Proceedings of the I·R·E

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Rochester Fall Meeting

November 13, 14, and 15, 1939

SECTION MEETINGS

DETROIT
November 17

ATLANTA November 17	**LOS ANGELES** November 21	**PITTSBURGH** November 21
CLEVELAND November 23	**PHILADELPHIA** December 7	**WASHINGTON** November 13

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The Institute of Radio Engineers serves those interested in radio and allied electrical-communication fields through the presentation and publication of technical material. In 1913 the first issue of the PROCEEDINGS appeared; it has been published uninterruptedly since then. Over 1500 technical papers have been included in its pages and portray a currently written history of developments in both theory and practice.

STANDARDS

In addition to the publication of submitted papers, many thousands of man-hours have been devoted to the preparation of standards useful to engineers. These comprise the general fields of terminology, graphical and literal symbols, and methods of testing and rating apparatus. Members receive a copy of each report. A list of the current issues of these reports follows:

Standards on Electroacoustics, 1938
Standards on Electronics, 1938
Standards on Radio Receivers, 1938
Standards on Radio Transmitters and Antennas, 1938.

MEETINGS

Meetings at which technical papers are presented are held in the twenty-one cities in the United States and Canada listed on the inside front cover of this issue. A number of special meetings are held annually and include one in Washington, D. C., in co-operation with the American Section of the International Scientific Radio Union (U.R.S.I.) in April, which is devoted to the general problems of wave propagation and measurement technique, the Rochester Fall Meeting in co-operation with the Radio Manufacturers Association in November, which is devoted chiefly to the problems of broadcast-receiver design, and the Annual Convention, the location and date of which are not fixed.

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Asymmetric-Sideband Broadcasting*

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Summary—*There is described a method of radio transmission designed to economize frequency space in broadcasting, wherein one sideband and the carrier are transmitted, together with the lower audio-frequency components of the other sideband. By retaining the lower frequencies of the second sideband there is minimized the distortion to which pure single sideband gives rise in the types of broadcast receivers now generally used. Instead of first setting up the usual double-sideband transmission and then filtering out the undesired high-frequency components of one sideband, the asymmetrical spectrum is produced by combining a double-sideband modulation of the lower frequencies only with a single sideband of the upper range of frequencies. Details of an experimental application of the method are given.*

ON QUESTION 21 before the CCIR at Lisbon in 1934 entitled "The Use For Broadcasting of the System of Transmission Comprising the Carrier Wave and Only One Sideband," a report was submitted by the Netherlands P.T.T. Administration[1] describing certain experimental single- and double-sideband transmissions made by Kootwijk-Radio. There was also submitted an additional report[2] concerning these experimental transmissions pointing out that actually the emission consisted of (a) the carrier wave, (b) one sideband, and (c) the lowest frequencies, up to about 300 or 400 cycles, of the other sideband. Strictly speaking, therefore, the experimental transmission by Kootwijk constituted a broadcast system with asymmetric sidebands, or in other words, with eccentric carrier. This system the Netherlands P.T.T. Administration considers worthy of being given serious study in order to meet the present difficulties of wavelength allocation.

The advantages that are inherent in broadcasting with asymmetric sidebands stand out clearest when this method is compared with the carrier-and-one-sideband method. The latter gives rise to distortion in ordinary broadcast receivers using linear detection. This distortion increases with the depth of modulation and rises from a practically insignificant amount to a value that is detrimental. In this connection reference may be made to the tests that have been carried out by the British Broadcasting Corporation and by the Marconi Company.[3] Furthermore, a single-sideband-and-carrier transmitter in which the lowest tones are present at their proper strength is not easily realized.

In the modulation system with asymmetric sidebands here recommended for consideration, the difficulties associated with the single-sideband system for the most part disappear. Here the modulating audio-frequency spectrum is divided by a filter into a high and a low band. If, in order to guide our thinking, it is assumed that the modulating frequencies are included between 0 and 7 kilocycles, then this band is divided into two separate bands including the frequencies 0 to 2 kilocycles and 2 to 7 kilocycles. These frequencies have been used in the experimental apparatus which will be described below. Of course these particular frequencies are subject to variation

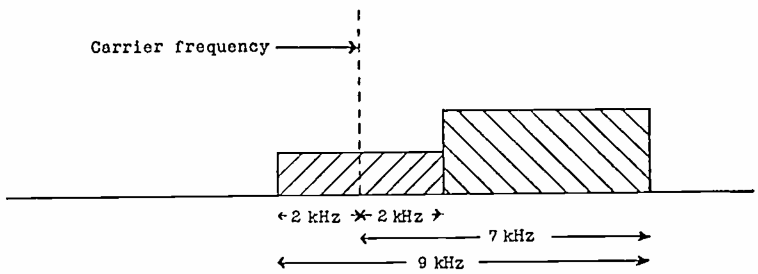

Fig. 1

after further consideration or later experiments, and in the temporary selection of these limits it is by no means intended to express a final opinion.

The resulting low-frequency band from 0 to 2 kilocycles is modulated on some carrier frequency according to the ordinary method of amplitude modulation, yielding two sidebands. The high-frequency band from 2 to 7 kilocycles is modulated on this same carrier frequency on the single-sideband basis. These two modulation products are then added. There results then a composite, which is represented schematically in Fig. 1, where the carrier is situated eccentrically and the two sidebands are disposed asymmetrically. In this figure the amplitudes of the high audio frequencies are drawn to be about twice as high as the low. In carrying out the above process it is arranged that the high frequencies are proportioned in this manner, since in ordinary receivers, employing for the most part linear detection, the low frequencies resulting from the two corresponding sidebands are in phase and are added, so that the high frequencies, which occur only with one sideband, must be made about twice as strong as would be required with two sidebands. It is possible, therefore, to adjust the strength of the high frequencies so that satisfactory reception is obtainable with ordinary commercial receivers.

In considering this method of modulation it must be taken into account that in ordinary speech and music the low frequencies below 2 kilocycles contain most of the energy, while frequencies above 2 kilocycles possess more of a quality-determining character and are of less strength. In speech, for example, the high-frequency band merely gives an intelligibility

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[1] Documents du CCIR, Lisbonne, 1934; vol. 1, p. 1208.
[2] Page 1222 of footnote reference 1.
[3] Page 1217 of footnote reference 1.

and distinctness correction, caused by the fact, among others, that the sibilant sounds are added.

If first the high and low audio frequencies are modulated in the above-described manner, by means of the modulation apparatus adapted for this purpose, and then the resultant, through a common intensity regulator, is modulated as deeply as the amplitude of the available carrier permits, that is to say, so deeply that in an ordinary receiver sufficient carrier is present for the detection to be properly accomplished, then the low frequencies are deeply modulated and the high frequencies will show a shallow modulation, in accordance with the relative strengths in which these frequencies generally occur in speech and music.

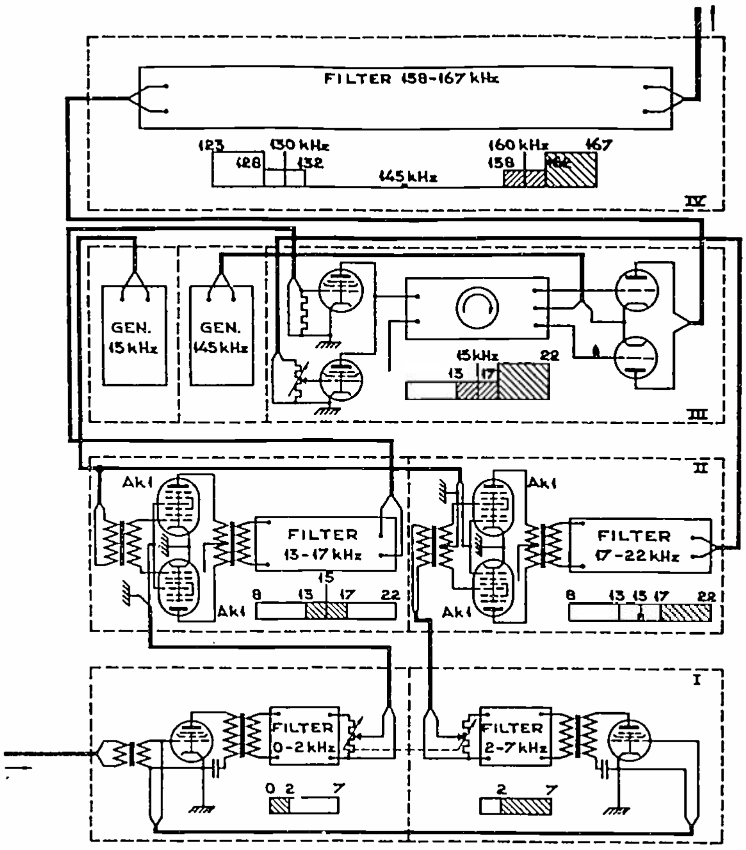

Fig. 2

The advantages possessed by the modulation method in question are as follows:

1. The low frequencies, which are strongest, occur in the modulation with two sidebands, so that with linear detectors practically no distortion arises. This circumstance is a favorable one because distorting harmonics of the strong low frequencies (a) are themselves strong, and (b) they would fall wholly within the audible-frequency spectrum.
2. The high frequencies, which are weak, are modulated, it is true, with only one sideband, yet they contain so little energy that the modulation depth is slight, so that the distortion is of practically no importance. In so far as still higher harmonics are formed, these for the most part fall outside the spectrum. Here, as under (1), for the sake of clarity, the occurrence of intermodulations between the low-frequency double-sideband components and the high-frequency single-sideband components has been left out of consideration, since these effects, with respect to magnitude at any rate, may be neglected.
3. The carrier wave is situated more in the middle, and not, as is the case in the single-sideband method, on the side of the radio-frequency band which must be selected out of the ether by the receiver. The band filter of the receiver will therefore better accept the carrier.
4. The modulation can be effected without special difficulties. The low audio frequencies give no trouble since they are modulated in the ordinary way with two sidebands, while the high audio-frequency band can be modulated without filter difficulties directly on a quite high frequency (up to 100 kilocycles for instance), on account of the fact that the low frequencies up to 2 kilocycles are missing.
5. As appears in Fig. 1, the frequency band of the transmissions is only 9 kilocycles wide and hence as wide as is permissible for each station to use with the present international wavelength allocation. Nevertheless, there occur frequencies up to 7 kilocycles in the modulation, in contrast with the 4.5 kilocycles that is now the limit. What this means is that, with the modulation method in question, the quality can be improved with retention of the present available band width (and hence with retention of the existing number of stations). It is evident also that with retention of the present quality, an audio frequency of 4.5 kilocycles now being the limit, a greater number of broadcast stations could be permitted with the modulation method proposed.

In Fig. 2 is given a schematic diagram of the experimental apparatus by means of which some preliminary tests of the proposed system were made. The dotted lines in the figure indicate the outlines of the four cabinets I, II, III, and IV, in which the various parts are placed. The design of this apparatus grew out of various circumstances and the description that follows is not intended as a recommendation that with it the best solution is to be obtained. It is only one solution of several that are possible.

It is assumed that the audio-frequency spectrum, originating from music or speech, contains frequencies between 0 and 7 kilocycles. This spectrum is divided into two parts with the aid of two filters, a low-pass filter, which passes all frequencies below about 2 kilocycles, and a high-pass filter, which passes only frequencies higher than about 2 kilocycles. This division takes place in cabinet I; the two frequency bands are shown in the figure. The amplitude of both frequency bands can be adjusted by means of two mechanically coupled volume controls. These two bands, which respectively contain the frequencies between 0 and 2 kilocycles and between 2 and 7 kilocycles, are each led to a first modulator shown in cabinet II; the modulation of the two bands is effected in different ways.

The modulator by means of which the 0 to 2-kilocycle band is modulated on a frequency of 15 kilocycles consists of a balanced circuit comprising two octodes connected so that the carrier frequency is in opposite phase on the corresponding control grids of the two octodes, while the modulating voltage is applied in phase to the other two corresponding control grids. As a result there exists at the terminals of a transformer inserted in the plate circuit a voltage that may contain all frequencies between 13 and 17 kilocycles. The carrier frequency is not suppressed in this circuit. In order to suppress harmonics of the carrier frequency or of the low-frequency signals that are not suppressed by the balanced circuit, there is inserted a filter with a pass band of 13 to 17 kilocycles.

The frequency band of 2 to 7 kilocycles is modulated on the same carrier frequency also by means of a balanced circuit of octodes. Here the carrier frequency is applied in phase to two corresponding control grids, the modulating frequencies in opposite phase to the other two corresponding control grids. By means of the balance connection the carrier wave is thus suppressed; the two sidebands and the low-frequency spectrum remain. After the modulator there follows a band filter with a pass of 17 to 22 kilocycles, so that only the upper sideband is let

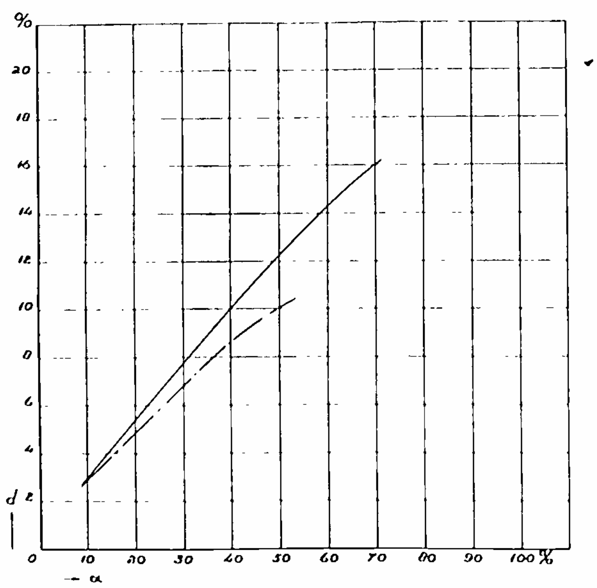

Fig. 3

through. The passed frequency bands are shown schematically in the figure.

After each of the two last-named filters there follows an amplifier tube, placed in cabinet III.

In order to be able to accomplish the previously discussed proportioning of the high audio frequencies, there is introduced a volume control for the 2- to 7-kilocycle band.

The plate circuits of the amplifier tubes are connected with each other, so that across a variable impedance in this common plate circuit there is a voltage which may contain all frequencies between

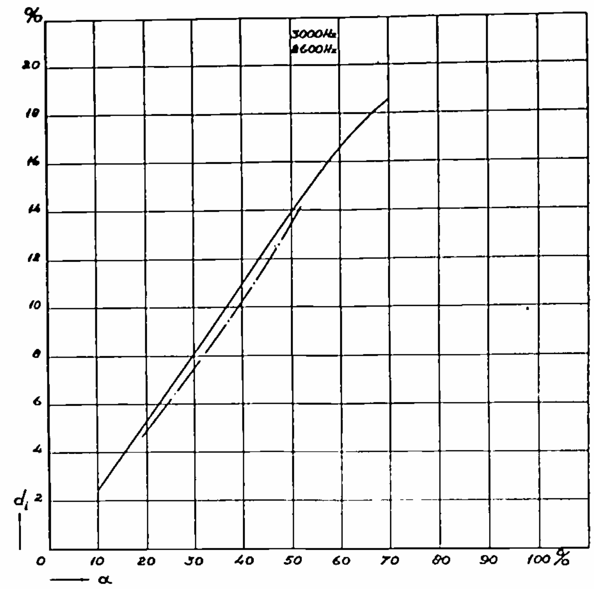

Fig. 4

13 and 22 kilocycles. The resulting spectrum is shown in the schematic.

Following this impedance is a second modulator by means of which modulation takes place at 145 kilocycles. This consists of a balance connection of triodes so that the 145-kilocycle frequency is suppressed. In the plate circuit there appear then the two sidebands, each embracing the spectrum 13 to 22 kilocycles.

After this there is a filter, in cabinet IV, which passes only the upper sideband, 158 to 167 kilocycles. The carrier frequency, which was originally 15 kilocycles, is now transformed to a frequency of 160 kilocycles, that is, the frequency of the Kootwijk broadcast transmitter, which the apparatus is intended to serve.

If the detected high-frequency signal is to give an exact reproduction of the original low-frequency spectrum, the division of the latter should be so arranged that after the recombination of the two parts all frequencies are present in the same relative amplitudes. For this to be accomplished the filters are so designed that they correctively overlap each other. In this connection account must be taken of attenuation in the pass bands, also of phase rotations.

Figs. 3, 4 and 5 represent distortion curves of low-frequency signals measured at a linear detector. The unbroken curves have been determined experimentally, while the dash-dot lines are roughly calculated.

In Fig. 3 the second harmonic in percentage of the fundamental d is shown as a function of the modulation depth α. Figs. 4 and 5 show, respectively, the interdetection d_i between two audio frequencies in the single-sideband channel and between a frequency in the single-sideband channel and a frequency in the double-sideband channel, as a function of the modulation depth. Here both frequencies were of equal amplitude; the frequency-difference component was measured and expressed as a percentage of one of the two frequencies.

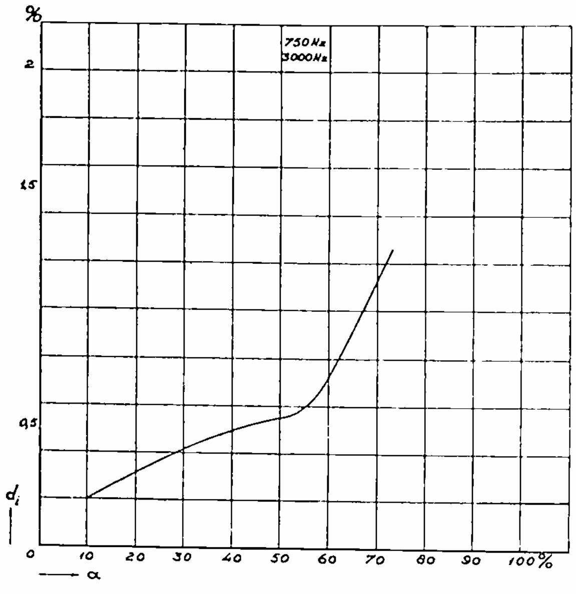

Fig. 5

Finally, it is to be repeated that this trial apparatus has acquired its described form under the influence of other objectives as well.

For a final design intended solely for the desired goal it seems preferable to choose a higher carrier frequency in the first modulator. One could go to 100 kilocycles here without difficulty.

Since the usual broadcast frequencies must be modulated a second time in any case, the first carrier frequency may be chosen a little lower than 100 kilocycles for the sake of filter simplicity, yet high enough to avoid any harmonics of the audio frequencies. The distortion factor can be diminished if for both the low-frequency band and the high-frequency band the first modulation is effected by a three-electrode tube using the customary Heising method of modulation. It is no more necessary to suppress the carrier (by balance) in the high-frequency band than it is in the low-frequency band. This can be selected out by the filter, and whatever remains can be permitted to combine with the carrier present in the modulator for the low-frequency band.

Supplement[4]

Various experiments were made to examine the distribution of energy in the low-frequency spectrum. For this purpose filters were constructed by means of which the separation between high and low audio frequencies could be made not only at 2000 cycles, but also at 1500 or 1000 cycles. As a measure of the energy represented in the high and low frequencies there was relied upon the indications given by two Braun tubes, in one of which the low frequencies were registered, in the other the high frequencies. If the separation between high and low frequencies is made at 2000 cycles, the high frequencies have only a quality-determining character; they represent only a small part of the total energy.

If the division occurs at 1500 cycles, there is still no distinct difference noticeable in the energy distribution in comparison with the separation at 2000 cycles. If it takes place at 1,000 cycles, then it is evident that too much of the total energy is represented in the high frequencies for these to be modulated by the single-sideband method without causing noticeable injury to the quality of the transmission.

A separation at 1500 cycles appears, therefore, to be indicated. If this frequency is chosen as the dividing point, there is to be obtained simultaneously both a good band width and a quality gain. If, for example, frequencies up to 5500 cycles are to be transmitted, then according to the asymmetric sideband method of modulation these can be accommodated in a total band width of 7000 cycles, namely, a sideband of 1500 cycles and one of 5500 cycles. According to the usual method of modulation, frequencies up to 4500 cycles are transmitted with a total band width of 9000 cycles. The gain realized, therefore, amounts to an extension of the high frequency range by 1000 cycles and a narrowing of the band width by 2000 cycles.

[4] Presented as a Supplement to the Report of the Netherlands P.T.T. Administration on Question 11 before the CCIR at Bucharest, 1937. Documents du CCIR, Bucarest, vol. 1, p. 706.

Stroboscopic-Light Source*

HEINZ E. KALLMANN†, ASSOCIATE, I.R.E.

Summary—A description of a high-speed stroboscope is given. A blocking-oscillator circuit is used to produce short pulses of several amperes of anode current in a high-vacuum tube, the anode of which is coated with fluorescent material.

STROBOSCOPIC-light sources are widely used for observation of fast movements. The use of mechanical light-chopping devices is restricted to low frequencies, and is generally inferior to gas-discharge lamps, which are very satisfactory up to flash frequencies of about 20 kilocycles but at higher flash frequencies seem to become impracticable because the inertia of gas molecules prevents an adequately quick discharge. Only spark discharges can at present be used for observation of the very fast movements of sound waves or of projectiles. An alternative method may thus be useful, offered by an arrangement using fluorescence light excited by short pulses of electric current. This should compare well

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† New York, N. Y.

with gas-discharge lamps as regards efficiency, maximum available light intensity and simplicity and it can be used at much higher flash frequencies, up to and exceeding 200 kilocycles, each flash lasting only a fraction of a microsecond.

The Circuit

Fig. 1 shows a simple circuit which produces short strong current pulses. It is based on a modification[1] of the well-known blocking-oscillator circuit. The grid circuit of a triode is tightly coupled to the anode circuit through a small iron-cored transformer. Inserted in the cathode circuit are a resistance R and a condenser C in parallel. The time constant $T=CR$ of this circuit may be adjusted by varying the resistance R to a value $T=1/f$ for a flash frequency f. Due to the tight reaction coupling, this circuit will start violent oscillations at the resonant frequency of the transformer winding. The first positive grid swing will cause strong grid and plate current, both passing through the resistance R and charging the condenser C through the then low resistance of the valve. When, after passing the positive peak, the grid starts to become negative, it will leave the cathode at a positive potential and thus interrupt the plate current. The transformer will continue a few damped oscillations which have no further effect on the tube, because their amplitude decays faster than the positive charge of the cathode condenser. Thus no plate current will flow until the condenser C has discharged through the resistance to a potential equal to the grid base of the triode; at which moment the plate current—and the oscillations—start afresh. Two alternating periods of the plate current are thus distinguishable: (1) The discharge period, equal to not more than a quarter cycle of the transformer resonance, and (2) The waiting period, during which no plate current flows, which is proportional to the time constant T of the cathode circuit. The first period may be as short as one thousandth of the second.

The Fluorescent Lamp

The triode of the circuit needs to differ from any normal tube only in that the bombarded side of the anode is coated with fluorescent material and that this anode is so arranged that the light of its fluorescence may radiate outward. Alternatively the electrons may pass through openings in the anode and impinge on a separate fluorescent screen, e.g. on the glass wall.

Frequency Limits

Whereas there is hardly any limit towards low frequencies, the limit towards high frequencies is set by the resonant frequency of the reaction-transformer winding. It is evidently necessary that the discharge time should be only a fraction of each flash period. Thus the flash frequency will at best be of the order of the resonant frequency of the transformer winding. Using transformers with reasonably small winding capacitances, flash frequencies of 0.25 megacycle have been reached without special measures, the discharges then lasting about 0.2 microsecond. The emission of light may last longer, due to a longer decay time of the fluorescent material; however several efficient fluorescent materials are known, the radiation of which decays in a fraction of a microsecond to negligible values.

[1] British Patent No. 471,737.

Tuning and Synchronizing

A single transformer size easily covers frequency ranges from 1 to 10; for larger frequency ranges switching is advisable. The same applies to the size of the cathode condenser C. Fine tuning is most easily done by varying the cathode resistance R. For synchronization a positive pulse may be applied to the grid, either direct or via an auxiliary transformer winding. This third winding may serve equally well to trigger an external circuit from the oscillations of the transformer. Alternatively a clean synchronization pulse can be derived from a small resistance in the cathode or anode path.

Efficiency

The light available from each current pulse is proportional to the product of anode current during the pulse, of anode voltage, and of the efficiency of the fluorescent screen material. The latter depends on the anode voltage and may, at a few thousand volts, be taken as about 2 candle power per watt. The amount of current available, even from a small receiver tube, is remarkably large. This fact may be demonstrated on the small indirectly heated triode Marconi-Osram MH.41, which has a heater consumption of 4 volts, 1 ampere, an amplification factor

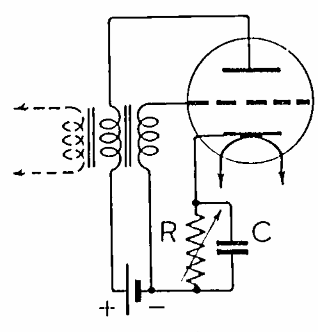

Fig. 1

of 80 and a mutual conductance of 6 milliamperes per volt. Its usual anode current is about 5 milliamperes. When this tube is used in the circuit, Fig. 1, with an anode voltage of 1000 volts, peak anode currents of about 5 amperes are regularly observed (with oscillograph on a small resistance in the anode circuit). No deterioration and no other signs of overload have been noticed. The current consumption drawn from the power supply is directly proportional to the flash frequency and the duration of each flash.

Thus it is, for 1000 flashes per second of each about 1 microsecond, found to be 5 milliamperes. The momentary anode load is very much larger, 5 kilowatts, 1000 volts and 5 amperes. Assuming an efficiency of 2 candle power per watt, such a small tube may thus be expected to produce flashes of 10,000 candle power.

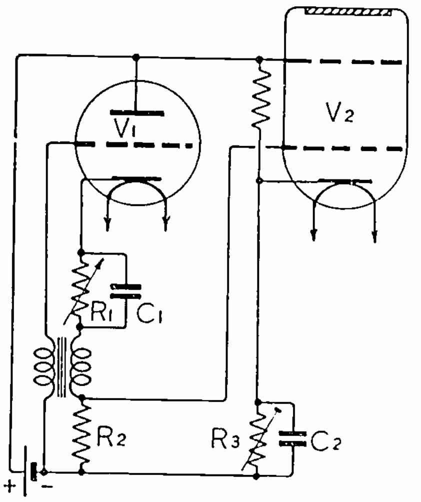

Fig. 2

Loss of efficiency due to saturation of the screen material need not be feared, as the anode current is not focused in a single spot, but may reach the anode surface with about evenly distributed current density. Thus the current density will hardly ever reach a value of 10 amperes per square centimeter as is usual (at 20,000 volts) in television projection tubes, without appreciable screen saturation.

The circuit Fig. 1, using a single tube only, is particularly suitable for driving the grid far in the positive region and thus offers an efficient way of producing strong anode-current pulses from a small cathode. This advantage is partly lost if the circuit is modified for use of two separate tubes, the one a triode to produce the pulses, the other a grid-controlled fluorescent lamp, Fig. 2. The latter circuit lends itself somewhat better to improvization with usual components, using a cathode-ray tube as the tube V_2. The variable resistance R_3 in Fig. 2, by-passed by a condenser C_2 for the pulse frequencies, then serves to bias the cathode-ray tube to cutoff between the pulses. A usual cathode-ray tube connected as the triode in the circuit Fig. 1 is seldom satisfactory.

Acknowledgment

Acknowledgment is due to the Electric and Musical Industries Limited for permission to publish this paper which is a by-product of television research carried out in their laboratories.

Transatlantic Reception of London Televison Signals*

D. R. GODDARD†, ASSOCIATE, I.R.E.

Summary—*The results of daily observations at Riverhead, L. I., N. Y., since September, 1938, of the English television transmissions on 41.5 and 45.0 megacycles are summarized and discussed. A photograph of one of the television images received during this period is shown. A résumé is made of the signal strengths observed since January, 1937. During each winter of this period signal strengths of between 10 and 500 microvolts per meter were frequently received.*

THE English transmitters are located at Alexandra Palace London, a distance of 5400 kilometers from Riverhead, L. I., N. Y. The frequency of the voice channnel is 41.5 megacycles per second and the picture channel 45 megacycles per second.[1,2]

The equipment used for observations at Riverhead was the same as that used the previous winter. The antenna consisted of a horizontal rhombic 45 feet above ground directed towards London. The length of each leg of this antenna was 400 feet. The major and minor axes were adjusted to give maximum response to a signal having a vertical arrival angle of about 6 degrees. The effective height of the antenna system was about 20 meters.

Fig. 1 shows the receiving equipment. In the foreground is a motor-driven motion-picture camera focused on the cathode-ray tube of a television receiver. Only the cathode-ray tube and video-frequency amplifier controls were used in this receiver. Directly behind the camera is the video-frequency receiver. This receiver provided automatic or manual volume control and a minimum noise equivalent of about 30 microvolts with a band width slightly less than 5 megacycles. It supplied a rectified signal to the television receiver just described. On the bench is the receiver and signal generator used for signal-strength measurements.

Most of the observations took place between 9:45 A.M. and 11:30 A.M. Eastern Standard Time as that corresponded approximately to the afternoon sched-

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† R.C.A. Communications, Inc., Riverhead, L. I., N. Y.

[1] H. O. Peterson and D. R. Goddard, "Field strength observations of transatlantic signals, 40 to 45 megacycles," Proc. I.R.E., vol. 25, pp. 1291–1299; October, (1937); *RCA Rev.*, vol. 2, pp. 161–170; October, (1937).

[2] D. R. Goddard, "Observations on sky-wave transmission above 40 megacycles," Proc. I.R.E., vol. 27, pp. 12–15; January, (1939); *RCA Rev.*, vol. 3, pp. 309–315; January, (1939).

ule of the British transmitters. On numerous occasions the transmitters continued until noon or later.

Fig. 2 shows the peak signal strengths measured during the winter of 1938–1939, in decibels above or below one microvolt per meter. As is indicated, the lower plot is the English voice channel while the one above it is the English picture channel. The small crosses indicate days on which no observations were made. The topmost curve is a plot of the F_2 virtual layer height and directly below it is a plot of the critical frequency of the F_2 ordinary ray, as measured near noon at Deal, N. J. The data for the latter two curves were supplied by J. P. Schafer and W. M. Goodall of the Bell Laboratories.

Inspection of Fig. 2 indicates that a strong signal on either channel is not necessarily accompanied by an exceptionally high critical frequency or low layer height. However, during November and December when the critical frequencies were consistently high, both signals were quite consistently strong. It is curious that on November 11 a very strong signal was measured on 45 megacycles while the 41.5-megacycle signal went unheard. During three winters' observations this is the only occasion on which the video-frequency channel has been heard unaccompanied by the sound channel. It might be added that on numerous occasions the period during which the video-frequency signal reached its maximum did not correspond to the period of maximum of the audio-frequency signal. In fact often the audio-frequency channel signal strength would drop from perhaps plus 5 or 10 decibels to minus 20 or 25 decibels during the period of maximum signal strength on the video-frequency channel. This effect was much more pronounced last winter than during either of the two previous winters.

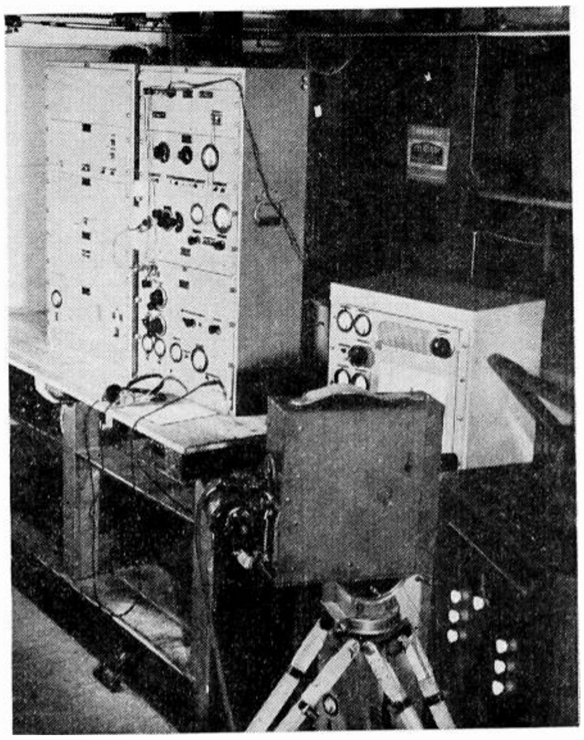

Fig. 1—Ultra-high-frequency receiving equipment used for field-strength measurements and visual monitoring.

Fig. 3 shows a comparison between signal strengths of the two signals and the predicted maximum usable frequencies as published each Wednesday by the National Bureau of Standards for conditions as measured at Washington, D. C. The lower half of this figure represents the maximum usable frequency interpolated for a distance of 2700 kilometers plotted for each Wednesday from October until March. Twenty-seven hundred kilometers represents half the

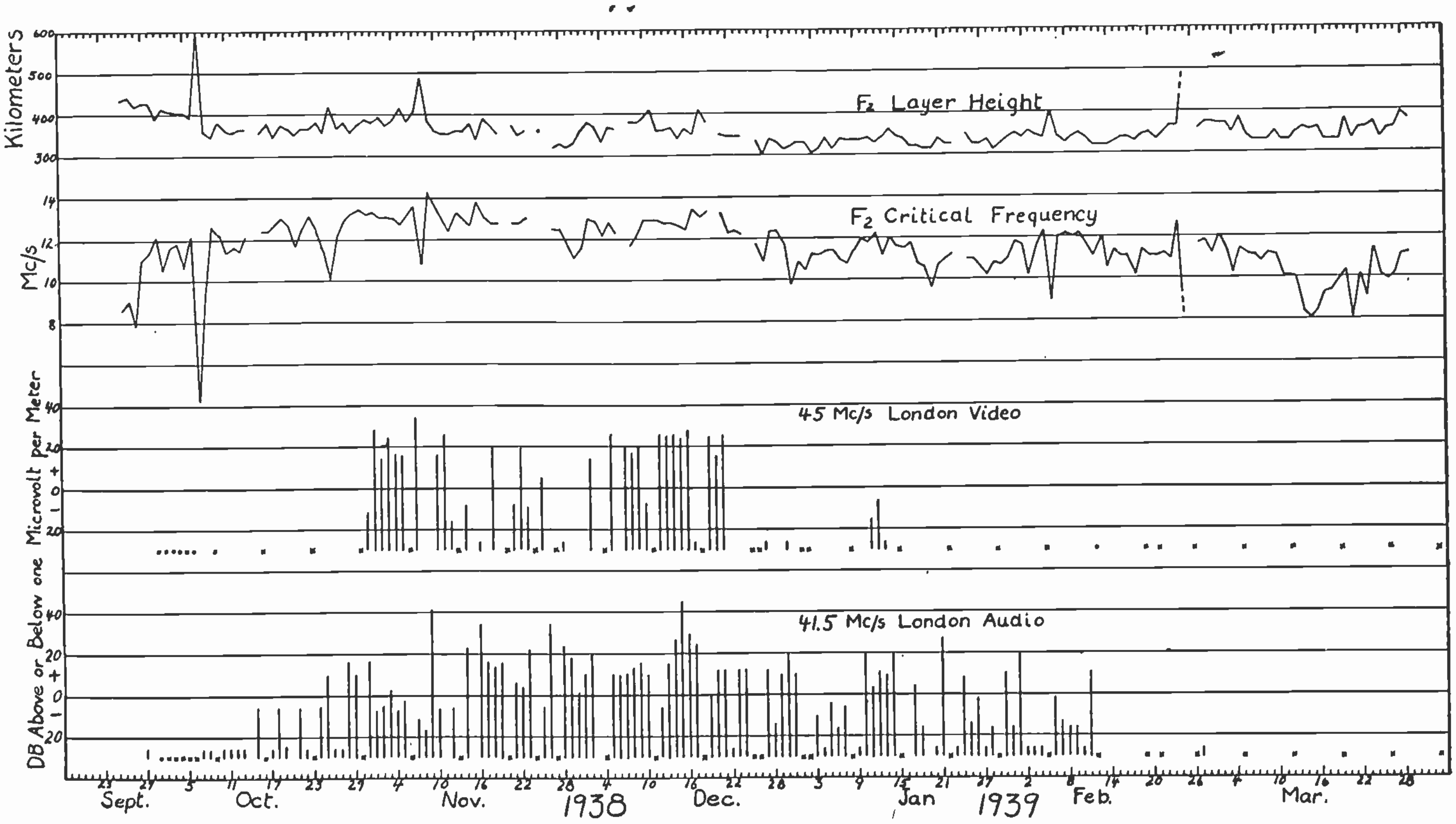

Fig. 2—Comparison of F_2-layer height and critical frequency to observed maximum signal strengths from London television audio- and video-frequency channels. The crosses indicate days on which no observations were made.

distance between Riverhead and London. The upper half of the figure is a plot of maximum signal strengths observed on the two English channels for the same

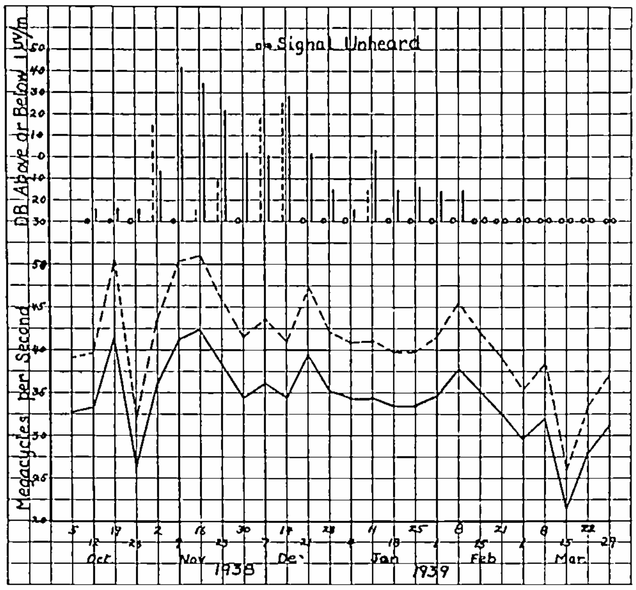

Fig. 3—Comparison of observed signal strengths to maximum usable frequencies interpolated for 2700 kilometers. Vertical solid lines of the upper plot represent maximum signal strengths observed from London on 41.5 megacycles. Broken lines represent the same for 45 megacycles. The broken-line plot represents the maximum-usable-frequency curve after applying the Lorentz correction term.

days that the ionosphere measurements were made. The solid vertical lines represent the 41.5-megacycle channel and the broken vertical lines represent the 45-megacycle channel. The small circles indicate no signal heard that day.

Examination of Fig. 3 shows that at no time was the 45-megacycle channel indicated as useful, while actually it was heard six times. The 41.5-megacycle channel was indicated as useful three times while actually it was heard eighteen times.

It has been suggested that possibly the Lorentz[3] polarization term should be included in the determination of maximum usable frequencies. If this is

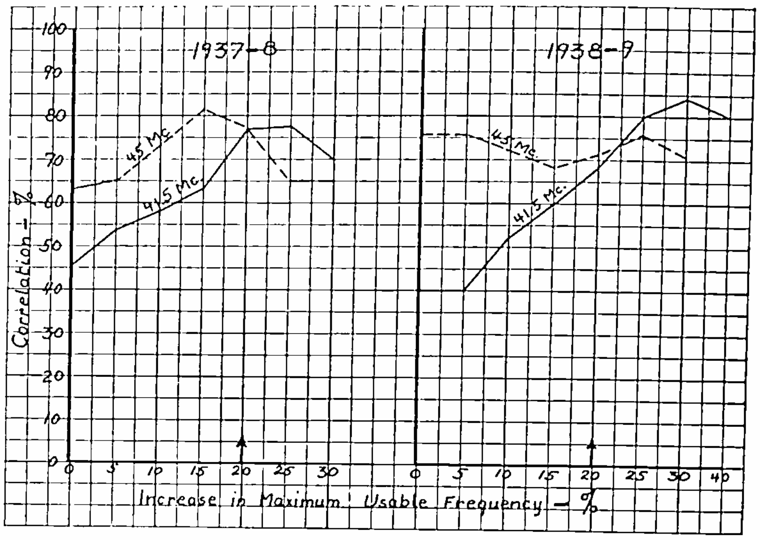

Fig. 4—Per cent correlation obtained by raising the maximum usable frequency curve by 5-per-cent increments. The arrows indicate the value determined from the Lorentz correction term.

done it would raise the maximum usable frequency curve by about 20 per cent. This is shown by the broken-line curve. In general an improvement in correlation results from this. The correlation of the 41.5-megacycle signal increases from 40 per cent correct to 68 per cent correct. The 45-megacycle channel, however, suffers somewhat from this change. It drops from 76 per cent correct to 72 per cent cor-

[3] H. G. Booker and L. V. Berkner, "Constitution of the ionosphere and the Lorentz polarization correction," *Nature*, vol. 141, pp. 562–563; March 26, (1938).

