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MATHEMATICS FOR THE ENGINEER

Great responsibility is incurred in setting any form of examination, especially when the study and training involved is a direct means of qualifying for advancement in a profession. The demand for such training also greatly influences educational policy in technical colleges and other training establishments.

There cannot, however, be any disagreement with the decision of the Council to include in the Graduateship Examination a three-hour paper in mathematics as a compulsory subject. A sound knowledge of mathematics is essential to any engineer, but more especially to the radio engineer. For this reason, the questions which have hitherto been set in the Graduateship Examination have required an understanding of mathematics up to at least Intermediate Degree standard. Records kept over the past few years have, however, made it abundantly clear that a great percentage of failures in the examination is attributable to lack of mathematical ability.

Normally, younger candidates enter for the complete examination ; a candidate who enters for only part of the examination, however, will not be permitted to take Part 4, the optional subject, unless he has previously passed or been exempted from both the Advanced Radio Engineering and Mathematics sections.

The syllabus will include Algebra, Trigonometry and Calculus to a standard comparable with that of the Pure Mathematics paper in the Intermediate B.Sc. examination. It must be stressed here that this is the minimum standard which the Council considers necessary and its scope may be widened at a later date.

Specimen examination papers are now available and the syllabus, with a revised list of exempting qualifications, is published in the 16th edition of the Regulations.

In preparing the syllabus, the Education and Examinations Committee has at all times considered the availability of instruction in the subject. In all technical colleges offering courses for the Institution's and similar examinations, mathematics of the required standard forms part of the normal course and candidates should not have any difficulty on this score.

This addition to the examination syllabus takes effect as and from the May 1951 Graduateship Examination and is a further step towards implementing the recommendations made in the report on Education and Training.* Members will be aware that this Report contained the main evidence of the Institution in favour of at least a Higher National Certificate in Radio Engineering. These proposals led to a meeting between representatives of the Institution, the Ministry of Education and other bodies ; whilst there was not total acceptance of the Institution's recommendations it was agreed that some subjects, including mathematics, are common to a number of Higher National Certificate courses, as well as to the City and Guilds of London Institute Examinations in Telecommunications Engineering.

The small number of students who take a Higher National Certificate with a Radio endorsement as at present constituted is in itself significant, but whatever the final outcome of official and industrial enquiries, there is no doubt that the standard of mathematical knowledge recommended by the Institution will be agreed as a minimum requirement.

E. E. Z.

* Post-war Development Report, Pt. II, Vol. 4, *J.Brit.I.R.E.*, 1944.

REPRESENTATIVES TO COUNCIL

Harry Gibson Henderson was born in Airdrie, Lanarkshire, in December 1903. He received his technical education at Glasgow University and served his apprenticeship at Metropolitan-Vickers Electrical Co., Ltd., Manchester.

In 1929 he was appointed to the staff of the Western Electric Co., Ltd., as an installation engineer of talking-picture equipment and subsequently became senior engineer. To-day he is the district supervisor for installation and maintenance of Western Electric equipment in Scotland.

Mr. Henderson was elected an Associate in 1935, transferring to Associate Membership in December, 1937, and to full Membership in June, 1942. He was the first Honorary Local Secretary of the Scottish Section and was instrumental in the formation of this Section in September 1945. In 1947 he was elected its Chairman and the success of the Scottish Section is largely due to his efforts.

Oswald Francis Mingay was born at Peak Hill, N.S.W., Australia, in July, 1895. He joined the Australian Post Office in 1911, and studied at Sydney Technical College. In the first world war he served with the A.I.F. in Egypt and in France with the Australian Engineers (Signals). He returned to the Post Office in 1919, but resigned in 1922 to enter the wireless industry

Mr. Mingay served as chief engineer and manager to several manufacturers, and in 1930 he founded Australian Radio Publications of which he is still the managing editor.

Elected a full member of the Institution in February, 1945, Mr. Mingay has recently been attending Council meetings as a representative of the members in Australia. He was Honorary General Secretary of the Australian Institution of Radio Engineers from 1932-1940, and has been an elected member of the Council since 1932. In 1946, Mr. Mingay was elected Vice-President and Deputy-President in 1947. During the last war Mr. Mingay served as a captain in the Australian Corps of Signals and, in May, 1948, was seconded to the Ministry of Munitions as radio production manager. In 1943 he was sent to the U.S.A. on radar and radio matters for the Ministry.

He is at present on a world tour studying radio manufacturing and research technique. Whilst in England he has addressed many trade and industrial meetings. Mr. Mingay leaves England for the U.S.A. in April next.

Ralph Aubrey Spears was born in Derby on November 2nd, 1911. He received his technical education at Derby Technical College, where he pursued a course of study in physics.

After general radio engineering experience in Derby, Mr. Spears joined the Automatic Telephone and Electric Co., Ltd., Liverpool, as a development laboratory engineer. He is now in charge of the development of quartz crystal design and radio frequency production testing equipment. He was recently a member of a delegation to America.

Elected an Associate Member of the Institution in December 1945, Mr. Spears actively assisted the North-Western Section Committee and was prominent in the petition to Council to form the Merseyside Section. He was elected its Chairman at the inaugural meeting in 1947 and has been re-elected each year.

Mr. Spears was the author of a paper on "Gold Film Electrodes for High Frequency Plates" which was published in the March-May 1946 issue of the Institution's Journal.

ELECTRONICS IN AIRCRAFT DESIGN*

by

A. L. Whitwell† (*Graduate*)

(*A paper read before the West Midlands Section on May* 25*th*, 1949, *the Scottish Section on October* 6*th*, 1949, *the London Section on December* 15*th*, 1949, *the South Midlands Section on November* 24*th*, 1949, *and the Merseyside Section on March* 1*st*, 1950.)

SUMMARY

The paper surveys the application of electronics as an aid to the measurement of factors influencing the design and performance of aircraft structures. Methods which may be applied to the measurement of strain, displacement, pressure, etc., are described and some comparison is made of the various possibilities. A number of individual recording equipments for use in vibration measurements and in flight work for both static and dynamic measurements are also described.

1. Introduction

1.1. The purpose of this paper is to review some of the applications of electrical and thermionic methods as an aid in the investigation and measurement of many of the details of performance which may be deduced from a proper study of the individual characteristics of aircraft structures, either statically by ground tests, or whilst actually in flight ; and in particular to examine their application in the development stages of aircraft design. The range of application is very wide, and it will not be possible in a paper of this nature to deal more than briefly with the many varieties of measurement and recording devices, but it is hoped that the material presented will lead to an awareness of the type of problem involved and the apparently wide scope for the increased use of thermionic methods in this field. It should also be realized that, while some of the requirements are peculiar to the aircraft industry, the majority are of equal application in the many other branches of structural engineering, and in some degree are already in use.

1.2. It is not necessary to look far in order to find the reasons underlying the rapid growth and search for new methods of making accurate measurements of physical quantities so far as aircraft are concerned. The introduction of jet propulsion and rocket techniques with the consequent practicability of supersonic aircraft speeds brings an entirely new range of problems before the designer, necessitating the application of comparatively untried theories which often require substantiation by measurement in order to consolidate and develop present knowledge in this field. A further extremely important factor which also requires more accurate and extensive methods of measurement than existed hitherto is a development which runs parallel, and is complementary to, the efforts to attain high speeds and superior performance. This is the strength and safety factor of aircraft structures in so far as they are a function of weight. Though there are many exceptions, it is true to say in a very general way that strength and weight go together. In the same manner, for a given engine power, performance is largely dependent on weight. The efforts of designers are therefore always partly directed to the problem of reducing the weight of material carried in an aircraft to a minimum while retaining a reasonable factor of safety. This factor should be adequate to meet all extreme conditions with a further reasonable allowance for perfect safety, but modern conditions demand that unreasonable and unnecessary degrees of safety should be avoided.

1.3. The chief advantage of electrical recording systems, of course, lies in the fact that all types of measurement may be made from remote and inaccessible positions with a minimum of complications. Whenever it is required to record extremely small displacements, or (what usually amounts to the same thing) to record the presence of force without undue

* Manuscript received 16th May, 1949.
† Boulton Paul Aircraft, Ltd.
U.D.C. No. 621.318.572:629.13.

mechanical loading of the " work," the pick-up units are invariably associated with electronic amplifying channels, and display is often achieved electronically.

At the present time a good deal of research work is being carried out on rocket- and jet-propelled aircraft exclusively by electronic methods. In this way aircraft may be subjected to endurance tests which no human being can withstand. Television cameras may be used to monitor instrument panels continuously while flying controls may be remotely operated either from the ground or from a parent aircraft. There is no space at present to investigate all these aspects, but they should be borne in mind.

1.4. In the following sections, firstly some of the most common measurement requirements are briefly reviewed ; next a number of typical instruments and devices used to translate the physical quantity concerned into an electrical equivalent are examined. Section 4 deals with some specific items of recording equipment, and the final Section includes a number of useful references.

2. Common Measurement Requirements

2.1. The type of measurements which are commonly required fall into two distinct classes. These are : (1) Laboratory or ground tests ; and (2) Airborne measurements. In general, similar methods are in use for both classes of measurement, but the problem of recording and the design of suitable apparatus is naturally more difficult in the airborne case ; it is always necessary to design to a minimum volume and weight and to maintain power consumption at a low value. Conflicting with these factors are the more severe problems of vibration and interference met with in an aircraft in flight.

2.2. Among the various physical quantities which commonly have to be recorded are the following :—

(*a*) Strains

(*b*) Displacements

(*c*) Forces

(*d*) Velocities

(*e*) Accelerations

(*f*) Pressures (liquid and gaseous).

The various devices which may be used for the above purposes are generally most useful when they are capable of directly translating the force or other physical quantity which is measured into an electrical equivalent. The name " pick-up " is loosely applied to indicate a device of this nature. In the following section a number of such tools is described.

2.3. The measurements to be described seldom call for extreme accuracy, so that tolerances of about 5 per cent are often permissible. This is particularly convenient when multiple measurements have to be made, as many standard pick-up devices can be relied on to have individual characteristics within this range, and the use of photographic means of recording from cathode ray tube or galvanometer presentations is then convenient and sufficiently accurate.

3. " Pick-up " Devices and Instruments

3.1. All pick-up devices should preferably be capable of consistent operation in extremes of temperature and humidity if they are to be of practical use for aircraft flight measurements. In the interests of safety to aircraft and personnel they should be operated from a low power source and incapable of causing sparks or generating dangerous heat under any conditions of misuse whatsoever. It is also often worth while to ensure that the design is such that the pick-ups may be connected to the recording apparatus in a Wheatstone bridge arrangement. This is generally the most sensitive manner of

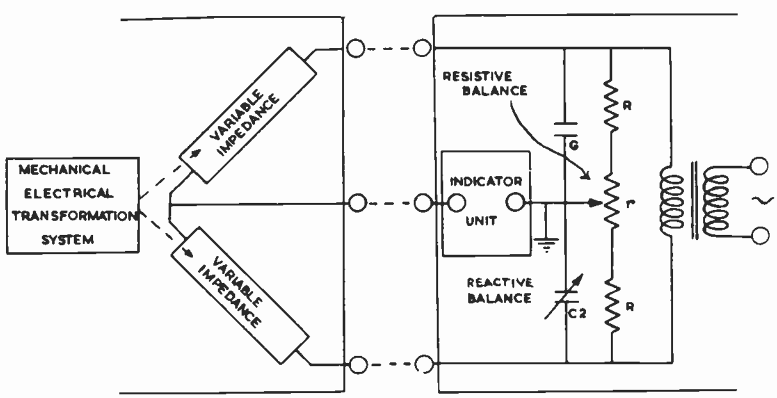

Fig. 1.—Variable impedance bridge.

operating a given type of pick-up element, but it is also subject to the usual difficulties associated with bridge circuits when an A.C. supply has to be used in connection with dynamic recording equipment. A further advantage is the standardization of connections to recording apparatus, which, in the case of a bridge with two external active arms, results in a three-wire connecting system. It is obviously necessary to cause a change of impedance in one arm or a differential change in both arms of the bridge by the property which has to be measured. Fig. 1 is a generalized representation of the ideal circuit arrangement. It will be observed that, in general, the principle of operation of the various pick-up devices depends on simple electro-mechanical phenomena which are constantly being applied in radio and allied sciences.

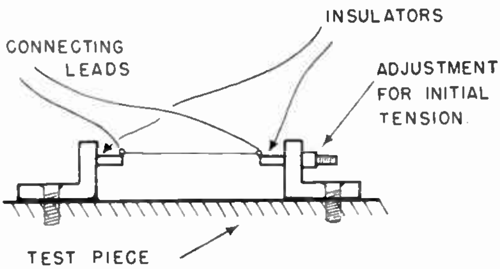

Fig. 2.—Simple resistance strain gauge.

3.2. *Measurement of Strain.*[11, 17, 19, 23]

Strain measurements may be required either on test pieces set up in the laboratory (and these may be whole aircraft frames or smaller components), or it may be necessary to determine the strains to which a component is subject in flight. Devices known as strain gauges are used for such measurements, and should be capable of detecting strains of 1×10^{-6}. There are several types of strain gauge, but that in most common use is the electrical resistance wire gauge. This type of gauge depends on the principle that, if a length of wire is subjected to a strain, there is an increase or decrease in the resistance of the wire in direct proportion to the strain, depending on whether the applied stress is tensile or compressive. (For most materials the resistance increases for tensile strain.) The increase in resistance is not entirely accounted for by the deformation which takes place, and is actually a little greater than the change which would be expected if it were assumed that the specific resistance of the material remained unchanged.

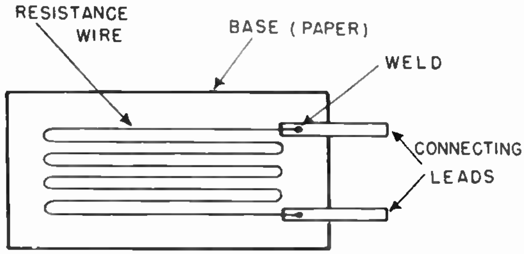

Fig. 3.—Bonded paper resistance strain gauge.

An early type of wire resistance strain gauge simply consisted of a thin resistance wire stretched between two brackets which were bolted to the test piece. The elongation due to the applied stress caused a corresponding change in the length of the resistance wire (Fig. 2). This is unsuitable for most purposes owing to its bulk and comparative insensitivity. It has been superseded by the bonded paper gauge which consists of several turns of fine resistance wire (0·001 or 0·002 in. dia.) lying parallel to each other and cemented to a paper base which is protected from damage by a felt or paper covering (Fig. 3). Gauges of this type have small dimensions—they may vary from approximately $\frac{1}{3}$ in. $\times$ $\frac{1}{4}$ in. to 2 in. $\times$ $\frac{1}{2}$ in.—and are comparatively very sensitive. The gauges are cemented to the surface under test with Durofix or a similar cement.

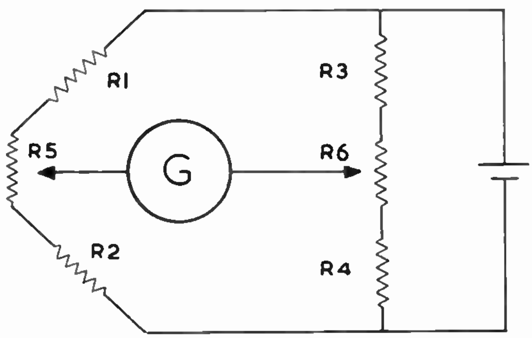

Fig. 4.—Strain gauge bridge circuit

There are several variations in the manner by which strains are measured, but all incorporate the strain gauge under test—called the " active " gauge—in some form of Wheatstone bridge circuit. This is necessary, for the change in resistance to be detected may only amount to a few parts in a million. The measurement of strain at one or two points is simple and calls for little apparatus. In the simple cases, the active gauge is incorporated into one arm of a bridge circuit while an identical non-active gauge is put in the other arm (Fig. 4). Small differences in the resistance of the two gauges are adjusted by a balancing potentiometer. The standard arms of the bridge are calibrated in terms of fixed resistance arm ratios, and the strains are determined by measuring the ratios necessary to obtain a null balance, and then applying a previously determined sensitivity factor. The bridge may be energized by D.C. for static measurements, and by A.C. for dynamic work. In the latter case, a sensitive cathode ray oscilloscope is a good indicating medium.

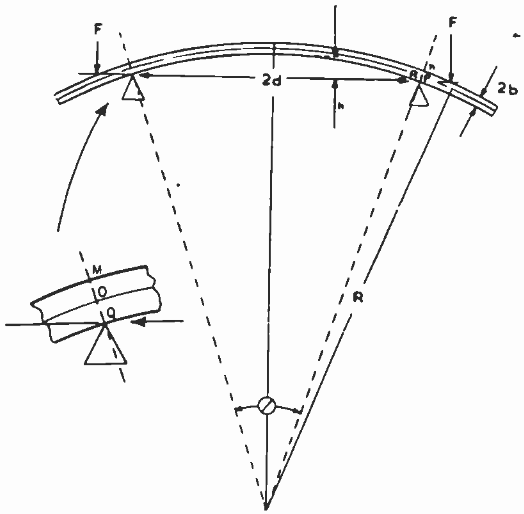

Fig. 5.—Calibrating beam.

The chief property of a strain gauge is the sensitivity factor which may be defined as the ratio of the percentage change in resistance to the percentage change in length due to stress.

Batches of manufactured strain gauges show very slight variations in sensitivity ; and it is usual to select a sample number from every batch received and to determine the average sensitivity factor and the amount of scatter which may be expected by testing the specimens on a calibration beam. This consists basically of a uniform rectangular bar mounted symmetrically on knife-edge supports, and loaded to an equal amount at each end. It can be shown that the increase in bending at the centre is proportional to the strain due to the load, and thus measurements may simply be made with the aid of a dial gauge or vernier microscope. A simplified theory to show the relationship between the stress resulting due to a change of applied force and the vertical movement of the centre of the beam is developed below. (See Fig. 5.)

$$\text{Distances, OP} = \text{R}\theta$$
$$mn = (\text{R} + b)\,\theta$$
$$qr = (\text{R} - b)\theta$$

Extension, $(\text{R} + b)\,\theta - \text{R}\theta = b\theta$.......... on top surface

Compression, $(\text{R} - b)\,\theta - \text{R}\theta = -b\theta$.... on bottom surface

$$\text{Strain} = \text{Extension/Original Length} = \frac{b\theta}{\text{R}\theta} = \frac{b}{\text{R}} \quad (1)$$

Since $d = h\,(2\text{R} - h) = 2h\text{R} - h^2$,

$\therefore$ If $h^2 \ll h\text{R}$, then $d \simeq 2h\text{R}$ $\therefore$ $\text{R} = d/2h$,

from (1), Strain $= b/\text{R} = 2hb/d$.

The sensitivity factor for most strain gauges is generally close to 2·0. Since the gauge current is usually limited to about 20 mA in order to reduce drift in the bridge balance due to temperature, and since a common type of gauge has a resistance of 100 or 200 ohms, the out-of-balance e.m.f. due to a strain of, say, 1×10^{-5} is of the order of 10^{-4} volts, so that a considerable gain is necessary in the amplifying stages of recording apparatus if it is required to measure such strains. The wire commonly used for strain gauge construction (Advance or Eureka) is chosen to have a minimum thermal coefficient of resistance, but thermo-electric e.m.f.'s are by no means negligible in comparison with the voltages caused by an average change in strain gauge resistance due to strain. The effects of thermo-electric e.m.f.'s can be neutralized to a large extent by careful construction of the bridge circuit to ensure that copper-nickelchrome junctions are kept as far as possible in high-resistance circuits, and that all junctions are balanced both physically and electrically in a symmetrical manner about the bridge circuit.

A further gauge property which must always be kept in mind is the fact that it is sensitive in a small degree to transverse strains in addition

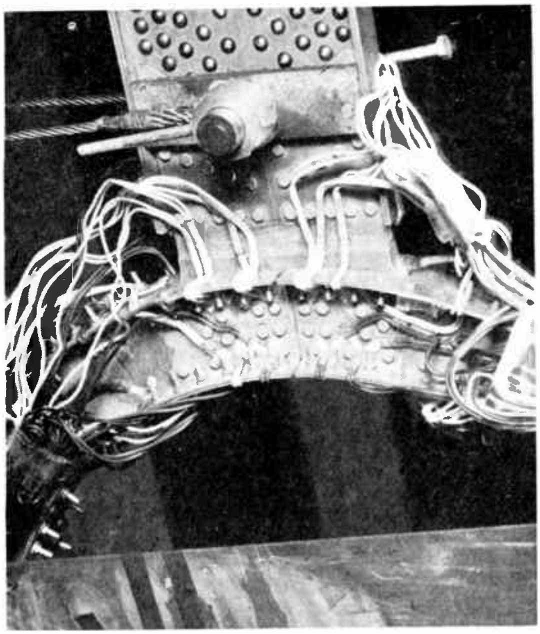

Fig. 6.—Strain gauge installation. The photograph shows a number of miniature type strain gauges fixed to a test piece and wired for the measurement of principal shear strains.

to axial strains. The cross sensitivity factor is usually two or three per cent. of the axial factor, but if there is a transverse strain large in comparison with the axial strain, a considerable error may be introduced. The positioning of strain gauges is thus a matter calling for expert consideration. A typical strain gauge installation is shown in Fig. 6.

It may seem strange that the operation of cementing a gauge to a test surface should result in an accurate strain detector with a consistently reproducible sensitivity factor for gauges of the same type, but there has been ample practical demonstration of this fact and the scatter in the sensitivity to strain of most gauges is not more than one or two per cent. from the mean value. With suitable precautions strain gauges can be protected from the detrimental effects of exposure to bad weather conditions, although it is usually rather difficult to achieve complete insensitivity to large temperature variations. Exposed gauges must be water-proofed with particular attention to correct sealing of the connecting leads.

3.3. *Measurement of Displacements*

There are obviously very many varieties and types of electrical circuit which may be designed for the purpose of converting linear and rotational displacements to electrical signals having corresponding magnitudes and signs. It is only proposed here to mention a few representative methods of the types which are suitable for straightforward test room use, and these are grouped according to the magnitudes which are to be measured. It will be noticed that a mechanical displacement is similarly contrived for all types of pick-up.

Displacements up to 0·01 in.

