SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

Vol. 65 No. 3 October 1972

1970–71 CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS

Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

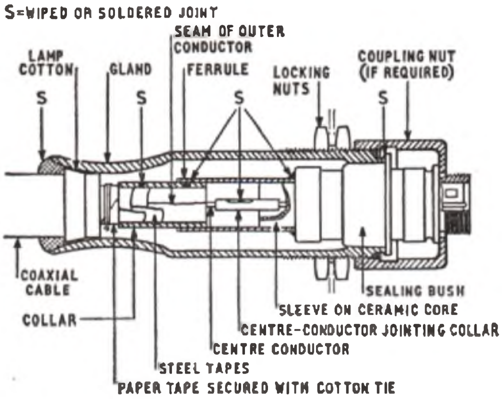
LINE PLANT PRACTICE B, 1971 (*continued*)

Q. 5. *Describe with sketches a method of terminating*

(*a*) *a* 2·6/9·5 *mm* (0·375 *in*) *coaxial pair,*
(*b*) *a cable containing four* 1·2/4·4 *mm* (0·174 *in*) *coaxial pairs.*

A. 5. (*a*) In order to maintain pressurization of coaxial cables of the 2·6/9·5 mm (0·375 in) type, a sealing end is fitted at the terminating point. Its construction consists of an inner ceramic body to which metal bushes are secured. The end accommodating the coaxial plug is soldered to one of these bushes with a high-melting-point solder which should not be adversely affected by soldering operations during the fitting of the sealing end.

The method of termination of a 2·6/9·5 mm coaxial pair (see sketch (*a*)) having polythene protection over lead sheathing may be summarized into the following operations.



(*a*)

(*i*) Remove the polythene protection and lead sheathing.
(*ii*) Secure the paper tape with a cotton tie near the lead sheath and remove the paper up to this tie to expose the steel tapes.
(*iii*) Cut the steel tapes.
(*iv*) Bind the exposed outer conductor with two turns of soft copper wire and remove surplus outer and inner conductors.
(*v*) Remove the wire tie and clean the outer conductor and exposed steel tapes with fine glass paper. Place the prepared collar over the outer conductor until the end traps the steel tapes and solder in place, feeding solder through the holes in the collar. Wipe off surplus solder with a clean dry rag.
(*vi*) Remove the surplus length of outer conductor and cut the inner conductor to the required length.
(*vii*) Replace the polythene disks by disks having a higher melting point to prevent their destruction during subsequent soldering operations.
(*viii*) Clean and tin the inner conductor. To prevent solder from passing along into the collar, the end disk should be protected with insulating paper and a cotton tie placed on the inner conductor.
(*ix*) Place the outer gland of the sealing end over the prepared cable, belled-end first, and pass the ferrule over the collar clear of the jointing area of the inner conductor.
(*x*) Fit the jointing collar on to the inner conductor, solder it in position and remove any surplus solder.
(*xi*) Slide the ferrule into place to bridge the gap between the outer conductor of the cable and the sleeve on the ceramic core. Solder into position.
(*xii*) Wrap four turns of clean, dry, insulating paper around the gap between the end of the cable sheath and the ferrule and tie in position.
(*xiii*) Place the outer gland in position and solder the end remote from the cable to the rim of the bush. Plug the gap between the bellmouth of the outer gland and the cable, with lamp cotton and seal it by plumbing with solder, using a Taft wipe. In order to retain air pressure within the cable, both of these joints must be completely airtight.

When the cable sheath is polythene a coaxial plug is connected, by soldering, to the inner and outer conductors. Approximately 165 mm ($6\frac{1}{2}$ in) of the cable sheath up to the stripped portion is taped with a 50 per cent overlap, first with a self-amalgamating tape and then with an adhesive plastic tape. A cable gland is then fitted over the sheath, building up the diameter of the cable, if necessary, with adhesive rubber tape. A short length of rubber hose is then secured with a hose clip around the cable gland and the joint encapsulated with an epoxy resin (see sketches (*b*) and (*c*)). When the resin has cured the rubber hose is removed and an identification label positioned on top of the encapsulated joint.

(*b*) The termination of a cable containing four 1·2/4·4 mm (0·174 in) coaxial pairs is basically the same as for the 2·06/9·5 mm coaxial pair described, except that there are four sealing ends and interstice pairs to be dealt with. The method of termination of this cable having protection and lead sheathing is as follows.

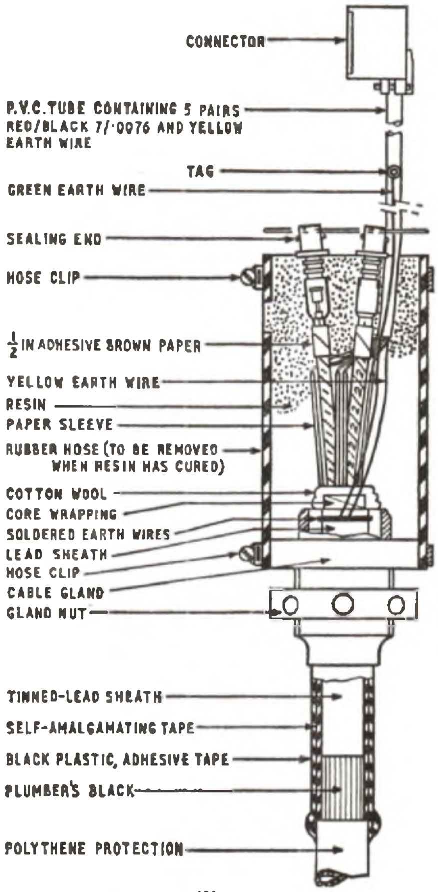
(*i*) Remove the polythene protection and lead sheathing.
(*ii*) Secure the paper tape with the cotton tie near the lead sheath and remove the paper up to this tie to expose the steel tapes.
(*iii*) Cut the steel tapes.
(*iv*) Lay back the interstice pairs.
(*v*) Cut the coaxial pairs to the required length.
(*vi*) Place a cotton tie round the numbered paper tape of a coaxial pair and remove the paper tape to this tie.
(*vii*) Cut the steel tapes to length and secure temporarily.
(*viii*) Cut to length, clean and tin the exposed inner and outer conductors.
(*ix*) Slide the outer-conductor ferrule over the outer conductor.
(*x*) Solder the inner conductor to the centre pin of the sealing end.
(*xi*) Position the ferrule correctly and solder it to the outer conductor at one end and to the shell of the sealing end at the other.
(*xii*) Secure the numbered paper tape and the steel tapes with a binding of brown adhesive paper tape.
(*xiii*) Repeat the above procedure for all the coaxial pairs.
(*xiv*) Joint the five interstice pairs to pairs formed from single p.v.c. covered wires (7/·0076 red and black).
(*xv*) Solder two wires to the lead sheath to provide earthing facilities. {One wire, (23/·0076 coloured green) is approximately 185 mm ($7\frac{1}{4}$ in) long and the second wire (7/·0076 coloured yellow) is the same length as the extended interstice pairs.}
(*xvi*) Place the five interstice pairs and the yellow earth lead within a p.v.c. tube for protection.
(*xvii*) Remove a further 130 mm ($5\frac{1}{4}$ in) of the polythene protection and treat 25 mm (1 in) of the lead sheath adjacent to the

polythene protection with plumbers' black. Tin the remainder of the exposed lead sheath with solder.

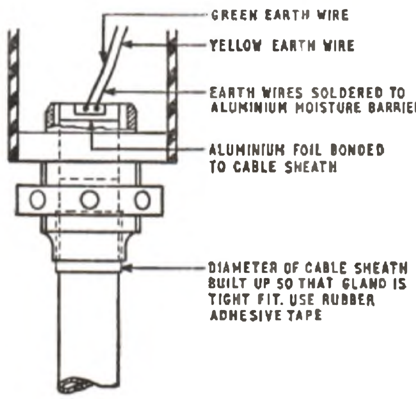
(*xviii*) Fix the gland in position on to the lead sheath.

(*xix*) Position a 180 mm (7¼ in) length of rubber hose over the gland and secure it with a hose clip. Place a second hose clip at the top of the rubber hose to maintain the hose in a circular form.

(*xx*) Place cotton wool round the coaxial and interstice pairs at the cable butt.



(*b*)



(*c*)

(*xxi*) Mix epoxy resin thoroughly and pour it into the hose until it is full; top up as necessary during the setting period.

(*xxii*) When the resin has cured (approximately 24 hours), remove the clips and rubber hose.

(*xxiii*) Wind a lapping of self-amalgamating tape, with a 50 per cent overlap, on the exposed lead sheath from the gland to the polythene protection. Wind a lapping of black plastic adhesive tape, with a 50 per cent overlap, over the self-amalgamating tape to a point 13 mm (½ in) over the polythene protection.

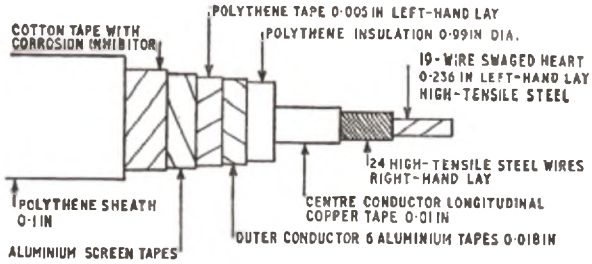
(*xxiv*) Place an identification label, in the form of a ring, over the top of the encapsulation.

The jointing procedure outlined above is the same for polythene-sheathed cables except that the earth wires are soldered to the aluminium moisture barrier. In addition, it is necessary to build up the diameter of the cable sheath with adhesive rubber tape to ensure that the gland is a tight fit. (See sketch (*c*).)

Q. 6. *Draw a sketch to show the make-up of a non-armoured (light-weight) deep-sea coaxial cable.*

Describe how a joint is made in a cable of this type.

A. 6. A section of lightweight coaxial cable is shown in sketch (*a*).



(*a*)

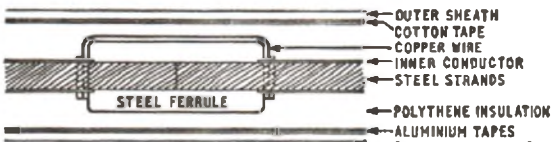
The high-tensile steel strand is torsionally balanced so that it has no tendency to twist under tension. The copper centre conductor is applied over the steel strand in the form of a tightly-applied, 10-mil tape closed with a folded box seam.

After a tight application of polythene to the centre conductor to 0·99 in diameter, the six 18-mil aluminium tapes are applied and held closely to the surface of the polythene with a 10-mil cotton tape. The tape is heavily impregnated with barium chromate which prevents the aluminium tapes from corroding. A tightly-extruded sheath of polythene over the cotton tape to 1·3 in diameter then completes the cable.

The joint in a lightweight deep-sea cable is made with an overlap of approximately one foot. The polythene sheath is removed for a distance of 7–8 in and the linen tape and aluminium tapes are laid back to expose the copper centre conductor and the included strands. Electrodes are connected, a few inches apart, on the copper tube and a large current is applied until the tube and the steel strands burn through. The heat causes the steel strands to become welded together which ensures that they all take a share of any tensile load applied to the completed joint. When the two cable ends have been treated they are allowed to cool before being cut back and filed to make a butt joint of circular cross-section.

The welded stubs are coated with a layer of silicon-carbide dust to provide extra grip; they are then slipped into a 4½ in steel ferrule. The dimensions are such that the ends of the copper tubes are just nipped by the ends of the steel ferrule. A press capable of exerting a force of 200 tons on a 2 in length of ferrule is used to swage the ferrule down on to the steel strands; the centre section is pressed first and then the two ends. After pressing, the ferrule is approximately 5 in long. Polythene insulation is applied over the joint by an injection moulding process then X-ray and high-voltage tests are applied.

When the joint is approved, the aluminium tapes are reapplied and overlapped for 2–3 in. A cold-pressure-welding process is employed to join the overlapped tapes together in four or five spots. The impregnated tape is wound over the jointed tapes and secured. The gap between the two cable sheaths is fitted with a polythene packing piece which is wrapped with a layer of transparent adhesive tape. A polythene sleeve, which was slid over one side of the cable before jointing commenced, is then eased over the joint so that a ¼ in of the packing piece is showing. A circumferential cut is made to remove the exposed packing piece, leaving a groove. After careful cleaning, it is put into a mould, heated to 200°C and molten polythene inserted into the gap. After cooling, the mould is removed and any surplus polythene trimmed off. This moulding operation is repeated at the other end of the sleeve and, thus, completes the jointing process. Sketch (*b*) shows the completed joint.

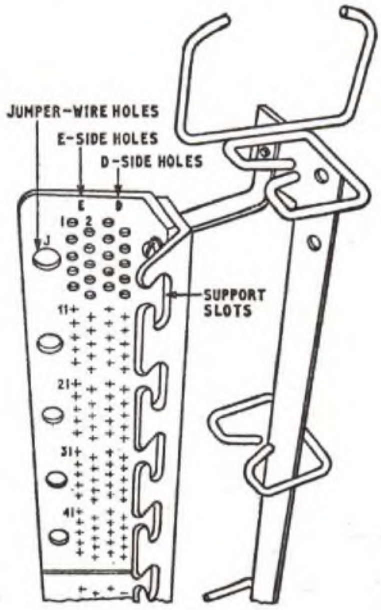


(*b*)

Q. 7. *Sketch and describe one type of assembly used in a cross-connexion cabinet.*

What is the function of the assembly described in a local-line network?

A. 7. The new type of assembly used in cross-connexion cabinets is shown in sketch (*a*). [*Note*: Those students who answered this question by describing the existing type, using terminal blocks, were marked accordingly.] The new type takes the form of a moulded plastic strip having two "wings" inclined at an angle to the back. The strip accommodates 100 exchange (E-side) and 100 distribution (D-side) pairs each arranged in 10 groups of 10. The left-hand wing, viewed from the front, includes a pattern of numbered holes, through which the pairs from the E-side and D-side cables are passed; the pair of E-side holes are positioned towards the front of the wing with the D-side holes occupying the rear, each hole taking one pair. The extreme edge of the left wing carries larger holes through which jumper wires are passed, there being one jumper-wire hole for each group of 10 pairs of E-side and D-side cables. The right-hand wing is formed into 10 supporting slots, one slot for each group of 10 holes on the opposite wing. The 10 cable pairs from each group of E-side and D-side holes are brought across the face of the strip and located in the associated supporting slot. The strip is fitted on to a mounting frame and installed in the cabinet in the normal way.



(*a*)

The cable usually enters the cabinet directly without the need for a joint in an adjacent jointing chamber. However, the strips may be pre-wired and tailed and the tails jointed into the cable network. Where the cable is brought into the cabinet without a joint, at least 1,200 mm (4 ft) of cable, measured from the cable seal in the base of the cabinet, is required to allow for correct threading and cross-connexion of the cable pairs. Where the strips are pre-wired, the type of cable used should be:

(*a*) Cable Polythene Unit Twin if to be connected to a pressurized, or proposed pressurized, cable or,

(*b*) Cable Polythene Twin if to be connected to a fully-filled or non-pressurized cable.

Note: Air blocks should be provided on all Cable Polythene Unit Twin as necessary.

After the cable has been installed in the cabinet, the duct entry is sealed. The cable sheath is then removed to a point 25 mm (1 in) above the lower bar of the mounting frame, the cable pairs numbered and the sheath securely tied to the lowest jumper ring of the mounting frame. The cable pairs are then passed through the appropriate numbered holes and neatly formed and tied at the rear of the strip so that the cable pairs extend beyond the right-hand supporting slot for 175 mm (7 in) to allow for wire connectors being cut off and subsequently reconnected. Where cross-connexion between E-side and D-side pairs occurs within the same group of 10, direct connexion between the pairs is made by jointing the conductors with wire connectors. If E-side and D-side pairs are not in the same group, cross-connexion is made using jumper wire and wire connectors. The jumper wires are taken through the appropriate jumper holes and rings. After jointing and jumpering, each group of 10 pairs of E-side and D-side pairs is straightened and located in the appropriate support slot and loosely tied.

The advantages of the new type of strip are:

(*a*) reduction in faults due to low insulation and corrosion in damp situations,

(*b*) elimination of disconnexion faults in locations where traffic vibrations results in loose pressure screws, and

(*c*) reduced maintenance by elimination of pressure screws—over-tightening of the pressure screws may result in the screw heads shearing.

The function of the assembly used in cross-connexion cabinets is to provide a flexibility point in the local-line network; it provides a ready means of interconnecting the main-cable pairs (E-side pairs) from the exchange to the cable pairs serving the various subscribers' distribution points (D-side pairs) in a local area. This arrangement leads to considerable economies in the provision and rearrangement of underground plant, mainly due to the economies that may be effected in the provision of expensive main cables between the exchange and the flexibility point.

Q. 8. *Explain how an earth fault on a high-voltage overhead power line can give rise to an induced voltage in nearby overhead and underground telephone line plant.*

What measures can be taken to protect telephone plant from these induced voltages?

Q. 9. *Describe in outline the principles of two types of cable-pressurization system.*

How would a leak in a cable sheath be localized in a cable route which employs one of the systems you have described?

A. 9. The two types of cable pressurization system are:

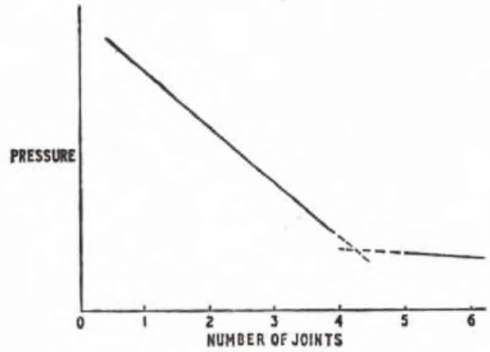
(*a*) the static system used on trunk and junction cables, and

(*b*) the continuous-flow system used on local cables.

In (*a*), the cable is completely sealed then pressurized with dry air supplied from either a compressor-dessicator or from storage cylinders. The pressure in the cable is set at approximately 9 lb/in^2 and monitored by alarm pressure gauges housed at repeater stations. At manholes on the route, a small tube is connected to the joint and terminated in a convenient place near the manhole entrance. This is used to connect a pressure gauge or manometer to the cable.

In (*b*), no attempt is made to seal the cable completely. Air is continuously pumped into the cable from a labyrinth-type compressor-dessicator. The air is dried by passing it over silica-gel beds from which the moisture is driven off, by electric heaters in one type or by dry air on a cycling basis in a second type. Monitoring is by pressure gauges and flow-meters at the exchange and contact pressure gauges housed in cabinets which are connected to a common alarm pair on any one particular cable.

The first indication of a leak on a trunk cable is given by loss of pressure which brings in an alarm at the nearest repeater station. The air pressure along the cable is measured by connecting a manometer to the tubes provided at manholes. The recorded readings are transferred to a graph and take the form shown in the sketch. The leak is at the point where the lines cross.



If the points on the graph where the broken lines cross show the fault to be in the length of cable between two joints then location within the length can be carried out by feeding SF_6 (Sulphurhexafluoride) gas into the cable at, say, joint number 4 and drawing a probe along the duct. The probe is connected to an instrument which measures the ratio of this gas to air.

If the curve shows the fault to be at a joint then the exact position of the hole can be found by coating the joint with a solution which bubbles as the air passes through it or by using an ultrasonic sound detector which produces either an amplified audible note by a frequency-changing system from the high pitched hiss of escaping air, or on some types, gives an indication on a meter.