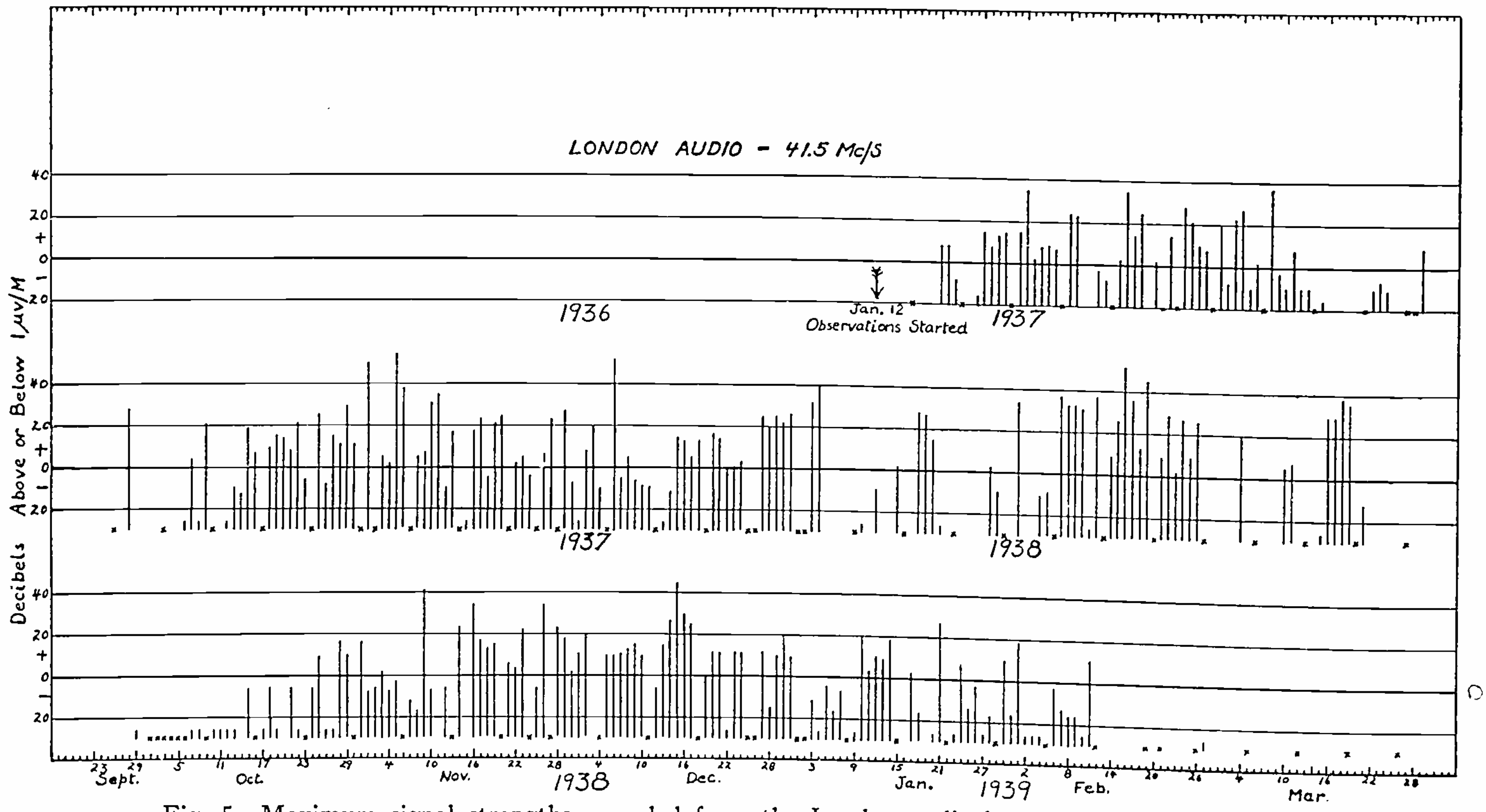

Fig. 5—Maximum signal strengths recorded from the London audio-frequency transmitter since January, 1937. The crosses indicate days on which no observations were made.

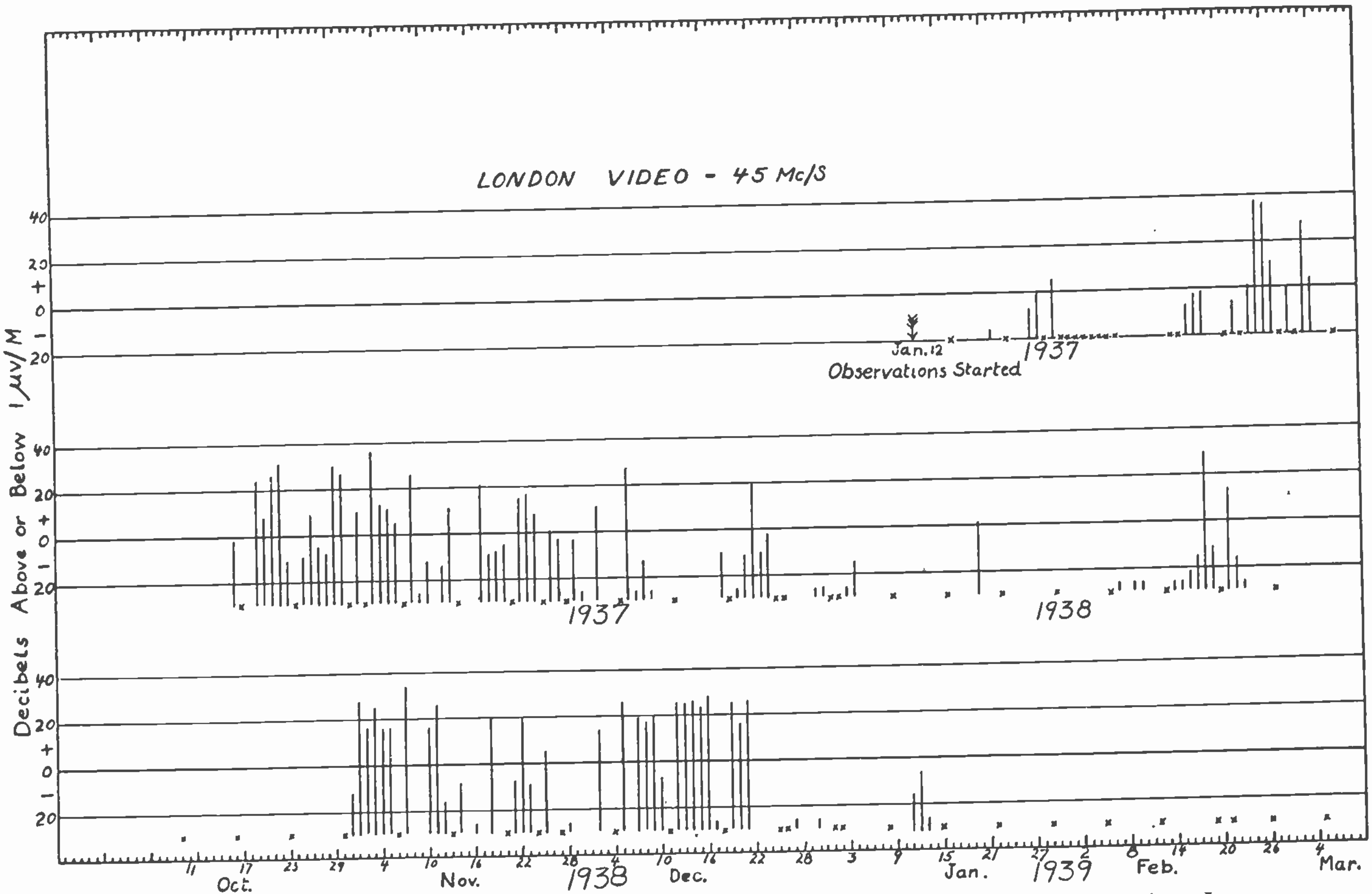

Fig. 6—Maximum signal strengths recorded from the London video-frequency transmitter since January, 1937. The crosses indicate days on which no observations were made.

rect. One reason for this is the large number of times during which this signal went unheard.

The effect on correlation by raising the maximum-usable-frequency curve by increments of 5 per cent is shown in Fig. 4. Data are given for both the winter of 1938–1939 and the winter of 1937–1938. Inspection shows that no one value of increase will produce the best correlation on all four curves shown. The small arrows on the abscissas mark the 20 per cent values indicated by the Lorentz polarization term.

Fig. 5 shows the maximum signal strength observed from the London audio-frequency channel for the last three winters. Each winter since 1937 the date on which the signal was last heard has advanced. This winter the signal was last heard about a month and a half earlier than during the winter of 1936–1937. By extrapolation we might predict that next winter this signal will disappear around the last part of December.

Fig. 6 shows similar data for the 45-megacycle London signal. The burst of signal that appeared in February of 1938 was absent this winter. The total period over which the signal was audible was very much shorter this winter than last.

The television images observed on the kinescope appeared to show selective fading as the contrast of the picture would often change between wide extremes. Multipath propagation tended to spoil the picture in two ways. First it would cause repetition of the subject matter and second it would result in more than one set of horizontal synchronizing pulses making it impossible to obtain a steady picture. These two factors were the chief obstacles to obtaining reasonably clear pictures as often as the signal strength was of sufficient strength and the variations in contrast were usually slow. However, on a number of occasions short glimpses were seen of the subject matter with sufficient clarity to recognize faces easily and perceive the nature of the scene.

Fig. 7 shows an enlargement of a frame selected from motion pictures taken of the London television images during the latter part of November and the first part of December. It indicates a couple dressed in the fashion of George Washington's time, standing before a doorway singing.

Fig. 7—Photograph of London television image taken at Riverhead. It shows a man and woman dressed in the fashion of George Washington's time, standing before a doorway singing.

Static Emanating from Six Tropical Storms and Its Use in Locating the Position of the Disturbance*

STEPHAN P. SASHOFF†, NONMEMBER, I.R.E., AND JOSEPH WEIL†, MEMBER, I.R.E.

Summary—*This paper describes the technique of locating the apparent source of atmospherics employing several stations located at a considerable distance from each other and equipped with cathode-ray-tube direction finders. It discusses the data obtained on six tropical storms and compares the results with the estimate of the position of the storm center as reported on the daily bulletins of the United States Weather Bureau. It presents some evidence of close connection between static and meteorological conditions. It suggests that further studies should be made along the same or similar lines to establish the feasibility of this method for locating the center of tropical storms by means of their associated static.*

Introduction

REPORTS of investigations on the nature of atmospherics have appeared in print from time to time. These investigations have dealt with static emanating from lightning and thunderstorms. While in most of the cases the observers have aimed at measuring the intensity, direction, and the wave pattern of the received static other investigators have attempted to develop some means of locating the position of the source of the arriving atmospherics.

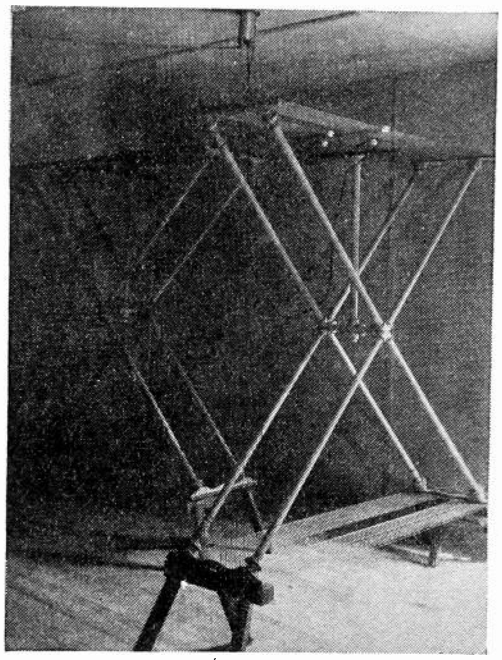

Fig. 1—Perpendicular rectangular loops used with cathode-ray-tube direction finder.

Fifteen years ago Watson Watt, Appleton, and Herd[1] found that atmospherics recorded on the Continent along with European time signals were coincident with the time of arrival of static recorded in America along with the same time signals. This led to the development of a method of locating the source of static by triangulation, using a cathode-ray-tube radiogoniometer or direction finder.

The University of Florida became interested in the study of atmospherics about four years ago. After a series of conferences with officials of the United States Weather Bureau, the National Bureau of Standards and other interested parties, it was decided to attempt such a study in order to determine how close a relationship, if any, exists between static and meteorological conditions. It was agreed that any new means for supplying information relative to the position and movements of tropical storms should be investigated, since if found reliable they would be of great value to the people of Florida and to those of the Caribbean countries because of the frequency of occurrence and the great loss of life and property which often follows in the wake of such storms. A systematic study of the nature and origin of atmospherics was undertaken, therefore, by the University of Florida and the University of Puerto Rico, Rio Piedras, to determine the feasibility of locating the center of tropical storms by means of their associated static. The necessary funds for this work were provided by the Works Progress Administration. Through co-operating agencies, equipment built by the National Research Laboratories of Great Britain, Slough, for the United States Navy was made available to the two schools. This equipment was remodeled and brought up to date and tests were begun in September, 1935. A report on the earlier observations has already been published.[2]

Triangulation for the Center of the Storm by Means of Static Associated with It

1. *Equipment and Method*

This paper describes the work on the oscillographic examination of the direction of arrival of atmospherics, the results of triangulation for the center of the static source, and the comparison of the position of the apparent source of the incoming static and that of centers of storms determined by other means. This series of tests began in August and continued through October, 1937. The method used in making the observations is in general that of Watson Watt and his associates.

Two perpendicular rectangular loops are located close to each other; one in the true East-West and the other in the true North-South directions as shown in Fig. 1. The outputs of these loops are fed

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† University of Florida, Gainesville, Florida.

[1] Appleton, Watson Watt, and Herd "On the nature of atmospherics," *Proc. Royal Soc.*, series A, vol. 3, pp. 654–677; June, (1926).

[2] University of Florida Engineering Experiment Station Bulletin No. 3, October, 1936.

into separate tuning networks. From the tuning networks the incoming signal passes into two amplifiers, one connected to each tuning unit. These amplifiers are so designed and adjusted that signals of equal magnitude and in phase will appear at the output terminals still equal and cophasal. The output of the East-West amplifier is then fed to the vertical plates of a cathode-ray tube while the North-South amplifier supplies the horizontal plates. The corresponding points of the compass are marked at the end of the tube as *E*, *W*, *N*, and *S*.

With this arrangement a static pulse arriving at the loops is passed through the tuners and the amplifiers and then appears on the screen of the tube in the form of a bright line. The orientation of this line with respect to the EWNS points at the end of the tube is that of the actual direction of arrival of static with respect to the loops. These lines, to be referred to from now on as static crashes, can be observed visually. Because of their short duration, however, visual observations are difficult. A camera is incorporated in the equipment, therefore, the crashes being photographed and a study of them made from the developed film.

The camera used in these studies was especially designed for this work by W. Mason and others. It uses 16-millimeter motion-picture film. The film may be kept stationary or moved at the rate of one inch per second. If the film is kept stationary while static is received a photograph is obtained which contains a large number of crashes. This type of photograph, Fig. 2, has been called a "composite." Photographs of static taken on moving film on the other hand have been designated as "synchronous." The name "synchronous" was derived from the fact that such photographs are usually made synchronously by all stations of the network. In order to synchronize the

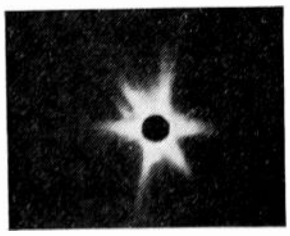

Fig. 2—Composite photograph. Exposure, 30 seconds at $f/11$.

films at the various recording stations, radio signals in the form of letters of the alphabet in the continental code, as shown in Fig. 3, are sent from the master station at Gainesville, Florida. The radio signals are picked up at the recording stations by short-wave receivers, the outputs of which are fed into neon glow lamps. The flashes of the lamp at each recording station correspond to the dots and dashes of the radio signal and are recorded on the film side by side with the static crashes. In addition to the synchronizing impulses and the record of the static the film record also contains the number of the test run, the time of day, and the date.

2. *Analysis of Data*

A study of the data thus obtained is made in the following manner. The film record of a given run is projected on a screen on which a compass rose and

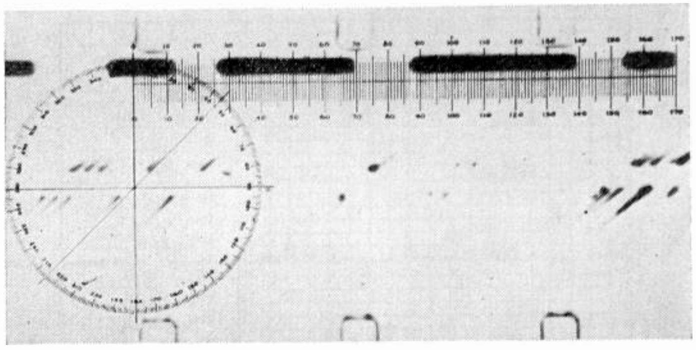

Fig. 3—Section of moving-film record superimposed on a compass rose and scale. Note direction of static crashes and synchronizing signal in continental code.

a scale having 170 divisions are drawn. The projector and the film are adjusted so that the point at which the line of the static crash cuts the east-west line, which also is the zero line, coincides with the center of the compass rose; the angle of the static crash is measured and the end of the letter of the synchronizing signal just above the crash noted on the scale. (Fig. 3.)

The readings obtained in the above manner are recorded on a form known as "the static record sheet." Such sheets are made for the individual runs, one for each of the recording stations. While analysis of the film records described in this paper were made separately at Gainesville and at Rio Piedras, the method may be used to read the film records at each station and then transmit the data by radio to a central point where triangulation may be made. In order to insure identical projection of the films and to compensate for any differences in film speeds, series of equally spaced dots are sent with each run. The person making the analysis is instructed always to project the film in such a manner that 5 of these dots cover 100 divisions of the scale.

The 2 recording stations already mentioned began making observations at the beginning of the 1937 hurricane season. Before the end of the period two additional stations, one at Miami and another at Pensacola, Florida, were put in operation.

The film records of the three Florida stations were developed in Gainesville, while G. W. Kenrick and his staff at the University of Puerto Rico developed their own films. Records were exchanged by mail.

When analysis of data first began it was our intention to look only for "synchronous crashes"; i.e., crashes which appear under corresponding points of the synchronizing signal. It became evident early in the work, however, that the number of such crashes coming from the direction of the storm is very small in comparison with the total number of crashes recorded. At first it was thought this may be due to some error possibly introduced by the unequal time intervals required for the synchronizing signal to reach each one of the recording stations, as well as by the unequal time delay in the various receiving

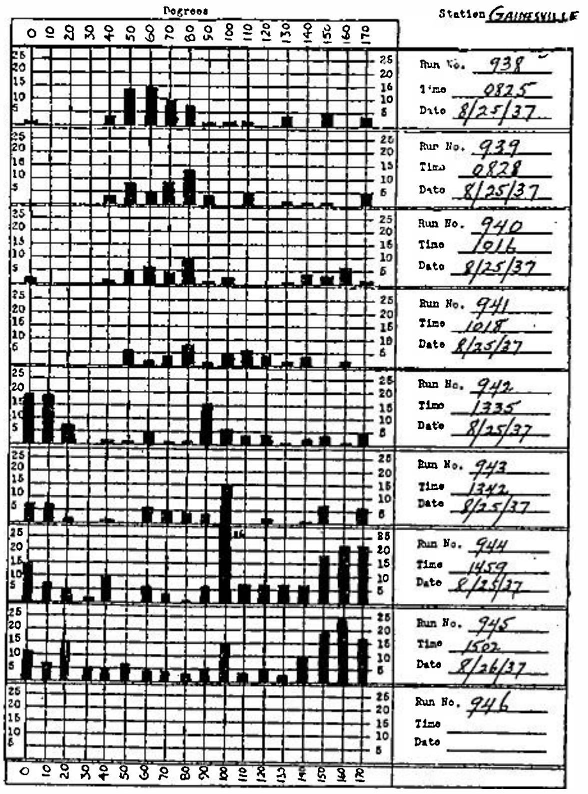

Fig. 4—Random distribution of static crashes. Storm at 120 degrees from Gainesville, Florida.

equipments. A thorough check for a possible correction factor, which should be a constant for each station, failed to produce any satisfactory results.

A further study of the data taken during days when a tropical storm was known to exist in a certain area, however, showed a considerable amount of static arrived at each station from the direction of the storm.

A new tabulation of the static crashes was then made. In this tabulation the crashes for each run were segregated according to the direction of their arrival in groups within ten degree angles. The listing of the data was made in 18 groups, the angles being measured from 355 to 175. Each group was designated by the angular position corresponding to the center of the group angle. The synchronizing signals were disregarded.

The graphs thus obtained were then plotted on "static continuation sheets", 9 graphs to a sheet. A study of the film records and the "static continuation sheets" leads to the following conclusions:

1. There is a random distribution of static crashes especially during the afternoon and early evening runs which suggests that most of the disturbances during this period are of local origin. (Fig. 4.)
2. Considerable amount of static arrives from the direction in which a storm is known to exist.
3. Although the intensity of static from such a direction varies, such static seems to persist on successive runs. (Fig. 5.)
4. The direction of the persisting static changes in close relationship to the shifting of position by the storm.
5. Static from directions like the above seems to disappear with the disappearance of the storm.
6. Many of the crashes in a given direction occur in groups of twos and threes. This supports the findings of Appleton and Chapman,[3] who concluded that if atmospherics originated in thundercloud discharges they may be expected to have the tendency to occur in groups.

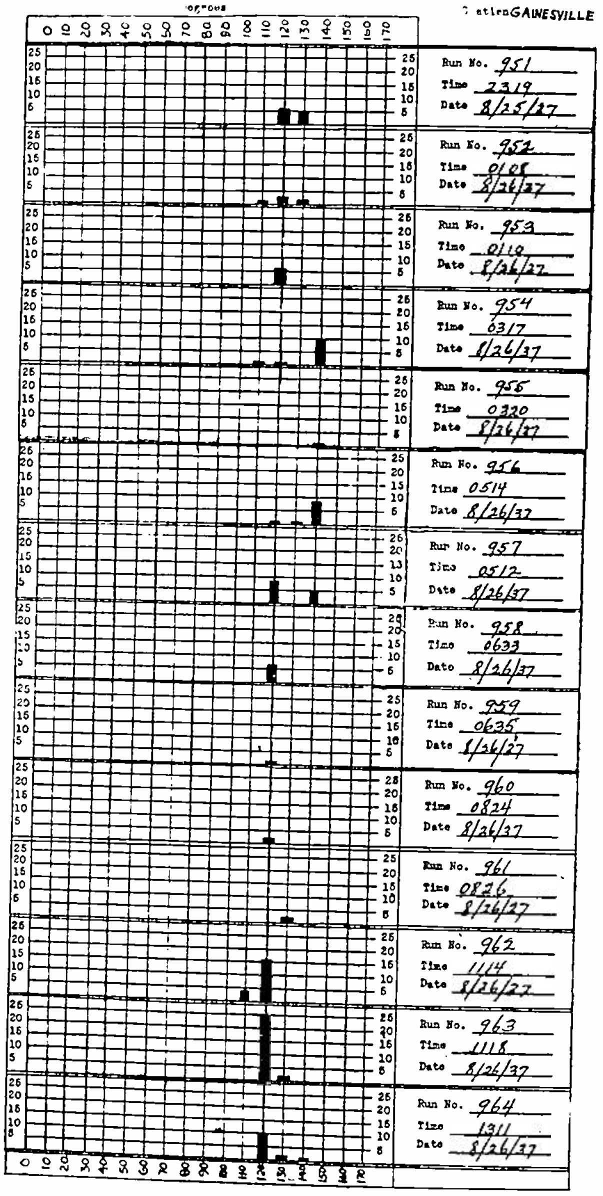

Fig. 5—Storm static persists on successive runs. Storm at 120 degrees from Gainesville, Florida.

7. The intervals between the individual crashes occurring in a group were found to be of the order of 5/100 second.
8. The best records were obtained between the hours of midnight and 12:00 noon.

[3] Appleton and Chapman, "On the nature of atmospherics," *Proc. Royal Soc.*, series A, vol. 158, pp. 1–22; January, (1937).

3. Paths of Six Tropical Storms

Having outlined the general method in securing the data and having listed the conclusions drawn from the compiled results, we shall view the results obtained on 6 tropical storms which occurred during the 1937 hurricane season, daily reports for the position of which were supplied by the United States Weather Bureau. At this time it should be pointed out that observations on these storms were made for periods of several days. At first runs were taken on these storms every 2 hours on the odd hour, but later because of interference from local static, the schedule was changed and runs were taken every hour on the hour from 7:00 P.M. until 9:00 A.M. of the next day. The observations were made with the equipment tuned to 10 kilocycles, this frequency having been found by previous investigators[4] to be the most common to this type of disturbance.

The first storm was detected by the United States Weather Bureau station located at San Juan, Puerto Rico, on August 24. The storm had its origin several hundred miles southeast of the island. The four stations of the network were put on a 24-hour schedule, making observations every other hour on the odd hour. The crashes received at the Gainesville station definitely indicated a continuity of static from the direction of the storm.

As the storm approached the Florida coast, the triangulation from Puerto Rico was of little value because the storm was nearly in line with the 2 stations.

As the storm passed the Florida shore line, squally weather was felt throughout the state. This caused a high level of local static to persist, making analysis difficult, but continuity of static was noted, and the direction of the storm followed by the Gainesville station until it dissipated itself near Lake City, Florida, on August 29. The path of the apparent source of static and that of the center of the storm as reported by the United States Weather Bureau are shown in Fig. 6.

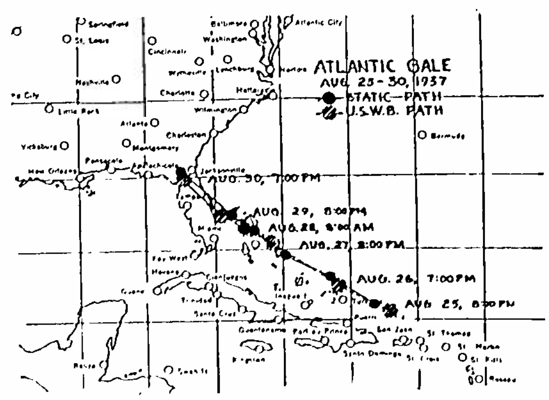

Fig. 6—Path of Atlantic gale, August 25–30, 1937.

A separate set of data was sent to the Puerto Rican station for independent analysis on this storm, and the results obtained there were markedly consistent with results obtained at Gainesville.

The second storm of the season occurred from September 9 to 13. It was brought into observation by the static-recording station a few hours after the United States Weather Bureau had approximated its position. This storm was of full hurricane force near the center, accompanied by heavy rains and heavy seas in the general area. The Gainesville observations as early as 9:07 A.M. on September 9 definitely indicated static emission from the general storm area. The storm was followed and the results plotted as shown in Fig. 7.

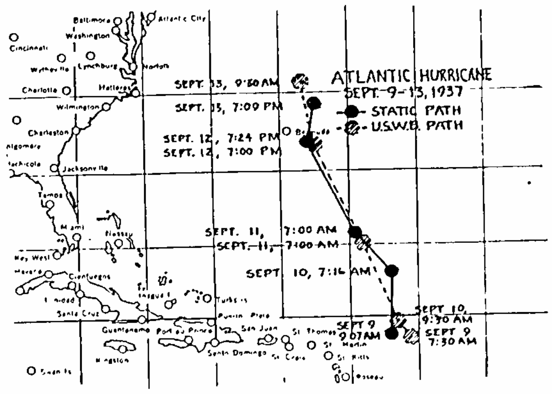

Fig. 7—Path of Atlantic hurricane, September 9–13, 1937.

The third storm persisted from September 14 through the 15th. It was located about 360 miles northeast of St. Martin, Leeward Islands, and was attended by winds of full gale to hurricane force. This disturbance did not cover a wide area and had very little if any movement. It dissipated itself in about 36 hours. The Gainesville and Puerto Rican stations definitely indicated emanation of static from the general storm area. (Fig. 8.)

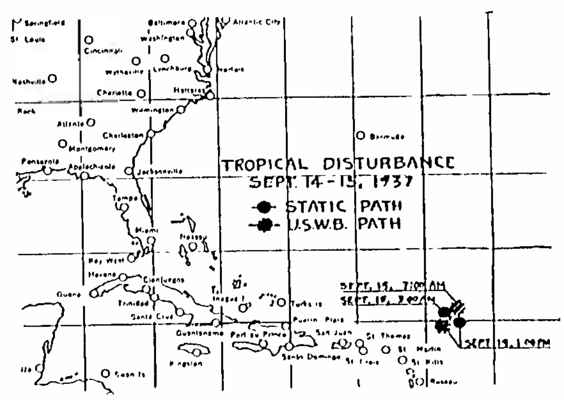

Fig. 8—Position of tropical disturbance, September 14–15, 1937.

The fourth storm was an Atlantic hurricane lasting from September 20 to September 25. This storm traversed a distance of about 2500 miles which made it of considerable interest. Apparently the equipment would record static emanating from a source several thousand miles distant. It was observed that while the addition of another thousand miles from the source of static to the Gainesville and Puerto Rican stations had little effect on the number of static crashes received, the amplitude of the crashes, as

[4] Harald Norinder, "Cathode-ray oscillographic investigations on atmospherics," PROC. I.R.E., vol. 24, pp. 287–304; February, (1936).

would be expected, showed a noticeable decrease. In general, for this as well as the other storms under observation, the static source corresponded favorably with the storm positions as plotted by the United States Weather Bureau. (Fig. 9.)

The fifth storm occurring from September 27 to 28 was an Atlantic gale, located in such a position

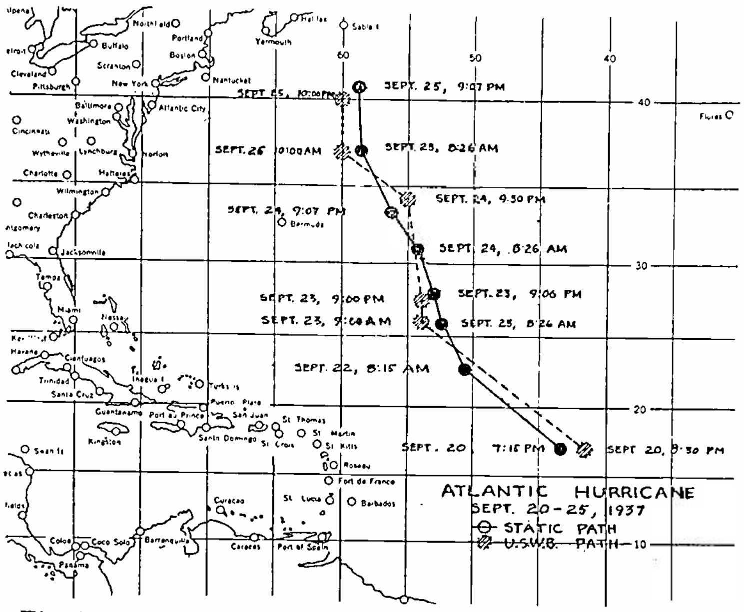

Fig. 9—Path of Atlantic hurricane, September 20–25, 1937.

that good triangulation was obtained from all the stations. About the only significance that could be placed on this disturbance is that static apparently emanates from squalls as well as hurricanes. Fig. 10 shows the course of this storm.

The sixth and last storm of the 1937 season lasted from October 1 to October 3. The results were quite consistent with those obtained by the United States Weather Bureau plottings. (Fig. 11.)

Conclusions

A study of 6 maps showing the apparent position of the static source and the corresponding position

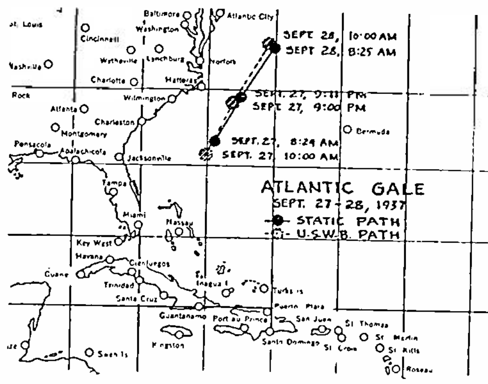

Fig. 10—Path of Atlantic gale, September 27–28, 1937.

of the center of the storm as indicated by the Weather Bureau shows that the two rarely coincide. Several explanations for the discrepancy may be offered. This may be due to one or all of the following:

a. Errors due to location of the equipment.
b. Errors due to improper location of the plates of the oscillograph tube.
c. Errors due to improper location of the film in the camera.
d. Errors in the reported position of the storm.

Another explanation suggested by some investigators and partly verified by Florida observers is that static does not apparently emanate from the center of the storm, but rather from its periphery. If this be the case, it remains for further studies to show just how far from the center of a storm static does emanate. When and if this is determined, there still remains to be found the exact part of the periphery from which the emission occurs. If these factors can be determined, perhaps a fairly accurate approximation of the storm center may be made.

The authors propose to continue the present investigation. Future tests are to include observation of the incoming static and triangulation for the position

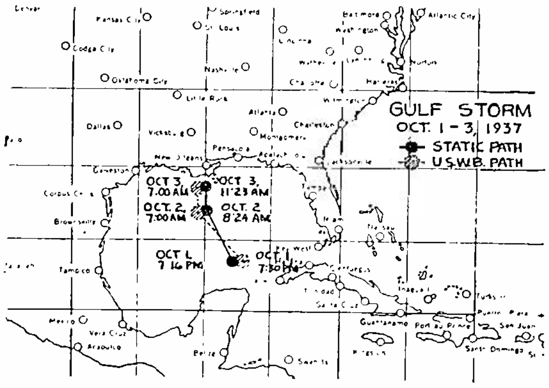

Fig. 11—Path of tropical storm in the Gulf of Mexico, October 1–3, 1937.

of the source without a knowledge as to the exact location of a storm but working only with information that a tropical storm does exist.

Further extension of the work will include a study of the wave shapes of atmospherics along the lines followed by other workers in this field. It is hoped that perhaps hurricane static may have some characteristics of its own to distinguish it from static caused by lightning and thundershowers. If such "fingerprinting" of hurricanes is found to be possible, then a more accurate and rapid location of the storm center by this means may be feasible when no other means of determining the position of the disturbance is available.

Acknowledgment

We take pleasure in expressing our gratitude to the staffs of the various recording stations in Florida and to Dr. G. W. Kenrick and his coworkers at the University of Puerto Rico, Rio Piedras, for their untiring co-operation in securing the necessary data; to the Works Progress Administration for making available the necessary funds for carrying on the work; to the United States Navy Department for making available to us the original Watson Watt equipment; and to all others who have helped with the work through suggestions and helpful criticism.

The Solar Cycle and the F_2 Region of the Ionosphere*

W. M. GOODALL†, MEMBER, I.R.E.

Summary—*This paper presents a method of analyzing F_2-region critical-frequency data in a way that shows in a clear-cut manner the correlation that exists between monthly average values of these critical frequencies for undisturbed days and solar activity as measured by the central-zone character figures for calcium flocculi. Curves are presented which show for each month the expected diurnal variation of $f^0_{F_2}$ for two values of solar activity. Other curves show for a number of different hours the expected seasonal variation of $f^0_{F_2}$ at a constant time of day for the same values of solar activity.*

MEASUREMENTS of the critical frequencies of the ionosphere have been made at Washington, D. C. since 1933.[1] During that time roughly half of an 11-year period of sunspot or solar activity has elapsed. The present paper deals with the correlation between the variations in solar activity and the variations in critical frequencies of the F_2 region during this period. In a previous paper[2] it was shown by the writer that a correlation exists between critical frequencies and smoothed sunspot numbers when the data are analyzed on a monthly basis. It was realized that a better measure of solar activity is needed. As a result of a number of attempts to find a better index than smoothed sunspot numbers, it has been found that the character figure for central-zone calcium flocculi gives a good correlation with the radio data. The radio data used in this study consist of monthly average values for undisturbed days of the O-component critical frequency for the F_2 region as obtained at Washington, D. C., by the National Bureau of Standards.[3] The central-zone calcium character figures used in this paper were obtained from the bulletins of the International Astronomical Union, which in addition to the indexes for calcium flocculi give a number of other indexes. The choice of the calcium character figure instead of one of the others was made in a preliminary study and has since been justified to a certain extent by comparisons between the character figure predicted from radio data and the other character figures.

In this paper the O-component critical frequency of the F_2 region is denoted by $f^0_{F_2}$, while the central-zone character figure for calcium flocculi is denoted by K_s and the corresponding radio character figure by K_r.

The method of analyzing data is the same as that used in the paper mentioned above. Each month is treated separately. For a given time of day, for example 1300 E.S.T., the monthly average value of the critical frequency of the F_2 region is obtained and plotted against the calcium character figure for that month. The result is the series of charts shown in Fig. 1 where each point represents the data from one month of one year. The slopes of the straight lines shown in these charts depend primarily upon the points plotted for each particular month. In view of limitations in data adjacent months have also been allowed some weight.

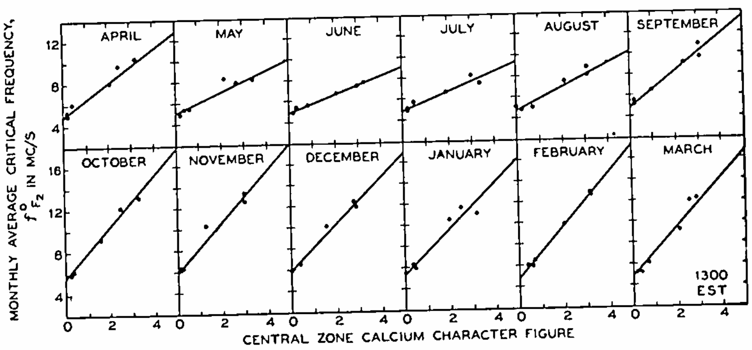

Fig. 1—Monthly correlation charts for 1300 E.S.T. K vs $f^0_{F_2}$ at Washington, D. C.

Having obtained the curves of Fig. 1, they may now be used in two ways: (1) to find the expected values of critical frequencies based on solar character-figure data or (2) to find the expected values of character figures based on radio measurements. The smooth curves shown in Fig. 2 are the expected

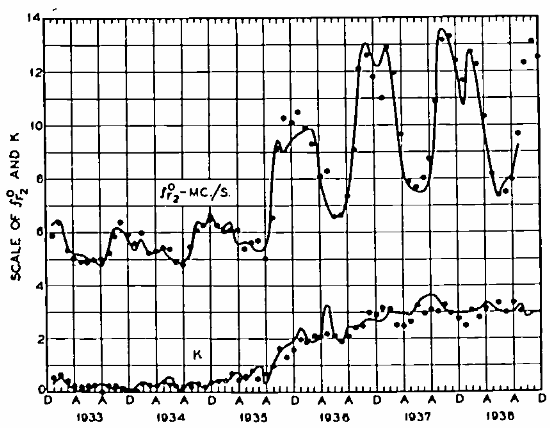

Fig. 2—Expected values of $f^0_{F_2}$ (for 1300 E.S.T.) and K are given by curves, data by points.

values while the data are plotted as points. There appears to be a tendency for the radio character figure to reach a maximum in 1937. In general, it is seen that the agreement is reasonably good.

It is possible to treat the data for other hours in a similar manner to that indicated above. This has

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[1] N. Smith, T. R. Gilliland, and S. S. Kirby, "Trends of characteristics of the ionosphere for half a sunspot cycle." *Jour. Res. Nat. Bur. Stand.*, vol. 21, pp. 835–846; December, (1938).

[2] W. M. Goodall, "On the ionization of the F_2 region," Proc. I.R.E., vol. 25, pp. 1414–1418; November, (1937).

[3] See footnote reference (1) for summary of data and various issues of the Proceedings for detailed values.

been done for the following hours: 0200, 0400, 0600, 0800, 1000, 1300, 1600, 1800, 2000, and 2300. These results are not given in the form of curves but a set of equations defining the lines obtained is given in Table I.

TABLE I
VALUES OF m AND b FOR $f^0 = mK + b$

Month		02	04	06	08	10	13	16	18	20	23 hour
Jan.	m	0.67	0.33	0.33	1.07	2.03	2.16	2.2	2.03	1.42	0.93
	b	2.4	3.0	2.8	4.4	5.0	5.3	4.8	3.4	2.4	2.0
Feb.	m	0.87	0.80	1.03	1.63	2.2	2.56	2.56	2.43	1.87	1.30
	b	2.3	2.4	2.2	3.8	4.7	5.0	4.5	3.5	2.7	2.1
Mar.	m	1.23	1.16	0.93	2.06	2.33	2.40	2.30	2.23	1.77	1.46
	b	2.3	2.1	2.6	3.5	4.3	4.8	4.7	4.1	3.6	2.6
Apr.	m	1.27	1.10	1.03	1.33	1.40	1.57	1.40	1.23	1.33	1.37
	b	2.3	2.1	3.1	4.3	4.8	4.9	5.6	5.7	4.7	2.9
May	m	1.17	0.97	0.87	0.80	0.87	1.00	0.90	0.87	0.93	1.27
	b	2.4	2.2	3.7	4.8	5.0	5.0	5.5	5.6	5.2	3.1
June	m	0.97	0.93	0.70	0.67	0.77	0.83	0.90	0.87	0.87	1.17
	b	2.6	2.1	3.7	4.8	4.9	4.9	5.1	5.3	5.2	3.3
July	m	1.00	0.90	0.70	0.93	0.87	0.90	0.87	0.87	0.80	1.17
	b	2.5	2.1	3.6	4.4	4.8	4.8	5.0	5.3	5.4	3.3
Aug.	m	1.27	1.00	0.80	1.10	1.10	1.13	1.10	1.07	0.83	1.27
	b	2.2	2.1	3.4	4.2	4.8	4.7	5.0	5.2	5.6	2.8
Sept.	m	1.30	1.10	0.90	1.40	1.53	1.77	1.67	1.50	1.07	1.27
	b	2.1	2.1	3.2	4.6	5.0	4.8	5.1	5.1	4.8	2.6
Oct.	m	0.77	0.87	0.83	1.80	2.26	2.43	2.26	1.97	1.77	1.37
	b	2.4	2.4	2.8	4.9	5.5	5.7	5.6	4.5	2.4	2.4
Nov.	m	0.77	0.53	0.47	1.40	2.2	2.4	2.26	2.06	1.63	1.13
	b	2.7	3.0	2.7	5.2	5.7	5.9	5.6	3.7	2.2	2.2
Dec.	m	0.60	0.40	0.30	1.17	2.06	2.2	2.2	1.93	1.47	0.83
	b	2.6	3.0	2.8	4.8	5.6	5.9	4.9	3.2	2.1	2.1

Using the results of these correlation charts it is possible to plot curves of expected critical frequencies for different conditions of solar activity as measured by the calcium character figures. One method of doing this is shown by the series of curves in Fig. 3 which give the diurnal characteristics for June and December for the different conditions of solar activity indicated. The general increase in critical-frequency values with solar activity is shown by these curves. It is seen that the largest increases occurred during the winter daytime and during the summer nighttime. The activity during the maximum of the present cycle has corresponded to the character figure of about[4] 3.0.