(*a*) *Strain Gauge Method* (Fig. 7). The displacement component, A, is in contact with a strip of metal, B, which is initially lightly stressed by the presence of the displacement component. Two strain gauges, " a " and " b," are cemented on each side of the strip B near the root of the member where the strain is a maximum. These are wired to a bridge circuit and balanced with A in its neutral position. Displacement of

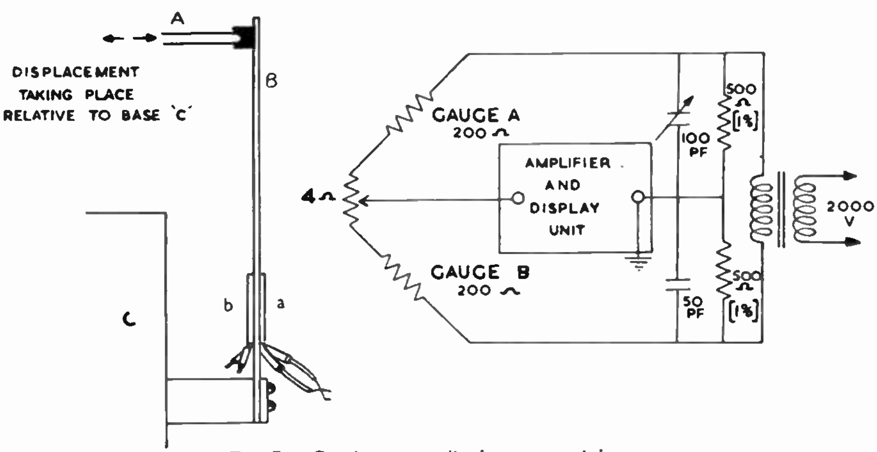

Fig. 7.—Strain gauge displacement pick-up.

A causes a differential change in the root strains of B, and hence an output signal from the bridge. This is linear when the displacement is small. Reference to Fig. 7 will show the circuit to be self-compensating for thermal effects. Under normal conditions, the thermal state of the strain gauges and connecting leads will be very similar, and thus equal thermo-electric e.m.f.'s will be generated in each active arm of the bridge; and similarly, providing the resistance of each gauge is equal, an identical change of resistance in each arm will be the result of a change in ambient or self (i.e. current heating effect) temperature. Under these conditions, the balance of the bridge will not be affected by temperature changes.

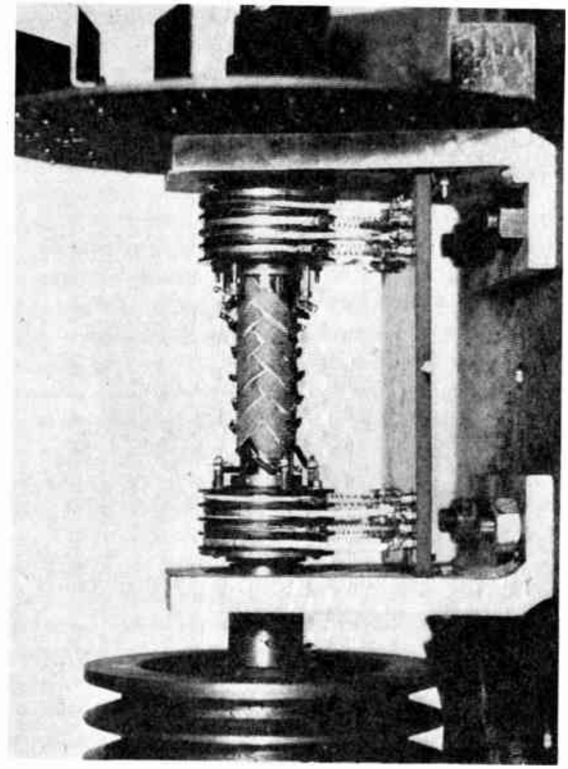

Fig. 8(b).—Strain torquemeter.

(*b*) *Impedance Variation method.* A physical displacement may be caused to change the magnitude of a capacitive or inductive reactance although a more sensitive pick-up is achieved by using the change of position to vary the balance of a bridge circuit with reactive balance arms.

A novel and sensitive form of displacement pick-up is described elsewhere.[4] It was developed for a special application where the maximum transmitted torque could not exceed more than one or two milligram millimetres. To the moving element was attached a small permanent magnet. The couple due to this and an electro-magnet mounted on the pick-up is transmitted to a piezo-electric crystal. Since the electro-magnet is supplied with A.C. there is an alternating torsional moment on the crystal which produces a relatively large e.m.f.

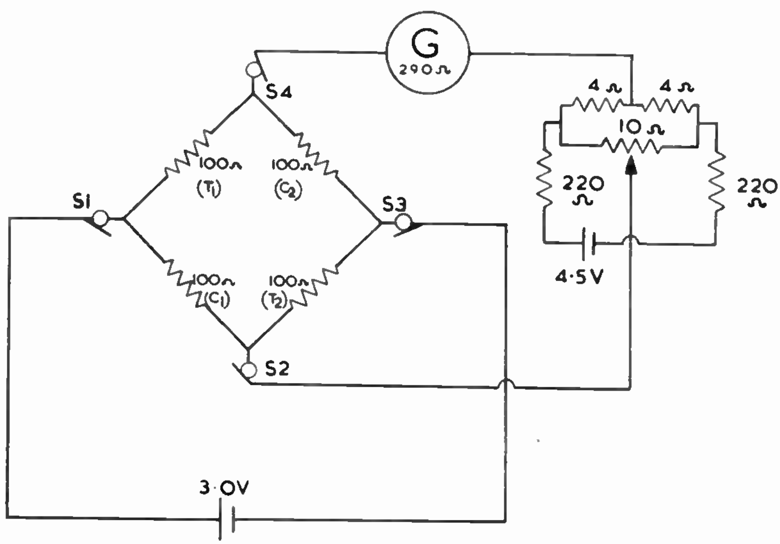

Fig. 8(a).—Strain torquemeter circuit.

The problem of recording the torque transmitted by a coupling shaft is often encountered. Torque may be measured electrically either by detecting the twist or "wind-up" of the shaft, or by measuring the strain at a point on the surface. The application of bonded wire strain gauges to this problem is a convenient practical solution,[15] and, with certain restrictions, a strain gauge torquemeter has a high order of accuracy and is capable of very consistent operation over long periods. The principal stresses lie at 45 deg. to the axis of the shaft, and so may be determined by means of strain gauges cemented to these positions. Connections to the gauges must be made by slip rings, and, due to contact difficulties, it is essential to ensure that the brush connections are in the external bridge leads. In this connection it is helpful to use high-resistance gauges. Fig. 8 shows an arrangement of this type employing a four-gauge bridge circuit with a "current" balancing arrangement replacing the usual resistance adjustment. The strain gauges are arranged to provide temperature compensation and to greatly reduce errors which might appear due to bending of the shaft. The use of electrical resistance strain gauges for torque measurements is limited chiefly by the temperature rise on a shaft, caused by adjacent slip-rings, when rotated at high speeds as thermo-electric e.m.f.'s become troublesome when large changes in temperature occur.

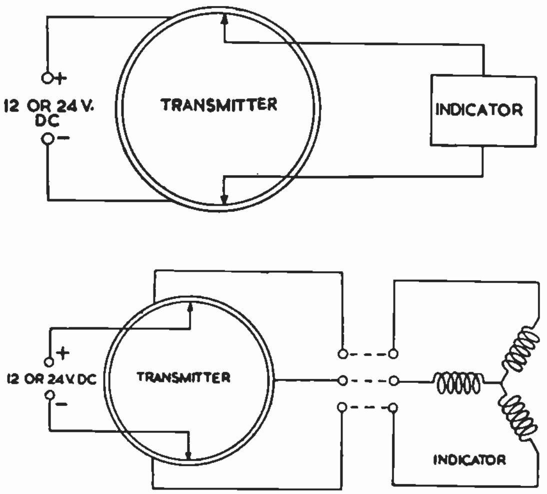

*Fig. 9(a).—Standard Desynn pick-up.**

*Smith's Aircraft Instruments

There are several methods of measuring the actual twist of a shaft; most of those which have been used depend on the measurement of a phase difference between the alternating outputs of two electromagnetic pick-ups. Each pick-up may be coupled to a toothed wheel or disc of magnetic material, the pick-ups being spaced as far apart as possible to allow for the greatest degree of wind-up. If the shaft speed is denoted by "w" and there are "n" teeth on each wheel it is clear that the effective angular frequency of the pick-up output is "nw" while the phase relationship between the two outputs is a function of the torque.

Larger Displacements. Some of the position pick-ups described above are adaptable for use in situations where the range of movement is relatively great; but, in these circumstances, more robust, coarser pick-up controls may often be used. Two D.C. instruments of great application within this range are the Desynn and Micro-Desynn transmitters. The former comprises a circular resistance element tapped at three equally spaced points which are connected to the three windings of a 3-phase star-connected indicator coil. The D.C. supply voltage is brought in at two diametrically opposite contacts resting on the resistance element. As the contacts rotate, the direction and distribution of the currents in the indicator windings are altered, so that the resultant field rotates synchronously with the transmitter. The indicator element is a pointer upon the axis of a permanent magnet rotor which thus takes up a position corresponding to the transmitter movement. The Micro-Desynn transmitter functions in an identical manner but has a very different physical structure since it is designed to require only very small operating forces (up to approximately 80 grams). This type of instrument is commonly used with the "automatic observor" type of recording where instrument readings are photographed at given intervals of time. When it is required to make a continuous recording, or to use a Desynn with, for instance, a multi-channel electronic recorder, the indicator may be removed and the transmitter connections rearranged as a bridge circuit, by making the supply connections permanent and taking the output from the wiper arms. (See Fig. 9.) This type of instrument is particularly useful for indicating control surface movements, rates of roll and pitch, stick positions, etc., where there is ample available operating torque and the simplest registering apparatus is required.

A further example, having a very wide range of possible applications is the differential transformer type of pick-up[16] (Fig. 10). Coils A and C are identical and are symmetrically disposed about coil B which is connected to an A.C. supply. A soft iron armature D couples A and C with B, and is mechanically attached to the displacement component. As, for all practical purposes, the total force acting on the displacement component is that due to the mass of the armature and frictional loads occurring between the armature and the coil former, the total loading may be made very small indeed. The outer coils are connected in series opposing so that, with D centred electrically, there is no resultant e.m.f. in the secondary circuit ; but a displacement of the armature in either direction from this position results in an " out-of-balance" e.m.f. with a 180-deg. change of phase. A well-designed pick-up of this type will have an accuracy better than 1·0 per cent. of its total range of linear output. The sensitivity of the pick-up may be greatly increased by tuning the secondary circuit to resonance with the supply frequency. Optimum conditions are obtained when this is in the audio range, i.e. one or two kc/s.

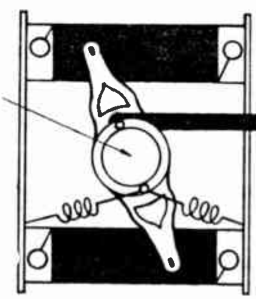

Fig. 9(b).—Diagrammatic plan.

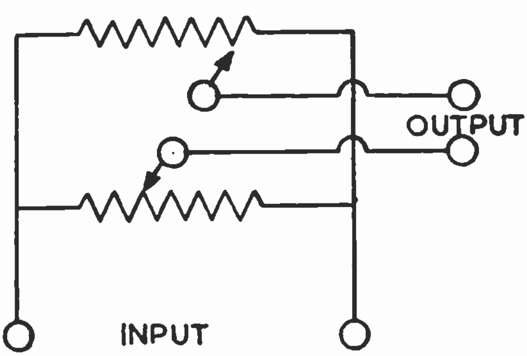

Fig. 9(b).—Micro-Desynn pick-up.

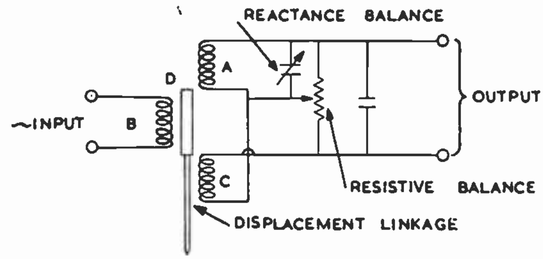

Fig. 10.—Variable differential transformer pick-up.

It may often be necessary to adapt standard instruments, having small maximum permissible displacements, to function as indicators of larger positional movements by means of reducing gears or levers. For simple linear displacements of large magnitude, an insulated contact wiper may be fixed to the moving element and maintained in contact with a long, finely wound, resistance element. Potentiometers, rheostats, etc., may be conveniently used to record rotational movements, as well as the well-known Selsyn and Magslip type of transmitters and receivers.

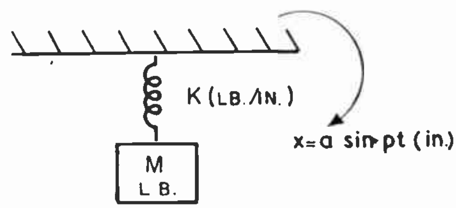

Fig. 11.—Elemental spring-mass system.

3.4. *Measurement of Velocities and Accelerations*

It is sometimes possible to make use of a displacement pick-up together with a differentiation circuit to record velocity, but more often devices which have come to be familiarly known as accelerometer-type pick-ups are used. These consist basically of a spring-mass system with damping. The relative motion of the mass (Fig. 11) with respect to the case of the pick-up is easily converted into an electrical signal. The general equation of motion shows that the output from the device may approximate either to the displacement or the acceleration depending upon which component is paramount. Actually this means that a displacement type of instrument will have a low natural frequency of vibration, and an accelerometer will have a high natural frequency. Both instruments suffer from resonance effects and have to be designed to respond to a range of frequencies only if they are required for use with straightforward test apparatus. They have the great advantage of portability. Thus, in vibration measurements, a pick-up may be held in the hand and fitted with a probe which is placed at any required point on the structure. Not only the amplitude but the phase of the vibration may be established using an amplifier and display unit for each pick-up and a phase reference to the master exciter. (See Section 4.2.) The addition of an integrator(s) will give displacement directly.

Commercial instruments of this type have been found to possess reasonably close natural frequencies and similar response characteristics. Thus, in one particular test where five velocity-type pick-ups were in use and measurements had to be made over the region in which resonances occurred, it was found possible to draw up a universal response curve which ensured accuracies to within 5 per cent of the true values. A constant correction factor could be applied to all the measured amplitudes at any one frequency. These instruments were damped to about 0·1 or 0·2 of the critical value.

A simplified theory for a vibration pick-up is developed below :—

With reference to Fig. 11, let A represent the case of the pick-up which is assumed to be subject to a sinusoidal oscillation,

$$x = a \sin pt \text{ (in.)}$$

Let mass of moving part = M (lb.)

Displacement of moving part = y (in.)

Stiffness of spring suspension = K (lb./in.)

The equation of motion is : $M\ddot{y} = -Kg(y-x)$

The complete solution of this is :

$$y = A \sin(\omega t + h) + a \sin pt / \{1 - (p/\omega)^2\}$$

The first term represents the free undamped vibration of the system and is neglected.

$$\therefore y = a \sin pt / \{1 - (p/\omega)^2\}$$

$$y - x = a \sin pt\,(p^2)/(\omega^2 - p^2)$$

If the exciting frequency is well below the natural frequency of the moving mass and suspension, i.e., if $p^2 \ll \omega^2$, then

$y - x \simeq (ap^2 \sin pt)/\omega^2$, and the pick-up satisfies the condition for it to function as an accelerometer since $(y - x)$ is proportional to $p^2 a \sin pt$. If ω is small compared with p, then ω may be neglected and $(x - y)$ is approximately proportional to $a \sin pt$, which satisfies the condition for the pick-up to produce a signal proportional to the displacement.

The properties developed above may obviously be applied to enable an electrical output proportional to the relative motion of the mass to be obtained. The principle of operation of some types of pick-up is similar to that of the moving coil loudspeaker, and, in others, a differential transformer or variable reluctance arrangement is used. There are also several types of thermionic accelerometers in which the electrodes and their suspensions form the spring-mass system and in which varying conductivity, as a function of electrode spacing, is used as the means of detection.

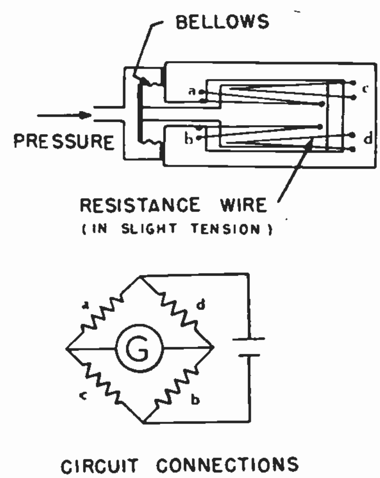

Fig. 12(a).—Statham pressure pick-up.

3.5. *Measurements of Pressure*

Pressure-recording devices generally incorporate a diaphragm or bellows which produce a displacement designed to have a linear, or nearly linear, relationship with the applied pressure. The various practical devices which have been developed have made use of the strain gauge principle, change in capacity between two electrodes, or the variable inductance, differential transformer, or piezo-electric effect, and there is therefore very little to add in this brief review to the details covered in the section on displacement pick-ups. Several schematic drawings are shown in Fig. 12, however, which illustrate the application of these principles.

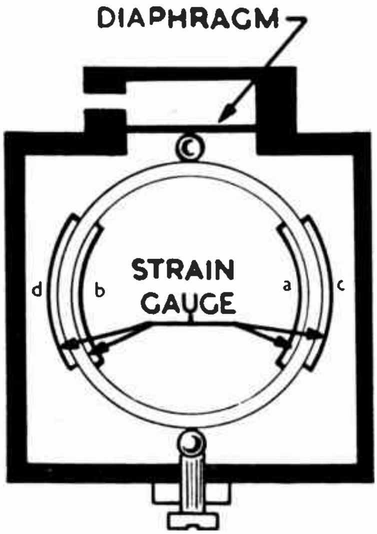

Fig. 12(b).—Strain gauge pressure pick-up.

3.6. *General*

Having thus surveyed a number of practical pick-ups grouped under the type of application for which each is required, it is now possible to make the following generalizations :—

(1) Of all the various electrical principles which have been applied to pick-ups, that involving a variation in inductance appears to have the greatest range of application owing to the relative ease of development and the high sensitivity which may be obtained.

(2) " Bridge " operation of any type of pick-up is commonly the most sensitive manner of use, but difficulties associated with balancing A.C. bridges must be fully considered during the design stage, and especially the fact that it may be necessary to operate a number of pick-ups from the same voltage supply.

(3) Finally it will probably help to fix ideas on pick-ups by considering the work required of them :—

(*a*) A linear response characteristic is desirable, and hysteresis effects should be as small as possible.

(*b*) Displacements (linear and rotational) which may require measurement on occasions range from a few ten-thousandths of an inch up to several feet.

(*c*) Pressure measurements may be required anywhere within the range 0-1 and 0-10,000 lb./in^2.

(*d*) The frequency range of structural vibrations of importance may be up to several hundred c/s.

(*e*) Acceleration measurements may be required up to 20 or 30 × " g " but normally chief importance will be attached to measurements up to about 6 × " g."

4. Recording Apparatus

4.1. *General Requirements*

In the following sections an attempt has been made to present as much useful information about the problems of recording aircraft test data as possible ; but, at the same time, it has been considered more constructive to describe a selected group of equipments in some detail rather than to try to review the whole field of aircraft measurements.

In strain measurement work, it is usually necessary to determine the strain distribution over the whole or a vital region of a structure ; and thus the strains at a number of points are required in order to enable a stress distribution pattern to be drawn up. When a relatively small number of points is required, i.e. up to about forty, it is generally satisfactory to use a manual selection board by which each strain gauge station is connected in turn to a standard Wheatstone bridge measuring set, which is adjusted for a null balance, but even with forty points the work becomes rather tedious and lengthy. In addition to the time factor, a practical difficulty is the continued maintenance of constant load conditions while measurements are being made, and another is the increased danger of drift occurring as time progresses due to temperature changes. Drift is commonly due either to circulating and changing external air conditions, or to the heat developed in the gauge circuits by the gauge current. That the error which is so introduced may be of considerable magnitude is soon visualized when it is recalled that a copper-cupro-nickel junction has a thermal coefficient of about 60×10^{-6} volt/deg. C, and that signals due to the straining of a gauge are not much larger. In order to reduce the time necessary for a large test to be completed, automatic high-speed bridge selecting and recording apparatus has been developed* in which selection of several hundred strain gauge stations in rotation is accomplished by a high-speed switch unit and uniselectors. Signals representing strains are displayed sequentially on an oscilloscope and recorded by a continuously running camera. Loading of the specimen under test is increased by suitable increments, and, at each increment, the strains are recorded in the space of a few seconds. The results achieved have good accuracy, and a useful reduction in the expense of large strain gauge tests is possible. Analysis of the recording is simplified by incorporating fixed out-of-balance ratios, in dummy bridge circuits, representing known amounts of strain. These calibration signals follow upon the test signals in the record. Two views of this recorder are shown in the photographs of Fig. 13.

It is only possible to make use of single-channel sequential recording methods in dynamic work when the switching rate is very high, and the changes in load conditions occur relatively slowly. These methods may suffice for the measurement of pressure and temperature changes, and for slow positional changes of control surfaces, etc., but high frequency vibrations cannot be dealt with in this manner.

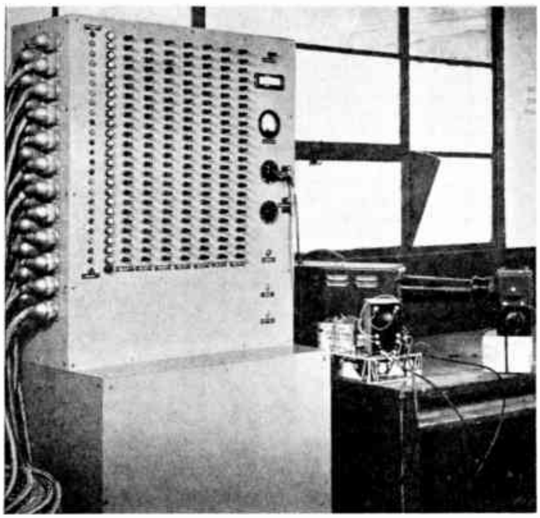

Fig. 13(a).—200-point strain gauge high-speed measurement set.