Where a leak is difficult to locate, the direction of the flow of air can be found by attaching two valves to the cable sheath as far apart as possible and measuring the pressure difference between them with a sensitive manometer. This will give localization within a length by measuring the air flow towards the leak from both ends. A temporary supplementary supply of air may have to be connected to the cable beyond the leak to create the flow.

Q. 10. *What types of anti-creepage devices are used with cables carrying coaxial pairs?*

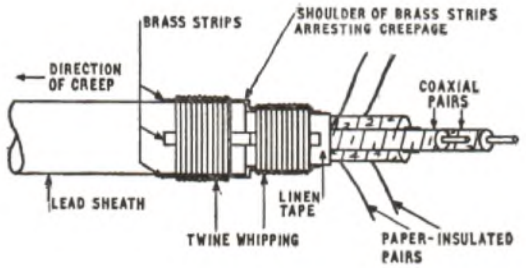
In a coaxial cable how is movement prevented between

(a) cable core and sheath,

(b) centre conductor and outer conductor?

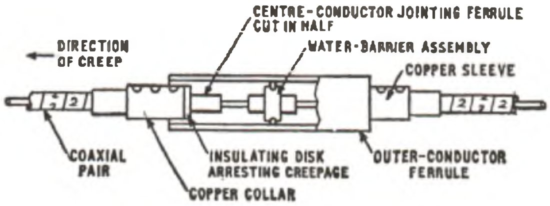
A. 10. The anti-creepage devices used for coaxial cables are usually cable anchors which reduce the effect of damage by constriction inherent in other types of anchor. The anchors are the same as those used for pair-type cables with lead or polythene sheath and were described in the answer to Q. 6, Line Plant Practice B, *Supplement*, Vol. 64, p. 65, Oct. 1971.

The methods devised to anchor the coaxial core and the centre conductors are as follows:



(*a*)

(*a*) Sketch (*a*) shows a method of anchoring the cable core. The lead sheath of the cable is removed and four specially-shaped brass strips are fitted as shown. The strips are held in position by twine whipping. The device has the advantage of simplicity and can be fitted in any joint, the relative movement between the core and sheath being practically halted by the shoulder of the brass strips against the edge of the cable sheath.



(*b*)

(*b*) Sketch (*b*) shows a method of anchoring the centre conductor of a coaxial cable. The device is fitted into a normal joint. A copper collar is fitted over the steel lapping and outer conductor and sweated on. A jointing ferrule is cut in half and soldered on to the centre conductor. The shoulder of this jointing abuts on to an ebonite insulating disk which in turn bears against the collar. The continuity of the outer conductor is thus maintained by the copper collar and outer-conductor ferrule.

TELECOMMUNICATION PRINCIPLES A, 1971

Students were expected to answer six questions, at least one being from questions 9 and 10.

Q. 1. *A series circuit takes* 1·2 *amps from a* 240 *volt* 50 *Hz a.c. supply.*

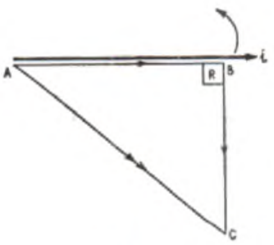
(*a*) *Calculate the resistance and capacitance of the circuit if the power factor is* 0·8 *leading.*

(*b*) *Calculate the resistance and inductance of the circuit if the power factor is* 0·8 *lagging.*

A. 1. The power factor of an a.c. circuit consisting of a resistance, R ohms, in series with a capacitor of reactance X ohms, is given by:

$$\text{power factor} = \frac{\text{resistance}}{\text{impedance}} = \frac{R}{\sqrt{(R^2 + X^2)}}.$$

The phasor diagram of sketch (*a*) illustrates this, where $\angle$ABC is a right angle.



(*a*)

For a series circuit, the current is the same in both the resistor and the capacitor. The voltage across the resistor per ampere is represented by phasor AB in phase with the current. The voltage across the capacitor of reactance X ohms is X volts per ampere, lagging by 90° in phase on the current. This is represented by phasor BC.

The total voltage drop per ampere across the circuit is represented by phasor AC of length Z where Z is the total impedance.

$$\therefore \text{ power factor} = \cos \angle \text{BAC} = \frac{\text{AB}}{\text{AC}} = \frac{R}{\sqrt{(R^2 + X^2)}}.$$

$$\text{Power factor can also be defined as } \frac{\text{real power (watts)}}{\text{apparent power (volts} \times \text{amps)}}.$$

Now, as the current is 1·2 amps on a 240 volt a.c. supply, the impedance numerically $= \frac{240}{1 \cdot 2} = 200$ ohms.

(*a*) When the power factor is 0·8 leading, the phasor diagram is as shown in sketch (*a*).

$$\text{Hence, } 0 \cdot 8 = \frac{\text{AB}}{\text{AC}} = \frac{R}{200}.$$

$\therefore R = 160$ ohms and is a pure resistance.

The reactance $X = \sqrt{(200^2 - 160^2)}$ because triangle ABC is right angled.

$\therefore X = \sqrt{(40 \times 360)}$,

$= 120$ ohms capacitive.

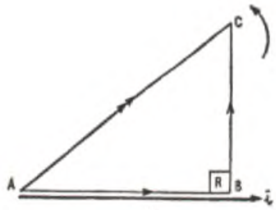
But, reactance of a capacitor $C = \frac{1}{2\pi f C}$ at frequency f Hz.

$$\therefore X = \frac{1}{2\pi f C}.$$

$$\therefore 120 = \frac{10^6}{100\pi C} \text{ where } C \text{ is in microfarads.}$$

$$\therefore \underline{C = 26 \cdot 6\pi \mu\text{F.}}$$

(*b*) When the power factor is 0·8 lagging the phasor triangle is ABC of sketch (*b*). This is the same shape as sketch (*a*), but inverted.



(*b*)

$\therefore R = 160$ ohms as in answer (*a*),

$X = 120$ ohms inductive.

For an inductance L henries, $X = 2\pi f L$ at frequency f Hz.

$$\therefore X = 2\pi 50 \times L.$$

$$\therefore L = \frac{120}{100\pi},$$

$$= \underline{0 \cdot 382 \text{ H}}.$$

Q. 2. (*a*) *Describe briefly, without experimental details, a method for finding the value of a resistance.*

(*b*) *For the circuit shown in Fig.* 1, *calculate, by any method,*

(*i*) *the current flowing in each branch,*

(*ii*) *the power dissipated in branch DB.*

The source is assumed to have negligible internal resistance.

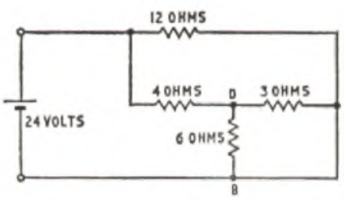
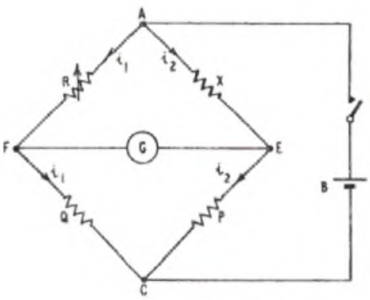


Fig. 1.

A. 2. The Wheatstone bridge offers a good method of resistance measurement. The basic circuit is shown in sketch (*a*). Of the four



(*a*)

resistances wired as a continuous loop, X is the unknown, R is adjustable and calibrated with known values, while P and Q, the resistances in the ratio arms, are accurately known. The source of voltage is the battery B, connected across AC, one diagonal of the bridge. The balance of the bridge is noted on the sensitive centre-zero galvanometer G, bridged across the other diagonal EF.

The bridge is said to be balanced when there is no deflexion on the galvanometer G, when the battery B is switched into circuit. Balance is obtained by adjusting resistance R to give the zero deflexion condition. The potential difference (p.d.) between the points E and F is then zero because no current is flowing between E and F. The current in resistances R and Q must then be the same, i_1, and similarly, the current in resistances X and P must be the same, i_2. The potential drop in resistance R is equal to that in resistance X because there is no p.d. between the points E and F.

Similarly, the p.d. across the resistances Q and P must be the same.

$$\therefore\ i_1 R = i_2 X, \qquad \ldots\ldots (1)$$

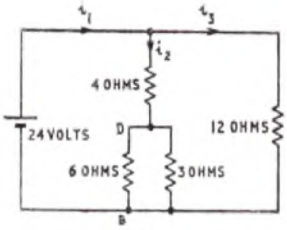
$$\text{and } i_1 P = i_2 Q. \qquad \ldots\ldots (2)$$

Dividing equation (1) by equation (2), gives

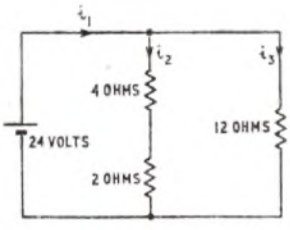
$$\frac{R}{P} = \frac{X}{Q} \text{ which is the condition for bridge balance.}$$

Any one of these four resistances can be calculated provided the other three are known.

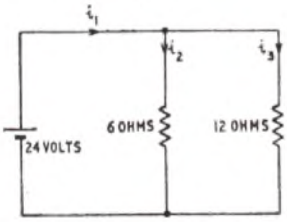
Since neither current nor voltage appears in this equation, the balance obtained is independent of the supply voltage. This is known as a *null* method because it depends on a zero reading of the galvanometer.



(*b*)



(*c*)



(*d*)

The circuit of Fig. 1 can be redrawn as shown in sketch (*b*). Using the relationship for parallel resistances R_1 and R_2 in terms of an equivalent resistance R, gives, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

It can be seen that sketch (*c*) is equivalent to sketch (*b*). Sketch (*d*) is clearly the equivalent of sketch (*b*) and therefore, resistances of 6 ohms and 12 ohms are in parallel across the 24 volt supply.

Then, for the 6-ohm resistance, $i_2 = \frac{24}{6} = \underline{4 \text{ amps}},$

and for the 12-ohm resistance, $i_3 = \frac{24}{12} = \underline{2 \text{ amps}}.$

Hence, the total battery current = 6 amps.

Refering back to sketch (*b*), the current i_2 equals 4 amps in the 4-ohm resistor. By inspection, this divides into the 6-ohm and 3-ohm resistances in the ratio 1 : 2.

$\therefore$ current in the 6-ohm resistance $= \frac{4}{3}$ amps, and

current in the 3-ohm resistance $= \frac{8}{3}$ amps.

Power dissipated in branch DB = (current)^2 × resistance,

$$= \left(\frac{4}{3}\right)^2 \times 6 \text{ W},$$

$$= \underline{10\tfrac{2}{3} \text{ W}}.$$

Q. 3. *Write a description, in the form of a laboratory report, of any experiment made, or demonstration witnessed, using either a transistor or a vacuum diode. The description should include aims, circuit arrangements, any special precautions that must be taken, typical results, graphs and conclusions.*

Q. 4. (*a*) *Explain what happens when an inductive coil is suddenly connected to a d.c. supply and, hence, describe what is meant by self-inductance.*

List factors affecting the self-inductance of,

(*i*) *an air-cored solenoid,*

(*ii*) *an iron-cored solenoid.*

(*b*) *A coil of resistance 8 ohms and inductance 1 H is connected to a 24 volt d.c. source having an internal resistance of 2 ohms.*

(*i*) *Calculate the electro-motive-force (e.m.f.) induced in the coil when the current is increasing at 20 amps/s.*

(*ii*) *Calculate the maximum energy stored in the coil.*

(*c*) *The presence of inductance in a circuit may be either an advantage or a disadvantage. Give a practical example of each case.*

A. 4. (*a*) A current commences to flow when an inductor is connected to a d.c. supply. The current rises slowly from zero. The rise is not instantaneous because the magnetic flux that is associated with the current must increase as the current rises. This changing flux induces a back e.m.f. in the turns of the coil in a direction that opposes the changing current producing the changing flux. The larger the flux-turns linkage, the larger the induced e.m.f. and the slower the rise of current. The property of a coil that leads to this induced e.m.f. is its self-inductance. The inductance L is given by

$$\frac{\text{induced e.m.f. (volts)}}{\text{rate of change of current (amps)}} = \text{inductance (henries)}.$$

(*i*) The inductance of an air-cored solenoid is proportional to the square of the number of turns and to the cross-sectional area of the coil.

(*ii*) Since inductance is related to the magnetic flux arising from a current in the coil, an iron core greatly increases the inductance. The high magnetic permeability of the iron core results in a larger flux for a given current compared with the air core.

(*b*) The circuit of the question has a loop resistance of 10 ohms and an inductance of 1 H.

(*i*) The current is increasing at 20 amps/s and

induced e.m.f. = − inductance × rate of change of current,

$$= \underline{-20 \text{ volts}}.$$

(*ii*) The maximum energy stored $= \frac{1}{2}Li^2$ joules, where i amps is the maximum steady current, given by Ohm's law.

But, the circuit resistance is 10 ohms.

$$\therefore\ i = \frac{24}{10} = 2 \cdot 4 \text{ amps}.$$

$\therefore$ stored energy $= \frac{1}{2} \times 1 \times (2{\cdot}4)^2$,

$$= \underline{2{\cdot}88\text{ J}}.$$

(*c*) Inductance is essential to some applications, e.g. a transformer for use with alternating currents.

It is a disadvantage in circuit wiring, where precautions must be taken to minimize it to reduce sparking on switches.

Q. 5. (*a*) *Comment briefly on the respective advantages and disadvantages of moving-coil and moving-iron electrical indicating instruments.*

(*b*) *What main precautions should be taken when connecting up any electrical indicating instrument to obtain an accurate result?*

(*c*) *A moving-coil electrical indicating instrument, giving full-scale deflexion (f.s.d.) for a current of 1 mA, is to be used to indicate at f.s.d.,*

(*i*) 100 *mA*,

(*ii*) 1 *volt*.

If the coil resistance is 5 ohms, calculate the value of any additional component required in each case. Sketch the circuit arrangements.

A. 5.

(*a*) *Moving-coil instrument*

A moving-coil instrument, such as a milliammeter, is sensitive to small currents. The coil can be made to have a low resistance, e.g. 20 ohms or less, and consequently has a low power consumption. The deflexion is proportional to the current in the coil so the scale has a linear law. Since a large permanent magnet surrounds the movement, external magnetic fields have little or no affect on the instrument. Disadvantages of the moving-coil instrument are that the movement is delicate and must not be overloaded, otherwise damage will occur that is expensive to rectify. The moving-coil meter can only respond to d.c., so that for a.c. measurements a suitable rectifier must be used with it.

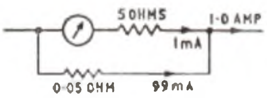
Moving-iron instrument

A moving-iron instrument is more robust than a moving-coil instrument and will stand some degree of overload. It operates on either d.c. or low-frequency a.c., such as 50Hz. It can be made to take higher currents than a moving-coil instrument but it requires more power to operate it. The fixed coil can be provided with taps so that alternative measuring ranges are practicable on one instrument. Basically, the moving-iron instrument operates to a square-law; the deflexion is proportional to the square of its current, but by shaping the moving vane some change in scale law can be produced. The moving-iron meter is susceptible to external magnetic fields unless a magnetic screening case is used to house the instrument.

(*b*) Connecting leads must be short and of low resistance compared to the meter resistance. Connexions must be good and secure.

(*c*) (*i*)

A shunt resistor is required to take 99mA of the current when 1 mA passes through the meter, the circuit for this is shown in sketch (*a*).



(*a*)

For full-scale deflexion, the voltage across the meter may be obtained using Ohm's law.

$$\therefore \text{ voltage} = 0{\cdot}001 \times 5,$$

$$= 0{\cdot}005 \text{ volts}.$$

If R ohms is the shunt resistance,

$$0{\cdot}005 = R \times 0{\cdot}099.$$

$$\therefore \underline{R = 0.0505 \text{ ohm}}.$$

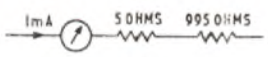
A practical value is 0·05 ohm in shunt.

(*ii*)

A series resistor is needed, of resistance R ohms such that the potential difference across the resistor is 0·995 volts when 1mA flows. This is shown in sketch (*b*).

$$\therefore 0{\cdot}995 = R \times 0{\cdot}001.$$

$$\therefore \underline{R = 995 \text{ ohms}}.$$



(*b*)

Q. 6. *For the circuit shown in Fig.* 2,

(*a*) *calculate the resultant capacitance of,*

(*i*) *the two series capacitors,*

(*ii*) *the two parallel capacitors,*

(*iii*) *the combination of all the capacitors.*

(*b*) *calculate,*

(*i*) *the total energy stored,*

(*ii*) *the potential difference across each capacitor,*

(*iii*) *the charge stored in the* 0·4μ*F capacitor.*

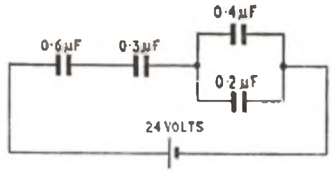


Fig. 2.

A. 6. When capacitors of values C_1 and C_2 are connected in series, the resultant capacitance, C, is given by

$$C = \frac{C_1 \times C_2}{C_1 + C_2}.$$

When the two capacitors are in parallel,

$$C = C_1 + C_2.$$

(*a*) (*i*) The equivalent capacitance of the 0·6μF and the 0·3μF capacitors in series is,

$$C = \frac{0{\cdot}6 \times 0{\cdot}3 \times 10^{-6} \times 10^{-6}}{(0{\cdot}6 + 0{\cdot}3) \times 10^{-6}} \text{ F},$$

$$= \underline{0{\cdot}2\mu\text{F}}.$$

(*ii*) The equivalent capacitance of the 0·4μF and the 0·2μF capacitors in parallel is,

$$C = (0{\cdot}4 + 0{\cdot}2)\,10^{-6} \text{ F},$$

$$= \underline{0{\cdot}6\ \mu\text{F}}.$$

(*iii*) The equivalent capacitance of the circuit is given by the equivalent capacitances of (i) and (ii) in series.

$$\therefore C = \frac{(0{\cdot}2 \times 0{\cdot}6)\,10^{-12}}{(0{\cdot}2 + 0{\cdot}6)\,10^{-6}} \text{ F},$$

$$= \underline{0{\cdot}15\mu\text{F}}.$$

(*b*) (*i*) The circuit is equivalent to a 0·15μF capacitor charged to 24 volts.

$$\therefore \text{ energy stored} = \tfrac{1}{2}CV^2 \text{ J}.$$

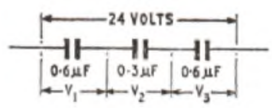
$$= \tfrac{1}{2} \times 0{\cdot}15 \times (24)^2\,10^{-6},$$

$$= \underline{43{\cdot}2\ \mu\text{J}}.$$

(*ii*) When capacitors are connected in series across a d.c. supply, the charge (Q) in each must be the same.

Now, $Q = CV$ coulombs.