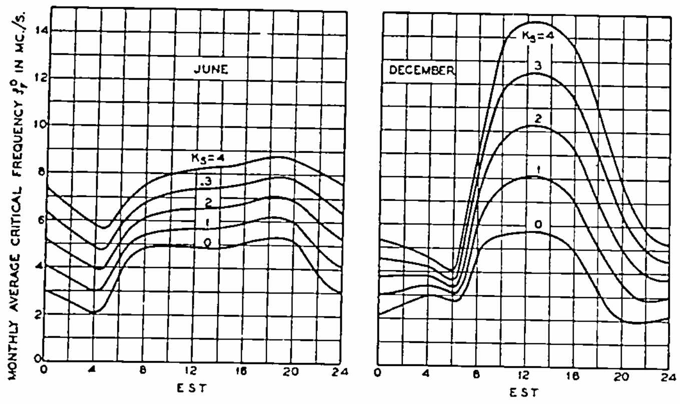

Fig. 3—Expected diurnal curves for June and December based on curves similar to those of Fig. 1 and values of K_s indicated.

[4] This corresponds to a smoothed average sunspot number of about 110.

It would be possible to construct a series of curves such as shown in Fig. 3 for all of the twelve months. In order to keep the figures presented in this paper down to a reasonable number a set of two curves for each month is plotted in Fig. 4. The dashed curves correspond to the character figure of 0 or minimum solar activity, while the solid curves correspond to the character figure of 3. When plotted in this manner it appears that, in general, the months from April to September, inclusive, are similar in diurnal characteristics to the month of June shown in Fig. 3 while the remaining months from October to March, inclusive, are similar to the month of December.

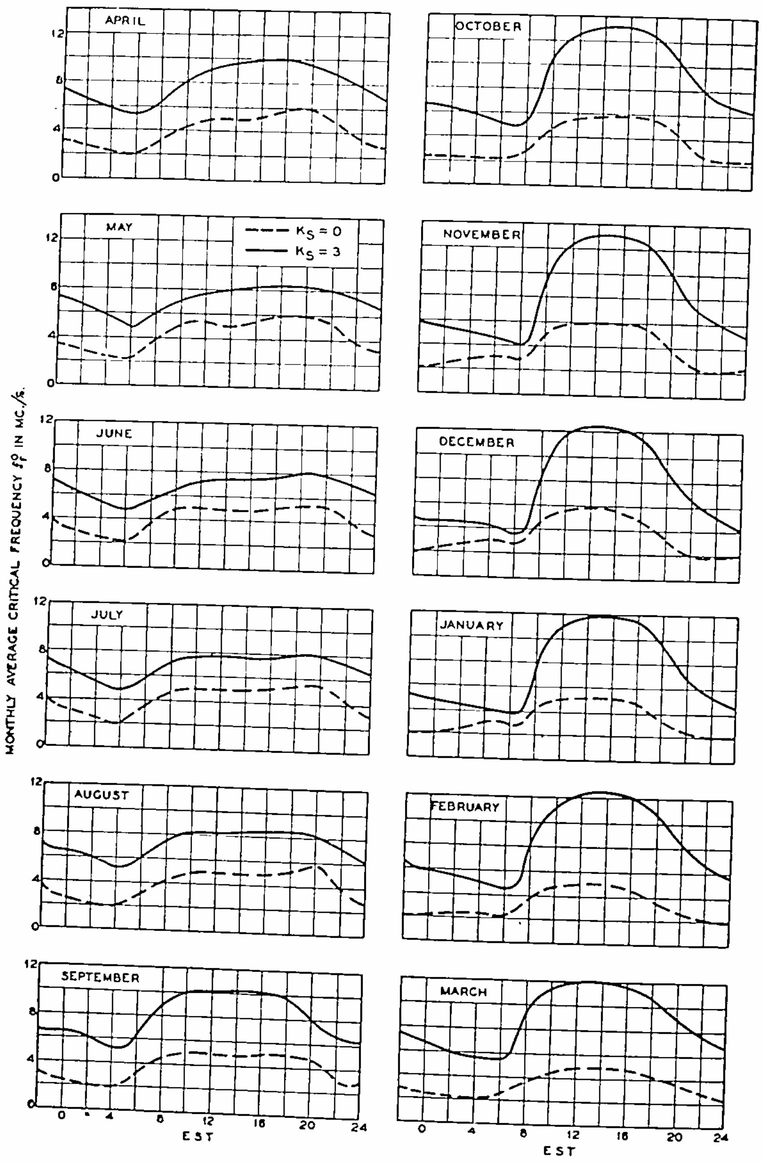

Fig. 4—Expected diurnal curves for each month. $K_s=0$ for dashed curves and $K_s=3$ for solid curves.

Another method of plotting the results obtained from the charts of the type shown in Fig. 1 is to plot the critical-frequency values expected for the different months of the year at a constant time of day. Two such sets of curves are shown in Fig. 5, one for 0400 and another for 1600. These hours were chosen because they represent on the average the time of minimum and maximum of the critical frequencies as obtained from the diurnal characteristics (see Fig. 4). In studying these curves we note that during a minimum of solar activity there is less variation in critical frequency with the months of the year than at other times. As the sunspot activity increases, however, it is seen that a pronounced variation occurs with a tendency for maxima in February or March and in October or November during the daytime and during March or April and August or September during the nighttime. It should be realized that these curves are based on data covering only half of a solar cycle and consequently the time of occurrence of the maxima is somewhat in doubt. It is interesting to note that the variations in magnetic activity also have two maxima at roughly corresponding times.

In Fig. 6 curves are plotted for constant time of day which are analogous to those shown in Fig. 4. As before the dashed curves correspond to a character figure of 0 and the solid curves to a character figure of 3. Here again there is a tendency for the curves to fall into two groups, one corresponding to daylight hours and one corresponding to nighttime conditions. The curves for the daylight hours are similar and indicate maxima during February and March and during October and November. It is also seen that the increase in critical-frequency values was much more pronounced during the months from October to March, inclusive than during the remainder of the year. This fact has been noted before but it is shown even more strikingly in these curves. Considering the curves for nighttime conditions it is also seen that there is a tendency for the increase to be greater in summer than in winter.

The data discussed above may be interpreted in terms of long-distance transmissions by converting the critical frequency into maximum usable frequency in accordance with the principles described,

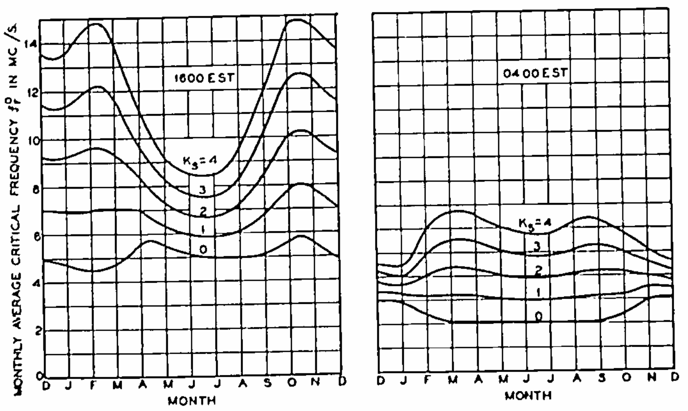

Fig. 5—Expected constant-time curves for 0400 and 1600 E.S.T. based on curves similar to those of Fig. 1 and values of K_s indicated.

for example, by N. Smith.[5] A rough rule is that the maximum usable frequency is about 3.0 times the critical frequency. The transatlantic transmission data presented by C. R. Burrows[6] have been considered in this manner and it was found that the critical frequencies corresponding to the transmission data agree in general with the expected values obtained from solar data.

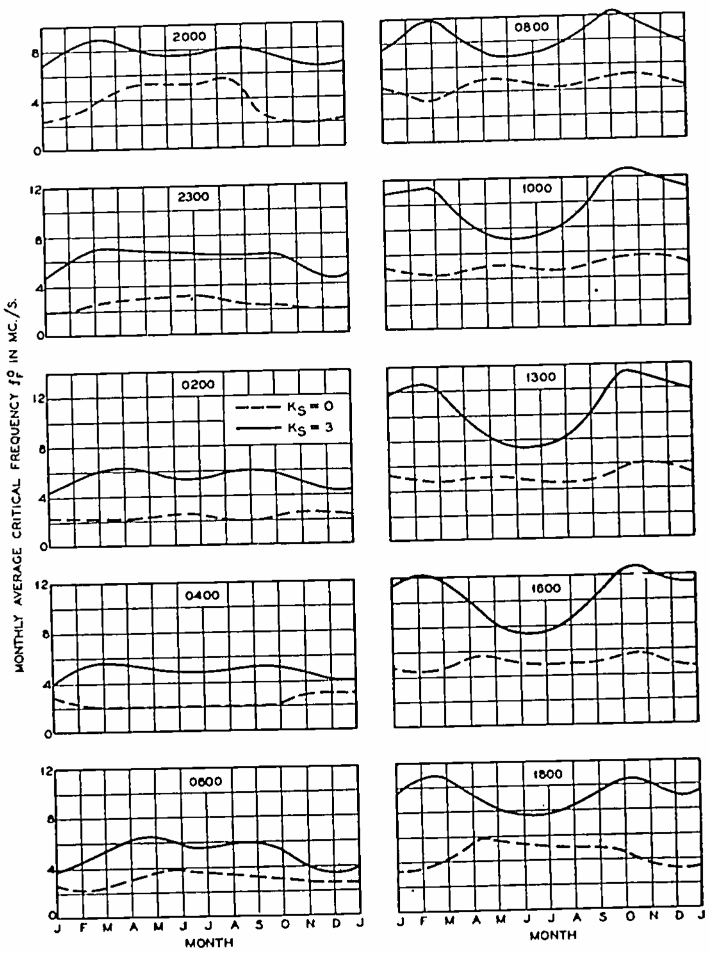

Fig. 6—Expected constant-time curves for hours indicated. $K_s=0$ for dashed curves and $K_s=3$ for solid curves.

Discussion

In addition to presenting the correlation between the solar and radio data it has been the object of this paper to present a useful method of analyzing the critical-frequency data for the F_2 region of the ionosphere and to present the expected values of critical frequencies for Washington, D. C., obtained by using this method of analysis.

It has been found that the method of analysis presented can be applied to data obtained at other locations than Washington and that the results obtained are useful in studies designed to show the characteristics of the F_2 region at other locations. So far sufficient data are not available to give a clear-cut picture of world-wide conditions.

An effort has been made *not* to introduce into this study theoretical ideas as to how the F_2 region should behave, since it is felt that such ideas can best be considered on the basis of a well-founded background of experimental data. It is hoped that the results presented in this paper will serve to summarize the experimental data in such a way as to facilitate further progress in the study of the F_2 region of the ionosphere.

[5] N. Smith, "Application of vertical-incidence ionosphere measurements to oblique-incidence radio transmission," *Jour. Res. Nat. Bur. Stand.*, vol. 20, pp. 683–693; May, (1938).

[6] C. R. Burrows, "The propagation of short waves over the north Atlantic," Proc. I.R.E., vol. 19, pp. 1634–1659; September, (1931).

Basic Economic Trends in the Radio Industry*

JULIUS WEINBERGER†, FELLOW, I.R.E.

Summary—*This paper considers the American radio industry from the standpoint of its long-time economic trends, with special reference to the past and probable future public demand for broadcast receivers and vacuum tubes. The history of the annual sales to each of the various markets for these is analyzed, equations are given for their long-time growth trends, and estimates of growth are projected ahead to 1947. Conclusions are stated regarding the probabilities of changes in the nature of the demand for initial and replacement receivers in the form of primary, secondary, automobile, and export receivers; and for vacuum tubes for initial equipment or replacement purposes.*

THE analysis of industries from the standpoint of their long-time trends has received the attention of a number of workers in the field of economic research in recent years.[1–5] These investigators have developed a statistical technique which is a powerful tool, and conclusions concerning the future prospects for an industry, resulting from applications of this technique, rest upon a basis of common sense as well as on mathematical calculations.

The radio industry has reached a stage where it becomes possible, and important, to study its long-time trends statistically. It is a commonplace saying of the industry that the demand for its products is now principally of the replacement type. When any industry reaches this point, particularly if it is one that manufactures durable luxury (or dispensable) goods, it becomes urgent for managements of the enterprises engaged in it to have some basis for estimating its future possibilities.

The Radio Market

Before any useful statistical analysis of the radio industry can be made, the nature of the markets which it supplies must be considered. Each market then must be treated separately, and separate conclusions drawn concerning its future prospects. The totals arrived at from these separate considerations may then be added, to reach a conclusion concerning the probable future for the total production of the industry.

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† License Laboratory, Radio Corporation of America, New York, N. Y.

[1] Carl Snyder, "Business Cycles and Business Measurements," The Macmillan Co., New York, N. Y., (1927).

[2] Simon S. Kuznets, "Secular Movements in Production and Prices," Houghton Mifflin and Co., Boston, Mass., (1930).

[3] A. F. Burns, "Production Trends in the United States Since 1870." National Bureau of Economic Research, New York, N. Y., (1934).

[4] Elmer C. Bratt, "Relations of Institutional Factors to Economic Equilibrium and Long-Time Trend." Lehigh University, Institute of Research, Bethlehem, Circular No. 126, (1937). (Contains an extensive bibliography.)

[5] Elmer C. Bratt, "Forecasting the growth of the steel industry," *The Iron Age*, p. 45, March 19, 1936. Additional articles by the same author on this subject in *The Iron Age*, p. 18, November 14, (1935), and *The Annalist*, p. 885, December 25, 1936. Also see "The timing of capital production and the need for forecasting," *Harvard Business Review*, Summer Number, 1937, p. 429.

The principal products of the radio industry are sound broadcast receivers and vacuum tubes for use therein. A small percentage of the total production consists of numerous special products, such as commercial and broadcast transmitters, commercial receivers, sound-amplifying equipment, and the like, and special types of vacuum tubes for use therein. The dollar value represented by this equipment is so small a proportion of the total (the 1935 Census showed 12 million dollars for the value of these products, as compared to 174 millions for broadcast receiving equipment and 207 millions for all radio apparatus) that it has been neglected in the present investigation. This has been confined to the market for broadcast receivers and tubes. Television receiving equipment is regarded as part of the replacement market of the somewhat distant future.

The distribution of the present products may be considered as taking place in response to the following types of demand:

A. Broadcast Receivers

(1) Receivers for initial equipment: These are sets purchased each year by persons or families who have not previously owned a radio set.

(2) Receivers for replacement purposes: These are sets purchased by persons who have previously owned a receiver, which has become useless, or has been traded in for a more recent model. The resale of trade-ins is believed to be of negligible proportions, and for the purposes of this analysis it has been disregarded.

(3) Secondary receivers: These are additional sets purchased by families who already own one, and who desire one or more additional sets for bedroom, kitchen, or portable use. The secondary receiver field became a prominent factor in 1933, when the low-priced "compact" types of sets where introduced.

(4) Receivers for other than family use: These are sets bought by retail stores and service establishments, restaurants, filling stations, churches, schools, clubs, and the like, for use in entertainment or instruction. This field has also assumed a more prominent position since the introduction of "compact" sets.

(5) Automobile receivers: These are sets permanently installed in automobiles, either at the factory or locally by a dealer. This field started in 1930 and has assumed substantial proportions in recent years.

(6) Exports: The radio export trade, while accounting for about 10 per cent of the total, has been until recently a growing field, subject to decidedly less fluctuation than the domestic trade.

B. Vacuum Tubes

(1) Tubes for initial equipment: These are vacuum tubes sold directly by the tube factories to producers of radio broadcast receivers, for resale with such receivers to the public. Roughly half of each year's tube production is sold for this purpose.

(2) Tubes for replacement purposes: These are tubes sold through dealers or service men, to replace tubes which have become worn out in service in receivers used by the public.

(3) Exports.

Annual Distribution of Broadcast Receivers

The most reliable series of data of the radio industry is that of total annual production. The breakdown of these figures, in order to determine the allocation of a suitable share of each year's totals to each of the various markets, can generally be accomplished only by means of estimates.

Table I gives the data for total unit production accepted by this writer as the most reliable, together with estimates of its probable distribution. Footnotes to the table indicate the sources of the data and methods of making estimates.

Total receiver production has risen to its present large volume in a series of waves or periods, each period being characterized by the attainment of new peaks in production and by inciting causes which brought new users of radio into the market, or induced replacements by those who had purchased receivers in earlier years. These periods may be described as follows:

1922–1926: This period constituted the first spurt in sales of initial equipment, when home-made receivers were being gradually displaced by factory-made products. Demand rose until 1925, then dropped off in 1926 because nothing having any particularly new sales appeal was being introduced.

1927–1932: In 1927 the first reasonably priced receivers operating directly from the electric power line (110 volts, alternating current) were introduced. Prior to that year, practically all receivers had been operated from batteries, which were costly, cumbersome, and nuisances to maintain. Introduction of the alternating-current-operated sets brought an immediate response from persons who formerly had not been interested in radio, and sales for initial equipment as well as for replacements increased from 1927 through 1929. Other technical and styling improvements, which were introduced in that period, helped in increasing sales, as well as the fact that general business conditions were prosperous. After 1929, a lack of new appeals in radio receivers, coupled with the onset of the depression, caused sales to decline.

1932–1936: During the years from 1930 to 1932, a number of events were taking place in the radio industry which set the stage for a revival of sales in 1933. The incorporation of the superheterodyne circuit in their products by the majority of manufacturers, occurred in these years; this was a technical improvement of considerable importance because of the congested condition of the radio-broadcast channels, which necessitated receivers of high selectivity. Again, prices were cut rapidly by improved production methods and design, so that the equivalent of a receiver costing $150 prior to 1929 could be bought in 1932 for half that price. New types of low-priced receivers between $20 and $50 retail were introduced, such as table and "compact" models, and automobile receivers received their initiation into the radio picture. The use of secondary receivers began, owing to the availability of sets retailing at prices from $6 to $25. All of these factors induced millions of new users, who had previously not been able to afford radio sets, to purchase their initial equipment, fostered replacements, and opened up new markets for additional sets.

Significant changes have taken place in the allocation of total production to the various markets, during these years. At first, sales for initial equipment for the home took practically all of the product. In 1930, however, initial equipment constituted only 59.4 per cent and replacements had risen to 33.5 per cent of the total. Sales to other markets still accounted for only a negligible proportion of the total. By 1936, initial equipment "home" sales had dropped to 19.8 per cent, replacement "home" sales were 30.3 per cent, and the new markets were absorbing receivers as follows: Secondary receivers, 15.7 per cent; sales for other than family use, 7.8 per cent; automobile receivers, 17.2 per cent; exports, 8.1 per cent.

It is clear from the foregoing that in order to learn more of what is happening, and what is likely to happen, to total radio production, we must study the separate growth characteristics of each of these markets. This is best done by an examination of the growth of ownership.

Long-Time Trends of the Broadcast-Receiver Market

From the data of Table I, and from other surveys, it is possible to estimate the ownership each year of radio receivers employed for various purposes. This has been done in Table II, and the data of this table are plotted in Figs. 1 and 2.

Trends have been fitted to these curves. After trial of several different types of equations[6] the writer decided that the Gompertz type of curve was best adapted to fit the data and also was best for utilization in forecasting the future trend.

[6] On the subject of fitting trends, see particularly the references cited in footnotes 1–5 and such standard statistical works as Brown and Brigham, "Laboratory Handbook of Statistical Methods," McGraw-Hill Book Co., New York, N. Y. (1931).

TABLE I

ANNUAL PRODUCTION AND DISTRIBUTION OF BROADCAST RECEIVERS (UNITS)

Year	Total Receivers Produced[1]	Domestic Sales to Families[2]				Sales for Other Than Family Use[6]	Automobile Receivers[7] (Initial Equipment)	Exports[8]
		Primary receivers		Secondary receivers[5]				
		Initial Equipment[3]	Replacement[4]	Initial Equipment	Replacement			
1922	100,000	100,000						
1923	190,374	190,374						
1924	1,400,000	1,100,000	300,000					
1925	2,345,790	1,945,790	400,000					
1926	1,750,000	1,250,000	500,000					69,482
1927	1,978,057	1,228,057	750,000					75,839
1928	3,187,343	2,277,343	850,000					96,341
1929	4,696,686	3,596,600	900,150					200,936
1930	3,837,979	2,275,000	1,285,887				31,900	245,192
1931	3,593,993	1,573,824	1,602,761	150,000			96,145	471,263
1932	2,443,771	1,015,049	595,049	420,000			123,000	290,673
1933	4,168,374	1,095,143	964,703	900,000			698,742	509,786
1934	4,478,967	996,442	1,217,441	800,000		100,000	753,000	612,084
1935	6,026,031	1,363,873	1,748,000	438,000		350,000	1,170,423	606,784
1936	8,248,755	1,631,000	2,499,000	1,292,000	100,000	644,000	1,412,000	635,984
1937	8,064,780	1,700,000	2,300,000	1,500,000	110,000	400,000	1,353,000	622,416
1938	7,107,400	1,400,000	2,600,000	1,000,000	550,000	300,000	800,000	450,000

[1] Source: RCA Licenses' reports, 1928 to date. Previous years from *Census of Manufactures*, (1923, 1925, 1927) and *Radio Retailing* (1922, 1924, 1926).

[2] Sales are assumed to be equal to production, since no year-end inventory data exist. "Families" are defined in accordance with census practice, as a group of persons living together, whether related or not. "One-person families" are included.

[3] Initial equipment sales of primary receivers estimated from data given in annual surveys of radio dealers conducted by *Radio Retailing*; from these surveys a percentage of total sales applicable to purchases of initial equipment was derived, which was applied each year to the "Total Receivers Produced" of column 1. Some adjustments were made so as to cause the figures thus obtained to check with the growth of ownership (Table II) subsequent to 1934.

[4] Replacement sales of primary receivers estimated as follows: For 1924–1928, it was assumed that each year 10 per cent of the families owning receivers at the beginning of the year made replacements. This percentage was based on experience of the period 1929–1936, in which replacements (estimated by other methods) ranged from 7 to 10 per cent of families owning radios at the beginning of each year. For 1929–1937, replacements were taken as the remainder after all other sales had been deducted from "Total Receivers Produced." Checks on the foregoing were obtained by the following methods: (a) From unpublished data (RCA) of sales of receivers by price classes for each year, a reasonable distribution of each price class to the various markets was estimated. (b) The Columbia Broadcasting System furnished the author with unpublished data based on quarterly surveys of radio ownership among 32,000 families, nationally distributed, in which the year of purchase of the principal receiver of each family was given. From these data, taken together with the known totals of families equipped and sales of initial equipment, some estimates of replacements were possible. (c) Plotting growth curves of each market, the points obtained by the preceding methods lie within reasonable distance of a smooth curve.

[5] Secondary receivers estimated from ownership surveys conducted by the Columbia Broadcasting System in which the percentage of all families possessing more than one receiver, each year, was determined. See also footnotes to Table II.

[6] Annual sales for other than family use estimated from an assumed ownership-growth curve having an asymptote at 2.6 millions (the total number of ultimate installations to be expected in this category). Checked at one point by data from a survey of radio ownership among retail dealers conducted by the Columbia Broadcasting System.

[7] Source: *Census of Manufactures*, 1931, 1933, 1935. Other years, *Radio Retailing's* estimates adjusted to conform to totals of column 1.

[8] Source: Bureau of Foreign and Domestic Commerce, Department of Commerce.

The equation determined for the trend line of "Families with One or More Receivers" is as follows:

$$\log y = 1.4793 - (0.345)^x (1.002)$$

where

y = number of families in millions having receivers on January 1 of the year x.

x = number of years after January 1, 1924, in 6-year units. That is, for 1924, $x=0$; for 1927, $x=0.5$; for 1930, $x=1$; for 1933, $x=1.5$; for 1936, $x=2$, for 1942, $x=3$.

This curve becomes asymptotic when $\log y=1.4793$, corresponding to an upper limit of 30,200,000 familes. The total number of families in the United States, deduced from Thompson and Whelpton's estimates of future population,[7] and estimated by the writer[8] is as follows:

Year	Estimated Families (Including One-Person Families)
1930	29,904,000
1940	34,300,000
1950	38,850,000
1960	39,850,000
1970	41,400,000
1980	41,600,000

[7] Warren S. Thompson and P. K. Whelpton, "Population Trends in the United States," McGraw-Hill Book Co., New York, N. Y., (1933).

[8] In accordance with a method suggested by Mr. Spiegelmann, population expert of the Statistical Bureau of the Metropolitan Life Insurance Company.

The upper limit of 30,200,000 families (which causes the curve to fit the past data and includes 88 per cent of the estimated 1940 families) seems a reasonable one for expectations of the next 10 to 30 years, since it may be assumed that there will always be a certain number of families so situated or so poor that they will not be able to afford a radio set. The cheapest

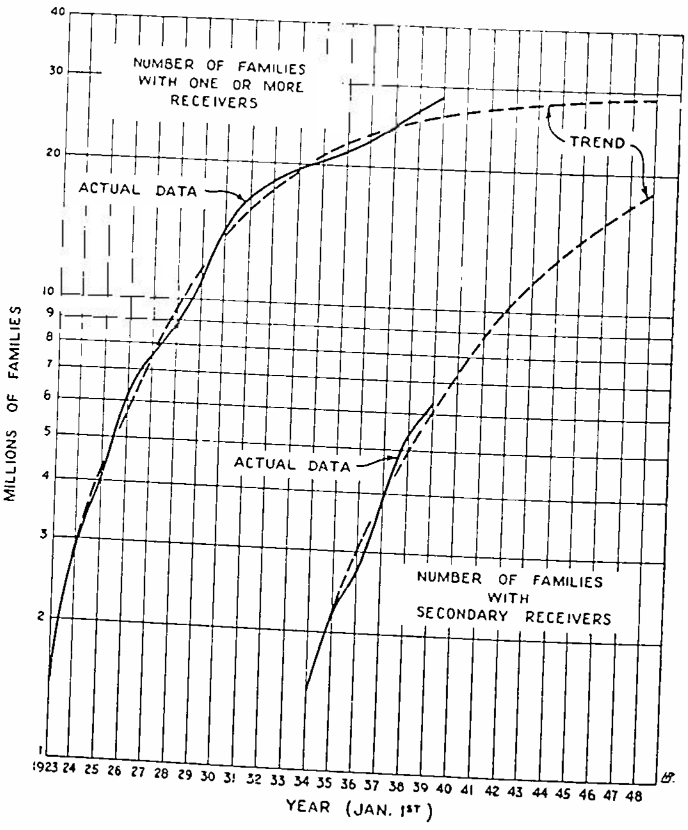

Fig. 1—Growth of installations of sound broadcast receivers in the home.

TABLE II
GROWTH OF OWNERSHIP OF BROADCAST RECEIVERS IN THE UNITED STATES
(As of January 1 of each year)

Year	Families with One or More Receivers	Families with Secondary Receivers	Installations Other Than in Families[1]	Automobiles Possessing Receivers	Total Number of Receivers Installed
1922	60,000				60,000
1923	1,500,000				1,500,000
1924	3,000,000				3,000,000
1925	4,100,000				4,100,000
1926	6,295,790				6,295,790
1927	7,545,790				7,545,790
1928	8,774,647				8,774,647
1929	10,901,000				10,901,000
1930	14,498,590				14,498,590
1931	16,773,590			31,900	16,805,490
1932	18,347,414	150,000		128,045	18,625,459
1933	19,362,463	570,000		251,045	20,183,508
1934	20,457,606	1,470,000		949,787	22,877,393
1935	21,456,000	2,270,000	100,000	1,702,787	25,528,787
1936	22,869,000	2,708,000	450,000	2,873,210	28,900,210
1937	24,500,000	4,000,000	1,194,000	4,285,210	33,979,210
1938	26,200,000	5,500,000	1,594,000	5,638,210	38,232,210
1939	28,266,500	6,500,000	1,894,000	6,438,210	43,098,710

Sources:
Families with One or More Receivers: For 1922–1924, estimates of *Radio Retailing*. For 1925–1933, obtained by adding data of "Primary Receivers Sold as Initial Equipment" (from Table I) for each year, to families possessing receivers in preceding year. For 1934, from joint survey of Columbia Broadcasting System and National Broadcasting Company. For 1935 and 1937, from surveys of Columbia Broadcasting System (1935 survey based on 80,000 interviews). For 1936, from Joint Committee on Radio Research. For 1939, from Joint Committee estimate as of January 1, 1938, plus estimate from N.B.C.-C.B.S. surveys in 1938.
Families with Secondary Receivers: For 1932–1934, estimated from probable distribution of known annual production of receivers retailing below $30 (which are the types principally used as secondary receivers). For 1935–1937, from surveys of Columbia Broadcasting System (88,000 interviews for 1935 and 32,000 each year for 1936 and 1937).
Other columns, by cumulative addition of annual production data from Table I.
[1] Groups included in this classification are as follows:
(a) Quasi-family groups: those living in hotels, schools, labor camps, military and naval posts, and in similar places, but excluding institutions. Total population in such groups was 1,428,827 in 1930 (U. S. Census).
(b) Commercial Users: retail sales and service establishments, hotels, amusement enterprises, factories, physicians, dentists, and similar enterprises. There are approximately 2,000,000 such enterprises in the United States.
(c) Community Enterprises: schools, churches and their associated interests, hospitals, clubs. Approximately 600,000 is the total number of such institutions.
(d) Miscellaneous: travelers' sets, summer-camp, and motor-boat equipment.

sets today require electric power for operation, and there are still many millions of homes unwired for electricity, while sets available to unwired homes, and using battery power, are sufficiently expensive in first cost and upkeep so as perhaps to be beyond the purchasing power of the poorest class of farm and city families.

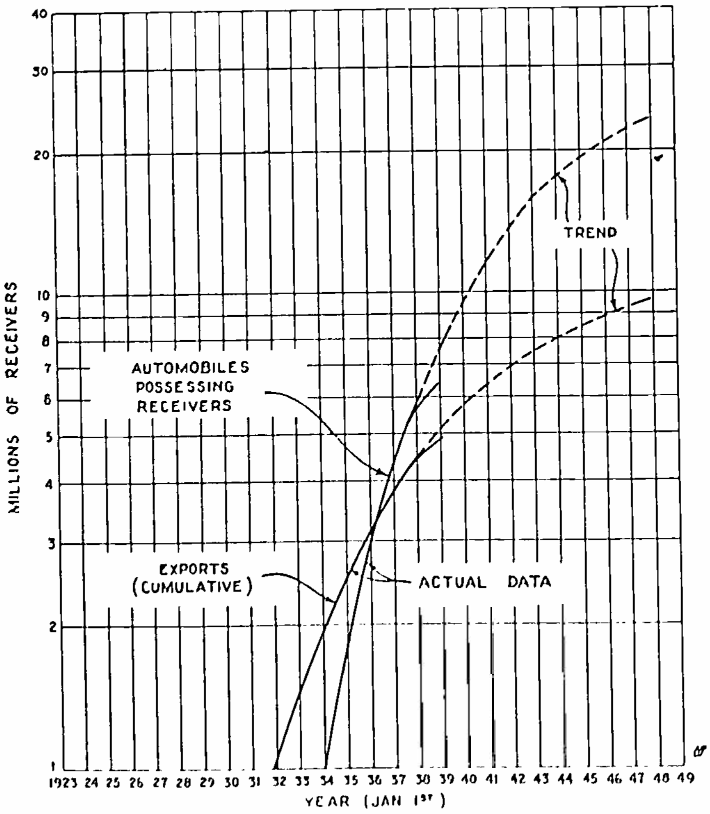

Fig. 2—Growth of installations of automobile and exported sound broadcast receivers.

The subject of *replacement demand* for primary home receivers warrants extended discussion. If a radio set had a definite life, like an automobile or a vacuum tube, it would be relatively simple to estimate the probable future annual replacements from this standpoint alone. Studies of the automobile industry have shown that the average buyer replaces his car roughly once every 3 years, and that the average life of a car from the time of production to final complete scrapping is now of the order of 8.25 years.[9] In a radio receiver, however, there are no moving parts to wear out, and while a few parts may deteriorate with usage, they can be replaced readily. Thus, it would seem that only through the appeal of the new service capabilities, better style, or better performance of new models of radio receivers, can owners of old sets be induced to replace them. Replacements are therefore optional, rather than compulsory purchases.

In view of this expectation it is rather astonishing to discover that replacements of radio sets seem to have been carried out by the general public in much the same way as replacements of automobiles, except that the time interval has been longer: this is indicated by the curves of Fig. 3.

In this chart two curves are shown. The left-hand curve is obtained by cumulating, or adding together, the data of primary receivers installed each year. This is the total of all receivers produced for initial equipment and replacement, as primary receivers, from the third and fourth columns of Table I. The right-hand curve is a cumulative curve of the receivers installed for replacements only (from the fourth column of Table I), and is equivalent to the receivers replaced or scrapped. Thus, the left-hand curve indicates the number of receivers that went into service, and the right-hand curve the number of

[9] "Automobile facts and figures," 1935 edition, page 15. Automobile Manufacturers' Association, New York, N. Y.

receivers that went out of service. The spacing between the curves indicates the number of years that receivers are retained, on the average.

The two curves start out with a spacing of about $4\frac{1}{2}$ years, in which period 100 per cent of the receivers were replaced. As the curves progress, the time interval between them gets longer. We can then regard the replacement time in two ways: (a) the time interval between the two curves along a horizontal line, or the time taken for 100 per cent of the public to make replacements; or (b) the time interval between the two curves where the sections of the actual data approximately parallel each other, or where

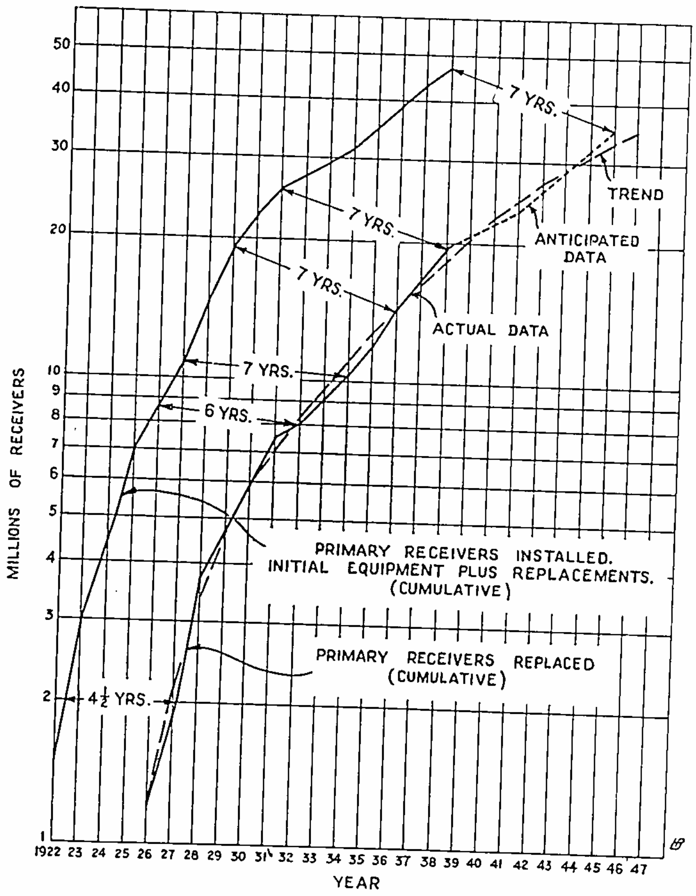

Fig. 3—Primary receivers installed and replaced.

similar peaks and valleys appear. In case we use the latter method we can speak of "partial replacement" after a given time interval, and in this case we find that, in each year since 1935, 75 per cent of the public replaced receivers bought 7 years previously. If we wished to consider 100 per cent of the public, the time interval would be of the order of 9 years or more, increasing in the future.

As a first approximation towards forecasting future replacement demand, we may determine what point on the right-hand curve corresponds to 75 per cent of one on the left-hand curve, with a time interval of 7 years. The data thus obtained has been projected on Fig. 3, and is designated "Anticipated Data."

It is obvious that replacement receivers sold in any given year are not used to replace only receivers which were precisely seven years old. Some families must have replaced receivers ten years old, others five years old, and others perhaps only two years old. The number of units sold for replacements in any given year is the total of replacements of receivers of all ages, with ordinarily a chance distribution of the percentages represented by each age. However, if there is any *predominant* replacement habit among purchasers, it will show up in a parallel relationship of two curves such as those given in Fig. 3. From this parallelism we are led to the conclusion that there is evidence of a predominating habit of replacing radio receivers about once every 7 years, at least up to the present.[10]

It is natural to inquire what will be the effect of the introduction of television upon the curve of replacement demand. We may arrive at some conclusions concerning this by considering the nature of the families which replaced receivers in the past and which currently constitute the replacement market.

The families which replaced their receivers when the first replacement demand occurred were those which were the early purchasers of radio, namely, the most prosperous and progressive groups. Those which replaced receivers in 1929, for example, were people who had bought their initial radio receivers between 1921 and say, 1926. In later years, replacement demand came from families which had bought their initial receivers about 7 years previously, on the average. It required marked price reductions and great improvements in merchandise, to bring about these replacements. The families constituting the future replacement market now fall into two groups:

(a) Those which purchased their initial equipments subsequent to 1930, composed of progressively lower income classes.

(b) Those which have already made at least one replacement purchase, composed of the middle and upper income classes.

In the author's opinion, the first group will serve to continue the "Cumulative Replacement" curve of Fig. 3 by purchasing new *sound* receivers as their income conditions permit, and as they become responsive to price, service, and style appeals. This group, however, cannot be expected to respond to the appeal of television receivers for a long time, because, having waited 10 years or more to be able to afford or to become interested in the purchase of a sound receiver at relatively moderate prices ranging from \$10 to \$70, it is unlikely that they will make a rapid response to television receivers having much higher prices.

The second group thus becomes the logical market for television receivers, which to them will constitute a *second replacement* of their present sound broadcast

[10] An extensive discussion of replacement demand in general is given in an article by T. M. McNiece, "The economic significance of replacement cycles in demand," *Trans. A.S.M.E.*, vol. 56; pp. 337–353; May, (1934).

receivers. Thus, we may assume that a *new cumulative replacement curve* will start, paralleling the curve of Fig. 3, when television is introduced, which we may term a curve of "Replacements of Sound by Television Receivers." However, the rate of growth of this curve cannot be expected to be as steep as that of the replacement curve shown in Fig. 3.

Secondary, Automobile, and Export Receivers

Secondary Receivers: The curve of the growth of ownership of secondary receivers is shown in Fig. 1. The development of the habit of using such receivers is still in its initial stages, and it is rather too early to attempt to derive an equation for the trend line. However, the rate of growth up to the present appears to be approximately the same as that of the early part of the curve of "Cumulative Replacements," of Fig. 3 and, as a first approximation, the curve of "Secondary Receivers" has been extrapolated in Fig. 1 in accordance with the trend shown for "Cumulative Replacements." It will be seen that there are many years of rapid growth ahead for this class of receivers, which is in accord with conclusions that may be derived from other considerations.

These small devices, retailing at prices from $6 to $30, appeal to a large number of persons who are not necessarily served by primary receivers already in possession of families. Some estimates may be made of the size of this market by attention to the following figures taken from the census of 1930:

Unmarried persons over 15 years of age	34,220,000
Children 10 to 14 years of age	12,005,000

The unmarried persons over 15 years of age are grown children still living with their families, or are single, widowed, or divorced people. In many cases they are gainful workers, contributing to the family income. For example, the census reported 13,812,000 gainful workers in families, in addition to the principal gainful worker. Most of these persons have rooms of their own, and constitute prospects for separate radio sets.

Children between the ages of 10 and 14 have their own special interests in radio programs, which are often quite different from the main family interest.

Assuming that 70 per cent of the unmarried group over 15 years of age, and 10 per cent of the children of 10–14 are prospects, we arrive at a conservative estimative for the ultimate sale of secondary sets, to such groups alone, of 25,000,000 sets. Therefore it does not seem unreasonable to assume, as a "saturation" value for the curve of secondary receivers, a total of about 30,000,000 sets.