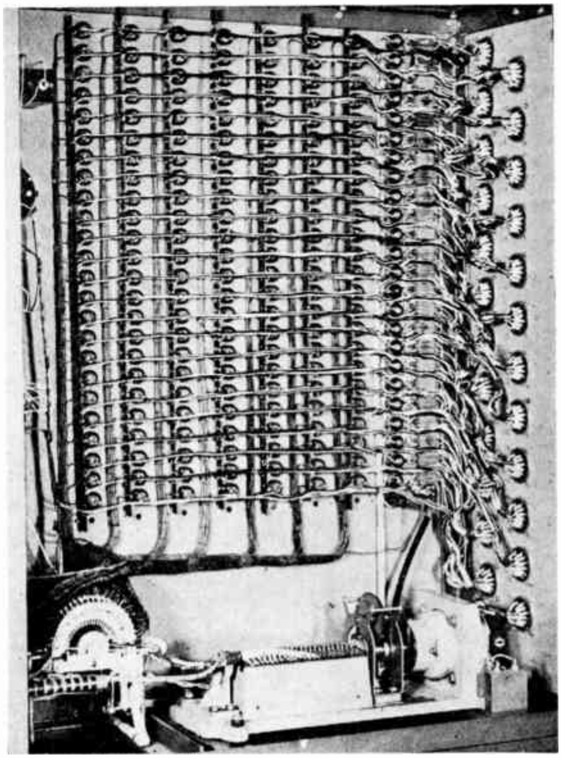

Fig. 13(b).—Internal wiring arrangement—200-point unit.

Since there are many occasions when it is imperative to record high-frequency vibrations and oscillatory strains at several points simultaneously, the need has arisen for the development of suitable multi-channel recording apparatus. Some early types of multi-channel systems made use of pick-up units connected to a number of separate meters mounted on a panel. The whole array was photographed at regular intervals by cine-camera and subsequent analysis was made by enlarging the film frames on a projector. This solved the problem of simultaneous measurement but of course was discontinuous. Additionally the method was insensitive in its simple form.

Later equipments have made use of thermionic amplifiers and either cathode ray oscilloscopes for presentation purposes or multi-element galvanometer units with pre-amplification of the various signals. For airborne purposes, and for dynamic work photographic recording is employed.

* With acknowledgments to the Royal Aircraft Establishment, who supplied information and advice upon which this design was based.

The chief requirements for recording equipments are summarized below :—

(1) Single-channel Equipment (Sequential selection).

(a) Wide-band amplifier response.
(b) Freedom from all types of distortion.
(c) Switching mechanism free from spark (input to amplifier preferably grounded during change-over).
(d) Freedom from " Microphony."
(e) Stable and accurate calibrating voltages.
(f) Continuous or frame camera recording (dependent on system employed).

(2) Multi-channel Equipment.

(a) Requirements (b), (d), (e) above.
(b) Small bulk and weight (particularly for flight work.
(c) Strength and resilience against shock.
(d) Versatility of application.

4.2. *Multi-channel Equipment*

In the development of multi-channel recording systems the possibility of making use of a single cathode ray tube presentation has often been considered. Most of the practical designs have made use of electronic switching methods. A straightforward four-channel arrangement of this type is shown schematically in Fig. 14. A disadvantage of this system is that RC coupling cannot be used since a change in the signal level on one channel would cause a shift in the apparent zero signal level and thus interact with the other channels. A design for a four-way switch unit incorporating a ring counter and gate amplifiers has also been described in the literature, and equipments having an even greater number of separate channels are available.

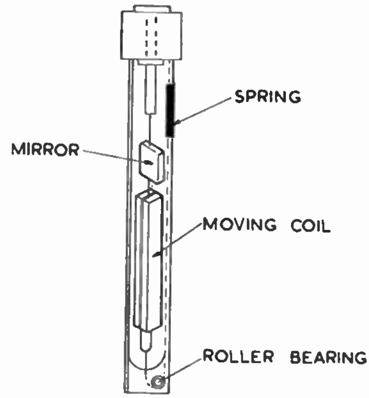

Fig. 15.—Diagram of miniature galvanometer element.

In view of the rather complex nature of these switching methods it is often advantageous to make use of separate recording channels with miniature type cathode ray tubes or galvanometer elements for display purposes. The design of the amplifier stages becomes simple, and more attention may generally be given to the quality of the amplifier response. At the same time good accuracy may be obtained, when carrier frequency operated equipment is in use, from an amplifier having a rather selective frequency response. This is a good point in view of the very large frequency range over which interference may be experienced from aircraft radio equipment, generator fields, vibration pick-up, static interference and so on.

Multi-channel galvanometer recording equipment has received a good deal of attention, particularly in America, and several reliable standard recording instruments having six, ten, sixteen channels, etc., are available for measurement work. Basically similar in

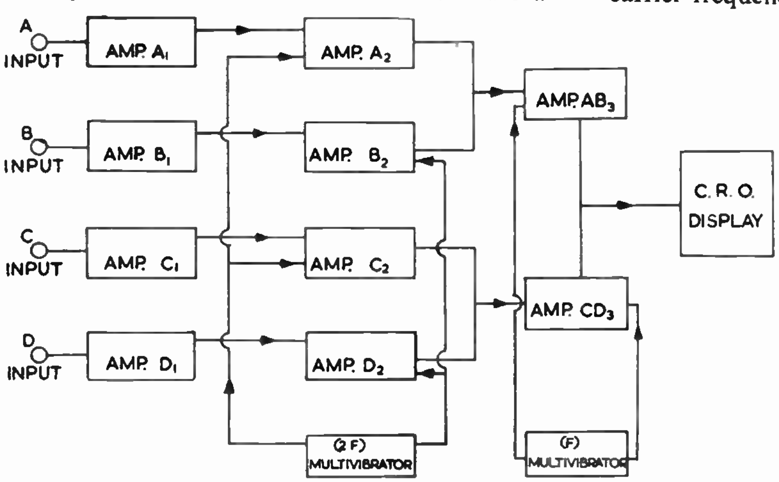

Fig. 14.—Schematic diagram of 4-way electronic switch.

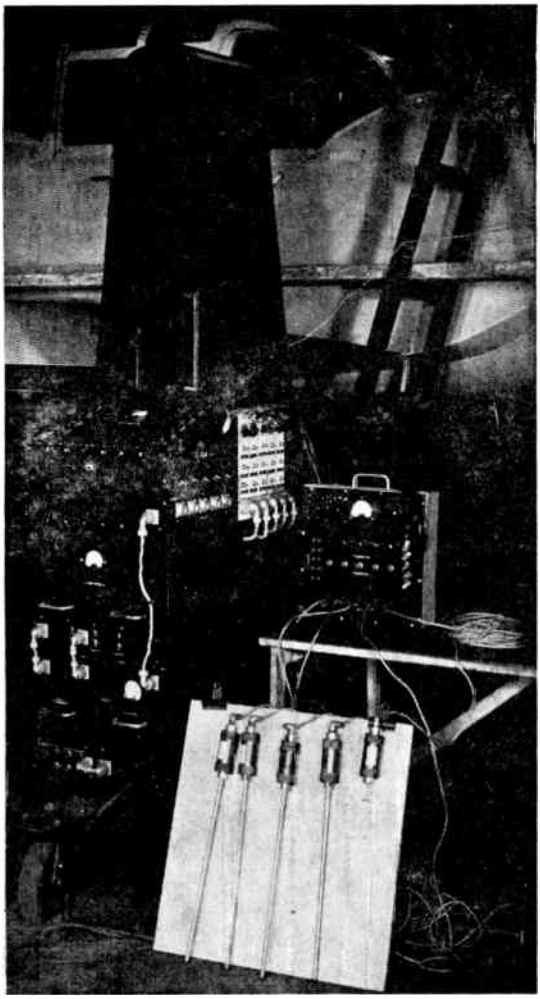

Fig. 16(a).—Ten-channel recorder.

conception, the galvanometer recording units consist of small cylindrical elements fitted between the pole pieces of a strong permanent magnet. Each element (Fig. 15) has a long narrow coil suspension with a small mirror attached to the moving element, and is usually immersed in an oil bath which provides damping. An optical system transmits the mirror movements to the surface of the recording paper or film. The chief disadvantage of galvanometer recorders is due to the natural periodicity of the moving coils which limits the useful frequency response of the equipment; and also, since the elements are rather fragile, their resistance to shock is not generally very great. On the other hand, the galvanometers are quite sensitive, so that a smaller degree of amplification is possible than may be necessary for cathode ray tube presentations.

A photograph of a ten-channel electronic recorder which makes use of cathode ray tube presentation and photographic recording is shown in Fig. 16(*a*). Each channel is independent of the remainder and consists of capacitive and resistive balance circuits for bridge operated pick-up units, a push-pull amplifier having a frequency response which is flat within 2 db between 0·1 and 10,000 c/s and with a gain of 10,000, and a display unit complete with brilliance, focus and shift controls using a $1\frac{1}{2}$-in. cathode ray tube. The amplifier and display unit are built into the same chassis frame. The complete unit is connected to the external balancing circuits and to power points by connectors on the front panel, and it is located in position in the main framework by a keyway along the top of each chassis. A common oscillator feeds all bridge circuits at either 5,000 or 2,000 c/s. Provision is made for a 50-c/s calibrating signal to be injected into the grids of the C.R.T. units, thus causing beam blanking to take place and assisting in subsequent analysis of the record. The bridge balancing potentiometers and capacity balances are located on the inclined front panel of the recorder. Self-energizing or other types of pick-up may be connected directly to the amplifiers. Photographic recording is employed, and a typical sample of recorded film, in which six channels are in use, is shown in the photograph (Fig. 16 (*b*)).

This recorder was developed chiefly for use in vibration measurement work. An example of the utility of this type of measurement is as follows :—

The liability of a structure to experience resonances due to particular frequency components of a complex vibration is well known, and may be determined in practice by forcing a small amplitude sinusoidal oscillation at a point in the structure and comparing the amplitude and relative phase of vibrations over the whole region. Vibrations originating from the engine are likely to be most troublesome;

and, since these frequency components can be readily found or calculated, it is possible to determine by a vibration test whether or not any particular mode is likely to cause serious weakness to any part of the airframe structure and to correct the design accordingly. In practice, a large number of points on the structure are selected as test stations. These may include points for measuring both lateral and vertical modes of vibration. The airframe is then oscillated at a fixed amplitude from a predetermined position, and measurements are made at all stations while the frequency of oscillation is maintained constant. The frequency of oscillation is then varied in small increments to cover the complete range of frequencies which are required, and, at each new frequency, the measurements are repeated ; in this manner complete and extensive information of the airframe response to mechanical excitation is obtained. The number of test stations may very well be large so that, in all, several thousand individual measurements may be necessary to complete a test. The advantage of multi-channel recording for this work is therefore quite obvious.

A small, compact recorder for use in flight tests is shown in Fig. 17. This apparatus has been recently developed by the Research Dept. of Boulton Paul Aircraft, Ltd. for strength tests on important components of an aircraft system in flight, and it is designed to function with strain gauge pick-up circuits, although self-energizing or other carrier-operated types of pick-up unit may be used if required. Severe space restrictions in the aircraft limited the design to a three-channel equipment, but a six-channel version is under development.

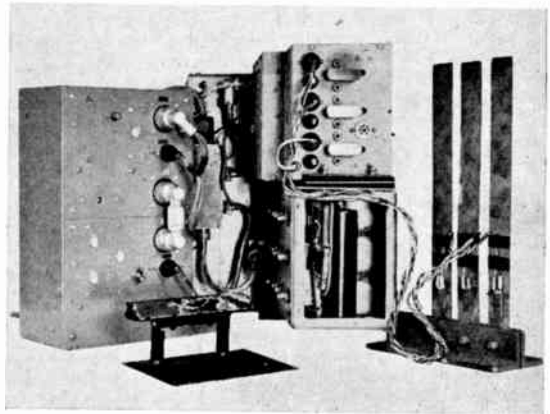

Fig. 17(a).—Three-channel flight recorder : amplifier, display, oscillator and discriminator units.

A schematic diagram of the recorder is given in Fig. 18 ; the power consumption is of the order of 500 watts, and the strain gauge bridge circuits are energized by a 2,000 c/s " carrier." The outputs are taken to three A.C. amplifying channels each with a gain of approximately 20,000. The three display tubes with the associated optical system and electrical controls occupy a space of 240 in.^{3} which does not compare too unfavourably with the space requirements of the equivalent magnetic oscillograph unit. A considerable saving in space is achieved by utilizing a mirror reflector so that the display tubes are positioned parallel to the optical path between the recording film surface and the tube face. Three lenses focus the display signals upon the film surface with a magnification of about 1·5 or 0·5, whichever is considered the most suitable.

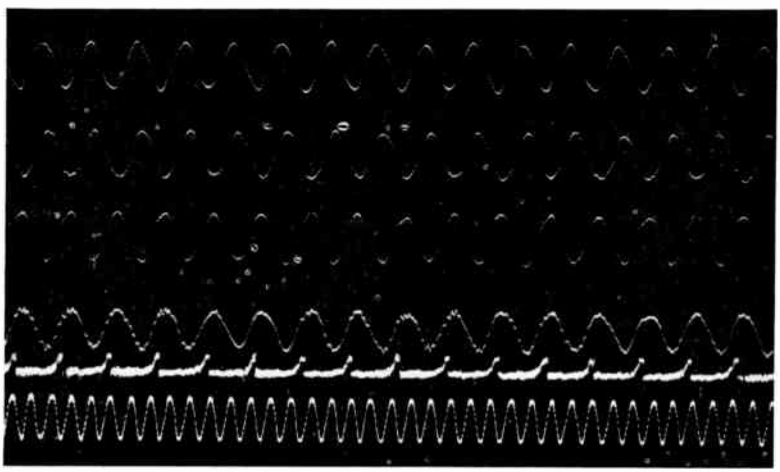

Fig. 16(b).—Typical record from 10-channel recorder.

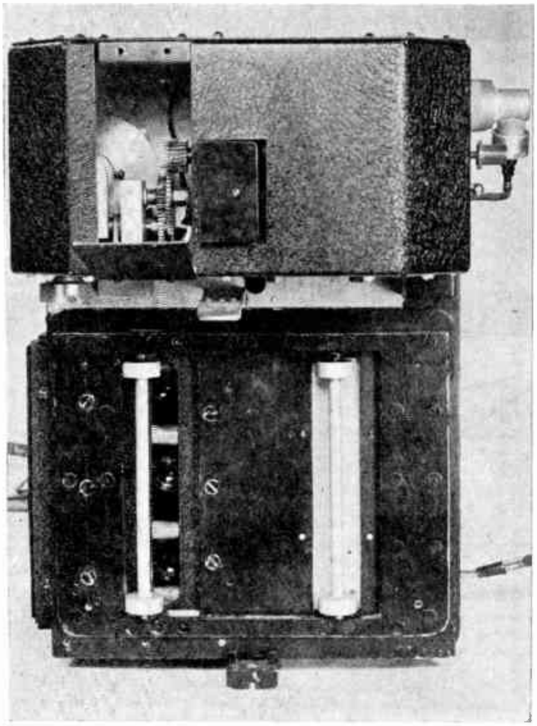

Fig. 17(b).—Three-channel flight recorder: magazine attachment and driving motor. View from rear of display unit.

The recording channels are arranged to be phase sensitive so that uncertainties in the identification of the sign of a modulation component or of the true waveform are eliminated. The possibility of uncertainty of the actual modulation waveform arises due to the effect of apparent 100 per cent. over-modulation which occurs in balanced bridge pick-up circuits. If the modulation frequency contains components other than the fundamental, then uncertainty may easily arise unless relative phase comparisons can be made. There are several methods of overcoming this difficulty. Firstly, the bridge may be given an initial out-of-balance, and normal demodulation methods may then be employed. This places a limit on the maximum modulation depth which can be accepted before distortion takes place, and also it is subject to attenuation of the higher frequency modulation components through the carrier wave filter section of the detector. (It should be noted that practical considerations require the lowest possible carrier frequency consistent with a good carrier/maximum signal frequency ratio and a ratio of 10/1 is usually considered the minimum.)

A phase discriminative rectifier amplifier[17] can be employed, but this requires at least one additional valve stage per channel. Discrimination in the flight recorder is obtained by applying a brightening pulse to the grids of each cathode ray tube. The pulse is derived from the oscillator, and wide control of its phase relative to the oscillator output waveform is obtained from a variable phase shift network. With resistance strain gauge circuits negligible phase shift is encountered in normal operation; and as long as the amplifiers are reasonably matched the total phase shift in each will be of the same order, so that a single discriminator may suffice for all channels while the problem of distortion occurring during rectification of the modulated carrier wave is eliminated. In setting up the recorder, a time base generator is

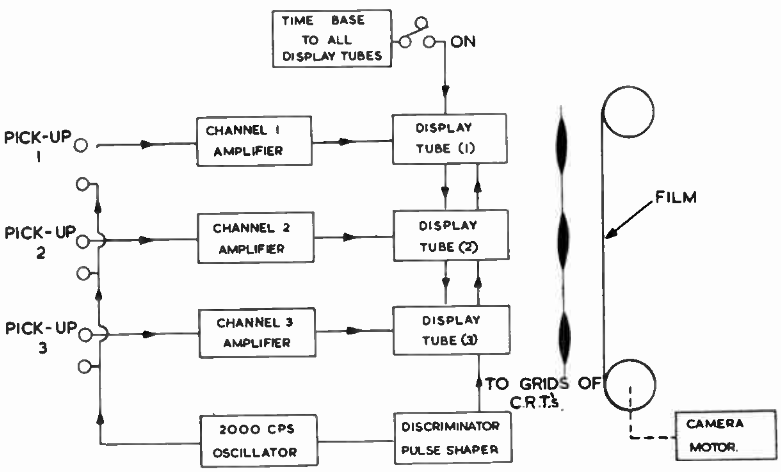

Fig. 18.—Schematic diagram of 3-channel recorder.

connected to all the X plates of the three display tubes, the brightness levels are adjusted to make the three traces visible, and each bridge circuit is given a slight out-of-balance by adjusting the balancing controls ; the discriminator " contrast control " is tuned to give the required degree of contrast between the brightening pulses and the general brilliancy level of the trace, and finally the phase control is adjusted to bring the brightening spots to the positive or negative peaks of the carrier wave. The time base is switched off for recording purposes.

In the interests of reducing the number of components in the recorder to a minimum, the three display tubes are energized from a common high voltage resistance network. Accurate calibration of the film records is possible ; the amplifier inputs are periodically switched, by means of a relay, to a resistance chain across the oscillator output which provides known percentage amounts of out-of-balance from what is in effect a dummy bridge circuit. The switching rate is relatively high, varying from approximately 10 and 60 operations per minute at the slowest and fastest camera speeds. This method ensures that accuracy is independent of slow changes in the amplitude of the voltage applied to the bridge circuits, and also of similar changes in amplifier response due to H.T. variations, temperature rise and fall, and so on. When great accuracy is not important, and a continuous signal recording is desirable, the calibration control relay may be manually operated at the beginning and end of a test run.

4.3. *Radio Telemetering*

It would be wrong to leave the subject of recording equipments without reference to the increasing use of radio transmission as a link in telemetering systems. This is particularly valuable when data has to be obtained of the performance of pilotless aircraft and missiles, and it will probably also find increasing application for piloted flight work as the technique of transmitting performance data by these means becomes more fully developed. There are some obvious advantages to be gained by radio telemetering. For instance, the weight and bulk of airborne recording apparatus may conceivably be reduced and, also, more complex, elaborate, ground recording equipment may be used ; viewing displays may enable vital preliminary information to be obtained in advance of the development of recording paper or film. The subject is already very extensive, and time and space unfortunately preclude a more detailed account. Several of the references given below deal with systems of this nature however.

4.4. *Equipment Design*

The designer of airborne electronic measurement equipment has to contend with some special difficulties. Aircraft equipments are generally powered from batteries which are charged by a D.C. generator. In many cases the capacity of the batteries alone is insufficient to satisfy the demands made on it, and the generator is run continuously with the batteries " floating." Intermittent operation of certain controls which cause, momentarily, very high loads on the supply, and other factors often mean that a variation in supply voltage of 25 per cent. is the rule rather than the exception. It is often difficult to reconcile this adverse state of affairs with the requirement of an overall accuracy of measurement of a few per cent., since the limited permissible weight and bulk of recording equipment often excludes the use of elaborate stabilizing systems. Wide variations in temperature and climatic conditions must be taken into account. Valve microphony is a further serious factor imposing limitations on the scope of design ; and the possibility of severe vibration and mechanical shock makes it necessary to exercise particular care in the choice of components which should preferably be subject to rigid inspection and mechanical endurance tests.

5.0. Conclusions

It has already been stated that the purpose of this paper is chiefly concerned with providing a general view of the manner in which electronics are being applied in connection with the design and test of aircraft structures, and that it is naturally impossible to review, even superficially, the whole aspect of measurements in this sphere. We have, at any rate, seen that electronics may in practice be applied in a practical and relatively simple manner to the measurement of almost any required factor, and also that, with suitable pick-ups, many recorders are adaptable to almost any requirement.

A short bibliography is appended which contains a number of useful references for those who wish to carry their investigations further.

6.0. Acknowledgment

In conclusion, the author wishes to express his thanks to the Management of Boulton Paul Aircraft, Ltd., for permission to publish this paper, in which several of the recording equipments developed in their Research Laboratory are described, and for all photographs. Thanks are also due to Dr. S. C. Redshaw for his kindly criticism and advice.

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GRADUATESHIP EXAMINATION, NOVEMBER 1949

ADDITIONAL PASS LIST

This list contains the results of the remaining overseas candidates which were not available for inclusion in the first pass list published in the February Journal.

The following Candidate passed the entire examination and is now eligible for election to Graduateship or higher grade of membership.

WOLFF, Henry Arnold	Launceston, Tasmania

The following Candidate passed Part I only

BREMNER, William (S)	Newcastle, N.S.W., Aust.

