WIRELESS ENGINEER

Vol. XXVII JANUARY 1950 No. 316

The Rationalized M.K.S. System

THIS was the title of the Editorial of November 1948. In it we made a statement which might cause some misunderstanding. We said "It should be noted that rationalization does not affect B, D or $\mathcal{E}$, but only H, μ and κ." We have been asked how κ could be affected if D and $\mathcal{E}$ are not—a very reasonable question. The difficulty arises over the definition of D; Maxwell, who introduced the conception of displacement, was quite definite about it, and stated that "the displacement through a given surface is the quantity which passes through it." The quantity passing through unit area is sometimes called the displacement density and sometimes simply the displacement. In the British Standards Glossary of Terms used in Electrical Engineering electric displacement is given as an alternative name for electric-flux density, which is defined as the quantity of electricity displaced across a given area in a dielectric.

In the m.k.s. system of units the unit of displacement density, is the coulomb per square metre, and the unit of electric force $\mathcal{E}$ is the volt per metre. Now neither of these would be affected by rationalization, for the simple reason that D was introduced by Maxwell in the rationalized form. If we write $D = \kappa_0\mathcal{E}$ for empty space then $\kappa_0 = 1/(36\,\pi \times 10^9)$ which is its rationalized value, but if for any reason one prefers to write $D = \kappa_0\mathcal{E}/(4\pi)$, then κ_0 will be increased to $1/(9 \times 10^9)$ which is its unrationalized value. In the c.g.s. electrostatic system the value of $\mathcal{E}$ at a distance r from a charge q was $q/(\kappa r^2)$ and the value of D was $q/(4\,\pi\,r^2)$, so that $D = \kappa\mathcal{E}/(4\,\pi)$. κ is here the unrationalized c.g.s. value; that is, unity. It can be rationalized by writing $D = \kappa_0\mathcal{E}$ where $\kappa_0 = \kappa/(4\pi)$.

The possible misunderstanding to which we referred is due to the fact that several authors of textbooks have modified Maxwell's definition of displacement and have de-rationalized it in order to bring it into line with the magnetic formula $B = \mu H$. Instead of following Maxwell and making a unit tube of displacement end on unit charge, they take as a unit tube, one ending on a charge of $1/(4\pi)$ of a unit. Those who have done this will, of course, have to retrace their steps when introducing rationalization.

Those who have de-rationalized Maxwell's displacement in order to assimilate the electric and magnetic expressions, obtain for the electric energy per unit volume the formula $D^2/(8\kappa_0\pi)$, where $\kappa_0 = 1/(9 \times 10^9)$, giving exact parallelism with the formula $B^2/(8\mu_0\pi)$ for the magnetic energy per unit volume. Using Maxwell's rationalized displacement the formula is $2\pi D^2/\kappa_0$ which will be very familiar to most of our readers. If we now complete the rationalization by rationalizing κ_0 the formula becomes $D^2/(2\kappa_0)$, where $\kappa_0 = 1/(36\pi \times 10^9)$. D is here in coulombs per square metre and the formula gives the energy in joules per cubic metre. Similarly, in the rationalized system the magnetic energy in joules per cubic metre is equal to $BH/2$, which for empty space becomes $B^2/(2\mu_0)$, where B is in webers per square metre, H in ampere-turns per metre, and $\mu_0 = 4\pi/(10^7)$.

G. W. O. H.

ELECTRICAL NOISE

Experimental Correlation between Aural and Objective Parameters

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1. Introduction

MANY types of electrical machinery cause radio interference because they make or break an electric current. The transient oscillation thus set up in neighbouring circuits may have Fourier components at audio and radio frequencies and these may be radiated or mains borne to audio- or radio-frequency receiving apparatus thus setting up a disturbance which, if the interference spectrum is uniform over the band of frequencies accepted by the receiver, is characterized mainly by the transmission properties of the receiver.

In order to suppress such interference it is necessary to decide subjectively how much of it is tolerable. The next step is to devise a measuring instrument capable of informing the engineer engaged in the suppression of interference when he has accomplished his task. This measuring instrument should thus have such characteristics as will enable its assessment of interference to simulate that of the person for whom the transmission is arranged. In the case of sound broadcasting it is the listener's ear which the interference-measuring device must simulate. The ideal interference-measuring set would thus consist of a typical receiver and loudspeaker and a human operator whose ear was tireless and never changing.

The tests to be described were undertaken with a view to ascertaining the law of variation of annoyance as the objective parameters of the interference were varied, and then the design or selection of a suitable measuring instrument.

In what follows the word loudness is invariably taken to mean the 'amount' of interference in the absence of broadcast programme. The word annoyance, on the other hand, is taken to mean the 'amount of disturbance' caused by the interference in the presence of a programme.

The impulsive type of interference, with which we shall usually deal, manifests itself in the receiver output in the form of successions of damped wavetrains. The time interval between them is usually much greater than their duration because the receiver bandwidth is much greater than the frequency of interruption of the current in normal types of electrical machinery such as motor-car ignition systems, electric motors, electric razors, thermostats, etc. It is thus convenient to idealize the interference into repeated Heaviside unit impulses. Such interference has only two parameters: the time integral, U, of its waveform and its pulse-recurrence frequency p.r.f.

The output of impulsive noise from the receiver will be a function not only of U and p.r.f., but also of the receiver modulation-frequency characteristic and any non-linear elements the receiver may contain. The shape of the receiver modulation-frequency characteristic has to conform to more stringent requirements than those which might be determined from impulsive-interference considerations and may be taken as more or less uniform over its acceptance band. We may now summarize and say that the significant parameters of the output noise are first, the peak or maximum value proportional to U and to receiver bandwidth; secondly, the duration of the noise phenomenon proportional to the reciprocal of receiver bandwidth, and thirdly the p.r.f. equal to that of the incoming interference. We have therefore to correlate annoyance with variation of each of these three parameters. The effect of the peak value on annoyance has been assumed a priori to be logarithmic so that we may start by measuring change of annoyance in decibels as units. This leaves bandwidth and pulse-recurrence frequency as suitable variables.

2. Experimental Arrangements

The experiments were based on a comparison between a standard noise and the noise whose parameters were varied. This method was chosen rather than working to an absolute value of annoyance such as adjusting the noise level until the annoyance conformed to some agreed standard such as 'tolerable' because it was thought to be quicker, to restrict the 'spread' or standard deviation of the listeners' results and to require less explanation to, and effort from, the listener himself. Fig. 1 shows a block schematic of the method adopted. Fig. 2 is a photograph of the 'listening room.' By means of a P.O. key the test listener could

MS accepted by the Editor, March 1949

obtain any one of a number of noises, each adjustable in level by him, and compare them one at a time with the reference noise. The listener was allowed to adjust the level of reference noise at the start of the test provided that his adjustment brought it between the limits of 'just perceptible' and 'disturbing.' Measured on a Tannoy Phon Meter type 800 these limits were 70 phons less 30 db and 70 phons less 60 db. The noise level of the 'listening room' was about 70 phons less 34 db but this did not interfere with listening to noise levels lower than this, presumably due to spectrum differences between the test noises and the room background noise. The programme level was also set to the listeners wishes and was invariably between 70 and 80 phons.

3. The Noises

The programme used during annoyance tests was not specially selected but consisted of any one of the broadcast programmes existing at the time. It was conveyed to the listening room from the broadcast studio by land line and was thus in itself reasonably free from noise and amplitude or harmonic distortion.

The test noises were seventeen in number, excluding the reference noise. This consisted of random fluctuation noise in a band from 250 c/s to 10 kc/s. The reason for the bass cut below 250 c/s was that hum from source amplifiers

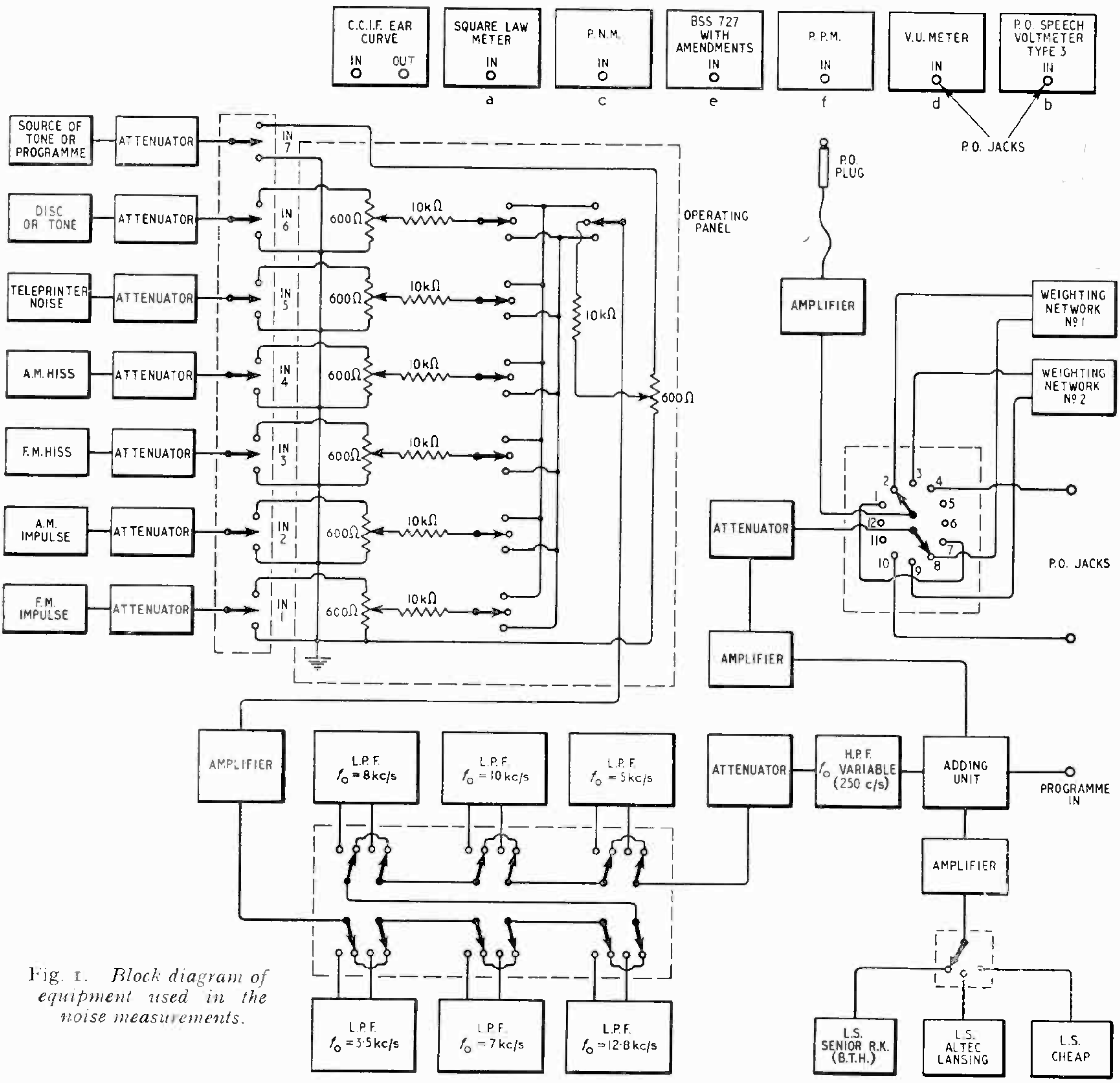


Fig. 1. *Block diagram of equipment used in the noise measurements.*

at the input of the a.f. chain caused trouble which was cured by the insertion of a 250-c/s high-pass filter. Fig. 3 (a) shews the spectrum. Noises one to eight were all restricted in spectrum to the band between 250 c/s and 10 kc/s.

tudes of successive output pulses have a random statistical distribution due to the random phase angles between each input pulse and a steady 'wanted' carrier wave.

Noise two was the output from a v.h.f. a.m.

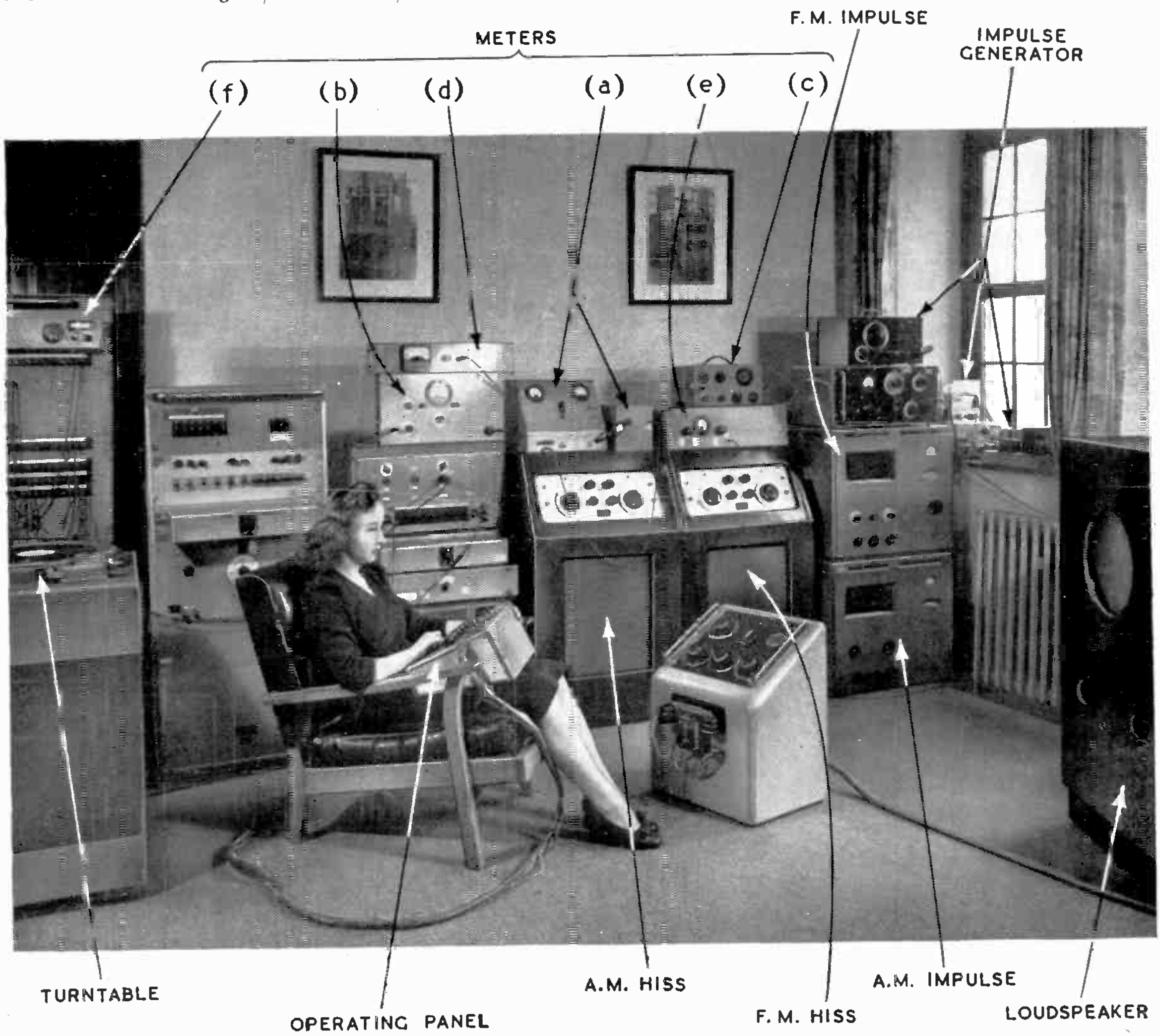


Fig. 2. *The listening room.*

Noise one consisted of the output from a v.h.f. f.m. receiver with 50 μs de-emphasis, the output being due to the application to its input of unit impulses repeated 100 times per second, Fig. 3(b). The receiver was operated just below the f.m./a.m. improvement threshold so that a small proportion of the output pulses had uniform rather than triangular spectra before de-emphasis, Fig. 3(c). Fig. 4 is a photograph of these output pulses, some being more or less unidirectional (uniform spectrum) and others bi-directional (triangular spectrum). The ampli-

receiver due to the same interference input as noise one. Its spectrum is shown in Fig. 3(a) and Fig. 5 is a photograph of it.

Noise three was random fluctuations emerging from a v.h.f. f.m. receiver with 50 μs de-emphasis. The spectrum is shown in Fig. 3(b).

Noise four was surface noise from a freshly-cut cellulose recording disc. The disc was cut in the absence of modulation on the cutter.

Noise five was teleprinter 'cross talk' obtained from a land line and recorded on a cellulose disc.

Noise six was 1000-c/s tone.

Noise seven was the same as noise one except that the p.r.f. was reduced from 100 c/s to 15 c/s.

Noise eight was the same as noise two but with a similar reduction of p.r.f.

Noises nine to seventeen were produced by feeding repeated unit impulses to a 30-c/s to 5-kc/s band-pass filter. The reference noise also had this bandwidth when these noises were used. The test arrangement was improved so that the hum was eliminated and the 250-c/s high-pass filter was no longer needed. The upper cut-off frequency of 5 kc/s was used in order to simulate conditions applying in the case of typical domestic broadcast receivers and also to be in close agreement with the bandwidth of a C.I.S.P.R. interference-measuring set, BS727. The spectrum of noises nine to seventeen is shown in Fig. 3(d). A photograph of noise nine is shown in Fig. 6. These noises differed only in p.r.f. Table I gives the p.r.f. of each noise.

4. Noise Meters

One way in which the law of variation of annoyance with variation of the objective noise parameters may be brought to light is to employ a number of different noise meters, each one having a known behaviour with respect to noise-parameter change. One might reasonably hope that one or more of the meters would closely follow the subjective results. This method was, in fact, adopted. Some of the meters were

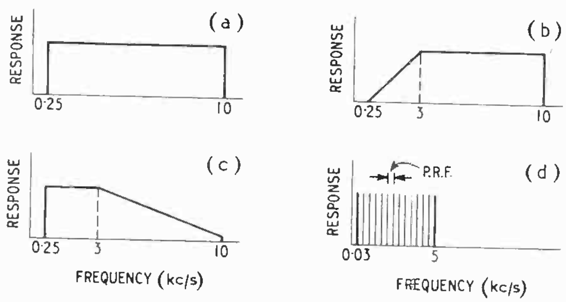


Fig. 3 (above). *Spectra of test noises; (a) reference noise and noise 2, (b) noises 1 and 3, (c) noise 1, (d) noises 9—17.*

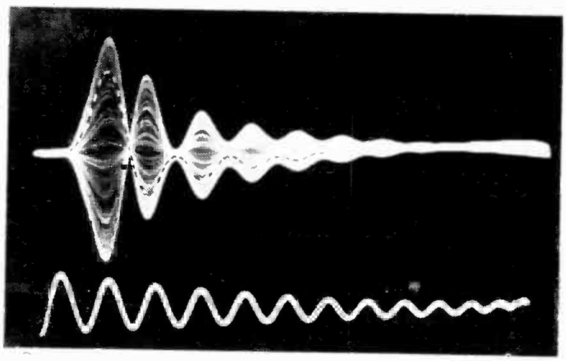
Fig. 4 (below). *Noise 1; dotted pulses are 'pops' due to operation of f.m. receiver just below improvement threshold. Untouched pulses are 'clicks' typical of f.m. reception. Amplitudes of successive pulses are random with time due to presence of a carrier. 10 kc/s timing wave shown below.*

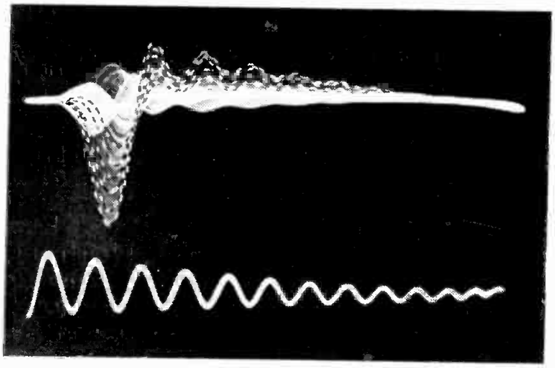
Fig. 5 (top right). *Noise 2; typical impulsive interference output from a.m. receiver. Amplitudes of successive pulses are random with time due to presence of a carrier. 10 kc/s timing wave shown below. One wavetrain dotted for clarification.*

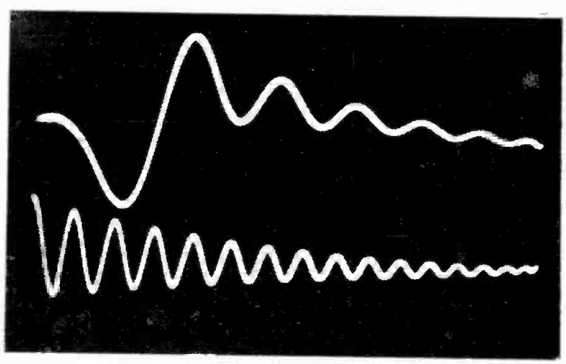
Fig. 6 (bottom right). *Noise 9; impulse response of 30-c/s to 5-kc/s bandpass filter. 10 kc/s timing wave below.*

TABLE I

Noise Number	P.R.F. (c/s)
9	16
10	8
11	4
12	2
13	1
14	0.5
15	0.25
16	0.125
17	0.1







preceded by aural-weighting networks, these having been chosen experimentally by pilot tests not described. Table II gives the noise-meter and weighting-network combinations adopted. All noise meters were used in conjunction with preceding attenuators, the attenuation being so adjusted as to maintain constant meter reading.