Automobile Receivers: The growth curve of ownership of automobile receivers has been almost indentical with initial equipment in the home, in its early stages. This is depicted on Fig. 2, and since the curve matches the lower part of the first curve on Fig. 1 so closely, we are justified in extending it tentatively in accordance with the same equation as that used for the home sets. Thus, the ownership of automobile receivers at the beginning of 1938 may be said to be at the same stage as was that of home receivers in the middle of the year 1925. The automobile receiver field should show continued growth for at least the next 10 years, if the past trend is maintained.

Exports: The ownership of radio receivers outside of the United States has grown at a slower rate than in our own country, and this is reflected in the cumulative curve of exported receivers shown in Fig. 2. The number of receivers exported annually increased until 1936, and since then has been decreasing.

Extrapolation of the curve for cumulative exports on Fig. 2 has been accomplished by derivation of an equation based on the data for 1929 to 1938, and the predicted future is shown in dashed form. This would appear to indicate that, if the past trend continues, we may expect the normal annual rate of growth, corresponding to an export trade of about 600,000 to 700,000 receivers, to decrease in the future.

World ownership of radio sets, outside of the United States, attained a total of 39,700,000 at the end of 1937. The annual accretion for the preceding 3 years was an average of 3,800,000 sets, of which U. S. exports account for an annual average of 650,000 or 17 per cent; a notable share of the total world trade in radio receivers.

Effects of Fluctuations in General Business Activity on Receiver Production

The annual distribution of radio receivers to each of the markets discussed in the foregoing has been affected to some extent by cyclical fluctuations in business activity. The effect generally has been for the production data to move above or below the trend-line values and in the same direction as the movement of the business cycle; an upward movement in business activity would cause the data to move toward or higher than the value normal for that year, and for certain years this would carry the figures above the normal values indicated by the trend lines of Figs. 1 and 3; while a decline in business activity would be accompanied by a decline of the data from supernormal towards the normal or to a subnormal value for the year.

These swings have been kept within moderate limits, for the maximum deviations from the trend values have not exceeded 10 per cent. The reason that unit volume was well maintained during the recent depression is that prices of radio receivers were cut drastically after 1929, and low-priced "midget" and table models were introduced during this period. A large part of the unit volume (50 per cent or more in some depression years) was composed of these low-priced sets.

Fig. 4 illustrates these points, by showing the following factors connected with deviations from the trend lines of Fig. 1: At the top there is shown the variation in the average retail list price of home-type receivers. This is a simple average obtained by divid-

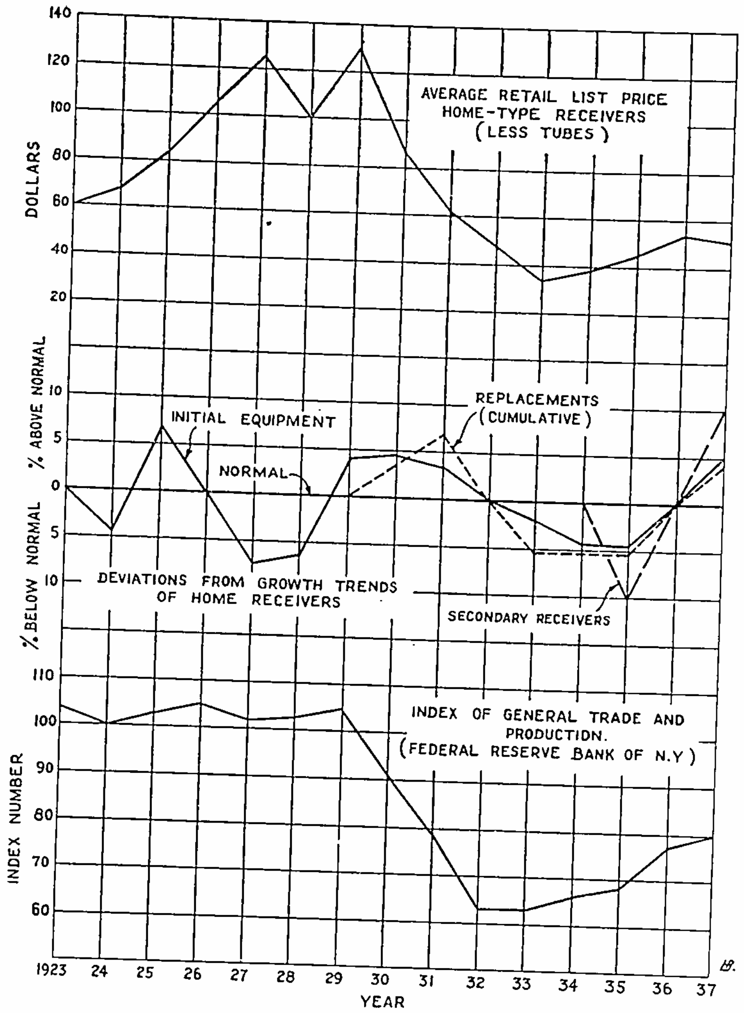

Fig. 4—Deviations from growth trends of home receivers, and related factors.

ing the total units of home-type sets produced each year into their estimated total retail value (less tubes). This average price, therefore, is influenced to some extent by the shift in demand from the more expensive console sets to cheaper sets after 1929, and indicates a fall in price from the 1929 average of $129.60 to $33.18 in 1933.

The middle portion of the chart shows the percentage that the cumulative initial equipment, replacement, and secondary-receiver sales departed from the trend line, or normal values; while the lower portion shows the movement of the "Index of General Trade and Production" of the Federal Reserve Bank of New York, which is an all-inclusive index of business activity.

Comparison of the middle and lower curves shows that these have maintained close correspondence as regards changes of direction, and comparison of the middle and upper curves illustrates the extent to which falling prices and the shift in demand have helped in avoiding a drastic departure from the normal trend of growth.

From this chart we may conclude that, without further production control, unit volume may be expected to swing above "normal" from 7 to 10 per cent in peak prosperity years, and perhaps from 7 to 10 per cent below "normal" in depression years, (the "normal" in each case being that of *ownership* of primary or secondary receivers, or *cumulative* replacements).

A 7 per cent deviation from these "normals," however, is very much more than 7 per cent of the annual production. For example, the theoretical "normal" for cumulative replacements, for 1938, was 18,500,000. The actual 1938 value of cumulative replacements was 19,500,000, which was 5.4 per cent above normal. This represented an excess of 1,000,000 sets. The actual sales for replacement purposes in 1938 were estimated as 2,600,000. Thus the excess over "normal" replacements, namely 1,000,000 sets, represented 38 per cent of these sales. This excess over normal replacements may well be followed in some future year by a corresponding drop to subnormal replacements, and may cause a severe decline in the annual production volume to which the industry has become accustomed in recent years.

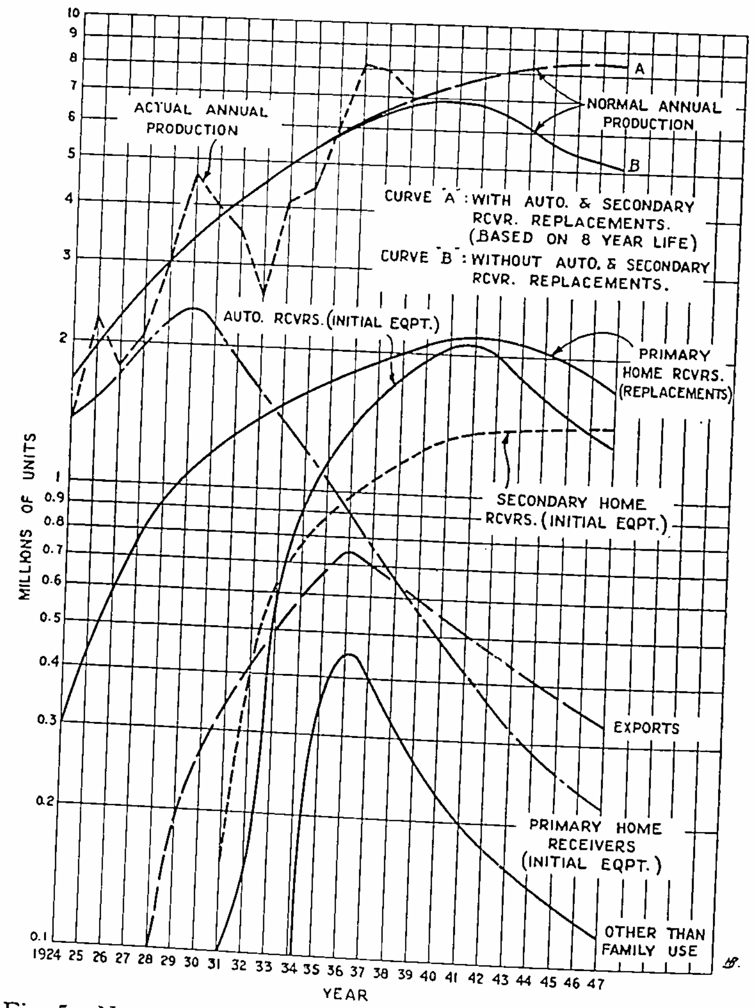

Fig. 5—Normal annual production (units) of broadcast receivers for various uses. Actual data: 1924–1937. Predicted data: 1938–1947.

Estimates of Future Normal Annual Demand for Sound Broadcast Receivers

From the long-time trends of Figs. 1, 2, and 3, by determining the yearly increases along each curve, we may derive normal values for the annual production that may be reasonably expected in the future. It is obvious that deviations from normal will occur, because of fluctuations in general business. But it is of interest to make these long-time annual estimates, in order to visualize the point which the industry has reached, and to determine its possibilities for future expansion or contraction in annual volume. Here again, no estimate is made of the modifications which may arise because of the introduction of television; but as pointed out previously, this is expected to grow relatively slowly, and for many years to come it is likely that the backbone of the industry will be the production of sound receivers.

Fig. 5 depicts the results arrived at. The past and anticipated future normal annual production is shown for each of the various types of demand previously referred to. At the top of the chart there is indicated the total normal production of units; this may lie between an upper limit (shown dashed) and a lower limit (shown solid) because of the unknown nature, at present, of the replacement demand for automobile and secondary receivers. Replacements in these fields have probably not yet begun to any substantial extent; but it may be anticipated that they will occur. For, as automobiles are scrapped every 7 or 8 years, the radio receivers which they bear probably will be scrapped with them, or will be quite obsolete; similarly, it may be anticipated that because of demise or obsolescence, replacements of secondary receivers will occur much as they have in the case of primary receivers. In order to care for these contingencies, the dashed portion of the curve has been shown on the assumption of an 8-year replacement, or life, period for these two types of receivers. In addition, there is shown in dotted form, across the early part of the curve, the actual annual production of the industry from 1924 to 1938.

The individual curves illustrate some points of practical interest. The first is that production of primary home-type receivers may be expected to decline at a continuing rate, in the near future; production of these for initial equipment has been declining for a number of years, and production for replacements seems to be approaching its peak. Secondary-receiver production, however, appears likely to continue to grow.

Automobile-receiver production for initial equipment still appears to be below its normal peak; it may grow for the next 2 or 3 years, and then commence to decline. At this point, however, replacements may make themselves felt as a substantial force, and maintain production volume. Replacement demand for secondary receivers also may make its appearance is substantial form in the early 1940's.

Thus, the principal conclusions to be drawn from these projected curves are the following:

(1) The character of sound-broadcast-receiver production should alter gradually, over the next 10 years from emphasis on primary home-type sets to greater emphasis on secondary and automobile models.

(2) The actual annual production in recent years has been in excess of normal, and it is possible that the industry faces several years of subnormal production. The cyclic fluctuations in production are undesirably large.

(3) The industry's production capacity should be geared to a normal annual production of about 7 million receivers. This may rise gradually to 8.5 million over the next 10 years, as the extent of replacement demand for automobile and secondary receivers warrants, or as future data tend to modify the trends deduced so far.

Long-Time Trends of the Vacuum-Tube Market

As pointed out earlier in this paper, discussion of the vacuum-tube market will be limited to the market for tubes used in broadcast receiving equipment. The subdivision of this into initial equipment, replacements, and exports has been described previously. Our object will be to determine long-time trends as indicated by the past history of production, and to project these into the future.

Table III gives the data for total unit production, and its breakdown into subdivisions. The sources of data for these subdivisions are somewhat better for vacuum tubes than for receivers, since the Census of Manufactures has collected biennial figures for tubes produced for initial equipment and replacements since 1931.

The average life of tubes used in receivers is an important factor in determining the number of tubes sold annually for replacements, and in order to find this it was necessary to go through the calculations given in Table IV. First it was necessary to calculate the total number of tubes in use in receivers at the end of each year. The tubes sold as replacements each year were estimated, as given in Table III. From the data of "Total Tubes in Use," and a 3-year moving average of tubes sold for replacements, the average age of the tubes replaced could be calculated by means of a formula given in the footnote to the table.

Tables III and IV furnish the basis of estimates of future production which are given in Table V. The footnotes to the tables explain the manner in which the data of each column were arrived at.

These data have been plotted in Figs. 6 and 7. Reference to the figures indicates the following situations:

Total Production increased sixfold from 1924 to 1938, reaching a peak of nearly 100 million tubes in 1936; extension of the data indicates another increase of 40 per cent above the 1936 peak, over the next 9 years, so that a normal annual production of 140 million vacuum tubes may be expected by 1947, for sound-broadcast reception alone.

TABLE III

ANNUAL PRODUCTION AND DISTRIBUTION OF VACUUM TUBES FOR BROADCAST RECEIVERS (UNITS)

Year	Total Production[1]	Domestic Sales[2]		Export Sales[3]		
		For Initial Equipment	For Replacements	Total Exports	Tubes Exported with Sets	Tubes Exported Separately
1922	1,000,000	1,000,000				
1923	4,687,400	4,687,400				
1924	12,000,000	8,000,000	4,000,000			
1925	23,444,035	11,720,000	11,725,035			
1926	30,000,000	11,290,000	17,559,000	1,151,000	486,000	665,000
1927	23,650,733	13,320,000	9,078,733	1,252,000	530,000	722,000
1928	50,200,000	21,475,000	27,492,000	1,233,000	675,000	558,000
1929	70,584,323	33,495,000	34,351,323	2,738,000	1,505,000	1,233,000
1930	40,212,891	20,178,000	16,788,891	3,246,000	1,472,000	1,774,000
1931	47,504,298	21,275,000	21,029,298	5,200,000	2,825,000	2,375,000
1932	47,047,138	12,517,000	29,319,138	5,211,000	1,453,000	3,758,000
1933	62,428,803	21,905,000	32,579,803	7,944,000	2,545,000	5,399,000
1934	63,247,423	25,731,000	27,465,423	10,051,000	3,369,000	6,682,000
1935	75,961,650	36,480,000	29,082,106	10,399,544	3,750,000	6,649,544
1936	98,304,208	51,430,000	34,664,564	12,209,644	3,820,000	8,389,644
1937	92,055,700	50,265,000	28,371,428	13,419,272	3,735,000	9,684,272
1938	74,690,500	34,215,107	30,491,393	9,984,000	2,700,000	7,284,000

[1] Source: *Census of Manufactures*, 1923, 1925, 1927. *Radio Retailing*, 1922, 1924, 1926, 1928. RCA Licensee reports, 1929 to date.

[2] Estimated as follows: For 1922–1929, an average number of tubes per receiver produced was assumed as given in Table IV, and initial-equipment sales taken as the product of "Receivers Produced for Domestic Sale" and this average, for each year. For 1930, 1932, and 1934, percentages given in *Radio Retailing's* annual statistical summary for initial-equipment sales were applied to the totals in the first column of the above table, and exports deducted. For 1931, 1933, and 1935, similar percentages derived from data of the *Census of Manufactures* were used in the same manner. For 1936 and 1937, initial-equipment sales were estimated from "Receivers Produced for Domestic Sale" multiplied by 6.7 tubes per receiver (the 1935 average, from Census reports). Replacements were taken as the remainder after deducting initial-equipment and export sales from total production. For 1938, estimated in the same manner as for 1934.

[3] Exports: "Tubes Exported with Sets" were estimated as the number of receiving sets exported multiplied each year by a reasonable average number of tubes per set. This average for each year was assumed as slightly smaller than the average for sets sold domestically, as given in Table IV. "Tubes Exported Separately" are the figures published each year by the Bureau of Foreign and Domestic Commerce under the Classification "Radio Receiving Tubes." These reports do not include tubes shipped with sets, unless declared, and so far as the writer is able to determine it is general practice in exporting radio sets not to declare tubes shipped as part of the sets.

TABLE IV

DOMESTIC VACUUM-TUBE INSTALLATIONS, REPLACEMENTS, AND AVERAGE TUBE LIFE
(All Data in Millions, except Third and Ninth Columns)

Year	Receivers Installed as Initial Equipment[1]	Average Number of Tubes per New Receiver[2]	Total Tubes in All Receivers Installed This Year	Total Receivers Operating in U. S. at End of Year[3]	Total Tubes in All Receivers at End of Year[4]	Tubes Sold for Replacements This Year[5]	Three-year Moving Average of Tubes Sold for Replacements	Average Age of Tubes Replaced[6] (in Years)
1922	1.50	3.00	4.50	1.50	4.50			
1923	1.50	4.00	6.00	3.00	10.50			
1924	1.10	5.00	5.50	4.10	16.60	4.00		1.65
1925	2.25	5.00	11.70	6.30	29.10	11.73	11.27	1.31
1926	1.25	6.72	8.40	7.55	38.80	17.56	12.97	1.63
1927	1.23	7.00	8.60	8.77	48.90	9.08	18.22	1.74
1928	2.13	7.00	14.90	10.90	65.00	27.49	23.64	1.75
1929	3.60	7.50	27.00	14.50	94.07	34.35	26.21	1.92
1930	2.31	5.65	13.05	16.81	107.12	16.79	24.06	2.42
1931	1.82	6.70	12.19	18.63	119.31	21.03	22.21	2.98
1932	1.55	5.73	8.86	20.18	128.17	29.32	27.37	3.52
1933	2.70	5.85	15.78	22.88	143.95	32.58	29.62	3.43
1934	2.65	6.50	17.21	25.53	161.16	27.47	29.71	3.72
1935	3.37	6.70	22.58	28.90	183.74	29.08	30.40	3.95
1936	5.08	6.70	34.05	33.98	217.79	34.66	30.71	3.92
1937	4.25	6.70	28.45	38.23	246.24	28.37	31.14	4.10
1938	3.50	5.00	16.50	43.10	262.74	30.49	(31.50)	4.58

[1] Includes all classes of receivers installed domestically: home primary and secondary, automobile, and "for other than family use."

[2] Estimated for 1922–1929. Computed for 1930 to date from ratio of tubes produced for initial-equipment sales to total production of receivers, using sources given with Table III.

[3] From Table II.

[4] Cumulatives of data in fourth column, but with allowance made for the effect on the tube total of replacement of old receivers by new ones having fewer or more tubes.

[5] From Table III.

[6] Computed from the following formula: Age of tubes replaced $=(B+CD)\div(A+C)$, where A = moving average of the number of tubes sold for replacement in the replacement year (say year "a"); B = number of tubes operating in all sets in some preceding year (say year "b"); C = number of tubes added to those in year "b," during the following year; D = number of years difference between years "a" and "b." The year "b" should be chosen from two to four years before year "a," depending on the approximate age of the tubes replaced.

TABLE V

NORMAL FUTURE ANNUAL PRODUCTION OF VACUUM TUBES FOR SOUND BROADCAST RECEPTION ONLY
(All Data in Millions, Except Seventh Column)

Year	For Initial Equipment[1]		For Domestic Replacements					Exports (Separate from Receivers)[6]	Total Annual Tube Production
	Normal Receiver Production[2]	Tube Production for Initial Equipment	Total Receivers in U. S. at End of Year[2]	Tubes Operating in All Receivers[1]	Tubes Subject to Replacement[3]	Avg. Age of Tubes to be Replaced[4] (Years)	Annual Tube Production for Replacements[5]		
1939	7.25	43.50	47.12	286.86	175.0	4.40	39.80	11.50	94.8
1940	7.50	45.00	51.23	311.30	202.0	4.46	45.30	12.30	102.6
1941	7.75	46.50	55.30	335.72	232.0	4.52	51.40	13.20	111.1
1942	8.05	48.30	59.22	359.24	252.5	4.58	55.10	14.00	117.4
1943	8.25	49.50	62.90	381.32	272.0	4.65	58.50	14.60	122.6
1944	8.30	49.80	66.33	401.90	294.5	4.71	62.40	15.20	127.4
1945	8.35	50.10	69.53	421.10	317.0	4.77	66.50	15.80	132.4
1946	8.40	50.40	72.64	439.76	340.0	4.83	70.40	16.50	137.3
1947	8.45	50.70	75.70	458.12	361.0	4.90	73.70	17.20	141.6

[1] Estimated on the basis of 6.0 tubes per receiver in the future.

[2] From data used in preparing Fig. 5.

[3] Obtained by plotting data in preceding column, and that of the sixth column in Table IV, and determining the points corresponding to the "Average Age of Tubes to be Replaced," given in the seventh column, above.

[4] Obtained by extrapolating a curve of the data of the last column of Table IV.

[5] Obtained by dividing the data of the sixth column by those of the seventh column, above.

[6] Estimated by extrapolation of the curve of exports from 1926 to 1937.

Tubes in Use in Domestic Receivers increased 15 times in the same period, reaching 262 million at the end of 1938; this may be expected nearly to double in the next 9 years, approximating 460 million tubes in use by 1947.

Production of Tubes for Initial Equipment is dependent upon the volume of receiver production. This grew about sixfold from 1924 to 1937, reaching a peak of slightly over 50 million tubes in 1936 and 1937. However, additional growth of the normal annual production beyond this peak is not anticipated in the future, because of the fact that the normal annual production of receivers probably will remain below 9 million during the next 10 years, as indicated in Fig. 4, and with a smaller average number of tubes per receiver, than heretofore.

Production of Tubes for Replacements grew approximately threefold, from 1925 to 1937; in the future a considerable expansion of production for this purpose may be anticipated, the normal rising from an average of 30 million to nearly 75 million tubes during the next nine years. This rise is inherent

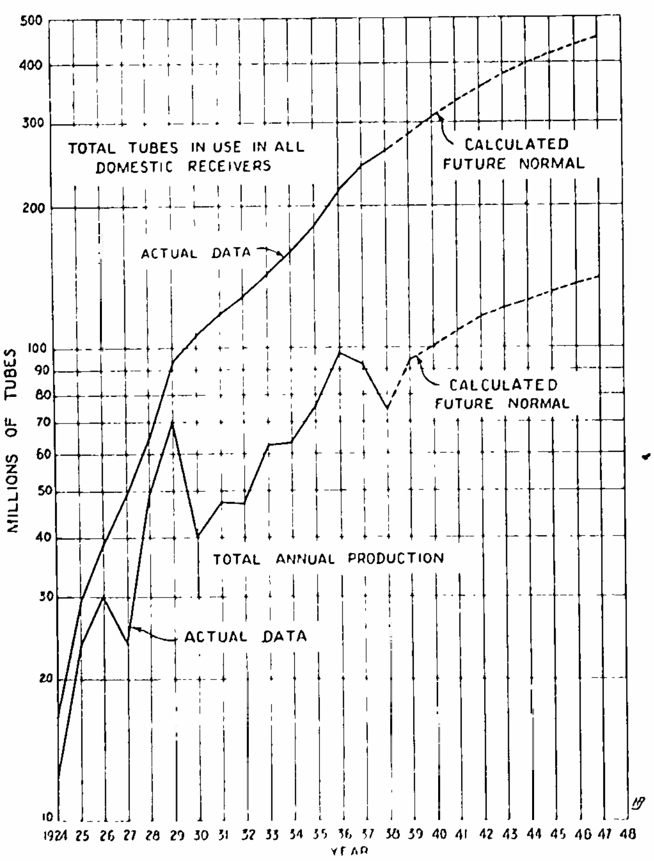

Fig. 6—Total annual production of vacuum tubes for broadcast receivers and total number of tubes in use in all domestic receivers.

in the fact that replacement demand from the large number of receivers sold for initial equipment since 1932 has not yet begun to be felt; and the additional demand arising from receivers to be sold for automobile and secondary use during the next five or six years will add substantially to the replacement market.

Exports have grown more rapidly than any other branch of the tube business, although the total volume is still small (about 10 million tubes apart from those sold with receivers). This has increased nearly tenfold in the 7 years from 1929 to 1937 and it has not shown the annual irregularity characterising the domestic demand. The present export market (for separate sales only) takes approximately 10 per

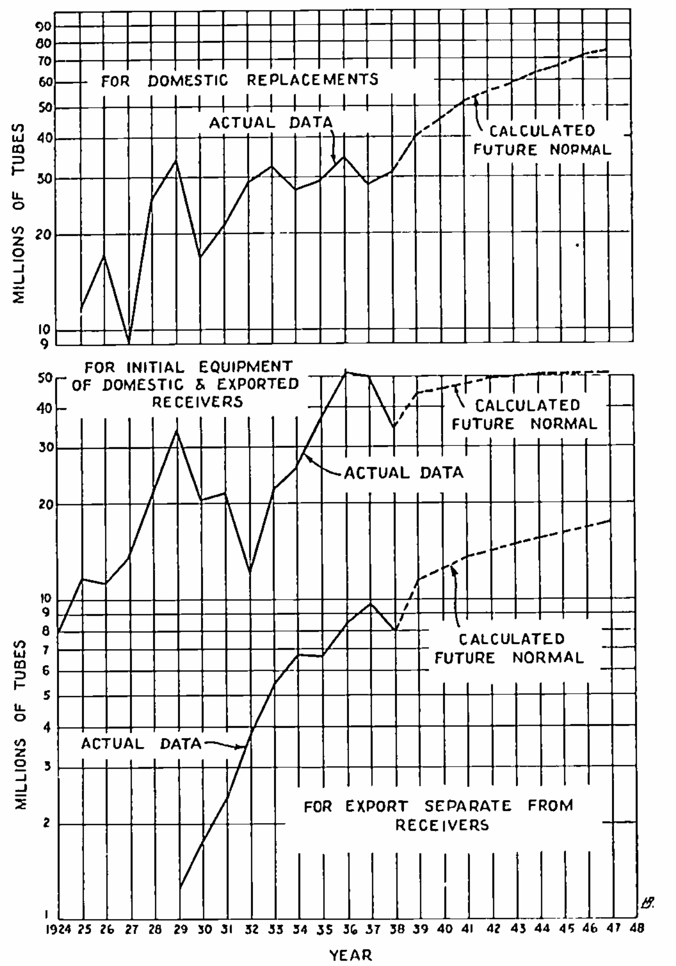

Fig. 7—Annual production of receiving vacuum tubes, for initial equipment, replacements, and export.

cent of the total production; if tubes exported with sets are included, the export market is taking 14.1 per cent of total production. This is a substantial proportion, and the total number of tubes exported may be expected conservatively to increase about 70 per cent more in the next 10 years.

The Average Life of Tubes Used in domestic receivers, given in the last column of Table IV, presents a rather startling condition. It has risen gradually from 1.65 years in 1924 to 4.58 years in 1938, a result which would appear contrary to the data published by the Columbia Broadcasting System on the number of hours of daily use of radio receivers. Investigations made by C.B.S. have indicated that the public are now using their receivers an average of 3.9 hours per day (for all urban radio sets), and that this extent of daily use has grown materially since 1933. With increasing hours of use of receivers, the life of tubes (in years) used in such receivers would be expected to decrease. Instead, it has become longer. However, the facts deduced from the tube-replacement sales are incontrovertible; and this can only mean that the life of tubes has been so greatly extended, by modifications in design and manufacture, that in spite of increasing use the tubes still live longer. There appears to be an upward trend in this direction, which if continued at the past rate would lead to an average tube life of 4.9 years, by 1947, as shown in Table V (seventh column). If the C.B.S. figures are correct, and sets on farms are included, it is probable that a national average of 4 hours of use daily may be considered as a fair estimate; this would imply 1460 hours of use per year, and with a tube life of 4.1 years of such use, (the 1937 figure) a total tube life of 5986 hours. This figure is far beyond the usually assumed life of 1000 hours, and indicates the extent to which the public allows its tubes to deteriorate before requiring their replacement. Just how good a tube of present-day manufacture is, which has run nearly 6000 hours, is a question which the writer is unable to answer; but it would provide an interesting field for research, and for educational material to be presented to the public.

To summarize the conclusions from this section of the paper:

(1) Total annual tube production is still in an upward trend, and an increase of at least 40 per cent above previous peaks may be expected during the next nine years.

(2) Practically all of the expansion in production should come from demand for tubes for replacement purposes and exports.

(3) The life of tubes, in receivers used by the public, is inordinately high. Tubes last an average of more than 4 years at the present time, corresponding to about 6000 hours of use. This factor appears still to be increasing, and may rise to 4.9 years by 1947 unless something is done to educate the public in the direction of more frequent replacement, or future developments involve tubes of more limited life.

Summary of Conclusions

Sound Broadcast Receivers

1. The character of the distribution of receivers to various markets has altered materially since the inception of broadcasting. Prior to 1927, sales for initial equipment to the home absorbed practically the entire annual production; by 1930, initial equipment for homes constituted only 59.4 per cent and replacements 33.5 per cent of all sales; by 1936, initial-equipment "home" sales were 19.8 per cent, replacement "home" sales were 30.3 per cent, and new sales channels were absorbing receivers as follows: Secondary receivers, 15.7 per cent; sales for other than family use, 7.8 per cent; automobile receivers, 17.2 per cent; exports, 8.1 per cent.

2. Extension of the trends of the past 14 years leads to the conclusion that further alterations in the character of the "normal" annual demand will occur; emphasis on home-type primary (or "living-room") sets should decrease and emphasis on secondary ("compact" or "extra") and automobile models should increase. Annual demand for primary receivers as initial equipment for homes should fall continuously; demand for replacements of primary receivers should pass its peak in a few years and fall slowly thereafter. At the same time, annual demand for secondary sets should continue to rise steadily and automobile-set demand for initial equipment should rise for the next 3 or 4 years, with a gradual decline thereafter. (A replacement demand for automobile receivers would tend to sustain total production after 1941.)

3. Total annual production of all types of receivers goes through wide cyclical fluctuations, above and below a "normal" trend line. At the present time, the "normal" is about 6.7 million receivers, and increasing at the rate of about 250,000 receivers per year. In the last 3 years, however, the industry has produced well over this normal amount (more than 8 million receivers during 1936 and 1937, and 7.1 million receivers in 1938). Thus, there has been an excess over "normal" production of about 4 million receivers in these 3 years. In view of this excess production, we may anticipate a sharp drop to subnormal demand sometime during the next few years, probably during the next business recession. During the next 9 years, the "normal" should rise gradually to about 8.5 million receivers.

4. The length of time that the public retains its receivers before purchasing replacements has been increasing. It would appear that in 1928, the average life of a receiver was about 5 years. This increased gradually, until now 75 per cent of receivers are being retained an average of 7 years. It is anticipated that this condition will continue to exist in the future.

5. Television receivers will constitute an inducement for the slow replacement of existing sound receivers. The growth curve of such replacements is not expected to be as steep as that of replacements of older types of sound receivers by newer types, for reasons given in the text. In terms of unit volume, annual production of sound receivers for the various markets, during the next 10 years, should considerably exceed production of television receivers; and for at least 5 years to come, it is likely that the backbone of the industry will be the production of sound receivers.

Vacuum Tubes

1. Vacuum tubes are produced for distribution to 3 markets: (a) initial equipment of receivers by receiver manufacturers; (b) Replacements of outworn tubes in receivers possessed by the public; (c) Exports. In 1938, the total production was divided between these as follows: Initial equipment, 49.4 per cent; replacements, 40.8 per cent; exports, 9.8 per cent.

2. The percentage distributions given above are expected to alter materially during the next 9 years. It is further anticipated that the total annual production will increase above the previous peak by about 40 per cent, with most of the increase arising from tubes sold as replacements. Thus, in 1947 the anticipated total production is 142 million tubes (compared with 98 million in 1936), distributed as follows: Initial equipment, 36 per cent; replacements, 52 per cent; exports, 12 per cent. This is exclusive of the demand for tubes to equip television receivers.

3. The export market for tubes is an important element. It has grown more rapidly than any other branch of the tube business, although the total volume is still small. It does not show the annual irregularity characterizing the domestic demand. The present export market (considering tubes sold separately from receivers) takes approximately 10 per cent of the total production; if tubes exported with sets are included, the export market is taking 13.3 per cent of the total production. The total number of tubes exported annually may be expected conservatively to increase about 70 per cent in the next 9 years.

4. The average life of tubes used in domestic receivers has risen gradually from 1.65 years (in 1924) to 4.58 years (in 1938), in spite of an increase in the daily use of receivers by the public. If this trend continues, the average tube life by 1947 will be 4.9 years. This means that the public is running its tubes over 6000 hours, on the average, before replacing them.

Generally speaking, the radio industry is still in the expansion stage. However, the long-range tendencies in demand for the various types of product must be studied carefully, since there are prospects for stationary or declining demand in some sections of the market, while further expansion appears possible in other sections.

I hope that the occasional study of these tendencies, by radio engineers, will help to give them a basis for the clearer understanding of economic factors which, in the long run, profoundly affect their own lives and fortunes.

The Application of Low-Frequency Circuit Analysis to the Problem of Distributed Coupling in Ultra-High-Frequency Circuits.*

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Summary—*The general problem of coupling between circuits which extend beyond the near zone is investigated theoretically. Integrals defining generalized coefficients of inductance are displayed and discussed for the case of the near zone and in general. As an application, the specific problem of determining the current distribution in two coupled circuits which extend beyond the near zone is examined, and solved for the special case of two loosely coupled sections of parallel line. It is proved that the electromotive force induced in the secondary by the primary oscillator may be treated as though concentrated at a point opposite the center of the oscillator provided the current distribution in the oscillator is symmetrical with respect to this center. The conclusions are generalized to include other circuits than the parallel line, in particular an antenna coupled to a symmetrical oscillator. Experimental curves are shown which verify the general theory.*

IN alternating-current circuits at all frequencies it is common practice to couple two or more parts of a network inductively. In low-frequency circuit analysis the voltage equations for two coupled circuits of which one contains a generator are written as follows:

$$I_1Z_{11} + I_2Z_{12} = V_1^e$$
$$I_1Z_{21} + I_2Z_{22} = 0. \tag{1}$$

Here Z_{11} and Z_{22} are the self-impedances of the primary and secondary, respectively, and $Z_{12}=Z_{21}$ is the mutual impedance between the two circuits. V_1^e is the amplitude of a harmonic electromotive force of the symbolic form,

$$V^e = V_1^e e^{j\omega t}. \tag{2}$$

If the two circuits are coupled inductively, the following relation obtains:

$$Z_{12} = Z_{21} = j\omega L_{12}. \tag{3}$$

Here L_{12} is the coefficient of mutual inductance. It is a constant depending only upon the geometrical configuration of the two circuits, and upon the permeability of the medium in which they are immersed. In particular, it is not a function of the frequency. The term $-I_1Z_{21}$ in the secondary is frequently called an induced electromotive force.

If the two circuits are excited by an ultra-high-frequency voltage V^e instead of a low-frequency one, the simple circuit analysis in terms of a constant mutual-inductance parameter cannot be used. This is due to the fact that two fundamental assumptions which are implied in all low-frequency theory are usually not justified at high frequencies. These are, (1) the current amplitude in all parts of a closed

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(series) circuit is the same and (2) the interaction of electric currents in different circuits is instantaneous.

The General Theory of Coupled Circuits

In order to investigate the problem of coupling two ultra-high-frequency circuits, such as an oscillator and a parallel line or an antenna, one may set up the general integrals which define the low-frequency coefficients of inductance before the two restricting conditions are imposed. This may be done by proceeding directly from the Maxwell equation,

$$\text{curl } E = -j\omega B. \tag{4}$$

In this relation E and B are, respectively, the amplitudes of harmonically varying electric and magnetic vectors written in the symbolic form. The application of this equation to the closed contour s, of a circuit of conductors is conveniently made by first integrating the normal component of each side over a cap surface S, which is bounded by the contour. The resulting surface integrals over the cap may then be transformed into line integrals around the contour of the circuit using Stokes' theorem and the definition of the vector potential A. This is

$$B = \text{curl } A. \tag{5}$$

In this way one readily obtains the integral equation,

$$\oint_{s(\text{line})} (E, ds) = -j\omega \int_{S(\text{cap})} (n, B)dS = -j\omega \oint_{s(\text{line})} (A, ds). \tag{6}$$

The value of E on the surface of a cylindrical conductor may be expressed in terms of the total current flowing in the conductor in the form

$$(E, ds) = z^i I ds. \tag{7}$$

Here the element ds and the current amplitude I are directed everywhere parallel to the axis of the conductor. z^i is the general internal-impedance function; it depends upon the conductivity, the permeability, and the radius of the conductor, as well as upon the frequency. It is usually convenient to assume that a return circuit and all coupled circuits are sufficiently far away so that cylindrical symmetry may be assumed in the current distribution across each section in the interior of the conductor.

The value of the vector potential A_2 at any point on the surface of the conductors forming circuit 2 depends upon the current in circuit 1 as well as upon the current in circuit 2 itself. Let A_{22} be the contribution to A_2 by the current I_2 in circuit 2; let A_{21} be the contribution to A_2 due to the current I_1 in circuit 1. For simplicity it is assumed that no other circuits are involved. With (7) one can write (6) as follows:

$$\oint_{s_2} I_2 z_2^i ds_2 = -j\omega\left\{\oint_{s_2} (A_{22}, ds_2) + \oint_{s_2} (A_{21}, ds_2)\right\}. \tag{8}$$

The values of the vector potential are given by the Helmholtz integrals,

$$A_{22} = \mu\Pi/4\pi \oint_{s_2} I_2'/r_{22}\cdot e^{-j\beta r_{22}} ds_2' \tag{9a}$$

$$A_{21} = \mu\Pi/4\pi \oint_{s_1} I_1'/r_{21}\cdot e^{-j\beta r_{21}} ds_1'. \tag{9b}$$

Here r_{22} and r_{21} are, respectively, the distances from the element ds_2 in circuit 2 where A_2 is being calculated, to the elements, ds_2' and ds_1' in circuits 2 and 1. I_2' is the current amplitude in the element, ds_2'; I_1' is the current amplitude in ds_1'. Π is the magnetic constant of empty space; its numerical value is, $\Pi = 4\pi\times 10^{-8}$ henrys per centimeter; $\beta = \omega/v = 2\pi f/v$.

Since the current amplitudes I_2 and I_1 are in general functions of the variable s, around the contour of each circuit, they can be written in terms of distribution functions $f(s)$. Thus,

$$I_2 = I_{20} f_2(s)$$
$$I_1 = I_{10} f_1(s). \tag{10}$$

The current amplitudes with subscript 0 are particular values at suitably chosen origins of the variable s in each circuit where $f(s) = f(0)$ is arbitrarily set equal to unity. Let the following symbols be introduced:

$$Z_2^i = \oint_{s_2} f_2(s) z_2^i ds_2 \tag{11a}$$

$$L_{22} - jN_{22}/\omega = \mu\Pi/4\pi \oint_{s_2} \left(ds_2, \oint_{s_1} f_2'(s)/r_{22}\cdot e^{-j\beta r_{22}} ds_2'\right) \tag{11b}$$

$$L_{21} - jN_{21}/\omega = \mu\Pi/4\pi \oint_{s_1} \left(ds_2, \oint_{s_1} f_1'(s)/r_{21}\cdot e^{-j\beta r_{21}} ds_1'\right). \tag{11c}$$

Here the L's are inductance, the N's radiactance[1] parameters. With (7) to (11), (6) becomes

$$I_{10}(j\omega L_{21} + N_{21}) + I_{20}(Z_2^i + j\omega L_{22} + N_{22}) = 0. \tag{12}$$

By finally setting,

$$Z_{21} = j\omega L_{21} + N_{21}, \tag{13a}$$

$$Z_{22} = Z_2^i + j\omega L_{22} + N_{22}, \tag{13b}$$

one has,

$$I_{10} Z_{21} + I_{20} Z_{22} = 0. \tag{14}$$

A similar equation may be written for the primary circuit by interchanging the subscripts 1 and 2 and

[1] The radiactance parameters are the same as radiation resistance parameters only in the case of uniform current distribution since they are factors of the current and not of the current squared.

by writing V_{1s}^e on the right instead of zero. It is assumed that V_{1s}^e is due to a point generator at some definite point s around the contour of the primary. In this way the general problem of two coupled circuits has been manipulated into precisely the low-frequency form. If one now imposes the following two conditions

$$f_2(s) = f_1(s) = 1 \tag{15a}$$

$$e^{-j\beta r_{22}} = e^{-j\beta r_{21}} = 1 \tag{15b}$$

the quantities defined in (11) become

$$Z_2^i = \oint_{s_2} z_2^i ds_2. \tag{16a}$$

$$N_{22} = 0; L_{22} = \mu\Pi/4\pi \oint_{s_2} \left(ds_2, \oint_{s_2} ds_2'/r_{22}\right) \tag{16b}$$

$$N_{21} = 0; L_{21} = \mu\Pi/4\pi \oint_{s_2} \left(ds_2, \oint_{s_1} ds_1'/r_{21}\right). \tag{16c}$$

These are precisely the low-frequency relations for the internal impedance, self, and mutual inductance. It is to be noted that the coefficients of inductance defined by (16b) and (16c) are constants of a particular geometrical configuration. An examination of the conditions (15), which define the low-frequency case, reveals that the first is simply a mathematical statement requiring currents of uniform amplitude at all points in each circuit. The second condition (15b) can be true exactly only if

$$\beta r_{22} = \beta r_{21} = 0. \tag{17a}$$

Since r_{22} and r_{21} cannot vanish everywhere, it follows that $\beta=\omega/v$ must vanish. Except for steady currents with $\omega=0$, this is mathematically possible only if $v=\infty$. An infinite velocity of propagation is equivalent to instantaneous action at a distance. However, this is not a sensible interpretation, because $v=c/\sqrt{\mu\epsilon}=3\times10^{10}/\sqrt{\mu\epsilon}$ centimeters per second, so that the velocity of propagation cannot become infinite. Instead of requiring (17a) to be satisfied, one can demand that

$$\beta r_{22} \ll 1; \; \beta r_{21} \ll 1. \tag{17b}$$

Then (17b) is satisfied if the exponential factors in (15b) are, to a close approximation, given by the leading term of unity in the power series expansions. Circuits of such dimensions and excited at such frequencies that (17b) is true are said to be in the near zone. The low-frequency equations of the quasi-steady state are then good approximations.