The following Candidates passed Part II only

BHATNAGAR, Brisnandan Saroop (S)	Lucknow, India
CHAN YORK CHYE (S)	Singapore
HSU SIN PO (S)	Singapore
LAJOIE, Marc Jean-Baptiste (S)	Mauritius
RAJEN, Chittoor Thyaga (S)	Bombay, India
RAMALINGAM, Gobi Ramiah	Jhansi, India

The following Candidate passed Parts II and IV only

LAJOIE, France Albert (S)	Mauritius

The following Candidate passed Parts III and IV only

REDFERN, Ralph Frederick (S)	Rotorua, N.Z.

A total of 192 candidates attended the November 1949 Graduateship Examination at 36 centres throughout the world (10 centres in the British Isles).

(S) indicates Registered Student.

Examiners' General Report

A more detailed analysis of the examination shows that 56 candidates sat the examination at the overseas centres, whilst 136 attended the 10 centres in the British Isles.

The percentage of candidates passing the various parts is as follows :

31 per cent. passed Part I.

62 per cent. passed Part II.

19 per cent. passed Part III.

41 per cent. passed Part IV.

The percentage of candidates passing Part I has dropped compared with previous examinations. The Examiners in this section remark that most of the candidates appear to have received inadequate instruction in Physics.

Candidates still find Advanced Radio Engineering the most difficult section of the examination. Last November the results were particularly poor and the Examiner's comments indicated that a general lack of fundamental knowledge is evident, "the average level of candidates' answers being lower than any reached in the last five examinations."

The most popular optional subject remains Radio Reception (receiver design and practice). This is because the syllabus is almost entirely covered by the normal technical college training in radio engineering. In the case of the other optional subjects, however, the candidate needs to have had specialized experience in the particular branch.

The committee hopes that every effort will be made by candidates to ensure that they are sufficiently prepared for the examination by being thoroughly familiar with the syllabus and standard. The detailed syllabus is in the Regulations and an indication of the standard required is shown in past examination papers.

PRACTICAL ASPECTS OF THE DESIGN OF INTERMEDIATE FREQUENCY TRANSFORMERS*

by

C. E. S. Ridgers† (*Associate Member*)

SUMMARY

The performance of i.f. transformers is investigated theoretically, and the results applied to "practical design."

Dissimilar primary and secondary damping is taken into account, without complicating the analyis or the design method by the introduction of a new term—" equivalent 'Q' of the transformer."

The emphasis, throughout the paper, is on "practicability" and comparisons are made between the calculated and observed performance of a particular design, and the paper concludes with some notes on unconventional i.f. transformer construction.

LIST OF SYMBOLS USED

Symbol	*Description*	*Units*
g_m	mutual conductance of valve	mA/V
r_a	valve anode impedance	ohms
μ	valve amplification factor	
L	primary and secondary coil inductance	henries
C	primary and secondary tuning capacity, i.e. capacity required to resonate L at frequency 'f_o'	farads
$f_o = \frac{\omega_o}{2\pi}$	centre frequency and resonant frequency of L and C when not coupled to another circuit	c/s
$\delta f = \frac{\delta\omega}{2\pi}$	small frequency deviation from 'f_o'	c/s
F_p	value of δf where the response is a maximum i.e. Peaks of response curve	c/s
P	pass band, i.e. highest value of δf which is attenuated by not more than 3 db	c/s
R_s	series losses of L when removed from circuit, at 'f_o'	ohms
R_t or R_T	total (equivalent) series losses of L when in circuit, at 'f_o'	ohms
R_e	an equivalent value for the losses of L (see analysis)	ohms
$Q_s = \frac{\omega_o L}{R_s}$	criterion of L (ex-circuit), i.e. measured on a Q-meter	
$Q_w = \frac{\omega_o L}{R_T}$	the working Q of L	
R_g	value of the damping across secondary	ohms
$\alpha = \frac{Q_s\omega_o L}{r_a}$	ratio of tuned circuit impedance to r_a	
$\beta = \frac{Q_s\omega_o L}{R_g}$	ratio of tuned circuit impedance to R_g	
$x = \frac{Q_{w1}}{Q_s}$	Q reduction factor of primary	
$y = \frac{Q_{w2}}{Q_s}$	Q reduction factor of secondary	
$Q_e = \frac{\omega_o L}{R_e}$	an equivalent value of the Q for L (see analysis)	
$D = \frac{\delta f}{f_o}$		
$z = \frac{Q_e}{Q_s}$		

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† Late of Avimo Ltd., Taunton.

Symbol	*Description*	*Units*
M	mutual inductance between primary and secondary coils	henries
K	general symbol for the coupling factor between primary and secondary	
$\lvert A_o \rvert$	amplification (stage gain) at the centre frequency 'f_o'	
$\lvert A_f \rvert$	amplification at a frequency of '$f_o \pm \delta f$'	
S	selectivity characteristic $= 20 \log_{10} \frac{\lvert A_o \rvert}{\lvert A_f \rvert}$ db	db
Sk	skirt selectivity i.e. an approximate expression for 'S' at frequencies away from resonance	
$\|$	denotes 'in parallel with'	

The suffixes, $_1$ and $_2$, are used to denote the primary and secondary, respectively, where required.

Introduction

The purpose of this paper is to present a practical method for the design of Intermediate Frequency transformers, based on the theoretical analysis, which the author has found invaluable in the past.

The method takes into account a factor which is often overlooked, i.e. the dissimilar primary and secondary working "Q"s. Unless this dissimilarity is taken into account, a considerable difference will be observed between the calculated and measured characteristics of a particular transformer.

Equations are developed for the various characteristics of a transformer, i.e. gain, selectivity, pass band, etc., and, in addition, an expression is obtained for the "skirt selectivity," (defined as the selectivity obtained at frequencies well away from the centre frequency). This expression is an approximation, and its value lies in the fact that it allows rapid comparisons to be made of the selectivity obtainable, between transformers having different values of the coupling coefficient, or different "Q"s.

Design Considerations and Definitions

The three most important points to be considered in any design are :—

(1) The gain at the centre frequency.

(2) The pass band.

(3) The overall response (or selectivity) at frequencies away from the centre.

(1) The centre frequency of the response curve may or may not be the resonant frequency of the transformer, since, if it is over-coupled there are two resonant frequencies, one above and one below the centre.

(2) The pass band is defined as the change in the frequency from the centre, at which the response has dropped by 3 db. This means, in effect, that the pass band represents the highest modulating frequency at which the amplification is not less than 3 db below that of the carrier frequency.

(3) The overall response of the stage considered is obtained by plotting the variation in amplification at any off-tune frequency with respect to that at the centre frequency.

The above three points are, possibly, laid down in the design specification, or alternatively, the designer may be called upon to reach a compromise in order to suit mass-production methods.

There is one other characteristic which may be of interest to the engineer, namely, the peak frequencies (if the transformer is over-coupled).

The analysis therefore falls into five distinct sections, thus :—

(A) GAIN (at the centre frequency).

(B) RESPONSE (Overall selectivity characteristic in db).

(C) PEAK FREQUENCIES (the frequencies at which the response is a maximum).

(D) PASS BAND (half the width of the response curve at 3 db down).

(E) SKIRT SELECTIVITY (an approximate expression for the response at frequencies well away from the centre).

It is assumed that :—

(1) The primary and secondary inductances and capacities are equal.

(2) The amplifying valve is a pentode with a high value of 'r_a' though not necessarily

much higher than the dynamic (i.e. parallel) impedance of the tuned circuit (i.e. $Q_s\omega_o L$).

(3) The initial value of the primary and secondary 'Q' is identical.

(4) R_s remains constant over the range of '$f_o \pm \delta f$.'

(5) All stray capacities in the primary and secondary are included in 'C,' and all parallel damping losses in 'r_a' or 'R_g.'

(6) The coupling impedance is purely reactive i.e. loss-less.

(7) 'δf' is small compared with 'f_o,' such that :—

$$\frac{fo \pm \delta f}{f_o} \simeq 1$$

The first assumption is usually true, within a few per cent., and is the aim in designs for mass-production. Assumption 7 limits the use of the analysis to intermediate frequencies around 450 kc/s or above. At an i.f. of 125 kc/s, large errors are likely, but, since the modern trend is to use an i.f. of 465 kc/s, the equations lose little of their value.

It is advantageous to consider, firstly, a single tuned amplifier circuit, for comparison purposes because some of the expressions are required, later, for the double tuned transformer investigation.

1.0. Single Tuned Amplifiers

The equivalent circuit is given in Fig. 1a and by means of Thevenin's Theorem can be reduced to that of Fig. 1b, thus :—

From Fig. 1b and noting that :

$$\omega = 2\pi f$$

where f is any frequency

$$\frac{V_1}{\mu V_g} = \frac{\dfrac{-j}{\omega C}}{r_a - \dfrac{j}{\omega C}} = \frac{-j}{\omega C r_a - j} = \frac{-j[\omega C r_a + j]}{\omega^2 C^2 r_a^2 + 1}$$

Now $\omega^2 C^2 r_a^2 \gg 1$

$$\therefore \frac{V_1}{\mu V_g} \simeq \frac{-j}{\omega C r_a} \text{ and } V_1 = \frac{-j r_a g_m V_g}{\omega C r_a}$$

$$\therefore V_1 = \frac{-j g_m V_g}{\omega C} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Equivalent internal impedance

$$Z_e = r_a \left\| \frac{-j}{\omega C} = \frac{-j r_a}{\left[r_a - \dfrac{j}{\omega C}\right]\omega C} = \frac{-j r_a}{[\omega C r_a - j]} \right.$$

$$= \frac{-j r_a[\omega C r_a + j]}{[\omega^2 C^2 r_a^2 + 1]} = \frac{-j\omega C r_a^2 + r_a}{[\omega^2 C^2 r_a^2 + 1]}$$

$$= \frac{r_a}{[\omega^2 C^2 r_a^2 + 1]} - \frac{j\omega C r_a^2}{[\omega^2 C^2 r_a^2 + 1]}$$

Again $\omega^2 C^2 r_a^2 \gg 1$

$$\therefore Z_e \simeq \frac{1}{\omega^2 C^2 r_a} - \frac{j}{\omega C}$$

So in Fig. 1b

$$R_T = R_s + \frac{1}{\omega^2 C^2 r_a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

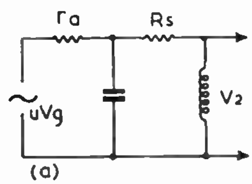

Fig. 1(a).—Theoretical circuit of tuned anode amplifying stage.

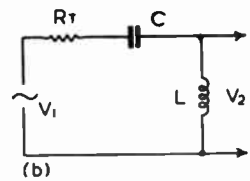

Fig. 1(b).—Thevenin's equivalent of Fig. 1(a).

From Fig. 1b, the overall gain can be calculated and is :—

$$\frac{V_2}{V_1} = \frac{j\omega L}{R_T + j\left[\omega L - \dfrac{1}{\omega C}\right]}$$

Using equation (1)

$$\frac{V_2}{V_g} = \frac{g_m \omega L}{\omega C\left[R_T + j\left(\omega L - \dfrac{1}{\omega C}\right)\right]} = \frac{g_m L}{CZ}$$

$$\ldots\ldots\ldots\ldots\ldots (3)$$

$$\text{where } Z = R_T + j\left[\omega L - \frac{1}{\omega C}\right]$$

Now Z is the impedance variation of a series tuned circuit, and for frequencies fairly close to resonance (i.e. 'δf' is small) it can be simplified as follows.

$$Z = R_T\left[1 + j\left(\frac{\omega L}{R_T} - \frac{1}{\omega C R_T}\right)\right]$$

$$= R_T\left[1 + j\left(\frac{[\omega_o + \delta\omega]L}{R_T} - \frac{1}{CR_T[\omega_o + \delta\omega]}\right)\right]$$

$$= R_T\left[1 + j\left(Q_w + \frac{\delta\omega L}{R_T} - \frac{1}{\frac{1}{Q_w} + \delta\omega C R_T}\right)\right]$$

Now

$$\delta\omega C R_T = \frac{\delta\omega R_T}{\omega_o^2 L} \text{ since } C = \frac{1}{\omega_o^2 L}$$

$$= \frac{\delta\omega}{\omega_o}\ \frac{1}{Q_w}$$

$$= \frac{\delta f}{f_o}\ \frac{1}{Q_w} \text{ or } \frac{D}{Q_w}$$

And

$$\frac{\delta\omega\,.\,L}{R_T} = \frac{\delta\omega}{\omega_o} Q_w = \frac{\delta f}{f_o} Q_w$$

since

$$\frac{L}{R_T} = \frac{Q_w}{\omega_o}$$

$$\therefore Z = R_T\left[1 + j\left(Q_w + \frac{\delta f}{f_o} Q_w - \frac{1}{\frac{1}{Q_w} + \frac{\delta f}{f_o Q_w}}\right)\right]$$

$$= R_T\left[1 + jQ_w\left(1 + \frac{\delta f}{f_o} - \frac{f_o}{f_o + \delta f}\right)\right]$$

$$= R_T\left[1 + jQ_w\left(\frac{\delta f}{f_o} + \frac{f_o + \delta f - f_o}{f_o + \delta f}\right)\right]$$

$$= R_T\left[1 + jQ_w\left(\frac{\delta f}{f_o} + \frac{\delta f}{f_o + \delta f}\right)\right]$$

Now $\delta f \ll f_o$ therefore $\frac{\delta f}{f_o + \delta f} \simeq \frac{\delta f}{f_o}$

$$\therefore \quad \simeq R_T\left[1 + j2Q_w \frac{\delta f}{f_o}\right]$$

$$\therefore Z = R_T[1 + j2\,Q_w D] \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Returning to equation (3)

$$\frac{V_2}{V_g} = A_f = \frac{g_m L}{CR_T(1 + jQ_w 2D)}$$

$$\therefore |A_f| = \frac{g_m Q_w \omega_o L}{\sqrt{1 + 4Q_w^2 D^2}} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

It is now convenient to examine the single tuned stage in the five previous sections—

1.1. *Gain*

At resonance, the imaginary term in equation 5 is zero.

$$\therefore |A_o| = g_m Q_w \omega_o L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

1.2. *Response*

By definition $S = 20 \log_{10} \frac{|A_o|}{|Af|}$ db

$$\therefore S = 20 \log_{10} \frac{g_m Q_w \omega_o L (1 + Q_w^2 4D^2)^{\frac{1}{2}}}{g_m Q_w \omega_o L}$$

from equations (5) and (6)

$$\therefore S = 10 \log_{10}\left(1 + 4Q_w^2 \frac{\delta f^2}{f_o^2}\right) \text{ db} \quad \ldots.(7)$$

By selecting values for δf in terms of Q_w and "f_o," the response given in Table 11 is constructed[1]. This table is widely known and used and is extremely valuable in practical engineering, since it enables the construction of the complete response curve of the amplifying stage, or the determination by measurement from the amplifier, of the working Q, i.e. Q_w of the tuned circuit.

1.3. *Peak Frequencies*

There will be only one peak in the response curve, at the centre frequency, "f_o".

1.4. *Pass Band*

By definition, "S" in equation (7) equals 3 when $\delta f = P$

$$\therefore 2 = 1 + \frac{4Q_w^2 \delta f^2}{f_o^2}$$

$$\therefore \delta f = P = \frac{f_o}{2Q_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

1.5. *Skirt Selectivity*

If in equation (7) $4Q_w^2 \frac{\delta f^2}{f_o^2} > 1$

Then

$$S_k = 20 \log_{10} \frac{2Q_w \delta f}{f_o} \text{ db}$$

$$S_k = \left[20 \log_{10} \frac{Q_w \delta f}{f_o} \right] + 6 \text{ db} \ldots\ldots (9)$$

Working " Q "

When using the equations above, it is emphasized that the value of " Q " which must be used, is the working " Q " i.e. " Q_w ".

This value is found in the following manner, from the value of " Q_s" and a knowledge of the parallel damping existing across the coil. In the case of a single coil, or the primary in double tuned circuits, the damping is " r_a " ; in the case of a secondary the damping is " R_g "

For an anode coil ;

$$Q_{w1} = \frac{\omega_o L}{R_{T1}}$$

where

$$R_{T1} = R_s + \frac{1}{\omega_o^2 C^2 r_a} \text{ (see equation (2)).}$$

For the secondary coil

$$Q_{w2} = \frac{\omega_o L}{R_{T2}}$$

$$\text{where} \quad R_{T2} = R_s + \frac{1}{\omega_o^2 C^2 r_g}$$

Primary ;

$$\frac{\omega_o L}{Q_{w1}} = \frac{\omega_o L}{Q_s} + \frac{\omega_o^2 L^2}{r_a}$$

Secondary ;

$$\frac{\omega_o L}{Q_{w2}} = \frac{\omega_o L}{Q_s} + \frac{\omega_o^2 L^2}{R_g}$$

Therefore :

$$\frac{1}{Q_{w1}} = \frac{1}{Q_s} + \frac{\omega_o L}{r_a}$$

$$\text{and} \quad \frac{1}{Q_{w2}} = \frac{1}{Q_s} + \frac{\omega_o L}{R_g}$$

$$\therefore \; Q_{w1} = Q_s \frac{r_a}{(r_a + Q_s \omega_o L)}$$

$$\text{and} \quad Q_{w2} = Q_s \frac{R_g}{R_g + Q_s \omega_o L}$$

$$\text{By definition } \alpha = \frac{Q_s \omega_o L}{r_a}$$

$$\text{and} \quad \beta = \frac{Q_s \omega_o L}{R_g}$$

$$\therefore \; Q_{w1} = Q_s \frac{1}{1 + \alpha} \ldots\ldots\ldots\ldots (10)$$

$$\text{and} \quad Q_{w2} = Q_s \frac{1}{1 + \beta} \ldots\ldots\ldots\ldots (11)$$

$$\text{By definition, } x = \frac{Q_{w1}}{Q_s}$$

$$\text{and} \quad y = \frac{Q_{w2}}{Q_s}$$

$$\text{Therefore,} \quad x = \frac{1}{1 + \alpha} \ldots\ldots\ldots\ldots\ldots (12)$$

$$\text{and} \quad y = \frac{1}{1 + \beta} \ldots\ldots\ldots\ldots\ldots (13)$$

From equations (10), (11), (12), (13) above, the importance of a high value of " r_a" or " R_g " can be clearly seen. The effect of the damping is to reduce the value of the Q of the coils, and since the gain and selectivity, in both single and double tuned amplifiers, depends on the working Q, then " r_a," "R_g " and " Q_s " should be as high as possible.

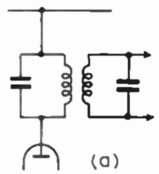

Fig. 2(a).—Doubled tuned transformer coupled amplifier.

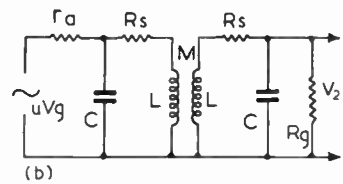

Fig. 2(b).—Theoretical circuit of Fig. 2(a).

2.0. Double Tuned Transformer Coupled Amplifiers

The circuit of a double tuned transformer connected in the anode circuit of a valve is shown in Fig. 2a, the equivalent circuit being that given in Fig. 2b. By the use of Thevenin's Theorem, the primary is reduced to the circuit shown in Fig. 3 (as was done in Fig. 1b) whilst the secondary losses are increased, due to the presence of " R_g," to a value given by equation (2) (" R_g " being substituted for " r_a ").

The primary damping (r_a) and the secondary damping (R_g) may be such that the working Q's are very different, and, under such conditions, the analysis is greatly complicated. But, by the introduction of the expression " equivalent Q," symbol " Q_e," it is a simple matter to derive practical expressions for the transformer characteristics.

Considering that, initially (i.e. ex circuit), the primary and secondary Q's are equal (Assumption 3) and the primary Q is reduced (by the presence of " r_a ") to Q_{w1}, and the secondary Q is reduced (by the presence of " R_g)" to Q_{w2}

Then from equation (12)

$$x = \frac{Q_{w1}}{Q_s} = \frac{1}{1+\alpha}$$

and from equation (13)

$$y = \frac{Q_{w2}}{Q_s} = \frac{1}{1+\beta}$$

Then, " Q_e " must be given a value, such that, in the particular expression containing xQ_s and yQ_s, " Q_e " can be substituted for each with no error.

For zero error, it is necessary that :—

$$x\,Q_s + y\,Q_s = 2\,Q_e$$

and $$x\,.\,Q_s\,.\,y\,.\,Q_s = Q_e^2$$

since, both the sum and the product of the primary and secondary working Q's are required.

There is no simple value for " Q_e " which will satisfy the above conditions, with negligible error, over a wide range of values for " x " and " y," but, if :—

$$Q_e = zQ_s = \sqrt{x\,.\,y\,.}\;Q_s \quad \ldots\ldots(14)$$

then the error of the product is zero, for all values of " x " and " y " ; and the error of the sum is zero, when " x " equals " y." When " x " and " y " differ, the error is not greater than 5 per cent., provided that the ratio of the two factors is not less than 0·523,

$$\text{i.e. } \frac{x}{y} \text{ or } \frac{y}{x} \text{ greater than } 0{\cdot}523 \quad \ldots\ldots\ldots(15)$$

The following analysis is derived, initially, using dissimilar values for the primary and secondary working Q's, namely, " Q_{w1} " and " Q_{w2}," and, when appropriate, " Q_e " is substituted. The only error caused by the substitution is in the case of the sum of " Q_{w1} " and " Q_{w2}," and, since the sum is of considerably less importance in the numerical evaluation of any of the expressions, the maximum allowable error, which is suggested, i.e., 5 per cent., results in a very small final error.

The error in the product of " x " and " y " must be a minimum, and, using the suggested values for "z," the error is zero for all values and all ratios, of " x " and " y."

The analysis is continued by deriving the relevant expressions for the equivalent transformer circuit shown in Fig. 3.

In Fig. 3, it is noted that " X_L " is used as a symbol to represent $(\omega L \pm X_m)$ to avoid cumbersome repetition.