The series circuit is shown in the sketch.



$$\therefore Q = V_1 \times 0{\cdot}6 = V_2 \times 0{\cdot}3 = V_3 \times 0{\cdot}6 = 24 \times 0{\cdot}15.$$

$$\therefore V_1 = \frac{24 \times 0{\cdot}15}{0{\cdot}6} = \underline{6 \text{ volts}},$$

$$V_2 = \frac{24 \times 0{\cdot}15}{0{\cdot}3} = \underline{12 \text{ volts}},$$

$$V_3 = \frac{24 \times 0{\cdot}15}{6} = \underline{6 \text{ volts}}.$$

(*iii*) The charge stored in the 0·4μF capacitor is given by $Q = CV_3$.

$$\therefore Q = 0{\cdot}4 \times 6 \times 10^{-6}\text{C},$$

$$= \underline{2{\cdot}4\ \mu\text{C}}.$$

Q. 7. (a) Explain the meaning of relative permeability.

(b) An iron ring has a mean circumference of 0·75 m and a cross-sectional area of 5 × 10⁻⁴m². The magnetization curve for the iron is,

B *teslas (webers per square metre)*	0·9	1·0	1·15	1·25	1·31	1·35	1·38	1·41	1·43
H *ampere-turns per metre (AT/m)*	300	450	800	1,300	1,800	2,300	2,800	3,300	3,800

Plot the B-H curve and from this determine,

(i) the μ_R-H curve, μ_R being the relative permeability,
(ii) the magneto-motive force (m.m.f.) to produce a flux of 650 μWb in the ring.

A. 7. (*a*) The relative permeability of a magnetic material, is the ratio of the flux-density, B_1, in that material when it is used as the core of a solenoid carrying a current, to the flux-density, B_2 in air when the magnetic material is removed. The current must, of course, be the same in each condition.

Relative permeability μ_R is defined by,

$$\mu_R = \frac{B_1}{B_2}.$$

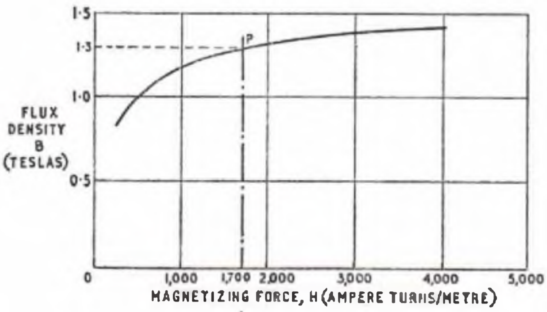
The magnetizing force, *H*, is related to the flux density by the follow-formulae

$$\mu_R = \frac{B}{\mu_0 H},$$

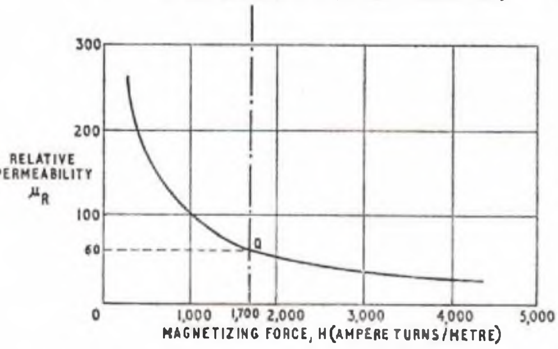
where, μ_0 is the constant magnetic permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7}\ \text{h/m}.$$

(*b*) (*i*) The values of μ_R corresponding to each entry in the given *B–H* table are calculated using the equation above and plotted in sketch (*b*) below the *B–H* curve plotted in sketch (*a*). The relative permeability is not a constant, but diminishes as the magneto-motive-force (m.m.f.) increases.



(*a*)



(*b*)

(*ii*) The flux is 650 μWb, *B* is the flux-density and the cross-sectional area, *a*, is 5 × 10⁻⁴ m².

Therefore, when the total flux = 650 μWb,

$$\text{flux-density } B = \frac{650 \times 10^{-6}}{5 \times 10^{-4}},$$

$$= 1\cdot 3\ \text{T}.$$

Reading from sketch (*a*), the point P gives *H* = 1,700 AT/m for *B* = 1·3 teslas, and from sketch (*b*), the point Q gives μ_R = 60 for *H* = 1,700 AT/m.

Now, for a closed magnetic circuit of length *l* carrying a flux:

$$\text{m.m.f.} = \text{flux} \times \text{reluctance of path},$$

$$= B \times a \times \frac{l}{\mu_0 \mu_R a},$$

$$= B \times a \times \frac{l}{\mu_0 \times \dfrac{B}{\mu_0 H} \times a},$$

$$= l \times H.$$

The length of path, *l*, is given as 0·75 m and *H* = 1,700 AT/m.

$$\therefore\ \text{m.m.f.} = 0\cdot 75 \times 1{,}700,$$

$$= \underline{1{,}280\ \text{AT}}.$$

Q. 8. (a) What is meant by an alternating current? Explain the meaning of,

(i) phase angle,
(ii) peak value,
(iii) r.m.s. value,
(iv) average value.

(b) The graph of Fig. 3 shows one cycle of an alternating electro-motive-force (e.m.f.) of frequency 50 Hz. If this e.m.f. is applied to a circuit having a resistance of 12·5 ohms, what will be (i) the average, and (ii) the r.m.s. value, of current in the circuit?

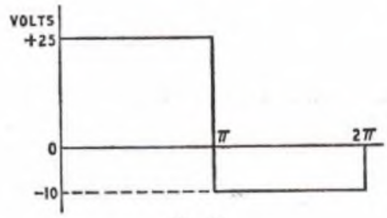


Fig. 3.

A. 8. An alternating current is one that varies cyclically in alternative directions at a regular frequency of repetition. The periodic time is the duration of one complete cycle without repetition. The frequency is the number of complete cycles per second.

(*i*) When an alternating current flows in an impedance, an alternating voltage is produced across the terminals of the impedance. The frequency of this alternating voltage is the same as that of the current producing the voltage. However, the instantaneous zeros of voltage will not occur simultaneously with the current zeros because of the cyclic changes of energy stored in the reactance. The amount by which the two are out of step is the phase difference. This phase difference, when expressed as a fraction of a complete circle, 360° or 2π radians, which represents one cycle of alternation, is known as the phase angle. The voltage can lag or lead on the current according to whether the impedance is capacitive or inductive.

(*ii*) The peak value of an alternating current is its largest instantaneous value. The peak value may be equal above and below the zero value, i.e. for a sinusoidal current, it is alternately positive and negative.

(*iii*) The root-mean-square (r.m.s.) value is the value of direct current that will produce the same heating effect in a resistor in the circuit as does the alternating current It can be calculated for alternating current by taking *n* equally-spaced ordinates on the current-time waveform, squaring them and taking the mean value. The alternate half-cycles both give positive values when squared and are, therefore, additive in a deduction of the heating effect over a whole cycle.

(*iv*) The average value of an alternating current, is taken as the average area between the waveform over one cycle and the zero axis. It is, therefore, the height of a rectangle erected on the time axis for the interval 360° or 2π, that gives an area equal to that between the a.c. waveform and the same intercept on the time axis. The average value of a sinusoidal alternating current must be taken over half a cycle since over a complete cycle the average value is clearly zero, because alternate areas are equally positive and negative.

(*b*) (*i*) The average value of the given waveform must be taken over a complete cycle. The positive and negative half-cycles are not equal, the positive half-cycle being the larger. As the waveform is constant over each half cycle,

$$\text{the average value} = \frac{25-10}{2} = 7\cdot 5\ \text{volts}.$$

By Ohm's law, the average current produced by 7·5 volts in a resistance of 12·5 ohms is

$$\frac{7\cdot 5}{12\cdot 5} = 0\cdot 6 \text{ amps.}$$

(*ii*) Since each half-cycle has a constant value, the r.m.s. value for a whole cycle will be equivalent to a succession of direct currents in the 12·5 ohm resistance.

$$\text{Therefore, r.m.s. voltage} = \sqrt{\left\{\frac{(25^2 + (-10^2)}{2}\right\}},$$

$$= \sqrt{\left(\frac{727}{2}\right)},$$

$$= 19\cdot 0 \text{ volts.}$$

$$\text{Therefore, r.m.s. current} = \frac{19}{12\cdot 5} \text{ amps,}$$

$$= \underline{1\cdot 53 \text{ amps.}}$$

Q. **9.** *Explain how the output resistance and current gain of a transistor can be derived from the collector-current/collector-voltage characteristics.*

Measurements on a transistor connected in common-emitter configuration are given in the following table.

Collector–emitter voltage V_{CE} (volts)	*Collector current I_C (mA)*		
	Base current (μA)		
	−50	−80	−110
2	3·2	4·7	6·3
6	3·9	5·7	7·5
10	4·6	6·7	8·7

(*a*) *Plot the collector-voltage/collector-current characteristics.*

(*b*) *Determine (i) the output resistance and (ii) the current gain, for a base current of −75 μA.*

A. **9.** The output resistance of a circuit is given by the ratio of the change in output voltage to the known change in output current producing this voltage change. This is basically an application of Ohm's law. For a transistor, connected in the common-emitter mode, the output is obtained across the collector–emitter connexion, and the output current is the collector current.

Therefore, the output resistance is given by the expression

$$\frac{\text{variation in collector–emitter voltage}}{\text{corresponding variation in collector current}}.$$

This ratio is the slope of the collector-voltage (V_{ce})/collector-current (I_c) characteristic for a specified value of base current.

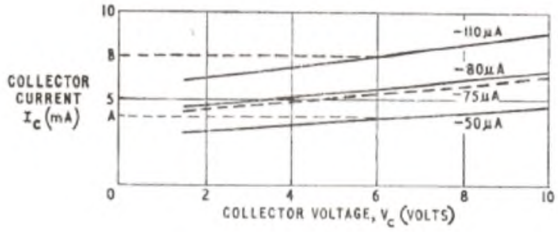
The current gain is the ratio of variation in output current to the variation of input current producing it, for a constant value of V_{ce}. In a common-emitter amplifier, the input is connected between base and emitter and the output is across the collector and emitter.

Therefore, the current gain is given by the expression

$$\frac{\text{variation in collector current}}{\text{corresponding variation in base current}}.$$

This can be derived from the V_{ce}/I_c curves when the base current values are also included.

(*a*) The curves of V_{ce} against I_c and I_b are plotted in the sketch.



The curve for a base current of -75μA is drawn in by extrapolation as shown. It is, for practical purposes, a straight line.

(*b*) (*i*) For the points where $V_c = 2$ volts and $V_c = 10$ volts the values of collector current are $I_c = 4\cdot 5$ mA and $6\cdot 3$ mA.

Therefore, output resistance = slope of the characteristic,

$$= \frac{10 - 2}{(6\cdot 3 - 4\cdot 5) \times 10^{-3}} \text{ ohms,}$$

$$= \frac{8}{1\cdot 8} \times 10^3 \text{ ohms.}$$

$$= \underline{4{,}400 \text{ ohms.}}$$

(*ii*) The current gain is almost constant over the range of collector voltages given, because the characteristics are almost parallel.

If values for $V_{ce} = 6$ volts are taken as typical,

$i_c = 7\cdot 5$ mA for base current of 110 μA,

and, $i_c = 4\cdot 0$ mA for base current of 50 μA.

$$\text{Therefore, current gain} = \frac{(7\cdot 5 - 4\cdot 0) \times 10^{-3}}{(110 - 50) \times 10^{-6}},$$

$$= \frac{3\cdot 5}{60} \times 10^3,$$

$$= \underline{54\cdot 3.}$$

Q. **10.** *Give a sectional sketch, and describe the principle of operation, of one type of telephone earpiece (receiver).*

Explain, with reasons, whether the following statement is correct: A telephone receiver loses sensitivity if the flux from its permanent magnet is reduced.

A. **10.** This is a standard bookwork question on a telephone earpiece. The answer can describe the stalloy diaphragm type, rocking armature or moving coil types.

The statement is correct. Sensitivity in any of the above types of telephone earpiece is reduced if the flux from the permanent magnet is reduced. The sensitivity depends on the magnitude of the displacement of the armature, diaphragm or moving coil for a given actuating current. The displacement depends on the force produced by the actuating current variations and this force, due to the reaction of the current on the permanent magnetic flux, is proportional to the square of the flux. Hence, if the flux weakens, the sensitivity falls.

TELECOMMUNICATION PRINCIPLES B, 1971

Students were expected to answer any six questions

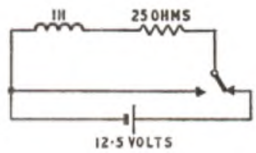
Q. **1.** (*a*) *A coil of* 10 *H with a resistance of* 25 *ohms has a p.d. of* 12·5 *volts across it from a d.c. supply of steady voltage. The supply is cut off and the coil is short-circuited immediately. Write down the current-time relation for the short-circuited coil.*

Sketch this curve. What is the value of current at zero time?

(*b*) *Calculate the energy in the circuit immediately after the battery is disconnected. Where has this energy been stored? In what form does it appear after the coil is short-circuited?*

(*c*) *Explain the meaning of 'time constant', calculate its value for this circuit, and explain its use in telecommunication engineering.*

A. **1.** The circuit is shown in sketch (*a*). The changeover switch disconnects the battery and immediately short-circuits the coil.



(*a*)

(*a*) By Ohm's law, the steady current from the battery $= \dfrac{12\cdot 5}{25}$,

$= 0\cdot 5$ amps.

The current i decays slowly according to the exponential law:

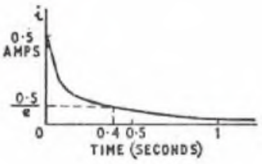
$$i = \frac{E}{R} e^{-Rt/L},$$

where E = battery voltage across circuit,
R = total resistance in ohms,
L = inductance in henries,
e = base of Naperian logarithms.

$$\therefore i = \frac{12 \cdot 5}{25} e^{-(25/10)t},$$

$$\text{or, } \underline{i = \tfrac{1}{2} e^{-2 \cdot 5t}}.$$

The curve is shown in sketch (b).



(b)

(b) The energy is stored in the magnetic field of the coil, owing to a current of i amperes flowing in the inductance of L henries.

$$\begin{aligned} \text{energy} &= \tfrac{1}{2} L i^2, \\ &= 5(0 \cdot 5)^2, \\ &= \underline{1 \cdot 25 \text{ joules.}} \end{aligned}$$

When the switch is operated to disconnect the battery, this energy is returned to the circuit as a flow of current. It is dissipated as heat in the resistance of 25 ohms.

Time constant is the time taken for the decaying current to fall to $\frac{1}{e}$ of its initial steady value.

(c) Time constant $= \dfrac{L}{R} = \dfrac{10}{25} = 0 \cdot 4$ seconds.

The time constant is a useful indication of the rate of change to be expected for the current in a circuit when the applied voltage is suddenly changed.

Q. 2. (a) *In connexion with amplitude modulation, explain the meaning of the terms 'carrier frequency' and 'sidebands'.*

(b) *Why is it usual to use a modulated radio frequency carrier in a radio transmission system?*

(c) *What advantages are obtained by using frequency translation in line transmission?*

Q. 3. *What do you understand by 'reactance'? Why is it sometimes negative?*

(b) *A capacitor of* 4·0 *µF is connected in series with a resistance of* 300 *ohms across an a.c. source. What other factor must be known before the impedance of the circuit can be calculated?*

This series circuit is connected across the 240 *volt a.c. domestic supply. Find, using phasor diagrams, the current flowing and its phase angle relative to the supply voltage. Check your answer by using operator j.*

An inductor of 2·5 *H and* 100 *ohms resistance is now connected in with the first circuit. Calculate by any method the magnitude and phase angle of the impedance across the supply.*

A. 3. (a) Reactance is the property of an inductor (inductance) or a capacitor (capacitance) that offers opposition to the passage of an alternating current. The value of reactance depends on the frequency of the a.c. By convention, inductive reactance is positive and the voltage is said to lead the current by 90° $\left(\frac{\pi}{2}\right)$. For a capacitance, reactance is negative, the current leading the voltage between the terminals by 90°. The phase change caused by an inductance is therefore, 180° (π) different from that caused by a capacitance.

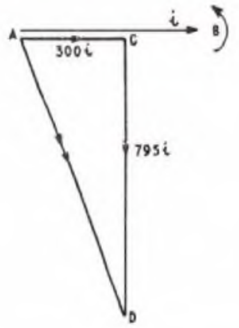
Reactance is frequency dependent.

At a frequency f Hz, inductive reactance $= 2\pi f L$ ohms,

$$\text{capacitive reactance} = \frac{-1}{2\pi f C} \text{ ohms.}$$

(b) The frequency of the a.c. supply must be known before reactance can be calculated.

The phasor diagram is shown in sketch (a). The current, i, in the series circuit is represented by AB.



(a)

The voltage drop in the resistance of 300 ohms = 300 i volts. This is in phase with the current i, represented by phasor AC. The reactance of 4·0 µF at the supply frequency 50 Hz,

$$\begin{aligned} &= \frac{-\mathrm{j}}{2\pi \times 50 \times 4 \cdot 0 \times 10^{-6}}, \\ &= \underline{-795 \text{ j ohms.}} \end{aligned}$$

The voltage drop in the capacitance is $-795\, i$ j volts. This is shown by phasor CD lagging 90° on the current.

The resultant voltage across the whole circuit is the diagonal AD.

$$\begin{aligned} \text{The impedance of the circuit} &= \sqrt{(300^2 + 795^2)}, \\ &= 850 \text{ ohms.} \end{aligned}$$

$$\begin{aligned} \text{The phase angle } \angle \text{CAD} &= \tan^{-1} \frac{795}{300}, \\ &= \tan^{-1} 2 \cdot 65, \\ &= 69° \, 20'. \end{aligned}$$

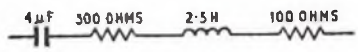
Using operator j as an alternative gives:

$$\begin{aligned} \text{Impedance } Z &= 300 - 795\,\mathrm{j}, \\ &= 300 - 800\,\mathrm{j} \text{ approximately.} \\ \therefore Z &= 100\sqrt{(9 + 64)}, \\ &= 855. \\ \theta &= \tan^{-1}\left(\frac{795}{300}\right), \\ &= \underline{69° \, 20'}. \end{aligned}$$

$$\text{The current, } i = \frac{\text{voltage}}{\text{impedance}} = \frac{240}{855} = 0 \cdot 28 \text{ amps.}$$

Therefore, a current of <u>0·28 amps</u> flows in the series circuit leading the supply voltage by <u>69° 20'</u>.

$$\begin{aligned} \text{The reactance of an inductor of } 2 \cdot 5 \text{ H} &= 2\pi f L \mathrm{j}, \\ &= 2\pi 50 \times 2 \cdot 5 \mathrm{j}, \\ &= \underline{784 \text{ j}} \text{ ohms.} \end{aligned}$$



(b)

The series circuit is shown in sketch (b).

$$\begin{aligned} \text{Total reactance} &= \frac{-\mathrm{j}}{2\pi f C} + \mathrm{j} 2\pi f L, \\ &= -795\,\mathrm{j} + 784\,\mathrm{j}, \\ &= -11\,\mathrm{j}. \\ \text{Total resistance} &= 400 \text{ ohms.} \\ \text{Therefore, impedance} &= 400 - 11\,\mathrm{j}. \\ \text{Magnitude of the impedance} &= \sqrt{(400^2 + 11^2)}, \\ &\simeq \underline{400 \text{ ohms.}} \end{aligned}$$

$$\begin{aligned} \text{Phase angle of the impedance} &= \tan^{-1} \frac{11}{400}, \\ &= \tan^{-1} 0 \cdot 028, \\ &= \underline{1° \, 36'.} \end{aligned}$$

Q. 4. When a transformer is considered as 'ideal' for calculation purposes, what practical features are ignored?

A transformer to be considered as 'ideal' operates from a 200-volt a.c. source to supply 5,000 W to a resistor of resistance 50 ohms. Calculate

(a) the voltage across the resistor,
(b) the turns of the transformer,
(c) the primary current on load,
(d) the equivalent input resistance that the loaded transformer presents to the supply.

