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## Can *H* and *D* be Measured Directly?

IN the Editorial of August 1946 we discussed the analogy between mechanical stress and strain, electrical $\mathcal{E}$ and $D$ and magnetic $H$ and $B$. This is a question that is continually arising, and on which divergent views are held. Strain can be measured directly but in his 'Theory of Elasticity' Professor Southwell expresses the opinion that stress has never been measured directly and "we can assert with some confidence that it never will." The stress can, however, be determined, at least theoretically, by indirect measurement, such as measurement of the strain produced in a standard material inserted at the point under consideration, analogous to the crevasse cavity experiment in magnetism. The stress is, however, usually calculated by dividing the load force by the cross-sectional area. In the Editorial referred to above we maintained that a similar state of affairs existed in the electric and magnetic fields; that in each case we have a directly measurable magnitude, $B$ or $\mathcal{E}$ and a magnitude that cannot be directly measured, $H$ or $D$. We showed that when a magnetic needle or solenoid is used to determine $H$, the measurement is primarily of the magnetic induction $B$ in which it is situated, and that $H$ is determined indirectly by what is really a cavity experiment. $H$ can be calculated by dividing a current or a number of ampere-turns by a length and similarly $D$ can be calculated by dividing a charge by an area.

This point of view has recently been discussed by P. Cornelius and H. C. Hamaker in *Philips Research Report* No. 4 of April 1949, and they maintain that both $H$ and $D$ can be directly measured. It depends largely on what is meant by the direct measurement of a magnitude which is really a condition existing at a given point. In a uniformly-wound toroid one can measure the current with an ammeter, and the length of the magnetic path in the toroid with a cm-scale, and divide one by the other to obtain the ampere-turns per cm. Can one call this a direct measurement of $H$? Cornelius and Hamaker say that $H$ may be measured, at least theoretically by means of a small solenoid of a single turn constructed as shown in Fig. 1, its diameter being small compared with its length. The material is assumed to be perfectly conducting and the coil is assumed to be short-circuited through an ammeter of negligible internal resistance. If the coil is brought from a field-free place to the point where $H$ is to be measured, a permanent current $I$ will be generated and $H$ in the direction of the axis is defined as the limit of $I/l$ as the length $l$ of the coil approaches zero about the point at which $H$ is to be determined. All this is very theoretical—one might almost say fictional—and tends to confirm the view that $H$ is not directly measurable. Moreover it is important to notice that the current set up in the short-circuited ideal solenoid produces a value of $H$ sufficient to maintain the flux through the solenoid, and therefore also $B$, at zero. This is not necessarily the same as maintaining the resultant $H$ at zero, unless the medium is free from hysteresis. If the medium had residual magnetism with $H = 0$, then on

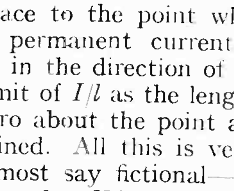


Fig. 1

introducing the short-circuited solenoid a current would be set up which would be a measure of the coercive force and not of $H$.

With the measurement of the displacement $D$ the authors seem to be on somewhat safer ground. They state that the method was indicated in Pohl's "Einführung in die Electricitätslehre." Assuming the medium to be gaseous or liquid, two metal discs are brought into contact face-to-face at the point where $D$ is to be measured, then separated and removed to a field-free place where they can be discharged through a ballistic galvanometer and the charge $Q$ thus determined. $D$ at the point will be equal to the quotient $Q/A$ as the area $A$ of the discs approaches zero. Here again, however, the magnitude $D$ at the point is not measured, but is assumed to be calculated from the limiting value of the quotient of a quantity and an area as the area approaches zero,—a very fictitious experiment. When two small metal discs are placed in contact face-to-face in the dielectric field, charges accumulate and distribute themselves over the external surfaces so as to make the resultant electric field $\mathcal{E}$ in the metal zero at every point. It might be thought at first sight that this charge would depend on the value of $\mathcal{E}$ and not of $D$ in the dielectric. It must be noted, however, that this is a cavity experiment; the metal discs are occupying a crevasse in the dielectric, and in the absence of the metal, the value of $\mathcal{E}$ in the vacuous crevasse would be $\kappa$ times that in the dielectric, since the displacement $D$ would be continuous across the crevasse. Hence the charge on the discs is a measure of $\kappa\mathcal{E}$, that is, of $D$ and not of $\mathcal{E}$ in the dielectric.

Professor Southwell did not say that the mechanical stress at a point could not be determined; he said that it could not be measured directly, and the arguments of Cornelius and Hamaker seem to confirm the view that the same is true of $H$ and $D$. G. W. O. H.

# The Movement of Electrons in a Transmitting Aerial

It is now generally known that the old idea that the current in a conductor is due to a rapid movement of electrons is quite wrong, and that even very large currents are associated with extremely slow drift of the electrons of the order of a few millimetres per second. If an alternating current of 1,000 amperes at a frequency of 50 cycles per second flows in a conductor of 1 square centimetre cross-section, the mean value of the current is about 900 amperes, and the total quantity crossing any cross-section in a half-cycle (i.e., in 1/100 second) is 9 coulombs. Assuming the number of free electrons to be equal to the number of atoms (i.e., $84 \times 10^{21}$ per cm$^3$) and the charge on each to be $1.6 \times 10^{-19}$ coulomb, the quantity of free electricity per cm$^3$ is $84 \times 1.6 \times 100$; i.e., 13 440 coulombs. In order that 9 coulombs may pass across the 1 cm$^2$ cross-section in the half-cycle, the average movement or drift of the electrons must be 9/13,440 cm. Hence we see that in a conductor carrying an alternating current of 1,000 amperes at 50 cycles per second the extreme travel of the free electrons does not exceed about a thousandth of a centimetre.

The results are even more surprising when we consider a radio transmitting aerial, for we feel sure that most people picture something rushing up and down the aerial. If the current is 100 amperes and the wavelength 10 metres, then assuming the aerial to have a cross-section of 0.5 cm$^2$, the average current density will be 180 A/cm$^2$, and the time of a half-cycle $1/(60 \times 10^6)$ second. The total charge of the free electrons in 1 cm length of the aerial will be 6 720 coulombs, and if their movement in the half-cycle is equal to $x$, then $6.720 \times x = i_{av} \times t = 90 \times 1/(60 \times 10^6)$ from which we find that the total movement $x$ of the electron is equal to $2.2/10^{10}$ cm. Far from rushing up and down the aerial, the free electrons—and they are the only things that move—have only a sub-microscopic movement.

During the quarter-cycle that the aerial is charging the average current is 90A and the duration of the quarter-cycle $1/(120 \times 10^6)$; hence the charge imparted to the aerial will be $75/10^8$ coulomb, and if the capacitance is say 100 pF, the aerial will be charged to a voltage of $(75/10^8) \times 10^{10} = 7{,}500$ volts. Hence, although the movement of the free electrons is sub-microscopic, their density is so great that the movement produces an appreciable change in the distribution of the charge and the consequent potential distribution.

The same reasoning would apply to the classical experiments of Hertz with his oscillator and spark-gap. Although the electron was then unknown, we feel sure that everybody at that time pictured the breaking down of the spark-gap as initiating a rushing to and fro of electricity in some form or other. This was doubtless true in the spark-gap, but in the adjacent conductors the movement of the electrons would be almost infinitesimally small.

G. W. O. H.

# HIGH-STABILITY OSCILLATOR

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**SUMMARY.**—An oscillator circuit which has been used extensively by the B.B.C. for some considerable time is discussed and analysed.

It is shown that with the circuit designed for maximum frequency stability, the stability obtained is for all practical purposes, a function only of the parameters of a single tuned circuit at series resonance.

Measured performance data are given for a practical form of the circuit which has been used for many years as a variable-frequency source of drive for B.B.C. short-wave transmitters.

## 1. Introduction

DURING the past year several references have been made in the American technical press to a particular form of oscillator circuit which bears a close resemblance to a Colpitts oscillator, the main difference being that a series LC circuit is substituted for the tuning inductance of the Colpitts circuit[1,2]. The circuit of this oscillator was conceived and developed independently in this country by the author some nine years ago, initially for use as a crystal-controlled oscillator, and in the same year as an LC controlled oscillator for use in variable-frequency transmitter-drive equipment.

Since its development, this circuit has proved to be highly satisfactory and has been used almost exclusively in B.B.C. transmitter equipment throughout the country.

Some aspects of the theoretical behaviour of this oscillator have been dealt with by J. K. Clapp, in an article published early last year.[1] It is, however, proposed to present here the author's original analysis of the circuit, since this is considered to be somewhat simpler than the treatment by Clapp, while it nevertheless demonstrates the many unique features of the circuit and provides the necessary design information for obtaining optimum performance.

Before discussing the circuit in question, it is considered worth while to examine briefly some of the fundamental principles which apply in general to all oscillators.

Any network that is capable of sustaining continuous oscillation at a fixed frequency must fundamentally comprise a frequency-controlling element and a source of energy. In order that a condition of stable equilibrium may be satisfied, any practical oscillator must also contain some non-linear element which is capable of ensuring that the amount of energy supplied to the frequency-controlling element will compensate exactly for the energy expended in the latter due to inherent losses. However, in theory at least, we are entitled to simplify the analysis by making the assumption that the equilibrium is obtained by an initial adjustment to the source of energy and thereafter that no changes occur in the value of the parameters involved, and for this idealized case we may regard all elements as being linear, and base our analysis on the assumption of a perfectly sinusoidal oscillation. While such an ideal arrangement cannot be realized in practice, we shall show later that it can be so closely approximated as to make all the theoretical conclusions substantially correct for the case of a practical circuit.

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Neglecting the possibility of compensating errors, it is quite obvious that the frequency stability of an oscillator can never exceed that of its frequency-controlling element; for example, the greatest accuracy with which a clock can record time is strictly limited to a figure which is dependent upon the physical stability of its pendulum. The accuracy obtained from the clock will probably be substantially less than this limiting accuracy, the reason being that it is impossible to impart to the pendulum the energy that is necessary to overcome the frictional losses, without altering to some extent its period of free oscillation. Thus, when comparing the merits of different oscillator circuits, it is convenient to assume in each case that the frequency-controlling element is intrinsically stable, and to examine the extent to which the 'pendulum' is upset by the presence of the maintaining element.

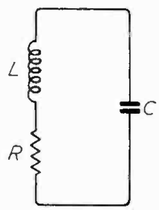



Fig. 1. *Basic series-resonance circuit.*

The simplest electrical frequency-controlling element is the tuned circuit comprising an inductor and a capacitor. As is well known, with such a circuit the loss of energy occurs mainly in the inductive limb, and Fig. 1 is a close approximation to a physically realizable closed resonant circuit.

In order to replace the energy dissipated in the loss resistance $R$, the simplest procedure, were it possible, would be to insert into the loop a two-terminal impedance of such a value that the circulating current $i$ would develop across it a voltage equal to that developed across the loss resistance, but of opposite sign as shown in Fig. 2(a). The value of this impedance would, of course, be $-R$, which introduces the concept of negative resistance. It is apparent that such an element would be providing energy and not dissipating it, and furthermore it would be a perfect maintaining element since it would have no effect upon the loop resonance frequency at which the oscillation would be maintained.

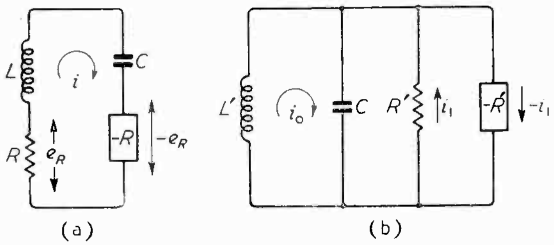



Fig. 2. *Zero-loss circuit obtained with series negative resistance (a), or shunt negative resistance (b).*

There is an alternative method by which energy could be supplied to the circuit. As is well known, we may express the series loss resistance as an equivalent parallel resistance $R'$ and arrive at the circuit of Fig. 2(b). The maintaining element may, therefore, be connected across the inductive limb, in which case, if expressed as a two-terminal impedance, it would need to have a value $-R'$.

It will be seen that there are two distinct disadvantages in this arrangement; first, the frequency of oscillation is no longer independent of the value of $R$, and secondly, the impedance across which the maintaining element is connected is relatively high and is, therefore, more susceptible to the effect of stray shunt capacitance. While it is true that the arrangement of Fig. 2(a) is equally susceptible to stray series inductance, the latter is far less troublesome at the frequencies for which such oscillators are normally used.

The ideal arrangement of Fig. 2(a) cannot be realized in practice, but circuits which will provide a complex impedance that includes a negative resistance are quite common and in fact form part of any oscillator circuit. Such a case is shown in Fig. 3, where the impedance of the maintaining circuit is represented by a negative resistor $R_g$ and a series capacitor $C_g$. It is clear that if $C_g$ is small and comparable with $C_0$, its effect upon frequency will be of a first order, and it must, therefore, be regarded as part of the frequency-controlling element. If it exists physically as a capacitor it may be possible to arrange for it to have the same order of stability as that of the controlling capacitance $C_0$, but if it merely represents a quadrature component of voltage or current, as, for example, in the case of Miller effect, then to stabilize its value may prove to be very difficult.

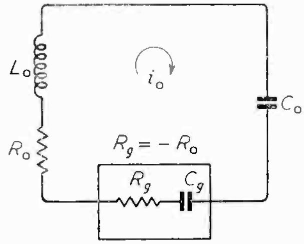



Fig. 3. *The impedance of the maintaining circuit is represented by $R_g C_g$.*

In either case, if $C_g$ is sufficiently large compared with $C_0$, although strictly speaking it will still form part of the frequency-controlling network, its effect upon frequency may be small enough for it to be neglected. In the analysis which is to follow, it will be shown that, in the circuit with which we are concerned, $C_g$ is a capacitance which exists physically, and also that it can be made sufficiently large compared with $C_0$ for the circuit to approximate closely to the ideal arrangement of Fig. 2(a).

## 2. Theoretical Analysis

A simplified form of the B.B.C. oscillator circuit is shown in Fig. 4(a). In practice a choke of suitable inductance would be connected across $C_2$ to serve as a d.c. path for the anode current of the valve, but since this component has virtually no effect on the behaviour of the circuit it has been omitted. If an inductance is substituted for the series LC element which is connected between terminals G and E, the circuit will become identical to a Colpitts oscillator.

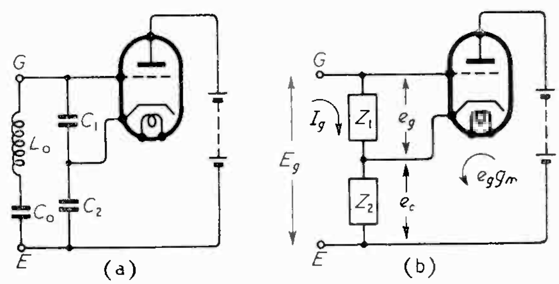



Fig. 4. *Simplified form of the practical circuit (a), and the generalized form of its oscillation maintaining elements (b).*

In Fig. 4(b) the series LC element has been removed, and the capacitors $C_1$ and $C_2$ have been replaced by the generalized impedances $Z_1$ and $Z_2$. It is shown in Appendix I that the impedance looking into the terminals G and E may be written

$$Z_g = Z_1 + Z_2 + g_m Z_1 Z_2 \quad .. \quad .. \quad (1)$$

From this it will be seen that if $Z_1$ and $Z_2$ are pure reactances, $jX_1$ and $jX_2$ respectively, the impedance becomes

$$Z_g = j(X_1 + X_2) - g_m X_1 X_2 \quad .. \quad (2)$$

of which the real part will be negative providing $X_1$ and $X_2$ are of the same sign. The analysis is therefore equally applicable to a Hartley circuit, similarly modified, in which $Z_1$ and $Z_2$ are inductances.

Returning to the circuit of Fig. 4(a) in which $Z_1$ and $Z_2$ are the reactances of $C_1$ and $C_2$ respectively, we shall obtain here, for the input impedance, a resistance.

$$R_g = -\frac{g_m}{\omega^2 C_1 C_2} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

in series with a capacitative reactance,

$$\frac{1}{\omega C_g} \text{ where } C_g = \frac{C_1 C_2}{C_1 + C_2} \quad \ldots \quad \ldots \quad (4)$$

It will be noted that $C_g$ is simply the series combination of the physically real capacitances $C_1$ and $C_2$.

With the series LC element connected across this impedance, it is apparent that the condition of loop resonance which will determine the frequency of oscillation is given by

$$\omega L_0 - 1/\omega C_0 - 1/\omega C_g = 0 \quad \ldots \quad \ldots \quad (5)$$

The energy equilibrium necessary for continuous oscillation will be satisfied by the condition

$$R_0 = -R_g = g_m/\omega^2 C_1 C_2 \quad \ldots \quad \ldots \quad (6)$$

By letting $G$ equal the maximum value of mutual conductance that can be obtained from the valve under prescribed conditions, we may allow for slightly more energy to be available than is actually required to maintain the oscillation, and write (6) as

$$R_0 \leqslant G/\omega^2 C_1 C_2 \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Now it is desirable that the total capacitance $C_g$ shall be as large as possible in order that the frequency of oscillation shall approximate closely to the resonant frequency of the controlling elements $L_0$ and $C_0$, but a limit to the maximum value is imposed by (7), which states implicitly that for a given value of mutual conductance and for a given frequency, unless the product $C_1 C_2$ is sufficiently small to satisfy the equilibrium, the energy supplied will be insufficient to maintain the oscillation.

For a given product of $C_1$ and $C_2$, the series capacitance $C_g$ is a maximum when $C_1 = C_2$, and by writing

$$C_1 = C_2 = 2\, C_g \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

we obtain from (7)

$$1/\omega C_g \geqslant 2\sqrt{(R_0/G)} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

This equation is important since it shows that, for oscillation to be maintained, the minimum permissible reactance of $C_g$ is a function only of the loss resistance of the frequency-controlling element and the valve mutual conductance.

At this stage in the analysis it is worth while examining the value that this reactance will have in a practical case. For example, we will assume that a high-slope pentode is used with a maximum mutual conductance of 8 mA/V and that it is operated at a value of 6 mA/V.

At a frequency of 1.0 Mc/s let

$$\omega L_0 = 1/\omega C_0 = 800\,\Omega \; (L \approx 127.5\,\mu\text{H}, C = 200\,\text{pF}) \text{ and } Q = \omega L_0/R_0 = 200$$

whence $R_0 = 4\,\Omega$

The required reactance of $C_g$, assuming that the oscillation frequency is 1.0 Mc/s, is thus

$$X_g = 2\sqrt{\{4/(6 \times 10^{-3})\}} = 51.6\,\Omega \quad \ldots \quad (10)$$

which corresponds to a capacitance,

$$C_g = 3100\,\text{pF approximately.}$$

This capacitance is in series with the frequency-controlling capacitance $C_0$, and therefore the oscillation will occur at a frequency slightly higher than 1.0 Mc/s. To a first approximation the actual frequency will be obtained by multiplying 1.0 Mc/s by a factor $\sqrt{(1 + C_0/C_g)}$.

Since the value of $C_0$ is approximately 200 pF we obtain for the actual frequency of oscillation, to a close approximation

$$\sqrt{(1 + 200/3100)} = 1.032 \text{ Mc/s} \quad \ldots \quad (11)$$

The error of 3.2% is sufficiently small to be neglected in the following analysis and henceforth we shall assume that the oscillation occurs at the series resonant frequency of $L_0$ and $C_0$.

In the practical case we have just examined, the capacitors $C_1$ and $C_2$ will each be approximately 0.006 $\mu$F, and at the frequency of oscillation the reactance of each will be of the order of 25 Ω. These figures indicate how relatively insignificant will be the effect upon frequency of stray impedance, due, for example, to the valve interelectrode capacitances, since the magnitude of such impedance at the frequency of oscillation will be of the order of thousands of ohms.

