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EDITORIAL

Alexander S. Popov *

IN his presidential address on "Earth, Stars and Radio" before Section A of the British Association at Dundee, Sir Edward Appleton referred to the jubilee of Marconi's invention and said "It is, of course, true that Marconi was not the first to use Hertzian waves for signalling. But, in addition to the introduction of various improvements in the details of the receiving apparatus, he was the first to insist on the importance of an earthed connection." A footnote has been added, however, saying that "Marconi's absolute priority in the use of an elevated antenna and an earth has recently been questioned as the result of careful study by V. A. Bailey and K. Landecker, who have shown that A. S. Popov published an account of his use of such devices for radio signalling some months before Marconi. Their conclusion is therefore 'that if Marconi has any claim to be an inventor of radio-communication, then Popov has prior claim over Marconi.' It will not be disputed, however, that the main stream of radio development has proceeded from the efforts of Marconi rather than from those of Popov."

This reference to Bailey and Landecker is based on an article by them which appeared in the *Australian Journal of Science* of February 1947 under the title "On the 'Inventor' of radio-communication."

In Professor Eric Ashby's recent book "Scientist in Russia" there is an interesting account of the efforts being made to boost the claim that Popov was the inventor of radio. "On May 7, 1945, the Bolshoi Theatre was packed with a distinguished audience . . . to celebrate the fiftieth anniversary of the invention of the radio by A. S. Popov. On the stage there sat scientists, marshals, admirals, commissars, leaders of the Party, and Popov's daughter. It was announced that in future 7 May would be celebrated as the 'Day of the Radio,' and Popov's name would be perpetuated by a monument in Leningrad, a gold medal to be awarded for inventions in the field of radio, plaques in the various houses where Popov lived and worked, an annual wireless exhibition in Moscow, a biography, a book entitled *Fifty Years of Radio*, and by naming a museum after Popov." Ashby is not carried away by these 'shrill festivities,' as he calls them, but suggests that there may be some substance in Popov's claim.

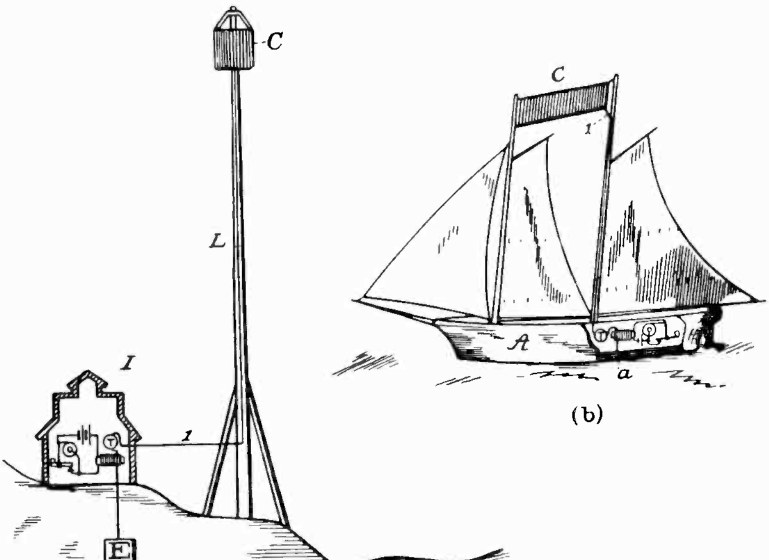
Pre-Hertzian Period

This raises some interesting questions. As long ago as 1842 Joseph Henry read a paper in which he described experiments with a decidedly Hertzian flavour. To see

* Whether one writes Popov or Popoff is largely a matter of taste. A German would pronounce the 'v' as an 'f' and every English dictionary tells us to pronounce 'of' as 'ov.'

how far the inductive effect of an intermittent current could be transmitted he placed a wire circuit in an upper room and excited it by a spark about an inch long from the prime conductor of an electrical machine. His secondary or receiving circuit was in a cellar thirty feet below with two intervening floors and ceilings each fourteen inches thick. The induced current was sufficiently strong to magnetize needles placed in a spiral coil in the circuit. *He also inserted such a spiral in a wire led from the roof down to his study, and found that the needles were magnetized by the current induced by distant lightning.* The wire was presumably earthed at the lower end. He also tried the inductive effect between two parallel wires 220 feet apart. With reference to these experiments he said in 1851 'as these are the results of currents in alternate directions, they must produce in surrounding space a series of plus and minus motions analogous to, if not identical with, undulations.' Maxwell was then a young man of twenty; Hertz was unborn.

In 1885 Edison took out a patent (U.S. 465971) for an inductive system of wireless telegraphy with a vertical earthed aerial



(a) (b)

for land stations and an inverted 'L' aerial for ship stations. He also proposed the use of balloons covered with conducting foil and connected through transmitting or receiving apparatus to earth. This was pre-Hertzian, and the aerial was energized by inserting in it the secondary of an induction coil, in the primary of which was a rotating make and break device which was normally short-circuited by the sending key. We reproduce one of the diagrams from the specification. The telephone receiver T was not of the Bell type, but was Edison's electromotograph receiver in which a rod fastened to the centre of the diaphragm carries a platinum point rubbing on a rotating cylinder of chalk mixed with potassium hydrate and mercuric acetate, and kept moist. 'When the electric waves are received, each wave as it passes through the chalk cylinder effects by electro-chemical decomposition more or less neutralization of the friction between the bar and the cylinder, according as the wave may be strong or weak. The diaphragm therefore undulates . . .' This quotation is dated 1879. It is noteworthy that this patent was acquired by the Marconi Company in 1903. Hence, without going back to Benjamin Franklin we find pre-Hertzian examples of the use of elevated aerials for the transmission and reception of signals, and for the reception of atmospherics. These were probably quite unknown to Marconi and Popov and do not detract from any credit due to them for inventing such devices.

It was in 1879 that Hughes gave a private demonstration of the transmission and reception of radio signals up to a distance of 60 feet. Those present included W. H. Preece and Sir William Crookes. By putting the receiving apparatus in his pocket and walking down the street with the telephone to his ear he received signals up to several hundred yards, but obtained no definite results at half a mile. On 20th Feb., 1880, he gave a demonstration to Professor Stokes and Professor Huxley, but the former is said to have pooh-poohed the results and said that they could all be explained by ordinary induction. This so disheartened Hughes that he did not pursue the matter any further. His transmitter was a spark-coil and his experiments were on somewhat similar lines to those of Hertz, but he had no conception of electromagnetic waves and ascribed his results to some mysterious conduction through the air. He never published an account of these experiments and always refused any recognition. In 1899, Sir William Crookes said "It is a pity that a man who was so far ahead of all other workers in the field of wireless telegraphy should lose all the credit due to his great ingenuity and prevision."

Post-Hertzian Period

The publication of Hertz's work in 1888 opened up a new world and caused physicists in all countries to repeat his experiments. Although Hughes, who had invented the microphone in 1878, used it as a detector in his unpublished experiments in 1879, it was Branly who in 1890 described the use of the coherer as a detector of Hertzian waves.

In 1892, in an article in the *Fortnightly Review*, Sir William Crookes said "Here is revealed the bewildering possibility of telegraphy without wires, posts, cables, or any of our present costly appliances . . . by concerted signals messages in the Morse code can thus pass from one operator to another . . . the correspondents must attune their instruments to a definite wave-length, say, for example, fifty yards." In 1894, Oliver Lodge transmitted and recorded signals across a distance of 60 yards. Although Popov had been experimenting with Hertzian waves, using mirrors, prisms, etc., it was Lodge's book on "The Work of Hertz" that gave a new direction to his work in 1894. He made various modifications in the apparatus described by Lodge, improving the coherer and the tapping arrangement, and he demonstrated his receiver to the St. Petersburg Physical Society in May 1895, in connection with a paper on "The Influence of Electrical Oscillations on Metallic Powders." The receiver was described in the Journal in Jan. 1896, under the title "Apparatus for Detecting and Recording Electrical Oscillations in the Atmosphere." Although only used as a lightning recorder *Popov expressed the hope that with certain improvements it might be employed for the reception of signals from a distance when a sufficiently strong source of oscillations had been discovered.* He arranged for the coherer tapper to ring a bell and he also connected in parallel with the tapper an electromagnetic recorder with a 12-hour time cylinder on which the lightning discharges were recorded. In the *Electrician* of 10th Dec. 1897, that is, eighteen months after Marconi had patented his invention, a letter from Popov dated 26th Nov. 1897, was published, giving translated extracts from his paper of Jan. 1896. Among other things he said "From July, 1895, until now my apparatus has worked very well as a lightning recorder. By using in the coherer tube a steel bead instead of iron filings I obtain a good coherer by which I can detect electromagnetic waves at a distance of 1 km if I work with Hertz's vibrator with 30-cm spheres and with the ordinary Siemens-Halske relay. With the Bjerknes vibrator of 90 cm diameter and a more sensitive relay I reach 5 km of good working." He then added: "From the foregoing remarks may be inferred that the arrangement of Marconi's receiver is a reproduction of my lightning recorder." It must not be inferred from this that Popov was accusing Marconi of copying his recorder; by reproduction he undoubtedly meant that the two were similar, as indeed they were to a large extent.

One must not picture Popov as an obscure young man working in the dark. He was sixteen years older than Marconi and in 1893 he had been one of the Russian delegates to the Chicago Exhibition, where he had been much impressed by the telautograph of Gray. He was born in the Urals in 1859, went to school at Perm and then went to St. Petersburg University, where, after graduation, he was an assistant in the physics laboratory. He then lectured in mathematics and physics at the Naval College at Cronstadt, where he later became head of the physics department, "Un cabinet de physique, en ces temps là le plus perfectionné de Russie, muni presque exclusivement d'appareils électriques et magnétiques de première qualité et de choix varié, dont le nombre était complété chaque année, grâce à des sommes assignées spécialement à cet effet. Il s'y trouvait aussi une bibliothèque, où l'on pouvait trouver la plupart des revues physiques et électrotechniques étrangères. C'est dans ce milieu favorable que se trouvait Popov, après avoir terminé ses études à l'Université de St. Pétersbourg."† On the other hand, we are told that "Popov travaillait dans les conditions fort primitives; il dut confectionner plusieurs objets de ses propres

† By his colleague Prof. Guéorguievsky in *Electritchestvo*, April 1925, p.10. Special French edition of Popov Memorial number.

mains,"‡ which suggests that there was a good supply of normal equipment but no workshop facilities. In 1901 he returned to St. Petersburg as Professor at the Electrotechnical Institute, and died in 1905.

In trying to assess the relative claims of Marconi and Popov one has to bear in mind that Marconi saw the enormous commercial possibilities of what he was trying to accomplish and realized the importance of withholding any publication until he had obtained a patent, whereas Popov had no such ideas and published his experimental work as opportunity offered. On the other hand, the publications being in Russian would probably remain unknown in other countries.

Apart from his published work Popov is reported to have collaborated with the Russian Navy in 1896 in experiments designed to test the possibility of signalling between ships by means of Hertzian waves, but as this was kept secret, it cannot be taken into account. Similarly Captain H. B. Jackson (later Admiral Sir Henry Jackson) in 1895 (i.e., before any publication by Marconi or Popov) had communicated by morse code between two ships by apparatus said to be very similar to that used by Marconi.

In the minutes of the meeting in May 1895 referred to above, at which Popov read his paper on "The Influence of Electrical Oscillations on Metallic Powders," there is a lengthy summary of his paper but no suggestion of radio telegraphy.§ Popov, as was his custom, spent the summer at Nijny-Novgorod, where he was in charge of the electric lighting of the Exhibition, and on his return he spent much of his time during the winter repeating the newly-discovered experiments of Röntgen. It was in January 1896 that he published the paper referred to above, and it was during this winter that he is said to have made experiments in conjunction with the Navy. It was in February 1896 that Marconi, who had been carrying out experiments at his father's country house at Pontecchio near Bologna, came to London, and in June took out the first patent ever granted for wireless telegraphy based on the use of electric waves. On March 12, 1896 Popov gave a second lecture to the Physical Society. He showed the transmission of Hertzian waves between different parts of the University buildings. The minutes report that "A. S. Popov showed apparatus for the public demonstration of the experiments of Hertz." Again no mention of radiotelegraphy or suggestion of anything of practical importance. As before, Popov put his experiments aside and spent the summer and autumn of 1896 at Nijny-Novgorod, and it was there in September that he read in the newspapers of Marconi's invention, which, as one of his colleagues said, shook him brusquely out of his torpor.

‡ By another colleague, Prof. Lébédinsky in the same journal, p.4.

§ *loc. cit*, p. 2.

A Misrepresentation of the Facts

There are several points on which we think that Professor Ashby may have been unwittingly misled by Bailey and Landecker, and consequently may have done less than justice to Professor Fleming. Ashby says "If he did in fact transmit and receive Hertzian waves over a distance of five kilometres, and publish that fact in January 1896, then it can be claimed that he used Hertzian waves *to transmit a message* before Marconi lodged his patent application." Whether he did or not, apparently. We have italicized some words which Ashby appears to regard as of no consequence, but which are very important. Ever since Hertz published his discovery various physicists had transmitted and received Hertzian waves over different distances, and Popov's experiments had been very successful, but what justification has Ashby for stating that he transmitted a message or even attempted to do so?

In his letter in the *Electrician* of Dec. 10, 1897 Popov quoted from his Russian publication of Jan. 1896, and after describing how the lightning discharge rang a bell and made a record on the time cylinder, he said: "In conclusion I can express my hope that my apparatus will be applied for signalling on great distances . . . as soon as there will be invented a more powerful generator of such vibrations." After this "in conclusion" he adds the statement already quoted above that "from July 1895 [he read his paper and gave the demonstration in May] until now my apparatus has worked very well *as a lightning recorder*. By using . . . I can detect electromagnetic waves at a distance of 1 km. and if I . . . I reach 5 km. of good working." It should be specially noted that there is here no suggestion of the transmission of a message. He was presumably recording artificially simulated lightning flashes produced by various types of Hertzian vibrators. Ashby says "How, then, has Popov's claim been overlooked? The answer to this question is to be found in one of the standard books on wireless telegraphy by J. A. Fleming. Fleming prints a translation of parts of Popov's paper, but he omits the part of Popov's report which refers to the transmission and receiving of waves over five kilometres, and he dismisses Popov's claim by saying: 'It is beyond question that the use he made of his apparatus was not the communication of intelligence to a distance, but for studying atmospheric electricity.' It appears that Fleming's account is quite misleading, and that before 1896 Popov, independently of Marconi, did transmit and receive Hertzian waves, and published an account of it six months earlier than Marconi."

This is certainly very misleading unless by the "communication of intelligence" one understands the recording of the fact that a lightning flash, or something simulating it, had occurred at a distance. Nobody questions the fact that Popov transmitted and received Hertzian waves and published an account of it six months before Marconi applied for his patent.

With regard to the omission by Fleming of the details at the end of Popov's letter, it is interesting to note that in the account of the paper given on p.1 of *Electritchestvo* of April 1925, we are told that "Popov *conclut* en ces termes: 'Je termine en exprimant l'espoir, que mon appareil moyennant certains perfectionnements pourra être employé pour la transmission des signaux à une certaine distance à l'aide de rapides oscillations électriques, aussitôt que sera trouvée la source d'oscillations électriques d'une puissance suffisante," and no more, so that if Fleming omitted anything of importance, the Popov memorial number did the same.

Having been shaken out of his torpor at Nijny-Novgorod in September 1896, Popov returned to St. Petersburg and resumed his experiments, and when in 1897 more detailed accounts of Marconi's experiment became available, Popov had extended his aerial to 18 metres and obtained transmission over about 5 km, which in 1898 was extended to 9.6 km.|| This makes one wonder whether the final part of his letter in the *Electrician* of Dec. 1897, with its reference to a range of 5 km was really a translation from his paper of Jan. 1896, or was added as an item of current information. In the latter case Fleming was doubly justified in omitting any reference to it. In 1899 Popov visited the embryo wireless stations in Germany and France.

Towards the end of 1899 he succeeded in communicating between two points 47 km apart by means of radio; by this means communication was established with a cruiser that had run on to a reef. According to his colleague, "Vers la fin de 1899 le service de la radiotélégraphie fut utilisé pour la première fois."

By this time the company formed to acquire the Marconi patents was already two years old, Marconi had established communication across the English Channel, had been to Italy to demonstrate his system to the King, and was beginning to cast his eyes across the Atlantic. We have it on the authority of his colleague Professor Lébédinsky that when Popov returned from his visit to the wireless stations in France and Germany in 1899 he remarked that "nous n'étions pas beaucoup en retard sur les autres."|| This, we feel, was a thoroughly justifiable statement. He was a pioneer in the development of devices for the reception and recording of electromagnetic waves of the type produced by lightning flashes, but to say that he has a prior claim over Marconi as an inventor of radio-communication is to show no conception of what constitutes radio-communication. One must honour his memory as that of a Russian scientist who did brilliant experimental work in connection with Hertzian waves, but he would be the first to disclaim precedence over Marconi as the inventor of radio-communication. He would say, as he did in 1899, "but I was not very far behind."

In this review of the work of Popov we have depended entirely on pro-Popovian literature and have given many of the quotations from the 1925 Popov Memorial number of the Russian *Electritchestvo* in the original French, so that the reader may be in a position to form his own opinion as to the justification for the propaganda being so feverishly pursued in some quarters with the object of having Popov recognised as the inventor of radio-telegraphy.

G.W.O.H.

|| *loc. cit.*, p. 7.

ELECTRONIC TUNING OF REFLECTION KLYSTRONS*

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SUMMARY.—A simple theory of the electronic frequency control of a reflection klystron is developed by which it is possible to calculate the frequency deviation $\pm (\Delta f)_m$ over which the power output does not fall below a certain fraction m of the power output at zero frequency deviation. It is shown that for a simple resonator of high impedance $(\Delta f)_m = \pm A\beta^2 I_0^2/CW_1$ where β = modulation factor of resonator gap, I_0 = beam current, C = resonator capacitance, W_1 = power drawn from the beam, and A is a Bessel Function expression whose maximum value is derived as a function of m. In particular the frequency range from half-power to half-power has a maximum value of $(\Delta f)_{\frac{1}{2}} = \pm 52\ \beta^2 I_0^2/CW_1$ Mc/s (I_0 in mA, C in $\mu\mu$F, and W_1 in mW) which occurs when the tube is loaded to deliver its maximum output. When the tube is used with a lightly-coupled load such as a crystal mixer the resonator may require to be loaded with some extra loss to obtain the maximum value of $(\Delta f)_{\frac{1}{2}}$.

