WIRELESS ENGINEER

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Alexander S. Popov

IN our Editorial of January 1948, we referred to the Popov memorial number of *Electritchestvo* published in Russia in 1925. It was published in both Russian and French, and in order that there should be no question of mistranslation, we gave several quotations in the original French. Among these was the statement which, according to his colleague Professor Lébédinsky, Popov made after visiting wireless stations in France and Germany in 1899; viz., 'nous n'étions pas beaucoup en retard sur les autres.' V. L. Rastorgoueff has drawn our attention to the fact that this is not strictly in agreement with the Russian account of what he said. This sentence by itself gives the impression that Popov was referring to something in the past, whereas the Russian statement refers to the present. Mr. Rastorgoueff, who has gone to considerable trouble in the matter, finds that the statement was made in a letter written by Popov from Paris to his assistant Rybkin, who quoted it in an article published in 1919. Mr. Rastorgoueff gives the following translation of the quotation: 'I have seen and learnt all that was possible. I have spoken to Slaby, seen his apparatus, and visited Blondel at the station in Boulogne. In a word, I have learnt all that was possible and I see that we have not fallen much behind the others.' This is supported by the fact that Lébédinsky in another part of his article says, 'Nous pouvons répéter encore ses paroles: "Nous ne sommes pas restés trop en arrière".' It is also supported by the wording, 'En 1899, ayant visité les premières installations, radiotélégraphiques d'Allemagne et de France, Popov en tira la conclusion "Que nous n'étion pas beaucoup en retard sur les autres".' We suspect that the quotation marks are here out of place and that this is Lébédinksy's account of of what Popov said and not a reproduction of his actual words.

There appears to be no doubt that Popov's remark referred to the state of affairs at the time of his visit and not in any way to any priority of invention three or four years before.

A Confusion of Dates

In the *Soviet News* of March 17 there is a reference to the 90th anniversary of the birth of Popov and to an interview with Professor Shatelen who as a third year University student in 1887 accompanied Popov on a solar-eclipse expedition. He is reported to have said, 'In the spring of 1895 Popov demonstrated his remarkable atmospheric discharge recorder at a sitting of the Russian Physical Chemical Society at St. Petersburg University. I was present at this historic sitting when a telegram was transmitted by wireless telegraph for the first time in the world.' This is not a quotation from Professor Shatelen but a reviewer's report, worded so as to give quite a false impression, by confusing two occasions. Popov certainly demonstrated his atmospheric apparatus in May 1895; in September 1895, he replaced the lightning flash recorder by a Morse inker and in March 1896, he demonstrated the transmission of the words 'Heinrich Hertz' from another part of the University about 250 metres away. This was the historic sitting referred to and it was in the spring of 1896 and not 1895. It was in February 1896, that Marconi came from Italy to London in order to take out his patent.

J. B. Thornton's Letter

In our April issue we published a letter by

J. B. Thornton of Australia containing some criticism of our Editorial of January 1948, but written, we suspect, without any knowledge of the subsequent Editorial of May 1948. If he turns to this second Editorial he will find that we went to great trouble to fix definitely the details and dates of the various steps in Popov's experiments and publications. We expressed our indebtedness to V. L. Rastorgoueff for supplying us with translations from Russian documents which threw considerable light on the subject. We feel sure that if he studies this second Editorial he will find that every effort was made to answer the very questions that he now raises. He will also see the reasons for the doubts and uncertainties in connection with Popov's contributions to the subject.

We can quite understand that Bailey and Landecker, writing in Australia, were at a disadvantage because of the difficulty of consulting Russian publications of fifty years ago. We do not doubt for a moment that their Paper was the result of careful study of the available information, but we still think it possible that Professor Ashby, who is a botanist, '*may* have been unwittingly misled by it.' There is one statement made by Ashby which we quoted in the January Editorial, to which we feel that some further reference should be made. It is, 'If he did in fact transmit and receive Hertzian waves over a distance of five kilometres, and publish that fact in January 1896, then it can be claimed that he used Hertzian waves to *transmit a message* before Marconi lodged his patent application.'

Now although it has nothing to do with the transmission and reception of Hertzian waves over a distance of five kilometres, or with anything published in January 1896, we showed in the May Editorial that there is fairly conclusive evidence that something did occur to justify the statement that he used Hertzian waves to transmit a message *before Marconi lodged his patent application*. Although Marconi came to England in February 1896, with the object of applying for a patent, the application was not lodged until June, and it was in March that Popov is reported to have transmitted the words 'Heinrich Hertz' from one part of the University to another, a distance of 250 metres, and recorded them on a Morse inker. Although the recording of distant lightning flashes or electric sparks can hardly be described as transmitting a message or even radio-communication, the transmission and recording of the words, 'Heinrich Hertz' certainly can. As we pointed out in the May Editorial, Popov's assistant, Rybkin, in a booklet on the subject, says that Popov was warned by the Naval authorities not to publish details of the experiment, and no mention is made of it in a long letter that he wrote to Ducrétet in 1897 describing his experiments. This is all the more surprising because Marconi's experiments and patent were then known. It is strange things like this that have made it so difficult to determine with certainty the dates and details of his experimental demonstrations.

There is an important point, however, that the advocates of Popov are apt to ignore, and that is, that a month before the transmission of 'Heinrich Hertz' over a distance of 250 metres, Marconi had brought his experiments in Italy to the stage where he felt justified in coming to England and taking out a patent.

We agree with the definition of radio-communication as 'the process (or method) of communicating intelligence at a distance by means of free Hertzian waves.' but if the reception of a lightning flash or spark is to be regarded as communicating intelligence—as it may be from one point of view—then the claim for Popov as against Marconi is very much strengthened on the ground of prior publication, but at the same time a number of other candidates enter the field. The reception of sparks by means of free Hertzian waves had been carried out by many experimenters since Hertz's publication in 1888, but perhaps the words 'at a distance' would be said to rule them out. We doubt, however, whether the ordinary individual would regard a lightning flash or a spark as the communication of intelligence or the transmission of a message. The transmission and reception of a spark—a simulated lightning flash—could, however, be a pre-arranged signal that something had occurred or was about to occur, and would then be correctly described as communicated intelligence or a message in a prearranged code. From this point of view our remark in the January Editorial that 'to say that Popov has a prior claim over Marconi as an inventor of radio-communication is to show no conception of what constitutes radio-communication' was certainly unjustified; we were using the term radio-communication in the more generally accepted sense.

As we said of Popov in the first Editorial, 'one must honour his memory as that of a Russian scientist who did brilliant experimental work in connection with Hertzian waves,' but whether 'he would be the first to disclaim precedence over Marconi as the inventor of radio communication' as we said, would depend on his interpretation of 'radio-communication.'

G. W. O. H.

CAR-IGNITION INTERFERENCE

By W. Nethercot, M.A., B.Sc., F.Inst.P.

(British Electrical and Allied Industries Research Association)

1. Introduction

IT is well known that ignition interference to radio reception can be severe and widespread in the short-wave bands and, in particular, to the television service. Its characteristics and its effect on receiving equipment have been studied extensively at frequencies up to 100 Mc/s and satisfactory methods of suppression have been developed.[1,2]

At one time it was thought that the strength of ignition interference decreased progressively with increase of frequency above 100 Mc/s. Recent measurements have shown, however, that although it varies considerably over relatively narrow frequency bands, the average level remains substantially constant up to 650 Mc/s, which is the highest frequency at which measurements have been made, at least so far as the author is aware.[3,4]

Eaglesfield has put forward a theory to explain the fact that the frequency spectrum of ignition interference is extremely wide and without gaps.[5] He finds the radiated field to be an impulse, short compared with the period of the carrier frequency up to frequencies of hundreds of Mc/s. In the author's opinion, however, there are serious objections to Eaglesfield's over-simplified treatment of the problem. Furthermore, some of the implications are at variance with experimental observation. An alternative theory is presented in this paper which is in at least qualitative agreement with experimental fact.

2. The Ignition Circuit

(2.1) *The Spark Current*:

Basically the ignition circuit is as shown in Fig. 1. L is the inductance of the h.t. winding of the coil or magneto and C_1 is its self-capacitance. l_1 and l_2 are the cables joining the coil to the distributor and the distributor to the sparking plug. l_1 is, of course, absent for magnetos with integral distributors. G_1 is the distributor gap and G_2 the sparking-plug gap. C_2 is the self-capacitance of the sparking plug. The capacitance associated with the distributor gap is not shown.

The circuit is shock excited by the breakdown of the gaps G_1 and G_2. G_1 breaks down at about 3 kV and G_2 between about 3 and 12 kV, depending on engine conditions. When the sparking-plug gap breaks down, the energy stored in the capacitances and the h.t. cables, all of which are charged to the breakdown voltage of the plug gap, is rapidly dissipated giving rise to the so-called capacitance component of the ignition spark. A succession of such discharges may occur, due to current chopping, before the final inductive or flame discharge occurs. The latter makes no contribution to the radio interference, except possibly at the lowest frequencies, and so can be neglected.

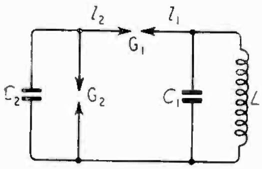

Fig. 1. *Basic circuit of ignition system.*

Referring to Fig. 1, it is seen that the ignition circuit, disregarding for the moment the distributor gap and the self-capacitance of the sparking plug, reduces essentially to a transmission line terminated by a capacitor at one end and a gap at the other with the engine block as an 'earth' plane. In many engines the h.t. cables run more or less parallel to the engine block and so, neglecting resistance and conductance, the line may be considered to have a surge impedance $Z = \sqrt{l/c}$ where l is the inductance and c the capacitance per unit length of line.

For a conductor parallel to an earth plane and whose distance from it is large compared to its diameter :—

$$l = \{2 \log_e (2h/r)\}\, 10^{-9} \text{ henry per cm}$$

and

$$c = 1/\{18 \log_e (2h/r)\}\, 10^{-11} \text{ farad per cm}$$

where h is the distance of the conductor from the earth plane and r is its radius.

$$\text{Hence } Z = 60 \log_e (2h/r) \text{ ohms} \quad .. \quad .. \quad (1)$$

As the h.t. cables do not run truly parallel to the engine block their surge impedances are not constant. However, from equation (1) it is clear that h can be varied considerably

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without causing much change in Z and so it is reasonable to consider Z as constant.

Now when the plug gap breaks down, current flows into it from the charged cable, each element of which contributes its share, starting from the plug end. In other words, a travelling wave is set up in the cable which moves from the plug towards the coil. The effect of the breakdown of the plug gap, assuming this to occur instantaneously, can be simulated mathematically by injecting a voltage $-E\mathbf{1}$ across the gap, where E is the breakdown value and $\mathbf{1}$ is the Heaviside step operator. This sends a current wave of magnitude $-E/Z$ down the cable as shown in Fig. 2 (a). When the wave reaches point B it is reflected back along the cable, the reflection operator being—

$$-\frac{1/pC_1 - Z}{1/pC_1 + Z} = 1 - \frac{2\alpha}{p+\alpha} \quad \ldots \quad \ldots \quad (2)$$

where $\alpha = 1/(ZC_1)$.

At A it undergoes another reflection, the reflection operator in this case being unity. Successive reflections occur at the terminations B and A and produce a current through the gap which is the sum of all the incident and reflected waves at A. This is shown by the lattice diagram in Fig. 2 (b). T is the time for the wave to travel along the cable and back and is equal to $2x/v$ where x is the length of the cable and v is the velocity of light.

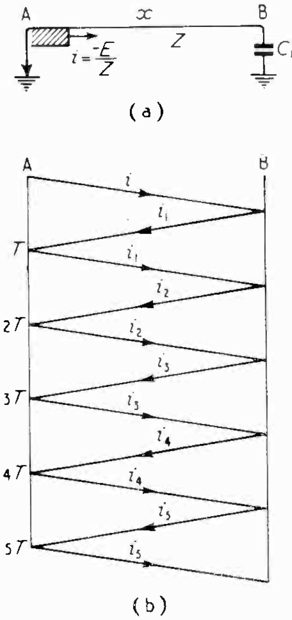

Fig. 2. *Reflection of waves along the ignition cable.*

The gap current is therefore :—

$$I = i + 2i_1\,(t-T) + 2i_2\,(t-2T) + \ldots + 2i_n\,(t-nT) \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where i is the initial current at gap breakdown and $(t - nT)$ denotes that the function starts at time $t = nT$, counting from the instant of gap breakdown.

Now—

$$i_1 = \{1 - 2\alpha\,/\,(p+\alpha)\}\ i,$$
$$i_2 = \{1 - 2\alpha\,/\,(p+\alpha)\}^2\, i,$$
$$i_n = \{1 - 2\alpha\,/(p+\alpha)\}^n i \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Writing $2\,\alpha\,/\,(p+\alpha) = \beta$, i_n becomes—

$$\left[1 - n\beta + \frac{n(n-1)}{2\,!}\beta^2 - \frac{n(n-1)(n-2)}{3\,!}\beta^3 + \ldots\right] i \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

It can be shown that :—

$$\{1 - 2\,\alpha\,/\,(p+\alpha)\}^n\,\mathbf{1} = 2^n\left[1 - e^{-\alpha t}\left\{1 + \alpha t + \frac{\alpha^2 t^2}{2\,!} + \frac{\alpha^3 t^3}{3\,!} + \ldots + \frac{\alpha^{n-1}\,t^{n-1}}{(n-1)\,!}\right\}\right]\mathbf{1} \quad \ldots \quad (6)$$

Since $i = -E/Z$ the gap current can be computed from Equations (3), (5) and (6) if the breakdown voltage of the gap and the parameters of the ignition system are known. Reasonable values to assume for the length of cable between the coil and sparking plug and its average distance from the engine block are 20 in and 3 in respectively. The diameter of the cable conductor is approximately 0.04 in. These give $Z = 300$ ohms and $T = 3 \times 10^{-9}$ sec. The self-capacitance of the coil is of the order of 50 pF.

Fig. 3(a) shows the current through the plug gap, using the above values and taking the breakdown voltage as 5 kV. The calculation becomes laborious after the first few reflections and so it has been taken only to the first current zero. It is seen that the current consists of a series of abrupt steps but that the envelope is oscillatory with a frequency of approximately 30 Mc/s. The current at the first maximum is 75 amperes.

The effect of shortening the h.t. cable is to reduce the time interval between the steps, increase the peak value of the current and increase the oscillation frequency of the envelope.

Fig. 3(b) shows the current when the length of the cable is reduced to 10 in. Here the peak current is approximately 100 amperes, and the oscillation frequency about 40 Mc/s.

A reduction in the surge impedance of the cable has the effect of increasing both the peak current and the oscillation frequency as is shown in Fig. 3(c) which is for a cable 20-in long and surge impedance of 100 ohms. As mentioned before, however, the distance between the cable and the engine block has to be varied considerably before much change occurs ; for example, if it is reduced to 0.5 in the surge impedance is reduced only to 200 ohms ; conversely increasing it to 6 in raises the surge impedance only to 350 ohms. The disposition and the lengths of the ignition cables in motor vehicles are such that an oscillation frequency of the current envelope of 30 to 50 Mc/s may be expected. The peak current is a function also of the breakdown voltage of the sparking plug and so will vary widely ; values up to 300 amperes are possible.

It is interesting to note that the peak current as recorded oscillographically by the author[2] on an ignition system whose parameters fall within the range considered above is of the same order as predicted by the theory.

So far no account has been taken of the attenuation of the waves along the cable. A reasonable assumption is to put the attenuation factor equal to $e^{-\gamma t}$ where γ is a constant. This results in the attenuation of the envelope in approximately the same way.

It is clearly impossible for an oscillograph with time sweeps of the order of a microsecond to resolve the individual steps in the current wave. What is recorded is the peaky oscillatory envelope.

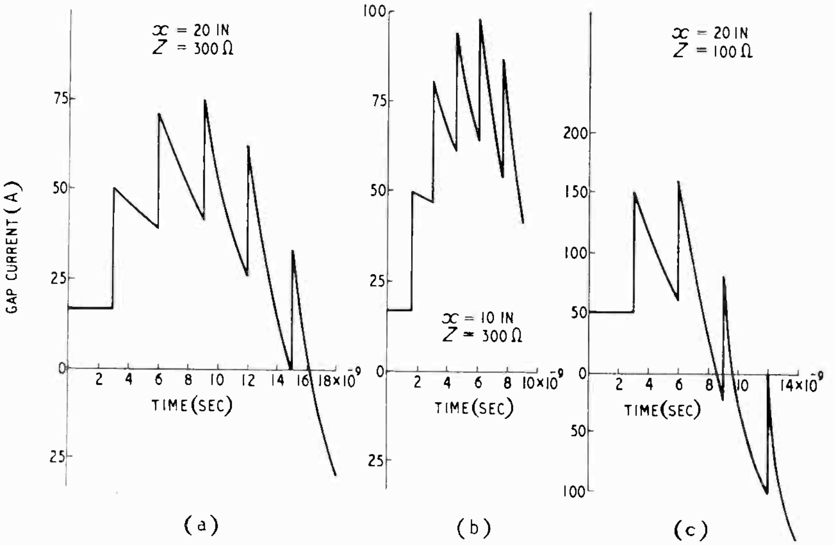

Fig. 3. *Current through the sparking-plug gap.*

2.2) *Radiation from the Spark Current :*

The radiation field from a current of the form shown in Fig. 3 is much more complex than Eaglesfield's simple pulse, as is also its effect on a bandpass receiver[6]. However, it is clear that the radiation will have a continuous frequency spectrum although its intensity will not be independent of the frequency. Peaks should occur at the envelope frequency, which has been shown to be in the region of 30-50 Mc/s, and also at the frequency corresponding to the time interval between successive steps in the current wave. For a cable 20-in long this time interval is 3×10^{-9} second so that the frequency is approximately 300 Mc/s.

The above treatment neglects the effect of the distributor gap. When this breaks down travelling waves emanate from it towards the coil and the sparking plug. These produce currents with different step intervals and envelope frequencies and so complicate the radiation spectrum. Also most motor vehicles have multi-cylinder engines in which the individual h.t. ignition cables are not all equal in length. Other factors not considered in the simplified treatment, such as the screening effect of the vehicle body, have also to be taken into account. The latter effect is not amenable to calculation and so it seems unprofitable to attempt a quantitative correlation between theoretically derived values of the radiation intensity and measurements made with interference-measuring equipment.

However the theory is in at least qualitative agreement with experimental observation as regards the frequency distribution of the radiation and in quantitative agreement as regards the magnitude and form of the current through the sparking plug at gap break-down.

The upper limit of the frequency spectrum of the radiation is determined by the rate at which the sparking plug and distributor gaps become conducting. Theoretical considerations indicate that this should take place in a very short time, probably of the order of 10^{-9} second. The collapse of voltage at breakdown is too rapid for accurate measurement, even with a high-speed oscillograph. Some tests made by the author on the breakdown under impulse voltages of a 1-mm gap (which is of the same order as a sparking-plug gap) have shown the time to be certainly less than 4×10^{-9} second.

3. Effect of Suppression Resistors

(3.1) *Single Resistor in the Coil-Distributor Cable*

A single resistor inserted in the coil-distributor cable is the simplest method of suppressing ignition interference. It is fitted at the distributor end of the cable but for the present argument it is assumed to be at the coil. Then from Fig. 2(a), neglecting the distributor gap, the reflection operator at the cable end remote from the sparking plug becomes—

$$-\frac{1 + pC_1(R - Z)}{1 + pC_1(R + Z)} \quad \dots \quad \dots \quad \dots \quad (7)$$

where R is the value of the resistor. Since

R is usually not less than 5,000 ohms, $R \gg Z$ and (7) becomes simply -1. This approximation is valid for low values of t. Fig. 4 shows the gap current for a cable 20-in long and 300-ohms surge impedance. The 30-50 Mc/s oscillation has disappeared and the current consists of a succession of square-topped pulses of alternating polarity. Although the magnitude of the peak current is reduced, that of the steps remains unaltered. This will produce continuous radiation over a wide frequency band, but some change in the intensity-frequency characteristic from that given without resistance is to be expected in view of the change in shape of the current wave. The disappearance of the oscillatory envelope suggests that the intensity of the radiation at 30-50 Mc/s will be reduced. This is what actually happens; extensive measurements have shown the single resistor to be often quite effective in suppressing ignition interference at frequencies up to that of television. Above this its efficiency falls off rapidly and it is usually completely ineffective at frequencies above 80-100 Mc/s.

