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REGIONAL DIRECTOR, 1951–1952

Paul Leslie Hoover was born in Mansfield, Ohio, on December 20, 1898. A graduate of the Carnegie Institute of Technology, he received the M.A. degree from Harvard University in 1923, studied abroad at the Universities of Berlin, Vienna, and Paris until 1924, and returned to Harvard where, in 1926, he was awarded the D.Sc. in electrical engineering. He stayed on there as a research fellow until 1927, when he was appointed assistant professor at the Case School of Applied Science, Cleveland, Ohio. After an interim of nine years of teaching at Rutgers University, he returned to Case in 1939, where he has remained as professor and head of the electrical engineering department.

Mr. Hoover is a registered professional engineer in the State of Ohio, and has been active for many years as a consulting engineer, dealing particularly with patent cases. The author of several technical articles, he has been granted six patents in the fields of electronics and instrumentation.

Mr. Hoover joined the Institute as a Member in 1942, and became a Senior Member in 1943. From 1942 to 1943 he served as Chairman of the Cleveland IRE Section. He is a Fellow of the American Institute of Electrical Engineers, and belongs to the American Association for the Advancement of Science, the American Optical Society, the Franklin Institute, the Cleveland Engineering Society, whose vice-president he was from 1945 to 1946, Sigma Xi, Tau Beta Pi, Theta Tau, Eta Kappa Nu, and Gamma Alpha.

> The growth of The Institute of Radio Engineers has depended in large measure on the energy and enterprise and upon the flexibility with which the Institute welcomed changes that improved its operations. Originally the activities of the Institute were concentrated in one city. Then came the great change of establishing many Sections, one after another. Years later the Regions were built up, thus forming an even larger, more democratic, and more effective Institute. Still more recently the Professional Groups were established, thus enabling members interested in any particular field to find full opportunity for conferences and publication. This opened up a new and even greater era for Institute activities and expansion.
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> The following guest editorial ably deals with this latest policy development in Institute affairs. It was written by a former Chairman of the Cedar Rapids, Iowa, Section of the IRE, the head of the Hunter Manufacturing Company in Iowa City, who has also been a member of the Policy Development Committee and of the Board of Directors.—*The Editor*.

IRE Growing Pains

T. A. HUNTER

Since its creation several years ago, the Policy Development Committee has been concerned with Institute problems requiring deliberate and prolonged study. One of the first problems upon which this Committee deliberated for many hours was that of how the Institute might better serve its members, and many opinions were sought from the various strata of the IRE membership. In general, the question of the PROCEEDINGS and publication of technical material received much attention, and was one of the first topics to be considered.

At that time the Professional Groups were in an embryonic state and in need of some sort of adhesive force. The problem of publishing technical material was also evidenced within this new structure of the Institute.

After a detailed study of the mass of material which was received in response to the Committee's provocative approach, one fact emerged very clearly: Many readers indicated that the PROCEEDINGS was not serving their needs fully. Further study also disclosed the belief of these same critics that the PROCEEDINGS should continue to publish the best technical and scientific information offered by the profession. These conflicting views could be reconciled only by the conclusion that more published material was needed. Minor changes were suggested from this study, and the Editorial Department made additional changes, with the result that the PROCEEDINGS is even more widely accepted as a fine technical journal. The Committee found a number of sound reasons why it was deemed inadvisable to start a second publication at this time.

For a number of years, the Cincinnati Section of the IRE has held a Television Conference, and has made available preprints of all papers given at the Conference. Several members of the Policy Development Committee have attended these Conferences regularly and have proposed that this inexpensive publication system could be adapted to the desires of each Professional Group. Thus the publication problem was approached from this viewpoint. A brief summary of the Committee's proposals follows:

1. Every member of each Professional Group must first be an Associate or higher-grade member of the Institute.

2. In order to encourage Professional Groups to grow, the Board of Directors will authorize up to $1,000 for each Group, for the purpose of publications or for conducting Group symposia, provided that Group raises an equal sum within a period of 18 months by means of assessments or by conducting Group symposia. Two dollars per member per year seems to be a reasonable assessment fee.

3. At present no paid advertising will be permitted in the publications sponsored by the Professional Groups. (This regulation is imposed to protect the advertising income of the Institute.) Studies are being conducted to see if ways can be found safely to modify this requirement.

During the 1951 IRE National Convention, many suggestions were made by members of the Professional Groups which called for the waiving of some of the requirements outlined above. This is as it should be. It seems to the author, however, that for a trial period of several years it would be wise to accept the fact that there are sound reasons for each of the requirements, and that many hours of thought and work have gone into the plan as it stands now. The rules are very flexible, and the Headquarters Staff offers many helpful suggestions through the able assistance of L. G. Cumming, Technical Secretary.

Just as in the beginning of the IRE there were Hogans, Marriotts, and Goldsmiths, so now there are many capable men in the profession who can provide the medicine to help kill the growing pains which plague our professional society at the present time. Be off with you in your microsecond chariots! Let's progress in the direction of service to the great foundation of the Institute—the average engineer. The cash register will tinkle as soon as the Professional Groups begin to send in technical material which helps the engineer obtain more engineering "know-how" in his specialized field of activity.

The time has come for action, not words.

1951 IRE Medal of Honor—Speech of Acceptance*

VLADIMIR K. ZWORYKIN†, FELLOW, IRE

I AM DEEPLY appreciative of the Award of the Medal of Honor by The Institute of Radio Engineers. It means to me more than recognition of my past efforts by the group of engineers who have shared in them and are best qualified to judge them. I take it as a challenge to those of us of the older generation to look ahead and join with the younger members of our profession in blazing new paths in the effective use of our knowledge and facilities.

Nothing could be more obvious than the universal application of radio engineering in present-day life. Radio, radar, television—these touch every one of us. This may well be a highly gratifying situation for the radio engineer. Yet, it is to be hoped that he will not be satisfied with self-admiration for his accomplishment. Essentially, in these three fields he is dealing with ground that has already been won. This applies even to color television, in spite of the numerous difficulties in design and production which remain to be solved.

It seems to me that it is the primary function of the engineer to map out new possibilities for growth of industry by

rendering possible new ways of fulfilling human needs. The radio engineer has done so in the past with great effectiveness in the fields of communication and entertainment. Far less attention has so far been paid by him to medicine, even though the promotion of health, the alleviation of suffering, and the lengthening of life represent objectives of primary importance.

The range of problems in medicine to which electronic methods may be applied is remarkably broad, embracing both diagnosis and therapy. As a matter of course, their effective application demands close co-operation between the biological scientist or physician and the radio engineer throughout. The proper distribution of effort will depend on the nature of the individual problem.

Even today, on this basis, electron microscopes and television microscopy provide tools of great value for the study of vital processes. Highly sensitive electrocardiographs and encephalographs are routine instruments for the diagnosis of heart conditions and brain injuries. Here also, as elsewhere in medicine, the application of television methods can lead to increased effectiveness: The television picture conveys a specific piece of information about every one of its component picture elements. By a suitable modification of the pickup mecha-

nism it may equally well provide a map of any other parameter, such as potential or temperature. In particular, maps of skin potentials, varying with time, could picture the character of a circulatory defect or locate a brain tumor with an immediacy not attainable with conventional instruments.

Television technique also makes possible notable advances in electronic blood-counting methods, promising increased speed and accuracy in blood analysis. A careful examination would doubtless reveal many other ways in which electronic instruments could relieve the medical technician of routine duties. In radiology, scintillation and Geiger counters even now warn against radiation injury, and image intensifiers protect patient and surgeon against overexposure in X-ray fluoroscopy.

Yet, I am sure, this is but a sample of what the electronic engineer might do, in collaboration with the medical practitioner and research worker, to advance health. Increased emphasis on this objective would enhance the service of our profession to mankind, broaden the base of the electronic industry, and add to the personal satisfaction derived from our work. As yet, the application of electronics in medicine is new ground. It meets a universally recognized human need. Let us cultivate it.

Polycasting*

RAYMOND M. WILMOTTE†, FELLOW, IRE

Summary—Polycasting is a method applicable to very-high-frequency and ultra-high-frequency bands for broadcasting a program from a number of low-powered stations instead of from a single high-powered one with the object of covering a continuous area. It may be regarded as a special application of satellite operation, making use of the fact that at high frequencies the signal intensity varies over a wide range from point to point so that the service must be evaluated on a statistical basis. The paper is a theoretical study of the service obtainable.

It is shown that to provide a large percentage of the locations in an area with good service requires far more power from a single sta-

tion than from several stations. This effective power gain is over and above the simple gain from better "illumination" of a large area when served by several sources instead of concentrating all the power at one point.

A special application of the system to serve rural areas is explained. It is there proposed to locate the component stations of the polycasting system in small towns, so located that a rural grade of service is provided by them in the area between them.

Estimates of service are given, taking into account ghosts and signal intensity required to overcome noise. There is also laid out tentative allocation of stations in two typical areas of the United States.

I. THE NATURE OF POLYCASTING

IN A PETITION presented to the Federal Communications Commission by Paul A. deMars and the author on November 30, 1948, it was pointed out that excessively large powers would be required to

serve an area with television, particularly in the ultra-high-frequency band. The petition was, in effect, a memorandum urging the consideration of providing service by a system termed "polycasting," involving the use of a number of low-powered stations on the same or different frequencies instead of a single high-powered station. The Federal Communications Commission responded to the petition by specifically naming polycasting as an issue in the present television hearings. Addi-

tional work by the author has evolved new concepts and improvements on the system described in the petition.

Another term that has been used for transmitting the same program from several stations rather than one is satellite operation, but this term implies that satellite stations are small relative to the main station, and generally indicates the case of a station some distance away from the main station and operated for the purpose of serving a new area with little or no regard to the service in the area between the stations. It has also been applied only at frequencies for which the signal intensity in a small area does not materially vary from location to location. Polycasting as the term is used by the author, on the other hand, is specifically concerned in serving a large, continuous area at frequencies at which the signal intensity varies over a wide range from location to location immediately adjacent to each other. There is, therefore, little relationship between service estimates for satellite operation at low frequencies and the operation studied in this paper.

The estimates given below show that there is a considerable saving in power if a polycasting system is used as against a single station. These estimates cover both city, or heavily built-up areas where the service has to be very good, and rural areas, where the obstructions are mainly hills, and the service does not have to be of the same high quality. Of special interest is the service to small towns and rural areas because of the obvious economic problem that arises of providing a costly service to a small population. The only solution appears to be to allow the operator in such a territory to serve a much larger area than is reasonably possible with a single station. With a polycasting system, that can be achieved. The component stations are located in selected small towns and designed so that the rural area in between these towns is adequately served. Such a system has a twofold advantage. It provides good service in towns too small to support a television station, and it provides service to a large sparsely populated rural area which it is uneconomical to serve without the support of the towns.

At low frequencies, such as the regular broadcast band, the service is helped by the fact that the signal intensity in a limited area is relatively constant. It may vary over a wide range in time, but not in location. That is an ideal condition for single-station operation. At high frequencies, such as the television very-high-frequency band, and even more so at the ultra-high-frequency band, a major difficulty arises in serving an area in that the signal intensity varies over a wide range from location to location. Polycasting makes a virtue of this handicap. In fact, the full advantage of polycasting relative to single-station operation is obtained only when the signal strength varies widely with location.

This unexpected result becomes clear on examining Fig. 1. There is shown, by the lowest line, a typical distribution curve of the percentage of locations within a limited area in which the signal exceeds a specified intensity. If there are two signals from two different stations serving the area, at a location where one signal is strong, the other signal may or may not be strong. Presumably the antenna will be directed to receive the stronger signal. Assuming that there is no correlation as to location between the signals from the two stations, and that each has the same distribution for the percentage of location versus the value of the signal strength that is exceeded, it is possible to calculate the new distribution with regard to location for the signal strength exceeded by the stronger of the two signals. Such a calculation results in the distribution shown by the second curve for the case when the two signals have the same median value. The other line corresponds to the distribution when there are four uncorrelated signals of equal median value. This line corresponds to the signal distribution at the center of a square with a station at each corner and indicates the probability of a certain signal intensity being exceeded by the strongest of the four signals at that point.

When considering a system for broadcast service, it will be designed to provide acceptable service to a large percentage of the locations. The addition of one station would be expected to produce the effect of doubling the power or an increase of 3 db. Similarly the addition of
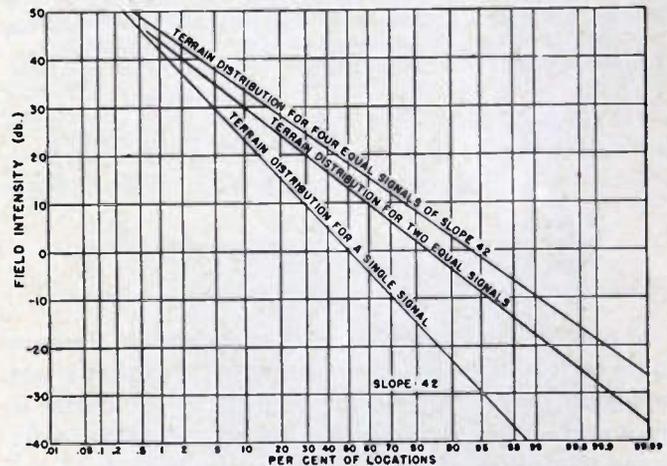
Fig. 1—Effective increase of signal strength with polycasting due to multiple signals at a location; slope 42.

three stations should increase the power four times or 6 db. But looking at Fig. 1, it is seen that for 90 per cent of the locations the signal intensity exceeded in the central area of the system is 14 db for two stations and 26 db for four stations above that which would be exceeded for one station. From a service point of view, that increase in signal represents an effective gain in the power received. It is seen that this effective gain is substantial and represents a large saving in power at the transmitter.

The difference between Figs. 1 and 2 lies in the slope of the distribution curve for location. In Fig. 1, the
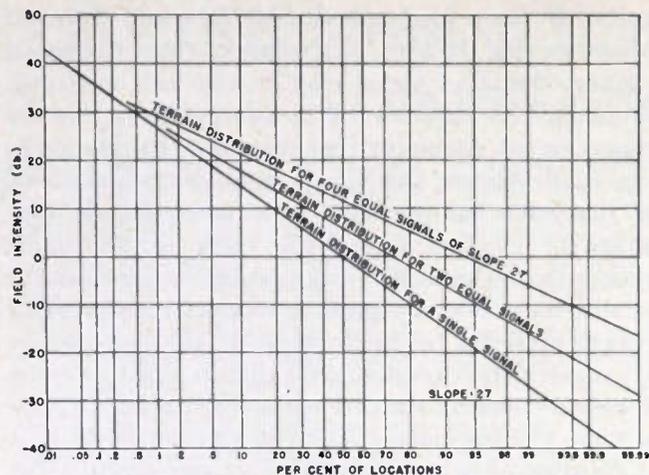
Fig. 2—Effective increase of signal strength with polycasting due to multiple signals at a location; slope 27.

slope is 42 (for built-up areas); in Fig. 2 it is 27 (for rural areas). The slope of the log normal distribution is defined as the ratio between the 99-per cent point and the 50-per cent point.

Table I gives the effective gain at the center of a square for a polycasting system consisting of four equal stations located at the corners.

TABLE I

EFFECTIVE POWER GAIN OF FIELD AT CENTER OF A POLYCASTING SYSTEM CONSISTING OF FOUR EQUAL STATIONS LOCATED AT THE CORNERS OF A SQUARE COMPARED WITH AN EQUAL STATION OF EQUAL TOTAL POWER CONCENTRATED AT ONE OF THE CORNERS

Percentage of Locations	Effective Gain	
	Slope of Location Distribution	
	Slope 42 (City)	Slope 27 (Rural)
90 per cent	20 db	10 db
70 per cent	15 db	7 db
50 per cent	12 db	5 db

At points other than the center of the square the gain would be less than that given in Table I, but that is not important because the signal intensity at the center is the lowest of any point within the square. This result means that either there is better service provided for a given amount of power, or the size of the square can be expanded to provide the same service but over a larger area. What is actually decided upon depends on the conditions. Generally when serving a city, emphasis would probably be placed on improving the service, but when serving a rural area emphasis would rather be put on the size of the area.

The arrangement of polycasting stations in the form of a square is convenient for discussion and description of operation. If the stations operated on the same frequency, there would be a problem of ghosts, particularly within the square itself. The worst section would in general be the area near the center of the square where the signal intensities of each of the four stations are most likely to be of the same order of magnitude. It can be shown that with amplitude modulation as contemplated

at present, the ghost problem would eliminate most of the locations near the center of the square as being satisfactory for reception. This difficulty can be partly overcome by using two channels. In the case of an arrangement in a square, each pair of co-channel stations would be located at the ends of a diagonal. Still better results would be obtained if more than two channels were used.

All the estimates given below are based on using two channels in each polycasting system.

The use of several channels to reduce ghosts in polycasting seems at first to the allocation engineer to be exhorbitantly costly in frequencies. But on further consideration that is not so, for the total power required to provide satisfactory service is only a small fraction of the power required to serve the same area with a single powerful station. The interference to other stations will be proportionately smaller, and closer spacing of stations becomes possible. The providing of two channels to serve the large service areas is not likely to reduce the total number of stations possible if polycasting is properly used. In fact, there is described below an allocation which indicates that the uhf band has potential room for multiple services while allowing two channels for each operator. Limitations of channels is a problem essentially limited at this time to the United States. In England, for instance, it may be possible to use three or even four channels to a program. The special problem of ghost of polycasting would not therefore exist and it would be expected that service could be provided with good quality and relatively cheaply by means of carefully located low-powered stations.

The possible advantages of polycasting may be listed as follows:

1. It provides a high and more uniform signal level over the area (a) because the radiated energy is more uniformly distributed than with a single station; and (b) because of the large gain in the effective power for large percentages of locations (explained above, see Table I).
2. It fills in shadows by locating stations so that obstacles receive signals from several directions.
3. It reduces interference because low powers are used in the available system.
4. It reduces blanketing because still lower powers are used at each transmitting point.
5. It provides conditions more satisfactory for "built-in" antennas because the signal is high and steel structures do not effectively shield uhf.
6. It makes it possible to pattern the service to the needs of the community, and plan expansion from time to time when and where the community needs it.
7. The initial cost is much lower for larger service areas.
8. There is less fading at limit of service area.

Against these advantages are to be considered two apparent disadvantages.

1. Ghosts. Ghosts are a problem when dealing with a single station. They effectively produce a number of locations of poor reception even when the signal is strong. With two or more stations on the same frequency the number of such locations will be increased. Will the number of additional locations rejected because of ghosts be greater or less than the number accepted because of increase in signal intensity due to polycasting operation?

2. Wider frequency band. If the present trend of thinking is retained, whereby the picture is transmitted by amplitude modulation, it appears necessary to use two frequency channels for one service. If the trend is reversed and frequency modulation is adopted, there is a very good possibility that circuits will be devised able to separate signals more nearly equal in intensity, thereby overcoming the ghost problem. In that case it may be necessary to increase the bandwidth beyond 6 mc, but probably not beyond 12 mc.

These two objections are constantly kept in mind in the discussion which follows. The effect of ghosts is considered in the estimates of service that can be rendered with polycasting. The effect of the use of a wide band or two bands for each service and the correlative compensation of low interference is considered in an allocation study for two important areas of totally different type.

Some cost figures are also given and a brief discussion is presented of what might be possible should our acceptance of amplitude modulation be questioned and the powerful weapon of frequency modulation become accepted for picture transmission.

II. GHOSTS

Although, to our eye, a ghost has a definite character, as far as the receiver is concerned, it is merely an interfering signal, like any other interfering signal. The removal of a ghost involves therefore the same methods as the removal of an interfering co-channel signal. The best method is that which separates signals having a ratio as close to unity as possible, provided that, in so doing, undesirable features are not developed or emphasized.

The four possible methods of separating co-channel signals are:

1. Using directional receiving antennas.
2. Adjusting the location of a simple receiving antenna to take advantage of the difference in the standing-wave patterns of the desired signal and the ghost signal.
3. Using two or three different frequency channels (in the case of polycasting).
4. Using FM or other type of modulation instead of AM.

The effect that polycasting has of increasing the ghost type of interference will not appreciably affect the size of the service area, for at the outer boundaries of the area the principal signal received will practically al-

ways be a great deal stronger than the signal of any other co-channel station of the system; but in the area within the stations, polycasting will reduce the number of acceptable locations because ghost type of interference will be worse than with a single station. In the estimates of service this problem has been taken into consideration. The assumptions used in making these estimates are given in the Appendix.

In estimating the effect of ghosts, it has been assumed that when the ratio from two co-channel stations of a polycasting system is more than 10 db, the picture may be considered acceptably free from ghosts. (Part of the reasoning leading to this figure given in the Appendix is based on obtaining a degree of separation of the signals by the use of directional receiving antennas.) The question may reasonably be raised as to why it has been assumed that an amplitude separation of 10 db is satisfactory for co-channel stations within a polycasting system and a separation of 20 or 40 db between polycasting systems. The figure of 10 db is, as explained, a guess. It is, I believe, a realistic guess, but there is no field experience to prove it. It is reasonable to expect that an owner of a polycasting system will endeavor to engineer the location of his stations with special consideration to providing the best service so that shadows will be eliminated as much as possible. No similar planning directed to reducing interference can be expected from the owner of a neighboring polycasting system. For the same reason, the accuracy and maintenance of offset carrier or synchronous carrier, or of the sync pulse, may be expected to be more accurate inside a single polycasting system than between polycasting systems. Generally speaking, it is easier to correct de-
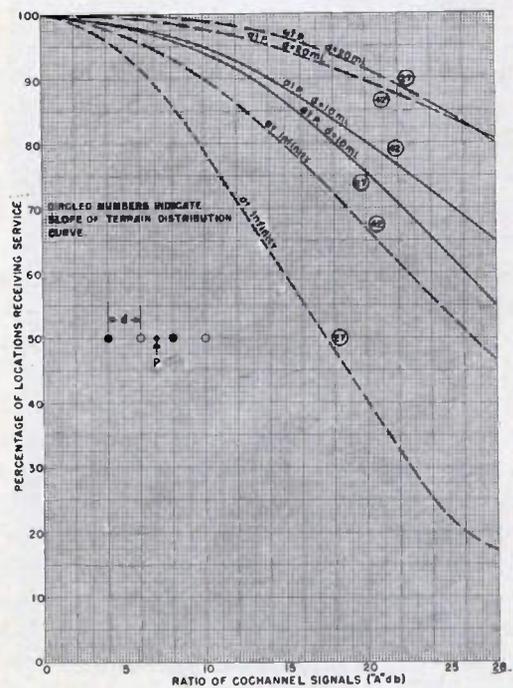
Fig. 3—Four-station polycasting system with the stations in a line. Percentage of locations receiving service at P versus required ratio of cochannel signals.

TABLE II
Comparative Powers of Polycasting and Single-Station Operation

Type of Area	Grade of Service	Area (Square Miles)	Polycasting (300 Feet)		Single Station		Fading at Extreme of Service	
			Number of Stations	Total Power	300 Ft	500 Ft	Poly-casting	Single Station*
Rural								
Small (Fig. 5)	A	1,400	4	8 kw	55 kw	30 kw		
	B	2,500		8 kw	80 kw	30 kw		
	C	4,100			120 kw	40 kw	9 db	15 db
Large (Fig. 6)	C	5,500	4	8 kw	1,000 kw	300 kw	7 db	20 db
Very Large (Fig. 7)	C	19,000	12	12 kw	300×10^6 kw	200×10^6 kw	7 db	72 db
City								
Circular (Fig. 8)	A	1,100	4	8 kw	1,100 kw	340 kw		*
	B	1,600			180 kw	50 kw		
	C	3,500			190 kw	40 kw	10 db	13 db
Elliptical (Fig. 9)	A	46×30	4	8 kw	4,700 kw	1,200 kw		
	B	60×44			1,800 kw	400 kw		
	C	76×58			850 kw	150 kw	7 db	17 db

* The fading is given for height of 300 feet. At 500 feet, the fading is 3 to 4 db less.

fects in one's own back yard than nuisances from one's neighbors. It is inherently appropriate, therefore, to specify a greater amplitude separation between polycasting systems than between stations within a polycasting system. More important is that such increased protection between polycasting systems frees each viewer to direct his attention more exclusively to reducing ghosts, whether they are caused by reflections or come from a polycasting station.

Figs. 5 through 9 show the estimated service areas of a number of examples of polycasting systems. The percentage of locations free from ghost type of interference are shown by the dashed lines and marked with the appropriate percentage figure. If the rejection figure of 10 db should prove incorrect, these percentage figures will change. The amount of the change can be roughly gauged from Figs. 3 and 4.

III. Estimates of Service

The arrangement of the stations of a polycasting system will depend primarily on the size of the area and the grade of service to be given to it. For a city service, the area is relatively restricted, but the quality or grade of service should be high. The component stations of the system should, therefore, be relatively close together, so that the signal may be strong over the whole of the densely populated area.

In a rural service, the reverse is the case. The grade of the service can be relatively low, first, because it is not economical to provide a strong signal where the population is sparse, and second, because isolated houses can effectively install antennas of good directivity and substantial gain.

Rural service by polycasting leads to the specially interesting possibility of selecting small towns for the location of the component stations of the system, so that these towns will be served with a strong signal. The rural area between them would be served by selecting these towns carefully and planning the radiation from the stations so that the whole area comprised within these towns is served with at least grade-C service. Such a service does not require the component stations to be very close together.

There would seem to be little value economically in putting the component stations of a system close together in a rural area. As will be seen from the estimates given in Table II, this operation provides some gain over a single-station operation, but that gain is small compared with the gain obtained for city service or large rural coverage.
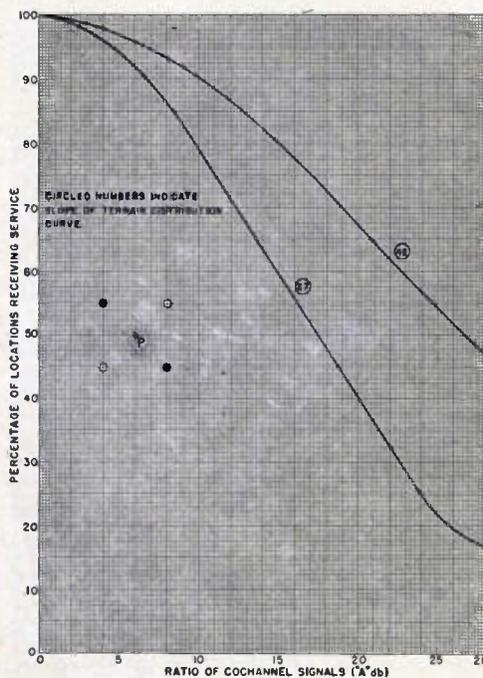
Fig. 4—Four-station polycasting system with stations at corners of a square. (Curves independent of distance between stations.) Percentage of locations receiving service at P versus required ratio of cochannel signals.
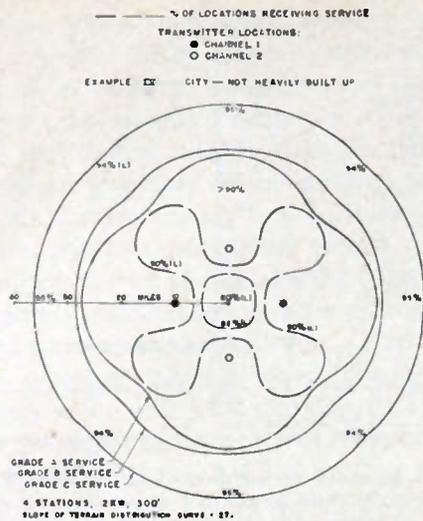
Fig. 5—Rural service, small area. Four stations, to kw, 300 feet; slope of terrain distribution curve, 27.
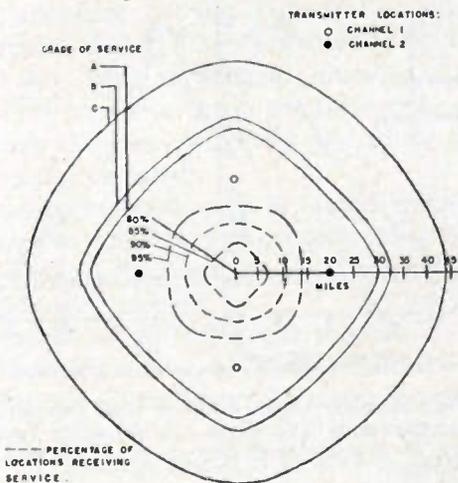
Fig. 6—Rural service, small area. Service by polycasting, 4 stations, 2 kw, 300 feet; slope of terrain distribution curve, 27.
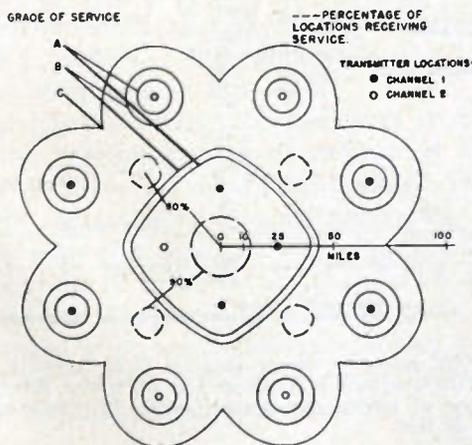
Fig. 7—Rural service, large area. Service by polycasting, 12 stations, 2 kw, 300 feet; slope of terrain distribution curve, 27. Service within contour is greater than minimum signal for class C service free from objectionable interference for more than 80 per cent of the locations.
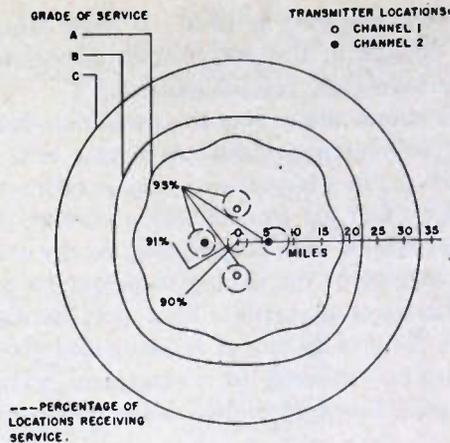
Fig. 8—City, heavily built up. Service by polycasting, 4 stations, 2 kw, 300 feet; slope of terrain distribution curve, 42.
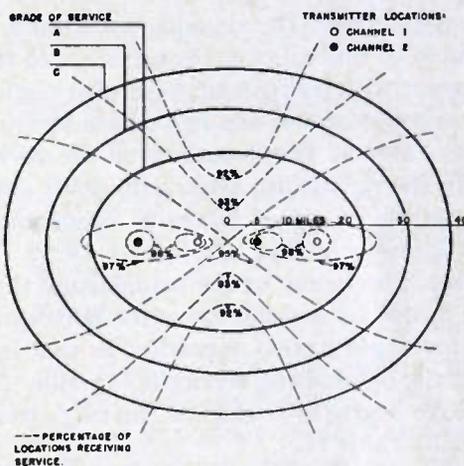
Fig. 9—City, heavily built up. Service by polycasting, 4 stations, 2 kw, 300 feet; slope of terrain distribution curve, 42.

The estimates given in Table II and in Figs. 5 through 9 are based on a polycasting system, one consisting of small stations radiating 2 kw of power at a height of 300 feet above surrounding terrain. Two channels are used with half the stations on each channel. The general arrangement is a square with a station at each corner, the co-channel stations being at the extremities of a diagonal. The arrangement of the stations is shown on the figures. The slope of the log normal distribution curve was taken as 27 for rural areas and 42 for city. The details of the data used are given in the Appendix. Table II shows the area of the polycasting service for different grades, and the estimated power required from a single high-powered station to serve the same area. In the figures the solid lines indicate the field-intensity contours. The dashed lines indicate contours for which a specified percentage of the locations is acceptably free from the ghost type of interference from the co-channel station of the polycasting system. The percentage figures in the dashed lines depend greatly on the assumptions made as to the possible signal ratio required to eliminate objectionable ghost type of interference due to co-channel operation within a system. As explained, this

ratio has been assumed as 10 db. If some other ratio is used, the change in the percentage value can be estimated approximately from Figs. 3 and 4.

Table II shows the saving in power that may be obtained by polycasting. Grade-A service in a city and grade-C service in a large rural area is estimated to require about 1,000 kw from a single station, compared with a total of 8 kw with polycasting. As the polycasting system is extended, the saving in power becomes fantastic. With twelve stations in a polycasting system radiating a total of 24 kw, it is estimated that class-C service may be rendered to a rural area which would require several hundred million kilowatts from a single station.

The polycasting service is likely to be better, relative to the single station operation, than shown by the diagrams, for three reasons:

1. Signal intensity. The signal is more uniformly distributed so that all over the service area the signal is greater with polycasting than with single-station operation, except in the immediate vicinity of the station and at the boundary of the service area where the signals are arbitrarily made equal. The comparison of signal intensity inside the service area is shown in Figs. 10 through 13.

2. Fading. The fading at the boundary of the service area (grade C) is much lower for polycasting than it is for single-station operation, so that in this respect the polycasting service is superior. The comparative degree of fading for the two cases is indicated in Table II.

3. Interference. Each channel with polycasting radiates very much less power than the single station. The difference will permit closer spacing of stations and also reduce the very long, distance interference conditions that occur spasmodically. In addition, the fact that the signal intensity is greater over the area with polycasting increases the desired-to-undesired ratio within the area.
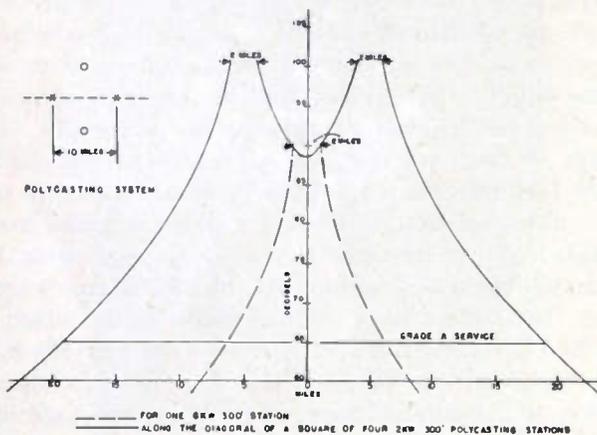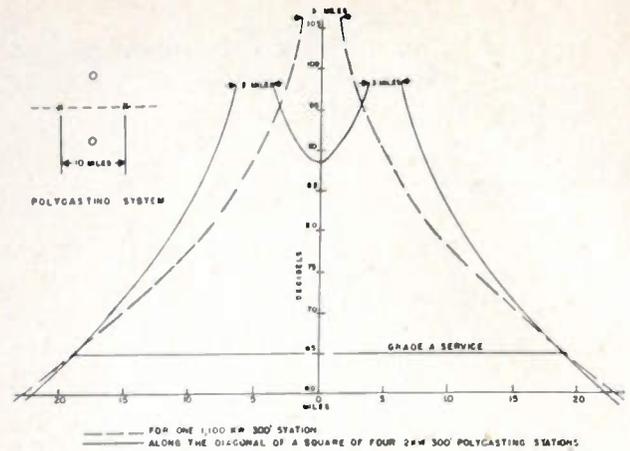
Fig. 11—Field intensity versus distance. Field intensity at 90 per cent of the locations in db above 1 μv per meter. Frequency, 600 mc; slope of terrain distribution curve, 42; receiving antenna height, 30 feet.
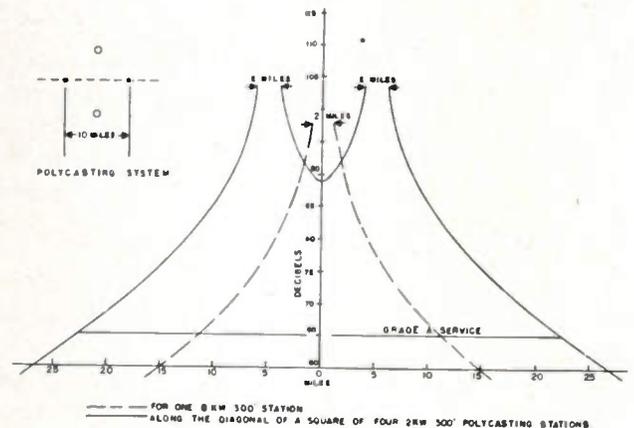
Fig. 12—Field intensity versus distance. Field intensity at 90 per cent of the locations in db above 1 μv per meter. Frequency, 600 mc; slope of terrain distribution curve, 27; receiving antenna height, 30 feet.

Fig. 13—Field intensity versus distance. Field intensity at 90 per cent of the locations in db above 1 μv per meter. Frequency, 600 mc; slope of terrain distribution curve, 27; receiving antenna height, 30 feet.
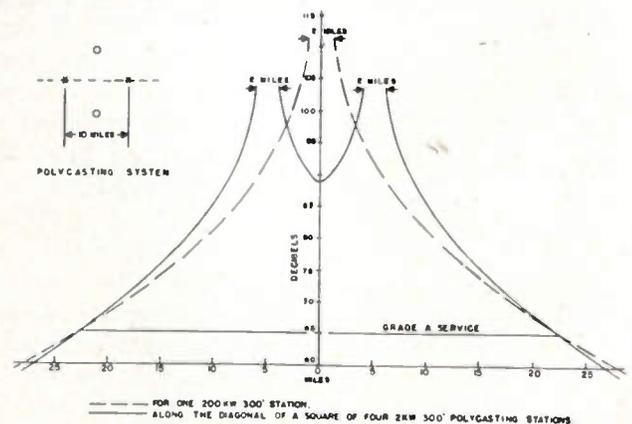
Fig. 10—Field intensity versus distance. Field intensity at 90 per cent of the locations in db above 1 μv per meter. Frequency, 600 mc; slope of terrain distribution curve, 42; receiving antenna height, 30 feet.

IV. Study of Allocation

A polycasting system is not limited to serving a circular area, nor is the area limited in size as it is with a

single-station operation. A polycasting system can be designed, therefore, to fit substantially with the great variety of social, cultural, economic, and political areas that make up the country. In any allocation that takes these basic factors into account, the polycasting systems used should vary greatly from one area to the next. However, to simplify the study of an estimated allocation pattern, a simple polycasting system, consisting of two channels with two stations having 2 kw of radiated power at a height of 300 feet on each channel, has been taken as the basic polycasting unit.

Two examples of possible allocations were studied. One is designed for a typical midwestern area and the other for the congested eastern area lying between New York City and Baltimore, Md.[1]

For the first example, the section of the country taken includes the cities of St. Louis, Mo., Kansas City, Mo., and Wichita, Kan. Each city in the area which has a population of more than 100,000 was assigned as many uhf polycasting services as are provided for it in the proposed allocation of the Federal Communications Commission, including those provided in both the vhf and uhf band. After an adequate number of channels was provided for these cities, the rest of the area was studied with the view of serving the whole area with grade-C service or better.

There are innumerable ways of providing this service. On the basis of the arrangement selected it was found that the whole of the rural area can be served with the use of only nine channels.

The purpose of providing uhf services in the large cities to the same number as is provided in the vhf band in the proposal of the Federal Communications Commission was to see if the uhf band was large enough to carry all the services required in a truly complete national television service in the area studied. The results indicated that several such national services appeared to be possible.

Table III shows the channels that were allocated in this study to cities of more than 100,000, and indicates also the estimated possible number of services that might be provided for cities and rural areas if all channels from 14 through 55 were used.

In making this study, the minimum separation of polycasting systems in Figs. 21 through 24 was maintained with only a few minor exceptions. Also in accordance with the concept explained previously, the individual stations of a polycasting system were located as far as possible in small towns. In some of the larger towns, such as St. Joseph, Mo., and Lincoln, Neb., it was arranged to have two stations belonging to different polycasting systems designed to serve areas lying in different directions from these towns.

[1] The maps showing the allocation that developed out of the study were too large to be conveniently reproduced. They are on record in the files of the Federal Communications Commission, Docket Nos. 8736, 8975, 9175, and 8976, Exhibits 290 and 291.

TABLE III
EXAMPLE OF POLYCASTING ALLOCATION MIDWESTERN AREA
Operation: Each service 4 polycasting stations, 2 kw radiated, 300 feet, two channels.

Area Served	Shown on Figure		Estimated Total Possible	
	Number of Services	Channels	Number of Services	Channels
Cities over 100,000				
St. Louis, Mo.	6	14–36	10	14–52
Kansas City, Mo.	4	15–29	10	15–53
Omaha, Neb.	3	14–24	6	14–36
Des Moines, Iowa	3	14–24	6	14–36
Wichita, Kan.	3	14–24	6	14–36
Davenport, Iowa	2	15–21	4	15–29
Peoria, Ill.	2	15–21	4	15–29
Other Areas	1–2	14–22	4–6	

The population within each polycasting system is given approximately in Table IV. It is seen that a substantial population is served by each system.

TABLE IV
APPROXIMATE POPULATION SERVED BY ONE POLYCASTING SYSTEM—RURAL SERVICE

State	Population
Illinois	100,000
Iowa	200,000
Kansas	105,000
Missouri	150,000
Nebraska	66,000

For the second example, the congested area lying between New York, N.Y., and Baltimore, Md., the allocation was designed to provide grade-B service over the whole area. Here again, all cities of over 100,000 population, of which there are sixteen, were provided with as many services as was proposed by the Federal Communications Commission for the vhf and uhf bands combined. An exception was made for New York, which was provided with ten channels, four more than were proposed by the Federal Communications Commission. For the rest of the area, polycasting stations were located, whenever possible, at the more important population centers.

A deviation from the simple basic polycasting system of locating the station at approximately the corners of a square was used in one case on Long Island, where the service from a number of stations in a line was an ellipse. To serve the Scranton—Wilkes–Barre, Pa., area, a similar arrangement would probably be used.

It would be desirable for some of the services in certain large cities to include the city proper plus an area extending in one direction, while other services will include areas in another direction. Such an arrangement would make it possible to bring adjacent polycasting systems closer to the city. For instance, in the case of New York City, if a polycasting system were established in New Jersey starting at Asbury Park and going south along the coast, a New York City system could be established on adjacent channels, but designed to spread its service toward Long Island or Westchester County

and deliberately restrict its service in New Jersey. For similar reasons, some New York services would spread into New Jersey to allow adjacent channel systems to serve Long Island or Westchester County.

The study indicates that few areas would be limited to two services only.

V. Operation

In the operation of a polycasting system the program would be originated at one of the stations which would become the "master" station. From that station it would be relayed to the "slave" stations. In relaying, there is no need to demodulate the signal. That would probably introduce more noise than is necessary. All that is necessary is to change the frequency and amplify. The operation may be achieved through the use of a radio relay link operating in the thousands of megacycles.

In many cases a simpler method would be available since in the polycasting system described two channels are used. A "slave" station on one of the channels could obtain its program by receiving the signal from one of the stations of the system operating on the other channel, changing the frequency to its own channel, and amplifying.

VI. Costs

In order to estimate costs, inquiries were made of the principal manufacturers of the cost of the equipment of a "slave" station. The request was worded as follows:

"I think we should assumed for the purposes of estimating a price that polycasting is an accepted mode of operation so that the small slave installations would be built in reasonable quantities. The engineering design cost should therefore be substantially eliminated.

"A slave station would operate by picking up a signal from another station by means of a dish on the 300-foot tower, changing the frequency to that of another television channel and amplifying to about 100 watts, and then carrying that power to a broadcasting antenna on the top of the same tower having a gain of some 16 db (this figure was reduced to 11 or 13 db in a subsequent letter) so as to radiate about 2 kw. The equipment would consist at each of these stations, of:
"1. a receiving dish
2. 350 feet of receiving transmission line
3. A frequency changer and amplifier with output of 200 watts
4. 350 feet of power transmission line
5. A high-gain broadcasting antenna."

The results from this letter are given in Table V.

TABLE V

Manufacturer	Price
Gates Radio Company	$10,800.00
General Electric Company	27,000.00
Raytheon Manufacturing Company	28,700.00
Westinghouse Electric Corporation	11,634.00–12,120.00

No indication was given as to the accuracy of these figures and how much study had been made in arriving at them except in the case of Westinghouse, who provided a detailed report with their estimate of price.

Gates and Westinghouse based their prices on the assumption that a production line of some fifty units could be established. Such a production seems reasonable, if polycasting were to become an accepted means of providing television service.

Taking the lower figures of around $11,500.00 and assuming that the installation will be made on a slight hill so that the cost of a tower to raise the antenna to 300 feet may be about $5,000.00, and putting a figure of $1,500.00 for the doghouse to house the equipment, the total cost may be taken to be between $18,000.00 and $20,000.00. Thus four stations would cost about $80,000.00—far less than the 1,000 kw needed by a single station to provide a similar service.

The cost of the master station, not including the studios and control, which are common to both polycasting and single-station operation will be about the same.

The "slave" stations should be automatic and not need permanent staffs of operators. A polycasting system would therefore represent little more in personnel cost than a single station.

VII. Acknowledgment

The author's appreciation is extended to Philip Rice, who carried out many of the calculations in this paper and made many valuable and original suggestions.

APPENDIX

Data and Method of Estimating Service

Since the signal intensity varies from location to location, the approach to any estimate of service must involve statistical considerations.

The procedure used here for estimating field intensity follows broadly the direction contained in the report of the "Ad Hoc" Committee, which was established late in 1948 by the Federal Communications Commission for that purpose.[2] It also adheres as closely as possible with the standards of service originally proposed by the Federal Communications Commission and released on July 11, 1949, in connection with the television hearing Docket No. 9175, which was associated with Dockets Nos. 8736, 8975, and 8976.

Four special problems arise in estimating service for a polycasting system. They are:

1. Ultra-high-frequency propagation. The "Ad Hoc" Committee report does not cover the uhf band. Information is to be sought from other sources.
2. Ghosts. Calculations must be made to take into account the effect of ghosts when more than one station operates on the same frequency.
3. Service Contours. Consideration must be given to the fact that the service does not deteriorate from a single point but from several, so that there may be weak contours within stronger ones.
4. Interference. The interference between polycasting systems and the required separation between them depends on the arrangement of the stations within each system.

[2] The report of the "Ad Hoc" Committee and the associated references were subject to criticism on the grounds that its analysis is too intensive for the data on which it is based. At this time it appears to be the most authoritative data available. Without it there would be no way of making even rough estimates.

The polycasting system selected for calculation comprises the following operating conditions:

Two channels.

Four stations, two on each channel. The stations are located at the corners of a square, those on the same frequency being located at opposite ends of a diagonal.

8 kw total power radiated, 2 kw at each station, making a total of 4 kw on each channel.

Height of antennas, 300 feet above surrounding terrain.

Stations on the same channel operated with carrier frequency offset by optimum amount.

A. Ultra-High-Frequency Propagation

There are three principal factors involved in the establishment of a signal:

1. The distribution of signal intensity with location in a limited area.
2. The median value of the signal intensity and its variation with distance.
3. Fading or the distribution of signal intensity with time.

A discussion of these factors follows.

1. Distribution of Signal Intensity with Location

Fig. 14 shows the nearest log normal distribution with location for different terrain conditions. The circles indicate measurements of the median fields as described in the literature on the uhf survey made in Washington, D. C., by the Radio Corporation of America on a frequency of 505 mc.[3] The data are in close agreement with the distribution curve proposed by the Federal Communications Commission. This curve is the same as $r(L)$ of Fig. 2 of the "Ad Hoc" report.

The crosses on Fig. 14 are taken from the published data of the Philco survey of the same operation and represent the terrain distribution factor of the same
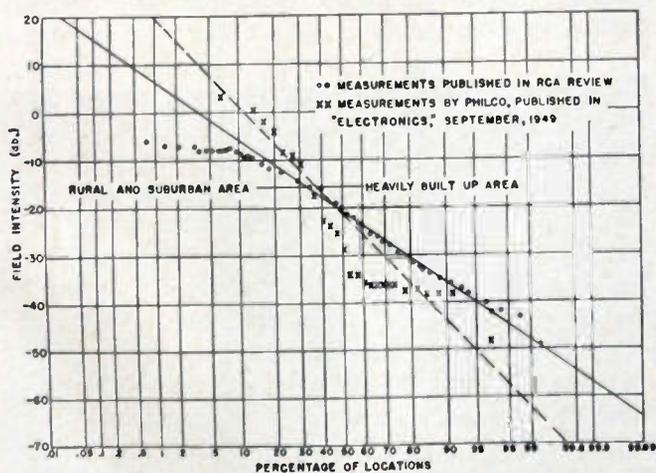
Fig. 14—Distribution of measured signals on 505 mc from transmitter at Wardman Park Hotel, Washington, D. C.

[3] G. H. Brown, "Field test of ultra-high-frequency television in the Washington area," *RCA Rev.*, vol. 9, pp. 565–584; December, 1948. See Figs. 8 through 15, pp. 574–577.

operation, with one exception.[4] In the case of the Philco data, the information is limited to the points lying only within a four-mile radius of the transmitter location.[5] It will be seen that the distribution of these points does not fit the previously suggested terrain factor. It is believed, therefore, that the dashed line on Fig. 14 represents more closely the terrain distribution factor for heavily built-up areas than the factor $r(L)$ of Fig. 2 of the "Ad Hoc" report.

In the two examples of rural service and the city example for "suburban" type of area, the $r(L)$ curve entitled "rural and suburban" was used. In the example for the "heavily built-up" type of city area, the dashed curve was used entitled "heavily built-up." Theoretically different log normal distribution should be used at different distances. To reduce the work involved, however, the same log normal distribution was used.

The slope of a log normal distribution is $E(99) - E(50)$, as defined in the "Ad Hoc" Committee report.

2. Median Value of Signal Intensity

Since the "Ad Hoc" Committee report did not include analysis of measurements in the uhf region, a 600-mc propagation curve was calculated following the correction procedure outline in reference C of the "Ad Hoc" report.

The first step was to calculate a curve for a smooth 4/3 earth radius using the curves and formulas of Norton.[6] This curve was based on a transmitting antenna height of 300 feet, a receiving antenna height of 30 feet and an effective power of 2 kilowatts radiated on 600 mc. At small distances where the smooth earth curves give values oscillating about the free-space value of the field, a constant free-space field distribution was assumed.

Figs. 15 and 16 show the propagation curves used for 600 mc. The $M(d, f)$ curve used, which corresponds to Fig. 1 of the "Ad Hoc" report, is shown in Fig. 17. This correction is applied to the theoretical propagation curve for a smooth sphere of 4/3 the earth's radius to obtain the curves in Figs. 1 and 2 up to forty miles from the transmitter. For distances beyond eighty miles, the propagation characteristics were obtained by extrapolating the "Ad Hoc" Committee curves to 600 mc as shown in Fig. 18. The propagation curves were completed by connecting these two sections with a smooth curve over the distance from forty to eighty miles. Receiving antenna heights are assumed to be 30 feet for all estimates.

The dotted curves in Figs. 15 and 16 represent the theoretical field intensity-versus-distance curve referred to as S in the "Ad Hoc" report. The distance d_0 beyond which S is always less than the free-space field is

[4] J. Fisher, "Field test of uhf television," *Electronics*, vol. 22, p. 106; September, 1949.
[5] See Fig. 4, p. 109, of footnote reference 4.
[6] K. A. Norton, "The calculation of ground-wave field intensity over a finitely conducting spherical earth," PROC. I.R.E., vol. 29, pp. 623–639; December, 1941.
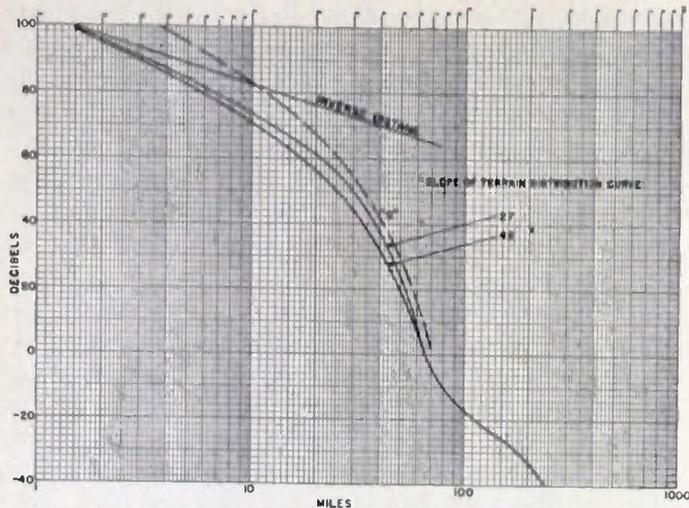
Fig. 15—Median field intensity versus distance. Frequency: 600 mc; transmitting antenna height: 300 feet; receiving antenna height: 30 feet; 1-kw radiated; field intensity in db above 1 μv per meter.

Fig. 16—Median field intensity versus distance. Frequency, 600 mc; transmitting antenna height, 500 feet; receiving antenna height, 30 feet; 1-kw radiated field intensity in db above 1 μv per meter.
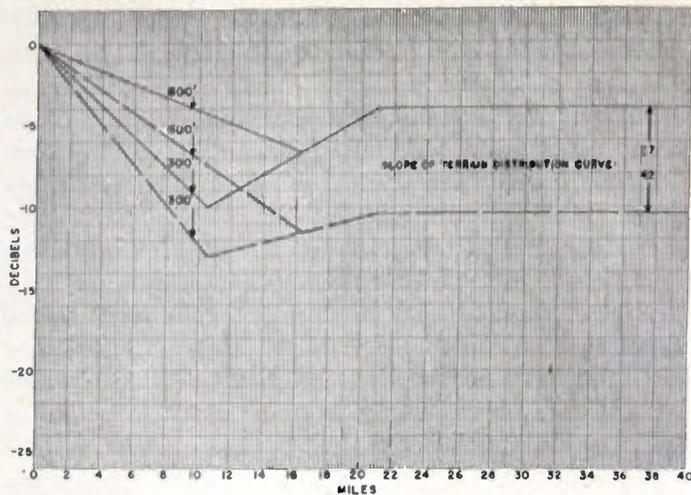
Fig. 17—Correction factor "$M(d,f)$." Transmitting antenna height as indicated. Receiving antenna height, 30 feet; frequency, 600 mc.
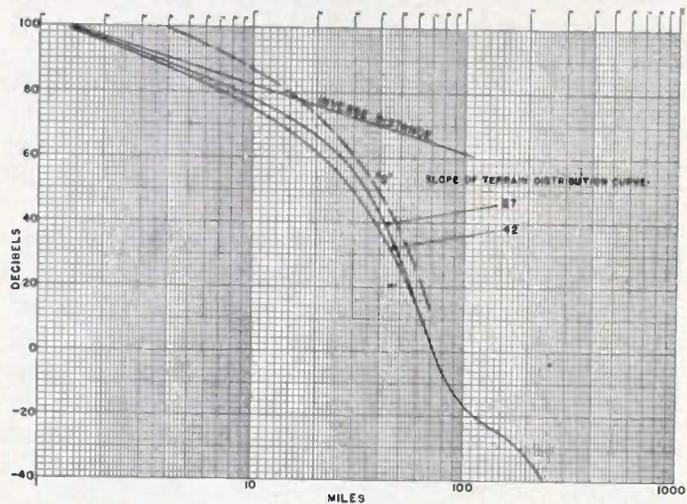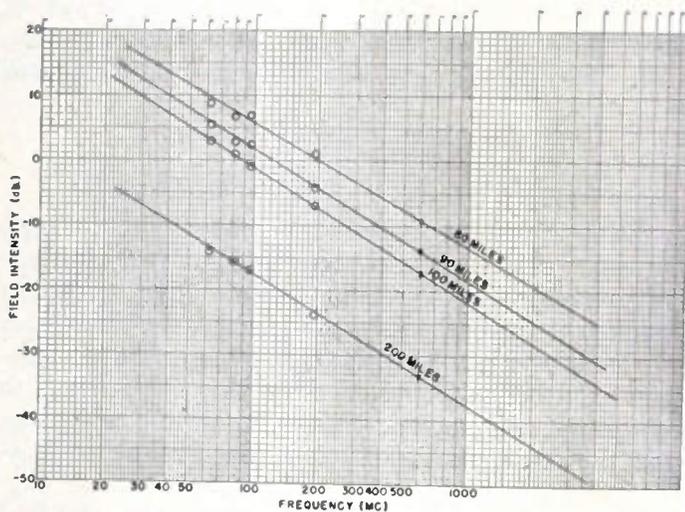
Fig. 18—Field intensity versus frequency for various distances. Field intensity in db above 1 μv per meter. Transmitting antenna, 1 kw, 300 feet.

Fig. 19—Median field intensity propagation curve. Frequency, 195 mc; transmitting antenna height, 500 feet; receiving antenna height, 30 feet; 1-kw radiated. Field intensity in db above 1 μv per meter.

10.5 miles for a transmitter height of 300 feet and 16.5 miles for a transmitter height of 500 feet.

For comparison, the propagation curve of the "Ad Hoc" Committee for 195 mc for a transmitter height of 500 feet and a receiver height of 30 feet is shown in Fig. 19.

3. Fading

The estimates of fading are based on Fig. 20. This Figure was obtained from data for 195 mc shown on Fig. 20 of reference D to the report of the "Ad Hoc" Committee for distances up to sixty miles. It is certain that fluctuations in the field will be as great at 600 mc at short distances as are experienced at 195 mc. At a large distance, these fluctuations will be greater at 600 mc as indicated by the scant published information. The fading at large distances was obtained by comparing the estimated $F(50, 50)$ fields with the $F(50, 1)$ fields of the "Ad Hoc" report. The published information leads to the assumption that the $F(50, 1)$ values of field are substantially independent of frequency at large distances.

Fig. 20—Fading at 600 mc.

Log normal distribution of fading was assumed and the values for E_{90}/E_{50} were interpolated from the E_{99}/E_{50} fading range. It will be seen that the fading is greater than that suggested in the standards proposed by the Federal Communications Commission, but at distances less than thirty miles the differences are quite small.

B. Ghosts

The effect of ghosts involves a statistical calculation of the probability that the ratio of the desired signal to the ghost signal is greater than a certain amount. The assumption was made by the "Ad Hoc" Committee that the terrain distribution for the signals in an area was independent of the location of the transmitter and that there was no correlation in the terrain distribution of two transmissions.

This latter assumption is not fully correct for probably signals on hills will tend to be stronger than signals in valleys. On the other hand, by intelligent selection of sites the reverse effect may occur, namely, that locations of poor signal for one station will tend to be the very locations where the signal from another station is strong. This reversed correlation will probably more than offset any natural correlation that may exist.

In reference F of the "Ad Hoc" Committee report, a method is developed for determining the percentage of locations receiving service on the basis of distribution of actual fields available as a function of hypothetical locations. To use this method the slope of the log normal distribution curve must be known for both the desired and undesired signals. In the case of ghosts in a polycasting system, the distribution curve of the two signals will have the same slope. The slopes used are 27 for rural and 42 for city areas. It appears probable that there are congested conditions in some areas because of rough terrain and for locations inside buildings; when indoor antennas are used, still steeper slopes may be obtained.

In this analysis, as applied to polycasting, a ratio of signals greater than 10 db was assumed to produce service. This ratio has been selected on the basis that with optimum offset carrier operation a ratio of about 18 db

is required between the desired and undesired (ghost) signal, and that at least an additional 8 db is obtainable by using directional receiving antenna or standing-wave system of separating the signals. The only factors defining each signal are the median field (F 50) and the terrain distribution factor $r(L)$. The solution of the problem of how many locations receive service is through statistical analysis based on the probability distribution curves. As an example, if the two signals are of equal median intensity, the signal A is interfered with by B at 71 per cent of the locations, and signal A interferes with B at 71 per cent of the locations. Then at 58 per cent of the locations either A or B produces an acceptable service.

C. Service Contours

In estimating service contours the definition proposed by the Federal Communications Commission was used as follows:

Grade	Minimum Signal Strength 90 per cent of Time
A	65 db at 90 per cent of the locations
B	65 db at 70 per cent of the locations
C	60 db at 50 per cent of the locations.

The signal strength in an area served by more than one station was taken to be the strength of the strongest signal.

The percentage of locations free from ghosts was then calculated. The pattern obtained was rotated through 90 degrees to represent the effect of the other two stations of the four-station polycasting system on the other diagonal of the square.

The two patterns were then combined into one to give the field contours of the strongest of the four stations and the percentage of locations free from ghosts. The limits of the service are controlled by the signal intensity to overcome noise, while, in the center, the limits may be due to the percentage of locations subject to ghosts.

D. Interference

In calculating the separation required between polycasting systems to give adequate protection to the service contours, a desired-to-undesired ratio of 40 db has been assumed between co-channel stations and of 6 db between adjacent channel stations. These are standards proposed by the Federal Communications Commission. When applying these ratios to protect specific grades of service, terrain factors have to be added to refer to percentages of locations other than 50 per cent.

The mileage separation between polycasting systems depends to some extent on the arrangement of the stations within each system.

Examples of the separation needed to protect co-channel operation of different services by 46 db and adjacent channel by 12 db for grade B, and by 40 db for co-channel and 6 db for adjacent channel for grade-C service are given in Fig. 21 for the case of grade-B service, and in Fig. 22 in the case of grade-C service.

The minimum separations of polycasting systems

were arrived at in accordance with the following specifications:

1. Permissible co-channel and adjacent-channel interference ratios as are given in the Federal Communications Commission proposed standards.

2. There are four groups of three systems each in Figs. 21 and 22, labeled (a), (b), (c), and (d). The limitations in Figs. 21(a) and 21(c), and 22(a) and 23(c) is co-channel interference, and in Figs. 21(b) and 21(d), and 22(b) and 22(d) is adjacent-channel interference. Station pairs in a polycasting system are shown either in line with a co-channel or adjacent-channel-station pair of another system with which interference is to be avoided, or else broadside to the co-channel or adjacent-channel-station pair of the other system. These two cases represent the extremes of the minimum possible and maximum necessary separation of systems.

3. If polycasting systems are placed other than corner to corner, as they are shown in Figs. 21 and 22, the required separation of systems is intermediate between the extremes shown at the left and right of each of the four groups of systems, (a), (b), (c), and (d).
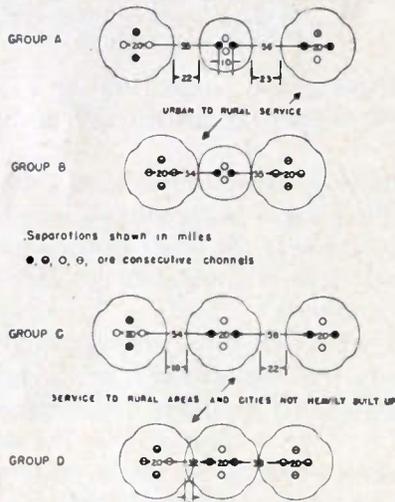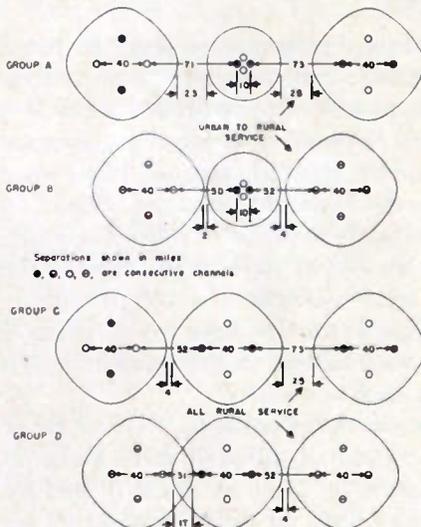
4. In making up Fig. 21, showing protection of grade-C service, the slope of the terrain distribution curve was taken to be 27, and for Fig. 22, showing protection of grade-B service, the two terrain distribution slopes of 27 and 42 db were used in the case of urban to rural service. Propagation from the urban stations to the limit of grade-B service was assumed to require a terrain distribution slope of 42 db, and for the rest of the area a slope of 27 was assumed to apply. The required separations are, however, found to be substantially independent of the slope of the terrain distribution curve.

Examples of the minimum separations required between single stations providing the same service as the polycasting systems illustrated in Figs. 21 and 22 are shown in Figs. 23 and 24, respectively. Sections (a) and (b) show minimum separations of a rural and a city service for co-channel and adjacent channel stations, and (c) and (d) show minimum separations of two rural services for co-channel and adjacent-channel stations. This corresponds to the presentation in Figs. 21 and 22.

Protection of grade-B service is illustrated in Fig. 23, and protection of grade-C service in Fig. 24. As in Figs. 21 and 22, a terrain distribution slope of 42 db is assumed in establishing the service contour of the heavily built-up urban area. For estimating interference and in establishing rural service contours, a slope of 27 db is assumed.

Fig. 21—Minimum separation of polycasting systems for protection of grade-B service.

Fig. 22—Minimum separation of polycasting systems for protection of grade-C service.
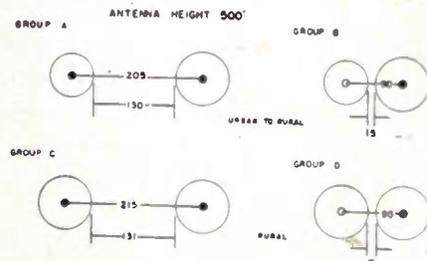
Fig. 23—Protection of grade-C service, minimum separation of single stations. ●, ⊙ are adjacent channels.
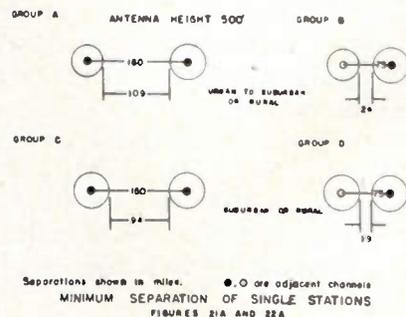
Fig. 24—Protection of grade-B service, minimum separation of single stations. Separations shown in miles. ●, ⊙ are adjacent channels.

Some Circuit Properties and Applications of *n-p-n* Transistors*

R. L. WALLACE, JR.† AND W. J. PIETENPOL†

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Summary—Shockley, Sparks, and Teal have recently described the physical properties of a new kind of transistor. Preliminary studies of circuit performance show that it is a stable, high-gain, quiet amplifier of considerable practical interest. This paper analyzes the performance of a few early experimental units.

INTRODUCTION

ALMOST two years ago Shockley[1,2] first published the theory of a transistor made from a single piece of germanium in which the conductivity type varies in such a way as to produce two rectifying junctions. Since that time Sparks, Teal, and others at the Bell Telephone Laboratories[3,4] have contributed notably to the physical realization of this device.

Recently Sparks has produced a number of *n-p-n* transistors and has found their behavior to be closely in accord with Shockley's theory.[4] Preliminary circuit studies on these devices have shown that in several respects their performance is remarkable. In view of this, our transistor development group has undertaken to produce small quantities of *n-p-n* transistors in a form suitable for incorporation in working circuits.

This paper will deal principally with the circuit aspects of the *n-p-n* transistor by presenting and analyzing the performance data on a small number of experimental units. For a discussion of the solid-state physics of its design and operation the reader is referred to the previouly mentioned works of Shockley, Sparks, and Teal.

OUTSTANDING PROPERTIES

To prevent getting lost in a maze of detail it seems worth while to list and mention briefly the salient features of this new transistor. They are:

1. *Relatively low noise figure.* Most of the units measured so far have a noise figure between 10 and 20 db at 1,000 cps.

2. *Complete freedom from short-circuit instability.* The input and output impedances are always positive whether the transistor is connected grounded emitter, grounded base, or grounded collector. This permits a great deal of freedom in circuit design and makes it possible, by choosing the appropriate connection, to obtain a considerable variety of input and output impedances.

3. *High gain.* Power gains of the order of 40 to 50 db per stage have been obtained.

4. *Power-handling capacity and efficiency.* The design can readily be varied to permit the required amount of power dissipation up to at least 2 watts. Furthermore, the static characteristics are so nearly ideal that class-A efficiencies of 48 or 49 out of a possible 50 per cent can be realized. The efficiencies for class-B and class-C operation are correspondingly high.

5. *Ruggedness and small size.* The germanium part of the transistor is enclosed in a hard plastic bead about 3/16 inch in diameter. Inside the bead three connections are mechanically as well as electrically fastened to the germanium and are brought out as pigtails through the bead. This gives a very sturdy unit.

6. *Freedom from microphonics.* Vibration tests in the audio-frequency range indicate that these devices are relatively free from microphonic noise.

7. *Limited frequency response.* Collector capacitance limits the frequency response at full gain to a few kilocycles. By using a suitable impedance mismatch, it is possible to maintain the frequency response flat to at least 1 mc while still obtaining a useful amount of gain.

8. *Operation with exceedingly small power consumption.* Perhaps the most remarkable feature of these transistors is their ability to operate with exceedingly small power consumption. The best example of this to date is an audio oscillator which requires only 6 microamperes at 0.1 volt for a power supply. This represents 0.6 microwatt of power, which contrasts sharply with the million or more microwatts required to heat the cathode of an ordinary receiving-type vacuum tube.

[1] W. Shockley, "The theory of *p-n* junctions in semiconductors and *p-n* junction transistors," *Bell Sys. Tech. Jour.*, vol. 28, p. 435; 1949.

[2] W. Shockley, "Electrons and Holes in Semiconductors," D. Van Nostrand Co., New York, N. Y.; 1950.

[3] F. S. Goucher, G. L. Pearson, M. Sparks, G. K. Teal, and W. Shockley, "Theory and experiment for a germanium *p-n* junction," *Phys. Rev.*, vol. 81, p. 637; 1951.

[4] W. Shockley, M. Sparks, and G. K. Teal, "*p-n* junction transistors," *Phys. Rev.*, vol. 83, p. 151; 1951.
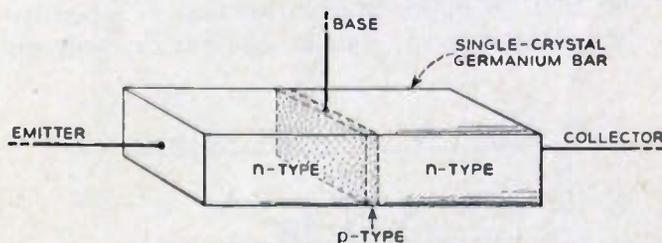
Fig. 1—The heart of an *n-p-n* transistor is a tiny bar of germanium to which three mechanically strong electrical connections are made.
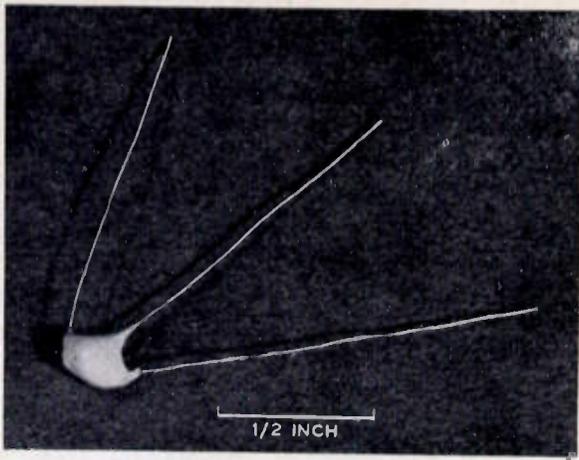
Fig. 2—A beaded *n-p-n* transistor.

Fig. 4—Static characteristics of an *n-p-n* transistor.

Physical Appearance and Construction

Fig. 1 shows schematically the configuration of an *n-p-n* transistor. The small bar of single crystal germanium contains, a thin layer of *p* type interposed between regions of *n* type. Mechanically strong ohmic connections are made to the three regions, as indicated, and are brought out through a hard plastic bead. A finished transistor is shown in the photograph of Fig. 2. It should be pointed out that Fig. 1 is not drawn to scale and that the *p* layer may be less than one-thousandth of an inch thick.

Static Characteristics

A great deal of information about the low-frequency performance of a transistor can be obtained from a set of static characteristics, such as those shown in Fig. 4. Curves of this sort are obtained simply by connecting suitable current sources to the emitter and collector circuits of the transistor and measuring the resulting voltages. The currents are called positive when they flow into the emitter and collector as shown, and the voltages are called positive when they have the signs indicated in Fig. 3.

Let us first examine these curves to find out what kind of voltage and current supplies are needed to bias the transistor into the range in which it can amplify. To make this easy, that part of the characteristics which lies within the normal operating range has been shown as solid lines, and that part of the characteristics corresponding to cutoff as dotted lines.

Note from the upper set of curves that V_c is positive in the operating range. This means that the collector
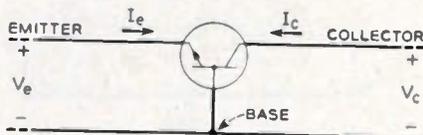
Fig. 3—The symbol for a (P-type) transistor on which the convention of signs for currents and voltages is indicated.

must be biased positive with respect to the base. For this particular transistor a bias voltage anywhere between about 0.1 volt and 35 volts is suitable. Note also that all the curves on this plot correspond to negative emitter currents. This means that the emitter must be biased in such a way that current flows out of the emitter into a suitable current supply. Furthermore, the collector current corresponding to any given emitter current can be seen to be almost equal in magnitude to the emitter current. Since these two currents are opposite in sign, this means that most of the current flowing into the collector leaves by way of the emitter, with the result that the current in the base circuit is very small.

Suppose that the collector is held at a constant positive voltage, as, for example, by connecting a battery between collector and base (with a transformer winding in series, perhaps). Now, if a negative current is forced into the emitter by a battery and resistance connected in series between emitter and base, the collector current can be controlled by varying the emitter current, to which it will always be approximately equal in magnitude. Suitable collector currents for this particular transistor range from about 20 microamperes to about 5 milliamperes.

The exact choice of collector current and voltage within the ranges mentioned above will be dictated

largely by the amount of power output required. The more required, the more current and voltage will be needed from the power supply. Since the collector circuit efficiency cannot exceed the theoretical limit of 50 per cent in class-A operation, the signal power output cannot exceed half the power supplied by the battery. This means, for example, that if the collector is worked at 20 volts and 2 milliamperes the class-A power output cannot exceed 20 milliwatts.

From the lower plot of Fig. 4 it is possible to obtain information about the bias voltage required for the emitter. Note first that the entire emitter-voltage plot corresponds to a very small range of emitter voltages near zero and, furthermore, that the part of the characteristics corresponding to the operating range covers only a few thousandths of a volt. This means that if the collector voltage is held constant, very small changes in emitter voltage will produce fairly large changes in collector current; or if the collector current is held constant, very small changes in emitter voltage will produce relatively enormous changes in collector voltage. This at once suggests the use of this transistor as a dc amplifier between a low-impedance source and a high-impedance load. In this application, voltage stepup of the order of 10,000 times is possible.

The very great sensitivity of the collector circuit to emitter voltage suggests, however, that for ac amplifiers a current source should be used as an emitter-bias supply. This can be obtained from a battery and a large resistance in series. Besides, since the emitter voltage is always nearly zero, the emitter current can be calculated in advance by dividing the battery voltage by the value of the series resistance (provided, of course, that the supply voltage is large compared with the few hundredths of a volt drop across the emitter circuit).

Some interesting conclusions can also be drawn from the static characteristics about the large signal operation of the transistor. If the load is resistive, the instantaneous operating point will swing up and down along a straight line, such as the load line shown in the upper plot of Fig. 4. This particular load line corresponds to an ac load resistance of 10,000 ohms. Suppose that the steady collector biases are 20 volts and 2 milliamperes so that the drain from the power supply is 40 milliwatts. Now consider the permissible swings of collector voltage and current. Since the collector characteristics are quite straight and are evenly spaced over a wide range of current and voltage values, the output signal can swing nearly down to zero collector volts and nearly up to zero collector current without distortion. The limit on the lower end is imposed by the fact that the collector characteristics begin to be curved when V_e is less than about 0.1 volt and the limit on the upper end is imposed by the fact that the collector current does not drop completely to zero when I_e drops to zero. The lower limit of collector current is, in this case, about 50 microamperes. Since this amount of current in

10,000 ohms corresponds to 0.5 volt, this means that the instantaneous collector voltage is limited to swings between 39.5 volts and 0.1 volt. Starting from a quiescent value of 20 volts, the permissible positive swing is then 19.5 volts and the permissible negative swing 19.9 volts. Reducing the quiescent voltage to 19.8 volts (and keeping the same load line) make it possible to obtain a peak swing of 19.7 volts, which corresponds to 19.45 milliwatts of signal delivered to the load. This gives a collector circuit efficiency of 48.5 per cent out of a possible 50 per cent. Some transistors take even less collector current when the emitter current is zero, hence permitting even higher efficiencies.

These computations of efficiency have all been based on the assumption of sinusoidal *current* applied to the emitter. It will be shown in a later section that the emitter resistance varies with emitter current, however, and this means that to realize high efficiency with low distortion it is necessary to drive the emitter from a high-impedance source.
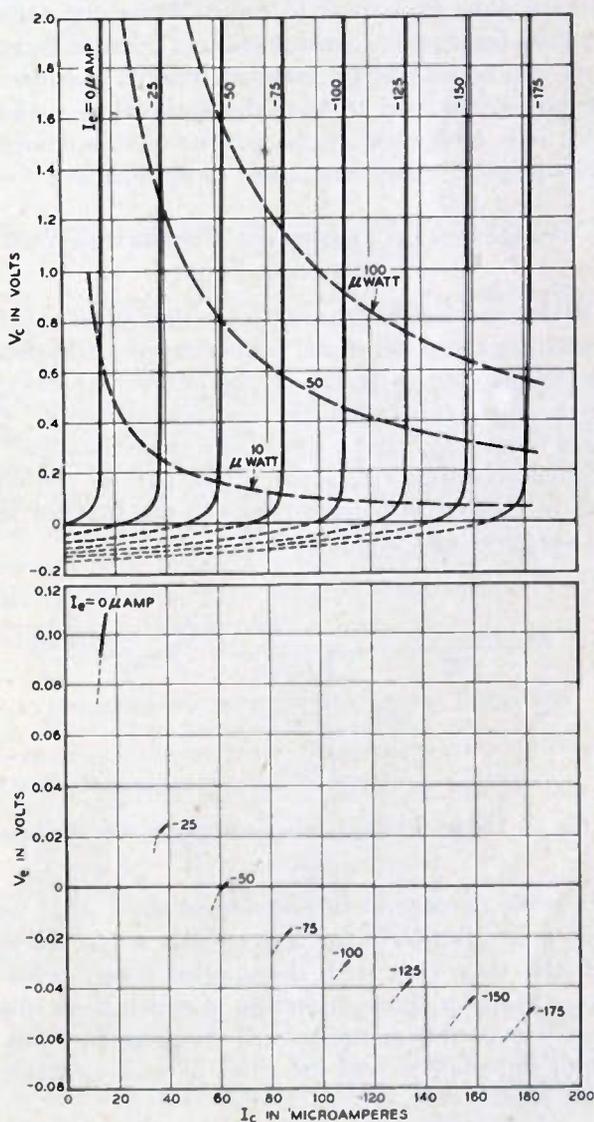
Fig. 5—Static characteristics showing behavior at very low applied voltages and currents.

OPERATION WITH SMALL POWER CONSUMPTION

For small signal applications the transistor represented by the characteristics of Fig. 4 can deliver useful gain at very much lower voltages and currents than those used in the example above. In order to show this, the characteristics of Fig. 5 have been plotted for a range of collector voltage extending up to only 2 volts and for a range of collector currents extending up to only 200 microamperes. It can be seen from the upper plot that the collector circuit characteristics are still quite usefully straight and are evenly spaced in this micropower range. In fact, for small signal operation it is sufficient to use a collector voltage only a little in excess of 0.1 volt and a collector current a little in excess of 10 microamperes. This means that the power required to bias the collector into the operating range amounts to only a few microwatts. Contours are shown for 10, 50, and 100 microwatts of power supply.

This ability of the transistor to work with extremely small power consumption is one of its most striking and perhaps most important features. When one considers that the total power consumption of a single transistor stage can be smaller by many thousands of times than the power required to heat the cathode in a vacuum tube, it is obvious that the advent of this device will make possible many new kinds of application.

VARIATION OF TRANSISTOR PROPERTIES WITH OPERATING POINT

Ryder and Kircher[5] have shown that it is convenient to analyze the small signal properties of a transistor at low frequencies in terms of the equivalent circuit of Fig. 6, where r_e is called the "emitter resistance," r_b the "base resistance," and r_c the "collector resistance." The internal generator $r_m i_e$ is the active part of the circuit and, in this respect, corresponds to the familiar μe_g of the vacuum-tube circuit theory.
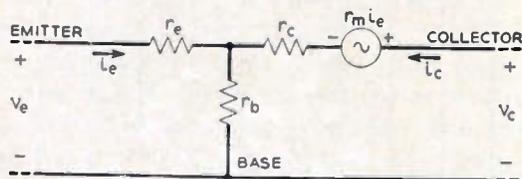
Fig. 6—The low-frequency equivalent circuit of a transistor.

It is the purpose of this section to show what values these quantities have for a particular *n-p-n* transistor and how they vary with the applied biases. This will form a basis for the next section in which these quantities will be used to compute such things as the input and output impedances and the gains of various transistor connections.

[5] R. M. Ryder and R. J. Kircher, "Some circuit aspects of the transistor," *Bell Sys. Tech. Jour.*, vol. 28, p. 367; 1949.

Ryder and Kircher have shown that these four *r*'s can be obtained directly from static characteristics, such as those shown in Figs. 4 and 5. In the case of *n-p-n* transistors, however, the magnitudes of these quantities are such that it is difficult to obtain satisfactory accuracy in this way, and it has been more convenient to measure the 4-pole *r*'s by ac methods.

These measurements have shown that all the *r*'s are, to a first approximation, independent of collector voltage so long as the collector voltage is above a few tenths of a volt, and so long as the total dissipation is small enough to prevent any appreciable heating of the transistor.

In view of this fact it is perhaps sufficient to show how these quantities vary with emitter current for a moderate fixed value of collector voltage. Figs. 7 and 8 show that r_e and r_m are very nearly equal and that they tend to decrease as I_e increases. Theoretically, r_m and r_c should both be infinite. The fact that they reach values as low as 10 megohms in this case is a measure of the imperfection in technique of fabricating the transistor. Values as high as 60 megohms have been achieved in the laboratory.
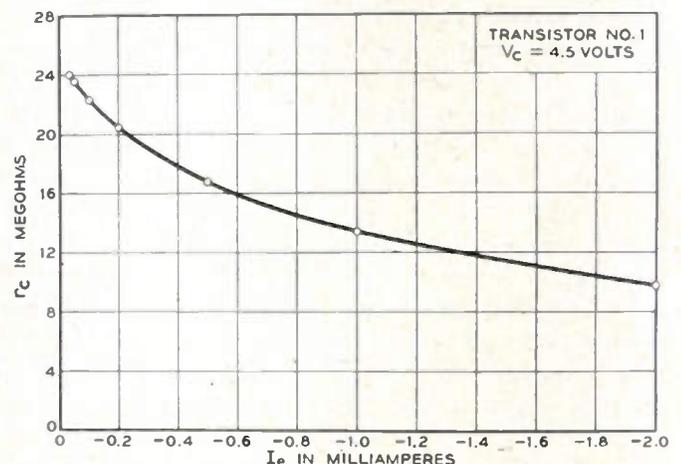
Fig. 7—The variation of collector resistance with emitter current at a fixed value of collector voltage.
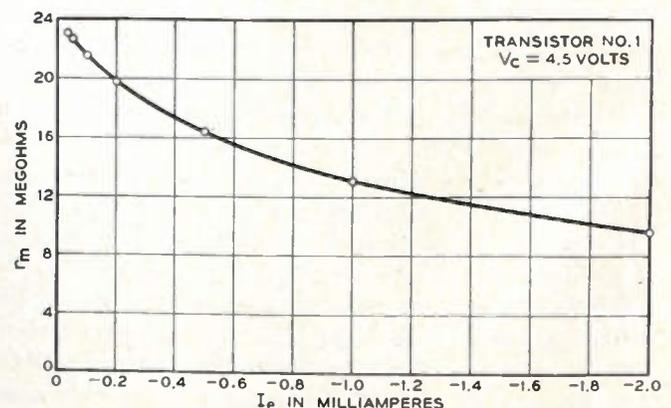
Fig. 8—Variations of r_m, with emitter current at fixed collector voltage.

Fig. 9 shows that r_b in this transistor is approximately 240 ohms and is independent of I_e.