1. Introduction

At centimetre wavelengths the receiving system normally employed consists of a crystal mixer, with a reflection klystron acting as the local oscillator. The frequency of oscillation of the klystron is primarily controlled by its sharply-tuned resonator, but it is possible to vary the frequency within narrow limits by changing the negative potential applied to the reflector electrode. This method is called "electronic tuning" and its importance lies in its use for the automatic-frequency control of receivers; it provides generally a more rapid and more convenient control than a mechanical system.

This "electronic tuning" depends on the interaction of the electron beam with the oscillating field in the resonator, and is thus intimately connected with the mechanism of the maintenance of oscillations. A brief description of this mechanism is therefore necessary.

In its passage through the alternating electric field of the resonator, the electron beam is modulated in velocity, some of the electrons being speeded up while those passing through half a period later are slowed down. After leaving the resonator, the beam approaches an electrode maintained at a negative potential and shaped so as to "reflect" the electrons back through the resonator. The time taken by an electron to return to the resonator depends on its velocity; since the faster electrons travel further towards the reflecting electrode their return to the resonator is delayed and coincides with that of slower electrons which started later. The periodic velocity modulation of the electron beam is thus transformed into a periodic density modulation, or "bunching." If the "bunches" of electrons return at an instant when their passage through the resonator is retarded by the alternating electric field, while the "rarefactions" are accelerated, there is a net transfer of energy from the electron beam to the alternating field. If this energy transfer exceeds the energy dissipated by resistive loss in the resonator, oscillations will be sustained.

If the potentials applied to the reflection klystron are such that the return of a bunch coincides with maximum retarding field in the resonator, the amplitude of oscillation is a maximum and the frequency of oscillation is the natural frequency of the resonator. If the bunch of electrons passes through the resonator before the retarding field has reached its maximum value, a small reactance is reflected into the resonator which slightly increases the frequency of oscillation. Similarly, if the bunches return after the retarding field has passed its maximum value, the frequency of oscillation is reduced. In this way an "electronic tuning" of the oscillator is produced. The time of return of the bunches can be varied by changing either the main accelerating potential (i.e., the initial velocity of the electron beam,) or by changing the potential of the reflecting electrode. The latter is generally more convenient since the beam current depends only indirectly on the potential of the reflector, and the latter takes no current.

* MS. accepted by Editor, October 1946.

When the frequency of oscillation is "pulled" in this way, the power extracted from the beam is diminished owing to the phase angle between the density modulation of the beam and the alternating electric field of the resonator. The amplitude of the oscillations, therefore, decreases steadily to zero as their frequency deviates more and more from the natural frequency of the resonator. In applications such as automatic-frequency control large variations in the local-oscillator power output are not permissible, since the conversion efficiency of the frequency changer is impaired. Thus it is only possible to utilize part of the electronic-tuning range of the valve, the requirement being that the power output shall not fall below a certain fraction (e.g., one-half) of the maximum.

In the following sections a simple theory is developed which explains the main features of electronic tuning. The variation of the strength of oscillation as the frequency is "pulled" from the natural frequency of the resonator is considered, and it is shown that there is an optimum resonator impedance which produces the greatest electronic-tuning range for a given variation in the amplitude of oscillation. The design of oscillators to produce maximum tuning range is considered, and a simple formula is derived showing how this range depends on the power output required, the beam current and the effective capacitance of the resonator.

2. Theory

If I_0 is the mean current in the electron beam, then the density-modulated current returning through the resonator may be written as

$$I = I_0\{1 + \alpha\cos(\omega t + \eth) + \text{terms in } 2\omega t, \text{etc}\}$$

Here α is a dimensionless factor determined by the degree of bunching of the beam, and $\eth$ is the phase angle between the density modulation and the resonator voltage V; $\cos \omega t$. $\eth$ is taken to be zero when the bunches of electrons suffer maximum retardation in their return through the resonator.

The power W being transferred to the resonator from the beam at any instant depends on the product of the oscillatory voltage and the term in the density modulation of the electron beam which varies with ωt. Thus we have

$$W = (\beta V \cos \omega t)\ \{\alpha I_0 \cos(\omega t + \eth)\}$$
$$= \beta V I_0 \alpha (\cos^2 \omega t \cos \eth - \cos \omega t \sin \omega t \sin \eth) \quad \ldots \quad (1)$$

Here β is a dimensionless factor which takes into account the fact that the voltage impressed on the electrons during their transit through the resonator gaps is less than the instantaneous voltage on the gap, owing to the finite transit time. The value of β lies between 0.5 and 1 in most oscillators.

At frequencies near resonance the impedance of the resonator will vary like that of a parallel-tuned circuit, of capacitance C and parallel resistance R, and one may write

$$1/Z = 2j\Delta\omega C + 1/R \quad \ldots \quad \ldots \quad (2)$$

where $\Delta\omega$ = deviation of the angular frequency from the resonant value. Hence the power in the resonator, when the oscillatory voltage is $V \cos \omega t$, will be

$$W' = V^2\left(\frac{\cos^2 \omega t}{R} - 2\Delta\omega C \cos \omega t \sin \omega t\right) \quad \ldots \quad \ldots \quad \ldots \quad (1a)$$

In the steady state the power dissipated in the resonator must equal the power extracted from the electron beam. Hence, equating W to W', one finds

$$\left.\begin{array}{l}\alpha\beta I_0 \cos \eth = V/R \\ \alpha\beta I_0 \sin \eth = 2\Delta\omega CV\end{array}\right\} \quad \ldots \quad \ldots \quad (3)$$

The bunching parameter α depends on the voltage swing V, the initial velocity of the beam, and the distribution of the electric field in the space between the resonator and the reflector electrode. By an analysis similar to that carried out by Webster[1] for the klystron, it may be shown that

$$\alpha = 2J_1(kV)$$

where k depends only the velocity of the electrons and the reflecting field. When the reflector voltage is changed in order to produce electronic tuning of the oscillator, the primary effect is a change in the mean time taken by the electrons to return to the resonator; i.e., a first order change in the phase constant $\eth$. In general the value of k will also be slightly changed but in the ideal case k may be regarded as a constant for a given set of oscillator conditions.† Insertion of this value of α in (3) yields

$$\left.\begin{array}{l}2\beta I_0 J_1(kV) \cos \eth = V/R \\ 2\beta I_0 J_1(kV) \sin \eth = 2\Delta\omega CV\end{array}\right\} \quad \ldots \quad (3a)$$

Elimination of $\eth$ gives

$$1/R^2 + 4C^2\Delta\omega^2 = 4\beta^2 I_0^2 J_1^2(kV)/V^2 \quad (4)$$

Since the value of $J_1(kV)/V$ falls steadily from $k/2$ to zero as (kV) increases from zero

† The main effect of a small variation in k, due to the change in reflector voltage, is to make the curves of power versus frequency pulling (Fig. 1.) asymmetrical.

to 3.83, it is obvious that the voltage swing in the resonator must decrease steadily to zero as $\Delta\omega$ is numerically increased; i.e., as the oscillating frequency is pulled further from the natural frequency of the resonator. If a detector is lightly coupled to the oscillator, the power registered by the detector is proportional to the square of the voltage swing V. It is interesting, therefore, to plot V^2 as a function of $\Delta\omega$, (Fig. 1) for various values of resonator impedance R. In the ideal case of a lossless resonator, ($R = \infty$), Equation (4) reduces to

$$(\Delta\omega') = \pm \frac{\beta I_0}{C} \frac{J_1 k(V)}{V} \quad \dots \quad \dots \quad (5)$$

that with a resonator of finite impedance a curve of (V^2, $\Delta\omega$) will be obtained lying wholly inside the ideal case shown dotted in Fig. 1. A set of typical curves may easily be constructed using the relation that, for a given value of V^2,

$$(\Delta\omega)^2 = (\Delta\omega')^2 - \left(\frac{R'}{R}\right)^2 (\Delta\omega')^2_{max} \quad \dots \quad (8)$$

obtained from Equations (4), (5), (6) and (7). Using R'/R as a dimensionless parameter, three curves have been drawn corresponding to values of 0.2, 0.5 and 0.8 for R'/R. The ideal dotted curve corresponds, of course, to $R'/R = 0$, while the curve for $R'/R = 1$ consists only of the singular point at the origin.

It will be seen that as the resonator impedance is lowered (i.e., R'/R is increased) the first effect is to reduce the sharp peak in V^2 without greatly effecting the tails of the curve. This flattening of the curve means that the power registered by the loosely-coupled detector will vary much less rapidly as the frequency is pulled by the electronic tuning. Thus when the reflection klystron is used as a local oscillator loosely coupled to the crystal frequency changer, more electronic tuning for a given variation in crystal current is obtained if the resonator is damped. From the curves of Fig. 1, it will be seen that, finally, as R'/R approaches unity the range of tuning decreases as the curve ultimately shrinks to a point at the origin. There is, therefore, an optimum resonator impedance which gives maximum electronic-tuning range for a certain permitted variation of crystal current.

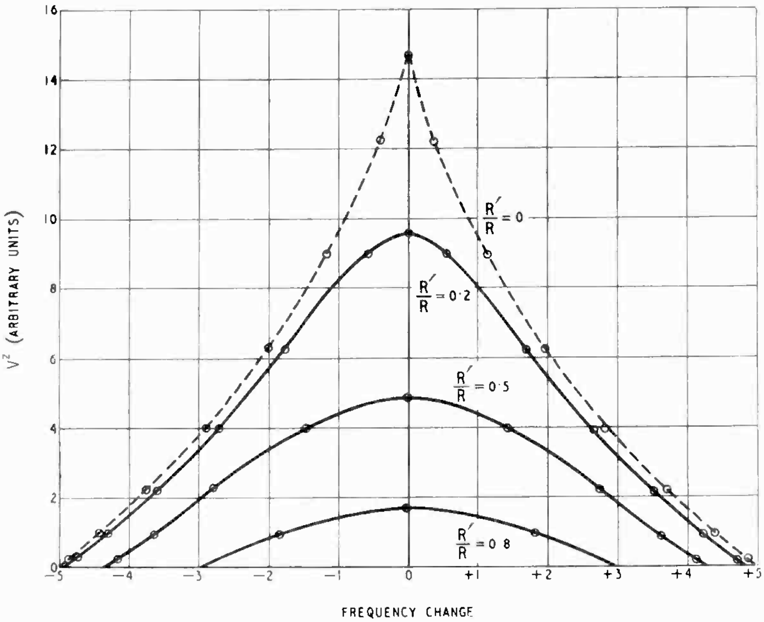

Fig. 1. V^2 is shown plotted against $\Delta\omega$ for various values of the resonator impedance. The dotted curve represents the ideal case (infinite resonator impedance).

In Fig. 1, this ideal case is shown as a dotted line. One may note that the maximum frequency change possible in this ideal case may be written

$$(\Delta\omega')_{max} = \pm \frac{1}{2CR'} \quad \dots \quad \dots \quad (6)$$

where R' is the lowest value of resonator impedance at which it is possible to maintain oscillations. For, since, $J_1(x)/x \to \frac{1}{2}$ as $x \to 0$, we have from (5)

$$(\Delta\omega')_{max} = \pm \frac{k\beta I_0}{2C}$$

and from (3a), putting $\partial = 0$

$$k\beta I_0 = \frac{1}{R'} \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

From the form of Equation (4) it is obvious

3. Design of Oscillators for Electronic Tuning

In the preceding section it has been tacitly assumed that ample power is available from the oscillator for use as a local oscillator. It will be seen later that in general a valve designed to give greater electronic tuning will give lower power output. For use as a local oscillator the valve must give a certain minimum power in order that sufficient drive may be applied to the mixer valve without coupling it strongly to the oscillator. It is necessary therefore to manipulate our equations so as to introduce a fixed quantity W_1, the power delivered to the resonator by the beam at the centre of the electronic tuning range‡. From Equation (3a), putting $\partial = 0$, we have

$$W_1 = V_1^2/2R = \beta I_0 V_1 J_1(kV_1) \quad \ldots \quad (9)$$

where V_1 is the voltage swing when $\partial = 0$.

The useful electronic tuning range is defined by the fact that the power must not fall below a certain fraction m of W_1. This means that the voltage swing V at the limit of the range must equal $V_1\sqrt{m}$. Equation (4) becomes now

$$\frac{1}{R^2} + 4C^2(\Delta\omega)^2 = 4\beta^2 I_0^2 J_1^2(kV_1\sqrt{m})/mV_1^2 \quad \ldots \quad (4a)$$

whence, putting $\Delta\omega = 0$,

$$\frac{1}{R^2} = 4\beta^2 I_0^2 J_1^2(kV_1)/V_1^2 \quad \ldots \quad (4b)$$

To obtain the maximum possible electronic tuning, it is convenient to take C, β and I_0 as fixed parameters, in addition to W_1 and m, which are fixed by factors external to the oscillator. The parameters R, V and k remain as variables related by Equations (9), (4a) and (4b); it is convenient to take (kV_1) as an independent variable as the equations can then be combined in the form

$$(\Delta\omega) = \pm \frac{\beta^2 I_0^2}{CW_1}\left[J_1(kV_1)\sqrt{\frac{J_1^2(kV_1\sqrt{m})}{m} - J_1^2(kV_1)}\right] \quad \ldots \quad (10)$$

To obtain the maximum electronic tuning, $(\Delta f)_m = (\Delta\omega)_m/2\pi$, the maximum value of the Bessel function expression in the square brackets must be found graphically. Then one may write

$$(\Delta f)_m = \pm A\beta^2 I_0^2/CW_1 \quad \ldots \quad \ldots \quad (11)$$

‡ The maximum power which can be drawn from the oscillator into an external load is less than W_1 owing to resistive losses in the resonator; but as W_1 cannot be measured directly it is customary to specify a certain power output. Consideration of the equivalent circuit shows, however, that it is the quantity W_1 which must be considerably greater than the power drawn by the crystal mixer, if the coupling between the oscillator and mixer is to be small enough to prevent material loss of signal power into the oscillator. For the effect of drawing power into the mixer may be represented by a resistance R'' placed in parallel with the resonator impedance R. If R'' is large compared with R the mismatch will prevent loss of signal into the oscillator; at the same time the power developed in R'' will be small compared with that in R, since the same voltage acts across both resistances. That is, W_1 is large compared with the power required by the crystal mixer. It is therefore justifiable as well as convenient to use W_1 in developing the theory.

where A is a pure number which is shown as a function of m in Fig. 2. The resonator impedance R_m required to give this maximum may be found by inserting the appropriate values of the Bessel function in (9) and (4b) which combine to give

$$R_m = BW_1/\beta^2 I_0^2 \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where B is again a pure number also shown as a function of m in Fig. 2.

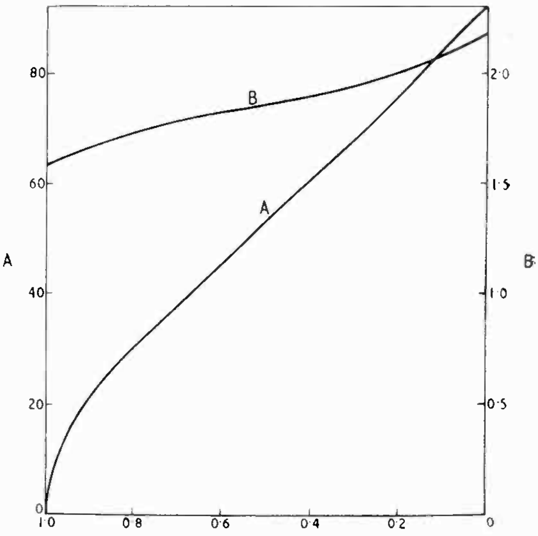

Fig 2. *Values of A and B used in Equations (11) and (12) are given as functions of m.*

The units chosen for A and B are such that if I_0 is in milliamperes, C in micro-micro-farads and W_1 in milliwatts, $(\Delta f)_m$ will be in megacycles per second and R in kilohms.

It will be seen from Fig. 2 that the value of R_m is never greatly different from $R_{\frac{1}{2}}$, the optimum value for pulling from half-power to half-power. The value of $(\Delta f)_m$ rises steadily, of course, as m is decreased.

The maximum value $(\Delta f)_m$, given by Equation (11), will only be attained by suitable adjustment of the bunching constant k. The required value of k can be found from (9), which may be written

$$W_1 = \frac{\beta I_0 (kV_1) J_1(kV_1)}{k}$$

whence

$$k = \frac{\beta I_0}{W_1}(kV_1) J_1(kV_1) \quad \ldots \quad \ldots \quad (13)$$

and the value of kV_1 is fixed as that required to make (Δf) a maximum.

On inserting the values of (kV_1) appropriate to the range $1 > m > 0$, one finds that k does not vary by more than 3 per cent over the entire range, having the mean value

$$k = 1.2\, I_0/W_1 \text{ (volts)}^{-1} \quad \ldots \quad \ldots \quad (13a)$$

where the units of I and W_1 are as before.

This means that the reflector field should be designed to give this value of k in order to obtain maximum frequency pulling for any value of m.

