

SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

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1971-72 CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELECOMMUNICATION PRINCIPLES C, 1972 (continued)

Q. 8. The circuit diagram of a simple three-winding transformer bridge is shown in Fig. 3. The turns ratios of the windings PQ, ST and

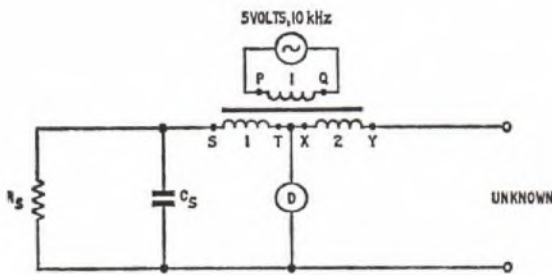


Fig. 3

XY are 1 : 1 : 2. Two unknowns are connected separately to the bridge. When the first is connected, balance is achieved with $R_s = 1.5 \text{ kohm}$ and $C_s = 1,200 \text{ pF}$. When the second is connected, balance is achieved with $R_s = 2 \text{ kohm}$ and C_s set to $3,000 \text{ pF}$ and moved to be in parallel with the unknown.

- Determine an equivalent circuit for each unknown.
- Deduce the current flowing in the primary winding of the transformer at balance in the second case.

A. 8. (a) (i) If the three windings have the same number of turns, then the equivalent circuit of the unknown is a capacitance of $1,200 \text{ pF}$ in parallel with a resistance of 1.5 kohm . Since the winding XY has actually twice the number of turns as ST, the unknown is a capacitance of $\frac{1,200}{4} = 300 \text{ pF}$ in parallel with a resistance of $4 \times 1.5 \text{ kohm} = 6 \text{ kohm}$.

(ii) In this case, the unknown is an inductor having an inductance L in parallel with a resistance of $4 \times 2 \text{ kohm} = 8 \text{ kohm}$.

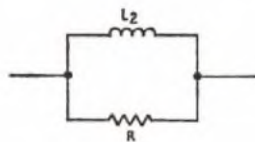
$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi \times 10 \times 10^3)^2 3,000 \times 10^{-12}}$$

$$\approx \frac{1}{40 \times 10^8 \times 3 \times 10^{-9}} = \frac{1}{12} = 0.083 \text{ H.}$$

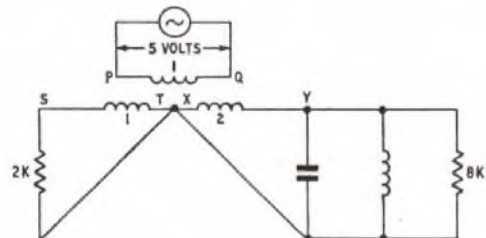
To find the equivalent series circuit, then with reference to sketches (a) and (b),



(a)



(b)



(c)

$$Z_1 = r + j\omega L_1,$$

$$\text{and } Z_2 = \frac{j\omega L_2 R}{R + j\omega L_2}$$

$$= \frac{\omega^2 L_2^2 R + j\omega L_2 R^2}{R^2 + \omega^2 L_2^2}$$

$$\therefore r = \frac{\omega^2 L_2^2 R}{R^2 + \omega^2 L_2^2}$$

$$= \frac{1}{\frac{R}{\omega^2 L_2^2} + \frac{1}{R}}$$

$$\text{or } \frac{1}{r} = \frac{1}{R} + \frac{R}{\omega^2 L_2^2}$$

$$= \frac{1}{8 \times 10^3} + \frac{8 \times 10^3 \times 144}{40 \times 10^8}$$

$$= 1.25 \times 10^{-4} + 2.88 \times 10^{-4} = 4.13 \times 10^{-4}$$

$$\text{i.e. } r = 2.42 \times 10^3.$$

$$\text{Again, } L_1 = \frac{L_2 R^2}{R^2 + \omega^2 L_2^2}$$

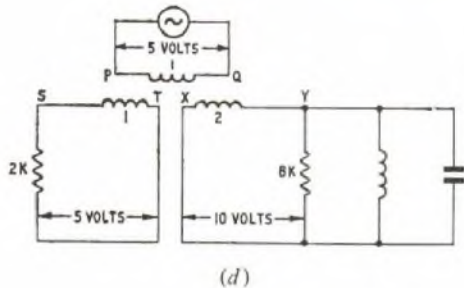
$$= \frac{L_2}{1 + \frac{\omega^2 L_2^2}{R^2}}$$

$$= \frac{1}{1 + 0.435}$$

$$= 0.058 \text{ H.}$$

Thus, the unknown is represented as an inductance of 0.058 H in series with a resistance of 2.42 kohm .

(b) At balance, there is no p.d. across D. Hence, the circuit conditions will not be affected if D is short-circuited, as shown in sketch (c), and the conditions will not be changed if the windings ST and XY are separated as shown in sketch (d).



Since the complete transformer is regarded as ideal,
 power dissipated in winding ST = $\frac{V^2}{R} = \frac{(1 \times 5)^2}{2,000} = 12.5 \text{ mW}$,
 power dissipated in winding XY = $\frac{(2 \times 5)^2}{8,000} = 12.5 \text{ mW}$,
 and power supplied by primary = $5 \times I_p = 25 \text{ mW}$.
 $\therefore I_p = 5 \text{ mA}$.

- Q. 9.** (a) Compare the relative merits of thermionic valves and transistors when used as power amplifiers.
 (b) A single-stage transistor amplifier is used to deliver 5 W to a transformer-coupled load. The stage has a collector efficiency of 40 per cent and the transistor requires no heat sink for dissipations up to 1 W. Two heat sinks are available. Sink A is suitable for dissipations up to 10 W and the more expensive sink B for dissipations up to 20 W. Deduce which sink should be used if the signal input to the amplifier is:
- continuous, and
 - intermittent, with long periods of no signal.

- A. 9.** (a) The advantages of transistors over thermionic valves when used in power amplifiers are that they:
- are of small size and weight,
 - have no filament to be heated and they operate from a much lower voltage (thus leading to simpler power supplies and greater overall efficiency),
 - have a life similar to that of other circuit components such as resistors, capacitors, etc.,
 - are robust,
 - are non-microphonic,
 - have very low inter-electrode capacitance, and
 - have no real delay in operation after switch-on.

The input resistance of the transistor is low and this is a disadvantage with power amplifiers.

(b) With a collector efficiency of 40 per cent, maximum collector power

$$= \frac{100}{40} \times 5 = 12.5 \text{ W.}$$

Power to be disposed of = $12.5 - 1 = 11.5 \text{ W}$.
 Hence, the heat sinks to be used for the two cases are:

- sink B, and
- sink A.

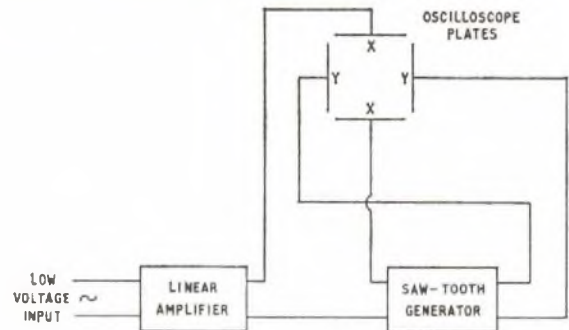
- Q. 10.** (a) With the aid of suitable sketches, explain the basic construction and operation of an electrostatic cathode-ray tube.
 (b) What is post-deflexion acceleration and what are its advantages?
 (c) It is required to display the waveform of a low-voltage signal on a cathode-ray tube. Indicate by means of a block diagram what additional units are required.

A. 10. (a) See A.8, Telecommunication Principles IV. Supplement Vol. 41, Jan. 1949.

(b) Post-deflexion acceleration, as the term implies, is a means of accelerating the electrons in the beam of a cathode-ray tube after the deflecting system and before they reach the screen. It is achieved by means of a broad graphite band or spiral inside the screen end of the tube flare, insulated from the coating of the screen but connected to a terminal through the glass. When a potential higher than that of the final anode (gun) is applied to this, the brilliance of the trace is increased, since the electron cloud formerly behind the screen has now been removed.

Alternatively, for a given brilliance, the gun potentials may be substantially reduced, thereby increasing the sensitivity, since the electron beam passes more slowly through the deflecting system.

(c) If an electrostatic cathode-ray tube is to be used to display the waveform of a low-voltage signal, a linear voltage amplifier would be required to feed the Y-plates and a saw-tooth generator, synchronized by the signal, connected to the X-plates (see sketch).



COMMUNICATION RADIO C, 1972

Students were expected to answer any six questions

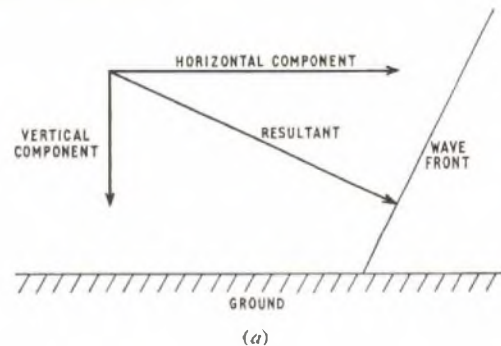
- Q. 1.** (a) State clearly what you understand by the term surface wave.
 (b) Why does the field strength for surface-wave propagation fall off approximately in inverse proportion to the distance over which the wave has travelled?
 (c) Up to what frequencies is the surface wave the main method of propagation?
 (d) Upon what two main ground constants does the strength of the propagating surface wave depend?
 (e) Is surface-wave propagation better over sea than over land?
 (f) How does the penetration of the induced surface-wave current in the ground vary with frequency?

- A. 1.** (a) When an electromagnetic wave is propagated above the surface of the ground, currents are induced to flow in the ground where, due to the resistivity of the earth, energy is dissipated. The transfer of energy from the electromagnetic (e.m.) wave to the ground is continuous so that the wave is propagated with horizontal and vertical components, as shown in sketch (a). The resultant is a wave tilting towards the earth's surface, over and through which it passes.
 (b) As the e.m. wave emanates from the aerial, the wave-front circumference expands. Since the total energy dissipated remains

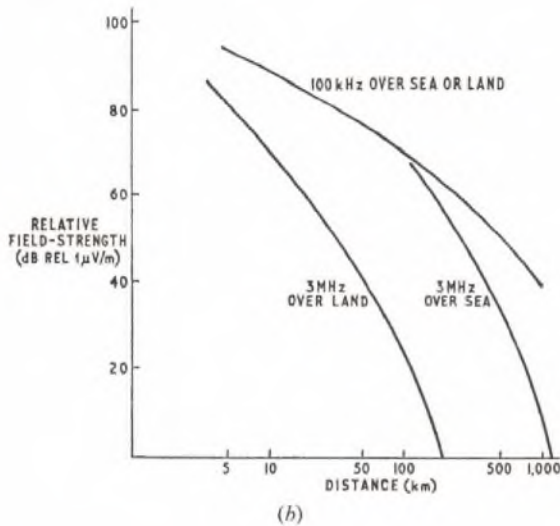
constant, the energy per unit length of the circumferential wave front (field strength) decreases.

(c) Although the surface wave can be propagated at frequencies up to 30 MHz, it is mainly used up to 5 MHz.

(d) The two main ground constants are conductivity and, above 1.5 MHz, the permittivity.



(e) There is little difference in propagation over sea or land at the lower frequencies (100 kHz) but there is a significant difference at higher frequencies (3 MHz), when propagation is better over sea (sketch (b)).



(f) The penetration of the induced surface-wave current into the ground is inversely proportional to the square root of the frequency.

Q. 2. (a) A superheterodyne communication receiver is required to cover exactly the frequency range 30-90 MHz. If the first intermediate frequency is to be 10.7 MHz, what range of local-oscillator frequency is needed?

(b) If the capacitance in parallel with a fixed inductance used in the local-oscillator frequency-determining circuit in this receiver can be varied over a range of 200 pF, calculate the minimum value of this capacitance.

A. 2. (a) The local oscillator could be tuned to a frequency either 10.7 MHz greater than, or 10.7 MHz less than the signal frequency. Possible ranges are 40.7-100.7 MHz or 19.3-79.3 MHz. However, the former has a frequency ratio (upper to lower limit) of approximately 2 whereas the latter has a frequency ratio of approximately 4. Since the frequency ratio has to be achieved by means of a variable inductor or capacitor changing its inductance or capacitance by the square of the same ratio, it is easier to use the lower ratio. Consequently, the oscillator having a range 40.7-100.7 MHz would be selected.

$$(b) f_r = \frac{1}{2\pi\sqrt{LC}}$$

where f_r = resonant frequency,

L = inductance, and

C = capacitance.

Let f_{min} be the minimum frequency to be produced by the local oscillator when the tuning capacitor is a maximum (C_{max}).

Let f_{max} be the maximum frequency to be produced by the local oscillator when the tuning capacitor is a minimum (C_{min}).

$$\text{Then, } \left(\frac{f_{min}}{f_{max}}\right)^2 = \frac{C_{min}}{C_{max}}$$

Since the range of the tuning capacitor is 200 pF,

then $C_{max} = C_{min} + 200$ pF,

where C_{max} and C_{min} are in picofarads.

Substituting for $f_{min} = 40.7$ MHz, $f_{max} = 100.7$ MHz gives:

$$\begin{aligned} \left(\frac{40.7}{100.7}\right)^2 &= \frac{C_{min}}{C_{min} + 200} \\ \therefore C_{min} \left(\frac{100.7}{40.7}\right)^2 &= C_{min} + 200 \\ \therefore C_{min} &= \frac{200}{\left(\frac{100.7}{40.7}\right)^2 - 1} \text{ pF} \\ \therefore C_{min} &= 39.05 \text{ pF} \end{aligned}$$

Q. 3. (a) State briefly the advantages of using a radio-frequency (r.f.) amplifier stage in a superheterodyne radio receiver.

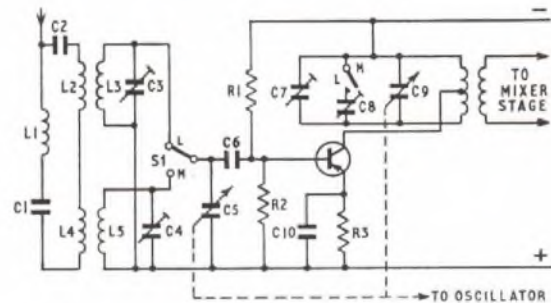
(b) Draw the circuit diagram of the r.f. amplifier stage of a transistor superheterodyne broadcast receiver, showing clearly how the aerial is coupled to the amplifier and how wave-changing is facilitated. Indicate which components would be ganged to the tuning control.

A. 3. (a) The radio-frequency (r.f.) amplifier stage improves:

- (i) the selectivity of the receiver,
- (ii) the image-channel rejection,
- (iii) the signal-to-noise ratio, and
- (iv) the automatic gain-control by increasing loop gain.

Furthermore, the r.f. amplifier attenuates any local-oscillator voltages that might otherwise reach the aerial and cause unwanted radiation.

(b) A circuit diagram of the r.f. amplifier stage in a transistor superheterodyne receiver is shown in the sketch.



L2 and L4 are the long- and medium-wave input transformers, the secondaries of which (L3 and L5) are shunted by pre-set trimmer capacitors (C3 and C4). Band selection is made by switch S1, which also selects the capacitors (C7 and C8) in the collector circuit.

Tuning is facilitated by capacitor C5 which is ganged to capacitor C9 in the collector tuned-circuit and the variable capacitor in the oscillator tuned-circuit. The aerial signal is fed to the input transformers via capacitor C2, whilst inductor L1 and capacitor C1 provide an intermediate-frequency (i.f.) trap for signals close to the i.f. used in the receiver.

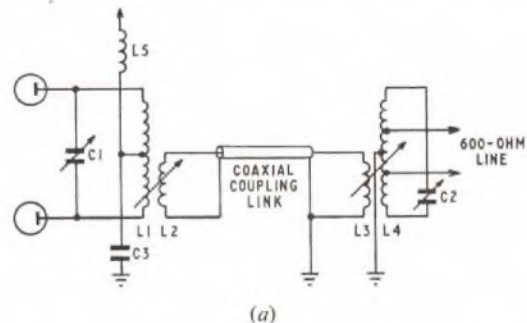
Q. 4. (a) Using circuit diagrams to illustrate your answers, show how:

- (i) a telephony transmitter with a class-B push-pull output stage would be coupled to a 600-ohm transmission line, and
- (ii) a transmitter output tank circuit would be connected to a coaxial aerial feeder.

Show how matching and tuning are facilitated.

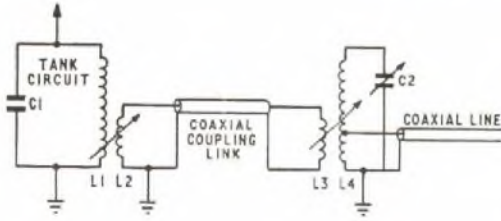
(b) Briefly explain how a high-power transmitting valve is protected in the event of a feeder or aerial failure.

A. 4. (a) (i) A telephony transmitter using class-B push-pull can be connected to a 600-ohm line as shown in sketch (a). Matching



of the load impedance to the transmitter is achieved by adjustment of the tapping points on inductor L4. The efficiency of the coupling is optimized by adjustment of the coupling between inductors L1 and L2, and L3 and L4. Tuning is achieved by means of capacitors C1 and C2, the coaxial coupling link being non-resonant. Radio-frequency decoupling in the transmitter is provided by inductor L5 and capacitor C3.

(ii) A transmitter output tank circuit can be connected to a coaxial feeder by the circuit shown in sketch (b). Tuning and matching are achieved in a similar manner to that described in (i) above.



(b)

The suppression of harmonics is achieved in both cases by arranging that the link inductors (L2 and L3) are at the earthy ends of inductors L1 and L4.

(b) A high-power transmitting valve can be protected by fitting a thermal fuse in the anode circuit. The fuse is mounted on the anode surface so that, in the event of a current overload, the rise in temperature at the anode melts the fuse thereby releasing a spring-loaded switch which disconnects the anode voltage.