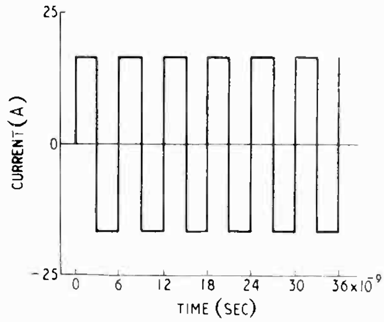

Fig. 4. *Effect of resistor in the coil-distributor cable on the current through the sparking-plug gap.*

The energy stored in the h.t. cable is much less than in the self-capacitance of the coil and so attenuation will cause the current waves in the cable to damp out quickly and leave the low-current, comparatively long time discharge of the coil self-capacitance through the suppression resistor. Thus the gap current, once the waves along the cable have died out, becomes

$$e^{-t/Rc}$$

3.2 *Resistor at the Sparking-Plug End of the H.T. Cable.*

In the case of a resistor inserted at the sparking-plug end of the h.t. cable the magnitude of the initial current wave along the cable is $EZ/(R + Z)$. As R is large compared with Z the initial current is small and so, therefore, are the amplitudes of the successive reflected waves. This means that the intensity of the radiation is reduced at all frequencies. The larger R becomes the smaller is the current and so theoretically the intensity of the radiation should decrease progressively with increase of R.

The current waves generated by the breakdown of the distributor gap can be reduced by inserting resistors at the distributor ends of the coil-distributor and distributor-sparking plug cables. This is why resistors have to be inserted at the distributor terminals as well as at the sparking plugs in order to obtain maximum suppression of ignition interference.

4. The Residual Spark-gap Radiation

It has been established experimentally that ignition interference decreases progressively with increase of resistance only up to a value of about 25,000 ohms, whereas the theory outlined above predicts a continuous decrease. However, the radiation from the spark gaps themselves has to be taken into account.

The self-capacitances of the electrode assemblies of the gap, although small, are not entirely negligible and they are charged to a high voltage. The very rapid collapse of voltage at breakdown is accompanied by high current discharges in the localized gap circuits and it is suggested that the radiation resulting from these currents is responsible for the fact that there is a limit to the suppression which can be obtained with resistors.

There is evidence to confirm this view for tests have shown that a further substantial reduction of the radiation can often be obtained by enclosing the distributor in a screening can after the maximum suppression has been obtained with resistors.

5. Conclusions

The wideband continuous radiation from the ignition circuit is due to travelling waves set up in the h.t. cables when the distributor and sparking-plug gaps break down. The current through the sparking-plug gap consists of a series of very steep fronted steps, the intervals between which are determined by the time the waves take to travel twice the length of the h.t. cables. The envelope of these current steps is oscillatory and its frequency lies between 30 and 50 Mc/s.

The theory is in at least qualitative agreement with experimental fact and there is satisfactory quantitative agreement between the theoretical and experimentally measured values of the spark current.

Resistors at the sparking-plug and distributor terminals should give suppression over the whole frequency band but a single resistor in the coil-distributor cable is effective only at frequencies up to about the television region.

Remanent interference comes from the local circuits in the sparking plugs and distributor and normally requires screening for its suppression.

6. Acknowledgment

The author wishes to thank the Director of the British Electrical and Allied Industries Research Association for permission to publish this paper.

REFERENCES

[1] A. J. Gill and S. Whitehead, "Electrical Interference with Radio Reception," *J. Instn elect. Engrs*, Vol. 83, p. 345, September 1938.

[2] W. Nethercot, "Radio Interference from the Ignition Systems of Motor Vehicles," *Wireless World*, October and December 1947.

[3] R. W. George, "Field-Strength of Motor-Car Ignition between 40 and 450 Megacycles." *Proc. Inst. Radio Engrs*, September 1940.

[4] B. G. Pressey and G. E. Ashwell, "Radiation from Car Ignition Systems," *Wireless Engineer*, January 1949, Vol. 26.

[5] C. C. Eaglesfield, "Motor-Car Interference," *Wireless Engineer*, October 1946, Vol. 23.

[6] S. Whitehead, "The Transient Response of Idealised Networks applied to Interference and Discharge Measurements." E.R.A. Report Ref. M/T80.

MEASUREMENT OF MICROWAVE-TRANSMISSION EFFICIENCY

By A. L. Cullen, B.Sc., A.C.G.I.

SUMMARY.—The transmission efficiency of any transmission device is defined as the ratio (power out)/(power in) when the device is inserted in an otherwise matched transmission system. It is shown that if the reflection coefficient for waves incident on the normal output end of the device, with the input end closed by a movable short-circuiting plunger, is plotted in the complex plane for several positions of the plunger, the points so obtained will lie on a circle the radius of which is equal to the transmission efficiency as defined above.

1. Introduction

ONE of the most important characteristics of any transmission device is its transmission efficiency, defined as above under normal operating conditions, or as the ratio of the powers supplied to the load of an otherwise matched system with and without the device inserted. It may be measured directly if two power indicators of known relative sensitivity are available, but this method is not always convenient, particularly if a band of frequencies is to be covered, for then the power indicators must be calibrated over the whole band. This difficulty does not arise with the present method.

2. Transmission Efficiency

As far as external characteristics are concerned any device inserted in a waveguide-transmission system can be completely specified by three complex quantities, which may conveniently be two reflection coefficients and a transmission coefficient. (The reciprocity theorem shows that only one transmission coefficient is needed, but in general the reflection coefficient will depend on the direction of the incident wave, and two reflection coefficients will be required.)

Referring to Fig. 1, if a wave E_1^+ is incident on end 1 of the device, the reflected field will be $E_1^- = \rho_1 E_1^+$, and the transmitted field at end 2 will be $E_2^+ = \tau E_1^+$, a pure travelling wave.*

If for simplicity, we assume that the incident wave carries unit power, the power in the reflected wave will be $|\rho_1|^2$, so that the net power input is $1 - |\rho_1|^2$, and the power output is $|\tau|^2$.

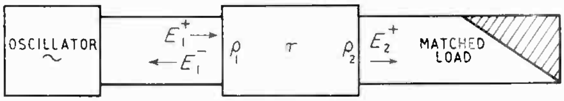

Fig. 1. *Reflection and transmission coefficients.*

Denoting the transmission efficiency in the direction $1 \to 2$ by η_1, we have :

$$\eta_1 = \frac{|\tau|^2}{1 - |\rho_1|^2} \quad \quad (1)$$

Similarly, the transmission efficiency η_2 in the direction $2 \to 1$ is given by :

$$\eta_2 = \frac{|\tau|^2}{1 - |\rho_2|^2} \quad \quad (2)$$

Thus if the magnitudes of the reflection and transmission coefficients are known, the transmission efficiencies can be found. We shall now consider the possibility of finding these quantities from the results of measurements of the *modified* reflection coefficients ρ_1' and ρ_2' when ends 2

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*The input and output waveguides need not be identical, provided that E is normalized in such a way that incident power varies as $|E|^2$ with the same constant of proportionality in both guides.

and 1 respectively are terminated by a movable short-circuiting plunger in place of the matched load.

3. Calculation of Modified Reflection Coefficient

We shall first calculate the modified reflection coefficient ρ_2' at end 2 of the device with a short-circuiting plunger connected to end 1.

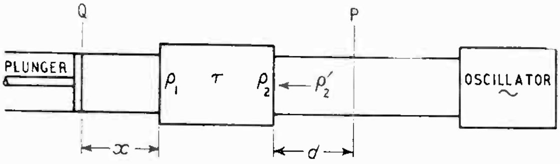

Fig. 2. *Arrangement for measurement of modified reflection coefficient ρ'_2.*

Referring to Fig. 2, let the plunger be situated at Q, distant x from end 1, and let P, distant d from end 2, be any point on the oscillator side of the device. If the field at P can be expressed in terms of d, x remaining constant, the effective reflection coefficient for that particular value of x can be found from the resulting formula. We shall calculate the field at P by use of the superposition theorem in the following way. Let us first imagine the plunger to be replaced by a non-reflecting termination and consider fields at P and Q resulting from a wave incident on end 2 from the right, whose value at $d = 0$ is E_2^+. If these are denoted by E_p' and E_q' respectively, then clearly :

$$\left.\begin{aligned} E_p' &= E_2^+ \,(e^{j\beta d} + \rho_2 e^{-j\beta d}) \\ \text{and } E_q' &= \tau E_2^+ e^{-j\beta x} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3)$$

Now interchange the oscillator and non-reflecting termination and consider the effect of a wave, incident on end 1 from the left, whose value at $x = 0$ is E_1^+. The fields E_p'' and E_q'' under these conditions are given by :

$$\left.\begin{aligned} E_p'' &= \tau E_1^+ \, e^{-j\beta d} \\ E_q'' &= E_1^+ \,(e^{+j\beta x} + \rho_1 e^{-j\beta x}) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (4)$$

It follows from the superposition theorem that any linear combination of equations (3) and (4) is also a possible solution, and in particular the sum is a solution, and represents a standing-wave pattern of the desired character. Accordingly, if E_p and E_q denote the total fields at P and Q respectively with the plunger at Q we can write :

$$\left.\begin{aligned} E_p &= E_p' + E_p'' \\ E_q &= E_q' + E_q'' \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

—and we must then arrange that $E_q = 0$. Substituting from (3) and (4) in the second equation (5), this condition gives the following relation between E_1^+ and E_2^+ :

$$E_1^+ = -\tau E_2^+/(e^{j2\beta x} + \rho_1) \quad \ldots \quad \ldots \quad (6)$$

Substituting this value of E_1^+ in equation (4), we get :

$$E_p'' = -\tau^2 E_2^+ \, e^{-j\beta d}/(e^{j2\beta x} + \rho_1)$$

—and combining this result with the expression for E_p' given in equation (3) we see that the total field E_p at the point P as given in equation (5) becomes :

$$E_p = E_2^+ [e^{+j\beta d} + \rho_2 e^{-j\beta d} - \tau^2 e^{-j\beta d}/(e^{j2\beta x} + \rho_1)]$$

This expression may be rewritten in the form :

$$E_p = E_2^+ [e^{+j\beta d} + \rho_2' \, e^{-j\beta d}]$$

—where

$$\rho_2' = \rho_2 - \tau^2/(e^{j2\beta x} + \rho_1) \quad \ldots \quad \ldots \quad (7)$$

and this modified equation clearly exhibits the total field at P in terms of the incident and reflected components, the effective reflection coefficient being given by equation (7) for any position x of the plunger. We note that ρ_2' may be regarded as a bilinear transformation of $e^{j2\beta x}$, and as this quantity describes a circle in the complex plane as x varies, it follows, by a well-known property of such transformations, that ρ_2' also describes a circle as x varies.

4. Circle of Modified Reflection Coefficient

In the first place, let us rewrite equation (7) thus :

$$\frac{1}{\rho_2' - \rho_2} = -\frac{\rho_1}{\tau^2} - \frac{e^{+j2\beta x}}{\tau^2} \quad \ldots \quad \ldots \quad (7a)$$

The locus of $\dfrac{1}{\rho_2' - \rho_2}$ in the complex plane is clearly a circle of radius $\left|\dfrac{1}{\tau}\right|^2$ and with centre at $-\dfrac{\rho_1}{\tau^2}$. If ρ_2 is determined by a subsidiary experiment with a matched load in place of the plunger, and ρ_2' measured as described above, this procedure would make it possible to determine $|\tau|$ and $|\rho_1|$, and the phase angle between ρ_1 and τ^2. If ρ_1 is also determined by a subsidiary experiment, the phase angle of τ can be found, so that all three quantities, ρ_1, ρ_2, and τ, are completely determined, with some additional information which may be useful as a check on the accuracy of the experimental results. However, the calculation of $1/(\rho_2' - \rho_2)$ for each experimental point is wasteful of time and it is desirable to be able to make use of the circular locus of ρ_2' directly.

We therefore invert the circle of equation (7), using the following well-known theorem in complex algebra :

If $w = 1/z$, and z describes a circle of radius γ with centre at α (γ real, α complex) then w describes a circle of radius σ with centre at β where :

$$\left.\begin{aligned} \sigma &= \frac{\gamma}{|\alpha|^2 - \gamma^2} \\ \beta &= \frac{1}{\alpha} \frac{|\alpha|^2}{|\alpha|^2 - \gamma^2} \end{aligned}\right\} \quad \ldots \quad (8)$$

Applying this result to (7a), it is seen that the locus of $\rho_2' - \rho_2$ as x varies in a circle of radius σ_2 and centre β_2 given by :

$$\sigma_2 = \frac{|\tau|^2}{1 - |\rho_1|^2} \quad \ldots \quad (9)$$

and

$$\beta_2 = \frac{\tau^2}{\rho_1} \cdot \frac{|\rho_1|^2}{1 - |\rho_1|^2} \quad \ldots \quad (10)$$

Comparing (9) with (1) we see that the radius of this circle is precisely equal to the transmission efficiency η_1 of the device, in the direction $1 \rightarrow 2$, which it is required to find. It is sufficient to plot ρ_2' in the complex plane, for the constant ρ_2 will not affect the radius of the circle, but only the position of its centre. Similarly, plotting ρ_1' gives a circle of radius η_2.

5. Experimental Verification

In order to verify the theory, measurements were made at a wavelength of about 10 cm in 1-in × 3-in rectangular waveguide carrying the H_{01} mode. The device measured consisted of a short length of the same size of waveguide with a 1-in × ½-in strip of wood about 6-in long, running diagonally across the broad face of the guide, and held in position by a 2 B.A. screw passing through a hole tapped in the centre of the broad face, and bearing on the ½-in edge of the wooden strip. A short length of 2 B.A. studding passing through a second tapped hole near one end of the guide penetrated into the guide sufficiently to ensure appreciable asymmetry of the device. The two ends of the device were arbitrarily designated 1 and 2, and end 1 was connected to a variable position short-circuiting plunger of the non-contact type, while end 2 was connected to the oscillator and a standing-wave indicator. The standing-wave ratio and position of the minimum of the standing-wave pattern were measured for various plunger settings, mostly in about 0.5-cm steps, with a few intermediate points where necessary. The modified reflection coefficient ρ_2' could be calculated from these results in the usual way.

The experiment was then repeated with the waveguide reversed, giving ρ_1', the modified reflection coefficient of end 1. Fig. 3 shows the experimental results, together with the best possible circles through the experimental points.

The asymmetry of the device is exhibited very clearly by these results, which show that in the direction $1 \rightarrow 2$ the transmission efficiency η_1 is 0.316, whilst in the direction $2 \rightarrow 1$ the transmission efficiency η_2 is 0.413.

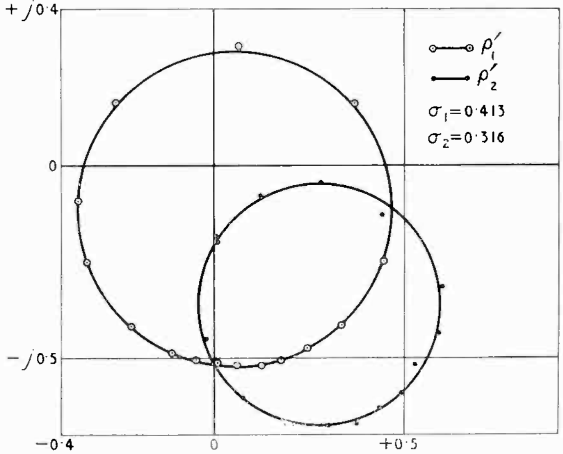

Fig. 3. *Experimentally-determined reflection-coefficient loci.*

As a check on the theory we note from (9) that

$$|\tau|^2 = \sigma_2(1 - |\rho_1|^2) = \eta_1(1 - |\rho_1|^2)$$

Similarly, $|\tau|^2 = \eta_2(1 - |\rho_2|^2)$, and these two values of $|\tau|$ should agree. A separate measurement of $|\rho_1|$ and $|\rho_2|$ with plunger replaced by a well-matched load gave $|\rho_1| = 0.539$ and $|\rho_2| = 0.572$. Substituting these figures in the above expressions for $|\tau|$ we get $|\tau|^2 = 0.275$, and $|\tau|^2 = 0{\cdot}278$ respectively in the two cases, which provides excellent confirmation of the theory.

6. Conclusion

A method of measuring transmission efficiency at microwave frequencies has been described, and shown by experiment to be capable of giving results of a high order of accuracy.

7. Acknowledgment

The author wishes to thank Professor H. M. Barlow for his interest and encouragement in this work.

REFLECTION CANCELLATION IN WAVEGUIDES

Junctions of Uniform and Tapered Sections

By L. Lewin

Introduction

A TAPER is a part of a waveguide system where the cross-section varies in a uniform manner. It is commonly used to provide a smooth transition from waveguides of one size of cross-section to another. It can also be used to enlarge the mouth of a radiating waveguide so as to provide a better termination, and also to control to some extent the radiation polar diagram. In order to be substantially reflection-free, the taper should be at least a half wavelength long, and the longer and more gradual it is, the better. At the join with the uniform waveguide, however, there will always be a small reflection, and the taper may have to be inconveniently long if this reflection is required to be very small. These reflections show up particularly when two different waveguides are joined by a tapered section, and the frequency of the source is varied. If the far waveguide is matched, the input standing-wave ratio of the near waveguide will vary rapidly with frequency, the variations being the more rapid, and their amplitude the smaller, as the length of the taper increases. Minima of total reflection occur when the taper length is an even number of quarter wavelengths, and maxima of reflection when it is an odd number. It follows that for narrow bands, the reflection can be kept low by appropriately choosing the taper length, but not so for broad bands. Hence it is important to know how long a taper must be in order to reduce the reflections at the joins below a given value, and also to know in what way these reflections can be cancelled, thus permitting the use of a shorter taper than would otherwise have been required. Both these effects are investigated.

Method of Analysis

Three separate cases are investigated and are shown in Figs. 1, 2 and 3. In Fig. 1, the narrow side of a rectangular waveguide, of cross-section $a \times b$, is flared from b to d in a taper of length L. The broad side, a, is constant. In Fig. 2, the broad side, a, is tapered to a width D in a taper of length L, the narrow side being unchanged. In Fig. 3, the cross-section $a \times b$ is tapered to $D \times d$ in a length L, it being supposed, for the time being, that the four tapered sides meet in a point, so that $a/b = D/d$. In each case, a dominant waveguide mode, with a reflected mode at the taper, is assumed to exist in the uniform guide, while an outgoing wave is assumed to exist in the taper. This outgoing wave is chosen to be the dominant mode for the natural co-ordinate space of the taper under consideration; i.e., cylindrical co-ordinates for tapers 1 and 2, and spherical co-ordinates for taper 3. The choice of only the dominant modes is permissible provided that the tapers are not too abrupt. In practice this means that

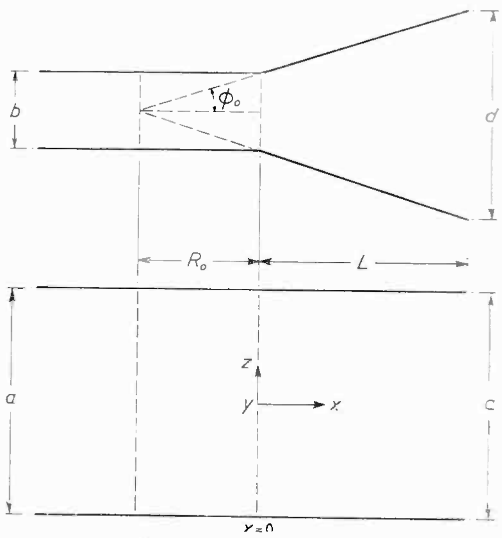

Fig. 1. *Taper in narrow side of waveguide*

the angles of the tapers are assumed small, so that the distance R_0 from the virtual apex of the taper to the cross-section at the join, is substantially constant in any one plane parallel to the plane of the taper. It follows that the equiphase surface defined by the outgoing wave in the taper coincides substantially with that defined by the waveguide dominant mode at the position of the join. It only remains to choose

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the various amplitudes appropriately in order to satisfy the boundary conditions, namely continuity of E and H at the join. These continuity conditions, which should hold, strictly, all over the cross-section, will be applied at the centre only. (The effect of applying it at other points is to bring into account just that variation of R_0 over the cross-section discussed above, and for which higher-order modes would have to be introduced. This is avoided by limiting the treatment to those cases for which R_0 is substantially constant over the cross-section; i.e., to moderate tapers only).

