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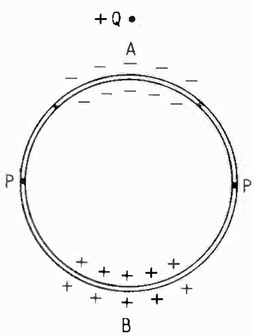
Relatively-moving Charge and Coil

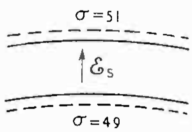
ON p. 140 of this issue we publish a letter from Professor Cullwick criticizing our December Editorial. His criticism appears, however, to be based partly on some misunderstanding. He gives an alleged summary of our argument, but his item (a) is no part of our argument but quite contrary to it. We said that 'all parts of the ring are at the same potential,' but far from maintaining that the fields cancel at all points in the plane of the coil within the loop, we showed in Fig. 1(b) how the electric field enters the ring both on the outside and on the inside, and in Fig. 1(a) the charges are shown on which the field terminates both on the outer and inner surface. Where we differed from Professor Cullwick was with regard to the sign of the charges. He has opposite charges on the outside and inside of the ring and it is fairly obvious that if such a state of affairs were possible the charges would to some extent cancel each other's magnetic effect when in motion. We maintained, and still maintain, that this is quite wrong. He says 'the polarity of the charges on the inner surface is immaterial. This will depend on the geometry of the system, and may be shown by Professor Howe in Fig. 1 of his Editorial or Fig. 2 of my letter in the same issue, according to circumstances.' He had, however, specified the geometry and circumstances as a single loop of thin wire, and for such a loop we maintain that his Fig. 2 was entirely wrong. The polarity of the charges on the inner surface is of very great importance and his failure to realise this is, we believe, the cause of much of his trouble.

With regard to item (b) of his letter, we showed in the September Editorial that, far from cancelling their magnetic effects, the positive and negative induced charges produce within the ring a magnetic field which by a rough approximation we showed to be equal to that produced when the charge was moving and the ring stationary. Although we did not attempt to prove it rigidly we said 'we can safely state that the magnetic field reaches the same maximum value in both cases.' In both cases the observer is regarded as stationary. We realize that this is quite contrary to Professor Cullwick's thesis that the adjacent electric charges are of opposite sign and cancel their magnetic effects, but, as we said above, we regard this as fundamentally wrong.

Under item (c) he correctly states our point of view, viz., that the induced e.m.f. in the ring is the same in both cases. The induced e.m.f. is a physical fact and not like magnetic and electric fields dependent on the motion of the observer. If a charge and a metal ring move relatively to one another, the e.m.f. induced in the ring cannot depend on the idiosyncrasy of any observer. Professor Cullwick admitted this in his letter in our December number but said that difficulties arise in the application of classical theory. Having come to the conclusion that the maximum magnetic flux through the ring is the same as when the ring was stationary and the charge moving, and that a moment later, when the ring has moved away, the flux is reduced to zero, we said 'Would Maxwell have agreed that this magnetic flux could be smuggled somehow in and out of the ring without inducing an e.m.f. in it?' This appears to be the main point of Professor Cullwick's argument, for he maintains that detailed application of Maxwell's theory gives zero e.m.f. We simply applied the formula $e = -d\Phi/dt$ and asked no question as to whether the resultant change of flux was due to the change of flux through a stationary path or to the motion of the ring through the magnetic field. The exact calculation of the two components would be very complicated and we did not consider it necessary. We know that the flux through the ring is reduced from Φ_{max} to 0, but to what extent this is due to a weakening of the field, and to what extent due to the movement of the ring in the field, we regarded as of no great importance in the present argument.

Professor Cullwick's strange statement that the two components are found to cancel, giving zero e.m.f. indicates that he believes that the magnetic flux can be smuggled out of the ring without inducing an e.m.f. in it, but he gives no suggestion of evidence in support of this statement. If he has calculated the two components and found them equal but of opposite sign, we suspect that one of the signs was wrong; it would be interesting to know if the components were each equal to $0.5\,d\Phi/dt$; if so, correcting the wrong sign would bring the result into agreement with Maxwell. We doubt, however, if any such detailed calculation has been made. The fundamental error in his Fig. 2 is due to overlooking the fact that the induced charge on the portion of the ring nearer $+Q$ must necessarily be entirely negative while that on the more remote part of the ring, not shown in his diagram, must must be entirely positive. The radial field $\mathcal{E}_s$ within the conductor at A, which exactly neutralizes the field due to Q and the charges on other parts of the ring, is not due to any positive charge on the inner surface of the ring, but is due entirely to the relatively small difference of surface density of the negative charge on the outer and inner surfaces. This is illustrated in the Figure which shows the charge Q and the thin wire ring. In order to neutralize the induced field at P the induced charges must be distributed roughly as shown. Let us assume that the charge at A corresponds to a surface density of say, 50 units. In order to neutralize the field, in the metal due to the charges, the surface density must be greater on the outer than on the inner surface. We show a portion of the ring at A with surface densities of 51 and 49, but the ratio of the difference to the mean value will depend on the thickness of the ring, and, for the thin wire specified by Professor Cullwick, no great error will be made if one neglects the difference and assumes the same surface density on the outer and inner surface when calculating the magnetic field. It is mainly due to this error that he is led to conclude that 'the problem seems to provide an exception to the accepted principle that Maxwell's theory does not require the assistance of Einstein except when velocities are comparable with the velocity of light.'





In the final paragraph of his letter Professor Cullwick deals with another problem, viz., that of the charge and short-circuited toroid in relative motion, and his criticism in this case is perfectly justified. In our Editorial we said that we could not follow his argument, but his letter now makes the point clear and we agree with him. As the charge moving along the axis approaches the toroid, which we assume to have negligible resistance, the induced current prevents the growth of magnetic flux within the toroid. One may picture two equal and opposite magnetic fluxes within the toroid, one, due to the moving charge, that would occupy the same space if the toroid were not there, and which therefore has no effect on the moving charge, and the other, due to the current, which by its growth induces an electric field tending to accelerate the approaching charge, as Professor Cullwick correctly stated. The energy that would otherwise have been in the magnetic field within the space occupied by the toroid will now be distributed throughout the surrounding space owing to the increased speed of the charge. After passing through the toroid the process will be reversed, but any ohmic losses in the toroid will presumably cause a permanent decrease in the speed of the charge. Our suggestion in the December Editorial that the approaching charge would be retarded was based on a misunderstanding, and the above considerations show that Professor Cullwick was quite right in maintaining that it would be accelerated.

In his previous letter Professor Cullwick mentioned that the ether plays a part in Maxwell's theory, but that nobody believes in it now. This is true, but the part that it played appears to have been taken over by space, which has the same number of letters and, if endowed with the same properties, seems to answer just as well. He also mentioned that many scientists maintain that the idea of a storage of energy in the space around the moving charge is not really tenable. The concepts of the magnetic and electric fields in space with their associated energies have certainly played an important part in the development of electromagnetic theory, and it is difficult to imagine how the subject can be developed without these concepts. Whether they are anything more than concepts is another matter.

G. W. O. H.

GRID CURRENT WITH RC COUPLING

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(Communication from the Research Staff of the M.O. Valve Company at the G.E.C. Research Laboratories, Wembley, England.)

SUMMARY.—A common feature of multivibrator circuits is that the valves forming them are driven into grid current during some part of each cycle. These currents produce, across the coupling capacitors, potentials that play a significant part in the operation and determine the minimum value that the coupling capacitance may be given. The following discussion attempts to analyse these effects in the general case of a valve driven into grid current by a square-wave through capacitance coupling.

THE coupling elements in Fig. 1 are linear and therefore, if the valves are operated over a reasonably linear part of their characteristics, it is a simple matter to calculate the output from the input waveform; similarly there is no difficulty in calculating the specifications of C, R, and r.

If, however, the second grid is driven into grid current, a non-linear element is introduced and the calculations become less obvious. This situation is frequently met, a good example being the circuit shown in Fig. 2. This circuit is based on the Schmitt trigger[1], and is required to produce square waves with very sharp fronts from a sine wave input, the frequency of which may be varied over a wide range. In this circuit, the voltage on the first anode has the form shown in Fig. 3; i.e., it is roughly a square wave. This voltage drives the second grid alternately to cut-off and then into grid current, thus producing the square-wave output. The merit of the circuit lies, of course, in the cathode coupling which results in extremely sharp wavefronts in the output.

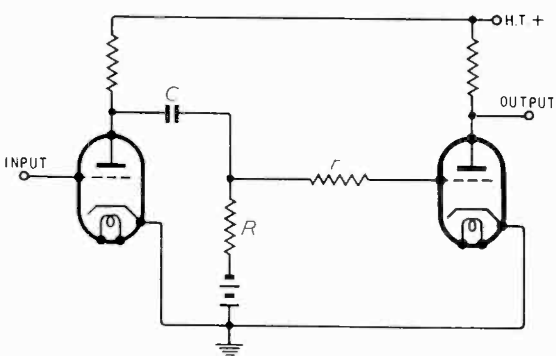


Fig. 1. *The elementary circuit.*

During the intervals 0–T, $2T$–$3T$, etc. (see Fig. 3), the grid drive current builds up a charge on C, so producing a potential difference which opposes the driving e.m.f.; consequently the drive current falls steadily until the anode voltage falls sharply at T, $3T$, etc., by the operation of the circuit. If the frequency is reduced, a value is ultimately reached at which the charge built up on C produces a potential great enough to stop the further flow of grid

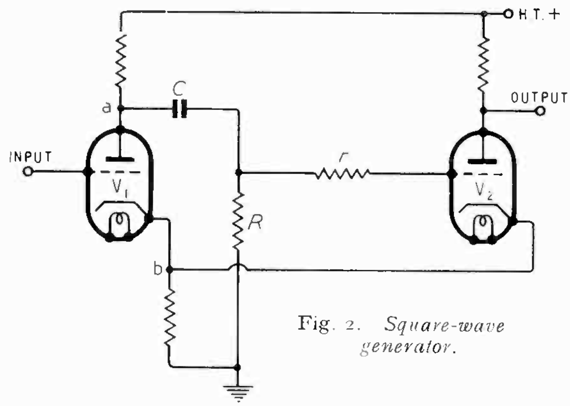


Fig. 2. *Square-wave generator.*

current against the cathode potential. A further reduction of frequency then produces a voltage on the driven grid as shown in Fig. 4(b) and the output is correspondingly distorted. The frequency minimum so reached is evidently a function of C, R, and r among other quantities and it has been thought worthwhile to investigate this relation as there seemed to be no other convenient way of finding the best values for these components.

As far as currents in the coupling elements are concerned, Fig. 5 is an equivalent of Fig. 2 provided that the voltage sources $V(t)$ and $E(t)$ produce the same potentials to earth as those at points a and b of Fig. 2. It is assumed therefore that $V(t)$ and $E(t)$ have the waveforms shown in Fig. 10(a) and (b) respectively when feeding into the load provided by the circuit of Fig. 5. The analysis may be simplified, without serious loss of equivalence, by assuming $V(t)$ and $E(t)$ to have the waveforms shown in Fig. 6.

MS accepted by the Editor, January 1948

The physical processes that are to be investigated are as follows. In the intervals 0–T, $2T$–$3T$, etc., the potential on the anode of V_1 charges C through the resistances R and r in parallel. In the intervals T–$2T$, etc., C discharges through R alone, because the rectifying action of the grid prevents current from flowing through r. But the mean current through C must be zero when the circuit is in steady operation, accordingly a steady charge is built up on C which produces a potential so biasing the voltage waveform that the mean current through C in each half cycle is the same, despite the unbalance between the charge and the discharge resistances.

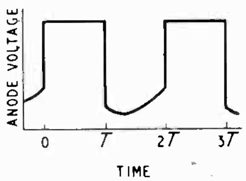


Fig. 3. *Waveform of voltage on driving anode.*

Applying Kirchhoff's Laws to the arrangement of Fig. 5 gives

$$V(t) = \frac{q}{C} + ri_2 + E(t) \quad \ldots \quad (1)$$

$$V(t) = \frac{q}{C} + R(i_1 - i_2) \quad \ldots \quad (2)$$

$$dq/dt = i_1 \quad \ldots \quad (3)$$

The solution of these equations is shown in Appendix 1 to lead to the equation

$$\frac{nE}{V} = \frac{\beta . F(\alpha, \beta)}{\beta - \alpha + \alpha . F(\alpha, \beta)} \quad \ldots \quad (18)$$

where

$$F(\alpha,\beta) = \frac{e^{-\beta/\alpha(\beta-\alpha)} - e^{-(2\beta-\alpha)/\alpha(\beta-\alpha)}}{1 - e^{-(2\beta-\alpha)/\alpha(\beta-\alpha)}}$$

$$\alpha = CR/T$$

and

$$\beta = \frac{CR}{T} + \frac{Cr}{T}$$

T is the duration of the half period of the frequency at which defective operation (as described above) appears; i.e., of the lower-frequency limit of the unit.

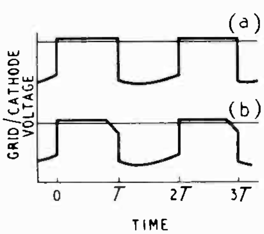


Fig. 4. *Voltage on second grid above (a) and below (b) the minimum frequency.*

Equation 18 states the relation between α, β, and nE/V for the diode current just to fall to zero at the end of the half-period T. Alternatively, if a stage developing a rectangular signal of amplitude V is required to drive the following valve into grid current for a full half-period against a bias nE, Equ. 18 gives the pairs of values of α and β that just suffice. The curves resulting from the evaluation of this equation are shown in Fig. 7; none of those plotted rises above 0.46 and it may be shown that Equ. 18 has a limiting value of 0.5, which it approaches when α, β, and (β–α) are very great. The form of the curves shows that, for a particular value of nE/V, there is minimum value of β that may be employed; this value being that which produces a curve with a maximum equal to the required value of nE/V. The values of α and β corresponding to the maxima in Fig. 7 are given in Table I; these values are obtained by difference methods from the tabular values used in plotting the curves and have a maximum error of 0.0005. The table also gives the corresponding values of Cr/T.

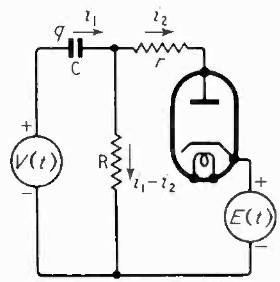


Fig. 5. *The equivalent arrangement.*

It should be noted that Fig. 7 shows that values of α and β may be chosen to meet any value of nE/V less than 0.5. If nE/V is 0.5 or greater, it is not possible to arrange that grid current persists for a full half-cycle.

Now let it be supposed that the coupling elements R, r and C of a particular equipment are to be investigated and that, as is usual, it is desired to reduce C to a minimum. T and nE/V will be part of the prescribed data and therefore Equ. 18 may be written

$$V/nE - 1 = \text{a constant} = f(\alpha, \beta) \quad \ldots \quad (18a)$$

Given that a pair of values of α and β have been chosen that satisfy Equ. 18a, the values of R, r, and C may be changed in any way that leaves α and β unaltered.

Since

$$\alpha = CR/T \text{ and } \beta = C(r + R)/T$$

it is clear that whatever the values of α and β,

TABLE I

Pairs of values of Cr/T and CR/T giving a minimum value of β for a given value of nE/V.

nE/V	β	α (CR/T)	β–α (Cr/T)
0.032	1	0.547	0.453
0.169	2	1.124	0.876
0.269	3	1.707	1.293
0.318	4	2.288	1.712
0.377	6	3.461	2.539
0.408	8	4.641	3.359
0.426	10	5.793	4.207

the smallest value of C is obtained by using the greatest value of $(R + r)$. However $(R + r)$ is the external grid-cathode resistance, a quantity for which valve makers invariably specify a maximum. Thus the value of $(R + r)$ is determined, it remains to divide the available resistance between R and r. But it has already been shown that the smallest value of β, and hence of C since $(R + r)$ is already fixed, is permitted when α lies on the curve defined by the values in Table I. Reverting to Fig. 7 the flatness of the curves in the neighbourhood of the maxima invites practical approximations, an obvious one being to take the maxima as given by values of Cr/T equal to those given by Table I for CR/T. This slightly increases β above the true minimum but makes R equal to r. The appropriate values are given in Table II and plotted in Fig. 8. It follows that, for all practical purposes, the smallest value of C is obtained by making both R and r equal to half the maximum permitted grid-cathode resistance.

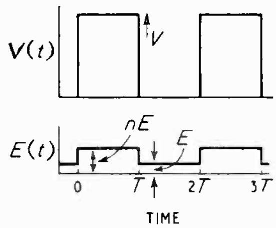


Fig. 6. *Waveforms in the equivalent circuit.*

TABLE II

Minimum values of CR/T for given values of nE/V when R is made equal to r.

nE/V	0.0313	0.1652	0.2585	0.3141	0.3498
CR/T	0.5	1.0	1.5	2.0	2.5
nE/V	0.3743	0.3921	0.4054	0.4160	0.4244
CR/T	3.0	3.5	4.0	4.5	5.0

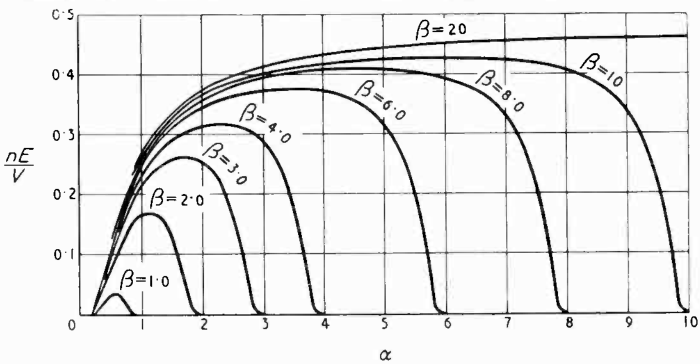


Fig. 7. *n/EV as a function of α and β.*

It is sometimes necessary to restrict the value of either R or r. For instance, it may be necessary to maintain at a low value the time constant formed by r and the input capacitance of the driven grid. A question then arises as to the value of R that permits the smallest value for C when r is restricted and the two remaining variables in Equ. 18 (i.e., T and nE/V) have some assigned value. Evidently T and nE/V become, for present purposes, any positive constants subject only to the restriction that nE/V must be less than 0.5. Equ. 18(a) may be applied to these conditions, and the problem becomes one of investigating the way in which C changes in this equation when r is held constant and R allowed to vary. This investigation is made in Appendix II,

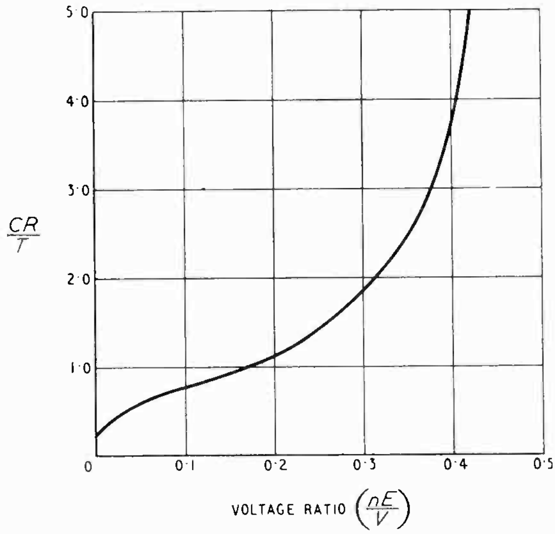


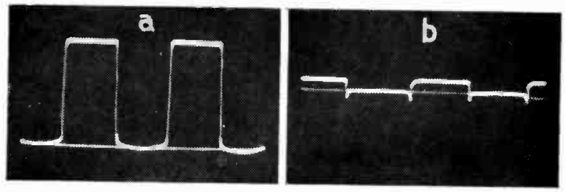
Fig. 8. *Minimum values of CR/T for given values of nE/V; R is made equal to r.*

where it is shown that, whatever the value assigned to r, the smallest C results from increasing R to the maximum possible extent. Similarly if R is assigned a value, the smallest C results from increasing r to the maximum extent. In other words, C is minimized by arranging R and r to be approximately equal and their sum to be as great as possible: if it is not possible to make R and r approximately equal, the smallest C is still obtained by making their sum as great as possible.