If the frequency is sufficiently high and the circuits are sufficiently extensive or far apart, the general expressions (11) must be used instead of the simpler ones (16). But the use of (11) instead of (16) implies more than a very considerable increase in mathematical complexity. Because the generalized inductance parameters defined in (11) are functions both of the frequency and of the contour variable s, they are no longer simply geometrical constants which may be evaluated or measured experimentally once and for all for each circuit element or combination of elements. Each circuit problem must be analyzed individually in terms of the current distribution peculiar not only to the geometrical configuration but also to each frequency. Thus the generalized inductance parameters are not useful in solving general circuit problems. Their evaluation for any particular circuit would carry with it only the solution of that circuit, nothing more comprehensive. But even that is usually out of the question because of the difficulty in solving a pair of simultaneous equations of the form (1) with the generalized impedances defined by (13) and (11). The problem is not even significantly simplified if one assumes a simple current distribution such as the sinusoidal.

There is evidently no other choice but to attempt to achieve simplification at the expense of generality by judicious specialization and restriction. A first step in this direction suggests itself at once. It is imposing such conditions as will make the primary equation corresponding to (1) independent of the secondary equation. It is accomplished by demanding that the following inequality be satisfied:

$$I_{10}Z_{11} \gg I_2Z_{12}. \tag{18}$$

This is a loose-coupling restriction. It means that the two circuits are sufficiently far apart, or otherwise so loosely coupled so that the reaction of the secondary on the primary may be neglected. Although this very considerably limits the scope of any solution which may be obtained, it does not exclude a large variety of important circuit arrangements. Moreover, closely coupled circuits often satisfy the conditions of the near zone.

If (18) is presumed to be satisfied, one has two independent equations of the form

$$I_{10}Z_{11} = V_1^e \tag{19a}$$

$$I_{21}Z_{22} = -I_{10}Z_{21} = V_2^i. \tag{19b}$$

The quantity V_2^i is the induced electromotive force. It is defined by the equation on the right in (19b). The relation (19a) defines the current distribution in the oscillator circuit. A solution by directly evaluating Z_{11} using the integrals (11b) leads to great mathematical complications even for the simplest circuit configurations. The current distribution may, however, be determined for a simple circuit like the parallel line as will be done below. A solution of the secondary problem contained in (19b) is vastly more complicated than the primary problem because an intricate induced electromotive force V_2^i takes the place of the simple generator electromotive force V_{1s}^e. Even if it is assumed that the primary-current-distribution factor $f_1(s)$ is known from the solution of (19a), the evaluation of Z_{21} from the general integral (11c) is no simple problem. And this is only the first step in the determination of the current distribution in the secondary using (19b).

The general theoretical investigation of coupled circuits has led to formidable mathematical expressions which are not readily evaluated directly even for simple circuits. They are, nevertheless, extremely valuable in that they reveal the form of the solution and the variables involved. In the case of coupled circuits of simple configuration both the equations (19) may be solved by other methods. These solutions are in themselves of considerable importance and the conclusions derived from them are readily extended to more general forms of coupled circuits including antennas. The simple coupled circuits which can be solved consist of a secondary which is a section of a parallel line terminated at each end by general impedances, and a primary which is another, shorter section of parallel conductors terminated by impedances of which at least one contains a point generator. Such a primary oscillator is readily approximated experimentally by connecting a triode to one or both ends of a section of parallel rods. Each triode is mathematically represented by a suitable impedance containing an hypothetical concentrated generator or point source of electromotive force of magnitude[2,3] μe_g. The primary and secondary are coupled by placing them parallel to each other and as near as possible, without violating the loose-coupling requirement which permits writing the two independent equations (19). These circuits will now be analyzed in turn.

Solution of the Primary Equation

The determination of the current distribution in a primary circuit consisting of a section of parallel conductors and suitable terminal impedances of which at least one contains a point generator may be analyzed by parallel-line theory suitably generalized to ultra-high frequencies. It has been shown[4] that by modifying the definitions of the circuit parameters the telegraphist's or long-line equations are a good approximation even at ultra-high frequencies. The general hyperbolic solutions of these equations for the amplitudes of the current in, and the potential difference between, the parallel wires at a co-ordinate $x=x$ along the line are

$$V_x = D\cosh Kx + E\sinh Kx \tag{20a}$$

$$I_xZ_c = -D\sinh Kx - E\cosh Kx. \tag{20b}$$

Here, $K=\alpha+j\beta$ is a complex constant involving all of the circuit parameters; Z_c is the characteristic impedance of the line; D and E are constants of integration.

In order to express the constants, D and E, as functions of the terminal impedances, as in the formally identical low-frequency problem, it is first necessary to define impedance at ultra-high frequencies. This may be done precisely in terms of general integrals like (11), which are obtained by integrating around that part of the circuit only, which forms a part of the impedance to be defined. If this is done, it is again necessary to introduce a current-distribution function $f(s)$ from which it may be concluded that one cannot, in general, assume a parallel line to be terminated by lumped impedances, that is, by impedances in which the current amplitude is the same at every point. Consequently it is quite meaningless to speak of the current flowing in a terminal impedance in any quantitative sense, unless reference is made to the amplitude at a particular point. The question is, which point shall be selected in defining impedance? It is to be noted that the problem of definition is in no way simplified if one seeks to avoid the specific problem of evaluating impedance in terms of geometrical configuration and frequency by writing

$$Z = V/I. \tag{21}$$

In order to determine the impedance Z of a coil or other device in terms of this definition, it is evidently necessary to measure the potential difference V between the terminals of the coil, and the current I flowing through it. Since no quantitative significance can be attached to the current flowing through a coil or other element unless it is the same at every point, (21) fails as a general definition of impedance when the current distribution is not uniform. As before, one must adopt a convention for locating a reference point for the contour variable s appearing in the distribution factor $f(s)$. Let this be done by specifying that the current I in (21) shall be the current entering the coil at one of its terminals. Let it be required, in addition, that $f(s)$ be symmetrical in the coil or element so that the instantaneous value of I at one terminal is always the negative of that at the other. Then (21) uniquely defines a quantity which is called the input impedance for symmetrical current distribution. That is, one can define the input impedance of a coil or network to be the ratio of the voltage amplitude between the terminals to the current amplitude at the one where the current is assumed to enter. The impedance so defined will be a complex quantity, just as the low-frequency impedance, if the symbolic notation is used. Since it is postulated in the case of a parallel line that the currents at opposite points in the two wires are always equal and oppositely directed, it follows that $f(s)$ for any device terminating a parallel line must have the

[2] Ronold King, "Wavelength characteristics of coupled circuits having distributed constants," Proc. I.R.E., vol. 20, pp. 1368–1400; August, (1932).

[3] Ronold King, "Capacitance at ultra-high frequencies," *Phil. Mag.*, series 7, vol. 20, p. 514; September, (1935).

[4] Ronold King, "The telegraphist's equations at ultra-high frequencies," *Physics*, vol. 6, p. 121; April, (1935).

same current amplitude at each terminal, so that one can properly define its input impedance. As a result of this fact, low-frequency-line theory may be retained formally unchanged at ultra-high frequencies for a pair of parallel conductors terminated at each end by perfectly general impedances. It is to be noted, however, that the mutual impedance between the terminating device and the two wires is necessarily included in the self-impedance of this device.

Let the network or element which is connected between the ends of a parallel line at $x=O$ have an input impedance

$$Z_0 = R_0 + jX_0. \tag{22a}$$

Its reciprocal is the input admittance of the element and is

$$1/Z_0 = Y_0 = G_0 - jB_0. \tag{22b}$$

At the other end, at $x=s$, the terminal impedance and admittance are defined by

$$Z_s = R_s + jX_s \tag{22c}$$

$$1/Z_s = Y_s = G_s - jB_s. \tag{22d}$$

In order to maintain a current in the closed circuit consisting of the parallel line and the two terminal impedances, it is necessary to provide induced or other driving electromotive forces concentrated at suitable points in the circuit. Let it be assumed that two identical point-generators are connected symmetrically at the end of the line at $x=0$ in such a way that the entire terminal impedance Z_0 is between them. The circuit is illustrated in Fig. 1. Each generator has an electromotive force equal to one-half V_0^e at a frequency $f=\omega/2\pi$. The two are exactly in phase from the point of view of a current flowing completely around the circuit. That is, they act to have equal and opposite currents flowing in the two conductors of the parallel line. The current I_0 flowing through each generator is the same as the current entering or leaving Z_0 and the current at $x=0$ in the line. This follows from the fact that the points $x=0$ locating the two generators are simultaneously the junction points of the parallel conductors with Z_0. The internal impedances of the generators may be assumed contained in Z_0. The potential difference across Z_0 is given by

$$V_0 = I_0 Z_0. \tag{23}$$

The potential differences between the ends of the parallel line are

$$x = 0, \quad V = V_0^e - V_0 = V_0^e - I_0 Z_0; \tag{24a}$$

$$x = s, \quad V = V_s = I_s Z_s. \tag{24b}$$

These boundary conditions for (20) are formally exactly like those in the low-frequency case of a parallel line terminated by lumped impedances Z_0 and Z_s with a generator of electromotive force V_0^e connected in series with Z_0. The solution of (20) subject to (24) for the current amplitude at any point x along the parallel line has already been studied and expressed in terms of conventional transmission-line theory.[5,6] The solution is,

$$I_x = (V_0^e Y_0/H)[Z_c Y_s \cosh K(s-x) + \sinh K(s-x)] \tag{25a}$$

$$H = (Z_c^2 Y_0 Y_s + 1) \sinh Ks + Z_c(Y_0 + Y_s) \cosh Ks. \tag{25b}$$

This relation is of the same form as (19a), which was to be solved, if written as follows:

$$I_{1x} Z_{11} = V_0^e. \tag{26}$$

In this case the contour variable along the line is x, and the reference point for the corresponding distribution factor $f(x)$ is the point x measured from the terminal impedance Z_0. (The fixed length s in

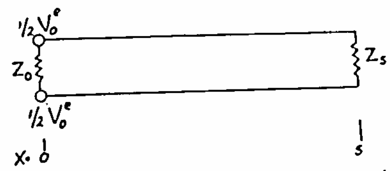

Fig. 1—Schematic diagram of an unsymmetrical oscillator.

(25) is not related to the contour variable s used in (10).) If it is desired to determine the current I_0 entering the terminals of Z_0 in terms of point generators symmetrically placed at $x=x$, this is easily done. By treating Z_0 (or Y_0) in (25) as a section of a parallel line of length s' extending beyond $x=0$, and terminated by an impedance $Z_{-s'}$ and then moving the origin of x to this impedance the following expression is obtained. It may also be derived by applying a general reciprocity theorem[7,8] to (25a).

$$I_0 = (V_x^e Y_0/H)[Z_c Y_s \cosh K(s-x) + \sinh K(s-x)] \tag{27a}$$

$$V_0 = I_0/Y_0. \tag{27b}$$

It is thus clear that a solution of (19a) has been derived for the parallel-line section of an oscillator terminated by general impedances and with point generators anywhere along the parallel line. By treating the section of line extending in either direction beyond the generators as a terminal impedance Z_0 and shifting the origin of x to the point generators, (25) gives the current distribution in the part of the line not included in Z_0.

If it is desired to determine the current distribution in a symmetrical oscillator which has identical generators and terminal impedances at each end of a section of parallel line, the following simple analysis may be used. Let a second oscillator, exactly like the one analyzed above, be placed end to end with this latter. Let the former extend from $x=s$ to $x=2s$.

[5] Ronold King, "Electrical measurements at ultra-high frequencies," Proc. I.R.E., vol. 23, pp. 885–934; August, (1935).
[6] Cohen, "Heaviside's Electrical Circuit Theory," p. 117, McGraw-Hill Book Co., New York, N. Y., (1928).
[7] Pierce, "Electric Oscillations and Electric Waves," p. 204. McGraw-Hill Book Co., New York, N.Y., (1920).
[8] Page 887 of footnote reference 4.

Let Z_0 of the first oscillator equal Z_{2s} of the second oscillator. The current distribution in the second oscillator is

$$I_x = (V_{2s}^e Y_{2s}/H)\left[Z_c Y_s \cosh K(x-s) + \sinh K(x-s)\right]. \quad (28)$$

Now let Z_s be so chosen that the current in both oscillators at corresponding points along their extension is exactly the same in magnitude and in direction. In particular,

$$I_s^{(1)} = I_s^{(2)}. \quad (29)$$

Upon solving (25a) and (28) simultaneously subject to (29) and with $V_0^e = V_{2s}^e$, one readily proves that

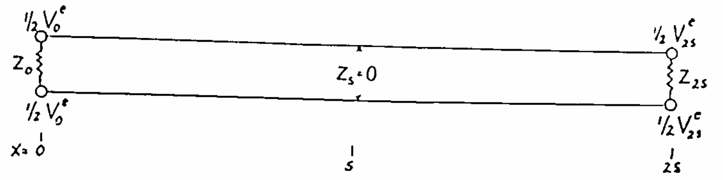

Fig. 2—Schematic diagram of a symmetrical oscillator.

Y_s terminating each oscillator must become infinite, or Z_s must vanish. The current distributions in the two oscillators are, then,

$$I_x^{(1)} = (V_0^e Y_0/H') \cosh K(s-x) \quad 0 \leqq x \leqq s \quad (30a)$$

$$I_x^{(2)} = (V_{2s}^e Y_{2s}/H') \cosh K(x-s) \quad s \leqq x \leqq 2s. \quad (30b)$$

With this current distribution the impedance at $x=s$ vanishes in both oscillators. Consequently each may serve as $Z_s=0$ for the other. One then has a symmetrical oscillator extending from $x=0$ to $x=2s$ with current distribution given by

$$I_x = (V^e Y/H') \cosh K(s-x). \quad (31a)$$

Here and above,

$$H' = Z_c Y \sinh Ks + \cosh Ks; \quad (31b)$$

also,

$$Y = Y_0 = Y_{2s}, \quad (32a)$$

$$V^e = V_0^e = V_{2s}^e. \quad (32b)$$

The circuit is illustrated in Fig. 2.

If the oscillator rods have sufficiently low resistance (31c) becomes,

$$I_x = (V^e Y/H'') \cos \beta(s-x) \quad (33a)$$

$$H'' = jZ_c Y \sin \beta s + \cos \beta s. \quad (33b)$$

The dependence of I_x upon x is thus seen to be cosinusoidal with a maximum amplitude at the center $x=s$ of the oscillator rods. The current-distribution problem contained in (19a) has thus been solved for the symmetrical parallel-rod oscillator as well as for the unsymmetrical one.

Solution of the Secondary Equation

The determination of the current distribution in a secondary parallel line (terminated by general impedances) which is loosely coupled to an oscillator of the simple type such as has just been analyzed is equivalent to a solution of (19b). Two steps are involved. The first is the evaluation of V_2^i as defined by the right side of (19b). The second is the solution of the left side of (19b) for the current distribution in the secondary in terms of V_2^i.

The calculation of V_2^i from Z_{21} as defined by an integral of the form (11c), is so complicated, that an alternative method must be used as follows. The defining relation for the scalar potential V in terms of the electric field E and the vector potential A is contained in the vector relation,

$$E = -\nabla V - j\omega A. \quad (34)$$

The components of E and A parallel to the conductors of the parallel line satisfy the following relation:

$$\partial V/\partial x = -E_x - j\omega A_x. \quad (35)$$

By integrating over the length of the secondary line, one obtains

$$V^i = V_0 - V_s = \int_0^s E_x dx + j\omega \int_0^s A_x dx$$

$$= \int_0^s (E_x + j\omega A_x) dx. \quad (36)$$

If E_x and A_x are calculated entirely in terms of the current and charge distribution in the oscillator, (36) gives the induced electromotive force V^i in one conductor of the secondary. The evaluation of the first integral in (36) is possible in reasonably simple form for the loosely coupled circuits here involved if one assumes the undamped current distribution (33a) to obtain in the oscillator. A curve for one rather typical case is shown in Fig. 3. The calculation

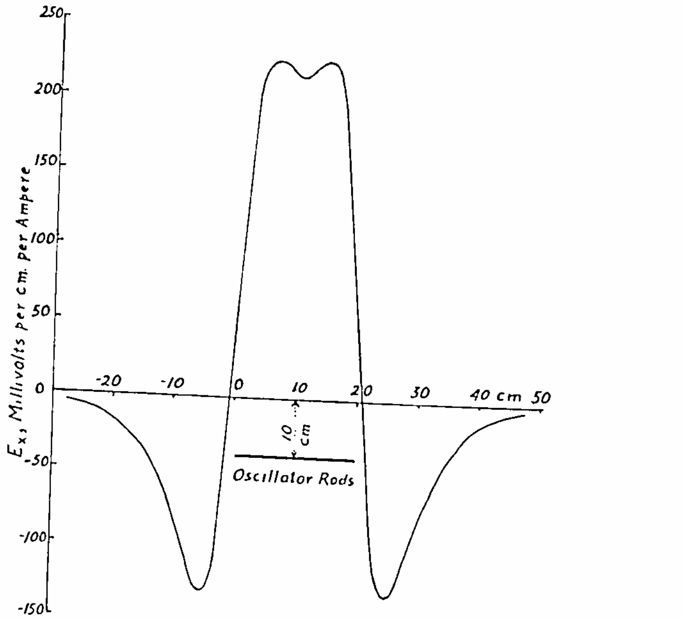

Fig. 3—Theoretical curve of the electric field along a conductor due to a loosely coupled oscillator. The oscillator length is 20 centimeters; the separation of its rods is 2 centimeters; it is coupled 10 centimeters from the parallel line. It is operating at a wavelength of 188.4 centimeters. A cosinusoidal current distribution is assumed.

of the second integral in (36) is much more laborious, and since it is actually not necessary to determine V_2^i from (36), it will not be carried out. All that is necessary is to draw certain general conclusions from (36), using the fact that both E and A as calculated for an element of a current-carrying conductor diminish at least as rapidly as $1/r$, where r is the distance from the element to the point of calcution. Furthermore, in the case of the parallel line, two conductors carrying oppositely directed currents and charges of opposite signs are separated a distance b which is small compared with a wavelength. That is,

$$(\beta b)^2 = (2\pi b/\lambda)^2 \ll 1. \tag{37}$$

The loose-coupling condition requires that the shortest distance from the oscillator rods to the coupled parallel line shall certainly satisfy the inequality

$$c^2 \gg b^2. \tag{38}$$

It follows that E_x and A_x in (36) must never be calculated nearer than the distance c. That is,

$$r^2 \geqq c^2 \gg b^2. \tag{39}$$

One can now conclude that the contribution to E_x or to A_x due to a pair of elements ds one in each conductor of the oscillator, can have no term which diminishes less rapidly than one which is proportional to

$$1/r_1 - 1/r_2 = (r_2 - r_1)/r_1r_2. \tag{40}$$

With (39) one can write, using the law of cosines,

$$r_1 = \sqrt{r_2^2 + b^2 - 2br_2 \cos (r_2, b)}$$
$$\doteq r_2 - b \cos (r_2, b). \tag{41}$$

Hence, $r_2 - r_1$ is of the order of magnitude of b, and the slowest possible decrease of E_x or A_x with distance is proportional to

$$1/r_1 - 1/r_2 \sim b/r_1r_2 \sim b/r^2. \tag{42}$$

Here r is the distance from the point of calculation to a point midway between the pair of elements ds (Fig. 4). Since the contribution to E_x and A_x of every pair of elements in the oscillator rods must diminish at least as rapidly as $1/r^2$, one can conclude that values of E_x and A_x computed at points beyond the ends of the oscillator rods, and due to the entire length of these, must also diminish at least as $1/r^2$, where now r is a mean value between the nearest and the farthest points on the oscillator to the point of calculation. It follows that the significant contributions to the integrals in (36) will be for a range of x not extending very far beyond points opposite the ends of the oscillator rods. This is verified by the curve for E_x (Fig. 3) which indicates a rapid decrease beyond the ends of the oscillator. If the integrand on the right in (36) is separated into its real and imaginary parts (both E_x and A_x are complex), each part could be plotted as a function of x. The areas under the curves so obtained would give the quadrature components of the induced electromotive force V_2^i. (In the case calculated for E_x alone in Fig. 3, the real part was negligible compared with the imaginary.) Each curve would necessarily be continuous and reasonably smooth, so that it might be represented to a good approximation by a Fourier series of the form

$$dV_2^i = \sum_{m=1}^{n} a_m \cos m\beta'(\bar{x} - x) \tag{43a}$$

$$\text{in the range,} \quad (\bar{x} - h) \leqq x \leqq (\bar{x} + h'). \tag{43b}$$

Here $\bar{x}$ is the co-ordinate of the center of the oscillator relative to the coupled line, h and h' are arbitrary lengths to be determined from the rapidity with which the represented curve approaches negligibly small values beyond the ends of the oscillator. If the curve is symmetrical, $h = h'$.

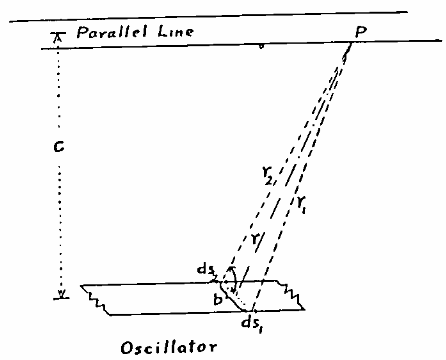

Fig. 4—Diagram to illustrate relative positions of the parallel conductors in the oscillator and in the parallel line. The electric and vector potential fields are to be calculated at P.

With V_2^i formally determined, it is now necessary to solve the final part of the problem, that of calculating the current distribution in the secondary line in terms of V_2^i. In order to do this it is only necessary to use the expression for the current amplitude, I_0, entering the terminal impedance, Z_0, in terms of point generators placed at $x = x$ along the line, given by (27). The distributed induced electromotive force, defined by (43), may be considered to be made up of an infinite number of point generators of suitable amplitude and phase placed end to end over the range given by (43b). An oppositely directed set may be arranged in the second conductor. Each pair of point generators contributes to I_0 an amount given by (27a). The contribution due to the entire distributed electromotive force is given by the integral, obtained by substituting (43a) for V_x^e in (27a) and integrating over the range (43b). Since (27a) involves hyperbolic functions, it is slightly more convenient to use the series

$$dV_1^i = \sum_{m=1}^{n} a_m \cosh mK'(\bar{x} - x) \tag{44}$$

instead of (43a). This reduces to (43a) with α' in, $K' = \alpha' + j\beta'$, set equal to zero.

Upon substituting (44) in (27a) and integrating, one has

$$I_0 = (Y_0/H) \sum_{m=1}^{n} a_m \int_{\bar{x}-h}^{\bar{x}+h'} [Z_c Y_s \cosh K(s-x) + \sinh K(s-x)] \cosh K'm(\bar{x}-x)dx. \quad (45)$$

This expression involves the following two integrals:

$$A = \int_{\bar{x}-h}^{\bar{x}+h'} \cosh K(s-x) \cosh K'm(\bar{x}-x)dx \quad (46)$$

$$B = \int_{\bar{x}-h}^{\bar{x}+h'} \sinh K(s-x) \cosh K'm(\bar{x}-x)dx. \quad (47)$$

These may be integrated after some manipulation to give

$$A = f_m' \cosh K(s-\bar{x}) + f_m \sinh K(s-\bar{x}) \quad (48a)$$

$$B = f_m' \sinh K(s-\bar{x}) + f_m \cosh K(s-\bar{x}) \quad (48b)$$

with,
$$f_m' = \left[\frac{\cosh F_m h - \cosh F_m h'}{2F_m} + \frac{\cosh G_m h - \cosh G_m h'}{2G_m}\right] \quad (49a)$$

$$f_m = \left[\frac{\sinh F_m h + \sinh F_m h'}{2F_m} + \frac{\sinh G_m h + \sinh G_m h'}{2G_m}\right]. \quad (49b)$$

Here,
$$F_m = K + mK' \quad (50a)$$
$$G_m = K - mK'. \quad (50b)$$

With (48), (46) becomes,

$$I_0 = \sum_{m=1}^{n} (Y_0 a_m/H)[f_m'\{\cosh K(s-\bar{x}) + Z_c Y_s \sinh K(s-\bar{x})\} + f_m\{\sinh K(s-\bar{x}) + Z_c Y_s \cosh K(s-\bar{x})\}]. \quad (51)$$

This expression reduces exactly to the form (27a) under the following conditions:

$$f_m' = 0 \quad (52a)$$

$$V_x^e = \sum_{m=1}^{n} a_m f_m. \quad (52b)$$

It thus appears that subject to (52) each quadrature component of an electromotive force which is induced over a large or smaller part of a parallel line, and hence the resultant induced electromotive force itself, may be treated exactly as if it consisted only of two point generators located at opposite points x in the parallel line.

Since the real parts of the hyperbolic functions are essentially positive and not periodic, (52a) will be satisfied in general only if

$$h = h'. \quad (53)$$

This condition requires that the reference point x must be at the center of the section, of length $h+h'=2h$, over which the induced electromotive force is distributed. An examination of the form of the assumed electromotive force (44), reveals that (53) is actually a symmetry condition demanding that the amplitude distribution of the induced electromotive force be symmetrical with respect to the center x of the section $2h$ over which it has significant values. This is equivalent to requiring the current distribution in the primary oscillator to be symmetrical with respect to its center.

Subject to (52), (51) becomes,

$$I_0 = (V_x^e Y_0/H)[Z_c Y_s \cosh K(s-\bar{x}) + \sinh K(s-\bar{x})]. \quad (54)$$

This is entirely like (44a), which defines the current amplitude at $x=0$ in terms of point generators at $x=x$. The only difference between (54) and (27a) is that the co-ordinate $\bar{x}$ locating the center of an extended symmetrical oscillator, appears in (54) whereas the co-ordinate x locating an hypothetical point generator appears in (27a). It is readily shown that if h is sufficiently small compared with a wavelength so that

$$\sinh (K + mK')h \doteq (K + mK')h \quad (55)$$

one has for (52b)
$$V_x^e \doteq \sum_{m=1}^{n} 2ha_m. \quad (56)$$

By allowing h to approach zero, the case of point generators is obtained. One must, of course, assume that a_m becomes infinitely large in the limit as h vanishes.

One may conclude that the following theorem is true.

Theorem: A distributed electromotive force induced in a section of a parallel line may be treated just as though it were concentrated at the center of the section, provided only it is symmetrical with respect to that center. The distribution may be as complicated as desired, but the symmetry condition must be satisfied.

It follows that if a symmetrical oscillator is coupled to a parallel line of about equal length, that the centers of the line and of the oscillator must be opposite each other In each case the secondary behaves just as if a pair of point generators of appropriate amplitude were placed at its center. If the oscillator is very short compared with the secondary line, and it is no farther from the line than its own length, it may be coupled anywhere along the line except near the ends. Wherever it is coupled, the secondary will behave just as though two point generators were connected opposite its (the oscillator's) center.

The current-distribution problem in the secondary for the case of a symmetrical oscillator is now solved in unexpectedly simple form. It has, in fact, been reduced precisely to the case of the primary with point sources of electromotive force. The fact that in the primary solution (25a), the point generators are at one end, whereas in the present case they are at any point along the parallel conductors, introduces no difficulties whatsoever. It will be recalled that the terminal impedance Z_0 was in no way restricted. It may evidently include any desired length of parallel line, which may, in turn, be terminated by perfectly general impedance. By simply choosing the origin of the co-ordinate x at the location of the equivalent point generators, instead of at Z_0, the section of parallel line extending in either direction together with its terminal impedance may be assumed to be a new terminal impedance Z_0. The current distribution in the part of the line in the other direction is then given by (25a). This is the desired solution.

Generalization of the Solutions

In solving both the primary and secondary problem, it was assumed that the oscillator, as well as the coupled secondary, consisted of sections of parallel conductors. It is now readily seen that other types of circuits may be handled in a similar manner. In so far as the oscillator is concerned, the only fundamental requirement is that it must be of a form such that the components of E and A parallel to the secondary and due to the primary current and charge distribution are symmetrical with respect to the center of the oscillator. For example, the oscillator rods may be two semicircular sections, or they may be two similar parallel coils. The form of the secondary cannot be generalized very much, since it is already very general in not restricting the shape, construction, or extension of the terminal impedances. The problem of determining the current distribution in these is beyond the scope of the present paper, unless they are also sections of parallel line. An important generalization may be made, however, to include the case of a single conductor instead of two parallel ones. If an oscillator is coupled to such a conductor, as for example to an antenna, the induced electromotive force may evidently be assumed concentrated at a point opposite the center of the oscillator, provided this is symmetrical.

If the oscillator is not symmetrical, it can be shown that by neglecting damping, each term of the Fourier series representing the quadrature components of (36) may be treated as though concentrated at a point. However, the points due to the several terms do not coincide. For the m^{th} term an equivalent point generator must be assumed placed at

$$x_m = \bar{x} + (1/\beta) \arctan (f_m'/f_m). \qquad (57)$$

Here f'_m and f_m are defined by (49) by setting $K \doteq j\beta$, $K' = j\beta'$ in (50). Since the E and A fields due to an unsymmetrical oscillator are likely to require many terms, and since, moreover, a knowledge of these involves the actual evaluation of (36), one can conclude that the unsymmetrical case, although theoretically solved, is not practically useful. Only in the symmetrical problem do all the points x_m defined by (57) fall together, and so permit the complicated problem of distributed coupling to be handled like the simple problem of two point generators.

Experimental Verification and Significance—Conclusion

The fundamental theory of distributed coupling developed above has been experimentally verified both in the case of a coupled parallel line, and in the case of a coupled antenna. The verifications proceed from a very simple relation involving the position along the line or antenna of point generators. It has

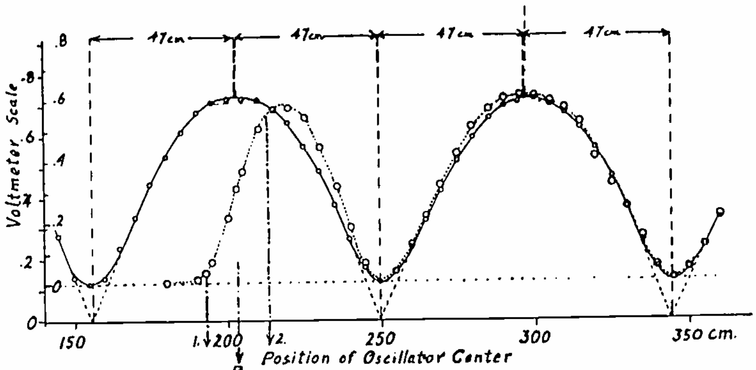

Fig. 5—Deflection of voltmeter at $x = 400$ centimeters as a symmetrical oscillator is moved along a parallel line. Correction for a lower curvature of the voltmeter characteristic is indicated. The solid curve was obtained with a bridge fixed at $x = 109.0$ centimeters; the dotted curve with a bridge fixed at 205.0 centimeters. Amplitudes were adjusted to be about equal in the two cases. Position 1 to the left of 200 centimeters gives the location of the oscillator center when the entire oscillator (except the one triode) was to the left of a plumb line dropped from the bridge. Position 2 indicates the location of the oscillator center when the entire oscillator (except one triode) was to the right of the plumb line from the bridge. The measured half wavelength was 94.0 centimeters.

been shown[9] that the voltage amplitude V_0 across a terminal impedance Z_0 is given by the following simple expression:

$$V_0 = [V_x^e/Q(s)] \cos \beta(x_m - x). \qquad (58)$$

Here x is the position of a pair of point generators measured from Z_0. If an extended oscillator is coupled to a parallel line instead of introducing point generators in it, the above relation should be valid if, and only if the extended oscillator is equivalent to a pair of point generators connected opposite its center. In this case the co-ordinate $\bar{x}$ of the center should appear in (58) in place of the coordinate x locating the point generators, but the relation should be otherwise unchanged. This is completely verified by Fig. 5 in the paper referred to in reference 5. It is also verified in Fig 5 for a parallel line, and in Fig. 6 for an antenna. It is found experimentally that the maxima of the curve defined by (58) occur exactly when the center of a symmetrical oscillator is below a true current

[9] Page 898, formula (3) of footnote reference 4.

loop in the parallel line. It is to be noted that the position of a bridge giving a maximum value for V_0 is not at a true current loop, but is shifted toward the voltmeter by the equivalent length of the bridge if this is inductive, away from the voltmeter if this is capacitive. This has been discussed in detail.[8]

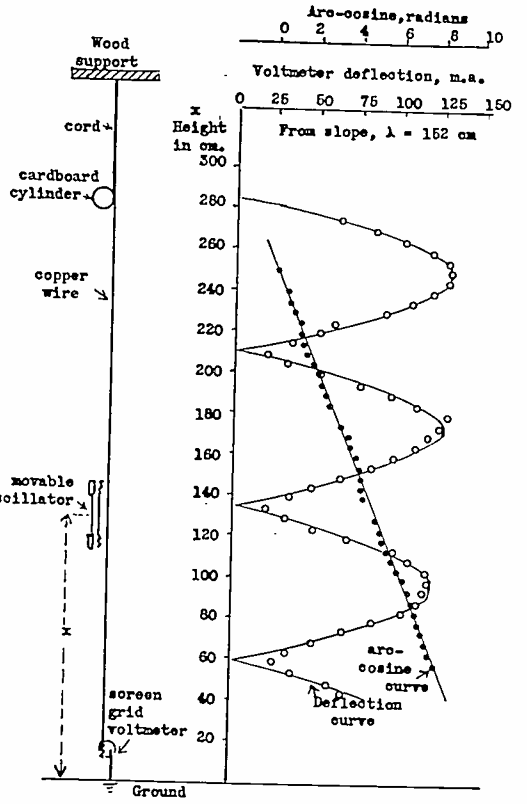

Fig. 6—Deflection of the voltmeter at the base of the vertical antenna as a symmetrical oscillator is hoisted along it.

In order to determine the rapidity with which the field due to a symmetrical oscillator diminishes beyond the ends of the oscillator, the dotted curve of Fig. 5 was plotted. In this case the oscillator was moved below a bridge with properly adjusted tandem, so that the electromotive force induced beyond the bridge did not contribute to the current amplitude in the parallel line proper. The amplitude decrease is seen to be extremely rapid, although the oscillator was very loosely coupled. It is to be noted that the amplitude of the dotted curve in Fig. 5 rises above that of the solid curve as the oscillator approaches the bridge moving toward the left. This indicates that as part of the oscillator field is cut off, a *greater* electromotive force is induced in the line. The reason is clear from Fig. 3 which is calculated for the oscillator used in obtaining the data of Fig. 5. The contribution of E_x to the induced electromotive force is seen to reverse very near the ends of the oscillator. If a part of this reverse field is cut off, the effective field is increased, and with it the induced electromotive force. The theory is thus very satisfactorily and convincingly verified.

The significance of the present solution of the fundamental problem of distributed coupling should not be underestimated because it has been taken for granted, or assumed as an approximation without proof. It is to be noted that major parts of various methods of electrical measurements at ultra-high frequencies, in particular a method for calibrating current and voltage devices,[10] depends directly upon the correctness of the assumption that a distributed induced electromotive force might be represented by point generators. This has been questioned recently, and proof is now given. The fact that induced electromotive forces due to extended oscillators with non-uniform current distribution can be represented in this simple way under very general conditions which are specified, reduces a formally perplexing problem to ordinary methods of circuit analysis familiar from low-frequency transmission-line theory. Aside from its more general application, the present paper provides a complete analysis of the problem of two loosely coupled sections of parallel line.

[10] Page 899 ff. of footnote reference 4.

Television Detail and Selective-Sideband Transmission*

STANFORD GOLDMAN†, ASSOCIATE, I.R.E.

Summary—*Recently selective-sideband transmission has been adopted in this country in order to increase the picture detail which can be transmitted in a frequency channel of given width. The present analysis is an examination of the theoretical justification for this practice.*

The effect of various degrees of unequal-sideband transmission on typical television signals is calculated by Fourier integral analysis. The conclusions reached are as follows:

(1) *For small percentages of modulation and small changes in the percentage of modulation selective-sideband transmission is equivalent to double-sideband transmission of twice the frequency pass band.*

(2) *For large changes in the percentage of modulation, selective-sideband transmission is equivalent to double-sideband transmission of twice the frequency pass band for the reproduction of symmetrical fine detail. However, at the edges of patterns where there is a great change of visual intensity, selective-sideband transmission introduces more or less false detail, not present in the original signal. The extent of this false detail increases with the sharpness of the rise of the frequency characteristic at the edge of the transmission band where the carrier is located.*

Introduction

In order to increase the detail observable in a television picture, it is necessary that higher and higher video frequencies be used. This in turn requires, among other things, that signal channels in television have extremely wide band widths. Thus, with the present standard 441-line pictures, television-channel widths are measured in megacycles, instead of kilocycles as is customary in sound broadcasting. Under these circumstances, it has been inevitable that serious efforts should have been made to find means of decreasing the required channel widths, in order to allow for more television channels in the available frequency spectrum. This need has given rise to the use of selective-sideband transmission[1] in television, which now has been tentatively standardized. It is the purpose of the present paper to make a theoretical analysis of selective-sideband transmission in television, to see how it affects the amount of picture detail which can be transmitted in a frequency channel of given width.

An important contribution to this study has already been made by Poch and Epstein.[2] These investigators found experimentally that, in a given frequency channel, improved picture detail could be obtained by the use of selective-sideband operation. They also verified these conclusions mathematically. However, their analysis was essentially of a steady-state character, in contrast with the transient analysis, which is to be given here. It is believed by the present author that the transient analysis gives a more fundamental understanding of the problem because video-frequency signals are essentially of a transient character.

We may point out here that the size of the scanning spot and the intensity distribution within it, both at the transmitter and the receiver, also affect the ultimate picture detail obtainable. This problem has been attacked by previous investigators,[3,4,5] but while the results which they found and the conclusions drawn from them are important in their own right, they do not greatly affect the characteristic features of selective-sideband transmission. Therefore, we shall not consider this subject at present.

Statement of the Problem

As is well known, the effect of the frequency spectrum on the detail observable in a television picture appears in the horizontal resolution. Due to the large number of picture elements which must be traced every second by the scanning spot, very high frequency components must be present in the television signal in order to show fine detail in the horizontal direction.

Let us suppose that we are looking at some action on a television screen, and that we can see a man's face. There is a certain amount of detail. With better detail, however, we can make out his nose, and with still better detail perhaps even a freckle on his nose. Now what does this all mean technically? Just this —when we see a detail in a certain region, it means that we can distinguish between the intensity of light at this point of detail and the intensity in the region around it. We may therefore specify a detail as shown in Fig. 1(b), where the ordinate is intensity and the abscissa is spatial extension.

The ability of the system to reproduce detail then depends on just how wide such a trough must be so that it is discernible in the final picture and is not indistinguishable from the surrounding region. In practical systems if the width BC is made too nar-

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[1] Selective-sideband transmission is the term applied to the type of transmission obtained when the position of the carrier in the pass band is that designated as position V in Figs. 4, 5, 6 and 8; i.e., the transmission of the carrier is 50 per cent. This is effectively single-sideband transmission for high frequencies, gradually changing over to double-sideband transmission at very low frequencies.

[2] Poch and Epstein, "Partial suppression of one sideband in television reception," Proc. I.R.E., vol. 25, pp. 15–31; January, (1937); *R.C.A. Rev.*, vol. 1, pp. 19–35; January, (1937).

[3] F. Gray, J. W. Horton, and R. C. Mathes, "The production and utilization of television signals," *Bell Sys. Tech. Jour.*, vol. 6, pp. 560–603; October, (1927).

[4] P. Mertz and F. Gray, "A theory of scanning and its relation to the characteristics of the transmitted signal in telephotography and television," *Bell Sys. Tech. Jour.*, vol. 13, pp. 464–515; July, (1934).

[5] H. A. Wheeler and A. V. Loughren, "The fine structure of television images," Proc. I.R.E., vol. 26, pp. 540–575; May, (1938).

row, the detail will be lost in the background of the surroundings after transmission.

In actual practice it turns out that a detail BC as defined in Fig. (1c) is actually a still better test pattern for determining the resolving power of a system. This will be evident after we have made our analysis. It might be supposed offhand that the slope after transmission of a line such as A in Fig. (1a) should be a good measure of the resolving power of a system. We shall find, however, that this is by no means true. In Fig. 1(d, e, f, and g) are some additional types of elementary details which are useful in analyzing the characteristics of a television system.