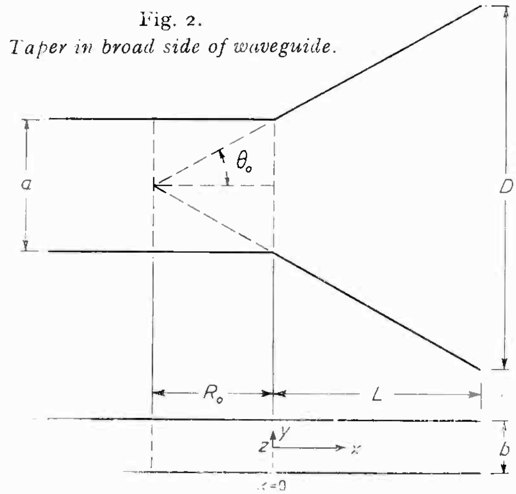

Fig. 2.
Taper in broad side of waveguide.

Taper in Narrow Side

Referring to Fig. 1, let ϕ_0 be the semi-angle of the taper, so that $R_0 = \frac{b}{2} \cot \phi_0$, R_0 being measured from the virtual apex to the join. Defining a reflection coefficient r, and a transmission coefficient t, we can write for region 1 (waveguide) and region 2 (taper)

$$\left.\begin{aligned} {}_1E_y &= \cos\frac{\pi z}{a}\left[e^{-jk'x} + re^{jk'x}\right] \\ {}_1H_z &= \cos\frac{\pi z}{a}\left[e^{-jk'x} - re^{jk'x}\right]\frac{k'}{k} \end{aligned}\right\} \quad .. \quad (1)$$

with $k = \frac{2\pi}{\lambda}$; $k' = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda}\sqrt{1 - \frac{\lambda^2}{4a^2}}$

$$\left.\begin{aligned} {}_2E_\phi &= t\cos\frac{\pi z}{a}\,\mathrm{H}_1{}^{(2)}(k'R) \\ {}_2H_z &= t\cos\frac{\pi z}{a}\,\mathrm{H}_0{}^{(2)}(k'R)\frac{jk'}{k} \end{aligned}\right\} \quad .. \quad (2)$$

Let us equate ${}_1E_y = {}_2E_\phi$ and ${}_1H_z = {}_2H_z$ at $x = 0$ $z = 0$, $R = R_0 = \frac{b}{2}\cot\phi_0$.

This gives
$$\left.\begin{aligned} 1 + r &= t\mathrm{H}_1{}^{(2)}\left(k'\frac{b}{2}\cot\phi_0\right) \\ 1 - r &= jt\mathrm{H}_0{}^{(2)}\left(k'\frac{b}{2}\cot\phi_0\right) \end{aligned}\right\} \quad .. \quad (3)$$

Hence
$$\frac{1+r}{1-r} = \frac{\mathrm{H}_1{}^{(2)}\left(k'\frac{b}{2}\cot\phi_0\right)}{j\mathrm{H}_0{}^{(2)}\left(k'\frac{b}{2}\cot\phi_0\right)}$$

Since ϕ_0 is assumed small, $k'\frac{b}{2}\cot\phi_0$ will in general be large enough to permit the use of the asymptotic expansions of the Bessel functions, giving, after some reduction

$$(1 + r)/(1 - r) \approx 1 - j/(k'b\cot\phi_0) \quad .. \quad (4)$$

If a reactance jX is put across a matched line, it can be shown that the reflection coefficient is $-1/(1 + 2jX/Z_0)$, so that $(1 + r)/(1 - r) = 1/(1 + Z_0/jX)$. This can be put equal to $1 + jZ_0/X$ if X is large. Comparing with Equ. (4) we see that the effect of the taper is to introduce a negative *shunt susceptance* of value $k'b$ cot ϕ_0 relative to the Z_0 of the waveguide. Hence the standing-wave ratio, given by $\rho = (1 - |r|)/(1 + |r|)$ is approximately

$$\rho = 1 - 1/(k'b\cot\phi_0) \quad .. \quad .. \quad (5)$$

Introducing the length of taper, L, we have $\cot\phi_0 = 2L/(d - b)$

Hence
$$\begin{aligned} \rho &= 1 - \lambda(d - b)/(4\pi Lb\sqrt{1 - \lambda^2/4a^2}) \\ &= 1 - \lambda_g(d - b)/4\pi Lb \quad .. \quad .. \quad .. \quad (6) \end{aligned}$$

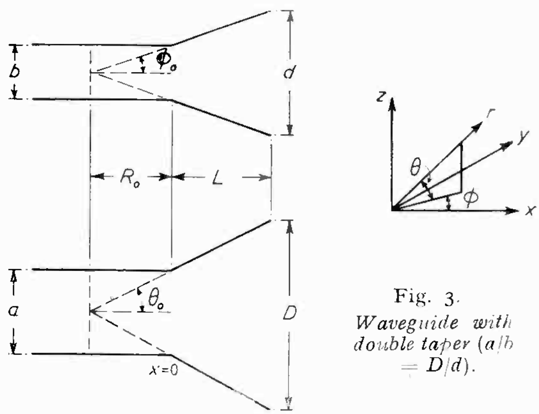

Fig. 3.
Waveguide with double taper ($a/b = D/d$).

(It should be noted that this formula assumes that the far end of the taper is, in some way, matched. If this is not so, additional reflections are caused which, as explained above, will interfere with the reflection at the join, causing beating effects).

As an example, assume a 3-in × 1-in waveguide, with the 1-in side tapered to 2-in in a distance of 4-in. Let $\lambda = 10$ cm, so that $\lambda_g = 13.3$ cm.

$$\rho = 1 - \frac{13.3}{2.54} \cdot \frac{2 - 1}{4 \times 1 \times 12.56} = 1 - 0.1 = 0.9$$

If a matched 3-in × 2-in guide is now joined at the far end of the taper, the reflection there, obtained by interchanging b and d in Equ. (6), will be, in this example, half that occurring at the 1-in end. Since 4 in is very nearly $3\lambda_g/4$, the reflections will add in phase, giving a total standing-wave ratio of $1 - 0.1 - 0.05 = 0.85$.

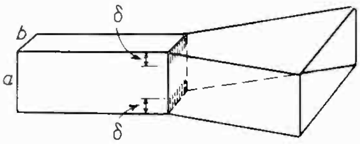

Fig. 4. *Expanding taper in narrow side compensated by inductive diaphragm.*

Reflection Cancellation

Since the reflection is the same as that from a shunt capacitance, the provision of an inductive diaphragm at the join should provide adequate compensation. This can conveniently take the form of two thin metal strips protruding into the waveguide from the narrow side by a distan e δ. The inductance (relative to Z_0) thereby introduced is given by[1]

$$X_L = (a/\lambda_g) \cot^2 (\pi\delta/a) \quad \ldots \quad \ldots \quad (7)$$

Equating this to $k'b \cot \phi_0$, the negative reactance introduced at the join, we get an equation for δ

$$\cot (\pi\delta/a) = \sqrt{4\pi bL/a(d - b)} \quad \ldots \quad \ldots \quad (8)$$

For moderate tapers, $\pi\delta/a$ will not be large, and we can write

$$\delta = (a/2\pi) \sqrt{a(d - b)/\pi bL} \quad \ldots \quad \ldots \quad (9)$$

This equation shows, since λ does not occur, that the compensation is, to the order we are considering, independent of frequency, and we can certainly expect the effect to be very broad-band in practice. As an example, let us find what depth of diaphragm we need to compensate for the 1-in to 2-in taper in a 3-in × 1-in guide, with a 4-in length of taper. Equ. (9) gives

$$\delta = \frac{3}{6.28} \sqrt{\frac{3}{3.14} \frac{2 - 1}{1.4}} = 0.234 \text{ in.}$$

The resulting arrangement is shown in Fig. 4.

At the far end of the taper, if we again go to a uniform waveguide, the sign of reflection reverses, as may be seen by interchanging b and d in Equ. (6). Hence the effect there is *inductive*, and a shunt capacitance is needed for compensation. This can take the form of a diaphragm of two strips protruding a distance δ into the guide from the broad side.

For such a diaphragm we have[1]

$$X_c = -\frac{\lambda_g}{4b} \Big/ \log_e (\sec \pi\delta/b) \quad \ldots \quad \ldots \quad (10)$$

so that the equation for δ becomes, since d now replaces b as the narrow side

$$\frac{\lambda_g}{4b} \Big/ \log_e (\sec \pi\delta/d) = (2\pi/\lambda_g)d \cot \phi_0 \quad \ldots \quad (11)$$

Here $\cot \phi_0 = 2L/(d - b)$ as before.

For moderate tapers, δ will be small, so that we can write $\log_e (\sec \pi\delta/d) \approx \log_e (1 + \frac{1}{2} (\pi\delta/d)^2) \approx \frac{1}{2}(\pi\delta/d)^2$

giving $\delta \approx (\lambda_g/2\pi)\sqrt{(d - b)/2\pi L} \quad \ldots \quad \ldots \quad (12)$

This differs from the previous case, Equ. (9), in that δ now depends on frequency, since λ_g occurs in Equ. (12). For example, if the uncorrected taper to a 3-in broad guide gives a reflection coefficient of 0.1 at $\lambda = 10$ cm, and this is corrected by a capacitive iris at this wavelength, then the resultant response will be as curve (b) in Fig. 5. Curve (a) shows the response of the uncorrected taper. It will be seen that compensation is possible over a reasonable band. Fig. 6 shows the arrangement of the diaphragm.

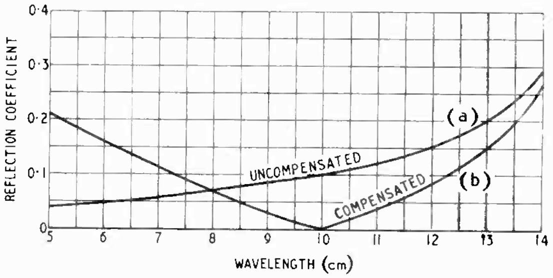

Fig. 5. *Theoretical frequency response of expanding-taper to waveguide combination ($d = 3$ in); taper in broad side.*

Taper in Broad Side.

Referring to Fig. 2, let θ_0 be the semi-angle of the taper, so that $R_0 = \frac{a}{2} \cot \theta_0$. Proceeding as before, we have:

$$\begin{aligned} E_y &= \cos \frac{\pi z}{a} (e^{-jk'x} + re^{jk'x}) \\ H_z &= \cos \frac{\pi z}{a} (e^{-jk'x} - re^{jk'x}) \frac{k'}{k} \end{aligned} \quad \ldots \quad (13)$$

$${}_2E_y = t \cos(\theta\psi) \, H_\psi^{(2)} (kR)$$

$${}_2H_\theta = jt \cos(\theta\psi) \, H_\psi^{(2)\prime} (kR)$$

where $\psi = \pi/2\theta_0$

The field forms in region 2 are chosen so as to give zero tangential electric field at $\theta = \pm \theta_0$; i.e., on the metal surface of the taper.

If we match the fields at $x = z = 0$, $\theta = 0$, $R = R_0 = (a/2) \cot \theta_0$ we get

$$1 + r = t \, H_\psi^{(2)} (\tfrac{1}{2}ka \cot \theta_0)$$

$$1 - r = jt \frac{k}{k'} H_\psi^{(2)\prime} (\tfrac{1}{2} ka \cot \theta_0) \quad \ldots \quad (15)$$

Hence

$$\frac{1 + r}{1 - r} = \frac{(k'/k)\,H_\psi^{(2)}\,(\frac{1}{2}ka\cot\theta_0)}{jH_\psi^{(2)\prime}\,(\frac{1}{2}ka\cot\theta_0)} \quad \cdots \quad (16)$$

Now for θ_0 small, $\dfrac{\frac{1}{2}ka\cot\theta_0}{\pi/2\theta_0} \approx 2\dfrac{\alpha}{\lambda}$ which is greater than, but of order, unity. Also, since $\pi/2\theta_0$ is large, this means that we are interested in the expansion of Bessel functions of large order, whose argument is greater than the order. An expression for $H_\nu(\nu\sec\beta)$ exists[2] when ν is large and β acute, from which can be derived the relation

$$\frac{H_\nu^{(2)}\,(\nu\sec\beta)}{H_\nu^{(2)\prime}\,(\nu\sec\beta)} \approx \frac{1 + j/(2\nu\sec\beta\sin^3\beta)}{-j\sin\beta}$$

Putting $\nu\sec\beta = x$, $\sin\beta = \sqrt{1 - \nu^2/x^2}$, we get

$$\frac{H_\nu^{(2)}\,(x)}{H_\nu^{(2)\prime}\,(x)} \approx j\,\frac{1 + j/\{2x\,(1 - \nu^2/x^2)^{\frac{3}{2}}\}}{\sqrt{1 - \nu^2/x^2}} \quad \cdots \quad (17)$$

In our case $\nu = \pi/2\theta_0$, $x = \frac{1}{2}ka\cot\theta_0$, $\nu/x \approx \lambda/2a$; substituting into (16) gives

$$\frac{1 + r}{1 - r} = 1 + \frac{j}{ka\cot\theta_0\,(1 - \lambda^2/4a^2)^{\frac{3}{2}}} \quad \cdots \quad (18)$$

This means that the discontinuity is equivalent to an inductive reactance whose value, relative to Z_0, is

$$X_L = ka\cot\theta_0\,(1 - \lambda^2/4a^2)^{\frac{3}{2}} \quad \cdots \quad (19)$$

Cancellation of the Reflection.

As in Equ. (10) the reflection can be cancelled by a capacitive diaphragm, whose insertion, δ, in the broad side is given by

$$\frac{\lambda_g}{4b\log(\sec\pi\delta/b)} = (2\pi a/\lambda)\,(1 - \lambda^2/4a^2)^{\frac{3}{2}}\cot\theta_0 \quad \cdots \quad \cdots \quad \cdots \quad (20)$$

Writing $\log(\sec\pi\delta/b) = \frac{1}{2}(\pi\delta/b)^2$ as before, and using the relation $\cot\theta_0 = 2L/(D - a)$ we get an approximate equation for δ

$$\delta \approx \frac{\lambda}{(1 - \lambda^2/4a^2)}\cdot\frac{1}{2\pi}\cdot\sqrt{\frac{(D - a)b}{2\pi aL}} \quad \cdots \quad (21)$$

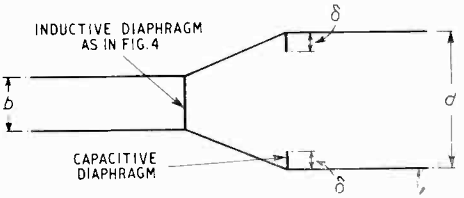

Fig. 6. *Compensation for narrow-side taper (expanding-taper to waveguide).*

The dependence of the response on frequency is shown in Fig. 7, for the example of a 3-in waveguide where the uncorrected taper gives a reflection of 0.1 at $\lambda = 10$ cm. In Fig. 7, (a) is the uncompensated response, and (b) the corrected with a capacitive diaphragm designed according to Equ. (21). Although the response is poorer than in Fig. (6), a reasonable bandwidth suitable for most purposes can now be covered. The diaphragm arrangement is shown in Fig. 8.

In the second case, when we go from taper to waveguide of dimensions $D \times b$, the reflection is as from a capacitance

$X_c = kD\cot\theta_0\,(1 - \lambda^2/4D^2)^{\frac{3}{2}}$ where $\cot\theta_0 = 2L/(D - a)$. The compensating inductive diaphragm has an insertion in the narrow side of amount δ given by

$$(D/\lambda_g)\cot^2(\pi\delta/D) = \frac{2\pi D}{\lambda}\,(1 - \lambda^2/4D^2)^{\frac{3}{2}}\,.\,2L/(D - a) \quad \cdots \quad (22)$$

or, approximately

$$\delta = (D/2\pi)\sqrt{(D - a)/[2L(1 - \lambda^2/4D^2)]} \quad \cdots \quad \cdots \quad \cdots \quad (23)$$

This arrangement, is very frequency insensitive. since in practice, D is considerably greater than $\lambda/2$.

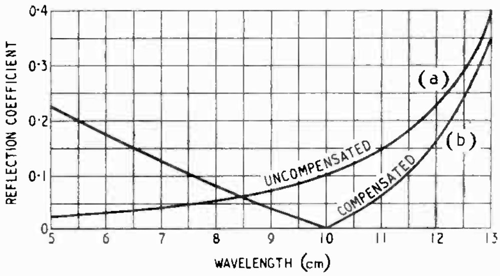

Fig. 7. *Theoretical frequency response of waveguide to expanding-taper combination ($a = 3$ in): taper in broad side.*

Double Taper

The difficulty in applying the method of the previous two cases to the double taper is that two contiguous sides of the double taper do not constitute a pair of orthogonal surfaces, so that the normal method of resolution of the wave equation into orthogonal co-ordinate systems is not applicable here. It does not seem worthwhile to go to generalized curvilinear co-ordinates in order to solve this problem, but it appears possible to get a reliable result by using spherical polar co-ordinates. This method suggests itself in the particular case in which the four sides of the taper meet at a point, the virtual apex of the taper. Actually, the surfaces appropriate to spherical co-ordinates, ϕ = constant and θ = constant are respectively planes and cones, but if the angle ϕ_0 is small, the small part of the conical surface considered will not differ sensibly from a flat surface. In fact, the replacement causes a dip at the centre of the horn aperture of $(D/2)(1 - \cos\phi_0)$, which is a second-order correction. Referring to Fig. 3, the four sides of the taper will be taken as $\theta = \pm\theta_0$ and $\theta = \pm\phi_0$ for the broad and narrow sides respectively.

