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CONVENTION BACKGROUND

As this issue of the *Journal* goes to press, the months of work spent on the planning and execution of the 1959 Convention reach their climax. The synopses of a further twenty-seven papers which are to be presented at the Convention are given on pages 326 to 332. Together with the thirteen papers quoted in the May *Journal* and some short formal contributions to discussions, this will result in about fifty subjects being reviewed in the course of three and a half days. A very full programme!

Exhibitions of apparatus will demonstrate many factors which cannot be dealt with in the course of lectures and discussions. This exhibition will be sited in a Laboratory adjacent to the lecture theatres and most demonstrations will be working throughout the Convention. One demonstration will serve a particularly important function for it is proposed to link three rooms—the Clerk Maxwell and the Rayleigh lecture theatres and the Demonstration Room—by a closed-circuit colour television system using a large screen projector.

The published Programme does not require further elaboration in these notes. It is the background to the formal meeting, however, that may be emphasized; there are several features which make the Convention more than a succession of meetings for the presentation and discussion of technical papers.

The provision of residential accommodation in the Colleges of the University of Cambridge has already been mentioned in the May *Journal*; the Convention handbook, which is being despatched to all corporate members shortly, describes the historical backgrounds to the Colleges, and gives an account of the work of the Cavendish Laboratory.

Due to the difficulty of finding a Hall this year which would be large enough to accommodate all the delegates and their ladies, it has been decided to hold a second banquet. Accordingly, approximately half the delegates will receive invitations to attend a banquet in Downing College on the evening of Thursday, July 2nd, and the others will meet at a similar function on the following evening in Gonville and Caius College. As already announced, Dr. Vladimir K. Zworykin will give the fourth Clerk Maxwell Memorial Lecture in the Cavendish Laboratory during the Convention, and this important meeting will now take place on the evening of the opening day of the Convention, Wednesday, July 1st.

Other social occasions have been deliberately kept to a minimum as it has always been realized that Institution Conventions in the precincts of Universities give unequalled opportunities for those participating to discuss matters both technical and otherwise with their fellow engineers all over the world. Delegates are expected to attend from at least a dozen countries outside Great Britain, and their formal and informal contributions will add to the value of the Convention proceedings.

Not the least important of the additional arrangements is the morning service for delegates in the University Church, Great St. Mary's. The new rector, Canon J. E. Fison, has kindly agreed to give a special address at this service.

Although the programme of the Convention is a concentrated one, there will be opportunities for delegates to see something of the Town and other Colleges of Cambridge, and it is with this in mind that Sunday morning has been made the official close of the Convention.

Synopses of Papers to be presented at the Convention

Industrial Television: a Survey of History, Requirements and Applications. SESSION 1

J. E. H. BRACE, B.SC. (*Marconi's Wireless Telegraph Co. Ltd.*)

Industrial television is considered, for the purposes of this paper, to embrace all applications of television not specifically concerned with entertainment. Since the first of such applications was established some ten years ago, progress has been much slower than was at first anticipated. The author seeks to explain this by a survey of the history and requirements of the technique as seen today which indicates that the initial approach was misguided and the results consequently discouraging.

The economic basis of industrial television is analysed and current and future trends in equipment design are briefly considered. A survey of typical established applications is also given although this is not claimed to be comprehensive.

Technical Considerations of Television in the International Field. SESSION 1

T. KILVINGTON, B.SC. (*Post Office Research Station*)

International aspects of all classes of telecommunications are dealt with by the I.T.U. (International Telecommunications Union) and its consultative committees, the C.C.I.R. (radio) and C.C.I.T.T. (telegraph and telephone). Over the past few years considerable discussions have taken place in these bodies on the standards for television and particularly on the requirements for the point-to-point transmission of television signals over long distances. A recommendation on the latter subject has recently been adopted by the C.C.I.R. and forms a useful basis for the design of microwave and coaxial-cable television relay links.

A Common Carrier Multi-channel Television Wire Broadcasting System SESSION 1

K. A. RUSSELL, B.SC., AND F. SANCHEZ, ASSOCIATE MEMBER.
(*British Relay Wireless and Television Limited.*)

The paper surveys the factors governing the choice and design of a television relay system and the history and background of the development of various systems. It then describes the basic technical features of a particular system.

Any television relay system basically depends on the cable employed for its performance and information is given regarding various types of cable and their specifications. Network jointing and matching fittings are described and the method of using the system characteristics to plan a network is explained with examples and an indication of the coverage obtained.

Various types of subscriber installations can be provided and their characteristics are defined. Typical examples of subscriber equipment are described and illustrated. The operation of the main receiving station and repeater equipment is explained in some detail and finally test methods and test equipment are described.

Subception. SESSION 1

N. F. DIXON, M.B.E., PH.D. (*Department of Psychology, University College, London.*)

The lecture will start with a brief defining of terms. What is subception? What are the awareness and recognition thresholds? What are the physiological implications of subception phenomena? This will be followed by a short history of experimental work on subception. Reference will be made to recent experiments involving both television and the cinema as the media for subliminal stimulation. The paper will conclude by saying something about those hypotheses and experimental methods in which the author is interested.

Printed Wiring in Development and Small Batch Production of Video Equipment.

SESSION 2

E. DAVIES. (*Marconi's Wireless Telegraph Co. Ltd.*)

Before embarking on a development programme for a range of television studio and outside equipment, the author investigated various aspects of development, drawing office, production, and test procedure that would be affected by the adoption of printed wiring.

An initial experiment showed that a video amplifier with input and output impedances of 75 ohms and 20 db overall gain, increased in bandwidth by 25% when printed wiring replaced conventional construction. This encouraging result led to new approaches in the mechanical design of equipment and the introduction of a new grade of labour to serve as a link between development engineers and the drawing office. The new type of staff, known as Translators, are usually female and educated to General Certificate of Education standards, and their function is to convert circuit diagrams and associated component schedules into wiring and component legend layouts for printed wiring boards.

The choice of standards, method of translation, and the type of machinery which was specially designed for the automatic punching of holes, assembly of components, and soldering of boards, will be discussed. The paper will conclude with brief comments on the range of equipment so far produced by the methods described.

The Combined Television-Radio Receiver and its Problems.

SESSION 2

R. S. HILDERSLEY. (*Murphy Radio Ltd.*)

A new class of domestic receiver has evolved since the introduction of frequency modulated sound transmissions in Band II, which can be switched to receive television or f.m. sound broadcasts at will.

However, because of the large number of frequency allocations in and around Band II compared with Bands I and III, a serious selectivity problem arises. The sound bandwidth of a television receiver is usually of the order of 500—1,000kc/s and since the frequency allocations of radio communications transmitters are liable to be within 500 kc/s of the B.B.C. transmitters, interference can occur. Interference can also occur between the various B.B.C. regional transmitters whose frequency separation is usually 400 kc/s, but can be as little as 200 kc/s. Intensive work has been carried out to find a solution and several alternative systems have been proposed as a result. The paper discusses the problem and solutions in detail.

The new problems that are posed by these solutions are also examined. Accurate methods of measuring the selectivity of a receiver are described and the paper concludes with a detailed description of the design of a commercially practicable double superheterodyne receiver which attempts to meet these various requirements.

Some Aspects of Television Reception on Band V.

SESSION 2

H. N. GANT, ASSOCIATE MEMBER. (*E.M.I. Electronics Ltd.*)

The problems facing the designer of a television receiver for use on Band V are considered. The major difficulties, namely oscillator stability and signal/noise ratio, are considered in some detail in respect of performance and complexity. Suitable valves available as oscillators are considered. The performance of various possible combinations of valves and crystals for the front end are discussed. Some experimental results achieved are quoted, using two valves type A2521 as r.f. amplifier and oscillator, and the construction of this unit, using normal mass production components and techniques, is described.

Good Practice Techniques in Television Scanning Circuits.

SESSION 3

K. E. MARTIN. *(Mullard Research Laboratories.)*

Using improved circuit techniques it is possible to use the same valves in the line scanning circuits for 110° picture tubes as were used for the 70° tubes. The circuit improvements which have made these advances possible are :—(1) third harmonic tuning; (2) improved ferrites; (3) shorted turn linearity and width adjuster; (4) better knowledge of design procedure. These techniques are discussed in detail.

Television Field Scan Linearization.

SESSION 3

H. D. KITCHIN. *(Mains Radio Gramophones Ltd.)*

Although the design of a field timebase is based on well established principles and theory, there are many points at which there is considerable divergency between the theoretical and practical results. Foremost of these is that of calculating the parameters of linearizing networks, particularly of the feedback type. The present paper examines conventional linearizing methods and it is shown that the deflection current produced is S shaped under certain conditions. The deflection current demanded by modern wide angle cathode ray tubes is of this form and it is shown that by the correct choice of operating conditions conventional linearizing methods can approximate to the desired shape sufficiently close for normal domestic receiver requirements.

Transistors in Video Equipment.

SESSION 4

P. B. HELSDON, ASSOCIATE MEMBER. *(Marconi's Wireless Telegraph Co. Ltd.)*

Principles involved in the design of iterated video amplifiers for television using transistors are discussed in terms of simplified hybrid-pi equivalent circuit. The importance of the current gain-bandwidth factor is emphasized. Transistor non-linearities at high levels are described in general terms. Noise at low levels is investigated with regard to camera head amplifier design. Finally, the design and performance is described for experimental equipment.

A Transductor Regulator for Stabilized Power Supplies.

SESSION 4

A. N. HEIGHTMAN, ASSOCIATE. *(Marconi's Wireless Telegraph Co. Ltd.)*

A new form of transductor regulator is described which, in replacing the series-valve regulators typically used in stabilized power supplies, enables improvements in efficiency and reliability to be gained. The slow response of the transductor necessitates the incorporation of a valve regulator to deal with rapid disturbances.

The transductor circuit is unusual in that only one core, and in the simplest case only one winding, is required for full-wave operation. A description of the mode of operation and an analysis of the circuit yielding design data are given. A high efficiency is obtained in the associated valve regulator by Class-B working; some design notes are included. A simpler power supply using a transductor but without an associated valve regulator is similarly treated.

The emphasis is on the needs of television studio and similar installations where power units delivering about one ampere output at 250 volts are required.

Some Aspects of Vidicon Performance.

SESSION 4

H. G. LUBSZYNSKI, DR.ING., J. WARDLEY, A.R.C.S., B.SC., AND S. TAYLOR, B.A.
(E.M.I. Electronics Ltd.)

Some properties of the vidicon have been measured. The general performance of the tube depends on these properties and on the operating condition. By matching the two, optimum performance can be obtained for various applications.

A Television Master Switcher. SESSION 4

B. MARSDEN, ASSOCIATE MEMBER. (*Associated Television Ltd.*)

A survey is made of the standard methods at present in use for switching video signals: the simple mechanical switch, electromechanical relays, and switching systems using thermionic relays. A method of video selection is then described in which the switch elements are made up of semi-conductor diodes. Both master control room and studio type switchers are discussed. Reference is made to current development work in which transistorized pulse generators are being used to achieve vision switching between successive frames of the television waveform.

Automation of Television Programme Switching. SESSION 4

G. E. PARTINGTON, B.SC.TECH. (*Marconi's Wireless Telegraph Co. Ltd.*)

A day's television programme is assembled from programme segments of various lengths. These usually follow the general pattern of extended transmissions from a studio, film or outside broadcast interspersed with short periods during which several different sources may be used to make announcements, show time and station identification, and to advertise. Typically the real programme segments range from 10 to 90 minutes and the "break" periods use three or four sources in two minutes. This programme assembly is usually handled by a team consisting of a programme controller, a vision mixer, a sound mixer, and possibly some assistants.

Normally the personnel involved behave, for the greater part of the time, as human servos reacting to the stimuli provided by reading the programme schedule and a clock. It is only during deviations from this that their faculties are fully extended. Accordingly it seems reasonable to devise a machine to carry out the routine operations. If, further, a simplification can be made in the manual control, then when necessary there can be a reduction in personnel, particularly as they will be relatively rested by not having to deal with the main routine operations.

The paper outlines a complete system under which the bulk of the operating is controlled by a punched paper tape derived directly from the preparation of the programme schedule. Simplified manual control facilities are provided for non-scheduled operation.

The term "Programation" has been coined to describe this system.

Operational Facilities in the R.C.A. Colour Television Tape Recorder. SESSION 5

A. H. LIND. (*Radio Corporation of America.*)

The design of the R.C.A. TRT-1A television tape recorder makes many new operating facilities available as integrated parts of the recorder. In addition, features such as the electrical delay adjustments are an advance in the art that make greater precision readily available and subsequently will aid in achieving tape interchangeability with highly acceptable performance.

(*To be presented by Dr. H. R. Lamont, European Technical Representative of R.C.A.*)

A Mobile Television Camera and Recording Vehicle. SESSION 5

AUBREY HARRIS, ASSOCIATE MEMBER. (*Ampex Corporation.*)

The paper describes the design and construction of a mobile television camera and recording unit mounting all the necessary apparatus to televise and record live scenes, even while the vehicle is in motion. The mobile unit, which is known as the "Videotape Cruiser," contains a television tape recorder, two image orthicon cameras, vision and sound mixing equipment, a synchronizing generator and a petrol-electric alternator set. Details are given of vision and sound facilities and power supply system; the mechanical arrangement of the vehicle and equipment mounting is also described.

The Development and Progress of Medical Colour Television. SESSION 6

R. D. AMBROSE, C.I.E., O.B.E., M.C., AND A. R. STANLEY. (*Smith, Kline & French Laboratories Ltd.*)

The history of colour television in medicine began in 1949 when a mobile colour television unit gave the first demonstration of surgical operations through this medium. Starting as a one-camera unit, it has now expanded to three complete units (two in the U.S.A. and one in England) each operating two or three cameras. The 12 in. receivers previously used have been changed to large projector screens, the largest of which is now 12 ft. x 8 ft. In America colour television is used in surgical demonstrations, demonstrations of patients, autopsies, laboratory and microscope demonstrations, fluoroscopic image intensification and deep therapy observation.

Perhaps the biggest future advance in the use of this medium will be in endoscopy—showing a colour picture of what is happening *inside* the body. Although this has been done by means of the endoscope tube in colour film, and in black and white television, present colour cameras are too heavy and too unwieldy in use. With the new transistorized colour camera now being developed for bronchoscopy and cystoscopy a large screen colour picture of the interior organs of the living body will be shown to hundreds of students at a time.

A Gating Circuit for Single-gun Colour Television Tubes. SESSION 6

K. G. FREEMAN, B.SC., GRADUATE. (*Mullard Research Laboratories.*)

The requirements of an ideal gating circuit for use with single-gun colour television tubes and the limitations of some existing circuits are discussed. A new type of gating circuit which employs low-level gating of the red, green and blue video signals in conjunction with a wideband amplifier is described. Such a circuit is believed to have a performance superior to that of most existing circuits and by fairly simple modification is applicable to either reversing colour sequence, continuous colour sequence or to colour difference operation.

The Use of Television for the Microscopical Examination of Radioactive Metals. SESSION 7

E. C. SYKES. (*Atomic Energy Research Establishment.*)

The paper describes the use of closed circuit television for relaying the image produced by remotely controlled microscopes enclosed within thick lead shields and used for the examination of highly radioactive irradiated fissile materials, e.g. uranium 235. Brief descriptions are given of two installations incorporating respectively C.P.S. Emitron and Image Orthicon camera chains, and also of a device enabling accurate measurements of microstructural features displayed on monitor screens. (The "electronic cursor.")

The advantages and disadvantages of television compared with optical viewing are discussed, and brief mention is made of experiments with television for general remote handling.

A High-grade Industrial Television Channel with reference to Infra-red Operation. SESSION 7

J. H. TAYLOR. (*E.M.I. Electronics Ltd.*)

The greatest effort in the television engineering field has undoubtedly been made in the design of equipment for entertainment, but there is a growing demand for television as an aid in many industrial processes, and for other purposes.

The paper will give some indication of the range and scope of the uses of television for these purposes. Some of the design requirements will be discussed and it will be shown how they have been met with reference to a particular television channel. In addition, two special applications are described, these being the use of this channel with infra-red and ultra-violet light.

The Application of Closed-circuit Television in the Nuclear Industry. SESSION 7

P. BARRATT, M.A., PH.D., AND I. M. WATERS. (*Pye Ltd.*)

The paper reviews some of the most significant ways in which standard and specially developed television equipment can be used by nuclear power stations and experimental establishments, and discusses how development is influenced by the environmental conditions to be encountered. For example, the use of closed circuit television to provide remote viewing in "hot" laboratories for routine fuel element inspection and control, and for observing the charge and discharge conditions in a reactor power station, etc. Details are given of a new application of closed-circuit television for the accurate positioning of re-fuelling machines over reactor-fuel tubes, permitting the entire operation to be carried out by remote control. Other applications dealt with include remote handling operations and general inspection methods. The overall equipment design philosophy is considered including the need for selected components to meet conditions of high gamma and neutron flux.

Phosphors for Cathode-ray Tubes in Industrial and Low Scanning Speed Display Systems. SESSION 7

M. D. DUDLEY, M.B.E., B.SC. (*Ferranti Ltd.*)

Phosphors for use in low-speed scanning tubes are considered from the point of view of persistence, light output and texture, when operated at writing speeds lying between 10^3 and 10^6 spot diameters per second. For photography of the received image, the colour, texture and persistence of phosphors are considered in the light of their effect upon photographic materials. The requirement of long-persistence displays for visual use are described, dealing in particular with the use of ZnS (Cu-Pb) phosphor whose afterglow is stimulated in infra-red radiation of approximately one micron wavelength: phosphors with this type of behaviour were first investigated by Urbach and Fonda and whilst originally intended as a means for detecting infra-red radiation, they show some promise as a simple means for displaying in its entirety an image which has been built up over an appreciable period of time. By excessive infra-red stimulation the erasure of each image can be achieved thus ensuring that the following image is not confused by long-persistent traces.

Photo-electric Image Techniques in Astronomy. SESSION 7

B. V. SOMES-CHARLTON. (*High Definition Television Ltd.*)

In introducing the subject the author explains briefly the underlying scientific principles of light detectors such as the human eye, photographic emulsion, or photo-electric device, and compares their quantum efficiencies in terms of signal-to-noise ratios. The paper discusses how television techniques can be applied to aid astronomical observations and help to overcome, in some measure, the fundamental problems presented by the existing enforced circumstances in having to observe the celestial objects through the semi-transparency of the Earth's atmosphere. A description is given of the television techniques exploited by research workers in this field, whose efforts are being directed towards the detection of threshold and extremely low light level stellar and planetary images respectively. Achievements to date are summarized and the results illustrated by photographs including those obtained by the author at the Lamont-Hussey Observatory, Bloemfontein, during the "favourable opposition" of Mars in 1956.

The paper concludes with a survey of the present trends in applying television techniques including proposals for the construction of a "space orbitting astronomical telescope" which will focus radiation from celestial objects on to an electronic scanning device for subsequent transmission to a ground station equipped for monitoring the signals and controlling the satellite's behaviour in space.

Transmitting Aerials for Television Broadcasting in the United Kingdom. SESSION 8

A. BROWN, ASSOCIATE MEMBER. (*British Broadcasting Corporation Research Department.*)

The development of transmitting aerials for television broadcasting in Band I is surveyed. Reference is made to the increase of gain and to the splitting of the aerial system for increased flexibility and reliability. Aerials having directional radiation characteristics have been required for filling gaps in the service areas of other stations, and giving adequate protection against interference with other services. Measurement techniques and the permissible variation of reflection coefficient and radiation pattern over the video band are considered. Future developments discussed include the use of higher frequencies to obtain higher gains. The problem of giving an adequate service in areas near the mast will thus be increased and the maximum possible useful gain which can be obtained from an aerial of given height, and the degree to which a practical aerial can approach this condition, are discussed. Measurements on scale models are also discussed.

Microwave Television Mobile Relay for Outside Broadcasting. SESSION 8

J. P. POLONSKY, I.E.G., E.S.E.R. (*Compagnie Generale de T.S.F., Paris.*)

A brief account will be given of the principal qualities required in the mobile links for Outside Broadcasting and in particular:

(a) Transmission of picture and sound without degradation of the quality and stability in time of the technical performances.

(b) Facility of operation, as regards: transport, installation monitoring and maintenance.

After reviewing the essential causes of distortion introduced in the transmission by a microwave link, the author indicates the solutions which have been used.

The problem of cross-talk between the picture and the sound channels and the transmission of a colour television programme are also dealt with. The last part of the paper gives a short description of equipment operating in the 6400-6900 Mc/s band.

Some Aspects of Television Transmission over Long Distance Cable Links. SESSION 8

H. MUMFORD. (*British Telecommunications Research Ltd.*)

An outline of the basic properties of 0·375 inch diameter coaxial cable and the combined or alternative multi-channel telephony/television systems based on it is given. Most of the required transmission limits for such systems have now been agreed internationally and a "hypothetical reference circuit" evolved for which such limits can be stated. Although these limits have tended to be agreed by national experiments on the various television systems, two basic subjective laws, the Weber-Fechner law and Riccô's law, on which such limits must ultimately be based, are illustrated and discussed. Television transmission is fundamentally a waveform matter but for design purposes it is necessary to use frequency responses. The paper describes two simples guides to estimating these firstly by reference to two simple minimum phase networks and secondly by an extension of the paired echoes method. The advantages of using a reduced carrier level are briefly pointed out and a number of synchronizing systems reviewed.

This list completes the details of the principal papers accepted for presentation during the eight sessions of the Convention. Synopses of thirteen papers were given in the May Journal; there will, in addition, be a number of short contributions, details of which will be circulated to those attending the Convention.

Members are reminded that to ensure accommodation in one of the Colleges their reservation forms must be returned to the Institution not later than Thursday, June 25th.

INSTITUTION NOTICES

Birthday Honours List

The Council has congratulated the following members whose names appear in Her Majesty's Birthday Honours List:

Group Captain George R. Scott-Farnie (Member) on his appointment as a Commander of the Civil Division of the Most Excellent Order of the British Empire. Group Captain Scott-Farnie is managing director of International Aeradio Ltd.

Squadron Leader Ronald Brickwood (Associate Member) on his appointment as an Ordinary Member of the Military Division of the Most Excellent Order of the British Empire. Sqdn. Ldr. Brickwood was chairman of the Merseyside Section from 1956-58. He is now attached to the Headquarters of Allied Air Forces, Central Europe.

Canadian Advisory Committee

During the course of his recent visit to North America, referred to the April issue of the *Journal*, Mr. G. A. Marriott, Immediate Past President, met many members of the Institution and took the chair at well attended meetings in Toronto and Montreal. He has brought back to the Council a request from Canadian members that a Canadian Advisory Committee should be formed which would be concerned with the establishment of Sections of the Institution in Toronto and Montreal.

Mr. L. H. Paddle (Member) who has been the Council's representative in Canada and has served as a Vice President, was elected Chairman of the proposed Advisory Committee. Mr. R. J. A. Turner (Member) was elected Honorary Secretary; members may get in touch with Mr. Turner at 66 Gage Avenue, Scarborough, Ontario.

Professor S. K. Mitra, F.R.S.

The Council feels sure that all members will share its regret that Professor S. K. Mitra will not be able to attend the Convention. Professor Mitra, who was to have represented the Indian members at the Convention, and to have given a lecture on Propagation, has been advised not to make the journey to Europe on medical grounds.

South East London Technical College

The Council has nominated Mr. S. R. Wilkins (Member) as its representative on the Electrical Engineering and Applied Physics Consultative Committee of the South East London Technical College. Mr. Wilkins represents the Institution on B.S.I. Technical Committee TLE/8 and was for many years one of the Institution's Examiners; he has also served on the Programme and Papers and Technical Committees. Mr. Wilkins is the Technical Director of Avo Limited.

Dr. V. K. Zworykin

The Council has announced its intention to confer Honorary Membership of the Institution on Dr. Vladimir K. Zworykin when he visits England next month to deliver the Clerk Maxwell Memorial Lecture at the Convention. A brief note on the distinguished career of Dr. Zworykin was published in the April issue of the *Journal*, and it is appropriate that the Institution should honour him for his contribution to Television Engineering at a Convention on this subject.

Dr. Zworykin's other honours include Fellowship of the American Institute of Radio Engineers, of the American Institute of Electrical Engineers, and the American Physical Society, and he is Honorary Member of the Society of Motion Picture and Television Engineers. He has received awards from these bodies and others in the United States as well as from professional and learned societies in this country, France and Belgium. In addition, his work has received official recognition from the Governments of the United States and of France.

Visit of the President of the I.R.E. Australia to England

Advice has been received from the Institution of Radio Engineers, Australia, that Mr. Graham G. Hall, President for 1959-60 is to visit London in July in the course of a tour of Europe and America. Mr. Hall is also chairman of the Australian Institution's Publications Board and a member of its Charter Committee. He is with Ducon Condenser Ltd. and presented a paper at the recent Convention in Melbourne.

Advertisements in the Journal

Reference has frequently been made in Annual Reports to the important revenue obtained from display advertisements in the Institution's *Journal*. This helps to offset the heavy costs of printing and paper, and it is therefore essential that advertisement revenue should increase to keep pace with these costs. Members who are in a position to do so are therefore urged to recommend to their companies the use of the *Brit.I.R.E. Journal* as an advertising medium. In this connection the certificate of the Audit Bureau of Circulations is a guide to the value of the publication.

Course on Transistor Theory

A Summer School on the Fundamentals of Transistor Theory and Applications will be held at the Borough Polytechnic, London, S.E.1, during the week 13th to 17th July (10.00 a.m. to 4.30 p.m. each day). This course is intended to give engineers familiar with the applications of thermionic devices an introduction to similar applications in the transistor field. A knowledge of basic circuit and network analysis (approximately H.N.C. standard will be assumed.

The lecturers responsible for the course, for which the fee will be £10, will be Mr. S. N. Ray, M.SC., B.SC.(ENG.), and Mr. D. N. Tilsley, B.SC. (ASSOCIATE MEMBER).

Montefiore Award

The Belgium engineering society, L'Association des Ingenieurs Electriciens Sortis de L'Institut Electrotechnique Montefiore, has recently announced that there will be an International Competition for the "Fondation George Montefiore" in 1960. This prize, which is awarded every five years, is for the best original work submitted on scientific advances and progress in the technical applications of electricity in any field. It is worth one hundred thousand Belgian francs (about £714), and the closing date for the receipt of the papers by the Board of Examiners will be 1st July, 1960.

Further information may be obtained from the Institution or from the Association at 31, Rue Saint-Gilles, Liege 1, Belgium.

