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The Patent Question

AN article under this title appeared in the American Journal *Electrical Engineering* of February, 1948. In it the author, L. A. Hawkins of the General Electric Company of Schenectady, discusses the question whether patent consciousness interferes with co-operation between industrial and university research laboratories. Apparently the suggestion had been made that the American patent system caused many industrial concerns to draw an iron curtain around their research laboratories, thus preventing the desirable interchange of ideas with other scientists. The author of the article disagrees entirely with this suggestion, and maintains that the American patent system, by protecting the inventor, makes it possible to operate a research laboratory "with doors wide open to the stimulating visits of outside scientists, without loss and with the great benefit of the mutual fertilization of ideas which such visits produce." If it were not for the patent system, secrecy would be essential, but as it is, the only restraint is that, if the work suggests a possible patentable practical application, it should be submitted to the patent attorney and further disclosure withheld until the necessary papers have been prepared. The author advocates the exchange of visits between various industrial research laboratories and between them and the universities, and maintains that it is the patent system that makes this possible. Without the patent system the industrialist would have the choice between complete isolation and the certainty of leakage and financial loss.

The patent system referred to throughout the article is, of course, that of the United States, and the question immediately arises whether the conclusions apply equally to the patent system in this country. There are important differences between them. In this country the applicant can file a provisional specification to be followed within twelve months by a complete specification with all the claims duly set out. In the United States there is no provisional specification ; the applicant must file the complete specification and claims with the application. This difference makes it easier for the industrialist in this country to protect immediately any discovery that suggests a practical application. When the complete specification has been filed the British patent office makes an examination to determine whether the invention claimed has been disclosed in any publication *within the United Kingdom* prior to the filing of the application. In America, however, the patent office combs the whole world in their search for prior publications.

Although in the U.S.A. it is necessary to file the complete specification, it may be informal in part and contain freehand sketches, the formal drawings complying with the Patent Office requirements being filed subsequently. Amendments can also be made, but no additions ; if it is desired to make additions a new application must be filed.

Although the British patent office requires the applicant to refer in his specification to any prior publication which it considers close enough to the invention claimed, it has no power to refuse the grant of a patent on the sole ground that there is insufficient subject matter in view of prior publications. In the United States the applicant is not required to refer to prior art, but he must clearly distinguish his claims from what is already known. Recently, however, the American Patent Office has instituted the practice of publishing at the end of the specification a list of all the prior art cited during the prosecution of its search. If the American patent office considers that there is insufficient new matter it may refuse to grant a patent. The recent Swan Committee recommended that our patent office should also be given this power.

In the U.S.A. the publication of an invention does not invalidate the patent application if it is filed within a year of the publication; before 1940 the allowable period was two years. In this country before being sealed the patent application is advertised as open to inspection, and it may be opposed by anyone on the grounds of prior publication, or of the invention having been obtained from the opponent. In the U.S.A. the public knows nothing about a patent application unless and until it has been accepted. It is then open to anyone to assert his rights by filing an application with the same claims. When the examiner finds that two or more applications are in conflict and claim the same invention, the Patent Office declares an "Interference" and in the procedure before the Patent Office great weight is given to evidence as to the date of conception of the invention and of its reduction to practice. A priority date of as much as two years prior to the application may be claimed. The proof of the date of conception usually involves the production of documents such as laboratory notebooks, and it has been said that the patent goes to the biggest liar. If an applicant is dissatisfied with the decision of the Patent Office, he can take the case to the courts, and it may drag on for years.

In the U.S.A. a patent runs for 17 years from the date of sealing; in this country it runs for 16 years from the filing of the complete specification, but after 4 years annual renewal fees have to be paid, failing which the patent lapses. Many useless patents are thus allowed to lapse, whereas in America, where there are no renewal fees, they exist for the whole period. In this country a patent may be revoked if the patentee can be shown to have abused his monopoly, for example, by refusing to grant licences on reasonable terms. In America there is no corresponding provision.

The only difference between the two systems that can affect the question of co-operation between industrial and university research laboratories appears to be that of the provisional application and, as we said above, this should make it easier to cultivate and maintain such co-operation without running the danger of leakage of patentable information.

On comparing the two systems there appears to be no point on which the British system is inferior to that of the United States. The power to refuse a patent on the ground of insufficient new matter adds considerably to the responsibility of the American Patent Office, and the lack of that power, although it may lead to some worthless patents, does not appear to be a very serious matter, especially in view of the renewal fees.

G. W. O. H.

We are indebted to two old friends, Cyril F. Elwell and Arthur H. Morse, for much information concerning American Patent Practice.

SHORT-CIRCUITED TURNS

By K. R. Sturley, Ph.D., M.I.E.E.

SUMMARY.—In broadcast transmitters as in other types, it is common practice to short-circuit turns of a coil in order to to change the wavelength. On the other hand short-circuited turns cannot be tolerated in the modulator, audio-frequency or power stages of a transmitter. The purpose of this article is to show that there is no fundamental theoretical difference between r.f. and a.f. short-circuits, but that the different practical results arise from differences in the values of the parameters, coupling coefficient, Q factor and inductance ratio of main coil to short-circuit section. The increase in power loss with short-circuit compared with that without short-circuit under normal operating conditions is shown to be 2 for the r.f. case and 12.5 for the a.f. or power case.

1. Introduction

In radio-frequency transmitting and receiving apparatus it is common practice to short-circuit turns of a coil in order to change its inductance or, if a tapped coil is employed, to short-circuit unused turns. On the other hand at power and audio frequencies short-circuited turns must be avoided, and many know from bitter experience the damage that can result from a short-circuited turn in a power transformer. The purpose of this article is to show that there is no fundamental difference between the action of a short-circuited turn in the r.f., coil and that of one in the power transformer. The practical difference arises from a difference in the value of certain coil and short-circuit parameters.

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2. Theory

The r.f. coil, or power transformer, with a short-circuited turn or turns can be represented by the circuit of Fig. 1, in which for convenience it is assumed that the short-circuit has zero resistance. L_1 and R_1 are the inductance and resistance of the active part of the coil, and L_2 and R_2 of the short-circuited section. In order to represent the power transformer an iron core needs to be shown, but since it has no material effect on the theoretical analysis it has

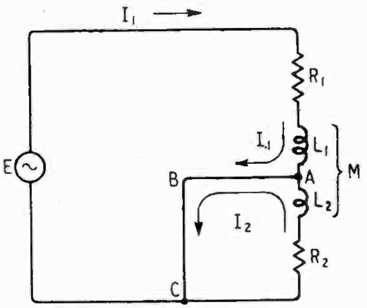


Fig. 1. *The r.f. coil or power transformer with short-circuited section.*

been omitted. If I_1 is the current taken from the source of driving voltage E, and I_2 the current circulating in the short-circuited section, the voltage and current relationships are as follows :

$$E = I_1 (R_1 + j\omega L_1) + I_2 j\omega M \quad .. \quad .. \quad (1)$$

$$0 = I_1 j\omega M + I_2(R_2 + j\omega L_2) \quad .. \quad .. \quad (2a)$$

From (2a) $$I_2 = \frac{-j\omega M \; I_1}{R_2 + j\omega L_2} \quad .. \quad .. \quad (2b)$$

and substituting this in (1)

$$E = I_1 \left[R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2} \right] \quad .. \quad (3)$$

Rationalizing (3) and collecting resistive and reactive terms

$$E = I_1 \left[\left(R_1 + \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} R_2 \right) + j\omega \left(L_1 - \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} \cdot L_2 \right) \right] \quad .. \quad (4a)$$

$$E = I_1[(R_1 + AR_2) + j\omega \; (L_1 - AL_2)] \quad (4b)$$

where $$A = \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} \quad .. \quad .. \quad .. \quad (5a)$$

This factor A is all important because it not only controls the value of the resistance and reactance reflected from the short-circuited turn into the main section, but its square root gives the ratio of the magnitudes of the circulating current and the current in the main section. This can be seen from (2b) because

$$\left|\frac{I_2}{I_1}\right| = \frac{\omega M}{\sqrt{(R_2^2 + \omega^2 L_2^2)}} = \sqrt{A} \quad .. \quad .. \quad (2c)$$

Expression (5a) can be converted into a more useful form as follows :

$$A = \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} = \frac{M^2}{L_2^2\left(\dfrac{R_2^2}{\omega^2 L_2^2} + 1\right)}$$

$$= \frac{M^2 \dfrac{L_1}{L_2}}{L_1 L_2 \left(\dfrac{1}{Q_2^2} + 1\right)} = \frac{Q_2^2 k^2 \dfrac{L_1}{L_2}}{1 + Q_2^2} \quad .. \quad (5b)$$

Where Q_2 is the Q-factor of the short-circuit section $\left(\text{i.e., } \dfrac{\omega L_2}{R_2}\right)$ and k is the coupling coefficient, $\dfrac{M}{\sqrt{(L_1 L_2)}}$, between the short-circuited section and the main coil.

In most practical cases Q_2 will not be less than 5 and, when very high accuracy is unnecessary, $1/Q_2^2$ may be neglected in comparison with unity, so that

$$A \approx k^2 L_1/L_2 \quad .. \quad .. \quad .. \quad .. \quad (5c)$$

This expression provides the clue to the different effect produced by a short-circuited turn at radio as compared with power and audio frequencies. In the former instance k is unlikely to exceed 0.6, and a probable value for L_1/L_2 is 5 rising to perhaps 40 with an accidental short circuit, whereas in the latter case k is unlikely to be less than 0.95, and L_1/L_2 may be of the order of 10^6. Such a high value of L_1/L_2 means a very large circulating current in the short-circuited turn with excessive localized heating and possible fusing of the wire.

To determine the ratio of power loss with and without the short-circuited turn it is convenient to make a separate examination of the two cases, because each allows certain simplifying assumptions to be made. For example, in the radio-frequency case the short-circuit will generally be intentional in order to reduce inductance, and the performance of the coil with short-circuited section should be compared with a coil of the same inductance obtained by using the correct number of turns. The performance of the power or audio-frequency transformer must, however, be compared with what it was before the short-circuit occurred.

3. R.F. Coil

Since the Q-factor is high, R may be neglected whenever it is associated with ωL, and $A = k^2 L_1/L_2$. The current in the main section is

$$I_1 = \frac{E}{(R_1 + AR_2) + j\omega(L_1 - AL_2)} \approx \frac{E}{j\omega(L_1 - AL_2)}$$

and this will equal the current which would flow in a coil of inductance $(L_1 - AL_2)$ obtained by winding the correct number of turns. Such a coil can be assumed to have a resistance of $\omega(L_1 - AL_2)/Q_1$, where $Q_1 = \omega L_1/R_1$, the effective Q-factor of the main coil. Hence the power loss ratio is

$$\frac{P_1}{P_0} = \frac{\text{Power loss with short-circuited section}}{\text{Power loss with correct coil}}$$

$$= \frac{I_1^2(R_1 + AR_2)}{I_1^2 \dfrac{\omega(L_1 - AL_2)}{Q_1}} = \frac{R_1 + AR_2}{L_1 - AL_2} \cdot \frac{L_1}{R_1}$$

$$= \frac{1 + \dfrac{AR_2}{R_1}}{1 - \dfrac{AL_2}{L_1}} \quad \ldots \quad \ldots \quad \ldots \quad (6a)$$

Replacing A in (6a) by $k^2 L_1/L_2$

$$\frac{P_1}{P_0} = \frac{1 + k^2 \dfrac{L_1}{R_1} \cdot \dfrac{R_2}{L_2}}{1 - k^2} = \frac{1 + k^2 \dfrac{Q_1}{Q_2}}{1 - k^2} \quad \ldots \quad (6b)$$

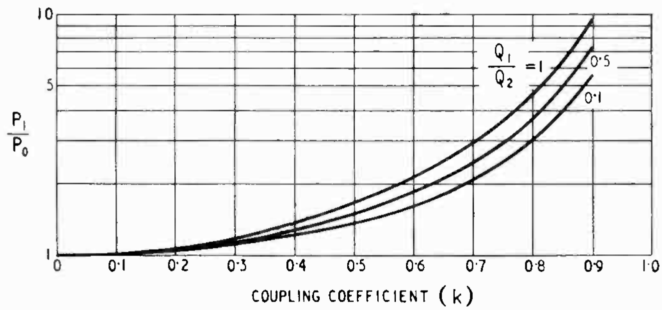


Fig. 2. *R.F. coil. Effect of variation of coupling coefficient, k, on power loss ratio: P_1/P_0 = power loss ratio with to without short-circuit; Q_1/Q_2 = Q factor ratio of main coil to short-circuited section.*

In transmitter coils the circuit will be loaded so that Q_1 will be less than Q_2, and this will reduce the power loss due to the short-circuited section. Fig. 2 shows the variation of power-loss ratio against k for selected values of Q_2/Q_1, where Q_1 is the variable and Q_2 is fixed and much greater than unity.

It is clear from the curves that with most r.f. circuits, for which k and Q_1/Q_2 normally never exceed 0.6 and 1 respectively the power-loss ratio due to the short circuited section is unlikely to be much greater than 2, and the convenience of securing a change of inductance outweighs this disadvantage.

A practical point arising from Fig. 1 is that the connection ABC common to the main coil and short-circuited section carries a current equal to the vector sum of I_1 and I_2, and care must be taken to see that it has a low resistance, otherwise considerable heating and loss may occur.

In receiver circuits it can be assumed that $Q_1 = Q_2$ so that

$$\frac{P_1}{P_0} = \frac{1 + k^2}{1 - k^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6c)$$

An important ratio in receivers is that of the Q-factors of the coil with short-circuited turns and with the correct number of turns. The effective Q with short-circuited turns is

$$Q' = \frac{\omega(L_1 - AL_2)}{R_1 + AR_2} = \frac{\omega L_1}{R_1} \cdot \frac{1 - A\dfrac{L_2}{L_1}}{1 + A\dfrac{R_2}{R_1}}$$

$$= Q_1 \cdot \frac{1 - k^2}{1 + k^2 Q_1/Q_2} \approx Q_1 \cdot \frac{1 - k^2}{1 + k^2} \quad \ldots \quad (7)$$

which gives a Q-factor ratio the inverse of the power-loss ratio.

A condition worth examination is that of an incomplete short-circuit while still maintaining the same value for the resultant inductance $(L_1 - AL_2)$. Assuming that

$$R_1 + AR_2 \ll \omega(L_1 - AL_2),$$

I_1 is unchanged by variation of R_2, and the power loss is therefore proportional to $R_1 + AR_2$.

Since $L_1 - AL_2 = L_1\,(1 - AL_2/L_1)$ = constant

$$\frac{AL_2}{L_1} = \frac{Q_2^2 k^2}{1 + Q_2^2} = \text{constant} = B$$

$$\therefore \text{ Power loss} \propto R_1\left(1 + A\frac{R_2}{R_1}\right)$$

$$= R_1\left(1 + B\frac{L_1}{R_1} \cdot \frac{R_2}{L_2}\right) = R_1\left(1 + B\frac{Q_1}{Q_2}\right)$$

From this it is clear that resistance in the short-circuit link, which decreases Q_2, increases the power loss for the same inductance value, and correct procedure is to have the lowest possible resistance in the short-circuiting link.

The above must be qualified if the coil can be tapped to give the required inductance; minimum power loss is realized when the unused turns are open-circuited. The danger associated with this condition is that stray capacitance across the unused turns may cause resonance at a frequency close to the tuning frequency of the main coil. The power loss will then be much greater than if the unused turns are short-circuited, and for this reason inactive turns of a r.f. coil are almost always short-circuited.

4. L.F. Transformer

For the power or audio-frequency transformer it is necessary to compare the power dissipated in the coil after the short-circuit has occurred with that dissipated before the short-circuit takes place, and the currents from the supply will now be unequal.

The current I_0 before the short-circuit occurs is

$$I_0 = \frac{E}{\sqrt{(R_1^2 + \omega^2 L_1^2)}} \quad \ldots \quad \ldots \quad (8)$$

and the current I_1 with a short-circuited turn is

$$I_1 = \frac{E}{\sqrt{[(R_1 + AR_2)^2 + \omega^2(L_1 - AL_2)^2]}} \quad (9)$$

Thus

$$\left[\frac{I_1}{I_0}\right]^2 = \frac{R_1^2 + (\omega L_1)^2}{(R_1 + AR_2)^2 + \omega^2(L_1 - AL_2)^2} \quad (10a)$$

Dividing top and bottom by R_1 and noting that $\omega L_1/R_1 = Q_1$

$$\left[\frac{I_1}{I_0}\right]^2 = \frac{1 + Q_1^2}{\left(1 + \frac{AR_2}{R_1}\right)^2 + Q_1^2\left(1 - \frac{AL_2}{L_1}\right)^2} \quad (10b)$$

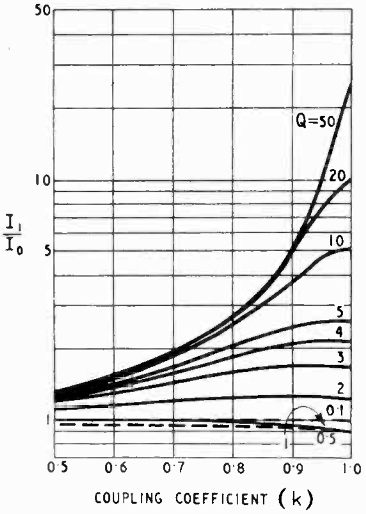
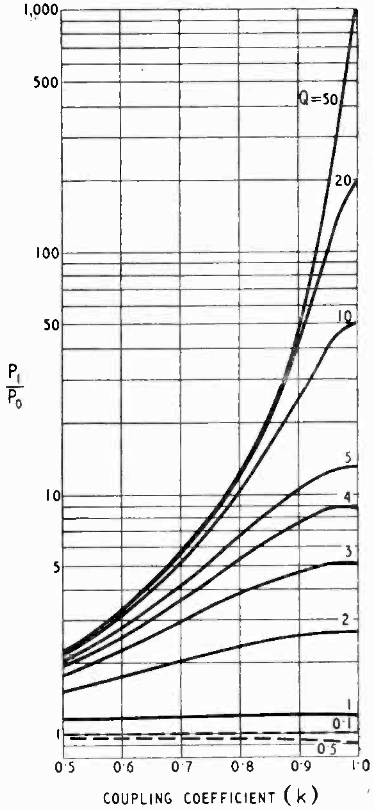


Fig. 3 (left). *Power transformer. Effect of variation of coupling coefficient, k, on the current ratio for different values of Q factor; I/I_0 = input current ratio with to without short-circuit; Q factors of main coil and short-circuit section assumed equal.*

Fig 4 (right). *Power transformer. Effect of variation of coupling coefficient, k, on the power loss ratio for different values of Q factor: P_1/P_0 = power loss ratio with to without short-circuit; Q factors of main coil and short-circuit section assumed equal.*



Since the short-circuited turn is in the main magnetic field, any load effect will act on it equally with the rest of the coil, and it may be assumed that

$$Q_1 = Q_2 = Q, \text{ and } L_2/L_1 = R_2/R_1$$

Hence

$$\left[\frac{I_1}{I_0}\right]^2 = \frac{1 + Q^2}{[1 + AL_2/L_1]^2 + Q^2[1 - AL_2/L_1]^2}$$

and by noting that $A = \dfrac{k^2 L_1/L_2}{1 + 1/Q^2}$

$$\left[\frac{I_1}{I_0}\right]^2 = \frac{1 + Q^2}{\left(1 + \frac{Q^2k^2}{1+Q^2}\right)^2 + Q^2\left(1 - \frac{Q^2k^2}{1 + Q^2}\right)^2} \quad \text{or} \quad \frac{I_1}{I_0} = \frac{1 + Q^2}{\sqrt{[(1 + Q^2 + Q^2k^2)^2 - 4Q^4k^2]}} \quad \ldots \quad (11)$$

The above expression is plotted in Fig. 3 against k for different values of Q. It increases rapidly with increase of Q when k is large but becomes less dependent on Q as k is decreased. An interesting feature is that expression (11) is not a maximum when k is unity if $Q > 1$, and the condition for maximum value, found by differentiating (11) with respect to k and equating to zero is

$$k_0 = \sqrt{(1 - 1/Q^2)}$$

Thus when $Q = 2$, $k_0 = 0.866$, and if the coupling coefficient is increased beyond this value I_1/I_0 decreases; an unexpected result which can, however, be demonstrated experimentally. For values of $Q \leqslant 1$, increase of k causes a reduction in I_1/I_0. These effects occur when the resistive component $(R_1 + AR_2)$ is much greater than the reactive component $[\omega(L_1 - AL_2)]$ in the denominator of expression (9). Increase of k, which increases the former and decreases the latter, influences expression (9) through $(R_1 + AR_2)$, and $\omega(L_1 - AL_2)$ has practically no effect. Hence increase of k decreases I_1, and hence

I_1/I_0. When R_1 and R_2 are comparable with ωL_1 and ωL_2 respectively, any increase in k gives $R_1 + AR_2$ more influence than $\omega(L_1 - AL_2)$ and leads to a decrease in I_1.