Experimental Verification

The above calculations were checked by experiment on a small pulse generator. The circuit (shown in Fig. 9) is an elaboration of the arrangement of Fig. 2. A diode has been inserted in the grid circuit of the second valve, so that the voltage across the cathode resistance of this valve appears as the minimum grid bias. The mean anode current may then be set to the rated maximum for the valve type by adjusting the value of this resistance which, incidentally, is too low to require a capacitive by-pass. A further change is that the anode load of this valve has been modified by adding an artificial line. Then, to a sufficiently rapid disturbance, the anode load presents the parallel resistance of R_0 and Z_0 for twice the delay time of the line and then collapses to approximately zero. Provided, therefore, that the grid signal persists longer than this time, the duration of the output impulse is controlled by the artificial line, and may be varied by manually switching-in lines of differing lengths. Finally, a short line has been inserted in the anode circuit of the first valve, because the initiating signal for a cathode-ray tube time base is derived from this point and the short delay between it and the following grid enables the rise of the output pulse to be studied.

In the first set of experiments, various values of the coupling components were inserted and the minimum frequency determined experimentally and compared with values deduced from the foregoing theory. Fig. 10 shows oscillograms of the potential on the first anode, and on the second cathode. Comparison with Fig. 6 shows a fair similarity and it is evident that the ratio nE/V may be measured on the traces. The potential zero line is not shown on the oscillograms and the actual measurements were made on the oscilloscope by the usual method of balancing the deflection against a steady shift voltage applied to the opposite oscilloscope plate. The minimum frequency was determined by watching the grid-cathode potential of the second valve while running down the input frequency, so finding the frequency at which this potential changes from the form of Fig. 4(a) to that of (4b). Fig. 11 shows oscillograms of the grid-cathode potential above and below the minimum frequency. The results of these experiments are given in Table III.



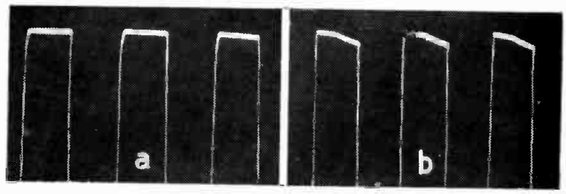


Fig. 10 (above). *Potential on the first anode (a); on the second anode (b).*

Fig. 11 (below). *Grid-cathode potential at frequencies above (a) and below (b) the minimum.*

The discrepancies between the measured and the calculated values seem reasonable enough, allowing for the differences between the waveforms of Figs. 6 and 10.

Experiments were then made to check the conclusion that, to meet given values of nE/V and the minimum frequency when the total grid-cathode resistance $(R + r)$ is fixed, r and R should equal each other if C is to be as small as possible. In these experiments the total grid-cathode resistance was fixed at 500 kΩ and r then varied over a wide range; at each value of r, C was adjusted to make the minimum frequency 300 c/s. The results are given in Table IV.

If, from these results, $(R - r)$ is plotted against C, it will be seen that, to the accuracy of the experiments, the minimum capacitance occurs when $(R - r)$ is zero.

The final experiments were made to confirm that, with given values of nE/V and the minimum frequency and when r is at some fixed value C is minimized by making R as

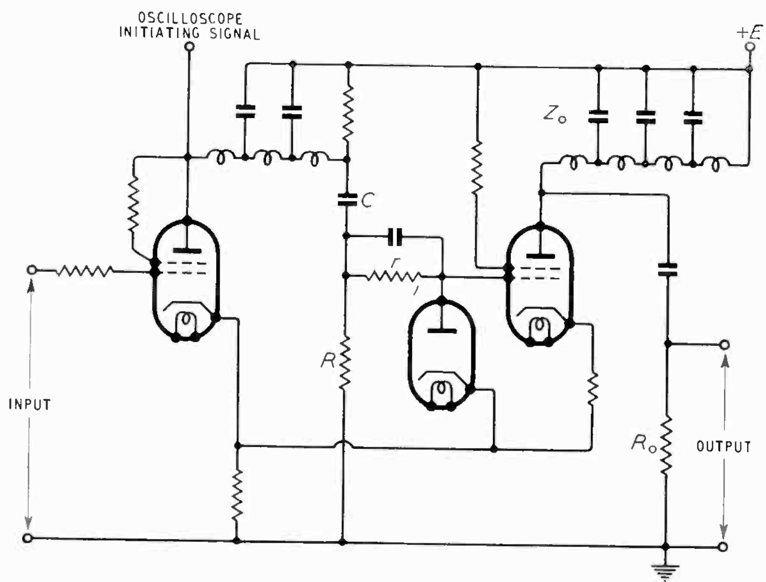


Fig. 9. *Small impulse generator.*

large as other considerations permit. In these experiments, r was fixed at 30 kΩ and R then varied over a wide range; at each value of R, C was adjusted to make the minimum frequency 300 c/s. Table V gives the results.

TABLE III

Comparison of experimentally and theoretically determined values of the minimum frequency, with R and r each of 50 kΩ.

C (μF)	nE/V	Minimum frequency (c/s)	
		Experimental	Theoretical
0.01	0.069	680	645
0.02	0.073	300	330
0.10	0.073	70	65.9

TABLE IV

Optimum division of the total grid-cathode resistance (500 kΩ) between R and r.

Minimum frequency (c/s)	r (kΩ)	R (kΩ)	C (μF)
300	30	470	0.018
300	50	450	0.010
300	100	400	0.006
300	200	300	0.0055
300	300	200	0.0055
300	400	100	0.009
300	450	50	0.020
300	470	30	0.030

TABLE V

Optimum value for R when r is fixed; minimum frequency = 300 c/s, r = 30 kΩ.

R (Ω)	C (μF)
30,000	0.030
50,000	0.025
100,000	0.020
470,000	0.018

These results evidently support the theoretical result.

Acknowledgment

In conclusion, the author desires to tender his acknowledgment to the M.O. Valve Co. Ltd., on whose behalf the work described in this publication was carried out.

REFERENCE

[1] "A Thermionic Trigger," by O. H. Schmitt. *Journal of Scientific Instruments*, 1938, Vol. 15, p. 24.

APPENDIX I

The solution of the circuit equations.

The equations to be solved are

$$V(t) = \frac{q}{c} + ri_2 + E(t) \quad \ldots \quad (1)$$

$$V(t) = \frac{q}{c} + R(i_1 - i_2) \quad \ldots \quad (2)$$

$$\frac{dq}{dt} = i_1 \quad \ldots \quad (3)$$

The Laplace transforms are therefore

$$[V] - [E] = + r[i_2] + \frac{[q]}{C}$$

$$[V] = R[i_1] - R[i_2] + \frac{[q]}{C}$$

$$- q^{(0)} = [i_1] \qquad - p[q]$$

where $[V]$ is the Laplace transform of $V(t)$, $[i_1]$ is the transform of i_1, and so on. Also $q^{(0)}$ is the value of q at the time 0, $q^{(T)}$ is the value of q at the time T, and similarly for the other variables.

It follows from the last three equations that

$$[i_1] = \frac{pC[V] - q^{(0)}}{1 + pCR_p} - \frac{pC(E]}{r} \cdot \frac{R_p}{1 + pCR_p} \quad \ldots \quad (4)$$

$$[i_2] = \frac{pC[V] - q^{(0)}}{r} \cdot \frac{R_p}{1 + pCR_p} - \frac{[E]}{r + R} \cdot \frac{1 + pCR}{1 + pCR_p} \quad \ldots \quad (5)$$

$$[q] = \frac{C[V]}{1 + pCR_p} + \frac{q^{(0)} CR_p}{1 + pCR_p} - \frac{C[E]}{r} \cdot \frac{R_p}{1 + pCR_p} \quad \ldots \quad (6)$$

Where

$$R_p = \frac{rR}{r + R}$$

Inspection of Fig. 6 shows that

$V(t) = V$ for $0 < t < T$

$V(t) = 0$ for $T < t < 2T$

$E(t) = nE$ for $0 < t < T$, where n is some positive constant.

and $\quad E(t) = E$ for $T < t < 2T$.

It follows that, in the interval $0 < t < T$,

$$[V] = V/p, \text{ and } [E] = nE/p.$$

Inserting these results in Equs. (4) and (5), and inverting the transforms gives (for the interval $0 < t < T$)

$$i_1 = \frac{V - q^{(0)} C}{R_p} \cdot e^{-t/CR_p} - \frac{nE}{r} \cdot e^{-t/CR_p} \quad \ldots \quad (8)$$

$$i_2 = \frac{(V - q^{(0)}/C - nE)}{r} \cdot e^{-t/CR_p} - \frac{nE}{r + R} \cdot (1 - e^{-t/CR_p}) \quad \ldots \quad (9)$$

$$\frac{q}{C} = V - \frac{nER}{r + R} - \left(V - \frac{nER}{r + R} - \frac{q^{(0)}}{C}\right) \cdot e^{-t/CR_p} \quad (10)$$

From Equ. 10, it is seen that the change in q during the interval $0 < t < T$ equals

$$q^{(T)} - q^{(0)} = C\left(V - nE \cdot \frac{R_p}{r} - \frac{q^{(0)}}{C}\right)(1 - e^{-T/CR_p}) \quad \ldots$$

whence

$$\frac{q^{(T)}}{c} = \left(V - nE\frac{R_p}{r}\right)(1 - e^{-T/CR_p}) + \frac{q^{(0)}}{C} \cdot e^{-T/CR_p} \quad \ldots \quad (11)$$

Before proceeding to the next interval, it is worth noting that if i_2 falls to zero at t_0, Equs. (8), (9), (10), and (11) are invalid for values of T greater than t_0. This is because the operation of the diode (see Fig. 5) isolates the branch containing r at this instant. However, since these greater values of T correspond to frequencies below the minimum (as defined earlier), there is no need to make provision for their study; in fact, the analysis may be confined to those cases in which i_2 just becomes zero at T; i.e., to the minimum frequencies. It follows that the fall of $V(t)$ at the intervals $T, 3T$, etc., must reverse the voltage across the diode and so isolate r.

Passing now to the interval $(T<t<2T)$, and noting from Fig. 6 that $V(t)$ is zero, it is evident (because the branch containing r is isolated) that

$$q^{(2T)} = q^{(T)} \cdot e^{-T/CR} \quad \ldots \quad (12)$$

but, because the operation is cyclic

$$q^{(2T)} = q^{(0)} \quad \ldots \quad (13)$$

Therefore, from Equs. (11), (12) and (13)

$$q^{(0)} = \left(CV - \frac{nE.CR}{r+R}\right) \cdot \frac{e^{-T/CR} - e^{-\frac{T}{CR} - \frac{T}{CRp}}}{1 - e^{-\frac{T}{CR} - \frac{T}{CRp}}} \quad (14)$$

Eliminating $q^{(0)}$ between Equs. (9) and (14) gives

$$ri_2 = \left(V - \frac{nER}{r+R}\right)\left(\frac{1 - e^{-T/CR}}{1 - e^{-\frac{T}{CR} - \frac{T}{CR_p}}} \cdot e^{-t/CRp}\right) - \frac{nE.r}{r+R} \quad \ldots \quad (15)$$

Now i_2 is zero when t equals T. Therefore

$$\frac{nEr}{V(r+R) - nER} = \frac{(1 - e^{-T/CR}) \cdot e^{-T/CRp}}{1 - e^{-\frac{T}{CR} - \frac{T}{CRp}}}$$

Re-arranging and also making the transformations

$$\alpha = CR/T \ (16), \text{ and } \beta = \frac{CR}{T} + \frac{Cr}{T} \quad \ldots \quad (17)$$

leads to
$$\frac{nE}{V} = \frac{\beta.F(\alpha, \beta)}{\beta - \alpha + \alpha F(\alpha, \beta)} \quad \ldots \quad (18)$$

where
$$F(\alpha, B) = \frac{e^{-\beta/\alpha(\beta-\alpha)} - e^{-(2\beta-\alpha)/\alpha(\beta-\alpha)}}{1 - e^{-(2\beta-\alpha)/\alpha(\beta-\alpha)}}$$

APPENDIX II

Optimum values for either of the elements R and r when the other is fixed.

Writing a for T/CR, and b for T/Cr, it may be shown from Equ. (18) that

$$\frac{V}{nE} - 1 = \frac{a}{a+b} \cdot \frac{e^{a+b} - 1}{1 - e^{-a}} \quad \ldots \quad (19)$$

The object of this study is to find the smallest value of C that may be used in a given equipment, nE/V is therefore part of the prescribed data. One may therefore write

$$\frac{V}{nE} - 1 = \phi = \text{a constant}$$

Substituting in Equ. (19) and re-arranging

$$\phi = \frac{a}{a+b} \cdot e^{a + \frac{1}{2}b} \cdot \frac{\sinh \frac{1}{2}(a+b)}{\sinh \frac{1}{2} a}$$

and
$$\frac{\partial \phi}{\partial a} = \frac{b}{(a+b)^2} e^{a+\frac{1}{2}b} \cdot \frac{\sinh \frac{1}{2}(a+b)}{\sinh \frac{1}{2} a} + \frac{a}{(a+b)} \cdot \frac{e^{a+\frac{1}{2}b}}{\sinh \frac{1}{2} a}\{\sinh \tfrac{1}{2}(a+b) - \tfrac{1}{2}\sinh \tfrac{1}{2} a\} \quad \ldots \quad (20)$$

Evidently $\partial\phi/\partial a$ is positive for all positive values a and b.

$$\text{Again } \frac{\partial \phi}{\partial b} = \frac{a}{a+b} \cdot \frac{e^{a+\frac{1}{2}b}}{2\sinh \frac{1}{2} a}\left\{\frac{-\sinh \frac{1}{2}(a+b)}{\frac{1}{2}(a+b)} + \sinh \tfrac{1}{2}(a+b) + \cosh \tfrac{1}{2}(a+b)\right\} \quad \ldots \quad (21)$$

By expansion, the quantity in the bracket is seen to be

$$\left\{\tfrac{1}{2}(a+b) + \frac{\{\frac{1}{2}(a+b)\}^2}{2!}(1 - \tfrac{1}{3}) + \frac{\{\frac{1}{2}(a+b)\}^3}{3!} + \frac{\{\frac{1}{2}(a+b)\}^4}{4!} \cdot (1 - \tfrac{1}{5}) + \ldots\right\}$$

Evidently $\partial\phi/\partial b$ is positive for all positive values of a and b.

Since $\phi = f_1(a, b) =$ a constant,

$$\frac{\partial b}{\partial a} = -\frac{\partial\phi/\partial a}{\partial\phi/\partial b} \quad \ldots \quad (22)$$

Equations (20), (21), and (22) show that $\partial b/\partial a$ is negative for all positive values of a and b.

Since this is an analysis of a physical problem, $f(a, b)$ may be assumed to be a continuous function of the two variables. It then follows from Equ. (22) that a small increase in b may always be neutralized by a small decrease in a, since it has been shown that $\partial b/\partial a$ is always negative. Thus, if T/Cr is increased by reducing C, the effect on Equ. (19) may always be nullified by a reduction in T/CR, achieved by increasing R. It follows that C is minimized by increasing R to whatever limit may be imposed by other considerations. The same argument may be employed to show an identical behaviour for r when R is fixed.

VALVES WITH RESISTIVE LOADS

Output Signal-Handling Capacity

By S. W. Amos, B.Sc.(Hons.), Grad.I.E.E.

(Engineering Training Department, British Broadcasting Corporation)

It is the purpose of this article to derive expressions for the maximum undistorted output voltage that can be delivered by a valve with a purely resistive load and a given value of h.t. supply. Although the article is written throughout in terms of cathode followers, many of the expressions derived apply equally to valves with loads connected in the anode circuit.

Because the output load is connected between cathode and negative h.t., a cathode-follower stage has 100 per cent voltage feedback. The improvement in linearity and the reduction in gain brought about by this feedback can be visualised by superimposing a load line on a family of I_a—V_{k0} characteristics, plotted for various values of V_{g0} (see Fig. 1). These characteristics can be plotted in the following way. The dotted lines in Fig. 2 represent the I_a—V_{ak} characteristics of a small triode for various values of V_{gk}. It is assumed that the valve has an h.t. supply of 200 volts and since, from Fig. 1, $V_{ak} + V_{k0} = 200$ volts, the horizontal axis may be calibrated in terms of V_{ak} or V_{k0} as indicated. At P, $I_a = 8.5$ mA, $V_{k0} = +50$ volts and $V_{gk} = -10$ volts. Since, from Fig. 1,

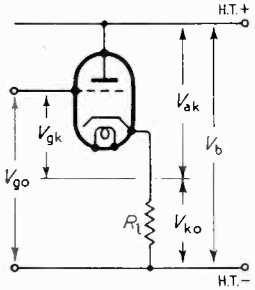
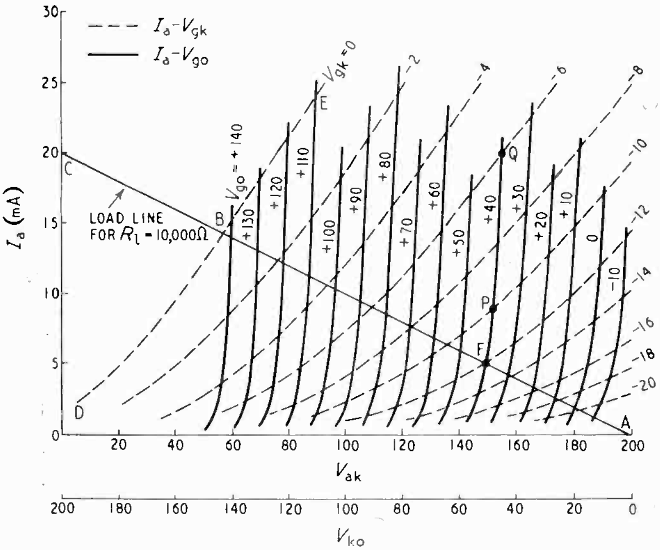


Fig. 1 (left). *Notation used in the text.*

Fig. 2 (right). *Effective characteristics of a triode connected as a cathode follower.*



$$V_{g0} = V_{gk} + V_{k0}$$

$$V_{g0} = -10 + 50 = +40 \text{ volts}$$

point P lies on the characteristic for $V_{g0} = +40$ volts.

Similarly, point Q, for which $I_a = 20$ mA and $V_{k0} = 46$ volts, also lies on the characteristic for $V_{g0} = +40$ volts. After determining other points in a similar manner, the characteristics for $V_{g0} = +40$ volts, and for other voltages, may be drawn in as shown by the solid lines in Fig. 2. The steepness of these characteristics illustrates the great reduction in effective r_a caused by the voltage feedback, and their more uniform spacing shows that the valve, used under these conditions, is a better approximation to the ideal than without feedback.