OBITUARY

The death took place on the 12th July last of Mr. Thirumalai Seshachari Rangachari (Associate Member) at the age of 58. Mr. Rangachari was a graduate of Madras University where he took an M.A. degree in Physical Sciences in 1925 and obtained the M.Sc. degree by a thesis in High Frequency Measurements in 1930. He subsequently held an appointment as a Research Assistant in Communication Engineering at the Indian Institute of Science, Bangalore. He then entered the Indian Radio Telegraph Company; from 1945 to 1947 he was engineer-in-charge of the Beam Wireless Transmitting Station at Poona.

In 1948 Mr. Rangachari was transferred to the Department of Civil Aviation as a Senior Technical Officer, and subsequently became Controller, Radio Construction and Development Unit; he represented the Government of India on committees of the International Civil Aviation Organization. In 1954 he was appointed Controller of Communications, Madras Region. He retired from Government service in 1956 and was for one year Professor of Electronics at Birla Engineering College, Pilani. Ill-health forced him to relinquish this position some months before his death.

Mr. Rangachari, who was elected an Associate Member of the Institution in 1955, was author of a number of papers in "Wireless Engineer" and in the journal of the Indian Institute of Science, "Electrotechnics," serving for a time as a member of its editorial board.

William Sinclair (Associate Member) died in April this year in Edinburgh Royal Infirmary after a brief illness. Until his retirement on medical grounds last autumn, he was at the Admiralty Hydroballistic Establishment on the Clyde; he first joined the Admiralty at the beginning of the war and had held appointments at various establishments in the United Kingdom and overseas. Elected an Associate Member in 1938, Mr. Sinclair died at the age of 58 years.

Theoretical and Experimental Characteristics of Random Noise in Television†

by

R. FATEHCHAND, B.SC.(ENG.), PH.D.‡

Summary: Consideration is given to certain aspects of random noise which may be important in television, namely that due to electron current fluctuations in valves or thermal agitation of electrons in resistances. The relationship between frequency spectrum and the time variation of random noise is shown. In particular, a fluctuating low-frequency envelope is associated with "quasi-triangular" noise, that is, noise peaked at the high-frequency end of the pass-band. A non-linear transfer characteristic is next considered. Low frequency components are then produced from quasi-triangular noise, these terms arising from the fluctuating envelope. Phase distortion after the non-linearity has appreciable influence on noise properties. The relevance of these effects to noise visibility on a picture tube is examined, and oscilloscope photographs are given. For random noise plus a sinusoidal signal, non-linearity gives rise to intermodulation products of the noise and signal. This is of special significance if the signal is near the top of the pass-band, and the noise is quasi-triangular, since in this case the presence of the low-frequency intermodulation products drastically increases the noise visibility on a picture tube. It is shown that this intermodulation effect may be produced by the non-linear characteristic of the picture tube.

1. Introduction

Recent studies[1, 2] in the television field have considered the subjective effect of random noise as determined by its position on the grey scale and the shape of its frequency spectrum. For one of these investigations[1], a theoretical treatment was given, based on the assumption that the small amplitude of the noise made it possible to neglect non-linear effects. However, it is important to know what may be expected when the influence of a non-linear transfer characteristic must be taken into account. The present paper describes some simple experiments carried out with this end in view. The noise to be considered is that which arises from electron current fluctuations in valves or from thermal agitation of electrons in resistances. The theoretical treatment is based on that of Rice.[3]

The following two types of noise spectrum are often encountered in television practice:

(a) "White," where the noise power is uniformly distributed over the frequency band, as for example the output of an image orthicon camera tube.

(b) "Quasi-triangular," where the noise power in a small frequency band increases with the mid-band frequency, as for example the output of a film-scanner subsequent to afterglow and aperture correction.

Section 2 of the paper describes the differences in noise characteristics which correspond to the above classification. Section 3 then shows how the effect of a non-linearity is related to these differences. The modification of noise frequency spectrum is considered, and the influence of phase distortion subsequent to the non-linearity. Finally, Section 4 examines the effect of non-linearity on noise plus a steady sinusoidal signal. This is of particular interest in the N.T.S.C. system of colour television, where the chrominance information is associated with a sinusoidal sub-carrier.

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2. Wave-shapes of Noise

2.1. *Auto-correlation Function*

It is possible to determine the likely wave-shape of the random noise provided its frequency spectrum is known. Here the *power* spectrum $w(f)$ must be considered, defined such that if the noise current $I(t)$ flows through a resistance of one ohm the average power dissipated by the components of frequency between f and $f+df$ is $w(f)\,df$. The Wiener-Khintchine theorem[3] then gives

$$\psi(\tau) = \int_0^\infty w(f) \cos 2\pi f\tau \, df \qquad \ldots\ldots\ldots(1)$$

The function $\psi(\tau)$ is known as the Auto-correlation Function. It will be normalized by assuming an r.m.s. noise current of unity, and then lies in the range -1 to $+1$. If the noise amplitude at an arbitrary time is known, the amplitude at a time τ later can be estimated by means of the function $\psi(\tau)$. For example, if $\psi(\tau_1) = +1$ or -1, then at times $\tau = 0$ and $\tau = \tau_1$, the amplitudes will have equal or equal and opposite values respectively. If $\psi(\tau_1) = 0$, the two amplitudes are completely independent. If $\psi(\tau_1)$ is neither 0 or ± 1, there will be some degree of dependence of one amplitude on the other. Thus if $\psi(\tau)$ is graphed against τ, some idea of the probable noise wave-shape can be obtained. This is explained more fully in Appendix 1.

In Fig. 1, $\psi(\tau)$ has been plotted against $2\pi f_c\tau$ for various power spectra characteristics. Here f_c represents either a sharp cut-off frequency or some nominal cut-off, and the noise band extends down to zero frequency. For white noise through a resistance-capacitance or Gaussian type of filter, $\psi(\tau)$ decays monotonically as τ increases. If white noise has a sharp cut-off at f_c, an oscillatory characteristic is imposed on $\psi(\tau)$, and this behaviour becomes considerably more marked for a quasi-triangular noise spectrum. For narrow-band noise confined within the frequency range $f_c - \Delta f$ to f_c, provided $\Delta f/2$ is much less than f_c,

$$\psi(\tau) = \frac{\sin 2\pi\tau\Delta f/2}{2\pi\tau\Delta f/2} \cos 2\pi f_c\tau \qquad \ldots\ldots\ldots(2)$$

The function $\psi(\tau)$ is now a sine wave of frequency f_c, with a low frequency envelope $\dfrac{\sin 2\pi\tau\Delta f/2}{2\pi\tau\Delta f/2}$ which reduces in frequency with

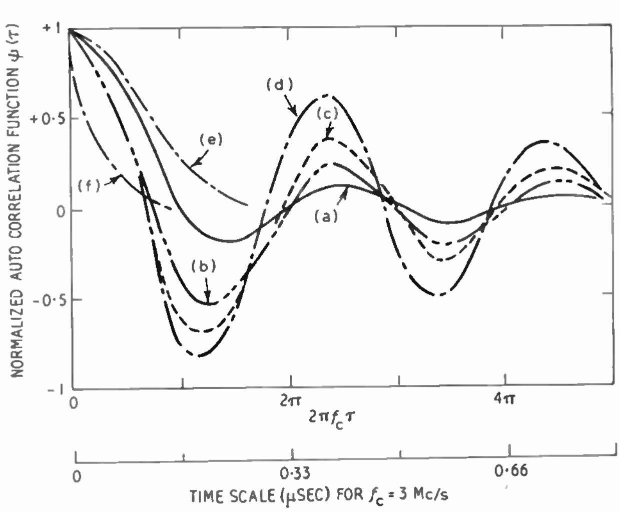


Fig. 1. Normalized auto-correlation function for various noise spectra. For each curve, $\psi(-\tau) = (\tau)$. The noise power spectra are: (a) white noise, (b) $W(f) = a_1 f$, (c) $W(f) = a_2 f^2$, (d) $W(f) = a_3 f^4$. For (a) to (d), the noise band extends from zero frequency to a cut-off frequency f_c. (e) $W(f) = a_4 \exp - 2(f/f_c)^2$, Gaussian. (f) $W(f) = a_5/[1+(f/f_c)^2]$, RC filter with $f_c = 1/2\pi RC$.

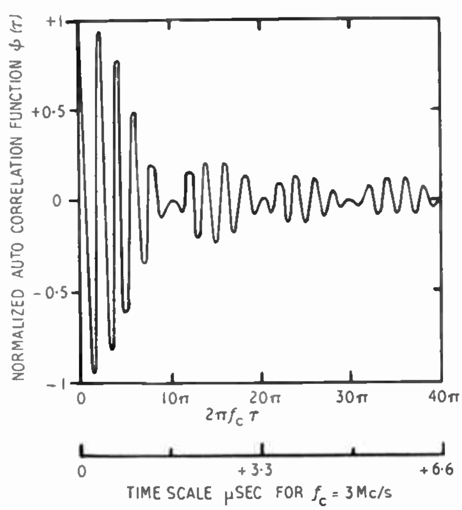


Fig. 2. Normalized auto-correlation function for narrow-band noise. For the curve, $\psi(-\tau) = \psi(\tau)$. The noise is solely in the band $f_c - \Delta f$ to f_c, where $\Delta f = f_c/5$.

Δf. This is shown in Fig. 2 for $\Delta f = f_c/5$. Thus, if the noise energy is increasingly concentrated at the high-frequency end of the pass-band, the envelope frequency of $\psi(\tau)$ will reduce.

2.2. *Observations*

Quasi-triangular noise was obtained by passing valve shot-noise current through a suitable network. This noise spectrum is (e) on Fig. 3. An illuminated photo-multiplier, which produced an output noise spectrum flat to well above 3 Mc/s, was used as the source of white noise. The noise band was sharply cut at an upper frequency f_c of 3 Mc/s, and extended down to around 50 c/s.

Single-sweep oscilloscope photographs of white and quasi-triangular noise are shown respectively in (a), (b) of Fig. 4, while (c), (d) of the same figure are photographs of the noise displayed on a television monitor. It will be noticed that the white noise has an extremely irregular wave-shape, and a visual appearance rather similar to film grain. The quasi-triangular noise, on the other hand, consists of bursts of sinusoidal oscillations of frequency in the neighbourhood of f_c. The monitor display shows that the envelope of the oscillations produces a low-frequency streaking effect, and that there is quite a good chance of vertical coincidence of bursts in two or more consecutive television lines. In Fig. 4, (a) and (b) bear a marked resemblance to the auto-correlation functions of Figs. 1 and 2. Appendix 2, which considers the noise frequencies in a narrow band, indicates how the low-frequency envelope of the noise arises.

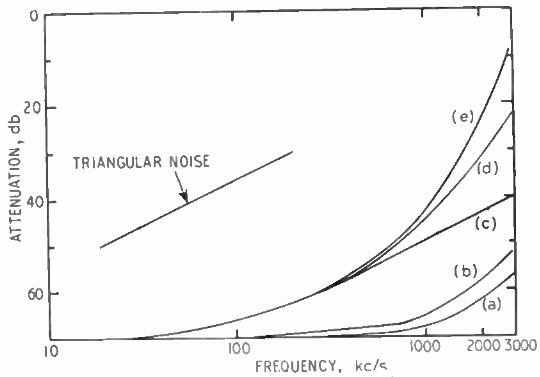
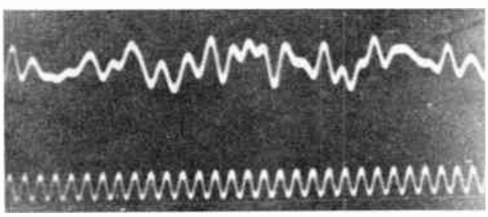


Fig. 3. Noise power spectra.

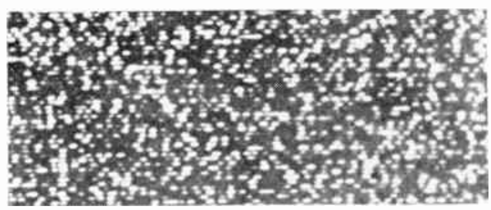
3. Effects of Non-linearity on Noise

In general, the noise fluctuations in television are assumed to be so small in comparison with the steady signal level that for noise excursions the transfer characteristic in linear, as in Fig. 5 (a). However, if this assumption is not valid, non-linearity may alter the noise properties. The effects to be considered are:—

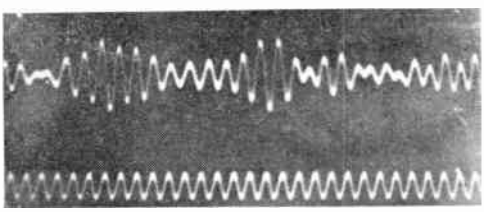
(a) Modification of noise frequency spectrum.



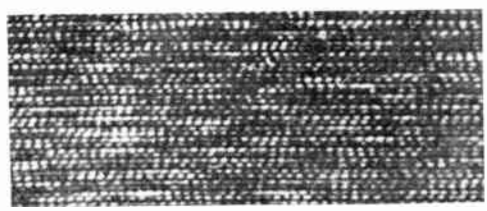
(a) white noise waveform.



(c) white noise.



(b) quasi-triangular noise waveform: timing wave 3 Mc/s.



(d) quasi-triangular noise: portion of picture monitor display.

Fig. 4. Wave-forms and picture monitor displays of noise.

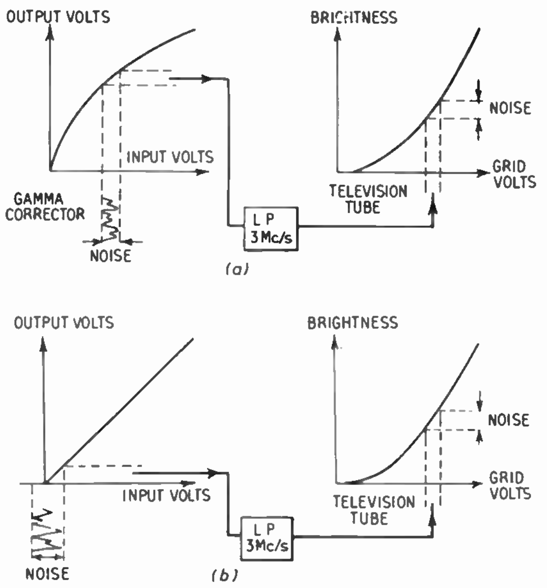


Fig. 5. Transfer characteristics: (a) typical linear transfer conditions for noise in television; (b) experimental non-linear transfer conditions for noise.

(b) Dependence of noise properties on the phase characteristic which follows the non-linearity.

As a convenient example of non-linearity, the case of a "linear" rectifier will be considered. Figure 5(b) shows the non-linearity transfer characteristic of the tests. The experimental arrangement is shown in Fig. 6. White or quasi-triangular noise, band-limited to 3 Mc/s, was applied to the input of a television picture channel. The quasi-triangular noise had the frequency spectrum of Fig. 3(e). The picture channel provided a convenient means for the insertion of non-linearity. It could be adjusted to operate on the noise either as a linear transfer device or as a half-wave linear rectifier. The output of the channel was normally passed through a phase-compensated 3 Mc/s low-pass filter LP2 which confined the noise spectrum to the usual television band, and the noise then displayed on a television monitor operated at a fixed contrast and mean brightness. The quasi peak-to-peak noise amplitude[2] of the 3 Mc/s band-limited noise at the monitor input was maintained at a fixed value irrespective of the picture channel operating condition. The quasi peak-to-peak noise is defined as that measured as follows. The oscilloscope time-base was switched off and the brightness increased. The amplitude of the resultant line was measured over the portion that appeared to be illuminated continuously. It has been shown[2] that the r.m.s. noise is between 17 and 18db lower than the peak-to-peak amplitude measured in this way. The following results should be regarded as descriptive of noise behaviour, but not as precise measurements.

3.1. *Modification of Noise Frequency Spectrum*

The phase-distortion network shown in Fig. 6 was by-passed and the picture channel followed by either of the filters LP2, LP3. The low-pass filter LP3 had a cut-off frequency of 270 kc/s, and was used to determine the low-frequency content of the noise. Figures 7(a) to (d) are oscilloscope photographs of the noise at the output of the 270 kc/s filter LP3 and exhibit the effect of changing from linear to non-linear operation of the picture channel.

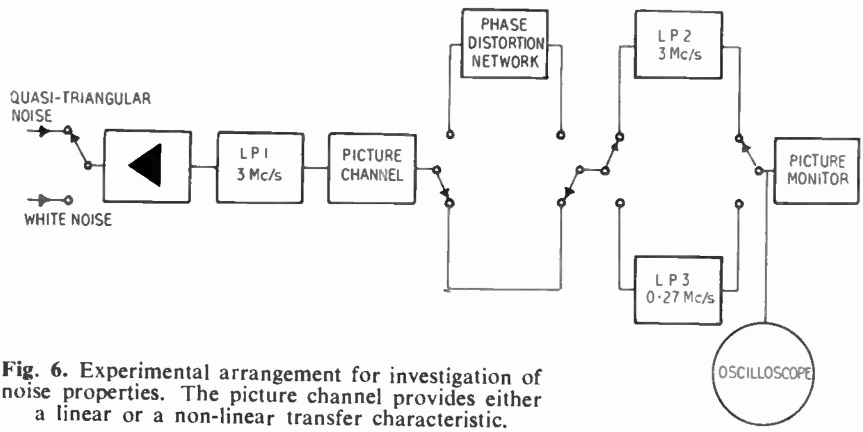
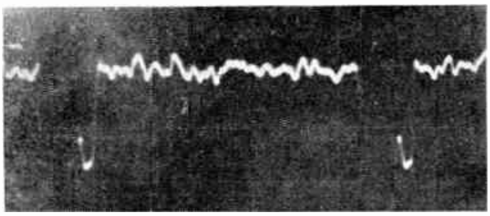
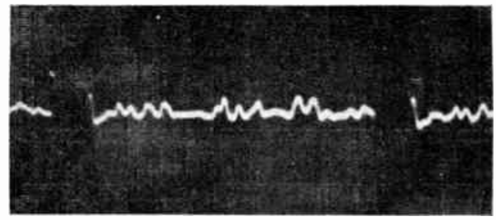


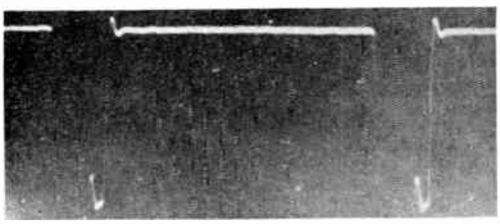
Fig. 6. Experimental arrangement for investigation of noise properties. The picture channel provides either a linear or a non-linear transfer characteristic.



(a) white noise.

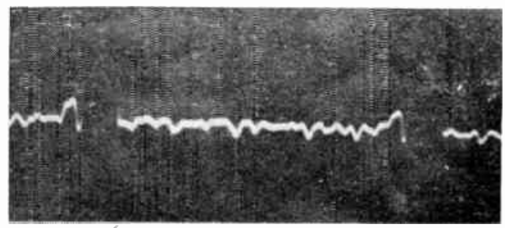


(c) white noise.



← 100 μ sec →

(b) quasi-triangular noise: picture channel as linear device.



← 100 μ sec →

(d) quasi-triangular noise: picture channel as half-wave rectifier.

Fig. 7. Noise: Effect of non-linearity on low-frequency content. Picture channel followed by 270 kc/s low-pass filter.

When the transfer characteristic was changed from linear to non-linear, the low-frequency content of white noise was not appreciably affected, but for quasi-triangular noise there was a large increase in its low-frequency content. With the 3 Mc/s LP2 substituted for LP3 the television monitor indicated that non-linearity produced little change in the visibility of the white noise, although the noise structure altered from granular to that of bright dots on a background of steady brightness. On the other hand, the effect of a non-linear transfer characteristic on quasi-triangular noise was to coarsen its appearance and greatly increase its visibility.

Appendix 3 considers the effect of a half-wave linear rectifier on noise band-limited as in the experiment above. For white noise there should be little change in the frequency spectrum. However, the non-linearity should increase the low-frequency content of quasi-triangular noise by demodulation of its fluctuating low-frequency envelope. It is known that low-frequency noise has greater visibility than equi-power high frequency noise.[2] The experimental results can therefore be reasonably explained.

It is interesting that recent work[1, 2] on noise visibility in television has produced results which may be due to similar non-linearity effects. It was found that the expected reduction in the visibility of narrow-band noise, for a fixed noise power, did not continue when the mid-band frequency was raised above 2·5 Mc/s. Instead, the noise visibility remained or even increased. As an explanation it was suggested that the non-linear characteristic of the display tube produced low-frequency intermodulation products which were much easier to see than the high-frequency noise. This is consistent with the results just discussed, and with some observed display-tube non-linear effects which will be considered in Section 4.

3.2. *Dependence of Noise Properties on Phase Characteristic*

The picture channel was followed by the phase distortion network whose characteristic is given in Fig. 8, and then the 3 Mc/s phase-

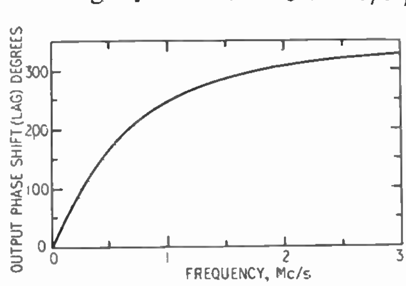
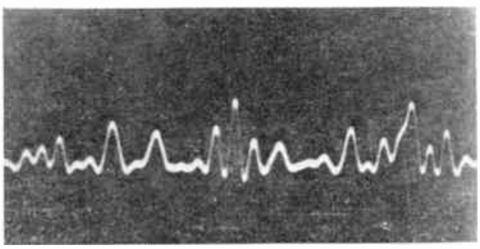


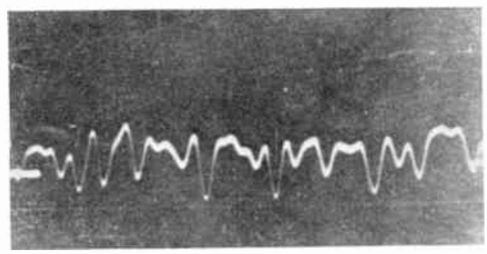
Fig. 8. Characteristic of phase distortion network.

compensated low-pass filter LP2. Insertion of the phase network did not affect the noise characteristics if the picture channel was used in the linear mode. This was to be expected since valve shot or thermal agitation noise can be represented by a large number of small amplitude sinusoidal components, all in random phase. However, when the picture channel was operated as a non-linear transfer device, the phase network had significant effect on the noise. The picture monitor display will be first considered. For white noise the bright pulses were lengthened and preceded by black-going dots. For quasi-triangular noise the streaks, comprising the noise bursts, appeared lengthened. However, there was little change in the visibility of either type of noise. The photographs (a) to (d) of Fig. 9 show the influence of phase distortion on the noise waveform and indicate that after a non-linearity. phase distortion tends to affect the individual pulses in the case of white noise, while for quasi-triangular noise the pulses are affected as groups.

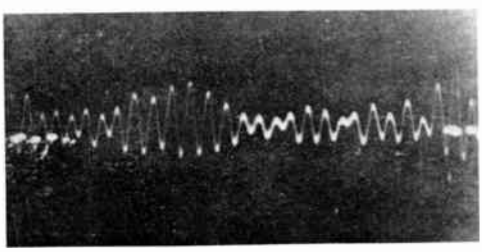
Although it is difficult to apply analytical treatment to the effect of phase distortion on white noise subject to non-linearity, consideration of a simple model is instructive. It is known[3] that white noise, confined to the frequency band 0 to f_c, has a mean frequency of $0{\cdot}58\, f_c$. That is, the noise current passes through zero $1{\cdot}16\, f_c$ times per second. A sine wave of frequency $\frac{1}{2} f_c$ will therefore be taken as a very simplified representation of white noise, where



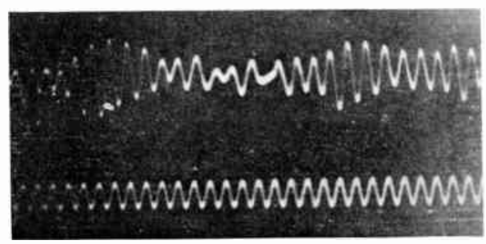
(a) white noise.



(c) white noise.



(b) quasi-triangular noise: no phase distortion.



(d) quasi-triangular noise: phase distortion.

Fig. 9. Noise: Effect of phase distortion subsequent to non-linearity. Picture channel operated as half-wave rectifier and followed by 3 Mc/s low-pass filter. The 3 Mc/s timing wave T is common to the four photographs.

$f_c = 3$ Mc/s. In Fig. 10, this time function is passed through a half-wave rectifier, then band-limited to a frequency slightly greater than f_c, and then passed through the phase characteristic of Fig. 8. It will be noticed that the effect of phase distortion is to increase the negative-going peaks. The correspondence between Fig. 10 and the photographs (a), (c) of Fig. 9 is sufficient to suggest that the simple analogy is of significance.

Since quasi-triangular noise consists essentially of a high-frequency wave with an envelope which fluctuates at a much lower frequency, it is easy to produce a plausible model. Figure 11 shows a 2·8 Mc/s sine wave. The envelope frequency is 200 kc/s, and the depth of modulation 90 per cent. This time function is passed through a half-wave rectifier, then band-limited to 3 Mc/s, and then phase distorted by the characteristic of Fig. 8. The non-linearity, followed by band-limitation, produces a low-frequency component (the modulation envelope) in addition to the original modulated wave. The subsequent distortion delays the low-frequency

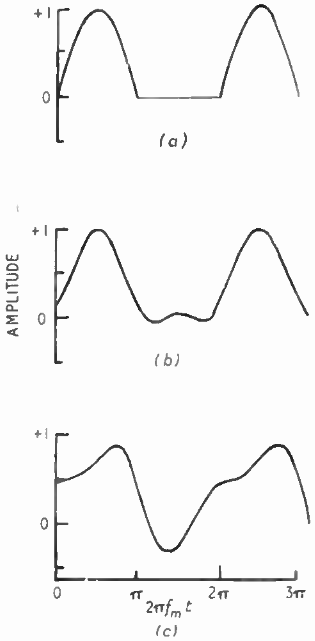


Fig. 10. White noise: Simplified illustration of effect of phase distortion after non-linearity. (a) half-wave rectification of sin $2\pi f_m t$, $f_m = 1{\cdot}5$ Mc/s. (b) as (a), but band-limitation to f_c (f_c just above $2f_m$). (c) modification of (b) by phase characteristics of Fig. 8.

component with respect to the modulated wave, which probably explains the increased duration of the noise streaks observed on a picture monitor when phase distortion is introduced following a non-linearity. The effect is shown on the noise photograph of Fig. 9(d).