In the waveguide we have as before

$$_1E_y = \cos\frac{\pi z}{a}[e^{-jk'x} + re^{jk'x}]$$

$$_1H_z = \cos\frac{\pi z}{a}[e^{-jk'x} - re^{jk'x}]\,k'/k \quad .. \quad (24)$$

and in the double taper

$$_2E_\phi = \frac{t}{R}[R^{\frac{1}{2}}\mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR)]\cos\phi\,[A\mathrm{P}_n'(\sin\theta) + B\mathrm{Q}_n'(\sin\theta)]$$

$$_2H_\theta = \frac{jt}{kR}\cdot\frac{d}{dR}[R^{\frac{1}{2}}\mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR)]\cos\phi\,[A\mathrm{P}_n'(\sin\theta) + B\mathrm{Q}_n'(\sin\theta)] \quad .. \quad (25)$$

Here A and B have to be chosen so that $A\mathrm{P}'_n(\sin\theta) + B\,\mathrm{Q}_n'(\sin\theta)$ is symmetrical about $\theta = 0$ and n is such that, when A and B have been so chosen, $A\mathrm{P}'_n(\sin\theta_0) + B\,\mathrm{Q}'_n(\sin\theta_0) = 0$. This latter is the condition for zero tangential electric field at the sides of the taper $\theta = \pm\theta_0$. A glance at curves of P'_n and Q'_n shows that, for small θ_0, n is large, so that it is permissible to use assymptotic expansions of P'_n and Q'_n. These expansions[3] permit us to write, for large n

$$A\mathrm{P}_n + B\mathrm{Q}_n \approx \sqrt{2/(\pi n\cos\theta)}\,\sin\,(n + \tfrac{1}{2}\theta),$$

which, when differentiated, has the required symmetry. Since n is large, and θ_0 small, the effect of the variation of $\cos\theta$ relative to that of $\sin\,(n + \frac{1}{2}\theta)$ can be ignored, so that the equation for n becomes

$$\cos\,(n + \tfrac{1}{2}\theta_0) = 0 \quad \text{or } n \approx \pi/2\theta_0 \quad .. \quad (26)$$

Equating the fields at $x = y = z = 0$, $\theta = \phi = 0$,

$$R = R_0 = \tfrac{1}{2}a\cot\theta_0 = \tfrac{1}{2}b\cot\phi_0$$

we get, after division

$$\frac{1+r}{1-r} = \frac{-j\,(k'/k)\sqrt{kR_0}\,\mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR_0)}{\dfrac{d}{d(kR_0)}[\sqrt{kR_0}\,\mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR_0)]} \quad (27)$$

n being given by Equ. (26).

Using now the expansion of Equ. (17), we find, after some reduction

$$\frac{1+r}{1-r} = 1 + \frac{j}{ka\cot\theta_0\,(1 - \lambda^2/4a^2)^{\frac{3}{2}}} - \frac{j\theta_0}{ka\,(1 - \lambda^2/4a^2)^{\frac{1}{2}}} \quad .. \quad .. \quad (28)$$

Since, to our order of approximation, $\cot\theta_0 = 1/\theta_0$, this means the effect is that of an inductive reactance of value, relative to Z_0, of

$$X_L = (\lambda^2/4a^2)\,(\lambda\theta_0/2\pi a)\,/(1 - \lambda^2/4a^2)^{\frac{3}{2}} \quad (29)$$

Corresponding to (21) we have, for the insertion, δ, of the compensating capacitive iris

$$\delta \approx \frac{\lambda}{2a}\cdot\frac{\lambda}{1 - \lambda^2/4a^2}\cdot\frac{1}{2\pi}\sqrt{\frac{(D-a)\,b}{2\pi aL}} \quad .. \quad (30)$$

General Double Taper.

Since the main use of a double taper will probably be to enlarge the mouth of a waveguide into a radiating horn, it is undesirable to be tied down to the spherical co-ordinate requirement, namely $a/b = D/d$. Moreover, Equ. (29) shows that there is nothing particularly virtuous about this combination as the taper is not self-compensating, as we would like.

Now Equ. (28) can be put in the form

$$\frac{1+r}{1-r} = 1 + \frac{j}{ka\cot\theta_0\,(1 - \lambda^2/4a^2)^{\frac{3}{2}}} - \frac{j}{k'b\cot\phi_0} \quad .. \quad .. \quad .. \quad (31)$$

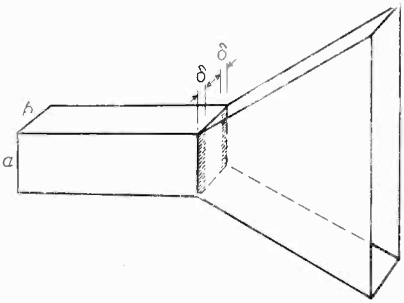

Fig. 8. *Expanding taper in broad side compensated by capacitive diaphragm.*

by virtue of the relation $R_0 = \frac{1}{2}a\cot\theta_0 = \frac{1}{2}b\cot\phi_0$ which has so far been assumed. Comparing (31) with (18) and (4) it is apparent that the total reflection is simply the sum of the reflections to be expected if the two tapers were combined independently. It seems reasonable, therefore, to assume this to be so in general, (provided only that the taper angles are small), and not limited to the special case $a/b = D/d$. Assuming this to be the case, Equ. (31) gives directly the reflection to be expected for the general taper. In particular, the taper should be self-compensating if

$$ka\cot\theta_0\,(1 - \lambda^2/4a^2)^{\frac{3}{2}} = k'b\cot\phi_0 \quad .. \quad (32)$$

$$\text{i.e., } \phi_0/\theta_0 = b/[a(1 - \lambda^2/4a^2)]$$

This relation is substantially frequency independent, except near to cut-off.

Since $\cot\theta_0 = 2L/(D - a)$ and $\cot\phi_0 = 2L/(d - b)$, this equation can be put in the form

$$(d-b)/(D-a) = b/\{a(1-\lambda^2/4a^2)\} \quad .. \quad (33)$$

which is independent of the length of the taper. (The taper length must not, of course, be so short that the flare angles become too large for the preceding analysis to remain valid).

Experimental Results

Some measurements have been made on a horn flared in the narrow side from 0.67 in to 2.75 in in a taper length of 2.625 in. The broad side, 2 in, was unflared. This arrangement gives a flare semi-angle of $\theta_0 = 22°$, which is rather large. The standing-wave ratio, taken between $\lambda = 5.5$ cm and 9 cm is shown in Fig. 9(a). It has to be noted that the reflection at the mouth may change considerably over this band, and some allowance has to be made for this. Nevertheless, the curve is seen to possess maxima and minima separated by the required $\lambda_g/4$ period. According to Equ. (9) the reflections should be compensated by an inductive diaphragm of depth

$$\delta = \frac{2}{2\pi}\sqrt{\frac{2}{2\pi}\,\frac{2.75 - 0.67}{0.67 \times 2.625}} = 0.195 \text{ in}$$

The curve when this diaphragm is inserted is shown in Fig. 9(b). Apart from the very slight ripple that remains, the curve correctly shows the variation in standing-wave ratio to be expected from the horn mouth. In particular, the violent peaks at the half-wavelength and wavelength positions have gone, and the minima centred at the quarter and three-quarter wavelength positions have been filled in. The drop near the quarter-wave position is a property of the horn itself, since the cut-off wavelength is very close. The dip at $\lambda = 5.7$ cm is common to both curves, and does not seem to be directly due to the taper.

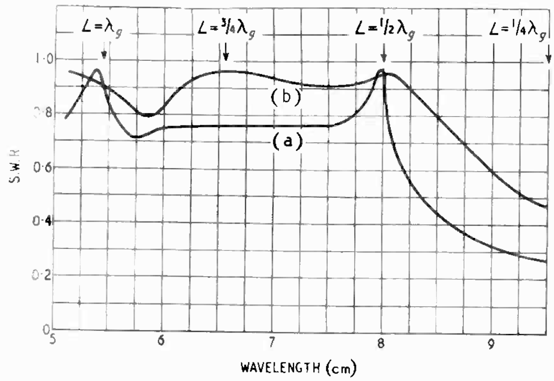

Fig. 9. *Experimental response curves from tapered horn, (a) without, and (b) with the theoretically-designed compensating diaphragm.*

Considering the rather abrupt change occurring at the taper in this horn, it is considered that the difference in the curves of Fig. 9 due to the insertion of the diaphragm, is adequate evidence that satisfactory compensation is achieved by the use of the formulae developed here for the insertion.

Conclusions

The formulae here obtained for the effects of small angle tapers would appear to be valid up to taper half-angles of about 20°. This probably covers the range of tapers most likely to be of use in practice. The diaphragm method of reflection compensation is seen to be sound, and the diaphragm sizes, as obtained from the formulae, seem near enough to the optimum to warrant their use directly, in most cases.

The field-fitting method, possible in the simple cases first treated, yields results which are capable of generalization, particularly to the general double taper. Although an independent derivation would be desirable, it is believed that the results thus obtained, and in particular, those for the self-compensating double taper, are substantially correct.

Summary of Formulae

Taper in Narrow Side.

Waveguide of dimensions $a \times b$ tapered to $a \times d$ in a taper length L. The half-angle ϕ_0, of the taper is given by $\cot \phi_0 = 2L/(d - b)$. The reflection at the taper is equivalent to that produced by a capacitance of value, relative to Z_0, given by

$$X_c = (2\pi b/\lambda_g) \cot \phi_0$$

If the rest of the system is matched, the resulting standing-wave ratio ρ, is

$$\rho = 1 - \lambda_g/2\pi b \cot \phi_0$$

The reflection can be matched out by an inductive diaphragm at the taper whose insertion from each of the narrow sides of the waveguide is given by

$$\delta = \frac{a}{2\pi}\sqrt{\frac{a}{\pi} \cdot \frac{d - b}{bL}}$$

When going from an expanding taper back to a waveguide of dimensions $a \times d$, interchange b and d and change the sign of ϕ_0 in the previous formulae. Since this reverses the phase of the reflection, a capacitive diaphragm is needed to provide compensation. The required insertion from each of the broad sides of the guide is given by

$$\delta = (\lambda_g/2\pi)\sqrt{(d - b)/2\pi L}$$

Taper in broad side.

Waveguide dimensions $a \times b$ tapered to $D \times b$ in a taper length L. The half angle of the taper is θ_0, so that

$$\cot \theta_0 = 2L/(D - a)$$

The reflection at the taper is equivalent to an inductive reactance, of value, relative to Z_0, given by

$$X_L = \frac{2\pi a}{\lambda} \cot \theta_0 \,.\, (1 - \lambda^2/4a^2)^{\frac{3}{2}}$$

If the rest of the system is matched, the resulting standing-wave ratio is

$$\rho = 1 - \lambda/[2\pi a \cot \theta_0 (1 - \lambda^2/4a^2)^{\frac{3}{2}}]$$

The reflection can be matched out by a capacitive diaphragm at the taper, whose insertion from each of the broad sides is given by

$$\delta = \frac{\lambda}{2\pi (1 - \lambda^2/4a^2)} \cdot \sqrt{\frac{b\,(D - a)}{2\pi a L}}$$

When going from an expanding taper back to a waveguide of dimensions $D \times b$, interchange D and a, and change the sign of θ_0 in the previous formulae. Since this reverses the phase of the reflection, an inductive diaphragm is needed to provide compensation. The insertion from each of the narrow sides is given by

$$\delta = (D/2\pi)\sqrt{(D - a)/[\pi L(1 - \lambda^2/4D^2)]}$$

Double Taper

Waveguide dimensions $a \times b$ tapered to $D \times d$ in a taper length L. The half angles in the narrow and broad side are

$$\cot \phi_0 = 2L/(d - b)\ ; \quad \cot \theta_0 = 2L/(D - a)$$

The effect of the taper is that of a susceptance given by

$$Y = \lambda/[2\pi a \cot \theta_0 (1 - \lambda^2/4a^2)^{\frac{3}{2}}] - \lambda_g/2\pi b \cot \phi_0$$

According to the sign of this quantity it can be matched out by a suitable diaphragm of an inductive or capacitive nature.

The taper is self-compensating when

$$\phi_0/\theta_0 = b/[a(1 - \lambda^2/4a^2)]$$

or $(d - b)/(D - a) = b/[a(1 - \lambda^2/4a^2)]$

Acknowledgment

The work described was carried out at Standard Telecommunication Laboratories Ltd., and permission to publish is gratefully acknowledged.

REFERENCES

[1] "Microwave Transmission Circuits", by G. L. Ragan, p. 211. McGraw-Hill Book Co.

[2] "A Treatise on the Theory of Bessel Functions," by G. N. Watson, p. 228. Cambridge University Press.

[3] "Tables of Functions", by Jahnke and Emde, p. 117.

TRANSIENT RESPONSE OF WIDEBAND AMPLIFIERS

Critically-Damped Two-Terminal Load of Infinite-Order

By W. E. Thomson, M.A.

(P.O. Research Station)

SUMMARY. A suitable two-terminal load for a wideband amplifier stage is the 'infinite-order critically-damped load,' derived in a previous article. This load gives the fastest unit-step response without overshoot. Any desired approximation to the compensating reactance can be obtained by a certain series of networks; the second approximation, consisting of one inductor and one capacitor, gives a result sufficiently near the ideal for most practical needs.

1. Introduction

A PREVIOUS article[1] included a discussion of the transient response of the circuit shown in Fig. 1, the criterion being the unit-step response (or indicial response); i.e., the voltage waveform appearing at the anode when a unit voltage step is applied to the grid. The valve is regarded as a constant-current generator so that the voltage step at the grid produces a current step through the valve and through the load. The resultant voltage across the load can be determined from its operational impedance (which can be obtained by substituting the operator p for $j\omega$ in the complex impedance) by interpreting this impedance as a function of time by general methods or by standard forms.[3] In order to obtain a unit-step response free from overshoot but with as fast a build-up as possible,

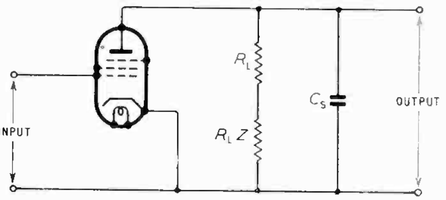

Fig. 1. *Effective circuit of wideband amplifier stage; R_L = nominal load, resistive; $R_L Z$ = compensating reactance; C_S = total shunt capacitance.*

the compensating impedance, in which dissipation is neglected, is chosen to give critical damping.

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The results of this investigation are given in Fig. 2, which shows the form of the compensating reactance for one, two and three components and the corresponding unit-step responses. The best possible unit-step response, which is approached as the number of components is increased indefinitely, is also shown. This circuit has also been discussed by Elmore,[2] who suggests that this infinite-order critically-damped load is in fact the best possible two-terminal load, with shunt capacitance, for achieving a unit-step response without overshoot.

This article deals with the problem of approximating to the infinite-order case by a finite compensating reactance, the resulting loads being not necessarily critically damped.

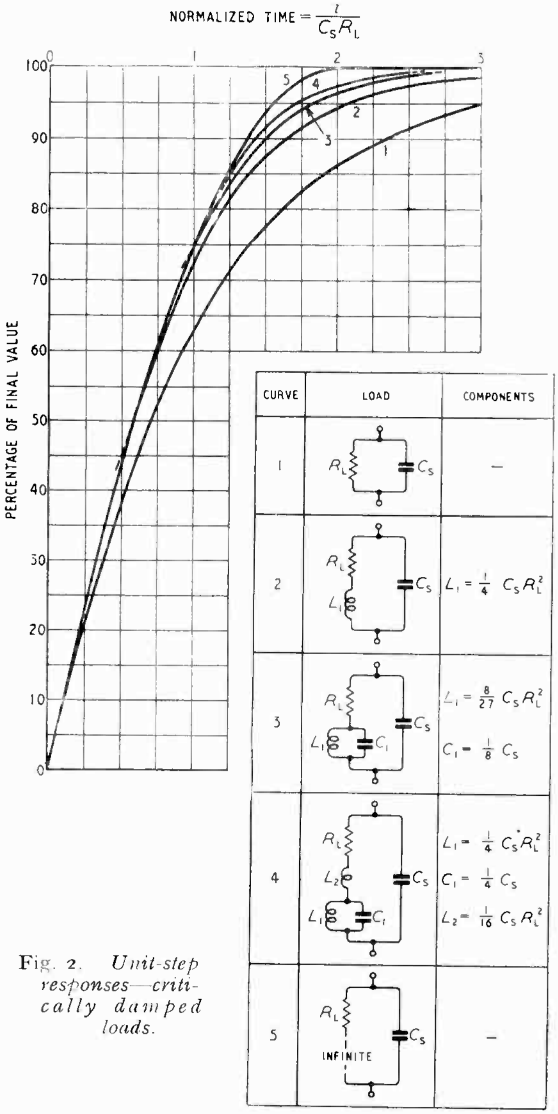

Fig. 2. *Unit-step responses—critically damped loads.*

2. Infinite-Order Compensating Reactance

The article referred to[1] showed that the normalized infinite-order compensating impedance is equal to

$$\coth x - 1/x$$

where $x = pC_SR_L$. This impedance may be expressed in terms of an infinite network of

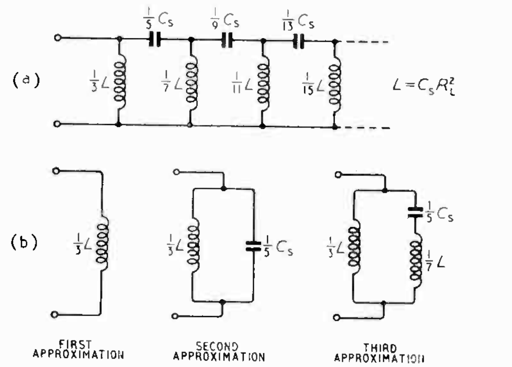

Fig. 3 (*a*) *Ladder form of infinite-order compensating reactance, and* (*b*) *successive approximations to circuit* (*a*).

inductors and capacitors by the methods of Foster and Cauer. Of the possible networks, that of Fig. 3(a), determined from the continued-fraction expansion of coth x, seems to lend itself best to approximation in terms of one, two, three, etc., components of the infinite network; the successive approximations are shown in Fig. 3(b).

3. Unit-Step Responses of Approximations

3.1 *First Order*

The normalized impedance of the effective load of the amplifier, and thus the normalized gain of the amplifier stage, is

$$\frac{1 + x/3}{1 + x + x^2/3} = \frac{x + 3}{x^2 + 3x + 3}.$$

This may be arranged as

$$1 - \frac{x(x + 3/2)}{(x + 3/2)^2 + (\sqrt{3}/2)^2} - \frac{(\sqrt{3}/2)x}{\sqrt{3}\{(x + 3/2)^2 + (\sqrt{3}/2)^2\}}$$

which can be regarded as the Laplace transform of the normalized unit-step response of the amplifier stage, and can be interpreted by standard forms[3] to give the unit-step response

$$1 - e^{-3\tau/2}\{\cos \tau\sqrt{3}/2 + (1/\sqrt{3}) \sin \tau\sqrt{3}/2\}$$
$$= 1 - (2/\sqrt{3})e^{-3\tau/2} \cos (\tau\sqrt{3}/2 - \pi/6)$$

where $\tau = t/C_SR_L$.

This response is shown in Fig. 4, where it may be compared with the ideal curve.

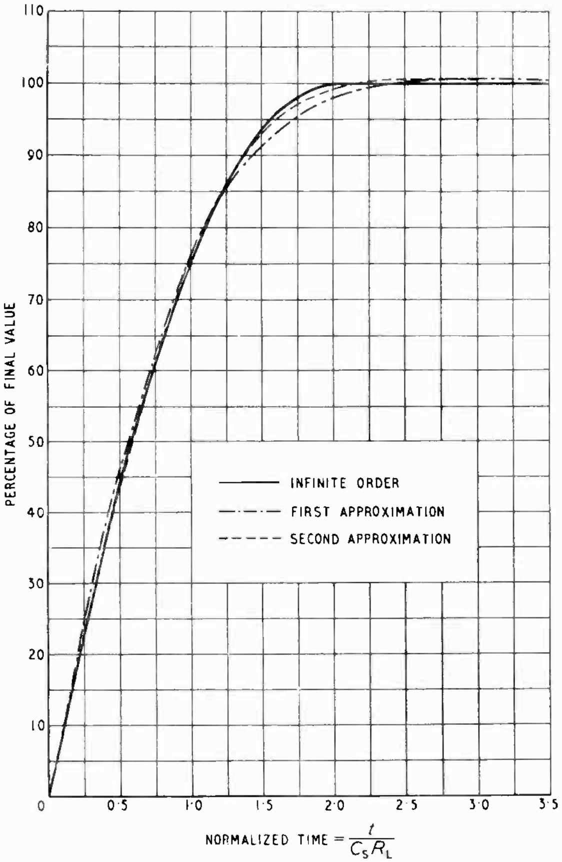

Fig. 4. *Unit-step responses, infinite-order critically-damped load and approximations.*

3.2 *Second Order*

This follows the same lines as the first order case, so only the expressions are given.