In addition to the presence of stray capacitance, the value of the impedance $Z_g$ will also be modified to some extent owing to the presence of harmonic components which, so far, we have chosen to ignore. Llewellyn has shown[3] that by the process of intermodulation these components can cause an additional phase-shift at the fundamental frequency. The significance of such phase-shift will be discussed in more detail later, but for the present, assuming that a modification to the impedance $Z_g$ results, it is apparent that it can be accounted for completely in terms of a change of $C_g$, since the real part of $Z_g$ must of necessity exactly equal the loss resistance $R_0$, which according to our premises is of constant value.

Thus the merit of the circuit, from the point of view of frequency stability, is conveniently expressed by the value of the differential coefficient, $df/dC_g$ and in Appendix 2 this has been deduced for the general case and is shown to be given by

$$\frac{df}{dC_g} = -\frac{4\pi f^2}{QG} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

For small changes of the order of a few parts in a million we may write this as

$$\frac{\Delta f}{f} = -\frac{4\pi f}{QG} \cdot \Delta C_g \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

or $$\Delta F = -\frac{4\pi F}{QG} \text{ cycles/megacycle/per pF} \quad (14)$$

where $F$ is expressed in megacycles per second.

As a generalized expression of the stability, the equation in this form is very useful, since we can obtain from it immediately the minimum value of the coefficient that can be obtained at any frequency, given the $Q$ of the controlling element and the maximum mutual conductance at which the valve can operate.

At first sight, however, this expression is likely to be misleading, since it would suggest that for circuits of similar $Q$ the coefficient is independent of the L/C ratio of the controlling element, and also that its magnitude is directly proportional to the frequency of oscillation. While this is quite correct, it is somewhat surprising on philosophical grounds, but the reason is bound up with the postulate of a constant $Q$. The time constant of an inductance is physically a more appropriate expression of its merit than is its $Q$, and if we substitute $\omega L_0/R$ for $Q$ in, equation (13) we obtain $\frac{\Delta f}{f} = -\frac{2}{\tau G} \cdot \Delta C_g$ where

$$\tau = L_0/R_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

This is entirely reasonable since it states simply that the stability depends only upon the loss factor of the controlling element and the mutual conductance of the valve, the latter of which parameters expresses in effect the degree of coupling which exists between the maintaining element and the controlling element.

Since in practice the value of $R_0$ depends upon such factors as skin effect and eddy-current loss, the effective value of $\tau$ at any particular frequency may be obtained conveniently only by steady-state measurements, and for this reason the coefficient expressed in terms of $Q$ is more useful. If it is found that over a frequency band the maximum value of $Q$ that we can obtain tends to remain constant, so that the fractional frequency change expressed in (14) increases with frequency, it is simply because we cannot produce such good inductors at high frequencies.

As a practical example, if we substitute in (14) the values quoted in the previous example, in which $F = 1.0$ Mc/s, $Q = 200$, $G = 6$ mA/V we shall obtain $\Delta F = \frac{4\pi}{200 \times 6 \times 10^{-3}} = 10.5$ cycles per second per pF change of $C_g$. A similar change in the value of $C_1$ or $C_2$ alone will produce one quarter of this change in frequency, which is approximately 2.6 cycles per second per pF.

For such small frequency changes to be of significance it would, of course, be necessary to stabilize the temperature of the controlling elements $L_0$ and $C_0$ to a relatively high degree of constancy, and if this is done, there is no reason why the capacitors $C_1$ and $C_2$ should not be included in the temperature-controlled chamber.

In this connection it is of interest to compare the frequency : temperature coefficient of the input capacitance $C_g$ and the frequency-controlling capacitor $C_0$. High-stability capacitors are available commercially with a capacitance temperature coefficient as low as 25 parts in $10^6$ per °C over a wide range of values. If the values of $C_1$ and $C_2$ are each changed by 1.0 pF simultaneously, the resulting change in frequency for the values quoted in the above example will be 5.25 c/s. For the frequency : temperature coefficient we thus have

$$\Delta f = \frac{5.25 \times 3\,000 \times 25}{10^6} = 0.4 \text{ cycles per}$$

second/°C.

For a tuned circuit at resonance, as is virtually true of the frequency-controlling element, the frequency : capacitance coefficient is given by

$$\Delta f/f = \Delta C_0/2C_0$$

which may be written as

$$\Delta f = 10^6/2C_0 \text{ cycles/megacycle/pF}$$

For the frequency : temperature coefficient, assuming the same temperature stability of capacitance, we obtain

$$\Delta f = \frac{10^6}{2C_0} \times C_0 \times \frac{25}{10^6} = 12.5 \text{ cycles per}$$

second/°C.

Comparing the figure of 12.5 cycles per second/°C for the controlling capacitor with that of 0.4 cycle per second/°C for the input capacitance it will be seen that the effect upon frequency of the maintaining circuit due to change of temperature may be disregarded.

In addition to the effect of temperature variations, we have also to consider the stability of the valve interelectrode capacitances. The significant interelectrode capacitances are the grid-cathode capacitance and the cathode-earth capacitance, both of which are usually of the order of 10 pF. Experience shows that when a valve is left operating over a long period of time the variation of these capacitances does not usually exceed the order of 2%. Such a change in both capacitances simultaneously would, in the example quoted, cause the relatively insignificant frequency change of approximately 0.2 c/s.

So far we have considered the frequency

stability of the maintaining circuit in terms of variations in the value of the circuit capacitances which physically exist. We have referred briefly to the existence of what is in effect additional reactance, which does not exist physically, but results from a phase shift caused by the intermodulation of harmonic components generated by the non-linear characteristic of the valve. As a means of examining the effect of such phase shift, suppose that the phase-angle of the input impedance $Z_g$ were caused to change slightly by adding to the valve current a small current with some arbitrary phase relationship. Since the real part of $Z_g$ must always be equal in magnitude to the loss resistance $R_0$ of the controlling element, we may relate such a phase shift to a change in the effective value of $C_g$ only, and from (12) we may deduce the resulting change in frequency.

In Appendix 3 this is shown to be given by:

$$\frac{df}{d\phi} = \frac{f}{2Q} + \frac{1}{\pi L_0 G} \quad \ldots \quad (16)$$

The first term on the right-hand side of this expression is the differential coefficient of frequency with respect to phase of a tuned circuit at resonance, while the second term is due to the effect of the maintaining circuit. Using the same values as in the previous examples, we find that only the second term is significant, this being approximately 100 times as great as the first. From this it will be seen that there is an advantage in using a high L/C ratio. It is not proposed to give any quantitative interpretation to the value of this coefficient, except to say that perhaps it is worth noting that the effect of an additional quadrature current flowing through $Z_g$ with a relative amplitude of $-60$ db would be to cause a phase change of approximately 3.5 minutes of arc and a frequency change of 250 c/s. Such a current might well be caused by unintentional feedback from a subsequent amplifying stage, and if the relation between the signal fed back and the oscillation signal is not strictly linear, a relatively large frequency change may result, for example, from a small change in the supply voltage.

A direct comparison with the Colpitts circuit is simple and instructive. Since in this circuit a single inductance $L_g$ is connected across $C_g$, we may from (5) and (9) immediately write the conditions

$$\omega = 1/\sqrt{(L_g C_g)} \quad \ldots \quad (17)$$

$$C_g \leqslant \frac{1}{2\omega}\sqrt{\left(\frac{G}{R_g}\right)} \text{ where } R_g = \frac{\omega L_g}{Q} \quad \ldots \quad (18)$$

Substituting (17) in (18) we obtain

$$C_g = L_g G/4R_g = QG/4\omega \quad \ldots \quad (19)$$

For a resonant circuit we know that $\Delta f/f = \Delta C/2C$

$$\text{whence } \frac{\Delta_f}{f} = \frac{4\pi f \Delta C_g}{QG} \quad \ldots \quad (20)$$

which is precisely the same as for the modified circuit as expressed in (13). This is to be expected since we have already shown that for the modified circuit this coefficient is independent of the $L_0/C_0$ ratio, and the Colpitts circuit is in effect the modified circuit with the $L_0/C_0$ ratio put to zero.

However, when we examine the value that $C_g$ would have in a practical case we find from (19) using the previous values, $\omega = 2\pi \times 10^6$, $Q = 200$, $G = 6$ mA/V

$$C_g = \frac{200 \times 6 \times 10^{12}}{4 \times 10^3 \times 2\pi \times 10^6} = 48000 \text{ pF}$$

while $L_g$ would be 0.53 $\mu$H.

The impracticability of using such values is obvious, but the comparison nevertheless demonstrates that the stability of the Colpitts circuit is theoretically as great as that of the modified circuit insofar as component changes are concerned. We have, however, shown that the effect upon frequency of a phase change caused by non-linearity is directly proportional to the L/C ratio, and in this respect the modified circuit will be of the order of 150 times as stable.

## 3. The Amplitude Limiter

If an oscillator of the type described were to be constructed with circuit values chosen accurately for operation at a frequency of the order of 1.0 Mc/s, the measured frequency would be found to differ from the exact design frequency by some 100 to 500 cycles per second according to the amplitude of oscillation. If the amplitude of oscillation were to be reduced by any means, for example by reducing the valve current, the discrepancy would decrease, and would in fact tend to zero as the amplitude of oscillation approached zero. A complete demonstration of this fact is difficult to perform, since at small amplitudes of oscillation a valve characteristic is too linear to provide a condition of stable equilibrium, but it is only too easy to demonstrate with any simple oscillator that the frequency is to some extent a function of the supply voltages!

As mentioned earlier, the phenomenon may be explained in terms of the phase shift which results from intermodulation products, but there is also a simple mechanical analogue, which, although not in all respects perfect, may nevertheless be found instructive.

Consider a freely suspended pendulum, the free period of oscillation of which has been accurately deduced from its known constants. Having displaced the pendulum from rest and allowed it to swing, we find that in order to maintain it in oscillation at a constant amplitude we must, after each return stroke, tap the bob so as to replace the energy that has been lost over the cycle due to friction. The action of tapping the bob must necessarily alter its velocity, since if this were not so, no force would be applied, and if while so maintained in oscillation, the frequency of the oscillation were measured, it would be found to be somewhat greater than the calculated frequency due to the increased velocity of movement on the forward stroke. The error could of course be corrected by adjusting the effective length of the pendulum, but the frequency would then only be exact for one amplitude of oscillation corresponding to one strength of 'tap.' The smaller the amplitude of swing, the more closely will the displacement of the bob follow simple harmonic motion, and the more accurately will the maintained frequency equal the 'free' frequency of the pendulum.

If we wish to construct an oscillator to have the maximum possible frequency stability, we must, therefore, include some form of limiter which will provide a condition of stable equilibrium when the actual amplitude of oscillation is small—sufficiently small to be confined to a substantially linear portion of the valve characteristic. A simple and effective method of limiting the oscillation is shown by the block schematic in Fig. 5. The signal output of the oscillator valve is amplified, rectified, and the d.c. signal so obtained is applied as a bias voltage to the oscillator grid.

By this means it is a simple matter to arrange that the oscillation will limit at any desired level.

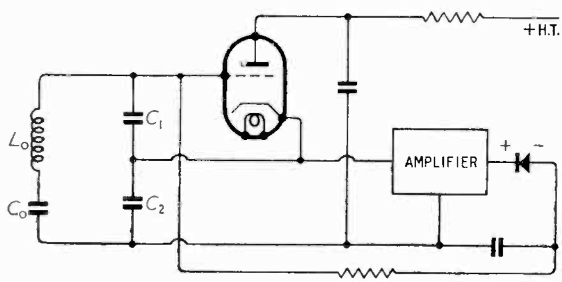
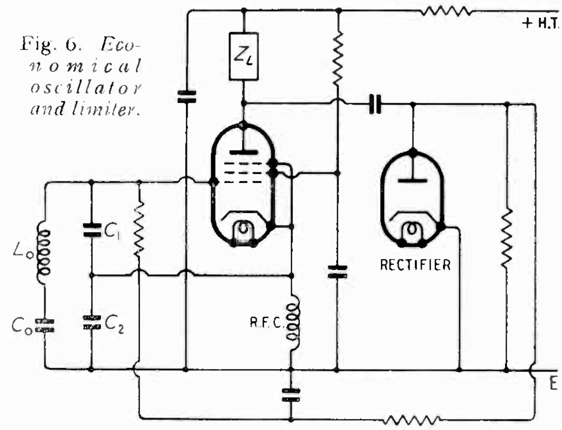



Fig. 5 *Simple oscillation amplitude-limiting circuit.*

For example, suppose that a slope of 6 mA/V is required of the oscillator valve to maintain the oscillation, and that from the valve characteristic we find that for a given anode voltage a grid bias of −3.0 volts will provide this slope. If we wish the oscillation to limit at a peak amplitude of, say, 0.1 V, measured between grid and cathode, an amplification of 30 will provide the required bias voltage. The amplified signal is of course available as an output signal, and in the example quoted above a 3.0-V peak signal would be obtained at the input to the rectifier.

As with most devices which provide automatic control by means of feedback, the circuit constants must be chosen to avoid instability. Instability in the form of a low-frequency relaxation oscillation will result if the time constant of the r.f. filter following the rectifier is too long, but it is not usually difficult to choose values that will provide adequate attenuation to the r.f. signal without introducing appreciable delay to the rectified control signal.

Fig. 6. *Economical oscillator and limiter.*




In one experiment, the author constructed a limiting circuit with a gain sufficient to cause the amplitude of oscillation to limit at a peak voltage of the order of 0.01 V. At such a small amplitude the harmonic distortion was, of course, negligible, and in order to prove that the frequency of oscillation was then precisely equal to the loop resonance of the controlling circuit and substantially unaffected by the valve parameters the oscillation was gradually reduced to zero amplitude by

(a) reducing the anode voltage ;

(b) reducing the heater voltage ;

(c) adding a negative bias voltage to the control grid.

In each case the frequency did not change by more than one part in $10^6$ before the oscillation ceased completely. It will be appreciated that a *low* amplitude of oscillation does not necessarily mean a 'weak' oscillation in the accepted sense of the term. The 'strength' of the oscillation is indicated by the built-up time, and depends upon the extent to which the energy available at the instant of switching on exceeds the energy required to maintain the oscillation.

A very simple and economical arrangement which will provide a useful control of the amplitude is shown in Fig. 6. By making $V_1$ a pentode, we may derive an amplified signal from the anode without seriously disturbing the operation of the circuit, provided the anode-load impedance, shown as $Z_L$, is not excessive. In practice, a gain of about 10 may be obtained without difficulty, and this is sufficient to operate the limiter to give a worth-while improvement in performance.

In a superior form of limiter, a separate valve is used to provide the necessary gain, in which case the oscillator valve may be connected as a triode with the anode decoupled directly to earth, and the signal output may be conveniently taken from the cathode, at which point, as has been shown, the impedance is very low.

## 4. Practical Performance

A practical form of the oscillator complete with limiter is shown in Fig. 7. For high-stability the series LC controlling circuit together with the two capacitors $C_1$ and $C_2$ are mounted in a temperature-controlled compartment or oven. Since the impedance measured at the grid and cathode is very low, stray capacitance introduced by the connections, which may be fairly long, is relatively unimportant.

If it is required to vary the frequency of the oscillator, the capacitors $C_1$ and $C_2$ must of course be chosen for the highest frequency, in which case the amplitude of the oscillation will increase as the frequency is reduced due to increased coupling.

For high stability over a wide frequency band it is, therefore, desirable to divide the total range into a number of smaller ranges; for example an octave may be conveniently divided into three ranges each of ratio approximately 1.26 : 1. The components $L_0$, $C_0$, $C_1$ and $C_2$ are then chosen for each range separately and a switching arrangement provided so that any one of the three complete assemblies may be connected to the oscillator valve. It will be noted that the switching takes place at the grid and cathode connections to the valve where the impedance looking towards the tuning elements is very low, while looking into the valve it is relatively high, so that neither stray shunt capacitance or stray series impedance will have a serious effect upon the frequency of oscillation.

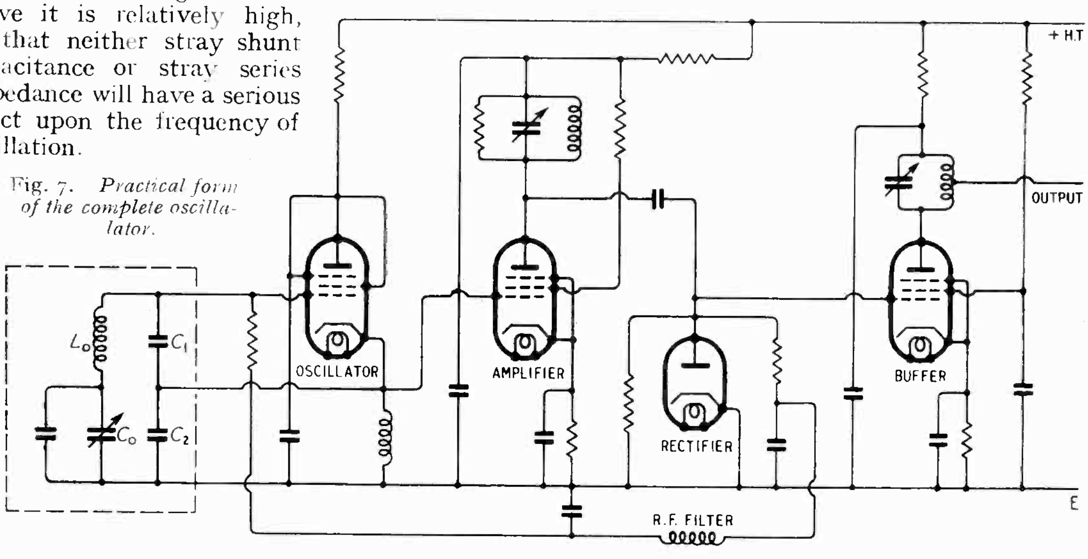



*Fig. 7. Practical form of the complete oscillator.*

A circuit of this form has been used for several years in B.B.C. short-wave transmitter-drive equipment with considerable success. In the actual circuit in use two high-slope pentodes are connected in parallel in order to obtain twice the mutual conductance of a single valve, thereby permitting the impedance measured across the series-tuned circuit to be approximately halved.

Below are quoted some measured performance data for this type of oscillator:

(1) H.T./frequency coefficient $\pm 2.5/10^6$ for $\mp 20\%$ h.t. voltage

(2) L.T./frequency coefficient $\pm 2.5/10^6$ for $\mp 5\%$ l.t. voltage

(3) Frequency Stability

(a) Short term (about 1 hour) $\pm 1/10^6$

(b) Med. term (about 1 day) $\pm 10/10^6$

(c) Long term (about 1 month) within re-setting accuracy of $\pm 30/10^6$

Needless to say, it was necessary to pay great attention to mechanical design and temperature stabilization in order to obtain this stability, but the results serve to illustrate the capabilities of the circuit.

As mentioned earlier, the circuit has been used with equal success as a crystal-controlled oscillator. Since the equivalent electrical circuit of a crystal approximates to a series LC circuit, it is only necessary to replace the series LC elements with a crystal for this purpose. A high-grade crystal cut for medium frequencies and carefully mounted usually has an equivalent series resistance of the order of 100 Ω and the appropriate value of the capacitors $C_1$ and $C_2$ is therefore considerably less than for an LC controlled circuit, for which the series resistance is usually much less than 10 Ω. However, due to the high $Q$ of a crystal as compared with an LC circuit, the effect upon frequency of a small change in the values of these capacitances is exceedingly small—in practice of the order of $2/10^9$ c/s per pF change. Again, in such circumstances it is doubtful if the maintaining circuit will contribute to any measurable extent to the overall instability of the oscillation.