It is interesting to note that when Q is variable and k is fixed, the curves show a minimum value for I_1/I_0 at a value of $Q < 1$. The latter may be found by differentiating expression (11) with respect to Q and equating to zero. This gives

$$Q_0 = \frac{1}{\sqrt{(3 - k^2)}}$$

This is plotted in Fig. 4 against k for the same values of Q as are used for Fig. 3. For values of $Q > 1$, the curves show a similar phenomenon to that of Fig. 3; i.e., the power-loss ratio reaches a maximum before k reaches unity. The condition for maximum obtained by differentiating (12) with respect to k and equating to zero is

$$k = \frac{\sqrt{[2Q\sqrt{(1 + Q^2)} - (1 + Q^2)]}}{Q} \quad \ldots \quad (13)$$

For values of $Q < 0.577$, increase of k decreases

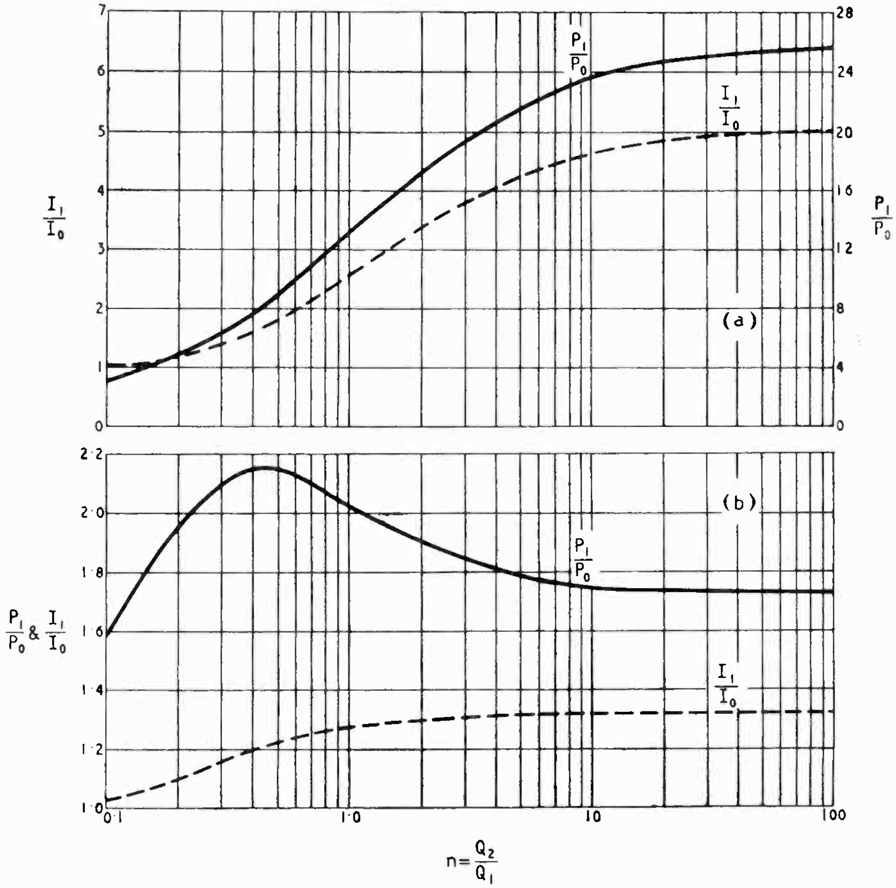


Fig. 5. *Power transformer. Effect of variation of the Q factor of the short-circuited turn: (a) coupling coefficient $k = 1$; and (b) $k = 0.5$; P_1/P_0 = power loss ratio with to without short-circuit; I_1/I_0 = current ratio with to without short circuit.*

The power loss ratio is obtained by multiplying the square of expression (11) by the factor.

$$B = \frac{R_1 + AR_2}{R_1} = 1 + \frac{Q^2k^2}{1 + Q^2} = \frac{1 + Q^2(1 + k^2)}{1 + Q^2}$$

$$\text{or } \frac{P_1}{P_0} = B\left(\frac{I_1}{I_0}\right)^2$$

$$= \frac{(1 + Q^2)\,[1 + Q^2(1 + k^2)]}{[1 + Q^2\,(1 + k^2)]^2 - 4Q^4k^2} \quad \ldots \quad (12)$$

the power-loss ratio, which is unity at $k = 0$ and less than unity for all other values of k.

Normal values for Q and k in an audio or power transformer are 5 and 0.95 respectively, and a typical value for the power loss ratio is 12.5, as compared with 2 for the r.f. coil.

To complete the analysis it is only necessary to consider the effect of an incomplete short-circuit, and this can be seen by starting from expression (10b), and replacing A by $\dfrac{Q_2^2k^2L_1/L_2}{(1+Q_2^2)}$.

Thus $$\left(\frac{I_1}{I_0}\right)^2 = \frac{1+Q_1^2}{\left[1+\dfrac{Q_1Q_2k^2}{1+Q_2^2}\right]^2 + Q_1^2\left[1-\dfrac{Q_2^2k^2}{1+Q_2^2}\right]^2}$$

$$= \frac{(1+Q_1^2)(1+Q_2^2)^2}{[1+Q_2^2+Q_1Q_2k^2]^2+Q_1^2[1+Q_2^2-Q_2^2k^2]^2} \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

or letting $Q_2 = nQ_1$

$$\frac{I_1}{I_0} = \sqrt{\frac{(1+Q_1^2)(1+n^2Q_1^2)^2}{(1+n^2Q_1^2+nQ_1^2k^2)^2+Q_1^2(1+n^2Q_1^2-n^2Q_1^2k^2)^2}} \quad \ldots \quad \ldots \quad (15)$$

$$\frac{P_1}{P_0} = \left(\frac{I_1}{I_0}\right)^2\left(1+A\frac{R_2}{R_1}\right)$$

$$= \left(\frac{I_1}{I_0}\right)^2\frac{1+Q_1^2(n^2+nk^2)}{1+n^2Q_1^2} \quad \ldots \quad (16a)$$

$$= \frac{(1+Q_1^2)(1+n^2Q_1^2)[1+Q_1^2(n^2+nk^2)]}{[1+Q_1^2(n^2+nk^2)]^2+Q_1^2[1+Q_1^2n^2(1-k^2)]^2} \quad \ldots \quad \ldots \quad \ldots \quad (16b)$$

Expressions (15) and (16b) are plotted against n for $k = 1$ and 0.5 in Fig. 5 (a) and (b). The curves for I_1/I_0 show that this function increases with increases of n (i.e., increase of Q_2), rising from unity when n is small and becoming asymptotic to a value

$$\sqrt{\left(\frac{1+Q_1^2}{1+Q_1^2(1-k^2)^2}\right)}$$

when n is large.

For $k = 1$, the curve shape for P_1/P_0 is similar to that for I_1/I_0, rising from unity when n is small and becoming asymptotic to

$$\frac{1+Q_1^2}{1+Q_1^2(1-k^2)^2}$$

when n is large.

For $k = 0.5$, P_1/P_0 has a maximum value at about $n = 0.5$, and this is due to the fact that the rate of increase of I_1^2/I_0^2 with increase of n is much less than the rate of increase of

$$1 + AR_2/R_1,$$

which has a maximum value when $n = 1/Q_1$; i.e., when $Q_2 = 1$. This merely states a well known fact that the resistance reflected into a coil from an LR circuit, coupled to it, is a maximum when the resistive and reactive components of the coupled circuit are equal.

Attempts to find the condition for the maximum value of P_1/P_0 by differentiating (16b) with respect to n lead to a very cumbersome expression not easy to solve. In most practical cases k will be nearly equal to unity, and then it is clear that maximum power loss occurs when the short-circuit is complete and Q_2 at its highest value. This is the reverse of what was found for the r.f. coil, but it must be noted that for the r.f. coil a given value of inductance is required, and any change in the short-circuit resistance must not be allowed to affect the factor A. If it does influence A, the effective inductance of the coil and I_1 are changed, and the result is similar to that obtained above for the power transformer.

5. Conclusion

The analysis has shown that there is no fundamental difference between the action of a short-circuited section at radio frequencies as compared with a short-circuit at power or audio frequencies, though the power-loss ratio is likely to be about 2 for the former as compared with 12.5 for the latter. The damaging effect of a short-circuited turn at power or audio frequencies is a result of the higher coupling coefficient between the short-circuited section and the main coil, and of the localized heating effect of the circulating current. The latter is due mainly to the transformer current step-up between the main coil of many turns and the short-circuit coil of few (or a single) turns. A low Q-factor, resulting from the lower operating frequency, also leads to a higher power dissipation. In the r.f. coil the high Q-factor of the short-circuited turns means that the induced voltage and circulating current are very nearly 90° out of phase and power dissipation is small.

TABLE

Ratio		Radio Frequency	Audio or Power Frequencies
Circulating current / Primary current	$= \dfrac{I_2}{I_1}$	$k\sqrt{\left(\dfrac{L_1}{L_2}\right)}$	$\sqrt{\left(\dfrac{L_1}{L_2}\right)}$
Primary current— with short-circuit / without short-circuit	$= \dfrac{I_1}{I_0}$	1	$\frac{1}{2}Q$
Power loss— with short-circuit / without short-circuit	$= \dfrac{P_1}{P_0}$	$\dfrac{1+k^2}{1-k^2}$	$\frac{1}{2}Q^2$

where $k = \dfrac{M}{\sqrt{L_1L_2}}$ and $Q = \dfrac{\omega L_1}{R_1} = \dfrac{\omega L_2}{R_2}$

The Table, which assumes that $Q_1^2 = Q_2^2 \gg 1$, and that k is very nearly unity at power and audio frequencies, summarizes the analysis by comparing approximate ratio values.

SMALL AERIALS IN DIELECTRIC MEDIA

By R. H. Barfield, D.Sc., M.I.E.E. and R. E. Burgess, B.Sc.

(Communication from National Physical Laboratory)

SUMMARY.—The paper contains an experimental and theoretical investigation of certain aspects of the effect of surrounding open aerials and loops with a mass of dielectric material, all dimensions being small compared with the wavelength. The original object of the investigation was to assess the possibility of improvement in the sensitivity of compact direction-finding systems by the use of dielectrics.

Four main experiments are described and a theoretical discussion of each is given :

(i) Inserting a small loop in a sphere of water has negligible effect on its pick-up.

(ii) Inserting a short dipole in a dielectric mass increases its capacitance and resistance but reduces the electric field acting on the dipole, resulting in an overall reduction of signal.

(iii) Comparison of the currents flowing in short rods of metal and of dielectric when placed parallel to the electric field of an incident wave.

(iv) The property of a dielectric introducing a field from an external signal into a screened box when inserted through a hole in it.

It is concluded that dielectric materials exhibit pick-up effects analogous to those of conductors ; a single formula for the current in an ellipsoidal receiving aerial is derived which applies generally to conducting or dielectric materials.

The sensitivity of an Adcock-type direction-finder can probably be increased by inserting it in a dielectric of low loss, though it is possible that such an increase could be more simply attained by enlarging the aerial system to utilize fully the space occupied by the dielectric.

1. Introduction

THE experiments described and discussed in this paper were started with the object of throwing light on a particular problem connected with the design of h.f. Adcock systems of small dimensions. This problem arose from the fact that the pick-up factor of such systems is very small compared with that of a loop aerial of the same dimensions. The reason for this is a fundamental one, namely, that a dipole aerial which is very short in comparison with the operating wavelength has a large negative reactance, and therefore requires a correspondingly large positive tuning reactance. Such a reactance must take the form of an inductance which, with its large self-capacitance, is itself close to resonance and introduces large loss into the circuit. The loop aerial on the other hand can be designed with sufficient external "lumped" capacitance to ensure its having a high magnification factor.

The following problem arose out of these considerations :

If a loaded dipole is totally immersed in a dielectric medium its aerial capacitance will be increased and will thus not need such a large loading inductance. To what extent can this arrangement be employed to improve the pick-up without reducing the e.m.f. induced in the aerial system or, in the case of an Adcock System, introducing unwanted pick-up which would destroy its directional properties.[2]

It will be recognized at once that the problem bears a close analogy to that of increasing the pick-up of a loop aerial by providing it with a core of high permeability magnetic material—a problem which has been discussed in a recent paper[1] by one of the authors.

Although as the work proceeded, the prospects became fainter of the immersed dipole system revealing itself as a practical solution of the pick-up problem, it nevertheless became apparent that the experimental results were of sufficient interest in themselves to justify the analysis and discussion which form the subject matter of the remainder of the paper.

MS accepted by the Editor, April 1947.

2. Screened-loop Aerial Immersed in Water

2.1 *Experiment*

A small screened loop, arranged as the receiving aerial of a well-screened sensitive receiver, was suspended at the centre of the interior of a large empty glass carboy. While signals were being received (frequency 3-8 Mc/s) the vessel was quickly filled to the top with tap water. There was no detectable change in the signal strength, as judged by ear. The d.f. null points were also apparently unaltered in sharpness. The dimensions of the experimental arrangements are shown in Fig. 1.

2.2 *Theoretical Discussion*

The decisive nature of this result justifies some theoretical discussion. If we view the loop in respect of its pick-up as a magnetic dipole, immersed in a medium of permeability not appreciably different from that of air, the result is as would be expected. Alteration of the resultant magnetic field in the space immediately surrounding the loop could only be brought about by a medium possessing a permeability definitely greater than that of air or by the presence of closed circulating currents of appreciable magnitude therein. Circulating displacement currents will undoubtedly exist, since the dielectric constant is much greater than that of air, but at low frequencies their effect on the loop pick-up will be of a second order only.

Nevertheless the experiment is not without value in providing a basic fact on which to build conjectures as to the effect of a dielectric medium on systems involving *open* aerials. The simplest of these is the dipole and less simple is the spaced dipole or Adcock system, the consideration of which when immersed in a dielectric gave rise to the whole series of experiments under review.

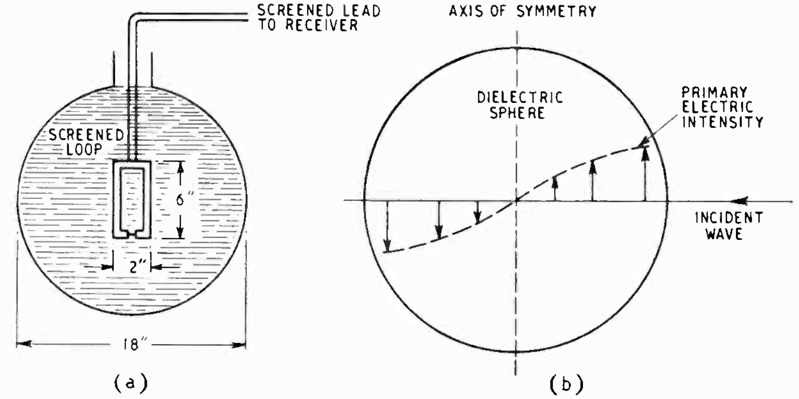


Fig. 1. (*a*) *Arrangement of loop immersed in water and* (*b*) *instantaneous primary field incident on dielectric sphere.*

In order that this aspect of the experiment may be made use of it is necessary to consider the pick-up of the loop as due to the effect of the electric fields acting on its sides.

Now the electric field at any instant in the interior of the medium near the centre will be the vector sum of the main primary field at that instant, and the secondary field produced by the surface charge distribution over the approximately spherical surface of the container. This charge produces a secondary field in opposition to the primary field E_0 such that the resultant field is given by

$$E = \frac{3E_0}{K + 2} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

and thus, in the case of a water sphere $E = E_0/27$ approximately. There is thus a very definite screening action, of a magnitude which would have been easily detectable in the experiments if the loop 'pick-up' were proportional to this resultant field.

Thus we are led to ask how it is that such a large alteration in the resultant electric field, acting on the vertical sides of the loop, does not affect the circulating e.m.f. in the loop.

The reason is not far to seek. It is best approached by considering the field distribution at the instant when the primary field, along the vertical axis of symmetry of the sphere is at its zero value [see Fig. 1(b)]. At this instant the primary field on either side of this line will not be zero but will be finite and opposite in sign on each side. Now screening is always caused by charge separation along the direction of screening and in this case owing to the fact that there is a reversal in sign of the primary field on crossing the axis of symmetry, charge separation along vertical lines would result in charge separation along horizontal lines. But this horizontal charge separation cannot exist in the dielectric as there is no horizontal force to maintain it. A circulating displacement current will, in fact, be set up by this process but this will have negligible secondary electric field associated with it. It is thus to be concluded that the vertical secondary field within the dielectric is *at all points* co-phasal with the primary field along the axis of symmetry. It follows, therefore at once that the circulating loop e.m.f. which has its maximum at the instant of zero primary field along the axis of symmetry will remain unaffected by the secondary field caused by the *vertical* displacement in the sphere.

It will, however, be affected by the circulation displacement current, but to an extent which is entirely negligible for spheres of dimensions small compared to the wavelength.

The physical picture presented gives a satisfactory explanation from the electric standpoint of the negative effect on the pick-up of immersing the loop in water. It is assumed, however, that the circulating displacement currents in the dielectric are too small to be of influence and this assumption is only valid for a dielectric mass very small in dimensions compared with the wavelength.

Thus, at higher frequencies the first-order effect of a dielectric sphere on the pick-up of a loop at its centre may be derived from the classical theory of diffraction of a plane wave by a sphere as presented for example by Stratton[3] It is found that the ratio of magnetic field at the centre of the sphere of radius a to that in the absence of the sphere is given by

$$\frac{H}{H_0} = 1 + \frac{K-1}{15}\left(\frac{2\pi a}{\lambda}\right)^2 \quad .. \quad .. \quad (2)$$

when $2\pi a < \lambda/\sqrt{K}$.