Fig. 10 shows that r_e decreases with increasing emitter current, ranging from about 500 ohms at 50 microamperes down to about 5 ohms at 5 milliamperes. Shockley[4] has shown that r_e should be given by

$$r_e = \frac{kT}{qI_e},\tag{1}$$

where q is the charge on an electron, k is Boltzman's constant, T is the Kelvin temperature, and I_e is the emitter current. When the temperature is about 80 degrees F, this reduces to

$$r_e = \frac{25.9}{I_e},\tag{2}$$

where I_e is measured in milliamperes. Within experiment error, values of r_e computed from this relation agree perfectly with the measured curve shown in Fig. 10.
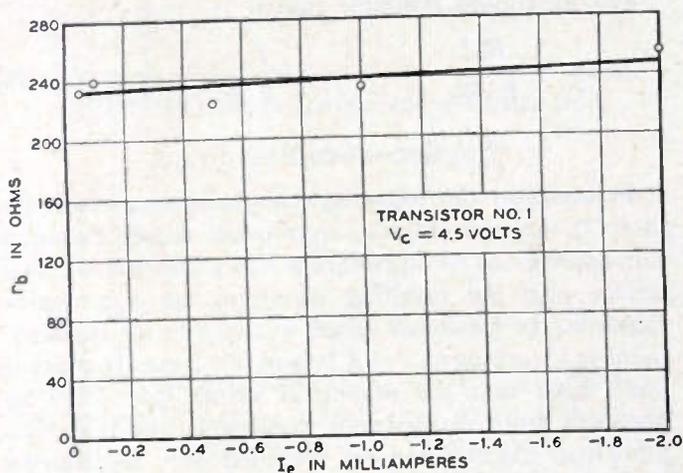
Fig. 9—Variation of base resistance with emitter current. Scatter of the data indicates that the measurements were not accurate.
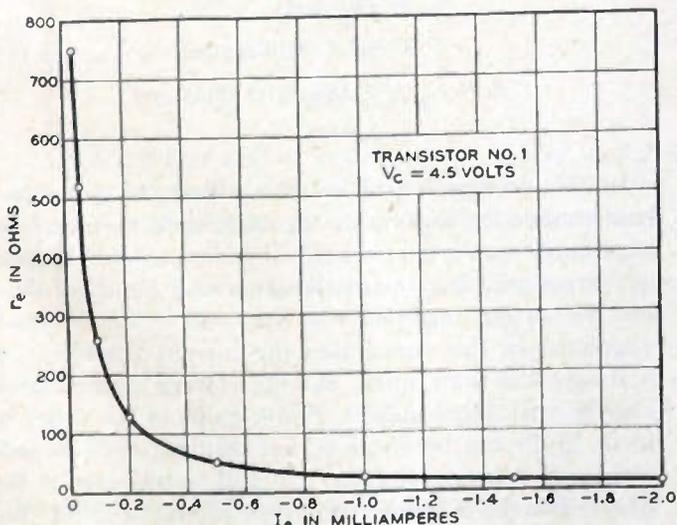
Fig. 10—The emitter resistance is inversely proportional to emitter current.

Fig. 11 introduces a new quantity, α, the current amplification factor of the transistor. This quantity is defined by the equation

$$\alpha = \frac{r_m + r_b}{r_c + r_b}.\tag{3}$$

Since r_m and r_c are both very large compared to r_b, α is approximately equal to to the ratio of r_m to r_c. It will be shown in a later section that this quantity is important in determining some of the circuit properties of the transistor and that many of the circuit properties become more desirable as α approaches unity.

It can be seen from Fig. 11 that in this transistor α is approximately equal to 0.98 and that it increases slightly with increasing emitter current. The highest value of α so far achieved is 0.9965.
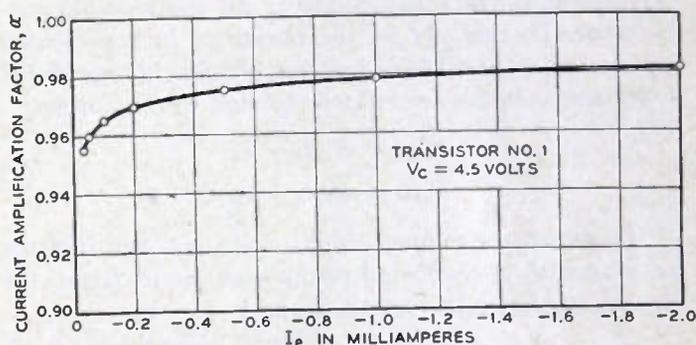
Fig. 11—The current amplification factor, α, increases slightly with increasing emitter current. Note the expanded scale for α.

Those units which have been made in the laboratory so far show considerable variation in some of the properties, but this is partly due to the fact that changes have been made deliberately to test one aspect or another of Shockley's theory. The data in Table I are presented to indicate what properties have been achieved to date. The collector capacitance C_c will be discussed in a later section.

TABLE I

CONSTANTS FOR VARIOUS TRANSISTORS MEASURED
AT $V_c = 4.5\text{v}$, $I_e = 1.0$ ma

Transistor No.		I	II	III	IV	V
r_e	(ohms)	25.9	31.6	33.1	30.2	38.8
r_b	(ohms)	240	44	300	3070	180
r_c	(megohms)	13.4	0.626	1.11	1.21	2.00
$r_c - r_m$	(megohms)	0.288	0.00387	0.0168	0.00422	0.0439
α		0.9785	0.9936	0.9848	0.9965	0.9780
C_c	($\mu\mu$f.)	7	7.7	18.9	27.9	21.2

GENERAL CONSIDERATIONS AND FORMULAS

It is a consequence of the fact that α is always less than unity in this structure that these transistors are unconditionally stable with all terminations. This means that stability considerations do not prevent matched terminations to be worked with. Furthermore,

it is possible to obtain a variety of input and output impedances by connecting the transistor as a grounded-emitter, grounded-base, or grounded-collector stage. It is the purpose of this section to give some idea of the characteristics of these various stages and to show in each case at least one way of supplying the required biases and couplings to the stage.
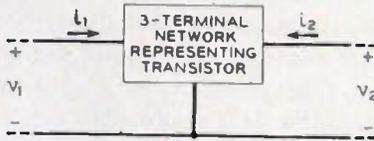
Fig. 12—A three-terminal network representing either grounded-emitter, grounded-base, or grounded-collector connection of a transistor. Note the convention of signs.

It will be convenient to begin by writing down general relationships which will apply to all the possible connections. To this end let the transistor be represented by the box in Fig. 12. At low frequencies the signal currents and voltages are related through the equations

$$R_{11}i_1 + R_{12}i_2 = v_1$$
$$R_{21}i_1 + R_{22}i_2 = v_2. \tag{4}$$

If a generator of open-circuit voltage v_g and internal resistance R_g is connected to the input terminals of the device, as shown in Fig. 13, then

$$v_1 = v_g - i_1 R_g. \tag{5}$$
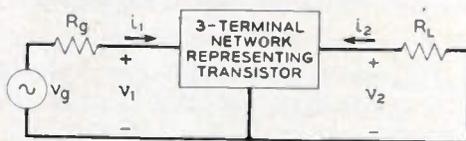
Fig. 13—The three-terminal network of Fig. 12 connected between a generator and a load.

If a load of resistance R_L is connected to the output terminals,

$$v_2 = - R_L i_2. \tag{6}$$

The equations for the circuit of Fig. 13 are, therefore,

$$(R_{11} + R_g)i_1 + R_{12}i_2 = v_g$$
$$R_{21}i_1 + (R_{22} + R_L)i_2 = 0. \tag{7}$$

Solving for the voltage developed across the load $(= - R_L i_2)$ gives

$$v_2 = \frac{R_L R_{21}}{(R_{11} + R_g)(R_{22} + R_L) - R_{12}R_{21}} v_g. \tag{8}$$

The power gain in the circuit is the power delivered to the load $(v_2{}^2/R_L)$, divided by the power available from the generator $(v_g{}^2/4R_g)$. From (8), this gives

$$G = \frac{4R_g R_L R_{21}{}^2}{[(R_{11} + R_g)(R_{22} + R_L) - R_{12}R_{21}]^2}. \tag{9}$$

The gain depends on R_g and R_L, and will be maximum when these are chosen to match the input and output impedance of the transistor stage. But the input impedance depends on R_L and the output impedance depends on R_g in the following way:

$$\text{input impedance} = R_i = R_{11} - \frac{R_{12}R_{21}}{R_{22} + R_L} \tag{10}$$

$$\text{and output impedance} = R_o = R_{22} - \frac{R_{12}R_{21}}{R_{11} + R_g}. \tag{11}$$

If $R_i = R_g$ and $R_o = R_L$, then impedances are matched at the input and output terminals and the gain is a maximum. The conditions are:
matched input impedance =

$$R_{im} = R_{11}\sqrt{1 - R_{12}R_{21}/R_{11}R_{22}}; \tag{12}$$

Matched output impedance =

$$R_{om} = R_{22}\sqrt{1 - R_{12}R_{21}/R_{11}R_{22}}; \tag{13}$$

and Maximum available gain =

$$\text{MAG} = \frac{R_{21}{}^2}{R_{11}R_{22}} \frac{1}{[1 + \sqrt{1 - R_{12}R_{21}/R_{11}R_{22}}]^2}. \tag{14}$$

The Grounded-Base Stage

In this and the following two sections we will put into (7) through (14) the appropriate 4-pole r's to obtain expressions for impedances and gains. We will substitute into the resulting equations, as a numerical example, the measured values of these r's for transistor number I working at $V_c = 4.5\text{v}$ and $I_c = 1$ ma. It must be understood that the numerical values may vary appreciably from transistor to transistor, and that these numerical calculations are intended only for illustration and not as a basis for final circuit design. The numerical values to be used are

$$r_e = 25.9 \text{ ohms}$$
$$r_b = 240 \text{ ohms}$$
$$r_c = 13.4 \ (10)^6 \text{ ohms} \tag{15}$$
$$r_c - r_m = 0.288 \ (10)^6 \text{ ohms and}$$
$$\alpha = 0.9785.$$

In this section it will be shown that the grounded-base connection is suitable for working between a low-impedance source and a high-impedance load. The input impedance may be of the order of a hundred ohms and the output impedance of the order of one or more megohms. In this connection the current amplification is always less than unity, but the voltage amplification may be very large indeed. Power gains of the order of 40 to 50 db can be obtained between matched impedances and appreciable gains can still be obtained if the load resistance is reduced to a few thousand ohms (because the current gain is then almost equal to unity). In this case the gain of the stage is almost completely

independent of those transistor properties which tend to vary from unit to unit. This sort of stage does not produce a phase reversal.

For the grounded-base stage, shown in Fig. 14,

$$R_{11} = r_e + r_b = 266 \text{ ohms}$$
$$R_{12} = r_b = 240 \text{ ohms}$$
$$R_{21} = r_m + r_b = 13.1 \,(10)^6 \text{ ohms}$$
$$R_{22} = r_c + r_b = 13.4 \,(10)^6 \text{ ohms}$$

$$\alpha = \frac{r_m + r_b}{r_c + r_b} \tag{16}$$

$$\doteq \frac{r_m}{r_c}$$

and

$$= 0.9785.$$
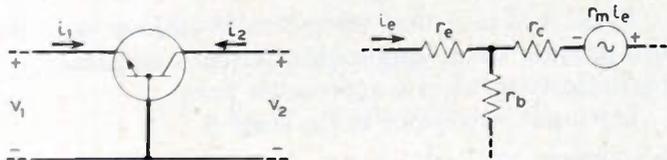
Fig. 14—The grounded-base connection of a transistor.

In this case, if r_b is neglected by comparison with r_m and r_c, (8) leads to

$$v_2 = \frac{\alpha R_L v_g}{(r_e + r_b + R_g)(1 + R_L/r_c) - \alpha r_b}. \tag{17}$$

Since for these transistors r_m/r_c $(\doteq \alpha)$ is always less than unity, the output voltage is in phase with the input voltage. Furthermore, if R_L is very high, the output voltage is enormous by comparison with the input voltage. For example, if $R_g = 0$ and R_L is infinite,

$$v_2 = v_g \frac{r_m}{r_e + r_b}. \tag{18}$$

For the numerical example, this is

$$v_2 = 4.93 \,(10)^4 \, v_g.$$

To achieve this stepup would require a load impedance very large compared with 13 megohms, but even with more modest values of load impedance the voltage stepup is large.

If R_L is small compared to r_c, the second part of (7) leads to

$$i_2 = -\frac{r_m}{r_c} i_1 \tag{19}$$

$$\doteq -i_1,$$

and the current delivered to the load is approximately equal to the current which the generator delivers to the transistor.

From (10) and (11), the input and output impedances are

$$R_i = r_e + r_b - \frac{r_b(r_m + r_b)}{r_c + R_L + r_b} \tag{20}$$

$$R_o = r_c + r_b - \frac{r_b(r_m + r_b)}{r_e + r_b + R_g}. \tag{21}$$

As the load impedance varies from zero to infinity, the input impedance varies from

$$R_i = r_e + r_b\left[1 - \frac{r_m + r_b}{r_c + r_b}\right] \text{ for } R_L = 0 \tag{22}$$

$$\doteq r_e + r_b(1 - \alpha)$$

$$= 31.1 \text{ ohms}$$

to

$$R_i = r_e + r_b = 266 \text{ ohms for } R_L = \infty. \tag{23}$$

When $R_g = 0$, the output impedance is

$$R_o = r_c - \frac{r_b}{r_e + r_b}(r_m - r_e) \tag{24}$$

$$\doteq r_c - \frac{r_b}{r_e + r_b} r_m \tag{25}$$

$$= 1.56 \,(10)^6 \text{ ohms.}$$

As R_g increases to infinity,

$$R_o = r_c + r_b = 13.4 \,(10)^6 \text{ ohms.} \tag{26}$$

From (12) and (13), the matched input and output impedances are approximately

$$R_{im} = (r_e + r_b)\sqrt{1 - \alpha r_b/(r_c + r_b)} \tag{27}$$

$$= 91 \text{ ohms}$$

$$R_{om} = (r_c + r_b)\sqrt{1 - \alpha r_b/(r_e + r_b)} \tag{28}$$

$$= 4.58 \,(10)^6 \text{ ohms.}$$

With matched impedances, the maximum available gain is

$$\text{MAG} = \frac{\alpha(r_m + r_b)}{r_e + r_b}\left[1 + \sqrt{1 - \alpha r_b/(r_e + r_b)}\right]^{-2} \tag{29}$$

$$= 2.7 \,(10)^4 \text{ or } 44.3 \text{ db.}$$

The matched output impedance of this stage is inconveniently high, but a useful amount of gain can be maintained if R_L is reduced to a more reasonable value. For example, if $R_L = 200,000$ and $R_g = 25$, (9) gives

$$G = 5.3 \,(10)^3 \text{ of } 37.2 \text{ db.}$$

If stages of this sort are to be cascaded, a step-down transformer must be used to couple each collector to the following emitter. Otherwise, since the current amplification factor of the transistor is slightly less than unity, the gain per stage will also be slightly less than unity.

One practical arrangement of a grounded-base stage would be as shown in Fig. 15. The required value of R will be approximately

$$R = \frac{E_{B1}}{i_c}, \tag{30}$$

where i_c is the desired collector current and E_{B1} is the voltage of the emitter-bias battery. For operating at $i_c = 1$ ma, for example, $E_{B1} = 1.5$ v and $R = 1,500$ ohms would be suitable.
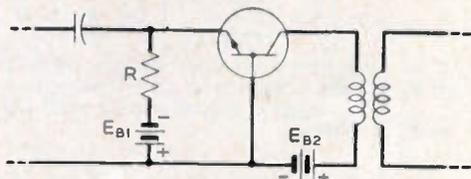
Fig. 15—One practical arrangement of a grounded-base amplifier stage.

THE GROUNDED-EMITTER AMPLIFIER

For many applications the grounded-emitter connection is more desirable than either of the other two. The power gains which can be obtained are high—of the order of 50 db—and the interstage coupling problem is simplified by the fact that the input impedance is somewhat higher than that of the grounded-base stage while the output impedance is very much lower. The input impedance may be of the order of a few hundred ohms and the output impedance of the order of a few hundred thousand ohms. Both voltage and current amplification are produced (with a phase reversal), and gains of the order of 30 db or more per stage can be obtained without the use of interstage coupling transformers. The input and output impedances depend very critically on α and may vary appreciably from unit to unit.

For this connection, which is indicated schematically in Fig. 16,

$$R_{11} = r_e + r_b = 266 \text{ ohms}$$
$$R_{12} = r_e = 25.9 \text{ ohms}$$
$$R_{21} = r_e - r_m = -13.1 (10)^6 \text{ ohms} \tag{31}$$

and

$$R_{22} = r_e + r_c - r_m = 0.288 (10)^6 \text{ ohms}.$$

Putting these values into (8) shows that v_2 is always opposite in sign compared with v_g; in other words, the
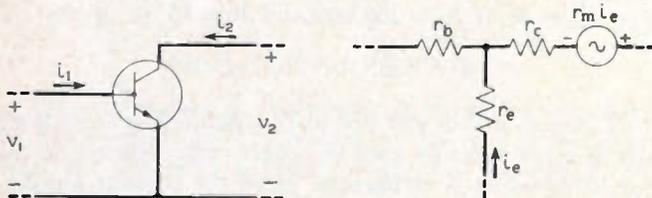
Fig. 16—The grounded-emitter connection of a transistor and the equivalent circuit.

grounded-emitter stage produces a phase reversal as does the grounded-cathode vacuum tube.

If R_L is infinite and $R_g = 0$,

$$v_2 = v_g \frac{r_e - r_m}{r_e + r_b}$$
$$= -4.93 (10)^4 v_g,$$

which is the same as for the grounded-base stage. But if $R_L = 0$,

$$i_2 = \frac{r_m - r_e}{r_e + r_c - r_m} i_1 \tag{32}$$

$$\doteq \frac{\alpha}{1 - \alpha} i_1 \tag{33}$$

$$= 45.5 i_1.$$

Thus, it is seen that the grounded-emitter amplifier can produce quite appreciable current amplification—particularly so when α approaches unity.

The input impedance to the stage is

$$R_i = r_e + r_b + \frac{r_e(r_m - r_e)}{r_e + r_c - r_m + R_L}. \tag{34}$$

When $R_L = 0$, this reduces to

$$R_i = r_b + r_e \frac{1}{\dfrac{r_e}{r_c} + 1 - \dfrac{r_m}{r_c}} \tag{35}$$

$$\doteq r_b + r_e \frac{1}{1 - \alpha} \tag{36}$$

$$= 1,440 \text{ ohms}.$$

As R_L increases to infinity, the input impedance decreases to $r_e + r_b$, which, for the numerical example, is 266 ohms.

The output impedance is

$$R_o = r_e + r_c - r_m + \frac{r_e(r_m - r_e)}{r_e + r_b + R_g}. \tag{37}$$

When $R_g = 0$, this gives

$$R_o = r_c - \frac{r_b}{r_e + r_b} (r_m - r_e) \tag{38}$$

$$\doteq r_c \left[1 - \frac{r_b}{r_e + r_b} \alpha \right] \tag{39}$$

$$= 1.56 (10)^5 \text{ ohms}.$$

As R_g increases to infinity, R_o decreases to

$$R_o = r_e + r_c - r_m \tag{40}$$

$$= 0.288 (10)^6 \text{ ohms}.$$

The matched input and output impedances are

$$R_{im} = (r_e + r_b)\sqrt{1 + r_e(r_m - r_e)/(r_e + r_b)(r_e + r_c - r_m)} \tag{41}$$

$$= 619 \text{ ohms}$$

and

$$R_{om} = (r_e + r_c - r_m)\sqrt{1 + r_e(r_m - r_e)/(r_e + r_b)(r_e + r_c - r_m)} \tag{42}$$

$$= 0.671 \ (10)^6 \text{ ohms.}$$

As α increases toward unity, the matched input impedance increases and the matched output impedance decreases. They approach the limits

$$R_{im} = \sqrt{(r_b + r_c)(r_e + r_b)} \tag{43}$$

$$R_{om} = r_c\sqrt{(r_b + r_c)/(r_e + r_b)} \tag{44}$$

$$\text{as } \alpha \to 1.$$

If r_m in the transistor of our numerical examples could be increased to exactly the value of $r_c(\alpha = 1)$, then the matched impedances would be

$$R_{im} = 59,700 \text{ ohms}$$

$$R_{om} = 5,800 \text{ ohms.}$$

From this example, it is seen that the impedances vary rapidly with α as α approaches unity.

With matched impedances, the maximum available gain from the grounded-emitter stage is

$$\text{MAG} = \frac{(r_e - r_m)^2}{r_e r_c}\left[\sqrt{\left(1 + \frac{r_b}{r_e}\right)\left(\frac{r_e}{r_c} + 1 - \frac{r_m}{r_c}\right)} \right.$$

$$\left. + \sqrt{1 + \frac{r_b}{r_e}\left(\frac{r_e}{r_c} + 1 - \frac{r_m}{r_c}\right)}\right]^{-2} \tag{45}$$

$$= 2.02 \ (10)^5 \text{ or } 53 \text{ db.}$$

When α is exactly unity, this expression reduces to r_c/r_e, provided r_e and r_b are small compared with r_c. For values of α which are enough smaller than unity so that

$$\frac{r_e}{r_c} \ll 1 - \alpha,$$

the expression for maximum available gain reduces to the approximate expression

$$\text{MAG} = \alpha(r_m/r_e)\left[\sqrt{(1 - \alpha)\frac{r_b}{r_e}} \right.$$

$$\left. + \sqrt{1 + (1 - \alpha)\frac{r_b}{r_e}}\right]^{-2}. \tag{46}$$

From the above equations it can be seen that gain does not increase rapidly with α when α is sufficiently near unity. In the case of our numerical example, increasing α from 0.9785 to unity increases the gain by only 4.1 db. The gain of the stage is approximately proportional to r_c and inversely proportional to r_e, and can, therefore, be increased by operating at higher emitter currents or by fabricating the transistor in such a way as to obtain higher values of r_c. In the latter case

it would be desirable also to increase α in order to keep the output impedance from becoming unreasonably high.

It has been seen that for the case of our numerical example the matched output impedance is large compared to the input impedance (671,000 ohms compared to 619 ohms). This means that if the maximum available gain is to be obtained in cascaded stages, step-down interstage transformers must be used. But an appreciable amount of gain can be obtained without interstage impedance matching. This is because of the short-circuit current amplification, previously mentioned, which amounts approximately to

$$\frac{\alpha}{1 - \alpha},$$

or 45.5 times in the case of our numerical example. For this transistor, then, the iterative gain per stage without impedance transformation would be 33.2 db. This gain increases very rapidly as α approaches unity, not only because the short-circuit current amplification increases but also because the output impedance decreases and the input impedance increases so that a better interstage impedance match is obtained. From (41) and (42) it is seen that the matched input and output impedances are equal when

$$r_c - r_m = r_b,$$

or when

$$1 - \alpha = \frac{r_b}{r_c + r_b}.$$

In this case the gain per stage would be approximately r_c/r_e. If r_m could be increased in the numerical example until

$$\alpha = 0.999821,$$

the gain per stage (without impedance transformation) would be increased to 57.1 db. Values of α this near to unity have not even been approached in transistors made to date. This unrealistic example is included only to indicate one of the reasons for seeking to make α very near unity.

Now consider one possible way of supplying biases to a grounded-emitter stage. Suppose that the collector is connected through a transformer winding to a fixed voltage supply, as indicated in Fig. 17. Since V_e is always a very small fraction of a volt (see Figs. 4 and 5),

the collector voltage will be approximately equal to the supply voltage. If no dc connection is supplied to the base, it will float at a potential above ground equal (in magnitude) to V_e (i.e., a very small fraction of a volt) and the collector current will be exactly equal to the emitter current. To find out approximately what this value of current will be, consider the upper set of static characteristics in Fig. 5. Note that when the emitter current is zero, the collector current is of the order of 20 microamperes (the exact value varies from
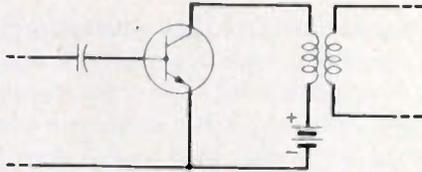
Fig. 17—One practical arrangement of a grounded-emitter stage.

1 to 30 microamperes in transistors tested so far). The collector current is then of the order of 20 microamperes greater than the emitter current. If the emitter current is now increased by ΔI_e, the collector current will increase by $\alpha \Delta I_e$. That is, the increments of collector current will be slightly smaller than the increments of emitter current. As the emitter current is increased, the emitter and collector currents will become more nearly equal. If α were perfectly constant, these currents would become exactly equal when

$$I_e = I_c = \frac{I_{co}}{1 - \alpha}, \tag{47}$$

where I_{co} is the collector current which flows when the emitter current is zero. Equation (47), then, gives the values of emitter (and collector) current which will flow in a grounded-emitter stage if no dc connection is made to the base. This current varies rapidly with α. For the transistor which has been considered numerically, this current would amount to about 465 microamperes, which is certainly within the range of suitable values for the transistor. In certain low-level applications, however, it might be desirable to work at a smaller current for the sake of decreasing battery-power consumption. This requires that a small current be drawn out of the base. The required current is small because the collector current will decrease by $1/(1-\alpha)$ microamperes for each microampere drawn from the base. One method of obtaining this base current is to provide a resistive path between base and ground, as shown, for example, in Fig. 18. Since the base floats at a positive potential with respect to ground, this circuit produces a base current of the right sign to decrease the collector current. As the value of the series resistance is decreased to zero, the collector current decreases to a value corresponding to zero emitter voltage. Still further decrease in collector current can be obtained by inserting resistance between emitter and ground.

In order to increase the collector current to values higher than that corresponding to zero base current, a high resistance path between base and the positive supply voltage may be used, as shown in Fig. 19. In this case the collector current will increase by $1/(1-\alpha)$ microamperes for each microampere which flows through the bias resistor. Since the current in the bias resistor will be approximately E_B/R, it is a simple matter to compute the required value of bias resistor once the desired collector current is known.
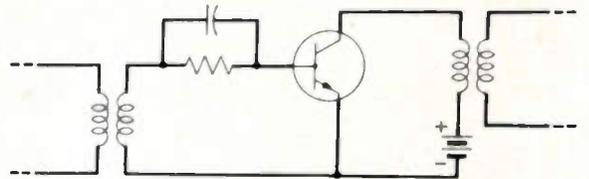
Fig. 18—Modification of Fig. 17 to obtain lower collector current.
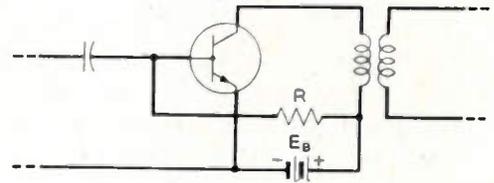
Fig. 19—Modification of Fig. 17 to obtain higher collector current.

Fig. 20 shows a two-stage audio amplifier which gives approximately 90-db gain. Circuit is shown in Fig. 21.
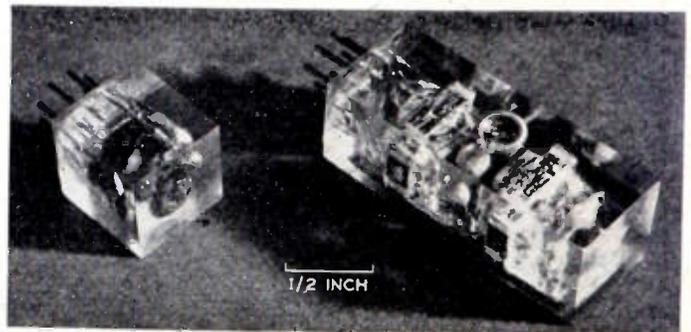
Fig. 20—A two-stage grounded-emitter amplifier which produces approximately 90-db power gain is shown on the right and a micropower audio oscillator is shown on the left.
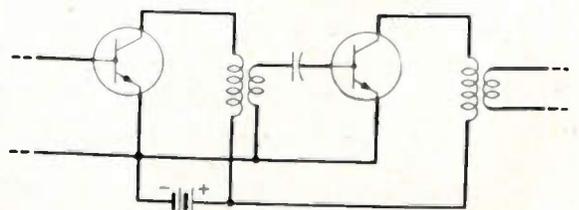
Fig. 21—Circuit of the amplifier shown in Fig. 20.

THE GROUNDED-COLLECTOR STAGE

Although the power gain obtainable from this connection is relatively low (of the order of 15 or 20 db), it has very interesting possibilities in producing very high input impedances or very low output impedances. If it is worked into a fairly high load impedance, the input impedance may be several megohms. If it is worked from a source of moderately low impedance (a few thousand ohms), the output impedance may be of the order of 25 ohms or lower.

For this type of stage, which is shown schematically in Fig. 22,

$$R_{11} = r_b + r_c = 13.4 \ (10)^6 \text{ ohms}$$
$$R_{12} = r_c - r_m = 0.288 \ (10)^6 \text{ ohms}$$
$$R_{21} = r_c = 13.4 \ (10)^6 \text{ ohms} \tag{48}$$

and

$$R_{22} = r_e + r_c - r_m = 0.288 \ (10)^6 \text{ ohms}.$$

If this stage is worked from a zero impedance generator into an infinite impedance load,

$$v_2 = v_g \left(\frac{r_c}{r_b + r_c} \right) \tag{49}$$

$$\doteq v_g.$$

Like a cathode follower, it gives less output voltage than input voltage, but in the same phase.
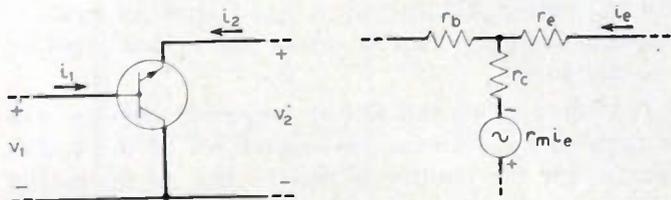
Fig. 22—The grounded-collector connection of a transistor and the equivalent circuit.

If the stage is operated into a short circuit,

$$i_2 = - i_1 \frac{r_c}{r_e + r_c - r_m} \tag{50}$$

$$\doteq - i_1 \frac{1}{1 - \alpha} \tag{51}$$

$$= - 46.5 i_1,$$

which indicates that the stage can give an appreciable current gain.

The input impedance is

$$R_i = r_b + r_c - \frac{r_c(r_c - r_m)}{r_e + r_c - r_m + R_L}. \tag{52}$$

When $R_L = 0$, this reduces to

$$R_i = r_b + r_e \frac{1}{\dfrac{r_e}{r_c} + 1 - \dfrac{r_m}{r_c}} \tag{53}$$

$$\doteq r_b + r_e \frac{1}{1 - \alpha} \tag{54}$$

$$= 1,445 \text{ ohms}.$$

When R_L is infinite,

$$R_i = r_b + r_c \tag{55}$$

$$= 13.4 \ (10)^6 \text{ ohms}.$$

With respect to input impedance, the grounded-collector stage is again seen to be like a cathode follower in that the input impedance is high when the load impedance is high.

The output impedance is

$$R_o = r_e + r_c - r_m - \frac{r_c(r_c - r_m)}{r_b + r_c + R_g}. \tag{56}$$

For $R_g = 0$, this reduces to

$$R_o = r_e + r_b \frac{r_c - r_m}{r_b + r_c} \tag{57}$$

$$\doteq r_e + r_b(1 - \alpha) \tag{58}$$

$$= 31.1 \text{ ohms}.$$

For R_g infinite,

$$R_o = r_e + r_c - r_m \tag{59}$$

$$= 0.288 \ (10)^6 \text{ ohms}.$$

The matched input impedance is

$$R_{im} = (r_b + r_c) \sqrt{\frac{r_b}{r_b + r_c} + \frac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}} \tag{60}$$

$$\doteq \sqrt{r_c [r_b + r_e/(1 - \alpha)]} \tag{61}$$

$$= 139,000 \text{ ohms}.$$

The matched output impedance is

$$R_{im} = (r_e + r_c - r_m) \sqrt{\frac{r_b}{r_b + r_c} + \frac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}} \tag{62}$$

$$\doteq (1 - \alpha) \sqrt{r_c [r_b + r_e/(1 - \alpha)]} \tag{63}$$

$$= 2,990 \text{ ohms}.$$

With matched impedance, the maximum available gain of the grounded-collector stage is

$$\text{MAG} = \frac{r_c^2}{(r_b + r_c)(r_e + r_c - r_m)} \left[1 + \sqrt{\frac{r_b}{r_b + r_c} + \frac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}} \right]^{-2}. \tag{64}$$

As α approaches unity, this approaches approximately

$$MAG = r_c/4r_e; \qquad (65)$$

but so long as $r_e \ll r_c - r_m$, a good approximation is

$$MAG = 1/(1 - \alpha) \qquad (66)$$

$$= 46.5 \text{ or } 16.7 \text{ db.}$$

The considerations involved in supplying biases to a grounded-collector stage are rather similar to those discussed already for the grounded-emitter case. If the base is allowed to float, the collector current will be given approximately by (47), as discussed for the grounded-emitter case. A resistance between base and the negative side of the supply battery in Fig. 24 will serve to decrease the collector current while a resistance between base and ground will serve to increase it. In applications where it is desired to make full use of the high input impedance, which this stage can afford, it may be most desirable to let the base float, as in Fig. 23.
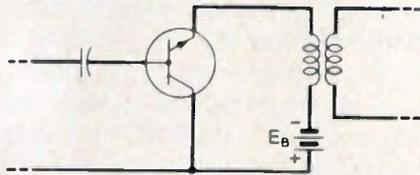
Fig. 23—One practical arrangement of a grounded-collector stage.
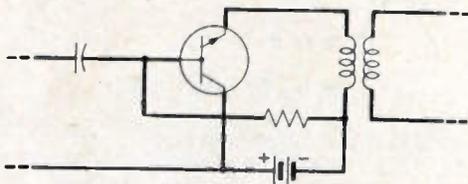
Fig. 24—Modification of Fig. 23 to obtain lower collector current. To raise collector current remove the resistance shown and connect a high resistance between the base and ground.

FREQUENCY RESPONSE—GENERAL REMARKS

Shockley has shown that there are several different physical considerations which lead one to expect a high-frequency cutoff in the response of *n-p-n* transistors. The frequency at which cutoff occurs depends in a theoretically understandable way on such things as the geometry of the transistor and the physical properties of the germanium from which it is made. If these factors could all be controlled and varied at will, it would then be possible to design a transistor to have a specified cutoff frequency.

One limitation comes about in the following way. In order to produce transistor action, the electrons which are injected into the *p* layer at the emitter junction must travel across this thin layer and arrive at the collector junction. They do this principally by a process

of diffusion and require a finite (but small) amount of time to make the journey. If this time were exactly the same for all electrons, the effect would be simply to delay the output signal with respect to the input and there would be no effect on frequency response. But there is a certain amount of dispersion in transit time which means that the electrons corresponding to a particular part of the input signal wave do not all arrive simultaneously at the collector. When this difference in time of arrival amounts to an appreciable part of a cycle there is a tendency for some of the electrons to cancel the effect of others so that the frequency response begins to fall off. As the signal frequency increases beyond this point, the effect becomes more and more pronounced and the response continues to fall with increasing frequency.

In terms of the equivalent circuit, this dispersion in transit time means that beyond a certain frequency, r_m (and hence α) begins to decrease with increasing frequency and so the transistor may be said to have a certain α cutoff which we will call $f_{c\alpha}$.

Shockley has shown that $f_{c\alpha}$ is inversely proportional to the square of the *p*-layer thickness and hence increases rapidly as the *p* layer is made thinner. For *n-p-n* transistors now available, this cutoff should occur at frequencies between 5 and 20 megacycles.

Another limitation on frequency response comes about from the fact that at sufficiently high frequencies, the emitter junction fails to behave as a pure resistance and is, in effect, shunted by a capacitance. In terms of the equivalent circuit, this means that r_e is shunted by a capacitance.

The effect which this has on frequency response can be reduced by reducing the impedance of the source from which the emitter is driven. But so far as the emitter junction is concerned, r_b is always in series with the source impedance, and so it is the value of r_b which ultimately determines the emitter cutoff frequency.

This capacitative reactance should begin to become appreciable with respect to emitter resistance at a frequency which may be of the same order as $f_{c\alpha}$. If r_b is high, the emitter cutoff frequency f_{ce} will then be of the same order of magnitude as $f_{c\alpha}$ and will increase as r_b is decreased.

A third cause for limited frequency response is the capacitance of the collector junction. The *n*-type germanium on one side of the junction behaves as one plate of a parallel-plate condenser and the *p*-type germanium on the other side behaves as the other plate. Since the transition from *n*-to-*p* type germanium may be made in an exceedingly small fraction of an inch, the plates of the condenser are very closely spaced and the capacitance may be appreciable.

Collector capacitance also depends on collector voltage, decreasing with increasing voltage. Theoretically, the capacitance should be in proportion to the negative one-third power of V_c.

Fig. 25 shows measured values of C_c as a function of collector voltage. For reasons which are not understood at present, these data show a departure from the usual inverse one-third power variation. At $V_c = 4.5$ volts the capacitance is seen to be approximately 7 micromicrofarads. In terms of the equivalent circuit, this capacitance is in shunt with the series combination of r_c and the generator, $r_m i_e$, as shown in Fig. 26. This can be shown to be equivalent to the circuit of Fig. 27 in which r_c has been replaced by

$$r_c' = r_c/(1 + jC_c r_c \omega) \qquad (67)$$

and r_m has been replaced by

$$r_m' + r_m/(1 + jC_c r_c \omega). \qquad (68)$$
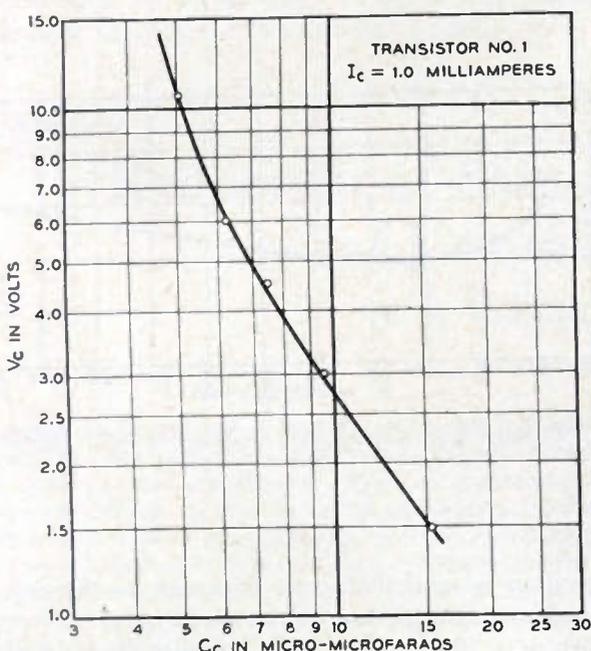
Fig. 25—Collector capacitance decreases as collector voltage is increased.
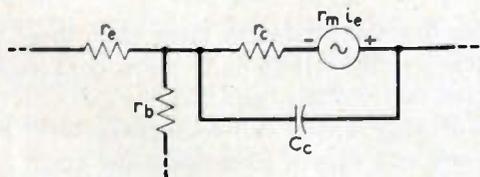
Fig. 26—The equivalent circuit of a transistor with collector capacitance shown.

cies f_{cc} will be computed. It will be shown that, at least for the transistor on which data are presented, collector capacitance tends to produce a cutoff frequency well below those to be expected from emitter cutoff or alpha cutoff. For this reason, only collector cutoff will be considered.
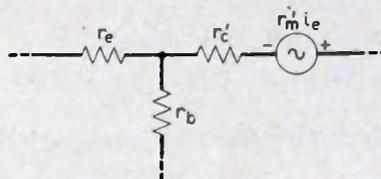
Fig. 27—The effect of collector capacitance is to modify r_m and r_c to r_m' and r_c'. See (67) and (68).

COLLECTOR CUTOFF IN THE GROUNDED-BASE STAGE

If the values of r_c' and r_m' from (67) and (68) are substituted for r_c and r_m in (16) and the resulting values of the R's are substituted into (8), the result is

$$v_2/v_g = \frac{\alpha R_L}{(r_e + r_b + R_g)[1 + R_L(1 + j\omega C_c r_c)/r_c]}. \qquad (69)$$

The cutoff frequency f_{cc} is defined as the frequency at which the voltage across the load has dropped 3 db compared to its low-frequency value. This is the frequency at which the imaginary part of the denominator of (69) is equal to the real part. Solving for f_{cc} gives

$$f_{cc} = \frac{1}{2\pi C_c}\left[\frac{1}{R_L} + \frac{1}{r_c} - \frac{\alpha r_b}{R_L(r_e + r_b + R_g)}\right]. \qquad (70)$$

Substituting into this equation $C_c = 7(10)^{-12}$ farad, numerical values of the r's from (16), and the values $R_g = 91$ and $R_L = 4.58 (10)^6$ ohms corresponding to maximum available gain gives

$$f_{cc} = 3,390 \text{ cps.}$$

With these terminations, the low-frequency gain is 44.3 db. If R_g and R_L are reduced to 25 and 200,000 ohms, respectively, f_{cc} is raised to 23,500 cps and the gain is lowered to 37.2 db. A further reduction of R_L to 20,000 ohms increases f_{cc} to 0.22 megacycle and reduces the gain to 27.8 db. This corresponds to a gain-bandwidth product of $1.2(10)^8$ cps, and shows that useful gain could be obtained at frequencies well above a megacycle, *provided* alpha and emitter cutoffs did not interfere.

COLLECTOR CUTOFF IN THE GROUNDED-EMITTER STAGE

The procedure described in the last section leads, in this case, to

$$v_2/v_g = \frac{- R_L r_m/r_c + (R_L r_e/r_c)(1 + jr_c C_c \omega)}{r_e + (r_b + R_g)(1 - r_m/r_c) + [r_e R_L/r_c + (r_b + R_g)(r_e + R_L)/r_c](1 + jr_c C_c \omega)}. \qquad (71)$$

The effect of collector capacitance can be computed by substituting r_c' and r_m' for the values of r_m and r_e (implicitly contained) in (8). In the sections which follow, this will be done for each of the three transistor connections and the resulting collector cutoff frequen-

For the transistor of our numerical example, the imaginary term in the numerator is completely negligible at frequencies below $(10)^9$ cps. Neglecting it leads to

$$f_{cc} = \frac{1}{2\pi C_c} \frac{1 + R_L/r_e + [(r_b + R_g)/r_e][1 - (r_m - r_e - R_L)/r_e]}{R_L + [(r_b + R_g)/r_e](r_e + R_L)}. \tag{72}$$

In this case the values of R_g and R_L (619 ohms and 671,000 ohms, respectively) which correspond to maximum available gain give

$$f_{cc} = 3,740 \text{ cps and}$$
$$\text{MAG} = 53 \text{ db.}$$

Reducing R_L to 100,000 and increasing R_g to 1,000 ohms gives

$$f_{cc} = 11,120 \text{ cps}$$
$$G = 50 \text{ db.}$$

For $R_g = 1,000$ and $R_L = 10,000$,

$$f_{cc} = 97,900 \text{ cps}$$
$$G = 41.3 \text{ db.}$$

and for $R_g = R_L = 1,000$ ohms,

$$f_{cc} = 943,000 \text{ cps}$$
$$G = 31.4 \text{ db.}$$

The gain-bandwidth product for this stage is $1.3(10)^9$ cps, as compared to $1.2(10)^8$ cps for the same transistor connected as a grounded base amplifier. It should be pointed out, however, that this stage is particularly sensitive to change in α and on this account alpha cutoff may influence the response at fairly low frequencies. For example, when the terminating resistances are both 1,000 ohms, reducing α from 0.9785 to 0.900 reduces the gain from 31.4 to 0.2 db.

COLLECTOR CUTOFF IN THE GROUNDED COLLECTOR STAGE

In this case

$$v_2/v_g = \frac{R_L}{[r_e + R_L + (r_b + R_g)(1 - r_m/r_c)] + (1/r_c)(r_b + R_g)(r_e + R_L)(1 + j\omega C_c r_c)} \tag{73}$$

and

$$f_{cc} = \frac{1}{2\pi C_c}\left[\frac{1}{r_c} + \frac{1}{r_b + R_g} + \frac{1 - r_m/r_c}{r_e + R_L}\right]. \tag{74}$$

For matched impedances ($R_g = 139,000$ ohms and $R_L = 2,990$ ohms),

$$f_{cc} = 320,000 \text{ cps}$$
$$G = 16.7 \text{ db.}$$

The cutoff frequency can be raised by decreasing either R_g or R_L. With $R_g = 139,000$ and $R_L = 25$ ohms

$$f_{cc} = 9.77 \text{ megacycles}$$
$$G = 1.8 \text{ db.}$$

The gain-bandwidth product in this case is $1.5(10)^7$.
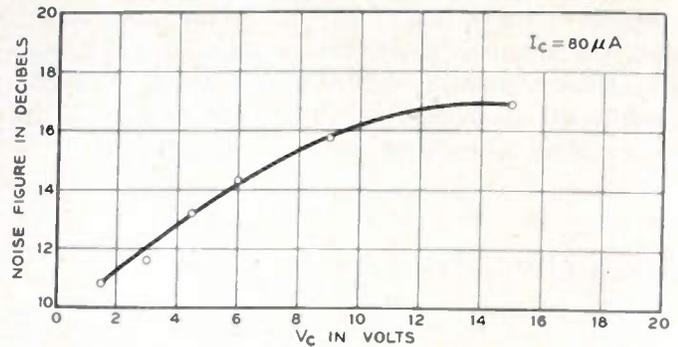
Fig. 28—Noise figure increases with increasing collector voltage.
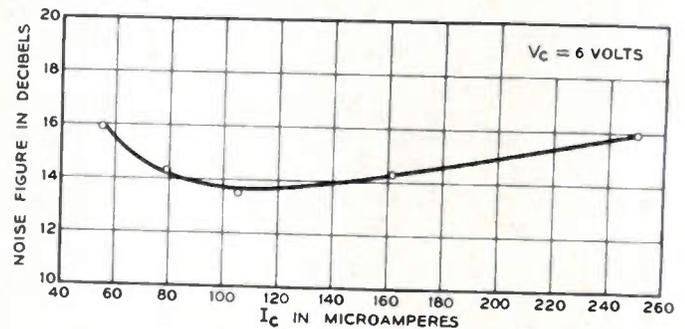
Fig. 29—Noise figure does not vary much with collector current.

NOISE

The data now available on noise are insufficient to give an adequate picture of the performance of n-p-n transistors in this respect. Such measurements as have been made, however, make it clear that these devices are very much quieter than early point-contact transistors reported on by Ryder and Kircher.

Transistor noise seems still to decrease with increasing frequency at a rate of something like 11 db per decade. It also decreases as the thickness of the p layer is decreased.

About half a dozen units of various dimensions have been measured at 1,000 cps and have shown noise figures as low as 8 db and as high as 25 db.

The dependence of noise figure on operating point has been measured for only one transistor. As indicated in Figs. 28 and 29, these data show that the noise figure improves as V_c is reduced and that it may be roughly independent of collector current. These data were taken on a grounded emitter stage with impedance

match at the input terminals. The noise figure for this connection varies slightly with source impedance and has been found to be a minimum when the source impedance is roughly equal to the input impedance of the stage.

It must be emphasized that this functional dependence of the noise figure on operating point and source impedance has been measured for only one transistor. Further measurements may show that these results are not typical.

FINAL COMMENTS

In this paper we have attempted to present what is known about the circuit performance of *n-p-n* transistors. Since these devices are still undergoing exploratory development and since only a limited number have been produced, it is obviously impossible to give statistical data on reproducibility or on such reliability factors as the effect of ambient temperature.

It is much too soon to know what properties may be achieved after further development, but the results obtained to date seem encouraging and worth reporting.

ACKNOWLEDGMENT

Reduction of Skin-Effect Losses by the Use of Laminated Conductors[*]

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Summary—It has recently been discovered that it is possible to reduce skin effect losses in transmission lines by properly laminating the conductors and adjusting the velocity of transmission of the waves. The theory for such laminated transmission lines is presented in the case of planar systems for both infinitesimally thin laminae and laminae of finite thickness. A transmission line completely filled with laminated material is discussed. An analysis is given of the modes of transmission in a laminated line, and of the problem of terminating such a line.

I. INTRODUCTION

IT HAS LONG been recognized that an electromagnetic wave propagating in the vicinity of an electrical conductor can penetrate only a limited distance into the interior of the material. This phenomenon is known as "skin effect" and is usually measured by a so-called "skin depth" δ. If y is measured from the surface of a conductor into its depth, the amplitude of the electromagnetic wave and the accompanying current density decreases as $e^{-y/\delta}$, provided the conductor is several times δ in thickness, so that for $y = \delta$ the amplitude has fallen to $1/e = 0.367$ times its value at the surface. The skin depth δ is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}, \qquad (1)$$

where σ is the conductivity of the material, μ is its permeability, and ω is 2π times the frequency f under consideration. Throughout this paper rationalized mks units are used.

From one point of view, skin effect serves a most useful purpose; for instance, in shielding electrical equipment or reducing cross talk between communication circuits. On the other hand, the effect severely limits the high-frequency performance of many types of electrical apparatus, including in particular the various kinds of transmission lines.

Surprisingly enough, it has been discovered that it is possible, within limits, to increase the distance to which an electromagnetic wave penetrates into a conducting material. This is done essentially by fabricating the conductor of many insulated laminae or filaments of conducting material arranged parallel to the direction of current flow. If the transverse dimensions of the laminae or filaments are small compared to the skin depth δ at the frequency under consideration, and if the velocity of the electromagnetic wave along the conductor is close to a certain critical value, the wave will penetrate into the composite conductor a distance great enough to include a thickness of conducting material many skin depths deep. Physically speaking, the lateral change of the wave through the conducting regions is very nearly cancelled by the change through the insulating regions.

In Fig. 1 there is shown a cross-section view of a coaxial cable with a laminated center conductor. The center conductor is formed of a nonconducting core surrounded by alternate layers of a conductor of thickness W and conductivity σ, and an insulator of thickness t and dielectric constant ϵ. The center conductor is embedded in an insulator of dielectric constant ϵ_1, which is in turn encased in the outer conductor. We will assume all the conductors and insulators to have the permeability μ_0 of free space.
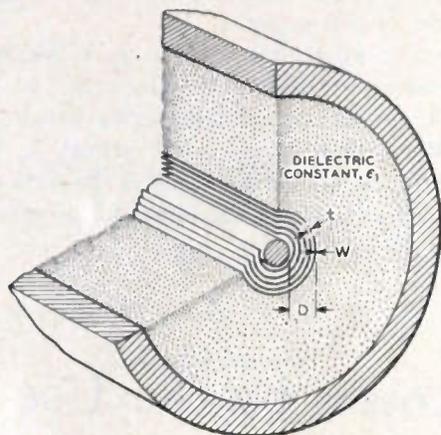
Fig. 1—Laminated transmission line.

We will associate with the inner laminated conductor an average dielectric constant for transverse electric fields given by[1]

$$\bar{\epsilon} = \epsilon \left(1 + \frac{W}{t} \right). \tag{2}$$

It will be shown in the following sections that the electromagnetic wave and the accompanying currents will penetrate most deeply into the center conductor if the wave travels through the line with a velocity

$$v = \frac{1}{\sqrt{\bar{\epsilon}\mu_0}}. \tag{3}$$

One way to make the wave assume this velocity is to let the dielectric constant ϵ_1 have the value

$$\epsilon_1 = \bar{\epsilon} = \epsilon \left(1 + \frac{W}{t} \right). \tag{4}$$

If the depth of the stack of laminations D is small compared to the distance between the stack and the outer conductor, and if the wave travels with the velocity given in (3), it will be shown that the wave decreases with distance into the center conductor as e^{-v/δ_w} where δ_w is given by

$$\delta_w = \sqrt{3}\,(1 + t/W)(\delta/W)\delta; \quad W \ll \delta. \tag{5}$$

Here $\delta = 1/\sqrt{\pi f \mu_0 \sigma}$ is the skin depth appropriate to the material of the conducting laminae and the frequency

[1] A similar average dielectric constant has been considered by Tokio Sakurai, *Jour. Phys. Soc.* (Japan), vol. 5, no. 6, pp. 394–398; November–December, 1950.

f under consideration. Let us now also associate with the center conductor an average longitudinal conductivity given by

$$\bar{\sigma} = \sigma \frac{W}{W + t}. \tag{6}$$

We will suppose for the present case that most of the attenuation of the transmission line results from the currents flowing in the inner conductor. It is easy to see that the attenuation of the line for very low frequencies will be $A/\bar{\sigma}D$ where A is a constant depending on the impedance of the line. As the frequency increases, δ_w decreases, and when δ_w becomes several times smaller than D it will be shown that the attenuation becomes
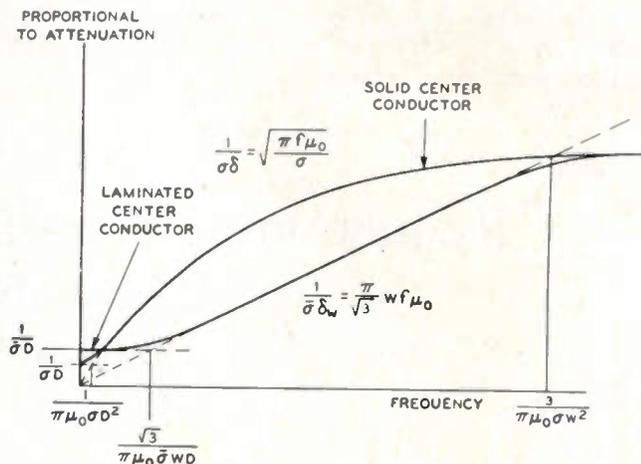
Fig. 2—Comparison of conventional and laminated transmission lines.

$A/\bar{\sigma}\delta_w$. At still higher frequencies δ will similarly become several times smaller than W, and the attenuation then becomes $A/\sigma\delta$. From these considerations, a qualitative picture of the attenuation of the laminated line can be sketched as in Fig. 2.

For comparison, we have also sketched in Fig. 2 the attenuation that would be obtained if the laminations in Fig. 1 were replaced with solid metal. At low frequencies, the attenuation of this line would clearly be $A/\sigma D$. When the frequency becomes high enough for δ to be several times smaller than D the attenuation will be shown to become $A/\sigma\delta$.

It will be observed how the attenuation of the unlaminated line remains constant over a low range of frequencies and then rises at a rate proportional to the square root of the frequency. The laminated line has a higher initial attenuation, but remains constant to higher frequencies. At high enough frequencies the attenuation of the laminated line rises at a rate directly proportional to frequency for a while, then eventually approaches the attenuation of the unlaminated line.

The frequency at which the attenuation of the laminated line begins to increase is greater than the corresponding frequency for the conventional line by a factor

$$\sqrt{3}\left(\frac{\sigma}{\bar{\sigma}}\right)\left(\frac{D}{W}\right).$$

This is accomplished with an increase in initial attenuation by a factor

$$\sigma/\bar{\sigma} = \left(1 + \frac{t}{W}\right),$$

which we will see later will be about 3/2 in a typical case. We might make a corresponding increase in the flat range of the conventional line by decreasing σ to a new value σ_1. In that case, the attenuation would be increased by a factor

$$\sqrt{3}\left(\frac{\sigma}{\bar{\sigma}}\right)\left(\frac{D}{W}\right),$$

which may be very large since $(\sigma/\bar{\sigma})$ (D/W) is just the number of laminations of conductor or dielectric used on the center conductor.

The flat range of the conventional line might be alternatively increased to equal that of the laminated line by decreasing D to a new value D_1. In this case the attenuation would be increased by a factor

$$\sqrt{\sqrt{3}\left(\frac{\sigma}{\bar{\sigma}}\right)\left(\frac{D}{W}\right)},$$

just the square root of the factor achieved by changing σ, but still a large number.

In the frequency range in which the attenuation of the laminated line is governed by the skin depth δ_ω, and is therefore increasing linearly with frequency, this attenuation is less than the attenuation of the conventional line by a factor

$$\frac{\sigma\delta}{\bar{\sigma}\delta_\omega} = \frac{1}{\sqrt{3}}\left(\frac{W}{\delta}\right). \tag{7}$$

It is interesting to note that the position of this region is governed by the conductivity of the conducting laminations, but that the attenuation is independent of the conductivity.

Considerable theoretical and experimental work has been carried out on laminated transmission lines by the author's colleagues. The following report therefore will be limited to bringing out some of the fundamental ideas in a simple way. We will, for instance, consider only planar systems so that the results will be only approximately applicable to real transmission lines. Other papers will more fully develop the formal theory, particularly for cylindrical systems, and discuss the practical and experimental aspects of the problem.

II. Skin Effect

We shall begin the discussion with this section by considering skin effect in various kinds of conducting media. We will first derive the skin depth equation (1) for an ordinary conductor like copper, and then discuss the behavior of a composite conductor made up of many thin, insulated conducting laminae. This second discussion will be first carried out for the case of infinitesimally thin laminae, and then in Section III the effects of the finite thickness of the conducting sheets will be considered.

Let us first set down and integrate Maxwell's equations in a form that will be useful in all our following discussions. Referring to the orthogonal co-ordinate system in Fig. 3, we shall be concerned with fields that
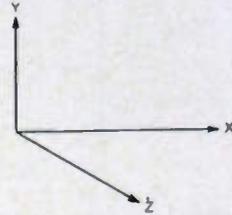
Fig. 3—Rectangular co-ordinate system.

have no variation along the z axis and for which the z component of electric field is zero. The only component of magnetic field is then H_z and the field equations become in rationalized mks units,

$$\frac{\partial H_z}{\partial y} = i\omega D_x + J_x, \tag{8}$$

$$-\frac{\partial H_z}{\partial x} = i\omega D_y + J_y, \tag{9}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega B_z, \tag{10}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = \rho. \tag{11}$$

In these equations H, B, D, E, J, and ρ all have their usual meanings. A positive time factor $e^{i\omega t}$ has been introduced.

Let us, for the moment, suppose that we are dealing with an anisotropic medium such that the following relations exist:

$$J_x = \sigma_x E_x; \qquad J_y = \sigma_y E_y \tag{12}$$

$$D_x = \epsilon_x E_x; \qquad D_y = \epsilon_y E_y \tag{13}$$

$$B_z = \mu_0 H_z. \tag{14}$$

Here the σ's are conductivities, the ϵ's dielectric constants, and μ_0 is the permeability of free space. Suppose also now that the fields all vary with x according to a factor e^{-ikx}. If k has a positive real part we will be dealing with a wave moving along the x axis in a positive direction, and a negative imaginary part will indicate that this wave is attenuated.

Using the above relations, one can easily find the following equations:

$$\frac{\partial^2 H_z}{\partial y^2} = \frac{i\omega\epsilon_x + \sigma_x}{i\omega\epsilon_y + \sigma_y}\left[i\omega\mu_0\sigma_y - \omega^2\mu_0\epsilon_y + k^2\right]H_z \tag{15}$$

$$E_z = \frac{1}{i\omega\epsilon_z + \sigma_z} \cdot \frac{\partial H_z}{\partial y} \tag{16}$$

$$E_v = \frac{ik}{i\omega\epsilon_y + \sigma_y} H_z. \tag{17}$$

Image that we have a semi-infinite volume of material arranged as in Fig. 4, where the z axis is pointing out
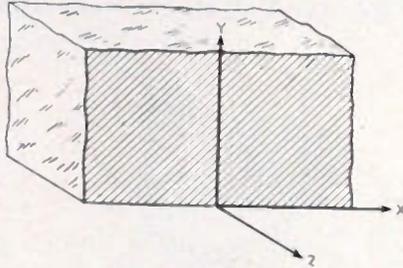
Fig. 4—Orientation of solid conductor.

of the paper. If H_{z0} is the value of H_z at $y = 0$, it is clear from (15) that H_z must depend upon y according to

$$H_z = H_{z0}e^{-\alpha y}, \tag{18}$$

where

$$\alpha = \pm \sqrt{\frac{i\omega\epsilon_z + \sigma_z}{i\omega\epsilon_y + \sigma_y} \left[i\omega\mu_0\sigma_y - \omega^2\mu_0\epsilon_y + k^2 \right]} \tag{19}$$

and the sign is chosen so the real part of α is positive.

We can now consider the case when the material is an ordinary conductor, such as copper or silver. In this case we must let $\sigma_z = \sigma_y = \sigma$ and $\epsilon_z = \epsilon_y = \epsilon$. Then α becomes (the subscript S stands for solid)

$$\alpha_S = \pm \sqrt{i\omega\mu_0\sigma - \omega^2\mu_0\epsilon + k^2}. \tag{20}$$

Now, under any practical circumstances the propagation constant k will certainly not be more than a factor 100 larger than the propagation constant of free-space $k = \sqrt{\omega^2\mu_0\epsilon_0}$. This applies also to the factor $\sqrt{\omega^2\mu_0\epsilon}$. Consider, then, the ratio

$$\frac{\omega\mu_0\sigma}{\omega^2\mu_0\epsilon_0} = \frac{\sigma}{\omega\epsilon_0}.$$

For the metal copper, for instance, $\sigma = 5.80 \times 10^7$ mhos/meter and the dielectric constant of free space $\epsilon_0 = 0.885 \times 10^{-11}$. If we consider frequencies as high as 10,000 megacycles, the ratio is still as great as 10^8. Thus, the second two factors under the square-root sign in (20) are entirely negligible, and we have

$$\alpha_S = \pm \sqrt{i\omega\mu_0\sigma} \tag{21}$$

$$= (1 + i)\sqrt{\frac{\omega\mu_0\sigma}{2}}. \tag{22}$$

Finally, we have

$$\text{real part } (\alpha_S) = \frac{1}{\delta} = \sqrt{\frac{\omega\mu_0\sigma}{2}}. \tag{23}$$

We can now turn our attention to the stack of laminations shown in Fig. 5. There are shown a series of conducting sheets of conductivity σ and thickness W, separated by a series of insulating sheets of thickness t
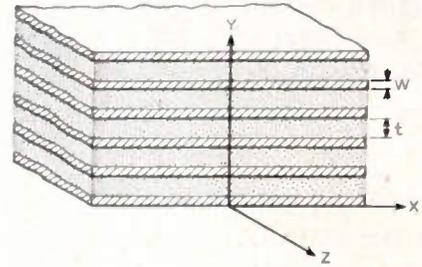
Fig. 5—Orientation of laminated conductor.

and dielectric constant ϵ. Suppose we let W and t approach zero while maintaining a constant ratio to obtain a homogeneous but anisotropic material to which we can apply (19). In order to obtain ϵ_y, σ_y, ϵ_z and σ_z, we can write

$$\frac{1}{\sigma_y + i\omega\epsilon_y} = \frac{1}{W + t}\left[\frac{W}{\sigma} + \frac{t}{i\omega\epsilon} \right] \tag{24}$$

and

$$\sigma_z + i\omega\epsilon_z = \frac{1}{W + t}\left[W\sigma + t i\omega\epsilon \right]. \tag{25}$$

One then has approximately, letting $\epsilon_y = \bar{\epsilon}$ and $\sigma_z = \bar{\sigma}$,

$$\bar{\epsilon} = \epsilon\left(1 + \frac{W}{t}\right) \tag{26}$$

$$\bar{\sigma} = \sigma\left(\frac{W}{W + t}\right) \tag{27}$$

$$\epsilon_z = \epsilon\left(\frac{t}{t + W}\right) \tag{28}$$

$$\sigma_y = \sigma\left(\frac{\omega\epsilon}{\sigma}\right)^2 \frac{W}{t}\left(\frac{W}{t} + 1\right). \tag{29}$$

As before, $\omega\epsilon_z$ is completely negligible compared to $\bar{\sigma}$, besides, σ_y is negligible compared to $\omega\bar{\epsilon}$. We have therefore for α (the subscript 0 stands for zero thickness).

This situation is rather surprising. Let us suppose conditions are such that most of the energy of the wave is flowing in the region outside the stack of laminations.

$$\alpha_0 = \pm \sqrt{\frac{\bar{\sigma}}{i\omega\bar{\epsilon}}\left[k^2 - \omega^2\mu_0\bar{\epsilon} + i\omega\mu_0\sigma\left(\frac{\omega\epsilon}{\sigma}\right)^2 \frac{W}{t}\left(\frac{W}{t} + 1\right) \right]}. \tag{30}$$

If this region is filled with an insulator of dielectric constant ϵ_1, k^2 will be very nearly equal to $\omega^2\mu_0\epsilon_1$. Then α_0 will be given by

$$\alpha_0 = \pm \sqrt{\frac{1}{i} \; \bar{\sigma}\omega\mu_0 \left[\left(\frac{\epsilon_1}{\bar{\epsilon}} - 1\right) + i\left(\frac{\omega\epsilon}{\sigma}\right)\left(\frac{W}{t}\right)\right]}. \quad (31)$$

If ϵ_1 is made equal to $\bar{\epsilon}$, α_t becomes

$$\alpha_0 = \frac{W}{W + t} \, k \quad (32)$$

$$= \frac{1}{1 + t/W} \frac{2\pi}{\lambda}, \quad (33)$$

where λ is the longitudinal wavelength. Thus, under these conditions, the wave will penetrate into the laminations to a depth $\lambda/2\pi(1 + t/W)$ before it has decreased by a factor $1/e$. This distance is, of course, enormous compared to ordinary skin depths.

We will see in the next section that the finite thickness of the laminae limits the penetration of the waves for $\epsilon_1 = \bar{\epsilon}$ to a distance much smaller than that implied in (33), but still large compared to conventional skin depths. In any case, we see in this simple way the suggestion of a method for obtaining great penetrations and consequently considerably reducing the attenuation of a transmission line.

The analysis of this section, carried out by assuming the medium to anisotropic but homogeneous, can be given more physical meaning by examining a little more closely how the fields vary through the laminations shown in Fig. 5. From (15) and (16), one finds for a general case

$$\frac{\partial E_z}{\partial y} = \frac{1}{i\omega\epsilon_y + \sigma_y} \left[i\omega\mu_0\sigma_y - \omega^2\mu_0\epsilon_y + k^2\right]H_z. \quad (34)$$

Let the value of H_z at the interface of a conducting lamina and a dielectric lamina be $(H_z)_0$. From (34), one finds just within the conducting lamina

$$\frac{\partial E_z}{\partial y} = i\omega\mu_0(H_z)_0, \quad (35)$$

while just within the adjacent dielectric lamina

$$\frac{\partial E_z}{\partial y} = i\omega\mu_0\left[1 - \frac{\epsilon_1}{\epsilon}\right](H_z)_0. \quad (36)$$

Thus, if the laminae are very thin, the change in E_z across the conducting lamina is

$$(\Delta E_z)_m = i\omega\mu_0(H_z)_0 W, \quad (37)$$

and the change in E_z across the dielectric lamina is

$$(\Delta E_z)_d = i\omega\mu_0(H_z)_0\left(1 - \frac{\epsilon_1}{\epsilon}\right)t. \quad (38)$$

Therefore, when $\epsilon_1 = \epsilon(1 + W/t)$, the change in E_z across the conducting lamina is just balanced by the change in

E_z across the dielectric lamina. This is the basic reason for the deep penetration of the fields into the laminated structure. When $\epsilon_1 = \epsilon$, there is no change in E_z across the dielectric lamina. In this case we note from (31) that $\alpha_0 = W/(W+t)\alpha_s$, and we see that the waves will penetrate into the laminae an increased distance that is just accounted for by the spacing of the laminae. Thus, for $\epsilon_1 = \epsilon$, the attenuation of a laminated line will be unchanged if the laminae are replaced by solid metal.

III. LAMINATIONS OF FINITE THICKNESS

Let us refer again to Fig. 5 where a stack of conducting laminae of thickness W and conductivity σ are shown separated by insulating laminae of thickness t and dielectric constant ϵ. First, we shall inquire as to how the fields change across the conducting laminations. According to (15) and (16) one has

$$\frac{\partial^2 H_z}{\partial y^2} = i\omega\mu_0\sigma H_z \quad (39)$$

$$E_z = \frac{1}{\sigma} \frac{\partial H_z}{\partial y}, \quad (40)$$

where $\omega\epsilon$ has been neglected against σ as before. Now, letting $\eta = \sqrt{i\omega\mu_0\sigma}$, we can write within any conducting lamina

$$H_z = Ae^{\eta y} + Be^{-\eta y} \quad (41)$$

$$E_z = \frac{\eta}{\sigma}\left[Ae^{\eta y} - Be^{-\eta y}\right]. \quad (42)$$

If H_0 and E_0 are the values of H_z and E_z at the lower surface of a particular lamination, and if H_1 and E_1 are their values at the upper surface of the lamination, one can find from (41) and (42)

$$H_1 = H_0 \cosh \eta W + E_0 \frac{\sigma}{\eta} \sinh \eta W \quad (43)$$

$$E_1 = H_0 \frac{\eta}{\sigma} \sinh \eta W + E_0 \cosh \eta W. \quad (44)$$

If we wish, this can be expressed as a matrix equation:

$$\begin{pmatrix} H_1 \\ \\ E_1 \end{pmatrix} = \begin{pmatrix} \cosh \eta W & \dfrac{\sigma}{\eta} \sinh \eta W \\ \\ \dfrac{\eta}{\sigma} \sinh \eta W & \cosh \eta W \end{pmatrix} \begin{pmatrix} H_0 \\ \\ E_0 \end{pmatrix}. \quad (45)$$

For the dielectric laminae, (15) and (16) become

$$\frac{\partial^2 H_z}{\partial y^2} = (k^2 - \omega^2\mu_0\epsilon)H_z \quad (46)$$

$$E_z = \frac{1}{i\omega\epsilon} \frac{\partial H_z}{\partial y}. \quad (47)$$

Just as for the conducting laminae, let H_1 and E_1 be the values of H_z and E_z at the lower surface of a dielectric

lamination, and let H_2 and E_2 be these values at the top surface. Then, if $\xi = \sqrt{k^2 - \omega^2 \mu_0 \epsilon}$, one has

$$\begin{pmatrix} H_2 \\ E_2 \end{pmatrix} = \begin{pmatrix} \cosh \xi t & \dfrac{i\omega\epsilon}{\xi} \sinh \xi t \\ \dfrac{\xi}{i\omega\epsilon} \sinh \xi t & \cosh \xi t \end{pmatrix} \begin{pmatrix} H_1 \\ E_1 \end{pmatrix}. \tag{48}$$

From (45) and (48), we can find the variation of H_z and E_z from the bottom surface of a conducting lamination designated as point zero, to the top surface of the adjacent dielectric lamination designated as point two. Thus,

$$\begin{pmatrix} H_2 \\ E_2 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} H_0 \\ E_0 \end{pmatrix}, \tag{49}$$

where the T's are given below:

$$T_{11} = \cosh \eta W \cosh \xi t + \frac{i\omega\epsilon}{\sigma} \frac{\eta}{\xi} \sinh \eta W \sinh \xi t \tag{50}$$

$$T_{12} = \frac{\sigma}{\eta} \sinh \eta W \cosh \xi t + \frac{i\omega\epsilon}{\xi} \cosh \eta W \sinh \xi t \tag{51}$$

$$T_{21} = \frac{\xi}{i\omega\epsilon} \cosh \eta W \sinh \xi t + \frac{\eta}{\sigma} \sinh \eta W \cosh \xi t \tag{52}$$

$$T_{22} = \frac{\sigma}{i\omega\epsilon} \frac{\xi}{\eta} \sinh \eta W \sinh \xi t + \cosh \eta W \cosh \xi t. \tag{53}$$

It is easy to verify from the above that

$$T_{11}T_{22} - T_{12}T_{21} = 1. \tag{54}$$

If we now designate the lower surface of each conducting lamination successively as points 0, 1, 2, 3, $\cdots$, we can write down the following simultaneous difference equations:

$$H_{n+1} = T_{11}H_n + T_{12}E_n \tag{55}$$

$$E_{n+1} = T_{21}H_n + T_{22}E_n. \tag{56}$$

The solutions of these difference equations are

$$H_n = A\beta^n + B^{-n} \tag{57}$$

$$E_n = A \frac{\beta - T_{11}}{T_{12}} \beta^n + B \frac{1/\beta - T_{11}}{T_{12}} \beta^{-n}, \tag{58}$$

where

$$\beta = \left(\frac{T_{11} + T_{22}}{2} \right) + \sqrt{\left(\frac{T_{11} + T_{22}}{2} \right)^2 - 1}. \tag{59}$$

Let us now proceed to determine the skin depth to be associated with the stack of laminae in Fig. 5. Since we have assumed the stack to be very deep, A must be taken zero in (57) and (58), and the fields vary into the stack according to a factor β^{-n}, so that

$$H_n = H_0\beta^{-n}. \tag{60}$$

If we now define

$$y_n = (W + t)n, \tag{61}$$

one has

$$H_n = H_0\beta^{-y_n/(W+t)}$$
$$= H_0 e^{-\ln\beta/(W+t)\,y_n}$$
$$= H_0 e^{-\alpha_w y_n}, \tag{62}$$

where (the subscript w indicates a thickness W for the conducting laminae)

$$\alpha_w = \frac{\cosh^{-1}\left(\dfrac{T_{11} + T_{22}}{2} \right)}{(W + t)}. \tag{63}$$

From (50) and (53) one has

$$\left(\frac{T_{11} + T_{22}}{2} \right) = \cosh \eta W \cosh \xi t$$
$$+ \frac{1}{2}\left(\frac{i\omega\epsilon}{\sigma} \frac{\eta}{\xi} + \frac{\sigma}{i\omega\epsilon} \frac{\xi}{\eta} \right) \sinh \eta W \sinh \xi t. \tag{64}$$

As a practical matter, only rarely will k^2 be greater than ten times $\omega^2\mu_0\epsilon$ and ϵ greater than $10\,\epsilon_0$. Hence, ξ will be no larger than $10\sqrt{\omega^2\mu_0\epsilon_0}$. Furthermore, we will see that t should not be very different in magnitude from W, which must be smaller than $\sqrt{2/\omega\mu_0\sigma}$. Thus, we can be sure that ξt is smaller than $100\sqrt{2\omega\epsilon_0/\sigma}$, a quantity which is, as before, much smaller than unity. Under these conditions, (64) becomes approximately

$$\left(\frac{T_{11} + T_{22}}{2} \right)$$
$$= \cosh \eta W + \frac{\xi t}{2}\left(\frac{i\omega\epsilon}{\sigma} \frac{\eta}{\xi} + \frac{\sigma}{i\omega\epsilon} \frac{\xi}{\eta} \right) \sinh \eta W \tag{65}$$
$$= \cosh \eta W$$
$$+ \frac{1}{2}\left(\frac{t}{W} \right)\left[\frac{i\omega\epsilon}{\sigma} + \left(1 - \frac{\epsilon_1}{\epsilon} \right) \right] \eta W \sinh \eta W, \tag{66}$$

where we have again let $k^2 = \omega^2\mu_0\epsilon_1$ as in (31).

Let us set

$$P = \frac{1}{2}\left(\frac{t}{W} \right)\left(1 - \frac{\epsilon_1}{\epsilon} \right) \tag{67}$$
$$= \frac{1}{2}\left[-1 + \left(1 + \frac{t}{W} \right)\left(1 - \frac{\epsilon_1}{\epsilon} \right) \right]. \tag{68}$$

Then, again neglecting $\omega\epsilon/\sigma$, we have for α_w

$$\alpha_w = \frac{1}{W + t} \cosh^{-1}\left[\cosh \eta W + P(\eta W) \sinh \eta W \right]. \tag{69}$$

By definition, $\eta W = (1+i)W/\delta$. We can therefore write approximately,

$$\cosh \eta W \simeq \left[1 - \frac{1}{6}\left(\frac{W}{\delta} \right)^4 \right] + i\left(\frac{W}{\delta} \right)^2 \tag{70}$$

$$(\eta W) \sinh \eta W \simeq -\frac{2}{3}\left(\frac{W}{\delta} \right)^4 + i2\left(\frac{W}{\delta} \right)^2. \tag{71}$$

Using expressions (70) and (71), (69) becomes

$$\alpha_w = \frac{1}{W + t}\cosh^{-1}\left(\left[1 - \frac{(1 + 4P)}{6}\left(\frac{W}{\delta}\right)^4\right]\right.$$

$$\left. + i(1 + 2P)\left(\frac{W}{\delta}\right)^2\right). \tag{72}$$

Provided that $(1 + 2P)(W/\delta)^2 \ll 1$, (72) can be expanded in orders of magnitude. Thus, for

$$\left|1 - \frac{\epsilon_1}{\bar{\epsilon}}\right| \ll \frac{W}{W + t}\left(\frac{\delta}{W}\right)^2 \tag{73}$$

we find approximately

$$\alpha_w = \frac{1}{(W + t)}\left(\frac{W}{\delta}\right)\{\sqrt{-f + \sqrt{f^2 + g^2}}$$

$$\pm i\sqrt{f + \sqrt{f^2 + g^2}}\}, \tag{74}$$

where

$$f = \frac{1 + 4P}{6}\left(\frac{W}{\delta}\right)^2 \tag{75}$$

and

$$g = 1 + 2P. \tag{76}$$

The plus sign is to be used when g is positive and the minus sign when g is negative.

Equations (74) for α_w and (31) for α_0 are very similar. With a little manipulation we can rewrite equation (31) as

$$\alpha_0 = \frac{1}{\delta}\sqrt{\frac{\bar{\sigma}}{\sigma}}\left(\sqrt{\sqrt{\left(\frac{\omega\epsilon}{\sigma}\right)\left(\frac{W}{t}\right) + \sqrt{\left(\frac{\omega\epsilon}{\sigma}\right)^2\left(\frac{W}{t}\right)^2 + \left(1 - \frac{\epsilon_1}{\bar{\epsilon}}\right)^2}}}\right.$$

$$\left. \pm i\sqrt{-\left(\frac{\omega\epsilon}{\sigma}\right)\left(\frac{W}{t}\right) + \sqrt{\left(\frac{\omega\epsilon}{\sigma}\right)^2\left(\frac{W}{t}\right)^2 + \left(1 - \frac{\epsilon_1}{\bar{\epsilon}}\right)^2}}\right) \tag{77}$$

Also, (74) can be written

$$\alpha_w = \frac{1}{\delta}\sqrt{\frac{\bar{\sigma}}{\sigma}}\left(\sqrt{-\left(\frac{W}{W + t}f\right) + \sqrt{\left(\frac{W}{W + t}f\right)^2 + \left(1 - \frac{\epsilon_1}{\bar{\epsilon}}\right)^2}}\right.$$

$$\left. \pm i\sqrt{\left(\frac{W}{W + t}f\right) + \sqrt{\left(\frac{W}{W + t}f\right)^2 + \left(1 - \frac{\epsilon_1}{\bar{\epsilon}}\right)^2}}\right). \tag{78}$$

Equation (77) is a very good approximation for α for a stack of infinitesimally thin laminae, but is inadequate for the discussion of situations where the finite value of W/δ is important. Equation (78) on the other hand is a good approximation to α when W/δ is appreciable, but the assumptions made in deriving (78) do not allow us to go correctly to the limit $W/\delta = 0$.

To estimate how small W must be before (78) fails, let us set $(-W/(W + t)f)$ equal to $(\omega\epsilon/\sigma)(W/t)$. We will also set $\epsilon_1 = \bar{\epsilon}$, since it is only then that these terms are important. In this case,

$$W = \frac{1}{\sigma}\sqrt{12\frac{\bar{\epsilon}}{\mu_0}}. \tag{79}$$

If we take $\bar{\epsilon}$ to be 5 times the dielectric constant ϵ_0 of free space, and take $\sigma = 5.8 \times 10^7$ mhos/meter for copper, we obtain $W = 3.5 \times 10^{-8}$ cm. Thus, for any practical purpose we can ignore the failure of (78) to have the proper limit for $W/\delta = 0$.

By definition, the fields decrease into the stack according to $e^{-\alpha_w y}$. Let us define the distance at which the fields have decreased by $1/e$ to be the effective skin depth δ_w. Then we have

$$\frac{1}{\delta_w} = \text{real part }(\alpha_w) = \frac{1}{\delta}\sqrt{\frac{\bar{\sigma}}{\sigma}}\sqrt{-\left(\frac{W}{W + t}f\right) + \sqrt{\left(\frac{W}{W + t}f\right)^2 + \left(1 - \frac{\epsilon_1}{\bar{\epsilon}}\right)^2}} \tag{80}$$

From (68) and (75) we find

$$-\left(\frac{W}{W + t}f\right) = \frac{1}{6}\left[\frac{W}{W + t} + 2\left(\frac{\epsilon_1}{\bar{\epsilon}} - 1\right)\right]\left(\frac{W}{\delta}\right)^2. \tag{81}$$

For $\epsilon_1 = \bar{\epsilon}$ (81) and (80) give us finally

$$\delta_w(\epsilon_1 = \bar{\epsilon}) = \sqrt{3}\left(1 + \frac{t}{W}\right)\left(\frac{\delta}{W}\right)\delta. \tag{82}$$

With the results obtained in Sections II and III, we can compare the curves of attenuation as a function of frequency for conventional and laminated lines. Let us consider a transmission line such as that shown in Fig. 1, where we may imagine the center conductor to be either laminated as shown or made of solid metal. Let us suppose as in the introduction, that most of the power loss is in the center conductor and that the distance between the stack and outer conductor is large compared to the depth of the stack. Clearly, the attenuation of the line will be proportional to the power per unit area flowing into the center conductor for a given power flow in the line. If H_z is the transverse magnetic field and E_x is the longitudinal electric field, the power flow into the center conductor per unit area will be given by

$$\tfrac{1}{2}|H_z| \cdot |E_x|\cos\Phi = \tfrac{1}{2}\text{ real part }(H_z \cdot E_z^*), \tag{83}$$

where Φ is the phase angle between H_s and E_z and (*) indicates the conjugate of a complex quantity. If C is the circumference of the center conductor, and Z is the characteristic impedance of the line, the attenuation of the line will be given by

$$\gamma = \frac{1}{2CZ \mid H_s \mid^2} \text{ real part } (H_s \cdot E_z^*). \tag{84}$$

First, let us suppose that the inner conductor is solid and that the frequency is very low. In that case, the uniform current density in the metal will be H_s/D, and therefore, E_z will be $H_s/\sigma D$. Hence, for this case the attenuation will be

$$\gamma_s \, (f \text{ small}) = \frac{1}{2CZ\sigma D} \cdot \tag{85}$$

Similarly, the attenuation of the line when the center conductor is laminated will be for very low frequencies

$$\gamma_w \, (f \text{ small}) = \frac{1}{2CZ\bar{\sigma}D} \cdot \tag{86}$$

Next, let us consider the solid conductor again, but for frequencies where $\delta \ll D$. Then we have from (16) and (22)

$$E_z = - \frac{1+i}{\sigma\delta} H_z. \tag{87}$$

Hence,

$$\gamma_s = \frac{1}{2CZ\sigma\delta} \cdot \tag{88}$$

Finally, we desire the attenuation γ_w of the laminated line at elevated frequencies. Therefore we need the relation between E_z and H_z at the first surface of the stack which, for definiteness, we can take to be a metal lamina. For a sufficiently deep stack, this ratio is the same at each successive corresponding point. Referring to (45) and (48), we wish to find

$$R = \frac{E_0}{H_0} = \frac{E_2}{H_2} \cdot \tag{89}$$

By eliminating among these equations and using the same approximations made previously, we obtain

$$R = \frac{1}{\sigma W} \frac{\eta W}{\sinh \eta W} \frac{1 - \beta \cosh \eta W}{\beta} \cdot \tag{90}$$

If use is now made of the relation $\beta = e^{\alpha_w(W+t)}$, one finds to first order

$$R = - \frac{\alpha_w}{\bar{\sigma}} \cdot \tag{91}$$

Therefore, for the attenuation one has,

$$\gamma_w = \frac{1}{2CZ\bar{\sigma}} \text{ real part } (\alpha_w^*),$$

$$= \frac{1}{2CZ\bar{\sigma}\delta_w} \tag{92}$$

which is equivalent to an expression used in the introduction with $1/2cZ$ being the value of the constant A.

We have compared the attenuation of conventional and laminated lines as a function of frequency for an insulator between inner and outer conductors having the critical dielectric constant $\epsilon_1 = \bar{\epsilon}$. It is also of interest to draw the comparison as a function of dielectric constant ϵ_1 at a fixed frequency. The ratio of the two attenuations will be

$$\frac{\gamma_w}{\gamma_s} = \frac{\sigma\delta}{\bar{\sigma}\delta_w} \cdot \tag{93}$$

This curve is drawn in Fig. 6 for $W/\delta = 1/3$ and $t/W = 1$.
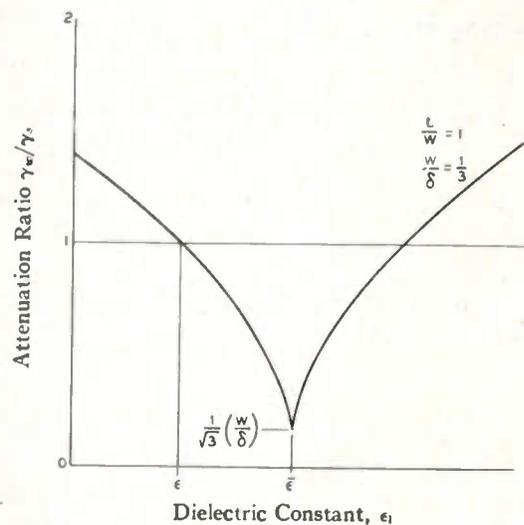
Fig. 6—Relative attenuation as a function of dielectric constant of material between stack and outer conductor.