For the purpose of our analysis we shall assume the scanning at both the camera tube and picture tube to be done by a spot of a height of exactly one line and infinitesimal width. In this way the details in Fig. 1 will transform into signals having exactly the same form as the functions in Fig. 1, only the abscissa in the case of the signals will be time. In a future paper we will show how the results can be generalized for spots of arbitrary form.

The problem of analyzing the effect of the transmission system on picture detail will thus consist of the following steps:

1. Choose the form of the detail to be transmitted.
2. Choose the characteristics of the transmission system.
3. Write down the chosen detail as modulating a radio-frequency carrier.
4. Find the expression for the transmitted modulated radio-frequency carrier after going through the chosen transmission system.
5. Determine the form of the envelope of the transmitted wave.

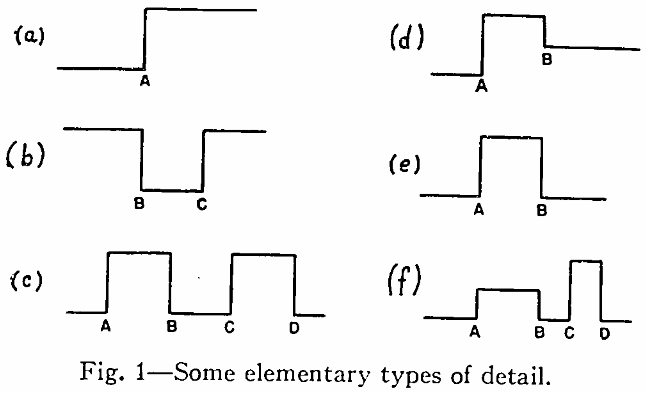

Fig. 1—Some elementary types of detail.

Mathematical Analysis of the Fundamental Case

Let us first consider the signal shown in Fig. 2(A), or rather the same signal modulating a radio-frequency carrier[6], $\sin \omega_c t$. Fig. 2(B). This signal may be expressed by

[6] In the case considered here, the radio-frequency signal is 100 per cent modulated. In a later section we shall indicate how the results about to be derived can, by simple modifications, be used to analyze cases with any percentage of modulation.

$$\left.\begin{aligned} \phi(t) &= \sin \omega_c t \text{ from } T_1 \text{ to } T_2 \\ \phi(t) &= 0 \text{ everywhere else} \end{aligned}\right\} \quad (1)$$

where $\omega_c = 2\pi$ times the carrier frequency shown in Fig. 2. Next we send this signal through a transmission system having the characteristics shown in Fig. 3. In this system we have

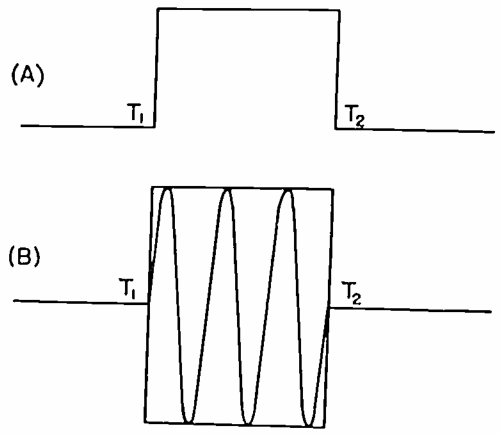

Fig. 2—The fundamental pulse
(A) As video frequency.
(B) As modulating a radio-frequency carrier.

$$\left.\begin{aligned} \text{transmission} &= (\omega - \omega_3)/(\omega_1 - \omega_3) \text{ from } \omega_3 \text{ to } \omega_1 \\ \text{transmission} &= 1 \text{ from } \omega_1 \text{ to } \omega_2 \\ \text{transmission} &= (\omega_4 - \omega)/(\omega_4 - \omega_2) \text{ from } \omega_2 \text{ to } \omega_4 \end{aligned}\right\}. \quad (2)$$

The flat top of Fig. 3 is assumed to signify unity transmission.

$$\left.\begin{aligned} &\text{Phase change} = \phi = G(\omega - \omega_0) \text{ in the pass band and edge bands} \\ &\text{Phase change} = \pm n\pi \text{ outside of the transmission band} \end{aligned}\right\}. \quad (3)$$

In (3), G is a constant, representing the slope of the phase-change line in Fig. 3.

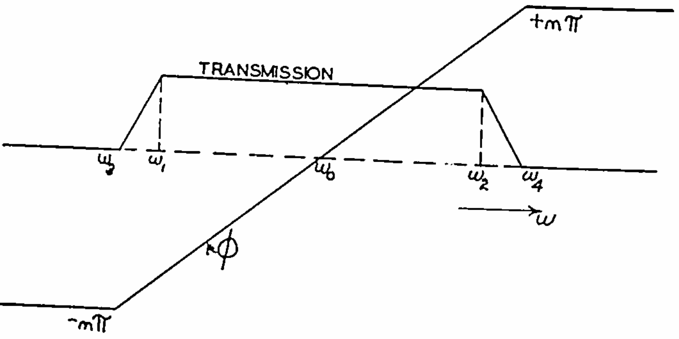

Fig. 3

The band-pass transmission system shown in Fig. 3 is an idealization of a present-day television system.[7]

In order to find the effect of the transmission system of Fig. 3, on the function $\phi(t)$, we must first express $\phi(t)$ in terms of its frequency components. To do this we make use of Fourier's integral theorem. Accordingly

[7] Fig. 7 of footnote reference 2.

$$\phi(t) = 1/\pi \int_0^\infty \left[\int_0^\infty \phi(t) \sin \omega t dt \right] \sin \omega t d\omega + 1/\pi \int_0^\infty \left[\int_0^\infty \phi(t) \cos \omega t dt \right] \cos \omega t dt. \quad (4)$$

Now, in this case

$$\int_{-\infty}^{+\infty} \phi(t) \sin \omega t dt = \int_{T_1}^{T_2} \sin \omega_c t \sin \omega t dt \quad (5)$$

$$= [\sin(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\sin(\omega-\omega_c)T_1/2(\omega-\omega_c)] - [\sin(\omega+\omega_c)T_2/2(\omega+\omega_c)] + [\sin(\omega+\omega_c)T_1/2(\omega+\omega_c).]$$

Likewise

$$\int_{-\infty}^{+\infty} \phi(t) \cos \omega t dt = \int_{T_1}^{T_2} \sin \omega_c t \cos \omega t dt \quad (6)$$

$$= -[\cos(\omega+\omega_c)T_2/2(\omega+\omega_c)] + [\cos(\omega+\omega_c)T_1/2(\omega+\omega_c)] + [\cos(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\cos(\omega-\omega_c)T_1/2(\omega-\omega_c)].$$

If, as we may assume, the carrier frequency is high with respect to the modulation-frequency range, then the terms with $(\omega+\omega_c)$ in the denominator become negligible with respect to the others. This can readily be verified in any specific case. Therefore approximately

$$\phi(t) = 1/\pi \int_0^\infty \{ [\sin(\omega - \omega_c)T_2/2(\omega - \omega_c)] - [\sin(\omega - \omega_c)T_1/2(\omega - \omega_c)] \} \sin \omega t d\omega + 1/\pi \int_0^\infty \{ [\cos(\omega - \omega_c)T_2/2(\omega - \omega_c)] - [\cos(\omega - \omega_c)T_1/2(\omega - \omega_c)] \} \cos \omega t d\omega. \quad (7)$$

Let us now send $\phi(t)$ through the transmission system shown in Fig. 3. Then the emerging signal $\phi'(t)$ may be expressed

$$\begin{aligned}
\phi'(t) = {} & 1/\pi \int_{\omega_3}^{\omega_1} \{ [\sin(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\sin(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} (\omega-\omega_3)/(\omega_1-\omega_3) \cdot \sin[\omega t - G(\omega-\omega_0)] d\omega \\
& - 1/\pi \int_{\omega_3}^{\omega_1} \{ [\cos(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\cos(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} (\omega-\omega_3)/(\omega_1-\omega_3) \cdot \cos[\omega t - G(\omega-\omega_0)] d\omega \\
& + 1/\pi \int_{\omega_1}^{\omega_2} \{ [\sin(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\sin(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} \cdot \sin[\omega t - G(\omega-\omega_0)] d\omega \\
& + 1/\pi \int_{\omega_1}^{\omega_2} \{ [\cos(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\cos(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} \cdot \cos[\omega t - G(\omega-\omega_0)] d\omega \\
& + 1/\pi \int_{\omega_2}^{\omega_4} \{ [\sin(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\sin(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} (\omega_4-\omega)/(\omega_4-\omega_2) \cdot \sin[\omega t - G(\omega-\omega_0)] d\omega \\
& + 1/\pi \int_{\omega_2}^{\omega_4} \{ [\cos(\omega-\omega_c)T_2/2(\omega-\omega_c)] - [\cos(\omega-\omega_c)T_1/2(\omega-\omega_c)] \} (\omega_4-\omega)/(\omega_4-\omega_2) \cdot \cos[\omega t - G(\omega-\omega_0)] d\omega. \quad (8)
\end{aligned}$$

By an involved series of transformations of integrals, (8) finally reduces to

$$\phi'(t) = M \sin(\omega^c t + G\omega_b) + N \cos(\omega_c t + G\omega_b) = \sqrt{M^2 + N^2} \sin(\omega_c t + G\omega_b + \theta) \quad (9)$$

$$\omega_b = \omega_0 - \omega_c$$

where

$$M = P(T_2 - t + G) - P(T_1 - t + G) \quad (10)$$

$$N = Q(T_2 - t + G) - Q(T_1 - t + G) \quad (11)$$

and

$$\theta = \tan^{-1} N/M.$$

The quantity $\sin(\omega_c t + G\omega_b + \theta)$ is just the carrier with a constant phase shift. The envelope function, which gives the wave shape of the emerging signal is thus $\sqrt{M^2+N^2}$. This is the quantity in which we are interested.

The functions P and Q of (10) and (11) are characteristic functions of the transmission network shown in Fig. 3 and of the position of the carrier. They are defined as follows:

$$\begin{aligned}
P(x) = {} & 1/[2\pi x(\omega_1-\omega_3)] \int_{(\omega_3-\omega_c)x}^{(\omega_1-\omega_c)x} \sin u du \\
& + [(\omega_c-\omega_3)/2\pi(\omega_1-\omega_3)] \int_{(\omega_3-\omega_c)x}^{(\omega_1-\omega_c)x} [(\sin \mu)/\mu] d\mu \\
& + 1/2\pi \int_{(\omega_1-\omega_c)x}^{(\omega_2-\omega_c)x} [(\sin \mu)/\mu] d\mu \quad (12) \\
& - 1/[2\pi x(\omega_4-\omega_2)] \int_{(\omega_2-\omega_c)x}^{(\omega_4-\omega_c)x} \sin \mu d\mu \\
& + [(\omega_4-\omega_c)/2\pi(\omega_4-\omega_2)] \int_{(\omega_2-\omega_c)x}^{(\omega_4-\omega_c)x} [(\sin \mu)/\mu] d\mu
\end{aligned}$$

$$\begin{aligned}
Q(x) = {} & 1/[2\pi x(\omega_1-\omega_3)] \int_{(\omega_3-\omega_c)x}^{(\omega_1-\omega_c)x} \cos \mu d\mu \\
& + [(\omega_c-\omega_3)/2\pi(\omega_1-\omega_3)] \int_{(\omega_3-\omega_c)x}^{(\omega_1-\omega_c)x} [(\cos \mu)/\mu] d\mu \\
& + 1/2\pi \int_{(\omega_1-\omega_c)x}^{(\omega_2-\omega_c)x} [(\cos \mu)/\mu] d\mu \quad (13) \\
& - 1/[2\pi x(\omega_4-\omega_2)] \int_{(\omega_2-\omega_c)x}^{(\omega_4-\omega_c)x} \cos \mu d\mu \\
& + [\omega_4-\omega_c/2\pi(\omega_4-\omega_2)] \int_{(\omega_2-\omega_c)x}^{(\omega_4-\omega_c)x} [(\cos \mu)/\mu] d\mu.
\end{aligned}$$

The curves of $P(x)$ and $Q(x)$ have been plotted for six different positions of the carrier in the pass band[8] as shown in Fig. 4. We see that the Q functions are symmetrical about the line $x=0$ while the P functions are antisymmetrical about this line.

With the aid of (9), (10), and (11) and Fig. 4, we can now find the signal emerging from the transmission system when an impulse of the type shown in Fig. 1(a) enters the network. In this case, we note that T_2 is infinite. Furthermore, as mentioned previously, we are considering the case of 100 per cent modulation. The results are shown[9] in Fig. 5, where we see that the signal rises most sharply when the carrier is at the center of the pass band. We would be apt to expect that the signal rise would be sharpest when the carrier is at the edge of the pass band, since in that case, higher video frequencies are passed. Of course to test this we should first normalize the six response curves to the same final steady-state value. This has been done in Fig. 6. Even in this case, however, the curve of case I rises as sharply as any except case VI, in which latter case the response is certainly far from a faithful reproduction

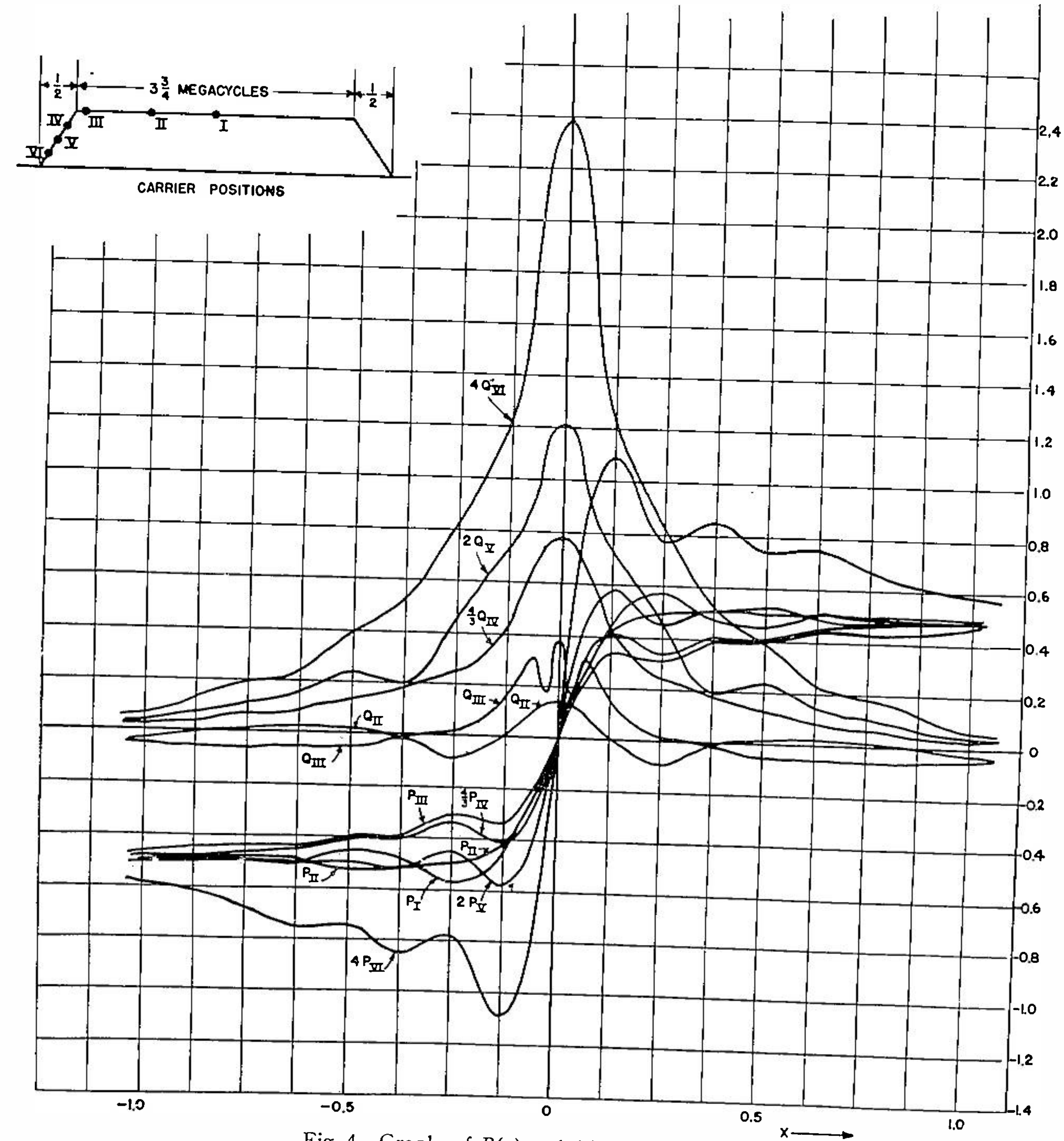

Fig. 4—Graphs of $P(x)$ and $Q(x)$, normalized.

[8] The numerical values

$$\omega_2-\omega_1=3\tfrac{3}{4} \text{ megacycles}$$
$$\omega_4-\omega_2=\omega_1-\omega_3=\tfrac{1}{2} \text{ megacycle}$$

were those commonly considered at the time this analysis was started. The present R.M.A. standard is, of course, different. The analytical results of the present paper and the foregoing equations apply, however, just as well to the present R.M.A. standard transmission characteristic, only the curves in the figures must be changed somewhat to bring them up to date.

[9] We see here that G is the time delay of the network. This time delay must be relatively long, since a large number of filter sections must be used to get the transmission characteristics of Fig. 3 and $G=2\pi n/(\omega_4-\omega_3)=n/4.75$ microseconds where n is the number of filter sections. If it were not for this time delay, there could, of course, be no signal in the negative region of $t-G$ as shown in Figs. 5 and 6.

of the signal.[10] We are, therefore, led to inquire whether there is, after all, any gain in sharpness of detail obtained by moving the carrier to the edge of the pass band. We shall shortly find, however, that detail really is improved by so doing, but first we must consider an important theorem.

A Theorem on the Addition of Component Functions

In (9) we showed that the response of a network to an impulse of the type shown in Fig. 2(b) may be expressed

$$\phi'(t) = [P(T_2-t+G) - P(T_1-t+G)] \sin(\omega_c t + G\omega_b) + [Q(T_2-t+G) - Q(T_1-t+G)] \cos(\omega_c t + G\omega_b). \quad (14)$$

due to any number of square impulses such as shown in Fig. 7 may be expressed

$$\begin{aligned}\phi'(t) = \{ & a[P(T_2-t+G) - P(T_1-t+G)] \\ & + b[P(T_4-t+G) - P(T_3-t+G)] \\ & + c[P(T_6-t+G) - P(T_5-t+G)] \\ & + \cdots \} \sin(\omega_c t + G\omega_b) \\ & + \{ a[Q(T_2-t+G) - Q(T_1-t+G)] \\ & + b[Q(T_4-t+G) - Q(T_3-t+G)] \\ & + c[Q(T_6-t+G) - Q(T_5-t+G)] \\ & + \cdots \} \cos(\omega_c t + G\omega_b). \quad (15)\end{aligned}$$

With the aid of (12), (13), and (15), we can now solve the response problem for any of the types of detail shown in Fig. 1.

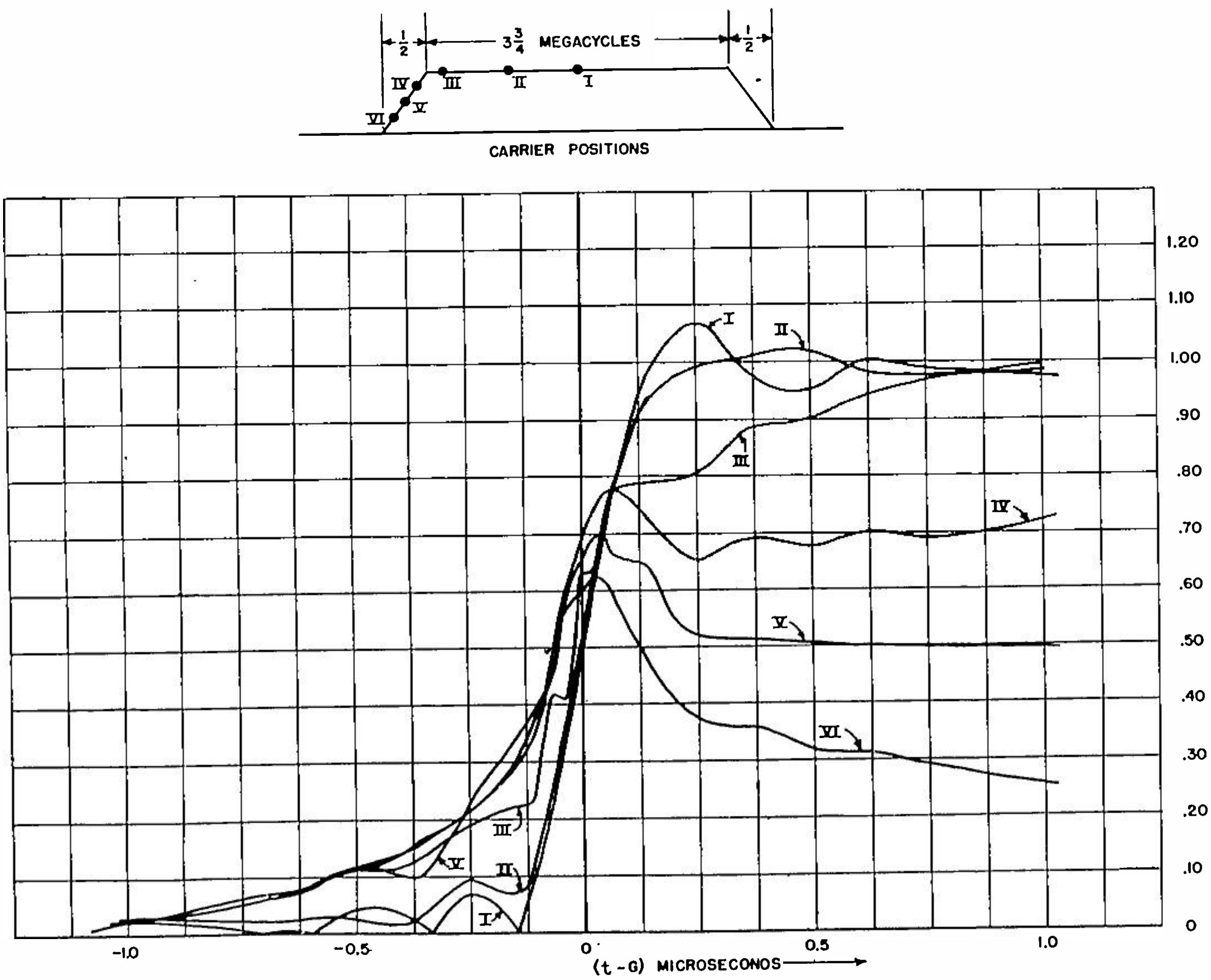

Fig. 5—Response to a unit-function signal.

Since the argument $(\omega_c t + G\omega_b)$ does not depend on the video-frequency signal, and since the response of the network is linear[11] it follows that the response

[10] The initial overshooting of the final value (commonly called a leading white) and the existence of signal ahead of $t-G=0$ (called anticipatory transients) are undesirable effects which are more pronounced in selective-sideband transmission than in the double-sideband case. Both of these effects become worse as the steepness of slope of the frequency characteristic is increased in the edge bands. They also rapidly become worse if the linear phase-versus-frequency characteristic is not maintained in the edge bands. While these effects are very important, we shall not consider them further in the present paper.

[11] Linearity in this case means that the magnitude of the response is directly proportional to the magnitude of the signal.

Analysis of Television Detail at 100 Per Cent Modulation

The above equations have been applied to the analysis of a detail of the type shown in Fig. 1(c), and the spacing was so chosen that the detail was just lost when the carrier was at the center of the pass band. The analysis was performed for a signal having 100 per cent modulation and was performed for six positions of the carrier in the pass band. The results are shown in Fig. 8.

In this figure we see that as the carrier is moved toward the edge of the pass band, the reproduction of detail progressively improves. *This is the real justification for selective-sideband transmission.* The reason that the selective-sideband detail shows up better in this case, even though no such superiority was shown in reproducing the unit function (Figs. 5 and 6) is that the Q functions cancel by symmetry in the center of a detail of the type shown in Fig. 8., while in the case of Figs. 5 and 6 the Q functions interfered with the sharpness of the rise at the edge.

A comparison[12] *of Fig. 8, case V, and Fig. 9(c) indicates that for the same width of pass band, selective-sideband transmission has almost a two-to-one superiority over double-sideband transmission for the reproduction of fine detail.* An analytical comparison of this type using a different elementary detail would very likely require some modification of the foregoing statement. The writer hopes to analyze additional cases at an early date. For the present, however, the tentative standardization of selective-sideband transmission appears to be well founded.

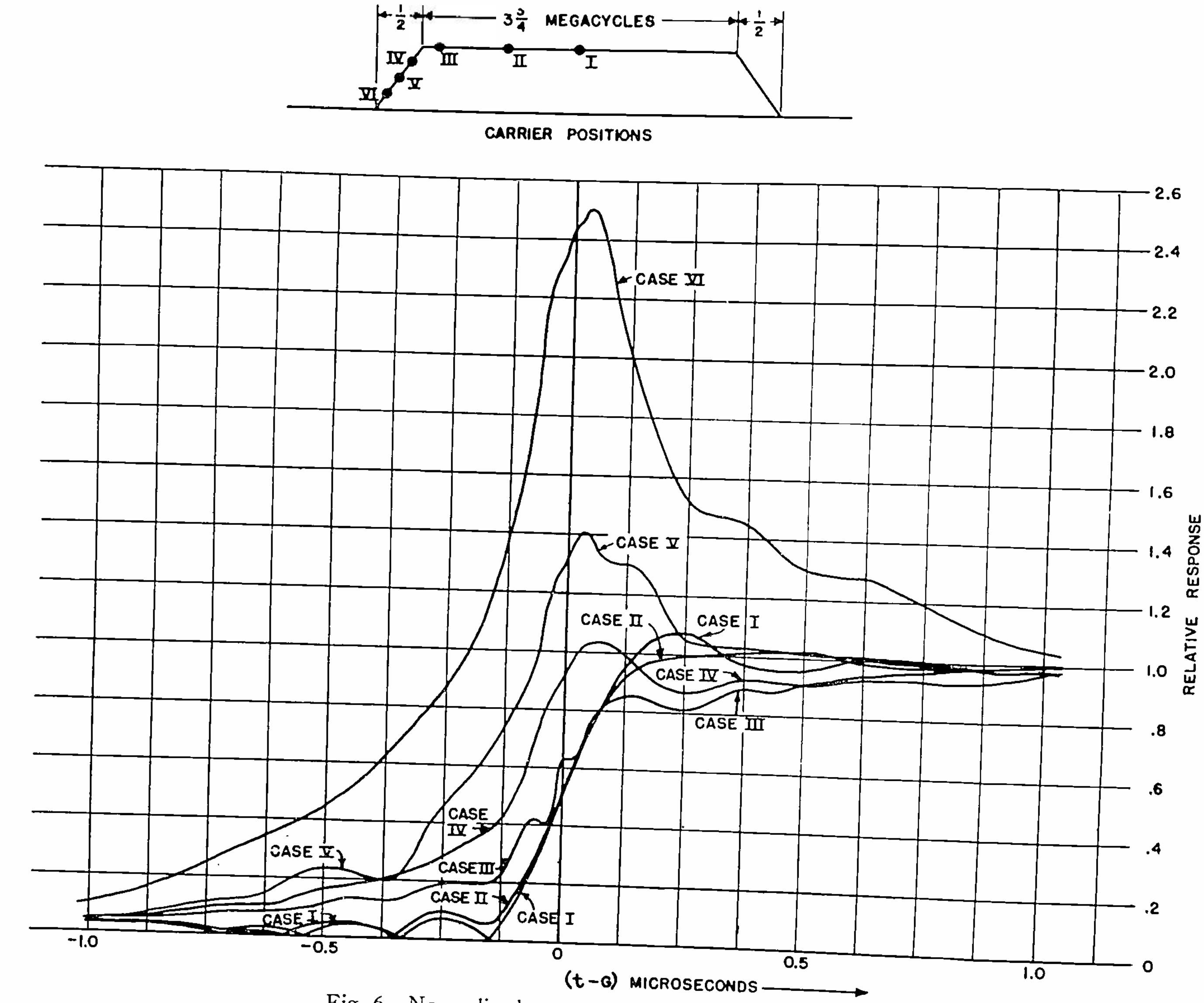

Fig. 6—Normalized response to a unit-function signal.

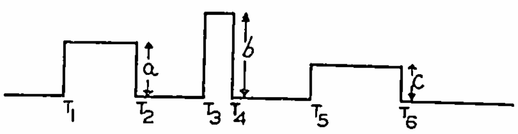

Fig. 7

As far as very fine detail is concerned, there seems to be little choice between positions IV, V, and VI. The choice in this case must be governed by the relative amount of lower-frequency response which is desired. For equally good reproduction of all frequencies, case 5 must be chosen. It may, at first, appear surprising that the response shown in Fig. 8 does not decrease progressively in magnitude as the carrier position is moved down the sloping side of the transmission band. This is a striking demonstration of the difference between the transient and steady-state cases. If we look into the matter closely, we can easily understand it. A Fourier integral analysis of the signal impulse shown in Fig. 8 will give a continuous distribution of its frequency components. There will not be components only at the carrier frequency and at sidebands corresponding to the fundamental and harmonics of the period of the square wave, as would be the case in the steady state. Therefore, if the carrier is partially (or even wholly[13]

[12] Taking proper account of their respective scales.

[13] If the carrier were moved slightly beyond ω_3, into the cutoff region, the response would still be quite similar to case VI.

removed) the initial response is not very greatly affected. The magnitude of the steady-state response, however, as shown for example in Fig. 5, is directly proportional to the fraction of carrier transmitted.

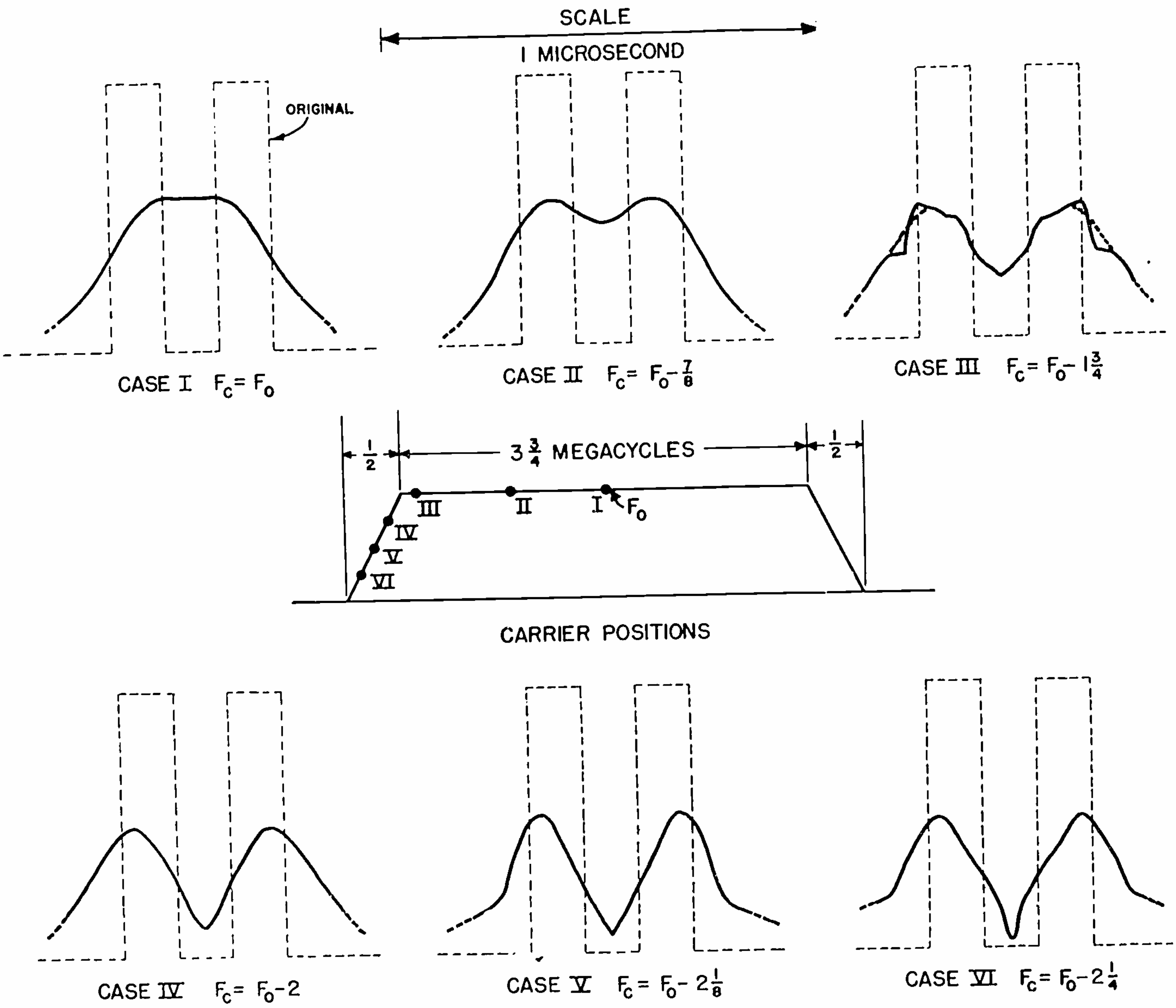

Fig. 8—Effect of carrier positions on the reproduction of detail.

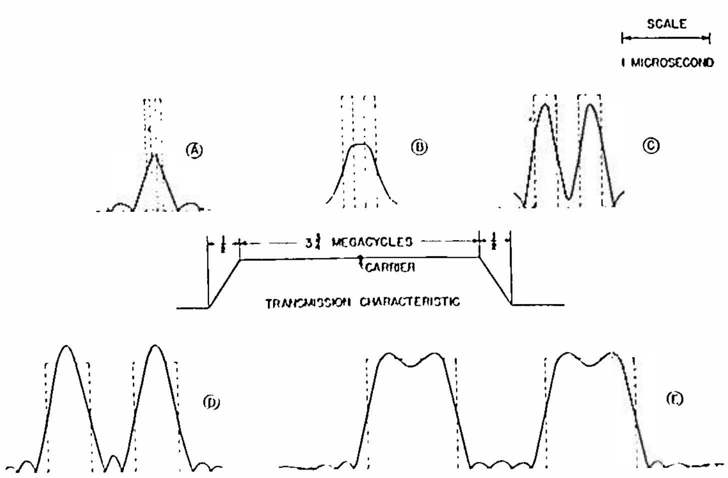

Fig. 9—Scale drawing showing improved detail as spacing is increased. Carrier at center of pass band.

In Fig. 9 are shown a group of picture details of various sizes, illustrating how the faithfulness of reproduction comes back as the detail size is increased. The carrier was considered at the center of the pass band in this case. Fig. 9(C) may be compared with case V of Fig. 8, showing that, as already stated, selective-sideband transmission has almost a two-to-one superiority over double-sideband transmission for the reproduction of fine detail.

Analysis of Television Detail for Any Range in the Percentage of Modulation

When this paper was first presented at the Rochester Fall Meeting, H. A. Wheeler pointed out that the above derived results hold for 100 per cent modulation but may not hold for smaller percentages of

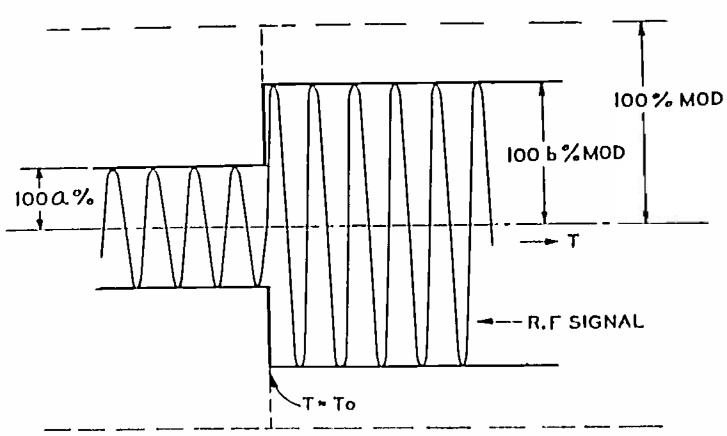

Fig. 10

modulation. Let us therefore see how these results are altered for smaller percentages of modulation, and for small changes in the percentage of modulation.

Consider a unit-function-type signal, such as shown in Fig. 10 where the percentage of modulation changes from 100a per cent to 100b per cent at $t=t_0$. What will such a signal be like after passing through the transmission network.

To solve this problem, we refer to (15). Accordingly we find

$$\begin{aligned}\phi'(t) = \{ & b[P(+\infty)-P(t_0-t+G)] \\ & +a[P(t_0-t+G)-P(-\infty)]\}\sin(\omega_c t+G\omega_b) \\ & +\{b[Q(+\infty)-Q(t_0-t+G)] \\ & +a[Q(t_0-t+G)-Q(-\infty)]\}\cos(\omega_c t+G\omega_b) \\ = & [(b+a)P(\infty)+(a-b)P(t_0-t+G)]\sin(\omega_c t+G\omega_b) \\ & +[(a-b)Q(t_0-t+G)]\cos(\omega_c t+G\omega_b) \qquad (16)\end{aligned}$$

since

$$P(-\infty)=-P(\infty) \quad \text{and} \quad Q(-\infty)=Q(+\infty)=0.$$

According to (16), the envelope function of the signal is

$$\Phi(t)=\sqrt{[(a+b)P(\infty)+(a-b)P(t_0-t+G)]^2 + [(a-b)Q(t_0-t+G)]^2}. \qquad (17)$$

If $a=0$, (17) reduces to the case depicted in Fig. 5. However, if $(a-b)$ is small in comparison with $(a+b)$ we can put (17) into the form

$$\begin{aligned}\Phi(t)=(a+b)[P(\infty)+(a-b)/(a+b)P(t_0-t+G) \\ +(\text{terms in }((a-b)/(a+b))^2]. \qquad (18)\end{aligned}$$

Therefore, in this case the quadrature functions become of negligible importance and selective-sideband transmission becomes for all purposes as good as double-sideband transmission of twice[14] the frequency pass band. As the relative value of $(a-b)$ is increased, the importance of the quadrature functions increases with it, and the characteristic peculiarities of selective-sideband transmission become more pronounced.

By application of (15) in a similar manner, in conjunction with Fig. 4, the effect of the transmission system on any type of detail and for all percentages of modulation can readily be determined.[15]

Acknowledgment

In conclusion the author wishes to thank Mr. I. J. Kaar and Mr. R. B. Dome for suggestions and encouragement in the course of the work.

[14] The reason for this is that P_5 (Fig. 4) is similar to P_2 but its variations occur in about one half the time. Therefore, for small variations in the percentage of modulation, since the Q terms are then negligible, selective-sideband transmission becomes for all purposes as good as double-sideband transmission of twice the frequency pass band.

[15] With the aid of (15) and the superposition theorem, the reproduction of detail by a scanning spot of arbitrary cross-sectional distribution can be analyzed. The labor of calculation, however, becomes so great in this case that it is hardly justified.

The Anode-Tank-Circuit Magnetron*

ERNEST G. LINDER†, NONMEMBER, I.R.E.

Summary—*A new type of magnetron is described in which the split cylindrical anode is made approximately one-quarter wave in length, the two segments being short-circuited at one end. The anode resonates and acts as a tank circuit. Thus difficulties due to interelectrode capacitance and tube lead inductance are circumvented and a much greater heat radiating area is provided. An output of 20 watts at 3750 megacycles (8 centimeters wavelength) and an efficiency of 22 per cent is obtainable. The theory of the anode tank circuit is developed, and expressions are given for wavelength, internal resistance, and logarithmic decrement.*

Introduction

In designing vacuum-tube oscillators for operation at very high frequencies, two of the principal limiting factors have been:

1. Circuit constants. It has been necessary to use electrodes of small size in order to avoid excessive interelectrode capacitance. Likewise, short leads, and small electrodes, have been employed to reduce inductance to the minimum.
2. Heat dissipation. The dissipative ability of the electrodes has been limited by their small surface area.

As examples, the dimensions of two typical split-anode magnetron tank circuits of the usual type are given in Fig. 1. Circuit (a) is that of a 9-centimeter oscillator similar to those previously described by the writer.[1] The split cylinder plate functions as a capacitance, and the wire loop as an inductance, the combination resonating at 9 centimeters. The 4-millimeter diameter is clearly near the maximum size. A further increase would result in troublesome capacitance between the wire loop and adjacent parts of the cylindrical plate, or require a larger loop, which would result in a longer wavelength. The length of the plates is 7 millimeters, and any increase in this also would result in increased capacitance and wavelength. The maximum output of this tube is 2.5 watts, and is limited by the small heat-dissipating

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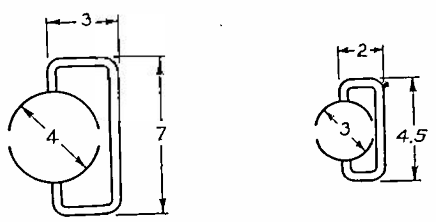
[1] E. G. Linder, "Description and characteristics of the end-plate magnetron," Proc. I.R.E., vol. 24, pp. 633–653; April, (1936).

area of the plates. Fig. 1 (*b*) shows a similar structure which oscillates at about 6 centimeters. Any substantial increase of either plate length or diameter would not be possible without an increase of wavelength. Structures of this type, operating at wave lengths as short as 0.64 centimeter, have been built.[2] This has been accomplished by greatly reducing all dimensions. However, the power outputs obtained have been exceedingly small.