The performance of the valve with a load in the cathode circuit, as shown in Fig. 1, can be assessed by drawing a straight line through the point where $I_a = 0$ and $V_{k0} = 0$ with a slope equal to $1/R_l$. The line in Fig. 2 is drawn for a load of 10,000 Ω From this diagram the cathode potential for any value of V_{gk} or V_{k0} can be obtained from the co-ordinates of the points of intersection of the load line with the appropriate characteristic. For example, suppose the grid is biased -12 volts with respect to the cathode: the characteristic for $V_{gk} = -12$ intersects the load line at F, where $V_{k0} = 52$ volts.

When the grid receives an alternating signal the operating point moves up and down the load line in a manner determined by the waveform of the signal. But, to obtain distortionless amplification, the extent of travel of the operating point must be limited at both ends of the load line. To avoid grid current and consequent harmonic distortion the operating point must

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not cross the characteristic for $V_{gk} = 0$; on the other hand the grid must not be driven more negative than the value of V_{gk} at point A or anode current is suppressed during part of the cycle of input potential and harmonic distortion results. The greatest input signal which can be handled by the valve without distortion and the corresponding output signal are as illustrated in Fig. 3. In this diagram the I_a—V_{k0} characteristic for $V_{gk} = 0$ is drawn as a straight line. Clearly, to obtain the performance illustrated in Fig. 3, the valve must be so biased that the quiescent point (i.e., the operating point for zero input signal) is at the centre of AB. The valve is then said to have optimum bias and a specific value of automatic bias resistor is necessary to provide such bias.

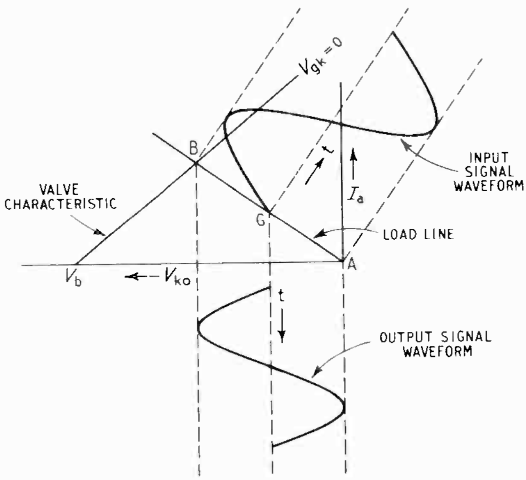

Fig. 3. *Illustrates the greatest output a signal cathode follower can supply without distortion.*

Optimum Value of Bias Resistor

In general the equation to an ideal valve characteristic is

$$I_a = \frac{V_{ak} + \mu V_{gk}}{r_a} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The equation to the I_a—V_{ak} characteristic for $V_{gk} = 0$ is

$$I_a = \frac{V_{ak}}{r_a}$$

But $\quad V_{ak} = V_b - V_{k0}$

where V_b = the voltage of the h.t. supply.

$$\therefore I_a = \frac{V_b - V_{k0}}{r_a} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The equation to the load line (Fig. 4) is

$$I_a = \frac{V_{k0}}{R_l} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

By solving (2) and (3) as simultaneous equation the co-ordinates of B may be obtained. They are

$$\left.\begin{aligned} I_a &= \frac{V_b}{r_a + R_l} \\ V_{k0} &= \frac{V_b R_l}{r_a + R_l} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

The greatest undistorted peak output signal, V_{out}, is given by half the value of V_{k0} at B.

$$\therefore V_{out} = \frac{V_b R_l}{2(r_a + R_l)} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

At G, the quiescent point, the anode current and cathode potential are half their values at B. At G

$$\left.\begin{aligned} I_a &= \frac{V_b}{2(r_a + R_l)} \\ V_{k0} &= \frac{V_b R_l}{2(r_a + R_l)} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Substituting these co-ordinates in (1) to find the value of V_{gk} at G,

$$\frac{V_b}{2(r_a + R_l)} = \frac{V_b}{r_a} - \frac{V_b R_l}{2(r_a + R_l)\, r_a} + \frac{\mu V_{gk}}{r_a}$$

from which

$$V_{gk} = -\frac{V_b}{2\mu} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

At G the anode current and grid-cathode potential are given by (6) and (7) respectively. From these, the value of automatic-bias resistor R_b necessary to give optimum value of grid bias is given by

$$R_b = \frac{V_{gk}}{I_a} = \frac{V_b}{2\mu} \cdot \frac{2(r_a + R_l)}{V_b} = \frac{r_a + R_l}{\mu} \quad \ldots \quad (8)$$

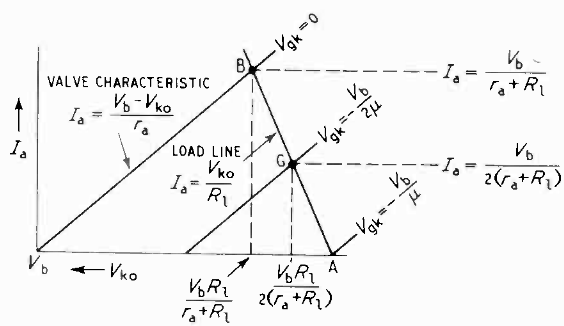

Fig. 4. *Determination of optimum value of* R_b.

This may be written in the alternative forms

$$R_b = \frac{R_l}{A} = \frac{1 + R_l/r_a}{g_m}$$

where $A = \dfrac{\mu R_l}{r_a + R_l}$ = stage gain of the valve with a load of value R_l connected normally.

To illustrate the way in which R_b varies with R_l, expression (8) is plotted in Fig. 5, in which R_b is expressed as a multiple of $1/g_m$ and R_l as a multiple of r_a. From Fig. 5 it is clear that the value of R_b, for a given valve, increases when R_l is increased and decreases asymptotically to $1/g_m$ as R_l approaches zero. This result applies when the valve load is connected in the anode circuit, as in a normal RC-coupled amplifier, and when it is connected in the cathode circuit as in a cathode follower.

If, in Fig. 1, the load R_l is equal to $1/g_m$, the output impedance of the cathode follower, the optimum value of grid bias is automatically obtained and the stage delivers the maximum undistorted output. If, however, R_l appreciably exceeds $1/g_m$, R_b should be increased to the value given by (8) and a circuit such as that illustrated

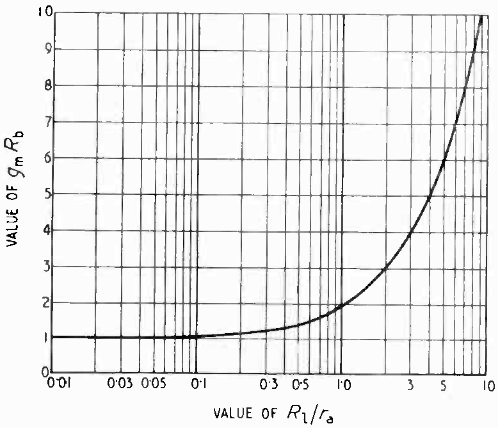

Fig. 5. *Variation of R_b with R_l.*

in Fig. 6(a) or 6(b) is necessary. For very small values of R_l (less than $1/g_m$), R_b is equal to $1/g_m$ and a circuit such as that illustrated in Fig. 7 is necessary. If R_b is not adjusted in accordance with (8), the maximum undistorted output of which the stage is capable is not obtained in practice. With small values of R_l the maximum undistorted output is very small even with optimum grid bias: with incorrect bias the output becomes even smaller.

The performance of cathode followers in which R_l is greater than, equal to or less than $1/g_m$ is examined in greater detail later.

Determination of V_{out}

From Fig. 4 it can be seen that V_{out}, the amplitude of the greatest signal which can be obtained from the valve without distortion is given by the potential co-ordinate at point G.

$$\text{Hence } V_{out} = \frac{V_b R_l}{2(r_a + R_l)}$$

$$\text{from which } \frac{V_{out}}{V_b} = \frac{R_l}{2(r_a + R_l)} \quad \ldots \quad \ldots \quad (9)$$

This expression is plotted in Fig. 8, which illustrates the way in which the maximum undistorted output varies with the value of the load resistor. As R_l approaches infinity V_{out} becomes asymptotic to $V_b/2$ and when R_l is small compared with r_a, V_{out} is directly proportional to R_l.

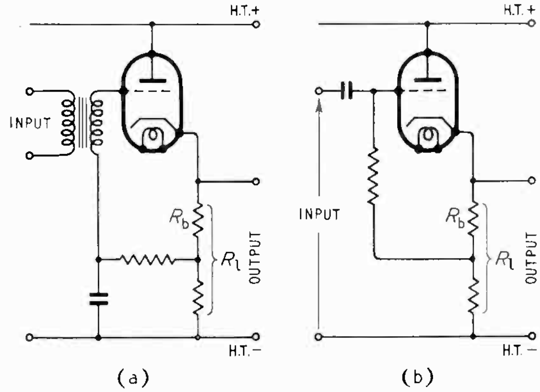

Fig. 6. *Two methods of connecting a cathode follower so that the cathode load exceeds the bias resistor.*

Case 1. *R_l greater than $1/g_m$.*

A cathode follower is sometimes used before a high-frequency amplifying stage so that the low output impedance of the former may reduce the loss in gain caused by the low input impedance of the latter. The a.c. and d.c. loads of such a cathode-follower stage are normally unequal because of the input capacitance of the following valve, but the reactive component of the load is neglected in the following simplified treatment. In such a cathode-follower stage R_l should be large enough to enable the valve to deliver without distortion the input signal required by the following stage and the bias for the cathode follower should be obtained by one of the methods illustrated in Fig. 6. On the other hand R_l should not be any larger than is necessary to supply the required undistorted signal, for large values of R_l cause large steady potentials between cathode and heater, which are undesirable.

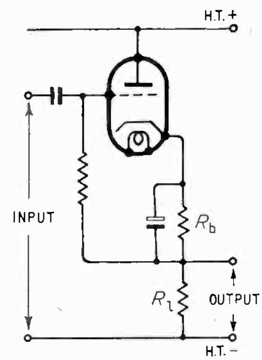

Fig. 7. *Method of connecting a cathode follower so that the cathode load is smaller than the automatic bias resistor.*

The value of R_l necessary to supply a particular undistorted output voltage, V_{out}, is, from (9), given by

$$R_l = \frac{2 V_{out}\, r_a}{V_b - 2 V_{out}} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

If V_{out} is small compared with V_b, this expression may be simplified as follows.

$$R_l = 2 r_a\, V_{out}/V_b$$

Substitution in (8) of the value of R_l given in (10) gives the optimum value of automatic bias resistor as

$$R_b = \frac{V_b}{g_m(V_b - 2 V_{out})} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

If V_{out} is small compared with V_b, R_b is thus equal to $1/g_m$.

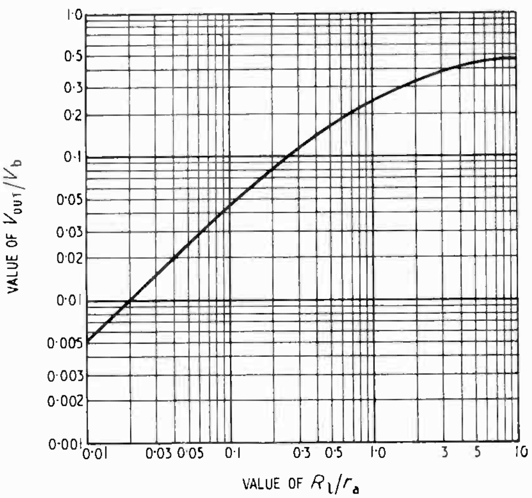

Fig. 8. *Variation of output voltage with load resistance.*

Numerical Example.

Suppose $\mu = 40$, $r_a = 20{,}000\ \Omega$, $g_m = 2$ mA/V and $V_b = 200$ V. What value of cathode resistor is necessary to supply a 10-V signal without distortion ?

From (10)

$$R_l = \frac{2 \times 10 \times 20{,}000}{200 - 2 \times 10}$$
$$= 2{,}200\ \Omega$$

What value of bias resistor is required ?

From (11)

$$R_b = \frac{200 \times 1{,}000}{2 \times (200 - 2 \times 10)}$$
$$= 550\ \Omega$$

If R_l is greater than $1/g_m$, and if the steady p.d. across R_l is used as grid bias, the grid bias obtained is more negative than the optimum value and the maximum undistorted output signal obtainable is correspondingly limited. The anode current of a valve with a resistor of value R_l in the cathode circuit is given by

$$I_a = \frac{V_a + \mu V_{gk}}{r_a + R_l}$$

But $V_{ak} \approx V_b$ and $V_{gk} = I_a R_l$

$$\therefore\ I_a = \frac{V_b}{r_a + (\mu + 1) R_l}$$

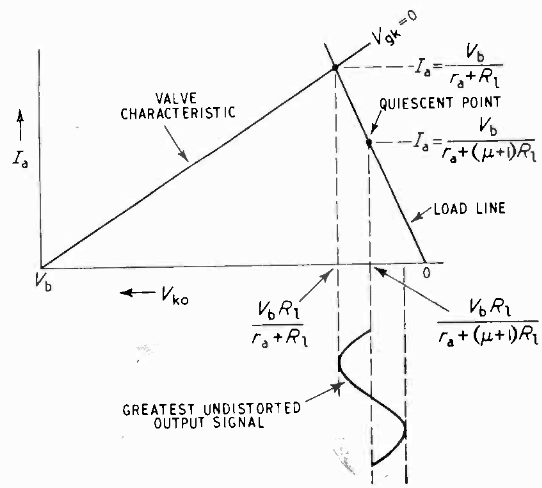

Fig. 9. *Performance of cathode follower with bias produced by cathode resistance greater than $1/g_m$.*

This is indicated in Fig. 9, from which it is clear that the maximum undistorted peak output is given by

$$V_{out} = \frac{V_b R_l}{r_a + (\mu + 1) R_l} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

This may be written in the form

$$V_{out} = \frac{V_b}{1 + \mu + r_a/R_l}$$

from which it is clear that the output increases asymptotically to $V_a/(1 + \mu)$ as R_l is increased. Cathode followers with this circuit cannot, therefore, be used to provide an output voltage greater than $V_b/(1 + \mu)$.

Case 2. $R_l = 1/g_m$.

If $R_l = 1/g_m$, the cathode follower works into a load equal to its own output impedance. The value of the maximum undistorted output is given by substituting r_a/μ for R_l in (9).

$$V_{out} = \frac{V_b r_a/\mu}{2 r_a(1 + 1/\mu)}$$
$$= \frac{V_b}{2\,(1 + \mu)}$$

By substituting $R_l = 1/g_m$ in (8) it can be shown that the optimum value of automatic bias resistor is given by $\frac{r_a}{\mu}\left(1 + \frac{1}{\mu}\right)$ which, if μ is very much greater than unity, is so nearly equal to $1/g_m$ that the valve may be considered perfectly biased by the cathode resistor.

Case 3. R_l small compared with $1/g_m$.

When R_l is small compared with $1/g_m$ the bias resistor should equal $1/g_m$ as shown by expression (8) and Fig. 5.

Since R_l is small compared with r_a/μ (i.e., $1/g_m$), it may be neglected in comparison with r_a and the maximum undistorted output voltage with optimum bias is, from (5), given by

$$V_{out} = \frac{V_b R_l}{2r_a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

For example, if $r_a = 20{,}000\ \Omega$, $\mu = 40$, $R_l = 100\ \Omega$ and $V_b = 200$ V

$$V_{out} = \frac{200 \times 100}{2 \times 20{,}000} = 0.5 \text{ V}$$

If the steady p.d. across R_l is used as bias, the grid is biased positively with respect to optimum bias and the undistorted output is correspondingly limited.

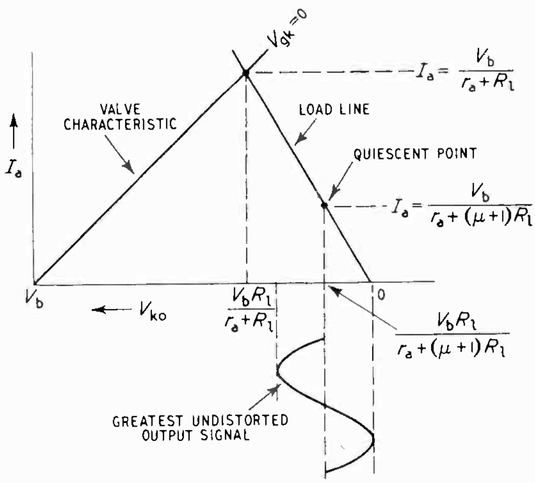

Fig. 10. *Performance of cathode follower with bias produced by cathode resistance smaller than $1/g_m$.*

From Fig. 10 the greatest undistorted output signal is given by

$$V_{out} = \frac{V_b R_l}{r_a + R_l} - \frac{V_b R_l}{r_a + (\mu + 1) R_l}$$
$$= \frac{\mu R_l\, 2V_b}{(r_a + R_l)\,[r_a + (\mu + 1) R_l]}$$

If R_l is neglected in comparison with r_a

$$V_{out} = \frac{\mu R_l^2 V_b}{r_a\,[r_a + (\mu + 1) R_l]}$$

which may be written

$$V_{out} = \frac{\mu n^2 V_b}{(\mu + 1)\, n + 1} \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where $n = R_l/r_a$.

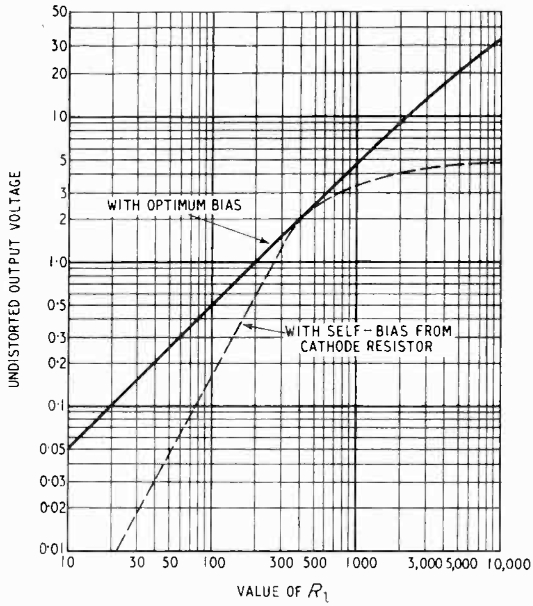

Fig. 11. *Variation of output voltage with R_l for a particular valve with optimum bias and bias produced by R_l.*

Repeating the previous numerical example:

$$n = \frac{100}{20{,}000} = \frac{1}{200}$$

$$V_{out} = \frac{40 \times 200/200^2}{41 \times (1 + 1/200)} = 0.17 \text{ V.}$$

The maximum undistorted output voltage is thus reduced to a third, for the particular values chosen, by the use of incorrect grid bias. In general, by comparison of (13) with (14), it may be shown that

$$\frac{\text{maximum undistorted output with incorrect bias}}{\text{maximum undistorted output with optimum bias}} = \frac{2}{1 + 1/\mu n}$$

Fig. 11, shows the maximum undistorted voltage as a function of R_l for a cathode-follower stage with self-bias and with optimum bias. In this, $V_b = 200$ V, $r_a = 20{,}000\ \Omega$ and $\mu = 40$.

H.F. TRANSMITTER FOR D.F. MEASUREMENTS

Determination of Polarization Error

By B. G. Pressey, M.Sc. (Eng.), Ph.D., A.M.I.E.E.

(Communication from the National Physical Laboratory)

SUMMARY.—The polarization error of a direction finder can be conveniently measured using an elevated transmitter which is capable of producing a test signal of known polarization. This paper describes the design, construction and operation of such a transmitter covering the frequency range 3-20 Mc/s. It has a loop aerial which can be rotated about a horizontal axis so as to produce any desired polarization and it is mounted at the top of a wooden tower or pole. Rotation of the transmitter so that it faces the direction finder, rotation of the aerial and change of frequency are all controllable from the ground.