For $\epsilon_1 = \bar{\epsilon}$ we have

$$\frac{\gamma_w}{\gamma_s} = \frac{1}{\sqrt{3}} \left(\frac{W}{\delta} \right). \tag{94}$$

It will also be observed that for $\epsilon_1 = \epsilon$

$$\frac{\gamma_w}{\gamma_s} = 1. \tag{95}$$

IV. Transmission Line with Laminated Conductors

In the preceding sections, we have considered the case of a transmission line with a depth of laminations small compared to the spacing of the conductors, yet large compared to the effective skin depth δ_w. In the following sections, we will deal with several situations in which the stacks are much smaller in depth than δ_w. The size of the stack, then, reflected in the imaginary part of the propagation constant k has more effect on α than anything else and we may consider W/δ and, all the more, σ_v to be zero.

Under these conditions, we shall calculate the attenuation of the parallel plane transmission line shown in

Fig. 7. In that figure we have two parallel plates or shields of conductivity σ separated a distance $2d$. Inside each plate there is a thickness s of laminated conductor of average conductivity $\bar{\sigma}$ and average transverse di-
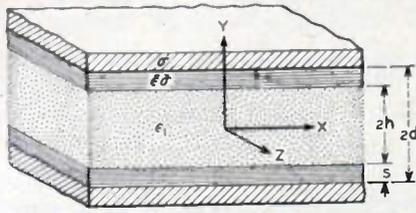
Fig. 7—Plane-parallel transmission line with laminated conductor.

electric constant $\bar{\epsilon}$. The interior of the line is filled with dielectric of thickness $2h = 2(d-s)$ and having a di-electric constant ϵ_1. The calculations to be made will be valid down to some low frequency at which the skin depth in outer shields becomes equal to their thickness.

With reference to (15) and (16), we can write down the following expressions for the fields in the various parts of the line:

in shield:

$$H_z = A e^{-\eta(y-d)} \tag{96}$$

$$E_z = -A \frac{\eta}{\sigma} e^{-\eta(y-d)} \tag{97}$$

$$\eta = \sqrt{i\omega\mu_0\sigma}; \tag{98}$$

in laminae:

$$H_z = B \cosh \zeta(y-d) + C \sinh \zeta(y-d) \tag{99}$$

$$E_z = \frac{\zeta}{\bar{\sigma}} \left[B \sinh \zeta(y-d) + C \cosh \zeta(y-d) \right] \tag{100}$$

$$\zeta = \sqrt{\frac{\bar{\sigma}}{i\omega\bar{\epsilon}} (k^2 - \omega^2\mu_0\bar{\epsilon})}; \tag{101}$$

in dielectric:

$$H_z = \cosh \xi y \tag{102}$$

$$E_z = \frac{\xi}{i\omega\epsilon_1} \sinh \xi y \tag{103}$$

$$\xi = \sqrt{k^2 - \omega^2\mu_0\epsilon_1}, \tag{104}$$

where A, B, and C are constants.

The fields H_z and E_z must match at the boundaries $y = h$ and $y = d$. Imposing these conditions, we find the characteristic equation for determining k to be

$$\tanh \zeta s = -\frac{1 + \left(\frac{\sigma}{i\omega\epsilon_1}\right) \frac{\xi}{\eta} \tanh \xi h}{\left(\frac{\sigma}{i\omega\epsilon_1}\right)\left(\frac{\bar{\sigma}}{\sigma}\right)\frac{\xi}{\zeta} \tanh \xi h + \left(\frac{\sigma}{\bar{\sigma}}\right)\frac{\zeta}{\eta}} \tag{105}$$

The constants are also determined to be

$$B = \frac{\cosh \xi h}{\cosh \zeta s + \frac{\bar{\sigma}}{\sigma} \frac{\eta}{\zeta} \sinh \zeta s} \tag{106}$$

$$A = B$$
$$C = -\frac{\bar{\sigma}}{\sigma} \frac{\eta}{\zeta} B. \tag{107}$$

Just as before, it is obvious from (101) that k must be nearly equal to $\omega^2\mu_0\bar{\epsilon}$ if the fields are to penetrate deeply into the laminations. Let us guess on physical grounds that this can be accomplished by setting $\epsilon_1 = \bar{\epsilon}$. Under these conditions from (104) and (101)

$$\xi = \sqrt{\frac{i\omega\bar{\epsilon}}{\bar{\sigma}}} \zeta \tag{108}$$

and

$$\xi h = \sqrt{\frac{i\omega\bar{\epsilon}}{\bar{\sigma}}} \left(\frac{h}{s}\right) (\zeta s).$$

Thus, if (ζs) is not to be very large and (h/s) is not greater than 100, ξh will be a very small number. We can therefore set $\tanh \xi h = \xi h$ and rewrite (105) as

$$(\zeta s) \tanh (\zeta s) + \frac{\left(\frac{h}{s}\right)(\zeta s)^2 + \left(\frac{\bar{\sigma}}{\sigma}\right)\eta s}{1 + \left(\frac{\bar{\sigma}}{\sigma}\right)\eta h} = 0. \tag{109}$$

From (98), $\eta h = (1+i)h/\delta$. We shall imagine that h is many times greater than the skin depth in the outer shield and may therefore reduce (109) to

$$\theta \tanh \theta + \frac{1-i}{2}\left(\frac{\delta}{s}\right)\left(\frac{\sigma}{\bar{\sigma}}\right)\theta^2 + \frac{s}{h} = 0, \tag{110}$$

where (ζs) has been set equal to θ. Now, if s is also many times the skin depth in the outer conductor, the second term in (110) may be neglected compared to the first term. We have then, finally, for the characteristic equation:

$$\theta \tanh \theta + \frac{s}{h} = 0. \tag{111}$$

For s much smaller than h, approximate solutions of (111) may be written

$$\theta_0^2 = -\frac{s}{h}, \tag{112}$$

and

$$\theta_n = in\pi \left[1 + \frac{s}{h} \frac{1}{(n\pi)^2} \right] \quad n = 1, 2, 3, \cdots. \tag{113}$$

The fundamental solution (112) obviously agrees with our assumption that (ζs) is not large. Similarly, the higher-order solutions (113) are consistent with that assumption if n is not taken too large.

We may now return to (101) and obtain approximate expressions for k.

$$k_0 = \sqrt{\omega^2\mu_0\bar{\epsilon}}\left[1 + \frac{1}{i\omega\mu_0\bar{\sigma}}\frac{1}{2sh}\right] \qquad (114)$$

$$k_n = \sqrt{\omega^2\mu_0\bar{\epsilon}}\left(1 + \frac{1}{i\omega\mu_0\bar{\sigma}}\left[\frac{1}{sh} + \frac{1}{2}\left(\frac{n\pi}{s}\right)^2\right]\right)$$
$$n = 1, 2, 3, \cdots . \quad (115)$$

We see that k^2 is indeed approximately equal to $\omega^2\mu_0\bar{\epsilon}$. The imaginary parts of (114) and (115) are negative and give us the desired attenuation for the fundamental and higher modes of the line in nepers per meter.

$$I(k_0) = -\frac{1}{2sh}\frac{1}{\bar{\sigma}}\sqrt{\frac{\bar{\epsilon}}{\mu_0}} \qquad (116)$$

$$I(k_n) = -\frac{1}{2sh}\left[2 + \frac{h}{s}(n\pi)^2\right]\frac{1}{\bar{\sigma}}\sqrt{\frac{\bar{\epsilon}}{\mu_0}}$$
$$n = 1, 2, 3, \cdots \quad (117)$$

where we have assumed $\delta \ll s \ll h$ and $\epsilon_1 = \bar{\epsilon}$.

Let us first comment on the fact that there exist several modes of transmission in this line. The fundamental mode with propagation constant k_0 corresponds to the ordinary mode of transmission that would exist between a pair of parallel plates such as shown in Fig. 8. The higher modes are waves that are confined almost entirely to the laminations and are not encountered in an ordinary transmission line. These modes will be more fully discussed in Section VI.

For comparison with (116) and (117), we shall next calculate the attenuation of the parallel plate transmission line in Fig. 8. For the fields we have again:
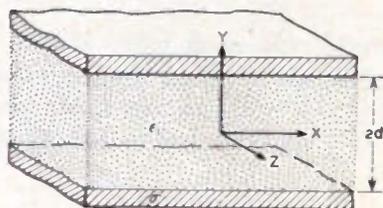
Fig. 8—Plane-parallel transmission line with solid conductors.

in shield:

$$H_z = Ae^{-\eta(y-d)} \qquad (118)$$

$$E_x = -A\frac{\eta}{\sigma}e^{-\eta(y-d)} \qquad (119)$$

$$\eta = \sqrt{i\omega\mu_0\sigma} \qquad (120)$$

in dielectric:

$$H_z = \cosh \xi y \qquad (121)$$

$$E_x = \frac{\xi}{i\omega\epsilon_1}\sinh \xi y \qquad (122)$$

$$\xi = \sqrt{k^2 - \omega^2\mu_0\epsilon_1}. \qquad (123)$$

By matching the fields at $y = d$ we readily obtain the characteristic equation

$$(\xi d) \tanh (\xi d) + \frac{i\omega\epsilon_1}{\sigma}\eta d = 0 \qquad (124)$$

which gives approximately

$$(\xi d)^2 = -\frac{i\omega\epsilon_1}{\sigma}\eta d. \qquad (125)$$

We also determine the constant A to be unity. Proceeding as before, we find for the propagation constant

$$k = \sqrt{\omega^2\mu_0\epsilon_1}\left[1 + \frac{1}{2d\sqrt{i\omega\mu_0\sigma}}\right], \qquad (126)$$

and for the attenuation.

$$I(k) = -\frac{1}{d}\sqrt{\frac{1}{8}\frac{\omega\epsilon_1}{\sigma}}. \qquad (127)$$

It is observed at once that the attenuation expressed in (116) is independent of frequency, while that given in (127) increases as the square root of the frequency. If we take the ratio of equation (127) to (116), we obtain

$$\frac{\text{attenuation of regular line}}{\text{attenuation of laminated line}}$$

$$= \left(\frac{h}{d}\right)\left(\frac{\bar{\sigma}}{\sigma}\right)s\sqrt{\frac{\omega\mu_0\sigma}{2}} \qquad (128)$$

$$= \left(\frac{h}{d}\right)\left(\frac{\bar{\sigma}}{\sigma}\right)\left(\frac{s}{\delta}\right), \qquad (129)$$

which is, of course, a thoroughly reasonable result. A sketch of the attenuation curves for the two lines is shown in Fig. 9.
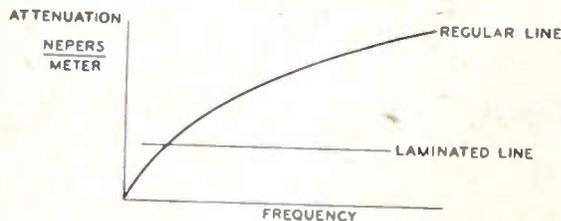
Fig. 9—Comparison of conventional and laminated lines.

As a final point, we observe that the attenuation given in (116) is proportional to $\sqrt{\bar{\epsilon}}/\bar{\sigma}$. Therefore, for stacks in which the laminations have the dimensions shown in Fig. 5, the attenuation is seen from (26) and (27) to be proportional to

$$\left(1 + \frac{t}{W}\right)\sqrt{1 + \frac{W}{t}}.$$

This expression has a minimum for $W/t = 2$. Thus, the optimum arrangement is for the conductor to be twice the thickness of the dielectric. This rule holds, of course,

only as long as the effective skin depth defined in (80) is greater than s. If use of the ratio $t/W = 1/2$ finds δ_w smaller than s, the best thing to do is to increase t/W until the effective skin depth δ_w becomes equal to s.

V. Transmission Line Filled with Laminations

In the last section, we have calculated the attenuation of the transmission line shown in Fig. 7. By reference to (116), it is seen that this attenuation decreases as s increases. Since we have assumed in deducing equation (116) that $s \ll h$, we cannot use that equation to find the attenuation for $s = d$. Nevertheless, the suggestion is evident that this case may be particularly interesting.

Accordingly, let us consider the transmission line in Fig. 10, where the space between the outer shields has been completely filled with laminations. As before, we can write down the following fields:

in shield:
$$H_z = A e^{-\eta(v-d)} \tag{130}$$

$$E_z = -A \frac{\eta}{\sigma} e^{-\eta(v-d)} \tag{131}$$

$$\eta = \sqrt{i\omega\mu_0\sigma}; \tag{132}$$

in laminae:
$$H_z = \cosh \zeta y \tag{133}$$

$$E_z = \frac{\zeta}{\bar{\sigma}} \sinh \zeta y, \tag{134}$$

where A is some constant. Matching fields at $y = d$, we obtain the characteristic equation

$$(\zeta d) \tanh (\zeta d) = -\frac{\bar{\sigma}}{\sigma} \eta d. \tag{135}$$

We might also have obtained this equation by placing $h = 0$ and $s = d$ in (105).
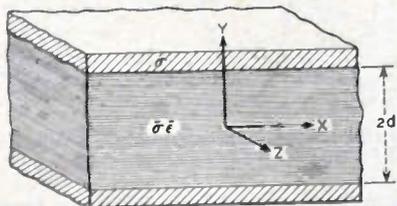
Fig. 10—Plane-parallel transmission line filled with laminations.

We can verify that an approximate solution of (135) is

$$(\zeta d) = \frac{n\pi i}{2}\left[1 - \frac{\sigma}{\bar{\sigma}} \frac{1}{\eta d}\right] \quad n = 1, 3, 5, \cdots . \tag{136}$$

Proceeding as before, we find for the propagation constant

$$k_n = \sqrt{\omega^2 \mu_0 \bar{\epsilon}}\left[1 + \frac{1}{2i\omega\mu_0\bar{\sigma}}\left(\frac{n\pi}{2d}\right)^2\right], \tag{137}$$

and for the attenuation

$$I(k_n) = -\frac{1}{2\bar{\sigma}} \sqrt{\frac{\bar{\epsilon}}{\mu_0}} \left(\frac{n\pi}{2d}\right)^2. \tag{138}$$

If we place $n = 1$ in (138) and compare the result with (116) we see that the attenuation of the transmission line has been indeed decreased by completely filling it with laminations, and without sacrifice of the frequency independent characteristic. Furthermore, it is no longer necessary to supply a dielectric with dielectric constant equal to $\bar{\epsilon}$. This case clearly represents something unfamiliar in the way of transmission lines. We might in fact consider the laminated material as a new kind of transmission medium.

In order to visualize this situation more completely, let us study the distribution of fields and currents inside the transmission line. We will be interested in H_z, E_v which can be obtained from (17) and (137), the current density $J = \bar{\sigma} E_z$, the Poynting vector $P = 1/2$ real part $(E_v H_z^*)$, the total current $I = |\int_0^d J\, dy|$, and the total power $W = \int_{-d}^d P\, dy$. From (136), we can take ζ equal approximately to $(n\pi i/2d)$ and obtain

$$H_z = \cos \frac{n\pi y}{2d} \tag{139}$$

$$E_v = \sqrt{\mu_0/\bar{\epsilon}} \cos \frac{n\pi y}{2d} \tag{140}$$

$$J = -\left(\frac{n\pi}{2d}\right) \sin \frac{n\pi y}{2d} \tag{141}$$

$$P = 1/2\sqrt{\mu_0/\bar{\epsilon}} \cos \frac{n\pi y}{2d} \tag{142}$$

$$I = 1 \tag{143}$$

$$W = 1/2\sqrt{\mu_0/\bar{\epsilon}}\, d. \tag{144}$$

For comparison let us write down these same quantities for the transmission line of Fig. 8. This can be done in an obvious way from (118) to (123) and by use of the characteristic equation (125). We have then approximately,

$$H_z = \frac{1}{\sqrt{2}} \tag{145}$$

$$E_v = \frac{1}{\sqrt{2}} \sqrt{\frac{\mu_0}{\epsilon_1}} \tag{146}$$

$$P = \frac{1}{2\sqrt{2}} \sqrt{\frac{\mu_0}{\epsilon_1}} \tag{147}$$

$$W = \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_1}}\, d, \tag{148}$$

and in the shield

$$J = -\frac{1}{\sqrt{2}} \eta e^{-\eta(v-d)} \tag{149}$$

$$I = \frac{1}{\sqrt{2}}. \tag{150}$$

If we let $\epsilon_1 = \bar{\epsilon}$, the second set of equations has been normalized to the same transmitted power as the first set. A comparison of these equations is shown in Fig. 11. The decreased attenuation of the laminated line is, of course, accounted for by its much smaller current density, even though its total current is bigger by a factor $\sqrt{2}$. Only the fundamental mode of the laminated line is considered in Fig. 11. The higher modes will be discussed in the next section.
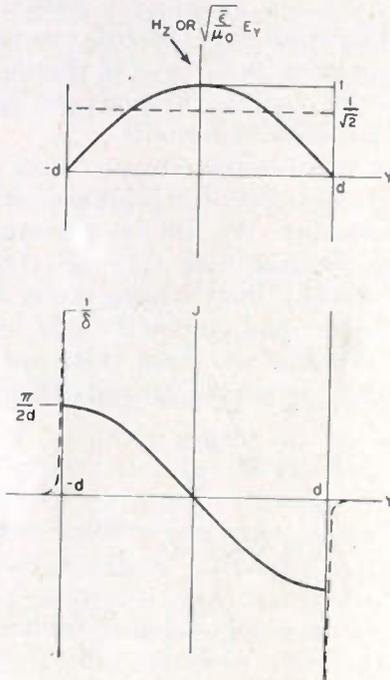
Fig. 11—Distribution of fields and current in transmission line filled with laminations.

VI. Modes of Transmission

We have seen that both the transmission line partly filled with laminations as in Section IV, and the completely filled line described in Section V, have fundamental and higher modes of transmission. Let us now examine this situation more closely.

First, in the case of the partially filled line, we can use the results of Section IV to find the following approximate expressions for the current density and magnetic field in the lamina:

$$J_0 = -\frac{1}{s} \tag{151}$$

$$J_n = (-1)^{n+1} \left(\frac{h}{s}\right)(n\pi)^2 \frac{1}{s} \cos\frac{n\pi}{s}(y-d) \tag{152}$$

$$(H_z)_0 = -\frac{1}{s}(y-d) \tag{153}$$

$$(H_z)_n = (-1)^{n+1} \left(\frac{h}{s}\right)n\pi$$
$$\cdot \sin\left[\left(n\pi + \frac{s}{h}\frac{1}{n\pi}\right)\frac{y-d}{s}\right]. \tag{154}$$

The current densities for $n = 0, 1, 2$ are shown in Fig. 12. We see that the current density for $n = 0$ is all of one phase, while for the higher modes the current density has one or more reversals of phase. For these higher modes, the net current in the laminae is essentially
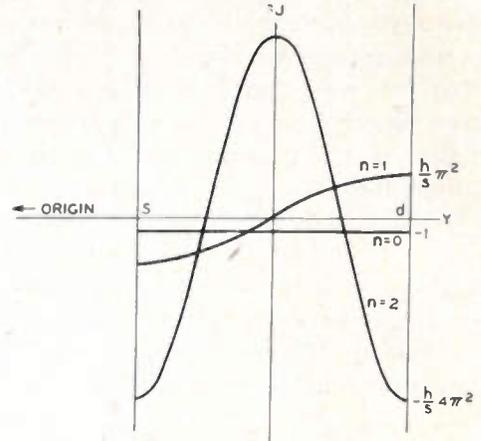
Fig. 12—Distribution of current in laminae of partially filled line for various modes.

zero, and consequently we should expect only small fields in the interior of the line. This supposition is confirmed in Fig. 13, where the magnetic fields in the laminae are drawn for $n = 0, 1, 2$. The fields for all the modes have a common value unity at $y = s$, but for the higher modes the fields in the interior of the laminae are much greater than unity.
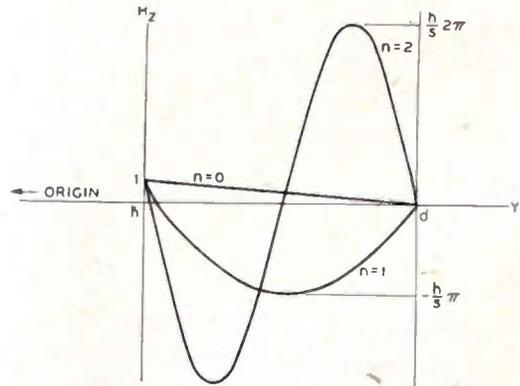
Fig. 13—Distribution of magnetic field in laminae of partially filled line for various modes.

For the completely filled line, we have from (141)

$$J_n = -\frac{n\pi}{2d}\sin\frac{n\pi y}{2d}. \tag{155}$$

These current densities are drawn in Fig. 14 for $n = 1$ and $n = 3$. It will be observed that in each case a net current flows in one sense on one side of the center line and in the opposite sense on the other side of the line.

We can now profitably compare the modes for the two types of lines. Clearly there should be a one-to-one correspondence of the modes, since the partially filled line can be made to approach the completely filled line continuously by adding more laminated material. This

correspondence is shown schematically in Fig. 15. The modes we have discussed to this point have all been anti-symmetric in current density about the center plane. It will be clear from Fig. 15 that there are, in addition, another set of modes symmetric in the current density.
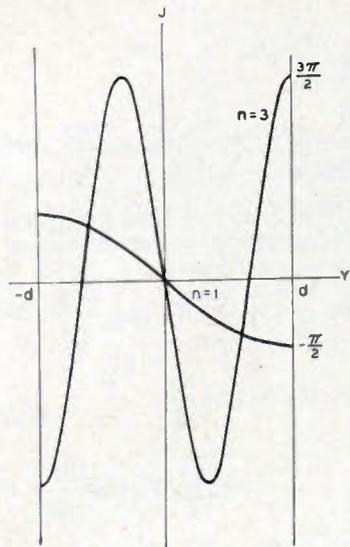
Fig. 14—Current distribution in completely filled line for various modes.

For the completely filled case, these are the modes $n = 2, 4, 6, \cdots$, and for the partially filled case the modes $n = 1', 2', 3', \cdots$.

(a)

(b)

(c)

(d)

(e)
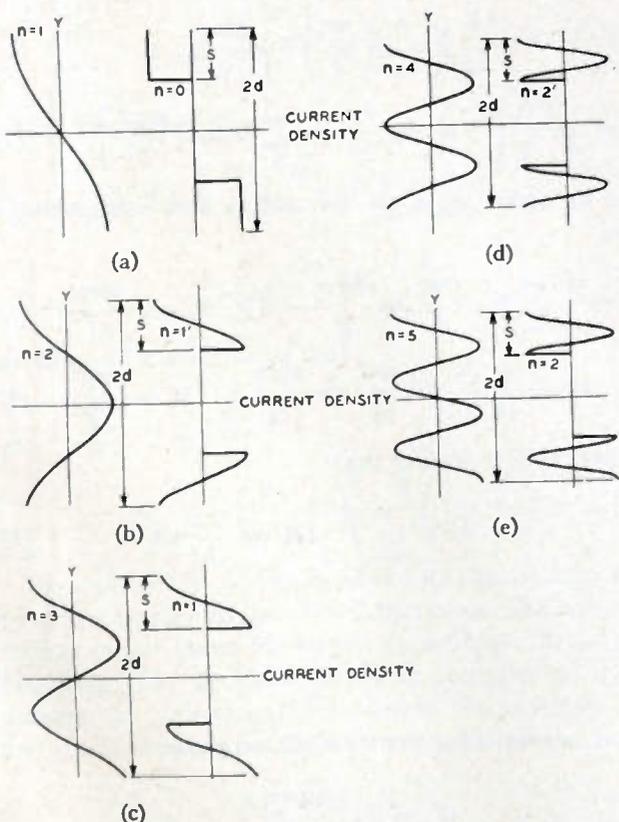
Fig. 15—Correspondence of modes between partially filled line and wholly filled line.

An important point can now be made. For the completely filled case, there are higher modes such as $n = 3$

where a net current flows on one side of the center plane. The corresponding mode however for the partially filled case with $s \ll h$ has nearly zero net current on either side of the center plane. Thus, for the partially filled case with s much smaller than h there are no modes except the fundamental with large fields in the interior of the line.

VII. Termination of a Laminated Line

The discussion of modes of transmission in the last two sections enables us now to consider what occurs at the junction of an unlaminated transmission line, and one partially or completely filled with laminated material. We will for simplicity consider mainly the case of the completely filled line, as shown in Fig. 16. To the left of $x = 0$ there is an unlaminated line such as shown in Fig. 8 filled with a dielectric of dielectric constant ϵ_1. To the right of $x = 0$ there is a line of the type con-
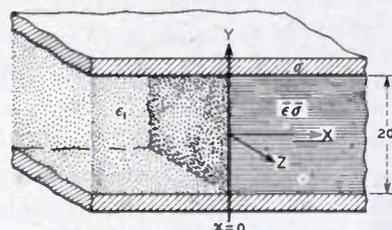
Fig. 16—Junction between two plane-parallel transmission lines, one of which is filled with laminations.

sidered in Section V, filled with laminated material of average transverse dielectric constant $\bar{\epsilon}$ and average longitudinal conductivity $\bar{\sigma}$. We shall consider separately what happens to a wave incident upon the boundary from left or right.

When expressing the fields in the unlaminated line, we shall have to include certain unpropagated modes which have not yet been discussed. These modes must attenuate to the left, and can be written

$$H_z = \cos \frac{n\pi y}{d} e^{(n\pi x)/d} \tag{156}$$

$$E_z = -\frac{1}{i\omega\epsilon_1}\left(\frac{n\pi}{d}\right)\sin \frac{n\pi y}{d} e^{(n\pi x)/d} \quad n = 1, 2, 3, \cdots \tag{157}$$

$$E_y = -\frac{1}{i\omega\epsilon_1}\left(\frac{n\pi}{d}\right)\cos \frac{n\pi y}{d} e^{(n\pi x)/d}. \tag{158}$$

It has been assumed that the wavelength in the dielectric of the frequency under consideration is much greater than d.

We shall first consider a wave incident upon the boundary from the left. From (145), (146), (156), and (158) we have for $x < 0$

$$H_z = Ae^{-ikz} + Be^{ikz} + \sum_m C_m \cos \frac{m\pi y}{d} e^{(m\pi x)/d} \tag{159}$$

$$E_y = \sqrt{\frac{\mu_0}{\epsilon_1}} \left[Ae^{-ikz} - Be^{ikz}\right]$$

$$-\frac{1}{i\omega\epsilon_1}\sum_m C_m\left(\frac{m\pi}{d}\right)\cos\frac{m\pi y}{d}\, e^{(m\pi z)/d}, \tag{160}$$

where $m = 1, 2, 3\cdots$ and k is given by (126). For $x > 0$ we have from (139) and (140)

$$H_z = \sum_n D_n \cos\frac{n\pi y}{2d}\, e^{-ik_n z} \tag{161}$$

$$E_y = \sqrt{\frac{\mu_0}{\bar\epsilon}}\sum_n D_n \cos\frac{n\pi y}{2d}\, e^{-ikz}, \tag{162}$$

where $n = 1, 3, 5\cdots$ and k_n is given by (137). At $x = 0$ the boundary conditions give

$$A + B + \sum_m C_m \cos\frac{m\pi y}{d} = \sum_n D_n \cos\frac{n\pi y}{2d} \tag{163}$$

$$\sqrt{\frac{\mu_0}{\epsilon_1}}(A - B) - \frac{1}{i\omega\epsilon_1}\sum_m C_m\left(\frac{m\pi}{d}\right)\cos\frac{m\pi y}{d}$$

$$= \sqrt{\frac{\mu_0}{\bar\epsilon}}\sum_n D_n \cos\frac{n\pi y}{2d}. \tag{164}$$

If $\epsilon_1 = \bar\epsilon$, it can be seen from (163) and (164) that $B = C_m = 0$. Thus there is no reflected wave and no unpropagated waves are needed. Let us consider only this case. The coefficients D_n are determined by

$$A = \sum_n D_n \cos\frac{n\pi y}{2d}, \tag{165}$$

which yields

$$D_n = \frac{4}{n\pi}(-1)^{(n-1)/2}A. \tag{166}$$

Referring to (144) we have for the power flow in the transmitted wave,

$$W = \frac{d}{2}\sqrt{\frac{\mu_0}{\bar\epsilon}}\sum_n D_n{}^2 \tag{167}$$

$$= A^2\sqrt{\frac{\mu_0}{\bar\epsilon}}\frac{8d}{\pi^2}\sum_n\frac{1}{n^2}. \tag{168}$$

Let us now find the ratio of the power transmitted in the fundamental mode $n = 1$ to the total power transmitted, which is also the total incident power as can be checked from (167). We have

$$\frac{\text{power in fundamental}}{\text{total power}} = \frac{8}{\pi^2}. \tag{169}$$

Thus, in exciting the fundamental mode of the laminated line we have a power loss which can be expressed in db as

$$\text{db loss} = 10\log\frac{\pi^2}{8}$$

$$= 0.912. \tag{170}$$

Let us next consider a wave composed of the fundamental mode of the laminated line incident upon the boundary from the right. For $x < 0$ we have in this case

$$H_z = Be^{ikz} + \sum_m C_m \cos\frac{m\pi y}{d}\, e^{(m\pi z)/d} \tag{171}$$

$$E_y = -B\sqrt{\frac{\mu_0}{\epsilon_1}}\, e^{ikz}$$

$$-\frac{1}{i\omega\epsilon_1}\sum_m C_m\left(\frac{m\pi}{d}\right)\cos\frac{m\pi y}{d}\, e^{(m\pi z)/d}, \tag{172}$$

where again $m = 1, 2, 3,\cdots$ and k is given by (126). For $x > 0$

$$H_z = Me^{ik_1 z}\cos\frac{\pi y}{2d} + \sum_n N_n \cos\frac{n\pi y}{2d}\, e^{-ik_n z} \tag{173}$$

$$E_y = \sqrt{\frac{\mu_0}{\bar\epsilon}}\left[-Me^{ik_1 z}\cos\frac{\pi y}{2d}\right.$$

$$\left. + \sum_n N_n \cos\frac{n\pi y}{2d}\, e^{-ik_n z}\right] \tag{174}$$

with k_n given by (137) and $n = 1, 3, 5\cdots$. At $x = 0$,

$$B = \sum_m C_m \cos\frac{m\pi y}{d}$$

$$= M\cos\frac{\pi y}{2d} + \sum_n N_n \cos\frac{n\pi y}{2d} \tag{175}$$

$$-B - \frac{1}{ik}\sum_m C_m\left(\frac{m\pi}{d}\right)\cos\frac{m\pi y}{d}$$

$$= \sqrt{\frac{\epsilon_1}{\bar\epsilon}}\left[-M\cos\frac{\pi y}{2d} + \sum_n N_n \cos\frac{n\pi y}{2d}\right]. \tag{176}$$

Let us again let $\epsilon_1 = \bar\epsilon$. By adding and subtracting we find

$$-\frac{1}{ik}\sum_m C_m\frac{m\pi}{d}\cos\frac{m\pi y}{d} = 2\sum_n N_n \cos\frac{n\pi y}{2d} \tag{177}$$

$$2B + \frac{1}{ik}\sum_m C_m\frac{m\pi}{d}\cos\frac{m\pi y}{d} = 2M\cos\frac{\pi y}{2d}. \tag{178}$$

From (178) it is clear that

$$2B = \frac{1}{2d}\int_{-d}^{d} 2M\cos\frac{\pi y}{2d}\, dy \tag{179}$$

or

$$B = \frac{2}{\pi}M. \tag{180}$$

Thus, we have for the transmitted power

$$W_t = d\sqrt{\frac{\mu_0}{\epsilon_1}}B^2$$

$$= d\sqrt{\frac{\mu_0}{\epsilon_1}}\left(\frac{2}{\pi}\right)^2 M^2 \tag{181}$$

and for the incident power

$$W_i = \frac{d}{2} \sqrt{\frac{\mu_0}{\bar{\epsilon}}} \, M^2.$$

The ratio of transmitted power to incident power is therefore

$$\frac{W_t}{W_i} = \frac{8}{\pi^2}, \tag{182}$$

just as in (169). There is thus the same power loss in crossing the boundary in either direction. It is interesting to note, however, that in the second case the non-propagating modes in the unlaminated line are excited.

From (177), (178), and (180) we can find

$$\sum_n N_n \cos \frac{n\pi y}{2d} = M\left(\frac{2}{\pi} - \cos \frac{\pi y}{2d}\right), \tag{183}$$

and consequently

$$N_1 = M\left(\frac{8}{\pi^2} - 1\right), \tag{184}$$

and

$$N_n = M \frac{8}{\pi^2} \frac{1}{n} (-1)^{(n-1)/2}, \quad n \neq 1. \tag{185}$$

The reflected power is found as before to be for the fundamental mode of the laminated line

$$W_{r1} = \frac{d}{2} \sqrt{\frac{\mu_0}{\bar{\epsilon}}} \, M^2 \left(\frac{8}{\pi^2} - 1\right)^2 \tag{186}$$

and for the higher modes

$$\sum_{n \neq 1} W_{rn} = \frac{d}{2} \sqrt{\frac{\mu_0}{\bar{\epsilon}}} \, M^2 \sum_{n \neq 1} \left(\frac{8}{n\pi^2}\right)^2,$$

$$= \frac{d}{2} \sqrt{\frac{\mu_0}{\bar{\epsilon}}} \, M^2 \frac{8}{\pi^2}\left(1 - \frac{8}{\pi^2}\right). \tag{187}$$

We can now easily check that

$$\frac{W_t}{W_i} + \frac{W_{r1}}{W_i} + \frac{1}{W_i}\sum_{n \neq 1} W_{rn}$$

$$= \frac{8}{\pi^2} + \left(\frac{8}{\pi^2} - 1\right)^2 + \frac{8}{\pi^2}\left(1 - \frac{8}{\pi^2}\right) = 1. \tag{188}$$

The case of the partially filled line can be studied in a manner similar to the above discussion, and will show smaller power losses for waves transmitted through the boundary. The problem is further complicated by the presence of unpropagated modes in the partially filled line similar to those in the unlaminated line.

APPENDIX A
PLANE WAVES

It is interesting to inquire about the waves that exist in a laminated medium of infinite extent. Let us return to (30). It is easy to show that α_0 is zero for k given approximately by

$$k = \sqrt{\omega^2 \mu_0 \bar{\epsilon}} \left[1 - i \, \frac{1}{2} \, \frac{W}{t}\left(\frac{\omega\epsilon}{\sigma}\right)\right]. \tag{189}$$

For $\alpha_0 = 0$, E_z is of course zero. We thus have a plane wave propagating through the medium with a wavelength appropriate to the average dielectric constant $\bar{\epsilon}$ and with a very small attenuation proportional to the square of the frequency. Equation (189) can be obtained from equation (45) in Sakurai's paper.[2]

If we wish next to observe the effect of finite thickness laminations, we can require α_w to be zero in (72). In this case we obtain

$$k = \sqrt{\omega^2 \mu_0 \bar{\epsilon}} \left[1 - i \, \frac{1}{12} \, \frac{W}{W + t}\left(\frac{W}{\delta}\right)^2\right]. \tag{190}$$

Again, the attenuation is proportional to the square of the frequency. The attenuation given by (190) is equal to that obtained from (189), if

$$W = \frac{1}{\sigma} \sqrt{12\left(1 + \frac{W}{t}\right)\frac{\epsilon}{\mu_0}}. \tag{191}$$

For copper, (191) requires W to be of the order of 10^{-8} cm. Under ordinary circumstances, therefore, the attenuation given in (189) is much smaller than that obtained when consideration is given to the finite thickness of the laminations.

APPENDIX B
TRANSMISSION LINE FILLED WITH LAMINATIONS OF FINITE WIDTH

Let us consider the transmission line shown in Fig. 17. As before we have a set of metal laminae of width W and conductivity σ separated by insulating laminae of
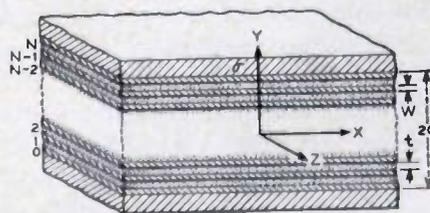
Fig. 17—Transmission line completely filled with laminations.

width t and dielectric constant ϵ. The laminae will be numbered as shown in the figure from 1 to N. Let us define r and p by the relations

$$r = \frac{\beta - T_{11}}{T_{12}}; \qquad p = \frac{1/\beta - T_{11}}{T_{12}}. \tag{192}$$

[2] See page 398 of footnote reference 1.

Then, from (57) and (58) we can write for the z component of magnetic field and the x component of electric field,

$$H_n = A\beta^n + B\beta^{-n} \tag{193}$$

$$E_n = -rA\beta^n + pB\beta^{-n}. \tag{194}$$

From (16) and (22) we clearly have

$$\frac{E_N}{H_N} = -R; \qquad \frac{E_0}{H_0} = R, \tag{195}$$

where $R = \alpha_s/\sigma$ and $\alpha_s = (1+i)/\delta$. It is easy now to obtain the characteristic equation

$$\begin{vmatrix} R\beta^N & R\beta^{-N} & \beta^N & \beta^{-N} \\ R & R & -1 & -1 \\ r & 0 & -1 & 0 \\ 0 & p & 0 & -1 \end{vmatrix} = 0. \tag{196}$$

After expansion and use of expressions (192), the characteristic equation is found to take the form

$$\coth (N \ln \beta) = \frac{R^2 T_{12} + T_{21}}{R\left(\dfrac{1}{\beta} - \beta\right)}, \tag{197}$$

which can be written, using the relation $\ln \beta = \alpha_w(W+t)$,

$$2 \sinh \alpha_w(W + t) \coth (2d\alpha_w) = RT_{12} + \frac{1}{R} T_{21}. \tag{198}$$

Let us now assume as in Section III that $|\xi t|$ and $|\eta W|$ are much smaller than unity. Equation (197) can then be written

$$(2\alpha_w d) \coth (2\alpha_w d)$$
$$= -\left(\frac{W}{W + t}\right)(\alpha_s d)\left(2(P + 1)\right.$$
$$\left. + i\frac{2}{3}(3P + 1)\left(\frac{W}{\delta}\right)^2\right). \tag{199}$$

The quantity on the right of (199) is of the order of d/δ. We write, therefore,

$$(2\alpha_w d) \coth (2\alpha_w d) = 0\left(\frac{d}{\delta}\right), \tag{200}$$

and

$$(2\alpha_w d) = \pi i \left[1 + 0\left(\frac{\delta}{d}\right)\right]. \tag{201}$$

Thus we have approximately

$$\cosh \alpha_w(W + t)$$
$$= 1 - \frac{1}{2}\left(\frac{\pi(W + t)}{2d}\left[1 + 0\left(\frac{\delta}{d}\right)\right]\right)^2. \tag{202}$$

Furthermore, from (72)

$$\cosh \alpha_w(W + t)$$
$$= 1 - \frac{1 + 4P}{6}\left(\frac{W}{\delta}\right)^4 + i(1 + 2P)\left(\frac{W}{\delta}\right)^2. \tag{203}$$

We can now equate (202) and (203), and after suitable manipulation obtain

$$k = \sqrt{\omega^2\mu_0\bar{\epsilon}}\left[1 - \frac{i}{2\left(1 + \dfrac{t}{W}\right)}\left[\frac{\pi^2}{8}\left(1 + \frac{t}{W}\right)^2\left(\frac{\delta}{d}\right)^2\right.\right.$$
$$\left.\left. + \frac{1}{6}\left(\frac{W}{\delta}\right)^2\right]\right], \tag{204}$$

where we have now dropped a term of order (δ/d) compared to unity.

Equation (204) is of considerable interest. It is observed that for $(W/\delta) \ll (\delta/d)$ the attenuation becomes that given in (137) for infinitely thin laminae. For $(W/\delta) \gg (\delta/d)$, on the other hand, the attenuation approaches that given in (190) for an unbounded array of finite laminae. This can be considered in two ways. Let us ask for the condition that the two terms contributing to the attenuation in (204) are equal. We find for this to be true that

$$d = \frac{\pi}{2}\sqrt{3}\left(1 + \frac{t}{W}\right)\left(\frac{\delta}{W}\right)\delta. \tag{205}$$

In other words, at a given frequency, the attenuation of the line can be little reduced by making d larger than the value given in (205) which will be recognized as approximately the skin depth given in (82).

Alternatively, we see that the attenuation as a function of frequency remains constant from low frequencies up to the point where δ satisfies equation (205). At higher frequencies, the attenuation increases parabolically. At frequencies where $\delta \ll W$, the attenuation will clearly correspond to propagation in a parallel-plate transmission line of width t, and will therefore increase with the square root of the frequency. This behavior is similar to that discussed in Section I for a line with a thin stack of laminations on its inner conductor.

A Simple Crystal Discriminator for FM Oscillator Stabilization*

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Summary—A crystal discriminator employing only one quartz crystal and very simple in adjustment is described in detail. The stability of center frequency determination in the presence of frequency modulation and variation of component values is calculated in terms of the circuit parameters, and is shown to be adequate for the requirements of broadcast transmitters. Experimental measurements confirm the results of the theoretical analysis.

The discriminator was designed for use in a television aural transmitter, and a brief description is given of the complete FM exciter unit.

Introduction

A FREQUENCY-MODULATED voltage of low noise and distortion content can readily be generated by means of a feedback-type oscillator and a reactance tube. The center frequency stability of such a generator is, however, entirely inadequate when it is employed as the modulator of an FM broadcast transmitter, and some form of center frequency stabilization is necessary. The frequency stabilizing system will, in general,[1,2] consist of a circuit for measuring the center frequency and a device for correcting any error detected by the measuring circuit. In one such system[3,4] the measuring circuit comprises a frequency discriminating network for which the measurement accuracy is determined primarily by one or more quartz crystals. This network, usually called a "crystal discriminator," can be designed to produce a dc output voltage which is a measure of the input frequency. The dc voltage can then be used to control the FM oscillator frequency by means of a reactance tube, as shown in the block diagram in Fig. 1. This provides an attractively simple system of center frequency stabilization.

Basic Discriminator Circuit

The crystal discriminator described in this paper makes use of the fact that the impedance of a quartz crystal changes very rapidly with change of frequency in the vicinity of the resonant frequency of the crystal. A measurement of frequency is thus obtained by comparing the crystal impedance with a reference imped-
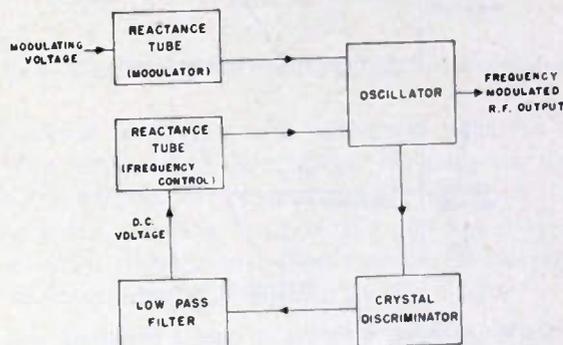
Fig. 1—Block diagram of stabilized FM oscillator.

ance which remains substantially constant over the range of measurement. A circuit for making this impedance comparison is shown in Fig. 2. The radio-frequency
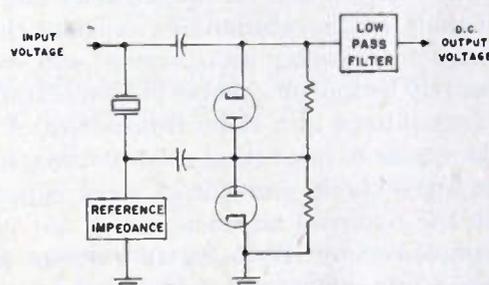
Fig. 2—Basic discriminator circuit.

input voltage of unknown frequency is applied to the crystal and reference impedance in series and the voltages appearing across these two components are rectified separately. The outputs of the two rectifiers are added in opposite polarity and filtered to produce a dc output voltage which is determined by the difference in magnitude of the crystal impedance and reference impedance.

The manner in which the impedance of a quartz crystal varies with frequency is well known[5] and is shown in Fig. 3(a). The impedance increases rapidly from a minimum of a few ohms at the crystal resonant frequency to a maximum of several megohms at the antiresonant frequency. It is evident that if the reference impedance is given a value approximately midway between the resonant and antiresonant impedances of the crystal, then the dc output voltage-input frequency curve for the discriminator will be of the general form shown in Fig. 3(b). This is the familiar "S curve" of a frequency discriminator, and it is usually convenient to arrange the

[1] N. Marchand, "Direct FM frequency control methods," *Communications*, vol. 26, pp. 30–35; July, 1946.
[2] R. R. Batcher, "FM systems engineering," *Electronic Ind.*, vol. 5, pp. 75–77 and 130–132; April, 1946.
[3] Marconi Co. and H. R. Cantello, British Patent No. 561954; 1942.
[4] J. L. Hollis, "Simplified automatic stabilization of a frequency-modulated oscillator," Proc. I.R.E., vol. 36, pp. 1164–1171; September, 1948.
[5] W. P. Mason, "Electro-Mechanical Transducers and Wave Filters," D. Van Nostrand Co., Inc., New York, N. Y.; 1942.

circuit so that the assigned center frequency is at the middle of the curve where it intersects the zero voltage axis.
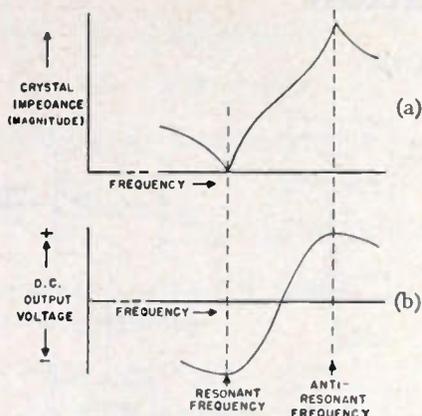
Fig. 3—Curves showing variation of crystal impedance and discriminator output voltage with frequency.

The indication of correct frequency, as given by zero dc output, is then independent of input voltage amplitude.

Before proceeding to a detailed analysis of the circuit, it is desirable to consider the nature of the reference impedance. The requirements for the reference impedance are that it shall remain substantially constant throughout the range of frequency measurement and shall not vary excessively because of changes in temperature, etc., normally encountered in a radio transmitter. A resistance would appear to meet these requirements, but owing to the unavoidable presence of stray capacity in shunt with the reference impedance, it is not possible to use a simple resistor. By using an inductor and resistor in series, the inductor can be tuned to resonate approximately with the stray capacity at the center frequency, and then the heavy damping provided by the series resistor makes the effective impedance a substantially constant resistance over the range of frequency measurement. In this way, it is readily possible to obtain a reference impedance which does not vary more than 0.5 per cent because of changes in conditions normally encountered in practice.

Equation for Discrimination Curve

The equivalent circuit of the discriminator is shown in Fig. 4. The crystal is represented by its equivalent
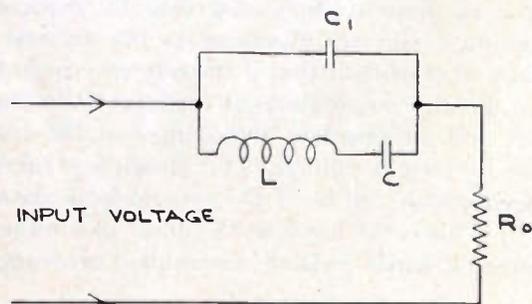
Fig. 4—Equivalent discriminator circuit.

electrical circuit LC and C_1 and the reference impedance is shown as the resistance R_0. The equivalent resistance of the crystal is neglected since it is small compared with the other impedances in the circuit, and experimental measurements show that it has no appreciable effect on the operation of the discriminator. The reference impedance is adjusted so that it is equal in magnitude to the crystal impedance at the assigned center frequency, and the dc output voltage is then zero. The crystal is chosen so that its resonant and antiresonant frequencies are an equal amount below and above the center frequency, respectively.

Let

F_0 = center frequency

$F_0 - f_1$ = resonant frequency of crystal

$F_0 + f_1$ = antiresonant frequency of crystal

$F_0 + f$ = input frequency

$\omega = 2\pi f$, $\omega_1 = 2\pi f_1$, $\omega_0 = 2\pi F_0$

Z = crystal impedance at frequency $F_0 + f$

V = dc output voltage at frequency $F_0 + f$

V_0 = a constant dc voltage.

Examination of the conditions at the resonant and antiresonant frequencies yields the following well-known equation:

$$\frac{C}{C_1} = \frac{4f_1}{F_0 - f_1}. \tag{1}$$

This shows that some adjustment of the value for f_1 can be obtained by varying the crystal shunt capacity C_1. In practice, the resonant frequency is determined when the crystal is ground and the antiresonant frequency is adjusted to the required value by varying C_1.

From Fig. 4, the crystal impedance Z can be calculated for any input frequency $F_0 + f$, i.e., for any frequency distant f from the center frequency. The frequency separation between the resonant and antiresonant frequencies of a crystal is always very small compared with the resonant frequency, and hence f_1 can be neglected in comparison with F_0. Provided f is also negligibly small compared with F_0, then the value of the crystal impedance is given by

$$Z = \frac{j}{\omega_0 C_1} \times \frac{\omega_1 + \omega}{\omega_1 - \omega}. \tag{2}$$

Since R_0 is equal to the crystal impedance at the center frequency when $\omega = 0$,

$$R_0 = \frac{1}{\omega_0 C_1}. \tag{3}$$

From equations (2) and (3), the voltages across the crystal and reference impedance can be evaluated, and since the difference between these two voltages is proportional to the dc output voltage V, then an expression can be obtained for V as given by (4), where V_0 is a constant dependent upon the amplitude of the input voltage and the rectifier characteristics.

$$V = V_0 \frac{2f}{\sqrt{2f_1^2 + 2f^2}}. \qquad (4)$$

This equation is true for values of f between $+f_1$ and $-f_1$ which covers the normal operating range of the discriminator.

The curve represented by (4) is shown by the broken line in Fig. 5, where suitable numerical values have been
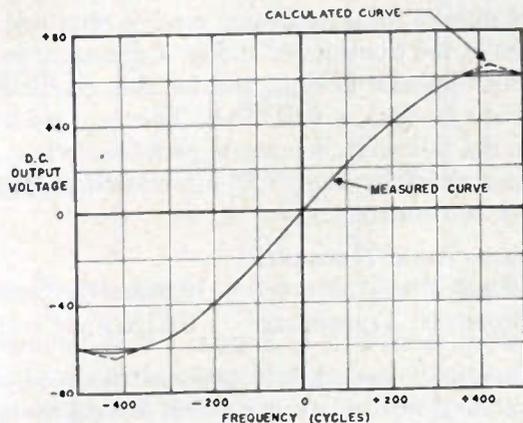
Fig. 5—Curves showing input frequency versus output voltage for discriminator. Center frequency = 299.65 kc.

assigned to the constants V_0 and f_1. The full line shows a curve obtained by measurement on an actual discriminator, and it is seen that there is close agreement between the two curves, except in the immediate vicinity of the resonant and antiresonant frequencies. It can be shown that the divergence at these points is due to the effective resistance of the crystal which was not taken into account for the calculated curve. The divergence is not important since these points are outside the normal range of deviation of the input frequency.

EVALUATION OF FREQUENCY ERRORS

A. *Error Due to Asymmetrical Modulation*

Since the curves of Fig. 5 are symmetrical about the center frequency, there will be no frequency measurement error introduced when frequency modulation of symmetrical waveform is applied to the input voltage. There will, however, be a measurement error due to nonlinearity of the curves when modulation of asymmetrical wave form is applied.

When the modulation frequency is very low, the discriminator input voltage at any instant can be considered to have a steady frequency of value equal to the actual instantaneous frequency. The maximum measurement error that can be produced can then be calculated by considering the effect of modulation by the extremely asymmetrical waveform shown in Fig. 6. It is apparent that when n is greater than unity, the positive part of the cycle extends further into the nonlinear portion of the curve than the negative part of the cycle, and the average dc output is negative. It is also apparent that if the maximum deviation ΔF is fixed and n is increased from unity to infinity, the average dc output
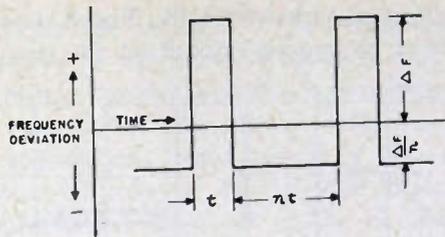
Fig. 6—Waveform of extremely asymmetrical modulation.

will increase from zero to a maximum negative value and then decrease to zero again. By substituting ΔF and $\Delta F/n$ for f in (4), values can be obtained for the output voltage during the positive and negative parts of the cycle, respectively. The average dc output can thus be determined and is given by V_{dc}, where

$$V_{dc} = \frac{V_0}{1+n} \left[\frac{2\Delta F}{\sqrt{2f_1^2 + 2\Delta F^2}} - \frac{2\Delta F}{\sqrt{2f_1^2 + 2\Delta F^2/n^2}} \right]. \qquad (5)$$

This dc voltage represents a frequency measurement error f_M, which can be evaluated by substituting f_M for f in (4)

$$V_{dc} = V_0 \frac{2f_M}{\sqrt{2f_1^2 + 2f_M^2}}. \qquad (6)$$

Equating the expressions for V_{dc} in (5) and (6) and neglecting f_M^2 in comparison with f_1^2,

$$\frac{f_M}{\Delta F} = \frac{f_1/\Delta F}{1+n} \left[\frac{1}{\sqrt{f_1^2/\Delta F^2 + 1}} - \frac{1}{\sqrt{f_1^2/\Delta F^2 + 1/n^2}} \right]. \qquad (7)$$

For any value of $f_1/\Delta F$, there will be a value of n for which $f_M/\Delta F$ is a maximum, and this value of n can be determined by differentiation or by graphical methods. By substituting these values of n in (7), the curve of Fig. 7 is obtained showing maximum values of $f_M/\Delta F$ in terms of $f_1/\Delta F$.
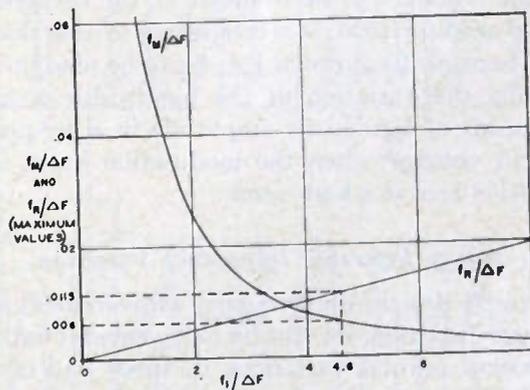
Fig. 7—Curves showing maximum values for $f_M/\Delta F$ and $f_R/\Delta F$ in terms of $f_1/\Delta F$.

To calculate the measurement error when the modulation frequency is not very low, it is necessary to take into account the extensive sideband spectrum of a frequency-modulated voltage. The computations become very complex and so the required results can more

easily be obtained experimentally. Fig. 8 shows the results obtained by measurements on the discriminator
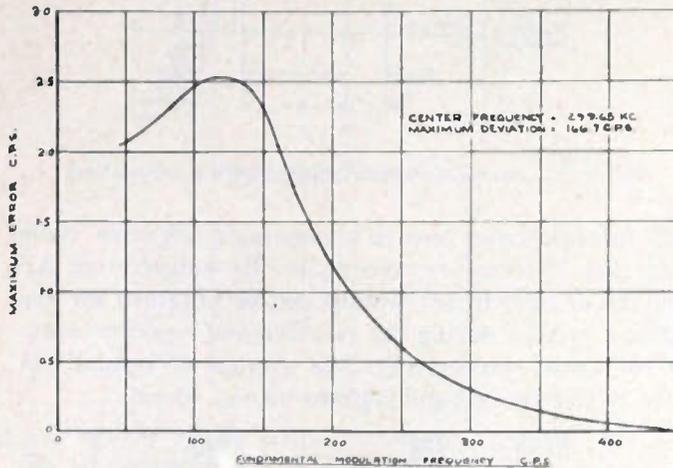
Fig. 8—Curve showing maximum frequency measurement error due to extremely asymmetrical modulation.

when the modulating wave form was of the extremely asymmetrical type in Fig. 6, and n was adjusted to give maximum frequency measurement error. For the conditions under which the tests were made, the theoretical error f_M for very low modulation frequencies was 2.9 cps, as determined from the curve of Fig. 7. It is seen that the actual error obtained at very low frequencies was about 2 cps and the error did not exceed 2.6 cps at any frequency. Hence it would seem that the frequency measurement error calculated for very low frequencies will not, in general, be exceeded in practice at any modulation frequency.

The somewhat lower experimental figure is probably due to the attenuation of the higher harmonics of the modulation frequency in the discriminator. This would cause the wave form of the modulation voltages appearing at the rectifiers to be rounded at the corners and hence less asymmetrical. It is interesting to note that the general shape of the curve of Fig. 8 can be predicted by considering the variation of the bandwidth occupied by sidebands of significant amplitude in a frequency-modulated voltage[6] when the modulation index is increased from zero to a high value.

B. *Error Due to Reference Impedance Variation*

By suitable design, the frequency measurement accuracy of the discriminator can be made substantially independent of normal variations in tubes and components other than the reference impedance. If the reference impedance changes, the crystal impedance must change an equal amount to re-establish a condition of zero dc output, and the frequency error represented by this impedance change can be evaluated from (2). Taking the previously suggested figure of 0.5 per cent as

[6] A Hund, "Frequency Modulation," McGraw-Hill Book Co., New York, N. Y., pp. 16–28; 1942.

being the maximum variation of the reference impedance, then the maximum frequency measurement error is given by f_R, where

$$\frac{f_R}{\Delta F} = 0.0025 \frac{f_1}{\Delta F}. \tag{8}$$

The curve represented by (8) is shown in Fig. 7 along with the curve for $f_M/\Delta F$.

The minimum total frequency error is obtained when the slopes of the two curves of Fig. 7 are equal in magnitude but opposite in sign, and for this condition the total error $(f_M + f_R)$ is $0.0175\Delta F$. This is considerably less than the following maximum permissible frequency errors prescribed by the FCC for television and FM broadcast transmitters:

Television Aural Transmitter
(channel 2) $0.048\Delta F(\Delta F = 25$ kc)
FM Broadcast Transmitter $0.027\Delta F(\Delta F = 75$ kc).

The margin is particularly good in the case of the television aural transmitter for which this discriminator was originally developed.

The figure calculated for the maximum frequency error is based upon a maximum reference impedance variation of 0.5 per cent, and actually a variation considerably less than this is readily obtainable with commercially available components. Even greater accuracy of frequency measurement can be obtained by increasing the stability of the reference impedance, such as by the use of temperature compensation or control.

CHOICE OF CRYSTAL FREQUENCY

Since the input voltage to the discriminator can be derived from the FM oscillator directly or by multiplication or division of frequency, the discriminator can be operated in any frequency range for which suitable crystals are available. A range of 236 to 360 kc was chosen, mainly because it was the highest convenient range for which the bar type of crystal could be obtained, and this type of crystal has several advantages, as compared with the plate type for this application.

Among these advantages are: (a) freedom from unwanted resonant modes close to the wanted mode is more easily obtained; (b) the ratio C/C_1 (1) can be made larger, thus facilitating the achievement of the required separation between the resonant and anti-resonant frequencies; and (c) the type of crystal required is similar to that used in filters for carrier telephony and the development of the bar type for this application has reached an advanced stage.

The discriminator crystal is specified by means of its resonant frequency which is independent of the type of circuit in which it is used. The specified resonant frequency is the required center frequency less f_1, and since there is some latitude in the choice of f_1, the accuracy of the crystal frequency need not be better than about 0.01 per cent.

The Complete Circuit

The schematic diagram of Fig. 9 shows the complete discriminator circuit. The class-A amplifier tube $V1$ supplies a voltage substantially free from harmonics to
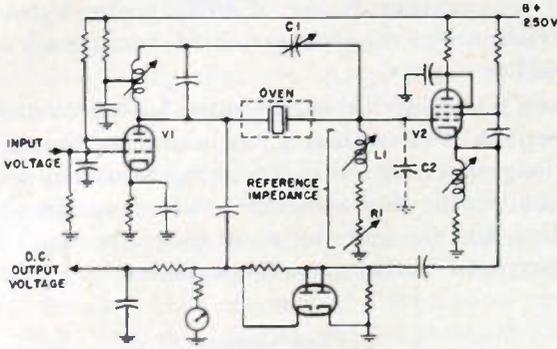
Fig. 9—Schematic of complete discriminator circuit.

one terminal of the crystal. A voltage in antiphase is supplied to the other terminal by capacitor $C1$, and this neutralizes part of the crystal shunt capacity. By adjustment of $C1$ the desired value can be obtained for the effective shunt capacity and hence for the frequency separation between the resonant and antiresonant frequencies. The reference impedance consists of inductor $L1$ and the partly variable resistor $R1$ in shunt with the

stray capacity $C2$. Inductor $L1$ is resonated with the stray capacity at the center frequency and $R1$ is adjusted to obtain the required effective resistance for the reference impedance. The cathode follower tube $V2$ is included to avoid the loading effect of the rectifiers on the reference impedance. By suitable choice of component values, the operation of the circuit has been made substantially independent of normal supply voltage and component variations, as illustrated by the fact that it is possible to replace any tube without producing a significant change of center frequency.

The FM exciter unit in, which the discriminator is used, is shown in schematic form in Fig. 10. The unit is conventional in design, and the schematic is largely self-explanatory. It will be noted that separate reactance tubes are used for modulation and frequency control, thus enabling the modulator to operate at minimum distortion irrespective of the frequency-control voltage. The three-tube frequency divider[7] "locks in" over a wide frequency range and practically eliminates any need for adjustment in service. To enable the unit to be used in both high-and low-band television transmitters, the output frequency range is 8.4 to 13.2 mc. Subsequent frequency multiplication by 8 covers tele-

[7] R. L. Miller, "Fractional frequency generators utilizing regenerative modulation," Proc. I.R.E., vol. 27, p. 446; July, 1939.
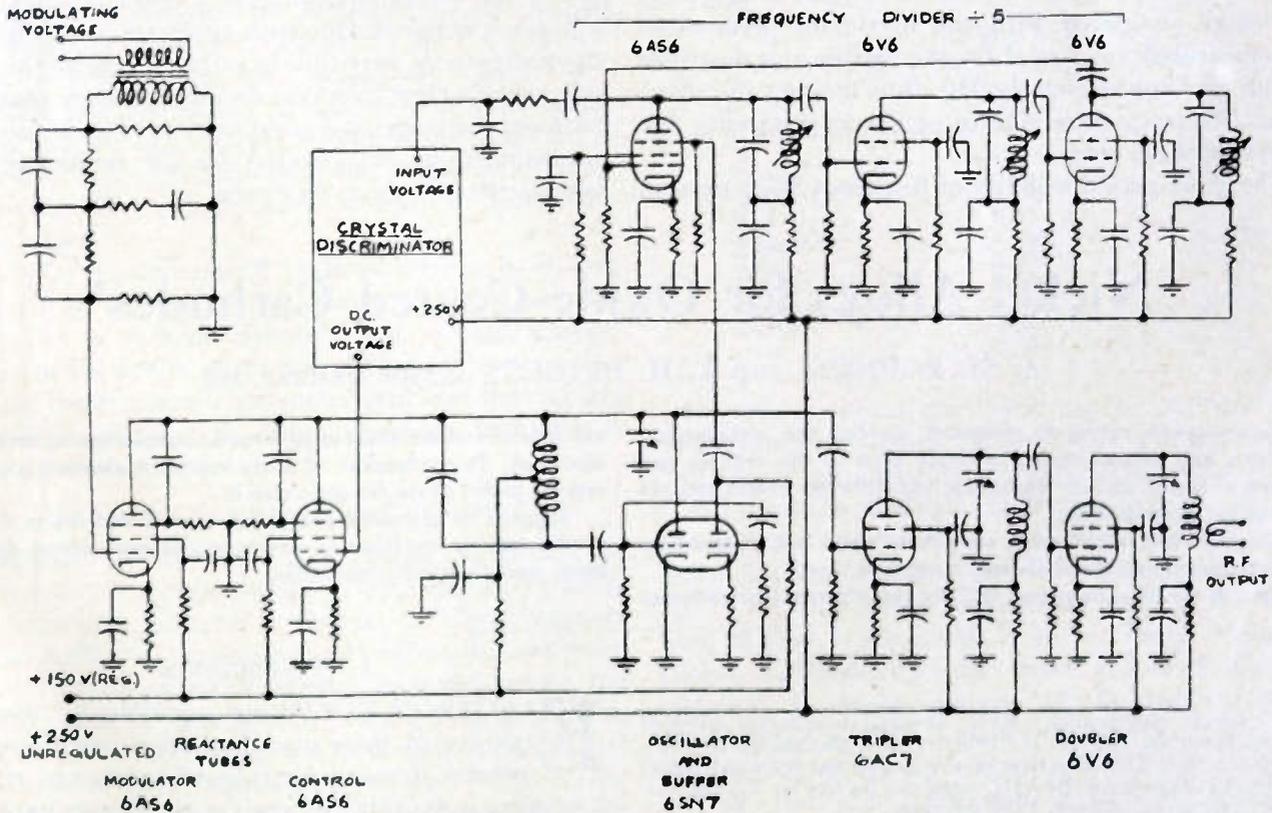
Fig. 10—Simplified schematic of FM exciter unit.

vision channels 2 to 6 and multiplication by 24 covers channels 7 to 13.

In practice, the initial adjustment of the discriminator can be performed very simply when it is in the transmitter with which it is to be used. With the exciter unit under control, the transmitter frequency is automatically adjusted to the value—which makes the dc output of the discriminator substantially zero. The discriminator can then be adjusted by observing the frequency indicated by the direct-reading frequency monitor with which the transmitter is normally equipped. Inductor $L1$ (Fig. 9) is tuned so that the frequency is a maximum; capacitor $C1$ is adjusted so that there is no frequency change when 100 per cent sinusoidal modulation is applied; and $R1$ is adjusted so that the center frequency has the assigned value.

Performance

The circuit comprising the oscillator, discriminator, and reactance tube is essentially a system with inverse feedback, wherein the inherent frequency stability of the oscillator is improved by the addition of a feedback loop. A practical figure for the inherent frequency stability of an FM oscillator is 0.1 per cent, and measurements on the exciter unit gave a figure somewhat better than this for normal ambient temperature change, including the change during the warm-up period. With frequency control, the oscillator stability is increased to $0.1/1+A$ per cent, where A is the over-all gain of the feedback loop. It is readily possible to make A so large that frequency variations due to oscillator instability are small compared with the maximum permissible. The measured value of A for the exciter unit described herein was approximately 250, thus making the maximum frequency error due to oscillator instability less than 0.0004 per cent.

The short-period stability of frequency measurement by the discriminator is determined largely by ambient temperature variation. An average frequency-temperature coefficient of -1.2 parts in 10^7 per °C was measured for the complete discriminator unit in the ambient temperature range 20°C to 70°C. For a normal ambient temperature change of 20°C during warm-up of the transmitter, the change in center frequency is thus 0.00024 per cent.

Several of these FM exciter units have been in service for periods of more than a year and have shown excellent long-period stability. There has been no noticeable frequency drift due to aging or deterioration of components, and the units have not generally required any readjustment during normal operation.

Conclusion

It can be concluded that it is quite practical to stabilize the center frequency of a television broadcast aural transmitter by means of a crystal discriminator in which the center frequency is measured by comparing the impedance of a quartz crystal with a reference impedance. The performance of the discriminator is readily amenable to calculation in terms of the circuit parameters, and experimental measurements have confirmed the calculated results. The unit is very simple to build on a production basis, and requires no critical adjustments during testing operation or maintenance.

Use of this crystal discriminator could readily be extended to other applications where an accurate measure of the center frequency of a frequency-modulated voltage is required. One such application for which the discriminator is particularly suited would be the maintenance of a close tolerance on the frequency separation between the visual and aural carriers of a uhf television transmitter, as necessitated by the requirements of intercarrier television receivers.

Nickel Alloys for Oxide-Coated Cathodes*

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Summary—A survey is presented covering the metallurgical problems and manufacturing methods used in the refining and melting of nickel, and its fabrication into indirectly heated cathode sleeves for electron tubes.

The important role of minor constituent alloys is discussed from both the metallurgical and electronic points of view.

Four of the most important cathode performance characteristics are defined and the effect of base-metal composition on each one is discussed. The influences of each important element present in cathode nickel alloys are enumerated.

Suggestions are made concerning present and future developmental cathode materials to meet new and more severe demands being placed upon electron tubes.

I. Introduction

ELECTRON TUBES are notorious for the complexity of their manufacturing operations. The present demand for tube efficiency and reliability makes it desirable to analyze the factors important to achieving these goals. Thus, we can better understand and employ the requisite controls.

Few technical products other than electron tubes have ever received as widespread development and use with as little clear understanding of basic theory. A great deal of basic research must still be completed before the mechanism of thermionic emission is explained satisfactorily. One reason for this situation is that many highly specialized skills enter into every part and process of tube manufacture. Another reason is that concise and critical tests on material or process changes are difficult because of the masking effect of the large number of uncontrolled variables.

The oxide-coated cathode assembly is the greatest unknown in the electron tube. It is an aggregate of chemical and metallurgical specialties. The shape and size of the carbonate crystals, the method of coating application, and the exhaust and seasoning schedules are of paramount importance in producing satisfactory results. The cathode base metal plays a secondary role, but it must not be underrated in view of the new and more severe demands now placed upon the electron tubes and ultimately upon the cathode.

A properly chosen base metal of uniform high quality goes a long way in providing greater latitude for cathode spraying and tube-processing tolerances.

Previously, the cathode sleeve was considered primarily as a support for the oxide coating, as a heat transfer medium, and as a portion of the electrical conducting circuit. Any effect of the metal in chemically reducing the coating oxides was only to be considered during initial activation.

Now it is known that the properties of the cathode base metal are important throughout life. The ways in which they enter the tube action are called "performance characteristics."[1] Specifically, they are classified as follows: rate of free barium evolution, rate of activation, rate of sublimation, and degree of interface impedance. They are discussed in detail in Section IV of this paper.

In order to understand cathode action, it is first necessary to review the metallurgy of nickel and the fabrication of cathode sleeves. Then, through consideration of the action of the nickel alloys in conjunction with the coating, it will become evident how the cathode assembly is affected by tube processing and how it enters into tube applications.

II. Nickel Refining and Processing

Compared with other major metals, economic deposits of nickel are quite scarce. Commercially valuable ores are widely scattered in Europe, on New Caledonia in the Pacific, in Venezuela, and in Canada, but there are only a very few major deposits. As is well known, the Sudbury area in Canada is by far the most important, while the New Caledonian ores belonging to France and the Petsamo Mine, now worked by the U.S.S.R., are smaller but significant. The New Caledonian ores are

[1] Reports #926 from Electronics Laboratory, Superior Tube Company, December, 1948 to May, 1951.

oxides and chemically basic, while the others are chiefly sulfides and carry varying amounts of cobalt, copper, iron, platinum, palladium, iridium, gold, and silver.

In the Canadian ores the nickel varies from less than 1 to $4\frac{1}{2}$ per cent, and the copper from as little as 0.10 to $3\frac{1}{2}$ per cent. These are smelted to a matte (impure nickel and copper sulfide) and treated with sodium nitrate. When allowed to cool in thimbles the copper matte separates from the nickel matte rather completely, the copper sulfide floating to the top and the heavier nickel sulfides solidifying at the bottom. This is known as the Orford process and its discovery was the key to the commercialization of the Canadian ores.

The "bottoms" (nickel sulfide matte) are separated, crushed, and leached to remove trapped copper and other impurities. The residue is roasted to an impure black oxide of nickel and for the production of pure nickel it is charged into large open hearth furnaces where the oxide is reduced to metal, cast into anodes, and made ready for electrolytic refining.

The electrolytic refining step is very similar to that used for copper and other metals, where the impure nickel anode is gradually dissolved in a sulfuric acid electrolyte as the pure metal is deposited on a cathode starter sheet. During this step the precious metals are removed as slimes and other impurities remain in solution in the electrolyte from which they must be continuously removed. The analysis of a single random sample of electrolytic nickel obtained in 1947 is shown in Table I.

TABLE I

	Electrolytic Nickel	Carbonyl Nickel
	per cent	*per cent*
Carbon	—	0.07
Copper	0.007	0.001
Iron	0.03	0.013
Manganese	0.0005	0.0005
Sulfur	0.001	0.003
Silicon	0.015	0.016
Magnesium	0.006	0.003
Cobalt	0.44	0.001
Lead	0.003	<0.001

In other countries the nickel may be purified by the Mond process in which carbonyl nickel of the formula $Ni(CO)_4$ is produced by first reducing roasted (oxidized) matte to metallic form by treatment in producer gas and then retreating in the same gas. Nickel carbonyl is volatile as low as 50°C. When heated above 200°C it decomposes to yield metallic nickel in the form of shot. Since cobalt is not similarly affected by carbon monoxide, carbonyl nickel is free of this sister element. A single analysis of European carbonyl nickel, made in 1947, is shown in Table I.

It is evident that by either method there is available a very pure nickel for melting and processing into cathodes. Up to this point the procedure is not a great deal different from that for other nonferrous metals,

nor is it a great deal more difficult. However, until 1870 nickel was not considered to be a malleable metal. It readily cracked under small amounts of either hot or cold work. It was the discovery of the magnesium deoxidation process in that year which permitted the melting and casting of nickel and high nickel alloys that could be hot and cold worked. Simple deoxidation with magnesium was not the entire answer since metal produced by this process was still "tender" or "short" (cracked easily during working) below 1,600°F (870°C), and did not permit hot piercing to seamless tubes and similar severe hot working.

In the United States nickel is melted in acid (silica) brick-lined open hearth furnaces of 6- to 20-ton capacity, in carbon arc electric furnaces lined with basic (magnesia) brick of 7- to 20-ton capacity, or in basic refractory-lined high-frequency furnaces holding from 400 to 10,000 pounds. In Europe the melting of such alloys is on a much smaller scale and induction melting is chiefly used. Vacuum melting of nickel containing aluminum for filamentary type cathodes is practiced to a small extent in Europe and such alloys are deoxidized by a small addition of calcium silicide only. Obviously nickel alloys containing magnesium cannot be vacuum-melted because of the volatility of that metal.

According to Mudge,[2] "The process of malleabilization includes four essentials: (1) deoxidation to remove harmful oxides; (2) degasification to remove carbon monoxide, nitrogen, or hydrogen; (3) fixing of harmful elements; [i.e., sulphur] and (4) no injurious effect from the malleabilizing materials added.

"As far as is known, no single addition element satisfied all four of these requirements. Aluminum will deoxidize, but will not fix harmful elements; calcium will deoxidize and fix sulphur, but an excess of it [even a trace] causes brittleness; phosphorous will convert a harmful element, such as calcium, to a harmless constituent, but will not remove gases; titanium will not provide adequate deoxidation; and magnesium although sufficient for deoxidation and fixation of sulphur will not always provide for adequate degasification, nor will it fix selenium or tellurium completely.

"The treatment for optimum malleability, therefore, requires the use of two or more addition elements . . . .

"Treatment of molten nickel with only magnesium gives a sound material that can be hot worked satisfactorily over the temperature range of approximately 1,600 to 2,250°F (870 to 1,235°C). The preferred treatment with small amounts of magnesium, titanium and boron gives a product that can be hot worked at all temperatures from approximately 1,100 to 2,350°F (600 to 1,290°C)."

Since the elements magnesium, titanium, and boron are added specifically for their scavenging effect, the difficulty of control of residual amounts of these ele-

ments must be self-evident.

And yet, present theory of the reducing action of nickel cathode on oxide coatings indicates that the control of residual amounts of these elements and the control of their state of combination with gases, impurities, and each other is the key to control of the "reducing potential" of the cathode alloy. If these functions are not mutually exclusive, they at least present an extremely difficult metallurgical problem and one which cannot be solved by an arbitrary rearrangement of composition limits whenever it is desired to slightly modify the activation characteristics of a cathode nickel. It is truly remarkable that such a degree of uniformity has been attained in the melting and processing of cathode nickel alloys.

But what of the other impurities in cathode nickel? Manganese, iron, copper, and silicon are all present in significant amounts and subject to specification. Manganese appears to enhance considerably the hot-working characteristics of nickel, particularly large masses of coarsely crystalline metal. For this reason manganese is purposely added to nickel when it is to be cast into large ingots, and a compromise must be arranged between the requirements of the metal producer and those of the electronic engineer with respect to this element. As with most compromises, neither can be completely satisfied.

Economic considerations require the reworking of a certain amount of scrap nickel into each new heat and some iron may be derived from this source. In addition, the manganese and other alloy additions carry small amounts of iron, so there is a practical lower limit below which iron cannot be reduced economically. Similarly, copper is derived from the original electrolytic nickel and from scrap, but little if any comes from the alloy additions. Nickel may be melted to closer limits and greater purity in 600- to 12,00-pound heats than in larger quantities, but such small heats would preclude the cathode melt approval method of production tube testing which is presently used by the electron tube manufacturers.

Whenever metal having an affinity for silicon is melted in siliceous refractories or under siliceous slags, chemical laws indicate a small amount of reduction of silicon to the metal. Given fixed furnace practices, it is evident that a relatively constant silicon residual will be present and it is not even necessary to try to control it. On the other hand, controlled additions of silicon to a nickel melt present a different problem, and this is a reason that small contents of silicon of the order of 0.02 per cent in nickel remain constant from heat to heat while larger quantities show variability.

Wise has reported the analyses of alloy additions for small experimental melts of nickel to be approximately as follows in Table II.[3]

[2] W. A. Mudge, "Non-ferrous melting practice," Institute of Metals Division, Symposium Series, American Institute of Mechanical Engineers, New York, N. Y.; 1946.

[3] E. M. Wise and R. H. Schaefer, "The Properties of Pure Nickel," Reinhold Publishing Corp., New York, N. Y., September to December, 1942.

TABLE II

Element	Form of Addition	Analysis
Mg	Ni Mg	68.3 per cent Mg, 31.5 per cent Ni, 0.16 per cent Fe
Si	Ni Si	24 per cent Si, 74.3 per cent Ni, 0.80 per cent Fe
Mn*	Electrolytic Mn	97.3 per cent Mn, 0.74 per cent Si, 1.5 per cent Fe

* (Note: Electrolytic manganese of 99.9 per cent purity is now available.)

These addition alloys are about as pure as can be commercially obtained, but it is evident that they do carry very small amounts of unwanted impurities.

The processing of cathode nickel ingots into fine tubing and strip is lengthy and in general believed to be without great effect on cathode characteristics providing the methods remain constant. However, they will be very briefly reviewed:

Seamless Tubing

Nickel ingots are mechanically overhauled to remove approximately $\frac{1}{4}$ inch from each surface, heated to approximately 2,250°F (1,135°C) and hot forged to billets. These are again machined, cut into short lengths, and reheated for extrusion. Extruded tubes, approximately $3\frac{1}{4}$ inches outside diameter (see Fig. 1), are annealed in a reducing atmosphere produced from natural gas,
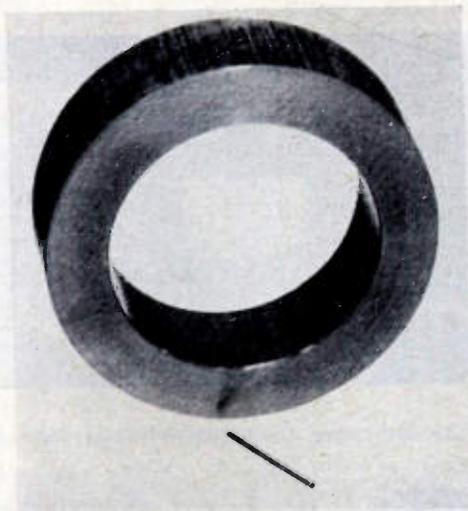
Fig. 1—A hot-finished seamless cathode nickel alloy tube, and seamless cathode drawn from it.

pickled, and again ground to remove defects. From this point onward they are cold reduced in tube-reducing machines and by bench drawing with intermediate annealing in controlled atmosphere. For the larger tubes the atmosphere is that produced from the partial burning of natural gas which is free of sulphur, while smaller tubing is annealed in cracked ammonia.

In cold drawing, tubing is drawn through tungsten carbide dies and over hardened and polished steel mandrel rods or fixed tungsten carbide plug mandrels. Lubricants vary from heavy viscous oils to light petroleum products, and care must be exercised constantly to prevent their contamination by sulphur or chlorine.

In "rod drawing" an expanding operation between hardened steel rolls follows each draw in order to free the tube from the mandrel rod. After degreasing in stabilized trichlorethylene the tubing is annealed continuously through muffle furnaces in a cracked ammonia atmosphere from which residual ammonia has been carefully removed. The temperature may vary from 1,450 to 1,600°F (790 to 870°C) depending upon tube size, prior cold reduction, and other detail fabricating requirements.

After as many as fifteen or more redrawing and annealing operations the tubing is finished in the hard temper, straightened, inspected, and gaged. It is then
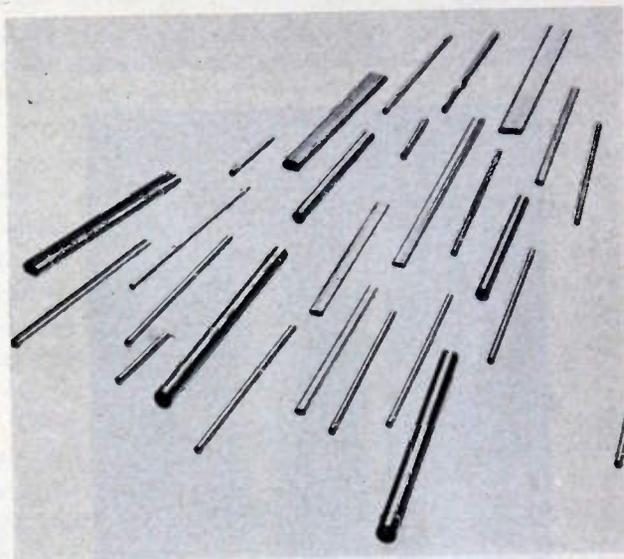
Fig. 2—Variety of seamless and lockseam special nickel-alloy cathodes.

ready for shipment in random lengths or for cutting and embossing as a finished part. The final operation is always a thorough degreasing treatment in trichlorethylene, and sometimes other solvents as well. (See Fig. 2.)

Strip for Seamed Cathodes

Ingots must be carefully prepared for strip as for tubing, and approximately $\frac{1}{4}$ inch is removed from all surfaces before the nickel ingots are heated to 2,250°F (1,232°C) and forged to square blooms. (Figs. 3 and 4.) These blooms are again chipped and ground, reheated to about 2,150°F (1,176°C) and hot rolled to billets. There follows another overhauling operation and a further reheating for hot rolling to coiled strip approximately $\frac{1}{8}$ inch thick. (See Figs. 5 and 6.)

The coiled strip is annealed in a reducing atmosphere, pickled and cold rolled to approximately 1/16 inch thickness. Such strip may be annealed and slit for conversion into welded tubing which, after welding by the inert arc process, is processed exactly the same as seamless tubing. For seamed cathodes, the strip is rerolled many times and reannealed in cracked ammonia atmospheres until reduced to a thickness of approximately

Fig. 3—Ingot of cathode nickel (499 alloy) being lathe-turned before hot rolling (Courtesy of Driver-Harris Company).
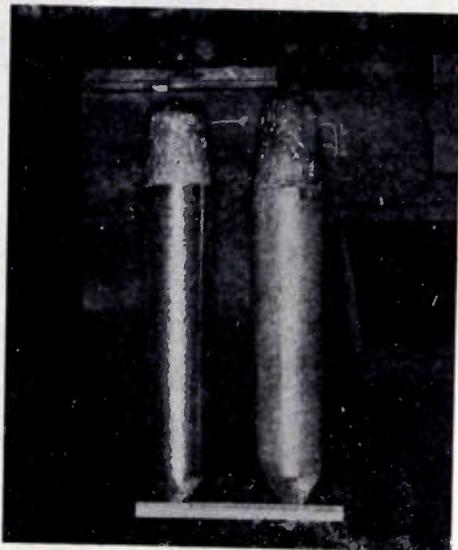
Fig. 4—300-pound ingots of cathode nickel (499 alloy) as cast, and after lathe turning to remove surface imperfections (Courtesy of Driver-Harris Company).