Thus for the experimental case in which $2\pi a\sqrt{K}/\lambda$ was at the most about 0.3 the fractional increase of field would be quite negligible.

At very high frequencies where the dimensions of the dielectric space are comparable with the wavelength, $\lambda/(\mu K)^{\frac{1}{2}}$, in it, there can arise special resonance effects giving enhanced pick-up. An example is discussed by Leigh Page[5] who considers the case of a long cylinder around whose centre a few turns of wire are wound. Even in the case of unity permeability there occurs a resonant increase of pick-up, the magnitude and frequency of which depend on the radius and the permittivity of the cylinder.

Another application of dielectrics at very high frequencies is in the 'polyrod' type of aerial[6]. This consists in effect of a dielectric waveguide from whose surface power is radiated by a process of leakage resulting in an "end-fire" characteristic.

These two types of phenomena are only manifest at relatively high frequencies and therefore lie outside the scope of the present paper but reference has been made to them in order to indicate their essential difference from the 'quasi-static' problems we are considering.

3. Dipole Aerial Immersed in Dielectric Medium

3.1 *Experiment*

A small dipole of 39 cm total length with flat discs 10 cm in diameter at each end, was loosely coupled, through a screened transformer, to the output circuit of a power oscillator. The current at the centre of the dipole was measured by means of a low-resistance non-contact thermojunction inserted at this point (see Fig. 2.)

A receiver was set up at a distance of 150 ft arranged so that the energy picked up from the dipole source could be measured by means of a valve voltmeter.

The experiment consisted in determining, both with and without a dielectric medium surrounding the transmitting aerial, the ratio of the e.m.f. induced in the dipole to the volts across the tuning capacitor in the receiving aerial circuit. A measurement of the current flowing in the dipole was made at the same time so that the resistance of the dipole aerial circuit could also be obtained.

These measurements were made, first with the dipole in air and then when it was located at the centre of a dielectric medium. The dielectric material chosen in this case was paper, chiefly because, when used in the form of books, a suitable surrounding medium could be conveniently built up round the dipole and it permitted changes in the dimensions thereof to be made quickly and easily.

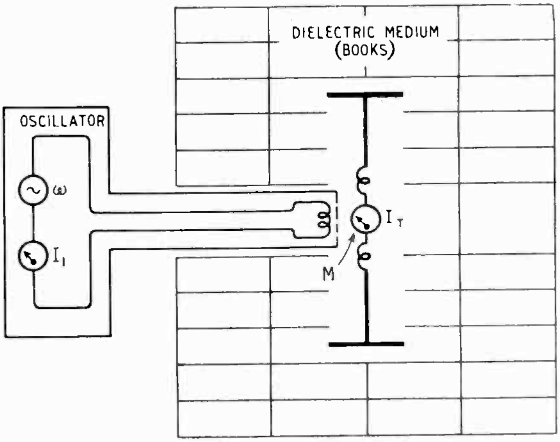


Fig. 2. *Dipole immersed in paper dielectric.*

Preliminary measurements were made of the dielectric constant and losses of the paper in book form and of the effect of different dimensions of the medium on the capacitance and resistance of the dipole circuit when immersed therein.

The permittivity was found to be between 2.5 and 3.0 as measured on a low-frequency bridge.

The results of the tests are shown in the Table where,

V_T = e.m.f. induced in the dipole circuit ($\omega M I_1$)

I_T = current flowing in the dipole circuit

V_R = volts across receiver capacitor

R_T = resistance of dipole $= \frac{V_T}{I_T}$

λ = operating wavelength (adjusted to give resonance of dipole aerial).

The interpretation of these results is as follows :—(i) the resistance of the dipole aerial circuit is increased to 1.8 times its air value by the presence of the dielectric and (ii) the ratio V_R/V_T is reduced to about 0.4 times its original value. Thus it is, clear that the change in resistance is not the *only* cause of the drop in received energy, part of which must therefore be due to other causes. The amount of this effect is shown by the last column V_R/I_T since this quantity allows for the change in resistance. It shows a net decrease of about 0.8 times. It must now be noted, however, that there is a difference in wavelength (about $7\frac{1}{2}\%$) in the two cases. This was unavoidable under the conditions of the test. It represents the change in resonant frequency of the dipole circuit brought about by the dielectric surround. Unfortunately it is not possible to make a reliable correction for this change. For, apart from the $1/\lambda$ term in the transmission formula (for which an accurate correction could be made) there is the pick-up factor of the receiving aerial, and this may be affected in more than one way by the wavelength.

In view of this uncertainty it will be best not to attempt to extract any more from the evidence than the conclusion that the drop in the overall transmission efficiency, represented by $\frac{V_R}{V_T}$, cannot all be ascribed to resistance increase and that the residual effect is almost certainly greater than could be accounted for by experimental error and therefore, in all probability, attributable to the dielectric constant of the paper.

3.2 *Theoretical discussion*

The rectangular dielectric block may be considered to approximate to a sphere of the same volume. Consider first of all that the process is reversed—the dipole acting as a receiving aerial under the influence of a sender at the place occupied by the receiver.

The resultant field at the centre of a sphere of paper for which $K = 3$ is, from equation (1) given by,

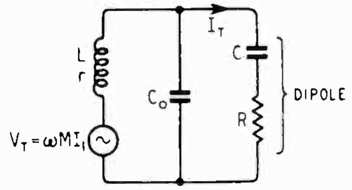
$$E = \frac{3}{5} E_0$$

Thus the e.m.f. induced in the dipole will be 3/5 of its value in the absence of the cube.

Applying the principle of reversibility it follows that, in the transmitting case described above, the ratio received volts/dipole current should decrease in the ratio of 0.6 when the dipole is surrounded by books, whereas the observed decrease has a probable maximum value of 0.8.

It is considered that this agreement is as close as could be expected from the nature of the experiment and of the theoretical assumption.

Fig. 3. *Equivalent circuit of transmitting dipole.*



The equivalent circuit of the system is shown in Fig. 3 where L, r represents the tuning coil and C_0 its self-capacitance, while C and R are the capacitance and resistance (radiation + ohmic + dielectric) of the dipole. Elementary analysis shows that the apparent resistance of the circuit when in tune is approximately.

$$R_T = \frac{V_T}{I_T} = r\left(1 + \frac{C_0}{C}\right) + \frac{R}{\left(1 + \frac{C_0}{C}\right)} \quad .. \quad (3)$$

The following rough values are consistent with the experimental system at a frequency of 9 Mc/s.

$L = 100\ \mu$H $\quad r = 40\ \Omega \quad \omega L/r = 140$

$C_0 = 1$ pF

Air dielectric $\begin{cases} C = 2 \text{ pF} \\ R = 0.1\ \Omega \text{ (radiation resistance)} \end{cases}$

Paper dielectric $\begin{cases} C = 2.7 \text{ pF} \\ R = 80\ \Omega \end{cases}$

The dipole loss factor ωCR at 9 Mc/s is about 10^{-5} for the air dielectric case and 10^{-2} for the paper dielectric. These figures are seen to be reasonable. It is particularly noted that the radiation resistance is only 0.1 Ω while the coil resistance is 40 Ω. The influence of the dielectric mass is to increase the dipole capacitance by about one-third but to increase its resistance to 80 Ω which

TABLE

Dielectric	λ (m)	V_T (V)	I_T (mA)	R_T (Ω)	V_R (V)	$\frac{V_R}{V_T}$ ($\times 10^{-2}$)	$\frac{V_R}{I_T}$ (Ω)
Air	32.5	3.81	60	63.5	0.15	3.92	2.50
	32.5	5.28	82	64.3	0.20	3.78	2.44
	32.5	3.80	61	62.2	0.15	4.05	2.46
Mean :				63.3		3.92	2.47
Paper	36.0	3.9	34	115	0.07	1.79	2.05
	36.0	5.52	47.5	116	0.09	1.63	1.89
	36.0	5.52	48	115	0.09	1.63	1.87
Mean :				115.7		1.68	1.94

lowers the overall efficiency. With a higher quality dielectric the total resistance would be almost entirely that of the coil and the full benefit of the capacitance increase would be produced.

For example if the dielectric has a loss factor of 10^{-4} and a permittivity of $K/3$, the dipole resistance is increased to only about $1\ \Omega$ due to dielectric loss and the ohmic resistance of the coil is then the only significant component. In this case the effects of the dielectric are to increase the aerial capacitance and to decrease the field acting at the aerial. The latter effect is approximately constant at $3/(K + 2)$ independently of the dimensions of the dielectric sphere; the increase of capacitance has an upper limit of K for a sphere of diameter large compared with the dipole length. Thus to obtain any benefit it is necessary to use a large dielectric sphere of small loss and it becomes questionable whether this is superior to a longer dipole of length equal to the diameter of the sphere.

4. Dielectric Cylinder Inserted Through Toroid

4.1 *Experiment*

An ebonite tube, 39 inches long, 2 inches diameter (internal) and $\frac{1}{8}$-inch wall thickness, was filled with distilled water. This was then inserted through a screened toroidal inductance connected by screened leads to the input of a well-screened and sensitive receiver. Before the dielectric cylinder was inserted no signals could be picked up by the receiver, but, on inserting the dielectric cylinder, loud signals were immediately obtainable (Fig. 4).

On withdrawing the water cylinder and inserting six short cylindrical porcelain insulators placed end to end (total length about 60 cm) no signals were obtainable. On repeating the test with six calit* bricks ($1\frac{1}{2}$ in $\times$ $1\frac{1}{2}$ in $\times$ 3 in) again no pick-up was detectable.

A brass tube of the same length and diameter as the ebonite tube, when inserted in the toroid, gave signals of a strength which indicated a pick-up of many times that of the water tube.

Measurements were then made of the relative "pick-up" of the water cylinder and of a brass tube.

A small-power portable transmitter having a rod aerial was set up at a distance of about 100 ft from the receiving system which has already been described. Signals were obtained on the water-tube aerial. This was then removed and brass tubes of varying length were inserted in the toroid until a length was found which gave the same output as the dielectric tube when this was substituted for it. In this way it was found that a brass tube of length 27 in and diameter 1 in was equivalent to the water tube 39-in long and diameter 2 in.

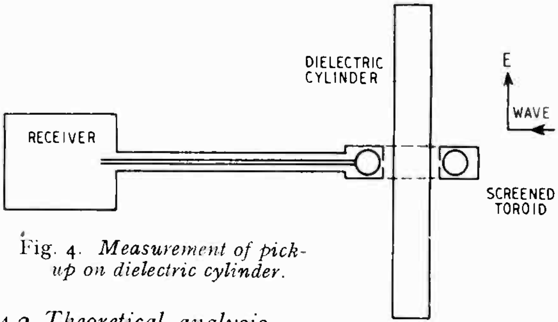


Fig. 4. *Measurement of pick-up on dielectric cylinder.*

4.2 *Theoretical analysis*

The effect of the dielectric cylinder in the toroid is assumed to be produced by the displacement current. A formula for this is obtainable based on the treatment of the cylinder as a prolate spheroid in which the field distribution is assumed to be identical with the static case.

If E_0 be the primary or inducing field in which the cylinder is placed, the resultant field E within the dielectric is:—

$$E = E_0 - 4\pi g P \qquad \qquad (4)$$

where P is the polarization or induced field and g is a constant depending on the geometrical configuration of the dielectric.

This depolarization constant g for a prolate spheroid with semi-axes a and b $(= c)$ is given by the formula

$$g = \frac{1 - e^2}{e^2}\left(\frac{1}{2e}\log\frac{1 + e}{1 - e} - 1\right) \qquad \qquad (5)$$

where e = eccentricity of ellipsoid $= \sqrt{1 + \frac{b^2}{a^2}}$.

For $e = 0$ (sphere) $g = \frac{1}{3}$.

Its values for a cylinder of the same ratio of height/cross section is not quite the same as for a spheroid and is not expressible by a simple formula but computations of this parameter have been made by Bozorth and Chapin[4] for the strictly analogous case of the demagnetizing factors of cylinders of magnetic material.

Continuing the analysis we have the well-known relationship:

$$4\pi D = KE$$

where D = electric displacement,
K = permittivity.

Also $D = \dfrac{E}{4\pi} + P$, hence $P = \dfrac{E\,(K - 1)}{4\pi}$

and $$E = \frac{E_0}{1 + g\,(K - 1)} \qquad \qquad (6)$$

* Calit is a ceramic insulating material of permittivity approximately 6.0 and low dielectric loss.

Now the equivalent current at the central cross-section A round which the toroid is placed is given by

$$I = A\,\frac{dP}{dt} = j\omega P \pi bc$$

$$= \frac{j\omega bc}{4} \cdot \frac{E_0}{g + 1/(K-1)} \quad \ldots \quad \ldots \quad (7)$$

where a is the axis parallel to E_0.

In the case of a prolate spheroid ($b = c < a$)

$$I = \frac{j\omega b^2}{4} \cdot \frac{E_0}{g + 1/(K-1)} \quad \ldots \quad \ldots \quad (8)$$

As the length of the major axis is increased the current at the centre tends to the limiting value appropriate to no depolarizing effect.

$$I = j\omega \,.\, \pi b^2 \cdot \frac{E_0\,(K-1)}{4\pi} \quad \ldots \quad \ldots \quad (9)$$

It is of interest to see what value for the current this formula leads to in the case of a perfectly conducting thin aerial in which K is put equal to infinity and the eccentricity e is nearly unity.

Then

$$I = \frac{j\omega b^2}{4} . \frac{E_0}{g}$$

$$\approx \frac{j\omega b^2}{4} \cdot \frac{E_0}{\dfrac{b^2}{a^2}\left(\log\dfrac{2a}{b} - 1\right)} \quad \text{[from equ. (4) for } g\text{]}$$

$$= j\omega \,.\, aE_0 \cdot \frac{a}{4\left(\log\dfrac{2a}{b} - 1\right)} \quad \ldots \quad \ldots \quad (10)$$

in which aE_0 is recognized as the effective height of a short aerial of length $2a$ and the last term is the capacitance at the centre of the aerial. It is therefore concluded that equation (7) is valid for conducting as well as for dielectric aerials. We can interpret K in the generalized sense of a complex quantity.

$$K = K_0 - \frac{2j\sigma}{f}$$

where σ = conductivity (e.s.u.)

The ratio of the pick-ups of two aerials of different materials and dimensions is thus

$$\frac{P_1}{P_2} = \left(\frac{b_1}{b_2}\right)^2 \frac{g_2 + 1/(K_2 - 1)}{g_1 + 1\,(K_1 - 1)} \quad \ldots \quad \ldots \quad (11)$$

This theory may be applied to the cases dealt with experimentally in which the length of water tube $2a_1 = 39$ in and the diameter $2b_1 = 2$ in giving $a_1/b_1 = 19.5$ and $g_1 = 0.004$. The distilled water had a conductivity of 5×10^7 e.s.u., and thus at 7 Mc/s $2j\sigma/f$ is $j14$ which may be neglected by comparison with the permittivity of 80, so that $K_1 = 80$.

The length of brass tube $2a_2 = 28$ in and diameter $2b_2 = 1$ in gave the same pick-up as the water tube. According to the theory, for equal pick-up on aerial-2 with $K_2 = \infty$ (brass)

$$g_2 = \left(\frac{b_2}{b_1}\right)^2 \left(g_1 + \frac{1}{K_1 - 1}\right) = 0.0041$$

corresponding to $a_2/b_2 = 24$. Thus the theory predicts for the length of the brass tube $2a_2 = 24$ in to be compared with the experimental value of 27 in.

The above formula also explains why the six calit bricks ($g \approx 0.004$, $K = 6$) gave an undetectable effect; viz., the relatively high value of $[g + 1/(K-1)]$ due to the low permittivity of the material, which results in a pick-up only about 1/16 of that of the water and brass tubes.

It may be concluded that within the errors of experiment and the approximation of the theory, the pick-up of the dielectric aerial has been satisfactorily interpreted and analysed.

5. Dielectric Rod and Screened Electrode

A sensitive receiver with tuned input circuit, was placed in a well-screened box. A circular brass plate ($2\frac{1}{2}$-in diameter) was fixed in a horizontal position about two inches below a circular aperture of the same diameter in the top of the box. This plate was connected to one terminal of the tuned input circuit of the receiver, the other terminal being connected to the screen (Fig. 5). The sensitivity and screening of the apparatus was such that a short length of wire touching the plate and projecting only a few inches through the aperture, produced strong signals from distant transmitters, whereas on withdrawing the wire the signals dropped below noise level.

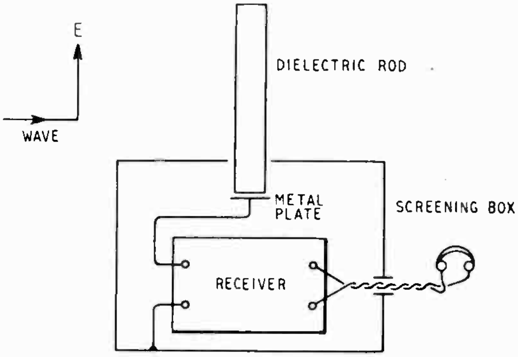


Fig. 5. *Pick-up due to dielectric rod.*

On resting one 'calit' brick (2 in $\times$ 2 in $\times$ 6 in) endways on the disc, so that it projected 4 in above the top of the box a definite pick-up was observed. An increase in pick-up was noticed when a second brick was placed on top of the first and a further increase on placing a third on top of the second, but the increase per brick grew less and less as still more were added up to six in number. On placing a small metal box on top of this pile a still greater pick-up was observed and signals from distant sources could be readily received.

Glass bottles containing oil and alcohol gave a large pick-up. Small ebonite tubes and porcelain insulators also gave easily detectable effects.

On suspending the water-filled ebonite tube, so that its base rested on the disc electrodes, a very large pick-up was observed ; much greater than that obtained with any of the preceding insulators.

No quantitative measurements were made in these experiments ; but they demonstrate in a striking way that an insulator may be made to behave as a receiving aerial. It is perhaps worth noting, as a corollary, that in cases where very efficient screening of a receiver or signal generator is required, a leakage may occur if insulating control handles are brought out through the screen. This may be considerably greater than would be caused by the hole alone through which the controls are passed. In such cases, therefore, both the dimensions of the controls and their dielectric constant should be kept as low as possible.

6. General Conclusion

Three main conclusions can be drawn from these tests :—

(a) The experiments give a clear demonstration that insulators can "pick-up" electromagnetic radiation and transfer the energy to a receiver, thus behaving like open aerials. Conversely it is obvious that the reverse process will be possible, that is, that they can be made to radiate energy. In Experiment No. 3 (with toroid) the displacement current flowing in the dielectric produces a magnetic field which links with the windings of the toroid and is strictly analgous to the effect of the conduction current flowing in an open aerial which passes through a toroid. In Experiment No. 4 the 'pick-up' is produced by the charge accumulating (due to the flow of displacement current) at or near the end of the dielectric rod. This charge acts on the receiver input electrode exactly as if it were on the top plate of a capacitor of which the electrode formed the bottom plate.

In the former case the quantitative relationship established between the effect of the dielectric and that of a conductor of similar shape indicates that though the effect of the dielectric at first increases rapidly with its length the rate of increase quickly diminishes so that as soon as the length becomes large compared with the cross-section, the effect tends to a constant value and becomes practically independent of the length. The same considerations apply to the 'pick-up' of the dielectric when, as in Experiment 4, it acts on the input electrode. This is because, in both cases, the pick-up depends on the total charge displacement across a given cross-section of the dielectric and not on the moment of this charge displacement which increases with the length. Thus it is not possible to increase the efficiency of pick-up of a dielectric rod indefinitely merely by increasing its length.

