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Diplex*

P. J. NOIZEUX†, ASSOCIATE, I.R.E., H. KRÄHENBÜHL†, ASSOCIATE, I.R.E., AND B. NOVIKS†, ASSOCIATE, I.R.E.

Summary—The communications companies are often faced with the problem of increasing their facilities and occupying more efficiently existing channels, usually by the installation of new transmitters.

To take advantage of existing equipment, use has been made of diplex (two messages in one direction), employing one transmitter whose output was commutated from one frequency to another at a rhythm superior to the keying rhythm.

The various radio-frequency components of modulated signals create a frequency-diversity effect at the receiver which attenuates somewhat the random effect of fading.

A particular way of switching is described, taking place when aerial current is zero, thus avoiding excessive sidebands due to abrupt wave front.

In the "synchronous" version, the keying is delayed, and stored for a fraction of a cycle of the modulating frequency, then released at the beginning of the next cycle. The rise and fall of the keying is then substantially sinusoidal and sidebands reduced.

The original diplex system used by Transradio since 1937 was later improved to decrease the bandwidth; several units of the improved diplex are now in common use.

It must be emphasized that no synchronism is required at both ends, neither keying speed bears any relation to the diplexing frequency.

A RELATIVELY simple diplex arrangement for transmitting two messages simultaneously with only one transmitting equipment has been developed by Transradio Internacional of Argentina. This arrangement has been in service continuously since 1937[1] and may be considered today as a reliable and convenient multiplex system.

The diplex system consists, in principle, of a means to commutate rapidly the transmitter from one output frequency to another. The commutation must be effected at a rhythm higher than the keying frequency, so that each emitted telegraphic dot contains several wave trains. In practice, the commutation is effected at the rate of 250 per second. Since telegraphic keying is generally less than 50 per second, each telegraphic dot will always contain at least 5 wave trains. The commutation is applied at a low level, i.e., in a buffer stage following the crystal oscillator. The two output frequencies must be sufficiently close together to be amplified equally in the subsequent stages of the transmitter. All of the transmitter stages are adjusted for the usual class C operation.

The output of a single transmitter is switched alternately to emissions on two distinct frequencies, for instance, LQB 17,690 kilocycles and LQC 17,600 kilocycles. Each one of these emissions can be keyed at any desired speed independently of the other. No special apparatus is required at the receiving end, since synchronization between transmitter and receiver, such as occurs in multiplex systems based on time division is not required. In fact, the correspondent is not aware that he is receiving a diplexed transmission. He will only observe a deep modulation of the interrupted-continuous-wave type. However, by tuning over the frequency band with a receiver, a diplex is easily distinguishable. Two transmissions separated by some 30 kilocycles are heard, both having the same intensity and the same characteristic modulation.

At the beginning and end of every commutation cycle, 250 per second, a click is produced, due to the abrupt signal front. This front contains numerous harmonics of low frequency, forming sidebands around the carrier and producing a "broad signal," like the old spark transmitters (see the Appendix and Fig. 13).

This defect of the first type of diplex has been corrected, as will be explained below, to fulfill the requirements of the international rules with regard to the width of the occupied channel.

It is evident that the energy emitted on each one of the frequencies, and integrated in a telegraph signal, is half what could be emitted with a pure wave on a single frequency, which could result in a decrease of the possible speed of transmission. The same conditions prevail in the case of ordinary interrupted-continuous-wave transmission. In practice, no such decrease takes place, as (when a single receiver is used) modulated signals produce a kind of frequency diversity which counterbalances, in some manner, for the random effect of fading. This diversity effect compensates largely for the reduction of energy and, in practice, diminution of the speed of transmission has not been noted. Even if the decrease of telegraphic speed should amount to 50 per cent, the diplex system would still be desirable, since it takes the place of two transmitters and allows the transmission of two simultaneous messages, directed to two different correspondents, independently.

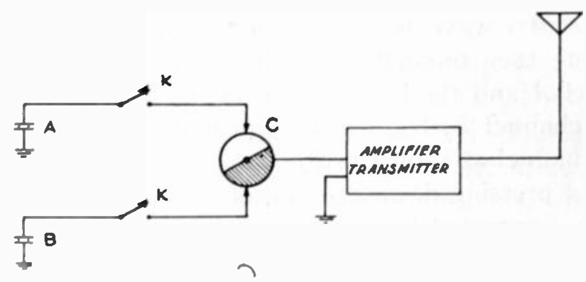


Fig. 1—Mechanical analogy of diplex system. A rotating commutator connects alternatively both oscillators to the transmitter. Each one can be keyed independently.

This diplex system may be considered as a particular case of combined frequency and amplitude modulation. The commutation from one frequency to another is frequency modulation and the telegraphic keying

* Decimal classification: R460. Original manuscript received by the Institute, April 16, 1941. Published in *Revista Telegrafica*, vol. 28, pp. 17–22, January, 1940.

† Transradio Internacional, Buenos Aires, Argentina.

[1] The authors had no knowledge at that time of a paper on the same subject by A. J. A. Gracie, *P. O. Tech. Jour.*, April, 1937.

between zero and the maximum is amplitude modulation.

In the mechanical representation of Fig. 1, two oscillators, represented schematically by two crystals A and B are connected alternately to the transmitter through a rotary commutator.

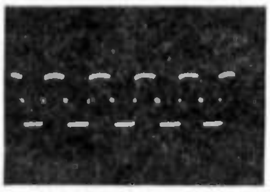


Fig. 2—Oscilloscope pattern of commutating voltage.

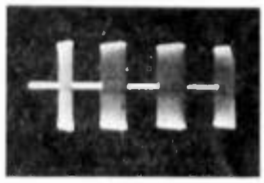


Fig. 3—Oscilloscope pattern of aerial current, one channel.

Each frequency is emitted only as the key is closed and only while the commutator allows it. When both keys are closed, both frequencies are emitted alternately, being commutated at the rhythm of rotation of the commutator. In the working model, electronic commutation is, of course, used instead of the mechanical type shown.

The over-all tuning curve of the transmitter has to be checked in order to locate the two frequencies to be used within adequate amplitude limits, so as to avoid parasitics due to lack of excitation. A separation of the order of 30 kilocycles has been chosen as a compromise.

The diplex high-frequency unit which excites the common power amplifier is composed of two identical exciters adjusted to the chosen radio frequencies. Each exciter contains a quartz oscillator and two buffers, the second buffer being controlled by keying and diplex commutation. The output of both exciters is then applied in parallel to the power stages.

To obtain the required abrupt front of commutation, a sinusoidal wave is transformed to a square wave, applying then one-half cycle (i.e., positive) to block channel A and the other half cycle (i.e., negative) to block channel B. Keying is applied independently to each channel as shown in Fig. 2.

Upon pressing down the key of channel A, a series of dots, separated by spaces equal to their length, is emitted at the commutation frequency. See Fig. 3.

Upon pressing down also the key of channel B, the empty spaces between dots are filled up, leaving only small intervals which correspond to the "shoulder" of the commutating wave form as shown in Fig. 4. This small interval is a protection against mixing of the two channels.

Since the main trouble is the exaggerated width of each channel, several ways to reduce the sidebands were tried, substituting the square dot and its abrupt front by a smoothly rising and decaying dot, following, if possible, a sinusoidal law.

It was observed that when slightly overmodulating a carrier by a sinusoidal audio frequency small spaces are obtained between each cycle. This envelope is equivalent to that resulting from diplexing (Fig. 5) when the two channels are on "dash," but the square front is substituted by a much smoother one. We then can use a cycle for channel A, the following for channel B, etc., *commutating them at the exact moment when the amplitude is zero.*

It is then deduced that the modulation frequency should be double the commutating frequency, and, moreover, to obtain perfect synchronism, the modulation and commutating source should be the same.

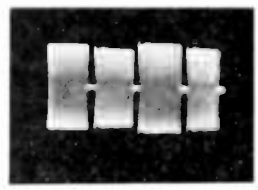


Fig. 4—Oscilloscope pattern of aerial current, both channels.

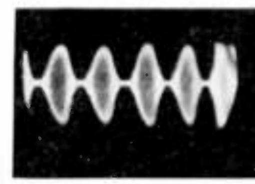
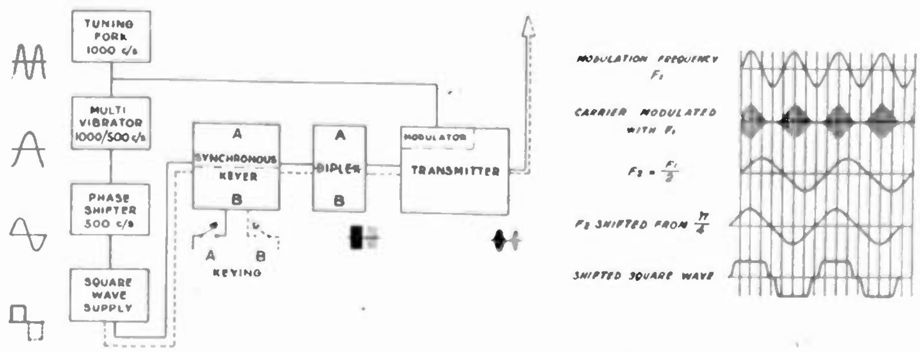


Fig. 5—Oscilloscope pattern of aerial current corresponding to overmodulated carrier.

By means of a multivibrator the modulating frequency is divided, obtaining the switching square-wave frequency which will block with the positive alternation, e.g., channel A, and the negative, channel B. Fig. 6B shows that the commutating frequency should be dephased, so that it coincides with the minimum modulation value and not with zero of the modulation frequency. This dephasing is effected before applying the frequency to the square-wave generator. The phase is properly adjusted, observing on an oscilloscope the radio wave form, in such a way that with the key pressed down on only one channel, there appears a group of sinusoidal dots with a space between them somewhat larger than the width of one of the dots. If the dephasing is correct, the switching is effected exactly when the modulation is zero.

The analysis of the radio-frequency spectrum shows



A B

Fig. 6—A. Block diagram of synchronous diplex. B. Wave forms.

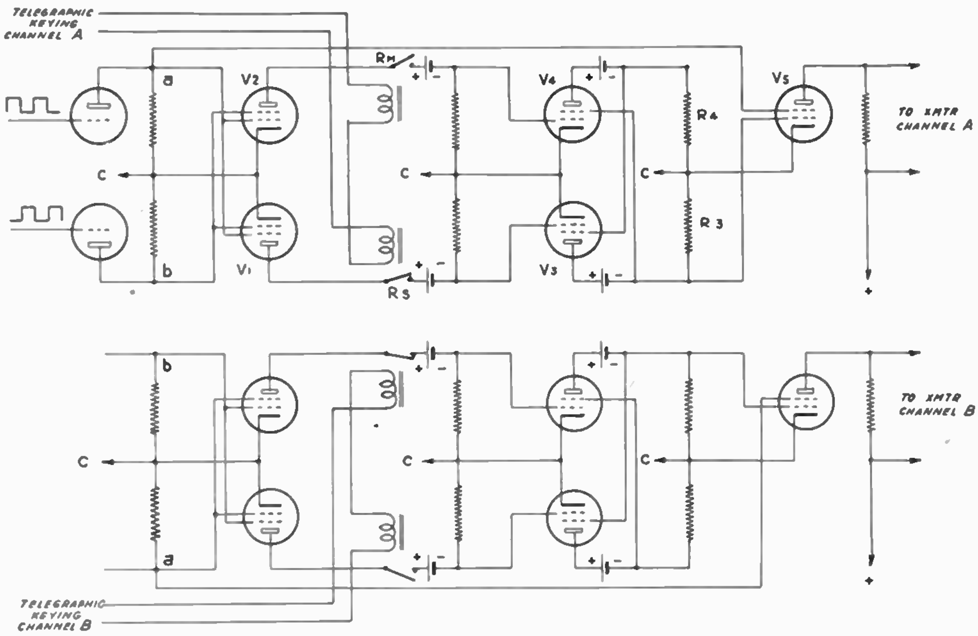


Fig. 7—Synchronous keyer, 1939.

the great improvement obtained through reduction of sidebands, the occupied channel being of reduced width (see the Appendix and Fig. 14).

In this way the reduction of the channel width would be effective, but for keying, which, when applied at any moment of the modulation cycle, would produce again the usual clicks.

As a matter of fact, upon pressing down the key when this channel is in working position, an abrupt front will be produced due to the sudden connection during a moment of maximum value. On the other hand, if the key is pressed down when the corresponding channel is blocked, no output will be obtained until it is switched on by the commutating process, the amplitude rising smoothly, with no accompanying click.

Taking advantage of this case, a device has been introduced which insures that keying begins and finishes only when the antenna current is zero. This is equivalent to the synchronous keyer (Fig. 7) which has been in operation on transmitter LSE/LSE2 since 1939, and whose mechanical analogy, shown in Fig. 8, works as follows:

(a) The square wave of Fig. 2 is applied to the grids of V_1 and V_2. Both grids are cross-connected so that short pulses appear in the plates of V_1 and V_2, as shown in Fig. 9.

(b) A telegraphic relay is connected in the plate circuit of each valve (R_s and R_m). Both relays, operated by telegraphic keying, are cross-connected so that one is open when the other is closed. Valve V_1 acts then on "spacing" and V_2 on "marking."

(c) Grids No. 1 of V_3 and V_4 will then receive impulses, alternately, at the rate of telegraphic keying.

(d) Assuming that the telegraphic relays are in the spacing position, the impulses will pass only through the spacing valve V_1.

(e) The front of the first impulse from V_1, reaching grid No. 1 of V_3, will block it. The plate current of V_3 ceases to flow, and V_4, whose grid No. 2 is now at cathode potential, becomes conductive, and the plate drop across R_4, applied on grid No. 2 of V_3, blocks V_3. V_3 then remains blocked as long as the telegraphic relay remains in the "spacing" position.

(f) Assuming now that the telegraphic relay is in the "marking" position, the short impulses now pass through V_2, and V_4 is then blocked by the front of the first "marking" impulse.

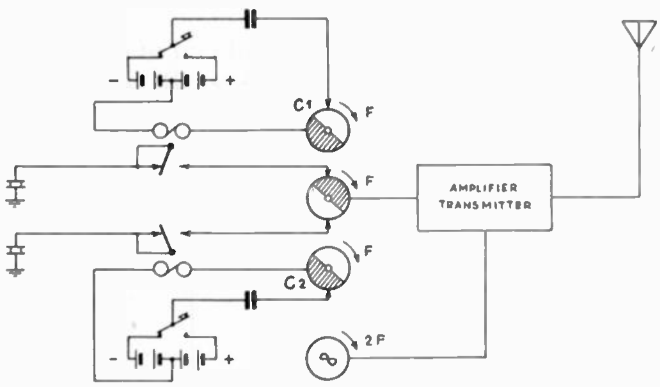


Fig. 8—Mechanical analogy of synchronous diplex.

The same process described above takes place, and V_4 remains blocked as long as the relays remain in the "marking" position. The current flows through R_3 during the entire marking period. Valves V_1 and V_3 perform as "spacing" valves and V_2 and V_4 as "marking" valves.

(g) Valve V_5 is a mixing valve, whose grid No. 1 is controlled by the telegraphic keying and grid No. 2 by the commutating square wave.

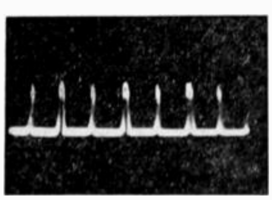


Fig. 9—Oscilloscope pattern of blocking impulses.

As the commutating square wave is derived from the basic modulating frequency, and as the keying can only begin and end during the impulses derived from

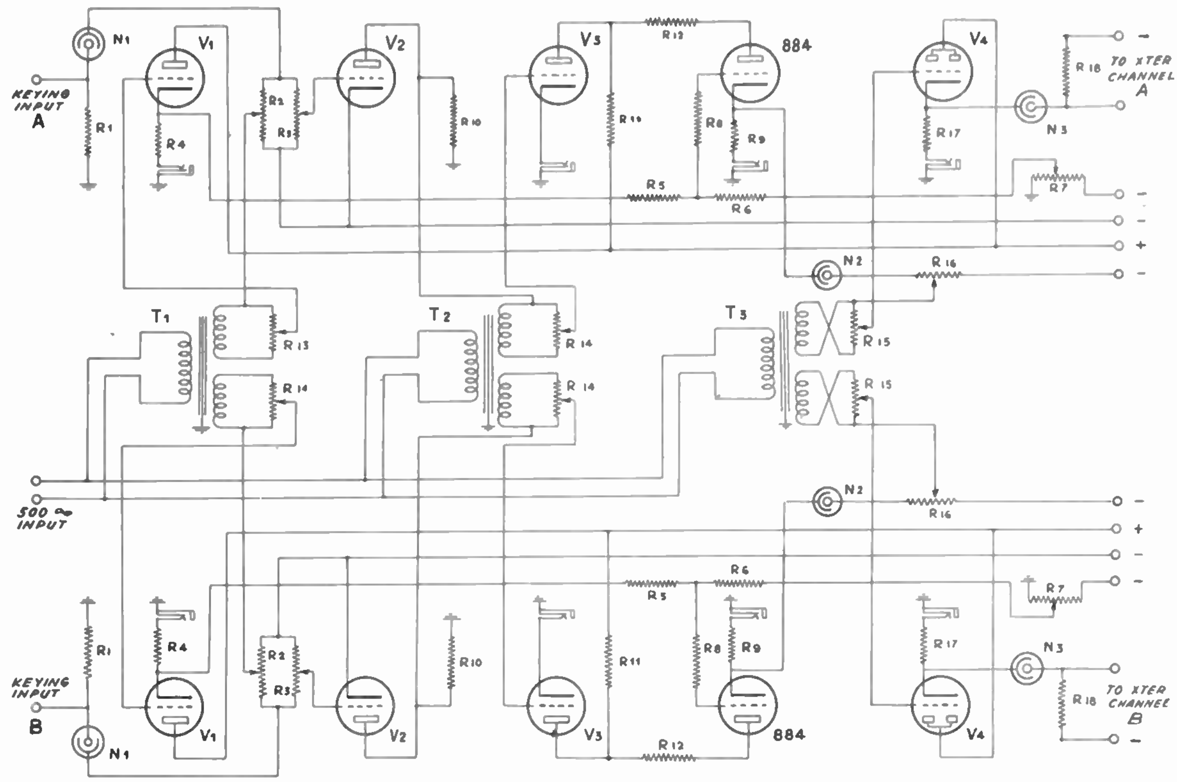


Fig. 10—Synchronous keyer, 1941.

the square wave, the beginning and end of every telegraph signal will always coincide with zero aerial current. Every diplex dot will then rise and fall smoothly, following a sinusoidal law.

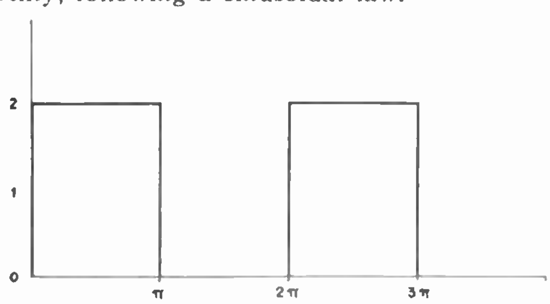


Fig. 11—Diplex wave form.

When everything is properly adjusted, and both channels are on a long "marking" dash, the aerial current is equivalent to a 100 per cent modulated carrier. Nevertheless, the bandwidth required is somewhat wider than in the case of a modulated carrier, as the sinusoidal diplex dots are separated by an interval equal to their duration. However, the bandwidth is very much narrower than in the case of the original diplex, as shown in the mathematical treatment of both (see the Appendix).

The synchronous diplex described above has been in use in Buenos Aires since 1939, with good results. Its design presented certain disadvantages, such as the two mechanical keying relays which should work in perfect synchronism, and also the difficulties inherent to the direct-current amplifiers needing individual, ungrounded plate supplies.

A new synchronous keyer has since been developed, using electronic commutation and keying, as shown in Fig. 10. Instead of impulses derived from the square wave, a sinusoidal frequency is used throughout in this synchronous keyer. Transformers T_1, T_2, and T_3 are fed from a common 500-cycle sinusoidal source, properly phased, as explained above, so that the positive peaks coincide with the positive peaks of the modulating frequency. Each transformer has two opposite secondary windings, one for each channel.

The keying voltage (marking +50 volts, spacing −50 volts) is applied to a 991 neon tube (N_1), whose negative bias is such that it will fire on the keying voltage difference. A positive voltage then appears across both potentiometers R_2 and R_3, which is applied to the grids of V_1 and V_2, respectively. Both valves are biased (about 70 volts) to neutralize in V_1 the 500-cycle peaks during "spacing" periods.

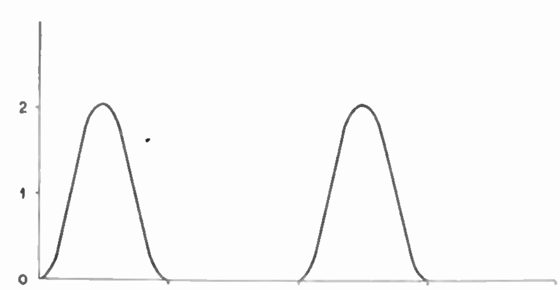


Fig. 12—Synchronous diplex wave form.

On "marking," the voltage across R_2 will decrease the bias on V_1, so that this will work only on positive 500-cycle peaks. These peaks, across the cathode resistance R_4 of V_1 will then fire the 884 thyratron. The thyratron cathode resistance R_9 will supply the necessary voltage to fire the 991 neon tube N_2. Across the load resistor of N_2 the "keying" voltage will remain constant until the thyratron is extinguished. This voltage was built up by coincidence of the beginning of the keying signals with the first peak of the 500 cycles in the synchronous keyer.

On the other hand, V_2, whose cathode is highly negative to ground, shows an increase of plate current during "marking" periods, thus blocking V_3. No 500-cycle current flows through V_3, so the thyratron is not affected.

On "spacing," V_2 grid bias disappears, and consequently V_3 grid bias decreases. The 500-cycle peaks flow now through V_3, and, due to the common R_{11} plate resistor, the voltage on the thyratron decreases, extinguishing it.

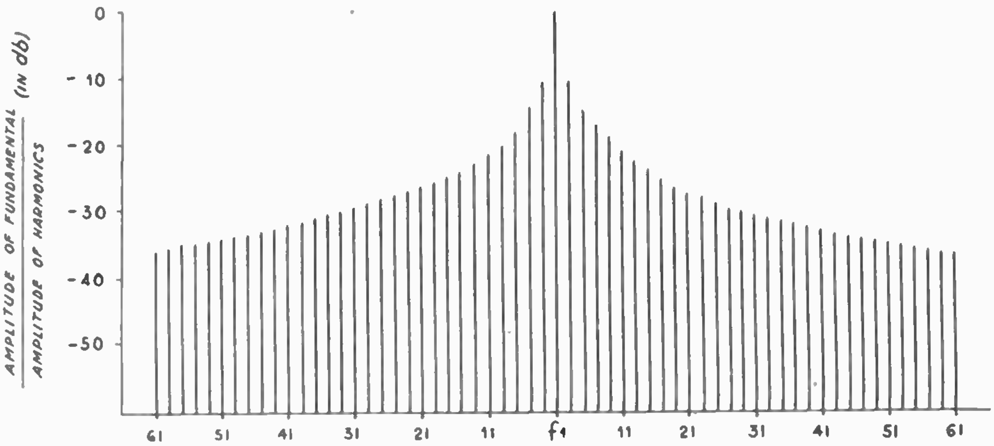


Fig. 13—Harmonic content of diplex wave.

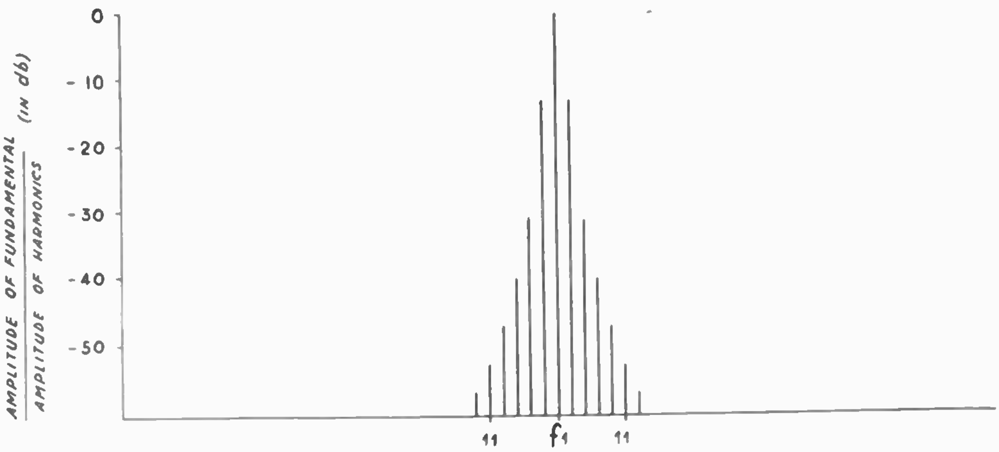


Fig. 14—Harmonic content of synchronous diplex wave.

The voltage across R_{16} then disappears, coinciding with the end of the keying signal and the last peak of the 500 cycles in the synchronous keyer. The voltage across R_{16} would vary in accordance with the keying, but is further interrupted at the diplex rate, both channels being alternately connected to the transmitter control. This commutation is performed by the diplexing valve V_4.

When the thyratron is fired, 500 cycles flow through V_4 and so the 991 (N_3) is also fired, controlling a low-power stage of the transmitter.

The 500 cycles applied to the grid of V_4 is 180 degrees out of phase with that applied to the previous grids of the synchronous keyer, so that diplex commutation takes place during the no-signal period of the transmitter.

As previously described, the second channel performs alternately in the same manner as the first one.

Attention must be drawn to the fact that, using a higher-order subharmonic of the modulating frequency in the synchronous keyer, a number of channels greater than two can be used.

Acknowledgment

The authors gratefully acknowledge the valuable contribution of C. A. Vaio for the development of the second type of synchronous keyer, and of J. Lafon and S. Cabrera for their work in the adaptation to power transmitters.

Appendix

The Fourier analysis of the square wave in the original diplex system (Fig. 11) is as follows:

$$e = 1 + 4/\pi\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \cdots\right).$$

The sinusoidal dots of the synchronous diplex system (Fig. 12) are expressed by the following series:

$$e = \frac{1}{2} + \frac{8}{\pi}\left(\frac{\sin x}{3} - \frac{\sin 3x}{3.5} - \frac{\sin 5x}{5.21} - \frac{\sin 7x}{7.45} \cdots\right).$$

Obviously, the harmonic content of the latter wave is much lower than that of the first one, as shown in Figs. 13 and 14, which demonstrate the calculated values of the harmonics of the above series.

Distortion Tests by the Intermodulation Method*

JOHN K. HILLIARD†, MEMBER, I.R.E.

Summary—*Although the term "intermodulation" has been used for a long time to describe a form of distortion, only recently has simplified testing equipment been made available which permits the commercial testing of audio, radio, and general sound-recording equipment by this method.*

In this paper there is described a compact piece of apparatus which is being used for routine maintenance tests on the sound recording and reproducing systems. A comparison is made between the harmonic and intermodulation methods of making such tests to show the advantage of the latter method. Circuits and descriptions of the apparatus for generating and analyzing the intermodulation test tone needed in making the tests are given.

Tests on radio transmission systems, sound disk and film recording apparatus, iron-core coupling devices, and general sound systems are described which show how the intermodulation method of measuring distortion is in many cases a more adequate indication of equipment performance than other methods commonly used, as well as being less laborious.

In radio transmission and reception and in motion-picture sound recording and reproduction, attempts are constantly being made to interpret and then to reduce or eliminate all types of distortion. This paper will explain a type of distortion, known as intermodulation,[1] describe equipment to measure such distortion, and outline tests which indicate the amount of intermodulation which the ear can tolerate.

Sound originating from a single musical instrument is usually harmonic in character and small relative amplitude changes in the harmonic structure are not easily detectable by ear. Hence, when such sounds are transmitted through a nonlinear system (which may be an amplifier or recording system), the resulting distortion will merely change the amplitude relations of the harmonic content and if not too severe will not be especially noticeable. However, where sound originates from a group of sources such as dialog with musical instruments, bells, sound effects, etc., the resulting signal consists of frequency components not necessarily harmonically related. When such signals are transmitted through a nonlinear system the resulting signal contains many new frequency components not present in the original signal. These new frequency components, known as intermodulation products, extend throughout the transmitted frequency band and are caused by the intermodulating of the signal frequencies with each other. Such a reproduced signal may be disagreeably harsh in quality and is usually easily detectable by the ear. It is not unusual to note in listening to sound programs that a clear quality is obtained when the sound originates from a single instrument but becomes harsh when more complex sounds are added.

For the past several years the harmonic-analysis method has been used to make distortion measurements on amplifiers, transmitting equipment, and recording equipment, etc. For this method a single fixed-frequency tone comparatively free of harmonics is "sent" into the apparatus under test. The output from this apparatus is then directed to a suitable harmonic analyzer and the amplitude of each harmonic present relative to the fundamental, determined. The per cent harmonics for the equipment under test is then determined in the usual manner from these data.

In making these harmonic-analysis tests on audio-frequency equipment there has been a tendency, somewhat augmented by necessity, to use a low-frequency test tone (in most cases 400 cycles per second) in order that a large number of the harmonics produced will lie within the transmission band and hence be available for measurement. As a result the equipment is not tested for distortion of the higher frequencies. In some cases this is not necessary as the low-frequency measurement is a satisfactory indication of the equipment's total performance. However, under certain conditions, distortion at high frequencies exists and although the resulting high-frequency harmonic components of the signal may not be important since they are inaudible, intermodulation products are produced and these must be considered as they may lie anywhere within the transmission band.

The intermodulation method of measuring distortion has, in many cases, certain inherent advantages over the harmonic-analysis method. Both methods, of course, make use of pure tones for measuring; the one employing two tones to generate intermodulation products when transmitted through a nonlinear system; the other using one pure tone to create harmonic components when transmitted through the system. An advantage of the intermodulation method can be seen at a glance. For instance, the intermodulation products which must be quantitatively measured always lie within the frequency band of the equipment under test; whereas, by the harmonic-analysis method, the harmonic frequencies produced may, and often do, lie completely outside of the transmission band and, moreover, the measuring equipment must be designed to perform over a wide frequency range in order to measure the harmonic content.

It has been the further experience of the writer that the intermodulation method involves much less labor than the harmonic method to secure practical results. By the latter method the harmonic components not only must be measured but they should be weighted with respect to the average ear response.

The following tests are an interesting comparison of

* Decimal classification: R148.1×R270. Original manuscript received by the Institute, March 27, 1941; revised manuscript received, July 29, 1941.

† Metro-Goldwyn-Mayer Pictures, Culver City, California.

[1] The I.R.E. Standards on Radio Receivers, 1938, page 5, 3R7 defines intermodulation as follows: Intermodulation is the production, in a nonlinear circuit element, of frequencies corresponding to the sums and differences of the fundamentals and harmonics of two or more frequencies which are transmitted through that element.

results which may be obtained from these two methods. A comparative listening test was made between two amplifiers, each of which had the same amount of harmonic content as measured by the harmonic-analysis method. The reproduction from the two amplifiers, however, sounded quite different. For the amplifier with the better sound reproduction, proper investigation showed that the harmonics from this amplifier were of low order, that is, second harmonics and a small amount of third harmonics. In the case of the amplifier having objectionable sound reproduction, it was found that in addition to the second and third harmonics, a small amount of fifth and higher harmonics (less than 0.1 per cent) were present. An intermodulation test revealed that the better of these amplifiers from a listening standpoint also gave better intermodulation-test results.

Following the trend of previous procedure, that is, to measure modulation products in terms of the per cent amplitude, intermodulation is also defined in per cent according to the accepted standards.[2]

Wherever it is possible to measure completely the amplitude and frequency of the harmonics generated from a single-frequency tone being transmitted through a piece of equipment, the amount of intermodulation, resulting from the transmission of two tones through the equipment, can be computed. However, this method is laborious and impracticable.

As a result of the restricted range involved in most equipment, harmonics of certain frequencies which may generate modulation products (the higher harmonics present) lie outside the range of the transmitting equipment. This is particularly true in recording apparatus where the signal-to-noise ratio is low at these suppressed frequencies. Consequently, the necessity of reducing or eliminating intermodulation products in film recording has created the demand for a device simple in operation which will accurately measure the magnitude of these modulation products.[3]

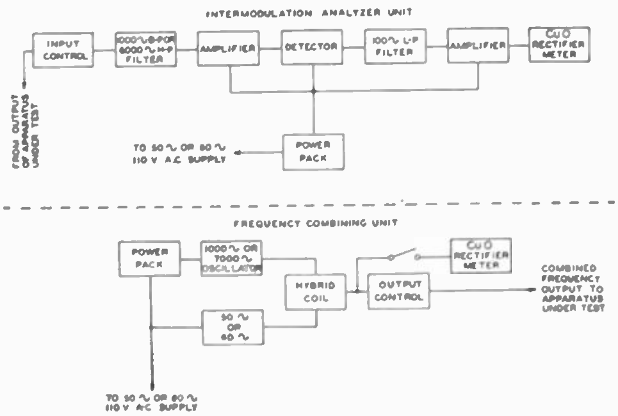


Fig. 1

[2] The per cent modulation or the modulation factor expressed in per cent is the ratio of the maximum departure (positive or negative) of the envelope of a modulated wave from its unmodulated value to its unmodulated value.

[3] Frayne and Scoville, *Jour. Soc. Mot. Pic. Eng.*, June, 1939.

This is accomplished by introducing into the equipment to be measured, two frequencies spaced several octaves apart, with the amplitude of the higher frequency adjusted to be 12 decibels down with respect to the lower frequency. The effect of changing the relative amplitude of the higher and lower frequencies is to change the sensitivity of the intermodulation test.

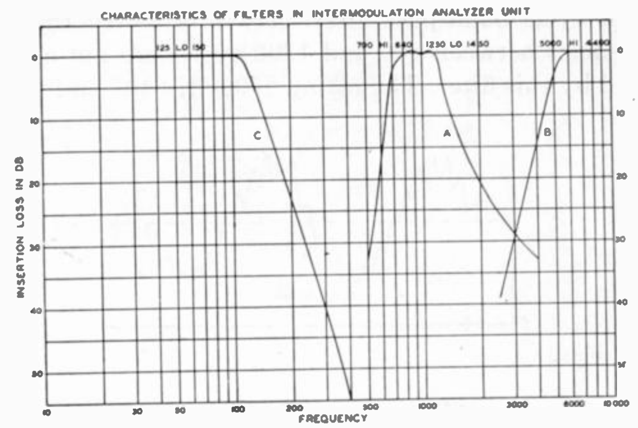


Fig. 2

For the purpose at hand it was found desirable to employ the 12-decibel differential mentioned. Since this relation affects sensitivity, the differential once selected must be maintained in order that comparative results may be secured. The generated modulation products developed as a result of nonlinearity in the equipment are then measured. This method, therefore, is not critical of any frequency suppression taking place at the output of the transmitting equipment.

The lower portion of Fig. 1 is a block schematic drawing of an intermodulation frequency-combining unit used in the film industry. Standardization has taken place to the extent that certain frequencies at definite relative amplitudes are used for testing. For the higher-frequency tone either 1000 or 7000 cycles per second is used depending upon where, in the frequency range, it is desirable to test the equipment. This supply is obtained from an oscillator of the resistance-capacitance type to measure stability of output over long periods of time. For the lower tone a frequency of either 50 or 60 cycles per second is used depending upon which of these is available from the commercial power source. This makes it unnecessary to supply a separate oscillator for this tone. In cases where a substantial amount of induced power hum is present in the equipment under test or the testing equipment, it may be advisable to use a frequency other than that of the power supply. To obtain accurate level adjustment with a small power loss (maximum of 3 decibels), the two frequencies are combined through a three-winding hybrid coil.[4] Where loss of power is no concern, the two frequencies may be combined by introducing the two voltages in series through a resistance.

The upper portion of Fig. 1 is a schematic diagram

[4] Albert, "Electrical Communication," pp. 434 and 435.

of a typical intermodulation analyzer unit. The input to the analyzer is controlled by an attenuator providing sufficient range for studio purposes. After this control, the signal is admitted by means of a selective key to either the 1000-cycle band-pass filter or the 6000-cycle high-pass filter, depending upon the frequency employed.

Curve *A* of Fig. 2 gives the characteristic of the 1000-cycle band-pass filter. It will be noted that frequencies between 800 and 1200 cycles are not attenuated by this filter. The rating "700-Hi-640" and "1250-LO-1450" are the 3- and 10-decibel loss points of the high- and low-pass filter which goes to make up the band-pass filter.[5]

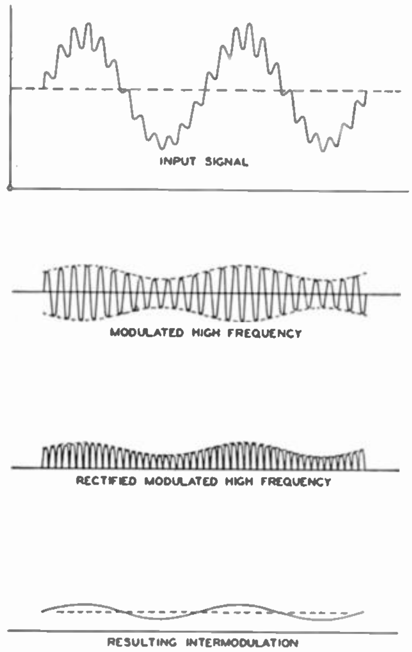


Fig. 3.

Curve *B* of Fig. 2 gives the characteristic of the 6000-cycle high-pass filter. Only those frequencies above 6000 cycles are passed.

Beyond the filters, the signal is amplified and applied to a full-wave detector. This signal is composed of the higher of the two superimposed input frequencies and its associated sidebands.

If the intermodulation is a result of curvature at only one end of the characteristic (as in the case of the toe of a vacuum tube or film H & D curve), the higher frequency is modulated at a rate equal to the lower-frequency signal. However, if the amplitude of the signal intrudes on both the toe and shoulder of the operating characteristic of the equipment under measurement, the higher frequency will be modulated at a rate of twice the lower frequency, since the compression of the signal occurs twice for each cycle of the lower frequency. The output of the detector, that is, the rectified components, is then applied to the low-pass filter which permits frequencies below 100 cycles to pass (curve *C* of Fig. 2).

This filter may well have a cutoff as low as about 300 cycles per second where the lower-frequency test tone is not greater than 150 cycles per second. This is because the intermodulation may be the result of compression at both the toe and shoulder of the characteristic and hence the low-pass filter must transmit frequencies up to twice the frequency of the lower of the two test tones. Where it is desired to measure intermodulation distortion of a very low amplitude, this filter must discriminate considerably against the upper-frequency test tone of 1000 or 7000 cycles per second. Usually a 60-decibel discrimination between the cutoff and the higher frequency will be sufficient. No useful purpose is served by having a cutoff higher than twice the fundamental frequency because of the rate at which this compression can take place.

The direct-current component of the modified signal is then removed and the alternating-current component is amplified and measured by means of a copper-oxide-rectifier meter. The phases of detection are shown in Fig. 3. The full schematic drawing of the frequency-combining unit is shown in Fig. 4 and the analyzer is shown in Fig. 5.

The intermodulation-test unit has a range from 0.1 of 1 per cent to 100 per cent intermodulation; that is, a range of 60 decibels in convenient steps.

Calibration of the intermodulation-test unit is made by sending two frequencies, such as 1000 and 1050, through the mixer unit or frequency combiner. The amplitudes of the two frequencies are adjusted so that one frequency is 20 decibels down with respect to the other. By definition, this is a condition of 10 per cent intermodulation and from this reference point a calibration curve can then be calculated.

The intermodulation-test unit is used extensively to check several phases of the recording process.

In film recording with a light valve, the amplitude of the higher-frequency component will be a minimum when the valve approaches its maximum spacing and will be a maximum when the ribbons approach the zero-spaced condition. Thus, the higher-frequency tone is modulated at the lower-frequency rate, resulting in the addition to the higher frequency of sum and difference tones. The intermodulation test has provided an excellent method of obtaining rapid quantitative data in the magnitude of such distortion changes which is not measurable by the harmonic-analysis method. With the latter method, only the average reduction in the amplitude of the fundamental is indicated.

Values of optimum exposure of the negative, printer light, and over-all gamma in variable-density recording are now arrived at through the intermodulation-test method. (See Fig. 6.) If a uniform contact between negative and print is not maintained because of printer

[5] Academy of Motion Picture Arts and Sciences' Research Council Technical Bulletin on Standard Nomenclature for Filters, August 10, 1937.

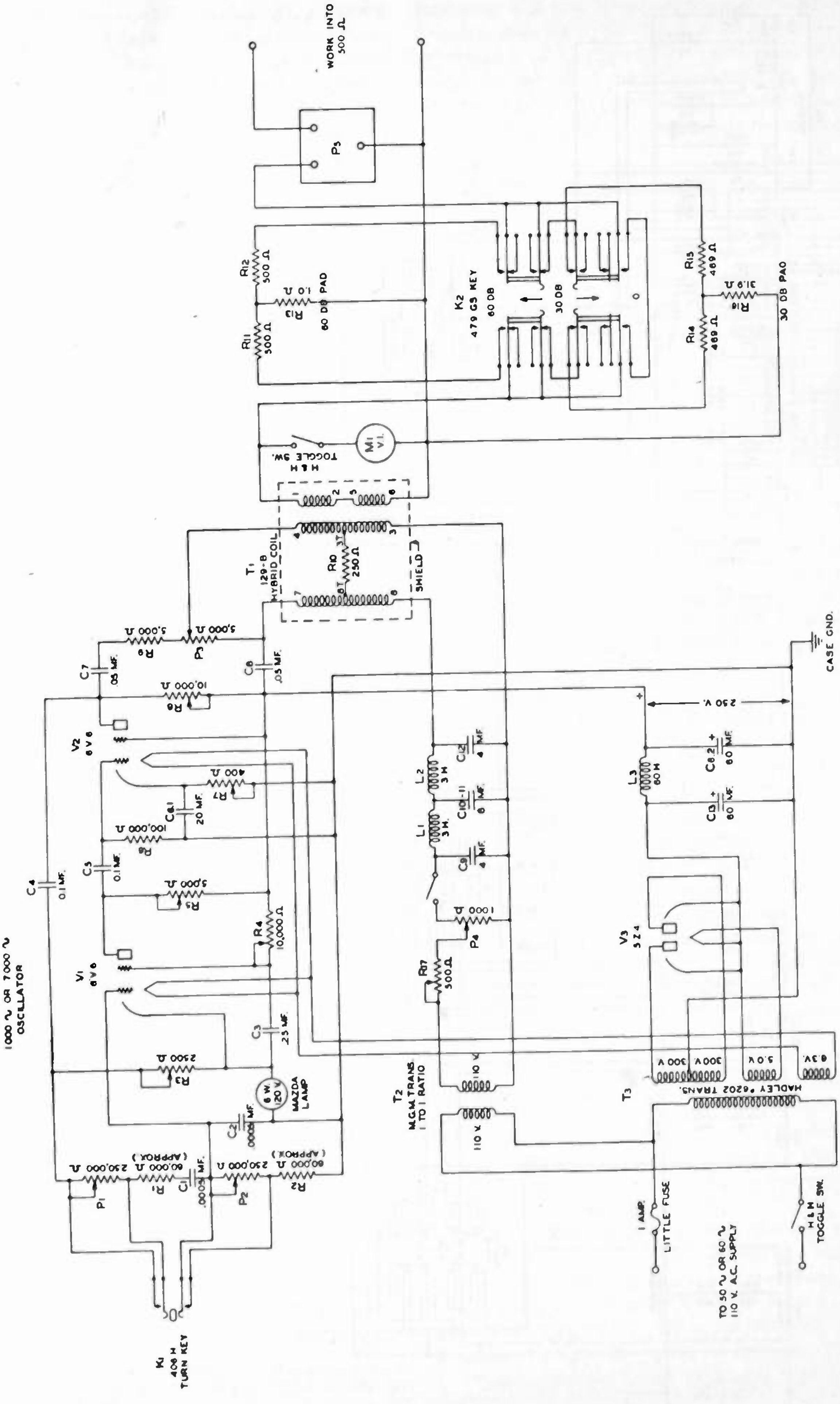

WORK INTO 500 Ω
P3
R12 500 Ω
R11 500 Ω
R13 1.0 Ω
60 DB PAD
K2 479 GS KEY
60 DB
30 DB
0
R15 489 Ω
R14 489 Ω
R16 31.6 Ω
30 DB PAD
H & H TOGGLE SW.
M1 V.I.
T1 129-B HYBRID COIL
R10 250 Ω
SHIELD
R9 5,000 Ω
P3 5,000 Ω
C7 .05 MF.
C8 .05 MF.
R6 10,000 Ω
CASE GND.
250 V.
V2 6V6
R7 400 Ω
C61 20 MF.
R8 100,000 Ω
C5 0.1 MF.
R5 5,000 Ω
C4 0.1 MF.
R4 10,000 Ω
V1 6V6
1000 ~ OR 7000 ~ OSCILLATOR
C3 .25 MF.
R3 2500 Ω
6 W. 120 V MAZDA LAMP
C2 .0005 MF.
P1 250,000 Ω
R1 60,000 Ω (APPROX.)
C1 .0005 MF.
P2 250,000 Ω
R2 60,000 Ω (APPROX.)
K1 406 H TURN KEY
L2 3 H
C12 4 MF.
C10-11 8 MF.
L1 3 H.
C9 4 MF.
P4 1000 Ω
R17 500 Ω
T2 M.G.M. TRANS. 1 TO 1 RATIO
110 V
L3 60 H
C8.2 80 MF.
C13 80 MF.
V3 5Z4
T3 HADLEY #6202 TRANS.
300 V
5.0 V.
6.3V.
1 AMP. LITTLE FUSE
TO 50 ~ OR 60 ~ 110 V. A.C. SUPPLY
H & H TOGGLE SW.

Fig. 4

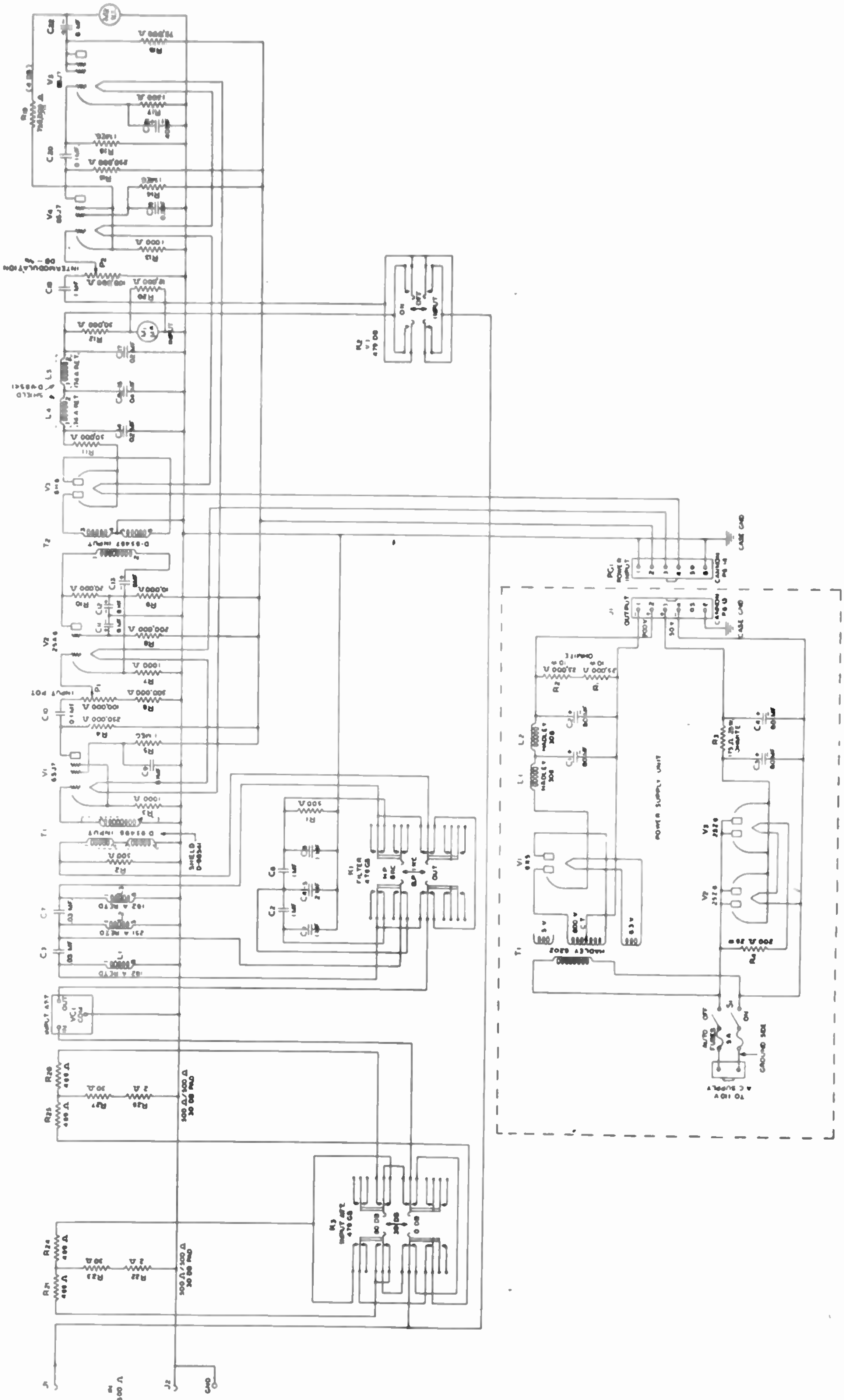


Fig. 5

slippage or chatter, the track is modulated, resulting in a different amount of high frequencies on the print at different positions. This result can be measured directly by printing a 7000-cycle loop and measuring the output in the same manner as when two frequencies are used, the slippage being the equivalent of the lower frequency normally used.

