

$$Z = a \pm jb \text{ ohms},$$

where a is the resistance of the circuit (Ω), and b is the reactance (Ω). The reactance is in quadrature to the resistance, and the imaginary term is positive if the reactance is inductive, and negative if it is capacitive.

(*b*) The equivalent impedance

$$= 4 + j + 2 - j3 = \underline{6 - j2\,\Omega}.$$

The phase angle

$$= -\tan^{-1}\frac{2}{6} = \underline{-18^\circ\,26'}.$$

(*c*) (*i*) An impedance of $6 - j2\,\Omega$ is a resistance of $\underline{6\,\Omega}$ in series with a capacitance of reactance $2\,\Omega$. Therefore, the capacitance, C farads, is given by

$$C = \frac{1}{2\pi f X_C} \text{ farads},$$

where f is the frequency (Hz), and X_C is the reactance (Ω),

$$= \frac{1}{2\pi \times 50 \times 10^3 \times 2}\text{ F} = \underline{1\cdot 59\ \mu\text{F}}.$$

(*ii*) For the circuit to be non-reactive, an inductor, L henrys, must be connected across it to give parallel resonance at a frequency of 50 kHz. Assuming the resistance of the inductor to be negligible (say, not more than about 1 Ω), the admittance of the parallel combination

$$= \frac{1}{6 - j2} + \frac{1}{jX_L} \text{ siemens},$$

where X_L is the reactance of the inductor (Ω),

$$= \frac{6 + j2}{6^2 + 2^2} - \frac{j}{X_L} \text{ siemens}.$$

At resonance, the algebraic sum of the imaginary terms is zero.

$$\therefore\ \frac{1}{X_L} = \frac{2}{6^2 + 2^2}\text{ S}.$$

$$\therefore\ X_L = 20\,\Omega.$$

$$\therefore\ L = \frac{20}{2\pi \times 50 \times 10^3}\text{ H} = \underline{63\cdot 7\ \mu\text{H}}.$$

MATHEMATICS B, 1975

Students were expected to answer any 6 questions

Q 1 (a) Calculate, to slide-rule accuracy, the values of i_1 and i_2 given by the equations:

$$2\cdot 83i_1 + 1\cdot 72i_2 = 5\cdot 90,$$
$$5\cdot 19i_1 - 2\cdot 07i_2 = 2\cdot 13.$$

(b) Find the co-ordinates of the points at which the straight line $2x + y = 1$ cuts the circle $x^2 + y^2 = 9$.

A 1 (*a*) $\underline{i_1 = 1\cdot 074;\ i_2 = 1\cdot 664}$.

(*b*) The co-ordinates of the points at which the straight line intersects the circle are $\underline{(1\cdot 727, -2\cdot 454) \text{ and } (-0\cdot 927, 2\cdot 854)}$.

Q 2 Given the formula $V = I\{R^2 + (\omega L - 1/\omega C)^2\}^{1/2}$,

(a) express L in terms of V, I, R, ω and C,

(b) derive an expression for the value of ω that makes the admittance, I/V, a maximum, assuming L, C, and R to be constant as ω varies, and

(c) calculate the admittance when $\omega = 10^4$, $R = 4\cdot 4$, $L = 2 \times 10^{-3}$, and $C = 4\cdot 2 \times 10^{-6}$.

A 2 (*a*)
$$V = I\{R^2 + (\omega L - 1/\omega C)^2\}^{1/2}.$$

Squaring the equation gives

$$V^2 = I^2\{R^2 + (\omega L - 1/\omega C)^2\}.$$

$$\therefore\ R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{V^2}{I^2}.$$

$$\therefore\ \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{V^2}{I^2} - R^2.$$

$$\therefore\ \omega L - \frac{1}{\omega C} = \pm\left(\frac{V^2 - I^2R^2}{I^2}\right)^{1/2}.$$

$$\therefore\ \omega L = \frac{1}{\omega C} \pm \frac{(V^2 - I^2R^2)^{1/2}}{I}.$$

$$\therefore\ \underline{L = \frac{1}{\omega^2 C} \pm \frac{(V^2 - I^2R^2)^{1/2}}{\omega I}}.$$