(b) The second conclusion relates to the question whether any improvement of pick-up efficiency can be gained from immersing an open aerial (dipole) in a dielectric medium. In the analogous case of a loop aerial it has been shown that a definite advantage is gained by inserting a core of high permeability through the coil[1].

The resultant electric force within a dielectric medium is less than the primary field which would exist at the same point of space in the absence of the dielectric medium. Hence, under no circumstances can there be any gain in the e.m.f. induced in the aerial. At the same time, for the case of a long thin dielectric surrounding the conductor, the diminution becomes very small and the inner and outer fields are almost identical.

There is, however, another way in which the presence of the dielectric might increase the pick-up efficiency of dipoles or conductors short compared with the wavelength. It is well known that a lossy circuit results when a short aerial is loaded with the inductance necessary to obtain resonance and for this reason the pick-up efficiency of a short open aerial is much lower in relation to its effective height than that of a long aerial. Now it is clear that there is an increase in the capacitance of the short dipole due to a medium of dielectric constant greater than that of air which will reduce the total loading inductance necessary to tune the aerial and thus tend to decrease the circuit resistance.

Unfortunately this effect is not likely to produce an appreciable gain in practice, for, unless the dielectric medium occupies an appreciable fraction of the space surrounding the aerial, its effect on the capacitance will be small. There will, however, also be a reduction in the interior electric force acting on the dipole due to the screening action of the surface charges in the medium as already mentioned.

Hence gain in circuit efficiency will be offset by loss in aerial e.m.f. Of course, if the shape of the medium is still kept long and thin by extending its length in proportion to its girth an increase of circuit efficiency will be duly gained, provided the dielectric losses can be neglected.

It appears, however, extremely improbable that any circumstances will arise which would make it worth while to set up the arrangement since the space occupied by the dielectric medium might just as well be utilized by increasing the length of the dipole aerial itself.

(c) For the third and last conclusion we revert to the case which formed the starting point of the whole investigation. Namely that of a spaced dipole or H-Adcock aerial system immersed in a dielectric medium.

Though no direct test of the effect of this has been possible, the information gained from the study and detailed explanation of the four sets of experimental results has made it possible to predict with reasonable certainty what the behaviour would be in such a case. Thus, in the first place, it appears quite certain that the dielectric mass surrounding the aerial will have a screening effect reducing the field acting on the aerial by an amount which will depend on the dielectric constant and on the geometrical shape of the medium.

In the case or a sphere of water of radius large compared with that of the aerial system the screening effect will be large—the field within will be reduced to about 1/27th of that outside. But the effect would become less and less as the length of the dielectric medium in the direction of the aerial was increased in proportion to its cross-section.

In spite of this pronounced screening effect the differential value of the field acting on the two aerials in opposite sense (and having its maximum at the instant of zero field along the axis of symmetry) will remain sensibly unchanged so that the effective or circulating e.m.f. in the aerial system will be the same as before.

The secondary field of force relating to each dipole will be partially in a dielectric of permittivity greater than that of air and will cause the capacitance of the aerial circuit to be increased so that the efficiency will be increased. In the case of aerials very short compared with the operating wavelength and a dielectric constant of the order of 100 it can be shown that the increase may be of the order of ten times.

There is, however, a further consideration to be taken into account which may affect this possible gain. This is the increased damping which will result from the dielectric loss of the medium ; in other words the efficiency, though increased by the higher ratio of aerial capacitance to total tuning capacitance will be diminished by the extra resistance produced by the conductivity of the dielectric.

Finally, it is also necessary to consider whether the presence of the dielectric medium will introduce any unwanted pick-up in the aerial system such as would re-introduce polarization errors and thus destroy or mar its directional properties.

The detailed analysis of the case of the loop aerial immersed in water seems to answer the question decisively in the negative (Experiment No. 1). The analysis brought out the fact that no charge accumulation took place having a component at right angles to the direction of the primary field.

It thus follows that horizontally polarized waves can only give rise within the medium to horizontal electric fields which, being at right angles to the aerial, cannot affect them. It may therefore be concluded that the presence of the dielectric will not affect the directional properties of the aerial system.

We may therefore conclude that a possibility exists of improving the pick-up efficiency of a spaced aerial pair (or of a four aerial system) by surrounding it with a dielectric of high permittivity. The conditions for this to be achieved are

(i) That the permittivity shall be great enough to increase appreciably the capacitance of the aerial circuit.

(ii) That the loss in the dielectric shall be small enough to prevent an undue increase in the aerial-circuit resistance.

It is again necessary to point out that such a gain can only be produced by means of a dielectric medium of dimensions considerably exceeding those of the aerial system which they surround. If such extra space is available therefore it might, in the majority of cases, be just as easy to obtain the required gain in pick-up by increasing the size of the aerial system by the same amount.

7. Acknowledgments

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REFERENCES

[1] R. E. Burgess, " Iron-Cored Loop Receiving Aerial," *Wireless Engineer*, 1946, Vol. 23, pp. 172–178.

[2] R. H. Barfield, " Some Experiments on the Screening of Radio Receiving Aerials," *J. Instn elect. Engrs*, 1924, Vol. 62, pp. 249–262.

[3] J. A. Stratton, " Electromagnetic Theory," (McGraw Hill, N.Y., 1941), pp. 563–573.

[4] R. M. Bozorth and D. M. Chapin, " Demagnetisation Factors of Rods," *J. appl. Phys*, 1942, Vol. 13, pp. 320–326.

[5] Leith Page, " Magnetic Antenna," *Phys. Rev.*, 1946, Vol. 69, pp. 645–648.

[6] J. A. Ratcliffe, " Aerials for Radar Equipment," *J. Instn elect. Engrs*, 1046, Part IIIA, No. 1, (Proc. of Radiolocation Convention) pp. 22–32.

PROPERTIES OF LOOP AERIALS

Non-uniform Current Distribution

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(Communication from the National Physical Laboratory)

SUMMARY.—A simple loop, excited at a high frequency, carries a current which is non-uniform in amplitude, unless the wavelength of excitation is very great compared with the loop perimeter. This non-uniform current distribution results in a correspondingly non-uniform field round any circle coaxial with the loop. The field of a loop with a perimeter not greater than a quarter-wavelength can be represented as that due to a uniform-current loop (a true magnetic dipole) together with a component due to an electric dipole of suitable location and excitation. A loop formed by combining two non-uniform coplanar loops and exciting them in parallel so that the magnetic dipoles are additive is similarly equivalent to a uniform-current loop together with two dipoles suitably spaced and excited.

The appropriate locations and excitations of the effective dipoles are discussed for circular and rectangular loops of the above simple and compound types.

It is also shown that the radiation resistance of a loop carrying a non-uniform current is materially different from the calculated value which would be obtained by assuming a uniform current distribution.

1. Introduction

THE field due to an alternating current of uniform amplitude flowing in a circular loop is necessarily uniform round any circle coaxial with the loop. This circular symmetry of the field applies very closely to a loop of any shape provided that the current is uniform and that the perimeter is very small compared with the wavelength. In most loops of practical dimensions, however, the amplitude of the current is not uniform and the field will show a corresponding degree of non-uniformity round a coaxial circle. This lack of uniformity may have practical significance as shown by Alford and Kandoian[1], and Ross[2]. For example, a horizontal loop intended to serve as an omni-directional aerial may radiate signals of very different intensities towards different azimuths. Also some types of spaced-loop direction finder depend for their operation on the behaviour of the loops as pure magnetic dipoles.

It was stated by Ross that for a small degree of non-uniformity in the loops the distant field can be represented as that due to a uniform current loop (a true magnetic dipole) together with a component due to an electric dipole of suitable location relative to the loop and of suitable excitation. A combination of two or more non-uniform loops, as suggested by Alford and Kandoian, is similarly equivalent to a uniform current loop plus two or more dipoles, suitably located and excited.

The locations of the effective dipoles and the ratios of their moments to those of the loops are the same for transmission and reception provided that the loops are in each case used with balanced circuits.

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The main purpose of these notes is to set out briefly the equivalent loop and dipole combinations for the more common types of loop. The results are used to derive the radiation resistances of the loops.

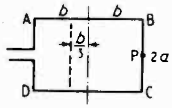
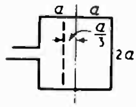
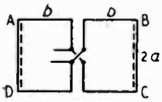
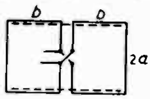
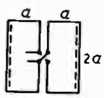
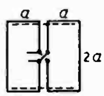
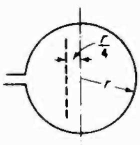
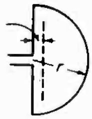
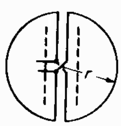
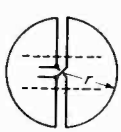
2. General Considerations

Two general types of loop will be discussed, the 'simple' loop consisting of a single conductor and the 'twin' loop built up from two coplanar simple loops. These types are illustrated in Table I. Loops may be screened or unscreened, but as regards the distribution of current in the radiating conductors there is no fundamental difference between the two types and the discussion is therefore limited to unscreened loops. The loops are considered small, though not infinitesimally so, compared with the wavelength, and the current is assumed to have a cosine-law distribution around the conducting members and to have the same phase at all points. In a balanced loop the current maximum occurs at the point furthest from the feed-point. As a rough guide it may be assumed that the discussion applies to a simple loop with a perimeter not greater than a quarter-wavelength and to a twin loop with a perimeter not greater than a half-wavelength.

It has been stated that a non-uniform loop can be represented as a uniform loop and a dipole or combination of dipoles because of the nature of the differences between the actual field distribution and that having ideal circular symmetry. These differences contain terms of order k, k^2, k^3 and so on, in diminishing magnitude compared with the ideal field, where k represents βa or βb (rectangular loop of sides $2a$, $2b$) or βr (circular loop of radius r), and β is 2π/wavelength. In a

TABLE I

Equivalent Circuits of Non-Uniform Loops

Type of Loop	Equivalent Aerial	Loop Moment	Dipole or Spaced-Dipole Moment
Rectangular Simple ($2a \times 2b$)		$4abi_0\left\{1-\frac{\beta^2}{6}(3a^2+6ab+4b^2)\right\}$	$2\beta a i_0(a+b)(a+2b)$
Square Simple (Side $2a$)		$4a^2i_0\left(1-\frac{13}{6}\beta^2a^2\right)$	$12\beta a^3 i_0$
Rectangular Twin ($2a \times 2b$)		$4abi_0\left\{1-\frac{\beta^2}{6}(3a^2+3ab+b^2)\right\}$	$\frac{2}{3}\beta^2abi_0(a+b)(2a+b)$
		$4abi_0\left\{1-\frac{1}{6}\beta^2a^2\right\}$	$-\frac{2}{3}\beta^2abi_0(a+b)(2a+b)$
Square Twin (Side $2a$)		$4a^2i_0\left\{1-\frac{7}{6}\beta^2a^2\right\}$	$4\beta^2a^4i_0$
		$4a^2i_0\left\{1-\frac{1}{6}\beta^2a^2\right\}$	$-4\beta^2a^4i_0$
Circular Simple (Radius r)		$\pi r^2 i_0\left\{1-\frac{\beta^2r^2\pi^2}{6}+\frac{\beta^2r^2}{4}\right\}$ $\left[\pi r^2 i_0\left\{1-1.9\beta^2r^2\right\}\text{approx.}\right]$	$2\beta\pi r^3 i_0$
Semicircular Simple (Radius r)	$\frac{3\pi r}{2(14+3\pi)}$ or nearly $\frac{r}{5}$ 	$\frac{\pi r^2 i_0}{2}\left[1-\frac{\beta^2r^2}{4}\left(\frac{\pi^2}{6}+1\right)\right]$	$\frac{\beta r^3 i_0}{6}(14+3\pi)$
Circular Twin (Radius r)		$\pi r^2 i_0\left\{1-\frac{\beta^2r^2}{4}\left(\frac{\pi^2}{6}+1\right)\right\}$	$\frac{\pi}{2}\beta^2r^4i_0$
		$\pi r^2 i_0\left\{1-\frac{\beta^2r^2}{4}\left(\frac{\pi^2}{6}-1\right)\right\}$	$-\frac{\pi}{2}\beta^2r^4i_0$

simple loop terms of orders k and k^2 only are considered and all smaller terms neglected. In a twin loop the largest difference term is of order k^2 and all smaller terms are assumed to be negligible.

The treatment of a rectangular loop (or any form of loop with straight sides) may be greatly simplified by assuming the current in each side to be uniform and equal to the mean value of the actual current in that side. It can be shown that in the types of loop under consideration the field derived from such a current distribution is identical with the actual field when terms smaller than those of order k^2 compared with the ideal field are neglected. Circular loops can be analysed more conveniently by determining the field due to the actual current distribution, with suitable approximations, using an integration process.

3. Nomenclature

The m.k.s. system of units will be adopted and the loop and dipole moments defined in the following terms. All currents, and therefore all moments, here and throughout the paper, are peak values with respect to time.

Loop moment $= M_L = Ai_m$

where A = area of loop

i_m = an effective uniform current which will be defined for each loop in turn.

The electric field at a large distance d in the plane of the loop has then the magnitude $30\beta^2 M_L/d$ volts per metre, if A is small.

Dipole moment $= M_D = il\beta$ where i is the mean current in the dipole and l is its length. The electric field at a point in the equatorial plane, at a great distance d is then $30\beta^2 M_D/d$ volts per metre.

For the treatment of twin loops it is necessary also to define the moment of a pair of spaced dipoles whose individual moments are equal but in opposite senses.

Spaced dipole moment $= M_{SD} = ils$ where i is the mean current in each dipole of length l and s is the spacing between the dipoles. The field at a point in both the equatorial plane of the dipole and in the plane of the dipole pairs is then $30\beta^2 M_{SD}/d$.

It can be seen that the field formulae are all of the same form, which simplifies the calculation of fields when the separate moments have been ascertained. The above formula for the field from a loop with uniform current is, however, valid only for a loop of small area and it will be necessary to consider the departures of the field from this simple form. Moreover the field due to a uniform current in a rectangular loop of practical dimensions is not in general circularly symmetrical. This must be taken into account in the analysis of a rectangular twin loop, for the departures from circular symmetry of the field from an assumed uniform current are of the same order as the departures due to the non-uniformity of the current. At a point in the plane of the loop the field due to a uniform current is

$$\frac{30\beta^2 M_L}{d}\left[1 - \frac{\beta^2}{6}(a^2\sin^2\psi + b^2\cos^2\psi)\right]$$

where $2a$ and $2b$ are the sides of the loop and ψ is the direction of transmission measured from the direction of the sides of length $2b$. In the special case of a square loop ($a = b$) the field has circular symmetry to the order considered.

A circular loop carrying a uniform current produces an electric field at a point in the plane of the loop, equal to

$$\frac{30\beta^2 M_L}{d}\left[1 - \frac{\beta^2 r^2}{8}\right]$$

The electric field at a great distance from a small loop (a pure magnetic dipole) is directed along circles coaxial with the loop. The electric field from an electric dipole lies in planes containing the dipole and is directed along circles centred on the dipole.

4. Equivalent Forms of Loops

(a) *Rectangular simple loop*

Consider the loop shown in the first figure of Table I. The maximum current i_0 occurs at the point P and the mean (peak) currents in the various sides are

$$\text{BC}: \; i_1 = i_0\left\{1 - \frac{\beta^2 a^2}{6}\right\}$$

$$\text{DA}: \; i_2 = i_0\left\{1 - \frac{\beta^2}{6}(7a^2 + 18ab + 12b^2)\right\}$$

$$\text{AB and CD}: \; i_3 = i_0\left\{1 - \frac{\beta^2}{6}(3a^2 + 6ab + 4b^2)\right\}$$

The loop may therefore be considered to carry a uniform current i_3 plus a current in BC equal to $(1/3)\beta^2(a + b)(a + 2b)i_0$ flowing in the same direction round the loop as the main current, and a current in AD equal to $(2/3)\beta^2(a + b)(a + 2b)i_0$ flowing in the opposite direction to the main current and therefore in the same direction ***in space*** as the excess current in BC. The non-uniform loop is therefore equivalent to a loop ABCD carrying a uniform current i_3 (which is therefore the effective mean current i_m of Section 3), plus a pair of spaced dipoles coincident with AD and BC, the dipole AD having a moment twice that of the dipole BC, and of the same sense. To the accuracy considered here (i.e., excluding terms smaller than order k^2 compared with the main field) these two dipoles may be replaced by a single dipole of moment $2\beta a(a + b)(a + 2b)i_0$ parallel to AD and distant $2b/3$ from it as shown. To summarize, the equivalent representation is defined by the following parameters.

$$M_L = Ai_3 = 4abi_0\left\{1 - \frac{\beta^2}{6}(3a^2 + 6ab + 4b^2)\right\}$$

$$M_D = 2\beta ai_0(a + b)(a + 2b)$$

Position of dipole :—parallel to AD and distance $2b/3$ from it.

It may be noted that the second-order term in the loop moment may usually be neglected by comparison with the dipole moment in computing fields but should be taken into account in a determination of the power radiated by the loop.

The analysis shows that in order to simulate correctly the field from the non-uniform-current loop by means of a uniform-current loop and a dipole, the *currents* in the dipole and in the loop side opposite the terminals must have the same phase when measured in the same sense. When the loop is used as receiving aerial the correct output e.m.f. is obtained by connecting the e.m.f. due to the dipole in series with that due to the loop. The rule for obtaining the correct phasing is that the *e.m.fs.* due to the dipole and due to the loop side opposite the terminals, measured in the same sense round the circuit, must have the same phase, neglecting small differences due to the spatial displacement between these members in the external field.

(b) *Rectangular twin loop*

The twin loop shown in the third figure of Table I can be analyzed from first principles as was the simple loop, or it can be regarded as two simple loops connected together. If the latter course is adopted it would seem natural to regard the twin loop as equivalent to a loop ABCD carrying a uniform current equal to the mean current in AB or CD plus two dipoles coincident with BC and DA. The other dipoles of the simple loops, being adjacent, have no resultant external field. This method of approach gives as the equivalent aerial a loop ABCD carrying a uniform current

$i_0\left\{1 - \frac{\beta^2}{6}(3a^2 + 3ab + b^2)\right\}$ plus a pair of spaced dipoles BC and DA of individual moments $(1/3)\beta a(a + b)(2a + b)i_0$ but of opposite senses in space, the currents being in the same direction as the main loop current. The combined spaced dipole moment is $(2/3)\beta^2 ab(a + b)(2a + b)i_0$.

This representation is certainly a valid one, but is not unique. For instance the assumption that the current is uniform in each side of the loop is sufficiently accurate only to determine the overall moment of the dipole pair; the individual dipole moments and the spacing of the dipoles cannot be derived separately without a more rigorous analysis. Such an analysis would be hardly worth the effort involved in view of the fact that the precise resolution of the spaced dipole moment affects only very small terms in any expressions for the field components.

Another equally valid representation is a loop ABCD carrying a uniform current equal to the mean current in AD or BC $[i_0(1 - \beta^2a^2/6)]$ plus a pair of spaced dipoles coincident with AB and DC and carrying currents opposed to the main loop current. The moment of the dipole pair has the same magnitude as before.