Using Kirchoff's equations in Fig. 3 :—

$$V_1 = i_1 Z_1 - jX_m i_2 \quad \ldots\ldots\ldots\ldots\ldots(16)$$

$$O = i_2 Z_2 - jX_m i_1 \quad \ldots\ldots\ldots\ldots\ldots(17)$$

$$V_2 = \frac{-ji_2}{\omega C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

where Z_1 and Z_2 are the primary and secondary series impedances respectively.

i.e. $Z_1 = R_{T1} + jX_1$

and $Z_2 = R_{T2} + jX_2$

From equation (17)

$$i_1 = \frac{i_2\,.\,Z_2}{j\,.\,X_m}$$

From equations (16) and (17)

$$V_1 = \frac{i_2(Z_1 Z_2 + X_m)}{j\,.\,X_m} \quad \ldots\ldots\ldots\ldots\ldots(19)$$

From equations (18) and (19)

$$\frac{V_2}{V_1} = \frac{-j\,.\,i_2\,.\,jX_m}{\omega C\,.\,i_2\,.\,(Z_1\,.\,Z_2 + X_m^2)}$$

$$= \frac{X_m}{\omega C(Z_1 Z_2 + X_m^2)}$$

and by the use of equation (1)

$$\frac{V_2}{V_g} = \frac{-j\,.\,g_m\,.\,X_m}{\omega^2 C^2 (Z_1 Z_2 + X_m^2)} = A_f \quad \ldots(20)$$

$$\therefore\; A_f = \frac{-j\,.\,g_m\,.\,X_m}{\omega^2 C^2\big((R_{T1} + jX)(R_{T2} + jX) + X_m^2\big)} \quad \ldots\ldots\ldots\ldots\ldots(21)$$

At resonance, the imaginary terms are zero, and therefore :—

$$|A_o| = \frac{g_m\,.\,X_m}{\omega_o^2 C^2 (R_{T1} R_{T2} + X_m^2)} \quad \ldots\ldots(22)$$

To find the condition for maximum gain, when X_m is varied, equation (22) is differentiated with respect to X_m, and equated to zero :—

$$\frac{d|A_o|}{dX_m} = \frac{g_m X_m(-1)(R_{T1} R_{T2} + X_m)^{-2} 2X_m}{\omega_o^2 C^2} + \frac{(R_{T1} R_{T2} + X_m)^{-1} g_m}{\omega_o^2 C^2}$$

$$\text{and } 0 = 1 - \frac{2X_m^2}{(R_{T1}R_{T2} + X_m^2)}$$

$$\therefore 2X_m^2 = R_{T1}R_{T2} + X_m^2$$

$$\text{and } X_m^2 = R_{T1}R_{T2}$$

Now

$$R_{T1} = \frac{\omega_o L}{Q_{w1}}$$

$$\text{and } R_{T2} = \frac{\omega_o L}{Q_{w2}}$$

By definition

$$x = \frac{Q_{w1}}{Q_s} \text{ and } y = \frac{Q_{w2}}{Q_s}$$

$$\therefore \quad R_{T1} = \frac{\omega_o L}{xQ_s} \text{ and } R_{T2} = \frac{\omega_o L}{yQ_s}$$

Therefore

$$X_m^2 = \frac{\omega_o^2 L^2}{Q_s x \,.\, y\,.}$$

$$\text{and } X_m = \frac{\omega_o L}{Q_s\sqrt{xy}} = \frac{\omega_o L}{Q_e} \quad \ldots\ldots\ldots\ldots (23)$$

Equation (23) gives the condition for maximum gain and is the so-called " critically coupled " condition. If the value of " X_m" is increased above this, the gain at resonance decreases and the response curve becomes flatter, finally exhibiting the phenomena of " double humps " or " rabbit ears," whilst the overall selectivity decreases. Below the above value for "X_m," the gain is less, but the selectivity is increased. These points are discussed later when limiting values are obtained.

Equation (23) is inconvenient and a more practical form is obtained later. Before this is derived, an examination of coupling factors and types of coupling in common use is advantageous.

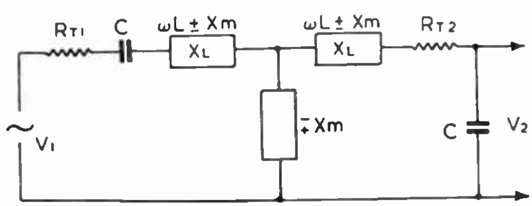

Fig. 3.—Thevenin's equivalent of Fig. 2(b).

3.0. Types of Xm and Coupling Factors

The previous analysis and value of Xm (equation 23) assumes that the coupling impedance is mutual, (i.e. common to both primary and secondary, as in Fig. 3), but it does not define or limit the type of impedance.

X_m may be capacitive, inductive, mutually inductive, or a mixture of both, and, in this section, these couplings are examined and the relevant expressions obtained.

In the case of mixed couplings comprising mutual inductance and bottom or top capacity (Figs. 4d and 4e), the sign of the mutual inductance is important. If it is required to be negative, this can be arranged by connecting the two coils (i.e. primary and secondary) in " series-aiding," i.e. when looking into the free ends of the coils (one end of each coil is " earthy " and therefore connected together so far as a.c. is concerned), the total inductance obtained is :—

$$L\text{ (total)} = L_1 + L_2 + 2M$$

The definition of " coupling factor " is:—

$$K = \frac{X_m}{X_1 X_2} = \frac{X_m}{X} \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

where

X_m = mutual coupling reactance

and $X_1 = X_2 = \omega_o L$ = total similar reactance of primary and secondary respectively.

The value of X_m for the critically coupled transformer was obtained in equation (23) ; and from equations (23) and (24) :—

$$X_m = \frac{\omega_o L}{Q_e}$$

therefore

$$K = \frac{1}{Q_e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The condition for critical coupling can therefore be given as :—

$$K\,.\,Q_e\,. = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

When dealing with i.f. transformers, they can be conveniently denoted as being tightly or loosely coupled. This refers to the value of the coupling required for the critically coupled condition, as given above.

4.0. Methods of Coupling Transformers

In Fig. 4, the actual and equivalent circuits for five methods of coupling are shown. These do not exhaust all possible arrangements but are the types commonly encountered.

For each circuit, the coupling factor is determined, as well as the condition for critical coupling, in terms of the circuit constants. The emphasis, in the analysis, is on simplicity, and this is particularly desirable in the case of the mixed couplings involving inductive and capacitive elements.

4.1. *Mutual Inductance Coupling* (*Fig. 4a*)

From equation (24)

$$K = \frac{X_m}{X}$$

Since $X_m = \omega M$

$$\therefore \quad K = \frac{M}{L} \quad \text{.....................(27)}$$

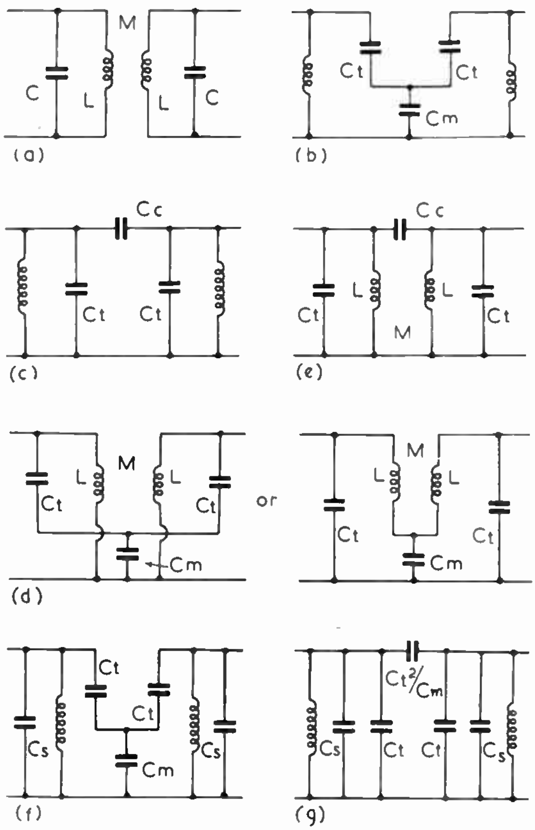

Fig. 4(a).—Mutual inductance.
Fig. 4(b).—Bottom capacity.
Fig. 4(c).—Top capacity.
Fig. 4(d).—Mutual inductance and bottom capacity.
Fig. 4(e).—Mutual Inductance and Top Capacity.
Fig. 4(f).—Modified Bottom Capacity.
Fig. 4(g).—Equivalent of Fig. 4(f).

and the condition for critical coupling is :—

$$M = \frac{L}{Q_e} \quad \text{...................(28)}$$

4.2. *Bottom Capacity Coupling* (*Fig. 4b*)

$$K = \frac{C_t}{C_t + C_m} \text{................(29)}$$

The actual tuning capacity, i.e. "C," is produced by "C_t" in series with "C_m," thus :—

$$C = \frac{C_t \, . \, C_m}{C_t + C_m}$$

Solving for $(C_t + C_m)$ and replacing in equation (29) gives :—

$$K = \frac{C}{C_m} \quad \text{...................(30)}$$

and the condition for critical coupling is :—

$$C_m = Q_e \, . \, C \text{ or } C_m = C_t(Q_e - 1) \text{..(31)}$$

4.3. *Top Capacity Coupling* (*Fig. 4c*)

$$K = \frac{C_c}{C_t + C_c} \quad \text{..............(32)}$$

and the condition for critical coupling is :—

$$C_c = \frac{C_t}{Q_e - 1} \quad \text{................(33)}$$

Note that the coil tuning capacity, i.e. "C," is equal to :—

C_t plus (C_c in series with C_t)

4.4. *Mutual Inductance and Bottom Capacity Coupling* (*Fig. 4d*)

This is a mixed coupling, and :—

$$X_m = \pm \, \omega M - 1/\omega C_m$$

$$\text{Therefore } K_t = \frac{\pm \, \omega M - 1/\omega C_m}{\omega L}$$

where "K_t" is the total coupling due to the inductive and capacitive effects.

$$\text{Now} \quad K_t = \frac{\pm \, \omega M}{\omega L} - \frac{1}{\omega L . \omega C_m}$$

$$= \pm \, K_m - \frac{1}{\omega^2 . L . C_m}$$

where "K_m" is the coupling factor calculated on the basis of mutual inductive coupling alone.

Now $\dfrac{1}{\omega^2 . L} = C$ where "C" is the coil tuning capacity,

$$\text{and } C = \frac{C_t \,.\, C_m}{C_t + C_m}$$

therefore

$$\frac{1}{\omega^2 \,.\, L \,.\, C_m} = \frac{C_t \,.\, C_m}{(C_t + C_m)\, C_m} = \frac{C_t}{(C_t + C_m)}$$

$$\text{and } \frac{C_t}{C_t + C_m} = K_c$$

where " K_c " is the coupling factor calculated on the basis of mutual capacity coupling alone.

Therefore $K_t = \pm K_m - K_c$ ....(34)

and the condition for critical coupling is: —

$$\pm K_m - K_c = \frac{1}{Q_c} \quad \text{....................(35)}$$

The importance of the sign of " M," therefore, in " K_m," can be seen from equation 34. The additional coupling due to the capacity may aid or oppose that due to the mutual inductance, with a consequent change in the final value of the combined coupling factor, " K_t."

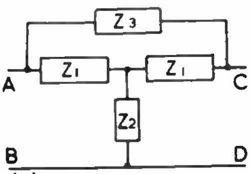

Fig. 5(*a*).—*Bridged T section.*

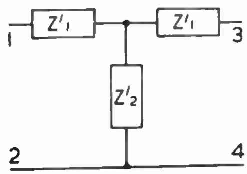

Fig. 5(*b*).—*Symmetrical T section.*

4.5. *Mutual Inductance and Top Capacity Coupling* (*Fig.* 4e)

This, again, is a mixed coupling, less amenable to manipulation than the previous example.

The analysis is simplified by transforming the bridged T-section (see Fig. 5a) into a symmetrical T-network (see Fig. 5b).

Referring to Fig. 5a and 5b :—

For the transform to be correct, the impedance looking into the terminals " A " and " B " (also " C " and " D," since similarly positioned impedances are identical) must equal that across " 1 " and " 2 " (similarly " 3 " and " 4 "). Also the impedance across " A " and " C " must equal that across " 1 " and " 3 "

Therefore, with " CD " and " 34 " open circuit :—

$$Z(AB) = [Z_1 // (Z_3 + Z_1)] + Z_2$$
$$Z(12) = Z_1' + Z_2'$$
$$Z(AC) = Z_3 // 2 \,.\, Z_1$$
$$Z(13) = 2 \,.\, Z_1'$$

Now

Z(AB) = Z12), and therefore :—

$$\frac{Z_1(Z_3 + Z_1) + Z_2(2 \,.\, Z_1 + Z_3)}{2 \,.\, Z_1 + Z_3} = Z_1' + Z_2' \quad \text{..............(1a)}$$

And

$$Z(AC) = Z(13)$$

Therefore

$$\frac{2 \,.\, Z_3 \,.\, Z_1}{2 \,.\, Z_1 + Z_3} = 2 \,.\, Z_1'$$

$$\therefore \; Z_1' = \frac{Z_3 \,.\, Z_1}{(2 \,.\, Z_1 + Z_3)} \quad \text{............(2a)}$$

From 1a and 2a

$$Z_2' = \frac{Z_1(Z_3 + Z_1) + Z_2(2 \,.\, Z_1 + Z_3) - Z_3 Z_1}{(2 \,.\, Z_1 + Z_3)}$$
$$= \frac{Z_1 Z_3 + Z_1^2 + 2 \,.\, Z_1 Z_2 + Z_2 Z_3 - Z_1 Z_3}{(2 \,.\, Z_1 + Z_3)}$$
$$= \frac{Z_2(2 \,.\, Z_1 + Z_3)}{(2 \,.\, Z_1 + Z_3)} + \frac{Z_1^2}{(2 \,.\, Z_1 + Z_3)}$$

Therefore

$$Z_2' = Z_2 + \frac{Z_1^2}{(2 Z_1 + Z_3)}$$

and

$$Z_1' = \frac{Z_1 Z_3}{(2 \,.\, Z_1 + Z_3)}$$

from equation 2a.

Considering Z_3 as a negative inductance, viewing Fig. 5b as a modified transformer and calculating the total coupling coefficient, gives :

$$K_t = \frac{\text{M equivalent}}{\text{L equivalent}} = \frac{Z_2'}{Z_1'}$$

Therefore

$$K_t = \frac{Z_2 + \dfrac{Z_1^2}{(2 \,.\, Z_1 + Z_3)}}{\dfrac{Z_1 Z_3}{(2 \,.\, Z_1 + Z_3)}}$$
$$= \frac{2 \,.\, Z_1 Z_2 + Z_2 Z_3 + Z_1^2}{Z_1 Z_3}$$
$$= \frac{Z_2}{Z_1} + \frac{(2 \,.\, Z_2 + Z_1)}{Z_3}$$

$$\text{and} \quad K_t = \pm K_m + \frac{(2 \,.\, Z_2 + Z_1)}{Z_3} \quad \text{....(3a)}$$

where " K_m " is the amount of coupling due to the transformer (i.e. inductive) effect alone. For ease of manipulation, the sign of " Z_2 " has been ignored until equation 3a. The equations are not affected by this, but K_m may of course be negative.

Rationalizing the last term in equation 3a :

$$\frac{2.Z_2 + Z_1}{Z_3} = \frac{\pm 2\omega M + \omega L}{-\dfrac{1}{\omega C_c}}$$

$$= -\omega^2 C_c(\pm 2.M + L)$$

$$= -\omega^2 C_c L\left(\pm 2.\frac{M}{L} + 1\right)$$

$$= -\omega^2 C_c L(\pm 2.K_m + 1)$$

In practice $2.K_m$ is much less than 1,

therefore $\dfrac{2.Z_2 + Z_1}{Z_3} \simeq -\omega^2 C_c L$

Now $\omega^2 L = \dfrac{1}{C_t + \dfrac{C_c C_t}{C_c + C_t}}$

$$= \frac{1}{\dfrac{C_c.C_t + C_t(C_c + C_t)}{(C_c + C_t)}}$$

$$= \frac{(C_c + C_t)}{C_t(C_t + 2.C_c)}$$

and, therefore :—

$$-\omega^2 C_c L = \frac{-C_c(C_t + C_c)}{C_t(C_t + 2.C_c)}$$

since $C_c < C_t$; then $(C_t + C_c) \simeq (C_t + 2.C_c) \simeq C_t$ and

$$-\omega^2 C_c L \simeq \frac{-C_c}{(C_t + C_c)} \simeq -K_c$$

(where K_c is the value of the coupling factor based on the top capacity coupling alone.)

Returning to equation 3a :

$$\therefore\ K_t = \pm K_m + \frac{(2.Z_2 + Z_1)}{Z_3} \quad \ldots\ldots (3a)$$

From above $\dfrac{(2.Z_2 + Z_1)}{Z_3} \simeq K_c$

replacing in equation 3a :

$$K_t = \pm K_m - K_c \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and for critical coupling,

$$\pm K_m - K_c = \frac{1}{Q_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The error in the second term of equation 36, i.e. " K_c," due to the two approximations is small and is of the order of the square of the numerical value of " K_c." In a practical case, " K " may be of the order of 0·01 ; and the error therefore is vanishingly small.

In practical transformers, the top capacity coupling " C_c," is often caused by the capacity existing between the coils or by the strays between the connecting leads to the transformer. Thus no physical component may be added, intentionally, and, the circuit may appear to be the same as that of Fig. 4a. In most cases of mutual inductive coupling, top capacity coupling is inevitable. The sign of " M " is important, and the actual value of the coupling factor is open to doubt, since it is a difficult matter to determine the value of the distributed stray capacities between the coils, and the external wiring strays will depend on the wiring layout.

The fact that top capacity may exist in an apparently, mutually inductively coupled transformer is not usually inconvenient, since its effect on the coupling factor can be compensated for by a proportionate change in " M." But when the design of variable band width transformers, using a variable mutual inductance, is being considered, the change in the value of the top capacity, due to the alteration in the spacing between the coils, will cause a change in the effective tuning capacity across the coils, resulting in de-tuning and an assymetrical response curve.[1] For such variable transformers the top capacity must be kept small or made zero by an electrostatic screen between the two coils.

4.6. *Modified Bottom Capacity Coupling* (*Fig.* 4f)

This coupling is essentially the same as that shown in Fig. 4b, but, due to the presence of stray capacities across each coil, i.e. "C_s," the actual value of " K " differs from that given in Fig. 4b.

The section of the circuit, between the dotted lines is now transformed into the equivalent circuit for top capacity coupling, Fig. 4c, but the value of " C_t " is maintained constant.

For the transform to be true :—

For bottom capacity coupling (Equation 29)	For top capacity coupling (Equation 32)

$$K = \frac{C_t}{C_t + C_m} = K = \frac{C_c}{C_t + C_c}$$

If " C_t " = constant, then :

$$C_c(C_t+C_m)= C_t(C_t + C_c)$$

and $C_c = \dfrac{C_t^2}{C_m}$

Thus, to obtain the same value of " K ", it is necessary to replace the bottom capacity " C_m", with a top capacity of C_t^2/C_m, the value of " C_t " remaining constant.

The equivalent circuit is now that shown in Fig. 4g, and the value of " K " for this type of circuit, is, by similarity with Fig. 4c, and from equation 32 :

$$K = \frac{C_c}{C_t + C_c}$$

But for Fig. 4g :

$$C_c = \frac{C_t^2}{C_m}$$

and C_t is increased to $(C_t + C_s)$

$$= S.C_t \text{ (say)}$$

where $S = \dfrac{(C_t + C_s)}{C_t}$ ....(38)

Therefore, K (for Fig. 4f) is given by

$$\frac{\dfrac{C_t^2}{C_m}}{S.C_t + \dfrac{C_t^2}{C_m}}$$

$$K = \frac{C_t}{C_t + S\,.\,C_m} \qquad \text{..............(39)}$$

Thus, equation 39 gives the value of the coupling factor for the circuit shown in Fig. 4f, the condition for critical coupling, again, being when equation 39 equals the reciprocal of " Q_e."

Note that in Fig. 4f the tuning capacity is :

$$C = C_s + \frac{C_t C_m}{C_t + C_m}$$

usually $C_m \gg C_t$

and $C = C_s + C_t$

5.0. General Derivation of Practical Expressions

From the several equations developed earlier, it is now possible to derive general expressions for the five sections given in the introduction, namely, Gain, Response, Peak frequencies, Pass band and Skirt selectivity.

In the following analysis the equivalent "Q," i.e. " Q_e," is introduced.

5.1. *Gain*

From equation 20

$$A_f = \frac{-j\,.\,g_m\,.\,X_m}{\omega^2C^2(Z_1Z_2 + X_m^2)}$$

Now $Z_1 = (R_{T1} + jX)$ and $Z_2 = (R_{T2} + jX)$

and by similarity with equations 2 and 4

$$Z_1 = R_{T1}(1 + jQ_1 2\,.\,D)$$

and $Z_2 = R_{T2}(1 + jQ_2 2\,.\,D)$

where " Q_1 " and " Q_2 " are the primary and secondary *working* " Q "s respectively.

From equation 24

$$K = \frac{X_m}{\omega_o L}$$

$$\therefore\ X_m = K\,.\,\omega_o L$$

$$\therefore\ A_f = \frac{-jg_m K\,.\,\omega_o L}{\omega^2C^2(R_{T1}R_{T2}(1+jQ_1 2.D)(1+jQ_2 2.D)+K^2\omega^2L^2)}$$

If " δf" is small in comparison with " f_o " (assumption 7) then

$$\omega (= \omega_o \pm \delta\omega) = \omega_o$$

and $\dfrac{\omega_o L}{R_{T1}} = \dfrac{1}{\omega C R_{T1}} = Q_1$

and $\dfrac{\omega_o L}{R_{T2}} = \dfrac{1}{\omega C R_{T2}} = Q_2$

Replacing the above values in the equation for " A_f," and dividing the denominator by "R_{T1}" and R_{T2}, gives :

$$A_f = \frac{-jg_m K\omega_o L Q_1 Q_2}{(1 + jQ_1 2\,.\,D)(1 + jQ_2 2\,.\,D) + K^2Q_1Q_2}$$

$$= \frac{-jg_m K\omega_o L Q_1 Q_2}{(1 - Q_1Q_2\,.\,4\,.\,D^2 + K^2Q_1Q_2) + j2\,.\,D(Q_1+Q_2)}$$

Q_1 is the primary working " Q," i.e. Q_{w1}, and from equation 12, it equals xQ_s.