0.002 inch and in just the right temper for forming into cathodes. (See Fig. 7.)

Effects of Processing

In the working of nickel to cathode form, by whatever process, there are several possibilities of slightly changing the original composition or state of oxidation of the reducing elements. Contamination with sulphur would render the nickel unfit for further working, so there is little likelihood that such contamination could ever reach the cathode user. There is a slight and measurable increase in surface-iron content because of the many contacts with steel tools. This can be and is removed by acid pickling close to finish. Some oxidation of such components as magnesium, silicon, titanium, and the like inevitably occurs at the surfaces of the nickel tube or strip even though highly reducing furnace atmospheres are used. Care must be used to keep such atmospheres to dew points in the neighborhood of −40°C or lower, but even so, small air infiltrations into the furnaces raise the dew point of the actual

furnace atmosphere sufficiently to cause some oxidation of surface silicon and magnesium. This is one reason why electropolishing and similar surface stripping methods applied to cathodes often improve their speed of activation.

Carbon content is also slightly changed. The original carbon content of most of the cathode alloys will run from 0.05 to 0.12 per cent with some of the special passive alloys going as high as 0.25 per cent. The many

Fig. 5—Hot rolling of nickel alloy rods for strip as used for lockseam cathodes.

Fig. 6—Further hot rolling of nickel-alloy rod as shown in Fig. 5.
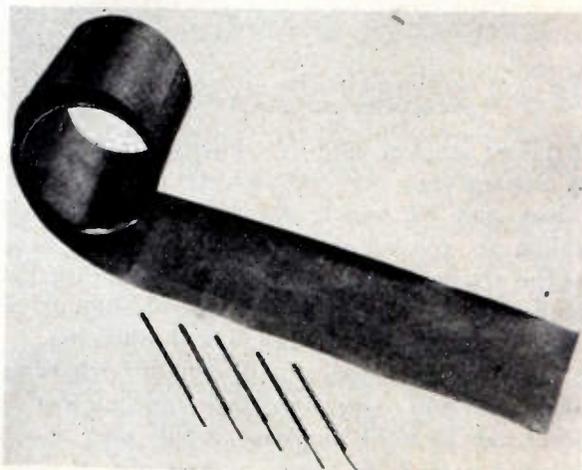
Fig. 7—0.002-inch thick cathode nickel-alloy strip and integral tab lockseam cathodes made from it.

successive anneals in highly reducing hydrogen atmospheres and the purposely applied decarburizing treatments in the case of "passive" alloys reduces carbon content to the neighborhood of 0.03 per cent to 0.05 per cent in the finished product. Little, if any, changes in cathode characteristics have been definitely shown to be caused by a small change in carbon content.

The Role of Reducing Elements from the Metallurgical Standpoint

The primary metallurgical reasons for the inclusion of various reducing or deoxidizing elements in nickel have been briefly explained. The metallurgist no doubt approaches the rôle of these elements in cathode activation from a somewhat different standpoint than does the physicist or electronic engineer.

There is a widespread opinion both in the United States and abroad that magnesium, silicon, and similar elements which can reduce the coatings are the primary source of emissive activity. Practically all of the recent work in this country has been on the basis of this hypothesis, and there is a long list of research papers which might be cited in support of the theory.

There are good reasons for separating the effects of reducing elements upon the initial activation of the cathode from their effects on emission properties during life. One investigator[4] believes that "active" nickel and "pure" nickel approach each other but do not actually coincide in efficiency as a base metal for an oxide cathode during life. If this is true, the effects of reducing elements would be chiefly in the direction of rapid activation which is of great commercial interest. Of course they also have an effect on interface impedence and evaporation rate. Consideration of the reducing elements as initial activators only would also explain the successful use of "passive" alloys in commercial electron tubes by some producers, while others deem the use of more active alloys to be essential for the proper operation of the same tube.

Assuming that the reducing elements contained in the immediate surface of the metal cathode are converted to their oxides during the activation cycle, the supply of these elements to the surface becomes dependent upon the rates of their diffusion in nickel. These diffusion rates for the different elements are dependent upon temperature, concentration, the melting point of the solute element, atomic size compared with interatomic distance in the solvent lattice, degree of solubility and by the peculiar and often anomalous effects of the presence of other elements. Knowledge of diffusion rates of the various constituents in complex alloys is still somewhat meager. A fundamental discussion of the subject may be found in the literature.[5]

Whether, under all of the conditions for diffusion noted above, a continuous supply of reducing elements

is available at the surface of the cathode after initial activation is being investigated. In this connection it is interesting to note that a German patent indicates that plating of the inside surface of a cylindrical cathode with pure nickel effectively reduces emission in the direction of the heater, thus increasing heater-cathode resistance. Although the nickel deposit is extremely thin, it would appear that the reducing elements do not diffuse through the coating sufficiently during life to re-establish the normal emission level of the inside surface of the unplated cathode. If this be true, then the concentration of activating elements on and near the surface of the metal is the only consideration of significance. However, this concept is complicated by the known fact, mentioned above, that the highly reactive elements near the surface of the cathode are often already combined with oxygen, nitrogen, or other elements when the cathode is put into service. The concept is supported, however, by a German investigation[4] which showed no detectable loss of magnesium in the cathode base metal during life. This same investigator showed a very considerable diffusion of barium into the base metal, approaching the limit of solubility of that element in nickel. (More recent research in this country has indicated magnesium loss of upwards of 30 per cent from the cathode during life.)

There have been many reports of electrical leakage and other troubles which presumably were caused by evaporation of such elements as copper, iron, manganese, and magnesium from the cathode under the conditions of operation of a vacuum tube. It would appear that after initial evaporation of these elements from the surface (assuming that they are not oxidized or otherwise chemically combined) their evaporation is directly dependent upon their rates of diffusion, which, in turn, are dependent upon the number of factors enumerated above. Two chemical laws are of interest in this respect: The vapor pressure of an alloy will be governed *in part* by a law analogous to Dalton's law of partial pressures,[6] thus: "*The total vapor pressure of an alloy is equal to the sum of the partial vapor pressures of its constituents, but the partial pressure of each component of the alloy is lower than the normal vapor pressure which the element would exert if it were not contained in an alloy.*" According to Raoult's law, as applied to dilute solutions, the vapor pressure of the solution is lower than that of the pure solvent by an amount which is proportional to the concentration of the solute.

That the cathode nickel alloys do not rigidly conform with these laws has been indicated by a number of experiments which are still in progress in the authors' laboratories. For instance, the rate of evaporation of nickel from a very pure cathode seems to be less than the rate from a cathode containing more of the alloying elements.

[4] Unpublished data.
[5] R. F. Mehl, "Diffusion in solid metals," *Trans. AIEE*, vol. 122, pp. 19–27; 1936.

[6] Allison Butts, "A Textbook of Metallurgical Problems," Mc-Graw-Hill Book Co., Inc., New York, N. Y.; 1932.

If one ignores diffusion constants, which is evidently unwarranted, and also any chemical combination of the alloying elements with nickel and with each other, which is just as unwarranted, one can rather easily calculate the vapor pressures of a series of elements commonly found in cathode nickel from vapor pressure data found in the literature. The difficulty lies with the credibility of the vapor pressure data in the temperature range in which we are interested. Based on vapor pressure data published by Espe and Knoll,[7] Kelley,[8] and Dushman,[9] such calculations have been made for a heat of #220 (normal alloy) cathode nickel which has been used for the past five years as a reference standard by Section A of Sub-Committee VIII, Committee B-4 of the ASTM. That is Superior Tube Company melt #66, International Nickel Company heat #N-6564A. These calculations appear in Tables III and IV, and may form a useful reference in checking the casual and sometimes uninformed statements which have been made concerning evaporation from vacuum-tube cathodes. Variations in cathode-operating temperatures in commercial vacuum tubes appear to be larger than suspected, and may be the cause of much of the evaporation trouble reported.

In this discussion we have ignored the effects of intermetallic compounds formed between the various elements present in cathode nickel alloys. That such compounds exist, and that they have a real effect upon "reducing potential" and evaporation characteristics is not to be doubted. Silicon combines with nickel to form the intermetallic compound Ni_3Si which is stable at room temperature. Similarly, nickel forms compounds with manganese, magnesium, boron, aluminum, and titanium to mention only a few. The reducing elements of chief interest in cathode nickels, magnesium and silicon, form a compound Mg_4Si which in the pure form is liquid just slightly above the operating temperature of a cathode. Copper forms numerous compounds with silicon and magnesium and the possible intermetallic compounds between all of the elements present are exceedingly numerous. Each such compound displays a free energy different from the elements which compose it; each displays its own peculiar melting point and vapor pressure characteristics; each displays its own characteristic diffusion rate. A really thorough study of the "performance characteristics" of cathode nickel alloys will include all of the diverse properties which have been mentioned, and probably others yet to be isolated.

III. Oxide-Coated Cathodes

Cathode sleeves are conventionally spray-coated with mixed crystals of $Ba \cdot Sr \cdot CO_3$.[10] Frequently, a small percentage of $CaCO_3$ is added. During exhaust the coating is decomposed to the corresponding oxides, and mixed $Ba \cdot Sr \cdot Ca \cdot O$ crystals are slowly formed during activation or "seasoning." The cathode assembly is then ready to deliver copious amounts of free electrons when operated at about 825°C.

The cathode structure is generally conceived as consisting of several strata (Fig. 8)[11] The estimated potential gradients are graphed below the respective layers.
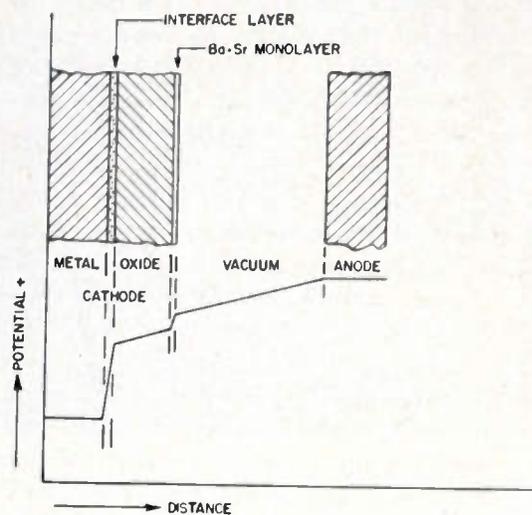
Fig. 8—Schematic view through a diode tube with potential distribution approximation.

The free Ba·Sr layer on the surface of the coating is constantly being depleted by evaporation and by chemical recombination. It must constantly be reformed to provide a low work function which, in turn, allows ready liberation of free electrons.

Ba evaporates from this monolayer more readily than Sr at operating temperatures. It deposits upon the grid wires to cause grid emission. The evaporation energy or temperature of Sr on SrO is lower than that for Ba on BaO. However, for the Ba, Sr layer in BaO·SrO mixed crystals the Ba apparently boils off more easily.

The energy of the electron beam is believed capable of decomposing films on positive electrodes.[12] These are fundamentally Ba·O compounds evolved during coating decomposition and activation. When the SrO⁻ negative ions are liberated by the electron beam energy they react easily to form compounds with the nearby layer of Ba++Sr++, reducing the activity of the cathode surface layer. Any other chemically active gases present in the tube also can be "gettered" by free Ba. These are the most serious causes for fluctuating and slumping tube characteristics.

The depletion of free Ba through evaporation and

[7] W. Espe and M. Knoll, "Werkstoffkunde der Hochvakuumtechnik," Verlag von Julius Springer, Berlin, Germany; 1936.
[8] K. K. Kelley, Bureau of Mines Bulletin #383; 1935.
[9] S. Dushman, "Vacuum Technique," John Wiley and Sons, Inc., New York, N. Y.; 1948.
[10] E. G. Widell and R. A. Hellar, "Effect of coating composition of oxide-coated cathodes on electron emission," *Jour. Appl. Phys.*, vol. 21, p. 1194; November, 1950.
[11] R. Loosjes and H. J. Vink, "Distribution of potential in the coating of an oxide during a pulse of great current density," *Le Vide*, vol. 5, p. 731; January, 1950.
[12] J. Cardell, et al., "Fundamental Research on Raw Materials Used for Electron Emissivity on Indirectly Heated Cathodes," Reports for ONR Contract N7 onr-389.

gettering action must be balanced by reactivation. This is partially controlled through correct selection of the base alloy. For long and reliable tube operation there is a premium upon the best possible vacuum and the cleanest metal mount parts. Unless these conditions are observed, there is a large uncontrolled variable present.

Obviously, means for regenerating the surface Ba layer of the coating must be provided during tube life. This is accomplished in part by the reducing action of certain elements in the cathode base metal.

The ASTM cathode committee[13] has classified cath-

[13] A.S.T.M. Specification B239-49T.

TABLE III

MAXIMUM VAPOR PRESSURES OF METAL COMPONENTS OF MELT 66, INCO #220 ALLOY, AT 800, 900, AND 1,000° BY 3 DATA SOURCES: DUSHMAN,[9] ESPE AND KNOLL,[7] AND KELLEY[8]

Melt 66	at %	Dushman		Espe & Knoll		Kelley	
			mm Hg		mm Hg		mm Hg
800°C							
Mg	0.09	40,000.	$\times 10^{-6}$	30,000.	$\times 10^{-6}$	20,000.	$\times 10^{-6}$
Ca	0.0004	3.	$\times 10^{-6}$	3.	$\times 10^{-6}$	3.	$\times 10^{-6}$
Mn	0.11	0.14	$\times 10^{-6}$	0.1	$\times 10^{-6}$	0.17	$\times 10^{-6}$
Al	0.01	0.0002	$\times 10^{-6}$	0.00003	$\times 10^{-6}$	0.007	$\times 10^{-6}$
Ni	98.54	0.00008	$\times 10^{-6}$	0.008	$\times 10^{-6}$	0.001	$\times 10^{-6}$
Cu	0.02	0.00002	$\times 10^{-6}$	0.00001	$\times 10^{-6}$	0.00004	$\times 10^{-6}$
Si	0.06	4.	$\times 10^{-12}$	4.	$\times 10^{-12}$	0.003	$\times 10^{-12}$
Cr	0.0009	3.	$\times 10^{-12}$	0.01	$\times 10^{-6}$	0.8	$\times 10^{-12}$
Fe	0.07	0.7	$\times 10^{-12}$	7.	$\times 10^{-12}$	3.	$\times 10^{-12}$
Ti	0.04	0.3	$\times 10^{-12}$	no data		no data	
B	0.02	0.3	$\times 10^{-12}$	no data		no data	
Co	0.74	0.078	$\times 10^{-12}$	no data		no data	
C	0.29	500.	$\times 10^{-30}$	16.	$\times 10^{-30}$	0.0004	$\times 10^{-30}$
900°C							
Mg		200,000.	$\times 10^{-6}$	90,000.	$\times 10^{-6}$	80,000.	$\times 10^{-6}$
Ca		15.	$\times 10^{-6}$	15.	$\times 10^{-6}$	14.	$\times 10^{-6}$
Mn		1.9	$\times 10^{-6}$	1.4	$\times 10^{-6}$	1.8	$\times 10^{-6}$
Al		0.003	$\times 10^{-6}$	0.0007	$\times 10^{-6}$	.08	$\times 10^{-6}$
Ni		0.005	$\times 10^{-6}$	0.2	$\times 10^{-6}$	.05	$\times 10^{-6}$
Cu		0.0005	$\times 10^{-6}$	0.0004	$\times 10^{-6}$	.0008	$\times 10^{-6}$
Si		0.0002	$\times 10^{-6}$	0.0002	$\times 10^{-6}$	.3	$\times 10^{-12}$
Cr		0.00008	$\times 10^{-6}$	0.06	$\times 10^{-6}$	.00002	$\times 10^{-6}$
Fe		0.00003	$\times 10^{-6}$	0.0001	$\times 10^{-6}$	.00009	$\times 10^{-3}$
Ti		9.	$\times 10^{-12}$	no data		no data	
B		10.	$\times 10^{-12}$	no data		no data	
Co		4.	$\times 10^{-12}$	no data		no data	
C		500.	$\times 10^{-27}$	200.	$\times 10^{-27}$	1,800.	$\times 10^{-27}$
1,000°C							
Mg		500,000.	$\times 10^{-6}$	300,000.	$\times 10^{-6}$	300,000.	$\times 10^{-6}$
Ca		60.	$\times 10^{-6}$	50.	$\times 10^{-6}$	50.	$\times 10^{-6}$
Mn		10.	$\times 10^{-6}$	10.	$\times 10^{-6}$	10.	$\times 10^{-6}$
Al		0.03	$\times 10^{-6}$	0.01	$\times 10^{-6}$	.6	$\times 10^{-6}$
Ni		0.1	$\times 10^{-6}$	3.	$\times 10^{-6}$	1.	$\times 10^{-6}$
Cu		0.008	$\times 10^{-6}$	0.006	$\times 10^{-6}$	0.01	$\times 10^{-6}$
Si		0.003	$\times 10^{-6}$	0.004	$\times 10^{-6}$	0.00002	$\times 10^{-6}$
Cr		0.001	$\times 10^{-6}$	0.3	$\times 10^{-6}$	0.0003	$\times 10^{-6}$
Fe		0.0006	$\times 10^{-6}$	0.003	$\times 10^{-6}$	0.002	$\times 10^{-6}$
Ti		0.0002	$\times 10^{-6}$	no data		no data	
B		0.0004	$\times 10^{-6}$	no data		no data	
Co		0.0001	$\times 10^{-6}$	no data		no data	
C		200.	$\times 10^{-21}$	300.	$\times 10^{-21}$	0.002	$\times 10^{-21}$

TABLE IV

SATURATED VAPOR PRESSURE AT 800°C OF ELEMENTS COMMONLY USED IN CATHODE NICKEL ALLOYS, ACCORDING TO DATA PUBLISHED BY DUSHMAN, KELLEY, AND ESPE AND KNOLL

Element	Dushman		Kelley		Espe & Knoll	
		mm Hg		mm Hg		mm Hg
Mg	410,000.	$\times 10^{-4}$	240,000.	$\times 10^{-4}$	280,000.	$\times 10^{-4}$
Ca	7,400.	$\times 10^{-4}$	7,200.	$\times 10^{-4}$	7,000.	$\times 10^{-4}$
Mn	1.3	$\times 10^{-4}$	1.6	$\times 10^{-4}$	*1.1	$\times 10^{-4}$
Cr	0.0035	$\times 10^{-4}$	0.00086	$\times 10^{-4}$	*13.	$\times 10^{-4}$
Al	0.027	$\times 10^{-4}$	1.2	$\times 10^{-4}$	0.0056	$\times 10^{-4}$
Cu	0.001	$\times 10^{-4}$	0.0024	$\times 10^{-4}$	0.0006	$\times 10^{-6}$
Fe	0.001	$\times 10^{-6}$	0.0038	$\times 10^{-6}$	0.01	$\times 10^{-6}$
Si	0.0066	$\times 10^{-6}$	0.000004	$\times 10^{-6}$	*0.0071	$\times 10^{-6}$
Ni	0.000085	$\times 10^{-6}$	0.0015	$\times 10^{-6}$	0.0080	$\times 10^{-6}$

Dushman's values were obtained by calculation from his recommended constants. Kelley's values were mathematically extrapolated from range 0.076 to 760 mm. Hg. Espe and Knoll values were taken from a plotted curve except for mathematical extrapolation for starred values.

ode alloys as "passive," "normal," or "active," depending upon the rate at which the reducing agents of the base metal react with the oxide coating. Table I summarizes the composition of the presently available commercial cathode alloys.

To obtain the designed level of electron emission with long life and without grid emission requires a careful balance between the reducing activity of the cathode, and the evolution of "poisons" from other tube parts.

The important alloy reducing agents presently used are Mg, Al, Si, W and C. In reacting with BaO they produce compounds at the core-coating interface.[1,12,14,16]

White[16] has reported a thermodynamical analysis of some oxide cathode reactions. He shows that at $1,273°K$ the rate of BaO loss by dissociation to gaseous products $(Ba^{++}(g)+O^{--}(g))$ is negligible compared to the loss of BaO molecules by evaporation. Consequently, to obtain Ba^{++} ions requires the BaO to be broken down by chemical reduction. SrO and CaO can thermally decompose theoretically to a slight extent. However, in practice, traces of oxygen inhibit this reaction. Therefore, they too must be chemically reduced.

The following tabulation (also from White[16]) shows the free energy in Kilocalories which must be added at a given temperature $(1,073°K)$ to obtain the products shown for these typical oxide cathode reactions:

	Kilocalories
$2BaO(s)=2Ba(g)+O_2$	255
$2SrO(s)=2Sr(g)+O_2$	254
$2CaO(s)=2Ca(g)+O_2$	280
$BaO(s)+H_2=Ba(g)+H_2O$	82
$BaO(s)+CH_4=Ba(g)+CO+2H_2$	73
$BaO+Ni(s)=Ba(g)+NiO(s)$	93
$BaO(s)+C(gr)=Ba(g)+CO$	78
$2BaO(s)+Si(s)=2Ba(g)+SiO_2(1)$	102
$3BaO(s)+Si(s)=2Ba(g)+BaSiO_3(s)$	77
$BaO+Mg(g)=Ba(g)+MgO(s)$	2

It is generally being found that those compounds remaining at the interface have semi-conductor properties and show an equivalent electrical circuit impedance. The resistance value may be almost zero or build up to hundreds of ohms. The capacity has a value less than a microfarad. These are dependent upon the composition, previous history of the cathode, and circuit application of the tube.[17-19]

By X-ray diffraction analysis some of the interface compounds have been identified as MgO, Ba_2SiO_4,

[14] D. A. Wright, "Electrical conductivity of oxide cathode coatings," *Phys. Rev.*, vol. 78, p. 355; May 1, 1950.
[15] A. S. Eisenstein, "Oxide Coated Cathodes," Advances in Electronics, Academic Press; 1948.
[16] A. H. White, "Application of thermodynamics to chemical problems involving the oxide cathode," *Jour. Appl. Phys.*, vol. 20, p. 856; September, 1949.
[17] A. S. Eisenstein, "The Leaking-Condenser Oxide Cathode Interface," ONR Technical Report #2, N7 onr-292; September, 1950.
[18] L. S. Nergaard, "Cathode Impedance and Tube Failure," Report of 10th Annual Conference, Physical Electronics, MIT; March, 1950.
[19] J. F. Weymouth, "Deterioration of Oxide Cathodes under Low Duty Factor Operation," Report of 10th Annual Conference, Physical Electronics, MIT; March, 1950.

TABLE V

COMPOSITION OF U. S. COMMERCIAL CATHODE ALLOYS[13]
(Maximum per cent by weight, unless range is shown)

A.S.T.M. Grade	Cu	Fe	Mn	Mg	Si	Ti
(Active Alloy Type)						
1	0.20	0.20	0.20	0.01-0.10	0.12-0.20	0.02
2	0.04	0.10	0.05	0.01-0.10	0.12-0.20	0.01
3	0.20	0.20	0.20	NS	0.15-0.25	NS
4	0.04	0.05-0.10	0.10	0.01	0.15-0.25	NS
5	0.04	0.05-0.10	0.10	0.05-0.15	0.05-0.15	NS
6	0.04	0.05	0.02	0.01	0.15-0.25	NS
(Normal Alloy Type)						
11	0.02	0.02	0.02	0.01-0.10	0.01-0.05	NS
(Passive Alloy Type)						
21	0.04	0.05	0.02	0.01	0.01	NS

NS = not specified.
Normally: S < .005 per cent.
C = 0.01 per cent per 0.001 inch thickness of nickel sleeve wall.

$BaWO_3$ $BaWO_4$, or even Ba_3WO_6. The thickness varies widely, but is of the order of 10^{-4} cm. It is possible to measure the interface impedance by pulsed techniques and to construct equivalent RC circuits which can be evaluated.

Through operation of tubes under cutoff conditions, with no cathode current drain, and for several hundred hours, the interface resistance buildup can be accelerated. This characteristic is of particular importance for application in computers and similar devices where there may be long stand-by periods of operation followed by pulse currents. The pulses are distorted or attenuated when the interface impedance is high.

Experience has shown that the silicon interface compounds are most likely to produce a high resistance. Aluminum, if held to relatively low values does not seem to develop a resistance.[1] None at all has been found when Mg, W, C are employed.

Another cathode characteristic is the tendency of nickel alloys to sublime at activation or operating temperatures. Metal ions are released from the cathode which condense upon cooler portions of the tube, resulting in electrical leakage, noise, or shifts in contact potential.[1,12] It is believed that sublimation occurs into the coating as well as into the vacuum directly from uncoated areas of the sleeve. This is particularly important in the disc cathodes used in television picture tubes, where the coated area is unusually small.

It has been shown that Mg sublimes readily and is the element causing most leakage and tube noise. As pointed out earlier, Mg is also a powerful reducing agent upon the oxide coating. Thus, as it sublimes away from the cathode, it is no longer available to regenerate free Ba, and the cathode emission falls off; the useful tube life may be shortened.

Analysis of the bulb deposits from thermionic cathodes shows that at operating temperatures $(825°C)$ it is predominantly Mg which sublimes in significant quan-

tity.[1] At somewhat higher temperatures Mn and Cu appear, and then large amounts of Fe, Co, Ni. The Mg, Mn, Cu are present in greater proportions in the deposit that in the original sleeve. The related elements Fe, Co, Ni are in the same proportions as originally. At higher temperatures (about 1,000°C) when the nickel contains very little Mg, Mn, Cu, then Ni and Co *do not sublime* appreciably. This offers the interesting possibility that presence of impurities provides some sort of mechanism for boiling off the Ni and Co. The process is probably quite complex and a theory has not yet been advanced.

In compensation for the loss of Mg, other reducing agents are available to the cathode coating from other tube parts. This has only recently been shown, and is not yet generally appreciated.[1,12] Migration occurs primarily from plates and getters, but may involve micas, lead wires, glass, and other parts. Spectrochemical analyses show reduction in Mg, Si, Cu from such tube parts, and a buildup of these elements upon the cathode can best be detected if it is a "passive" (high purity) nickel alloy.

The design and production of tubes requires the proper choice of all materials to obtain control of electron emission over a long life period.

Fig. 9 is a time-temperature relationship for obtaining a given amount of evaporated metal for a very pure and for a commercial grade of cathode nickel ("passive" and "normal"). The normal operating temperature for cathodes is at the knee of this curve, and hence changes in temperature are most critical. A change of 70°C will alter the time for a fixed amount of sublimation by about one order of magnitude. Temperature control is seen to be much more effective than differences between most cathode alloys. This is especially true for uncoated cathode sleeve areas. In cathode ray tubes such sublimation deposits upon ceramic insulators cause loss of grid control, resulting in tube failure.

The following data have been obtained on a wide range of cathode alloys. Comparisons were made between cathodes sleeves removed from standard diodes after 5,000 hours of life and identical sleeves which had not been mounted. Only the central portion of the metal under the oxide coating was exposed to the spectrograph arc.

Element	Per Cent Loss	Valid Over Composition Range— Percentage
Co	0	0.1–0.7 by weight
Ti	0	0.002–0.03
Si	10	0.008–0.042
Mg	0	0.005 (a possible measure of free Mg,
Mg	30	0.04 capable of sublimation)
Cu	70	0.007–0.022
Mn	60	0.005 (probably due to compounds)
Mn	90	0.11
Fe	50 GAIN	0.0–0.02 (migration from getter?)

Each element in the base metal has a migration rate dependent upon the atom size, concentration, solubility, chemical activity, and sleeve temperature. Thus the processes inside the cathode metal are merely those

normal to metallurgy for alloys at elevated temperature. Use is made of these same properties once the elements reach the metal surface. The molecules either react in the desired manner to keep the coating activated and form an interface layer of varying quality; or they continue to sublime through the coating and into the vacuum in a generally undesired manner.

As these agents diffuse to the surface, their concentration, and consequently, their diffusion rate, is reduced. A more constant supply over a longer period of time— for longer life—can only be obtained by increasing the mass of metal (wall thickness) or by receiving the reducing agents through migration from external tube parts (anode, grid, getter).

The base metal reducing agents and the coating react at a rate dependent upon the materials and temperature. The desired rate of activation, free Barium evolution, sublimation, and interface resistance should determine the amount and type of reducing agents in the base metal.

As pointed out in the preceding section on the metallurgy of nickel, some portion of the reducing agents— particularly near the sleeve surface—can be compounded during processing. In general, these are stable compounds, such as MgO. They are larger molecules, have lower mobilities and cannot react to reduce the coating. In processing cathodes, only the uncompounded reducing agents can be considered as active. This requires the hydrogen cleaning treatment to be conducted

N—NORMAL CATHODE NICKEL 220–66
L—GASEOUS 'L' NICKEL
S—STEAM-ANNEALED 220–66
P—PASSIVE CATHODE NICKEL DH-999
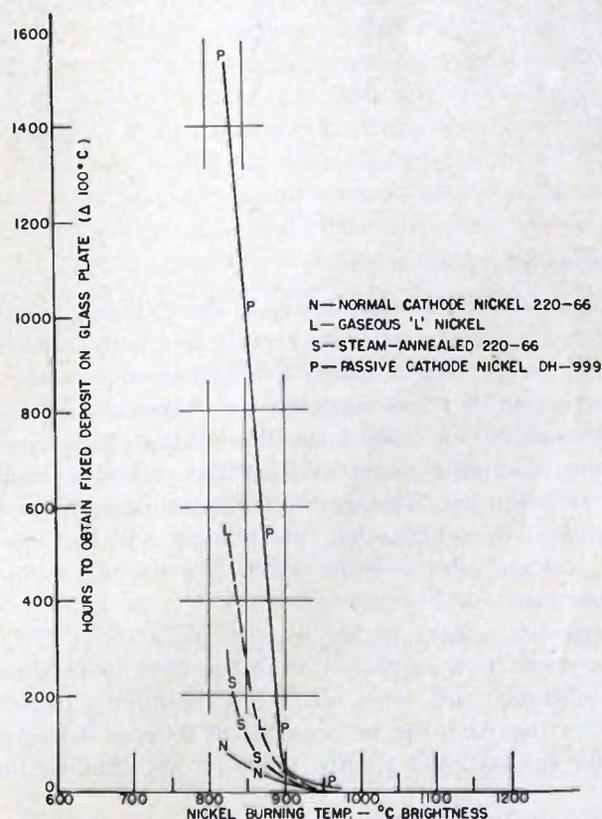
Fig. 9—Bulb deposit absorption time versus burning temperature for four alloys of nickel.

with a low dew point $(< -40°)$ in the furnace muffle itself.

Until now, there has been no accurate means for determining what portions of the cathode sleeve reducing agents are still uncombined and able to react with the coating. Solution of this testing problem is the greatest obstacle to forecasting the emission behavior of a given melt of cathode base metal. When this information does become available it will be possible to rate alloys and their melts for any desired type of service. At present, only the inaccurate, expensive, time consuming prove-in method of actually making production tube tests is possible.

IV. CHARACTERISTICS OF CATHODE ALLOYING ELEMENTS

With the previous background concerning the mechanism of thermionic emission, it is now possible to consider the effects of each common additive to nickel base cathode alloys. This data is summarized in Table VI and discussed in the following paragraphs.

TABLE VI

TENTATIVE RATING OF CATHODE ALLOYS FOR PERFORMANCE CHARACTERISTICS. LOWEST FIGURE IS BEST, HIGHEST FIGURE IS POOREST. * = VERY POOR

A.S.T.M. Grade	Activation	Sublimation	DC Emission Level (ohrs)	Interface Impedance	Gm Life
(Active)					
1	3	3	3	3	
2	3	3	3	3	
3	2	3	2	5*	High Silicon Alloy Probably Poorest
4	4	2	4	4*	
5	1	4*	1	?	
6	5	2	4	4*	
(Normal)					
11	6	3	5	2	
(Passive)					
21	7*	1	6*	1	

A. Reducing Agents

Magnesium is the most active of the reducing agents employed. Cathodes activate rapidly to a high emission level. However, Mg is readily oxidized or compounded, so that some of it, as reported by chemical analysis, is not available for coating reaction. Due to high vapor pressure it rapidly migrates from the cathode, giving short emission life. The interface resistance of MgO is negligible. The sublimation rate is high, and bad electrical leakage effects occur when Mg exceeds about 0.07 per cent.

Aluminum is next to Mg in reducing activity. Filamentary emitters employ 1 to 2 per cent to produce high emission and long life.[20] For indirectly heated cathodes the Al must be reduced to 0.05 to 0.10 per cent for similar good results. (Patents are pending for compositions of this type.) The interface compound is not yet positively identified but may be Al_2O_3 or $BaAlO_3$, having negligible resistance. It does not show any sublimation at operating or activating temperatures. To date, it has only been used experimentally for cathode sleeves, but it appears to be unusually promising.

Silicon is a major variable in cathode alloys for altering emission activity. It is intermediate in speed of cathode activation, and can be present in a wide range of alloy composition (0.02 to 0.25 per cent) with significant effects upon dc emission and related tube characteristics. The emission life with high Si content is long, but there is evidence that it has a depressing effect upon Gm life. The interface compound has been identified as Ba_2SiO_4. Under certain conditions (cutoff operation) this develops an ohmic resistance of hundreds of ohms! The sublimation rate is low.

Titanium acts as a reducing agent upon the coating, but is not now added for this purpose. Usually Ti content is kept as low as possible, since it forms a heavy black interface of high resistance, which radiates heat so well that the cathode temperature is reduced—even to the extent of effecting heater current. It shows no sign of sublimation or migration.

Tungsten has a slight reducing action upon BaO. It has long been used for directly heated filaments, either alone or in combination with Al in a nickel base alloy. When tested in cathode sleeves (4 per cent W, bal. mainly Ni) it produced a high dc and pulsed emission level and exceptionally long life.[1] The interface compound may be $BaWO_3$, which is found to have little, if any, interface resistance. There is no sign of sublimation.

Carbon has been shown to be a slow reducing agent for BaO,[21] but it is exceedingly difficult to control in the completed cathode since it reacts to form CO and CO_2, which are gaseous. Some belief exists that it is effective only in pure nickel cathodes, relatively free from the other reducing agents, known as "passive" grade alloys. Great care must be exercised not to confuse the reducing power of carbon with the action of other agents migrating in from external elements. No visible or measurable interface layer is formed, and there is no sublimation of a metallic leakage film.

Cobalt cathodes have been reported to behave as normal nickel cathodes.[21,22] However, high Co-Ni alloys seem to have good activity. Meager reports indicate good life, low interface resistance, and low sublimation.

B. Other Elements

Chromium is never intentionally present in significant quantities in cathode alloys. It has been reported as a reducing agent, but it is also harmful, since the heavy black interface reduces the expected operating

[20] M. Benjamin, "The influence of impurities in the core metal on the thermionic emission from oxide-coated nickel," *Phil. Mag.*, ser. 7, vol. 20, July, 1935.

[21] E. G. Widell, "Cathode for Electron Discharge Device,"—U. S. Patents 2,179,110, November 7, 1939, and 2,192,491, March 5, 1940.
[22] Confidential Report.

temperature, and sublimation is probably fairly rapid.

Copper and *iron* are harmless to emission if their percentage is confined to low values by usual good metallurgical practices. Surface iron is definitely detrimental to emission. Copper can be harmful since it migrates and sublimes with greater ease than Fe, Ni, and Co. Cu and Fe probably have no effect upon interface resistance, since they do not enter into any of the compounds which have been determined by X-ray diffraction.

Manganese is purposely present due to previously mentioned metallurgical requirements. The amount is dependent upon the size of the ingot cast at melting and practices of the particular melter. The almost universal experience of tube engineers is that Mn is undesirable from an emission standpoint.[22] Less than 0.05 per cent should be present, although 0.15 per cent is used when the reducing agent content is increased to offset it.[23] The explanation may lie in the fact that Mn·O compounds, which may be formed during metal processing, have a higher vapor pressure than Mn alone. Chemically indicated Mn migrates from the cathode sleeve at a very high rate. We believe that a Mn·O compound introduces excess oxygen into the coating to the detriment of emission. There seems to be no interface resistance effect due to this element.

Lead, tin, and *zinc* are rarely, if ever, found in significant quantities in cathode alloys. If present, they rapidly sublime to create intolerably high levels of electrical leakage on tube insulator surfaces.

V. Present and Future Cathode Alloys

The strength of cathode sleeves requires improvement to enable tubes to operate under severe mechanical conditions. This can be accomplished by use of recognized hardening agents. Most promising candidates are W, Co and Mo. A certain amount of mechanical processing may be needed to enhance their hardening properties. Future and further investigation are warranted.

At present, there are nine commercial and important cathode alloys in the United States. England and the European countries each have their own favorite and different cathode alloy compositions.[22]

From the comments in the previous sections, it is easily seen that most existing U. S. cathode alloys have serious faults. They were developed to meet needs of their times. Current requirements, which are far more severe, demanding tube efficiency, reliability, and new operation conditions, have outmoded these alloys with startling rapidity. Recent advances in the knowledge of cathode behavior have provided information permitting evolution of new and improved alloys.

We believe that four alloys of three grades will fill the needs of the radio receiving-tube industry for some time

to come. They must have uniformly low levels of impurity content: such elements as Cu, Fe, Mn, and Ti. Suggested specifications for the reducing agents are tabulated below.

Alloy & Grade	Aluminum	Magnesium	Silicon	Tungsten
	(per cent composition by weight)			
Active-Premium	0.03 max	0.02–0.07	0.03 max	3.75–4.25
Active-Commercial	0.03–0.10	0.02–0.07	0.05 max	—
Normal	0.02 max	0.02–0.07	0.02–0.06	—
Passive	0.01 max	0.01 max	0.01 max	

(Carbon should average about 0.02 per cent per 0.001-inch of strip or wall thickness.)

These alloys would all have low rate of sublimation and low interface impedance values. They would provide three levels of saturated emission (both dc and pulsed), and three levels of rate of free Ba evolution. Their rates of activation can be controlled by tube exhaust and processing schedules.

It is likewise important that these future alloys be capable of commercial melting and metallurgical processing. In addition, they should be closely reproducible from melt to melt or produced in sufficiently large melts to provide a continuity of tube production not requiring major tube process changes. It is believed that these suggested future alloys are capable of meeting these requirements.

During the past ten years tremendous strides have been made in acquiring knowledge concerning the mechanism of thermionic emission from oxide cathodes. Some of this progress has resulted through considering the effects of each assembly constituent as though it was an independent entity. In this manner the basic concepts have been clarified concerning the effect of nickel analysis and treatment upon activation, emission level, and life expectancy. It is necessary occasionally to stop and take stock, to recognize the omissions and simplifications, and then to go forward more carefully considering all of the combined factors bearing on the problem.

Acknowledgments

Bibliography

1. "The Romance of Nickel," International Nickel Company, New York, N. Y.; 1947.
2. B. Stoughton and A. Butts, "Engineering Metallurgy," McGraw-Hill Book Publishing Co., Inc., New York, N. Y.; 1938.
3. D. M. Liddell, "Handbook of Non-Ferrous Metallurgy," McGraw-Hill Book Publishing Co., Inc., New York, N. Y.; 1926.
4. W. A. Mudge, "The Melting of Nickel," American Institute of Mining and Metallurgical Engineers, New York, N. Y.; 1946.
5. R. F. Mehl, "Diffusion in solid metals," *Trans. AIME*, vol. 122; 1936.
6. "The Iron Age," Chilton Co., Inc., Philadelphia, Pa., vol. 161, no. 9.
7. "Metals Handbook," 1948 Ed. Am. Soc. for Metals, Cleveland, Ohio, p. 28.

[23] F. Violet and J. Riethmuller, "Contribution to the study of oxide cathodes," *Ann. Radioelect.*, vol. 4, pp. 148–215; July, 1949.

A 1.5-Kw 500-Megacycle Grounded-Grid Triode*

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Summary—A plane-electrode triode is described which may be operated as a grounded-grid amplifier to obtain 1.5 kw at frequencies up to 700 mc. It operates with 3,000 volts on the plate, is water cooled and has an oxide-coated equipotential cathode. The grid is of an unusual design, made of copper and is nonemitting. The performance at 500 mc is 1.5-kw output, apparent efficiency 55 per cent, power gain 6.5 times.

INTRODUCTION

POWER AMPLIFIERS for frequencies in the region of 500 mc with useful output of the order of one kw are currently of considerable interest for the services assigned to this portion of the spectrum such as ultra-high-frequency television and possibly mobile telephone systems. There have been tubes of the "negative-grid" type described which are capable of satisfactory performance at lower power levels,[1-3] and one tube capable of considerably higher power which was a continuously pumped device.[4]

It is in this frequency range that the klystron tube begins to have some advantages over the negative-grid amplifier. However, because of the usual relatively low efficiency of the klystron, and other considerations, there is great temptation to push the design of negative grid tubes because of their simplicity and greater inherent efficiency. Transit-time effects, heat dissipation, and circuit difficulties are the principal limitations. If wide-band amplifiers are desired, it is important that tube capacitances, particularly those appearing in the output circuit, be kept to a minimum. Circuit considerations will also require that tube lead inductances and the inductances of the electrodes themselves be kept to a minimum.

TUBE GEOMETRY

There are two basic arrangements of electrodes which are practical for triodes: (1) the electrodes may be arranged as concentric cylinders, or (2) as parallel planes. There are advantages and disadvantages for each type from either mechanical or electrical viewpoints. It appeared to the writers that the parallel plane geometry had more to offer in the solution of our problem. Electrodes in the form of parallel disks with connections to their peripheries would certainly have a minimum of

inductance. The capacitances will depend mainly on electrode areas and spacings in any case. The leads will become relatively large diameter cylinders for cathode and anode and a large diameter disk for the grid. This arrangement is most suitable for coaxial line circuits. It also requires the use of an equipotential cathode.

The tube about to be described, was designed with the considerations given above in mind. A picture of the tube is shown in Fig. 1 and the three basic parts: cathode, grid and anode are shown in Fig. 2. The cathode is an indirectly heated oxide-coated disk.
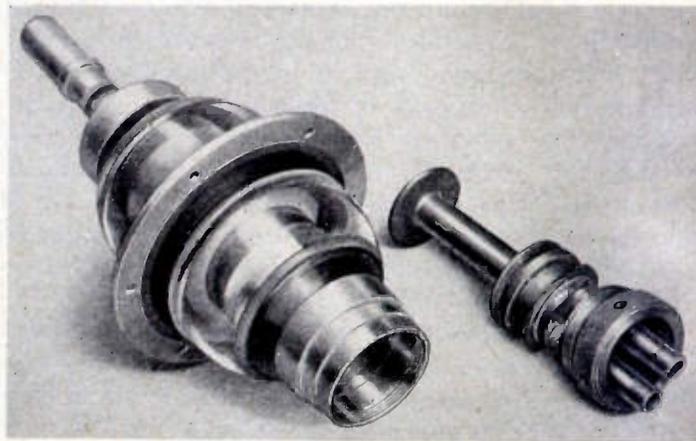
Fig. 1—The 1.5-kw 500-mc triode with water-cooling plug for the anode.
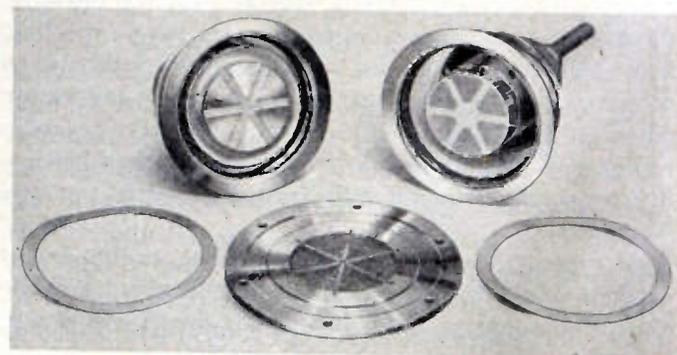
Fig. 2—The assembled components of the triode ready for sealing.

THE GRID

One of the most difficult problems in this development was that of the grid. High radio-frequency charging currents, electron bombardment of the grid support members and active elements, and heat radiation by the cathode all contribute to heating of the grid. In the presence of a cathode which evaporates active material, the grid may become contaminated by this material and emit electrons thermionically at relatively low temperatures. Appreciable thermionic emission by the grid results in reduced efficiency of operation. There are

* Decimal classification: R339.2. Original manuscript received by the Institute, April 20, 1950; revised manuscript received, November 2, 1950.
† Bell Telephone Laboratories, Inc., Murray Hill, N. J.
[1] S. Frankel, J. J. Glauber, and J. P. Wallenstein, "Medium power triode for 600 megacycles," *Elec. Commun.*, vol. 24, pp. 179–186; June, 1947.
[2] W. G. Wagener, "500-Mc transmitting tetrode design considerations," PROC. I.R.E., vol. 36, pp. 611–619; May, 1948.
[3] W. P. Bennett, E. A. Eshbach, C. E. Haller, and W. R. Keyes, "A new 100-watt triode for 1,000 megacycles," PROC. I.R.E., vol. 36, pp. 1296–1302; October, 1948.
[4] W. W. Salisbury, "The Resnatron," *Electronics*, vol. 19, pp. 92–97; February, 1946.

two obvious methods of attack upon this difficulty: one, to make the grid of a material which resists activation; and the other, to insure that the grid will operate at a temperature low enough that primary electron emission will be negligible. Copper is a material well suited to use in both of these methods. Its high thermal conductivity will serve to conduct heat away from the structure and its ability to absorb most thermionically active materials at relatively low temperatures makes it difficult to activate. Its principal disadvantage is its structural weakness in the soft state.

The grid of this vacuum tube consists of radial copper spokes attached to the lead-out copper annulus together with a spiral winding of copper wire swaged and brazed to the spokes, as shown in Fig. 2. The spokes are $1/8 \times 1/16$ inch copper bars and the spiral winding is 0.010-inch diameter copper wire wound 28 turns per inch. This grid has been found to have negligible emission when subjected to more than 50 watts of direct bombardment power plus rf resistance losses and cathode radiation incidental to ultra-high-frequency operation. Expansion of the parts of this grid takes place in the plane of the grid. The use of the same material for all parts eliminates any bi-metallic effect which could cause distortion normal to its plane.

The Cathode

The use of the conventional barium-strontium oxide-coated cathodes in power tubes has been restricted in the past to relatively low voltage operation with maximum plate voltages usually below 1,000 volts. It was found during the war, however, that the oxide-coated cathode could be used with reasonable success in various types of pulsed oscillators and modulators where the voltages were of the order of ten thousand. The principal

limitation seemed to be sparking which resulted in the removal of coating material from the cathode. On some of the magnetrons, schemes were employed for anchoring the coating more firmly to the cathode to enable it to withstand sparking. One of these schemes which was developed at Bell Laboratories was the use of a porous nickel matrix or sponge sintered to the base metal of the cathode.[5] The active coating material was soaked into the nickel matrix so as to remain firmly held after the usual exhaust and activation. These cathodes showed considerable superiority from the life standpoint where the possibility of sparking was serious. A cathode of similar type is used in this tube. The cathode is indirectly heated by means of a pancake tungsten wire spiral heater mounted directly behind the cathode disk. One end of the heater is tied to the cathode and the other end is connected to a terminal which projects from the end of the cathode terminal cylinder. This heater terminal also serves to cover the exhaust tubulation.

As can be seen in Fig. 2, the active coating is applied in "islands" with blank spaces in between. The grid is aligned with the cathode so that the grid support spokes register above the blank spaces in order that they will not collect electrons when the grid is positive with respect to the cathode.

The Anode

The anode, as shown in Fig. 2, is a copper disk which is slotted to correspond to the grid support spokes. This is necessary because the thickness of these spokes is comparable to the grid anode spacing, and sufficient clear-

[5] J. B. Fisk, H. D. Hagstrum, and P. L. Hartman, "The magnetron as a generator of centimeter waves," *Bell. Sys. Tech. Jour.*, vol. 25, pp. 167–348; April, 1946.
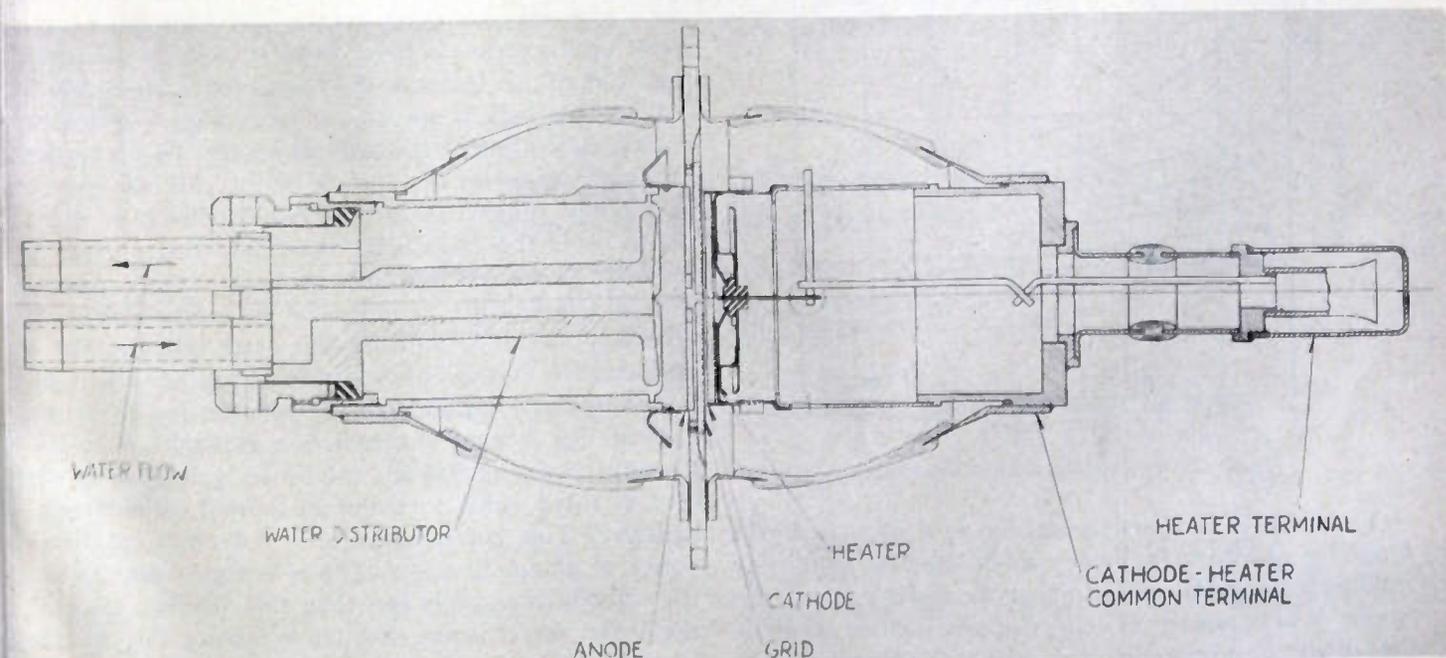
Fig. 3—Cross-section drawing of the 500-mc triode.

ance must be maintained to prevent voltage breakdown and excessive capacitance. The anode is cooled by means of water directed against its reverse face by the mushroom fitting shown in Fig. 1. With a cooling water flow of 0.7 gallon per minute, 2,500 watts can be dissipated. A cross-section drawing of the tube is shown in Fig. 3.

CHARACTERISTICS

With the cathode heated, the nominal spacing from cathode to grid is 0.015 inch and the spacing from grid to anode is 0.075 inch. The latter is greater than that which would be considered optimum in an ordinary tube. It was made greater mainly in the interest of higher voltage breakdown and lower plate-grid capacitance. The characteristics are still quite acceptable for the purpose for which the tube was designed. The total active cathode area is approximately 9 square centimeters. The diameter of the grid terminal disk is $4\frac{1}{4}$ inches and the over-all length from the end of the anode cylinder, exclusive of the cooling plug, to the end of the heater terminal is 9 inches.

The characteristic data and tentative maximum ratings are as follows:

Heater Voltage	12.6 volts
Heater Current, nominal	9.0 amperes
Amplification Factor	40
Tentative Maximum Ratings	
Max. dc Plate Voltage	3,000 volts
Max. dc Plate Current	1.25 amperes
Max. dc Grid Current	0.25 ampere
Max. Plate Dissipation	2.5 kilowatts
Nominal Inter-electrode Capacitances	
Grid to Cathode (Cathode Hot)	30 $\mu\mu f$
Grid to Anode	15 $\mu\mu f$
Anode to Cathode	0.4 $\mu\mu f$.

The static characteristics of the tube are shown in Fig. 4.
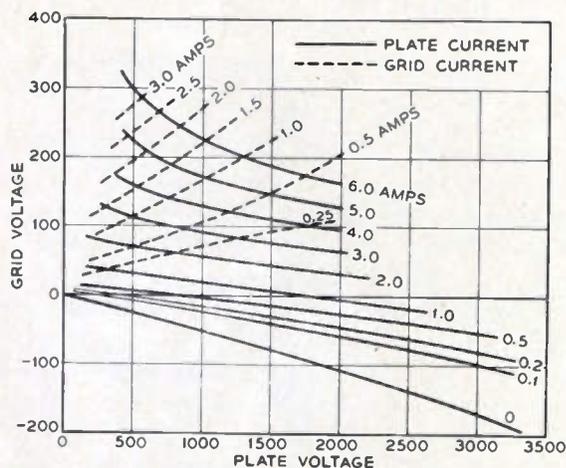
Fig. 4—Static characteristics of the triode.

The tube is intended primarily as a grounded-grid amplifier of either frequency modulated or amplitude modulated signals. In addition to water-cooling the anode, it is necessary to deliver approximately 50 cubic feet of air per minute directed against the glass envelope at both the cathode and anode ends of the tube.

Typical performance of the tube as a grounded-grid amplifier in the range from 400 to 700 mc is shown in Fig. 5. Although full output is obtainable at frequencies above 700 mc, the power gain in this region is so low that the value of the amplifier is questionable.
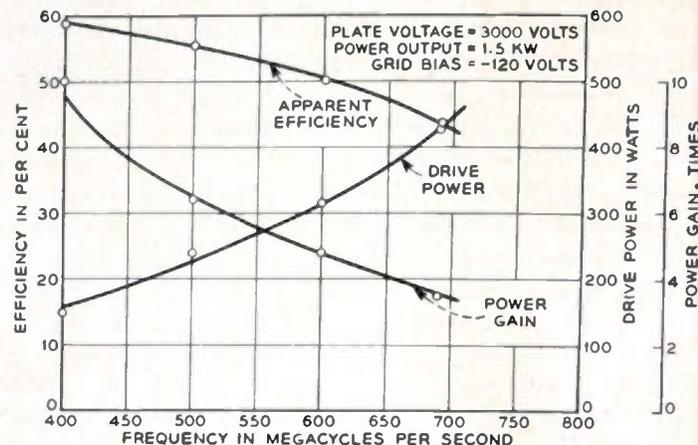
Fig. 5—Performance of the triode as a grounded-grid amplifier.

As the frequency is increased, transit-time effects and circuit losses increase rapidly. The power output recorded is that measured calorimetrically in a water-cooled resistor. The driving power is that measured in a coaxial line feeding the input circuit of the amplifier. Three-quarter wave coaxial circuits were used for both input and output.

Some positive feedback is unavoidable. It results mainly from inductance of the grid which is common to both input and output circuits. It is necessary that this feedback be kept to a minimum in order to assure stability. Such feedback reduces the drive required and increases the power gain of the amplifier. Although this feedback is usually greater at high frequencies than at low frequencies, its effect here is overshadowed by the more rapid increase in drive required because of transit time and circuit losses as the frequency is increased.

When the tube is operated at ultra-high frequencies, additional heating of the cathode occurs. This is caused by ohmic losses on the surface of the cathode and by back-bombardment of the cathode by electrons which fail to reach the grid during the conducting portion of the cycle. Since it is desirable to operate the cathode at a fixed temperature, a reduction in heater voltage is necessary to compensate for this extra heating. Fig. 6 indicates the amount of reduction in heater voltage required under the conditions of operation for which the data of Fig. 5 were obtained. For example, when delivering 1.5 kw at 500 mc, the heater voltage is 87 per cent of rated value to maintain normal cathode temperature. This corresponds to a reduction in heater power of about 30 watts. The rf energy dissipated in the cathode is actually less than this amount, since all the heater power is not effective in heating the cathode in this tube. The amount of this rf heating of the cathode depends to a considerable extent on the circuit
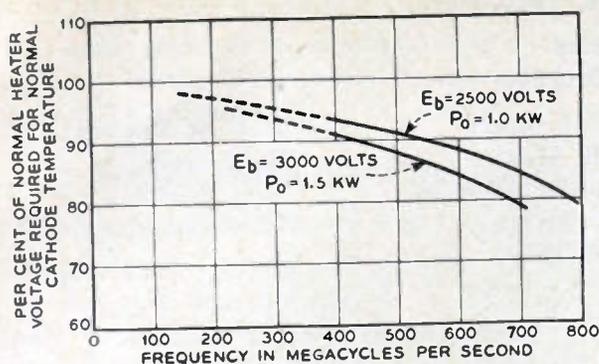
Fig. 6—The reduction in heater voltage required to compensate for cathode heating in uhf operation.

adjustment. The circuit adjustments for which these data apply were those which resulted in maximum efficiency for the conditions noted and in general resulted in about the minimum rf heating of the cathode.

No bandwidth measurements were made in these tests. Bandwidths calculated from the output capacitance and load resistances appropriate for the tube ratings are of the order of 10 mc. In an experimental transmitter using some early models of this tube in a two-stage amplifier at 200 mc, a bandwidth of 17 mc was obtained with a relatively simple circuit arrangement.

The foregoing describes results obtained with an experimental tube in the laboratory. The tube is not available on a commercial basis, and there is as yet no definite plan to make it so. The results are of interest mainly to show what has been done toward making higher power available in the 500-mc range, and to indicate a method of solution of some of the problems. No extensive life test data are available but indications are that performance in excess of 1,000 hours should not be difficult to attain.

ACKNOWLEDGMENT

As with any such project, many people contributed to its success, and the authors gladly acknowledge these contributions.

Standards on Radio Receivers: Open Field Method of Measurement of Spurious Radiation from Frequency Modulation and Television Broadcast Receivers, 1951*

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1. INTRODUCTION

Modern broadcast radio receivers of the superheterodyne type are frequently sources of spurious radiation from the local oscillator, which radiation may cause severe interference with other services. In addition, in the case of television broadcast receivers, there may be radiation of power from other sources beside the local oscillator. This Standard describes the potential sources of spurious radiation from frequency modulation and television broadcast receivers and sets up methods of measurement whereby the strength of some of these radiations may be determined. Where the methods for the two classes of receivers differ, the specifications for each are outlined.

2. SOURCES OF SPURIOUS RADIATION

2.1 The local oscillator fundamental and harmonics.

2.2 Intermediate-frequency amplifiers which may radiate spurious signals at the fundamentals, and harmonics or combinations of the intermediate frequencies.

2.3 In some television receivers, the high voltage for the cathode-ray tube is obtained by means of a radio-frequency oscillator, which is then a potential source of radiation at its fundamental and harmonic frequencies.

2.4 In television receivers the sweep circuits may radiate harmonics of their fundamental frequencies.

2.5 In television receivers the video amplifier and any nonlinear circuit elements which may produce signals by demodulation of the radio or intermediate-frequency signals.

3. MODES OF RADIATION OF SPURIOUS SIGNALS

3.1 Spurious signals emanating from any source, such as those described in Section 2, may appear on the receiver antenna terminals and be radiated from an antenna system attached thereto. Such signals may be balanced or unbalanced to ground.

3.2 Radiation from the sources enumerated in Section 2 may occur in the vicinity of the receiver due to direct electric and/or magnetic fields created by the components assembled on or within the chassis, or by cavity resonances of the chassis itself.

3.3 Radiation from any of the sources may be propagated through the power supply cord. Such radiation may be due to balanced or unbalanced currents flowing in the line cord. This Standard does not cover measurement of spurious radiation from this source.

4. METHODS OF MEASUREMENT

4.1 General

This Standard establishes a system whereby the radiation described above may be measured, and sets up operating conditions and specifications for equipment for accomplishing these measurements. In this method, the receiver and measuring equipment are set up under reproducible conditions as described below, and the field strengths of the various radiations are measured under specified conditions.

The method described herein is primarily applicable to the measurement of spurious radiation in the uhf and vhf bands from those sources producing an ap-

preciable field at 100 feet. For sources producing radiation in other bands or those producing comparatively insignificant fields at 100 feet other methods of measurement may be necessary.

4.2 Equipment Required and Setup Details

4.2.1 Receiver Platform and Antenna

The platform upon which the receiver under test is to be mounted shall be located on relatively level ground, free of any obstruction to a distance of at least 100 feet from both the receiver and the field strength meter, and away from reflecting objects. This platform shall carry a nonconducting rotatable antenna mast. For convenience, it may be desirable to have the mast and the platform rotatable as a unit. This mast shall carry at a height of 30 feet a single straight dipole antenna constructed of $\frac{1}{2}$-inch od tubing. A transmission line of the type and characteristic impedance for which the receiver under test is designed, shall be attached to this dipole at its center and shall be vertically disposed under the center of the dipole with the plane of the maximum cross sectional dimension of the transmission line parallel to the dipole. This transmission line shall be secured to the pole at sufficiently frequent intervals to insure mechanical stability against movement.

When testing receivers designed for use with unbalanced shielded transmission line, the antenna may be connected to the transmission line through a balanced-to-unbalanced transformer of the type, if any, specified by the manufacturer. If the receiver is designed to operate from either an unbalanced transmission line or a 300-ohm balanced line, the latter shall be used in these measurements.

If the receiver under test is a table model, it shall be placed upon a table, the top of which is 48 inches above ground. If the receiver is a floor model, it shall be located on a platform, the top of which is 18 inches above ground. In the case of those receivers where the 'lead-in does not reach the antenna terminals when mounted in accordance with Section 4.2.1.3, the receiver shall be raised the necessary amount. The plane of the front panel of the cabinet shall be parallel to the dipole, and the center of the cabinet shall be directly below the center of the dipole.

4.2.1.1 For vhf television receivers the dipole antenna shall measure 88 inches from end to end. For uhf television receivers the length shall be 12 inches from end to end.

4.2.1.2 For frequency modulation receivers having terminals for an external dipole, the length shall be 58 inches from end to end.

4.2.1.3 Disposition of Transmission Line. The transmission line shall be 28 feet in length and shall proceed horizontally directly back from the antenna terminals of the receiver, then vertically upward to a height of 84

inches above ground, and finally, again horizontally, to a position vertically disposed under the center of the dipole; and after the first bend, shall not approach closer than six inches to the cabinet at any point. Fig. 1 illustrates a possible method of routing the transmission line.
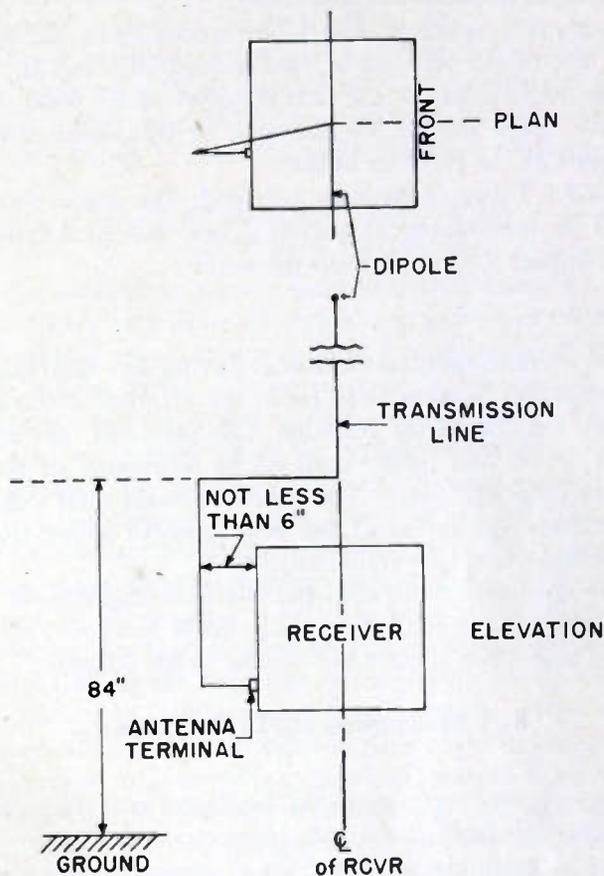
Fig. 1—Possible method of routing transmission line.

4.2.2 Field-Strength Meter

A suitable field-strength meter shall be provided, equipped with an antenna capable of being adjusted for horizontal or vertical polarization. The field-strength-meter antenna shall be located at a distance of 100 feet horizontally from the antenna of the receiver under test.

The transmission line from the field-strength-meter antenna shall be disposed horizontally for a distance of 24 inches in a direction directly away from the receiver under test, then dropped approximately vertically to the field strength meter.

Provision shall be incorporated whereby the height of the antenna above ground may be varied from seven feet to twenty feet.

4.2.3 Power Supply

Power lines to both the receiver under test and the field strength meter shall be buried to a depth of 12 inches or more, and the power outlet at each location

shall be at the top of a metal conduit which extends not more than 18 inches above the level of the ground, and at the receiver location shall be not more than 12 inches from the vertical axis of the antenna. The power cord of the receiver under test shall be short and as vertical as possible, with any excess length bundled up as close as possible to the plug end.

Adequate isolation shall be incorporated in the supply line to the receiver so that a negligible amount of signal will appear at the field strength meter from this source. Each side of the line shall be by-passed to the conduit at the receiver outlet.

4.2.3.1 Unless otherwise specified, the line voltage shall be maintained at within 2 per cent of specified line voltage during all measurements.

4.2.4

No extraneous metallic object having any dimension in excess of 6 inches shall be in the vicinity of the set under test, or of its antenna. The platform, antenna mast, guys, and tables shall all be fabricated of non-conducting materials. The antenna mast and other associated equipment at the field strength meter shall likewise be free of metallic objects.

The ground at the site shall be as homogeneous as possible, with special effort being made to avoid gravel paths and other similar low-conductivity features.

4.3 Measurements To Be Made

4.3.1

The receiver under test shall be tested with the transmission line from the dipole connected directly to its antenna terminals. A second set of measurements shall be made either with these connections reversed or, in the case of coaxial lines, with the receiver antenna rotated 180 degrees with respect to the receiver chassis.

4.3.2

If the receiver is supplied with a built-in antenna, measurements shall also be made with this antenna connected to the antenna terminals. Any adjustment provided for tuning this built-in antenna shall be set at that position producing maximum radiation. If it can be demonstrated that the disconnected standard dipole antenna and transmission line mounted on the platform affect the field strength, they shall be removed for this test. In the case of receivers utilizing the power line as an antenna, separate tests, other than the open field measurements, may be necessary to determine the amount of potential radiation due to voltages existing on or between the wires of the cord.

4.3.3

As a means of measuring direct chassis radiation, the receiver shall be tested with the antenna terminals terminated in a noninductive resistor equal to the impedance for which the receiver is designed. A precaution with respect to the disconnected antenna, similar to that in the above Section, shall be observed.

4.4 Method of Measurement

The receiver under test shall be checked over the frequency range to which it is intended to be tuned. The number of test frequencies shall be adequate to insure measurement of maximum radiation within this range.

With the field-strength meter tuned to the frequency of the spurious radiation being measured, and with its antenna aligned broadside to the receiver under test and 20 feet above ground, the receiver under test and its dipole antenna shall be rotated together in a horizontal plane until maximum signal at the field-strength meter is obtained. The antenna of the field-strength meter is now varied in elevation from twenty feet to seven feet, holding it broadside to the receiver under test, and the maximum reading of field strength so obtained is recorded as the radiation strength of the receiver under test. These tests are now repeated with the receiver antenna reversed, as in Section 4.3.1.

The above tests shall be repeated with the field-strength-meter antenna aligned for vertical polarization.

If reversing the antenna of the field strength meter produces an appreciable difference, the results so obtained shall be averaged.

CORRECTION

Rudolf C. Hergenrother, co-author with B. C. Gardner of the paper, "The Recording Storage Tube," which appeared on pages 740–748 of the July, 1950, issue of the PROCEEDINGS OF THE I.R.E., has requested that the editors publish the following correction:

The coefficient in equation (5), page 747, should read 8.85×10^{-14}, instead of 8.85×10^{-10}.

A Simple Logarithmic Receiver[*]

JOSEPH CRONEY†

Summary—A receiver with a logarithmic input-output law can be used with advantage on radar equipment, but it has hitherto been difficult to obtain such a law without the addition of a considerable number of tubes. One system by which such a law can be obtained is described and a circuit is given which shows how the method may be simplified to produce a receiver which can be either logarithmic or linear by the operation of a single switch. A method of predicting the shape of the input-output curve of such a receiver is described. The measured input-output curves for three logarithmic receivers of this simplified type are shown and the relevant circuit values specified. Some design notes are added.

A note on the use of the logarithmic receiver as an anticlutter device is followed by consideration of the display of signals from a logarithmic receiver, and a brief discussion on bandwidth variations.

I. Introduction

A RECEIVER which has a logarithmic relationship between output volts and input volts over its working range of inputs may be called, perhaps loosely, a logarithmic receiver. Receivers of this type can be used with advantage in place of linear receivers in radar equipments.

For instance:

(a) A logarithmic receiver preserves amplitude discrimination at its output between all signals fed to its input, throughout the working range of inputs (usually about 80 db). For this reason no gain control is necessary on logarithmic receivers, and the receiver output can be calibrated directly in decibels, the calibration not being dependent upon some arbitrary setting of gain control.

(b) A radar equipment being used over sea will receive random returns from the sea surface (sea clutter). Such signals produce, at the output of the receiver, a waveform to some extent similar to random tube and circuit noise in which the peak value of the impulses bears a constant ratio k to the mean value. Returns from rain clouds bear an even closer resemblance to the receiver random noise structure as displayed on a cathode-ray tube. If, therefore, noise plus sea clutter or noise plus rain clutter is fed to a logarithmic receiver which is followed by a simple RC differentiating circuit to remove the mean value, the peak output level of noise plus clutter is reduced to a constant value across the cathode-ray tube trace.[1] Since the receiver cannot saturate, signals which have an amplitude greater than the peak clutter amplitude, then appear quite clearly above this constant level of noise plus clutter on the display. The logarithmic receiver may thus replace the linear receiver fitted with a

sensitivity time control (STC) with the advantage that no adjustment is needed as with the STC, to allow for differing degrees of roughness of the sea.

There are several methods by which a receiver may be made to give a logarithmic relationship between input and output.[1,2] The author's work has been devoted entirely to the successive-detection method.

II. The Successive-Detection Logarithmic Amplifier

An ordinary linear intermediate-frequency (IF) amplifier has a chain of amplifying tubes the final one of which feeds a detector. Over a small range of input the output bears a linear relation to the input, but linearity rapidly ceases to hold and very soon an input is reached beyond which no further increase in output is possible, the final IF tube being saturated.

The successive-detection amplifier differs from this in that it has a detector after every IF stage so that each stage besides feeding an IF input to the next stage in the normal way, can make an independent contribution to the video output of the amplifier, the output of all the detectors being combined. The input to such an amplifier can clearly be increased until every IF amplifying tube in the chain is saturated before the total output reaches a saturation value. This means that if we have two amplifiers, each of n stages with stage gain m, the one linear and the other of the successive-detection type, the latter amplifier can accept an input m^{n-1} times as great as can the linear amplifier before saturation takes place.

The following simple consideration of the successive-detection amplifier with n stages, of gain m, in which the output of the detectors are all added directly, will show that the input-output relation is approximately logarithmic.

Let us assume the saturation output from each of the IF stages to yield a rectified voltage V from the detector attached to the stage. We can imagine a signal generator to be connected to the input of the amplifier and its attenuator adjusted until the saturation output V is just derived from the last detector at, say, a signal generator input of v. If the generator input is now turned up m times to mv, we shall get an additional output V derived from the penultimate $(n-1)$ IF stage which now saturates giving a total output $2V$ from the last two detectors. If the generator output is now turned up a further m times to m^2v, we get an additional output V from the $(n-2)$th IF stage giving $3V$ in all. It is clear that this step-by-step process can be continued until the first IF tube in the successive-detection

¹ R. V. Alred and A. Reiss, "An anti-clutter radar receiver," *Jour. IEE* (London), vol. 95, part 3; November, 1948.

² S. N. Van Voorhis, "Microwave Receivers," *Rad. Lab.*, vol. 23, pp. 583–606, McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

amplifier saturates giving a total output from the amplifier of nV for a signal generator input $m^{n-1}v$. The signal generator inputs for all these steps will be in geometric progression and the amplifier outputs in arithmetic progression, hence an approximate logarithmic input-output law has been achieved.

Approximate because in the simple argument above the input steps taken are from the condition for saturation of one IF tube to saturation of its predecessor in the chain, and clearly each of these points lies on a perfect logarithmic curve. Whether or not intermediate points are on the curve will depend upon the law connecting input and output voltage for any single typical IF stage, up to the input which saturates the stage. This point is given further consideration in Section V.

III. THE ORIGINAL LOGARITHMIC AMPLIFIER CIRCUIT

The author's first experiments were with a 10-stage amplifier having a bandwidth of 8 mc centered at 30 mc. The amplifier incorporated band-pass coupled circuits of conventional type. From each control grid a detector (diode), fed video signals to a load resistance placed between grid and ground of a buffer video amplifying tube. Each stage in the logarithmic amplifier, therefore, required three tubes (one IF amplifier, one detector, one video amplifier). The outputs of all the video amplifiers were combined by giving them a common anode load. The anodes were connected together through an artificial delay line having one section between each anode, the delay per section being equal to the delay per stage of the IF amplifier. The delay line was terminated at each end by its characteristic impedance, the common anode load of the video amplifiers being formed by the parallel value of the two terminating resistances.

The video delay line is necessary in all amplifiers of this type when used for pulse reception, because the output pulse is built up from elements derived from each IF stage in the amplifier each element being therefore subject to a different IF delay. The delay line by bringing all the elements together at one time at the output, prevents the output pulse from having a stepped leading edge.

This type of amplifier is, of course, too costly in tubes and space to be of general application, and the author has derived a much simpler version. Before passing to this, however, some of the results derived from the above type of amplifier will be of interest.

Two exactly similar 10-stage amplifiers were constructed, since it was desired to divide the signal in one channel by the signal in another by subtracting the amplifier outputs. Miniature pentodes were used for the IF and video amplifying tubes and diodes as the detectors. No selection of tubes was necessary to match the amplifier characteristics. Fig. 1 shows the input-output curve for the amplifiers, the readings for one amplifier being represented by dots and those for the other by crosses. Fig. 2 shows the curves repeated after

a complete change of tubes in the amplifier, and without any preselection of tubes.
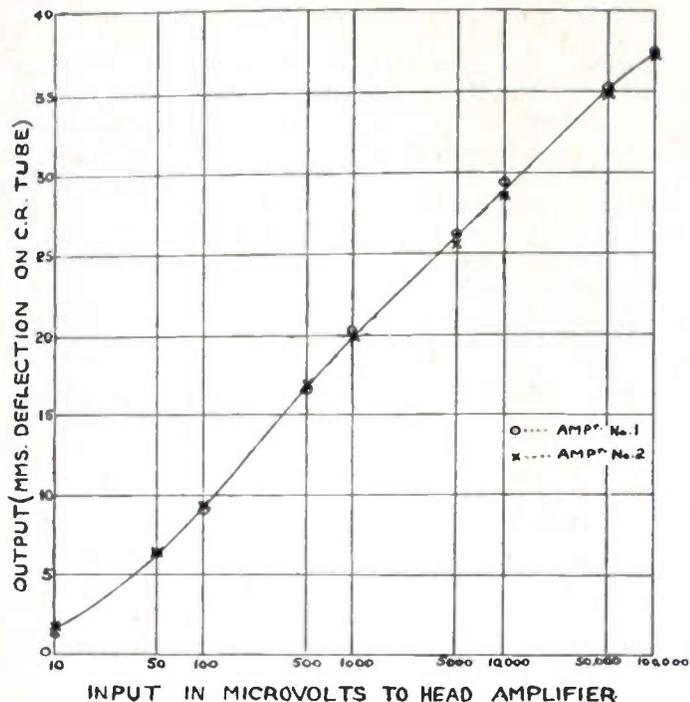
Fig. 1—Input-output curve for matched pair of receivers with log amplifiers of type using video buffer tubes.
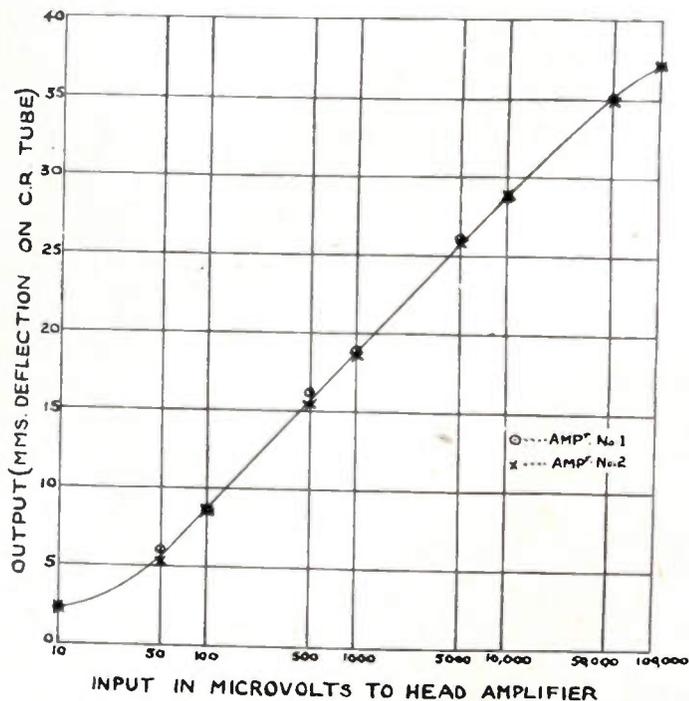
Fig. 2—Curve of Fig. 1 repeated after complete change of tubes, and with no attempt at tube selection.

IV. THE SIMPLIFIED LOGARITHMIC AMPLIFIER CIRCUIT

Fig. 3 shows the circuit of the simplified amplifier. To save space, three stages only are shown. The diode detectors have been replaced by germanium crystals and the chain of buffer video amplifying tubes has been
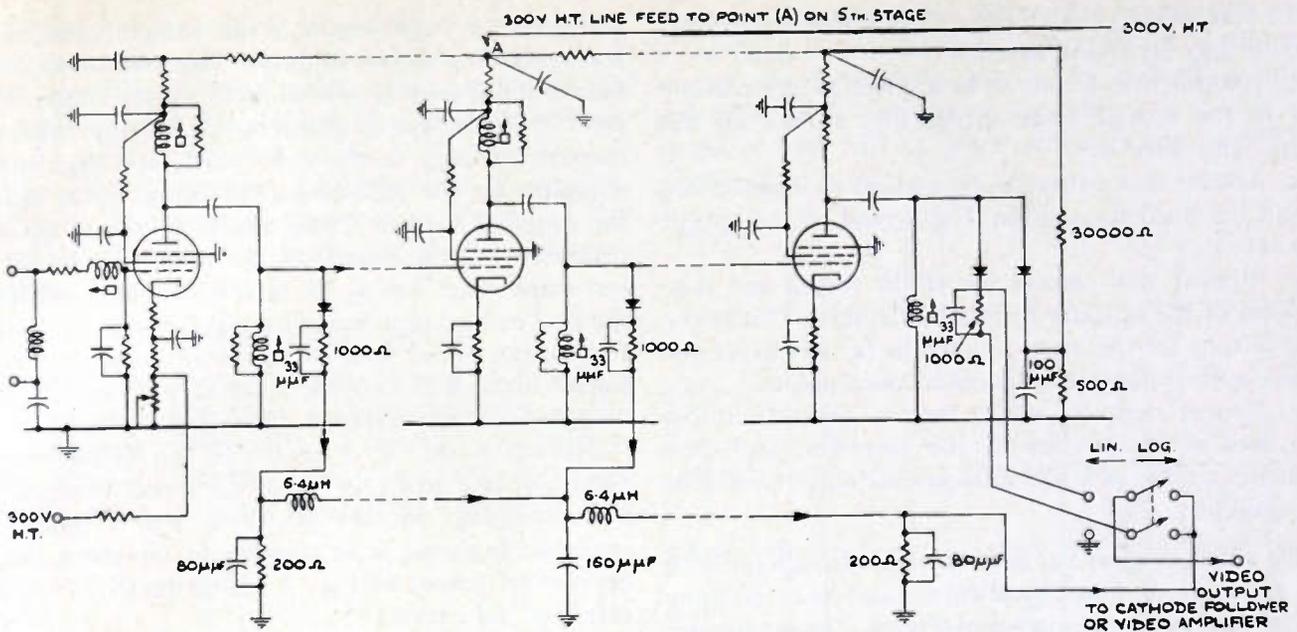
Fig. 3—Circuit of first and tenth stages of successive-detection logarithmic amplifier together with a typical intervening state. IF stages band-pass "top-end" capacity-coupled.

cut out. Isolation of the detectors from each other has been achieved sufficiently well by giving all the crystal detectors an individual load, and combining the load currents in a small common section formed by the terminating impedances of the delay line. The biasing effect of one detector upon its predecessor is now tapped down by the ratio of individual load to common load and can be made small enough not to matter. The purpose of the extra crystal shunting the last crystal detector will be explained in the next section.

A useful feature of this type of amplifier is that it can be switched to be an ordinary linear amplifier by a simple two-pole two-way switch as shown in Fig. 3. In the linear position the delay line is left isolated (but still terminated), and the amplifier output taken from the upper end of the last detector load which has its lower end switched to ground instead of to the delay line. In this condition, the noise output from the amplifier will, of course, be much greater since it is no longer tapped down by a potentiometer network, but a preset gain control can be included in one of the earlier stages to readjust the noise level to the working value.

Experiments were performed on three amplifiers of this type: (a) a 10-stage amplifier of 8-mc bandwidth centered at 45 mc; approximate gain, 8 db per stage; (b) a 5-stage amplifier of 4-mc bandwidth centered at 60.5 mc; approximate gain, 18 db per stage; and (c) a 4-stage amplifier of 0.5-mc bandwidth centered at 13.7 mc; approximate gain, 20 db per stage.

V. The Input-Output Relationship

Fig. 4 shows the curves relating input and output for a typical single IF stage of the amplifier of 8-mc bandwidth, and the amplifier of 4-mc bandwidth. Using
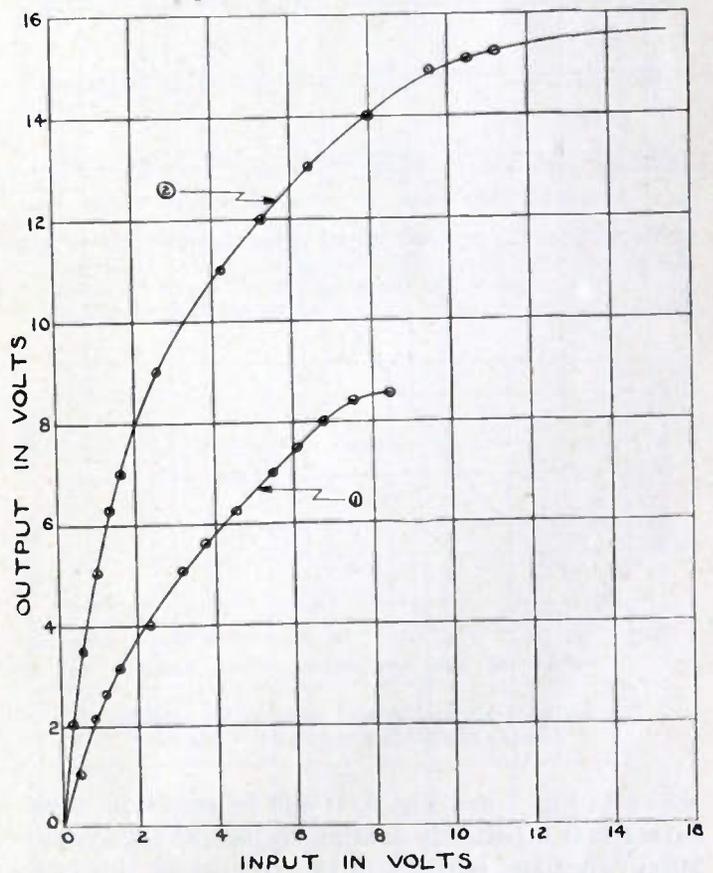
Fig. 4—Input-output curves for typical single IF stages of log amplifiers of 8-mc bandwidth (curve 1) and 4-mc bandwidth (curve 2).

these curves it is possible to construct input-output curves for the complete IF amplifiers in the following way. Taking the case of the 10-stage amplifier:

(a) Assume a small input to be applied to the grid

of the first tube in the amplifier. From Fig. 4 (curve 1) read off the output and note it.