(*b*)
$$\frac{I}{V} = \frac{1}{\{R^2 + (\omega L - 1/\omega C)^2\}^{1/2}}.$$

The maximum value of I/V occurs when the denominator is a minimum. Since R^2 is constant, the denominator is a minimum when $(\omega L - 1/\omega C)$ is zero.

$$\therefore\ \omega L = \frac{1}{\omega C}.$$

$$\therefore\ \omega^2 = \frac{1}{LC}.$$

$$\therefore\ \underline{\omega = \frac{1}{\sqrt{(LC)}}}.$$

(*c*)
$$\frac{I}{V} = \frac{1}{[4\cdot 4^2 + \{10^4 \times 2 \times 10^{-3} - 1/(10^4 \times 4\cdot 2 \times 10^{-6})\}^2]^{1/2}},$$

$$= \frac{1}{\{19\cdot 36 + (20 - 23\cdot 81)^2\}^{1/2}},$$

$$= \frac{1}{33\cdot 88^{1/2}},$$

$$= \underline{0\cdot 1718}.$$

Q 3 The number of faulty switch contacts recorded each day in a telephone exchange are summarized in the table for a period of 105 d.

x (*faults/d*)	1	2	3	4	5	6	7	8
y (*number of days on which x faults/d occurred*)	12	18	20	15	14	10	9	7

Assuming the formula $y = axe^{-kx}$ connects these figures, plot suitable variables to obtain a straight-line graph. Hence, estimate the constants a and k.

A 3 Taking logarithms of $y = axe^{-kx}$ does not give the equation of a straight-line graph, since one of the terms on the right-hand side will be $\log_e x$. Therefore, it is necessary to alter the form of the original equation as follows:

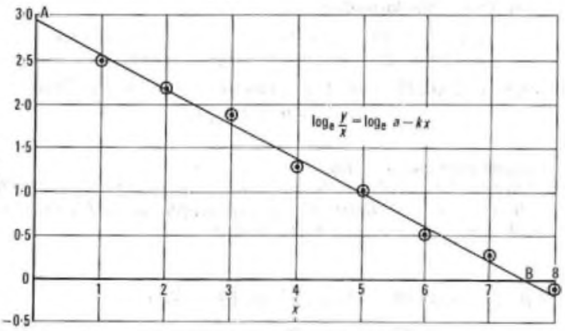
$$\frac{y}{x} = ae^{-kx}.$$

Taking logarithms of the rearranged equation gives

$$\log_e \frac{y}{x} = \log_e a - kx.$$

This is now of the general form for a straight-line graph, $y = mx + c$. The graph is shown in the sketch, constructed from the values derived in the table.

x	1	2	3	4	5	6	7	8
y	12	18	20	15	14	10	9	7
$\frac{y}{x}$	12	9	$6\cdot\dot{6}$	3·75	2·8	$1\cdot\dot{6}$	1·286	0·875
$\log_e \frac{y}{x}$	2·485	2·197	1·897	1·322	1·030	0·511	0·252	−0·134



It can be seen that the plotted points lie reasonably close to a straight line. Hence, the data conform approximately to the law $y = axe^{-kx}$.

When $x = 0$,

$$\log_e \frac{y}{x} = \log_e a.$$

Hence, the value of $\log_e a$ is given by the intercept of the graph on the $\log_e (y/x)$ axis. From the graph, when $x = 0$ (point A),

$$\log_e \frac{y}{x} = \log_e a = 2\cdot95$$

$$\therefore\ \underline{a = 19\cdot11.}$$

The gradient of the graph, $-k$, is obtained from the co-ordinates of points A and B. Thus,

$$-k = -\frac{2\cdot95}{7\cdot5},$$

since the gradient of a graph that slopes down from left to right is negative.

$$\therefore\ \underline{k = 0\cdot393.}$$

Q 4 (a) If $z = 3 - j2$, express

(i) $(z - 2)(2z + j3)$ in the form $a + jb$, and

(ii) $\frac{z+2}{z - j2}$ in the polar form $r\angle\theta$.