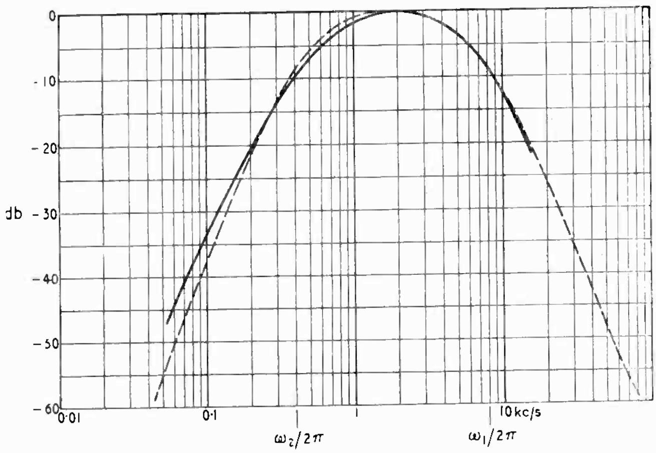


Fig. 7. *C.C.I.F. aural weighting network. The solid-line curve is the specified frequency characteristic; the dash-line is the curve of the experimental network.*

Meter (a) indicates the mean-square value of any input waveform less the mean value (usually negligible or zero). This meter will read mean-square values in a linear (undistorted) manner for all waveforms having crest factors not greater than 40 db.

Meter (b) is similar to meter (a) but is not linear for waveforms having crest factors much above 14 db.

Meter (d) indicates the full-wave rectifier mean value of a waveform applied to it.

Meters (c), (e), and (f) are all based upon the same principle, namely charging a capacitor through one resistor and discharging it through another. They are completely defined by the charge and discharge time constants if the waveform being measured is repeated with sufficient frequency for the needle of the indicating instrument to give a steady reading. This can always be assured by making both time constants large enough, and this will not affect the behaviour of the meter to varying pulse-recurrence frequencies. These meters give characteristic curves of response against p.r.f. or bandwidth (determining pulse height and width). Both these characteristic curves are unaltered if the ratio of discharge to charge times is kept constant; but they are altered if this ratio is changed. The alteration in response characteristic against p.r.f. for a multiplication of the ratio by a number N is a translation $1/N$ along the p.r.f. axis, Figs. 9 and 10. The actual shape of the response against p.r.f. characteristic is constant whatever the charge or discharge times and is due to the manner in which a capacitor charges and discharges. A very important practical remark results from the foregoing. It is that if a certain discharge-to-

TABLE II

	Noise Meter	Best Available Aural Weighting Network
(a)	B.B.C. Mean Square Meter	C.C.I.F. Aural Network for Broadcast Circuits, Fig. 7
(b)	G.P.O. Speech Voltmeter	C.C.I.F. Aural Network for Broadcast Circuits
(c)	B.B.C. Portable Noise Meter PNM/1	C.C.I.F. Aural Network for Broadcast Circuits. (This circuit is included inside the PNM/1)
(d)	V.U. Meter	7-kc/s l.p. filter in cascade with A.S.A. weighting network Z 24.3–1944 curve B
(e)	A.F. version of C.I.S.P.R. interference measuring set. Charge time :—1 ms. Discharge times switchable :—500 ms or 160 ms	5 kc/s l.p. filter
(f)	B.B.C. Peak Programme Meter PPM/6	New A.S.A. weighting network for Broadcasting Circuits, Fig.8

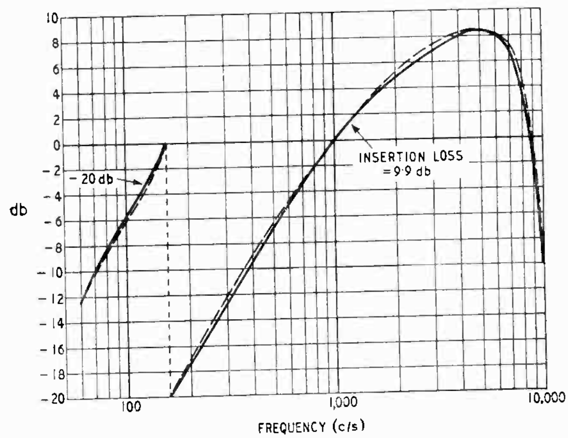


Fig. 8. *Meter (f) has the weighting curve shown by the dash-line while the solid line indicates the A.S.A. weighting curve.*

charge ratio is found to simulate the aural annoyance against p.r.f. over a certain range of the latter, then perfectly steady meter-needle readings may be obtained, however low the p.r.f., by multiplying both charge and discharge times by a sufficiently big number. The above remarks have only been confirmed experimentally for discharge-to-charge time ratios greater than 10 and for values of p.r.f. less than a fifth of the circuit cut-off frequencies. These conditions are, however, eminently practical.

The significant characteristics of the meters for noise measurements are thus as given in Table III.

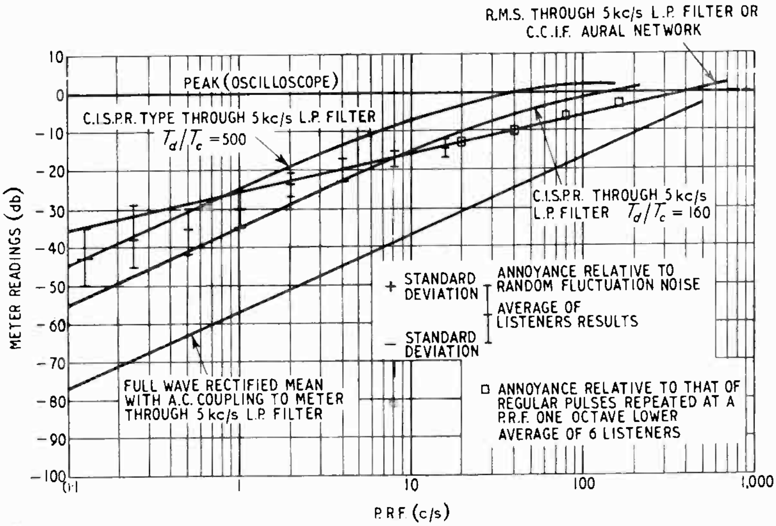


Fig. 9 (*above*). *Experimental results showing the meter readings with regular pulses. All meters were adjusted on random fluctuation noise applied through a 5-kc/s l.p. filter. The peak amplitude of regular impulses was made equal to the peak amplitude of random fluctuation noise, both being fed through a 5-kc/s l.p. filter.*

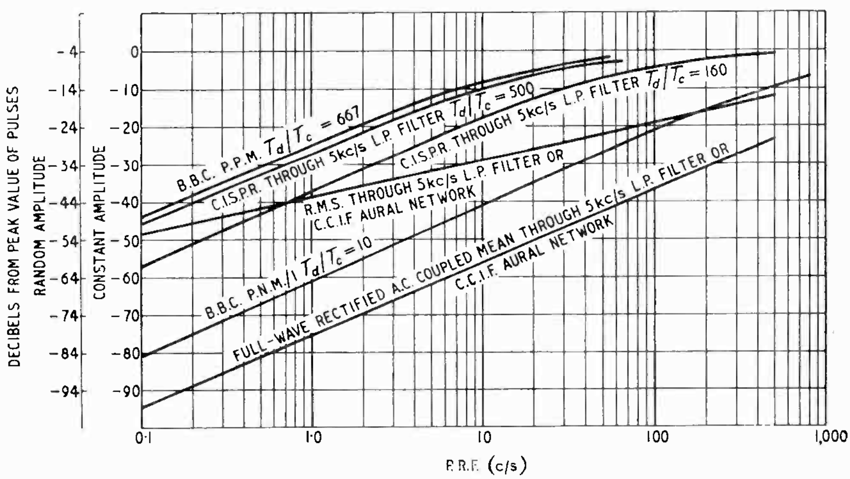


Fig. 10 (*right*). *Calculated meter readings for regular pulses. All meters adjusted on a sine wave, taking crest and form factors into account.*

TABLE III

Noise Meter	Characteristics
(e)	Charge time . . . . 1 ms Discharge time . . 160 ms or 500 ms
(c)	Charge time . . . . 150 ms Discharge time . . 1500 ms
(f)	Charge time . . . . 1.5 ms Discharge time . . 1000 ms

5. The Experiments

5.1 *Annoyance and Loudness as Functions of Receiver Bandwidth (Random Amplitude Pulses)*

Impulsive interference repeated at 100 c/s was fed to a v.h.f. a.m. receiver of overall bandwidth ± (30 c/s to 25 kc/s). An unmodulated carrier was also present. The output was passed to a loudspeaker (B.T.H. Senior R.K. with internal spider. An Altec-Lansing speaker was also tried but did not alter the laws of annoyance and loudness variation) through a de-emphasis circuit of variable time constant. Programme was fed directly to the loudspeaker from a land line. The test listener was asked to equalize by means of a calibrated attenuator the annoyance and loudness of the random amplitude output pulses emerging from the different bandwidths to that from the narrowest of them. No meter was used. The results are shewn in Fig. 11.

5.2. *Annoyance as a Function of P.R.F. (Random Amplitude Pulses)*

This experiment was similar to the previous one except that the bandwidth was fixed at 30 c/s to 5 kc/s, and the p.r.f. was varied. The test listener was asked to equalize again, with a calibrated attenuator, the annoyance at each p.r.f. to that obtaining when the p.r.f. was 30 c/s. The results are shown in Fig. 12.

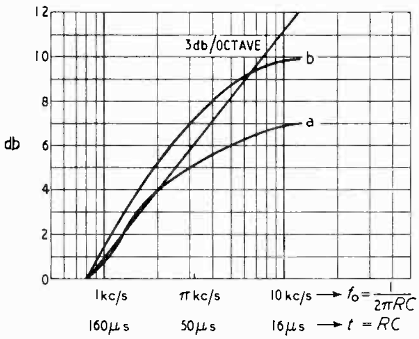


Fig. 11. *Relative annoyance and loudness levels of impulsive interference with varying bandwidth. Average curves of 7 listeners (a) no programme (loudness), (b) with programmes of speech and music (annoyance) ± 3 db deviation from mean.*

5.3. *Annoyance as a Function of P.R.F. (Constant Amplitude Pulses)*

The noises used for this test had spectra as shewn in Fig. 3(d). The test listener was asked to equalize with a calibrated attenuator the annoyance of pulses at a given p.r.f. to that obtaining for a p.r.f. one octave lower. The results are shown in Fig. 9 as □. The results of six listeners were all within ± 5 db. Again no meter was used but the results are placed upon the response characteristic of meter (a) at 10 c/s.

5.4. *Annoyance and Loudness as Read on Different Noise Meters*

The noises used for this test were those numbered one to eight. The test listener was asked to equate annoyance and loudness to that caused by the reference noise. When the test listener had equated these properties of the noises to one another and to the reference noise, the results were measured on meters (a) to (f). Tables IV, V and VI summarize the results.

Of 40 people only 3 were rejected as giving results very far from the average; 6 of the listeners were women and 30 were radio engineers. The people giving most consistent results were the least familiar with acoustic and engineering problems. Engineers with most acoustic experience gave the least consistent results. Fig. 13 shows the average change in meter readings between reference noise and each of the eight noises. Fig. 14 shows statistical frequency distributions of the test listeners' results on the various meters. Meter (a) appears to be the most

TABLE IV Loudness

Noise	1	2	3	4	5	6	7	8
No. of observations..	18	18	28	19	19	18	9	9
Average of readings of meter (a) in db ..	2.81	0.805	−0.125	−0.105	1.6	16.6	7.71	5.83
Standard deviation in db	4.71	2.19	0.893	2.23	3.64	6.16	3.07	2.6

TABLE V Annoyance

Noise	1	2	3	4	5	6	7	8
No. of observations	28	28	43	27	26	25	14	14
Average readings of meter (a) in db ..	−0.518	−1.79	−0.221	1.89	−0.260	14.8	1.04	0.428
Standard deviation in db	3.07	2.74	1.07	2.87	4.84	6.67	2.9	2.32

TABLE VI Loudness and Annoyance

Noise	1	2	3	4	5	6	7	8
No. of observations	46	46	71	46	45	44	23	23
Average readings of meter (a) in db ..	0.32	−0.77	−0.18	1.06	0.523	15.2	4.03	2.54
Standard deviation in db	4.17	2.84	0.997	2.8	4.47	6.8	4.18	3.6

promising. This would seem to indicate that Ohm's Law of Hearing (D. Gabor, "New Possibilities in Speech Transmission," *J. Instn elect. Engrs*, Vol. 94, Part III, No. 32, November 1947) applies with adequate accuracy to wide spectrum type noises ; that is, the ear is insensitive to small changes of phase. It is interesting to note, in this connection, that there is very little aural difference between random amplitude pulses repeated at a sufficient rate for the individual clicks to have merged into a continuous sound and random fluctuation noise. The two energy spectra can be identical, the only difference being that all pulses are in phase at zero time while random fluctuation noise may be regarded as pulses having a random statistical phase distribution. There is a difference in sound (that between frying and hissing) but it is not great.

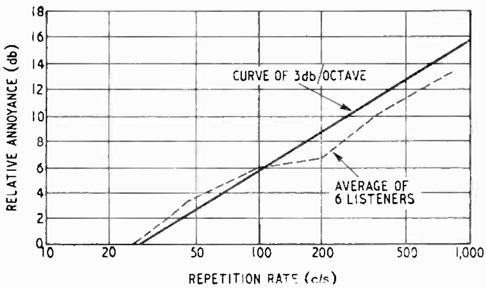


Fig. 12. *Relative annoyance of impulsive interference of random amplitude.*

5.5. *Variation of Difference in Annoyance between Random and Constant Amplitude Pulses as a Function of P.R.F.*

Noises of the type nine to twelve were used on the one hand with random amplitudes, and on the other as reference noises with constant amplitudes. The test listener was asked to adjust the level of the random-amplitude noise so that it caused the same annoyance as that produced by the same type of noise except that the amplitudes of successive pulses were constant. Table VII summarizes the results of readings of meter (a) upon the subjectively-adjusted noises.

TABLE VII Annoyance

Noise	9	10	11	12
No. of observations	14	18	9	8
Average readings of meter (a) in db.	0.822	−0.194	0.778	0
Standard Deviation in db	1.85	1.08	2.2	1.5

The meter was first 'set-up' on constant-amplitude pulses. The average change of meter reading when connected to the output of random-amplitude pulses is shown in the Table. As in 5.4 we note that meter (a) gives readings quite close to the subjective results.

5.6. *Annoyance as a Function of P.R.F. (Constant Amplitude Pulses of very Low P.R.F.)*

This test is a variation and extension of test 5.3. Instead of comparing annoyance at one p.r.f. with that caused by pulses repeated at a frequency an octave lower, the annoyance of the constant-amplitude pulses was compared with that of random fluctuation reference noise. The results are given in Table VIII.

These figures are also plotted in Fig. 9 as ⫯. They are plotted in such manner as to show the average test listener's ear in the form of a noise meter. Thus, for example, at a p.r.f. of 1 c/s the M.S. meter (a) would read $4\frac{1}{3}$ db higher than the human noise meter, whilst the C.I.S.P.R. meter with a discharge time of 160 ms reads 5 db lower than it. The V.U. meter (d) reads 27 db lower and the C.I.S.P.R. meter with 500 ms discharge time reads $5\frac{1}{3}$ db higher than it. It may be seen from Fig. 9 that at a p.r.f. below about 2 c/s the rate of fall-off of annoyance with decrease in p.r.f. appears to increase from 3 db per octave to about 6 db per octave. This would seem to shew that the ear can store energy for about a half second but not longer. It should be stated that for a p.r.f. lower than 1 or 2 c/s none of the meters used could copé with the crest factors of the impulsive waveforms present. The method adopted was to increase the p.r.f. from that listened to by the test subject to a value of 2 c/s, allowing an increase of 3 db per octave increase of p.r.f. on meter (a).

5.7. *Measurement of Programme-to-Noise Ratios.*

It seems from the foregoing that at least for the types of noises used in this series of experi-

TABLE VIII Annoyance

Noise	9	10	11	12	13	14	15	16	17
No. of observations	10	12	14	17	18	18	17	18	8
Average readings of meter (a) in db	0.625	0.25	− 0.054	1.22	4.33	7.06	5.93	7.25	7.23
Standard deviation in db	2.2	1.99	2.89	2.79	4.56	6.12	8.73	7.27	7.05

ments a M.S. meter indicates annoyance adequately. Thus two major experiments remain to be done. First, is it possible to measure programme loudness with a M.S. meter? If it is, secondly, can it measure programme-to-noise ratios which have been subjectively adjusted to fit certain designations?

were then measured with meters (a) and (f), each measurement being done in a manner suitable to the particular instrument in use. Table IX shows that for the three test listeners used, the M.S. meter (a) gives a more constant reading of signal-to-noise ratio than does the peak programme meter (f).

TABLE IX

	Average r.m.s. signal-to-noise Ratio (db)	Maximum Deviation from Average r.m.s. (db)	Average signal-to-noise Ratio meter (f) (db)	Maximum Deviation from Average meter (f) (db)
Speech	30.3	1.7	32.3	1.7
Music	30	0	26.6	— 0.6

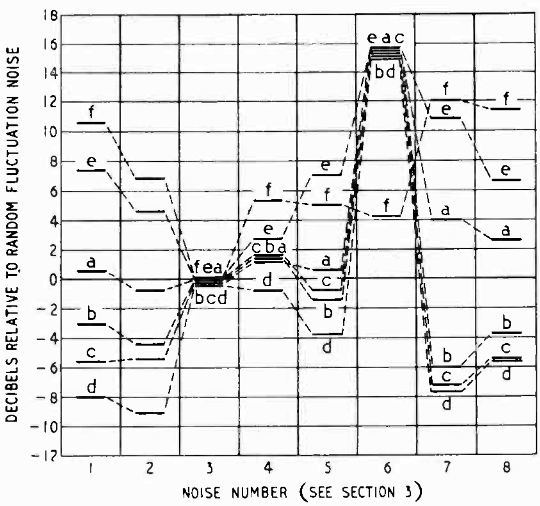


Fig. 13. *Experimental results; average readings for each noise. All meters aligned on random fluctuation noise; the letters refer to the meters of Table II.*

The method adopted for the measurement and comparison of programme-to-noise ratios consisted in the adjustment of a speech programme to have the same loudness as a music programme, both in the presence of a random fluctuation noise background. The signal-to-noise ratios

5.8. *Designated Programme to Noise Ratios.*

Programme was adjusted to a level of 75 phons in the absence of noise. Random fluctuation noise was then switched on and each of four test listeners was asked in turn to adjust the noise level to the description given in the first column of Table X.