4. Effects of Non-linearity on Noise Plus Signal

If noise and a steady sinusoidal signal are applied to a non-linear transfer characteristic, the resulting output contains frequencies produced by the intermodulation of noise and signal. The redistribution of noise spectrum in such cases has been treated theoretically[3], and the change in noise spectrum can be simply demonstrated by experiments such as in Section 3. For the particular case of a square-law characteristic, consider the application to the non-linearity of a sinusoidal signal of frequency f_1 and noise of small band-width Δf centred on f_1. The intermodulation products will then be the sum and difference frequencies of signal and noise and will increase in amplitude with the signal. The output of the non-linear device therefore contains, in addition to the original signal and noise, a low-frequency noise spectrum between 0 and $\Delta f/2$, and a noise band of width Δf centred on $2f_1$. Two television examples which illustrate this effect will now be considered.

When Test Card "C" is displayed on a television monitor, the noise is often most visible in the region occupied by the 3 Mc/s bars. The results of some simple experiments suggest that this is probably because of intermodulation between the 3 Mc/s signal and quasi-triangular noise, the necessary non-linearity being provided by the square-law characteristic of the monitor. A relatively noise-free video Test Card "C" signal was mixed in turn with quasi-triangular

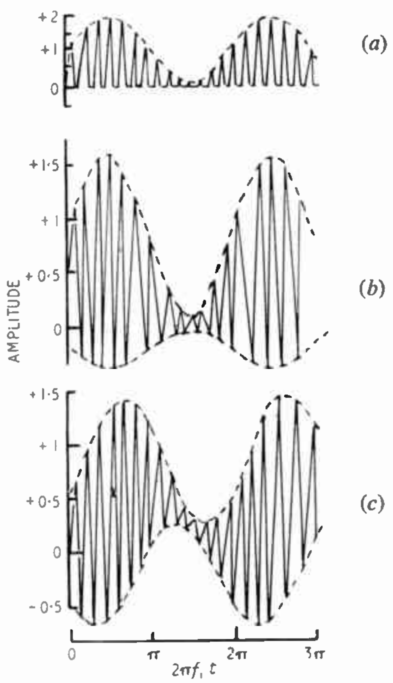


Fig. 11. Quasi-triangular noise: Simplified illustration of effect of phase distortion after non-linearity. (a) half-wave rectification of $(1+0{\cdot}9 \sin 2\pi f_1 t) \sin 2\pi f_m t$, where $f_1 = 200$ kc/s and $f_m = 2{\cdot}8$ Mc/s. (b) as (a), but band-limitation to 3 Mc/s. (c) modification of (b) by phase characteristic of Fig. 8.

(Fig. 3(e)) and white noise, band-limited to 3 Mc/s, and displayed on a television picture monitor. In the quasi-triangular case, quite coarse streaking noise could be observed in the 2·5 and 3 Mc/s bar regions. This noise remained if the 2·5 and 3 Mc/s bars were completely defocused on the picture monitor, as was to be expected if the beam current contained relatively low-frequency intermodulation products. The intermodulation effect could not be observed with white noise, presumably because the low-frequency products appeared where noise frequencies of appreciable power were already present.

It follows that if a steady sinusoidal signal and noise are applied to the input of a television monitor, the frequency and level of the signal should affect the noise visibility. The results which follow have been briefly reported elsewhere.[4] The monitor was set up so that a 0·7V d.a.p. signal produced a brightness excursion between subjective black and peak white. The r.m.s. value of the noise was obtained by subtracting 17 db from the quasi peak-to-peak level[2], and was set at 40 db down on 0·7V d.a.p. as this was considered a representative value. The noise band had a sharp upper cut at 3 Mc/s, and extended down to around 50 c/s. A sinusoidal signal was mixed in with the noise, zero level signal being 0·7V d.a.p. With white noise, the noise visibility was unaffected by the sinusoidal signal, although the signal level was increased up to 0 db, and the frequency varied between 100 kc/s and 3 Mc/s. With quasi-triangular noise the noise visibility could be drastically increased by application of the sinusoidal signal. The visibility increased with signal level, and for a fixed level a signal frequency f_s existed which produced maximum degradation. This frequency was in the 2·5 to 2·9 Mc/s region and increased as the noise spectrum was increasingly weighted towards the upper frequencies. If the signal frequency was shifted towards the maximum degradation point from either above or below, the noise appearance became progressively coarser. This presumably meant that as the signal approached the mean "carrier" frequency of the noise one of the intermodulation products tended to become the low-frequency envelope of the noise (Figs. 2 and 4(b)). The noise visibility did not vary rapidly over the grey scale. Some results are given in the Table.

Noise plus Signal

Minimum signal level for visible increase in noise db	Optimum signal frequency f_s Mc/s	Noise frequency spectrum
−22	2·55	(c) of Fig. 3
−30	2·7	(a) of Fig. 3
−32	2·7	(b) of Fig. 3
−35	2·75	(d) of Fig. 3
−40	2·9	(e) of Fig. 3

5. Conclusions

If the noise-power spectrum is known, the probable noise waveform can be deduced. The predicted characteristics are in general agreement with observation. If the noise is band-limited, and its high frequency components increasingly emphasised, the waveform becomes nearly sinusoidal with a low frequency envelope.

Some simple experiments were carried out for 3 Mc/s band-limited noise applied to a non-linearity provided by a half-wave rectifier, and followed by a 3 Mc/s low-pass filter. The measurements were performed for a fixed quasi peak-to-peak value of the noise at the filter output. For quasi-triangular, but not for white noise, the non-linearity produces a sharp increase in the low-frequency content, and in the noise visibility on a television monitor. For both types of noise the monitor display becomes dependent on the phase characteristic which follows the non-linearity. The effects can be reasonably explained, and are shown on oscilloscope photographs.

If band-limited noise and a sinusoidal signal are mixed in a non-linear device, the redistribution of the noise spectrum is affected by the signal amplitude. In this connection, some simple experiments were carried out for noise band-limited to 3 Mc/s, and using a television monitor to provide the non-linearity. A sinusoidal signal in the neighbourhood of 3 Mc/s makes quasi-triangular noise both more visible and coarser, but has no influence on white noise. The effect becomes stronger as the high-frequency components of the noise are increasingly emphasized, and is attributed to demodulation of the noise low-frequency envelope in the presence of the 3 Mc/s signal.

There are at least two cases where these noise properties may be of significance:

(a) The N.T.S.C. System of Colour Television. For U.K. Television, the chrominance information is associated with a 2·66 Mc/s sub-carrier, so that low-frequency noise is likely to be produced by any non-linearity which follows a point where this sub-carrier and quasi-triangular noise are present.

(b) A Television Recording/Rescanning chain. There are many non-linearities in tandem which may considerably increase low-frequency noise.

6. Acknowledgments

The author wishes to thank the Director of Engineering of the British Broadcasting Corporation for permission to publish this paper, and the Head of the Designs Department under whose direction the work was carried out.

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8. Appendix 1

Relation between the Correlation Function and the Amplitude Probability Distributions

Suppose the amplitude I_1 of the noise current at zero time is known. Then the value of the normalized auto-correlation function $\psi(\tau)$ fixes the probability distribution of the noise current I_2 at time τ. The r.m.s. noise current is assumed to be unity. Then, provided that the noise amplitudes have a Gaussian probability distribution, it can be shown[3] that the probability density function $p(I_1, I_2)$ is given by:

$$p(I_1, I_2) = \frac{1}{2\pi\sqrt{1-\psi^2}} \exp \frac{-(I_1^2 + I_2^2 - 2\psi I_1 I_2)}{2(1-\psi^2)} \qquad \ldots\ldots\ldots(3)$$

where $p(I_1, I_2)\, dI_1\, dI_2$ is the probability that I_1, I_2 will be simultaneously in the ranges I_1 to $I_1 + dI_1$ and I_2 to $I_2 + dI_2$. A simple transformation of eqn. (3) gives

$$p(I_1, I_2) = \left[\frac{1}{\sqrt{2\pi}} \exp - I_1^2/2\right] \times \left[\frac{1}{\sqrt{2\pi(1-\psi^2)}} . \exp - (I_2 - \psi I_1)^2/2(1-\psi^2)\right] \qquad \ldots\ldots\ldots(4)$$

and hence

$$p(I_1, I_2)\, dI_1\, dI_2 = \left[\frac{1}{\sqrt{2\pi}} \exp - I_1^2/2\, dI_1\right] \times \left[\frac{1}{\sqrt{2\pi(1-\psi^2)}} \exp - (I_2 - \psi I_1)^2/2(1-\psi^2)\, dI_2\right] \qquad \ldots\ldots\ldots(5)$$

The first bracket is the probability that I_1 is in the range I_1 to $I_1 + dI_1$. This follows from the assumption that the noise amplitudes have a Gaussian probability distribution. The second bracket then represents the conditional probability $p_{I_1}(I_2)\, dI_2$ that I_2 is in the range I_2 to $I_2 + dI_2$, given I_1. Therefore,

$$p_{I_1}(I_2)\, dI_2 = \frac{1}{\sqrt{2\pi(1-\psi^2)}} \exp - (I_2 - \psi I_1)^2/2(1-\psi^2)\, dI_2 \qquad \ldots\ldots\ldots(6)$$

Here, the r.m.s. or standard deviation σ of I_2 is $\sqrt{(1-\psi^2)}$ about its mean ψI_1. The probability P_σ that I_2 will be in the range $\psi I_1 - \sigma$ to $\psi I_1 + \sigma$

$$\text{is } P_\sigma = \int_{\psi I_1 - \sigma}^{\psi I_1 + \sigma} p_{I_1}(I_2)\, dI_2 = 0{\cdot}68 \qquad \ldots\ldots\ldots(7)$$

Thus, in 68 per cent. of the trials I_2 will be within this range. It is now easy to indicate how the $\psi(\tau)$ curve predicts the noise amplitudes at time τ subsequent to an initial value I_1 at zero time. $I_1\psi(\tau)$ is plotted, the two limit curves $I_1\psi(\tau) - \sigma$ and $I_1\psi(\tau) + \sigma$ are drawn, and at any particular value of τ the noise current I_2 has a 68 per cent. chance that it will be between these two limits.

The preceding is illustrated in Fig. 12. Here, $I_1\psi(\tau)$ was obtained for triangular noise from curve (c) of Fig. 1. For an r.m.s. noise current of unity, the initial currents I_1 were taken as 0, 0·5, 1, and 2. The subsequent current waveform depends upon whether I_1 is high or low compared to the r.m.s. value. For a high initial amplitude the subsequent noise current tends to follow the $\psi(\tau)$ curve shape, whereas if the initial noise amplitude is low the wave shape soon becomes indeterminate.

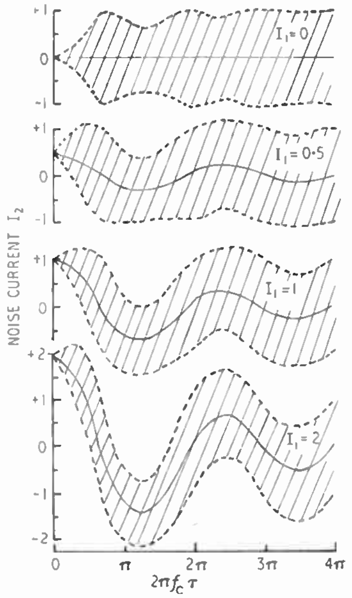

Fig. 12. Predicted variation of noise current with time, for triangular noise spectrum. Noise current has r.m.s. value of unity. I_1 is initial current at time $\tau = 0$. I_2 is current at time τ, and has probability of 0·7 that it will be between shaded limits.

9. Appendix 2
Narrow Band Noise

Let the noise cover the frequency range $f_c - \Delta f$ to f_c, and f_m be a representative frequency inside the band. Since the noise current I may be considered as the sum of a large number N of small amplitude sinusoidal components in arbitrary phase,

$$I \doteq \sum_{n=1}^{N} C_n \cos(2\pi f_n t - \emptyset_n) \quad \text{.........(8)}$$

$$= \sum_{n=1}^{N} C_n \cos\left\{(2\pi f_n t - 2\pi f_m t - \emptyset_n) + 2\pi f_m t\right\} \quad \text{.........(9)}$$

$$= I_c \cos 2\pi f_m t - I_s \sin 2\pi f_m t \quad \text{.........(10)}$$

Let $\left.\begin{array}{l} I_c = R\cos\theta \\ I_s = R\sin\theta \end{array}\right\}$ where $R = \sqrt{I_c^2 + I_s^2}$

Hence $I = R\cos(2\pi f_m t + \theta)$ ....(11)

Thus, the noise current behaves approximately as a sinusoidal current of frequency equal to f_m, with a fluctuating envelope of maximum frequency Δf.

10. Appendix 3
Noise Through a Half-wave (Linear) Rectifier

10.1. *White Noise*

It may be shown by known methods[3] that, if white noise limited to the frequency range 0 to f_c is applied to a half-wave linear rectifier, a good approximation to the output power spectrum is

$$w(f) = a[1 - 0{\cdot}12\, f/f_c], \text{ for } 0 \leqslant f \leqslant f_c \quad \text{.........(12)}$$

Thus, at f_c the value of $w(f)$ has fallen by 12 per cent. with respect to zero frequency, so that in the band 0 to f_c the frequency spectrum of the output noise is little different from that of the input.

10.2. *Quasi-triangular Noise*

A simplified analysis is possible if the noise energy is concentrated towards the upper frequencies. Consider noise of narrow band-width Δf, and f_m a frequency within the band. Appendix 2 and the oscillogram of Fig. 4(b) indicate that the noise is reasonably represented by $R \sin 2\pi f_m t$, where R is positive, and the fluctuating phase angle of eqn. (11) neglected. For convenience, the cosine representation of eqn. (11) is replaced by a sine expression. Then the output given by a half-wave linear rectifier is

$$R\sin 2\pi f_m t\left[\tfrac{1}{2} + \frac{2}{\pi}\sin 2\pi f_m t + \frac{2}{3\pi}\sin 3{\cdot}2\pi f_m t + \ldots\right] \quad \text{.........(13)}$$

If the rectifier output is passed through a low-pass filter of cut-off frequency $f_m + \Delta f$, the result is

$$\tfrac{1}{2}\left[R\sin 2\pi f_m t + 2R/\pi\right] \quad \text{.........(14)}$$

The filter output is thus the quasi-triangular noise plus a low-frequency component which corresponds to the fluctuating envelope of the original noise. A rigorous analysis leads to the same conclusions.

DEVELOPMENTS IN TELEVISION

Barclays Bank Install Closed Circuit Television

An announcement a few days ago by the Institution's bankers of their permanent closed-circuit television installation reveals an interesting application of television techniques to a particular problem of office organization. The object was to remove the ledger posting machines from valuable ground floor space to comparatively remote upstairs accommodation, while keeping the records produced by those machines immediately available to management and cashiers. The equipment now in operation at the City Office of Barclays Bank in Fenchurch Street, London, is already proving itself successful in meeting their need.

In the book-keeping department on the upper floor three Television Document Viewing Consoles are installed. Each console is, in essentials, a light-box with a lid at the top which gives access to a glass screen upon which the document to be viewed is placed. Inside the console a Marconi television camera is housed, together with its control unit; the camera is arranged to view, via a solenoid-operated rotatable mirror, one-half of the document area. Above the consoles, three 14 in. monitors are installed, each connected to one console to enable the operator to obtain a picture of maximum quality by adjustment of the simple controls.

On the ground floor, eight viewing monitors are installed in key positions, such as managerial offices. When an executive wants details of a customer's account he telephones the console operator who selects the appropriate ledger page from the files and places it in one of the three consoles. The operation of a push button presents the image of the half-page on to the executive's screen on the ground floor. Should the information required be on the lower half of the page operation of a push-button on the small remote control panel re-orientates the mirror in the Document Viewing Console upstairs and puts the second half page on the screen. Pressing another button removes the screen presentation and provides visual indication to the operator that the display is finished with.

Facsimile Transmission of a Newspaper

The Japanese newspaper "Asahi Shimbun", which has one of the largest circulation figures in the world, has recently inaugurated facsimile editions using equipment produced by the British firm of Muirhead.

Simultaneously with the normal printing of the edition in Tokyo, the complete pages are being transmitted by microwave to Sapporo in the northern island of Hokkaido 500 miles distant where the facsimile received negatives are printed down to offset plates and rolled off in the usual manner, all in the space of 75 minutes. Previously by boat and rail it took nearly two days for the Tokyo copies to reach this northern populated area of 750,000 people.

The value of the facsimile equipment supplied was over £85,000 and it had to be capable of reproducing in detail the intricate Japanese characters. By using a system of double channel transmission, the designers have arranged the actual machines to work in pairs—this achieves two major objects: Firstly, the printer receives the two complete news-sheets together which he requires for his rollers; secondly, in case of say a power breakdown in any machine, a standby can immediately take over, with hardly any disruption in production. The equipment will be in full use printing on schedule five editions daily.

Extension of Television in Northern Ireland

The Independent Television Authority has stated that in the area to be served by its new station at Black Mountain near Belfast there is a population of over one million. Practically all these people will be in the primary service area which extends southwards to the borders of Eire; Londonderry is however well outside the fringe area.

The construction of the station is proceeding rapidly. The mast is now about 200 ft. high and construction of the station building well advanced. It is hoped that it will be possible to begin full power test transmissions in August. Signals will be sent out on channel 9 (191·25 Mc/s sound, 194·75 Mc/s vision) with horizontal polarization.

APPLICANTS FOR ELECTION AND TRANSFER

As a result of its May meeting the Membership Committee recommended to the Council the following elections and transfers.

In accordance with a resolution of Council, and in the absence of any objections, the election and transfer of the candidates to the class indicated will be confirmed fourteen days after the date of circulation of this list. Any objections or communications concerning these elections should be addressed to the General Secretary for submission to the Council.

Direct Election to Associate Member

GODEL, Norman Allen, B.Sc.(Eng.). *Portsmouth.*
HORWINSKI, Elwood Robert, B.S. *Paris.*
KENWARD, Anthony John, B.Sc. *London, W.4.*
LEWISOHN, Bruce Maxwell. *Morden, Surrey.*
PANWAR, Birendar Singh, Lt. Col. *Mahendargarh, Punjab, India.*
PRICE, Robert. *Liverpool, 14.*
SHARP, Frank William. *Cheam.*

Transfer from Graduate to Associate Member

COFFEE, Ronald Alan. *London, S.W.16.*
SAVAGE, John Wilfred. *Wells, Somerset.*

Direct Election to Associate

SIDDON, Jack. *Chesterfield.*

Direct Election to Graduate

HICKEY, Major Douglas, R.A. *Gravesend.*
MOHAN, Virendra, B.Sc.(Hons.). *Enfield.*
OWEN, Arthur Neville. *Vancouver 8, Canada.*
SLINGERLAND, Gysbert. *Eindhoven, Netherlands.*
TURNER, William Albert, Lieut. B.A.(Cantab.), R.N. *Gosport, Hants.*
WILLIAMS, Jeffrey. *Manchester.*

Transfer from Associate to Graduate

KNIGHT, Ronald Newbrook. *Auckland, New Zealand.*

Transfer from Student to Graduate

BAYFIELD, Alan John. *West Drayton, Middx.*
JEYASINGH, Rajamoney Daniel. *Madras State.*
SYLVESTER, Anthony Bradbury. *Marlow, Bucks.*

STUDENTSHIP REGISTRATIONS

The following 18 Students were registered at the April meeting:—

*CHOUDHURY, Sudhir Kumar. *Gosport.*
DAWSON, John Sydney. *Wallsend.*
HANBURY, David John, B.Sc. *Greenwood, Canada.*
HOOD, Gordon S. *Edgware, Middlesex.*
INSON, Eric Gilbert, B.Sc. *Cardiff.*
LAWSON, Alan John William. *Montreal.*
PEREIRA, Xavier Alphonse. *Singapore.*
PRASHANTA, Roy, B.Sc. *Varanasi, India.*
RADHAKRISHNAN, T., M.Sc.(Eng.), B.E.(Elec.). *Madras.*
*RAJANAYAGAM, Davy Peter, B.Sc. *Somaliland Protectorate.*
REVITCH, Alexander. *Ramat-Gan, Israel.*
SARKER, Arun Kumar. *Bombay, India.*
SHARP, Robert W. D. *Cranham, Essex.*
SHARPE, Robin Neville. *Leicester.*
SUPPLE, Leslie Evan. *Manchester.*
*UPADHYAY, Sisir Kumar. *Calcutta.*
WELGEMOED, Christo Johan. *Grahamstown, South Africa.*
WONG YUI KEONG, Alan. *Singapore.*

The following 35 Students were registered at the May meeting:—

ACTON, Kenneth Eldin. *Hockley, Essex.*
ATTWELL, David Brian. *Lagos, Nigeria.*
BIRNHACK, Gideon. *London, N.W.2.*
CHAUDHRI, Bashir Uddin Ahmad. *Lahore, W. Pakistan.*
CHHABRA, Surinder Lal, Plt. Off. M.Sc., B.A., B.Sc. *India.*
DHARAM PAL, M.Sc., B.Sc. *Muzaffaznacar, U.P. India.*
HAQUE, Mohammad Sami Ul. *London, S.W.4.*
HOOTON, Arthur. *Stockton-on-Tees.*
HUNT, Gordon. *Reading.*
KAPOOR, Gopal Krishna, M.Sc., B.Sc. *Shahjahanpur, U.P. India.*
LIDDIARD, Ralph Remo. *Maraisburg, South Africa.*
LOCK, Peter John. *Stockport.*
MALL, Madan Mohan, B.Sc. *Azamgarh, India.*
MARTIN, Joseph William, B.Sc. *New Delhi, India.*
NIFLAND, Ivor Russell Julian, Plt. Off., R.A.F. *Thorpe Bay, Essex.*
PASTAKIA, Minoo Barjorji. *Bombay 14, India.*
PENNY, Raymond William, B.Sc. *Carshalton.*
PEREIRA, Jose Santana. *Entebbe, Uganda.*
PURI, Joginder Pal Singh, M.A. *London, W.11.*
ROSS, John Gary. *Lower Hutt, New Zealand.*
SADLER, Alan. *Newton-Le-Willows, Lancs.*
SAYAL, Nardev Pal. *Singapore.*
SINGH, Darshan. *Amritsar (PB), India.*
SIVAGNANAM, Kathiravelu. *Kedah, Malaya.*
SMITH, Peter John. *West Bank, Lancaster.*
STEPHEN, William John. *Surrey Hills, Victoria.*
STEWARD, Robert. *Harwich, Essex.*
*SUBBARAO, Addepalli, B.Sc. *Kurnool, S. India.*
VENKATESWARA RAO, Simhadri, B.Sc. *Avanigadda, A.P.*
VENUGOPALAN, Mangazohi, B.Sc. *Trichur, Kerala State, India.*
VOGEL, Dov. *Holon, Israel.*
VOJDANI, Mostafa. *Abadan, Iran.*
WADHAWAN, Pran Nath, B.A. *New Delhi.*
WEATHERILL, Flt. Lt. Louis, R.A.F. *Saffron Walden, Essex.*
WELLS, Alan Alfred. *Lightwater, Surrey.*

* Reinstatement.

Theory of the Mode Spectra of Cylindrical Waveguides containing Gyromagnetic Media†

by

R. A. WALDRON, B.A., ASSOCIATE MEMBER‡

Summary: A waveguide is considered of circular cross-section and radius a. The interior is divided into two regions by a concentric cylinder of radius b. Two cases are considered; in one, the region $0 < r < b$ contains a gyromagnetic medium, and the region $b < r < a$ contains a dielectric, while in the other it is the region $b < r < a$ that contains the gyromagnetic medium and the region $0 < r < b$ that contains dielectric. In each case, by putting the phase constant equal to zero in the characteristic equation a pair of equations is obtained which are called the cut-off equations; they may be related to E- and H-modes in the empty guide. The cut-off equations may be solved for any parameter if the others are all given; the solution marks a critical value of the parameter in question, such that to one side of the critical value propagation takes place while to the other it does not. In this way it is possible to separate out the normal modes and to devise a system of nomenclature. Comparisons are made between the cut-off equations and their solutions in the ferrite-centred and dielectric-centred cases in the light of the study of the ferrite-centred case previously made by the author. Numerical results are given for the dielectric-centred case for certain of the lower modes.

List of Symbols

Symbols which are used throughout the paper are defined below, with their usual meanings. Some symbols which are used only in one place are defined where they occur. As far as possible the same notation has been used as in the author's previous paper[1], but in some cases alterations have been necessary; these have been made in such a way as to minimize any likelihood of confusion.

r, θ, z	Cylindrical co-ordinate system.
μ_0, ε_0	Permeability and permittivity, respectively, of the dielectric medium, supposed isotropic, which occupies that part of the waveguide which does not contain the gyromagnetic medium.
μ, α, ε	Respectively the diagonal and off-diagonal elements of the relative permeability tensor and the relative permittivity of the gyromagnetic medium; these quantities are relative to the properties of the dielectric medium.
λ_0	Wavelength of electromagnetic waves in an infinite extent of the dielectric medium. Thus $\lambda_0 = 2\pi/\omega\sqrt{\varepsilon_0\mu_0}$.
a	Radius of the waveguide.
b	Radius of the surface, concentric with the waveguide, which is the outer or inner limit of the ferrite-filled region.

It is sometimes convenient to normalize a and b; we then write

A	$= 2\pi a/\lambda_0$; $B = 2\pi b/\lambda_0$.
β	Phase constant in the waveguide, $= 2\pi/\lambda_g$, λ_g being the wavelength in the guide.

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‡ Marconi's Wireless Telegraph Co. Ltd., Research Department, Great Baddow, Essex.

U.D.C. No. 621.372.852.22

$\bar{\beta}$ — Normalized phase constant, $= \lambda_0\beta/2\pi = \lambda_0/\lambda_g$.

E, H, D, B — Electric and magnetic fields and inductions.

∇_t — Transverse component of ∇, regarded as a formal vector.

$A_p, B_p, C_p, D_p, F_p, G_p$—arbitrary constants.

p — Integer giving the θ dependence of electromagnetic fields.