It should be borne in mind that the postulated rectangular loop carrying a uniform current will not produce a circularly symmetrical field if the loop is not square, but a third equivalent circuit can be assumed with a true loop moment of circular symmetry. The spaced dipole moment must then be modified to include the previous departures from symmetry of the equivalent uniform current loop, which have the same form of polar diagram as the spaced dipole moment. This representation is, however, somewhat artificial.

(c) *Circular simple loop*

By consideration of the polar diagram of the circular simple loop of radius r it can be shown that the equivalent aerial is a loop carrying a uniform current plus a dipole as illustrated in Table I.

(d) *Circular twin loop*

The case of a circular twin loop can also be analyzed by consideration of the polar diagram. As with the rectangular twin loop two representations are equally valid, with the dipoles parallel to either of two orthogonal lines. If the dipoles are parallel to the straight sides of the half-loops the moment of the equivalent uniform-current loop is $\pi r^2\left[1 - \frac{\beta^2r^2}{4}\left(\frac{\pi^2}{6} + 1\right)\right] i_0$ and the field from the spaced dipoles augments the field from the loop at all distant points in space. With the dipoles in the orthogonal position the equivalent uniform loop moment is $\pi r^2\left[1 - \frac{\beta^2r^2}{4}\left(\frac{\pi^2}{6} - 1\right)\right]$ and the field from the spaced dipoles opposes the field from the loop at all points. In either case the magnitude of the spaced-dipole moment is $\pi\beta^2 r^4 i_0/2$.

It is of interest to note that with either half of the twin loop taken separately the equivalent dipole is parallel to the straight side of the loop and spaced $\dfrac{3\pi r}{2(14 + 3\pi)}$ or very nearly $r/5$ from it.

5. Impedance of the Loops

The reactances of simple loops, screened and unscreened, balanced and unbalanced, have been discussed by previous workers[3, 4]. The results can be extended without difficulty to the various types of twin loop and no further discussion of reactance will be given here.

Formulae for the resistive components of loop impedances are given in various text books, but usually apply only to loops which are extremely small compared with the wavelength. In loops of more practical dimensions the radiation resistance may be considerably modified by the non-uniform distribution of current, and the foregoing analysis of non-uniform loops can be readily applied to the determination of the radiation resistance.

Only unscreened loops of the types illustrated in Table I will be discussed. The output impedance of a screened loop may be obtained from that of an unscreened type by transforming the loop impedance along the transmission-line system formed by the combined inner and outer conductors.

In simple loops, owing to the phase relationships between the fields from the effective loop and dipole moments respectively, the power radiated by the loop as a whole is very nearly the sum of the powers radiated by these elements separately. A small correction is required, however, due to the displacement of the dipole from the centre of the loop, so it is preferable to derive the radiated power from the actual field distribution. This procedure is necessary also for the analysis of twin loops. The representation of the non-uniform loops as a combination of uniform-current loops and dipoles greatly assists in the formulation of the polar diagrams.

The formulae for the output resistances (including the ohmic loss components) will now be given for the four types of loop discussed in previous sections.

(a) *Rectangular simple loop.*

By consideration of the polar diagram of the loop it can be shown that the power radiated is

$$10\beta^4\left[M_L^2\left\{1 - \frac{2}{15}\beta^2(a^2+b^2)\right\} + M_D^2 - M_L M_D\frac{\beta b}{3}\right] \text{ watts,}$$

the first bracketed term being due to the uniform loop, the second to the effective dipole and the third to the displacement $b/3$ of the dipole from the centre of the loop. The values of M_L and M_D previously determined may be substituted in this expression, but as the derived expressions are rather cumbersome for a rectangular loop, the loop will, for purposes of illustration, be assumed to be a square of side $2a$. The power loss is then

$$W = 160\beta^4 a^4 i_0^2\left\{1 + \frac{17}{5}\beta^2 a^2\right\} \text{ watts,}$$

where i_0 is the peak current with respect to time. Now the terminal current is $i_t = i_0 \cos 4\beta a \approx i_0\left\{1 - 8\beta^2 a^2\right\}$ amperes. The radiation resistance is therefore

$$R_r = \frac{2W}{i_t^2} \approx 320\beta^4 a^4\left\{1 + \frac{97}{5}\beta^2 a^2\right\} \text{ ohms.}$$

To the radiation resistance must be added the ohmic loss component which may be calculated from the known current distribution and the

TABLE II

Resistive Components of Loop Impedances

R_0 = ohmic resistance per unit length

Type of Loop	Radiation Resistance (ohms)	Ohmic Loss Resistance (ohms)
Square Simple	$320\beta^4 a^4\left\{1 + \frac{97}{5}\beta^2 a^2\right\}$	$8aR_0\left\{1 + \frac{32}{3}\beta^2 a^2\right\}$
Rectangular Twin	$80\beta^4 a^2 b^2\left\{1 + \frac{\beta^2}{10}(32a^2 + 35ab + 7b^2)\right\}$	$R_0(2a+b)\left\{1 + \frac{2}{3}\beta^2(2a+b)^2\right\}$
Square Twin	$80\beta^4 a^4\left\{1 + \frac{37}{5}\beta^2 a^2\right\}$	$3R_0 a\left\{1 + 6\beta^2 a^2\right\}$
Circular Simple	$20\beta^4\pi^2 r^4\left\{1 + \beta^2 r^2\left(\frac{19}{5} + \frac{2\pi^2}{3}\right)\right\}$	$2\pi r R_0\left\{1 + \frac{2}{3}\pi^2\beta^2 r^2\right\}$
Circular Twin	$5\beta^4\pi^2 r^4\left\{1 + \frac{\beta^2 r^2}{60}(21 + 60\pi + 10\pi^2)\right\}$	$rR_0\left(\frac{\pi}{2} + 1\right)\left\{1 + \frac{2}{3}\beta^2 r^2\left(\frac{\pi}{2} + 1\right)^2\right\}$

resistance per unit length R_0 of the conductor. The increase in the output resistance of the loop due to copper loss is $8aR_0\left(1 + \frac{32}{3}\beta^2a^2\right)$ ohms.

(b) *Rectangular twin loop.*

Power radiated by the loop

$$= 16\beta^4a^2b^2\left[1 - \frac{\beta^2}{10}(8a^2 + 5ab + 3b^2)\right] \text{ watts.}$$

Terminal current

$$= 2i_0\left\{1 - \frac{\beta^2}{2}(4a^2 + 4ab + b^2)\right\} \text{ amperes.}$$

Radiation resistance

$$= 80\beta^4a^2b^2\left\{1 + \frac{\beta^2}{10}(32a^2 + 35ab + 7b^2)\right\} \text{ ohms.}$$

For a square loop of side $2a$ the radiation resistance becomes $80\beta^4a^4\left(1 + \frac{37}{5}\beta^2a^2\right)$ ohms.

Ohmic loss resistance

$$= R_0\,(2a + b)\left\{1 + \frac{2}{3}\beta^2(2a + b)^2\right\} \text{ ohms.}$$

or for a square loop

$$= 3R_0a\left\{1 + 6\beta^2a^2\right\} \text{ohms.}$$

(c) *Circular simple loop.*

Power radiated

$$= 10\beta^4\pi^2r^4i_0^2\left\{1 + \beta^2r^2\left(\frac{19}{5} - \frac{\pi^2}{3}\right)\right\} \text{ watts.}$$

Terminal current

$$= i_0\left\{1 - \frac{\pi^2}{2}\beta^2r^2\right\} \text{ amperes.}$$

Radiation resistance

$$= 20\beta^4\pi^2r^4\left\{1 + \beta^2r^2\left(\frac{19}{5} + \frac{2\pi^2}{3}\right)\right\} \text{ ohms.}$$

or $20\beta^4\pi^2r^4\left\{1 + 10.4\beta^2r^2\right\}$ ohms approximately.

Ohmic loss resistance

$$= 2\pi rR_0\left\{1 + (2/3)\,\pi^2\beta^2r^2\right\} \text{ ohms.}$$

(d) *Circular twin loop.*

Power radiated

$$= 10\beta^4\pi^2r^4i_0^2\left\{1 - \frac{\beta^2r^2}{60}(39 + 5\pi^2)\right\} \text{ watts.}$$

Terminal current

$$= 2i_0\left\{1 - \frac{\beta^2r^2}{2}\left(\frac{\pi}{2} + 1\right)^2\right\} \text{ amperes.}$$

Radiation resistance

$$= 5\beta^4\pi^2r^4\left\{1 + \frac{\beta^2r^2}{60}(21 + 60\pi + 10\pi^2)\right\} \text{ ohms.}$$

or

$$5\beta^4\pi^2r^4\left\{1 + 5.1\,\beta^2r^2\right\} \text{ ohms approximately.}$$

Ohmic loss resistance

$$= rR_0(\pi/2 + 1)\left\{1 + \frac{2}{3}\beta^2r^2\left(\frac{\pi}{2} + 1\right)^2\right\} \text{ohms}$$

or

$$2.57rR_0\left\{1 + 4.4\beta^2r^2\right\} \text{ohms approximately.}$$

6. Conclusions

It has been shown that a simple loop, with the non-uniform current distribution which results from the existence of standing-wave conditions, may be represented, so far as the external field is concerned, as a combination of a uniform-current loop and a dipole. Similarly a twin loop may be represented as a combination of a uniform-current loop and a pair of spaced dipoles with moments in opposite senses. The spaced dipoles may be represented as being parallel to either of two orthogonal lines in the plane of the loop.

Even with loops which are small compared with the wavelength the radiation resistance may be substantially greater than the value which would be calculated on the assumption of uniform current. The increase is about 100% for a loop of which the side (square simple loop) is about $\lambda/14$ or the diameter (circular simple loop) is about $\lambda/10$ where λ is the wavelength. The increases for corresponding twin loops are about 40% and 50% respectively. The percentage increases in the ohmic loss resistances are of comparable magnitude.

The main characteristics of the various loops discussed are summarized in Tables I and II.

7. Acknowledgments

The work described in this paper was carried out as part of the programme of the Radio Research Board and is published by permission of the Department of Scientific and Industrial Research. The author wishes to acknowledge the help received in discussions with Mr. W. Ross.

REFERENCES

[1] A. Alford and A. G. Kandoian, "Ultra-high Frequency Loop Antennas," *Trans. Amer. Inst. elect. Engrs*, 1940, Vol. 59, p. 843.

[2] W. Ross, "The Development and Study of a Practical Spaced Loop Radio Direction-Finder for High Frequencies," *J. Instn elect. Engrs*, 1947, Vol. 94, Part III, No. 28, pp. 99–107.

[3] F. M. Colebrook, "The Application of Transmission-Line Theory to Closed Aerials," *J. Instn elect. Engrs*, 1938, Vol. 83, p. 403.

[4] R. E. Burgess, "Reactance and Effective Height of Screened Loop Aerials," *Wireless Engineer*, 1944, Vol. 21, p. 210.

RADIATION FROM SHORT AERIALS

New Method of Calculation

By R. G. Medhurst, B.Sc.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England).

SUMMARY.—By the use of a theorem based on what is believed to be a new trigonometrical approximation, it is shown how the radiation patterns from linear radiators shorter than a half-wavelength may be combined, the radiators being spaced in any manner and carrying currents having arbitrary phase differences. Examples of the application of the method are given.

1. Introduction

THE magnitude of the field strength due to a straight, thin, radiating rod of arbitrary length at a point whose distance is very much greater than the length of the rod can be readily calculated from well-known formulae both exact and approximate[1, 2, *]. These formulae are derived by considering the rod to be made up of a large number of very short elements ; the radiation field at a distant point is then obtained by adding vectorially the components at this point due to the currents in each of the elements of the rod, account being taken of the phase differences between these components. Phase differences arise in two ways, (a) because the currents flowing through the elementary sections of the radiator will not, in general, be in phase, and (b) because these sections will be at different distances from the point under consideration, so that the field component due to each section will arrive either in advance of or lagging behind all the other components.

Now, suppose we have a number of radiating rods spaced in some manner and we wish to find the resultant radiation pattern of the combination. We know the magnitude and phase angle of the field at a distant point due to each of the rods, both of these quantities depending on the angle that the direction of the point makes with the direction of the rod. The phase angle will usually be given in terms of the phase of the current at some point in the rod. Knowing the configuration of the rods it is now theoretically possible to work out the resultant field due to the whole system, appropriate phase shifts being introduced to take account of the distances between the phase reference points of the individual rods. The algebra demanded by this procedure and the subsequent numerical computation is likely to be severe, the expression for the phase angle of the field due to a single rod being already, in general, a complicated function of the inclination of the rod to the direction under consideration.

In the special case of rods an exact number of half-waves long it can readily be shown (if it is not obvious from symmetry considerations) that the field component due to each rod may be taken as originating at the centre of the rod, with the phase appropriate to the current passing through the centre. Hence, knowing the relative disposition of the rods the calculation of the relative phases of the field components at a distant point, and hence the resultant field is straightforward. The radiation patterns of groups of rods n half-waves long arranged in parallel and in V formation have been worked out in this way[1].

When the length of the rod is not an exact number of half-waves it is still possible to regard the radiation as originating at a point on the rod, with a phase appropriate to the phase of the current flowing through this point. This origin of radiation, however, will now have to be moved up and down the rod as we consider various directions of propagation. We shall proceed to show that in the special case of a rod shorter than one half-wavelength the deviation of this point from its mean position is so small that to a close approximation we can consider it to be fixed. A radiating rod shorter than a half-wavelength will in general form part of an aerial system. The remainder may be, for example, the other half of a V or an image in a reflecting ground. If the apparent origin of radiation may be taken as fixed, radiation patterns due to such systems may be calculated as easily as in the half-wave rod case.

2. Thin Straight Isolated Rod

We assume, to begin with, that the standing wave on the rod is sinusoidal, with a zero of current at the free end.

* In these references and in the present article it is assumed that the current distribution along the rod is sinusoidal. This assumption is discussed in Appendix 2.

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Let l (metres) be the length of the rod,

r_o (metres) the distance of a point P from the free end, O, of the rod, P being so far removed that straight lines from P to any points of the rod may be regarded as parallel,

θ (radians) the inclination of OP to the direction of the rod.

ω (radians/sec) the angular frequency of oscillation,

λ (metres) the wavelength,

I (amp) the amplitude (peak) of the current on the rod at a current loop.

Then, if e (volts/metre) is the field strength at the point P, it may be shown[1] that

$$e = \frac{30I}{r_o \sin\theta}\left\{\left[1 - \cos L \cos (L \cos\theta) - \cos\theta \sin L \sin (L\cos\theta)\right] \cos\left[\omega\left(t - \frac{r_o}{c}\right)\right] + \left[\cos L \sin (L\cos\theta) - \cos\theta \sin L \cos (L\cos\theta)\right] \sin\left[\omega\left(t - \frac{r_o}{c}\right)\right]\right\} \quad \ldots \quad (1)$$

where $L = \dfrac{2\pi l}{\lambda}$ radians $= \dfrac{360\, l}{\lambda}$ degrees.

This can be written

$$e = \frac{30I}{r_o \sin\theta} F(L, \theta) \cos\left\{\omega\left(t - \frac{r_o}{c}\right) + \tan^{-1}\left[\frac{\cos\theta \sin L \cos (L\cos\theta) - \cos L \sin (L\cos\theta)}{1 - \cos L \cos (L\cos\theta) - \cos\theta \sin L \sin (L\cos\theta)}\right]\right\} \quad \ldots \quad (2)$$

where F (L, θ) is a function of L and θ.

We have assumed that the current distribution on the rod is sinusoidal, so that if the length of rod is less than a half-wave the current will be in phase at all points along the rod. Consequently if the field at P in Fig. 1 had originated at a point G in the aerial, distant x from the free end, with a phase angle corresponding to the phase of the current at G, the time-variable part of the field-strength formula would be

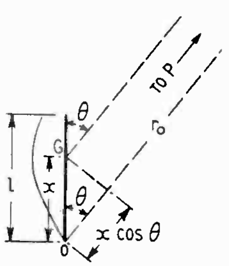

Fig. 1. *Calculation of field for an aerial of length less than $\lambda/2$.*

$$\cos\left[\omega\left(t - \frac{r_o}{c}\right) + \frac{2\pi x}{\lambda}\cos\theta\right]$$

This is equivalent to the time-variable portion of the right hand side of equation (2) if

$$\frac{2\pi x}{\lambda}\cos\theta = \tan^{-1}\left[\frac{\cos\theta \sin L \cos (L\cos\theta) - \cos L \sin (L\cos\theta)}{1 - \cos L \cos (L\cos\theta) - \cos\theta \sin L \sin (L\cos\theta)}\right]$$

We shall show in Appendix I that the right-hand side of this equation may be replaced by the approximate expression $\dfrac{\sin L - L\cos L}{1 - \cos L}\cos\theta$ the error not exceeding 0.54 per cent in the ranges $-1 \leqslant \cos\theta \leqslant 1$ and $0 \leqslant L \leqslant \pi$; i.e.,, for any θ and for a length of rod between zero and a half-wave.

Thus, $\dfrac{2\pi x}{\lambda}\cos\theta = \dfrac{\sin L - L\cos L}{1 - \cos L}\cos\theta$ very nearly and hence

$$x = \frac{\lambda}{2\pi}\,\frac{\sin L - L\cos L}{1 - \cos L} \quad \ldots \quad (3)$$

which is independent of θ.

The approximation that we have just made may not seem so far-fetched if we notice the physical significance of the right-hand side of (3). We have to imagine that the aerial has a mass distributed along its length, the mass per unit length at any point being proportional to the amplitude of the standing wave at that point. Then, the right-hand side of (3) represents the distance of the centre of gravity of this distributed mass from the free end. This centre of gravity we shall call, from now on, the "current centre of gravity" (abbreviated to "current c.g.").

Thus, we have established a theorem which may be stated in this way: *the phase of the radiated field at a distant point, due to a straight isolated thin aerial of length less than or equal to a half-wave, is to a close approximation the same as that of the field due to the current flowing through a short element of the aerial at its current centre of gravity.*

It is evident that when $\theta = \pi/2$ (i.e., at right angles to the aerial) the theorem will be exactly true, since all points of the aerial are then at the same distance from P. As θ becomes smaller the apparent origin of radiation moves further away from the free end, having its maximum displacement when $\theta = 0$. This maximum displacement varies with L, the electrical length of the aerial. It is maximum when L is about 120° being then about 0.54 per cent of the distance of the current c.g. from the free end (see Fig. 8.).

So far, we have concerned ourselves only with the phase of the field radiated by our straight isolated rod. The amplitude of this field is also a complicated function of L and θ. From equation (1) it is given by

$$E = \frac{30I}{r_0 \sin\theta}\sqrt{1 + \cos^2 L + \cos^2\theta \sin^2 L - 2\cos L \cos(L\cos\theta) - 2\cos\theta \sin L \sin(L\cos\theta)} \quad .. \quad (4)$$

It is well known[3] that the radiation pattern due to a straight rod shorter than half a wavelength can be approximated by a sinusoid, the approximation being better the shorter the rod. In fact, equation (4) can be written, approximately,

$$E = \frac{30I}{r_0}(1 - \cos L)\sin\theta \quad .. \quad .. \quad (5)$$

For an L of $\pi/2$ (i.e., for a quarter-wave rod) the error, which increases with decreasing θ, is 5% at $\theta = 30°$ and 8% at $\theta = 10°$. This may not be very serious when the rod forms part of an array, because the error is increasing in those directions in which the field strength component due to the rod is decreasing, so that the approximate formula is most accurate in those directions in which the aerial is making its largest contribution to the total field.