TABLE X

Noise level Designation	Average r.m.s. Programme to Noise ratio (db)	Maximum Deviation from Average (db)
Just perceptible ..	60.6	3
Perceptible	50	0
Slightly disturbing ..	42	2
Disturbing	31	1

During these tests the listener could switch the programme on and off at will for cross-checking purposes. Meter (a) was used in this test.

5.9. *Effect of Noise Level on Comparison of Annoyance of Different Types of Noise.*

Test listeners were asked to adjust the levels of noises one to six so as to equate the annoyance

TABLE XI Annoyance

Noise	1	2	3	4	5	6
Number of Observations	6	6	6	6	6	6
Average increase in meter reading in db when noise level decreased by 10 db ..	5.5	2.9	0.17	2.4	8.3	— 0.42
Standard Deviation in db	4.86	4.58	1.07	4.77	9.00	6.56

to that caused by the random-fluctuation reference noise. This experiment was undertaken for two values of the reference noise level, those corresponding with 'slightly disturbing' and 'disturbing.' Table XI shews the average change in listening criterion resulted in changes of annoyance considerably less than the actual change in level. This ceases to be true for very poor signal-to-noise ratios. Meter (c) was used in this test.

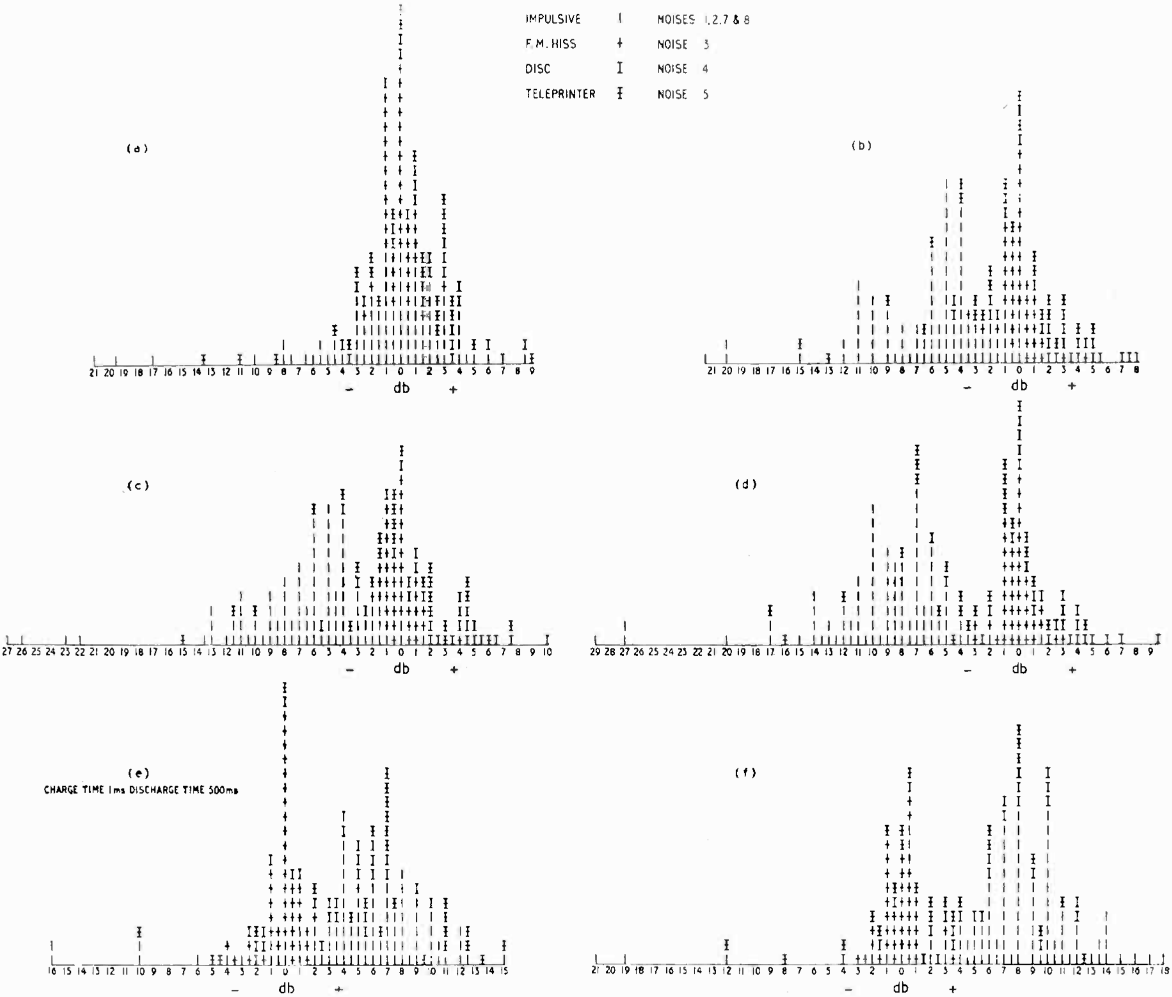


Fig. 14. *Statistical frequency distributions of results. The letters refer to Table II. Each symbol represents one observation. Abscissae show meter indications of annoyance of test noises relative to reference noise, the listener having adjusted the test and reference noises to equality of annoyance.*

(taken as positive decibels for an increase) of meter readings when the reference noise was changed from 'disturbing' to 'slightly disturbing.'

This Table would seem to indicate that the relative annoyance of impulsive noise (principally noises one, two and five) vis-à-vis random-fluctuation noise decreased slightly with decrease in level or increase in signal-to-noise ratio. The standard deviations are rather large, however, so that broadly speaking one may say that for reasonably good signal-to-noise ratios the change

TABLE XII Loudness

Noise	1	2	3
No. of observations . .	4	4	4
Average increase in meter reading in db when noise level decreased by 20 db	0.63	0.25	0
Standard deviation in db	1.08	2.28	1.37

5.10. *Effect of Noise Level on Comparison of Loudness of Different Types of Noise.*

This experiment differed from the previous one only in that programme was not used, and that a 20-db change in reference-noise level was used instead of a 10-db change. The noise levels used were 70 phons less 30 db and 70 phons less 50 db. Table XII shews virtually no change in the relative loudness of impulsive noise vis-à-vis random fluctuation noise.

6. Conclusions

The subjective effects of typical kinds of electrical interference have been studied, and a meter suitable for their measurement has been designed. The present meter, the M.S. meter (a), is, however, not suitable for p.r.f. below 1 c/s emerging from a C.C.I.F. aural network because the crest factor of such pulses would exceed 40 db and the meter would depart from true M.S. readings. It is further evident from Fig. 9 that a rate of fall of annoyance per octave decrease in p.r.f. of 6 db is required for values of p.r.f. below about 2 c/s. This characteristic could be arranged as follows: a meter of the type (a) with an improved crest factor characteristic followed by a de-emphasis circuit of time constant about half a second (to take the average of the noise waveform over a $\frac{1}{2}$-second interval) followed by a 'square-rooting' device to the output of which would be connected a d.c. indicating instrument of time constant somewhat longer than the reciprocal of the lowest p.r.f. likely to be encountered in practice. Such a device should follow the 3-db per octave law down to a p.r.f. of about 2 c/s and then change gradually to a 6-db per octave law.

7. Acknowledgments

The authors wish to express their thanks to Mr. H. Bishop, Chief Engineer of the B.B.C., for kind permission to publish this work which was undertaken at the request of Mr. H. L. Kirke, Head of the B.B.C. Research Dept.

The authors also wish to express their appreciation of the valuable advice received from Messrs. West, McMillan and Thorne of the Research Station and Radio Branch of the G.P.O.

DEVELOPMENT IN PULSE-MODULATION CIRCUITS

By F. Butler, B.Sc., M.I.E.E., M.Brit.I.R.E.

SUMMARY.—A description is given of some circuits for the generation of rectangular pulses of variable duration. They are applicable to speech transmission and a derivative of the basic circuit serves to convert the length or width modulation into pulse position or displacement modulation. The equipment is primarily intended for working over a single-channel v.h.f. radio link, but if desired, it can be arranged to provide several channels on a time-allocation multiplex basis.

The apparatus is sufficiently simple to be built into signal generators or test equipment, and the pulse parameters are readily variable.

Introduction

PULSES of high peak power are commonly produced by amplification of the output of a low-power primary generator. The process is inefficient, since certain valves in the chain must dissipate energy except during the short period in which the pulse is generated. This disadvantage has been pointed out [2,3] and a very satisfactory system has been described for the direct production of high-level pulses.[2] This circuit has been used as a basis for the present work. The following is an outline of its mode of operation :—

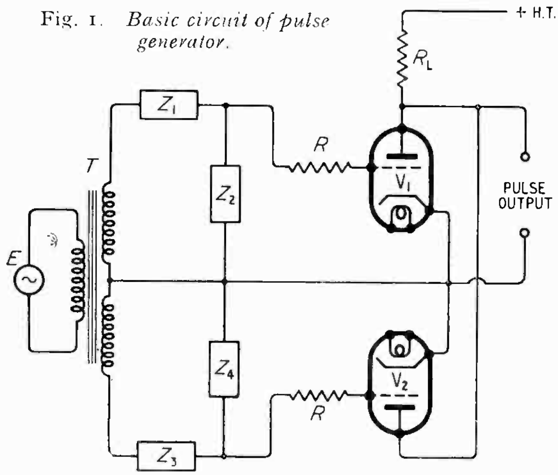
In Fig. 1, an alternator E maintains a sinusoidal voltage across the primary of a transformer T and develops a high voltage across the ends of the centre-tapped secondary. The frequency is that of the desired pulse-recurrence frequency. Ignoring first the impedances Z_1, Z_2, Z_3 and Z_4 and supposing the secondary output to be applied in push-pull, through the high resistances R, to the grids of the valves V_1, V_2, then during the half-cycle of applied voltage which makes the grid of one valve go negative, this valve is driven to anode current cut-off. The other valve conducts heavily, though the presence of the high grid resistance prevents dangerous over-running. This state of affairs is reversed during the next half-cycle of the driving voltage and the valve which was previously cut off now conducts. There is negligible output voltage developed across the common anode load resistance of the two valves except during the brief change-over periods when the instantaneous

MS accepted by the Editor, March 1949

total anode current is less than its steady value.

If the phase of the grid-cathode voltage of one valve is advanced or retarded with respect to that of the other, there will be periods during which both grids are simultaneously negative and the consequent reduction in anode current produces a high-amplitude positive pulse across the load resistor. It is the function of the impedances Z_1, Z_2, Z_3 and Z_4 to introduce this small phase difference. Normally, one pair of the impedances is redundant but all four are included in the discussion for the sake of generality. Variation of the phase angle of one pair advances or retards in time the leading edge of the output pulse. Variation of the other pair of impedances similarly affects the trailing edge.

Fig. 1. *Basic circuit of pulse generator.*



Modulation of the pulse duration will be achieved if it is possible to vary the phase angle of one of the two pairs of impedances in accordance with the signal voltage. This can be accomplished by making use of the properties of a reactance modulator valve.

Reactance Modulators

A reactance modulator is essentially a thermionic valve with associated circuits so arranged that the anode-cathode impedance has a reactive component. The magnitude of the reactance is a function of the mutual conductance of the valve, which can be varied by a change in the instantaneous electrode potentials.

The two commonest reactance modulators are shown in Fig. 2.

Considering Fig. 2 (a) and ignoring the effect of the d.c. blocking capacitor in series with R and of the grid resistor in parallel with C, it is possible to calculate the anode-cathode impedance of the valve at any given frequency. At frequencies for which the reactance of C is small compared with the resistance of R, this impedance is given by :—

$$Z_i = \frac{1}{g_m} + j\omega\frac{CR}{g_m} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\text{if } \mu \gg 1 \text{ and if } (\mu+1)\frac{1}{j\omega C} \gg R.$$

The valve thus simulates an inductance $L = CR/g_m$ in series with a resistance $1/g_m$.

The Q-factor $\left(\text{ratio } \frac{\text{reactance}}{\text{resistance}}\right)$ is ωCR.

Subject to the validity of the approximations made, it is interesting to note that the Q of the reactance modulator is the reciprocal of that of the associated phase-shifting circuit RC. The expression for the input impedance given in Equ. (1) does not take into account the effect of the phase-shifting circuit permanently connected in parallel with the valve. In practice, the approximation is justifiable, because the impedance of the RC circuit is very much greater than the impedance of the valve with which it is in parallel.

An exact analysis of the linear reactance modulator, taking into account the phase shifting circuit, has been given elsewhere by the writer.[1]

In the case of the circuit shown in Fig. 2 (b), the input impedance is given by :—

$$Z_i = \frac{1}{g_m} + \frac{R}{j\omega Lg_m} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

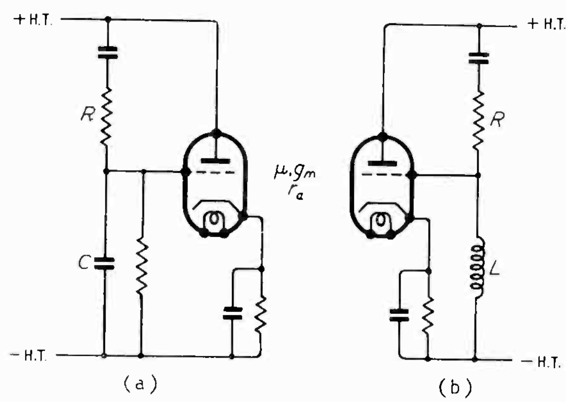


Fig. 2. *Two forms of reactance valves are shown here.*

In this case $Q = R/\omega L$ and the valve simulates a capacitance $C = g_m L/R$ in series with a resistance $1/g_m$. The series combinations of resistance and reactance given by Equs. (1) and (2) can be expressed as equivalent parallel combinations. In practice $Q \nless 10$ and the effect of the series resistance $1/g_m$ is equivalent to a parallel resistance Q^2/g_m, the error being about 1 per cent for $Q = 10$. To the same order of accuracy, the reactance values are unchanged

The expression for the valve reactance includes the term g_m. In the case of a triode valve, the mutual conductance is a function of the control-grid voltage. For a pentode or a hexode, g_m is also dependent on the suppressor- or mixer-grid potentials.

Referring now to Fig. 1 and assuming Z_2 and Z_4 to be replaced by reactance modulators, the possibility of pulse-length modulation is apparent. The mean value of the modulator impedance Z_i decides the average pulse duration. Variations of this impedance under the influence of the modulation voltage cause corresponding changes in pulse duration. By proper choice of operating conditions, a linear relationship can be secured between the amplitude of the modulating signal and the mutual conductance of the reactor valve. It remains to see if this condition satisfies the requirement for proportionality between pulse duration and the amplitude of the signal voltage.

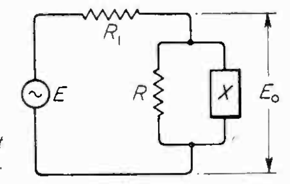


Fig. 3. *Equivalent circuit of phase-shifting network.*

Modulation Linearity

Replacing Z_1 in Fig. 1 by a pure resistance R_1 and Z_2 by the equivalent parallel circuit of a reactance modulator, the simple network shown in Fig. 3 is obtained. In this circuit,

$$\frac{E}{E_0} = \frac{R + R_1}{R} - j\,\frac{R_1}{X}. \qquad \ldots \qquad (3)$$

Let ϕ = phase difference between E and E_0. Then :—

$$\tan\phi = -\frac{RR_1}{R+R_1}\cdot\frac{1}{X} = -\frac{R_0}{X} \text{ where } R_0 = \frac{RR_1}{R+R_1}.$$

For a linear reactance modulator, X is inversely proportional to the mutual conductance of the valve, which, with a proper choice of operating voltages, is proportional to the instantaneous modulating signal. Short pulses are always used, so that $X \gg R_0$ and without appreciable error, $\tan\phi = \phi$. In this system of modulation, amplitude distortion is therefore small.

Variations, during modulation, of the parallel resistance of the reactor valve have been neglected. The mean value of this resistance R is, in practice, much higher than the series resistance R_1. The parallel combination of R and R_1 is, therefore, scarcely affected.

Practical Modulator Circuit

Fig. 4 is the theoretical circuit of a complete modulator with its associated pulse amplifiers, shaping circuits, audio-modulating equipment and filters.

The primary oscillator employs a pair of beam tetrode valves V_1, V_2 (Type 807) in a push-pull circuit arranged to generate a sinusoidal output of frequency 12,500 c/s. This is applied to a push-pull output transformer T with separate secondary windings. Each half-secondary supplies one grid of a 6SN7 twin-triode valve V_4, V_5 having both anodes paralleled and connected to a common load resistance R_{20}. A fixed phase-shift is applied to one of the half-secondary outputs. In the circuit of the other is included a 6SA7 valve V_3 connected as a reactance modulator, capable

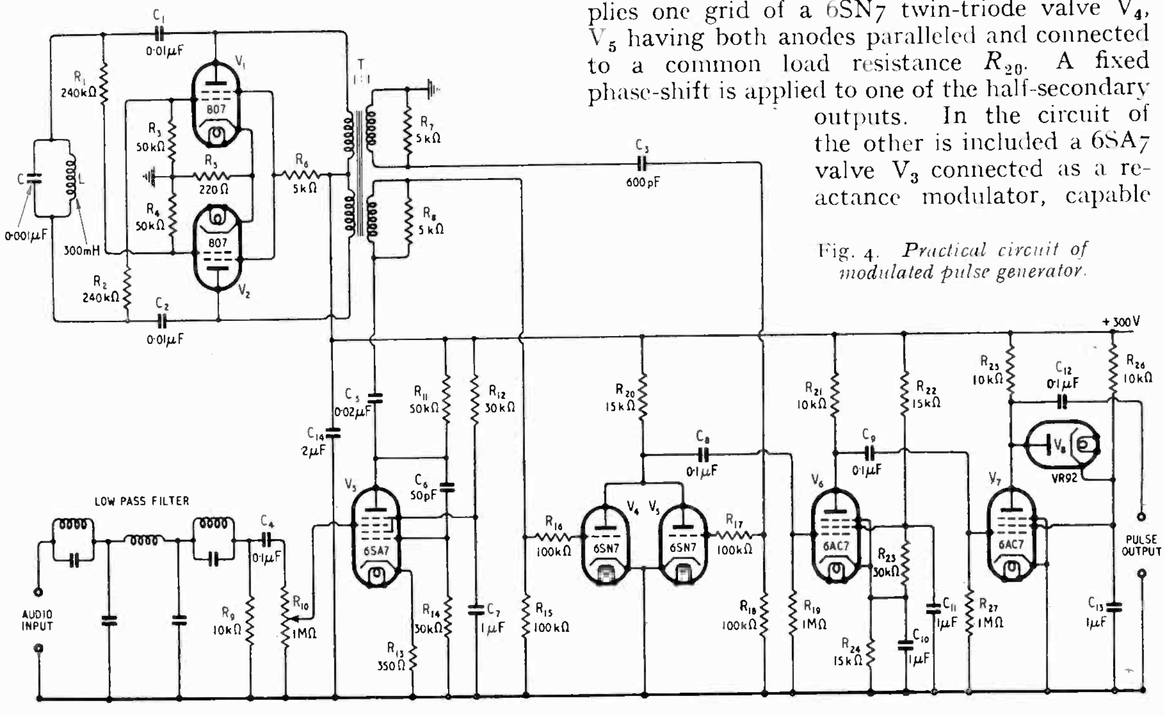


Fig. 4. *Practical circuit of modulated pulse generator.*

of simulating a variable reactance and so of causing a variable phase-displacement of the voltage developed between grid and cathode of the other half of the 6SN7 valve, referred to above. Audio-modulating voltages, applied to the grid of V_3, through a low-pass filter cutting off at 3,500 c/s, cause variations of reactor valve impedance and consequent changes in phase of the voltage derived from the primary-oscillator transformer. These phase changes cause corresponding variations in duration of the voltage pulse developed across the 6SN7 valve anode load. The remaining valves in the circuit are amplifiers and pulse-shaping stages. The output from the equipment is in the form of positive pulses, amplitude 100 volts and mean duration 1.5 microseconds.