Q. 5. (a) Why does a f.m. system need a greater bandwidth than an a.m. system to transmit a given sinusoidal tone?

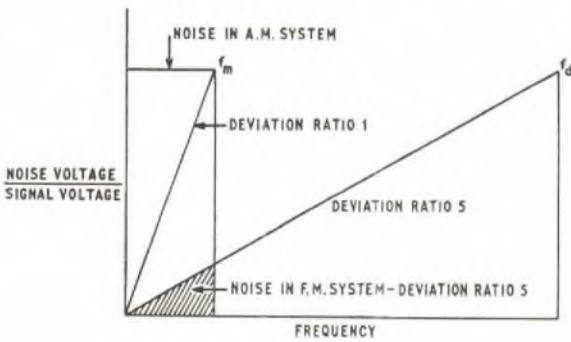
(b) Why are pre- and de-emphasis used in f.m. systems?

(c) Draw the circuit diagram of: (i) a pre-emphasis, and (ii) a de-emphasis, circuit.

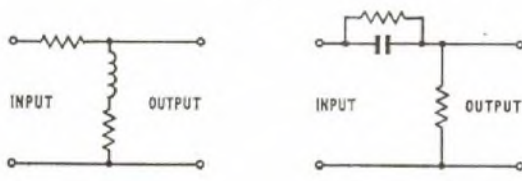
A. 5. (a) In an amplitude-modulated (a.m.) system, two sidebands are produced requiring a pass band of $2f_m$, where f_m is the modulating frequency.

In a frequency-modulated (f.m.) system, an infinite number of sidebands are produced. Fortunately, not all these sidebands need be transmitted for faithful reproduction of the modulation signal at the receiver, and it can be shown that a pass band of $2(f_d + f_m)$ is acceptable, where f_d is the rated system deviation.

Assuming a deviation ratio $\frac{f_d}{f_m(max)} = 1$, and a maximum modulation



(a)



(b)



(c)

lating frequency f_{max} of, say, 15 kHz, then the a.m. pass band is 30 kHz, while the f.m. pass band is 60 kHz. Also, considering the noise spectrum inherent in a.m. and f.m. systems, see sketch (a), it can be seen that the a.m. spectrum is independent of frequency, while the f.m. spectrum is a function of frequency and deviation ratio. If the advantage of improved signal-to-noise ratio is to be gained by using a deviation ratio of, say, 5, then the required f.m. bandwidth is $2(75 + 15) = 180$ kHz.

(b) A modulating signal, in an f.m. system, which has the amplitude maintained constant while the frequency is varied over the modulating range, reproduces at the receiver a signal that is of constant amplitude over the modulation frequency range. However, the noise appearing in the output is proportional to the modulating frequency (see sketch (a)) so that the signal-to-noise ratio decreases with increase in modulating frequency. To improve the signal-to-noise ratio, a pre-emphasis network is used which results in the low modulating frequencies being attenuated with respect to the higher frequencies.

Correction of the signal is undertaken at the receiver by use of a de-emphasis network having an inverse transfer characteristic to the pre-emphasis network.

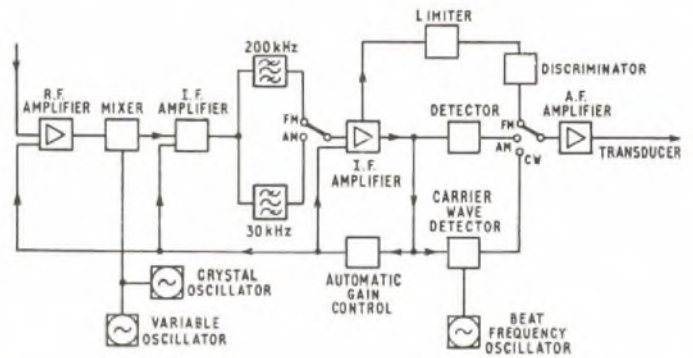
(c) Typical pre-emphasis and de-emphasis networks are shown in sketches (b) and (c) respectively.

Q. 6. (a) Draw the block diagram of a v.h.f. communications receiver operating over switched bands for the reception of both a.m. and f.m. signals.

(b) Explain briefly the function of each block.

(c) What manual controls would you expect to find on such a receiver?

A. 6. (a) A block diagram of a very high frequency (v.h.f.) receiver operating over switched bands for both amplitude modulation (a.m.) and frequency modulation (f.m.) is shown in the sketch.



(b) The radio-frequency (r.f.) amplifier provides selectivity, image-channel rejection and buffering of oscillator from aerial. Conversion down to the intermediate frequency (i.f.) is achieved by the mixer driven by either a variable oscillator, providing a variable tuning facility, or a crystal-controlled oscillator for fixed-frequency reception. The 10.7 MHz i.f. stages provide selectivity and gain, followed by selected filters (30 kHz for a.m., 200 kHz for f.m.). An amplitude-modulated signal is then amplified before the audio signal (information content) is extracted by the detector. The audio signal is then amplified to develop sufficient power to drive a transducer.

An f.m. signal is amplified, and amplitude-limited to reduce the noise level, before the original modulating signal is reproduced by the discriminator. Detection of a plain sinusoid is facilitated by the carrier-wave detector, driven by the beat-frequency oscillator.

Automatic gain control (a.g.c.) is provided by the a.g.c. unit and applied to the i.f. and r.f. amplifiers.

(c) The following controls are probably provided: r.f. gain, i.f. gain, audio gain, tuning, a.m./f.m. switch, muting, squelch, tone, automatic frequency control, on/off switch.

Q. 7. A 120 kHz carrier, 75 per cent amplitude modulated with a 5 kHz tone, is applied to the circuit shown in Fig. 1. The tank circuit

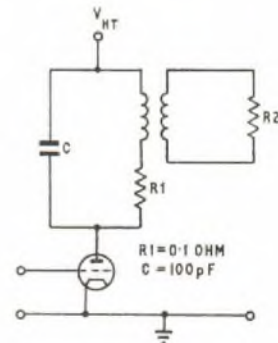


Fig. 1

is resonant at the carrier frequency. The circuit has a loaded Q of 13.26. Calculate:

- (a) the unloaded Q of the tank circuit,
- (b) the percentage of the carrier power delivered to the tank circuit that reaches the load, and
- (c) the effective depth of modulation at the load.

A. 7. (a) The unloaded Q , Q_u , of the tank circuit shown in the sketch is given by $Q_u = \frac{1}{\omega CR_1}$.

Substituting for $f = 120$ kHz,

$$C = 100 \text{ pF, and}$$

$$R_1 = 0.1 \text{ ohm, gives}$$

$$Q_u = \frac{10^{12}}{2\pi \times 120 \times 10^3 \times 100 \times 0.1}$$

$$\therefore Q_u = \frac{10^7}{24\pi} = 132.6 \times 10^3.$$

(b) The percentage of carrier power η delivered to the tank circuit that reaches the load R_2 is given by:

$$\eta = \left(1 - \frac{Q_l}{Q_u}\right) \times 100,$$

where $Q_l^2 =$ loaded Q of the tank circuit = 13.26, and

$$Q_u = \text{unloaded } Q \text{ of the tank circuit} = 132.6 \times 10^3.$$

$$\therefore \eta = \left(1 - \frac{13.26}{132.6 \times 10^3}\right) \times 100 = 99.99 \text{ per cent.}$$

(c) At frequencies off resonance, the gain of the amplifier is decreased by the same ratio as the impedances of the tuned circuits, at resonance, Z , and off resonance, $Z_{\delta f}$. The effective depth of modulation at the load is reduced by the same ratio.

\therefore Effective depth of modulation = $\frac{Z_{\delta f}}{Z}$ \times (depth of modulation at the tank circuit).

$$\frac{Z_{\delta f}}{Z} = \frac{1}{\sqrt{1 + Q_l^2 \left(\frac{2\delta f}{f_0}\right)^2}}$$

where $f_0 =$ frequency at resonance, and

$\delta f =$ frequency deviation from resonance.

$$\therefore \frac{Z_{\delta f}}{Z} = \frac{1}{\sqrt{1 + (13.26)^2 \left(\frac{2 \times 5 \times 10^3}{120 \times 10^3}\right)^2}}$$

$$= 0.67.$$

\therefore Effective depth of modulation at the load is given by:
 $\frac{75}{100} \times 0.67 \times 100 = 50 \text{ per cent.}$

Q. 8. (a) Why is the use of frequency modulation confined to the v.h.f. band and above as a general rule?

(b) Briefly describe what you understand by the term capture effect.

(c) (i) Indicate typical values of frequency deviation and highest modulating frequency used in f.m. broadcasting services operating in the v.h.f. band.

(ii) Using these values, calculate the maximum percentage band occupancy when the minimum carrier spacing is 2.2 MHz for transmitters serving the same area.

A. 8. (a) The bandwidth required for the transmission of frequency-modulated (f.m.) waves is very much greater than that required for amplitude-modulated (a.m.) waves. The use of f.m. in the medium-frequency or high-frequency bands would fully occupy the available spectrum with few channels, so use is made of the larger very-high-frequency (30–300 MHz) band and part of the ultra-high-frequency (300–3,000 MHz) band.

(b) Capture effect is that whereby the ratio of wanted signal to interfering signal at the output is greater than at the input of the receiver.

(c) (i) Typical frequency deviation (f_d) used for broadcast transmissions is 75 kHz for a maximum modulating frequency (f_m) of 15 kHz.

(ii) Bandwidth of f.m. transmission = $2(f_m + f_d) = 180$ kHz.

$$\text{Percentage band occupancy} = \frac{\text{f.m. bandwidth} \times 100}{\text{carrier spacing}}$$

$$= \frac{180 \times 10^3 \times 10^2}{2.2 \times 10^6}$$

$$= 8.2 \text{ per cent.}$$

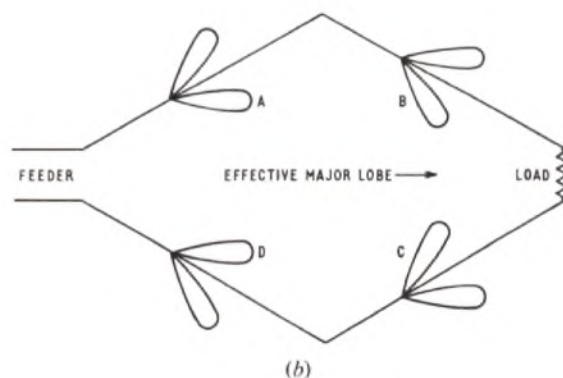
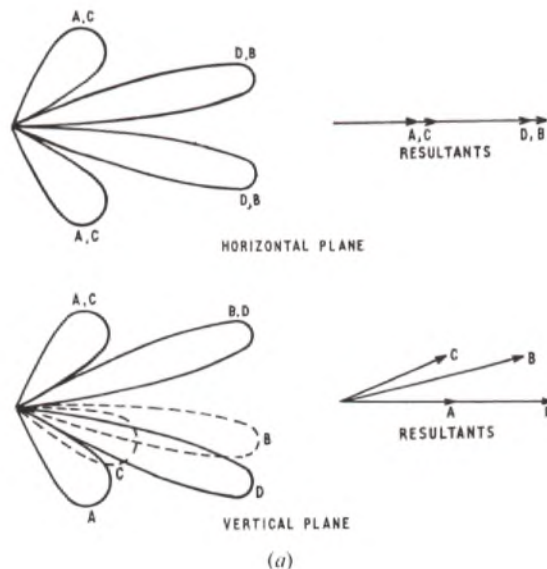
Q. 9. (a) A current-carrying conductor parallel to the ground is fed at one end and terminated in a matched impedance at the other. Sketch the approximate radiation patterns parallel with, and perpendicular to, the ground for each of the following conditions:

- (i) when the length of the wire is 2 wavelengths and the height of the wire above the ground is 10 or more wavelengths,
- (ii) when the length is 10 or more wavelengths and the height is less than one wavelength,
- (iii) when the length is 2 wavelengths and the height is less than one wavelength, and
- (iv) when the length and the height are both 10 or more wavelengths.

(b) From your sketches show how four such wires can be combined to form a rhombic aerial, and indicate how the radiation pattern of the combination varies with frequency.

(c) What do you understand by the statement that the rhombic is a travelling-wave aerial?

A. 9. (a) The radiation patterns, for a current-carrying conductor parallel to the ground, in the vertical and horizontal planes are shown in sketch (a).

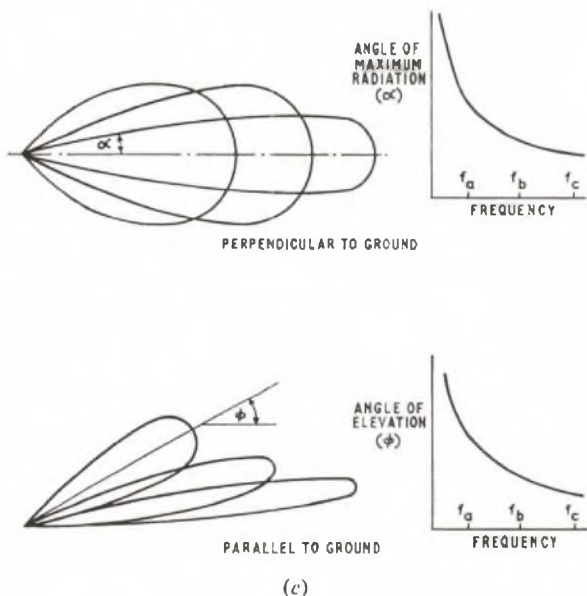


The patterns shown represent the situations when the length of the wire:

- (i) is 2 wavelengths and the height of the wire above the ground is 10 or more wavelengths (pattern A),
- (ii) is 10 or more wavelengths and the height is less than one wavelength (pattern B),

COMMUNICATION RADIO C, 1972 (continued)

- (iii) is 2 wavelengths and the height is less than one wavelength (pattern C), and
 - (iv) when the length and the height are both 10 or more wavelengths (pattern D).
- (b) A rhombic aerial can be formed by connecting four equal-length wires arranged in the form of a rhombus as shown in sketch (b). The radiation pattern for each wire is shown and it can be seen that the lobes ABC and D reinforce each other, producing an effective pattern with the major lobe in the direction shown. As the signal frequency is increased, the horizontal and vertical radiation patterns vary as shown in sketch (c).



(c) A travelling-wave aerial is one in which the steady-state current distribution in any element arising from the sinusoidal excitation of the aerial can be regarded as due to a current wave which traverses the elements in one direction only.

- Q. 10. (a) Define the term noise factor.
 (b) List the equipment required to measure the noise factor of a superheterodyne receiver.

- (c) Describe the measurement technique in a series of single-operation steps.
 (d) A communication receiver has a noise figure of 10 dB at 30 MHz. In your opinion, is this a poor, typical or good performance?

A. 10. (a) The noise factor (n.f.) can be defined as the ratio of total output noise of a receiver to the output noise due to source noise only, i.e.

$$n.f. = \frac{N_a + N_r}{N_a}$$

where N_a = source noise at output, and

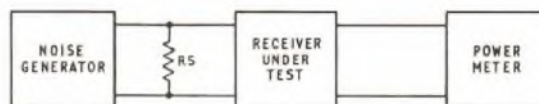
N_r = receiver noise at output,

or, $n.f. = \frac{\text{signal-to-noise ratio at input}}{\text{signal-to-noise ratio at output}}$

(b) The noise factor of a receiver can be measured using the following equipment:

- (i) a noise generator having a noise-diode current meter,
- (ii) a power meter, and
- (iii) a resistance (R_s) equal to the resistance of the source to which the receiver is connected.

The equipment is connected as shown in the sketch.



(c) With the noise generator connected in circuit, but with the noise diode disconnected, the gain of the receiver under test is adjusted to give a convenient reading on the power meter. The noise diode is switched in circuit and the diode current adjusted, and recorded when the power-meter reading is doubled. The noise factor is then determined by the formula:

$$n.f. = 20I_dR_s = 10 \log (20 I_d R_s) \text{ dB,}$$

where I_d = noise diode current, and

R_s = source resistance.

(d) A communication receiver having a noise figure of 10 dB can be regarded as poor. A typical figure for such a receiver is 6 dB, while a good figure is 3 to 4 dB.

TELEGRAPHY C, 1972

Students were expected to attempt any six questions

- Q. 1. (a) Describe a method of measuring the receive margin of a teleprinter.
 (b) Why is it important that the early and late margins have equal value?
 (c) With the aid of a timing diagram, explain how the margin of a teleprinter is affected by a motor which is not running at the correct speed.
- Q. 2. Explain the purpose of the following equipment provided at a subscriber's teleprinter station. Draw a diagram showing where each item is connected in the circuit and give typical values for each component.
 (a) A low-pass filter.
 (b) A shunted capacitor.
 (c) A spark-quench circuit.
- Q. 3. (a) What are the functions of the following in a semi-automatic torn-tape message-relay centre?
 (i) A teleprinter.
 (ii) A printing reperforator.
 (iii) A serial-numbering transmitter.
 (iv) A triple-ganged automatic transmitter.
 (b) Draw a diagram to illustrate the layout of equipment in a typical semi-automatic torn-tape message-relay centre.
 (c) Give details of the format of a typical message and explain the purpose of each group of characters.

A. 3. (a) (i) A teleprinter is provided as part of a torn-tape semi-automatic message-relay centre as a safeguard against lost or mutilated messages. A copy of all outgoing messages on any particular circuit

may be recorded on a monitor teleprinter, together with the serial number or preamble to the message. If an enquiry is made from the distant terminal, lost or mutilated messages may be retransmitted using the teleprinter copy.

(ii) Printing reperforators are used to receive incoming messages on paper tape. Each character received causes the machine to perforate the tape according to the five elements; for a negative (or mark) element, one hole is punched in the paper whilst the paper remains untouched for a positive (or space) element. The character is also printed on the tape, displaced from the perforations by about eight characters, due to the mechanical limitations of the perforating and printing mechanisms.