Impedance :

$$\frac{x^2 + 5x + 15}{x^3 + 6x^2 + 15x + 15}$$

$$= 1 - \frac{1.142x}{x + 2.322} + \frac{0.142x(x + 1.839)}{(x + 1.839)^2 + 1.754^2} - \frac{0.7928 \times 1.754x}{(x + 1.839)^2 + 1.754^2}$$

Unit-step response :

$$1 - 1.142e^{-2.322\tau} + 0.142e^{-1.839\tau} \cos 1.754\tau - 0.7928e^{-1.839\tau} \sin 1.754\tau$$

$$= 1 - 1.142e^{-2.322\tau} - 0.8054e^{-1.839\tau} \sin (1.754\tau - 0.177)$$

and is shown in Fig. 4.

4. Conclusions

The two networks dealt with in the previous section give responses which are slightly less than critically damped, and it seems reasonable to suppose that this will be true of all networks in the series. The overshoot is very small, being 0.6% for the first order and 0.4% for the second order.

The second-order response differs from the ideal by less than 1% (of the final value) except for a small region where it lies between 1 and 1.5%. This second approximation, in which the compensating reactance consists of an inductor, value $C_s R_L^2/3$, in parallel with a capacitor, value $C_s/5$, is near enough to the ideal for most practical purposes, and is recommended as suitable for an amplifier stage whose unit-step response must be as fast as possible without appreciable overshoot.

5. Acknowledgments

The author is indebted to the Engineer-in-Chief, G.P.O., for permission to publish this work.

REFERENCES

[1] Thomson, W. E. "Transient Response of V.F. Couplings", *Wireless Engineer*, January 1947.

[2] Elmore, W. C. "The Transient Response of Damped Linear Networks with Particular Regard to Wideband Amplifiers", *J. appl. Phys.*, January 1948.

[3] McLachlan, N. W. "Complex Variable and Operational Calculus", Cambridge, 1939.

INSTITUTION OF ELECTRICAL ENGINEERS

Professor E. B. Moullin, M.A., Sc.D., has been elected President of the Council of the Institution of Electrical Engineers, while J. Eccles, B.Sc., and Sir John Hacking have been elected Vice-Presidents. They take office on 30th September.

As a result of the ballot, H. Nimmo becomes Honorary Treasurer and the new Ordinary Members of the Council are Professor Willis Jackson, D.Sc., D.Phil., and Sir George H. Nelson (Members) ; J. E. L. Robinson, M.Sc., and J. A. Saxton, Ph.D., B.Sc. (Associate Members) and Sir Cyril Hurcombe, G.C.B., K.B.E. (Companion).

The Chairman of the Radio Section Committee will be R. T. B. Wynn, C.B.E., M.A., and the Vice-Chairman, D. C. Espley, D.Eng. Five Ordinary Members of the Committee have been elected : A. W. Cole, H. G. Hopkins, Ph.D., J. S. McPetrie, Ph.D., D.Sc., R. A. Smith, M.A., Ph.D., and H. Stanesby.

NATIONAL PHYSICAL LABORATORY

Professor E. C. Bullard, M.A., Ph.D., F.R.S., has been appointed by the Lord President of the Council to be Director of the National Physical Laboratory in succession to Sir Charles Darwin, K.B.E., M.C., Sc.D., F.R.S. Professor Bullard is Professor of Physics in the University of Toronto and is expected to take up the appointment in January 1950.

IMPEDANCE OF COMPOSITE CONDUCTORS

By A. Rosen, A.C.G.I., Ph.D., M.I.E.E.

(*Siemens Brothers & Co. Ltd., Woolwich*)

Introduction

COMPOSITE conductors, consisting of a layer of one material on another have many electrical applications: examples in which the high-frequency impedance is important are conductors of tinned copper and copper-clad steel. Even for homogeneous conductors the calculations are elaborate, except in a few special cases, and this makes it difficult to form an over-all picture of their behaviour with alternating currents: the extension to composite conductors makes the calculations still more complex, and tends still further to obscure the over-all picture. Nevertheless, by considering the cases most amenable to calculation, one obtains useful working formulae, and may infer the probable effects of changing from homogeneous to composite conductors in more general cases.

The circuit which lends itself most readily to mathematical treatment is one consisting of coaxial tubular conductors, and a theorem for calculating the impedance of composite coaxial conductors is given by Schelkunoff in his paper on "The Electro-magnetic Theory of Coaxial Transmission Lines and Cylindrical Shields"[1]; the problem has also been studied by Carsten[2] and some results based on Carsten's work have been published by Mildner.[3] In the following, the author has developed Schelkunoff's theorem into serviceable formulae, and shows that it can be deduced by an extension of the method devised by Howe,[4] which may be readily followed by those familiar with telephone-line transmission formulae.

LIST OF SYMBOLS

R	effective resistance of conductor*
L	internal inductance of conductor†
ωL	internal reactance of conductor*
a	inner radius of tube in metres
b	outer radius of tube in metres
r	radius of surface facing return conductor in metres
t	thickness of tube or flat conductor in metres
w	width of flat conductor in metres
l	length of conductor in metres
η	intrinsic impedance of solid metal in ohms
$\lvert\eta\rvert$	modulus of η in ohms
σ	intrinsic propagation constant of solid metal in nepers/metre
α	intrinsic attenuation constant of solid metal in nepers/metre
g	conductivity in mhos/metre
μ	permeability†
μ'	relative permeability
f	frequency in cycles per second
ω	angular frequency in radians per second
j	$\sqrt{-1}$
$I_0(\sigma x)$, $K_0(\sigma x)$	modified Bessel functions of zero order and respectively of the first and second kind
$I_1(\sigma x)$, $K_1(\sigma x)$	modified Bessel functions of the first order and respectively of the first and second kind
Z_{aa}	surface impedance, when return is internal*
Z_{bb}	surface impedance, when return is external*
Z_{rr}	surface impedance of surface of radius r*
Z_{ab}	transfer impedance*
R_{ab}	transfer resistance*
L_{ab}	transfer inductance†
Z_{aa}'	surface impedance of composite tube when return is internal*
Z_{bb}'	surface impedance of composite tube when return is external*
Z_{ab}'	transfer impedance of composite tube*
L	inductance of hypothetical line†
G	leakance of hypothetical line in mhos/metre
W, X	series arms of equivalent T network
Y	shunt arm of equivalent T network
Z'	impedance of composite conductor of negligible curvature*
R'	resistance of composite conductor of negligible curvature*
L'	inductance of composite conductor of negligible curvature†
S	surface material of composite conductor
B	base material of composite conductor
θ	$\tanh^{-1} (\eta_B/\eta_S)$
θ'	$\tanh^{-1} (\eta_S/\eta_B)$
h	g_S/g_B
R_B	resistance of thick slab of base material*
L_B	inductance of thick slab of base material†
R_S	resistance of thick slab of surface material*
L_S	inductance of thick slab of surface material†
R_{cu}	resistance of thick slab of copper*
L_{cu}	inductance of thick slab of copper†

* ohms/metre † henrys/metre

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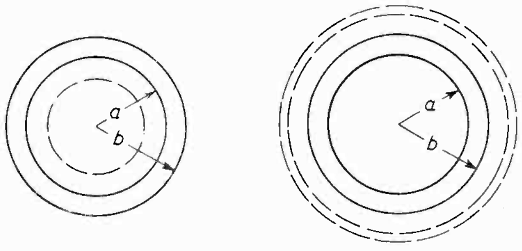
Homogeneous Tubular Conductors

Consider a tube of inner radius a metres and outer radius b metres forming one conductor of a coaxial pair; the return conductor may be internal as in Fig. 1 (a) or external as in Fig. 1(b). If R is the effective resistance and L the internal inductance of the conductor under consideration, then $R + j\omega L$ is equal to the surface impedance, defined by Schelkunoff[1] as the ratio of the longitudinal electromotive intensity on the surface facing the return conductor to the total current carried by the conductor. For a homogeneous material, the surface impedance with internal return, Z_{aa}, is given by

$$Z_{aa} = \frac{\eta}{2\pi a D}[I_0(\sigma a).K_1(\sigma b) + K_0(\sigma a).I_1(\sigma b)] \text{ ohms/metre} \quad \ldots \quad (1)$$

The surface impedance with external return, Z_{bb}, is given by

$$Z_{bb} = \frac{\eta}{2\pi b D}[I_0(\sigma b).K_1(\sigma a) + K_0(\sigma b).I_1(\sigma a)] \text{ ohms/metre} \quad \ldots \quad (2)$$



(a) (b)

Fig. 1. *Tube of internal radius a and external radius b with internal return conductor (a) and external return conductor (b).*

The transfer impedance, defined by Schelkunoff as the ratio of the longitudinal electromotive intensity on the surface remote from the return conductor to the total current flowing in the conductor, is the same whether the return be internal or external. For a homogeneous tube it is given by

$$Z_{ab} = Z_{ba} = \frac{1}{2\pi\, abgD} \text{ ohms/metre} \quad \ldots \quad (3)$$

In the above,

$$D = I_1(\sigma b).K_1(\sigma a) - I_1(\sigma a).K_1(\sigma b) \quad \ldots \quad (4)$$

η, the intrinsic impedance of solid metal

$$= \sqrt{j\omega\mu/g} = |\eta| \Big/ \frac{\pi}{4} \text{ ohms} \quad \ldots \quad (5)$$

$|\eta|$, the modulus of the intrinsic impedance

$$= \sqrt{2\pi f \mu / g} = 2\pi\sqrt{2f\mu' \times 10^{-7}/g} \text{ ohms} \quad (6)$$

σ, the intrinsic propagation constant of solid metal

$$= \sqrt{j\omega\mu g} = \alpha(1 + j) \text{ nepers per metre} \quad (7)$$

α, the intrinsic attenuation constant of solid metal

$$= \sqrt{\pi f \mu g} = 2\pi\sqrt{f\mu' g \times 10^{-7}} \text{ nepers per metre} \quad \ldots \quad (8)$$

g = conductivity in mhos/metre
μ = permeability = $4\pi\mu' \times 10^{-7}$ henrys per metre
μ' = relative permeability
f = frequency in cycles per second
$\omega = 2\pi f$
$j = \sqrt{-1}$

$I_0(\sigma x)$, $K_0(\sigma x)$ denote modified Bessel functions of zero order and respectively of the first and second kind

$I_1(\sigma x)$, $K_1(\sigma x)$ denote modified Bessel functions of the first order, and respectively of the first and second kind.

The modified Bessel function may be evaluated by the aid of the following relations:—

$$I_0(\sigma x) = \text{ber}(\sqrt{2}\,\alpha x) + j\,\text{bei}(\sqrt{2}\,\alpha x) \quad (9)$$

$$K_0(\sigma x) = \text{ker}(\sqrt{2}\,\alpha x) + j\,\text{kei}(\sqrt{2}\,\alpha x) \quad (10)$$

$$I_1(\sigma x) = \frac{\text{ber}'(\sqrt{2}\,\alpha x) + j\,\text{bei}'(\sqrt{2}\,\alpha x)}{\sqrt{j}} \quad (11)$$

$$K_1(\sigma x) = -\frac{\text{ker}'(\sqrt{2}\,\alpha x) + j\,\text{kei}'(\sqrt{2}\,\alpha x)}{\sqrt{j}} \quad (12)$$

The ber, bei, ker, kei functions and their derivatives have been tabulated[5], and thus the impedances can be calculated.

Schelkunoff's Theorem for Composite Conductors

The impedance of a composite conductor is given by Schelkunoff[1] in the following theorem :— 'Let two conductors, both of which may be made up of coaxial layers, fit tightly one inside the other. Any surface self-impedance of the compound conductor equals the individual impedance of the conductor nearest to the return path diminished by the fraction whose numerator is the square of the transfer impedance across this conductor and whose denominator is the sum of the surface impedances of the two component conductors if each is regarded as the return path for the other. The transfer impedance of the compound conductor is the fraction whose numerator is the product of the transfer impedances of the individual conductors and whose denominator is that of the self-impedance.'

If the composite conductor consist of an inner tube 1 having radii a_1 b_1 and an outer tube 2 of radii a_2 b_2 ($a_2 = b_1$), then with internal return, the surface impedance in ohms per metre is

$$Z_{aa}' = Z_{aa}^{(1)} - \frac{[Z_{ab}^{(1)}]^2}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad (13)$$

With external return, the surface impedance is

$$Z_{bb}' = Z_{bb}^{(2)} - \frac{[Z_{ab}^{(2)}]^2}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad (14)$$

The transfer impedance in both cases is

$$Z_{ab}' = Z_{ba}' = \frac{Z_{ab}^{(1)} \cdot Z_{ab}^{(2)}}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad (15)$$

Approximate Formulae for Special Cases

Schelkunoff[1] gives a number of simplified expressions for the surface and transfer impedances of homogeneous conductors in special cases, and others may be readily deduced.

(a) *Thick tubes.* When σa is small, equation (2) for external return reduces to

$$Z_{bb} = \frac{\eta\, I_0(\sigma b)}{2\pi\, b I_1(\sigma b)} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

The impedance of a solid inner conductor is given exactly by (16) of which it is the limiting case.

When σb is large, equation (1) for internal return reduces to

$$Z_{aa} = \frac{\eta K_0(\sigma a)}{2\pi\, a K_1(\sigma a)} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

For a thick shell whose surface facing the return has a radius r metres, we have, when σr is large (e.g., $\alpha r > 4$)

$$Z_{rr} = \frac{\eta}{2\pi r}\left(1 \pm \frac{1}{2\,\sigma r}\right) \quad \ldots \quad \ldots \quad (18)$$

where $\frac{1}{2\,\sigma r}$ is the correction for curvature, the sign being positive or negative according as the return is external or internal.

$$\text{Whence } R = \frac{|\eta|}{\sqrt{2}} \cdot \frac{1}{2\pi r}\left(1 \pm \frac{1}{2\,\alpha r}\right) \text{ ohms/metre} \quad (19)$$

$$\omega L = \frac{|\eta|}{\sqrt{2}} \cdot \frac{1}{2\pi r} \text{ ohms/metre} \quad (20)$$

$$L = \frac{\mu'}{\alpha r} \times 10^{-7} \text{ henrys/metre} \quad (21)$$

$$\frac{R}{R_0} = \frac{\alpha r}{2} \pm \frac{1}{4} \quad \ldots \quad \ldots \quad (22)$$

where R_0 is the d.c. resistance of a solid wire of radius r.

(b) *Thin tubes.* When the tube wall is thin $\left(\text{e.g., the thickness } t = b - a < \frac{r}{4}\right)$, we obtain

$$Z_{rr} = \frac{\eta}{2\pi r}\left[\coth \sigma t \pm \frac{1}{2\,\sigma r}\right] \quad \ldots \quad \ldots \quad (23)$$

$$Z_{ab} = \frac{\eta}{2\pi\sqrt{ab}} \operatorname{cosech} \sigma t \quad \ldots \quad \ldots \quad (24)$$

where the sign is positive or negative according as the return is external or internal.

Whence

$$R = \frac{|\eta|}{\sqrt{2}}\,\frac{1}{2\pi r}\left[\frac{\sinh 2\,\alpha t + \sin 2\,\alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t} \pm \frac{1}{2\,\alpha r}\right] \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$\omega L = \frac{|\eta|}{\sqrt{2}}\,\frac{1}{2\pi r}\left[\frac{\sinh 2\,\alpha t - \sin 2\,\alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad (26)$$

$$R_{ab} = \frac{|\eta|}{\sqrt{2}}\,\frac{1}{\pi\sqrt{ab}}\left[\frac{\sinh \alpha t \cos \alpha t + \cosh \alpha t \sin \alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad (27)$$

$$\omega L_{ab} = \frac{|\eta|}{\sqrt{2}}\,\frac{1}{\pi\sqrt{ab}}\left[\frac{\sinh \alpha t \cos \alpha t - \cosh \alpha t \sin \alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad (28)$$

If the electrical thickness is large ($\alpha t > 2.5$), $\coth \alpha t$ approaches 1, and the equations (18) to (22) may be used. In this case we have for the transfer resistance and inductance

$$R_{ab} = \frac{|\eta|}{\pi\sqrt{ab}}\, e^{-\alpha t} \cos\left(\alpha t - \frac{\pi}{4}\right) \quad (29)$$

$$\omega L_{ab} = \frac{|\eta|}{\pi\sqrt{ab}}\, e^{-\alpha t} \cos\left(\alpha t + \frac{\pi}{4}\right) \quad \ldots \quad (30)$$

(c) *Large thin tubes.* When the tube becomes electrically large (e.g., $\alpha r > 50$), the conductor approximates to a sheet with parallel plane faces. Then for a strip of width w on the surface facing the return conductor, we have from (23) and (24),

$$Z_{rr} = \frac{\eta}{w} \coth \sigma t \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$Z_{ab} = \frac{\eta}{w} \operatorname{cosech} \sigma t \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

Whence

$$R = \frac{|\eta|}{\sqrt{2}\,w}\left[\frac{\sinh 2\,\alpha t + \sin 2\,\alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad \ldots \quad (33)$$

$$\omega L = \frac{|\eta|}{\sqrt{2}\,w}\left[\frac{\sinh 2\,\alpha t - \sin 2\,\alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad \ldots \quad (34)$$

$$R_{ab} = \frac{\sqrt{2}|\eta|}{w}\left[\frac{\sinh \alpha t \cos \alpha t + \cosh \alpha t \sin \alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$\omega L_{ab} = \frac{\sqrt{2}|\eta|}{w}\left[\frac{\sinh \alpha t \cos \alpha t - \cosh \alpha t \sin \alpha t}{\cosh 2\,\alpha t - \cos 2\,\alpha t}\right] \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

All the values in equations (23) to (36) inclusive are in ohms per metre.

Application of Howe's Method

Prof. Howe has shown[4] that the resistance of certain conductors is equal to the 'real' part of the sending-end impedance of a hypothetical 'transmission line': the 'conductors' of the hypothetical 'line' are formed by infinitesimally-thin resistance-free planes normal to the direction of the real current, and the 'dielectric' is composed of the substance of the conductor. Considering the 'line' parameters, the resistance is zero, and for any substance of reasonably good conductivity the capacitance is negligibly small; the inductance and leakance depend upon the shape of the actual conductor. In general the 'line' parameters will not be constant and the 'line' will not be uniform along its 'length.' However, in the special case when both radii of curvature of the actual conductor are large, the 'line' approximates to the uniform line familiar in telephone practice; its 'length' corresponds to the thickness of the actual conductor.