One edge only of the output pulse is modulated, and there is thus a spurious form of pulse position modulation caused by this asymmetric operation.

Symmetrical modulation of both leading and trailing pulse edges can be achieved by the connection of a second reactor valve in series with the upper half-secondary of the primary oscillator output transformer. To achieve this, both reactor valves must be modulated by the audio signal, the polarity of which must be arranged to advance one edge of the output pulse as it retards the other.

Parallel-Tuned Phase-Shift Circuit

Examination of the impedance and phase characteristics of a parallel-tuned LC circuit shows that, near resonance, the phase angle of the impedance varies very rapidly with changes of inductance, capacitance or frequency. Such a circuit may be used as the impedance Z_2 or Z_4 in Fig. 1. The slight mistuning required to cause a phase-displacement, during modulation, can be provided by a parallel-connected reactance valve controlled by the signal voltage. Near resonance, the phase characteristic is steep and linear and a given variation of output pulse duration is achieved with a much lower audio-modulating signal than in the case of a reactance valve used alone, as in Fig. 4.

Time-Allocation Multiplex Working

The equipment shown in Fig. 4 is for single-channel working. Arrangements for two-channel operation are shown in Fig. 5.

In Fig. 5 (a) the requisite time delay between the pulse trains corresponding to Channel 1 and Channel 2 is obtained by phase-shifting bridges connected between the common primary oscillator and the two ouput transformers associated with the separate channels. A different arrangement is shown in Fig. 5 (b), where the phase-shifting circuits for interlacing the two pulse-trains are connected on the secondary side of the output transformer, coupled to a single primary oscillator.

Several other multiplex arrangements are possible. If two separate reactance valves are used, one to modulate the leading edge of the output pulse and one to modulate the trailing edge, then by feeding separate audio signals to the two reactor valves, a single pulse train may be used to carry two separate programmes.

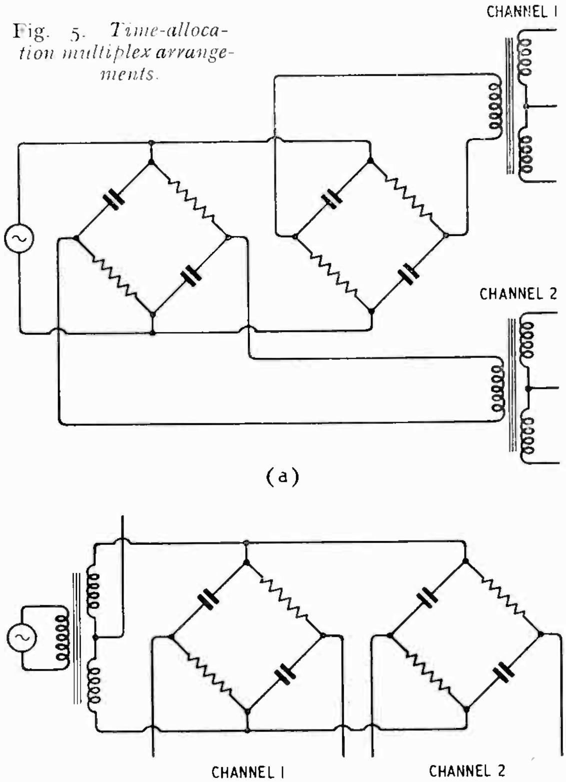


Fig. 5. *Time-allocation multiplex arrangements.*

If distortion is to be avoided in pulse-transmission systems, the highest modulation frequency is theoretically limited to one-half of the pulse-recurrence frequency. It is possible, by the employment of a band-filtering and inversion scheme, to transmit a total bandwidth equal to the pulse recurrence frequency, provided that both leading and trailing edges of the pulse train are modulated. Suppose that it is desired to transmit the frequency band 0–7 kc/s using a pulse-recurrence frequency of 7 kc/s. The sequence of operations necessary to achieve this is as follows :—

(a) Filter out the band 0–3.5 kc/s and use this to modulate the trailing edge of the pulse train as already described.

(b) Heterodyne the residue 3.5–7 kc/s against the 7-kc/s primary oscillator and filter out the sideband 0–3.5 kc/s, which will contain all the intelligence in the range 3.5–7 kc/s, although the component frequencies will have suffered inversion in the process.

(c) Use this new band of frequencies 0–3.5 kc/s to modulate the leading edge of the primary pulse train.

(d) At the receiving end, reverse these operations, separating the two channels using the technique to be described in the succeeding section. The original signal is thus recovered.

The operation is much simplified since there is available, at the receiving terminal, a train of pulses from which may be derived the precise heterodyne frequency required to reconstitute the original signal.

By an extension of the practice of band splitting, filtering and inversion, it is not difficult to evolve an elementary privacy device which, if not of high security, calls for a considerable addition to the plant at the intercepting station.

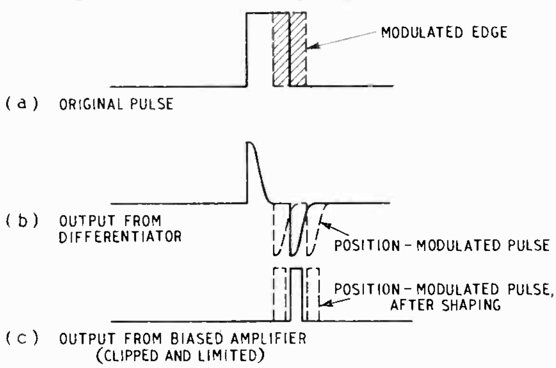


Fig. 6. *Conversion of pulse duration-modulation to position-modulation.*

It is important to realize that in all equipment used to generate pulses from sine waves, the exact definition of the start and finish of a pulse is subject to some uncertainty. If it is desired to engineer a multiplex system capable of handling some hundreds of separate channels, then the method indicated in Fig. 5 is insufficiently precise. The arrangement provides a means of generating a small number of separate pulse trains, with an adjustable time separation between each train. This facility is often useful, and the result is achieved very simply.

Displacement - Modulated Pulses

If the output of the equipment shown in Fig. 4 is passed through a short *CR* differentiator circuit, the result shown in Fig. 6 is observed.

This operation is common in radar practice, and results in the production of two exceedingly short pulses of opposite polarities from each primary pulse. The separation of the two trains is effected by biased amplifier valves.

Recurrence-Frequency Modulation

If a reactance-modulator valve is joined in parallel with one of the valves in the primary oscillator, then an audio signal applied to the reactor valve will vary the oscillator frequency and so change the recurrence frequency of the pulses. This system is not of much use in communications but has other applications.

Practical Results

The modulator circuit of Fig. 4 has been built and tested on speech transmissions. Using an audio-filter circuit cutting off at 3,500 c/s there is naturally some degradation of speech quality, but there is no noticeable difference between the signal direct from the first filter output and that after transmission, demodulation and subsequent filtering. No measurements have yet been made of the distortion of tone signals. A rough comparison of input and output waveforms at 1,000 c/s by viewing on a cathode-ray oscilloscope shows that unless the pulses are deeply modulated, such distortion is not serious, though with the circuit shown, it becomes worse at lower frequencies for some reason which has not so far been determined. Background noise is low and the 12,500-c/s tone is easily filtered out at the receiver.

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ABSTRACTS AND REFERENCES INDEX

The Index to Abstracts and References published during 1949 is in course of preparation and will, it is hoped, be available in February, priced 2s. 8d. (including postage). It includes a list of journals abstracted, together with the addresses of their editorial or publishing offices. As supplies will be limited our Publishers ask us to stress the need for early application for copies.

REFLECTION COEFFICIENT OF SNOW AND ICE AT V.H.F.

By J. A. Saxton, Ph.D., B.Sc., A.M.I.E.E.

(Communication from the National Physical Laboratory)

SUMMARY.—A review is given of the nature and composition of snow, and of the experimental knowledge of the dielectric properties of ice at very high radio frequencies. From this information an estimate has been made of the dielectric properties of snow. The presence of a layer of ice or snow on the earth's surface will produce a modification in the resultant reflection coefficient of the surface as a consequence of multiple reflections within the layer. A general formula is given in the paper by means of which the resultant reflection coefficient for plane waves in the presence of such a layer may be calculated. The formula is in terms of several basic parameters which have been determined for frequencies of 30, 300, 3,000 and 30,000 Mc/s. To illustrate the effect of these layers, the resultant reflection coefficients of a stratum of ice on sea water and of one of snow on land have been calculated for frequencies of 300 and 3,000 Mc/s, at angles of incidence of 0° (normal incidence), 45° and 80°, and for radiation polarized with the electric vector either parallel to or in the plane of incidence. It thus appears, for example, that the vertical-coverage diagram of a very high frequency radio transmitter may be appreciably modified by the presence of layers of ice or snow in depths likely to occur in practice.

1. Introduction

IN some parts of the world it is not unusual at times to have layers of snow or ice an appreciable fraction of a wavelength—or even several wavelengths—thick when the radio frequency concerned is very high, say of the order of 100 Mc/s or greater. The purpose of the present work is to calculate the effect of such layers on the reflection coefficient of the earth's surface for very high frequency radio waves. The general manner in which the problem should be tackled is obvious, it being simply a question of evaluating the effect of multiple reflections within the layer. As far as the author is aware, few such calculations have previously been made in detail, although mention should be made of the work by Pfister and Roth[1] who have considered several types of stratification including water on asphalt, ice on water and very dry sand overlying wet clay soil. The results obtained by Pfister and Roth are not readily applicable to the present problem: no investigation was made of the effects of layers of snow; furthermore a value of 5 was used for the relative permittivity of ice, a figure which is now known to be too high. In view of the possible use of very high frequencies for various purposes under conditions where ice and snow layers are likely to exist, it seems therefore that a basic knowledge of the phenomena likely to be encountered would now be desirable.

It is, of course, not possible to give in this paper a treatment of all circumstances likely to arise, since the number of independent parameters is so large, but the fundamental components of the complete solution are given together with certain selected examples to show the importance of the effects produced by ice and snow layers overlying sea water and land respectively. Any other specific case may quickly be worked out from the formulae given below for the resultant reflection coefficient and the basic data provided in Figs. 2 to 9.

It is throughout assumed that we are dealing with plane surfaces. In practice, of course, the types of surface encountered may often be rough, and when irregularities are comparable in dimensions with the wavelength we may expect the effects of multiple reflections to be somewhat blurred: even in such cases the fundamental information provided by the present calculations should be useful in the examination of snow and ice layer effects.

MS accepted by the Editor, March 1949

2. Composition and Nature of Snow

Snow consists of ice crystals; it is, however, of a loose texture when lying on the ground, since there are considerable air spaces. Thus, to determine the effect of a layer of snow on the reflecting properties of land, besides having a knowledge of the dielectric properties of ice we require to know also the amount of ice in a given depth of snow.

Seligman[2], in a description of Alpine landscapes, says that: "When snow falls it lies on the ground in the loosest and most impalpable condition, and its specific gravity is 0.06, but under very calm conditions and at low temperatures the specific gravity may be only 0.01. The snow immediately begins to settle: the sharp points of the rays (i.e., of the crystal structure) evaporate and the flakes break up, so that the powder of broken flakes sinks together and becomes firmer. Its specific gravity will now have risen to 0.3. Then, under the influence of alternate heating and cooling a crust begins to form on the surface, and then grows in depth. This hard crust is called 'firn snow,' it is not true ice, which is far rarer than commonly believed: in firn snow there is always some air remaining between the ice crystals." It is thus evident that the composition of a layer of snow depends much on its past history, but it would appear that generally the specific gravity of snow lying on the ground is not likely to be much greater than 0.3. This view is supported by Geddes[3] who states that on an average newly fallen snow gives a water equivalent of about one-tenth its own depth, but that snow which has lain for some time may give a yield as high as three-tenths its thickness.

3. Dielectric Properties of Ice and Snow at V.H.F.

The electrical properties of any given medium may in general be described by the relations:—

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon' - 2j\sigma/f = (n - jk)^2 \quad .. \quad (1)$$

where
ϵ = complex relative permittivity,
σ = conductivity in e.s.u.,
f = frequency in c/s,
n = refractive index,
k = absorption coefficient.

As is well known, the water molecule is electrically polar, so that ice, like liquid water, shows the property of anomalous dispersion at radio frequencies. Whereas this dispersion occurs in liquid water in the frequency range[4] 10^3 to 10^6 Mc/s, in ice the region of maximum dispersion is near to 6 kc/s (wavelength 50 km) at a temperature of $-2°$ C, and occurs at even lower frequencies as the temperature decreases. The most recent observations, by Lamb[5], of the relative permittivity (ϵ') of ice at a frequency of 10,000 Mc/s (wavelength 3 cm) give a value of 3.05, and he found no variation from this value over the temperature range 0° C to $-40°$ C. Lamb also measured the dielectric properties of ice in the frequency range 8 kc/s to 1.25 Mc/s, and his results, together with those of Smyth and Hitchcock[6], indicate that no further absorption band occurs as the frequency is increased up to 30,000 Mc/s other than that corresponding to the region of dispersion mentioned above. Consequently at frequencies of 30 Mc/s and higher, through to the centimetre waveband, the relative permittivity (ϵ') of ice may be assumed constant and equal to 3.05.

The measured[5] value of the absorption coefficient (k) of ice at 10,000 Mc/s and at 0° C is about 10^{-3}, and at $-50°$ C it is 10^{-4}. This means that waves of this frequency will suffer an attenuation (due to absorption) of 2 db per metre of path when transmitted through ice at 0° C. According to Granier[7] the absorption coefficient of ice at $-12°$ C and at a frequency of 10 Mc/s is 10^{-2}, which means an attenuation of 0.02 db per metre. There is no reason to suppose that the variation of k between the frequencies of 10 Mc/s and 10,000 Mc/s is not monotonic, so that the value at intermediate frequencies may be estimated if required.

As far as the author is aware, there are no direct measurements of the dielectric properties of snow, so that an attempt must be made to assess them from the known properties of ice and the density of the snow. If we assume that snow is a more or less homogeneous mixture of ice and air, it may be shown (see Appendix I) that the effective relative permittivity (ϵ') of snow of specific gravity 0.3 is about 1.4, while for a specific gravity of 0.1 the relative permittivity may be taken as about 1.15. It may further be assumed, to sufficient accuracy, that the attenuation in snow arising from absorption in the ice is simply proportional to the amount of ice per unit volume. In this way it is found that for a specific gravity of 0.3 the attenuation is probably not greater than 0.7 db per metre and 0.007 db per metre at frequencies of 10,000 Mc/s and 10 Mc/s respectively.

In view of the relatively low absorption loss in ice and snow, and having regard to the thicknesses of ice and snow layers likely to occur in practice (which, save in exceptional circumstances, will not exceed about 1 metre) this absorption will be neglected in the following calculations of reflection coefficients. The assumption that ice and snow both behave as perfect dielectrics, with $\epsilon' = 3.05$ and $\epsilon'' = 0$ for ice, will not lead to any serious error in the values so obtained for the reflection coefficients, and it has the great advantage that from the dielectric point of view we may regard the behaviour of ice and snow as being independent of the frequency of the radiation.

An important point to be borne in mind is that if the ice or snow begins to melt, then the presence of even a small amount of liquid water will be of great importance from the point of view of absorption[4], and also to a less extent in relation to the reflectivity of a given layer. The increased absorption will be particularly significant at centimetre and decimetre wavelengths. It is difficult to see how this matter can be handled satisfactorily in a precise quantitative manner, and it must therefore be remembered that the present calculations can be expected to indicate only the order of magnitude of effects likely to occur with layers of dry ice and snow.

4. Dielectric Properties of Land and Sea

Before we can proceed to the calculation of layer effects we have still to consider what values should be assumed for the dielectric properties of soil and sea water at very high frequencies. Whereas we have assumed that for present purposes we may regard both ice and snow as pure dielectrics, a similar assumption is not possible for land and sea. The relative permittivity of soil varies over quite a large range[8, 9], depending largely upon the amount of water present, and at very high frequencies usually lies somewhere between 5 and 30. The conductivity of soil also shows appreciable variation with water content, and may have values between rather less than 10^8 e.s.u. and about 10^9 e.s.u. Both components of the complex permittivity of soil, and particularly the conductivity, may be expected to decrease as the temperature of the soil falls below 0° C, but it is difficult to make a precise estimation of the magnitude of the effect in any given case. In most circumstances, however, and at any rate for frequencies not less than about 200 Mc/s, the relative permittivity of soil is several times as large as the conductivity term ($2\sigma/f$); therefore, since the relative permittivity is appreciably greater than unity whether the temperature be above or below 0° C, the reflection coefficient at the interface between air (or snow) and soil will only change slowly with changes in the relative permittivity of the soil. If an attempt were made to take account of the fact that the soil is frozen to a certain depth, it would be necessary to consider the combined effects of multiple reflections in two layers, snow and frozen soil, interposed between air and unfrozen soil. In practice the boundary between frozen and unfrozen soil may not be too well defined, and it is not likely to represent such a large discontinuity in permittivity as the snow-soil boundary: also, absorption in the soil will tend further to reduce the importance of the possible discontinuity in the soil itself. It is therefore proposed to ignore this factor: to include it would greatly increase the complexity of the calculations and, in view of the considerations given above, would be likely to lead to only a small change in the final results. In the next section formulae are given [equations (3) to (7)] for the reflection coefficient at a single plane interface separating two media, and what is required in these equations is the (complex) permittivity of the second medium relative to that of the first. Thus, suppose we have soil of conductivity (σ) 5×10^8 e.s.u. at a frequency of 300 Mc/s, and of permittivity (ϵ') 11 relative to vacuum (or to air with sufficient accuracy) the complex permittivity of such soil relative to air is $11 - 3.5j$. Relative to snow, where $\epsilon' = 1.4$ (relative to air) the complex permittivity of the soil becomes $(11 - 3.5j)/1.4$, or $8 - 2.5j$.

The range of frequency to be considered here is from 30 Mc/s to 30,000 Mc/s, and in arriving at reasonable probable values of the complex permittivity of soil to cover the whole of this range it is necessary to bear in mind the dispersion which occurs in water at very high radio frequencies[1]. We therefore expect that at the highest frequencies under consideration the permittivity (ϵ') of soil will decrease somewhat, while the conductivity will increase relative to the values obtaining at the lower frequencies. We therefore arrive at the complex permittivities of land relative to snow (specific gravity 0.3) given in Table I as a function of frequency. To obtain the corresponding values relative to air it is simply necessary to multiply throughout by 1.4.

TABLE I

Frequency (Mc/s)	Complex Permittivity	
	Land relative to Snow	Sea Water Relative to Ice
30	$8 - 25j$	$29 - 660j$
300	$8 - 2.5j$	$29 - 66j$
3 000	$8 - 0.8j$	$28.5 - 15j$
30 000	$4 - 0.4j$	$10 - 14j$

Table I also includes the values which have been used of the complex permittivity of sea water (at 0° C) relative to ice, and to obtain the corresponding values relative to air it is necessary to multiply throughout by 3.05. For example, the complex permittivity of sea water relative to air at a frequency of 300 Mc/s is $3.05\ (29 - 66j)$, or $88 - 200j$. The dipolar dispersion in water at very high radio frequencies is here of major importance in determining the total effective conductivity[4]. It is known that the presence of appreciable quantities of salt in water tend to reduce the permittivity (ϵ') somewhat, but for the concentration of salt occurring in sea water the effect is relatively not very great, and for the purpose of the present calculations this effect has been neglected: the inclusion of this factor would not materially change the magnitude of the calculated reflection coefficients.