The most commonly used value of m is $\frac{1}{2}$, for which the maximum occurs when $(kV_1) = 2.4$, and we have then

$$(\Delta f)_{\frac{1}{2}} = \pm\, 52\, \beta^2 I_0{}^2/CW_1 \text{ Mc/s} \quad \ldots \quad (11a)$$

This formula gives immediately the maximum possible value of $(\Delta f)_{\frac{1}{2}}$ when β, the beam current, the capacitance of the resonator and the power output required are determined. The resonator impedance $R_{\frac{1}{2}}$ required to give this maximum value is

$$R_{\frac{1}{2}} = W_1/2\beta^2 I_0{}^2 J_1{}^2\,(2.4) = 1.8\, W_1/\beta^2 I_0{}^2 \text{ k}\Omega \quad \ldots \quad (12a)$$

Now the power delivered to the resonator at its natural frequency is, by equation (9), a maximum when

$$d/dV_1\,\{V_1 J_1\,(kV_1)\} = kV_1 J_0\,(kV_1) = 0$$

The appropriate root of this equation is $kV_1 = 2.4$, that is, the same value as required to make $(\Delta f)_{\frac{1}{2}}$ a maximum. The frequency pulling from half-power to half-power is, therefore, a maximum when maximum power is being delivered to the resonator. This concurrence depends on the fact that we have taken the value of $\frac{1}{2}$ for m. If a wider variation of power output can be permitted (i.e., $m < \frac{1}{2}$) $(\Delta f)_m$ is a maximum for a value of (kV_1) greater than 2.4; i.e., the resonator should be rather less damped than for $m = \frac{1}{2}$. For $m > \frac{1}{2}$, the resonator should be more heavily damped to obtain the maximum value of $(\Delta f)_m$.

4. Discussion

Equation (11) shows that for a given resonator shape (i.e., given values of β and C), and a given current the electronic-tuning range can only be increased by reducing the power output W_1 of the oscillator. This limitation was discovered experimentally early in the design of reflection klystrons, and the reason for it can be seen to be as follows.

Under these conditions the electronic tuning can only be increased by increasing the bunching constant k; i.e., by increasing the degree of bunching of the beam for a given voltage swing in the resonator. This may be achieved, for instance, by withdrawing the reflector so as to increase the time taken by the electrons to return to the resonator after their first transit. It follows that complete bunching will be obtained at a lower voltage swing in the resonator, and this reduced voltage means that the power is less, corresponding to the presence of W_1 in the denominator of Equation (11).

If W_1 is fixed, as is usual, by the demands of the crystal mixer, then the electronic-tuning range can only be increased by improving the β factor, increasing the beam current or reducing the capacitance of the resonator. These requirements are conflicting, since most of the capacitance resides in the small gap in the resonator through which the beam flows. Once the limit of electron-gun design has been reached the current can only be increased by increasing the area of the gap, which will, of course, also increase the capacitance. On the other hand, the capacitance cannot be reduced by enlarging the separation of the resonator trumpets since this reduces the β factor of the gap. Thus the best performance can only be obtained by a compromise in the design of the resonator which gives the maximum value of $(\beta^2 I_0{}^2/C.)$ The reflector electrode must then be designed to give the correct value of the bunching constant k, as well as the maximum efficiency in reflecting the electrons back through the resonator. Finally, the resonator impedance must be adjusted to the value demanded by Equation (12a).

The curves of Fig. 1 strongly suggest that the optimum value of $R_{\frac{1}{2}}$ is connected with the "starting impedance" R' by a simple numerical factor. Combination of equations (7), (12a) and (13a) yields the relation

$$R_{\frac{1}{2}} = 2.2R' \quad \ldots \quad \ldots \quad \ldots \quad (12b)$$

i.e., the optimum resonator impedance is little more than twice the lowest impedance at which oscillations can be sustained in the resonator. In most oscillators the unloaded impedance of the resonator is considerably greater than the starting impedance. From the point of view of power output this is a desirable feature since it means that most of the power delivered by the electron beam can be extracted into an external load.

The loading at which the power output is a maximum is practically the same as that required to extract greatest power from the electron beam, and the frequency pulling to half-power will then also be a maximum.

If the klystron is used to supply local-oscillator power to a mixer it is, in general, very lightly loaded. Artificial damping of the resonator is thus required to bring its impedance down to the optimum value required by Equation (12b). The increase in the electronic-tuning range obtained by such damping was first observed experimentally. It should be noted that this increase is not due to the fact that the resonance curve of the cavity is broadened (i.e., its "Q" is lowered), but to the reduction in the voltage swing in the resonator to the point at which maximum power is extracted from the electron beam; i.e., the point at which the interaction between the beam and the resonator is greatest. As far as the resonator itself is concerned, what is required to increase the electronic-tuning range is not simply a lower "Q," but a smaller capacitance C. That is, a reduction in the reactive component of the resonator impedance when off tune.

5. Acknowledgment

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TRANSIENT RESPONSE OF SYMMETRICAL 4-TERMINAL NETWORKS*

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SUMMARY.—This paper gives a simple method of calculating the transient currents which flow at the input and output of a symmetrical 4-terminal network. It uses equations hitherto neglected, and otherwise laborious problems are thereby easily solved; in particular an exact solution for the transient response to a unit-step voltage of a 6-element band-pass filter connected between resistive terminations can be obtained.

The equations can be applied to steady-state problems, but the simplification which they produce is not so marked. Using the method, the synthesis of circuits of a given transient response is sometimes facilitated.

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1. Introduction

THE transient response of a network can be calculated from a consideration of its circuit equations, but the differential equations to be solved are often of high order and their solution is very difficult. For example, the differential equation for a 6-element band-pass filter is of the sixth order. The solution of an equation of this order is very laborious even assuming that the roots of the auxiliary equation can be found.

It is shown in this paper how, from a consideration of the fundamental transmission equations of a 4-terminal network, two simple equations, differing from each other only by the sign of one of the terms, can be obtained, describing completely the input and output currents of the network in terms of the responses of two 2-terminal networks.

2. General Equations of a Network

The input and output currents I_s and I_r respectively, of any symmetrical 4-terminal network of transfer constant ϕ, and image impedance Z_0, terminated with an impedance

* MS. accepted by the Editor, December 1946.

R_0 and connected to a generator of e.m.f. E and internal impedance R_0, are given by[1,†].

$$I_s = E\,\frac{Z_0\cosh\phi + R_0\sinh\phi}{2Z_0R_0\cosh\phi + (Z_0^2 + R_0^2)\sinh\phi} \quad \ldots \quad (1)$$

$$I_r = E\,\frac{Z_0}{2Z_0R_0\cosh\phi + (Z_0^2 + R_0^2)\sinh\phi} \quad \ldots \quad (2)$$

Now $\sinh\phi = \dfrac{2}{\coth\phi/2 - \tanh\phi/2}$

and $\cosh\phi = \dfrac{\coth\phi/2 + \tanh\phi/2}{\coth\phi/2 - \tanh\phi/2}$

$$\therefore I_r = \frac{E}{2}\cdot\frac{Z_0\coth\phi/2 - Z_0\tanh\phi/2}{R_0(Z_0\coth\phi/2 + Z_0\tanh\phi/2) + Z_0^2 + R_0^2} \quad \ldots \quad (3)$$

But $Z_0^2 = Z_0\tanh\phi/2\,.\,Z_0\coth\phi/2$

since $\tanh\phi/2\,.\,\coth\phi/2 = 1$

$$\therefore I_r = \frac{E}{2}\cdot\frac{Z_0\coth\phi/2 - Z_0\tanh\phi/2}{R_0Z_0\coth\phi/2 + R_0Z_0\tanh\phi/2 + Z_0\tanh\phi/2.\,Z_0\coth\phi/2 + R_0^2} \quad \ldots \quad (4)$$

$$= \frac{E}{2}\cdot\frac{Z_0\coth\phi/2 - Z_0\tanh\phi/2}{(R_0 + Z_0\tanh\phi/2)(R_0 + Z_0\coth\phi/2)} \quad \ldots \quad (5)$$

$$= \frac{E}{2}\cdot\left(\frac{1}{R_0 + Z_0\tanh\phi/2} - \frac{1}{R_0 + Z_0\coth\phi/2}\right) \quad \ldots \quad (6)$$

Similarly

$$I_s = \frac{E}{2}\cdot\frac{Z_0\coth\phi/2 + Z_0\tanh\phi/2 + 2R_0}{R_0Z_0\coth\phi/2 + R_0Z_0\tanh\phi/2 + Z_0\tanh\phi/2.\,Z_0\coth\phi/2 + R_0^2} \quad \ldots \quad (7)$$

$$= \frac{E}{2}\cdot\left(\frac{1}{R_0 + Z_0\tanh\phi/2} + \frac{1}{R_0 + Z_0\coth\phi/2}\right) \quad \ldots \quad (8)$$

No earlier presentation of equations (6) and (8) has been found in the literature.

Consider now the lattice network of Fig. 1 having series and lattice arms of impedances Z_1 and Z_2 respectively, terminated by an impedance R_0, and connected to a generator of e.m.f. E and internal impedance R_0. The image impedance Z_0 and transfer constant ϕ of such a network are given by the equations[2,3,4]

$$Z_1 = Z_0\tanh\phi/2 \quad \ldots \quad (9)$$

$$Z_2 = Z_0\coth\phi/2 \quad \ldots \quad (10)$$

Therefore, from equations (8) and (6), the input and output currents I_s and I_r of a lattice network, are given by :—

$$I_s = \frac{E}{2}\left(\frac{1}{R_0 + Z_1} + \frac{1}{R_0 + Z_2}\right) \quad \ldots \quad (11)$$

$$I_r = \frac{E}{2}\left(\frac{1}{R_0 + Z_1} - \frac{1}{R_0 + Z_2}\right) \quad \ldots \quad (12)$$

Inspection of Fig. 2 will show that equations (8) and (6) also give the input and output currents of a lattice network with series and lattice arms of impedances $(R_0 + Z_1)$ and $(R_0 + Z_2)$ respectively, short circuited at the output, and connected to a zero

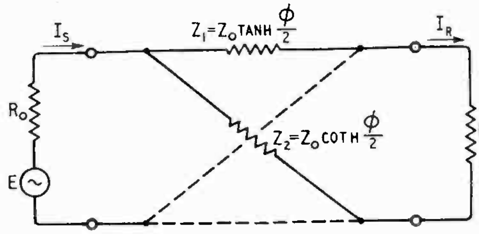


Fig. 1. *Basic lattice network.*

impedance generator of e.m.f. E at the input.

Equations (8), (6), (11), and (12) can be proved by another method. This method is given below, and was the way in which the author originally obtained these equations.

In lattice-network filter design, use is made of a transformation[3,4,5] whereby impedances common to the series and lattice arms of the network can be removed, and a single impedance of the same value connected in series with the input, and another of the same value in series with the output. The opposite transformation can be made, whereby equal impedances in series with the input and output of the network, can be connected in series with the series and lattice arms. Following this argument, the transformation of a lattice network from the form shown in Fig. 1 to that in Fig. 2 is apparent.

The lattice network of Fig. 2 is connected to a zero-impedance generator, and is short-circuited at the output. The currents in the series and lattice arms of the network are

† See p. 185 of. Ref. (1).

therefore independent of each other. Calling these currents I_1 and I_2 respectively, the input and output currents of the network are given by :—

$$I_s = I_1 + I_2 \quad \ldots \quad (13)$$
$$I_r = I_1 - I_2 \quad \ldots \quad (14)$$

By redrawing the network as Fig. 3, it is at once obvious that :—

$$I_1 = \frac{E}{2(R_0 + Z_1)} \quad \ldots \quad (15)$$
$$I_2 = \frac{E}{2(R_0 + Z_2)} \quad \ldots \quad (16)$$

Whence :—

$$I_s = \frac{E}{2}\left(\frac{1}{R_0 + Z_1} + \frac{1}{R_0 + Z_2}\right) \quad \ldots \quad (17)$$
$$I_r = \frac{E}{2}\left(\frac{1}{R_0 + Z_1} - \frac{1}{R_0 + Z_2}\right) \quad \ldots \quad (18)$$

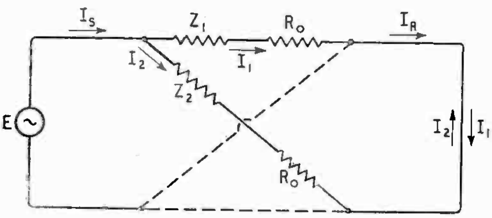


Fig. 2. *Short-circuit currents of lattice network.*

Since the lattice is the general form[4] of all symmetrical 4-terminal networks, the equations (8) and (6) follow, by the reverse argument of that used in obtaining equations (11) and (12).

By using Bartlett's Bisection Theorem[2, 3, 4] the values $Z_0 \tanh \phi/2$ and $Z_0 \coth \phi/2$ in equations (8) and (6), can be obtained for any symmetrical 4-terminal network having image impedances Z_0 and transfer constant ϕ, in terms of the impedances of which the network is made. With the values of $Z_0 \tanh \phi/2$ and $Z_0 \coth \phi/2$ substituted in equations (8) and (6), the currents flowing into and out of the network can be calculated.

The equations (8) and (6) are fundamental, and can be used either in steady state calculations, for which R_0, Z_0 and ϕ are functions of $j\omega$, or in transient problems, where R_0, Z_0, and ϕ are functions of the differential operator $p = \frac{d}{dt}$. In solving the latter problems, use of the operational calculus may be necessary.

In the study of transients, it is often convenient to redraw the network under consideration, with the configuration of Fig. 2. If the transient responses of the impedances $(R_0 + Z_1)$ and $(R_0 + Z_2)$ are known, the response of the network can be written down immediately.

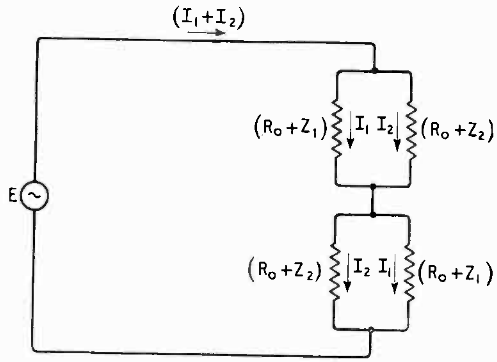


Fig. 3. *Equivalent circuit of Fig. 2.*

Thus if

$$I_1(t) = \text{transient response of } (R_0 + Z_1) \quad (19)$$
$$I_2(t) = \text{transient response of } (R_0 + Z_2) \quad (20)$$

Then

$$I_s = \tfrac{1}{2}\{I_1(t) + I_2(t)\} \quad \ldots \quad (21)$$
$$I_r = \tfrac{1}{2}\{I_1(t) - I_2(t)\} \quad \ldots \quad (22)$$

Equations (21) and (22) will hereafter be written in the form :—

$$I_{s.r} = \tfrac{1}{2}\{I_1(t) \pm I_2(t)\} \quad \ldots \quad (21), (22)$$

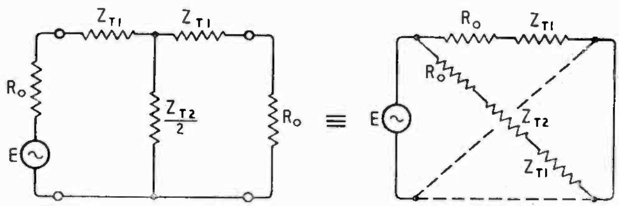


Fig. 4. *Equivalent T and lattice networks.*

where $I_{s,r}$ means that I_s is given by taking the +ve sign and I_r by taking the —ve sign.

3. Solutions for Well-known Networks

Figs. 4, 5, and 6 show the transformation of " T," " π," and " Bridged-T " networks to the lattice form of Fig. 2.

The equations for the input and output currents in terms of the impedances of these networks are given hereafter :—

" T " *Network.*

$$I_{s,r} = \frac{E}{2}\left(\frac{1}{R_0 + Z_{T1}} \pm \frac{1}{R_0 + Z_{T1} + Z_{T2}}\right) \quad .. \quad .. \quad .. \quad (23), (24)$$

" π " *Network*

$$I_{s,r} = \frac{E}{2}\left(\frac{1}{R_0 + \dfrac{Z_{\pi1}\,Z_{\pi2}}{Z_{\pi1} + Z_{\pi2}}} - \frac{1}{R_0 + Z_{\pi2}}\right) \quad .. \quad .. \quad .. \quad (25), (26)$$

" *Bridged*-T " *Network*

$$I_{s,r} = \frac{E}{2}\left(\frac{1}{R_0 + \dfrac{R_1 Z_{B1}/2}{R_1 + Z_{B1}/2}} \pm \frac{1}{R_0 + 2Z_{B2} + R_1}\right) \quad .. \quad .. \quad .. \quad (27), (28)$$

applied voltage of $\frac{E}{2}$, which is the current of the series arm of the network Fig. 7(b), is then given by :—

$$I_1 = \frac{E}{2}\cdot\frac{1}{2}\left(\frac{1}{R_0} + \frac{1}{R_0 + Z_{B1}}\right) \quad .. \quad (29)$$

The current I_2 in the lattice arm of Fig. 7(b) is given by :—

$$I_2 = \frac{E}{2}\cdot\frac{1}{2}\left(\frac{1}{R_0 + Z_{B2}}\right) \quad .. \quad .. \quad (30)$$

whence the input and output currents I_s and I_r respectively of the complete network are :—

$$I_{s,r} = \frac{E}{4}\left(\frac{1}{R_0} + \left[\frac{1}{R_0 + Z_{B1}} \pm \frac{1}{R_0 + Z_{B2}}\right]\right) \quad .. \quad .. \quad .. \quad (31), (32)$$

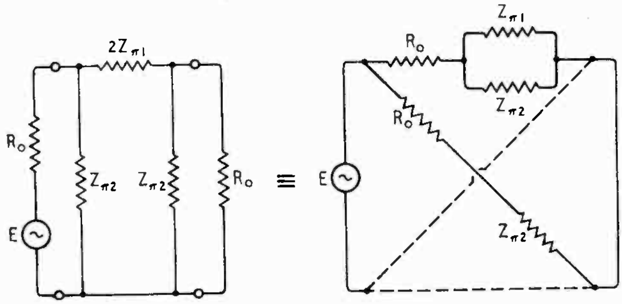


Fig. 5. *Equivalent π and lattice networks.*

4. Transient Response of Low- and High-pass Filters connected between Resistive Terminations

The simplicity of equations (8) and (6) is brought out in a striking manner in this problem. The case for the low-pass filter has been treated, using a different method, by Tucker[8], who obtains a similar result to that obtained here. The low- and high-pass filters, together with their equivalent forms, are shown in Figs. 8 and 9.