The paper includes a discussion of the principles underlying the design of the aerial, an account of the performance of the instrument and a description of the method of measuring polarization error.

1. Introduction

In determining the performance of a direction finder one of the most important quantities to be measured is the extent to which the instrument is subject to polarization error. This error varies with the angle of polarization and the angle of incidence of the wave and is very conveniently measured in the field by using a test wave of known angle of incidence and angle of polarization. At the National Physical Laboratory this test wave is produced by a low-power transmitter mounted on a wooden tower or pole and having an aerial which is rotatable about a horizontal axis and thus capable of producing waves of any desired polarization. An early model of such a transmitter suitable for operation in the high-frequency band has been described by Barfield[1]. The transmitter described in this paper differs essentially in the type of aerial used, the dipole aerial employed in the early model having been replaced by a loop aerial. There are also a large number of mechanical improvements including the provision of a remotely-controlled tuning mechanism which shortens very considerably the time required to make error measurements over a frequency range.

2. General Description

The transmitter, complete with batteries, is contained in a small metal box which, by means of a rope is hauled up into a carriage at the top of a wooden tower or pole. A screened loop aerial is fixed to this box and is capable of rotation about a horizontal axis. The fixture at the top of the tower is rotatable about the vertical axis of the tower so that the aerial may face in any direction. Both these movements together with the switching of the transmitter supplies are controllable from the ground. The remotely-controlled tuning device enables the frequency to be set to a number of predetermined values without the necessity of lowering the transmitter. The transmitter is modulated and covers a frequency band of 3-20 Mc/s in three ranges.

MS accepted by the Editor, December 1947.

3. Principles of Design

The chief consideration in the design of the transmitter is that of the type of aerial to be used. In the earlier model a dipole was used but it has since been shown by American workers at the National Bureau of Standards that such an aerial, when placed horizontally, is not capable of giving a purely horizontally-polarized wave except at points in its horizontal or equatorial planes. Since the direction-finding aerials, the horizontal pick-up of which is being measured, lie outside these planes, a signal is introduced into the aerial system by this unwanted vertical component of the field and gives rise to erroneous measurements. The magnitude of this so called 'parallax error' is dependent upon the distance between the transmitter and the direction finder, decreasing with increasing distance. The amount of error which can be tolerated depends also upon the magnitude of the polarization error being measured so that a minimum distance cannot be specified but, broadly speaking, it is not advisable to ignore the parallax effect at distances less than ten wavelengths. Such a distance is usually impracticable when working in the h.f. band and since theoretical corrections for shorter distances may lead to dubious results this type of aerial is no longer used.

With a loop aerial, however, there are no such defects, since when set horizontal the radiated field is everywhere horizontally polarized. When set in the vertical plane, however, the field is not purely vertically polarized but its horizontal component is everywhere parallel to the plane of the loop so that horizontal conductors normal to this plane have no voltage induced in them. This desirable performance of the loop is dependent, however, upon two conditions, (a) that the size of the loop is small compared with the wavelength and (b) that its axis of rotation, is exactly horizontal.

Considering first the size of the loop, it has been shown[2] that, due to the non-uniform distribution of current, a loop is equivalent to a combination of a magnetic dipole and an electric dipole, the latter being in the plane of the loop and normal to the line joining the gap in the screen to the centre of the opposite side. Since the transmitter is normally set up for polarization-error measurements so that this line is horizontal and directed towards the direction finder, it is important that the equivalent electric dipole shall be small, otherwise an appreciable parallax error will be present. The ratio K of the moment of the electric dipole to that of the magnetic dipole can be shown to be $3\pi a/\lambda$ for a square loop of side a. This is a first approximation and assumes that a is not greater than about $\lambda/25$. Now it can also be shown that for a given angle of incidence and a given angle of polarization the parallax error from a dipole is approximately proportional to λ/d, where d is the distance to the direction finder. Thus the error introduced by the equivalent electric dipole of the loop is independent of wavelength and inversely proportional to the distance. It is found that the parallax error from a dipole is negligible at distances greater than ten wavelengths, so that if the loop transmitter is to be usable at practicable distances, say, down to one wavelength, then K must be not greater than 1/10. Thus the maximum value for a will be equal to $\lambda/30\pi$, which for the wavelength range of 15-100 m is approximately 1 m.

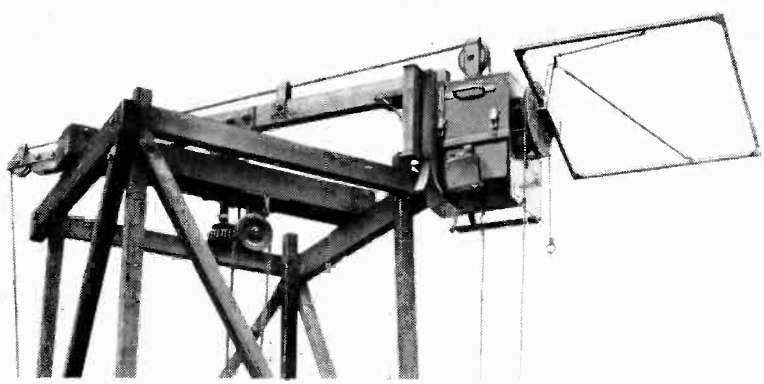

Fig. 1. *View of elevated loop transmitter.*

It can be shown[3] that, if the axis of rotation of the loop tilts up or down by an angle β to the horizontal in the plane of propagation, then the ratio of the pick-up due to this tilt to the normal pick-up on the vertical aerials is equal to

$$\frac{\beta}{(\pi/2-\alpha)} = \frac{\lambda}{2\alpha d}$$

where α=angle of polarization of loop (to vertical) and β and $(\pi/2 - \alpha)$ are assumed to be small angles. If observations to an accuracy of 1° are to be made this ratio should be less than 0.0175 at the shortest distance used, which is taken as one wavelength. Thus the maximum permissible value for β is given by

$$\frac{\beta}{(\pi/2-\alpha)} = \frac{1}{10}$$

and is 0.6° for $\alpha = 84°$, which is the largest value of α for which the transmitter is designed.

4. Details of Construction

The transmitter in position at the top of the tower is shown in Fig. 1.

4.1 *Tower-head Fixture.*

This fixture consists of a horizontal 6-ft wooden boom which is supported centrally on a vertical axle and carries at one end the transmitter carriage. The boom is rotated by means of a worm-drive pulley which is operated by an endless halyard hanging down the centre of the tower. The axle bearing and the drive are carried by a pair of transverse members bolted to the top of the tower. The transmitter is hauled into position by means of a rope passing over a pulley at each end of the boom and is located by rollers on the back of the transmitter box fitting in the guides of the carriage. The carriage is rigidly and accurately fixed to the boom so that when the transmitter is in position the axis of rotation of the aerial is horizontal. Observations made from the ground showed that this was achieved to within ½°. A transmitter counter-balance weight of 30 lb is permanently attached to the rope and the transmitter is retained in the carriage by the addition of a second weight.

For measurements on direction finders which have already been erected on their own site a portable type of fixture has been constructed. It is designed to fit on the top of a pole and consists of a carriage, similar to that described above, which is clamped to the pole and a short boom which is attached to a cap fitting over the top of the pole and carries the pulley wheels for the raising rope. The pole is usually about 40-50-ft high and is fitted with steps. The fixture is fitted on the pole after the latter's erection, the boom being fitted first so that the carriage can be hauled up on the rope. The carriage is clamped to the pole so that it faces the direction finder and by means of wedges inserted between the clamps and the pole its position is adjusted with the transmitter in place so that the axis of rotation of the loop is horizontal as indicated by a spirit level.

4.2 *Transmitter Unit.*

The transmitter itself together with its h.t. batteries is contained in the metal box which measures 12 in × 12 in × 10 in. The l.t. battery is slung on the outside of the box in such a way that it still remains in an upright position when the unit is inverted for adjustment to the tuning mechanism which is attached to the underside of the box. The complete unit weighs approximately 35 lb.

The loop aerial is 2-ft square and consists of

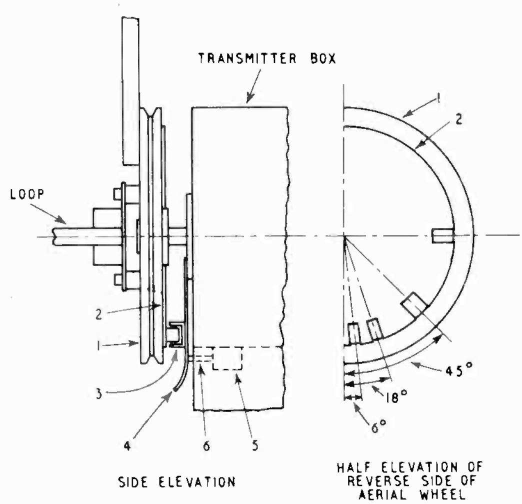

Fig. 2. *Aerial locating mechanism.*

a light-weight copper screening tube enclosing a two-turn air-spaced winding. It is attached to the front face of a wooden wheel which is rotatable about a horizontal stub axle fixed to the front of the transmitter box as shown by the drawing in Fig. 2. On the reverse face of the wheel (1) is mounted a metal disc (2) with radial slots cut in it at specific angles. A roller (3) carried on a sprung arm (4) fixed to the box fits into these slots and thus locates the loop. The angular positions of the loop which are obtainable are 0°, 72°, 84°, 96°, 108° and 180° to the vertical. In addition there are two further slots at 45° and 135° which are wider than the others and whose purpose is explained in Section 4.3. The wheel is rotated by a cord running in a groove cut in its periphery.

The construction of the tuning mechanism is shown in Fig. 3. Coupled to the tuning capacitor spindle is a shaft (1) carrying three gear wheels. One of these (2) meshes with the driving pinion of a clockwork motor (3). The teeth of the other two (4) are used as locating slots for six stop-bars (5) of which only one is shown. These bars are held in position by radial springs (6), and may be fitted into any of the 133 teeth of the wheels. The rotation of the shaft is prevented by one of the stop-bars engaging with a two-pronged escapement (7). This escapement is attached to a spring-loaded rod (8) which may be moved axially. The two prongs of the escapement (see enlarged inset) are staggered in such a way that when the rod is pulled downwards the stop-bar is disengaged from the lower prong (9) and re-engaged on the upper (10). When the rod is returned to the normal position the stop-bar is completely disengaged and the shaft is driven round until the next stop-bar engages with the escapement. The mechanism is operated from the ground by means of a cord attached to the escapement rod. The teeth of the lower wheel are numbered so that the stop-bars may be set for any desired frequency by reference to a calibration chart.

4.3 *Electrical Circuits.*

The h.f. oscillator circuit is of the push-pull shunt-fed Hartley type using the screened loop as the tuning inductance. The loop is wound with two turns, each of which are brought out to a range-change switch. The frequency band of 3–20 Mc/s is covered in three ranges. For the lowest range the two turns of the loop are connected in series with each other and with an auxiliary coil; for the middle range the loop turns are connected in parallel and the auxiliary coil is switched out of circuit; and for the highest range the loop turns and the auxiliary coil are all connected in parallel. The oscillator is anode modulated by a 1,000-c/s oscillator. The power supplies consist of a small capacity 2-V accumulator and a 120-V dry battery.

In addition to the usual hand-operated on-off switches there is a three-position switch in the heater circuits of the h.f. oscillator and the modulator valves which is operated by the rotation of the aerial. This switch (5, Fig. 2) is actuated by a spring-loaded rod (6), one end of which bears on the arm carrying the locating roller. When the roller is riding on the surface of the disc only the h.f. oscillator is switched on. As the roller falls into a slot in the disc the actuating rod moves forward and allows the contacts in the modulator circuit to close. Thus the modulation note is heard in the receiver only when the aerial is properly located in a known position. The forward movement of the roller arm is controlled by the width of the slot, and when the roller falls into one of the two wider slots at the 45° and 135° positions the switch rod moves into its third position, and in doing so switches off both the h.f. oscillator and the modulator and also an indicator lamp in the oscillator heater circuit.

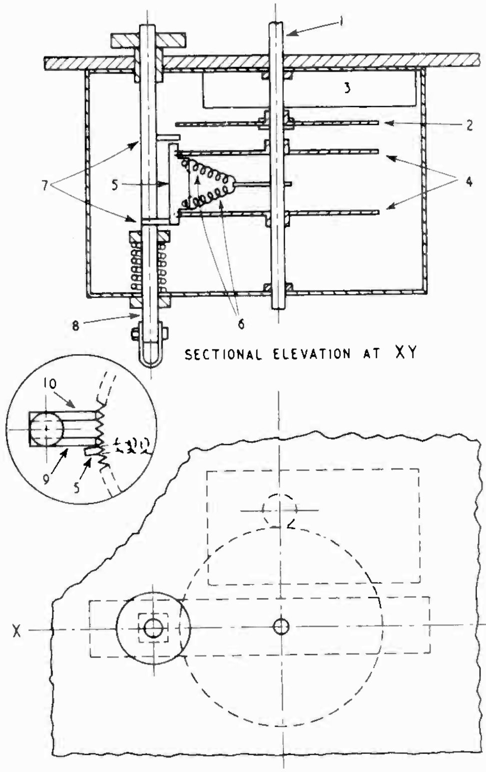

Fig. 3. *Tuning mechanism.*

5. Performance

Prior to the use of the transmitter for polarization-error measurements some tests as to its suitability were made. The first test was designed to determine the presence of any spurious radiation either due to leakage through the screening box or to asymmetry of the loop. The transmitter was mounted on a turn-table so that the axis of rotation of the loop was vertical and coincided with that of the turn-table. The radiation was received on a vertical aerial, and by rotating the transmitter the two orientations for minimum signal were found. From their relative positions and the sharpness of the minima it was deduced that the spurious radiation was less than one-hundredth of the wanted radiation over the whole frequency band.

For the second test, the purpose of which was to determine the ratio of the magnetic and electric dipole moments, the transmitter was mounted on the turntable so that the loop was in the vertical plane and its axis of rotation horizontal. In this position the equivalent electric dipole is vertical, and therefore gives a constant radiation irrespective of the orientation of the turntable. The relative receiver output was then measured with the loop in the plane of propagation and with it normal to the plane. The ratio of the output in the former case to that in the latter is equal to the ratio of the sum of the magnetic and electric dipole moments to the electric dipole moment. If it is assumed that the two moments are in phase quadrature then the ratio of the electric to the magnetic moment can be deduced from the measurement. The validity of this assumption was demonstrated by the fact that the receiver output when the loop was in the plane of propagation was substantially the same whether the loop was directed towards the receiver or away from it; i.e., whether the sum or the difference of the moments was measured. The measured ratio of the moments at frequencies of 5.6 and 11.1 Mc/s was 0.098 and 0.18 respectively, which is in good agreement with the theoretical values of 0.10 and 0.20.

6. Measurement of Polarization Error

A general discussion on the measurement of polarization error is given elsewhere,[3] so that this section will be confined to a description of the procedure of measurement used with this transmitter.

The direction finder is erected on a suitable site at a distance of 50 to 100 metres from the 70-ft transmitter tower. The actual distance chosen should be greater than the longest wavelength used but small enough to give a reasonably large angle of elevation to the transmitter. If the direction finder is already erected on its own site, then a pole 40–50ft high is usually erected at the appropriate distance and the transmitter affixed to it as described in Section 4.1.

The transmitter tuning mechanism is set for the required frequencies. The mechanism can be set for frequencies on only one range at a time, since the range switch is manually operated, but if the settings are carefully chosen suitable frequencies will be obtained on all three ranges without further adjustment. Thus in making measurements over the whole frequency band the transmitter need be lowered only twice for operation of the range switch. The transmitter is then pulled up into the carriage and the boom rotated so that the axis of rotation of the loop is in line with the direction finder.

On each frequency a set of three measurements is made, the first with the loop vertical, the second with it inclined at a positive angle to the vertical and the third with it inclined at an equal negative angle. Positive angles are obtained by clockwise rotation and negative angles by anti-clockwise rotation. The actual angle used is either 72° or 84°, whichever gives the more suitable reading on the direction finder. It is usually found that rotation of the loop produces a change in both the position and the sharpness of the bearing because the pick-up due to the horizontal component of the field is not in phase with that due to the vertical component. Since it is the actual bearing error which is the main interest it is necessary to make the measurement in such a way that its maximum value (i.e., its value when the two pick-up voltages are in phase) is obtained. It can be shown that if θ_1 is the observed bearing error, θ_2 the error equivalent to the flattening and K the ratio of the receiver input signal at the minimum pick-up position of the aerials or goniometer to that at the maximum pick-up position then the total error θ_m is given by

$$\cos 2\,\theta_m = \cos 2\theta_1 \,.\, \cos 2\theta_2$$

where $\tan\,\theta_2 = K$. If θ_1 and θ_2 are small the expression can be simplified to

$$\theta_m = \sqrt{\theta_1^2 + \theta_2^2}$$

This approximate formula is found to be sufficiently accurate for practical purposes if θ_1 and θ_2 do not exceed 20°.

The value of θ_1 can be observed directly but θ_2 must be obtained from a measurement of the ratio K. This is done most conveniently by connecting an output meter to the receiver, measuring the minimum output signal and then noting the angle through which the aerials or goniometer must be rotated in order to increase the input signal to $\sqrt{2}$ times its original value. If this angle is γ, then $K = \sin\gamma$. It is assumed above that with the loop in the vertical plane the signal at minimum is negligible. If, however, this is not so, then unequal values of K are obtained with polarization angles of equal but opposite sign. The required value of K can still be found, however, for if half the sum and half the difference of these two measured values are evaluated it is found that one result is equal to the value of K for vertical polarization and the other result is the required value of K. In practice γ is small and it is sufficiently accurate to take θ_2 equal to γ and so simplify the above calculations.

The results of the measurement described above give the total polarization error for a wave having a particular polarization and angle of incidence. Although for some purposes this is adequate, it often happens that a more general expression for the polarization error, such as the Standard Wave Error or the Pick-up Ratio, is required. These quantities may be readily deduced from the measured polarization error using the well-known propagation formulae for the calculation of the field components.

7. Conclusion

In the paper a practical form of transmitter for the measurement of the polarization error of direction finders has been described and the performance measurements have shown it to be capable of producing the required test field with sufficient accuracy. The transmitter has been used for numerous measurements on various types of direction finders[4, 5] and has been found to be very satisfactory in operation. The introduction of remote controls, particularly the frequency one, has facilitated very considerably the measurement of the error over a wide frequency range by reducing to a minimum the number of times which the transmitter has to be lowered to the ground. It has also been found possible for the tuning mechanism to be operated from the direction finder at a distance of 100 yards or more, thus making measurements by a single operator feasible.

8. Acknowledgments

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NOISE SPECTRUM OF TEMPERATURE-LIMITED DIODES

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1. Introduction

THE production of noise currents in diode thermionic valves has been the subject of much theoretical and experimental work. Schottky[1] was the first to derive an expression for the fluctuation current as $\overline{i^2} = 2eI_0\Delta f$, where $\overline{i^2}$ is the mean-square value of the fluctuation current, I_0 the direct current as measured on an ammeter, e the electron charge in coulombs and Δf the band of frequencies within which the noise power is required. Such an expression is sufficiently accurate where transit times are negligibly small but, with application at increasingly higher frequencies, discrepancies soon become apparent between observed and predicted results. The reason for this is the fact that in all the derivations of the above formula the assumption has been made, implicitly or otherwise, that the transfer of an electron from cathode to anode may be regarded as an instantaneous event. Obviously, at high frequencies this assumption is invalid and the transit time must be taken into account if the true magnitude of the fluctuation or noise current is to be obtained.