$k_0^2 = \omega^2\varepsilon_0\mu_0 - \beta^2 = \omega^2\varepsilon_0\mu_0(1-\bar{\beta}^2)$

$k_1^2 = \omega^2\varepsilon_0\mu_0\varepsilon\mu - \beta^2 = \omega^2\varepsilon_0\mu_0(\varepsilon\mu - \bar{\beta}^2)$

$f = k_1^2/\mu \quad g = \omega\varepsilon_0\beta.\alpha\varepsilon/\mu$

$c = k_1^2 - \omega^2\varepsilon_0\mu_0.\alpha^2\varepsilon/\mu = \omega^2\varepsilon_0\mu_0(\varepsilon\mu - \bar{\beta}^2 - \alpha^2\varepsilon/\mu)$

$d = \omega\mu_0\beta.\alpha/\mu$

f and d are also used as suffices to indicate the value of a field component in the gyromagnetic or dielectric medium respectively.

$$s^2 = \tfrac{1}{2}\left\{(f+c) \pm \sqrt{(f-c)^2 + 4gd}\right\}$$

s_1^2 takes the + sign, s_2^2 the − sign.

$$S = \frac{J_p'(k_0b).\sqrt{\mu_0/\varepsilon_0}}{\sqrt{1-\bar{\beta}^2}} \qquad T = \frac{\lambda_0\bar{\beta}^2 J_p(k_0b).\sqrt{\mu_0/\varepsilon_0}}{2\pi b(\varepsilon\mu - \bar{\beta}^2)(1-\bar{\beta}^2)}$$

$$L = \sqrt{\frac{\varepsilon_0/\mu_0}{1-\bar{\beta}^2}} \qquad M = \frac{\varepsilon^2\mu^2.\sqrt{\varepsilon_0/\mu_0}}{(\varepsilon\mu - \bar{\beta}^2)^2\sqrt{1-\bar{\beta}^2}}$$

$$R = \frac{\bar{\beta}}{(\varepsilon\mu - \bar{\beta}^2)\sqrt{1-\bar{\beta}^2}}$$

$$P = \sqrt{\frac{\varepsilon/\mu}{1-\alpha^2/\mu^2}} \qquad Q = \frac{\alpha/\mu^2}{(1-\alpha^2/\mu^2)}$$

$$\chi = \sqrt{\varepsilon\mu(1-\alpha^2/\mu^2)}$$

$$\Delta_1 = \begin{vmatrix} J_p(A) & Y_p(A) \\ J_p'(B) & Y_p'(B) \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} J_p(A) & Y_p(A) \\ J_p(B) & Y_p(B) \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} J_p'(A) & Y_p'(A) \\ J_p'(B) & Y_p'(B) \end{vmatrix} \qquad \Delta_4 = \begin{vmatrix} J_p'(A) & Y_p'(A) \\ J_p(B) & Y_p(B) \end{vmatrix}$$

Δ_1' and Δ_2' are obtained from Δ_1 and Δ_2 by replacing A and B by $A\chi$, $B\chi$, respectively. Δ_3' and Δ_4' are obtained from Δ_3 and Δ_4 by replacing A and B by $A\sqrt{\varepsilon}$, $B\sqrt{\varepsilon}$, respectively.

In the following, the symbol defined takes the suffix 1 or 2 according as s takes the suffix 1 or 2.

$$\tau = -jg/(s^2 - f)$$

$$\xi = \alpha\beta\varepsilon s - j\sqrt{\frac{\mu_0}{\varepsilon_0}}\left\{\mu(\varepsilon\mu - \bar{\beta}^2) - \varepsilon\alpha^2\right\}\tau s$$

$$\eta = j\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{\alpha p\bar{\beta}^2}{a}.\tau$$

$$K = \omega\varepsilon_0\varepsilon\beta/(s^2 - f)$$

$$P = \frac{\lambda_0\bar{\beta}}{2\pi b}\left\{\frac{1-\varepsilon\mu}{(1-\bar{\beta}^2)(\varepsilon\mu - \bar{\beta}^2)}\right\}$$

$$Q = \frac{2\pi\bar{\beta}.\varepsilon\mu.s}{\lambda_0(s^2-f)(\varepsilon\mu - \bar{\beta}^2)}$$

$$U = \frac{\varepsilon s}{\omega\mu_0(\varepsilon\mu - \bar{\beta}^2)}$$

$$V = \frac{\lambda_0\varepsilon\sqrt{\varepsilon_0/\mu_0}}{2\pi b(\varepsilon\mu - \bar{\beta}^2)^2(1-\bar{\beta}^2)(s^2-f)}\left\{\begin{array}{l}\omega^2\varepsilon_0\mu_0\bar{\beta}^2(1-\varepsilon\mu)(\varepsilon\mu - \bar{\beta}^2) - \\ \quad - \varepsilon\mu(1-\bar{\beta}^2)(s^2-f)\end{array}\right\}$$

$$W = \frac{\omega^2\varepsilon_0\mu_0\bar{\beta}^2\varepsilon^2\mu\lambda_0 s\sqrt{\varepsilon_0/\mu_0}}{2\pi(\varepsilon\mu - \bar{\beta}^2)^2(s^2-f)} \qquad X = \frac{\omega^2\varepsilon_0\mu_0\bar{\beta}^2\varepsilon^3\mu^2\lambda_0\sqrt{\varepsilon_0/\mu_0}}{2\pi b(\varepsilon\mu - \bar{\beta}^2)^2(1-\bar{\beta}^2)(s^2-f)}$$

1. Introduction

The first microwave Faraday rotation devices consisted of waveguides of circular cross-section containing longitudinally-polarized concentric ferrite rods. Such systems are limited in use to low powers by the difficulty of keeping the ferrite cool when working at high power. The difficulty is lessened by placing a tube of ferrite in the guide, instead of a rod; the outer radius of the tube is made equal to that of the guide, so that ferrite and guide are in contact. The waveguide, being metal and therefore a good conductor of heat, helps to keep the ferrite cool.

A study of the normal modes of the ferrite-centred case has previously been made by the author.[1] In the present paper, the modes for the dielectric-centred case will be studied, and relationships between the two cases pointed out. (The dielectric may, of course, be air or vacuum, as well as a solid material.) The method is the same as that used previously, namely to derive the cut-off equations, which can be solved for any parameter if the other parameters are given. The solution divides values of the parameter into two ranges, in one of which propagation can take place and in the other not. The parameters in question are the properties of the ferrite (ε, μ, α) and the geometry of the system (b/a, a/λ_0). It is convenient to assign values to ε, μ, α and a/λ_0, and to solve for b/a.

The permittivity and permeability of the medium in the guide other than ferrite, supposed isotropic and homogeneous, are denoted by ε_0, μ_0, respectively. μ_0 is the permeability of free space; ε_0 may, in special cases, be the permittivity of free space. The permittivity of the ferrite is then $\varepsilon\varepsilon_0$, and its permeability elements are $\mu\mu_0$ and $\alpha\mu_0$. ε is thus the ratio of the dielectric constant of the ferrite to that of the dielectric; if the latter is free space (or air) ε is the dielectric constant of the ferrite. ε_0 may be many times the permittivity of free space, so that fairly small values of ε are possible—even values less than unity may be of interest.

2. The Characteristic Equation for the Dielectric-centred Case

The wave equations for propagation in the z direction, in a ferrite polarized in the z direction, may be taken as the starting point for the present treatment. They were shown in the previous work[1] to be

$$\left.\begin{aligned}(\nabla_t^2+s_1^2)(\nabla_t^2+s_2^2)H_z &= 0\\(\nabla_t^2+s_1^2)(\nabla_t^2+s_2^2)E_z &= 0\end{aligned}\right\}\qquad\ldots\ldots\ldots(1)$$

We shall not require the solution for the ferrite-centred case; for the dielectric-centred case the solutions are

$$\left.\begin{aligned}E_{zf} &= \{A_pJ_p(rs_1)+B_pY_p(rs_1)+C_pJ_p(rs_2)+D_pY_p(rs_2)\}\\H_{zf} &= \{\tau_1[A_pJ_p(rs_1)+B_pY_p(rs_1)]+\tau_2[C_pJ_p(rs_2)+D_pY_p(rs_2)]\}\end{aligned}\right\}\qquad\ldots\ldots\ldots(2)$$

As in the previous work, it can be shown that

$$\tau_1=\frac{-jg}{s_1^2-f};\qquad \tau_2=\frac{-jg}{s_1^2-f}$$

The wave equations for the isotropic region of the waveguide are

$$\left.\begin{aligned}(\nabla_t^2+k_0^2)E_z &= 0\\(\nabla_t^2+k_0^2)H_z &= 0\end{aligned}\right\}\qquad\ldots\ldots\ldots(3)$$

and in the dielectric-centred case the solutions are

$$\left.\begin{aligned}E_{zd} &= F_pJ_p(k_0r)\\H_{zd} &= G_pJ_p(k_0r)\end{aligned}\right\}\qquad\ldots\ldots\ldots(4)$$

In equations (2) and (4), the factors $e^{-jp\theta}$, $e^{-j\beta z}$, $e^{j\omega t}$, are understood.

Expressions for the other field components in the isotropic region in terms of E_{zd} and H_{zd}, and in the ferrite region in terms of E_{zf} and H_{zf}, were given in the previous work. Applying the boundary conditions to the fields, six linear homogeneous equations in A_p, B_p, C_p, D_p, F_p, G_p are obtained. The condition for consistency is that the determinant of the coefficients shall vanish. We thus obtain for the characteristic equation

$$
\begin{vmatrix}
J_p(s_1a) & Y_p(s_1a) & J_p(s_2a) & Y_p(s_2a) & 0 & 0 \\
[\xi_1 J_p'(s_1a)+\eta_1 J_p(s_1a)] & [\xi_1 Y_p'(s_1a)+\eta_1 Y_p(s_1a)] & [\xi_2 J_p'(s_2a)+\eta_2 J_p(s_2a)] & [\xi_2 Y_p'(s_2a)+\eta_2 Y_p(s_2a)] & 0 & 0 \\
J_p(s_1b) & Y_p(s_1b) & J_p(s_2b) & Y_p(s_2b) & -J_p(k_0b) & 0 \\
\frac{\alpha K_1}{\mu p} J_p(s_1b) & \frac{\alpha K_1}{\mu p} Y_p(s_1b) & \frac{\alpha K_2}{\mu p} J_p(s_2b) & \frac{\alpha K_2}{\mu p} Y_p(s_2b) & 0 & J_p(k_0b) \\
\left\{\begin{matrix} P_1 J_p(s_1b)+ \\ +\frac{\alpha}{\mu p} Q_1 J_p'(s_1b)\end{matrix}\right\} & \left\{\begin{matrix} P_1 Y_p(s_1b)+ \\ +\frac{\alpha}{\mu p} Q_1 Y_p'(s_1b)\end{matrix}\right\} & \left\{\begin{matrix} P_2 J_p(s_2b)+ \\ +\frac{\alpha}{\mu p} Q_2 J_p'(s_2b)\end{matrix}\right\} & \left\{\begin{matrix} P_2 Y_p(s_2b)+ \\ +\frac{\alpha}{\mu p} Q_2 Y_p'(s_2b)\end{matrix}\right\} & \frac{-\alpha R}{p} J_p'(k_0b) & \left\{\begin{matrix} S+ \\ +\alpha pT\end{matrix}\right\} \\
\left\{\begin{matrix} \left[U_1-\frac{\alpha^2 W_1}{\mu^2}\right] J_p'(s_1b)+ \\ +\frac{\alpha p}{\mu}\left[V_1+\frac{\alpha^2 X_1}{\mu^2}\right] J_p(s_1b)\end{matrix}\right\} & \left\{\begin{matrix} \left[U_1-\frac{\alpha^2 W_1}{\mu^2}\right] Y_p'(s_1b)+ \\ +\frac{\alpha p}{\mu}\left[V_1+\frac{\alpha^2 X_1}{\mu^2}\right] Y_p(s_1b)\end{matrix}\right\} & \left\{\begin{matrix} \left[U_2-\frac{\alpha^2 W_2}{\mu^2}\right] J_p'(s_2b)+ \\ +\frac{\alpha p}{\mu}\left[V_2+\frac{\alpha^2 X_2}{\mu^2}\right] J_p(s_2b)\end{matrix}\right\} & \left\{\begin{matrix} \left[U_2-\frac{\alpha^2 W_2}{\mu^2}\right] Y_p'(s_2b)+ \\ +\frac{\alpha p}{\mu}\left[V_2+\frac{\alpha^2 X_2}{\mu^2}\right] Y_p(s_2b)\end{matrix}\right\} & \left\{\begin{matrix} -L+ \\ +\frac{\alpha^2 M}{\mu^2}\end{matrix}\right\} & 0
\end{vmatrix}
= 0 \qquad \text{.........(5)}
$$

An equivalent equation to this has recently been published by Rizzi.[2]

3. The Sign Convention

The so-called normal modes of a cylindrical waveguide are actually degenerate mode-pairs. Each mode of a pair is helically polarized, i.e. a point in the wave-front moves along a helical path. A helically-polarized wave is characterized by a factor $e^{-jp\theta}$ or $e^{jp\theta}$ according as the helical path is right-handed or left-handed; thus eqns. (2) and (4) refer to right- or left-handed helical waves according as p is positive or negative respectively. Normally, the two helically-polarized waves exist together with equal amplitudes, and the factors $e^{-jp\theta}$, $e^{jp\theta}$, combine together to give $\cos p\theta$ or $\sin p\theta$. When the guide contains ferrite, however, the phase constants of the helical components are different, and they must be treated separately. This may be done by distinguishing between positive and negative values of p, but it is preferable instead to adopt the following sign convention.

Examination of eqn. (5) shows that α only occurs to an odd power if multiplied or divided by an odd power of p, and p only occurs to an odd power if multiplied or divided by an odd power of α. Thus a change in sign of p is equivalent to a change in sign of α. If the polarizing field is defined as positive when it is in the positive z direction, and if it is smaller than the value required to give ferromagnetic resonance, α is negative for a positive polarizing field and positive for a negative polarizing field. It is convenient to think of p as always positive (or zero), and to take α to be either positive or negative although the direction of the polarizing field does not change. The polarizing field can then be taken as defining the positive z direction.

Right- and left-handed helically polarized waves are now defined with respect to the polarizing field; thus negative α indicates a right-handed wave and positive α a left-handed wave, irrespective of the direction of travel, which only affects the sign of β. Examination of eqn. (5) shows that the value of the determinant, and hence its zeros, are independent of the sign of β, so that a backward wave has the same wavelength as a forward wave for the same values of parameters. It will be sufficient in the present paper to consider β to be always positive.

4. The Cut-off Equations

Equation (5) may be solved for the normalized phase constant, $\bar{\beta}$, for given values of the parameters a/λ_0, b/a, ε, μ, and α. A value must be assigned to p, and there may be more than one value of $\bar{\beta}$ that satisfies equation (5). Let us consider a single mode, and suppose that with a/λ_0, b/a, ε, μ, and α fixed the frequency $\omega/2\pi$ is increased steadily from zero. As ω increases from zero, moving out along the real frequency axis, $\bar{\beta}$ moves inwards from infinity along the imaginary $\bar{\beta}$ axis; at some point $\omega=\omega_0$ on the frequency axis, $\bar{\beta}$ reaches the origin of the $\bar{\beta}$ plane, and as ω increases further $\bar{\beta}$ moves out from zero along the real axis. When $\bar{\beta}$ is imaginary, the guide is said to be cut off; when $\bar{\beta}$ is real, waves can propagate freely in the guide in the mode in question. The critical angular frequency ω_0 thus divides the frequency axis into two regions; for $\omega < \omega_0$, propagation does not take place, while for $\omega > \omega_0$ it does. Similarly, for any of the other parameters, if we know its value when $\bar{\beta}=0$, we can say that to one side of this value propagation takes place, while to the other it does not. (For the ferrite-centred case this was found to be not quite true for H modes, and we may expect this to apply also in the dielectric-centred case. However, a study of the condition $\bar{\beta}=0$ is still a good basis for understanding the mode spectrum.)

In the five-dimensional space with dimensions a/λ_0, b/a, ε, μ, and α, if a given set of values of these parameters satisfies the characteristic equation when $\bar{\beta}=0$ we shall refer to the set as a cut-off point. It is not necessary to take the frequency as a sixth dimension; this is taken care of by expressing the guide radius in terms of the wavelength λ_0. Let us call this five-dimensional space β-space. In β-space, a/λ_0 and ε are limited to positive values, and b/a to the range from 0 to 1. μ and α may take any values, but in this paper we shall only consider values of μ of the order of unity and values of α such that $|\alpha| < |\mu|$. These values are typical of a ferrite in a polarizing field much smaller than that required to give ferromagnetic resonance, which is the normal working condition. It is assumed, of course, that the polarizing field is sufficiently great to saturate the ferrite. It is fortunate, also, that these values of μ and α are the ones most amenable to theoretical treatment.

Putting $\beta=0$ in eqn. (5), the left-hand side reduces to a product of two factors, either of which may be equated to zero. Thus we obtain the two cut-off equations for the dielectric-centred case:

$$J_p(B)\left\{P\Delta_1' - \frac{pQ}{B}\Delta_2'\right\} - J_p'(B)\Delta_2' = 0 \quad \ldots\ldots\ldots(6)$$

$$J_p(B)\Delta_3' - \sqrt{\varepsilon}.J_p'(B)\Delta_4' = 0 \quad \ldots\ldots\ldots(7)$$

The cut-off equations for the ferrite-centred case were given in the earlier work. They are

$$J_p(B\chi)\Delta_1 - \left\{PJ_p'(B\chi) - \frac{pQ}{B}J_p(B\chi)\right\}\Delta_2 = 0 \quad \ldots\ldots\ldots(8)$$

and

$$J_p(B)\sqrt{\varepsilon})\Delta_3 - \frac{1}{\sqrt{\varepsilon}}J_p'(B\sqrt{\varepsilon})\Delta_4 = 0 \quad \ldots\ldots\ldots(9)$$

The solutions of eqns. (6) and (7) can be made to depend on those of eqns. (8) and (9) by means of the transformation we shall now give. Quantities appropriate to eqns. (6) and (7) will be indicated by dashes; thus ε', α', etc., are the values of these quantities, for the dielectric-centred case, for which ε, α, etc., are the appropriate values in the ferrite-centred case. The transformation of eqn. (7) into eqn. (9) is independent of μ and α, and we have merely.

$$\varepsilon' = 1/\varepsilon; \quad a'/\lambda_0 = a\sqrt{\varepsilon}/\lambda_0 \quad \ldots\ldots\ldots(10)$$

For the transformation of eqn. (6) into eqn. (8), we have

$$\left.\begin{aligned} &\varepsilon' = 1/\varepsilon; \qquad a'/\lambda_0 = a\chi/\lambda_0; \qquad a/\lambda_0 = a'\chi'/\lambda_0 \\ &\mu' = \frac{\mu(1-\alpha^2/\mu^2)}{\{\mu^2(1-\alpha^2/\mu^2)^2-\alpha^2/\mu^2\}}; \quad \mu = \frac{\mu'(1-\alpha'^2/\mu'^2)}{\{\mu'^2(1-\alpha'^2/\mu'^2)^2-\alpha'^2/\mu'^2\}} \\ &\alpha' = \frac{-\alpha/\mu}{\{\mu^2(1-\alpha^2/\mu^2)^2-\alpha^2/\mu^2\}}; \quad \alpha = \frac{-\alpha'/\mu'}{\{\mu'^2(1-\alpha'^2/\mu'^2)^2-\alpha'^2/\mu'^2\}} \end{aligned}\right\} \ldots\ldots\ldots(11)$$

These transformations are not of much use for deriving solutions of eqns. (6) and (7) from those of eqns. (8) and (9), because the solutions of most interest for both the ferrite-centred and dielectric-centred cases are those for $\varepsilon > 1$, whereas the transformations relate values of ε in the one case to the reciprocal values in the other case. However, if computations are to be made on an electronic machine, only one pair of equations need be programmed, the other pair being solved by using the same programme and transforming the parameters. It would be a valuable help if solutions of eqns. (6) and (7), for a given value of ε, could be related simply to solutions of eqns. (8) and (9) for the same value of ε, but this does not seem to be possible. Such a relationship would imply that solutions of eqns. (8) and (9), for a given value of ε, are simply related to their solutions when ε has the reciprocal value, but numerical checks of the most obvious guesses fail, and, as we shall see in Section 6, it is extremely unlikely that any simple relationship exists. Of course, there must be *some* relation, but if it is not a simple one it is of no help in solving the equations—they might just as well be solved directly.

It has already been shown[1] that, on putting $\mu = 1$, $\alpha = 0$, and $b/a = 1$, eqns. (8) and (9) reduce to the cut-off equations for the dielectric-filled guide for E and H modes respectively, and that there is a 1 – 1 correspondence between the modes in the dielectric-filled guide and the guide partly filled with ferrite. Equations (6) and (7) reduce similarly, when $b/a = 0$, which again corresponds to the filled guide, and the same 1 – 1 correspondence can be demonstrated. The modes for the dielectric-centred case can thus be designated H_{pq}, E_{pq}, according to the mode which they reduce to in the case of a homogeneously isotropically filled guide, as was done before for the ferrite-centred case. True E and H modes do not, of course, exist in an homogeneously-filled guide, as was shown by Kales[3]; however, the nomenclature is convenient as long as this point is borne in mind.

5. Special Cases of the Cut-off Equations

For special values of the parameters, the cut-off equations take simpler forms. Examination of these forms sheds light on the general behaviour of the cut-off curves; this will become apparent in Section 6, where these curves are discussed.

5.1. *Special Forms of Equation* (7)

(a) When $b = a$, the guide is empty of ferrite and we have

$$J_p'(A) = 0 \qquad \text{.........(12)}$$

(b) When $b = 0$, the guide is filled with ferrite, and we have

$$J_p'(A\sqrt{\varepsilon}) = 0 \qquad \text{.........(13)}$$

(c) When $\varepsilon = 0$, eqn. (7) reduces to $\Delta_3' = 0$, which is satisfied only by

$$b = a \qquad \text{.........(14)}$$

(d) When $\varepsilon = \infty$, eqn. (7) reduces to $J_p'(B) = 0$, which is satisfied only by

$$b = a \qquad \text{.........(15)}$$

5.2. *Special Forms of Equation* (9)

(a) When $b = 0$, the guide is empty of ferrite and we have

$$J_p'(A) = 0 \qquad \text{.........(16)}$$

(b) When $b = a$, the guide is filled with ferrite and we have

$$J_p'(A\sqrt{\varepsilon}) = 0 \qquad \text{.........(17)}$$

(c) When $\varepsilon = 0$, eqn. (9) reduces to

$$\Delta_4 = 0 \qquad \text{.........(18)}$$

which is satisfied by a value of b/a less than 1 provided that A is greater than the value which satisfies equation (16) for the mode in question.

(d) When $\varepsilon = \infty$, eqn. (9) reduces to $\Delta_3 = 0$, which, for values of A less than that which satisfies eqn. (16) for the mode in question, is satisfied only by

$$b = a \qquad \text{.........(19)}$$

Equations (16), (17), and (19) are the obvious counterparts of eqns. (12), (13), and (15), but eqn. (18) is not the counterpart of eqn. (14). This is demonstrated by the fact that eqn. (18) has a non-trivial solution, illustrated in Fig. 2(a) for the H_{11} mode.

5.3. *Special Forms of Equation* (6)

(a) When $p = 1$, $\mu = 1$, $\alpha = 0$, equation (6) becomes identical with eqn. (7) for $p = 0$.

(b) When $\alpha^2 = \mu^2$, eqn. (6) reduces to $b/a = 1$.

(c) When $b = a$, the guide is empty of ferrite and we have

$$J_p(A) = 0 \qquad \text{.........(20)}$$

(d) When $b = 0$, the guide is filled with ferrite and we have

$$J_p(A\chi) = 0 \qquad \text{.........(21)}$$

(e) When $\varepsilon = 0$, equation (6) reduces to $\Delta_2' = 0$, which requires

$$b = a \qquad \text{.........(22)}$$

(f) When $\varepsilon = \infty$, equation (6) reduces to $J_p(B) = 0$, which requires

$$b = a \qquad \text{.........(23)}$$

5.4. *Special Forms of Equation* (8)

(a) When $p = 1$, $\mu = 1$, $\alpha = 0$, equation (8) becomes identical with equation (9) for $p = 0$.

(b) When $\alpha^2 = \mu^2$, equation (8) reduces to $b/a = 1$.

(c) When $b = 0$, the guide is empty of ferrite and we have

$$J_p(A) = 0 \qquad \text{.........(24)}$$

(d) When $b = a$, the guide is filled with ferrite and we have

$$J_p(A\chi) = 0 \qquad \text{.........(25)}$$

(e) When $\varepsilon = 0$, equation (8) reduces to

$$\Delta_1 + \frac{pQ}{B}\Delta_2 = 0 \qquad \text{.........(26)}$$

which has a non-trivial solution.

(f) When $\varepsilon = \infty$, equation (8) reduces to $\Delta_2 = 0$, which requires

$$b = a \qquad \text{.........(27)}$$

Equations (24), (25), and (27) are the obvious counterparts of equations (20), (21), and (23), respectively. It is also apparent that there is an analogy between the special forms of equations (6) and (8) on the one hand and the special forms of equations (7) and (9) on the other; in particular, it may be noted that equation (26), analogously to equation (18), has a non-trivial solution.

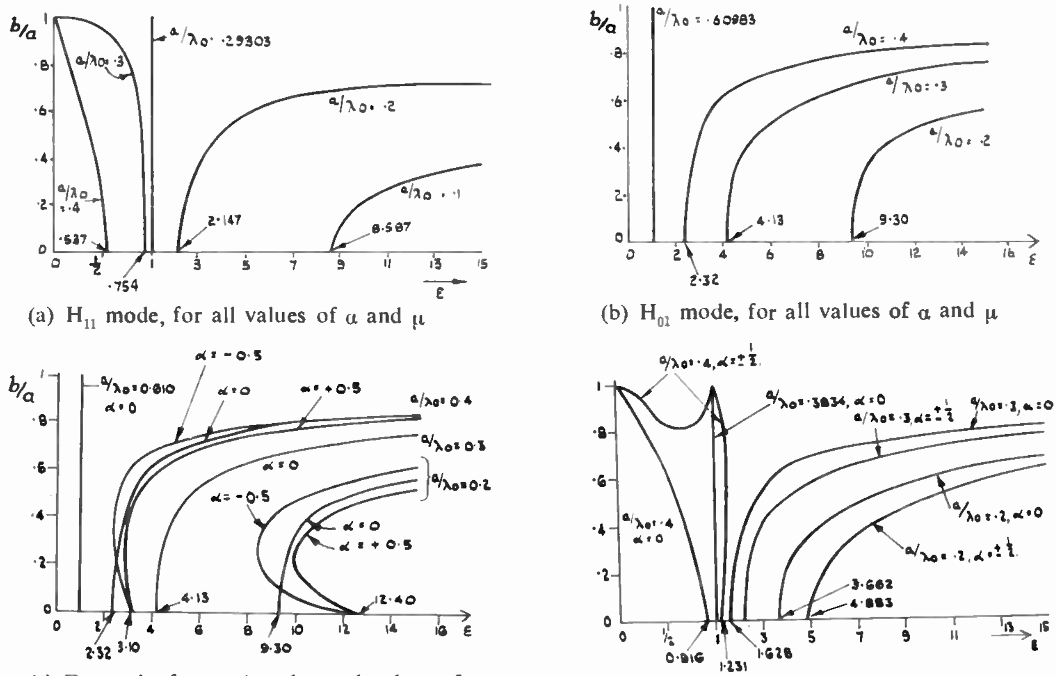
6. Solutions of the Cut-off Equations

It is convenient to assign values to a/λ_0, μ, and α, for a given mode, and to solve the cut-off equations for b/a, regarded as a function of ε.