5. Calculation of Reflection Coefficients

The reflection coefficients determined here are those relating to the incidence of plane waves at plane surfaces, and it is supposed that the ice or snow layers overlying sea or land are uniform. The formulae for the reflection coefficient at a single interface between two media under such conditions were originally given by Fresnel, and they have been arranged by McPetrie[10] in a form suitable for computation in applications such as the present.

The reflection coefficient depends not only on the angle of incidence but also on the wave polarization, and there are two basic cases to be considered, one in which the electric vector of the wave is in the plane of incidence, and one in which this vector is perpendicular to the plane of incidence. Any other state of polarization may be dealt with by resolution into these two basic components.

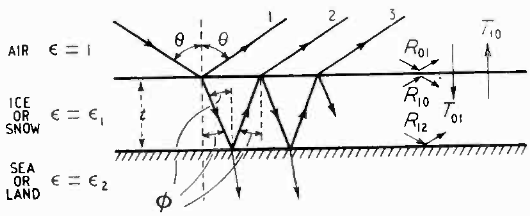


Fig. 1. *Illustration of the production of multiple reflections in ice or snow layers.*

In general a wave train suffers on reflection a change both in amplitude and phase, so that the reflection coefficient, R, is of the form :—

$$R = |R|\, e^{j\psi} \qquad (2)$$

where ψ is the advance in phase after reflection. Or, alternatively, we may write :—

$$R = K_1 + jK_2 \qquad (3)$$

so that $|R| = (K_1^2 + K_2^2)^{\frac{1}{2}}$ (4)

and $\psi = \tan^{-1}(K_2/K_1)$ (5)

For the case of a single plane discontinuity between two media McPetrie[10] has derived from the original Fresnel equations the following relations for K_1, K_2, the angle of incidence being θ :—

$$\left.\begin{aligned}
K_{1H} &= \frac{\cos^2\theta - (c^2 + d^2)}{\cos^2\theta + (c^2 + d^2) + 2c\cos\theta} \\
K_{2H} &= \frac{-2d\cos\theta}{\cos^2\theta + (c^2 + d^2) + 2c\cos\theta} \\
K_{1V} &= \frac{(\epsilon'^2 + \epsilon''^2)\cos^2\theta - (c^2 + d^2)}{(\epsilon'^2 + \epsilon''^2)\cos^2\theta + (c^2 + d^2) + 2(\epsilon' c - \epsilon'' d)\cos\theta} \\
K_{2V} &= \frac{-2(\epsilon' d + \epsilon'' c)\cos\theta}{(\epsilon'^2 + \epsilon''^2)\cos^2\theta + (c^2 + d^2) + 2(\epsilon' c - \epsilon'' d)\cos\theta}
\end{aligned}\right\} \qquad (6)$$

$$\left.\begin{aligned}
\text{where } c &= \frac{1}{\sqrt{2}}\{[(\epsilon' - \sin^2\theta)^2 + \epsilon''^2]^{\frac{1}{2}} + (\epsilon' - \sin^2\theta)\}^{\frac{1}{2}} \\
d &= -\frac{1}{\sqrt{2}}\{[(\epsilon' - \sin^2\theta)^2 + \epsilon''^2]^{\frac{1}{2}} - (\epsilon' - \sin^2\theta)\}^{\frac{1}{2}}
\end{aligned}\right\} \qquad (7)$$

Here the suffixes H and V refer to polarization perpendicular to and in the plane of incidence respectively. The permittivity concerned is that of the second medium relative to that of the first.

Now consider the case where we have a relatively thin layer of medium 1 interposed between the air and medium 2. Medium 1 will be either ice or snow and medium 2 either sea water or land. It will be supposed that medium 2 extends in a uniform manner indefinitely in a direction perpendicular to the interface between media 1 and 2 : thus no energy entering medium 2 returns to the air to contribute to the resultant reflected wave. The situation is as shown in Fig. 1. Since we propose to neglect the effects of absorption in medium 1, the resultant reflection coefficient, R, for an angle of incidence θ may be written (see Appendix II) as :—

$$R = |R|\, e^{-j\Psi} = P + jQ \qquad (8)$$

(N.B. It is more convenient to work in terms of a retardation rather than an advance in phase.)

where :

$$P = |R_{01}|\cos\psi_{01} + \left\{\frac{A\cos y - AB\cos(z - y)}{1 + B^2 - 2B\cos z}\right\} \qquad (9)$$

$$Q = -|R_{01}|\sin\psi_{01} - \left\{\frac{A\sin y + AB\sin(z - y)}{1 + B^2 - 2B\cos z}\right\} \qquad (10)$$

$$-\psi = \tan^{-1} Q/P \qquad (11)$$

$$|R| = \sqrt{P^2 + Q^2} \qquad (12)$$

(ψ must be located in the correct quadrant by inspection of the signs of P and Q separately)

and where :

$A = |T_{01}\, T_{10}|\; |R_{12}|$; $\quad B = |R_{12}|\; |R_{10}|$;

$y = \delta + \psi_{12}$; $\quad z = y + \psi_{10}$;

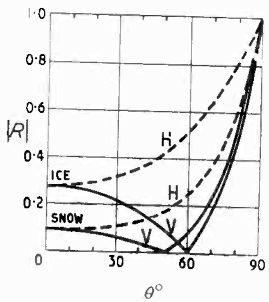
$\delta = (4\pi t\sqrt{\epsilon_1}\cos\phi)/\lambda$; $\sqrt{\epsilon_1} = \sin\theta/\sin\phi$;

$R_{01} = |R_{01}|\, e^{-j\psi_{01}}$; $\quad R_{10} = |R_{10}|\, e^{-j\psi_{10}}$;

$R_{12} = |R_{12}|\, e^{-j\psi_{12}}$;

The suffixes 0, 1, 2 refer to air and media 1 and 2 respectively. Thus R_{01} is the reflection coefficient, T_{01} the transmission coefficient and ψ_{01} the phase retardation on reflection in the case of a wave incident at the interface between air and medium 1, and similarly for the other symbols. δ represents the phase retardation of a wave in a single excursion in a thickness t of medium 1 from the air surface to medium 2 and back to the air surface again.

Thus the basic parameters required, as a function of angle of incidence and of frequency, are R_{01}, R_{10}, R_{12} and T_{01}, T_{10}, though, for the assumptions we have made concerning the dielectric behaviour of ice and snow in the frequency range concerned, only one of these quantities, R_{12}, is a function of frequency. When the above basic parameters are known it is then possible to determine R for any angle of incidence and any thickness of medium 1—snow or ice, by substitution in equations (9) to (12).



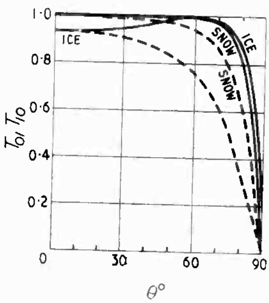


Fig. 2 (left). *Reflection coefficients of air-ice and air-snow boundaries (R_{01}) as a function of angle of incidence (θ).*

Fig. 3 (right). *Product of transmission coefficients ($T_{01} \times T_{10}$) for one transit each way across air-ice and air-snow boundaries as a function of angle of incidence on the air side (θ).*

NOTE. *Throughout Figs. 2-13 broken lines refer to polarization perpendicular to the plane of incidence (H) and full lines to polarization in the plane of incidence (V).*

6. Results and Discussion

Fig. 2 shows the variation of $|R_{01}|$ with the angle of incidence (θ) for air-ice and air-snow (specific gravity = 0.3) boundaries, and for the two basic types of wave polarization. Throughout the series of Figs. 2 to 13 the dotted lines refer to waves polarized perpendicular to the plane of incidence (horizontally polarized) and the full lines to the case of polarization in the plane of incidence (vertically polarized). Since we are assuming that ice and snow behave as perfect dielectrics (with $\epsilon' > 1$) it follows that at all angles of incidence $\psi_{01} = \pi$ for horizontally-polarized waves, and further that for vertically-polarized waves and angles of incidence less than the Brewster Angle $\psi_{01} = 0$, while when θ exceeds the Brewster Angle $\psi_{01} = \pi$. The Brewster Angle—at which $R_v = 0$—occurs at angles of incidence given by $\tan^{-1}\sqrt{\epsilon_1}$; i.e., about 50° and 60° for air-snow and air-ice boundaries respectively.

We may also determine $|R_{10}|$ from Fig. 2. To do this we remember the well-known property that :—

$$(R_{01} \text{ at angle of incidence } \theta) = -(R_{10} \text{ at angle of incidence } \phi) \quad .. \quad (13)$$

where $\sin\theta/\sin\phi = \sqrt{\epsilon_1}$

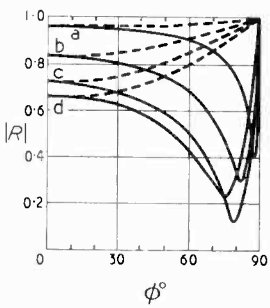
ϕ is the angle of refraction and, as is seen from Fig. 1, it is also the angle of incidence at the snow (or ice)–air boundary and the snow (or ice)–land (or sea) boundary. Thus in any specific example $|R_{01}| = |R_{10}|$. It must be noted, however, that for horizontally-polarized waves ψ_{10} is zero for all angles of incidence, and that for vertically-polarized waves when ϕ is less than the Brewster Angle $\psi_{10} = \pi$, but that when ϕ exceeds this angle $\psi_{10} = 0$; i.e., in any given case $\psi_{10} = (\psi_{01} - \pi)$. When considering reflection inside medium 1 at the boundary of medium 0 (air) it must be noted that the Brewster Angle is now given by $\tan^{-1} 1/\sqrt{\epsilon_1}$; i.e., it is about 40° and 30° for snow-air and ice-air boundaries respectively.

In order to determine $T_{01}\,T_{10}$ we make use of the further property that :—

$$T_{01}\,T_{10} = 1 - |R_{01}|^2 \quad .. \quad .. \quad (14)$$

The phase change to be associated with $T_{01}\,T_{10}$ is always zero. Fig. 3 shows $T_{01}\,T_{10}$ as a function of θ for air-snow, air-ice boundaries, and for both types of polarization.

As stated earlier, the data in Figs. 2 and 3 may be taken to cover all frequencies in the range under consideration, namely 30 to 30 000 Mc/s.



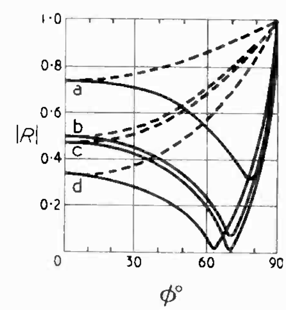
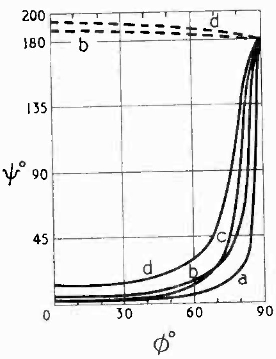


Fig. 4 (left). *Reflection coefficient of ice-sea boundary (R_{12}) as a function of angle of incidence (ϕ) and at frequencies of 30, 300, 3,000 and 30,000 Mc/s in curves a, b, c and d respectively.*

Fig. 5 (right). *Reflection coefficient of snow-land boundary (R_{12}) as a function of angle of incidence (ϕ) and at the frequencies given in Fig. 4.*

For the computation of R_{12} four frequencies have been chosen, viz: 30, 300, 3 000 and 30 000 Mc/s. Figs. 4 and 5 show $|R_{12}|$ as a function of the angle of incidence—in this case ϕ—for ice-sea and snow-land boundaries respectively, while Figs. 6 and 7 give the corresponding values of ψ_{12}. It will be observed that since ϵ'' is no longer equal to zero the Brewster Angle for vertically-polarized waves is not as well defined as when $\epsilon'' = 0$. $|R_{12}|$ never reaches zero, and

ψ_{12} changes continuously from 0 to π—although very rapidly in the neighbourhood of the pseudo Brewster Angle—and not discontinuously as in the pure dielectric case.



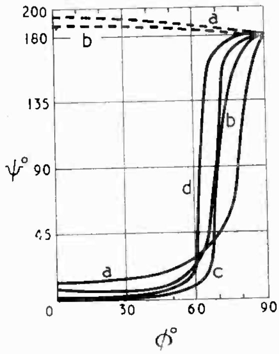


Fig. 6 (left). *Phase retardation (Ψ_{12}) on reflection at ice-sea boundary. Frequencies as in Fig. 4; for horizontally-polarized waves (Ψ_{12}) does not exceed 185° and 183° for frequencies of 3,000 and 30 Mc/s respectively.*

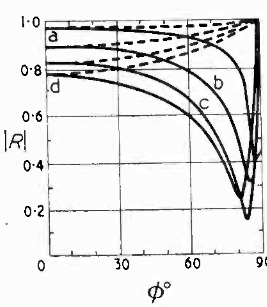
Fig. 7 (right). *Phase retardation (Ψ_{12}) on reflection at snow-land boundary. Frequencies as in Fig. 4: for horizontally-polarized waves (Ψ_{12}) does not exceed 183° and 184° for frequencies of 3,000 and 30,000 Mc/s respectively.*

Figs. 8 and 9 illustrate the reflection coefficient of air-sea and air-land boundaries as a function of frequency, wave polarization and angle of incidence, and enable a comparison to be made to indicate the modifications produced by ice and snow layers in any given instance.

The basic parameters provided in Figs. 2 to 7 have been used to calculate the effects of layers of snow and ice, up to one wavelength thick, overlying land and sea respectively, and for the two frequencies of 300 and 3 000 Mc/s. The calculations have been made for both types of polarization, but for three angles of incidence only: $\theta = 0$ (normal incidence, where it is unnecessary to distinguish between the two kinds of polarization), $\theta = 45°$ and $\theta = 80°$. To cover all possible cases would require a prohibitive amount of computation and number of graphs, and the above selected examples serve well to show the order of magnitude of the effects to be expected. Since it is assumed that no absorption occurs in either ice or snow the resultant reflection coefficients show a cyclic variation as a function of layer thickness, and the values for $t/\lambda > 1$ may readily be deduced from those at appropriate points within the range $0 < t/\lambda < 1$. This process must not be extended too far, however, since although absorption in the layer may be small the cyclic variation is in fact damped, and the damping may become appreciable for layer thicknesses much exceeding a wavelength, particularly in the case of ice.

Figs. 10(a), (b) and (c) show the modulus of the resultant reflection coefficient, and the phase retardation (ψ) on reflection for a layer of ice on sea water for $\theta = 0°$, 45° and 80° respectively, and for a frequency of 3 000 Mc/s. Figs. 11(a), (b) and (c) cover similar conditions but for a frequency of 300 Mc/s. The case of air-snow-land reflection phenomena at the same angles of incidence is illustrated by Figs. 12(a), (b) and (c) for a frequency of 3 000 Mc/s, and by Figs. 13(a), (b) and (c) for a frequency of 300 Mc/s. In all the Figs. 10 to 13 short horizontal lines extending from the ordinate axis indicate what would have been the reflection coefficient in the absence of the ice or snow layer. It should be noted that the phase retardations are relative to that of the wave incident at the first boundary; i.e., the air-snow or air-ice interface.

Figs. 10 to 13 are self-explanatory, but attention might perhaps be drawn to one or two points. In the first place it will be observed that although quite large variations occur in the modulus of the resultant reflection coefficient for horizontally-polarized waves, they are such as in general to reduce $|R|$ below the value which would have obtained in the absence of the layer: only in a few instances does $|R|$ exceed the latter value, and then by a negligibly small amount.



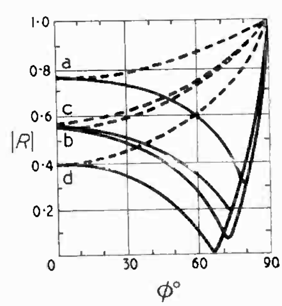


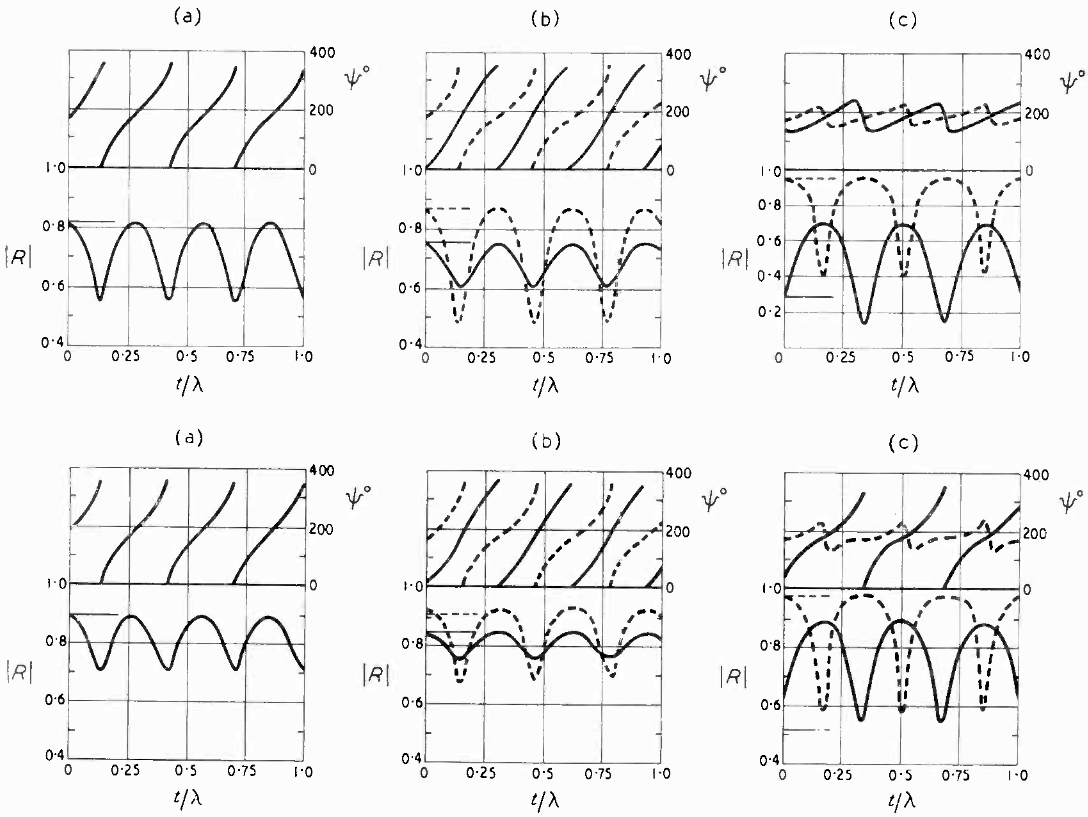
Fig. 8 (left). *Reflection coefficient of air-sea boundary. Frequencies as in Fig. 4.*

Fig. 9 (right). *Reflection coefficient of air-land boundary. Frequencies as in Fig. 4.*

In some of the examples chosen vertically-polarized waves show the same kind of variation as horizontally-polarized waves, but on the other hand we see also that with vertical polarization considerable variations in $|R|$ may occur both above and below the value obtaining in the absence of the layer. This different behaviour on the part of vertically-polarized waves is, of course, a result of the Brewster phenomenon. Thus, the angle of incidence might be such that, in the absence of an ice or snow layer, it would be near to the pseudo Brewster Angle for sea or land, and so vertically-polarized waves would have only a quite small reflection coefficient. The interposition of a layer of ice or snow, however, materially changes the angle at which the waves are incident to the sea or land surface, and can thus give a resultant reflection coefficient considerably greater than that without the layer if the layer thickness is such as to produce a system of multiply reflected waves which are in phase when finally emerging into the air again.