(b) Assume this output to be applied as input to the grid of the second stage. From Fig. 4 read off the output and note it.

(c) Assume this output to be applied as input to the grid of the third stage. From Fig. 4 read off the output and note it.

(d) Repeat this process for all 10 stages and take the sum of the outputs from all 10 stages. This is the total output for the input applied in (a) and gives one point on the input-output curve of the amplifier.

(e) Repeat steps (a) to (d) for any desired number of values of input volts to the amplifier, and thus construct a table of input volts against output volts for the amplifier.

The above process has been worked through for both the amplifier of 8-mc bandwidth, and that of 4-mc bandwidth, using the curves of Fig. 4. The results are
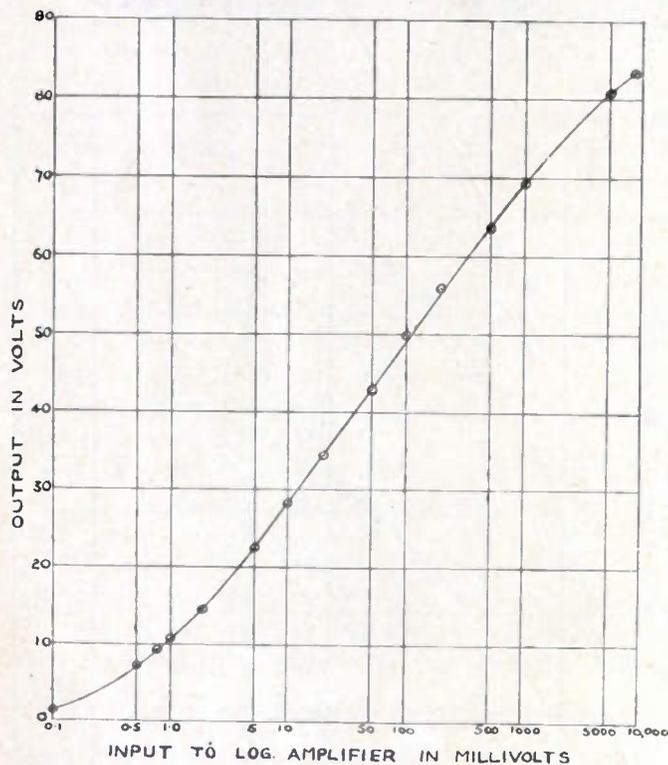
Fig. 5—Predicted input-output curve for log amplifier of 8-mc bandwidth, using curve 1 of Fig. 4.

shown in Fig. 5 and Fig. 6. It will be seen from these curves that a perfectly continuous logarithmic law relating input and output can be expected for this type of amplifier, and this disposes of the point left unanswered in Section II.

It is essential that all stages in the amplifier should be similar not only with regard to tubes but to operating conditions, within the normal limits allowed by component tolerances.

The curves of Fig. 4 show that the IF stages are amplifying linearly for small inputs, the stage gain

falling off for larger inputs, until the saturation output is reached with a gain of unity. This saturation output level is influenced by several factors, such as the curvature of the dynamic characteristic of the tube, grid current damping in the tube itself, and grid current damping by the following tube acting back through the coupling circuits. These effects will be of the same magnitude for all stages in the amplifier, except the last stage which has no IF tube following it to provide the grid current damping effect. To prevent the saturation output of the final IF tube from being as a result higher than all the others, a suitably biased crystal is used to limit the output to the required level (Fig. 3).

The curves of Fig. 4 cannot be conveniently represented by one equation except by a polynominal with a considerable number of terms. For a qualitative analysis, however, it is possible to represent the experimental curves of Fig. 4 approximately by parabolas, e.g., for curve 1:

$$(v_{n+1} - 8.43) = - 0.111(v_n - 8.73)^2. \qquad (1)$$

Assuming a parabolic law, the output voltage of the amplifier which is given by the sum

$$u = \sum_1^{n+1} v_n$$

can be expressed analytically as a function of input voltage v_0. This has been done and the result shows u to be a linear function of log v_0 over a restricted range of v_0, whereas experimentally this holds over a considerably wider range. The agreement, therefore, is of a qualita-
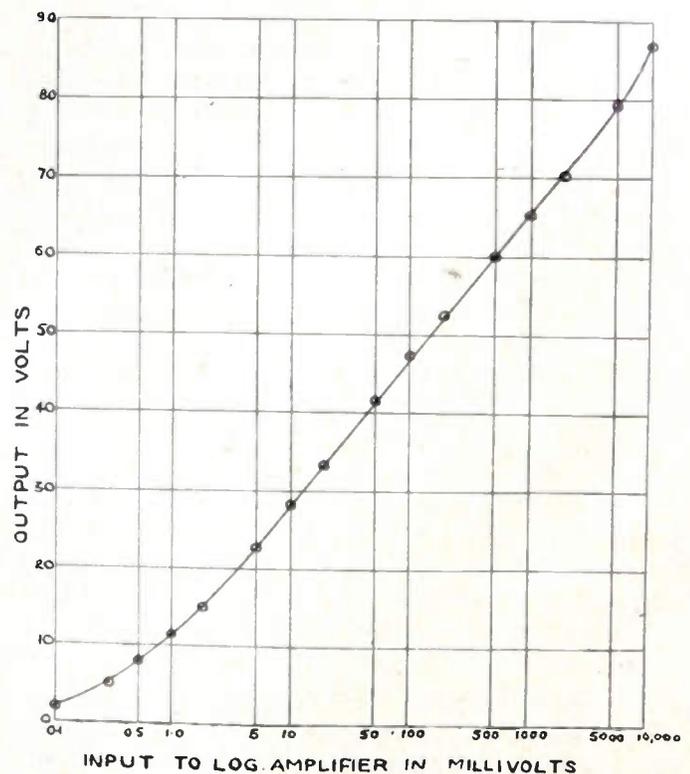
Fig. 6—Predicted input-output curve for log amplifier of 4-mc bandwidth, using curve 2 of Fig. 4.

tive nature only, as is to be expected since a parabola is only a crude approximation to the experimental curves, especially for small values of v_n. Attempts have been made to solve this problem by finding expressions which fit the experimental curves of Fig. 4 more accurately, but the algebra rapidly became intractable. The graphical solution is, therefore, submitted as the only practical method of predicting the over-all performance.

VI. Experimental Results

The experimental results were obtained by feeding a 10-microsecond pulse from a signal generator to the head amplifier of the logarithmic receivers and measuring the height of the output pulse on a cathode-ray tube.

Fig. 7 (curve 1) is an input-output curve for the
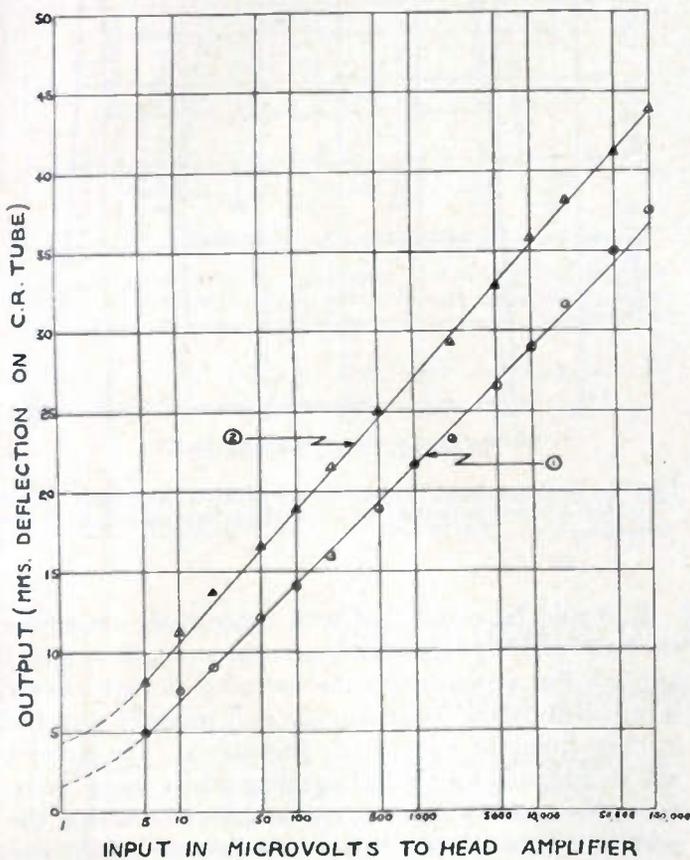
Fig. 7—Input-output curves for log receiver of 8-mc bandwidth. Individual detector loads of log amplifier = 1,000 ohms. Curve 1, common detector load 100 ohms, detectors on all stages. Curve 2, common detector load 250 ohms, detectors on alternate stages.

amplifier of 8-mc bandwidth, having the detector circuit values of Fig. 3. The law is closely logarithmic over an input range of 86 db. Curve 2 is taken for the same amplifier using a common load of 250 ohms in place of the 100 ohms, and with crystal detectors applied to alternate stages only. It is, therefore, possible to obtain a smooth logarithmic response from a multistage amplifier by using crystals on every other stage.

Fig. 8 shows the input-output curves for the amplifier of 4-mc bandwidth. Curve 1 was obtained with crystals on every stage, and curve 2 with crystals on alternate stages. The individual loads were 5,600 ohms, and the

common load, 340 ohms. Although a satisfactory result is obtained in this instance with crystals on alternate stages, the practice is not to be recommended on amplifiers with 5 stages or less, since the failure of a single crystal will be more serious when so few are employed.
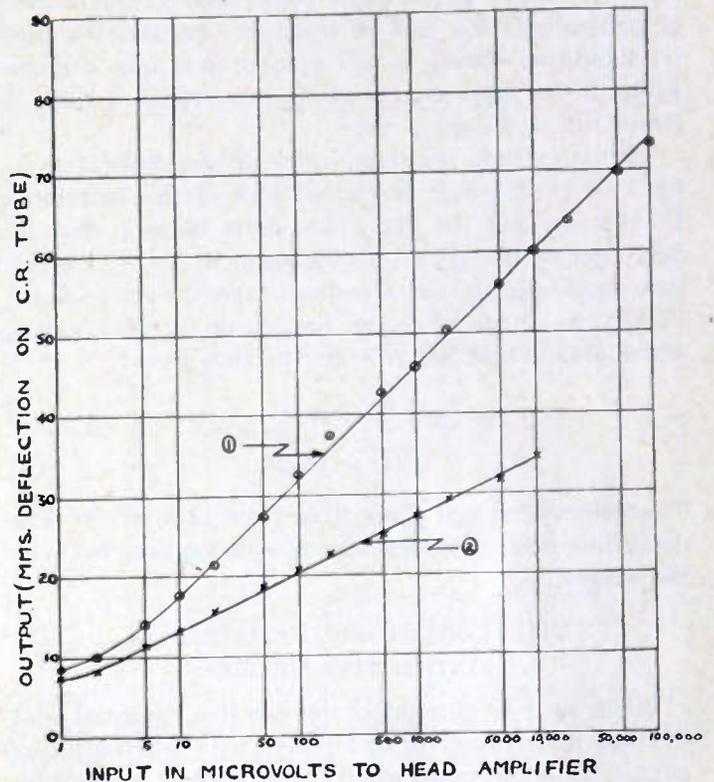
Fig. 8—Input-output curves for log receiver of 4-mc bandwidth. Individual of log amplifiers = 5,600 ohms. Common detector load = 340 ohms. Curve 1, detectors on all stages. Curve 2, detectors on alternate stages.
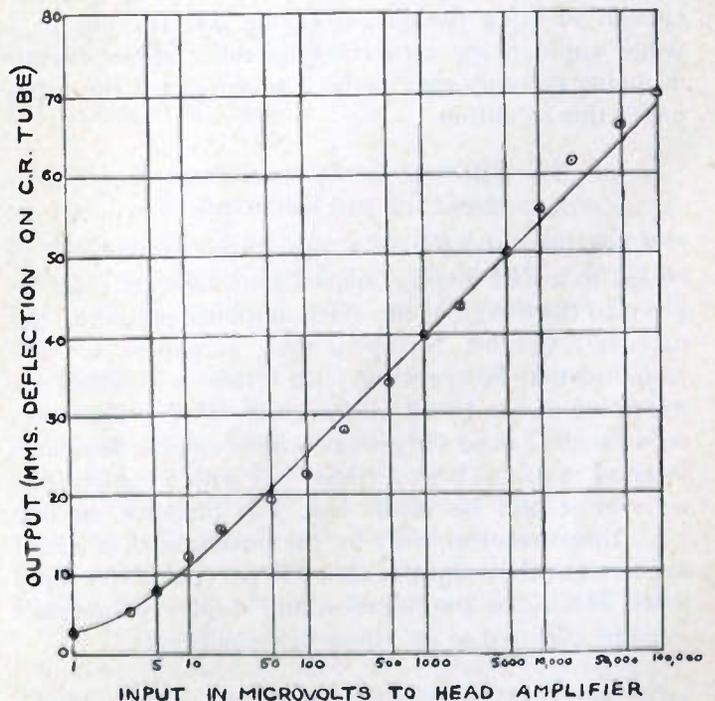
Fig. 9—Input-output curve for log receiver of 0.5-mc bandwidth. Individual detector loads 15,000 ohms. Common detector load 3,000 ohms.

Fig. 9 shows the input-output curve for the amplifier of 0.5-mc bandwidth. The individual loads were in this case 15,000 ohms and the common load 3,000 ohms.

VII. Design Notes

The value decided upon for the crystal individual loads (R) will be governed by the normal considerations of pulse distortion and IF loading. The common load (r) should be chosen as not greater than one-fifth the value of the individual load, and one-tenth is quite a reasonable fraction to take.

The delay time per stage in the IF amplifier is given by $T_D = 1/\pi B$ where $B =$ bandwidth of the individual IF circuits, and the delay line must be such that its delay per section $T_s = \sqrt{LC}$ is equal to T_D. ($L =$ series inductance per section, $C =$ shunt capacity per section.) Finally we must have the line correctly terminated, which means that $\sqrt{L/C} = 2r$. We thus have:

$$\sqrt{L/C} = 2r. \tag{2}$$

$$\sqrt{LC} = 1/\pi B. \tag{3}$$

The values of L and C are fixed from (2) and (3). The delay line itself provides adequate decoupling between the stages.

VIII. Logarithmic Receivers as Anticlutter Devices

When used as anticlutter devices it is essential that the logarithmic receiver be followed by a differentiating circuit.[1] Only one point will be made here. It is essential, when used as an anticlutter measure, for the receiver to be logarithmic not only up to the maximum signal level received, but also down to below noise level. This means that the gain of the whole receiver must be enough to bring the output of the last IF tube to a value approaching saturation on noise alone. Screen dropping resistors may be used to safeguard the tubes under this condition.

IX. Display of Signals from a Logarithmic Receiver

When the output from a logarithmic amplifier is to be fed to a PPI display, special consideration must be given to the design of the video amplifier preceding the cathode-ray tube. Normally such amplifiers are adjusted so that full painting on the tube is obtained for echoes of power 10 db above noise. With a linear receiver such an echo will give an output amplitude signal-to-noise ratio of about 3 times, but with a logarithmic receiver it will be much less. For instance, in one logarithmic receiver used by the author such a signal appeared at the output at about 50 per cent above noise level. The video amplification and display parameters must be adjusted to suit these new conditions.

X. The Bandwidth of Logarithmic Amplifiers

Figs. 10, 11, and 12 show response curves for the three amplifiers. These were obtained by varying the

input level and frequency of input voltage to maintain the output constant at a level of 40 db below the saturation output of the receiver, that is, with the amplifier working at approximately the middle of its logarithmic range.
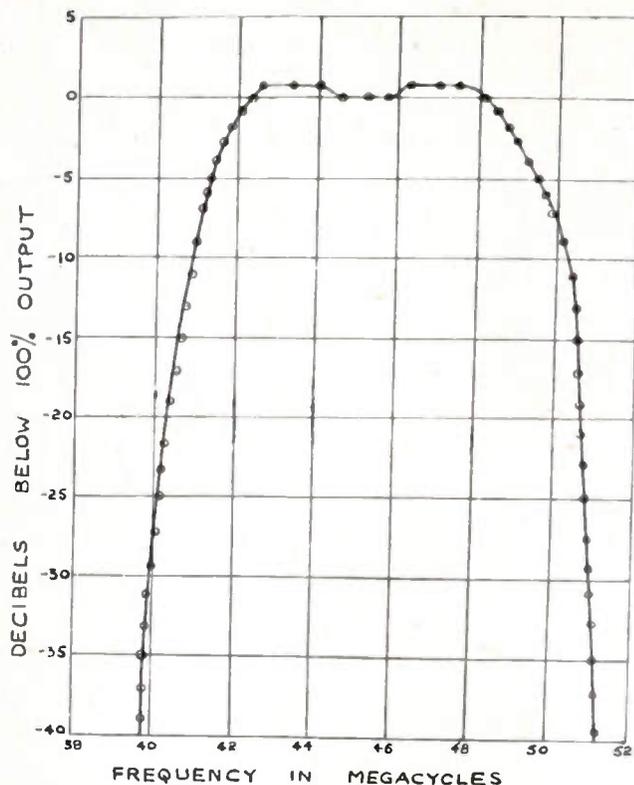
Fig. 10—Response curve for log receiver of 8-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.

It should be noted that with logarithmic amplifiers the bandwidth is dependent upon the strength of signal applied. For weak signals the damping on each circuit is provided by the resistance across it together with the loading from the tube input impedance. The circuits will be adjusted for critical coupling under these conditions. Now as the input to the amplifier increases, the grid circuits of the band-pass pairs become one-by-one more heavily damped due to grid current than under the weak signal condition. Measurements on the amplifiers of 8- and 4-mc bandwidth showed the damping to increase by some 20 per cent through this effect. As a result the coupling drops below critical, and the bandwidth for strong signals, therefore, tends to be reduced.

Another consideration is that strong signals at the output of the amplifier will be built up by contributions from every detector stage, and each contribution will have passed through a different IF bandwidth gate.

The situation regarding bandwidth of logarithmic amplifiers is, therefore, a little obscure, and possibly some new definition of bandwidth is required in relation to them. However, several of the amplifiers described in this paper have been in continuous use on radar equipments, and when compared with linear amplifiers
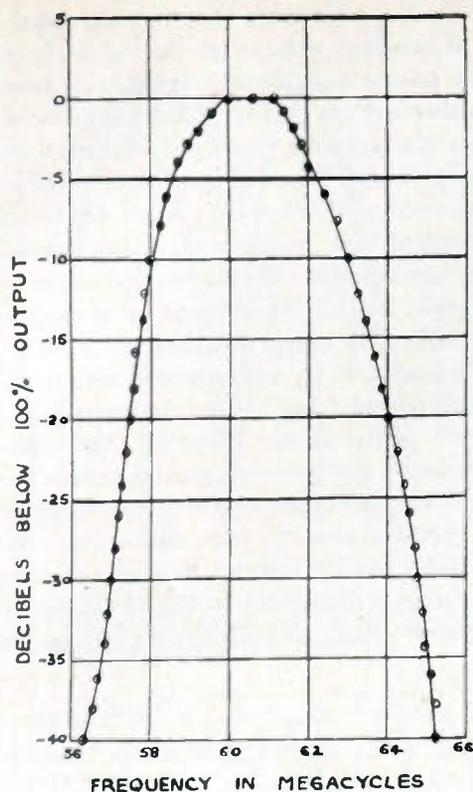
Fig. 11—Response curve for log receiver of 4-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.
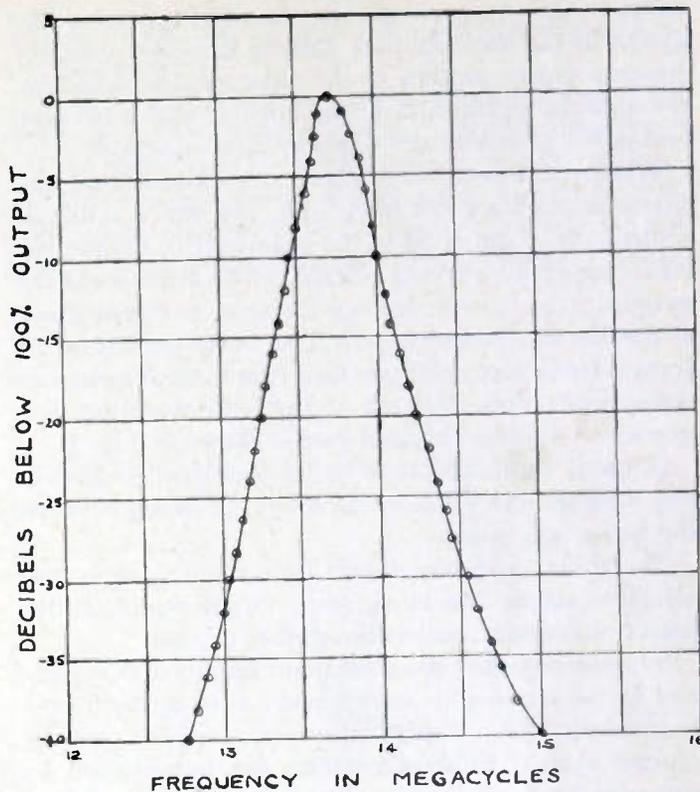
Fig. 12—Response curve for log receiver of 0.5-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.

of similar bandwidth, no noticeable deterioration of pulse shape has been observed.

XI. Acknowledgments

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Resolution in Radar Systems*

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Summary—The problem of azimuthal resolution in radar systems is discussed. It is shown how the resolution is related to the antenna beamwidth and thus in turn to the size of aperture. The possibilities of improving resolution beyond the normal capabilities is considered and the limitations are presented from two aspects. The first aspect is concerned with the attempt to build a "supergain" antenna. The second aspect is concerned with the attempt to achieve a similar result through the utilization of filter equalization techniques. It is shown that no essential improvement in resolution appears to be obtainable in a practical manner.

I. Introduction

RESOLUTION in a radar system is defined as the ability to distinguish between closely spaced objects. In the ordinary radar system, the ability to separate and distinguish between objects, or targets as they are called, is provided by two different techniques. The first is dependent on the pulse width of the

system and provides resolution in range. This type of resolution will not be treated here. The second is dependent on the radiation field pattern and provides angular resolution either in azimuth, elevation, or both. This form of resolving power will be discussed in this paper.

In order to simplify the considerations, resolution in one plane only will be covered, namely, resolution in the azimuthal plane. Such resolution is normally provided by a fan beam having a narrow beam angle in the horizontal plane and rotating at a uniform angular velocity about an axis perpendicular to the horizontal plane. The analysis and conclusions may be applied to elevation angular coverage and also to the combination of both.

The field-amplitude pattern of an antenna with a uniformly illuminated aperture is a $(\sin x)/x$ function. The scale is related to the aperture. In the normal radar system, the same antenna is used for transmitting and receiving. The received signal field will then be propor-

tional to the square of the one-way field patterns. The square of the one-way field pattern coincides with the one-way power pattern of the antenna. By definition, the antenna beamwidth is the angular width between half-power points on this one-way power pattern.

When the antenna is rotating at a uniform rate and passes through a point target (i.e., one which subtends a small angle compared to the beamwidth), then in the ideal case a time-varying voltage can be available at the output of the receiver having the same envelope shape as the one-way power pattern. Due to the pulsing of the transmitted energy, this voltage is not continuous, but consists of pulses repeated at the pulse-repetition frequency. A typical idealized case is shown in Fig. 1.

A usual combination of pulse-repetition frequency and rotation rate will provide about ten pulses between the half-power points.

The actual case may depart from the ideal to a considerable extent due to changing target aspect, multipath transmission, and various other causes.

Suppose now that two ideal point targets of equal size and at the same range were separated in azimuth by a beamwidth. The resultant receiver output voltage plotted against angle of rotation can be obtained by summing two envelopes of the shape shown in Fig. 1 but separated by a beamwidth. The result is shown in Fig. 2(a). It is to be noted that for this ideal case there is no pronounced valley or notch in the center of the curve which might indicate that it is the sum of two distinct curves.
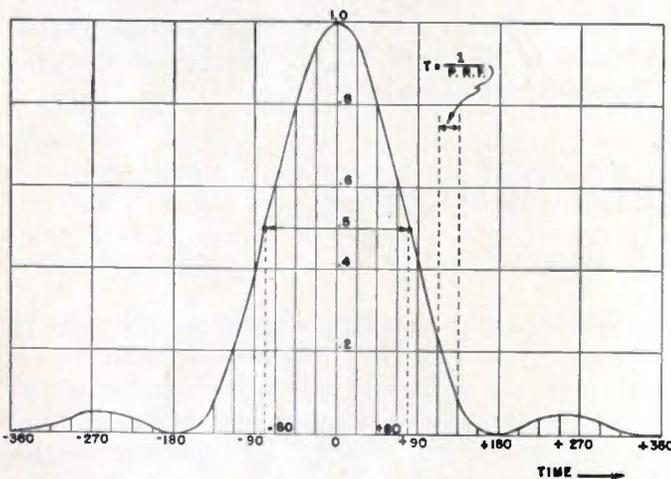
Fig. 1—Idealized voltage output from receiver due to scanning antenna passing over a point target.

The type of curve shown in Fig. 2(a) can be achieved if the two ideal targets are at precisely the same range to a small fraction of a wavelength or displaced in range by an integral number of half wavelengths. This is so because the curve is based on "in-phase" addition of the voltages due to both targets. At the high frequency of operation of radar systems, this separation cannot persist for any important period of time. A wandering in phase will probably occur sufficient to bring the voltages in and out of phase. Fig. 2(a) represents an extreme con-

dition. Fig. 2(c) represents the other extreme. If a large number of readings were taken during the interval that the beam passes over both targets, the average integrated value shown in Fig. 2(b) would be displayed. Practical radar systems would only deliver a small number of samples (less than 10) for this interval so that the statistical result will not apply. What will be seen on the display system may possibly be a merging of the two targets on one rotation. On the next rotation the targets may separate and so on. This merging and separation is also modified by target scintillation.

Targets separated by a beamwidth may be considered to be resolved only due to the random-phase phenomenon which likely exists. However, the separation of the targets need not be much greater than a beamwidth to provide a pronounced notch in the receiver voltage curve in the ideal case. In fact, this separation need be only 1.3 times the beamwidth to give a half-amplitude notch and this is illustrated in Fig. 3. Again due to the random-phase effect this ideal case represents an ex-
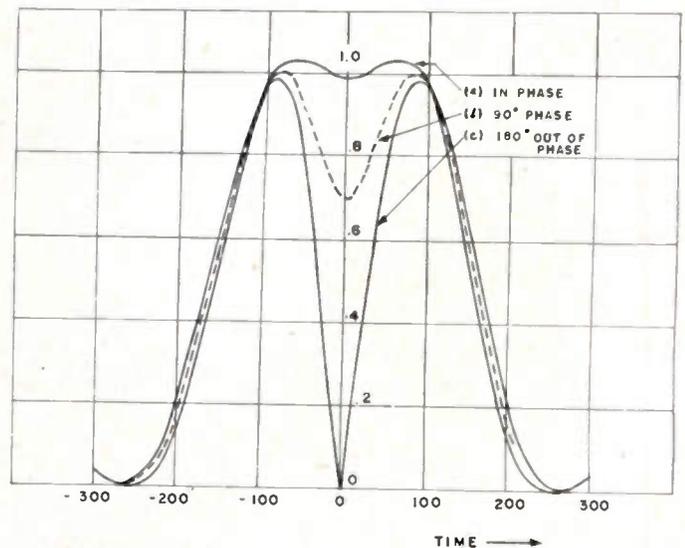
Fig. 2—Idealized voltage output of receiver for two targets separated by one beamwidth when the return from targets is (a) in phase, (b) 90° out of phase, (c) 180° out of phase.

treme. The other extreme of out-of-phase addition is shown in this figure. The average value of the notch (for 90°) is also indicated.

The device normally used for display (a B scope or PPI scope) is responsive to power.[1]

Note—Actually Payne-Scott has shown that the screen brightness is proportional to the third power of the receiver output voltage for a magnetically focused tube. The incremental brightness is something less and is shown to be proportional to the received power for small signals. Hence, this more conservative result has been used.

Thus the ultimate curve of intensity versus angular position is the receiver output voltage squared or the

[1] R. Payne-Scott, "Visibility of small echoes on PPI displays," PROC. I.R.E., vol. 36, pp. 180–196; February, 1948.
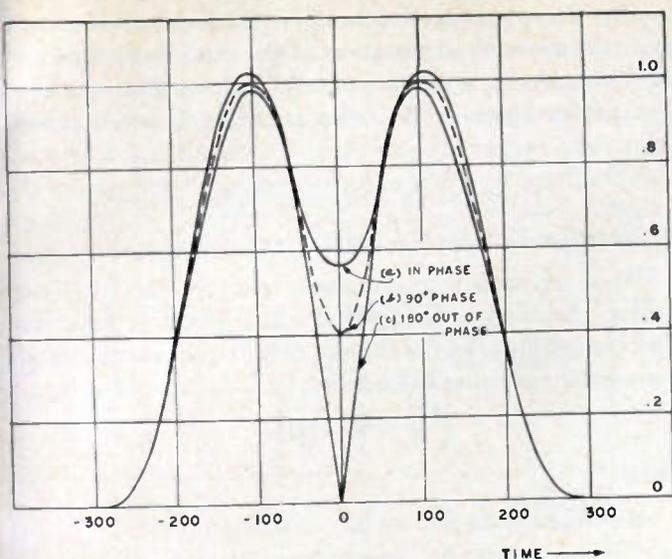
Fig. 3—Idealized voltage output of receiver for two targets separated by 1.3 beamwidths when the return from the targets is (a) in phase, (b) 90° out of phase, (c) 180° out of phase.

one-way power pattern squared provided that linearity is maintained. In the case of two targets, the receiver output voltage must first be obtained and then the squaring process applied. It is to be noted that the effect of the squaring process is to emphasize any notch if it does indeed exist in the summed voltage curve. This is illustrated in Fig. 4 where the two extremes are shown.

A series of curves can be plotted similar to those shown in Figs. 2 and 3, except for target separations varying from zero to several beamwidths. It will be noted that for target separations of less than one beamwidth, the resultant curve will have an envelope much like that shown in Fig. 1. For target separations of a beamwidth or greater the curve will have an envelope much like that of Fig. 3. It is possible to plot the ratio of the power at the true target positions to the power at the center of gravity of the two targets versus the target separation from data obtained from these curves. Such a plot is shown in Fig. 5. There is a region between one and two beamwidth separations where the resolution may be disputed depending on the observer.

An arbitrary definition may be set up similar to that for beamwidths or bandwidths which states that two targets are resolved by the antenna system when the power at the center of gravity is one half the power at the true target position. For the antenna illumination considered, this point occurs at a target separation of 1.2 beamwidths. It is interesting to compare this result with the criterion established by Lord Rayleigh for the resolving power of a diffraction grating. Lord Rayleigh has shown that in order to separate two spectrum lines, the distances between the central maxima of their diffraction images (which are also $(\sin^2 x)/x^2$ functions) must be at least as great as the distance of the first minima from the central maximum.[2] This represents a

[2] R. W. Wood, "Physical Optics," The Macmillan Co., New York, N. Y.; 1914.

spacing of 180° or, by our definition, a separation of 1.12 beamwidths.

Thus, to a first approximation, this system may be said to have an azimuth resolution of one beamwidth.

This result can be significantly modified by non-linearities in the over-all receiving and display system and, indeed, often is. Both the receiver and the display system have a limited dynamic range. Through the use
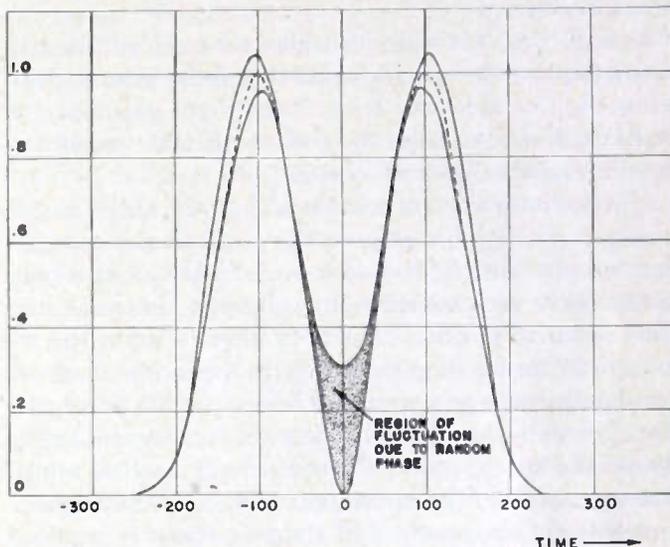
Fig. 4—Display intensity (receiver output voltage squared) for two targets separated by 1.3 beamwidths.
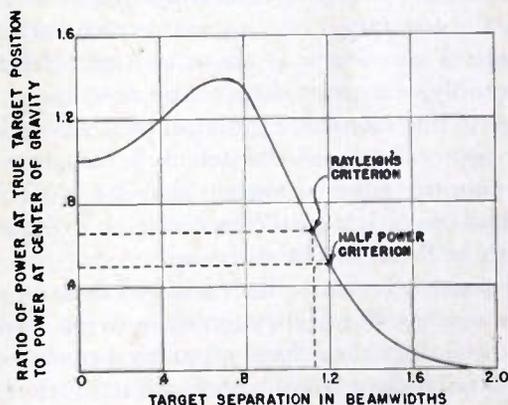
Fig. 5

of gain and intensity controls, the threshold of the display is normally set at the noise level in order to detect the weakest signals. There is a considerable variation possible in signal level due to two major causes:

1. Size and shape of targets
2. Range of the target $(P_r = K/R^4)$
   where P_r = received power
        R = range
        K = constant for the system.

Experimentally determined variations in target level of 80 db have been noted due to size differences between targets. These extreme variations are due to fixed targets. Variations of as much as 20 db may occur between

aircraft targets. Such variations are considerably beyond the dynamic range of most systems and the net effect is clipping or saturation on the stronger signals. This non-linearity has a serious effect on the resolution capabilities of the system. Consider, for example, the case of two targets separated by 1.2 beamwidths. Assume that the system has a dynamic range of 6 db and that the targets have a power level of 9 db above noise. It is clear from this that the two targets may no longer be resolvable.

As a further example, consider a target sufficiently strong to place the saturation level of the system slightly below the first side-lobe level. This target need only be about 30 db above noise to yield an ultimate resolution of 4 beamwidths instead of one.

A little consideration will show that not only can the presence of side lobes coarsen the resolution of the system, but also false information can be introduced. Many devices have been considered to alleviate this situation. Time sensitivity control has been introduced in the receiver as a means of eliminating the variation of target signal amplitude with range. This correction is a function of elevation angle and is only correct for one angle. Instantaneous gain control has also been used to eliminate saturation. A little thought will show that a prior knowledge of the presence of a strong signal is required to retain the resolution and that this is not possible in a simple system.

It is worth while to mention that even in a linear system, the effect of target scintillation may coarsen the result, since one target can appear as two targets and two as one. This effect is random and uncorrelated for the interval between rotations of the antenna.

The techniques available for improving resolution fall into two general classes, one which is concerned with operation on the antenna system and one which is concerned with operation on the receiving and display system. Both of these will be discussed.

Before passing on to further considerations on resolution it is worth while to distinguish between resolution and accuracy since these have often been confused. It is entirely possible for a system having a resolution of one beamwidth to be able to determine the position of a target to one tenth of a beamwidth or even less. This fine position determination is only possible for a single isolated target. Consider for example, the ideal envelope shown in Fig. 1. By various means, the center of gravity of this envelope may be found to an order of accuracy determined fundamentally by the number of pieces of data making up the envelope. For the case illustrated, the center might be determined to one tenth of a beamwidth. If the scanning rate were decreased, more pulses would be available and an even more precise determination of target angle would be possible. In the practical case, target scintillation and system instabilities modify the results and place a top limit on accuracy.

If the same accurate type of angular determination were attempted in the presence of two targets separated by less than a beamwidth, then the net result would be an accurate determination of the center of gravity of the two targets. This result follows from the fact that the system behaves as if the two targets were indeed one.

II. Antenna System Resolution

A. Relation Between Beamwidth and Side Lobes

For a uniformly illuminated aperture, as has been stated, the oneway field pattern is a $(\sin x)/x$ function. The beamwidth, as previously defined, is related to the size of the aperture as follows:

$$\theta = K(\lambda/D)$$

where

θ = angle bounded by half-power points
λ = wavelength of radiation
D = length of aperture
K = constant depending on the type of aperture.

There is then an inverse relation between the aperture and beamwidth. The resolution of the system can be improved by utilizing choice of wavelength and size of aperture. It is important to note that the level of the side lobes, which cannot be ignored, will remain unchanged with these variations provided that the aperture illumination remains uniform. A number of investigations have been made to determine means of reducing side lobes and these are well known and extensively treated in the literature. The method used is to taper the illumination over the aperture. The field patterns for many different types of illumination are given in the references. Handelsman[3] has taken the results of Ramsay[4] and plotted a curve relating beamwidth and side-lobe level for a fixed aperture size. This curve has been normalized and replotted as the lower curve in Fig. 6.
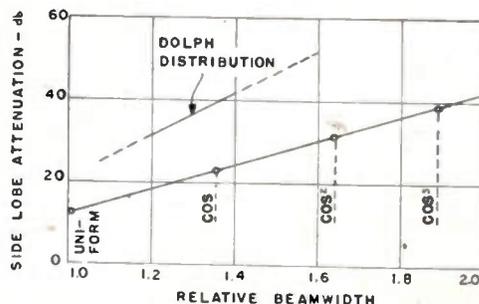
Fig. 6—Comparison of beamwidths versus first side-lobe attenuation for illuminations yielding constant side-lobe level (Dolph) and illuminations yielding diminishing side-lobe levels.

While there is no rigorous justification for connecting the points given by the data in the curve, the resulting curve has meaning in the light of our experience with

[3] M. Handelsman, "Calculation of the Current Distribution on a Linear Antenna to Produce a Given Radiation Pattern," thesis, Ohio State University, 1948.
[4] J. F. Ramsay, "Fourier transforms in aerial theory," *Marconi Rev.*, vol. 83, p. 139; October–December, 1946.

antenna construction. The implications of this curve are indeed significant for it is immediately apparent that, for a given aperture, if we wish to suppress side lobes, we must sacrifice beamwidth. This result, of course, is well known. Apparently side-lobe suppression is a mixed blessing, but if not carried to extremes is justifiable considering the limited dynamic ranges of our system. In order to illustrate this, there is shown in Fig. 7 the an-
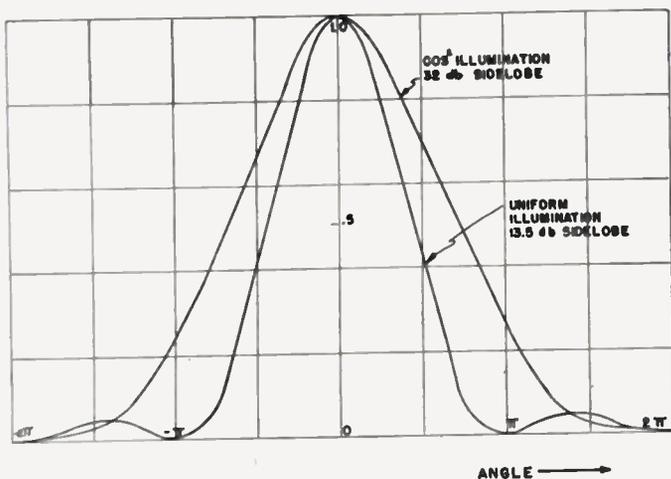
Fig. 7—Comparison of antenna power-gain patterns for a uniform illuminated aperture (13.5-db side-lobe) and a $(\cos)^2$ illuminated aperture (32-db side-lobe).

tenna field patterns for a uniformly illuminated aperture and a tapered illuminated aperture with first side lobes of 32 db.

B. Super-Gain Antenna

As was previously stated, there is no justification for connecting the points in Fig. 6 in a smooth curve other than that they represent types of illumination which can and, in some cases, have been constructed as practical antennas. Certainly it is conceivable that other functions exist which, for a given aperture, will give a pattern which has smaller beamwidth than obtainable for uniform illumination. These functions may be grouped in two classes:

Class I includes those functions where the side-lobes are greater than for uniform illumination and whose points will fall on an extension of the lower curve given in Fig. 6.

Class II includes the functions which do in fact give narrower beams without sacrifice of the side-lobe level.

Ramsay[4] has treated a case (Class I) where the side-lobe amplitudes are on the order of 35 per cent. This point has not been included in the curve given in Fig. 6. It should be noted that the beamwidth is only decreased to 0.8. A further extension of the lower curve will indicate that decrease of the beamwidth to 1/2 will involve side lobes on the order of the main lobe.

Class II has been treated by a number of investigators.[6-9]

[5] W. W. Hansen, "Notes on Microwaves" (A series of lectures de-

It has been shown that on a theoretical basis such functions do indeed exist. Unfortunately, as has been commented by Riblet,[5] no antenna has been built to date which exceeds the performance of a uniformly illuminated aperture. Schelkunoff[8] speculated that this situation is explained on the basis of the high currents required which are impossible to achieve due to ohmic losses. More recently, Chu[7] has shown that, at least in the case of an omnidirectional antenna, any attempt to attain "super gain" to even a small degree is accompanied by an extremely rapid rise in the Q. This effect is of course related to the high currents noted by Schelkunoff.

Taylor[9] has also arrived at these conclusions in a note on this subject. His conclusions are that any attempt to alter the excitation in such a way as to increase the resolving power appreciably is accompanied by severe dissipation in the antenna itself. This dissipation may be attributed to the extremely rapid rise in Q as noted by Chu.

Recently considerable attention has been focused on a type of illumination which apparently allows more efficient utilization of antenna aperture than is possible with uniform illumination.[10,11] This type of illumination makes use of a point rather than a continuous distribution of power across the aperture and is based on the properties of the Tchebyscheff polynomials. A characteristic of this type of distribution, and this should strike a familiar note with those acquainted with filter design, is that the side-lobe level is constant for all side-lobes. Now a characteristic of all types of illumination, for which the results are shown in the lower curve of Fig. 6, is that the side lobes decay. Thus, for practically all cases the first side lobe is the one of the most significance. For the Dolph[10] distribution this is not so. In effect, a threshold has been created since all side lobes are of equal level. For this type of design, for a signal sufficiently strong to saturate down to the first side lobe, no resolution at all will be obtained! This is indeed an unfortunate situation and radar designers seeking an increased aperture efficiency should bear this well in mind. The relation between beamwidth and side lobes for the Dolph distribution is shown as the upper curve in Fig. 6.[12]

livered at the MIT Radiation Laboratory, 1941–1944). Notes compiled by the Sperry Gyroscope Company.
[5] H. J. Riblet, "Note on the maximum directivity of an antenna," Proc. I.R.E., vol. 36, pp. 620–623; May, 1948.
[7] L. J. Chu, "Physical Limitations of Omnidirectional Antennas," Technical Report 64, Research Laboratory of Electronics, Massachusetts Institute of Technology, May, 1948.
[8] S. A. Schelkunoff, "A mathematical theory of linear arrays," Bell Sys. Tech. Jour., vol. 22, pp. 80–107; January, 1943.
[9] T. T. Taylor, "A discussion of the maximum directivity of an antenna," Proc. I.R.E., vol. 36, p. September, 1135; 1948.
[10] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beamwidth and side-lobe level," Proc. I.R.E., vol. 34, pp. 335–347; June, 1946.
[11] H. J. Riblet, "Discussion on reference 10," Proc. I.R.E., vol. 35, pp. 489–492; May, 1947.
[12] Data for curve obtained from an unpublished paper by M. Handelsman, Watson Laboratories, Red Bank, N. J.

From this figure it is possible to see that for an antenna design having the first side lobe attenuated by 32 db the improvement in beamwidth due to increased efficiency of aperture utilization is about 25 per cent.

From the preceding discussion, it is apparent that if we wish to describe the resolving capabilities of an antenna in a radar system, it is insufficient to state the beamwidth—a description of the side lobes must also be included. To give this description in terms of the first side-lobe level only is inadequate to completely define the antenna system. However, once the side lobe level is stated, the practically attainable beamwidth for a given aperture is fairly well defined and possible variations are confined to a small spread as is indicated by the bounded region of Fig. 6. There are many degrees of freedom available in defining the side lobes (i.e., rate of decay, spacing, and the like). Arbitrary choice of these controls allows a varying aperture efficiency, but the amount of this variation is small as is indicated by the bounded region in Fig. 6, and may not be considered to be super gain.

A super-gain antenna would imply a curve lying well above the bounded region. This has been shown to be theoretically possible. However, it is also generally concluded that such an antenna cannot be practically constructed without an enormous sacrifice in efficiency even to obtain only a small improvement in resolving power.

III. Receiving-System Resolution

It is possible to conceive methods of increasing angular radar resolution by means other than increasing the antenna size and raising the operating frequency.

One such method was proposed by Laemmel of the Polytechnic Institute of Brooklyn. This method was based on filter techniques. It is assumed that a suitable storage medium is available to store the data as they are collected on a PPI scope. A method is then assumed to be available which will remove the signal by a reading beam sweeping in azimuth. A time-varying voltage is then available which is a replica of the system angular-selectivity pattern for point targets. This output may be considered similar to that available from a low-pass filter (i.e., the high-frequency fluctuations due to closely spaced targets are eliminated by the broad antenna beam). It was proposed to improve the resolution by amplifying the attenuated high frequencies using an amplifier having the inverse frequency characteristic of the antenna system. This type of technique is well known in telephone systems under the name of equalizing.

Some comments are made by Hansen[13] concerning the maximum frequency present in a scanning system. He points out that the antenna gain pattern is the Fourier transform of the aperture illumination. (This subject is covered in detail in footnote reference 4.) The voltage

output of the antenna system as it scans across a point target is a function of the antenna pattern. The time axis is determined by the scanning rate of the antenna which is assumed to be uniform. The frequency spectrum of this time-varying voltage will then be the Fourier transform of the antenna pattern. But the Fourier transform of a Fourier transform is the function itself. So that, except for a question of scale, the frequency spectrum of the scanning output voltage of the antenna will be the aperture illumination function. This, of course, is a simplification of the problem since the antenna pattern enters into the process twice, once during transmission and once during reception.

In the case of a uniformly illuminated aperture, the antenna scanning output voltage will have the form

$$\left(\frac{\sin t}{t}\right)^2.$$

The Fourier transform of this function (with due attention to phase which is not indicated) is triangular in shape. The frequency spectrum is then triangular in shape. If it is desired to produce an effective time-varying voltage of the same shape and half the width being obtained, then the triangular spectrum must be modified by some equalization process to have twice the base line. This is obviously impossible. It does, however, appear possible to modify the frequency spectrum to produce the equivalent of varying the antenna illumination. This result is interesting for it suggests that filter equalization techniques may be applied to vary the effective antenna illumination instead of varying the illumination itself. Since present-day storage techniques are awkward to apply, it is not likely that this will have any immediate practical application.

For other examples of illumination, the conclusions are not so obvious but for all practical types of illumination it turns out that extremely large amplification, well beyond ordinary methods, is required even to decrease the effective beamwidth by 50 per cent.

There is thus a striking parallel between the Q or kva required in the "super-gain" antenna and the extreme amplification required in the "super-gain" corrective network.

IV. Conclusions

The resolution capabilities of a scanning antenna have been shown to be roughly on the order of a beamwidth. The beamwidth of an antenna system has been related to the size of the antenna aperture.

The beamwidth of an antenna may be decreased by increasing the aperture size in wavelengths.

Other means of decreasing the beamwidth or improving the resolution have been discussed. These consist of building a "super-gain" antenna or applying correction filters to the antenna output. These techniques have been shown to be impractical since they involve extremely large (approaching infinite) antenna currents or equalized gains.

[13] W. W. Hansen, "C-W Radar Systems," chap. 5, "Radar System Engineering," edited by L. W. Ridenour, McGraw-Hill Book Co., New York, N. Y.; 1947.

A Note on a Bridged-T Network*

PETER G. SULZER†, ASSOCIATE, IRE

Summary—A bridged-T network previously employed for the measurement of resistance is analyzed, and is shown to be useful as the frequency-determining element of a resistance-tuned oscillator. The resistance-capacitance form of the network permits a wide tuning range and simple switching between ranges, while the resistance-inductance network suggests the possibility of building an oscillator having decade frequency dials. With both forms of the network a moderate value of equivalent Q can be obtained, which is desirable for accuracy and stability of frequency calibration.

THE NETWORK of Fig. 1(a) has been used for the measurement of resistance at radio frequencies.[1] It is the purpose of this note to show that the network also has application in a dual-feedback RC sinusoidal oscillator.

(a)

(b)

(c)
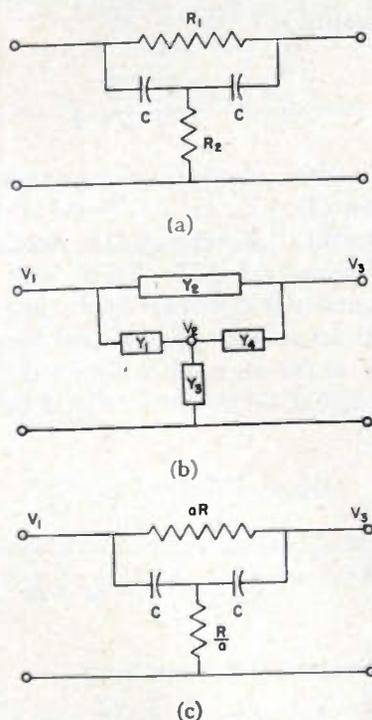
Fig. 1—Bridged-T networks.

Considering the general bridged T of Fig. 1(b), and writing the equations for the node voltages V_2 and V_3 in terms of the component admittances Y_1, Y_2, Y_3, Y_4,

$$V_1Y_1 - V_2(Y_1 + Y_3 + Y_4) + V_3Y_4 = 0, \quad (1)$$

$$V_1Y_2 + V_2Y_4 - V_3(Y_2 + Y_4) = 0. \quad (2)$$

Solving for the voltage ratio V_3/V_1,

$$\frac{V_3}{V_1} = \frac{Y_1 + Y_2 + \dfrac{Y_1Y_2}{Y_4} + \dfrac{Y_2Y_3}{Y_4}}{Y_1 + Y_2 + \dfrac{Y_1Y_2}{Y_4} + \dfrac{Y_2Y_3}{Y_4} + Y_3}. \quad (3)$$

Employing the circuit of Fig. 1(c) for convenience, and substituting the appropriate admittances in (3),

$$\frac{V_3}{V_1} = \frac{\dfrac{2}{a} + j\omega RC + \dfrac{1}{j\omega RC}}{\left(\dfrac{2}{a} + a\right) + j\omega RC + \dfrac{1}{i\omega RC}}, \quad (4)$$

$$= \frac{\dfrac{2}{a} + ju}{\dfrac{2}{a} + a + ju}, \quad (5)$$

where $u = (\omega/\omega_0) - (\omega_0/\omega)$, and $\omega_0 = 1/RC$. Thus a voltage-ratio minimum, accompanied by zero phase shift, is obtained at ω_0. Fig. 2 is a plot of the magnitude and phase angle of (5), which shows that the sharpness and depth of the minimum increase with a.

It is of interest to obtain the equivalent Q of the network, which is done by defining $Q = (\omega_0/\Delta\omega) = 1/u$, where the limits of $\Delta\omega$ occur when the network response is 3 decibels higher than its minimum value.

Equating the expression for the magnitude of the right-hand side of (5) to $\sqrt{2}$-times its minimum value, and solving for u,

$$u = \frac{2}{a\sqrt{1 - \dfrac{8}{(2 + a^2)^2}}};$$

and therefore,

$$Q_e = \frac{a}{2}\sqrt{1 - \frac{8}{(2 + a^2)^2}} \quad (6)$$

$$\approx \frac{a}{2} \quad \text{for} \quad a \gg 1. \quad (7)$$

Thus a fairly selective network can be obtained with this RC bridged T. Although the sharpness of the response is not comparable to that obtained with the parallel T[2] a smaller number of circuit elements is required, while variable-frequency operation is conveniently accomplished with a dual-section variable capacitor. A simple audio-frequency oscillator is produced[3] if the network is placed in the negative-feedback path of an amplifier having both positive and negative feedback. In this application the possibility of obtaining a moderate value of Q indicates superiority over a previously used network,[4] which has a Q of about 1/3.

[1] P. M. Honnell, "Bridged-*T* measurement of high resistance at radio frequencies," PROC. I.R.E., vol. 28, pp. 88–90; February, 1940.
[2] H. H. Scott, "A new type of selective circuit and some applications," PROC. I.R.E., vol. 26, pp. 226–235; February, 1938.
[3] P. G. Sulzer, "Wide-range *RC* oscillator," *Electronics*, vol. 23, pp. 88–89; September, 1950.
[4] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, "Some applications of negative feedback with particular reference to laboratory equipment," PROC. I.R.E., vol. 27, pp. 649–655; October, 1939.

It is of interest to determine the effect of adding a small capacitance bC across R/a, as shown in Fig. 3(a). Substituting the appropriate admittances in (3),

$$\frac{V_3}{V_1} = \frac{\frac{1}{a}(2+b) + ju}{\frac{1}{a}(2+b) + a + j\left(u + b\frac{\omega}{\omega_0}\right)}. \tag{8}$$
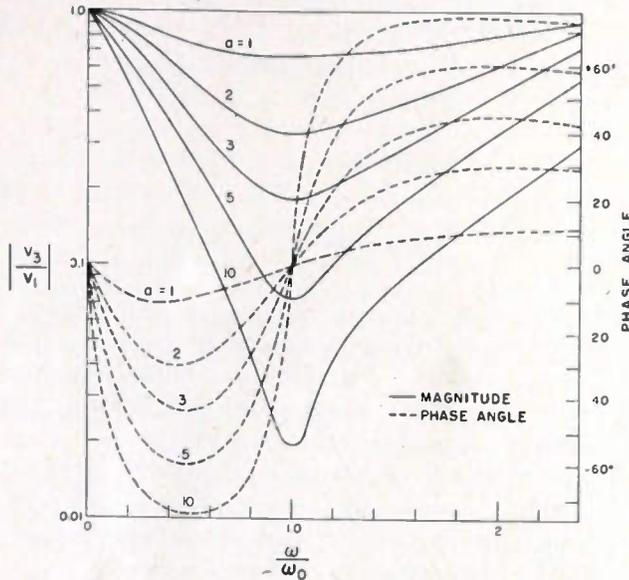
Fig. 2—Amplitude and phase characteristics of the bridged-T network of Fig. 1(c).
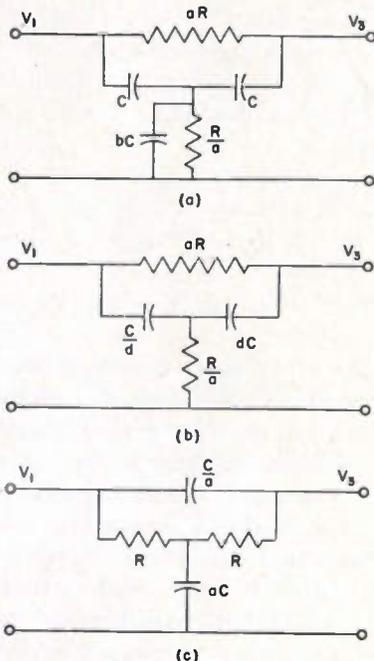
Fig. 3.—Modified bridged-T networks.

A new resonant frequency ω_1 is defined when the phase angle of (8) becomes zero, which occurs when

$$\frac{u}{\frac{1}{a}(2+b)} = \frac{u + b\frac{\omega}{\omega_0}}{\frac{1}{a}(2+b) + a}. \tag{9}$$

Solving for the angular-frequency ratio,

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{1}{1 - b\left(\frac{2+b}{a^2}\right)}. \tag{10}$$

It is seen from (10) that the effect of the additional capacitor is to increase the resonant frequency of the network. Furthermore, the amount of increase depends upon the capacitance ratio b. Consequently, if variable-capacitance tuning is employed, the additional capacitor will have its greatest effect at the low-capacitance end of a tuning range, which is of considerable practical value when one dial calibration must suffice for two or more decade ranges.

One might suspect that such a trimming process would alter the attenuation of the network. Substituting (10) in (8) and obtaining the magnitude, the new voltage ratio at resonance is

$$\left|\frac{V_3}{V_1}\right| = \frac{2+b}{a^2 + 2 + b}. \tag{11}$$

To consider some practical values, suppose $a = 4$, and $u = b = 0$. From (5), $\left|V_3/V_1\right| = 1/9 \approx 0.111$. Letting $a = 4$, $u = 0$, and $b = 0.1$, $\left|V_3/V_1\right| \approx 0.116$ (from 11), while, from (10), $(\omega_1/\omega_0) \approx 1.0065$. Thus with $b = 0.1$ the resonant frequency is increased approximately 0.65 per cent, while the attenuation is decreased about 5 per cent.

A variation of the network is obtained if the capacitors C are unequal, as shown in Fig. 3(b). Employing (3) as before,

$$\frac{V_3}{V_1} = \frac{\frac{1}{a}\left(d + \frac{1}{d}\right) + ju}{\frac{1}{a}\left(d + \frac{1}{d}\right) + ad + ju}, \tag{12}$$

while the equivalent Q is given by
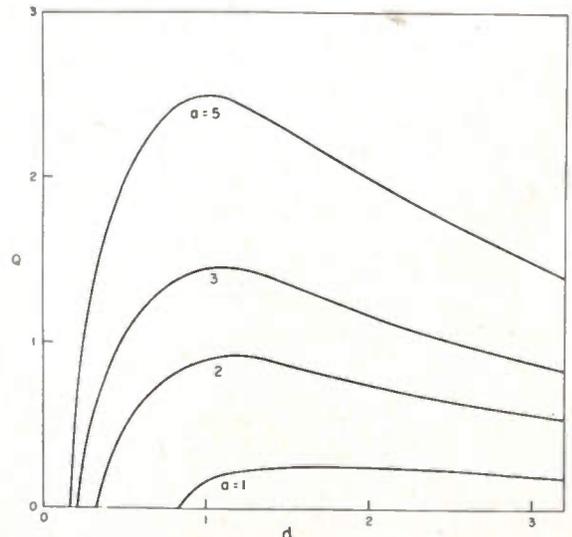
Fig. 4—Q_e versus d for the unsymmetrical bridged-T network.

$$Q = \frac{a\sqrt{d^2(a^4 + 2a^2 - 1) + 2(a^2 - 1) - \dfrac{1}{d^2}}}{d^2(a^2 + 1) + (a^2 + 2) + \dfrac{1}{d^2}} \cdot \quad (13)$$

Fig. 4 contains a plot of (13) showing that a slight increase in Q for low values of a is obtained by using the unsymmetrical bridged T.

A symmetrical variation is obtained if the capacitors and resistors of Fig. 1(c) are interchanged, as shown in Fig. 3(c). It can be shown that the properties of this network are the same as those of Fig. 1(b). The use of equal resistances is convenient when resistance tuning is to be obtained with a dual potentiometer.

(a)

(b)
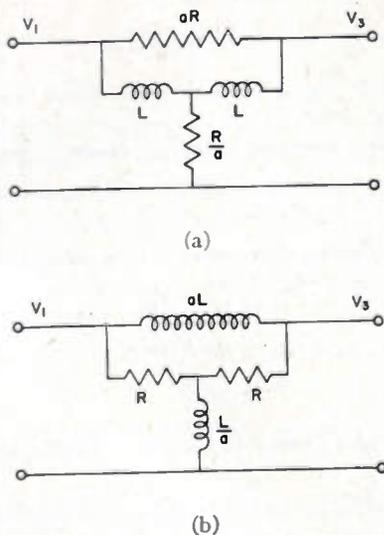
Fig. 5—*RL* bridged-T networks.

The two additional networks of Fig. 5 are obtained by using resistors and inductors. Considering Fig. 5(a) and employing (3),

$$\frac{V_3}{V_1} = \frac{\dfrac{2}{a} + ju}{\dfrac{2}{a} + a + ju}, \quad (14)$$

where V_3, V_1, and u are defined as before, and $\omega_0 = R/L$. Thus ω_0 is proportional to R, which presents the very interesting possibility of a decade-frequency oscillator, with frequency selected by decade dials, as are resistance or capacitance in the familiar "decade boxes." For (14) to apply, it is required that the magnitude of L be independent of frequency, and that L be lossless. This first requirement can be met if L is operated well below its self-resonant frequency; however, the second requires some consideration.

Substituting in (3) as before, and allowing for a finite inductor Q,

$$\frac{V_3}{V_1} = \frac{\dfrac{\dfrac{1}{Q} - j1}{\omega L\left(\dfrac{1}{Q^2} + 1\right)} + \dfrac{2}{aR} + \dfrac{j\omega L}{R^2} + \dfrac{\omega L}{QR^2}}{\dfrac{\dfrac{1}{Q} - j1}{\omega L\left(\dfrac{1}{Q^2} + 1\right)} + \dfrac{2}{aR} + \dfrac{j\omega L}{R^2} + \dfrac{\omega L}{QR^2} + \dfrac{a}{R}} \cdot \quad (15)$$

If, as before, ω_1 is defined as the angular frequency producing zero phase angle,

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{1}{1 + \dfrac{1}{Q_2}} \cdot \quad (16)$$

With an inductor Q of 10 the resonant frequency is decreased approximately $\frac{1}{2}$ per cent, which is not serious in most applications.

The Effects of Anisotropy in a Three-Dimensional Array of Conducting Disks*

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Summary—This microwave delay lens medium is shown to have both magnetic and electric anisotropy, which necessitates an analysis describing its refractive properties for obliquely incident waves.

A simple linear transformation is applied to the field equations such that the transformed system is magnetically isotropic. Classical solutions from studies in optics provide the ray velocity surfaces in that system. An inverse transformation yields the ray velocity surfaces in the original medium.

Huyghens construction is employed, for two particular arrays, to determine the refracted wave direction after oblique incidence.

INTRODUCTION

THE USE OF an array of conducting elements to focus microwaves has received increasing attention since its introduction by Kock[1] in 1948.

When the conducting elements are spherical particles, their symmetry permits the evaluation[2] of averaged dielectric and permeability coefficients which include the effects of interaction between the spheres. That same symmetry makes the coefficients independent of

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† The Institute for Advanced Study, Princeton, N. J.

[1] W. E. Kock, "Metallic delay lenses," *Bell Sys. Tech. Jour.*, vol. 27; pp. 58–83; January, 1948.
[2] L. Lewin, "Electrical constants of spherical conducting particles in a dielectric," *Jour. IEE*, vol. 94, part III, p. 65; January, 1947.

the polarization of the external fields; hence the sphere array is isotropic. However, circulating currents induced on the spheres by a varying magnetic field result in a relative permeability coefficient less than unity and thereby decrease the index of refraction of the medium. To avoid this decrease and to simplify the construction of a lens, Kock investigated the use of an array of thin conducting disks. By restricting his analysis to the case of propagation normal to the disk faces, he obtained a simple formula for the index of refraction of the medium which indicated a distinct improvement over that of the sphere array. By restricting the analysis further to the case of a single type of polarization of the incident wave, Kock evolved the strip lens.

Kock's pioneering work has been extended by means of a transmission line analogue by Cohn[3] in America and Brown[4] in England. They established the frequency dependence of the refractive index of the strip lens medium and included the effect of proximity of the conducting elements. The above papers did not consider the evident anisotropy of the disk and strip arrays.[5]

The primary purposes of this paper are to show how the electric and magnetic properties of the disk loaded lens medium vary with direction and to present an analysis of propagation in the anisotropic medium. These results are then applied to the problem of lens design.

The discussion begins by establishing the dielectric, permeability and conduction coefficients of the disk array in three principal directions. These coefficients permit an explicit statement of the matrix constituitive equations which display both electric and magnetic anisotropy. A simple linear transformation is applied and the magnetic anisotropy is mathematically eliminated in the transformed system. The transformed field equations turn out to be precisely those which have received extensive treatment in studies of crystal optics where the relative permeability coefficients are assumed to be unity.

After a brief summary of relevant aspects of wave propagation in crystals, surfaces representing the directional variation of the velocity of energy propagation in the original disk medium are obtained by means of the same linear transformation. These "ray velocity surfaces" permit the application of Huyghens Principle to determine the angle of refraction of an obliquely incident ray. With this analysis as a foundation, the requirements for simple lens design are then evaluated.

The Constitutive Equations

An investigation of electromagnetic wave propagation in the disk array of Fig. 1 requires the specification

[3] S. B. Cohn, "Metal strip delay structure for microwave lenses," *Jour. Appl. Phys.*, vol. 20, p. 257; April, 1949.
[4] J. Brown, "The design of metallic delay dielectrics," *Jour. IEE*, vol. 97, part III; pp. 45–48; January, 1950.
[5] The author has been informed by Dr. Cohn that he carried out some analytical work on this problem. Unfortunately the report containing it has not been available for comment.
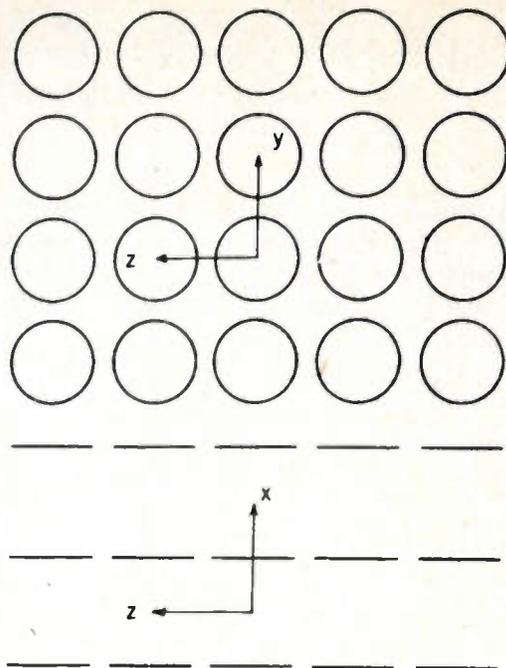
Fig. 1—Orientation of the conducting disk array.

of the quantities in the constitutive equations,

$$\overline{D} = \epsilon_0 [K_e] \overline{E}, \tag{1a}$$

$$\overline{B} = \mu_0 [K_m] \overline{H}, \tag{1b}$$

$$\overline{J} = [\sigma] \overline{E}, \tag{1c}$$

where $[K_e]$, $[K_m]$, and $[\sigma]$ are in general matrix quantities.

When the electric field is parallel to the disk faces, the relative dielectric constant of the array in air is given by,[1]

$$K_{ey} = K_{ez} = 1 + (16Na^3/3). \tag{2}$$

This formula was obtained with the following assumptions:

(I) N, the number of disks per unit volume, is small enough to neglect interaction.

(II) a, the disk radius, is small compared to the wavelength of the applied field.

(III) The disks are perfectly conducting.

When the electric field is perpendicular to the disk faces, the relative dielectric constant of the array in air is unity.

When the magnetic field is parallel to the disk faces, the relative permeability coefficient of the array in air is equal to unity. When the magnetic field is perpendicular to the disk faces, circulating currents are induced. An analysis by the author[6] shows that a summation of the effects of these circulating currents results in a relative permeability coefficient less than unity in the direction normal to the disk faces given by,

$$K_{mz} = 1 - (8Na^3/3). \tag{3}$$

[6] G. Estrin, "The effective permeability of an array of thin conducting disks," *Jour. Appl. Phys.*, vol. 21, pp. 667–670; July, 1950.

The restrictions on N and a are the same as those stated above.

Since the disks are insulated from one another, the array is nonconducting and $[\sigma] = [0]$.

Referred to the co-ordinate system of Fig. 1, the above description of the disk medium may be explicitly summarized by means of the matrix expressions,

$$
\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 + (16Na^3/3) & 0 \\ 0 & 0 & 1 + (16Na^3/3) \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \tag{4a}
$$

$$
\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \mu_0 \begin{bmatrix} 1 - (8Na^3/3) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}, \tag{4b}
$$

$$
[J] = [0]. \tag{4c}
$$

If a co-ordinate system were chosen such that the x-axis made some oblique angle with the disk faces, the dielectric and permeability matrices could have nine nonzero components. The co-ordinate orientation which produces the diagonal matrices defines the principal dielectric and permeability axes of the medium. The ensuing analysis sacrifices generality for clarity by using this favored orientation.

There now remains the task of solving the field equations,

$$
\nabla \times \overline{E} + \mu_0 [K_m] \frac{\partial \overline{H}}{\partial t} = [0], \tag{5a}
$$

$$
\nabla \times \overline{H} - \epsilon_0 [K_e] \frac{\partial \overline{E}}{\partial t} = [0], \tag{5b}
$$

where $[K_m]$ and $[K_e]$ are the diagonal matrices explicitly stated in (4).

Wave propagation in anisotropic media has received intensive study in the field of crystal optics but successful solutions have always been preceded by the assumption that the relative permeability could be approximated by a unit matrix. That approximation is obviously not applicable here.

In a set of notes prepared at New York University in 1948, R. K. Luneburg[7] attacked the problem of wave propagation in anisotropic media with utmost generality and he did not introduce the assumption of unity permeability until late in his analysis. As a result, he was able to indicate a method for solution of the field equations in materials having both electric and magnetic anisotropy. A great deal of the generality in Luneburg's work has been eliminated here to simplify its application to this medium and to make a brief discussion possible.

A simple linear transformation is defined by

$$
\overline{E}' = [\alpha]\overline{E}, \tag{6a}
$$

$$
\overline{H}' = [\alpha]\overline{H}, \tag{6b}
$$

$$
\nabla' = [\alpha]\nabla, \tag{6c}
$$

[7] R. K. Luneburg, Lecture notes on "Propagation of Electromagnetic Waves," New York University, 1947–1948.

where the transformation matrix, $[\alpha]$, is defined as

$$
[\alpha] = \frac{[K_m]^{1/2}}{\sqrt{|K_m|}}
$$

$$
= (K_{mx}K_{my}K_{mz})^{-1/2} \begin{bmatrix} (K_{mx})^{1/2} & 0 & 0 \\ 0 & (K_{my})^{1/2} & 0 \\ 0 & 0 & (K_{mz})^{1/2} \end{bmatrix}. \tag{7}
$$

The transformed quantities in (6) are substituted into the field equations of (5). With the aid of the matrix identity,

$$
[A]\overline{V} \times [A]\overline{W} = |A|[A]^{-1}\{\overline{V} \times \overline{W}\}, \tag{8}
$$

the field equations reduce to the following form

$$
\nabla' \times \overline{E}' + \mu_0 \frac{\partial \overline{H}'}{\partial t} = [0], \tag{9a}
$$

$$
\nabla' \times \overline{H}' - \epsilon_0 [\gamma] \frac{\partial \overline{E}'}{\partial t} = [0], \tag{9b}
$$

where

$$
[\gamma] = [K_m]^{-1}[K_e]
$$

$$
= \begin{bmatrix} K_{ex}/K_{mx} & 0 & 0 \\ 0 & K_{ey}/K_{my} & 0 \\ 0 & 0 & K_{ez}/K_{mz} \end{bmatrix}. \tag{10}
$$

The significance of the simple linear transformation of (6) and (7) may now be evaluated. The original field equations described the field variations in a medium having both magnetic and electric anisotropy. The transformed field equations describe the field variations in a transformed medium in which the magnetic anisotropy has been eliminated. The $[\gamma]$ matrix assumes the role of the relative dielectric coefficients of the transformed system.

The transformed field equations in (9) are now in a form where the results of studies in crystal optics may be readily applied. Elimination of $\overline{H}'$ between (9a) and (9b) leads to solutions for the "ray velocity surfaces" representing propagation in the transformed system. This analysis is carried out in detail in the treatise, "Optik" by Born.[8] The highlights of that treatment are utilized in the following section.

Assuming plane wave propagation in a crystalline medium, Born develops an equation in terms of the components of the velocity of energy propagation, i.e., the ray velocity, in the form,

$$
\frac{(s_x')^2}{(c/c_e')^2 - \gamma_x} + \frac{(s_y')^2}{(c/c_e')^2 - \gamma_y} + \frac{(s_z')^2}{(c/c_e')^2 - \gamma_z} = 0, \tag{11}
$$

[8] M. Born, "Optik," Chap. V., Julius Springer, Berlin; 1933.

where

$\bar{s}' = \bar{S}/|S'| =$ the unit vector in the direction of energy propagation

$\bar{S}' = \bar{E} \times \bar{H}' =$ the Poynting vector

$c_s' = |S'|/W' =$ the speed of energy propagation in the direction, $\bar{s}'$

$W' =$ the total energy density of the electromagnetic field.

$c =$ the velocity of light in free space.

A three-dimensional surface may be drawn representing the ray velocity c_s', as a function of direction. It is a two-sheeted surface. One sheet is the surface of an ellipsoid of revolution; the other is the surface of a sphere. The cross sections of these surfaces in the coordinate planes, $z'=0$, $y'=0$, and $x'=0$, plotted in Fig. 2, can be expressed most conveniently in terms of the components of the ray velocity normalized to the free space velocity of light.
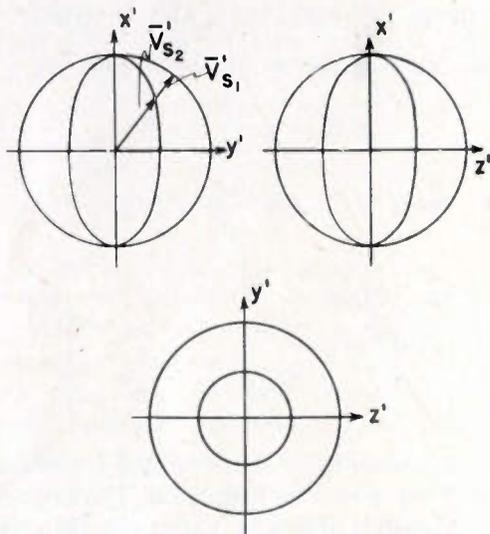
Fig. 2—Co-ordinate plane cross sections of the ray velocity surfaces representing the transformed system.

In the plane $z'=0$,

$$(c_z')^2 + (c_v')^2 = 1/\gamma_z, \quad (12a)$$
$$\gamma_y(c_z')^2 + \gamma_z(c_v')^2 = 1, \quad (12b)$$

where

$$\bar{s}\frac{c_s'}{c} = \bar{V}_s' = \bar{i}c_x' + \bar{j}c_v' + \bar{k}c_z'. \quad (13)$$

The corresponding equations in the planes $x'=0$, $y'=0$ are obtained by permuting the subscripts in the order $xyzxyz \ldots$ The magnitude of the normalized velocity of radiation in any direction is given by the length of the radius vector to the plotted surface. The two ray surfaces have a double point of tangency at the points of intersection with the x' axis. The surface of equal phase, the wave front, is not in general propagated in the same direction or with the same velocity as the energy. The direction of propagation of the wave front, the wave normal, is always perpendicular to the ray velocity surface. In the direction of the x' axis of Fig. 2,

the energy and phase fronts are, propagated with equal velocity. This characteristic defines the x' direction as the optic axis of the transformed system.

These analytical results are in essential agreement with the early observation that a plane wave impinging upon the surface of quartz or calcite crystals split into two waves within the medium. The wave associated with the spherical ray velocity surface is the ordinary wave and is characterized by the fact that its electric field is perpendicular to the plane containing the direction of energy propagation and the optic axis. The wave whose velocity depends on direction is the extraordinary wave and has its electric field polarized parallel to the plane just described.

TRANSFORMATION BACK TO ORIGINAL MEDIUM

It is now desirable to see what information about propagation in the actual disk medium may be derived from the above discussion.

A simple relation between the transformed ray velocity V_s' and the actual ray velocity V_s is derived in the Appendix, showing that,

$$\bar{V}_s = [\alpha]\bar{V}_s'. \quad (14)$$

The transformation in (14) is substituted into equation (12) resulting in the equations of the cross sections of the actual ray velocity surfaces in the coordinate planes of the disk array of Fig. 1.

In the plane, $z=0$,

$$K_{my}K_{mz}C_z^2 + K_{mz}K_{mz}C_v^2 = 1/\gamma_z, \quad (15a)$$
$$\gamma_y K_{my}K_{mz}C_z^2 + \gamma_z K_{mz}K_{mz}C_v^2 = 1. \quad (15b)$$

The corresponding equations in the planes $x=0$, $y=0$ are obtained by permutation of the subscripts in the sequence $xyzxyz \ldots$. Equations (15) are plotted in Fig. 3. Instead of one ordinary ray and one extraor-
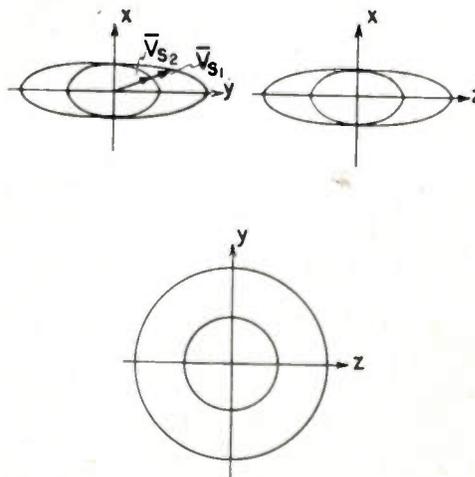
Fig. 3—Co-ordinate plane cross sections of the ray velocity surfaces representing the actual disk array.

dinary ray, two extraordinary rays are propagated in the medium with electric and magnetic anisotropy. When the rays lie in one of the co-ordinate planes, one ray has its electric field perpendicular to the plane and the electric field of the other is parallel to the plane.

HUYGHENS CONSTRUCTION OF REFRACTED WAVES

The ray velocity surfaces are the essential tools for construction of the waves refracted at a surface of the disk medium, when a plane wave impinges at oblique incidence. If the optic axis does not lie in the plane of incidence, the rays are not in general refracted according to Snell's Law, and three-dimensional representation is necessary.

Fig. 4 illustrates a particular case, representable in two dimensions. A semi-infinite array of disks is
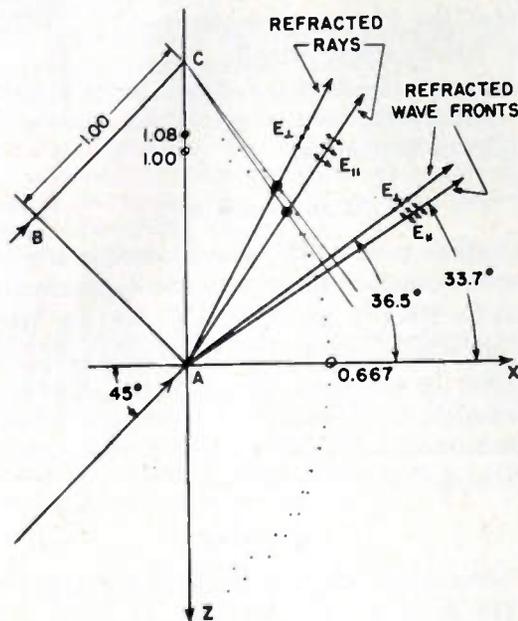
Fig. 4—Huyghens construction of the refracted rays and wave normals.

bounded by the plane $x = 0$, with the disk orientation described by Fig. 1. The size and spacing of the disks are chosen such that

$$K_{ez} = K_{ey} = 1 + (16Na^3/3) = 2.25. \qquad (16)$$

It was previously indicated that this formula was derived under conditions satisfied by a wave normally incident at the boundary, $x = 0$. The dotted cross section of the ray velocity surface in Fig. 4 shows that in the x direction, the velocity of the ray is slowed to 0.667 of its free-space velocity.

Fixing the value of the dielectric coefficients automatically fixes the value of the permeability coefficient K_{mz}, since they depend on the same variables. In this case,

$$K_{mz} = 1 - (8Na^3/3) = 0.375. \qquad (17)$$

A plane wave is now assumed incident at an angle of 45°. When Huyghens Construction is applied, the illustration shows two wave fronts refracted at 33.7° and 36.5°. This result is to be compared with the angle of refraction of 28.1° which would be predicted if a refractive index of 1.5 were assumed in all directions.

The polarization of the refracted rays and wave fronts is indicated by the symbols $E_\perp$ or $E_\parallel$ and by the associated dots and arrows.

Fig. 5(a) shows a plot of the predicted variation of the angle of refraction of the wave normals with changing angle of incidence in Fig. 4. The curve shows that the perpendicularly polarized wave should not be transmitted at angles of incidence greater than 66.3°.

Fig. 5(b) shows a similar curve for a disk medium whose dielectric coefficients in the y and z directions are designed to be 1.21, corresponding to an array used at the University of Wisconsin for the construction of an experimental prism. For incidence in the $x - z$ plane no critical angle occurs at which the perpendicularly polarized wave should not be transmitted.
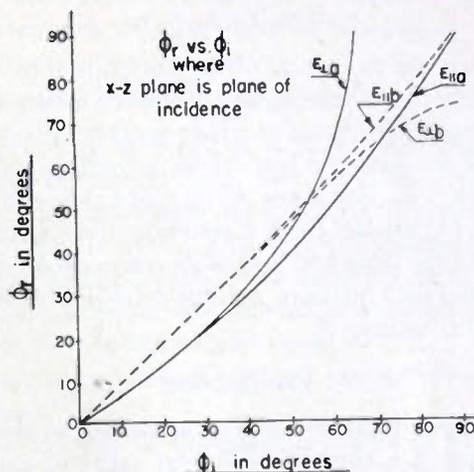
Fig. 5—Variation of the angle of refraction of the wave normals with changing angle of incidence in the plane of Fig. 4.
(a) $K_{mz} = 0.375$; $K_{ey} = K_{ez} = 2.25$.
(b) $K_{mz} = 0.895$; $K_{ey} = K_{ez} = 1.21$.

LENS DESIGN

Now that some of the important characteristics of the anistropic array of conducting disks have been investigated, it is desirable to establish the lens shape required to produce a flat phase front from a divergent one.

Fig. 6 shows the cross-sectional contour of a plano-convex lens. The optic axis (x axis) is perpendicular to the plane surface of the lens. The incident rays are assumed to diverge from a point on the x axis to the left of the convex surface. The conditions to be fulfilled at

$$OQ = OP + n_x(PP')$$
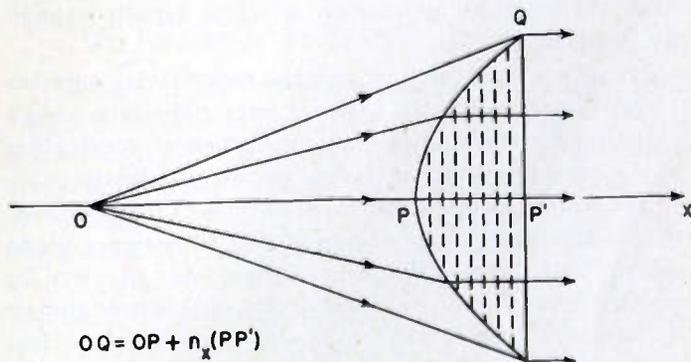
Fig. 6—Cross-sectional contour of plano-convex lens.

the convex surface are very different from those considered in Figs. 4 and 5, since both the angle of incidence and the angle between the optic axis and the normal to the boundary are changing along the lens surface. At

each point on the lens surface the incident ray is to be refracted so that it leaves the convex surface in the direction of the optic axis. For the usual case of a spherical wave front, this is easily achieved by making the convex surface a surface of revolution about the x axis. The optic axis always lies in the plane of incidence and Snell's Law holds for refraction of the rays, i.e.,

$$\sin \phi_i = \eta(\phi_r) \sin \phi_r, \tag{18}$$

where n is the reciprocal of the normalized ray velocity in the lens medium, and is a function of ϕ_r. The curvature of the lens cross section must be designed such that

$$\sin \phi_i / \sin \phi_r = \eta_z, \tag{19}$$

where n_z is the reciprocal of the normalized ray velocity in the direction of the optic axis. Since n_z is a constant, (19) is precisely the relation that must be satisfied in the design of an isotropic lens whose refractive index in all directions is n_z. For a one surface lens the required contour is a hyperbola. Such a simplification cannot be extended to the design of a doubly convex lens where the rays in the lens medium are not refracted in the direction of the optic axis.