(b) A sinusoidal alternating current, defined by the phasor $(40 - j25)$ mA, feeds 2 parallel circuits, B and C. Circuit B takes a current of $(24 + j5)$ mA.

(i) Show the 2 currents on a phasor diagram, and show, on the same diagram, the current flowing in circuit C.

(ii) Calculate the r.m.s. value of the current in circuit C. What is its peak value?

A 4 (*a*) (*i*) $(z - 2)(2z + j3) = (3 - j2 - 2)(6 - j4 + j3),$

$$= (1 - j2)(6 - j),$$

$$= 6 - j13 - 2,$$

$$= \underline{4 - j13.}$$

(*ii*)

$$\frac{z+2}{z-j2} = \frac{3 - j2 + 2}{3 - j2 - j2},$$

$$= \frac{5 - j2}{3 - j4},$$

$$= \frac{\sqrt{(5^2 + 2^2)}\angle \tan^{-1}(-2/5)}{\sqrt{(3^2 + 4^2)}\angle \tan^{-1}(-4/3)},$$

$$= \frac{5\cdot385}{5}\angle -21^\circ 48' - (-53^\circ 8'),$$

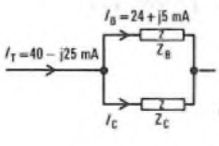
$$= \underline{1\cdot077\angle 31^\circ 20'.}$$

(*b*) The circuit is shown in sketch (*a*), with the total current, I_T milliamperes, feeding parallel impedances Z_B and Z_C, carrying currents I_B and I_C milliamperes respectively.

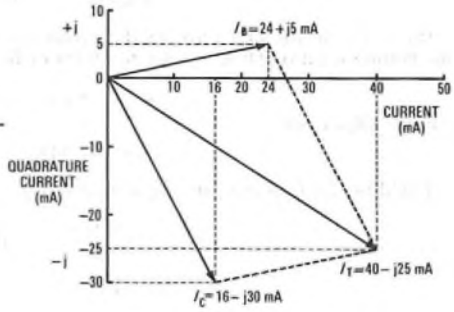
Now, $I_T = I_B + I_C$ milliamperes.

$$\therefore\ I_C = 40 - j25 - (24 + j5)\text{ mA},$$

$$= 16 - j30\text{ mA}.$$



(*a*)



(*b*)

(*i*) The phasor diagram is shown in sketch (*b*). The total current can be seen to be the resultant of phasors I_B and I_C.

(*ii*) The r.m.s. value of the current in circuit C, $I_{C(r.m.s.)}$, is represented by the magnitude of phasor I_C.

$$\therefore\ I_{C(r.m.s.)} = \sqrt{(16^2 + 30^2)}\text{ mA},$$

$$= \underline{34\text{ mA}.}$$

Since the alternating current is sinusoidal, the peak value, $I_{C(pk)}$, is $\sqrt{2}$ times the r.m.s. value.

$$\therefore\ I_{C(pk)} = \sqrt{2} \times 34 = \underline{48\cdot08\text{ mA}.}$$

Note: The question does not state whether the given values of current are r.m.s. or peak values. However, in accordance with normal electrical engineering practice, it has been assumed that r.m.s. values were intended. If it is assumed that peak values are given, $I_{C(pk)}$ becomes 34 mA and $I_{C(r.m.s.)}$ becomes $34/\sqrt{2} = 24\cdot04$ mA.

Q 5 (a) The number of subscribers connected to a rural exchange doubles in 10 years.

(i) What percentage increase is this per annum, assuming a compound-interest law of growth?

(ii) Assuming the same percentage increase each year, after how many years will the number have trebled?

(b) Calculate the sum to infinity of the geometric progression in which the second term is -12 and the fifth term is $+6\cdot 144$.