## 5. Acknowledgment

The author wishes to thank the Chief Engineer of the British Broadcasting Corporation for granting permission to publish this article,


### APPENDIX 1

*Input Impedance*

The input impedance of the circuit will be analysed with reference to Fig. 4(b).

Assuming the valve to have an infinite impedance (e.g., a perfect pentode) we may write :—

Current flowing through $Z_2 = I_g + e_g g_m$ .. (1a)

Voltage across $Z_2 = Z_2 (I_g + e_g g_m) = e_c$ .. (2a)

and as $E_g = e_c + e_g$

$$E_g = Z_2 (I_g + e_g g_m) + e_g = I_g Z_2 + e_g (1 + g_m Z_2) \quad \text{(3a)}$$

but $e_g = I_g Z_1$

$$\therefore E_g = I_g Z_2 + I_g Z_1 (1 + Z_2 g_m) \quad \text{(4a)}$$

whence $E_g = Z_g = Z_1 + Z_2 + g_m Z_1 Z_2$ .. (5a)

We have assumed a perfect pentode but in fact it is only necessary that the valve impedance should be much greater than $Z_2$. Since, as is shown in the text, the impedance of $Z_2$ is, in practice, of the order of 100 Ω, the analysis is valid for any normal triode.

### APPENDIX 2

*Evaluation of the Coefficient* $\frac{df}{dC_g}$

Solving (5) for frequency, we obtain

$$f = f_0 \sqrt{(1 + C_0/C_g)} \quad \text{(6a)}$$

where $f_0$ is the resonant frequency of $L_0 C_0$.

Differentiating (6a) with respect to $C_g$ we obtain

$$\frac{df}{dC_g} = -f_0 \cdot \frac{C_0}{C_g^2} \cdot \frac{1}{2\sqrt{(1 + C_0/C_g)}} \quad \text{(7a)}$$

and since $C_0$ is small compared with $C_g$, and also $f_0$ is very nearly equal to $f$, we have approximately

$$\frac{df}{dC_g} = -\frac{fC_0}{2C_g^2} \quad \text{(8a)}$$

Also since $\omega L_0 \approx 1/\omega C_0$ we may write

$$Q = \frac{\omega L_0}{R_0} = \frac{1}{\omega C_0 R_0} \text{ or } R_0 = \frac{1}{\omega C_0 Q} \quad \text{(9a)}$$

By substituting this value for $R_0$ in (9) and taking the maximum permissible value of $C_g$, we obtain

$$C_g = \frac{1}{2\omega} \sqrt{GQ\omega C_0} \quad \text{(10a)}$$

and therefore $\frac{C_0}{C_g^2} = \frac{4\omega}{QG}$ .. (11a)

From (8a) and (11a) we therefore have

$$\frac{df}{dC_g} = -\frac{4\pi f^2}{QG} \quad \text{(12a)}$$

### APPENDIX 3

*Evaluation of the Coefficient* $\frac{df}{d\phi}$

The phase angle of the input impedance, $Z_g$, is given by $\phi = \tan^{-1} \frac{1}{\omega C_g R_g}$ .. (13a)

as shown in Fig. (8)

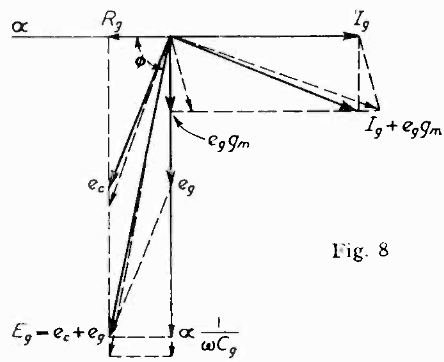



Fig. 8

Since $R_g$ is at all times numerically equal to the loss resistance $R_0$ (but of opposite sign) any change in the phase angle $\phi$ must be accounted for by a change in the reactive component only, and it is therefore equivalent to a change in the value of $C_g$. This is illustrated by the dotted vectors in Fig. (8) which indicate the effect of a phase displacement originating in the valve current vector.

From (13a) we obtain by differentiation

$$\frac{dC_g}{d\phi} = -\left[\frac{1}{\omega R_g} + \omega R_g C_g^2\right] \quad \text{(14a)}$$

or

$$\frac{dC_g}{d\phi} = \frac{1}{\omega R_0} + \omega R_0 C_g^2 \quad \text{(15a)}$$

since $R_g = -R_0$

From (9) we obtain

$$C_g^2 = \frac{1}{4\omega^2} \frac{G}{R_0} \quad \text{(16a)}$$

which by substitution in (15a) gives

$$\frac{dC_g}{d\phi} = \frac{1}{\omega R_0} + \frac{G}{4\omega} \quad \text{(17a)}$$

Also from (12) we have

$$\frac{df}{dC_g} = \frac{4\pi f^2}{QG} \quad \text{(18a)}$$

whence

$$\frac{df}{d\phi} = \frac{dC_g}{d\phi} \cdot \frac{df}{dC_g} = \frac{4\pi f^2}{QG}\left[\frac{1}{\omega R_0} + \frac{G}{4\omega}\right] \quad \text{(19a)}$$

$$= \frac{2f}{QGR_0} + \frac{f}{2Q} \quad \text{(20a)}$$

and since $Q = \omega L_0/R_0$

$$\frac{df}{d\phi} = \frac{1}{\pi L_0 G} + \frac{f}{2Q} \quad \text{(21a)}$$

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[1] "An Inductance-Capacity Oscillator of Unusual Frequency Stability," J. K. Clapp, *Proc. Inst. Radio Engrs*, March 1948, Vol. 36, No. 3.

[2] For example, see *Q.S.T.*, May 1948, p. 42, August 1948, p. 26, January 1949, p. 43.

[3] "Constant-Frequency Oscillators," Llewellyn. *Proc. Inst. Radio Engrs*, 1931, Vol. 19, pp. 2063–2094.

# IMPEDANCE MATCHING NETWORKS

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**SUMMARY.**—A method is described of designing a four-terminal network whose image impedance at one pair of terminals is a constant resistance and at the other pair of terminals a complex quantity varying with frequency.

## Introduction

It is sometimes necessary to connect equipment, such as an unloaded cable whose characteristic impedance $W'$ is a complex quantity varying with frequency, to some other equipment whose impedance is nearly a constant resistance $R$ over the working frequency band. To overcome the bad effects of reflection due to connecting two such dissimilar impedances together an impedance-matching network is inserted between the two pieces of equipment. This network is designed to have image impedances $W$ and $R$; $W$ being nearly equal to $W'$ over the frequency band.

Let the network of Fig. 1 (a) fulfil these conditions. If two such networks are connected back to back at the terminals whose impedance is $W$, then the resulting network is a symmetrical constant-resistance network and so may be transformed into a lattice whose series and lattice arms $Z_1$ and $Z_2$ are inverse with respect to $R$; i.e., $Z_1 Z_2 = R^2$

The most general method of design is therefore to start with a constant-resistance lattice network, expand it into a ladder network and bisect it.

If the lattice is capable of being expanded into a ladder network, then the arms $Z_1$ and $Z_2$ will contain a common series or shunt impedance. Let the former be the case, then

$$Z_1 = Z'_1 + Z_a$$

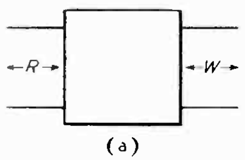
and

$$Z_2 = Z'_2 + Z_a$$

The common impedance $Z_a$ can be taken outside the lattice in series with both the input and the output as shown in Fig. 1 (b).

The remaining arms $Z'_1$ and $Z'_2$ will now have a common shunt impedance $Z_b$ and this can be taken outside the network in shunt with the input and output as shown in Fig 1 (c).

This process may be repeated taking out successive common series and shunt impedances until one pair of arms becomes either infinite or zero. The resulting ladder network is symmetrical and so may be bisected down the middle.



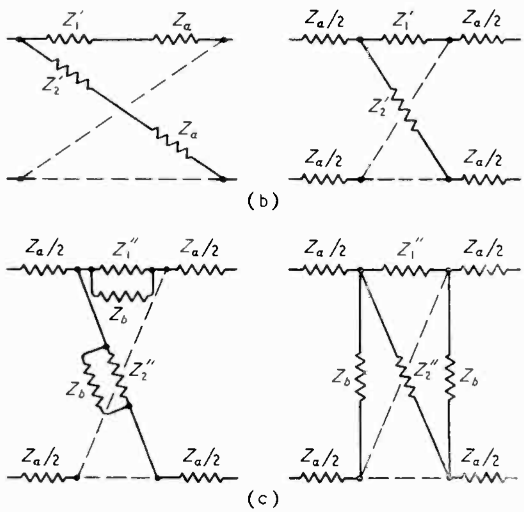




Fig. 1. *Basic network (a) and its expansion as lattice structures (b) and (c).*

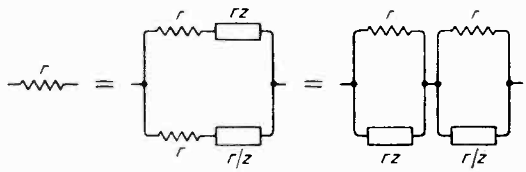


Fig. 2. *Equivalent networks, types 1 and 2 respectively.*

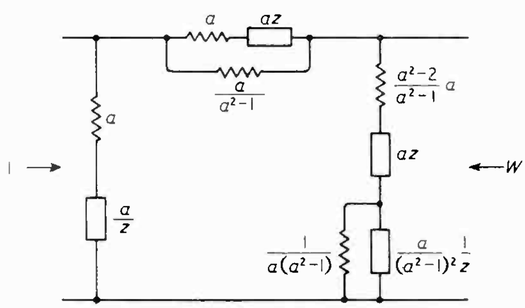



Fig. 3. *Network type 1.2.*

MS accepted by the Editor, July 1949.

An important relation which will be used in all the succeeding expansions is that a resistor $r$ may always be replaced by either of the networks of Fig. 2 where $rz$ is any two-terminal network containing resistors, inductors, and/or capacitors, and $r/z$ is its inverse with respect to $r$.

## Impedance-Matching Networks

*Type* 1.1

The simplest type of constant-resistance lattice network which can be expanded as a ladder structure is one containing a single resistor in each arm. The straightforward expansion of this

TABLE 1

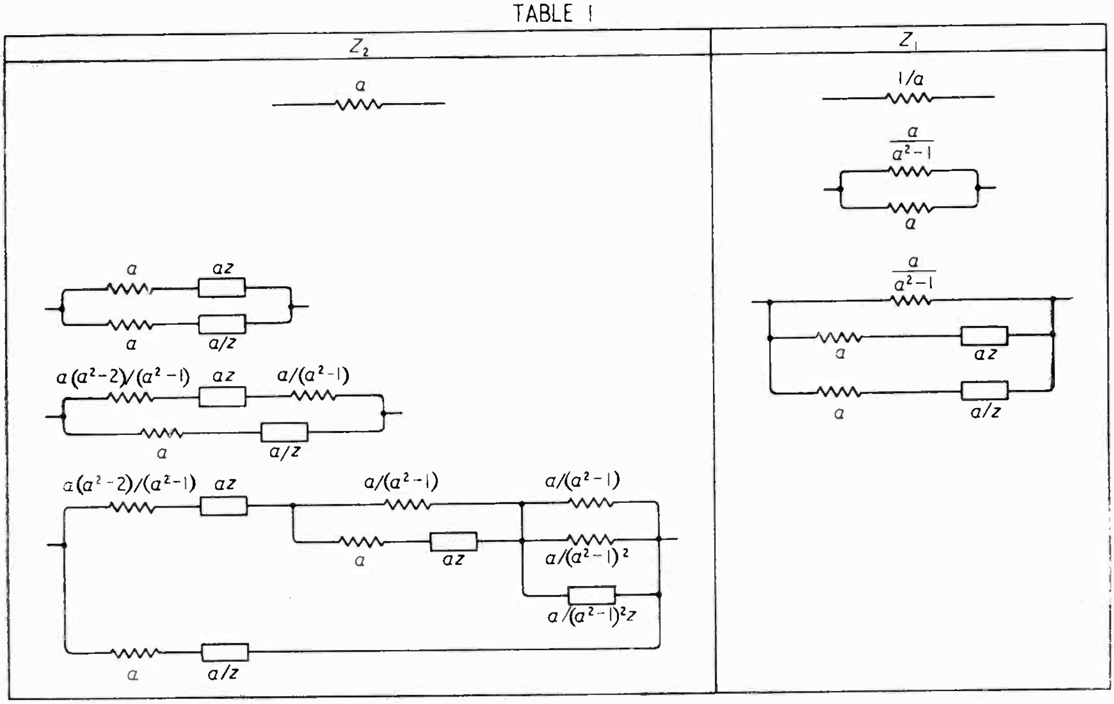
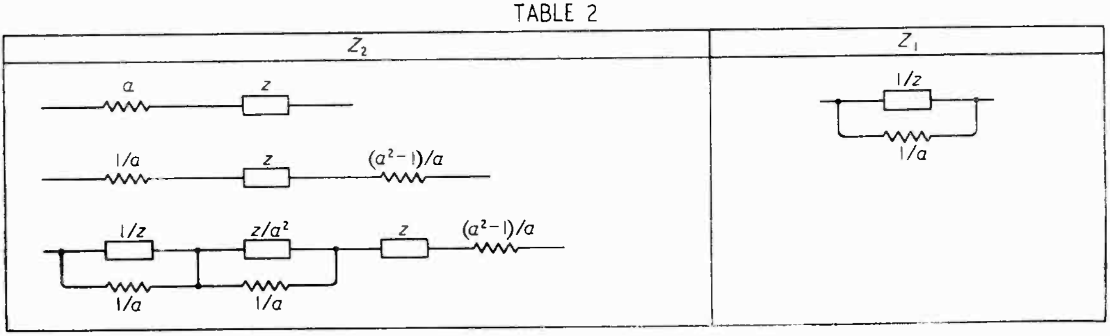



TABLE 2




*Explanation of Table* 1.

*Step* 1. *$Z_1$ is split into two resistors in parallel one of which has the value 'a'.*

*Step* 2. *The resistors 'a' in both $Z_1$ and $Z_2$ are replaced by an equivalent network, type* 1.

*Step* 3. *The resistor 'a' of $Z_2$, which is in series with 'az' is split into two resistors in series, one of which is equivalent to the shunt resistor of $Z_1$.*

*Step* 4. *This is replaced by an equivalent network type 2 part of which is equal to the two upper shunt arms of $Z_1$.*

*Explanation of Table* 2.

*Step* 1. *The resistor 'a' of $Z_2$ is split into two resistors in series one of which has the same value as the shunt resistor of $Z_1$.*

*Step* 2. *This resistor in $Z_2$ is replaced by an equivalent network type 2 part of which is the same as the whole of $Z_1$.*

network leads to the trivial case of an attenuator pad which matches the constant resistance $R$ to another constant resistance $nR$.

*Type* 1.2

In this and in subsequent networks the value of $R$ will be taken as unity for simplicity. If $Z_1$ and $Z_2$ are again purely resistive then each of them may be built out as shown in Table 1 by successive applications of the equivalent-network relationships of Fig. 2. If the value of $Z_2$ is $a$ then the value of $Z_1$ is $1/a$.

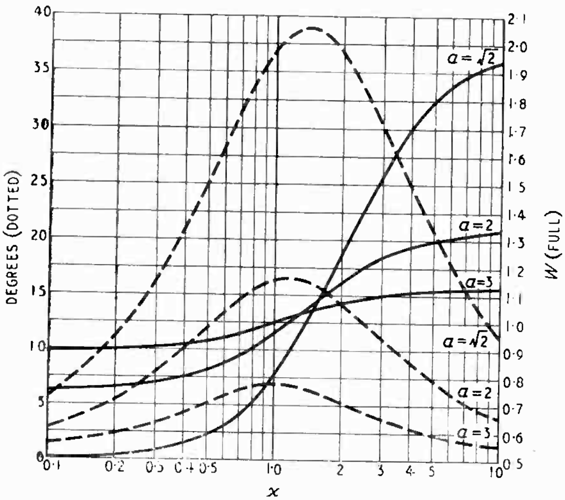



Fig. 4. *Modulus and argument of W for network type 1.2 when $z = jx$.*

The impedance-matching network obtained by expanding and bisecting the lattice is shown in Fig. 3.

For this network to be physically realizable the value of $a$ must be greater than or equal to $\sqrt{2}$. Since it is derived from pure resistors it gives a constant loss at all frequencies.

The impedance $W$ is given by :—

$$W = \frac{a^2(a^2-1)(1+z)^2}{\{a^2+(a^2-1)z\}^2}$$

$$= \left\{ \frac{1+z}{\dfrac{a}{\sqrt{(a^2-1)}} + \dfrac{\sqrt{(a^2-1)}}{a}\cdot z} \right\}$$

If $\dfrac{a}{\sqrt{(a^2-1)}} = b$ and $z = jx$

Then

$$W = \frac{(1+jx)^2}{(b+jx/b)^2}$$

$$= \frac{(b - jx/b + jbx + x^2/b)^2}{(b^2+x^2/b^2)^2}$$

$$= \frac{\{(b+x^2/b) + jx(b-1/b)\}^2}{(b^2+x^2/b^2)^2}$$

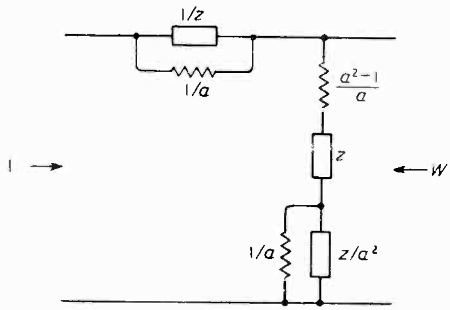



Fig. 5. *Network type 2.1.*

Therefore Mod.

$$W = \frac{(b+x^2/b)^2 + x^2(b-1/b)^2}{(b^2+x^2/b^2)^2}$$

$$= \frac{b^2 + 2x^2 + x^4/b^2 + b^2x^2 - 2x^2 + x^2/b^2}{(b^2+x^2/b^2)^2}$$

$$= \frac{(b^2+x^2/b^2)(1+x^2)}{(b^2+x^2/b^2)^2}$$

$$= \frac{1+x^2}{(b^2+x^2/b^2)}$$

and Arg.

$$W = 2\tan^{-1}\frac{x(b-1/b)}{b+x^2/b}$$

$$= 2\tan^{-1}\frac{x(b^2-1)}{b^2+x^2}$$

The modulus and argument of $W$ for the type 1.2 network are shown in Fig. 4 for several values of $a$.

*Types* 1.3, *etc.*

More complicated networks may be built up by developing the arms $Z_1$ and $Z_2$ in different ways; e.g., the resistance $a$ in series with $az$ in the arm $Z_2$ may be divided into two parts,

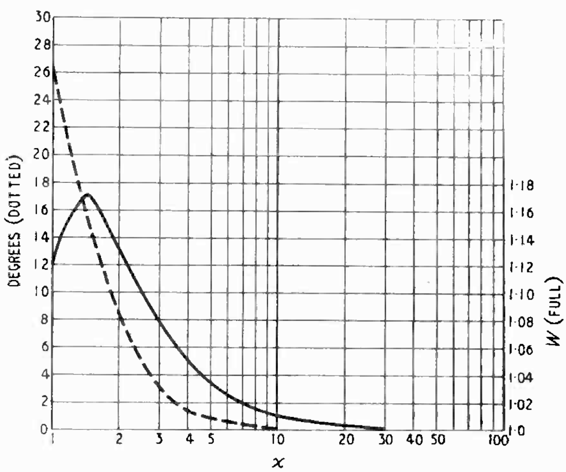



Fig. 6. *Modulus and argument of W for network type 2.1 when $z = jx$ and $a = 1$.*

one of which is less than $a/(a^2 - 1)$. This is replaced by its equivalent network as before and then the arm $Z_1$ is developed a stage further in such a way that the whole network may be built out as a ladder structure.