Some results are given in Figs. 1(a), (b), (c), and (d) of the present paper for the dielectric-centred case. In Fig. 1, the region in which propagation takes place is that bounded by the cut-off curve and that part of the ε axis to the right of its intersection with the cut-off curve. We shall not discuss these curves in detail, because their behaviour may be readily deduced from that of the solutions for the ferrite-centred case, which were discussed in the earlier work. Thus the nomenclature is obtained by reducing α to zero, μ to 1, b/a to 1, and comparing the limiting equations so obtained (eqns. (13) and (21) of the present paper) with those for a guide filled homogeneously with dielectric. As in the ferrite-centred case, the E modes for the dielectric-centred case are of two kinds, which we call primary or secondary according as $|\mu|$ is greater or less than $|\alpha|$, respectively. The secondary E modes are difficult to deal with, and probably of little practical interest; we shall only consider primary E modes further in this paper. Again, E_{p0} modes, for $p \neq 0$, occur trivially; we shall not discuss them here.

The mode which is of greatest interest is the H_{11}, at least over the ranges of values of parameters that we have studied, which are the values normally used in practice. The H_{11} mode is not necessarily the lowest—its cut-off curves cross those of the E_{01} mode, so that sometimes one mode is the lowest, sometimes the other. However, cylindrical waveguides containing gyromagnetic media are mainly of interest for their property of Faraday rotation, and the rotation observed for a given mode is not affected by the presence of a mode for which the field patterns have circular symmetry, i.e. the E_{01} and H_{11} modes. Thus as long as some energy travels in the H_{11} mode, it does not matter if the E_{01} mode is also excited. The modes to be rejected are those, other than the H_{11} mode, for which $p \neq 0$; the next one of these after the H_{11} mode is the E_{11} mode. Rejection of the E_{11} mode will also involve rejection of the H_{01} mode in many cases, by virtue of the relation (a) of Sections 5.3 and 5.4.

For the ferrite-centred case, it was found that the H_{11} mode and E_{11} mode cut-off curves, in the



(a) H_{11} mode, for all values of α and μ

(b) H_{01} mode, for all values of α and μ

(c) E_{11} mode, for $\mu=1$ and stated values of α

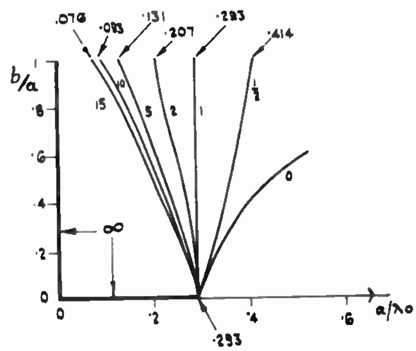
(d) E_{01} mode, for $\mu=1$ and stated values of α

Fig. 1. Cut-off curves in the b/a, ε plane in the dielectric-centred case.

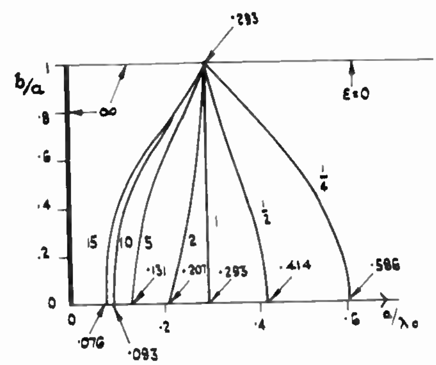
b/a, ε plane, cross for sufficiently low values of (a/λ_0) and for sufficiently high values of ε (See Figs. 6, 7, 8, of reference 1). No actual crossing is exhibited in the curves that have been plotted for the dielectric-centred case, but examination of Figs. 1(a) and 1(c) of this paper indicates that if the curves for $a/\lambda_0 = 0{\cdot}2$ are extended to slightly higher values of ε crossing will occur. For values of ε above the cross-over value, the E_{11} mode is the lowest (not counting E_{0q} and H_{0q} modes), and its study is therefore not without practical interest. For a range of values of ε on either side of the cross-over value, the values of b/a at cut-off for the E_{11} and H_{11} modes are fairly near, so that it is likely that both modes will be excited in devices made to work under these conditions. This will make for difficulty in obtaining a required performance, so that such conditions are probably best avoided. This means that the values of a/λ_0 and ε for a device must be chosen together; the choice of one governs the choice of the other.

It is instructive to plot cut-off curves in the b/a, a/λ_0 plane, when the relationships between the ferrite-centred and dielectric-centred cases become evident. This has been done in Figs. 2(a) and 2(b) for the H_{11} mode; similar curves may be drawn for any H mode, and analogous curves for the E modes. We shall now discuss the H_{11} mode cut-off curves in the b/a, a/λ_0 plane; a similar discussion will apply to the other H modes.

When the guide is empty of ferrite, b/a is 1 for the dielectric-centred case and 0 for the ferrite-centred case. The value of a/λ_0 for cut-off is then given by eqns. (12) and (16); the same value apples, of course, in both cases. When the guide is filled with ferrite, the cut-off value of a/λ_0 is given by eqns. (13) and (17); for the same value of ε, this value of a/λ_0 is again the same in both cases. It was seen in Section 4 that the cut-off equations for the two cases are related by simple transformations; these enable the cut-off curves in the b/a, a/λ_0 plane for one case to be obtained from those for the other case. In Fig. 2(a), consider any one of the curves for, say, $\varepsilon = \rho$. From this, we obtain the curve in Fig. 2(b) for $\varepsilon = 1/\rho$ by taking values of a/λ_0 for given b/a, equal to $\sqrt{\rho}$ times the value of a/λ_0 in Fig. 2(a). For example, in Fig. 2(a) a curve is given for $\varepsilon = 2$. When $b/a = 1$, the value of a/λ_0 on this curve is $0{\cdot}2073$. The corresponding curve in Fig. 2(b) is that for $\varepsilon = 1/2$, and when $b/a = 1$ the value of a/λ_0 on this curve is $0{\cdot}29303$. The ratio of the values of a/λ_0 is $0{\cdot}29303/0{\cdot}2073 = \sqrt{2}$. For $b/a = 0$, the ratio of the values of a/λ_0 is $0{\cdot}4144/0{\cdot}29303 = \sqrt{2}$. For intermediate values of b/a, the same ratio of $\sqrt{2}:1$ applies.



(a) Ferrite centred case.



(b) Dielectric-centred case.

Fig. 2. Cut-off curves in the b/a, a/λ_0 plane for the H_{11} mode, for all values of μ and α. The values of ε are given for the individual curves

In the limiting case of $\varepsilon = 0$ in Fig. 2(a), the corresponding curve in Fig. 2(b) is that for $\varepsilon = \infty$. The values of a/λ_0 in Fig. 2(a), for all values of b/a except 1, are finite, and in Fig. 2(b) the corresponding values of a/λ_0 should be $\sqrt{0}$ times the values in Fig. 2(a), i.e. should be zero. This is seen to be the case, conforming with eqn. (15). For $\varepsilon = \infty$ in Fig. 2(a), a/λ_0 is zero for all values of b/a except 1, and the corresponding values of a/λ_0 in Fig. 2(b) should be $\sqrt{\infty}$ times as great. This is indeterminate; however, the loci shown in Fig. 2(b) as the solution of eqn. (14) are in accordance with our expectation.

The fact that the limiting values of ε (0 and ∞) give values that do not correspond, due to the anomalous form of eqn. (18), suggests that a search for a simple relationship between a curve in Fig. 2(a) and the curve in Fig. 2(b) for the same value of ε is likely to be fruitless.

Some unexpected and rather peculiar behaviour is found for the E_{01} mode when a/λ_0 is sufficiently large for propagation to take place in this mode in the guide with no ferrite, and when α is sufficiently large for the cut-off value of ε in the ferrite-filled guide to be greater than unity. In the ferrite-centred case, the cut-off curve is similar in form to that for $\alpha = 0$, and passes smoothly through some value of b/a at $\varepsilon = 1$. In the dielectric-centred case, however, only a trivial solution for b/a exists when $\varepsilon = 1$, giving curves of the form shown in Fig. 1(d), for $a/\lambda_0 = 0{\cdot}4$ and $\alpha = 1/2$. For such curves, there are two propagation regions, one above the scoop-shaped curve for $\varepsilon < 1$, the other to the right of the quasi-vertical line for $\varepsilon > 1$. The different kinds of behaviour of the dielectric-centred and ferrite-centred systems at $\varepsilon = 1$ are probably not unrelated to the behaviour at $\varepsilon = 0$, when the cut-off curve for the dielectric-centred system goes to $b/a = 1$, while that for the ferrite-centred system goes to a value of b/a between 0 and 1.

7. Acknowledgment

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The Application of Magnetic Resonance to Solid State Electronics†

by

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In the Chair: Dr. T. B. Tomlinson, Associate Member.

Summary: The paper first describes the basic principles and techniques of magnetic resonance experiments, and then outlines four recent applications which are of particular interest in electronics. The concepts involved in these new solid-state electronic devices are introduced by outlining the basic theory of magnetic resonance phenomena. The actual applications are then summarized under the separate section headings of "Hyperfine Structure and the Analysis of Impurities;" "Cyclotron Resonance of Current Carriers;" "Maser Developments from Paramagnetic Resonance;" and "Mavar Developments from Ferromagnetic Resonance."

1. Introduction

Magnetic resonance is a branch of research that has only come into existence since the end of the last war, and it is one of the classic examples of how advances in applied research have sometimes opened up an entirely new field of fundamental investigation. The techniques involved in present-day high-sensitivity and high-resolution magnetic resonance experiments would not have been possible without the vast amount of wartime research that produced all the microwave generators and components that are now available. Since the trend in research is normally in the other direction, with applied research taking over where fundamental studies cease, it has been very interesting to see this striking example of a return of the results of applied research to fundamental work.

During the last year, or so, however, the wheel has turned full circle, since the latest results of magnetic resonance are now finding entirely new and far-reaching practical applications. Thus the experiments on the resonance of various paramagnetic salts, undertaken initially as a pure study in magnetism, are now being used as a basis for the development of solid-state "masers" for amplification with very low noise figures in the microwave region. It is the aim of this present paper to summarize these new developments in which the results of microwave physics are being rapidly applied to produce new solid-state electronic devices.

It may be noted that experiments in magnetic resonance can be divided into two broad groups: (1) those in nuclear magnetic resonance where radio-frequency fields cause nuclear spins to change their orientation, and (2) those in electron resonance where radio-frequency or microwave fields cause electron spins to change their orientation. So far only the latter have found any real practical application in new electronic devices and hence this paper will be confined to considerations of electron resonance, although the general principles also apply to the nuclear case. The basic principles and experimental techniques of electron resonance will be outlined first and then the various applications summarized in detail. These can be conveniently grouped under the four distinct headings given below, which also follow the chronological order of their development.

(1) Impurity analysis in semiconductors.
(2) The study of current carriers by cyclotron resonance.
(3) Maser design from paramagnetic resonance.§
(4) Mavar design from ferromagnetic resonance.§

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§ "Maser" = Microwave Amplification by Stimulated Emission of Radiation (see Section 6); "Mavar" = Microwave Amplification by Variation of Reactance (see Section 7).

2. Basic Principles of Electron Resonance

Electron resonance can only be observed in material possessing unpaired electrons. The normal binding between atoms in the solid state is effected by pairing electrons, either in the outer shells of ionic solids or in the electron-pair bonds of covalent compounds. As a result of this the great majority of compounds are diamagnetic and possess no unpaired electrons. There are however certain types of material for which this is not true and they may be grouped as (a) salts of the transition metal atoms where the unpaired electrons are located in inner electron shells and produce the phenomena of para- and ferro-magnetism; (b) metals and semi-conductors, in which a certain proportion of the conduction electrons remain unpaired; (c) impurity atoms in semi-conductors where the abnormal binding produces an odd number of resultant electrons and (d) molecular compounds, known as "free radicals," in which the unpaired electrons are spread over the whole molecule instead of being closely associated with any one atom. It will be noticed that all of these groups contain compounds of practical importance and the application of electron resonance is therefore more widespread than might at first have been thought.

If attention is first confined to the case where the atom or molecule only contains one unpaired electron, the electron resonance condition can be represented as in Fig. 1. In the absence of an external magnetic field all the electrons will have the same energy, whatever the orientation of their spins and associated magnetic moments. If an external magnetic field is applied across the specimen, however, the electron spins and magnetic moments will align themselves so that they are either parallel or anti-parallel to the direction of the field (no other orientation being permissible by quantum conditions). Those aligned anti-parallel to the field will have more energy than those aligned parallel, as can be seen by analogy with the unstable and stable positions of a simple bar magnet in a magnetic field. The electrons are therefore split into two energy groups, as shown in the figure, and the difference between these is given by $g\beta H$, where β is the Bohr magneton, and g is a factor which effectively measures the amount of orbital and spin momentum possessed by the unpaired electron, and has a value of about 2·0 for a free electron possessing no orbital motion.

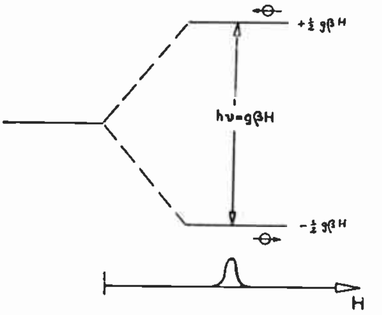


Fig. 1. Basic condition for electron resonance. For a free electron $\nu = 2{\cdot}8 \times 10^6 . H$ c/s.

The essential feature of an electron resonance experiment is to separate the electrons into two such groups by the application of an external magnetic field, and then place the sample in a field of electromagnetic radiation of frequency, ν, such that

$$h\nu = g\beta H \qquad \text{.........(1)}$$

The incident quanta are then of just the right energy to excite the electrons from the lower to the upper energy level, reversing their spins and moments in the process. Substitution of numerical values into this equation gives a resonance condition of

$$\nu = 1{\cdot}4.10^6 . g.H \text{ c/s} \qquad \text{.........(2)}$$

and it can be seen that if the applied field is 10 gauss the resonance frequency will be about 30 Mc/s, whereas if the applied field is 10,000 gauss the resonance field will be about 30,000 Mc/s.

There is no reason, in principle, why electron resonance should not be performed at any frequency, provided that the corresponding magnetic field is applied, but there is one important practical reason why most measurements are made at as high a magnetic field strength and frequency as possible. This concerns the intensity of the observed resonance signal and hence the sensitivity of the apparatus. In the same way that the electromagnetic power is absorbed in raising electrons from the lower to the upper level, so it is emitted when electrons fall from the upper to the lower level. The coefficients of absorption and stimulated emission are equal and the only reason why any net absorption signal is obtained is because there are more electrons in the lower than in the upper level. This difference is determined by the Boltzmann statistics, and the ratio of n_1 (the number in the upper level) to n_2 (that in the lower level) is

$$\frac{n_1}{n_2} = \exp\left(-\frac{h\nu}{kT}\right) \qquad \text{.........(3)}$$

It will be seen from this that the larger the value of the resonance frequency, ν, and the lower the temperature, T, the larger will be the differences in population, and hence the larger the intensity of the resonance signals. It is for this reason that most experiments in electron resonance are performed at microwave frequencies and often at low temperatures.

There is another important concept that follows from a consideration of the simple energy level diagram of Fig. 1. This concerns the "relaxation" of the spins back to their normal population distribution. If the processes of absorption and stimulated emission were the only means whereby the electron spins could exchange energy, the net absorption would rapidly cease since the electromagnetic radiation would very quickly equalize the numbers in the two energy levels. This would only occur in a completely isolated spin system, however, and in practice, the spins are also coupled to the thermal vibrations of the whole solid and can lose extra energy by sharing with these. This type of interaction is termed "spin-lattice" interaction and it has the effect of trying to restore the normal population distribution given by the Boltzmann statistics for the ambient temperature. If this spin-lattice interaction is strong the populations of the levels will be given by eqn. (3), whatever the power level of the incident electromagnetic radiation. If the spin-lattice interaction is weak, however, the difference between n_1 and n_2 will be steadily decreased, as the incident power level is raised, and there will be a reduction in the intensity of absorption, as compared with the first case. This phenomenon is known as "saturation" of the energy levels by the microwave power, and will be seen to play a very important role in the action of maser amplifiers and oscillators. It may be noted that interaction with the lattice always becomes weaker as the temperature is decreased, and "saturation effects" will therefore be more pronounced at the lower temperatures.

The above concepts cover the basic principles of electron spin resonance, and will be considered in more detail when the different practical applications are summarized in the later sections. There is, however, one other quite different type of resonance experiment that can also be performed with unpaired electrons, namely "cyclotron resonance." In this the *electric* field of the electromagnetic radiation is allowed to interact with the charge of the electron, instead of the magnetic vector of the radiation interacting with the magnetic moment of the electron, as in electron spin resonance. The principle of cyclotron resonance is shown in Fig. 2 where an electron is shown moving in a circular orbit under the influence of an external

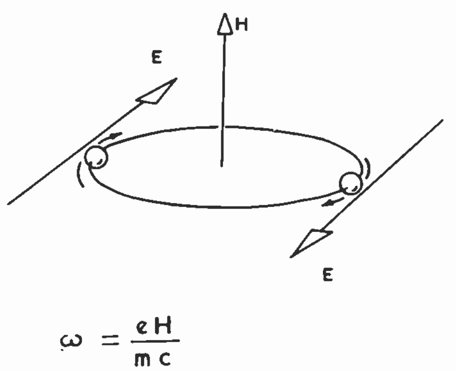


Fig. 2. Cyclotron resonance. The orbit of the electron is in a plane normal to the direction of the applied magnetic field.

magnetic field applied normal to the plane of its orbit. If an oscillating electric field is applied in the plane of the orbit, and if its period is equal to that of the rotating electron, it can interact so that it always accelerates a given electron. This is, of course, the same principle as used in a cyclotron accelerator, and the resonance condition is given by

$$\omega = \frac{eH}{mc} \qquad \text{.........(4)}$$

In the case of cyclotron resonance in solids the resonance condition is detected by the resultant absorption of the microwave power and, for a free electron, substitution of numerical values in eqn. (4) shows that at any given frequency such absorption will occur at the same field strength as that for the *spin* resonance of the free electron.

When conduction electrons are moving in the solid state, however, they are very far from "free" and interact strongly with the periodic potential waves of the lattice. This interaction can be best represented by a reduction of the electron's mass to an "effective mass" which may be a tenth, or less, of its real mass. The actual value of the effective masses of different current carriers is a very important parameter in semi-conductor theory, and cyclotron resonance is one of the very few methods of direct measurement.

The essential experimental difference between cyclotron resonance and spin resonance is that in the former case the sample is placed in the maximum concentration of the *electric* vector of the microwave field, whereas in the latter, the sample is placed in the maximum concentration of the *magnetic* component of the oscillating field. In all the other main features the experimental requirements are very similar and these are briefly summarized in the next section, before the actual applications of these two new techniques are described in detail.

3. Experimental Techniques

In both electron spin resonance and cyclotron resonance greater absorption and higher sensitivity are obtained by working at as high a magnetic field strength and frequency as possible. The upper limit is set by the purely practical considerations of the availability of magnets with sufficient field homogeneity, and microwave generators of sufficient power. For this reason very little work is carried out at frequencies greater than 36,000 Mc/s ($\lambda = 8$ mm, resonant field for $g = 2$ is 13,000 gauss), and most resonance spectrometers work with X-band microwave equipment at about 9,000 Mc/s ($\lambda = 3{\cdot}2$ cm, resonant field = 3,200 gauss).

The four basic components of any spectrometer are (a) the microwave source (b) the absorption cell, (c) a microwave detector and (d) a display system. These are illustrated for the simplest type of spectrometer in Fig. 3. The klystron feeds the radiation along the waveguide run to the cavity resonator, which acts as the

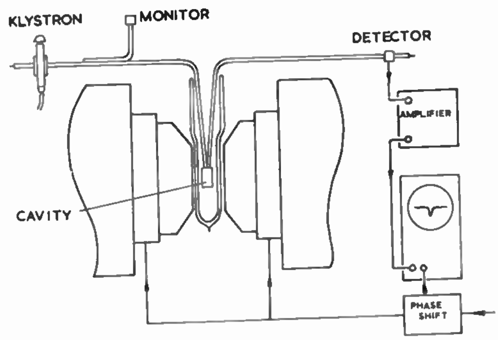


Fig. 3. Block diagram of a crystal-video E.S.R. spectrometer.

absorption cell. In this way the sample can be placed in a highly concentrated microwave field, and the cavity containing the sample can be inserted between the pole pieces of an electromagnet. In the transmission system shown, a second waveguide then leads from the cavity to a crystal detector where any change in signal level produced by absorption in the cavity, is passed on to the display system. The resonance absorption is obtained by varying the strength of the magnetic field until a dip occurs in the detected microwave power. If a small a.c. magnetic field modulation is also applied the resonance absorption will be swept through twice in a cycle, and may be amplified and displayed on an oscilloscope as shown. It can also be seen that measurements may be carried out at low temperatures by placing the cavity in a vacuum flask containing liquid oxygen, nitrogen, hydrogen or helium, the whole assembly being inserted into the magnet gap.

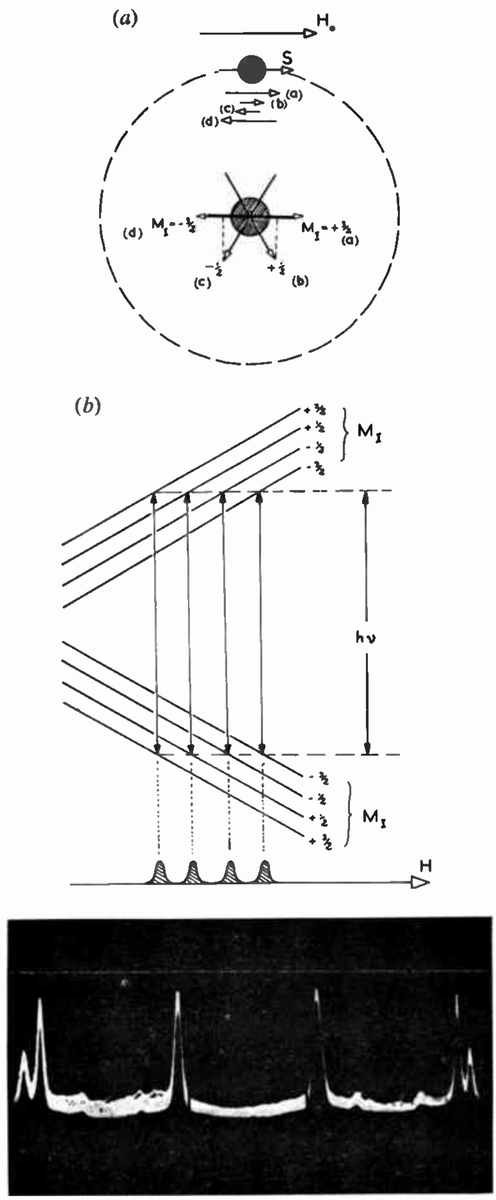
Although this basic type of equipment contains all the essential features of any electron-resonance spectrometer its sensitivity is not very high since the noise from the detecting crystal is large at audio frequencies. In practice this is overcome by either using superheterodyne detection or high-frequency magnetic field modulation.[1] Phase-sensitive detection is also employed to reduce the final noise band-width, and the signal is obtained as a trace on a pen-recorder. Under these optimum conditions it is then possible to detect down to 10^{12} unpaired electrons in a sample when measurements are made at room temperature, and considerably less at lower temperatures.

The actual type of cavity resonator that is employed varies with the particular specimen that is to be studied. If single crystals are to be investigated these are usually placed centrally on the bottom of an H_{111} cylindrical cavity, whereas powders or liquids are usually contained in glass or silica tubes and these are inserted axially through the narrow side of a rectangular cavity operating in the H_{012} mode. It will be seen later that cavity design can be of particular importance in maser construction where two or more resonant frequencies must exist at the same time.

The more important practical applications of magnetic resonance studies will now be outlined in some detail, and some of the more specific experimental techniques will be included in these sections. It will be seen that the first two of these applications are concerned with *materials* which are of importance to the electronic industry, while the last two are concerned with new electronic *devices*.

4. Hyperfine Structure and the Analysis of Impurities

In the analysis of section 2 it was assumed that the unpaired electron was only affected by the externally-applied magnetic field. It is possible, however, for it to be also affected by magnetic fields arising within the compound itself and, in particular, by those originating from the nuclei of the atoms around which the unpaired electrons are moving. Many nuclei have magnetic moments, and these can produce additional magnetic fields at the electrons of the order of 100 gauss. These additional field increments will also be quantized, as can be seen by reference to Fig. 4. In Fig. 4(a) the unpaired electron is shown in an orbit around a nucleus with a spin of $\frac{3}{2}$ (e.g. a copper atom). The nuclear spin and magnetic moment will also align themselves with respect to the externally applied field, H, and the quantum condition allows only orientations which have resolved components differing by unity. There are therefore four possible orientations in this case (with resolved spin components of $+\frac{3}{2}$, $+\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{3}{2}$. Hence there are four different incremental values of magnetic field produced by the nuclei at the electron, and each of the single electronic levels of Fig. 1 will therefore be split into four sub-components, as shown in Fig. 4(b). The resonance conditions is now fulfilled for four slightly different values of magnetic field



(c)

Fig. 4. Hyperfine splitting of resonance spectra. (a) Incremental fields produced by the four orientations of a nucleus with spin, $I = \frac{3}{2}$. (b) Resultant splitting of the electronic energy levels and absorption line. (c) Observed hyperfine splitting in a copper salt. ($I = \frac{3}{2}$ for both Cu^{63} and Cu^{65}.)

strength, and hence the single absorption line is now split into four hyperfine components. (For a nucleus of spin I there will be $(2I+1)$ components). The actual trace obtained from a copper salt[2] is shown in Fig. 4(c). The outermost lines are split into two components here, because copper has two isotopes Cu^{63} and Cu^{65} each with a spin of $\frac{3}{2}$ but with slightly different magnetic moments. This separation of such closely-related isotopes illustrates the very high resolving power available in measurements.