A 5 (*a*) (*i*) Let n be the initial number of subscribers, and $r\%$ be the increase per annum. The compound-interest law of growth states that, after x years, the number of subscribers

$$= n\left(1+\frac{r}{100}\right)^x.$$

When $x = 10$, n has doubled.

$$\therefore\ 2n = n\left(1+\frac{r}{100}\right)^{10}.$$

$$\therefore\ \left(1+\frac{r}{100}\right)^{10} = 2.$$

$$\therefore\ 10\log_{10}\left(1+\frac{r}{100}\right) = \log_{10} 2 = 0\cdot 3010.$$

$$\therefore\ \log_{10}\left(1+\frac{r}{100}\right) = 0\cdot 0301.$$

$$\therefore\ 1+\frac{r}{100} = 1\cdot 072.$$

$$\therefore\ \frac{r}{100} = 0\cdot 072.$$

$$\therefore\ \underline{r = 7\cdot 2\%.}$$

(*ii*) For treble the number of initial subscribers,

$$3n = n\left(1+\frac{7\cdot 2}{100}\right)^x.$$

$$\therefore\ 1\cdot 072^x = 3.$$

$$\therefore\ x = \frac{\log_{10} 3}{\log_{10} 1\cdot 072},$$

$$= \frac{0\cdot 4771}{0\cdot 0302} = \underline{15\cdot 8 \text{ years.}}$$

(*b*) Let a be the first term of the geometric progression, and r be the common ratio. Then, the second term of the progression

$$= ar = -12,$$

and the fifth term

$$= ar^4 = 6\cdot 144.$$

Dividing the fifth term by the second gives

$$r^3 = -\frac{6\cdot 144}{12} = -0\cdot 512.$$

$$\therefore\ r = -0\cdot 8.$$

Hence, the first term

$$= a = \frac{-12}{r} = \frac{-12}{-0\cdot 8} = 15.$$

The sum to infinity

$$= \frac{a}{1-r} = \frac{15}{1-(-0\cdot 8)} = \underline{8\cdot \dot{3}.}$$

Q 6 (*a*) *Derive a formula for expressing a logarithm to the base 2 as a logarithm to the base 10. Hence, evaluate $\log_2 1000$ to 3 significant figures.*

(*b*) *The discharge current, i amperes, flowing in a resistive-inductive circuit t seconds after the source of e.m.f. is removed, is given by $i = i_0 e^{-Rt/L}$.*

(*i*) *Express t in terms of i, i_0, R and L.*

(*ii*) *If $i_0 = 0\cdot 5$ A, $R = 400\ \Omega$ and $L = 10$ H, calculate, to the nearest milliampere, the current flowing after 45 ms.*

A 6 (*a*) Let the logarithm of any number, N, to the base 2, be p.

$$\therefore\ \log_2 N = p,$$

or

$$N = 2^p.$$

Taking logarithms to the base 10 gives

$$\log_{10} N = \log_{10} 2^p,$$

$$= p\log_{10} 2.$$

$$\therefore\ p = \frac{\log_{10} N}{\log_{10} 2}.$$

$$\therefore\ \underline{\log_2 N = \frac{\log_{10} N}{\log_{10} 2}.}$$

This formula enables the logarithm of a number to the base 2 to be derived from common logarithms. Hence,

$$\log_2 1000 = \frac{\log_{10} 1000}{\log_{10} 2},$$

$$= \frac{3}{0\cdot 3010} = 9\cdot 967,$$

$$= \underline{9\cdot 97}, \text{ to 3 significant figures.}$$

(*b*) (*i*)

$$i = i_0 e^{-Rt/L}.$$

$$\therefore\ e^{-Rt/L} = \frac{i}{i_0}.$$

$$\therefore\ -\frac{Rt}{L} = \log_e \frac{i}{i_0}.$$

$$\therefore\ \underline{t = -\frac{L}{R}\log_e \frac{i}{i_0}.}$$

(*ii*) Substituting the given values,

$$i = 0\cdot 5e^{-400\times 0\cdot 045/10} \text{ A},$$

$$= 0\cdot 5e^{-1\cdot 8} \text{ A},$$

$$= 0\cdot 5 \times 0\cdot 1653 = 0\cdot 08265 \text{ A},$$

$$= \underline{83 \text{ mA}}, \text{ to the nearest milliampere.}$$

Q 7 (*a*) *Using the formulae*

$\cos(A+B) = \cos A\cos B - \sin A \sin B$, and
$\sin(A+B) = \sin A\cos B + \cos A\sin B$,