*Type* 2.1

A constant-resistance lattice structure which is a step more complicated than the one with a single resistor in each arm is one containing a resistor in series with a network $z$ in one arm and a resistor in parallel with a network $1/z$ in the other arm. Each arm can be built out as shown in Table 2 and from this is obtained the impedance matching network of Fig. 5.

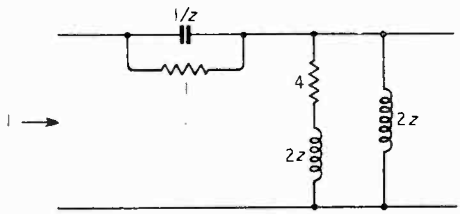



Fig. 7. *Special case of type* 2.1.

For this network

$$W = \frac{(1 + a + z)\,(a^2 - 1 + 2az + az^2)}{(a + z)\,(a^2 + a + z + 2a^2z + az^2)}$$

The modulus and argument of $W$ are shown in Fig. 6 for $z = jx$ and $a = 1$.

In the special case of the above where $a = 1$ and $z$ is an inductor then the network may be built as shown in Fig. 7.

An advantage of this network is that the inductor appearing in shunt across the right-hand terminals may be wound as a transformer, thus altering the original impedance transformation by a constant multiple.

*Type* 2.2

The same basic lattice arms may be built up into more complicated structures as shown in Table 3. The resulting impedance-matching network is shown in Fig. 8. The expression for the impedance $W$ is rather complicated and is best worked out for particular values of $a$, $b$, and $z$.

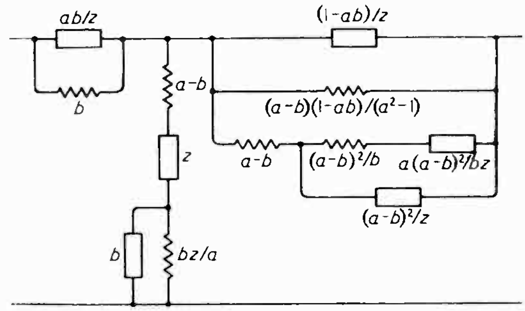



Fig. 8. *Network type* 2.2.

For the network to be physically realizable we must have :—

| | | |
|---|---|---|
| $a > b$ | .. .. .. .. | 1 |
| $1 > ab$ | .. .. .. .. | 2 |
| $a^2 > 1$ | .. .. .. .. | 3 |

i.e.,

$$a > 1 > 1/a > b$$

*Type* 3.1

The basic network of this type contains two resistors in each arm. It may be developed as shown in Table 4, resulting in the impedance-matching network of Fig. 9.

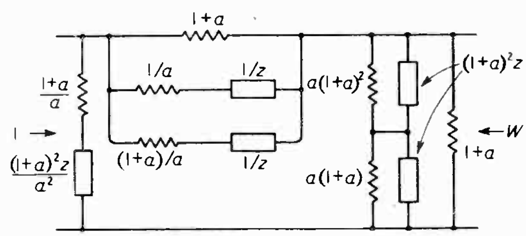



Fig. 9.—*Network type* 3.1.

*Explanation of Table* 3 (opposite).

*Step* 1. *The resistor 'a' of $Z_2$ is split into two resistors in series one of which has the value 'b', and $Z_1$ is split into two parts one of which contains a resistor 'b'.*

*Step* 2. *The resistor 'b' of $Z_2$ is replaced by an equivalent network type* 2 *part of which is the same as that part of of $Z_1$ which contains the resistor 'b'. The other resistor in $Z_1$ is replaced by two resistors in parallel one of which has the value 'a − b'.*

*Step* 3. *The resistor 'a − b' is replaced by an equivalent network type* 1, *part of which is the same as that part of $Z_2$ which was not previously common to $Z_1$.*

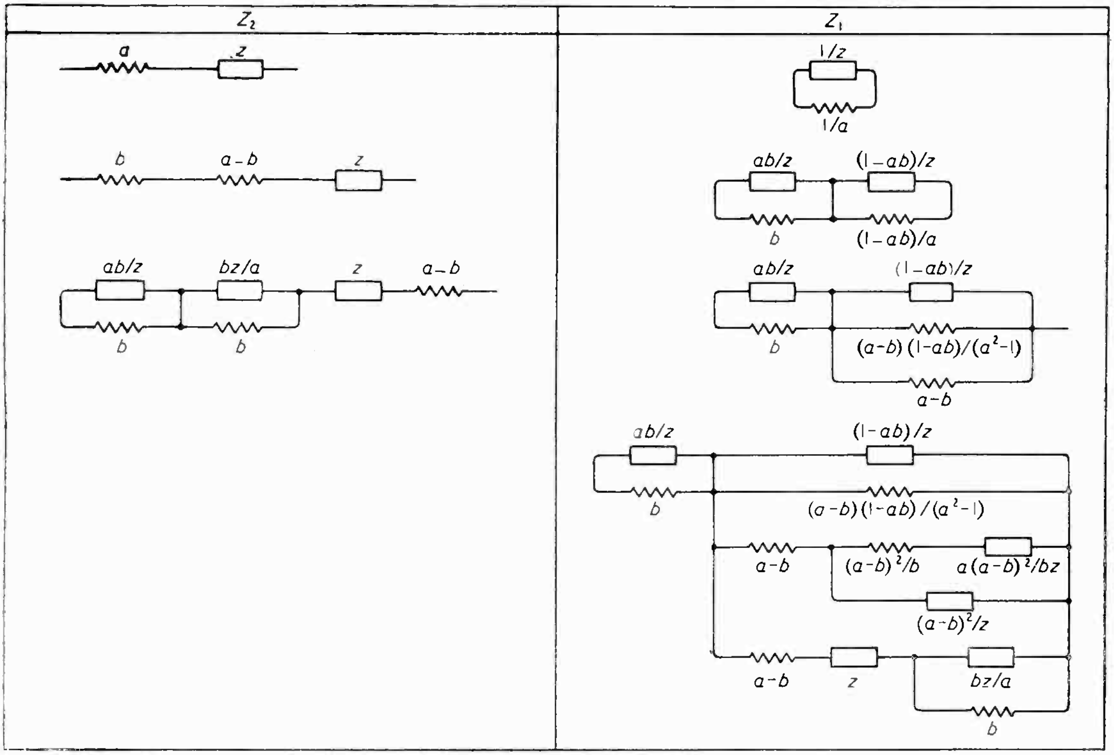
*Explanation of Table* 4 (opposite).

*Step* 1. *The whole of $Z_2$ is replaced by an equivalent network with a shunt resistor. The resistor* 1 *in $Z_1$ is replaced by two resistors in parallel one of which is equivalent to the shunt resistor of $Z_2$.*

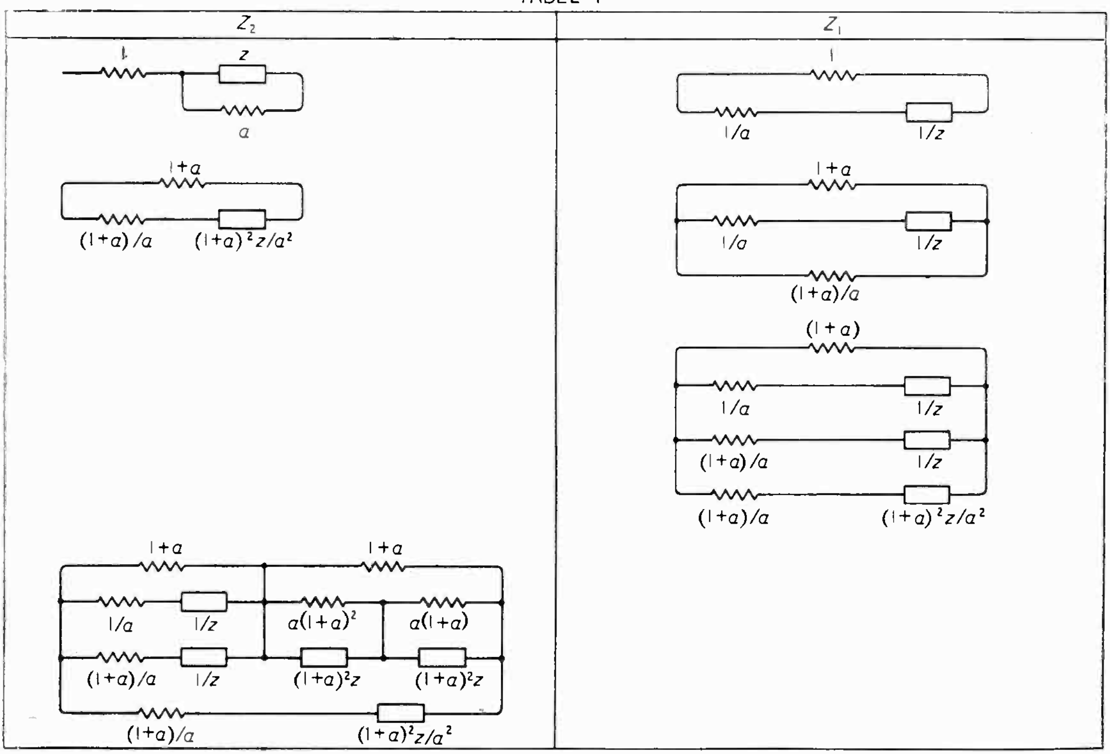
*Step* 2. *The other parallel resistor of $Z_1$ is replaced by an equivalent network type* 1, *one arm of which is the same as that of $Z_2$.*

*Step* 3. *The resistor* $1 + a$ *of $Z_2$ is replaced by an equivalent network type* 2 *part of which is the same as the whole of $Z_1$ except for the arm mentioned in step* 2.

## TABLE 3




## TABLE 4



The impedance $W$ of the above network is given by

$$W = \frac{s(t-u)}{s-t-u}$$

Where

$$s = \frac{a(1-a)z\{2a-(2-a)z\}}{(1-4a-2a^2)z^2-a(2-5a-a^2)z-a^2(1-a)}$$

$$t = \frac{(1-a)(a-z)\{a-(1-a)z\}}{(1-a)^3z^2-(2-3a-2a^2)az-a^2}$$

$$u = \frac{(1-a)\{a-(1-a)z\}}{(1-a)^2z-a(1-2a)}$$

*Further Types*

There is no limit to the number of types of networks that may be constructed by the method outlined, but they will be successively more complex and will eventually cease to be of any practical value.

To each of the networks already described there also exists a dual which may be obtained from the original by the standard method of constructing dual networks*.

**Acknowledgment**

The author is indebted to Dr. Sturley for reading the manuscript of the paper.

* A method of doing this is described in Guillemin's "Communication Networks," Vol. II, page 246.

# THE MAGNETIC AMPLIFIER

## *Transductor Theory*

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**SUMMARY.**—The transductor is the vital element of a magnetic amplifier circuit. The conditions for obtaining optimum linear control of a transductor with free magnetization with no load and sinusoidal voltage are: a transductor core having a magnetization curve with a sharp bend at the knee and a high initial permeability, and a transductor voltage of such magnitude that the peak value of the alternating magnetic-flux density in the transductor core is equal to the value of flux density at the knee of the magnetization curve. The control sensitivity on the linear part of the transductor characteristic is substantially equal to the turns ratio between control winding and load winding of the transductor, apart from a factor of 2, accounting for parallel connection of the transductor load coils in the case of the parallel-type transductor.

### Introduction

MAGNETIC amplifiers have in recent years increasingly been coming into practical use for special applications as reliable substitutes for vacuum-valve amplifiers.[1] The principle upon which a magnetic amplifier is built has been known for about fifty years and has in various forms been utilized in heavy-current control and measuring techniques.[2] It was, for an experimental period, also applied in the technique of wireless communication,[3] and there is at present an interest in its use, in modified form, in high-frequency technique.[4]

The transductor is the vital element of a magnetic amplifier circuit. It is the purpose of this paper to present the theory of a transductor characteristic for practical evaluation.

### Basic Circuit of Magnetic Amplifier

Fig. 1 shows the basic arrangement of a magnetic amplifier circuit. A source 1 of alternating current of fixed voltage $V_l$ supplies current, $I_l$, to a load 2 which is connected in

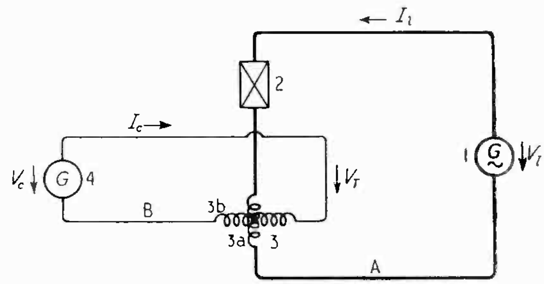


Fig. 1. *Basic circuit arrangement of a magnetic amplifier; A = Load circuit, B = Control circuit, 1 = Alternating-current source of fixed voltage, 2 = Load, 3 = Transductor, 3a = Load winding, 3b = control winding, 4 = Source of control current.*

series with a transductor 3 containing a 'load winding' 3a and a 'control winding' 3b, the control winding carrying the control current, $I_c$ supplied from a direct-current source 4 of variable voltage $V_c$. 'Transductor' is the short technical term for a ferromagnetic-core inductor with variable direct-current bias-magnetization.

The graphical symbol used in Fig. 1 to denote a transductor shows two windings crossing one

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another at right-angles. The symbol means that the two windings are placed on a common core but have no transformer interaction.

The magnetic amplifier, Fig. 1, is operated by varying the control current $I_c$ and obtaining a corresponding change of the load current $I_l$. The transductor acts in the load circuit as a variable-impedance element controlled by $I_c$, and is so designed that a small change of $I_c$ causes a magnified change of $I_l$.

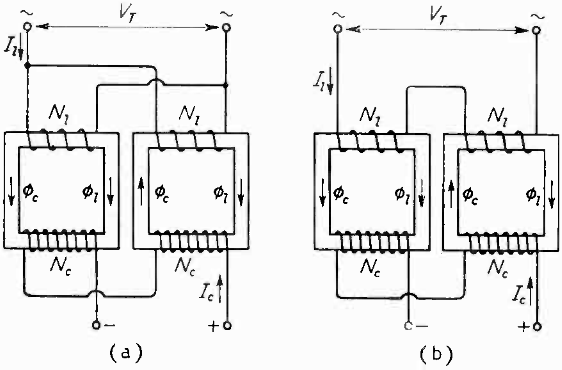



Fig. 2.—*Transductor with electric elimination of transformer interaction between load winding and control winding, (a) parallel-type and (b) series-type.*

## Basic Transductor Structure

A transductor is a modification of an iron-core transformer. Transformer interaction between load winding and control winding is avoided by using either an electric or a magnetic method.

Fig. 2 illustrates the electric method of avoiding transformer interaction. The transductor consists of two 2-limb cores (or ring-shaped cores) with the control winding and the load winding split into two coils each. The two control coils are electrically connected in series-opposition with the result that the fundamental components of the e.m.fs induced in them cancel one another in the control circuit. The two coils of the load winding are connected either in parallel, see Fig. 2(a), or in series, see Fig. 2(b).

The magnetic method of avoiding transformer interaction is shown in Fig. 3. A 3-limb core carries the control winding as a single coil on the middle limb and the load winding as two coils on the outer limbs the two load coils being connected either in parallel, as in Fig. 3(a), or in series, as in Fig. 3(b). The fundamental components of the alternating magnetic fluxes excited by the load coils are in series-opposition in the middle limb of the core and cancel one another, the control circuit is thus free from a corresponding e.m.f.

In a practical transductor either the electric method of Fig. 2 or the magnetic method of Fig. 3, or a modification of either of the two, is used for avoiding transformer interaction.

## Mechanism of Transductor Operation

The control mechanism of a transductor is based on the magnetic saturability of the core material. Figs. 4 and 5 demonstrate the effect of direct-current bias-magnetization on the shape of the magnetization loop of a transductor core made of Permalloy C. The oscillograms of Fig. 4 were taken with a large-amplitude sinusoidal voltage at the transductor load winding and the oscillograms of Fig. 5 with a small-amplitude voltage. Fig. 4 shows that increasing bias-magnetization makes the magnetization loop increasingly unsymmetrical, and Fig. 5 illustrates how the magnetization loop is 'lifted' over the knee of the magnetization characteristic into the upper region of magnetic saturation.

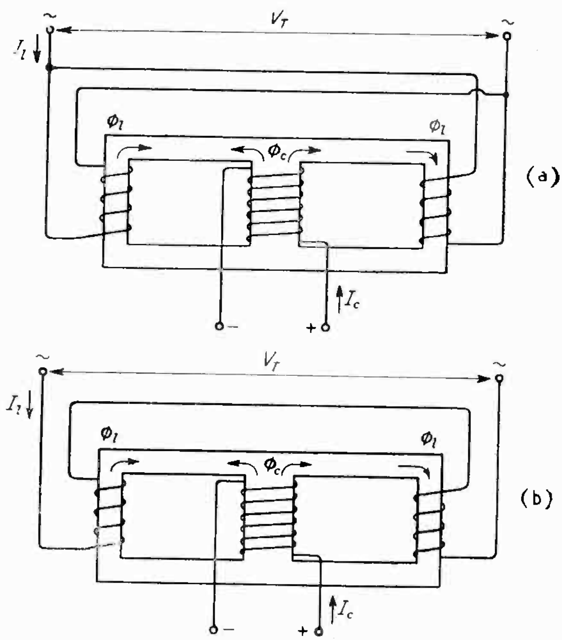



Fig. 3. *Transductor with magnetic elimination of transformer interaction between load winding and control winding, (a) parallel-type, (b) series-type.*

The change of the magnetization loops in Figs. 4 and 5 means in terms of the circuit arrangement of Fig. 1 that an increase of control current $I_c$ causes an increase of load current $I_l$. The quantitative relation between the changes of $I_c$ and $I_l$ is determined by the electric characteristic of the transductor and the impedance of the load, and by the mode of transductor magnetization.

There are two principal modes of transductor magnetization, respectively denoted free magnetization and constrained magnetization.[5] In a transductor with free magnetization part of the a.c. magnetization current flows outside the load circuit (A in Fig. 1) so that the load current, $I_l$, is smaller than the total a.c. magnetization current of the transductor. In the case of constrained magnetization $I_l$ is identical with the total a.c. magnetization current.

ductor has no load (i.e., when in Fig. 1 the load 2 is short-circuited); the load current contains odd harmonics and is, therefore, non-sinusoidal. On the other hand, we have sinusoidal load current and non-sinusoidal transductor voltage when the transductor is in series with a resistive or inductive load of high impedance. The characteristics differ from one another in the two extreme conditions of waveforms. In normal load condition the transductor characteristic is intermediate between the two extremes.