Conclusion

This paper began by taking cognizance of the evident anisotropy of a three dimensional array of conducting disks. For the simplest case where interaction between disks may be neglected, the dielectric, permeability and conduction coefficients along favored cartesian axes are known. The constitutive equations are thus completely specified. With the aid of a constant transformation matrix, it is possible to utilize prior studies in crystal optics to obtain a quantitative solution for the velocity of energy propagation in any direction in the disk array. Due to the rotational symmetry of the array, there is one direction in which the ray velocity is independent of polarization. This medium differs from the usual uniaxial optical crystal insofar as neither of the two rays associated with the incident polarization has a velocity independent of direction. This characteristic results from the magnetic anisotropy which is usually negligible in optical media.

Huyghens construction uses the ray velocity surfaces to predict the refracted rays for any angle of incidence and thus provides a basis for experimental verification of the foregoing analysis. It also permits the justification of the simple formula which uses only the refractive index in the direction of the optic axis to design a plano convex lens. Such a design technique cannot safely be applied to arrays of conducting elements whose shapes do not have the symmetry of the circular disk unless more stringent conditions on the polarization of the incident rays are established.

The severest limitation on the analysis presented in this paper is the assumption that interaction effects may be neglected in the derivation of the constitutive coefficients. If the number of disks per unit volume is

increased until

$$1 + (16Na^3/3) > 3,$$

the formula, (3), for K_{mz} would indicate a negative permeability coefficient. Since the energy density of the electromagnetic field must have a positive definite form, that is an impossible condition and the formulas can no longer be true. Reference to a method for including interaction effects in derivation of the dielectric coefficients was made in the introduction. If such a correction factor can be determined for the permeability coefficient, the analysis of propagation in the disk array may be greatly strengthened.

An experimental study of this medium is now being carried out by the author and V. C. Rideout at the University of Wisconsin.

Appendix

The normalized velocity of energy propagation may be written, from (15) and (11), as

$$c\overline{V}_s' = \overline{S}'/W'$$

in the transformed system and as

$$c\overline{V}_s = \overline{S}/W$$

in the original system. The relations between the energy densities and the Poynting vectors in the two systems are derived by utilizing the transformations defined in (6).

$$W' = U_M' + U_E'.$$

$$U_M' = \frac{1}{2}\left\{ [\alpha]\overline{H} \cdot \mu_0 [\alpha]\overline{H} \right\}$$

$$= \frac{1}{2}\mu_0 \frac{[K_m]}{|K_m|} \overline{H} \cdot \overline{H} = \frac{U_M}{|K_m|}.$$

$$U_E' = \frac{1}{2}\left\{ [\alpha]\overline{E} \cdot \epsilon_0 [\gamma][\alpha]\overline{E} \right\}$$

$$= \frac{1}{2}\epsilon_0 \frac{[K_e]}{|K_m|} \overline{E} \cdot \overline{E} = \frac{U_E}{|K_m|}.$$

$$W' = W/|K_m|.$$

$$\overline{E}' \times \overline{H}' = |\alpha| [\alpha]^{-1} \{\overline{E} \times \overline{H}\} = [\alpha]^{-1}\{\overline{E} \times \overline{H}\}/|Km|.$$

$$c\overline{V}_s' = \frac{\overline{E}' \times \overline{H}'}{W'} = [\alpha]^{-1}\frac{\overline{E} \times \overline{H}}{W} = [\alpha]^{-1}c\overline{V}_s.$$

$$\overline{V}_s = [\alpha]\overline{V}_s'.$$

The Theory of Biconjugate Networks*

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Summary—The properties possessed in common by biconjugate networks are derived. It is shown that all biconjugate networks possess the following properties:

(1) Of the six possible transfer impedances only three are independent, one of them being infinite;

(2) The magnitudes of the reflection coefficients at each resistance are equal;

(3) The phase angles of the transfer impedances are not independent but must satisfy equation (4) of the text.

It is shown that biconjugate networks can be divided into two classes depending upon the phase relationship existing between pairs of transfer impedances. One class of networks is such that the responses to two driving voltages consist of one output proportional to the sum of the driving voltage and of one output proportional to the difference of the driving voltages.

Waveguide networks, such as hybrid circles, magic tees, and directional couplers, are examples of quasi-biconjugate networks.

The $7/2$-λ_0 hybrid circle is analyzed in detail. Computations yielding all the driving-point and transfer impedances have been made. The results are plotted.

Two quantities measuring the deviation of the $7/2$-λ_0 hybrid circle from the ideal behavior are defined and evaluated. One of these quantities measures the cross coupling between pairs of quasi-conjugate arms. The other quantity measures the degree to which the network fails to take sums and differences.

I. INTRODUCTION

Scope

BICONJUGATE NETWORKS play an important role in radar systems. It is the purpose of this paper to discuss these networks in general terms. The characteristics which they exhibit in common will be pointed out. A specific waveguide network—the $7/2$-λ_0 hybrid circle will be analyzed in detail. At the design center frequency, this network exhibits the properties of biconjugate networks. At other frequencies, the hybrid circle has properties differing from those of the ideal biconjugate network. The nature of these changes in characteristics will be analyzed.

The properties of biconjugate networks are of interest in the design of microwave communications systems and of radar systems since magic tees, hybrid rings, and directional couplers play important roles in the radio-frequency portions of these systems.

There are three major types of biconjugate waveguide[1-4] structures:

[1] W. A. Tyrell, "Hybrid circuits for microwaves," PROC. I.R.E., vol. 35, pp. 1244–1306; November, 1947.
[2] H. J. Riblet, "A mathematical theory of directional couplers," PROC. I.R.E., vol. 35, pp. 1307–1313; November, 1947.
[3] W. W. Mumford, "Directional couplers," PROC. I.R.E., vol. 35, pp. 160–166; February, 1947.

(1) hybrid rings
(2) hybrid junctions (magic tees)
(3) directional couplers.

The best configuration of hybrid ring electrically is the shortest ring. For mechanical reasons a hybrid ring whose mean circumference is $7/2$-λ_0 is a good compromise. The analysis of Section III has as its object the determination of the properties of this ring.

Statement of the Problem—Definitions

A biconjugate network is a linear network of any complexity containing four, and only four, resistances such that the resistances are conjugate in pairs.

Given a linear network N, two resistances A and B are said to be conjugate if the insertion of an electromotive force in series with one of the resistors causes no current to flow in the second resistor.

A biconjugate network contains two sets of conjugate resistances.

The network to be discussed is shown in Fig. 1. The four resistances are shown explicitly. The portion of the network contained within the box of Fig. 1 contains reactive elements only.
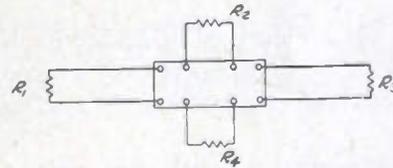
Fig. 1—General passive network containing four resistances.

For definiteness it is assumed that R_1 is conjugate to R_3 and that R_2 is conjugate to R_4.

II. THEORY OF BICONJUGATE NETWORKS

The properties of general linear network are described by Campbell and Foster[5] in terms of coefficients t_{jk}. The coefficient t_{jk} is defined as follows: Let an electromotive force be inserted in series with the resistance R_j and let the current flowing in R_k be measured, then t_{jk}^2 is the ratio of the volt-ampere product at R_k divided by the maximum possible volt-ampere product. If S_{jk} represents the transfer impedance from the loop containing R_j to the loop containing R_k, then

[4] H. T. Budenbom, "Analysis and performance of waveguide hybrid rings for microwaves," *Bell Sys. Tech. Jour.*, vol. 27, p. 473; July, 1948.
[5] G. Campbell and R. M. Foster, "Maximum output networks for telephone substation and repeater circuits," *Trans. AIEE*, vol. 39, pp. 231–280; February, 1920.

$$t_{jk} = \frac{2\sqrt{R_jR_k}}{S_{jk}}. \tag{1}$$

From the definition of the t_{jk} it is evident that the magnitude of these quantities must be between zero and unity.

A general linear network containing four resistances satisfies equations of the following form:

$$t_{ik} + t_{ik}' = \sum_{q=1}^{4} t_{iq}t_{qk}' \tag{2}$$

$$i, k = 1, 2, 3, 4.$$

In (2) the prime accent denotes the complex conjugate. There are ten equations of the form (2) for a network containing four resistances. These equations are written explicitly in Appendix I.

If biconjugacy is assumed (for definiteness let it be assumed that R_1 is conjugate to R_3 while R_2 is conjugate to R_4), then

$$t_{13} = t_{24} = 0. \tag{3}$$

Imposing the condition (3) on (2) results in the following set of relations:

$$\begin{aligned}
|\rho| &= |t_{11} - 1| = |t_{22} - 1| = |t_{33} - 1| = |t_{44} - 1| \\
&|t_{12}|^2 = |t_{34}|^2 \\
&|t_{14}|^2 = |t_{23}|^2 \\
&t_{12}t_{23}' = -t_{14}t_{34}'.
\end{aligned} \tag{4}$$

Equations (4) are derived in Appendix I. Equations (3) and (4) describe the properties of a biconjugate network. Some of these properties are stated explicitly below, namely:

(1) A biconjugate network has only three independent transfer impedances, one of which is infinite;

(2) The magnitude of the reflection coefficients ρ at each of the resistances are equal. The fact that $t_{jj}-1$ is a reflection coefficient is shown in Appendix I;

(3) The phase angles of the transfer impedances must satisfy the last of relations (4). The implications of this relationship will be discussed in Section II, part 3.

The relations (3) and (4) apply to any biconjugate network with 4 resistances. In particular, these relations apply to waveguide circuits such as (1) hybrid junctions (magic tees), (2) hybrid rings, and (3) directional couplers to the extent that these circuits possess the biconjugacy condition.

The networks enumerated above approximate the biconjugacy condition over a band of frequencies.

Classification of Biconjugate Networks

Biconjugate networks can be divided into two classes in accordance with the conditions necessary for zero output in one resistor when two coherent signals are fed into the network in series with the two resistors which are nonconjugate to the resistor with zero output.

The basis for this classification is developed below.

Let electromotive forces E_1 and E_3 be inserted in series with R_1 and R_3, respectively, in Fig. 1. Let e_2 and e_4 represent the voltage drop across R_2 and R_4 due to the simultaneous application of E_1 and E_3. Then

$$\begin{aligned}
e_2 &= R_2 \left[\frac{E_1}{S_{12}} + \frac{E_3}{S_{23}} \right] \\
e_4 &= R_4 \left[\frac{E_1}{S_{14}} + \frac{E_3}{S_{34}} \right].
\end{aligned} \tag{5}$$

The relations (5) show that the conditions necessary to produce zero voltage drop across R_2 and R_4, respectively, are

$$\alpha_2 = \frac{E_1}{E_3} = -\frac{S_{12}}{S_{23}} = -\sqrt{\frac{R_1}{R_3}}\frac{t_{23}}{t_{12}} \tag{6}$$

and

$$\alpha_4 = \frac{E_1}{E_3} = -\frac{S_{14}}{S_{34}} = -\sqrt{\frac{R_1}{R_3}}\frac{t_{34}}{t_{14}}. \tag{7}$$

Equations (6) and (7) furnish a basis for dividing biconjugate networks into two classes, namely

(1) Class I—Networks which produce zero output in R_2 or R_4 for real values of α_2 or α_4;

(2) Class II—All other cases.

1. Biconjugate Networks of Class I: Hybrid junctions and hybrid rings approximate in their characteristics the networks of Class I, while equal-arm directional couplers resemble networks of Class II.

It will be shown in part 3 below, however, that a 90° phase shifter in series with one arm of an equal-arm directional coupler will transform the resultant network into a network of Class I, while a 90° phase shifter in series with one arm of a hybrid circle will transform the resultant network into a network of Class II.

Since the networks of Class I which are of interest in this report are hybrid junctions and hybrid circles, a further restriction satisfied by these networks may be imposed, namely that the values of α_2 and α_4 have the value plus or minus unity. It can be shown by taking the ratio of α_2 and α_4 and using the last of relations (4) that the sign of α_2 is opposite to that of α_4. For definiteness, let it be assumed that α_4 has the positive value. This involves no loss of generality. This means that reversing the phase of one of the inputs E_1 or E_3 interchanges the arms with zero and nonzero output.

If all resistances are assumed equal, as is the case for matched loads on each of the arms external to the hybrid structures, the combination of (6) and (7) with (4) gives the result that

$$|t_{12}|^2 = |t_{14}|^2 = |t_{23}|^2 = |t_{34}|^2$$
$$t_{34} = -t_{14} \tag{8}$$
$$t_{23} = t_{12}.$$

The first of equations (4) shows that the reflection coefficients at each resistance of a biconjugate network are equal. Normally the design is such that the value of this coefficient is zero (matched impedances at each resistance). For this case

$$|t_{12}| = |t_{14}| = |t_{23}| = |t_{34}| = \sqrt{\tfrac{1}{2}}. \qquad (9)$$

For the case that ρ does not equal zero, the equivalent relation is

$$|t_{12}| = |t_{14}| = |t_{23}| = |t_{34}| = \sqrt{\frac{1 - |\rho|^2}{2}}. \qquad (10)$$

2. Biconjugate Networks of Class II: An example of a biconjugate waveguide structure of Class II is an equal-arm directional coupler. Let a directional coupler with
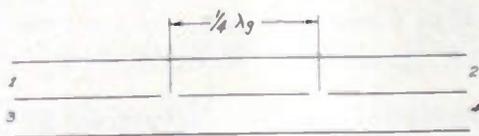
Fig. 2—Equal-arm directional coupler.

four equal arms be assumed (see Fig. 2). The symmetry of the structure indicates that

$$t_{12} = t_{34} = |t_{12}| e^{i\phi_1}$$
$$t_{14} = t_{23} = |t_{14}| e^{i\phi_2}. \qquad (11)$$

Substitution of relations (11) into the last of relations (4) requires that

$$\phi_2 - \phi_1 = \pi/2. \qquad (12)$$

This means that in a directional coupler a generator inserted in series with one arm produces two outputs which differ in phase by 90°. This compares with networks of Class I in which a single generator produces two outputs which are in phase, or differ in phase by 180° (see (8)).

If K represents the magnitude of t_{12}, then

$$|t_{34}| = |t_{12}| = K$$
$$|t_{23}| = |t_{14}| = \sqrt{1 - K^2 - |\rho|^2}. \qquad (13)$$

If the directional coupler is designed to see a match at each arm, then the second equation of (13) becomes

$$|t_{14}| = \sqrt{1 - K^2}. \qquad (13a)$$

3. Reciprocal Phase Relationships: Equations (6) and (7) contain the conditions which produce zero output in resistors R_2 or R_4 due to the simultaneous application of two coherent electromotive forces. These equations may be rewritten as follows:

$$\begin{aligned} E_1 t_{12} &= -E_3 t_{23} \\ E_1 t_{14} &= -E_3 t_{34}. \end{aligned} \qquad (14)$$

or

These equations show that it is possible to produce

zero output in arm 2 or 4 by adjustment of the amplitude and phase of the electromotive forces.

Let these equations be applied specifically to a directional coupler. Since the difference in phase between t_{12} and t_{23} and between t_{14} and t_{34} is a phase shift of 90° in the proper direction in E_1 or E_3 or in any one of the quantities t_{12}, t_{14}, t_{23} or t_{34} will result in zero output in R_2 or R_4.

The discussion of the above paragraphs indicates that the phase associated with the transfer impedances or with the generators may be interchanged. Equation (14) illustrates this point since each term is the product of an electromotive force by one of the t_{jk}.

III. ANALYSIS OF THE $7/2\lambda_0$ SERIES OF HYBRID RING

There are many forms which a hybrid ring may take. The specific ring to be analyzed is shown in Fig. 3. The junctions of the ring to external circuits are all of
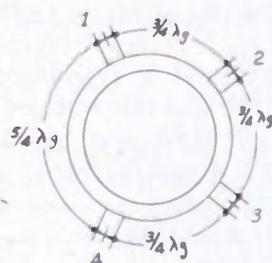
Fig. 3—$7/2$-λ_g series hybrid ring.

the series type. The mean circumference of the ring is $7/2 \, \lambda_{g_0}$, where λ_{g_0} is the guide wavelength at the design center frequency. As shown in Fig. 3, the spacing between junctions is $3/4 \, \lambda_{g_0}$, $3/4 \, \lambda_{g_0}$, $3/4 \, \lambda_{g_0}$, and $5/4 \, \lambda_{g_0}$. The equivalent circuit for the structure of Fig. 3 is shown in Fig. 4.
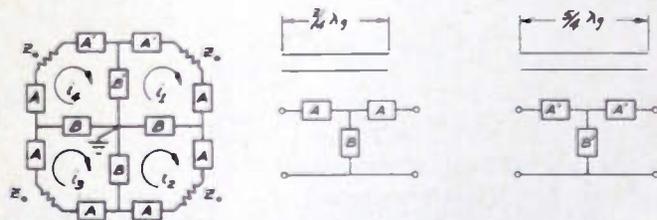
Fig. 4—Equivalent circuit for a series hybrid circuit.

Fig. 4 is based on an analysis by Budenbom.[4] In his report, Budenbom considers the behavior of networks of the type shown by Fig. 4 at the design center frequency and points out how to extend the analysis at other frequencies.

It is the purpose of this section to extend the analysis to all frequencies. At the design center frequency the network will posses the property of biconjugacy. At this frequency the results of Section II apply.

For frequencies other than the design frequency, the network will not possess the biconjugacy property except as an approximation. It is a further purpose of this section to determine the degree of this approximation.

When the network becomes quasi-biconjugate at frequencies removed from the design values, two types of quantities are important as a measure of the departure from biconjugacy. One type of quantity measures the transmission between the pairs of quasi-conjugate arms. The second type of quantities measures the degree to which the network fails to take sums and differences. The first type of these quantities will be referred to as the cross coupling between the quasi-conjugate arms. The second quantity will be referred to as the unbalance of the network.

The cross coupling X_{jk} between a pair of quasi-biconjugate resistors is defined as the value of t_{jk} where j and k refer to a pair of quasi-conjugate resistors. This makes $|t_{jk}|$ equal to zero for perfect isolation and equal to unity if the coupling has the maximum possible value.

Because of the symmetry of Fig. 3, it is true that

$$X_{13} = X_{24} = |t_{13}| = |t_{24}|. \qquad (15)$$

The unbalance U_{jk} of a quasi-biconjugate network is defined as the ratio of the voltages e_j/e_k across one pair of quasi-conjugate resistors due to the application of two coherent electromotive forces in series with the other pair of resistors when the electromotive forces are such that zero output would result across e_j if the network were biconjugate.

From the definition of unbalance

$$U_{13} = \left(\frac{e_1}{e_3}\right)_{E_2 = E_3} = \frac{t_{12} + t_{14}}{t_{23} + t_{34}}. \qquad (16)$$

Similar expressions can be derived for U_{31}, U_{24} and U_{42}.

Because of the symmetry of the hybrid circle $t_{12} = t_{34}$. Hence, only two of the four unbalances are independent, namely:

$$U_{13} = U_{42} = \frac{t_{12} + t_{14}}{t_{23} + t_{34}}$$

$$U_{31} = U_{24} = \frac{t_{23} - t_{34}}{t_{12} - t_{14}}. \qquad (17)$$

To evaluate the quantities X_{jk} and U_{jk}, it is necessary to analyze the network of Fig. 4. The analysis is made in Appendix II.

Let Δ represent the determinant of the network shown in Fig. 4 and let Δ_{jk} represent the minor of Δ obtained by striking out the jth row and the kth column of Δ. The equations of the text involving t_{jk} may be expressed in terms of Δ and its minors by making use of relations (1) and the fact that

$$S_{jk} = \frac{\Delta}{\Delta_{jk}},$$

so that

$$t_{jk} = \frac{2\sqrt{R_j R_k}}{S_{jk}} = 2\sqrt{R_j R_k}\, \frac{\Delta_{jk}}{\Delta}. \qquad (18)$$

If all the R's are equal and of value Z_0, where Z_0 is the characteristic impedance of the external arms of the waveguide, then

$$t_{jk} = 2Z_0 \frac{\Delta_{jk}}{\Delta}. \qquad (19)$$

It is shown in Appendix II that $\overline{Z} = Z_0/\sqrt{2}$ is the characteristic impedance which the hybrid ring must have in order to present an impedance of Z_0 to each of the external arms at the design center frequency.

The determinants Δ and Δ_{jk} have been evaluated for frequencies within 6 per cent of the design center frequency. The normalized values of all transfer impedances and driving-point impedances as well as the cross coupling and unbalances have been computed. These quantities are plotted in Figs. 5 through 7. In plotting Figs. 5 and 6 the impedance was normalized to the value the quantity has at the design center frequency.

The normalized transfer impedances $S_{12} = S_{34}$ are shown in Fig. 5. The magnitude and phase angle of this impedance are plotted against frequency. It will be noted from the magnitude curve that over the band shown, the normalized transfer impedance varies from a maximum value of about 1.08 to a minimum value of 1.0. The phase angle increases from about $-125°$ to about $-55°$ passing through $-90°$ at the band center.

Fig. 5 also contains plots of the normalized impedances S_{14} and S_{23}. The magnitude plots for these impedances are very nearly equal. The value of the magni-
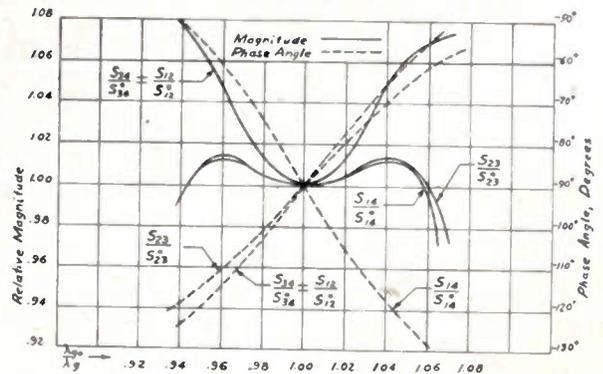
Fig. 5—Transfer impedance.

tude varies between 0.964 and 1.012. The phase angle for S_{14} varies from about 50° to about 135°. The phase angle for S_{23} varies from about $-120°$ to about $-58°$.

Fig. 6 shows the normalized driving-point impedances for $S_{11} = S_{44}$ and for $S_{22} = S_{33}$, respectively. The magnitude of $S_{11} = S_{44}$ varies between 0.954 and 1.002. Its phase varies between about $-1.0°$ to $+1.0°$. The magnitude of $S_{22} = S_{33}$ varies between 1.0 and 0.860. Its phase angle varies between $+5°$ and $-5°$.

Fig. 7 shows the cross coupling. The magnitude of the cross coupling varies from zero at the center frequency
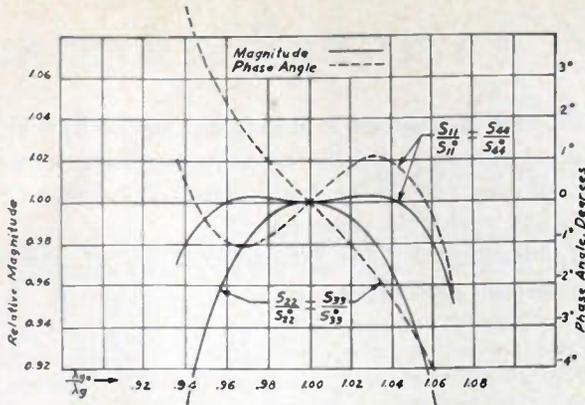
Fig. 6—Driving-point impedance.

to about 0.06 at the band edges. The phase angle jumps discontinuously from $+90°$ to $-90°$ in passing through
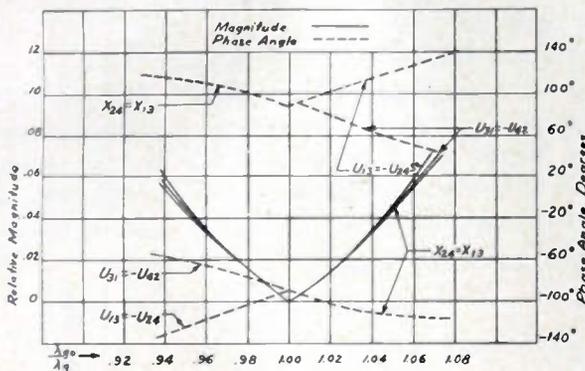
Fig. 7—Cross coupling and unbalance.

the design center frequency. The phase angle for frequencies less than the design frequency varies from about 115° to 90°. The phase angle for frequencies greater than the design frequency varies from $-115°$ to $-90°$.

Fig. 7 also shows the unbalances of the network. The magnitudes of the two unbalances are seen to be very nearly equal to each other. The common magnitude, moreover, is very nearly equal to the cross coupling.

Appendix I

For a network with four and only four resistances, (2) becomes

$$|t_{11} - 1|^2 + |t_{12}|^2 + |t_{13}|^2 + |t_{14}|^2 = 1$$

$$|t_{22} - 1|^2 + |t_{21}|^2 + |t_{23}|^2 + |t_{24}|^2 = 1$$

$$|t_{33} - 1|^2 + |t_{31}|^2 + |t_{32}|^2 + |t_{34}|^2 = 1$$

$$|t_{44} - 1|^2 + |t_{41}|^2 + |t_{42}|^2 + |t_{43}|^2 = 1$$

$$(t_{11} - 1)t_{12}' + t_{12}(t_{22}' - 1) + t_{13}t_{23}' + t_{14}t_{24}' = 0$$

$$(t_{11} - 1)t_{13}' + t_{12}t_{23}' + t_{13}(t_{33}' - 1) + t_{14}t_{34}' = 0 \tag{20}$$

$$(t_{11} - 1)t_{14}' + t_{12}t_{24}' + t_{13}t_{34}' + t_{14}(t_{44}' - 1) = 0$$

$$t_{12}t_{13}' + (t_{22} - 1)t_{23}' + t_{23}(t_{33}' - 1) + t_{24}t_{34}' = 0$$

$$t_{12}t_{14}' + (t_{22} - 1)t_{24}' + t_{23}t_{34}' + t_{24}(t_{44}' - 1) = 0$$

$$t_{13}t_{14}' + t_{23}t_{24}' + (t_{33} - 1)t_{34}' + t_{34}(t_{44}' - 1) = 0.$$

In the text, it is assumed that resistors R_1 and R_3 form one conjugate pair while R_2 and R_4 form the second conjugate pair. If equations (3) are substituted into (20), these equations become

$$|t_{11} - 1|^2 + |t_{12}|^2 + |t_{14}|^2 = 1$$

$$|t_{22} - 1|^2 + |t_{12}|^2 + |t_{23}|^2 = 1$$

$$|t_{33} - 1|^2 + |t_{34}|^2 + |t_{23}|^2 = 1$$

$$|t_{44} - 1|^2 + |t_{34}|^2 + |t_{14}|^2 = 1$$

$$(t_{11} - 1)t_{12}' + t_{12}(t_{22}' - 1) = 0$$

$$t_{12}t_{23}' + t_{14}t_{34}' = 0 \tag{21}$$

$$(t_{22} - 1)t_{23}' + t_{23}(t_{33}' - 1) = 0$$

$$(t_{11} - 1)t_{14}' + t_{14}(t_{44}' - 1) = 0$$

$$t_{12}t_{14}' + t_{23}t_{34}' = 0$$

$$(t_{33} - 1)t_{34}' + t_{34}(t_{44}' - 1) = 0.$$

To prove the first equation of (4) let the fifth equation (21) be written as

$$(t_{11} - 1)t_{12}' = -t_{12}(t_{22}' - 1). \tag{22}$$

If the absolute values of both sides of this equation are taken, the result is evident, since $|t_{12}|$ is not zero, that

$$|t_{11} - 1| = |t_{22} - 1|. \tag{23}$$

Similarly, by considering the seventh and tenth of (21), the desired result is obtained, namely

$$|\rho_{jj}| = |t_{11} - 1| = |t_{22} - 1| = |t_{33} - 1|$$
$$= |t_{44} - 1|. \tag{24}$$

The quantity

$$\rho_{jj} = t_{jj} - 1 \tag{25}$$

is the reflection factor at the resistance R_j. By definition of t_{jj} (see (1))

$$\rho_{jj} = \frac{2R_j}{S_{jj}} - 1 = \frac{R_j - (S_{jj} - R_j)}{R_j + (S_{jj} - R_j)}. \tag{26}$$

Since S_{jj} is the total impedance faced by a generator in series with R_j, then the impedance faced by R_j is $S_{jj} - R_j$. It will be noted from the form of (26) that ρ_{jj} is a reflection coefficient. This completes the proof that the magnitudes of all the reflection coefficients of a biconjugate network are equal.

The fourth of (4) of the text is simply the sixth of (21) with one of the terms transferred to the right-hand side.

To derive the second of (4) let the third of (21) be subtracted from the second of (21). The result using (24) is

$$|t_{12}|^2 = |t_{34}|^2. \tag{27}$$

Similarly, by subtracting the second of (21) from the first of these equations and using (26), it can be shown that

$$|t_{14}|^2 = |t_{23}|^2. \tag{28}$$

APPENDIX II

To analyze the waveguide structure of Fig. 3, the network shown in Fig. 4 must be considered. In Fig. 4, the tees consisting of arms A, B, A; or A'; B'; A' represent equivalent circuits for waveguide of length $3\lambda_g/4$ and $5\lambda_g/4$, respectively.

The quantities A', B', A', and B have the values

$$A = -j\bar{Z}\left[\cot\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g} - \csc\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right]$$

$$B = -j\bar{Z}\left[\csc\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right]$$

$$A' = -j\bar{Z}\left[\cot\frac{5\pi}{2}\frac{\lambda_{g0}}{\lambda_g} - \csc\frac{5\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right]$$

$$B' = -j\bar{Z}\left[\csc\frac{5\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right]$$

(29)

where $\bar{Z}$ is the characteristic impedance of the ring.

The mesh equations for Fig. 4 are

$$
\begin{aligned}
E_1 &= i_1(Z_0 + A + B + A' + B') - i_2 B && - i_4 B' \\
E_2 &= -i_1 B && + i_2(Z_0 + 2A + 2B) - i_3 B \\
E_3 &= && -i_2 B && + i_3(Z_0 + 2A + 2B) - i_4 B \\
E_4 &= -i_1 B' && && -i_3 B && + i_4(Z_0 + A + B + A' + B').
\end{aligned}
$$

(30)

In these equations, Z_0 is the characteristic impedance of the waveguide external to the ring proper.

The determinant Δ of the system of equations, is, therefore,

$$
\Delta =
\begin{vmatrix}
Z_0 + A + B + A' + B' & -B & 0 & -B' \\
-B & Z_0 + 2A + 2B & -B & 0 \\
0 & -B & Z_0 + 2A + 2B & -B \\
-B' & 0 & -B & Z_0 + A + B + A' + B'
\end{vmatrix}
$$

(31)

In the text, following (19), it is stated that the choice

$$\bar{Z} = \frac{Z_0}{\sqrt{2}}$$

(32)

causes each waveguide arm external to the hybrid ring to face a match. To verify this statement, let the values of Δ, and Δ_{11}, and Δ_{22} be evaluated at the design center frequency with this value for Z.

At this frequency the quantities A, B, A', and B' of (30), assume the values

$$
\begin{aligned}
A &= -j\bar{Z} \\
B &= j\bar{Z} \\
A' &= j\bar{Z} \\
B' &= -j\bar{Z}
\end{aligned}
$$

(33)

and

$$
\begin{aligned}
\Delta &= Z_0^4 + 4Z_0^2\bar{Z}^2 + 4\bar{Z}^2 = 4Z_0^4 \\
\Delta_{44} &= \Delta_{11} = Z_0^3 + 2Z_0\bar{Z}^2 = 2Z_0^3 \\
\Delta_{33} &= \Delta_{22} = Z_0^3 + 2Z_0\bar{Z}^2 = 2Z_0^3.
\end{aligned}
$$

(34)

Hence

$$S_{44} = S_{11} = \frac{\Delta}{\Delta_{11}} = \frac{4Z_0^4}{2Z_0^3} = 2Z_0$$

$$S_{33} = S_{22} = \frac{\Delta}{\Delta_{22}} = \frac{4Z_0^4}{2Z_0^3} = 2Z_0.$$

(35)

This verifies the fact that for impedance match the characteristic impedance of the ring should be $Z_0/\sqrt{2}$.

Synthesis of Passive *RC* Networks with Gains Greater than Unity*

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Summary—This paper illustrates the fact that three-terminal passive resistor-capacitor networks are capable of having unity voltage gain at zero frequency and higher than unity gain over a prescribed frequency band. A general design procedure for synthesizing a transfer function with these properties using, insofar as possible, known methods of synthesis is outlined. Together with suggestions for further work, two examples are presented to illustrate the method.

I. INTRODUCTION

IN THE SYNTHESIS of three-terminal passive resistor-capacitor networks to obtain a specified transfer function, the voltage gain at zero frequency is never greater than unity. It is possible, however, to synthesize three-terminal passive resistor-capacitor networks with unity voltage gain at zero frequency and higher than unity gain over some pass band.[1] A typical asymptotic plot[2] of the transfer function versus frequency obtainable with such networks is shown in Fig. 1. The graph plots the following function on logarithmic
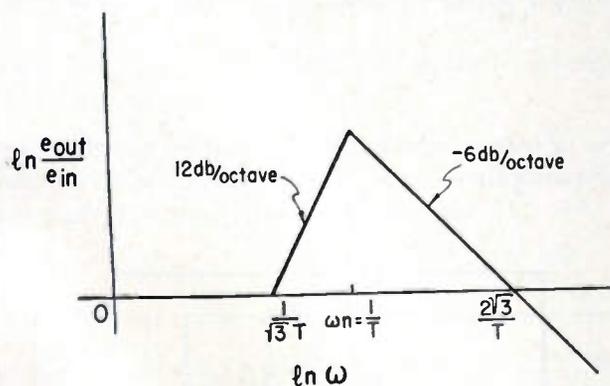
Fig. 1—Asymptotic logarithmic plot of a transfer function.

scales, assuming that no section of the network loads the preceding sections:

$$\frac{e_{out}}{e_{in}} = \frac{3T^2 s^2 + 3Ts + 1}{(Ts)^3 + 3(Ts)^2 + 3(Ts) + 1}, \quad (1)$$

where s is the Laplace operator, and equals $j\omega$ for plotting purposes;

$$T = \frac{1}{\omega_n}.$$

[1] C. L. Longmire, "An *R-C* circuit giving over unity gain," *Tele-Tech*, vol. 6, pp. 40–41; April, 1947.
[2] H. M. James, N. B. Nichols, and R. S. Phillips, "Theory of Servomechanisms," p. 169 ff., McGraw-Hill Book Co., New York, N. Y.; 1947.

II. DEVELOPMENT OF SYNTHESIS PROCEDURE

Fig. 2 shows the two possibilities in the output connections with a specific input connection for three-terminal networks. Output$_a$ at terminals 2–2' is the
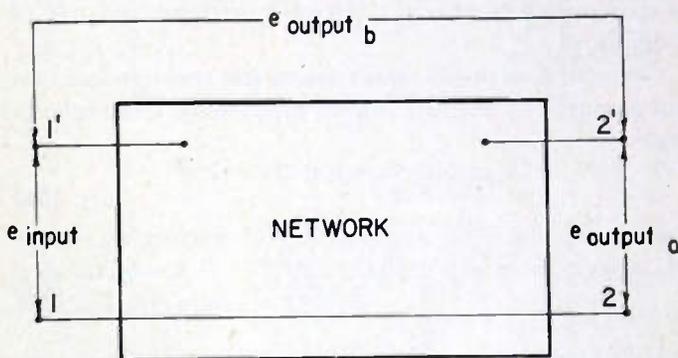
Fig. 2—General three-terminal network.

usual output connection. In this case, the common terminal 1–2 is ground, with the voltage transfer ratio given by

$$\frac{e_{out_a}}{e_{in}} = \frac{f(s)}{F(s)}, \quad (2)$$

where
  $f(s) =$ polynomial in s
  $F(s) =$ polynomial in s of equal or higher degree than the numerator.

For resistor-capacitor networks in general, the roots of the denominator must be simple, real, and negative. In certain cases, a good approximation of the transfer function can be made with multiple poles in the denominator. This is equivalent, for instance, to an assumption that the sections in a ladder network do not load one another. The roots of the numerator, however, can lie anywhere in the complex plane. In other words, nothing can be specified about the location of the zeroes of the transfer function of a resistor-capacitor network in general.

The one other possibility of taking the output from the network of Fig. 2 is indicated by output$_b$ across terminals 1'–2'. Assume that output$_b$ is connected to an infinite load such as the grid of a vacuum tube. The common lead 1' may be grounded. This type of connection changes the entire character of the network. Output$_b$ is now, in effect, the difference between the input and output$_a$;

$$e_{out_b} = e_{in} - e_{out_a}. \quad (3)$$

If, for instance, a resistor-capacitor ladder network pro-

duces a phase shift of π radians at some frequency, output_a is now expressed as

$$e_{\text{out}_a} = -ke_{\text{in}}, \tag{4}$$

where $0 < k < 1$. Thus, if the $1'$–1 terminals have polarity marks $+$, $-$, respectively, then the $2'$–2 terminals have polarity marks $-$, $+$, and output_b is then greater than the input

$$e_{\text{out}_b} = (1 + k)e_{\text{in}}. \tag{5}$$

Thus a gain greater than unity is obtained from a passive resistor-capacitor network without the use of amplifiers.

Generally, with (2) representing the transfer from input to output_a, output_b is then expressed by the following:

$$\frac{e_{\text{out}_b}}{e_{\text{in}}} = 1 - \frac{f(s)}{F(s)} \tag{6a}$$

$$= \frac{F(s) - f(s)}{F(s)}. \tag{6b}$$

It is convenient and now possible to obtain a synthesis procedure based upon known methods. Therefore, suppose a certain specified transfer function is to be synthesized and is given as the ratio of two polynomials

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{f_1(s)}{F(s)}. \tag{7}$$

This function is specified as having unity gain at zero frequency and a gain greater than unity in some band. It is desired to synthesize this transfer ratio with as little, if any, additional amplification as is possible. From (6b) and (7), using output_b in Fig. 2,

$$\frac{e_{\text{out}_b}}{e_{\text{in}}} = \frac{f_1(s)}{F(s)} = \frac{F(s) - f(s)}{F(s)} \tag{8a}$$

or

$$f(s) = F(s) - f_1(s). \tag{8b}$$

This equation states that $f(s)$ is the numerator and $F(s)$ is the denominator of a transfer function given in (2), which is to be synthesized using known methods; for instance, the Bower-Ordung synthesis procedure.[3] The numerator of the function to be synthesized is the difference between the denominator and numerator of the specified transfer function, and the denominator is the same as the specified transfer function. The passive resistor-capacitor network then used in connection b of Fig. 2 satisfies the specified transfer function, gives gains greater than unity, and requires less over-all additional amplification.

The method involves taking the specified transfer function, and according to (8), changing it to another transfer ratio which is synthesized by usual means. It is

[3] J. L. Bower and P. F. Ordung, "The synthesis of resistor-capacitor networks," Proc. I.R.E., vol. 38, pp. 263–269; March, 1950.

evident that the derived function which is actually synthesized is never more complicated and is often simpler in form than the specified transfer function. Thus, not only is there a saving in amplification, but possibly also in the number of components.

III. GAIN CONSIDERATIONS

The theoretical maximum voltage gain obtainable from a general type of RC ladder network is two. Each branch of the general ladder may consist of a parallel combination of a resistor and condenser. If the network consists of n sections, $n > 2$, with all the sections having the same resistor-capacitor product, and if it is assumed that each succeeding section does not load the previous section, a vector diagram yields the following as the gain at some frequency:

$$\frac{e_{\text{out}}}{e_{\text{in}}} = 1 + \cos^n\left(\frac{\pi}{n}\right) \qquad n > 2 \tag{9a}$$

where the total phase shift due to the network is π radians. A phase shift of π radians can be shown to yield maximum gain. Since the cosine has a maximum value of unity at $\pi/n = 0$ and has some value between 0 and 1 for other values of π/n, and since a fraction raised to an integer power n is less than the original value, then

$$\frac{e_{\text{out}}}{e_{\text{in}}} \leq 2. \tag{9b}$$

For a comparison to this maximum, the theoretical maximum gain of a specific four section ladder shown in Fig. 3 is 1.25, assuming no loading and $\pi/4$ phase shift
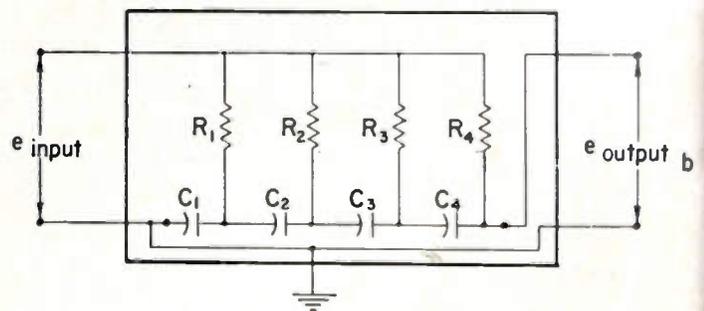
Fig. 3—Four-section ladder network.

from each section. On the other hand, the same type of circuit, made up of four identical sections, having the same impedance as the first section in the previous case has a gain magnitude of only 1.02 due to loading effects. This is calculated on the basis of zero source impedance and infinite load impedance. This comparison figure is arrived at by simply calculating the ratio of the output over the input voltage from the transfer determinant for a four-mesh ladder.

It has been shown that it is possible to obtain a saving in gain by a factor of two in designing so-called "lead" networks. It should be pointed out that a much greater

saving in gain is possible in the synthesis of so-called "lag" networks having unity gain at zero frequency.

IV. EXAMPLES

As the first example, let the specified transfer function be that stated in (1) and shown graphically in Fig. 1. After the synthesis procedure is carried out, the result is a three-stage RC network of the type shown in Fig. 3, with all the RC products being made equal. The impedance level of each section should be raised by about a factor of ten. Note that this network yields complex zeroes which are impossible to obtain with the conventional connection of the ladder network. Furthermore, Fig. 3 indicates that the network may be nonminimum phase. A ladder of this sort, even in the connection shown, is still a minimum phase network as seen from an inspection of its transfer function (1).

The second example will show another simple case, often arising in servo synthesis, in which a passive resistor-capacitor network has unity gain at zero frequency and a gain greater than unity over a specified band.

Transfer function to be synthesized is given in (10).

$$\frac{e_{\text{outb}}}{e_{\text{in}}} = \frac{(1 + T_1 s)}{(1 + T_2 s)(1 + T_3 s)} = \frac{f_1(s)}{F(s)} \quad (10)$$

where $T_1 > T_2 > T_3$. Following the synthesis procedure, a new function is formed

$$\frac{f(s)}{F(s)} = \frac{F(s) - f_1(s)}{F(s)}$$

$$= \frac{s\left[1 + \frac{T_2 T_3}{T_2 + T_3 - T_1} s\right](T_2 + T_3 - T_1)}{(1 + T_2 s)(1 + T_3 s)} \cdot \quad (11)$$

In this case, a network is designed for $T_2 + T_3 > T_1$. With the condition imposed upon the T's in (10) it can be shown that

$$T_a = \frac{T_2 T_3}{T_2 + T_3 - T_1} > T_1. \quad (12)$$
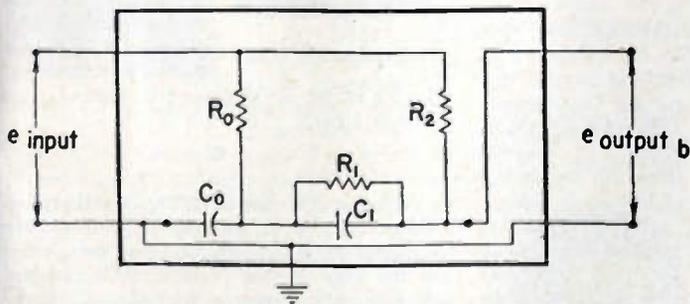
Fig. 4—Synthesized network satisfying (10).

$$T_3 = R_0 C_0$$
$$T_a = R_1 R_2 C_1$$
$$T_2 = \frac{R_1 R_2 C_1}{R_1 + R}.$$

The usual synthesis procedure yields the result shown in Fig. 4, with the impedance level of the second section raised to minimize loading effects. An exact synthesis of (11) carried out according to the Bower-Ordung procedure yields the network shown in Fig. 5. The values
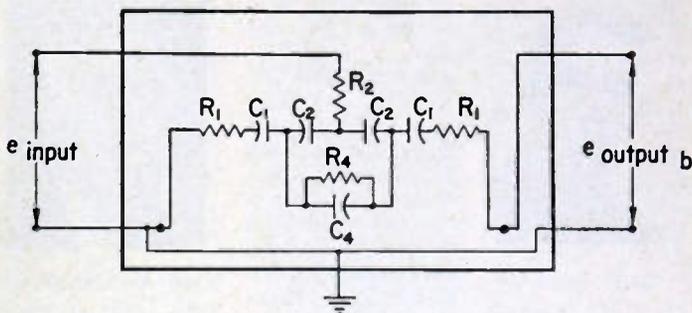
Fig. 5—Exact synthesis satisfying (10).

of the parameters in this network depend upon the specific values of T_a, T_2, and T_3 with the same restrictions on the T's as given in (10)

$$T_a > T_2 > T_3.$$

V. SUGGESTIONS FOR FURTHER WORK

While there are only two possible output connections in a three-terminal network, there are five such possibilities in a four-terminal network with only one of these connections usually used. These possibilities could be investigated in order to ascertain any advantages that may be derived from their use.

Although it seems intuitively evident that the maximum gain possible through the use of Fig. 2 and any network is two, a rigorous proof is not evident, particularly if loading is taken into account.

Perhaps the most important link missing in the synthesis procedure is a method which enables one to directly recognize a specified transfer function as being realizable by means of a particularly connected three- or four-terminal network. A general investigation of the different types of connections and their limitations in the two types of networks may point to a solution of this problem.

VI. CONCLUSIONS

It is possible to obtain a passive resistor-capacitor network having a gain greater than unity over a band of frequencies with unity gain at zero frequency. This is achieved very often with a saving in the number of components when compared to other methods. Furthermore, complex zeroes in the transfer ratio are easily obtainable even using ladder-type networks in a connection as shown in Fig. 2. The general resistor-capacitor network can also be made to perform as a tuned LC element by judicious choice of the zeroes in the transfer ratio. It is important to recognize the fact that the proper location of the zeroes of a transfer function can aid materially in synthesis problems.

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❖

Thomas H. Briggs (A'45–SM'45) was born in Illinois. He received the B.S. degree in physics from Wesleyan University in 1927, and the M.S. from California Institute of Technology in 1928. Mr. Briggs was employed in tube development and production engineering successively with Westinghouse Lamp Company during 1928–1930; the Raytheon Manufacturing Co. from 1930–1939; and RCA Victor Division of RCA during 1939–1945. He was chief engineer of the special purpose tube factory of RCA at Harrison, N. J. From 1945–1950 he headed the electronics laboratory of Superior Tube Company, specializing in problems of cathodes for radio tubes.

THOMAS H. BRIGGS

Since 1950, Mr. Briggs has been head of the tube laboratory of the Research Division, Burroughs Adding Machine Company, Philadelphia, Pa. He is currently chairman of Committee B-4 VIII-A of the American Society for Testing Materials, concerned with radio tube cathodes.

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For a photograph and biography of RAYMOND M. WILMOTTE, see page 1220 of the October, 1950, issue of the PROCEEDINGS OF THE I.R.E.

❖

A. M. Clogston (SM'51) was born on July 13, 1917, in Boston, Mass. He received the B.S. degree in physics from the Massachusetts Institute of Technology in 1938. During the period 1938 to 1941, he was a graduate student and teaching fellow in physics at MIT, where he received the Ph.D. degree in 1941.

Dr. Clogston worked on magnetron research at the Radiation Laboratory of MIT from 1941 to 1946, when he joined the technical staff of the Bell Telephone Laboratories, Inc. At the present time, he is engaged in research on electron tubes.

A. M. CLOGSTON

❖

Joseph Croney was born in London, England, on May 24, 1912. He received the B.Sc. (Engineering) degree of London University in 1938, with first class honors, and until the outbreak of war was on the staff of the Northampton Polytechnic, in London.

He subsequently joined the experimental section of H.M. Signal School (now, Admiralty Signal and Radar Establishment), where he was engaged in the development of radar receivers. He took part in the early naval experiments with centimeter equipments for which, later on, he was the first to devise anti-clutter measures.

JOSEPH CRONEY

Although still with the Admiralty Mr. Croney has been, since 1948, one of an advisory group associated with the Marine Safety Division of the British Ministry of Transport. This group is concerned with improvements to electronic navigational aids, and in this connection he has been actively interested in improved anti-clutter measures for navigational radars.

❖

Louis J. Cutrona (M'46–SM'47) was born on March 11, 1915, in Buffalo, N. Y. He received the B.A. degree in physics from Cornell University in 1936 and the Ph.D. degree in physics from the University of Illinois in 1940.

From 1940 to 1942 he served as an instructor in physics and electrical engineering at the Duluth Junior College. He was a member of the technical staff of Bell Telephone Laboratories, Inc., from 1943 to 1946.

LOUIS J. CUTRONA

During the period from 1946 to 1948 he was employed as a senior project engineer at Federal Telecommunications Laboratories. Dr. Cutrona was also a senior project engineer at the Sperry Gyroscope Company from 1948 to 1949. In the fall of 1949 he accepted his present position as supervisor of electronics and controls at the Aeronautical Research Laboratory of the University of Michigan.

❖

Gerald Estrin (S'48) was born on September 9, 1921, in New York, N. Y. In 1942 he was employed as an engineering assistant by the Kurman Electric Company, Long Island City, L. I., N. Y. After serving as radio mechanic in the Signal and Air Corps from 1942 to 1945, he entered the Engineering College at the University of Wisconsin in March, 1946. He received the B.S. degree in February, 1948; the M.S. in June, 1949; and the Ph.D. in January, 1951, all in electrical engineering.

GERALD ESTRIN

Dr. Estrin was awarded a Graduate Research Fellowship by the Wisconsin Alumni Research Foundation from February, 1948, to June, 1949. From July, 1949, to August, 1950, he was granted the RCA Research Fellowship in Electronics. At the present time Dr. Estrin is associated with the Electronic Computer Project at the Institute for Advanced Study in Princeton, N. J. He is a member of Sigma Xi, Tau Beta Pi, and Eta Kappa Nu.

❖

Clifford E. Fay (A'26–SM'45) was born on December 2, 1903, in St. Louis, Mo. He received the B.S. degree in electrical engineering from Washington University in 1925, and the M.S. degree in electrical engineering and physics in 1927, also from Washington University.

Since 1927 Mr. Fay has been a member of the technical staff of the Bell Telephone Laboratories, Inc., where he is engaged in the development of high-vacuum power tubes.

CLIFFORD E. FAY

❖

Jerome Freedman (A'48) was born in New York, N. Y., on August 16, 1916. He received the B.E.E. degree from the College of the City of New York in 1938 and has done graduate work at the School of Technology, C.C.N.Y., the Moore School of Electrical Engineering, University of Pennsylvania, and the Polytechnic Institute of Brooklyn.

From 1939 until 1942 he was employed by the U. S. Army Signal Corps as a radio engineer. In 1942 he was commissioned in the Army of the United States. He attended the Radar School at Harvard University and the Massachusetts Institute of

Technology, and then served as a radar officer, first with the Signal Corps, and then with the United States Air Forces, until 1946. In 1946 he resumed his civilian employment as a research and development engineer with the U. S. Air Forces, at Watson Laboratories. He is now concerned with design and development of new radar techniques and also research in the field of communication theory in connection with his doctoral studies at the Polytechnic Institute of Brooklyn.

JEROME FREEDMAN

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Douglas A. S. Hale was born on July 31, 1904, in Kent, England. He was awarded the Certificate of the Institution of Mechanical Engineering (England) in 1927, from the Gillingham Technical Institute.

In 1927 Mr. Hale joined the research department of the Bell Telephone Laboratories, Inc. After working on general research problems for two years, he was transferred to the electronic apparatus development department, engaged in the mechanical design of electron tubes. During World War II he worked on magnetrons for radar equipment and the rotating cathode-ray tube speech visualizing equipment. Since the war Mr. Hale has been mainly concerned with power tube development.

DOUGLAS A. S. HALE

❖

Raymond J. Kircher (A'30–M'40–SM'43) was born on November 2, 1907, in El Paso, Tex. He received the B.S. degree in electrical engineering from the California Institute of Technology in 1929, and the M.S. degree in communication engineering from Stevens Institute of Technology in 1941.

In 1929 Mr. Kircher joined the radio research department of Bell Telephone Laboratories, Inc., engaged in the development of short-wave transmitters and receivers. During the second World War he worked on radar and counter-measure projects. In 1944 he transferred to the electronic apparatus development department, where he was mainly concerned with the development of ultra-high-frequency power tubes. More recently, he has joined a group working on the development of transistors.

R. J. KIRCHER

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For a biography and photograph of PETER G. SULZER, see page 60 of the May, 1950, issue of the PROCEEDINGS OF THE I.R.E.

William J. Pietenpol was born in Boulder, Colo., on July 15, 1921. He received his early schooling there, and then attended the University of Colorado, from which he received the B.S. degree in electrical engineering in 1943. Moving east, he became an employee of the Radio Corporation of America in Lancaster, Pa. In 1946 he left RCA to continue his studies at Ohio State University, where he held three different fellowships: Eastman-Kodak, 1946; Westinghouse, 1947; and Research Foundation, 1948. In 1949 he received the Ph.D. degree.

W. J. PIETENPOL

The next month Dr. Pietenpol joined Bell Telephone Laboratories, Inc., where his special interest is transistor development. He is a member of the American Physical Society, Phi Gamma Delta, Sigma Xi, Phi Beta Kappa, and Tau Beta Pi.

❖

John Ruston (M'48) was born near Sheerness, England, on October 22, 1911. He received the B.Sc. degree in electrical engineering from London University in 1932, and completed postgraduate study in communications engineering at the same institution in 1933. From 1933 to 1938 he was engaged in the design of sound film recording and reproducing equipment with British Acoustic Films Ltd., in London. He joined the Plessey Company in England as a senior engineer in 1938, and was responsible for the design of radio transmitters, remaining until 1947, with the exception of two years during the war when he was working on the design of radio communication equipment for the Ministry of Supply.

JOHN RUSTON

Since 1947 Mr. Ruston has been employed in the television transmitter division of Allen B. DuMont Laboratories, Inc., at Clifton, N. J., and is at present a project engineer on the development of television transmitters. He is an associate member of the Institution of Electrical Engineers, London.

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Herman Epstein (A'47) was born in New York, N. Y., on June 1, 1926. He received the B.E.E. degree from the College of the City of New York in 1947, the M.S. degree in electrical engineering from Columbia University in 1948, and the Ph.D. degree from Yale University in 1950.

Dr. Epstein spent a year in the Air Force, teaching radar at Boca Raton, Fla. From June, 1947, to September, 1948, he was a part-time member of the electrical

HERMAN EPSTEIN

engineering staff of CCNY. At present, he is employed as a research engineer in the research division of the Burroughs Adding Machine Company in Philadelphia, Pa.

Dr. Epstein is an associate member of the American Institute of Electrical Engineers and a member of Sigma Xi, Tau Beta Pi, and Eta Kappa Nu.

❖

Robert L. Wallace, Jr., was born in Callina, Texas, on February 21, 1916. He attended Westminister Junior College and then the University of Texas, from which he received the B.A. degree in physics and mathematics in 1936, followed by the M.A. in physics in 1939. He then attended Harvard University, where he combined teaching and studying. From 1941 to 1945 he was a special research associate at Harvard, in the field of military communications.

R. L. WALLACE, JR.

In 1946 Mr. Wallace joined Bell Telephone Laboratories, Inc., where he has been concerned with some problems in magnetic recordings and with transistors. He is a member of the Acoustical Society of America, Phi Beta Kappa, and Sigma Xi.

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Vladimir K. Zworykin (M'30–F'38) was born in Mourom, Russia, in 1889. He received the E.E. degree from the Petrograd Institute of Technology in 1912, the Ph.D. from the University of Pittsburgh in 1926, and the D.Sc. from Brooklyn Polytechnic Institute in 1938.

Dr. Zworykin came to the United States in 1920 and joined the research staff of Westinghouse Electric and Manufacturing Company. In 1930 he became associated with the Radio Corporation of America as director of the electronic research laboratory, and in 1947 he was elected vice-president and technical consultant of the RCA Laboratories Division.

V. K. ZWORYKIN

In World War II he served on the Scientific Advisory Board to the Commanding General of the United States Army Air Forces, the Ordnance Advisory Committee on Guided Missiles, and three subcommittees of the National Defense Research Committee, also directing important research work.

Dr. Zworykin received the IRE Morris Liebmann Memorial Prize in 1934, as well as numerous other awards for basic contributions to television, electron microscopy, and other phases of radio and electronics. He is a Fellow of the American Institute of Electrical Engineers, the American Physical Society, and the American Association for the Advancement of Science, and a member of the Electron Microscope Society of America, and Sigma Xi.

Correspondence

An Experimental Investigation of the Electron Orbits in a Magnetron*

In order to see what type of electron orbits is to be found in a nonoscillating magnetron with space charge, we have used a method for the examination of electron orbits proposed by Gerhard Müller,[1] with some modifications found necessary when dealing with a magnetron. Fig. 1 shows a section of the tube used, together with a surrounding coil producing the homogeneous axial field for the magnetron. In this figure, C and A are the cylindrical cathode and anode of the magnetron. The electrodes a and f
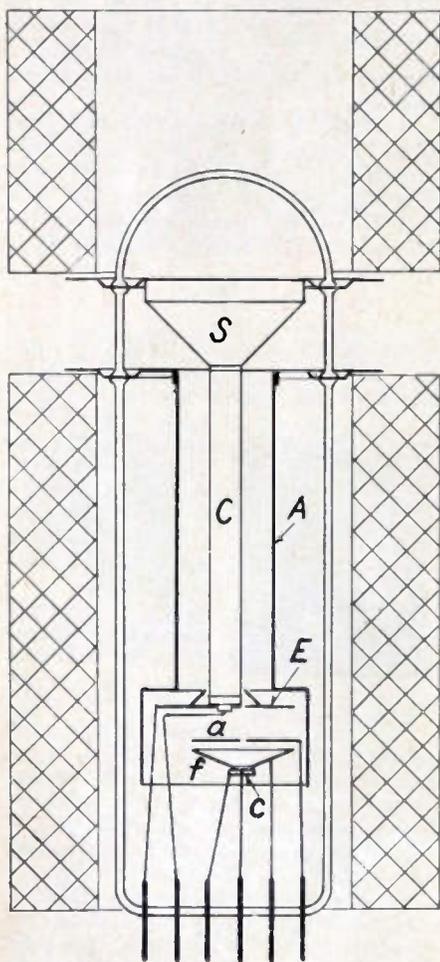
Fig. 1—Sectional drawing of tube for investigation of electron orbits in a nonoscillating magnetron.

and the cathode c constitute an electron gun projecting an electron beam towards the lower end-shield E of the magnetron. Part

of the beam passes through a very small hole in the shield forming a thin beam just grazing the cathode and parallel to it at the entrance to the magnetron. Here the electrons of the beam are subjected to exactly the same forces as are the electrons of the magnetron and, as they enter it at the cathode with no component of velocity in a plane perpendicular to the axis, the projection of their path on such a plane coincides with part of the electron orbits of the magnetron starting from the same generator on the cylindrical cathode. How great a part of this orbit the electrons in the beam describe before striking the fluorescent screen S at the upper end of the magnetron depends upon the axial velocity, and by varying this periodically (through variation of the voltages of the electron gun relative to the magnetron) one can visualize part of the orbit on the screen.

Fig. 2 is a photograph of the tube. The dimensions of the magnetron are as follows: length of anode = 50 mm, diameter of anode = 15 mm, diameter of cathode = 5 mm.
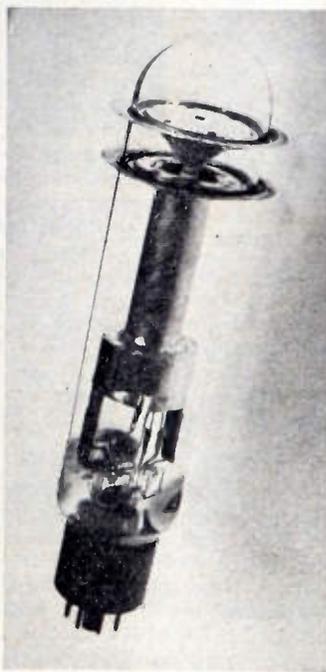
Fig. 2—Photograph of tube for investigation of electron orbits in a nonoscillating magnetron.

Fig. 3 is a photograph of a trace on the screen of the tube shown in Fig. 2. The anode voltage of the magnetron was about 400 volts and the flux density about 200 gausses; the velocity of the axial beam was varied from 150 to 1,700 volts at 50 cycles. The trace corresponds evidently to electron orbits of cycloidal character, and from the position of the hole relative to the trace it could be inferred that the corresponding part of the electron orbit constitutes from $\frac{1}{2}$ to $3/2$ of a cycloid.

Some details about the tube are perhaps worth mentioning. At the ends of the magnetron section the electrons in the axial beam pass through fields which are different from the field within the magnetron; and in order to minimize the error arising from this deficiency, the magnetron section has been made long relative to the diameter and the voltages were adjusted so that the transit time in the end-spaces was small, in

Fig. 3—Photograph of electron trace on conical fluorescent screen as obtained with tube shown in Fig. 2.

comparison with the time it takes an electron to go through the part of an orbit observed on the screen. Besides the obvious reason of facilitating the observation of the trace on the screen, the cone angle of 45° was chosen because this angle would not disturb the field near the cathode if a single stream prevailed in the magnetron.

The double-disk seal construction was used in order to get sufficient cooling of the anode and the screen.

We have made measurements with a nonoscillating magnetron only, but we believe it possible to get valuable information about an oscillating magnetron as well. Suppose some anode structure of a magnetron to be investigated replacing the cylindrical anode A of Fig. 1 and the magnetron so formed oscillating in such a mode that the electromagnetic field is independent of the axial position (this is the case with most magnetrons). Then electrons starting from one point on the cathode no longer follow the same path, but move in orbits depending on the phase of the hf field at their time of departure, and after a given time of flight they will have reached different points in the cathode-anode space, these points constituting a curve. A set of curves corresponding to known times of flight may be visualized on the screen by using in succession a number of known axial velocities of the beam. To push it still further, things could be arranged so that the point where the axial beam enters the magnetron could be shifted around the cathode.

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A Note on Autocorrelation and Entropy*

There is a simple relation between the information content per symbol of a message and its autocorrelation function which does not seem to have been noted in the literature. The autocorrelation function of a message provides an immediate upper bound to its information content. This is useful, for there are many random processes which are of interest as possible messages which have autocorrelation functions which are easy to measure, while a direct measurement of their information content may involve measuring joint probability distributions of such high order that the experiment is not feasible. Two- and three-dimensional processes, of interest in facsimile and television transmission, are of this nature.

Consider a sequence of N message terms, $m_{i-1} \cdots m_{i-N}$, occurring in order at successive sampling intervals. Let $\phi(j)$ be the autocorrelation function of the message, which is assumed to be ergodic:

$$\phi(j) = \overline{m_i m_{i-j}},$$

where the bar indicates a time or ensemble average. Knowing $\phi(j)$, we know all of the second moments of the sequence of N terms: the moment $A_{ij} = \overline{m_i m_j}$ is just:

$$A_{ij} = \phi(|i-j|),$$

where the absolute value signs are permitted because of the symmetry of the autocorrelation function. Now Shannon has shown that the normal random process having the second moments A_{ij} has the maximum entropy of all processes having the same set of second moments.[1] Thus the entropy of this process provides a direct upper bound for the possible total entropy of the process whose autocorrelation function is given. Shannon evaluates this: it is just

$$NG_N = \log \left[(2\pi e)^{N/2} |A_{ij}|^{1/2} \right]. \quad (1)$$

This total entropy of the N-message terms has been labeled NG_N, since it is N times the average entropy per symbol of a group of N message terms selected from the normal random process having the same autocorrelation function as the given message. G_N is then one of Shannon's upper bounds for the true entropy per symbol of the message: H the true entropy per symbol, is the limit of G as N increases without bound. A better estimate is, not the average entropy of the N terms, but the entropy added by the Nth term. This is Shannon's $F_N = NG_N - (N-1)G_{N-1}$. Evaluating this gives

$$F_N = \tfrac{1}{2} \log \left[(2\pi e) |A_{ij}^N| / |A_{ij}^{N-1}| \right], \quad (2)$$

where the superscripts in the determinants indicate their order.

In any particular case, evaluating F_N or G_N for large N involves evaluating one or two large determinants. In cases where the given autocorrelation function is, or may be approximated by, a simple mathematical expression, which gives a continuous function $\phi(t)$ for the series $\phi(j)$, the determinants may be avoided by using Wiener's procedure for finding the best linear predictor for the message from its autocorrelation function, and using a formula he gives for evaluating

the mean-square error of the prediction, when the prediction is made one sampling time interval in advance.[2] Now a knowledge of the second moment of a distribution sets a bound on its information content: using (1) above for $N=1$, we have

$$F_N \leq \tfrac{1}{2} \log (2\pi e \sigma^2),$$

where the equality sign holds if the error distribution of the prediction is in fact normal.

Wiener's formula may or may not be more convenient to use in the case of time series, depending on the nature of the autocorrelation function. But in the case of two- or three-dimensional messages, his explicit solution of the best predictor problem does not hold, and his formula thus cannot be used for a message which is defined, e.g., on the points which are the intersections on a piece of graph paper. However the autocorrelation function is still defined in this case: it is now a function of a vector argument, but it still determines the second moments A_{ij} for any particular collection of N such intersections, and the formulas (1) and (2) still apply in this case: the only difference is that the moment matrix will now, in general, be less symmetric than it is in the case of time series.

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The Permittivity of Air at a Wavelength of Ten Centimeters*

Phillips[1] has reported the results of measurements of the permittivity of dry air, and of moist air for various concentrations of water vapor, at a frequency of 3,000 mc. The authors measured, in 1949, the permittivity of dry air at this frequency, and obtained a result agreeing exactly with that of Phillips. Since the authors' result was based on a statistical analysis of a considerable number of varying determinations, and since the final result differed appreciably from those of other workers, it had been decided not to publish these results until modifications had been made to the apparatus. These modifications are designed to reduce the effects causing the random scatter of the results, thereby producing a more reliable result. However, in view of the result obtained by Phillips, which supports the value originally obtained, it seemed desirable to submit this communication.

The following is a brief outline of the method used by the writers to measure the dielectric constant of dry air. A block diagram is given in Fig. 1.

Klystrons A and B were type-707B klystrons operated from stabilized power packs. Klystron A was frequency-modulated by a sawtooth voltage extracted from the oscilloscope sweep circuit. As its frequency swept through the resonance of the cavity,

Fig. 1—Diagram of equipment for measuring the change in cavity resonance.

the cavity response curve was shown on one trace of a double-beam oscilloscope. The receiver was a calibrated, narrow-band receiver, and its output, shown on the other trace of the oscilloscope, gave a sharp pip when the frequency of klystron A was such that its beat with klystron B equaled the receiver frequency.

The cavity was mounted in a chamber which could be evacuated. Leads to the cavity were coaxial cables passing through airtight sockets mounted in the wall of the chamber.

The receiver was tuned, with the cavity evacuated, until the receiver pip was central over the cavity response curve. Dry air was then admitted to the cavity, at an observed temperature and pressure, and the change in receiver tuning necessary to restore the coincidence of the pip and cavity response curve was noted. The mean frequency of klystron A was read on a calibrated cavity wavemeter. All results were reduced to S. T. P. for comparison.

Unfortunately, if the frequency of klystron B varied between the reading with the cavity evacuated and the reading with air in the cavity, the movement on the oscilloscope of the cavity response curve was unaffected, but the movement of the receiver pip was affected. Such changes of frequency of the klystron did occur, so that individual results varied very considerably. As these changes of frequency should be quite random, they should be governed by a normal error curve, and the mean of a large number of results should be independent of that effect.

A large number of readings were obtained and analyzed. The fit to a normal error curve was very good. The mean value at S.T.P. was

$$(K - 1) \times 10^6 = 606,$$

which is the value obtained by Phillips. This value was later verified by equipment designed to eliminate errors introduced by a shift in frequency of klystron B.

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Perturbation and Correlation Methods for Enhancing the Space Resolution of Directional Receivers*

Considerable study has been devoted recently to the statistical and correlation properties of stationary time series, and to the development of automatic equipment for rapid computation of correlation functions. It has also been suggested that correlation procedures offer a powerful method for detecting periodic signals buried in noise.[1] The performance of correlation devices has sometimes been characterized as "merely" achieving in the time domain the same results which could otherwise be achieved by previously known methods of filtering in the frequency domain. But recognizing this fact should not imply that these new tools are any the less useful to the communications engineer; on the contrary, it represents an important new degree of freedom to be able to exchange integration time for frequency space, and to be able to specify with precision optimum methods for allowing random noise to average itself out.

It is the purpose of this note to call attention to the existence of an interesting class of signal functions which are stationary, in the statistical sense, with respect to variables other than time alone. In the present context, correlation analysis usually starts with an electrical signal that can be characterized completely by specifying a scalar voltage as a function of time. However, if the signal S is an electromagnetic or acoustic wave originating or scattered at some localized position in space, its proper characterization must involve at least four variables; thus $S = S(t, \alpha, \beta, r)$, where α, β, and r represent the horizontal and vertical position angles and the distance to the point of origin of the signal. The operational test for significance in considering additional variables in the signal function is to inquire as to whether or not it is possible to discriminate at the receiving point with respect to these variables. Since it *is* possible to effect such discrimination with respect to the space co-ordinates of the point of origin of a signal, it seems worthwhile to assess the applicability of techniques for correlation with respect to the space variables as well as with respect to the time variable.

The use of correlation techniques in connection with a space variable can be illustrated by a simple example: Let a desired signal source be located in the horizontal plane, and for the sake of simplicity consider correlation with respect to the azimuth variable only. We can assume that the signal is to be received by a directional system whose beam pattern in the horizontal plane is $K(\phi)$, where ϕ is the angle of deviation from the beam axis at which the normalized response K occurs. We designate by θ the instantaneous horizontal position angle of the beam axis.

Suppose, now, that a periodic perturbation, such as might be represented by

writing

$$\theta = \theta_B + \theta_p \cos \omega_p t, \qquad (1)$$

has been superimposed on this position angle. The effect of this systematic perturbation of the directional beam will be to *modulate* the signal received from a source localized at α_0 in the horizontal plane, in accordance with a modulation function $M_s(t)$ determined by the shape of the receiving beam through the relation

$$M_s(t) = K(\theta_B - \alpha_0 + \theta_p \cos \omega_p t). \qquad (2)$$

Since the only signal variables being taken into account in this example are time and one position angle, the signal can be represented as $S(t, \alpha_0)$. Then it follows that the desired signal E_{s1} received via this channel will be

$$E_{s1} = M_s(t) \cdot S(t, \alpha_0). \qquad (3)$$

Note, now, that if the perturbation angle θ_p is of the order of magnitude of the half-width of the primary beam, the modulation of the desired signal is substantially complete; but the signal amplitude and the waveform and phase of the modulation depend on whether the oscillating beam is, on the average, pointed toward the wanted signal, i.e., on the sign and magnitude of $(\theta_B - \alpha_0)$. Application of the "correlation principle" would suggest at once that such a distinctive property of the signal known in advance at the receiving point ought to be usable in discriminating the signal against a noise background. Since the periodic angular perturbation of the beam is controlled at the receiving point, the perturbation frequency, at least, is known accurately and some suitable local signal having the same frequency (or frequencies) can presumably be generated for cross correlation with the received signal.

Consider explicitly now the addition of noise in this situation. If the limiting noise sources are distributed in some manner throughout the transmission medium, the received noise, as well as the desired signal, would need to be considered as a function of time and all the space variables. It will be simpler, however, in this illustrative example, to assume that the noise in the medium is completely isotropic, or, what is equivalent, to assume that the limiting noise originates within the receiving apparatus. The noise can be written then as $N(t)$, and it will have the same statistical character for any orientation or perturbation of the receiving beam. This noise term can be added to (3) to give the total received signal as

$$E_{s1} = M_s(t) \cdot S(t, \alpha_0) + N(t). \qquad (4)$$

Consider now the generation of a suitable local signal E_R for cross correlation with E_{s1}. Suppose, for example, that the directional receiving system makes available, in addition to the perturbed signal given by (4), an unperturbed signal corresponding to normal directional reception. If this unperturbed signal is now subjected locally to symmetrical modulation $M_R(t)$ at the perturbation frequency, there will be available in this reference channel a signal,

$$E_R = M_R(t)\{K(\theta_B - \alpha_0)S(t, \alpha_0) + N(t)\}. \qquad (5)$$

We may proceed at once to multiply the

signals in these two channels and average their product over the effective integration time T; thus

$$R_T(\theta_B - \alpha_0) = \frac{1}{T} \int_0^T [M_s(t) \cdot S(t, \alpha_0) + N(t)]$$
$$\cdot [M_R(t)\{K(\theta_B - \alpha_0) \cdot S(t, \alpha_0) + N(t)\}]dt. \qquad (6)$$

The finite-time cross-correlation coefficient R_T can be written as the sum of one desired signal term and three noise terms. As T increases, two of the noise terms go to zero because $N(t)$ is not coherent with $S(t, \alpha_0)$; and the third, containing $N^2(t)$, goes to zero by virtue of the zero average of the symmetrical reference modulation function $M_R(t)$. The desired signal term, however, does *not* vanish for large T, by virtue of the like periodicity of $M_s(t)$ and $M_R(t)$, and it can be made to have its maximum value when $\theta_B = \alpha_0$. It follows that the effective resolution of the directional receiving system, or its effective gain in discriminating a localized source against a uniform noise background, can be made to increase without limit as the effective integration time is made progressively longer.

It is notable that the cross correlation, represented by (6), can be carried out for almost any reasonable assumption about the time variation of S, provided only that the desired signal S has a vanishing cross correlation with the limiting ambient noise. Thus, although the periodic angular perturbation of the receiving beam is transformed into a time variation of the signal envelope, a cross correlation in time at the perturbation frequency is intrinsically a correlation with respect to the angular variable and is effectively independent of the regular or statistical variation of the signal with time. It follows then that the auto-correlation techniques discussed in the literature[1] can be applied independently, subject to suitable choice of parameters, in order to utilize any distinguishing time properties of the signal in discriminating it from the noise background. For example, if the signal were periodic, its autocorrelation function would preserve the periodicity of the signal while affording a substantial degree of noise suppression; moreover, the amplitude of the autocorrelation function would preserve the modulation envelope imposed by angular perturbation of the beam so that the cross-correlation procedure, described above, could be applied in a second operation without substantial modification. It is interesting to speculate as to whether the finite integration times found adequate for correlation with respect to each variable must necessarily be additive, or whether methods might not be devised for "averaging" simultaneously with respect to several variables.

An experimental program using acoustic waves is being undertaken to explore further the basic features of correlation with respect to space variables. A semicircular array of small loudspeakers makes it possible to establish in our anechoic chamber an isotropic noise field and to simulate superposition of a localized source in such a noise background.

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Experimental Results of Continuous-Wave Navigation Systems*

It has been observed experimentally[1] and verified mathematically[2] that reradiation effects may cause erroneous position measurements in a navigation system which measures the phase angle of arrival of continuous-wave signals. This effect was first noted during operational tests of Raydist[3] equipment which was constructed by the Hastings Instrument Company, Inc., for the Photographic Charting and Mapping Branch of the U. S. Air Force in 1946. A series of studies was conducted in connection with propagation research and operational tests of other Raydist systems which indicate what the practical limits of the expected effects may be.

A mathematical analysis which was part of the above-mentioned propagation research was made of the ability of a grounded quarter-wave vertical element to produce reradiation effects. If the losses in the element are assumed to be zero, that is, all of the power absorbed is reradiated, and if the element is placed at random on the circumference of a circle whose center is at a receiving antenna, then the maximum random effect on the phase of the received wave will be 3 degrees, if the radius of the circle is 5 wavelengths. In most of the tests conducted, three degrees of phase measurement represented a position measurement of two feet or less.

Stability tests have been performed with the Raydist equipment where random reradiation effects of large magnitude were to be expected. In these tests the system was installed with all components held stationary. One of the station sites was selected near the end of a dock facing Hampton Roads. Four times an hour a ferry boat one-hundred feet long and forty feet in height above the water line passed within one hundred feet of this site. Observations were made on frequencies of 1.820, 2.398, and 4.805 mc during the month of January, 1950. The results of these observations indicate that the magnitude of the phase oscillations that were produced by the presence of the moving ferry boat were worse on the higher frequency only if phase angle was the criterion, rather than position sensitivity. The magnitude of the oscillations when referred to position measurement in feet was very nearly identical on all frequencies, and the maximum excursion was eight feet.

In the past it has been observed that a phase change of 360 degrees may be produced by the nullification of one signal in the system through signals arriving out of phase over two paths. Normal signal fluctuations are taken care of by volume leveling

which is incorporated in Raydist receiving equipment; however, under very special but easily avoided conditions, the resultant of signals received over more than one path may cancel so completely as to be lost in the noise level. In such a case, the phase meter may be driven one or more revolutions by the unaffected signal and the count may be lost.

In reviewing the articles referred to above,[1,2] it appears that two simple precautions which must be applied in the use of a successful phase-measuring navigation system have been omitted.

The first of these precautions is that of not moving the mobile equipment under an overhead structure. Whether the overhead structure is a bridge or a power line, the chances are that the signals arriving at the receiver over and under the structure will balance at some point during the process and a signal null will appear. Even in the case that the signal does remain above the noise level, the change from one transmission path to another is sure to cause a phase anomaly.

The other precaution which appears in practice is a consideration of the inefficiency of casual parasitic elements. In both articles referred to above, the values of induced current in the parasitic elements was assumed to be larger than is to be expected under normal operating conditions. It seems reasonable that any navigation system station sites should be chosen such that they are removed at least 5 wavelengths from any protruding object that might act as a parasitic element. In the case investigated by Hufford, it appears from other sources[4] that the current induced in the parasitic element will be somewhat less than half the current of the driven element, when they are separated 4 wavelengths. Even if it is assumed that 100 per cent of the received signal is reradiated, the current in the parasitic element may be assumed only one-tenth of that which appears in the driven element. Mr. Hufford's integration over a path which passes within 1 wavelength of a parasitic element appears to be somewhat extreme, for practical purposes. Instead, it appears reasonable to restrict any measurement path such that it may pass no closer than 5 wavelengths to a casual parasitic element, under which conditions it will be found that all phase measurements to any point by more than one path will result in the same number of degrees of phase angle.

FRED S. HOWELL
Hastings Instrument Co., Inc.
Box 1275
Hampton, Va.

* Received by the Institute, March 13, 1951.
[1] Ross Bateman, E. F. Florman, and A. Tait, "A source of error in radio phase-measuring systems," PROC. I.R.E., vol. 38, pp. 612–614; June, 1950.
[2] George A. Hufford, "An analysis of some anomalous properties of equiphase contours," PROC. I.R.E., vol. 38, pp. 614–618; June, 1950.
[3] Charles E. Hastings, "Raydist—A radio navigation and tracking system," *Tele-Tech*, June, 1947.
[4] G. H. Brown, "Directional antennas," PROC. I.R.E., vol. 25, pp. 78–145; January, 1937.

Telepathic Communication*

In his recent correspondence[1] Bibbero points out that up to now no telepathic

* Received by the Institute, March 17, 1951.
[1] R. J. Bibbero, "Telepathic communication," PROC. I.R.E., vol. 39, p. 290; March, 1951.

interference in the entire field of the electromagnetic spectrum has ever been reported. This is only partly true. About 25 years ago a spurious electromagnetic brain radiation, not an electromagnetic telepathy, was observed by the Italian professor Cazzamalli.[2-8] In order to avoid any misinterpretation we must strictly distinguish between Berger's encephalography, which measures and registers the fluctuating action potentials of an active brain picked up by contact electrodes, and Cazzamalli's brain radiation, which is detected by a sensitive vhf receiver. Unfortunately, Cazzamalli's papers are somewhat mysterious in the light of modern radio engineering, but the brain is clearly proven to emit vhf oscillations which produce audible beat pulses in a separate receiver by heterodyning with a local oscillator. These pulses occur under strong emotional stresses and are depicted in numerous oscillograms. Since in Cazzamalli's time the vhf technique was rather crude, it is open to conjecture whether this brain radiation was heterodyning with the fundamental frequencies between 60 and 400 mc of the local oscillator or heterodyning with more or less high harmonics.

Be that as it may, Cazzamalli's experiments—which as far as the author knows and notwithstanding some unpublished efforts were never checked by other scientists—are worth repeating with the aid of modern microwave technique, thus emphasizing Bibbero's suggestion.

A possible verification and elucidation, however, would not solve the problem of telepathic communication which, of course, is not only a matter of an emitter, but also of an extraordinarily sensitive receiver, of a noise figure, of energy propagation, and so on. In other words, it is very doubtful whether telepathic communication altogether fits into our physical field of transmission of intelligence. Even modern physicists and engineers should be very careful and open-minded, so that they will not be subject to the smiles of future generations as the generations of the past have been to us.

HANS E. HOLLMANN
105 North G St.
Oxnard, Calif.

[2] F. Cazzamalli, "Fenomeni telepsichici e radio-onde cerebrali," *Neurologica*, vol. 42, pp. 193–218; 1925.
[3] F. Cazzamalli, "Esperienze, argomenti e problemi di biofisica cerebrale," *Quaderni di Psichiatria*, vol. 16, pp. 81–105; 1929.
[4] F. Cazzamalli, "Fenomeni elettromagnetici radianti dal cervello umano . . . ," *Giornale di Psichiatria e di Neuropatologia*; 1933.
[5] F. Cazzamalli, "Di un fenomeno radiante cerebropsichico come mezzo di esporazione psicobiofisica," *Giornale di Psichiatria e di Neuropatologia*, vol. 63, pp. 45–56; 1935.
[6] F. Cazzamalli, "Phénomènes électromagnétiques du cerveau humain en activité psychosensorielle intense et leur demonstration par des complexes oscillateurs-révélateurs à triodes pour ondes ultra-courtes," *Archives de Neurologie*, vol. 54, pp. 113–142; 1935.
[7] F. Cazzamalli, "Fenomeni elettromagnetici irradianti dal cervello umano . . . ," *Archivio Generale di Neurologia*, vol. 17, pp. 74–75; 1936.
[8] F. Cazzamalli, "Di un nuovo apparato radio-elettrico rivelatore dei fenomeni elettromagnetici radianti dal cervello umano," *L'Energia Elettrica*, vol. 18, pp. 28–42; 1941.

Institute News and Radio Notes

TECHNICAL COMMITTEE NOTES

The Standards Committee under the Chairmanship of J. G. Brainerd held meetings on April 26 and 27, at which the formation of a new **IRE Technical Committee on Servosystems** was announced. This Committee will deal with recommended standard definitions of terms within the field, and will encourage the formation of a Professional Group on Servosystems. The Standards on Radio Receivers: Open-Field Method of Measurement of Spurious Radiation from Frequency Modulation and Television Broadcast Receivers, 1951, presented by the Receivers Committee, are published in this issue of the PROCEEDINGS. Reprints of this material will be available within a short time, and may be purchased from Headquarters for a nominal charge.

The **Antennas and Waveguides Committee** convened on May 8, A. G. Fox, Chairman, presiding. This Committee is reviewing transducer definitions.

The **Committee on Measurements and Instrumentation**, under the Chairmanship of Ernst Weber, held a meeting on March 9. Its scope and activities were reviewed by the Standards Committee, and it was agreed that in order to assure the best possible co-ordination of all activities, involving standardization of measurements and measurement techniques, it would be necessary to appoint a Measurements Co-ordinator whose responsibility it will be to correlate efforts of standardization of the various Technical Committees with those of the basic Committee on Measurements and Instrumentation. The Executive Committee has appointed Professor Weber to this post. He will be replaced as Chairman of the Measurements and Instrumentation Committee for the period beginning with May 1, 1951, and ending with April 30, 1952, by Francis J. Gaffney. Comprehensive reports on the activities of the various Subcommittees were given.