The occurrence of such hyperfine splitting was first employed to determine unknown nuclear spins and magnetic moments[3, 4] but this technique can now be used in reverse as a powerful tool for analytical studies. Thus it is known that a hyperfine pattern of $(2I+1)$ lines must be produced by a nucleus which has a spin of I, and this fact, together with the magnitude of the observed splitting, enables the nucleus to be identified. It therefore follows that the nature of unknown impurity atoms present in a solid can be identified directly by a simple analysis of the hyperfine pattern that they produce. The quantity of the impurity present can also be estimated accurately from the integrated intensity of the spectrum and hence electron resonance provides a very powerful tool for non-destructive quantitative analysis.

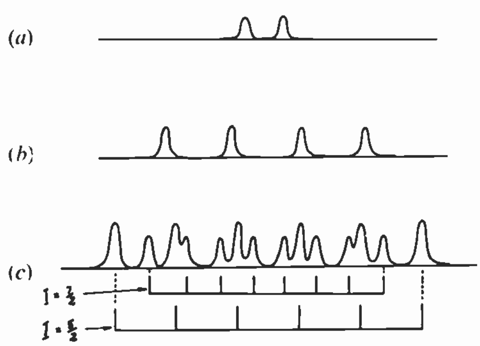


Fig. 5. Hyperfine structure analysis of different impurities in silicon. (a) phosphorus ($I=\frac{1}{2}$), (b) arsenic ($I=\frac{3}{2}$), and (c) antimony ($I=\frac{5}{2}$ and $\frac{7}{2}$).

One of the best examples of the use of this is in the analysis of semi-conductor material where impurity concentrations play a vital role in the determination of suitable electronic properties. The type of results that can be obtained is shown in Fig. 5, which summarizes the hyperfine patterns that are observed from different impurities in silicon.[5] A simple doublet is obtained from phosphorus impurity atoms, a quartet of 73 gauss splitting from arsenic (compare the 120 gauss splitting of the copper quartet), and overlapping six and eight line patterns from the two isotopes of antimony. This method of analysis is, of course, being applied to a large number of other types of material, but the study of semi-conductors is probably that most directly related to the electronics industry.

5. Cyclotron Resonance of Current Carriers

The basic principle of cyclotron resonance has been illustrated in Fig. 2 and it has been seen that this is one of the few direct means of determining the equivalent mass of current carriers in solid material. Both holes and electrons will give rise to cyclotron resonance, and each may have effective masses which vary with direction inside a crystal. The physical reason for this variation, and for the possibility of negative equivalent masses, can be best understood by representing the moving electrons by a wave-motion of the appropriate DeBroglie wavelength, and considering the interaction of this with the periodic array of atoms within the crystal. When the wavelength is nearly equal to the distance between successive atoms, constructive interference will take place (in the same way as Bragg reflection occurs with X-rays) and the electron motion will tend to be "reflected" by the regular atomic array. This effect will vary with direction because the atoms will not necessarily have the same spacing in all directions. It is evident that the magnitudes of the equivalent masses are intimately connected with both the atomic structure and the energy level system of the material, and a precise measurement of their values is of very great help in predicting other electrical properties.

Semi-conductor material is again one upon which a large amount of attention has been focused, and Fig. 6 shows a typical plot of the cyclotron resonances observed from silicon[6] at 4°K. It will be noticed that there are three distinct resonances obtained from the electrons, showing that there are three kinds of conduction electron in silicon. The effective masses of all

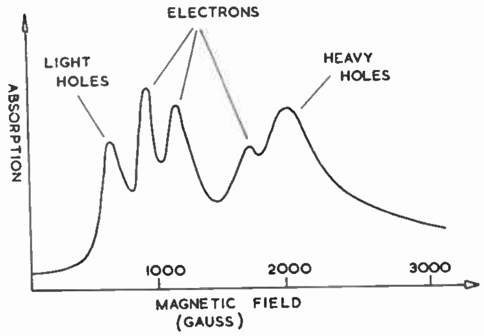


Fig. 6. Cyclotron resonance peaks from silicon at 4°K.

three vary strongly with direction, the three patterns of angular variation being mutually orthogonal. These experiments have to be conducted at low temperatures since the thermal vibrations of the lattice will otherwise scatter the electrons before they complete many rotations and abstract any microwave energy.

Cyclotron resonance can also be used to study the current carriers in metallic conductors. This initially proved to be a very difficult problem, since the much larger number of conduction electrons present caused a phase defocusing of the orbits, due to their mutual repulsion, and no sharp resonance lines could be obtained. A new experimental technique has recently been suggested, however, by a Russian, Azbel.[7] This eliminates the defocusing effect by applying the external magnetic and oscillating electric fields accurately *parallel* to the surface of the metal, as illustrated in Fig. 7. The rectangular block

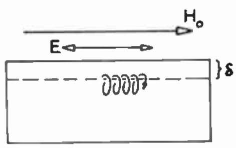


Fig. 7. Experimental arrangement for observing cyclotron resonance in metals. *H* is the d.c. magnetic field, *E* the microwave electric field, and δ the skin depth.

represents a piece of metal with skin depth δ as indicated. The magnetic field, *H*, is applied parallel to the top of the surface so that electrons will be rotating in orbits in a plane normal to that of the paper. One such orbit is shown at such a depth that only its top portion reaches into the skin depth and is effected by the oscillating electric field. This field is now applied in the same direction as the magnetic field, and its effect on such an electron will therefore be a lateral movement from left to right. The electron's orbit is thus converted into a spiral, instead of an enlarged circle with phase-focused electrons. Under resonance conditions the given electron will be acted on by the same force each time it comes up within the skin depth, and will also be kept apart from similarly interacting electrons. A whole series of sub-harmonic resonance frequencies are predicted in such an experiment, since electrons rising into the skin depth after two or three periods will also have the correct phase to interact. Recent experiments on very pure samples of tin and copper have strikingly confirmed this theory, with sub-harmonic resonance peaks stretching back into low fields.

6. Maser Developments from Paramagnetic Resonance

In all the resonance experiments so far discussed there has been a net *absorption* of radiation, as is to be expected if there is a normal population distribution between the energy levels. If, by some artificial means, it is possible to invert this population distribution so that there are more electrons in the upper energy level than in the lower, it follows that microwave *emission* instead of absorption will be obtained. It is this artificial inversion of the populations of the energy levels that is the essential idea behind the maser. This acronym stands for "*M*icrowave *A*mplification by *S*timulated *E*mission of *R*adiation" and it is a device which employs the principles of electron resonance in reverse so that microwaves are emitted (or amplified) on resonance, instead of being absorbed. There is a variety of methods whereby this artificial inversion of the energy level populations can be obtained, some requiring pulse operation and rapid change of field strengths.[8] The most promising method produces a continuously inverted population system, however, by a relatively simple method and is designated the "three-level maser." This was first suggested by Bloembergen[9], and Basov and Prokhorov[10], and the method affords a very good example of general maser principles.

Before this method of operation is discussed in detail the concept of the "electronic splitting" of an electron resonance absorption must be introduced. In Section 2 consideration was limited to atoms containing only one unpaired electron, but many transition group atoms contain two, three or more unpaired electrons, and these will add their individual spins and magnetic moments to produce a resultant S of 1, $\frac{3}{2}$ or larger. If the particular case of an atom with two unpaired spins is considered then the net spin of $S=1$ will be able to align itself with components of $+1, 0$, or -1 with respect to the externally applied field. The splitting of the levels corresponding to these three orientations is shown in Fig. 8(a). It is seen that, if all the levels have the same energy in zero-magnetic field, then the two resonance transitions induced by the microwave radiation $h\nu$, will occur at the same magnetic field strength, and the two absorption lines will superimpose to give a single trace. In a solid, however, the strong internal crystalline electric fields often split the three levels, even in zero-magnetic field, to give an energy-level diagram as shown in Fig. 8(b). It is seen that in this case the microwave radiation requires two different field strengths for resonance, and hence two absorption lines are obtained, and the splitting between them is termed "an electronic splitting." The magnitude of this splitting normally varies quite markedly with angle between the applied magnetic field and the crystalline axes.

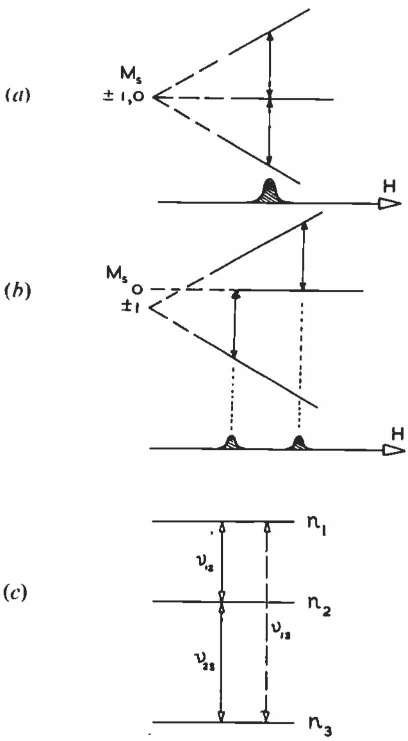


Fig. 8. Splitting of energy levels for $S=1$. (a) With no zero-field splitting. (b) With a zero-field splitting, which produces two separate resonance lines. (c) Splittings and resonant frequencies at a given magnetic field strength.

The higher-energy transition between the two extreme levels, as shown by the dotted arrow in Fig. 8(c) is normally forbidden, since the selection rules state that the resolved component of spin must only change by unity (not 2 as in this case). There are second-order terms of some magnitude, however, which tend to "mix" the quantum assignment of the energy levels, and these remove the restriction on the dotted transition to some extent. It will be seen that the existence of these cross-terms is essential for maser operation. As a result a system of three energy levels is produced, and at any given magnetic field there are three different splittings between them, with corresponding frequencies of absorption or emission.

These three frequencies are designated ν_{12}, ν_{23} and ν_{13} in Fig. 8(c), as shown, and the population of the three levels as n_1, n_2 and n_3 respectively.

Under normal experimental conditions $n_1 < n_2 < n_3$ and *absorption* can be observed at three different resonant frequencies for any given field strength. Maser action can be obtained, however, by feeding in a high level of power at frequency ν_{13} and thus "saturating" this transition. It was seen in Section 2 that under such conditions the population of the two levels will become more or less equal ($n_1 = n_3$), and as a result n_1 will become greater than n_2. If this difference is sufficiently large, continuous *emission* at ν_{12} will therefore be obtained, and the system will act as an oscillator. If the difference is not quite so great, the stimulated emission will still be larger than the absorption and an incoming signal at ν_{12} will be amplified. It is therefore possible to operate such a device as either a microwave amplifier or oscillator by suitably adjusting the level of the pumping power. It will be evident that the relaxation times associated with the different transitions play an important part in determining the equilibrium values of n_1, n_2 and n_3 and it is usually necessary to work at low temperatures for efficient maser action.

The great advantage of the maser as a microwave amplifier is its inherently low noise figure. No semi-conductor contacts· involving the large excess flicker noise of silicon detectors, are present, nor are hot electron beams, as in normal valve devices. The noise, in fact arises solely from the Johnson noise of the effective resistance at the temperature of operation, and since this is usually 4°K, the inherent noise figure of the amplifier can be made extremely small. Many different compounds can be used for the active element, since the only essential requirement is the existence of three or more electronic levels with suitable second-order cross-terms to couple them. The first three-level maser[11] employed gadolinium in lanthanum ethyl sulphate, and three of its eight electronic levels were selected. Chromium atoms, as found in artificial ruby, also form a very suitable working medium, with four electronic levels available.

The basic experimental apparatus required for maser operation is very similar to that of a low-temperature electron resonance spectrometer, with the additional complexity that the cavity resonator must be resonant at two frequencies at the same time, i.e. the pump frequency and the signal frequency. This can be relatively easily accomplished by using strip waveguide to form a resonant transmission line at the longer wavelength, and placing this inside a cavity which is itself tuned to the higher pumping frequency. The waveguide or coaxial leads are usually made of thin stainless-steel or nickel-silver to reduce the heat conduction as much as possible.

One of the drawbacks of maser operation has been the necessity to operate at 4°K. since the much weaker spin-lattice interaction present at the low temperature allows saturation to be more easily achieved. Quite recently[12], however, it has been demonstrated that three-level solid state maser action can be carried out quite successfully at 60°K, a temperature that can be reached with liquid oxygen. This was achieved by the use of a "double-pumping" system, illustrated schematically in Fig. 9. If a diluted chromium salt is taken (as in artificial ruby), at certain angles between the applied magnetic field and the crystal axes the splitting between the top two and bottom two levels will be equal, as shown in Fig. 9. The bottom three levels can be used for normal maser operation, the pump

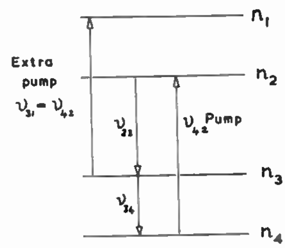


Fig. 9. Maser operation at 60°K by "double pumping." The energy levels are for Cr^{+++} as in ruby.

frequency being ν_{42}, and the signal frequency ν_{23} It is now also possible, however, to reduce the population of level 3 by pumping between this level and level 1. It follows that there is a new mechanism whereby the population of level 3 can be lowered as well as that of level 2 raised and this mechanism is not dependent on variation of relaxation times with temperature. A maser using this principle has recently been operated successfully at the Royal Radar Establishment at Malvern as an X-band amplifier.[12]

This successful operation of the solid-state maser at liquid oxygen temperatures holds great promise for the development of relatively simple and portable amplifiers. It would appear, however, that the main applications of the maser as such will be to systems where the incoming radiation has a low temperature, such as from outer space as in radio astronomy or satellite tracking. Only in such cases can the greatest advantage be taken of the inherently low noise figure of the low temperature solid-state masers.

7. Mavar Developments from Ferromagnetic Resonance

The term "mavar" stands for "*M*icrowave *A*mplification by *V*ariation of *R*eactance" and is used to describe a family of new electronic devices which also hold very great promise as low noise microwave amplifiers.

It is very interesting that, although the basic operation of the mavar is quite different from that of the maser, the construction of the first successful mavar was also a direct result of experiments in magnetic resonance. The general principle of the mavar, or parametric amplifier, is illustrated in Fig. 10. Three tuned circuits are connected across a variable reactance, here

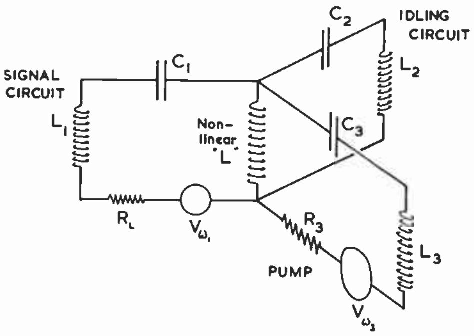


Fig. 10. Equivalent circuit of mavar or parametric amplifier.

drawn as an inductance, and the essential requirement for operation is that this should have a non-linear relation between the flux and the current flowing through it. The circuit tuned to the highest frequency, ω_3, is fed with relatively high level power and acts as the "pump," while the small signal to be amplified is fed into the circuit resonant at ω_1. The third circuit resonant at ω_2 (such that $\omega_2 = \omega_3 - \omega_1$) acts as an idling circuit,to effect a power balance. If a detailed analysis of this circuit is carried out[13, 14] it can be shown that it is possible to transfer power from the pump circuit to the signal circuit, and thus to amplify signals at ω_1, at the expense of power at ω_3.

The general possibility of such amplification has been known for some time[15, 16], but it was only recently that Suhl[17] pointed out that the results of ferromagnetic resonance could be applied to design such an amplifier in the microwave region. He showed that not only would a crystal of ferrite material act as a suitable non-linear inductance, if held at its ferromagnetic resonance condition, but that the magnetostatic modes within the ferrite itself could be used as the signal and idler resonant circuits. These magnetostatic modes arise because of the uneven distribution of magnetic flux through the specimen, and each corresponds to a different spatial configuration of microwave field within the ferrite. Suhl's idea was that, by selecting two such magnetostatic modes with frequencies which added to the pump frequency ω_3, the ferrite itself could be made to form both the signal and idling circuits. It would then only need to be placed in a cavity resonant to the pump power. It can be shown that such "magnetostatic" operation is very efficient and requires relatively little pumping. It is, however, very difficult to couple with only the two specified magnetostatic modes, without exciting a large number of others, and, for this practical reason, "magnetostatic" operation of the Suhl type of mavar has not yet been effected.

It is possible, however, to place the ferrite in a cavity-transmission line system so that the signal and idler circuits are provided externally. Operation of the Suhl type mavar in this "electromagnetic" mode was in fact achieved quite rapidly by Weiss[18], but a pump power of 15 kW peak was required. This high value of pump power is necessary because the non-linear terms in the effective inductance only become appreciable at the higher power levels of the ferromagnetic resonance frequency (in the same way as "saturation" is required in the case of the maser). The required pump power depends markedly however, on the line-width of the ferromagnetic resonance absorption. Weiss employed ferrites with a width of about 50 gauss in his original Mavar[18]. but widths of less than 1 gauss are now being achieved, with a corresponding reduction in required pumping power, since this varies with the square of the resonance line width.

For this reason most of the research and development on the Suhl type mavar is being concentrated on the production of ferrites with very small line widths. Yttrium iron garnet is of particular promise in this connection and widths of less than a quarter of a gauss have been reported. It should be noted that variation of inductance is not the only means of operating a parametric amplifier in the microwave region, and silicon diodes with varying capacitance have also been used to produce successful mavar amplifiers.[19] It would appear to have been more than a coincidence, however, that the magnetic resonance measurements on ferrites came before Suhl's conception of a variable inductance at microwave frequencies, and this, in turn, seems to have stimulated all the new ideas on parametric amplification by other devices.

8. Conclusion

It will be seen from the different applications considered in the previous sections that magnetic resonance has played a very important role in the development of several recent electronic techniques. It is probably not so surprising that it should have been applied to analysis of physical properties, such as impurity concentrations and mass measurements of current carriers. On the other hand, the development of the maser and mavar field from this fundamental work would have been very difficult to foresee a few years ago, and its rapid application to the design of new microwave amplifiers and oscillators has been most stimulating.

Much yet remains to be done in the way of fundamental measurements, especially in the determination of relaxation times and their variation with temperature, so that the most suitable type of material can be chosen for maser operation. A better understanding of the properties of ferrites and semiconductor material will also be necessary for the accurate design of mavars and parametric amplifiers. Such fundamental work is already being carried out with a greater awareness of its immediate practical application, and the whole of this field is a very striking example of the great benefits to be obtained from a close co-operation between fundamental and applied research.

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# of current interest . . .

Standard Industrial Classification

For the purpose of comparing official statistics of various government departments, a standard industrial classification was introduced by the Central Statistical Office in 1948. In the first edition, Radio and Electronic Engineering was given scant mention under the heading of "Wireless Apparatus and Gramophones."

This state of affairs has been remedied to a considerable extent in the 1958 revision of "Standard Industrial Classification" which has recently been issued by the Central Statistical Office (published by H.M.S.O., price 2s. 6d.).

"Radio and Other Electronic Apparatus" is now a "minimum list heading," with the following sub-sections:—

1. Valves:

Manufacturing electronic valves (including cathode ray tubes) and equivalent crystal devices. (Glass envelopes are excluded and classified elsewhere.)

2. Radio and other electronic equipments and gramophones:

Manufacturing radio and television transmitters and receivers, radar and electronic navigational aids, X-ray apparatus, electronic computers, electronic measurement apparatus, high frequency heating apparatus, etc., and sound reproducing and recording equipment including gramophones, gramophone records and tape recordings. Components and assemblies not elsewhere specified are also included. Electro-medical equipment other than X-ray apparatus is excluded.

The classification takes into account the International Standard Industrial Classification issued by the United Nations and follows the same general principles.

Whitworth Foundation Scheme

The Minister of Education has decided to double the value of the Whitworth Fellowship —for long the premier award in engineering— from £500 to £1,000 a year from 1960. The rules have also been altered so that the award in future will be given not to newly qualified graduates but to practising engineers who have shown themselves to be extremely able and likely to benefit from additional study and training. The original purpose of the scheme was to provide an opportunity for the *intending* engineer of particular merit to obtain further qualifications and experience, but this has now been largely superseded by the growth of assistance from industry and public funds.

From next year up to three Whitworth Fellowships of £1,000 a year will be offered. These may be supplemented by allowances for dependants and for travelling and subsistence. In addition up to three Whitworth Exhibitions of £100 will be awarded to unsuccessful candidates whose work deserves recognition.

Applicants must be over the age of 25 years and be in possession of a university degree in engineering, a Diploma in Technology (Engineering), a Higher National Diploma or a Higher National Certificate in Engineering with at least two distinctions, or a qualification approved by the Minister as of equivalent standard. They must have been subsequently engaged as practising engineers for not less than three years. Applications for the 1960 competition must be submitted to the Ministry of Education by 31st July, 1959. Full details are obtainable from H.M.S.O., price 4d.

"Photo-Emission"—a New Science Film

Produced by Mullard Educational Service in conjunction with the Educational Foundation for Visual Aids, "Photo-Emission" is the latest addition to the Advanced Science Series for sixth forms and technical colleges. It runs for 18 minutes, on 16mm black and white sound film, and is backed-up by comprehensive teaching notes.

The film starts by describing the working and construction of a simple photo-emissive cell, and then shows some of the early experiments of Elster, Geitel and Millikan. The theory of photo-emission is concisely and simply explained, and the established laws of the effect are set out. The explanations are put over effectively by animated diagrams and easily-followed experiments. A short survey of the many practical and laboratory applications of photo-emissive cells is given in the closing sequences.

"Photo-Emission" is available on hire from the E.F.V.A. Film Library, Brooklands House, Weybridge, Surrey.

Multiplicative Receiving Arrays†

by

V. G. WELSBY, PH.D., MEMBER and Professor D. G. TUCKER, D.SC., MEMBER‡

Summary: The performance of arrays in which the outputs from two groups of elements or sections are multiplied together is studied from the points of view of directional patterns, directional discrimination and signal/noise performance. It is shown that whereas, in ordinary linear arrays, these three considerations are usually all determined by the directional pattern, yet in multiplicative arrays they are all three quite distinct and often unreconcilable with one another. A comparison with super-directive arrays is also made, and an example is given of a possible practical application of the multiplicative array in an underwater echo-ranging system using electronic sector scanning.

List of Symbols

θ	bearing angle.
λ	wavelength in medium.
d	spacing of elements.
d_1	spacing of groups.
l	length of array.
x	$=\frac{\pi l}{\lambda}\sin\theta$. This is used in dealing with continuous arrays or apertures.
p	$=\frac{\pi d}{\lambda}\sin\theta$. This is used for point arrays.
q	$=\frac{2d_1}{d}$
n	$=(n_1+n_2)$ number of elements.
DF	directivity factor.
NF	noise factor.
R	signal/noise voltage ratio.
RF	rejection factor.
x'	$=\frac{x}{2}$
$\frac{1}{s}$	signal/interference ratio in medium.
$D_0(p)$	normalized directional function.
$D_1(p)$; $D_2(p)$	ditto for groups of elements.
$D_m(p)$	ditto for complete multiplicative system.

1. Introduction

The principle of multiplying together the outputs from two receivers (e.g. radio aerials or electro-acoustic transducers) is well known; its applications include, for example, radio astronomy. Several papers[1, 2, 3] have discussed some aspects of the subject, but no treatment of the problems of synthesis of directional patterns of multiplicative arrays appears to have been published. The present paper is intended to give a fairly general treatment of the design and properties of multiplicative arrays when used to receive coherent tone or narrow-band noise signals.

The system considered in this paper is that in which a line array of point receivers, or alternatively a continuous aperture or transducer divided into equal sections, is divided into two groups of elements and the combined output of one group is multiplied by the combined output of the other. Following multiplication the output signals are smoothed so that addition frequencies are removed and the required signal, for any particular direction of signal incidence, is uni-directional or, more briefly, "d.c."

The circumstances in which it may be advantageous to use the multiplicative array instead of the ordinary additive kind are very varied, and it is not the object of the present paper to discuss them. Nevertheless the performance of multiplicative arrays will be at all

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stages compared with the best that can be obtained with additive arrays, so that the problem of assessing the relative merits in any particular practical circumstances will not be difficult.

Since the synthesis of directional patterns of multiplicative arrays is to be discussed, it must be mentioned that use will be made of two methods which are well known in their application to ordinary linear arrays: one is the method of suitable location of zeros on the angular scale, published by Schelkunoff[4], and the other is that of superposition of basic patterns, published by Woodward[5] and further discussed by one of the present authors[6]. By the use of these methods directional patterns can be obtained for linear arrays which have advantages over that of the plain linear array with all its elements of equal sensitivity. A practical way in which the array is made to give these patterns is the variation of the sensitivity of the elements over the length of the array. The relation between sensitivity and position in the array is often called the "taper function." This will, in general, be a complex function in the sense that additional phase-shifts or even complete phase reversals may be applied to the output of the elements before they are finally added together.

A manifestation of improvement in the directional pattern may be a reduction in the magnitude of secondary lobes, or the narrowing of the main beam, or both. When both these improvements are obtained together, the array usually becomes "super-directive"[7]; the meaning and significance of this will be discussed in Sect. 7. It is found that any improvement in the directional pattern, as compared with that of the plain uniform linear array, is accompanied by the degradation of the noise factor of the array. The term "noise factor" was introduced by one of the authors[3] as a convenient parameter for the comparison of the noise performance of various types of array. It is defined as the worsening of signal-to-noise ratio in the output of the array relative to the signal-to-noise ratio obtained when the same signal and noise are received on a plain uniform linear array. The noise concerned is defined as any noise which is uncorrelated between one element or section of the array and all the others. A more detailed discussion of this is given in Sect. 6.2.

It will now be shown that improved directional patterns can be obtained by *multiplying* together the outputs from two groups of elements, and processes of synthesis are applied which are developments of those, mentioned above, which are normally used with ordinary linear arrays. It is found that the signal-to-noise performance of such a system, although inferior to that of a plain array, nevertheless compares favourably with that obtained if the principle of superdirectivity is applied to the array to obtain a comparable beamwidth.

The way in which the multiplicative principle can be applied to obtain higher angular accuracy in detection, in an acoustic echo-ranging system using electronic sector-scanning, is shown in Appendix 3.

2. Directional Patterns of Multiplicative Arrays in terms of Suitable Location of Zeros

In this section it will be assumed that the array is made up of uniformly spaced point receivers of equal sensitivity. If, in fact, the array is really continuous, but divided into sections, the final results may be easily adjusted to take account of this by multiplying the directional patterns by the directional pattern of one section of the array. The more general case, in which the elements are not of equal sensitivity, is considered in Appendix 2.