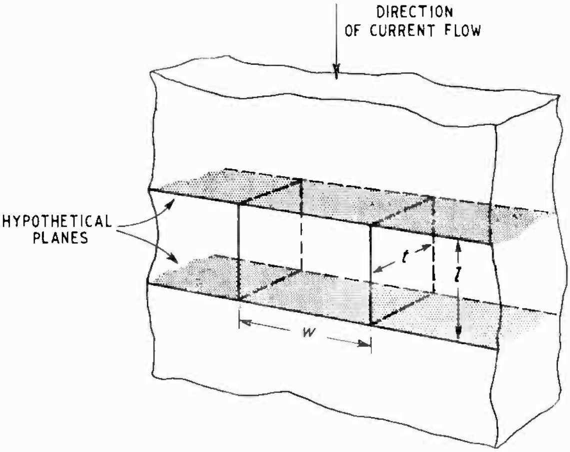


Fig. 2. *Conductor with plane parallel faces t metres apart. The return conductor is not shown.*

Fig. 2 shows a conductor consisting of a slab with plane parallel faces t metres apart; the return conductor, which is a similar slab parallel to it, is not shown. Consider a portion of l metres in length measured in the direction of current flow, and w metres in width; the 'conductors' of the corresponding hypothetical 'line' will be two parallel strips w metres wide and l metres apart. The inductance L and leakance G of the 'line' are respectively $4\pi \times 10^{-7}\mu' \frac{l}{w}$ henrys and $\frac{gw}{l}$ mhos per metre of thickness. The propagation constant of the 'line' $= \sqrt{j\omega LG} = \sqrt{j\omega 4\pi \times 10^{-7}\mu' g} = \sigma$ per metre of thickness. The characteristic impedance of the 'line' $= \sqrt{\frac{j\omega L}{G}} = \frac{l}{w}\sqrt{j\omega 4\pi \times 10^{-7}\mu'/g} = \frac{l\eta}{w} = \frac{\eta}{w}$ ohms per metre of length, where σ and η are the intrinsic propagation constant and intrinsic impedance respectively as defined by Schelkunoff.

The sending-end impedance of the 'line' is $\frac{\eta}{w}\coth \sigma t$ ohms/metre, which is the same as the surface impedance in equation (31). The transfer impedance, defined as the ratio of received voltage to sent current, is $\frac{\eta}{w}\,\text{cosech}\,\sigma t$ ohms per metre, and is the same as the transfer impedance in equation (32).

Consideration of the more general case of coaxial tubular conductors yields a similar result, and we may infer that the sending-end impedance of Howe's hypothetical 'line' has the same value as Schelkunoff's surface impedance, and its tranfer impedance is equal to Schelkunoff's transfer impedance; thus the 'real' part of the sending-end impedance is equal to the effective resistance R of the conductor, and the 'imaginary' part equals the internal reactance ωL.

We may further represent the hypothetical 'transmission line' by its equivalent network. In Fig. 3 (a), A A′, B B′ represent a portion of the tubular conductor shown in Fig. 1 between two planes normal to the axis and unit distance apart: A A′ are on the inner surface and B B′ on the outer surface. If the equivalent network is a T having series arms W, X and shunt arm Y, then the sending-end impedance looking at the inner surface from the axis is

$$W + Y = Z_{aa} \quad \ldots \quad (37)$$

The sending-end impedance looking at the outer surface from the outside is

$$X + Y = Z_{bb} \quad \ldots \quad (38)$$

The transfer impedance is

$$Y = Z_{ab} \quad \ldots \quad (39)$$

Applying this method to the composite conductor, let the equivalent T networks be as

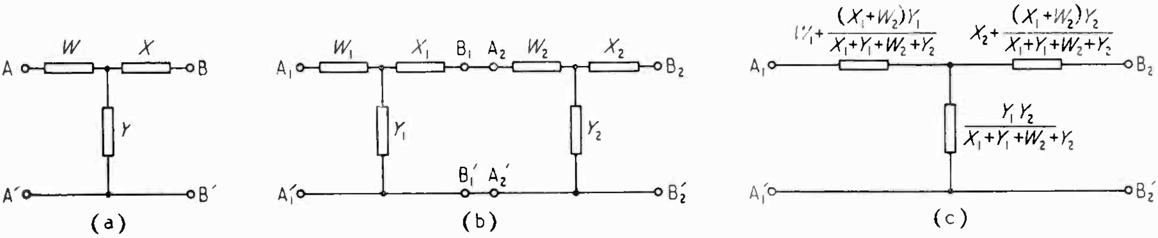


Fig. 3. *The equivalent network of a portion of the tubular conductor of Fig.* 1 *is shown at* (a). *A similar equivalent for a composite conductor is given at* (b) *which can be transformed by the star-delta theorem to the form* (c).

shown in Fig. 3 (b) where $A_1 A_1'$, $B_1 B_1'$ represent the surfaces of the conductor 1, and $A_2 A_2'$, $B_2 B_2'$ the surfaces of conductor 2. Using the star-mesh conversion theorem, the equivalent networks may be converted to the T shown in Fig. 3 (c). The sending-end impedance looking into surface $A_1 A_1'$ is

$$Z_{aa}' = W_1 + \frac{(X_1+W_2)Y_1}{X_1+Y_1+W_2+Y_2} + \frac{Y_1\, Y_2}{X_1+Y_1+W_2+Y_2}$$

$$= W_1 + Y_1 - \frac{Y_1^2}{X_1+Y_1+W_2+Y_2}$$

$$= Z_{aa}^{(1)} - \frac{[Z_{ab}^{(1)}]^2}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad \ldots \quad (40)$$

Similarly the sending-end impedance looking into surface $B_2 B_2'$ is

$$Z_{bb}' = X_2 + \frac{(X_1+W_2)Y_2}{X_1+Y_1+W_2+Y_2} + \frac{Y_1\, Y_2}{X_1+Y_1+W_2+Y_2}$$

$$= Z_{bb}^{(2)} - \frac{[Z_{ab}^{(2)}]^2}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad \ldots \quad (41)$$

The transfer impedance is

$$Z_{ab}' = Z_{ba}' = \frac{Y_1\, Y_2}{X_1+Y_1+W_2+Y_2}$$

$$= \frac{Z_{ab}^{(1)} \cdot Z_{ab}^{(2)}}{Z_{bb}^{(1)} + Z_{aa}^{(2)}} \quad \ldots \quad \ldots \quad (42)$$

These are the same results as are given by Schelkunoff's theorem, see Equations (13), (14) and (15), and are a further justification of the generalization regarding the Howe hypothetical 'line' given earlier. The extension to a composite conductor formed of a number of different layers follows similar lines. It is interesting to observe that a conductor is represented by the same network irrespective of whether the return is internal or external.

Simplification of Formulae for Composite Conductors

The formulae for the impedance of composite conductors do not in general lend themselves to simplification. However, when the radii of the conductor are electrically large, and the curvature may be neglected, some useful simplifications may be made.

Let the composite conductor consist of a surface material S and a base material B, S being nearer the return conductor; S is on the concave side when the return is internal and on the convex side when the return is external; subscripts S and B will be used to distinguish the characteristics of the two materials.

Since the curvature is being neglected, the surface impedance, denoted by Z', will be the same whether the return is internal or external. From equations (13), (31) and (32) we have

$$Z' = \frac{\eta_S}{w}\coth\sigma_S t_S - \frac{\left(\dfrac{\eta_S}{w}\operatorname{cosech}\sigma_S t_S\right)^2}{\dfrac{\eta_S}{w}\coth\sigma_S t_S + \dfrac{\eta_B}{w}\coth\sigma_B t_B}$$

$$= \frac{\eta_S}{w}\left(\frac{\eta_S + \eta_B\coth\sigma_S t_S\coth\sigma_B t_B}{\eta_S\coth\sigma_S t_S + \eta_B\coth\sigma_B t_B}\right) \text{ ohms/metre} \quad (43)$$

If now the layer of material B is taken to be electrically thick, $\coth\sigma_B t_B \to 1$, and we obtain

$$Z' = \frac{\eta_S}{w}\tanh(\sigma_S t_S + \theta) \text{ ohms/metre} \quad \ldots \quad (44)$$

$$\text{where } \tanh\theta \equiv \frac{\eta_B}{\eta_S} = \sqrt{\frac{\mu_B g_S}{\mu_S g_B}} \quad \ldots \quad \ldots \quad (45)$$

From the relation $\tanh(x + jy)$

$$= \frac{\sinh 2x + j\sin 2y}{\cosh 2x + \cos 2y}$$

we may write

$$\tanh(\sigma_S t_S + \theta) \equiv (P + jQ) \quad \ldots \quad \ldots \quad (46)$$

(a) If $\frac{\eta_B}{\eta_S} < 1$, θ is wholly real,

$$\text{then } P = \frac{\sinh 2(\alpha_S t_S + \theta)}{\cosh 2(\alpha_S t_S + \theta) + \cos 2\alpha_S t_S} \quad \ldots \quad (47)$$

$$Q = \frac{\sin 2\alpha_S t_S}{\cosh 2(\alpha_S t_S + \theta) + \cos 2\alpha_S t_S} \quad \ldots \quad (48)$$

(b) If $\frac{\eta_B}{\eta_S} > 1$, $\theta = \theta' + j\frac{\pi}{2}$,

$$\text{where } \theta' = \tanh^{-1}\left(\frac{\eta_S}{\eta_B}\right) \quad \ldots \quad \ldots \quad (49)$$

$$\text{then } P = \frac{\sinh 2(\alpha_S t_S + \theta')}{\cosh 2(\alpha_S t_S + \theta') - \cos 2\alpha_S t_S} \quad \ldots \quad (50)$$

$$Q = \frac{-\sin 2\alpha_S t_S}{\cosh 2(\alpha_S t_S + \theta') - \cos 2\alpha_S t_S} \quad \ldots \quad (51)$$

From (44) and (46) we have

$$Z' = \frac{\eta_S}{w}(P+jQ) = \frac{|\eta_S|}{\sqrt{2w}}\Big[P - Q + j(P+Q)\Big]$$

$$\text{ohms/metre} \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

$$\therefore R' = \frac{|\eta_S|}{\sqrt{2w}}(P - Q) \text{ ohms/metre} \quad \ldots \quad (53)$$

$$\omega L' = \frac{|\eta_S|}{\sqrt{2w}}(P + Q) \text{ ohms/metre} \quad \ldots \quad (54)$$

It is convenient to compare the composite conductor with a uniform conductor composed wholly of material S or material B; the former is more suitable where the layer of S is thick, and the latter where the layer of S is thin.

Comparing with an electrically thick slab of material S for which

$$R_S = \omega L_S = \frac{|\eta_S|}{\sqrt{2w}} \quad \text{ohms/metre} \quad .. \quad (55)$$

we have

$$\frac{R'}{R_S} = P - Q \quad .. \quad .. \quad .. \quad .. \quad (56)$$

$$\frac{L'}{L_S} = P + Q \quad .. \quad .. \quad .. \quad .. \quad (57)$$

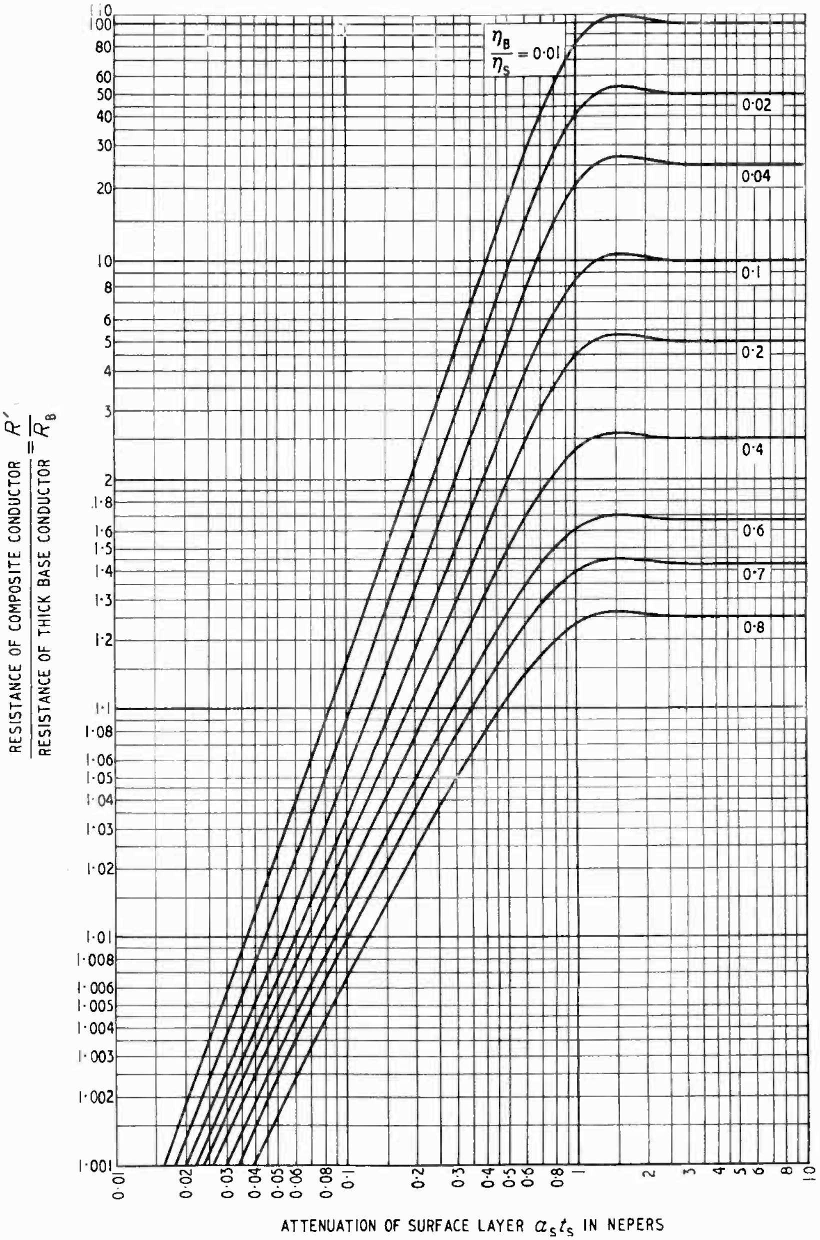
Again comparing with an electrically thick slab of material B for which

$$R_B = \omega L_B = \frac{|\eta_B|}{\sqrt{2w}} \quad \text{ohms/metre} \quad .. \quad (58)$$

we have

$$\frac{R'}{R_B} = \frac{\eta_S}{\eta_B}(P - Q) \quad .. \quad .. \quad .. \quad (59)$$

$$\frac{L'}{L_B} = \frac{\eta_S}{\eta_B}(P + Q) \quad .. \quad .. \quad .. \quad (60)$$



A convenient parameter for calculations is the attenuation thickness of the surface layer, $\alpha_S t_S = 2\pi t_S \sqrt{f \mu_S' g_S \times 10^{-7}}$ nepers. In Fig. 4, the ratio $\frac{R'}{R_B}$ is plotted against $\alpha_S t_S$ for values of $\frac{\eta_B}{\eta_S} < 1$, and in Fig. 5 $\frac{R_B}{R'}$ is plotted against $\alpha_S t_S$ for values of $\frac{\eta_B}{\eta_S} > 1$. The scales for the ordinates are 'pseudo-logarithmic', obtained by plotting $\frac{R'}{R_B} - 1$ and $\frac{R_B}{R'} - 1$ on logarithmic scales and re-numbering the ordinates to show the net values.

When the layer of material S is electrically thick (e.g., $\alpha_S t_S + \theta > 1.2$) equations (56) and (57) may be simplified.

From (44) we have, if $\frac{\eta_B}{\eta_S} < 1$,

$$Z' = \frac{\eta_S}{w} \tanh (\sigma_S t_S + \theta)$$

$$\approx \frac{\eta_S}{w} \{1 - 2 \exp(-2[\alpha_S t_S + \theta])\}$$

$$= \frac{|\eta_S|}{\sqrt{2w}} (1 + j)\{1 - 2 \exp(-2[\alpha_S t_S + \theta])(\cos 2\alpha_S t_S - j \sin 2\alpha_S t_S)\}$$

Fig. 4. *Relation between the attenuation thickness of the surface layer and the ratio of the resistances of the two materials is shown here for values of $\eta_B/\eta_S < 1$.*

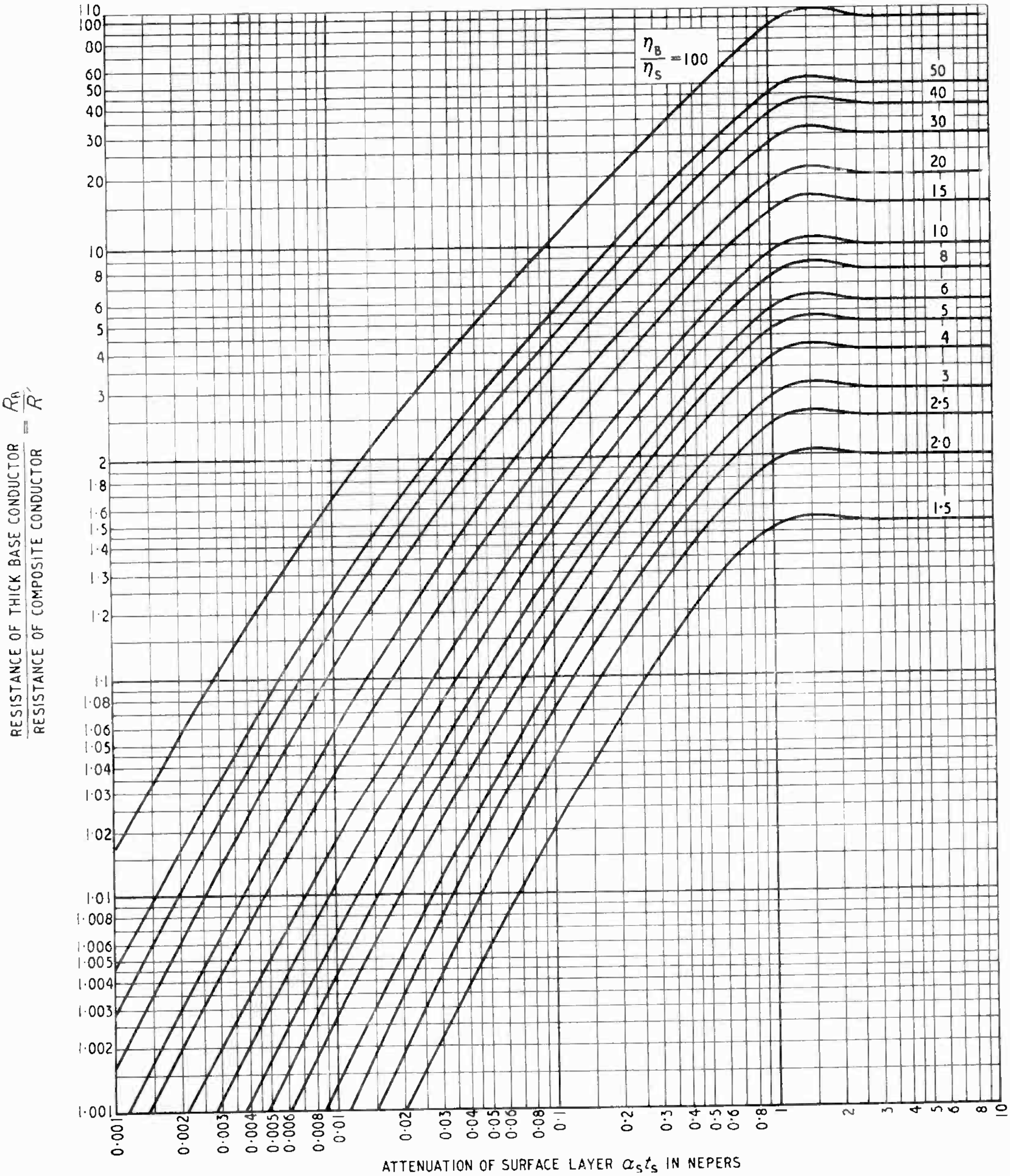


Fig. 5. *Relation between the attenuation thickness of the surface layer and the ratio of resistance of the two materials is shown here for values of $\eta_B/\eta_S > 1$.*

Whence

$$\frac{R'}{R_s} = 1 - 2\exp\left(-2[\alpha_s t_s + \theta]\right)(\cos 2\alpha_s t_s + \sin 2\alpha_s t_s) \quad \ldots \quad (61)$$

$$\frac{L'}{L_s} = 1 - 2\exp\left(-2[\alpha_s' s + \theta]\right)(\cos 2\alpha_s t_s - \sin 2\alpha_s t_s) \quad \ldots \quad (62)$$

Similarly if $\frac{\eta_B}{\eta_S} > 1$ and $\theta' = \tanh^{-1}\left(\frac{\eta_S}{\eta_B}\right)$,

$$\frac{R'}{R_s} = 1 + 2\exp\left(-2[\alpha_s t_s + \theta']\right)(\cos 2\alpha_s t_s + \sin 2\alpha_s t_s) \quad \ldots \quad (63)$$

$$\frac{L'}{L_s} = 1 + 2\exp\left(-2[\alpha_s t_s + \theta']\right)(\cos 2\alpha_s t_s - \sin 2\alpha_s t_s) \quad \ldots \quad (64)$$

When the layer of material S is electrically thin ($\alpha_s t_s < 0.5$) equations (59) and (60) may be converted to forms convenient for calculation with the slide-rule.