In disk recording the representation of distortion computations for two-tone signals is quite complex. The intermodulation tests are a measure of the tracking distortion over a wide band which is principally nonharmonic in character.[6] Amplitudes for pre-equalized or high-frequency-emphasis recorded disks may be determined for an accepted distortion.

To measure the intermodulation resulting from hum or power-line pickup introduced in an amplifier, the equipment is set up in the regular manner except the lower of the two test frequencies is removed and the induced current used instead. The indicated intermodulation is then a result of the induced hum which acts as the lower frequency.

Approximate relation between the intermodulation and harmonic terms may be arrived at under certain assumptions regarding the form of distortion present. Experimental data indicate that the intermodulation terms are approximately four times the harmonic terms, that is, if a certain apparatus is found to have 1 per cent total harmonics, an intermodulation test will show intermodulation products of approximately 3 to 4 per cent where the higher frequency is adjusted 12 decibels down in amplitude from the lower frequency.[3]

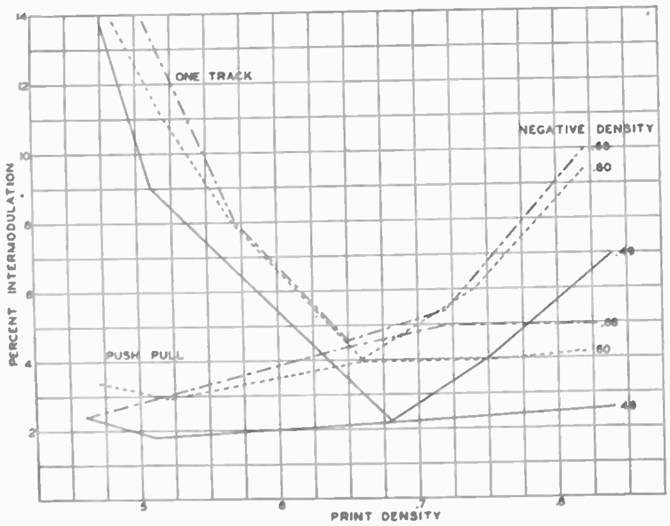


Fig. 6—Over-all intermodulation including electrical and photographic characteristics of variable-density light-value recording system, using a 10–1 cylindrical-lens image reduction.

The intermodulation-test method is extremely useful in other fields. It may be employed in the adjustment and operation of radio transmitters as experience has often shown that even after careful adjustment of a transmitter on a steady-state basis, satisfactory performance will not be secured on a semitransient or dynamic state with speech and music. However, proper adjustment of the modulator and class B amplifier stages, by the use of the intermodulation-test method, resulted in an adjustment under test conditions which can be relied upon for program performance. In the

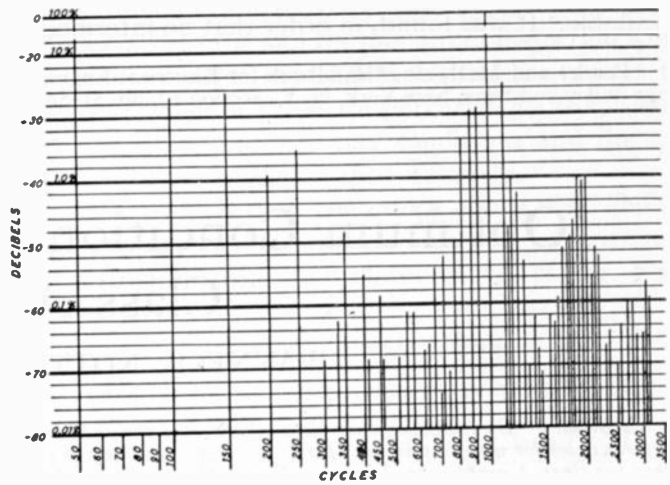


Fig. 7

case of the single-tone steady-state measurement the apparatus under test is forced to assume a new characteristic dependent upon the degree of modulation present. This new characteristic is maintained so long as the single frequency is held at constant amplitude. However, in the case of the multiple tone, such as is used in the intermodulation test, if the modulation is sufficient to shift the static characteristic to a new point this shift does not remain constant as in the case of a single frequency, but varies at the rate of, or twice the rate of the lower of the two impressed tones and thus a dynamic state is created.

In a great number of cases, intermodulation in audio amplifiers is due to a lack of low-frequency carrying capacity and the intermodulation test method immediately shows the degree of magnetic modulation in transformers and filters. Where a single-frequency test wave is applied to a ferromagnetic-core coil only the main hysteresis loops are utilized but with complex signals a magnetized force of different character is involved since other loops appear.[7]

Although normal impedance ratings and frequency characteristics of iron-core coupling devices are given with reasonable accuracy, these data are not necessarily an indication of the amount of modulation products of intermodulation generated by this apparatus when subjected to the wide range in current values and frequencies of a complex wave.

An amplifier was set up with the test modulation tones of 50 and 1000 cycles to obtain a slightly overloaded condition. The frequency spectrum up to 3150 cycles per second was analyzed by means of a quartz-filter tuned analyzer. With the 50-cycle component considered as 100 per cent modulation, Fig. 7 shows

[6] W. D. Lewis and F. V. Hunt, "Theory of tracing distortion in sound reproduction from phonograph records," *Jour. Acous. Soc. Amer.*, vol. 12, pp. 348–365.

[7] "Motion Picture Sound Engineering," D. Van Nostrand Company, New York, N. Y., pp. 97–115.

the frequency and magnitude of the measurable generated terms. It will be noted that in some cases these terms are not harmonically related to the original sources or to each other.

An amplifier system was used for the pickup and recording of a voice and flute duet. This material was then reproduced using the highest-quality horn system available.[8] It was found, in order that no intermodulation be detected by ear, the intermodulation term had to be less than 2 per cent under test conditions exclusive of the horn system.

[8] Pender and McIlwain, "Handbook for Electrical Engineers," John Wiley and Sons, New York, N. Y., section 10, pp. 51–56.

Consequently, as the intermodulation test is approximately four times as sensitive as the harmonic-analysis method, it approaches the sensitivity of the ear in detecting intermodulation effects, and it is a very valuable tool with which to measure distortion. By comparison, other methods are inadequate and inconvenient, as well as more laborious.

Optimum Conditions for Maximum Power in Class A Amplifiers*

WAYNE B. NOTTINGHAM†, ASSOCIATE, I.R.E.

Summary—*By following a simple analysis, it is shown that there are three cases for which optimum operating conditions may be established for class A amplifiers. These are for (I) the small signal, (II) the fixed quiescent plate voltage, and (III) the fixed quiescent plate dissipation. There is a "best" operating condition, which might be considered as a fourth case, if both the quiescent plate voltage and the quiescent plate dissipation are fixed. For cases (I) and (II) the results are definite and give* $R=r_p$ *and* $R=2r_p$ *but for (III)* $R \doteq 20r_p$ *and for the fourth condition* $R \doteq 8r_p$. *In the last two conditions R is not exactly the same for all tube types but depends to some extent on the tube characteristic. The undistorted power delivered to the load for these cases varies by a factor of nearly 3.*

Introduction

THE FIRST optimum condition for class A power amplifiers was established long ago[1] for the case of a small amplitude of grid swing and resulted in the relation $R=r_p$ where R is the load resistance and r_p the plate resistance of the tube. The second relation applies in case the grid swing is limited only by the distortion properties of the tube and the quiescent plate voltage is specified. Brown[2] showed that the maximum power is delivered to the load when $R=2r_p$ assuming that the no-signal plate dissipation is adequate. The third case, which so far as the author is aware, has never been published, applies if the quiescent plate dissipation is specified and no limitations are placed on the plate voltage. The grid swing is assumed to be limited by the distortion properties of the tube just as in the second case above. The optimum plate load for the third case turns out to be of the order of fifteen to twenty times the plate resistance r_p and depends on the quiescent plate dissipation and the minimum plate current permitted by the tube characteristics. Since the treatment of all three of these cases can be unified so easily, it will be presented here as briefly as possible.

* Decimal classification: R132. Original manuscript received by the Institute, September 26, 1940; revised manuscript received, August 25, 1941.

† George Eastman Laboratory of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] H. J. Van Der Bijl, "The Thermionic Vacuum Tube," McGraw-Hill Book Company, New York, N. Y., 1920, p. 188.

[2] W. J. Brown, *Proc. Phys. Soc.* (London), vol. 36, p. 218; 1924.

Symbols

The symbols to be used can best be defined in terms of the idealized tube characteristic curves shown in Fig. 1. The static tube characteristics over the range of approximate linearity are represented by

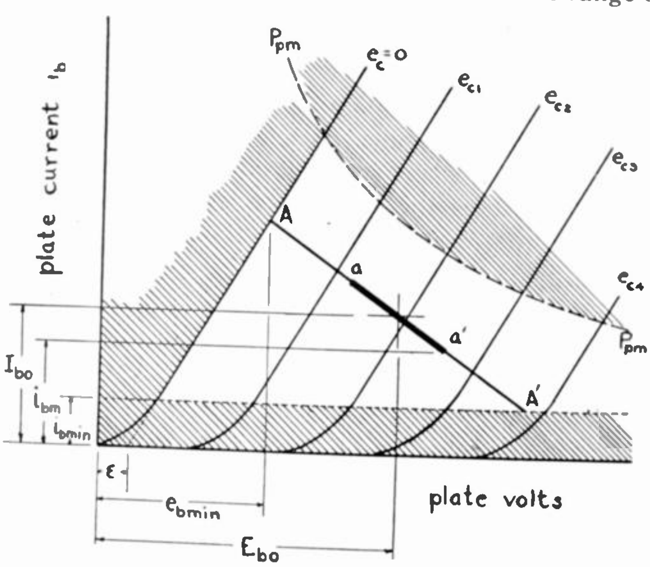


Fig. 1—Idealized tube characteristics to illustrate symbols used. a–a' is load line for small signal; A–A' is load line for large signal always terminated at zero grid volts at A and a minimum plate current at A'. Maximum plate dissipation requires quiescent point to lie on line P_{pm}–P_{pm} or below it.

$$i_b = \frac{1}{r_p}(\mu e_c + e_b - \epsilon) \qquad (1)$$

where

i_b = instantaneous total plate current

e_c = instantaneous total grid voltage

e_b = instantaneous total plate voltage

r_p = plate resistance over the linear part of the characteristic

μ = amplification factor

ϵ = intercept on the voltage axis of the extrapolated $e_c=0$ curve

In the equations below the following symbols have the meanings:

E_{c0} = quiescent grid voltage
I_{b0} = quiescent plate current
E_{b0} = quiescent plate voltage
$P_{p0} = I_{b0}E_{b0}$ = quiescent plate dissipation
P_{pm} = maximum quiescent plate dissipation
i_{bm} = minimum total plate current for a specified grid signal
$i_{b\min}$ = minimum total plate current permitted as set by distortion
$I_{pm} = I_{b0} - i_{bm}$ = maximum value of varying component of the plate current
E_{gm} = maximum value of the varying component of grid voltage
P = alternating-current component of the power delivered to the load R
R = load resistance.

General Equations

A general equation for the power delivered to the load R is

$$P = \tfrac{1}{2}I_{pm}{}^2R = \tfrac{1}{2}(I_{b0} - i_{bm})^2R. \tag{2}$$

If the power is maximized with respect to R then $(dP/dR) = 0$ and we have

$$I_{pm} + 2R(dI_{pm}/dR) = 0 \tag{3a}$$

or

$$(I_{b0} - i_{bm}) + 2R\frac{d(I_{b0} - i_{bm})}{dR} = 0. \tag{3b}$$

The maximum value of the varying component of the plate current is related to the grid signal by

$$\mu E_{gm} = I_{pm}(r_p + R) = (I_{b0} - i_{bm})(r_p + R). \tag{4}$$

Application to Specified Cases

As mentioned above, there are three specific cases of interest which are (*I*) small but fixed grid signal, (*II*) specified quiescent plate voltage, and (*III*) specified quiescent plate dissipation. These will be discussed in this order.

Case I

With a fixed grid signal $(d\,E_{gm}/d\,R) = 0$, and the optimum load resistance is obtained by differentiating (4) with respect to R and combining this with (3a).

$$0 = (r_p + R)(dI_{pm}/dR) + I_{pm} = I_{pm} + 2R(dI_{pm}/dR). \tag{5}$$

From this we obtain

$$R = r_p \tag{6}$$

for the optimum load resistor.[3]

Case II

In order to limit the distortion produced in a power amplifier it is common practice to specify that $-E_{co} = E_{gm}$ (that is the grid must not go positive) and the minimum value of the plate current should never fall below some arbitrary minimum current $i_{b\min}$ which is determined in the practical case by the true characteristics of the tube and the amount of distortion which can be tolerated. If, in addition to these limitations, the quiescent value of the plate potential E_{b0} is specified, then the optimum operating conditions and the load resistance may be determined uniquely.

The first and second conditions above yield the following two general equations which may be written directly from an inspection of the curves of Fig. 1 and the use of (1):

$$r_p(2I_{b0} - i_{b\min}) = e_{b\min} - \epsilon \tag{7}$$

since $(2I_{b0} - i_{b\min})$ is the current at point A of Fig. 1.

$$R = \frac{E_{b0} - e_{b\min}}{I_{b0} - i_{b\min}} = \frac{E_{b0} - r_p(2I_{b0} - i_{b\min}) - \epsilon}{I_{b0} - i_{b\min}}. \tag{8}$$

If we differentiate (8) and remember that both $(d\,E_{b0}/d\,R)$ and $(d\,i_{b\min}/d\,R)$ are zero, then

$$(I_{b0} - i_{b\min}) + R\frac{dI_{b0}}{dR} = -2r_p\frac{dI_{b0}}{dR}. \tag{9}$$

With the use of (3b) it follows that the optimum value of the load resistance is given by

$$R = 2r_p, \tag{10}$$

the optimum quiescent plate current is

$$I_{b0} = \tfrac{3}{4}i_{b\min} + \frac{E_{b0} - \epsilon}{4r_p} \tag{11}$$

and the optimum quiescent grid voltage is

$$E_{c0} = -\frac{3r_p}{\mu}(I_{b0} - i_{b\min}) = -\frac{3}{4\mu}(E_{b0} - \epsilon - r_p i_{b\min}). \tag{12}$$

If the conditions of this case are adhered to and R made equal to $2r_p$, then the quiescent plate dissipation P_{p0} may exceed P_{pm} which is the maximum permitted without tube damage. This results if the value of E_{b0} is too large. It is clear that under these conditions the problem is "overspecified" and it is not necessary to determine the optimum load resistance by differentiation since it may be determined directly from (8) because the maximum value of I_{b0} is determined by the specified P_{pm} and E_{b0}. The other constants of the equation may all be determined from the tube characteristics. If the value of R calculated from (8) is greater than $2r_p$, then this is the best value consistent with a specified maximum E_{b0} and P_{pm} but if this value of R is less than $2r_p$ then a choice of $R = 2r_p$ is the optimum and P_{p0} will be less than P_{pm}.

The calculation of the maximum value of E_{b0}, for which *Case II* can be said to apply without exceeding

[3] There is a value of I_{b0} for which the greatest amount of power can be delivered to the load of resistance $R = r_p$ without exceeding the quiescent plate power rating P_{pm}. This may be calculated using the equation

$$I_{b0} = [(P_{pm}/3r_p) + (i_{b\min}^2/9)(\epsilon/2r_p i_{b\min} - 1)^2]^{1/2} - (i_{b\min}/3)(\epsilon/2r_p i_{b\min} - 1) \tag{6a}$$

a specified maximum P_{pm}, is straightforward and the result is

$$(\max)\ E_{b0} = \tfrac{1}{2}\{\epsilon - 3i_{b\min}r_p + \sqrt{16r_pP_{pm} + (\epsilon - 3i_{b\min}r_p)^2}\}. \tag{13}$$

A consideration of the usual magnitudes of constants of the above equation leads to the approximation

$$(\max)\ E_{b0} \doteqdot 2\sqrt{r_pP_{pm}}. \tag{13a}$$

If the maximum value of E_{b0} permitted by the tube manufacturer is greater than that given by (13a), then (8) determines the best value of R unless this value exceeds that determined under *Case III* below. Under those circumstances the value of R determined by the equations of *Case III* is the optimum.

Case III

If the limits set by distortion are again restricted by the two conditions (1) the grid must not go positive and (2) the minimum value of the plate current must not fall below $i_{b\min}$ and in addition the maximum quiescent plate dissipation is specified as P_{pm}, then a unique solution to the problem of finding the optimum operating conditions is available.

Since $(dP_{pm}/dR) = 0$ it follows that

$$\frac{d(I_{b0}E_{b0})}{dR} = E_{b0}\frac{dI_{b0}}{dR} + I_{b0}\frac{dE_{b0}}{dR} = 0. \tag{14}$$

Equation (8) may be differentiated to obtain

$$(I_{b0} - i_{b\min}) + R\frac{dI_{b0}}{dR} = \frac{dE_{b0}}{dR} - 2r_p\frac{dI_{b0}}{dR}\cdot \tag{15}$$

Substitutions from (3b) and (14) result in the relations

$$R = 2r_p + \frac{E_{b0}}{I_{b0}} = 2r_p + \frac{P_{pm}}{I_{b0}^2}\cdot \tag{16}$$

The elimination of R between (8) and (16) gives the final relationship by which the optimum value of I_{b0} may be determined as follows:

$$\frac{P_{pm}i_{b\min}}{I_{b0}^2} = 4r_pI_{b0} - 3r_pi_{b\min} + \epsilon. \tag{17}$$

This equation is a cubic in I_{b0} depending only on known quantities[4] P_{pm}, $i_{t\min}$, r_p, and ϵ. It may be solved by plotting the left-hand side of the equation as a function of arbitrarily chosen values of I_{b0} and finding the intersection of this curve with the stright line which represents graphically the value of the right-hand side of the equation for the same range in I_{b0}. Since the relations here are so simple, it is also an easy matter, with the help of a slide rule, to find the solution to (17) by trial. After having found the optimum value of I_{b0}, the optimum load resistance R may be determined by (16) and the corresponding value of E_{b0} may be found from the relation

$$E_{b0} = \frac{P_{pm}}{I_{b0}}\cdot \tag{18}$$

Equation (19) serves to determine the optimum value of the quiescent grid voltage E_{c0}

$$E_{c0} = -\frac{(r_p + R)}{\mu}(I_{b0} - i_{b\min}). \tag{19}$$

Numerical Examples

In order to illustrate the use of these equations, a numerical example has been computed for a tube characteristic which is approximately that of the RCA 845 tube. The basic constants are as follows:

Maximum quiescent plate dissipation	$P_{pm} = 100$ watts
Plate resistance	$r_p = 1700$ ohms
Maximum quiescent plate voltage	$E_{b0} = 1250$ volts
Amplification factor	$\mu = 5.3$
Estimated intercept	$\epsilon = 85$ volts
Estimated minimum plate current	$i_{b\min} = 10\times10^{-3}$ ampere.

TABLE I

Case	E_{b0} Volts	I_{b0} Ampere	P_{pm} Watts	R Ohms	(R/r_p)	P Watts	I_{b0} Equation	R Equation
I	740	135×10^{-3}	100	1700	1.0	13.3	(6a)	(6)
II	840	119×10^{-3}	100	3400	2.0	20.0	(11)	(10)
Limited E_{b0}	1250	80×10^{-3}	100	13,000	7.65	31.9	(18)	(8)
III	1950	51×10^{-3}	100	41,500	24.4	35.3	(17)	(16)

Four examples have been worked out using the above constants and the results are tabulated in Table I. Circuit constants have been chosen so that the quiescent plate dissipation is 100 watts in every case and the power delivered to the load is computed to be the maximum consistent with the two limitations that the grid must not swing positive and that the plate current must not swing to a value less than 10×10^{-3} ampere. In the last two columns of the table, the numbers of the equations used to compute the values of I_{b0} and R are recorded. The value of E_{b0} comes from (18) in every example.

Discussion of Results

The maximum voltage for which *Case II* applies to the 845 tube is 840 volts and yet, since the manufacturer's maximum rating is between 840 volts and 1950 volts, the best operating conditions for this tube are not determined by the true maximization with respect to power delivered. The fact that the power delivered with $E_{b0} = 1250$ volts is less than 10 per cent below the absolute maximum which can be obtained under operating conditions as defined, shows that the manufacturer's rating is a practical and reasonable compromise.

In making the above calculations, $i_{b\min}$ was assumed

[4] A brief discussion of the selection of $i_{b\min}$ comes later in this paper but here it seems desirable to interpret (17) for the indeterminate case of $i_{b\min} = 0$. Here $I_{b0} \rightarrow 0$ and $E_{b0} \rightarrow \infty$ in such a way that $I_{b0}E_{b0} = P_{pm}$. Of course $R \rightarrow \infty$ at the same time.

to have the same value for every case whereas experience shows that the higher the value of the load resistance the lower the value of $i_{b\min}$ for a given distortion. There are no simple rules by which the harmonic distortion can be determined for a given tube characteristic although the graphical methods explained by Reich[5] are helpful and would serve for estimating the harmonic distortion expected after the optimum load resistance has been determined for a definite assumed value of $i_{b\min}$. If the resultant harmonic distortion is greater or less than the amount considered tolerable, then a new value of $i_{b\min}$ would have to be assumed and a new value of load resistance determined using the methods outlined here. These results are applicable to circuits with resistance feed, choke feed, or transformer feed of the plate power as long as the input impedance to the plate load is dominated by its resistance component.

[5] H. J. Reich, "Theory and Application of Electron Tubes" McGraw-Hill Book Company, New York, N. Y., 1939, p. 248.

The Calculation of Ground-Wave Field Intensity Over a Finitely Conducting Spherical Earth*

K. A. NORTON†, MEMBER, I.R.E.

Summary—Equations and curves are presented which simplify the calculation of ground-wave field intensity over a finitely conducting spherical earth for transmitting and receiving antennas of arbitrary heights and polarization.

WITHIN the last few years many theoretical papers[1–9] have appeared which deal with the calculation of ground-wave field intensity. It is the purpose of this report to summarize the results obtained in these papers and to present graphical methods for the computation of ground-wave field intensity which may be used easily by the engineer. The ground wave is here considered to be the portion of a radio wave that is propagated through space and is, ordinarily, affected by the presence of the ground; the ground wave does not include any portion of the wave reflected from anything other than the ground; it is thus exclusive of ionospheric waves (sky waves) and of tropospheric waves. The ground wave is, however, slightly refracted by the gradual decrease with height of the dielectric constant of the lower atmosphere. In the following discussion the earth is assumed to be a perfect sphere with uniform ground constants surrounded by an atmosphere in which the dielectric constant decreases uniformly with height. It has been found experimentally[10] that the departures from these idealized conditions, such as hills, buildings, trees, etc., cause large variations from the calculated values at particular distances but the theory does provide an excellent guide to the *average* fields encountered in practice.

The transmitting antenna is assumed to be either a vertical electric doublet or a vertical magnetic doublet[11] with a free-space field intensity at a unit distance equal to E_0 in the equatorial plane of the doublet. The theory is applicable for other transmitting antennas provided an appropriate value of E_0 is used and the directivity considered; for example, with 1 kilowatt radiated from a half-wave dipole in free space, $E_0 = 137.6$ millivolts per meter at 1 mile in the equatorial plane of the dipole; this value becomes equal to 97.3 for a vertical quarter-wave antenna at the surface of a perfectly conducting earth. The equations to be given are for the vertical electric field as received on a vertical electric doublet or for the horizontal electric field as received on a vertical magnetic doublet; the

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† Federal Communications Commission, Washington, D. C.

[1] Balth. van der Pol and H. Bremmer, "The diffraction of electromagnetic waves from an electrical point source round a finitely conducting sphere, with applications to radiotelegraphy and the theory of the rainbow," Part I, *Phil. Mag.*, vol. 24, pp. 141–176; July, 1937; Part II, *Phil. Mag.*, vol. 24, pp. 825–864, supplement, November, 1937.

[2] Balth. van der Pol and H. Bremmer, "Ergebnisse einer Theorie über die Fortpflanzung elektromagnetischer Wellen über eine Kugel endlicher Leitfähigkeit," *Hochfrequenz. und Elektroakustic*, Band 51, Heft 6, pp. 181–188; June, 1938.

[3] Balth. van der Pol and H. Bremmer, "The propagation of radio waves over a finitely conducting spherical earth," *Phil. Mag.*, vol. 25, pp. 817–834; June, 1938.

[4] Balth. van der Pol and H. Bremmer, "Further note on the propagation of radio waves over a finitely conducting spherical earth," *Phil. Mag.*, vol. 27, pp. 261–275; March, 1939.

[5] T. L. Eckersley and G. Millington, "Application of the phase integral method to the analysis of the diffraction and refraction of wireless waves round the earth," *Phil. Trans. Roy. Soc.*, vol. 237, pp. 273–309; June, 1938.

[6] T. L. Eckersley and G. Millington, "The diffraction of wireless waves round the earth," *Phil. Mag.*, vol. 27, pp. 517–542; May, 1939.

[7] T. L. Eckersley and G. Millington, "The experimental verification of the diffraction analysis of the relation between height and gain for radio waves of medium lengths," *Proc. Phys. Soc.*, vol. 51, pp. 805–809; September, 1939.

[8] Marion C. Gray, "Horizontally polarized electromagnetic waves over a spherical earth," *Phil. Mag.*, vol. 27, pp. 421–436; April, 1939.

[9] K. A. Norton, "The propagation of radio waves over the surface of the earth and in the upper atmosphere," Part I, PROC. I.R.E., vol. 24, pp. 1367–1387; October, 1936; Part II, PROC. I.R.E., vol. 25, pp. 1203–1236; September, 1937.

[10] E. W. Chapin and K. A. Norton, "Field intensity survey of ultra high frequency broadcasting stations," Federal Communications Commission mimeograph No. 40004. Presented, Hearing before the Federal Communications Commision in the Matter of Aural Broadcasting on Frequencies above 25,000 Kilocycles," March 18, 1940.

[11] A vertical magnetic doublet is a small loop antenna with its axis vertical; such an antenna radiates only horizontally polarized waves.

theory is applicable for other receiving antennas provided the directivity relative to the doublet is appropriately considered.

The ground-wave field intensity at short distances may be calculated by means of the following formula which was derived on the assumption that the earth may be considered to be a plane:

$$E = \frac{E_0}{d}\Big[\overbrace{\overbrace{\cos^3\psi_1 e^{i2\pi r_1/\lambda}}^{\text{direct wave}} + \overbrace{R\cos^3\psi_2 e^{i2\pi r_2/\lambda}}^{\text{ground-reflected wave}}}^{\text{space wave}} + \overbrace{(1-R)f(P,B)\cos^2\psi_2 e^{i[2\pi(r_2/\lambda)+\phi]}}^{\text{surface wave}}\Big]. \qquad (1)$$

The quantities d, r_1, r_2, h_1, h_2, ψ_1, and ψ_2 are defined in Fig. 1; $i\equiv\sqrt{-1}$; E and E_0 are to be expressed in the same

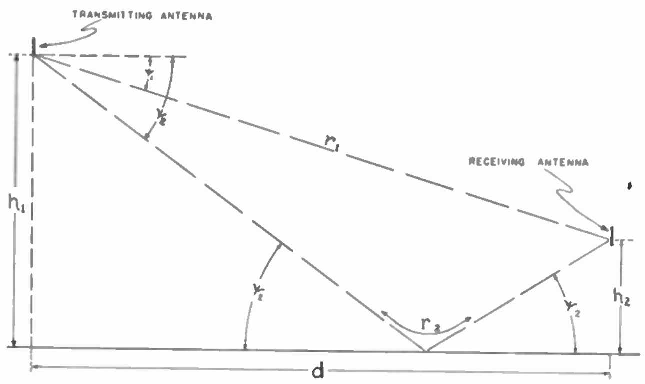


Fig. 1—Geometry for plane-earth calculations.

units; the distances, d, r_1, and r_2, and λ, the wavelength, are to be expressed in the same units used for the transmitting- and receiving-antenna heights h_1 and h_2.

$$\left.\begin{array}{ll} \tan\psi_1 = \dfrac{h_1 - h_2}{d} & \tan\psi_2 = \dfrac{h_1 + h_2}{d} \\[2ex] r_1 = \dfrac{d}{\cos\psi_1} & r_2 = \dfrac{d}{\cos\psi_2} \end{array}\right\}. \qquad (2)$$

The heights of the antennas h_1 and h_2 are to be measured from the average level of the terrain in their vicinity up to the mid-point of the antenna. The first term in the square brackets of (1) corresponds to the "direct wave," the second term to the "ground-reflected wave" and the third term to the "surface wave." The sum of the first two waves, i.e., the direct wave and the ground-reflected wave, is called the "space wave."[12] R is the plane-wave reflection coefficient of the ground. It is given by the following formula:

$$R = \frac{\dfrac{(q_1+q_2)}{2p}e^{i(\pi/4-b/2)} - 1}{\dfrac{(q_1+q_2)}{2p}e^{i(\pi/4-b/2)} + 1}. \qquad (3)$$

[12] A more complete discussion of these space and surface waves is given in the paper by the author "Physical reality of space and surface waves in the radiation field of radio antennas," PROC. I.R.E., vol. 25, pp. 1192–1202; September, 1937.

In the above equation p is the "numerical distance," a parameter introduced by Sommerfeld in 1909, while q_1 and q_2 are the numerical antenna heights of the transmitting and receiving antennas.

$$p = \pi\frac{r_2}{\lambda}\frac{\cos^2 b''}{x\cos b'} \quad \text{(vertical polarization)} \qquad (4a)$$

$$p = \pi\frac{r_2}{\lambda}\frac{x}{\cos b'} \quad \text{(horizontal polarization)} \qquad (4b)$$

$$b = 2b'' - b' \quad \text{(vertical polarization)} \qquad (5a)$$

$$b = 180° - b' \quad \text{(horizontal polarization)} \qquad (5b)$$

$$q_{1,2} = \frac{2\pi h_{1,2}}{\lambda}\left[\frac{\cos^2 b''}{x\cos b'}\right]^{1/2} \quad \text{(vertical polarization)} \qquad (6a)$$

$$q_{1,2} = \frac{2\pi h_{1,2}}{\lambda}\left[\frac{x}{\cos b'}\right]^{1/2} \quad \text{(horizontal polarization)} \qquad (6b)$$

$$x = \frac{1.79731\cdot 10^{15}\,\sigma_{\text{e.m.u.}}}{f_{mc}} \qquad (7)$$

$\sigma_{\text{e.m.u.}}$ = ground conductivity expressed in electromagnetic units

f_{mc} = frequency expressed in megacycles per second

$$\tan b' = (\epsilon - \cos^2\psi_2)/x \qquad (8)$$

$$\tan b'' = \epsilon/x \qquad (9)$$

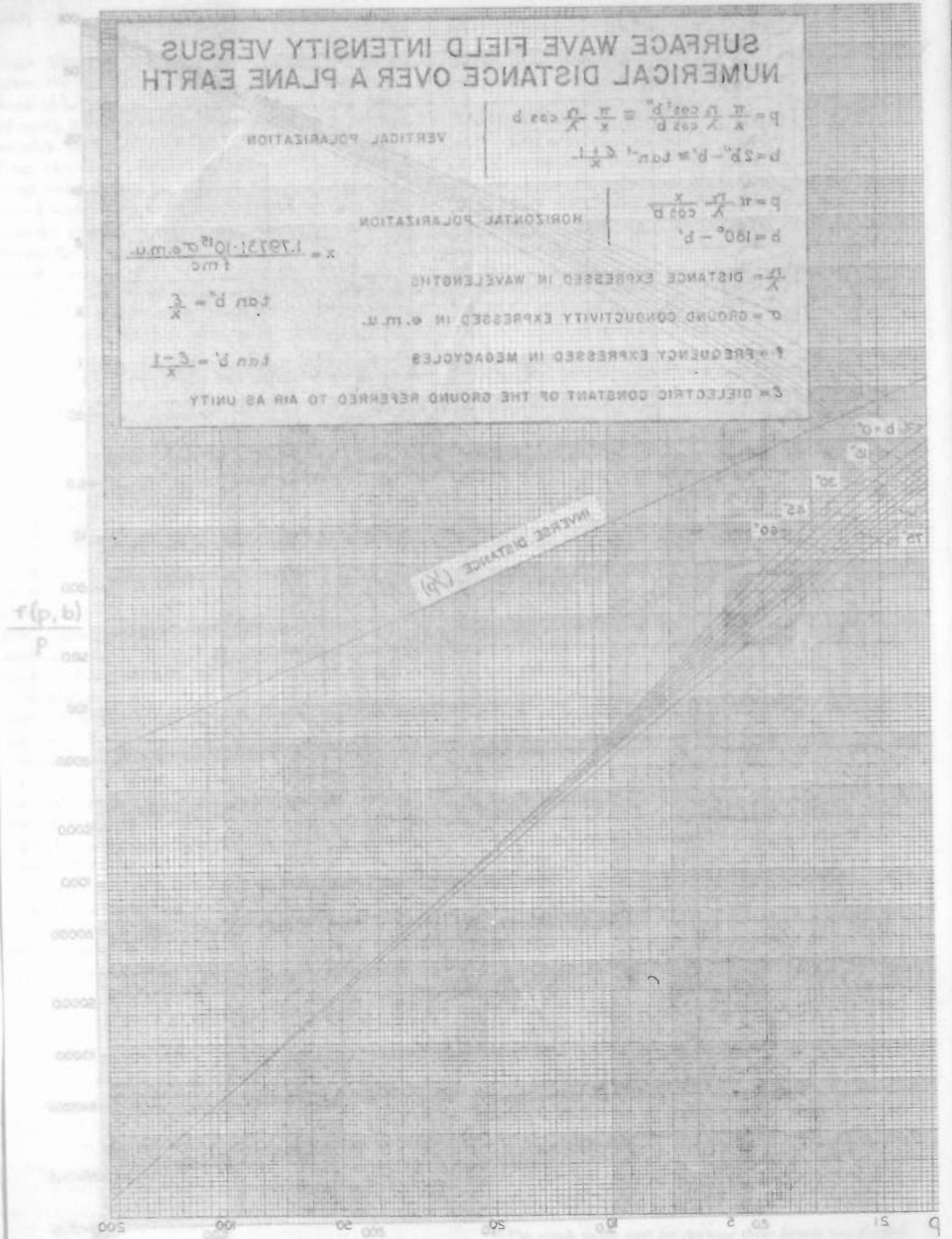
ϵ = dielectric constant of the ground referred to air as unity.

When the transmitting and receiving antennas are elevated above the earth, the "numerical distance" p and the angle b become equal to P and B as defined in the following equation:

$$Pe^{iB} \equiv p\left[1 + \frac{(q_1+q_2)}{2p}e^{i(\pi/4-b/2)}\right]^2 e^{ib} \equiv \frac{4pe^{ib}}{(1-R)^2}. \qquad (10)$$

Thus P and B are the values of p and b corresponding to elevated transmitting and receiving antennas. The function $f(P,B)\,e^{i\phi}$, appearing in the third term of (1), is the surface-wave attenuation function; the values of $f(P,B)$ and of ϕ for antennas on the surface of the earth may be obtained from Figs. 2 and 3, which are graphs of $f(p,b)/p$ versus p and b, and of ϕ versus p and b. For elevated transmitting and receiving antennas the values of $f(P,B)$ and of ϕ may be obtained from Figs. 2 and 3 by using the parameters P and B in place of p and b.

Equation (1) is a general formula applicable for short distances (such that the earth may be considered to be a plane) at any radio frequency, for any set of ground constants encountered in practice, and at any point in space such that the transmitting and receiving antennas are greater than a wavelength apart and are not too far above the earth; the modifications of this formula introduced by the curvature of the earth at


SURFACE WAVE FIELD INTENSITY VERSUS NUMERICAL DISTANCE OVER A PLANE EARTH
VERTICAL POLARIZATION
HORIZONTAL POLARIZATION
INVERSE DISTANCE (1/p)
RELATIVE FIELD INTENSITY
f(p, b) / p
DISTANCE

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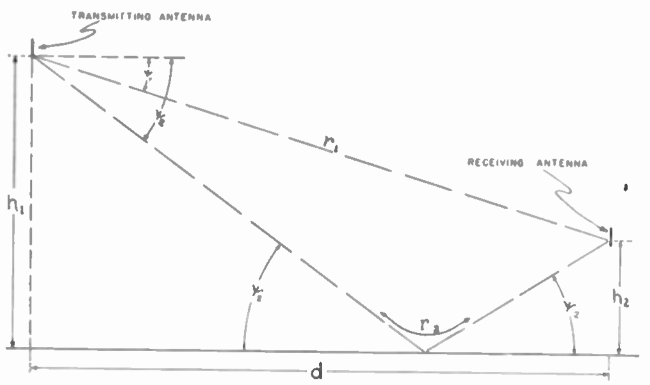

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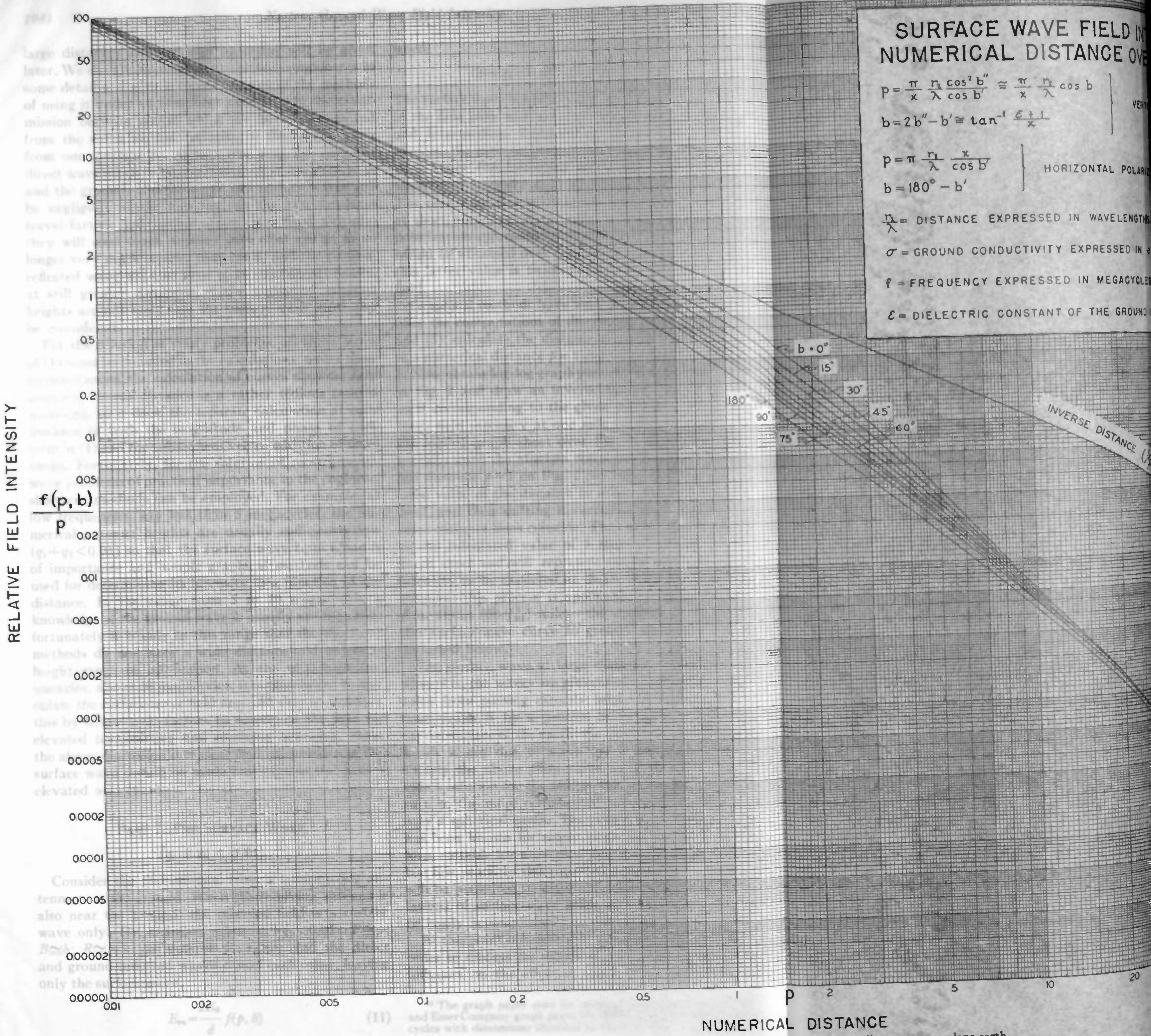
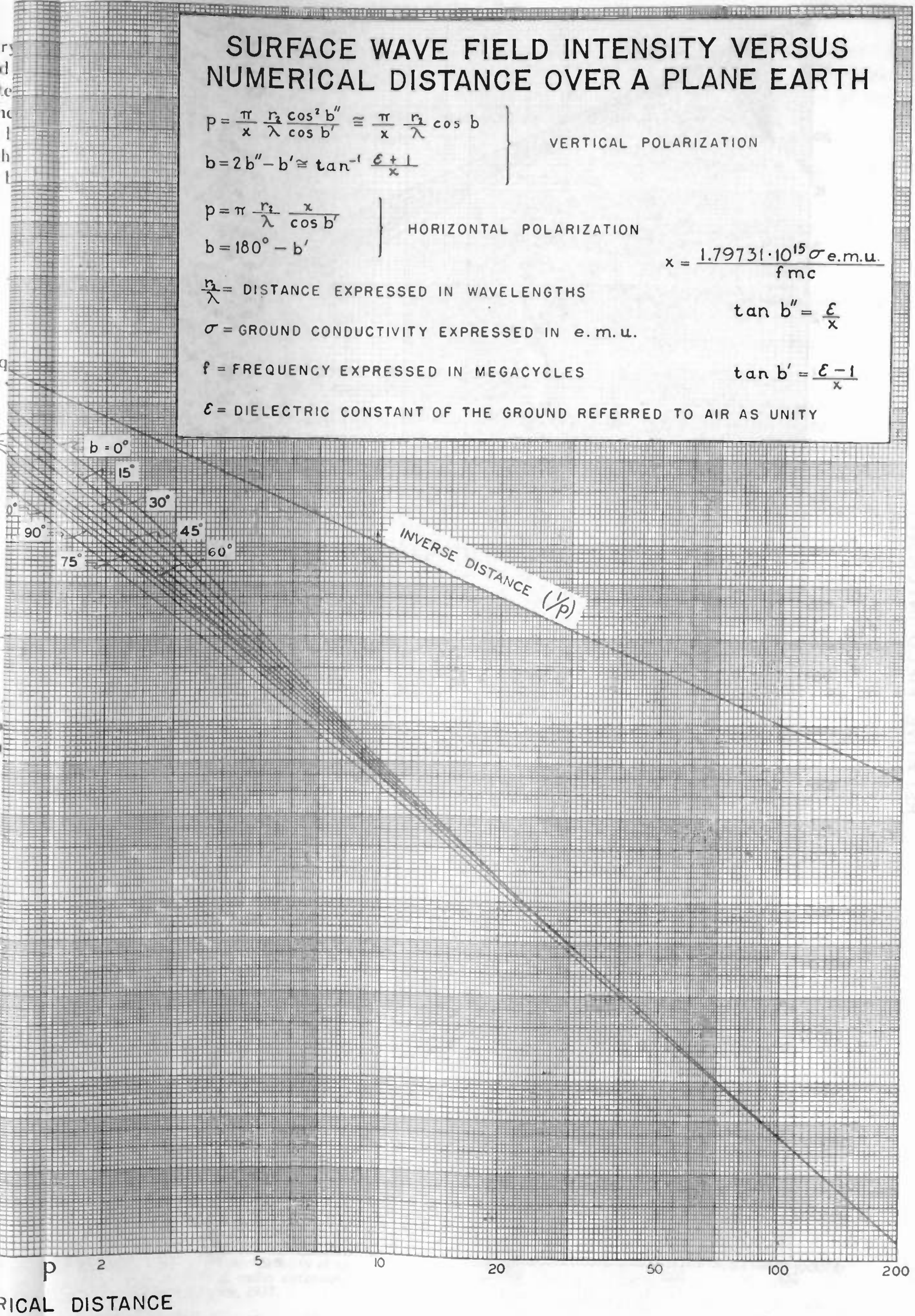


Fig. 2—Surface-wave field intensity versus numerical distance over a plane earth.



ity versus numerical distance over a plane earth.

large distances and for high antennas will be given later. We shall now consider its various components in some detail and give several examples of the methods of using it under various circumstances. In the transmission between points which are both far removed from the earth as, for example, in the transmission from one airplane to another one near by, only the direct wave needs to be considered since $\cos \psi_1 \gg \cos \psi_2$ and the ground-reflected wave and surface waves will be negligibly small; however, as the two airplanes travel farther and farther apart (at a fixed altitude) they will soon reach a point such that $\cos \psi_2$ is no longer very much smaller than $\cos \psi_1$ and the ground-reflected wave will also need to be considered; finally, at still greater distances, if the numerical antenna heights are not too great, the surface wave must also be considered.

For the solution of many problems, all three terms of (1) must be included in the calculations; under these circumstances, the calculation of curves showing field intensity versus distance is a rather tedious process involving, as it does, the separate calculation at each distance of both the magnitude and phase of each term in (1) and the subsequent vector addition of these terms. Fortunately, for the solution of most ground-wave problems of practical importance to the engineer, short-cut methods can be employed. For example, at low frequencies, say less than 5 megacycles, the numerical antenna heights are usually sufficiently low $(q_1+q_2<0.01)$ so that the surface-wave term alone is of importance and simple graphical methods can be used for determining its intensity as a function of the distance. In the range from 5 to 20 megacycles, a knowledge of the ground wave is usually not required; fortunately it is only in this range that the short-cut methods do not have a wide distance and antenna-height range of application. At the ultra-high frequencies, above 20 megacycles, it is convenient to calculate the surface-wave field first and then to multiply this by height-gain factors to determine the field for elevated transmitting and receiving antennas. From the above discussion it is clear that calculations of the surface wave should be made first even in the case of elevated antennas.