A. 4. An ideal transformer has no energy losses in its winding or iron core and has no magnetic leakage between its coils. It is a useful theoretical concept to help simplify the study of transformers. Let R be the load and V the voltage across it.

(*a*) Then the power into the load $= \frac{V^2}{R}$,

$= 5{,}000$ watts.

Since $R = 50$ ohms, $\underline{V = 500 \text{ volts.}}$

(*b*) In an ideal transformer, the turns ratio (n) between the primary windings and the secondary windings is the input-to-output voltage ratio.

$$\text{Therefore, } n = \frac{200}{500},$$

$$\text{or, } \underline{n = 2{:}5.}$$

(*c*) The input power equals the output power in an ideal transformer. For a 200-volt input and input current i,

$$200\, i = 5{,}000.$$

$$\text{Therefore, } \underline{i = 25} \text{ amps.}$$

(*d*) The impedance ratio of a transformer is the square of the turns ratio.

$\therefore$ equivalent input resistance $= (0{\cdot}4)^2 \times 50$,

$= \underline{8 \text{ ohms.}}$

Q. 5. A laminated iron core is constructed of semi-circular iron stampings having mean radius 50 mm with two equal air gaps as in Fig. 1.

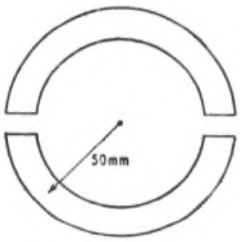


Fig. 1

The core has a square cross-section 10 mm by 10 mm. The B/H curve for the iron is given in the following table.

B teslas (webers per square metre)	0·5	0·75	0·85	1·03	1·20	1·29	1·35
H ampere-turns per metre	50	150	250	500	1,000	1,500	2,000

A coil of 2,400 turns is wound uniformly round the core and the leakage factor is 1·2 when each air gap is 1 mm long.

(a) Plot the B/H curve.
(b) Find the relative permeability when the flux in each air gap is 65 μWb (microwebers).
(c) Calculate the current in the coil to give this flux.

A. 5. The flux in each air gap is 65 μWb and the cross-sectional area of each is 100 mm^2.

$$\text{Therefore, flux density} = \frac{65 \times 10^{-6}}{100 \times 10^{-6}},$$

$$= 0{\cdot}65 \text{ T.}$$

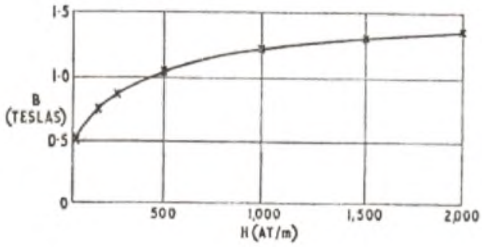
The leakage factor $= 1{\cdot}2$.

The flux-density in the iron

$= 1{\cdot}2 \times 0{\cdot}65$,

$= 0{\cdot}78$ T,

(*a*) From the curve plotted in the sketch,



$$H = 155 \text{ AT/m, when } B = 0{\cdot}78 \text{ T}$$

where H is the magneto-motive force and B is the flux-density.

Now, $\mu_r\mu_0 = \frac{B}{H}$, where μ_r is the relative permeability and μ_0 is the permeability of free space $= 4\pi \times 10^{-7}$ H/m.

(*b*) The relative permeability $\mu_r = \frac{B}{\mu_0 H}$,

$$= \frac{0{\cdot}78}{4\pi \times 10^{-7} \times 155},$$

$$= \underline{3{,}950.}$$

(*c*) It is necessary to find the reluctance of the magnetic path. This is the sum of the reluctance of the iron plus the reluctance of the air.

$$\text{Reluctance} = \frac{l}{\mu_r\mu_0 a}.$$

Where l is the length of magnetic path in metres, a is cross-sectional area in square metres, μ_r is relative permeability, μ_0 is magnetic permeability of free space.

Path length (l) of the half-circle in iron, neglecting the 1 mm due to the air gap,

$= \pi \times 50 \times 10^{-3}$ m,

$= 0{\cdot}157$ m.

$a = 100 \times 10^{-6}$ m^2.

$$\text{Reluctance of iron path} = \frac{0{\cdot}157}{3{,}950 \times 4\pi \times 10^{-7} \times 10^{-4}},$$

$$= 0{\cdot}3165 \times 10^6.$$

$$\text{Reluctance of one air gap} = \frac{0{\cdot}001}{4\pi \times 10^{-7} \times 10^{-4}},$$

$$= 7{\cdot}96 \times 10^6.$$

Total reluctance of whole ring of two air gaps plus two semicircles of iron

$= 2 \times 10^6 (0{\cdot}3165 + 7{\cdot}96)$,

$= 2 \times 10^6 \times 8{\cdot}28$,

$= 16{\cdot}56 \times 10^6$.

But, in a magnetic circuit, flux $\times$ reluctance $=$ magneto-motive force (m.m.f.).

Therefore, m.m.f. $= 78 \times 10^{-6} \times 16{\cdot}56 \times 10^6$,

$= 1{,}290$ Ampere turns.

Since the coil has 2,400 turns,

$$\text{current} = \frac{1{,}290}{2{,}400},$$

$$= \underline{0{\cdot}537 \text{ amps.}}$$

Q. 6. (a) Define the decibel.

Explain why a statement of decibels is often related to a value of power.

(b) In the circuit shown in Fig. 2 an a.c. source maintains 10 volts r.m.s. across AB. Calculate

(i) the r.m.s. voltage across CD
(ii) the ratio in decibels between the power in the 150-ohm resistor in Fig. 3 and the 150-ohm resistor CD in Fig. 2

(c) If the 150 *ohm resistor in Fig.* 3 *were changed to* 300 *ohms, what would be the answer to (b) (ii)?*

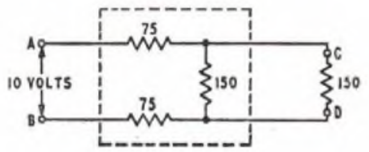


Fig. 2

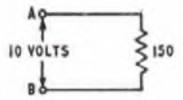
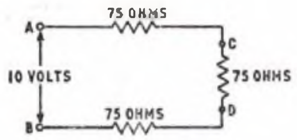


Fig. 3

A. 6. (*a*) Ten times the logarithm to base 10 of the ratio of two powers is the difference in decibels between the powers.

It is usual to refer to powers by their decibel difference with relation to 1 watt, or milliwatt for small powers (dB relative to 1mW.). The decibel notation then offers a way of stating a precise power value because it is a ratio between the power value and a known power.

(*b*) The circuit of Fig. 2 is equivalent to that in the sketch because the two 150 ohm resistors are in parallel.



(*i*) The voltage across CD is then $\frac{1}{3}$ of that across AB, i.e. $\frac{10}{3}$ volts.

(*ii*) The power in the 150 ohms resistor in Fig. 3 $= \frac{10^2}{150}$,

$= 0 \cdot 67$ W.

Because the voltage across CD $= \frac{10}{3}$ volts,

the power in the 150-ohm resistor in Fig. 2 $= \left(\frac{10}{3}\right)^2 \frac{1}{150}$,

$= \frac{100}{1{,}350}$,

$= 0 \cdot 074$ W.

The ratio in decibels $= 10 \log \frac{0 \cdot 67}{0 \cdot 074}$,

$= 10 \log 9 \cdot 0$,

$= 9 \cdot 54$ dB.

(*c*) If the resistor in Fig. 3 was changed to 300 ohms,

$$\text{Power} = \frac{10^2}{300},$$

$$= 0 \cdot 33 \text{ W}.$$

$$\therefore \text{Ratio in decibels} = 10 \log \frac{0 \cdot 33}{0 \cdot 074},$$

$$= 10 \log 4 \cdot 5,$$

$$= -\,6 \cdot 53 \text{ dB}.$$

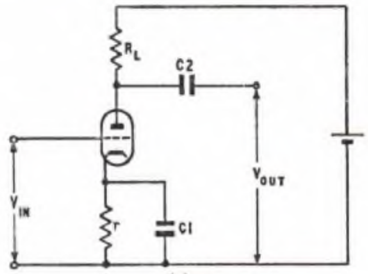
Q. 7. Draw the circuit of a single-stage valve amplifier with a resistive load, and explain its operation.

Give a simple equivalent circuit and explain the reason for each element in it.

Hence, deduce an expression for the amplification in terms of relevant parameters.

What conditions must be satisfied for this expression to apply?

A. 7. The basic circuit of a single-stage valve amplifier is shown in sketch (*a*). The anode of the valve has a load resistance R_L shown in series with the battery, positive terminal towards the anode. This anode battery provides the d.c. path through the valve via the cathode resistor r. The input voltage is applied between the grid and the negative side of the cathode resistor. The d.c. voltage drop across r gives the positive bias necessary to keep the grid–cathode p.d. negative for all permissible values of alternating input voltage. The capacitor C1 which is large in value, e.g. 50 μF, gives a relatively low-impedance

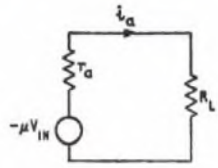


(*a*)

to alternating voltages. This ensures that resistor r only produces an effective p.d. to the steady d.c. in the cathode circuit. The capacitor C2 isolates the a.c. from the d.c. in the anode circuit but offers low impedance to the amplified alternating signal which passes on to the output circuit.

Conduction through the valve occurs by virtue of a stream of electrons emitted from the cathode. When an alternating potential, the input signal, is supplied to the valve between grid and cathode the grid–cathode potential varies strictly in sympathy with the input signal. This causes the number of electrons allowed to pass to the anode to vary in sympathy. The varying current that emerges into the anode circuit generates an alternating voltage across the output. This voltage is much larger than the input because it is developed across a large anode load resistor (e.g. 10 k ohms). Hence, the valve acts as a voltage amplifier.

A simple equivalent circuit in the form of a constant voltage generator in series with r_a, the a.c. resistance of the valve and R_L, the external



(*b*)

load resistance is shown in sketch (*b*). The valve is a generator of e.m.f. μV_{in}, where μ is the amplification factor under the working conditions specified. The equivalent circuit, therefore, represents the impedances in the closed loop of the anode-load–cathode–anode path, with a generated voltage $-\mu V_{in}$ to replace the action of the valve in amplifying the input voltage V_{in}. The negative sign denotes the phase reversal introduced by the valve between the grid circuit and the anode circuit.

Then, by Ohm's law, anode current $i_a = \frac{-\mu V_{in}}{r_a + R_L}$.

$$\text{The output voltage} = i_a R_L = \frac{-\mu V_{in} R_L}{r_a + R_L}.$$

$$\text{Amplification} = \frac{V_0}{V_{in}} = \frac{-\mu R_L}{r_a + R_L}.$$

But, $\mu = r_a g_m$ where, g_m is the mutual conductance of the valve.

$$\text{Therefore, amplification} = \frac{-r_a R_L g_m}{r_a + R_L},$$

which is expressed in terms of common valve parameters. This expression will only apply for small signal conditions when the valve operates as a linear amplifier.

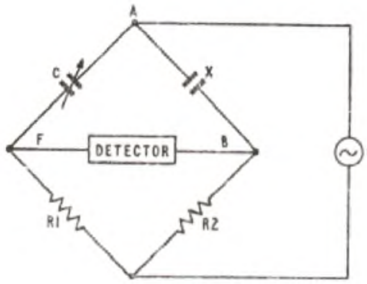
Q. 8. Give the circuit of an a.c. bridge suitable for measuring loss-free capacitors.

How would this be modified to measure capacitors having electrical loss?

Derive the conditions for balance of each arrangement.

Explain why an accurate measurement is more difficult to obtain in the second case.

A. 8. Sketch (*a*) shows the circuit of an a.c. bridge suitable for measuring high-grade capacitors with negligible loss. The basic principle of the a.c. bridge is similar to that of the Wheatstone bridge but the effect of inductance and capacitance is due to their reactance. In the simplest capacitance bridge, there are two resistive arms and two capacitive arms. C is a calibrated adjustable low-loss capacitor, X is the unknown high-grade capacitor. R1 and R2 are stable resistors of known values. The source of a.c. (O) is an oscillator of the frequency



(*a*)

at which the capacitor is to be measured. The detector can be headphones, or a tuned amplifier and a detector, with a pass frequency equal to that of oscillator O.

Reactance of capacitor C $= \frac{1}{2\pi fC}$,

where C is the capacitance of capacitor C.

Similarly, that of capacitor X $= \frac{1}{2\pi fX}$,

where X is the capacitance of capacitor X.

Then for balance, there must be no input across the detector FB. Hence, the voltages across capacitors C and X are equal as are those across resistor R1 and R2. Also the current (i_1) in capacitor C and resistor R1 must be equal, because there is no current in FB. Similarly the current i_2 in capacitor X and resistor R2 is equal.

Therefore, $\dfrac{\frac{i_1}{2\pi fC}}{R1\, i_1} = \dfrac{\frac{i_2}{2\pi fX}}{R2 i_2}$.

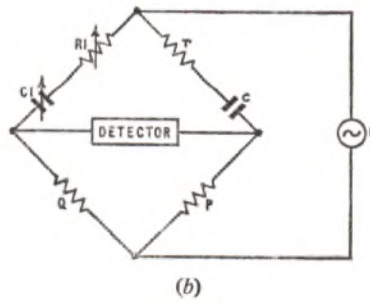
Where $R1$ is the resistance of resister R1 and $R2$ is the resistance of resistor R2.

The currents and frequency f cancel out leaving as the balance condition

$$X = \frac{R1C}{R2}.$$

This simple bridge circuit is only satisfactory if a good balance can be obtained, i.e. the detector can be brought to zero. For this condition in an a.c. bridge the voltages at F and B must be in phase as well as equal in amplitude, which implies that capacitors X and C have only pure capacitance, and no equivalent resistance.

For the measurements of capacitors with some loss the most convenient circuit depends on the degree of magnitude of the equivalent loss resistance in the unknown capacitor. The bridge circuit in sketch (*b*) can be used if the capacitor is of poor quality so that it can



(*b*)

be considered as equivalent to a capacitance C in series with a resistance r that is not too large. The adjustable calibrated resistor R1 (resistance $R1$) which will be of high value, is required to provide phase correction between F and B. Balance is obtained by adjustment of capacitor C1 and resistor R1 in alternate steps until voltages equal in magnitude and phase exist between F and B.

At balance,

$$\frac{R1 - \frac{j}{2\pi fC1}}{Q} = \frac{r - \frac{j}{2\pi fC}}{P}.$$

$$\therefore PR1 - \frac{j}{2\pi f}\frac{P}{C1} = Qr - \frac{j}{2\pi f}\frac{Q}{C}.$$

Equating real terms gives;

$$PR1 = Qr.$$

$$\therefore r = R1\frac{P}{Q}.$$

Equating imaginary terms and dividing throughout by $\frac{1}{2\pi f}$ gives:

$$\frac{P}{C1} = \frac{Q}{C}.$$

$$\therefore C = \frac{Q}{P}C1.$$

An accurate null in the detector is more difficult to obtain in this a.c. bridge because during the adjustment for balance the variables C1 and R1 are inter-related. Small adjustments of capacitor C1 and resistor R1 must be made in turn in order to approach a balance by small steps.

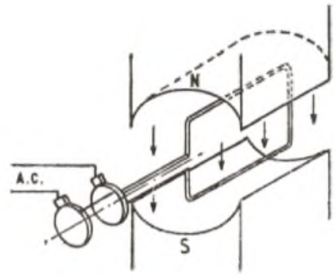
Q. 9. *Describe briefly the essential parts of a simple rotary a.c. generator.*

Assuming that the e.m.f. induced in each conductor is sinusoidal, deduce an expression for the r.m.s. output voltage given by such a machine in terms of the numbers of armature conductors, the air-gap flux, the number of poles in the field system and the speed of rotation.

How is the air-gap flux produced,

(*a*) *in a very small machine,*
(*b*) *in a large alternator?*

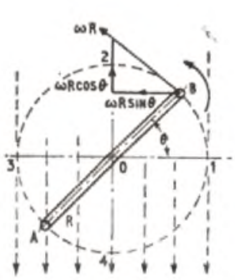
A. 9. The essential parts of a simple a.c. generator are shown in sketch (*a*) and are:

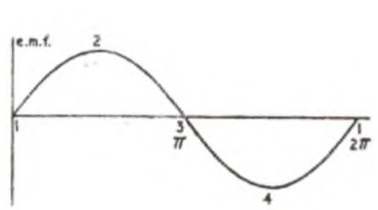


(*a*)

(*i*) The field magnets giving alternate north and south poles that set up a strong magnetic flux between poles.
(*ii*) A rotating coil turning between the field poles so as to cut the lines of force. This coil on its shaft comprises the armature.
(*iii*) Two insulated slip rings on the armature shaft, to which each end of the coil in connected.
(*iv*) Insulated carbon brushes fixed to the frame of the machine acting as current collectors by sliding contacts, one with each slip ring.

As the coil, with the armature, rotates, it cuts the lines of magnetic flux. An e.m.f. must, therefore, be induced in the coil, according to Faraday's Law of magnetic induction. In a practical generator there will be many conductors, n, in the armature coil and several pairs p, of field poles—alternately north and south. However, the simple theoretical machine can be taken as in sketch (*b*) which shows the end view of one turn on the armature, with conductors in section A and B, rotating about a centre O. Let a magnetic field, considered here as parallel and uniform over the area of the coil, be as shown. Let the plane of the armature turn denoted by AB make an angle θ with the





(*b*)

horizontal. The motion of the conductor velocity, v m/s, will be tangential to the circle through which it rotates. This tangential motion can be resolved into $v \sin\theta$ at right angles to the magnetic field and $v \cos\theta$ parallel to it. The component $v \cos\theta$ will not cause any induced e.m.f. The component $v \sin\theta$ will give rise to an e.m.f. in each conductor given by $e = v \sin\theta \times l \times B$, where l is the length of the conductor in metres and B is the magnetic field strength in teslers.

For a coil of N turns, the e.m.f. induced instantaneously $= 2lnBv\sin\theta$. The tangential velocity $= 2\pi R$ m/s where R is the coil radius in metres.

Hence, the instantaneous e.m.f., e, is $2NRBl \sin\theta$, or if there were p pairs of poles,

$$e = 2\ NRpBl \sin\theta.$$

This is only an approximation in practice as the magnetic flux is not uniform, nor is it a parallel field.

(*a*) The air gap flux is produced in very small machines by permanent magnets.

(*b*) In large a.c. generators electro-magnetic field systems are necessary, energized by d.c. The d.c. is normally provided by a d.c. generator on a shaft extension of the main armature. It is then convenient to make the field poles, the "rotor", rotate within the air-gap and the conductors in which the e.m.f. is induced are embedded in slots on the inner surface of the stationary outer cylinder, the "stator".

Q. 10. *Describe a method of calibrating any form of a.c. voltmeter, setting out your answer in the form of a report of a laboratory experiment. Comment on the limits of accuracy for the calibration and the effect of the waveform and frequency on the accuracy obtained.*

A. 10. *Experiment*: To calibrate a rectifier-type a.c. voltmeter on a frequency of 50 Hz. over the range 0–10 volts.

Equipment required:

(*a*) Mains supply at 50 Hz with a transformer to give an adjustable voltage of 0–20 volts at a rating of not less than 20 W.

(*b*) A slide-wire resistor of maximum value about 24 ohms. The slider must provide a smooth voltage-tap over the whole length of the resistor. It must be rated at about 10 W maximum and have low temperature co-efficient wire.