Q_2 is the secondary working " Q," i.e. Q_{w2}, and from equation 13 it equals yQ_s. Therefore, from equation 14 and the discussion preceding it, $(Q_1\,.\,Q_2)$, and $(Q_1 + Q_2)$ in the above expression for A_f, can be replaced by Q_e^2 and $2\,.\,Q_e$ respectively where, from equation 14 :

$$Q_e = zQ_s = \sqrt{x\,.\,y}\,.\,Q_s$$

It is apparent from the expression for " A_f " that, for the error in the sum of $(Q_1 + Q_2)$ when replaced by $2.Q_e$ to materially affect the numerical evaluation of "A_f," the term :

$$j2.D(Q_1+Q_2)$$

must be greater than

$$(1-Q_1Q_2.4D^2+K^2Q_1Q_2)$$

or, replacing with Q_e, and considering the modulus :

$$16.Q_e^2.D^2$$

must be greater than

$$(1-4.Q_e^2.D^2+K^2.Q_e^2)^2$$

For the second term to be a minimum, $K.Q_e$ must be made to approach zero, and 'D' ($=\delta f/f_0$) is given a value, such that :

$$1-4.Q_e^2.D^2=1$$

$$\text{Thus } D=\frac{1}{Q_e\sqrt{2}} \quad \left(\text{'}\delta f\text{'}=0{\cdot}707.\frac{f_0}{Q_e}\right)$$

With the above, the error in $|A_f|$ is 5 per cent., for a 6 per cent. error in the sum ($Q_1+Q_2=2.Q_e$) at a value of $K.Q_e=O$, and 'D' $=1/Q_e\sqrt{2}$.

If 'D' is increased to $1/Q_e$, and the error in the sum is still 6 per cent., the final error in $|Af|$ becomes 1·8 per cent.

At critical coupling, i.e. '$K.Q_e$' $=1$, with 'D' $=1/Q_e$, a 6·5 per cent. error in the sum results in a 5 per cent. error in $|A_f|$, but increasing 'D' to $2/Q_e$, means that there will be an error, in the sum, of 20 per cent., for the same error in $|A_f|$.

Therefore, from practical considerations, the maximum error of 5 per cent. suggested for the sum (equation 15), corresponding to a ratio of 'x' to 'y' of 0·523, results in small overall errors.

These errors will normally be masked by those caused by making the assumptions given after the Introduction, and by those inherent in the general method of the analysis.

Returning to 'A_f', the modulus is therefore :

$$|A_f|=\frac{g_m.K.\omega_0L.Q_e^2}{\sqrt{(1-4.Q_e^2.D^2+K^2.Q_e^2)^2+16.Q_e^2D^2}} \quad \ldots\ldots\ldots\ldots\ldots(40)$$

Equation 40 represents the gain at any value of $K.Q_e$ (or K) and at any off-tune frequency 'δf'

At resonance, 'δf' is zero, and putting 'D' $=$ O in equation 40, the gain, i.e. $|A_o|$, of the transformer and valve at the centre frequency 'f_0' will be obtained, at any value of '$K.Q_e$.'

$$\therefore |A_o|=\frac{g_m.Q_e.\omega_0L.K.Q_e}{(1+K^2.Q_e^2)} \quad \ldots\ldots\ldots\ldots(41)$$

5.2. *Response*

The response has been defined earlier and is

$$S=20\log_{10}\frac{|A_o|}{|A_f|}\text{ db}$$

The gain at resonance is given by equation 41, i.e. for $|A_o|$.

The gain at ($f_0\pm\delta f$) is given by equation 40 i.e. for $|A_f|$.

$$\therefore S=20\log_{10}\frac{\sqrt{(1+K^2Q_e^2-4.Q_e^2.D^2)^2+16.Q_e^2.D^2}}{(1+K^2.Q_e^2)}$$

$$\therefore S=10\log_{10}\left(1+\frac{8.Q_e^2.D^2(1-K^2.Q_e^2)+16.Q_e^4.D^4}{(1+K^2.Q_e^2)^2}\right) \quad \ldots\ldots\ldots\ldots\ldots(42)$$

5.3. *Peak Frequencies*

For values of $K.Q_e$ greater than unity, equation 40 will have two maxima, representing the peak frequencies, 'F_p.' Equation 40 is symmetrical about 'f_o,' and the two peaks will therefore be equi-spaced on either side of 'f_o.'

By differentiating equation 40 with respect to 'δf' (or 'D') and equating to zero, the maxima can be determined, thus :

$$|A_f|=\frac{g_mKQ_e^2\omega_oL}{\sqrt{(1+K^2Q_e^2-4.Q_e^2.D^2)^2+16.Q_e^2.D^2}}$$

$$\frac{d|A_f|}{dD}=\frac{\frac{1}{2}g_m\omega_oLKQ_e^2((1+K^2Q_e^2-4.Q_e^2D^2)^2+16Q_e^2D^2)^{-\frac{1}{2}}(-16Q_e^2D(1+K^2Q_e^2-4.Q_e^2D^2)+32Q_e^2D)}{((1+K^2Q_e^2-4.Q_e^2.D^2)^2+16.Q_e^2.D^2)}$$

Equating to zero :

$$O = -16.Q_e^2.D(1 + K^2Q_e^2 - 4.Q_e^2D^2) + 32.Q_e^2.D$$

$$O = -(1 + K^2Q_e^2 - 4.Q_e^2.D^2) + 2$$

$$\therefore\ 4.Q_e^2D^2 = (K^2.Q_e^2 - 1)$$

and since

$$D = \frac{\delta f}{f_o}$$

$$\therefore\ \delta f = \pm \frac{f_o}{2.Q_e}\sqrt{K^2.Q_e^2 - 1} \quad \ldots\ldots\ldots(43)$$

and the peak frequencies are :

$$F_p = f_o\left\{1 \pm \frac{1}{2.Q_e}\sqrt{K^2.Q_e^2 - 1}\right\} \quad \ldots(44)$$

Equation 44 gives the Peak frequencies in terms of the centre frequency and the equivalent, 'Q,' i.e. 'Q_e' of the transformer, for any value of the coupling.

The heights of the peaks, i.e. the amplification, at the frequencies given by equation 44 can be obtained, simply, by giving 'δf' (or 'D') in equation 40 the value found from equation 43. thus :

Amplification at peaks,

$$|A_p| = \frac{g_m K Q_e^2 \omega_o L}{\sqrt{\left(1 + K^2Q_e^2 - 4.\dfrac{Q_e^2(K^2Q_e^2 - 1)}{4.Q_e^2}\right)^2 + 16.Q_e^2\dfrac{(Q_e^2K^2 - 1)}{4.Q_e^2}}}$$

$$= \frac{g_m K.Q_e^2.\omega_o L}{\sqrt{4.Q_e^2.K^2}}$$

$$\therefore\ |A_p| = \frac{g_m.Q_e.\omega_o L}{2}$$

The value of $|A_p|$ is identical with that in equation 41 when $K.Q_e$ equals unity, i.e. the gain at the centre frequency at critical coupling.

The amplification at the peaks, therefore, in an over-coupled transformer, cannot be greater than the amplification for the same transformer at the centre frequency with critical coupling.

In a later section of the paper, tables are given for the response of a transformer against varying values of $K.Q_e$; and the response at the peaks, for the over-coupled condition, is given as positive, since the response is referred to the gain at the centre frequency at the particular value of $K.Q_e$ considered.

5.4. *Pass Band*

The pass band has been defined earlier, and therefore by definition, is equal to the value of 'δf' satisfying equation 42, when 'S' = 3 db

$\therefore$ From equation 42

$$16.Q_e^4.D^4 + 8.Q_e^2.D^2(1 - K^2.Q_e^2) - (1 + K^2.Q_e^2) = 0$$

$$\therefore\ Q_e^2.D^2 = \frac{K^2.Q_e^2 - 1 \pm \sqrt{2 + K^2.Q_e^2(1 + K^2.Q_e^2)}}{4}$$

$$\therefore\ \delta f = P = \frac{f_o}{2.Q_e}\sqrt{K^2.Q_e^2 - 1 + \sqrt{2 + K^2.Q_e^2(1 + K^2.Q_e^2)}} \quad \ldots\ldots\ldots\ldots\ldots(45)$$

Only the positive values of equation 45 need be considered, and the negative roots are ignored, since no significance can be attached to a negative pass band.

It is pointed out that equation 45 gives the value of 'δf' at which the response is 3 db below that at the centre frequency for the given value of $K.Q_e$. If the transformer is overcoupled, the peaks may rise to a considerable height above 0 db, and the usefulness of the above expression is therefore limited in estimating the Pass Band.

The physical basis of the term, Pass Band, and the reason for its use, is that the human ear does not notice a change in sound intensity corresponding to 3 db. Equation 45, will thus, fulfil its purpose for lower values of $K.Q_e$; but with greatly overcoupled circuits, the rise in amplification at the peaks will become noticeable. This can be overcome by arranging that the overall response curve of the receiver is substantially flat, i.e., within 3 db, by utilizing another amplifying stage to place a third peak, of the same height, at 'f_o,' i.e. midway between those due to the over-coupled transformer.[2]

5.5. *Skirt Selectivity*

The expression for the response can, under certain conditions, be simplified, thus enabling quick and easy comparisons between the selectivity curves obtained at different values of 'Q_e' and/or coupling, $K.Q_e$.

If, in equation 42, 'δf' is sufficiently far from resonance, that :

$$\frac{16 . Q_e^4 . D^4}{(1 + K^2 Q_e^2)^2} \gg \frac{8 . Q_e^2 . D^2(1 - K^2 . Q_e^2)}{(1 + K^2 . Q_e^2)^2} \gg 1$$

Then

$$Sk = 10 \log_{10} \frac{16 . Q_e^4 D^4}{(1 + K^2 . Q_e^2)}$$

$$\therefore Sk = \left(40 \log_{10} Q_e \frac{\delta f}{f_o} - 20 \log_{10}(1 + K^2 . Q_e^2)\right) + 12 \text{ db} \quad \dots\dots\dots(46)$$

The accuracy of the above expression can be judged from the tables given towards the end of the paper. It is intended, only, as a method for quick comparison of the selectivity obtained at higher values of 'δf,' although it retains considerable usefulness at values of 'δf' fairly close to the centre frequency, 'f_o.'

5.6. *Derivation of Constants*

The five expressions derived above can be used to determine the constants of a required design of transformer coupled stage. By giving selected values to K.Q_e, in terms of the value for the critically coupled condition i.e., 1, very simple equations for the five sections are obtained. Further, by giving 'δf' values in terms of 'f_o' and 'Q_e,' it is possible to plot the complete response curves for a transformer which has a selected value of K.Q_e.

Tables are given later, with the numerical evaluation for the five sections, using the following values of K.Q_e. :

$$K . Q_e = 1 \qquad K . Q_e = 2 \qquad K . Q_e = 4$$
$$K . Q_e = \tfrac{1}{2} \qquad K . Q_e = 3 \qquad K . Q_e = 5$$

As an example the, method of calculating the values for the critically coupled case is detailed below :

Critical coupling K.Q_e = 1

5.6.1. *Gain at 'fo'*

From equation 41

$$|A_o| = \frac{g_m Q_e \omega_o L}{2}$$

5.6.2. *Response*

From equation 42

$$S = 10 \log_{10} (1 + 4 . Q_e^4 . D^4) \text{ db}$$

$$\therefore S = 10 \log_{10} \left((1 + 4 . Q_e^4 \frac{\delta f^4}{f_o^4}\right) \text{ db} \quad (47)$$

Suppose

$$\delta f = \frac{f_o}{Q_e}$$

then $S = 10 \log_{10} (1 + 4)$ db
$= 7$ db

Therefore, considering a transformer having a Q_e of 80, and centre frequency, 'f_o,' of 465 kc/s, the attenuation will be 7 db at $\pm$ 5·82 kc/s from 'f_o,' i.e. at 470·82 and 459·18 kc/s.

5.6.3. *Peak Frequencies*

Solving equation 44 when K.Q_e = 1, gives $F_p = f_o$. There is thus one peak, at the centre frequency.

5.6.4. *Pass Band*

Putting

K . Q_e = 1 in equation 45 gives :

$$P = 0{\cdot}707 \frac{f_o}{Q_e}$$

5.6.5. *Skirt Selectivity*

From equation 46, when K.Q_e = 1

$$Sk = \left(40 \log_{10} \frac{Q_e . \delta f}{f_o}\right) + 6 \text{ db} \quad \dots(48)$$

5.7. *Graphical Presentation of Results*

For general use, it is advantageous to derive various expressions which can be plotted as a series of curves, and this has been done in Figs. 6, 7, and 9.

The variation in the gain at the centre frequency, with respect to K.Q_e., is plotted in Fig. 6, using the gain at critical coupling as the reference level, i.e., 100 per cent.

The numerical value of the response (equation 42) is plotted in Fig. 7 for values of K.Q_e and 'δf.' The curves are drawn for 'spot' values of 'δf,' and infinitely variable values of K.Q_e, and will be found of greater use than the conventional method of drawing such curves, since K.Q_e rarely has a 'spot' value.

Note, in Fig. 7, that the attentuation at any value of K.Q_e is referred to the gain at 'f_o' at the particular value of K.Q_e considered, and, therefore, all curves have 0 db as their origin.

The reduction in the gain at 'f_o,' due to K.Q_e having some value other than unity, can be obtained by reference to Fig. 6.

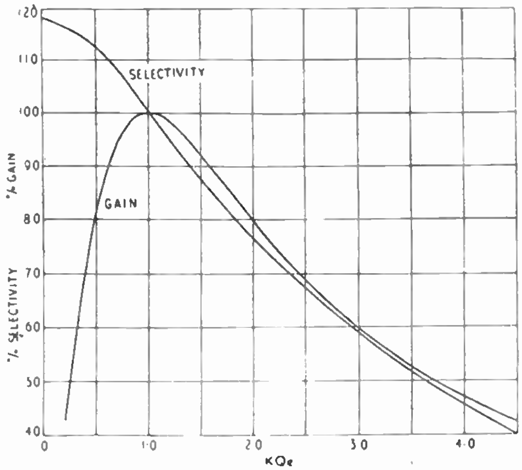

Fig. 6.—Selectivity and gain at "f_o", versus coupling.

The expression for the Pass Band (equation 45) can be expressed in the form :

$$P = p\,\frac{f_o}{Q_e}$$

where

$$p = \tfrac{1}{2}\sqrt{K^2Q_e^2 - 1 + \sqrt{2 + K^2Q_e^2(1+K^2Q_e^2)}}$$

and in this form, it is plotted in Fig. 9; the ordinates being, of course, 'p' and 'K.Q_e.'

6.0. Single Transformer Selectivity Limitations

Using a single, double tuned transformer coupled stage, there is a definite limit to the selectivity obtainable by decreasing the coupling factor.

From the point of view of the selectivity alone, a decrease in the value of the coupling factor, is identical with an increase in 'Q_e' whilst maintaining critical coupling.

Considering an off-tune frequency

$$`\delta f\text{'} = 5\,.\,\frac{f_o}{Q_e}$$

it can be shown (from equation 42) that the apparent increase in Q_e is very nearly $\sqrt{2}$, when the coupling is reduced to zero.

A more practical method of estimating the increase of selectivity is by taking the expression for the response, equation 42, and plotting the variation of 'S' against K.Q_e for the same value 'δf' given above. This curve is drawn in Fig. 6 and shows that the selectivity increases to a limiting value of 118 per cent. of that obtained at critical coupling (i.e. K.Q_e = 1) and that there is no material advantage in reducing K.Q_e below 0·25.

Plotted in the same figure is the variation in the gain, at the centre frequency, for the same values of K.Q_e, against the gain at critical coupling. It will be seen from both curves that the best compromise between the gain and selectivity, is obtained with a value of K.Q_e between 0·25 and 0·5.

There are other methods of increasing the selectivity obtainable from a double tuned transformer, though these are only applicable when the initial value of 'Q_e' is low (i.e. when the primary and secondary damping is considerable).

Fundamentally, methods used, increase the value of 'Q_e,' and this can be achieved by a reduction of 'L,' or by tapping the coils.

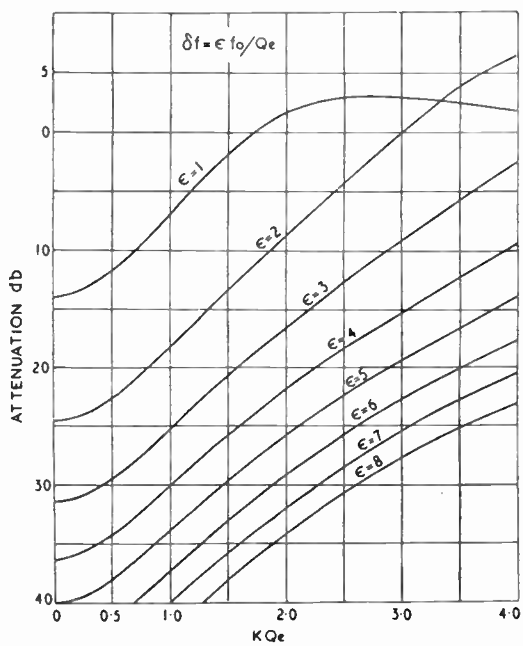

Fig. 7.—Universal transformer response curves.

As was seen earlier, 'Q_e' is dependent on the relative values of the shunt damping across the coils (r_a and R_g) and the parallel impedance of the tuned circuits at the centre frequency

($Q_s\omega_oL$). Since the valve 'r_a' cannot be increased except by negative feedback and the secondary damping is not usually capable of variation, the only methods of increasing 'Q_e' are, the obvious one of increasing 'Q_s' or reducing the value of $Q_s\omega_oL$. In order to reduce the latter, 'L' is the only parameter which can be reduced, either by an actual reduction or by tapping the coil.

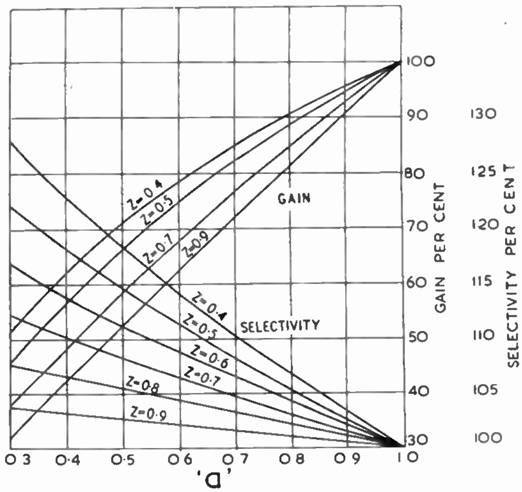

Fig. 8.—Selectivity and gain versus coil inductance.

Tapped coils are only dealt with briefly in this paper, since the analysis entails the use of a factor which is either unknown or subject to wide variation, i.e. the coupling factor between the two sections of the tapped coil. This will vary according to the type of winding used, whether one tapped coil is used, or two separate 'pies' etc.

Some indication of the method of approach is given later in the section dealing with practical applications.

The method using a reduction of the coil inductance is, however, amenable to analysis; and curves are drawn in Fig. 8 showing the variation in the gain and the selectivity against the reduction in 'L' at $KQ_e = 1$. It is noted that the initial value of 'L' is given as unity and the values of 'z' quoted are the initial values obtained before the reduction of 'L' is made.

The methods used to obtain the curves of Fig. 8 are interesting, and are illustrated below.

Considering, firstly, the response at an off-tune frequency given by:

$$\delta f = 5 \, . \, \frac{f_o}{Q_e}$$

$$\text{thus } D = \frac{5}{Q_e}$$

And, at critical coupling, from equation 47

$$S = 10 \log_{10}(1 + 4 \, . \, Q_e^4 \, D^4) \text{ db}$$

At the above value of 'D,'

$$4 \, . \, Q_e^4 D^4 \gg 1$$

$$\therefore \; Sk = 40 \log_{10} Q_e D \sqrt{2} \text{ db}$$

This equation, for the Skirt selectivity, is in a slightly different form from that given in equation 48.

$$\text{Putting } D = \frac{5}{Q_e}$$

$$\therefore \; Sk = 40 \, . \log_{10} 5 \, . \sqrt{2} \text{ db}$$

This is the initial value of the selectivity at the given value of 'δf,' for critical coupling.

If, now, 'Q_e' is increased to, say, $b \, . \, Q_e$., then

$$Sk_1 = 40 \, . \log_{10} b \, . \, 5 \, . \sqrt{2}$$

From Sk and Sk_1, therefore, the selectivity ratio is:

$$\frac{Sk_1}{Sk} = \frac{40 \, . \log_{10} b \, . \, 5\sqrt{2}}{40 \, . \log_{10} 5\sqrt{2}}$$

$$\therefore \; \frac{Sk_1}{Sk} = 1 + \frac{\log_{10} b}{\log_{10} 5\sqrt{2}} = 1 + \frac{\log_{10} b}{0{\cdot}85}$$

$$\text{Now, } Q_e = Q_s \, . \, z = Q_s \sqrt{x.y}.$$

from equation 14

$$\text{and } \; b \, . \, Q_e = Q_s \, . \, z_1 = Q_s \sqrt{x_1 \, . \, y_1}$$

$$\therefore \; b = \frac{Q_s \, . \, z_1}{Q_s \, . \, z} = \frac{\sqrt{x_1 \, . \, y_1}}{\sqrt{x \, . \, y}}$$

From equations 12 and 13:

$$x = \frac{1}{1 + \alpha} \text{ and } y = \frac{1}{1 + \beta}$$

$$\text{where, } \; \alpha = \frac{Q_s\omega_oL}{r_a} \text{ and } \beta = \frac{Q_s\omega_oL}{R_g}$$

If 'L' is decreased to, say, 'aL,' then

$$x_1 = \frac{1}{1 + a\alpha} \text{ and } y_1 = \frac{1}{1 + a\beta}$$

$$\therefore\ b = \sqrt{\frac{(1+\alpha)(1+\beta)}{(1+a\alpha)(1+a\beta)}}$$

Considering, secondly, the variation in gain due to the variation in 'L' it is again noted that at critical coupling, from equation 41

$$|A_o| = \frac{g_m K . Q_e Q_e \omega_o L}{(1+K^2 Q_e^2)}$$

$$= \frac{g_m Q_e \omega_o L}{2}\ (\text{at } K . Q_e = 1)$$

If 'L' is reduced to 'aL,' then 'Q_e' is increased to '$b . Q_e$.'

$$\therefore\ |A_{o1}| = \frac{g_m Q_e b \omega_o L a}{2}$$

Therefore, the gain ratio is :

$$\frac{|A_{o1}|}{|A_o|} = \frac{g_m . Q_e . b . \omega_o L . a . 2}{2 . g_m . Q_e . \omega_o L .} = a . b .$$

It is seen that the increase in gain and selectivity is a function of 'b'; and 'b' itself, is dependent on the values of 'α,' 'β' and 'a.' For the curves to be of real practical value, it is preferable that they should be a function of the initial value of 'z' (Note that $z = Q_e/Q_s$) rather than of 'α' and 'β'. The reason for this is that 'α' and 'β' may have any values, whilst still giving a constant value for 'z', and therefore curves drawn for stated values of 'α' and 'β' will apply only to a transformer having these values, and not necessarily to another transformer having an identical value of 'z,' but different values of 'α' and 'β.'