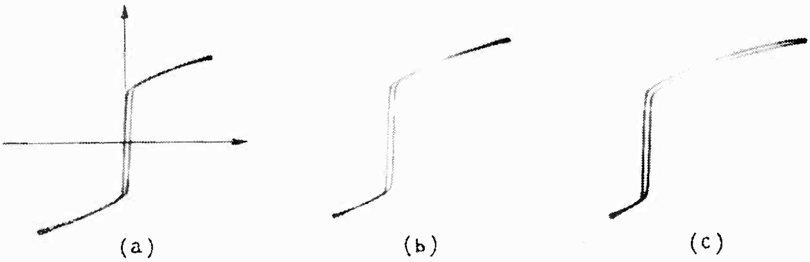



Fig. 4 (left). *Large-amplitude magnetization loops of a Permalloy C core and varying direct-current bias-magnetization. (a) No bias-magnetization. (b) Slight bias magnetization (c) Bias-magnetization increased.*

We have, for example, free magnetization in the parallel-transductor [see Figs. 2(a) and 3(a)] with sinusoidal voltage, $V_T$. The even current harmonics of a.c. magnetization form a circulating current in the two coils of the load winding and are thus absent from the load circuit; i.e., $I_l$ is smaller than the total a.c. magnetization current.

The series-transductor [see Figs. 2(b) and 3(b)] with sinusoidal voltage $V_T$ operates either in free or in constrained condition, or somewhere in between. The criterion for the condition of operation is given by the impedance of the load circuit (B in Fig. 1) in relation to the impedance of the load circuit (A in Fig. 1). Suppose that the control circuit impedance is very small: each transductor core has sinusoidal a.c. magnetization with the even current harmonics required flowing freely in the control circuit. Part of the a.c. magnetization current thus flows outside the load circuit; i.e., the load current, $I_l$, is smaller than the total a.c. magnetization current. We have free condition. In the other extreme, assume that the impedance of the control circuit is very high so that no alternating current component can flow in the control circuit: the a.c. magnetization of each transductor core is forced to be non-sinusoidal. The load current, $I_l$, provides the total a.c. magnetization; i.e., the transductor operates in condition of constrained magnetization.

## Transductor Characteristic

The electrical characteristic of a free transductor depends on the waveforms of transductor voltage and load current. The voltage is sinusoidal, like the supply voltage, when the trans-

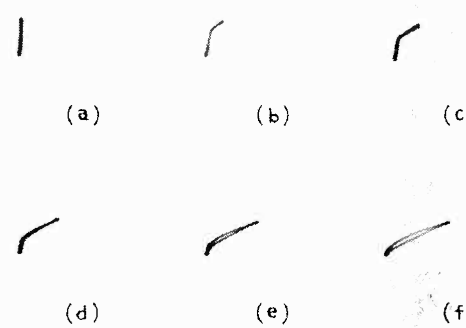



Fig. 5. *Small-amplitude magnetization loops of the transductor used for Fig. 4. (a) No bias-magnetization. (b) Slight bias-magnetization. (c-f) Bias-magnetization increased in stages.*

It is proposed to calculate the characteristic for sinusoidal transductor voltage. The assumptions for calculation are a transductor of Fig. 2 with free magnetization, sinusoidal transductor voltage $V_T$ (see Figs. 1 and 2), a simplified magnetization curve consisting of straight lines (see Fig. 6), perfect magnetization of the core (i.e., no leakage flux) and disregard of iron and copper losses in the transductor.

Fig. 7 illustrates the basis of calculation. The transductor voltage

$$v_T = -V_T\sqrt{2}\sin\omega t \quad \ldots \quad \ldots \quad (1)$$

and the bias-magnetization current $I_c$ produce, in each transductor core, Fig. 2, a magnetic flux density of instantaneous value

$$b = B_p + B_r\sqrt{2}\cos\omega t \quad \ldots \quad \ldots \quad (2)$$

where $B_p$ is the mean value of magnetic-flux density in a core and $B_r$ is related to $V_T$ by the equation

$$B_r = k_v \frac{V_T}{\omega A N_l} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

with $k_v = 1$ for Fig. 2(a) and $k_v = 0.5$ for Fig. 2(b); $A$ denotes the cross-section of a core, $\omega = 2\pi f$ the angular frequency of the supply voltage, and $N_l$ designates the number of turns of a load coil.

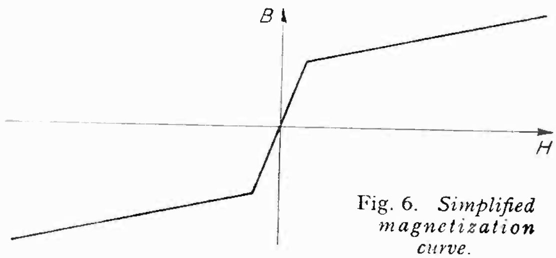



Fig. 6. *Simplified magnetization curve.*

$B_p$ is related to $I_c$ and $B_r$. Let $H_0$ be the mean value of magnetizing force in a transductor core, $l$ the mean length of magnetic path in a core assumed to be of uniform cross-section $A$, and $N_c$ the number of turns of a control coil. Then

$$H_0 = I_c N_c / l \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

The magnetizing force, $H_p$, corresponding to $B_p$ (see Fig. 7), is less than $H_0$ because the convex shape of the magnetization curve produces a magnetic rectification effect of a negative sign. To evaluate $H_p$ and hence $B_p$ in terms of $H_0$, see equation (4), and of $B_r$ is of no direct interest for the calculation of the transductor characteristic.

To facilitate calculation of the transductor characteristic it is proposed to consider $B_p$ as

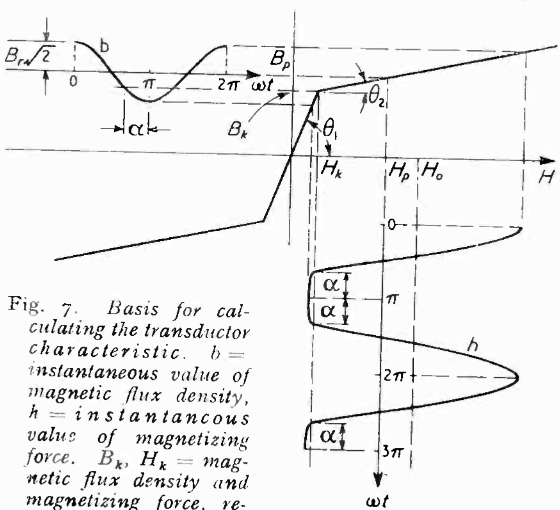



Fig. 7. *Basis for calculating the transductor characteristic. $b$ = instantaneous value of magnetic flux density, $h$ = instantaneous value of magnetizing force. $B_k$, $H_k$ = magnetic flux density and magnetizing force, respectively, at the knee of the magnetization curve, $H_0$ = mean value of magnetizing force, $B_p$ = mean value of magnetic flux density, $H_p$ = fictitious magnetizing force corresponding to $B_p$.*

an unknown auxiliary quantity used to define the unknown parameter $\alpha$, see Fig. 7, in the relation

$$\cos\alpha = \frac{B_p - B_k}{B_r\sqrt{2}} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

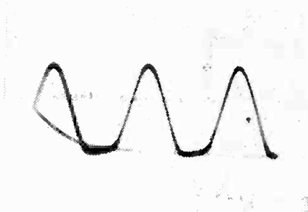
in which $B_k$ denotes the magnetic-flux density at the knee of the magnetization curve. We have then with the notation of Fig. 7 and with equations (2) and (5) for the instantaneous value of magnetizing force the expressions

$$h\Big|_{0}^{\pi-\alpha}\Big|_{\pi+\alpha}^{2\pi} = H_k + \frac{b - B_k}{\mu_2\mu_0} = \frac{B_k}{\mu_1\mu_0} + \frac{B_p + B_r\sqrt{2}\cos\omega t - B_k}{\mu_2\mu_0} = \frac{1}{\mu_1\mu_0}\Big[B_k + \frac{\mu_1}{\mu_2}B_r\sqrt{2}\,(\cos\alpha + \cos\omega t)\Big] \quad (6)$$

and

$$h\Big|_{\pi-\alpha}^{\pi+\alpha} = H_k - \frac{B_k - b}{\mu_1\mu_0} = \frac{B_k}{\mu_1\mu_0} + \frac{B_p + B_r\sqrt{2}\cos\omega t - B_k}{\mu_1\mu_0} = \frac{1}{\mu_1\mu_0}\Big[B_k + B_r\sqrt{2}\,(\cos\alpha + \cos\omega t)\Big] \quad (7)$$

Fig. 8. *Oscillogram of the magnetizing force in a transductor core with sinusoidal transductor voltage and direct-current bias-magnetization.*



where $\mu_0 = 1.257 \times 10^{-6}$ henry/metre denotes the absolute permeability of free space expressed in the rationalized form of the m.k.s. system of units, and $\mu_1 = (\tan\theta_1)/\mu_0$ and $\mu_2 = (\tan\theta_2)/\mu_0$, see Fig. 7, are the relative incremental permeabilities of the high- and low-permeability regions, respectively, of the magnetization curve. The oscillogram of Fig. 8 verifies the typical wave form of $h$ shown in Fig. 7. The Fourier analysis of $h$ gives the series expression

$$h = H_0 + \sum_{n=1}^{\infty} H_n\sqrt{2}\cos n\,\omega t \quad \ldots \quad (8)$$

with the coefficients

$$H_0 = \frac{1}{\mu_2\mu_0}\Big[B_r\sqrt{2}\Big(\frac{\pi - \alpha}{\pi}\cos\alpha + \frac{\sin\alpha}{\pi}\Big) + \frac{\mu_2}{\mu_1}\Big(B_k + B_r\sqrt{2}\,\frac{\alpha\cos\alpha - \sin\alpha}{\pi}\Big)\Big] \quad (9)$$

for $n = 1$:

$$H_1\sqrt{2} = \frac{B_r\sqrt{2}}{\mu_2\mu_0}\Big[1 - \frac{2\alpha - \sin 2\alpha}{2\pi}\Big(1 - \frac{\mu_2}{\mu_1}\Big)\Big] \quad \ldots \quad \ldots \quad (10)$$

and for $n > 1$:

$$H_n\sqrt{2} = (-1)^n \frac{B_r\sqrt{2}}{\mu_2\mu_0}\left[\frac{2\sin n\alpha\cos\alpha - 2n\cos n\alpha\sin\alpha}{n(n^2-1)\pi}\right]\cdot\left(1-\frac{\mu_2}{\mu_1}\right) \quad \ldots \quad (11)$$

It is the practice to measure $I_l$ either with a r.m.s. instrument, $I_{l,\,rms}$, or with a rectifier instrument, $I_{l,\,rect}$. We obtain with equations (10) and (11) in the first case

$$I_{l,\,rms} = k_i \frac{l}{N_l}\sqrt{H_1{}^2 + H_3{}^2 + H_5{}^2 + \ldots} \quad (12)$$

and in the second case

$$I_{l,\,rect} = k_i \frac{l}{N_l}\frac{2\sqrt{2}}{\pi}\left(H_1 - \tfrac{1}{3}H_3 + \tfrac{1}{5}H_5 - + \ldots\right) \quad \ldots \quad (13)$$

where for Fig. 2(a) : $k_i = 2$ .. .. (14a)

and for Fig. 2(b) : $k_i = 1$ .. .. .. (14b)

$N_l$ denotes the number of turns of a load coil, see Fig. 2.

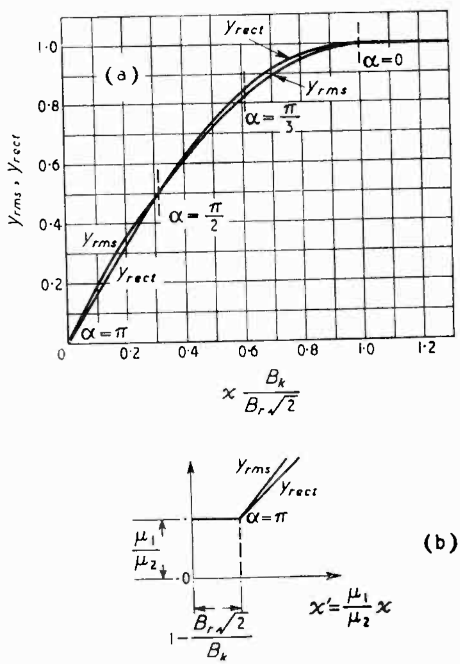



Fig. 9. *Calculated transductor characteristic for sinusoidal transductor voltage; (a) $\mu_1 \gg \mu_2$ (i.e., $\mu_2/\mu_1 = 0$), (b) Curves near the origin of the co-ordinate system for $\mu_2/\mu_1 \neq 0$.*

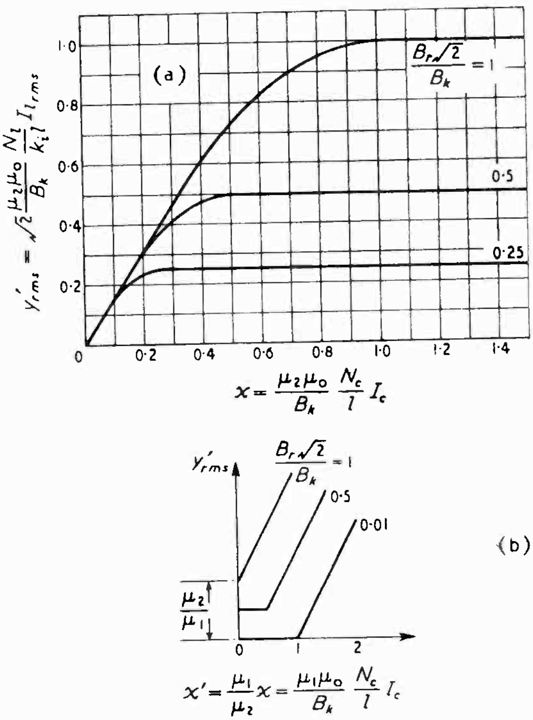



Fig. 10. *Family of calculated transductor characteristics for load current measured with a r.m.s. instrument; (a) $\mu_1 \gg \mu_2$ (i.e., $\mu_2/\mu_1 = 0$), (b) Curves near the origin of the co-ordinate system for $\mu_2/\mu_1 \neq 0$.*

In the transductor characteristic the load current, $I_{l,\,rms}$ or $I_{l,\,rect}$, is related to the control current $I_c$ derived from equation (4):

$$I_c = \frac{l}{N_c}H_0 \quad \ldots \quad (15)$$

in which $H_0$ is substituted from equation (9). For generalized presentation of the transductor characteristic it is convenient to give the co-ordinates of the characteristic as dimensionless quantities. We write with equations (15) and (9):

$$x = \frac{\mu_2\mu_0}{B_k}\frac{N_c}{l}I_c = \frac{B_r\sqrt{2}}{B_k}\left(\frac{\pi-\alpha}{\pi}\cos\alpha + \frac{\sin\alpha}{\pi}\right) + \frac{\mu_2}{\mu_1}\left(1 + \frac{B_r\sqrt{2}}{B_k}\frac{\alpha\cos\alpha - \sin\alpha}{\pi}\right) \quad \ldots \quad (16)$$

with equations (12), (10), (11) and (14):

$$y_{rms} = \frac{\mu_2\mu_0}{B_r}\frac{N_l}{k_i l}I_{l,\,rms}$$

$$= \sqrt{\left[\left\{1 - \frac{2\alpha - \sin 2\alpha}{2\pi}\left(1-\frac{\mu_2}{\mu_1}\right)^2\right\} + \left(1-\frac{\mu_2}{\mu_1}\right)^2 \sum_{n=3,5,7\ldots}\left\{\frac{2\sin n\alpha\cos\alpha - 2n\cos n\alpha\sin\alpha}{n(n^2-1)\pi}\right\}^2\right]} \quad (17)$$

and with equations (13), (10), (11) and (14) :

$$y_{rect} = \frac{\mu_2\mu_0}{B_r}\frac{N_l}{k_i l}\frac{\pi}{2\sqrt{2}}I_{l,\,rect}$$

$$= 1 - \left(1 - \frac{\mu_2}{\mu_1}\right)\left[\frac{2\alpha - \sin 2\alpha}{2\pi} + (-1)^{\frac{n-1}{2}} \sum_{n=3,5,7\ldots} \frac{2 \sin n\alpha \cos \alpha - 2n \cos n\alpha \sin \alpha}{n^2(n^2-1)\pi}\right] \quad \ldots \quad (18)$$

Fig. 9(a) shows the calculated curves of $y_{rms}$ and $y_{rect}$ as functions of $x\,.\,B_k/B_r\sqrt{2}$ for

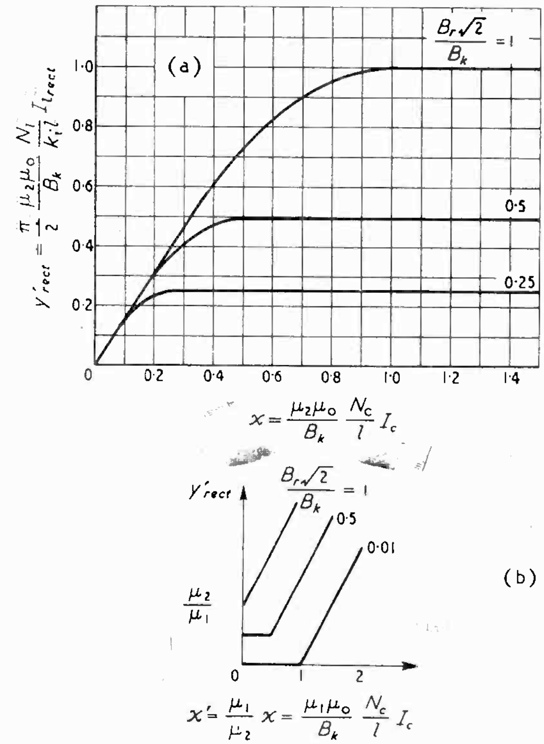



Fig. 11 (above). *Family of calculated transductor characteristics for load current measured with a rectifier instrument; (a) $\mu_1 \gg \mu_2$ (i.e., $\mu_2/\mu_1 = 0$), (b) Curves near the origin of the co-ordinate system for $\mu_2/\mu_1 \neq 0$.*

Fig. 12 (right). *Experimental transductor characteristics.*

$\mu_1 \gg \mu_2$; i.e., $\mu_2/\mu_1 = 0$ in equations (16), (17) and (18). The $y_{rect}$ curve rises virtually linearly with $x$ until $\alpha = \pi/2$ (i.e., $x\,.\,B_k/B_r\sqrt{2} = 0.32$), and then gradually bends to a constant value, $y_{rect} = 1$ for $x\,.\,B_k/B_r\sqrt{2} \geqslant 1$. The $y_{rms}$ curve is slightly above $y_{rect}$ for $\alpha > \pi/2$ and slightly below for $\pi/2 > \alpha > 0$.

The curves for $y_{rms}$ and $y_{rect}$ in Fig. 9(b) are near the origin of the co-ordinate system and are calculated for $\mu_2/\mu_1 \neq 0$. For $0 \leqslant x' \leqslant (1 - B_r\sqrt{2}/B_k)$ we have $y_{rms} = y_{rect} = \mu_2/\mu_1$ = const. provided that $B_r\sqrt{2}/B_k < 1$. The incapacity to control the load current in this region is explained in terms of Fig. 7: direct-current bias-magnetization is small (i.e., $H_0 < H_k$), and the entire cycle of alternating magnetization is below the knee of the magnetization characteristic; i.e., $h < H_k$ throughout a magnetization cycle.