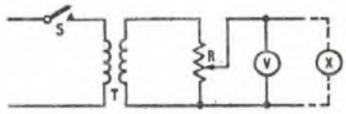
(*c*) A sub-standard a.c. voltmeter calibrated over the range 0–10 volts on a sinusoidal supply.

(*d*) A 240-volt on-off switch.

Method:

The transformer T, switch S and resistor R are wired as shown in the sketch. One side of the resistor is connected to the sub-standard voltmeter V and to the voltmeter X to be calibrated. The two voltmeters are connected in parallel, the other terminal going to the slider of resistor R.

The switch is closed. The slider on resistor R is adjusted until the reading on voltmeter V reads 10 volts. The voltmeter X will now also deflect to 10 volts so a mark is made on its scale at this 10 volt point. If voltmeter X is already scaled, the error in its reading is also noted. The slider is re-adjusted until voltmeter V reads 9 volts. The error on voltmeter X is again noted and its scale is marked. This is repeated, down to a reading of 1 volt.



Results:

A graph is drawn showing the true voltage reading vertically against the scale reading of voltmeter X horizontally. If the existing scale on voltmeter X has no errors, the result is a straight line at 45° between the two axes.

Sources of error:

The effect of a non-sinusoidal voltage wave-form on the rectifier meter being calibrated can give an error. The transformer voltage output should be a pure sine wave and the load resistor R gives the optimum condition for this. This is a comparison method so the stability of the load resistor R generating the test voltage is not primary importance. For the same reason, the voltmeter currents will not cause any error. The precise frequency of the supply is unimportant provided both the a.c. voltmeters have low input reactance.

Accuracy:

The accuracy of calibration of the sub-standard voltmeter V sets the basic limit to accuracy in the experiment. A 2 per cent accuracy at mid-scale would be a good practical figure. A high-grade instrument could indicate to 1 per cent at full-scale deflexion. The readings on voltmeters X and V could be taken to one tenth of a volt if the parallex errors were eliminated, for example if both voltmeters V and X were fitted with mirrors behind their pointers to ensure that readings were taken at right angles to the scales every time.

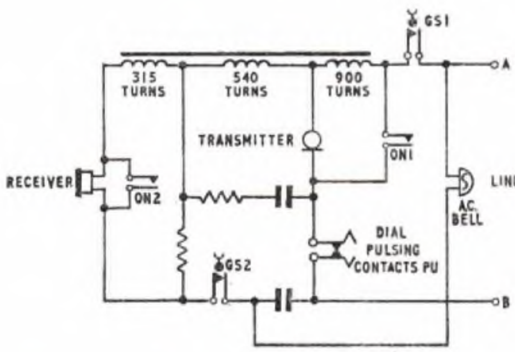
TELEPHONY AND TELEGRAPHY A, 1971

Students were expected to answer any six questions

Q. 1. *Sketch a simplified circuit of a telephone instrument suitable for a subscriber connected to an automatic exchange.*

Explain the functions of the various contacts when a call is originated. Which of the contacts is subject to the greatest wear? Give reasons.

A. 1. The circuit of a telephone instrument, suitable for connexion to an automatic exchange, is shown in the sketch. There are two groups of contacts in the instrument.



(*a*) GS1 and GS2 are the gravity-switch contacts which are operated when the handset is removed from the rest and broken when the handset is restored. When operated, GS1 extends the A-and B-wires of the line into the instrument and provides the calling-signal loop via the A-wire, GS1, induction-coil winding, transmitter, dial-pulsing contacts to the B-wire. GS2 completes the receiver circuit and, when the handset is on the rest, prevents d.c. flowing from the A-wire via the bell-coils, receiver circuit, induction coil, transmitter, PU contacts to the B-wire.

(*b*) ON1, ON2 and PU contacts are associated with the dial. ON1 and ON2 operate whenever the dial is moved from its rest position and remain operated until the dial returns to its rest position. ON1 provides a low-resistance dialling loop by short-circuiting the transmitter and induction-coil winding. ON2 short circuits the receiver thus preventing any impulse noise reaching the telephone earpiece during dialling. The dial-pulsing contacts PU make and break the low-resistance loop provided by ON1 to produce loop-disconnect signalling to operate the exchange equipment. The number of operations of the contacts depends upon the digit dialled.

GS1 and GS2 are operated at the beginning of a call and remain operated for the duration of the call; ON1 and ON2 operate and break once for each operation of the dial and PU makes and breaks between 1 and 10 times for each operation of the dial depending upon the digit dialled. As a result, PU is operated many more times than GS1, GS2, ON1 and ON2 and suffers considerably more wear. In addition, the dial-pulsing contacts are operated in the low-resistance loop prepared by ON1 and the amount of current in the PU contact circuit is more than that through the other contacts, adding further to the wear.

Q. 2. *A telephone relay has two windings, each having the same number of turns and resistance. It can be connected across the exchange battery in different arrangements as follows:*

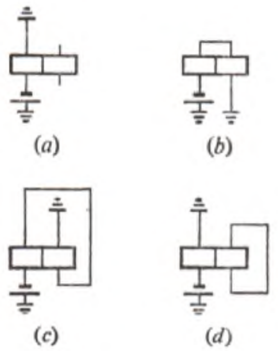
(*a*) *using only one winding, leaving the second disconnected,*
(*b*) *using both windings connected in series aiding, or*
(*c*) *using both windings connected in series opposition.*

Explain why the relay would perform differently under each of the three conditions.

What would be the effect of applying a short-circuit across the second winding in arrangement (*a*)?

A. 2. Let the two coils of the relay each have N turns, resistance R and inductance L. Let the exchange battery voltage be E. When connected as shown in sketch (*a*), with only one coil in circuit, the relay operates with a current equal to $\frac{E}{R}$ amps flowing through N turns and with tractive force on the armature proportional to $\frac{E \times N}{R}$ ampere-turns.

With both coils connected in series-aiding as in sketch (*b*), the relay operates but with a current equal to $\frac{E}{2R}$ flowing through $2N$ turns, and ampere-turns equal to $\frac{2 \times E \times N}{2R}$ to provide the tractive force on the armature. Therefore, the magneto-motive force and the force on the armature are the same as in (*a*). The introduction of a second coil into the circuit and the mutual inductance between the coils increases the time-constant of the circuit. The rise in current through the coils, therefore, takes longer to reach the operate value than in (*a*), and circuit (*b*) will be slower to operate than circuit (*a*).



(*a*) (*b*) (*c*) (*d*)

When the coils are connected in series opposition as in sketch (*c*), the current through the coils is again $\frac{E}{2R}$, but the resultant magnetomotive forces from each coil cancel thus preventing a tractive force being created to operate the relay. The application of a short-circuit across one coil, as in sketch (*d*), produces both operate and release delay. Eddy currents are set up in the winding during both the rise and decay of the flux due to the change in the current in the main winding. The eddy currents produced by the rise in the main flux oppose that rise; similarly the eddy currents produced by the fall in the main flux oppose that fall. Thus, the operation and release of the relay are delayed. The short-circuited winding is, in effect, working in a similar manner to a slug.

Q. 3. *Describe the 5-unit telegraph code.*
Why are start and stop signals added for teleprinter working?
Why is a letter/figure shift necessary on a teleprinter?

Q. 4. (*a*) *Define*

(*i*) *the erlang,*
(*ii*) *busy-hour calling-rate, and*
(*iii*) *grade of service.*

(*b*) *In the territory covered by a group of exchanges there are more circuits between exchanges and subscribers than there are between the exchanges. Explain why this should be so.*

Explain why an average junction circuit costs more than an average subscriber's circuit.

A. 4. (*a*) (*i*) *Erlang*

The average number of calls simultaneously in progress during a period is known as the traffic flow, or intensity, and is measured directly in erlangs (sometimes known as traffic units or T.U.). Thus, if one call occupies a circuit for one hour, the traffic flow is one erlang; alternatively, if 10 calls during one hour each occupy the circuit for an average of 6 minutes, the traffic flow carried is again one erlang. For groups of circuits, the definition is applied to the average occupancy, or holding time, for the total number of calls on all circuits within the hour period. The traffic is given by the expression $A = C \times T$ where A is the traffic in erlangs, C is the total number of calls in the hour and T is the average holding time of the calls in hours.

(*ii*) *Busy-hour calling-rate*

This is defined as the average number of calls originated per line during the busy hour, i.e. during the hour when most calls are originated. Typical busy-hour calling rates are 1·5 and 0·5 for business and residential exchanges, respectively.

(*iii*) *Grade of service*

For economic reasons, the traffic carried by a group of circuits during the busy hour is deliberately allowed to be less than the traffic offered. The difference between the traffic offered to the group and the traffic carried by the group is the traffic lost at the first attempt from the last outlet of the group. The grade of service (B) is defined as the ratio of the traffic lost to the traffic offered, i.e. the proportion of calls allowed to fail at the first attempt during the busy hour due to limitations of plant. For a typical grade of service, where $B = 0{\cdot}002$, two calls are lost in 1,000 calls offered.

(*b*) Individual subscribers require a permanent circuit to the exchange in order to be able to make and receive calls both to and from other subscribers on the same exchange and subscribers on other exchanges. There are, therefore, as many circuits between the exchange and the subscribers as there are subscribers. (Shared-service subscribers share the same line plant but require separate circuits in the exchange.) Circuits between exchanges are provided in relation to the average number of calls made by subscribers between the exchanges and have no direct relationship to the number of subscribers connected to the exchanges. Since the inter-exchange circuits, junctions, are only used occasionally by individual subscribers, one circuit may be shared by many subscribers and, therefore, the number of circuits is considerably lower than the number of circuits between subscribers and the exchange.

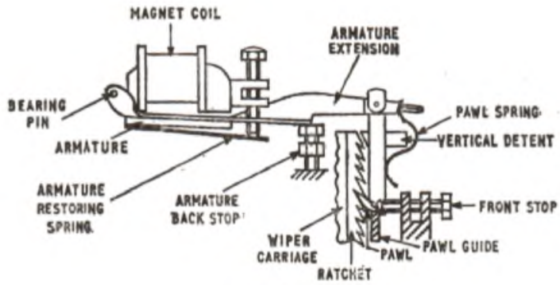
Junction-circuit costs

The main factor causing a junction circuit to be more costly than the average subscriber's circuit is the length of the circuit. The distance between adjacent exchanges is at least twice the length of the longest subscriber's circuit and considerably longer than the length of the average subscriber's circuit. Apart from the increased length, involving greater costs for additional cable, duct, etc., heavier-gauge conductors may be required to meet transmission and signalling limits, further adding to the junction-circuit costs. On very long junction circuits, it may also be necessary to introduce amplifing equipment.

Q. 5. *Describe with the aid of simplified sketches either the rotary or the vertical stepping mechanism of a two-motion selector. Explain how the momentum of the moving wiper shaft or carriage is prevented from causing over-stepping.*

A. 5. *Vertical stepping*

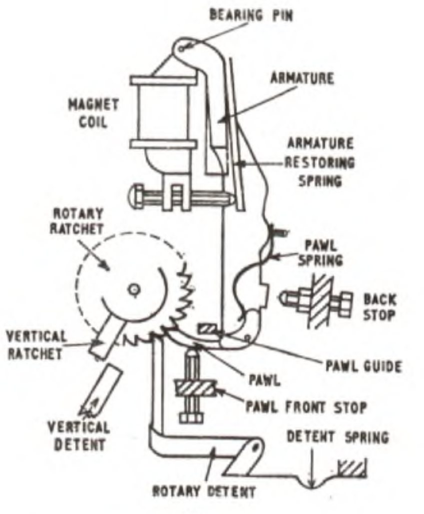
Sketch (*a*) shows the arrangement of the vertical magnet, armature, pawl and vertical ratchet for a two-motion selector.



(*a*)

When at the normal position, the armature is held firmly against the back stop by the restoring spring. When the magnet is energized, the armature is attracted and the pawl, under the influence of the pawl spring, is directed by the pawl guide into the root of the ratchet notch and the wiper carriage is lifted. The detent slides over the first tooth of the ratchet and, just as it engages the next notch, the pawl is wedged between the front stop and the vertical ratchet, so arresting the vertical movement of the carriage. The armature has not yet made contact with the pole faces of the magnet and, consequently, the armature extension is slightly bowed as the armature stroke is completed.

When the current ceases to flow in the magnet coil, the armature straightens and then restores to the normal position under the combined

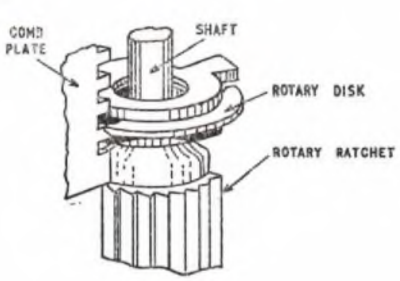


(*b*)

forces of gravity and the tension of the armature-restoring spring. The vertical detent, in engagement with a notch in the vertical ratchet, holds the wiper carriage in the raised position. The wedging of the pawl between the front stop and the ratchet decides the amount of stepping at each armature stroke and is controlled by the adjustment of the front stop.

Rotary stepping

Sketch (*b*) shows the arrangement of the rotary magnet, armature pawl and rotary ratchet for a two-motion selector. When at the normal position, the armature is held firmly on the back stop by the restoring spring. When the magnet is energized, the armature is attracted and the pawl, under the influence of the pawl spring, is directed by the pawl guide into the root of the ratchet notch and the wiper carriage is rotated. The detent slides over the first tooth of the ratchet at the same time as the pawl strikes the front stop, the adjustment of the front stop ensuring that overstepping does not take place. During the first rotary step, the vertical ratchet leaves the vertical detent but, at the same time, the rotary disk fitted on the wiper carriage enters the appropriate notch in the comb plate (sketch (*c*)) and supports the carriage during rotary stepping. When the current is removed from the rotary-magnet coil, the rotary detent, engaged in a notch in the rotary ratchet, holds the carriage against the tension of the rotary restoring spring. The armature returns to the normal position under the tension of the armature-restoring spring.



(*c*)

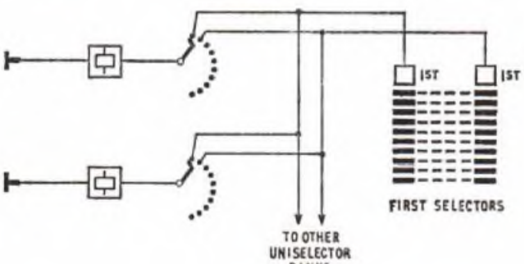
Q. **6.** *Describe the sequence of operations on an originated call for each of the following methods of connecting callers to first selectors:*

(*a*) *full uniselector hunter system,*
(*b*) *full linefinder system, and*
(*c*) *composite linefinder plus uniselector system.*

Explain why it is necessary to use system (*a*) *for an exchange with a high calling rate.*

A. 6. (*a*) *Full uniselector-hunter system*

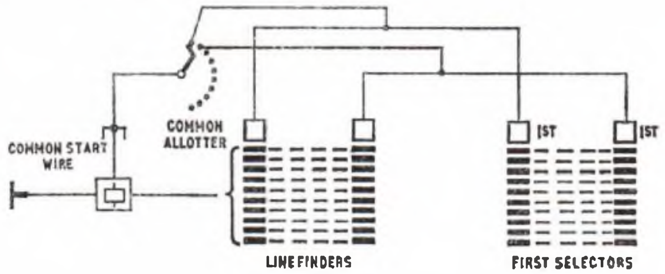
In this system, each subscriber has a uniselector with the wipers connected to his line. The bank of the uniselector is wired to first selectors, the first selectors being multipled to a number of subscribers' uniselectors, as shown in sketch (*a*). When the subscriber removes the telephone handset to originate a call, the calling loop causes the uniselector wiper assembly to hunt over the bank contacts, testing each outlet in turn until a free first selector is found. The selector returns dial tone when the subscriber is connected through and dialling can then commence.



(*a*)

(*b*) *Full linefinder system*

In the full linefinder system, the subscribers are connected to the bank contacts of two-motion switches the wipers of which are connected directly to first selectors (sketch (*b*)). When the subscriber originates a call by removing the telephone handset, the loop extended to the exchange causes a condition to be placed on the subscriber's linefinder bank contacts to identify the calling line and a start signal to be given via the common start wire and allotter to indicate that a call is to be set up. The wipers of the linefinder that has been allocated are then caused to step up the level and into the contact to which the calling subscriber is connected. The subscriber is then extended to the first selector associated with that particular linefinder. The first selector then returns dial tone to the subscriber indicating that dialling can commence.

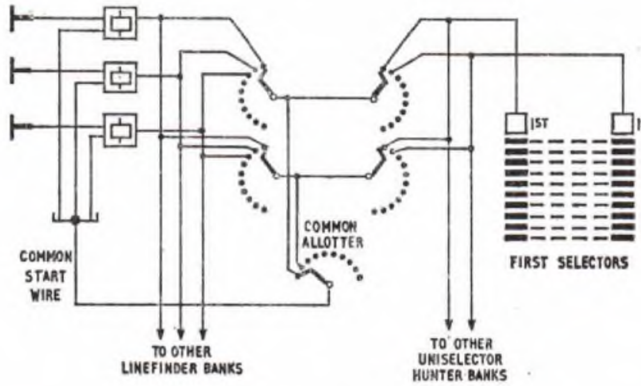


(*b*)

(*c*) *Composite linefinder plus uniselector system*

The composite system (sketch (*c*)) is similar to the full linefinder system in that the subscribers are connected to the banks of the linefinder (uniselectors in the example shown) but, instead of the wipers being directly connected to the first selectors, the wipers are connected to the wipers of uniselector hunters, the banks of which are multipled to first selectors.

When a call is originated by a subscriber, a free linefinder plus uniselector hunter combination is allocated to the caller and the wiper assembly of the linefinder is caused to step until the bank contact of the calling subscriber is reached. At the same time, the associated uniselector hunter is caused to step round the bank contacts until it finds a free first selector. When the connexion between the subscriber and the first selector has been completed, dial tone is returned and dialling can commence.



(*c*)

The ability of a subscriber to make a call is always dependent upon the existence of one or more disengaged first selectors (which are provided in sufficient quantity to ensure that only a few calls fail due to all selectors being engaged. With a full linefinder system, using two-motion selectors with 200 bank contacts, the number of linefinders provided to serve the group of 200 subscribers connected to the banks is equal to the number of first selectors. Hence, if the calling rate is high, the number of simultaneous calls is also high and the number of linefinders necessary to give an adequate grade of service is high and may approach the number of subscribers. The high cost of a large number of linefinders (plus allotters) to serve 200 high-calling-rate subscribers may, therefore, exceed the cost of providing 200 subscriber's uniselectors. The latter are cheap (only 25 bank contacts) and have the added advantage of simplicity and speed.

Q. 7. *A group of* 800 *subscribers originates* 20 *erlangs in the busy-hour. Their uniselectors are graded to* 30 *first selectors.*

(*a*) *If the average call duration is* 3 *minutes, what is the average busy-hour calling rate?*
(*b*) *Why is the number of first selectors considerably in excess of the number of erlangs to be carried?*
(*c*) *If half the subscribers were to double their calling rate, what effect would this have on outgoing service?*

What steps could be taken to restore outgoing service to normal?

A. 7. (*a*) The traffic A in erlangs carried by a group of circuits is given by $A = C \times T$ where C is the total number of calls in the hour and T is the average holding time of the calls expressed in hours.