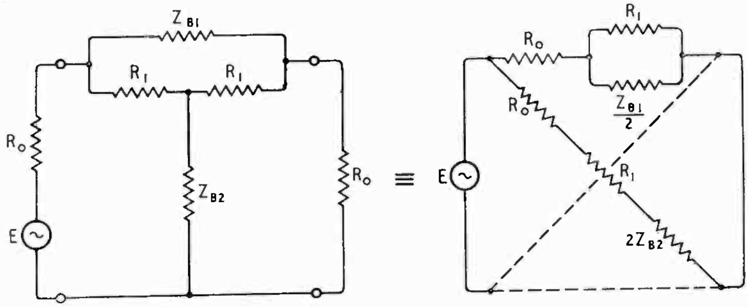


Fig. 6. *Equivalent bridged-T and lattice networks.*

An interesting case occurs for the " bridged-T " network of Fig. 6 if $R_0 = R_1$. If, further, $Z_{B1}Z_{B2} = R_0^2$ and R_0 is a resistance, the network is one of a general type of constant image-impedance attenuation equalizer, described by Zobel[6]. However, the relationship $Z_{B1}Z_{B2} = R_0^2$ is not essential in the discussion which follows. Putting $R_1 = R_0$ the network of Fig. 6 can be redrawn as shown in Fig. 7(a). The series arm of this network is a " T " network, and can therefore be redrawn in its equivalent lattice form, as shown in the network of Fig. 7(b).

The input current I_1 of this lattice, for an

The transient responses of the low-pass filter to a unit-step voltage can now be written down, using the known transient responses of the arms of the network of Fig. 8 (b). Thus the transient response of the series arm of this network is :—

$$I_1(t) = \frac{1}{R_0}\left(1 - e^{-\frac{R}{mL_K}t}\right) \quad .. \quad (33)$$

and that of the lattice arm is :

$$I_2(t) = \frac{1}{\sqrt{\dfrac{L_K}{m^2C_K} - \dfrac{R_0^2}{4}}} e^{-\frac{mR_0}{2L_K}t} \sin\sqrt{\frac{1}{L_KC_K} - \frac{m^2R_0^2}{4L_K^2}} \cdot t \quad (34)$$

If the resistance R_0 is equal to the design resistance of the filter, then :—

$$\frac{L_K}{C_K} = R_0^2, \ \frac{1}{L_KC_K} = \omega_0^2 \quad .. \quad (36)$$

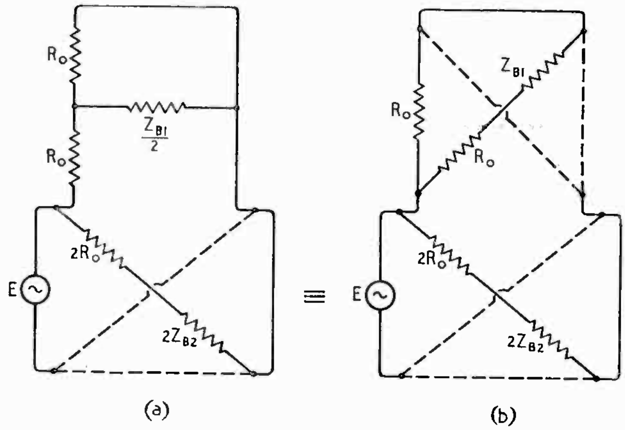


Fig. 7. *Further equivalent circuits to Fig. 6.*

where $\omega_0 = 2\pi \times$ cut-off frequency of the filter.

$$\therefore L_K = \frac{R_0}{\omega_0}, \ C_K = \frac{1}{R_0\omega_0} \quad .. \quad (37)$$

The transient response of the filter is then given, after substituting the values for L_K and C_K, by :—

$$I_{s,r} = \frac{1}{2R_0}\left(1 - e^{-\frac{\omega_0}{m}t} \pm \frac{e^{-\frac{m\omega_0}{2}t}}{\sqrt{\dfrac{4-m^2}{4m^2}}} \sin\sqrt{\frac{4-m^2}{4}} \cdot \omega_0 t\right) \quad (38), (39)$$

Noting the similarity between the lattice arm of the network in Fig. 8 (b) and that of the network in Fig. 9 (b), the solutions for the transient response of a high-pass filter can now be written down :—

$$I_{s,r} = \frac{1}{2R_0}\left(e^{-\frac{\omega_0}{m}t} \pm \frac{e^{-\frac{m\omega_0}{2}t}}{\sqrt{\dfrac{4-m^2}{4m^2}}} \sin\sqrt{\frac{4-m^2}{4}} \cdot \omega_0 t\right) \quad .. \quad (40), (41)$$

where $\omega_0 = 2\pi \times$ cut-off frequency .. (42)

5. Transient Response of 6-Element Band-Pass and Band-Stop Filters connected between Resistive Terminations

The 6-Element Band-Pass Filter

The transient response of a 6-element band-pass filter connected between resistive terminations to an alternating voltage, has been treated by Tucker[8] and by Eaglesfield[9], whose solutions have been obtained after making an approximation. The exact solution in the case where a unit-step voltage is applied to the filter is treated below. Although the solution for a unit-step voltage is exceedingly cumbersome, the transient response for the case when an alternating voltage is applied can be calculated from the result obtained below, by using Duhamel's Integral.

The circuit of the 6-element band-pass filter with resistive terminations is shown in Fig. 10 (a). The filter is treated fully elsewhere[5], and the notation of this reference will be used here.

$$\left.\begin{array}{lll} f_0 = \text{Mid-band frequency;} & & \omega_0 = 2\pi f_0 \\ f_B = \text{Bandwidth;} & & \omega_B = 2\pi f_B \\ n = \dfrac{\omega_B}{\omega_0} = \dfrac{f_B}{f_0} & ; & \omega_\theta^2 = \dfrac{1}{L_KC_K} \end{array}\right\} \quad (43)$$

If the filter is connected between resistances equal to its design resistance, then :—

$$R_0^2 = \frac{L_K}{C_K}, \ L_K = \frac{R_0}{\omega_0}, \ C_K = \frac{1}{R_0\omega_0} \quad (44)$$

The equivalent lattice of the filter is given in Fig. 10 (b).

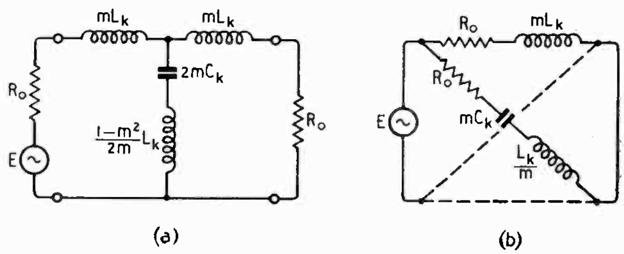
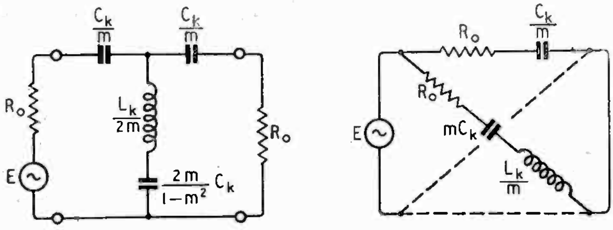


Fig. 8. *T and lattice low-pass filters.*

Referring to the network of Fig. 10 (b) Let $I_1(t)$ = transient response of series arm to a unit step voltage, and $I_2(t)$ = transient response of lattice arm to a unit step voltage.



(a) (b)

Fig. 9. *T and lattice high-pass filters.*

$$I_1(t) = \frac{1}{R_0}\frac{1}{\sqrt{\frac{m^2}{n^2}-\frac{1}{4}}}\, e^{-\frac{\omega_B}{2m}t} \sin\sqrt{\omega_0^2-\frac{\omega_B^2}{4m^2}}\,.\,t \qquad (48)$$

The solution for $I_2(t)$ is much more complicated and has been worked out in the Appendix. Quoting the result from the Appendix (Equation 17) :—

$$I_2(t) = \frac{m\omega_0}{2R_0}\left\{\left(1+j\sqrt{\frac{m^2}{4-m^2}}\right)\left(\frac{\exp.-\left(\frac{a_1-\sqrt{a_1^2-4b_1^2}}{2}\right)t-\exp.-\left(\frac{a_1+\sqrt{a_1^2-4b_1^2}}{2}\right)t}{\sqrt{a_1^2-4b_1^2}}\right)\right.$$
$$\left.+\left(1-j\sqrt{\frac{m^2}{4-m^2}}\right)\left(\frac{\exp.-\left(\frac{a_2-\sqrt{a_2^2-4b_2^2}}{2}\right)t-\exp.-\left(\frac{a_2+\sqrt{a_2^2-4b_2^2}}{2}\right)t}{\sqrt{a_2^2-4b_2^2}}\right)\right\} \qquad (49)$$

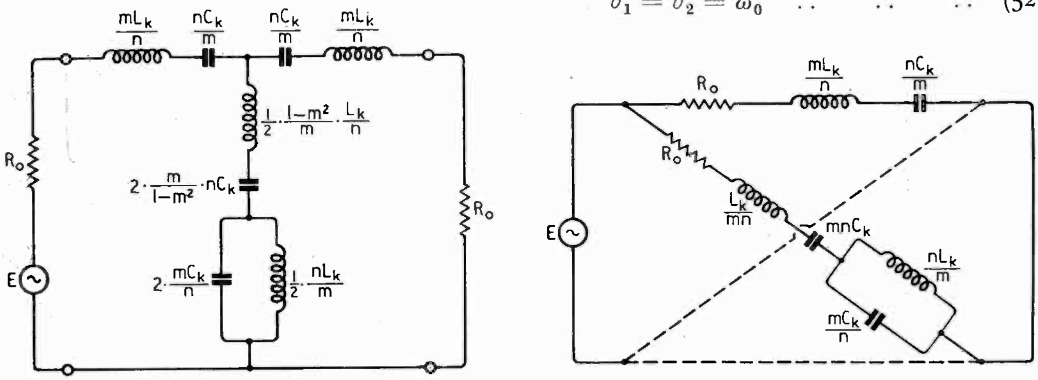
Then the input and output currents I_s and I_r of the band-pass filter, when a unit step voltage is applied, are given by :—

$$I_s = \tfrac{1}{2}\{I_1(t)+I_2(t)\} \quad \ldots \quad \ldots \quad (45)$$
$$I_r = \tfrac{1}{2}\{I_1(t)-I_2(t)\} \quad \ldots \quad \ldots \quad (46)$$

where $a_1 = -\omega_B\left(-\frac{m}{2}+j\sqrt{1-\frac{m^2}{4}}\right)$ (50)

$$a_2 = -\omega_B\left(-\frac{m}{2}-j\sqrt{1-\frac{m^2}{4}}\right) \qquad (51)$$

$$b_1 = b_2 = \omega_0 \quad \ldots \quad \ldots \quad \ldots \quad (52)$$



(a) (b)

Fig. 10. *6-element T band-pass filter (a) and its lattice equivalent (b).*

$I_1(t)$ is the response of a series-tuned circuit to a unit-step voltage and, for this case, is given by :—

$$I_1(t) = \frac{1}{\sqrt{\frac{m^2}{n^2}\cdot\frac{L_K}{C_K}-\frac{R_0^2}{4}}}\, e^{-\frac{nR_0}{2mL_K}t} \sin\sqrt{\frac{1}{L_KC_K}-\frac{n^2R_0^2}{4m^2L_K^2}}\,.\,t \qquad (47)$$

and substituting the relations expressed above :

By combining equations (48) and (49) according to equations (45) and (46), the solution for the transient response of the filter is obtained.

The 6-Element Band-Stop Filter

Fig. 12 (a) is the circuit of a band-stop filter, and its equivalent network is Fig. 12 (b).

The notation is that used in Reference (5) and :—

$$\left.\begin{aligned} f_0 &= \text{"mid-stop" frequency}\ ;\ \omega_0 = 2\pi f_0 \\ f_B &= \text{stop bandwidth}\ ;\ \omega_B = 2\pi f_B \\ n &= \frac{\omega_B}{\omega_0} = \frac{f_B}{f_0}\ ;\ \omega_0 = \frac{1}{L_K C_K} \end{aligned}\right\} \quad (53)$$

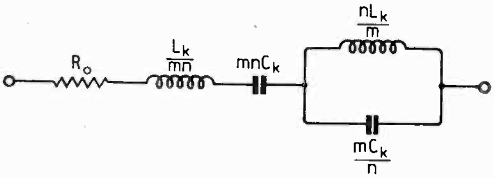


Fig. 11. *2-terminal network used in determining the transient response of the circuit of Fig. 10.*

If the filter is connected between resistances equal to its design resistance R_0, then:—

$$\frac{L_K}{C_K} = R_0^2,\ L_K = \frac{R_0}{\omega_0},\ C_K = \frac{1}{R_0\omega_0} \quad \cdots \quad (54)$$

The transient response, for a unit-step voltage, of a 2-terminal network consisting of a resistance R_1, in series with a parallel combination of an inductance L_1, and a capacitance C_1, is given by :—

$$\frac{1}{R_1}\left(1 - \frac{e^{-\frac{t}{C_1R_1}}}{\sqrt{\frac{R_1^2C_1}{L_1} - \frac{1}{4}}} \sin\sqrt{\frac{1}{L_1C_1} - \frac{1}{4C_1^2R_1^2}}\cdot t\right) \quad \cdots \quad (55)$$

The transient response $I_1(t)$ of the series arm of the lattice network, Fig. 12 (b), is therefore :—

$$I_1(t) = \frac{1}{R_0}\left(1 - \frac{e^{-\frac{mn}{2C_KR_0}t}}{\sqrt{\frac{R_0^2L_K}{L_Km^2n^2} - \frac{1}{4}}} \sin\sqrt{\frac{1}{L_KC_K} - \frac{m^2n^2}{4C_KR_0}}\cdot t\right) \quad \cdots \quad (56)$$

and substituting the relations of equations (53)

$$I_1(t) = \frac{1}{R_0}\left(1 - \frac{e^{-\frac{m\omega_B}{2}t}}{\sqrt{\frac{1}{m^2n^2} - \frac{1}{4}}} \sin\sqrt{\omega_0^2 - \frac{m^2\omega_B^2}{4}}\cdot t\right) \quad \cdots \quad (57)$$

The lattice arm of Fig. 12 (b) is exactly similar to that of Fig. 10 (b). The transient response $I_2(t)$ of this arm of the network is, therefore, given by Equation (49). The transient currents, I_s and I_r, flowing at the input and output of the band-stop filter due to a unit-step voltage, can then be calculated as in the case of the band-pass filter.

6. Transient Response of Coupled-Circuit Filters

Coupled-circuit filters are used extensively in radio engineering. Two types of circuit can be distinguished, viz., "series" type, Fig. 13 (a), and "parallel" type, Fig. 14 (a).

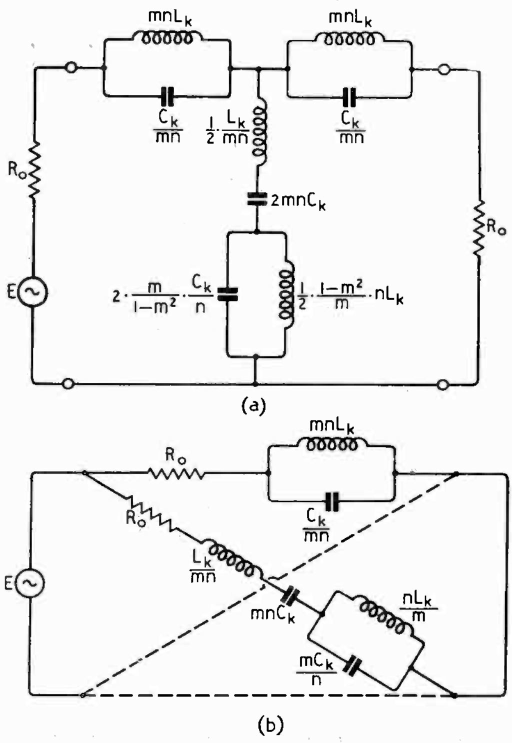


Fig. 12. *6-element T band-stop filter (a) and its lattice equivalent (b).*

Series Type of Coupled-Circuit Filter

The circuit consists of a transformer having equal primary and secondary inductances L, and with a mutual inductance M between the windings, and connected as shown in Fig. 13 (a). It can be transformed to an equivalent "T" network, as shown in Fig. 13 (b), and the equivalent lattice for the whole network is given in Fig. 13 (c).

The equations for the transient currents flowing in the network on application of a unit-step voltage follow simply from a knowledge of the transient responses of series-tuned circuits :—

$$I_{s,r} = \frac{E}{2}\left(\frac{1}{\sqrt{\frac{L-M}{C}-\frac{R^2}{4}}}\, e^{-\frac{R}{2(L-M)}\cdot t} \sin\sqrt{\frac{1}{(L-M)C}-\frac{R^2}{4(L-M)^2}}\cdot t \pm \frac{1}{\sqrt{\frac{L+M}{C}-\frac{R^2}{4}}}\, e^{-\frac{R}{2(L+M)}\cdot t} \sin\sqrt{\frac{1}{(L+M)C}-\frac{R^2}{4(L+M)^2}}\cdot t\right) \quad \ldots \ldots \ldots (58), (59)$$

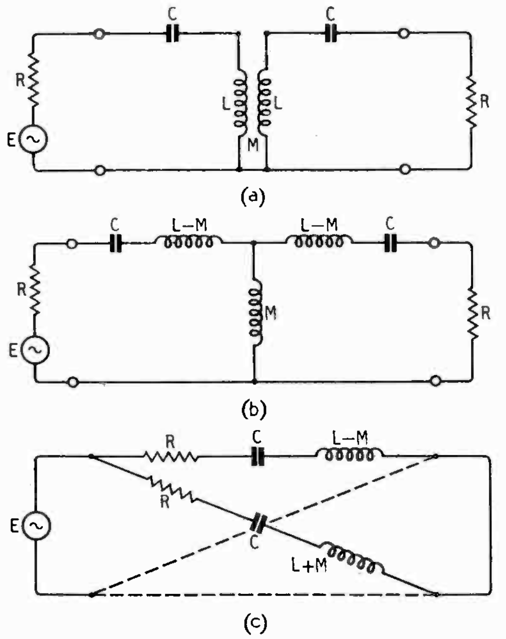


Fig. 13. *Series-type coupled filter circuit as transformer (a), T-network (b) and lattice (c).*

"Parallel" Type of Coupled-Circuit Filter

The circuit of a parallel-coupled filter is given in Fig. 14 (a), and the successive stages in its transformation into the equivalent lattice are shown in Fig. 14 (b) and (c).