This paper is concerned with presenting a simple and completely general derivation of the formula for the fluctuation currents in temperature-limited diodes. Account is taken explicitly of the transit time, and it will be found that in the case where the transit time is negligible, the general expression reduces to that given above.

2. Basic Physical Concepts

It is known that when a current impulse is applied to a circuit, disturbances at all frequencies in the spectrum are manifested in that circuit. Whether this is an intrinsic property of the circuit, or whether a pulse of current can be assumed to be made up of an infinite number of components, is a difficult question to answer. In either case, the mathematical devices employed lead to identical expressions in the result and, as the latter gives a more direct and simple approach, it is the one used here.

A current pulse represented by the equation $i = f(t)$, where $f(t)$ may be continuous or have finite discontinuities, may also be represented by a Fourier integral thus,

$$i = f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G(\omega)\epsilon^{j\omega t}\, d\omega \quad \ldots \quad (1)$$

where

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t)\epsilon^{-j\omega t}\, dt \quad \ldots \quad (2)$$

$G(\omega)$ is the Fourier transform of $f(t)$ and is generally complex in nature; it represents both the phase and amplitude distribution of that part of the continuous frequency spectrum lying in the range $\omega \pm \frac{1}{2}\, d\omega$. As soon as there is any regularity or periodicity of the function $f(t)$, the Fourier integral degenerates into the Fourier series, with its harmonically related components.

While noise is the resultant effect of the random emission of a large number of electrons from the cathode, it is first necessary to determine the effect of the current impulse due to a single electron. An electron in motion between the cathode and anode of a diode induces certain charges on the two electrodes. If the electrodes are connected by an external circuit, a current is induced, whose magnitude has been given by Ramo[2] as

$$i_x = ev_x E_x \quad \ldots \quad (3)$$

where i_x is the current corresponding to electron position x, e the electron charge, v_x the electron velocity at the point x in a direction normal to the electrodes, and E_x the field at that point if a unit potential difference existed between the cathode and the anode. If the time of transit is τ, then the current pulse due to an electron exists only in the interval from $t = t_0$ to $t = t_0 + \tau$, where t_0 is the time of emission. After the latter time, the current in the external circuit is determined solely by the electrical characteristics of that circuit. It is sufficient, however, to note that the current pulses may be considered as generated by a source of infinite internal impedance, for the case of temperature-limited diodes.

As the electron possesses a charge e,

$$\int_{t_0}^{t_0+\tau} f(t)dt = e.$$

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Also, as $f(t)$ is zero outside the interval from t_0 to $t_0 + \tau$, it follows that

$$\int_{-\infty}^{+\infty} f(t)\,dt = \int_{t_0}^{t_0+\tau} f(t)\,dt = e \qquad \ldots \qquad (4)$$

Equation (4) is important in establishing the function $f(t)$ once the boundary values of the particular problem are known.

To determine the resultant effect of all the electrons emitted from the cathode, it is necessary to consider the conditions and manner of such an emission. Electron emission is a completely random process—that is, the emission of one electron in no way affects the emission of any other. Likewise, under temperature-limited conditions, the space charge is negligible and exerts no influence on the emission of further electrons. It has been shown by Rice[3] and Fry[4] that the emission is both individually and collectively random, and therefore calculable by the Poisson probability expression. The probability $p(n)$ that the emission rate will have any particular value n, is given by the expression[3,4]

$$p(n) = \frac{n_0{}^n \epsilon^{n_0}}{n!} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

where n_0 is the mean rate. n_0 is simply determined by measuring the direct current I_0,

$$n_0 = I_0/e \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (6)$$

The mean-square deviation σ^2 (where σ is the standard deviation) may be shown[4,5] to be equal to the mean rate.

$$\therefore \sigma^2 = \overline{(n - n_0)^2} = I_0/e \qquad \ldots \qquad \ldots \qquad (7)$$

Hence the fluctuation of emission rate, or measure thereof, is a function only of the direct current, and may be measured in terms of it.

3. Derivation of Noise Formula

It has been suggested[5] that the noise in thermionic valves arises from variations in the emission velocities of the electrons, as given by the Fermi-Dirac statistics. However, when account is taken of the magnitude of such velocity variations in relation to the potentials normally applied between the electrodes to ensure temperature-limited operation, it is seen that variations in the transit time, and consequently the induced current, are negligibly small—probably of the order of 1-2%. While the variation of emission velocity must be taken into account in determining the reduction of noise due to space charge limitations, it may be neglected when dealing with temperature-limited conditions.

The mean-square fluctuation current is a function of both the mean-square fluctuation of emission rate and the current pulse, and Campbell[6] has shown that it may be written as,

$$\overline{i^2} = \overline{(n - n_0)^2} \int_{-\infty}^{+\infty} \{f(t)\}^2\,dt \qquad \ldots \qquad \ldots \qquad (8)$$

It may be further shown[7] that

$$\int_{-\infty}^{+\infty} \{f(t)\}^2\,dt = \int_{-\infty}^{+\infty} |G(\omega)|^2\,d\omega \qquad \ldots \qquad \ldots \qquad (9)$$

and hence, that

$$\overline{i^2} = \overline{(n - n_0)^2} \int_{-\infty}^{+\infty} |G(\omega)|^2\,d\omega \qquad \ldots \qquad \ldots \qquad (10)$$

$G(\omega)$ is now identified as the spectrum associated with the noise current, and is a function of the current pulse as given by equation (2).

As $|G(\omega)|^2$ is an even function, the complete expression for the noise of fluctuation current may be written

$$\overline{i^2} = \frac{2I_0}{e} \int_0^{\infty} |G(\omega)|^2\,d\omega \qquad \ldots \qquad \ldots \qquad \ldots \qquad (11)$$

$|G(\omega)|^2$ may also be written $G(\omega) \,.\, G^\star(\omega)$ where the star indicates the complex conjugate of the function. In certain expressions this form is more easily evaluated than the modular form.

Where only the noise in a narrow bandwidth Δf is required, equation (11) may be written, assuming the value of $G(\omega)$ to remain constant in that bandwidth, as

$$\overline{i^2} \approx \frac{2I_0}{e} 2\pi\Delta f |G(\omega)|^2 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (12)$$

where Δf is the bandwidth being considered.

Thus, noise in a temperature-limited diode may be accurately calculated at all frequencies from a knowledge of the current-pulse shape and the frequency and bandwidth under consideration. The application of this formula to particular cases will now be considered.

4. Planar Diode

In a planar diode operating under temperature-limited conditions there is a uniform accelerating field between the cathode and anode. The current pulse is then shown in Fig. 1, where $i = kt$.

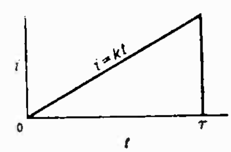

Fig. 1. *Current pulse in planar diode.*

As the total charge transferred is e,

$$\int_0^{\tau} i\,dt = \int_0^{\tau} kt\,dt = e$$

$$\therefore k = 2e/\tau^2$$

and $i = kt = \dfrac{2e}{\tau^2} t$.

As $G(\omega) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{+\infty} f(t) \epsilon^{-j\omega t}\,dt$, and as $f(t)$ exists only in the interval 0 to τ,

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \frac{2e}{\tau^2} \int_0^\tau t\, \epsilon^{-j\omega t}\, dt$$
$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{2e}{\tau^2} \left\{ \frac{\epsilon^{-j\omega t}}{\omega^2} + j\frac{\tau}{\omega}\epsilon^{-j\omega t} - \frac{1}{\omega^2} \right\}.$$

Using equation (11),

$$\overline{i^2} = \frac{2I_0}{e}\frac{4e^2}{2\pi\tau^4}\int_0^\infty \left| \frac{\epsilon^{-j\omega\tau}}{\omega^2} + j\frac{\tau}{\omega}\epsilon^{-j\omega\tau} - \frac{1}{\omega^2} \right|^2 d\omega.$$

For a limited bandwidth Δf, and making use of equation (12),

$$\overline{i^2} = \frac{4e^2 I_0}{\pi e \tau^4 \omega^4} \Delta\omega \mid \epsilon^{-j\omega\tau} + j\omega\epsilon^{-j\omega\tau} - 1 \mid^2$$
$$= 2eI_0\Delta f \frac{4}{(\omega\tau)^4}\{(\omega\tau)^2 + 2(1 - \cos\omega\tau - \omega\tau\sin\omega\tau)\}$$
$$= 2eI_0\Delta f\, \phi(\omega\tau).$$

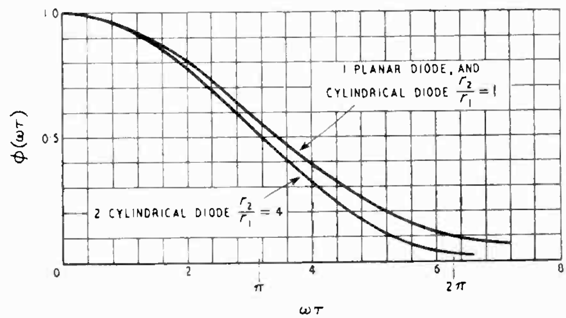

Fig. 2. *Transit time correction $\phi(\omega\tau)$.*

It will be noticed, that this expression is the same as Schottky's, multiplied by a correction factor $\phi(\omega\tau)$ to take account of transit time, where $\omega\tau$ is the transit angle. Values of $\phi(\omega\tau)$ for the range 0 to 2π are shown in Fig. 2.

5. Cylindrical Diode

In temperature-limited cylindrical diodes, the accelerating field between the electrodes is non-uniform. If the radii of the inner emitting electrode and the outer electrode are r_1 and r_2 respectively, then the transit time to any point r between the electrodes is given by

$$t = 2r_1\sqrt{\frac{m}{2eV}}\, a\int_0^z \exp.(z^2)\, dz,$$

where $z^2 = \log_\epsilon r/r_1$, $a^2 = \log_\epsilon r_2/r_1$, V is the potential applied between the electrodes and e and m the charge and mass respectively of the electron. By substituting r_2 for r the complete transit time τ is obtained,

$$\tau = 2r_1\sqrt{\frac{m}{2eV}}\, a\int_0^a \exp.(z^2)\, dz.$$

(The integral may be evaluated from Fig. 3, which covers values of r_2/r_1 from 1 to 50).

Using equation (3), the induced current at any point r, due to a single electron is given by

$$i = \frac{e}{r_1}\sqrt{\frac{2eV}{m}}\frac{1}{a^3}\frac{z}{\exp.(z^2)}.$$

It may be seen that it is not possible to write an explicit expression for i in terms of t, and hence that it is impossible to proceed with the determination of $G(\omega)$ in the same manner as for the planar diode. However, it is possible to draw a graph of i versus t for various values of the parameter a, and this is shown in Fig. 4, where values of r_2/r_1 are marked off along the curve. The pulse shape and its magnitude and duration are determined by cutting off the curve at the appropriate value of r_2/r_1 and inserting the corresponding values of a and A. As there is no analytical expression for the pulse shape so obtained, it is now necessary to obtain an empirical expression for $i = f(t)$ which approximates to the desired shape. Once this is done, $G(\omega)$ may be calculated, and the noise determined as for the planar diode.

For the value $r_2/r_1 = 4$, the expression $i = k(0.8t - 0.43t^2)$ gives a good approximation to the shape of the actual current pulse within the limits $t = 0$ to $t = \tau$. The correction factor $\phi(\omega t)$ obtained by using this expression is shown in Fig. 2.

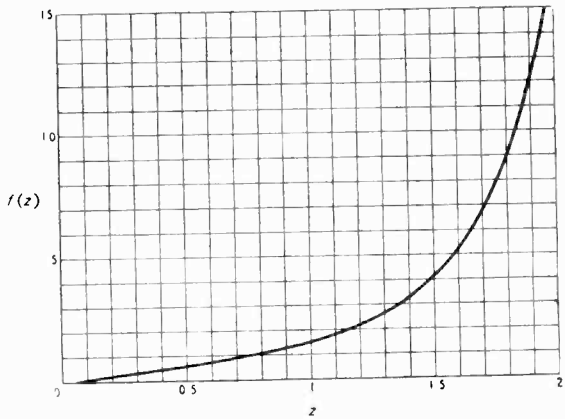

Fig. 3. *Values of $f(z) = \int_0^z \exp(z^2)\, dz$ obtained from Jahnke and Emde, "Tables of Functions."*

6. Conclusion

The correction factor obtained for the planar diode is identical with that due to Rack,[8] who gives a lengthy derivation in terms of electron dynamics. The curve given by Spenke[9] obtained by considering in-phase and quadrature noise components, shows somewhat higher values at large transit angles than those in Fig. 2. Spenke also considered the cylindrical diode, and gave curves for several values of r_2/r_1. His work was later extended by Ashcroft and Hurst [10] and the curve in Fig. 2 for the case $r_2/r_1 = 4$ is in close agreement with their tabulated values.

As mentioned earlier, a small percentage of the noise current arises from the variation of emission velocities of the electrons, but with the voltage needed between the electrodes to ensure temperature-limited emission, it would never be more than 1 to 2 per cent. There is, in addition, noise due to secondary emission from the anode but it, too, is only a small percentage of the total. It can thus be seen that, unless the magnitude of noise from these sources is accurately determined, there is little point in endeavouring to obtain the factor $\phi(\omega\tau)$ to a high degree of accuracy. As regards the approximation to the pulse shape for cylindrical diodes, it appears that this need not be extremely good, as errors up to 5 per cent will have a negligible effect on the factor $\phi(\omega\tau)$.

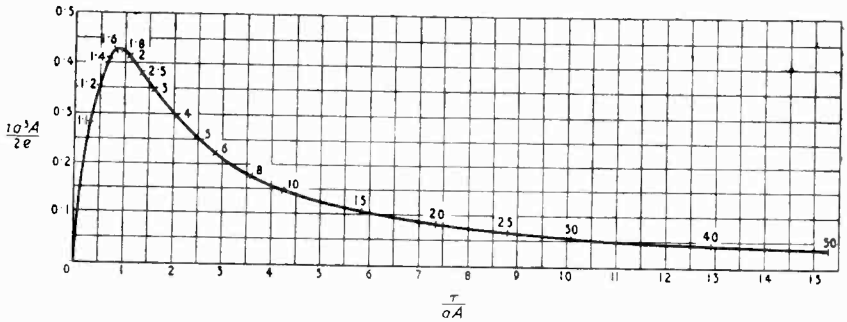

Fig. 4. *Current pulse shape for cylindrical diodes; the figures marked on the curve are values of* r_2/r_1; $A = 2r_1\sqrt{m/2eV}$; $a^2 = \log_e r_2/r_1$.

The method of determining noise as discussed in this paper may be very easily applied to a wide range of problems involving the generation of noise by electron movements. All that is required is a knowledge of the frequency range under consideration and the shape of induced current pulses.

Since this work was originally carried out (May, 1946),[11] a report has been published by Sard[12] dealing with noise in electron-multiplier tubes, in which he makes use of the Fourier transform. However, his paper is limited to the low-frequency case where transit time may be neglected.

The work discussed in this report was carried out as part of a joint research programme of the Divisions of Radiophysics and Electrotechnology, Council for Scientific and Industrial Research, Commonwealth of Australia.

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THE ENGINEERS' GUILD

A discussion meeting, open to all Chartered Civil, Mechanical and Electrical Engineers, on "The Guild as an Association of Professional Engineers," is to be held at The Lecture Hall, City Museum, Park Row, Leeds, at 6 p.m. on 7th April. The speakers are to be E. W. Greensmith and C. V. S. Anderson and the discussion will be open to all.

After the discussion the North-Eastern Branch of the Engineers' Guild will be inaugurated.

INSTITUTION OF ELECTRICAL ENGINEERS

A Radio Section meeting will be held on 12th April, at which Dr. R. H. Barfield will give an informal lecture on "Rado-Frequency Heating." The 40th Kelvin Lecture is on "Semi-Conductors and Rectifiers," by Professor N. F. Mott, on 21st April, while Professor G. W. O. Howe's lecture to the Measurements Section on 3rd May is on "Some Electromagnetic Problems." The lectures start at 5.30 at the Institution, Savoy Place, Victoria Embankment, London, W.C.2.

PUBLICATIONS RECEIVED

Climate and Durability Tests for Radio Components

A specification covering the procedure and general conditions for testing radio components for durability has been published by the Radio Industry Council, 59, Russell Sq., London, W.C.1. It is intended for the use of the British Radio Industry and has not yet been considered by the British Standards Institution.

The specification, RIC/11, covers vibration, damp heat, and salt-spray tests and includes information about the apparatus needed. Cold test cycles and mould growth tests are also included.

Silica Gel

A booklet describing the properties of Silica Gel and the methods of applying it as an adsorbent and catalyst support has been prepared by Joseph Crosfield & Sons Ltd., Warrington, Lancs. Entitled "Sorbsil Brand Silica Gel" (Technical Publication No. 21), it is handled in London by Lever Bros. & Unilever Ltd., Unilever House, London, E.C.4.

GROUND ABSORPTION

With Elevated Vertical and Horizontal Dipoles

By R. E. Burgess, B.Sc.

(Communication from the National Physical Laboratory)

SUMMARY.—The fraction of the power radiated from an elevated dipole which is absorbed in the ground is calculated for dipole heights which are large compared with the wavelength. The dipole is assumed to have a sine-law polar diagram and the energy flow into the ground is evaluated using the Fresnel reflection coefficients which depend upon the wave polarization. The absorption factors are evaluated and expressions are derived for limiting ground permittivities, which are respectively large and nearly equal to unity. The ground absorption for a vertical dipole is a very slowly varying function of the electrical properties of the ground and is very nearly 0.4 for most practical conditions. It is appreciably greater than that of a horizontal dipole on account of the Brewster phenomenon. The results obtained are compared with those derived earlier by Strutt, Niessen, and Sommerfeld and Renner.

The noise in an aerial system due to thermal radiation from the ground is evaluated directly using thermodynamical principles and is shown to be consistent with the value derived by application of the Reciprocity Principle.

Brief consideration is given to the ground absorption in aerial arrays.

1. Introduction

THE behaviour of dipole radiators in free space and over perfectly reflecting ground has received considerable theoretical and experimental attention and may be regarded as fully investigated.

The problem of an aerial over imperfectly reflecting ground has also been the subject of theoretical investigations but these have led to a variety of conflicting conclusions while the mathematical nature of the work has tended to obscure the physical and engineering implications.

In the case of the imperfect ground the quantities which are of interest are (i) the total power which is radiated by the aerial for unit current (i.e., the radiation resistance) and (ii) that fraction of the power which is absorbed by the ground. The difference between these powers is the power radiated upwards (both directly and after ground reflection) which may be termed the useful power.

There are two obvious methods of calculating these powers. The power entering the ground may be calculated by integrating the Poynting vector directed vertically downwards over the whole surface of the earth. The useful power passing upwards may be calculated by integrating the Poynting vector over a hemisphere of very large radius such that only the space wave is significant at its surface and can be determined from the Fresnel reflection coefficient.

In 1929 Strutt[1] was the first to make an analysis using the Reciprocity Principle and the Fresnel reflection coefficient to calculate the field at points distant from a dipole and so obtained the polar diagram at distances very large compared with the wavelength. Integration of the corresponding power flux over the surface of a hemisphere gave the useful power, and over the complete sphere gave the total power in the case of a dielectric ground. The quantities derived were computed for a few special cases.