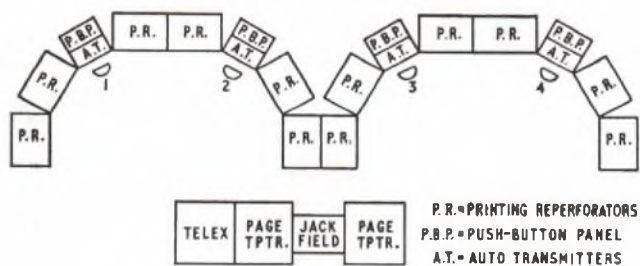
The reperforators are normally mounted in a cabinet which houses three machines in vertical formation. As each message is received, the perforated tape is fed from a hole in the front of the cabinet; the operator reads the tape, selects the end-of-message marker, and tears off the message for onward transmission.

(iii) A serial-numbering transmitter is not normally under the control of the operator but is employed to apply, automatically, a serial number as a prefix to each message transmitted over a particular outgoing circuit. The operator loads the message tape in the automatic transmitter associated with the outgoing circuit appropriate to the message destination and operates the RELEASE key. The serial-numbering transmitter automatically transmits a fixed preamble, together with the next serial number, to the line; at the end of the numbering transmission, the message transmitter is energized and the message is transmitted. Serial numbering is required when heavily-used circuits are worked duplex, i.e., messages are sent in both directions at the same time. The originator has no assurance that messages have reached the distant terminal and relies upon the distant operator

checking the sequence of the serial numbers attached to messages on that channel.

Time-and-date injection units may be associated with the numbering transmitter and connexions are provided for correcting and re-setting the date and the serial number, and for suppressing the serial number if desired.

(iv) Triple-ganged automatic transmitters are provided to give more efficient circuit occupancy and to save space in the operating suite. One motor is employed to clutch-drive three separate transmit mechanisms which may be employed in several ways according to the station requirements. For a push-button tape-relay system, two such machines, offering six message heads, are used at each transmit position. The operator reads the destination of any message, inserts the message in a free transmitter head, operates a push-button for the selected route until the DEMAND-STORED lamp lights, and the message is automatically transmitted when the circuit is disengaged. In this way, the operator can deal with up to six separate messages at a time.



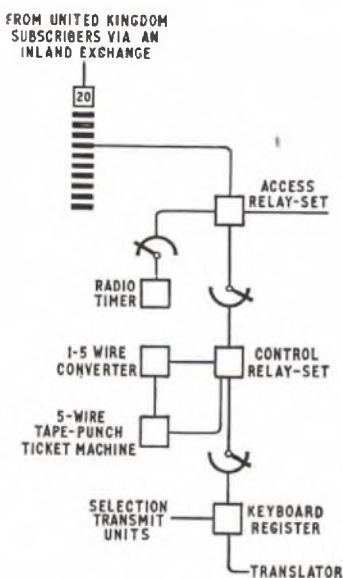
(b) The sketch shows the layout of equipment at a typical push-button torn-tape message centre.

(c) For details of the format of a typical message, see A. 8., Telegraphy C, 1968. Supplement, Vol. 62, p. 38, July 1969.

Q. 4. (a) Draw a block diagram and explain the operation of a system of telex call charging by automatic ticketing.

(b) Why, for certain types of call, is this system preferred to the system of call charging by operating a subscriber's totalling meter?

A. 4. (a) The sketch shows the equipment provided for telex call charging by automatic ticketing. The call-charging information is produced as two lengths of perforated tape by the ticketing machine,



the two tapes being known as the head sequence and the tail sequence. The former contains the information pertinent to the setting-up of the call, such as the calling-subscriber's answer-back code and a record of the selection signals indicating the destination of the call; the tail sequence contains information on the duration of the call. Both sequences contain four carriage-return characters at the start (which separate the information on that sequence from the preceding entry), a reference code for the call (which enables the two sequences to be associated later for charging), and the time of day in case of later dispute. The perforated tapes are collected and processed by computer to enable the subscriber to be charged for the call.

The subscriber gains access to the equipment by dialling the digits 207 to seize an access relay-set. This contains the mechanism for timing the call and is associated with the call for the duration of the call. A control relay-set with the associated 1-5 wire converter and the 5-wire tape-punch ticketing machine are seized by the access relay-set. The entry-separation characters are automatically transmitted to the tape machine to form part of the head sequence, and these are followed by the reference code of the call which consists of two characters identifying the access relay-set and two characters identifying the ticket reperforators. Four characters representing the time of day are also recorded on the perforated tape. The reference code and time are transmitted to the calling subscriber. The who-are-you (WRY) signal is then transmitted to the subscriber and the answer-back code recorded on the ticket reperforator. The control relay-set hunts for a free register and, when this has been seized, the characters KEY+ are transmitted to the subscriber as an invitation to transmit the remaining selection digits from the teleprinter keyboard. These are recorded as part of the head sequence to identify the destination of the call.

The register, translator and selection transmit units route the call through the intercontinental exchange to the outgoing trunk circuit and the distant terminal. The control relay-set and associated ticketing machine are released to accept further head and tail entries for other calls, but the control relay-set remains marked by the access relay-set to be seized for the tail entry when the call is terminated.

The access relay-set remains held throughout the call and records the duration of the call in 6 s steps on uniselectors. If the call is connected over an error-corrected radio path, the radio timer is used to detect the 6 s release pulses from the error-correcting radio equipment and the elapsed time is stored accordingly. When the call has terminated, the access relay-set seizes the marked control relay-set when this is free, and the tail entry is transmitted to the ticketing machine. The duration of the call is recorded on the perforated entry, firstly, as three of ten characters of a self-checking group to be used for automatic processing and, secondly, in numerical form for ease of monitoring. A call is restricted to a maximum of 99.9 minutes; if the call exceeds this time, it is forcibly released. An ineffective call is recorded in a similar manner with the elapsed time recorded as zero.

(b) There are a number of reasons for using an automatic ticketing system in preference to operating a subscriber's meter and these are listed below.

(i) Expensive, long-distance calls are traditionally charged individually. This enables the subscriber to keep a check on them.

(ii) For the more expensive call, the meter pulses would require to be at a faster rate than the subscriber's meter could tolerate. In addition, the time-zone equipment would be overloaded.

(iii) If the meter pulses were returned from the international exchange, the method of signalling the pulses over the reverse path would have to be resolved.

(iv) The provision of ticketing information enables the international account to be properly credited. In addition, this information could be used to obtain statistics, for example, to determine the efficiency of the radio circuits.

Q. 5. (a) Why are semi-automatic routine testers provided in a telex exchange?

(b) List the tests which this type of tester would make on a group selector and describe the indications given to the operator of the progress of the tests and of a test failure.

(c) What connexions are necessary between the tester and a rack of selectors?

A. 5. (a) Semi-automatic routine testers normally consist of trolley-mounted equipment with plugs and cords for connexion to the equipment under test. With the appropriate connexions made, and the equipment to be tested disengaged, the operation of the START key causes a series of tests to be applied automatically. The progress of the cycle of tests is indicated by a series of lamps, one of which lights during the progress of each test. In this way, one tester quickly applies tests which would otherwise have to be made, possibly one at a time, with the aid of box testers or calibrators. The semi-automatic tester normally only requires one connexion to be made to each item of main equipment, and a complete cycle of tests is completed in about one minute. The tester, thus, eases the work of the maintenance staff and improves the service by enabling more frequent and regular checks of main equipment to be made. The tester is used at frequent intervals to detect incipient faults and enables equipment to be repaired before it causes a break in service. This is particularly useful for common equipment where a faulty switch may affect a number of calls before the fault is noticed. Testers of this type are also used for acceptance testing new equipment when a new exchange is installed or an existing one is extended.

(b) The tester checks that:

(i) the selector is disengaged,

(ii) the potentials on the S-wire are correct,

(iii) the operation of relay A to minimum and maximum current value is satisfactory,

- (iv) the release times of relays B and CD are within limits,
 (v) the selector steps vertically to 50 per cent and 80 per cent break pulses at 12 pulses/s.,
 (vi) the selector switches to the HA choice on outlet 19 of level 9 and to the HB choice on outlet 20,
 (vii) backward P-wire guarding is provided,
 (viii) the time to reach outlet 20 on level 9 and the time to return to normal from this contact are within limits, and
 (ix) the service signals are transmitted from the 11th step.

The lamps on the tester indicate which test is in progress; if a fault is found, the tester stops and the appropriate lamp indicates the test which has failed. An additional lamp indicates that the cycle of tests has been successfully completed.

(c) Plugs and cords are used to give the tester access to the equipment to be tested. One cord is connected to the group selector under test to give connexion to the test points within the selector. Further cords are connected to the rack for access to power supplies (-50 volts and ± 80 volts), service signals (OCC, NP, etc.), phasing pulses and test lines. The latter are circuits connected to contacts 18, 19 and 20 of level 9 so that the tester may gain an indication that vertical stepping and switching has been satisfactory.

Connexions are also provided on the tester for a relay-timing tester so that the tester timing elements may be adjusted before use.

Q. 6. (a) A telex subscriber is connected to an automatic telex exchange by a short physical circuit. List the signalling potentials which are present on each wire during the progress of a normal call initiated and cleared by that subscriber. Explain the function of each signal, and indicate the timing of each signal or group of signals.

(b) What arrangements are made to inform a caller that the subscriber's office is closed and the power to the station has been switched off?

(c) How do these arrangements differ if the subscriber is connected to the exchange over a multi-channel voice-frequency circuit?

A. 6. (a) When the station is idle and no call is connected, positive 80-volt potential is connected to the send (S) wire at the subscriber's station. This potential is looped at the exchange and is connected to the receive (R) wire to the outstation.

When an outgoing call is to be made, the DIAL key on the subscriber's unit is operated, connecting negative 80-volt potential to the S-wire. A rectifier in series with the circuit at the exchange prevents the connexion of the negative potential to the R-wire, which remains at zero potential until a free group selector and time-zone equipment are seized, at which time earth potential is connected to the R-wire. This is interrupted by the *proceed-to-select* signal which consists of negative potential connected for 50–100 ms and indicates to the caller that dialling may begin.

When the outstation CALL lamp lights in response to the *proceed-to-select* signal, the dial may be operated. The selection signals from the dial are transmitted on the S-wire and consist of double-current (± 80 -volt) signals of nominal 60/40 positive/negative ratio at 10 pulses/s. When the connexion has been established, negative potential is connected to the station R-wire by the exchange equipment. With the call established, either station may transmit double-current signals; when either station is idle, negative 80-volt potential is connected to line by that station.

To release the call, the station CLEAR key is operated, connecting positive 80-volt potential to the S-wire. If the key is operated for at least 325 ms, this is recognized as a *clear* signal by the exchange equipment and the switches are released. Earth potential is connected to the outstation R-wire so that current ceases to flow in that wire and the outstation control-unit releases. The release of the control-unit reconnects positive potential to the S-wire to provide the normal disengaged condition to the exchange.

(b) When it is required to close the station to incoming traffic, e.g. when changing paper, or because the subscriber wishes to prevent messages arriving at the station when it is unattended, the power supply to the station unit is switched off. With the disconnection of the positive potential on the S-wire, the subscribers' line equipment in the exchange hunts for a free group selector. This switch cannot hold, however, as negative potential is not present on the S-wire and, in the subscriber's line circuit, the 700-ohm negative potential connected to the PX-wire of the final-selector multiple is disconnected. When any call is routed to that subscriber, the final selector detects the absence of potential on the PX-wire and connects the service signal ABS to the calling subscriber. This is followed by positive potential to clear the call.

(c) For subscribers connected to the exchange over a multi-channel voice-frequency circuit, the disconnection of the power at the subscriber's station causes a negative potential calling signal to be applied over the v.f. circuit to the subscriber's line circuit. A group selector and the associated time-zone equipment are seized in the normal way, but no dialled digits are received by the time-zone equipment. After 12–24 seconds, the circuit is force-released; the group selector and time-zone equipment restore to normal and the subscriber's line equipment

disconnects the potential on the PX-wire. A calling subscriber receives the ABS signal and positive potential as before.

Q. 7. (a) Distinguish between the use of a regenerative repeater and an automatic-error-correcting (ARQ) system on the international telex network.

(b) With the aid of a diagram, explain briefly the method of operation of one of the above types of equipment.

(c) What arrangements are made in each of the above types of equipment for the transmission of telex selection signals?

A. 7. (a) A regenerative repeater is inserted in a telegraph circuit when it is expected that the received signal may be so distorted as to prevent accurate recording of the message on the terminal machine or equipment. The repeater receives the signal and retransmits each character with negligible distortion. On the telex network, regenerative repeaters are inserted in the circuit between a high-frequency radio circuit and the associated land-line or, on trunk circuits, where a switched connexion may exceed the permissible number of links in tandem. One repeater is used for each direction of transmission. A regenerative repeater is capable of accepting signals carrying up to 45 per cent distortion and retransmitting them correctly.

An automatic-error-correcting (ARQ) system is normally used on high-frequency radio routes where fading, multipath propagation, noise and interference may cause telegraph signals to be mutilated. At the transmitter, the 5-unit telegraph character is converted to a 7-unit character containing a 3 : 4 ratio of Z-to-A (mark to space) elements, so that the receiver may easily detect a mutilated character. The receive station automatically requests a retransmission of a character in error, and the cycle is repeated until a correct character is received. The ARQ system, thus, consists of equipment at the two ends of the circuit, operating on both directions of transmission. Each character is regenerated and re-transmitted with negligible distortion. The equipment is synchronous and is normally arranged as a 2-channel multiplex system.

(b) For the operation of a regenerative repeater, see A. 1, Telegraphy C, 1965. Supplement, Vol. 59, p. 4, July 1966. For the operation of ARQ equipment, see A. 8, Telegraphy C, 1971. Supplement, Vol. 65, p. 37, July 1972.

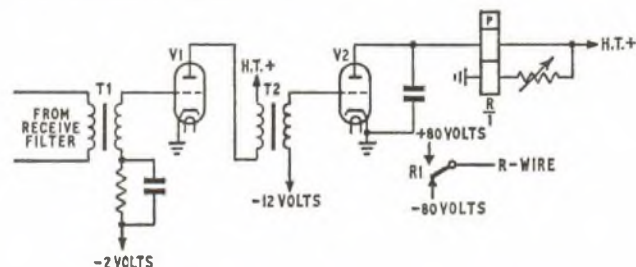
(c) Telex selection signals normally consist of dial pulses at a nominal rate of 10 pulses/s. These signals could not pass through a regenerative repeater without mutilation as they consist of double-current pulses of nominal duration 100 ms and ratio 60 : 40, whereas a teleprinter character has a nominal duration of 150 ms. To prevent mutilation, the repeater is switched out of circuit whilst dial pulses are being transmitted and is only switched into the circuit when a steady negative potential has persisted on one or both transmission paths for 125 ± 25 ms. The repeater is switched out of circuit when the clearing signal (steady positive potential) has been detected on both transmission paths for 300 ± 60 ms.

Since an ARQ system cannot deal with dial pulses, selection signals must consist of keyboard characters. To achieve this, the circuits routed over ARQ systems are normally operator-controlled with the selection signals transmitted in 5-unit code from the operator's keyboard.

Q. 8. (a) Draw a circuit diagram and explain the operation of equipment for converting voice-frequency alternating-current signals to direct-current telegraph signals.

(b) List the tests which would be necessary to ensure that the equipment is functioning correctly.

A. 8. (a) The sketch shows the operation of the receive equipment in converting amplitude-modulated voice-frequency alternating signals to direct-current telegraph signals.



For stop polarity (mark) signals, tone is received from line; for start polarity (space) signals, there is no tone. The signals from the receive filter are applied through the voltage step-up transformer T1 to the amplifier/detector V1. The amplified signals are fed through transformer T2 to valve V2 which has a -12 -volt bias and which acts as a lower anode-bend rectifier. The output current of V2 is connected to one winding of the polarized relay R. The windings of this relay are so arranged that the steady current in the lower, or bias, winding produces a magnetic flux equal

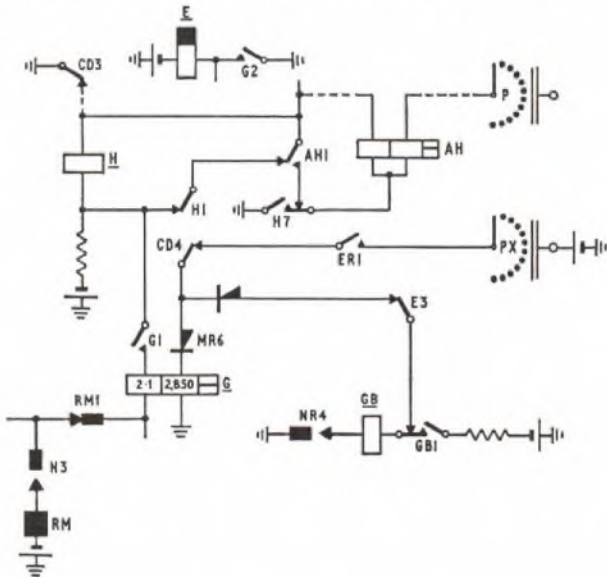
to one half of that produced when current flows in the winding connected to the anode of V2. In this way, equal forces act on the tongue of the relay when V2 conducts for a stop polarity element, or when V2 does not conduct when a start polarity element is received. The relay tongue connects -80 volts and +80 volts to the line for stop and start polarity elements, respectively.

(b) For a list of the tests to ensure that the equipment is functioning correctly, see A. 3, Telegraphy C, 1967. *Supplement*, Vol. 61, p. 35, July 1968.

Q. 9. (a) With the aid of a diagram, describe the operation of a telex final selector when switching to an intermediate line in an auxiliary group of circuits.

- (b) What arrangements are made to inform the calling subscriber
 (i) that all the lines in the auxiliary group are engaged, and
 (ii) that a fault has prevented answer-back signals from being transmitted from the called teleprinter?