(*i*) *express $3\sin(\theta+60°) + 2\cos(\theta-30°)$ in the form*

$a\cos\theta + b\sin\theta$,

and

(*ii*) *prove that $\cot A - \tan A = 2\cot 2A$.*

(*b*) *Sketch the graph of the sinusoid $v = 8\sin(100\pi t + \pi/6)$ from $t = -0\cdot 01$ to $t = +0\cdot 01$. From the graph, or otherwise, estimate the smallest positive value of t at which $v = 2$.*

A 7 (*a*) (*i*) $3\sin(\theta+60°) + 2\cos(\theta-30°)$

$$= 3(\sin\theta\cos 60° + \cos\theta\sin 60°) + 2\{\cos\theta\cos(-30°) - \sin\theta\sin(-30°)\},$$

$$= 3\sin\theta\times\frac{1}{2} + 3\cos\theta\times\frac{\sqrt{3}}{2} + 2\cos\theta\times\frac{\sqrt{3}}{2} + 2\sin\theta\times\frac{1}{2},$$

$$= \frac{5}{2}\sin\theta + \frac{5\times\sqrt{3}}{2}\cos\theta,$$

$$= \underline{4\cdot 33\cos\theta + 2\cdot 5\sin\theta.}$$

(*ii*) The left-hand side of the identity

$$= \cot A - \tan A,$$

$$= \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A},$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A\cos A}.$$

But, from the given formulae,

$$\sin 2A = \sin(A+A) = 2\sin A\cos A,$$

and

$$\cos 2A = \cos^2 A - \sin^2 A.$$

Hence, the left-hand side of the identity

$$= \frac{\cos 2A}{\frac{\sin 2A}{2}},$$

$$= \underline{2 \cot 2A}. \qquad \text{QED.}$$

(*b*) The graph is sinusoidal with a maximum amplitude of ± 8.
When $t = -0{\cdot}01$,

$$v = 8 \sin(-\pi + \pi/6),$$
$$= -8 \sin(\pi/6) = -4.$$

When $t = +0{\cdot}01$,

$$v = 8 \sin(\pi + \pi/6),$$
$$= -8 \sin(\pi/6) = -4.$$

The graph passes through zero when $\sin(100\pi t + \pi/6) = 0$; that is, when

$$100\pi t + \pi/6 = \ldots -\pi, 0, \pi, 2\pi \ldots,$$

$$\text{or } 100t = \ldots -\frac{7}{6}, -\frac{1}{6}, \frac{5}{6}, \frac{11}{6} \ldots,$$

$$\text{or } t = -0{\cdot}001\dot{6} \text{ or } 0{\cdot}008\dot{3},$$

within the given range.
The maximum positive value of v occurs when $\sin(100\pi t + \pi/6) = 1$; that is, when

$$100\pi t + \pi/6 = \ldots -3\pi/2, \pi/2, 5\pi/2, 9\pi/2 \ldots,$$

$$\text{or } 100t = \ldots -\frac{3}{2} - \frac{1}{6}, \frac{1}{2} - \frac{1}{6}, \frac{5}{2} - \frac{1}{6} \ldots,$$

$$\text{or } t = 0{\cdot}00\dot{3},$$

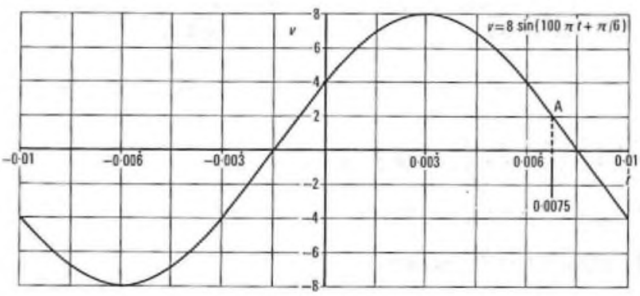
within the given range.
Similarly, the maximum negative value of v occurs when

$$100\pi t + \pi/6 = \ldots -5\pi/2, -\pi/2, 3\pi/2 \ldots,$$

$$\text{or } 100t = \ldots -\frac{1}{2} - \frac{1}{6} \ldots,$$

$$\text{or } t = -0{\cdot}00\dot{6},$$

within the given range.
The graph is constructed from these salient values, and is shown in the sketch.