Plotting the two expressions developed above, for varying values of 'α,' 'β' and 'a', will result in an infinity of curves, but, by manipulation of the expression for 'b,' this can be avoided, thus :

$$b = \frac{\sqrt{x_1 . y_1}}{\sqrt{x . y .}} = \frac{\sqrt{x_1 . y_1 .}}{z}$$

$$= \sqrt{\frac{(1+\alpha)(1+\beta)}{(1+a.\alpha)(1+a\beta)}}$$

$$= \sqrt{\frac{(1+(\alpha+\beta)+\alpha\beta}{(1+a(\alpha+\beta)+a^2\alpha\beta)}}$$

When the term 'Q_e' was obtained, an approximation was made, which can, with advantage, be applied here.

Therefore, by similarity with equations 12, 13 and 14 and noting the limit imposed by equation 15 :

Let

$$(\alpha+\beta) = 2\sqrt{\alpha\beta}$$

$$\therefore\ b = \sqrt{\frac{(1+2\sqrt{\alpha\beta}+\alpha\beta)}{(1+2.a.\sqrt{\alpha\beta}+a^2\alpha\beta)}}$$

$$= \frac{(1+\sqrt{\alpha\beta})}{(1+a.\sqrt{\alpha\beta})} = \frac{1}{z.(1+a\sqrt{\alpha\beta})}$$

Since $z = \dfrac{1}{(1+\sqrt{\alpha\beta})}$

$$\therefore\ \sqrt{\alpha\beta} = \frac{1}{z} - 1 = \frac{1-z}{z}$$

$$\therefore\ b = \frac{1}{z . \left\{1 + \dfrac{a.(1-z)}{z}\right\}}$$

$$= \frac{\mathbf{1}}{\{z + a.(1-z)\}}$$

There are now only two variables, 'a' and 'z', contained in a greatly simplified expression, and using the above expression for 'b,' the ratios of the gain and selectivity increases are plotted in Fig.8 for the values of 'a' from 1·0 to 0·3, and 'z' from 0·9 to 0·4.

It is apparent that the advantage in performance gained by a reduction of 'L' is dependent on the initial value of 'Q_e,' and any consideration of the relative advantages of the two systems (i.e. reduction in K.Q_e or reduction in 'L') in gain or selectivity must be made in relation to the particular transformer in question. But, in general, where 'Q_e' (or rather 'z') is high, K.Q_e should be reduced. Where 'z' is low, 'L' should be reduced.

For maximum possible selectivity from any particular transformer, a combination of both methods can be used, in which case reference must be made firstly to Fig. 8 and thence to Fig. 6.

For example, it is assumed for a transformer that 'z' = 0·4 and K.Q_e = 1. Referring to Fig. 8 it is seen that by reducing 'L' to 0·9 L, the gain is reduced to 85·5 per cent. and the selectivity is increased to 110 per cent. of that obtainable when 'a' = 1 at 'δf' = 5 .f_o/Q_e.

Referring to Fig. 6, it is assumed that K.Q_e is reduced to 0·75, the gain is reduced to 96 per cent. and the selectivity increased to 106·5 per cent. Multiplication shows that where ' L ' is reduced to 70 per cent. and $K.Q_e$ to 0·75, the final gain is 82 per cent. and the selectivity 117 per cent. of the performance at $K.Q_e = 1$ and ' L ' = ' L.' The combination results in a better performance than if $K.Q_e$ or ' L ' are reduced separately.

In order to obtain the curves of Fig. 8, Q_s was assumed to be constant with decreasing ' L.' This is not necessarily true. Q_s is partly a function of the coil diameter, and may therefore drop as ' L ' is reduced, though not necessarily in the same ratio[1]. Because of the doubt in the variation of ' Q_s,' the curves in Fig. 8 are not taken below ' *a* ' = 0·3 since, below this value, the results would tend to become ambiguous.

A useful practical method of keeping ' Q_s ' constant with decreasing ' L,' is to maintain the coil diameter constant but decrease the coil width. (It is assumed that coils of the wave wound type are considered.)

7.0. Determination of the Value of $K.Q_e$ (of a transformer)

It is frequently necessary to know the actual value of the coupling of a particular transformer as, for example, when a prototype is being developed, or in the process of "quality-control" on mass-production lines.

Any method of measuring the coupling must be made on the actual transformer ' in situ,' i.e.

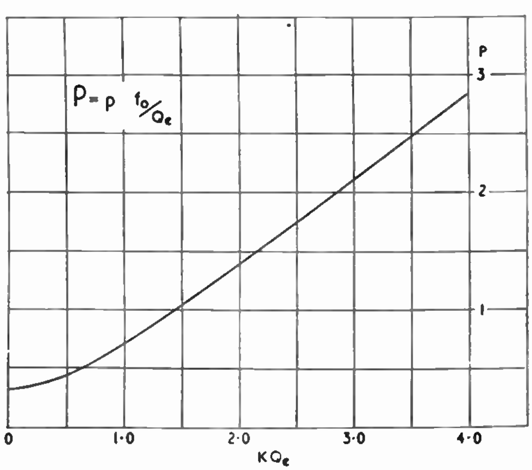

Fig. 9.—Pass band [P = pf_o/Q_e] *versus coupling.*

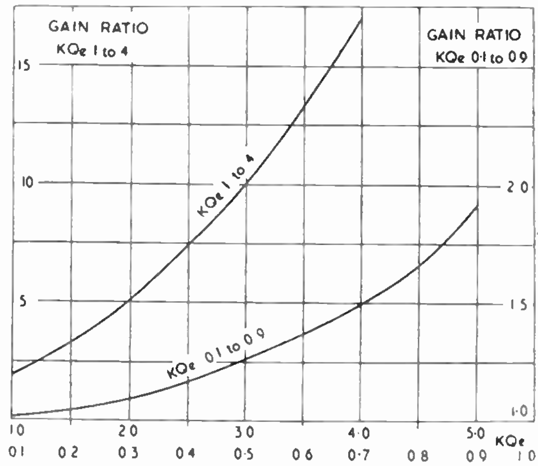

Fig. 10.—Determination of "K.Qₑ."

interest centres on the value of $K.Q_e$ and not on K ; and the following method has been found satisfactory.

The secondary is open circuited and any coupling is removed, apart from mutual inductance between the primary and secondary coils (i.e. top or bottom capacity).

The gain is measured (with a valve voltmeter) to the anode of the valve, at the centre frequency, at the same time ensuring that the coil is accurately tuned. Let this gain be $|A_{as}|$ (i.e. gain at the anode with a single tuned circuit).

Any couplings removed are replaced, the secondary circuit is completed, and both coils are accurately aligned. (It is advisable to leave the valve voltmeter in circuit for this.)

The gain at the *anode* of the valve at the centre frequency is measured as before. Let this gain be $|A_{ad}|$ (i.e. gain at the anode with the double tuned circuit).

Then

$$K.Q_e = \sqrt{\frac{|A_{as}|}{|A_{ad}|} - 1} \quad \ldots\ldots\ldots\ldots(49)$$

Equation 49 is plotted in Fig. 10 for reference purposes. Examination shows that the slope is reasonably low and thus reasonably accurate results can be expected.

The precautions necessary when using the above method can be seen more clearly by a perusal of the proof given below.

Referring to Fig. 3 :

$$V_1 = i_1 Z_1 - j\,\omega M\, i_2 \qquad \ldots\ldots\ldots\ldots(1b)$$

$$O = i_1 j\,\omega M - i_2 Z_2 \qquad \ldots\ldots\ldots\ldots(2b)$$

$$V_o = i_1 j\,\omega L \qquad \ldots\ldots\ldots\ldots(3b)$$

where V_o is the voltage across the primary inductance.

From 2*b*

$$i_2 = i_1 j \frac{\omega M}{Z_2}$$

∴ From 1*b*

$$V_1 = i_1 Z_1 - i_1 \frac{(j\omega M)^2}{Z_2}$$

$$= i_1 \left(\frac{Z_1 Z_2 + \omega^2 M^2}{Z_2} \right) \qquad \ldots\ldots(4b)$$

From 3*b* and 4*b*

$$\frac{V_o}{V_1} = \frac{i_1 j\omega L Z_2}{i_1 (Z_1 Z_2 + \omega^2 M^2)}$$

and with equation 1

$$\frac{V_o}{V_g} = A_{af} = \frac{i_1 j\omega L Z_2 (-j g_m)}{i_1 \omega C (Z_1 Z_2 + \omega^2 M^2)}$$

$$= \frac{g_m \omega L}{\omega C \left(Z_1 + \dfrac{\omega^2 M^2}{Z_2} \right)}$$

at resonance $Z_1 = R_{t1}$ and $Z_2 = R_{t2}$ and $X_1 = X_2 = 0$

$$\therefore |A_{ad}| = \frac{g_m \omega L}{\omega C \left\{ R_{t1} + \dfrac{\omega^2 M^2}{R_{t2}} \right\}}$$

$$= \frac{g_m \omega L}{\omega C \left\{ \dfrac{\omega L}{Q_1} + \dfrac{\omega^2 M^2 Q_2}{\omega L} \right\}}$$

where 'Q_1' = primary working 'Q'
and Q_2 = secondary working 'Q'

$$\text{Now } K = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L} \text{ and } \omega L = \frac{1}{\omega C}$$

$$\therefore |A_{ad}| = \frac{g_m \omega L}{\omega C \left(\dfrac{\omega L}{Q_1} + \dfrac{\omega^2 K^2 L^2 Q_2}{\omega L} \right)}$$

$$= \frac{g_m \omega L}{\left(\dfrac{1}{Q_1} + K^2 . Q_2 \right)} = \frac{g_m Q_1 \omega L}{(1 + K^2 Q_1 Q_2)}$$

Now

$$Q_1 Q_2 = Q_e^2$$

$$\therefore |A_{ad}| = \frac{g_m Q_1 \omega L}{(1 + K^2 . Q_e^2)} \qquad \ldots\ldots\ldots\ldots(5b)$$

From equation 6

$$|A_{as}| = g_m Q_1 \omega L$$

where Q_1, is the anode coil working 'Q'

From equation 5*b* and 6

$$\frac{|A_{as}|}{|A_{ad}|} = (1 + K^2 . Q_e^2)$$

$$\text{and } K . Q_e = \sqrt{\frac{|A_{as}|}{|A_{ad}|} - 1} \qquad \ldots\ldots\ldots\ldots(49)$$

Precautions must be taken to ensure that the measurement of the gain for the case of the single, anode, coil circuit, is accurate (i.e. the effect of the secondary must be negligible). In the case of capacitively coupled transformers, it is a simple matter to disconnect the secondary completely, and in the case of mutual inductance coupling, the effect of opening the secondary is to increase its impedance to a high value, and thus the effect on the primary is small.

8.0. Method of Design Using Expressions

Design methods are best illustrated by means of examples, and two such examples are given below.

Example 1 (1*st* i.f. *transformer*)

Consider that the following information is given :

Valve	6K8 frequency changer.
Conversion conductance	0·36 mA/V.
Anode impedance ..	0·6 megohm.
Intermediate frequency ..	465 kc/s.
Coil tuning capacity ..	100 pF (fixed condenser).
Coil 'Q' i.e. 'Q_s' ..	150 (Coil/iron-cored tuned).
Required Pass Band, appx.	5 kc/s.
Secondary damping ..	Infinity.
Required to find	Gain at 465 kc/s. General selectivity characteristic. Value of K.Q_e. Peak frequencies, if any.

Thus :

$$\text{Tuning capacity} = 100 + (\text{say})\ 8\ (\text{valve holder strays}) + (\text{say})\ 12\ (\text{wiring strays}).$$
$$= 120 \text{ pF}.$$
$$\therefore \omega L = 1/\omega C = 2{\cdot}87.\ 10^3 \text{ ohms}.$$
$$\text{and } \alpha = 0{\cdot}717.$$
$$\text{and } \beta = 0$$
$$\text{Thus } x = 0{\cdot}583 \text{ (see equation 12).}$$
$$y = 1 \quad \text{(see equation 13).}$$
$$z = 0{\cdot}763 \text{ (see equation 14).}$$
$$\text{and } Q_e = 114{\cdot}4 \text{ (see equation 14).}$$

Since the coil is tuned by an iron core, the value of 'Q_s' will tend to vary with the position of the slug, and, in such cases, it is advisable to use a value of 'Q_s' some 10 per cent. lower than the maximum.

This has not been done above, but the necessary changes in the calculations are obvious.

$$\text{Now } `\delta f\text{'} = \frac{f_0}{Q_e} = 4{\cdot}07$$

and

$$\frac{\text{Pass Band}}{4{\cdot}07} = \frac{5}{4{\cdot}07} = 1{\cdot}23 = p.$$

From Fig. 9, with $p = 1{\cdot}23$:

The required value of $K.Q_e = 1{\cdot}7$ (approx.) From equation 41 :

The gain at 'f_0' $= 51{\cdot}6$.

The response can be calculated from equation 42, or obtained from Fig. 7 and is :

kc/s off tune	Attenuation
$f_0/Q_e = 4{\cdot}07$	0·5 db
$2 f_0/Q_e = 8{\cdot}14$	11·5 db
$3 f_0/Q_e = 12{\cdot}21$	19·5 db
$4 f_0/Q_e = 16{\cdot}28$	24 db
$5 f_0/Q_e = 20{\cdot}35$	28·5 db

The advantage of the unconventional method of plotting response curves as in Fig. 7 is now apparent.

From equation (44), the peak frequencies can be obtained :—

$$\text{and } F_p = 465 \pm 2{\cdot}79 \text{ kc/s}$$

Example 2 (*2nd i.f. Transformer*)

Valve 6K7 R.F. pentode

Mutual conductance	1·45 mA/V
Anode impedance	0·8 megohm
Intermediate frequency ..	465 kc/s
Coil tuning capacity	100 pF (fixed condenser)
Coil Q (i.e. Q_s) ..	120 (coil—iron-cored tuned)
AVC diode load ..	1 megohm
Signal diode load ..	0·5 megohm

Gain required to be maximum ($KQ_e = 1$)

It is required to find :—

General selectivity characteristic

Gain at 465 kc/s

Pass band

The circuit of the above transformer is of the usual type and is given in Fig. 11(*a*) for reference.

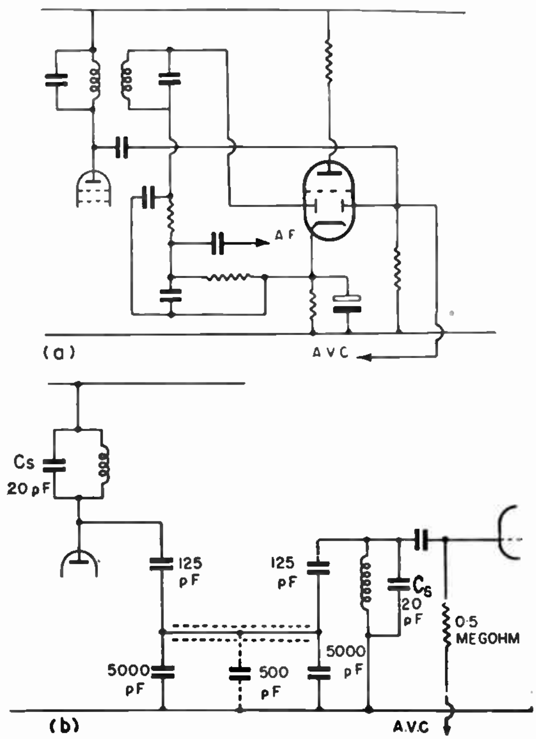

Fig. 11(*a*).—*Circuit of 2nd i.f. transformer.*
Fig. 11(*b*).—" *Capacity link* " *coupled 1st i.f. transformer.*

Tuning capacity $= 100 + 20$ pF (using the same value for strays as before)

$= 120$ pF

$$\therefore \omega L = \frac{1}{\omega C} = 2{\cdot}87 \times 10^3 \text{ ohms}$$

$$\text{and } Q_s \omega_o L = 0{\cdot}344 \times 10^6 \text{ ohms}$$

The primary damping is the resultant of the anode resistance in parallel with the input resistance of the diode.

For the AVC diode (shunt load resistance)

$$R_i = \frac{R_L}{(1 + 2 \,.\, n)} \text{ (approx.)}$$ where " n " is the diode rectification efficiency[2].

Normally " n " lies between 1·0 and 0·6, and taking " n " as 0·9, then the primary damping is:

$$R_1 = \frac{R_L}{(1 + 2 \,.\, n)} \| r_a = 0{\cdot}247 \times 10^6 \text{ ohms}$$

Therefore $\alpha = 1{\cdot}39$, (see equation 12)

$x = 0{\cdot}418$ and $\sqrt{x} = 0{\cdot}646$

(see equation 12)

The secondary damping is due to the input resistance of the signal diode only.

For the signal diode

(Series load resistance) $R_i = \dfrac{R_L}{2 \,.\, n}$ (approx.)

Taking " n " as 0·9, then the secondary damping is :—

$R_i = R_2 = 0{\cdot}278 \; 10^6$ ohms

Therefore $\beta = 1{\cdot}24$

$y = 0{\cdot}447$ and $\sqrt{y} = 0{\cdot}668$

(see equation 13)

and $z = 0{\cdot}43$

$Q_e = 51{\cdot}6$ (see equation 14)

$\therefore$ " δf " $= 9{\cdot}0$ kc/s

From Fig. 9, at $K.Q_e = 1$, $p = 0{\cdot}707$

Therefore the Pass band $= 6{\cdot}37$ kc/s

The gain is calculated from equation (41) and is

Gain at " f_o " $= A_o = 107{\cdot}5$

From equation (42) or Fig. 7, the response is :—

kc/s off tune	Attenuation db
9·0	7
18	18·13
27	25·12
36	30·1
45	34

The overall selectivity obtainable in a receiver using the above transformers can be determined by plotting the individual response curves, and adding the attenuation at equal values of " δf ".

The total response for the above examples, at $\pm$ 20 kc/s is 48 db approx.

The overall Pass band can be found in a similar manner.

It is apparent that the primary and secondary damping in a conventional i.f. transformer, feeding the signal and A.V.C. diodes, is considerable. This results in poor selectivity. The selectivity in the above example could be improved by reducing the value of the coil inductances, and reference to Fig. 8 shows that, by taking the mean of the curves for " z " $= 0{\cdot}4$ and 0·5, for a reduction in " L " to 60 per cent., the new gain is 77 per cent. of 107·5 (= 82·8) and the selectivity is 112 per cent. of that originally obtained at " δf " $= 5 f_o / Q_e$.

The new selectivity is, thus, 38 db at 45 kc/s off tune.

The selectivity can also be increased by tapping the coils, and an indication of the results to be expected are discussed after the next section.

9.0. Practical Comparison of Method

In order to illustrate the accuracy of the equations, the calculated and measured characteristics of a particular transformer are given below.

The circuit of the transformer in question is given in Fig. 11b. This particular transformer coupling is a form of " link " coupling, except that capacities are used, and it is identical with Fig. 4f.

Valves and Transformer details

Valve	6A8 frequency changer	
Conversion conductance ..	0·55 mA/V	at 250 V H.T.
Anode impedance ..	0·36 megohm	
Intermediate frequency ..	465 kc/s	

Coil "Q_s"	130 (at working position of core)
Tuning capacity ..	125 pF (fixed condenser)
Secondary damping	
R_g	0·5 megohms
Coupling capacity ..	5000 + 5000 + 500 pF (The 500 pF is due to distributed capacity of link)

Total tuning capacity = 125 + (strays) 20 pF (effect of Cm ignored)

$= 145$ pF

$\therefore\ \omega_0 L = 1/\omega_0 C = 2{\cdot}36.\ 10^3$ ohms

Therefore $\alpha = 0{\cdot}854$

and $\beta = 0{\cdot}613$

$x = 0{\cdot}54$ and $\sqrt{x} = 0{\cdot}735$

$y = 0{\cdot}62$ and $\sqrt{y} = 0{\cdot}787$

Therefore $z = 0{\cdot}578$

and $Q_e = 75$

And "δf" $= f_0/Q_e = 6{\cdot}2$ kc/s

To calculate the value of K, reference is made to Fig. 4*f* and equation (39).

$$\therefore\ S = \frac{145}{125} = 1{\cdot}16$$

$$\text{and } K = \frac{125}{125 + 1{\cdot}16\ .\ 10{\cdot}5.10^3}$$

$$= 0{\cdot}0106$$

$$\therefore\ K.Q_e = 0{\cdot}76$$

Calculating the gain, at "f_0," from equation (41) :—

$$|A_0| = 46{\cdot}8$$

From Fig. 9 or equation (45)

$$\text{Pass band} = 0{\cdot}583\, f_0/Q_e$$

$$= 3{\cdot}6 \text{ kc/s}$$

From equation (42) or Fig. 7, the response is :—

From Fig. 7

kc/s off tune	Attenuation db
6·2	9·5
12·4	20·5
18·6	27·2
24·8	32·2
31·0	37·0

The actual response curve of the transformer is plotted in Fig. 12, and below, the measured and calculated results are compared directly.