The transient response for a unit-step voltage of a 2-terminal network consisting of a resistance R_1, in series with a parallel combination of an inductance L_1 and capacitance C_1, is given by Equation (55).

The transient response of the parallel coupled filter is therefore :—

$$I_{s,r} = \frac{e^{-\frac{t}{2CR}}}{2R}\left(\frac{\sin\sqrt{\frac{1}{(L+M)C}-\frac{1}{4C^2R^2}}\cdot t}{\sqrt{\frac{R^2C}{L+M}-\frac{1}{4}}} \pm \frac{\sin\sqrt{\frac{1}{(L-M)C}-\frac{1}{4C^2R^2}}\cdot t}{\sqrt{\frac{R^2C}{L-M}-\frac{1}{4}}}\right) \quad \ldots \ldots \ldots (60), (61)$$

7. The Synthesis of Networks

An important example of the synthesis of a 4-terminal network will now be given. The problem is to determine the 4-terminal network which, when short-circuited at the output and energized by the application of a unit-step voltage to its input, has input and output transient currents given by :—

$$I_s = \frac{1}{Rl}\left(1 + 2\sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2}{CRl^2}t}\right) = \frac{1}{Rl} + \frac{2}{Rl}e^{-\frac{\pi^2 t}{CRl^2}} + \frac{2}{Rl}e^{-\frac{4\pi^2}{CRl^2}t} + \ldots \quad \ldots \ldots \ldots (62)$$

$$I_r = \frac{1}{Rl}\left(1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-\frac{n^2\pi^2}{CRl^2}t}\right) = \frac{1}{Rl} - \frac{2}{Rl}e^{-\frac{\pi^2}{CRl^2}t} + \frac{2}{Rl}e^{-\frac{4\pi^2}{CRl^2}t} - \ldots \quad \ldots \ldots \ldots (63)$$

These equations are for a transmission line of length l, having distributed constants C F/unit length, and R ohms/unit length.

Our problem is therefore to design an artificial transmission line with the same electrical properties as a line having these distributed constants. The effects of inductance and leakage can be included, but for simplicity these quantities will be ignored here.

Now the terms in Equations (62) and (63) can be regrouped in the form shown below :

$$I_s = \left[\frac{1}{Rl} + \frac{2}{Rl}e^{-\frac{4\pi^2}{CRl^2}t} + \frac{2}{Rl}e^{-\frac{16\pi^2}{CRl^2}t} + \ldots\right] + \left[\frac{2}{Rl}e^{-\frac{\pi^2}{CRl^2}t} + \frac{2}{Rl}e^{-\frac{9\pi^2}{CRl^2}t} + \ldots\right] \quad (62a)$$

$$I_r = \left[\frac{1}{Rl} + \frac{2}{Rl}e^{-\frac{4\pi^2}{CRl^2}t} + \frac{2}{Rl}e^{-\frac{16\pi^2}{CRl^2}t} + \ldots\right] - \left[\frac{2}{Rl}e^{-\frac{\pi^2}{CRl^2}t} + \frac{2}{Rl}e^{-\frac{9\pi^2}{CRl^2}t} + \ldots\right] \quad (63a)$$

But each of the terms in the above equations, except the first, represents the transient response of a capacitance of value $\frac{2Cl}{n^2\pi^2}$ in series with a resistance of value $\frac{Rl}{2}$. (The first term is simply the response of a resistance of value Rl.)

A lattice network can thus be built up, in which the response of the series arms is represented by the terms in the first pair of square brackets of Equations (62a) and (63a), and the response of the lattice arms is represented by the terms in the second pair of square brackets. The arms of the network will each consist of an infinite number of 2-terminal networks connected in parallel, each 2-terminal network being a resistance in series with a capacitance.

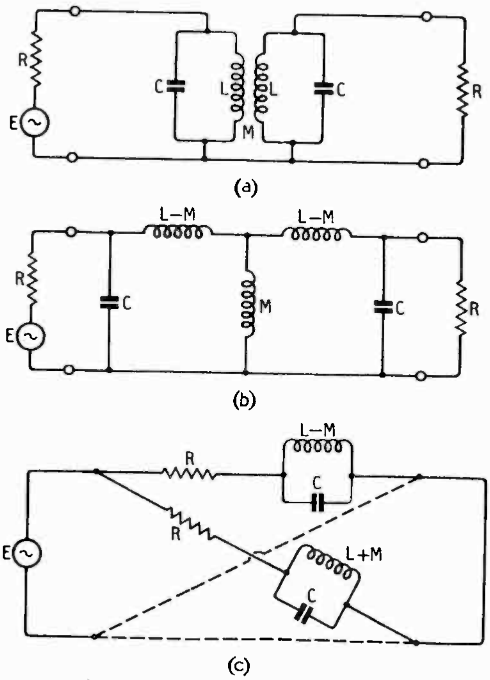


Fig. 14. *Parallel-type coupled-filter circuit as transformer (a), π-network (b) and lattice (c).*

The resultant network is shown in Fig. 15. The transient responses of this network to a unit-step voltage are given exactly by Equations (62) and (63). The network of Fig. 15 can be obtained by a different method[2,4].

8. Conclusion

A simplified method has been described, by means of which the transient response of a symmetrical 4-terminal network can be determined in terms of that of two 2-terminal networks.

The method has been applied to several problems of interest to radio engineers. An example of the synthesis of a network has also been given.

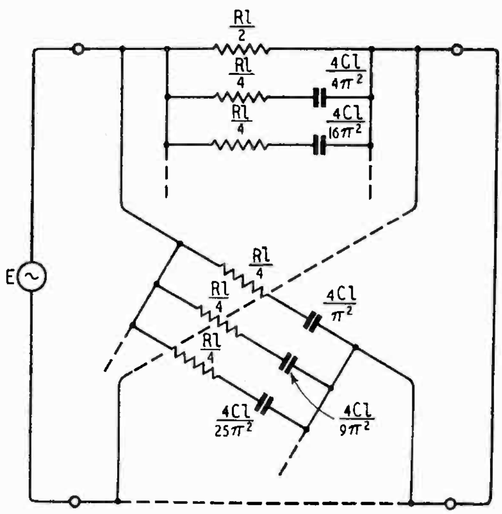


Fig. 15. *Lattice network with transient response simulating that of a transmission line.*

APPENDIX

Determination of the transient response $I_2(t)$ of the 2-terminal network shown in Fig. 11.

The operational impedance $Z(p)$ of this network is given by :—

$$Z(p) = R_0 + \frac{pL_k}{mn} + \frac{1}{pmnC_K} + \frac{1}{\dfrac{1}{\dfrac{pnL_K}{m}} + \dfrac{pmC_K}{n}} \quad \ldots \quad (1)$$

$$= R_0 + \frac{1}{mn}\left(\frac{1 + p^2L_KC_K}{pC_K}\right) + \frac{n}{m}\left(\frac{pL_K}{1 + p^2L_KC_K}\right) \quad \ldots \quad (2)$$

But $L_KC_K = \frac{1}{\omega_0^2}$, $R_0^2 = \frac{L_K}{C_K}$, $n = \frac{\omega_B}{\omega_0}$ .. .. (3)

$$\therefore L_K = \frac{R_0}{\omega_0} \text{ and } C_K = \frac{1}{R_0\omega_0} \quad \ldots \quad (4)$$

$$\therefore Z(p) = R_0 + \frac{1}{mn}\left(\frac{1 + \dfrac{p^2}{\omega_0^2}}{\dfrac{p}{R_0\omega_0}}\right) + \frac{n}{m}\left(\frac{\dfrac{pR_0}{\omega_0}}{1 + \dfrac{p^2}{\omega_0^2}}\right) \quad (5)$$

$$= R_0\left(1 + \frac{p^2 + \omega_0^2}{pm\omega_B} + \frac{p\omega_B}{m(p^2 + \omega_0^2)}\right) \quad \ldots \quad (6)$$

$$= R_0 \frac{(p^2 + \omega_0^2)^2 + pm\omega_B(p^2 + \omega_0^2) + p^2\omega_B^2}{pm\omega_B(p^2 + \omega_0^2)} \quad (7)$$

$$= R_0 \frac{(p^2 - \lambda_1 p\omega_B + \omega_0^2)(p^2 - \lambda_2 p\omega_B + \omega_0^2)}{pm\,\omega_B(p^2 + \omega_0^2)} \quad (8)$$

where

$$\lambda_1 = \left(-\frac{m}{2} + j\sqrt{1 - \frac{m^2}{4}}\right) \quad .. \quad .. \quad (9)$$

$$\lambda_2 = \left(-\frac{m}{2} - j\sqrt{1 - \frac{m^2}{4}}\right) \quad .. \quad .. \quad (10)$$

The solution for the current $I_2(t)$, which flows when a unit-step voltage is applied to the circuit of Fig. 11, is then :—

$$I_2(t) = \frac{1}{2\pi j}\int_{\mathrm{BR1}} \frac{e^{pt}}{pZ(p)}\, dp \quad .. \quad .. \quad (11)$$

where the contour BR_1, is a Bromwich Contour[7].

$$\text{Put } H\left(\quad\right) \equiv \frac{1}{2\pi j}\int_{\mathrm{BR1}}\left(\quad\right) dp \quad .. \quad (12)$$

$$\text{Then } I_2(t) = H\left(\frac{e^{pt}}{pZ(p)}\right) \quad .. \quad .. \quad .. \quad (13)$$

(This symbolism is used for simplicity, and implies that the indicated integration is to be performed, or alternatively that the equation for the current is to be solved using Heaviside's Expansion Theorem, or other methods.)

Then

$$I_2(t) = H\left(\frac{e^{pt}\, p\, \omega_B m\,(p^2 + \omega_0^2)}{p\,(p^2 - \lambda_1 p\omega_B + \omega_0^2)\,(p^2 - \lambda_2 p\omega_B + \omega_0^2)}\right)\frac{1}{R_0} \quad .. \quad .. \quad (14)$$

$$= H\left(\frac{m\omega_B}{2R_0}\left[\left(1 + j\sqrt{\frac{m^2}{4 - m^2}}\right)\frac{e^{pt}}{p^2 - \lambda_1 p\omega_B + \omega_0^2} + \left(1 - j\sqrt{\frac{m^2}{4 - m^2}}\right)\frac{e^{pt}}{p^2 - \lambda_2 p\omega_B + \omega_0^2}\right]\right) \quad (15)$$

The components of this equation are of the standard form : $H\left(\frac{e^{pt}}{p^2 + ap + b^2}\right)$, whose solution is :—

$$\frac{1}{\sqrt{a^2 - 4b^2}}\left(\exp. -\left(\frac{a - \sqrt{a^2 - 4b^2}}{2}\right) t - \exp. -\left(\frac{a + \sqrt{a^2 - 4b^2}}{2}\right) t\right) \quad .. \quad .. \quad (16)$$

$$\therefore I_2 = \frac{m\omega_B}{2R_0}\left[\left(1 + j\sqrt{\frac{m^2}{4 - m^2}}\right)\left(\frac{\exp. -\left(\frac{a_1 - \sqrt{a_1^2 - 4b_1^2}}{2}\right) t - \exp. -\left(\frac{a_1 + \sqrt{a_1^2 - 4b_1^2}}{2}\right) t}{\sqrt{a_1^2 - 4b_1^2}}\right) + \left(1 - j\sqrt{\frac{m^2}{4 - m^2}}\right)\left(\frac{\exp. -\left(\frac{a_2 - \sqrt{a_2^2 - 4b_2^2}}{2}\right) t - \exp. -\left(\frac{a_2 + \sqrt{a_2^2 - 4b_2^2}}{2}\right) t}{\sqrt{a_2^2 - 4b_2^2}}\right)\right] \quad .. \quad .. \quad (17)$$

where

$$a_1 = -\omega_B\left(-\frac{m}{2} + j\sqrt{1 - \frac{m^2}{4}}\right) \quad .. \quad .. \quad (18)$$

$$b_1 = \omega_0 \quad .. \quad .. \quad (19)$$

$$a_2 = -\omega_B\left(-\frac{m}{2} - j\sqrt{1 - \frac{m^2}{4}}\right) \quad .. \quad (20)$$

$$b_2 = \omega_0 \quad .. \quad .. \quad .. \quad .. \quad (21)$$

This equation contains real and complex terms, but if it is expanded, the complex terms will cancel out. The equation has been fully worked out in terms of the constants of the network, but the solution is too cumbersome to be given here.

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GAIN AND NOISE FIGURES AT V.H.F. AND U.H.F.*

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SUMMARY.—Optimal stage gain of amplifiers and of mixers at v.h.f. and u.h.f. is derived for narrow- and wide-band conditions. Five useful properties of noise figures attending amplifiers and mixers are applied to obtain considerable reductions of noise figures, in some cases amounting to 15 db at v.h.f. (e.g., 100 Mc/s f.m. reception) in mixer stages†.

1. Gain Figures of Amplifier and of Mixer Stages

BOTH amplifier and mixer stages can be considered as four-terminal networks. With amplifiers the frequency at the input terminal pair is equal to that at the output pair, but with mixer stages the frequency is different at the two pairs of terminals. The concept of the power gain of a four-terminal device or "four-pole" is most easily realized by introducing the idea of "available power" at a terminal pair. This available power is the largest average power which may be obtained from the terminals under consideration when the conditions of the external circuit are optimum for this.

Assuming the usual conditions for linearity (that momentary signal currents are exactly proportional to signal voltages and that impedances are independent of both) the properties of any two-terminal device may be expressed by an equivalent voltage generator (of zero internal impedance) in series with an impedance $Z_s = R_s + jX_s$, $j = +\sqrt{-1}$. If the generated r.m.s. voltage is V_s, the available power is: $P_a = V_s^2/4R_s$. It is obtained if an external impedance $Z = R_s - jX_s$ is connected to the terminal pair and is, in this case dissipated, in the resistance R_s.

Let us now connect a two-terminal device of the kind considered to the input terminals of a four-pole. If the available power of the input device is P_a and the available power at the output of the four-pole is P_0, its gain is $A = P_0/P_a$. From this definition of gain we may conclude that its value does not depend on the impedance connected to the output terminals of the four-pole. It is, however, dependent on the impedance connected to the four-pole's input terminals, P_a as well as P_0 being dependent on this impedance. Optimum gain is obtained if the four-pole is matched at its input. The input impedance of the four-pole should in this case be $R_s - jX_s$, matching the impedance $R_s + jX_s$ of the voltage source.

The properties of any four-pole may be completely derived from four quantities, which may be assumed to be admittances. If the currents and voltages at the four-poles' terminals are given positive directions

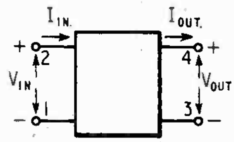


Fig. 1. *The positive directions of voltages and currents in a four-pole are shown here.*

as indicated by arrows in Fig. 1, the relations between the said currents and voltages may be written as:

$$I_{out} = a\,V_{in} - b\,V_{out}$$
$$I_{in} = c\,V_{in} - d\,V_{out} \quad \ldots \quad (1)$$

If, by some means or other, the feed-back admittance d is made zero (e.g., by the use of proper screen-grid valves or by neutralization), the input-admittance of the four-pole is c, its output admittance b and its trans-admittance a. The susceptances of b and c are in most cases absorbed into the attached resonant circuits and hence b, as well as c, may be assumed to be pure conductances b_r and c_r in these cases. If a generator of voltage V_s and of internal resistance R_s is connected to the input terminals 1, 2 of Fig. 1, we obtain:

$$\frac{I_{in}}{V_s} = \frac{c_r}{1 + c_r R_s}, \quad V_{in} = \frac{I_{in}}{c_r} = \frac{V_s}{1 + c_r R_s}$$

Assuming $I_{out} = 0$ in the first Equ. (1) the ratio of output voltage V_{out} to generator voltage V_s becomes:

$$\frac{V_{out}}{V_s} = \frac{a}{b_r}\,\frac{1}{1 + c_r R_s}$$

* MS. accepted by the Editor, November 1946.

† The substance of this article coincides with a lecture, delivered by the author at the Swiss Federal Institute of Technology at Zurich on May 18th, 1946, by invitation.

Hence the available output power P_0 is:

$$P_0 = \tfrac{1}{4}\,|V_{out}|^2\, b_r = \tfrac{1}{4}\,\frac{|a|^2}{b_r}\left(\frac{1}{1 + c_r R_s}\right)^2 |V_s|^2,$$

vertical dashes indicating moduli. As the available input power is $P_a = |V_s|^2/4R_s$ the gain works out as

$$A = \frac{R_s|a|^2}{b_r}\left(\frac{1}{1 + c_r R_s}\right)^2 \quad .. \quad .. \quad (2)$$

Optimum gain A_0 is obtained if $c_r R_s = 1$ (matching condition at input) and hence is:

$$A_0 = |a|^2/4b_r c_r \quad .. \quad .. \quad .. \quad (3)$$

2. Optimal Gain of v.h.f. and u.h.f. Amplifier Stages

With both amplifier and mixer stages it is of practical importance to consider the gain not merely for a single-frequency signal but within a frequency band of angular width ω_0, centred round the angular frequency ω. In amplifier stages ω has the same value at the input and output terminals. We shall discuss the dependence of a, b and c of Equ. (3) on ω in some outstanding cases. At v.h.f., tetrodes and pentodes are often used in a so-called grounded-cathode circuit. This is the usual circuit in which the input terminals are control grid and cathode and the output terminals anode and cathode, the last being the electrode common to input and output. By theory as well as by experiments b_t and c_t (i.e., the conductance parts of the valve admittances) are proportional to ω^2 at v.h.f. (30 to 300 Mc/s) and in the lower part of the u.h.f. range (300–3,000 Mc/s) for valves of suitable construction whereas $|a_t|$ is practically independent of ω in these ranges [14].