From (59) we have when $\frac{\eta_B}{\eta_S} < 1$,

$$\frac{R'}{R_B} = \frac{\sinh 2\alpha_S t_S \cosh 2\theta + \cosh 2\alpha_S t_S \sinh 2\theta - \sin 2\alpha_S t_S}{\cosh 2\alpha_S t_S \cosh 2\theta + \sinh 2\alpha_S t_S \sinh 2\theta + \cos 2\alpha_S t_S} \cdot \frac{1}{\tanh \theta}$$

Expanding the functions of $2\alpha_S t_S$ and taking the first two terms of each series, we have, after rearrangement

$$\frac{R'}{R_B} = 1 + \frac{2(\alpha_S t_S)^2\left[1 - \left(\frac{\eta_B}{\eta_S}\right)^2\right]\left[1 + \frac{2}{3}\alpha_S t_S\left(\frac{\eta_S}{\eta_B}\right)\right]}{1 + 2\alpha_S t_S\left(\frac{\eta_B}{\eta_S}\right) + 2(\alpha_S t_S)^2\left(\frac{\eta_B}{\eta_S}\right)^2 + \frac{4}{3}(\alpha_S t_S)^3\left(\frac{\eta_B}{\eta_S}\right)} \quad \ldots \quad \ldots \quad (65)$$

Similarly
$$\frac{L'}{L_B} = 1 + \frac{2\alpha_S t_S\left[1 - \left(\frac{\eta_B}{\eta_S}\right)^2\right]\left(\alpha_S t_S + \frac{\eta_S}{\eta_B}\right)}{1 + 2\alpha_S t_S\left(\frac{\eta_B}{\eta_S}\right) + 2(\alpha_S t_S)^2\left(\frac{\eta_B}{\eta_S}\right)^2 + \frac{4}{3}(\alpha_S t_S)^3\left(\frac{\eta_B}{\eta_S}\right)} \quad \ldots \quad (66)$$

The same results are obtained when $\frac{\eta_B}{\eta_S} > 1$, and hence these expressions are valid for all values of $\frac{\eta_B}{\eta_S}$.

When both materials are non-magnetic,

$$\left(\frac{\eta_B}{\eta_S}\right)^2 = \frac{g_S}{g_B} \equiv h \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (67)$$

Then

$$\frac{R'}{R_B} = 1 + \frac{2\alpha_B^2 t_S^2 h(1-h)(1 + \frac{2}{3}\alpha_B t_S)}{1 + 2\alpha_B t_S h + 2\alpha_B^2 t_S^2 h^2 + \frac{4}{3}\alpha_B^3 t_S^3 h^2} \quad (68)$$

$$\frac{L'}{L_B} = 1 + \frac{2\alpha_B t_S(1-h)(1 + \alpha_B t_S h)}{1 + 2\alpha_B t_S h + 2\alpha_B^2 t_S^2 h^2 + \frac{4}{3}\alpha_B^3 t_S^3 h^2} \quad (69)$$

If material B is copper (subscript *cu*), $\alpha_{cu} = 15.3\sqrt{f}$ nepers/metre.

$$\therefore \frac{R'}{R_{cu}} = 1 + \frac{470 f t_S^2 h(1-h)(1 + 10.2 f^{\frac{1}{2}} t_S)}{1 + 30.5 f^{\frac{1}{2}} t_S h + 470 f t_S^2 h^2 + 4740 f^{\frac{3}{2}} t_S^3 h^2} \quad (70)$$

$$\frac{L'}{L_{cu}} = 1 + \frac{30.5 f^{\frac{1}{2}} t_S(1-h)(1 + 15.3 f^{\frac{1}{2}} t_S h)}{1 + 30.5 f^{\frac{1}{2}} t_S h + 470 f t_S^2 h^2 + 4740 f^{\frac{3}{2}} t_S^3 h^2} \quad (71)$$

where f is in cycles per second and t_S in metres or f is in Mc/s and t_S in millimetres.

Alternatively,

$$\frac{R'}{R_{cu}} = 1 + \frac{0.301 f t_S^2 h(1-h)(1 + 0.259 f^{\frac{1}{2}} t_S)}{1 + 0.775 f^{\frac{1}{2}} t_S h + 0.301 f t_S^2 h^2 + 0.0777 f^{\frac{3}{2}} t_S^3 h^2} \quad (72)$$

$$\frac{L'}{L_{cu}} = 1 + \frac{0.775 f^{\frac{1}{2}} t_S(1-h)(1 + 0.388 f^{\frac{1}{2}} t_S h)}{1 + 0.775 f^{\frac{1}{2}} t_S h + 0.301 f t_S^2 h^2 + 0.0777 f^{\frac{3}{2}} t_S^3 h^2} \quad (73)$$

where f is in cycles per second and t_S is in inches or f is in Mc/s and t_S in mils.

In Table I on next page, the constants of some common conductors are listed, and the data are employed in the two examples which follow.

Conclusion

The identity of the impedance equations for conductors and for the hypothetical transmission line permits us to deduce the general behaviour of compound conductors from the known properties of composite transmission lines, it being noted that current density and voltage gradient in conductors correspond to current and voltage in transmission lines. The characteristic feature is the reflection that takes place at a boundary between dissimilar structures, the magnitude of the reflected wave being determined by the relative values of the impedances on either side of the boundary, and by the attenuation of the section between the origin and the boundary. For example, it is well-known that a hollow homogeneous conductor has an optimum thickness for a given frequency at which its resistance is a minimum, and a similar effect occurs with composite conductors, as the thickness of the surface layer is varied. However, as the attenuation constant of metals is equal to the phase constant, the reflection effects are relatively limited, corresponding to the series-resistance shunt-capacitance type of line, rather than to the series-inductance shunt-capacitance type, in which reflection effects are marked.

Although the equations which have been studied in detail refer only to composite conductors in which the curvature is negligibly small, they cover many of the cases which occur in practice. When the curvature of a composite conductor is small but not negligible, it can be shown that for electrically-thin surface layers the correction is the same as for the base conductor, and equations (59) and (60) should be used; when the surface layer is electrically thick, the correction is the same as for the surface conductor and equations (56) and (57) apply. In general the reflection phenomena in curved conductors are of the same nature as those in which the curvature is negligible.

Example 1

A copper tube, used as the outer conductor of a

TABLE I

	Copper	Iron	Lead	Tinning Alloy	Sea-water
Conductivity, g mhos/metre	5.90×10^7	8.3×10^6	4.7×10[illegible]	7.4×10^6	4
Relative permeability μ'	1	say 100	1	1	1
Intrinsic impedance at 1 Mc/s, microhms	366	9,800	1,300	1,030	1.4×10^6
Attenuation α at 1 Mc/s, nepers/metre . .	15,300	57,000	4,300	5,400	4

coaxial pair, is coated on the inside with a layer of tin 2.5×10^{-3} mm thick, the internal diameter being 1 cm; to find the effect at 20 Mc/s. Applying equations (70) and (71), $h = 0.125$, whence $R'/R_{cu} = 1.0069$, $L'/L_{cu} = 1.29$. Alternatively using the curves in Fig. 6, $\alpha_s = 2.42 \times 10^4$ nepers/metre, $\alpha_s t_s = 0.061$ neper, $\eta_B/\eta_s = 0.355$ whence $R'/R_b = 1.0068$. Using equation (19), as $2\alpha r = 684$ nepers, the correction for curvature is negligible, and $R_{cu} = 36.9$ ohms/kilometre. From equation (21) $L_{cu} = 0.293$ μH/kilometre. Thus the resistance added by the layer of tin is 0.25 ohm/kilometre, and the internal inductance is increased by 0.09 μH/kilometre.

It will be seen from this example that the effect on the internal inductance is relatively large compared with the effect on the resistance. However, as the internal inductance is small compared with the total inductance of the usual coaxial pair, the inductance change is of less interest than the resistance change.

Example 2

A solid circular iron conductor coated with a copper sheath 0.1 mm thick has an external diameter of 1 mm; to find the resistance and internal inductance at 1 Mc/s. We have $\alpha_s t_s = 1.53$ nepers, $\eta_B/\eta_s = 26.8$, $\theta' = 0.0374$, and from equations (63) and (64), $R'/R_s = 0.921$, $L'/L_s = 0.907$. Applying equations (19) and (21), $2\alpha r = 15.3$ nepers, correction for curvature $= 6.5\%$; $R_s = 87.8$ ohms/kilometre, $L_s = 13.1$ μH/kilometre, whence $R' = 80.9$ ohms/kilometre, $L' = 12.0$ μH/kilometre. This example is a case in which, due to the reflection effect mentioned above, the replacement of the interior of a solid copper conductor by iron causes a *decrease* in the effective resistance.

REFERENCES

1 S. A. Schelkunoff: "The Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shields," *Bell Syst. tech. J.*, 1934, Vol. 13. p.532.

2 H. Carsten: "The Effective Resistance of a Bi-metallic Conductor," 1941, Unpublished memorandum, B.I. Cables, Ltd.

3 R. C. Mildner: "The Power Rating (Thermal) of Radio-Frequency Cables," *J. Instn elect. Engrs*, 1946, Vol. 93, Part III, p. 296.

4 G. W. O. Howe: "The Application of Telephone Transmission Formulae to Skin-effect Problems," *J. Instn elect. Engrs*, 1916, Vol. 54. p. 473.

5 British Association Tables, 1912. pp. 57–68; 1915. pp. 36–38; 1916, pp. 108–122. "Tables of Functions," By Jahnke and Emde (book).

NEW BOOKS

Terrestrial Radio Waves

By H. BREMMER. Pp. 344 + x, with 91 illustrations. Cleaver-Hume Press, Ltd., 42a, South Audley St., London, W.1. Price 36s.

No treatise of the scope of the one under review has previously appeared, for it is based on the researches of its author originating in the classic papers written in collaboration with Dr. van der Pol. As it contains all the mathematical rigour associated with their solution of the spherical-earth problem, it inevitably makes difficult reading.

It will appeal most to the mathematical physicist who can follow the general line of the analysis, but there are some sections of direct use to the engineer, in particular the brief description of the forecasting of ionospheric conditions and the curves of ground-wave and sky-wave field-strengths. The author has been at great pains to show the physical significance of the complicated mathematics and is to be commended for the clarity with which he has developed the subject and stated the fundamental ideas.

In places, however, the adoption of a rigorous mathematical standard has obscured the essential physical simplicity of the argument. This is especially so in the discussion of the height-gain factor. Its initial form, independent of the order of the mode, appears as a result of a difficult mathematical manipulation, whereas it is a fundamental property of Fresnel reflection, as likewise is the nature of the forward component of field at the surface of an imperfectly conducting earth.

Of special interest are the discussions on the transition from the Sommerfeld flat-earth formula to the residue series in the diffraction region, and on the failure of a simple Huyghens treatment of diffraction round a perfectly-conducting flat screen to account for the exponential attenuation with distance. One of the most useful features of the book is the erudite discussion of the effects near caustics and foci, dealing with the field at the edge of the skip zone in sky-wave propagation and at the antipodes.

The second part of the book takes account of an inhomogeneous but radially-symmetrical atmosphere including both the troposphere and the ionosphere.

It is nearly all original work done by the author during the war years, though it naturally overlaps with similar work done in other countries. Here again many familiar results could be more simply derived, but it is also here that such a treatment as the present is most needed. It is one of its greatest merits that by its rigidity it provides a final court of appeal in questioning the validity of many approximate methods.

The final chapter on magneto-ionic theory is based almost entirely on ray theory with the help of the W.K.B. approximation and serves as a salutary reminder of the immense difficulties in the way of a full wave treatment taking account of the variations in the ionosphere and in the direction of the magnetic field over the surface of the earth. Similarly, in the ground-wave theory the assumption of an ideal smooth homogeneous earth is in effect maintained, and no attack is made on the problem of transmission over composite land and sea paths apart from a passing reference to coastal refraction.

The book is remarkably up-to-date and contains references to the work of most contemporary writers which has only been accessible to the author since the war. The printing is excellent, and only in one or two trivial instances does the grammar reveal that English is not the author's native tongue.

Though it will probably have a very limited number of readers who can penetrate through the mathematics to its physical and engineering implications, this book will be a mine of information and an inspiration to the few to whom will fall the task of tackling the outstanding problems of propagation theory. G. M.

An Introduction to the Laplace Transformation

By J. C. Jaeger. Pp. 132. Methuen & Co., 36, Essex St., London, W.C.2. Price 7s. 6d.

The book contains the substance of a course of lectures delivered to engineers and physicists at the National Standards Laboratory, Sydney, in 1944.

Chapter 1 contains fundamental theory and includes a table of the transforms of simple functions. The usefulness of the transforms in initial-value problems is pointed out. The technique for expressing a rational function in partial fractions is discussed in some detail. An interesting comparison with the ordinary method of solving linear differential equations with constant coefficients, and with Heaviside's operational method, is included.

Chapter 2 deals with the fundamental equation of an LRC circuit and its solution, and with networks, circuits containing triodes operating on the linear portion of their characteristic and filter circuits. Mechanical systems and their equivalent electrical networks, and servomechanisms, are also discussed. The solution of numerical equations and the behaviour of a particular circuit containing variable parameters are considered in some detail; in this connection a recent paper by Porter & Mack† should be noted.

Chapter 3 states and explains general theorems, namely Heaviside's shifting theorem, Heaviside's series expansion, Thévenin's theorem, Duhamel's or the 'Superposition' theorem, and the inversion theorem. The connection with Fourier's integral theorem and Parseval's theorem is also discussed.

Chapter 4 deals with the reduction of partial differential equations to ordinary ones by means of Laplace transformations, and uniform, semi-infinite and finite transmission lines (including those with terminal impedances) are discussed in detail. A table of transforms of functions required for these problems is given on page 126.

". . . The book contains as little theory as possible; it is, in fact, largely a collection of worked examples illustrating the methods of solution of the various types of problem commonly arising in circuit theory." Proofs of theorems have been omitted where they involve advanced pure mathematics, but limitations affecting the practical use of the theorems are clearly stated. Examples for the reader are included.

Only a few minor misprints can be found, such as one in equation (2.6) on p. 7, where the factor $(a_r + a_{r-1})$ in the denominator should read $(a_r - a_{r+1})$. In some equations, such as that preceding (11.3) on page 37, the alignment is unsatisfactory. These few misprints merely serve to emphasize the clear and concise presentation of the subject as a whole.

The Laplace transform is in essence a purely mathematical device, but one capable of suggesting the solution to many practical problems. This book is very successful in showing how those whose interests are mainly practical can use the transform with advantage.

J. W. H.

† A. Porter and C. Mack. "New Methods for the Numerical Solution of Algebraic Equations." *Phil. Mag.*, May 1949, Vol. 40, No. 304, pp. 578–585.

Studien über Impulsmodulation

By Walter Bachmann. Pp. 69, with 48 illustrations. Verlag Leemann, Zürich. Price, 9 fr. (Swiss).

This is a description of work done at the Institut für Hochfrequenztechnik of the Hochschule in Zürich. The first half is devoted to the theory of the various methods of impulse modulation, and the second to their practical applications. Apparatus is described which was constructed and tested in the laboratory, and the results obtained are discussed. The method adopted in the laboratory was the position or phase modulation of the impulse, as this was considered the most suitable for multi-channel operation. A bibliography of the subject is given.

G. W. O. H.

Table for Use in the Addition of Complex Numbers

By Jørgen Rybner & K. Steenberg Sørensen. Pp. 92 + xiv. In Danish and English. Jul. Gjellerups Forlag, Copenhagen. Price Kr. 20.

The table gives the modulus R to 5 places of decimals and amplitude α in degrees to 3 places of decimals of the complex number

$$R\epsilon^{j\alpha} = 1 + r\epsilon^{j\phi}$$

for $0 \leqslant r \leqslant 1$ and $0 \leqslant \phi \leqslant 180^\circ$, at intervals of 0.01 for r and 1° for ϕ.

By means of this table, the addition of two complex numbers in polar form can be performed with considerably fewer operations than are required for the normal reduction to Cartesian form and reconversion of the result to polar form. If

$$z_1 = r_1\epsilon^{j\phi_1}, \quad z_2 = r_2\epsilon^{j\phi_2}, \quad (r_1 > r_2)$$

we write

$$z_1 + z_2 = r_1\epsilon^{j\phi_1}\,[1 + (r_2/r_1)\epsilon^{j(\phi_2 - \phi_1)}]$$

and use the tables to reduce the contents of the square brackets to a single complex number. An introduction gives specific examples of the use of the tables, and discusses problems of interpolation in detail.

As the polar form of a complex number is undoubtedly more convenient than the Cartesian for all operations of arithmetic except addition and subtraction, and is the form in which the significance of electrical quantities is most obvious, the tables are likely to prove well worth the trouble of becoming accustomed to an indirect method of attack upon what appears at first sight to be a problem too simple to require the help of tables.

J. W. H.

A Dictionary of Mathematics

By C. H. McDowell, A.M.I.C.E., F.R.G.S. Vol. II, pp. 63. Mathematical Dictionaries, 72, Victoria St., London, S.W.1. Price 5s.

The Principles of Scientific Research

By Paul Freedman. Pp. 222 + x. Macdonald & Co., Ltd., 19, Ludgate Hill, London, E.C.4. Price 15s.

Tables of Sines and Cosines to 15 Decimal Places at Hundredths of a Degree

Pp. 95. National Bureau of Standards, U.S. Dept. of Commerce, Washington 25, D.C., U.S.A. Price 40 c. (Postage + 33⅓%).

Electrical Standards for Research and Industry

This is a 204-page catalogue of the products of H. W. Sullivan, Ltd., Lea St., Peckham, London, S.E.15. It is a very full and detailed catalogue of high-precision measuring equipment and standards. Standard and secondary inductors, capacitors and resistors are included, as well as inductance and capacitance bridges.

Wavemeters of both the absorption and self-oscillating types are listed in addition to audio- and carrier-frequency oscillators of precision types.

Voltage Control with Variacs

This 32-page catalogue (V-549), produced by Claude Lyons, Ltd., 180, Tottenham Court Rd., London, W.1, contains details of the range of Variacs marketed by this firm. In addition, general information is given about the characteristics and applications of such variable transformers.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Valve Noise and Transit Time

Sir,—In the May 1948 issue of your journal, Campbell, Francis and James[1] criticized some conclusions reached by me in an article, "Fluctuations and Electron Inertia," which I wrote as long ago as 1941 in the Dutch journal *Physica*.[2]

First, Campbell, Francis and James criticize the formula I derived for the induced grid-current fluctuations in a pentode high-frequency amplifying valve.[3] They state that the arguments by which I derive this formula are so condensed and employ conceptions so different from those they regard as significant that they are unable to determine how, if at all, my physical assumptions differ from theirs.

I must agree that my 1941 paper was written in a condensed form and I was about to produce a more detailed treatment when I became aware of E. J. Schremp's interesting article on amplifier sensitivity, in which he arrives at my formula[4]; as he gives an extensive derivation of this formula I consider it is sufficient to refer to his article.