At a meeting of the Standards Committee on May 11, at which Axel G. Jensen presided, the scopes and activities of all IRE Technical Committees were reviewed. A revised Standards Manual will be drawn up for distribution to members of the Committee, and for the Chairmen of all the Technical Subcommittees who might have need for it. The Chairman also announced that an Administrative Committee of the Standards Committee had been organized to cope with items which would ordinarily come before the Standards Committee. This Administrative Committee will meet when necessary, prior to the meetings of the Standards Committee, and its actions will be incorporated in the minutes of the Standards Committee meetings.

A Subcommittee of the **Television Systems Committee** has been activated for the purpose of preparing an adequate set of definitions relating to color television. Millard W. Baldwin, Jr., of Bell Telephone Laboratories will serve as Chairman.

Calendar of
COMING EVENTS

International Conference on Beneficial Uses of Atomic Energy, Oxford, England, July 16–21

Conference on Auroral Physics, University of Western Ontario, London, Ont., Canada, July 23–26

Emporium Section Summer Seminar, Emporium, Pa., August 17–18

AIEE Pacific General Meeting, Portland, Ore., August 20–23

1951 IRE Western Convention, San Francisco, Calif., August 22–24

4th Conference on Gaseous Electronics, General Electric Research Laboratory, Schenectady, N. Y., October 4–6

1951 National Electronics Conference, Edgewater Beach Hotel, Chicago, Ill., October 22–24

AIEE Fall General Meeting, Cleveland, Ohio, October 22–26

Radio Fall Meeting, King Edward Hotel, Toronto, Ont., Canada, October 29–31

1952 IRE National Convention, Waldorf-Astoria Hotel and Grand Central Palace, New York, N. Y., March 3–6

PROFESSIONAL GROUP NOTES

A petition has been received at IRE Headquarters for the formation of an **IRE Professional Group on Engineering Management**, promoted by Ralph I. Cole of the Watson Laboratories. The scope of the Group, if approved, would include the dissemination of information upon, as well as promotion of discussion of those problems confronting engineering managers that especially require the combination of engineering and management know-how in the direction of engineering projects and programs.

The **Professional Group on Antennas and Propagation** will sponsor three sessions at the IRE West Coast Convention: Session I, Propagation and Optics, and Session II, Linear Array Antennas, will be held on August 23, and Session III, Antenna Applications, on August 24. The Group will publish the papers which are presented in its own Transactions, and abstracts of these papers will appear in the PROCEEDINGS. Members will be assessed $2.00 each to cover publication costs.

The **Professional Group on Audio** has issued Newsletter No. 6 which has been distributed to the entire membership along with a copy of the Group's revised bylaws. In addition, the Group has published two papers, "The Application of Damping to Phonograph Reproducer Arms," by William S. Bachman, and "A Single-Ended Push-Pull Audio Amplifier," by Arnold Peterson and Donald B. Sinclair. Members will be assessed a fee of $2.00 annually to cover the cost of distribution of papers such as these. A paper entitled "Amplitude and Phase Measurements on Loudspeaker Cones," by M. S. Corrington and M. C. Kidd, procured by the Group, will be published in a forthcoming issue of the PROCEEDINGS. Recent elections held by the Group resulted in the following Administrative Committee for the term April 1, 1951, to March 31, 1952: B. B. Bauer, Chairman; J. K. Hilliard, Vice-Chairman; and S. L. Almas, L. L. Beranek, A. M. Wiggins, and J. J. Baruch, members.

The Administrative Committee of the **IRE Professional Group on Broadcast and Television Receivers** has recently elected Dorman D. Israel to succeed Virgil Graham as the Group's Chairman.

The new **Professional Group on Industrial Electronics** held a meeting in Chicago on May 8, at which time a proposed constitution was drawn up.

A paper procured by the **IRE Professional Group on Nuclear Science**, "What is Nuclear Engineering?" by A. M. Weinberg, is scheduled for early publication in the PROCEEDINGS.

A number of members to serve on the Administrative Committee of the **Professional Group on Quality Control** has been increased to 15, and a revised constitution has been submitted to IRE Headquarters. The Group hopes to sponsor a session at the Joint RTMA/IRE Meeting in Toronto next autumn. Plans are being made for participation in the National Electronics Conference in the fall, and in the National AIEE and ASQC Meeting in Syracuse in 1952.

A recent election conducted by the **IRE Professional Group on Vehicular Communications** has appointed the following persons new members of the Administrative Committee: F. T. Budelman, R. V. Dondanville, C. M. Heiden, Newton Monk, and Waldo Shipman. A paper entitled "Experimental Radio Telephone Service for Train Passengers," by Newton Monk, has been procured by the Group, and will appear in a forthcoming issue of the PROCEEDINGS.

OAK RIDGE SYMPOSIUM PROGRAM COMPLETED

The program for the 1951 Oak Ridge Summer Symposium, sponsored by the Oak Ridge National Laboratory and the Oak Ridge Institute of Nuclear Studies, on "The Role of Engineering in Nuclear Energy

Development" has been completed. The symposium will be held at Oak Ridge from August 27 to September 7, with Labor Day a holiday. Sessions will be unclassified in nature, and will be held in the downtown portion of Oak Ridge. They are open to all interested professional engineers and others. Symposium subjects and speakers are as follows:

"The Contribution of Engineering to Nuclear Energy Development," by J. A. Lane, Oak Ridge National Laboratory; "Elementary Nuclear Concepts," by A. M. Weinberg, Oak Ridge National Laboratory; "Simplified Approach to Reactor Calculations," speaker to be announced; "Economic Aspects of Atomic Power," by Sam H. Schurr, U. S. Bureau of Mines; "Stable Isotope Separations," by Manson Benedict, MIT; "Standards of Radiological Protection and Control," by Karl Z. Morgan, Oak Ridge National Laboratory; "Environmental Problems of Radioactive Waste Materials," by R. J. Morton, Oak Ridge National Laboratory; "The Treatment of Radioactive Wastes," by Warren K. Eister, Oak Ridge National Laboratory; "Thermal Problems in Reactor Design," by R. N. Lyon, Oak Ridge National Laboratory; "Instrumentation and Control of Reactors," by J. D. Trimmer, University of Tennessee; "The Chemical Problems in the Development of Reactors," by J. A. Swartout, Oak Ridge National Laboratory; "Review of Reactor Types," by Neal F. Lansing, Oak Ridge National Laboratory; "Hazards Associated with Low-Power Research Reactors," by M. M. Mills, North American Aviation Corp.; "The Materials of Reactor Construction," by G. E. Evans, Oak Ridge National Laboratory.

R. C. Ernst, Dean of the Speed Scientific School of the University of Louisville, will be chairman of a panel discussion on Nuclear Engineering Education, to be held on the last day of the symposium. Members of the panel and their subjects will be: A. M. Weinberg, "The Scope of Nuclear Engineering Education"; K. C. Beck, North Carolina State College, "The Nuclear Engineering Curriculum at North Carolina State College"; L. M. K. Boulter, Dean, University of California at Los Angeles, "The Impact of Atomic Energy Developments on the Training of Professional Engineers"; and F. C. Vonderlage, Oak Ridge National Laboratory, "The Oak Ridge School of Reactor Technology."

Additional information on the symposium may be obtained from the University Relations Division, Oak Ridge Institute of Nuclear Studies, P.O. Box 117, Oak Ridge, Tenn.

TRUMAN NAMES SCIENTISTS TO ADVISORY COMMITTEE

Eleven of the nation's top scientists were named recently by President Truman to a Science Advisory Committee of the Office of Defense Mobilization which will advise the President and Mobilization Director Charles E. Wilson in matters relating to scientific research and development for defense.

Chairman of the group will be Oliver E. Buckley, who for the last ten years has been president of Bell Telephone Laboratories, and now becomes chairman of the board of Bell Laboratories.

The other members of the committee are Detlev W. Bronk, president of Johns Hopkins University and of the National Academy of Sciences; William Webster, chairman of the Research and Development Board; Alan T. Waterman, Jr., director of the National Science Foundation; Hugh Dryden of the Interdepartmental Committee on Scientific Research and Development; James B. Conant, president of Harvard University; Lee DuBridge, president of the California Institute of Technology; James R. Killian, president of the Massachusetts Institute of Technology; Robert F. Loeb of the College of Physicians and Surgeons, Columbia University; J. Robert Oppenheimer, director and professor of physics at the Institute of Advanced Study, Princeton; and Charles A. Thomas, executive vice-president of the Monsanto Chemical Co.

NADC SEEKS SCIENTISTS

The U. S. Naval Air Development Center at Johnsville, Pa., is accepting applications for engineering and other technical positions from high-grade engineering and scientific personnel with education, training, or experience in the fields of electrical and electronics engineering, or in the field of physics. Salaries for these positions range from $4,600 to $7,600 per annum.

Under the jurisdiction of the Navy's Bureau of Aeronautics, the NADC functions as one of the Navy's major aviation activities devoted to the application of the latest developments in the above-mentioned fields to specific Navy projects essential to the national defense program.

Inquiry regarding the above positions should be made, as soon as possible, by letter addressed to the Industrial Relations Officer, U. S. Naval Air Development Center, Johnsville, Pa., or by personal visit to the NADC Industrial Relations Office any week-day except Saturday between 8:30 A.M. and 3:30 P.M.

IN COMMEMORATION OF THE WESTERN UNION TELEGRAPH COMPANY CENTENARY

April 30, 1951

Mr. W. P. Marshall, President
The Western Union Telegraph Co.
60 Hudson Street
New York 13, N. Y.

Dear Mr. Marshall:

The Executive Committee of the Board of Directors of The Institute of Radio Engineers has taken note of the fact that The Western Union Telegraph Company, founded in 1851 as the Mississippi Valley Printing Telegraph Company, is this month celebrating its 100th anniversary.

Amid the array of means of electrical communication now available to serve every conceivable purpose, from a private wire message or conversation to a television network or a world-wide broadcast, engineers engaged in the business remember that it was in Morse telegraphy that electricity first emerged from the physical laboratory into the workaday world to begin to bestow its manifold benefits on mankind. To great degree all succeeding forms of use of electricity are thus indebted to Morse as the inventor, and to the founders of Western Union as the early entrepreneurs of the electric telegraph.

The Institute of Radio Engineers, therefore, wishes to felicitate Western Union upon the attainment of the 100th milestone, and to extend very best wishes for continued technical progress and prosperity as the company enters upon its second century of service.

Very truly yours
s/s I. S. Coggeshall
President

BRITAIN PLANS INTERNATIONAL ATOMIC ENERGY CONFERENCE

The largest international conference for the exchange of information on beneficial uses of atomic energy yet to be held in Britain will take place at Oxford from July 16 to July 21.

The conference will be devoted to reports by users of radioactive isotopes on the techniques employed and results obtained. Medical topics will be discussed at the beginning of the week, and the second half of the week will be devoted to industrial applications, and agricultural and other research uses of isotopes.

Invitations to the conference have been sent to users of isotopes in industry, in medical, scientific, and agricultural departments of universities, hospitals, medical research institutes, and government research establishments. It is expected that Australia, Belgium, Canada, Denmark, France, Holland, Italy, Norway, Sweden, Switzerland, and the United States will be among the countries represented.

Visitors will live at various Oxford colleges, and tours may be arranged to Harwell and the radiochemical center at Amersham to see some of the nonsecret work carried out at these stations.

There will be a conference fee of three guineas in addition to the residence charges. Inquiries should be addressed to the Secretary, Isotope Conference, A.E.R.E., Harwell, Didcot, Berks., England.

PERMISSION SOUGHT FOR NEW RADIO RELAY ROUTE

The long lines department of the American Telephone and Telegraph Company has filed application with the Federal Communications Commission for permission to construct a microwave radio relay system connecting Washington, D. C., Charlotte, N. C., and Atlanta, Ga. The route would join the present New York-Washington microwave system at Garden City, Va., a point just across the Potomac River from Washington.

The proposed radio relay route will augment existing coaxial cable, and other cable and wire facilities presently providing communications between the Southeast and the rest of the nation. Built primarily to provide additional telephone circuits, microwave systems can also be equipped to handle television, if required.

CANADA AURORAL PHYSICS CONFERENCE SCHEDULED

A Conference on Auroral Physics, jointly sponsored by the physics department of the University of Western Ontario and the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories, Cambridge, Mass., will be held July 23 to 26 at the University of Western Ontario, at London, Ont., Canada.

The entire field of auroral physics will be given consideration, with important theoretical papers to be presented by internationally known scientists. General topics to be discussed will deal with the formation of the aurora, mechanisms of solar corpuscular streams, and excitation mechanisms in the ionosphere (80–400 km) and in the mesosphere (400–1,000 km).

Several papers will be presented on the identification and interpretation of the emission spectra of the ionosphere and mesosphere and other observational studies.

Industrial Engineering Notes[1]

AURAL BROADCAST INCOME RISES

An increase in the total income (before Federal income taxes) of the aural broadcast services in 1950 was indicated by preliminary estimates released by the FCC. This represents a sharp reversal in the trend of aural broadcast income which had been declining steadily since 1944, the industry's peak income year, the FCC said.

Total 1950 AM and FM revenues (time sales plus other revenues) amounting to $447.7 million were the highest on record. Together with the estimated total revenue of television networks and stations for 1950 of $105.8 million, the grand total revenues of the broadcasting industry passed the half-billion mark for the first time. As in past years, almost all of the FM-only stations reported unprofitable operations.

COMPANIES RECEIVE RTMA PREFERRED VALUES REPORTS

A booklet giving information contained in a report on preferred values series used for radio and electronic components, prepared by Paul S. Smith of Motorola, Inc., has been distributed to the chief engineers of all member companies by the RTMA engineering department. The booklet explains reasons for preferred values, which are those selected values that are based on a series of preferred numbers. It is designed to aid in the understanding of how such a series came about, what it is, and why it is used. Additional copies of the booklet may be obtained by member companies from the RTMA Engineering Department, 489 Fifth Ave., New York 17, N. Y.

INDUSTRY STATISTICS

Amid increasing reports of radio and television set production cutbacks, RTMA's first quarter production estimates indicate that the output of radio and TV sets increased 27 and 37 per cent, respectively, over the corresponding 1950 period. Preliminary figures for the second quarter, however, indicate that set production is declining, and it was pointed out that the greatest proportion of last year's record TV output was manufactured in the third and fourth quarters. The quarterly production report revealed that 95 per cent of all TV sets manufactured had picture tubes of 16 inches or larger, and that 3,672 sets were equipped with picture tubes 22 inches or over. Eighty-four per cent of all television picture tubes sold to receiver manufacturers in the first quarter of 1951 were rectangular, according to further RTMA reports.

SPECTRUM ALLOCATION BILL INTRODUCED

Senator Edwin Johnson (D.Colo.) has introduced a bill (S.1378) which would have the FCC allocate the entire spectrum. The action would include all government frequencies. The bill, which is intended to eliminate the "dog-in-the-manger" attitude of users who waste frequencies, has been referred to the Senate Foreign Relations Committee and the Interstate Commerce Committee.

COLOR TV COMMITTEE DISBANDED

W. R. G. Baker, Director of the RTMA engineering department, this week disbanded the Committee on Color Television of the engineering department. Dr. Baker said the action was taken because of the establishment of the second National Television System Committee. In the future, after the work of the NTSC has progressed to a point where it approaches completion, it may be necessary to establish another RTMA committee on color TV, Dr. Baker added.

TELEVISION NEWS

The FCC has set September 17 as the date to commence hearings in Washington on petitions proposing allocation of frequencies and promulgation of rules and regulations for a theater television service. Earlier, the FCC issued a notice of proposed rule making and received notices of appearances from some parties. Other parties desiring to participate in the proceedings are given until August 15 to file statements listing witnesses and subjects for which testimony and evidence will be offered. . . . FCC Chairman Wayne Coy stated recently that the **FCC could not start granting construction permits for new television stations before the first of December.** . . . Curtis B. Plummer, until now Chief Engineer of the Federal Communications Commission, will head **the recently created Broadcast Bureau**, according to an FCC announcement. The new Bureau, which began operations on June 4, 1951, consists of an office of the chief, and the following five divisions: aural facilities, television facilities, renewal and transfer, hearing, and rules and standards. Its function will be to unify work pertaining to radio broadcasting which has heretofore been handled by various legal, accounting, and engineering units within the Commission. This will mean the abolishment of the separate broadcast divisions now under the General Counsel, Chief Accountant, and Chief Engineer, and the transfer of their personnel to the new bureau. The effect will be that a single Broadcast Bureau will be responsible to the Commission for discharging legal, accounting, and engineering functions in connection with all broadcast services, the FCC states. The aural facilities division will administer to the standards (AM) and frequency modulation (FM) services, and the television facilities division will supervise television matters, with the exception of renewal and transfer, hearing, and rules and standards functions, which will be discharged by the new Bureau's other divisions mentioned above, all under the co-ordinated direction of the Bureau's Chief.

Western IRE Convention and Pacific Electronic Exhibit

CIVIC AUDITORIUM, SAN FRANCISCO, CALIF.—AUGUST 22–24, 1951

"Behind the Scenes in Electronics" will be the theme of the 1951 Western IRE Convention and the 7th Annual Pacific Electronic Exhibit. The setting for this event will be the Civic Auditorium in San Francisco on August 22, 23, and 24.

The technical program has been carefully planned so that it will tie together work in selected fields, as well as present the newest developments and indicate the direction of research efforts. Three of the technical sessions are being sponsored jointly with URSI and the IRE Professional Group on Antennas and Propagation. The preliminary program of the technical sessions is listed below.

Industrial, governmental, and educational activities in the electronic field will be well represented by the Electronic Exhibit, when some 300 lines of electronic equipment will be on display.

Registration fees covering both the technical sessions and exhibits have been announced as follows: IRE members, exhibitors—$2; nonmembers—$3; IRE Student members—$1; nonmember students—$1.50.

Details of social events, field trips, and women's activities may be found on page 65A of this issue.

PROGRAM

Wednesday, 10:00 A.M., August 22

BROADCAST AND TELEVISION

Chairman, R. A. Isberg,
Radio Station KRON

"Utilization of Microwave Radio Relay in Bell System Radio and Television Services—a Progress Report," F. M. Ryan, American Telephone & Telegraph Co.
"Klystron Transmitting Tube Suitable for UHF Television," Wayne Abraham, Varian Associates.
"A New High Power Transmitter and High Gain Antenna for UHF Television," F. P. Barnes, General Electric Co.
"Establishing Subjective Tolerances for Color Television Pictures," W. E. Evans and C. M. Kelley, Stanford Research Institute.

Wednesday, 1:30 P.M., August 22

PROPAGATION AND OPTICS

Chairman, J. B. Smyth, U. S. Naval
Electronics Laboratory

"A Correlation Computer and Applications to Radio Propagation," F. E. Brooks, Jr., and H. W. Smith, Electrical Engineering Research Laboratory, University of Texas.
"A Quasi-Static Solution for Diffraction of a Plane Electromagnetic Wave by a Spheroid," by C. T. Tai, Stanford Research Institute.
"Strip Transmission Line Study," by N. A. Begovich, Hughes Aircraft Co.
"The Zero Phase-Front in Microwave Optics," by J. E. Eaton, Naval Research Laboratory.

NOISE AND COMMUNICATION THEORY

Chairman, O. G. Villard, Stanford
University

"The Generation and Measurement of Low Frequency Random Noise," by R. R. Bennett and A. S. Fulton, Hughes Aircraft Co.
"Probability Distributions of the Zero-Crossing Periods of Filtered Random Noise," by C. R. Gates, Jet Propulsion Laboratory, Cal. Inst. of Tech.
"Applications of Communication Theory to Radio Navigation Systems," by M. Leifer, Sylvania Electric Products Inc.
"An Octave-Band Noise Analyzer," by Dr. Arnold Peterson, General Radio Co.

Thursday, 9:30 A.M., August 23

MEASUREMENTS

Chairman, J. R. Pettit,
Stanford University

"A Direct Reading VHF Frequency Meter," by Leonard Cutler, Gertsch Products, Inc.
"A Precision Frequency Measuring Equipment," by D. A. Pitman, Marconi Instruments, Ltd.
"Digital Frequency Measurement up to 10 MC," by A. Bagley, Hewlett-Packard Co.
"Precision Time Delay Generation," by J. S. Johnson, Rutherford Electronics Co.

VACUUM TUBE APPLICATIONS

Chairman, W. E. Evans, Stanford
Research Institute

"The Coaxial Tetrode as a Wideband Power Amplifier at VHF and UHF," by D. H. Preist, Eitel-McCullough, Inc.
"Subminiature Tubes Developed for 26.5-Volt Operation," by William R. Wheeler, Sylvania Electric Products Inc.
"Reduction of Nonlinear Distortion in Broad-Band Amplifiers," by W. E. Ayer, Stanford University.
"A Single-Ended Push-Pull Audio Amplifier," by Arnold Peterson and D. B. Sinclair, General Radio Co.

Thursday, 1:30 P.M., August 23

LINEAR ARRAY ANTENNAS

Chairman, T. J. Keary, U. S. Naval
Electronics Laboratory

"Optimum Patterns for Arrays of Nonisotropic Sources," by George Sinclair, University of Toronto, and Frank V. Cairns, National Research Council.
"Slot Radiators and Arrays at X-Band," by Robert J. Stegen, Hughes Aircraft Co.
"Factors Affecting the Performance of Linear Arrays," by L. L. Bailin and M. J. Ehrlich, Hughes Aircraft Co.
"Thickness Effects in Slots Located in Various Positions in Rectangular Waveguide," by L. Felsen, H. Kurss, N. Marcuvitz, and A. A. Oliner, Polytechnic Institute of Brooklyn. (Paper to be presented by A. A. Oliner.)

CIRCUITS

Chairman, D. F. Tuttle,
Stanford University

A paper on circuits, Title to be announced, by D. L. Trautman, University of California at Los Angeles.
"Frequency Analysis of Control Systems Employing Digital Computers," by J. M. Ham, J. M. Salzer, and W. K. Linvill, Massachusetts Institute of Technology.
"Synthesis of Cascaded 3-Terminal R-C Networks with Transfer Ratios that Contain Complex Zeros," by H. L. Krauss, F. Hopkins, E. L. Sparrow, and P. F. Ordung, Yale University.
"Network Synthesis Using Tchebycheff Polynomial Series," by S. Darlington, Bell Telephone Laboratories, Inc.

Thursday, 7:45 P.M., August 23

THE HANDLING AND UTILIZATION OF INFORMATION

Chairman, F. E. Terman,
Stanford University

"The Auditory Cortex and Information Theory," by A. R. Tunturi, University of Oregon Medical School.
"Exploitation of Message Statistics," by B. M. Oliver, Bell Telephone Laboratories, Inc.
"The Performance of Communication Systems in the Presence of Noise," by R. Fano, Massachusetts Institute of Technology.

Friday, 9:30 A.M., August 24

VACUUM TUBES

Chairman, Karl Spangenberg,
Stanford University

"Electron Tube Reliability," by E. Finley Carter, Sylvania Electric Products Inc.
"Storage Tubes, Review of Progress," by Andrew V. Haeff, Hughes Aircraft Co.
"Review of Recent Developments in Klystrons," by Russell H. Varian, Varian Associates.
"Survey of Traveling-Wave and Related Vacuum Tube Devices," by L. M. Field, Stanford University.

SPECIAL EQUIPMENT

Chairman, W. J. Warren,
University of Santa Clara

"The Bennett Radio-Frequency Mass Spectrometer," by G. S. Hewitt, University of Arkansas.
"A High Speed Camera with Electronic Controls," by R. Bowersox, Jet Propulsion Laboratory, California Institute of Technology.
"A High Speed Recording Potentiometer," by C. Nylkema and R. Bowersox, Jet Propulsion Laboratory, California Institute of Technology.
"A Single Sideband Communications System," by John F. Honey, Stanford Research Institute.

Friday, 1:30 P.M., August 24

ANTENNA APPLICATIONS

Chairman, L. C. Van Atta,
Hughes Aircraft Co.

"An Organ Pipe Scanner," by G. D. M.
Peeler, K. S. Kelleher, and H. H. Hibbs,
Naval Research Laboratory.

"A Broadband Microwave Quarter-Wave
Plate," by Alan J. Simmons, Naval Re-
search Laboratory.

"Modified Magic Tee Phase-Shifter," by
Richard H. Reed, Hughes Aircraft Co.

"Polarization Switch and Universal Horn,"
by Sanford Hershfield, The Glenn L.
Martin Co.

COMPUTERS

Chairman, P. L. Morton, Computer Labo-
ratory, University of California

"The ERA 1101 Digital Computer," by
John L. Hill, Engineering Research
Associates.

"Man-Machine Communication in the Com-
puter Field," by B. S. Benson, Benson-
Lehner Corp.

"Reduction of von Neumann's Constant in
Electronic Computing Equipment," by
R. G. Canning, Naval Air Missile Test
Center.

"The Maddida 44A Computer," by C. B.
Dennis, Northrop Aircraft.

"Performance of Electronic Components in
the SWAC," by Edward Lacy, National
Bureau of Standards, Los Angeles.

IRE People

Gilbert Bryan Devey (S'45–A'48–M'50),
employed by the Office of Naval Research,
Washington, D. C., has been named Navy
Department repre-
sentative and con-
sultant to the
Physical Security
Equipment Agency.

Born in Swissvale,
Pa., in 1921, Mr.
Devey received the
B.S. degree from the
Massachusetts Insti-
tute of Technology in
1946 after a war-
interrupted college
course, which had be-
gun at the Carnegie Institute of Technology
in 1938. During 1944 and 1945 he studied in
engineering electronics at the Postgraduate
School, U. S. Naval Academy.

G. B. DEVEY

Mr. Devey has been engaged in elec-
tronics for the Navy since 1941. From 1941
to 1945 he served as electronics officer, 30
months of which time he spent on the *USS
Saratoga*. In 1946 he joined the Electronics
Design Division of the Navy Bureau of Ships
as project engineer specializing in antenna
development. From 1947 to 1949 he was
antenna-system engineering supervisor at
the U. S. Navy Underwater Sound Labora-
tory, New London, Conn. Since March,
1949, he has been staff electronics engineer
with the Undersea Warfare Branch of ONR.

Mr. Devey has been responsible for
carrying out an extensive program designed
to improve the reliability of electronic de-
vices; he has been instrumental in bringing
about widespread interest in the problem of
reliability. In addition, he has fostered the
concept of machine assembly of electronic
equipments, and has served as consultant to
the National Bureau of Standards in this
matter.

❖

Robert G. Scott (A'48), senior engineer,
has been appointed the new head of the
commercial engineering department of
Allen B. DuMont Laboratories, Inc. He
joined the DuMont tube division engi-
neering staff in 1948 as a senior engineer in
design and development of television-type
cathode-ray tubes, and later transferred to
manufacturing engineering. During the war
he was active in the Navy's sonar develop-
ment program.

Mr. Scott was graduated from Hamilton
College with the B.S. degree in physics.

Edwin Lloyd Powell (A'14–
M'29–SM'43), 58, pioneer radio en-
gineer and long-time Government
employee, died at Eau Gallie, Fla.,
recently, it has been learned.

A native of Natrona, Pa., Mr.
Powell was retired for disability in
January of this year. At the time of
his retirement he was an electronic
scientist with the Naval Research
Laboratory, where he had spent
more than 15 years.

Mr. Powell's Government career
began in 1909 when he became a
messenger with the Patent Office in
the Department of the Interior.
From 1912 to 1918, he was with the
Geological Survey, transferring in
January, 1918, to the Fuel Admin-
istration, and again, in the same
year, to the Naval Gun Factory. He
joined the Naval Research Labora-
tory when it was organized in 1923.
For more than seven years, he was
in charge of designing all naval
radio receivers for marine and land
station use. In 1924, he installed a
multiple radio-receiver equipment
on the *USS Leviathan*.

Mr. Powell resigned from the
Laboratory in 1928 to enter the
commercial radio field. He was as-
sociated with Charles R. Speaker
and Company, the DeForrest Radio
Company, the Hygrade Sylvania
Company, the National Electrical
Supply Company, the Radio Re-
search Company, and the Bendix
Radio Corporation, successively.

In 1939 he returned to the
Naval Research Laboratory as a
radio engineer, remaining there un-
til his retirement. He received a
Meritorious Civilian Service Award
at the close of World War II for
"outstanding work in the develop-
ment of standardized shipboard
radio antenna distribution systems
and accessories."

Mr. Powell held several radio
operator's licenses and joined the
American Radio Relay League
during the first year of its organiza-
tion.

❖

Mervin J. Kelly (M'25–F'38), formerly
executive vice-president of Bell Telephone
Laboratories, Inc., was recently elected
president of that or-
ganization.

A native of
Princeton, Mo., he
was graduated from
the Missouri School
of Mines and Metal-
lurgy in 1914, re-
ceived the master's
degree from the Uni-
versity of Kentucky
in 1915, and the
Ph.D. from the Uni-
versity of Chicago in

MERVIN J. KELLY

1918. He then joined the engineering de-
partment of the Western Electric Company,
later incorporated by Bell. Dr. Kelly served
as director of vacuum-tube development
from 1928 to 1934, and as development di-
rector of transmission instruments and elec-
tronics until 1936, when he was made
director of research. In 1944 he became ex-
ecutive vice-president, and held both posi-
tions until 1946, when he relinquished the
duties of director of research to concentrate
on his major administrative responsibilities.

During World War II, Dr. Kelly de-
voted most of his time to war activities at
Bell Laboratories. He received the Presi-
dential Certificate of Merit, which cited him
"for outstanding fidelity and meritorious
conduct in aid of the war effort." For the
past year, at the request of the Secretary of
the Air Force, Dr. Kelly has acted as an
advisor to the Department of the Air Force
to assist in research and development or-
ganization.

A member of numerous professional
societies, Dr. Kelly has served on various
Institute Committees and Subcommittees
concerned with vacuum tubes, and in 1939
represented the IRE on the ASA Committee
on Vacuum Tubes.

❖

Henry B. Yarbrough (M'45) was ap-
pointed manager of Air Force sales at the
Bendix Radio Division of the Bendix Avia-
tion Corporation, it was announced re-
cently. Mr. Yarbrough, who has been as-
sociated with Bendix Radio since 1945,
formerly was supervisor of radar sales. As a
lieutenant colonel during World War II, he
held the posts of Chief of the Communica-
tions Branch, Signal Corps Aircraft Radio
Laboratory, at Wright Field, Chief of the
Airborne Radio Section of the Office of the
Chief Signal Officer and Deputy Chief of the
Radio and Radar Section, Headquarters,
U. S. Air Force.

Richard H. Dorf (A'47) has joined the staff of the Brach Manufacturing Division of the General Bronze Corporation as an electronics project engineer, according to an announcement made by Ira Kamen (M'48), director of electronics at that organization.

Mr. Dorf is the author of a book on disk recording, and co-author with Mr. Kamen of "TV Master Antenna Systems," which was recently published by John F. Rider, Publisher, Inc. He has written over 50 articles for the leading electronic publishers on various development projects, the most recent of which is one concerning electronic musical instruments. He also reviews new audio patents each month. Mr. Dorf has served as television consultant to leading realty organizations and system manufacturers.

Prior to entering full-time consulting practice, Mr. Dorf was associate editor of the magazine *Radio-Electronics*, and before that served for some 10 years in the broadcast industry. During the last war, he was an Air Force communications officer.

❖

Adolph H. Rosenthal (A'40) was elected vice-president and director of research and development of the Freed Radio Corporation, New York, N. Y., it was announced recently.

Dr. Rosenthal, who joined Freed Radio in 1949, has served up to now as director of physics. In his new capacity he will supervise important government development projects for which the corporation has been building enlarged laboratory facilities.

A. H. ROSENTHAL

Prior to his association with Freed Radio, Dr. Rosenthal was engaged for many years in electronic research for American and British concerns. His career includes work at the Einstein Institute in Potsdam, Germany, where he engaged in research applying television methods to solar observations.

Dr. Rosenthal is a Fellow of the Royal Astronomical Society, and a member of the American Physical Society, and of the Society of Motion Picture and Television Engineers.

❖

Frank G. Marble (A'35–SM'43) has been appointed sales manager of the Boonton Radio Corporation, Boonton, N. J., designers and manufacturers of direct-reading electronic instruments.

Born in Leland, Miss., Mr. Marble attended Mississippi State College, where he received the degree of B.S. in electrical engineering in 1934. In 1935 he received the M.S. degree in electrical engineering from the Massachusetts Institute of Technology.

From 1935 to 1940 Mr. Marble was employed as a development engineer in the television research department of the Philco Corporation in Philadelphia. During the period beginning with 1940 and ending with 1944, he was associated with the electrical

research products division of the Western Electric Company, and the Bell Telephone Laboratories, Inc., as a member of the technical staff. In this capacity he was responsible for development work on various measuring instruments, and acted as coordinator on several projects for the armed services. From 1944 to 1948 he was in charge of developing and applying instrumentation for the installation engineering department at Pratt and Whitney Aircraft, East Hartford, Conn. For the past three years Mr. Marble has been sales manager of the Kay Electric Company, Pine Brook, N. J.

Edmund Rich Morton (SM'50), a pioneer in the development of electronic controls, died recently at Brooklyn Hospital, where he had been confined during a brief illness.

The holder of over 50 patents on electronic controls and other devices, Mr. Morton contributed to the development, during World War II, of radar, sonar, and other equipment developed by Bell Laboratories for military use.

Mr. Morton was born on May 28, 1896, in Swanton, Vt. He was educated at Stevens Institute, from which he received the master of engineering degree in 1917, and at Harvard University, where he studied during 1919–1920.

He joined the technical staff of Bell Laboratories in 1923, engaging chiefly in the design and development of power apparatus. During the 1920's, he worked on the motors for the first television system developed by the Laboratories, and also on those for early sound-picture recording and reproducing. From 1936 to 1940, he designed special-purpose relays used in telephone and other communications systems.

In World War I, Mr. Morton served with the Signal Corps and Air Service; at that time he developed antenna camera mounts which were resistant to vibration, and which contributed to the advances of antenna reconnaissance.

He belonged to the American Cryptogram Association, the New York Cipher Society, whose president he was in 1949, the Aircraft Owners and Pilots Association, and the Telephone Pioneers of America.

Joshua Sieger (SM'49) has recently established himself as an independent technical consultant in the fields of radar, communications, and television, it has been learned.

During World War II, Mr. Sieger was divisional head of the engineering department of the Telecommunications Research Establishment, the British Government radar research organization. Before that, he was for many years executive engineer of Scophony Ltd., where he was responsible for the large-screen, theater-size television

pictures installed in various theaters in London.

Mr. Sieger is a member of the technical committees of the Radio-Television Manufacturers Association, and belongs, also, to the National Television System Committee.

❖

William P. Short (SM'44) has been named chief engineer of the General Precision Laboratory, Pleasantville, N. Y., in which capacity he will supervise production of electronic and optic equipment developed by the company's research division.

Born in Newark, N. J., in 1906, Mr. Short received the M.E. degree from the Stevens Institute of Technology in 1928, and, at a later date, attended the radar schools at Harvard and the Massachusetts Institute of Technology.

WILLIAM P. SHORT

He has been active in the electronics field for 22 years. While employed by the International Telephone and Telegraph Corporation in Europe, he was responsible for the design of an important line of radio broadcast receivers which were the first to embody the use of the superheterodyne principle in European practice. As a Navy lieutenant in World War II, he made valuable contributions to radar training practices; following this he was asked to take charge of the engineering at the Research Construction Company, Inc., in Cambridge, Mass. For his achievements as that company's chief engineer, he was awarded the Presidential Certificate of Merit.

❖

John Ward Dawson (A'39), formerly in charge of equipment engineering for the Stanford Research Institute, became chief engineer of the electronics division, Sylvania Electric Products Inc., according to a recent announcement.

Born in Sumner, Iowa, in 1901, Mr. Dawson received the B.S. degree from Iowa State College in 1926, and the M.S. from the University of Pittsburgh in 1938.

JOHN W. DAWSON

Prior to joining the staff of the Raytheon Manufacturing Company, from 1926 to 1937, he was associated with Westinghouse Electric. From 1949 until his acceptance of the Sylvania appointment he was in charge of equipment engineering at the Stanford Research Institute. He is noted for his work on the design of welding equipment and on the principles and applications of gaseous discharge tubes. In the course of his engineering career, he was granted some 80 patents for various inventions and devices.

Mr. Dawson is a member of Tau Beta Pi, Eta Kappa Nu, and the American Institute of Electrical Engineers.

Books

Microwave Electronics by John C. Slater

In this book, Dr. Slater has presented one of the few unified accounts of electromagnetic physics. It is a record of much of the wartime and prewar work in this field which contains most of the basic treatment of such devices. The treatment is not detailed and the texts cannot be used as a handbook from which calculations of specific problems can readily be made. It presupposes some familiarity with the subject, and although the first three chapters deal with classical electromagnetic theory, the treatment is too concentrated for the beginner. However, this was not the intent of the text and should not be construed as a criticism of a well organized discussion which carries the reader's thinking smoothly from a basic consideration of klystrons into linear accelerators, and from magnetrons into cyclotrons.

Chapters 5 through 7 represent one of the few consolidated treatments of coupled circuits, and together with Chapter 13, contain much of the material which the authors contributed to magnetron theory during the war.

The chapters on klystrons and traveling-wave tubes include the standard discussions of these subjects published previously and apparently are included here for continuity.

In summation, the experienced worker or teacher will find parts of this book a valuable reference which he will want to have available.

E. D. McArthur
General Electric Co
Schenectady, N. Y

Modern Oscilloscopes and Their Uses by Jacob H. Ruiter, Jr.

There are certain fundamentals essential to a good technical book. The purpose, scope, and intended reader qualification should be clearly and correctly delineated in the preface or introduction. The subject matter and its basic terminology should be uniquely defined in the first chapters. The context should have high tutorial value. The stated purpose should be fulfilled.

"Modern Oscilloscopes and Their Uses," by Jacob H. Ruiter, Jr., fulfills these requirements in a most satisfactory manner. The purpose: to fulfill the need for "a book which would serve as general instructions to all users" of oscilloscopes; the scope: modern oscilloscopes, their history, principles of operation, detailed analysis of component parts, associated circuitry and auxiliary equipment, applications in the electronics industry, in servicing all types of radio receivers, in servicing and monitoring radio transmitters, in teaching and in other industries, and cathode-ray pattern photography. In the term "modern oscilloscope" is imposed a restriction to cathode-ray tube presentation, although one introductory chapter is devoted to mechanical oscillographs. The scope is fully covered.

The intended reader qualification: a general background in physics. Even the most elementary physics training is adequate for a full understanding of this thoroughly non-mathematical treatment of the subject. As to definition of subject matter and terminology, not only in the early chapters, but throughout the book, nothing is left to the imagination. Meticulous attention to this matter in the text is augmented by a glossary of about 200 carefully defined technical terms. This contributes to the tutorial value, which is extraordinarily high. The clarity and direct simplicity of the language, and of the many illustrations by which the reader is led from the most elementary physical concepts of charge and motion to the interpretation of almost every conceivable type of cathode-ray pattern, puts a working knowledge of the whole subject within the grasp of any grade-school graduate with a predilection toward things scientific.

The value of the book is by no means high only to the novice. The carefully outlined step-by-step procedures for use of the oscillograph in many applications, with the "why" as well as the "how" in the text, make it an invaluable manual for general electronic servicing. Even the experienced engineer or technician, if he will take the trouble to look, is likely to find many a term he has never heard before, while the research scientist, through perusal of these stimulating pages, may find his oscilloscope to be a research instrument more versatile than he had dreamed.

In the opinion of this reviewer, the purpose is admirably met. No modern reference library for electronics, public or private, large or small, can be considered adequate without this book. Every electronics service man should have a copy, or have access to one. Every electronics laboratory should consider it an engineer's tool. The only one to regret its publication is the instructor in oscillography. It makes his job less essential.

R. M. Page
U. S. Naval Research Laboratory
Washington 25, D. C.

Design of Electrical Apparatus, Third Edition, by John H. Kuhlmann, assisted by N. F. Tsang.

In the third edition of this book, the general plan and method of presentation used in the previous two editions have not been changed.

The book is divided into four sections: direct-current machines, synchronous machines, inductive motors, and transformers. The method of treatment adopted for each type of apparatus discussed is, first, to explain the construction of the apparatus to be designed; second, to explain the formulas and procedure; third, to give the design limits established by practice; and fourth, to illustrate with complete sample calculations.

In this third edition the complete design calculations are given for two direct-current machines, two synchronous machines, one polyphase induction motor, two fractional horse-power single-phase induction motors, and four transformers. The design constants and limits have been revised in line with the latest practices. The characteristic curves of electrical sheet steels are plotted on semilogarithmic paper, and the characteristic of cold-reduced electrical sheet steels, used in rolled-core transformer construction, have been added. The copper tables have been enlarged to include the new wiring insulating materials.

In the direct-current design section, the sample design calculations have been revised in accordance with the revised output constants. New values of insulation thickness and clearance for the armature slots have been adopted. A discussion of multiplex windings used on large-capacity machines has been added.

The method of calculating the field current for a specific load and power factor for synchronous machines has been changed to agree with the procedure adopted by the American Standards Association. The equivalent circuit of a synchronous motor with single and double squirrel-cage rotor winding is included, and a method of predetermining starting torque, pull-in torque, and starting kva is given with sample calculations for single- and double-cage starting winding.

The section on induction motor design has been largely rewritten. New methods for determining motor dimensions are given, and design constant curves are plotted on semilogarithmic paper. A method of determining the effect of saturation in the leakage flux paths, and of calculating the deep bar effect of squirrel-cage rotors has been added.

A method for the design of small transformers used in control circuits and for power supply in electronic devices has been included with a sample design.

This book is invaluable to the professional design engineer, since the procedure and methods employed throughout are those used by the professional designer. The book is also suitable as a text for college-level courses. The design sheets used in the text are similar to those used by the commercial designer, and contain all the data necessary to construct the apparatus.

Henry F. Cooke
Syracuse University
Syracuse, N. Y.

TV and Other Receiving Antennas, Theory and Practice by Arnold B. Bailey

Published (1950) by John F. Rider Publisher, Inc., 480 Canal St., New York 13, N. Y. 581 pages +10-page index. 310 figures. 5⅛×8⅜. $6.00.

This book on antennas by a specialist in the field is very readable and rather interestingly written. It dispels much of the mystery and mumbo-jumbo connected with the design and capabilities of television receiving antennas. Satisfactory answers will be found to most of the familiar questions raised by the experimentally inclined television receiver owner, and the nonspecialist in the antenna field.

After a review of definitions used in antenna practice, an analysis is made of the television signal and of the conditions that determine signal levels at the receiver. Antenna theory and its practical applications are treated next. Performance data on most of the simple types of antennas are included at the end of the book.

The purist, though he will find details with which to quarrel, will recognize that a certain amount of poetic license is frequently necessary to convey a complicated idea to the layman.

There are a few explanations relating to antenna theory which are at a considerable variance with this reviewer's belief and experience. For example, circuit Q is defined simply as the ratio of input reactance to resistance, and is accordingly stated to be zero at resonance. On this basis, Q varies widely with frequency near resonance. This apparently unorthodox definition is bound to be confusing to the student who has accepted the definition for Q of a resonant circuit as the ratio of the reactance of one element to the total series resistance.

Furthermore, this reviewer does not share the belief that receiving antennas under all conditions reradiate into space at least half of the energy impinging upon them, nor that half-wave parasitically excited dipole reflectors will enhance the gain of the zero decibel source dipole by from 4 to 5½ db, as implied in the tabulation of antenna characteristics at the end of the book.

It should be clearly understood, however, that these criticisms are not intended to detract from the over-all usefulness of this well-written, practical treatise of an involved subject. The easy style is refreshing, and it is evident that a great deal of careful thought has gone into the preparation of this book.

PHILLIP H. SMITH
Bell Telephone Laboratories, Inc.
Whippany, N. J.

Practical Television Engineering by Scott Helt

Published (1950) by Murray-Hill Books Inc., 232 Madison Ave., New York 16, N. Y. 664 pages +13-page index +30-page glossary +xv pages. 387 figures. 6¼×9¼. $7.50.

This is a very full volume which covers the whole field of television, including principles, cameras, studio equipment, transmitters, and receivers. In addition, there is a chapter on television–broadcasting techniques, and a 30-page glossary of terms. Within the limitation imposed by 664

pages, the subject matter is covered in acceptable detail.

The book should appeal to those who have an engineering background. A particularly good feature is its adequate references to the literature, and a student of television will find the book valuable for this reason alone.

In the main, the information is clearly presented. There is some tautology, however, and some errors occur, as is perhaps to be expected with a new book not yet subjected to critical review. But the errors are not of sufficient magnitude to affect the value of the book.

Especially interesting chapters are those covering the camera chain and the television transmitter. Several commercial designs are discussed and compared in these sections. There is also a long chapter on synchronizing generators.

The book represents a sincere effort on the part of the author to cover an extremely wide field. He has succeeded in producing a volume of considerable merit that should appeal both to the practicing engineers and to students.

F. J. BINGLEY
Philco Corporation
Philadelphia, Pa.

Ultra-High-Frequency Engineering by Thomas L. Martin, Jr.

Published (1950) by Prentice-Hall, Inc., 70 Fifth Ave., New York, N. Y. 439 pages +12-page index +5-page appendix +viii pages. 320 figures. 5⅜×8⅜. $6.35.

This is the first edition of a book in which the author relates the fundamentals of electricity and electron tubes to the many units which comprise a modern ultra-high-frequency broad-band communications system. The purpose is not to analyze or design any one system, but rather to study the generic types used in all systems.

The major divisions of the book are: (1) nonsinusoidal periodic waveforms, including means for generating and modifying these waves; (2) signal amplifiers of the types commonly known as video and IF amplifiers; (3) transmission lines, waveguides, and elementary cavity resonators; (4) ultra-high-frequency generators including special triods, klystrons, and magnetrons; and (5) propagation of electromagnetic waves through the troposphere and ionosphere. Each topic is introduced with a derivation of the elementary theory and is followed by illustrations of the applications of the theory to produce practical components. Because the use of advanced mathematical tools is avoided, the development of the theory is sometimes cumbersome. However, a clear physical concept for each phenomenon is usually presented, and this fulfills the main purpose of the book. The groups of problems given in the text provide adequate exercise for the student, while a bibliography of book and periodical references points the way for additional study.

In a volume of such large scope, it is difficult to be general and yet to cover all the phases of the ultra-high-frequency system. However, this reviewer feels that two additional topics should have been included: One is the role played by the silicon crystal diode as a frequency converter; the second, a discussion of radiation with a description

of the elementary antenna types peculiar to the ultra-high-frequency spectrum.

This book should prove useful in a course for undergraduates in electrical engineering, to familiarize them with the basic elements of the modern wide-band communications system. It is not written as a reference book for the professional engineer.

R. A. DEHN
General Electric Co.
Schenectady, N. Y.

Mathematical Engineering Analysis by Rufus Oldenburger

Published (1950) by the Macmillan Co., 60 Fifth Ave., New York 11, N. Y. 414 pages +xiv pages +11-page index. 220 figures. 6¼×9¼. $6.00.

The avowed purpose of the book is expressed by the author as follows:

"One of the most difficult tasks in industrial research is that of expressing physical situations in the form of equivalent mathematical relations. A good background in mathematics or engineering, or both, does not insure that the research engineer and scientist will have a facility for making the transition from physical engineering situations to the corresponding mathematics. This book was written to make the task of transition easier for the reader.

"To operate at maximum efficiency the man in charge of an industrial research program today should have not only a thorough understanding of the experimental aspects of his problems but also should be able to set up his problems in mathematical form so that he can determine when something can be done more economically by mathematical computation than by experimentation, or vice versa. With the mathematics as guide he can better interpret experimental results and plan future physical studies."

This reviewer would take exception to the thesis that such attention to detail is either desirable or possible for the man in charge of an industrial research program, particularly in the larger industrial research organizations. The activities for which this book should prove useful are those which are usually carried out by an active working group leader. I do not take issue with the author on other aspects of the book.

The ability to see the main problem, to sift out the nonessentials and to set up a solvable mathematical problem which teaches a little more of the basic facts of the problem, is a gift denied to all but a few fortunate individuals. The book should prove valuable to those who have this ability.

A large portion of the field of mathematical physics is covered by the author in sections dealing with mechanics of rigid bodies, electricity and magnetisms, heat, elasticity, and fluid mechanics. For one not already skilled in the mathematical manipulations usually associated with theoretical physics, this book will prove to be heavy going. Those whose interest is in the direction of the book material will find the author's style and treatment is refreshing. Dr. Oldenburger has brought about a nice balance between the mathematical approach and the physical reality.

GEORGE H. BROWN
RCA Laboratories
Princeton, N. J.

Sections*

Chairman		Secretary
J. S. Hill Box 298 Hudson, Ohio	AKRON (4)	R. L. Burtner Goodyear Aircraft Corp. 1210 Massillon Rd. Akron, Ohio
E. S. Lammers, Jr. Westinghouse Electric Corp. Box 4808 Atlanta 2, Ga.	ATLANTA (6)	D. B. Buchanan 50 7 St., N.E. Atlanta, Ga.
J. V. Lebacqz Johns Hopkins University Baltimore 18, Md.	BALTIMORE (3)	G. R. White Bendix Radio Div. Towson 4, Md.
Eric D. Coburn Box 793 Nederland, Texas	BEAUMONT- PORT ARTHUR (6)	C. B. Trevey 2555 Pierce St. Beaumont, Texas
W. H. Hauser WBZ, WBZ-FM, WBZ- TV 1170 Soldiers Field Rd. Boston, Mass.	BOSTON (1)	Frank D. Lewis General Radio Co. 275 Mass. Ave. Cambridge 39, Mass.
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The Annual Index to these Abstracts and References, covering those published in the PROC. I.R.E. from February, 1950, through January, 1951, may be obtained for 2s.8d. postage included from the *Wireless Engineer*, Dorset House, Stamford St., London S.E., England. This index includes a list of the journals abstracted together with the addresses of their publishers.

ACOUSTICS AND AUDIO FREQUENCIES

534.213.4   **1290**
Acoustic Waveguides—Y. Rocard. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 485–487; February 5, 1951.) Theory is given relative to the propagation of sound waves in a medium filling an infinitely long cylinder; the velocity of sound in the medium being c_1 and in the material of the cylinder c. If $c_1 < c$, the waves will be propagated along the axis of the tube without exciting any appreciable radial propagation in the tube wall. Such conditions are easily realized approximately in practice by means of a series of small equidistant disks strung on a central rod, the surrounding medium being air. Experiments carried out in the Bell System laboratories have shown that such a system acts as an acoustic waveguide, even when the rod is curved.

534.321.9:534.232   **1291**
Ultrasonic Energy—A. E. Crawford. (*Electronic Eng.* (London) vol. 23, no. 275, pp. 12–18; January, 1951.) The various methods available for producing ultrasonic energy are described briefly and compared. An outline of the elements of wave propagation at these frequencies includes tabulated values of the speed of longitudinal waves in common solids, liquids, and gases.

534.321.9:534.25:621.315.616.9   **1292**
Ultrasonic Lenses of Plastic Materials—D. Sette. (*Nuovo Cim.*, vol. 6, pp. 135–147; March 12, 1949.) The focusing properties of certain plastics for ultrasonic radiation were studied. When a planocylindrical or planospherical plastic lens is used, the energy required from the quartz generator, for the same intensity of radiation over a given region, may be reduced to one 10th or one 100th, respectively, of its original value. See also 3006 of 1949.

534.84   **1293**
Vibrations of Enclosed Spaces with Deformable Elastic Walls—T. Vogel. (*Jour. Phys. Radium*, vol. 11, pp. 627–632; November, 1950.) Hypotheses used in simple geometric theory of acoustic resonance are not compatible with the wave equation in the case where a velocity potential exists. Under certain simplifying assumptions based on elasticity theory, the velocity potential is expressed as a convergent series of integral powers of a small parameter related to the absorption of the walls; the coefficients being the solutions of a succession of Neumann problems extended to the equation $(\Delta_2 + \lambda)u = 0$. A shorter version was noted in 800 of May.

534.846.3/.4   **1294**
Recent Developments in Speech-Reinforcement Systems—P. H. Parkin and W. E. Scholes. (*Wireless World*, vol. 57, pp. 44–50; February, 1951.) Experimental results obtained in an open-air theater confirm the conclusions reached in a review of previous investigations, which indicated that, to achieve realism and maximum intelligibility in a sound-reinforcement system, the time delay between hearing the direct and the amplified sounds should be 5 to 35 ms, and that the amplitudes should not differ by more than 7 db.

538.613:621.395.625.3   **1295**
Magneto-optic Transducers—A. W. Friend. (*RCA Rev.*, vol. 11, pp. 482–507; December, 1950.) A magneto-optical playback system and head have been developed for use with magnetic-tape records. Signal amplitude is constant with frequency and linearity over the desired amplitude range is satisfactory, but the maximum signal noise ratio available is only about 30 db. Means of improving this ratio are discussed. The effect of using more active magneto-optical material and of variations of the system parameters can be evaluated from a theoretical treatment of the system.

621.395.623.41   **1296**
The Ring-Armature Telephone Receiver—E. E. Mott and R. C. Miner. (*Bell Sys. Tech. Jour.*, vol. 30, pp. 110–140; January, 1951.) "A new type of telephone receiver is described in which the permanent magnet, the pole piece, and the armature, which drives a light weight dome, are all ring-shaped parts. This structure exhibits a substantially higher grade of performance than present receivers of the bipolar type, with regard to efficiency, frequency range, leakage-noise level, and response when held off the ear. In addition to showing the characteristics of this new receiver, an analysis of the various losses is given, and ideal performance limits are established. The advantage of providing an auxiliary path for the air-gap flux is indicated, and other applications of the device as a transducer are described."

621.395.813:621.39.001.11   **1297**
Application [to signal and information theory] of Means Used for Assessing the Quality of Telephone Transmission—Chavasse. (*See* 1486.)

534.75.001.11   **1298**
Theory of Hearing [Book Review]—E. G. Wever. Publishers: Wiley and Sons, New York, N. Y., and Chapman and Hall, London, Eng., 1949, 484 pp., $6.00. (*Proc. Phys. Soc.* (London), vol. 64, p. 90; January 1, 1951.) "The volume can be recommended unreservedly to all those interested in the subject of hearing in man and animals."

ANTENNAS AND TRANSMISSION LINES

621.392.09   **1299**
Predetermination of the Conditions of Propagation along a 3-Phase Symmetrical Line of a High-Frequency Wave applied between One Conductor and Earth, the Other Two Conductors being Loaded at the Ends with Equal Impedances—A. Chevallier. (*Compt. Rend. Acad. Sci.*, (Paris), vol. 232, pp. 490–491; February 5, 1951.)

621.392.09   **1300**
Surface-Wave Transmission Line—H. M. Barlow. (*Wireless Eng.*, vol. 28, p. 67; February 1951.) Comment on 563 of March (Rust).

621.392:621.314.2:621.3.015.7†   **1301**
Design of Exponential-Line Pulse Transformers—E. M. Williams and E. R. Schatz. (PROC. I.R.E., vol. 39, pp. 84–86; January, 1951.) Design procedures are described for pulse transformers consisting of exponential transmission-line sections, and data are included for an experimental unit with a nominal impedance ratio of 1:4. This unit will develop its full voltage ratio of 1:2 with a pulse of duration about 3 mμs. Its peak power capacity with dielectric of powdered Ba-Sr titanate is about 12.5 mw, but about 500 mw with nitrobenzene as dielectric.

621.392.26†   **1302**
Expendability of a Wave-Guide Field in Terms of Normal Modes—J. Van Bladel. (*Jour. Appl. Phys.*, vol. 22, pp. 68–69; January, 1951.) The fields in a waveguide can be expanded as a linear combination of normal modes. This is proved by showing, (a) that this expansion is possible for the z components of the fields and modes, and (b) that in three dimensions the fields as a whole are equal to a linear expansion of modes with precisely the same coefficients as were found in the expansion of the z components.

621.392.26†   **1303**
Slow Electric Waves in Gas-Filled Metal Tubes—W. O. Schumann. (*Z. Phys.*, vol. 128, pp. 629–634; December, 1950.) Low-velocity waves observed in high frequency-excited

metal tubes filled with rarefied gas are related to the surface waves along the boundary of a plasma layer at the inner surface of the metal. The natural frequency of the plasma lies approximately between ω and $\sqrt{2}\omega$ for the regions in which this slow propagation is possible, the waves being represented by the expression $e^{j(\omega t - \alpha z)}$.

621.392.26†:621.3.09   **1304**
Electromagnetic Waves in Wave Guides—In 565 of April, for "1851 of 1950," please read "1861 of 1950."

621.392.26†:621.396.67   **1305**
Theory of the Circular Diffraction Antenna—H. Levine and C. H. Papas. (*Jour. Appl. Phys.*, vol. 22, pp. 29–43; January, 1951.) "The circular diffraction antenna consists of a coaxial waveguide fitted with an infinite-plane conducting baffle, and open to free space. An equivalent-circuit description, appropriate to principal-mode propagation in the coaxial region, is investigated theoretically. Variational expressions for the circuit parameters are derived and used for accurate numerical evaluation."

621.396.67   **1306**
Electric Dipoles in the Presence of Elliptic and Circular Cylinders—W. S. Lucke. (*Jour. Appl. Phys.*, vol. 22, pp. 14–19; January, 1951.) Application of the Green's-function method yields expressions for the dipole field in the form of an integral in the complex plane, which can be simplified by imposing the known far-zone conditions. Typical polar diagrams are calculated for the E and H fields for dipoles parallel and perpendicular to strips and cylinders.

621.396.67   **1307**
Radiation from Wide-Angle Conical Antennas Fed by a Coaxial Line—C. H. Papas and R. King. (*Proc. I.R.E.*, vol. 39, pp. 49–51; January, 1951.) An approximate expression for the radiation from spherically capped conical antennas is derived by the Fourier-Lamé eigenfunction method. Diagrams of the far-zone field as a function of zenith angle are shown for antennas of various lengths and flare angles.

621.396.67   **1308**
Applications of Potential Theory to the Design of Linear Arrays—T. T. Taylor and J. R. Whinnery (*Jour. Appl. Phys.*, vol. 22, pp. 19–29; January, 1951.) Schelkunoff's polynomial formulation of the linear-array problem, studied from the potential-theory point of view, leads to an electrical analogy which has made possible the construction of a successful analogue computer. This has been used for the synthesis of new arrays and the evaluation of existing ones. Several theorems concerning the roots of the polynomials are discussed.

621.396.67.011.2   **1309**
Mutual Impedance of Parallel Aerials—R. G. Medhurst and S. D. Pool. (*Wireless Eng.*, vol. 28, p. 67; February, 1951.) Comment on 648 of 1949 (Barzilai).

621.396.67:621.316.761.2   **1310**
Wide-Band Aerials and Resonant Circuits with Simple and Double Compensation—O. Zinke. (*Fernmeldetech. Z.*, vol. 3, pp. 454–458; December, 1950.) *See* 818 of 1950.

621.392.26†   **1311**
Principles and Applications of Waveguide Transmissions [Book Review]—G. C. Southworth. Publishers: D. Van Nostrand Co., New York, N. Y., 1950, 689 pp., $9.50. (*Electronics*, vol. 24, pp. 136–144; January, 1951.) "Most of the emphasis is placed on the applications and this book will be invaluable to all development engineers employing or developing microwave components." See also 2973 of 1950.

CIRCUITS AND CIRCUIT ELEMENTS

621.3.016.35   **1312**
Concerning a Criterion of Stability—M.

Parodi. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 204–206; January, 1951.) The following empirical criterion is frequently applied; a square matrix $M = (a_{ii})$, of order n, with real elements, is in general positive definite if its determinant $\|M\|$ is positive and if min $(a_{ii}) > \max |a_{ij}|$, where $i \neq j$ and i, $j = 1$, 2, $\cdots$, n. Owing to the excellence of the results given by use of this criterion, the question arises as to whether it is exact in all circumstances. It is here shown that this is not the case, but that the criterion can be applied to a large class of matrices. See also 3348 of 1949 (Korn) and 1443 of 1950 (Raymond).

621.314.2:621.3.015.7†:621.3.012.8   **1313**
Some Studies of Pulse-Transformer Equivalent Circuits—C. K. Hadlock and D. Lebell. (*Proc. I.R.E.*, vol. 39, pp. 81–83; January, 1951.) The equivalent circuit for a pulse transformer is derived from data obtained by direct tests on a typical transformer. The differential equations for the equivalent circuit are solved by means of a mechanical differential analyzer for various values of the circuit parameters. Waveforms of transformer output voltage obtained from the analyzer assist in the design of pulse transformers.

621.314.222.017.14   **1314**
The Determination of the Leakage Reactance of Transformers—J. Lagasse. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 48–50; January 3, 1951.) The resonance method of measuring transformer leakage reactance consists of varying the capacitance in the secondary circuit and noting values of this capacitance at which harmonic resonance occurs in the primary circuit. For simplicity of operation, the primary-current variation is observed with a direct-reading ammeter. By inserting an autotransformer between the mains and the unknown transformer and varying the applied voltage, it is found that the leakage reactance is practically unaffected by saturation.

621.314.3†   **1315**
Saturable-Reactor Considerations—F. H. Shepard, Jr. (*Proc. Radio Club Amer.*, vol. 27, no. 3, pp. 3–9; 1950.) Elementary discussion of saturable-reactor characteristics and of methods of demonstrating them.

621.316.761.2:621.396.67   **1316**
Wide-Band Aerials and Resonant Circuits with Simple and Double Compensation—O. Zinke. (*Fernmeldetech. Z.*, vol. 3, pp. 454–458; December, 1950.) *See* 818 of 1950.

621.318.572:621.385.832   **1317**
Deflection of Cathode-Ray Tubes in Sequence—G. W. Gray and A. S. Jensen. (*RCA Rev.*, vol. 11, pp. 527–533; December, 1950.) A circuit is described which enables a parallel-connected system of ten beam-deflection storage tubes, with different bias voltages, to be operated sequentially as the signal amplitude varies.

621.318.572:621.387.422   **1318**
A Magnetic Scaling Circuit—H. Hertz. (*Jour. Appl. Phys.*, vol. 22, pp. 107–108; January, 1951.) Description of the operation of a circuit including two thyratrons coupled by coils wound on a ring-shaped iron core. Pulses, applied to the grid of the first thyratron, trigger the discharge of a capacitor through one of the coils on the iron core, thus producing a voltage pulse on the grid of the second thyratron. Circuit component values are chosen so that the second thyratron fires only after n such pulses. Stable operation is obtained for values of n up to about 10.

621.385.2:546.289   **1319**
The Inverse-Voltage Characteristic of a Point Contact on n-Type Germanium—Hunter. (*See* 1530.)

621.385.2:546.289   **1320**
Pulse Measurement of the Inverse-Voltage Characteristic of Germanium Point Contacts—Bennett and Hunter. (*See* 1531.)

621.385.3.012.8   **1321**
Triode Transmission Networks under Linear Negative-Grid Conditions—A. W. Keen. (*Wireless Eng.*, vol. 28, pp. 56–66; February, 1951. Correction, see vol. 28, p. 98; March, 1951.) "The small-amplitude alternating-current signal behaviour of the negatively-biased triode tube is represented by a Π configuration in which the grid-anode element is allowed to assume negative values in its real component, in order to avoid the use of fictitious voltage or current generators. The value of the grid-anode impedance is computed for the six possible orientations of the tube in three types of circuit configuration, and the validity of the representation checked by computing the driving-point impedance and voltage gain for all eighteen cases. An appendix gives the matrix expressions corresponding to the transmission equivalents of three basic networks."

621.392:083.71;   **1322**
Standards on Circuits: Definitions of Terms in Network Topology, 1950—(*Proc. I.R.E.*, vol. 39, pp. 27–29; January, 1951.) Reprints of this Standard, 50 IRE 4.S1, may be obtained, while available, from IRE at fifty cents per copy.

621.392.5   **1323**
Theory of the Negative-Impedance Converter—J. L. Merrill, Jr. (*Bell Sys. Tech. Jour.*, vol. 30, pp. 88–109; January, 1951.) A relatively new approach to the solution of negative-impedance problems relating to tube circuits consists in reducing the circuit of a device for producing negative impedance to an electrically equivalent four-terminal network, from which the stability and operation of the device can be predicted accurately. A negative-impedance repeater, such as the recently developed Type E1, can be connected in series with a voice-frequency telephone line to provide a transmission gain which is ample for many purposes.

621.392.5:621.3.012   **1324**
New Conductance Diagrams for Passive Linear Quadripoles—H. Kafka. (*Arch. elekt. Übertragung*, vol. 4, pp. 446–454; November, 1950.) All currents and powers are referred to either the input or the output voltage; the corresponding admittance determines the current and the apparent power on the side of the network referred. A "calculation" admittance is introduced and the current and apparent power in the other side of the network are derived by a geometrical construction. All the usual transmission quantities, including currents, active and reacting power, phase angle, efficiency, and voltage ratio, can be obtained in a simple manner from the conductance diagram.

621.392.52   **1325**
Design of High-Pass, Low-Pass and Band-Pass Filters Using RC Networks and Direct-Current Amplifiers with Feedback—C. C. Shumard. (*RCA Rev.*, vol. 11, pp. 534–564; December, 1950.) Operation of a dc amplifier near the (1, 0) point of the Nyquist diagram results in controlled regeneration without oscillation. When such an amplifier is used with an external RC filter network, the combination gives pass bands comparable with those of LC filters at higher frequencies. Cut-off frequencies down to 0.005 cps can be obtained.

621.392.52   **1326**
Design of Reactance Filters with Only One Finite Terminal Resistance by Use of Zobel X-Terminations—J. M. Linke. (*Arch. elekt. Übertragung*, vol. 4, pp. 465–474; November,

1950.) Analysis shows the close connection between this type of filter (e.g. one having effectively zero source or output impedance) and the separator-type filter, for the design of which the Zobel method is useful. This makes use of divided circuits; the impedance of the input branch being suitably modified by added reactors (X terminations). The general construction of an unloaded filter by this method is described, and its behavior in the pass band and the stop band is investigated. In the pass band mismatches have a much greater effect on the attenuation curve than is the case for normal filters with finite terminations at both ends; but for given attenuation requirements in the stop band, fewer circuit elements are required. The practical design of a no-load low-pass filter is outlined.

621.392.52.018.8   1327
The Effect of Circuit Capacitances on Filter Attenuation Curves, and Its Elimination—W. Herzog. (*Arch. elekt. Übertragung*, vol. 4, pp. 462–464; November, 1950.) The effects of capacitance in a bridge-type filter are analyzed; they can be minimized by use of trimmers and completely eliminated by balancing the input and output circuit capacitances.

621.396.4:621.396.82   1328
Theoretical Study of Cross-Modulation Resulting from the Simultaneous Excitation of an Aerial by Two Transmitters—Familier. (*See* 1507.)

621.396.6:665.3   1329
Printed-Circuit Production and Assembly Techniques—R. G. Peters. (*TV Eng.*, vol. 1, pp. 20–23, 44; November, 1950.) A detailed description of the techniques developed at the Bureau of Standards for the mass production of printed circuits. The main limitations are imposed by the difficulty of printing satisfactory resistors. These difficulties are overcome by using resistors stamped out of coated asbestos-paper tape, which are cemented to the printed-circuit assemblies.

621.396.611.1   1330
Sinusoidal Variation of Inductance in a Linear Series *RLC* Circuit—E. I. Hawthorne. (PROC. I.R.E., vol. 39, pp. 78–81; January, 1951.) A comprehensive theoretical treatment. The solution is simply related to that for capacitance variation, and damping is taken into account. Typical differential-analyzer solutions are presented.

621.396.611.1:517.942.932   1331
Forced Oscillations in Nonlinear Systems—Cartwright. (*See* 1417.)

621.396.615.029.53/.55   1332
A 300–4000-Kilocycle Electrically Tuned Oscillator—A. I. Pressman and J. P. Blewett. (PROC. I.R.E., vol. 39, pp. 74–77; January, 1951.) Tuning is accomplished by varying the degree of saturation of a toroidal ferroxcube core in the inductor of an *LC* circuit. The output frequency is stable to within 0.1 per cent when the temperature of the inductor is controlled to within 0.01° C; the amplitude is constant to within 4 per cent. A greater frequency range (100 to 9,500 kc) can be obtained by using the saturable inductor in an *LR* Wien-bridge circuit.

621.396.645:621.396.822   1333
Background Noise in Amplifiers—G. Lehmann. (*Rev. gén. élét.*, vol. 59, pp. 543–553; December, 1950.) Classical and modern theoretical approaches are combined in studying the origin, nature, and transmission of the irregular residual currents observed in amplifier circuits in the absence of applied voltage. The calculation of current and voltage fluctuations due to thermal agitation of electrons is given briefly. Amplifier-tube, anode-current fluctuations, and the noise they produce in connected transducers are considered. The use of correlation functions enables the transmission of the

fluctuations through practical circuits to be investigated; in a well designed amplifier only the input circuit and first tube contribute significantly to the noise. Experiments carried out on an amplifier with a gain of 170 db are described.

621.396.645.029.3   1334
An Amplifier with Very High Fidelity—N. Mikhnewitch and M. Alixant. (*Radio Tech. Dig.* (France), vol. 4, no. 6, pp. 337–350; 1950.) Complete circuit details of the French version of Williamson's design (see 335 of 1950 and back references).

621.396.645.029.42   1335
Amplifiers for Slowly Varying Very Low Voltages—H. Doizelet. (*Radio Tech. Dig.* (France), vol. 4, no. 6, pp. 353–358; 1950.) Circuits and brief description of two amplifiers for 12 and 24 cs input, based on the design of Aiken and Welz (381 of 1948).

621.396.645.211   1336
Resistance-Coupled Amplifier Bandwidth—B. A. Lippmann. (*Electronics*, vol. 24, pp. 192, 200; January, 1951.) The gain characteristic of the coupling network of a *RC* amplifier is shown to be equivalent to that of a single-tuned circuit.

621.314.2   1337
Transformers [Book Review]—F. C. Connelly. Publishers: Pitman and Sons, London, Eng., 490 pp., 35s. (*Wireless Eng.*, vol. 28, p. 68; February, 1951.) "Although the major part of it is devoted to the 'mains transformer,' audio-frequency types, both output and intertube, instrument transformers and even television scanning transformers are covered, although rather less thoroughly . . . . An extremely good book which should be of the greatest assistance to everyone concerned with small transformers."

GENERAL PHYSICS

534.01   1338
Parametric Excitation—N. Minorsky. (*Jour. Appl. Phys.*, vol. 22, pp. 49–54; January, 1951.) "This paper outlines a theory of excitation of oscillations under the rather slow variations of a parameter on which a system depends. The basic equation is a Mathieu equation. The basic method consists in reducing the system to polar co-ordinates and applying perturbations."

534.01:621.3.015.3   1339
Transients in Multiply Periodic Nonlinear Systems—F. E. Bothwell. (*Quart. Appl. Math.*, vol. 8, pp. 247–254; October, 1950.) Analysis of transient oscillations in the general dynamical system with *n* degrees of freedom.

535.3:539.23   1340
Research on the Propagation of Sinusoidal Electromagnetic Waves in Stratified Media: Application to Thin Layers: Part 2—F. Abelès. (*Ann. Phys.* (Paris), vol. 5, pp. 706–782; November to December, 1950.) Detailed study of the characteristics of single and multiple homogeneous thin layers on a transparent or absorbent support. New formulas relative to an absorbent layer above a transparent medium are derived for the case of total reflection. Phase displacement in an absorbent layer is considered, and the necessary conditions for phase constancy with varying amplitude are derived. Optical methods of examination of transparent layers are described and experimental results quoted. The method of calculation outlined in part 1 (857 of May), is applied to determine the transmission coefficient for a system of alternate thin layers with different refractive indexes n_1, n_2.

535.37:621.32   1341
Electroluminescence—A New Light Source—E. C. Payne, E. L. Mager, and C. W. Jerome. (*Sylvania Technologist*, vol. 4, pp. 2–5;

January, 1951.) Theory is given of the production of light by the direct transfer of energy from a fluctuating electric field to a suitable phosphor embedded in the dielectric material of a capacitor. The intensity of illumination of such a source increases rapidly with voltage at any given frequency, and with frequency at any given voltage. The relation between light output and power consumption is linear over a range of frequencies. Possible applications as low-level, large-area light sources are mentioned.

537.311.1   1342
The Electron and Electrical Conduction in Solids—J. Malsch. (*Elektron Wiss. Tech.*, vol. 4, pp. 348–355; October to November, 1950.) Nonmathematical treatment of the theory of the electron gas and the modern theory of conduction. The latter is examined by considering, (a) interaction between lattice ions and electrons; (b) wave motion of electrons. Examples are given of the energy-level representation of atomic structure, and its application in explaining the characteristic properties of conductors, semiconductors, and insulators are discussed.

537.311.33   1343
Graphical Determination of the Fermi Level in a Simple Impurity Semiconductor—K. Lehovec and H. Kedesdy. (*Jour. Appl. Phys.*, vol. 22, pp. 65–67; January, 1951.) The position of the Fermi level is determined for the case of zero space charge, and the resulting graph is applied to determine the effective electron mass in SiC. Measurements by Busch and Labhart (1820 of 1947) indicate that the effective masses of electrons and holes in SiC crystals are of the order of mass of a free electron.

537.523/.527].4   1344
Recent Research on Spark Discharges in Gases—J. D. Craggs and J. M. Meek. (*Research* (London), vol. 4, pp. 4–10; January, 1951.) A review dealing with breakdown in gaps subjected to unidirectional or hf voltages.

537.525   1345
On the Theory of Double Layers in Low-Pressure Gas Discharges—T. Wasserrab. (*Z. Phys.*, vol. 128, pp. 575–585; December 7, 1950.)

537.533/.534:538.122   1346
Consequences of the Radiation from Very Fast Particles in a Magnetic Field—B. Kwal. (*Jour. Phys. Radium*, vol. 11, pp. 685–690; December, 1950.) The emission of photons by charged particles moving in a magnetic field is discussed on the basis of work done by Arzimovich and Pomeranchuk. The formulas derived are applied to simple cases of motion of electrons, mesons, and protons in the magnetic fields of the earth and of the sun.

537.533.8   1347
A Pulse Method of Determining the Energy Distribution of Secondary Electrons from Insulators—K. G. McKay. (*Jour. Appl. Phys.*, vol. 22, pp. 89–94; January, 1951.) Description of a method based on analysis of the transient resulting from pulse bombardment. The analysis is simplest when leakage through the target is negligible, but the effect of such leakage is considered. Space-charge limitation of the emission is assumed to be negligible.

538.122:537.525.8   1348
Improvements in Visual Depiction of Magnetic Lines of Force by Means of a Gas Discharge—F. Blaha and J. A. Schedling. (*Jour. Appl. Phys.*, vol. 22, pp. 11–13; January, 1951.) Description of a method basically similar to that described by Lutz and Tetenbaum (2375 of 1949). The use of perforated screens of dielectric material covering the pole-pieces enables the starting points of the lines of force displayed to be predetermined.

538.311:621.318.423:513.647.1   **1349**

The Electromagnetic Field Produced by a Helix—R. S. Phillips. (*Quart. Appl. Math.*, vol. 8, pp. 229–246; October, 1950.) It is shown that the phase velocity of a monochromatic field is the same as that which would be obtained if the wave travelled along the helix with the free-space velocity. The radiation of energy from the helix is discussed.

538.311:621.318.423:513.647.1   **1350**

Calculation of the Field of a [travelling-wave] Helix—É. Roubine. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 221–222; January 15, 1951.) The method previously used (see 3036 of 1947) to calculate the E_z component of the electric field at a point on the axis of an endless helix traversed by a progressive current wave Ie^{-jkz} is applied to determine the components of the $\vec{E}$ and $\vec{H}$ fields, at any point of the axis, in a finite form involving only known transcendental functions.

538.56:530.12   **1351**

On the Relativistic Electromagneto-Ionic Theory of Wave Propagation—V. A. Bailey. (*Phys. Rev.*, vol. 77, pp. 418–419; February 1, 1950.) The equation of dispersion is modified to allow for the occurrence of electron velocities comparable with that of light. The omission of the relativistic terms from the equation previously given (see 3406 of 1949) is in general unimportant, but leads to erroneous conclusions in certain special cases.

538.56:535.13   **1352**

The Reflection of an Electromagnetic Plane Wave by an Infinite Set of Plates: Part 3—A. E. Heins. (*Quart. Appl. Math.*, vol. 8, pp. 281–291; October, 1950.) Complete results are presented for the cases in which (a) the magnetic vector is parallel to the edges of the plates, (b) there is one transmitted mode and two reflected waves. More complicated cases are discussed very briefly. Parts 1 and 2: 2756 (Carlson and Heins) and 3504 (Heins and Carlson) of 1947.

538.561:621.315.612   **1353**

Free Oscillations of Dielectric Rings—H. Wigge. (*Arch. elekt. Übertragung*, vol. 4, pp. 455–461; November, 1950.) Theoretical analysis of the quasi-stationary condition in an oscillating ring of high-permittivity material. Maxwell's equations are expressed in toroidal co-ordinates and integrated; from the limiting conditions the free oscillations can be calculated. The distribution of the electric field over the cross section of the ring is determined. The results obtained indicate that the ring may be regarded as a ring-shaped series arrangement of an infinite number of elementary capacitors possessing a self-inductance which can be calculated from the formulas given. The combination of two rings, one of dielectric and the other of magnetic material, constituting a simple electrical oscillatory system, is briefly considered. Experimental results for rings made from Condensa C (permittivity 76 to 80) are noted. Oscillations in the wavelength range 50 to 100 cm were obtained.

538.561.029.65†:535.42   **1354**

Evanescent Microwaves Generated by Diffraction—M. Schaffner and G. Toraldo di Francia. (*Nuovo Cim.*, vol. 6, pp. 125–130; March 12, 1949.) Experiments demonstrating the existence of these waves are described, the wavelength being 32 mm. A diffraction grating made of metal strips was used. One of the first-order waves was transformed into an ordinary plane wave by passing it through a paraffin prism. On plotting the logarithm of the wave power against the distance between prism and grating, a straight line is obtained, whose slope gives the attenuation constant of the surface wave, as predicted by theory.

538.569.4:061.3   **1355**

International Congress on Spectroscopy at Radio Frequencies, Amsterdam, September 1950—M. Soutif. (*Jour. Phys. Radium*, vol. 11, pp. 90S–92S; December, 1950.) A brief report in which the subjects discussed are grouped under five headings: absorption spectra at centimeter wavelengths, the method of atomic jets, electronic and nuclear paramagnetic resonances, and ferromagnetic resonance. A practical application of resonance absorption mentioned is an "ammonia clock" constructed by H. R. L. Lamont in the GEC laboratories, Wembley. This operates as follows: a fm oscillator produces two pulses, one as it passes through the resonance frequency corresponding to a spectral line of ammonia, the other when it produces a given beat with the fixed oscillator to be stabilized; the time difference between the two pulses controls the system regulating the fixed oscillator. See also 1983 of 1949.

538.569.4:532   **1356**

On the Absorption of U.H.F. Radio Waves in Organic Liquids at Different Temperatures —S. N. Sen. (*Indian Jour. Phys.*, vol. 23, pp. 495–502; November, 1949.) Absorption maxima were found in the frequency range 250 to 500 mc for acetone, ether, and methyl ethyl ketone at temperatures above $0°$ C. The maxima shift to lower frequencies at lower temperatures.

538.632:621.315.592†   **1357**

The Hall Coefficient of Semiconductors— H. Jones. (*Phys. Rev.*, vol. 81, p. 149; January 1, 1951.) An inconsistency is noted in the theory of Johnson and Lark-Horowitz (2491 of 1950) which takes into account both thermal and impurity scattering of the electrons. The mobilities quoted by these authors (662 of April) should, as a consequence, be reduced. The variation in Hall coefficient is shown as a function of the ratio of the resistance due to impurity scattering to total resistance.

GEOPHYSICAL AND EXTRATERRESTRIAL PHENOMENA

523.72:621.396.822   **1358**

Study of Low-Intensity Solar Radio Storms—É. J. Blum, J. F. Denisse, and J. L. Steinberg. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 387–389; January 29, 1951.) A report of observations (at frequencies near 164 mc) obtained with recording equipment of small time constant, allowing rapid fluctuations of solar rf radiation to be followed easily. On days of low activity, intensity jumps occur, generally well separated from one another and with durations of 0.1 to 0.4 second. When the activity is greater (variability 1 on the URSI scale), the radiation level may fluctuate for many hours, returning from time to time to the level for calm periods. The results indicate that this type of disturbance is due to the superposition of a large number of the intensity jumps observed on calmer days. See also 627 of April (Blum and Denisse).

523.72:621.396.822   **1359**

The Interpretation of Solar-Noise Jumps— É. J. Blum, J. F. Denisse, and J. L. Steinberg. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 483–485; February 5, 1951.) The jumps in the intensity level of solar rf radiation previously reported (1358 above) are only observed on relatively low frequencies, below about 500 mc. The energy is confined to a narrow frequency band, at most a few mc wide. The radiation is almost completely circularly polarized. The jumps last, on the average, about 0.2 sec, and the power radiated during a jump is of the order of 5×10^{-22} wm² per cs at a frequency of 164 mc. Possible solar phenomena which could account for the existence of such jumps are considered. The plasma theory of Bohm and Gross (88 and 89 of 1950) can provide a basis for a satisfactory explanation of the observed effects. A mechanism is proposed which can also explain the radio storms which often accompany chromospheric eruptions.

523.72:621.396.822   **1360**

A Mechanism of Coronal R.F. Emission— Y. Rocard. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 598–600; February, 1951.) A mechanism is described which could possibly explain the origin of the jumps in the level of solar rf radiation recently described by Blum, Denisse, and Steinberg (1358 and 1359 above).

523.746"1949.10/.12"   **1361**

Provisional Sunspot-Numbers for October to December, 1949—M. Waldmeier. (*Jour. Geophys. Res.*, vol. 55, p. 93; March, 1950.)

523.75:537.591   **1362**

On the Influence of Solar Activity on the Intensity of Cosmic Rays—I. L. Chakraborty and S. D. Chatterjee. (*Indian Jour. Phys.*, vol. 23, pp. 525–534; December, 1949.) One theory of the origin of cosmic rays assumes that they are generated on or near the sun and are kept near the solar system by the action of magnetic fields. The validity of this theory is examined in the light of measurements of cosmic-ray intensity and magnetic activity made during the week following the solar flare on January 25, 1949, at Calcutta. An initial increase of cosmic-ray intensity unexpected for the low geomagnetic latitude of 12° was observed. These and other results mentioned support the view that the paths by which the rays can arrive at the earth depend on the combination of magnetic fields prevailing on the sun and the earth.

523.854/.855:621.396.822]+523.5   **1363**

The New Science of Radio Astronomy— A. C. B. Lovell. (*Nature* (London), vol. 167, pp. 94–97; January 20, 1951.) Radio "telescopes" have resulted in important advances in astronomical knowledge. The following aspects of these advances are briefly discussed, with references: (a) the measurement of meteor velocities and the conclusion that the interstellar origin of meteors is unlikely; (b) the discovery of day-time meteor streams; (c) the possible existence of stars which can only be detected on radio frequencies; (d) structure of the galaxy; (e) radio emissions from extragalactic nebulae; (f) the effect of the ionosphere on extraterrestrial radiation.

523.854:621.396.822]+537.591   **1364**

Origin of the Radio-Frequency Emission and Cosmic Radiation in the Milky Way— A. Unsöld. (*Z. Astrophys.*, vol. 26, pp. 176–199; December 14, 1949.) A survey with 72 references. See also 2217 of 1949.

523.854:621.396.822   **1365**

Origin of the Fluctuations in the Intensity of Radio Waves from Galactic Sources— F. G. Smith. (*Nature* (London), vol. 165, pp. 422–423; March 18, 1950.) A report of observations by Cambridge workers on a wavelength of 6.7 m, using receivers with spacings up to 170 km. The two types of fluctuation observed are described, one involving variations of intensity both above and below the mean value, the other having the appearance of individual "bursts" of large amplitude and of duration 10 to 20 seconds. It seems likely that two separate mechanisms are responsible for the observed fluctuations; one appears to be related to variations of the emission from the sources, the brief duration of the bursts suggesting sources of stellar dimensions, while the other appears to be due to diffraction in a comparatively local region.