Part I. The Surface Wave

$$(q_1 + q_2 < 0.01)$$

Consider the transmission from a transmitting antenna near the ground. When the receiving antenna is also near the ground, the received field is a surface wave only; for example, when $q_1+q_2<0.01$: $P\cong p$; $B\cong b$; $R\cong -1$; $\cos \psi_2 \cong \cos \psi_1$; $r_2 \cong r_1$, and the direct and ground-reflected waves cancel each other, leaving only the surface wave,

$$E_{su}=\frac{2E_0}{d} f(p, b) \tag{11}$$

where

$$(q_1+q_2<0.01);\ d<(50/f_{mc}^{1/3}) \text{ miles.}$$

The surface wave is a practically important component of the field only with vertical polarization and at low and intermediate frequencies; under these circumstances, the inverse-distance field intensity is referred to 2 times the free-space field and the inverse-distance field intensity at a unit distance is thus $2\ E_0$. The reason for this is the surface-wave field intensity at short numerical distances $(p<0.001)$ with vertical polarization approaches very closely to the value which would be obtained over a perfectly conducting plane; this latter value is twice the value of the free-space field.

Graphical methods have been developed which simplify the computations of the surface wave. Using (4) and (5), calculate the distance in miles corresponding to a numerical distance $p=1$ and calculate the value of b. Now obtain log-log graph paper[13] similar to that used in Fig. 2 and draw an inverse-distance line $(2\ E_0/d)$ on it corresponding to the given value of inverse-distance field intensity at one mile $(2\ E_0)$. Superimpose the log-log graph sheet over Fig. 2, shifting it horizontally until the abscissa corresponding to a numerical distance $p=1$ on Fig. 2 coincides with the calculated value of the distance in miles corresponding to $p=1$, and then shifting it vertically until the two inverse-distance lines coincide. The curve corresponding to the calculated value of b may be traced on the graph sheet and will represent the ground-wave field intensity to be expected at short distances from the antenna; this process is applicable for distances less than about $(50/f_{mc}^{1/3})$ miles—the method of extending this surface-wave curve to greater distances will be discussed below.

The surface wave at large distances is a diffracted wave; i.e., the waves are prevented by the bulge of the earth from passing directly into this region and so must reach it by a process of bending around the curved surface of the earth; the portion of this bending which is not due to refraction is called diffraction. Clearly the above plane-earth methods will not provide a solution for the field in this region and we must turn to the more complicated equations for the field over a spherical earth. The solution of this problem has been known for many years but it was put in a form suitable for numerical interpretation only in the last few years. In this report the more recent solutions will be presented in graphical form so that the calculations of surface-wave fields at large distances may be easily accomplished.

At this point it is desirable to digress for a moment in order to discuss the effects of air refraction and their influence on the effective value of the radius of the

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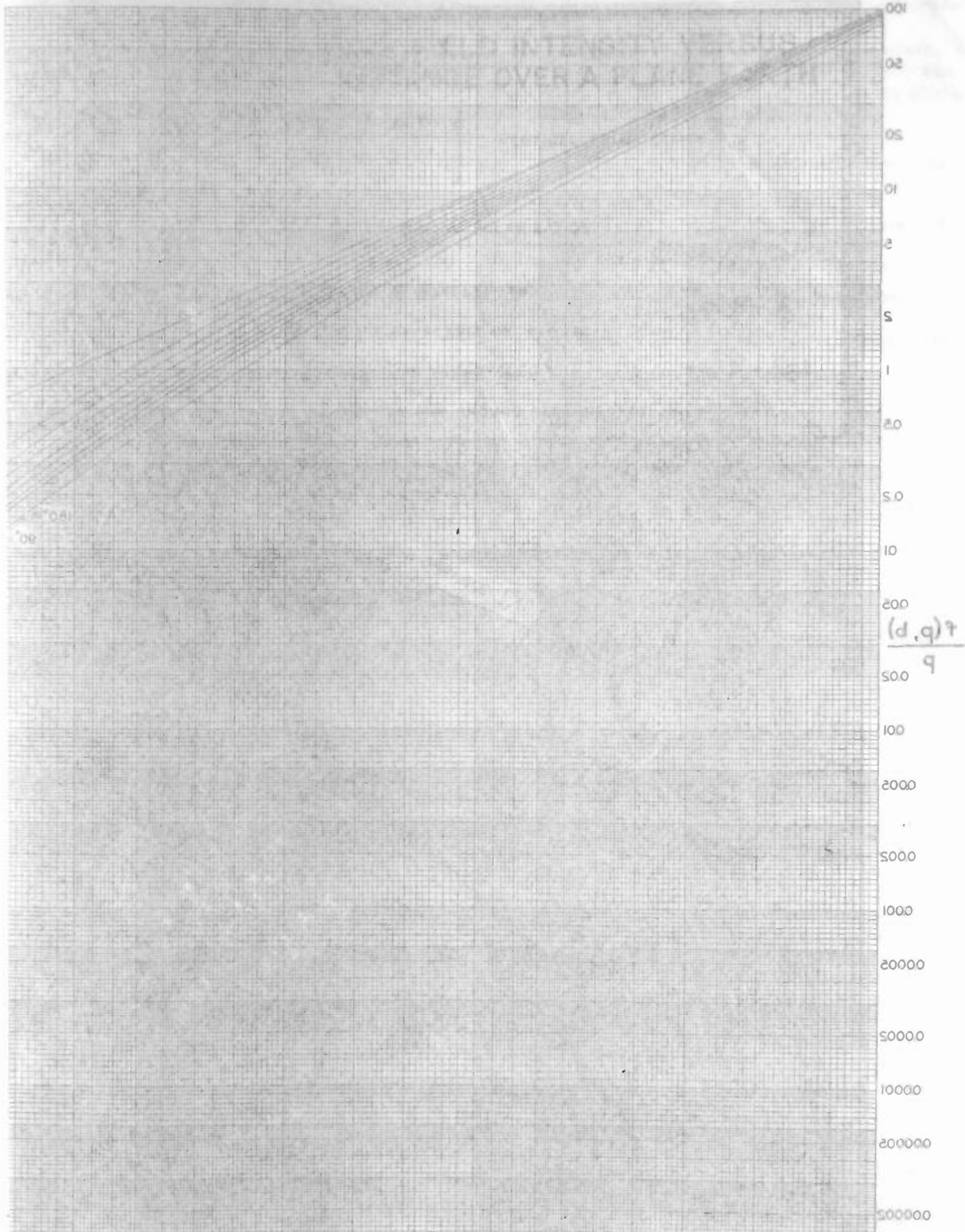

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Fig. 2—Surface-wave field-intensity ver

large distances and for high antennas will be given later. We shall now consider its various components in some detail and give several examples of the methods of using it under various circumstances. In the transmission between points which are both far removed from the earth as, for example, in the transmission from one airplane to another one near by, only the direct wave needs to be considered since $\cos\psi_1 \gg \cos\psi_2$ and the ground-reflected wave and surface waves will be negligibly small; however, as the two airplanes travel farther and farther apart (at a fixed altitude) they will soon reach a point such that $\cos\psi_2$ is no longer very much smaller than $\cos\psi_1$ and the ground-reflected wave will also need to be considered; finally, at still greater distances, if the numerical antenna heights are not too great, the surface wave must also be considered.

For the solution of many problems, all three terms of (1) must be included in the calculations; under these circumstances, the calculation of curves showing field intensity versus distance is a rather tedious process involving, as it does, the separate calculation at each distance of both the magnitude and phase of each term in (1) and the subsequent vector addition of these terms. Fortunately, for the solution of most ground-wave problems of practical importance to the engineer, short-cut methods can be employed. For example, at low frequencies, say less than 5 megacycles, the numerical antenna heights are usually sufficiently low $(q_1+q_2<0.01)$ so that the surface-wave term alone is of importance and simple graphical methods can be used for determining its intensity as a function of the distance. In the range from 5 to 20 megacycles, a knowledge of the ground wave is usually not required; fortunately it is only in this range that the short-cut methods do not have a wide distance and antenna-height range of application. At the ultra-high frequencies, above 20 megacycles, it is convenient to calculate the surface-wave field first and then to multiply this by height-gain factors to determine the field for elevated transmitting and receiving antennas. From the above discussion it is clear that calculations of the surface wave should be made first even in the case of elevated antennas.

Part I. The Surface Wave

$$(q_1 + q_2 < 0.01)$$

Consider the transmission from a transmitting antenna near the ground. When the receiving antenna is also near the ground, the received field is a surface wave only; for example, when $q_1+q_2<0.01$: $P\cong p$; $B\cong b$; $R\cong -1$; $\cos\psi_2\cong\cos\psi_1$; $r_2\cong r_1$, and the direct and ground-reflected waves cancel each other, leaving only the surface wave,

$$E_{su}=\frac{2E_0}{d}f(p,b) \tag{11}$$

where

$$(q_1+q_2<0.01);\ d<(50/f_{mc}^{\frac{1}{3}})\text{ miles.}$$

The surface wave is a practically important component of the field only with vertical polarization and at low and intermediate frequencies; under these circumstances, the inverse-distance field intensity is referred to 2 times the free-space field and the inverse-distance field intensity at a unit distance is thus $2E_0$. The reason for this is the surface-wave field intensity at short numerical distances $(p<0.001)$ with vertical polarization approaches very closely to the value which would be obtained over a perfectly conducting plane; this latter value is twice the value of the free-space field.

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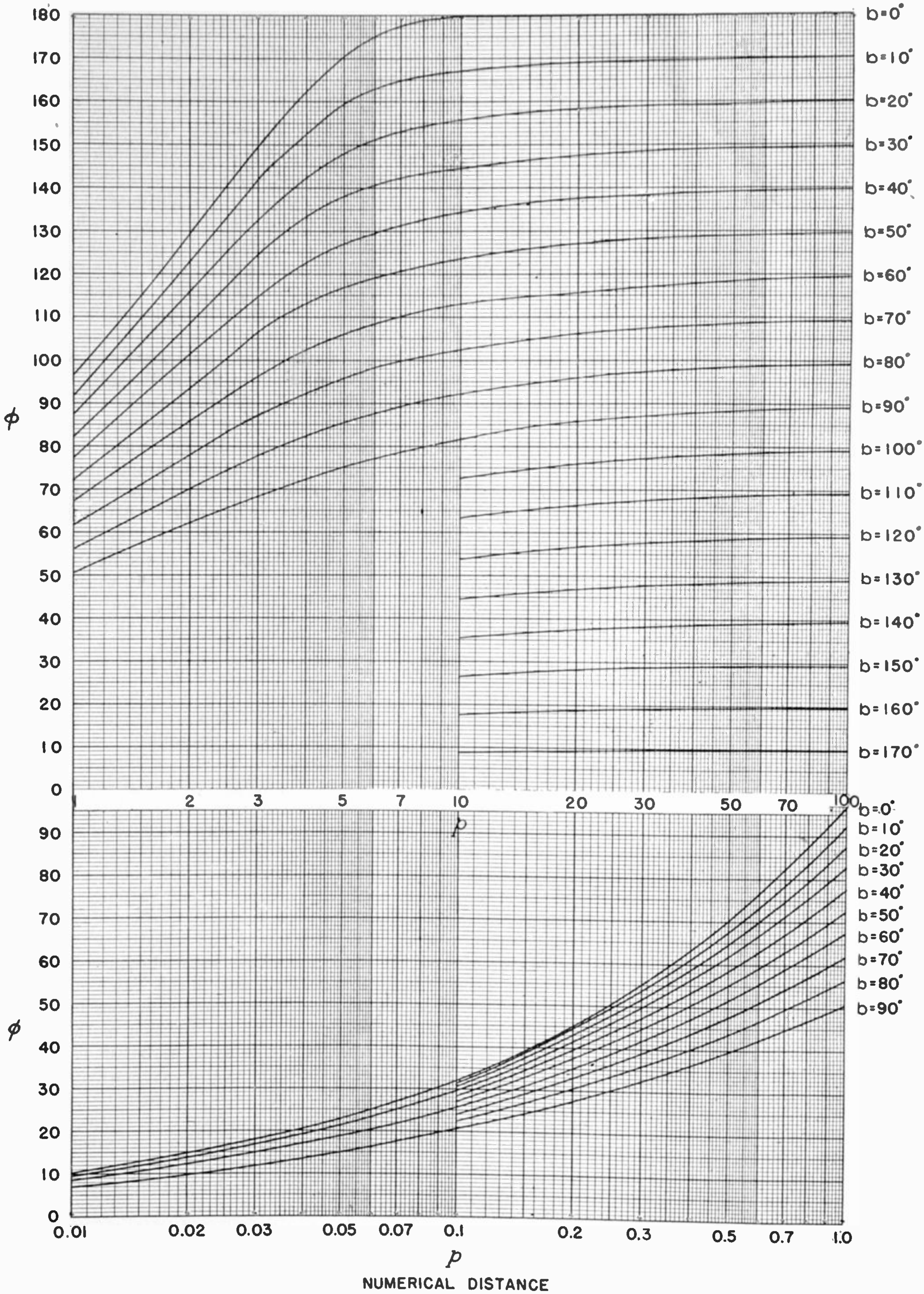

Fig. 3—The phase of the surface-wave attenuation function.

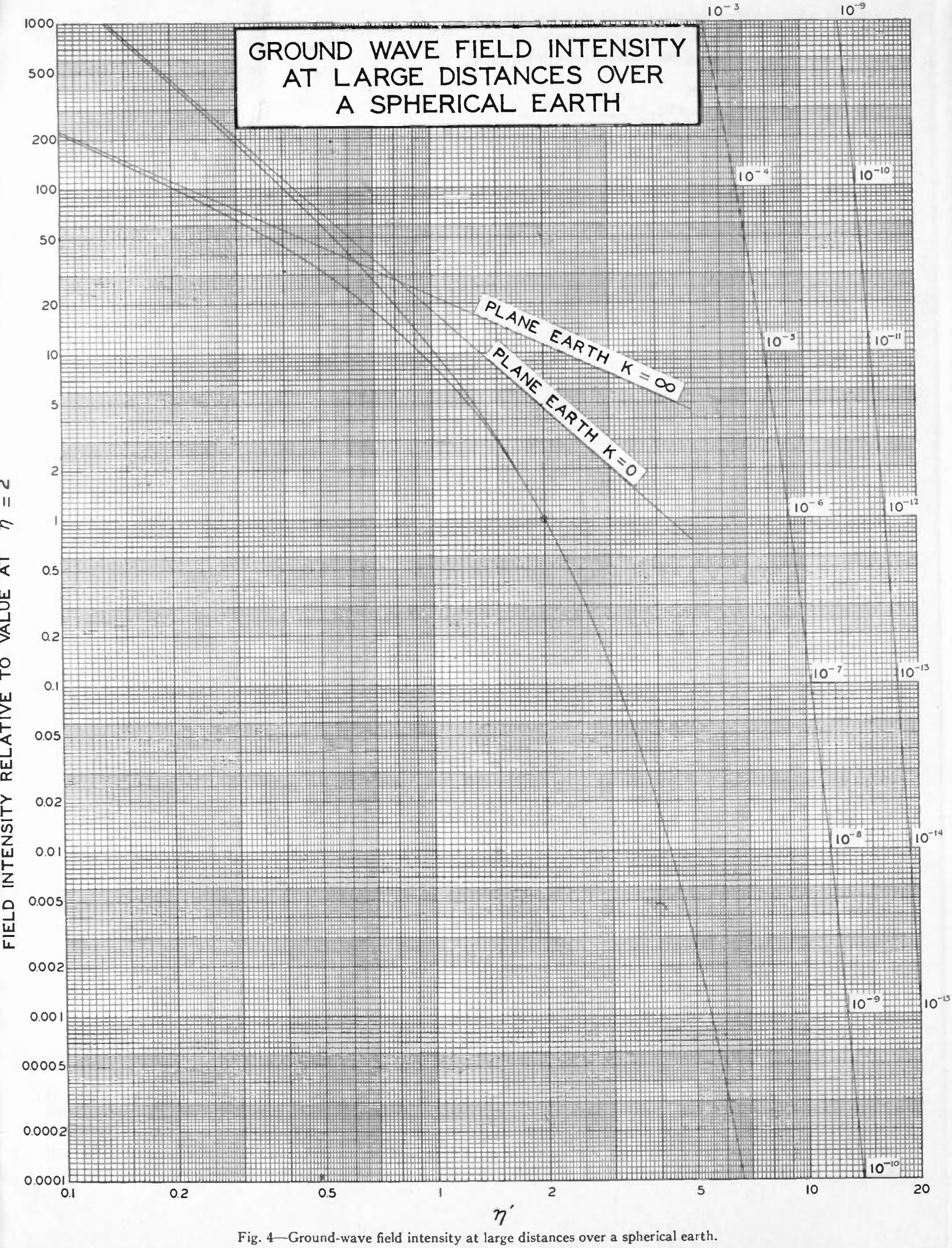

Fig. 4—Ground-wave field intensity at large distances over a spherical earth.

earth. It is known that the refractive index n of the air decreases with the height h above the earth and this has the effect of refracting the radio waves downward

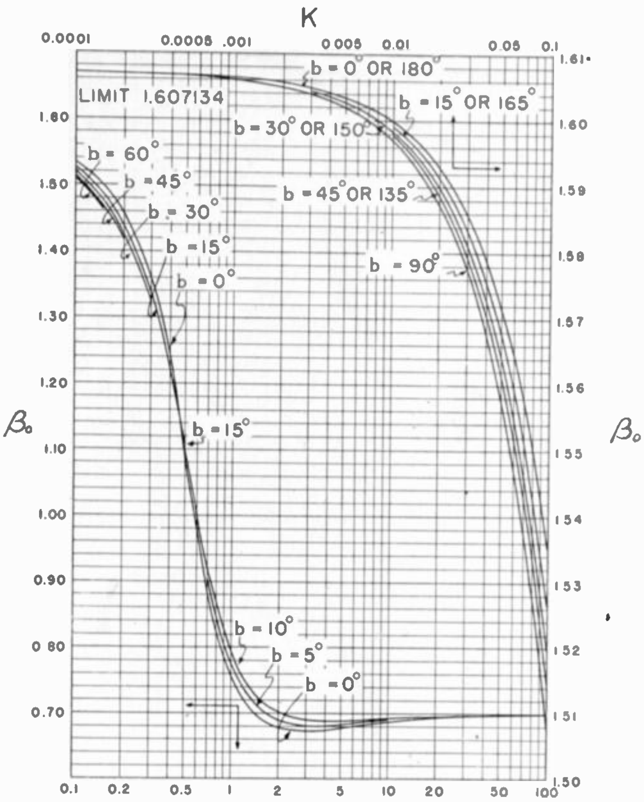

Fig. 5—The parameter β_0 as a function of K and b. (See also Appendix III.)

toward the earth; this systematic effect of air refraction in the lower atmosphere may be approximately included in the calculations by using an effective radius of the earth which is k times the actual radius. Thus, if we assume that the refractive index of the air changes with height above the earth at the uniform rate dn/dh, then k may be expressed as

$$k = 1/\left(1 + \frac{a}{n}\cdot\frac{dn}{dh}\right) \tag{12}$$

where a is the actual radius of the earth expressed in the same units used for the height h. There is some experimental evidence (both from the standpoint of wave propagation and the analysis of meteorological data) to indicate that k has a value equal to about 4/3 on the average so that the effective radius of the earth ka is equal to about 5280 miles; however, due to variations in the gradient of the dielectric constant of the air with weather changes, k may be expected to be variable from hour to hour, day to day, and season to season and may range from less than 1 up to an infinitely large value.

The nature of the surface-wave field at large distances is determined as a function of frequency and ground constants primarily through the parameters K and b, the latter being defined in (5a) and (5b).

$$K = \left[\frac{\lambda}{2\pi ka}\right]^{1/3}\cdot\left[\frac{x\cos b'}{\cos^2 b''}\right]^{1/2} \quad \text{(vertical polarization)} \tag{13a}$$

$$K = \left[\frac{\lambda}{2\pi ka}\right]^{1/3}\cdot\left[\frac{\cos b'}{x}\right]^{1/2} \quad \text{(horizontal polarization).} \tag{13b}$$

It will be found that K is always very much smaller for horizontal polarization than for vertical polarization.

Fig. 4 shows the relative values of field intensity at large distances over a spherical earth as a function of the parameter η' where

$$\eta' = \beta_0\eta_0 d \tag{14}$$

$$\eta_0 = (k^2a^2\lambda)^{-1/3} \tag{15}$$

and β_0 is given graphically as a function of K and b in Fig. 5. Fig. 4 is to be used in the following way to extend the calculations of the surface wave to large distances. First calculate the field intensity corresponding to $\eta' = 2$ by means of the following formula:

$$E_{su(\eta'=2)} = 2E_0\eta_0\gamma. \tag{16}$$

The parameter γ is given graphically as a function of K and b in Fig. 6; the formula for γ is given in (52) in Appendix II. Next calculate the distance d corresponding to $\eta' = 2$ by means of the following formula:

$$d_{(\eta'=2)} = \frac{2}{\beta_0\eta_0}. \tag{17}$$

Now plot the value of field intensity as given by (16) at the distance given by (17) on the transparent log-

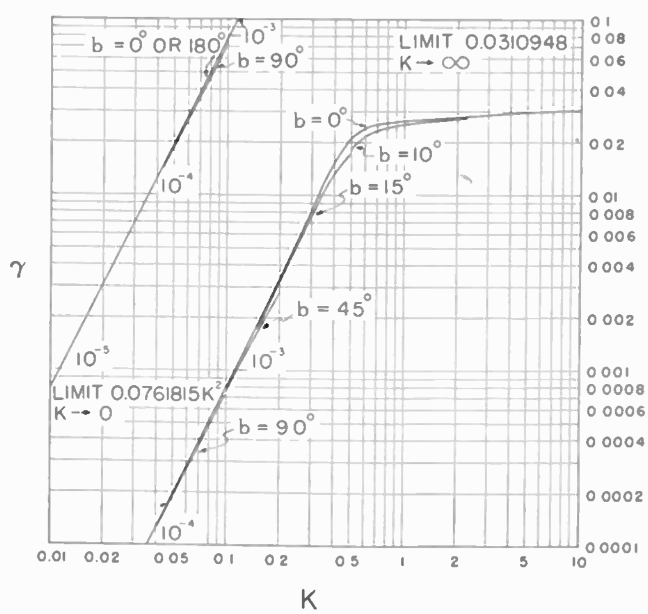

Fig. 6—The parameter γ as a function of K and b. (See also Appendix III.)

log graph sheet containing the surface-wave curve as determined at short distances by the plane-earth method. Finally, superimpose the log-log graph sheet over Fig. 4 shifting it vertically and horizontally until the plotted point coincides with the curve on Fig. 4 at

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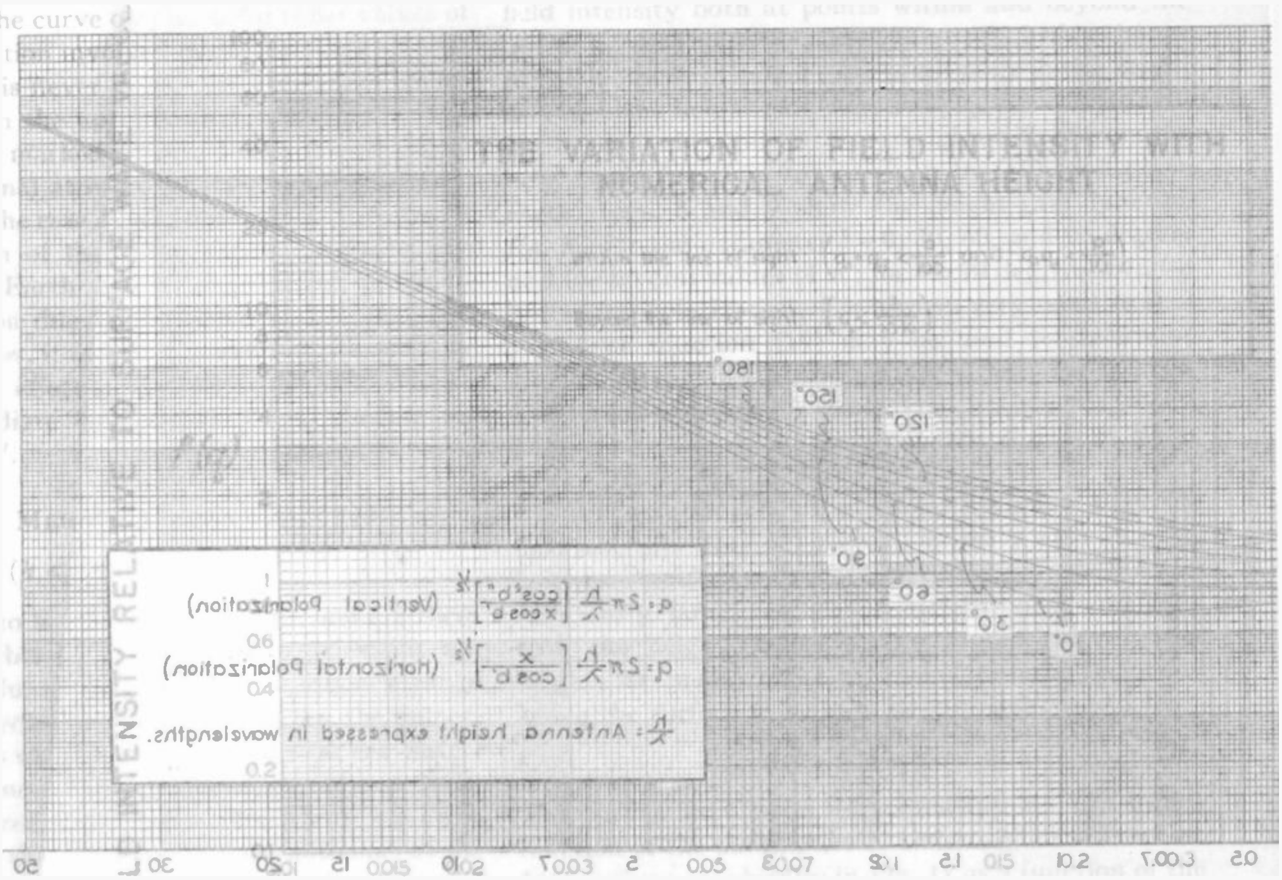

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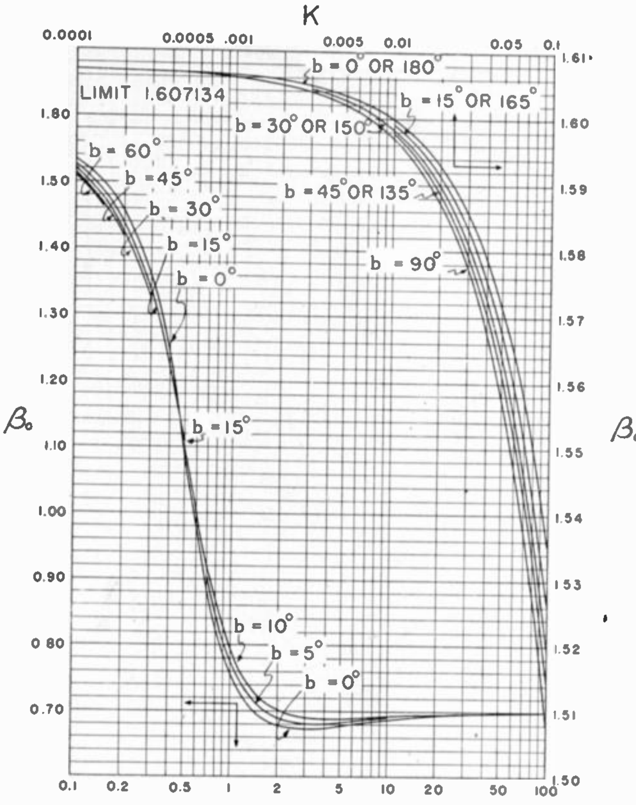

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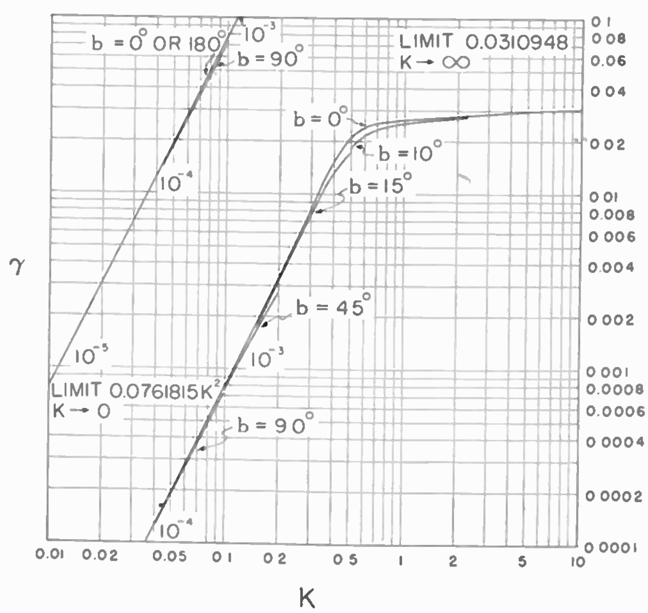

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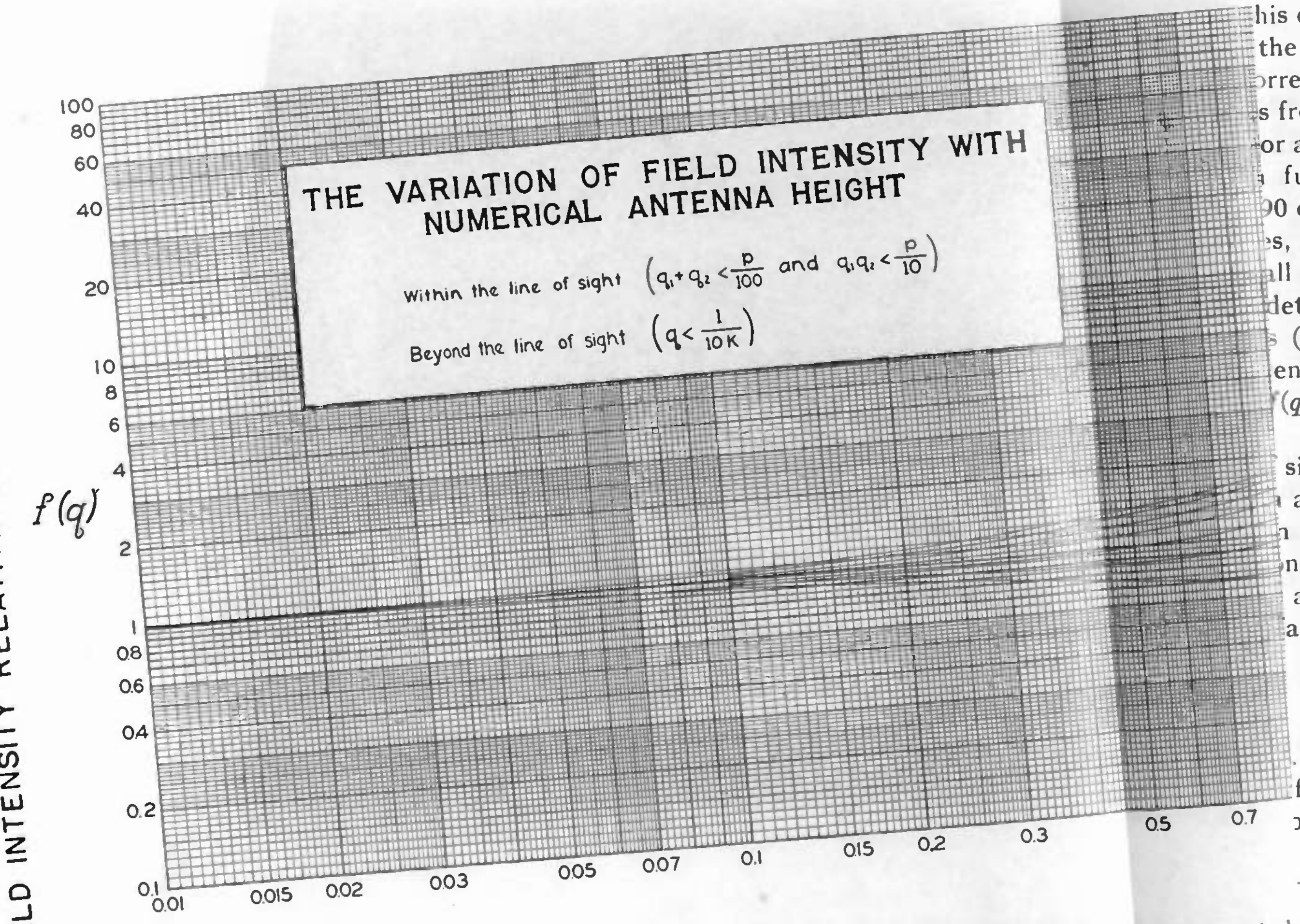

Fig. 7—The variation of field intensity with numerical

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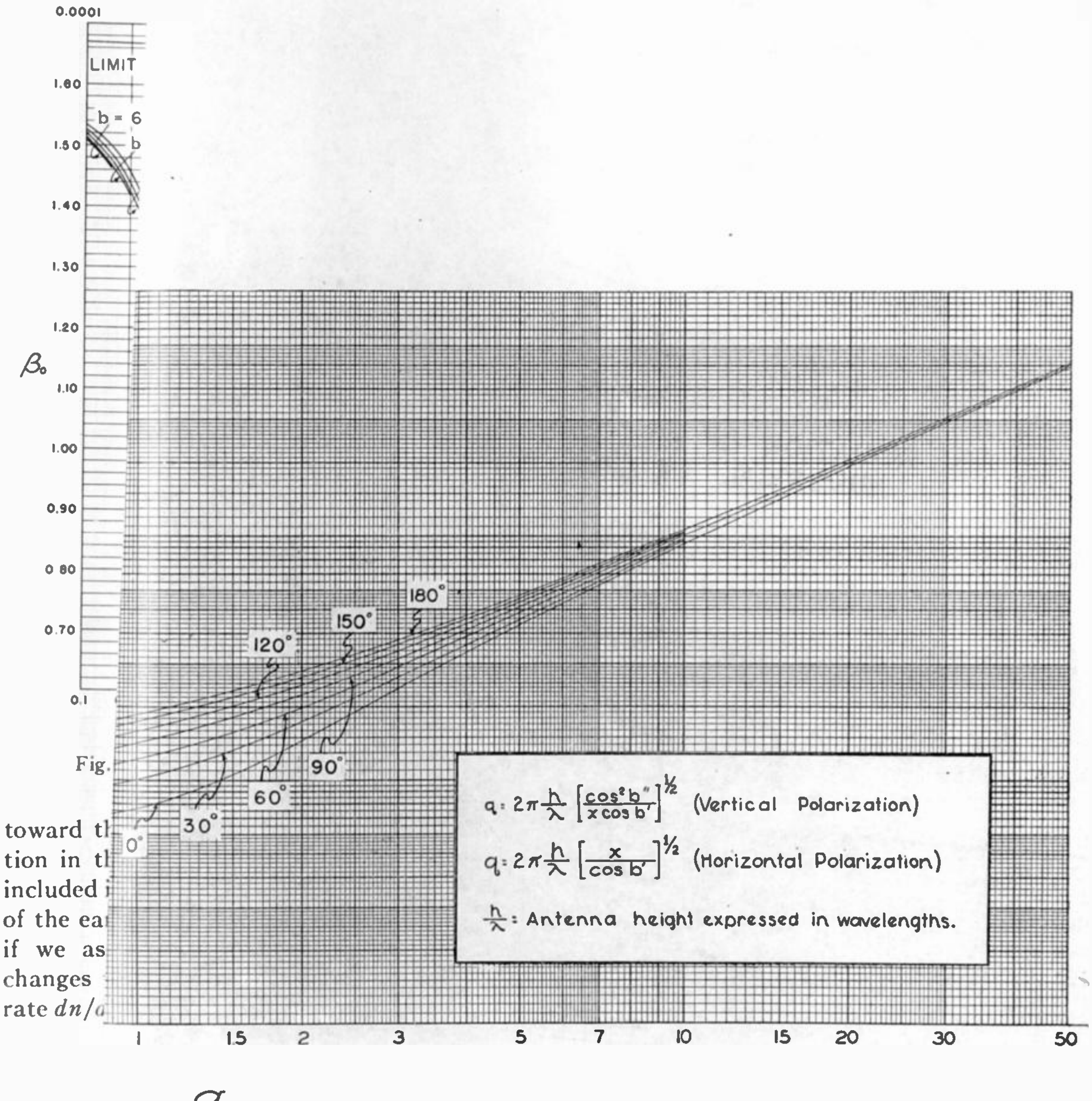

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Part II. Medium Antenna Heights

$(h<(2000/f_{mc}^{\frac{1}{3}})$ feet)

The methods to be outlined in this section are theoretically applicable at all radio frequencies but are particularly useful in the ultra-high-frequency range above 20 megacycles. At these frequencies, the numerical distance p is usually large (except over sea water) when the distance d is greater than, say, 1 mile. Thus, at distances such that $p>20$, $p>10\ q_1q_2$, and $p>100\ (q_1+q_2)$, the field intensity for antennas at the numerical antenna heights q_1 and q_2 may be expressed by the following simple formula:

$$E = E_{su}\cdot f(q_1)\cdot f(q_2) \tag{18}$$

where

$$h < (2000/f_{mc}^{\frac{1}{3}}) \text{ feet}; \quad p > 20;$$

$$p > 10 q_1 q_2; \quad p > 100(q_1+q_2)$$

$$f(q) = \left[1 + q^2 - 2q\cos\left(\frac{\pi}{4}+\frac{b}{2}\right)\right]^{1/2}. \tag{19}$$

The derivation of (18) for the case of a plane earth is given in Appendix I. This equation is also applicable, however, to calculation over a spherical earth at sufficiently small heights (determined approximately by the relation $h<(2000/f_{mc}^{\frac{1}{3}})$ feet) such that the height-gain function is independent of the parameter K. (See Part III on High Antennas and equations (53) and (54) in Appendix II.) Thus, once the curve of field intensity versus distance for the surface wave E_{su} has been properly determined by the methods of Part I, it is only necessary to multiply these values by the height-gain functions $f(q_1)$ and $f(q_2)$ for the transmitting and receiving antennas in order to determine the field at these heights. The function $f(q)$ is given graphically for various values of b in Fig. 7.

Part III. High Antennas

$(h>(2000/f_{mc}^{\frac{1}{3}})$ feet)

When the transmitting or receiving antenna is high, the curvature of the earth affects the calculations of field intensity both at points within and beyond the line of sight. The antenna height at which this change in the methods of calculation takes place is the height at which the height-gain function $f(q, K)$ corresponding to points beyond the line of sight, differs from the height-gain function $f(q)$, as given by (19) for a plane earth. Figs. 8 and 9 show the height-gain function $f(q, K)$ for the parameter $b=0$, 30, 60, and 90 degrees and for several values of K; for $b>90$ degrees, i.e., for horizontal polarization, K is always very small so that Fig. 7 or equation (19) may be used to determine $f(q, K)$ for small numerical antenna heights $(q<50)$. The formula for $f(q, K)$ at small heights is given in (53) in Appendix II; it will be noted that $f(q)=f(q, K)$ at all heights for $K=0$.

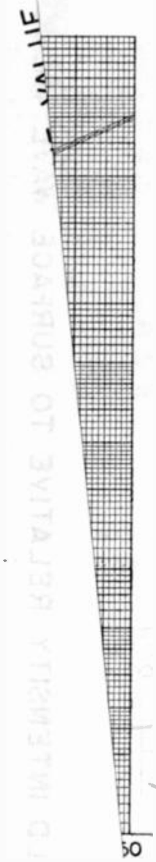
At large antenna heights beyond the line of sight the field intensity does not increase linearly with antenna height as would be indicated by (19) and, in fact, at sufficiently large heights, it increases exponentially with the height; thus at large numerical antenna heights we may write for the height-gain factor by which the surface wave is to be multiplied

$$f(q, K) = \delta f_1(\bar{h}). \tag{20}$$

The parameter δ is given graphically in Fig. 10 as a function of the parameters K and b while the function $f_1(\bar{h})$ is given graphically in Fig. 11 as a function of the parameters $\bar{h}$, K and b. The formulas for δ and for $f_1(\bar{h})$ are given in Appendix II. The parameter $\bar{h}$ is defined by the following equation:

$$\bar{h} = h\beta_0^2/(ka\lambda^2)^{1/3} = (2\pi)^{-2/3}\beta_0^2 Kq. \tag{21}$$

In order to determine the factor $f(q, K)$ graphically for all values of the antenna height below the line of sight, the following procedure may be followed. First, calculate the antenna height in feet corresponding to $q=1$ by using (6a) or (6b). Then, using transparent log-log graph paper similar to that of Figs. 7, 8, and 9, label the ordinates from 0.1 upward and the abscissas in terms of antenna height expressed in feet. Now superimpose the log-log chart over Fig. 7, shifting it vertically until the corresponding ordinates are lined up and shifting it horizontally until the value of antenna height calculated for $q=1$ coincides with that value. Now a curve may be traced on the log-log



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$$\bar{h} = h\beta_0^2/(ka\lambda^2)^{1/3} = (2\pi)^{-2/3}\beta_0^2Kq. \tag{21}$$

In order to determine the factor $f(q, K)$ graphically for all values of the antenna height below the line of sight, the following procedure may be followed. First, calculate the antenna height in feet corresponding to $q=1$ by using (6a) or (6b). Then, using transparent log-log graph paper similar to that of Figs. 7, 8, and 9, label the ordinates from 0.1 upward and the abscissas in terms of antenna height expressed in feet. Now superimpose the log-log chart over Fig. 7, shifting it vertically until the corresponding ordinates are lined up and shifting it horizontally until the value of antenna height calculated for $q=1$ coincides with that value. Now a curve may be traced on the log-log

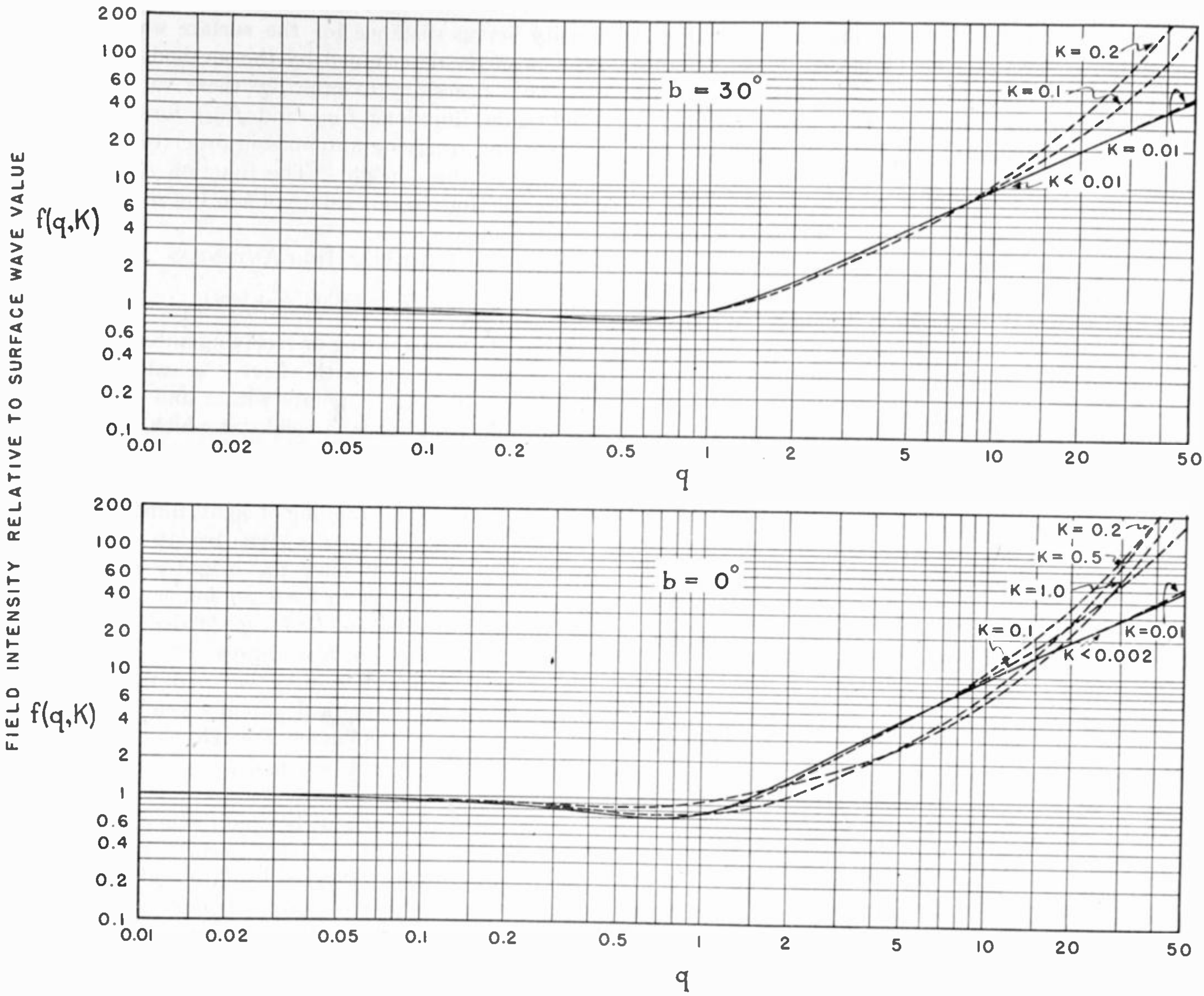

Fig. 8—The variation of field intensity with numerical antenna height at points beyond the line of sight ($b=0°$ and $30°$).

graph paper showing the height-gain factor $f(q)$ as a function of the antenna height h, expressed in feet. In order to determine the modifications of this curve at the larger antenna heights due to the curvature of the earth, determine by means of (21) the antenna height corresponding to $\bar{h}=1$ and at this height plot the appropriate value of δ as ordinate on the log-log graph sheet at this height. Now superimpose the log-log graph sheet over Fig. 11, shifting it vertically and horizontally until the point plotted on the log-log graph sheet corresponds to the point on Fig. 11 at which the unit ordinate and the unit abscissa cross. Now a curve may be traced from Fig. 11 which will merge with the $f(q)$ curve already drawn for a plane earth. The height at which these two curves merge determines the maximum height for which the methods of Part II may be used. For larger heights the following modified procedure must be used both for points within and beyond the line of sight.

The distance d_0 to the optical horizon for an antenna at a height h is given by the following formula which includes the systematic effects of air refraction by using the effective earth's radius ka.

$$d_0 = \sqrt{2kah} \qquad [h < 20{,}000 \text{ feet}] \tag{22}$$

so that the distance d_L to the line of sight is

$$d_L = \sqrt{2kah_1} + \sqrt{2kah_2}. \tag{23}$$

Now at points within the line of sight, the curvature of the earth has three effects on the propagation of radio waves for high transmitting or receiving antennas. First, the plane-wave reflection coefficient of the ground-reflected wave is different for the curved surface of the earth than for a plane surface; this effect is of little importance, however, under the circumstances normally encountered in practice and will be neglected in this discussion. Second, since the ground-reflected wave is reflected against the curved surface of the earth, its energy is diverged more than would be indicated by the inverse-distance law $(1/r_2)$ and we must multiply the ground-reflected wave by a divergence factor D.