In a series of papers in 1935–38 Niessen[2–6] attacked the problem by integrating the Poynting vector over two horizontal planes situated at an infinitesimal distance above and below the aerial (regarded as a point source) so obtaining respectively the useful power and the power absorbed in the ground. However, Niessen's analysis contains errors some of which were pointed out by Sommerfeld and Renner[7] in 1942 when these authors investigated the problem afresh using Niessen's method. However, some even more serious errors in Niessen's work on the horizontal dipole were not corrected and, furthermore, certain of the results obtained by Sommerfeld and Renner are incorrect.

The object of the present paper is to re-examine the problem using Strutt's method, to indicate the errors in later papers and to present the results in a graphical form suitable for practical use together with a physical discussion of their significance.

2. General Considerations

The ground is assumed to be uniform with an infinite plane surface. If its relative permittivity is κ and its conductivity is σ (mho/m) its effective relative permittivity at a wavelength λ in metres is

$$\kappa' = \kappa - j\,60\sigma\lambda \quad \ldots \quad (1)$$

MS accepted by the Editor, December 1947

or in terms of a complex refractive index of magnitude n and phase angle $-\beta$:

$$\sqrt{\kappa'} = ne^{-j\beta} \quad \ldots \quad (2)$$

with

$$n = [\kappa^2 + (60\sigma\lambda)^2]^{1/4} \text{ and } \beta = \tfrac{1}{2}\tan^{-1}\frac{60\sigma\lambda}{\kappa} \quad (3)$$

Typical values for very dry soil, damp soil and sea water are given in Table I.

TABLE I

Typical Ground Constants

Ground	κ	σ (mho/m)	λ (m)	n	β (degs)
Very dry soil	5	10^{-4}	1	2.2	0.035
			10	2.2	0.35
			100	2.2	3.4
Damp soil ..	25	10^{-2}	1	5.0	0.7
			10	5.1	6.7
			100	8.1	33.7
Sea Water ..	80	4	1	15.9	35.8
			10	49	44.0
			100	155	44.9

It is seen that soil can be regarded substantially as a dielectric ($\beta \approx 0$) at the two shorter wavelengths while in the case of sea water at the two longer wavelengths ($\beta \approx 45°$) the conductivity is the more important factor.

It will be assumed that the dipole is short compared with the wavelength so that the radiation pattern follows a sine law. Thus if α is the angle between the axis of the dipole and the direction of a point at large distance R ($\gg \lambda$) the free-space field intensity at that point due to a central current of I amperes is given by

$$E = \frac{60\pi Il}{\lambda R}\sin\alpha \quad \text{volt/m} \quad \ldots \quad (4)$$

where l is the effective length of the aerial. It should be noted that the factor of 60 which occurs in this formula and in equation (1) is really $2c/10^7$ where $c \approx 3 \times 10^8$ m/sec. Since $R \gg \lambda$ there will be no radial component of field and the wave will be sensibly plane. The total power radiated by such an aerial in free space is

$$P_0 = 80\pi^2 I^2 l^2/\lambda^2 \quad \ldots \quad (5)$$

and this is a convenient parameter in terms of which other powers may be expressed.

The case of a dipole at a height large compared with the wavelength will be treated and the power absorbed in the ground will be calculated in terms of its (power) absorption coefficient

$$A = 1 - |r|^2 \quad \ldots \quad (6)$$

where r is the reflection coefficient (for the electric intensity) appropriate to the angle of incidence θ and polarization of the wave at the ground. For waves polarized in the vertical plane of incidence,

$$r_V = \frac{\kappa' u - \sqrt{\kappa' - 1 + u^2}}{\kappa' u + \sqrt{\kappa' - 1 + u^2}} \quad \ldots \quad (7)$$

and for horizontally-polarized waves

$$r_H = \frac{u - \sqrt{\kappa' - 1 + u^2}}{u + \sqrt{\kappa' - 1 + u^2}} \quad \ldots \quad (8)$$

where $u = \cos\theta$.

Writing

$$\frac{\sqrt{\kappa' - 1 + u^2}}{\kappa'} = a + jb,$$

$$\sqrt{\kappa' - 1 + u^2} = c + jd \quad \ldots \quad (9)$$

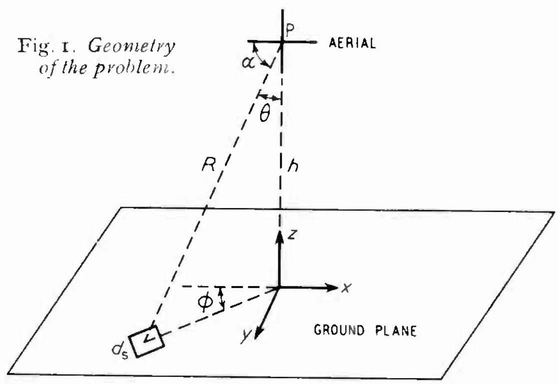
gives

$$A_V = \frac{4ua}{(u+a)^2 + b^2}$$

$$\text{and } A_H = \frac{4uc}{(u+c)^2 + d^2} \quad \ldots \quad (10)$$

An ideal black body has $A_V = A_H = 1$ at all values of θ, a condition which is realized only when $\kappa' = 1$. In practice the absorption coefficients are less than unity reaching their respective maxima for horizontal polarization at vertical incidence and for vertical polarization at the Brewster angle, which is given by $\tan\theta_b = \sqrt{\kappa} = n$ for a pure dielectric.

Fig. 1. *Geometry of the problem.*



3. Absorption Factor for Vertical Dipole

If a vertical dipole is fed with unit current at the centre (Fig. 1) the power radiated in the elementary solid angle $d\Omega$ ($= du.d\phi$) is

$$\frac{1}{120\pi}\left(\frac{60\pi\, l \sin\theta}{\lambda R}\right)^2 . \; R^2\, d\Omega$$

$$= \frac{30\pi\, l^2}{\lambda^2}(1 - u^2)\, . \, du.d\phi$$

The total power absorbed in the ground is obtained by multiplying this by A_V (since the wave is everywhere polarized in the plane of incidence) and by integrating over θ (0 to $\pi/2$) and ϕ (0 to 2π):

$$P_g = \frac{60\pi^2 l^2}{\lambda^2}\int_0^1 A_V(1-u^2)\,.\,du$$

The power which is absorbed in the ground expressed as a fraction of the free-space radiation power P_0 is therefore

$$\frac{P_g}{P_0} = f_V = \frac{3}{4}\int_0^1 A_V(1-u^2)\,.\,du$$

$$= 3\int_0^1 \frac{au\,(1-u^2).}{(u+a)^2+b^2}du \quad .. \quad (11)$$

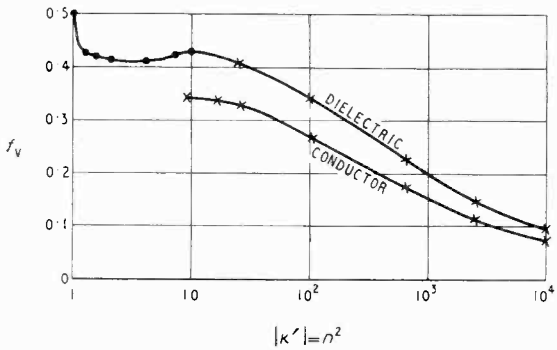


Fig. 2. *Fractional ground absorption from vertical dipole; $|\kappa'| = n^2$ = effective dielectric constant of ground.*

Now when the ground refractive index n is large compared with unity the quantities a and b [cf. equation (9)] vary only slightly with u and may, therefore, to a first approximation be taken as constant giving

$$f_V = 3a\left[2a - \tfrac{1}{2} + \frac{1-3a^2+b^2}{2}\log\frac{(1+a)^2+b^2}{a^2+b^2} - \frac{a}{b}(1-a^2+3b^2)\tan^{-1}\frac{b}{a+a^2+b^2}\right] \quad (12)$$

This may also be expressed in terms of n and β:

$$f_V = \frac{3\cos\beta}{n}\left[(\log n - \tfrac{1}{2} - \beta\cot\beta) + \frac{4\cos\beta}{n} - ..\right] \quad .. \quad .. \quad (13)$$

It is not useful to give the terms in higher powers of $1/n$ as these are modified by the dependence of a and b on u which has been neglected, but introduces terms in f_V of the order of $1/n^3$.

Niessen[1, 2] and Sommerfeld and Renner[7] have derived equivalent approximate expressions for f_V. However, in the latter authors' formula it is necessary to drop the terms arising from their equation (4) as this vanishes at large aerial heights instead of involving height logarithmically as is suggested by their equation (46).

The cases of a pure dielectric and of a pure conductor have been evaluated and plotted using equation (12) for values of $|\kappa'|$ greater than 10. For smaller values of κ in the case of a dielectric, numerical integration of the exact equation (11) has been used to continue the curve back to $\kappa = 1$. In the immediate neighbourhood of $\kappa = 1$ it is found that (see Appendix 2):

$$f_V = \tfrac{1}{2} - \tfrac{1}{5}\sqrt{\kappa - 1} \quad .. \quad .. \quad (14)$$

which implies a very steep decrease of f_V from the value of $\tfrac{1}{2}$ at $\kappa = 1$ as κ increases.

It is seen from the curves in Fig. 2 for the dielectric that as κ increases from 1 there is a sharp initial decrease in f_V followed by an almost constant region when f_V is very nearly constant at 0.41 for $\kappa = 1.5$ to 40. Within the accuracy of the numerical integration there is the indication of a slight maximum of 0.425 in the region of $\kappa = 10$. For most practical cases of substantially pure dielectrics the absorption factor of a vertical dipole can be taken as 0.4. Only over substantially conducting ground (which in practice implies a large value of n as well) will f_V be significantly smaller; e.g., for sea water at $\lambda = 10$ m it is equal to 0.11. Nevertheless f_V decreases very slowly with increasing n and this implies that even over ground of large κ' the fractional absorption of the radiated energy from a vertical dipole is remarkably high.

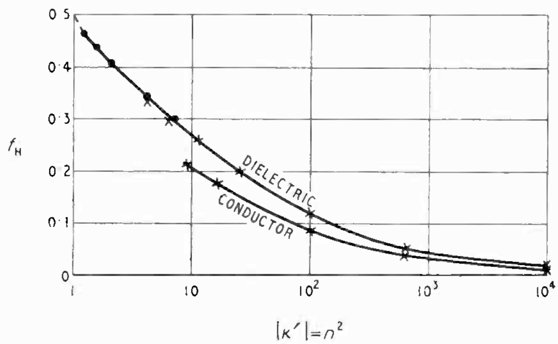


Fig. 3. *Fractional ground absorption from horizontal dipole; $|\kappa'| = n^2$ = effective dielectric constant of ground.*

4. Absorption Factor for a Horizontal Dipole

The field produced at the ground by a horizontal dipole parallel to the x-axis (Fig. 1) will be of mixed polarization and for unit current the components are given by

$$E_H = \frac{60\pi l}{\lambda R}\sin\phi \qquad E_V = \frac{60\pi l}{\lambda R}\cos\phi\cos\theta$$

the resultant being

$$E = \sqrt{E_V^2 + E_H^2}$$
$$= \frac{60\pi l}{\lambda R} \sqrt{\sin^2\phi + \cos^2\phi \cos^2\theta} = \frac{60\pi l}{\lambda R} \sin\alpha$$

in agreement with equation (4).

The power absorbed in the ground is thus

$$P_g = \frac{30\pi l^2}{\lambda^2} \int_0^{2\pi} d\phi \int_0^{\pi/2} (A_V \cos^2\phi \cos^2\theta + A_H \sin^2\phi) \sin\theta \, d\theta$$
$$= \frac{30\pi^2 l^2}{\lambda^2} \int_0^1 (A_V u^2 + A_H) \, du$$

The absorption factor is therefore

$$f_H = \frac{3}{8} \int_0^1 (A_V u^2 + A_H) \, du$$
$$= \frac{3}{2} \int_0^1 \left[\frac{a u^3}{(a+u)^2 + b^2} + \frac{c u}{(c+u)^2 + d^2} \right] du \quad \ldots \quad \ldots \quad (15)$$

When $|\kappa'| \gg 1$ so that a, b, c, and d, can be taken as constant we find

$$f_H = \frac{3a}{2} \left[\frac{1}{2} - 2a + \frac{3a^2 - b^2}{2} \log \frac{(1+a)^2 + b^2}{a^2 + b^2} + \frac{a}{b}(3b^2 - a^2) \tan^{-1} \frac{b}{b^2 + a^2 + a} \right]$$
$$+ \frac{3c}{2} \left[\frac{1}{2} \log \frac{(1+c)^2 + d^2}{c^2 + d^2} - \frac{c}{d} \tan^{-1} \frac{d}{d^2 + c^2 + c} \right] \quad \ldots \quad \ldots \quad (16)$$

Expressing it as a series in $1/n$ we have, as far as terms in $1/n^2$

$$f_H = \frac{3 \cos\beta}{2n} \left[1 - \frac{8 \cos\beta}{3n} + \ldots \right]$$
$$= \frac{3a}{2} \left(1 - \frac{8a}{3} + \ldots \right) \quad \ldots \quad \ldots \quad (17)$$

and thus, to this degree of approximation, is determined by a alone.

It is seen from equation (15) that in the case of dielectric ground of large permittivity the contributions of the two polarizations to the ground loss are equal since each term in the integrand behaves as u/n.

Fig. 3 shows f_H as a function of $|\kappa'|$ for a pure dielectric and a pure conductor, numerical integration being used for $|\kappa'|$ less than 10. It is seen that for a dielectric the absorption factor falls steeply from 0.5 as κ increases from unity and there is no flat region as in the case of the vertical dipole. In the immediate neighbourhood of $\kappa = 1$ the absorption factor behaves as

$$f_H = \frac{1}{2} - \frac{1}{10} \sqrt{\kappa - 1} \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Neissen's formula for this region has an error of sign and predicts an increase of f_H above 0.5. Clearly this is erroneous because 0.5 is the upper limit for the absorption factor since only half the radiated power is incident upon the ground. Niessen's results for large values of n are also in error and, in fact, his expressions for the case of the horizontal dipole are incorrect; it has not been considered profitable to attempt to trace the point in his treatment at which the error is introduced.

5. Comparison of Vertical and Horizontal Dipoles

For ground of large refractive index it is seen that the ratio of the power absorption for a vertical aerial to that for a horizontal dipole tends to

$$\frac{f_V}{f_H} \to 2 \left(\log n - \tfrac{1}{2} - \beta \cot\beta \right) \quad \ldots \quad \ldots \quad (19)$$

and not to 4 as erroneously stated by Niessen[6].

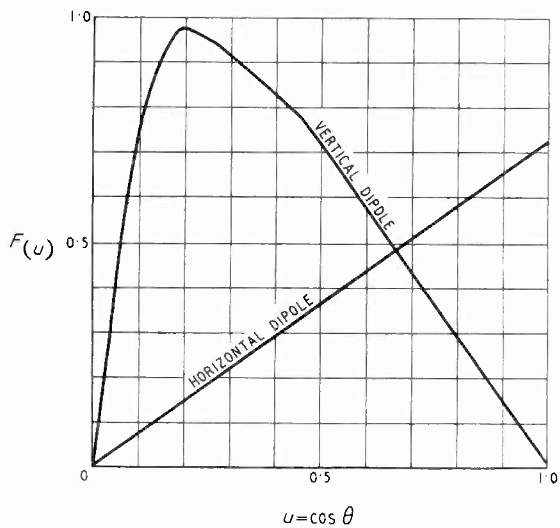


Fig. 4. *Ground absorption as a function of angle of incidence and polarization for $\kappa = 10$.*

This ratio can be quite large; e.g., for sea water when $\lambda = 10$ m it is 5.2. This difference is explicable in terms of the Brewster phenomenon and Fig. 4 may help to illustrate the effect. It shows, for the case of $\kappa = 10$, the function $F(u)$ which when multiplied by $(60\pi^2 l^2/\lambda^2) du$ gives the power absorbed from the radiation in a conical segment u to $u + du$ from a vertical or a horizontal dipole carrying unit current. From equations (11) and (15)

$$F_V(u) = A_V (1 - u^2)$$

and

$$F_H(u) = \tfrac{1}{2}(A_V u^2 + A_H)$$

The peak in the case of the vertical dipole occurs at the angle of incidence for which the ground reflection coefficient vanishes. The area under either curve is proportional to the relevant absorption factor since :

$$f = \frac{3}{4}\int_0^1 F(u)\,.\,du$$

This effect shows that, other things being equal, horizontal dipoles are preferable to vertical dipoles if minimum ground absorption is required. It should also be noted that the Brewster angle corresponds to very nearly grazing incidence for large values of n; for example, over sea water at $\lambda = 10$ m maximum absorption occurs for rays at about 1° to the horizontal. In these conditions homogeneity and levelness of the site are likely to exercise appreciable influence on the magnitude of the ground absorption.

6. Thermal-noise Radiation from the Ground

It follows from the reciprocity principle[8] that a dipole above the ground at absolute temperature T will receive thermal noise from the ground corresponding to a temperature fT where f is the absorption factor calculated above (Kirchhoff's Law.).

This relation may also be proved by a detailed consideration of emission of radiation from the ground. In the first place the ratio of the ground emissivity e to that of a black body e_0 is equal to the absorptive power of the ground :

$$\frac{e}{e_0} = A \qquad \ldots \quad \ldots \quad \ldots \quad (20)$$

It must be remembered that all three quantities e, e_0 and A are functions of wavelength, temperature, angle of incidence, and polarization and the above relation holds only when these parameters are specified. In a frequency bandwidth B the Rayleigh-Jeans law gives

$$e_0 = 2\pi kTB/\lambda^2 \qquad \ldots \quad \ldots \quad \ldots \quad (21)$$

Consider the radiation from an element of ground surface dS at (θ, ϕ) which passes through unit surface located at P, (the centre of the aerial) and normal to the direction of dS. (Fig. 5). Then the power flux at P is

$$dF = \frac{e\,dS\cos\theta}{\pi R^2} = \frac{2kTB}{\lambda^2 R^2}A\,dS\cos\theta \,\ldots \quad (22)$$

and when integrated over the upper hemisphere gives $e\,dS$ as required. Now we can put $d\Omega = dS\cos\theta/R^2 = \sin\theta.d\theta.d\phi$, the solid angle subtended at P by dS and thus the power flux is

$$dF = \frac{dE^2}{120\pi} = \frac{e}{\pi}.d\Omega = \frac{e_0 A}{\pi}.d\Omega$$
$$= \frac{2kTB}{\lambda^2}\cdot\frac{A_V + A_H}{2}\cdot\sin\theta.d\theta.d\phi \quad \ldots \quad (23)$$

where dE^2 is the mean-square electric intensity in the field at P, and $A = \frac{1}{2}(A_V + A_H)$ since the radiation is randomly polarized.