If the traffic in the busy hour is 20 erlangs and the average call duration or holding time is 3 minutes, then $20 = C \times \frac{3}{60}$, therefore $C = 20 \times \frac{60}{3} = 400$, i.e. there are 400 calls. Since there are 800 subscribers, then the average busy-hour calling-rate $= \frac{800}{400} = 0{\cdot}5$.

(*b*) For the number of first selectors to be equal to the number of busy-hour erlangs means that each selector is occupied for the full hour. This could only be so if the subscribers' calls are offered to the selectors one after the other, i.e. in a perfectly smooth manner. Traffic generated by subscribers is very far from being smooth; the calls originate in a random manner and with varying durations. The result is that selectors, in theory, each capable of carrying one erlang, are, in practice, restricted to carrying considerably less than one erlang. Additional selectors are, therefore, required to ensure an adequate grade of service is provided.

(*c*) If half of the subscribers increased their calling rate from 0·5 to 1·0 the resultant average busy-hour calling-rate would be

$$\frac{400 \times 0{\cdot}5}{800} + \frac{400 \times 1{\cdot}0}{800} = \frac{0{\cdot}5}{2} + \frac{1{\cdot}0}{2} = 0{\cdot}75.$$

Assuming the average call duration remains at 3 minutes the traffic generated by the 800 subscribers in the busy hour now becomes

$$A = 800 \times 0{\cdot}75 \text{ (calls)} \times \frac{3}{60} \text{(duration)} = 30 \text{ erlangs.}$$

This represents a 50 per cent increase in the traffic offered to the selectors and results in a greater loss of calls, i.e. a lower grade of service. In order to restore the outgoing service to the original standard, it would be necessary to provide additional first selectors and, at the same time, rearrange the grading to the selectors. The regrading is necessary to adjust for the increased number of first selectors and also to cater for the increased traffic from some groups of subscribers.

Q. **8.** *State the functions of a main distribution frame (m.d.f.).*

Estimate the floor space required for the m.d.f. of a 10,000-line exchange.

How are the terminations numbered?

What arrangements are made to secure an orderly array of jumpers?

Q. **9.** *Why is the battery a necessary part of the power plant at an exchange?*

What factors determine

(*a*) *the required capacity of the battery, and*
(*b*) *the required rating of the a.c.–d.c. conversion equipment?*

What advantage is gained by dividing the battery capacity between two groups of cells?

Why is one pole of the battery earthed?

A. **9.** Power supplies for telephone exchanges are derived from the public a.c. mains using a.c.–d.c. conversion equipment. It is necessary to include batteries in the power plant for two reasons.

(*i*) In the event of a failure of the public a.c. mains supply, a d.c. supply must be available during the changeover to the stand-by mains supply. If no stand-by supply is provided, the batteries are required to supply the power until the public mains supply is restored or until an alternative a.c. mains supply is provided.
(*ii*) An efficient smoothing circuit is required for the d.c. power supplies produced by rectification of the a.c. mains. The batteries, together with the power-plant filter, provide such a smoothing circuit.

(*a*) The capacity of the battery provided depends upon the factors described in (*i*) above. The battery must be capable of providing power (at the level required during the busy hour) for the length of time it is expected that it will be necessary to maintain the exchange on batteries alone. This may be a short period when alternative mains supplies are provided (either as a second public mains supply or from diesel-driven alternators) or a considerable time when the duration of the breakdown of the public supplies is long and an alternative source is not immediately available.

(*b*) The equipment used for a.c.–d.c. conversion must be capable of providing the full load of the exchange in the busy hour and, in addition, must cater for any losses in the conversion equipment. Losses in the equipment relate to the conversion efficiency which is typically 65 per cent for motor generators and up to 95 per cent for modern rectifier systems. The rating of the conversion equipment is, therefore, always in excess of the exchange busy-hour load requirements.

Battery cells require maintenance and replacement. By dividing the battery into two groups of cells connected in parallel, one group of cells may be maintained and, if necessary, replaced, whilst ensuring that a battery is available for smoothing and stand-by. The group of cells can be removed from service by using circuit-breakers.

One pole of the battery is connected to earth to permit the use of earth-return line-signalling systems where these are appropriate.

Q. **10.** (*a*) *Define*

(*i*) *full availability, and*
(*ii*) *limited availability.*

(*b*) *Design a grading for* 21 *trunks from a level of* 100-*outlet group selectors.*

Why do the trunks that are individual to each group in a grading carry more traffic than those which are shared between all the groups?

A. **10.** (*a*) (*i*) *Full availability* is said to exist when a selector has access to all the trunks in the particular group to the next switching stage. A selector with 10 outlets has full availability when the number of trunks to the next switching stage is 10 or less.

(ii) *Limited availability* is the condition where a selector has access to a limited number only of the trunks on a given route, e.g. a selector level with 10 outlets connected to only 10 out of a group of 17 trunks.

(*b*) *Design of grading*

Using the formula $n = \frac{ga}{2}$ where n is the number of trunks, g is the number of grading groups and a is the availability, for the example,

$$21 = \frac{g \times 10}{2} \text{ or } g = \frac{2 \times 21}{10} = 4{\cdot}2 = \text{number of grading groups.}$$

Taking 4 as the nearest even number of grading groups to give a balanced grading the factors of 4, i.e. 4, 2 and 1 indicate that there will be individuals, pairs and full commons in the grading.

Let a be the number of individuals,
b be the number of pairs, and
c be the number of full commons.

Then, the sum of the number of individuals, pairs and full commons is equal to the availability.

Therefore

$$a + b + c = 10. \qquad \ldots\ldots (1)$$

Also, the total number of trunks is given by

$$4a + 2b + c = 21. \qquad \ldots\ldots (2)$$

The grading can now be completed by finding values for a, b and c which satisfy equations (1) and (2) simultaneously.

Subtracting (1) and (2),

$$3a + b = 11, \text{ therefore } a = \frac{11 - b}{3}. \qquad \ldots\ldots (3)$$

Substituting values for b in (3) will produce values for a which must be equal to zero or a positive whole number.

Grading	1	2	3
a b c	3 2 5	2 5 3	1 8 1
Sum of successive differences	4	5	14

Note: c found by using equation (1).

The smoothest grading is that which produces the smallest sum of successive differences, i.e. grading 1 in the above table. This produces a grading as shown in the sketch.

The trunks which are individual to each group are also early choices in each group and are, therefore, offered more traffic than the later choices, i.e. outlets 6–10 which share trunks between all groups. The combined traffic on the shared late-choice trunks is less than that on the individual trunks.

MATHEMATICS B, 1971

Students were expected to answer any six questions.

Q. **1.** *A ball projected vertically upwards at 36 ft/s reaches a height s ft in t seconds, given by $s = 36t - 16t^2$. Express s in the form $a - b(t - c)^2$ and hence, or otherwise, find the greatest height reached by the ball.*

At what times will the ball be 8 ft above the point of projection?

A. 1. $s = 36t - 16t^2$.

The form of the expression required for s indicates completion of the square as follows:

$$s = -16\left(t^2 - \frac{36}{16}t\right),$$
$$= -16\left(t^2 - \frac{9}{4}t\right),$$
$$= -16\left\{t^2 - \frac{9}{4}t + \left(\frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2\right\},$$
$$= -16\left\{\left(t - \frac{9}{8}\right)^2 - \frac{81}{64}\right\},$$
$$= \frac{81}{4} - 16\left(t - \frac{9}{8}\right)^2,$$
$$\therefore\ s = \frac{81}{4} - 16\left(t - \frac{9}{8}\right)^2.$$

The value of s is determined by the constant term $\frac{81}{4}$ and variable term $16\left(t - \frac{9}{8}\right)^2$ and will, therefore, be greatest when the variable term is least. Since $\left(t - \frac{9}{8}\right)^2$ is a square its least value must be zero, i.e. when $t = \frac{9}{8}$ and, hence, the greatest value of s will be $\frac{81}{4}$ or $20\frac{1}{4}$ ft.

When, $s = 8$ ft,

$$8 = \frac{81}{4} - 16\left(t - \frac{9}{8}\right)^2,$$
$$\text{or, } 16\left(t - \frac{9}{8}\right)^2 = \frac{81}{4} - 8,$$
$$= \frac{49}{4}.$$
$$\therefore\ \left(t - \frac{9}{8}\right)^2 = \frac{49}{64},$$
$$= \left(\pm\frac{7}{8}\right)^2.$$
$$\therefore\ t - \frac{9}{8} = \pm\frac{7}{8},$$
$$\text{or, } t = \frac{9}{8} \pm \frac{7}{8}$$
$$= 2 \text{ or } \frac{1}{4}\text{s}.$$

Thus, the ball is 8 ft above the point of projection after $\frac{1}{4}$ or 2s.

Note: The ball is projected upwards to reach a height of 8 ft after $\frac{1}{4}$s and a maximum height of $20\frac{1}{4}$ ft after $\frac{9}{8}$s. It then falls under gravity to a height of 8 ft again after 2 s. The time of ascent from 8 ft is $\frac{7}{8}$s and equals the time of descent to the same height.

Q. **2.** *For a non-linear device, the current I mA passing for different values of the applied voltage V is given by the table*

V	30	75	120
I	10·2	13·8	18·6

Assuming a formula $I = a + bV + cV^2$, calculate the constants a, b, c. [*In solving the simultaneous equations involved, a useful substitution is $c = k \times 10^{-3}$.*]

A. 2. $I = a + bV + cV^2$.

Substituting the data given in the table, the following equations are obtained:

$$10{\cdot}2 = a + 30b + 900c. \qquad \ldots\ldots (1)$$
$$13{\cdot}8 = a + 75b + 5{,}625c. \qquad \ldots\ldots (2)$$
$$18{\cdot}6 = a + 120b + 14{,}400c. \qquad \ldots\ldots (3)$$

Making the substitution of $k = 1{,}000c$ suggested in the question, the equations become:

$$10{\cdot}2 = a + 30b + 0{\cdot}9k. \qquad \ldots\ldots (4)$$
$$13{\cdot}8 = a + 75b + 5{\cdot}625k. \qquad \ldots\ldots (5)$$
$$18{\cdot}6 = a + 120b + 14{\cdot}4k. \qquad \ldots\ldots (6)$$

Subtracting equation (4) from equation (5), gives:

$$3{\cdot}6 = 45b + 4{\cdot}725k. \qquad \ldots\ldots (7)$$

Subtracting equation (5) from equation (6), gives:

$$4{\cdot}8 = 45b + 8{\cdot}775k. \qquad \ldots\ldots (8)$$

Subtracting equation (7) from equation (8), gives:

$$1{\cdot}2 = 4{\cdot}05k.$$
$$\therefore\ k = \frac{1{\cdot}2}{4{\cdot}05} = 0{\cdot}2963.$$

Substituting for $k = 0{\cdot}2963$ in equation (7) gives:

$$3{\cdot}6 = 45b + 4{\cdot}725 \times 0{\cdot}2963,$$
$$\text{or, } 45b = 3{\cdot}6 - 1{\cdot}400,$$
$$= 2{\cdot}2.$$
$$\therefore\ b = \frac{2{\cdot}2}{45} = 0{\cdot}04889.$$

Substituting for $b = 0{\cdot}04889$, and $k = 0{\cdot}2963$ in equation (4) gives:

$$10{\cdot}2 = a + 30 \times \frac{2{\cdot}2}{45} + 0{\cdot}9 \times 0{\cdot}2963,$$
$$= a + 1{\cdot}4\dot{6} + 0{\cdot}26\dot{6}.$$
$$\therefore\ a = 10{\cdot}2 - 1{\cdot}73\dot{3},$$
$$= 8{\cdot}4\dot{6}.$$
$$\text{But, } c = k \times 10^{-3},$$
$$= 0{\cdot}0002963.$$

Hence, $a = 8{\cdot}467$, $b = 0{\cdot}04889$ and $c = 0{\cdot}0002963$.

Q. **3.** *The signal power P mW at a distance x km from the sending end of a transmission line is given by the table*

x	20	40	60	80	100
P	10·05	6·74	4·52	3·03	2·03

Assuming a formula $P = ae^{-kx}$, plot suitable variables to test this assumption, and from the graph obtain the constants a and k.

What physical significance has the constant a?

A. 3. $P = ae^{-kx}$.

Taking logarithms to base e,

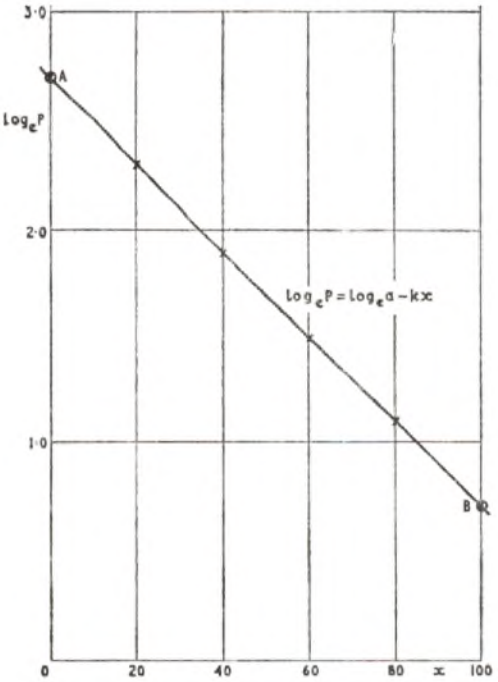
$$\log_e P = \log_e a - kx.$$

Since a and k are constants, the logarithmic form of the formula is that of a linear law $y = mx + c$, where $\log_e P$ is the variable corresponding to y, $\log_e a$ is a constant corresponding to c and $-k$ the constant corresponding to m.

The values of $\log_e P$ are given in the table.

x	20	40	60	80	100
P	10·05	6·74	4·52	3·03	2·03
$\log_e P$	2·308	1·908	1·509	1·109	0·708

The graph of $\log_e P$ plotted against corresponding values of x is shown in the sketch, from which it is seen that the plotted points lie almost exactly on a straight line. Hence, the assumption that the formula $P = ae^{-kx}$ applies to the given data, is true.



When $x = 0$, $\log_e P = \log_e a$ and hence, by extrapolating the graph slightly to cut the axis of $\log_e P$ the value of $\log_e a$ may be read from the sketch as 2·7.

$\therefore \log_e a = 2{\cdot}7,$

or, $a = 14{\cdot}88$ from a table of natural logarithms.

The gradient of the graph measured from two widely separate points A and B, will give the value of $-k$.

Reading from the graph,

$$-k = \frac{0{\cdot}708 - 2{\cdot}7}{100 - 0},$$
$$= -\frac{1{\cdot}992}{100},$$

or, $k = 0{\cdot}0199.$

Thus, $a = \underline{14{\cdot}88}$, and $k = \underline{0{\cdot}0199.}$

When $x = 0$, $P = ae^{-kx} = a$ and hence, $a = 14{\cdot}88$ mW is the signal power applied at the sending end of the line.

Q. **4.** (*a*) *Derive an expression, in the form of phasor* $a + jb$, *for the admittance in siemens equivalent to an inductive impedance* $50 \angle 20°$ *ohms.*

(*b*) *The current in a circuit component is* $(3{\cdot}6 + j1{\cdot}2)$ *mA when the applied voltage is* $(3 - j2)$ *volts.*

Calculate as a phasor the impedance of the component.

What is the phase relationship between voltage and current?

A. 4. (*a*) If $r \angle \theta$ denotes the polar form of the impedance Z, then,

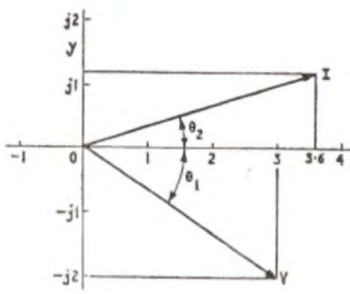
$$Z = r \angle \theta,$$
$$= r(\cos\theta + j \sin \theta).$$
$$\text{Admittance} = \frac{1}{Z},$$
$$= \frac{1}{r \angle \theta},$$
$$= \frac{1}{r(\cos\theta + j\sin\theta)},$$
$$= \frac{1}{r} \times \frac{\cos\theta - j\sin\theta}{(\cos\theta + j\sin\theta)(\cos\theta - j\sin\theta)},$$
$$= \frac{1}{r} \times \frac{\cos\theta - j\sin\theta}{\cos^2\theta + \sin^2\theta},$$
$$= \left(\frac{1}{r}\right) \angle -\theta,$$
$$= \frac{1}{50}(\cos 20° - j\sin 20°),$$
$$= 0{\cdot}02(0{\cdot}9397 - j \times 0{\cdot}3420),$$
$$= 0{\cdot}0188 - j0{\cdot}00684 \text{ S}.$$

Thus, the phasor for the admittance is $\underline{0{\cdot}0188 - j0{\cdot}00684 \text{ S}}$.

Note: An alternative, but longer, method would be to evaluate Z in the form $a + j\,b$ first and then to find its reciprocal by normal conversion methods.

(*b*) $$\text{Impedance} = \frac{\text{voltage}}{\text{current}},$$
$$= \frac{3 - j2}{10^{-3}(3{\cdot}6 + j1{\cdot}2)}.$$
$$\therefore \text{Impedance} = \frac{10^3(3 - j2)(3{\cdot}6 - j1{\cdot}2)}{(3{\cdot}6 + j1{\cdot}2)(3{\cdot}6 - j1{\cdot}2)},$$
$$= \frac{10^3(10{\cdot}8 - j7{\cdot}2 - j3{\cdot}6 - 2{\cdot}4)}{12{\cdot}96 + 1{\cdot}44},$$
$$= \frac{10^3(8{\cdot}4 - j10{\cdot}8)}{14{\cdot}4} \text{ ohms}.$$

Thus, impedance $= \underline{10^3(0{\cdot}58\dot{3} - j0{\cdot}75) \text{ ohms}}.$



The phase relationship between voltage and current is shown in the Argand diagram of the sketch and is such that the current leads the voltage by an angle $(\theta_1 + \theta_2)$, where, θ_1 and θ_2 are, respectively, the angles of voltage and current.

$\tan\theta_1 = \frac{2}{3}$ whence, $\theta_1 = 33° 41'$, and

$\tan\theta_2 = \frac{1{\cdot}2}{3{\cdot}6} = \frac{1}{3}$ whence, $\theta_2 = 18° 26'$.

Thus, $\theta_1 + \theta_2 = 52° 7'$ and, hence, the current leads the voltage by $\underline{52° 7'}$.

Note: This angle could, alternatively, be calculated directly from the impedance phasor as $\tan^{-1}\frac{0{\cdot}75}{0{\cdot}58\dot{3}}$, the negative sign indicating that the current leads the voltage.

Q. **5.** *An investor is offered two alternatives,*

(*a*) *to invest £200 annually at 6 per cent simple interest,*

(*b*) *to invest £200 annually at 4½ per cent compound interest.*

Find which investment is the more profitable if he withdraws all his money at the end of ten years, having paid in ten instalments, one at the beginning of each year.

[Log_{10} (1·045) = 0·019116 *to five significant figures.*]

A. 5. The total capital invested over the 10-year period will be £200 × 10, i.e. £2,000 for each method and, since this is all withdrawn, it may be ignored in assessing which method is more profitable. The assessment, therefore, rests solely on the amount of interest.

(*a*) At the end of the first year, the interest due at 6 per cent simple interest $= \frac{6}{100} \times 200 = £12.$

As the first year's investment of £200 attracts the same interest for a total of 10 years, the total interest from this will be 12 × 10 = £120.