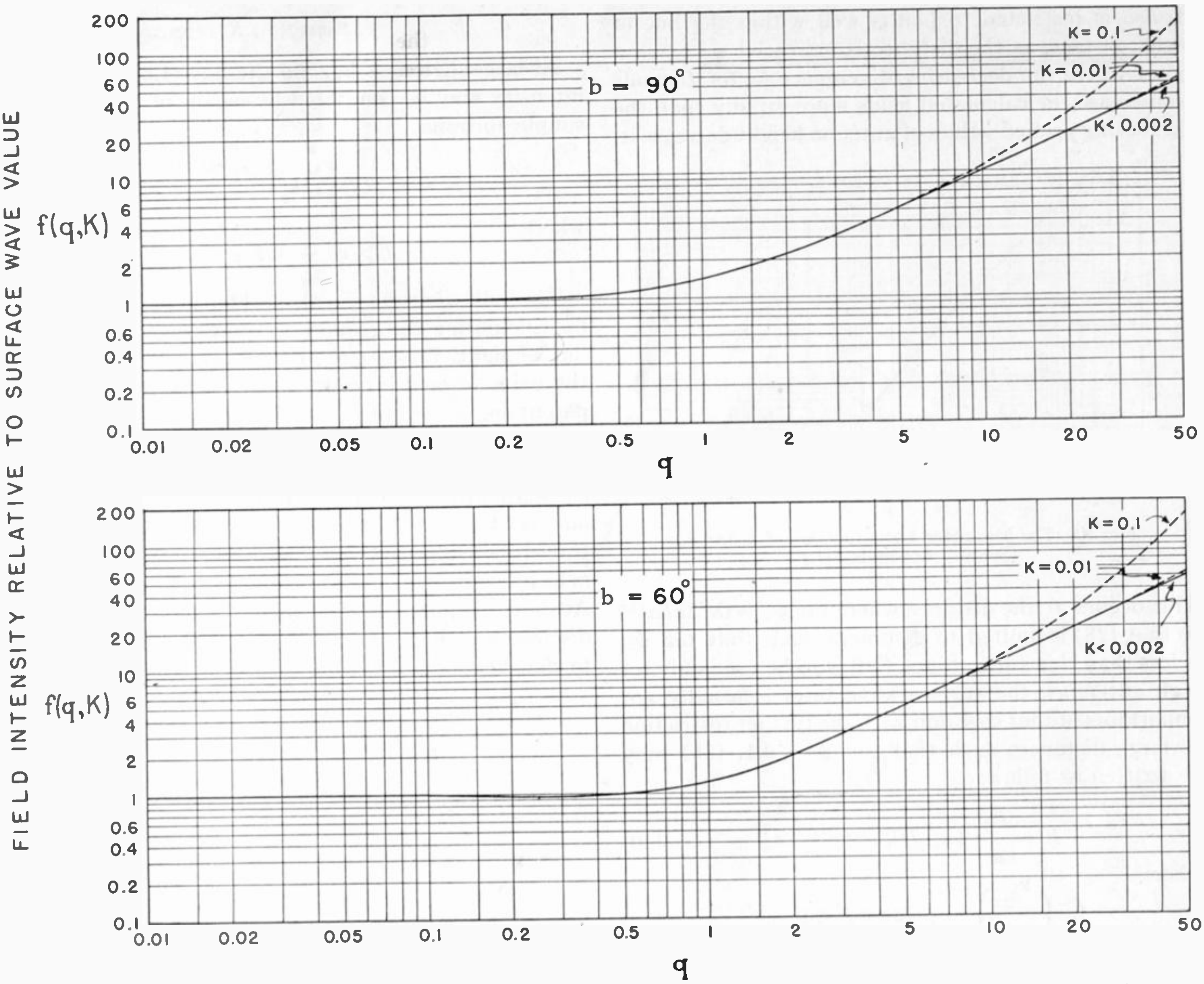

Fig. 9—The variation of field intensity with numerical antenna height at points beyond the line of sight ($b=60°$ and $90°$).

$$D = \left[1 + \frac{2d_1 d_2}{kad \tan \psi_2'}\right]^{-1/2}. \tag{24}$$

The distances d_1 and d_2 and the angle ψ_2' are defined in Fig. 12. Finally, for a spherical earth, the heights of the transmitting and receiving antennas h_1' and h_2' above the plane which is tangent to the surface of the earth at the point of reflection of the ground-reflected wave, are less than the antenna heights h_1 and h_2 above the surface of the earth.

$$h_1' = h_1 - \frac{d_1^2}{2ka} \tag{25}$$

$$h_2' = h_2 - \frac{d_2^2}{2ka} \tag{26}$$

$$\tan \psi_2' = \frac{h_1' + h_2'}{d} = \frac{h_1'}{d_1} = \frac{h_2'}{d_2}. \tag{27}$$

Thus, with high antennas, (1) must be modified as follows to determine the ground wave at points within the line of sight:

$$E = \frac{E_0}{d} \left| \cos^2 \psi_1' e^{i2\pi r_1'/\lambda} + DR' \cos^2 \psi_2' e^{i2\pi r_2'/\lambda} + (1 - R') f(P, B) \cos^2 \psi_2' e^{i[2\pi(r_2'/\lambda)+\phi]} \right| \tag{28}$$

where

$$\tan \psi_2' > (\lambda/2\pi ka)^{1/3}.$$

The following expressions for the reflection coefficient are convenient for calculations in this case:

$$R' = \frac{\sin \psi_2' \left[\dfrac{x \cos b'}{\cos^2 b''}\right]^{1/2} e^{i(\pi/4 - b/2)} - 1}{\sin \psi_2' \left[\dfrac{x \cos b'}{\cos b''}\right]^{1/2} e^{i(\pi/4 - b/2)} + 1} \quad \text{(vertical polarization)} \tag{29a}$$

$$R' = \frac{\sin \psi_2' \left[\dfrac{\cos b'}{x}\right]^{1/2} e^{-i(\pi/4 - b'/2)} - 1}{\sin \psi_2' \left[\dfrac{\cos b'}{x}\right]^{1/2} e^{-i(\pi/4 - b'/2)} + 1} \quad \text{(horizontal polarization).} \tag{29b}$$

Equation (28) needs to be used in place of (1) only for high antennas, as discussed earlier in this section, and it also is restricted to points well within the line of sight; in fact, as the distance is increased we reach a point where the decreasing divergence factor D tends to increase the calculated fields more rapidly than the decreasing primed values of antenna height decrease it.

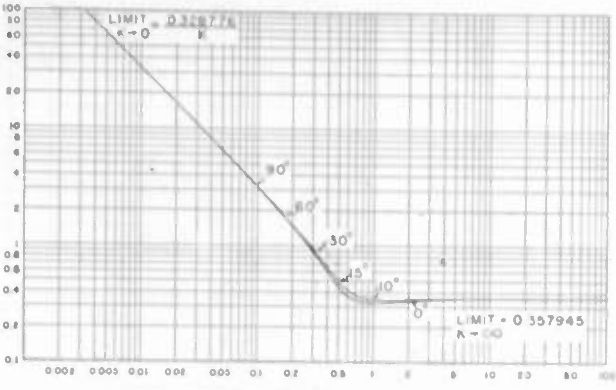

Fig. 10—The parameter δ as a function of K and b. (See also Appendix III.)

This occurs at the distance where $\tan \psi_2' = (\lambda/2\pi ka)^{1/3}$ so that (28) is limited to distances such that $\tan \psi_2'$ is less than this value. Since (28) is to be used only for high antennas, the surface-wave term is usually not important; in that case and with the further restriction to large distances such that $\tan \psi_2' < 0.1$, (28) may be written as follows:

$$E = \frac{E_0}{d}\left| 1 + DR' e^{i4\pi h_1' h_2'/\lambda d} \right| \tag{30}$$

where

$$\left(\frac{\lambda}{2\pi ka}\right)^{1/3} < \tan \psi_2' < 0.1.$$

Equation (30) shows that the received field oscillates with distance between the values

$$E = \frac{E_0}{d}\left[1 + D\left| R' \right|\right] \tag{31a}$$

and

$$E = \frac{E_0}{d}\left[1 - D\left| R' \right|\right]. \tag{31b}$$

Equations (31a) and (31b) thus define the envelope of the oscillations of the received field.

If we write $R' = -\left| R' \right| e^{-ic}$ and $\theta = 4\pi h_1' h_2'/\lambda d$, then (30) becomes

$$E = \frac{E_0}{d}\left[1 + (D\left| R' \right|)^2 - 2D\left| R' \right| \cos(\theta - c)\right]^{1/2} \tag{32}$$

where

$$\left(\frac{\lambda}{2\pi ka}\right)^{1/3} < \tan \psi_2' < 0.1.$$

The above completes the discussion of the region within the line of sight. At points beyond the line of sight the following procedure may be followed. First calculate the field intensity at the distance corresponding to $\eta' = 2$ by the following formula:

$$E_{(\eta'=2)} = 2E_0\eta_0\gamma f(q_1, K)\cdot f(q_2, K). \tag{33}$$

At large distances beyond the line of sight the field intensity may be computed by means of the following simple forumla:

$$E = E_{(\eta'=2)}\left[\frac{56.6626 e^{-(2\pi)^{1/3}\eta'}}{\sqrt{\eta'}}\right] \tag{34}$$

where

$$d > d_L + 1.5/\beta_0\eta_0.$$

The following graphical method may also be used for calculating the ground wave at points beyond the line of sight. First at the distance given by (17) plot the value of field intensity as given by (33) on transparent log-log graph paper like that of Fig. 4. Next superimpose this log-log graph sheet over Fig. 4, shifting it vertically and horizontally until the point calculated from (17) and (33) coincides with the curve on Fig. 4 at $\eta' = 2$. Now the curve on Fig. 4 may be traced for distances greater than $(d_L + 1.5/\beta_0\eta_0)$ to give field intensity versus distance at very large distances. At short distances within the line of sight the methods discussed in the first part of this section may be used to determine the field out to the distance such that

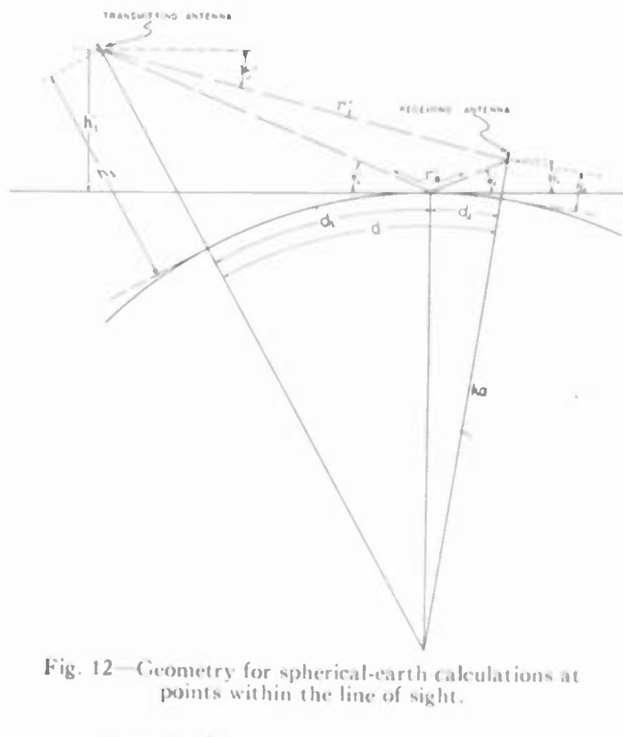

Fig. 12—Geometry for spherical-earth calculations at points within the line of sight.

$\tan \psi_2' = (\lambda/2\pi ka)^{1/3}$; from this point out to the distance $(d_L + 1.5/\beta_0\eta_0)$ a smooth transition curve may be drawn. This transition curve will always lie below the point determined by (17) and (33). Thus a complete determination of field intensity versus distance has been obtained in this high-antenna case; the only approximations lie in drawing the intermediate-distance curve and this may be done with considerable accuracy in practice. The alternative to the above approximate graphical method of solution for the field in this intermediate range of distances consists of a long

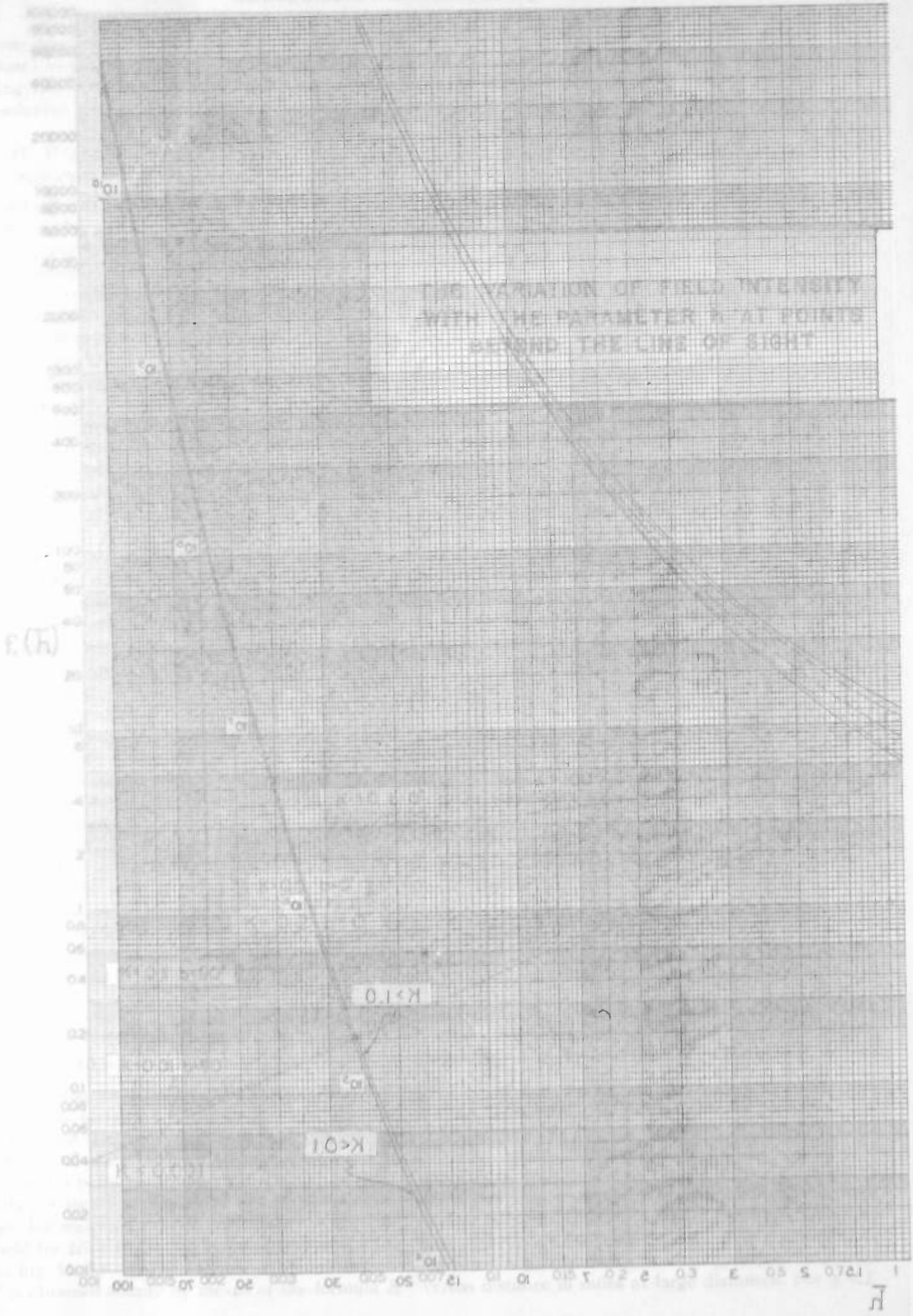

Fig. 11—The variation of field intensity with the parameter k at points beyond the line of sight.

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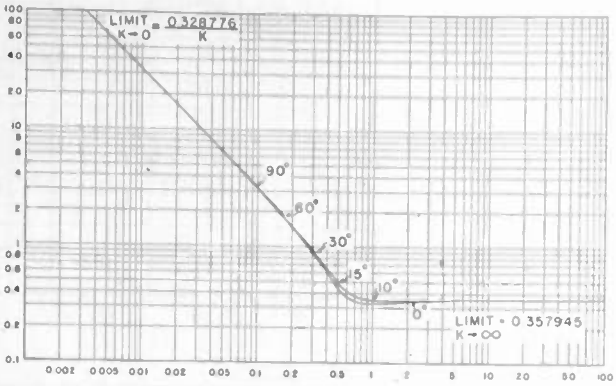

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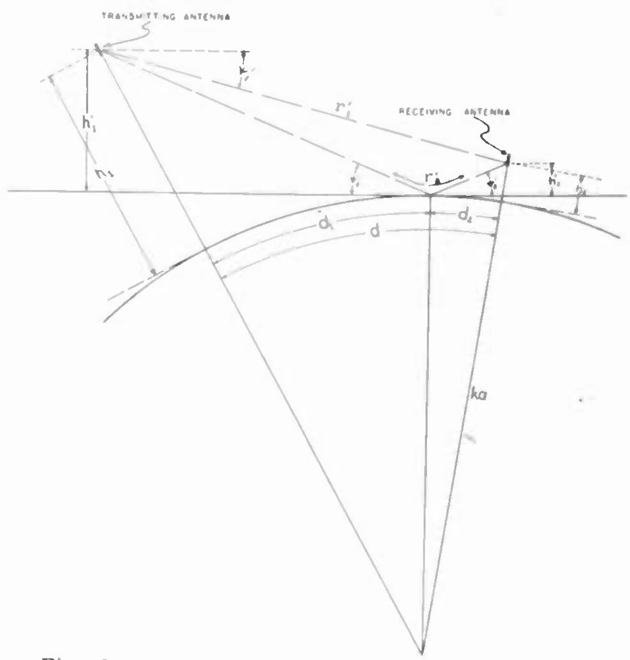

Fig. 12—Geometry for spherical-earth calculations at points within the line of sight.

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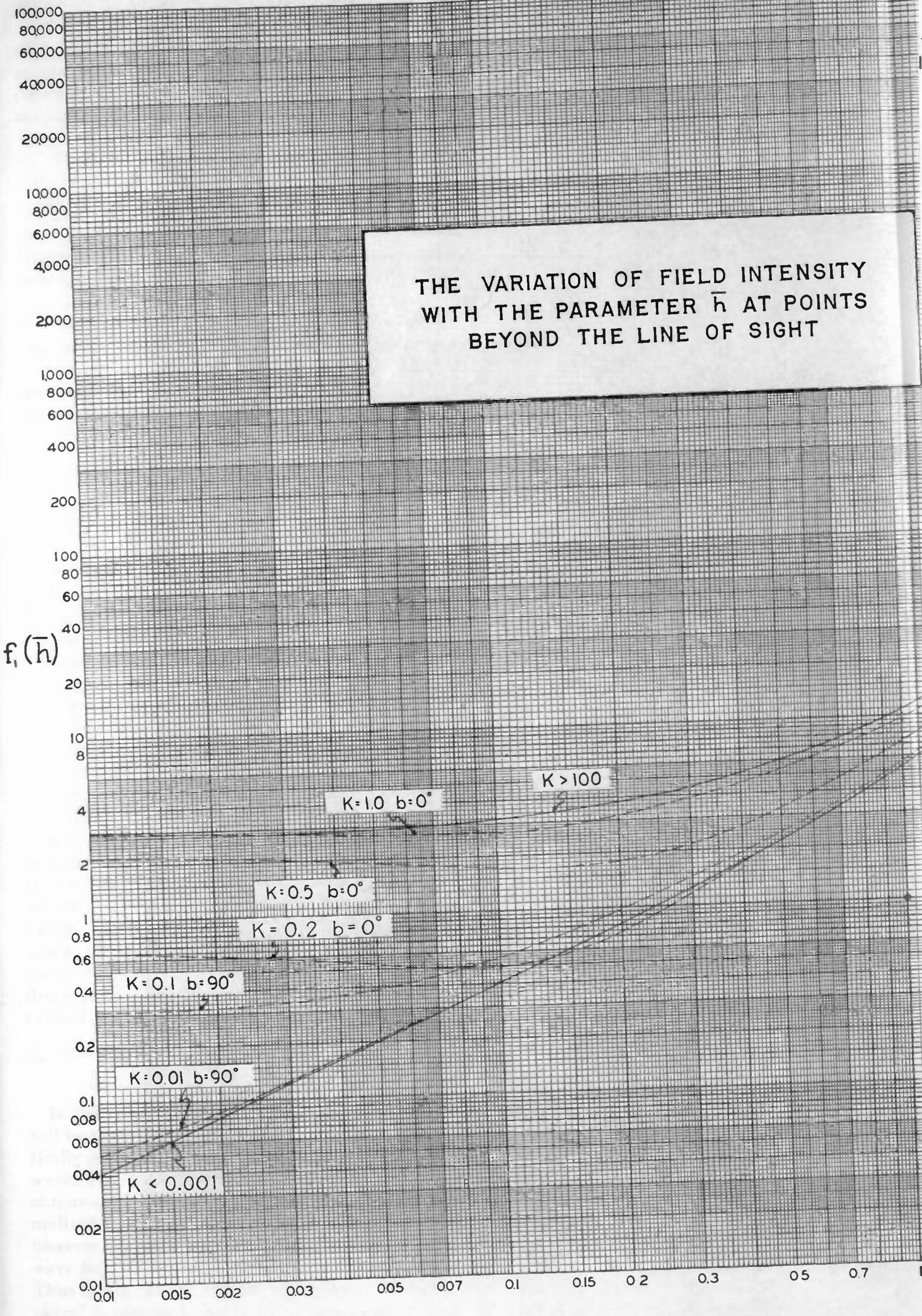

Fig. 11—The variation of field intensity with the para

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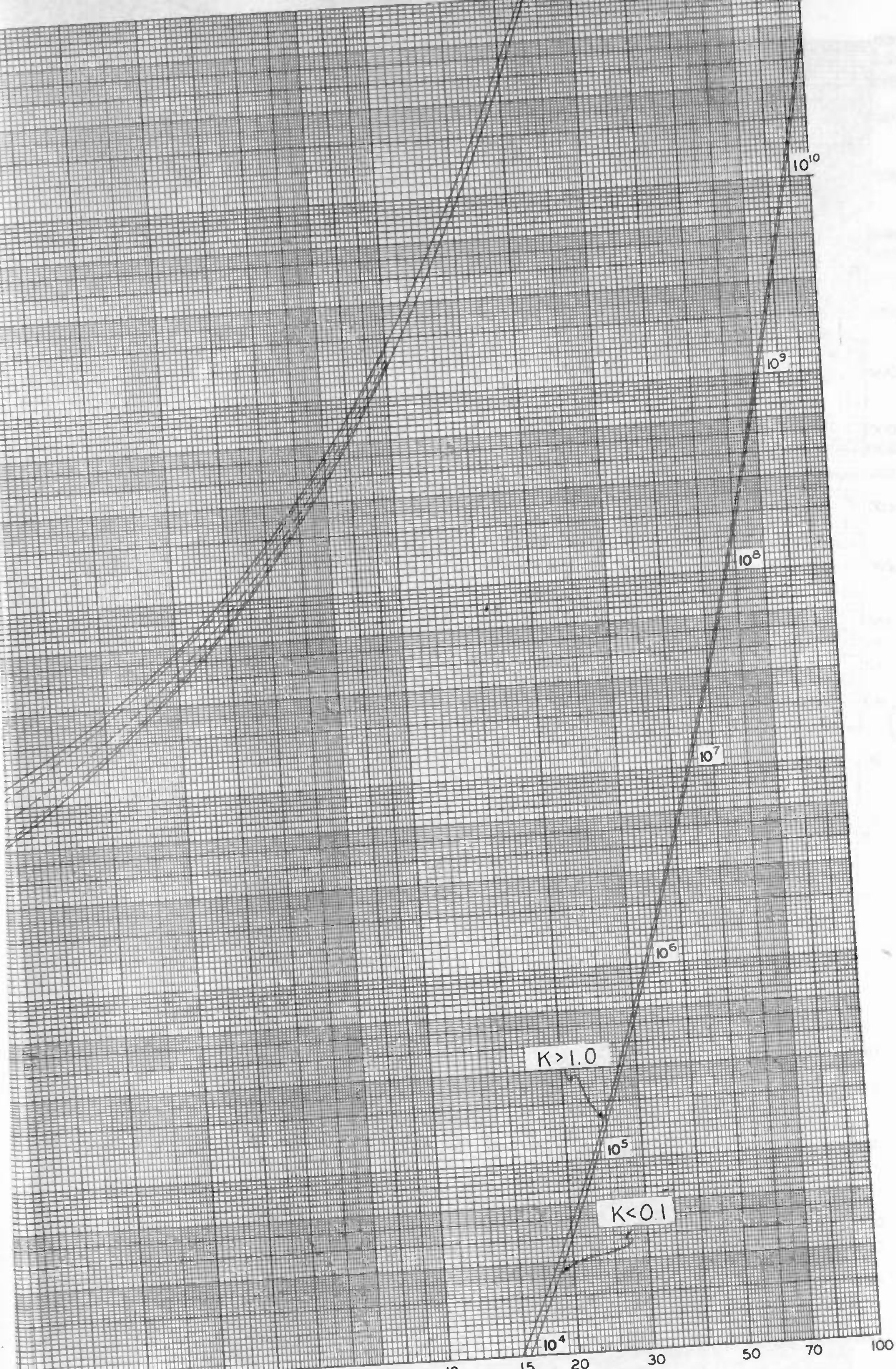

arameter $\bar{h}$ at points beyond the line of sight.

and tedious process of computing and adding (in proper phase) long series of terms, this latter method constituting the only other means of arriving at an accurate solution.

Part IV. Examples of the Calculation of Ground-Wave Field Intensities

The graphical methods to be described below are all carried out on log-log paper. This has several advantages. For example, in plotting field intensities, it is usually desirable to plot a value of 1000 millivolts per meter to the same *percentage* accuracy as a value of 1 millivolt per meter; this can only be done on a logarithmic field-intensity scale since a given percentage error, say 10 per cent at the 1000-millivolt-per-meter level represents the same linear distance on the logarithmic graph paper as 10 per cent at the 1-millivolt-per-meter level. Furthermore, if, after having determined a field-intensity curve for one value of the radiated power, we wish to determine it for some other value of the power, the curve of field intensity versus distance will have exactly the same shape for the two powers when plotted on a logarithmic scale; thus it will only be necessary to determine the effect of the increased power at a single distance, say 1 mile, and then shift the entire curve a linear distance upward on the logarithmic scale until it passes through the appropriate field intensity at one mile. Also, if we use a logarithmic distance scale as well as a logarithmic field-intensity scale, then curves of field intensity plotted as a function of the distance expressed in wavelengths, for example, will have the same shape as the same values of field intensity plotted as a function of the distance expressed in miles, or kilometers, or feet.

Several examples of the computations necessary when using the graphical method will now be given. In these examples the effective radius of the earth is assumed to have its average value, i.e., 4/3 the actual radius, so that $ka = 5280$ miles; the velocity of light taken as 299,776 kilometers per second; the frequency f_{mc} expressed in megacycles per second; the distance d expressed in miles; and all antenna heights expressed in feet.

(a) 500 *Kilocycles over Sea Water*

(10 miles to 2000 miles)

In this case the transmitting and receiving antennas will be near the surface of the ground and will be vertically polarized. Thus we are dealing with the surface wave. For 1 kilowatt radiated from a short vertical antenna over a perfectly conducting earth $2E_o = 186.3$ millivolts per meter at 1 mile; it is more convenient, however, for many purposes to calculate the ground-wave field for $2E_o = 100$ millivolts per meter at 1 mile. Thus in Fig. 13 the straight line marked "Inverse Distance" is obtained simply by means of the formula E (Inverse Distance) $= 100/d$. The ground conductivity for sea water $\sigma = 5 \times 10^{-11}$ electromagnetic unit and the dielectric constant $\epsilon = 80$. Thus we obtain

$$x = \frac{1.79731 \cdot 10^{15}\, \sigma_{\text{e.m.u.}}}{f_{mc}} = 179{,}731$$

$$\tan b' = (\epsilon - 1)/x = 0.0004395;$$

$$b' = 0°1'30''; \qquad \cos b' = 1$$

$$\tan b'' = \epsilon/x = 0.000445;$$

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$$\frac{x \cos b'}{\cos^2 b''} = 179{,}731$$

$$d_{(p=1)} = \frac{0.0592922}{f_{mc}} \frac{x \cos b'}{\cos^2 b''} = 21{,}300 \text{ miles}$$

$$b = 2b'' - b' = 0°1'30''.$$

From the above we see that, since the distance at which the numerical distance $p = 1$ is 21,300 miles, p will be very small (less than 0.1) throughout the entire range of distances under consideration, i.e., from 10 to 2000 miles. Reference to the $b = 0°$ curve on Fig. 2 will show that there is practically no plane-earth attenuation at distances less than 2000 miles since the $b = 0°$ curve is practically coincident with the inverse-distance line for $p < 0.1$. This plane-earth attenuation curve is nevertheless shown as a dotted line on Fig. 13 in order to illustrate this point. It was obtained by superimposing the log-log graph sheet of Fig. 13 over Fig. 2, shifting it horizontally until the abscissa corresponding to a numerical distance $p = 0.01$ on Fig. 2 coincides with $d_{(p=0.01)} = 213$ miles and shifting it vertically until the two inverse-distance lines coincide. Now the curve corresponding to $b = 0°$ was traced on Fig. 13.

Since there is practically no plane-earth attenuation we turn to calculations showing the effect of the curvature of the earth.

$$K = \frac{0.0177737}{f_{mc}^{1/3}} \left[\frac{x \cos b'}{\cos^2 b''}\right]^{1/2} = 9.49$$

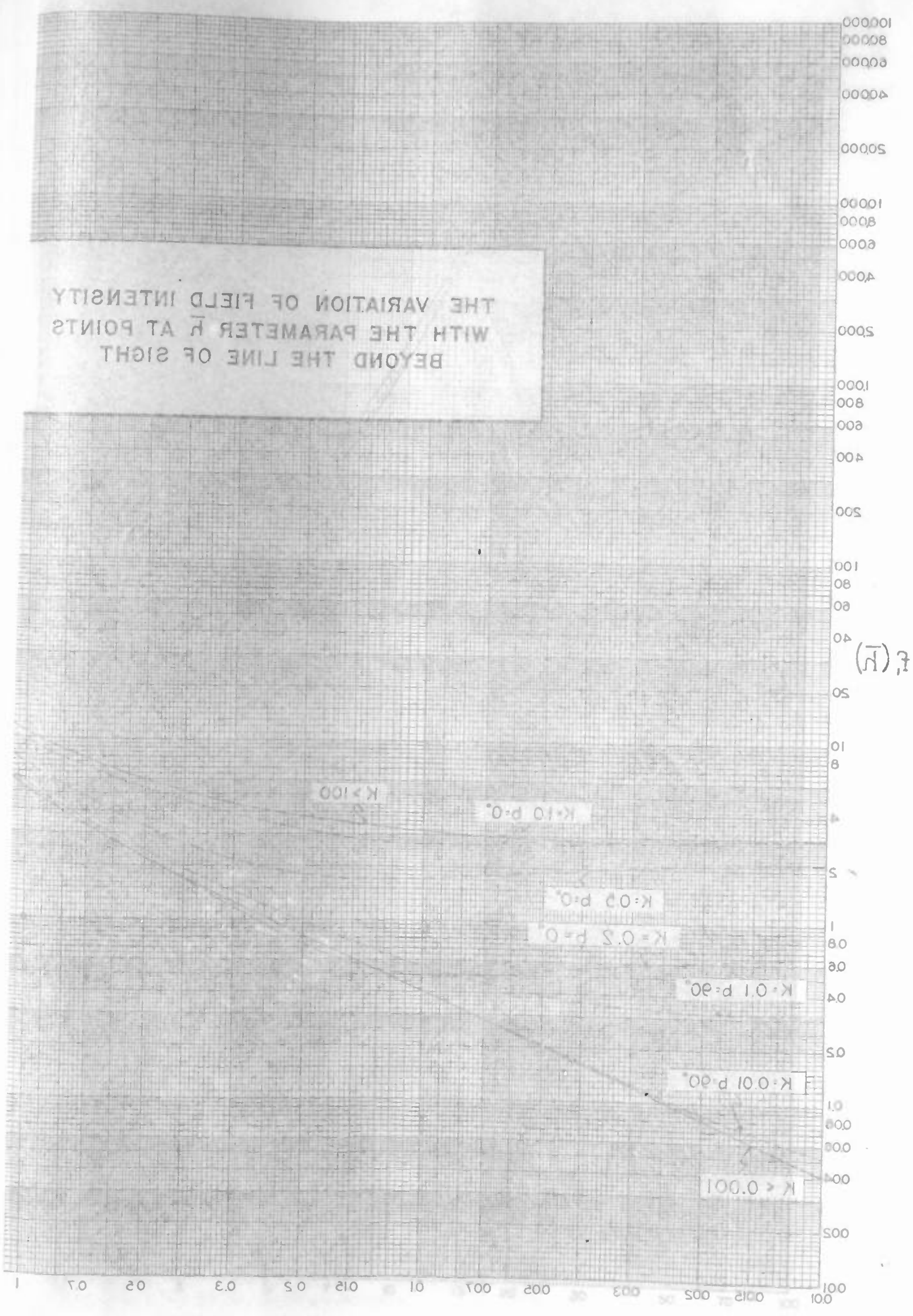
$$\eta_0 = 5.77469 \cdot 10^{-3} f_{mc}^{1/3} = 0.00458 \text{ (mile)}^{-1}.$$

Referring to Figs. 5 and 6 and using the above values of K and b we obtain $\beta_o = 0.6855$ and $\gamma = 0.0302$. Thus we can calculate the field intensity and the distance at which $\eta' = 2$.

$$E_{(\eta'=2)} = 2E_0\eta_0\gamma = 0.01384 \text{ millivolt per meter}$$

$$d_{(\eta'=2)} = 2/\beta_0\eta_0 = 636.5 \text{ miles}.$$

After plotting this point on Fig. 13 it is only necessary to superimpose it on Fig. 4 and shift it vertically and horizontally until the plotted point coincides with the curve on Fig. 4 at the point $\eta' = 2$. For $\eta' > 2$ the single curve on Fig. 4 is then traced to give field intensity versus distance in miles at large distances. For $\eta' < 2$



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(*a*) 500 *Kilocycles over Sea Water*

(10 miles to 2000 miles)

In this case the transmitting and receiving antennas will be near the surface of the ground and will be vertically polarized. Thus we are dealing with the surface wave. For 1 kilowatt radiated from a short vertical antenna over a perfectly conducting earth $2E_o=186.3$ millivolts per meter at 1 mile; it is more convenient, however, for many purposes to calculate the ground-wave field for $2E_o=100$ millivolts per meter at 1 mile. Thus in Fig. 13 the straight line marked "Inverse Distance" is obtained simply by means of the formula E (Inverse Distance) $=100/d$. The ground conductivity for sea water $\sigma=5\times10^{-11}$ electromagnetic unit and the dielectric constant $\epsilon=80$. Thus we obtain

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the lower curve on Fig. 4 should be traced since K is comparatively large. It will be noted that the straight line on Fig. 4 marked (Plane Earth $K=\infty$) is very

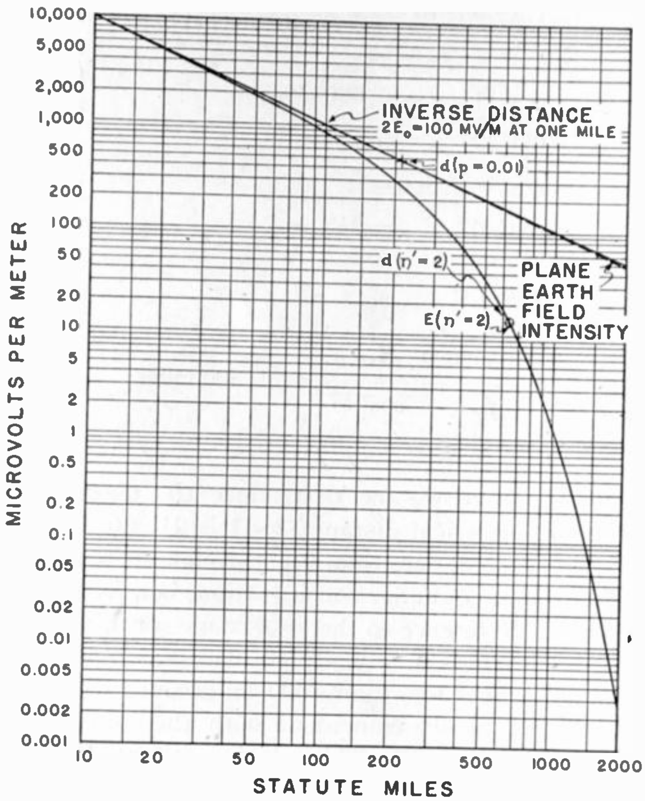

Fig. 13—Example illustrating the graphical method of calculating ground-wave field intensity versus distance; frequency: 500 kilocycles, ground conductivity: $\sigma=5\times10^{-11}$ electromagnetic unit, ground dielectric constant: $\epsilon=80$; these ground constants are those of sea water.

nearly coincident with the inverse-distance line; this will always be the case for large values of K.

(*b*) 1120 *Kilocycles over Land*: $\sigma=9\times10^{-14}$ *electromagnetic unit*, $\epsilon=21$

(1 mile to 200 miles)

In this case we proceed in exactly the same manner as before and calculate the following quantities: $x=144$; $\tan b'=0.1385$; $b'=7°\ 53'$; $\cos b'=0.9905$; $\tan b''=0.1454$; $b''=8°\ 16'\ 20''$, $\cos b''=0.9896$; $x \cos b'/\cos^2 b''=146$; $d_{(p=1)}=7.73$ miles; $b=8°\ 39'\ 40''$; $K=0.2069$; $\eta_o=0.005997(\text{mile})^{-1}$; $\beta_o=1.44$; $\gamma_o=0.00348$; $E(\eta'=2)=0.002087$ millivolt per meter; $d_{(\eta'=2)}=231.6$ miles.

On Fig. 14 an inverse-distance line is drawn as before corresponding to $2E_o=100$ millivolts per meter at 1 mile and the distance at which $p=1$ marked off. Now by superimposing this figure over Fig. 2 we can trace a curve which lies approximately half way between the curves corresponding to $b=0°$ and $b=15°$. This gives a curve showing the expected field over a plane earth.

Next, by plotting the computed value of the field intensity at $\eta'=2$ we can use Fig. 4 to determine the field intensity over a spherical earth. For distances greater than 200 miles, the single curve on Fig. 4 can be traced. For shorter distances, it is necessary to draw a smooth curve between the plane-earth curve and the spherical-earth curve. This curve is shown as a solid line between 20 and 200 miles. Also shown on this figure for reference are the plane-earth curve and the two branches of the spherical-earth curve at short distances. The field at intermediate distances does not coincide with either branch of the spherical-earth curves of Fig. 4 since K is neither very small nor very large.

The solid curve on Fig. 14 gives the surface-wave field intensity to be expected over an earth of conductivity $\sigma=9\times10^{-14}$ electromagnetic unit and a dielectric constant $\epsilon=21$. In order to determine whether this field intensity might be expected to change with the height (not the length) of the transmitting or the receiving antenna we compute the height at which $q=1$.

$$h_{(q=1)} = \frac{156.531}{f_{mc}}\left[\frac{x\cos b'}{\cos^2 b''}\right]^{1/2} = 1689 \text{ feet.}$$

Referring now to Fig. 7 we see that the field intensity will not change appreciably with receiving-antenna

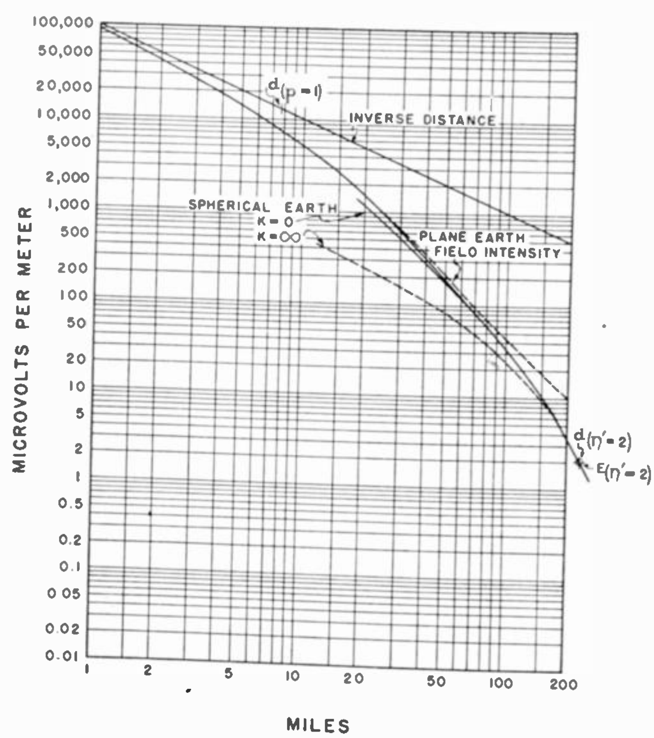

Fig. 14—Example illustrating the graphical method of calculating ground-wave field intensity versus distance at the broadcast frequency 1120 kilocycles for the ground constants $\sigma=9\times10^{-14}$ electromagnetic unit and $\epsilon=21$.

height over a very extended range when $b=8°\ 39'\ 40''$. For example, $q=0.0592$ at 100 feet and at this height the field intensity is only about 2 per cent lower than the value at the surface.

(*c*) 46 *Megacycles over Land*: $\sigma = 5 \times 10^{-14}$ *electromagnetic unit*, $\epsilon = 15$, *vertical polarization*

(1 mile to 200 miles)

Although the surface wave is a negligible component of the received field intensity in this case, it is simplest to determine the surface wave first and then to use a height-gain curve to determine the field intensity for elevated transmitting and receiving antennas. Thus we compute the following quantities: $x = 1.954$; $\tan b' = 7.166$; $b' = 82° \ 3' \ 20''$; $\cos b' = 0.1382$; $\tan b'' = 7.678$; $b'' = 82° \ 34' \ 50''$; $\cos b'' = 0.129$; $x \cos b'/\cos^2 b'' = 16.19$; $d_{(p=1)} = 0.02086$ miles; $b = 83° \ 6' \ 10''$.

Since the distance at which $p = 1$ is very small, we see that p is very large even at a distance of 1 mile. When p is greater than about 20, we see by Fig. 2 or by reference to (48) that the attenuation factor for the surface wave over a plane earth may be expressed simply as $1/2p$. Thus, the field intensity versus distance for the surface wave over a plane earth may be expressed as

$$E = \frac{2E_0}{d} \cdot \frac{1}{2p} = \frac{E_0 \cdot d_{(p=1)}}{d^2} \quad \text{(plane-earth surface wave; } p > 20).$$

If we take $E_0 = 137.6$ millivolts per meter which is the free-space field intensity at 1 mile corresponding to a power of 1 kilowatt radiated from a half-wave dipole, we obtain the following expression for the plane-earth surface-wave field intensity:

$$E = \frac{2.87}{d^2} \text{ millivolts per meter}$$

(plane-earth surface wave; $d > 0.4$ mile).

This is shown on Fig. 15 as a straight line. To determine the effect of the curvature of the earth on the surface wave we determine the following quantities:

$K = 0.01996$

$\eta_c = 0.02069 \ (\text{mile})^{-1}$

$\beta_c = 1.587$

$\gamma = 3.014 \cdot 10^{-5}$

$E_{(\eta'=2)} = 0.0001716$ millivolt per meter

$d_{(\eta'=2)} = 60.9$ miles.

The effect of the earth's curvature is obtained, as before, by tracing Fig. 4 on Fig. 15 with the field intensity at $\eta' = 2$ lined up on the two sheets. This time, since K is very small, the upper branch of the curve on Fig. 4 may be traced for $\eta' < 2$ and the plane-earth curve marked $K = 0$ on Fig. 4 will be found to be practically coincident with the plane-earth surface-wave curve already drawn on Fig. 15.

In order to extend the above results to elevated transmitting and receiving antennas, it is necessary to determine the height-gain function. For this purpose we need to compute the following quantities:

$h_{(q=1)} = 13.69$ feet

$\delta = 16.38$

$h_{(\bar{h}=1)} = 29{,}987.7/\beta_0^2 f_{mc}^{2/3} = 927$ feet.

The height-gain function for a plane earth was determined on Fig. 16 in the following way. After labeling the ordinates and abscissas as shown, Fig. 16 was superimposed over Fig. 7 and shifted horizontally

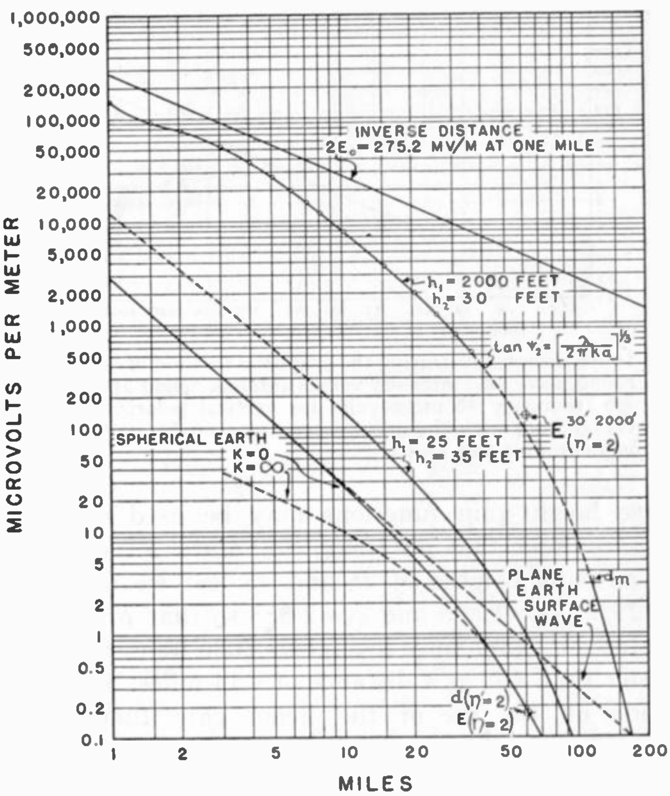

Fig. 15—Example illustrating the graphical method of calculating ground-wave field intensity versus distance at the ultra-high frequency 46 megacycles for vertical polarization over average land with the ground constants $\sigma = 5 \times 10^{-14}$ electromagnetic unit and $\epsilon = 15$. (Note that the curves for the various transmitting-antenna heights are drawn for a fixed value of $2E_0 = 275.2$ millivolts per meter at one mile and not for a fixed value of radiated power; for the lower antenna heights $2E_0$ would be less than 275.2 for 1 kilowatt radiated from a half-wave dipole.)

until the height 13.7 feet on Fig. 16 coincided with the abscissa $q = 1$ on Fig. 7. Then a curve was traced corresponding to $b = 83$ degrees. This curve gives $f(q)$, i.e., the field intensity relative to the surface-wave value over a plane earth as a function of the antenna height in feet. Next, in order to extend this curve to the case of a spherical earth, we plot the value $\delta = 16.38$ at the height $h_{(\bar{h}=1)}$ on Fig. 16 and superimpose it over Fig. 7. Fig. 16 is then shifted vertically until the ordinate on Fig. 16 at $\delta = 16.38$ coincides with the ordinate $f_1(\bar{h}) = 1$ on Fig. 11 and shifted horizontally until the abscissa $h_{(\bar{h}=1)} = 927$ feet coincides with the abscissa $\bar{h} = 1$ on Fig. 11. Then the curve corresponding to $K = 0$ is traced to give $f(q, K)$, i.e., the field intensity relative to the surface-wave value versus antenna height over a spherical earth. For example, to determine the received field for a transmitting-antenna height of 25 feet and a receiving-antenna height of 35 feet we find that the factors on Fig. 16 are 2.02 and 2.67 so that the field at these heights will be $2.02\times 2.67=5.393$ times the values given for the surface wave on Fig. 15. A curve is drawn on Fig. 15 to show these values.

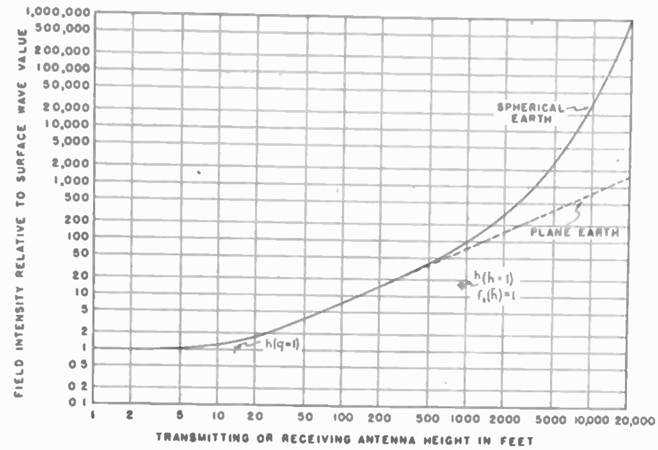

Fig. 16—Example illustrating the graphical method of calculating ground-wave field intensity versus antenna height at the ultra-high frequency 46 megacycles for vertical polarization over average land with the ground constants $\sigma=5\times 10^{-14}$ electromagnetic unit and $\epsilon=15$.