A. 9. (a) The circuit arrangement is shown in the sketch.



Lines forming part of an auxiliary group are marked by the connexion of positive potential to the PX-wire of all lines except the last one of the group; this line is marked with negative potential. If the first line is engaged, relay AH does not operate on the release of contact CD3. Relays E and ER are slow to release, and relay G operates to the positive potential on the PX-wire through contacts CD4 released and ER1 operated. Contact G2 maintains relay E which, in turn, maintains relay ER; contact G1 connects earth potential from contacts CD3, AH1, H1 to the rotary magnet through the 2.1-ohm coil of relay G. The operation of the rotary magnet causes the switch to step to the next outlet and the rotary-magnet springs RM1 disconnect the circuit to the rotary-magnet coil. Relay G remains operated as the PX-wiper moves between outlets. If the next outlet corresponds to an engaged line, relay G remains operated to the positive potential on the PX-wire, the rotary magnet again operates to repeat the cycle and the switch steps to the third line in the group.

When a free line is found, relay AH operates and disconnects the rotary magnet circuit at AH1; the switch remains on this line. Contact

AH1 also removes the short-circuit across relay H which operates and switches the circuit to the called subscriber. A contact of relay H also disconnects relay ER which releases relay G at contact ER1.

(b) (i) For the last line of an auxiliary group, the PX-wire is connected to negative potential and if all lines, including the last line, are engaged, relay GB operates and causes the engaged service signal occ to be returned to the calling subscriber. Relay G is a high-speed relay to ensure that contact G1 breaks and disconnects the rotary drive circuit when the rotary springs RM1 re-make on a free outlet or on the last outlet.

(ii) When a disengaged line is seized by the selector, the negative calling signal is connected from the preceding switch to the subscriber's station circuit. If the teleprinter is not being used "in local", negative potential is connected to the send line at the outstation. This call-connected signal operates relay CT in the final selector and a contact of this relay connects the call-connected negative potential to the preceding switches. The time-zone (TZ) equipment recognizes this signal and transmits the who-are-you (WRU) signal to the subscriber's teleprinter. If the subscribers answer-back signals are not transmitted in response within 1.8 s of the end of the WRU signal, the TZ equipment connects the service signal occ to the calling subscriber, and clears the connexion.

Q. 10. (a) Describe the operation of a photo-electric cell in converting light signals to electrical signals in facsimile telegraph equipment.

(b) Draw a diagram to illustrate the location of the cell in the optical system of such equipment.

(c) In such equipment, why is it necessary to employ a carrier frequency when transmitting the electrical signals over a line?

(d) What is the standard frequency of such a carrier and what bandwidth is required for the transmission of the facsimile signals?

A. 10. (a) A photo-electrical cell used for facsimile equipment consists of an evacuated or gas-filled envelope containing an anode and a cathode. The latter is covered with a very thin deposit of caesium-silver which reacts to the presence of light by emitting electrons at a rate proportional to the intensity of the light. The anode, which is at a positive potential with respect to the cathode, attracts the electrons, causing a current to flow in the external anode-cathode circuit. A load resistor in the external circuit is used as a source of voltage which is directly related to the intensity of the light incident upon the cathode. The polarizing voltage is of the order of 150 volts but, as the cell produces a very low current (about 130 mA/lumen), it is necessary to amplify the signal, and the components of the circuit must be screened to prevent interference.

A device frequently used is a multiplier type of vacuum cell housing auxiliary electrodes, called dynodes, which are at progressively higher potentials from the cathode to the anode. As the emitted electrons from the preceding stage strike each dynode, a greater number of electrons are emitted to bombard the succeeding stage. This gives the required amplification within the cell. A 9-stage multiplier cell of this type has a sensitivity of about 10 amps/lumen, with a maximum anode current of 1 mA.

(b) (d) For the location of the cell in the optical equipment and for a list of standard frequencies and bandwidths see A. 9, Telegraphy C, 1971. *Supplement*, Vol. 65, p. 37, July 1972.

(c) The output from the photoelectric cell consists of direct current with a superimposed alternating current, the frequency ranging from zero to a maximum determined by the speed of scanning and the definition required. The normal range of frequency is 0-550 Hz. A telephone line is not suitable for the transmission and amplification of signals at these frequencies, and it is necessary to employ a carrier frequency which is amplitude or frequency modulated by the signals from the photo-electric cell.

MATHEMATICS B, 1972

Students were expected to answer any six questions

Q. 1. (a) Solve for z the simultaneous equations

$$\begin{aligned} 15x - 5y &= -4, \\ 25x + 10(y + z) &= -2, \\ 5y - 20z &= 8. \end{aligned}$$

(b) Find values for p and q which satisfy the equations $3p - 2q = 1$, $(p - q)^3 = 3(p - q)$.

A. 1. (a)

$$\begin{aligned} 15x - 5y &= -4 && \dots\dots (1) \\ 25x + 10(y + z) &= -2 && \dots\dots (2) \\ 5y - 20z &= 8 && \dots\dots (3) \end{aligned}$$

From equation (1),

$$\begin{aligned} 15x &= 5y - 4, \\ \text{or, } x &= \frac{5y - 4}{15}. \end{aligned}$$

Substituting for x in equation (2) gives

$$\frac{5}{3}(5y - 4) + 10y + 10z = -2,$$

or, $25y - 20 + 30y + 30z = -6$, on multiplying by 3.

$$\therefore 55y + 30z = 14 \quad \dots\dots (4)$$

MATHEMATICS B, 1972 (continued)

Multiplying equation (3) by 11 gives

$$55y - 220z = 88 \quad \dots\dots (5)$$

Subtracting equation (5) from equation (4) gives

$$\begin{aligned} 250z &= -74, \\ \text{or, } z &= -\frac{74}{250}, \\ &= -0.296. \end{aligned}$$

Thus, $z = -0.296$.

$$(b) \quad 3p - 2q = 1 \quad \dots\dots (1)$$

$$(p - q)^3 = 3(p - q) \quad \dots\dots (2)$$

From equation (1), $3p = 1 + 2q$,

$$\text{or, } p = \frac{1 + 2q}{3} \quad \dots\dots (3)$$

From equation (2),

$$(p - q)^2 = 3, \quad \text{or, } p - q = 0.$$

Substituting for p from equation (3) gives

$$\left(\frac{1 + 2q}{3} - q\right)^2 = 3, \quad \text{or, } \frac{1 + 2q}{3} = q \quad \dots\dots (4)$$

Taking the first equation in q ,

$$\begin{aligned} \left(\frac{1 - q}{3}\right)^2 &= 3, \\ \text{or, } 1 - 2q + q^2 &= 27, \\ \therefore q^2 - 2q - 26 &= 0. \end{aligned}$$

From the general solution to a quadratic equation,

$$\begin{aligned} q &= \frac{2 \pm \sqrt{(4 + 104)}}{2}, \\ &= \frac{2 \pm 10.39}{2}, \\ &= 6.195 \text{ or, } -4.195. \end{aligned}$$

From the second of the equations in (4),

$$\begin{aligned} 1 + 2q &= 3q, \\ \text{or, } q &= 1. \end{aligned}$$

The values of p corresponding to the three values obtained for q are now found from equation (3).

$$\text{When } q = 1, \quad p = \frac{1 + 2}{3} = 1.$$

When $q = 6.195$,

$$p = \frac{1 + 12.39}{3} = 4.463.$$

When $q = -4.195$,

$$\begin{aligned} p &= \frac{1 - 8.39}{3}, \\ &= -2.463. \end{aligned}$$

Thus, $p = 1, q = 1$,

$$p = 4.463, q = 6.195,$$

$$\text{and, } p = -2.463, q = -4.195.$$

Note.—Each pair of values should be checked in the original equations.

Q. 2. (a) The current capacity of a busbar is 120 amperes per square inch of its cross-section. Express this figure in milliamperes per square millimetre, given that $1 \text{ m} = 39.37 \text{ in}$.

(b) A rectangular sheet of paper is a millimetres long and b millimetres wide. When cut into two equal rectangles each has the same length-to-width proportions as the original sheet. Prove that $a : b = \sqrt{2} : 1$.

A. 2. (a) $1 \text{ m} = 39.37 \text{ in}$.

$$\therefore 1 \text{ in} = \frac{1,000}{39.37} \text{ mm},$$

$$\begin{aligned} \text{and, } 1 \text{ in}^2 &= \left(\frac{1,000}{39.37}\right)^2 \text{ mm}^2, \\ &= \frac{10^6}{39.37^2} \text{ mm}^2. \end{aligned}$$

$$120 \text{ amperes/in}^2 = 120,000 \text{ mA/in}^2,$$

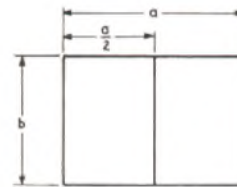
$$= \frac{120,000}{10^6/39.37^2} \text{ mA/mm}^2,$$

$$= \frac{120,000 \times 39.37^2}{10^6} \text{ mA/mm}^2,$$

$$= 0.12 \times 39.37^2 \text{ mA/mm}^2,$$

$$= 186.0 \text{ mA/mm}^2.$$

(b) The sketch shows the rectangle divided into halves. The proportions are such that the longer side, a , must be bisected in order to produce the shorter side of each half-rectangle.



Then, $a : b = b : \frac{a}{2}$,

$$\text{or, } \frac{a}{b} = \frac{2b}{a},$$

$$\therefore a^2 = 2b^2,$$

$$\text{or, } a = \sqrt{2}b.$$

$$\text{i.e. } a : b = \sqrt{2} : 1.$$

Q.E.D.

Q. 3. An airports authority reports the annual number of passengers carried in four successive years as follows

Year	1967	1968	1969	1970
Passengers in millions (x)	14.4	15.6	17.1	18.5

Assuming the growth law $x = ae^{kt}$, where t is measured in years,

(a) plot suitable variables to obtain a straight-line graph to verify this assumption,

(b) from the graph drawn under (a) obtain estimates of the constants a and k ,

(c) hence, show the annual growth rate is approximately 9 per cent.

A. 3. (a) $x = ae^{kt}$.

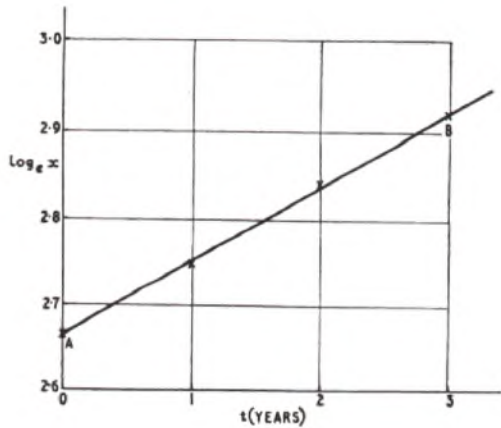
Taking logarithms to base e ,

$$\log_e x = \log_e a + kt.$$

Since a and k are constants, the logarithmic form of the growth law is of the linear form $y = ax + b$, the variables being $\log_e x$ and t .

Taking 1967 as the base year, i.e. $t = 0$, the graph of $\log_e x$ against t may be plotted from the following table of values.

Year	1967	1968	1969	1970
t	0	1	2	3
x	14.4	15.6	17.1	18.5
$\log_e x$	2.667	2.747	2.839	2.918



The graph is shown in the sketch, from which it is seen that a straight line has been drawn to pass through, or very close to, all the plotted points.

Thus, the data given are in accordance with the growth law $x = ae^{kt}$.

(b) When $t = 0$, $\log_e x = \log_e a$, or, $x = a$.

Hence, the value of a may be found from the graph by reading off the value of $\log_e a$ when $t = 0$.

From the graph, at $t = 0$,

$$\log_e a = 2.667.$$

$$\therefore a = 14.4.$$

k is the gradient of the straight line and may be obtained from two widely-separated points, A and B , on the graph. Since the graph passes almost exactly through A and B , the appropriate values of $\log_e x$ may be read accurately from the table of values.

$$\therefore k = \frac{\log_e x \text{ at } B - \log_e x \text{ at } A}{3 - 0},$$

$$= \frac{2.918 - 2.667}{3},$$

$$= \frac{0.251}{3},$$

$$= 0.0836.$$

Thus, $a = 14.4$, and $k = 0.0837$.

(c) If x_1 and x_2 are the numbers of passengers, in millions, after times t_1 and t_2 years respectively, the percentage growth rate is given by

$$\begin{aligned} 100 \times \frac{x_2 - x_1}{x_1} \times \frac{1}{t_2 - t_1} &= 100 \left(\frac{x_2}{x_1} - 1 \right) \times \frac{1}{t_2 - t_1}, \\ &= 100 \left(\frac{ae^{kt_2}}{ae^{kt_1}} - 1 \right) \times \frac{1}{t_2 - t_1}, \\ &= 100(e^{k(t_2 - t_1)} - 1) \times \frac{1}{t_2 - t_1}. \end{aligned}$$

For an annual growth rate, $(t_2 - t_1) = 1$.

$$\begin{aligned} \therefore \text{Percentage annual growth rate} &= 100(e^k - 1), \\ &= 100(e^{0.0837} - 1), \\ &= 100(1.087 - 1), \\ &= 8.7. \end{aligned}$$

Thus, the annual growth rate is approximately 9 per cent.

Q. 4. (a) The first two terms of a series are 3.6 and 2.4. Calculate to three significant figures the sum of 10 terms of the series if it

- (i) an arithmetical progression,
- (ii) a geometrical progression.

(b) The amplitude of the first oscillation of a pendulum is 12° . Assuming each succeeding oscillation has 0.94 times the amplitude of the preceding one, after how many oscillations will the amplitude first be less than 2° ?

A. 4. (a) (i) If the series is an arithmetical progression, it will take the form

$$3.6, 2.4, 1.2 \dots$$

since the common difference will be -1.2 , the difference between the first two terms given.

The sum of n terms of an arithmetical progression is

$$S_n = \frac{1}{2}n(2a + (n-1)d),$$

where a is the first term and d the common difference.

$$\begin{aligned} \therefore S_{10} &= \frac{1}{2} \times 10(2 \times 3.6 + 9 \times (-1.2)), \\ &= 5(7.2 - 10.8), \\ &= 5 \times (-3.6), \\ &= \underline{-18}. \end{aligned}$$

(ii) If the series is a geometrical progression, the ratio r will be $\frac{2.4}{3.6} = \frac{2}{3}$ and the series becomes

$$3.6, 2.4, 1.6 \dots$$

The sum of n terms is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ where } a \text{ is the first term.}$$

$$\begin{aligned} \therefore S_{10} &= \frac{3.6 \left\{ \left(\frac{2}{3} \right)^{10} - 1 \right\}}{\frac{2}{3} - 1}, \\ &= \frac{3.6(0.01734 - 1)}{-\frac{1}{3}}, \\ &= (-10.8) \times (-0.98266), \\ &= 10.61, \\ &= \underline{10.6} \text{ to three significant figures.} \end{aligned}$$

(b) Since the amplitude of each oscillation is 0.94 times that of the preceding oscillation, the amplitudes will form a geometrical progression whose terms are

$$12, 12 \times 0.94, 12 \times 0.94^2 \dots$$

If a denotes the first term and r the ratio 0.94, the n th term will be,

$$ar^{n-1}.$$

Equating this expression to 2° ,

$$2 = 12 \times 0.94^{n-1},$$

$$\text{or, } 0.94^{n-1} = \frac{1}{6}.$$

$$\therefore (n-1) \log_{10} 0.94 = \log_{10} \left(\frac{1}{6} \right)$$

$$\text{or, } n-1 = \frac{-\log_{10} 6}{\log_{10} 0.94},$$

$$= \frac{-0.7782}{-0.0269},$$

$$= \frac{0.7782}{0.0269},$$

$$= 28.93.$$

$$\therefore n = 29.93.$$

Thus, the amplitude will first be less than 2° after 30 oscillations.

Q. 5. (a) (i) If $\log_2 N = 3$, what is N ?

(ii) If N, a, b are all positive, show that $\log_a N = \log_b N \times \log_a b$.

(b) If V_1 volts are applied to an amplifier the output voltage is V_2 ; the voltage gain of the amplifier may be defined either as $\log_e \left(\frac{V_2}{V_1} \right)$ nepers or as $20 \log_{10} \left(\frac{V_2}{V_1} \right)$ decibels. Using these definitions, convert into nepers a voltage gain of 24 dB.

A. 5. (a) (i) $\log_2 N = 3$.

$$\therefore N = 2^3 = 8 \text{ by definition.}$$

(ii) Let $\log_b N = p$.

$$\therefore N = b^p \text{ by definition.}$$

$$\therefore \log_a N = \log_a (b^p),$$

$$= p \times \log_a b.$$

$$\text{But, } p = \log_b N.$$

$$\therefore \log_a N = \log_b N \times \log_a b.$$

Q.E.D. (c)

Now admittance, $Y = \frac{1}{Z}$, where Z is the impedance.

$$\therefore Y = \frac{I}{V},$$

$$= \frac{(18 + j7) \times 10^{-3}}{240} \text{ siemens,}$$

$$= (0.075 + j0.02917) \times 10^{-3} \text{ siemens.}$$

Thus, admittance = $(75 + j29.17) \times 10^{-6}$ siemens.

$$Z = \frac{1}{Y},$$

$$= \frac{240}{(18 + j7) \times 10^{-3}} \text{ ohms,}$$

$$= \frac{10^3 \times 240(18 - j7)}{18^2 + 7^2} \text{ ohms,}$$

$$= 10^3 \times \frac{240}{373}(18 - j7) \text{ ohms,}$$

$$= (11.58 - j4.504) \text{ kohms.}$$

The phase relationship between the applied voltage and the total current is that denoted as angle ROV in the sketch.

$$\tan \angle ROV = \frac{7}{18},$$

$$= 0.3889.$$

$$\therefore \angle ROV = 21^\circ 15'.$$

Hence, the current leads the voltage by $21^\circ 15'$.

Note: This angle is the same, in magnitude, as that of the impedance or admittance.