523.854:621.396.822   **1366**

Origin of the Fluctuations in the Intensity of Radio Waves from Galactic Sources—C. G. Little and A. C. B. Lovell. (*Nature* (London), vol. 165, pp. 423–424; March 18, 1950.) An account of observations at Jodrell Bank, Cheshire, in collaboration with F. G. Smith at Cambridge, 210 km away. Simultaneous observations were made of the radiation from the sources in Cygnus and Cassiopeia on a wave-

length of 3.7 m. The results show that the radiation is generally either steady at both sites or fluctuating at both sites on a given night. There are about 10 per cent significant exceptions when fluctuations were observed at one site and not at the other. When fluctuations occurred at both sites, there was no correlation between the disturbances. Further simultaneous observations at sites near Jodrell Bank show that there is complete correlation between the observed fluctuations for a site separation of 100 m. For a separation of 3.9 km the correlation is not complete but remains high, with a correlation factor ranging from 0.5 to 0.95. This indicates that the origin of the fluctuations must be fairly local, and probably lies in the earth's atmosphere or ionosphere rather than in the interstellar medium. Considerations of the Fresnel zone theory indicate that the reversal of the phase of an appreciable fraction of a zone, due to localized changes in the refractive index of the ionosphere, could cause fluctuations of the type observed. If such a mechanism is responsible for the fluctuations, the effect may be expected to increase in prominence with increasing angle of incidence and to show some correlation with abnormal ionospheric effects.

523.854:621.396.822   **1367**
Microwave Sky Noise—A. E. Covington. (*Jour. Geophys. Res.*, vol. 55, pp. 33–37; March, 1950.) Observations made during the period April, 1946 to June, 1947 and reported in 721 of 1948 are examined further for correlation between microwave-noise storms on the one hand and magnetic disturbances and auroral displays on the other. The correlation is concluded to be not so close as implied in the earlier communication. The absence of correlation with auroral displays is probably due to the phenomena occurring outside the acceptance cone of the fixed antenna used for the observations.

537.533/.534:538.122   **1368**
Consequences of the Radiation from Very Fast Particles in a Magnetic Field—Kwal. (*See* 1346). (*Jour. Phys. Radium*, vol. 11, pp. 685–690; December, 1950.) The emission of photons by charged particles moving in a magnetic field is discussed on the basis of work done by Arzimovich and Pomeranchuk. The formulas derived are applied to simple cases of motion of electrons, mesons, and protons in the magnetic fields of the earth and of the sun.

550.38:551.52   **1369**
On the Relation between Variations of the Earth's Magnetic Field and Variations of the Large-Scale Atmospheric Circulation—O. R. Wulf and M. W. Hodge. (*Jour. Geophys. Res.*, vol. 55, pp. 1–20; March, 1950.) Comparison of geomagnetic data with observations of circulation in the lower atmosphere confirms the view that large-scale movements in the electrically conducting region of the atmosphere affect the value of the earth's magnetic field.

550.38"1949.07/.09"   **1370**
International Data on Magnetic Disturbances, Third Quarter, 1949—J. Bartels and J. Veldkamp. (*Jour. Geophys. Res.*, vol. 55, pp. 91–92; March, 1950.)

550.38"1949.10/.12"   **1371**
Cheltenham Three-Hour-Range Indices K for October to December, 1949—R. R. Bodle. (*Jour. Geophys. Res.*, vol. 55, p. 93; March, 1950.)

550.384   **1372**
'Sudden Commencements' in Geomagnetic Field Variations—S. K. Chakrabarty. (*Nature* (London), vol. 167, p. 31; January 6, 1951.) The records of Alibag observatory over the period 1905 to 1944 have been analyzed. The results provide data additional to those recently given by Newton (2235 of 1948) and

by Ferraro and Parkinson (1142 of 1950). The first type of sudden commencement, characterized by a rise in H and a fall in V, is very prominent in the Alibag records, but the second type described by Ferraro and Parkinson, in which the increase in H is preceded by a small decrease, is absent. The "inverted" third type occurs relatively infrequently. Curves for the diurnal variation of *sc* incidence indicate minima at 0400 to 0700 and about 1700 local mean time, with a more pronounced maximum at 0900 to 1300. Prominent sudden commencements noted at Alibag but not at Abinger, and vice versa, are tabulated.

550.384.3(68)   **1373**
Secular Variation of the Magnetic Elements in South Africa, 1939–1948—A. M. van Wijk and J. A. N. Burger. (*Jour. Geophys. Res.*, vol. 55, pp. 57–64; March, 1950.)

550.385+551.594.5   **1374**
The Theory of Magnetic Storms and Auroras—D. F. Martyn. (*Nature* (London), vol. 167, pp. 92–94; January 20, 1951.) The Chapman-Ferraro theory postulates a neutral stream of ionized particles of solar origin which envelops the earth during a magnetic storm. Analytical difficulties prevent full use being made of this theory in the quantitative explanation of observed storm phenomena. The formal analogy between familiar hydrodynamical problems and that of a tenuous ionized gas moving in a magnetic field is here applied to the estimation of the dimensions of the hollow formed by the particle stream round the earth, the width and latitude of the auroral zones, the currents flowing in these zones, the formation and stability of the equatorial ring current, and F_2-region disturbances.

550.385   **1375**
Principal Magnetic Storms [April–Dec. 1949]—(*Jour. Geophys. Res.*, vol. 55, pp. 94–95; March, 1950.

551.510.4   **1376**
Further Determinations of the Vertical Distribution of Ozone—E. Regener, H. K. Paetzold, and G. Pfotzer. (*Naturwiss.*, vol. 37, pp. 559–560; December, 1950.)

551.510.535   **1377**
Statistics of the Night-Time Abnormal E Layer—E. A. Lauter. (*Z. Met.*, vol. 4, pp. 234–240; July and August, 1950.) Statistical presentation of results of night observations on a wavelength of 1,250 m and of the occurrence of magnetic disturbances between November, 1947 and May, 1950 shows a marked correlation between times of strong absorption and cosmic bursts. Conditions specially favorable for electrically charged corpuscular radiation evidently exist 3 to 4 days after a magnetic disturbance, at which time the night-time ionization below the normal E layer is observed to reach a maximum.

551.510.535   **1378**
Observations of the Ionosphere in the Arctic Region—O. Burkard. (*Arch. Met. Geophys. Bioklimatol.*, vol. 1, pp. 93–99; June 30, 1948.) Observations on the F layer obtained at Tromso during the period June, 1944 to April, 1945 are reported and discussed. The region is traversed by many radiocommunication lines. During the summer f_oF_2 was almost constant; during the winter it varied with the sun's height. Magnetic disturbances cause a reduction of f_oF_2; when they are very powerful, the reflected wave from the F layer disappears altogether. This is only partly due to the screening effect of the lower absorbing layers; the other factor is a reduction of ionization which is particularly marked during morning hours after magnetic disturbances. Even on magnetically undisturbed days, communication lines using the F layer can be relied on only during the daytime. Reference is made to the appearance of the sporadic F layer (see also 390 of 1949).

551.510.535   **1379**
The Approximate World Distribution of F_2-Layer Ionization—K. Rawer. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 98–100; January 3, 1951.) The ionization distribution may be represented sufficiently closely by the results of observations in two regions, an eastern and a western [see 2794 of 1949 (Oboril and Rawer)], a linear interpolation being made for the two intermediate regions. The eastern zone corresponds roughly with the land mass of Europe, Africa, and Asia, and the western zone with America. The apparent influence on the F_2-layer ionization of the disposition of the continents is attributed to a probable relation between the latter and the magnetic inclination.

551.510.535:522.1(481)   **1380**
The Ionospheric and Radio Wave Propagation Observatory at Kiruna, 67° 50′ N, 20° 14.5′ E—O. E. H. Rydbeck. (*Tellus*, vol. 1, pp. 61–64; November, 1949.) A short account of the lay out and equipment of the station.

551.510.535:523.75:621.396.11   **1381**
Ionospheric Effects of Solar Flares—R. Lindquist. (*Acta Polyt., Stockholm, Elec. Eng. Series*, vol. 2, no. 9, 1950. *Chalmers Tekn. Högsk. Handl.*, no. 95, 11 pp., 1950. In English.) Presentation of fade-out observations recorded during the period July, 1948 to June, 1949 at Gothenburg. See also 619 of 1950 (Rydbeck and Stranz).

551.510.535:621.3.087.4   **1382**
The Panoramic Ionospheric Recorder—Lindquist. (*See* 1425.)

551.510.535:621.396.11   **1383**
Longitudinal and Transverse Propagation in Canada—Scott. (*See* 1471.)

551.510.535:621.396.11.029.51:535.312   **1384**
Ionospheric Reflection of Very Long Radio Waves—Stanley. (*See* 1474.)

551.578.1:621.396.9   **1385**
The Scattering of Ten-Centimetre Radio Waves by Rain—R. C. Langille. (*Jour. Geophys. Res.*, vol. 55, pp. 51–52; March, 1950.) Rain distribution in showers has been determined from observations of reflected 10-cm radio waves, using modified height-finding radar equipment.

551.593.9   **1386**
Study of the Emission Spectrum of the Night Sky from 6800 to 9000 Å—J. Dufay and M. Dufay. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 426–428; January 29, 1951.) All the spectrum lines observed in this region belong to the vibration-rotation bands of OH and to the (0, 1) band of O_2. In agreement with Meinel's measurements, the rotation temperature is found to be 255 to 270°K for OH and about 130°K for O_2. The intensity of the bands probably has a seasonal variation.

551.594.6   **1387**
Variation of Intensity of Distant Atmospherics with Frequency Channels—S. R. Khastgir and A. Sen. (*Indian Jour. Phys.*, vol. 23, pp. 483–494; November, 1949.) A cathode-ray tube direction finder was used to investigate the intensity E of distant atmospherics in the frequency (f) range 170 to 204 kc. During the day and occasionally at night, $E \propto 1/f^3$. Generally at night $E \propto e^{-mf}$, where m is a constant. Sometimes, during the day or night, the variation of E suggested a combination of the inverse and exponential types. The inverse type apparently occurs when the source of atmospherics is not very distant and the pulse travels by the ground path. The exponential type is associated only with sky waves.

LOCATION AND AIDS TO NAVIGATION

621.396.9   **1388**
Sunderland Shore-Based Radar Station—

Electronic Eng. (London), vol. 23, p. 29; January, 1951.) A short account of the facilities available, which include Kelvin Hughes Type-1A, Series-2 Marine Radar, with ranges of 1 to 5, 10, 15, and 25 miles, a 2-frequency simplex AM system for communication with pilots on frequencies of 163.1 and 158.6 mc, and single-frequency simplex equipment for communication on the international marine frequency of 156.8 mc. See also *Wireless World*, vol. 57, p. 34; January, 1951.

621.396.9   **1389**
Angular Jitter in Conventional Conical-Scanning Automatic-Tracking Radar Systems—C. E. Brockner. (PROC. I.R.E., vol. 39, pp. 51–55; January, 1951.) Four sources of jitter are discussed and each is expressed in terms of range and the system parameters. The existence of a range interval of optimum tracking is pointed out, and the importance of beamwidth in determining the magnitude of angular jitter is stressed.

621.396.9:519.2   **1390**
The Statistical Properties of Noise Applied to Radar Range Performance—S. M. Kaplan and R. W. McFall. (PROC. I.R.E., vol. 39, pp. 56–60; January, 1951.) The relations between detection probability, the clipping level, the pulses per element, the false-alarm time, and other factors are presented both in equation form and graphically. From the graphs the detection range for any desired detection probability and also the probability of false target echoes can be calculated. The influence of various system parameters on the radar performance is also indicated.

621.396.932.1   **1391**
New Developments in Radar for Merchant Marine Service—C. E. Moore. (*RCA Rev.*, vol. 11, pp. 465–481; December, 1950.) The general features of 3-cm radar equipment specifically designed for small vessels are described. Technical factors in design and details of the complete installation are illustrated with diagrams and photographs. General trends in commercial marine radar are considered.

621.396.933.2   **1392**
Distance-Measuring Equipment for Civil Aircraft—D. G. Lindsay, J. P. Blom and J. D. Gilchrist. (*Proc. IRE* (Australia), vol. 11, pp. 307–315; December, 1950.) A description of the development of a ground-beacon interrogator-responder system. Requirements and circuitry are outlined. Beacon reply pulses are delayed by 12.4 μs and actuate a locking system in the aircraft, which requires 3 to 5 successive correct pulses to operate. Range is displayed by meter.

621.396.933.2   **1393**
Hyperbolic [aerial navigation] Systems—W. Stanner and H. C. Freiesleben. (*Elektron Wiss. Tech.*, vol. 4, pp. 417–426; December, 1950.) The principles of pulse displacement and phase-displacement measurement are illustrated in a review and description of American and British systems. The location and service area of stations in Europe are indicated.

621.396.9   **1394**
Radar Simply Explained [Book Review]—R. W. Hallows. Publishers: Chapman and Hall, London, Eng. 2nd edn, 190 pp., 10s.6d. (*Elec. Radio Trading*, vol. 23, p. 141; January, 1951.) Includes information previously withheld. "It has been translated into six European languages and, it is claimed, is recognized as the best available introduction to radar."

MATERIALS AND SUBSIDIARY TECHNIQUES

533.5   **1395**
An Ionization Pump—R. Champeix. (*Le Vide*, vol. 5, pp. 912–913; November, 1950.) Description of the principle of an apparatus for obtaining very low pressures. A 1-kv discharge takes place between two nickel-mesh electrodes sealed inside a tube connected to a backing pump. The gas molecules are ionized, the ions traveling to the electrodes, where they are subjected to the action of the backing pump and are evacuated. The discharge is interrupted after some thirty seconds and the process repeated. For another account see *Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 40–42; July 3, 1950.

535.37   **1396**
Luminescence Efficiency of Organic Solutions and Crystals—P. D. Johnson and F. E. Williams. (*Phys. Rev.*, vol. 81, p. 146; January 1, 1951.)

537.311.3:539.234   **1397**
The Release of Electrons on Transition from Metal Atoms to the Compact Metal—H. Mayer. (*Elektron Wiss. Tech.*, vol. 4, pp. 341–347; October and November, 1950.) Evaporation processes for producing very thin layers of different metals are outlined; experimental curves showing variation of resistivity with layer thickness are analyzed and discussed with reference to Herzfeld's criterion for the free-electron state, namely, $R > Ms$, where R is the molecular refractive power for the gaseous state, M the molecular weight, and s the layer thickness.

537.311.33:546.289   **1398**
Mobilities of Electrons in High Electric Fields—E. J. Ryder and W. Shockley. (*Phys. Rev.*, vol. 81, pp. 139–140; January 1, 1951.) An investigation of the current-density electric-field characteristic for n-type Ge samples, using a pulse technique. The resistivity is constant for fields up to about 6×10^2 v cm, corresponding to a region of constant mobility. For higher fields, the mobility $\propto F^{-1}$ in a region of constant drift velocity and almost constant current. Hole injection or generation is thought to occur for values of $E > 2 \times 10^4$ v cm. The predicted drift velocity depends on "acoustical" and "optical" scattering, the former predominating at low temperatures.

537.533.8   **1399**
Secondary Emission of Nickel-Barium Mixtures and Rhenium when Bombarded by Electrons with Energies from 50 to 8,000 Electron-Volts—H. E. Farnsworth and M. J. Lun. (*Jour. Appl. Phys.*, vol. 22, pp. 77–79; January, 1951.) For a cast alloy of Ni containing 1.5 per cent Ba, the secondary-emission ratio (δ) has a maximum value of 2.8 for electrons of 800 to 900 ev. δ_{max}, depends on the previous heat treatment; this is illustrated graphically. For Rh, δ_{max}, is 1.3 at 900 ev and the most probable energy of low-speed secondary electrons is 5 ev.

537.58:621.385.032.213   **1400**
Certain Refractory Compounds as Thermionic Emitters—D. L. Goldwater and R. E. Haddad. (*Jour. Appl. Phys.*, vol. 22, pp. 70–73; January, 1951.) Several carbides, nitrides, and borides of Zr, Th, Ti, and Ta were tested after aging in vacuo. The results are shown graphically and discussed. Only ZrC has practical possibilities. Its emissivity is less than 40 per cent of that expected from thoria, but there is no decay of emission when high-density emission currents are continuously drawn. Two ZrC cathodes were operated for several hours at 1,500°C, with a continuous space-charge-limited emission current of 0.7A cm².

538.221   **1401**
Magnetic Properties of Zinc Ferrite ($Fe_2O_3 \cdot ZnO$) in relation to Its Structure—C. Guillaud and M. Sage. (*Jour. Phys. Radium*, vol. 11, p. 4E; December, 1950.)

538.221:621.3.011.5   **1402**
Relation between Dielectric Constant and Loss Angle of Ferroelectric Materials—M. Kornetzki. (*Z. angew. Phys.*, vol. 2, pp. 446–448; November, 10, 1950.) Measurements show that the Rayleigh relation for ferromagnetic materials is equally applicable to ferroelectric materials.

538.632+537.311]:546.87:539.23   **1403**
Hall Coefficient and Resistivity of Evaporated Bismuth Layers—W. F. Leverton and A. J. Dekker. (*Phys. Rev.*, vol. 81, pp. 156–157; January 1, 1951.)

546.431.82:537.226.2   **1404**
The Dielectric Constant of Barium Titanate—M. Kornetzki. (*Z. Phys.*, vol. 128, pp. 605–613; December 7, 1950.) A formula for calculating the permeability of ferromagnetic materials, based on displacements of the Bloch wall, is used to estimate the dielectric constant of polycrystalline $BaTiO_3$ at room temperature.

621.315.61   **1405**
Research Devlopments in Dielectrics—A. E. Javitz. (*Elec. Mfg.*, vol. 45, pp. 80–85, 184; January, 1950.) Report of the 1949 Annual Conference on Electrical Insulation. Prepared papers were grouped under the headings: electrical properties of matter; insulating materials; dielectric and related measurements; insulated wire and cable. At informal sessions the most widely discussed subjects were: service performance and trends in the use of insulation; high-temperature organic and organo-inorganic insulation; ferroelectric materials. The recently developed tetrafluoroethylene suspensoids are expected to find wide application. The production of "bonded" and "integrated" mica was described. Semiconductor synthesis and the rectifying properties of metal, semiconductor interfaces were also discussed.

621.315.61   **1406**
The Interpretation of Dielectric Measurements Using the Cole-Cole Plot—J. G. Powles. (*Proc. Phys. Soc.* (London), vol. 64, pp. 81–82; January 1, 1951.) Plots of the imaginary part against the real part of the dielectric constant, using frequency and temperature, respectively, as parameters, are of practical (though limited) use for estimating dielectric properties under conditions for which measurements are not available. See also 3022 of 1950 (Stark).

621.315.613.1   **1407**
'Samica' Mica Paper—H. George and L. Metzger. (*Rev. gén. élect.*, vol. 59, pp. 519–524; December, 1950.) A brief account is given of the invention of "mica paper" by the French chemist J. Bardet; thermal and chemical treatments are involved. A method of manufacturing the material in continuous sheet form from mica pulp has been developed, using modified paper-making machinery. The product has a density about half that of natural mica; the thickness is from some thousandths to some hundredths of a millimeter, and is very uniform. The pulp can be dried, ground, and moulded into required shapes, but the sheet form is of most importance in the electrical industry and can be used as an economical substitute for mica splittings in insulation. Results of measurements of dielectric and mechanical properties are reported and compared with figures for corresponding products made from splittings.

621.316.8:621.396.822   **1408**
Noise in Unidirectional Conductors—W. Kroebel. (*Fernmeldetech. Z.*, vol. 3, pp. 466–470; December, 1950.) An account is given of experiments on semiconductor rectifiers (galena, carborundum and Ge point types, and a "Sirutor" disk type). The values found for dc, hf, and equivalent noise resistances as functions of applied dc voltage are shown in curves. An explanation of the variation of equivalent noise resistance is based on Schottky's theory of variation of barrier-layer thickness with applied dc voltage, and on the

assumption that discharges take place within the barrier layer, on account of the high field strengths present. The difference, in the course of the equivalent-noise-voltage curve for point rectifiers for forward and backward voltages, is due partly to the difference of operative volume in the two cases. The measurement arrangements are described briefly.

621.317.3:538.569.4.029.64+621.317.335.3†
1409

Microwave Measurements of the Dielectric Properties of Gases—Birnbaum, Kryder, and Lyons. (*See* 1426.)

666.3.056:621.791.3
1410

Metal/Ceramic Sealing with Manganese— H. J. Nolte and R. F. Spurck. (*TV Eng.*, vol. 1, pp. 14–18, 49; November, 1950.) The use of a metallizing mixture of 20 per cent mn and 80 per cent mo for forming a bonding layer on ceramic materials enables reproducible vacuum-tight ceramic metal seals to be made efficiently. Details are given of techniques used in various industrial applications of the process.

669.27:621.42:548.53
1411

On the Recrystallization and Grain Growth in Tungsten Wire—H. A. DeVincentis and J. H. Dedrick. (*Sylvania Technologist*, vol. 4, pp. 6–8; January, 1951.) A brief survey is made of the important factors affecting recrystallization and grain growth in tungsten, with emphasis on the manufacturing problems in the tube and incandescent-lamp industries. Methods of controlling these two phenomena are discussed.

669.27:621—42:621.385.032.213
1412

Preliminary Treatment of Tungsten Wire for Electronic Valves—G. Mesnard and R. Uzan. (*Le Vide*, vol. 5, pp. 896–904; November, 1950.) Discussion of the preparation of tube filament wire of diameter <1 mm. Microphotographs reproduced indicate that, for decarbonizing, the soda treatment is generally the most effective; the concentration and temperature of the solution should be as high as possible. Other methods considered are heat treatment in a hydrogen atmosphere, and electrolysis. Details are given of an electrolytic method of polishing and of pyrometric determination of the operating temperature of the wire.

533.5
1413

Le Vide et ses Applications [Book Review] —L. Dunoyer. Publishers: Presses Universitaire de France, Paris, France, 1950, 114 pp., 90 fr. (*Proc. Phys. Soc.* (London), vol. 64, p. 93; January 1, 1951.) "A book particularly suitable for the physics student, or for the layman with some mathematical knowledge and an intelligent interest in science."

621.315.211:614.841
1414

A Comparison of the Flame-Retardant Properties of Plastics-Insulated and Rubber-Insulated Electric Cable [Book Review]— Publishers: British Plastics Federation, 47–48 Picadilly, London, W. 1, Eng., 1950, 2s.6d. (*Electrician*, vol. 145, p. 1603; December 29, 1950.) The comparison is made for the two cases of heating due to an external source and heating due to overload current, respectively. The plastics used were polythene and pvc.

MATHEMATICS

517.942.72:621.396.11
1415

Associated Legendre Polynomial Approximations—R. Landauer. (*Jour. Appl. Phys.*, vol. 22, pp. 87–89; January, 1951.) "Approximations for the associated Legendre Polynomials are derived by a phase-integral method. The method is an extension of the WBK method, applicable to separable multidimensional wave propagation problems."

517.942.9:519.21
1416

Random Walks and the Eigenvalues of

Elliptic Difference Equations—W. Wasow. (*Bur. Stand. Jour. Res.*, vol. 46, pp. 65–73; January, 1951.) Discussion of the application of random-walk procedure to the solution of differential equations of the form $\Delta^2 u + g(x, y) \cdot u = 0$. It is shown that under suitable conditions the Monte Carlo solution converges and gives the solution of the difference analogue of the above equation.

517.942.932:621.396.611.1
1417

Forced Oscillations in Nonlinear Systems— M. L. Cartwright. (*Bur. Stand. Jour. Res.*, vol. 45, pp. 514–518; December, 1950.) Shows how the approximate form of the solutions of a certain nonlinear differential equation may be obtained from certain general results which are proved in detail. The method could be applied to any equation of the form

$$\ddot{x} + k f(x)\dot{x} + g(x) = k p(t)$$

where the period of $p(t)$ is $2\pi/\lambda$, and $\int_0^t p(t)dt$ is bounded for all values of t, $f(x) \geqq 1$ for $|x| \geqq a$, and $g(x)/x \geqq 1$ for $|x| \geqq a$. See also 2740 of 1948.

517.948
1418

An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators—C. Lanczos. (*Bur. Stand. Jour. Res.*, vol. 45, pp. 255–282; October, 1950.) A systematic method is described for finding the latent roots and the principal axes of a matrix without reducing its order. The method has wide applicability and great accuracy, since the accumulation of rounding errors is avoided by use of a method of "minimized iterations." The method leads to a rapidly convergent analytical iteration procedure by which the solution of integral equations of the Fredholm type and of the eigenvalue problems of linear differential and integral operators can be obtained.

681.142
1419

An Analogue Multiplier—B. O. Marshall, Jr. (*Nature* (London), vol. 167, pp. 29–30; January 6, 1951.) Short description of a method based on the relation $xy = [(x+y)^2 - (x-y)^2]/4$. The equipment required is relatively simple and gives results to an accuracy well within 1 per cent.

681.142
1420

Automatic Computing Equipment at the N.P.L.—(*Engineering* (London), vol. 171, pp. 6–8; January 5, 1951.) A general description, with illustrations, of the pilot model electronic computer known as ACE. It employs the binary system for the coding of data and instructions, the information being fed into standard Hollerith, punched-card equipment. Calculating circuits are not described, but an account is given of the data-storage system, employing acoustic delay lines to circulate pulses continuously until required. Results are produced as punched cards or printed numerals. The equipment can deal with 900 million binary digits in one quarter of an hour. Programing and coding are relatively slow, but when these procedures have been carried out, extended computations may thereafter be performed with great rapidity. Equipment failure is usually readily recognizable, and servicing is facilitated by assembly in replaceable units.

681.142:511.2.004.4
1421

Digital Information Storage in Three Dimensions Using Magnetic Cores—J. W. Forrester. (*Jour. Appl. Phys.*, vol. 22, pp. 44–48; January, 1951.) An operating mode is suggested which depends on the ability of the magnetic-material to discriminate between two magnetizing forces having a ratio of 2:1. Existing metallic-magnetic-materials have switching times of 20 to 10,000 μs and are too slow, but nonmetallic-magnetic materials of the ferrite type, with switching times <1 μs, are promising. See also 1422 below.

681.142:511.2.004.4
1422

Static Magnetic Memory for Low-Cost Computers—M. Kincaid, J. M. Alden, and R. B. Hanna. (*Electronics*, vol. 24, pp. 108–111; January, 1951.) A new principle in storage systems is described, employing a magnetic material, Deltamax. The hysteresis loop of this material is flat at top and bottom, giving two stable states suitable for the storage of information in binary form. A method is described by which binary digits may be fed serially into a set of Deltamax solenoid cores. No power is required during storage, nor is reshaping of pulses necessary. The method is also useful for transferring pulses between systems operating at different speeds, for use in pulse-counting systems, and for interconversion of parallel and serial sets of pulses.

518.3
1423

The Nomogram [Book Review]—H. J. Allcock and J. R. Jones. Publishers: Pitman and Sons, London, Eng., 4th edn 1950, 238 pp., 18s. (*Wireless Eng.*, vol. 28, p. 68; February, 1951.) "Little change has been made from the third edition published in 1941, except for the addition of a new chapter showing the connection between intersection and alignment nomograms . . . . Strongly recommended to engineers, designers, and research workers."

MEASUREMENTS AND TEST GEAR

389.2(94):621.396.91
1424

Time Signals for Scientific and Industrial Use—(*Proc. IRE* (Australia), vol. 11, p. 316; December, 1950.) Commonwealth Observatory time signals, broadcast by Belconnen Naval Wireless Station, are now available by day and by night. Mean Time signals, of the American type, are emitted and measured during the Australian daytime, at 0025 to 0030 UT. They consist of a dot for every second of Mean Time. Modified rhythmic type time signals are emitted during the Australian night, at 1025 to 1030, 1455 to 1500, and 1855 to 1900 UT. They give equally spaced intervals of 60 to 61 parts of one second of Mean Time and are comparable in accuracy with the Mean Time signal except that they are usually unattended during the transmission. Estimates of errors are given.

621.3.087.4:551.510.535
1425

The Panoramic Ionospheric Recorder— R. Lindquist. (*Tellus*, vol. 1, pp. 37–43; November, 1949.) Description of a recorder covering the frequency range 1 to 20 mc in 30 seconds with an output pulse power varying from 16 kw on the lowest frequency to 6 kw on the highest frequency. The results are presented on the screen of a 12-inch cathode ray tube and recorded on 16-mm film. The short sweep time is particularly suitable for investigations in the polar regions, where ionospheric conditions often change rapidly. Only a single variable capacitor is used in the frequency sweep. Automatic tracking between receiver and transmitter is secured by means of a heterodyne system. Details are given of the different units of the equipment, with sample records obtained at Kiruna since the beginning of observations in July, 1948.

621.317.3:538.569.4.029.64+621.317.335.3†
1426

Microwave Measurements of the Dielectric Properties of Gases—G. Birnbaum, S. J. Kryder, and H. Lyons. (*Jour. Appl. Phys.*, vol. 22, pp. 95–102; January, 1951.) The resonance curves of two identical cavities were observed simultaneously on a cro. The change in resonance frequency caused by the admission of a gas into one of the resonators was measured directly by superposing frequency markers on the resonance curves. The rms error in measuring $\epsilon' - 1$ for a lossless gas is estimated as 0.4 per cent. The loss factor was measured by observing either the bandwidth of the cavity or the change in peak amplitude.

Results obtained on O_2, N_2, CO_2, Ar, He, air, and NH_3 at 9 kmc are given.

621.317.329 1427

Plotting Electric Fields with the Aid of Semiconducting Films—W. Claussnitzer and H. Heumann. (*Z. angew. Phys.*, vol. 2, pp. 443–446; November 10, 1950.) Discussion of a method using semiconducting sheets of graphite-impregnated paper on which copper electrodes are deposited by spraying through a suitable mask. Equipotential points are marked directly with a probe. Experimental results for a parallel-plate capacitor with plates of different sizes are in good agreement with theory.

621.317.329:548.311 1428

A Method for the Mapping of Vector-Potential Distributions in Axially Symmetrical Systems—G. Liebmann. (*Phil. Mag.*, vol. 41, pp. 1143–1151; November, 1950.) Measurements in two electrolyte tanks, one with inclined bottom, can be used for the experimental determination of the magnetic field in a rotationally symmetrical system containing iron. From the basic equations of the system, the form of the resistance networks required to set up the appropriate fields is derived. The networks are then used to determine the vector-potential distribution. The correctness of the method has been confirmed by experiments with a linear resistance strip. The method should have applications in the design of betatron and synchrotron magnets and in problems of machine design. See also 1988 of 1949 (Peierls and Skyrme).

621.317.335.3.029.6.073.4† 1429

Principles of a Null Method for the Determination of Dielectric Constants at U.H.F.—M. Bruma. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 42–44; January 3, 1951.) Theory of the method depending on the change of the resonance frequency of a cavity resonator when the test sample is introduced, the initial resonance frequency being restored by deformation of the resonator wall.

621.317.335.3.029.6 1430

Equipment for the Determination of Dielectric Constants at U.H.F. by a Null Method—M. Bruma. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 219–221; January 15, 1951.) The general formula previously given (1429 above) can be simplified considerably in certain cases. A formula giving results accurate to within 1 per cent is derived for the case in which the cavity is cylindrical, vibrating in the E_{010} mode, and the sample of dielectric, also cylindrical, is introduced along the axis. The initial resonance frequency before insertion of the dielectric is restored by displacement of a small metal cylinder projecting through the cavity wall either axially or in a direction normal to the axis.

621.317.42 1431

A Simple Means of Measuring Large Magnetic Fields—G. K. T. Conn and B. Donovan. (*Jour. Sci. Instr.*, vol. 28, pp. 7–9; January, 1951.) Difficulties in applying the magnetoresistance effect in Bi to the measurement of magnetic fields have been overcome by using thin fibers of Bi drawn down in soda glass. The anisotropy of the Bi fibers limits the accuracy obtainable, but by etching the fibers and winding in tight coils the anisotropy is largely eliminated.

621.317.7.001.4:621.385.3:546.289 1432

Transistor Measurement Technique—Mataré. (*See* 1538.)

621.317.733 1433

A Method of Decreasing the Effect of Ear Admittances in A. C. Bridges—C. G. Mayo and G. H. Rayner. (*Jour. Sci. Instr.*, vol. 28, p. 24; January, 1951.) Comment on 1973 of 1950 (Rayner and Willmer) and author's reply.

621.317.755.029.5/.6 1434

An Experimental "Stroboscopic" Oscilloscope for Frequencies up to about 50 Mc/s Part 1—Fundamentals—J. M. L. Janssen. (*Philips Tech. Rev.*, vol. 12, pp. 52–59; August, 1950.) Describes the principle of stroboscopic examination of a hf voltage by mixing with phase-modulated pulses whose central repetition frequency is a submultiple of the frequency of the voltage to be examined. The pulses of anode current of the mixer tube produce "snapshots" of the examined voltage; the low-frequency components of the anode current are filtered out and used for vertical trace deflection in a cathode ray tube. The phase modulation scans the voltage over in adjustable region, and a stationary picture with linear time scale is obtained by using the same time function for the horizontal deflection as for the phase modulation, preferably a sinusoidal function at mains frequency. The effects of the gaps between pulses and of the finite pulse duration are considered.

621.317.755.029.5.6 1435

An Experimental "Stroboscopic" Oscilloscope for Frequencies up to about 50 Mc/s: Part 2—Electrical Build-Up—J. M. L. Janssen and A. J. Michels. (*Philips Tech. Rev.*, vol. 12, pp. 73–82; September, 1950.) Part 1: 1434 above. See also 949 of May 1 (Janssen).

621.317.755:621.385.012 1436

Electronic Tracing of Tube Characteristics—J. Arnold. (*Sylvania Technologist*, vol. 4, pp. 14–17; January, 1951.) Description, with block diagrams, of equipment for displaying in a few seconds on a cro complete families of tube characteristics.

621.396.615.001.4:621.396.611.21 1437

Progress in Development of Test Oscillators for Crystal Units—L. F. Koerner. (Proc. I.R.E., vol. 39, pp. 16–26; January, 1951.) Early test oscillators were principally duplicates of the actual equipment in which the crystals were to be used, but it is now recognized that a knowledge of the equivalent electrical elements of the crystal unit is essential. Modern test oscillators, with frequency and capacitance measuring apparatus as auxiliary equipment, which are used to determine the magnitudes of these elements and also the frequency characteristics are described. The transmission measuring circuit is proposed as a standard reference circuit for comparison with the test oscillator.

OTHER APPLICATIONS OF RADIO AND ELECTRONICS

531.787.6:621.38 1438

Diaphragm-Type Micromanometer for Use on a Mass Spectrometer—V. H. Dibeler and F. Cordero. (*Bur. Stand. Jour. Res.*, vol. 46, pp. 1–4; January, 1951.) A detailed description, with full circuit details, of the instrument noted in 952 of May.

531.787.6:621.38 1439

An Electronic Circuit for Measuring the Displacement of Pressure-Sensitive Diaphragms—M. L. Greenough and W. E. Williams. (*Bur. Stand. Jour. Res.*, vol. 46, pp. 5–10; January, 1951.) Description of apparatus depending on the change in the mutual inductance of a pair of coils due to a displacement of the diaphragm near which the coils are mounted. See also 1438 above.

534.321.9:534.232 1440

Ultrasonic Energy—Crawford. (*See* 1291.)

534.321.9:620.179.16 1441

Supersonic Examination of Boiler Plate and Welded Seams—(*Engineering*, (London), vol. 171, pp. 29–30; January 5, 1951.) Description of technique under investigation for routine testing in the works of Babcock and Wilcox, Renfrew. Ultrasonic pulses of frequency 2.25

mc are sent through the plate under test and flaws are revealed by the existence of echo pips on the cathode ray tube trace, in addition to that corresponding to the echo from the back face. During the development stage of the technique, radiographic tests have also been carried out. The relative merits of the two methods are discussed briefly.

538.71 1442

Airborne Equipment for Geomagnetic Measurements—L. H. Rumbaugh and L. R. Alldredge. (*Trans. Amer. Geophys. Union*, vol. 30, pp. 836–848; December, 1949.) The equipment described in an AN ASQ-3A magnetometer modified to permit mapping the contours of the total magnetic field over wide areas. The main features of the instrument system are: (a) a self-oriented magnetometer using the output of a system of saturable inductors to provide a dc signal whose noise level and drift rate are low; (b) instruments for helping the pilot to follow predetermined lines of a grid covering the area to be explored; (c) an interlocked group of recorders for automatically correlating a continuous magnetometer record with airplane position and altitude. The system has been used extensively by The United States Government services. A shorter account was noted in 697 of 1950 (Schonstedt and Irons).

621.317.073.7:629.13.00.141 1443

A Multichannel P.A.M.-F.M. Radio Telemetering System—J. P. Chisholm, E. F. Buckley and G. W. Farnell. (Proc. I.R.E., vol. 39, pp. 36–43; January, 1951.) A 1950 IRE National Convention paper. The system, designed primarily for flight testing of pilotless aircraft, can provide up to 64 channels with a bandwidth per channel >800 c, or fewer channels with a correspondingly increased bandwidth. Frequency modulation of the transmitter is effected by a reactance switching technique and an overall linearity to within ± 1 per cent is obtained.

621.38 + 534.321.9].001.8 1444

Electronic Guiding Aids for Blind People—R. L. Beurle. (*Electronic Eng.* (London), vol. 23, pp. 2–7; January, 1951.) Desirable characteristics of guiding aids are discussed and various experimental aids are described, all of which provide aural indication. The instruments fall into three classes: (a) operating by light pulses, distances of obstacles being derived by simple scanning of the transmitted beam, the receiver "looking" in a fixed direction; (b) operating by ultrasonic waves of frequency 17 to 150 kc; binaural reception for direction indication was tried; (c) operating by sound waves, an audible click being transmitted and the unaided ears used to detect the echoes. The results of the tests carried out indicate that no device which can be envisaged at present is likely to become popular with blind people.

621.38.001.8:786.6 1445

Gas-Diode Electronic Organ—R. M. Strassner. (*Electronics*, vol. 24, pp. 70–74; January, 1951.) Details are given of the construction of an organ which uses Type-NE2 neon tubes in chains of sawtooth-wave generators, whose frequencies are stabilized by twelve separately tuned master oscillators. Good tone quality is obtained by filtering out undesired harmonics.

621.394.6† 1446

The Importance of Gas Scattering in Particle Accelerators—L. Riddiford. (*Proc. Phys. Soc.* (London), vo. 64, pp. 10–12; January 1, 1951.)

621.384.6† 1447

Questions of Direction and Phase in Recently Developed Particle Accelerators—R. Kollath. (*Elektron Wiss. Tech.*, vol. 4, pp. 383–392; October and November, 1950.)

621.384.611.1/.2† 1448

A Note on Resonance Damping, at Injection, in Betatrons and Synchrotrons—S. E. Barden. (*Proc. Phys. Soc.* (London), vol. 64, pp. 85–86; January 1, 1951.)

621.384.612.1/.2† 1449

Cyclotron and Synchrocyclotron—W. de Groot. (*Philips Tech. Rev.*, vol. 12, pp. 65–72; September, 1950.) A general account, introductory to a series of articles on the Amsterdam synchrocyclotron.

621.385.73:621.386.1 1450

An Electrostatic Focusing System and Its Application to a Fine-Focus X-Ray Tube—W. Ehrenberg and W. E. Spear. (*Proc. Phys. Soc.* (London), vol. 64, pp. 67–75; January 1, 1951.)

621.385.833 1451

Schlieren Optics in the Electron Microscope—H. König. (*Naturwiss.*, vol. 37, pp. 486–490; November, 1950.) An account of techniques particularly suitable for investigating crystal structure and magnetic-field patterns, including the methods developed by Marton and Lachenbruch (199, 967, and 1211 of 1950).

621.385.833 1452

Spider-Thread Grids for Electron Microscopy—H. Dessens and C. Fert. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 606–608; February 12, 1951.) A description is given of the technique used in the preparation of a spider-thread grid supported on a small capsule with a central hole of diameter 0.5–1.0 mm for the passage of the electron beam. The diameter of the threads is sometimes <100 Å; the use of these grids facilitates the detection and examination of very small particles.

621.385.833 1453

A Condenser Aperture Device for Electron Microscopes—J. D. Boadway. (*Jour. Appl. Phys.*, vol. 22, pp. 104–105; January, 1951.) Description of a device for reducing spherical aberration without causing electron bombardment of large areas of the specimen under examination.

621.385.833 1454

Calculation of Optical Parameters of Magnetic Electron Lenses from the Dimensions of the Pole-Pieces and the Operating Characteristics—F. Lenz. (*Z. angew. Phys.*, vol. 2, pp. 448–453; November 10, 1950.) See also 705 of April.

621.385.833 1455

Theory of the Independent Electrostatic [electron] Lens with Thick Central Electrode—É. Regenstreif. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 604–606; February 12, 1951.) Extension of theory given previously (1213 and 1743 of 1950).

621.385.833 1456

Potential and Field of a Particular Type of Cylindrical [electron] Lens—E. Durand. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 314–316; January 22, 1951.) A series of conformal transformations is used to calculate the potential and field of an electrostatic lens formed by two parallel half planes located between two infinite planes parallel to the first two but maintained at a different potential.

621.385.833 1457

Permanent-Magnet Electron Miscroscope—J. H. Reisner and S. M. Zollers. (*Electronics*, vol. 24, pp. 86–91; January, 1951.) A simple and reliable instrument in which the magnetic fields required for focusing the electron beam are provided by permanent magnets of Alnico v. Changeable pole pieces enable magnifications of 1,500, 3,000 or 6,000 to be used. A resolution of 100 Å is obtained using an accelerating voltage of 50 kv, stabilized to one part in

10⁴. The construction of the microscope is described in detail, with circuit and photographs.

621.385.833 1458

The Philips 100-kV Electron Microscope—A. C. van Dorsten, H. Nieuwdorp, and A. Verhoeff. (*Philips Tech. Rev.*, vol. 12, pp. 33–51; August, 1950.) Describes in detail the construction and operation. The tube (20-cm screen) is mounted at an angle on a desk containing all the auxiliary equipment. Five water-cooled magnetic lenses, supplied with maximum currents of 100 ma in some cases and 400 ma in others, include a diffraction and an intermediate lens providing magnification variable from 1,000 to 60,000 times and quick change from a normal image to a diffraction diagram. The object plane lies in the gap between the pole pieces, thus permitting simple adjustment of the specimen. Forty photographs on 35 mm film are obtainable; a wobbling beam facilitates focusing. Accelerating voltage is 40, 60, 80, or 100 kv and is derived from a stabilized tube oscillator. Various attachments simplify the operation of the instrument.

621.385.833:[537.212+538.122 1459

Experimental Determination of Fields by Marton's Electronic Method—F. Bertein. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 491–493; February 5, 1951.) An explanation of Marton's method (199 and 967 of 1950) is given, based on the convergence properties of low-power thin lenses.

621.387.4† 1460

Oversize Pulses in Vapour-Filled Counter Tubes—E. Fünfer and H. Neuert. (*Z. Phys.*, vol. 128, pp. 530–537; November 16, 1950.) See also 3121 of 1950.

621.387.4† 1461

The Properties of Spark Counters of the Rosenblum Type—R. D. Connor. (*Proc. Phys. Soc.* (London), vol. 64, pp. 30–44; January 1, 1951.)

621.387.4†:621.3.089.6 1462

Current Integration Circuits for Counter Tubes—K. H. Lauterjung. (*Z. angew. Phys.*, vol. 2, pp. 433–437; November 10, 1950.) Discussion of calibration methods for the thyratron- and the multivibrator-driven integrator.

621.398 1463

A New Inductance Method for Remote Indication of Tank Level—J. M. Marzolf. (*Instruments*, vol. 23, pp. 562–563; June, 1950.) Apparatus developed at the United States Naval Research Laboratory is described. The tank unit comprises a set of 48 coils wound in series on a vertically mounted plastics tube, the squares of the numbers of turns on successive coils forming an arithmetic progression. The inductance of the system varies linearly with the position of a cylindrical steel float which moves inside the tube. The measuring circuit is described and illustrated. Application is particularly for use on board ship.

PROPAGATION OF WAVES

538.566:535.51 1464

A Note on Ionospheric Radio Wave Polarization—A. L. Alden, J. T. deBettencourt, and A. T. Waterman, Jr. (*Jour. Geophys. Res.*, vol. 55, pp. 53–56; March, 1950.) Early papers on the magneto-ionic theory are cited as giving the impression that the major axis of the polarization ellipse for the ordinary ray is perpendicular to that for the extraordinary ray; this is not necessarily the case unless the effect of collisional friction is neglected. Analysis is presented to clarify the point.

538.566.1 1465

Theory of the Propagation of Electromagnetic Waves in an Atmospheric Duct—T.

Kahan and G. Eckart. (*Ann. Phys.* (Paris), vol. 5, pp. 641–705; November and December, 1950.) Complete analysis, with mathematical theory in two appendixes (pp. 656 to 705). The principal results have been published previously (see 1466 and 1467 of 1949). The region above the duct is also considered. Here, for large angles of elevation, the field varies as $1/R$, where R is the distance from the dipole source, and tends linearly towards zero as $z \rightarrow h$, h being the height of the duct. In the region immediately above the dividing plane the field decreases exponentially with height. These two phenomena account for the existence of zones of silence.

538.566.21:551.52 1466

Some Meteorological Aspects of Radio-Duct Formation—E. Knighting. (*Proc. Phys. Soc.* (London), vol. 64, pp. 21–30; January 1, 1951.) Three problems of meteorological interest concerning the formation of radio ducts are treated. The first concerns the rate of growth of ducts, with special reference to ducts over the sea; ducts form quickly at first and then more slowly as the maximum duct width is approached. The second problem concerns the formation of ducts under conditions of nocturnal cooling; there is a limit to the height which the duct may attain. The third problem concerns the width of a radio duct necessary to enclose the track width of a mode associated with a propagated signal of wavelength λ, assuming a power law of modified refractive index with height; the necessary width is proportional to $\lambda^{2/3}$, and the factor of proportionality, which depends upon the index in the assumed power law, is calculated.

The three problems have a common meteorological basis, for each is concerned with turbulent motion near the earth's surface. The law of turbulent diffusion assumed here is that the coefficient of eddy diffusion is proportional to a power of the height, following the theory developed by O. G. Sutton and others.

621.396.11 1467

Asymmetry in Wave Propagation: Study of Nonreciprocity observed on the Two-Way Paris/Algiers Link—P. Niguet. (*Onde élect.*, vol. 30, pp. 533–541; December, 1950.) Curves of the received field strength throughout the day at the two ends of the link, for nearly equal operating wavelengths (12.105 mc in the southward and 12.12 mc in the northward direction), show marked differences, which vary in magnitude with the season, being large in September and decreasing with the approach of winter. The discrepancy is investigated theoretically; formulas are developed for determining the paths of the refracted and reflected beams in an ionosphere of complex structure, taking into account that the ionization gradient has a horizontal as well as a vertical component. In the vertical plane the lines of equal ionization are deformed parabolas on a circular axis, the arms extending southward and approaching asymptotically the limit circles of the ionized region; this is shown to cause the asymmetry in propagation.

621.396.11:517.942.72 1468

Associated Legendre Polynomial Approximations—Landauer. (*See* 1415.)

621.396.11:523.75:551.510:535 1469

Ionospheric Effects of Solar Flares—R. Lindquist. (*Acta Polyt., Stockholm, Elec. Eng. Series*, vol. 2, No. 9; 1950. *Chalmers Tekn. Högsk. Handl.*, No. 95, 11 pp.; 1950. In English.) Presentation of fade-out observations recorded during the period July, 1948, to June, 1949, at Gothenburg. See also 619 of 1950 (Rydbeck and Stranz).

621.396.11:551.510:535 1470

Ionospheric Storms and Radio-Circuit Disturbances—C. M. Minnis. (*Wireless Eng.* vol. 28, pp. 43–51; February, 1951.) "The im-

portance of distinguishing between forecasts of radio-circuit disturbances and ionospheric or magnetic storms is emphasized. Varying amounts of advance warning are given by different precursors of storms, and the consequent logical division of storm warnings into the long- and short-range and immediate-warning categories is explained. The more important storm precursors are discussed, but it is concluded that none of them alone can be used as a reliable basis for making forecasts. The problem of forecasting reduces to a statistical one which cannot immediately be solved because the necessary data do not exist. Empirical methods must therefore be used; these would become more reliable if more were known of the physical processes in the sun responsible for the emission of storm-causing radiation and of the subsequent development of the storm effects both in time and over the earth's surface."

621.396.11:551.510:535   **1471**
Longitudinal and Transverse Propagation in Canada—J. C. W. Scott. (*Jour. Geophys. Res.*, vol. 55, pp. 65–84; March, 1950.) The Canadian ionosphere is complicated by the presence of the geomagnetic pole and the auroral zone. The apparent terrestrial magnetic field in the ionosphere, as calculated from measured critical-frequency differences for the extraordinary and ordinary modes, shows large diurnal and seasonal variations. The longitudinal mode, which is often observed in high magnetic latitudes, gives a field consistently different from that of the transverse mode. These results are explained as due to deflection of the energy path from the vertical in conjunction with variations of the horizontal ionization gradients. The direction of the complex Poynting vector is calculated for the different modes. The westward component of the deflection of the longitudinal ordinary mode is shown to be as large as the north-south deflection.

621.396.11:621.317.353.3†   **1472**
Some Further Investigations of Ionospheric Cross-Modulation—I. J. Shaw. (*Proc. Phys. Soc.* (London), vol. 64, pp. 1–20; January 1, 1951.) Measurements of the phase and percentage depth of the modulation transferred to an unmodulated-wanted wave by a modulated disturbing wave, together with simultaneous measurements of the height of reflection of the wanted wave, lead to a value of 1.4×10^6 per second for the collision frequency of electrons with gas molecules at a height of 92 km. Calculations of the percentage depth of transferred modulation to be expected with given pairs of transmitters, using formulas derived theoretically, are in most cases in good agreement with experimental values, both for vertical and for oblique incidence. No increase in cross modulation is observed when the disturbing transmitter operates on the local gyromagnetic frequency. At sunrise the percentage depth of transferred modulation is observed to decrease in all cases. These facts are discussed with reference to a modification of the theory previously given [3219 of 1948 (Ratcliffe and Shaw)].

621.396.11.029.45/.51:551.594.6   **1473**
The Use of Atmospherics to Study the Propagation of Very Long Radio Waves—F. F. Gardner. (*Phil. Mag.*, vol. 41, pp. 1259–1269; December, 1950.) The mean level, (integrated over one minute) of atmospherics received on narrow-band receivers tuned to frequencies between 3.5 and 50 kc, was recorded at Cambridge during 1948 and 1949. The results are analyzed so as to eliminate the unknown spectral distribution of energy at the source. The attenuation suffered by radio waves from a distant source is deduced; this is large for frequencies below 10 kc and is greater during daylight than at night for all frequencies. The quasi-sinusoidal waveform often assumed by

atmospherics is related to the frequency cutoff at 10 kc.
The level of atmospherics on frequencies below 10 kc decreases during sudden ionospheric disturbances. This contrasts with the known increase on frequencies above about 20 kc.

621.396.11.029.51:535.312:551.510.535   **1474**
Ionospheric Reflection of Very Long Radio Waves—J. P. Stanley. (*Canad. Jour. Res.*, vol. 28, Sec. A, pp. 549–557; November, 1950.) The simplified model of the long-wave-reflecting region of the ionosphere previously considered (974 of May) is used to calculate the theoretical variation of the sky-wave reflection coefficient with angle of incidence and with the angle of dip of the earth's magnetic field. The curves obtained agree with experimental results. The assumption of a vertical magnetic field should not lead to errors in estimates of the vertical-incidence sky-wave reflection coefficient greater than about 10 per cent, even though the field is actually inclined to the vertical at an angle of as much as 23°.

621.396.11.029.62   **1475**
Effective Earth's Radius for Radiowave Propagation Beyond the Horizon—William Miller. (*Jour. Appl. Phys.*, vol. 22, pp. 55–62; January, 1951.) The ideas of geometrical optics, on which calculations of the equivalent radius of the earth for radio-wave propagation are based, are extended to nonhorizontal rays. The appropriate wave equation for a nonuniform, but spherically symmetric, region is derived and a solution obtained in terms of a Green's function. The formal solution includes the known solutions for a uniform medium. For the nonuniform case, the solutions of the radial equation are found by a Langer technique. The functions involved account for the variation of refractive index near the surface of the earth without making unwarranted assumptions about the variation at greater heights. The formal series obtained is summed by the Watson technique. The first term in this series (lowest mode) alone determines the field at large distances and indicates that standard atmospheric refraction may be accounted for by assuming an effective radius for the earth.

621.396.81:621.3.018.41(083.74)   **1476**
¡ Reception of WWV Standard-Frequency Transmissions—W. Ebert. (*Tech. Mitt. schweiz. Telegr.-Teleph. Verw.*, vol. 28, pp. 457–482; December 1, 1950. In German and French.) Observations were made at Châtonnaye, Switzerland, of the reception of WWV standard-frequency transmissions over the period February 15, 1946 to July 8, 1949, for 2.5, 5, 10, and 15 mc, and December 22, 1946, to July 8, 1949, for 20, 25, 30, and 35 mc. The receiving site and equipment are described. Signal strengths were measured at every even hour on all the frequencies; results are presented in charts. Short-wave propagation conditions are discussed with reference to the structure of the ionosphere. Maximum usable and optimum working frequencies are determined for distances >4,000 km, and observed and predicted values are compared.

621.396.812.5   **1477**
Investigation of Phenomena connected with Mögel-Dellinger Effects—F. Schindelhauer and E. A. Lauter. (*Z. Met.*, vol. 4, pp. 243–245; July and August, 1950.) Analysis of observations of sudden short-wave fadeouts and associated geomagnetic disturbances, particularly those of June 28, and September 13, 1949, indicates that, (a) in most cases long-wave reflection occurs only a few minutes after the onset of short-wave fading; (b) some of the effects observed [see 716 of 1950 (Lauter)] are not connected with short-wave fadeouts, but are rather traceable to reflection of long waves

and slight increases in the limiting frequency for the *D* layer.

RECEPTION

621.396.11.029.45/.51:551.594.6   **1478**
The Use of Atmospherics to Study the Propagation of Very Long Radio Waves—Gardner. (*See* 1473.)

621.396.621:621.396.619.13   **1479**
Detector Circuits for Frequency-Modulation Receivers: Part 2—C. J. Boers. (*Fernmeldetech. Z.*, vol. 3, pp. 458–465; December, 1950.) Particular types of discriminator circuit are described. Part 1: 2880 of 1950.

621.396.621:621.396.931   **1480**
Adjacent-Channel-Rejection Receiver—H. Magnuski. (*Electronics*, vol. 24, pp. 100–104; January, 1951.) Flat-topped selectivity curves for adjacent-channel rejection may be obtained by the use of a passive bandpass filter between a low-gain input and mixer unit and a nonselective high-gain amplifier. Advantages include high stability of gain and selectivity for a wide range of input signal amplitudes, freedom from intermodulation and image interference, and good signal noise ratio.

621.396.621.54   **1481**
Gain-Doubling Frequency Converters—V. H. Aske. (*Electronics*, vol. 24, pp. 92–96; January, 1951.) Approximately twice the normal conversion transconductance can be obtained from a pentode mixer by applying the signal to the control grid and the local oscillation to the suppressor, with the tuned IF circuit between the anode and screen grid. Performance figures verifying this are given for push-pull and single-tube mixer circuits. The method also gives more effective rejection of IF signals and a smaller equivalent noise resistance. The principle can be applied in a converter tube combining the functions of mixer and local oscillator. The design of such a stage for the broadcast band is described and its performance characteristics are compared with those of a conventional circuit.

621.396.621.54   **1482**
Comments on Interelectrode Feedback in Frequency Changers: Part 1—B. G. Dammers and J. Otte. (*Philips Tech. Commun.* (Australia), no. 5, pp. 13–19; 1950.) Discussion of the general formulas for frequency changers and also of the influence of anode-grid capacitance of the modulator section on amplification and selectivity.

621.396.82:621.315.1   **1483**
A New Locator of Sources of R.F. Interference Affecting Overhead Electrical Lines—J. Meyer de Stadelhofen. (*Tech. Mitt. schweiz. Telegr.-Teleph. Verw.*, vol. 28, pp. 482–485; December 1, 1950. In French.) A locating device which indicates the direction of propagation of interfering energy is based on determination of the sign of the vector expression $|\vec{U}+\vec{I}| - |\vec{U}-\vec{I}|$, where $\vec{U}$ is the interfering hf voltage between earth and the group of conductors considered, and $\vec{I}$ is the current produced in a section of the line by $\vec{U}$. Voltages proportional to $\vec{U}$ and $\vec{I}$ are obtained by means of a CR potential divider and a loop pickup, respectively. The arrangement can be used also as a reflection-coefficient indicator. A laboratory prototype is described; this has given excellent service over several months under varied conditions.

STATIONS AND COMMUNICATION SYSTEMS

621.39.001.11   **1484**
Communication Theory and Physics—D. Gabor. (*Phil. Mag.*, vol. 41, pp. 1161–1187; November, 1950.) Full paper. Abridged version abstracted in 208 of February.

621.394/.395:621.396.619.13   1485
Frequency Modulation for Line Transmission—G. Hässler. (*Fernmeldetech. Z.*, vol. 3, pp. 445–454; December, 1950.) Two important applications are: (a) telephony via hv lines; (b) ac telegraphy. Interference and distortion in these systems are investigated quantitatively. The advantages obtainable from the use of fm within the restricted frequency band available are discussed.

621.395.813:621.39.001.11   1486
Application [to signal and information theory] of Means Used for Assessing the Quality of Telephone Transmission—P. Chavasse. (*Ann. Télécommun.*, vol. 5, pp. 427–440; December, 1950.) Paper included in the symposium on Signal and Information Theory held in Paris, April to May, 1950. The transmission of ideas is discussed to some extent subjectively. On the basis of established phonetic theory, it is possible to predict the quality of a transmission from a knowledge of the characteristics of the telecommunication system used and of the properties of the human voice and ear. The CCIF is experimenting with methods for designating by a single number the effect of a complex of factors on the ease with which a telephone conversation can be carried on. The intelligibility of different languages is considered.

621.396.619.16:519.283   1487
An Application of Auto Correlation Analysis—E. R. Kretzmer. (*Jour. Math. Phys.*, vol. 29, pp. 179–190; October, 1950.) Analysis of the spectrum of the interference produced in pulse-time modulation systems (e.g., pulse-duration and pulse-position modulation) by random disturbances. The relation between the auto-correlation function and power-densit spectrum is established and applied to a pulse sequence in which one or both edges of each pulse undergo random time shifts, or in which pulses are randomly missing. Besides giving the power and spectral distribution of the noise, the analysis also throws some light on the nature of random time modulation. For example, it shows that the time-shift noise is generated at the expense of the hf components of the pulse-train spectrum and permits exact calculation of this exchange of power. The auto-correlation method used should prove very useful in the increasing applications of statistical theory to communications.

621.396.619.16:621.3.018.078†   1488
Quantization Distortion in Pulse-Count Modulation with Nonuniform Spacing of Levels—P. F. Panter and W. Dite. (PROC. I.R.E., vol. 39, pp. 44–48; January, 1951.) A 1949 IRE National Convention paper. The distortion introduced in a pulse-count-modulation system due to quantization can be minimized by nonuniform spacing of levels. Equations are derived for an arrangement of levels resulting in minimum distortion. When the crest factor of the signal is >4, minimum distortion is significantly less than distortion resulting from uniform quantization.

621.396.65:621.396.619.13:621.3.018.78†   1489
Distortion in Multichannel Frequency-Modulation Relay Systems—L. E. Thompson. (*RCA Rev.*, vol. 11, pp. 453–464; December, 1950.) Design formulas are given relating intermodulation to the harmonic distortions produced by the modulator and discriminator and by phase nonlinearity in tuned circuits, with experimental confirmation and an example of their application.

621.396.712.006   1490
Optimum Spacing of Broadcast Transmitters—D. C. Espley. (*Wireless Eng.*, vol. 28, pp. 37–39; February, 1951.) "It is known that the interference range of a radio transmitter extends, to some extent, beyond the limits of the effective service area. This note gives the unique solution of the geometrical problem in

which it is desired to know the number of different stations necessary to cover an unlimited territory for a given ratio of service range to interference range. The shape and extent of a finite territory have some effect on this number."

621.396.97:621.396.66   1491
The Broad Principles in the Design of Automatic Monitors—H. B. Rantzen, F. A. Peachey, and C. Gunn-Russell. (*Electronic Eng.* (London), vol. 23, pp. 19–27; January, 1951.) The purpose of an automatic monitor is to call attention to a transmission imperfection of such magnitude that a normal listener would be disturbed by it, while neglecting minor defects which are normally corrected by routine maintenance tests. Two types of equipment are described. The simpler type, the Minor, is used on program links, where both the reference program and the program to be monitored are available at one point. The Major type is designed for use on long-distance circuits where the reference and monitored programs are only available at opposite ends of the system. The operation of the two types is described with the aid of block diagrams. The installation of these monitors has already resulted in a considerable economy in skilled personnel; they have particular advantages for unattended stations. Some experiments on the application of automatic monitoring to recording processes are briefly described.

SUBSIDIARY APPARATUS

621.316.722.078.3   1492
A Low-Power Alternating-Voltage Stabilizer—D. J. R. Martin and A. J. Maddock. (*Jour. Sci. Instr.*, vol. 28, pp. 1–3; January, 1951.) "A unit is described providing an ac output at 15 v (pre-set within the range 12 to 18 v) and possible current drain of 2 ma; its main use would be as a source of stable reference voltage, for the efficiency of the system is inherently low and the output voltage is sensitive to load variations. A change of alternating input voltage between 190 and 270 v (i.e. 230 ± 17 per cent) produces ± 0.015 v (± 0.1 per cent) change in output voltage. Temperature compensation is provided and the frequency coefficient and waveform distortion are both small."

621.355.2 + 621.355.8   1493
Accumulators for Very Low Temperatures—H. Mandel. (*Elektron Wiss. Tech.*, vol. 4, pp. 412–416; December, 1950.) German version of 2904 of 1950.

TELEVISION AND PHOTOTELEGRAPHY

621.397.335   1494
Improved Vertical Synchronizing System—R. C. Moses. (*Electronics*, vol. 24, pp. 114–118; January, 1951.) The conventional integration method for segregation of frame synchronizing pulses does not achieve accurately-timed triggering of the line-deflection oscillator. A new method described uses a *RC* differentiating network. The relatively long frame-synchronizing pulses undershoot the pulse baseline after differentiation. Subsequent rectification and amplification provide a train of sharply rising pulses which can be used for frame synchronization at a rigidly controlled instant.

621.397.5 + 778.5   1495
Some Comparative Factors of Picture Resolution in Television and Film Industries—H. J. Schlafly. (PROC. I.R.E., vol. 39, pp. 6–10; January, 1951.) Discussion of the meaning given to the term "resolution" in the television and film industries. Limiting values of resolution, however defined, should not be regarded as the sole measure of picture quality.

621.397.5:621.396.619.16   1496
Television by Pulse Code Modulation—W. M. Goodall. (*Bell Sys. Tech. Jour.*, vol.

30, pp. 33–49; January, 1951.) A 1949 IRE National Convention paper. In a PCM system, the information signal is periodically sampled and its instantaneous amplitude described by a group of pulses according to a pre-set code. These pulse groups occur at the sampling rate and constitute the transmitted signal. In this process an operation known as amplitude quantization is required. Time sampling, amplitude quantization, binary coding, and decoding of a television signal, and the operation of the equipment used to perform these functions, are described. The results obtained with an experimental system for different numbers of digits (i.e., maximum number of pulses per group) from one to five are illustrated by photographs. The television signal used in these tests was obtained from a special low-noise film scanner. As was expected, the number of digits required depends upon the amount of noise in the test signal.

621.397.5:621.396.68(083.74)   1497
Standards on Television: Methods of Measurement of Electronically Regulated Power Supplies, 1950—(PROC. I.R.E., vol. 39, pp. 29–35; January, 1951.) Reprints of this Standard, 50 IRE 23.S3, may be obtained, while available, from IRE at seventy-five cents per copy.

621.397.611.2   1498
The Formation of the Signal in Pickup Tubes. The Conductron—W. Veith. (*Le Vide*, vol. 5, pp. 887–895; November, 1950.) In the conductron a magnetically deflected scanning beam is passed through a focusing and retarding electric field on to a semiconducting target of CdS, backed by a thin transparent metal plate which receives the light. The semiconducting layer is photosensitive, its resistance increasing with brightness. Optimum layer thickness and operating conditions are discussed. Sensitivity and storage capacity are compared with those of the superisonoscope, the image orthicon and the vidicon [2040 of 1950 (Weimer et al)].

621.397.611.2:537.533.8   1499
Secondary Emission and Application to the Design of a Television Analyzer—R. Barthélemy. (*Onde élect.*, vol. 30, pp. 499–509; December, 1950.) Methods are described for the measurement of secondary emission from conducting and from insulating targets, and the distribution of secondary-emission velocities is investigated. An analytical expression is derived for the variation with time of the potential of the target in a capacitive signal-plate assembly. The processes by which the signal voltage is produced in a superisonoscope are examined and the operating conditions under which space charge becomes effective are discussed. The reasons for the relatively recent introduction of thin insulating targets are explained; beyond certain limits further reduction of thickness produces no advantage. An expression is derived from which the sensitivity can be stated in ua lumen; sensitivity depends on the mean voltage at which the target is maintained.

621.397.611.21:537.533.8   1500
Measurement of the Secondary Emission from Insulators—Barthélemy. (*See* 1511.)

621.397.621.2   1501
Low-Reflection Picture Tubes—G. S. Szegho, M. E. Amdursky, and W. O. Reed. (*Electronics*, vol. 24, pp. 97–99; January, 1951.) A process is described by which unwanted reflections from the face plates of metal-cone cathode ray tubes can be much reduced. The plate surface is first etched, either chemically or by spraying with a liquid in which abrasive material is suspended. It is then treated with hydrofluoric acid to restore satisfactory transmission characteristics. Results are given of comparative tests of the reflection of treated and untreated surfaces.

621.397.621.2   **1502**

The Optimum Use of Picture Tubes—W. B. Whalley. (*Sylvania Technologist*, vol. 4, pp. 9–13; January, 1951.) The principal circuit factors affecting the performance of cathode ray television tubes are discussed and methods of investigation are outlined which assist in attaining optimum performance.

621.397.8 : 535.371.07   **1503**

Flicker in Television Pictures—J. Haantjes and F. W. de Vrijer. (*Wireless Eng.*, vol. 28, pp. 40–42; February, 1951.) The effects of (a) distance from picture to observer (measured in frame-height units), (b) frame frequency, (c) phosphor decay time, on critical high-light brightness for flicker to be observable, are shown in curves. About ten different observers were used. With normal pictures at 50 frames per sec on a sulphide screen with 0.1-ms decay time, critical brightness was about 12 foot lamberts at 6 frame heights, and 30 at 14, about 5 times higher than with blank raster. With a screen of willemite (decay time 13 ms) with admixture of a short-persistence blue phosphor, these values were improved to about 56 and 136, respectively. At 60 frames per sec on a sulphide screen, critical brightness was about 4 times higher than at 50 frames per sec, and about 5.5 times higher for a silicate screen with decay time 5 ms. Ambient illumination had little influence on the results.

621.397.828   **1504**

Interference Suppression Circuits—W. I. Flack. (*Elec. Radio Trading*, vol. 23, pp. 75–78; January, 1951.) Reviews basic and popular ignition-interference suppression circuits used in the vision and sound channels of television receivers. Sound-channel interference suppression is considered most important. Various circuits used in Murphy receivers are discussed.

621.397.9   **1505**

Industrial Television—V. K. Zworykin. (*Electronic Eng.* (London), vol. 23, pp. 8–11; January, 1951.) A general article indicating the broad scope of television applications within industry. The design of standardized equipment suitable for many applications is outlined. It comprises a highly sensitive and compact camera, connected by 500 feet of cable to a receiver monitor which includes controls and power supplies for the camera. The possibilities of stereoscopic and color television for industrial purposes are briefly mentioned.

TRANSMISSION

621.396.4   **1506**

Simultaneous Feeding of an Aerial from Two Transmitters of 100 and 20 kW [respectively] at the Toulouse-Muret Station—M. Merlet. (*Onde élect.*, vol. 30, pp. 522–527; December, 1950.) The reconstruction of the French transmitting-station network to provide alternative programs has been proceeding since 1944, and in view of the high cost of antenna installations (e.g. 25 million francs in 1950 for a 220-m pylon) a single antenna is being used for two or more transmitters where possible. A trial was made at the Muret station, where a 100 kw transmitter was already operating on 913 kc with a 120-m mast; a new 20-kw transmitter on 1.339 mc was added. The feeder arrangements are described in detail, measurements indicate that they satisfy the conditions, (a) that the impedance presented by the antenna to one of the transmitters at its operating frequency must not vary by more than 2 per cent when the other transmitter is connected, and (b) that the voltage induced by one transmitter in the oscillating circuit of the other must not exceed a thousandth of the carrier voltage of the latter. Difficulties encountered in the actual installation are described. The cabins provided for the matching and filter networks should be spacious to reduce undesired capacitances. An appendix gives a simple graphical method for determining the impedance of two elements connected in parallel.

621.396.4 : 621.396.82   **1507**

Theoretical Study of Cross-Modulation resulting from the Simultaneous Excitation of an Aerial by Two Transmitters—V. Familier. (*Onde élect.*, vol. 30, pp. 528–532; December, 1950.) The cross modulation is calculated by the following steps: (a) derivation of an expression for the composite voltage e_c applied to the class-C power tube of the disturbed transmitter, (b) simulating the tube characteristic by a broken straight line, part of which coincides with the voltage axis, (c) representation of this characteristic by a finite power series of e_c, (d) determination of the coefficients of this series from a set of linear equations. From the general formula thus established, the cross modulation is shown to vary approximately as the square of the ratio of undesired to desired voltage; this is fairly well confirmed by experiment.

TUBES AND THERMIONICS

537.533.8   **1508**

A Pulse Method of Determining the Energy Distribution of Secondary Electrons from Insulators—McKay. (*See* 1347.)

537.533.8   **1509**

Secondary Emission of Nickel-Barium Mixtures and Rhenium when Bombarded by Electrons with Energies from 50 to 8,000 Electron-Volts—Farnsworth and Lun. (*See* 1399.)

537.533.8 : 621.397.611.2   **1510**

Secondary Emission and Application to the Design of a Television Analyser—Barthélemy. (*See* 1499.)

537.533.8 : 621.397.611.21   **1511**

Measurement of the Secondary Emission from Insulators—R. Barthélemy. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 20–22; January 3, 1951.) A method is described which has been used for determining the distribution of initial velocities of secondary electrons emitted by the target of a television pickup tube.

537.58 : 621.385.032.213   **1512**

Certain Refractory Compounds as Thermionic Emitters—Goldwater and Haddad. (*See* 1400.)

621.383.42   **1513**

Characteristics of Selenium Photocells—P. T. Landsberg. (*Proc. Phys. Soc.* (London), vol. 64, pp. 82–83; January 1, 1951.) Preston's (3199 of 1950) experimental reverse-current characteristics for Se with a sputtered layer of *ZnO* are in excellent agreement with those obtained when Landsberg's theory (465 of 1950) is applied to a Schottky barrier. Analysis of Preston's results shows that the mobility of current carriers decreases with increasing thickness of the *ZnO* layer.

621.385 : 537.525.92   **1514**

The Distribution of Space Charge in Thermiomic Valves—M. Matricon and S. Trouvé. (*Onde élect.*, vol. 30, pp. 510–521; December, 1950.) Langmuir's rigorous derivation of formulas expressing cathode current as a function of anode voltage for simple electrode systems is summarized; the current at the cathode surface can be expressed in a form common to parallel-plane, cylindrical, and spherical arrangements by introducing a term λ, which depends on the proportions rather than the absolute dimensions of the electrodes. The formulas derived may be applied to systems with any configuration by adjusting the value of λ. The case of a filamentary cathode, parallel to but not coaxial with the anode, is examined, making the simplifying assumption that the shapes of the equipotentials are the same in the absence and presence of space charge, though the levels differ; simple conformal transformations are used to determine these shapes. Experimental results are reported for this case; they are in satisfactory agreement with the approximate formulas. Other commonly used cathode arrangements are also investigated theoretically.

621.385 : 621.318.572.004.6   **1515**

Electrode Deterioration in Transmit-Receive Tubes—J. C. French. (*Bur. Stand. Jour. Res.*, vol. 45, pp. 310–315; October, 1950.) The glow discharge maintained in a Type-1B24TR tube develops momentary arcs which disintegrate the cathode, so that deposits of cathode material short circuit cathode and anode. This effect was studied with actual tubes and experimental diodes. Recommendations are: to reduce distributed capacitance, to avoid pulsed operation when permissible, to use low cathode-current density, to avoid water vapour or active gas, and to use 18-8 stainless steel in place of kovar as cathode material.

621.385.012 : 621.317.755   **1516**

Electronic Tracing of Tube Characteristics—Arnold. (*See* 1436.)

621.385.012.6   **1517**

Electron Tube Performance with Large Applied Voltages—A. E. S. Mostafa. (PROC. I.R.E., vol. 39, pp. 70–73; January, 1951.) An analytical treatment based on the transconductance characteristic instead of on the anode-current, grid-voltage characteristic. By choosing a certain function and its Fourier expansions over a suitable interval of the characteristic, the analysis of tubes with large applied voltages is simplified. Experimental verification is carried out for a triode.

621.385.029.63/.64   **1518**

Theory of the Travelling-Wave Valve—W. Frey and F. Lüdi. (*Z. angew. Math. Phys.*, vol. 1, pp. 237–247; July 15, 1950. In German.) An equivalent circuit is used to deduce the characteristic impedance of the helix, for purposes of comparison with the klystron and the triode. With the klystron and the traveling-wave tube a shot effect is induced in the amplifier input, but not with the triode, so that increase of their transductance does not result in a reduction of the noise factor as in the case of the triode. For the same beam current the noise factor is about the same for the klystron and the traveling-wave tube, but the latter has the advantage that for the same beam length the gain is greater, as it increases exponentially with the beam length, while in the klystron it increases linearly. It should be possible to improve considerably the signal, noise ratio of the klystron type of amplifier by increasing the beam length, e.g., by magnetic means. A resulting advantage would be the decoupling of input and output; in the case of the traveling-wave tube this leads to oscillation if the helix is not matched so as to avoid reflections.

621.385.029.64/.65   **1519**

Medium-Power Traveling-Wave Tube Type 5929—J. H. Bryant. (*Elect. Commun.* (London), vol. 27, pp. 277–279; December, 1950.) A helix type of tube for operation in the band 4.4 to 5.0 kmc, with a gain of 20 db and power rating of 10 w. Waveguide input and output systems are used.

621.385.029.64   **1520**

A Survey of Modern Radio Valves: Part 6—Valves for use at Frequencies above 3,000 Mc/s—W. J. Bray. (*P.O. Elec. Eng. Jour.*, vol. 43, pp. 148–153 and 187–191; October, 1950 and January 1951.) The principles of operation, methods of construction, and characteristics of uhf low-power oscillator and amplifier tubes are described, including klystron oscillators and amplifiers, traveling-

wave and electron-wave amplifiers, grounded grid triode amplifiers, and cw magnetrons. The suitability of the oscillators for AM, FM, or PM is considered. Part 5: 3201 of 1950 (white).

621.385.029.64/.65   1521
Amplification at 6-Millimeter Wavelength—J. B. Little. (*Bell Lab. Rec.*, vol. 29, pp. 14–17; January, 1951.) Describes a traveling-wave tube amplifier with a gain of about 4 db near 6 mm wavelength, designed by scaling down a 4-kmc tube. Axial spacing rods were eliminated, input and output waveguides made part of the tube envelope, and a special matching method was used. The helix was wound with 0.003-inch wire on a 0.030-inch mandril and stretched to 0.0065-inch pitch. Preliminary tests are briefly described and a gain-wavelength curve is shown.

621.385.032.21:539.16   1522
The Use of Radioactive Elements in the Study of Oxide Cathodes—J. Debiesse, J. Challansonnet, and G. Neyret. (*Compt. Rend. Acad. Sci.* (Paris), vol. 232, pp. 602–604; February 12, 1951.) Experiment shows that it is possible to prepare cathodes emitting β and γ radiation (period 5.3 years) by subjecting Ni electrodes containing a small proportion of Co to neutron irradiation. The resultant activity affords an extremely sensitive quantitative method of estimating the amount of Co in Ni alloys.

621.385.032.213   1523
Emissivity Changes of Thoria Cathodes—O. A. Weinreich. (*Jour. Appl. Phys.*, vol. 21, pp. 1272–1275; December, 1950.) The variations with time of thermionic emission and spectral emission of a thoria-coated tungsten filament are dependent on its previous heat treatment and on the electron current which is being drawn from it. Experimental results are presented in a series of curves. The relation between emissivity and thermal activation is discussed in terms of impurities produced during the heat treatment, such as free thorium and possibly a thorium sub oxide.

621.385.032.213:669.27:621–42   1524
Preliminary Treatment of Tungsten Wire for Electronic Valves—Mesnard and Uaan. (*See* 1412.)

621.385.032.216   1525
Optical Pyrometry of Oxide Cathodes: Measurement of Spectral Emissive Power—R. Champeix. (*Le Vide*, vol. 3, pp. 469–479; July and September, 1948.) A reflectometer method is described. See also 2973 of 1948.

621.385.032.216   1526
Deterioration of Oxide-Coated Cathodes under Low-Duty-Factor Operation—J. F. Waymouth, Jr. (*Jour. Appl. Phys.*, vol. 22, pp. 80–86; January, 1951.) The oxide coatings were deposited on a Ni alloy containing about 0.1 per cent Si. At zero duty factor, the operation of these cathodes favoured the development of high resistance in a layer of Ba_2SiO_4, located at the interface between the oxide and the core. This layer was present in cathodes aged without or with electron emission. A possible explanation of the observations is suggested. Evidence is presented which indicates that an "active" Ni alloy may exist which does not lead to the development of undue interface resistance.

621.385.032.216   1527
Oxide-Cathode Base-Metal Studies—R. Forman and G. F. Rouse. (*Bur. Stand. Jour. Res.*, vol. 46, pp. 30–37; January, 1951.) The effect of small traces of Mg in the Ni base metal of an oxide-coated cathode was investigated experimentally. Experimental double-diode tubes are described having two plane cathodes with a common heater, one cathode having a pure Ni base and the other a base of Ni with a trace of Mg. The cathodes

were processed under identical conditions before insertion in the envelope. The Mg-Ni cathode was found to have the greater emission decay in dc life tests, and an earlier departure from the space-charge voltage, current curve in pulse tests. The phenomenon of pulse-current decay, for pulses lasting about 250 μs, is discussed. Experiments indicate that this effect does not occur if the cathode and anode are correctly processed and aged, and it may thus provide a test of the initial quality of a tube.

621.385.2:546.289   1528
A Note on the Decay of Current in Germanium Diodes—J. R. Tillman and H. Yemm. (*Phil. Mag.*, vol. 41, pp. 1281–1283; December, 1950.) From measurements of crosstalk voltages in time-division multiplex systems, it is found that the current in a Ge diode decays with a time constant of the order of 1 μs. The time constant varies considerably for different diodes; no correlation of the observed crosstalk with "turnover" voltage or with reverse resistance could be found.

621.385.2:546.289   1529
Reverse Current of Germanium Diodes at High Voltages—P. Aigrain. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 1047–1048; November 13, 1950.) By assuming that a strong normal electric field [see 3198 of 1950 (Stuetzer)] modifies the surface conductance, the characteristics of Ge diodes can be explained quantitatively. In particular, the increase of reverse current at high voltages is explained without recourse to a field emission theory. Experimental results support the theory.

621.385.2:546.289   1530
The Inverse-Voltage Characteristic of a Point Contact on n-Type Germanium—L. P. Hunter. (*Phys. Rev.*, vol. 81, pp. 151–152; January 1, 1951.) A qualitative explanation of the effect of self-heating on the inverse-voltage characteristic. A method is shown of deriving a thermal-equilibrium curve, resembling the measured characteristic. The theory of Aigrain (1305 and 1306 of 1950) is discussed in relation to the relatively large currents found at low voltages with some point contacts.

621.385.2:546.289   1531
Pulse Measurement of the Inverse-Voltage Characteristic of Germanium Point Contacts—A. I. Bennett and L. P. Hunter. (*Phys. Rev.*, vol. 81, p. 152; January 1, 1951.) The isothermal pulse characteristics are almost linear up to voltages of more than twice the dc inverse breakdown voltage. Some experimental details are shown graphically and the linear results are discussed in terms of Aigrain's theory (1305 and 1306 of 1950). A qualitative relation apparently exists between the thermal-equilibrium dc characteristic and the isothermal pulse characteristic, back-voltage breakdown being really a self-heating effect.

621.385.2:621.314.632   1532
The Impedance of Crystal Rectifiers with Heavy D.C. Bias in the Blocking Direction—E. Spenke. (*Z. Phys.*, vol. 128, pp. 586–604; December 7, 1950.) The diffusion theory of crystal rectifiers with ac loading was previously limited to frequencies lower than the relaxation frequency of the high-resistance boundary layer of the semiconductor (see 532 of 1942). The theory is extended to high frequencies, small compared with the relaxation frequency of the low-resistance interior of the semiconductor. For "exhausted" boundary layers, the equivalent circuit of resistance with parallel capacitance is strictly applicable. For "reserve" layers, the approximate validity of the so-called "fixed layer" hypothesis is established, at least for high frequencies. See also 349 of 1950 (Bardeen) and 2917 of 1949.

621.385.3:546.289:621.317.7.001.4   1533
Transistor Measurement Technique—H. F. Mataré. (*Elektron Wiss. Tech.*, vol. 4, pp. 368–379; October and November, 1950.) An account of the present state of development of the transistor based on information given at the Reading conference on semiconductors, July, 1950. Forms and types of transistor are classified and the best method of specification is outlined. Test equipment discussed includes: (a) a cro circuit for displaying the characteristic modulation waveform obtained by modulating collector and emitter currents at different frequencies, the width of the wave envelope indicating transistor quality; (b) an arrangement for connecting the transistor as an amplifier for measuring amplification factors; (c) a saturated-pentode circuit for loading the transistor without altering the working point, to determine the characteristic cross resistances of the transistor; (d) an ac bridge operating at 5 kc to determine the interaction coefficient γ. Examples of measured values and typical characteristics are shown.

621.385.3.012.8   1534
Triode Transmission Networks under Linear Negative-Grid Conditions—Keen. (*See* 1321.)

621.396.822   1535
A Note on Induced Grid Noise and Noise Factor—I. A. Harris. (*Jour. Brit. IRE*, vol. 10, pp. 396–400; December, 1950.) A theoretical expression is given for induced grid noise which gives better agreement with experimental data than previous theoretical formulas. The noise factor of a common-cathode triode circuit is calculated by a novel method which ensures that account is taken of transit-time effects. The results confirm the possibility of reducing the noise factor by detuning the input circuit, with or without neutralization.

621.385.832   1536
New Developments in Storage Tubes—H. Klemperer. (*Arch. Elektrotech.*, vol. 40, pp. 45–48; 1950.) The operation of signal-storage cathode ray tubes is described on the basis of the secondary-emission processes involved. When operated with appropriate voltage relations between cathode target, and collector, such tubes are capable of distinguishing between irregular signals and periodically repeated signals, and hence can be used as filters for transmitting the former while suppressing the latter. With tubes at present available, a separation of 40 db can be effected, for example, a variable input signal is amplified up to the level of a steady input signal 100 times as large.

669.27:621–42:548.53   1537
On the Recrystallization and Grain Growth in Tungsten Wire—DeVincentis and Dedrick. (*See* 1411.)

MISCELLANEOUS

551.510.535:522.1(481)   1538
The Ionospheric and Radio Wave Propagation Observatory at Kiruna, 67° 50′ N, 20° 14.5′ E—Rydbeck. (*See* 1380.)

621.396.6.002.2.004.13.004.5/.6   1539
The Relation between Production, Operation and Maintenance of Service Radio Equipment—(*Proc. IEE* (London), Part III, vol. 98, p. 68; January, 1951.) Report of an IEE Radio Section discussion, April 24, 1950.

621.396.029.63/.64   1540
Microwave Electronics. [Book Review]—J. C. Slater. Publishers: D. Van Nostrand Co., New York, N. Y., 1950, 406 pp., $6.00. (*Rev. Sci. Instr.*, vol. 21, p. 877; October, 1950.) " . . . a logical, thorough, basic, and easy to understand development . . . a reader familiar with Maxwell's theory and equipped with the mathematical background of a graduate student of physics or engineering will find himself well prepared for its study."