The present article is concerned with a new type of magnetron called the anode-tank-circuit magnetron, which completely avoids limitation (1), and greatly extends (2).

Description of Tube

The essential features of the oscillating elements of the tube are illustrated in Fig. 2, which shows the anode tank circuit with attached transmission line and load. Other parts of the tube, such as envelope, cathode, etc., are similar to those of conventional split-anode magnetrons, of which this may be regarded as an improved form. An idea of the arrangement of the parts may be gained from Fig. 3, which shows a photograph of a typical experimental tube.



(a) (b)

Fig. 1—Typical tank circuits for older type magnetrons.

In Fig. 2, A is the anode tank circuit, comprising a tantalum cylinder split lengthwise along almost its complete length.

The dimensions of such an anode to operate between 8 and 9 centimeters are: length of slot 2.3 centimeters, width of slot 0.0635 centimeter (0.025 inch) and diameter of cylinder 0.7 centimeter.

The cylinder, thus slotted, is equivalent to a section of two-conductor transmission line, short-circuited at end a, and connected to a load at end b, as shown at A'. The load at b consists of the actual load, at the outer terminal of the transmission line as seen from the input end at b. When in operation this slotted cylinder resonates at the operating frequency, which is determined principally by its characteristics as a line, and by the impedance of the load, as shown later on. A standing wave forms on the cylinder, with a voltage node at a, and a voltage maximum near b.

It is the functioning of the anode as a section of transmission line, which circumvents limitation (1), listed above. The large capacitance between the cylinder halves, which, if used in a circuit of the old type, such as shown in Fig. 1, would result in a much longer wavelength, in the present case affects only the characteristic impedance of the line, and thereby the impedance required for optimum loading. Provided a load of proper impedance be used, the inter-

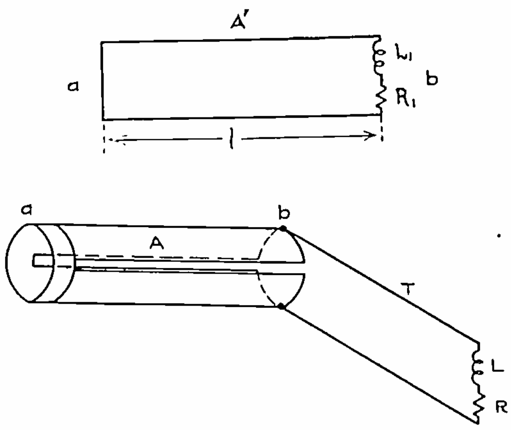


Fig. 2—Anode tank circuit with transmission line and load.

electrode capacitance has no effect upon wavelength. As an example of this, an anode was constructed by drilling a 4-millimeter diameter hole lengthwise through a copper block 1.2 centimeters square and 3 centimeters long. Slots were then cut only 0.005 centimeter wide. The width of the slot edges was, therefore, 4 millimeters and the capacitance per unit length of the transmission line thus formed was 14 micromicrofarads. Yet, when unloaded, this circuit oscillated at the same wavelength as circuits having about one hundredth the capacitance. When loaded, the behavior is somewhat different. The relationship between load impedance, line characteristic impedance, and wavelength will be discussed later.

In limitation (2), the heat dissipation is greatly extended because of the larger area of the anode tank circuit. The tube shown in Fig. 3. has an anode area over six times that of the structure shown in Fig. 1 (a). It is possible to use anodes of greater diameter.

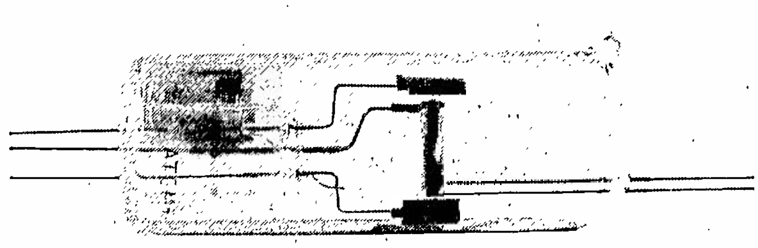


Fig. 3—Photograph of the experimental anode-tank-circuit magnetron for 8 centimeters wavelength.

The upper limit of diameter (i.e., surface) for a given length has not been determined, however, an anode as large as 1.7 centimeters in diameter and 2.3 centimeters long was made to oscillate at 9 centimeters.

To increase heat dissipation still further the use of low-melting-point metals was avoided. The anode

[2] C. E. Cleeton and H. N. Williams, "The shortest continuous radio waves," *Phys. Rev.*, vol. 50, p. 1091; December, (1936).

was made of tantalum, as were also the two rectangular shields for protecting the glass walls, which are visible in Fig. 3. The transmission line, cathode, cathode supports, and anode support were all of tungsten. The use of these refractory materials, combined with degassing at unusually high temperature, permitted the safe operation of the tube at anode temperatures as high as 1800 degrees Kelvin.

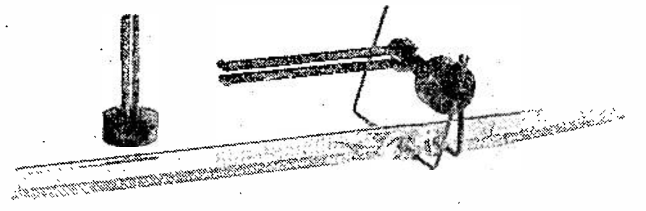


Fig. 4—Electrode structure as mounted for use in demountable magnetron.

Most of the data given below were not obtained with sealed-off tubes, such as that of Fig. 3, but with a demountable tube. This greatly facilitated the testing of a large number of different types of structures. The demountable tube consisted of a large glass cylinder supported between the poles of an electromagnet. It was connected to a vacuum system at one end, and at the other had a large ground joint and plug having lead-in wires and supports suitable for holding the desired parts. In Fig. 4 is shown an electrode structure ready for insertion and test in the demountable tube. In this particular case the shorted end of the anode was embedded in a cylindrical copper block to facilitate mounting and to aid cooling. A transmission line consisting of rectangular copper bars also is shown attached to the load end of the anode. Both of these features were discarded in later tests. An extra tank circuit is also shown. The figure illustrates the method of mounting structures on a tubular glass support having a tapered ground joint at one end for the purpose of holding the assembly in place in the demountable tube.

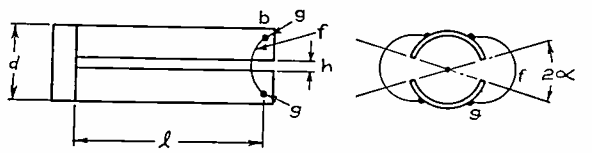


Fig. 5—Anode tank circuit showing load filaments.

Data and Discussion

1. *Measurement of Power Output*

The power output was measured in all cases by heating load filaments, which were calibrated in terms of optical-pyrometer readings. The loads were used in several ways, among which were direct connection across the tank circuit, connection across the end of a transmission line, and use in a special type of load lamp either evacuated or hydrogen-filled. The filaments were of tungsten wire, an arbitrary length of 1.2 centimeters being selected as standard. The diameters varied from 0.001 centimeter (0.0004 inch) to 0.005 centimeter (0.002 inch), according to the resistance desired. Frequently several were used in parallel. The direct-current calibration for power was used, the accuracy being considered sufficient without corrections of any kind.

Most of the measurements were made in the demountable tube, and in these cases the load filaments were usually connected directly across the end *b* of the anode tank circuit, as shown in Fig. 5. The load filaments *f* were formed into half loops and cemented to the anode with small droplets of colloidal graphite *g*, as shown. If several filaments were used, as nearly as possible equal numbers were cemented on opposite sides. Care was taken that no part of the filaments extended beyond the end of the anode, as in that case, they might be subjected to electron bombardment.

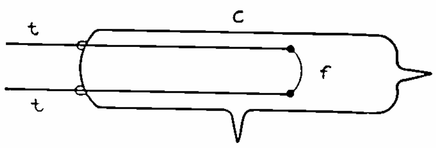


Fig. 6—Hydrogen-filled load lamp.

Any errors due to such electron bombardment were shown to be small by tests made with load filaments on the distant end of a transmission line, as shown in Fig. 2, and by many tests made with sealed-off magnetrons having a completely external load.

The mounting of load filaments directly on the tank circuit has proved a very convenient and simple method for measuring output power and testing the effects of various kinds of loads. When used in conjunction with a demountable tube it provided a simple, quick means for changing the load, and avoided all complications arising from the use of a transmission line.

The demountable tube was made sufficiently long so that, if desired, transmission lines up to about 30 centimeters length could be connected to the tank circuit without the necessity of sealing them through the glass wall. In tests with lines, the loads were cemented directly across the end of the line.

For measurements with sealed-off tubes special load lamps were constructed, as shown in Fig. 6. The lead-in wires *t* were made of the same size and spacing as the transmission line from the oscillator, and thus formed an extension of that line. The load filament *f* was of the same dimensions as those used in the demountable tube. For the sake of permanence it was welded to the wires *t* instead of being cemented.

Some of these lamps were evaucated, others were filled with hydrogen at atmospheric pressure, depending upon the amount of power it was desired to dissipate. Increases of several hundredfold in dissipation may be obtained with small-diameter filaments by immersion in a hydrogen atmosphere instead of in a vacuum. A filament 1.2 centimeters in length and 0.001 centimeter in diameter will dissipate 30 watts and have 120 ohms resistance at 2800 degrees Kelvin. Hydrogen-filled lamp resistors have been discussed by the writer elsewhere,[3] and no further details will be given here. The use of these lamps was found necessary to provide a load capable of dissipating the maximum output of the anode-tank-circuit magnetron, and provide a suitable load resistance of about 100 ohms. Filaments of this resistance when used in vacuo, as in the demountable tube, were capable of dissipating but a small portion of the maximum tube output.

2. Optimum Operating Conditions

The maximum output obtained with tubes such as that of Fig. 3 is 20 watts at an efficiency of 22 per cent, efficiency being here defined as the ratio of measured output power to plate input power.

An output of 20 watts requires driving the tube at a high plate temperature. An output of 15 watts may be considered as more normal. Approximate operating conditions for a particular case are given below. It should be borne in mind that these conditions vary somewhat from tube to tube.

Plate potential	3300 volts
Magnetic field	1500 gauss
Plate current	20 milliamperes
Tilt	0-10 degrees
Wavelength	8-9 centimeters
Plate load resistance	80-140 ohms
Output power	13 watts
Efficiency	20 per cent

In order to obtain the above-mentioned power output it was necessary to use a plate-current regulator to maintain the plate current at a constant value. Without some such regulator it was not possible to obtain outputs greater than three or four watts. The necessity for the regulator arises from instability due to cathode bombardment, an effect which occurs in all common types of magnetrons.[4] Bombardment causes the cathode temperature to rise above that due to the heating current alone, and if not regulated, the emission rises to very large values, stopping oscillations, and sometimes destroying the tube. In the regulator employed by the writer, the plate current of the magnetron was fed back to the filament supply unit in such a way that when the plate current tended to increase the filament-heating current tended to decrease. In the absence of some such a regulator outputs of three or four watts may be obtained, but in this case a resistance of 100,000 ohms should be used in series with the plate.

3. Relation between Wavelength and Load

In Fig. 2, the equivalent circuit of the anode tank circuit is shown. It consists of a section of two-conductor transmission line of length l and characteristic impedance Z. It is short-circuited at one end, and terminated at the other by a load of inductance L_1 and resistance R_1. The capacitance per unit length of line will be denoted by C; the resistance will be considered negligible. A transverse conductance G per unit length also will be introduced due to the presence of the electron swarm. The nature of this conductance will be discussed later.

In determining the behavior of this system, it will be considered as force-free, and not as a driven system. According to this viewpoint, there is no applied electromotive force. Oscillations occur when the conductance G has the proper negative value, as will be shown below. Actually G is probably a function of distance along the line, because of the variation of voltage and current, and the change in the nature and position of the electron swarm due to tube tilt in the magnetic field. We shall make the simple assumption that G possesses the same value all along the line. This enables us to make use of the usual transmission-line equations and their solutions in which G is considered constant. The supposition of a constant G is equivalent to assuming that the conduction current between the anode halves is proportional to the voltage and is therefore sinusoidal, and zero at the shorted end.

As will be shown later, G has a negative value and current through it results in a delivery of power. Since the power delivered is proportional to the product of this current and the voltage, it is evidently small at the short-circuited end and reaches a maximum near the load where the voltage is highest. Although we have no experimental evidence for or against this distribution, it seems a reasonable assumption.

Guillemin[5] has given the expression for force-free behavior for a transmission line,

$$Z_0(Z_S + Z_R)/(Z_S Z_R + Z_0^2) = -\tanh \gamma l, \quad (1)$$

where Z_0 is the characteristic impedance, Z_S and Z_R are the terminating impedances, and $\gamma^2 = (R+Lp)(G+Cp)$, where R, L, G, and C are, respectively, resistance, inductance, transverse conductance, and capacitance per unit length of line. For sustained oscillations p must be a pure imaginary; i.e., $p = j\omega$.

In the present case, $Z_S = 0$, $R = 0$,

$$Z_R = R_1 + j\omega L_1, \text{ and } Z_0 = \sqrt{j\omega L/(G + j\omega C)}.$$

[3] E. G. Linder, "The use of gas-filled lamps as high-dissipation, high-frequency resistors, especially for power measurements," *RCA Rev.*, vol. 9, pp. 83-88; July, (1939).

[4] E. G. Linder, "Excess-energy electrons and electron motion in high-vacuum tubes," Proc. I.R.E., vol. 26, pp. 346-371, March, (1938).

[5] E. A. Guillemin, "Communication Networks," vol. II, p. 559, John Wiley and Sons, New York, N. Y., (1935).

Putting $1/LC=v^2$, and $\lambda=2\pi v/\omega$, then expanding by Taylor's theorem, yields

$$Cv(R_1+j\omega L_1)(1-jG/2\omega C+1/8\cdot G^2/\omega^2C^2+\cdots) \tag{2}$$
$$=-j\tan 2\pi l/\lambda-\pi lG/\lambda\omega C\sec^2 2\pi l/\lambda$$
$$+j(\pi lG/\lambda\omega C)^2(\tan 2\pi l/\lambda-\lambda/4\pi l)\sec^2 2\pi l/\lambda+\cdots,$$

where third and higher-order terms have not been written.

In the experimental work the load resistance was small compared to the load reactance, i.e., $R_1/\omega L_1 \ll 1$. As a consequence of this it follows that G has a possible value such that $|G/\omega C| \ll 1$.

This is evident from (2) since as R_1 approaches zero, G also may approach zero. Other values of G, not small, may exist which satisfy the equation, but these will be ignored, since the small value evidently corresponds to the case of the conventional negative-resistance oscillator in which, when the load resistance R_1 is small, the internal negative conductance must be small also.

Equation (2) may now be rewritten, first multiplying out, neglecting terms of higher than second order, and equating real and imaginary terms.

Also put $Z=1/Cv$, and $G=1/r$, then

$$1/G=r=-L_1/2CR_1-lZ^2/2R_1\sec^2 2\pi l/\lambda, \tag{3}$$

and

$$\omega L_1-\lambda R_1Z/4\pi r+\lambda L_1Z/16\pi Cr^2=-Z\tan 2\pi l/\lambda$$
$$+(lZ/2r)^2(\tan 2\pi l/\lambda-\lambda/4\pi l)\sec^2 2\pi l/\lambda. \tag{4}$$

In these expressions λ is the wavelength in air or on a dissipationless line, Z is the characteristic impedance of the tank circuit when $R=G=0$. These meanings are evident from the manner in which λ and Z were introduced above.

If, as in the present experiments, the load resistance is so small that even the second-order terms are unimportant, (3) will be unaffected, but (4) may then be written

$$\omega L_1=-Z\tan 2\pi l/\lambda. \tag{5}$$

The left-hand side is the reactance of the load, the right-hand side is the reactance of the line. Thus (5) states that the condition for oscillation is that the reactance of line and load shall be equal and opposite in sign, as is to be expected.

In comparing (5) with experiment, it is necessary to know the inductance of the load filaments, and also the characteristic impedance of the anode tank circuit. The load inductances L_1, given in Table I were computed by considering the load filaments as straight round wires of length 1.2 centimeters, and 0.005 centimeter diameter, and applying the usual formula[6]

$$L_1=0.002S[\ln 2S/a-1] \tag{6}$$

where S is the wire length and a its radius, and L_1 is in microhenrys.

The characteristic impedance of the anode tank circuits were calculated by first determining the capacitance in micromicrofarads per unit length by means of the equation[7]

$$C=(0.11/\pi^2)\ln(x+\sqrt{x^2-1}) \tag{7}$$

where $x=\cot\alpha/2$,
the angle 2α being the angle subtended at the cathode by one of the slots (see Fig. 5). The characteristic impedance Z was then found by use of the relation

$$ZC=1/v, \tag{8}$$

where v is the velocity of light. The results of these computations are given in Table I, which includes data for tank circuits of several lengths, diameters, and slot widths, as well as for loads of various reactances.

TABLE I
EFFECT OF LOAD REACTANCE ON WAVELENGTH

Test	1 (cms)	Z Eq. (9) (ohms)	ωL_1 (ohms)	λ Expt. (cms)	λ Calc. (cms)
834	1.58	93	∞	6.5	6.3
837	1.58	93	249	5.3	5.1
838	1.58	93	185	4.8	4.9
841	1.30	93	287	4.6	4.3
844	2.30	74	167	7.9	7.3
848	2.30	79	103	6.4	6.5
958	2.30	79	171	7.7	7.3
960	2.30	79	322	8.2	8.0

In Fig. 7 the values of λ computed from (5) are plotted against the observed values. The agreement is considered satisfactory in view of the several sources of error. Important among these is the fact that the wavelength is affected to some extent by other factors than those included in (5). It is known that the electron swarm constitutes a complex load[8,9] on the anode segments, its value depending upon such factors as magnetic field strength, plate voltage, and space charge. In making the above calculation only the resistive component was considered. If the load is treated as having a real and imaginary part, the real part is taken care of by the G which has been used, and the imaginary part will change the effective distributed capacitance of the tank circuit. Since this reactance can be either negative or positive the capacitance of the tank circuit can be either increased or decreased with resultant effect on the generated frequency.

A further source of error arises from the difference between the actual line length and the effective length. Differences of as much as a millimeter would account for the scattering of the points in Fig. 7.

A quantitative demonstration of the variation of wavelength with anode length, and showing an ex-

[6] Bureau of Standards Circular C74, p. 243, 1937.

[7] A. A. Slutzkin, et al., *Phys. Ziet. der USSR*, vol. 6, p. 150, (1934).

[8] S. Benner, "Über die Eigenschwingung freier Elektronen in einem konstanten Magnetfeld," *Die Naturwissenschaften*, vol. 17, p. 120, (1929).

[9] A. Giacomini, "Anomalous dispersion in the magnetron," *La Ricerca Sci.*, vol. 1, No. 11–12, (1934).

ample of differences between actual length and effective length, is given in Fig. 8. The wavelength is plotted against actual line length (slot length), for an unloaded tank circuit, of 0.6 centimeter diameter. The wavelengths given are those corresponding to maximum amplitude of oscillation.

Within the accuracy of the measurements the wavelength is four times the anode length until the anode length becomes close to its diameter. The wavelength is then larger than four times the anode length, probably due to end effects. These data are more accurate than those of Fig. 7, since they apply to tank circuits having no load filaments, and hence uncertainty as to the correct value of l is removed.

From (3) it is possible to compute the effective negative internal resistance per unit length of the oscillator. This has been done for the cases listed in Table II, and the results are given in the last column.

4. *Effect of Load Resistance on Efficiency*

The resistance of the load has an important effect upon the oscillator efficiency. The efficiencies found for loads of various resistances are given in Table II and Fig. 9. These measurements were made with a sealed-off magnetron feeding load lamps through a transmission line. The wavelength was maintained at 8 centimeters by adjusting the magnetic field and transmission-line length. The plate voltage was 3300 and the plate current 4 milliamperes.

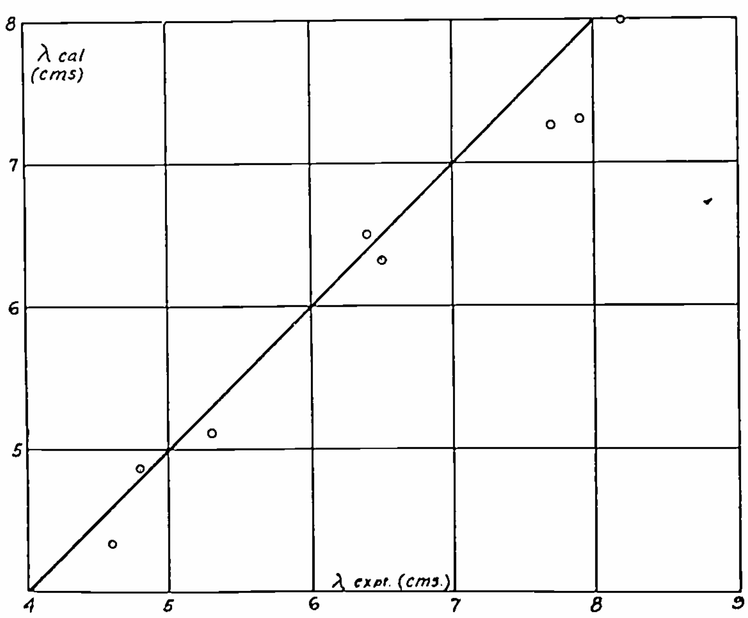


Fig. 7—Comparison of observed and calculated values of wavelength.

The measurements were made at a low power output in order that identical operating conditions could be used in all cases without danger of burning out the load filament. The loads consisted of tungsten-filament lamps such as described above.

The determination of the load resistance was complicated somewhat by skin effect, by large temperature variations, and by the transmission line. The effect of the first two factors was computed by the usual formula[10] for the high-frequency resistance of a straight, round conductor, the resistivity corresponding to the operating temperature being used in

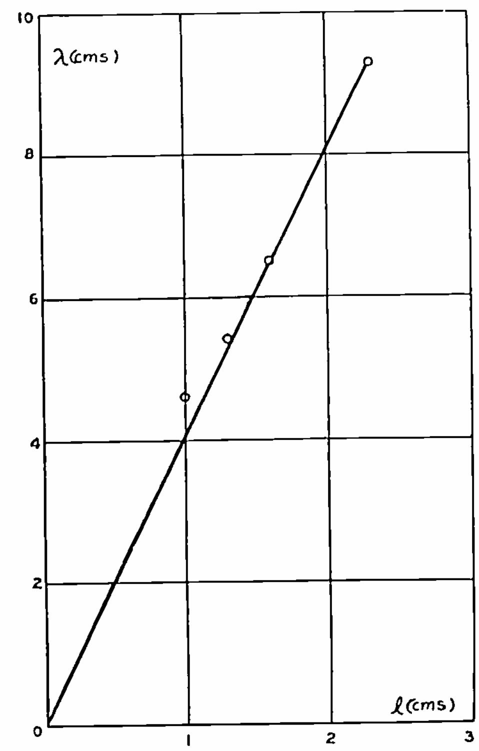


Fig. 8—Variation of wavelength with anode length.

each case. These values are given under R_{hf} in Table II.

TABLE II
OSCILLATOR EFFICIENCY AND LOAD RESISTANCE

Test	No. of filaments	Diameter (cms)	E %	W (watts)	R_{hf} (ohms)	R_{eff} (ohms)	$-r$ (ohms/cms)
980	3	0.005	3.2	0.42	2.4	3.9	37,200
975	2	0.005	5.1	0.67	3.9	5.6	25,600
976	1	0.005	7.0	0.93	9.1	9.1	15,700
983	1	0.0025	16.4	2.16	27.0	27.0	5,300
989	1	0.001	20.0	2.64	43.0	43.0	3,320
999	1	0.001	22.0	20.0	115.0	115.0	1,240

It was difficult to determine the effect of the transmission line, since the effective electrical length of the line was unknown, due to its passing through the glass envelopes of the magnetron and load lamp. As a first approximation, it was assumed first that the line was uniform but of unknown length. This led to the following computation for input resistance. (The effect of the glass walls will be estimated later.)

Since all measurements of the effect of load resistance on efficiency were made at a wavelength of 8 centimeters, this indicates according to (5) that the reactance of the load on the tank circuit, i.e., the reactive component of the input impedance of the transmission line was the same in all cases, and equal to approximately 320 ohms. A wavelength of about 8 centimeters is obtained also with a single load filament connected directly across the tank circuit with no intervening transmission line. Thus, it follows that in the case where a single load filament was used, the line (assumed uniform) must have been an integral number of half waves in length, and func-

[10] Page 299 of footnote reference 6.

tional as a one-to-one transformer. In cases where several load filaments were used in parallel the line length must have been such that the reactive component of input impedance was still that corresponding to an 8-centimeter wavelength. i.e., 320 ohms. In these cases the line acted no longer as a one-to-one transformer and both the resistive and reactive components were affected.

The expression for the input impedance of a uniform line is

$$Z_1 = Z_0 \frac{Z_R + jZ \tan 2\pi S/\lambda}{Z + jZ_R \tan 2\pi S/\lambda}.$$

By introducing the known values of λ, Z, and Z_R, it is possible to determine values of S such that the input reactance will be 320 ohms, and then, knowing

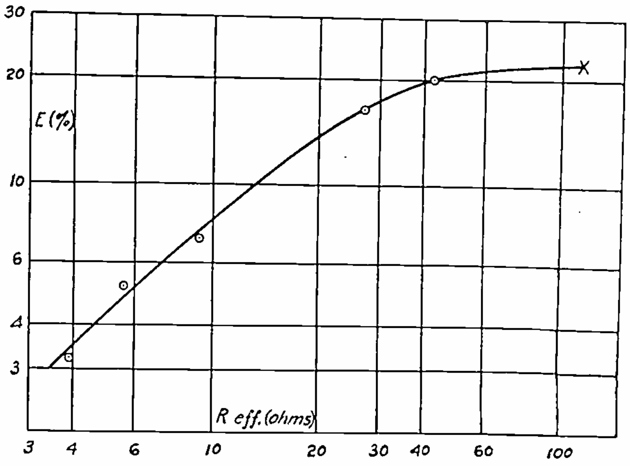


Fig. 9—Variation of efficiency with load resistance.

S, compute the input resistance. Values of resistance thus computed are listed in Table II under R_{eff}, and represent the load resistance on the magnetron anode tank circuit computed on the basis of a uniform line. It is seen that the transmission line affected the load resistance only in tests number 975 and 980.

To estimate the effect of the passage of the line through the glass walls of the tubes, computations were made on the effect of placing a condenser, across the line having a reactance sufficient to obtain the approximate shortened length of line. The line shortening was determined by measuring the distance between voltage nodes on opposite sides of the glass. It was assumed in each case that the line length was adjusted so that the line input reactance was 320 ohms. Thus it was possible to plot a curve showing the complete range of variation of input resistance as the condenser position changed. It was found that a maximum variation by a factor of two might occur. Thus the resistance values R_{eff} in Table II and Fig. 9 may be inaccurate by some such factor.

The curve of Fig. 9 illustrating the variation of efficiency with load resistance, shows a continuous rise as the resistance increases varying from 3.2 per cent at 3.9 ohms to 20.0 per cent at 43 ohms. Further extension of the curve was not feasible due to lack of higher-resistance loads. It appears, however, that the optimum resistance has been nearly reached since the curve seems to be flattening off. For a different set of magnetron operating conditions yielding greater output, and thus higher-temperature loads and greater resistance, efficiencies of about 22 per cent for load resistances of about 100 ohms have been obtained. This point is shown as a cross in the figure.

5. *Q of the Anode Tank Circuit*

The Q of the loaded tank circuit may be computed from (1). It will be assumed that the resistance R and transverse conductance G are negligible in comparison with the load resistance. The load consists of a resistance R_1 in series with an inductance L_1. Hence, $Z_R = R_1 + pL_1$, $\gamma^2 = p^2 LC$, and $p = \delta + j\omega$ where δ is the decay constant.

Substituting in (1), and rewriting and equating real and imaginary terms, gives

$$R_1 + \delta L_1 - \omega L_1 \tanh \delta l/v \tan 2\pi l/\lambda = -Z \tanh \delta l/v, \qquad (9)$$

and

$$\omega L_1 + (R_1 + \delta L_1) \tanh \delta l/v \tan 2\pi l/\lambda = -Z \tan 2\pi l/\lambda. \qquad (10)$$

These two equations may be used for the determination of the two quantities δ and ω. However, (5) has already been derived for the determination of ω, hence either of the above equations may be used for the computation of δ. Expression (9) is the more accurate in the present case since (10) contains the difference of two large and nearly equal terms, ωL_1 and $-Z \tan 2\pi l/\lambda$ (see (5)).

In cases of interest at present $\delta l/v$ is small. Hence put $\tanh \delta l/v = \delta l/v$. Also from (5) put $\omega L_1 = -Z \tan 2\pi l/\lambda$. Equation (9) may then be solved for δ/f, which is the negative logarithmic decrement. Hence we have

$$Q = \frac{\pi}{\text{decrement}} = \frac{\pi l/\lambda [Z^2 + \omega^2 L_1^2 + (v/l) Z L_1]}{Z R_1}. \qquad (11)$$

As an example take test No. 999 from Table II, which represents the conditions giving nearly maximum output and efficiency. The numerical values are $Z = 80$, $R_1 = 115$, $\omega L_1 = 320$, $l/\lambda = 0.28$. Introducing these in (11) gives $Q = 11.5$. This corresponds to a decrement of 0.27, which is near the maximum value for stable oscillation in circuits of the feedback type.

Acknowledgment

The writer is indebted to Dr. Irving Wolff for many valuable discussions and helpful suggestions.

Characteristics of the Ionosphere at Washington, D.C., September, 1939*

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DATA on the critical frequencies and virtual heights of the ionospheric layers during September are given in Fig. 1. Fig. 2 gives the monthly average values of the maximum usable frequencies for undisturbed days, for radio transmission by way of the regular layers. The F_2 and F layers

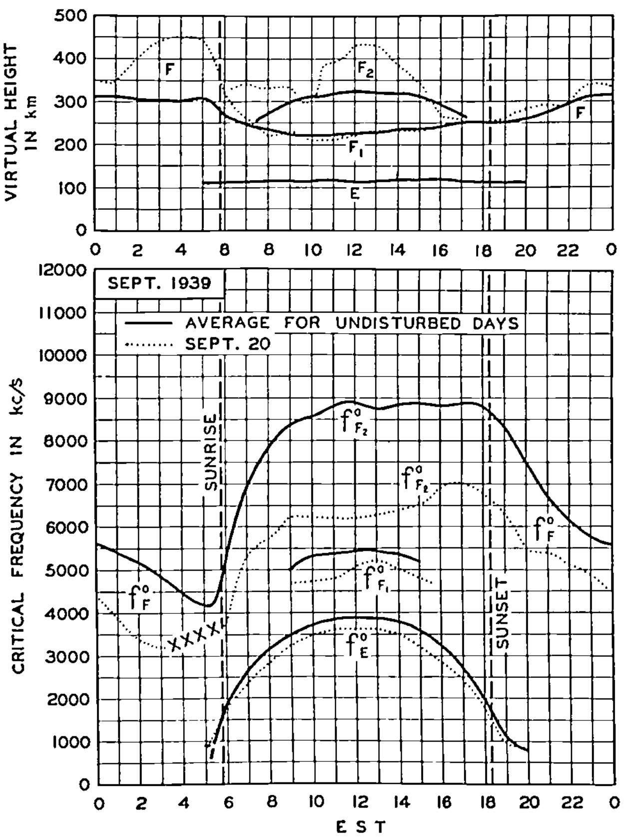


Fig. 1—Virtual heights and critical frequencies of the ionospheric layers, September 1939. The solid-line graphs are the averages for the undisturbed days; the dotted-line graphs are for the ionospheric storm day of September 20. The crosses represent the times on September 20 when the F-layer reflections were so diffuse that the critical frequencies could not be determined.

ordinarily determined the maximum usable frequencies during the day and night, respectively. Fig. 3 gives the distribution of hourly values of F and F_2 data about the undisturbed average for the month. Fig. 4 gives the expected values of the maximum usable frequencies for radio transmission by way of

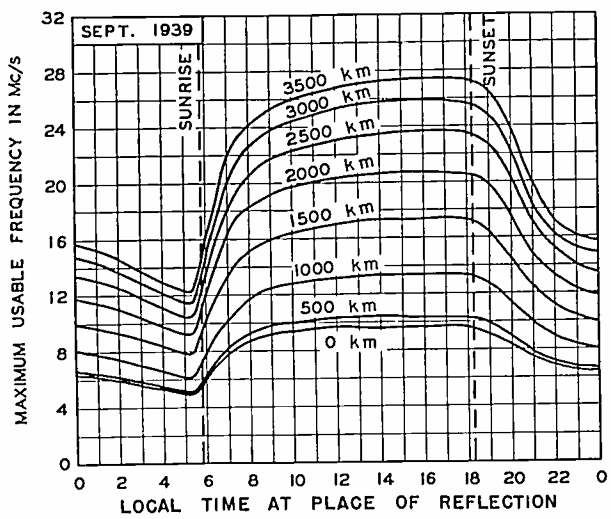


Fig. 2—Maximum usable frequencies for dependable radio transmission via the regular layers, average for undisturbed days for September, 1939.

the regular layers, average for undisturbed days, for December 1939. Ionospheric storms and sudden iono-

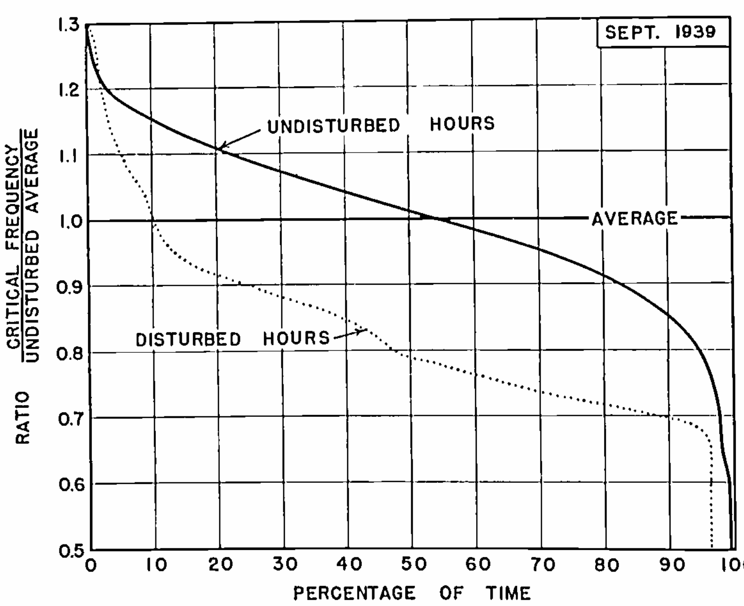


Fig. 3—Distribution of F- and F_2-layer ordinary-wave critical frequencies (and approximately of maximum usable frequencies) about monthly average. Abscissas show percentage of time for which the ratio of the critical frequency to the undisturbed average exceeded the values given by the ordinates. The solid-line graph is for 425 undisturbed hours of observation; the dotted graph is for 59 disturbed hours of observation listed in Table I.

* Decimal classification: R113.61. Original manuscript received by the Institute, October 11, 1939. These reports have appeared monthly in the PROCEEDINGS starting in vol. 25, September, (1937). See also vol. 25, pp. 823–840, July, (1937). Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

† National Bureau of Standards, Washington, D.C.

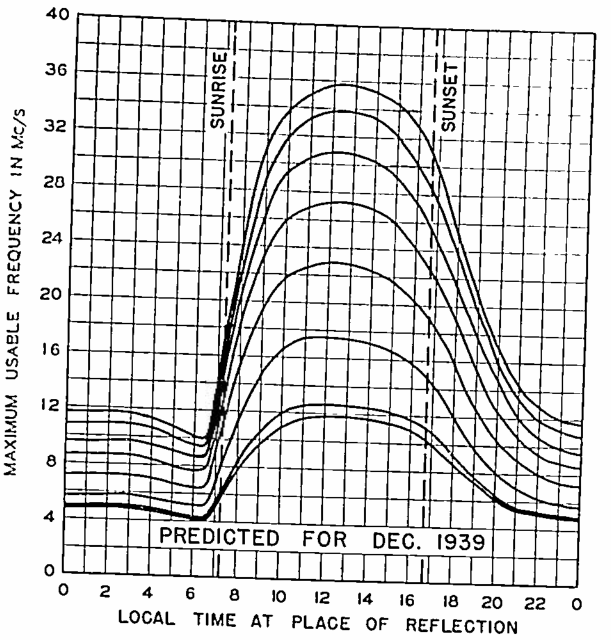


Fig. 4—Predicted maximum usable frequencies for dependable radio transmission via the regular layers, average for undisturbed days, for December, 1939.

TABLE I
IONOSPHERIC STORMS (APPROXIMATELY IN ORDER OF SEVERITY)

Day and hour E.S.T.	h_F before sunrise (km)	Minimum f_F^0 before sunrise (kc)	Noon $f_{F_2}^0$ (kc)	Magnetic character[1]		Ionospheric character[2]
				00–12 G.M.T.	12–24 G.M.T.	
Sept.						
19	334	3100	7900	0.8	1.0	0.7
20	416	diffuse	6200	1.0	0.6	1.3
21 (until 0500)	338	2900	—	0.2	0.3	0.5
26 (0800 to 1800)	—	—	7100	0.9	0.4	0.5
2 (after 2300)	—	—	—	0.1	0.8	—
3 (until 1600)	330	4500	7700	1.4	0.8	0.
For comparison: Average for undisturbed days	305	4175	8900	0.2	0.2	0.0

[1] American magnetic character figure, based on observations of seven observatories.
[2] An estimate of the severity of the ionospheric storm at Washington on an arbitrary scale of 0 to 2, the character 2 representing the most severe disturbance.

spheric disturbances are listed in Tables I and II, respectively. During September few strong vertical-

TABLE II
SUDDEN IONOSPHERIC DISTURBANCES

Day	G.M.T. Beginning	G.M.T. End	Locations of Transmitters	Relative intensity at minimum[1]	Other phenomena
Sept.					
2	1426	1500	Ohio, Mass., Ont., D.C.	0.05	
3	1531	1600	Ohio, Mass., Ont., D.C.	0.0	
3	1606	1620	Ohio, Mass., Ont., D.C.	0.01	
3	1741	1810	Ohio, Mass., Ont.	0.05	
5	1717	1800	Ohio, Mass., Ont.	0.0	
5	1820	1850	Ohio, Mass., Ont.	0.05	
5	1911	1950	Ohio, Mass., Ont.	0.0	
6	1441	1530	Ohio, Ont.	0.0	
6	2003	2200	Ohio, Mass., Ont.	0.0	Ter. mag. pulse[2] 2003 to 2120
7	1520	1630	Ohio, Mass., Ont., D.C.	0.01	
7	1756	1840	Ohio, D.C.	0.01	
7	2049	2120	Ohio, Mass., Ont.	0.01	
8	1142	1210	Ohio, Mass., Ont., D.C.	0.0	Ter. mag. pulse 1142 to 1154
8	1445	1530	Ohio, Mass., Ont.	0.1	
9	2032	2130	Ohio, Mass., Ont., D.C.	0.0	
11	1707	1740	Ohio, Mass., D.C.	0.0	
12	1828	1930	Ohio, Mass., Ont., D.C.	0.0	
13	1538	1630	Ohio, Mass., Ont., D.C.	0.0	Ter. mag. pulse 1538 to 1551
13	1657	1730	Ohio, Ont.	0.01	
15	1328	1355	Ohio, Mass., Ont.	0.05	
15	2019	2100	Ohio, Mass., Ont.	0.01	
16	1504	1600	Ohio, Mass., Ont.	0.02	
16	1832	1910	Ohio, Mass., Ont.	0.01	
17	1534	1620	Ohio, Mass., Ont.	0.0	
27	1859	1940	Ohio, Mass., Ont., D.C.	0.02	
27	2001	2040	Ohio, Ont., D.C.	0.00	Ter. mag. pulse 2001 to 2100
28	1523	1600	Ohio, Ont., D.C.	0.05	
28	1712	1750	Ohio, Mass., Ont.	0.01	
29	1605	1630	Ohio, Mass., Ont., D.C.	0.1	
30	1757	1820	Ohio, Mass., Ont.	0.01	
Aug.					
29	1952	2018	Ohio, Mass., Ont.	0.05	Ter. mag. pulse[3] 1954 to 2008

[1] Ratio of received field intensity during fade-out to average field intensity before and after; for station WLWO, 6060 kilocycles, 650 kilometers distant.
[2] Terrestrial magnetic pulse, observed on magnetograms from Cheltenham Observatory of the United States Coast and Geodetic Survey.
[3] Fade-out data from August report repeated to show data on terrestrial magnetic pulse not then available.

incidence sporadic-E reflections were observed. They were observed up to 8 megacycles during one hour and up to 6 megacycles during two hours.

Institute News and Radio Notes

Board of Directors

The regular monthly meeting of the Board of Directors was held on October 4, 1939, and those present were R. A. Heising, president; Melville Eastham, treasurer; Ralph Bown, F. W. Cunningham, Alfred N. Goldsmith, Virgil M. Graham, O. B. Hanson, C. M. Jansky, Jr,. I. J. Kaar, F. B. Llewellyn, Haraden Pratt, B. J. Thompson, H. M. Turner, A. F. Van Dyck, and H. P. Westman, secretary.