Next, Campbell, Francis and James criticized the experiments which I carried out to test my formula. They remarked that the electrode configuration of the EF50 valve which I investigated diverges so widely from plane parallelism that the theory is not applicable. The cathode is a circular cylinder (ϕ1.30 mm) and the surrounding grid is of rhombic form. However, as most of the current passes through the grid at the points where it is nearest to the cathode (about 0.16 mm) it is, in my opinion, quite correct to suppose that a plane parallel configuration is a good approximation.

Consequently, I consider that my measurements are a convincing proof of the correctness of my theory. I am glad to note that Dr. Stumpers, who has repeated my early measurements and extended them considerably, comes to the same conclusion. (See following letter to the Editor).

The criticism of Francis, Campbell and James as to the correctness of my measurements was strongly supported by Houlding in a letter in the November 1948 issue of *Wireless Engineer*.[5] He considers that I made a serious mistake by neglecting the background fluctuations due to the receiver, and I must agree that my paper was not completely clear in this respect. However, in a private correspondence with Mr. Houlding, I have explained my method of measurement and he has expressed his satisfaction with it.

He has, however, drawn attention to a slight correction needed in my measurements to allow for the effective elimination of input conductance as well as induced grid noise when the test valve is biased back. As Dr. Stumpers has used the same procedure as I did and proved again that the correction is negligible, I need not refer to it here.

Zeeman-Laboratorium,
Amsterdam University, Holland.
C. J. Bakker.

REFERENCES

[1] N. R. Campbell, V. J. Francis and E. G. James, *Wireless Engineer*, Vol. 25, pp. 148, 1948.
[2] C. J. Bakker, *Physica*, Vol. 8, p. 23, 1941.
[3] Also, North and Ferris, *Proc. Inst. Radio Engrs*, Vol. 29, p. 49, 1941, derived the same formula
[4] "Vacuum Tube Amplifiers," Vol. 18, M.I.T. Radiation Laboratory Series, p. 568. McGraw-Hill.
[5] N. Houlding, *Wireless Engineer*, Vol. 25, p. 372, 1948.

Measurements of Induced Grid Noise

Sir,—In view of N. Houlding's criticism of the measurements of induced grid noise by Prof. Bakker, it may be worthwhile to give a summary of some further experimental results. Since Prof. Bakker's original publication[1] may not be known to your readers, we shall first discuss the procedure. The schematic arrangement is given in the figure. The amplifier output is measured by means of a thermocouple connected to a galvanometer. The linearity of the amplifier is tested by variation of the diode current.

First the test valve is completely cut off and no diode current flows. The resonant circuit is detuned or short-circuited. The galvanometer current is I_1.

Then the LC circuit is tuned, and the maximum galvanometer current is I_2.

Now we apply a diode current i_1 in order to reach a galvanometer current $I_3 = 2I_2 - I_1$.

The test valve is given a negative voltage corresponding to the condition in which we wish to measure it, and the tuned circuit is brought into resonance again. No diode current flows and the galvanometer current is I_4.

By means of a diode current i_2 an output current $I_5 = 2I_4 - I_1$ is reached.

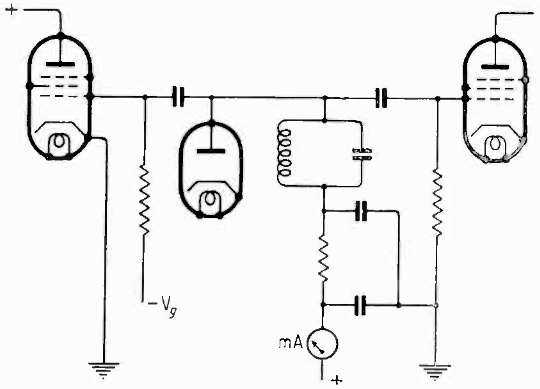
If we introduce $i_D = i_2 - i_1$ it can be shown that the induced grid noise in a frequency interval $\Delta\upsilon$ is given to a first approximation by

$$\overline{i_g^2} = 2ei_D \Delta\upsilon.$$

Prof. Bakker has remarked that theoretically it is necessary to subtract a term

$$4kTR_{n2}\, G_{L2}^2\, \Delta\upsilon.$$

from the right-hand member, but he has also shown that for the frequencies he used, the correction was negligible.



Mr. Houlding has drawn attention to another correction term to be subtracted from the right-hand side in the above formula; viz.,

$$8kTR_{n1}\, G_{L1}\, (G_{L2} + G_\tau)\, \Delta\upsilon.$$

where:

R_n = equivalent (triode) noise resistance of valve;
G_L = input conductance due to cathode-lead inductance;
G_τ = test-valve input conductance as a transit-time effect.

The suffix 1 refers to the first amplifier valve, and the suffix 2 to the test valve.

In the earlier experiments it was shown that a substantial increase in G_{L2} has no appreciable influence on the noise measurement. Moreover, in so far as this correction-term depends on the input-valve, only the product $L_k C_{gk}$ is of importance and not the transconductance. For this reason the acorn valve 4672 (as used by Bakker above 30 Mc/s), the EF51 and 6AK5 (both with two cathode leads) tend to make the correction small. A calculation has confirmed that in the region 20–100 Mc/s this term remains within 1% of the experimental value for the 6AK5. For this valve we found at 50 Mc/s:

I_a = 7.5 mA; g_m = 5.5 mA/V; i_D = 4.83μA; $G_\tau = 18.7 \times 10^{-6}$ mho;
$G_L = 7.6 \times 10^{-6}$ mho.
I_a = 5 mA; g_m = 4.0 mA/V; i_D = 3.52μA; $G_\tau = 13.6 \times 10^{-6}$ mho;
$G_L = 5.6 \times 10^{-6}$ mho.

For the calculation of G_τ we used Bakker's formula:

$$\overline{i_g^2} = 1.43 \times 4kT_c G_\tau \cdot \Delta\upsilon$$

The total conductance is measured by the width of the resonant circuit and by subtraction we get G_L.

Since G_L is defined by the equation

$$G_L = g_m \omega^2 L_k C_{gk}$$

and C_{gk} can be measured, we have an indirect determination of L_k. We used it to check the formula. From the above values we calculated for the 6AK5 a cathode-lead inductance of 0.4×10^{-8} H, in agreement with a value obtained by Mr. van Hofweegen of this laboratory by determining the cathode-grid resonance. (The two cathode-leads were used in parallel.)

For the measurements with the EF51 input valve the correction is somewhat larger. The experimental points straying 5% about the line of best fit, this is hardly noticeable. In this way we got the following results for an EF42 at 50 Mc/s:

I_a = 10 mA; g_m = 9.0 mA/V; i_D = 25.65μA; $G_\tau = 113 \times 10^{-6}$ mho; $G_L = 99.0 \times 10^{-6}$ mho.
I_a = 5 mA; g_m = 6.0 mA/V; i_D = 17.4μA; $G_\tau = 70 \times 10^{-6}$ mho; $G_L = 67 \times 10^{-6}$ mho.
I_a = 2 mA; g_m = 5.3 mA/V; i_D = 8.65 μA; $G_\tau = 55 \times 10^{-6}$ mho; $G_L = 33 \times 10^{-6}$ mho.

From this we calculated a cathode-lead inductance of 1.5×10^{-8} to 1.6×10^{-8} H. This value was confirmed by resonance methods. For an EF42 with shortened cathode-leads we got:

I_a = 10 mA; g_m = 10 mA/V; i_D = 28.5μA; $G_\tau = 110 \times 10^{-6}$ mho; $G_L = 59 \times 10^{-6}$ mho.
I_a = 8 mA; g_m = 9.55 mA/V; i_D = 26.2μA; $G_\tau = 101 \times 10^{-6}$ mho; $G_L = 56 \times 10^{-6}$ mho.
I_a = 5 mA; g_m = 8 mA/V; i_D = 20μA; $G_\tau = 77 \times 10^{-6}$ mho; $G_L = 56 \times 10^{-6}$ mho. The values of the cathode-lead inductance are 0.75×10^{-8} to 0.93×10^{-8} H. The first value is confirmed by resonance.

In the region 20–100 Mc/s our measurements are thus in satisfactory agreement with Bakker's theory.

A fuller account of these and other measurements will be published in Philips Research Reports.

F. L. H. M. Stumpers.

Philips Research Laboratories,
Eindhoven, Holland.

REFERENCE

[1] C. J. Bakker: "Fluctuations and Electron Inertia," *Physica*, Vol. 8 p. 23—43. 1941.

Transit-Time Effects in U.H.F. Valves

Sir,—Dr. Diemer and Dr. Knol of the Philips Research Laboratories, Eindhoven, have been kind enough to point out to me an error in my paper with the above title which appeared in the June issue of *Wireless Engineer*. Owing to a slip in my algebra, the equation in the appendix

$$\frac{dx/dt}{v_o} = -1 + \frac{2aX}{\theta} - \frac{2a^2}{\theta^2}\left(Y - \frac{X^2}{\theta^2}\right)$$

should have read

$$\frac{dx/dt}{v_o} = -1 + \frac{2aX}{\theta^2} - \frac{2a^2}{\theta^2}\left(Y - \frac{X^2}{\theta^2}\right)$$

This considerably simplifies the expression for $f(\theta)$, making it, in fact, $F(\theta)$.

Consequently, wherever in the article $f(\theta)$ appears, it should be replaced by $F(\theta)$, and the curve of Fig. 6 should be disregarded. The important features of equation (16) are unaltered. The maximum conductance varies inversely with the square of the repelling voltage, and directly with the total emission.

I much regret that the earlier work of the Philips group,[1] establishing the resistance of the non-conducting diode, had escaped my notice. The matter has been taken a stage further recently by Begovich[2].

Greenwich. J. Thomson.

[1] *Physica*, Vol. II p. 683 and Vol. V. p. 325.
[2] *J. App. Phys.*, Vol. 20, p. 457 (1949).

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

609 309.—Microphonic system, including a contract-compression amplifier, and a cathode-ray indicator, for analyzing and controlling the detonation conditions in an internal-combustion engine.

Sperry Gyroscope Co. Inc. Convention date (U.S.A.) 22nd March, 1944.

609 541.—A.F. amplifier having a volume control in the grid-cathode circuit which is arranged to apply negative reaction so that the lower notes are attenuated less than the higher notes, as the volume is reduced.

Philips Lamps, Ltd. Convention date (Netherlands) 29th July, 1941.

611 330.—Two-valve circuit for stabilizing the operation of a compression-and-expansion type of a.f. amplifier.

Svenska Akt. Gasaccumulator. Convention date (Sweden) 28th October, 1944.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

609 813.—Automatic frequency-control for stabilizing the local oscillator of a pulsed radiolocation receiver, and a gate circuit to exclude interfering signals.

Sperry Gyroscope Co. Inc. Convention date (U.S.A.) 7th April, 1944.

610 465.—Rotary switch of the capacitance type for controlling the cyclic transmission of short-wave signals from a radio beacon.

Standard Telephones and Cables Ltd. (assignees of F. J. Lundberg). Convention date (U.S.A.) 9th April 1945.

610 551.—Navigational system in which the position of a moving craft is continuously shown on a cathode-ray indicator by periodic pulsed signals of relaxation, as distinct from sinusoidal, type.

Sadir-Carpentier. Convention date (France) 20th March, 1945.

610 866.—Direction finder in which a pentode valve with a phase-shifting network serves to offset the 'vertical effect' of a loop aerial.

Philips Lamps Ltd. Convention date (Netherlands) 20th November 1941.

610 054.—Visual indicator for the dot-dash signals of a directional system of the overlapping-beam or equi-signal type.

The Decca Record Co. Ltd. and W. J. O'Brien. Application date 15th April, 1946.

611 568.—Stabilizing-circuit for a pulse modulator, say for radiolocation equipment installed in aircraft, where the primary voltage source is liable to fluctuate.

Standard Telephones and Cables Ltd. (assignees of D. D. Grieg). Convention date (U.S.A.) 19th May, 1944.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

609 228.—Switching system for the remote control of a radio receiver from a number of spaced points, wherein exclusive control is given to the point first operated.

Collins Radio Co. Convention date (U.S.A.) 12th January, 1945.

609 496.—Pentagrid circuit of the 'locked-in oscillator' type for receiving frequency-modulated signals, wherein a network of given time-constant is used to increase the normal range of operation.

Marconi's W. T Co. Ltd. (assignees of M. S. Corrington). Convention date (U.S.A.) 14th November, 1944.

609 520.—Receiver for frequency-modulated signals, wherein the discriminator circuits are off-tuned to different harmonics of the carrier, so that their response curves do not overlap.

Marconi's W. T. Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 7th March, 1945.

610 119.—Bracket for mounting a radio set in a motor-car comprising a fixed baseplate supporting two parallel arms for carrying the set and allowing it to be moved relatively to the baseplate.

Transreceivers Ltd. and L. W. Hermes. Application date 26th March, 1946.

611 078.—Receiver for frequency-modulated signals, wherein an oscillator valve is synchronized with the incoming carrier wave, to facilitate the correct tuning of the set.

Philco Radio and Television Corporation (assignees of D. B. Smith). Convention date (U.S.A.) 28th February, 1945.

611 203.—Receiver provided with a suppressor circuit which is automatically cut-out on the receipt of a carrier wave, or of an S.O.S. or other characteristic calling-signal.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 25th April 1945.

611 335.—Receiver for frequency-modulated signals in which a squelch-tube circuit is provided to facilitate tuning, and to reduce inter-channel noise.

Radio Corporation of America (assignees of W. LaV. Carlson). Convention date (U.S.A.) 27th April, 1945.

611 564.—Receiver for frequency-modulated signals in which a locked-in oscillator is automatically controlled in order to reduce common-channel interference,

Marconi's W. T. Co. Ltd. (assignees of W. F. Sands). Convention date (U.S.A.) 2nd May, 1945.

611 631.—Adjusting-wheel, of the projecting-periphery type, and snap-action switching device, for setting a clock-controlled radio receiver.

The British Vacuum Cleaner and Engineering Co. Ltd. and H. O. Hamilton. Application dates 19th March and 9th May, 1946.

611 701.—Push-pull mixing circuit for centimetre waves, in which input and output damping is reduced by using a valve comprising two separate triodes and a double diode.

Philips Lamps Ltd. Convention date (Netherlands) 30th May, 1940.

611 841.—Utilizing the inherent valve noise for automatically muting the loudspeaker of a set, in the absence of an incoming signal.

The General Electric Co. Ltd., L. C. Stenning and A. J. Bayliss. Application date 10th May, 1946.

TELEVISION CIRCUITS AND APPARATUS

For Transmission and Reception

609 755.—Television receiver including gain-control means for regulating background contrast, an energy-storing circuit, and a limiter device for suppressing interference.

Marconi's W.T. Co. Ltd. (assignees of A. C. Schroeder). Convention date (U.S.A.) 12th March, 1945.

609 839.—Television receiver with a safety device for de-focusing the electron stream in the event of failure of one or both of the scanning oscillations.

A. C. Cossor Ltd., A. H. A. Wynn, and M. Franco. Application date 7th March, 1946.

610 288.—Automatically controlling the grid bias in accordance with the average illumination, in television cameras using a low-velocity scanning beam.

Marconi's W.T. Co. Ltd. (assignees of R. R. Thalner). Convention date (U.S.A.) 9th January, 1945.

610 528.—Selective system of remote control based on the television of predetermined code symbols, and the use of corresponding and adjustable photo-electric targets at the receiving end.

H. W. K. Jennings (communicated by J. H. Homrighous). Application date 30th August, 1945.

610 884.—Optical system for ensuring optimum illumination, with correct progressive displacement, when scanning a cinema film for television.

The General Electric Co. Ltd. and D. C. Espley. Application date 17th April, 1946.

610 977.—Combined transmitter and receiver installation with automatic waveguide switching-circuits, for the two-way relaying of television or other signals.

Cinema-Television Ltd., M. Morgan and G. E. G. Graham. Application date 18th April, 1946.

611 222.—Regulating the background or overall intensity of illumination in a television camera of the Orthicon type using a low-velocity scanning beam.

Cie pour la Fabrication des Compteurs et Materiels d' Usines à Gaz. Convention dates (France). 8th and 21st June, 1944.

611 751.—Circuit for generating impulses that are timed to coincide with the duration of an integral number of subordinate impulses, say for framing in television.

E. L. C. White and E. A. Newman. Application date 20th April, 1946.

612 376.—Method of mounting and intercoupling the viewing screen and associated reflector in the cabinet of a television receiver.

Marconi's W.T. Co. Ltd. (assignees of G. M. Daly). Convention date (U.S.A.) 23rd May, 1945.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

609 748.—Modulating system wherein the original signal is variably clipped to decrease the ratio of peak-to-average voltage, and is selectively attenuated to accentuate the higher frequencies.

Marconi's W.T. Co. Ltd. (assignees of E. R. Shenk). Convention date (U.S.A.), 29th April, 1944.

609 861.—Half-wave Lecher-wire circuit for preventing parasitic oscillations in a neutralized push-pull amplifier operating at a high level of power.

Philips Lamps Ltd. Convention date (Netherlands) 30th June, 1941.

609 970.—Two-valve circuit arrangement, simulating a straight-line reactance, applicable to phase or frequency modulation.

Philips Lamps Ltd. Convention date (Netherlands) 17th September, 1941.

610 599.—Frequency-changing circuit, comprising at least two mixing-stages coupled in cascade, for controlling the synchronous operation of broadcasting stations using the same wavelength. [Divided from 610 532.]

The General Electric Co. Ltd. and L. C. Stenning. Application date 12th October, 1945.

611 083.—Unmanned relay station, with a local step-by-step frequency control, for connecting any one of a number of signalling channels to a selected channel.

Marconi's W.T. Co. Ltd. (assignees of R. W. Bumstead). Convention date (U.S.A.) 18th January, 1944.

611 639.—Testing the circuit parameters of an unmanned relay station, by the cyclic transmission of selected supervisory signals from a remote point.

Cinema-Television Ltd., G. E. G. Graham and M. Morgan. Application date 30th April, 1946.

611 931.—Modulating the amplitude of centimetre waves by varying the transmission coefficient of a waveguide through which they are travelling.

Cie Generale de Telegraphie Sans Fil. Convention date (France) 6th February, 1945.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

609 789.—Multiplex system of pulsed signalling, in which provision is made for broadening the normal width of selected channels for messages demanding high fidelity.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.), 2nd April, 1945.

610 653.—Cutting out extraneous noise in the time intervals of a pulsed-signalling system using phase or frequency modulation.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 17th April, 1945.

610 774.—Minimizing cross-talk interference in a multiplex system of pulsed signalling by suitably interleaving the sequence of pulsed trains.

Standard Telephones and Cables Ltd., P. K. Chatterjea and A. H. Reeves. Application date 16th April, 1946.

SUBSIDIARY APPARATUS AND MATERIALS

609 824.—The use of a 'voltage-sensitive' capacitor, having a ceramic dielectric made of a mixture of titanium and alkaline-earth compounds, in a variety of tuned circuits for the purpose of automatic frequency control.

P. R. Coursey, L. J. Snell, Steatite and Porcelain Products Ltd., and Dubilier Condenser Co. (1925) Ltd. Application date 2nd October, 1945.

610 024.—Device for measuring ultra-high-frequency fields comprising an evacuated bulb containing a dipole pick-up and a thermocouple located at a current loop and connected to an external meter.

Philco Radio and Television Corporation (assignees of W. E. Bradley). Convention date (U.S.A.) 31st December, 1943.

612 316.—Electro-mechanical arrangement for automatically tuning a circuit to an applied frequency, through a motor-driven capacitor and two interlocked control relays.

The General Electric Co. Ltd. and E. P. Fairbairn. Application date 18th July, 1945.