A much better approximation† to (4) is given by

$$E = \frac{30I}{r_0}(1 - \cos L) \times$$
$$\{[1 - G(L)]\sin\theta - G(L)\sin 3\theta\} \quad .. \quad (6)$$

where G (L) is a function of L given by the Table.

TABLE

L	G(L)	L	G(L)
0°	0.0000	100°	0.0212
20°	0.0008	120°	0.0303
40°	0.0034	140°	0.0402
60°	0.0077	160°	0.0492
80°	0.0136	180°	0.0537

For a half-wave rod (the worst case, in our range) the deviation of (6) from (4) is equal to or less than 0.5%. It is thought that (6) is worth giving because, though rather cumbersome, it should be considerably less trouble to work out, for a particular L and a range of θ, than the right-hand side of (4). It will also be more amenable to the kind of analytical work required for combining the radiation patterns of a number of spaced rods. In general, however, the very simple expression of equation (5) will be sufficiently accurate, in view of the probable error implicit in the assumption of sinusoidal current distribution.

† This is worked out from (4) by some straightforward but rather tedious algebra. As a matter of interest, the analytical expression for G(L) is

$$\frac{1}{4}\left[1 - \frac{\sqrt{\sin^2 L - 2L\sin L\cos L + L^2}}{2(1 - \cos L)}\right]$$

Summarizing the results of this section, we find that the field strength at distance r_0 from a straight rod shorter than a half-wave, in a direction making an angle θ with the direction of the rod, is given by

$$e = \frac{30I}{r_0}(1 - \cos L)\left\{[1 - G(L)]\sin\theta - G(L)\sin 3\theta\right\}\cos\left[\omega\left(t - \frac{r_0}{c}\right) + \Psi\right]$$

where L is the electrical length

$$= \frac{30I}{r_0}\left(1 - \cos L\right)\sin\theta\cos\left[\omega\left(t - \frac{r_0}{c}\right) + \Psi\right]$$

approximately, and Ψ is a phase angle obtained by assuming that the field originates from the current flowing through a short element of the rod at its current c.g. If the rod has a free end at which the current is zero, the distance of the current c.g. from the free end is $\frac{\lambda}{2\pi}\,\frac{\sin L - L\cos L}{1 - \cos L}$. This takes the value $\frac{2}{\pi}l$ for a quarter-wave aerial, and approximates to $2l/3$ when the aerial is very short. If the sinusoidal standing wave does not go to zero on the straight portion of the radiating system, it is necessary to imagine the rod to be extended to the point at which the current would become zero, and to subtract the field due to the added portion from the field due to the extended rod.

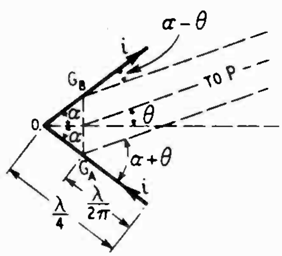

Fig. 2. *Diagram used for a quarter-wave V aerial.*

3. Examples

Quarter-Wave V Aerial in Free Space.

This aerial is shown in Fig. 2. It consists of two quarter-wave rods forming a V of angle 2α, fed in antiphase at the vertex O. This arrangement, mounted on an aeroplane, has been used as an approximately "all-round-looking" receiver for horizontally-polarized waves coming in at some not very large angle to the plane (horizontal) of the V. Polar diagrams taken when the aeroplane is in flight will be very much more irregular than the calculated patterns and those measured over a well-conducting plane earth, owing to complicated reflections from the body of the aeroplane (see Montgomery's measurements in the similar U case[5]). However, the free-space pattern, to begin with, has to be as nearly circular as possible.

The positions of the current centres of gravity of the two rods are as shown in Fig. 2 (G_A and G_B). Using the simple sinusoidal approximation for the amplitudes of the fields due to each rod, the resultant field at a distant point P in the plane of the V is given by

$$e = \frac{30I}{r_o} \sin(\theta + \alpha) \cos(\omega t - \beta) - \frac{30I}{r_o} \sin(\theta - \alpha)$$
$$\cos\left[\omega t - \beta + \frac{2\pi}{\lambda}\left(\frac{\lambda}{4} - \frac{\lambda}{2\pi}\right) 2 \sin\alpha \sin\theta\right]$$

where r_o = OP and θ is the inclination of OP to the bisector of the V.

After some simplification, this gives

$$E = \frac{30I}{r_o}\sqrt{1 - \cos 2\theta \cos 2\alpha - (\cos 2\alpha - \cos 2\theta)\cos[(\pi - 2)\sin\alpha \sin\theta]} \quad \ldots \quad (7)$$

where E is the amplitude of e.

When $\theta = 0$ (i.e., along the bisector)

$$E = \frac{60I}{r_o} \sin\alpha$$

When $\theta = \pi/2$ (i.e., along the perpendicular to the bisector)

$$E = 30\sqrt{2}\,\frac{I}{r_o}\sqrt{1 - \cos[(\pi - 2)\sin\alpha]}\,\cos\alpha$$

These expressions are plotted in Fig. 3, and the polar diagrams in the plane of the V for several values of α in Fig. 4. The field strengths in Fig. 4 are given as multiples of the field strength in the direction of maximum radiation due to a half-wave dipole having the same loop-current amplitude.

When α = 45° (i.e., the optimum angle for reception in the direction given by $\theta = \pi/2$) equation (7) reduces to

$$E = \frac{30I}{r_o}\sqrt{1 + \cos 2\theta \cos\left[\frac{\pi - 2}{\sqrt{2}}\sin\theta\right]}$$
$$= \frac{30I}{r_o}\sqrt{1 + \cos 2\theta \cos(0.807 \sin\theta)} \qquad (8)$$

The right-angled V is called by N. Wells[4] the "Quadrant Aerial." He has carried out the somewhat formidable rigorous computation of the theoretical polar diagram, over a range of lengths of arm, on the basis of an expression equivalent to that of equation (1). In the quarter-wave case, the deviation of the simple expression of equation (8) from the theoretically exact polar diagram as worked out by Wells' method is between 0 and 4%, according to the value of θ chosen.

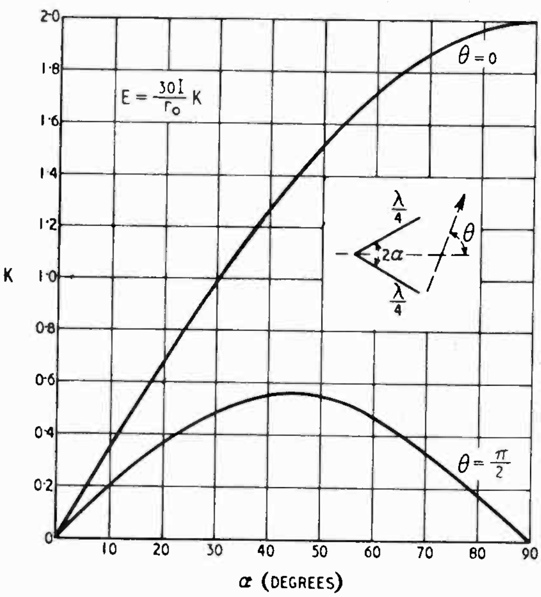

Fig. 3. *Plots of equation (7) for $\theta = 0$ and $\theta = \pi/2$.*

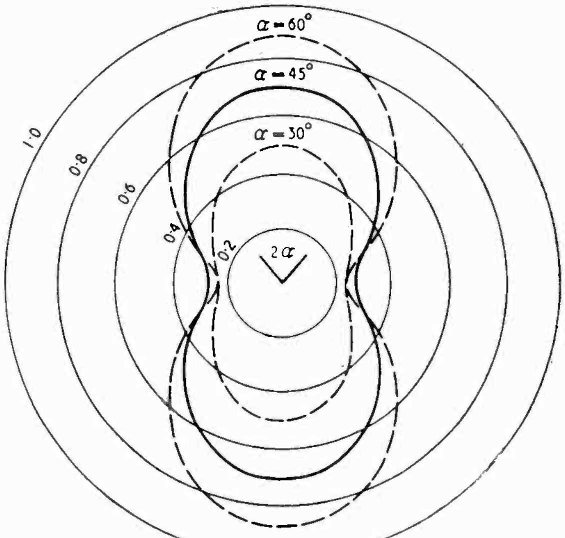

Fig. 4. *Polar diagrams of V aerial for several values of α.*

At points not in the plane of the V the analysis is more complicated, because the components due to the two arms will not be polarized in the same direction. We have now to consider a distant point P such that OP = r_o, the direction of OP being at angle γ to the plane of the V and the projection of OP on the plane of the V at angles θ, θ_a and θ_b respectively to the bisector of the V and its two arms. Then $\theta_a = \theta + \alpha$, $\theta_b = \theta - \alpha$.

Suppose OP is inclined at angles ϕ, ϕ_a and ϕ_b respectively to the bisector of the V and its two arms. Let e_a and e_b be the field components at P due to the two arms, and Ψ_a and Ψ_b the inclinations of e_a and e_b to the direction of horizontal polarization at P. The situation with respect to one of the arms is shown in Fig. 5, the " a " arm being supposed to lie along OX. The other arm is in the plane OXY, at an angle 2α to OX.

Now, if e is the horizontally-polarized component of the field at P due to the whole aerial system, we have

$$e = e_a \cos \Psi_a + e_b \cos \Psi_b.$$

From the spherical triangle PAX,

$$\cos \Psi_a = \frac{\sin \theta_a}{\sin \phi_a}$$

$$\text{so } e = \frac{\sin \theta_a}{\sin \phi_a} e_a + \frac{\sin \theta_b}{\sin \phi_b} e_b$$

$$= \frac{30I}{r_0}\left\{\sin \theta_a \cos (\omega t - \eta_a) - \sin \theta_b \cos (\omega t - \eta_b)\right\},$$

using equation (5), where η_a and η_b are angles to be determined.

The phase angle between radiation from a and $b = 2\pi \Delta d$ where Δd is the projection of $G_A G_B$ (Fig. 2) on OP, so $\Delta d = \frac{4\pi}{\lambda} g \sin \alpha \sin \theta \sin \gamma$ where $g = OG_A$.

Hence,

$$e = \frac{30\,I}{r_0}\left\{\sin \theta_a \cos (\omega t - \eta_a) - \sin \theta_b \cos\left(\omega t - \eta_a + \frac{4\pi}{\lambda} g \sin \alpha \sin \theta \sin \gamma\right)\right\}$$

Then, if E is the amplitude of e, we have, after some simplification,

$$E = \frac{30I}{r_0}\sqrt{1 - \cos 2\theta \cos 2\alpha + (\cos 2\theta - \cos 2\alpha) \cos [(\pi - 2) \sin \alpha \sin \theta \cos \gamma]}$$

since for a quarter-wave rod $\frac{4\pi}{\lambda} g = \pi - 2$.

This is the amplitude of the horizontally-polarized component of the field at P when the aerial is functioning as a transmitter. By the reciprocity theorem, the same expression represents the polar diagram when the aerial is receiving horizontally-polarized radiation.

This expression is the same as that of equation (7) with the exception of the $\cos \gamma$ term. This term, when γ is greater than 0, causes the polar diagram to contract in the $\theta = \pi/2$ direction while leaving the field strength in the $\theta = 0$ direction unaffected.

Quarter-wave U Aerial in Free Space.

This aerial, illustrated in Fig. 6, has been used[5] for the same purpose as the quarter-wave V. It consists of a semi-circle COD and two tangential straight arms CA and DB. The sides OCA and ODB are fed in phase opposition at O. Each side is a quarter-wave long, so that if $2s$ is the distance between the straight arms, AC and BD are each of length $\frac{\lambda}{4} - \frac{\pi}{2} s$.

Assuming a sinusoidal current distribution, the contribution of the two straight arms to the

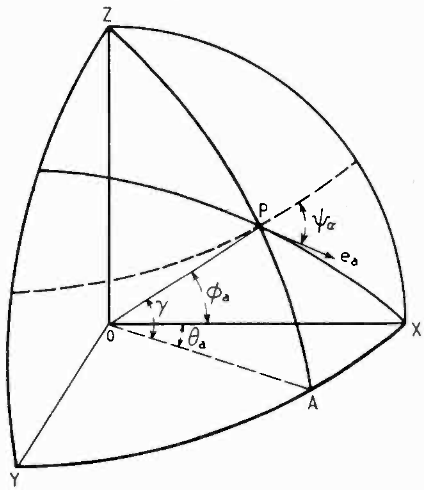

Fig. 5. *Calculation of V aerial for points out of the plane of the V.*

resultant field in the plane of the aerial can be written down immediately. It is, after a little rearrangement,

$$\frac{60I}{r_0}\left(1 - \cos L\right) \sin \theta \sin\left(\frac{2\pi s}{\lambda} \sin \theta\right) \sin\left[\omega\left(t - \frac{r_0}{c}\right) + \frac{L - \sin L}{1 - \cos L}\right] \quad \ldots \quad (9)$$

where r_0 is the distance from Q, the mid-point of CD, θ is measured with respect to the direction OQ, $L = \dfrac{2\pi\left(l - \frac{\pi}{2} s\right)}{\lambda}$ where l is the length of the whole arm OCA, $= \lambda/4$ in the present case.

The contribution of the semi-circular portion has to be worked out by integrating round the semi-circle. After simplifying, it becomes

$$\frac{60\,I}{r_0}\left[A \cos \omega\left(t - \frac{r_0}{c}\right) + B \sin \omega\left(t - \frac{r_0}{c}\right)\right] \quad (10)$$

$$\text{where } A = \frac{2\pi s}{\lambda} \cos\left(\frac{\pi}{2} \cdot \frac{\pi s}{2l}\right) \times$$

$$\left[\frac{J_0 - J_2}{1 - \left(\frac{\pi s}{2l}\right)^2} \cos \theta + \frac{3(J_2 - J_4)}{9 - \left(\frac{\pi s}{2l}\right)^2} \cos 3\theta + \ldots\right]$$

and $B = \frac{2\pi s}{\lambda} \cdot \frac{\pi s}{2l} \sin\left(\frac{\pi}{2} \cdot \frac{\pi s}{2l}\right) \times$

$$\left[\frac{J_1}{\left(\frac{\pi s}{2l}\right)^2} + \frac{J_1 - J_3}{4 - \left(\frac{\pi s}{2l}\right)^2}\cos 2\theta + \ldots\right]$$

The J terms are Bessel functions of argument $\frac{2\pi s}{\lambda}$ which can vary between 0 and 1, and in this range all terms beyond the first two may be neglected. When $l = \frac{\lambda}{4}$, $\frac{\pi s}{2l}$ becomes $\frac{2\pi s}{\lambda}$.

The resultant field is now obtained by combining (9) with (10) which can best be done by expanding (9) into two terms in $\cos \omega\left(t - \frac{r_0}{c}\right)$, and $\sin \omega\left(t - \frac{r_0}{c}\right)$, adding to the corresponding terms in (10) and taking the modulus. Values of the field strength in the plane of the U for $\theta = 0$ and $\pi/2$ are plotted in Fig. 7 for various values of s ranging from $s = 0$ (which is the case of two parallel adjacent straight rods) to $s = 1/2\pi$ (the case of a semi-circle). Comparison of Figs. 7 and 3 shows that the U can be made more "all-round-looking" at the quarter-wave frequency than the V particularly if considerations of matching and mechanical construction will permit of working at a low value of s/λ.

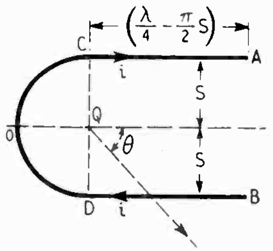

Fig. 6. *Quarter-wave U aerial.*

Montgomery[5] gives a measured polar diagram of a U whose s/l ratio had the value 0.21. His curve (which unlike the theoretical diagram, is unsymmetrical in the forward and backward directions) deviates from the calculated curve by 1 db or less, except over a region covering some 40° about each of the $\theta = \pm \pi/2$ directions. Here the measured curve bulges inwards by as much as 3 db from the calculated values. A possible cause of this discrepancy is that the semi-circular part of the U is encased in a polystyrene slab, for support. This will reduce the velocity of the radiation from the semi-circle while it passes through the polystyrene-filled region, and hence cause an additional phase-shift relative to the phase of the radiation from the straight arms. The same consideration would account for the asymmetry in the forward and backward directions.

APPENDIX 1

The approximation we used in Section 2 for establishing the current centre of gravity theorem was,

$$B \cos\theta = \tan^{-1}\left[\frac{\cos\theta \sin L \cos(L\cos\theta) - \cos L \sin(L\cos\theta)}{1 - \cos L \cos(L\cos\theta) - \cos\theta \sin L \sin(L\cos\theta)}\right]$$

approximately,

where $B = \frac{\sin L - L\cos L}{1 - \cos L}$

When $L = \pi$ (i.e., for a half-wave rod) the equation is exact, since both sides become $\frac{\pi}{2}\cos\theta$. When L is small the approximation can readily be shown by expanding the right-hand side as a Fourier series in θ.

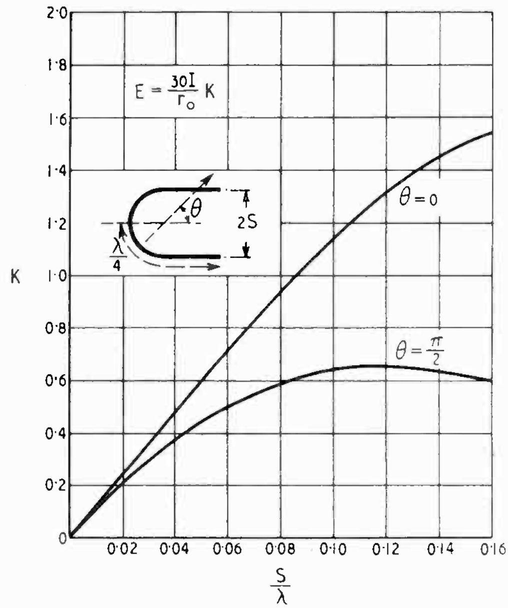

Fig. 7. *Field strength of a U aerial in the plane of the U for $\theta = 0$ and $\theta = \pi/2$.*

No analytical method has occured to the writer, up to this moment, for dealing with intermediate values of L. However, for particular values of θ the approximation can be established over the full range of L from 0 to π by direct computation, and from the nature of the physical problem we may plausibly assume that there will be no unexpected deviations for intermediate values of θ.

Fig. 8 shows the percentage deviation of the right-hand side of the approximation from the left-hand side for $\theta = 0$, $\pi/3$ and $\pi/2$, over the range $0 \leqslant L \leqslant \pi$. The approximation becomes exact when $\theta = \pi/2$, and the deviation is maximum for a particular L when $\theta = 0$.

For particular values of the variables the approximation takes some interesting special forms.

Putting $\cos\theta = A$, when $L = \pi/2$ we have

$$\tan A = \frac{A\cos\frac{\pi}{2}A}{1 - A\sin\frac{\pi}{2}A}$$

with a maximum error of about 0.4% in the range $-1 \leqslant A \leqslant 1$.