From the graph, the smallest positive value of t at which $v = 2$ (point A) is $\underline{0{\cdot}0075}$.

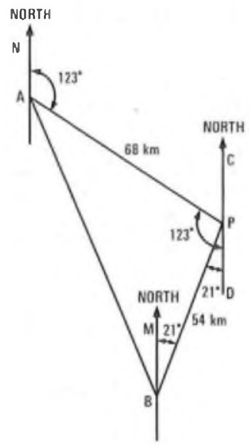
Note: By calculation (that is, by putting $\sin(100\pi t + \pi/6) = 0{\cdot}25$), the accurate value for t is obtained as $\underline{0{\cdot}00753}$.

Q 8 Radar equipment at airport A detects an approaching aircraft at a distance of 68 km on a bearing of 123°. Airport B at the same instant detects the same aircraft 54 km away on a bearing of 21°. All bearings are east of north. Calculate the distance and bearing of airport A from airport B.

A 8 The sketch shows the ground plan of airports A and B. Point P represents the position of the aircraft at the instant of the radar sightings.
Since AN and DPC are parallel, and AP is transverse to them, the alternate angles, NAP and APD, are equal.

$$\therefore \angle\text{APD} = 123°.$$

Similarly, alternate angles MBP and BPD are equal.



$$\therefore \angle\text{BPD} = 21°.$$

$$\therefore \angle\text{APB} = \angle\text{APD} - \angle\text{BPD},$$
$$= 123° - 21° = 102°.$$

Applying the cosine rule to triangle APB gives

$$\text{AB}^2 = \text{AP}^2 + \text{PB}^2 - 2 \times \text{AP} \times \text{PB} \times \cos\angle\text{APB},$$
$$= 68^2 + 54^2 - 2 \times 68 \times 54 \times \cos 102°,$$
$$= 4624 + 2916 - 7344 \times (-0{\cdot}2079),$$
$$= 9067.$$
$$\therefore \text{AB} = 95{\cdot}22 \text{ km}.$$

Applying the sine rule to triangle APB gives

$$\frac{\sin\angle\text{ABP}}{\text{AP}} = \frac{\sin\angle\text{APB}}{\text{AB}}.$$

$$\therefore \sin\angle\text{ABP} = \frac{68 \sin 102°}{95{\cdot}22},$$
$$= 0{\cdot}6985.$$
$$\therefore \angle\text{ABP} = 44°\ 19'.$$
$$\therefore \angle\text{ABM} = 44°\ 19' - 21° = 23°\ 19',$$

and the bearing of A from B is given by the reflex angle, MBA, thus:

$$\angle\text{MBA} = 360° - 23°\ 19' = 336°\ 41'.$$

Hence, the distance of A from B is $\underline{95{\cdot}22 \text{ km}}$, and its bearing is $\underline{336°\ 41'}$ east of north.

Q 9 (*a*) *Derive, from first principles, an expression for dx/dt when*

$$x = ut + \tfrac{1}{2}at^2,$$

where u and a are constants.
(*b*) *Sketch the graph of $y = x^3 - 3x$, showing both positive and negative values of x. Find the critical values of x for which $dy/dx = 0$, and state their significance.*

A 9 (*a*) $\quad x = ut + \frac{1}{2}at^2$.

Let t increase by a small amount, δt, and let the corresponding change in x be δx. Then

$$x + \delta x = u(t + \delta t) + \tfrac{1}{2}a(t + \delta t)^2.$$

$$\therefore \delta x = ut + u\delta t + \tfrac{1}{2}a\{t^2 + 2t\delta t + (\delta t)^2\} - (ut + \tfrac{1}{2}at^2),$$

since $x = ut + \frac{1}{2}at^2$.