(b) For a gain of 24 decibels (dB), the voltage ratio is given by

$$24 = 20 \log_{10} \left(\frac{V_2}{V_1} \right),$$

$$\text{or, } \log_{10} \left(\frac{V_2}{V_1} \right) = 1.2.$$

For the same voltage ratio, the number of nepers

$$= \log_e \left(\frac{V_2}{V_1} \right),$$

$$= \log_{10} \left(\frac{V_2}{V_1} \right) \times \log_e 10 \text{ from part (i) of the question,}$$

$$= 1.2 \times 2.3026,$$

$$= 2.763 \text{ to 3 decimal places.}$$

Thus, a voltage gain of 24 dB is equivalent to 2.763 nepers.

Note: It may easily be deduced that the multiplying factor to convert from decibels to nepers is $\frac{2.3026}{20} = 0.1151$.

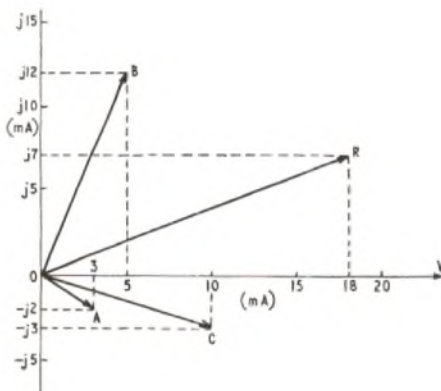
Q. 6. The currents, in milliamperes, in three parallel branches of a network at a certain frequency may be expressed in phasor form as $3 - j2$, $5 + j12$ and $10 - j3$ mA, referred to the common applied voltage of 240 volts.

(a) Display these phasors on a complex plane diagram and show on the same diagram the total current in the form $a + jb$.

(b) Calculate, in phasor form $a + jb$, the admittance (the current/voltage ratio) of the network at this frequency.

(c) Derive the impedance (the reciprocal of the admittance) as a phasor, and state in words the phase relationship between the voltage and the total current.

A. 6. (a) The three current phasors are shown in the sketch as OA, OB and OC representing $3 - j2$, $5 + j12$ and $10 - j3$ mA, respectively.



Since the currents are in three parallel branches, the total current, i_t , will be obtained as the phasor sum of the individual currents.

$$\therefore i_t = 3 - j2 + 5 + j12 + 10 - j3 \text{ mA,}$$

$$= 18 + j7 \text{ mA.}$$

This is shown in the sketch as OR.

(b) The common applied voltage, V , may be expressed in phasor form as $240 + j0$ and has been included in the sketch.

Q. 7. (a) Using the $\cos(A + B)$ formula, show that

$$(i) \sqrt{2 + 2 \cos \theta} = 2 \cos \frac{\theta}{2},$$

$$(ii) \frac{1 + \tan^2 A}{1 - \tan^2 A} = \sec 2A.$$

(b) Evaluate $\sin 105^\circ$ without using trigonometrical tables.

(c) Use trigonometrical tables to calculate the remaining angles and side of the triangle PQR, given $QR = 17.2$ mm, angle $PQR = 123^\circ$, and $PQ = 9.6$ mm.

A. 7. (a) (i) $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

$$\text{If } A = B = \frac{\theta}{2},$$

$$\cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}.$$

$$\text{But, } \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1,$$

$$\text{or, } \sin^2 \frac{\theta}{2} = 1 - \cos^2 \frac{\theta}{2}.$$

Hence, substituting for $\sin^2 \frac{\theta}{2}$ in the expansion for $\cos \theta$ gives

$$\cos \theta = \cos^2 \frac{\theta}{2} - \left(1 - \cos^2 \frac{\theta}{2} \right),$$

$$= 2 \cos^2 \frac{\theta}{2} - 1,$$

$$\text{or, } 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}.$$

$$\text{Now, } \sqrt{2 + 2 \cos \theta} = \sqrt{(2)(1 + \cos \theta)},$$

$$= \sqrt{(2)} \sqrt{\left(2 \cos^2 \frac{\theta}{2} \right)} \text{ from above,}$$

$$= 2 \cos \frac{\theta}{2}.$$

Q.E.D.

$$(ii) \quad \frac{1 + \tan^2 A}{1 - \tan^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 - \frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}$$

But, $\cos^2 A + \sin^2 A = 1$.

and, from the expansion for $\cos(A + B)$, if $A = B$, then,
 $\cos 2A = \cos^2 A - \sin^2 A$.

$$\text{Hence, } \frac{1 + \tan^2 A}{1 - \tan^2 A} = \frac{1}{\cos 2A},$$

$$= \sec 2A.$$

(b) $\sin 105^\circ = \sin(60^\circ + 45^\circ)$.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$.

Hence, from this expansion,

$$\sin 105^\circ = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ,$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}},$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}},$$

$$= \frac{2.732}{2.828},$$

$$= 0.9663.$$

Note: From a 4-figure table of sines, $\sin 75^\circ = 0.9659$ which shows appreciable divergence from the answer obtained. This is because of the approximations inherent in 4-figure tables.

(c) The obtuse-angled triangle PQR is shown in the sketch.



As two sides and the included angle constitute the given data, it is necessary to use the cosine rule, which may be stated as

$$c^2 = a^2 + b^2 - 2ab \cos C,$$

where a, b and c are the sides of the triangle and C is the angle opposite side c .

Applying the rule to triangle PQR gives

$$PR^2 = QR^2 + PQ^2 - 2 \times QR \times PQ \times \cos \angle PQR,$$

$$= 17.2^2 + 9.6^2 - 2 \times 17.2 \times 9.6 \times \cos 123^\circ,$$

$$= 295.8 + 92.16 - 330.24 \times (-\cos 57^\circ),$$

$$= 387.96 + 330.24 \times 0.5446,$$

$$= 567.9.$$

$\therefore PR = 23.83 \text{ mm}.$

Using the sine rule,

$$\frac{\sin \angle QPR}{17.2} = \frac{\sin 123^\circ}{PR},$$

or, $\sin \angle QPR = \frac{17.2 \times 0.8387}{23.83},$

$$= 0.6052.$$

$\therefore \angle QPR = 37^\circ 15'.$

Hence, $\angle PRQ = 180^\circ - (123^\circ + 37^\circ 15'),$

$$= 19^\circ 45'.$$

Thus, the remaining angles are $19^\circ 45'$ and $37^\circ 15'$, and the third side is $23.83 \text{ mm}.$

Q. 8. (a) Sketch the graph of the current

$$i = 30 \sin \left(\omega t + \frac{\pi}{3} \right) \text{ milliamperes,}$$

from $t = 0$ to $t = \frac{2\pi}{\omega}$ seconds.

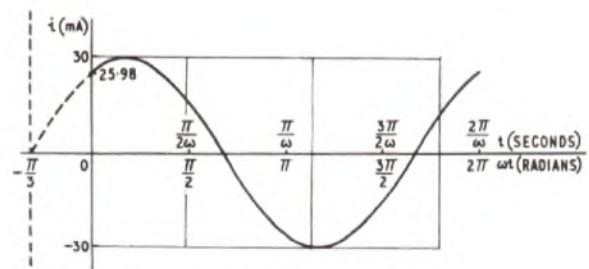
(b) Express i in the form $i_1 \sin \omega t + i_2 \cos \omega t$.

(c) If $\omega = 10^3 \text{ rad/s}$, find in milliseconds the first instant after $t = 0$ when the current is -15 mA .

(d) If the same current flows through a 4 mH inductor of negligible resistance, calculate the voltage $\left(L \frac{di}{dt} \right)$ across this inductor at $t = 0$.

A. 8. (a) $i = 30 \sin \left(\omega t + \frac{\pi}{3} \right) \text{ mA.}$

The time period from 0 to $\frac{2\pi}{\omega}$ seconds is equivalent to an angular period of 2π radians, during which one complete cycle of the current waveform is formed. When $t = 0, i = 30 \sin \frac{\pi}{3} = 25.98 \text{ mA}$, and hence, the wave commences at this value, reaches a maximum of 30 mA and subsequently a minimum of -30 mA as shown in the sketch. The effect of the introduction of the constant $\frac{\pi}{3}$ in the angle



is simply to displace the origin by the same amount, as shown by the dotted axis.

(b) $i = 30 \sin \left(\omega t + \frac{\pi}{3} \right),$

$$= 30 \left\{ \sin \omega t \cos \frac{\pi}{3} + \cos \omega t \sin \frac{\pi}{3} \right\},$$

$$= 30 \left\{ \frac{1}{2} \sin \omega t + 0.866 \cos \omega t \right\},$$

$$\therefore i = 15 \sin \omega t + 25.98 \cos \omega t.$$

i.e. $i_1 = 15$ and $i_2 = 25.98$.

(c) If $\omega = 10^3 \text{ rad/s}$ and $i = -15 \text{ mA}$, then,

$$-15 = 30 \sin \left(10^3 t + \frac{\pi}{3} \right),$$

or, $\sin \left(10^3 t + \frac{\pi}{3} \right) = -\frac{1}{2},$

$$\therefore 10^3 t + \frac{\pi}{3} = \pi + \frac{\pi}{6} \text{ or } 2\pi - \frac{\pi}{6},$$

or, $10^3 t = \left(\frac{6 + 1 - 2}{6} \right) \pi$ for the first instant,

$$= \frac{5\pi}{6},$$

$$\therefore t = \frac{5\pi}{6} \text{ ms,}$$

$$= 2.618 \text{ ms.}$$

Thus, the current is first equal to -15 mA after $2.618 \text{ ms}.$

Note: The angle $\frac{5\pi}{6}$ rad at which the instantaneous current becomes -15 mA can be seen from the sketch.

(d) $i = 30 \sin \left(\omega t + \frac{\pi}{3} \right) \text{ mA,}$

$$= 0.03 \sin \left(\omega t + \frac{\pi}{3} \right) \text{ amps.}$$

$$\therefore \frac{di}{dt} = \omega \times 0.03 \cos\left(\omega t + \frac{\pi}{3}\right).$$

If $t = 0$ and $\omega = 10^3$,

$$\begin{aligned} \frac{di}{dt} &= 10^3 \times 0.03 \cos \frac{\pi}{3}, \\ &= 30 \times \frac{1}{2} = 15. \end{aligned}$$

Voltage across 4 mH inductor at $t = 0$,

$$\begin{aligned} &= L \frac{di}{dt}, \\ &= 4 \times 10^{-3} \times 15 \text{ volt}, \\ &= \underline{0.06 \text{ volt}}. \end{aligned}$$

Q. 9. (a) The rate of working of an electric motor is given by the formula $P = IV - I^2r$ where V and r are constants. Obtain $\frac{dP}{dI}$ from first principles, and hence, determine the value of I for which $\frac{dP}{dI}$ is zero.

(b) Find the slope of the tangent to the curve $y = x^2 + \frac{16}{x}$ at the point $(4, 20)$, and find the minimum value of the function $\left(x^2 + \frac{16}{x}\right)$.

(c) Sketch the curve $y = x^2 + \frac{16}{x}$.

A. 9. (a) $P = IV - I^2r$.

Let I increase by a small amount δI and suppose that P changes by a corresponding amount δP .

$$\begin{aligned} \text{Then, } P + \delta P &= V(I + \delta I) - r(I + \delta I)^2, \\ \text{or, } \delta P &= V(I + \delta I) - r(I + \delta I)^2 - (IV - I^2r), \\ &= VI + V\delta I - IV - r\{I^2 + 2I\delta I + \delta I^2 - I^2\}, \\ &= V\delta I - r(2I\delta I + \delta I^2). \end{aligned}$$

$$\therefore \frac{\delta P}{\delta I} = V - 2Ir - r\delta I.$$

In the limit, as $\delta I \rightarrow 0$, $\frac{\delta P}{\delta I} \rightarrow \frac{dP}{dI}$.

$$\therefore \frac{dP}{dI} = \underline{V - 2Ir}.$$

When $\frac{dP}{dI} = 0$,

$$V = 2Ir,$$

$$\text{or, } I = \frac{V}{2r}.$$

(b) $y = x^2 + \frac{16}{x}$.

$$\therefore \frac{dy}{dx} = 2x - \frac{16}{x^2}.$$

When $x = 4$,

$$\begin{aligned} \frac{dy}{dx} &= 2 \times 4 - \frac{16}{4^2}, \\ &= 7. \end{aligned}$$

Hence, the slope of the tangent to the curve at the point $(4, 20)$ is $\underline{7}$.

At the minimum value, the slope of the tangent to the curve is zero,

i.e. $\frac{dy}{dx} = 0$.

$$\therefore 2x - \frac{16}{x^2} = 0,$$

$$\text{or, } 2x = \frac{16}{x^2}.$$

$$\therefore x^3 = 8.$$

$$\therefore x = 2.$$

$$\begin{aligned} \text{Hence, } y &= 2^2 + \frac{16}{2}, \\ &= 4 + 8, \\ &= 12. \end{aligned}$$

Thus, the minimum value of the function is 12.

$$(c) \quad y = x^2 + \frac{16}{x}.$$

As $x \rightarrow 0$, the first term becomes very small and the second very large. The value of y , therefore, approaches $-\infty$ as x increases from a small negative value to zero, and approaches $+\infty$ as x decreases from a small positive value to zero.

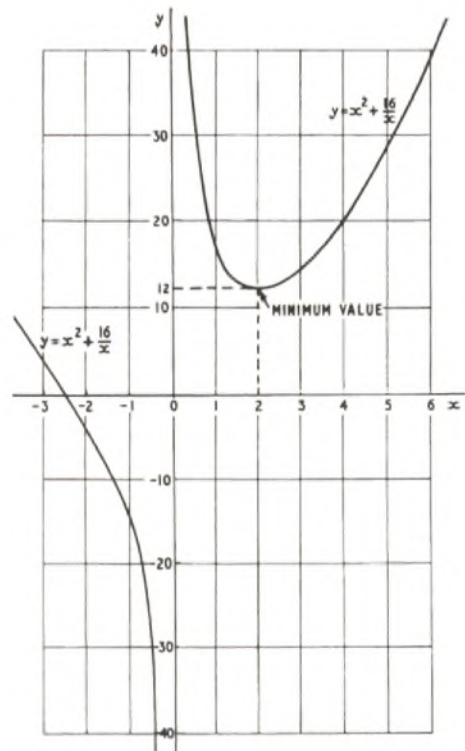
As x goes increasingly negative, the positive x^2 term predominates over the smaller $\frac{16}{x}$ term and when x is fairly large, $y \approx x^2$. The curve will cross the x -axis when $y = 0$, i.e.

$$x^2 = -\frac{16}{x},$$

$$\text{or } x^3 = -16.$$

$$\therefore x = -\sqrt[3]{16} = -2.52.$$

As x increases from zero, the value of the function first decreases to its minimum of 12 at $x = 2$ and, thereafter, increases until it again approximates to x^2 at large values of x .



The graph, therefore, assumes the shape shown in the sketch, the y -axis being an asymptote to both parts of the curve.

Q. 10. (a) If $\frac{dy}{dx} = 6x^2 - 7$, and the graph of y passes through the point $(2, 5)$, derive an equation for y in terms of x .

(b) Evaluate the integral $\int_2^5 (x - 1)^2 dx$ and deduce the mean value of the function $(x - 1)^2$ over the range $x = 2$ to $x = 5$.

A. 10. (a) $\frac{dy}{dx} = 6x^2 - 7$.

$$y = \int \frac{dy}{dx} \cdot dx,$$

$$= \int (6x^2 - 7) dx,$$

$$= \frac{6x^3}{3} - 7x + c,$$

where c is a constant.

Since the graph passes through the point (2, 5),

$$5 = 2 \times 2^3 - 7 \times 2 + c,$$

or, $c = 5 - 16 + 14,$
 $= 3.$

Hence, the equation for y is

$$y = 2x^3 - 7x + 3.$$

$$(b) \int_2^5 (x-1)^2 dx = \int_2^5 (x^2 - 2x + 1) dx,$$

$$= \left[\frac{x^3}{3} - x^2 + x \right]_2^5.$$

$$= \frac{5^3}{3} - 5^2 + 5 - \left(\frac{2^3}{3} - 2^2 + 2 \right),$$

$$= \frac{125}{3} - \frac{8}{3} - 25 + 5 + 4 - 2,$$

$$= 39 - 18,$$

$$= 21.$$

The mean value of y between $x = 2$ and $x = 5$ is given by

$$\text{mean value} = \frac{\int_2^5 (x-1)^2 dx}{5-2},$$

$$= \frac{21}{3},$$

$$= 7.$$

COMPUTERS B, 1972

Students were expected to answer any six questions

Q. 1. Convert the following numbers into binary, showing all working.

- (a) 1,376.36 (denary),
- (b) 477361 (octal)
- (c) A3EB8F9 (hexadecimal - radix 16).
- (d) 96531 (binary coded decimal weighted 8421).

A. 1. (a) To convert the denary (decimal) number 1,376.36 into binary, it is advisable to consider the integer and fractional parts separately.

Quotient	Remainder	Integer	Fraction
2) 1,376			0.36 × 2
688	0	0	0.72
344	0	1	0.44
172	0	0	0.88
86	0	1	0.76
43	0	1	0.52
21	1	1	0.04
10	1	0	0.08
5	0	0	0.16
2	1	0	0.32
1	0	0	0.64
0	1	1	0.28

∴ 1,376.36₁₀ = 10 101 100 000.010 111₂ correct to six binary places.

Table 1

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

$$\therefore A3EB8F9_{16} = 1010\ 0011\ 1110\ 1011\ 1000\ 1111\ 1001_2$$

$$= 1\ 010\ 001\ 111\ 101\ 011\ 100\ 011\ 111\ 001_2.$$

(b) To convert 477361 (octal) into a binary number, it is necessary to convert each octal digit into a three-bit binary representation.

$$4\ 7\ 7\ 3\ 6\ 1$$

$$100\ 111\ 111\ 011\ 110\ 001$$

$$\therefore 477361_8 = 100\ 111\ 111\ 011\ 110\ 001_2.$$

(c) To convert A3EB8F9 (hexadecimal) into a binary number it is necessary to convert each hexadecimal digit into a four-bit binary representation.