Whether these valve admittances can be substituted for the stage admittances a, b_r and c_r of Equ. (3) depends on the conditions of stage operation and, predominantly, on the angular bandwidth ω_0. If ω_0 is sufficiently small compared with the angular centre frequency ω, the resonant circuits of the stage may be given resonant impedances which are large compared with either $1/c_r$ or $1/b_r$. Under these conditions b_t may be substituted for b_r and c_t for c_r in Equ. (3). If feedback is suitably suppressed, a_t may be substituted for a in Equ. (3) and thus we obtain approximately an optimum stage gain under the specified conditions of:

$$A_0 = |a_t|^2/4b_t c_t \quad .. \quad .. \quad .. \quad (4)$$

In reality A_0 will be slightly less owing to: $b_r > b_t$, $c_r > c_t$ and $|a| < |a_t|$. This optimal gain is inversely proportional to ω^4. Hence an angular frequency exists, at which $A_0 = 1$. Let it be ω_b. Then:

$$A_0 = (\omega_b/\omega)^4 \quad .. \quad .. \quad .. \quad (5)$$

With television pentodes and tetrodes the border frequency f_b is often between 300 and 600 Mc/s. In the former case we obtain an optimal gain of 625 at the television frequency of 60 Mc/s and in the latter case 10,000, whereas at 100 Mc/s (f.m. transmission) the corresponding figures are 81 and 1,296.

At an angular bandwidth ω_o between half-power points the resonant impedance of a tank circuit or cavity is:

$$Z_{res} = 1/\omega_o C$$

C being its lumped shunt capacitance at the resonant frequency. With any valve stage the lumped shunt capacitance at the input is larger than the input capacitance C_{in} of the valve and, similarly, the lumped shunt output capacitance is larger than the output capacitance C_{out} of the valve. Hence the assumptions that the resonant impedance of the input circuit is $1/\omega_o C_{in}$ and that of the output circuit is $1/\omega_o C_{out}$ are somewhat too favourable, the impedances being lower in reality.

With present-day valves b_t is often much smaller than c_t; e.g., 10 times smaller. Hence if ω_o is increased, the resonant output impedance will become comparable to $1/b_t$ before the resonant input impedance becomes comparable to $1/c_t$. There are thus three different cases to consider:

(a) $c_t \gg \omega_0 C_{in}$, $\quad b_t \gg \omega_0 C_{out}$,
(b) $c_t \gg \omega_0 C_{in}$, $\quad b_t \ll \omega_0 C_{out}$,
(c) $c_t \ll \omega_0 C_{in}$, $\quad b_t \ll \omega_0 C_{out}$,

The first case has been dealt with already. In the second case (b) we may identify c_r of Equ. (3) with c_t, and b_r with $\omega_0 C_{out}$. Thus:

$$A_0 = \frac{|a_t|^2}{4c_t\,\omega_0\,C_{out}} = \left(\frac{\omega_{b0}}{\omega}\right)^2 \quad .. \quad (6)$$

ω_{b0} being the angular border frequency at which A_0 is unity, corresponding to the centre of the band ω_0. Under the present conditions A_0 is hence inversely proportional to the square of the centre frequency ω. In the third case (c) we have:

$$A_0 = \frac{|a_t|^2}{4\omega_0^{\,2} C_{in} C_{out}} = \left(\frac{\omega_{b1}}{\omega_0}\right)^2, \quad .. \quad (7)$$

ω_{b1} being the angular border bandwidth at which A_0 is unity. Optimum gain is independent of the angular centre frequency ω in this case and inversely proportional to the bandwidth squared.

If the angular bandwidth ω_0 is fixed and its angular centre frequency ω is increased continuously, we go through the three cases discussed in reverse order, the third one being met at low values of ω/ω_0, then the second one and lastly the first one. Hence optimum gain versus centre-frequency curves have the general character illustrated in

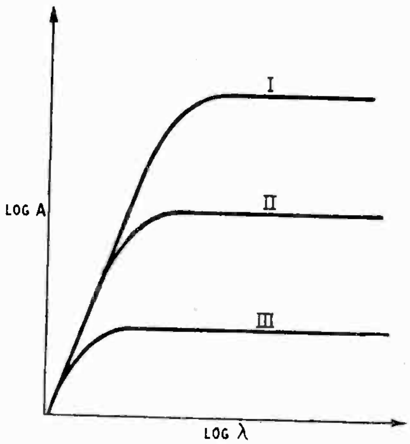


Fig. 2. *General trend of log gain versus log wavelength curves at three consecutive bandwidths; the bandwidth of curve III being larger than of curve II and this being larger than of curve I.*

Fig. 2. These relations have been well confirmed by experiments. At and beyond the angular border frequency at which $A_0 = 1$, the valve in question, if used in an oscillator stage, will fail to start oscillations. This property offers an easy means of determining this critical frequency experimentally and thence the entire A_0 versus frequency curves for any particular valve. The values ω_b, ω_{b0} and ω_{b1} constitute universal figures of merit for valves under different conditions of operation as specified.

At v.h.f. and u.h.f. triodes have been used in so-called grounded grid circuits, the stage input terminals being connected to cathode and grid, while the stage output terminals are connected to anode and grid[2,9]. The difference between grounded-cathode and grounded-grid stages is shown in Fig. 3. In such grounded-grid stages, if the amplification factor of the triode is sufficiently large (e.g., 20 or more), the values of a_t and b_t are substantially equal to the corresponding values of a grounded-cathode stage using the same valve. The value of c_t is, however, considerably larger in the grounded-grid stage at the lower end of the v.h.f. range with most triodes. With increasing frequency a point occurs at which both values of c_t are equal and beyond this frequency c_t is smaller in a grounded-grid than in a grounded-cathode stage. Hence optimal gain, in case (a) above, is smaller in a grounded-grid stage than in a grounded-cathode stage below the critical frequency while it is larger beyond. Still, grounded-grid stages are being used even below this critical frequency as their unneutralized feedback admittance is much smaller than with grounded-cathode stages, approximately in the ratio of the reversed amplification factor. By u.h.f. electronics, excluding all contributions to c_t other than those due to electron motion and assuming the electron-transit time between grid and anode to be small compared with the transit time t between cathode and grid, we obtain for the real parts of the input admittances the values:

grounded cathode:

$$c_t = \frac{1}{20} g_0 (\omega t)^2 + \ldots\ldots$$

grounded grid:

$$c_t = g_0 \left\{ 1 - \frac{7}{300} (\omega t)^2 + \ldots \right\} \qquad (8)$$

The dots stand for terms proportional to $(\omega t)^4$ and g_0 is the r.f. mutual conductance.

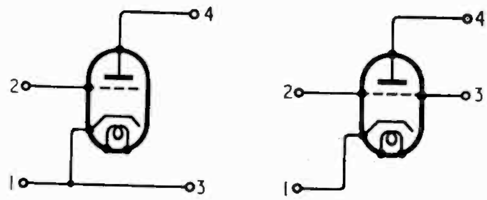


Fig. 3. *Illustrating the grounded-cathode (left) and grounded-grid (right) circuits.*

At r.f. the conductance values of a_t, b_t and c_t of grounded-grid stages are: $a_t = g_0$, $b_t = g_0/\mu$ and $c_t = g_0$; hence $A_0 = \mu/4$, in the first of the three cases of the ratio ω_0/ω, μ being the amplification factor. The critical frequency f_c at which the conductance c_t is equal for both types of stage corresponds to ωt a little larger than $5\pi/4$. This value $\omega t = 5\pi/4$ corresponds to a frequency:

$$f_c = \frac{12.5\sqrt{V_p}}{d} \text{ (Mc/s)} \qquad \ldots \quad \ldots \quad (9)$$

V_p being the equivalent steady potential at the average grid surface in volts and d the cathode-grid distance in cm. As a practical example, $V_p = 2$ volts and $d = 0.01$ cm, when $f = 1{,}750$ Mc/s. Two curves illustrating the conductance parts of c_t for grounded-cathode and for grounded-grid stages, exclusively due to electron motion, are shown in Fig. 4.

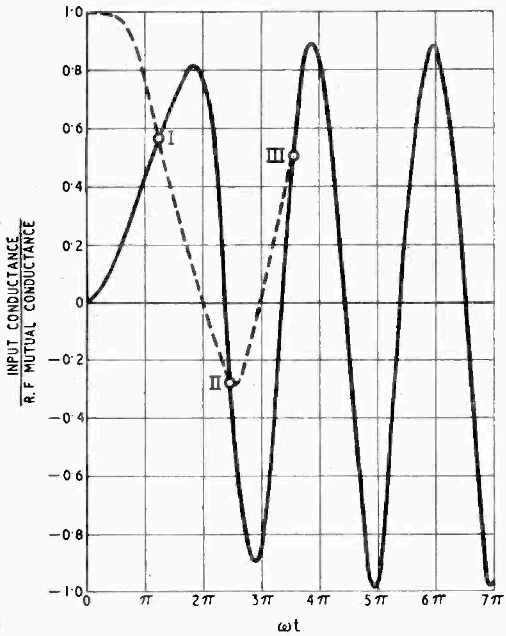


Fig. 4. *Vertical scale: ratio of input conductance to r.f. transconductance. Horizontal scale: product of angular frequency and electron-transit time t between cathode and adjacent control grid. Full curve: grounded-cathode stage; broken curve: grounded-grid stage. The transit time between the control grid and a subsequent electrode is assumed to be negligible compared with t.*

The dependence of A_0 on angular centre frequency ω is somewhat different with grounded-grid than with grounded-cathode stages. Thus, within the range of validity of Equ. (8) we have approximately:

case (a) $A_0 \approx \mu/4$,

case (b) $A_0 \approx |a_t|/4\omega_0 C_{out}$,

case (c) $A_0 \approx |a_t|^2/4\omega^2 C_{out} C_{in}$

Thus in all cases A_0 is practically independent of ω under the specified conditions.

3. Operation of v.h.f. and u.h.f. Mixer Stages

We shall discuss the use of triodes and multi-grid valves as well as of diodes and crystals. With triodes and multi-grid valves three types of mixer circuits will be considered: single-grid mixers, in which the local-oscillator and the input voltages are applied to the same pair of electrodes and two types of double-grid mixers, in which the local-oscillator voltage is applied to the cathode and the so-called oscillator electrode, while the input voltage is applied to the cathode and the input-signal grid (see Fig. 5). As the output frequency of the mixer stage is in most cases much lower than its input frequency, feedback is often negligible. Hence grounded-grid triode mixer stages are usually of no practical importance except, perhaps, at the highest part of the u.h.f. range, where input conductance may be lower than with corresponding grounded-cathode stages. We shall for this reason only consider the latter type of stage.

With single-grid mixers the bias voltage of the control grid is varied by the local oscillator between a value corresponding to top mutual conductance, avoiding grid current, and a largely negative value at which the mutual conductance is substantially zero. This variation is effected at a rate corresponding to the local oscillator angular frequency[6] ω_{osc}. The resulting mutual admittance (with respect to signals of input frequency) versus time curve is periodic at this rate and may be expressed as a series of corresponding harmonic components:

$$Y = Y_0 + 2Y_1 \cos \omega_{osc} t + 2Y_2 \cos 2\,\omega_{osc} t + \ldots \quad (10)$$

The admittance sign Y has been used in

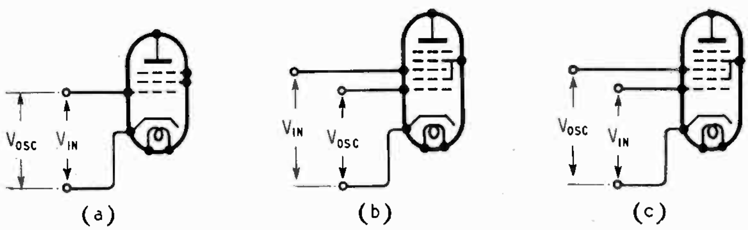


Fig. 5. *Three different types of mixer valves are shown. Diagram (a): single-grid pentode mixer; diagram (b): double-grid heptode mixer; diagram (c) as (b) but with oscillator and input voltages reversed.*

Equ. (10) instead of a mutual conductance symbol as the ensuing transadmittance corresponding to v.h.f. or u.h.f. input signals is complex with most valves. Thus Y, Y_0, Y_1, etc., will in general be complex. If a signal voltage $V_{0in} \exp(j\omega_{in} t)$ is applied to the tube's input, the ensuing output current in a short-circuit connection of anode and cathode is obtained by multiplication of

Y by the signal mentioned. Carrying this out we see that the current component of angular frequency $|\omega_{osc} - \omega_{in}|$ is of amplitude $|Y_1 V_{0in}|$. This is the reason for the indication of Y_1 as "conversion transadmittance," the desired output or intermediate angular frequency being either $\omega_{in} - \omega_{osc}$ or $\omega_{osc} - \omega_{in}$. We are primarily interested in estimating the modulus of Y_1 in relation to the modulus of top transadmittance at normal amplifier operation of the same tube. This ratio depends on the

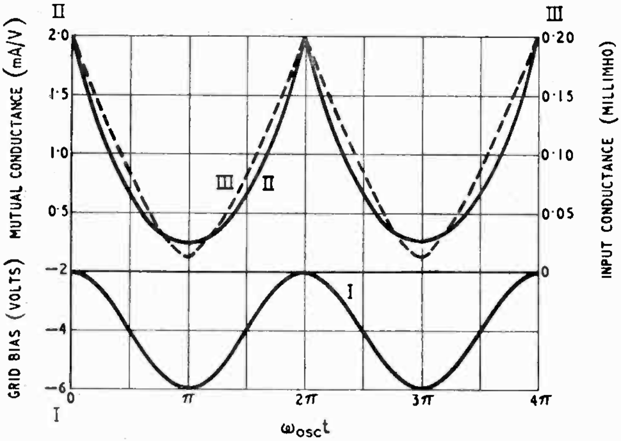


Fig. 6. *Curve I: grid-bias volts (vertical scale) as dependent on time (horizontal scale showing product of angular oscillator frequency ω_{osc} and time t). Curve II: ensuing mutual conductance as dependent on time. Curve III: ensuing input conductance at approximately 70 Mc/s as dependent on time.*

shape of the transadmittance versus time curve of Equ. (10). In many cases it is 1/3 to 1/4 with favourable applied local oscillator voltages (see curve II of Fig. 6).

Besides the transadmittance, the valve's input conductance is also varied during each local-oscillator voltage swing. An example is given in Fig. 6, showing the mutual conductance, as well as the input conductance, of a conventional r.f. pentode versus control grid bias. As there is no phase correlation between the input-signal frequency and the local-oscillator voltage, the effective input conductance at the former frequency is the average, or arithmetic mean, value of the input conductance versus time curve ensuing from the local-oscillator action. This average value is in many cases 1/3 to 1/4 of the value at top transconductance (avoiding grid current). The output conductance of a mixer valve is varied in a similar way, by the oscillator swing. At the intermediate angular frequency ω_{out} its average value is effective, as there is no phase correlation between output voltages at ω_{out} and the local-oscillator voltages at ω_{osc}. This average value of output conductance is often 1/3 to 1/4 times the value at top transconductance. These operative data are sufficient to obtain general estimates of single grid mixer gain figures, as will be shown in Section 4.

We now turn to double-grid mixers, of which two types exist corresponding to the diagrams (b) and (c) of Fig. 5. As regards transadmittance and output conductance both types show a very similar behaviour to the single-grid type discussed above. Hence no further discussion of these items seems necessary. With double-grid mixers of the type (b) of Fig. 5 the input conductance is in most cases negative and remains so at increasingly negative bias of the oscillator grid, its modulus decreasing, however. Hence the average input conductance at ω_{in} remains negative, its modulus increasing in proportion to ω_{in}^2. With type (c) mixers of Fig. 5, however, conditions are different. Input conductance increases with increasing bias in this case, as illustrated in Fig. 7. This is largely due to electrons reversing their motion in front of the oscillator grid and returning to the vicinity of the input signal grid. These electrons induce charges on the input grid and thus cause an increased input conductance. By this effect the input conductance at ω_{in} is often 2 to 4 times the value at top transconductance conditions corresponding to zero or positive bias of the oscillator grid. These considerations are sufficient to obtain gain figures of double-grid mixers[13] in Section 4.

4. Gain Figures of Mixer Stages

The output frequency being in most cases much lower (e.g., 1/10 or less) than the input frequency, the ratio of output conductance to input conductance of mixer valves is still lower than that of amplifier valves and may be between 1/100 and 1/1000 depending on the ratio ω_{out}/ω_{in}. Due to this fact, the first case mentioned in Section 2 (i.e., $c_t \gg \omega_0 C_{in}$ and $b_t \gg \omega_0 C_{out}$ is hardly of any importance in mixer operation, even with r.f. input signals. This discussion may thus be confined to the second and third cases ; i.e., to $c_t \gg \omega_0 C_{in}$, $b_t \ll \omega_0 C_{out}$, and to $c_t \ll \omega_0 C_{in}$, $b_t \ll \omega_0 C_{out}$. The symbols a_t, b_t and c_t will be assumed to have the same significance as in Section 2, thus denoting the valve's transadmittance, output conductance and input conductance at top amplifier operation, avoiding signal-grid current, a_t and c_t being related however to ω_{in} and b_t to ω_{out}. By the discussion of Section 3 the values of a and c_r to be inserted in Equ. 3 for optimal gain are approximately : $|a| = \frac{1}{4}|a_t|$, $c_r = \frac{1}{4} c_t$ with mixers of type (a) Fig. 5 and $|a| = \frac{1}{4}|a_t|$, $c_r = 2\, c_t$ with mixers of type (c) Fig. 5. These multipliers 1/4 and 2 are, of course, not of general validity, being dependent on valve characteristics and local-oscillator operation, but they often represent a fair average under favourable conditions of mixer operation. Introducing these values we obtain in the second case mentioned above :

Type (a) mixers :

$$A_0 = \frac{|a_t|^2}{16 c_t \omega_0 C_{out}} = \left(\frac{\omega_1}{\omega_{in}}\right)^2$$

Type (c) mixers : .. (11)

$$A_0 = \frac{|a_t|^2}{128\, c_t\, \omega_0\, C_{out}} = \left(\frac{\omega_2}{\omega_{in}}\right)^2$$

With these two types of mixers optimal gain is thus inversely proportional to ω^2_{in} because c_t is so. The angular border frequency ω_1 is in general much higher than ω_2 under comparable conditions due to the multipliers 16 and 128 in the denominators of Equ. (11). Subsequently, in the third case we obtain :

Type (a), (b) and (c) mixers :

$$A_0 = \frac{|a_t|^2}{64\omega_0{}^2\, C_{in}\, C_{out}} \quad \ldots \quad (12)$$

With type (b) mixers of Fig. 5 only this third case is of practical interest due to the negative value of c_t. The effective input susceptance of Equ. (3) is given by $\omega_0 C_{in}$ in this case. As $\omega_0 C_{in}$ is assumed to be much less than c_t for mixers of type (c) in the second case, it is seen that in this case type (b) mixers are superior to type (c) as regards optimal gain. This is in harmony with the general application of type (b) mixers in f.m. reception (e.g., 100 Mc/s input frequency). In wideband reception corresponding to the third case no considerable difference in optimal gain exists between the three types of mixers of Fig. 5. Generally, in such wideband reception, the figure of merit of amplifier, as well as of mixer, valves is $|a_t|^2/(C_{in}\, C_{out})$; i.e., the ratio of modulus of top amplifier transadmittance squared to the product of input and output valve capacitances. In narrow-band amplifier operation corresponding to the first case the relevant figure of merit is $|a_t|^2/c_t b_t$, while in the intermediate second case of amplifier operation, as well as in mixer operation corresponding to Equs. (11), the figure is $|a_t|^2/(c_t\, C_{out})$.