It is desirable for practical evaluation to have the transductor characteristic presented with $B_r\sqrt{2}/B_k$ as parameter. We therefore introduce, with equations (17) and (18),

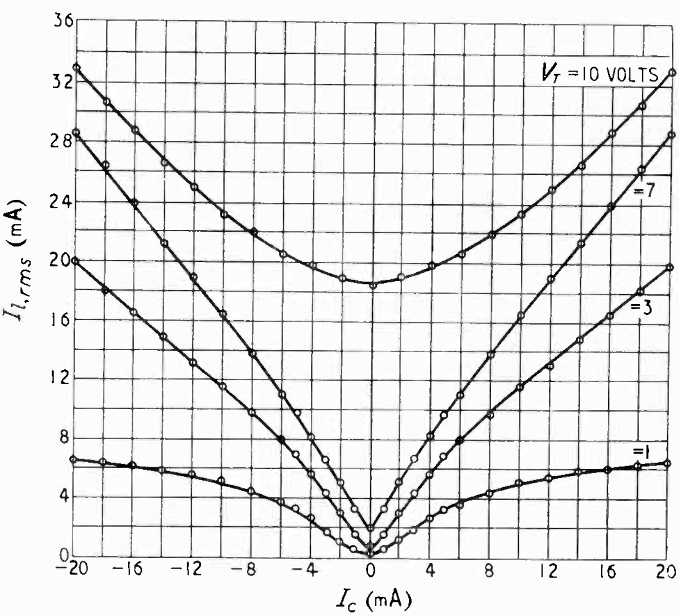
$$y'_{rms} = \frac{B_r\sqrt{2}}{B_k}y_{rms}$$

$$= \sqrt{2}\frac{\mu_2\mu_0}{B_k}\frac{N_l}{k_i l}I_{l,\,rms} \quad \ldots \quad \ldots \quad (19)$$

and $y'_{rect} = \frac{B_r\sqrt{2}}{B_k}y_{rect}$

$$= \frac{\pi}{2}\frac{\mu_2\mu_0}{B_k}\frac{N_l}{k_i l}I_{l,\,rect} \quad \ldots \quad \ldots \quad (20)$$

The families of characteristics shown in Figs. 10(a) and 11(a) represent $y'_{rms}$ and $y'_{rect}$, respectively, plotted as functions of $x$, see equation (16), for $\mu_1 \gg \mu_2$ (i.e., $\mu_2/\mu_1 = 0$), and Figs. 10(b) and 11(b) give $y'_{rms}$ and $y'_{rect}$, respectively, near the origin of the co-ordinate system for $\mu_2/\mu_1 \neq 0$. It is interesting to note from the curves of Figs. 10(a) and 11(a) that the condition $B_r\sqrt{2}/B_k = 1$ gives the largest range of 'linear' control. Figs. 10(b) and 11(b) further show that there is no 'backlash' of control at zero control



current when $B_r\sqrt{2}/B_k = 1$. The initial load current, at $I_c = 0$, can be made small by having a core material with a high value of $\mu_1$.

The characteristics of Fig. 10 are verified by the experimental curves of Fig. 12 obtained with the transductor voltage $V_T$ as parameter the control current $I_c$ having positive and negative polarity. The curve for $V_T = 7$ volts corresponds to $B_r\sqrt{2}/B_k = 1$ and shows a sharp-cornered V-form. The curve for $V_T = 1$ volt is rounded at the bottom, short in the linear range, and bends to a constant value as predicted by the general form of the curves of Figs. 10(a) and 10(b). In the case of $V_T = 10$ volts we have a magnetization loop stretching into both regions of magnetic saturation (see, e.g., Fig. 4a) with a consequent high value of load current when $I_c = 0$.

## Transductor Control Sensitivity

Let $\Delta I_c$ be a small change of control current and $\Delta I_{l,\,rect}$ the corresponding change of load current. The transductor control sensitivity, $S$, is then defined by the relation

$$S = \frac{\Delta I_{l,\,rect}}{\Delta I_c} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

or, with equations (16) and (18),

$$S = \frac{\dfrac{2\sqrt{2}}{\pi}\,\dfrac{k_i l}{N_l}\,\dfrac{B_r}{\mu_2\mu_0}\,\dfrac{\Delta y_{rect}}{\Delta x}}{\dfrac{l}{N_c}\,\dfrac{B_k}{\mu_2\mu_0}} \qquad \ldots \quad \ldots \quad (22)$$

On the linear part of the $y_{rect}$ curve, Fig. 9(a), and for $\mu_1 \gg \mu_2$ (i.e., $\mu_2/\mu_1 = 0$), we have with equations (16) and (18)

$$\frac{\Delta y_{rect}}{\Delta x} = \frac{y_{rect}}{x} \text{ at } \alpha = \pi/2$$

$$= \frac{\pi}{2}\,\frac{B_k}{B_r\sqrt{2}} \qquad \ldots \quad \ldots \quad \ldots \quad (23)$$

Equation (23) substituted in (22) gives, with equation (14),

$$S = k_i \frac{N_c}{N_l} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

This means that in the case of $\mu_1 \gg \mu_2$ the current control sensitivity on the linear part of the transductor characteristic is determined only by the turns ratio between control winding and load winding, apart from the factor $k = 2$ if the transductor is of the parallel type.

## Conclusion

The transductor control characteristic is determined by the shape of the magnetization curve of the transductor core and by the ratio between the peak value of alternating magnetic-flux density and the value of magnetic-flux density at the knee of the magnetization curve. The control characteristic has an optimum form in regard to range of linear control, low initial load current and absence of backlash of control at zero control current if the magnetization curve has a sharp bend at the knee and a high value of initial permeability, and when the peak value of alternating magnetic-flux density equals the value of magnetic flux density at the knee of the magnetization curve.

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# TRANSMISSION FACTOR OF DIFFERENTIAL AMPLIFIERS

By D. H. Parnum, A.R.C.S., Ph.D., F.Inst.P.

**SUMMARY.**—It is shown that the quantity 'discrimination factor' (anti-phase/in-phase gain ratio) is insufficient to define the performance of a differential amplifier, and that a quite different quantity, the 'transmission factor,' sets the limit to the interference/signal ratio that can be tolerated. The likely values of this quantity are worked out for typical input stages and found to be insufficient for many purposes; an adjustable control is described which is capable of giving a very high value. Further analysis shows that the discrimination and transmission factors of all stages must be considered to obtain the overall transmission factor for a complete amplifier, and this leads to recommendations for the design of the first two stages.

An excellent paper by Johnston[1] appears to show confusion on a highly practical point which has seldom been discussed by other writers. It concerns the so-called 'discrimination factor' of balanced or differential amplifiers. For clarity a brief recapitulation of the function and principles of these amplifiers will first be given.

### The Coupled-cathode Input Stage

Fig. 1 shows a differential input stage of the coupled-cathode type. An 'anti-phase' (or 'differential') signal is one applied between the two grids; the signal currents cancel in $R_c$, there is no negative feedback, and an output appears at full gain between the anodes. An 'in-phase' signal is one applied between earth and the two grids in common; negative feedback occurs in $R_c$ and the anode output is much less than in the first case. If the two halves of the stage are identical, moreover, the in-phase outputs are equal and the voltage difference between the anodes is zero; i.e., there is no anti-phase output.

This arrangement is used chiefly in medical or biological applications, where the input grids are connected to electrodes placed on (or in) the subject. The small voltages generated by the subject, which it is required to measure, are then applied between the grids as an anti-phase signal. Generally, however, there is stray capacitance coupling to the electrodes from an interfering source (most commonly 50 c/s), which gives rise to unwanted in-phase signals. The *balanced* property of the amplifier (i.e., equality of the two halves) means that these signals will not cause an anti-phase output and so mask the wanted signals. The difference in anti-phase and in-phase *gain* is required for a different reason. Interference signals are commonly many times (100—1,000) the full drive wanted voltage, and would hopelessly overload the later stages if amplified at full gain. These two properties, of balance and gain ratio, are therefore different in nature and required for different purposes.

MS accepted by the Editor, September 1949

Many circuits have been devised for securing a high gain ratio, that of Fig. 1 being one of the simplest and most effective. For this stage Johnston quotes the well-known formulae:

$$\text{Anti-phase gain, } M = \mu R/(r_a + R), \quad (1)$$

$$\text{In-phase gain, } \overline{M} = \mu R/\{r_a + R + 2R_c(\mu + 1)\} \quad (2)$$

He then calls the ratio $M/\overline{M}$ the discrimination factor, $F$:

$$F = \{r_a + R + 2R_c(\mu + 1)\}/(r_a + R)^* \quad (3)$$

In what follows it will be convenient to take average figures for $\mu$, $r_a$ and $R$; the numerical results are not vastly different over the range of triodes available. We shall take $\mu = 40$, $r_a = 20$ kΩ, $R = 100$ kΩ.

It is seen from (3) that $F$ approaches infinity as $R_c$ is made very large. If $R_c$ is a resistor, it cannot be increased indefinitely without causing

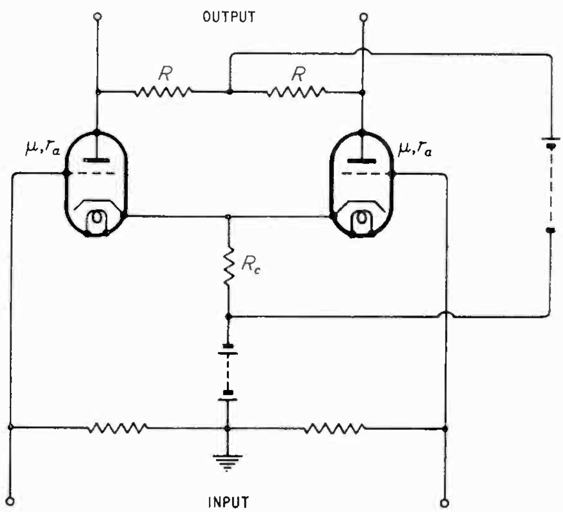


Fig. 1. *Cathode-coupled input stage.*

* In Johnson's paper the denominator of this expression is misprinted.

an excessive static voltage drop; the value of $R_c$ seldom exceeds 0.5$R$. We then find that $F = 35$. This is not very large, and Johnston describes an ingenious method in which $R_c$ is the slope resistance of a pentode. In this way a value of 5 MΩ can be obtained with a static drop of only 70 V; and $F$ becomes 3,400.

## Discrimination and Transmission Factors

Johnston's paper suggests that the attainment of high $F$ is all that is needed to eliminate interference troubles. Thus, he criticises the low value obtained with a resistor for $R_c$, and advocates the pentode as a means of raising $F$ to a satisfactory value. The implication is that if, say, $F = 3{,}000$, interference can be tolerated which is 3,000 times the minimum wanted signal. This is not the case. The point which Johnston does not mention is that *an in-phase input can produce an anti-phase output.* He does not, in fact, consider the previously-mentioned distinction between the 'balance' and 'gain ratio' properties. His formulae (1)—(3) involve mean parameters and so cannot take account of inequalities between the two halves of the stage. It is these inequalities, which are inevitable in a stage that has no adjustable control, which cause the spurious output to occur. Once it has occurred no discrimination or rejection process in the rest of the amplifier can remove it, because it is an anti-phase voltage and is indistinguishable from a genuine signal.

Before considering the analysis of a stage with unequal halves, as shown in the equivalent circuit of Fig. 2, it is useful to look at the matter physically and see how the anti-phase output can arise. It is obvious that if $R_c = 0$, the two halves function without interaction; they amplify the equal input voltages slightly differently, and the difference at the anodes is an anti-phase voltage. But if $R_c$ approaches infinity, so that the in-phase output is zero, the situation is at first sight not the same. No in-phase current can flow in an infinite $R_c$; hence no such current can flow in $R_1$ and $R_2$. From this it is an easy, but false, step to the conclusion that no signal current of any kind flows in $R_1$ and $R_2$, so that there is no output voltage of any kind. The fallacy of this reasoning is that signal current can circulate round $R_1$, $R_2$, and the valves without flowing through $R_c$ at all, and in fact it will do so if the halves are not identical. This can be seen from Fig. 2 by considering that $\mu_1$ and $\mu_2$ are not equal; the fictitious voltages in the two valves are then unequal and the resultant voltage must act round the closed circuit of $R_1$, $R_2$ and the valves. The current thus driven round the circuit gives rise to a pure anti-phase output. The mean value of this output (with respect to earth) is zero, which agrees with the statement that there is no in-phase output.

The term 'transmission factor' is suggested for this property of a stage (or complete amplifier) and is defined in the following way. Suppose that an in-phase input $e$ gives rise to an anti-phase output $Ke$. The anti-phase input which would produce the same output is $Ke/M$. An in-phase signal therefore has to be $M/K$ times as large as an anti-phase to produce the same effect, and this ratio is defined as the transmission factor, $H$:

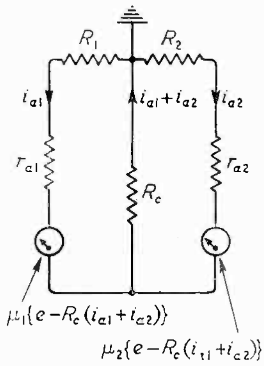


Fig. 2. *Equivalent circuit of Fig. 1 with unequal halves. In-phase input between each grid and earth = e.*

$$H = M/K \quad \ldots \quad (4)$$

The name 'transmission factor' has been proposed because 'discrimination factor' is by now probably firmly attached to the anti-phase/in-phase gain ratio. It is not ideal, but suggests to the writer, at least, the concept of the stage 'transmitting' a fraction of an in-phase voltage as anti-phase. If an amplifier is perfectly balanced, an in-phase voltage applied at the start does not appear in the final output at all; i.e., it is simply not transmitted. In an article by the writer[2] the term 'output ratio' (to contrast with 'gain ratio') is used; but this term now seems to be insufficiently descriptive.

Turning now to the analysis of Fig. 2, the result for $K$ is:

$$K = \frac{\mu_1 R_1 r_{a2} - \mu_2 R_2 r_{a1} + (\mu_1 - \mu_2)\{R_1 R_2 + (R_1 + R_2) R_c\}}{(r_a + R)\{r_a + R + 2R_c(\mu + 1)\}} \qquad (5)$$

The numerator of this expression is accurate; the denominator is symmetrical in $R_1$, $R_2$, etc., and these quantities have been replaced by a common value $R$, etc.

To cast (5) into a useful form we put

$R_2 = R$, $R_1 = R + \Delta R$, etc.

It is then found that $M$ is a factor of $K$, and we finally have from (4) the approximate value

$$H = \frac{r_a + R + 2R_c(\mu + 1)}{r_a} \Big/ \left[\frac{\Delta R}{R} - \frac{\Delta r_a}{r} + \frac{\Delta\mu}{\mu}\left(\frac{r_a + R + 2R_c}{r_a}\right)\right] \quad \ldots \quad (6)$$

The approximation is that $\Delta\mu/\mu$ should also be multiplied by $(1 + \Delta R/R)$; this gives rise to a second-order product. It is shown later that the $\Delta\mu/\mu$ term is nearly always predominant, and in that case the second-order product causes only a small change in this predominant term.

A number of useful deductions can be made from (6) by considering numbers and certain special cases. Assumptions will be made about the likely values of $\Delta\mu/\mu$, etc.; those chosen are:

(i) $\Delta\mu/\mu$, $\Delta r_a/r_a$, and unintentional values of $\Delta R/R$ do not exceed 10%.

(ii) $R/r_a$ is not less than 3.

Three special cases are of interest.

*Case A*. $R_c = 0$.

This will substantially be so when the stage has a bias resistor only; i.e., no attempt is made at discrimination.

It may happen that the square bracket in (6) is zero, so that $H = \infty$ and the stage is perfect; but we are concerned with the lower limit of $H$ in the worst case. This occurs when $\Delta\mu/\mu$, etc., are all numerically equal but such that they all add up. The square bracket then becomes

$$\frac{3r_a + R}{r_a} \cdot \frac{\mu}{\Delta\mu},$$

and we get for $H$:

$$H = \frac{r_a + R}{3r_a + R} \cdot \frac{\mu}{\Delta\mu}.$$

On the assumption made about $R/r_a$, the coefficient of $\mu/\Delta\mu$ lies between 2/3 and 1. As we only want to know the order of $H$ in the worst case, it is convenient and sufficient to say that approximately

$$H = \mu/\Delta\mu.$$

On the assumption made about $\Delta\mu/\mu$, this may be as low as 10.

*Case B*. $R_c = 0.5R$.

This value is not likely to be exceeded when $R_c$ is a resistor.

Again taking the most unfavourable combination in the bracket, we get

$$H = \frac{\mu + 2 + r_a/R}{2 + 3r_a/R} \cdot \frac{\mu}{\Delta\mu}.$$

The numerator of the coefficient is effectively $\mu$; the denominator lies between 2 and 3. Following the course taken for Case A, it is again sufficient to say that approximately

$$H = \tfrac{1}{2}\,\mu^2/\Delta\mu.$$

This is a considerable improvement on Case A; for $\mu = 40$, $H$ is 20 times as large. The likelihood of a fortuitous high value is decreased, however. The coefficient of $\Delta\mu/\mu$ in (6) is now not less than 7, so that unless $\Delta\mu/\mu$ is less than 1/7 of the other inequalities the term involving it will predominate. A study of (6) then shows that the lower limit deduced for $H$ becomes the actual value. At the best, then, $H$ may not exceed 200.

*Case C*. $R_c = \infty$.

This is substantially approached when a pentode is used for $R_c$.

It is at once seen from (6) that, without any approximations,

$$H = \mu^2/\Delta\mu.$$

This is only twice as good as Case B, and is an improvement which hardly justifies the trouble of using a pentode; other reasons, however, are given below which necessitate its use in the first stage. The value of $H$ may be no higher than 400; if this is compared with the corresponding figure for $F$ of 3,400, it is seen what a false idea $F$ can give of the performance of the stage. It should also be noted that, when $R_c$ is really infinite, we have a case where the in-phase gain is zero, but there is still a finite anti-phase transmission given by the above value of $H$.

As a tailpiece to this section, the following paragraph from Johnston may be quoted:

"The discrimination factor is increased in proportion to the cathode load. If adjustments are made to the inequalities of characteristics in a pair of valves with differential input and output, it is possible to realise a factor of 100,000 : 1; using components of commercial tolerance 3,000 : 1 is obtained in the circuit of Fig. 10(a) without further adjustment."

This paragraph appears to show complete confusion between discrimination and transmission factors. "Adjustments to inequalities" cannot alter $F$ (which depends on mean parameters only), but can alter H, and the figure of 100,000 : 1 must be what we have defined as $H$. The figure of 3,000 : 1 is then stated to refer to the same quantity, so that it must also be $H$; yet it is really $F$. It is impossible to believe that Johnston is unaware of the distinction, but the writer can find no interpretation of the quoted paragraph which is self-consistent.

## Attainment of High Transmission Factor

The values of $H$ required are commonly very high. In a practical case of the writer's experience, the observed interference level (across 1-MΩ grid resistors) was 500 mV, while the minimum desired signal was 10 μV; thus $H$ had to be at least 50,000. None of the above values approach this. It is, therefore, natural to enquire whether an adjustable control can be added to the stage to raise $H$ to any given value. It is seen from (6) that the problem is to make the square bracket approach zero.

Control of $\Delta R$ by making the anode loads variable is tempting; it has been used by some writers with possibly this end in mind, but examination of (6) shows it to be generally impracticable. When $R_c = 0.5R$ the $\Delta\mu/\mu$ term is nearly 1, and one anode load would have to be nearly zero to compensate for it; when $R_c = 5$ MΩ the term is 50 and control by $\Delta R$ is impossible. The same remarks apply to $\Delta r_a$. It is apparent from these examples that the $\Delta\mu/\mu$ term is dominant and the correct course is to vary $\Delta\mu$.

Fortunately, this can be done very easily by the circuit of Fig. 3. Each valve has a variable shunt from anode to cathode. By Thévenin's theorem, when a valve of parameters $\mu$, $r_a$ is shunted by $S$ it behaves like one of parameters $\mu.S/(S + r_a)$ and $r_a S/(S + r_a)$. The variation of $\Delta\mu$ is thus produced; the associated variation of $\Delta r_a$ is, as just explained, of little effect in comparison.