The conversion table is shown in Table 1.

(d) In the binary-coded-decimal system, four bits are required to form each decimal digit. The bits are weighted such that the least significant bit has the decimal value 1, the second significant bit the value 2, the third bit the value 4, and the most significant bit the value 8. Therefore, the decimal numbers 0 to 9 are represented as shown in Table 2.

Table 2

decimal digit	binary coded decimal (b.c.d.)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

$$\therefore 96531_{\text{b.c.d.}} = 10\ 010\ 110\ 010\ 100\ 110\ 001_2.$$

Q. 2. (a) If $f_1 = (\bar{A}.B \vee C) \vee (A \vee \bar{B}.C)$, write down the simplified expression for \bar{f}_1 .

(b) (i) Minimize the following Boolean expression

$$f_2 = B.\bar{C} \vee \bar{A}.B.C \vee A.\bar{B}.\bar{C} \vee A.C.$$

(ii) Show how this minimized expression could be represented using NOR logic elements only.

[Key: the sign $\bar{}$ means logic NOT; \cdot means logic AND; \vee means logic OR.]

A. 2. (a) The simplified expression for the complement of f_1 can be obtained as follows:

$$f_1 = (\bar{A}.B \vee C) \vee (A \vee \bar{B}.C).$$

$$\therefore \bar{f}_1 = \overline{(\bar{A}.B \vee C) \vee (A \vee \bar{B}.C)},$$

$$= \overline{(\bar{A}.B \vee C)} \cdot \overline{(A \vee \bar{B}.C)} \text{ using De Morgan's theorem,}$$

$$\begin{aligned}
 &= ((\overline{A \cdot B}) \cdot \overline{C}) \cdot (\overline{A} \cdot (\overline{B \cdot C})), \\
 &= (A \vee \overline{B}) \cdot \overline{C} \cdot \overline{A} \cdot (B \vee \overline{C}), \\
 &= \overline{C} \cdot \overline{A} \cdot (A \cdot B \vee A \cdot \overline{C} \vee \overline{B} \cdot B \vee \overline{B} \cdot \overline{C}), \\
 &= \overline{C} \cdot \overline{A} \cdot (A \cdot B \vee A \cdot \overline{C} \vee \overline{B} \cdot \overline{C}) \text{ as } \overline{B} \cdot B = 0, \\
 &= \overline{C} \cdot \overline{A} \cdot A \cdot B \vee \overline{C} \cdot \overline{A} \cdot A \cdot \overline{C} \vee \overline{C} \cdot \overline{A} \cdot \overline{B} \cdot \overline{C}, \\
 &= \overline{A} \cdot \overline{B} \cdot \overline{C}, \text{ as } \overline{A} \cdot A = 0 \text{ and } \overline{C} \cdot \overline{C} = \overline{C}.
 \end{aligned}$$

(b) The Boolean expression f_2 can be simplified as follows:

$$(i) f_2 = B \cdot \overline{C} \vee \overline{A} \cdot B \cdot C \vee A \cdot \overline{B} \cdot \overline{C} \vee A \cdot C.$$

Using a Karnaugh map (sketch (a))



(a)

Therefore, $f_2 = A \vee B$.

(ii) The logical expression $A \vee B$ can be written in Boolean algebra as $A + B$. This expression can be represented using NOR logic elements as shown in sketch (b) using inputs A and B .

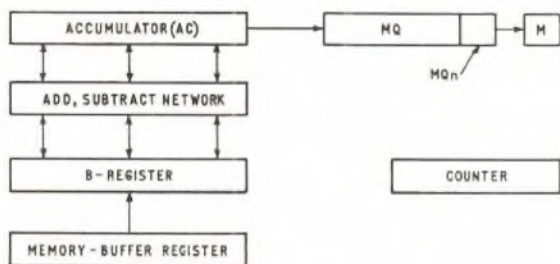


(b)

Q. 3. (a) With the aid of a block diagram, explain the operation of the section of a digital computer which will perform multiplication by hardware.

(b) Verify the operation of the block diagram you have drawn for (a) by illustrating how the unit would perform the following calculation $(10\ 010)_2 \times (00\ 101)_2$.

A. 3. (a) Multiplication by hardware can be performed by a digital computer using an algorithm invented by A. D. Booth. The method is widely used and affords a remarkably simple and direct way to multiply two binary numbers which are presented in 2's complement form. The binary numbers are multiplied directly, complete with sign digits, and the product is stored in 2's complement form. With reference to the sketch, the multiplier quotient register (MQ), has the



necessary logical circuitry such that the combined accumulator (AC) contents and the contents of MQ can be shifted right, the right-most bit of AC being shifted into the left-most bit of MQ.

The multiplier operates as follows. The contents of AC, which will be the multiplier, are transferred into MQ leaving zero in AC. The multiplicand is read from the memory into the memory-buffer register. The multiplicand is transferred into the B register, and, at the same time, the step counter is set to a binary number equal to the number of digits in the computer word. To the right of MQ is a flip-flop, designated M, into which the right-most bit of MQ is shifted when the contents of AC and MQ are shifted right. To begin with, M is set to zero. At each step of the algorithm, the right-most bit of MQ (MQ_n) is compared with the contents of M (M) and one of the following instructions is carried out until the counter is zero.

(i) If $MQ_n = 0$, and $M = 0$ or, if $MQ_n = 1$ and $M = 1$, then shift the contents of AC and MQ right one digit and decrement counter by 1.

(ii) If $MQ_n = 1$ and $M = 0$, subtract contents of B register from contents of AC then shift the contents of AC and MQ right and decrement counter by 1.

(iii) If $MQ_n = 0$ and $M = 1$, add contents of B register to contents of AC then shift the contents of AC and MQ right and decrement counter by 1.

(b) The multiplier could perform the following calculation as shown below:

$$(10\ 010)_2 \times (00\ 101)_2,$$

becomes

$$(010\ 010)_2 \times (000\ 101)_2$$

with the addition of sign bits, assuming both numbers are positive. Therefore, 000 101 is stored in the B register.

AC	MQ	M	Comment	Count
000000	010010	0	Start	6
000000	001001	0	Shift right	5
111011	001001	0	Subtract B from AC	5
111101	100100	1	Shift right	4
000010	100100	1	Add B to AC	4
000001	010010	0	Shift right	3
000000	101001	0	Shift right	2
111011	101001	0	Subtract B from AC	2
111101	110100	1	Shift right	1
000010	110100	1	Add B to AC	1
000001	011010	0	Shift right	0

Note: When shifting right, the most significant bit, i.e. sign bit, does not change, but is shifted into the vacated position.

The result, read as a double word from the AC and MQ registers, is 000 001 011 010₂. The sign of the product is given by the sign bit of AC.

Q. 4. (a) What is (i) a differentiating circuit, (ii) an integrating circuit?

(b) The pulse in Fig. 2 is applied to input A of the circuit shown in

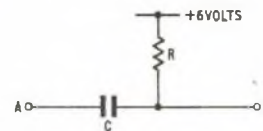


Fig. 1

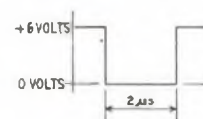


Fig. 2

Fig. 1. For each of the following draw a graph of the resulting waveforms at point B. (All three graphs should be to the same scale).

- (i) $C = 0.001\ \mu F, R = 250\ \text{ohms}$.
- (ii) $C = 0.001\ \mu F, R = 500\ \text{ohms}$.
- (iii) $C = 0.001\ \mu F, R = 1\ \text{k ohm}$.

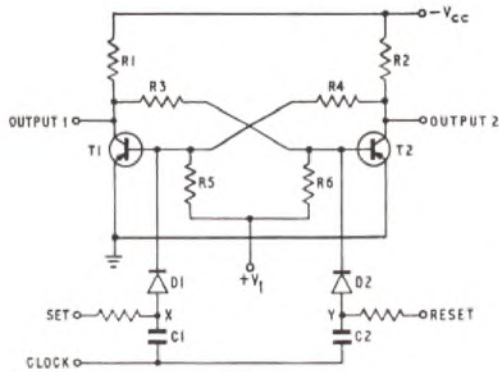
Q. 5. (a) With the aid of a circuit diagram, explain the operation of a bistable device which has gated inputs for typical use in a register.

(b) Explain briefly the meaning of the terms master and slave when applied to a clocked bistable device.

A. 5. (a) The common "flip-flop" circuit is used as the basis of a gated bistable device used in binary registers (see sketch (a)). The circuit can be in either of the two stable states representing binary 1 or 0.

Consider the case where transistor T1 is ON. The collector voltage is equal to approximately 0 volts. Transistor T2 is held in the OFF state by the positive voltage applied to its base produced by the action of the potential divider formed by resistors R3 and R6 and voltage $+V_1$. The collector voltage of transistor T2 is, therefore, approximately equal to $-V_{cc}$. Suitable values of resistors R4 and R5 are chosen to produce a negative bias on the base of transistor T1 to hold it ON. The circuit is, therefore, in a stable state.

If a positive pulse of sufficient amplitude and duration is applied to the base of the transistor which is in the ON state, it is turned OFF and

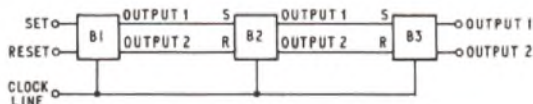


(a)

the circuit is switched to the other stable state i.e. transistor T1 OFF and transistor T2 ON.

The Set and Reset inputs are either $-V_{cc}$ or 0 volts, obtained from the outputs (output 1 and output 2) of the previous stage in the register. The clock line is normally held at the negative voltage $-V_{cc}$. If the Set input is 0 volts, capacitor C is charged to the potential of the clock line i.e. $-V_{cc}$, whereas the Reset input is set as $-V_{cc}$ volts and capacitor C2 is not charged. When the clock line is set to 0 volts, a positive pulse is applied at point X and diode D1 conducts. Transistor T1 is turned OFF if it was in the ON condition and output 2 changes from 0 volts to $-V_{cc}$ volts. The potential at point Y, does not change significantly and diode D2 does not conduct. The state of the circuit, therefore, remains unaltered if the Set and Reset voltages are the same as output 1 and output 2 respectively, otherwise there is a change of state and the outputs become equal to the inputs.

(b) Shift registers are created by connecting the outputs of a previous bistable circuit to the input of the next bistable circuit as shown in sketch (b).

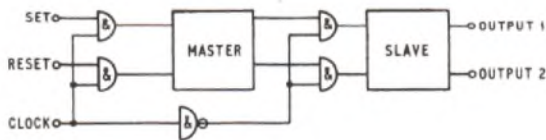


(b)

However, if the clock pulse is long, the Set and Reset voltages may alter during this period and the outputs of each stage will not depend upon the original inputs to the stage. In order to prevent this occurring, master-slave bistable circuits are used.

The master-slave bistable circuit consists of two bistable circuits as shown in sketch (c). When the clock voltage goes positive, the state of the master bistable circuit is adjusted. The state of all slave bistable circuits in the register are unaffected during this period and all outputs are steady.

At the end of the clock pulse, each slave bistable circuit is adjusted by its master.



(c)

Q. 6. (a) Draw a block diagram of a typical modern digital computer, showing the main registers and their interconnections.

(b) Use the diagram you have drawn for (a) to explain the sequence of operations and flow of information for a jump (branch) instruction.

A. 6. (a) A block diagram of a typical modern digital computer is shown in the sketch. The main registers are described below.

(i) Accumulator (AC)

The accumulator is associated with the arithmetic unit and consists of a special register which retains the result of arithmetic, or logical operations.

(ii) B-Register (B)

This register is used to store the word which is to amend the contents of the accumulator in accordance with the instruction being performed e.g. add, subtract.

(iii) Instruction Counter (IC)

This counter holds the address of the next instruction to be executed in the program. Normally, during each instruction cycle, the counter is incremented by one.

(iv) Operation-Code Register (OC)

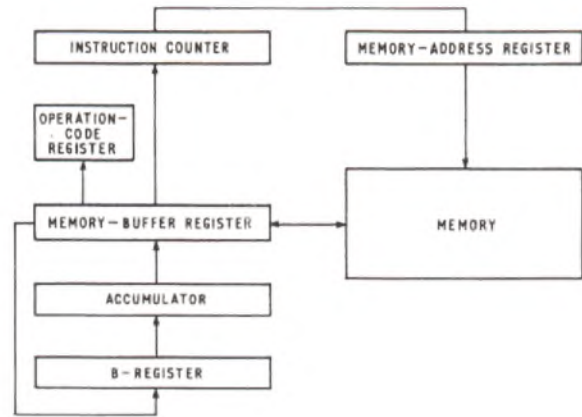
When an instruction word is read from memory, the operation-code section of this word is stored in order to determine which operation is to be performed.

(v) Memory-Address Register (MA)

This register contains the address of a word which is to be read from, or written to, the memory.

(vi) Memory-Buffer Register (MB)

This register is used to store, temporarily, the information read from, or written to, the memory.



(b) The basic pattern or sequence of operations for most instructions in a digital machine of the single-address type consists of an alternation of a time period called the instruction cycle, followed by a time period called the execution cycle. At the beginning of the instruction cycle, the contents of the IC are transferred into the MA register. The memory then uses the address in the MA register to obtain the next instruction, which is subsequently placed into the MB register. The operation code part of the instruction is transferred to the OC register and the address part is transferred to the MA register. If the instruction is an unconditional jump instruction, then the contents of the MA register are transferred to the IC. Therefore, the instruction performed in the next cycle will be the one indicated by the address part of the jump instruction.

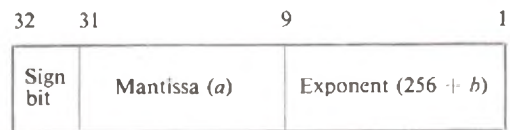
Q. 7. (a) Explain the meaning of the term "normalization" ("standardization") and say why it is used in most modern digital computers.

(b) Show how the following numbers can be represented in binary on a digital computer which has a 32-bit word length and is capable of floating-point representation.

- (i) $+\frac{7}{8}$,
- (ii) $+22\frac{1}{10}$,
- (iii) $-3\frac{1}{2}$.

A. 7. (a) Normalization is used in modern digital computers to maintain the accuracy of floating point numbers during arithmetic operations. Following any operation involving floating point numbers, the binary representation is adjusted to hold the maximum number of significant bits before continuing with further operations.

(b) Numbers can be represented in floating-point form using the format below.



Floating-point numbers are of the form $A = a \times 2^b$, where a is the mantissa

$0.5 \leq a < 1$ for positive numbers, and

$-1 < a < -0.5$ for negative numbers,

and b is the exponent, $+255 \geq b \geq -256$.

The exponent part is stored as $(256 + b)$ since the exponent part does not accommodate a sign bit.

(i) $+\frac{7}{8} = +0.875$.

The binary representation of $+0.875$ is obtained by repeated multiplication as shown.

Integer Part	
	0.875×2
1	0.75
1	0.50
1	0.0

$\therefore +0.875_{10} = 0.111_2$.

In floating-point form, $+0.875 = 0.111 \times 2^0$.

The bit pattern is shown in sketch (a).



(a)

(ii) $+22\frac{1}{8} = 22.6875$.

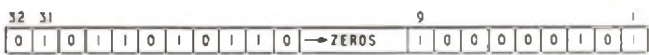
The binary representation of 22.6875 is obtained as follows.

Integer	remainder	Fraction Part
22		0.6875×2
11	0	0.3750
5	1	0.7500
2	1	0.5000
1	0	0.0000
0	1	0.0000

Therefore, $+22.6875_{10} = 010110.1011_2$

In floating-point form, $+22.6875 = 0.101101011 \times 2^5$.

The bit pattern is shown in sketch (b).



(b)

(iii) $-3\frac{3}{4} = -3.75$.

Binary representation of $+3.75$ is 011.11 .

The binary representation of -3.75 is stored as the two's complement of $+3.75$.

011.11
100.00 complement
1 + 1
100.01 two's complement.

Therefore, the floating point representation of -3.75 is 1.0001×2^2 . The bit pattern is shown in sketch (c).



(c)

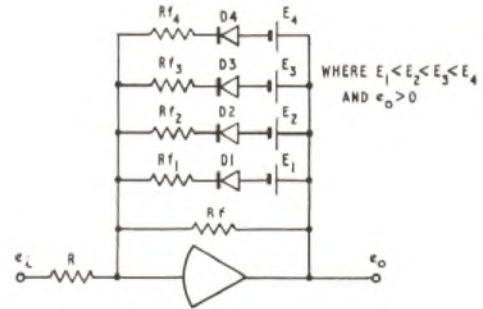
Q. 9. (a) Draw a diagram of a diode function generator which has four break-points and can represent a positive function only.

(b) Explain how the generator is adjusted to represent particular functions.

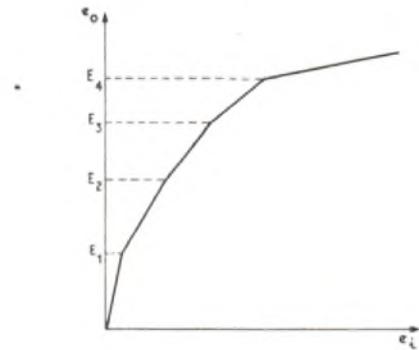
(c) How can a more accurate approximation be obtained?

A. 9. (a) A circuit of a diode function generator which has four break-points is shown in sketch (a).

A typical function produced by this circuit is shown in sketch (b).



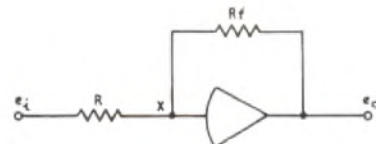
(a)



(b)

(b) It can be seen from sketch (a) that the feedback branches containing diodes are effectively open-circuited as long as the diodes are reverse biased i.e. $(e_0 - E_i) < 0$.

If $e_0 < E_1$, the simplified circuit is as shown in sketch (c).