The second year's investment, however, will attract interest for nine years only, the third year's investment for eight years and so on. Hence, the total amount of simple interest

$$= £(120 + 108 + \cdots + 12).$$

This is an arithmetic progression of 10 terms and common difference -12.

$\therefore$ Sum of arithmetic progression $= \frac{1}{2} \times$ (number of terms) $\times$ (sum of first and last terms),

$$= \tfrac{1}{2} \times 10 \times (120 + 12),$$

$$= 660.$$

$\therefore$ Total amount of simple interest $= £660$.

(*b*) Considering the first £200 invested, this will attract interest of $£\frac{4 \cdot 5}{100} \times 200$ by the end of the year. Hence,

amount by the end of the first year $= 200 + (0 \cdot 045 \times 200)$,

$$= £200 \times 1 \cdot 045.$$

At the end of the second year the interest on this amount will be $£\frac{4 \cdot 5}{100} \times 200 \times 1 \cdot 045$ and thus,

amount by the end of the second year $= 200 \times 1 \cdot 045 + 0 \cdot 045 \times 200 \times 1 \cdot 045$,

$$= 200 \times 1 \cdot 045(1 + 0 \cdot 045),$$

$$= £200 \times 1 \cdot 045^2.$$

In like manner, it may easily be deduced that, amount by the end of the tenth year, interest $= £200 \times 1 \cdot 045^{10}$.

The second £200 invested will bear compound interest for nine years only and hence, the amount accrued at the end of the 10-year period will be, $£200 \times 1 \cdot 045^9$.

Similarly, the next investment of £200 will be worth $£200 \times 1 \cdot 045^8$ at the end of the 10-year period and so on. Hence, total amount accruing from the compound interest investment will be

$$£(200 \times 1 \cdot 045^{10} + 200 \times 1 \cdot 045^9 + \ldots + 200 \times 1 \cdot 045)$$

$$= £200(1 \cdot 045 + 1 \cdot 045^2 + \ldots + 1 \cdot 045^{10}).$$

The series within the brackets is a geometric progression with a ratio of $1 \cdot 045$. Hence, it sum to 10 terms will be

$$S_{10} = \frac{1 \cdot 045(1 \cdot 045^{10} - 1)}{1 \cdot 045 - 1},$$

$$= \frac{1 \cdot 045}{0 \cdot 045}(1 \cdot 045^{10} - 1).$$

$$\text{Log}_{10}\, 1 \cdot 045^{10} = 10 \log_{10} 1 \cdot 045,$$

$$= 10 \times 0 \cdot 019116,$$

$$= 0 \cdot 19116.$$

$$\therefore\ 1 \cdot 045^{10} = \text{antilog}_{10}\, 0 \cdot 19116 = 1 \cdot 553.$$

$$\therefore\ S_{10} = \frac{1 \cdot 045 \times 0 \cdot 553}{0 \cdot 045},$$

$$= 12 \cdot 84.$$

Hence, total amount $= £200 \times 12 \cdot 84$,

$= £2{,}568$.

Total investment $= £10 \times 200$,

$= £2{,}000$.

$\therefore$ Amount of interest $= £568$.

Since this is less than the interest obtained under part (*a*), investment at 6 per cent simple interest is the more profitable arrangement.

Q. 6. (*a*) *State the compound angle formulae for* $\cos(A + B)$ *and* $\sin(A + B)$.

Hence, show that,

(*i*) $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$, *and derive a similar formula for* $2 \sin A \sin B$,

(*ii*) $\cot 75° = 2 - \sqrt{(3)}$ *without using trigonometrical tables.*

(*b*) *If* $\frac{P}{W} = \frac{\sin(\alpha + \beta)}{\cos \alpha}$, *calculate* α, *if* $W = 1{,}000$, $P = 367$, $\beta = 29°$, *where* $0 < \alpha < 180°$.

A. 6. (*a*)

$$\cos(A + B) = \cos A \cos B - \sin A \sin B.$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

(*i*) The compound angle formulae are true for all values of A and B. Hence, substituting $(-B)$ for B gives:

$$\sin(A - B) = \sin A \cos(-B) + \cos A \sin(-B).$$

But, $\cos B = \cos(-B)$,

and $\sin B = -\sin(-B)$.

$$\therefore\ \sin(A - B) = \sin A \cos B - \cos A \sin B. \quad \ldots\ldots (1)$$

Also, $\sin(A + B) = \sin A \cos B + \cos A \sin B. \quad \ldots\ldots (2)$

Subtracting equation (1) from equation (2) gives:

$$\underline{\sin(A + B) - \sin(A - B) = 2 \cos A \sin B.} \qquad \text{Q.E.D.}$$

Now, $\cos(A + B) = \cos A \cos B - \sin A \sin B. \quad \ldots\ldots (3)$

Substituting $(-B)$ for B as before gives:

$$\cos(A - B) = \cos A \cos(-B) - \sin A \sin(-B),$$

$$= \cos A \cos B + \sin A \sin B. \quad \ldots\ldots (4)$$

Subtracting equation (3) from equation (4) gives:

$$\cos(A - B) - \cos(A + B) = 2 \sin A \sin B.$$

Thus, $2 \sin A \sin B = \underline{\cos(A - B) - \cos(A + B).}$

(*ii*)

$$\cot 75° = \frac{\cos 75°}{\sin 75°},$$

$$= \frac{\cos(45° + 30°)}{\sin(45° + 30°)},$$

$$= \frac{\cos 45° \cos 30° - \sin 45° \sin 30°}{\sin 45° \cos 30° + \cos 45° \sin 30°},$$

$$= \frac{\frac{1}{\sqrt{(2)}} \times \frac{\sqrt{(3)}}{2} - \frac{1}{\sqrt{(2)}} \times \frac{1}{2}}{\frac{1}{\sqrt{(2)}} \times \frac{\sqrt{(3)}}{2} + \frac{1}{\sqrt{(2)}} \times \frac{1}{2}},$$

$= \frac{\sqrt{(3)} - 1}{\sqrt{(3)} + 1}$ since $2\sqrt{(2)}$ is a common denominator for each term,

$$= \frac{(\sqrt{(3)} - 1)^2}{(\sqrt{(3)} + 1)(\sqrt{(3)} - 1)},$$

$$= \frac{3 - 2\sqrt{(3)} + 1}{3 - 1},$$

$$= \frac{4 - 2\sqrt{(3)}}{2},$$

$$= \underline{2 - \sqrt{(3)}.} \qquad \text{Q.E.D.}$$

(*b*)

$$\frac{P}{W} = \frac{\sin(\alpha + \beta)}{\cos \alpha},$$

$$= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha},$$

$$= \tan \alpha \cos \beta + \sin \beta.$$

$$\therefore\ \tan \alpha \cos \beta = \frac{P}{W} - \sin \beta,$$

$$\text{or, } \tan \alpha = \frac{P}{W \cos \beta} - \tan \beta.$$

When, $W = 1{,}000$, $P = 367$ and $\beta = 29°$,

$$\tan \alpha = \frac{367}{1{,}000 \times 0 \cdot 8746} - 0 \cdot 5543,$$

$$= \frac{367}{874 \cdot 6} - 0 \cdot 5543,$$

$$= 0 \cdot 4197 - 0 \cdot 5543,$$

$$= -0 \cdot 1346.$$

Now, $\tan(180° - \theta) = -\tan \theta$.

$$\therefore\ \alpha = 180° - 7° 40',$$

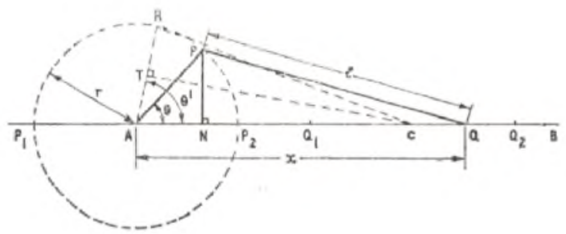
$$= 172° 20'.$$

Thus, $\alpha = \underline{172° 20'}$, where $0 < \alpha < 180°$.

Q. 7. *Rods AP, PQ are 1·2 m and 3·2 m long respectively, and are jointed at P, in such a way that P moves on a circle centre A while Q moves along a fixed line AB in the plane of the circle. If angle PAQ is θ and AQ = x metres, derive an expression for x in terms of θ.*

Calculate the acute angle θ for which Q is midway between its extreme positions along the line AB.

A. 7. The crank and connecting-rod mechanism is shown in the sketch, with P positioned so that $\angle PAQ = \theta$ is acute.



When P moves to P_1, on BA produced, Q moves to Q_1 such that $P_1Q_1 = PQ$. Similarly, when P moves to its other extreme position P_2, Q moves to Q_2.

For convenience let AP, radius of circle with centre A, be equal to r and let PQ equal l. Drop the perpendicular PN from P on to AB.

$$\begin{aligned} \text{Now, } AQ = x &= AN + NQ, \\ &= r\cos\theta + NQ. \end{aligned}$$

From the right-angled triangle NPQ,

$$\begin{aligned} NQ^2 &= PQ^2 - PN^2, \\ &= l^2 - r^2\sin^2\theta, \\ \text{or, } NQ &= \sqrt{(l^2 - r^2\sin^2\theta)}. \\ \therefore\ x &= r\cos\theta + \sqrt{(l^2 - r^2\sin^2\theta)}, \\ &= 1{\cdot}2\cos\theta + \sqrt{(3{\cdot}2^2 - 1{\cdot}2^2\sin^2\theta)}. \\ \text{Thus, } x &= \underline{1{\cdot}2\cos\theta + \sqrt{(10{\cdot}24 - 1{\cdot}44\sin^2\theta)}}. \end{aligned}$$

Suppose that C is the position of Q midway between its extremes Q_1 and Q_2, and that R is the corresponding position of P.

$$\begin{aligned} \text{Now, } P_1Q_2 &= l + 2r, \\ \text{and } P_1Q_1 &= l. \\ \therefore\ Q_1Q_2 &= P_1Q_2 - P_1Q_1, \\ &= 2r. \end{aligned}$$

Since C is the mid-point of Q_1Q_2,

$$\begin{aligned} Q_1C &= CQ_2 = r. \\ \text{Also, } AC &= AQ_1 + QC, \\ &= l - r + r, \\ &= l. \end{aligned}$$

∴ Triangle RAC is isosceles and,

$$\angle RAC = \angle ARC = \theta',$$

where θ' denotes θ when P moves to R.

If CT is the perpendicular bisector of RA in the isosciles triangle RAC,

$$\begin{aligned} \cos\theta' &= \frac{AT}{AC}, \\ &= \frac{\frac{r}{2}}{l}, \\ &= \frac{0{\cdot}6}{3{\cdot}2}, \\ &= 0{\cdot}1875. \\ \therefore\ \theta' &= 79^\circ\ 12'. \end{aligned}$$

Hence, the value of θ for which Q is midway between its extreme positions is $\underline{79^\circ\ 12'}$.

Note: There are several ways of obtaining this value of θ. One likely to have been used is to obtain the value of x (i.e. AC) as 3·2 m and then to substitute this in the formula for x already obtained. Although, at first sight this might appear cumbersome, it works out fairly easily and would be quite acceptable.

Q. 8. *Derive from first principles the gradient of the curve*

$$y = 2x^3 - \tfrac{2}{3}x + 7,$$

at the point where $x = \frac{2}{3}$.

Find the points on the curve at which the tangent to the curve is parallel to the x-axis.

A. 8. $$y = 2x^3 - \tfrac{2}{3}x + 7.$$

For any point (x, y) on the curve, suppose x increases by a small amount δx and that the corresponding change in y is δy.

$$\begin{aligned} \text{Then, }\quad y + \delta y &= 2(x + \delta x)^3 - \tfrac{2}{3}(x + \delta x) + 7, \\ &= 2(x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3) \\ &\quad - \tfrac{2}{3}x - \tfrac{2}{3}\delta x + 7. \\ \therefore\ \delta y &= 2x^3 + 6x^2\delta x + 6x\delta x^2 + 2\delta x^3 - \tfrac{2}{3}x \\ &\quad - \tfrac{2}{3}\delta x + 7 - 2x^3 + \tfrac{2}{3}x - 7, \\ &\quad \text{since } y = 2x^3 - \tfrac{2}{3}x + 7, \\ &= 6x^2\delta x + 6x\delta x^2 + 2\delta x^3 - \tfrac{2}{3}\delta x. \\ \therefore\ \frac{\delta y}{\delta x} &= 6x^2 + 6x\delta x + 2\delta x^2 - \tfrac{2}{3}. \end{aligned}$$

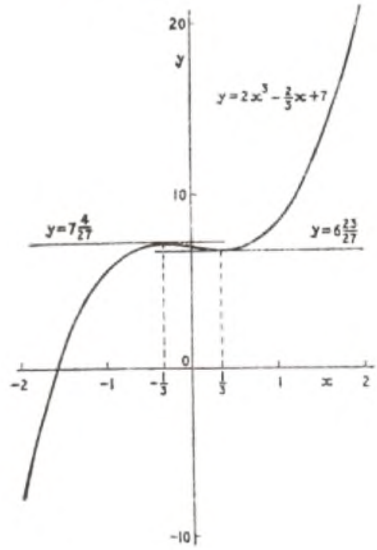
$$\begin{aligned} \text{The gradient of the curve} &= \frac{dy}{dx}, \\ &= \lim_{\delta x \to 0} \frac{\delta y}{\delta x}, \\ &= 6x^2 - \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{When, } x &= \frac{2}{3}, \\ \frac{dy}{dx} &= 6\left(\frac{2}{3}\right)^2 - \frac{2}{3}, \\ &= 2. \end{aligned}$$

∴ Gradient of the curve where $x = \frac{2}{3}$ is $\underline{2.}$

Since the gradient of the curve at any point is the gradient of the tangent at that point, the tangent will be parallel to the x-axis where $\frac{dy}{dx} = 0$.

$$\begin{aligned} \therefore\ 6x^2 - \frac{2}{3} &= 0, \\ \text{or, } 6x^2 &= \frac{2}{3}. \\ \therefore\ x^2 &= \frac{1}{9}, \\ \text{or, } x &= \pm\frac{1}{3}. \end{aligned}$$



When, $x = \frac{1}{3}$, $y = 2\left(\frac{1}{3}\right)^3 - \frac{2}{3}\left(\frac{1}{3}\right) + 7,$

$$= \frac{2}{27} - \frac{2}{9} + 7,$$

$$= \frac{185}{27}.$$

When, $x = -\frac{1}{3},$

$$y = 2\left(-\tfrac{1}{3}\right)^3 - \tfrac{2}{3}\left(-\tfrac{1}{3}\right) + 7,$$

$$= -\frac{2}{27} + \frac{2}{9} + 7,$$

$$= \frac{193}{27}.$$

Hence, the points on the curve at which the tangent to the curve is parallel to the x-axis are $\underline{\left(-\frac{1}{3}, \frac{193}{27}\right)}$ and $\underline{\left(\frac{1}{3}, \frac{185}{27}\right)}$.

Note: Although strictly speaking not required by the question, a sketch of the curve is useful to illustrate the second part of the answer and is shown.

Q. 9. The potential difference v volts at time t seconds across a choke of inductance L henries and series resistance R ohms is given by $v = L\frac{di}{dt} + iR$ *where i amperes is the current through the choke. If* $i = 3 \sin 100\pi t$, $L = 5 \times 10^{-3}$ *and* $R = 2$, *find the voltage across the choke at,*

(*a*) $t = 0$,
(*b*) $t = \frac{1}{300}$.

Find in milliseconds one value of t when the voltage is zero.

A. 9. $$v = L\frac{di}{dt} + iR,$$

and, $i = 3 \sin 100\pi t.$

$$\therefore \frac{di}{dt} = 3 \cos 100\pi t \times (100\pi),$$

$$= 300\pi \cos 100\pi t.$$

Hence, $v = 5 \times 10^{-3} \times 300\pi \cos 100\pi t + 3 \sin 100\pi t \times 2,$

$$= 1{\cdot}5\pi \cos 100\pi t + 6 \sin 100\pi t,$$

for the values given in the question.

(*a*) When, $t = 0$,

$\cos 100\pi t = \cos 0 = 1,$

and, $\sin 100\pi t = \sin 0 = 0.$

$$\therefore v = 1{\cdot}5\pi = \underline{4{\cdot}712 \text{ volts.}}$$

(*b*) When, $t = \frac{1}{300}$,

$$\cos 100\pi t = \cos\left(100\pi \times \frac{1}{300}\right),$$

$$= \cos\frac{\pi}{3},$$

$$= \frac{1}{2},$$

and, $\sin 100\pi t = \sin\frac{\pi}{3},$

$$= 0{\cdot}866.$$

$$\therefore v = 1{\cdot}5\pi \times \tfrac{1}{2} + 6 \times 0{\cdot}866,$$

$$= 2{\cdot}356 + 5{\cdot}196,$$

$$= \underline{7{\cdot}552 \text{ volts.}}$$

When, $v = 0$,

$$1{\cdot}5\pi \cos 100\pi t + 6 \sin 100\pi t = 0,$$

or, $6 \sin 100\pi t = -1{\cdot}5\pi \cos 100\pi t.$

Dividing by $6 \cos 100\pi t$ gives,

$$\tan 100\pi t = -\frac{\pi}{4},$$

$$= -0{\cdot}7854.$$

Since, $\tan(180° - \theta) = -\tan\theta,$

$$\tan^{-1}(-0{\cdot}7854) = 180° - 38°\,9',$$

$$= 141°\,51',$$

$$= 2{\cdot}4758 \text{ rad.}$$

$$\therefore 100\pi t = 2{\cdot}4758,$$

or, $t = \frac{2{\cdot}4758}{100\pi}$ s,

$$= \frac{24{\cdot}758}{\pi} \text{ ms},$$

$$= 7{\cdot}881 \text{ ms.}$$

Thus, one value of t, in milliseconds, when the voltage is zero, is $\underline{7{\cdot}881}$.

Note: In general, v will be zero when $t = 7{\cdot}881 \pm 10\,n$ ms, where $n = 0, 1, 2$ etc.

Q. 10. (*a*) *Evaluate* $\int_1^4 (3x^2 - x)dx$ *and deduce the mean value of* $(3x^2 - x)$ *from* $x = 1$ *to* $x = 4$.
(*b*) *Sketch the curve* $y = 2x^2 - 5x + 2$, *and calculate the area enclosed between this curve and the x-axis.*

A. 10. (*a*) $$\int_1^4 (3x^2 - x)dx = \left[\frac{3x^3}{3} - \frac{x^2}{2}\right]_1^4,$$

$$= \left(4^3 - \frac{4^2}{2}\right) - \left(1 - \frac{1}{2}\right),$$

$$= 64 - 8 - \tfrac{1}{2},$$

$$= \underline{55\tfrac{1}{2}}.$$

The mean value of $(3x^2 - x)$ from $x = 1$ to $x = 4$ is given by,

$$\frac{\int_1^4 (3x^2 - x)\,dx}{4 - 1} = \frac{55\frac{1}{2}}{3} \text{ from the preceding,}$$

$$= \underline{18\tfrac{1}{2}}.$$

(*b*) $$y = 2x^2 - 5x + 2.$$

This is the equation of a parabolic curve and, since the index of x^2 is positive, it will have a maximum value on its axis of symmetry.

$$\frac{dy}{dx} = 4x - 5.$$

For maximum or minimum values, $\frac{dy}{dx} = 0.$

$$\therefore 4x - 5 = 0,$$

or, $x = \frac{5}{4}$ gives the value of x at which the minimum value occurs.