When $A = 1$, $\tan B = \dfrac{2L - \sin 2L}{1 - \cos 2L} = L \operatorname{cosec}^2 L - \cot L$

(maximum error 0.54% in range $= \; -\pi \leqslant L \leqslant \pi$)

When $A = \dfrac{1}{2}$, $\tan \dfrac{B}{2} = \dfrac{\sin^3 \left(\dfrac{L}{2}\right)}{1 - \cos^3 \left(\dfrac{L}{2}\right)}$ (maximum error 0.13% in range $-\pi \leqslant L \leqslant \pi$).

For small L, these last two approximations become,

$$\left.\begin{aligned} \tan\left(\frac{2}{3}L\right) &= L \operatorname{cosec}^2 L - \cot L \\ \tan\left(\frac{1}{3}L\right) &= \frac{\sin^3\left(\frac{L}{2}\right)}{1-\cos^3\left(\frac{L}{2}\right)} \end{aligned}\right\} \text{approximately.}$$

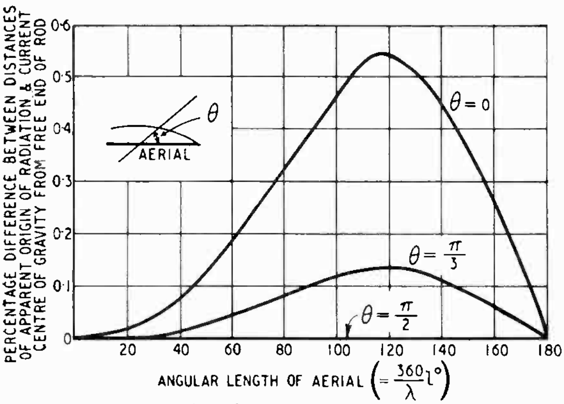

Fig. 8. *Percentage difference between distances of apparent origin of radiation and current centre of gravity from free end of rod as a function of the angular length of the aerial.*

APPENDIX II

On the assumption that the distribution of current along a thin uniform aerial is sinusoidal we can predict polar diagrams which are in good enough agreement with experiment to form a useful guide to the performance of a proposed aerial array, both when the component rods are many wavelengths long and when they are shorter than one wavelength[1, 4, 6]. An end correction is necessary, the electrical length of the aerial having to be assumed somewhat greater than the physical length. In the case of aerials shorter than a half-wave, this correction may be as much as 20%[4, 5].

The present work consists of the development of an approximation method for simplifying the calculation of polar diagrams for certain types of aerials on the assumption of a sinusoidal distribution of current. The deviation of the field strength thus calculated from those calculated rigorously on the sinusoidal assumption is well within the deviation of the latter from measured values. The approximation we have introduced thus fulfils the necessary criterion for an approximation to a physical theory. It seems to be of interest, however, to go over briefly the evidence for a non-sinusoidal current distribution and to consider how such a distribution would effect the procedure developed above.

Experimental work on the current distribution along aerials is somewhat sparse and the results not in very good accordance. Wilmotte[7] constructed a uniform vertical aerial in the form of a cage and placed a number of thermo-ammeters inside it, the ammeters being read by a telescope. After correcting for the impedances of his thermo-ammeters he found current distributions which were quite closely sinusoidal over aerial lengths from about $\frac{1}{4}$ to $\frac{3}{4}$ of a wavelength, even when the aerial was bent to form an inverted L. Gihring and Brown[8] measured the pick up of a small coil held close to the aerial and moved up and down its length. They found current distributions that were closely sinusoidal for uniform aerials (over about the same range of lengths as used by Wilmotte) but markedly non-sinusoidal for tapering aerials. Morrison and Smith[9], however, found non-sinusoidal distributions, using the same method of measurement, even with uniform aerials. Objection has been made to Gihring and Brown's procedure[10]. More recently, Palmer and Gillard[11] measured substantially sinusoidal current distributions on aerials several wavelengths long by moving a vacuum thermo-junction along the aerial.

The conclusion we can draw from these measurements is that in so far as the amplitude of the standing wave departs from a sinusoidal shape it does so at current nodes (except at the node at the free end.) In Morrison and Smith's results, these nodes become finite minima instead of zeros. When dealing with aerials considerably shorter than a half-wave this effect will not concern us.

Besides a possible departure in amplitude of the standing wave from a sinusoidal distribution, however, we have also to consider that its phase along the rod may not show the behaviour characteristic of a sinusoid. The attempts that have been made to apply transmission-line theory to aerials [10] lead to the conclusion that there will be a progressive phase shift along the length of the aerial, in place of a phase of constant magnitude changing sign at the nodes. This would cause a shift of the apparent origin of radiation along the aerial away from the current centre of gravity. Unfortunately, no experimental work to determine the magnitude of this effect has been attempted, and the transmission-line theory of aerials is itself still sub-judice [12]. Consequently, in the present state of knowledge we are obliged to dispense with the refinement (which, in any case, is not likely to be of large magnitude) which would follow from the use of a physically more exact current distribution.

REFERENCES

1 Carter, Hansell and Lindenblad, "Development of Directive Transmitting Antennas," *Proc. Inst. Radio Engrs*, 1931, Vol. **19**, p. 1773.

2 F. E. Terman, "Radio Engineers' Handbook," McGraw-Hill, 1943.

3 Brown, Lewis and Epstein, "Ground Systems as a Factor in Antenna Efficiency," *Proc. Inst. Radio Engrs*, 1937, Vol. **25**, p. 756.

4 N. Wells, "The Quadrant Aerial: An Omni-directional Wide-band Horizontal Aerial for Short Waves," *J. Instn elect. Engrs*, 1944, Vol. **91**, Pt. III, p. 182.

5 B. E. Montgomery, "A Very High Frequency Aircraft Antenna for the Reception of 109 Megacycle Localiser Signals," *Proc. Inst. Radio Engrs*, 1945, Vol. **33**, p. 767.

6 G. H. Brown, "The Turnstile Antenna," *Electronics*, April 1936, Vol. **9**, p. 15.

7 R. M. Wilmotte, "The Distribution of Current in a Transmitting Antenna," *J. Instn elect. Engrs*, 1928, Vol. **66**, p. 617.

8 H. E. Gihring and G. H. Brown, "General Considerations of Tower Antennas for Broadcast Use," *Proc. Inst. Radio Engrs*, 1935, Vol. **23**, p. 311.

9 J. F. Morrison and P. H. Smith, "The Shunt-Excited Antenna," *Proc. Inst. Radio Engrs*, 1937, Vol. **25**, p. 673.

10 W. L. McPherson, "Electrical Properties of Aerials for Medium and Long Wave Broadcasting." *Elect. Commun.*, 1938–39, Vol. **17**, p. 44.

11 L. S. Palmer and K. G. Gillard, "The Distribution of Ultra-High-Frequency Currents in Long Transmitting and Receiving Antennae, *J. Instn elect. Engrs*, 1938, Vol. **83**, p. 415.

12 S. A. Schelkunoff, "Theory of Antennas of Arbitrary Size and Shape," *Proc. Inst. Radio Engrs*, 1941, Vol. **29**, p. 493.

BOOK REVIEWS

Practical Five-Figure Mathematical Tables

By C. ATTWOOD. Pp. 74. Macmillan & Co., Ltd., St. Martin's St., London, W.C.2. Price 3s.

The author is the principal of the Ford Motor Co.'s Trade School at Dagenham. He has undoubtedly succeeded in producing a reliable set of tables without imposing extra labour upon the user. All the the normal logarithmic, trigonometrical and exponential functions are included. The addition of cologarithms and the direct tabulation of trigonometrical ratios of angles expressed in hundredths of a radian should prove to be valuable labour-saving devices. Constants and conversion factors are tabulated on pp. 1 and 62. Cubes and cube roots, and higher powers and roots of integers up to 99 are included as well as the usual squares and square roots.

The author claims that 'more than 95 per cent of the 160,533 possible combinations of mean proportional parts contain either no error or an error of one unit in the last figure, while less than 0.6 per cent produce an error in excess of three units'. This remarkable result has been achieved mainly by adjusting the interval of tabulation to suit the rapidity of change of the tabulated function; e.g., reciprocals are tabulated for every 0.001 between 1.000 and 1.509, while between 1.50 and 2.00, as many as four rows of mean proportional parts have been provided for each row of the main table. Italics are used in other cases where the use of mean proportional parts could give rise to unusually large errors; such errors are systematically tabulated on p. 71. Red figures are used when differences have to be subtracted—a notable improvement. The reliability of mean proportional parts is fully discussed in extensive notes on the use of the tables on pp. 63–74. The printed values of mean proportional parts have been adjusted to minimize the errors due to rounding off the entries in the body of the tables. This sometimes gives the appearance of a misprint. Inverse, backward and linear interpolation are illustrated by examples and Bessel's formula for second-difference interpolation is quoted and explained.

The problem of the regions where mean proportional parts are untrustworthy has thus been resolutely tackled: no blanks appear in Mr. Attwood's difference columns unless there are very clear instructions as to how they can be filled. This book of tables therefore represents a notable advance and can be thoroughly recommended. J. W. H.

Principles of Radar

By DENIS TAYLOR, Ph.D. and C. H. WESTCOTT, Ph.D. Pp. 141 + x, with 52 figures and 5 plates. Cambridge University Press, 200, Euston Road, London, N.W.1. Price 12s. 6d.

This is another of the series on "Modern Radio Technique," of which Huxley's "Principles and Practice of Wave Guides" and R. A. Smith's "Radio Aids to Navigation" have already been reviewed. The authors assume that readers who need detailed information on radar techniques will refer to other volumes in the series, and have therefore concentrated on expounding the general principles peculiar to radar. The formulae derived are, however, illustrated by a number of numerical examples typical of different types of equipment. A book of this compass is likely to give the reader a clearer general picture of essentials than the more elaborate works on the subject.

After an introductory chapter, the factors determining radar performance are identified. Measurement of the co-ordinates is then considered in turn—range, azimuth, elevation, and combined azimuth and elevation. The problem of unwanted echoes has a chapter to itself; and in a penultimate chapter the numerical characteristics of representative radar equipments are summarized. Except for the last chapter, devoted to secondary radar (using responders on the target), only primary or true radar, involving reflection from the target, is considered. Radar navigation is not within the scope, being covered by Smith's volume.

The subject of the target receives considerable attention, not only in the body of the book, but in Appendix 2, devoted to the calculation of absorbing, scattering and echoing areas. Appendix 1 is a list of the symbols and units used, and the most important formulae derived. The authors wisely work in metres throughout. The few departures from B.S. nomenclature (for example, 'c./sec.' for 'c/s') seem unnecessary. In view of the emphasis laid on working in terms of power rather than field strength, it is rather surprising to find that policy reversed in a departure from the term 'cosec^2 aerial,' in general use, to 'cosec aerial.'

Since the series of monographs of which the present volume is one is announced as "dealing with advances in radio technique made during the war" it is perhaps unjust to criticize it for doing that, even though by now one may have had some opportunity to study 1939–1945 achievements and be looking for guidance on future or at least present-day practice. But one might fairly expect that, even in an exposition of wartime radar, attention would be directed mainly to those types which emerged at the end, earlier systems being only briefly mentioned to indicate the logic of development. But in the present book, the majority of space is given to types which were obsolete or obsolescent in 1945.

With this qualification, "Principles of Radar" can be recommended as a clear, concise presentation of the subject. M. G. S.

Crystal Rectifiers

By HENRY C. TORREY and CHARLES A. WHITMER. 443 + viii pp., 218 illustrations. (Vol. 15, M.I.T. Radiation Laboratory Series). McGraw-Hill Book Co., Ltd., Aldwych House, London, W.C.2. Price 36s. (in U.K.).

In 1940 it became clear, through work at Birmingham University, that the klystron and the multi-resonator magnetron were capable of giving large transmitter powers at centimetre wavelengths. There was, however, no corresponding vacuum tube which would make a sensitive receiver, either as a frequency changer or as a radio-frequency amplifier. As a stop-gap the crystal was used, and experiments soon showed that as a frequency changer (with a reflex klystron as local oscillator) it would make a sensitive receiver.

The "stop-gap" has not yet been superseded, and this book contains a comprehensive account of the intensive research on and development of the crystal rectifier up to the end of the war. After a brief introduction, the fundamental properties of the crystal rectifier are described, and the criteria defined—conversion loss and noise temperature—by which its performance as a frequency converter (mixer) is judged. This section of the book ends with two chapters in which the electrical properties of semi-conductors are interpreted in terms of modern physical theory of the solid state, and the process of rectification at the contact between the semi-conductor and a metal is discussed.

The major portion of this book is concerned with the crystal converter, and an extensive mathematical treatment of the theory of the conversion loss and noise temperature is followed by an excellent chapter on the measurement of these important quantities. The standard test equipment developed for this purpose at three wavelengths in the centimetre band is described in some detail. The slow or sudden deterioration of the electrical properties (burnout), which baffled designer and user of pulsed radar sets until the discovery of the " spike " of energy in the TR-cell break-through power, is discussed, together with the test procedure for eliminating crystals of poor resistance to burn-out. After a review of manufacturing techniques, the book concludes with a section on special types, the low-level detector for centimetre wavelengths, the high inverse-voltage rectifier with germanium instead of silicon, and the welded-contact germanium unit whose conversion *gain* caused such a flutter until it was found that the excessive noise made it inferior to standard types as a converter for centimetre wavelengths.

" Crystal Rectifiers " is Volume 15 of the series which the great wartime Radiation Laboratory at M.I.T. undertook in order to make available the enormous body of technical information collected during the war One cannot deny that the authors, who had presumably been told to 'pack everything in,' have made an excellent job. But the book suffers from the fact that it is directed at no particular class of reader and this reviewer is baffled to know who will read it. He is saddened also by the three scanty ' honourable mentions ' of British work in the text of a book which purports to cover the whole field ; in over two hundred footnote references to wartime reports—available to few readers of this book—there is but one to a British report. It is perhaps invidious to conclude this review by mentioning that the crystal cartridge, used in service equipment throughout the war, was designed in this country.

B. B.

Network Analysis and Feedback Amplifier Design

By H. W. Bode, Ph.D. Pp. 551 + xii (including index, pp. 21 and list of symbols, pp. 6), with 429 illustrations and 20 charts. D. Van Nostrand Company Inc., 250 Fourth Avenue. New York. Handled in U.K. by Macmillan & Co., Ltd., St. Martin's St., London, W.C.2. Price 42s.

Dr. Bode is such an outstanding authority in his field that this book cannot but be of interest to those concerned with networks or feedback systems. Not all of the material is original and much of it has appeared before in articles, patents, and to some extent in text-books, but the collection of this material into one volume makes a welcome addition to the range of textbooks on these subjects.

Although network analysis has been included in this book primarily in relation to the needs of feedback amplifier design, the field that has had to be covered is so wide that the treatment of network topics not necessarily connected with feedback takes up thirteen out of the nineteen chapters. Compared with existing textbooks, the book does not give much space to the design of, say, filters and equalizers, but is more concerned with the characteristics, and the limitations. of physically possible two-terminal and four-terminal networks.

The analysis of networks, active and passive, by the usual mesh and nodal systems is reviewed and the superiority of the nodal system, especially for amplifier stages, is stressed : the duality of the two systems is emphasized and most of the theorems and results of the work are framed so as to be capable of interpretation in either a mesh or a nodal analysis. Network functions are first dealt with as functions of the complex variable p and the properties of functions corresponding to stable systems are established. The theory of contour integration, a brief introduction to which is given, is then used to convert these properties into characteristics of the network functions as functions of $j\omega$, such as Nyquist's criterion of stability.

A general discussion of two-terminal networks includes the properties of minimum resistance, conductance, susceptance, or reactance networks and the synthesis of networks by Brune's method. A roughly parallel treatment of four-terminal networks introduces the conception of minimum loss and minimum phase-shift systems and shows how any physically realizable function can be represented by a number of basic constant-resistance lattice sections in tandem, although this representation may have to include an additional constant loss.

Various formulae dealing with real and imaginary components of network functions as functions of frequency are introduced : these include the phase-area theorem and the phase-shift as a function of attenuation slope, both associated with the name of Bode. A series of graphs is presented to facilitate the calculation of attenuation-phase relations.

The subjects of equalizers, input and output circuits for repeaters, and inter-stage coupling networks are discussed in a fairly general way.

Feedback-amplifier design is covered by four general chapters in which the conceptions associated with the symbols 'μ' and 'β' are abandoned in favour of quantities better adapted to a precise mathematical treatment. This part is extended to multiple-loop and conditionally stable amplifiers, which were to have been included in the chapters on design methods, but these chapters have had to be confined, because of ' invincible fatigue ' on Dr. Bode's part, to a general discussion of, and illustrative designs for, single-loop absolutely stable amplifiers.

This book will undoubtedly become a standard textbook. The material has been used as a text in various courses, before publication, and can therefore be expected to be reasonably free from errors and obscurities. It is not a book for the occasional designer, to be consulted for a ready-made solution to a particular problem, but is for the specialist, to be read and assimilated. The process of assimilation will not be easy ; the book is well packed with solid theory. The earlier chapters in particular fairly bristle with new terms ; one must beware, too, that ' output impedance,' about as often as not, is synonymous with 'load impedance.' Nevertheless, the designer, whether of networks in general or of feedback amplifiers in particular, who gets the material of this book at his finger-tips will be well repaid for his labour.

W. E. T.

Application of Statistical Methods to Industrial Standardization and Quality Control, B.S. 600. New printing of the 1935 edition. British Standards Institution, 24, Victoria St., London, S.W.1. Price 12s. 6d.

REPRODUCERS AND AMPLIFIERS LTD.

We are asked to correct an error which occurred in this firm's advertisement in the June issue of *Wireless Engineer*. The tolerance on the bore of the rear face was given as ± 0.005 instead of ± 0.0005 in.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Standard Terms and Abbreviations

SIR,—For any attempt to overcome the present misuse of symbols to be successful, I consider that it is necessary both to familiarize people with the correct symbols and to enlist the aid of schools and colleges. I have observed that even graduates differ widely in the symbols which they use for common quantities and this would seem to indicate that the schools and colleges differ also.

This can be remedied only by getting all connected with scientific education interested in the subject. I believe that the British Standards Institution has a committee concerned with it and I would like to suggest the desirability of extending its membership to include representatives of all interested bodies, including the teaching profession.

International standardization is also important. I recently saw in an American journal, which shall be nameless, the following inconsistencies on a single page: (a) M and m both used for mega, (b) m used for both mega and micro, (c) m and μ both used for micro, (d) Kw and kw both used for kilowatts.

There is certainly a need for an international campaign to get the wholehearted support of scientists and publishers of scientific works all over the world for the use of a revised list of standard abbreviations. If we in Britain can put our own house in order we shall have a very strong case to put before any international committee that may be formed to deal with the matter.

Teddington, Middx. A. WILKINSON.

SIR,—I have only just read E. D. Hart's letter on this subject in your May issue, but I should like, even at this belated date, to deal with the one point in it with which I am not in agreement.

Mr. Hart says that he cannot agree with the American form a-c with a hyphen as the abbreviation for 'alternating current,' even when the abbreviation is used adjectivally, because 'surely the hyphen is not normally used when the words are written in full.'