(c)

Assuming the potential at point X is zero,

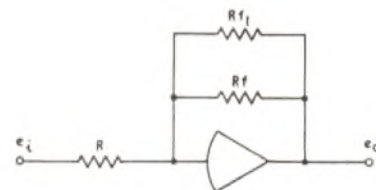
$\frac{e_i}{R} = \frac{e_0}{R_f}$ by Kirchoff's first law which states that there should be no net current at a junction.

$\therefore e_0 = \frac{R_f}{R} e_i$.

The slope of the function over the range, $0 < e_0 < E_1$ is

$\frac{de_0}{de_i} = \frac{R_f}{R}$.

The first break point occurs when $e_0 = E_1$. Diode D1 starts to conduct, switching in another feedback resistance R_{f1} . The simplified circuit is shown in sketch (d).



(d)

The slope of the function over this range $E_1 < e_0 < E_2$ is, therefore, $\frac{de_0}{de_i} = \frac{R_{f1}'}{R}$, where $R_{f1}' = R_f$ in parallel with R_{f1} .

Similarly, it can be shown that over the range $E_2 < e_0 < E_3$ that $\frac{de_0}{de_1} = \frac{R_f' 2}{R}$, where $R_f' 2 = R_f$ in parallel with R_{f1} in parallel with R_{f2} .

The remaining changes in slope are produced by adding further feedback resistors R_{f3} and R_{f4} in parallel.

The slope of the function at a particular point is, therefore, dependant on the total effective feedback resistance divided by the input resistance, R . By choosing the appropriate values for the feedback resistances, R_f , R_{f1} , R_{f2} , R_{f3} and R_{f4} the generator can be adjusted to represent particular functions.

(c) The accuracy of the approximation to a function can be increased by increasing the number of break points.

Q. 10. With the aid of a diagram, explain the operation of an integrator (as used in a general-purpose analog computer) which has the facility of setting initial conditions.

A. 10. See A. 10, Computers B, 1969. Supplement, Vol. 63, p. 33, July 1969.

BASIC MICROWAVE COMMUNICATION C, 1972

Students were expected to answer any six questions

Q. 1. (a) In what units are (i) the electric field components, and (ii) the magnetic field component of an electromagnetic wave measured?

(b) By reference to these units, explain what is meant by the statement that free space has an impedance of 120π ohms.

(c) What is the importance of this impedance in the efficient functioning of aerials?

A. 1. (a) (i) The electric field component of an electromagnetic wave E is measured in volts per metre, in the direction of polarization.

(ii) The magnetic field component H is measured in amperes per metre in a direction at right angles to both the direction of propagation and the electric field.

(b) The ratio $\frac{E}{H}$ is in the units $\frac{\text{volts/m}}{\text{amps/m}} = Z$ ohms. This is known as the wave impedance of the propagation medium.

The impedance of free space can be derived from that of a transmission line consisting of two parallel strips, one metre wide and spaced at a distance of d metres.

The voltage V between the strips is given by:

$$V = |E|d,$$

and the charge Q per metre on one of the strips is

$$Q = \epsilon|E|,$$

where ϵ is the permittivity of free space.

The capacitance per unit length of line is, therefore,

$$C = \frac{Q}{V} = \frac{\epsilon}{d} \text{ F/m.}$$

The current is at right angles to the magnetic field, and

$$I = |H|.$$

In free space,
$$Z_0 = \frac{V}{I} = \frac{|E| \cdot d}{|H|} = d \sqrt{\left(\frac{\mu}{\epsilon}\right)},$$

where V and I are both sinusoidal quantities.

$$\therefore \frac{|E|}{|H|} = \sqrt{\left(\frac{\mu}{\epsilon}\right)}.$$

In free space,
$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m,}$$

and
$$\epsilon_0 = \frac{10^{-9}}{4\pi \times 9} \text{ F/m.}$$

$$\therefore Z_0 = \sqrt{\left(\frac{4\pi \times 10^{-7} \cdot 4\pi \times 9}{10^{-9}}\right)},$$

$$= 120\pi \text{ ohms.}$$

(c) The power P radiated from an aerial can be calculated for a given feed current I_0 supplied to the aerial input, and the radiation resistance R_{rad} calculated by equating $\frac{1}{2}I_0^2 R_{rad}$ to the power.

In a half-wave dipole for instance, this gives $R_{rad} = 73$ ohms. The aerial behaves as a transformer which matches a 73-ohm input into the impedance of space of 120π ohms. The aerial losses R_{loss} can be considered as a resistance in series with the radiation resistance, so that the aerial efficiency $\eta = \frac{R_{rad}}{R_{rad} + R_{loss}}$.

If the transmitter or receiver is not matched to the aerial input impedance, power is reflected and the overall efficiency further reduced.

Q. 2. A low-loss coaxial cable of characteristic impedance 50 ohms is terminated in a resistive load of 75 ohms. The peak voltage across the load is found to be 30 volts.

(a) Calculate:

- (i) the reflexion coefficient of the load,
- (ii) the amplitude of the forward and reflected voltage waves,
- (iii) the amplitude of the forward and reflected current waves, and
- (iv) the v.s.w.r.

(b) By means of a sketch representing $\frac{3\lambda}{4}$ length of cable from the load, indicate the positions and magnitudes of the voltage and also the current maxima.

A. 2. (a) (i) The reflexion coefficient ρ of the load is given by:

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0},$$

$$\therefore \rho = \frac{75 - 50}{75 + 50},$$

$$= \frac{1}{5}.$$

(ii) The reflexion coefficient is the ratio of the reflected voltage V_r to the forward voltage V_f ,

$$\text{i.e., } \frac{V_r}{V_f} = \frac{1}{5},$$

and since

$$V_{load} = V_f + V_r = 30,$$

then the forward voltage $V_f = 5/6 \times 30 = 25$ volts,

and the reflected voltage $V_r = \frac{1}{5} V_f = 5$ volts.

(iii) The ratio of the reflected current I_r to the forward current I_f ,

$$\text{i.e. } \frac{I_r}{I_f} = -\frac{V_r}{V_f} = -\frac{1}{5},$$

$$I_{load} = I_f + I_r = \frac{V_{load}}{R_{load}} = \frac{30}{75} = 0.4 \text{ amp.}$$

$$\text{Since } I_r = -\frac{1}{5} I_f,$$

$$I_f - \frac{1}{5} I_f = I_{load}.$$

$$\therefore I_f = \frac{5}{4} \times 0.4 = 0.5 \text{ amp,}$$

$$\text{and } I_r = -\frac{1}{5} I_f = -0.1 \text{ amp.}$$

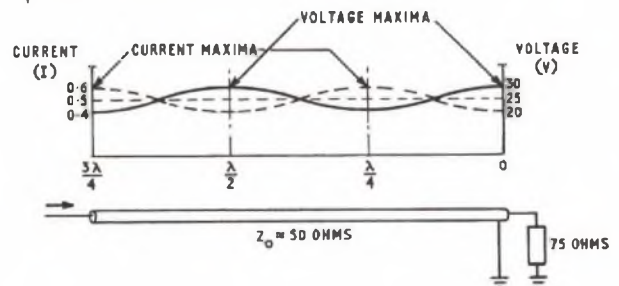
(iv) The voltage-standing-wave ratio (v.s.w.r.) can be defined as:

$$\frac{|V|_{min}}{|V|_{max}} = \frac{|V_f| - |V_r|}{|V_f| + |V_r|},$$

$$= \frac{25 - 5}{25 + 5} = 0.67.$$

(Note: The v.s.w.r. is sometimes given as $\frac{|V|_{max}}{|V|_{min}} = 1.5$).

(b) The positions and magnitudes of the voltage and current maxima of a $\frac{3\lambda}{4}$ length of cable are shown in the sketch.



Q. 3. (a) State, with reasons, the factors to be taken into account in deciding whether to employ waveguide or coaxial cable to interconnect two parts of a 6 GHz equipment.

(b) A klystron energizes, in the dominant mode, a rectangular waveguide having internal dimensions 2.86×1.27 cm. The wavelength in the guide is found to be twice the cut-off wavelength. Calculate the klystron frequency.

A. 3. (a) The following are the main factors to be taken into account.

(i) *The length of the connexion.* Coaxial cable of practical dimensions, and sufficiently small to avoid transmission of higher-order modes, has a much greater attenuation for a given length than waveguide. To minimize losses, coaxial cable is only feasible for very short connexions, and waveguide must be used for longer connexions.

(ii) *Flexibility.* Flexible waveguide connectors are available and give a degree of flexibility, but for very short connexions, coaxial cable gives more flexibility. This avoids the need for accurate positioning of the parts to be connected, and connexions can be made and broken easily.

(iii) *Matching.* Coaxial cables and connectors are available in standard impedance values, e.g. 50 ohms and 75 ohms, so an approximate match is obtained immediately. Precise matching is more difficult than with waveguide, and standing waves are likely to occur. The stability of a match can be affected by variations in the degree of twist in a coaxial-cable connexion.

(iv) *Compatibility with equipment input or output connexions.* The overall complexity can be reduced by using the same form of connexion as those provided in the equipment, as far as possible.

(v) *Power-carrying capacity.* For high-power work, waveguide, with its large internal dimensions, is generally able to withstand higher voltages.

(vi) *Cost.* The cost of coaxial cable is generally less than for waveguide.

(vii) *Group-delay.* The group-delay variations with frequency are negligible along a coaxial cable. This is not the case along a waveguide, where the dispersion can cause difficulties with wideband systems.

(b) It can be shown that for propagation along a waveguide:

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

where λ_g = guide wavelength,
 λ_0 = free-space wavelength, and
 λ_c = critical wavelength at cut-off frequency.

In the dominant mode, (see sketch (a) of A.4),

$$\lambda_c = 2a,$$

where a = maximum internal dimension of waveguide.

$$\therefore \frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{4a^2}$$

In the example,

$$\lambda_g = 2\lambda_c$$

$$= 4a,$$

and $a = 2.86$ cm.

$$\therefore \frac{1}{\lambda_0^2} = \frac{1}{16a^2} + \frac{1}{4a^2}$$

$$\text{and } \frac{1}{\lambda_0} = \frac{\sqrt{5}}{4a}$$

But, frequency f of klystron is given by:

$$f = \frac{c}{\lambda_0}$$

where $c = 3 \times 10^{10}$ cm.

$$\therefore f = \frac{3 \times 10^{10} \sqrt{5}}{4 \times 2.86} \text{ Hz,}$$

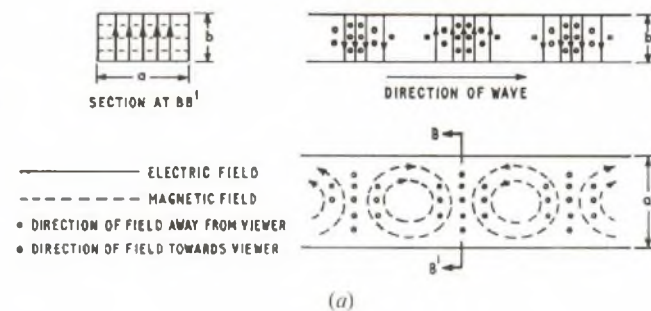
$$= 5.87 \text{ GHz.}$$

Q. 4. (a) With the aid of a diagram, explain what is meant by a higher-order mode in a rectangular waveguide.

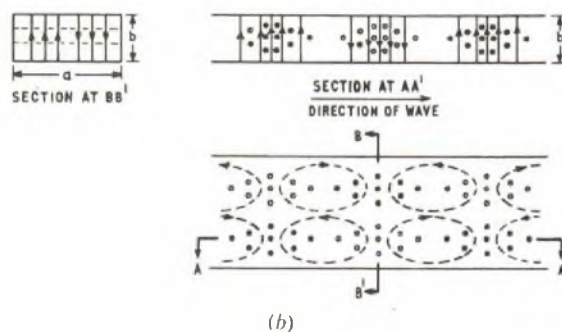
(b) Why is the presence in a waveguide of more than one mode undesirable?

(c) How, in practice, is it possible to ensure the presence of only the dominant mode?

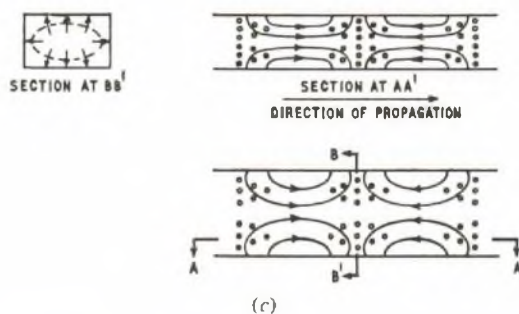
A. 4. (a) Higher-order modes can be transmitted by a waveguide with dimensions which exceed the free-space wavelength. For example, two sets of the H_{10} mode wave patterns [sketch (a)] can be



transmitted alongside each other as shown in sketch (b), forming the



H_{20} mode. When the waveguide is even larger, it also becomes possible to transmit the transverse-magnetic (E) series of modes, the simplest of which is the E_{11} mode shown in sketch (c).



(b) The presence in a waveguide of more than one mode makes the design of an efficient system very difficult as the different modes travel with different velocities. The resultant differential delays vary with frequency, causing distortion in frequency-modulated systems and an increase in the error rate of digital systems.

(c) The dominant mode has the lowest cut-off frequency, and so the guide dimensions can be chosen so that over a limited frequency range only the dominant mode (H_{10}) is present.

The critical wavelength of the H_{10} mode is given by:

$$\lambda_c = 2a,$$

where λ_c = critical wavelength at cut-off frequency, and
 a = the larger of the two internal waveguide dimensions.

The critical wavelength of the nearest other modes are:

$$\lambda_c = a \text{ (H}_{20} \text{ mode),}$$

$$\text{and } \lambda_c = 2b \text{ (H}_{01} \text{ mode),}$$

where b = the smaller of the two internal waveguide dimensions.
 Thus, if the H_{10} mode only is to be propagated,

$$\lambda/2 < a < \lambda,$$

and

$$b < \lambda/2 \text{ to avoid the H}_{01} \text{ mode.}$$

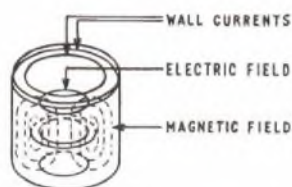
In practice, b is $\approx \frac{1}{2}a$, which gives the highest breakdown voltage compatible with avoidance of higher-order modes.

Q. 5. (a) Explain why, in the case of a cylindrical-cavity wavemeter, it is unnecessary for the tuning piston to make contact with the cavity walls.

(b) With the aid of diagrams, describe how a tunable cavity may be used as:

- (i) an absorption wavemeter, and
- (ii) a transmission wavemeter.

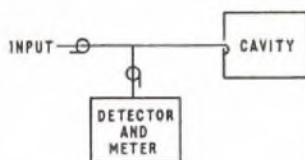
A. 5. (a) A cylindrical-cavity wavemeter using the H_{01n} mode can be considered as a length of circular waveguide n half-wavelengths long, and short-circuited at each end. The H_{011} electric and magnetic fields are shown in sketch (a). The currents in the walls of the cavity



(a)

are, everywhere, at right angles to the adjacent magnetic field, and, therefore, flow in concentric circles centred on the axis of the cylinder. A gap can be introduced between the tuning piston and the cavity walls without interrupting the current flow, and the effect on the resonant frequency is negligible. A gap is, in fact, desirable as it also suppresses unwanted modes whose wall-currents would flow across the gap.

(b) (i) The principle of an absorption wavemeter is shown in sketch (b). (b). When the cavity is tuned through resonance, power is absorbed



(b)

in the cavity and a sharp minimum occurs in the meter reading which otherwise has a steady value.

(ii) Sketch (c) shows the principle of a transmission wavemeter.



(c)

When the wavemeter is off-tune, no energy is transmitted to the detector. When it is tuned through resonance, the cavity is excited and transmits some of the energy to the detector, which shows a sharp maximum above the normal zero reading.

Q. 6. (a) On what factors do the following properties of an aerial using a parabolic reflector depend?

- (i) Beamwidth.
- (ii) Size of side lobes.
- (iii) Bandwidth.

(b) The parabolic reflectors used for both aeriels of a microwave communication link are replaced by others having diameters 1.5 times that of the original. Calculate in decibels the overall gain improvement which would be expected from this modification.

(c) Why, in practice, might the overall gain improvement be somewhat greater than this estimate?

A. 6. (a) (i) The beamwidth of a well-focussed aerial using a parabolic reflector is approximately $\frac{\lambda}{D}$ rad,

assuming uniform illumination of the reflector by the feed,

where λ = wavelength, and

D = diameter of the reflector.

An illumination tapered towards the edges widens the beamwidth. The illumination distribution depends upon the radiation pattern of the feed, and the angle subtended by the reflector to the feed which is placed at the focus of the parabolic reflector surface.

Irregularities in the reflector surface broaden the beam to some extent. For general purposes, the surface of the reflector must be accurate to $\pm \frac{\lambda}{16}$.

(ii) The relative size of the side lobes to the main beam can be reduced to about 20 dB or more by tapering the illumination towards the edges. Conversely, the side-lobe size is increased by inverse tapering.

Irregular surfaces, badly placed feed waveguide and supports, and inaccurate centering of the feed all cause large, badly-shaped and asymmetric side lobes. Power scattered from the supports also increases the general side-lobe radiation.

(iii) The bandwidth of a parabolic reflector is very wide, and the overall bandwidth is determined mainly by the feed design. A dipole feed, for example, would limit the bandwidth to a few per cent, though thick elements would have a wider bandwidth than thin ones. A horn-type waveguide feed gives a wider bandwidth, which is limited by the quality of match, over the working bandwidth between the waveguide and the feed.

(b) The theoretical gain of a parabolic reflector is proportional to the aperture area and hence to D^2 , where D = diameter of reflector. If D is increased by 1.5 times the original, the gain of each aerial will be increased by $1.5^2 = 2.25$, assuming the distribution of illumination of the feed is the same before and after the modification. The overall gain improvement is, therefore, $2 \cdot 25^2 = 5.01$, and the gain in decibels = $10 \log_{10} 5.01 = 7.0$ dB.