It would appear that in the case of normal incidence the reflection coefficient is likely at most to be reduced by some 20% to 35% relative to the undisturbed value; but at higher angles of incidence, such as 45° and 80°, much bigger changes are possible. As the angle of incidence (θ) approaches 90°, however, the effects of multiple reflection must become increasingly less significant, for at grazing incidence the reflection coefficient of both air-ice and air-snow boundaries, and for both types of polarization, is unity with a phase retardation of π: no energy then enters the layer, and multiple-reflection phenomena are not possible. At $\theta = 88°$, for instance, we find that for an air-snow boundary $|R_V| = 0.85$ and $|R_H| = 0.89$, while for an air-ice boundary $|R_V| = 0.86$ and $|R_H| = 0.95$.

Suppose we now consider for a moment an overland transmission link in which the frequency of operation is 300 Mc/s. Let us suppose that the transmitter is at a height of at least a few wavelengths, and that the receiver is sufficiently high for the angle of incidence of waves reflected at the ground to be about 80°. We further assume that the distance from the transmitter to the receiver is sufficiently large for the difference in length of the paths of the direct ray and the ground-reflected ray to be negligible, except in the determination of the phase difference between these two rays. Under such conditions the receiving terminal will be well up in the lobe structure of the polar diagram (in the vertical plane) of the transmitting aerial. Now suppose the receiver to be moved in height over an interval to embrace the nearest maximum and minimum signals; at the above elevation this interval will be a relatively small fraction of the total height, and to a first approximation we may

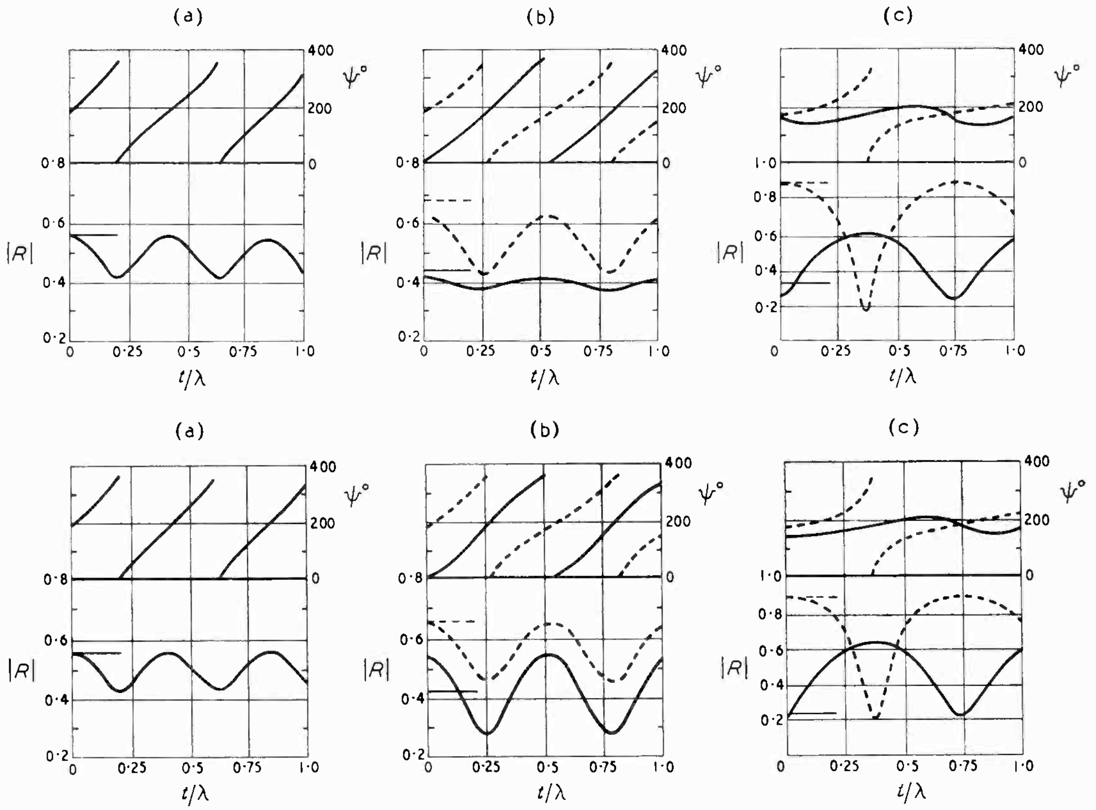


Figs. 10 and 11. *Resultant reflection coefficient with a layer of ice on sea water as a function of layer thickness in terms of the wavelength (t/λ); (a) $\theta = 0°$, (b) $\theta = 45°$, (c) $\theta = 80°$. Fig. 10 (top) frequency = 3,000 Mc/s. Fig. 11 (bottom) frequency = 300 Mc/s.*

assume tha: the reflection coefficient remains constant. In the absence of ice or snow we should expect, for horizontally-polarized waves, the maximum field strength to be of the order of $(1 + |R_H|)$, or 1.9—on some arbitrary scale—and the minimum to be $(1 - |R_H|)$, or 0.1 on the same scale. Now suppose that a layer of snow of specific gravity 0.3 exists to a depth of 0.36λ (i.e., 36 cm or 14 inches): the maximum and minimum signals will now be 1.2 and 0.8, since the resultant reflection coefficient with such a layer is only 0.2 when $\theta \approx 80°$. Compared with the situation in the absence of the snow, therefore, the maximum signal is reduced by nearly 4 db and the minimum signal increased by about 18 db. Under similar conditions vertically-polarized waves would give a maximum and a minimum of 1.25 and 0.75 respectively in the absence of snow, and of 1.65 and 0.35 with a snow layer of the above thickness (0.36λ). Thus, now, the presence of such a layer would increase the maximum signal by 2.5 db and decrease the minimum signal by 6.5 db.

An inspection of Figs. 11(a), (b) and (c) will similarly show that, whereas the reflection coefficient of sea water for horizontally-polarized waves at a frequency of 300 Mc/s, and at angles of incidence of 0°, 45° and 80°, is 0.90, 0.92 and 0.98, with a layer of ice of thickness 0.16λ (16 cm or 6.3 inches) the resultant reflection coefficients at these angles become 0.72, 0.68 and 0.58. There are, of course, variations in phase retardation, ψ, to be considered as well, as shown in the figures. For vertically-polarized waves of this frequency, instead of the reflection coefficients of 0.90, 0.85 and 0.52 at $\theta = 0°$, 45° and 80° which occur with no layer, we obtain values of 0.72, 0.75 and 0.90 with the ice of thickness 0.16λ.

Many similar examples might be worked out, but it is apparent from the few given that the vertical-coverage diagram of a transmitter operating at very high frequencies may be appreciably modified by the presence of a layer of snow or ice, except near to the ground and at such distances that grazing incidence occurs, and when therefore $R \rightarrow -1$ in any case for both horizontally- and vertically-polarized waves.



Figs. 12 and 13. *Resultant reflection coefficient with a layer of snow on land as a function of layer thickness (t/λ); (a) $\theta = 0°$, (b) $\theta = 45°$, (c) $\theta = 80°$. Fig. 12 (top) frequency = 3,000 Mc/s. Fig. 13 (bottom) frequency = 300 Mc/s.*

7. Conclusions

The effects of multiple reflection phenomena at very high radio frequencies (300 and 3 000 Mc/s) in layers of ice and snow overlying sea water and land respectively have been investigated theoretically. The resultant reflection coefficients of the ice-sea and snow-land systems have been calculated, and it appears that marked variations may occur from the reflection coefficient which would obtain in the absence of such ice or snow layers. As a consequence the vertical coverage diagram of a very high frequency transmitter may be appreciably changed by the presence of these layers in depths likely to occur in practice frequently in some parts of the world, and occasionally in others.

8. Acknowledgments

The work described above was carried out as part of the programme of the Radio Research Board. This paper is published by permission of the Director of the National Physical Laboratory, and the Director of Radio Research of the Department of Scientific and Industrial Research. Mr. J. A. Lane, B.Sc., assisted with the computations.

APPENDIX I

Suppose we have a homogeneous mixture of two media having dielectric constants of ϵ_1 and ϵ_2, and that there are n_1 and n_2 molecules per c.c. of these two media respectively. Then it may be shown that, if ϵ is the resultant dielectric constant of the mixture :*

$$\left(\frac{\epsilon - 1}{\epsilon + 2}\right)\left\{\frac{c_1 M_1 + c_2 M_2}{\rho}\right\} = \left(\frac{\epsilon_1 - 1}{\epsilon_1 + 2}\right) \cdot \frac{c_1 M_1}{\rho_1} + \left(\frac{\epsilon_2 - 1}{\epsilon_2 + 2}\right) \cdot \frac{c_2 M_2}{\rho_2} \quad \ldots \quad (15)$$

where $c_1 = \dfrac{n_1}{n_1 + n_2}$, $\qquad c_2 = \dfrac{n_2}{n_1 + n_2}$

and M_1 = Molecular Weight of medium 1,

M_2 = ,, ,, ,, ,, 2,

ρ_1 = Density of medium 1,

ρ_2 = ,, ,, ,, 2,

ρ = ,, ,, mixture.

To estimate the dielectric properties of snow we assume it to be a uniform mixture of air and ice. Air is itself, of course, a mixture, but for the present purpose it is sufficiently accurate to consider it as a gas of molecular weight 30 and of density 0.0013 grams/c.c. at 0°C and 76 cm pressure, and having a relative permittivity of 1.0007. The molecular weight appropriate to ice is 18, its density is 0.9 grams/c.c., and $\epsilon = 3.05$.

If now we work out c_1 and c_2 for snow of specific gravity 0.3 and substitute in equation (15) we find that $\epsilon = 1.4$: similarly if the specific gravity is 0.1, then $\epsilon = 1.15$.

* See, for example, "Polar Molecules" by P. Debye, 1929, The Chemical Catalog Company, Inc.

APPENDIX II

The various symbols have already been defined in the main text, Section 5. If we refer to Fig. 1 we see that the retardation in phase in ice (or snow) due to the extra path traversed in a 'unit' multiple reflection is given by :—

$$\delta = \frac{4\pi t \sqrt{\epsilon_1} \cos\phi}{\lambda} \quad \ldots \quad (16)$$

and therefore that the total phase retardation between successive reflected rays is :—

$$\{\delta + \psi_{12} + \psi_{10}\} = z \quad \ldots \quad (17)$$

there being no phase change to be associated with a double crossing of the air-medium 1 boundary, once in each direction.

Now the amplitude of reflected ray 1 = $|R_{01}|$

while ,, ,, ,, ,, 2 = $|T_{01}\, T_{10}\, R_{12}| = A$ (say)

and ,, ,, ,, ,, 3 = $|T_{01}\, T_{10}\, R_{12}^2\, R_{10}| = AB$ (say)

,, ,, ,, ,, ,, 4 = $|T_{01}\, T_{10}\, R_{12}^3\, R_{10}^2| = AB^2$,,

.... etc.

thus $B = |R_{12}\, R_{10}|$

(Note.—As explained in the text we assume absorption in medium 1 to be negligible: if this were not the case it would be necessary to include an exponential attenuation factor in term B to allow for the absorption in each unit multiple reflection.)

Referring phases to that of the incident wave at the air-medium 1 boundary, we may now write the resultant reflection coefficient, R, in the form of the infinite series :

$$R = |R_{01}|e^{-j\psi_{01}} + Ae^{-j(\delta+\psi_{12})} + ABe^{-j(\delta+\psi_{12}+z)} + AB^2e^{-j(\delta+\psi_{12}+2z)} + \ldots \quad (18)$$

i.e.,

$$R = |R_{01}|e^{-j\psi_{01}} + Ae^{-jy}\{1 + Be^{-jz} + B^2e^{-2jz} + B^3e^{-3jz} \ldots\} \quad \ldots \quad (19)$$

where $\quad y = (\delta + \psi_{12})$

and, summing to infinity the geometrical progression in equation (19), we obtain :—

$$R = |R_{01}|e^{-j\psi_{01}} + \left(\frac{Ae^{-jy}}{1 - Be^{-jz}}\right) \quad \ldots \quad (20)$$

If now $R = P + jQ$ it is easy to show from equation (20) that :—

$$P = |R_{01}| \cos\psi_{01} + \left\{\frac{A\cos y - AB\cos(z - y)}{1 + B^2 - 2B\cos z}\right\} \quad (21)$$

$$\text{and } Q = -|R_{01}| \sin\psi_{01} - \left\{\frac{A\sin y + AB\sin(z - y)}{1 + B^2 - 2B\cos z}\right\} \quad (22)$$

Equations (21) and (22) are the same as (9) and (10) of the main text.

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RC-AMPLIFIER FILTERS

By C. H. Miller, B.E. (Tas.)

(*The Physical Chemistry Laboratory, Oxford*)

SUMMARY.—Simple expressions are derived for the approximate cut-off frequencies of amplifiers which use resistance-capacitance feedback networks to produce high-pass, low-pass, or band-pass frequency characteristics similar to those of conventional filters. The design of such amplifiers is discussed, and illustrations are given of the results obtained with simple amplifiers of this type.

Introduction

IT has been known for some time that at frequencies in and below the audio range an entirely RC-coupled feedback amplifier which has a high-pass, low-pass, or band-pass frequency response can have useful advantages compared with amplifiers incorporating conventional filters. In particular the omission of inductances makes for compactness, low cost, and freedom from inductive pick-up.

The feedback amplifier of Fig. 1 has a high-pass characteristic with a sharp cut-off, as has been shown by Fritzinger[1]. This can be seen by considering the vector diagram of the feedback voltages given in Fig. 2. The amplifier itself produces a phase-change of an odd multiple of 180° between its input and output terminals, and the transmission of the feedback network R_1C_1, R_2C_2 increases from zero amplitude and $-180°$ phase change at very high frequencies to unity amplitude and zero phase change at very low frequencies. Hence the total transmission around the feedback loop is such that there is small positive feedback at high frequencies, and large negative feedback at low frequencies.

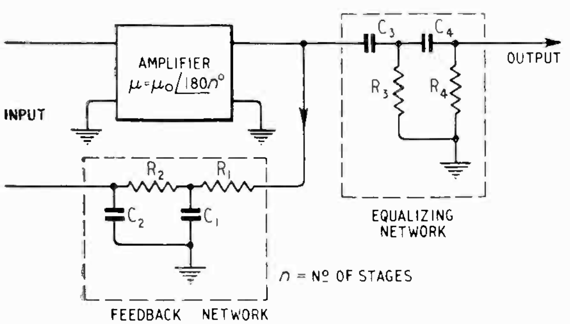


Fig. 1. *High-pass feedback amplifier.*

It follows on substituting the values of $1 - \mu\beta$ so obtained in the usual feedback amplifier expression*

$$\mu' = \mu/(1 - \mu\beta) \qquad \qquad (1)$$

that as the frequency decreases the effect of feedback is first of all to increase, and then greatly to decrease amplifier gain. Further there is one particular frequency f_0, corresponding to the point L in Fig. 2, at which the gain is a maximum and below which it rapidly decreases.

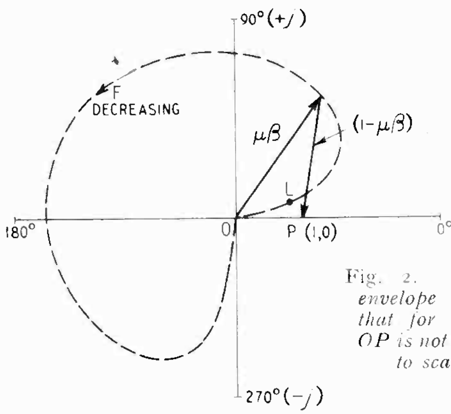


Fig. 2. *Vector envelope (note that for clarity OP is not drawn to scale).*

The resulting frequency response for an amplifier of this type is given in Fig. 3, curve (a), the response of the amplifier without feedback being very nearly flat over the frequency range considered. The amplifier may be followed by a simple equalizing network R_3C_3, R_4C_4 which has the response given by curve (b), and which makes the overall response, curve (c), of the complete amplifier, flat to within $\pm$ 1 db down to the cut-off frequency.

An amplifier with a corresponding low-pass response is obtained by interchanging R and C in each of the feedback and equalizer networks of Fig. 1, although different component values will be required for the same cut-off frequency. A band-pass response may be obtained by connecting high-pass and low-pass feedback networks in parallel, providing that their respective cut-off frequencies are sufficiently separated to prevent mutual interaction.

Cut-off Frequency

As can be seen from Fig. 3 the frequency of maximum gain, f_0, is very near the frequency of cut-off (response -3 db), and so can be used as an approximation for it. This has the advantage that f_0 can be simply calculated, whereas a plot

MS accepted by the Editor, February 1949

[1] G. H. Fritzinger, "Frequency Discrimination by Inverse Feedback," *Proc. Inst. Radio Engrs*, 1938, Vol. 26, p. 207.

* μ and β are, of course, both vector quantities.

of the $\mu\beta$ vector envelope would otherwise be required to obtain the cut-off frequency.

The calculation of f_0 is simplified, and remains sufficiently accurate, if the following assumptions are made. First, that the amplifier, and the networks R_1C_1 and R_2C_2, have high input and low output impedances, so that mutual loading effects can be neglected. Secondly, that over the frequency range being considered the amplifier gain is given by

$$\mu = \mu_0 \angle 180n^\circ \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

where $n = 1, 3, \ldots,$ the number of amplifier stages.

A further aid to calculation is to have the reciprocal time constants of the two feedback networks equal; i.e.,

$$\omega_1 = 1/R_1C_1 = \omega_2 = 1/R_2C_2 \qquad \ldots \quad (3)$$

although this is not a necessary condition for correct operation.

It follows that the transmission of the feedback network is given by

$$\beta = \beta_1\beta_2 = \left\{\frac{1 - j(\omega/\omega_1)^2}{1 + (\omega/\omega_1)^2}\right\} \quad \ldots \quad \ldots \quad (4)$$

where $\omega = 2\pi f$.

On substituting from Equs. (2) and (4) we have

$$1 - \mu\beta = \frac{\{(\omega_1/\omega)^2 + 1\} + \mu_0(\omega_1/\omega)^2\{(\omega_1/\omega)^2 - 1\} - j \,.\, 2\mu_0(\omega_1/\omega)^3}{\{(\omega_1/\omega)^2 + 1\}^2} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

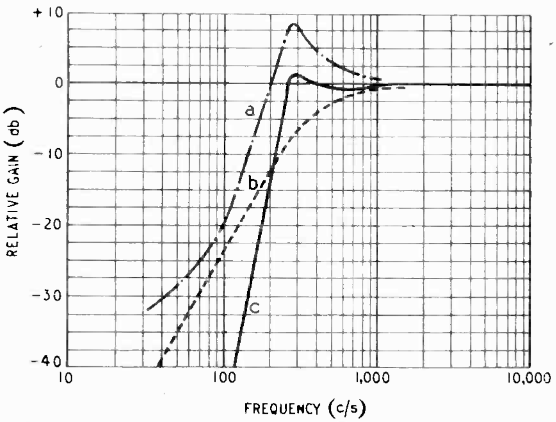


Fig. 3. *Typical high-pass amplifier response curves; (a) is for the amplifier alone, (b) is for an equalizer and (c) shows the two in combination.*

It can be seen from Equ. (1) that differentiation of $|1 - \mu\beta|^2$ with respect to ω will give the condition for the maximum value of $|\mu'|$, which is

$$(\mu_0 + 3)(\omega_1/\omega)^6 + (2\mu_0 + 5)(\omega_1/\omega)^4 + (\mu_0 + 1)(\omega_1/\omega)^2 - 1 = 0 \quad \ldots \quad (6)$$

For the large values of μ_0 likely to be used in practice (>100 say) this reduces with small error to

$$\omega_0 = \omega_1\sqrt{\mu_0} \qquad \ldots \quad \ldots \quad \ldots \quad (7)$$

$$\text{giving } f_0 = \frac{\sqrt{\mu_0}}{2\pi R_1C_1} \qquad \ldots \quad \ldots \quad \ldots \quad (8)$$

It follows similarly for the corresponding low-pass amplifier that

$$\omega_0 = \omega_1/\sqrt{\mu_0} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\text{and } f_0 = 1/2\pi R_1C_1\sqrt{\mu_0} \qquad \ldots \quad \ldots \quad (10)$$

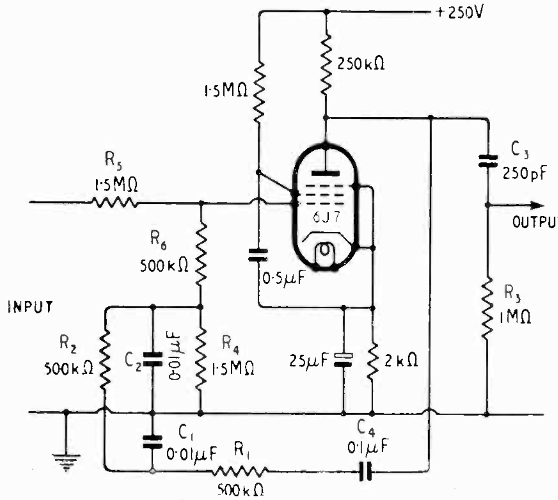


Fig. 4. *Single-stage high-pass amplifier.*

Amplifier Design

The requirements for successful design are chiefly that the assumptions made in the previous section are reasonably fulfilled. As with all feedback amplifiers care must be taken that at no time is the point P (1, 0) in Fig. 2 enclosed by the $\mu\beta$ vector envelope. This can be avoided if the condition of Equ. (2) is closely met, which is not difficult if the amplifier without feedback is designed to have a truly flat response over a sufficiently wide frequency range.