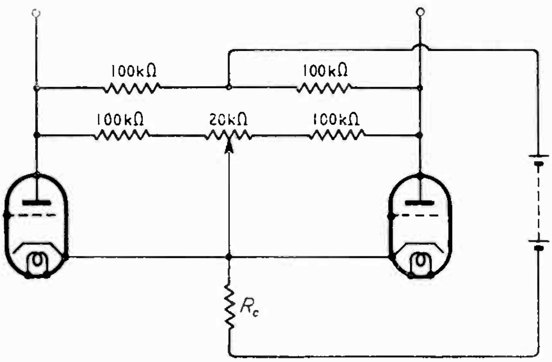


Fig. 3. *Elements of shunt control circuit to raise value of H.*

The values shown in Fig. 3 have been used with an ECC35. For this valve $r_a$ is about 30 kΩ; the reduction of $\mu$ from one end of the control to the other therefore ranges from 77% to 92%. The mean is 74.5%; thus the total range of variation is 18% of the mean, and the circuit will cover differences of this amount. There is some loss of anti-phase gain, about 20%; this is a small price to pay for the advantages gained. The valve is working into a relatively low anode load, but this is immaterial as the anti-phase signal level is far too small to cause distortion.

One practical point which must be mentioned is that the shunt circuit upsets the static conditions, and allowance must be made for this when setting-up the stage. Theoretically a blocking capacitor avoids this trouble; but the compensation then fails progressively as the frequency is lowered, and it is impossible to maintain a large $H$, even at 50 c/s, without an impracticably large capacitance.

On test the circuit has given very satisfactory results. A value of $H = 50{,}000$ was easily attained. The sensitivity of the detecting circuit following the stage did not permit any greater value of $H$ to be measured, and no attempt was made to set up a special circuit for doing so. The balance point did not appear to drift with time. It is probably sensitive to supply voltage; but this is unimportant because the supplies to a first stage of this nature must always be stabilized for other reasons.

The control naturally requires re-adjustment when the valve is changed. It is, however, independent of the input circuit conditions, and is therefore suitable for any application.

## Overall Transmission Factor

The attainment of a large $H$ in the first stage is a necessary condition for success, but not a sufficient one. Even if $H$ is made infinite by careful adjustment, the stage still has a finite in-phase gain, and will pass some in-phase voltage. This will be applied to stage 2; if $H$ for this stage is finite, it will give rise to an anti-phase output. Similarly stage 2 will pass some in-phase voltage to stage 3, which will in turn give rise to an anti-phase output, and so on. Clearly the overall effect depends on the transmission and discrimination factors of each stage. In general, the anti-phase output from one particular stage will predominate, and the design problem is to ensure that this one does not exceed the permitted amount.

The way in which the overall transmission factor $H$ is built up is as follows. We shall write:

Anti-phase gain of stage $n$ : $M_n$.

Discrimination factor of stage $n$ : $F_n$.

Transmission factor of stage $n$ : $H_n$.

It follows that:

In-phase gain of stage $n = M_n/F_n$.

Considering an in-phase input of 1 V to stage 1, we have the conditions of Table 1.

The last column shows the equivalent anti-phase voltages at the input which would give rise to the actual anti-phase transmission from each stage. We can therefore add all these terms to give the total equivalent input which would produce the total effect in the whole amplifier; this total input is by definition $1/H$. Thus

$$1/H = 1/H_1 + 1/F_1H_2 + 1/F_1F_2H_3 + \dots \quad (7)$$

Exact summation of this expression is unimportant. It is only necessary to examine the denominators to ensure that the smallest is sufficiently large, and that none of the others is less than about twice the smallest. To continue with our example for which $H$ was required to be 50,000, suppose we make $H_1 = 50{,}000$ by adjustment of the control in the first stage; then $F_1H_2$ must be not less than about 100,000. If stage 1 has a resistor for $R_c$, $F_1$ will be about 30, so that $H_2$ must be about 3,000. This is not possible unless stage 2 has specially selected valves or an adjustable control, and either of these courses is inconvenient. But if $R_c$ is a pentode, $F_1$ will be about 3,000, and $H_2$ need be no more than 30. This cannot be guaranteed if stage 2 has no cathode resistor, but can be if it has one of no more than about $R/5$, which is about the lowest value likely to be used. Thus the combination which gives a satisfactory result and is the least trouble is: pentode and shunt control for stage 1, resistor for stage 2.

The remaining terms in (7) are now automatically large enough. For $R_c = R/5$ we get $F_2 = 13$, so that $F_1F_2$ is about 40,000. This occurs as a factor in all the remaining terms; they also each contain an $H$ which cannot be less than 10, even with no cathode resistor; thus they cannot be less than 400,000.

The question of overloading by in-phase interference is also settled. The maximum expected interference is 50,000 times the minimum signal; but the overall in-phase gain is at the most $1/F_1F_2 = 1/40{,}000$ of the anti-phase. Thus the total in-phase voltage arriving at the output stage can be very little more than the *minimum* signal, so that overloading or cross-modulation is utterly impossible.

It should be mentioned finally that when the shunt control is fitted to the first stage, it is possible to obtain a high overall $H$ without the above precautions. By slight misadjustment of the control, the anti-phase transmissions of the various stages can obviously be made to cancel. Such a condition, however, is likely to be more sensitive to drift, and the control will require re-setting when *any* of the valves are changed. The proposed arrangement is sensitive to conditions in the first stage only.

## Conclusions

Without an adjustable control in the first stage of a balanced amplifier, in-phase interference will in general produce an intolerable output which appears to be due to a spurious signal. The use of a cathode pentode is insufficient to prevent this, and may result in a transmission factor of no more than 400. A successful control has been tested which permits $H$ to be at least 50,000. Further consideration shows that it is still necessary to use a pentode in the first stage, and a cathode resistor in the second, if this value of $H$ is not to be dominated by the effects of the second and later stages. If these precautions are adopted, the remaining stages need no special attention, and overloading by in-phase interference is impossible.

**TABLE 1**

| | In-phase input | In-phase output | Anti-phase output | Gain of preceding stages | Equiv. input to stage 1 |
|---|---|---|---|---|---|
| Stage 1 . . . . | 1 ↙ | — $M_1/F_1$ | $1/H_1$ | 1 | $1/H_1$ |
| Stage 2 . . . . | $M_1/F_1$ ↙ | — $M_1M_2/F_1F_2$ | $M_1/F_1H_2$ | $M_1$ | $1/F_1H_2$ |
| Stage 3 . . . . | $M_1M_2/F_1F_2$ | $\frac{M_1M_2M_3}{F_1F_2F_3}$ | $\frac{M_1M_2}{F_1F_2H_3}$ | $M_1M_2$ | $1/F_1F_2H_3$ |

## Acknowledgment

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### REFERENCES

[1] D. L. Johnston, "Electro-Encephalograph Amplifier" *Wireless Engineer*, August 1947, Vol 24, pp. 231–242; September 1947, Vol. 24, pp. 271–277; October 1947, Vol. 24, pp. 292–297.

[2] D. H. Parnum, "Biological Amplifiers." *Wireless World*, November 1945, Vol. 51, pp. 337–340; December 1945, Vol. 51, pp. 373–376.

## ERRATA

In the article "Phase-Shift Oscillator," by W. C. Vaughan, in the December 1949, issue, two errors occurred. In Equ. (16), p. 393, the last term should be '6,' not '1' and Equ. (18), p. 393, the denominator of the second fraction should be '$6 + n$,' not '$4 + n$.' The equations appear correctly in Table I.

Two errors occurred in the letter from R. Fürth, "Spontaneous Fluctuations in Double-Cathode Valves," on p. 33 in the January issue. Equation (1) should read:

$$(\delta I)^2_f = 4\, kTg_a\, \beta \Delta f$$

and Equation (2) $g_a = \left(\frac{\partial I}{\partial V_a}\right)_{I_s}$

An error occurred in the advertisement of H. W. Sullivan on p. 16 of the March issue. The range of frequencies over which the power factor of this firm's variable air condensers is below 0.00001 was given as $10^3$ to $10^2$ c/s instead of $10^3$ to $10^5$ c/s.

# CORRESPONDENCE

*Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.*

### Non-Linear Effects in Rectifier Modulators

SIR,—The non-linear operation of an ideal ring modulator was analysed in *Wireless Engineer*, Vol. 26, May 1949, p. 177, and it was stated that similar results hold for a Cowan modulator. This was criticized by D. G. Tucker and E. Jeynes in Vol. 27, February, 1950, p. 66, who raised various objections and presented many interesting new points. These finer points of the theory can only be examined on the basis of a more elaborate analysis taking into account the finite passing blocking conductances, $g_s$ and $g_o$, of the rectifiers and the finite carrier source resistance $r$. The results of this analysis are outlined below for (1) a ring modulator, (2) a Cowan modulator with a complementary rectifier connected in opposite polarity, (3) a single Cowan modulator. In cases (1) and (2) the results obtained by making $g_s = \infty$ and $g_o = 0$ coincide with those of May 1949, whatever the carrier source resistance may be. On the other hand, in case (3) the previous result is only valid for $r = 0$, and, for finite values of $r$, the new effects mentioned by Tucker and Jeynes are quantitatively explained. Thus, there is no reason to question the validity of a theory based on a discontinuous rectifier law. Even if it were proved that the exact shape of the rectifier characteristic in the transition interval has an important role in intermodulation processes, it would be interesting to separate this effect from similar ones already produced with a rectifier characteristic nearer to the ideal.

In any case, in practical modulators, additional complications are caused by selective termination and unbalance effects.

The ring modulator is assumed to work between resistances $R$ and the instantaneous signal e.m.f. is denoted by $2v_s$. If $e_c$ is the instantaneous carrier the carrier voltage in linear operation is given by

$$v_c = e_c/f\,;\; f = 1 + 2r\,(g_o + g_s) \quad \ldots \quad (1)$$

and the voltage $v_c$ so defined will be considered as the reference variable. When overload occurs, the modulator may be unbalanced and, if $r$ is not zero, the actual carrier voltage $v_c'$ will differ from $v_c$ and contain a signal component. A first overload point occurs when the signal voltage is slightly larger (due to the shunt effect of the other blocking rectifier) than the carrier voltage across one nominally blocking rectifier; i.e., for $v'_c = \pm a v_s$, where

$$a = (1 + g_oR)^{-1} \quad \ldots \quad (2)$$

A second overload point occurs when the signal current is slightly larger than the carrier current in a nominally passing rectifier; i.e., for $v'_c = \pm b v_s$, where

$$b = (1 + g_sR)^{-1} \quad \ldots \quad (3)$$

The signal leak into the carrier source can be calculated and $v'_c$ expressed in terms of $v_c$ and $v_s$. This does not alter the first overload condition but changes the second one into $v_c = \pm b' v_s$, where

$$b' = bh/f\,;\; h = \frac{1 + 2r\,(g_s + g_o + 2Rg g_{ms})}{1 + \frac{1}{2}(g_o + g_s)\,R} \quad \ldots \quad (4)$$

It is convenient to represent the condition of the rectifiers in the plane of co-ordinates $v_s$, $v_c$. This is done in Fig. 1 where $\pm a$ and $\pm b'$ denote the slopes $v_c/v_s$ of the straight lines separating the various modulator configurations. Passing rectifiers are shown in full and blocking rectifiers in dotted lines. When $v_c$ and $v_s$ are harmonic functions with amplitudes $V_c$ and $V_s$, the point representing the instantaneous configuration of the modulator traces a Lissajous figure inscribed in a rectangle of co-ordinates ($\pm V_c$, $\pm V_s$). The instantaneous output voltage is computed for each configuration by elementary network theory and all the voltages are combined into a single expression by using the discontinuous function of two variables*

$$F\,(x, y) = \tfrac{1}{2}x\,[\text{sign}(y + x) + \text{sign}(y - x)] + \tfrac{1}{2}y\,[\text{sign}(y + x) - \text{sign}(y - x)] \quad \ldots \quad (5)$$

studied in connection with the ideal modulator. The output voltage is

$$v = p[F(v_c,\, av_s) + qF(v_c,\, b'v_s)] \quad \ldots \quad (6)$$

where

$$p = \tfrac{1}{2}(g_s - g_o)\,Rf/z \quad \ldots \quad (7)$$

$$z = 1 + \tfrac{1}{2}(g_o + g_s)R + z(3g_s + g_o) + g_s rR\,(3g_o + g_s) \quad \ldots \quad (8)$$

$$q = (1 + 4rg_s)/h \quad \ldots \quad (9)$$

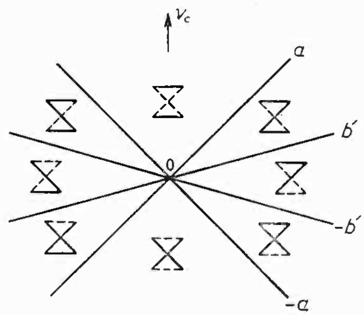



Fig. 1.

For an ideal modulator, $a = p = 1$, $b' = 0$ and the second overload occurs at infinite signal voltage; thus $v = F(v_c,\, v_s)$ as obtained previously. When $v_c$ and $v_s$ are harmonic functions, the double Fourier series expansion of (5) permits calculation of all intermodulation products. In particular, overload characteristics for the main sidebands are expressible in terms of elliptic integrals and can be computed numerically using, for instance, the tables of Jahnke-Emde, Dover, 4th Ed., p. 85. In practice, the second overload point occurs at very high signal voltages and the second term in (6) is reduced to its linear part $qb'v_s$. It can easily be checked that in linear operation the main sideband output takes the conventional value

$$V_{11} = \frac{2}{\pi}\,V_s k\,;\; k = p(a + qb') = \frac{(g_s - g_o)\,R}{(1 + g_oR)\,(1 + g_sR)} \quad \ldots \quad (10)$$

Assuming that the matching condition $R = (g_og_s)^{-\frac{1}{2}}$ holds and neglecting higher powers of the rectifier ratio $\alpha = (g_s/g_o)^{\frac{1}{2}}$, the intermodulation margin for small signal-to-carrier ratio is given by

$$20 \log \left|\frac{V_{11}}{V_{13}}\right| = 40 \log \left|\frac{V_c}{V_s}\right| + \left(\frac{3 + 8rg_s}{1 + 2rg_s}\right) 8.7 + 27.6 \text{ decibels} \quad \ldots \quad (11)$$

For $\alpha = 0$ (ideal modulator), this checks with the formula published previously and the additional dissipative

* Sign $x$ = the signature of $x$ and is defined as sign $x = +1$ when $x$ is positive, sign $x = 0$ when $x = 0$, and sign $x = -1$ when $x$ is negative.

term increases the intermodulation margin from $26\alpha$ for $r = 0$ to $35\alpha$ for $r = \infty$. For a matched carrier source $r = 1/2g_s$, and this term is $30.5\alpha$. The signal leaking into the carrier circuit has also been calculated: the voltage of the main products $f_c \pm f_s$ across the carrier source at impedance matching is approximately

$$3\alpha V_s^3/8\pi V_c^2;$$

the leakage attenuation, taking into account the different signal and carrier impedances, is thus

$$40 \log |V_c/V_s| + 16 + 10 \log (1/\alpha) \text{ decibels}$$

and is infinite for an ideal modulator.

The analysis of a Cowan modulator with a fifth rectifier in opposite connection leads to quite similar results if the signal impedances are denoted by $2R$ and the carrier source resistance by $2r$. The carrier voltage and e.m.f. are still related by (1) in linear operation. The modulator configurations are shown on Fig. 2. The first overload occurs as for the ring modulator but there are two further overload points characterized by the coefficient $b$ defined in (3) and by

$$b'' = \frac{r(g_s - g_o)}{[1 + \frac{1}{2}R(g_o + g_s)]f} \quad \ldots \quad (12)$$

As the configuration of Fig. 2 is not antisymmetrical with respect to $V_c$, intermodulation products of the type $mf_c \pm nf_s$ with $m$ even or zero and $n$ odd will also appear. These unimportant products complicate the discussion but are eliminated by considering only that part $v'$, of the output voltage which is antisymmetrical with respect to $v_c$; i.e.,

$$v' = \tfrac{1}{2}[v(v_c) - v(v_c)].$$

This gives

$$v' = \tfrac{1}{2}p[F(v_c, av_s) + (1 - z'/z)F(v_c, b''v_s) + \frac{z'}{z} F(v_c, bv_s)] \quad \ldots \quad (13)$$

where $p$ and $z$ have the signification given above and $z'$ is deduced from $z$ by interchanging $g_o$ and $g_s$. For all practical purposes, only the linear part of the last two terms is of importance. This is

$$(1 - z'/z)\, b''v_s + z'bv_s/z = qb'v_s$$

and is identical to the linear part of the single second overload term of the ring modulator. Thus, with this approximation, (6) and (13) only differ by the factor 1/2 (i.e., by 6 db) in all the products.

Using the same notation (impedances $2R$ and $2r$) for the single Cowan modulator, the positive carrier voltage in linear operation is

$$v_c = e_c/f'; \quad f' = 1 + 2rg_s \quad \ldots \quad (14)$$

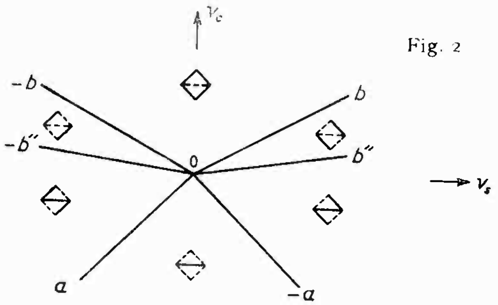



Fig. 2

The modulator configurations are defined in Fig. 3 with

$$c = \left(\frac{1 + 2g_o r}{1 + 2g_s r}\right) a \quad \ldots \quad (15)$$

This defines the first overload point which is strongly dependent on $r$. The second overload is of the same order as before but cannot *a priori* be neglected with respect to the first one. The antisymmetrical output voltage is

$$v' = \tfrac{1}{2}p'[F(v_c, cv_s) + F(v_c, bv_s)] \quad \ldots \quad (16)$$

with

$$p' = \frac{\frac{1}{2}R(g_s - g_o)f'}{1 + \frac{1}{2}R(g_o + g_s) + r(g_s + g_o + 2Rg_og_s)} \quad (17)$$

In linear operation this gives

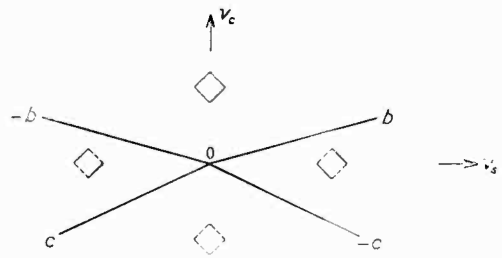
$$V_{11} = \frac{1}{\pi} p' (c + b) V_s = \frac{1}{\pi} V_s k,$$

as expected, and the intermodulation ratio is

$$\frac{V_{13}}{V_{11}} = \frac{1}{24}\left(\frac{V_s}{V_c}\right)^2 \lambda \quad \ldots \quad (18)$$

for small signal, with

$$\lambda = \frac{c^3 + b^3}{c + b} = b^2 [(c/b)^2 - c/b + 1] \quad \ldots \quad (19)$$




As $b$ is independent of $r$, this function will be a minimum for $c/b = 1/2$; i.e., for

$$r = \frac{\frac{1}{2} + R(g_s - 1\frac{1}{2}g_o)}{g_s - 2g_o - g_sg_oR} \quad \ldots \quad (20)$$

which tends to $R$ for good modulators. Assuming the usual matching condition $R = (g_og_s)^{-1/2}$ and keeping only significant powers of $\alpha = (g_o/g_s)^{1/2}$, the function $\lambda$ takes the following values for various values of $r$:

| $r$ | 0 | $1/2g_s$ (matching) | $R/2$ ($b = c$) | $R$ (optimum) | $\infty$ |
|---|---|---|---|---|---|
| $\lambda$ | 1 | 1/4 | $\alpha^2$ | $0.75\alpha^2$ | $\alpha^2$ |


V. Belevitch

Brussels.