These height-gain functions may be used only for distances such that $p>100(q_1+q_2)$ and $p>10q_1q_2$. For the antenna heights of 25 and 35 feet we find that $q_1=25/h_{(q=1)}=1.825$ and $q_2=2.555$ so that p must be greater than 438. Since $d_{(p=1)}=0.02086$ mile, p will be greater than 438 at a distance of 9.14 miles. This limitation on the use of the height-gain function is caused by the fact that, for these antenna heights, it is not permissible to set the ground reflection coefficient equal to -1 at distances less than 9.14 miles. Thus it becomes necessary at shorter distances to use the exact formula as given by (1). As an example of the method of computation by means of (1), a value of E will be determined for a distance of 1 mile for the antenna heights, 25 and 35 feet. At 1 mile $p=47.94$ and we can compute R by means of (3). Thus,

$$R=\frac{0.04568\,\underline{|3°26'55''}-1}{0.04568\,\underline{|3°26'55''}+1}=\frac{-0.9544\,\underline{|-0°9'50''}}{1.0457\,\underline{|0°9'0''}}$$

$$=-0.9128\,\underline{|-0°18'50''}$$

$$1-R=2/1.0457\,\underline{|0°9'0''}=1.9128\,\underline{|-0°9'0''}.$$

By (10) it can be determined that $P=52.41$, $B=83°24'\,10''$, so that

$$(1-R)f(P,\,B)e^{i\theta}=-0.01825\,\underline{|-83°33'10''}.$$

Now, since $\cos\psi_2\cong 1$, and since $2\pi(r_2-r_1)/\lambda$ corresponds to an angle of $2°\,47'\,25''$ we obtain by (1)

$$E=137.6\,\big|\,1-0.9128\,\underline{|-0°18'50''+2°47'25''}$$
$$-0.01825\,\underline{|-83°33'10''+2°47'25''}\,\big|$$
$$=137.6\,|0.08512-i0.02158|$$
$$=12.08\text{ millivolts per meter.}$$

Comparing this value with the surface-wave value at 1 mile (2.87 millivolts per meter) we find that their ratio is 4.21; thus the effect of raising the antennas at a distance of 1 mile is somewhat less than at the larger distances. A dashed curve is drawn on Fig. 15 between 1 mile and 9 miles to indicate that it was obtained by (1).

Consider next a transmitting-antenna height of 2000 feet and a receiving-antenna height of 30 feet. It should be noted on Fig. 16 that the field intensity at 2000 feet relative to the surface-wave value is greater for a spherical earth than for a plane earth. Whenever we are dealing with high antenna heights so that the height-gain function for either the transmitting or receiving antenna is different over a spherical earth than over a plane earth, then the methods of Part III must be used.

First we find that the distance to the line of sight may be determined as follows:

$$d_L=\sqrt{2h_1}+\sqrt{2h_2}=\sqrt{60}+\sqrt{4000}=71\text{ miles}$$

so that Fig. 4 may be traced for distances greater than

$$d_m=d_L+1.5/\beta_0\eta_0=117\text{ miles.}$$

From Fig. 16 we find that the height-gain factors $f(q,\,K)$ for 30 feet and for 2000 feet are 2.353 and 310.3 so that the field intensity, at the distance $\eta'=2$, is $E(\eta'=2)=0.0001716\times 2.353\times 310.3=0.1253$ millivolt per meter. This field is plotted at the distance 60.9 miles and Fig. 15 is then superimposed over Fig. 4 and the curve traced for the distance $d>117$ miles.

Next, at points within the line of sight, the field intensities are calculated by means of (32). The results obtained by this process are given on Fig. 15 as small circles for the distance range from 1 to 41 miles. The process followed in calculating these points will be outlined below for the distance 20.333 miles. h_1' and h_2' are first calculated as follows.

First, arbitrarily set $d_1=20$ miles and calculate h_1', $\tan\psi_2'$, h_2', d, and D.

$$h_1'=h_1-\frac{d_1^2}{2}=2000-200=1800\text{ feet}$$

$$(5280\tan\psi_2')=h_1'/d_1=90$$

$$\tan\psi_2'=0.017045$$

$$d_2=[(5280\tan\psi_2')^2+2h_2]^{1/2}$$
$$-5280\tan\psi_2'=0.3327\text{ mile}$$

$$h_2'=h_2-\frac{d_2^2}{2}=29.945\text{ feet}$$

$$d=d_1+d_2=20.333\text{ miles}$$

$$D=\left[1+\frac{2d_1d_2}{d(5280\tan\psi_2')}\right]^{-1/2}$$

$$\cong 1-\frac{d_1d_2}{d(5280\tan\psi_2')}=0.9964$$

$$\theta = 1.3865 \cdot 10^{-4} h_1' h_2' f_{mc}/d$$
$$= 16.907 \text{ degrees.}$$

If we write

$$R = -|R| e^{-ic}$$

then we can calculate for $\tan \psi_2' = 0.017045$ and for $x = 1.9536$, $\epsilon = 15$:

$$|R| = 0.871, \; c = 0.48°, \; \theta - c = 16.427°$$

$$E = \frac{E_0}{d}[1 + (D|R|)^2 - 2D|R| \cos(\theta - c)]^{1/2}$$
$$= 1.98 \text{ millivolts per meter.}$$

Calculations similar to these may be carried out for distances out to the point at which $\tan \psi_2' = [\lambda/2\pi ka]^{1/3}$. This occurs at a distance of 41 miles in this case and it is evident that the calculated field is slightly too high at this point. Between the point at 36 miles at which the above process fails and the point at the distance d_m, a smooth curve must be traced since no simple methods of calculation are available for this region. This curve will always lie below the point $E_{(\eta'=2)}^{h_1h_2}$ when the height-gain function for either h_1 or h_2 is substantially different in the plane- and spherical-earth cases.[14]

Acknowledgment

The writer wishes to acknowledge the assistance in the preparation of this paper given by Mr. Edward W. Allen, who computed the values of several of the parameters and made many helpful suggestions regarding the method of presentation.

Appendix I

The Height-Gain Function for a Plane Earth

At the higher frequencies, the numerical distance p is usually quite large. Thus, when $p > 20$ and $p > 100(q_1 + q_2)$, we can write

$$\frac{2\pi}{\lambda}(r_2 - r_1) \cong \frac{4\pi h_1 h_2}{\lambda d} = \frac{q_1 q_2}{p} \tag{35}$$

$$f(P, B)e^{i\phi} \cong -\frac{1}{2P}e^{-iB} \cong -\frac{1}{2p}e^{-ib} \tag{36}$$

$$R = -1 + \frac{(q_1 + q_2)}{p} e^{i(\pi/4 - b/2)} \tag{37}$$

$$1 - R \cong 2. \tag{38}$$

At the large distances at which these relations apply, we also have $\cos \psi_1 \cong \cos \psi_2 \cong 1$, and when the above relations are substituted in (1) we obtain

$$E \cong \frac{E_0}{d}\left| 1 + \left\{ -1 + \frac{q_1 + q_2}{p} e^{i(\pi/4 - b/2)} - \frac{1}{p} e^{-ib} \right\} e^{i(q_1 q_2/p)} \right| \tag{39}$$

where

$$p > 20 \quad \text{and} \quad p > 100\,(q_1 + q_2).$$

Now at sufficiently large numerical distances such that $p > 10\, q_1 q_2$, (39) becomes

$$E \cong \frac{E_0}{d}\left| \frac{q_1 + q_2}{p} e^{i(\pi/4 - b/2)} - \frac{1}{p} e^{-ib} - i\frac{q_1 q_2}{p} \right| \tag{40}$$

$$E \cong \frac{2E_0}{d}\left| \left(-\frac{1}{2p} e^{-ib} \right) [1 - (q_1 + q_2) e^{i(\pi/4 + b/2)} + q_1 q_2 e^{i(\pi/2 + b)}] \right|. \tag{41}$$

Finally, if we write

$$f(q) \equiv |1 - q e^{i(\pi/4 + b/2)}|$$
$$= \left[1 + q^2 - 2q \cos\left(\frac{\pi}{4} + \frac{b}{2}\right)\right]^{1/2}, \tag{42}$$

equation (41) becomes

$$E \cong \frac{2E_0}{d} f(p, b) \cdot f(q_1) \cdot f(q_2). \tag{43}$$

where $p > 20$; $p > 100(q_1 + q_2)$ and $p > 10 q_1 q_2$. This equation shows that, for sufficiently large numerical distances over a plane earth, the field intensity may be computed as the product of three factors: (1) The surface-wave field intensity $(2E_0/d)\, f(p)$, (2) the height-gain function $f(q_1)$ corresponding to the transmitting antenna height h_1, and (3) the height-gain function $f(q_2)$ corresponding to the receiving-antenna height h_2.

Appendix II

This appendix contains the formulas and series expansions for the functions which were given graphically in the various figures. These equations are not required for obtaining the solution by the graphical method but are given to show the basis for the graphs and to make possible more accurate computations if that is considered desirable.[15]

$$f(p, b)e^{i\phi} = 1 + i\sqrt{\pi p_1}\, e^{-p_1} \operatorname{erfc}(-i\sqrt{p_1}) \tag{44}$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du \tag{45}$$

$$p_1 = p e^{ib}. \tag{46}$$

Note that when q_1 or q_2 are appreciably different from zero, p should be replaced by P and b by B (see equation (10)). Equation (44) has the following series expansions:

$$f(p, b)e^{i\phi} = 1 + i\sqrt{\pi p_1}\, e^{-p_1} - 2p_1 + \frac{(2p_1)^2}{1 \cdot 3} - \frac{(2p_1)^3}{1 \cdot 3 \cdot 5} + \cdots. \tag{47}$$

[14] The effect of the curvature of the earth is considered in a somewhat different manner in a recent paper by C. R. Burrows and M. C. Gray, "The effect of the earth's curvature on ground-wave propagation," Proc. I.R.E., vol. 29, pp. 16–24; January, 1941. In this paper the field intensity is determined by multiplying together several factors: a free-space field factor, a plane-earth attenuation factor, a shadow factor, and two height factors. Methods are not given for calculating the field in those cases where this type of separation into factors cannot be made.

[15] A table of the function $f(p, b)$ is given in Part I of the paper referred to in footnote 9.

The above series is applicable for all values of p but converges very slowly for large values of p; the following asymptotic expansion may be used for large values of p:

$$f(p, b)e^{i\phi} = -\frac{1}{2p_1} - \frac{1\cdot 3}{(2p_1)^2} - \frac{1\cdot 3\cdot 5}{(2p_1)^3} - \frac{1\cdot 3\cdot 5\cdot 7}{(2p_1)^4} - \cdots. \quad (48)$$

The parameter $\tau_0 = \alpha_0 + i\beta_0$ is defined to be the root of the following equation:

$$H_{\frac{1}{3}}^{(1)}\left[\tfrac{1}{3}(-2\tau_0)^{3/2}\right] = -K\sqrt{-2\tau_0}\, e^{i(5\pi/12 - b/2)} H_{\frac{2}{3}}^{(1)}\left[\tfrac{1}{3}(-2\tau_0)^{3/2}\right]. \quad (49)$$

The functions $H_{\frac{1}{3}}^{(1)}$ and $H_{\frac{2}{3}}^{(1)}$ are the Hankel functions of orders $\frac{1}{3}$ and $\frac{2}{3}$, respectively. The following series expansions may be derived from (49):

$$\begin{aligned}\tau_0 = {} & 1.8557585\, e^{i(\pi/3)} - Ke^{i(3\pi/4 - b/2)} \\ & - 1.2371723K^3 e^{i(7\pi/12 - 3b/2)} \\ & + 0.5K^4 e^{i(\pi - 2b)} - 2.7550715K^5 e^{i(5\pi/12 - 5b/2)} \\ & + 2.8867355K^6 e^{i(5\pi/6 - 3b)} + 6.5896390K^7 e^{i(5\pi/4 - 7b/2)} \\ & + 13.3161793K^8 e^{i(2\pi/3 - 4b)} + \cdots. \end{aligned} \quad (50)$$

The above equation is useful for small values of K while the following equation may be used for large values of K.

$$\begin{aligned}\tau_0 = {} & 0.8086165174e^{i(\pi/3)} - \frac{0.61834008}{K} e^{i(b/2 - 13\pi/12)} \\ & - \frac{0.23641890}{K^2} e^{i(b - \pi/2)} + \frac{0.053338757}{K^3} e^{i(3b/2 - 11\pi/12)} \\ & - \frac{0.0022553514}{K^4} e^{i(2b - 4\pi/3)} + \frac{0.0024693571}{K^5} e^{i(5b/2 - 3\pi/4)} \\ & + \frac{0.00039940653}{K^6} e^{i(3b - \pi/6)} \\ & + \frac{0.0002120150}{K^7} e^{i(7b/2 - 7\pi/12)} \\ & - \frac{0.00046148795}{K^8} e^{i4b} + \cdots. \end{aligned} \quad (51)$$

The parameter γ is defined by the following equation:

$$\gamma = 0.030046482 \cdot \left[\frac{\beta_0}{[\alpha_0 + \sin b/2K^2]^2 + [\beta_0 - \cos b/2K^2]^2}\right]^{1/2}. \quad (52)$$

At points beyond the line of sight, the height-gain function $f(q)$ also depends upon K. Thus we may write for small values of K

$$\begin{aligned} f(q, K) = \Bigg| & 1 - qe^{i(\pi/4 + b/2)} + q^2K^2\tau_0 \\ & - \frac{q^3K^3}{3}\left(1 + \frac{\tau_0}{K} e^{i(\pi/4 + b/2)}\right) + \frac{q^4K^4}{6}\left(\tau_0^2 + \frac{5}{12\tau_0} \right. \\ & \left. + \frac{e^{i(\pi/4 + b/2)}}{K} - \frac{19e^{i(\pi/4 + b/2)}}{8K\tau_0^3}\right) + \cdots. \end{aligned} \quad (53)$$

For large values of q, the parameter $\bar{h}$ is more convenient for the graphical solution and we may write

$$f(q, K) = \delta \cdot f_1(\bar{h}) \quad (54)$$

$$f_1(\bar{h}) = \left| (y/\beta_0)^{1/2} \cdot H_{\frac{1}{3}}^{(1)}[(y)^{3/2}/3] \right| \quad (55)$$

$$y = 2Kq - 2\tau_0 = \frac{2(2\pi)^{2/3}\bar{h}}{\beta_0^2} - 2\tau_0 \quad (56)$$

$$\delta = \left| (\beta_0/2\tau_0)^{1/2} / H_{\frac{1}{3}}^{(1)}\left[\tfrac{1}{3}(-2\tau_0)^{3/2}\right] \right|. \quad (57)$$

The following asymptotic expansion for $f_1(\bar{h})$ may be used at the large heights where (53) is no longer applicable.

$$\begin{aligned} f_1(\bar{h}) = \Bigg| & (6/\pi\beta_0)^{1/2} y^{-1/4} e^{iy^{3/2}/3} \left[1 - \frac{i0.20833333}{y^{3/2}} \right. \\ & - \frac{0.33420138}{y^3} + \frac{i1.02581257}{y^{9/2}} \\ & \left. + \frac{4.66958443}{y^6} + \cdots \right] \Bigg|. \end{aligned} \quad (58)$$

Series expansions are also available for the Hankel function appearing in (57)

$$\begin{aligned} & H_{\frac{1}{3}}^{(1)}\left[\tfrac{1}{3}(-2\tau_0)^{3/2}\right] \\ & = 2.001475Ke^{i(\pi/4 - b/2)}(1 + 0.26943162Ke^{i(5\pi/12 - b/2)} \\ & \quad - 1.7468684K^2 e^{i(5\pi/6 - b)} \\ & \quad + 0.11776928K^3 e^{i(\pi/4 - 3b/2)} + \cdots). \end{aligned} \quad (59)$$

The above series is applicable for values of K less than about 0.3 while the following series may be used for values of K greater than 1.

$$\begin{aligned} & H_{\frac{1}{3}}^{(1)}\left[\tfrac{1}{3}(-2\tau_0)^{3/2}\right] \\ & = 1.8383752e^{-i\pi/6}\left[1 + \frac{0.38234445}{K} e^{i(b/2 - 17\pi/12)} \right. \\ & \quad \left. - \frac{0.056298160}{K^2} e^{i(b - (11\pi/6))} + \cdots \right]. \end{aligned} \quad (60)$$

Appendix III

The following tables of the values of the parameters α_0, β_0, γ, and δ will be useful for engineers who may wish to prepare their own graphs of these variables.

TABLE I
$b=0$ degrees

K	α_0	β_0	γ	δ
0.001	0.9285863	1.6064269	$7.616501\cdot10^{-8}$	328.7031
0.0015	0.9289399	1.6060733	$1.713533\cdot10^{-7}$	219.1109
0.002	0.9292934	1.6057198	$3.045959\cdot10^{-7}$	164.3156
0.003	0.9300005	1.6050127	$6.852004\cdot10^{-7}$	109.5181
0.005	0.9314147	1.6035984	$1.902594\cdot10^{-6}$	65.68014
0.007	0.9328291	1.6021838	$3.727726\cdot10^{-6}$	46.89191
0.01	0.9349506	1.6000609	$7.603803\cdot10^{-6}$	32.79994
0.015	0.9384869	1.5965234	$1.709646\cdot10^{-5}$	21.83806
0.02	0.9420238	1.5929823	$3.037687\cdot10^{-5}$	16.35583
0.03	0.9491006	1.5858885	$6.818058\cdot10^{-5}$	10.87093
0.05	0.9632712	1.5716285	$1.898278\cdot10^{-4}$	6.476817
0.07	0.9774730	1.5572238	$3.731241\cdot10^{-4}$	4.587858
0.1	0.9988499	1.5344475	$7.677931\cdot10^{-4}$	3.163627
0.15	1.034680	1.496843	$1.771490\cdot10^{-3}$	2.044263
0.2	1.07060	1.45536	$3.266602\cdot10^{-3}$	1.474433
0.3	1.1396	1.3571	$8.045880\cdot10^{-3}$	0.8845394
0.45	1.19855	1.16363	0.01828846	
0.5	1.195376	1.095261	0.02097513	0.4418851
0.55	1.18127	1.03250	0.02288201	
0.7	1.1039	0.8907	0.02551241	
1	0.94973	0.7588	0.02658905	0.3244638
1.5	0.787243	0.69393	0.02727269	0.3320993
2	0.6965244	0.677461	0.02781785	0.3363081
3	0.6014975	0.6726738	0.02859621	0.3426977
5	0.5233519	0.6776176	0.02942886	0.3488246
7	0.4894830	0.6822038	0.02985080	0.3515397
10	0.4639841	0.6866289	0.03019482	0.3535486
15	0.4441111	0.6906600	0.03047936	0.3550714
20	0.4341654	0.6928699	0.03062734	0.3558143
30	0.4242154	0.6952112	0.03077920	0.3565423
50	0.4162533	0.6971762	0.03090353	0.3571123
70	0.4128405	0.6980444	0.03095758	0.3573532
100	0.4102809	0.6987057	0.03099844	0.3575323
150	0.4082901	0.6992261	0.03103038	
200	0.4072947	0.6994882	0.03104643	
300	0.4062992	0.6997517	0.03106250	

TABLE II
$b=5$ degrees

K	α_0	β_0
0.5	1.167620	1.099345
0.55	1.15408	1.04106
0.7	1.0825	0.907
1	0.9386253	0.7774
1.5	0.7831507	0.70905
2	0.6948939	0.689518
3	0.6014343	0.6810541
5	0.5238493	0.6827698
7	0.4899851	0.6859098
10	0.4644213	0.6892338
15	0.4444476	0.6924011
20	0.4344348	0.6941770
30	0.4244064	0.6960823
50	0.4163734	0.6976998
70	0.4129280	0.6984186
100	0.4103430	0.6989678
150	0.4083320	0.6994008
200	0.4073262	0.6996192
300	0.4063204	0.6998360

TABLE III
$b=10$ degrees

K	α_0	β_0	γ	δ
0.5	1.14176	1.10364	0.01832457	0.483146
0.55	1.12854	1.04886	0.02012058	
0.7	1.0622	0.922	0.02322641	
1	0.92744	0.7951	0.02530199	0.34981
1.5	0.778609	0.72367	0.02660990	0.344526
2	0.6928323	0.701304	0.02737463	0.3453370
3	0.6010364	0.6893457	0.02832479	0.3483445
5	0.5240884	0.6879045	0.02927127	0.3519657
7	0.4903287	0.6896165	0.02973849	0.3536957
10	0.4647464	0.6918466	0.03011291	0.3550096
15	0.4447085	0.6941512	0.03042648	0.3560200
20	0.4346472	0.6954926	0.03058753	0.3565161
30	0.4245592	0.6969611	0.03075254	0.3570036
50	0.4164704	0.6982279	0.03088745	0.3573825
70	0.4129990	0.6987960	0.03094610	0.3575477
100	0.4103937	0.6992321	0.03099039	0.3576679
150	0.4083661	0.6995770	0.03102501	
200	0.4073520	0.6997514	0.03104159	
300	0.4063376	0.6999272	0.03105947	

TABLE IV
$b=15$ degrees

K	α_0	β_0	γ	δ
0.01	0.9339666	1.5991993		
0.015	0.9370101	1.5952296		
0.02	0.9400530	1.5912571		
0.03	0.9461371	1.5833005		
0.05	0.9582938	1.5673141		
0.07	0.9704229	1.5511821		
0.1	0.9885273	1.5265833	$7.607370\cdot10^{-4}$	3.164489
0.15	1.018245	1.484027	$1.707340\cdot10^{-3}$	2.049241
0.2	1.04683	1.43873	$3.143361\cdot10^{-3}$	1.483285
0.3	1.0958	1.3369	$7.488214\cdot10^{-3}$	0.9040418
0.45	1.1248	1.1633	$1.550383\cdot10^{-2}$	
0.5	1.1175	1.1082	$1.727935\cdot10^{-2}$	0.503777

TABLE V
$b=30$ degrees

K	α_0	β_0	γ	δ
0.001	0.9283792	1.6062680		
0.0015	0.9286292	1.6058350		
0.002	0.9288792	1.6054019		
0.003	0.9293792	1.6045359		
0.005	0.9303791	1.6028038		
0.007	0.9313790	1.6010714		
0.01	0.9328786	1.5984726		
0.015	0.9353771	1.5941400		
0.02	0.9378743	1.5898050		
0.03	0.9428622	1.5811247		
0.05	0.9527994	1.5637015		
0.07	0.9626566	1.5461554		
0.1	0.9772092	1.5195065	$7.528173\cdot10^{-4}$	3.169213
0.15	1.000470	1.473872	$1.696156\cdot10^{-3}$	2.059477
0.2	1.02173	1.42628	$3.019590\cdot10^{-3}$	1.498425
0.3	1.0537	1.3250	$6.676632\cdot10^{-3}$	0.9267893

TABLE VI
$b=45$ degrees

K	α_0	β_0	γ
0.01	0.9317050	1.5978944	
0.015	0.9336162	1.5932734	
0.02	0.9355250	1.5886506	
0.03	0.9393331	1.5793981	
0.05	0.9468900	1.5608527	
0.07	0.9543270	1.5422310	
0.1	0.9651474	1.5141109	$7.447599\cdot10^{-4}$
0.15	0.981841	1.466660	$1.657935\cdot10^{-3}$
0.2	0.99609	1.41857	$2.725817\cdot10^{-3}$

TABLE VII
$b=60$ degrees

K	α_0	β_0	γ	δ
0.01	0.9304662	1.5974747		
0.015	0.9317575	1.5926440		
0.02	0.9330460	1.5878130		
0.03	0.9356117	1.5781480		
0.05	0.9406722	1.5588012		
0.07	0.9455917	1.5394241		
0.1	0.9525869	1.5102899	$7.370080\cdot10^{-4}$	3.188159
0.15	0.962780	1.4615719	$1.622116\cdot10^{-3}$	2.091854

TABLE VIII
$b=90$ degrees

K	α_0	β_0	γ	δ
0.001	0.9278792	1.6061340		
0.0015	0.9278792	1.6056340		
0.002	0.9278792	1.6051340		
0.003	0.9278792	1.6041340		
0.005	0.9278791	1.6021341		
0.007	0.9278788	1.6001342		
0.01	0.9278781	1.5971346	$7.593007\cdot10^{-6}$	32.77774
0.015	0.9278756	1.5921361	$1.705996\cdot10^{-5}$	21.82045
0.02	0.9278707	1.5871389	$3.028018\cdot10^{-5}$	16.34229
0.03	0.9278512	1.5771507	$6.790917\cdot10^{-5}$	10.86555
0.05	0.9277491	1.5572117	$1.8660123\cdot10^{-4}$	6.487586
0.07	0.9275276	1.5373482	$3.617659\cdot10^{-4}$	4.614538
0.1	0.9268802	1.5077633	$7.241406\cdot10^{-4}$	3.213921

TABLE IX
$b=135$ degrees

K	α_0	β_0
0.01	0.9240532	1.5978964
0.015	0.9221394	1.5932799
0.02	0.9202268	1.5886662
0.03	0.9164031	1.5794503
0.05	0.9087653	1.5610894
0.07	0.9011474	1.5428670
0.1	0.8897751	1.5158991

TABLE X
$b=180$ degrees

K	α_0	β_0
0.001	0.9271714	1.6064256
0.0015	0.9268178	1.6060720
0.002	0.9264643	1.6057185
0.003	0.9257572	1.6050114
0.005	0.9243431	1.6035972
0.007	0.9229291	1.6021830
0.01	0.9208086	1.6000619
0.015	0.9172759	1.5965272
0.02	0.9137459	1.5929931
0.03	0.9066971	1.5859281
0.05	0.8926687	1.5718196
0.07	0.8787747	1.5577460
0.1	0.8582890	1.5367475

General Amplitude Relations for Transmission Lines with Unrestricted Line Parameters, Terminal Impedances, and Driving Point*

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Summary—*Subject to no other restrictions and approximations than those actually implied in the derivation of the transmission-line equations and their application to a terminated line, general expressions in simple form are derived for the current amplitude in, and the potential difference between, the two conductors of a uniform transmission line at any point along its extension. The terminal impedances are assumed to be perfectly general, and the line may be driven by a loosely coupled generator which maintains a sine-symmetrical, a cosine-symmetrical, or an unsymmetrical field along any section of the line. General resonance conditions are obtained and expressions written down for extreme values of the amplitudes. The special cases of low line damping and low over-all damping are considered.*

The transmission line is commonly used to connect an active network to one containing a power-consuming device. As a consequence, the usual mathematical treatment is restricted to the low-loss line driven at one end. The present analysis is concerned with the solution of the transmission-line equations subject to much more general boundary, operating, and driving conditions than are considered in this conventional case. Although the desired solution will be derived specifically as a generalization of methods for making a variety of electrical measurements at ultra-high frequencies, it is well to bear in mind that it is not restricted to this particular application.

The Transmission Line and Short-Wave Measurements

The conventional methods for making wavelength measurements at ultra-high frequencies make use of an apparatus originally due to Lecher.[1] It consists of a parallel line, usually with low resistance and negligible leakage conductance, one end of which is coupled to a generator, while a movable bridge containing a current or voltage detector is provided for locating resonance points. This form of line with simple terminations and operating conditions is the subject of an investigation by Hund[2] into its usefulness for determining ultra-high frequencies. A modification, which makes possible the measurement of low reactance as well as of wavelength, substitutes a movable short-circuiting bar or a coil or condenser of low reactance for the detector. This latter is then arranged in an auxiliary circuit which is loosely coupled to the line and which serves merely as a resonance indicator.[3] A more flexible arrangement makes use of a generator which is freely movable along the entire extension of the line[4,5] and of a current or voltage indicator at one end. A bridge consisting of an unrestricted impedance may be moved along the wires with a properly adjusted tandem bridge. With one termination of the line made available in this way,[6] it is possible to measure not only wavelengths and reactances, but also resistances and permeabilities[5] and dielectric constants.[7,8,9] The movable oscillator provided means for calibrating current and voltage instruments.[5] The theory of coupling to a line an oscillator with a nonuniform current amplitude distribution has been investigated,[10,11] and it has been proved that except when coupled near the terminations of the line, a loosely coupled symmetrical oscillator is analytically equivalent to one or two pairs of equal and opposite point generators inserted in the line opposite the center of the oscillator. One pair of point generators is sufficient if the oscillator sets up a cosine-symmetrical field, two pairs are required if the field is sine-symmetrical, while three pairs are needed in the unsymmetrical case.[12,13]

The specific purpose of the present paper is to lay the theoretical foundations for a further development in transmission-line technique which makes both ends of the line freely available by providing a movable detector as well as a movable oscillator. This arrangement

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[1] E. Lecher, "Eine Studie über electrische Resonanzerscheinungen," *Wied. Ann.*, vol. 41, pp. 850–870; 1890.

[2] A. Hund, "Theory of determination of ultra-radio frequencies by standing waves on wires," Scientific Paper of the Bureau of Standards, No. 491, 1924.

[3] L. S. Nergaard, "A survey of ultra-high frequency measurements," *RCA Rev.*, vol. 3, pp. 156–195; October, 1938.

[4] R. King, "Standing waves and resonance curves," *Rev. Sci. Instr.*, vol. 1, pp. 164–180; March, 1930.

[5] R. King "Electrical measurements at ultra-high frequencies," Proc. I.R.E., vol. 23, pp. 885–934; August, 1935.

[6] A different and in many respects equally useful way is that used by R. A. Chapman in "A resonance curve method for the absolute measurement of impedance at frequencies of the order of 300 Mc/second," *Jour. Appl. Phys.*, vol. 10, pp. 27–38; January, 1939.

[7] R. King, "An absolute method for measuring dielectric constants of fluids and solids at ultra-high frequencies," *Rev. Sci. Instr.*, vol. 8, pp. 201–209; June, 1937.

[8] H. R. L. Lamont, "Theory of resonance in microwave transmission lines with discontinuous dielectric," *Phil. Mag.*, ser. 7, vol. 29, pp. 521–540; June, 1940.

[9] H. R. L. Lamont, "The use of the wave guide for measurement of microwave dielectric constants," *Phil. Mag.*, ser. 7, vol. 29, pp. 1–15; July, 1940.

[10] R. King, "The application of low-frequency circuit analysis to the problem of distributed coupling in ultra-high-frequency circuits," Proc. I.R.E., vol. 27, pp. 715–724; November, 1939.

[11] A. Alford, "Coupled networks in radio-frequency circuits," Proc. I.R.E., vol. 29, pp. 55–69; February, 1941.

[12] R. King, "A variable oscillator for ultra-high-frequency measurements," *Rev. Sci. Instr.*, vol. 10, pp. 325–331; November, 1939.

[13] R. King, "A generalized coupling theorem for ultra-high-frequency circuits," Proc. I.R.E., vol. 28, pp. 84–87; February, 1940.

not only simplifies methods of measurement already described, such as, for example, the measurement of dielectric constants, but makes possible new measurements using a technique heretofore unavailable. An example is the selective separation of the fundamental and the second harmonic.[11] Use of the arrangement has also been made to determine the voltage amplitude distributions along feeder lines to ultra-high-frequency antennas. Investigations involving the measurement of the radiation resistance of transmission lines and the input impedance of antenna systems are in progress using the new apparatus.

It follows directly from the general coupling theorem for a loosely coupled, symmetrical oscillator[13] and the Rayleigh-Carson reciprocal theorem,[14,15] that a loosely coupled and extended, but symmetrical, detector used as a voltmeter across the line may be treated exactly as though coupled to the line at a point opposite its center, provided it is not too near the ends of the line. The mathematical problem thus reduces to calculating the potential difference between the two conductors of a transmission line of arbitrary length, driven by point generators connected anywhere along its extension, and terminated at both ends in general impedances. The reactive part of the characteristic impedance of the line will not be neglected. It has been pointed out recently,[16] and quite correctly, that the common practice of treating the characteristic impedance as a pure resistance may lead to considerable error.

In carrying out the mathematical investigation, use will be made of the conventional transmission-line equations so that approximations and restrictions implied in their derivation and in their application to terminated lines will apply to the present analysis. However, no other conditions will be imposed. The transmission-line or telegraphist's equations are

$$-\partial V/\partial z = (r + j\omega l)I \qquad (1a)$$

$$-\partial I/\partial z = (g + j\omega c)V. \qquad (1b)$$

These equations are accurate only for lines with no radiation. Moreover, they are derived for an infinite line, so that their use for a terminated line necessarily involves some error. For a line which is very long compared with the distance between the two conductors this is significant only near the ends of the line.[17,18] In the case of an open end, the error is termed an "end-correction"; for any other termination it is usually absorbed in the terminal impedance.

At ultra-high frequencies the separation of a parallel line is frequently not a sufficiently small fraction of a wavelength to permit neglecting radiation terms completely in deriving[19] (1). Since these do not lead to a radiation parameter per unit length which is even approximately independent of the current amplitude distribution along the line, (1) cannot actually be written in this more general case. However, if the average radiation resistance per unit length $\bar{r}_r$ is so small that

$$(\bar{r}_r/\omega l)^2 \ll 1, \qquad (2)$$

(i.e., if it is of the same order of magnitude as ohmic resistance per unit length in a low-loss line), the current amplitude distribution is not significantly affected by radiation, and a reasonably good approximation is obtained by substituting an effective average resistance per unit length r' for r in (1a). In this case r' is defined by

$$r' = r + \bar{r}_r. \qquad (3)$$

After (1) has been integrated and the total effective resistance due to a section of line of length s is obtained in the form

$$r's = rs + \bar{r}_r s, \qquad (4)$$

then

$$R_r = \bar{r}_r s, \qquad (5)$$

is the radiation resistance of the section. Equation (5) is actually the definition of the average radiation resistance per unit length, i.e., of $\bar{r}_r$. In this case, the problem reduces to a solution of (1) with r' written[18] for r. For a concentric line $\bar{r}_r$ is usually negligible compared with r.

With no other restrictions or approximations than those implied in writing (1) for a terminated transmission line, let these be solved for the line shown in Fig. 1. The oscillation generator is loosely coupled with its center opposite $z=x$; it is assumed to be symmetrical so that it may be represented by one pair (for cosine symmetry) or by two pairs (for sine symmetry) of equal and opposite point generators placed in the two conductors at or very near $z=x$. If one pair is used, it is assumed that each point generator maintains an electromotive force of amplitude $V_x^e/2$. If two pairs are used, each point generator must maintan an electromotive force of amplitude $W'/2$ with $W_x^e = 2W' \sinh Kx_1$. By making W' sufficiently large, the separation x_1 of each pair may be made as small as desired. The oscillator is assumed to fulfill the conditions requisite to permit the application of the general coupling theorem, in particular, that it be loosely coupled.

Although it is assumed that the detector is also loosely coupled at z, its impedance Z_s is shown in the

[14] Lord Rayleigh, "Theory of Sound," The Macmillan Company, London, England, 1877, vol. 1, p. 155.

[15] J. R. Carson, "A generalization of the reciprocal theorem," *Bell Sys. Tech. Jour.*, vol. 3, pp. 393–399; July, 1924.

[16] L. S. Nergaard and B. Salzberg, "Resonant impedance of transmission lines," Proc. I.R.E., vol. 27, pp. 579–583; September, 1939.

[17] E. M. Siegel, "Wavelength of oscillations along transmission lines and antennas," University of Texas Publication, No. 4031, August 15, 1940.

[18] A rigorous analysis of the application of the transmission-line equations to a line of finite length is in preparation. Approximations and restrictions implied in the conventional equations will be considered in detail. The effect of radiation on the current amplitude distribution will be examined.

[19] It must be emphasized that any method which assumes the transmission-line equations valid and uses the current amplitude distribution calculated from them in determining the radiation resistance of the line treated as two parallel antennas must be used with caution. If radiation is not small, it is also significant in determining the current amplitude distribution.

figure. Later it will be required that Z_z be so large that it plays no important part in determining the current and voltage amplitude distributions. It is included explicitly in the analysis in order to discover of what order of magnitude it (or any other impedance placed across the line at z) must be to fulfill this requirement.

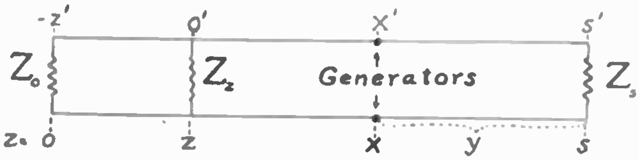

Fig. 1—Schematic diagram of the transmission line.

The terminal impedance Z_0 at $z=0$ and Z_s at $Z=s$ are general. They may consist of coils, condensers, antennas, or additional sections of transmission line.

The General Expressions for Current and Voltage Amplitudes

In order to derive an expression for the potential difference across Z_s, let an auxiliary, primed system of co-ordinates with origin at Z_z be defined tentatively (see Fig. 1). The terminal impedance $Z_{0'}$ is then made up of Z_z in parallel with the entire section of line to the left of 0′. The general solution of (1) for the potential difference $V_{0'}$ across $Z_{0'}$ due to one pair of equal and opposite point generators at x' has been given in equation (5) of reference 5 to be

$$V_{0'} = \frac{V_{x'}^{\,e}[Z_cY_s \cosh Ky' + \sinh Ky']}{(Z_c^2Y_{0'}Y_s + 1)\sinh Ks' + Z_c(Y_{0'}Y_s)\cosh Ks'} \equiv \frac{P_1(y', Y_s)}{Q'(s')}. \qquad (6a)$$

For two pairs of equal and opposite point generators, symmetrically placed with respect to x, the numerator in (6a) must be replaced by

$$P_2(y', Y_s) = W_{x'}^{\,e}[Z_cY_s \sinh Ky' + \cosh Ky']. \qquad (6b)$$

The following notation has been used:

$$y' = s' - x' \qquad (7a)$$

$$Y_{0'} = 1/Z_{0'}; \quad Y_s = 1/Z_s. \qquad (7b)$$

The characteristic impedance Z_c and the general complex propagation constant K are defined below in terms of the parameters r, l, c, and g of the line.

$$Z_c = \left[\frac{r + j\omega l}{g + j\omega c}\right]^{1/2} = R_c(1 - j\phi) \qquad (8)$$

$$K = [(r + j\omega l)(g + j\omega c)]^{1/2} = \alpha + j\beta. \qquad (9)$$

The characteristic resistance R_c, the parameter ϕ, the attenuation constant α, and the phase constant β, are expressed in most general terms using the $f(h)$ and $g(h)$ functions defined and tabulated by Pierce.[20] Thus,

[20] G. W. Pierce, "A table and method of computation of electric wave propagation, transmission line phenomena, optical refraction, and inverse hyperbolic functions of a complex variable," *Proc. Amer. Acad. Arts and Sci.*, vol. 57, pp. 175–191; April, 1922.

$$R_c = +\left[\frac{\omega^2 lc + rg}{\omega^2c^2 + g^2}\right]^{1/2} f(h_z);$$

$$\phi = g(h_z)/f(h_z) \qquad (10)$$

$$h_z = \frac{\omega(rc - lg)}{\omega^2 lc + rg} \qquad (11)$$

$$\beta = +[\omega^2 lc - rg]^{1/2} f(h_k);$$

$$\alpha = +[\omega^2 lc - rg]^{1/2} g(h_k) \qquad (12)$$

$$h_k = \frac{\omega(rc + lg)}{\omega^2 lc - rg}. \qquad (13)$$

With subscript z or k, one can compute $f(h)$ and $g(h)$ from

$$f(h) = +\left[\frac{+[1 + h^2]^{1/2} + 1}{2}\right]^{1/2};$$

$$g(h) = +\left[\frac{+[1 + h^2]^{1/2} - 1}{2}\right]^{1/2} \qquad (14)$$

or obtain them directly from Pierce's tabulation. One notes that for all finite values of h_k and h_z

$$\alpha < \beta; \quad \phi < 1. \qquad (15)$$

The input admittance $Y_{0'}$, across the terminals of Z_z, is

$$Y_{0'} = Y_z + Y_i. \qquad (16)$$

Here Y_i is the input admittance of the section of line with its termination to the left of Z_z (Fig. 1). It is readily obtained from an expression like (6a) by setting $x'=0$, writing z' for s', Y_0 for Y_s, and requiring $Y_{0'}$ to vanish.

$$Y_i = \frac{i_{0'}}{V_{0'}^{\,e}} = \frac{1}{Z_c}\left[\frac{Y_0Z_c \cosh Kz' + \sinh Kz'}{Y_0Z_c \sinh Kz' + \cosh Kz'}\right] = \frac{1}{Z_c}\cdot\frac{N}{D}. \qquad (17)$$

Combining (14) and (15), one can write

$$Z_cY_{0'} = Z_cY_z + N/D. \qquad (18)$$

Upon substituting (18) in the denominator $Q'(s')$ of (6), and after inserting the explicit expressions for N and D as defined in (17), and combining hyperbolic functions, one readily obtains

$$\begin{aligned} Q'(s') = 1/D[&Z_c^2Y_0Y_s + 1)\sinh K(s' + z') \\ &+ Z_c(Y_0 + Y_s)\cosh K(s' + z')] \\ &+ Z_cY_z[Z_cY_s \sinh Ks' + \cosh Ks']. \end{aligned} \qquad (19)$$

It is now readily verified, using Fig. 1, that,

$$\left.\begin{aligned} s = s' + z', \qquad z = z', \\ y = s - x = y' = s' - x'. \end{aligned}\right\} \qquad (20)$$

By defining the following symbols

$$\begin{aligned} Q(s) = (&Z_c^2Y_0Y_s + 1)\sinh Ks \\ &+ Z_c(Y_0 + Y_s)\cosh Ks \end{aligned} \qquad (21a)$$

$$P_1(y, Y_s) \equiv V_z^s[Z_cY_s \cosh Ky + \sinh Ky] \quad (21b)$$

$$P_2(y, Y_s) \equiv W_z^s[Z_cY_s \sinh Ky + \cosh Ky] \quad (21c)$$

$$D_2(z, Y_0) \equiv Z_cY_0 \sinh Kz + \cosh Kz \quad (21d)$$

$$M(s', Y_s) \equiv Z_cY_s \sinh Ks' + \cosh Ks' \quad (21e)$$

and using (20), (5) may be written as follows:

$$V_{0'} = V_s = \left[\frac{P(y, Y_s)D_2(z, Y_0)}{Q(s) + Z_cY_sM(s', Y_s)D_2(z, Y_0)}\right]. \quad (22)$$

For a generator which maintains a cosine-symmetrical field the subscript 1 must be used with $P(y,Y_s)$; a sine-symmetrical field requires the subscript 2.

With the definition

$$\Psi \equiv \frac{Z_cY_sM(s', Y_s)D_2(z, Y_0)}{Q(s)} \quad (23)$$

one can write (22) in the form

$$V_s = \left[\frac{P(y, Y_s)D_2(z, Y_0)}{Q(s)[1 + \Psi]}\right]. \quad (24)$$

It will be postulated at this point that the admittance Y_s of the voltage-measuring device is sufficiently small so that $|\Psi| << 1$. This is equivalent to

$$|Z_c| \gg \left|\frac{Z_cM(s', Y_s)D_2(z, Y_0)}{Q(s)}\right|. \quad (25)$$

Subject to (25), one can write

$$V_s = \left[\frac{P(y, Y_s)D_2(z, Y_0)}{Q(s)}\right]; \quad \begin{cases} y = s - x \\ 0 \leq z \leq x. \end{cases} \quad (26a)$$

The general expression for the complex current amplitude I_z is obtained readily using (1) and (9). Thus,

$$I_z = -\frac{1}{KZ_c}\frac{\partial V}{\partial z}$$

or,

$$I_z = -\left[\frac{P(y, Y_s)D_1(z, Y_0)}{Z_cQ(s)}\right]; \quad \begin{cases} y = s - x \\ 0 \leq z \leq x \end{cases} \quad (26b)$$

with

$$D_1(z, Y_0) \equiv Z_cY_0 \cosh Kz + \sinh Kz. \quad (27)$$

Throughout the analysis it has been assumed that the point x (locating the equivalent point generators) lies to the right (Fig. 1) of z (where current and voltage are calculated). Consequently, (26a) and (26b) are correct only for points z which lie between the terminal impedance Z_0 and the point x. However, since Z_0 and Z_s are both perfectly general, the expressions for V_z and I_z for points between x and the terminal impedance Z_s are easily obtained by interchanging the subscripts 0 and s on the admittances, and writing x for $y(=s-x)$ and $s-z$ for z. Thus,

$$V_z = \left[\frac{P(x, Y_0)D_2(w, Y_s)}{Q(s)}\right]; \quad \begin{cases} w = s - z \\ x \leqq z \leqq s \end{cases} \quad (28a)$$

$$I_z = \left[\frac{P(x, Y_0)D_1(w, Y_s)}{Z_cQ(s)}\right]; \quad \begin{cases} w = s - z \\ x \leqq z \leqq s. \end{cases} \quad (28b)$$

General Amplitude Functions

The real and imaginary parts of the several functions appearing in (26) and (28) may be separated using the conventional complex notation for impedances and admittances. Thus, with the subscript j standing for 0, z, or s and with (8), factors of the following type may be written down:

$$Z_cY_j = R_c(1 - j\phi)(G_j - jB_j). \quad (29)$$

For convenience let the following symbolism be introduced:

$$g_j \equiv R_cG_j; \quad b_j \equiv R_cB_j; \quad (30)$$

$$\bar{g}_j \equiv g_j - \phi b_j; \quad \bar{b}_j \equiv b_j + \phi g_j; \quad \bar{y}_j^2 = \bar{g}_j^2 + \bar{b}_j^2; \quad (31)$$

$$\bar{g}_1 \equiv \bar{b}_0\bar{g}_s + \bar{b}_s\bar{g}_0; \quad \bar{g}_2 \equiv \bar{g}_0 + \bar{g}_s; \quad (32)$$

$$\bar{b}_1 \equiv 1 + \bar{g}_0\bar{g}_s - \bar{b}_0\bar{b}_s; \quad \bar{b}_2 \equiv \bar{b}_0 + \bar{b}_s. \quad (33)$$

With (30) to (33) one can write (21) and (29) as follows:

$$Q(s) = (\bar{b}_1 - j\bar{g}_1) \sinh Ks + (\bar{g}_2 - j\bar{b}_2) \cosh Ks \quad (34a)$$

$$P_1(y) = V_z^s[\sinh Ky + (\bar{g}_s - j\bar{b}_s) \cosh Ky] \quad (34b)$$

$$P_2(y) = W_z^s[(\bar{g}_s - j\bar{b}_s) \sinh Ky + \cosh Ky] \quad (34c)$$

$$D_1(z) = \sinh Kz + (\bar{g}_0 - j\bar{b}_0) \cosh Kz \quad (34d)$$

$$D_2(z) = (\bar{g}_0 - j\bar{b}_0) \sinh Kz + \cosh Kz \quad (34e)$$

$$M(s') = (\bar{g}_s - j\bar{b}_s) \sinh Ks' + \cosh Ks'. \quad (34f)$$

From these expressions it is clear that all functions may be obtained directly from $Q(s)$ by an appropriate change of variable and by assigning proper values to $\bar{b}_1$, $\bar{g}_1$, $\bar{b}_2$, and $\bar{g}_2$.