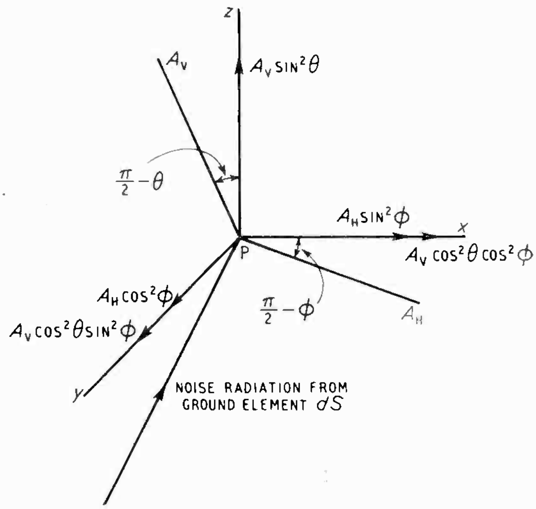


Fig. 5. *Resolution of noise field at P; the Cartesian components shown are proportional to the mean-square intensities in the three directions.*

Now the thermal-noise radiators in dS are randomly polarized; i.e., the mean-square values of the x and y components are equal on the surface. When the field E at P is resolved into the components X, Y and Z parallel to the three axes, it is seen that

$$\frac{dX^2}{A_V\cos^2\theta\cos^2\phi + A_H\sin^2\phi} = \frac{dY^2}{A_V\cos^2\theta\sin^2\phi + A_H\cos^2\phi}$$
$$= \frac{dZ^2}{A_V\sin^2\theta} = \frac{dE^2}{A_V + A_H}$$

Integrating to obtain the noise field at P from the whole ground surface gives

$$\left.\begin{aligned} X^2 = Y^2 &= \frac{120\pi^2 kTB}{\lambda^2}\int_0^1 (A_V u^2 + A_H)\,.\,du \\ Z^2 &= \frac{240\pi^2 kTB}{\lambda^2}\int_0^1 A_V(1 - u^2)\,.\,du \\ E^2 &= \frac{240\pi^2 kTB}{\lambda^2}\int_0^1 (A_V + A_H)\,.\,du \end{aligned}\right\} \quad (24)$$

In general the field components X, Y and Z are uncorrelated :

$$\overline{XY} = \overline{YZ} = \overline{ZX} = 0 \qquad \ldots \quad \ldots \quad (25)$$

since, although correlation is exhibited between the components of X, Y and Z from any given element of surface, the complete fields have zero cross-product when integrated over ϕ.

The effective noise temperature of a vertical dipole is given by

$$T'_{\mathrm{V}} = \frac{Z^2 l^2}{4kB(80\pi^2 l^2/\lambda^2)}$$

$$= \frac{3}{4} T \int_0^1 A_{\mathrm{V}} (1 - u^2).du = f_{\mathrm{V}} T \quad \ldots \quad (26)$$

Similarly for a horizontal dipole

$$T'_{\mathrm{H}} = f_{\mathrm{H}} T \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

so establishing that the absorption factor and the noise temperature ratio are identical, as expected from Kirchhoff's Law.

If the ground were a black body then $f_{\mathrm{H}} = f_{\mathrm{V}} = \frac{1}{2}$ and a dipole with any orientation would exhibit an effective temperature of $\frac{1}{2}T$.

7. Ground Absorption for Aerial Arrays

In general, an aerial array will have a radiation pattern which is sharper than that of a single dipole and the polarization will also be different.

Let the free-space radiation pattern of the array be defined by two functions $G_{\mathrm{V}}(u, \phi)$ and $G_{\mathrm{H}}(u, \phi)$ such that:

$$\frac{G_{\mathrm{V}}}{120\pi}\left(\frac{60\pi}{\lambda}\right)^2 d\Omega \text{ and } \frac{G_{\mathrm{H}}}{120\pi}\left(\frac{60\pi}{\lambda}\right)^2 . d\Omega$$

are the powers radiated in the solid angle $d\Omega = du.d\phi$ for unit aerial current and corresponding respectively to the vertically and horizontally polarized components.

The power absorbed in the ground is then

$$P_g = \frac{30\pi}{\lambda^2}\int_0^1 du \int_0^{2\pi} (G_{\mathrm{V}} A_{\mathrm{V}} + G_{\mathrm{H}} A_{\mathrm{H}}) . d\phi$$

while the total radiated power (which will be numerically equal to the radiation resistance) is

$$P = \frac{30\pi}{\lambda^2}\int_{-1}^{+1} du \int_0^{2\pi} (G_{\mathrm{V}} + G_{\mathrm{H}}) . d\phi$$

The ratio P_g/P is the absorption factor f as before.

It is instructive to consider two special cases of practical interest: a broadside array of horizontal or of vertical dipoles arranged in a horizontal row.

In a sharply-directional broadside array whose elements are parallel to the x-axis the functions $G(u, \phi)$ can be resolved into $G_1(u)\, G_2(\phi)$ where $G_1(u)$ will be the vertical pattern of the single aerial and $G_2(\phi)$ will be an azimuth function with a sharp maximum at $\pi/2$.

In the case of the array of vertical dipoles, the wave incident at the ground is everywhere vertically polarized and hence

$$f = \frac{\int_{-0}^{1} du\, G_{\mathrm{V}1}(u) A_{\mathrm{V}} \int_0^{2\pi} G_{\mathrm{V}2}(\phi) . d\phi}{\int_{-1}^{+1} du\, G_{\mathrm{V}1}(u) \int_0^{2\pi} G_{\mathrm{V}2}(\phi) . d\phi} = \frac{\int_0^1 du\, G_{\mathrm{V}1}(u) A_{\mathrm{V}}}{\int_{-1}^{+1} du\, G_{\mathrm{V}1}(u)}$$

which is identical with the absorption factor of a single vertical dipole since $G_{\mathrm{V}1} = (1 - u^2)$.

In the case of the array of horizontal dipoles the polarization will be substantially horizontal, since only radiation broadside to the dipoles is significant. With this simplification and remembering that $G_{\mathrm{H}1}(u)$ is a constant, the absorption factor is given by

$$f = \frac{\int_0^1 du\, G_{\mathrm{H}1} A_{\mathrm{H}}}{\int_{-1}^{1} G_{\mathrm{H}1}\, du} = \frac{1}{2}\int_0^1 A_{\mathrm{H}} . du$$

$$= 2c\left[\frac{1}{2}\log\frac{(1+c)^2 + d^2}{c^2 + d^2} - \frac{c}{d}\tan^{-1}\frac{d}{c + c^2 + d^2}\right]$$

for ground of $|\kappa'| > 10$. For a purely dielectric ground ($d = 0$)

$$f = 2n\left[\log\left(1 + \frac{1}{n}\right) - \frac{1}{1+n}\right] \rightarrow \frac{1}{n} \text{ for } n \text{ large}$$

compared with $3/2n$ for the single horizontal dipole. Thus the effect of directivity in the broadside array is to reduce the ground loss by a factor of 2/3.

8. Acknowledgments

The work described above was carried out in the Radio Division of the National Physical Laboratory as part of the programme of the Radio Research Board, and this paper is published by permission of the Department of Scientific and Industrial Research.

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APPENDIX 1

Relation of the Refracted Power to the Poynting Vector

Niessen and Sommerfeld and Renner use the Poynting vector directed into the ground as the measure of the absorbed power. Indeed, this is the only correct formulation of the absorption which is valid for all heights of aerial and wavelength. For large values of h/λ it is equivalent to the method using the Fresnel coefficients which has been adopted here.

Let E_ρ and E_ϕ be the horizontal radial and tangential components of the electric intensity at an elementary surface dS at any point on the ground and H_ρ and H_ϕ the corresponding horizontal components of magnetic intensity. Then the Poynting vector directed vertically into the ground is given by

$$F = \text{Re}\,(E_\rho H^*_\phi - E_\phi H^*_\rho)$$

where Re denotes 'real part of' and the * denotes the conjugate. Now in terms of the components E_V and E_H of electric force in the incident wave front, we have by simple resolution and addition

$$E_\rho = E_V\,(1 - r_V)\cos\theta \qquad E_\phi = E_H\,(1 + r_H)$$

$$H_\rho = -E_H\sqrt{\frac{\kappa_0}{\mu_0}}\,(1 - r_H)\cos\theta \quad H_\phi = E_V\sqrt{\frac{\kappa_0}{\mu_0}}\,(1 + r_V)$$

$$\text{Thus } F = \sqrt{\frac{\kappa_0}{\mu_0}}\cos\theta.\,\text{Re}\left[E^2_V\,(1 - r_V)\,(1 + r^*_V) + E^2_H\,(1 + r_H)\,(1 - r^*_H)\right]$$

$$= \frac{\cos\theta}{120\pi}\left[E^2_V\,(1 - |r^2_V|) + E^2_H(1 - |r^2_H|)\right]$$

The elementary power entering dS is thus

$$dP = F.\,dS = \frac{R^2.\,d\Omega}{120\pi}\,(E^2_V\,A_V + E^2_H\,A_H)$$

where $d\Omega = dS.\cos\theta/R^2$.

This is the basic equation used in Sections 3 and 4.

APPENDIX II

Absorption Factors for κ slightly greater than Unity

If the ground be a pure dielectric with $\kappa = 1 + \Delta\kappa$ where $\Delta\kappa \ll 1$ it is seen that A_V and A_H are both very closely equal to unity except near grazing incidence where they drop steeply to zero. It is convenient to write A_V and A_H in the form

$$A_V = 1 - \left(\frac{u - a}{u + a}\right)^2 \qquad A_H = 1 - \left(\frac{u - c}{u + c}\right)^2$$

and using the substitution $u = \sqrt{\Delta\kappa}\sinh v$ we find

$$a = \frac{\sqrt{\Delta\kappa + u^2}}{1 + \Delta\kappa} \approx \sqrt{\Delta\kappa}\cosh v$$

$$c = \sqrt{\Delta\kappa + u^2} = \sqrt{\Delta\kappa}\cosh v \text{ exactly.}$$

Whence $A_V \approx 1 - e^{-4v}$

$A_H = 1 - e^{-4v}$ exactly.

$$\text{Thus } f_V = \frac{1}{2} - \frac{3}{4}\int_0^1 (1 - A_V)\,(1 - u^2)\,.\,du$$

$$= \frac{1}{2} - \frac{3}{4}\int_0^{v_1} e^{-4v}\,(1 - \Delta\kappa\sinh^2 v)\,\sqrt{\Delta\kappa}\cosh v.\,dv$$

Since the major contribution to the integral occurs for small values of v (near grazing incidence) it is permissible to simplify to

$$f_V \approx \frac{1}{2} - \frac{3}{4}\sqrt{\Delta\kappa}\int_0^\infty e^{-4v}\cosh v\,.\,dv \approx \frac{1}{2} - \frac{1}{5}\sqrt{\Delta\kappa}$$

Similarly

$$f_H = \frac{1}{2} - \frac{3}{8}\int_0^{v_1}[(1 - A_H) + (1 - A_V)u^2]\,.\,du$$

$$\approx \frac{1}{2} - \frac{1}{10}\sqrt{\Delta\kappa}$$

These approximate expressions are used to deduce the behaviour of f_V and f_H near $\kappa = 1$.

NEW BOOKS

Waveforms

Edited by B. Chance, V. Hughes, E. F. MacNichol, D. Sayre and F. C. Williams. (Vol. 19, M.I.T. Radiation Laboratory Series). Pp. 785 + xxii. McGraw Hill Publishing Co., Ltd., Aldwych House, London, W.C.2. Price 60s. (in U.K.)

This book differs from most others in the series by being far more descriptive than analytic. It is relatively non-mathematical. This is brought about by the fact that the book deals mainly with non-linear circuits which are notoriously difficult to analyse. However, one of the main objects in the design of equipment is to obtain operating conditions which closely simulate a switching action in order to minimize the effects of variations in valves and voltages. This is done basically by ensuring that the input signal voltage is very large compared with the grid base of the valve.

Under such conditions reasonable accuracy of circuit calculation is not difficult, for the circuit can be treated as having a succession of different linear regimes, the changes from one to another being brought about by the opening or closing of switches. This method is described in some detail in Chapter 2, but it is applied very little in the rest of the book.

Most of the circuits needed for generating waveforms, from sine-waves to hyperbolic and parabolic waves, and including pulses and linear sweeps, are described but are not analysed. On many circuits component values are given, but they are naturally ones suited to some particular radar application and little or no indication is given of what must be done to make the circuit suitable for some other purpose.

The designer seeking ideas will find plenty, but having found several possible ways of doing what he wants he will have to carry out a detailed analysis of each in order to find the best. While one does not expect to find everything in a book of this kind, one is left with the feeling that it is not nearly as helpful as it might have been.

It is interesting to read that the major differences between the range-measuring techniques of American and British radar equipment arose out of the American use of the bootstrap circuit and the British use of the Miller integrator when basically the two circuits are identical! The fundamental difference is really only that 'whereas in America the plate was *grounded*, in England the cathode was *earthed*.' This simple difference results in the one giving a positive-going saw-tooth with respect to earth and the other a negative-going saw-tooth. This difference in its turn greatly affected the design of subsequent apparatus, such as pick-off circuits which are here disguised under the name of amplitude comparators.

There are many peculiarities of terminology; a long-tailed pair becomes a monostable multivibrator comparator, a flip-flop a monostable multivibrator and the ordinary multivibrator is an astable multivibrator. It is, therefore, a good point that a glossary is included.

The book is not confined to purely valve circuits and the use of synchros and potentiometers for producing waveforms is dealt with. In addition, frequency multipliers and dividers are included as well as counting circuits.

W. T. C.

Pulse Generators

Edited by G. N. Glascoe and U. V. Lebacqz. (Vol. 5, M.I.T. Radiation Laboratory Series). Pp. 741 + xiv, with 500 illustrations. McGraw-Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price 54s. (in U.K.).

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

"Relatively-moving Charge and Coil"

SIR,—With reference to Professor Howe's Editorial in the December 1948 issue, his argument concerning the induction of an e.m.f. in a moving coil by a stationary electric charge may be summarized as follows :

(a) the charges induced on the surface of the wire set up an electrostatic field which exactly cancels, at *all points* in the plane of the coil within the loop, the electrostatic field of the acting charge ;

(b) therefore these surface charges, by their motion with the coil, cause a magnetic field across the internal plane which is identical with that caused by the acting charge when the latter moves and the coil is stationary ;

(c) and therefore that the induced e.m.f. must be the same in the two cases.

Now even if (a) and (b) were generally true (i.e., true independently of the geometry of the coil), (c) would not logically follow. One must remember that the component of magnetic field caused by the motion of the surface charges with the loop exists only for a stationary observer. That is, it exists only in the frame of reference in which the acting charge is stationary. It is this stationary observer who must calculate, in his frame of reference, the e.m.f. and he must therefore, if he uses Maxwell's theory, calculate *two* components : one due to the rate of change of flux through a stationary path momentarily coincident with the moving loop, and one due to the motion of the loop *through* this field. These two components are found to cancel, giving zero e.m.f.

It further happens that (a) is not in general true. Suppose that the components of electrostatic field at any point are $\mathcal{E}_o$ due to the acting charge and $\mathcal{E}_s$ due to the induced surface charges. Then it is easily seen that $\mathcal{E}_s$ must be equal and opposite to $\mathcal{E}_o$ *within the conductor*, for, as Professor Howe says, the whole of the conducting coil must be at the same potential. If, then, $\mathcal{E}_s$ is also to be equal and opposite to $\mathcal{E}_o$ at all points *outside* the conductor on the plane of the loop and within it, it follows that $\mathcal{E}_s$ cannot suffer a discontinuity as one passes through the inner surface of the conductor from copper to air. This would mean that there could be no induced charge anywhere on the inner surface of the coil along the line of intersection with the plane, which is certainly not true in the general case. It is, in fact, true only in those special cases in which the coil provides a complete electrostatic screen for points within it, such as would be obtained with a closed conducting surface or a long solenoid. The polarity of the charges on the inner surface is immaterial. This will depend on the geometry of the system, and may be as shown by Professor Howe in Fig. 1 of his Editorial or in Fig. 2 of my letter in the same issue, according to circumstances. What *is* material is that any inner surface charge whatever invalidates Professor Howe's argument as to (a).

A reference to Maxwell's original work will show that his theory of electromagnetic forces and induction is related to experimental facts concerning *closed circuits* only. It has been reconciled with experiments (such as Bucherer's) with moving charges only with the aid of Einstein's 'restricted theory of relativity.' There is therefore nothing really surprising in the conclusion that the theory does not give a consistent result for the e.m.f. induced in a coil by a charge in relative motion.

The statements which I made, in my letter in the December issue, concerning a charge and toroid in relative motion may all be verified by ordinary mathematical analysis. The *attraction* of the charge by the growing current which it induces in the toroid may readily be verified by using Lenz's law. It is the *induced current* in the toroid which produces this force and not the magnetic field of the acting charge which is naturally changing at stationary points, both within and without the toroid. Professor Howe's argument would seem to entail the retardation of an isolated moving charge by its own magnetic field. Now one of the acknowledged inconsistencies in classical electromagnetic theory lies in the fact that it leads to the conclusion that an isolated charge in uniform motion can *accelerate* itself. A revision of classical theory which removes this difficulty has recently been proposed by Eliezer (*Proc. roy. Soc.*, A, 194, pp. 543–55, 1948).

Dept. of Defence, Ottawa. E. G. CULLWICK.

Radio as a Course for Graduation

SIR,—Thoughtful engineers will subscribe to much of the leading article in your February issue over the not wholly unrecognizable initials "E.B.M." But how difficult it is to be logical when discussing this complex subject !

All will agree that engineering undergraduates require the background science together with a 'correct and precise knowledge of some subject' and a 'severe discipline in the numerical solution of the widest possible variety of academic problems having some bearing on engineering apparatus' (and some practical problems, too). Does any branch of Engineering provide for these requirements better than Electronics and Telecommunications ?

Moreover, if an undergraduate acquiring a 'correct and precise knowledge of some subject' such as Chemical, Civil or Mechanical Engineering is allowed to claim that his Degree is in one or other of those subjects, why may not one choosing Electronics or Telecommunications be given his Degree in Radio (or any other title appropriate to this work) ?

A properly trained Engineer in Electronics and Telecommunications requires at least as complete a scientific education as any other Engineer and so has at least as good a claim to the title of Professional Engineer—possibly even a better claim than many—in spite of the fact that many members of the older branches of Engineering are loath to agree.

C. L. FORTESCUE.

Professor Emeritus, Imperial College and University of London.

Signal/Noise Ratio in Radar

SIR,—My attention has only recently been drawn to Mr. Levy's letter in your July 1948 issue, in which he comments on my earlier letter relating to his paper on "Signal/Noise Ratio in Radar." Although somewhat belated, I would like to correct Mr. Levy's apparent misinterpretation of my argument.

I willingly admit that the sweep of a uniformly-illuminated disc referred to in my imaginary experiment would produce a trace in which the brilliance decreases progressively outwards ; to obtain a uniformly-lit horizontal band the beam-spot should be taken as a rectangle with vertical sides, or a vertical line. The manner in which such a band could be obtained is, however, of no importance to the main argument put forward in my previous letter, which is briefly this:—

A comparison of the merits of a two-dimensional display, in which the signal is presented as a deflection at right angles to the time base, with a one-dimensional presentation in which the signal alters the brilliance of the base line, cannot be justifiably performed by selecting for consideration a single line from the 2-dimensional display (in this case the base line) and proving that the brilliance distribution along this line is less advantageous for the detection of signals than in the one-dimensional display compared. The example given in my letter proves the fallacy of the above procedure by *reductio ad absurdum*. The important point is, that in a two-dimensional display not only does the brilliance disappear from the base line, but it appears somewhat *outside this line instead*. This re-appearance contributes to the possibility of detecting the signal and therefore, the brilliance distribution in the base line gives only a part of the answer.

The decrease of brilliance towards the edges of a pattern drawn by a uniformly illuminated disc of a finite size, is caused by the fact that the cross-section of the disc parallel to the base line diminishes with the distance from this line. This phenomenon, however, contrarily to what is implied in Mr. Levy's letter, has no relation to the effect discussed in his paper, in which an infinitely small (or practically so) spot is randomly deflected by the receiver-noise voltage.