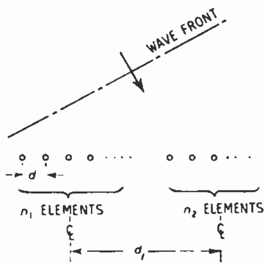

Fig. 1. Diagram representing an array of equally-spaced elements, divided into two groups.

Figure 1 represents a uniform array of n equally-spaced point elements, divided into two groups containing n_1 and n_2 elements respectively, so that $n = n_1 + n_2$. The spacing between adjacent elements is d and the distance between the centres of the two groups is d_1.

The directional patterns of the two groups, expressed in terms of the parameter $p = \frac{\pi d}{\lambda} \sin \theta$ are proportional to the functions

$$D_1(p) = \frac{\sin n_1 p}{n_1 \sin p} \qquad \text{.........(1)}$$

$$D_2(p) = \frac{\sin n_2 p}{n_2 \sin p} \qquad \text{.........(2)}$$

In general, the product of two alternating voltages $A \cos \omega t$ and $B \cos (\omega t + \psi)$ is

$$AB \cos \omega t \cos (\omega t + \psi)$$

$$= \frac{AB}{2} \cos (2\omega t + \psi) + \frac{AB}{2} \cos \psi \qquad \text{.........(3)}$$

The first term of the right-hand expression can be removed by filtration leaving only a term representing a "d.c." voltage which is proportional to the product of the amplitudes of the inputs and to the cosine of the phase angle between them. The phase of the output from each group of elements will be the same as that from an element situated at the centre of the group concerned or will differ from it by π radians. The phase difference between the outputs, due to the separation of the centres of the groups, will be

$$\psi = \frac{2\pi d_1}{\lambda} \sin \theta$$

$$= \frac{2d_1}{d} p \qquad \text{.........(4)}$$

The modulus of the directional function after multiplication is then proportional to

$$D_m(p) = \left(\frac{\sin n_1 p}{n_1 \sin p}\right)\left(\frac{\sin n_2 p}{n_2 \sin p}\right) \cos pq \qquad \text{.........(5)}$$

$$\text{where } q = \frac{2d_1}{d}$$

This expression† remains valid if $(n_1 + n_2)$ lies between n and $2n$, i.e. if the groups overlap so that some of the elements are shared between them. It can be shown however that an arrangement of this type is unsatisfactory from noise considerations. Assuming therefore that there is no overlapping of the groups, $n_1 + n_2 = n$ and $q = n$. Thus, for non-overlapping groups,

$$D_m(p) = \left(\frac{\sin n_1 p}{n_1 \sin p}\right)\left[\frac{\sin (n - n_1)p}{(n - n_1) \sin p}\right] \cos np \qquad \text{.........(6)}$$

The two parameters controlling $D_m(p)$ are

(*a*) the total number of elements

(*b*) the number of elements in one of the groups.

The design of multiplier arrays can be approached by making use of a theorem which applies to each of its constituent arrays.

Theorem

The directivity pattern of an array with a finite number of elements, expressed as a function of p, is completely defined by the positions of the zeros of this function in the complex plane of p within each repetition period of the pattern.

A proof of this theorem, in the general case, is given in Appendix 2 but, in the particular case of a uniform additive array, it is evident that $|D(p)|$ is equal to unity when $p = 0$ and passes through points spaced equally along the p axis at intervals of π/n. There are $(n - 1)$ such zero points in the interval between $p = 0$ and $p = \pi$ and the pattern is then repeated in the interval up to $p = 2\pi$, and so on.

If the array is divided into two groups and the multiplicative principle applied, the directivity pattern will therefore have, from the two polynomials relating to its component groups, a total of $(n_1 - 1) + (n_2 - 1) = (n - 2)$ zeros. This is one less than the number available for a simple array. If both the groups of elements are uniform (i.e. if all the elements have identical sensitivities), the zeros of one group will be spaced equally along the p axis at intervals of π/n_1 and those of the other group will be spaced at intervals of π/n_2. The directional function given by eqn. (6) also contains the factor $\cos np$ which introduces a further n zeros into the function $D(p)$ in the interval between $p = 0$ and $p = \pi$.

It can be concluded therefore that a uniform multiplicative array with $n_1 + n_2 = n$ elements will have a directional pattern with $(n - 2 + n) = 2(n - 1)$ zeros instead of the $(n - 1)$ which would be available for a simple n-element array.

† At this stage the directivity pattern is assumed to be simply a plot of the modulus of the total output of the system against the parameter p as a point signal source moves at a constant distance (very large compared with the length of the array) from the array. The exact significance of the directivity pattern will be discussed in Sect. 6.3.

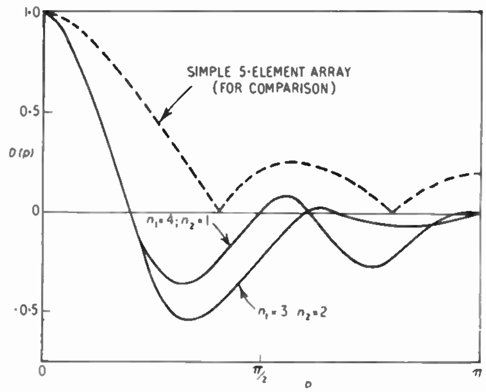

Fig. 2. (a) Directional patterns for 5-element multiplicative arrays.

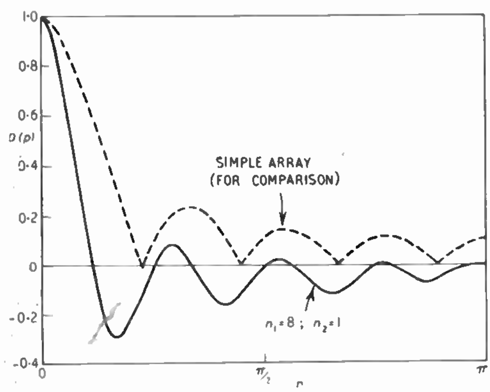

Fig. 2. (b) Directional patterns for 9-element arrays ($n_1 = 8$; $n_2 = 1$).

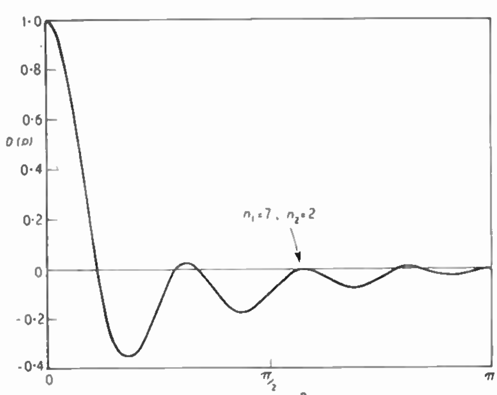

Fig. 2. (c) Directional patterns for 9-element arrays ($n_1 = 7$; $n_2 = 2$).

Of these zeros, n have their locations fixed by the number of elements and the locations of the remainder are fixed by the grouping, i.e. by the values of n_1 and n_2. A uniform multiplicative array with non-overlapping groups thus provides a choice of as many directional patterns as there are ways of arranging the grouping.

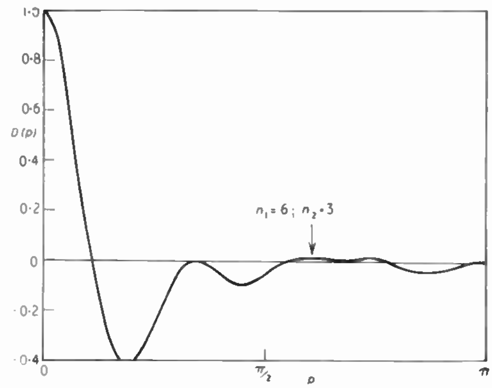

Fig. 2. (d) Directional patterns for 9-element arrays ($n_1 = 6$; $n_2 = 3$).

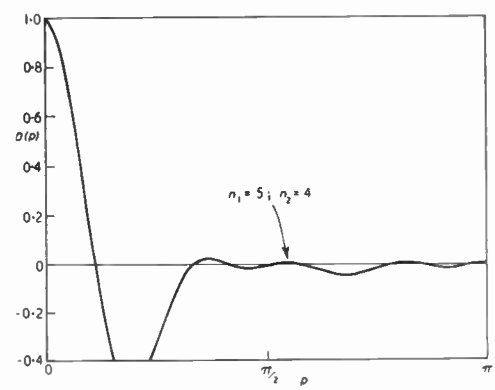

Fig. 2. (e) Directional patterns for 9-element arrays ($n_1 = 5$; $n_2 = 4$).

As an example, Fig. 2 shows curves of the directional function for the various possible multiplicative arrangements of uniform 5-element and 9-element arrays. It will be noted that the width (between zeros on the "p" scale) of the main lobe is determined in each case by the cos np factor so that it is just half that of the corresponding simple array. Although, as illustrated by these examples, the smallest adjacent secondary lobe is generally obtained when one of the groups contains one element only, the choice of grouping is not critical, particularly if alternate lobes of the pattern are rendered ineffective by the device described in Sect. 5. Apart from the directional pattern, the final choice may be influenced to some extent by the signal/noise considerations discussed in Sect. 6.

3. Deflection of the Multiplicative Beam by means of Phase-Shifters

It was shown in Section 2, eqn. (5), that the directional pattern of a uniform multiplicative array is made up of three factors: these are the directional patterns of the two portions of the array and the interference term cos np. If, therefore, it is desired to deflect the main lobe, or beam, of the composite pattern by electronic means, it is necessary to deflect all three constituents by means of suitable phase-shifters. The deflection of the individual patterns of the two parts of the array is simply effected by connecting each element to a corresponding tapping point of one of two uniformly-tapped delay lines[6], the output from each delay line forming one of the inputs to the multiplier: the deflection of the interference pattern is easily effected by connecting a phase-shifter in the input to the multiplier circuit on one side or the other. When all these phasing arrangements are correctly made, the whole directional pattern of the multiplicative array is moved bodily along the p-scale.

A small amount of deflection of the pattern can be obtained more simply merely by the use of a single phase-shifter on one side of the multiplier input—this deflects the cos np term—and if the shift in the p-scale is much less than π radians, the distortion of the pattern due to the other two factors remaining stationary will not be serious for most purposes. If the added phase-shift is increased to π radians—i.e. the polarity of one input is reversed, then the pattern merely returns to its original form and position, but with its polarity reversed.

4. Synthesis of Directional Patterns of Multiplicative Arrays by Superimposition of Basic Patterns

As stated earlier, a well-known alternative method of synthesis of directional patterns of ordinary arrays is the superposition of a number of patterns all of the same shape, namely, that derived from the plain uniform linear array, but deflected to various positions in the angular scale. This method is most simply applied to arrays of continuous form in which the basic pattern is of the (sin x)/x type. Allowance for non-continuous form can then readily be made. This method of synthesis has advantages and disadvantages as compared with the zero method previously described. Unless it is made very complicated, it requires uniformly spaced zeros on the x-scale, but on the other hand its results are much more directly obtained, since the synthesis is done directly in terms of pattern and not by means of a mathematical process.

This principle of superposition of basic patterns can be applied to multiplicative patterns most conveniently where the basic pattern used is that obtained from the multiplication of the outputs of two halves of the array. The directional pattern is then given by

$$D(x) = \left[\frac{\sin(x/2)}{x/2}\right]^2 \cos x \qquad \ldots\ldots\ldots(9)$$

As can be seen from Figs. 2 and 3, the pattern obtained from this arrangement has the peculiar feature that the first secondary lobe has the same width between zeros as the main lobe. It is therefore clear that additional patterns, one deflected by an angle $+\pi$ together with another deflected by an angle $-\pi$, each of peak amplitude $-0{\cdot}40$ can be used to give an approximate cancellation of the major secondary lobes of the original pattern†. The resultant directional pattern is shown in Fig. 3 and will be seen to be of a much better shape.

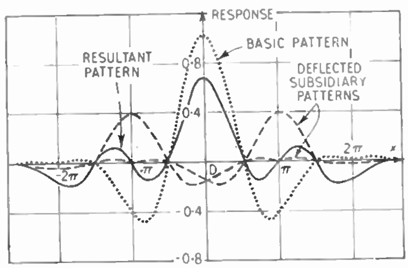

Fig. 3. Synthesis of directional pattern of multiplicative array by superposition.

† It must be appreciated that this method of synthesis, while similar to that used for ordinary arrays, lacks many of the advantages of the latter; in particular the amplitude of the main peak is not unaffected by the superposition of the additional patterns, and the physical realization is complicated.

especially as the largest secondary lobes are now well-removed from the proximity of the main beam. In any case, superposition of additional patterns could reduce these too. Other patterns could be synthesized by an extension of the principle.

The way in which the individual patterns can be realized has been discussed in the previous section. Three sets of separate connections do not, however, have to be taken from the elements of the array—two sets are sufficient; one is used to feed multiplier circuit no. 1 directly, and the other to tap on to a pair of delay lines, one for each half of the array. The outputs of the left-hand ends of the delay-lines are taken to multiplier no. 2, which has a phase-shifter inserted in one of its inputs; the outputs of the right-hand ends of the delay-lines are taken to multiplier no. 3, which has a similar phase-shifter inserted in the input opposite to that used in multiplier no. 2. The outputs of the three multipliers are then added together. It is clear the process is somewhat complicated, and unlikely to prove attractive in many practical applications.

5. Rectification of Multiplier Output

The multiplicative system has a further advantage over a plain linear array which may not be immediately apparent. The directional pattern of the latter represents the amplitude of the carrier signal, and reversal of polarity of the pattern signifies a reversal of carrier phase. The rectifier which must be used to extract the envelope is not affected by the phase of the carrier and so is unable to discriminate between lobes of different polarity. In the multiplicative system however, the polarity of the "d.c." signal after filtration depends on the sign of cos np and also on the signs of the outputs from the two groups of elements. The sign of cos np reverses each time its modulus passes through zero. Furthermore, the output of each group, represented by the modulus of a function such as $\left(\dfrac{\sin n_1 p}{n_1 \sin p}\right)$, suffers a phase jump of π radians at each of its zeros. It can be seen therefore that the sign of the "d.c." output will reverse at each zero of the complete function $D(p)$, whether the zero happens to be due to one of the groups or to the cos np factor. This simply means that, provided none of the zeros are coincident, the signs will alternate for successive lobes. At a double zero there will be no reversal of sign; an example of a double zero occurs at $\pi/2$ when $n_1 = 4$ and $n_2 = 1$. (See Fig. 2(a)).

This alternation of sign from lobe to lobe suggests a simple method of suppressing at least some of the unwanted subsidiary lobes of the pattern merely by arranging that the receiver unit will respond only to unidirectional voltages. A rectifier can be used if necessary to effect this.

It can easily be seen from Fig. 2 how great is the improvement in directivity obtained by suppression of negative lobes of the pattern in this way.

The removal of secondary lobes by this process may not often be of real value, however, for one of the big advantages of the multiplier—namely, that only correlated signal gives a d.c. output, uncorrelated noise giving only a.c.—is lost by the use of a rectifier, which, of course, produces d.c. from the noise.

6. Noise Factor and its significance with regard to Directional Patterns

Previous sections of this paper have considered the way in which particular patterns have been obtained in the graph of output signal versus direction. When the practical problem is to obtain the most accurate delineation of a single target or to make the most accurate measurement of its direction, then the question of the narrowness of the beam is of prime importance. When weak signals have to be detected, however, when adjacent targets have to be resolved, or when for some reason there is a strong background, then the question of discrimination against noise (and other coherent signals) is of greatest importance, and quite different factors enter into consideration.

It was shown in ref. 3 that, for additive linear arrays (i.e. ordinary arrays) of length at least several wavelengths, the "directivity factor," DF, which leads to the well-known "directivity index" of $10 \log_{10}(DF)$, is related to the noise factor, NF, thus:

$$DF = K/(NF)^2 \qquad \qquad \ldots\ldots\ldots(8)$$

where K is a numerical factor involving the length of the array. The usual definition of DF is

$$DF = \frac{[D(0)]^2}{\frac{\pi}{2}\int_{-\pi}^{+\pi} [D(\theta)]^2 \, d\theta} \qquad \text{.........(9)}$$

where $D(\theta)$ is the directional pattern expressed as a function of θ, instead of x, and $D(0)$ is its value when $\theta = 0$. Noise factor is defined as the signal/noise ratio of a plain linear array divided by that of the array being considered, both arrays being made up from the same elements, having the same overall length, and having the same received signal. Thus, for such arrays, the better the noise factor, the better is the directivity factor, which is clearly better for narrower beams and smaller secondary lobes. So, in these cases, there is no conflict between directional pattern (as interpreted by DF) and noise factor.

It was also shown in ref. 3, however, as also in more general form below, that multiplicative arrays have a poorer noise factor than the plain linear array in spite of the narrower beam in the directional pattern. The multiplier in which two halves of the array are used gives, for example, a noise factor of 3 db. The reason why noise factor and directional pattern appear to diverge as performance criteria in multiplicative arrays is that when noise is received as well as the signal, the multiplication introduces cross-products of noise and signal—i.e. the process is no longer linear.

The analysis of noise factor for uniform multiplicative arrays is given below. The assumption made is that the noise is uncorrelated between the two groups of the array. The significance of this assumption is analysed in Section 6.2, but for now it may be said that the noise, by this assumption, includes thermal-agitation noise arising in the dissipation resistance of the array elements and elsewhere and also, when the array is long compared with the wavelength, it includes noise arising in the medium if it is of uniform distribution.

6.1. *Calculation of noise factor*

Consider the effect of uncorrelated background noise on the reception of a single coherent signal. It can be shown (see Appendix) that multiplying together two coherent signal voltages, each of which is accompanied by uncorrelated noise, results in a r.m.s. signal/noise ratio R (defined as the ratio of d.c. to a.c.) where

$$R = \frac{R_1 R_2}{\sqrt{\frac{1}{2}(1 + R_1^2 + R_2^2)}} \qquad \text{.........(10)}$$

and R_1 and R_2 are respectively the r.m.s. signal/noise ratios of the two inputs.

Often R_1 and R_2 will be sufficiently large for this to be taken as

$$R \cong \frac{R_1 R_2}{\sqrt{\frac{1}{2}(R_1^2 + R_2^2)}} \qquad \text{.........(11)}$$

where $R_1^2 + R_2^2 \gg 1$.

Suppose that all the elements of the array have the same sensitivity and that the background noise is completely uncorrelated. Let R_0 denote the r.m.s. signal/noise ratio for each element. For a group of n_1 elements, the signal output will be increased n_1 times but the noise output only $\sqrt{n_1}$ times, so that the signal/noise ratio becomes $R_0\sqrt{n_1}$. Applying eqn. (11) it is seen that the resultant signal/noise ratio of the system will be R, where

$$R = R_0 \sqrt{\frac{2n_1 n_2}{n_1 + n_2}}$$

$$= R_0\sqrt{n} \sqrt{\frac{2}{\left(\frac{n}{n_1}\right) + \left(\frac{n}{n_2}\right)}} \qquad \text{.........(12)}$$

Thus the noise factor of the multiplier system, i.e. the ratio of the signal/noise ratio of the plain array to that of the new system, is

$$NF = \sqrt{\frac{1}{2}\left[\left(\frac{n}{n_1}\right) + \left(\frac{n}{n_2}\right)\right]} \qquad \text{.........(13)}$$

$$= \sqrt{1 + \frac{1}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right)}$$

Figure 4 shows the noise factor plotted as a function of the ratio n_1/n_2, assuming $R_1^2 + R_2^2 \gg 1$ as explained above. It is clear that the noise factor is at least $\sqrt{2}$ and becomes progressively greater as the ratio between the numbers of elements in the two groups is increased. As far as uncorrelated noise is concerned, the ideal condition would appear to be that in which the two groups are as nearly as possible equal.

The noise factor tends to a minimum as (n_1/n_2) approaches unity and, at the other extreme when $n_2 = 1$, it is given by

$$NF = \sqrt{1+\frac{1}{2}\left[\frac{1}{n-1}+(n-1)\right]} \quad ....(14)$$

which tends to $\sqrt{(n/2)}$ when n is large.

Thus, when $n_2 = 1$, the noise factor becomes progressively worse as the number of elements

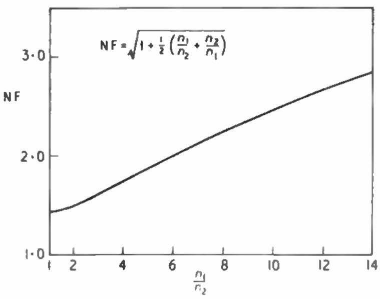

Fig. 4. Noise factor of multiplicative array as a function of the ratio (n_1/n_2), assuming $R_1^2+R_2^2 \gg 1$.

in the array is increased. (This assumes that the intrinsic signal/noise ratio of each element is kept constant.) The reason for this is clear: the signal plus noise of the single-element array is always one of the factors multiplied, and the output signal/noise ratio cannot be better than that of one element.

6.2. *Types of noise for which the Noise Factor is valid*

It is clear that the noise allowed for in the calculation of noise factor includes thermal-agitation noise arising in the dissipation resistance of the elements of the array. The calculations will also be valid in respect of noise arising in the medium if this produces uncorrelated waveforms in the outputs of the two groups of the array.

We assume for simplification that the noise originates in uncorrelated sources in a plane containing the length of the array; they are of equal mean power, spaced uniformly on the x-scale but confined to real angles and at a constant large distance from the array. Let the directional functions of the two groups of elements be $D_1(x')$ and $D_2(x')$ respectively. The centres of the groups are d_1 apart, and

$$x' = \frac{d_1}{d}p = \frac{x}{2}\,.$$

Consider first the rth noise source, and let D_{1r} and D_{2r} be the directional response of each part of the array in this particular direction. Assume the frequency spectrum of the noise sources is very narrow, so that x' may be considered the same for all frequency components; let the upper and lower limits of the frequency, ω_q, be ω_1 and ω_2. Then the rth source may be considered to give an output from one group of

$$n_1 D_{1r} \sum_{\omega_q=\omega_1}^{\omega_q=\omega_2} V_{rq} \cos(\omega_q t + \alpha_{rq} + x'_r) \quad ....(15)$$

and from the other group

$$n_2 D_{2r} \sum_{\omega_q=\omega_1}^{\omega_q=\omega_2} V_{rq} \cos(\omega_q t + \alpha_{rq} - x'_r) \quad ....(16)$$

The correlation factor between the two outputs, allowing for all the noise sources in the range of θ from $-\pi$ to $+\pi$, i.e. from $x' = -\frac{d_1\pi}{\lambda}$ to $x' = +\frac{d_1\pi}{\lambda}$, is readily shown to be

$$\rho = \frac{\sum_r [D_{1r}\, D_{2r} \cos 2x'_r]}{\sqrt{[\sum_r D_{1r}^2]\,[\sum_r D_{2r}^2]}} \quad ....(17)$$

where the summation includes all values of r; and if the sources are infinitesimal, with their number tending to infinity, then we may write

$$\rho = \frac{\int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} D_1(x')\, D_2(x') \cos 2x'\, dx'}{\sqrt{\left[\int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} [D_1(x')]^2 dx'\right]\left[\int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} [D_2(x')]^2 dx'\right]}}$$
$$.........(18)$$

We are concerned with the conditions which make $\rho = 0$, and therefore with the conditions which make the numerator zero. The easiest case to consider is when each group of the array is merely a single point receiver, having $D(x')$ uniform throughout the range of integration. Here ρ will be zero when the range comprises an integral number of cycles of $2x'$, but will in any event approach zero if $d_1/\lambda \to \infty$. Provided

$D(x')$ varies but little over each cycle of $2x'$, these conditions will still largely apply. In the other limit, when the array is continuous, and the two groups are equal, then $D(x')$ will have a beamwidth (between zeros) of only 2π on the x'-scale, but here the integral in the numerator of ρ quickly approaches zero as d_1 is increased and becomes much smaller than the denominator, since

$$D(x') = \frac{\sin x'}{x'} \qquad \text{.........(19)}$$

and $$\int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} \frac{\sin^2 x'}{(x')^2} \cos 2x' \, dx'$$

$$= 2\int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} \frac{\sin^2 2x'}{(2x')^2}\, dx' - \int_{-d_1\pi/\lambda}^{+d_1\pi/\lambda} \frac{\sin^2 x'}{(x')^2}\, dx' \qquad \text{.........(20)}$$

which can be seen to be very small if $d_1/\lambda > 1$, and approaches zero as $d_1/\lambda \to \infty$, since on substituting $y = 2x'$

$$2\int_{-\infty}^{+\infty} \frac{\sin^2 2x'}{(2x')^2}\, dx' = \int_{-\infty}^{+\infty} \frac{\sin^2 y}{y^2}\, dy = \int_{-\infty}^{+\infty} \frac{\sin^2 x'}{(x')^2}\, dx' \qquad \text{.........(21)}$$

A graph of ρ against d_1/λ is shown in Fig. 5.

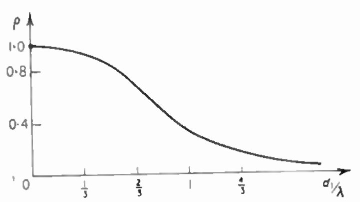

Fig. 5. Correlation of uniformly-distributed noise arising in the medium, between the two halves of a continuous array, plotted against the distance d_1 between centres.

It thus appears that the Noise Factor of a multiplicative array is a reasonable measure of its directional discriminaton against noise which is of uniform angular distribution in the medium provided the length of the array is several wavelengths. In these circumstances the main response of each half of the array (unless it is made up of point receivers) is confined to a range of angles where $\sin\theta \simeq \theta$ and so x' is proportional to θ. The assumption of uniform distribution of noise in the x'-scale as opposed to the θ-scale then causes no difficulty.

6.3. *Significance of directional patterns*

In Sect. 2 it was shown that the use of a multiplicative system leads to a considerable improvement of the directional pattern as compared with that for a corresponding simple array with the same dimensions. It has already been pointed out however, that although the directional pattern and its associated directivity factor give a practical measure of the directional properties of an additive array, this is not necessarily the case when a multiplicative process is used.

Consider, for example, the effect of a coherent interfering signal which may be assumed to originate in a distant point source on a bearing differing from that of the wanted signal source, which is assumed to be on the normal axis. Suppose that the signal/interference ratio in the medium, at the array, is $1/s$ and that the ordinates (corresponding to the bearing of the unwanted signal) of the directional patterns of the two groups of elements are D_1 and D_2 respectively. The outputs from the two groups will be proportional to

$$\frac{n_1}{n}\left[\cos \omega t + sD_1 \cos(\omega_1 t + \phi_1)\right] +$$
$$\text{and } \frac{n_2}{n}\left[\cos \omega t + sD_2 \cos(\omega_1 t + \phi_2)\right] \qquad \text{.........(22)}$$

where ω_1 is the frequency of the interfering signal and ϕ_1 and ϕ_2 represent the phase differences, in the medium, between the unwanted and wanted signals at the centres of the respective groups.