With (9) and the abbreviations,

$$C \equiv \cosh \alpha s; \; S \equiv \sinh \alpha s; \; c \equiv \cos \beta s; \; s \equiv \sin \beta s; \quad (35)$$

one readily obtains the following expression for (34a):

$$Q(s) = [(\bar{b}_1S + \bar{g}_2C)c + (\bar{g}_1C + \bar{b}_2S)s] + j[(\bar{b}_1C + \bar{g}_2S)s - (\bar{b}_2C + \bar{g}_1S)c]. \quad (36)$$

The real amplitude is

$$|Q(s)| = [A_s^2c^2 + D_s^2s^2 - 2F_scs]^{1/2}. \quad (37)$$

with

$$A_s^2 = \bar{y}_1^2S^2 + \bar{y}_2^2C^2 + 2(\bar{b}_1\bar{g}_2 + \bar{b}_2\bar{g}_1)CS, \quad (38a)$$

$$D_s^2 = \bar{y}_1^2C^2 + \bar{y}_2^2S^2 + 2(\bar{b}_1\bar{g}_2 + \bar{b}_2\bar{g}_1)CS, \quad (38b)$$

$$F_s = \bar{b}_1\bar{b}_2 - \bar{g}_1\bar{g}_2; \quad (38c)$$

and

$$\bar{y}_1^2 = \bar{g}_1^2 + \bar{b}_1^2; \quad \bar{y}_2^2 = \bar{g}_2^2 + \bar{b}_2^2. \quad (39)$$

The angle is obtained from

$$\tan \Theta_s = \left[\frac{(\bar{b}_1C + \bar{g}_2S)s - (\bar{b}_2C + \bar{g}_1S)c}{(\bar{b}_1S + \bar{g}_2C)c + (\bar{b}_2S + \bar{g}_1C)s}\right]. \quad (40)$$

The expression (37) can be put into a more convenient and symmetrical form in terms of the following parameters which characterize the terminal impedances.

$$\bar{\gamma}_0 \equiv \frac{\bar{g}_0}{1+\bar{y}_0^2}; \qquad \bar{\gamma}_s \equiv \frac{\bar{g}_s}{1+\bar{y}_s^2} \tag{41a}$$

$$\bar{\delta}_0 \equiv \frac{\bar{b}_0}{1-\bar{y}_0^2}; \qquad \bar{\delta}_s \equiv \frac{\bar{b}_s}{1-\bar{y}_s^2}. \tag{41b}$$

After extensive but simple transformations, using standard trigonometric and hyperbolic identities, one obtains

$$|Q(s)| = (1+\bar{y}_0^2)(1+\bar{y}_s^2)\sqrt{\tfrac{1}{2}(1-4\bar{\gamma}_0^2)(1-4\bar{\gamma}_s^2)}\,\{\cosh 2(\alpha s+\rho)-\cos 2(\beta s+\Phi)\}^{1/2} \tag{42a}$$

$$2\Phi = \tan^{-1}\left[\frac{-2(\bar{\delta}_0+\bar{\delta}_s)}{1-4\bar{\delta}_0\bar{\delta}_s}\right] \tag{42b}$$

$$2\rho = \tanh^{-1}\left[\frac{2(\bar{\gamma}_0+\bar{\gamma}_s)}{1+4\bar{\gamma}_0\bar{\gamma}_s}\right]. \tag{42c}$$

With $|Q(s)|$ determined, the other functions defined in (34) are quickly written down in a form similar to (42). The expressions so obtained as well as (42) are then easily rewritten as follows:

$$|Q(s)| = (1+\bar{y}_0^2)(1+\bar{y}_s^2)\sqrt{(1-4\bar{\gamma}_0^2)(1-4\bar{\gamma}_s^2)}\,\{\sinh^2(\alpha s+\rho)+\sin^2(\beta s+\Phi)\}^{1/2} \tag{43a}$$

$$|D_2(z, Y_0)| = (1+\bar{y}_0^2)\sqrt{1-4\bar{\gamma}_0^2}\,\{\sinh^2(\alpha z+\rho_0)+\cos^2(\beta z+\Phi_0)\}^{1/2} \tag{43e}$$

$$|M(s', Y_s)| = (1+\bar{y}_s^2)\sqrt{1-4\bar{\gamma}_s^2}\,\{\sinh^2(\alpha s'+\rho_s)+\cos^2(\beta s'+\Phi_s)\}^{1/2}. \tag{43f}$$

Here,

$$y = s-x;\ s' = s-z \tag{44}$$

$$\Phi = \tfrac{1}{2}\tan^{-1}\left[\frac{-2(\bar{\delta}_0+\bar{\delta}_s)}{1-4\bar{\delta}_0\bar{\delta}_s}\right]; \quad \Phi_0 = \tfrac{1}{2}\tan^{-1}[-2\delta_0]; \quad \Phi_s = \tfrac{1}{2}\tan^{-1}[-2\bar{\delta}_s]; \tag{45a}$$

$$\rho = \tfrac{1}{2}\tanh^{-1}\left[\frac{2(\bar{\gamma}_0+\bar{\gamma}_s)}{1+4\bar{\gamma}_0\bar{\gamma}_s}\right]; \quad \rho_0 = \tfrac{1}{2}\tanh^{-1}[2\bar{\gamma}_0]; \quad \rho_s = \tfrac{1}{2}\tanh^{-1}[2\bar{\gamma}_s]. \tag{45b}$$

It is easily proved that

$$\Phi = \Phi_0 + \Phi_s \tag{46}$$

$$\rho = \rho_0 + \rho_s. \tag{47}$$

The expressions for the unrestricted amplitudes $|V_z|$ and $|I_z|$ as obtained from (26) using (43) now may be written. For a cosine-symmetrical oscillator at x (one pair of point generators), one has for $0 \leqq z \leqq x$

$$|V_z| = |V_z^e| \left\{\frac{[\sinh^2(\alpha y+\rho_s)+\sin^2(\beta y+\Phi_s)][\sinh^2(\alpha z+\rho_0)+\cos^2(\beta z+\Phi_0)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2} \tag{48a}$$

$$|I_z| = \left|\frac{V_z^e}{Z_c}\right| \left\{\frac{[\sinh^2(\alpha y+\rho_s)+\sin^2(\beta y+\Phi_s)][\sinh^2(\alpha z+\rho_0)+\sin^2(\beta z+\Phi_0)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2}. \tag{48b}$$

$$|P_1(y, Y_s)| = V_z^e(1+\bar{y}_s^2)\sqrt{1-4\bar{\gamma}_s^2}\,\{\sinh^2(\alpha y+\rho_s)+\sin^2(\beta y+\Phi_s)\}^{1/2} \tag{43b}$$

For a sine-symmetrical oscillator at x (two pairs of opposing point generators symmetrically placed at x) one has for $0 \leqq z \leqq x$

$$V_z = |W_z^e| \left\{\frac{[\sinh^2(\alpha y+\rho_s)+\cos^2(\beta y+\Phi_s)][\sinh^2(\alpha z+\rho_0)+\cos^2(\beta z+\Phi_0)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2} \tag{48c}$$

$$I_z = \left|\frac{W_z^e}{Z_c}\right| \left\{\frac{[\sinh^2(\alpha y+\rho_s)+\cos^2(\beta y+\Phi_s)][\sinh^2(\alpha z+\rho_0)+\sin^2(\beta z+\Phi_0)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2}.$$

$$|D_1(z, Y_0)| = (1+\bar{y}_0^2)\sqrt{1-4\bar{\gamma}_0^2}\,\{\sinh^2(\alpha z+\rho_0)+\sin^2(\beta z+\Phi_0)\}^{1/2} \tag{43c}$$

$$|P_2(y, Y_s)| = W_z^e(1+\bar{y}_s^2)\sqrt{1-4\bar{\gamma}_s^2}\,\{\sinh^2(\alpha y+\rho_s)+\cos^2(\beta y+\Phi_s)\}^{1/2} \tag{43d}$$

The corresponding formulas for $z > x$ are given by (28) and (43) with appropriate changes in variable and admittance as indicated in the functions. For the cosine-symmetrical case with $x \leqq z \leqq s$ and $w = s-z$ one obtains

$$|V_z| = |V_z^e| \left\{\frac{[\sinh^2(\alpha x+\rho_0)+\sin^2(\beta x+\Phi_0)][\sinh^2(\alpha w+\rho_s)+\cos^2(\beta w+\Phi_s)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2} \tag{49a}$$

$$|I_z| = \left|\frac{V_z^e}{Z_c}\right| \left\{\frac{[\sinh^2(\alpha x+\rho_0)+\sin^2(\beta x+\Phi_0)][\sinh^2(\alpha w+\rho_s)+\sin^2(\beta w+\Phi_s)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2}. \tag{49b}$$

For the sine-symmetrical case with $x \leqq z \leqq s$ and $w = s-z$ one has

$$|V_z| = |V_z^e| \left\{\frac{[\sinh^2(\alpha x+\rho_0)+\cos^2(\beta x+\Phi_0)][\sinh^2(\alpha w+\rho_s)+\cos^2(\beta w+\Phi_s)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2} \tag{49c}$$

$$|I_z| = \left|\frac{V}{Z_c}\right| \left\{\frac{[\sinh^2(\alpha x+\rho_0)+\cos^2(\beta x+\Phi_0)][\sinh^2(\alpha w+\rho_s)+\sin^2(\beta w+\Phi_s)]}{[\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)]}\right\}^{1/2}. \tag{49d}$$

Using (29) to (31) and with the subscript j signifying either 0 or s, (45a) and (45b) become

$$\Phi_j=\tfrac{1}{2}\tan^{-1}\left[\frac{-2\bar{b}_j}{1-\bar{y}_j^2}\right];\quad \rho_j=\tfrac{1}{2}\tanh^{-1}\left[\frac{2\bar{b}_j}{1+\bar{y}_j^2}\right]. \tag{50}$$

The expressions (48) and (49) are the final formulas for the amplitudes of current and potential difference at any point along a line of length s between the terminal impedances Z_0 and Z_s. No conditions have been imposed other than those implied in the derivation and application of (1) to a finite line. The parameters per unit length, r, l, c, g, as well as the impedances Z_0 and Z_s, are in no way restricted. The driving field has been assumed to be symmetrical—either sine or cosine—but a simple combination of the solutions for the two types of symmetry yields the general case. The field may be distributed along any part of the line.

The condition (26) upon the impedance of a voltage detector is not a restriction on the generality of (48) and (49), but simply an inequality which must be fulfilled if (48) and (49) are to obtain under experimental conditions in which a voltmeter is connected across the line. Using (43), this condition (26) may be expanded into

$$|Z_z| \ll |Z_c| \left\{\frac{[\sinh^2(\alpha s'+\rho_s)+\cos^2(\beta s'+\Phi_s)][\sinh^2(\alpha z+\rho_0)+\cos^2(\beta z+\Phi_0)]}{[\sinh^2(\alpha s'+\rho_s+\alpha z+\rho_0)+\sin^2(\beta s'+\Phi_s+\beta z+\Phi_0)]}\right\}^{1/2}. \tag{51}$$

Since the extreme magnitudes of the trigonometric functions are 0 and 1, a fair estimate of the maximum value of the expression on the right in (51) (insofar as the trigonometric functions are concerned) is obtained by writing

$$\beta z+\Phi_0=\frac{m\pi}{2};\qquad \beta s'+\Phi_s=\frac{m'\pi}{2} \tag{52}$$

with m and m' odd integers. This leads to

$$|Z_z| \gg |Z_c| \left\{\frac{[\sinh^2(\alpha s'+\rho_s)+1][\sinh^2(\alpha z+\rho_0)+1]}{\sinh^2(\alpha s'+\rho_s+\alpha z+\rho_0)}\right\}^{1/2}. \tag{53}$$

If the voltage detector is placed at either end of that part of the line included in the length s, its impedance Z_z may be included in the terminal impedance. In this case no restriction need be imposed on Z_z and (53) is superfluous. A typical point well removed from the terminations is at the center of the line. Then,

$$z=s'=s/2 \tag{54}$$

and

$$|Z_z| \gg |Z_c| \left\{\frac{[\sinh^2(\tfrac{1}{2}\alpha s+\rho_s)+1][\sinh^2(\tfrac{1}{2}\alpha s+\rho_0)+1]}{\sinh^2(\alpha s+\rho_0+\rho_s)}\right\}^{1/2}. \tag{55}$$

This inequality is conveniently studied in three typical cases. These are

$$(\alpha s+\rho_0+\rho_s)<1;\quad |Z_z| \gg \left[\frac{|Z_c|}{\alpha s+\rho_0+\rho_s}\right] \tag{56a}$$

$$(\alpha s+\rho_0+\rho_s)\doteq 1;\quad |Z_z| \gg |Z_c| \tag{56b}$$

$$(\alpha s+\rho_0+\rho_s)>1;\quad |Z_z| \gg |Z_c|\, e^{\alpha s+\rho_0+\rho_s}. \tag{56c}$$

The significance of these conditions may be illustrated in the case of a line with low damping terminated in impedances with negligible losses. In this case (56a) reduces to

$$|Z_z| \gg \frac{|Z_c|}{\alpha s}. \tag{57a}$$

For a quarter-wave line, αs may be of the order of magnitude of 10^{-3} and Z_c of magnitude 10^2. Then one must have

$$|Z_z| \gg 10^5. \tag{57b}$$

One may expect, therefore, that current and voltage distribution will be disturbed if a voltmeter (or other high impedance) is placed across a line with low damping unless its impedance exceeds a megohm at the operating frequency.[21]

Extreme Amplitudes and Resonance

Resonance in a terminated transmission line occurs when the length of the line and the terminal impedances are so adjusted that the denominator in (48) and (49) assumes a minimum value. Since periodic functions are involved, multiple resonances are to be expected.

The conditions leading to extreme values of the function

$$q(s)\equiv\{\sinh^2(\alpha s+\rho_0+\rho_s)+\sin^2(\beta s+\Phi_0+\Phi_s)\}^{1/2}, \tag{58}$$

for given terminal impedances are obtained by differentiation with respect to s and equating to 0. This gives the following general condition defining the extremizing lengths s_n:

$$\frac{\sin 2(\beta s_n+\Phi_0+\Phi_s)}{\sinh 2(\alpha s_n+\rho_0+\rho_s)}=-\frac{\alpha}{\beta}. \tag{59}$$

This may be written in the alternative form

$$\cos 2(\beta s_n+\Phi_0+\Phi_s) = \pm\sqrt{1-\frac{\alpha^2}{\beta^2}\sinh^2 2(\alpha s_n+\rho_0+\rho_s)}. \tag{60}$$

Here the upper sign leads to the minimum value of $|q(s)|$ and hence to resonance. The lower sign maximizes

[21] In a recent paper, for example, it is stated without proof that "resistances as low as 8000 ohms can be placed across the line without disturbing the voltage distribution." A method for measuring resistance is based upon this statement, using equations which neglect all damping in the line.

$q(s)$ and hence minimizes the amplitudes in (48) and (49).

It follows directly from (59) that extreme values are possible only if

$$\frac{\alpha}{\beta} \sinh 2(\alpha s_n + \rho_0 + \rho_s) \leqq 1. \tag{61}$$

For values of s which do not satisfy (61) the positive slope of the hyperbolic function in (58) is greater than the maximum negative slope of the trigonometric function, so that no actual maxima or minima are formed, though there will be fluctuations about the hyperbolic function.

The optimum points for coupling the oscillator with respect to $|V_z|$ and $|I_z|$ with $0 \leqq z \leqq x$ are obtained by maximizing the functions

$$p_1(y) = \left[\sinh^2(\alpha y + \rho_s) + \sin^2(\beta y + \Phi_s)\right]^{1/2} \tag{62a}$$

$$p_2(y) = \left[\sinh^2(\alpha y + \rho_s) + \cos^2(\beta y + \Phi_s)\right]^{1/2}. \tag{62b}$$

The extremizing conditions for $\left\{\begin{smallmatrix} p_1 \\ p_2 \end{smallmatrix}\right\}$ are

$$\frac{\sin 2(\beta y + \Phi_s)}{\sinh 2(\alpha y + \rho_s)} = \mp \frac{\alpha}{\beta}. \tag{63a}$$

The maximizing conditions are

$$\cos 2(\beta y + \Phi_s) = \mp \sqrt{1 - \frac{\alpha^2}{\beta^2} \sinh^2 2(\alpha y + \rho_s)}. \tag{63b}$$

The upper sign in each case refers to p_1 (cosine symmetry), the lower to p_2 (sine symmetry). The minimizing conditions are given by (63) with $\pm$ written instead of $\mp$. For $x \geqq z \geqq s$ one must write x for y and the subscript 0 for s in (63).

The extreme amplitudes of $|V_z|$ (upper sign) and $|I_z|$ (lower sign) with $0 \leqq z \leqq x$ are obtained at points along the line which satisfy the conditions

$$\frac{\sin 2(\beta z + \Phi_0)}{\sinh 2(\alpha z + \rho_0)} = \pm \frac{\alpha}{\beta}. \tag{64a}$$

Maximum amplitudes occur at

$$\cos 2(\beta z + \Phi_0) = \pm \sqrt{1 - \frac{\alpha^2}{\beta^2} \sinh^2 2(\alpha z + \rho_0)}. \tag{64b}$$

Minimum amplitudes are obtained from (64b) by writing $\mp$ instead of $\pm$. For $x \leqq z \leqq s$ one must write $w(=s-z)$ for z and the subscript s for 0 in (64).

The Special Cases of Low Damping

In important special cases of the general theory one is justified in writing one or both of the following conditions defining low damping. The condition for moderately low damping per unit length of the line is

$$\left(\frac{\alpha}{\beta}\right) < 1. \tag{65a}$$

The condition for low over-all damping on the entire line including the terminal impedances is

$$(\alpha s + \rho_0 + \rho_s)^2 \ll 1. \tag{65b}$$

The relation (65a) will be satisfied if the line parameters obey the following inequalities:

$$\left(\frac{r}{\omega l}\right) < 1; \quad \left(\frac{g}{\omega c}\right) < 1. \tag{65c}$$

In this case (10) to (14) give

$$h_z^2 \ll 1; \quad h_k^2 \ll 1 \tag{65d}$$

so that

$$f(h) \doteq 1; \quad g(h) \doteq \frac{h}{2} < 1. \tag{65e}$$

Then

$$\beta \doteq \omega\sqrt{lc}; \quad \alpha \doteq \frac{\sqrt{lc}}{2}\left(\frac{r}{l} + \frac{g}{c}\right) < 1 \tag{65f}$$

$$R_c \doteq \sqrt{\frac{l}{c}}; \quad \phi \doteq \frac{h_s}{2} = \frac{1}{2\omega}\left(\frac{r}{l} - \frac{g}{c}\right) < 1. \tag{65g}$$

Subject to (65b) alone, (48a) and (48b) for the cosine-symmetrical field, with $0 \leqq z \leqq x$, reduce to

$$|V_z| = |V_z^c| \left\{\frac{[(\alpha y + \rho_s)^2 + \sin^2(\beta y + \Phi_s)][(\alpha z + \rho_0)^2 + \cos^2(\beta z + \Phi_0)]}{(\alpha s + \rho_0 + \rho_s)^2 + \sin^2(\beta s + \Phi_0 + \Phi_s)}\right\}^{1/2} \tag{66a}$$

$$|I_z| = \frac{|V_z^c|}{|Z_c|} \left\{\frac{[(\alpha y + \rho_s)^2 + \sin^2(\beta y + \Phi_s)][(\alpha z + \rho_0)^2 + \sin^2(\beta z + \Phi_0)]}{(\alpha s + \rho_0 + \rho_s)^2 + \sin^2(\beta s + \Phi_0 + \Phi_s)}\right\}^{1/2}. \tag{66b}$$

Similarly, (48c) and (48d) for the sine-symmetrical field for $0 \leqq z \leqq x$ become

$$|V_z| = |W_z^s| \left\{\frac{[(\alpha y + \rho_s)^2 + \cos^2(\beta y + \Phi_s)][(\alpha z + \rho_0)^2 + \cos^2(\beta z + \Phi_0)]}{(\alpha s + \rho_0 + \rho_s)^2 + \sin^2(\beta s + \Phi_0 + \Phi_s)}\right\}^{1/2} \tag{67a}$$

$$|I_z| = \frac{|W_z^s|}{|Z_c|} \left\{\frac{[(\alpha y + \rho_s)^2 + \cos^2(\beta y + \Phi_s)][(\alpha z + \rho_0)^2 + \sin^2(\beta z + \Phi_0)]}{(\alpha s + \rho_0 + \rho_s)^2 + \sin^2(\beta s + \Phi_0 + \Phi_s)}\right\}^{1/2}. \tag{67b}$$

Corresponding expressions for $x \leqq z \leqq s$ may be written down directly from (69).

The general nature of (66) and (67) is most clearly displayed by noting that except near vanishingly small values of the trigonometric functions the terms in α and ρ contribute a negligible amount. Thus, one can write for (66)

$$|V_z| \doteq |V_z^e| \left\{\frac{\sin(\beta y + \Phi_s)\cos(\beta z + \Phi_0)}{\sin(\beta s + \Phi_0 + \Phi_s)}\right\}; \quad \begin{cases} (\beta s + \Phi_0 + \Phi_s) \neq n\pi \\ (\beta y + \Phi_s) \neq n\pi \\ (\beta z + \Phi_0) \neq \dfrac{2n+1}{2}\pi \end{cases} \tag{68a}$$

$$|I_z| \doteq \frac{|V_z^e|}{|Z_c|}\left\{\frac{\sin(\beta y + \Phi_s)\sin(\beta z + \Phi_0)}{\sin(\beta s + \Phi_0 + \Phi_s)}\right\}; \quad \begin{cases} (\beta s + \Phi_0 + \Phi_s) \neq n\pi \\ (\beta y + \Phi_s) \neq n\pi \\ (\beta z + \Phi_0) \neq n\pi. \end{cases} \tag{68b}$$

Similarly for (87)

$$|V_z| = |V_z^e| \left\{\frac{\cos(\beta y + \Phi_s)\cos(\beta z + \Phi_0)}{\sin(\beta s + \Phi_0 + \Phi_s)}\right\}; \quad \begin{cases} (\beta s + \Phi_0 + \Phi_s) \neq n\pi \\ (\beta y + \Phi_s) \neq \dfrac{2n+1}{2}\pi \\ (\beta z + \Phi_0) \neq \dfrac{2n+1}{2}\pi \end{cases} \tag{69a}$$

$$|I_z| \doteq \frac{|V_z^e|}{|Z_c|}\left\{\frac{\cos(\beta y + \Phi_s)\sin(\beta z + \Phi_0)}{\sin(\beta s + \Phi_0 + \Phi_s)}\right\}; \quad \begin{cases} (\beta s + \Phi_0 + \Phi_s) \neq n\pi \\ (\beta y + \Phi_s) \neq \dfrac{2n+1}{2}\pi. \\ (\beta z + \Phi_0) \neq n\pi \end{cases} \tag{69b}$$

The symbol $\neq$ used in (68) and (69) stands for, "not too near."

The extreme values of (66) and (67) are obtained directly from the conditions of the general case using (65a) and (65b). They are listed below together with the extremizing values of s, y, and z.

$$|V_z|_{\max} \doteq \frac{|V_z^e|}{(\alpha s + \rho_0 + \rho_s)}; \quad \begin{cases} s = 1/\beta[n\pi - \Phi_0 - \Phi_s] \\ y = 1/\beta\left[\dfrac{2n+1}{2}\pi - \Phi_s\right] \text{ for cosine symmetry} \\ y = 1/\beta[n\pi - \Phi_s] \qquad \text{for sine symmetry} \\ z = 1/\beta[n\pi - \Phi_0] \end{cases} \tag{70a}$$

$$|I_z|_{\max} = \frac{|V_z^e|}{|Z_c|(\alpha s + \rho_0 + \rho_s)}; \quad \begin{cases} s \text{ and } y \text{ as above} \\ z = 1/\beta\left[\dfrac{2n+1}{2}\pi - \Phi_0\right] \end{cases} \tag{70b}$$

$$|V_z|_{\min} = V_z^e(\alpha y + \rho_s)(\alpha z + \rho_0); \quad \begin{cases} s = 1/\beta\left[\dfrac{2n+1}{2}\pi - \Phi_0 - \Phi_s\right] \\ y = 1/\beta[n\pi - \Phi_s] \qquad \text{for cosine symmetry} \\ y = 1/\beta\left[\dfrac{2n+1}{2}\pi - \Phi_s\right] \text{ for sine symmetry} \\ z = 1/\beta\left[\dfrac{2n+1}{2}\pi - \Phi_0\right] \end{cases} \tag{71a}$$

$$|I_z|_{\min} = \frac{|V_z^e|}{Z_c}(\alpha y + \rho_s)(\alpha z + \rho_0); \quad \begin{cases} s \text{ and } y \text{ as above} \\ z = 1/\beta[n\pi - \Phi_0]. \end{cases} \tag{71b}$$

Special Terminal Impedances

The terminal impedances are contained in the general formulas (48) and (49) in phase factors ϕ and ρ. These are defined by (50) with (29) and (30). Three simple cases will be treated without detailed discussion merely as examples. The subscript j may stand for 0 or s.

Example 1

Open end:

$$X_j = 0, \quad R_j = \infty. \tag{72a}$$

In this case,

$$B_j = 0, \quad G_j = 0; \tag{72b}$$

so that

$$\bar{b}_j = 0; \quad \bar{g}_j = 0; \quad \bar{y}_j = 0. \tag{72c}$$

Accordingly

$$\Phi_j = \tfrac{1}{2}\tan^{-1}\left[\frac{-0}{1}\right] = 0 \qquad (73)$$

$$\rho_j = \tfrac{1}{2}\tanh^{-1}\left[\frac{0}{1}\right] = 0. \qquad (74)$$

Example 2

Inductively bridged end of low impedance:

$$X_j = \omega L_j \ll R_c; \quad R_j \ll X_j. \qquad (75a)$$

In this case

$$B_j \doteq \frac{1}{\omega L_j}; \quad G_j \doteq \frac{R_j}{\omega^2 L_j^2} \ll 1; \qquad (75b)$$

so that

$$\bar{b}_j \doteq \frac{R_c}{\omega L_j} \gg 1; \quad \bar{g}_j = \left(\frac{R_c}{\omega L_j}\right)\left(\frac{R_j}{\omega L_j} - \phi\right). \qquad (75c)$$

Accordingly,

$$\Phi_j = \tfrac{1}{2}\tan^{-1}\left(\frac{-2\omega L_j}{-R_c}\right). \qquad (76a)$$

The angle $2\phi_j$ is in the third quadrant and the argument of the inverse trigonometric function is small. Hence

$$\Phi_j \doteq \left(\frac{\omega L_j}{R_c}\right) - \pi/2. \qquad (76b)$$

Similarly,

$$\rho_j \doteq \left(\frac{\omega L_j}{R_c}\right)\left(\frac{R_j}{\omega L_j} - \phi\right) \ll 1. \qquad (77)$$

In the case of low damping per unit length of line, as defined by (65a) alone, (65f) and (65g) permit writing

$$\frac{\omega L_j}{R_c} \doteq \frac{\omega\sqrt{lc}\cdot L_j}{l} = \beta k \ll 1. \qquad (78)$$

The equivalent bridge length k is defined by

$$k \equiv \frac{L_j}{l}. \qquad (79)$$

In the limiting case of

$$X_j = 0, \quad R_j = 0, \qquad (80)$$

one has

$$\Phi_j = -\pi/2, \quad \rho_j = 0. \qquad (81)$$

Example 3

End-bridged in the characteristic resistance

$$X_j = 0; \quad R_j = R_c.$$

In this case,

$$B_j = 0, \quad G_j = \frac{1}{R_c}; \qquad (82a)$$

so that

$$\bar{b}_j = \phi \bar{g}_j = \phi; \quad \bar{g}_j = 1. \qquad (82b)$$

Accordingly,

$$\Phi_j = \tfrac{1}{2}\tan^{-1}\left[\frac{-2\phi}{-\phi^2}\right]. \qquad (83a)$$

Since the angle $2\phi_j$ is in the third quadrant, one has

$$\Phi_j = \tfrac{1}{2}\left[\tan^{-1}\frac{2}{\Phi} - \pi\right] = -\tfrac{1}{2}\left[\tan^{-1}\frac{\phi}{2} + \frac{\pi}{2}\right]. \qquad (83b)$$

Similarly,

$$\rho_j = \tfrac{1}{2}\tanh^{-1}\frac{2}{2+\phi^2}. \qquad (84)$$

In the special case of low line damping defined by (65a) alone, (65g) gives $\phi^2 \ll 1$, so that one can write

$$\Phi_j \doteq -1/4(\phi + \pi); \quad \rho_j \rightarrow \text{very large}. \qquad (85)$$

Since for large values of the argument in hyperbolic functions one can write

$$x \geqq 3; \ \sinh x \doteq \cosh x \doteq \tfrac{1}{2}e^x \geqq 10. \qquad (86)$$

Considerable simplification in the general expressions (48) and (49) is possible if either end of the line is terminated in the characteristic resistance. Thus, if $X_s = 0$ and $R_s = R_c$, (48a) reduces to the following form. It must be recalled that $y = s - x$, that ρ_s is very large according to (85), while ρ_0 is unrestricted.

$$|V_z| = |V_z^e| e^{-(\alpha z + \rho_0)}\{\sinh^2(\alpha z + \rho_0) + \cos^2(\beta z + \Phi_0)\}^{1/2}. \qquad (87a)$$

The voltage distribution along the line between 0 and x (Fig. 1) is thus unchanged if $Z_s = R_c$, but its amplitude is small and it can never be a resonance amplitude. If $X_0 = 0$, $R_0 = R_c$ instead of $X_s = 0$, and $R_s = R_c$, one has from (48a)

$$|V_z| = |V_z^e| e^{-[\alpha(s-z)+\rho_s]}\{\sinh^2(\alpha y + \rho_s) + \sin^2(\beta y + \Phi_s)\}^{1/2}. \qquad (87b)$$

If both $X_0 = 0$, $R_0 = R_c$ and $X_s = 0$, $R_s = R_c$, one has

$$V_z = \tfrac{1}{2}|V_z^e| e^{-[\alpha(s-z+z)+\rho_0+\rho_s]}. \qquad (87c)$$

Conclusion

The extreme generality and surprising simplicity of the expressions (48) and (49) with (50) for the amplitudes of current and potential difference along a transmission line, and of the resonance and extremizing conditions (59), (63), and (64) should make these valuable in all transmission-line problems for which less exact formulas are inadequate. These include, in particular, many transmission-line methods for making electrical measurements at ultra-high frequencies.

Civilian Receiver Design in 1942*

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Summary—While obviously not of direct combat use, the propaganda, social, and economic contribution to the cause places civilian radio strategically in the defense program. A minimum of 7½ million such receivers are needed annually to maintain a satisfactory morale-building propaganda machine. The engineer has a rare opportunity to contribute handsomely to the expedition of this work. Such a program is outlined and a typical example cited.

THE radio industry finds itself thrust involuntarily into the most difficult program in its short but eventful history. Small consolation may be gained from the knowledge that hundreds of other industries are in the same or worse predicament. One hears so many opinions, numerous wild rumors, and such pessimistic prognoses that it is easy to be bowled over by the complex uncertainty that we face.

In the course of my preparation of this paper and, incidentally, trying to work at my own job, I have discussed the subject with men in government bureaus, military posts, and business-management capacities. These interviews have been of great help because they taught me that, regardless of details, each and every one of us has two primary aims in common. We have a war to win and an enemy to defeat. If we shall but look at all our problems in the light of these two aims, we shall be amazed at the clarity with which much of our course defines itself.

Note that I recited a double-barreled aim. "We have a war to win and an enemy to defeat." I did not give it that way merely for emphasis of restatement. They are two almost separate but significantly equally important tasks.

To "win a war" we devote every available resource of material, mental, and physical nature to the task of waging a combat sure enough to overpower any opposition that the other side can summon at the moment of contact. This is now an herculean job for the democracies and their ally.

To "defeat an enemy" is even more of a challenge to our ability and unity. It would be a sad sequel to a successful war if the victor gets spoiled. It we lose all the rights and liberties for which we are going to make such large sacrifices, our enemies will be the sly victors and we the ironically vanquished no matter how the shooting comes out.

All of this sermonizing might seem to be quite foreign to engineering and particularly to my subject "Civilian Receiver Design in 1942" but believe me, it's most pertinent. Let us realize that every problem we solve and decision that we make must contribute its share to winning the war and defeating the enemy. If only we can stick to this course we shall emerge from this ordeal as an industry aglow with the knowledge of having contributed intelligently to the cause and its vindication.

Remembering this we can begin to see light on our most important question, "Is there going to be any civilian radio business in 1942?" Most certainly, there should be if we are to do even one, let alone both of our tasks. The obvious propaganda value of radio has never been questioned by even the most hard-shelled militarist or bureaucrat who would recommend the abolition of all civilian industry. Just how many radios this country needs and what their useful life is becomes one of the moot questions that will determine the fate of our own civilian industry.

Let's make some "guesstimates." Forty million is generally conceded to be a good approximation of the number of families in this country. If we provide each family with a home radio, allow for an additional 10 million necessary for portable and office use, and then equip 10 of the 20 million passenger automobiles, we have a basic minimum quantity of 60-million civilian radios that must be kept in operation strictly as a propaganda unit. In peacetime we have learned that the average life of civilian receivers is not over six and perhaps more like four years. Allowing for an intensification of service work, the life cycle might be extended to eight years. This develops the simple arithmetic conclusion that our propaganda machine requires that at least 7½ million new civilian radios be produced per year. Actually, we all know that one radio per family will not be adequate in many instances because the additional income that is now and will continued to be enjoyed by some will provoke the natural desire for multiple radios.

It may be proposed that large installations in public squares and auditoriums will take care of tens of thousands of families at one time but the very vision of this smacks of the un-American way. The family unit is the foundation of democracy and regimented listening cannot help but develop into enforced listening which is just a raised arm away from sig heiling or its American equivalent.

"Winning the war" then requires that the way be paved for the annual production of at least 7½ million radios for civilian use. How does this tally with our second aim "defeating the enemy?" A lesson gained from the depressed thirties was that the public will go without much if it can have its radio entertainment. In spite of all the criticism, the American sponsored-program system has demonstrated that it gives the public the most entertainment for the least apparent cost. If the masses do not get this diversion from a

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nerve-racking daily (and perhaps nightly) grind, what is there to take its place? Perhaps soapbox orators on the street corner? "Social" clubs that teach ideologies of hatred? "Free" bulletins and handbills of subversion? Surely the civilian radio with its great mass appeal and publicly aired programs is the unchallenged leader in wholesome American diversion and entertainment.

The production of at least $7\frac{1}{2}$ million civilian receivers annually is needed to defeat the enemy in another way. No matter how hard we try, it is a recognized fact that we can never employ all of this country's workers in defense industry. True, many of the present radio workers have skills that are needed and must be utilized for defense. This transition is going to take considerable time as anyone familiar with defense-radio-production schedules well knows and the workers must get along meanwhile. But even more significant is the fact that civilian radio production can employ thousands of workers who cannot be absorbed in any defense work and yet need to eat, sleep, and become useful to society just as must their more skilled fellow Americans. An engineer who was in England through all the 1940 blitzes summed it up aptly when he said to me, "Morale cannot be maintained if one half of the workers get $1.50 per hour and the other half don't work."

Returning again to our aims, let us consider, what the radio engineer will be called upon to do so that the war will be won and the enemy defeated. In particular, the engineer can fall into a narrow, self-centered philosophy in these times. After years of irregular employment and not too flattering material compensation, the engineer suddenly finds his scientific talents in great demand. Many will have their heads turned by frantic offers to purchase their skill. When they do this, they will make more money for a while but what will they have done toward "defeating the enemy?" It is my firm conviction that an individual with the skill of the engineer owes it to mankind to make that skill serve the maximum number of people. If he finds himself in active defense work his course is pretty well set for him. However, if he is in any vestige of civilian work he has an opportunity to go a long way down the road toward "defeating the enemy."

To do this the radio engineer in civilian radio work must consider these four policies as virtually mandatory:

1. Standardize with a verve such as has never before prevailed.

   The economies in material and labor to be derived from standardization are of long-recognized value. The present emergency demands that normally important but now petty commercial and manufacturing idiosyncrasies be abandoned for the common cause. It has been most gratifying to see this plan in actual operation within the specialist groups of the Radio Manufacturers Association Materiel Bureau.

2. Meet changing conditions as they occur.

   We have already seen aluminum get tight only to find that it is currently easing up and copper is the headache. A while back there was a lot of newspaper talk about replacing metals with plastics while those of us in the industry knew that bakelite resin would surely become scarce. This same type of thing is going to happen over and over again for the duration. It behooves the engineer to keep informed so that he can even anticipate changes and be prepared to meet the conditions in his stride.

3. Co-operate with and assist in the co-ordination of government and management agencies.

   As W. R. G. Baker pointed out,[1] 1942 is going to require closer co-operation between business and government than we have ever known. Already our industry is forming an advisory board for collaboration with the Office of Production Management. Panels of this group will be named. The role of the engineer will be to feed management and thence the government with pertinent data and conclusions that will lead to sound operation of our civilian industry.

4. Make design decisions with due regard to the obligation of the engineer to society.

   On the heels of meeting changing conditions and co-operating with business and government comes this potent social contribution of the engineer. Daily he will have to face the challenge of the times. The easy course will be for him to keep himself busy with each problem as it arises and let someone else worry about other details. The trouble with this is that regardless of whether the war is won or lost this myopian soul will find that the enemy is the victor. The long-range approach is for the engineer so to plan his designs that he creates employment for the maximum number of workers and furnishes the public with the greatest amount of radio enjoyment from the materials allotted to civilian industry.

Naturally, we are anxious at this time to learn of our 1942 allotments and try to set our course. Unfortunately, we don't secure information that easily. The task of allotting strategic materials to civilian industry is of such magnitude and has such varied aspects that those in government charged with this task have not been able to make firm decisions yet on all materials. I believe that progress is being made and that by the end of 1941 some tangible status will be worked out for our industry.

[1] W. R. G. Baker, Annual message of Radio Manufacturers Association Director of Engineering, November 11, 1941.

It is the hope of the industry that we shall be able to get allotments of basic materials in contrast to an allotment specifying how many radio units may be made. When all is said and done, the allotment of units implies an allotment of basic materials. Just imagine a 75 per cent unit allotment with no other stipulation. The engineer would be likely to get the request from his management that everything up to and perhaps including the kitchen sink be tossed into the design. Why not? "The government said that the number of units be cut to 75 per cent and we must be patriotic." A ridiculous extreme, yes, but some may go part of the way. Certainly, it's not a setup that calls for engineering ingenuity and contribution. There cannot be an incentive for the engineer to conserve strategic materials. On the contrary, he is encouraged to waste them if he can get away with it.

If we are given our allotments in the form of basic-material tonnage, we will be faced with a real challenge to our ingenuity. If given a free hand to use the allotted materials to best advantage, we are almost certain to produce more than the apparent percentage of units from the allotted materials. Furthermore, the government agencies need not stipulate allotments on the basis of discrete proportions between various materials. Rather, they can allocate on the logical basis of materials available after due planning of direct defense needs. In this way the engineer can go still farther toward using available materials for the maximum service. Thus it might be said that allotments of basic materials are the result of and further encourage an exchange of confidences between business and government.

Unquestionably, this allocating of basic materials to various industries is a gigantic undertaking. The benefits to civilian industry are such though that it will be well worth our while to assist the government in all possible ways to promote and expedite such a system.

Now let's get down to something specific. I propose to discuss typical but not necessarily all the design changes that are now possible as a result of work done by the Radio Manufacturers Association Materiel Bureau and others. As an example I shall use a conventional five-tube broadcast-band ac-dc superheterodyne compact receiver. This design forms the backbone of American receivers and is familiar to all of us. I shall, furthermore, consider this receiver for 1942 in comparison with its 1940 construction. 1941 has been a year in which certain transitional effects were present to a greater or lesser degree.

TABLE I

	Iron and Steel			Brass, Copper, and Bronze			Aluminum			Tin			Lead			Nickel			Cobalt			Bakelite			Silver		
	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B	1940	1942 A	1942 B
Loud Speaker	1.15	1.28	1.28	0.24	×	×	—	0.02	0.02							—	0.04	0.04	—	0.05	0.05						
Variable Condenser or Permeability Tuner	0.38	0.68	0.39	×	×	×	0.10	—	—	×	×	×	×	×	×							×	×	×			
Loop Antenna				0.08	—	—																			—	0.08	—
Chassis	0.82	×	×																								
Brackets and Hardware	0.24	0.15	0.15										—	0.20	0.20												
Intermediate-Frequency Transformer	0.03	0.04	0.04	0.03	—	—	0.05	0	0	×	×	×	×	0.03	0.03	×	×	×				×	×	×	—	0.02	0.02
Other Coils				×	—	—																			—	0.01	0.03
Hookup Wire and Line Cord				0.06	0.06	0.06				×	×	×	×	×	×												
Electrolytics	×	×	×	×	×	×	0.014	0.03	0.03																		
Paper Condenser and Resistors				0.04	0.04	0.04	0.026	—	—	×	×	×	×	0.12	0.12												
Sockets	—	0.03	0.03	0.03	—	—																×	×	×			
Controls	0.21	0.26	0.26	0.05	×	×																					
Cabinet and Knobs	×	×	×																			1.8	—	—			
Remainder and ×	0.02	0.02	0.02	0.01	0.01	0.01	—	—	—	0.06	0.06	0.06	0.04	0.04	0.07	0.003	0.003	0.003				0.2	0.2	0.2			
Total	2.85	2.46	2.17	0.53	0.11	0.11	0.19	0.05	0.05	0.06	0.06	0.06	0.04	0.39	0.42	0.003	0.043	0.043	—	0.05	0.05	2.0	0.2	0.2	—	0.11	0.05
Decrease or *Increase*		0.39	0.68		0.42	0.42		0.14	0.14		0	0		*0.35*	*0.38*		*0.04*	*0.04*		*0.05*	*0.05*		1.8	1.8		*0.11*	*0.05*

I have prepared Table I for reference. With one exception it typifies what is now in sight and indicated for the receiver under consideration. All figures are in pounds per receiver. Details are bound to differ between different designers and their products but in general the trend is as indicated.

The 1940 receiver used a steel chassis weighing 0.82 pound and an aluminum-plate variable condenser. The 1942A chassis is similar but uses all steel plates in the gang condenser. If aluminum rotor end plates are necessary then a slight amount must be added but since the total rotor- and stator-plate weight was only one tenth of a pound in 1940, the two or four plates won't amount to much if needed. 1942B chassis differs in tuner and circuit. Permeability tuning is indicated and along with it, the elimination of the loop antenna. A lead-foil capacitance plate is substituted and an antenna coil used.

Both A and B 1942 receivers shown may use something other than a steel chassis. Perhaps it will be ceramic or it may be a pressed board or heavy mache. Lead foil or spray will be used for shielding within the chassis.

Under present conditions, it is hoped that aluminum, nickel, and cobalt will be allotted to the industry in the extremely small quantities necessary to permit the elimination of the electromagnetic field using 0.24 of a pound of copper in 1940. If alnico V can be procured a 0.2-pound magnet (including scrap) will work admirably and use only 0.02 of a pound of aluminum, 0.04 of nickel and 0.05 of cobalt. Additional filter capacitance using 0.016 of a pound of aluminum foil should be used but the net saving of aluminum in the receiver will, nevertheless, be 0.14 out of a total of 0.19 used in 1940 or almost 75 per cent reduction.

It is probable that if alnico V is used, the steel used in the speaker will decline abruptly with redesign of the magnetic circuit. This is the exception that has not been allowed in the chart as a saving. On the contrary, the amount of steel in the speaker rises in 1942 on our table because of the slight iron content of the magnet. Nevertheless, other chassis savings bring the net amount of steel shown in the table down for the entire set. Once the weight of the pot structure is brought down substantially through the use of alnico V, the frame might conceivably become some other material of nonstrategic nature.

At the risk of being branded as feebly dramatic, I propose in all seriousness, since we have eliminated the 0.24 of a pound of copper in the 1940 electrodynamic speaker field, we have maneuvered ourselves to stage a real "coup de copper." I find that silver wire is about 12 times the cost of copper wire. The condenser-tuned radio uses 0.08 of a pound of copper in the loop antenna. By using iron cup intermediate-frequency cores we can design the two intermediate-frequency transformers to use 0.02 of a pound of wire. Adding 0.01 for the oscillator coil this totals 0.11 of a pound of silver and our cost rises about 30 cents per set. This program and the electromagnetic-field saving gives us a reduction of 0.42 of a pound out of a 1940 usage of 0.52 of a pound of copper equivalent to about 80 per cent.

The set tabulated used a bakelite cabinet. This has been eliminated in 1942 in favor of wood or one of the many other alternate but admittedly more expensive materials. The small amount of bakelite that may be allotted to civilian radio in 1942 will not justify volume usage for cabinets.

We can go on, cutting down on bracket and dial-face steel through further use of crude ceramics and available low-grade plastics and even glass. Socket springs can be made of plated steel; lead foil may be used in paper condensers instead of aluminum; steel shafts instead of brass for controls and the like. All these minor elements add up and perform amazing results in the end.

You will observe that I suggest that copper wire be retained for hookup wire and the line cord. There are several good and sound reasons for this.

1. They are loose elements around a factory and petty thievery would certainly develop if silver were used. Not that anyone would get rich or a manufacturer go broke but these wires will be covered with insulation composed of more or less scarce materials and applied to the wire with precious machine hours. The first move by the thief would be to burn off the insulation so that the wire can be turned into cash. For the amount of wire used, it is my opinion that the industry should be allotted the copper for this reason alone.
2. The underwriters have certain basic and sound requirements for line cords and while an alternate for copper might be found, it now seems that the war may be over before it is approved. Perhaps a reconsideration will be allowed.
3. The insulation-braiding equipment available is at a great premium. In fact, direct defense needs may use all of this country's capacity. At any rate, the wire to be insulated must have such mechanical properties that the machines work at peak efficiency and with freedom from breakdown. I am told that this is not the case with steel wire.
4. For hookup wire use, assuming that steel or iron wire can be insulated suitably, it must be plated to permit easy and effective soldering by workers of indifferent skill. Furthermore, thorough and elaborate rustproofing is necessary or the service failures will be terrific.

The symbol "X" on the chart is used to designate that a very small amount of the material indicated is used. The actual amount is not given in the individual column because it would be confusing. However, this amount is included in the column designated "Remainder and X" so that the "total" column includes every bit of the indicated material.

The designation "—" in the chart is to indicate that the material used in one case is not used in another variation of the receiver. In other words, "—" indicates the complete elimination of the specific material.

Tubes are not listed in the chart but remarkable things have been and are being accomplished by the tube engineers. About half of the nickel used has been replaced by iron. Copper side rods are becoming silver-alloy side rods. Nickel-plated brass base rings are giving way to the use of less strategically important materials. An alternate for nickel-plated-brass base pins has not been found in spite of painstaking effort so our industry must plead for consideration and an allocation in this matter.

Receiver engineers should co-operate by avoiding the peacetime luxury of tube loading in their designs. Simple multiple-purpose tube structures should be used to conserve tube materials but tricky tubes causing high production shrinkage in tube manufacture are a waste.

I want to repeat that the tabulation shown is just one example of hundreds of designs. I hope that those who have not taken the pains to prepare similar ones for their own products will now do so because amazing things show up. Certainly, it is the orderly approach to our problem.

Civilian radio design in 1942 will surely be a formidable challenge to the engineer's ingenuity. Nevertheless the replenishment at the minimum rate of 7½ million new radios per year is his obligation to our propaganda unit. With the help of basic-material allocations he will come through. The happiness of those workers who will thereby earn their living and all those who will enjoy the use and companionship of these radios should be a thrilling gratification to such engineers.

High-Frequency Radio Transmission Conditions, October, 1941, with Predictions for January, and February, 1942*

NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.

THE radio transmission data herein are based on observations at Washington, D. C., of long-distance reception and of the ionosphere. Fig. 1 gives the October average values of maximum usable frequencies, for quiet days (hitherto called undisturbed days) for radio transmission by way of the regular layers of the ionosphere. The regular-layer maximum usable frequencies were determined by the F layer at night and by the F_2 layer during the day. Figs. 2 and 3 give the expected values of the maximum usable frequencies for radio transmission by the way of the regular layers, average for quiet days, for January and February, 1942, respectively.

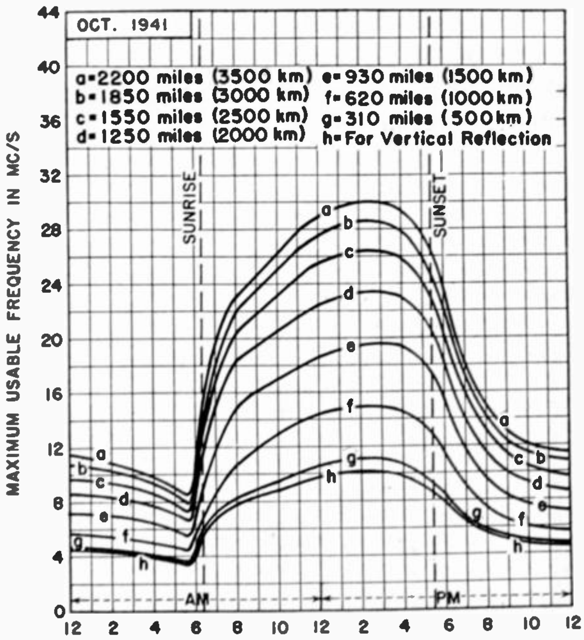

Fig. 1—Maximum usable frequencies for dependable radio transmission via the regular layers, average for quiet days, for October, 1941. These curves and those of Figs. 2 and 3 also give skip distances, since the maximum usable frequency for a given distance is the frequency for which that distance is the skip distance.