In the case of the low-pass amplifier in which f_0 is a high audio frequency care must be taken that additional phase shifts due to stray capacitance to ground are kept at a minimum. Their effect is to produce an additional lag in the feedback voltage, thus rotating the $\mu\beta$ vector away from the point P and reducing the sharpness of cut-off.

Instability can occur in a high-pass amplifier of three or more stages when the interstage RC couplings combine with the feedback network and its series d.c. blocking capacitor to act as a phase-shift oscillator at some frequency well below cut-off. This corresponds in Fig. 2 to the enclosure of the point P by the $\mu\beta$ vector from the fourth quadrant (270°–360°). This oscillation can be avoided if the loss of the interstage couplings and the feedback network at the frequency at which they give a total phase change of 180° is sufficient to reduce the total gain around the feedback loop to less than unity.

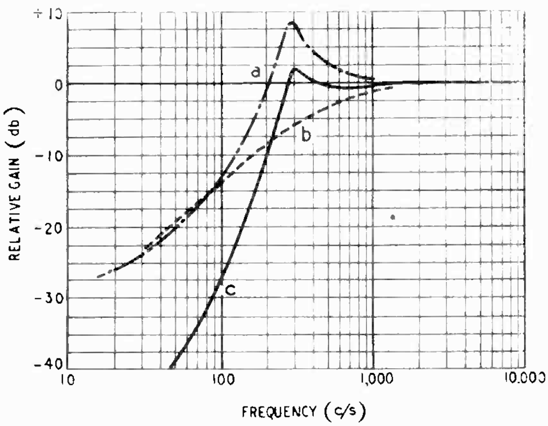


Fig. 5. *Single-stage high-pass amplifier response curves; (a) and (b) respectively are for the amplifier and equalizer and curve (c) for the two together.*

It is usually necessary to have the equalizing network follow the amplifier, even though this reduces the maximum output voltage available from the complete amplifier at frequencies near cut-off. Its connection directly into the amplifier input circuit is not usually possible because of the considerable phase shifts which would be introduced into the feedback loop.

The frequency response required from the equalizing network can be most speedily determined by measurement of the amplifier response after feedback has been applied. It can be seen from Fig. 2 that a small rotation of the $\mu\beta$ vector at the point L, such as is possible from unwanted phase shifts, can cause considerable variation in the value of $(1 - \mu\beta)$. For this reason the calculated increase in amplifier gain at f_0 is liable to be inaccurate, although it can be seen that such rotations of the $\mu\beta$ vector will have small effect on the accuracy of the calculated value of f_0.

Practical Amplifiers

The high-pass amplifier already referred to in Fig. 3 consists of three 6J5G triode stages which have additional negative feedback applied from unbypassed cathode-bias resistances. The gain over the feedback path has been adjusted to 54 db, and as can be seen from Fig. 3 the rate of attenuation beyond cut-off is approximately 30 db per octave.

A single RC-coupled pentode stage with a gain of the order of 40 db can provide a rate of attenuation beyond cut-off which is sufficient to be useful for many purposes. The circuit of the high-pass case of such an amplifier is given in Fig. 4, and it can be seen that for simplicity a single-section equalizing network is used. The series resistance R_5 in the grid circuit is necessary to prevent shunting of the feedback voltages by the input signal source. R_4 is used to ensure a d.c. return from grid to ground. The response curves for this amplifier are given in Fig. 5, from which it can be seen that the rate of attenuation beyond cut-off is approximately 17 db per octave. The use of a two-section equalizing network would increase this to over 20 db per octave, and would also produce a slightly flatter response down to the cut-off frequency.

The measured stage gain is found to be 77 (i.e., 37.7 db), and substituting in Equ. (8):

$$f_0 = \frac{\sqrt{77}}{2\pi \times 0.5 \times 10^6 \times 0.01 \times 10^{-6}} = 280 \text{ c/s.}$$

This agrees fairly closely with the measured value of 295 c/s, as shown in Fig. 5.

In the low-pass case of this single-stage amplifier, similar rates of attenuation are obtained for cut-off frequencies of up to about 1000 c/s.

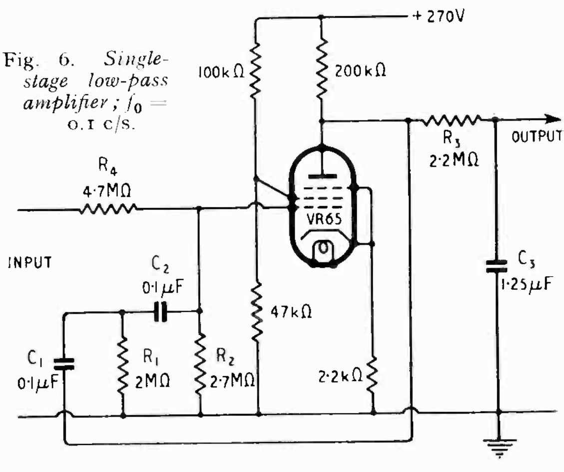


Fig. 6. *Single-stage low-pass amplifier; f_0 = 0.1 c/s.*

When higher cut-off frequencies are required it is necessary to use a high-slope pentode so that the same stage gain with less phase shift at high frequencies is obtained by the use of lower values of anode load resistance. Similarly it is necessary to use lower values of grid resistance to reduce phase shift in the input circuit.

The application of this type of amplifier to low frequencies is illustrated by the single-stage low-pass amplifier of Fig. 6 in which f_0 is 0.1 c/s. The resulting response curves are given in Fig. 7, which shows that the rate of attenuation beyond cut-off is approximately 20 db per octave. This particular amplifier is used to drive a pen-recording milliammeter via a cathode follower connected to its output. In this application a very narrow bandwidth filter with a sharp cut-off is required in order to record information with the maximum signal-to-noise ratio for a given speed of response.

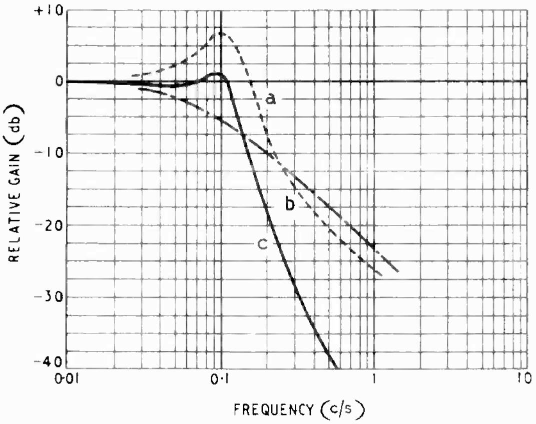


Fig. 7. *Single-stage low-pass amplifier frequency response; curves (a) and (b) are respectively for the amplifier and equalizer and curve (c) shows their combined performance.*

It is perhaps worth noting that considerable information regarding the operation of a low-pass amplifier of this type may be obtained from its response to a step voltage, or to a square wave in cases where f_0 is much over 1 c/s. As is shown in Fig. 8, the degree of overshoot indicates whether or not the peak at f_0 is correctly equalized, and also the frequency of this damped oscillation is very nearly f_0. This enables the response to be checked at frequencies for which a sine-wave generator may not be available. By this means it was checked that when C_1 and C_2 were increased to 1 μF and then to 10 μF that f_0 decreased to 0.01 c/s and 0.001 c/s respectively, as would be expected from equation (10). In the latter case equalization was obtained with R_3 increase to 28 MΩ and C_3 to 10 μF.

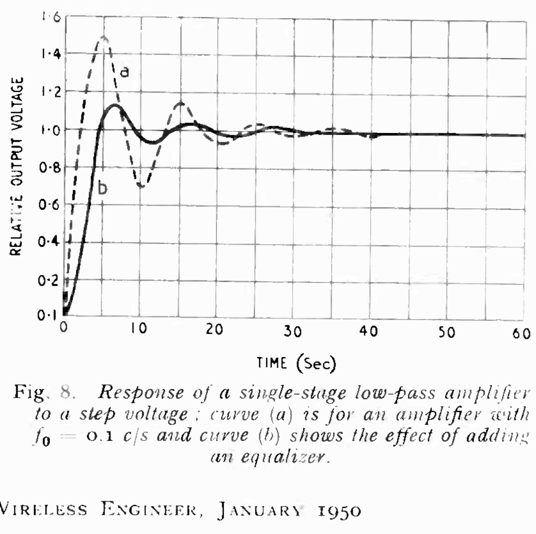


Fig. 8. *Response of a single-stage low-pass amplifier to a step voltage; curve (a) is for an amplifier with f_0 = 0.1 c/s and curve (b) shows the effect of adding an equalizer.*

As has been previously pointed out a band-pass response may be obtained by connecting low-pass and high-pass feedback networks in parallel over the same stages of an amplifier, provided that the upper and lower cut-off frequencies are sufficiently separated. Normally they have to be in a ratio of about 5 to 1. In cases where a narrower pass-band is required low-pass and high-pass amplifiers may be connected in cascade and the upper and lower cut-off frequencies adjusted independently. The response curve of a simple example of this type of amplifier is given in Fig. 9. Two RC-coupled pentode stages using VR65 valves with unbypassed cathode resistors provide an overall gain of 60 db for the two stages, including the loss in the interstage couplings. The gain around each feedback loop is approximately 40 db.

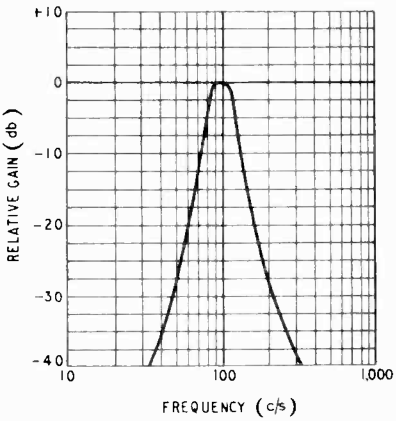


Fig. 9. *Response of a two-stage band-pass amplifier.*

Conclusion

It is evident that the advantages of this method of obtaining frequency discimination are greatest for cut-off frequencies of the order of 1000 c/s and below. At these frequencies the elimination of inductances can be a considerable advantage, and is in fact a practical necessity at frequencies below the audio range. At the higher audio frequencies there is, in many cases, little to choose between this method and the use of conventional filters as special precautions are required for optimum performance; also the inductances required for conventional filters at these frequencies are not unduly bulky or expensive.

Acknowledgment

The majority of the experimental work covered in this paper was carried out in the Electrical Engineering Laboratory of the University of Tasmania, and thanks is expressed for the assistance then obtained under the Ormsby Hamilton Research Scholarship.

BOOK REVIEWS

The Technique of Radio Design

By E. E. ZEPLER, Ph.D. Pp. 394 + xv, with 283 illustrations. Chapman and Hall Ltd., 37 Essex Street, London, W.C.2. Price 25s.

This is a second revised edition, the book having been first published in 1943. The author was formerly engaged in receiver design with the Telefunken Co. but is now head of the Radio Engineering Department of University College, Southampton. As stated in the preface, the book deals mainly with those problems which are closely linked with the daily routine work of an engineer in the development and testing of radio-receiving apparatus of all types. A more suitable title would have been "Radio Receiver Design." The author says that "complicated mathematics are avoided and approximations suitable to the problem in hand have been made whenever possible." In this new edition the chapter on receiver noise has been rewritten, and much more space given to negative feedback; 85 pages and 23 diagrams have been added.

Successive chapters deal with the various parts of the receiver, aerial coupling, amplifiers, detection, selectivity, noise, gain control, screening, hum, distortion, parasitic resonances, power supply, routine measurements and fault finding. Numerical examples are worked out wherever suitable, and the book is well illustrated with diagrams and graphs. Frequency modulation is only just mentioned, and the reader is referred to published papers for any further information. In the list of symbols the a.c. resistance is referred to as the impedance but is represented by R_a and not Z_a as an impedance should be. The treatment of real and reactive power on p.2 is very unsatisfactory and seems rather out of place in a book of this type. This section concludes with the following paragraph. "To think in power, either real or reactive, has often the advantage of quickly arriving at results which otherwise require an appreciable amount of calculation. It is easy to see that two different loads which, for a given E.M.F., absorb the same watts are interchangeable, watts being used in the general sense as the *sum of real and reactive power*. As the watts are the product EI the statement just made seems self-evident, because it is only another expression for the identity of two impedances. It will, notwithstanding, prove useful." We doubt it. The italics are ours.

G. W. O. H.

Cathode Ray Tube Traces

By HILARY MOSS. Pp. 66 with 52 figs. and 153 photographs. "Electronic Engineering," 28, Essex St., Strand, London, W.C.2. Price 10s. 6d.

This book has an unusually large page ($7\frac{1}{4}$in $\times$ $9\frac{5}{8}$in) and is set in small type, so that it contains much more material than its number of pages would lead one to suppose. The author starts by considering Lissajous figures on a geometrical basis and discusses their properties in considerable detail. In the second chapter he treats linear time bases and approximations thereto and shows that the centre part of a sinusoidal wave is adequate for many purposes. Circular and spiral time bases are discussed and there is a chapter on complex waveforms and another on modulated waves. There are appendices in which the responses of two-element series and parallel networks to various waves are discussed and in which there are useful notes on the characteristics and use of c.r. tubes. Finally, data is given on the circuits used for the production of the traces used to illustrate the book, together with some notes on linear time bases.

The photographs of c.r.-tube traces are excellent and in both their quality and their number they form an outstanding feature of the book.

W. T. C.

Die Systemtheorie der elektrischen Nachrichtenübertragung.

By K. KÜPFMÜLLER. Pp. 386 with 474 illustrations. S. Hirzel Verlag, Stuttgart.

This book is based on lectures given by Professor Küpfmüller at the Technische Hochschule in Berlin from 1937 to 1943. It is a systematic investigation of the transmission of electrical signals of all kinds under all possible conditions. It deals with telegraphic signals, speech, amplitude, frequency and pulse modulation, and all the various types of distortion. The amount of material in the book is very impressive and the treatment is very thorough and detailed and well illustrated with numerous diagrams. When any lesser known mathematical functions are introduced they are carefully explained and illustrated with numerical examples and diagrams. This is certainly a book that can be unreservedly recommended to anyone with a knowledge of German.

G. W. O. H.

Aerials for Metre and Decimetre Wavelengths

By R. A. SMITH. Pp. 218+xii. Cambridge University Press, 200, Euston Rd., London, N.W.1. Price 18s.

This book is one of a new series of monographs on Modern Radio Techniques, published under the general editorship of J. A. Ratcliffe, and dealing with the rapid advances made in the radio field during the past few years, particularly in the application of ultra-high frequencies. The aerial systems described are mainly those which have been developed for the wavelength range of 12 metres to 1 metre, and only one short chapter is devoted specifically to decimetre-wave aerials. The upper wavelength limit has been determined by two considerations: first, rigid dipoles, for the principal elements of the systems discussed, become cumbersome at greater wavelengths and are therefore not much used, and secondly 12 metres is the longest wavelength used for radar. When the wavelength falls much below 1 metre new principles of design appear. It is, of course, well known that the higher definition radar equipments, evolved during the progress of the late war were based on the use of centimetre waves; the types of aerial employed at such wavelengths, however, form the subject of another book of the series.

As Dr. Smith says in his preface: "This volume does not attempt a comprehensive description of the great variety of aerial systems used for short-wave radio applications. Indeed, most radio applications have aerial systems with their own characteristic features, so that the number of different arrangements which have been used is enormous. All that has been possible is a selection of a few applications as illustrating general principles. These illustrations have largely, though not entirely, been taken from radar applications, since it is here that perhaps the most outstanding developments in aerial design have taken place." It is, however, worth emphasizing that wavelengths in the metre-wave band have been used, and are likely to be used widely in the future, for purposes other than radar, such as in television and in radio communication links; the B.B.C. also intends to establish a high-quality broadcasting service on a wavelength of just over 3 metres (frequency 90 Mc/s).

It was already appreciated before the war, as a result of experienced gained with television, amongst other things, that just any piece of wire would not suffice as a satisfactory receiving aerial at very short wavelengths, as had been the case to a large extent at longer wavelengths. It was also realized that accurate matching of both transmitting and receiving aerials was desirable in order to avoid spurious reflections which, with even the very short delays normally involved in the passage of the signal from aerial to receiver or transmitter, may still be of significance. The same considerations also apply in radar, where matching is also an essential feature of aerial design. The developments described in this book will, therefore, be of great interest to all workers concerned with systems operating at metre wavelengths.

One of the ways in which technique at very short wavelengths differs from that at longer wavelengths is that more efficient and highly directive aerials become possible. At centimetre wavelengths the design of aerial systems is often based on quasi-optical principles; i.e., one finds that curved metal surfaces are used as mirrors and that lenses of various types can be made to focus radio beams. Such methods are hardly practicable at metre wavelengths, although paraboloidal reflectors have found application at wavelengths at least as great as 50 cm. In general, however, in the wavelength band with which we are here concerned, aerial systems are based on various spatial arrangements of dipoles, some or all of which may be directly connected to the transmitter or receiver, and some of which may be parasitic.

The author gives a comprehensive and critical discussion of the theory underlying the behaviour of the fundamental element, the dipole. The extent to which the simple 'sinusoidal' (i.e., as regards current and voltage distributions along the length of the element) theory is valid and useful is shown, and the differences introduced by the more elaborate approaches of Hallén, Schelkunoff and King are considered. It must not be imagined that Dr. Smith is content to leave the matter on a theoretical plane for, as one who, during the war, had to design aerials for many urgent practical applications, he is careful to show how the predictions of theory fit in with such accurate measurements as have been made on typical systems. This comparison between theory and experiment is particularly important and valuable, for the theories—even the most elaborate ones—introduce certain idealizations, such as the manner of coupling the generator (or receiver) to the aerial, which it is not possible to realize in practice. It is evident that there is still room for considerable fundamental experimental work in this field.

The most common way in which aerial directivity and gain are obtained at metre wavelengths is by the use of arrays, broadside or end-fire, or of parasitic elements, directors and reflectors, as in Yagi systems. The mutual interaction between these various elements is, therefore, an important matter, and it is discussed by the author in some detail. Plane reflecting curtains, often of wire mesh, also find an application, particularly in association with broadside arrays. It is frequently necessary to support elements of an array by means of insulators, and the effect of these on the electrical lengths of such elements is described.

Other topics discussed are the special features of the design of aerials for high-power transmitters, the properties of long-wire and rhombic aerials, and the problems connected with the installation of aerials in aircraft. There is also a chapter on the design and construction of slot aerials involving the application of Babinet's principle.