After multiplication and filtration, the output will be proportional to

$$1 + s^2 D_1 D_2 \cos(\phi_1 - \phi_2) +$$
$$+ sD_1 \cos[(\omega_1 - \omega)t + \phi_1] + sD_2 \cos[(\omega_1 - \omega)t + \phi_2] \qquad \text{.........(23)}$$

In this expression the first two terms represent the anticipated "d.c." signal and interference outputs respectively, the ratio between them being given by

$$\begin{aligned} & s^2 D_1 D_2 \cos(\phi_1 - \phi_2) \\ &= s^2 D_1 D_2 \cos pq \\ &= s^2 D_m \end{aligned} \qquad \text{.........(24)}$$

where D_m is the appropriate ordinate of the directional function of the multiplicative array.

It is evident therefore that the true signal/interference ratio of the system, for the type of interference under consideration, cannot be derived from the directional pattern unless the effect of the third and fourth terms in the above expression (i.e. the effect of intermodulation between wanted and unwanted signals) can be made negligible. Provided the "difference" frequency ($\omega_1 - \omega$) is above the highest modulation frequency of the signal, this can be achieved quite simply by filtration. The worst case will occur when the wanted and unwanted signals are at exactly the same frequency, the output then being proportional to

$$1 + sD_1 \cos \varnothing_1 + sD_2 \cos \varnothing_2 + s^2 D_1 D_2 \cos pq \qquad \text{.........(25)}$$

It is convenient here to introduce what will be termed the "Rejection Factor," defined as the ratio between the output signal/interference ratios for the new array and the original simple array, respectively. The rejection factor will, of course, be a function of p and may be expressed in the form

$$RF = \frac{D_0}{D_1 \cos \varnothing_1 + D_2 \cos \varnothing_2 + sD_m} \qquad \text{.......(26)}$$

where D_0 and D_m are the ordinates, expressed as functions of p, of the normalized directional patterns of the simple and multiplicative arrays, respectively.

The values of $\varnothing_1$ and $\varnothing_2$ are unknown even when $\omega = \omega_1$ and they will effectively vary with time if ω and ω_1 differ slightly; but the minimum value which the rejection factor can have for any particular bearing angle is given by

$$RF_{(min)} = \frac{|D_0|}{|D_1| + |D_2| + s|D_m|} \qquad \text{.........(27)}$$

It can be seen from this expression that except for values of p for which D_1 and D_2 vanish simultaneously, the minimum rejection factor will be given approximately, for small values of s, by

$$RF_{(min)}\ [s \ll 1] \cong \frac{|D_0|}{|D_1| + |D_2|} \qquad \text{........(28)}$$

It is obviously desirable to keep this minimum value of RF as large as possible and it is therefore of interest to consider how it is likely to depend on the relative numbers of elements in the two groups of the multiplicative system. The two extreme cases will be considered represented respectively by $n_2 = 1$ and $n_1 = n_2 = n/2$.

In the former case, the quantity under discussion will be given by

$$\frac{|D_0|}{|D_1| + 1}$$

which will tend to

$$\frac{|D_0|}{|D_0| + 1} = \frac{1}{1 + \left|\dfrac{n \sin p}{\sin np}\right|} \qquad \text{.........(29)}$$

when n is large.

In the second case,

$$\frac{|D_0|}{|D_1| + |D_2|} = \frac{1}{2}\frac{|D_0|}{|D_1|}$$

$$= \frac{1}{2}\left|\frac{\sin np}{n \sin p}\right| \Bigg/ \left|\frac{\sin \frac{n}{2}p}{\frac{n}{2}\sin p}\right|$$

$$= \frac{1}{2}\left|\cos \frac{n}{2} p\right| \qquad \text{.........(30)}$$

The general forms of (29) and (30), plotted as functions of p, are shown in Fig. 6 (calculated in this case for $n = 9$). This leads to the conclusion that, as far as rejection of a low-level interfering signal on a bearing adjacent to the main lobe of the directional pattern of the multiplicative array is concerned, the ratio of n_1 to n_2 does not appear to be important, the minimum rejection factor then being approximately $\frac{1}{2}$. If the bearing of the interfering signal is well outside the main lobe however, the best results will be obtained when the array is divided into two equal groups.

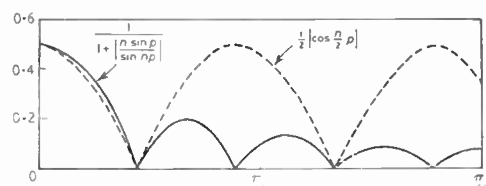

Fig. 6. Curves relating to the variation of the minimum rejection factor of a multiplicative array with the bearing of the interfering signal (see sect. 6.3).

Suppose now that the interfering signal is at a relatively high level so that, from eqn. (27), the minimum rejection factor tends to

$$\frac{|D_0|}{s|D_m|} \qquad \text{.........(31)}$$

except at values of p for which $|D_m|$ approaches zero. (At these points, $s|D_m|$ can be neglected and the conditions will be the same as those discussed above for small values of s.)

It will be noted that the minimum rejection factor is now approximately proportional to

$$\frac{|D_0|}{|D_m|} \qquad \text{.........(32)}$$

as would have been expected from the directional patterns, but that it is also proportional to $1/s$. This is due of course to the fact that the interfering signal has an effect on the multiplicative system which is proportional to the *square* of its amplitude and not directly proportional as for the plain array. Thus, when s exceeds unity, any apparent advantage due to the narrower beam width of the multiplicative directional pattern is likely to be outweighed by the "square-law" behaviour of the system with respect to the level of the interfering signal.

7. Comparison of Multiplicative and Superdirective Arrays

Although a multiplicative array is "superdirective" in the sense that it has a directional pattern with a narrower main beam than that of the plain uniform linear array, this term is generally reserved for linear additive arrays in which the sensitivity is not uniform over the array but follows an oscillatory "taper function." A superdirective array obtains its improved directional pattern at the expense of substantial response in the range of complex angles, i.e. in the range of x corresponding to $|\sin \theta| > 1$. The consequence of this is that the superdirective array can be designed only after its length has been determined, since the range of x corresponding to real values of θ is dependent on this factor. In the multiplicative array, on the other hand, the narrowing of the main beam is obtained without any substantial response in angular ranges remote from the main beam; consequently there is no restriction on the length and relative angular scale of such an array. It is clear immediately that, while superdirective arrays are not practicable for beam scanning applications[8], on the other hand, multiplicative arrays are suitable, at least in principle; an example of how they may be applied to an echo-ranging system with electronic sector-scanning is shown in Appendix 3.

In comparing the signal/noise properties of the two types of array, it must be remembered that the superdirective array is an additive type so that its ability to reject interfering signals and noise arising in the medium is given directly by its directional pattern and its rejection of uncorrelated background noise, i.e. mainly noise arising in the array itself, is characterized by its noise factor. For the multiplicative array however, it has been shown that the directional pattern may be misleading as far as single-signal rejection is concerned; and the noise factor, although giving an approximate measure of the rejection of background noise can be applied only to arrays which are several wavelengths long, when the noise concerned arises in the medium.

The signal/noise problem is evidently very complex but, nevertheless, some useful general conclusions can be reached. For example, it is clear that the multiplicative system offers no advantages if it is necessary to discriminate between a wanted signal and one or more coherent interfering signals, or noise, at the same or nearly the same frequency but approaching the array on different bearings. For background noise arising in the array itself however, where the noise factors can be used as a basis for comparison, it will almost certainly be found that the multiplicative array will have a better noise factor than a superdirective arrangement giving the same beamwidth.

This is illustrated in Fig. 7, where the directional response of a multiplicative array, in which the outputs from two equal portions of a continuous array are multiplied together, is compared with the superdirective response of very similar characteristics which was given and discussed in a previous paper.[7] Since the superdirective array has a large response for $|x|$ greater than π, it has to be assumed that its physical length must not exceed one wavelength. No such restriction attaches to the multiplicative array. The noise factor of the latter is only 3 db (for high input signal/noise ratios) whereas that of the superdirective array is 20·5 db.

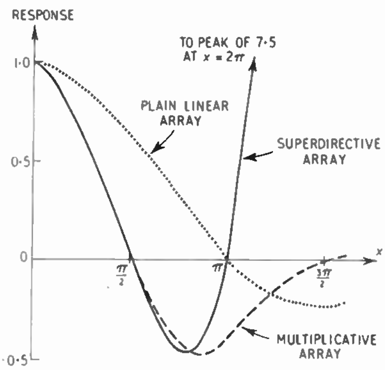

Fig. 7. Comparison of directional patterns of multiplicative and super-directive arrays.

This is of course only a particular example and must not be regarded as a general case. The improvement of superdirective patterns can in principle be taken to much greater lengths than can that of multiplicative patterns. It must, however, not be overlooked that, as pointed out in previous papers[5, 7], the price paid for superdirectivity in terms of practical difficulty is very great.

8. Conclusions

A general analysis of the properties of multiplicative receiving arrays, with coherent-tone signals, has been carried out, and two methods of synthesis of desired directional patterns have been described. The discrimination of such arrays against interfering coherent-tones and against narrow-band noise which is uncorrelated between the two portions of the array has been examined and the results compared with those given by linear additive arrays. The main conclusions are

(i) the directional patterns of linear arrays (non-superdirective), of multiplicative arrays, and of superdirective linear arrays always indicate the variation of output signal strength (received from a remote point source) as the array is rotated, assuming background interference is negligible.

(ii) the well-established Directivity Factor, based on the directional pattern, measures the signal/noise performance of all linear arrays (whether superdirective or not) as far as uniformly distributed narrow-band noise, arising in the medium, is concerned. It is meaningless for multiplicative arrays. Discrimination against interfering coherent signals of similar frequency to the wanted signal is similarly determined by the directional pattern for all linear arrays, but not for multiplicative arrays.

(iii) the more recently-introduced Noise Factor, based on a consideration of signal/noise ratio on each portion of the array, measures the performance of all arrays in respect to noise arising in the dissipation resistance of the array itself; and it also measures the signal/noise discrimination of multiplicative and linear (non-superdirective) arrays in respect to noise arising in the medium, providing the arrays are at least several wavelengths long.

(iv) For linear (non-superdirective) arrays of at least several wavelengths, the directional pattern, the Directivity Factor, and the Noise Factor are all precisely related, and all features of performance can therefore be related to the directional pattern. For superdirective arrays, angular variation of signal and discrimination against interference arising in the medium are both defined by the directional pattern, but for noise arising in the array itself the Noise Factor must be used.

For multiplicative arrays, only angular variation of signal is defined by the directional pattern. Discrimination against other coherent signals arising in the medium requires special analysis, and discrimination against uniformly-distributed noise arising in the medium is measured by the Noise Factor if the array is at least several wavelengths long. Discrimination against noise arising in the array itself is measured by the Noise Factor whatever the length of the array.

(v) The multiplicative array gives a directional pattern with a main beam of half the width of that of an ordinary linear array with uniform and equi-phase sensitivity. A similar reduction of beamwidth can also be given by a superdirective array but with poorer noise factor although with better discrimination against interfering signals and noise arising in the medium.

9. References

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10. Appendix 1: Effect of multiplication on signal/uncorrelated-noise ratio

The method and symbols used are exactly the same as those used by Tucker, (reference 2; Appendix 6.1), except that the signal/noise ratios of the two inputs will now be assumed to be unequal. It is evident that the r.m.s. noise output will be given by

$$\frac{1}{\sqrt{2}} \cdot \frac{V_1^2}{2} [nx^2 + ny^2 + n^2x^2y^2]^{\frac{1}{2}}$$

instead of $\frac{1}{\sqrt{2}} \cdot \frac{V_1^2}{2} [2nx^2 + n^2x^4]^{\frac{1}{2}}$

The input signal/noise ratios R_1 and R_2 are given by

$$R_1 = \sqrt{nx^2} \text{ and } R_2 = \sqrt{ny^2}$$

and the output signal/noise ratio R will thus be

$$R = \sqrt{nx^2} \cdot \left[\tfrac{1}{2}(1 + \frac{y^2}{x^2} + ny^2) \right]^{-\frac{1}{2}}$$

$$= R_1 \left[\tfrac{1}{2}(1 + \frac{R_1^2}{R_2^2} + \frac{1}{R_2^2}) \right]^{-\frac{1}{2}}$$

$$= \frac{R_1 R_2}{\sqrt{\tfrac{1}{2}(1 + R_1^2 + R_2^2})} \qquad \ldots\ldots\ldots(33)$$

11. Appendix 2: Theory of non-uniform multiplicative arrays

The proof of the theorem stated in Sect. 2 of the paper follows from the fact that the modulus of the output of an n-element additive array is proportional to that of the following function[4]

$$1 + \gamma_1 e^{2jp} + \gamma_2 e^{4jp} + \ . \ . \ . \gamma_{n-1} e^{2j(n-1)p} \qquad \ldots\ldots\ldots(34)$$

This is a polynomial in the variable e^{2jp} with complex coefficients γ_1, γ_2, etc. Since it is of the $(n-1)$th degree, it will have $(n-1)$ zeros and, apart from a numerical factor, its modulus will be defined completely by the values of e^{2jp} at which these zeros occur; i.e. by $(n-1)$ values of p, within each half-period of the function, for which the latter is zero.

In the perfectly general case, where an array has a random distribution of sensitivities among its elements, some or all of the $(n-1)$ zeros may occur for complex values of p. (A complex value of p cannot correspond to any physical value of θ and is, of course, simply a mathematical abstraction introduced for the purposes of analysis).

In a directional system, where the problem is to obtain a pattern with a narrow major lobe and small minor lobes, it is usually found that the best results are obtained when the pattern is symmetrical about $p = 0$ and when the largest possible number of non-coincident zeros occur for real values of p; i.e. when $D(p)$ is a real function whose graph oscillates about the p-axis, cutting it at $(n-1)$ points in the interval between $p = 0$ and $p = \pi$.

Before proceeding to a study of the effect of multiplication it will be convenient to normalize the directional function of each group by expressing its output in terms of the output of a hypothetical element, of unit sensitivity, located at the geometric centre of the group and then adjusting the magnitude of the resulting function so that its modulus is unity when $p = 0$.

In general, the total output from the group will not be in phase with that from the reference element so that $D(p)$ will be complex.

$$D(p) = |D(p)| \underline{/ \emptyset(p)} \qquad \ldots\ldots\ldots(35)$$

Equation (6) in Sect. 2 of the paper will now have to be replaced by

$$|D_m(p)| = |D_1(p)| \, . \, |D_2(p)| \, . \cos\,[np + \emptyset_1(p) - \emptyset_2(p)] \qquad \ldots\ldots\ldots(36)$$

where $D_1(p)$ and $\emptyset_1(p)$ refer to one group and $D_2(p)$ and $\emptyset_2(p)$ to the other.

The $(n-2)$ zeros produced by the $D_1(p)$ and $D_2(p)$ terms may be chosen arbitrarily and may even occur for complex values of p. Furthermore, this may be done without specifying the way in which the array is to be sub-divided into groups; in other words, there will be as many choices of distributions of element values for a given arrangement of the $(n-2)$ zeros as there are choices of grouping (for non-overlapping groups). Each of these choices corresponds to a different arrangement of the zeros produced by the term $\cos[np+\emptyset_1(p)-\emptyset_2(p)]$. It is finally concluded therefore that $(n-2)$ zeros may be chosen arbitrarily and there is then a further choice of $n/2$ (for n even) or $(n-1)/2$ (for n odd) predetermined ways in which the remaining zeros can be placed.

It must be emphasized that the multiplication principle does not provide as many additional degrees of freedom for the directional pattern as might appear from the number of zeros in its repetition interval. What it does do is to produce patterns which have more zeros than that of the additive array and which can be made to have generally narrower major lobes.

It can be shown that, if all the zeros of the pattern of an additive array are to occur for real values of p and, furthermore, if the pattern is to be symmetrical with respect to $p=0$, then $D(p)$ must necessarily be real. Under these conditions $\emptyset_1(p)$ and $\emptyset_2(p)$ are both zero and

$$|D_m(p)| = D_1(p) \,.\, D_2(p) \cos np \quad \text{.........(37)}$$

In this case the $(n-2)$ zeros produced by the groups themselves can be chosen arbitrarily, provided their positions are symmetrically disposed with respect to $p=0$, but the n zeros produced by the cosine term are fixed, irrespective of the grouping.

It is interesting to note that a chosen arrangement of the $(n-2)$ zeros may be realized in several different ways, depending on which particular zeros are allocated to each group. Since the noise factor of each group depends on the relative values of the sensitivities of the elements within the group, and since the noise factor of the whole system depends on the values of (n_1/n) and (n_2/n), (see Sect. 6.1 of the paper), it can be seen that a non-uniform multiplicative array does not necessarily have a unique value of noise factor associated with a given directional pattern. This is a point which would have to be taken into account when choosing the optimum design to fulfil given requirements.

12. Appendix 3: Application of the multiplicative principle to an acoustic underwater echo-ranging system with electronic sector-scanning

The multiplicative principle can be applied without difficulty to the method of electronic sector scanning in an echo-ranging system which was described in a recent paper in *J. Brit.I.R.E.*[8] It was explained in Section 3 of the present paper that deflection of the multiplicative beam requires the use of a delay line for each half of the array together with a phase-shifter in one input to the multiplier. In order to obtain

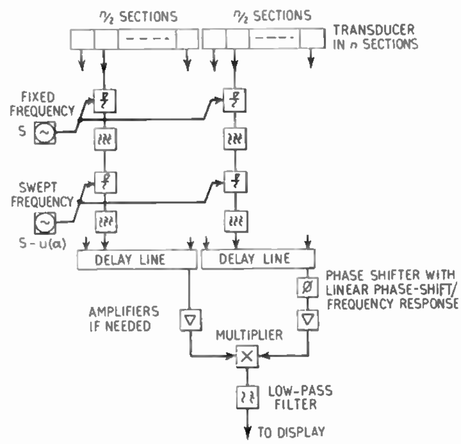

Fig. 8. Echo-ranging system with multiplicative receiver and electronic sector scanning. (Compare with Fig. 9 of reference 8.)

sector-scanning, rapid beam swinging is produced by sweeping the frequency of the signal applied to the delay lines and phase-shifter, which have characteristics such that different deflection angles are obtained at different frequencies. The block schematic of a multiplicative version of the sector-scanning system described in Fig. 9 of the previous paper is shown in Fig. 8. For a given length of array, this version will enable the angular accuracy of location of a target to be doubled at the expense of worse discrimination against unwanted signals and noise, and correspondingly reduced range of initial detection.

Radio Engineering Overseas . . .

The following abstracts are taken from European and Commonwealth journals received in the Library of the Institution. Members who wish to borrow any of these journals should apply to the Librarian, stating full bibliographical details, i.e. title, author, journal and date, of the paper required. All papers are in the language of the country of origin of the journal unless otherwise stated. The Institution regrets that translations cannot be supplied.

PRODUCTION OF MILITARY EQUIPMENT

For armed service equipment manufacture, the customer has a say in planning and production procedures as well as performance of the complete equipment. Specifications are written by the Service Departments to cover their precise requirements. These specifications will cover: (i) Security. (ii) Quality of materials used. (iii) Types and supply of components. (iv) Inspection prior, during, and after manufacture. (v) Particular production processes. (vi) Performance testing. (vii) Packaging. (viii) Drawings and equipment manuals. Industrial production systems are organized to suit various manufacturers and/or products. The systems very rarely need specifications to cover all the comprehensive requirements as listed above and therefore require changes to meet these needs. Service requirements follow a regular pattern and a recent Australian paper aims to provide a knowledge of the salient parts of this pattern. Emphasis has been given to the particular problems, which, from the author's experience, cause the most trouble in achieving specification limits for service electronic equipment.

"The production of Service electronic equipment." H. I. Millar, *Proceedings of the Institution of Radio Engineers, Australia*, **20**, pp. 185-190, April 1959.

TRANSISTORS AND "PULSACTORS"

In a paper from C.S.I.R.O. in Australia, a regenerative circuit is described which uses a power transistor to control the charging of a pulse-forming network through a transformer having a sharply saturating core. The transformer is designed to saturate upon completion of the charging interval, whereupon the stored energy is switched to a load in the form of a steep-fronted pulse. The peak pulse power can be much higher than the peak transistor power, and the pulse rise time is limited only by the core properties and not by the transistor "turn-on" time. One or more additional transfer stages may also be used to compress the pulse duration and increase its peak power still further.

"Pulse modulators using transistors and switching reactors." B. F. C. Cooper and W. J. Payten. *Proceedings of the Institution of Radio Engineers, Australia*, **20**, pp. 148-152, March 1959.

FREQUENCY MODULATED OSCILLATORS

A German paper deals with the problem of distortion-free frequency modulation of oscillators for any desired modulating frequencies. A tuned circuit is assumed whose oscillations are excited by way of a non-linear characteristic, and the system may also degenerate into one producing relaxation oscillations. The conditions for such an ideal frequency modulator can be derived in a simple and general manner. It is found that every system of modulation that allows a tuned circuit to be modulated in its natural frequency without distortion and incidental amplitude modulation remains an ideal frequency modulator also for the non-linear case of the excited circuit. With the tuned-circuit oscillator the simultaneous control of both elements of the tuned circuit is here required. For the relaxation oscillator which is reduced to its three basic functions, namely feedback, limitation, and integration, there results one controllable element. Only with exact integration is the relaxation oscillator an ideal frequency modulator; in the case of an integration approximated by finite time constants it will contain lock-in ranges.

"On the frequency modulation of tuned circuit and relaxation oscillators." Ernst Kettel. *Archiv der Elektrischen Ubertragung*, **13**, pp. 95-100, March 1959.

MICROWAVE AERIAL MEASUREMENTS

A paper from the National Research Council of Canada describes the use of an anechoic aerial as a coupling device for the comparative measurement of the combined loss due to internal reflections and attenuation in microwave aerial systems. Being nearly reflectionless, it can be placed very close to the aperture of the aerial under test without the usual effects of mutual reflections. An outline of the general requirements and the performance of some practical broadband designs for low power applications is given. Comparative measurements can be made with an estimated accuracy of $\pm\frac{1}{2}$ db if reasonable precautions are taken.

"An anechoic antenna for comparison of losses in microwave antenna systems." K. A. Steele. *Transactions of the Engineering Institute of Canada*, **3**, No. 1, pp. 9-12, April 1959.

PROPERTIES OF FERRITES

Investigations have been carried out in Poland on the production of ferrite materials having small temperature coefficients of initial permeability. On the basis of compiled literature data determining the relation between magnetic susceptibility and magnetization intensity, anisotropic constant, magneto-striction, internal stress, etc., it was decided to investigate the influence of the following factors on temperature phenomena pertaining to initial permeability:

(a) changes of relative concentration of basic components, including oxygen,
(b) introduction of impurities,
(c) crystallite growth in sintered ferrite.

In addition to measuring samples in temperature range between 20 and 350°C, samples having various O_2 contents have been prepared, this latter being analysed using methods described by Gorter. The form of the curve $\mu_p = f(T)$ depends also upon the oxygen content in the ferrite material. The influence of alkaline earth cations on the temperature curve of permeability for ferrite

$$(MeO)_x . \; (Mn_{0 \cdot 6}Zn_{0 \cdot 4} \; Fe_2O_{4 \pm \gamma})_{1-x}$$

is clearly visible for $x = 0 - 0{\cdot}02$.

It is shown that by introducing potassium ions the lowering of temperature coefficients of initial permeability and the increasing of T_c were possible without disturbing other properties of the samples when proper sintering and cooling process takes place. The slope of curve $\mu_p = f(T)$ grows with the increase of sintered ferrite crystallites. This enables the increasing of TK_μ or changing of its value from a negative into a positive one. This also shows that the influence of various factors on the temperature dependence of permeability of ferrites cannot be considered disregarding conditions of a synthesis.

"Temperature coefficients of permeability of Mn-Zn ferrites," A. Braginski, J. Kulikowski and S. Makolagwa. *Prace Instytutu Tele- I Radiotechnicznego (Warsaw)*, **3**, pp. 38-40, 1959.

TELEVISION IN FINLAND

Finland's television service, which operates under the General Broadcasting Company of Finland, commenced its official transmissions on January 1st, 1958. Prior to this time, technical experimental transmissions had been broadcast for about a year. Up to the spring of 1959 the transmissions have been sent out from the Broadcasting Company's former station building at Pasila, where two studios of about 120 m² aggregate floor area could be arranged. The studio equipment has been supplied almost in its entirety by a German firm. For normal transmissions, three image orthicon cameras are available, and also one vidicon camera for stills. There are two 16 mm and two 35 mm projectors for film transmission, each one provided with its own vidicon camera. Programs from outside the studio are picked up by an outside broadcast vehicle provided with three image orthicon cameras and the requisite sound equipment. From the studio as well as from the O.B. van, the program is relayed to the transmitter by means of microwave links operating in the 7000 Mc/s band.

"The studio procedures of Finland's television." P. Pesari. *Teknillinen Aikakauslehti*, **9**, pp. 247-254, May 1959.

A.F. AMPLIFIERS

When an amplifier is overloaded, energy can be stored in reactive coupling elements; in a resistance-capacitance coupled amplifier, charge can build up in the coupling capacitors. An Australian paper shows that after the overload is removed, the amplifier will be incorrectly operated, and can thus continue to produce distortion for an appreciable time until the stored energy is dissipated. The effect is aggravated by negative feedback, but can be reduced by suitable choice of coupling reactance and grid "stopper" resistance. Tables are included which give some quantitative estimate of the effect. Several other aspects of design against overload are considered.

"Recovery of amplifiers after overload." A. N. Thiele. *Proceedings of the Institution of Radio Engineers, Australia*, **20**, pp. 257-261, May 1959.

SCINTILLATION COUNTERS

The ratio between the height of scintillation pulses corresponding to a certain absorbed energy and the discriminator level at which no more than, e.g., one noise pulse per second is counted, the "signal-to-noise ratio," depends on the decay constant of the scintillation effect (τ_0), on the time constant of the anode circuit of the photomultiplier tube (τ_1) and on the frequency response of the amplifier. If the latter can be approximately represented by only one time constant (τ_2), it is possible to solve, at least numerically, the problem of what combination yields the best signal-to-noise ratio.

The time constants also determine the pulse duration, but, for a given τ_0, various combinations of τ_1, τ_2, can yield the same pulse duration. The best signal-to-noise ratio is obtained when $\tau_1 = \tau_2$. The numerical value is set by τ_0 and the required pulse duration.

"Signal-to-noise ratio and dead time of a scintillation counter." J. A. W. van der Does de Bye. *Philips Technical Review*, **20**, pp. 263-268, No. 9, March 1959.