While in the first case a single-stroke sweep will give a pattern without definite edges and having exactly the same brilliance distribution as the statistical result of several repeated strokes (differing only in intensity), in the second case a single-stroke display will give a pattern for which the upper and lower edges (i.e., the envelope) are quite definite. The gradual decrease of brilliance observed on the oscilloscope is, in the latter case, a purely statistical effect of overlapping of a number of patterns, the envelopes of which vary at random.

In the opening sentence of my previous letter I have pointed out that the display described by Mr. Levy is that of presentation of 'positive or negative pulses against a background of unrectified random noise,' I avoided the term 'radar display' which Mr. Levy ascribes to me in his letter, since I was aware that the type of display analysed in the paper is, in fact, never encountered in radar practice. Only in his letter does Mr. Levy extend his consideration to the genuine case of radar display in which both noise and signal are rectified. This correction does not affect, however, the main point of my argument that a one-dimensional analysis of a two-dimensional display cannot give a conclusive answer.

Haslemere, Surrey. S. DE. WALDEN.

Alexander S. Popov

SIR,—As a dabbler in the history of physical science, I was interested in the Editorial which appeared in the January 1948 issue of your journal. I would be grateful if you would allow me to comment on it, and if you would forgive the lateness of these comments!

As you will recall, some of the circumstances of your Editorial were:

1. Professor Eric Ashby in a book called 'Scientist in Russia' described some 'shrill festivities' in the Bolshoi Theatre on May 7, 1945, 'to celebrate the fiftieth anniversary of the invention of radio by A. S. Popov.'

2. V. A. Bailey and K. Landecker in a short paper published in the *Australian Journal of Science* for February 1947, described their examination of the evidence supporting the contrary claims made for Popov and Marconi. They concluded that, so far as the evidence available to them went the 'Russians are correct in regarding Popov as "the inventor of radio" in one widely accepted sense of this phrase.'

3. Your Editorial contended that 'Professor Ashby may have been unwittingly misled by Bailey and Landecker'; and that 'to say that he (Popov) has a prior claim over Marconi as an inventor of radio-communication is to show no conception of what constitutes radio-communication.'

I should like, if you will allow me the space, to point out that, whether the conclusions of Bailey and Landecker are true or false, they are at least not *misleading*—that is, they *do* follow from the premises which they state with some care; and that it is most improper to impute to Bailey and Landecker an ignorance of 'what constitutes radio-communication.' The fact is that Bailey and Landecker did explicitly define just that term, and your Editorial did *not* consider the validity of their definition. Their definition may be sound or unsound, but at any rate it was given and used, and deserved consideration.

It is true that Bailey and Landecker base some of their argument on the assumption that Popov in his letter published in the *Electrician* of Dec. 10th, 1897 correctly reports his own article published in the *Journal of the Russian Physical and Chemical Society* for January 1896. In so doing, Bailey and Landecker point out that, to their knowledge, the validity of this letter (as a report of the earlier article) had not hitherto been questioned. The Editorial article *does* question whether the *letter* is an accurate report of the *article*; but having raised the question, fails to answer it. Once raised, of course, the question can be properly answered only by referring to the *Journal of the Russian Physical and Chemical Society* for January 1896. Bailey and Landecker did try to secure that Journal, but found it unavailable in Australia. It is reasonable to suppose that the Editor of *Wireless Engineer* would have better opportunities than they to secure it, and to settle the question unequivocally. In any case, when the Editor claims that the *indirect* accounts he uses are 'pro-Popovian,' he does not assist his argument. There is no reason to suppose that an indirect pro-Popovian account must be more reliable than Popov's own account.

The Editor asks 'what justification has Ashby for stating that he (Popov) transmitted a message or even attempted to do so?' The justification for the statement is, of course, to be found in Popov's own letter. As to this, the Editor comments 'there is here no suggestion of the transmission of a message. He was presumably recording artificially simulated lightning flashes produced by various type of Hertzian vibrators.' Does the Editor require that every inventor of a device for communication shall begin by asking some one else the historic question 'What hath God wrought?' He allows that Popov did record 'artificially simulated lightning flashes'; and it is not debated that Popov published an account of this work before Marconi made any publication. Now surely we must suppose Popov to have been able to distinguish between the records of his own artificially produced sets of damped oscillations, and the records of those sets of damped oscillations emanating from 'atmospheric electric disturbances' and not artificially produced. To deny Popov the right to claim that he received *messages* is to assume an untenable distinction between messages and signals. There is no 'misconception of what constitutes radio-communication' in supposing this work of Popov's to fit Bailey and Landecker's definition of radio-communication as

'The process (or method) of communicating intelligence at a distance by means of free Hertzian waves.'

It seems to me that a fair summary of the order of events would be:

(a) Many investigators made contributions to the development of wireless telegraphy;

(b) One of Popov's special contributions was the use of a grounded antenna in the *receiver*;

(c) One of Marconi's special contributions was the use of grounded antennae in both the *receiver and transmitter*; and that there is some ground for believing that Popov's publication preceded that of Marconi. In any case, now that the validity of Popov's letter is challenged, the priority issue can be settled only by reference to Popov's paper of January 1896. This is a question not of 'propaganda . . . feverishly pursued,' but of ascertainable facts; and we in Australia, deprived of the facilities available to the Editor of *Wireless Engineer* would be glad if he would now settle that question of fact in the proper way. Meantime the writer at least feels that the charge made against Bailey and Landecker that their paper is misleading and shows no conception of what constitutes radio-communication, ought to be withdrawn.

Sydney, Australia. J. B. THORNTON

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

600 884.—High-gain deaf-aid amplifier wherein the signal is applied to at least three grids in succession, and is subjected to a double reversal of phase.

O. B. Sneath. Application date 21st November, 1945.

602 758.—Audio-frequency amplifier wherein the relative use of positive and negative feedback is automatically controlled by the input signal, for the purpose of securing contrast expansion.

E. P. Rudkin and Standard Telephones and Cables Ltd. Application date 23rd October, 1945.

603 277.—Automatic volume or compression control circuit for a Dictaphone or other sound-recording or reproducing amplifier.

Dictaphone Corporation. Convention date (U.S.A.) 29th December, 1944.

AERIALS AND AERIAL SYSTEMS

601 737.—Construction of a rod aerial and reflector, with supporting and housing means for an impedance-matching and coupling device.

D. Jackson and Pye Ltd. Application date 27th September, 1945.

602 068.—Dipole type of aerial in which each limb comprises two or more parallel half-wave wires, these being short-circuited, over a substantial part of their length.

Philips Lamps Ltd. Convention date (Netherlands) 20th July, 1940.

602 689.—Construction of highly-directive aerials of the electromagnetic horn or slotted-waveguide type.

O. H. Böhm, J. Jedrychowski, L. G. Reynolds and C. S. Wright. Application date 9th November, 1945.

603 358.—Means for mounting and insulating a dipole aerial from a thin-walled building.

W. J. H. Lambert trading as the Standard Electrical Engineering Co., and A. C. Snell. Application date 18th October, 1945.

603 465.—Directive aerial-array comprising eight vertical aerials arranged symmetrically at the circumferential points of a Maltese cross, and coupled to a common radio-goniometer.

A. F. L. Rocke, P. G. Redgment and C. S. Wright. Application date 2nd November, 1945.

603 676.—Beam aerial, consisting of a curved mirror coupled to an open waveguide, a small plate of dielectric being mounted close to the mouth of the waveguide to prevent the formation of standing waves.

E. Wild and C. S. Wright. Application date 27th October, 1945.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

600 889.—Waveguide switching device of the gas-discharge type, as used in radiolocation equipment.

The M-O Valve Co. Ltd. and N. L. Harris. Application date 30th November, 1945.

600 945.—Radiolocation equipment in which the receiver is switched on and off by a periodic 'gating' voltage at a frequency slightly different from the pulse-recurrence frequency.

Sperry Gyroscope Co. Inc. Convention date (U.S.A.) 9th June, 1944.

601 096.—Capacitance-coupling control, for sense-determination, in a directional aerial designed for vertically-polarized waves.

C. Crampton, W. Struszynski and C. S. Wright. Application date 2nd November, 1945.

601 136.—Arrangement for generating a stepped wave to represent the cyclic displacement of the exploring beam in a pulsed radiolocation system.

A. J. H. Oxford. Application date 10th August, 1945.

601 171.—Cathode-follower circuit, particularly adapted for amplifying the video signals and limiting the noise level, say in radiolocation indicators.

E. Parker and J. Buckingham. Application date 30th June, 1945.

601 269.—Box-like device for coupling or sealing the end of a waveguide feeder, say to a parabolic aerial.

L. B. Mullett. Application date 14th August, 1945.

601 280.—Directive aerial of the waveguide-horn type, fitted with strip or iris impedance-matching elements to compensate for reflection.

E. Wild and C. S. Wright. Application date 25th October, 1945.

601 353.—Combination of a dipole aerial and a line-balancing device for coupling the symmetrical radiator to an earthed coaxial-line feeder.

Marconi's W.T. Co. Ltd. (assignees of G. H. Brown *and O. MacD. Woodward*). *Convention date* (*U.S.A.*) *3rd May*, 1944.

601 401.—Navigational system in which pulsed responder signals are utilized to maintain, say a homing plane in an orbital course at a given distance from the aerodrome beacon.

L. C. Barber. Application date 5th September, 1945.

601 532.—Direction-finding system, of the kind using a constantly-rotating frame aerial, and a cathode-ray indicator which is fed by a time-base voltage synchronized with the aerial.

Standard Telephones and Cables Ltd. (*assignees of A. G. Richardson, F. O. Chesus and F. G. Thomas*). *Convention date* (*U.S.A.*) *20th February*, 1943.

601 536.—Radiolocation system of the interrogator-responder type in which the operating frequencies are constantly varied over a predetermined cycle, for the sake of secrecy, and to frustrate deliberate jamming.

Standard Telephones and Cables Ltd. (*assignees of E. M. Deloraine*). *Convention date* (*U.S.A.*) *26th October*, 1943.

601 641.—Navigational beacon transmitter, in which a directional and an omni-directional field are separately modulated to produce signals which mark out a given course-line.

Sadir-Carpentier. Convention date (*France*) *18th October*, 1943.

601 733.—Radiolocation apparatus in which a selected part of the time base of the cathode-ray indicator can be expanded and supplied with an appropriate calibration.

D. S. Watson, C. A. Laws and J. Buckingham. Application date 8th June, 1945.

602 030.—Cathode-ray display-system for radiolocation equipment in which the doppler effect is used.

W. S. Elliott, H. Pursey and R. S. Webley. Application date 2nd May, 1945.

602 037.—Radiolocation system in which a saw-toothed characteristic is utilized to clarify the indication of a moving target relatively to ground or like 'clutter.'

A. E. Bailey, F. E. J. Girling and J. W. Pletts. Application date 13th September, 1945.

602 501.—High-frequency bridge coupling of the transmission-line or waveguide type, for use in radiolocation and duplex communication systems.

Sperry Gyroscope Co. Inc. Convention date (*U.S.A.*) *4th October*, 1944.

602 717.—Direction finder in which two crossed aerials are connected through separate amplifying channels to the deflecting plates of a c.r. indicator and to the two fixed coils of a radio-goniometer, the moving coil of which is coupled to a telephone.

C. Crampton, C. S. Wright, The Plessey Co. Ltd., J. O. G. Barrett and J. E. Rhys-Jones. Application date 12th November, 1945.

603 292.—Time-base circuit including a calibration potentiometer for measuring the critical time-intervals in radiolocation.

Western Electric Co. Inc. Convention date (*U.S.A.*) *16th September*, 1943.

603 302.—Aircraft-training equipment arranged when stationary to simulate the response of standard radio-navigational and radiolocation systems in actual flight

Western Electric Co. Inc. Convention date (*U.S.A.*) *3rd February*, 1944.

603 328.—Arrangements to facilitate the testing of d.f. installations for quadrantal error, and for general accuracy.

P. G. Redgment, W. Struszynski, S. de Walden and C. S. Wright. Application date 12th November, 1945.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

601 069.—Super-regenerative receiver, particularly for short-wave signals, in which the quenching frequency is automatically controlled by two circuits having different time-constants.

Philco Radio and Television Corp., (*assignees of W. E. Bradley*). *Convention date* (*U.S.A.*) *15th July*, 1944.

601 830.—Tuning device in which the inductance is decreased as the capacitance is increased over congested sections of the short-wave range, in order to expand the indicator scale.

B. Tenenbaum. Application date 4th October, 1945.

602 147.—Superheterodyne type of receiver, adapted to be used for indicating and calibrating any selected one of a series of high signalling-frequencies, say for aircraft communications.

F. B. Dehn, (*communicated by Collins Radio Co.*) *Application date 9th January*, 1946.

602 488.—Receiving system in which static and like interference is eliminated by balancing-out the offending sidebands against corresponding but locally-produced sidebands.

I. M. Jones. Application date 31st July, 1945.

602 492.—Method of producing an electrically-conducting pattern, or circuit, on an insulating surface, say in the manufacture of radio receivers or the like.

Sir E. T. Fisk and W. Soby. Application date 23rd August, 1945.

602 634.—Resistance element forming part of the anode or screen-grid circuit of a pair of push-pull short-wave amplifiers, for controlling their high-frequency gain.

Philips Lamps Ltd. Convention date (*Netherlands*), *22nd June*, 1940.

602 785.—Bandpass filter arrangement of two resonant circuits, wherein the degree of coupling between the circuits, and the reaction applied to one of them, are automatically controlled by the applied signals.

E. P. Rudkin and Standard Telephones and Cables Ltd. Application date 7th December, 1945.

602 846.—Circuit arrangement for the balanced diodes of a receiver for frequency-modulated signals, designed to minimize the effect of amplitude variations and inter-station noise.

Radio Corporation of America. Convention date (*U.S.A.*) *14th June*, 1945.

603 010.—Clock-controlled device for switching a radio-receiver on and off at pre-determined times.

E. K. Cole Ltd., and A. W. Martin. Application date 16th November, 1945.

603 367.—Super-regenerative receiver in which additional damping-means are provided to prevent 'carry-over' between the quenching periods.

Hazeltine Corp., (*assignees of B. D. Loughlin*). *Convention date* (*U.S.A.*) *30th November*, 1944.

603 576.—Cathode-ray indicator for indentifying the stations received by a 'panoramic' set, the tuning of which is constantly varied over a wide band of frequencies.

Standard Telephones and Cables Ltd. (*assignees of H. G. Busignies*). *Convention date* (*U.S.A.*) *17th July*, 1941.

603 594.—System of facsimile telegraphy in natural colours, in which the received image is reproduced by the electrolytic decomposition of an arocuatic amine into an azo dye.

Marconi's W. T. Co. Ltd. (assignees of H. G. Greig). Convention date (U.S.A.) 14th October, 1944.

TELEVISION CIRCUITS AND APPARATUS
For Transmission and Reception

601 060.—Television receiver in which interference is eliminated by a process of periodic suppression and reformation, particularly for use with mobile transmitters (addition to 517 170).

Electric and Musical Industries Ltd. and E. L. C. White. Application date 14th March, 1945.

601 227.—Automatic gain-control system for giving independent regulation, both of the audio and video signals in a television receiver.

W. W. Triggs (communicated by Farnsworth Television and Radio Corp.). Application date 28th November, 1944.

602 033.—Television system in which the sound signals are radiated as a modulated train of pulses on the same carrier wave as the video and synchronizing signals.

A. V. Lord and Pye Ltd. Application date 14th July, 1945.

602 053.—Method of scanning the multiple-strip target-electrode used for reproducing television signals in natural colour.

Marconi's W. T. Co. Ltd. (assignees of V. K. Zworykin). Convention date (U.S.A.) 13th October, 1944)

602 341.—Large-scale projection system for television wherein the picture on a normal-sized c.r. screen is magnified to a limited degree by an uncorrected lens.

J. L. Baird. Application date 10th April, 1945.

TRANSMITTING CIRCUITS AND APPARATUS
(See also under Television)

601 164.—Regulating or modulating the frequency of a crystal-controlled valve-oscillator by means of variable reactances inserted both in series and in shunt with the crystal.

Marconi's W.T. Co. Ltd. (assignees of P. D. Gerber). Convention date (U.S.A.) 6th May, 1944.

601 397.—Modulation circuit in which one sideband is either wholly or partly suppressed, in order to produce a desired balance between the high and low signal-frequencies.

Philips Lamps Ltd. Convention date (Netherlands) 12th June, 1940.

601 791.—Construction and chassis arrangement, with provision for cooling, of a radio set which may be completely enclosed and hermetically sealed.

The British Thomson-Houston Co. Ltd. and W. S. Melville. Application date 7th November, 1945.

601 848.—Three-valve transformer device for coupling an unbalanced or earthed transmission-line to a balanced circuit.

Standard Telephones and Cables Ltd. and B. B. Jacobsen. Application date 5th October, 1945.

602 982.—Variable-reactance circuit in which negative reaction is applied to one of the electrodes of a multigrid valve to stabilize its performance say as a frequency modulator.

J. R. Tillman. Application date 14th November, 1945.

603 319.—Electro-mechanical device for use as an impedance-transformer or line-balancer, particularly for coupling a coaxial transmission-line to an aerial.

H. Selvidge. Convention date (U.S.A.) 21st October, 1944.

603 540.—Combined transmitting and receiving set, particularly for frequency-modulated signals, and including a push-pull l.f. amplifier which also serves as a variable-reactance frequency control.

R. F. Hill, H. B. Hadden and E. Forster. Application date 23rd October, 1945.

603 584.—Enlarging the service-area of communication to mobile receivers by a method of radiating ultra short-wave signals simultaneously on different carrier waves from two or more spaced aerials.

J. R. Brinkley. Application date 2nd May, 1945.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

601 931.—Multiplex pulsed signalling system, in which each of the operating channels is controlled and synchronized by a common saw-tooth oscillation generator.

Marconi's W.T. Co. Ltd. (assignees of W. D. Haughton). Convention date (U.S.A.) 1st July, 1944.

601 644.—Telegraphic system utilizing time-modulated pulses which are always separated by 50 per cent ' mark ' or spacing, and so occupy only a narrow waveband.

Marconi's W.T. Co. Ltd. (assignees of H. O. Peterson). Convention date (U.S.A.) 5th January, 1944.

602 849.—Multi-channel system of pulsed signalling which does not depend upon a time-selection of the desired channel at the receiver, and is therefore suitable for broadcasting.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 7th April, 1944.

602 969.—Gating arrangement for the selective reception of a desired signal in a multi-channel system of pulsed signalling.

T. J. McDermott and E. Coop. Application date 13th November, 1945.

603 188.—Electronic pulse-generating device suitable for use in a time-modulated multi-channel system of pulsed signalling.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 9th December, 1944.

603 196.—Three-valve triggered circuit for generating time- or phase-modulated pulses suitable for multi-channel signalling.

T. J. McDermott and E. Coop. Application date 13th November, 1945.

603 614.—Electronic-beam device of the cathode-ray type for detecting and separating the different messages in multi-channel pulsed signalling systems.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 25th November, 1944.

SUBSIDIARY APPARATUS AND MATERIALS

601 385.—Frequency-measuring or counting devices, particularly for use in radio-altimeters and drift-indicators.

Marconi's W.T. Co. Ltd. (assignees of R. C. Sanders). Convention date (U.S.A.) 27th February, 1941.

601 920.—Angle-bracket with pin-and-slot device for securing a radio chassis in a cabinet, so as to allow a quick release.

A. G. Imhof. Application date 11th October, 1945.