Finally we shall deal with diode and crystal-mixer stages. Unlike the preceding cases, based on Equ. (3), feedback cannot be ignored in this case. With simple assumptions as to the diode or crystal stage, ignoring image response, Equs. (1) are simplified as $b = c = Y_0$, $a = d = Y_1$, the significance of Y_0 and Y_1 being similar to that in Equ. (10). They denote the average and

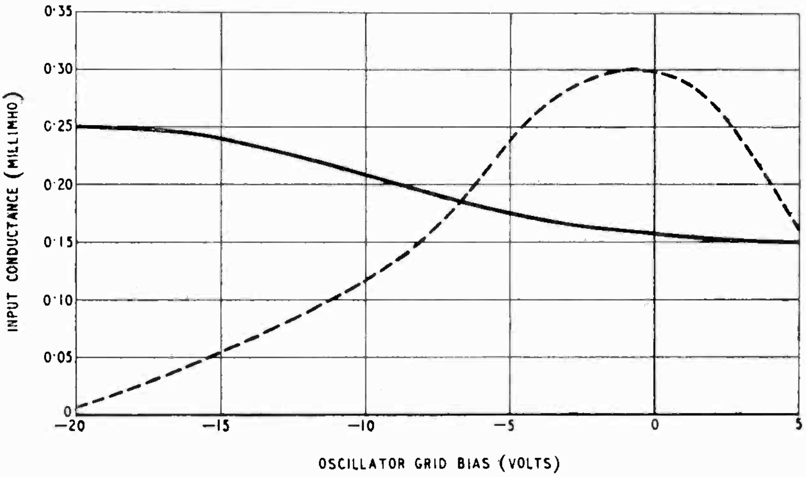


Fig. 7. *Input conductance of double-grid mixers as dependent on bias voltage of oscillator grid. Full curve : type (c) mixer of Fig. 5. Broken curve : type (b) mixer of Fig. 5. In the latter case the input conductance is negative and the curve corresponds to its modulus. Frequency : 42 Mc/s.*

fundamental component of diode or crystal conductivity caused by the oscillator swing. We shall assume that Y_0 and Y_1 are both real, thus ignoring electron-transit time effects. Assuming a voltage generator of internal resistance R_s (tuned tank circuit or cavity at ω_{in}) to be connected to the input terminals of Fig. 2 we obtain a gain figure :

$$A = \frac{Y_1^2}{(Y_0R_s + 1)^2 \; Y_{out}/R_s}, \quad \ldots \quad (13)$$

Y_{out} denoting the conductance looking into the four-pole from its terminals 3,4 (see Fig. 1). This conductance is :

$$Y_{out} = Y_0 - \frac{Y_1^2}{Y_0 + 1/R_s}. \quad \ldots \quad (14)$$

Optimal gain is obtained in the present case if $Y_0R_s \gg 1$, when :

$$A = \frac{Y_1^2}{Y_0R_s(Y_0^2 - Y_1^2) + Y_0^2} \quad \ldots \quad (15)$$

As Y_1 is always smaller than Y_0 this figure is less than unity and there is a loss instead of a gain. As with the mixer stages discussed previously we may again assume three cases if the resonant circuit or cavity admittances at input and output are ω_0C_{in} and ω_0C_{out} respectively, C_{in} and C_{out} being substantially the diode or crystal capacitance and ω_0 the angular bandwidth corresponding to half-power points. First : $\omega_0C_{in} = 1/R_s \ll Y_0$ and $\omega_0C_{out} \ll Y_{out}$, when Equ. (15) is valid. Secondly : $\omega_0C_{in} = 1/R_s \ll Y_0$ and $\omega_0C_{out} \gg Y_{out}$, when we obtain :

$$A = \frac{Y_1^2}{(Y_0/\omega_0C_{in})^2 \, \omega_0^2 C_{in} C_{out}} \quad \ldots \quad (16)$$

This figure is substantially equal to Y_1^2/Y_0^2. Thirdly, $\omega_0C_{in} \gg Y_0$ and $\omega_0C_{out} \gg Y_{out}$, when :

$$A = Y_1^2/\omega_0^2 C_{in} C_{out} \quad \ldots \quad \ldots \quad (17)$$

Whilst (15) and (16) need only be slightly less than unity, Equ. (17) yields a value considerably below unity. Thus wideband mixing using a diode stage leads to a considerable loss.

5. Noise Figures

In Section 1 we have given a definition of available power which may be applied to noise as well to signals. Noise power is in most cases uniformly distributed over a given frequency interval f_0. We shall assume that the gain A of a four-pole is uniform throughout this band f_0 and zero outside. Its available output-noise power corresponding to f_0 is denoted by P_{no}. Now a signal generator is connected to its input terminals at a frequency within the said band. The available power of this signal generator P_{si} is adjusted until the ratio of available output-noise power to available output-signal power of the four-pole is unity : $AP_{si} = P_{no}$. This signal power P_{si} is then divided by KTf_0 and the result is indicated as the noise figure of the four-pole [3, 4, 5, 10, 12].

$$N = \frac{P_{si}}{K\,T\,f_0} = \frac{P_{no}}{g\,K\,T\,f_0} \quad \ldots \quad \ldots \quad (18)$$

The symbol K denotes Boltzmann's constant (1.38×10^{-23} joule per degree Kelvin) and T the room temperature in degrees Kelvin (about 291 degrees). In some cases N is expressed in db, this number being $10 \log_{10}N$. The signal generator mentioned above inevitably emits noise as well as signal power. Its available noise power depends on its internal structure and may always be described by the expression KT_sf_0 within the bandwidth f_0, where T_s denotes the effective temperature of its internal resistance. At the terminals of a passive resistor of temperature T_s the available noise power is given by the above expression KT_sf_0. The available noise power KT_sf_0 of the signal generator creates an available noise power at the output terminals of the four-pole, given by AKT_sf_0. Hence the part of the overall available output noise power P_{no} due to the four-pole itself is $P_{no}—AKT_sf_0$ and the noise figure corresponding to the four-pole proper is thus :

$$N_0 = \frac{P_{no} - AKT_sf_0}{AKTf_0} = N - \frac{T_s}{T} \quad (19)$$

It is useful to apply these ideas to a simple four-pole, consisting of a single shunt impedance Z. Assuming the signal generator used in the determination of N, as well as the impedance Z, to be of effective temperature T we obtain :

$$A = \frac{1}{1 + \dfrac{RR_s}{|Z|^2}}, \quad N = 1 + \frac{1 + \dfrac{RR_s}{|Z|^2}}{1 + \dfrac{|Z|^2}{RR_s}},$$

where R is the real part of Z and R_s is the output impedance of the generator, assumed to be real. If the impedance Z consists of a tank circuit or of a resonant cavity, the highest value of $|Z|$ is obtained at the tuning position, when $R \approx |Z_{res}|$. At this resonant frequency the noise figure N obviously attains a minimum and A a maximum value if compared with neighbouring frequencies. Assuming the effective bandwidth f_0 to be small compared with the half-power width of the resonance curve of $|Z|$, we may centre f_0

round the resonant frequency or round some other frequency. We infer that in the former case N attains its lowest value.

We shall now evaluate the noise figure of amplifier or mixer stages using single and multi-grid valves. With such valve stages, short-circuited at their input and output terminals, the mean-square noise current I_n^2 through the short-circuit output lead may be written as :

$$I_n^2 = 4KT_t|Y_0|f_0 \quad \ldots \quad (20)$$

Y_0 being the average transadmittance as defined and used in Equ. (10) and T_t denoting the effective noise temperature of the valve. Its value is of the order of magnitude of the cathode temperature with triodes, and larger with multigrid valves, due to so-called "partition" fluctuations of the electrons passing the screen grids at positive steady voltages. The available output noise power due to the valve itself may be obtained from Equ. (20) by dividing by $4b_r$, where b_r denotes the effective output conductance. Applying the procedure of Equ. (18) in order to obtain the corresponding valve noise figure N_t, the result is :

$$N_t = \frac{T_t}{T}\,\frac{|Y_0|}{b_r A}, \quad \ldots \quad (21)$$

A being the gain according to Section 1. If the valve is considered as part of a stage the overall stage noise figure is, of course, larger than N_t due to additional stage noise and to interaction between the valve and the input circuit[7,11]. In the present case the value of Equ. (2) for A must be used, assuming $R_s c_r \ll 1$ and hence :

$$N_t = \frac{T_t}{T}\,\frac{|Y_0|}{R_s|a|^2} \quad \ldots \quad (22)$$

The value of $|a|$ is equal to $|Y_0|$ in the case of amplifiers and is $|Y_1|$ with mixers. As $|Y_1| < |Y_0|$, the value of N_t is larger in the latter than in the former case, using comparable valves. With proper oscillator operation this difference is not considerable, however, using triodes.

6. Properties of Noise Figures

We shall consider a four-pole I of Fig. 8 having at its output terminals 3,4 the available noise power P_{no} and the available signal power P_{so}, whilst its gain is A. We may connect a two-terminal device to its input terminals supplying the available noise power P_{no}/A and the available signal power P_{so}/A. Thus the four-pole I need not contribute any noise at all and still the available output powers remain as stated. Now we take a second four-pole II and connect it between the output terminals 3,4 and the input terminals 1,2 of the four-pole I. In doing so, the overall gain between the terminal pairs 1,2 and 3,4 is altered. The circuit under discussion constitutes a definite feedback between the output and the input of the four-pole I. We shall assume that the four-pole II does not contain any sources of appreciable noise. This condition is satisfied if II consists of a network of passive

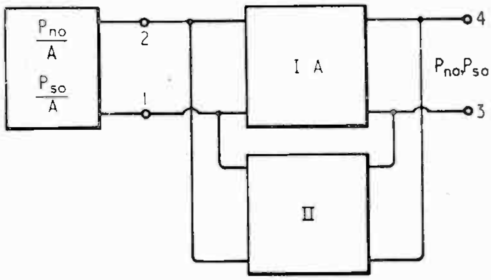


Fig. 8. *Four-pole I of gain A, available output noise power P_{no} and available output signal power P_{so}, connecting a two-terminal device to the input terminals 1,2 having an available noise power P_{no}/A and an available signal power P_{so}/A, the four-pole I is free of noise or signal sources. By the application of feedback using a second four-pole II containing no appreciable sources of noise and signal, the ratio P_{no}/P_{so} at the output terminals 3,4 is not altered.*

reactances. The ratio of available signal-to-noise power at the terminals 3,4 is obviously the same under these conditions as before the insertion of the four-pole II. By the definition of noise figure, its value remains unaltered in this case. Hence we have obtained :

Property A.—If feedback is applied to a four-pole in such a way that the effects of the additional noise-sources of the feedback circuit are negligible, the noise figure of the circuit including feedback is equal to the original noise figure of the four-pole without feedback.

Coupling of the signal (and noise) source to the input of a four-pole may be chosen so that the resulting noise figure attains a minimum value. When this is achieved and feedback according to property A is applied to the four-pole the noise figure remains unaltered. By its definition this noise figure is dependent on the impedance of the signal source presented to the input-terminals. Hence this impedance remains unaltered by the feedback and so does the coupling coefficient. This may be expressed as :

Property B.—If coupling between the signal source and the input of a four-pole is chosen so as to correspond to a minimum noise figure, the most favourable coupling-coefficient remains unaltered by feedback according to property A.

It may be well to state that coupling corresponding to maximum gain is in general affected by feedback, but such coupling does not as a rule correspond to minimum noise figure.

We now consider a $2q$-terminal device, having one pair of input-terminals and $q-1$ pairs of output-terminals. This may be regarded as $q-1$ separate four-poles with their input-terminals connected in parallel[16]. Each of these four-poles has its own noise-figure of which one is assumed to be the lowest. We apply feedback from the output of this particular four-pole to its input, according to property A. This feedback is assumed to be such that a very large gain figure of the four-pole in question results and is indicated as "nearly critical feedback," critical feedback corresponding to an infinitely large gain, to which our "linear" theory does not apply. By this nearly critical feedback the available noise power at the output as well as the input of this particular four-pole is increased to such a high level as to drown completely the available noise power from any other sources at the said input-terminals. The same reasoning applies to the available signal power. Hence the noise ratios at the outputs of all the other four-poles become nearly equal to that of the selected one and so do the noise figures. Thus we are led to :

Property C : By applying nearly critical feedback from the output of lowest-noise figure to the input of a $2q$-terminal stage, the noise figures corresponding to all the other outputs may be made to approach that of the output mentioned first.

At this point the concept of correlation between noises will be introduced. Noises issuing from the same random fluctuations and reaching a pair of terminals are said to be completely correlated. If their individual available noise powers at the said terminals are N_1 and N_2 the resulting noise power is $N_1 + N_2 \pm 2\sqrt{N_1 N_2}$. Noises issuing from physically different random fluctuations are said to be completely uncorrelated. The above addition of available noise powers results in $N_1 + N_2$ in this case.

In the discussion leading up to property C no correlation was assumed between the noise-powers at the separate output pairs of terminals of the $2q$-terminal stage. In special cases of practical interest such correlation does exist. For simplicity we shall assume complete correlation between the noise at two pairs of output terminals. By proper feedback from one of these pairs to the input an additional noise may be created at the other output pair of terminals which is exactly counterphase to the original noise, thus extinguishing the noise at these output terminals completely. Making the further assumption that by this feedback the signal power at the latter output terminals is not also extinguished, the noise figure corresponding to it can be made zero. Thus we obtain the

Property D : If the noise at two pairs of output terminals belonging to a $2q$-terminal device is completely correlated, feedback may be applied from one of these pairs to the input so that the noise at the other pair is extinguished. If by this feedback the signal at the latter pair is not also extinguished its corresponding noise figure becomes zero.

We consider two four-poles such that the output terminals of the first one are connected to the input terminals of the second one. What is the noise figure of this combination if the individual noise figures of the four-poles are N_1 and N_2 and their gains are A_1 and A_2 respectively? The noise contribution of the second four-pole, according to Equ. (19), is equivalent to $N_2 - T_s/T$. This value has to be divided by A_1 if transferred to the input of the first four-pole. Hence the overall noise-figure N becomes :

$$N = N_1 + \frac{N_2 - \dfrac{T_s}{T}}{A_1} \quad \ldots \quad \ldots \quad (23)$$

A cascade combination of more than two four-poles may be dealt with similarly.

Now we apply feedback from the output of the first four-pole to its input, increasing its gain from A_1 to A'_1. If critical feedback is approached, A'_1 is very much larger than A_1. Assuming that no appreciable additional noise is involved in this feedback, we may formulate :

Property E : By application of nearly critical feedback, involving no appreciable additional noise, from the output of the first of two successive four-poles to its input, the overall noise figure may be made to approach that of the first four-pole.

7. Applications to Amplifiers

As already mentioned in connection with Equ. (20) of Section 5, electron-partition fluctuations occur in screen-grid valves due to the element of chance involved in electrons landing on the screen-grid or not. A screen-grid valve is shown in Fig. 9. We shall consider only the partition fluctuations and exclude from the discussion all fluctuations due to other sources. As indicated in Fig. 9 the valve may be regarded as a six-terminal device with the input terminal pair 1, 2 and the output pairs, 3, 4 and 5, 6. By the above

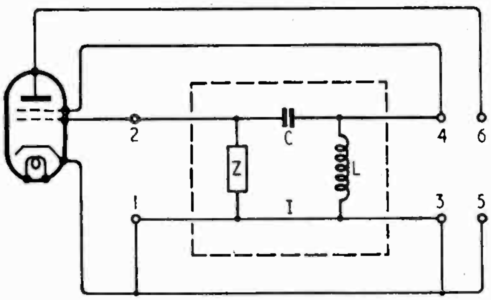
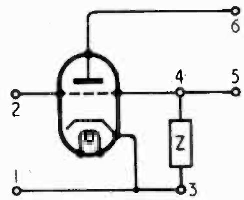


Fig. 9. *A tetrode is considered as a six-terminal device, the input-terminals being 1,2 and the two pairs of output terminals 3,4 and 5,6. A four-pole I is used for the establishment of feed-back between 3,4 and 1,2.*

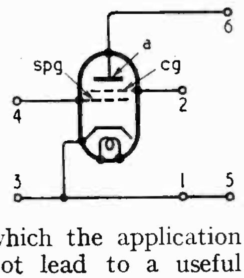
restriction to a single type of fluctuation, the noise at both pairs of output terminals is completely intercorrelated. Hence, property D may be applied, stating that by a proper feedback from the output terminal pair 3, 4 to the input pair 1, 2 the available noise power at the second output pair 5, 6 may be made zero. A four-pole I is indicated in Fig. 9, consisting of an inductance L, an impedance Z and a capacitance C. By the connection of this four-pole between the terminal pairs 3, 4 and 1, 2 the said annihilation of partition fluctuations at the output 5, 6 may be achieved, if L, Z and C are properly chosen in connection with the trans-admittances of the valve. At the same time it may be shown that the gain is not annihilated simultaneously. Hence the noise figure corresponding to partition fluctuations at 5, 6 is reduced to zero by the circuit mentioned. The possibility of suppressing the partition fluctuations at 5, 6 completely is a direct deduction from property D. The fact that no equivalent reduction of gain is involved comes about because the partition fluctuations are of opposite sign at the screen and the anode, whereas the transadmittances from 1, 2 to 3, 4 and to 5, 6 are substantially of the same sign. The reduction of partition fluctuations has been fully confirmed by experiments at 300 Mc/s.