(c) The waveguide and supports cause some shadowing of the beam, and the shadowing is less in proportion with the larger reflector. This might slightly increase the gain improvement over the theoretical figure. The beamwidth of the larger reflector is narrower, and in a practical system gives smaller ground reflexions. This could either increase or reduce the overall improvement, depending on the relative phase and amplitude of the reflected signal. Other factors such as the increased difficulty in maintaining the surface tolerance of a larger reflector, and the higher pointing-accuracy required, all affect the gain improvement obtained in practice.

Q. 7. (a) Explain why the sampling rate determines the highest modulation frequency that a pulse-modulation system can carry.

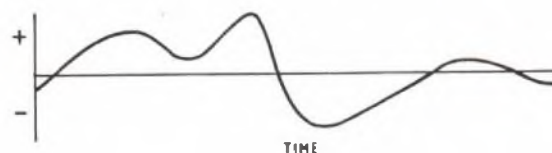
(b) A train of equal mark-space ratio pulses, of duration $40 \mu s$, is used in a pulse-amplitude-modulation system. What is the upper limit to the modulation frequency that the system can handle?

(c) Briefly describe how the information carried could be extracted from the modulated pulse train described in (b) above.

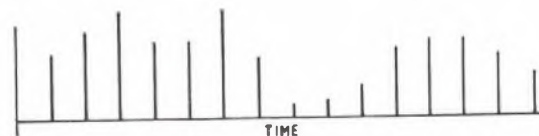
A. 7. (a) When a sampling-pulse train, as shown in sketch (a), is used to sample a complex waveform [sketch (b)], a pulse-amplitude-modulation signal, as shown in sketch (c), results. The frequency



(a)

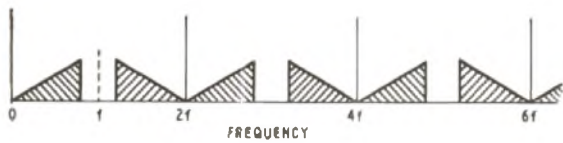


(b)



(c)

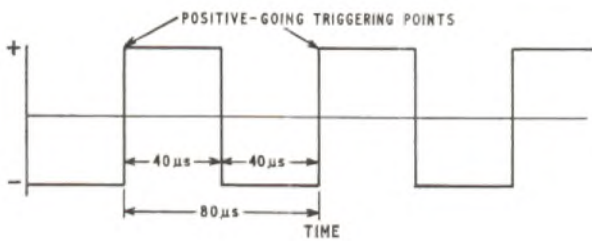
spectrum of the amplitude-modulated pulses consists of a series of lines spaced at intervals of $2f$, where f is the maximum modulating frequency. Each has sidebands as shown in sketch (d), including an



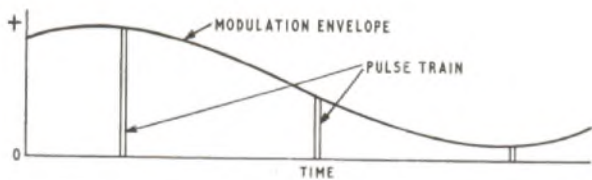
(d)

erect sideband on the upper side of zero frequency. In demodulation, this can be extracted by passing the sampling signal through a low-pass filter which cuts off frequencies above f . If the modulation waveform contains frequencies above f , the sidebands overlap and distortion occurs in the demodulated signal. Thus, if the sampling frequency is $2f$, the maximum modulating frequency is f , or half the sampling frequency.

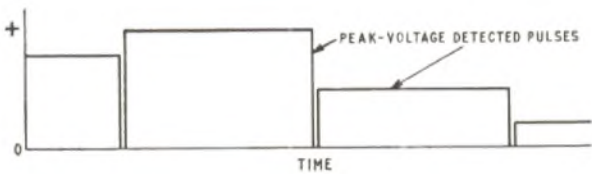
(b) The sampling rate of the train of pulses is $\frac{1}{80 \times 10^{-6}} = 12,500$ samples/s, as shown in sketch (e). The upper limit to the modulation frequency is, therefore, $\frac{12,500}{2}$ Hz = 6.25 kHz.



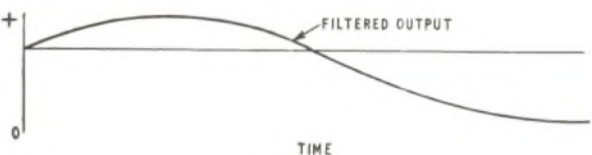
(e)



(f)



(g)



(h)

(c) The precise sampling point is at some chosen point of the $40 \mu s$ pulse, for example, at the rising edge as shown in sketch (f). The sampling pulses are of shorter duration, but still spaced at $80 \mu s$ intervals. To extract the modulation signal, the pulses are superimposed on a direct voltage to make them all of the same sign, and lengthened by applying the pulses to a peak-voltage detector which holds the voltage at the amplitude of each pulse for nearly $80 \mu s$.

This voltage is then dumped immediately before the next pulse is applied to the circuit [sketch (g)]. The signal now contains the original modulating waveform and a series of frequencies as shown in sketch (d). The latter can be removed by passing the waveform through a low-pass filter with a cut-off frequency of 6.25 kHz [sketch (h)].

Q. 8. (a) Prove that the noise power at the output of an intermediate-frequency amplifier having a noise factor F , a gain G and a bandwidth B is proportional to FGB .

(b) Redesign of a microwave equipment improves the receiver input signal-to-noise ratio by 3 dB and reduces the receiver noise factor from 12 to 6.5. Calculate the overall resulting improvement in the output signal-to-noise ratio.

(c) Briefly explain why the mixer stage contributes substantially to the overall noise factor of a microwave receiver.

A. 8. (a) The noise factor F of a device may be defined as:

$$F = \frac{\text{input signal-to-noise power ratio}}{\text{output signal-to-noise power ratio}}$$

$$\therefore F = \frac{\text{output noise power}}{\text{input noise power}} \times \frac{\text{input signal power}}{\text{output signal power}}$$

$$\therefore \text{Output noise power} = FG \times \text{input noise power,}$$

where G = gain of device.

Now, the input noise power = $KT B$,

where K = Boltzman's constant,

T = absolute temperature ($^{\circ}K$), and

B = bandwidth (Hz).

Assuming that T is constant, input noise power is proportional to FGB , and therefore output noise power is proportional to FGB .

(b) The noise factor before modification = 12. Therefore, by definition, old input signal-to-noise ratio = $12 \times$ old output signal-to-noise ratio. After the modification, the noise factor is reduced to 6.5 and the input signal-to-noise ratio is improved by 3 dB, i.e. the power ratio is doubled.

Improvement in output signal/noise ratio

$$= \frac{\text{new output signal-to-noise ratio}}{\text{old output signal-to-noise ratio}}$$

$$= \frac{\text{new input signal-to-noise ratio} \div 6.5}{\text{old input signal-to-noise ratio} \div 12}$$

$$= \frac{2}{1} \times \frac{12}{6.5} = \underline{3.70},$$

or, $10 \log 3.70 = \underline{5.7 \text{ dB}}$.

(c) The overall noise factor F of two stages is given by:

$$F = F_1 + \frac{F_2 - 1}{G_1},$$

where F_1 = the mixer stage noise factor,

F_2 = the intermediate amplifier noise factor, and

G_1 = the gain of the mixer stage.

The noise factor of the mixer stage (first term of the above expression) contributes directly to the overall noise factor. In the second term of the expression, G_1 is the gain of the mixer and is < 1 . This increases the value of $\frac{F_2 - 1}{G_1}$ considerably when compared with F_2 of the intermediate amplifier. The overall noise factor cannot be improved by adding succeeding amplification stages. The contribution of the mixer can be reduced by using a low-noise microwave amplifier in front of the mixer. If the two terms of the formula now refer to the microwave amplifier and the mixer, the contribution of the noise factor of the mixer F_2 to the second term will be reduced by the gain G_1 of the microwave-amplifier.

Students were expected to answer any six questions

- Q. 1.** (a) Explain what is meant by the statement that "the grade of service given at the group-selector outlets is 0.002".
 (b) Explain why more than one circuit is generally provided to carry one erlang of traffic.
 (c) If four switches in a full-availability group are offered one erlang of traffic, calculate
 (i) the grade of service given,
 (ii) the traffic lost, and
 (iii) the traffic carried by the first switch.

A. 1. (a) Grade of service (B)
 = the probability of a call (or traffic) being lost,
 = $\frac{\text{calls lost}}{\text{calls offered}}$ (during the busy hour).

When calls are considered to have equal holding time,

$$B = \frac{\text{traffic lost}}{\text{traffic offered}}$$

Also $B =$ probability of finding all circuits engaged.

$$\text{Therefore, } B = \frac{\text{number of seconds for which all circuits are simultaneously engaged}}{\text{number of seconds in one hour}}$$

$$\text{Thus, } 0.002 = \frac{1}{500} = \frac{\text{calls lost}}{\text{calls offered}}, \text{ and also}$$

$$0.002 = \frac{\text{number of seconds for which all circuits are simultaneously engaged}}{3,600}$$

So, 0.002 means that

- (i) one call in 500 is lost, and
 (ii) all circuits are engaged for $3,600 \times 0.002 = 7.2$ s.

(b) As the maximum amount of traffic that a circuit can carry is one erlang (one circuit engaged for one hour[busy]), then it is certain that traffic will be lost if only one circuit is provided to carry each erlang of traffic. This certainty will be reduced if more than one circuit is provided to carry each erlang of traffic, and the greater the number of circuits, the lower the probability of a call being lost.

$$(c) (i) B = \frac{A^N}{N!} \cdot \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

where $A =$ traffic offered in erlangs,
 $N =$ number of circuits.

$$\therefore B = \frac{1^4}{1 + 1 + \frac{1}{2} + \frac{1}{6}} = \frac{1}{\frac{16}{6}} = \frac{1}{64} = 0.016.$$

(ii)

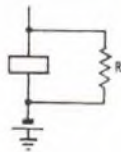
$$\text{Traffic lost} = \text{traffic offered} \times B = 1 \times 0.016, \\ = 0.016 \text{ erlangs.}$$

$$(iii) \text{Traffic carried by the first switch} = \frac{A}{1 + A} = \frac{1}{1 + 1} = \frac{1}{2}, \\ = 0.5 \text{ erlangs.}$$

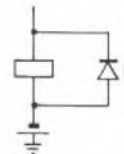
Q. 2. (a) Describe three methods by which the operate and release lags of a telephone relay may be increased.

(b) Explain how timing characteristics are utilized in pulse-control and ring-trip circuit elements. Use sketches to illustrate your answer.

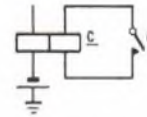
A. 2. (a) Sketches (a), (b) and (c) show three methods of increasing the release lag of a relay i.e. a resistor, or diode, or short-circuit winding. Each makes use of the fact that, when current ceases to flow through the coil, the collapsing magnetic field induces a back e.m.f., with the resulting induced current flowing through the resistor, diode or short circuit. Thus, the magnetic field takes longer to collapse and, hence, the armature releases more slowly. Sketch (d) shows a method of increasing the operate lag of a relay by a short-circuit winding. When current starts to flow, the build up of the



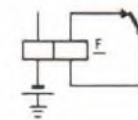
(a)



(b)



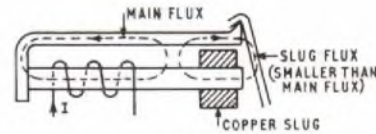
(c)



(d)

magnetic field is slowed down by the induced current flowing in the secondary coil. It should be noted that, if the secondary coil were permanently short circuit, the relay would be slow operate and release.

Relays may also be given slow-operate and -release characteristics by use of copper slugs. Sketch (e) shows a relay with an armature-end

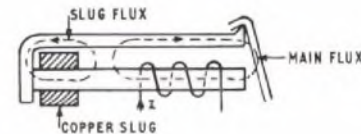


(e)

slug. When current is switched on, the winding creates a main flux which induces a current into the copper slug which, in turn, creates its own flux in opposition to the main flux. When the current reaches maximum, there is no induced current in the slug and so the slug flux in opposition dies away and the armature operates.

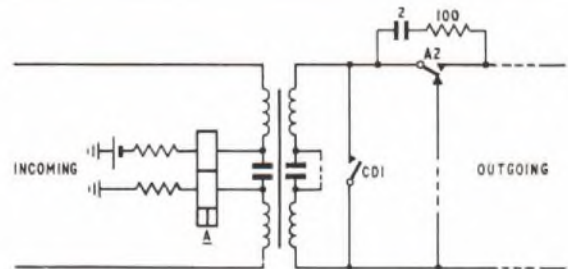
When the current is switched off, the collapsing main flux induces a current into the slug which again creates its own flux in opposition to the main flux and, thus, the slug current tries to build up the flux again until all the flux dies away and the armature releases. This relay is slow to operate and release.

Sketch (f) shows a relay with a heel-end slug. The relay operates normally as the main flux goes through the armature but the release is slow for the same reasons as described for the slow-operate and -release relay with the armature-end slug.



(f)

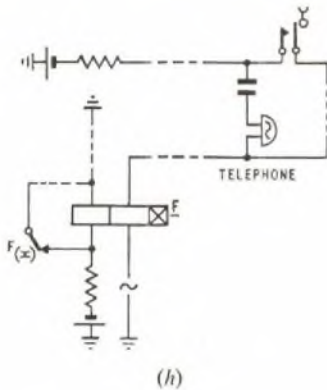
(b) Sketch (g) illustrates pulse control with slow-release relays.



(g)

Upon seizure, relay A operates and contact A1 operates relay B. During pulsing, contact A2 repeats the loop-disconnect pulses forward. Contact A1 short circuits relay B during each break pulse but relay B holds as the short circuit gives it a slow-release characteristic. Relay CD holds during each make pulse as the contact A1 short circuit gives it a slow-release characteristic. Relay CD releases during the interdigital pause.

Sketch (h) illustrates a ring-trip circuit with a slow-operate relay.



Relay F does not operate to the alternating ringing current applied to the telephone as it has a very slow-operate characteristic due to the armature-end slug and short-circuit winding. When the line is looped, there is a d.c. path and relay F operates slowly. The F_x contact then locks in relay F and other contacts of the relay disconnect relay F from the line and connect the line to the transmission bridge.

Q. 3. (a) What advantages has the motor uniselector over a two-motion selector when used as a group selector?

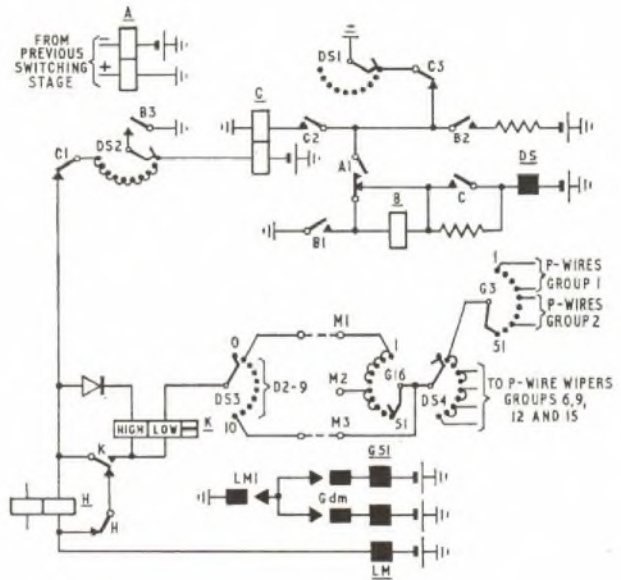
(b) Draw a circuit for controlling the wiper position of a motor uniselector from a train of dial pulses and explain the circuit operation.

A. 3. (a) In a motor uniselector group-selector (m.u.g.s.) circuit the pulse counting and selecting functions are separate, and this gives flexibility for the provision of outlet groups of different availability.

The m.u.g.s. can be used more efficiently where the groups of circuits served vary widely in size, since the availability can be chosen to suit the number of circuits in a group, thus, reducing the number of unused outlets. In a two-motion group-selector, the limit of availability is 20 per level and this can require up to 10 per cent more circuits when the number of circuits in a group is more than 20.

Economy can be made in large circuit groups by using a high availability, thus reducing the number of circuits for a given amount of traffic.

Another advantage of the m.u.g.s. is that it can test up to about 250 outlets in less than 300 ms, whereas the two-motion selector can only test up to around 20 outlets in the same period.



(b) A circuit for positioning the wipers of a m.u.g.s. in response to a train of dial pulses is shown in the sketch. Uniselector DS is a digit switch whose function is to count the dialled pulses and to select the part of the motor-uniselector bank in which testing is required.

On seizure, relay A operates to the dial loop, contact A1 operates relay B, and contact B3 operates relay C. When dialling occurs, relay A releases and operates in response to the dial pulses, and causes the digit switch to step over the appropriate number of contacts. The operate circuit for relay C is disconnected by wiper DS2, but relay C holds during pulsing on its other coil. Relays B and C are alternately short-circuited by contact A1 during the pulse break-periods and make-periods, respectively, but hold by virtue of their release lags. At the end of the pulse train, relay C remains short-circuited by contact A1 and releases. Contact C1 energizes the motor-uniselector latch-magnet and causes the motor-uniselector to hunt; it also connects earth to the testing circuit, the earth passing via relay K and arcs DS3, G16 and DS4 in series to the particular motor-uniselector P-wiper selected by the position of wiper DS4. The group of outlets tested in the selected motor-uniselector arc is determined by cross-connections between the marking tags M1-M3 and the digit tags D1-D0. If a digit tag is cross-connected to marking tag M1, testing takes place in outlet group 1, and if it is cross-connected to tag M2, group 2 is tested. A greater availability can be provided by cross-connecting to marking tag M3, as testing then takes place over both groups 1 and 2. When the motor-uniselector P-wiper reaches a free outlet, which is marked by a resistance battery, relay K operates. A contact of relay K removes the short-circuit from the operate coil of relay H to connect the coil in series with the latch magnet, which releases and arrests the wipers on the selected outlet.

(to be continued)

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