	Calculated		*Measured*	*Error*
GAIN :				
	46·8		48	2·5%
RESPONSE :				
	db down	*kc/s off tune*	*db down*	
	9·5	6·2	9·2	3·0%
	20·5	12·4	21·6	5·4%
	27·2	18·6	28·5	5·0%
	32·2	24·8	34	5·8%
	37	31	—	—
PASS-BAND :				
	kc/s		*kc/s*	
	3·6		3·75	4·0%

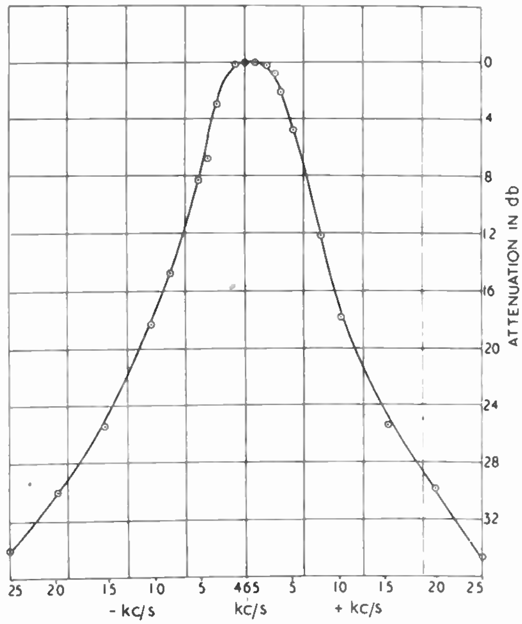

Fig. 12.—A measured response curve.

Since the measured response curve is slightly asymmetrical, the above values of the attenuation for the kc/s off tune are taken for the mean of those frequencies, i.e., the attenuation is taken at the point where the response curve has a width equal to twice the value of the off tune frequencies given.

The above type of transformer is particularly amenable to calculation and, generally speaking, an accuracy of the order of 5 to 10 per cent. is more usual.

10.0 Tapped Coils[1]

It is proposed to deal briefly with transformers having tapped coils, since, as mentioned earlier, a full analysis would require an implicit knowledge of the value of the coupling factor, say "k," between the two sections of the coil. This factor is normally difficult to measure at intermediate frequencies, since the coupling due to the stray capacities between the two sections may mask that due to the mutual inductance coupling.

Tests have shown that the coupling reaches a high value for a "wave-wound" coil, if the two sections are wound as one coil (i.e. on top of one another). If the coil is enclosed in a complete dust-iron "pot" with a dust iron core, the coupling may reach 0.9 plus. Without the "pot," the coupling is of the order of 0·85 to 0·9, provided that the core is not removed.

Since unity coupling factor is assumed, the following must be regarded as a discussion on the possibilities of tapped transformers, and not as an analysis.

Nevertheless, the expressions can be used as a basis for design purposes.

Considering any tapped coil, with unity coupling factor between the two sections.

Looking into the tap, let the ratio of the number of turns up to the tap, to the number of turns on the whole coil, be T.

Thus, if a coil has a total of 100 turns and is tapped at 50 turns, $T = \frac{1}{2}$.

Viewing the coil as an auto-transformer, a voltage applied to the tap will appear as 1/T volts across the whole coil. A voltage applied to the whole coil, will appear as T volts at the tap.

A resistance R applied across the tap, will appear as R/T^2 across the whole coil, or a resistance R applied across the whole coil, will appear as $R.T^2$ across the tap.

Figure 15 shows the circuit of a transformer having both primary and secondary coils tapped, and, for this circuit, there is a step up of voltage on the primary of $1/T_1$ volts, and a step down of voltage on the secondary of T_2 volts. Therefore, if $T_1 = T_2$, the overall (auto) transformer action is unity.

For the purpose of this discussion, only two cases are considered,

(1) When only the primary is tapped at T.

(2) When both primary and secondary are tapped at $T_1 = T_2 = T$.

10.1. *Effect on Q_e. CASE 1.*

For the secondary coil, the value of "β" and "y" is calculated in the usual manner. (Equation 13).

For the primary coil, the primary damping is determined in the usual manner (i.e. "$r_a \|$" external losses, if any). This value, say R1, appears as $R1/T^2$ to the coil, and therefore, "α" and "x" are calculated using the latter value.

Q_e is then calculated from $Q_s\sqrt{x.y}$. and the selectivity characteristic is determined as usual.

Effect on Gain

From the valve anode a stepped down impedance is seen when looking into the tap, and the signal voltage through the transformer is subjected to a step up of 1/T :—

Therefore, modifying equation (41)

$$|A_o| = \frac{g_m . K . Q_e . T^2 . Q_e . \omega_o L}{(1 + K^2 . Q_e^2)} \cdot \frac{1}{T}$$

$$\therefore |A_o| = \frac{g_m . K . Q_e . T . Q_e . \omega_o L}{(1 + K^2 . Q_e^2)} \quad \ldots\ldots(50)$$

Thus, the effect of tapping the primary coil, say halfway, (i.e. $T = \frac{1}{2}$) is to reduce the gain, apparently, by half. But the increase in Q_e must be considered. If, when the coil is not tapped, Q_e, or rather z, is low, then the increase in Q_e may be considerable and the reduction in the gain will be much less than at first indicated.

For use with second i.f. transformers, the tapped coil method offers many advantages, since the damping across both coils, and particularly the primary, is considerable.

10.2. *Effect* on *Q_e. CASE 2*

For both the primary and secondary, the values of "x" and "y" are calculated as indicated in 10.1 for the primary.

Effect on Gain

From the valve anode a stepped down impedance is seen and due to the two taps, the overall auto-transformer action is unity.

$$\therefore |A_o| = \frac{g_m . K . Q_e . T^2 . Q_e . \omega_o L}{(1 + K^2 . Q_e^2)} \quad \ldots\ldots(51)$$

The gain is thus reduced by the square of the tap, but, again, the increase in Q_e will partly compensate for the reduction.

As an illustration, the relevant information is obtained for the second i.f. transformer, the constants of which have previously been given, in Example 2.

Two conditions are considered :—

(i) The primary, only, tapped at $T = \frac{3}{4}$

(ii) Primary and secondary tapped at $T = \frac{1}{2}$

(i) This is an example of 10.1. Therefore :—

For the secondary :—

y = constant = 0·447

$\therefore \sqrt{y} = 0{\cdot}668$

For the primary :—

The damping is $0{\cdot}247 \times 10^6$ ohms

This appears to the coil as

$0{\cdot}247 \times 10^6 \times 16/9$ ohms

$= 0{\cdot}439.\ 10^6$ ohms

Therefore

$\alpha = 0{\cdot}784$

$x = 0{\cdot}561$

$\sqrt{x} = 0{\cdot}749$

$\therefore z = 0{\cdot}5$

and $Q_e = 60$

$\delta f = 7{\cdot}55$ kc/s

Thus the Pass band $= 0{\cdot}707 \times 7{\cdot}55$

$= 5{\cdot}46$ kc/s

(Note that $K . Q_e = 1$).

The gain is determined from equation 50 :—

Gain = 94

From Fig. 7, the response is :—

kc/s off tune	Attenuation db
7·55	7
15·1	18·13
22·65	25·12
30·2	30·1
37·75	34

These results should be compared with those of Example 2. It is noted that the selectivity has been improved (e.g. at approx. 37 kc/s off tune the attenuation is + 4 db whilst the gain is reduced by 1·2 db).

(ii) This is an example of 10.2, and therefore :—

For the primary :—

The damping of $0{\cdot}247 \times 10^6$ ohms appears to the coil as $0{\cdot}247 \times 10^6 \times 4$ ohms

$= 0{\cdot}988 \times 10^6$ ohms

$\therefore \alpha = 0{\cdot}349$

$x = 0{\cdot}742$ and $\sqrt{x} = 0{\cdot}862$

For the secondary :—

The damping is $0{\cdot}278 \times 10^6$ ohms and appears to the coil as $0{\cdot}278 \times 10^6 \times 4$ ohms

$= 1{\cdot}112 \times 10^6$ ohms

$\therefore \beta = 0{\cdot}31$

$y = 0{\cdot}763$ and $\sqrt{y} = 0{\cdot}873$

Therefore $z = 0{\cdot}753$

and $Q_e = 90$

and $\delta f = 5{\cdot}17$ kc/s

Therefore, Pass band = 3·65 kc/s

From equation (51),

The gain = 75

From Fig. 7 the response is :—

kc/s off tune	Attenuation db
5·17	7
10·34	18·13
15·51	25·12
20·68	30·1
25·85	34

Comparing these results with those of Example 2, it is seen that there is an increase in selectivity of about 11 db at 27 kc/s for a decrease in the gain of 3·1 db.

It is apparent therefore that considerable improvement in the selectivity of a transformer can be obtained, if, initially, the value of "z" for that transformer is low.

11.0. Effect of "k" on "T"

The following is an approximate analysis of the effect on "T" if the coupling factor between the two sections of the coil, i.e. "k", is less than unity.

Considering Fig. 13 :—

Let $L_o = L_1 + L_2 + 2 . M \quad \ldots\ldots\ldots\ldots(1c)$

where L_o = the total coil inductance

L_1 = the inductance to the tap

and M = mutual inductance between L_1 and L_2

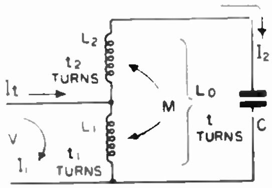

Fig. 13.—Equivalent circuit of tapped coil.

In practice, L_1 and L_2 will be similar coils, i.e. wound on the same former, using the same type of winding and wire. The size of the coils will not differ greatly (it is assumed that the coils are wave wound). Therefore, a reasonable assumption is that :—

$$\frac{L_2}{L_1} = \frac{t_2^2}{t_1^2}$$

where t_2 = number of turns on L_2

t_1 = number of turns on L_1

Now, if in the whole coil,

L_o, there are t turns

Therefore, to the tap, L_1,

there are $T.t$ turns $= t_1$

and in L_2 there are $(t - T.t)$ turns $= t_2$

where "T" has the same definition as previously.

$$\therefore \frac{t_2}{t_1} = \frac{t(1-T)}{t.T} = \frac{(1-T)}{T} \quad \ldots\ldots\ldots(2c)$$

$$\text{and } \frac{L_2}{L_1} = \left(\frac{t_2}{t_1}\right)^2 = \frac{(1-T)^2}{T^2} \quad \ldots\ldots\ldots(3c)$$

From $k = \dfrac{M}{\sqrt{L_1 . L_2}}$ $\quad M = k.\sqrt{L_1 . L_2}$

and with equation (3c).

$$M = \frac{k.L_1.(1-T)}{T} \quad \ldots\ldots\ldots(4c)$$

Returning to Fig. 13 :—

$$V = I_1(R_1 + j\omega L_1) - j\omega M . I_2 \quad \ldots(5c)$$

$$\text{and } V = I_2\left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) - j\omega M I_1$$

$$\therefore I_1(R_1 + j\omega L_1 + j\omega M) = I_2\left(R_2 + j\omega L_2 - \frac{j}{\omega C} + j\omega M\right)$$

if

$$I_1 . Z_1 = I_2 . Z_2$$

$$\therefore I_1 = I_2 \frac{Z_2}{Z_1} \quad \ldots\ldots\ldots(6c)$$

Now $It = I_1 + I_2$

$\therefore I_2 = It - I_1$

and with equation (6c)

$$I_1 = \frac{Z_2}{Z_1}(It - I_1)$$

$$= \frac{It}{1 + \dfrac{Z_1}{Z_2}} \quad \ldots\ldots\ldots(7c)$$

and similarly

$$I_2 = \frac{It}{1 + \dfrac{Z_2}{Z_1}} \quad \ldots\ldots\ldots(8c)$$

From equations (5c), (7c) and (8c)

$$V = \frac{It(R_1 + j\omega L_1)}{1 + \dfrac{Z_1}{Z_2}} - \frac{It(j\omega M)}{1 + \dfrac{Z_2}{Z_1}}$$

$$\text{and } Zi = \frac{V}{It} = \frac{R_1 + j\omega L_1}{1 + \dfrac{Z_1}{Z_2}} - \frac{j\omega M}{1 + \dfrac{Z_2}{Z_1}}$$

$$= \frac{Z_2(R_2 + j\omega L_1) - Z_1 . j\omega M}{(Z_1 + Z_2)}$$

where Zi is the impedance seen looking into the tap.

Now $Z_1 + Z_2$

$$= R_1 + j\omega L_1 + j\omega M + R_2 + j\omega L_2 + j\omega M - \frac{j}{\omega C}$$

$$= R_t + j\omega L_o - \frac{j}{\omega C}$$

At resonance, the imaginary terms are zero:

$$\therefore \omega^2 = \frac{1}{L_o . C}$$

Therefore, at resonance, and ignoring the resistive terms in the numerator, since these are relatively small :

$$\therefore Zi = \frac{\left(\omega L_2 + \omega M - \dfrac{1}{\omega C}\right)\omega L_1 - \omega M(\omega L_1 + \omega M)}{R_t}$$

$$= \frac{\dfrac{L_1}{C} - \omega^2 L_1 . L_2 + \omega^2 M^2}{R_t}$$

Using

$$\omega^2 = \frac{1}{L_o . C}$$

$$\text{Then } Zi = \frac{L_o . L_1 - L_1 . L_2 + M^2}{L_o . C . R_t}$$

$$= \frac{L_o . L_1 - L_1 . L_2 + M^2}{L_o^2} \cdot \frac{L_o}{C . R_t}$$

$$= \frac{L_1(L_1 + L_2 + 2.M) - L_1.L_2 + M^2}{L_o^2} Q\omega_o L_o$$

$$= \frac{(L_1 + M^2)^2}{L_o^2} Q\omega_o L_o$$

$$\therefore Zi = T^2_{(actual)} Q\omega_o L_o \quad \ldots\ldots\ldots\ldots\ldots (9c)$$

Note that

$$\frac{L_o}{C . R_t} = Q\omega_o L_o = \text{dynamic impedance of the tuned circuit at resonance.}$$

From equations (1c), (3c), (4c) and (9c)

$$T_{(actual)} = \frac{\left(L_1 + k . L_1 \frac{(1 - T)}{T}\right)}{\left(L_1 + \frac{(1 - T)^2}{T^2} + 2.k.L_1 \frac{(1 - T)}{T}\right)}$$

$\therefore T_{(actual)}$

$$= T \frac{(T + k(1 - T))}{(1 - 2.T(1 - T)(1 - k))} \quad \ldots (52)$$

where T is the tap based on the turns ratio.

It will be noted in equation 52 that, when $T = 1$ or $0 \cdot 5$, then $T_{(actual)} = T$, for any value of k.

The above analysis is not absolutely accurate, but is in order for general practical use.

12.0 Note on the Constancy of " Q_s "

In the preceding pages, it has been assumed that "Q_s" is known, initially, and not determined with respect to the required characteristics of the transformer. Or, in other words, a coil has been wound and re-wound until the maximum value of " Q_s ", which can be maintained in production, has been obtained. This value is then used for design purposes.

This is usual practice ; the determination of " Q_s " max., being the starting point of most designs, and owing to the several variables involved, it is usually done by " trial and error " methods.

13.0. Modifications to Standard i.f. Transformer Design

To those engaged in the development of i.f. transformers, there are two variations which may have possibilities for certain types of receivers.

The first is the case of a transformer having a mixed coupling the value of the composite coupling factor being given by equations 34 or 36.

If the value of the coupling due to the mutual inductance be increased, a corresponding change in the coupling due to the capacity will maintain the composite coupling constant, if the mutual inductance is positive.

The suggestion is, therefore, that the coupling due to the mutual inductance should be increased to the maximum possible, i.e. " K_m " = 1. The coils are connected in such a manner that " K_m " is positive, and then the top or bottom capacity is added in order to reduce the total coupling to that normal in i.f. transformers.

Equations 34 and 36 will not now necessarily apply, but they could be used to find the order of the required capacity.

The author built a transformer on the above lines in the following manner :—

The primary and secondary coils were (wave) wound on top of each other on a ¼-in. former. The two halves of a dust-iron "pot" were placed over the composite coil, and a 0·21-in. dust-iron core was inserted through the hollow former and glued in a symmetrical position (in relation to the pot). A midget double trimmer was used for tuning the primary and secondary, and a large value mica condenser was used as bottom capacity coupling.

Results upon measurement were very encouraging in view of the fact that the transformer measured only 1-in. cube.

The second possibility is drawn in Fig. 14. It is developed from a standard type, tapped, second i.f. transformer.

The purpose of the design is to allow the A.V.C. diode to operate with a series load resistance, giving a consequent reduction in the input resistance of the diode.

The circuit drawing is self-explanatory, but the usual component values are appended to facilitate the illustration.

The effect of the transform of Fig. 14 is that the diode input resistance is increased, with a resultant higher value for "z" for the transformer. For best results, the tap should be adjusted in conjunction with the value of the auto-transformer coupling factor "k," so that equal voltages appear across each coil section. The secondary should also be tapped, otherwise the voltage available for A.V.C. will be less than that applied to the signal diode.

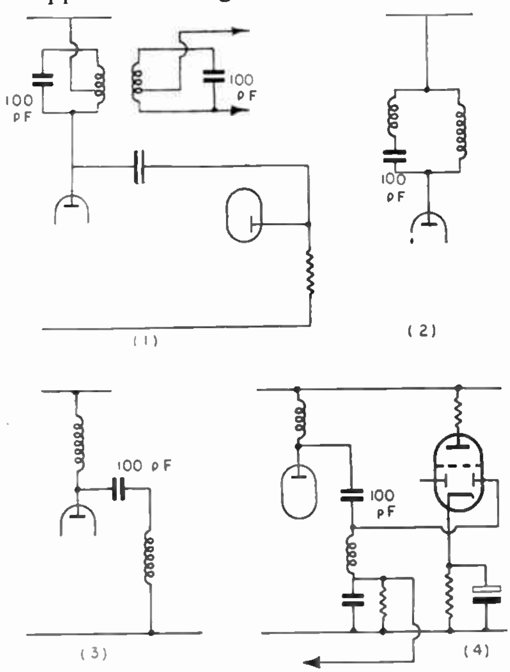

Fig. 14.—*Modification of standard design to reduce damping due to A.V.C. diode.*

No practical information regarding the performance of a transformer of this type is available. It still awaits an experimental check.

Possibly, after the publication of this paper, the investigation of the practicability of the design may be undertaken.

14.0. General Discussion and Limitations of Method

Most of the factors influencing the accuracy of the derived expressions can be discerned from a perusal of the analysis, but some of the points are emphasised here.

The equation for the response (equation 42), gives a symmetrical curve for positive and negative values of "δf," but, in practice, an asymmetrical response is usual. Assumptions 4, 7 and possibly 6, if untrue, all affect the symmetry of the response curve, and, since assumption 7 can never be true, some errors are to be expected in expressions based on the assumption.

Apart from any possible errors in the method used, errors in the value of the transformer "constants," i.e., in "L," "C," "Q_s," "g_m," "r_a" external damping and stray capacities inevitably occur. These values are rarely known to a greater accuracy than 5 per cent. and often it is as much as 10 per cent. Therefore, in many instances, there is an overall discrepance between calculated and measured results of more than 5 per cent.

The "practical comparison" given earlier, is, perhaps, less practical than it appears, since every constant was carefully measured with a view to checking the accuracy of the method of design.

With reference to the design of i.f. transformers in general, there are several important points which have not been discussed in this paper, but which must be considered. Notable among these are :—

(1) Effective input impedance of a valve, over a range of frequencies, with a tuned circuit anode load.[1]

(2) Consideration of the lower limit of "C" in relation to the de-tuning consequent upon A.V.C. action.[1]

(3) Consideration of the upper limit of the signal diode load resistance with respect to the clipping of the rectified signal, etc.[1]

With regard to these points, perusal of the literature listed at the end of this paper, is suggested.

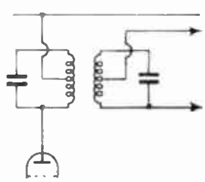

Fig. 15.—*Tapped i.f. transformer.*

15.0. Bibliography

1. K. R. Sturley, "Radio Receiver Design," Part I. Chapman & Hall.
2. F. E. Terman, "Radio Engineering Handbook." McGraw-Hill Publishing Co., Ltd.

TABLE 1

Value of K . Q_e	Gain at "f_0" $\times g_m Q_e \frac{\omega_0 L}{2}$	Pass Band $\times \frac{f_0}{Q_e}$	Skirt Selectivity $\left[40 \log_{10} \frac{\delta f Q_e}{f_0}\right] +$
1	1·0	0·707	+ 6 db
0·5	0·8	0·42	+10 db
2	0·8	1·[illegible]	− 2 db
3	0·6	2·27	− 8 db
4	0·47	2·8	−1[illegible]·6 db
5	0·385	3·52	−1[illegible]·3 db
	Single Tuned Circuit		
	$g_m Q_e \omega_0 L$	0·5	$\left[20 \log_{10} \frac{\delta f Q_e}{f_0}\right] + 6$ db

TABLE 2

S and SK in db

	Single Tuned		Double Tuned Circuits											
$\delta f / f_0$	Single Tuned		$KQ_e = 1$		$KQ_e = 0·5$		$KQ_e = 2·0$		$KQ_e = 3·0$		$KQ_e = 4$		$KQ_e = 5$	
	S	SK	S	SK	S	SK	S	SK	S	SK	S	SK	S	SK
$1/Q_e$	7	6	7	6	11·8	10	+1·68	—	+2·84	—	+2·0	—	+1·32	—
$2/Q_e$	12·3	12·02	18·13	18·04	22·55	22·04	8·7	—	0	—	+6·52	—	+6·16	—
$3/Q_e$	15·68	15·54	25·12	25·08	29·37	29·08	16·5	17·08	9·14	11·08	2·43	—	+4·44	—
$4/Q_e$	18·13	18·04	30·1	30·08	34·3	34·08	21·75	22·08	14·7	16·08	9·315	—	4·02	—
$5/Q_e$	20	20	34	34	38·13	38	25·76	25·96	19·3	20	14·03	15·4	9·4	11·7
Height of peaks above "f_0"			—		—		+2 db		+4·45 db		+6·57 db		+8·3 db	
Peak frequencies			f_0		f_0		$f_0 \pm 0·877 f_0/Q_e$		$f_0 \pm \sqrt{2} f_0/Q_e$		$f_0 \pm 1·94 f_0/Q_e$		$f_0 \pm 2·45 f_0/Q_e$	

16.0. Acknowledgments

It is realized that there are no new principles given in this paper except, possibly, the conception of "Q_e," and the construction of Figs. 7, 13 and 14. Full acknowledgment is given to books of reference containing information on the design of Intermediate Frequency Transformers, notably those listed.