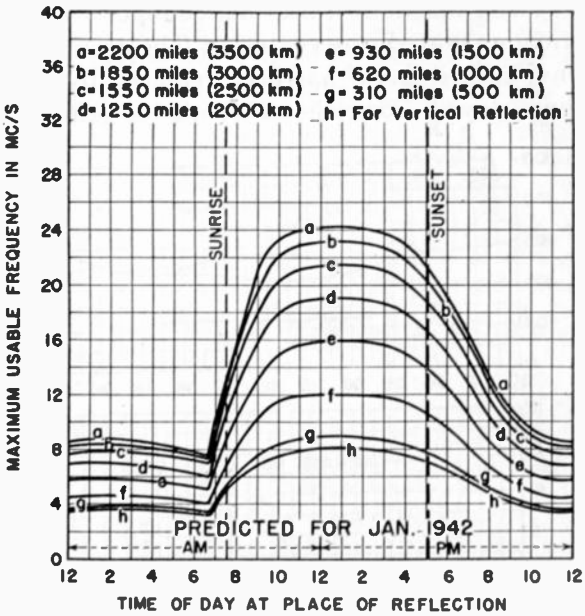

Fig. 2—Predicted maximum usable frequencies for dependable radio transmission via the regular layers, average for quiet days, for January, 1942. For information on use in practical radio transmission problems, see the pamphlets "Radio transmission and the ionosphere" and "Distance ranges of radio waves," obtainable from the National Bureau of Standards, Washington, D. C., on request.

Average critical frequencies and virtual heights of the ionospheric layers as observed at Washington, D. C., during October are given in Fig. 4. Critical frequencies for each day of the month are given in Fig. 5.

Beginning this month, Table I includes all days of the month and not merely stormy days (i.e., iono-

* Decimal classification: R113.61. Original manuscript received by the Institute, November 19, 1941.

spherically stormy days). Average ionospheric and magnetic character figures are given for each Greenwich half day. Ionospheric character figures are based on deviations of ionospheric characteristics from aver-

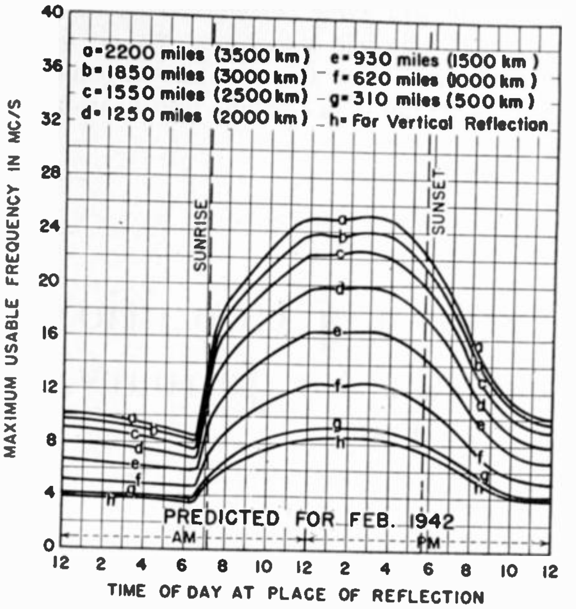

Fig. 3—Predicted maximum usable frequencies for dependable radio transmission via the regular layers, average for quiet days, for February, 1942.

age. The magnetic character figures are, as usual, based on ranges of magnetic variation, as reported by seven American-operated observatories. The term "average for undisturbed days" has been replaced by "average for quiet days." For radio transmission studies, days with character figures of 0, 1, or 2 are considered "quiet" and days with character figures 3 or greater are considered "stormy" and are not included in the average for quiet days.

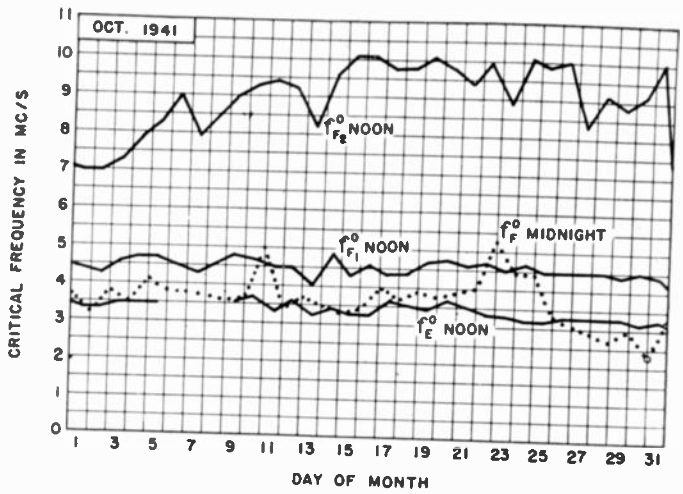

Fig. 5—Midnight and noon critical frequencies for each day of October. Open circle indicates the critical frequency observed during the ionospheric storm.

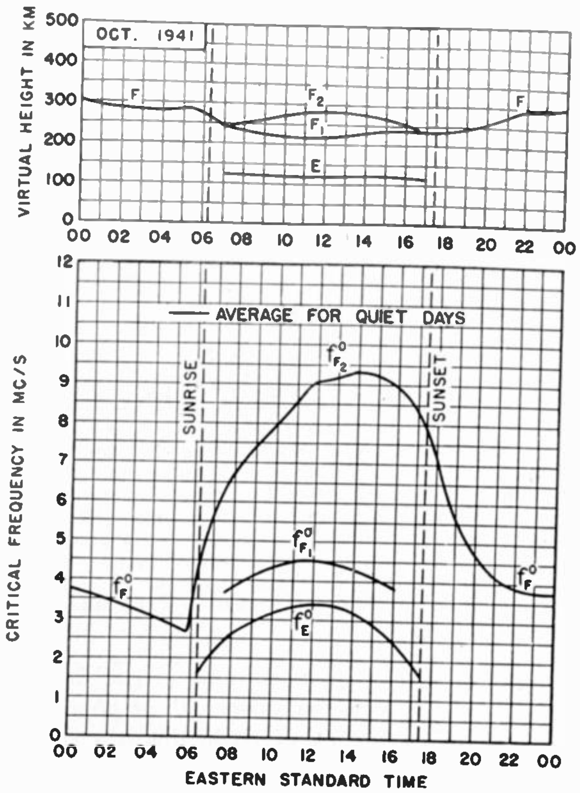

Fig. 4—Virtual heights and critical frequencies of the ionsopheric layers, observed at Washington, D.C., October 1941.

The following periods were considered stormy (all times E.S.T.):

0100 through 0500 October 12
0100 through 0600 October 13
0300 through 0600 October 15
2300 October 30 through 0600 October 31
1600 October 31 through 0600 November 3.

The mild ionospheric storm in progress at 0000

TABLE I

Ionospheric Storminess

Day	Ionospheric character[1] 00–12 G.M.T.	Ionospheric character[1] 12–24 G.M.T.	Magnetic character[2] 00–12 G.M.T.	Magnetic character[2] 12–24 G.M.T.
October				
1	2	2	2	1
2	2	2	1	2
3	2	2	1	0
4	2	1	1	2
5	2	1	2	1
6	2	0	2	2
7	1	1	1	2
8	1	2	2	2
9	1	1	1	2
10	1	1	1	2
11	1	2	3	4
12	3	0	3	2
13	3	2	2	1
14	2	1	2	2
15	3	1	3	2
16	2	1	2	3
17	2	2	1	1
18	2	1	2	1
19	2	1	2	2
20	1	1	2	1
21	2	1	2	1
22	2	2	2	5
23	1	1	2	2
24	1	1	2	3
25	1	1	2	1
26	1	1	2	2
27	2	2	2	1
28	2	2	1	2
29	1	2	1	1
30	1	2	2	2
31	3	2	3	4

[1] Ionospheric character figure (I figure) for ionospheric storminess at Washington during 12-hour period, on an arbitrary scale of 0 to 9, 9 representing the greatest disturbance.

[2] Average for 12 hours of American magnetic K figure determined by seven observatories, on an arbitrary scale of 0 to 9, 9 representing the most severe disturbance.

October 31 is indicated by the open circle in Fig. 5.

No sudden ionospheric disturbances were observed during the month.

Table II gives the approximate maximum usable frequencies for good radio transmission by means of sporadic-E reflections (as observed at Washington; sporadic-E conditions are patchy, not uniform over wide areas).

TABLE II

Approximate Maximum Usable Frequencies in Megacycles, for Radio Transmission by Means of Strong Sporadic-E Reflections, at Washington[1]

Day	Hour, E.S.T.																							
	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Oct.																								
9																				21	21	17		
10	16	17	15	16	16	15								29	33							16		
12		15	18	15	15														17	22				
13		15	16	15	17	18	16												15	15	15			
14			15	19	16													15	15					
16	16	17	19	19	18	15												15	15			18	15	17
17	18	18	15	←				NR			→													
18																			14	18	15	16		15
19	16	15	15	15												32	33	23	21	21	33			
20		15	16	22	18														15	23	44	24	24	27
21	28	18	18	19	16	17		18			31	33	43	27					38	34	18	17		15
22		17	17	13	15										23		17	22	21	25	34	16	15	
23	17	16	15						45										16	15	22	23	34	23
24	18	15																				15	17	
25		14	14	15																			15	14
26	14	15																22	17	23	27	45	33	24
27	18				15				17	23								15	15					
28																		23	15				14	15
29																		16	16	23				
30			15	15																				19
31																								

[1] "NR" indicates recorder not operating.

Books

Fourier Series and Boundary Value Problems, by Ruel V. Churchill

Published by McGraw-Hill Book Company, 330 West 42nd St., New York, N. Y. 201 pages+4-page index+ix. 8 figures. 6¼"×9¼". Price, $2.50.

The study of fields is of fundamental importance to the radio engineer. The mathematical relations in field theory require a knowledge of the solution of boundary value problems. This knowledge is of particular importance for the understanding of the techniques in the microwave field where wave guides and cavity resonators are extensively used to provide the boundary conditions, and ordinary circuit theory is inadequate.

The literature in this field which can be used by an engineer for independent study is meager and this volume is a contribution to a real need. Training in advanced calculus and ordinary differential equations is assumed and the rigor of the development will require that it be digested slowly by most engineers.

Examples from the field of physics are used to introduce the reader to the concept of orthogonal sets of functions and to the basic ideas of the use of such functions in representing arbitrary functions. The Fourier series, Fourier integral, Bessel functions, and Legendre polynomials are treated and problems in the appropriate co-ordinate systems are introduced to show their application. The omission of Fourier transforms is to be regretted since electrical problems yield best to this form of the Fourier integral.

The book would be an excellent text for the introduction of graduate students in physics and electrical engineering to this subject.

W. L. Everitt
The Ohio State University
Columbus, Ohio

Vacuum Tube Voltmeters, by John F. Rider

Published by John F. Rider Publisher. Inc., 404 Fourth Avenue, New York, N. Y, 176+xi pages+3-page index. 111 figures, 5¼"×8¼". Price, $1.50.

This book should be an extremely useful practical manual for users of vacuum-tube voltmeters. The first few chapters present simple, elementary, clearly expressed and illustrated descriptions of the circuit behavior of the common types of vacuum-tube voltmeters. Circuit diagrams, including magnitudes of circuit elements, are given for many typical arrangements used for direct-current, audio alternating-current, and radio-frequency alternating-current measurements. Comparisons of operation and use of peak, average, square-law, and other types of wave-form response are included, also discussions of linear, square-law, logarithmic, and decibel scales. Toward the end there is a useful summary of design and construction considerations, such as choice of tubes and resistors, leakage currents, input impedance limitations, by-passing, calibration stability, frequency errors, etc., also a useful chapter on calibration methods. The descriptions cover very satisfactorily the range of common types and uses of vacuum-tube voltmeters, including those used for signal tracing and those employed for ultra-high-frequency work. The author is occasionally careless in strictly electronic terminology, as for example in applying the term "contact potential" to small potential differences caused by high-initial-energy electron emission.

W. G. Dow
University of Michigan
Ann Arbor, Michigan

Radio Facsimile (Volume I, 1938)

Published by RCA Institutes Technical Press, 75 Varick Street, New York, N. Y. 353+xi pages. 219 figures. 6" x 9". Price $1.00 paper bound.

This is a collection of papers by engineers of the RCA Laboratories on the radio transmission and recording of images. As explained in the preface, it "may be regarded as the first book, or thesis, devoted exclusively to radio facsimile." The papers it includes have been arranged chronologically under four general headings that effectively cover the history of facsimile development and the applications to communication and broadcasting.

Separately, these papers describe major contributions in the field of radio facsimile; collectively they provide an interesting record of progress and an indication of development trends that will stimulate the imagination.

R. K. Potter
Bell Telephone Laboratories, Inc.
New York, N. Y.

Institute News and Radio Notes

THE INSTITUTE MOVES AHEAD IN 1941

The year 1941 has been an active one for the Institute. The more important developments and changes that have taken place are here summarized for this final issue of the 1941 volume of the PROCEEDINGS.

PROCEEDINGS: At the beginning of the year the PROCEEDINGS was $2\frac{1}{2}$ months behind schedule. This final issue of the 1941 volume comes from the press on time—a result of many hours of hard and effective work put in for the Institute by Dr. Goldsmith, Dr. Shackelford, Mr. Harold Wheeler, Mr. Dorman Israel, Dr. Everitt, and numerous others. Furthermore, the average thickness of each issue has been increased by nearly 15 per cent with the result that the first ten issues of the 1941 volume contained approximately the same number of pages as the entire 1940 volume. The PROCEEDINGS is still not as thick as desired, but the changes made are in the right direction.

A special effort has been made, and with some success, to increase the number of engineering papers that would be of practical assistance to the radio engineer in his daily work. Effort has also been successfully made to increase the number of published papers that relate to fields not adequately covered by papers voluntarily submitted, such as the broadcast transmitter and the receiver fields. The scope of acceptable papers has been widened to include papers that contain material, which, while not new, would be of interest and value to a considerable number of Institute members. Several such tutorial papers appear in the 1941 volume, and others have been promised that will be published in 1942.

Advertising: Beginning with the January, 1941, issue certain changes were made in the cover and arrangement of the PROCEEDINGS to facilitate the acceptance of color advertisements. This has increased the net income from advertising during 1941 by about \$2000.

Early in the year an increased advertising effort began to be made on a volunteer basis by some old friends of the Institute. This led, late in 1941, to an arrangement whereby the Institute obtained the services of an experienced advertising man, William C. Copp, on terms within the scope of the Institute budget.

The advertising billings for the last several issues of 1941 are up over 60 per cent above the average for 1940 before color began to be accepted, and are more than 30 per cent greater than for the early issues of 1941 (which contained color). Further increase is expected as a result of Mr. Copp's efforts.

More advertising revenue means a better and bigger PROCEEDINGS and more service to Institute members without extra expense to them.

Yearbook: Work was initiated on a new Yearbook, the first since 1937. This will appear in the spring of 1942. It will list the more than 6000 members of Junior and higher grade as of January 20, 1942, and will be the only "Who's Who of Radio Engineering" in existence.

Survey of Membership Preferences: For the first time in Institute history a poll was made to determine membership preference on editorial and other matters.

Results tabulated to date indicate that the present mathematical level of the PROCEEDINGS is approved and that most members would like the space devoted to committee and section activities reduced so that more papers could be published; also, of those who did express an opinion on the New York meetings, well over two-thirds felt that they should be called meetings of the "New York Section," but that slightly over half of those marking the questionnaire had no opinion on this subject.

Results on editorial preferences are now being compiled and will be used in guiding the editorial policy of the 1942 volume of the PROCEEDINGS.

Membership: The Institute paid membership at the end of 1941 will be about 7000, an all-time high, and a gain of about 1300 over 1940. This gain has been the result of a positive policy of membership promotion, combined with more than average effort to retain members. The result has been a large increase (over 50 per cent) in the number of new members elected, an even greater increase (over 150 per cent) in the number of former members reinstated to membership, and the lowest number of members of Associate and higher grades dropping out of the Institute in many years.

Study of the membership statistics indicates that improved business conditions in the radio industry, while an important contributing factor in this gain in membership, have not been the most important factor. The explanation of this year's gains largely lies in the hundreds of letters that have been written to prospective and past members of the Institute during the year, in personal contacts with such individuals, and in the systematic follow-up of those

FORTHCOMING MEETINGS

Winter Convention
New York, N. Y.
January 12, 13, and 14, 1942

Summer Convention
Cleveland, Ohio
June 29, 30, and July 1, 1942

who did not pay their 1941 dues promptly.

Numerous individuals participated in this work. The many names suggested by I.R.E. members in connection with the Yearbook questionnaire were especially valuable, as they have proved to be the best prospect list the Institute has been able to find.

Student and University Relations: The program begun three years ago to develop the possibilities of the student grade of membership has been continued and expanded. The close of this year will see approximately 950 student members, as compared with 314 at the end of 1938, and in this October alone the number of student applications was 153, compared with 130 for the entire year of 1938!

A system was put in operation in March whereby a faculty member is appointed as official Institute Representative at each university which has shown an interest in I.R.E. student membership. Along with this, a membership drive was carried on among college professors, which resulted in some sixty teachers of radio joining the Institute.

Students are now becoming an important source of Associate members. During 1941 approximately 180 will become Associate members, compared with 81 in 1940 and still fewer in previous years.

New Sections: During the year the Dallas-Fort Worth, Kansas City, and Twin Cities sections were established, and one section that had been dormant for the two previous years became active again. The possibility of petitioning for a section is now being discussed by several additional groups.

This expansion of section activity is largely the result of a new policy adopted by the Board of Directors whereby the formation of a new section is encouraged whenever it appears that a membership in excess of 25 Associates, Members, and Fellows can be maintained.

Office Operation: Trouble has been experienced in the past in the handling of Institute correspondence, from slow reviewing of papers, and by clerical delays in preparing standards reports. These difficulties have lost the Institute friends and dulled the interest of volunteer workers.

In a first attempt to remedy this situation, $1400 worth of dictating equipment was purchased and a new employee added to the headquarters staff. When these measures failed to give the hoped-for result, a business expert was employed to study the office and its operations. His report on this investigation will be made to the Board of Directors at the December meeting.

Executive Committee: The Executive Committee has been developed into an effective instrument for carrying out the policies established by the Board of Directors, and for relieving the Board of details so that it may have more time to consider policy matters.

The new Executive Committee setup, originated largely by Past-President Heising, also places in a definite way, responsibility for the proper functioning of each major activity of the Institute. Thus one member of the Executive Committee is responsible for office operations, another concentrates on technical committees, etc. This is a great advance over the past, where the Board of Directors attempted to exercise all management functions as well as to establish policies, but failed to function effectively in the former respect, because of being too large. What is 21 individuals' business tends to become no one's business.

Regional Representation on Board of Directors: The Board of Directors recognized the desirability of insuring that members some distance from New York City participate in the Institute management through representatives of their own choosing, and the desirability of placing the responsibility for the Institute welfare in each region in a definite way upon one in close contact with the area. To this end, there was submitted to the voting members a constitutional amendment for the establishing of regional Directors.

This amendment received 74.5 per cent of the votes cast on the question, but was 8 votes short of obtaining the three-fourths majority required by the constitution.

This problem accordingly still remains to be faced. It must be solved, since the wishes of nearly three fourths of the membership cannot be ignored on a vital matter without undermining the confidence of the membership in the organization.

Frederick Emmons Terman
President, 1941

Board of Directors

A regular meeting of the Board of Directors was held on November 5 and was attended by F. E. Terman, president; Haraden Pratt, treasurer; Alfred N. Goldsmith, editor; Austin Bailey, A. B. Chamberlain, I. S. Coggeshall, Melville Eastham, H. T. Friis, R. A. Heising, L. C. F. Horle, C. M. Jansky, Jr., B. J. Thompson, H. M. Turner, A. F. Van Dyck, H. A. Wheeler, and H. P. Westman, secretary.

There will be included in the 1942 Yearbook the names of all paid members as of December 31, 1941 and, in addition, the names of newly elected members whose entrance fees and dues have been paid by January 20, 1942.

President-elect Van Dyck was appointed a member of the Executive Committee.

The Tellers Committee reported on the election for officers: A. F. Van Dyck was elected president and W. A. Rush vice-president for 1942. A. B. Chamberlain, W. L. Everitt, and B. J. Thompson will serve as directors for the period 1942–1944.

The following report was submitted by the Tellers Committee on the ballots cast in the voting on proposed changes in the Constitution. In such balloting, at least 20 per cent of all voting members must

	Votes received	Per cent of voting members	Votes required to approve	Yes	No
Regional Directors	1577	41%	1183	1175	402
Member Grade, Art. II, Sec. 3 (b)	1612	42%	1209	1451	161
Member Grade, Art. II, Sec. 3 (c-1)	1595	41%	1197	1424	171
Equalize Member and Fellow Entrance Fees	1606	41%	1205	1266	340
Appointment of Committees	1602	41%	1202	1408	194
Appointment of Directors	1598	41%	1199	1528	70

participate and at least three quarters of the votes cast must be affirmative to adopt a proposal.

It will be noted that all amendments, with the exception of that for regional directors, were approved and become effective December 5, 1941. The proposal to establish regional directors failed by 8 votes.

Section 16 of the Institute By-laws was amended to read as follows.

Sec. 16—A bill shall be sent to each member not later than December 1 covering his dues for the following year. A second bill shall be mailed on or about February first to each member whose dues remain unpaid. Not later than March first, each member whose dues remain unpaid shall be so notified by the Secretary and informed that, in accordance with Article III, Section 7, of the Constitution, should his dues remain unpaid after March 31, his membership will terminate and he will lose the right to vote and to receive the publications of the Institute. On April first, the name of each member whose dues remains unpaid shall be removed from the roll of membership and such member shall be sent a notice to the effect that according to Article III, Section 7, his membership in the Institute has in fact terminated. The list of such terminated memberships shall be turned over to the Membership Committee.

To clarify certain reports to prospective advertisers, the following two new Bylaws were adopted.

Sec. 53—The price of a single annual subscription to the PROCEEDINGS for a Fellow, Member, or Associate shall be $5.00, to be included in his annual dues as specified in Article IV of the Constitution.

Sec. 54—The price of copies of the PROCEEDINGS supplied to a newly admitted Fellow, Member, or Associate in advance of the period for which dues are payable shall be included in his entrance fee which is specified in Article IV of the Constitution.

Another new section of the Bylaws was approved and follows.

Sec. 00—The Secretary shall, as soon as possible after the end of each year, present to the Board of Directors a report giving the number of members of each grade at the close of the year, and the gains and losses in each grade of membership during the year as a result of elections, transfers, reinstatements, deaths, resignations, failure to pay dues, and all other causes. This report shall also give the number of newly elected members whose elections become void during the year through failure to pay entrance fee and initial dues, and the number of newly elected members who at the end of the year have not yet paid entrance fee and initial dues but whose election is still effective as of that date.

The Appointments Committee will comprise A. F. Van Dyck, chairman; A.B. Chamberlain, B. J. Thompson, H. M. Turner, and H. A. Wheeler. It submits to the Board of Directors a list of candidates for secretary, treasurer, editor, and appointed directors, and of the chairmen and members of the Awards, Constitution and Laws, Executive, Nominations, and Tellers Committees.

On the recommendation of the Awards Committee, the Medal of Honor for 1942 was awarded to Dr. A. H. Taylor.

Also on the recommendation of the Awards Committee, the following were advanced to Fellow grade: W. L. Barrow, G. H. Brown, Geoffry Builder, A. B. Chamberlain, E. D. Cook, H. A. Knowles, W. P. Mason, H. O. Peterson, and G. C. Southworth.

Last August, questionnaires were mailed to the membership. Tabulations are available on about 3700 of the questionnaires concerning the PROCEEDINGS and New York meetings. On the question of the mathematical level of the PROCEEDINGS, 69 per cent thought it about right and 25 per cent considered it too high. These views will be observed in the scheduling of future material for the PROCEEDINGS.

In regard to the publication of committee and section activities reports, 55 per cent considered it more desirable to use this space for technical papers while 37 per cent preferred a continuation of the present reports. In view of these votes, the future PROCEEDINGS will carry only a tabulation of the committee meetings which are held and the section meetings which occur. Summaries of the papers presented at section meetings will be omitted.

The returns on the questions of establishing a New York section or continuing in New York with "Meetings of the Institute" was considered indecisive in that 60 per cent indicated no opinion.

PEDER OLUF PEDERSEN
1874–1941

Peder Oluf Pedersen was born on June 19, 1874, in Sig, Denmark. He was graduated with honor in civil engineering in 1897 from the Royal Technical College. His education was made possible through the interest of the King of Denmark.

He developed an interest in electrical research work soon after his graduation. In 1899 he became associated with Valdemar Poulsen and assisted in the development of the telegraphone and the arc transmitter.

He was appointed Assistant Profesor in 1909 and Professor in 1912 in Telegraphy, Telephony, and Radio at the Royal College in Copenhagen. In 1922, he became Principal of that college. The University of Copenhagen conferred a Ph.D. degree on him in 1929.

His contributions to the technical press on his experimental researches in electrophysics and electrotechnics were extensive.

Dr. Pedersen received the Gold Medal of the Royal Danish Society of Sciences in 1909, and was awarded the C. H. Oersted Medal in 1927. The Institute's Medal of Honor was awarded to him in 1930.

He held membership in many technical societies and was president of several. He joined the Institute as a Fellow in 1915 and served as Vice-President during 1939.

His death occurred in Copenhagen on August 30, 1941.

Executive Committee

A meeting of the Executive committee was held on November 4 and was attended by F. E. Terman, president; Melville Eastham, Alfred N. Goldsmith, editor; Haraden Pratt, treasurer; B. J. Thompson, and H. P. Westman, secretary.

Applications for Member grade numbering 14 for admission and 17 for transfer were approved.

Approval was granted of 99 applications for Associate, 11 for Junior, and 153 for Student membership.

A report was received from H. B. Richmond, chairman of the Engineering Registration Committee, on a law recently signed by the Governor of Massachusetts. This law does not require the registration of engineers practicing within the state. It provides a means whereby Massachusetts engineers practicing outside of the state may obtain registration and, as a result, such reciprocity as may be available from the state in which they are operating.

Special Papers

The Committee on Special Papers expresses its appreciation of the co-operation of the following authors who have contributed invited papers to the PROCEEDINGS during 1941:

Dudley Foster	Kenneth A. Norton
Alan M. Glover	Ralph A. Powers
Peter C. Goldmark	E. H. Scott
Pierre Mertz	H. P. Thomas
Garrard Mountjoy	Sidney K. Wolf

Committee and Section Reports

In the answers to the recent questionnaire sent to the membership of the Institute, the members indicated their preference for *more space devoted to papers.* The membership approved reduction in space devoted to other matters, including information on section and committee meetings.

Accordingly, the new policy desired by the membership has been put into immediate force. This issue of the PROCEEDINGS contains one more paper than was originally planned. The inclusion of this paper therefore marks the first application of the editorial policy desired by the membership and adopted for the PROCEEDINGS.

Section Meetings

Buenos Aires

"Acoustic Problems in Modern Architecture," by Frederico G. Malvarez, August 29.

"Single-Sideband Transmission," by R. H. Scott, September 12

"Discussion on Propagation," September 19.

"Second Discussion on Propagation," October 2.

"Third Discussion on Propagation," October 16.

"Illumination," by Salvador Masson, Chief of Power Plants and Telephones, Republic of Uruguay, October 24.

Cincinnati

"Auditory Phenomena in Radio Transmission," by L. A. de Rosa, National Cash Register Company, October 21.

"Historical Sketch, New Developments, and Unusual Features of the Lake Ship-to-Shore Communication System," by H. E. Hageman, D. A. Heisner, and H. P. Bosweau, Lorain County Radio Corporation, October 30.

Detroit

"Radio as Used by the State of Michigan," by Ralph O. Williams, Emergency Services, September 26.

"Klystron Oscillator," by G. P. Brewington, Lawrence Institute of Technology, October 17.

Emporium

"Inspection Trip Through the Acme Electric and Manufacturing Company Plant, October 18.

"Color Television," by John N. Dyer, Columbia Broadcasting System, October 30.

Indianapolis

"Design and Production Considerations in Frequency Modulation Receivers," by Marvin Hobbs, E. H. Scott Laboratories, October 24.

Los Angeles

"Directional Antennas, with Special Reference to the KECA Installation," by George Curran, Earl C. Anthony, Inc., October 21.

Philadelphia

"Magnetic Alloys in Communication Apparatus," by V. E. Legg, Bell Telephone Laboratories, Inc., October 2.

Pittsburgh

"Central Distribution of Music Over Leased-Wire Services," by V. B. Bretzin, Voco Tele Music Corporation, October 13.

San Francisco

"Current Division in Plane-Electrode Triodes" (Review of previously published article), by Karl Spangenberg, Stanford University, October 22.

"Manufacture and Operation of Glass Lathes for the Construction of Vacuum Tubes," by Charles V. Litton, Litton Engineering Laboratories, October 22.

"Design and Adjustment of Broadcast Antenna Arrays," by Norman Webster, McClatchy Broadcasting System, November 14.

Toronto

"Electricity—The Handmaiden of the Air Force," by Group-Captain C. H. Keith, Royal Air Force Bombing and Gunnery School, October 10.

Washington

"Design Features of a Modern Television System," by J. D. Schantz, Farnsworth Television & Radio Corporation, November 10.

Committee Meetings

Admissions—November 5
Awards—November 5
Convention—October 22, November 3 and 14
Convention Papers—November 13
Facsimile, Subcommittee on Transmission October 3 and 30
Publications (Formerly Co-Ordinating)—November 7
Television—October 17

Membership

The following indicated admissions and transfers of memberships have been approved by the Admissions Committee. Objections to any of these should reach the Institute office by not later than December 31, 1941.

Transfer to Member

Fox, R. A., 1311 Terminal Tower, Cleveland, Ohio
Leonard, S. E., 2651 Kerwick Rd., University Heights, Cleveland, Ohio
Pierce, R. M., 1080 Sylvan Ave., Cleveland, Ohio
Scott, H. J., Department of Electrical Engineering, University of California, Berkeley, Calif.
Sear, A. W., General Electric Co., 1285 Boston Ave., Bridgeport, Conn.
Smeby, L. C., 4801 Connecticut Ave., Washington, D. C.
Troeglen, K., 646 Grandview, Topeka, Kan.

Admission to Member

Busignies, H. G., 742 Burns St., Forest Hills, L. I., N. Y.
Dumeresque, J. S., 26 Fairview Circle, Birmingham, Ala.
Hoover, P. L., Case School of Applied Science, Cleveland, Ohio
Loughridge, D. H., University of Washington, Seattle, Wash.
Quarles, L. R., Proffit, Va.
Quayle, J. C., "Grassendale," Manley, Helsby, via Warrington, Lancs., England
Smith, F. W., 5 Sullivan Rd., Easton, Pa.
Van Atta, L. C., 79 Wildwood St., Winchester, Mass.

The following indicated admissions and transfers of memberships were approved by the Board of Directors on November 5, 1941.

Transfer to Member

Baldwin, C. F., 254 Bradley Blvd., Schenectady, N. Y.
Cochran, L. B., University of Washington, Seattle, Wash.
Flynn, R. M., 2915 Lovers Lane, Dallas, Texas
Gluyas, T. M., Jr., 6727 Montgall Ave., Kansas City, Mo.

Hauck, V. D., 427 Alabama Rd., Towson, Md.
Hunt, C. M., Radio Station WJSV, Earle Bldg., Washington, D. C.
Kallmann, H., 417 Riverside Dr., New York, N. Y.
Keen, A. W., Hygrade Sylvania Corp., Emporium, Pa.
Moody, R. C., 9023 Lloyd Pl., West Hollywood, Calif.
Nelson, W. H., 910 Spencer Ave., Marion, Ind.
Novy, J. F., 153 E. Quincy Rd., Riverside, Ill.
Plotts, E. L., 2509 E. 76th St., Chicago, Ill.
Reich, H. J., University of Illinois, Urbana, Ill.
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Winter Convention
January 12, 13, and 14, 1942
New York, N. Y.

The Winter Convention of the Institute will be held at the Commodore Hotel, New York City, from Monday morning, January 12, 1942, through Wednesday evening, January 14. Advance notice of the dates was mailed to the membership in October. A folder giving the program of technical sessions in detail will be distributed in December.

Since the year 1942 marks the thirtieth anniversary of the founding of the Institute in 1912, that milestone will be suitably marked at the banquet, Tuesday evening, January 13. Characteristically, however, the Convention's outlook will be forward, not backward. The phrase "Radio's Expanding Role" has been chosen to set the keynote, as fitting for this transitional period when radio's peacetime technique is being adapted to military purposes. Special speakers will be heard at the opening session Monday morning and at the banquet to inform the Institute how the expansion of radio is being made to fit the larger needs of government in both domestic and international fields of operation.

At the end of this article are synopses of the papers to be presented at the technical sessions, of which there will be six, in accordance with the following brief summary of the program.

PROGRAM

Monday, January 12

Morning
- Registration
- Report of the retiring president
- Introduction of the president for 1942
- Special speaker

Afternoon
- Dutch-treat luncheon
- Technical session

Evening
- President's reception
- Buffet dinner and party
- Technical session

Tuesday, January 13

Morning
- Technical session, trip

Afternoon
- Dutch-treat luncheon
- Technical session, trip

Evening
- Banquet

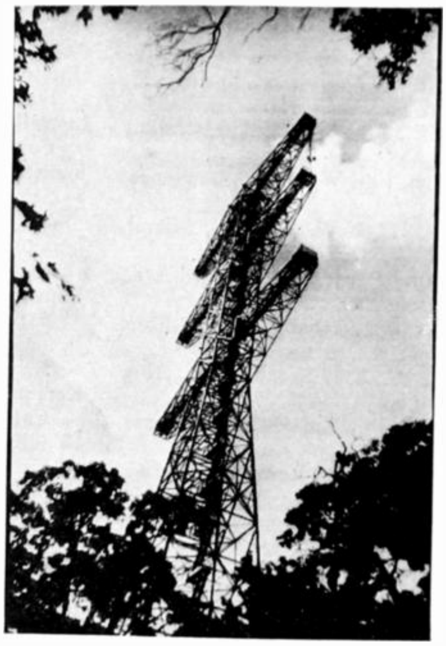
Wednesday, January 14

Morning
- Unscheduled time
- College technical session
- Meeting of Sections Committee

Afternoon
- Technical session

Evening
- Technical session

The following notes have been supplied by the chairman of the Convention Committee, I. S. Coggeshall.



A. Tennyson Beals

The "skyhook" at Alpine.

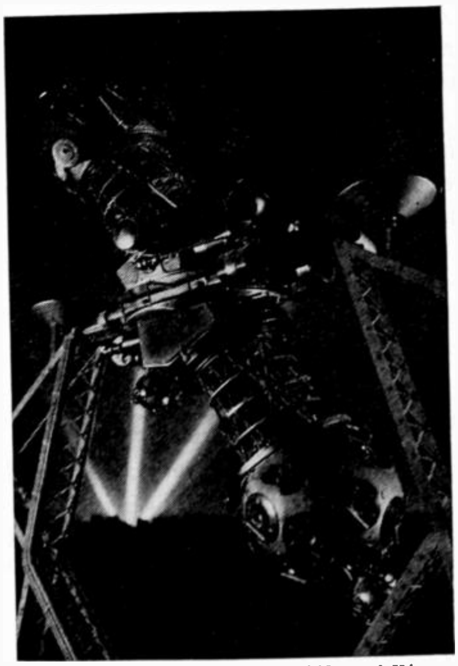
The objective has been set to make this convention notable for its friendliness to members coming to New York from out of town. By January, the Institute's membership will probably have approached, if not in fact reached, an all-time high of 7000. In "Radio's Expanding Role" it is essential that the Institute increase in solidarity as well as in size. An air of camaraderie should pervade this convention. Friendliness, especially to those members who desire to acquire more contacts with fellow radio engineers, will be evident, we believe, beginning from the moment of registration. Austin Bailey has the registration and welcoming in hand, and what with badges and greeters, everyone will be made to feel at home and introduced to engineers worth knowing.

On the program are two "Dutch-treat luncheons"—

the first on Monday, the other on Tuesday noon. A number of well-known New York Members and Fellows, including some past-presidents and directors, have pledged themselves not to eat luncheon with their usual companions so long as out-of-towners or strangers desire their company. The registration desk will supply particulars.

Another departure will be the buffet dinner and party, to which the women are invited, on Monday evening, designed, under the aegis of F. R. Lack's committee, to supply a means of "mixing things up" between technical sessions. The price will be $2.00 a ticket, all of which goes to the hotel, the taxes and tips being taken out of other convention receipts.

Ladies registering at the convention will meet Mrs. A. F. Van Dyck, who is honorary head of the Women's Committee and who has shared with Miss Helen M. Stote of the Institute staff, executive chairman, the pleasure of making arrangements for entertaining



American Museum of Natural History

Origin of the planetarium star.

them. Past conventions have set a mark in women's activities, and those attending will be kept busy again as the following outline of their program indicates.

After registration on Monday morning, the women will spend the afternoon at the Museum of Modern Art where the work of "New American Leaders" will be on exhibition. In addition, "Frontier Photographs" will be displayed. Tea will be served at the museum.

On Tuesday, the New York Historical Society will be visited. This is one of the more compact historical collections in New York City. A "behind-the-scenes" tour of the mammoth American Museum of Natural History will be followed by luncheon at the Museum and a lecture at the Planetarium where the "turning on" of man-made stars leaves the visitor breathless. In the evening—the banquet.

Wednesday morning will be unscheduled to allow co-ordinated activities of wives and husbands, shopping, or just seeing the town. "It Happens on Ice" will be viewed at the Center Theater in Radio City that afternoon.

Returning to the men's program, we have acted upon a suggestion that a regular technical session be omitted at some point in the three-day schedule so that those who wished to, might, without sacrificing attendance upon the meetings, take half a day to visit the offices of friends, or go shopping, or go sight-seeing. No regular technical session has been scheduled for Wednesday morning. However, the omission affords an opportunity we have been glad to seize of scheduling a college technical session. Under the chairmanship of F. E. Canavaciol, a program is being arranged for the participation of delegations from colleges in and within "thumbing distance" of New York. The students will be addressed by leaders in radio, at their own special session (to which all registrants are cordially invited), and will attend other technical sessions.

On the same Wednesday morning is scheduled a meeting of the Sections Committee of the Institute, with the Membership Committee also in attendance. This meeting will be of importance to all those interested in the Institute's organizational activities. At the opening session, on Monday morning, Dr. Terman will give a comprehensive report of his administration during 1941. At the banquet, Tuesday evening, President Van Dyck will disclose his objectives for 1942.

The banquet, which is being arranged by J. R. Poppele, will be held in the Commodore and, as has been usual, will be for men and women; dress is optional. President Van Dyck will be toastmaster. It is expected that one or more of our Canadian and South American officers will find it possible to attend. The Medal of Honor will be awarded to Past-President A. Hoyt Taylor of the United States Navy. Nine members will be made Fellows of the Institute. As noted previously, this is the Institute's big 30th birthday jubilation. The price of tickets has been held down to $3.50 each, in order to stimulate attendance.

The Committee on Trips, under the chairmanship of C. E. Sholz, will offer a variety of possibilities of interesting informal trips to points in New York, in literature to be supplied upon registration. Tickets to broadcasts will be available. The feature of radio engineering interest is the operation of buses all day Tuesday from the Commodore to Major Armstrong's frequency-modulation station at Alpine, New Jersey. By limiting the total number to 300 men every visitor will be assured a thorough look at things. Cost of transportation will be borne by the Institute.

Not the least informative feature of the convention will be the exhibits, which will occupy the East Ballroom. These are being arranged by J. D. Crawford, and should be of special interest in these days of making parts out of priority-free materials.

Other chairmen of committees not yet mentioned are E. J. Content, music, radio, and public-address apparatus; O. H. Caldwell, publicity; Haraden Pratt, finance; and H. P. Westman, general program. H. A. Affel heads the committee on technical sessions, and supplies the following abstracts of papers now in hand. It should be noted that copies of these papers are not available from the Institute and may never be published in the PROCEEDINGS.

New York Historical Society

Early American apothecary shop at the New York Historical Society.

AN ULTRA-HIGH-FREQUENCY TWO-COURSE RADIO RANGE WITH SECTOR IDENTIFICATION

A. ALFORD AND A. G. KANDOIAN
(International Telephone and Radio Manufacturing Corporation, New York, N. Y.)

The primary purpose of a radio range for aircraft use is to provide a reliable aural or visual indication to the pilot of an airplane as to his location with respect to a predetermined course. In addition it is very desirable to identify quickly and positively the sector in which the airplane is at any given time, i.e., whether it is east or west of an east-west radio range.

The basis of this present radio-range design is the two-course localizer used in instrument landing. Thus a group of three loop radiators provide the two overlapping mirror-image patterns modulated at 90 and 150 cycles, respectively. A cross-pointer instrument, the vertical pointer of which is actuated by the 90- and 150-cycle modulation provides the pilot with the necessary information for orienting his plane.

A second set of similar radiators, but at right angles to the first group, provides the keyed signal for aural sector identification. Except for the carrier radiation, which is common to both the aural and visual signals, the two systems are entirely independent.

The theory of the antenna system will be discussed in this paper, paying particular attention to the problem of interaction between the aural and visual radiating systems. Experimental and flight data on the installation at the Civil Aeronautics Administration experimental station in Indianapolis will be presented.

DIRECT-READING WATTMETERS FOR USE AT RADIO FREQUENCIES

GEORGE H. BROWN, J. EPSTEIN, AND D. W. PETERSON
(RCA Manufacturing Company, Inc., Camden, N. J.)

The principle upon which the operation of these wattmeters is based has been known for many years. The contribution of this paper lies in the application of this principle to a practical operating instrument for the measurement of high power.

Two instruments are described. The first is useful in the range of frequencies from 500 to 2000 kilocycles. This instrument contains circuits which permit operation at any frequency in this band with no tuning or other change in the instrument.

The second instrument operates in the region near 50 megacycles. It is inherently a single-frequency device constructed from sections of transmission lines.

The theory of operation is discussed, as well as calibration methods. Test data taken with the instruments with loads having a wide range of power factors are compared with power measurements made on water-cooled loads.

SPACE-CHARGE RELATIONS IN THE MAGNETRON WITH PLANE ELECTRODES

E. A. CONDON
(Westinghouse Electric and Manufacturing Company, East Pittsburgh, Penna.)

This paper discusses the effect of space charge on the one-dimensional potential distribution between a plane cathode and a parallel plane anode, as affected by a uniform magnetic field parallel to the plane of the electrodes.

BIOELECTRIC RESEARCH APPARATUS

HAROLD GOLDBERG
(Formerly, University of Wisconsin; now, Stromberg-Carlson Telephone Manufacturing Company, Rochester, N. Y.)

This paper describes a complete amplifying and cathode-ray-tube system suitable for most bioelectric research applications. Three independent amplifying channels, working into a three-trace cathode-ray tube allow the recording of three independent, simultaneous phenomena. The three traces may be partially or wholly superimposed as desired. Each amplifying channel consists of a battery-operated three-stage direct-current amplifier coupled to a power-line-operated direct-current output stage. All channels operate from a common battery and power-line supply.

Cathode-ray-tube sweep circuits are direct-current coupled and entirely power-line operated. Individual control of centering and sweep speed for each trace is provided. An associated stimulating circuit, synchronized with the sweep, provides stimuli for biologic specimens under study. Because of the direct-current amplification employed throughout the system, any event, with any given setting of the controls, will always appear at the same position of the cathode-ray-tube screen whenever repeated.

The amplifier input is either single-ended or differential as desired. Response is flat within 1 decibel from 0 to 7000 cycles with a maximum voltage gain of 131 decibels. A maximum voltage gain of 126 decibels may be attained with a response flat within 1 decibel from 0 to 14,000 cycles. The sweep amplifiers provide an undistorted output of approximately 700 volts, sufficient for full-scale deflection of the Western Electric 330C cathode-ray tube operating at 3 kilovolts accelerating voltage. Sweep frequencies range from 1 per minute to 20,000 per second.

AN ANALYSIS OF THE SIGNAL-TO-NOISE RATIO OF ULTRA-HIGH-FREQUENCY RECEIVERS

E. W. Herold
(RCA Manufacturing Company, Inc., Harrison, N. J.)

This paper presents an elementary analysis of the effect of the various sources of fluctuation noise on the signal-to-noise ratio of radio receivers. Because the noise induced in negative grids at high-frequencies is included, the work is particularly applicable at ultra-high frequencies. It is found that the signal-to-noise ratio depends on the antenna noise; in addition, when bandwidth is not a consideration, it depends on the ratio of equivalent noise resistance to input resistance of the first tube, and, when bandwidth is a major consideration, on the product of input capacitance and equivalent noise resistance. The coupling from antenna to first tube is an important variable in receiver design and an optimum coupling is found which results in an improvement in signal-to-noise ratio. This optimum condition is often considerably different from the adjustment for maximum gain and, by its use, the noise induced in the grid becomes relatively unimportant. The noise from the second stage of the receiver is also evaluated. It is shown that the thermal noise from a wide-band interstage circuit may be made negligible by concentrating all the damping on the secondary side. Calculations of typical receiver arrangements using triode and pentode mixers are given for 300, 500, and 1000 megacycles.

THE ABSOLUTE SENSITIVITY OF RADIO RECEIVERS

D. O. North
(RCA Manufacturing Company, Inc., Harrison, N. J.)

The total random noise originating in a receiver has customarily been described in terms of the equivalent noise voltage at the receiver input terminals. A comparison of the signal-to-noise ratios of two receivers working out of identical antennas is thereby facilitated, but only so long as the coupling between the antenna and receiver input is extremely loose.

Cosmo-Sileo

Scene from "It Happens on Ice" at the Center Theater, Radio City.

This paper describes a method for rating and measuring the noise in complete receiving systems, antenna included. The proposed rating appears particularly applicable to ultra-high-frequency services and, more generally, to any service in which signal-to-noise ratio is made a prime consideration in receiver design and operation.

A portion of the study deals with the properties of receiving antennas, yielding as a by-product an alternative derivation of Nyquist's theorem concerning thermal fluctuations in passive networks.

DIRECTION FINDING AT MEDIUM HIGH FREQUENCIES AND THE UNITED AIR LINES GROUND-STATION DIRECTION FINDER

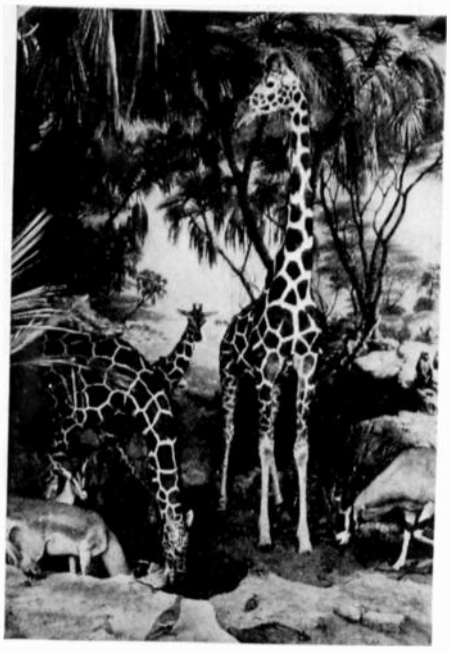
P. C. Sandretto and E. P. Buckthal
(United Air Lines Transport Corporation, Chicago, Ill.)

Direction finders of many types have been generally available on the market; however, all of these were possible only because they used long waves which propagate directly from the transmitter to the receiver via the great-circle path. Aircraft, however (in order to obtain long-distance transmissions with low power and low-efficiency antennas), utilize medium-high frequencies which reflect from the ionosphere. Since this ionosphere is continually undergoing changes, it is difficult to take bearings on waves propagated via this medium. This paper discusses the characteristics of the ionosphere and its effect on the propagation of radio waves. It further discusses the action of these waves on various direction finders and the United Air Lines equipment designed specifically for taking bearings on these peculiarly propagated waves.