It may be true that the words are not normally used with the hyphen when they are written in full, but they certainly should be so used where they are used adjectivally. In the term 'alternating-current circuit' the word 'circuit' is the noun and it is not qualified by the adjective 'alternating' nor by the adjective 'current,' but by the adjective 'alternating-current.' Thus the word 'alternating-current' is here a simple, unitary, composite, adjective, identified as such by the hyphen. Correspondingly, the abbreviation should also be written with a hyphen as a-c, or perhaps as a.-c.

Mr. Hart refers to the abbreviation a-c as being 'the American form,' but, as far as I can see, it is not the common American form, though it appears to have been regularly used by The Westinghouse Electrical and Manufacturing Company, who use it only for the adjectival form, reserving a.c. for 'alternating current' when this is a noun. This use of the two forms of the abbreviation seems to me to be correct practice.

Ealing, W.5. ROBERT H. NISBET.

Intensity-Distance Law of Radiation

SIR,—We have read with interest D. A. Bell's letter in the June issue of *Wireless Engineer*, in which he suggests a criterion for determining the point beyond which the radiation intensity should vary inversely as the square of the distance from a highly-directive radio aerial. While at first sight the basis of Mr. Bell's argument seems reasonable, there is little doubt that his criterion is too severe, and that as a consequence his "critical-distance" parameter is considerably too great. We are led to this belief by experimental results we obtained several years ago with systems working on wavelengths of 9 and 3 cm, and we would like to suggest that the following treatment of the problem is perhaps more satisfactory than Mr. Bell's: it is certainly in better accord with the experimental facts.

In this particular problem the region of space concerned is that confined to within a few degrees of the axis of the main lobe; and it is true that with an aerial having a large gain we may consider that the radiation is mainly confined to a cone, the apex angle of which is determined by the beam-width of the main lobe. Further the distance at which the inverse-square law becomes operative should be that at which the finite aperture source may be replaced by a point source. We suggest that this latter condition would better be specified by defining it as that for which the distances to a point in the radiation field, from the edge and the centre of the aperture respectively, differ by much less than a wavelength. Thus, for a source of aperture diameter d, and wavelength λ, since we need consider only points relatively near the axis, our suggested criterion becomes $d^2/8r \ll \lambda$: this is to be compared with Mr. Bell's $d^2/0.52r \ll \lambda$, r being the distance from the source and λ the wavelength in question. If we define a 'critical-distance' parameter, r_0, in the same manner as Mr. Bell we have $r_0 = d^2/8\lambda$.

In the experiments referred to above the transmitting aerial used at the wavelength of 9 cm consisted of a paraboloidal mirror of aperture diameter 1.2 metres irradiated by a half-wavelength dipole situated at the focus. A similar mirror was used at the wavelength of 3 cm, but excitation was by means of a waveguide feed. It was found that the inverse-square law was followed at distances greater than about 8 to 10 metres for $\lambda = 9$ cm, and greater than 20 to 25 metres for $\lambda = 3$ cm.

Now, on our definition of $r_0 = d^2/8\lambda$, for a source of aperture diameter 1.2 metres r_0 should be about 2 and 6 metres respectively for the wavelengths of 9 and 3 cm. On an empirical basis, therefore, we might suggest that the inverse-square law for radiation intensity may be assumed to obtain where $r \gg 4r_0$, or $r \gg d^2/2\lambda$, that is, when the path difference between rays arriving from the edge and from the centre of the aperture is not greater than $\lambda/4$. This relation is also supported by experimental observations by Bach.[1]

For an aerial over the aperture of which the phase of the radiation is not uniform, r_0 may have to be increased to counteract increased phase difference, and in making measurements on aerial gain and radiation patterns we have usually made the distance such that the path difference is about $\lambda/16$. This latter is the criterion used also by the Bell Telephone Laboratories.[2]

Research Laboratories of The General Electric Co., Ltd., Wembley. H. R. L. LAMONT.

National Physical Laboratory, Teddington. J. A. SAXTON.

[1] W. Bach, *Hochfrequenztech. u. Elektroakust.*, 1939, Vol. 53, p. 115.
[2] C. C. Cutler, A. P. King and W. E. Kock, *Proc. Inst. Radio Engrs*, 1947, Vol. 35, p. 1462.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationary Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

AERIALS AND AERIAL SYSTEMS

590 629.—Arrangement of a vertical aerial, and a symmetrically-arranged array of aerials for simultaneously radiating two distinct signals on different carrier waves.

The British Broadcasting Corporation and H. L. Kirke. Application date 13th February, 1945.

591 053.—Aerial systems comprising various combinations of a single resonant slot backed by a sheet-reflector.

H. G. Booker, C. H. Westcott and R. J. Lees. Application date 23rd March, 1944.

591 625.—Aerial system of the slotted-waveguide type for "end fire" radiation and reception.

Sperry Gyroscope Co. Inc. (assignees of M. H. Johnson, W. H. Ratliff Jr. and W. W. Hansen). Convention date (U.S.A.) 31st March, 1942.

591 637.—Construction and arrangement of a wire-mesh screen for minimizing polarization effects in directional aerials of the Adcock type.

W. Ross. Application date 19th January, 1945.

592 120.—Various arrangements of slotted waveguides serving as aerials for the transmission or reception of ultra-short waves.

Western Electric Co. Inc. Convention date (U.S.A.) 28th November, 1941.

591 987.—A pair of dipoles so arranged and fed from separate transmission lines as to form an aerial system having a broad resonance characteristic.

Marconi's W.T. Co. Ltd. (assignees of G. H. Brown). Convention date (U.S.A.) 23rd February, 1942.

592 162.—Various arrangements and arrays of waveguides which are terminated in such a way as to permit the gradual radiation of energy in predetermined phase-relation, and to provide a directional aerial system (divided from 592 120).

Western Electric Co. Inc. Convention date (U.S.A.) 28th November, 1941.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

591 041.—System designed to allow several aircraft to interrogate a common land-beacon, wherein, by the use of pulses of "wobbled" repetition-frequency, each craft receives clear-cut indications of its own distance and direction from the beacon-station.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies). Convention date (U.S.A.) 1st May, 1944.

591 130.—Radiolocation system in which an interrogating wave triggers a distant radio transmitter, and in which a cathode-ray tube is used to indicate the critical time interval.

R. A. Watson Watt. Application date 15th September, 1936 (secret patent, published 31st May, 1947).

591 668.—Direction-finding system, say of the switched-cardioid type, in which both phase and amplitude modulation are utilized to secure specified advantages.

Standard Telephones and Cables Ltd. and C. W. Earp. Application date 15th August, 1944.

591 802.—Valve switching arrangement, operating over a given cycle, particularly for controlling the response or identification signals used in radiolocation systems.

Ferranti Ltd., M. K. Taylor, F. C. Williams and R. H. A. Carter. Application date 22nd May, 1945.

591 867.—Arrangement for testing and balancing the transmission lines and antenna elements in a directional aerial array.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies and A. G. Richardson). Convention date (U.S.A.) 23rd April, 1942.

591 870.—Resistance-capacitance device for rapidly extinguishing spark-over in radiolocation equipment using grid-pulsed transmitting valves.

Philco Radio and Television Corp. (assignees of W. A. Stewart). Convention date (U.S.A.) 30th November, 1943.

591 874.—Protective device against inter-electrode arcing, particularly for the pulse-generating valves used in radiolocation.

Philco Radio and Television Corp. (assignees of R. G. Clapp). Convention date (U.S.A.) 2nd November, 1943.

592 029.—Aerial system with a parabolic reflector to which an oscillating as well as a rotating motion is imparted, for scanning a field of observation in radiolocation.

Nash and Thompson Ltd., A. G. Frazer-Nash, A. Whitaker and C. Fox. Application date 11th May 1942.

592 030.—Radiolocation equipment in which a given area is scanned and the target is shown in plan position, corrected for any pitching or rolling of the vessel on which the transmitter is carried.

Hazeltine Corpn. (assignees of H A. Wheeler). Convention date (U.S.A.) 3rd March, 1942.

592 269.—Automatic gain-control system for responding to the pulsed interrogating-signals used say in radiolocation equipment.

Hazeltine Corpn. (assignees of B. D. Loughlin and J. A. Hansen). Convention date (U.S.A.) 11th February, 1944.

592 271.—Means for controlling the sensitivity, or band-width response, of a super-regenerative set, as used for receiving the pulsed interrogating signals in radiolocation equipment.

Hazeltine Corpn. (assignees of H. A. Wheeler). Convention date (U.S.A.) 11th February, 1944.

592 274.—Gain-control system for a super-regenerative circuit which is arranged to respond at definite intervals to pulsed interrogating signals and to be kept inactive during the intervening periods.

Hazeltine Corpn. (assignees of B. F. Tyson). Convention date (U.S.A.) 19th April, 1944.

592 346.—Blocking systems for protecting the receiving circuits during the transmission of exploring pulses in radiolocation sets.

Standard Telephones and Cables Ltd. (assignees of E. Labin). Convention date (U.S.A.) 4th January, 1943.

592 367.—Cathode-ray device for indicating simultaneously, on a substantially-plane surface, three variable quantities, such as the range, azimuth, and elevation of an aircraft, as determined by radiolocation equipment.

E. Parker and C. S. Wright. Application date 2nd May 1945.

592 447.—Direction-finding system using a number of sectionalized aerial elements, and an injection antenna, to neutralize any error due to horizontally-polarized components.

Marconi's W.T. Co. Ltd. (assignees of L. E. Norton). Convention date (U.S.A.) 31st December, 1942.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

591 026.—Coaxial-line resonator with a screw-controlled disc, coacting with the end of the inner conductor, to give a straight-line tuning-control over a wide frequency-range.

M. C. Goodall and C. S. Wright. Application date 21st April, 1945.

591 301.—Automatically-controlled discriminator circuit for receiving phase- or frequency-modulated signals.

Marconi's W.T. Co. Ltd. (assignees of W. L. Carlson). Convention date (U.S.A.) 3rd May, 1944.

591 692.—Heterodyne or frequency-mixing circuit in which provision is made to prevent the so-called "pulling-in" effect.

Standard Telephones and Cables Ltd. and I. R. Worsley. Application date 4th May, 1945.

591 706.—Tuning system arranged to give a variable band-spread effect over selected short-wave sections of the overall tuning range.

E. K. Cole Ltd., L. W. D. Sharp and H. Hunt. Application date 10th May, 1945.

591 965.—Arrangement for maintaining a super-regenerative receiver in a state of constant sensitivity to signals transmitted on a continuous carrier-wave over a wide range of frequency.

Ferranti Ltd., F. C. Williams and J. R. Whitehead. Application date 15th March, 1945.

592 168.—Radio chassis which is assembled and wired by mass-production methods from previously-prepared sub-units.

Standard Telephones and Cables Ltd. (assignees of S. H. M. Dodington). Convention date (U.S.A.) 4th July, 1942.

592 556.—Receiving circuit, employing the principle of "carrier exaltation" or side-band attenuation, and adapted to detect either phase-modulated or amplitude-modulated signals.

Marconi's W.T. Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 18th May, 1945.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

590 468.—Method of applying periodic blanking-impulses to a television tube, fitted with an electron-multiplier, without heavily modulating the primary scanning-stream.

Marconi's W.T. Co. Ltd. (Assignees of O. H. Schade). Convention date (U.S.A.) 29th August, 1942.

590 778.—Two-stage pentode amplifying unit for mixing the blanking and synchronizing impulses with the video signals in television.

Farnsworth Television and Radio Corpn. Convention date (U.S.A.) 1st November, 1943.

591 686.—Valve circuit for controlling time-base voltages, and the coordination of video and synchronizing signals in television.

J. W. Pletts. Application date 2nd May, 1945.

591 707.—Television system in which the video signals are frequency-modulated and the synchronizing signals are amplitude-modulated.

W. S. Percival. Application date 10th May, 1945.

TRANSMITTING CIRCUITS AND APPARATUS

(*See also under Television*)

590 764.—Oscillation-generating system, say for frequency-modulation, comprising one valve which operates as an inductive impedance connected in parallel with a second valve operating as a capacitive impedance.

Marconi's W. T. Co. Ltd. (assignees of V. D. Landon.) Convention date (U.S.A.) 19th April, 1944.

590 886.—Multivibrator device for automatically monitoring and controlling the stability of a frequency-modulated radio transmitter.

The British Thompson-Houston Co. Ltd. Convention date (U.S.A.) 27th February, 1943.

591 369.—Phase-adjusting device for a waveguide, comprising a strip of dielectric material mounted to be moved, under control, into the interior of the guide through a slot in one of the walls.

L. B. Mullett. Application date 19th April, 1945.

590 351.—Method of "double-bending" a waveguide so that any elliptical polarization set-up at the first bend is neutralized or corrected at the second.

Western Electric Co. Inc. Convention date (U.S.A.) 25th January, 1944.

592 126.—Terminal construction for a coaxial transmission line, designed to facilitate coupling it to a balanced load, such as a centre-fed dipole aerial.

R. G. Garfitt. Application date 30th April, 1945.

592 302.—Waveguide with a folded or looped terminal section to facilitate matched-impedance coupling to a transmission line of the coaxial type.

Western Electric Co. Inc. Convention date (U.S.A.) 17th March, 1942.

592 349.—Modulating or switching system in which the balance of a looped transmission line is controlled by varying the tuning of a resonant circuit coupled to one of its branches.

Standard Telephones and Cables Ltd. (assignees of C. B. Watts, Jr.). Convention date (U.S.A.) 4th July, 1942.

592 509.—Radio transmitter and receiving set, primarily designed as a portable unit, for use under war conditions, and to be immune from shock, and from severe changes of temperature and climate.

Convention date (U.S.A.) 8th December, 1942.

592 513.—Permeability-tuned oscillator unit, provided with accurate compensation for temperature changes, particularly for use under war conditions.

A. Thomson. Convention date (U.S.A.) 8th December, 1942.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

590 472.—Signalling system in which two distinct messages are carried by phase-displaced portions of a single carrier-wave.

Marconi's W.T. Co. Ltd. (assignees of S. W. Seeley). Convention date (U.S.A.) 29th September, 1942.

591 098.—Pulsed system of signalling in which the frequency of each train, and the initial amplitude of

each train, are separately modulated by the same signal.
Standard Telephones and Cables Ltd., P. K. Chatterjea and L. W. Houghton. Application date 16th December, 1941.

591 357.—Facsimile signalling system in which varying shades are converted into equivalent audio tones, which are then applied to vary the frequency of one of a pair of oscillation-generators.
Press Wireless Inc. Convention date (U.S.A.), 18th September, 1943.

591 884.—Two-way system of signalling on two interlaced trains of pulses using the same carrier frequency and no send-receive switch.
Standard Telephones and Cables Ltd. (assignees of I. A. Krause). Convention date (U.S.A.) 20th April, 1944.

591 898.—Telegraphic signalling system in which constant short pulses are transmitted in a predetermined sequence and are converted into morse dots and dashes in the receiver.
Marconi's W.T. Co. Ltd. (communicated by The Radio Corpn. of America). Application date 31st July, 1944.

591 968.—Two-way signalling system utilizing the same carrier-wave frequency, and two interlaced pulse-trains to provide one-way control.
Ace Electronics Ltd., L. C. Welch and R. J. Cook. Application date 5th April, 1945.

591 976.—Pulsed signalling system utilizing a timed base-wave to control multi-channel communication.
Standard Telephones and Cables Ltd. (assignees of D. D. Grieg). Convention date (U.S.A.) 12th June, 1944.

592 328.—Gating or synchronizing circuit for receiving and converting time-modulated pulsed signals into equivalent width-modulated signals.
T. J. McDermott. Application date 17th May, 1945.

592 412.—Selecting and receiving circuit for a multiplex signalling system employing a number of modulated pulse-trains having different repetition-frequencies.
Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 6th February, 1943.

592 414.—Pulsed signalling system using time-modulation free from phase-modulation, so as to ensure secrecy and freedom from jamming.
Standard Telephones and Cables Ltd. (assignees of E. Labin). Convention date (U.S.A.) 1st April, 1941.

592 779.—Receiver for a multichannel signalling-system using pulses of distinctive duration for each channel.
Standard Telephones and Cables Ltd. (assignees of D. D. Grieg). Convention date (U.S.A.), 29th July, 1944.

592 789.—Circuit for separating signalling-pulses having a given duration from a complex of pulses.
Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 15th May, 1943.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

590 463.—Construction of electron-discharge tube wherein a series of electrodes surrounding the cathode produce a rotating electric feed, for generating h.f. oscillations, or pulses, or for high-speed switching.
Western Electric Co. Inc. Convention date (U.S.A.) 29th August, 1942.

590 486.—Short-wave electron discharge tube with a concentric annular cathode, grid and anode system, and with grid-cathode and grid-anode cavity-resonators.
Standard Telephones and Cables Ltd. (assignees of C. V. Litton). Convention date (U.S.A.) 23rd November, 1942.

592 109.—Arrangement for holding a metal shielding-member firmly to the glass base of an electron-discharge tube.
Standard Telephones and Cables Ltd. and J. R. Hunt. Application date 18th May, 1945.

592 229.—Construction of double-beam cathode-ray tube with an electrostatic storage screen for use as a recording and reproducing device.
Western Electric Co. Inc. Convention date (U.S.A.) 12th October, 1943.

592 401.—Construction of a magnetron-oscillator embedded in the walls of a tubular waveguide.
Western Electric Co. Inc. Convention date (U.S.A.) 3rd May, 1941.

592 402.—Construction and operation, as an amplifier generator, or frequency-changer, of an electron discharge tube, particularly of the grounded-grid type, enclosed within a looped and shielded transmission-line.
Western Electric Co. Inc. Convention date (U.S.A.) 27th September, 1941.

592 860.—Process for depositing the sensitive layer on the fluorescent screen of a cathode-ray tube.
Cinema-Television Ltd. and *R. B. Head. Application date 1st February, 1945.*

SUBSIDIARY APPARATUS AND MATERIALS

590 574.—Signal-generator unit for testing and setting the trimming of tuned circuits, particularly the i.f. stages of a superhet receiver.
A. C. Cossor Ltd., L. H. Bedford and D. A. Bell. Application date 2nd January, 1945.

590 901.—Heterodyne system, including a phase-operated tripping circuit, for indicating, comparing and controlling the frequency of electrical oscillations.
Marconi's W.T. Co. Ltd. (assignees of G. L. Usselman). Convention date (U.S.A.) 27th August, 1943.

591 210.—Device for measuring the power flowing in, or the terminating impedance of, a coaxial cable or waveguide, comprising a slotted conductor adapted to be inserted into the channel and to energize an associated resonant chamber.
L. B. Turner and C. S. Wright. Application date 2nd May, 1945.

591 258.—Short-wave tuning device in which both the inductance and capacity are simultaneously varied.
" Patel Hold " Patentverwertungs and Elektro-Holding A.G. Convention date (Switzerland) 12th May, 1944.

592 097.—High-frequency switching device controlled by the movement of a conductor to and from the open end of a transmission-line (addition to 564 988).
The General Electric Co. Ltd. and D. C. Espley. Application date 6th March, 1946.

592 664.—Push-pull oscillation-generator, with Lecher-wire couplings, and automatic control under varying loads, particularly for the h.f. treatment of plastic materials.
H. Bruton, P. G. Parrish and Rotol Ltd. Application date 26th September, 1944.

593 187.—Valve frequency-multiplier with a two-part tuned input-circuit to minimize the adverse effect of feedback through the valve electrodes.
Standard Telephones and Cables Ltd. (assignees of R. B. Hoffman). Convention date (U.S.A.), 14th July, 1942.