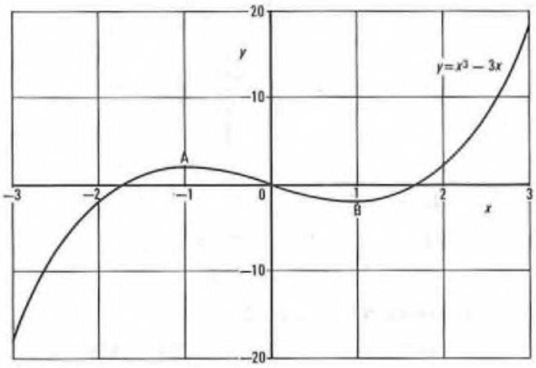
$$\therefore \delta x = u\delta t + at\delta t + \tfrac{1}{2}a(\delta t)^2.$$

$$\therefore \frac{\delta x}{\delta t} = u + at + \tfrac{1}{2}a\delta t.$$

$$\therefore \frac{dx}{dt} = \lim_{\delta x \to 0} \frac{\delta x}{\delta t},$$
$$= \underline{u + at}.$$

(*b*) The graph is shown in the sketch, constructed from the values derived in the table.

x	-3	-2	-1	0	1	2	3
x^3	-27	-8	-1	0	1	8	27
$-3x$	9	6	3	0	-3	-6	-9
y	-18	-2	2	0	-2	2	18



Below $x = -3$, the curve descends rapidly and, above $x = 3$, it ascends rapidly, since, for both cases, the term $3x$ becomes increasingly negligible compared with x^3.

Now,
$$\frac{dy}{dx} = 3x^2 - 3 = 0.$$
$$\therefore\ 3x^2 = 3$$
$$\therefore\ x = \pm 1.$$

Thus, the critical values of x for which $dy/dx = 0$ are $+1$ and -1.

At these values, the curve has turning points, marked A and B in the sketch. At point A, where $x = -1$, y has a maximum value of 2. At point B, where $x = 1$, y has a minimum value of -2.

The significance of a turning point is that the function, y, at that point, changes from an increasing value to a decreasing value (or vice versa), and is momentarily stationary. Thus, just below point A, y is increasing, whereas, just after point A, y is decreasing. The function therefore reaches a maximum value precisely at point A. At point B, the trends are reversed, and the function reaches a minimum value.

Q 10 (a) Evaluate $\int_0^4 x(3 - x)\,dx$.

(b) A curve in the x-y plane passes through the point (3, 10). At every point on it, $dy/dx = 4x^3 - 8x$. *Find the equation of this curve, and show that it cuts the x-axis at only 2 points.*

A 10 (*a*)
$$\int_0^4 x(3 - x)\,dx = \int_0^4 (3x - x^2)\,dx,$$
$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^4,$$
$$= 24 - \frac{64}{3} - 0,$$
$$= 2\tfrac{2}{3}.$$

(*b*) If
$$\frac{dy}{dx} = 4x^3 - 8x,$$
then
$$y = \int \frac{dy}{dx}\,dx = \int (4x^3 - 8x)\,dx,$$
$$= x^4 - 4x^2 + c,$$
where c is the constant of integration.

But the curve passes through the point (3, 10).
$$\therefore\ 10 = 3^4 - 4 \times 3^2 + c.$$
$$\therefore\ c = 10 - 81 + 36 = -35.$$

Hence, the equation of the curve is $y = x^4 - 4x^2 - 35$.

When the curve crosses the x-axis, $y = 0$.
$$\therefore\ x^4 - 4x^2 - 35 = 0.$$

Let $z = x^2$, so that $x = \pm\sqrt{z}$. Then
$$z^2 - 4z - 35 = 0.$$
$$\therefore\ z = \frac{4 \pm \sqrt{\{4^2 - 4 \times (-35)\}}}{2},$$
from the general formula for the solution of a quadratic equation,
$$= \frac{4 \pm 12{\cdot}49}{2} = 8{\cdot}245 \text{ or } -4{\cdot}245.$$
$$\therefore\ x = \pm\sqrt{8{\cdot}245} = \pm 2{\cdot}871.$$

Since $\sqrt{(-4{\cdot}245)}$ is imaginary, there can be only the 2 values of x given by $\pm\sqrt{8{\cdot}245}$ at which $y = 0$. Thus, the curve cuts the x-axis at only the 2 points $x = -2{\cdot}871$ and $x = +2{\cdot}871$.

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