In the final chapter the author discusses the way in which noise, from all sources, terrestrial and extra-terrestrial, may enter a receiver via the aerial, and summarizes his conclusions as follows: "For efficient resonant aerials sensitivity (of the receiving system) is determined by aerial noise for wavelenths greater than 5 metres. Using the best grounded-grid valves for the first stage of the receiver sensitivity is affected by aerial noise in certain directions (i.e., in relation to the Galaxy) down to wavelengths of 3 metres, but is generally determined by receiver noise. However, under conditions of abnormal sunspot activity the sensitivity may be greatly reduced at wavelengths more than a metre, particularly when directional aerials are used looking towards the sun. For shorter wavelengths sensitivity is entirely determined by receiver noise."

Dr. Smith is to be congratulated on presenting a most lucid and valuable account of the subject. The book is well produced, and liberally illustrated, and contains a wealth of practical information concerning the design of aerials. It should prove of great assistance to all who wish to construct aerials for the ever increasing applications of radio in the metre-wave band, and it may be unreservedly recommended. J. A. S.

Radio-Frequency Heating

By L. Hartshorn, D.Sc., A.M.I.E.E., A.R.C.S., D.I.C. Pp. 237. George Allen & Unwin Ltd., Ruskin House, Museum St., London, W.C.2. Price 21s.

This book covers both induction and dielectric heating and is concerned mainly with the principles of r.f. heating. Following a short introductory chapter in which the nature of heating processes is explained there is a discussion of methods of generating r.f. power. This is fairly elementary and occupies some 30 pages only.

Induction heating is then treated in two chapters covering principles and applications. There is a detailed theoretical treatment of the production of heat in conductors of simple geometrical shape by induction and of the effect of the constants of the work coil and its coupling to the work. Localized, as well as general, heating is treated and the methods of dealing with special problems are indicated. The chapter on applications is far from being merely a resumé of common applications of r.f. heating. It runs to 60 pages and contains a great deal of useful information about materials, methods and power requirements.

Dielectric heating is similarly divided between two chapters. The principles are very thoroughly covered and under applications the author points out some of the difficulties which are encountered when attempts are made to heat inhomogeneous material. The cooking of food by r.f. heating is mentioned as an extreme example.

In discussing applications, the author deals with the advantages and disadvantages of r.f. heating in a very balanced way and gives a very fair account of the possibilities. After some of the extravagant claims which have been made for it in the past, it is refreshing to read that "It is fairly safe to say that radio-heating should not be used for any process in which the actual cost of the power is a main consideration, for alternative methods will be cheaper."

A good point about the book is that it deals mainly with just those points upon which general information is lacking—the heating coil or electrode and the material itself. The power generator is not treated in any detail for, basically it is in no way peculiar to r.f. heating. Each chapter contains an extensive bibliography and the book undoubtedly forms a useful contribution to the literature.

W.T.C.

GRAPHICAL STATISTICAL METHODS

An error occurred in this article in the December 1949 issue. In the second column of page 403 the first paragraph should read "The method of treating the data is to divide the diagram into suitable horizontal strips . . ."

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Gain of Aerial Systems

Sir,—Mr. J. Brown's letter (*Wireless Engineer*, December 1949) explains why it is difficult to make direct practical application of the 'end-fire theorem' to radiators of the waveguide type, but I fear that I must have singularly failed to make clear the more general points which arise from wire radiators.

In the first place it was known that there is a reasonably close correspondence between the gain of a broadside array of current-carrying conductors and the gain of a uniformly illuminated aperture of the same area. It is also known that if the same array of wires, which as a broadside radiator corresponded with the illuminated aperture, is fed with a different phase distribution so as to act as an end-fire radiator, the gain is doubled. Let me repeat, the *same* aerial system so far as mechanical structure is concerned, and with no question of introducing or removing an infinite plane, gives double the gain when the phasing is correctly adjusted. The general end-fire theorem was found to correspond in a number of special cases to the phasing condition which had been found to be optimum as a result of the specific calculations applicable separately to each particular case (Yagi, rhombic, polyrod, etc.).

It is possible to say that whereas the optical analogue of a broadside array of dipoles is an illuminated aperture, the optical analogue of an end-fire array is a source of the same dimensions plus a plane reflector, but I object to introducing an additional surface into the analogue when no additional component is introduced into the aerial itself.

The second question is that of an *unlimited* increase of gain, not merely 2 : 1, for an aerial system of fixed dimensions.

This is the case which inevitably conflicts with the normal ideas of optical resolution, and is one which Mr. Brown has not discussed. Subject to the limitations of conductor loss, unlimited gain should be achievable at the relatively long wavelengths where arrays of wires can be used so that drastic control of phase to fit any desired distribution function is possible.

It is not possible in the traditional type of optical device where phase is controlled by the propagation velocities through the various media employed, and the media are homogeneous over areas of a wavelength or more in diameter.

Likewise the possibility of a train of narrow pulses (*not* a single narrow pulse) through a circuit arises from analogy with the 'unlimited gain' aerial. There are functions for intensity distribution across an aperture which Fourier-transform into more peaky distributions of radiated energy versus angle than we should have thought possible before the work of authors such as Bouwkamp and de Bruijn. If those same functions are used for spectral distributions of energy, they should then Fourier-transform into amplitude versus time functions which are more peaky than we should have expected from the width of spectrum which was used.

University of Birmingham. D. A. Bell.

Phase-Shift Calculation

Sir,—The curve showing the image phase-shift coefficient β, of a multi-section filter as a function of frequency f, is often of interest in network investigations; e.g.,

1. For calculating the time-delay,

2. For calculating the effect of dissipation on the insertion loss,

3. In insertion loss calculations or in the design of Zobel filters and some insertion parameter filters.[1]

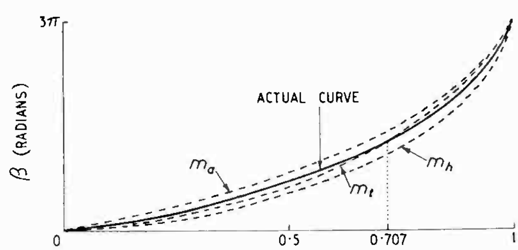
Whereas in 1 and 2 the slope of the β, f curve is the information required, in 3 the requirements are the values of f at which β is a multiple of $\pi/2$ or has some other specified value.

It is well known that the image phase-shift coefficient of a multi-section Zobel filter may be obtained as a function of frequency by adding the phase-shift coefficient curves of the individual sections. However, it is sometimes desirable to obtain a good approximation to the β, f curve more quickly.

The suggestion[2] has been made to use, as such an approximation, the phase curve of a reference filter having the same number of sections but equal m-values for all sections, the particular value of m being a suitable compromise value. It has further been suggested that the harmonic mean of the actual m-values is a suitable compromise value to be used for frequencies near the cut-off frequency(ies) (see also ref. 3).

We have examined a few compromise values and think that the results obtained may be of some general interest.

Let us, as usual in filter analysis, consider low-pass pass filters only. Let x be the normalized frequency, so that the pass band is defined by $0 \leqslant x \leqslant 1$. Let the filter consist of n half-sections with image phase-shift coefficients $\beta_\lambda/2$ and m-values m_λ. Then $\tan(\beta_\lambda/2) = m_\lambda y$ where $y = x/\sqrt{(1-x^2)}$ and the total phase-shift coefficient of the filter is $\sum_{\lambda=1}^{n} \beta_\lambda/2$. The compromise value of m will be called m_0 and the corresponding phase-shift coefficient $\beta_0/2$. Then the total phase-shift coefficient



of the compromise filter will be $n\beta_0/2$ and we are comparing $n\beta_0/2$ with $\Sigma\beta_\lambda/2$.

It is obvious that at $x = 0$ and $x = 1$, $n\beta_0/2$ and $\Sigma\beta_\lambda/2$ will be equal whatever value is chosen for m_0. However, for the cases x near to 0 and x near to 1, it can be shown (see Table I) that the best approximation of $n\beta_0/2$ to $\Sigma\beta_\lambda/2$ is obtained if m_0 is made equal to the arithmetic mean m_a or the harmonic mean m_h, respectively, of all the m-values.

A quantitative investigation shows that the harmonic mean, as a compromise value, gives rather large deviations from the actual β, x curve at the lower end of the range; the arithmetic mean, as a compromise value, leads to rather large deviations at the upper end of the range. In view of the fact that any value of m_0 leads to correct values of $n\beta_0/2$ at $x = 0$ and $x = 1$ (as distinct from x near to 0 and x near to 1), it seemed promising to choose the compromise value so that a correct value of $n\beta_0/2$ would also be obtained in the interior of the range $0 \ldots x \ldots 1$, in the hope that this would give a better overall approximation to the actual curve of $\Sigma\beta_\lambda/2$. A convenient way of achieving this is to select the 'tangent mean,' m_t, of the m_λ terms; i.e., a value

$$m_t = \tan\left[\frac{1}{n}(\tan^{-1}m_1 + \tan^{-1}m_2 + \ldots + \tan^{-1}m_n)\right]$$

Then the curve obtained by using this compromise value $m_0 = m_t$ will cross the actual β, x curve at $x = 0.707$ (see Table I), which is near the middle of the range.

TABLE I

x	y	β_λ	$\Sigma\beta_\lambda$	$n\beta_0$
$x \to 0$	$y \to 0$	$\beta_\lambda/2 \to m_\lambda y$	$\Sigma\beta_\lambda/2 \to y\Sigma m_\lambda$	$n\beta_0/2 \to ynm_0$
$x = \frac{1}{2}\sqrt{2}$	$y = 1$	$\tan\beta_\lambda/2 = m_\lambda$	$\Sigma\beta_\lambda/2 = \Sigma\tan^{-1}m_\lambda$	$n\beta_0/2 = n\tan^{-1}m_0$
$x \to 1$	$y \to \infty$	$\pi/2 - \beta_\lambda/2 \to 1/m_\lambda y$	$n\pi/2 - \Sigma\beta_\lambda/2 \to (1/y)(\Sigma 1/m_\lambda)$	$n\pi/2 - n\beta_0/2 \to (1/y)(n/m_0)$

Two examples, both three section filters, with m-values 0.4038, 0.7454, 0.3379 and 0.5528, 1.0, 0.7806, respectively, have been investigated. The results confirm that the approximation for x near to 0 is best if $m_0 = m_a$, for x near to 1 if $m_0 = m_h$, and that the maximium deviations δx and $\delta\beta/2$ in the x and β directions for the whole x-range $0 \ldots x \ldots 1$ are smallest if $m_0 = m_t$. A schematic drawing of a set of curves for one filter is shown, but with exaggerated deviations because of the small scale; δx and $\delta\beta/2$ are given in Table II.

TABLE II

1st Filter			2nd Filter		
m_0	δx	$\delta\beta/2$	m_0	δx	$\delta\beta/2$
$m_a = 0.5257$	0.015	0.12	$m_a = 0.7778$	0.0175	0.07
$m_h = 0.4742$	0.035	0.10	$m_h = 0.7337$	0.020	0.06
$m_t = 0.5143$	0.009	0.06	$m_t = 0.7620$	0.005	0.03

These results suggest that m_0 should always be chosen with reference to the part of the x-range which is of chief interest.

W. SARAGA.
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REFERENCES

[1] S. Darlington, Bell Monograph B-1186.
[2] This suggestion is due to Mr. H. J. Orchard (P.O. Research Station).
[3] V. Belevitch, *Electrical Communication*, June 1947.

Spontaneous Fluctuations in Double-Cathode Valves

SIR,—In a recent letter[1] K. S. Knol and G. Diemer report on the results of fluctuation measurements on electron tubes with two hot electrodes (double cathode valves) which indicate that the ratio β of 'fluctuation temperature' to true electrode temperature is equal to 1 for a valve with indirectly-heated electrodes in the range 1100–1300° K and equal to about 0.9 for a valve with directly-heated electrodes in the range 2250–2550° K. The latter result is in disagreement with experiments by D. K. C. MacDonald,[2] who observed an increase of β from 1 at 1900° K to about 2 at 2450° K. In a note to *Nature*[3] I endeavoured to show that MacDonald's results could be explained on the basis of my unified theory of spontaneous electrical fluctuations,[4] and Knol and Diemer therefore feel justified in rejecting my theory. In actual fact the new experimental results neither disprove MacDonald's findings nor my theory in any way.

My theory, applied to the present case, yields the following formula for the mean square of the current fluctuation in the frequency range $f \ldots f + \Delta f$

$$(\delta I)^2_f = 4kT_{ga}\beta\Delta f \quad \ldots \quad (1)$$

where
$$g_a = \left(\frac{\delta I}{\partial V_a}\right)_{I_s} \quad \ldots \quad (2)$$

is the differential tube conductivity for a fixed saturation current I_s (i.e., a fixed electrode temperature T,) and

$$\beta = \frac{1 + \alpha^2}{1 + \alpha}, \quad \alpha = g_c/g_a; \quad \ldots \quad (3)$$

here
$$g_c = \epsilon . \frac{e I_s}{kT} \quad \ldots \quad (4)$$

with
$$\epsilon = \left(\frac{\partial I}{\partial I_s}\right)_{V_a} \quad \ldots \quad (5)$$

The quantities g_a and g_c will, in general, vary with the operating conditions of the tube (i.e., with inter-electrode voltage V_a and electrode temperature T) and they can easily be evaluated from the $I - V_a$ characteristics for a set of T.

It is seen from (3) that $\beta = 1$ for $\alpha = 0$ and $\alpha = 1$ and that it has a minimum ≈ 0.8 at $\alpha = 0.4$; for large values of α one has $\beta = \alpha$, and thus β becomes large. The critical temperature at which β starts to become greater than unity depends entirely on the valve characteristics and can be very different for tubes of different construction.

For the valve used by MacDonald I found from the characteristics that ϵ did not appreciably vary with T and for the limit $V_a \to 0$ I found by extrapolation $\epsilon_0 \approx 0.001$. With this value I deduced a critical temperature for that particular valve of about 2100° K. However, it is seen from (4) that a valve with a smaller value of ϵ_0 and the same $I_s - T$ dependence might easily raise the critical temperature to a value well outside the range accessible to experiment. In such a case one would expect β to have a value between 0.8 and 1 which would explain the observations of Knol and Diemer with the directly-heated electrodes. For still smaller values of ϵ, β should again become equal to 1 as pointed out above.

Thus the results of Knol and Diemer, as far as they are set out in their note, cannot be used as evidence for or against my theory. MacDonald's criticism against this theory will be met elsewhere in the near future.

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R. FÜRTH.

REFERENCES

[1] K. S. Knol and G. Diemer, *Wireless Engineer*, October 1949.
[2] D. K. C. MacDonald, *Wireless Engineer*, Vol. 24, p. 30, 1947.
[3] R. Fürth, *Nature*, Vol. 60, p. 832, 1947.
[4] R. Fürth, *Proc. roy. Soc.* A. Vol. 192, p. 593, 1948.
[5] D. K. C. MacDonald, *Proc. roy. Soc.* A. Vol. 195, p. 225, 1948.

Network Theorem

SIR,—The theorem discussed by E. R. Wigan in the correspondence section of December has been proved by W. Cauer* and applied to various problems in filter design.

Brussels.

V. BELEVITCH.

* W. Cauer, "Theorie der linearen Wechselstromschaltungen", Becker & Erler, Leipzig, 1941, p. 379.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

625 729.—Production of a saw-tooth deflection current, wherein the coil is shunted by a diode and a variable unbypassed resistor to improve linearity.

Marconi's Wireless Telegraph Co. Ltd. (assignees of O. H. Schade). Convention date (U.S.A.) January 18th, 1945.

625 741.—Saw-tooth generator of the capacitance charge type using a multi-electrode valve, one electrode of which is biased during the charging cycle to prevent leakage from the capacitor such as might destroy linearity.

C. L. Faudell. Application date August 13th, 1946.

626 132.—Frame synchronization is effected, in a system in which the frame synchronizing pulses are interrupted at double the line-pulse frequency, by an impedance network of specified characteristics.

Philips Lamps Ltd. Convention date (Netherlands) June 15th, 1945.

626 714.—A method of television reception using a screen consisting of transparent crystalline material which becomes opaque under radiation wherein the radiation produces discolouration dependent on scanning intensity.

Scophony Ltd. and G. Wikkenhauser. Application date August 17th, 1946. *(Addition to No.* 513 776.)

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

625 063.—Hum is reduced in mains-operated transmitters by recording the hum during non-signal conditions and playing back to suppress or reduce the hum in the output.

Philips Lamps Ltd. Convention date (Netherlands) November 9th, 1945.

625 268.—A reactance-valve frequency- or phase-modulating circuit wherein feedback is applied in respect of the modulation but not of the carrier, to secure linearity of the overall characteristic.

Philips Lamps Ltd. Convention date (Netherlands) February 25th, 1943.

625 791.—Phase-shift or modulation is effected by a multiple-electrode valve to the first grid of which an alternating input is applied while a control or modulating potential is applied to a second control grid, the output being taken from the anode.

Radio Corporation of America (assignees of W. C. Morrisson). Convention date (U.S.A.) June 13th, 1946.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

625 478.—A multi-channel time-sharing communication system for moving vehicles having terminal and repeater stations wherein the received signals at the repeater stations are timed correctly to prevent interference.

Standard Telephones and Cables Ltd. (assignees of E. M. Deloraine and D. D. Grieg). Convention date (U.S.A.) March 28th, 1946.

625 623.—Selective call system for radio intercommunication using any of several frequencies in which a frequency not so used is employed to transmit a calling code and a code indicating the frequency to be used.

Standard Telephones and Cables Ltd. (assignees of Le Materiel Telephonique, S.A.). Convention date (France) September 1st, 1945.

CONSTRUCTION OF ELECTRON-DISCHARGE DEVICES

624 731.—A valve grid is formed of a platinum alloy containing 0.1% beryllium, apparently to reduce secondary emission.

The B.T.H. Co. Ltd. Convention date (U.S.A.) July 3rd, 1945.

625 259.—Velocity-modulation device in which the volume of the resonator is controlled by axial displacement of a stepped outer sleeve.

Standard Telephones and Cables Ltd. (assignees of Le Material Telephonique, Societe Anonyme). Convention date (France) December 22nd, 1943.

625 271.—A thermionic valve of the beam-deflection type with magnetic and electrical fields to cause the electrons to travel in a succession of looped paths to reduce the average speed through the valve and increase deflection sensitivity.

G. H. Metson. Application date August 1st, 1946.

625 275.—Magnetron for continuous-wave operation at high output having an anode and a coaxial cathode around it, the two being surrounded by a cylindrical housing divided into separated concentric parts.

Westinghouse Electric International Company. Convention date (U.S.A.) August 28th. 1945.

625 901.—U.h.f. amplifier valve including two aligned cavity resonators communicating by a tube and each having a secondary-emission cathode.

Compagnie Generale De Telegraphie Sans Fil. Convention date (France) February 14th, 1941.

SUBSIDIARY APPARATUS AND MATERIALS

624 767.—A closed-core inductor having a U-shaped iron-dust core and a bridge secured with a 'magnetic' adhesive (c.f. Spec. No. 616 249).

Communication Engineering Pty. Ltd. Convention date (Australia) September 9th, 1944.

625 188.—A piezo-electric crystal mounting having three metal spots around the periphery, of such thickness that when clamped between metal plates they function as energizing electrodes but are not in physical contact with the crystal.

The General Electric Company Ltd., G. M. Wells and L. Rollin. Application date March 3rd, 1947.

625 378.—Measurement of power flowing in one direction in a u.h.f. transmission line, by means of an auxiliary transmission line coupled to a rectifier and galvanometer.

Sperry Gyroscope Co. Inc. Convention date (U.S.A.) August 18th, 1943.

625 983.—A signal generator for producing 'pip' signals rich in harmonics comprises an oscillator of the desired recurrence frequency and including a highly-damped inductance having a resonant frequency much higher than the frequency of the oscillator.

W. J. O'Brien. Convention date (U.S.A.) August 27th, 1945.