### Are Transit Angle Functions Fourier Transforms?

Sir,—Despite the time which has elapsed since the writer's letter on this subject appeared, no one has shown any signs of interest. As there are now many workers in the field, I feel that this is no accident, and that the difficulty remains unsolved. In this event it may be of some use to place on record the writer's further thoughts on the subject, particularly as the whole question of sound engineering of high-frequency devices is intimately bound up with the transit-angle functions associated with the various gaps or paths of their electrodes.

First, it may be said that transit-angle functions *are* Fourier Transforms, but that perturbation of the direct-current conditions by alternating forces renders it difficult to arrive at a true result. The fact that the Fourier Integral of a linear saw-tooth wave is convertible to the correct 'transit-time' solution is believed to be due to the fact that, when such a saw-tooth is perturbed in a certain way its Fourier Transform remains unchanged.

Secondly, in regard to the solution for, say, the distance travelled in a temperature-limited diode, it must be remembered that this is obtained as a result of a double integration. The double integral involved may in well-known manner be converted to a single integral, of the form

$$\sum_{n=0} \int_{t_0}^{t} (t - u)\, e^{nj\omega u}\, du \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where $n = 0$ gives the d.c. term and $n = 1$ the fundamental. Putting $n = 0$, we see that we only get a 'parabolic' pulse when the limits are inserted after integration; and this 'pulse' is in terms of $t - t_0$, not of $t' - t_0$ where $t'(= u)$ is the intermediate value of $t$. When $n = 1$, the integral for this term may be transformed to

$$e^{nj\omega t} \int_0^{\tau} \sigma\, e^{-nj\omega\sigma}\, d\sigma = \frac{\alpha^2}{2(j\omega)^2} \,.\, Y_3(\alpha) \,.\, e^{j\omega t} \quad \ldots \quad (2)$$

Where $\sigma$ is the 'unelapsed' part of the transit time, and $\alpha = j\omega\tau$. This correct solution can be obtained, if one pleases, by regarding (1) as the 'incomplete' Fourier Transform of the unelapsed transit time $t - t'$.

I wish I were able to report success of a similar nature with the space-charge limited diode. A Fourier solution for the electron velocity has the same form as (2), but whereas this is correct, subject to allowance subsequently being made for 'variation time,' the temperature-limited diode velocity is already correct (including variation time effects), so this is not good enough. The reason why (2) is also correct is that when $x$ is placed equal to a constant the variation time comes from the two equations

$$x = Y \frac{\tau^2}{2} [1 + \epsilon\tau_3(\alpha)]$$

$$x = Y \frac{\tau_0^2}{2}$$

[in other words $\tau_1\tau_0 = -\frac{1}{2}\tau_0^2\epsilon Y_3(\alpha)$], so (2) is thus, in effect a solution for variation time; only $v$ need be corrected, and, in the space-charge case, $dv/dt$. This brings us to the discrepancy in the space-charge case for the acceleration, and for the electric field, obtained as a first integration of the equation

$$d^3x/dt^3 = \frac{d^2v}{dt^2} = C\left(1 + \sum_1^{\infty} M_n e^{nj\omega t}\right)$$

where $C$ is proportional to the space current per $cm^2$ of plane cathode. Transit-time metnod (a single integration, and subsequent 'variation time' correction applied) gives (for the first 2 terms)

$$\frac{dv}{dt} = C\left[\tau_0 + M_1 \frac{1 - Y_3(\alpha)}{j\omega} e^{j\omega t}\right]$$

where

$$\tfrac{3}{2}(1 - Y_3(\alpha)) = \alpha(1 - \tfrac{3}{8}\alpha + \tfrac{1}{10}\alpha^2 - \tfrac{1}{48}\alpha_3 + \tfrac{1}{280}\alpha^4 - \ldots)$$

while the Fourier Transform of a saw-tooth pulse (proportional to $\tau'$) gives a solution with the function $\frac{1}{2j\omega}\alpha Y_2(\alpha)$, where

$$\alpha Y_2(\alpha) = \alpha(1 - \tfrac{1}{3}\alpha + \tfrac{1}{12}\alpha^2 - \tfrac{1}{60}\alpha^3 + \tfrac{1}{360}\alpha^4 - \ldots)$$

exactly as for the velocity in the temperature-limited case. The differences are far greater than can be gauged by mere inspection of the power series expansions, moreover the Fourier method as above applied fails since the first term $\frac{1}{2}\alpha$ is here wrong—it has to be $\frac{2\alpha}{3}$, as is given by $1 - Y_3(\alpha)$, in order to satisfy 'low frequency' conditions. If, however, it is noted that transit-time dynamical methods are as a rule as much simpler than the manipulation of Fourier Integrals, the engineer need have no qualms about using the less classical procedure. As an example of great simplification by using 'transit time' may be mentioned the classical theory of radiation-collisions of electrons with atoms producing electromagnetic pulses as a result of sudden deceleration. The problem, dealt with by Lorentz, Larmor and Thomson,[2] comes out much more simply if Fourier Integrals are dispensed with entirely, at any rate, in case $t_1 \ll t_2$. The connection between transit-time effects and electromagnetic radiation is, thus, incidentally, quite close—you cannot have the one without the other.

Fordcombe, Kent. W. E. BENHAM.

[1] *Wireless Engineer*, June 1949, p. 210.

[2] "Corpuscular Theory of Matter," by J. J. Thomson, 1907, p. 92. Constable.

## Gain of Aerial Systems

SIR,—The point which I wished to make clear in my letter (*Wireless Engineer*, December 1949, ) was that the end-fire theorem, stated by Mr. Bell in his paper in the September issue, did not apply to apertures in the diffraction sense. Practical cases of apertures in this sense are the mouths of mirrors and electromagnetic horns, and the outer surfaces of lenses. The argument advanced in my previous letter referred entirely to apertures in this sense, and not as Mr. Bell has implied in his reply (*Wireless Engineer*, January 1950), to arrays of conducting elements. As far as such arrays are concerned, I am in agreement with Mr. Bell that the gain may be doubled by using the array to produce an end-fire beam instead of a broadside beam. In view of the confusion which has arisen, it is worth while considering in what respects apertures of the diffraction type differ from arrays of conducting elements.

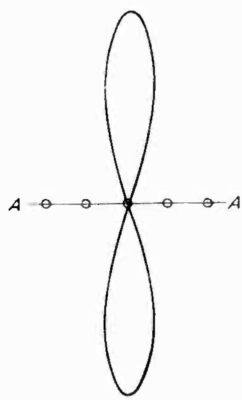



Fig. 1

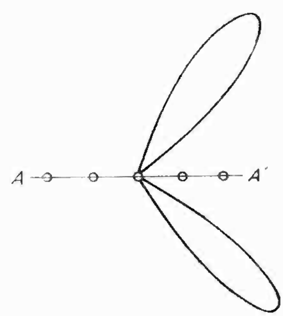
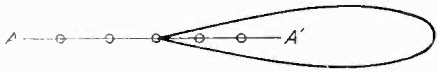



Fig. 2

When the currents in the conducting elements of an array are all of the same phase, two beams will be radiated in opposite directions as shown in Fig. 1. Now, if the phases of the currents are suitably altered, it will be possible to swing each beam round towards the direction of the axis AA' as in Fig. 2. Eventually, a position will be reached when these two beams will overlap completely as in Fig. 3. The array is now acting as an end-fire radiator, and in view of the fact that the *two* broadside beams have coalesced into the *one* end-fire beam, it is reasonable to expect the gain of the system to be increased, as predicted by the end-fire theorem.

When the phase across a diffraction aperture is constant, however, there will be only one radiated beam; this is immediately obvious for any system such as a mirror, a lens or a horn. There is therefore no possibility of doubling the gain by overlapping two beams. In fact, as shown in my last letter, the gain of the end-fire beam will be twice that of the broadside beam, only if the radiating aperture is surrounded by an infinite conducting sheet.

Fig. 3 

My objection to Mr. Bell's statement of the end-fire theorem is that it is so phrased as to appear valid for either of the above types of aerial system. The true position is that the theorem is valid for an array of conducting elements, but only for an aperture in the diffraction sense, if it is surrounded by an infinite conducting plane. In the second case, it is the presence of the conducting plane, rather than the fact the aperture is acting as an end-fire radiator, which leads to the doubling of the gain.

Radar Research and Development Establishment. J. BROWN

## Phase-Shift Calculation

SIR,—In a recent letter (*Wireless Engineer*, January 1950) W. Saraga and J. Freeman describe a rapid, approximate, calculation of image phase-shift in multi-section Zobel filters. It is of interest to note that a more exact calculation can easily be carried out in the special case where all minima in the image attentuation are equal to each other.

Cauer has shown a method for determining the optimum distribution of the attenuation peaks in a low-pass filter where the equal minima of the image attenuation will fall in the frequency band $f' \leqslant f \leqslant \infty$ ($f' > f_2$, the cut-off frequency). The frequencies for the peaks are calculated by means of the elliptic sn-function. It is not very difficult to rearrange the optimal distribution to the band $f' \leqslant f \leqslant f'' < \infty$ with suitable frequency transformations. The $m$-values for a Zobel filter can then be calculated from the peak frequencies. Presumably this has been done by a great many involved in actual filter design.

A slightly different method also permits the phase-shift in the pass-band to be calculated very easily for the resulting filter, as well as the image attenuation for $f > f''$ or $f < f'$ and the effect of dissipation on filter performance.

First, the following frequency transformations of a well-known type are used:

$$w = u + jv = \sqrt{1-(f_2/f)^2} \text{ for low-pass}$$

$$w = u + jv = \sqrt{1-(f/f_1)^2} \text{ for high-pass}$$

$$w = u + jv = \sqrt{\frac{f_2}{f_1}\cdot\frac{f_1{}^2-f^2}{f_2{}^2-f^2}} \text{ for band-pass, general case.}$$

$f_1$, $f_2$ are the cut-off frequencies.

$w = u$ corresponds to an attenuation band

$w = jv$ corresponds to a pass band.

The frequency band $f' \leqslant f \leqslant f''$ is now transformed to $u' \leqslant u \leqslant u''$, where the image attenuation $A \geqslant A_{min}$ for $N$ ($>1$) full-section peaks in the filter. By means of the rapidly convergent series-expansions for the Jacobi Theta-functions the following relations are established. (The infinite series can be terminated after only three terms or less with sufficient accuracy).

(1) Given: $u'$, $u''$, $N$.

$$A_{min} = N.\log_e \frac{1}{q} - \log_e 2 \text{ nepers}$$

$$q = \epsilon + 2\epsilon^5 ; \epsilon = \tfrac{1}{2}(1 - \sqrt{u'/u''}) / (1 + \sqrt{u'/u''})$$

(2) Given: $A_{min}$, $N$.

$$q = \exp [-(A_{min} + \log_e 2)/N$$

$$\sqrt{u'/u''} = (1 - 2q + 2q^4) / (1 + 2q + 2q^4)$$

For the part of the attenuating range in question ($u' \leqslant u \leqslant u''$) we have:

$$A = A_{min} - \log_e |\cos Nz|$$

$$u = \sqrt{u'u''}(1 - 2q \cos z + 2q^4 \cos 2z) / (1 + 2q \cos z + 2q^4 \cos 2z)$$

where $0 \leqslant z \leqslant \pi$.

The attenuation peaks are placed at

$$u_n = u(z_n),\ z_n = (2n - 1)\ \pi/2N,\ n = 1, 2 \ldots N$$

(For the low-pass and high-pass case $u_n$ will be equal to the Zobel $m$-values).

By choosing a suitable value of $z$, corresponding values of $u$ and $A$ are readily calculated.

The image attenuation and phase-shift are now completely specified and can be determined in the ordinary way by adding the contributions from the separate filter sections. The following relations will, however, give easier calculations with little loss of accuracy.

In the *attenuating* band where $A < A_{min}$ ($u'' < u < \infty$ or $0 < u < u'$) we get:

$$A = A_{min} - \log_e\left\{\cosh\left[\frac{2y}{\pi}(A_{min} + \log_e 2)\right]\right\}$$

$$(0 < y < \pi/2)$$

$$y = y_0 + 2 \exp (\pi^2/\log_e q) \sin 2y_0$$

$$\cos y_0 = u/u' \text{ or } \cos y_0 = u''/u \text{ respectively.}$$

[$\exp (\pi^2/\log_e q)$ is a small correction factor].

In the *pass* band ($0 \leqslant v \leqslant \infty$) the image phase-shift can be written as:

$$B = N(B_0 + q^2 \sin 2B_0)$$

where $B_0$ is the image phase-shift for a full-section filter with an attenuation peak at $u = \sqrt{u'u''}$.

The following equation can be used for calculation of the dissipation losses in the pass band etc.

$$\omega\frac{dB}{d\omega} = \frac{1}{s} N(1 + 2q^2 \cos 2B_0)\ |\sin B_0|$$

In the attenuation range the peaks will be rounded off to finite values $A_n$ due to filter losses:

$$A_n = A_{min} + \log_e\left(\frac{1}{d_L + d_C}\cdot\frac{8sq \sin z_n}{N(1 - 4q^2 \cos^2 z_n)}\right)$$

$d_L$, $d_C$ = the dissipation factor for the coils and capacitors respectively.

In the equations above we have

$$s = \left|\frac{w}{f}\cdot\frac{df}{dw}\right|$$ which can be expressed in several ways, for instance.

$$s = \frac{f_2 - f_1}{f_2 + f_1}\cdot|\cosh^2 \Gamma_k|$$

$\Gamma_k = A_k + jB_k$ is the complex image attenuation for an ideal half-section constant k-filter.

$|\cosh^2 \Gamma_k| = \cos^2 B_k$ in a pass band

$|\cosh^2 \Gamma_k| = \sinh^2 A_k$ in an attenuation band.

Stockholm, Sweden. NILS OLOF JOHANNESSON

## R.E.C.M.F. EXHIBITION

The 7th exhibition of components, materials, test gear and valves opens at Grosvenor House, Park Lane, London, W.1. on 17th April. It is to be open for three days from 10 a.m. to 6 p.m. and admission is restricted to holders of invitation cards from the Radio & Electronic Component Manufacturers' Federation, 22, Surrey St., London, W.C.2.

# NEW BOOKS

### The Mathematics of Circuit Analysis

By ERNST A. GUILLEMIN, Pp. 590 + xiv. John Wiley & Sons, New York and Chapman & Hall, 37 Essex St., London, W.C.2. Price 60s.

About 20 years ago the staff of the Department of Electrical Engineering at the Massachusetts Institute of Technology undertook to revise its entire presentation of the basic principles of electrical engineering. This revision resulted in the publication in 1943 of three volumes entitled "Electric Circuits" (a first course in circuit analysis for electrical engineers), "Magnetic Circuits and Transformers" (a first course for power and communication engineers), and "Applied Electronics," (a first course in electronics, valves and associated circuits). The volume under review was intended to be part of the same series, but publication was delayed by the war. Basic mathematical processes and methods are discussed; the chapter headings are Determinants, Matrices, Linear Transformations, Quadratic Forms, Vector Analysis, Functions of a Complex Variable, and Fourier Series and Integrals.

The object of the book is to explain how these basic methods and processes work; applications are deliberately kept for the companion volumes mentioned above. It is intended to stimulate interest and convey ideas rather than to be rigorous, and to develop a background of general understanding upon which the student whose mathematical ability is above the average may later build more carefully. Its scope is much wider than that of a book designed merely to help the student pass examinations.

Under the present British conditions, the time is hardly ripe for so fundamental a revision of the structure of electrical engineering courses as that undertaken at the M.I.T. While this may make it difficult to use the book directly as a text-book in this country, it contains a valuable collection of reference material chosen as the result of Prof. Guillemin's long practical experience of teaching electrical engineers. It therefore deserves careful study and should be available in libraries.

J. W. H.

### Television for Radiomen

By EDWARD M. NOLL. Pp. 595 + xii, with 356 illustrations. Macmillan & Co., Ltd., St. Martin's Street, London, W.C. 2. Price 52s. 6d.

This is an American book and deals solely with television equipment designed for the American standards. Its utility in this country is thus strictly limited by this fact.

It covers all aspects of television receivers in an elementary manner and includes transmitting material only in just enough detail to give the reader a picture of the complete system.

The method of treatment is usually to take a section of a receiver and to discuss it in general terms. Various alternative circuits are illustrated and described in detail and then a number of typical circuits, as used in American receivers, are given as examples of the way in which they take practical form. On these last circuits values of components are given.

As an example, the chapter on "Sweep Systems" starts with the basic method of generating a saw-tooth wave by the alternate charge and discharge of a capacitor. Blocking-oscillator and the cathode-coupled multi-vibrator discharge circuits are then described, then synchronizing methods, waveform defects and their correction and, finally, deflection amplifiers. There are about 12 pages of general discussion on amplifiers and some 7 extra pages on line-scanning amplifiers in which attention is mainly directed to diode and triode damping methods. There are then 9 pages of typical commercial systems.

The book includes over 50 pages on projection television and a useful feature is a set of 48 oscillograms illustrating the waveforms and their amplitudes at various points, mainly in the time-base circuits, of a particular commercial receiver, the complete circuit of which is given. Although it is a projection receiver this has little effect on the circuitry and the waveforms are thus typical of most television sets. A point of particular interest is that the set includes a fly-wheel sync circuit and the waveforms in this are given.

In spite of the length of the book the treatment is not always as detailed as one could wish and there are quite a number of superficialities. Except in the last chapter, mathematics are excluded and the book concludes with a set of mathematical tables.

The book should prove extremely valuable to those for whom it is intended—that is, those who have to maintain and repair American television receivers. There is a good deal of material in it useful to those who handle British receivers. The book, however, can hardly be recommended to those without a fair knowledge already, for if they are beginners they will undoubtedly have difficulty in knowing which parts are relevant to British practice and which are not.

W. T. C.

### Einführung in die Siebschaltungstheorie der elektrischen Nachrichtentechnik.

By R. FELDTKELLER. Pp. 160 + viii, with 121 illustrations. S. Hirzel Verlag, Stuttgart. Price DM 12.

Dr. Feldtkeller is Director of the Telecommunications Institute of the Stuttgart Technische Hochschule. This is the third edition of this book on filter theory; the second edition in 1942 was a reprint of the first, but the present edition has been largely rewritten. In the introduction the author points out that this volume does not deal with high frequencies; he has another book "Theorie der Rundfunksiebschaltungen," the second edition of which appeared in 1944, which deals with high-frequency filters. The book starts with simple oscillatory circuits and gradually develops the various types of band-pass filters and filter chains. Their characteristics are calculated and plotted in a number of curves. Several sections are devoted to the *m*-derived filters of Zobel, and a final chapter deals with the effect of losses. The book concludes with an extensive bibliography but mainly of German publications. The treatment throughout is very thorough and the book is undoubtedly a valuable addition to the literature of the subject.

G. W. O. H.

### Radio Servicing Equipment

By E. J. G. LEWIS. Pp. 371 + xi, with 194 illustrations. Chapman & Hall, 37 Essex St., London, W.C.2. Price 25s.

### Electrical Measurements and the Calculation of the Errors Involved

By D. KARO. Part I. Pp. 191, with 106 illustrations. Macdonald & Co. (Publishers) Ltd., 19 Ludgate Hill, London, E.C.4. Price 18s.

### The Practical Electrical Reference Book

Pp. 384. Odhams Press Ltd., Long Acre, London, W.C.2. Price 9s. 6d.