In elections to membership, forty-two Associates, one Junior, and eight Students were admitted.

On the recommendation of the Sections Committee, a new section with headquarters in Baltimore, Md., was established. This requires a revision of the territory allotted to the Washington Section.

A petition for the establishment of a section in Buenos Aires, Argentina, was granted. This is the first section to be established outside of the United States and Canada.

Fourteenth Annual Convention

Our Fourteenth Annual Convention was held in New York City on September 20–23. Two papers were added to the program published in the September PROCEEDINGS. They were "Radio or Short-Wave Therapy and Its Interference with Radio Communications " by Lee deForest of Lee deForest Laboratories, Los Angeles, California, and "Large-Tube Television Receivers Using Electrostatic Deflection," by T. T. Goldsmith, Jr., of Allen B. DuMont Laboratories, Passaic, New Jersey.

At the Awards Luncheon on September 21, the Institute Medal of Honor was presented to Sir George Lee, recently retired Engineer-in-Chief of the British Post Office. Through the courtesy of the British Post Office, the American Telephone and Telegraph Company, and the Columbia Broadcasting System, the presentation speech by President Heising was transmitted to Sir George Lee in London and his reply heard by everyone at the luncheon in New York.

The Morris Liebmann Memorial Prize was given to H. T. Friis of Bell Telephone Laboratories by President Heising at the luncheon. Photographs and biographical data on the award recipients appear in this issue.

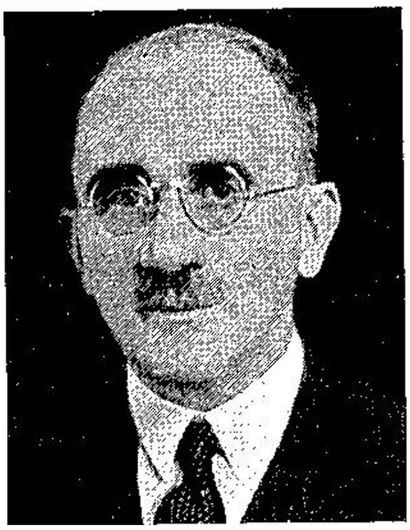
September 23 was designated "de Forest Day" at the New York World's Fair 1939 and several receptions were given Dr. deForest and the official party at the Fair. In the evening, a dinner, at which a testimonial scroll was presented to Dr. deForest, was attended by about two hundred.

The arrangements for "deForest Day" were made by the Veteran Wireless Operators Association, and the Institute acted as one of several co-sponsors.

Registration for the convention totaled 1615 men and 53 women.

ALBERT GEORGE LEE

The Institute Medal of Honor was presented to Sir George Lee at the Awards Luncheon on September 21 in recognition of his accomplishments in promoting international radio services and in fostering advances in the art and science of radio communication. Although unable to attend the luncheon, he acknowledged the presentation from London over the radiotelephone



system in the design and construction of which he played a leading part.

Albert George Lee was born in Conway, England, on May 24, 1879. A B.Sc. degree was conferred on him by London University in 1905. He entered the British Post Office engineering department in 1903 and until 1914 worked on cables, telephone, and telegraph systems.

At the end of 1914 he received a commission in the Royal Engineers and for a large part of the war he was in command of a telegraph construction company. He later became Officer-in-Charge of the General Headquarters Signal Area and Second-in-Command of a Signal Battalion. He received the Military Cross and at the close of the war held the rank of Major. He is now Lieutenant-Colonel in the Royal Corps of Signals (Supplementary Reserve).

Returning to the Post Office, he was first assigned to radio work in 1920. He became Assistant Engineer-in-Chief in 1927 and Engineer-in-Chief in 1931, retiring in 1939. For his services in radio, he was created an Officer of the Order of the British Empire in 1927. He was knighted in 1937.

Sir George became a Member of the Institute in 1927 and a Fellow in 1929. He served as Vice President in 1930.

Committees

Sections

The annual meeting of the Sections Committee was held during the Fourteenth Annual Convention at the Hotel Pennsylvania in New York City on September 21. The list of those present which follows gives also the sections represented. Where no section is indicated, the individual is a member at large.

R. A. Heising, President
E. D. Cook, Chairman
V. J. Andrew, Chicago
W. F. Cotter, Rochester
J. H. Dellinger, Washington
B. V. K. French, Indianapolis
E. L. Gove, Cleveland
Virgil M. Graham, Emporium
G. J. Irwin, Toronto
R. K. McClintock, Emporium
J. H. Miller
W. P. Place, Pittsburgh
E. H. Rietzke, Washington
C. E. Scholz
H. D. Seielstad, Detroit
R. L. Snyder, Philadelphia
R. E. Stark, Pittsburgh
H. P. Westman, Secretary

A tabulation of data on the meetings held by our sections during 1938 and their membership figures for the last few years was examined.

Petitions for the formation of sections at Baltimore, Md., and Buenos Aires, Argentina, were considered. In both cases it was felt that a sufficiently large membership existed and there were ample sources of papers for presentation to insure successful operation. Both petitions were favorably recommended to the Board of Directors.

The financial reports covering the calendar year, 1938, were reviewed.

President Heising, who visited most of the Institute sections during the past few months, led a discussion on the operation of sections with particular emphasis on the development of programs of papers to be presented at section meetings.

It was agreed that an attempt would be made to prepare a section operation manual for the use of the officers of the sections. A long list of items which might well be treated in this manual was prepared and a subcommittee appointed to assist in drafting the document.

New York Program

I. S. Coggeshall, chairman; Austin Bailey, Beverly Dudley (representing Keith Henney), G. T. Royden, and H. P. Westman, secretary, attended a meeting of the New York Program Committee held on October 2 in the Institute office. Plans for the New York meetings during the Fall were prepared.

Technical Committees

Electronics

Large High-Vacuum Tubes

The Subcommittee on Large High-Vacuum Tubes of the Technical Committee on Electronics met on September 21. Those present were E. L. Chaffee, chairman; K. C. DeWalt, H. E. Mendenhall, I. E. Mouromstseff, Alexander Senauke, E. E. Spitzer, C. M. Wheeler, and H. P. Westman, secretary.

The major portion of the meeting was devoted to a discussion of methods of measuring static characteristics of large high-vacuum tubes over both their negative and positive grid-voltage regions.

In preparing for the annual review, the various publications in the field were assigned to the members of the committee for examination as to new material published during the year.

Electronics Conference

The Electronics Conference Subcommittee met on September 29. F. R. Lack, chairman; F. B. Llewellyn, G. A. Morton, R. W. Sears, B. J. Thompson, and H. P. Westman, secretary; were present.

An amended mailing list of those to receive announcements of the Conference was prepared. Arrangements were made for an informal dinner to be held on October 20. The final list of subjects to be discussed at the meeting was approved.

Ultra-High Frequency

The subcommittee preparing the ultra-high-frequency program of the Electronics Conference met on September 19. Those present were F. B. Llewellyn, Chairman; E. L. Bowles, L. S. Nergaard, D. O. North, A. L. Samuel, Irving Wolff, and H. P. Westman, secretary.

This meeting was devoted to a consideration of names to be added to the list of those to receive invitations to attend the conference and the subjects to be treated.

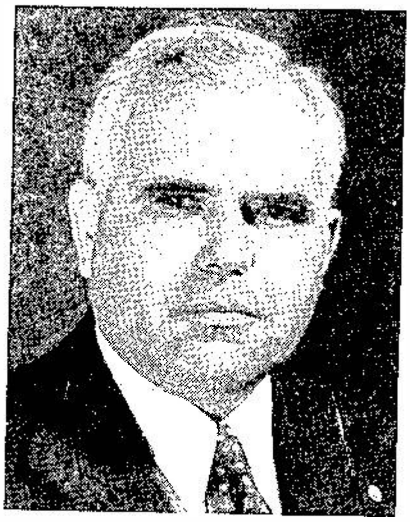
Wave Propagation

The Technical Committee on Wave Propagation met on September 19. Those present were J. H. Dellinger, chairman; S. L. Bailey, L. V. Berkner, C. R. Burrows, Harry Diamond, W. A. Fitch, G. D. Gillett, S. S. Kirby, H. R. Mimno, K. A. Norton, H. O. Peterson, and H. P. Westman, secretary.

A draft of definitions prepared at the previous meeting and the reports of several subcommittees appointed to revise various portions of that draft were reviewed.

HARALD TRAP FRIIS

The recipient of the Morris Liebmann Memorial Prize for 1939 was Harald Trap Friis. It was presented to him at the Awards Luncheon on September 21 for his investigations in radio transmission including the development of methods of measuring signals and noise and the creation of a receiving system for mitigating selective fading and noise interference.



Mr. Friis was born in Denmark on February 22 1893. He was educated there and received an Electrical Engineering degree from the Royal Technical College at Copenhagen in 1916. He continued at the College for the remainder of that year as an Assistant to Professor Pedersen, now Vice-President of the Institute. During 1917 and 1918 he served as a technical advisor at the Royal Gun Factory in Copenhagen.

In 1919 he was made a Fellow of the American Scandinavian Foundation and did graduate work at Columbia University. He joined the research engineering staff of the Western Electric Company in 1920. This became Bell Telephone Laboratories in 1925 and his work has continued in that organization.

He became an Associate of the Institute in 1918 transferring to Member in 1926 and Fellow in 1929. He is the author of numerous papers which have appeared in the PROCEEDINGS *and other technical journals.*

Sections

Atlanta

C. E. Davis, television research engineer of the RCA Manufacturing Company, presented a television demonstration. He presented first a description of the theory of operation and the construction of the equipment used in demonstrating television to the general public.

The pickup equipment was installed in a small studio and a group of entertainers, provided by Rich's Department Store in which the demonstration was held, were televised. Receivers located in several rooms permitted the audience to observe the reproduction of the program. Later, members of the audience were invited to be televised.

August 9, 1939, Ben Akerman, chairman, presiding.

Boston

"The Mathematical Theory of Coupling in Ultra-High-Frequency Circuits and its Application to Electrical Measurements" was presented by R. W. P. King of the department of physics at Harvard University.

January 20, 1939, H. W. Lamson, chairman, presiding.

E. M. Deloraine, European technical director of the International Electrical Corporation, presented a paper entitled "General Aspects of Telephone and Telegraph Communication in Europe with Particular Emphasis upon the Radio Technique Employed."

He described the communication networks used in Europe and discussed the types of line construction, including the coaxial type, used in Germany, England, and France. Pictures were shown of some of the important radio stations including the television installation in the Eiffel Tower in Paris. It was pointed out that communication problems are complicated by the languages used in the various foreign countries and somewhat by the lack of standardization of systems and equipment.

An interesting communication channel which was described is the ultra-high-frequency service between England and France. It is giving exceptionally reliable service under all weather and meteorological conditions.

February 17, 1939, H. W. Lamson, chairman, presiding.

President Heising visited the section and presented his paper on "Radio Extension Links to the Telephone System." This paper was summarized in the July, 1939, issue of the PROCEEDINGS.

February 24, 1939, H. W. Lamson, chairman, presiding.

"Some Recent Results in Power-Tube Investigations" was the subject of a paper by E. L. Chaffee, professor of physics and communication engineering at Harvard University, and R. I. Sarbacher of the Harvard Engineering School.

Dr. Chaffee discussed methods of measuring transmitting-tube characteristics. The possibilities of increasing the efficiency of the tube when used as a power amplifier were then considered. The introduction of a third harmonic of the fundamental, supplied by an auxiliary tube and circuit in the proper phase, results in an increase in the efficiency of a power-amplifier tube to over ninety per cent.

Mr. Sarbacher gave a demonstration of the operation of a circuit including the harmonic generator described above and showed its effectiveness in increasing the efficiency of operation.

March 17, 1939, H. W. Lamson, chairman, presiding.

W. L. Barrow, of the electrical engineering department of the Massachusetts Institute of Technology, presented a paper on "Horns for Radio Waves."

Professor Barrow presented a theoretical analysis of the operation of the electromagnetic horn antenna. Tapered pipe lines may be used for impedance matching between pipes of different cross section. Transmission constants have been calculated and a general discussion of electromagnetic horn radiators, concluded the paper.

April 21, 1939, H. W. Lamson, chairman, presiding.

E. H. Armstrong, Professor of Electrical Engineering at Columbia University, presented a paper on "Frequency Modulation."

A brief outline of the principles of frequency modulation was first given. Equipment used in frequency-modulated-wave transmitters was described and illustrated by photographs taken at Alpine, N. J., and the Yankee Network station being constructed at Asnebumskit Hill in Paxton, Mass.

A demonstration of the system was given by Paul DeMars, chief engineer of the Yankee Network. Transmission was from a station in Boston operating on a quiescent frequency of approximately 150 megacycles. This station will be used to transmit programs from Boston to Asnebumskit Hill.

The fine quality of reproduction resulting from the use of wide-band frequency-modulated-wave transmission clearly demonstrated the value of a wide acoustic frequency range and the enormous dynamic range that the system permits.

May 26, 1939, H. W. Lamson, chairman, presiding.

Cincinnati

"Recent Transformer Developments of WLW" was the subject of a paper by A. P. Foster, transformer engineer for the Crosley Corporation.

There was first presented a short résumé of the fundamental theory of transformer design. This was followed by a description of the development and constructional details of a special transformer to couple a pair of F-125-A vacuum tubes operating in class AB1 to their class C load which was the final stage of the 50-kilowatt WLW transmitter, a pair of F-124-A tubes. Methods of securing sufficient primary inductance with sufficiently low leakage inductance and input capacitance to insure a wide-band response-frequency characteristic were described. The final design and performance of the transformer were covered.

The special modulator tubes mentioned above operate with zero grid current and give an output of over forty kilowatts of audio-frequency power. They eliminate the conventional water-cooled driver stage and were developed by the Federal Telegraph Company to specifications provided by the WLW engineering department.

The next transformer discussed was for audio-frequency service to couple the 849-A driver tubes, class A operation to the grids of F-125-A modulator tubes, operating class AB1. As the amplifier was to operate with inverse feedback of about thirty decibels, the transformer must have a very small phase rotation. The final design resulted in a phase rotation at 16,000 cycles of only two degrees. Interlaced primary and secondary windings were used and the secondary coils were tapped to permit individual adjustment of the drive of the modulator tubes to obtain dynamic balance.

A high-leakage-inductance type of transformer for filament-heating purposes was next described. This type of transformer limits the starting current to a value which will not damage the tube filaments during the warm-up period. They eliminate the usual step-start relays, contactors, and resistors required for warming-up the filaments of large transmitting tubes.

The paper was closed with a description of a small output transformer covering a range from 20 to 50,000 cycles for use in an audio-frequency oscillator.

Detroit

"Some Recent Developments in Short-Wave Antennas" was the subject of a paper by J. D. Kraus of the University of Michigan.

A number of fundamental antenna concepts were first discussed. A method was then outlined for computing both the gain and radiation patterns of various antenna arrays, considering such factors as height above ground, tilt, and losses. A number of new multiwire doublet antennas were then described. These antennas possess a relatively high feed-point radiation resistance. Their application to several beam antenna systems was also shown. A brief description of the corner type of reflector system concluded the paper.

September 15, 1939, H. D. Seielstad, chairman, presiding.

Portland

A paper on "The Klystron Ultra-High-Frequency Oscillator" was presented by J. R. Woodyard, research associate at Stanford University.

September 26, 1939, H. C. Singleton, chairman, presiding.

Washington

"A Demonstration of Aerological Radio Sounding Equipment" was presented by Harry Diamond of the National Bureau of Standards.

The speaker first discussed the developments of the radio sonde equipment and by means of a series of demonstrations portrayed the development of the present type of equipment. A number of especially prepared models were employed. Following the paper and the preliminary demonstrations, a current model of automatic weather-reporting station was put into operation. The balloon carrying the transmitter and control mechanism was released from the roof of an adjacent building. Pulses from the portable transmitter attached to the balloon actuated a recorder which made a graphic record of the temperature and humidity conditions of the atmosphere at altitudes up to 50,000 feet.

September 11, 1939, L. C. Young, vice chairman, presiding.

Membership

The following indicated admissions to membership have been approved by the Admissions Committee. Objections to any of these should reach the Institute office by not later than November 30, 1939.

Admission to Associate (A), Junior (J), and Student (S)

Atchley, D. W., Jr., (S) 56 Concord Ave., Cambridge, Mass.
Atkin, F. W. T., (A) 25 Hayfield Cres., Frecheville, Sheffield, England.
Banerji, R. K., (J) D40/5 Lachmanpura, Benares, U.P., India.
Blake, F. G., Jr., (S) 32 Irving St., Cambridge, Mass.
Bluethenthal, H., Jr., (A) 1704 Market St., Wilmington, N. C.
Boghosian, E., (S) 6218 Catherine St., Philadelphia, Pa.
Callahan, G. F., (A) 108 W. 5th St., Owensboro, Ky.
Cassman, V. R., (S) 101 S. Montgomery Ave., Atlantic City, N. J.
Castle, C. X., (A) 415 Kanawha Country Club Rd., S. Charleston, W. Va.
Chesna, J., (A) 29 Barons St., Rochester, N. Y.
Coe, L., (S) 544 N. 4th St., Corvallis, Ore.
Crow, E., Jr., (J) 30 W. Chicago Ave., Chicago, Ill.
Dickey, M. B., (S) 2920 Woodside Pl., Cincinnati, Ohio.
Frantz, D. R., (A) 610 Lexington Pl., N.E., Washington, D. C.
Gow, K. P., (A) Provo, S. D.
Guest, J. W., (A) 1336 Glenwood Rd., Glendale, Calif.
Hance, H. V., (S) 304 Waverly St., Palo Alto, Calif.
Harris, A. S., (A) 310 Madison St., Fort Wayne, Ind.
Harris, E. R., (S) 487 Commonwealth Ave., Boston, Mass.
Hayes, K. J., (S) 4614 N. 6th St., Milwaukee, Wis.
Hilliard, D. M., (A) 329 Linden Ave., Glenside, Pa.
Hlavaty, E. M. J., (S) c/o Mrs. W. R. McCarty, R.R. 5, Lafayette, Ind.
Huggins, W. H., (S) Weatherford Hall, Corvallis, Ore.
James, C. L., (A) U.S.S. *Goff* (247), New York, N. Y.
Kennedy, R. L., (A) 10 Marion Rd., Upper Montclair, N. J.
Kniazuk, M., (A) Merck Institute of Therapeutic Research, 50 Lawrence St., Rahway, N. J.
Kres, A. J., (A) 3705 E. 52nd St., Cleveland, Ohio.
Laisk, E., (A) Kohleri 2-4, Tallinn, Esthonia.

Lazo, F. A. M., (A) c/o Andes Copper Mining Co., Chanaral-Barquito, Chile.
Lipscomb, J. L., (A) 2917–28th St., N.W., Washington, D. C.
Lloyd, L. H., (A) Box 345, American Falls, Idaho.
Marshall, L. J., (A) CBK Transmitter, Watrous, Sask., Canada.
McMullin, R. W., (A) 1110–4th St., Jackson, Mich.
Moore, K., (A) 1515 W. Monroe St., Chicago, Ill.
Pentilla, W., (A) 325 S. Colorado St., Butte, Mont.
Radom, S. R., (A) 2014 N. Berendo St., Hollywood, Calif.
Ranneft, J. E. M., (A) Netherlands Legation, Washington, D. C.
Roush, G. E., (S) 1514 Oxford St., Berkeley, Calif.
Reveal, P. A., (A) 65 Tonnele Ave., Jersey City, N. J.
Schindler, J. A., (A) 826 S. Wabash Ave., Chicago, Ill.
Shearer, R. W., Jr., (S) 780 West St., Reno, Nev.
Stiefel, K., (A) Worcester Polytechnic Institute, Worcester, Mass.
Winternitz, T. W., (S) 295 Beacon St., Boston, Mass.

Personal Mention

The following members have recently informed us of changes in their company affiliations or titles to those given below.

Badenack, R. M.; Supervising Engineer, Department of Civil Aviation, Melbourne, Australia.
Beard, D. C.; Lieutenant, U.S.N., U.S.S. *Boise*, San Pedro, Calif.
Cameron, J. A.; Radio Engineer, Automatic Winding Company, 900 Passaic Avenue, East Newark, N. J.
Dieringer, F. A.; Chief Engineer, The WFMJ Broadcasting Company, Youngstown, Ohio.
Distad, Merril; U. S. Naval Research Laboratory, Anacostia, D. C.
Guest, W. T.; Captain, U.S.A., Office of Chief Signal Officer, War Department, Washington, D. C.
Hall, L. S.; Radio Inspector, Department of Civil Aviation, Melbourne Australia.
Hickley, T. J.; U. S. Coast and Geodetic Survey, Washington, D. C.
Jaffe, D. L.; Television Engineer, Columbia Broadcasting System, 485 Madison Ave., New York, N. Y.
Johnson, R. M.; Lieutenant, U. S. Signal Corps School, Fort Monmouth, Oceanport, N. J.
Kitchin, H. W.; Lieutenant Commander, U.S.N., U.S.S. *Argonne*, San Pedro, Calif.
Laport, E. A.; Manager, Transmitter Engineering Department, RCA-Victor Company Limited, Montreal, P. Q., Canada.
Martin, C. A.; R.C.A. Communications, Rocky Point, N. Y.
Metz, H. I.; Civil Aeronautics Authority, Indianapolis, Ind.
Morris, H. B.; RCA Communications, Kahuku, Oahu, Territory of Hawaii.
O'Neill, H. N.; Standard Electrica, Lisbon, Portugal.
Powell, E. L.; U. S. Naval Research Laboratory, Anacostia, D. C.
Roorda, Jurjen, Jr.; Chief Engineer, Radiolaboratory, Vander Heem N. V. Maanstraat 250–260, The Hague, Holland.
Schuetz, R. F.; National Broadcasting Company, Hollywood, Calif.
Uhrane, F. F.; Lieutenant, Aircraft Radio Laboratories, Wright Field, Dayton, Ohio.

Books

Theory and Applications of Electron Tubes, by Herbert J. Reich

Published by McGraw-Hill Book Company, 330 West 42nd Street, New York, N. Y. 642 pages plus 1-page appendix and 26-page index. $6\frac{1}{4}$ by $9\frac{1}{4}$ inches. Price, $5.00.

A careful reading of this book, will convince one that it contains a large amount of useful information pertaining to various types of electron tubes. It is a valuable contribution to the literature both as a textbook and as a reference. The presentation is, for the most part, clear, orderly, and comprehensive. Considerably more space is devoted to amplifiers than to any other phase of the subject. The author purposely omits a discussion of class C amplifiers because he thinks they are adequately treated elsewhere. Other important sections are those on electrical conduction in gases, glow- and arc-discharge tubes, and light-sensitive cells. Considerable information on these applications is included which is not readily available to the engineer in convenient form. There are other interesting sections to which reference cannot be made here. An unusually valuable feature is the list of references which enables the reader to follow the development of the different phases of the subject.

Typical of minor criticisms are the following:

On Page 9 is the statement—"At several hundred volts the emission passes through a maximum and then falls with a further increase of the accelerating voltage," but the author fails to explain the phenomenon.

On Page 27—"If the emitted electrons pass from the cathode to the anode instantaneously, then there would be no electrons in the intervening space, and an infinitesimal voltage would suffice to draw all of the emitted electrons to the anode." It would be equally reasonable to assume that an infinite voltage would be required to move the electrons across this space instantly.

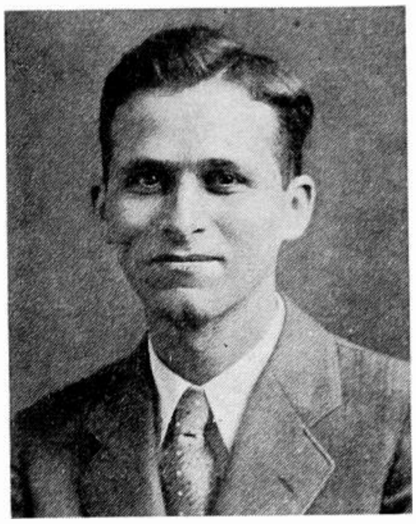
Figs. 3–20 indicates that the root-mean-square value of an alternating current is an ordinate to a point on the curve. While it is equal in magnitude to a particular ordinate this is not significant nor is it in accordance with the fundamental definition of the root-mean-square value.

There is no indication of the magnitude of the error introduced by the simplifying assumptions which made possible certain mathematical solutions.

Since the book possesses so much real merit these criticisms are of no great moment.

H. M. Turner
Yale University
New Haven, Conn.

Contributors



Stanford Goldman

Stanford Goldman (A'36) was born on November 14, 1907, at Cincinnati, Ohio. He received the B.A. degree from the University of Cincinnati in 1926; from 1928 to 1930 he was a graduate student at Harvard University where he received his Ph.D. degree in 1933. Dr. Goldman was a development engineer for RCA Photophone, Inc., from 1930 to 1931; a lamp engineer for the Hygrade Sylvania Corporation from 1933 to 1934; and employed in the Radio Engineering Department of the General Electric Company from 1935 to date.

Heinz E. Kallmann (A'38) was born on March, 10 1904, at Berlin, Germany. He received his Ph.D. degree from the University of Goettingen in 1929. He was a research engineer in the laboratories of the C. Lorenz A. G. from 1929 to 1934, and from 1934 to 1939 he was an engineer in the Research and Design Department of Electric and Musical Industries, Ltd.

H. E. Kallmann

Ronald King (A'30) was born on September 19, 1905, at Williamstown, Massachusetts. He received the A.B. degree from the University of Rochester in 1927, the M.S. degree in 1929, and the Ph.D. degree from the University of Wisconsin in 1932. Dr. King was an American-German Exchange Student at Munich from 1928–1929; a White Fellow in Physics at Cornell University from 1929 to 1930; and a Fellow in electrical engineering at the University of Wisconsin from 1930 to 1932. He continued at Wisconsin as a research assistant from 1932 to 1934. From 1934 to 1936 he was an instructor in physics at Lafayette College, serving as an assistant professor for the next two years.

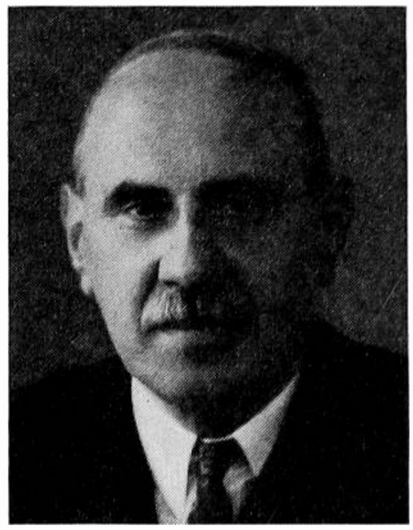
He was a Guggenheim Fellow at Berlin during 1937 and 1938. In 1938 he became

Ronald King

instructor in physics and communication engineering at Harvard University, advancing to assistant professor in 1939.

N. Koomans, was born on December 18, 1879, at Delft, The Netherlands. He received the D.Sc. degree in 1908. He was an engineer for the Technical Service, Netherlands P.T.T. in 1903; Extraordinary Member of the Dutch States Patent Office in 1918; Chief of Radio Laboratory Dutch, States Telegraph in 1926; and Engineer-in-Chief in 1928. He has served as Extraordinary Professor of Electrotechnics at Technische Hoogeschool; advisor in radio matters for the League of Nations; and a Member of the Board of the Nederlandsch Radiogenootschap.

Ernest G. Linder was born in 1902 at Waltham, Massachusetts. He received the B.A. degree from the University of Iowa in 1925, and the M.S. degree in 1927. He was an instructor in physics at the State University of Iowa from 1925 to 1927; an



N. Koomans

instructor at California Institute of Technology from 1927 to 1928; a Detroit Edison Fellow at Cornell University from 1928 to 1932, where he received his Ph.D. degree in 1931. Dr. Linder was employed in the Research Division of the RCA Victor Company from 1932 to 1935; and the RCA Manufacturing Company, RCA Victor Division from 1935 to date. He is a Member of the American Physical Society.

Stephan P. Sashoff was born on September 22, 1901, at Drenovo, Bulgaria. He received the B.S. degree in electrical engineering from Purdue University in 1925, and the M.S. degree in electrical engineering from the University of Pittsburgh in 1929. Mr. Sashoff joined the Westinghouse Electric and Manufacturing Company in 1925 and attended its Engineering and Design Schools; he was a relay engineer for Westinghouse from 1926 to 1928; with the Westinghouse Research Laboratories from 1928 to 1932; with the RCA Television Research Laboratory in

E. G. Linder

S. P. SASHOFF

1932. In the Fall of 1932 he became a member of the faculty of the College of Engineering, University of Florida, where at present he is an associate professor of electrical engineering and director of the Electronics and Communications Laboratory.

❖

Julius Weinberger (A'13–F'25) was born on July 22, 1893, in New York, New York. He received the B.S. degree from the College of the City of New York in 1913. Mr. Weinberger was laboratory assistant in the Radio Section of the National Bureau of Standards from 1914 to 1916; research assistant for the General Electric Company and the Marconi Wireless Telegraph Company of America from 1916 to 1919. From 1919 to 1928 he was in charge of research in the Research and Technical and Test Departments of the Radio Corporation of America becoming manager of the Research Department in 1929. He was placed in charge of research for RCA Photophone, Inc.; in 1930 and from 1932 to 1934 was in charge of acoustical research for the RCA Manufacturing Company. From 1935 to date he has been with the RCA License Laboratory devoting most of his time to economic studies of radio and allied industries. He is a Member of the Acoustical Society of America.

JULIUS WEINBERGER

❖

Joseph Weil (M'37) was born on July 19, 1897, at Baltimore, Maryland. He received the B.S. degree in engineering from Johns Hopkins University in 1918, and the M.S. degree from the University of Pittsburgh in 1926. He has held positions with the U. S. Consultory Board, Westinghouse Electric and Manufacturing Company, and has served as a consulting engineer for various municipalities, radio stations, and state and governmental agencies. In 1921 he became a member of the faculty of the University of Florida. He has been head of the Electrical Engineering Department and Chief Engineer of WRUF. He is now Dean of the College of Engineering and Director of the Engineering Experiment Station. Mr. Weil is a Fellow of the Florida Engineering Society and the American Institute of Electrical Engineers, and a member of Sigma Tau, Phi Kappa Phi, and the Society for the Promotion of Engineering Education.

JOSEPH WEIL

❖

For biographical sketches of T. R. Gilliland, D. R. Goddard, S. S. Kirby, and N. Smith, see the PROCEEDINGS for January, 1939; for W. M. Goodall, see the PROCEEDINGS for March, 1939.

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Current Literature

New books of interest to engineers in radio and allied fields—from the publishers' announcements.

A copy of each book marked with an asterisk (*) has been submitted to the Editors for possible review in a future issue of the Proceedings of the I. R. E.

* CATHODE MODULATION: For Economical Radiotelephony. By Frank C. Jones, San Francisco: Pacific Radio Publishing Company, Inc., 1939. 86 pages, illustrated, 8½×11 inches, paper. $1.00.

* SIMPLIFIED FILTER DESIGN. By J. Earnest Smith, Central Office, Engineering Laboratory, R.C.A. Communications, Inc. New York: RCA Institutes Technical Press, August, 1939. 57+6 pages of charts, illustrated, 8½×11 inches, paper-board. $1.00.

Booklets, Catalogs and Pamphlets

The following commercial literature has been received by the Institute.

CONDENSERS • • • *Aerovox Corporation, New Bedford, Massachusetts. Catalog, No. 9850, 28 pages, 8½×11 inches.* Specifications on the complete line of Aerovox condensers, resistors, and testing equipment.

CRYSTALS • • • *Bliley Electric Company, Union Station Building, Erie, Pennsylvania. Catalog G-11, 16 pages, 8½×11 inches* Operating data on piezo-electric quartz crystals and holders.

ELECTRICAL STEEL SHEETS • • • *Carnegie-Illinois Steel Corporation, Carnegie Building, Pittsburgh, Pennsylvania. Technical Bulletin, No. 1. 117 pages+cover, 8½×11 inches.* Characteristic curves and other application data for electrical sheet, with a description of measuring methods and commercial practices.

INSTRUMENTS • • • *Roller-Smith Corporation, Bethlehem, Pennsylvania. Catalog No. 123, 20 pages, 8½×11 inches.* General and technical data on a-c and d-c portable indicating instruments

INSTRUMENTS • • • *General Radio Company, 30 State Street, Cambridge, Massachusetts.* General Radio Experimenter, *October, 1939, 8 pages, 6×9⅛ inches.* Contains two brief articles on "radio-frequency distortion measurements with an audio-frequency wave analyzer" and on "extending the field of application of the variac."

RADIO NOISE-SOURCE LOCATION • • • *Tobe Deutschmann Corporation, Canton, Massachusetts. Bulletin, 4 pages, 9×12 inches.* Description of equipment for locating "radio noise."

RACKS • • • *Par-Metal Products Corporation, 35-25 41st Street, Long Island City, N. Y. Catalog No. 40, 28 pages+cover, 8½×11 inches.* Racks, cabinets, panels, and accessories.

RADIO COMPASS • • • *Western Electric Company, 195 Broadway, New York, N. Y. Bulletin, 2 pages, 8×11 inches.* Description of a radio-compass unit for marine use in conjunction with ship-to-shore radio-telephone installation.

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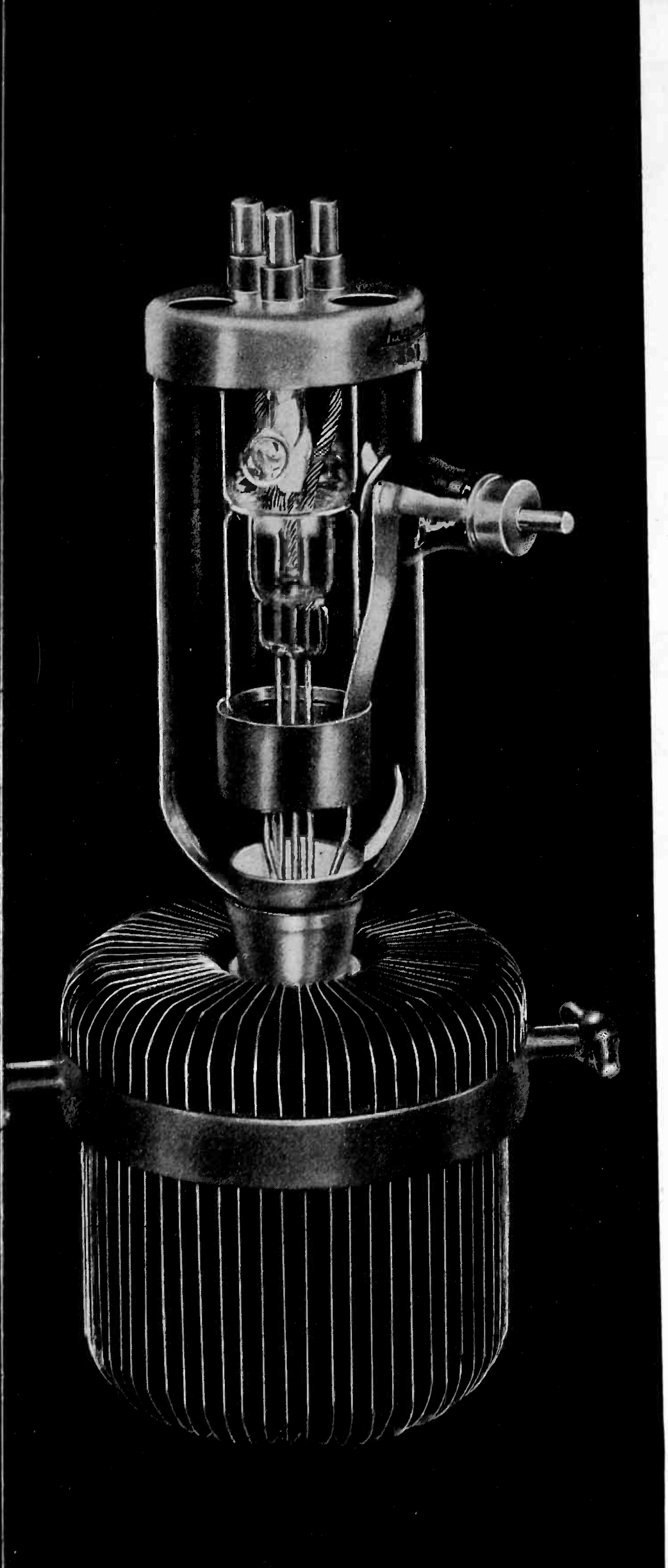

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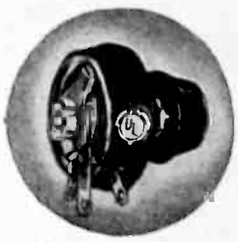

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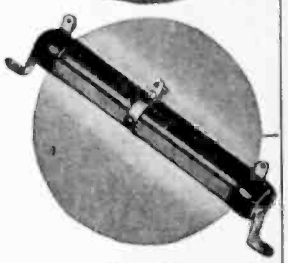

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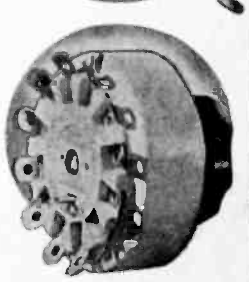

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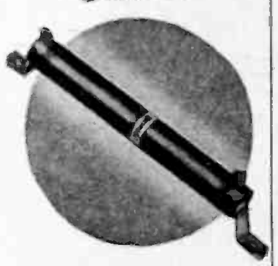

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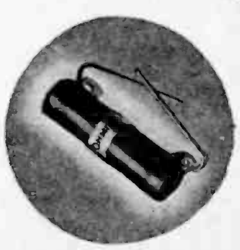

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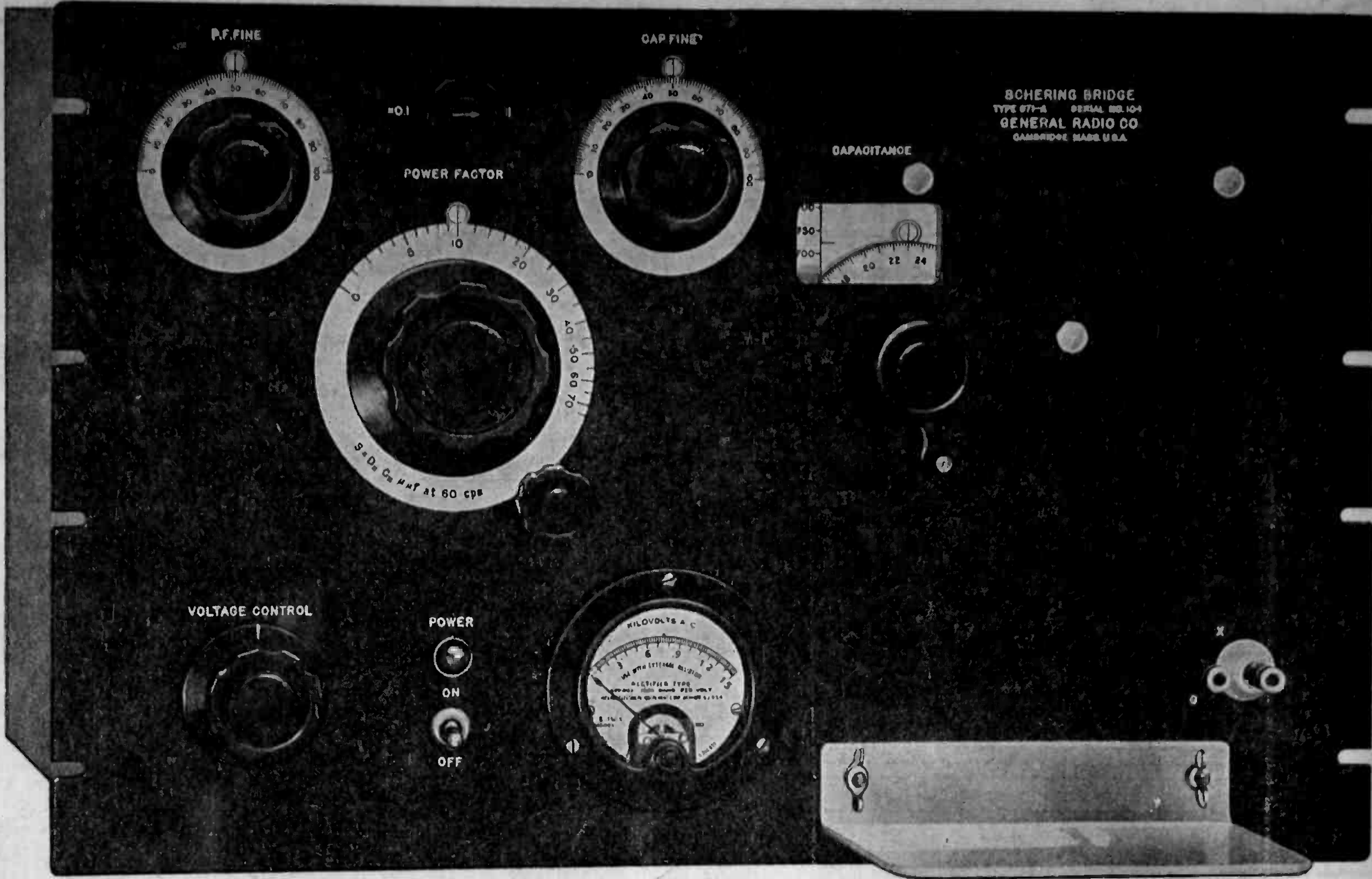

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