A WIDE-RANGE, LINEAR, UNAMBIGUOUS, DIRECT-READING PHASEMETER

J. E. Shepherd
(Formerly, Harvard University, Cambridge, Mass.)

Of the many methods proposed for the measurement of phase angle, none has combined in one instrument all the desiderata of an ideal measuring instrument.



American Museum of Natural History
Part of the water-hole group in the Akeley African Hall in the American Museum of Natural History.

A close approach has been achieved by the author in an instrument which, by using a time ratio as a parameter, delivers a direct current proportional to phase angle. The resulting instrument has the advantages that it performs over a wide range of frequencies and magnitudes of voltages without requiring adjustment, involves no null balance, has a linear scale, exhibits no quadrantal ambiguity, and is readily adapted to the operation of direct-reading meters, recording instruments, and servo mechanisms. Furthermore, it is self-calibrating in that no external standards are required for calibration.

VARIABLE-FREQUENCY BRIDGE-STABILIZED OSCILLATORS

W. G. Shepherd and R. O. Wise
(Bell Telephone Laboratories, Inc., New York, N. Y.)

The results of a theoretical and experimental investigation of two types of bridge-stabilized oscillators incorporating a thermal device for amplitude control are given. One circuit employs resistances and capacitances in the frequency-determining network and is well adapted to low-frequency operation, and the other circuit uses an inductance-capacitance network which adapts itself to the higher-frequency range. The conditions for optimum stability and the variation of the stability with frequency to be expected have been determined and general experimental and theoretical agreement has been obtained.

AUTOMATIC RADIO RELAY SYSTEMS FOR FREQUENCIES ABOVE 500 MEGACYCLES

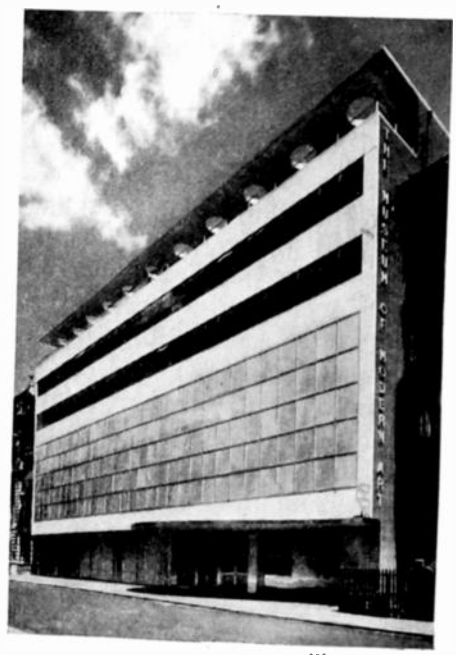
J. Ernest Smith
(R.C.A. Communications, Inc., New York, N. Y.)

This paper describes the problems involved in the design of a system of radio stations in a point-to-point relay network. Particular reference is made to experiments in which television signals were successfully transmitted through several unattended radio repeaters without demodulation and remodulation in the repeater equipment. General fidelity criteria concerning very wide-band transmission requirements, such as those imposed by present-day television standards, will be discussed.

THE DYNETRIC BALANCING MACHINE

M. P. Vore
(Westinghouse Electric & Manufacturing Company, East Pittsburgh, Penna.)

A rotating body may be statically and dynamically balanced by the addition of weights in two arbitrarily chosen planes perpendicular to the axis of rotation. In the apparatus described, a velocity-type electromag-



Wurts Brothers
Marble, tile, and glass are used in large areas in the facade of the new building of the Museum of Modern Art.

netic pickup measures the vibration at each bearing. The output of the pickups is fed into a network which separates the effects of unbalance in each plane and then into an amplifier and meter which indicates the amount of unbalance. The angular position of the unbalance is shown either by a stroboscopic light or by means of a dynamometer-type meter whose field is excited by an adjustable-phase alternator directly coupled to the rotor.

Production and portable balancing machines have been developed. Their principles of operation and design features will be described.

SIMULTANEOUS AURAL AND PANORAMIC RECEPTION

Marcel Wallace
(President, Panoramic Radio Corporation, New York, N.Y.)

The panoramic radio adaptor is a device which operates in conjunction with a conventional radio receiver. It permits the uninterrupted observation of a relatively wide band of the frequency spectrum, above and below the station to which one is listening.

All signals within that band are simultaneously reproduced on a screen, showing their relative signal strengths, frequencies, and modulation characteristics.

Because it permits simultaneous comparison of a number of signals, which may originate at different points, it becomes an important new tool in the hands of the radio technician, finding an increasing number of applications not only in the laboratory, but also in the practical field of communication and navigation.

IONOSPHERIC INVESTIGATIONS AT HUANCAYO MAGNETIC OBSERVATORY (PERU) WITH APPLICATION TO WAVE-TRANSMISSION CONDITIONS

H. W. Wells
(Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.)

A brief description is given of the principle and design of the automatic multifrequency ionospheric equipment developed by the Carnegie Institution of Washington, Department of Terrestrial Magnetism, which sweeps through a frequency range of 16.0 to 0.516 megacycles per second every fifteen minutes and automatically records the radio reflections which are returned from the different layers of the ionosphere. Three complete units of this type are now in operation under the auspices of this Department; one at Huancayo, Peru, one at Watheroo, Western Australia, and the last which has recently been installed at College, Alaska. The results of observations at Huancayo, Peru (12 degrees south latitude), where this apparatus has been in continuous operation since 1937, are representative of average ionospheric conditions in equatorial regions. An analysis of these results in terms of maximum usable frequencies for radio-wave propagation over various distances completes the paper.

THE RCA 10-KILOWATT FREQUENCY-MODULATED TRANSMITTER

E. S. Winlund and C. S. Perry
(RCA Manufacturing Company, Inc., Camden, N. J.)

A commercial 10-kilowatt frequency-modulated transmitter design is completely described. Distortion curves are presented with particular reference to the effect of proper tank-circuit design on high audio frequencies, the effect of low average distortion of 0.4 per cent on the bulk of program material and the effect of proper modulator design on low frequencies. Response curves are presented for both flat and pre-emphasis connection. The effect of power output change between 3 and 10 kilowatts upon distortion and response is shown in detail. All other guarantee specifications are given along with the actual measurements obtained.

A full mechanical description is included with photographs showing provisions for operating convenience, accessibility, and appearance.

Contributors

John K. Hilliard (A'25-M'41) received degrees from Hamline University and the University of Minnesota. After leaving college he was a consulting radio engineer. Joining a Hollywood studio sound department he has been engaged in that type of work for fifteen years, the last eight of which have been spent as sound engineer at the Metro-Goldwyn-Mayer studios. As transmission engineer his work has been chiefly concerned with the development of new apparatus, both for recording and reproducing, which includes the two-way loudspeaker system, now the accepted standard for theater reproducing systems. Mr. Hilliard is a member of the American Institute of Electrical Engineers and the Society of Motion Picture Engineers, and an active participant in the work of the Research Council of the Academy of Motion Picture Arts and Sciences, heading the Theater Standardization Program and the Vacuum Tube Standards Committee.

John K. Hilliard

❖

Dorman D. Israel (A'23-M'30) was born on July 21, 1900, at Newport, Kentucky, He received his E.E. degree in 1923 at the University of Cincinnati. He has been chief development engineer of Crosley Radio Corporation and chief engineer of Grigsby-Grunow Corporation. Since 1936 he has been chief engineer of Emerson Radio and Phonograph Corporation. In 1931 Mr. Israel was chairman of the Cincinnati section of the Institute of Radio Engineers. He was an instructor in radio engineering at the evening college of the University of Cincinnati for three years.

Dorman D. Israel

P. J. Noizeux

Wayne B. Nottingham

❖

H. Krähenbühl (A'39) was born in Switzerland in 1903. After being graduated from the Neuchâtel School of Technology he worked from 1924 to 1927 in cable measurements and at the Marconi receiving station at Bern. In 1927 Mr. Krähenbühl joined the technical staff of Transradio Internacional in Buenos Aires, where at present he is chief of the engineering department.

❖

P. J. Noizeux (A'30) was born in France in 1900. In 1923 he joined Transradio Internacional, Buenos Aires, of which he is now assistant general manager. Mr. Noizeux is chairman of the Buenos Aires section of the Institute of Radio Engineers.

❖

Kenneth A. Norton (A'29-M'38) was born February 27, 1907, at Rockwell City, Iowa. He received the B.S. degree in physics from the University of Chicago in 1928. During 1929, he was in the inspection development laboratory of the Western Electric Company. From 1929–1930, and 1931–1934 Mr. Norton was in the Radio Section of the National Bureau of Standards and at Columbia University from 1930–1931. Since 1934 he has been with the Federal Communications Commission. He is a member of American Association for the Advancement of Science, American Institute of Electrical Engineers, American Physical Society, American Statistical Association, and the Institute of Mathematical Statistics.

❖

Wayne B. Nottingham (A '37) was born at Tipton, Indiana, on April 17, 1899. He received the B.S. degree in electrical engineering in 1920 and in 1929 received the E.E. degree from Purdue University. After a year of study at Uppsala University at Uppsala, Sweden, as the Benjamin Franklin Fellow in Physics of the American Scandinavian Foundation, he worked on "carrier telephone" development at the Bell Telephone Laboratories until he entered Princeton University as a graduate student in 1925. Princeton awarded him the degrees of M.A. in 1926 and Ph.D. in 1929. Between 1926 and 1931 he was a Research Fellow at the Bartol Research Foundation. Since that time he has been in research and teaching in the department of physics at the Massachusetts Institute of Technology. He is a Fellow of the American Physical Society, and a member of the Optical Society of America, the American Association of Physics Teachers, Sigma Xi, and Eta Kappa Nu.

❖

B. Noviks (A'39) was born on January 1, 1910, at Riga, Russia. He received the degree of Diplomingenieur (M.S.) in 1935 from the Polytechnic Institute of Berlin. From 1935 to 1937 he was an engineer at the A. Leibovics radio factory in Riga. In 1938 Mr. Noviks joined the technical staff of the Transradio International, Buenos Aires; since January, 1940, he has been engineer-in-charge at the receiving station in Villa Elisa.

❖

For a biographical sketch of Ronold King, see the Proceedings for August, 1941.

H. Krähenbühl

Kenneth A. Norton

B. Noviks

Proceedings *of the* I·R·E

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GENERAL INFORMATION

The Institute

The Institute of Radio Engineers serves those interested in radio and allied electrical-communication fields through the presentation and publication of technical material.

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The PROCEEDINGS has been published without interruption from 1913 when the first issue appeared. Over 2000 technical contributions have been included in its pages and portray a currently written history of developments in both theory and practice. The contents of every paper published in the PROCEEDINGS are the responsibility of the author and are not binding on the Institute or its members. Material appearing in the PROCEEDINGS may be reprinted or abstracted in other publications on the express condition that specific reference shall be made to its original appearance in the PROCEEDINGS. Illustrations of any variety may not be reproduced, however, without specific permission from the Institute.

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In addition to the material published in the PROCEEDINGS, Standards on Electroacoustics, Electronics, Radio Receivers, and Radio Transmitters and Antennas were published in 1938. These are available to members free of charge as long as the supply lasts; others may purchase them for fifty cents each.

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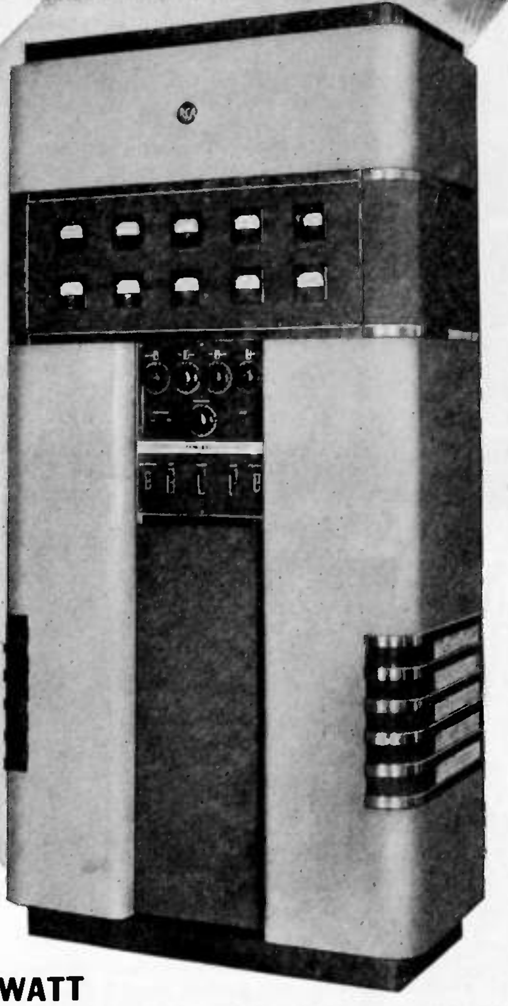
Sections Established

CHOICE OF 84 AMERICAN RADIO STATIONS

Why?



RCA 250-WATT
BROADCAST TRANSMITTER MODEL 250-K

ON THE MARKET for less than two years, the RCA Type 250-K Broadcast Transmitter has already been accorded an acceptance far beyond that of any other 25-watter produced by *any* manufacturer! The initial production-order, incidentally, was sold out *sight unseen* before the first 250-K ever came out of the factory.

It's only natural to ask, "*Why—?*"

Part of the answer lies in the extreme flexibility of the 250-K. It affords outstanding operation at 100 or 250 watts—and is easily adapted to 1,000-watt operation at any time by the simple and inexpensive addition of RCA amplifier unit and power supply Type MI-7187.

Another part of the answer lies in the *economy* of the 250-K. For example, it draws only 1625 watts from your power-line at average modulation on a 250-watt carrier... economy that helps keep power-bills *low*, thanks to its RCA-engineered high-level Class B modulation. Tube-costs are low. And installation is economical—economical because *simple*.

Program quality is part of the answer, too—audio is flat within 1½ db. from 30 to 10,000 cycles.

Dependable—? Ask any of the 84 stations who have bought the 250-K! For the RCA way is to work for dependability from the first line on the drawing-board to the last bolt in the final assembly!

Write for complete data and literature ... *today*. As we go to press, the 250-K is still available for immediate delivery—but it may not remain so for long.

OUTSTANDING ACCEPTANCE!

These American Broadcasting Stations—including 1,000-watt stations using it as an exciter unit—have chosen the RCA Model 250-K... all within the past two years. And the list does not include still others to foreign stations, American police installations, and stations now under construction!

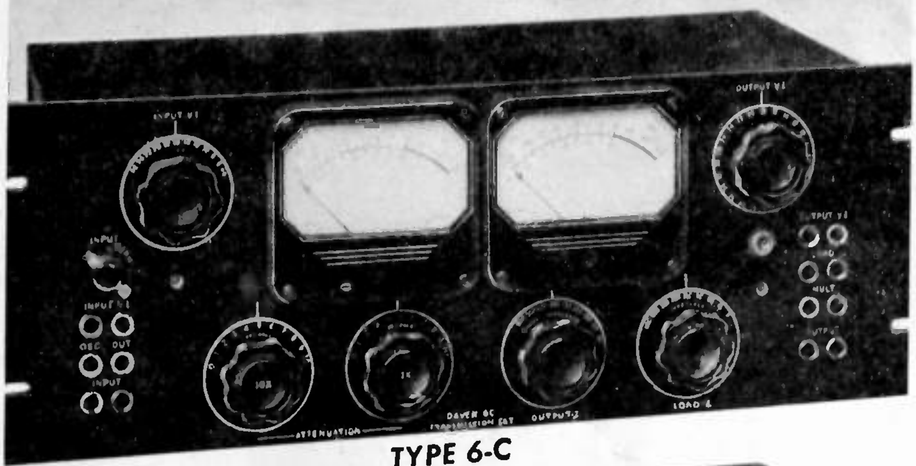
KANA	KBIX	KBUR	KBWD	KFBG	KFIO
KFIZ	KFMB	KFPW	KFXM	KGLO	KHAS
KHON	KLS	KLUF	KRJF	KROD	KSKY
KSRO	KUJ	KVFD	KVOE	KWIL	KWRC
KYCA	KYOS	WAJR	WARM	WATN	WBIR
WBML	WBOC	WBTA	WCBI	WCED	WCRS
WDAK	WDAS	WDEF	WFDF	WFIG	WFPG
WGAC	WGOV	WGTC	WHBQ	WHKY	WHUB
WHYN	WINX	WISR	WIZE	WKIP	WKMO
WKPA	WKWK	WLAV	WLBJ	WLKO	
WMJM	WMOB	WMOG	WMRN		
WORD	WOSH	WSAV	WSGN		
WSLB	WSOC	WSOO	WSRR		
WTHT	WTJS	WTMA	WWNY		

RCA *Broadcast Equipment*

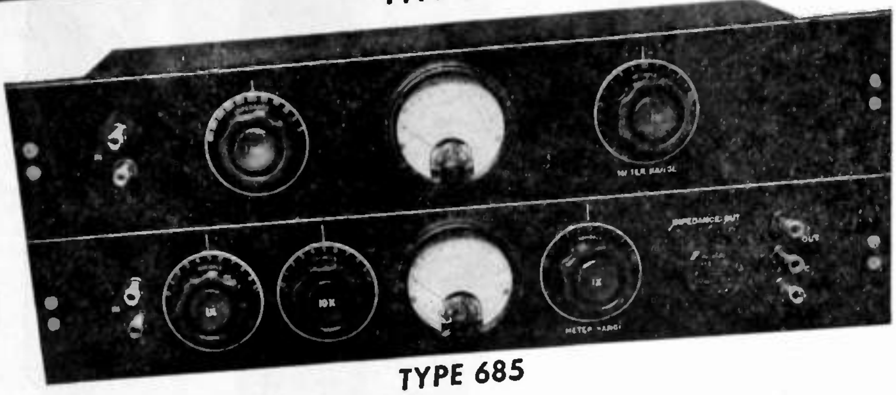
RCA Manufacturing Co., Inc., Camden, N. J. • A Service of Radio Corporation of America • In Canada, RCA Victor Co., Ltd., Montreal

New York: 411 Fifth Ave. Chicago: 589 E. Illinois St. Atlanta: 530 Citizens & Southern Bank Bldg. Dallas: Santa Fe Bldg. San Francisco: 170 Ninth St. Hollywood: 1016 N. Sycamore Ave.

DAVEN
TRANSMISSION MEASURING SETS



TYPE 6-C



TYPE 685

TYPE 6-C Designed in co-ordination with the General Engineering Department of the Columbia Broadcasting System, the 6-C Transmission Measuring Set consists of complete transmission and load units assembled on a single rack type panel. With a frequency range from 30 to 17,000 cycles, this set provides an accurate and rapid method for measuring the transmission characteristic of networks at audio frequencies.

The reference level is the new standard of 1 mw. across 600 ohms. New Weston Type 30 meters are employed. The attenuation range is from Zero to 110 db. in steps of 1 db. Power range is calibrated from —16 to +45 db. Dial selection of useful network input and load impedances. No correction is required when changing impedances. Overall accuracy is ±2% $325

TYPE 685 An unusually flexible, universal gain measuring instrument for rapid and accurate measurement of overall gain, frequency response and power output of audio amplifiers, this assembly has a useful frequency range from 30 to 17,000 cycles.

It is direct reading in decibels and does not require correction factors or calibration charts. All networks meters and associated apparatus are shielded and carefully balanced, matched for uniform accuracy over this wide frequency range.

Attenuation range is +10 db. to —120 db. in steps of 1 db. Power measuring range is —20 db. to +36 db. Eleven load impedance values, ranging from 5 to 600 ohms are available. Output impedances may be changed from "balanced" to "unbalanced" and to any loss impedance by means of plug-in type matching networks. Overall accuracy is ±2% $225

THE DAVEN catalog lists the most complete line of precision attenuators in the world; "Ladder," "T" type, "Balanced H" and Potentiometer networks—both variable and fixed types—employed extensively in control positions of high quality program distribution systems and as laboratory standards of attenuation.

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Long before the state of emergency was proclaimed, the Triplett Company was getting ready to do its part in building our national security. We knew that we must meet important new responsibilities. At the same time, we felt keenly our continuing obligations to our customers—old friends with whom we have had happy business relations through many years.

We doubled—then tripled—our output to fill the needs of our old accounts. We added to our production facilities . . . hired many more men . . . are working extra shifts at time-and-a-half.

All this has not been enough. We have been called on to produce more and more for national defense. We are proud of the job we are doing to help meet the emergency, but it is difficult not to be able to serve our old friends equally as well. In the face of these conditions, the Triplett Company has adopted these policies "for the duration."

FIRST: We will continue to serve you by our service to our mutual responsibility —the national emergency.

SECOND: We will continue to do everything we can to fill orders from our regular customers, even though some deliveries may be temporarily delayed. No business from new accounts has been nor will be accepted until after our old friends have been served, except where priorities make it impossible to do so.

THIRD: Our engineering and research departments will continue to work on the development of superior equipment and improved methods to serve you still better when we can resume normal operations.

The present emergency is incidental and as we work towards the future, we will do our best to continue to merit your confidence and loyalty.

R. L. Triplett

President
The Triplett Electrical Instrument Company

Manufacturers of Precision Electrical Instruments

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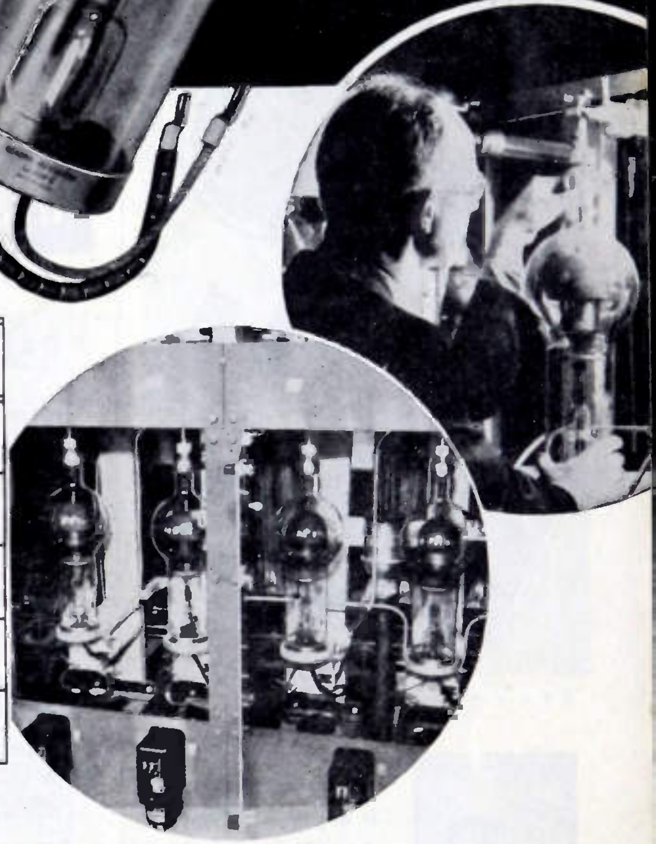
G-E Tubes Mean Peak Efficiency

Here are a few ways GL-857B's meet your high-power, high-voltage rectifier requirements



CIRCUIT	MAXIMUM A-C INPUT VOLTS* (RMS)	APPROXIMATE D-C OUTPUT VOLTS TO FILTER	MAXIMUM D-C LOAD CURRENT AMPERES
SINGLE-PHASE FULL-WAVE (2 tubes)	7750	7000	20
SINGLE-PHASE FULL-WAVE (4 tubes)	15500 total	14000	20
THREE-PHASE HALF-WAVE	9000 per leg	10500	30
THREE-PHASE DOUBLE-Y PARALLEL	9000 per leg	10500	60
THREE-PHASE FULL-WAVE	9000 per leg	21000	30

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THE exceptionally rigid filament structure in this tube assures long cathode life. Arcback has been greatly reduced. The low voltage drop and low power loss between electrodes — characteristics inherent in this type of tube — assure peak efficiency and great dependability.

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When you sign your next tube order specify General Electric tubes—proved in the laboratory, checked at our own broadcast stations, and verified by the long list of satisfied users throughout the radio industry. For your requirements in standard broadcasting, FM, or television see your G-E representative first, or write *General Electric, Radio and Television Department, Schenectady, N. Y.*

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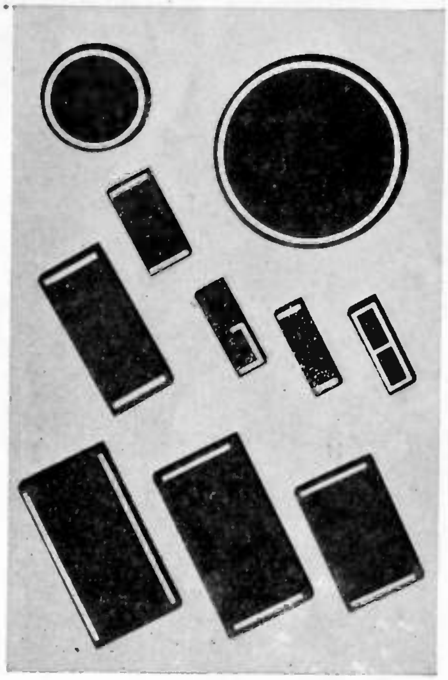

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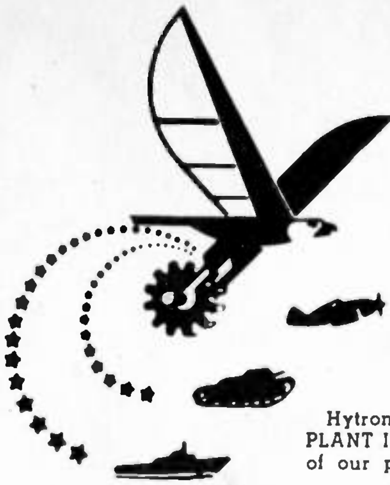

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PART OF THE ARSENAL OF DEMOCRACY

Hytron is authorized to display this official DEFENSE PLANT IDENTIFICATION, signifying that more than 50% of our production is devoted to the needs of defense.

Measured in radio tubes, this figure represents more than a casual devotion to Defense. Important, indeed, are the tasks which Hytron tubes will perform during long, routine hours and in sudden, crucial moments.

To loyal Hytron users—to all who know of the many basic improvements developed here—it will be no surprise to find these better tubes taking key positions in the vast military mosaic of America and Britain. What should be emphasized now is:—*Defense is getting what it needs from Hytron, with no serious effect upon our ability to supply the radio industry, nor any slightest relaxation in quality.*

The needs of our regular customers, increasing heavily, have put Hytron capacity to a double test. That additional trial is also being met—in a plant whose flexibility astonishes even ourselves—by a master-combination of men, women and machines that seems ever capable of doing more and more!

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HYTRON CORP. . . . SALEM, MASS.

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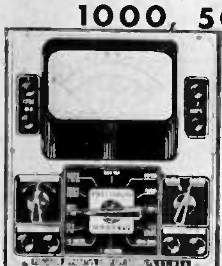
RADIO? COMMUNICATIONS? ELECTRIC POWER?

There is a PRECISION INDUSTRIAL CIRCUIT TESTER to meet your INDIVIDUAL SENSITIVITY REQUIREMENTS

Ranges to 6000 Volts—60 Amps—10, 20 or 60 Megs—70 DB
1000, 5000 or 20,000 ohms per volt!



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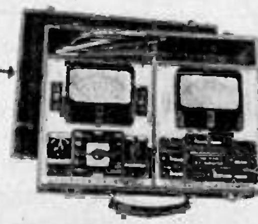

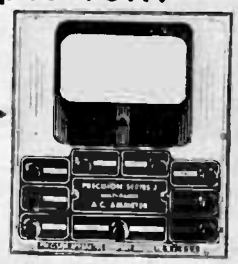

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★ **Series 856** 20,000 ohms/volt DC Plus 1000 ohms/volt AC and DC

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CONSOLIDATES AT A

NEW ADDRESS

★ ★ ★

FOR BETTER SERVICE to our patrons, Terminal Radio Corp. three years ago opened its doors at 68 West 45th Street. This move placed us in a better position to render quick and more convenient service to a greater number of customers in different parts of the city. The present emergency now dictates another move to maintain our record of service to the radio industry.

FOR BETTER SERVICE . . . we are now consolidating the stocks of radio parts and equipment from our two stores into new and larger quarters at

85 CORTLANDT STREET

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★ ★ ★

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85 Cortlandt Street

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HAM SHACK: A rendezvous for hams to congregate amid all the newest developments in radio communication.

RECORD DEPARTMENT: A complete stock of records and recording equipment.

70 West 45th Street

After January 1st, a completely modernized store devoted exclusively to radio sets, records, and accessories, under the management of Jack Haizen.

Enlargement of facilities providing more and larger listening booths, larger stock of records and a complete line of all radio sets, phonographs, and accessories. No radio parts and equipment will be available at this store.

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TERMINAL RADIO CORP.

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ATR
VIBRATORS
"for DEFENSE"

PROVEN UNITS
of the
HIGHEST QUALITY

Engineered to perfection, ATR Vibrators set high standards of performance and construction. Available for any operating input voltage from 6 volts DC to 220 volts DC in a wide variety of designs for practically any application.

In addition to the most complete line of vibrators, ATR offers a very extensive line of Inverters and other Vibrator-Operated Power Supplies for changing one DC voltage to another DC voltage or to invert DC to AC.

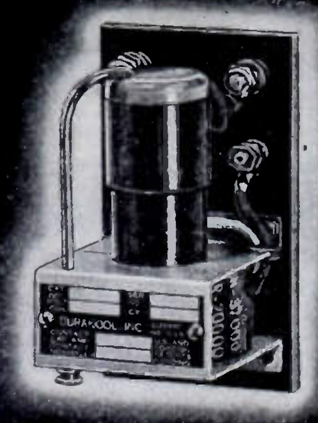
● ● ●

ATR vibrators, the heart of vibrator-operated power supplies, are proven units of the highest quality, engineered to perfection. They are backed by more than ten years of vibrator design and research, development and manufacturing—ATR pioneered in the vibrator field. American Television & Radio Co. has consistently devoted its efforts and energies to the perfection and production of vibrators and associated equipment, and today, after ten years of painstaking, persistent and diligent work resulting in steady development and progress, is considered the World's leader in its field. All ATR Products incorporate only the best materials and workmanship and are carefully manufactured under rigid engineering inspections and tests, making them the finest that can be built.

● ● ●

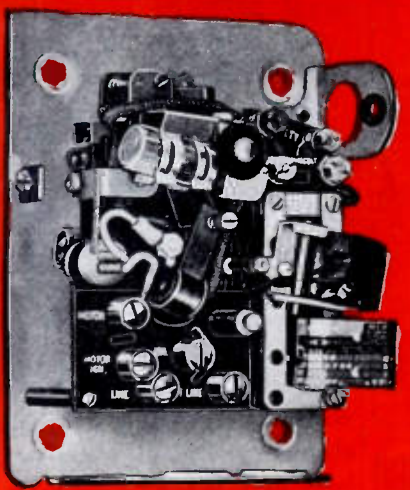
For Additional Information Address ATR Vibrator Division of

AMERICAN TELEVISION & RADIO CO.
ST. PAUL, MINN., U.S.A.



DURAKOOL

ELECTRIC SWITCH CORP.



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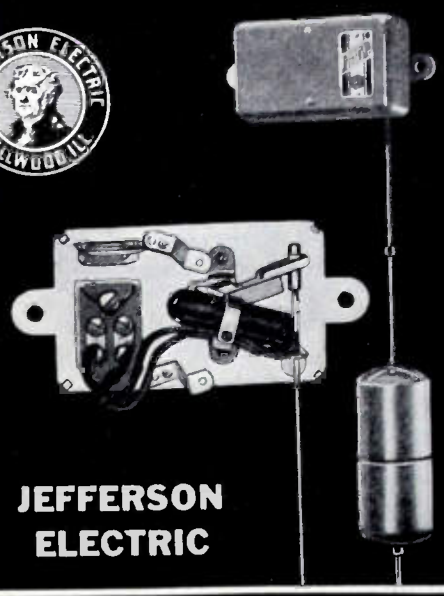
for MERCURY SWITCHES, RELAYS *and* CONTROLS

American Lava Corporation has developed several ceramic compositions which are well suited for mercury switches, relays and controls. They are not attacked or "wet" by mercury. They do not oxidize, corrode, dust or flake. They resist erosion. They are absolutely and permanently rigid, are shock-resisting, withstand heat and arcing, have high mechanical and dielectric strength. Parts are accurately custom-made to the blue prints of the manufacturer.

For permanently trouble-free insulation, follow the leaders . . . specify ALSIMAG.



JEFFERSON ELECTRIC

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ALSIMAG FROM CERAMIC HEADQUARTERS

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AMERICAN LAVA CORPORATION • CHATTANOOGA • TENNESSEE

CHICAGO • CLEVELAND • NEW YORK • ST. LOUIS • LOS ANGELES • SAN FRANCISCO • BOSTON • PHILADELPHIA • WASHINGTON, D. C.

Placed 18 inches apart, the 80 million metal radio tubes sold since 1935 would reach around the world! More important, these 80 million tubes attest the tremendous acceptance accorded by the industry to the finer performance of metal-envelope tubes!

. . . and RCA is now producing metal tubes at fastest pace in history!

CURRENT PRODUCTION of metal tubes is the greatest in history—for the amazing *acceptance* accorded to metal tubes is still *increasing!* 80,000,000 metal-envelope tubes have been sold since 1935 . . and today *four* of the *six* largest-selling tube types are *metal* types!

Will They Be Available—?

If you have found difficulty or delay in obtaining RCA Metal Tubes, remember the reason: *it is not because production is low but because both defense and commercial demands are high.* And defense comes *first!*

Naturally, priority requirements in materials may limit the general availability of *all* types of tubes—for all types require valuable and limited metallic materials in their *internal* structure.

RCA BROADCAST EQUIPMENT
From microphone to antenna

12 REASONS WHY
METAL Tubes are BETTER Tubes!

- Complete Self-Shielding
- Greater Flexibility in Design
- Greater Precision and Uniformity
- Lower Interelectrode Capacitances
- No Envelope Emission Troubles
- Freedom of Placement on Chassis
- Higher Getter Efficiency
- Simple, Efficient Grounding
- Single-ended Construction
- Large Pin-Contact Area
- Lower Socket Costs
- More Rugged Construction

RCA METAL TUBES

RCA Manufacturing Company, Inc., Camden, New Jersey
A Service of the Radio Corporation of America • In Canada: RCA Victor Company, Ltd., Montreal

POSITIONS OPEN

The following positions of interest to I.R.E. members have been reported as open on December 1. Make your application in writing and address to the company mentioned or to

Box No. . . . .

PROCEEDINGS of the I.R.E.
330 West 42nd Street, New York, N.Y.

JUNIOR COMMUNICATIONS OPERATORS

The U. S. Civil Service Commission has issued an amended announcement for Junior Communications Operators on high speed radio equipment. Additional information can be secured from the Commission at Washington, D.C., or from any first- or second-class post office.

RADIO MECHANIC-TECHNICIANS

Announcement No. 134 of the U. S. Civil Service Commission (obtainable at the Commission's offices in Washington or at any first- or second-class post office) has just been released. Salaries range from $1440 to $2300 per year.

TRANSMITTING-TUBE DESIGNER

A west coast manufacturer of radio transmitting tubes has an opening for an engineer with background in transmitting tube design and manufacture. Applicants should be engineering graduates and have had one to five years of experience in vacuum-tube work. Box 258.

VACUUM-TUBE AND RECEIVER ENGINEERS

(1) *Electronic Research*—Vacuum tube engineer having experience on design of magnetrons or velocity modulated tubes and associated circuits.
(2) *Vacuum-Tube Development Engineer* having experience on design and manufacture of high-power transmitting vacuum tubes.
(3) *Engineer* with experience on ultra high-frequency, especially on transmitter development.
(4) *Receiver Design Engineer* familiar with aircraft receivers, preferably including ultra high frequencies. Experience with cathode ray tubes and circuits desirable.
(5) *Receiver Engineer* experienced medium band communication receivers of high gain. Experience with cathode ray tubes and circuits desirable.
Only American citizens need apply. International Telephone and Radio Laboratories, 67 Broad St., New York, N.Y.

SENIOR RECEIVER ENGINEER

We have an opening for an engineer who is familiar with chassis layout and circuit design. At least four years of experience in radio manufacturing necessary. Executive experience desirable. Present staff knows of this opening. For personal and confidential interview, write Mr. A. A. Leonard, Philips Export Corporation, Hotel Roosevelt, Madison Avenue & 45th St., New York, N.Y.

RADIO ENGINEERS, ALL BRANCHES

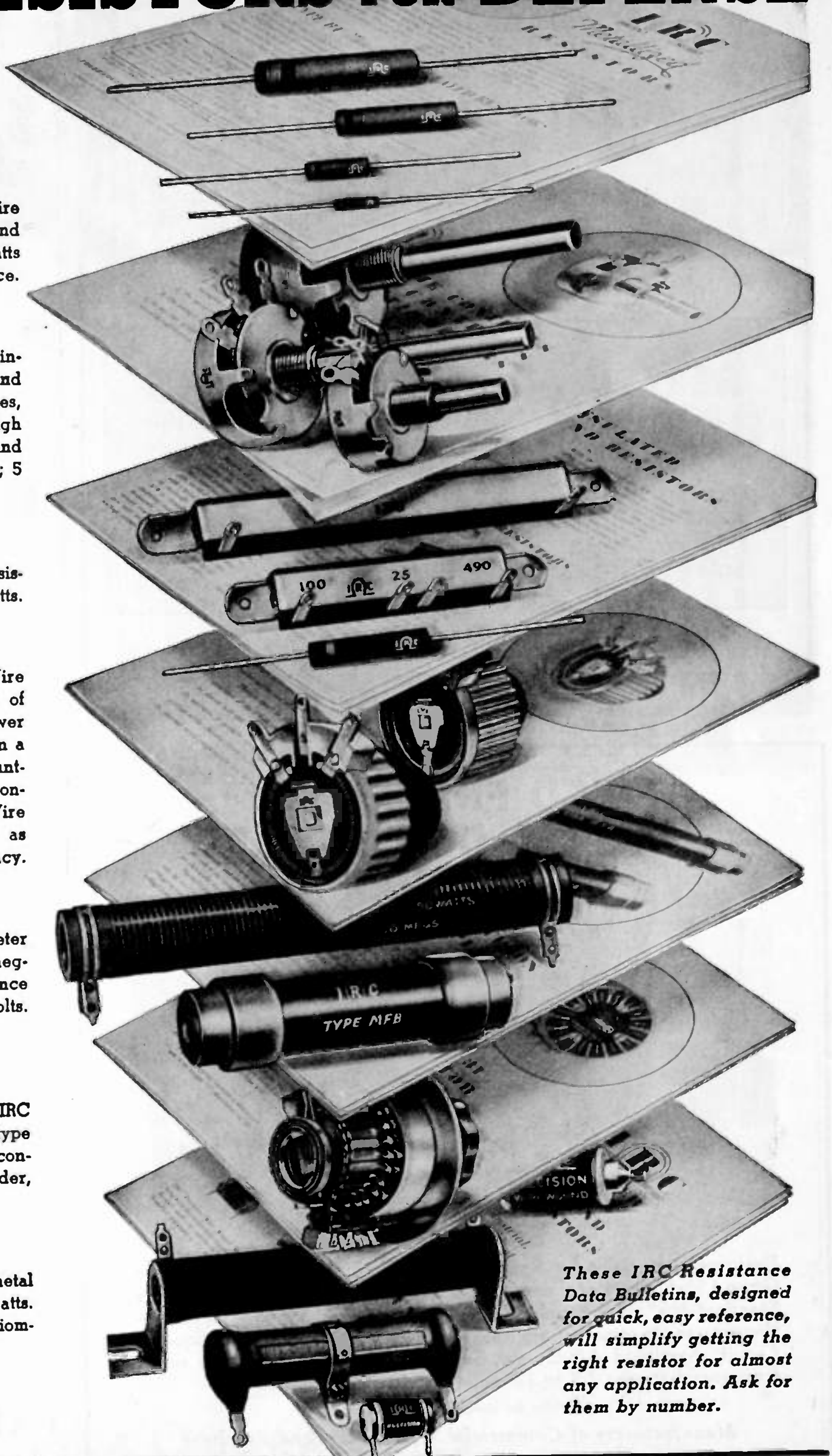
This department receives frequent urgent requests for engineers for national-defense work from various military and civilian branches of the U.S. government. Members and others interested are urged to write to Box 260 for application blanks. Applicants not now engaged in defense work are preferred.

Attention Employers . . .

Announcements for "Positions Open" are accepted without charge from employers offering salaried employment of engineering grade to I.R.E. members. Please supply complete information and indicate which details should be treated as confidential. Address: "POSITIONS OPEN," Institute of Radio Engineers, 330 West 42nd Street, New York, N.Y.

The Institute reserves the right to refuse any announcement without giving a reason for the refusal.

IRC RESISTORS FOR DEFENSE



BULLETIN I

Two sizes Metallized and Wire Wound Volume Controls and Potentiometers up to 2 watts and 20 megohms resistance.

BULLETIN II

Metallized-type Resistors: 4 insulated sizes, ⅓-, ½-, 1- and 2-watts; 10 high frequency sizes, ⅓- to 150-watts; 4 ultra-high range sizes; 5 high voltage and high frequency power sizes; 5 suppressor sizes.

BULLETIN III

Insulated Wire Wound Resistors: 7 sizes from ½- to 20-watts.

BULLETIN IV

Power and Precision Wire Wound Resistors: 53 sizes of fixed and adjustable power types from 10- to 200-watts; in a wide variety of shapes, mountings, etc. Inductive and non-inductive. 14 Precision Wire Wound Resistor types to as close as 1/10 of 1% accuracy.

BULLETIN IV-B

Sealed Precision Voltmeter Multipliers, 2 sizes, 1.0 megohm to 5 megohms resistance and 1 kilovolt to 5 kilovolts. Impervious to moisture.

BULLETIN V

Attenuators: Unique new IRC molded motor commutator type 20-step Attenuator; also, conventional 30-step units. Ladder, potentiometer or bridge T.

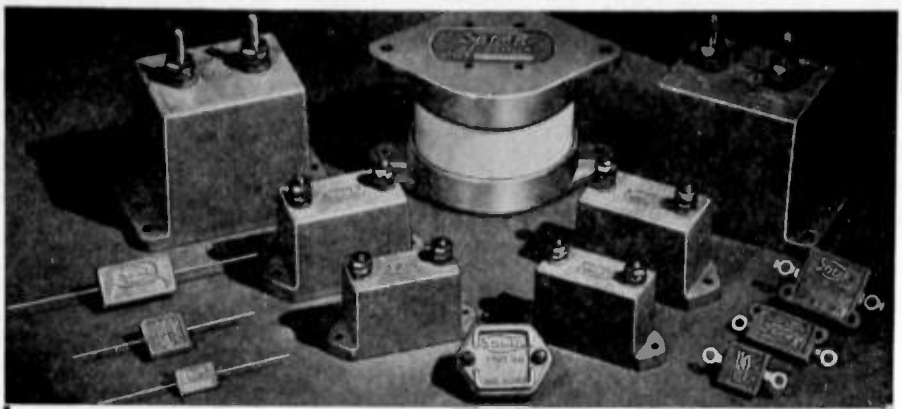
BULLETIN VI

Quick heat dissipating all-metal Rheostats, 25- and 50-watts. 2-watt Wire Wound Potentiometer and Rheostat.

These IRC Resistance Data Bulletins, designed for quick, easy reference, will simplify getting the right resistor for almost any application. Ask for them by number.

LET'S GO! USA KEEP 'EM FLYING!

INTERNATIONAL RESISTANCE COMPANY 431 NORTH BROAD STREET PHILADELPHIA, PENNA.



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MICA CAPACITORS

add vital dependability to radio and communications equipment for the Armed Service Branches of the Government. The "Quality Above All" incorporated in these units is evolved from a wealth of experience. If Mica Capacitors are part of your problem, consult Solar for a ready solution.

Ask for Special Catalog 12-E, available on letterhead request.

SOLAR MFG. CORP., Bayonne, N. J.

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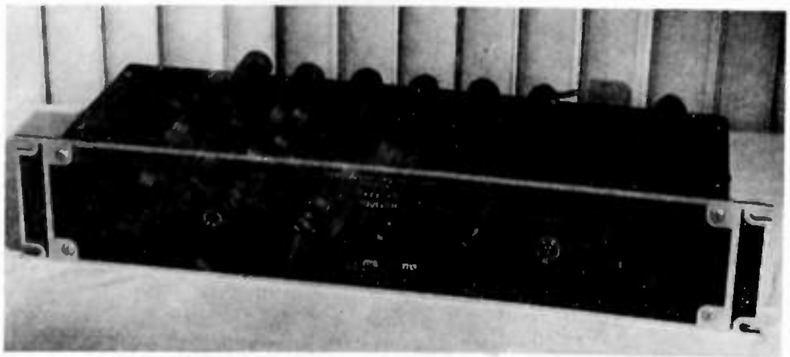

MODEL 82-A

Designed especially for Police, Airport Traffic Control, and Airline Ground Stations • Local or remote operation • Compact—Mounted on Standard $3\frac{1}{2}''$ Rack Panel • Aligning and Tubes changed from Rear—Unnecessary to remove from Rack • Excellent image rejection, Squelch Circuit, Amplified A.V.C., etc. Supplied for operation on any single frequency between 1.5 to 12 mc. • Moderately priced.

Write for complete information.

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INDEX

"Consider a Capacitor's Dependability"

THERE'S EXTRA DEPENDABILITY IN LONGER LASTING C-D CAPACITORS

Sound advice to engineers and purchasing agents is the capacitor recommendation of Canadian Colonial Airways. Consider dependability. You get *extra* dependability when you specify Cornell-Dubilier capacitors, because the broadest experience in the industry is behind them — more than 31 years of specialization in capacitor manufacturing.

Cornell-Dubilier superiority is hidden to the eye. But it's there in the capacitor ingredients . . . in the quality of parts, in the skillfulness of fabrication — and it shows in service. So much so that Cornell-Dubilier capacitors are the unanimous choice today of America's leading airlines for all aircraft and ground transmitting and receiving communications equipment. Whatever your capacitor requirements, it pays to specify C-Ds.

MICA • DYKANOL • PAPER • WET & DRY ELECTROLYTIC CAPACITORS

CAPACITORS MAY LOOK ALIKE BUT . . .

There is extra long life, extra uniformity and dependability built into C-Ds. Next time you specify capacitors look for the Cornell-Dubilier seal of experienced engineering. And get the hidden extras at no extra cost. Send for Catalog. Cornell-Dubilier Electric Corporation, 1012 Hamilton Blvd., South Plainfield, New Jersey.

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. . . MORE IN USE TODAY THAN ANY OTHER MAKE . . .

We Don't Want To Grow Too LARGE!

VISITORS to the laboratories and factory of General Radio are very often surprised at our size. Some think we must occupy a hole-in-the-wall, others that we are spread out over acres.

Happily, we occupy a position between both of these undesirables. Our total floor space is 75,000 square feet, divided between three four-story buildings and occupying about a half a city block. Our total personnel is 287, of which number 30 are engineers.

G-R does not want to grow large; only by following the basic idea upon which the company was founded in 1915 can we continue to serve our customers in the instrumentation field. That idea was to have an organization large enough to get instruments turned out, in peace time, in sufficient quantity to satisfy our customers and give us a reasonable profit; and at the same time small enough to enjoy the flexibility essential to adapting research, engineering and manufacture to the ever rapidly changing developments in the electronic art.

The type of equipment manufactured by G-R does not lend itself to production-belt methods; G-R design will never be cheapened to make mass production possible.

As soon as we grow to be a large company, we lose most of the essential direct contact between engineers and customers, and between engineers and the shop; ideas when diluted by eighteen in-betweens in an organization lose some of their sparkle and much of their originality.

Fundamentally we have only one thing to sell: engineering ideas wrapped up in cabinets with control panels. Many concerns can manufacture more economically than we; few have such a large percentage of idea-developing engineers.

If G-R grows large . . . if it grows so large that to change a machine screw from a 6-32 to an 8-32 requires a design conference, a thousand dollars in drafting time and a month's delay for tooling . . . we will cease to perform the function for which the company was established: to design and manufacture precision electrical measuring apparatus at a price consistent with both the quality of the product produced and the type of persons employed.

SEE OUR EXHIBIT

I. R. E.

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AND
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SHOW

NEW YORK
JANUARY 12-14

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