When $x = \frac{5}{4}$, $y = 2 \times \left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 2,$

$$= -1\frac{1}{8}.$$

$\therefore$ The point $\left(\frac{5}{4}, 1\frac{1}{8}\right)$ locates the axis of symmetry and the minimum value.

When $y = 0$,

$$2x^2 - 5x + 2 = 0,$$

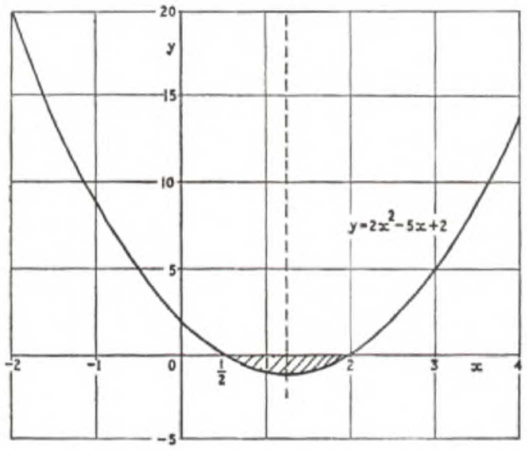
or, $(2x - 1)(x - 2) = 0.$

i.e. $x = \frac{1}{2}$ or 2.

Hence, the curve crosses the x-axis at these two values of x.

When $x = 0$, $y = 2$ and hence $(0, 2)$ is a point on the curve.

The curve may now be drawn approximately from the information obtained and is shown in the sketch. To determine its shape somewhat more accurately, additional values of y at say, $x = -2$ and $x = 4$ may be determined and the points plotted.



The parabola is shown in the sketch.

The area enclosed between the curve and the x-axis is shown shaded and may be obtained as

$$\int_{1/2}^{2} y\,dx = \int_{1/2}^{2} (2x^2 - 5x + 2)\,dx,$$

$$= \left[\frac{2x^3}{3} - \frac{5x^2}{2} + 2x\right]_{1/2}^{2},$$

$$= \frac{2}{3} \times 8 - \frac{5}{2} \times 4 + 4 - \left(\frac{2}{3} \times \frac{1}{8} - \frac{5}{2} \times \frac{1}{4} + 1\right),$$

$$= \frac{16}{3} - 10 + 4 - \left(\frac{1}{12} - \frac{5}{8} + 1\right),$$

$$= -\frac{2}{3} - \frac{11}{24},$$

$$= -\frac{9}{8}.$$

Thus, the required area is $1\frac{1}{8}$ square units.

Note: The negative sign arises from the fact that all the ordinates over the range concerned are negative.

MATHEMATICS C, 1971

Students were expected to answer any six questions

Q. 1. The resonant frequency ω_o of a circuit is given by $\omega_o^2 = \frac{1}{LC} - \frac{R^2}{L^2}$. Rearrange this formula as a quadratic equation in L, and show that it is satisfied by a single value of L only if $\omega_o = \frac{1}{2CR}$. When $C = 2 \times 10^{-6}$ and $R = 20$, calculate the possible values of L to give resonance at $\omega_o = 10^4$.

A. 1.

$$\omega_o^2 = \frac{1}{LC} - \frac{R^2}{L^2}.$$

$$\therefore\ \omega_o^2 L^2 = \frac{L}{C} - R^2,$$

$$\text{or, } \omega_o^2 L^2 - \frac{L}{C} + R^2 = 0.$$

From the general formula for the solution of a quadratic equation,

$$L = \frac{\frac{1}{C} \pm \sqrt{\left(\frac{1}{C^2} - 4\omega_o^2 R^2\right)}}{2\omega_o^2}.$$

For L to be single-valued, the discriminant must be zero.

$$\therefore\ \frac{1}{C^2} - 4\omega_o^2 R^2 = 0,$$

$$\text{or, } \omega_o^2 = \frac{1}{4C^2R^2}.$$

$$\therefore\ \omega_o = \frac{1}{2CR}. \qquad \text{Q.E.D.}$$

When $C = 2 \times 10^{-6}$, $R = 20$ and $\omega_o = 10^4$,

$$L = \frac{\frac{1}{2 \times 10^{-6}} \pm \sqrt{\left(\frac{1}{4 \times 10^{-12}} - 4 \times 10^8 \times 20^2\right)}}{2 \times 10^8},$$

$$= \frac{\frac{10^6}{2} \pm \sqrt{\left(\frac{10^{12}}{4} - 16 \times 10^{10}\right)}}{2 \times 10^8},$$

$$= \frac{10^6 \pm 10^6\sqrt{(1 - 0{\cdot}64)}}{4 \times 10^8},$$

$$= \frac{1 \pm \sqrt{(0{\cdot}36)}}{400},$$

$$= \frac{1 \pm 0{\cdot}6}{400},$$

$$= 0{\cdot}004 \text{ or } 0{\cdot}001.$$

Thus, the possible values of L to give resonance at $\omega_o = 10^4$ are 0·001 and 0·004.

Q. 2. Write down and simplify the first four terms of the binomial expansion of $(1 + 3x)^{1/3}$ in ascending powers of x, and give an expression for the term in x^n. For what range of x is this expansion justified? Use this expansion to calculate the cube root of 0·97 to five decimal places.

A. 2.

$$(1 + 3x)^{1/3} = 1 + \tfrac{1}{3} \times 3x + \frac{\frac{1}{3}(\frac{1}{3} - 1)}{1 \times 2}(3x)^2 + \frac{\frac{1}{3}(\frac{1}{3} - 1)(\frac{1}{3} - 2)}{1 \times 2 \times 3}(3x)^3 + \ldots,$$

$$= 1 + x - \frac{\frac{1}{3} \times \frac{2}{3}}{1 \times 2} 9x^2 + \frac{\frac{1}{3} \times \frac{2}{3} \times \frac{5}{3}}{1 \times 2 \times 3} 27x^3 - \ldots,$$

$$= 1 + x - x^2 + \frac{5x^3}{3} - \ldots.$$

The term in x^n may be written as

$$\frac{(-1)^{n-1}\, \frac{1}{3} \times \frac{2}{3} \times \frac{5}{3} \ldots \frac{\{2 + 3(n-2)\}}{3}}{1 \times 2 \times 3 \ldots n}(3x)^n,$$

$$= (-1)^{n-1}\frac{1 \times 2 \times 5 \ldots \{2 + 3(n-2)\}}{n!}x^n,$$

since there are n divisions each equal to 3 which will cancel the 3^n from the term $(3x)^n$.

Thus, the term in x^n may be expressed as

$$(-1)^{n-1}\frac{1 \times 2 \times 5 \ldots \{2 + 3(n-2)\}}{n!}x^n.$$

As the index of the binomial expansion is not a positive integer, the series is infinite.

The series converges when $1 > 3x > -1$,

i.e. when $\frac{1}{3} > x > -\frac{1}{3}$.

Thus, the expansion is justified for the range of x values from $\frac{1}{3}$ to $-\frac{1}{3}$.

$$\sqrt[3]{(0{\cdot}97)} = (1 - 0{\cdot}03)^{1/3} = \{1 + 3(-0{\cdot}01)\}^{1/3}.$$

Since x in this case is $-0 \cdot 01$, the expansion is justified.

$$\therefore \sqrt[3]{(0 \cdot 97)} = 1 - 0 \cdot 01 - (-0 \cdot 01)^2 + \tfrac{5}{3}(-0 \cdot 01)^3$$

$$+ (-1)^3 \frac{1 \times 2 \times 5 \times 8}{4!}(-0 \cdot 01)^4 + \ldots,$$

$$= 1 - 0 \cdot 01 - 0 \cdot 0001 - \tfrac{5}{3} \times 0 \cdot 000001$$

$$- \tfrac{10}{3} \times 0 \cdot 00000001 - \ldots.$$

It is clear that the last term and any subsequent terms will not affect the fifth decimal place and hence,

$$\sqrt[3]{(0 \cdot 97)} \simeq 1 - 0 \cdot 01 - 0 \cdot 0001 - \tfrac{5}{3} \times 0 \cdot 000001,$$

$$= 0 \cdot 9899 - 0 \cdot 000001\dot{6},$$

$$= 0 \cdot 989898\dot{3}.$$

$$\therefore \sqrt[3]{(0 \cdot 97)} = \underline{0 \cdot 98990} \text{ correct to five decimal places.}$$

Q. **3.** *By drawing suitable sketch-graphs verify that there is a value of x (measured in radians) between $x = \frac{\pi}{2}$ and $x = \pi$ which satisfies the equation $\tan x = x - \frac{3\pi}{2}$.*

By graphical enlargement (or otherwise) obtain this value to three significant figures.

A. 3. $\operatorname{Tan} x = x - \frac{3\pi}{2}$.

The solution to this equation may be obtained graphically by plotting the curve $y = \tan x$ between the appropriate limits and also the straight line $y = x - \frac{3\pi}{2}$ between the same limits and obtaining the intersection point.

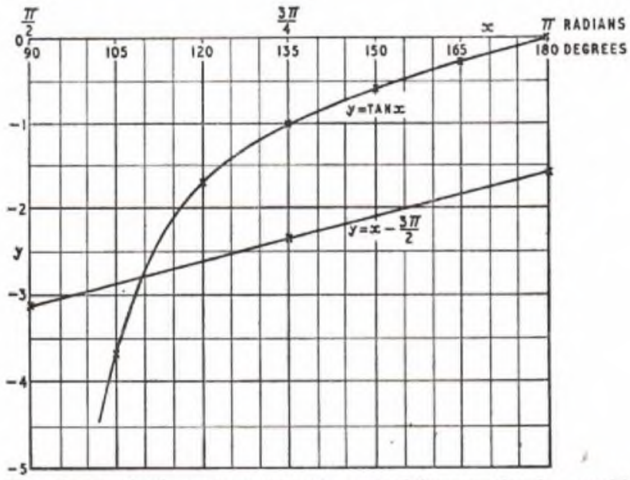
The straight line is obtained from the following values, using three rather than two to provide a check:

x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = x - \frac{3\pi}{2}$	$-\pi$ $= -3 \cdot 142$	$-\frac{3\pi}{4}$ $= -2 \cdot 356$	$-\frac{\pi}{2}$ $= -1 \cdot 571$

The curve is obtained from the following values for $\tan x$:

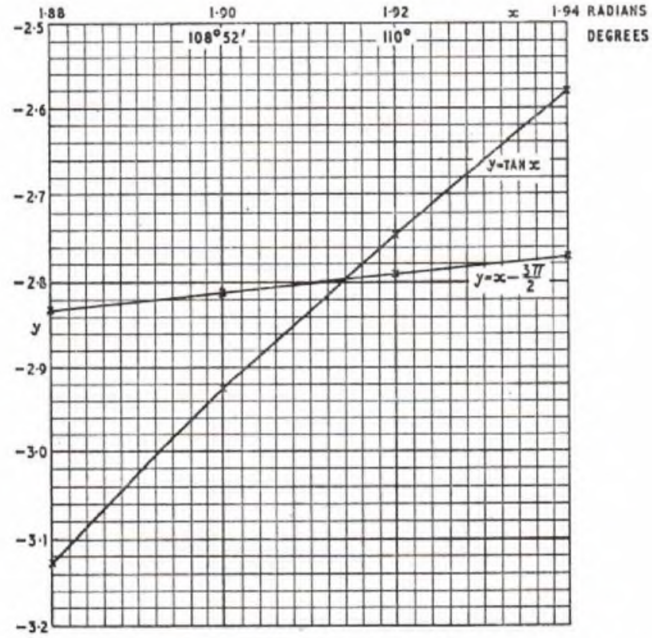
x	radians	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
	degrees	90	105	120	135	150	165	180
$y = \tan x$		$\pm\infty$	$-3 \cdot 732$	$-1 \cdot 732$	-1	$-0 \cdot 577$	$-0 \cdot 268$	0

The graphs are shown in sketch (*a*) from which it is seen that they intersect where $x \simeq 109\tfrac{1}{2}°$ or $1 \cdot 91$ radians.



In order to obtain a more accurate root of the equation by graphical enlargement, it is necessary to plot, to a much larger scale, in the vicinity of $x = 1 \cdot 91$. The accuracy obtained from the first graph should be at least within the range 108° to 111° or, in radians, from $x = 1 \cdot 885$ to $x = 1 \cdot 937$. Hence, an appropriate range for plotting a second and more accurate graph will be from $x = 1 \cdot 88$ to $x = 1 \cdot 94$ and appropriate values are tabulated below.

x	radians	$1 \cdot 88$	$1 \cdot 90$	$1 \cdot 92$	$1 \cdot 94$
	degrees	107° 43′	108° 52′	110° 0′	111° 9′
$y = \tan x$		$-3 \cdot 1307$	$-2 \cdot 9266$	$-2 \cdot 7475$	$-2 \cdot 5847$
$y = x - \frac{3\pi}{2}$ $= x - 4 \cdot 7124$		$-2 \cdot 8324$	$-2 \cdot 8124$	$-2 \cdot 7924$	$-2 \cdot 7724$



The graphs are shown in sketch (*b*) from which it is seen that they intersect very close to $x = 1 \cdot 9145$. It will be noted that, although $y = \tan x$ is a curve, the plotted points have been joined by straight lines because the discrepancy between a curve and a straight line is almost small enough to be negligible. It so happens in this instance that, had a curve been drawn with the aid of a few more plotted points, the intersection point would have given a slightly smaller value of x and, thus, lessened any possible doubt about proximity to the value $x = 1 \cdot 915$ in which case the answer would have been $1 \cdot 92$ to three significant figures instead of $1 \cdot 91$. The closeness of the answer obtained emphasizes the need for great accuracy and considerable forethought in plotting the second and "accurate" graph.

Thus, from the graph, the value of x in the range $x = \frac{\pi}{2}$ to $x = \pi$ to satisfy the equation $\tan x = x - \frac{3\pi}{2}$ is $x = \underline{1 \cdot 91}$ correct to three significant figures.

Note: The value of this root to four significant figures is $1 \cdot 914$ which is very close to the graphical value.

Q. **4.** *(a) Prove the identity $\tan x + \cot x = 2 \operatorname{cosec} 2x$.*

(b) Sketch the voltage-time graph of the waveform $v = 6 \cos \omega t - 2 \cos 3\omega t$ to display one complete wave (t in seconds).

Find the first maximum of v to occur after $t = 0$. If $\omega = 10^4$ radians find when this maximum occurs.

A. 4. (*a*) $\tan x + \cot x = 2 \operatorname{cosec} 2x$.

The left-hand side (L.H.S.) $= \tan x + \cot x$,

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x},$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}.$$

But, $\sin^2 x + \cos^2 x = 1$.

$$\therefore \text{L.H.S.} = \frac{1}{\sin x \cos x},$$

$$= \frac{2}{\sin 2x} \text{ since } \sin 2x = 2 \sin x \cos x,$$

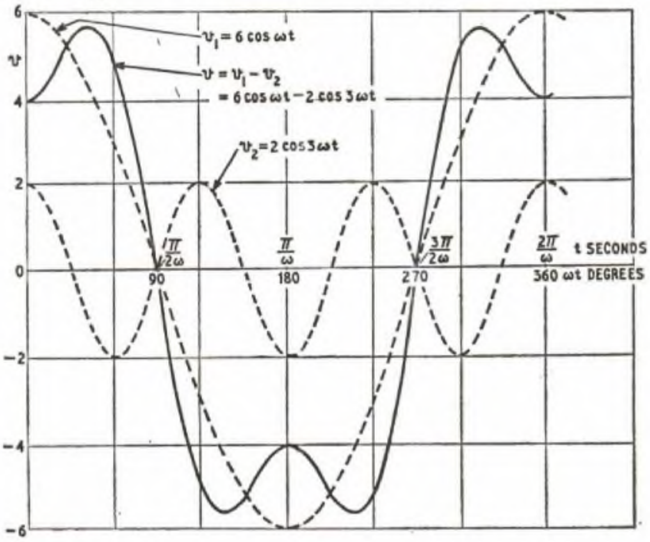
$$= 2 \operatorname{cosec} 2x,$$

$$= \text{right-hand side of identity.} \qquad \text{Q.E.D.}$$

(*b*) $$v = 6 \cos \omega t - 2 \cos 3\, \omega t.$$

The voltage waveform comprises two terms, one of angular velocity ω or frequency $f = \frac{\omega}{2\pi}$ and the other of three times this frequency. As the amplitude of the first term is three times that of the second, the frequency of the complete waveform of v will be that of the fundamental term and, hence, one complete wave occurs in t where $\omega t = 2\pi$ radians.

The curve may be sketched most easily by separately drawing the two cosine curves $v_1 = 6 \cos \omega t$ and $v_2 = 2 \cos 3\, \omega t$ and then combining them by subtracting values of v_2 from those of v_1 at corresponding times. At $t = 0$ both cosines equal unity and, hence, the instantaneous values of v_1 and v_2 are 6 and 2 respectively. One complete wave of v_1 occurs in $\frac{2\pi}{\omega}$ seconds, but three complete waves of v_2 occur in the same



time as shown in the sketch, their maxima occurring at 0°, 120°, 240° and 360°. The waveform of v, shown by the full-line sketch, is characterized by the peaks and troughs which correspond in time approximately with the minima and maxima respectivity of v_2.

Note: The curve reproduced here has been drawn accurately but only a freehand sketch would be expected from students, provided the salient features were included, i.e. scale points along both axes and a reasonable interpretation of the cosine waves and the combined wave of v. Some students might find it easier to plot the curve of $-v_2$ (thus producing the mirror-image of the curve in the sketch) and to add values to those of v_1.

$$\frac{dv}{dt} = -6\,\omega \sin \omega t + 6\,\omega \sin 3\,\omega t.$$

For maxima or minima to occur, $\frac{dv}{dt} = 0$.

$$\therefore 6\,\omega \sin \omega t = 6\,\omega \sin 3\,\omega t,$$

$$\text{or, } \sin \omega t = \sin 3\,\omega t.$$

$$\text{But, } \sin 3\,\omega t = 3 \sin \omega t - 4 \sin^3 \omega t.$$

$$\therefore \sin \omega t = 3 \sin \omega t - 4 \sin^3 \omega t,$$

$$\text{or, } 4 \sin^3 \omega t = 2 \sin \omega t.$$

$$\therefore \sin^2 \omega t = \tfrac{1}{2} \text{ or, } \sin \omega t = 0.$$

$$\therefore \sin \omega t = \pm 0{\cdot}7071 \text{ or } 0.$$

$$\therefore \omega t = 0,\ \pi,\ \pm \frac{\pi}{4} \text{ or } \pm \frac{3\pi}{4},$$

within the range of the graph.

From the graph, it is clear that the first maximum of v to occur after $t = 0$ is that at $\omega t = \frac{\pi}{4}$,

i.e. at $t = \frac{\pi}{4\omega}$ s.

When $\omega t = \frac{\pi}{4}$,

$$v = 6 \cos \omega t - 2 \cos 3\,\omega t,$$

$$= 6 \cos \frac{\pi}{4} - 2 \cos \frac{3\pi}{4},$$

$$= 6 \times \frac{1}{\sqrt{(2)}} - 2 \times \left(-\frac{1}{\sqrt{(2)}}\right),$$

$$= \frac{8}{\sqrt{(2)}} = 5{\cdot}6568.$$

(*to be continued*)

Published by *The Post Office Electrical Engineers' Journal*, 2–12 Gresham Street, London, EC2V 7AG, and printed in Great Britain at the Gresham Press by Unwin Brothers Limited, Woking and London.