Fig. 10. *A grounded-grid triode is considered as a six-terminal device, the input terminals being 1,2 and the two pairs of output terminals 3,4 and 5,6.*



A second application of property D is connected with the grounded-grid triode amplifier circuit shown in Fig. 10. Here again, one pair of input terminals 1,2 is present in conjunction with two pairs of output terminals 3,4 and 5,6. We shall only discuss fluctuation noise due to the random motion of electrons in the triode. If it is of ideal construction, the fluctuations due to this motion at the terminal pairs 3,4 and 5,6 are completely intercorrelated. Hence property D may be applied. The feedback in question is in this case simply constituted by a properly chosen impedance Z connected between 3 and 4 as shown in Fig. 10. By this feedback, triode fluctuations at the output terminals 5,6 may be reduced to zero whilst the gain from 1,2 to 5,6 is not annihilated. Hence the noise figure of the triode may be so much reduced that it is inappreciable in an amplifier stage. Here again, satisfactory experimental confirmation was obtained at 300 Mc/s. In these experiments a grounded-cathode circuit was used, but it can easily be shown that property D applies to both types of circuits,[15] interchanging cathode and grid in the circuit of Fig. 10, connecting the cathode to 2,4 and 5 and the grid to 1 and 3.

Fig. 11. *Space-charge grid valve* (spg *space-charge grid,* cg *control grid,* a *anode) as a six-terminal device.*



As an example in which the application of property D does not lead to a useful circuit we shall consider a space-charge grid valve as pictured in Fig. 11. The electrodes are denoted by spg (space-charge grid of positive steady voltage), cg (control grid of negative bias) and a (anode), the input terminals being 1,2 and the two pairs of output terminals 3,4 and 5,6 respectively. Considering partition noise exclusively, its contribution at the pair 5,6 may be annihilated by the application of proper feedback from 3,4 to 1,2 according to property D. But in doing so, the gain from 1,2 to 5,6 is practically annihilated too, thus leaving us with a stage devoid of any usefulness. This is due to the transadmittance from control grid to space-charge grid being of substantially opposite sign to the transadmittance from control grid to anode. The same sign relation obtains for the partition fluctuations.

8. Applications to Mixers

Noise figure reduction in triode-mixer circuits is possible by application of property C of Section 5. This reduction is such that the noise figure of a comparable triode amplifier stage under equal oscillator operation may be approached. As was mentioned in connection with Equ. (22), this reduction is hardly worth while in most cases of efficient oscillator operation.

We shall turn our attention to multi-grid mixer stages, in which much larger reductions of noise figures are possible in some specific cases. As an example we shall discuss the mixer circuit illustrated schematically in Fig. 12, involving a multi-grid valve of type (c) of Fig. 5. The partition fluctuations circulate in the leads connecting the grids numbered 2 and 4 with the points A and C and the anode a as well as grid No. 3. The lead connecting the cathode c with the point marked B is free of partition fluctuations. Due to the oscillator voltage V_{osc} active between the point A and grid No. 3 appreciable currents of output angular frequency ω_{out} circulate only in the leads connecting grids Nos. 2, 3 and 4 with No. 5 and the anode a. Hence only output pairs of terminals obtained by interruption of these leads of Fig. 12 show any appreciable available signal power of angular frequency ω_{out}. In contrast to this, any pair of terminals obtained by interrupting any lead of an electrode, including the cathode c and the anode a, shows available noise powers corresponding to a bandwidth f_0 centred round any frequency. At such a terminal pair these available noise powers are exactly equal in amount. But at terminal pairs in the leads connecting grids Nos. 2, 3 and 4 with the electrodes 5 and a the available noise powers mentioned are much larger in relation to the available signal powers at ω_{in} than in the lead from c to B, due to the partition fluctuations.

We may now construct a six-terminal device, the input terminals being A and grid No. 1, and the output terminal pairs being cathode c and point B as well as a second pair obtained by interrupting the lead connecting the point D with the anode a. At the first pair of output terminals there is

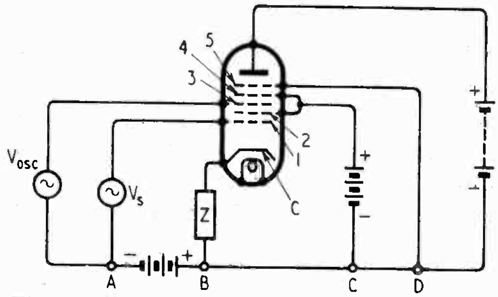


Fig. 12. *Mixer valve of type (c) of Fig. 5. The impedance Z in the cathode lead contributes feedback calculated to reduce the output noise figure.*

an available signal power corresponding to the angular frequency ω_{in} as well as an available noise power corresponding to a bandwidth f_0 centred round ω_{in}. From these powers we may deduct a noise figure N_1 at the said terminal pair. Similarly we may evaluate a noise figure N_2 at the second pair of output terminals, corresponding to an angular centre frequency ω_{out}. By the preceding reasoning N_2 is much larger than N_1 due to the partition fluctuations; e.g., 10 to 20 times N_1 in practical cases. As the partition fluctuations are dominant with N_2, only little correlation exists between the noise at the first and the second pairs of output terminals. Assuming complete non-correlation we may apply property C and establish a proper regenerative feedback from the first output to the input. When this feedback approaches its critical value the noise figure N_2 can be made to approach N_1. Thus a considerable reduction of noise figure (e.g., 15 db in practical cases) may be obtained. A simple feedback of this kind is obtained by inserting a proper impedance Z into the lead connecting c and B. This impedance may consist of a tank circuit or cavity tuned to an angular frequency slightly above ω_{in}.

The gain is very much increased by this regenerative feedback and there is danger of instability and oscillation. By application of property A we may restore complete stability without impairing the low noise figure obtained above. At the second pair of output terminals an available signal power of angular frequency ω_{in} exists. Hence a *degenerative* feedback active at this frequency may be established from the said output to the input, counteracting any tendency to instability in the mixer stage. According to property A the low noise figure obtained previously is not altered appreciably by this second feedback. The degenerative feedback in question may be established in several ways, using resistance-capacitance circuits.

No a.v.c. by bias control should be applied to the mixer valve in question as this might upset the proper functioning of the circuit. If a.v.c. at this stage is imperative, different means, such as, electro-mechanical variation of coupling capacitances, might be suitable.

Finally, we shall consider a diode- or crystal-mixer stage followed by an i.f. stage and preceded by a passive coupling four-pole between the aerial output and the mixer inputs. If the noise figures of these stages are N_m, N_{if} and N_c respectively, the overall noise figure has the value :

$$N = N_c + \frac{N_m - 1}{A_c} + \frac{N_{if} - 1}{A_c A_m}$$

where A_m denotes the gain of the mixer stage and A_c the gain of the coupler, both being always less than unity. Effective temperatures are all equal to the room temperature. By application of property E a reduction of N may be obtained by an increase of A_c. This may be achieved by connecting a feedback triode circuit to the input terminals of the mixer. Thereby A_c is increased and N may be made to approach N'_c which is the modified value of N_c taking the triode noise into account. We may apply property C to this stage, considering the input of the mixer as one pair of output terminals of the coupler stage, and two terminals obtained by interrupting the anode lead of the triode as another pair of output terminals of the modified coupler stage. By nearly critical feedback from the latter terminal pair to the input, the noise figure N'_c can be made to approach that of the triode's output. This is a useful result if the resulting value of N'_c is smaller than the original figure N.

9. Conclusion

From the gains evaluated in the earlier part of this paper figures of merit related to the different kinds of amplifier and mixer valves and devices have been derived. With narrow-band amplification and mixing these figures of merit are equal to the modulus of the transadmittance squared divided by the product of effective input and output conductances. With wide-band stages the latter product must be replaced by the product of effective input and output capacitances. The gain of grounded-grid stages is discussed. From the five main properties of noise figures derived in the second part of this paper a number of ways of reducing such figures have been indicated. These methods have in common the utilization of proper feedback, and the careful selection of the terminal pairs from which such feedback is established. Important results are obtained with grounded-grid triode amplifier stages and multi-grid mixer stages, with which considerable reductions of noise figures are possible.

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CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Comparison of A.M.-F.M.

SIR,—As one who was intimately connected with the design and subsequent road tests of one of the first f.m. double superheterodynes designed in this country for police v.h.f. communication, I can confirm the statements of Messrs. Scroggie and Macdiarmid in the December *Wireless Engineer* that f.m., from a noise point of view, is not superior to a.m. with carrier-controlled noise-limiter circuits.

I personally tested two receivers in very busy traffic conditions in a car having instantaneous f.m.-a.m. switching using two identical receivers at the same carrier frequency and having equal bandwidths. Signals were picked up from two transmitters, one f.m. and one a.m., having common modulation. The a.m. receiver held its own every time and had the obvious advantage of easy alignment, a point which was not brought out by either of your correspondents.

I can also confirm Mr. Macdiarmid's observation of a 1-kc/s de-tuning of the carrier causing a marked deterioration of impulse-noise performance on a 12.5-kc/s modulated system. Frequently having aligned the first i.f. accurately the discriminator secondary tuning capacitor was easily aligned by adjusting for minimum noise with an unmodulated carrier present. This was always the method used for final trimming on radio tests after all other trimmers had been locked accurately to the mid-band frequency.

I personally am convinced that the advantages of f.m. have been grossly over-rated and that for point to point v.h.f. work an a.m. system with a good noise limiter on the receiver is hard to beat. The only advantage of f.m. on such a system is the good a.g.c. characteristic obtained. Since the carrier powers used are so small, the reduced size of the modulator is not an advantage.

For high-quality broadcasting, modulator simplicity and the reduction of surges does enter into the argument of the advantages of f.m. over a.m. as will the extended range of sound intensities capable of being transmitted. Here again many of the advantages of good quality accorded to an f.m. system are largely due to the increased bandwidth position available on v.h.f. carriers which are apparent to anyone hearing the television sound programmes on a good receiver.

D. R. PARSONS.

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Potential Distributions in Screen-to-Anode Space of Beam Tetrodes

SIR,—It is known from the equations derived by Gill[1] that over a range of current densities there are apparently two steady-state solutions for the potential distribution in the screen-to-anode space of beam tetrodes. In addition to this, however, the transient conditions may also be studied.

Let it be assumed that at any instant a charge density σ is induced on the screen; then the acceleration of an electron which has taken a time τ_x to reach a plane at a distance x from the screen is

$$\ddot{x} = 4\pi \frac{e}{m}\left[-\sigma + I\tau_x\right] \quad \ldots \quad (1)$$

where I is the current density, and therefore $I\tau_x$ is the quantity of negative charge instantaneously interposed between the x-plane and the screen.

The choice of σ is not arbitrary but depends on the distribution of charge density ρ in the screen to anode space according to the equation—

$$\sigma = -\int_0^{x_a}\left(1 - \frac{x}{x_a}\right)\rho\, dx \quad \ldots \quad (2)$$

where x_a is the screen to anode gap.

When the screen and anode are at the same potential a self-consistent solution may be obtained by putting

$$\sigma = -\left[\frac{I\tau_a}{2} + \sigma_0 \epsilon^{-kt}\right] \quad \ldots \quad (3)$$

The solution shows that k is given by the equation

$$12\left[\frac{\alpha(1+\epsilon^{\alpha}) + 2(1-\epsilon^{\alpha})}{\alpha^3} - \frac{1}{6}\right] = \frac{3 - 2T_a}{T_a - 1} \quad (4)$$

in which $\alpha = kT_a$, and T_a is the "steady-state" time of flight, with the unit of time so chosen that T_a = unity for an electron moving from screen to anode in the absence of space charge.

When α is small, the l.h.s. is approximately α, so that

$$k = \frac{3 - 2T_a}{T_a(T_a - 1)} \quad \ldots \quad (5)$$

Now if k is positive, $\sigma_o\epsilon^{-kt}$ will ultimately die away to zero, and a steady-state solution will be attained.

If k is negative, $\sigma_a\epsilon^{-kt}$ will not decay and the corresponding steady state will not be possible. Hence it may be inferred that the states for which T_a is greater than 3/2 are not possible final steady states.

By comparison with the two solutions given by Gill's equation, it may be shown that it is precisely the lower of the two voltage distributions which has to be excluded.

S. RODDA.

New Barnet, Herts.

[1] Gill. *Phil. Mag.*, May 1925, pp. 993-1,005.

Harmonics in Oscillators

SIR,—In his paper on "Valve Oscillator" in your December issue Dr. Tillman repeats Groszkowski's "energy balance" explanation of the frequency change produced by harmonics in an oscillator. The "energy balance" theory has been quoted by so many writers and is so widely believed that it seems worth while to point out that it has no foundation in fact.

Groszkowski derived an accurate analytical formula for the frequency change produced by harmonics but, not content with this, he sought also to give a physical explanation.

There is, according to Groszkowski, a balance both of real and "imaginary" power (i.e., power and reactive volt-amps) between the oscillating circuit and the maintaining system. This is obvious and beyond dispute, but Groszkowski went on to make the plausible but erroneous claim that the maintaining system, because it is a non-linear resistor, cannot supply reactive currents and so the balance of "imaginary" power must exist between the reactive elements in the circuit itself.

It can be easily shown, however, that when a periodic voltage with harmonic components is applied to a non-linear resistor the currents at the various frequencies are not, in general, in phase with the voltages; i.e., the impedance of the resistor at the fundamental or a harmonic frequency appears to have a reactive component. The same is true of the transadmittance of a four-terminal non-linear resistor, such as a valve amplifier, as Dr. Tillman's analysis shows. It is in this effect that the true explanation of the frequency change is to be found. The circuit operates at a frequency such that the phase shift in the circuit at the fundamental frequency is equal and opposite to that produced in the amplifier owing to the presence of the harmonics.

The falsity of the "energy balance" theory is easily demonstrated by taking as an example the simple LC oscillator. Let the voltage across the LC circuit be $\Sigma V_n \cos(n\omega t + \phi_n)$. Then the mean electrostatic energy in the capacitance is $\frac{1}{4}C\Sigma V_n^2$ and, neglecting the resistance of the coil, the mean magnetic energy is $\frac{1}{4}L\Sigma V_n^2/n^2\omega^2L^2$. If the two energies are equal then $\omega^2/\omega_0^2 = \Sigma V_n^2/n^2\Sigma V_n^2$ where $\omega_0^2 LC = 1$. But, from Groszkowski's analytical formula, the true expression is $\omega^2/\omega_0^2 = \Sigma V_n^2/\Sigma n^2 V_n^2$.

Although, in this particular example, the "energy balance" theory has given the correct sign, though not the correct magnitude, of the frequency change, it is not difficult to find circuits for which the theory gives also the incorrect sign.

A. S. GLADWIN.

University of London.

Transmission Line Calculations

SIR,—For engineers requiring a simple theory of transmission lines, leading directly to the polar form of the circle diagram, I would recommend the following additional reading:—

1. "Short-Wave Aerial Systems," by E. Green, *Wireless Engineer*, June 1928, Vol. 5, p. 304.
2. Discussion on the paper by Jackson and Huxley, *J. Instn. elect. Engrs*, 1944, Vol. 9, Part III, pp. 110.

Writtle, Essex. E. GREEN.

NEW BOOKS

Frequency Modulation Engineering

By CHRISTOPHER E. TIBBS. Pp. 310 + x, with 172 illustrations. Chapman & Hall, Ltd., 37, Essex, St., London, W.C.2. Price 28s.

The author opens his preface by saying that "This book is intended to provide students, engineers and all those interested, with a concise and readily digestible survey of the whole field of frequency modulation engineering." It may fairly be said that the author has succeeded in his intention, and the book does provide a very large amount of useful information about frequency modulation.

The explanations are clear and the mathematics are neither too involved nor excessive in amount. Bessel functions are necessarily involved in the expressions for the sidebands but they need cause no-one any difficulty since their values are presented in both tabular and graphical forms.

After explaining the essential differences between amplitude, phase and frequency modulation, the author goes on to deal with interference and noise. This is well done and he brings out clearly the essential factors involved. It is followed by chapters on propagation and aerials. While these undoubtedly contain a large amount of information most of it is not peculiar to frequency modulation, and is equally applicable to any other system of modulation.

The author is undoubtedly right in explaining why frequency-modulation broadcasting is confined to frequencies above some 40 Mc/s, but it is a debatable point whether he should have gone into such detail about the propagation characteristics of such frequencies. Similarly with aerials. There is a lot of useful information, but none of it is peculiar to frequency modulation. One has the feeling that the engineer or student might reasonably be expected to turn to specialized books on these subjects.

Chapters on transmitters and receivers follow and are mainly concerned with modulators on the one hand and limiters and discriminators on the other.

Each chapter has a list of selected references for further reading. Unfortunately, there are no detailed references in the text to specific items in the bibliography, so that it is not always clear where one should look for further information about a particular point.

The book is unusually free from errors, but it is doubtful if full justice is done to the capabilities of amplitude modulation in comparisons of the two systems. There is no mention of the use of noise limiters with amplitude modulation. Furthermore, it is not adequately stressed that there is nothing inherent in a frequency-modulation system which leads either to a wider modulation-frequency response or more linear amplitude characteristics than with amplitude modulation. These are minor points of criticism against an excellent book, however.

W. T. C.

Wireless World Diary, 1948

Week to an opening, with 80 pages of reference material; $3\frac{1}{8}$in by $4\frac{1}{2}$in. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 3s. 8d. (including purchase tax), postage $1\frac{1}{2}$d.

The reference data includes base connections of 400 valves, wire tables, resistor wattage tables, coil winding abacs, predicted optimum frequencies for short-wave transmissions and addresses of radio organizations.

Manual of Hire Purchase Law (2nd Edition)

By A. C. CRANE and A. C. CRANE, Jnr., B.A. (Oxon.). Pp. 74. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 5s.